ONE-DIMENSIONAL TREATMENTS OF THE FLOWS IN HIGH PRESSURE GAS EJECTORS

By

W.T. Hanbury, B.Sc.

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Department of Aeronautics and Fluid Mechanics,

University of Glasgow.

SUMMARY.

Possible methods of describing the flows in constant area ejectors in terms of one-dimensional theories are examined and discussed. Both sonic and supersonic injection are considered, but the analysis is limited to the mixing of two streams of the same perfect gas. Three simple one-dimensional theories describing the primary expansion and a single theory describing the mixing process are presented initially. These theories allow for choking to occur either at the end of the primary expansion or at the end of the mixing duct. Some numerical results comparing the predictions of these theories with each other and experiment are given. The results from an extensive numerical survey of the total solution according to the combined Polytropic Two Stream theory and Straight Mixing theory are also presented.

A generalised theory of choking based on small disturbance velocities is put foreward. This theory is applicable to non-uniform flows. It considers choking to be a property of the flow as a whole, rather than a local phenomenon. A parameter B, pertaining to a cross-section of flow as a whole, is defined such that it is positive for "supersonic", negative for "subsonic" and zero for choked flow.

A one-dimensional method of describing the mixing process is proposed which, when combined with the general theory of choking, may be used to predict choking within the mixing process.

An experimental programme designed to investigate the applicability of the one-dimensional theory and the general phenomenon of choking is described and discussed. A plane flow air-air ejector is used. The full programme is not completed but some initial experimental results are given.

It is indicated that the Polytropic Two Stream theory may give reasonably good predictions of the choked flows in ejectors with high total pressure ratios, low total enthalpy ratios and high primary injection Mach numbers. Flows in ejectors of the opposite characteristics are liable to become choked within the mixing process and thus may not be predicted by any of the simple one-dimensional theories. The Straight Mixing theory is found only to be of limited use in that flows are not usually found to be completely mixed by the end of the duct. An alternative theory is suggested.

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1. INTRODUCTION.

Research into the physical performance of ejectors and the evolution of theories to describe the phenomena observed have been in progress for nearly half a century. In general ejectors can be divided into classes according to the properties of the fluids with which they operate. The type of fluid flowing through an ejector will not only affect the sort of physical phenomena to be expected but will also influence the theoretical approach to predicting its performance. Broadly speaking, there are in this context three main classes of fluid; incompressible compressible and vapours. Confining the subject to single stage ejectors, that is those with one driving stream and one driven stream, work has been published at some time or another on ejectors employing most of the possible fluid type combinations. A fairly extensive bibliography upon the subject is available, see reference 10. The work to be reported in this document concerns the performance of ejectors driven by and entraining streams of compressible fluids, and in the main the work is further restricted to the case in which both streams consist of the same gas. That is not to say that some of the general implications of the work cannot be generalised and applied to other cases.

Most theoretical approaches to the performance of compressible fluid ejectors are restricted to the consideration of two particular types of ejector geometry. This is largely because of the simplifications that these geometries introduce into the theory. The two ejector types are those in which the mixing takes place at constant pressure and those in which the mixing takes place at constant cross-sectional area. In the former case the mixing duct geometry, if it is fixed, is only likely to be correct for one design condition Under off-design conditions the mixing will probably no longer take place at constant pressure. In the constant area mixing case, however, the theory developed will apply under a much greater range of conditions. It is probably this prospect that has prompted workers, who have not been limited to the consideration of any one particular system, to develop their theories, at least in the initial stages, for ejectors with constant area mixing ducts.

Two distinct methods of approach to the general problem are available. These are the development of simple one-dimensional theories or the use of more exact but more complex two-dimensional methods. Which approach is chosen in any instance will largely depend on whether it is required to obtain limited results on a large number of flows or more exact solutions for a few flows. In the latter case where a detailed knowledge of a few well defined flows is desired, it may be advantageous to use a two-dimensional description of the flow may be including the effects of viscosity. However, it is the purpose of this work to investigate the simpler one-dimensional approaches through which the likely behaviour of ejector flows as a whole may be studied.

One-dimensional theories of compressible mixing were not unnaturally among the first to appear. The simplest of these, the Straight Mixing Theory, considers only the effects of the mixing. It has appeared in the literature in may forms. Basically it assumes that the mixed stream at the exit of the mixing duct may be represented by a single uniform state which may be expressed in terms of three parameters which may be determined by applying the laws of conservation of mass, momentum and energy. The theory is set out in full in Section 2.1. The Straight Mixing Theory has been shown to have a moderate degree of success in predicting the effect of back pressure on ejector flows in which no choking occurs. One of the major problems in compressible flow ejector theory is predicting the maximum mass flow that may pass through a system with given stagnation conditions, in other words the prediction of the choked condition. The Straight Mixing Theory appears to be of little help here. The only mode of choking that it predicts, a sonic mixed stream, does not often occur in practice. This is due mainly to two reasons, firstly the flows usually choke at a smaller mass flow than that corresponding to this solution and secondly at high mass flows the mixing tends not to be complete by the end of the duct.

It was realised early on (see reference 5) that the expansion of the primary stream within the mixing tube, when the injection pressures are not equal, might well be the cause of the flow choking. But it was not until the mid nineteen fifties that any theoretical attempts to describe such a process were published. The first of these was the one-dimensional theory of Fabri (reference 1) in which it was assumed that the primary stream became over-expanded and that the secondary stream choked sonically at the cross-section of maximum primary expansion. Both streams were assumed to behave isentropically. Good experimental agreement was claimed for this theory, but subsequently this has been shown not to have been entirely justified (see reference 3 et al). A theory making the same basic assumptions as Fabri but this time describing the expansion of the primary stream by a two-dimensional method of analysis was put forward and used by Messrs. Chow and Addi (reference 3) to describe the flows in supersonic ejectors (those with convergent-divergent primary injection nozzles). Messrs. Chow and Addi also included a boundary layer type analysis of the mixing region and this combined theory gave very good experimental predictions especially in the Base Pressure Regime (flows with very small secondary mass flows). It has, however, the disadvantage of being very complex and demanding considerable computer time to solve even a single solution.

Another approach to the question of choking in ejectors was made by Messrs. Pearson, Holliday and Smith, reference 2, in which they introduced, to the subject, the concept of general choking. In considering ejectors with short mixing ducts they treated the primary expansion one-dimensionally and assumed that no mixing took place. In contrast to Fabri, they allowed the primary stream to behave irreversibly and did not assume it to become over-expanded. The choking was assumed to occur when, towards the end of the expansion, both streams approached a common static pressure in a tangential manner. Such cross-sections of uniform static pressure are found to contain both supersonic and subsonic flow. The common wave velocity was shown to be stationary at such cross-sections. The theory was found to have reasonable success in predicting experimental flows in ejectors with short mixing ducts.

The concept of generalised choking has been discussed in a number of publications since the appearance of Pearson's paper, in particular in references 4 and 11. However, no satisfactory general theory of choking seems to have emerged yet, with the possible exception of a paper by Messrs. Hoge and Segars, reference 4. In this paper a general theory of choking, based on stationary points in a momentum function, is proposed. It is felt, however, that phenomena such as maxima or minima in momentum functions are more likely to be the effect of choking rather than the cause.

The major objectives of the work to be reported in this thesis are as follows :- It is proposed to investigate the possibilities of describing the flows in constant area mixing ejectors in terms of one-dimensional theories. Both sonic and supersonic injection are to be considered. An attempt to produce a generalised theory of choking is to be made. Numerical surveys are to be made of the solutions predicted by the various one-dimensional theories. An experimental programme is to be carried out aimed firstly, at obtaining some experimental results with which to compare the predictions of the theory, and secondly, at investigating the modes of choking that are found to occur.

2. THE ONE-DIMENSIONAL THEORY OF EJECTORS WITH CONSTANT AREA MIXING.

There are four basic phenomena that may occur in any ejector flow. Firstly there will occur some degree of mixing between the two streams. Secondly if the streams do not enter the mixing duct at the same static pressure there will occur some process by which this pressure difference is dissipated. Thirdly there will be a boundary layer along the walls of the mixing tube, together with the associated frictional drag. Lastly, if the back pressure into which the ejector exhausts is low enough, a process will occur whereby the ejector becomes choked. In general a one-dimensional theory splits the flow up into specific regions in which each of the phenomena accounted for in the theory may be assumed to take place separately. All the theories assume that each stream enters the mixing duct in a state uniform across the plane of injection. The majority of one-dimensional theories take into account the pressure equalising process and the mixing between the two streams and always consider them to take place in that order. Most theories neglect the effect of the boundary layer on the mixing tube walls. The main differences between the various theories lie in the ways that they describe the pressure equalising process, and this, in turn, leads them to predict different modes of choking. To avoid confusion, an ejector will be defined to have become choked when the secondary mass flow has become independent of the back pressure against which the ejector exhausts. The two injected streams need not necessarily contain the same gases, but it will be assumed that all the gases have constant specific heats. Most theories can be applied equally well to both sonic and supersonic injection ejectors, although the solutions naturally become more complex in the supersonic case.

2. 1. EJECTORS WITH SONIC INJECTION.

Several one-dimensional theories will now be presented in the context of sonic injection. The type of ejector under consideration is illustrated in Figure 1. For a fixed set of upstream stagnation conditions and a fixed ejector geometry, there will exist a continuous set of solutions dependent on the back pressure against which the ejector exhausts. The stagnation conditions and the geometry may be expressed by the following non-dimensional parameters:-

z, the ratio of the primary injection area to the cross-sectional area of the mixing duct.

h, the ratio of the primary total enthalpy to that of the secondary.

π, the ratio of the primary total pressure to that of the secondary.

These three ratios will hereafter be referred to as the control parameters.

For any fixed set of control parameters the corresponding continuous set of solutions is bounded by two extreme conditions. Firstly the back pressure may be so high that there is zero mass flow in the secondary stream. This is termed a Base Pressure solution. Secondly the back pressure may fall below the value at which the ejector becomes choked, in which case the solution enters the Supersonic Regime. The entire solution between these extremes is dependent on the back pressure. It may usually be divided into two sections, the Subsonic Regime and the Mixed Regime. In the Subsonic Regime both streams are injected subsonically and the injection pressures are assumed to be equal. In the Mixed Regime the primary becomes choked at the plane of injection and its injection pressure is therefore fixed at the critical value. The secondary injection pressure, however, is still free to fall as the back pressure is reduced, and will thus be less than that of the primary. The conditions at the injection plane may be fixed by defining a value for the secondary injection pressure, irrespective of which regime the solution happens to be in. Since it is not usually possible to find an explicit solution for the conditions upstream in terms of those downstream, the problem has to be tackled the other way round. Values are assumed for the secondary injection pressure and the corresponding downstream conditions determined. Several ways in which this may be done will now be explained.

2. 1. 1. Straight Mixing.

The simplest theory on ejector flows is one that considers only the mixing between the two streams. It assumes that the mixing is complete by the end of the duct, so that the state of the mixed stream is uniform. This state may then be represented by a total pressure, a total enthalpy and a Mach number which may be determined simply by equating the mass, momentum and energy fluxes in and out of the duct. Using the notation in Figure 1, these conditions give rise to the following equations when considering two streams composed of the same gases:-

Mass.

$$\rho_1 A_1 v_1 + \rho_2 A_2 v_2 = \rho_3 A_3 v_3 \qquad \ldots.1$$

Energy.

$$\rho_1 A_1 v_1 H_1 + \rho_2 A_2 v_2 H_2 = \rho_3 A_3 v_3 H_3 \qquad \ldots.2$$

Momentum.

$$A_1(p_1 + \rho_1 v_1^2) + A_2(p_2 + \rho_2 v_2^2) = A_3(p_3 + \rho_3 v_3^3) \qquad \ldots.3$$

Using the mass flow and momentum functions, derived in Appendix 1, together with the following substitutions

$$h_3 = \frac{H_3}{H_2}, \quad \pi' = \frac{P_1}{P_2}, \quad \pi'' = \frac{P_3}{P_2} \quad \text{and} \quad z' = \frac{A_1}{A_3},$$

equations 1, 2 and 3 may be rewritten in the non-dimensional form :

$$y_2 + \frac{z'}{1 - z'} \pi' y_1 h^{-\frac{1}{2}} = \frac{1}{1 - z'} \pi'' y_3 h_3^{-\frac{1}{2}} \quad \text{....1a}$$

$$y_2 + \frac{z'}{1 - z'} \pi' y_1 h^{\frac{1}{2}} = \frac{1}{1 - z'} \pi'' y_3 h_3^{\frac{1}{2}} \quad \text{....2a}$$

$$f_2 + \frac{z'}{1 - z'} \pi' f_1 = \frac{1}{1 - z'} \pi'' f_3 \quad \text{....3a}$$

Given that all the quantities on the left hand sides of these equations are known, there remain three equations and three unknowns. The unknowns are π'', h_3 and ω_3 the quantities y_3 and f_3 being the mass flow and momentum functions of ω_3. Eliminating π and h_3 between equations 1a 2a and 3a, there emerges a relation which gives a quadratic solution for for the state of the fully mixed stream.

$$\left[\frac{y_3}{f_3}\right]^2 = \frac{y_2^2 + \frac{z'}{1 - z'} y_2 y_1 \pi' (h^{\frac{1}{2}} + h^{-\frac{1}{2}}) + \left[\frac{z'}{1 - z'} \pi' y_1\right]^2}{(f_2 + \frac{z'}{1 - z'} \pi' f_1)^2} \quad \text{....4}$$

Equation 4 will in general yield two roots for ω_3. If they are real, one will be a subsonic solution and the other, if it is finite and positive, will indicate a supersonic solution. If the two roots are equal, there is only one solution and that is that the mixed stream is sonic and choked. If the roots are imaginary then the assumed secondary mass flow is greater than that at which the mixed stream becomes choked and therefore such a flow cannot exist. The only modes of choking predictable by this theory are the choking of the fully mixed stream and the sonic choking of the secondary injection nozzle. Once the values of ω_3 are known the corresponding values for the total pressure and the total enthalpy may be determined by substitution back into equations 1a and 2a. An algol procedure to solve equation 4 is set out in Appendix 3.

This simple theory deals fairly well with the Subsonic Regime, but is found inadequate to describe the phenomena that occur in the Mixed and Supersonic Regimes of ejector flow. However, it may be combined with various theories on the initial pressure adjustment to provide some more satisfactory flow descriptions.

2.1.2. The Two Stream Theories.

These theories deal with the flows in the Mixed and Supersonic Regimes and they all divide the flow into to two distinct regions. In the first region both streams accelerate, the primary expanding and the secondary contracting, in such a way as to reduce the inequality of the injection pressures without actually mixing. This process is assumed to take place rapidly and to be over by the time any appreciable mixing could have taken place. The mixing takes place in the second region and may be considered to be described by the Straight Mixing theory dealt with in the last section.

The ways in which it is possible to describe the pressure adjustment process in terms of one dimensional equations will now be discussed with reference to figure 2. . Here is depicted a simple constant area mixing ejector for which the conditions at the injection plane are already known. The primary stream is choked and the secondary injection pressure is less than that of the primary. The expansion will be considered to end at a plane further downstream where both streams will again be considered to have reached uniform states. These states may, therefore, be represented in a one dimensional manner.

The secondary stream being subsonic is generally assumed to behave isentropically during the expansion. The primary stream may also be assumed to behave isentropically, or in other words it may be assumed that there is no loss of total pressure. The momentum exchange between the two streams must be equal and opposite. The static pressures of the two streams should be equal at the end of the expansion. With all these conditions in operation the one dimensional solution becomes over specified by one constraint. Therefore, in order to obtain a solution one of the conditions must be relaxed. The differences between the various two stream theories lie in which of these conditions they choose to relax. In Fabri's solution the static pressure at the end of the expansion are not assumed to be equal. In the Isentropic Two Stream Theory it is the momentum balance that is neglected and in the Polytropic Two Stream Theory the primary stream is assumed to expand irreversibly. The equations describing this expansion process will now be derived according to these three different theories. The notation used is explained in Figure 2.

(i) The Primary Expansion According to Fabri's Theory.

The continuity equation for the secondary stream may be written :-

$$P_{21}A_{21}H_{21}^{-\frac{1}{2}}y_{21} = P_{22}A_{22}H_{22}^{-\frac{1}{2}}y_{22}$$

The energy equation may be written :-

$$H_{21} = H_{22}$$

Since it is isentropic,

$$P_{21} = P_{22}$$

Hence the contraction of the secondary stream may be described by the relation :-

$$A_{21}\, y_{21} = A_{22}\, y_{22} \qquad \ldots .5$$

Similarly for the primary stream,

$$A_{11}\, y_{11} = A_{12} y_{12} \qquad \ldots .6$$

The duct is of constant cross-sectional area, hence

$$A_{11} + A_{21} = A_{12} + A_{22} \qquad \ldots .7$$

The momentum exchange between the streams is equal and opposite and thus the momentum fluxes across the two planes are equal.

$$A_{11}\, P_{11}\, f_{11} + A_{21}\, P_{21}\, f_{21} = A_{12}\, P_{12}\, f_{12} + A_{22}\, P_{22}\, f_{22} \qquad \ldots .8$$

These equations may be expressed in the non-dimensional form :-

$$z\, y_{11} = z' y_{12} \qquad \ldots .5a$$

$$(1 - z)\, y_{21} = (1 - z')\, y_{22} \qquad \ldots .6a$$

$$\pi\, z\, f_{11} + (1 - z) f_{21} = \pi\, z' f_{12} + (1 - z') f_{22} \qquad \ldots .8a$$

Using 5a to substitute into 6a and 8a for z' we get the following two equations in functions of the unknowns ω_{21} and ω_{22}.

$$\pi z\, f_{11} + (1 - z) f_{21} = \pi z\, y_{11} \frac{f_{12}}{y_{12}} + (1 - z) y_{21} \frac{f_{22}}{y_{22}} \qquad \ldots .9$$

and

$$(1 - z)\, y_{21} = \left[1 - z \frac{y_{11}}{y_{12}} \right] y_{22} \qquad \ldots .10$$

A method of solution for these equations would be to choose successive values for ω_{22} in such a way that the two values of ω_{12} derived from the equations tend to converge.

Another mode of choking becomes feasible when the primary expansion process is described by this theory. As the secondary mass flow increases in the Mixed Regime the difference in injection pressures also increases. Hence the primary expansion becomes greater and so does the secondary contraction. This causes the secondary Mach number at the end of the contraction to increase rapidly as the secondary injection pressure falls. The situation reaches a limit when the secondary stream chokes in plane 2 (Figure 2) and thus causes the ejector to choke as a whole. The injection pressure at which this happens may be determined from equations 9 and 10, by putting in the critical value of ω_{22} as a known quantity and treating ω_{12} and ω_{21} as the unknowns. Then values for ω_{21} may be chosen in such a way that the corresponding values of ω_{12} determined from equations 9 and 10 tend to converge. An algol procedure to carry out this process is given in Appendix 3.

(ii) The Primary Expansion According to the Isentropic Two Stream Theory.

In this theory both streams are assumed to behave isentropically and the expansion is assumed to end with the static pressures in each stream equal. It is the momentum balance that is ignored.

The behaviours of the streams may then again be described by the equations :-

$$A_{21}\, y_{21} = A_{22}\, y_{22} \qquad \ldots.5$$

$$A_{11}\, y_{11} = A_{12} y_{12} \qquad \ldots.6$$

The equation of static pressures at the end of the proces may be written :-

$$p_{12} = p_{22} \qquad \ldots.11$$

Again the equations may be put in a non-dimensional form

$$z\, y_{11} = z'\, y_{12} \qquad \ldots.5a$$

$$(1 - z) y_{21} = (1 - z') y_{22} \qquad \ldots.6a$$

$$\omega_{12} = \frac{\omega_{22}}{\pi} \qquad \ldots.11a$$

z' may be eliminated between equations 5a and 6a to give

$$(1 - z)\, y_{21} = \left[1 - z\, \frac{y_{11}}{y_{12}} \right] y_{22} \qquad \ldots.12$$

Equations 11a and 12 are two equations in the two unknowns ω_{12} and ω_{22}
For solution they should be rewritten in the form :-

$$y_{12} = y\left[\frac{\omega_{22}}{\pi}\right] \qquad \text{....11b}$$

$$y_{12} = \left[\frac{z\ y_{11}}{1 - (1 - z)\ \frac{y_{21}}{y_{22}}}\right] \qquad \text{....12a}$$

For known injection conditions, successive values of ω_{22} should be chosen in such a way that the two values for y_{12} given by equations 12a and 11b tend to converge.

This theory also produces its own special mode of choking. It is probably best described with reference to a diagram such as that in Figure 3. Plotted in this diagram is the behaviour of the streams in the pressure-area field. The parameter chosen to represent the area is z', the fraction of the cross-sectional area of the mixing tube occupied by the primary stream. Thus an increase in z' represents an increase in the primary cross-sectional area but also represents a reduction in that of the secondary. The vertical scale of the graph is the pressure scale and it is normalised with respect to the secondary total pressure.

In the following explanation the primary expansion and the secondary contraction will be assumed to follow distinct isentropic curves. This assumption is more constricting than is absolutely necessary, and is only made in order to clarify the explanation of the choked solution. The only assumption that is in fact necessary in order to obtain the solution is that the flows towards the end of the expansion may be described by those portions of the curves in the vicinity of their intersection.

The primary stream is choked at the injection plane and its mass flow is, therefore, fixed. Thus the primary expansion may be represented in the pressure-area field by an isentropic curve of the form of AB in Figure 3.

The line z' = z represents the injection plane. The secondary injection condition will, therefore, be represented by a point on this line somewhere between the points A and C, C being at the secondary critical pressure. Choosing a particular value for the secondary injection pressure such as D, the corresponding isentropic pressure-area relation, DE, may then be drawn in. The end of the expansion occurs then at the intersection of the two curves AB and DE , where the static pressures have become equal. Now there exists a whole family of such secondary curves (FG, HI, etc.,) each corresponding to particular secondary injection pressures. As the secondary injection pressure falls in the Mixed Regime, it may be seen that these curves move to the left along the z' axis, and the degree of the primary expansion becomes greater. If the secondary injection pressure falls far enough there will come a stage when the two curves will only touch tangentially, and at even lower pressures there will be no real solution at all. This tangential solution is, therefore, the maximum mass flow solution compatible with this theory. It will be seen from the diagram that the secondary stream is always subsonic even in this maximum flow case. It is assumed that the tangential solution provides a mechanism of choking to prevent the secondary injection pressure from dropping any further. This choked solution may be determined from equations 12a and 11b above by choosing successive values for ω_{21} in such a way that the two values of y_{12} corresponding to a succession of values for ω_{22} tend to converge in a tangential manner.

(iii) The Primary Expansion According to the Polytropic Two Stream Theory.

This theory takes into account both the momentum balance between the streams and the equality of their static pressures at the end of the expansion process. As before the secondary stream is considered to behave isentropically. The primary stream, however, is not assumed to behave isentropically, though both energy and mass are considered to be conserved within the stream.

The secondary contraction may still be described by equation 5a.

$$\frac{1 - z'}{1 - z} = \frac{y_{21}}{y_{22}} \qquad \text{....5a}$$

For the irreversible expansion of the primary, continuity states that

$$\rho_{11} A_{11} v_{11} = \rho_{12} A_{12} v_{12}$$

Which may be combined with the energy equation $H_{11} = H_{12}$ and written in the form

$$z P_{11} y_{11} = z' P_{12} y_{12} \qquad \text{....13}$$

The equality of the static pressures at the end of the expansion may be expressed

$$p_{12} = p_{22} \qquad \text{....14}$$

or

$$P_{12} \omega_{12} = P_{22} \omega_{22} \qquad \text{....14a}$$

Since the two streams are contained in a parallel sided duct, the momentum exchange between them must be equal and opposite, and hence the momentum fluxes across planes 1 and 2 must also be equal. That is

$$A_{11}(p_{11} + \rho_{11} v_{11}^2) + A_{21}(p_{21} + \rho_{21} v_{21}^2) = A_{12}(p_{12} + \rho_{12} v_{12}^2) + A_{22}(p_{22} + \rho_{22} v_{22}^2) \quad \ldots.15$$

Using the momentum function $f(\omega)$ and the other substitutions mentioned above, equation 15 may be written in the form

$$z P_{11} f_{11} + (1 - z)P_{21} f_{21} = z' P_{12} f_{12} + (1 - z')P_{22} f_{22} \quad \ldots.15a$$

Since the secondary stream is isentropic it suffers no loss in total pressure. Further from equation 13,

$$P_{12} = \frac{z\, y_{11}}{z' y_{12}} P_{11} \quad \ldots.13a$$

Substituting this into equation 15a,

$$z \pi f_{11} + (1 - z)f_{21} = z \frac{y_{11}}{y_{12}} \pi f_{12} + (1 - z')f_{22} \quad \ldots.15b$$

and into 14a,

$$\pi z\, y_{11} \frac{\omega_{12}}{z'\, y_{12}} = \omega_{22} \quad \ldots.14b$$

Now from equation 5a,

$$z' = 1 - (1 - z) \frac{y_{21}}{y_{22}} \quad \ldots.5b$$

Substituting into equation 15b for z',

$$\frac{\pi z}{1 - z} f_{11} + f_{21} = \frac{\pi z}{1 - z} y_{11} \frac{f_{12}}{y_{12}} + y_{21} \frac{f_{22}}{y_{22}} \quad \ldots.15c$$

and into 14b,

$$\pi z\, y_{11}\, \omega_{12} = y_{12}\, \omega_{22} \left[1 - (1 - z) \frac{y_{21}}{y_{22}} \right] \quad \ldots.14c$$

The degree of irreversibility in the primary expansion may be judged by the loss of total pressure, which in turn may be found using equation 13a,

$$\frac{P_{11} - P_{12}}{P_{11}} = 1 - \frac{z\, y_{11}}{z' y_{12}} \quad \ldots.16$$

Equations 14c and 15c are two equations relating functions of the three variables ω_{21}, ω_{12} and ω_{22}. For given injection conditions, that is when ω_{21} is known, these equations may be solved to find the conditions in each of the streams at the end of the expansion. Again this may be done by choosing successive values for ω_{22} in such a way that the two corresponding values for ω_{12} derived from equations 14c and 15c tend to converge. An algol procedure designed to find the values of ω_{12} and ω_{22} for given values of ω_{21} is given in Appendix 3.

This theory provides a mode of choking very similar to that of the Isentropic Two Stream Theory. The solution in the choked case is again a tangential one. The possible contractions of the secondary stream may again be represented by a family of isentropic pressure-area curves. The primary expansions may also be represented by a family of curves each corresponding to a specific secondary injection pressure and each having a slightly different degree of irreversibility.

As in the last section, the assumption that both streams behave rigidly according to the given curves during the expansion is, in fact, unnecessary as long as they approach this condition towards the end of the expansion.

As the secondary injection pressure falls a tangential solution is approached at which the flow is assumed to choke. The secondary stream is again entirely subsonic, even in the choked condition. An algol procedure to find this choked solution is presented in Appendix 3.

2.1.3. Complete One-Dimensional Theories.

Given the injection conditions, each of the above two stream theories may be used to predict the states of the two streams at the end of the primary expansion. These states may in turn be used as the initial states for the Straight Mixing Theory in order to determine the state of the fully mixed stream. Thus using any one of the two stream theories in conjunction with the Straight Mixing Theory we now have a complete one-dimensional theory of ejector performance.

To obtain the complete set of solutions corresponding to a particular set of control parameters a process along the lines of the following should be carried out. The complete set of solutions will in general contain flows in the Subsonic, Mixed and Supersonic Regimes. As explained above each solution in the set corresponds to a particular value of the secondary injection pressure. The range in which this pressure may lie is restricted at the high end by the Base pressure solution and at the low end by the fully choked solution. Starting at the base pressure end, with a value of the secondary injection pressure just below that of the total pressure, the solutions should be determined for series of successively smaller injection pressures until it is found that the ejector has become choked. This may happen either due to the two stream choking effect or due to the sonic choking of the mixed stream. Which occurs first will depend on the actual control parameters in use.

The value of the secondary injection pressure at which the primary stream becomes choked in the injection plane may be determined from the equation

$$\omega_{21} = \pi \frac{2}{(\gamma + 1)^{\gamma/(\gamma - 1)}} \qquad \ldots .17$$

Flows with secondary injection pressures above this value will be in the Subsonic Regime. The injection pressures will be equal and their solutions may be found using the Straight Mixing theory by itself.

For values below this, however, the solution will be in the Mixed regime and the Straight Mixing theory must be used in conjunction with one of the two stream theories in order to get a more comprehensive description of the flow. In cases in which either the Polytropic Two Stream theory or Fabri's theory is being used for a flow in the Mixed Regime, it should be noted that if the only object in finding a solution is to determine the state of the mixed stream, this may be achieved by using the Straight Mixing theory alone; for in both these two stream theories mass, momentum and energy are all conserved. In the case of the Isentropic Two Stream theory, however, such a step would lead to a different answer because in the latter theory momentum is not necessarily conserved.

2.1.4. <u>The Two Stream Modes of Choking.</u>

Each of the Two Stream theories produces its own special mode of choking. In Fabri's theory the secondary stream chokes sonically at the end of the primary expansion and in doing so provides a readily acceptable mechanism for the choking. The other two theories on the other hand both choke the secondary while it is still subsonic and are, therefore, somewhat in need of an explanation. In the case of the Polytropic Two Stream theory it has been shown by Pearson et. al. (Reference 2) that the relation between the states at the end of the primary expansion is the same as that for a stationary wave velocity in a two stream system. This generalised concept of choking will be discussed further in a following section.

2. 2. ONE-DIMENSIONAL THEORY APPLIED TO SUPERSONIC EJECTORS.

The theories discussed above may be extended to cover certain flow regimes in supersonic ejectors. The set up referred to as a supersonic ejector in this paper is illustrated in Figure 4. It consists of two nozzles discharging into a constant area mixing duct. One of the nozzles is purely convergent and the other is convergent-divergent. The latter is referred to as the primary injection nozzle and the stream that passes through it is called the primary stream. To give an idea of the physical context into which any theory on supersonic injection must fit, a brief description of the various flow regimes that are found to exist will be given.

2. 2. 1. A Qualitative Description of the Flows Found to Occur in Supersonic Ejectors.

When the flow throughout an entire ejector system is subsonic the ejector is said to be operating in the Subsonic Regime. Such flows occur in ejectors operating at low total pressure ratios and exhausting into relatively high back pressures. The injection pressures are equal and the total pressures of the two injected streams and the back pressure are all of the same order. The whole flow is dependent on the back pressure against which the system exhausts.

In ejectors operating at higher total pressure ratios and lower back pressures the primary stream will become choked at the throat of the primary injection nozzle. The primary mass flow will thus become independent of the back pressure. The secondary stream, however, remaining entirely subsonic, will still be back pressure dependent. All flows in which the primary is choked and the secondary is not are classified in the Mixed Regime. The Mixed Regime may be subdivided according to the behaviour of the streams on entry to the mixing duct. If the primary is overexpanded on entry to the duct, that is if the primary injection pressure is less than that of the secondary, as depicted in Figure 4b, then the flow is said to be in the Supersaturated Mixed Regime. On the other hand, when the secondary injection pressure is less than that of the primary and the primary enters the mixing duct in an underexpanded state, the flow is said to be in the Supersonic Mixed Regime. Such a flow is illustrated in Figure 4d. The condition between these two, when the injection pressures are equal, is known as the Saturated Condition and is shown in Figure 4c. In all the Mixed Regimes the flows are always back pressure dependent.

There occurs, for any given ejection configuration, a certain limit in the back pressure below which the total mass flow becomes independent of the back pressure. The regimes of the solution in which this occurs are termed the Supersonic Regimes and they may again be distinguished from one another by the behaviour of the two unmixed streams on injection. Flows in which the primary stream is overexpanded on injection are classified in the Supersaturated Supersonic Regime and those in which it is underexpanded on injection are classified in either the Supersonic Regime or the Saturated Supersonic Regime. The boundary between the Supersaturated Supersonic Regime and the Saturated Supersonic Regime is independent of the theory used, but different theories for the Supersonic and Saturated Supersonic Regimes lead to different boundaries between them. Theories, in which only one solution with equal injection pressures occur, place all flows in which the primary is under-expanded in the Supersonic Regime; while theories, in which two solutions with equal injection pressures occur, place all flows between these two solutions in the Saturated Supersonic Regime whether the primary is under-expanded or not. In the latter case it may be seen that the Saturated Supersonic Regime is a finite regime, whereas in the former case it consists of only a single flow or solution. These three Supersonic Regimes are illustrated in Figures 4e to h.

2.2.2. The Possible Modes of Choking.

The flows in the Supersonic Regimes have been sub-classified above according to the behaviour of the two injected streams on injection. These flows, by definition, all contain some sort of choking. To a certain extent the way in which any particular flow chokes will depend on the regime to which it belongs. In the context of supersonic ejectors, the possible modes of choking may be divided into four major groups. Firstly if the total enthalpy ratio between the streams is large enough or the streams are composed of different gases the flow may become choked across the end of the mixing duct. The actual mechanism of this choking will naturally depend on the degree of mixing having taken place within the duct. Going to the other extreme, the ejector will choke if both streams choke in their respective injection nozzles. In this case the secondary will be sonic on injection and the primary will be supersonic on injection. Two intermediate modes of choking are possible. In one of these, the flow chokes at the end of the primary expansion by which stage little or no mixing will have taken place. In the other the choking takes place at some stage during the mixing process. This last possibility includes a continuum of possibilities between the case of choking at the end of the primary expansion and the case of choking at the mixing duct exit. It should be noted that two out of the four modes of choking are dependent on the mixing process and two are not. The two modes that are independent of the mixing achieve their independence by occurring before any mixing has taken place. Given the right conditions the mixing dependent modes of choking may occur in flows in any of the Supersonic Regimes. One such condition is a high total enthalpy ratio between the two injected streams. However, if a flow does not choke due to one of the mixing dependent processes the actual mode of choking will depend upon the reaction between the two injected streams and will, therefore, correspond directly to the regime in which the flow is classified. For an example, flows in the Supersonic Regime will choke due to some process at the end of the primary expansion, while flows in the Supersaturated Supersonic Regime will choke due to the choking of the secondary injection nozzle. The one-dimensional theories discussed above may be used to predict and provide descriptions of three of these modes of choking. Since these theories do not take into account the actual process of the mixing they cannot predict any event taking place during the mixing.

Hence if ejector flows are to be described by these theories alone, the predictable modes of choking are :- the choking of the fully mixed stream at the ejector exit, the choking of the flow at the end of the primary expansion, and the choking of the unmixed streams in their injection nozzles.

2.2.3. The Theories Used in Previous Work.

Of the modes of choking, mentioned at the end of the last section as being predictable by one-dimensional theory, the choking at the end of the primary expansion has aroused the most controversy. This is the pre-mixing mode of choking that occurs in flows in which the primary stream is injected in an under-expanded state. There are two rival modes that have been used to explain the mechanism of this choking. In one of these the primary stream is said to expand until the secondary becomes sonic and choked at the section of maximum primary expansion. In this state of maximum expansion the primary stream is usually well over-expanded. That is it has expanded beyond the stage at which the static pressures of the two streams would have become equal and uniform. In the second theory., on the other hand, the primary expands until the static pressures of both streams are equal. The choking in this case is provided by a two stream choking process taking place in this plane of uniform static pressure

Two methods of analysis based on the sonic choking theory have been put forward in previous works. The first of these was a one-dimensional treatment of supersonic ejection by Fabri (Ref. 1) in which both streams are assumed to behave isentropically on injection and the expansion is assumed to continue until the momentum exchange between the two streams has become equal and opposite. Choking takes place when the secondary mass flow reaches a value such that the secondary stream chokes at the cross-section of maximum primary expansion. The analysis corresponding to this theory was given in the section on sonic ejection. The other method of analysis based on the sonic choking theory is that of Messrs. Chow and Addi (Ref. 3). In this method both streams are again assumed to behave isentropically and the choking is again assumed to be caused by the secondary stream becoming sonic at the end of the primary expansion. But this time a one-dimensional treatment of the secondary stream is used as a boundary condition in the two-dimensional treatment of the primary expansion by the method of characteristics.

The two stream choking concept was first introduced by Messrs. Pearson, Holliday and Smith in their paper on sonic ejectors with short mixing ducts (Ref. 2). They used the method of analysis designated the Polytropic Two Stream Theory, earlier in this paper, in the section on sonic injection. The primary stream is assumed to expand polytropically until its static pressure matches that of the secondary. The choking process takes place across this plane of uniform pressure, where they show that the plane wave velocities for small disturbances are stationary.

Messrs. Hoge and Segars (Ref. 4) in their paper on choked flow assume that a two stream choking process takes place further downstream than the end of the primary expansion and that even by this stage no appreciable mixing has taken place. Both streams are assumed to behave isentropically when passing through the section at which the choking occurs. They do not state whether they consider the initial primary expansion to be isentropic or not.

Messrs. Hoge and Segars also give a general theory of choking which could be used to describe modes of choking that occur during the mixing process. More will be said about this in the section on the theory of general choking.

These four approaches, in References 1, 2, 3 and 4 represent the only really successful attempts using one-dimensional or quasi-one dimensional methods to obtain a better theory than the Straight Mixing Theory for supersonic ejector performance. Needless to say, there have been a considerably larger number of papers published dealing with the various aspects of the Straight Mixing Theory. A few examples are given in References 5 to 8.

2.2.4. Supersonic Ejector Operation According to One-Dimensional Theory.

The performance of a typical supersonic ejector will now be discussed assuming that it may be predicted by one-dimensional theories similar to those used in case of sonic injection. Consider a supersonic ejector such as that shown in Figure 4. Let its geometry be fixed and consider the stagnation conditions in the two injected streams to be constant. The performance of this ejector will then be a single valued function of the back pressure into which the ejector exhausts. Let us consider, then, the performance of the ejector as the back pressure is lowered from a reasonably high value. It should be noted, at this point, that, although the ejector performance in terms of mass flows, injection pressures, etc., may be dependent upon the back pressure, from the point of view of performing the necessary calculations the problem has to be tackled the other way round. Values for the injection pressures or mass flows have to be assumed and the corresponding values of the back pressure determined from them.

(i) The Subsonic, Mixed and Supersaturated Supersonic Regimes.

At back pressures somewhat greater than the secondary total pressure, the ejector will operate in the Subsonic Regime provided that the total pressure ratio π between the two injected streams is not too great. Here the flow is subsonic throughout and the injection pressures of the two streams are equal. If the total pressure ratio π is at all high the flow may never be subsonic throughout and still maintain a positive secondary mass flow. That is the Mixed Regime and the Base Pressure Regime may overlap to the exclusion of the Subsonic Regime. Thus, in general, at the highest back pressures to give positive secondary mass flows, ejectors will operate in either the Mixed or the Subsonic Regimes depending mainly upon the value of the total pressure ratio π .

For flows in the Subsonic Regime, lowering the back pressure from this maximum value causes the mass flows in both streams to increase until the primary injection nozzle chokes and the flow enters the Mixed Regime. Within the Subsonic Regime the performance of the ejector will be given by applying the Straight Mixing Theory together with the condition that the injection pressures are equal.

Lowering the injection pressure beyond the value at which the primary becomes choked results in the formation of a normal shock within the injection nozzle as in the case of the ordinary Laval nozzle. Assuming that no separation occurs within the nozzle this shock moves steadily downstream as the back pressure falls. Its exact location is determined by the equality of the injection pressures. Meanwhile the lowering of the back pressure continually increases the secondary mass flow. The connection between the ejector performance and the back pressure at this stage may again be found by a straightforward application of the Straight Mixing Theory for equal injection pressures, but this time an allowance must be made for the loss of primary total pressure over the normal shock. Eventually this normal shock will reach the injection plane and become oblique, the injection pressures will no longer be equal and the primary stream will contract upon injection as shown in Figure 4b. The primary injection pressure now being fixed and there being no loss of primary total pressure within the nozzle, the ejector performance is given simply by the Straight Mixing Theory alone. The Mixed Regime flows mentioned so far are all classified in the Supersaturated Mixed Regime.

Now, at any time during the process just described, the secondary stream will become choked if the secondary injection pressure falls to the critical value. This is most likely to occur in systems with low values of the total pressure ratio π. It is also possible of course that at any stage the flow may become choked at the end of the mixing process. When the flow chokes due to either of these causes and the primary stream is still under-expanded on injection the flow is said to enter the Supersaturated Supersonic Regime. An illustration is given in Fig. 4c.

However, considering those flows which have not yet choked, the oblique shock, originating from the point of initial stream contact, gets weaker as both the back pressure and the secondary injection pressure fall. During this process in which the normal shock becomes an oblique shock and grows gradually weaker, the difference between the injection pressures, and the degree of the primary contraction rise to a maximum and then fall back to zero as the weak shock vanishes. This last condition, where the primary stream is neither over nor under-expanded, is known as the Saturated condition and it represents the boundary between the Supersaturated Mixed Regime and the Supersonic Mixed Regime. For certain flows it happens that either the secondary injection pressure becomes critical or the flow chokes at the end of mixing simultaneously with the vanishing of the oblique shock, in which case the flow is said to choke in the Saturated Supersonic Regime. Such a case is depicted in Figure 4f.

Considering those cases in which flow is still unchoked in the Saturated Condition, further lowering of the back pressure will reduce the secondary injection pressure but not that of the primary. Thus the primary stream will be injected in an under-expanded state. The streams will then react so as to reduce the resulting pressure difference; that is the primary stream will expand and the secondary will contract as shown in Figure 4d. This region of the solution is called the Supersonic Mixed Regime. The ejector performance in this regime may be determined by using one of the two stream theories in conjunction with the Straight Mixing Theory.

(ii) The Supersonic and Saturated Supersonic Regimes.

All flows in the Supersonic Mixed Regime will become choked in one way or another as the back pressure continues to fall. Some will choke at the end of mixing and others will choke due to some process connected with the primary expansion. Let us neglect for the moment the choking at the end of the mixing process and discuss the pre-mixing modes of choking as though they were the only ones likely to take place. Now the various theories available to describe the initial primary expansion lead to distinctly different modes of behaviour in the Saturated Supersonic and Supersonic Regimes.

It may be instructive to look at these solutions in the following way. Consider, as above, a supersonic ejector of fixed geometry exhausting straight into a vacuum and thus operating entirely within the Supersonic Regimes. Now consider the effect of varying the total pressure ratio π For low values of π the ejector will be operating in the Supersaturated Supersonic Regime and the secondary injection nozzle will be choked. As the total pressure ratio is increased the primary injection pressure gradually rises eventually becoming equal to that of the choked secondary. At this stage the flow enters the Saturated Supersonic Regime. Let the particular value of the total pressure ratio at which this happens be denoted by π_1 Now let us tackle the problem from the other end. Starting with a high total pressure ratio, consider the effect of lowering it. Assuming that the initial value of the total pressure ratio is not high enough to cause the ejector to operate in the Base Pressure Regime, the ejector will be operating in the Supersonic Regime. Now as mentioned above there are a number of different methods of analysing the primary expansion and each leads to a different description of the Saturated Supersonic and Supersonic Regimes, and hence also to a different boundary between the two. The Two Stream Theories, given in the section on sonic injection, all lead to similar types of predictions for these two regimes, but the two-dimensional approach of Messrs. Chow and Addi leads to a definitely different prediction. This will now be illustrated as these regimes are first discussed with reference to the two-dimensional theory and then with reference to the one-dimensional theories.

As the total pressure ratio is lowered from the high value selected above, the degree of the primary expansion gradually decreases. Now if the process is being described by the two-dimensional theory, the secondary stream will choke sonically at the end of the primary expansion and, as the injection pressures become equal and the expansion tends to vanish, the secondary injection nozzle will tend to become choked. Thus this theory seems to provide a smooth continuous solution right through to the Supersaturated Supersonic Regime. This solution is shown as a continuous curve in Figure 5. On the other hand if the primary expansion is being described by one of the one-dimensional theories then the Supersonic Regime will not be smoothly connected to the Supersaturated Supersonic Regime. Let us consider first the predictions of the Polytropic and Isentropic Two Stream Theories. In both these theories the secondary stream chokes subsonically due to the two stream choking process. Thus as the total pressure ratio falls and the primary expansion tends to vanish the secondary injection condition will approach some subsonic condition rather than the sonic condition. The injection pressures will become equal and the expansion will vanish at π_2, a higher value of the total pressure ratio than π_1, that corresponding to the boundary of the Supersaturated Supersonic Regime. This type of solution is illustrated in Figure 5 in the form of a dottled line. This leaves a finite gap in the solution corresponding to flows with total pressure ratios between π_1 and π_2. The Saturated Supersonic Regime then embraces a continuous set of solutions as opposed to the single point solution predicted by the two-dimensional theory. In Fabri's theory, on the other hand, the secondary stream chokes sonically at the end of the primary expansion and so one might expect the solution to be similar to that due to the two-dimensional method of Messrs. Chow and Addi, but this is not found to be the case. As the total pressure falls the two injection pressures become equal while the secondary injection conditions are still well subsonic, but since the pressures at the end of the expansion are not considered equal, the primary expansion is still finite at this stage. This is rather a weak point in this particular theory because it requires the static pressures of the two streams to diverge from a common value at injection in order to choke the secondary stream sonically at the end of the expansion. Since both streams are assumed to behave isentropically this is an impossible situation. Thus it appears that this theory may only be considered for use for flows with relatively high total pressure ratios where primary injection pressure is considerably higher than that of the secondary and some degree of over-expansion might be expected to occur.

Going back to the case of the Polytropic and Isentropic Two Stream Theories, it should be obvious, on reflection, that the solution for which the primary expansion vanishes will be common to both theories, for a zero order primary expansion will be isentropic in both case and momentum will also be conserved in both cases. The boundaries of the Saturated Supersonic Regime will, therefore, be the same according to both theories. The Point D on Figure 5 corresponding to the total pressure ratio π_2 is thus the same for both theories.

If one-dimensional theories are to be applicable over all the Supersonic Regimes, it is then necessary to produce some theories on the flows in the Saturated Supersonic Regimes, as defined by the one-dimensional methods above.

(iii) One-Dimensional Theories for the Saturated Supersonic Regime.

Assuming that the one-dimensional theories proposed above for the Supersaturated Supersonic Regime and the Supersonic Regime are applicable up to boundaries corresponding to π_1 and π_2 respectively, there remains a need to produce some theory as to the mode of choking that will occur in the Saturated Supersonic Regime, in between π_1 and π_2. To get an idea of the context into which any such theory must fit it is appropriate to consider the effect of back pressure on an ejector with stagnation conditions corresponding to choking in the Saturated Supersonic Regime. It is, therefore, proposed to approach the problem by considering the behaviour of the flow in such an ejector as the back pressure is gradually reduced to the value at which the system chokes.

Consider an ejector with stagnation conditions such that, as the back pressure falls, the flow reaches the Saturated Condition in the Mixed Regime just before the whole system chokes. In such a system the total pressure ratio would, by definition, be somewhere between the values π_1 and π_2. When the back pressure is relatively high and the system is operating in the Supersaturated Mixed Regime, the secondary injection pressure is higher than that of the primary, but the primary, being supersonic, is able to alleviate the pressure difference instantaneously by passing through an oblique shock wave. On the other hand, when the flow has passed through the Saturated Condition the secondary injection pressure has fallen below that of the primary, there is no mechanism equivalent to a shock wave through which the subsonic secondary stream may pass in order to instantaneously increase its static pressure to match that of the primary. It would appear, in view of the difference in the injection pressures, that the primary must now expand on injection. However, when considering the behaviour of both stream according to any one of the one-dimensional theories it is found that such an expansion would lead to the divergence of the static pressures of the two streams. This is in contrast to the convergence found in similar treatments of flows in the Supersonic Regime. Faced with these facts we are left with the following choice; either the attempt to describe these flows in terms of a one-dimensiona theory must be abandoned - for, as it has already been pointed out, a two-dimensional method such as that of Messrs. Chow and Addi will provide theoretically acceptable solutions for the flows in question - or, it must be assumed that both streams in fact decelerate, thus allowing their respective static pressures to converge. The major objection to this last suggestion is that it requires the stream with the higher injection pressure to contract allowing that with the lower injection pressure to expand (in the sense of occupying a greater cross-sectional area). However, in the context of one-dimensional theory, this is the only way in which the static pressures of the two streams may converge.

Admitting the fact that the only available one-dimensional theory does have this serious drawback and that eventually one may have to resort to a two-dimensional approach, it is proposed to follow through with the one-dimensional theory to discover just what it does predict. For, in spite of its inherent weakness, it may produce some results of value. This is borne out to a certain extent by the very limited numerical results available at the time of writing. (See Section 3).

Any theory of flows in the Saturated Supersonic Regime must obviously involve some mode of choking. If the primary stream is going to contract and the secondary stream is not choked on injection a process involving the subsonic choking of the secondary must inevitably be assumed. This eliminates using a theory such as that of Fabri which involves sonic choking of the secondary. There thus remain the Polytropic Two Stream Theory and the Isentropic Two Stream Theory which may be used to describe the primary contraction. It is found, from numerical results, that use of the Polytropic theory always involves a loss of entropy within the primary stream. It could, therefore, be considered inadmissable on the grounds of contravening the second laws of thermodynamics. The isentropic theory, on the other hand, on acceptance of its inherent assumption of neglecting the overall momentum balance, does produce some slightly less untennable solutions. In the choked solutions, the streamwise static pressure gradients as well as the static pressures themselves tend to converge towards the end of the contraction whichever theory is used. This is a condition under which a two stream choking process is liable to take place (see Section 2.3). If choking does take place in the manner indicated it will involve a positive static pressure gradient in both streams during the contraction. This turns out to be compatable with the mixing processes that follow, in that they also involve positive streamwise static pressure gradients. For, reversing the procedure adopted above and considering the effect of raising the back pressure on a flow that is already choked, the back pressure at which the ejector would first become unchoked is in most cases substantially higher than either of the injection pressures and thus a positive static pressure gradient is indicated.

When numerical examples are undertaken it is found that the two one-dimensional theories in fact give very similar results. In other words the degree of irreversibility in the Polytropic theory and the momentum imbalance in the Isentropic theory are both found to be small. Consider again the effect of varying the total pressure ratio π in an ejector of fixed geometry exhausting into a vacuum. Let us now compare the type of solution offered by this one-dimensional approach with that of the two-dimensional approach of Messrs. Chow and Addi. Such solutions may be represented by curves in the $M_{21} - \omega_{21} / \pi$ field, where M_{21} is the secondary injection Mach number and ω_{21} / π is the ratio of the secondary injection pressure to the primary total pressure. The general form of these solutions is illustrated in Figure 5. The Supersaturated Supersonic Regime is represented by the line $M_{21} = 1.0$ on the right hand side of the figure and the point A on the intersection of this line with the line $p_{21} = p_{11}$ represents the boundary with the Saturated Supersonic Regime. Going to the other extreme the Base Pressure Regime is represented by a single point in each of the one-dimensional theories and by a curve with very low values of M_{21} in the two-dimensional case. The Supersonic Regime according to the two-dimensional theory is represented by a curve such as BA in Figure 5. It joins smoothly into the Base Pressure solution at the left hand end and approaches the line $p_{21} = p_{11}$ tangentially at the right hand end finally becoming concurrent at the point A. Thus in this case the Saturated Supersonic Regime is represented by the single point A. According to the one-dimensional theory, however, the Supersonic Regime is represented by a curve CD originating from the Base Pressure point, C, and tangentially touching the line $p_{21} = p_{11}$ at a point, D, below A. The point D corresponds to the total pressure ratio π_2 defined in the discussion above. The one-dimensional Saturated Supersonic Regime consists of two parts. In the first of these the primary contracts on injection and the system chokes at the end of the contraction. This portion of the solution results in a curve of the form of DE in Figure 5. The degree of the primary contraction increases as the total pressure ratio π falls and the solution point moves along the curve from D to E. The secondary mass flow increases and eventually the secondary injection nozzle becomes choked. So as the point E the flow is as it were doubly choked. In the second half of this regime the secondary is choked sonically on injection, but the primary contraction must still take place in order to equalise the static pressures. The degree of this contraction decreases as the total pressure ratio falls further and the solution point traverses the line EA in Figure 5. The contraction vanishes and the injection pressures become equal as the solution point approaches the point A.

Thus it may be seen that the main difference between the solutions supplies by the one and two-dimensional theories lie in their predictions for the Base Pressure Regime and the Saturated Supersonic Regime. In the Base Pressure Regime the one-dimensional theory predicts that the secondary mass flow will become zero at a much lower value of the total pressure ratio π than does the two-dimensional approach. The two-dimensional theory limits the Saturated Supersonic Regime to a single point considering the secondary stream to choke sonically at the end of the primary expansion for all higher values of the total pressure ratio π. The one-dimensional theories, on the other hand, predict two solutions in which the injection pressures have become equal. The regime of flows in between these solutions are then all considered part of a finite Saturated Supersonice Regime. It is fairly well established that the two dimensional theory gives the best results in the Base Pressure Règime, but not enough experimental work has been done with flows in the Saturated Supersonic Regime to come to any definite conclusions on the relative merits of the theories in this regime.

2.3. THE CONCEPT OF GENERALISED CHOKING.

It has been shown by a number of workers during the past decade that the wave velocity of a small pressure disturbance in a streamtube surrounded by elastic walls may be other than the normally recognised speed of sound. This concept has been used to explain the choking of two gas streams of different velocities flowing side by side in a duct without mixing. Other criteria such as those of maximum mass flow, minimum secondary total pressure, minimum momentum flux and maximum momentum flux density, have all been used as the basis of theories to explain this type of choking. The last collection of phenomena may occur as the result of choking in many cases, but they are not thought to be the cause. It must surely be a more correct approach to base a general theory of choking on what is likely to be the cause, rather than on an effect. In this case an exact definition of what is meant by choking must be given.

2.3.1. Definition of Choking.

The state of the flow at any cross-section in a streamtube is defined to become choked when it and the states immediately upstream of it first become insensitive to a drop in static pressure at the cross-section immediately downstream, while still remaining sensitive to a small rise in static pressure in the cross-section immediately downstream.

When choking is defined in this way as the loss of pressure communication through a cross-section, it seems logical to base the theory of choking on the criteria of small disturbance wave velocities. For it is well known that a drop in static pressure may only be transmitted upstream by an expansion wave; expansion waves must be continuous and isentropic in order to obey the second law of thermodynamics and, therefore, travel at the local small disturbance velocity. A generalised theory of choking, based on the wave velocity argument, will now be presented.

2.3.2. The Streamwise Disturbance Velocity in a Single Streamtube.

It is well known that the wave velocity in a steady streamtube with elastic inertialess sides is given by the following expression :-

$$q = a\left[1 + \frac{\rho a^2}{A}\left.\frac{\partial A}{\partial p}\right|_i\right]^{-\frac{1}{2}}$$

where $\left.\frac{\partial A}{\partial p}\right|_i$ expresses the elasticity of the streamtube walls.

However, for the sake of completeness this relation will now be derived.

For one-dimensional motion along a streamtube of variable area the laws of conservation of mass and momentum state that

$$\frac{\partial}{\partial x}(\rho vA) + \frac{\partial}{\partial t}(\rho A) = 0 \qquad \text{....18}$$

and

$$\frac{\partial}{\partial x}(\rho v^2 A) + \frac{\partial}{\partial t}(\rho vA) = -A\frac{\partial p}{\partial x} \qquad \text{....19}$$

or more fully

$$vA\frac{\partial \rho}{\partial x} + \rho A\frac{\partial v}{\partial x} + \rho v\frac{\partial A}{\partial x} + A\frac{\partial \rho}{\partial t} + \rho\frac{\partial A}{\partial t} = 0 \qquad \text{....18a}$$

$$Av^2\frac{\partial \rho}{\partial x} + 2\rho Av\frac{\partial v}{\partial x} + \rho v^2\frac{\partial A}{\partial x} + Av\frac{\partial \rho}{\partial t} + \rho v\frac{\partial A}{\partial t} + \rho A\frac{\partial v}{\partial t} = -A\frac{\partial p}{\partial x}$$

$$\text{....19a}$$

Now let the elasticity of the walls at the cross-section under consideration be given by

$$\left.\frac{\partial A}{\partial p}\right|_i = r$$

Also since the flow in the streamtube is assumed to be adiabatic

$$\frac{\partial p}{\partial \rho} = a^2$$

Hence the partial differentials in equations 18a and 19a may be written

$$\frac{\partial p}{\partial x} = a^2\frac{\partial \rho}{\partial x}, \quad \frac{\partial A}{\partial x} = r\frac{\partial p}{\partial x} = ra^2\frac{\partial \rho}{\partial x} \qquad \text{etc.}$$

Equations 18a and 19a may thus be rewritten

$$(A + \rho a^2 r) v \frac{\partial \rho}{\partial x} + \rho A \frac{\partial v}{\partial x} + (A + \rho a^2 r) \frac{\partial \rho}{\partial t} = 0 \qquad \ldots .18b$$

$$(A (v^2 + a^2) + \rho v^2 a^2 r) \frac{\partial \rho}{\partial x} + 2\rho A v \frac{\partial v}{\partial x} + (A + \rho a^2 r) v \frac{\partial \rho}{\partial t} + \rho A \frac{\partial v}{\partial t} = 0 \qquad \ldots .19b$$

Now in order to consider the propagation of small disturbances within the streamtube it is necessary to transform these equations to new axes, (y, T), stationary with respect to the fluid in which the disturbance is taking place. The timescale remains unchanged but the distance parameter, x, becomes y which is given by $y = x - Ut$ where U is the stream velocity at the cross-section under consideration. This transformation leaves the equations in the form

$$\frac{\rho A}{(A + \rho r a^2)} \frac{\partial v}{\partial x} + \frac{\partial \rho}{\partial t} = 0 \qquad \ldots .18c$$

$$\frac{a^2}{\rho} \frac{\partial \rho}{\partial x} + \frac{\partial v}{\partial t} = 0 \qquad \ldots .19c$$

It should be noted that the velocity, v, only appears in these equations in the form of a derivative which makes the equations independent of the actual velocity U used in the transformation. Either ρ or v may be eliminated from these equations to give an equation of the form

$$\frac{\partial^2 \phi}{\partial t^2} - \frac{1}{\frac{1}{a^2} + \frac{\rho r}{A}} \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad \ldots .20$$

where ϕ may represent any of the static flow parameters.

Now considering a small disturbance from the steady state in any of the variables ϕ, it may be written in the form

$$\phi = \phi_1 + \phi'$$

where ϕ_1 is the steady state and ϕ' the disturbance.

Substituting this into equation 20 and neglecting small quantities the following linear equation in ϕ' may be evolved,

$$\frac{\partial^2 \phi'}{\partial t^2} - \frac{1}{\frac{1}{a_1^2} + \frac{\rho_1 r_1}{A_1}} \frac{\partial^2 \phi'}{\partial y^2} = 0 \qquad \ldots.21$$

where a_1, r_1, A_1 etc., are the steady state conditions.

Hence it is shown that streamwise disturbances travel with a velocity given by,

$$q = a_1 \left[1 + \frac{\rho_1 r_1 a_1^2}{A_1} \right]^{-\frac{1}{2}} \qquad \ldots.22$$

2.3.3. Stationary Wave Velocities and Choking in a Single Streamtube.

It will now be shown that if the flow at any particular cross-section along a streamtube has the freedom to react in an isentropic manner to accommodate a small disturbance then the wave velocity is stationary at that cross-section.

In the context of a single streamtube, having the freedom to react isentropically means that r_1 or $\left.\frac{\partial A}{\partial P}\right|_1$ must possess the value corresponding to an isentropic departure from the state at the cross-section under consideration. This value will now be determined.

For adiabatic flow along a streamtube the equations of motion may be written :-

Continuity, $\frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0$

Momentum, $v\,dv = -\frac{dp}{\rho}$

Entropy, $\frac{dp}{d\rho} = a^2$

These equations may be combined to give :-

$$\frac{1}{A}\left.\frac{\partial A}{\partial P}\right|_s = \frac{1}{\rho a^2}\left[\frac{1}{M^2} - 1\right] \qquad \ldots.23$$

where M is the Mach number.

Substituting this expression into equation 22 we find the speeds of the stream and the disturbance waves are equal

$$q = a\left[1 + \frac{1}{M^2} - 1\right]^{-\frac{1}{2}} = v \qquad \ldots.24$$

Thus if the value of $\left.\frac{\partial A}{\partial p}\right|_i$ imposed on the streamtube by its enviornment is the same as the isentropic value $\left.\frac{\partial A}{\partial p}\right|_s$, the wave velocity is then stationary, and the flow may be considered choked.

2.3.4. The Use of a Parameter Analogous to the Mach Number.

It is useful at this stage to introduce a parameter, similar in concept to the Mach number, by which we can tell whether the flow at any particular cross-section is "supersonic" or not. The word "supersonic" here is meant with reference to the wave velocity, q. We could choose the parameters v/q, but this involves finding the value of $\left.\frac{\partial A}{\partial p}\right|_i$ at the cross-section in question and then substituting it into equation 22 to find q. In the context of the theory to follow it is of more use to define a parameter, b, as the difference between the imposed value of $\left.\frac{\partial A}{\partial p}\right|_i$ and the isentropic value $\left.\frac{\partial A}{\partial p}\right|_s$. With this definition, a positive value of b indicates "supersonic" flow and a negative value indicates "subsonic" flow. The main use of this parameter comes when the concept is widened to include flows containing two or more different streamtubes.

$$b = \left.\frac{\partial A}{\partial p}\right|_i - \left.\frac{\partial A}{\partial p}\right|_s = \left.\frac{\partial A}{\partial p}\right|_i - \frac{A}{\rho a^2}\left[\frac{1}{M^2} - 1\right] \qquad \ldots.25$$

2.3.5. The Proposition of a Generalised Theory of Choking.

It is suggested that a general flow will choke at a particular cross-section if, as a whole, it possesses the freedom to react in an isentropic manner to any small changes in pressure at that cross-section. As an example of the application of this theory consider a cross-section, of area a', in a general steady shearflow with a non-zero normal pressure gradient such as that shown in Figure 74. The flow is bounded on one side by a fixed wall and on the other side by the intertialess flexible wall which has an elasticity at the cross-section under consideration given by :-

$$B_i = \left.\frac{\partial a}{\partial p}\right|_i \qquad \ldots.26$$

In the case of a fixed boundary Bi = 0. Now the pressure disturbance over this cross-section may be non-uniform since the normal static pressure gradient is non-zero. Thus any disturbance must be defined with respect to a reference pressure, say p_1 . Let the reference pressure in this case be the static pressure at the flexible wall. The disturbance at any point on the cross-section due to a change δp_1 in p_1 is given by

$$\delta p = \left.\frac{\partial p}{\partial p_1}\right|_D \delta p_1$$

Considering a small element of area A, the isentropic area change due to δp_1 may be written,

$$\delta A = \left.\frac{\partial A}{\partial p}\right|_s \delta p = \left.\frac{\partial A}{\partial p}\right|_s \left.\frac{\partial p}{\partial p_1}\right|_D \delta p_1 .$$

Hence the change in a', the total area of the stream caused by p_1 is

$$\delta a' = \sum_{a'} \delta A = \int_{a'} \frac{\delta A}{A} da' = \delta p_1 \int_{a'} \frac{1}{A}\left.\frac{\partial A}{\partial p}\right|_s \left.\frac{\partial p}{\partial p_1}\right|_D da'$$

Which, using equation 23 and letting δp_1 tend to zero may be written

$$B_s = \left.\frac{da'}{dp_1}\right|_s = \int_{a'} \left.\frac{\partial p}{\partial p_1}\right|_D \frac{1}{\gamma p}\left[\frac{1}{M^2} - 1\right] da' \qquad \text{....27}$$

We may now define a parameter, B, for the whole cross-section, similar in concept to "b" defined above for a single streamtube.

$$B = B_i - B_s = \left.\frac{\partial a'}{\partial p_1}\right|_i - \int_{a'} \frac{1}{\gamma p}\left.\frac{\partial p}{\partial p_1}\right|_D \left[\frac{1}{M^2} - 1\right] da' \qquad \text{....28}$$

$\left.\frac{\partial p}{\partial p_1}\right|_D$ is a function of both the flow and the type of disturbance being considered and its value must be derived or assumed with regard to the situation under consideration.

As before, when B is positive the flow is "supersonic" with respect to the common wave velocity, and conversely when B is negative the flow is "subsonic".

Equation 28 may be simplified in two ways. For flows in which the normal static pressure gradient is zero the expression $\left.\frac{\partial p}{\partial p_1}\right|_D = 1$ and hence

$$B = \left.\frac{\partial a'}{\partial p}\right|_i - \frac{1}{\gamma p} \int_{a'} \left[\frac{1}{M^2} - 1 \right] da'$$

and for flows in ducts with fixed walls, though not necessarily with a constant cross-sectional area,

$$\left.\frac{\partial a'}{\partial p}\right|_i = 0 \text{ , therefore } \quad B = -\frac{1}{\gamma p} \int_{a'} \left[\frac{1}{M^2} - 1 \right] da' \qquad \ldots.29$$

Equation 29 may be used, for an instance, to find the value of B at any stage during the constant area mixing that takes place in ejector flows.

It is assumed that choking will occur at any cross-section for which the parameter B is found to be zero.

2.4. ALTERNATIVE ONE-DIMENSIONAL THEORIES OF MIXING.

In Section 2.1 above it was shown how the various theories on the initial pressure adjustment in ejector flows could be combined with the Straight Mixing Theory to provide complete theories of ejector operation. The Straight Mixing Theory, with its assumption of uniform conditions across the exit of the mixing duct, is the weaker link in what could be a more satisfactory theory. It should, therefore, be worth considering what improvements can be made without resorting to a two-dimensional description of the flow. In order to get an idea of what might be worth trying it is necessary to make a few generalisations based on results from experimental ejector flows.

The first and most important of these is that, except for a small section of the flow just after the two streams first come into contact, the static pressure is uniform across every cross-section of the flow. The word cross-section here is used in the sense of cross-sections normal to the direction of flow. It is thus quite in order to assume that the static pressure is uniform across the exit of the mixing duct. The total pressure on the other hand, virtually never becomes uniform, for if the mixing duct is long enough for the mixing to approach completion the effect of the boundary layers on the walls becomes very significant. A similar statement can be made concerning the total enthalpy. It is noticed that while parts of either stream remain unmixed they retain their total pressures and total enthalpies although the static pressure may change. It is also noticed that the total pressure distribution across the mixing region may in most cases be reasonably approximated by a linear variation between the two bounding values, while there remains some unmixed stream on either side. For flows in which the mixing region fills the duct at its exit it is also found that the total pressure distribution, outside the boundary layer, may in many cases be approximated by a linear expression.

A one-dimensional theory may only connect the states of the flow at cross-sections at arbitrary distances apart; it, therefore cannot, of itself, predict any rate of change in any of the flow parameters. The states at any two cross-sections are connected by the three relations of conservation of mass, momentum and energy. Hence, assuming that the state is known at one cross-section, a one-dimensional theory provides three equations to solve for three unknown parameters at the other cross-section this if the unknown state is at the duct exit and only a single solution is required, this state must be expressed in terms of three parameters for which the one-dimensional theory may provide a solution.

It is also possible to set up a model for the mixing process itself, this time expressing the state of the flow at any cross-section in terms of four parameters one of which must indicate in some way the degree of mixing having taken place. The last parameter may then be varied to observe the effects of mixing on the states at a series of cross-sections. Such an approach may give a better idea of the effect of mixing on ejector flows, but it cannot predict any rate of mixing. It may be useful, however, if the rate of mixing is either known or can be assumed with any confidence.

Two theories based on the observations of actual flows given above will now be presented in the context of plane flow. One provides a method of predicting the state of the flow at the exit of medium length mixing ducts. The other provides a method of predicting the effect of mixing within the duct.

2.4.1. The Linear Mixing Theory.

For given conditions at the end of the primary expansion, this theory gives a solution for the state of the mixed stream on leaving the mixing duct. It assumes a linear distribution for both the total pressure and the total enthalpy at the duct exit. Both injected streams are assumed to consist of the same gases. Without making any further assumptions five parameters are necessary to specify the state at the duct exit. These are the static pressure, the two boundary values of the total pressure and the two boundary values of the total enthalpy at the duct walls. Since there are only three equations, only three unknowns may be determined. It is, therefore, necessary to make some further assumptions appropriate to the flows being investigated, in order to reduce the number of unknowns to match the number of available equations. It will be evident that there are many ways in which this may be done and that different assumptions will suit different types of flow. An example will now be given.

For flows in which the mixing region fills the duct exit and for which the total enthalpy ratio between the injected streams is unity, the energy and continuity equations are no longer independant and, therefore, only two unknowns may be determined. One of these must be the static pressure and thus there is only one parameter left with which to define the total pressure distribution. Either one of the boundary values or the total pressure gradient, must therefore be assumed. It was suggested that the boundary value, on the side of that stream which occupies the larger portion of the duct at the end of the primary expansion, should be assumed to take the value it had on injection. In the case where the primary stream occupies the greater area after expanding, the total pressure at the primary wall in the exit plane is assumed to be given by π_3 . A solution of this type is illustrated in figure 31. The state at the duct exit is expressed in terms of two parameters, p_3 , the static pressure, and π_3 , the total pressure at the secondary wall divided by the initial primary total pressure P_{12} . The total pressure distribution is then given by :-

$$\pi'' = \frac{P_3}{P_{22}} = \pi'(1 - s(1 - \pi_3)) \qquad \ldots.30$$

where $s = y/d$ the fractional distance across the duct from the primary walls, and π' is the total pressure ratio between the two streams after the primary expansion. The static pressure, which is uniform and given by p_3, may be used in the non-dimensional form $\omega_3 = p_3/P_{22}$

Using this distribution the equations of conservation of mass and momentum may be written respectively :-

$$\pi' z' y_{12} + (1 - z') y_{22} = \int_0^1 \pi'(1 - s(1 - \pi_3)) y \left[\frac{p_3/P_{22}}{\pi'(1 - s(1 - \pi_3))} \right] ds \quad \ldots 31$$

$$\pi' z' f_{12} + (1 - z') f_{22} = \int_0^1 \pi'(1 - s(1 - \pi_3)) f \left[\frac{p_3/P_{22}}{\pi'(1 - s(1 - \pi_3))} \right] ds \quad \ldots 32$$

Equations 31 and 32 are two equations in the two unknowns ω_3 and π_3 . Neither ω_3 nor π_3 may be eliminated between the equations with any ease, so solutions have to be found by numerical methods using these equations in the form that they stand. Since the equations involve the double valued function "y", it would be expected that in general there would emerge two alternative solutions. In analogy to the Straight Mixing Theory it would be expected that one of these solutions would indicate a "subsonic" flow and the other, if it is finite and positive, would indicate a "supersonic" flow. It would also be expected that two coincident solutions would indicate a choked flow and the value of B, found from equation 29 would be zero for the cross-section of the flow at the duct exit. Such assertions may only be tested by doing some numerical examples.

It should be noted that theories such as this need not be limited to linear distributions of the stagnation variables. Any suitable distributions may be used provided that they allow the state at the duct exist to be expressed uniquely by three parameters in terms of which the one-dimensional equations may be written.

2.4.2. A One-Dimensional Description of the Mixing Process.

As mentioned above a model of the mixing process may be formulated using one-dimensional theory. In such a model the state at any cross-section may be defined in terms of four parameters, one of which must indicate in some way the degree of mixing having taken place. The other three are then connected by the one-dimensional theory. Thus the behaviour of the flow may be studied as the mixing progresses. Using the observations of experimental flows mentioned above, the following model of the mixing process was formulated. The model is described with reference to figure 32.

The mixing process is assumed to start at the end of the primary expansion and the state at this cross-section, being taken as known, is used as the reference state for the rest of the process. The static pressure is assumed to be uniform over each cross-section during the mixing. The proportions of the mixing tube occupied by the unmixed streams are determined by the primary expansion process and are given by, for the primary $\frac{A_{12}}{A_3} = z'$ and for the secondary $\frac{A_{22}}{A_3} = 1 - z'$ 33

The mixing process is shown diagrammatically in figure 32, the boundaries of the mixing region being shown in the form of dotted lines. The flows outside this region are assumed to retain their stagnation properties and thus will only experience changes in state due to changes in static pressure. (Within the mixing region). The stagnation conditions are assumed to vary linearly between the boundary conditions. If the mixing is assumed to be largely turbulent then the rates of mixing with respect to the total pressure and the total enthalpy will be the same and hence the mixing region shown in figure 32 will be the same for both quantities. This assumption reduces the number of equations provided by the one-dimensional theory because under these conditions the continuity and energy equations are no longer independent of each other. The problem is thus reduced to one of three parameters.

As can be seen from figure 32 the mixing process can be divided up into three successive regions. During the initial stages the mixing region lies between the two unmixed flows and there are as it were three different components of the flow, the two unmixed streams and the mixing region. At some stage further downstream the mixing region will envelope the whole of one of the injected streams and the composition of the flow will be reduced to two components. Which of the streams is totally enveloped first will differ from flow to flow. In the case of the example shown in figure 32 it is the secondary stream which becomes totally enveloped first. The theory to follow will be developed for this case. Eventually the mixing region will fill the whole mixing duct, toally enveloping both injected streams. This is the third region shown in figure 32.

Consider the state of the flow at a cross-section, n, at any stage before the mixing region envelopes the whole of either stream. The parameter chosen to represent the degree of mixing is x_{1n}, the fraction of the primary mass flow that has entered the mixing region by the plane n. The two remaining parameters are p_n the static pressure at the cross-section n and x_{2n} the fraction of the secondary mass flow to have entered the mixing region by the plane n. Thus the state at the cross-section may be defined by stating the values of p_n, x_{1n} and x_{2n}. Assuming a value for x_{1n} the corresponding values of p_n and x_{2n} may be found using the one dimensional theory. The relevant equations will now be derived.

The condition of conservation of mass may be written :-

$$x_{1n}\,\dot{m}_{12} + x_{2n}\,\dot{m}_{22} = \dot{m}_n \qquad \ldots.34$$

where $\dot{m}_{12}$ and $\dot{m}_{22}$ are the mass flow rates of the primary and secondary streams respectively, and $\dot{m}_n$ is the mass flow within the mixed region at the cross-section n.

Assuming that both streams are composed of the same gas this equation may be written :-

$$A_{12}x_{1n}P_{12}H_{12}^{-\frac{1}{2}}y_{12} + A_{22}x_{2n}P_{22}H_{22}^{-\frac{1}{2}}y_{22} = \int_{A_m} P_nH_n^{-\frac{1}{2}}y_n \, dA \qquad \ldots.34a$$

where A_m is the area occupied by the mixing region.

If p_n is the static pressure at the cross-section n, then the area A_m is given by

$$A_m = A_{12} + A_{22} - (1 - x_{1n})A_{12}\frac{y_{12}}{y_{1n}} - (1 - x_{2n})A_{22}\frac{y_{22}}{y_{2n}}$$

Substituting this into equation 34, the equation may be written in the non-dimensional form

$$\frac{z'}{1 - z'}\pi' h^{-\frac{1}{2}}y_{12}x_{1n} + y_{22}x_{2n} = \left[\frac{1}{1 - z'} - \frac{z'}{1 - z'}\frac{y_{12}}{y_{1n}}(1 - x_{1n}) - \frac{y_{22}}{y_{2n}}(1 - x_{2n})\right]\int_0^1 \left[\frac{1 + s(\pi' - 1)}{\sqrt{1 + s(h - 1)}}\right] y\left[\frac{p_n/P_{22}}{1 + s(\pi' - 1)}\right] ds \qquad \ldots.34b$$

where s is the fractional distance across the mixing region.
In a similar manner the momentum flux equation may be written :-

$$\frac{z'}{1 - z'}\pi' f_{12} + f_{22} = \pi'\frac{z'}{1 - z'}\frac{y_{12}}{y_{1n}}(1 - x_{1n})f_{1n} + \frac{y_{22}}{y_{2n}}(1 - x_{2n})f_{2n} + \left[\frac{1}{1-z'} - \frac{z'}{1-z'}\frac{y_{12}}{y_{1n}}(1 - x_{1n}) - \frac{y_{22}}{y_{2n}}(1 - x_{2n})\right]\int_0^1 (1 + s(\pi' - 1))f\left[\frac{p/P_{22}}{1 + s(\pi'-1)}\right] ds \qquad \ldots.35$$

Now for a known value of x_{1n} equations 34b and 35 may be solved to give solutions for x_{2n} and p_n. In fact x_{2n} may be eliminated fairly simply between equations 34b and 35 to give a single functional equation in p_n. This equation would obviously have to be solved numerically. In view of its large size but comparatively simple derivation it will not be written down here. A description of the mixing process has now been provided for the first region of flow in Figure 32.

Consider, now, the region of flow in which there only remains some of the primary stream outside the mixing region, that is the second region in Figure 32. This region is bounded by the two cross-sections at which the parameters x_{1n} and x_{2n} become unity. In this particular case it is x_{2n} that becomes unity first and has to be replaced by another parameter. The total pressure, P_{32}, on the boundary between the mixing region and the secondary wall of the duct will now be greater than that of the secondary on injection. We may define a parameter, π_3 , as P_{32}/P_{22} . It is this parameter that now replaces x_{2n} as the third parameter necessary to define the state over any cross-section. Hence the necessary parameters are x_{1n}, p_n and π_3 . The boundary value of the total enthalpy on the secondary wall will no longer be H_{22} but since the rates of mixing are considered equal the parameter $h_3 = \frac{H_{32}}{H_{22}}$ is dependent only on π_3 and is given by the expression :-

$$h_3 = \frac{\pi_3 - 1}{\pi' - 1}(h - 1) + 1 \qquad \ldots.36$$

Considering again a specific cross-section n, defined by $x_1 = x_{1n}$ the appropriate equations of conservation of mass and momentum may be derived. The conservation of mass leads to the equation :-

$$\frac{z'}{1 - z'}\pi' h^{-\frac{1}{2}} y_{12} x_{1n} + y_{22} = \left[\frac{1}{1 - z'} - \frac{z'}{1 - z'}\frac{y_{12}}{y_1}(1 - x_{1n})\right] \int_0^1 \left[\frac{\pi_3 + s(\pi' - \pi_3)}{\sqrt{h_3 + s(h - h_3)}}\right] y \left[\frac{p_n/P_{22}}{\pi_3 + s(\pi' - \pi_3)}\right] ds \qquad \ldots.37$$

and the conservation of momentum to :-

$$\frac{z'}{1-z'}\pi' f_{12} + f_{22} = \pi'\frac{z'}{1-z'}\frac{y_{12}}{y_{1n}}(1-x_{1n})f_1 + \left[\frac{1}{1-z'} - \frac{z'}{1-z'}\frac{y_{12}}{y_{1n}}(1-x_{1n})\right] \int_0^1 (\pi_3 + s(\pi' - \pi_3)) f\left[\frac{p_n/P_{22}}{\pi_3 + s(\pi' - \pi_3)}\right] ds \qquad \ldots.38$$

Equations 37 and 38 are two equations in the two unknowns p_n and π_3. Neither of these quantities may be eliminated with any ease, so the equations must be solved numerically in the form that they stand.

Turning now to the last region of flow, both x_{1n} and x_{2n} have become unity and thus the boundary values of the total pressure on the duct walls are both variable. The process of mixing will tend to make these values become equal. It is convenient to retain π_3 as the boundary value on the secondary wall and to define the total pressure at the primary wall, using the parameter π_3' as

$$\frac{P_{31}}{P_{22}} = \pi_3 + \pi_3'$$

The linear distribution across the mixing region is then given by

$$\frac{P}{P_2} = \pi_3 + s\pi_3'$$

Making the assumption that the rates of mixing are the same for both the total pressure and the total enthalpy, the total enthalpy distribution may be expressed

$$\frac{H}{H_{22}} = 1 + \frac{h - 1}{\pi' - 1} (\pi_3 + s(\pi_3' - 1))$$

Considering a particular cross-section, n, corresponding to a particular value of π_3 the equations of the conservation of mass and momentum may be written in terms of π_3, π_3' and p_n. Conservation of mass results in the equation :-

$$\frac{z'}{1-z'} \pi' h^{-\frac{1}{2}} y_{12} + y_{22} = \frac{1}{1-z'} \int_0^1 \left[\frac{\pi_3 + s\,\pi_3'}{\sqrt{1 + (h-1)(\pi_3 + s(\pi_3'-1))/(\pi'-1)}} \right] y \left[\frac{p\ /P_{22}}{\pi_3 + s\,\pi_3'} \right] ds \qquad ....39$$

and the conservation of momentum in :-

$$\frac{z'}{1 - z'} \pi' f_{12} + f_{22} = \frac{1}{1 - z'} \int_0^1 (\pi_3 + s\pi_3')\ f \left[\frac{p_n / P_{22}}{\pi_3 + s\,\pi_3'} \right] ds \qquad ....40$$

Equations 39 and 40 may again be solved numerically for π_3 and p_n.

In the last region of flow the parameter π_3', starts off with a positive value at the cross-section where the mixing region first fills the duct and is gradually reduced as the mixing proceeds. The final state is reached when π_3' becomes zero and the conditions are uniform across the duct.

To sum up, the mixing process as a whole may be described, in terms of the parameters p_n, x_{1n}, x_{2n}, π_3 and π_3', as follows. The mixing is assumed to start at the end of the primary expansion where x_{1n} and x_{2n} are zero. p_n is always considered uniform across the cross-section to which it corresponds. As the mixing proceeds both x_{1n} and x_{2n} rise until one of them reaches unity from whenceforth that one is replaced by a parameter of the form of π_3 as the third parameter. When both x_{1n} and x_{2n} have become unity the problem is then specified in terms of p_n π_3 and π_3', the last of these falling to zero as the mixing reaches completion.

Considering a particular ejector with fixed stagnation conditions there exists a continuous series of possible secondary mass flows. For any one particular mass flow a complete study of the flow may be made using one of the two Stream Theories for the initial pressure adjustment and the theory above for the mixing process.

The above sets of equations describing the state of the flow at any cross-section, n, will provide two solutions for each cross section.

These solutions may in general be either real or imaginary. Any flow resulting in imaginary roots must be assumed to be unable to exist if the mixing is allowed to proceed any further than the cross-section at which the roots first become imaginary. When the roots are real, in analogy to the Strain Mixing Theory, it would be expected that one of them would represent a "subsonic" solution and the other, if it is positive and finite, would represent a "supersonic" solution. Similarly if the roots are equal it might be expected that the flow would be choked. These theories can only be checked by finding the values of B, for equation 29, corresponding to the solutions in question. To do this, actual numerical examples will have to be undertaken. It is suggested, however, that if two such real solutions do exist the flow will only be able to assume the "supersonic" solution if it has already passed through a cross-section that is choked; that is for which the value of B is zero.

Going back to consider the ejector with fixed stagnation conditions it may be found that for different mass flow rates the system becomes choked at cross-section corresponding to different degrees of mixing. In the context of a known rate of mixing this would indicate that the mode of choking of the system might be dependent upon the length of the mixing tube.

Regarding this theory as a whole, two points arise, firstly, it is not strictly necessary to limit the theory to linear variations across the mixing region. Distributions of any suitable shape may be used provided that they allow the state at any unknown cross-section to be specified uniquely in terms of three parameters when the mixing rates for both the total pressure and total enthalpy are assumed equal, and uniquely in terms of four parameters when these rates are not assumed to be equal. The second point is that the theory does not include any allowance for the boundary layer build-up along the mixing duct walls. However, if the relation between the rates of mixing and the rate of boundary layer growth is either known or can be assumed with any confidence, there is no reason why this theory could not be adapted to include this phenomenon.

3. NUMERICAL INVESTIGATIONS OF THE SOLUTIONS PREDICTED BY SOME OF THE ONE-DIMENSIONAL THEORIES.

3.1. SONIC INJECTION, THE BACK PRESSURE DEPENDENT SOLUTIONS.

The majority of the numerical work to be reported in this paper consists of predictions of the flows in sonic ejectors, that is those with convergent injection nozzles. Three of the one-dimensional theories have been used to predict these flows. They are the Straight Mixing Theory, Fabri's Two Stream Theory and the Polytropic Two Stream Theory. All three of these theories are based on the conservation of mass, momentum and energy and, therefore, given a fixed state at the injection plane, they will all predict the same state for the fully mixed stream, provided that the flow is not choked. Thus the behaviour of these flows, in terms of externally observable quantities such as the mass flow rate and the overall pressure ratio is independent of which theory is used while the flow is still back pressure dependent. The only way in which the predictions of these particular theories differ is in the stage of the solution at which they predict that the flow will become choked. Thus in order to study the back pressure dependent solutions in terms of the external quantities it is only necessary to use one of the theories. As explained in Section 2.1.3 when the two stream theories are used to predict flows they are used in conjunction with the Straight Mixing Theory. It was, therefore, decided to use the Polytropic Two Stream Theory to investigate the flows in the Subsonic and Mixed Regimes of flow. The advantages of using one of the two stream theories instead of the Straight Mixing Theory by itself is that some information is obtained with regard to the likely internal behaviour of the flow.

The results from a fairly extensive investigation of the solutions for sonic injection predicted by the Polytropic Two Stream Theory are described and discussed in Section 3 of the paper "The Performance of an Air-Air Ejector According to a Quasi-One-Dimensional Theory". (Ref. 9) submitted as part of this thesis. The full back pressure dependent solutions are discussed in Sections 3.1, 3.2 and 3.3.1. In this last section the results from a fairly complete survey of the back pressure dependent solutions are presented. The ranges of the control parameters investigated were as follows :

	Range Investigated	Theoretically Possible Range
z, the geometric ratio	from 0.2 to 0.8	0.0 to 1.0
π, the total pressure ratio	from 1.7 to 2.0	1.0 to infinity
h, the total enthalpy ratio	from 1.0 to 4.0	1.0 to infinity

The ranges of z and h were reasonably comprehensive, but the total pressure ratio range was rather restricted. It did, however, include the critical value = 1.89 which marks the boundary between those solutions which have a Subsonic Regime and those that do not. At this critical value the primary injection nozzle becomes unchoked at the same time as the secondary mass flow vanishes.

Further to these results the performance of a sonic ejector with the same characteristics as that used in the experimental work was also evaluated. This ejector configuration consisted of a total anthalpy ratio of unity and an injection area ratio of unity. The total pressure ratio was varied from 1.0 to 2.0. The results from the full back pressure dependent solutions are presented in Fig. 48 in terms of the relations between the exhaust pressure and the secondary injection pressure for various fixed total pressure ratios. The truncation of these curves represents the choking of the flows as predicted by the Polytropic Two Stream Theory. The lower arms of these curves are the supersonic solutions and will only be allowed to exist physically if a choking process other than that here predicted causes the flow to choke at a higher secondary injection pressure. Otherwise the only physically significant parts of these solutions are upper subsonic solution arms and the vertical truncations representing the choked solutions. The ringed points on the subsonic solutions represent the stage at which the primary injection nozzle becomes choked and thus they form the boundary between the Subsonic and Mixed Regimes.

3.2. CHOKED FLOWS, RESULTS FROM THE POLYTROPIC TWO STREAM THEORY.

Turning to the consideration of the choked flows in sonic ejectors, the results from a fairly extensive survey of choked solutions as predicted by the Polytropic Two Stream Theory are presented in Section 3.3.2. of Reference 9, the A.R.C. paper presented as part of this thesis. This survey includes a determination of the boundary between those flows that choke due to the two stream process and those in which the fully mixed stream becomes choked at the end of the mixing duct. The resulting boundary is depicted in Figure 22.

The Supersonic Regime solutions were investiaged over the following ranges in the control parameters :-

z	0.01	to	0.9
π	1.1	to	6.0
h	1.1	to	5.0

The state of the mixed stream may be represented by the exhaust pressure. In those solutions in which the mixed stream is choked definition of the exhaust pressure fixes the state of the flow at the duct exit in one-dimensional terms, but in flows choking due to the two stream choking process two parameters are required to define the state of the mixed stream for it is no longer choked. The exhaust pressures corresponding to these solutions are given in Reference 9, Figures 23 to 28 as surfaces of constant ejector geometry in the π-h-p_3 field. Complementary to these results are the exhaust Mach numbers, and the secondary injection pressures corresponding to these flows. The Mach number results provide the second parameter necessary to define the state of the flow on completion of the mixing, in those flows that choke due to the two stream effect. These results are given in the form of constant ejector geometry surfaces in the π-h-m_3 field in Figures 68 to 73 in this report.

Irrespective of whether the mixing reaches completion by the end of the duct the mass flow rates of these flows that choke due to the two stream effect will remain constant. Going even further the secondary injection pressure at which the two stream choking takes place is independent of the total enthalpy ratio, provided that the latter is not large enough to cause the flow to choke at the end of the mixing process instead. Thus the surface representing the relation between the secondary injection pressure, the geometric ratio z and the total pressure ratio π may be considered to represent the entire solution for choking according to the Polytropic Two Stream Theory. This surface is depicted in Figure 61.

As mentioned above the secondary injection pressure in independent of the total enthalpy ratio while the latter is small, but as h increases it causes the flow to become choked at the end of the mixing process and thus the secondary injection pressure becomes dependent on the total enthalpy ratio. If the performance of an ejector with fixed geometry and fixed total pressure ratio is plotted in terms of the secondary injection pressure versus the total enthalpy ratio the graph would consist of a straight line parallel to the h axis for the low values of h. This represents the two stream choking. At the point where the flow becomes choked at the end of the mixing tube there is a discontinuous change in the gradient and the curve beyond this point is dependent on and rises with the total enthalpy h. This phenomenon is illustrated in Figures 62 to 67 where the secondary injection pressures corresponding to choked flows are plotted as lines of constant total pressure ratio in the p_{21}-h field for various values of the geometric parameter z.

3.3. COMPARISON OF PREDICTIONS FROM OTHER ONE-DIMENSIONAL THEORIES.

Fabri's Theory and the Polytropic Two Stream Theory were found to give fairly similar results. The main difference was that Fabri's Theory predicted that the flows would choke at a slightly lower secondary mass flow and, therefore, at a slightly higher secondary injection pressure. According to the Straight Mixing Theory, on the other hand, the flows do not choke until the secondary mass flows are considerably greater than those predicted by the other theories. In fact for the cases in which the total enthalpy ratio is unity it may be shown that the secondary injection nozzle becomes sonically choked simultaneously with the sonic choking of the fully mixed stream.

Some results illustrating these points are given in Figure 59 where the experimental performance of an ejector with the confirugation z = 0.5 and h = 1.0 is compared with the predictions of the various theories.

3.4. RESULTS FOR SUPERSONIC INJECTION, CHOKED FLOWS.

A limited attempt has been made to test the applicability of the Polytropic Two Stream Theory to flows with supersonic injection In the absence of any experimental results of our own it was decided to apply the Polytropic Theory to two ejector configurations for which results had already been published. The configurations chosen were taken from the work of Messrs. Chow and Addi, Reference 3. This had the additional advantage of making possible a comparison of the predictions of the Polytropic Theory with those of the two-dimensional theory of Messrs. Chow and Addi. The results of this exercise are given in Figure 49, where the secondary injection Mach number is plotted against the ratio of the secondary injection pressure over the primary total pressure. In these figures the total pressure ratio, π , decreases as the solutions are traversed from the bottom left hand ends of the curves towards the top right hand ends. As explained in Section 2.2, the lower left hand portions of the curves correspond to the Supersonic Regime and the extreme right hand portions correspond to the Supersaturated Supersonic Regime. In the Supersonic Regime both the Polytropic Theory and the Two-dimensional Theory are found to give predictions that agree fairly well with the experimental results. Fabri's Theory on the other hand does not give satisfactory predictions. The predicted mass flows are again too small. For the case in which z = 0.1937 the Polytropic Theory gives slightly closer predictions than the two-dimensional theory for the flows in the Supersonic Regime, but in the case where z = 0.3025 the two-dimensional theory comes off best.

The results concerning the flows in the Saturated Supersonic Regime, however, are by no means so simple or conclusive. As the total pressure ratio falls the experimental points pass through a maximum in the ratio p_{21}/p_1 just below condition of equal injection pressures. This ratio then appears to fall again until the secondary injection Mach number becomes unity and the injection nozzle becomes choked. Now as to whether this phenomenon is due to boundary layer effects within the injection nozzles as suggested by Messrs. Chow and Addi or to a form of choking similar to that suggested in Section 2.2, it is not really possible to decide without further experimental investigations. The solution according to the Polytropic Theory is shown in Figure 49. It must be remembered that this theory used in this context predicts a slight loss of entropy and, therefore, the results should be treated with caution. They do, however, give a better prediction of the form of the experimental performance found to occur in this regime.

4. THE EXPERIMENTAL PROGRAMME.

4.1 INTRODUCTION.

The experimental programme connected with this work has two major objectives. They lie in finding the answers to the following questions:- Firstly, to what degree can ejector flows be successfully predicted by one-dimensional theories? And secondly, does some mechanism of choking occur other than that due to a flow becoming sonic in the normally accepted sense, and, if so under what conditions does such a phenomenon take place? Since most one-dimensional theories are more easily applied to ejectors with constant area mixing it is proposed that the work be limited, at least in the initial stages, to ejectors with mixing ducts of constant cross-sectional area, although, as will be seen below, provision is made in the apparatus used to vary the mixing duct cross-section. It must be mentioned that, at the time of writing, the experimental programme originally invisaged is nowhere near completion. This situation is due in the main to delays in the manufacture, supply and erection of the necessary hardware. However, it is proposed to describe the complete original programme in this report as this may be of use to other workers. The limited experimental results that were obtained in the short time available after the experimental ejector was commisioned are presented and discussed. Although they can only be said to represent a small pilot programme, they do indicate some definite conclusions.

4.2 THE OBJECTIVES OF THE COMPLETE PROGRAMME.

The first question posed above, concerned as it is by the general success of the one-dimensional theories, must be considered within the context of the test facilities available. Two important restrictions arise immediately. The first is that air is the only gas readily available, thus in order to run a large number of tests with an ejector of reasonable dimensions we are restricted to an air-air system. The second restriction is that there is at present no facility available to vary the total enthalpy ratio, hence all experiments will have to be restricted to a total enthalpy ratio of unity. Within these restrictions it is hoped to make as wide a survey as possible of the performance of the one-dimensional theories of ejector operation. Considering the parameters in terms of which the one-dimensional theories are formulated the above restrictions leave us free to vary the following parameters:-

the injection area ratio between the two streams,
the design injection Mach number of the primary injection nozzle,
the total pressure ratio between the two streams,
the overall pressure ratio (the back pressure over the primary total pressure).

In the experimental system proposed the geometrical parameters defining the are ratio between the two injection nozzles and the design Mach number of the primary injection nozzle are varied by the insertion of different nozzles (see Figure 33). Thus these parameters may only be varied by discrete amounts, whereas the two pressure ratio parameters are infinitely variable. We are also free to vary such parameters as the mixing duct length and the injection nozzle geometries which do not appear specifically in the one-dimensional theories, but again these parameters may only be varied by discrete amounts.

With regard to the question of the success of any particular theory a careful distinction must be made between two degrees of success. Firstly, a theory may provide reasonably accurate predictions of overall ejector performance in terms of external relations such as those between the mass flows and overall pressure ratios, even though the flow within the ejector is not found to be exactly that predicted by the theory. Such a theory must be said to be "empirically" successful and may be of use in that limited sense. A theory with a more complete degree of success will be found to give correct predictions for the flows within the ejector as well as for the overall predictions for the performance. In order to establish, in this sense, the complete success of any theory it is necessary to verify, as far as is possible, all the assumptions inherent in the theory. It is, therefore, intended to list all the assumptions relevant to the one-dimensional theories and to find out experimentally to what degree they are justified. The main assumptions connected with the one-dimensional theories are concerned with the following points :-

a) the uniformity of each gas stream on injection,
b) the equality of the injection pressures in the Subsonic Regime,
c) the reversibility of the primary expansion in the Mixed and Supersonic Regimes,
d) the degree of mixing during the primary expansion,
e) the uniformity of conditions at the end of the expansion,
f) the degree of over-expansion at the end of the expansion,
g) the degree of mixing reached at the end of the mixing duct,
h) the boundary layer effect,
i) the mechanisms of choking.
j) the uniformity of static pressure over each cross-section downstream of the primary expansion.

Points b) and j) can be verified simply by static pressure measurements. Points a), g) and h) may be investigated using pitot traverse techniques connected with static pressure measurements. For points c), d), e), f) and i) it is necessary to use some form of flow visualisation technique in conjunction with the pitot and static pressure measurements. Naturally, even better results would be obtained if an interferometric technique could be used to find the general density distribution throughout the flow. For, if it is found that the normal static pressure gradient is zero throughout a majority of the flow and hence the static pressure distribution is known, this will provide a method of finding the total pressure distribution without disturbing the flow by the insertion of any probes.

In order to establish which, if any, of the one-dimensional theories give accurate descriptions of the flows in the various regimes it will be necessary to make detailed studies of the flows corresponding to several sets of control variables - at least one set corresponding to each mode of choking. Such an investigation involves making complete studies of the flows corresponding to a particular set of stagnation conditions as the back pressure is allowed to traverse the entire range from the Base Pressure flows through to the fully choked flows. If this is done, it should be possible in each case to tell from the results exactly where and how the flow becomes choked. For, as soon as the secondary mass flow becomes independent of the back pressure, only the flow downstream of the choked cross-section will be affected by any further drop in the back pressure.

Once the boundaries between the regimes found to exist physically have been determined experimentally and the observed modes of choking have been examined and classified, it is intended to make limited studies of a large number of flows to determine the overall performance of each theory in each regime to which it applies. To summarise, the objectives of this experimental programme are to test the applicability of the one-dimensional theories of ejector operation to the flows in sonic and supersonic ejectors with constant area mixing and also to make a thorough investigation of the various modes of choking. The programme, however, will be subject to the following limitations:-

- it will be limited to air-air systems,
- the total enthalpy ratio will be restricted to unity,
- only one geometric form will be used (the experimental ejector will be a plane flow system).

A description of the experimental set-up will now follow.

4.3 THE EXPERIMENTAL EJECTOR.

To carry out the required programme an air-air ejector of a fairly flexible nature was designed. The main requirments were:-

- to be able to vary the two stagnation pressures and the back pressure independently,
- to be able to vary the area ratio of the injection nozzles,
- to be able to vary the length of the mixing tube,
- to be able to measure the static pressure along both the streams as they mix,
- to be able to observe the flow optically.

The facilities to provide a difference in the total enthalpy between the two streams were not available so this could not be attempted.

An experimental air-air ejector was designed and built to meet the above requirments as closely as possible, see Figure 34. It was decided that a plane flow system would make the best use of the flow visulisation techniques and also be the simplest from the construction point of view. The facilities available for running the system consist of a compressor and 240 cubic feet of compressed air storage tanks with a maximum working pressure of 250 p. s. i. The air line from the tanks to the laboratory is a 2 inch nominal diameter line. The whole system was designed to a maximum working pressure of 250 p. s. i. The basic size of the ejector is a compromise between having it large enough to make details of the flow visible and small enough to get a reasonable running time with the air storage facilities available. The experimental system is shown in Figures 33 to 36. The system consists of two reservoirs maintained at constant pressure by two Fisher control valves. These reservoirs exhaust via the injection nozzles into the mixing duct.

The 2 inch pipe from the storage vessel is connected to a 2 inch hand valve which controls the air supply to the whole system. The pipe is then split into two $1\frac{1}{2}$ inch lines which lead to the Fisher control valves. Just downstream of the hand valve a small $\frac{1}{4}$ inch line is lead off to supply the control system for the Fisher valves. These valves are of the diaphragm type and are controlled by a double servo system. The system is designed to keep the total pressures constant in the reservoirs just upstream of the injection nozzles. Pressure tappings are taken from the reservoirs and fed into the first part of the control system (the Wizard Controllers) and also to dial pressure gauges for the information of the operators. The Wizard Controllers compare these pressures with the desired values and give appropriate output pressures, proportional to the errors, which are in turn fed into the Positrol system. These devices give an output signal dependent on the Wizard Controllers output and the control valve setting. This output is fed to the diaphragm of the valve. This double system allows the sensitivity of the system to be altered and also helps to cure any undesirable feed back effects.

After the valves there is a short straight section of $1\frac{1}{2}$ inch pipe to allow each flow to settle, then a six inch long transition section changing the flow cross-section from round to square. The flows then enter the diffusers attached to the upstream side of the reservoirs, as shown in Figure 34. Wire mesh screens are placed between the downstream end of the diffusers and the begining of the reservoirs or settling chambers. These help to keep the flow uniform and also control turbulance.

The rest of the system, the reservoirs, the injection nozzles and the mixing chamber, are incorperated in one assembly as a plane flow system. This consists of two heavy steel plates with a one inch space between them. They are held apart by liners forming the walls to the various chambers, see Figure 35. The reservoirs are both 5 inches wide at their upstream ends and curve smoothly into the injection nozzles' contours. Two access holes are provided in each reservoir; one through which to measure the pressure and the other through which to insert a thermocouple to measure the total temperature. The two reservoirs are seperated from each other by one of a set of needles, which may easily be exchanged so as to vary the area ratio betweem the injection nozzles and the design Mach number of the primary injection nozzle. Several such needles are illustrated in Figure 33.

The contours of the injection nozzles were designed with the object of producing reasonably uniform flows in the plane of injection. The shapes of the primary and secondary liners were designed so that the spaces between them and the dividing needle formed the two injection nozzles. The needle corresponding to an injection area ratio of unity was used for the design. In this case the primary surface of the needle is flat - as it is shown in Figures 34, 41 and 47. The contours of the primary liner were designed so that the space between the liner and the needle formed a symmetrical half of a nozzle the shape of which had been determined using potential flow analysis. The throat of this nozzle was extended by a parallel walled section of duct half an inch in length. Since the sharp end of the needle would not be supported by the side plates it had to be designed with a finite angle at the apex in order to avoid undue deflection while under load. The secondary stream therefore had to be injected at a slight angle to the primary. The contours of the secondary liner were designed so that the space between it and the secondary surface of the needle formed a nozzle with approximately the same axial distribution of cross-sectional area as the primary nozzle.

The mixing duct has a cross-section $1\frac{1}{2}$ inches high by 1 inch wide and has a maximum length of 20 inches. The front and back plates contain Schlieren windows, 16 inches in length, which run from just upstream of the duct entrance to within 4 inched from the end of the duct. This enables optical methods to be used over the major part of the flow. The bottom wall of the duct is formed by a fixed liner 1 inch in depth with pressure tappings along its full length. The top wall however consists of a thin brass tongue attached to the top liner at the upstream end and supported on adjustable jackes at regular intervals down the duct. In all experiments carried out so far the tongue has been kept in a constant position such that the cross-section of the mixing duct does not vary throughout its whole length. 'O' rings are used to seal the gaps between the plates and the liners but the flexible tongue is fitted with an inflateable seal maintained at a pressure 20 p. s. i. higher than either of the reservoir total pressures. Illustrations of the mixing duct are given in Figures 34, 35, 36 and 41.

At the end of the mixing duct there is an adjustable centre body which may be used to restrict the exit area of the duct and thus simulate changes in the back pressure upon a system with fixed upstream conditions. A pitot rake is built into this centre body in order to determine the degree of mixing that has occured by the end of the duct. This assembly is illustrated in Figure 36.

At the time of writing the control systems governing the total pressures in the reservoirs have not been fully perfected in that they do not hold the flow steady at high pressures. Total pressures of about 60 p. s. i. g. are found to be the maximum values at which the control systems will hold the flows reasonably steady. Above these values the flows tend to oscillate.

In the absence of any diffuser at the end of the mixing duct this maximum value of the primary total pressure also defines a minimum value for the overall pressure ratio of the order of 0.2. If the back pressure is going to be allowed to fall far enough to reach the fully supersonic version of the choked solution a maximum limit in the total pressure ratio between the two streams is also indicated. This value is of the order of 2.0 if it is assumed that the back pressure corresponding to the fully supersonic flow is of the order of 4/5 ths. of the secondary total pressure. Higher total pressure ratios may of course be used provided it is not required to reach the fully supersonic solutions. It is hoped that this situation will be improved by making some simple modifications to the control systems.

4.4. DIAGNOSTIC TECHNIQUES.

4.4.1. Pressure Measurements.

All pressure measurements are made using multitube mercury manometers. Two manometers were built; one ten foot, twenty tube manometer (see Figure 37), and one five foot, one hundred and twenty tube manometer. The first of these, the tall manometer, is used to measure the total pressures and is provided with a clamping mechanism to hold the readings while they are read. The short manometer on the other hand, has no clamp but is fitted with a transparent graduated screen which is illuminated from the rear thus allowing the readings to be recorded photographically. An illustration of this manometer and a specimen recording are given in Figures 37 and 38.

The air supply pressure and the two stagnation pressures are displayed on dial pressure gauges in the main control panal (Figure 39). The stagnation pressures are also measured on the tall mercury manometer. These pressures are in fact the static pressures within the reservoirs, but on investigation they are found to be well within one percent of the true stagnation pressures. Static pressure tappings are situated at regular intervals down the mixing duct both in the bottom liner and in the flexible tongue. The surface apertures of these tappings are all twenty thousandths of an inch in diameter. Their exact locations are shown in Figure 41.

Total pressure surveys are made using small pitot tube rakes mounted on stings. Twenty two gauge stainless steel tubes are used to construct these rakes. The internal diameter of these tubes is of the order of sixteen thousandths of an inch. A hollow sting through which the pitot tubes may pass out of the ejector is formed by a larger stainless steel tube one eighth of an inch in diameter. A specimen pitot rake is illustrated in Figure 42.

4.4.2. Total Temperature Measurements.

The stagnation temperatures of the two streams are measured by means of thermocouples situated in the reservoirs as shown in Figure 41. Both thermocouples have been calibrated against a standard supplied by the National Physical Laboratory. Their outputs are measured and recorded at half second intervals throughout any experiment by an automatic digital voltmeter. This device gives a printed digital output on continuous paper. The hot ends of the thermocouples are supported on porcelain rods glued into brass studs. This is illustrated in Figure 43. The cold ends are immersed in an ice bath.

At the time of writing this temperature measurement system has not been fully perfected and therefore was not used in the pilot experimental programme for which the results are given in this report.

4.4.3. Optical Techniques.

A small experimental programme to investigate and develope some of the new optical techniques made available by the introduction of laser light sources is being run concurrently with this work on ejector flows. A major objective of this programme is to set up a holographic interferometer using a Q-switched solid state ruby laser. The laser, at present in use, provides a twenty nano-second flash which corresponds to a gas partical movement of less than one thousandth of an inch during exposure, even at gas velocities in excess of 2,000 feet per second. (Laser on loan from Barr and Stroud Ltd.) Once developed it is hoped to use this interferometer to measure the density variations within the ejector flows. In connection with this work a large, vibration insulated, optical bench has been constructed upon which the various optical systems may be set up and tested. The bench is illustrated in Figure 44.

Meanwhile, however, a double pass Schlieren system was set up on the optical bench mentioned above and was used in the pilot experimental programme. The Q-switched ruby laser was used as the light source and the knife edge was used parallel with the direction of flow. This system is illustrated in Figure 45, and a specimen Schlieren picture is shown in Figure 46.

4.4.4. Mass Flow Measurement.

To measure the mass flows in the injected streams it was decided to make a detailed calibration of the injection nozzles themselves. For not only would such studies provide a means of measuring the respective mass flows, but it would also provide a detailed knowledge of the flows within these nozzles and this would undoubtedly be of value in the understanding of the phenomena that take place in the initial stages of the mixing. Once the nature of the flow within a nozzle is known a calibration between the mass flow and the pressure ratio across the nozzle may be evolved.

An investigation of an injection nozzle must first of all provide details of the flow at the injection plane which should ideally include a full knowledge of the total pressure and static pressure distributions. These distributions may be obtained from pitot static surveys across the plane. When the flow is choked or nearly choked in the injection plane, these results may not be very reliable because the static pressure is so sensitive to small geometrical changes when the flow is sonic. This means that the siting of the static pressure tappings that are supposed to indicate the injection pressures may be rather critical. It is therefore felt necessary to investigate the flow at an additional cross-section upstream of the nozzle throat. This would provide a double check on the mass flow calibrations.

4.5. THE PILOT EXPERIMENTAL PROGRAMME.

As explained above only a very short time was available between the commisioning of the experimental ejector and the writing of this report in which any experimental programme could be carried out. It was therefore decided to run a very restricted pilot programme investigating the flows in a sonic ejector of fixed geometry. The needle corresponding to an injection area ratio of unity was therefore fixed into the ejector, giving equal injection areas to each nozzle. The full available length (20 inches) of the constant area mixing duct was used. The total temperature measuring equipment was not ready in time for this programme, so no total temperatures were measured. But since both streams originated from the same pressure vessel and the same supply pipe. it is considered a fairly safe assumption that the total temperature ratio between the two streams would have been very close to unity. The holographic interferometer was also not ready in time for the initial programme. so a Schlieren system had to be used instead. Within this restricted programme the number of variable control parameters is reduced to two; the total pressure ratio and the back pressure against which the ejector exhausts.

Details of approximately one hundred seperate ejector flows were recorded throughout the programme. For each ejector flow these details consisted of the following:-

The injection area ratio,
The setting of the blocking centrebody, (corresponding to back pressure).
The total pressures of the two streams,
The primary injection pressure. measured at the first pressure tapping point in the primary wall of the mixing duct, (see Figure 41)
The secondary injection pressure, measured at a pressure tapping within the secondary injection nozzle,
The static pressure distribution along the primary wall of the mixing duct (27 tappings),
The static pressure distribution along the secondary was of the mixing duct (12 tappings),
The total pressure distribution across the end of the mixing duct in the plane of symmetry (5 equally spaced pitot tubes),

The atmospheric pressure,
And a Schlieren photograph of the initial mixing region.

The pilot programme consisted of six experiments; two of these were calibration experiments and the other four were studies of ejector flows. Before considering these experiments in detail it is proposed to give a brief description of the experimental procedure involved in recording a single ejector flow.

4.5.1. The Experimental Procedure for Recording an Ejector Flow.

The experimental procedure may be split up into three separate stages, preparation of the experiment, operation of the ejector and recording of the results. When carrying out a series of tests the preparation of the next experiment may be made while the compressor is restoring the air storage pressure after the last experiment. This usually took between ten and fifteen minutes.

Preparations for an experiment included the following. The total pressures to give the desired flow were calculated and the corresponding "target" manometer readings were estimated. The mercury levels in the manometers were adjusted according to the pressure differences expected. The camera recording the readings on the short manometer was wound on the reset. The Schlieren windows were cleaned both inside and outside. (Considerable difficulty was experienced with moisture and dirt in the air supply system due to a failure in the air drying apparatus. Hence many of the Schlieren pictures obtained were of a rather poor quality). The position of the blocking centre body was set to give the desired exit area to the mixing tube. The camera used to take the Schlieren pictures was automatic and, therefore, did not require rewinding or resetting. When the air storage pressure was fully restored and the capacitor bank in the laser power supply was charged the ejector was ready to go.

As mentioned above the control systems governing the total pressures in the two reservoirs have not yet been made to work completely satisfactorily. With the sensitivity of these controls set so as to eliminate any oscillatory tendencies it was found that the controlled pressures were not kept constant but gradually fell with the falling air storage pressure, even, though the settings remained unaltered. In order to maintain constant total pressures it was found necessary to make continuous manual adjustments to the control settings. This situation led to the following running procedure being adopted.

The control settings for both streams were simultaneously adjusted to give total pressures of about 10 p.s.i.g. in both streams. This was done in order to avoid developing highly sub-atmospheric pressures within the ejector and thus being in danger of sucking mercury into the system from the manometers. The primary total pressure was then raised to about ten percent in excess of the desired value and allowed to settle while the secondary total pressure was also raised to a value just in excess of that required. Minor adjustments were then made so as to induce a situation such that the total pressures, while gradually falling without any manual adjustment, both passed through their desired values at the same instant. At this instant the following actions were taken simultaneously :- The tall mercury manometer was clamped, the short manometer was photographed and the laser was fired to take a Schilieren picture. The ejector was then closed down and the atmospheric pressure and the tall manometer readings were recorded. The average run time per experiment, that is the time that elapses between initial raising of the secondary total pressure and the instant of recording, was of the order of 30 seconds. At the instant of recording the rates of fall in total pressure were of the order of one half percent per second. It is, therefore, considered that these flows should approximate fairly well to the true steady state.

The results, for each flow recorded, were processed and presented on single pages as illustrated in Appendix 4. On these sheets the particulars of a flow are given followed by a Schlieren picture of the initial mixing region, plots of the static pressure distributions along the primary and secondary walls of the mixing duct and the total pressure distribution across the end of the duct.

4.5.2. The Calibration Experiments.

In view of the limited time available the full calibration of the injection nozzles as suggested above was not feasible. It was, therefore, decided to carry out a limited calibration on each injection nozzle. This calibration consisted of recording flows through the nozzle for various total pressures and various mass flows. For each flow the following measurements were made ;

The static pressure in reservoir was measured (tapping position shown in Figure 41).
The total pressure in the reservoir was measured at three points in the plane of symmetry (see Figure 41).
The total pressure in the injection plane was measured using a four tube pilot rake in the plane of symmetry (see Figure 47).
The injection pressure was measured at one of the tappi ngs indicated in Figure 41.

The results from these experiments will not be given in full here but the main findings and the conclusions drawn from them will be presented briefly. These are largely the same for both nozzles. It was found that total pressure measurements in the reservoir were all within one percent of each other and also within one percent of the static pressure within the reservoir. Thus it was concluded that the static pressure measurement would be sufficient to indicate the reservoir total pressure in subsequent experiments. It was found that the total pressure measurements in the plane of injection were also within one percent of each other and within one percent of the reservoir total pressure. From this it may be concluded that the flow within the injection nozzles is reasonably isentropic. Since there was no measurement of the static pressure distribution across the injection plane there is little more that can be said with regard to the uniformity of the steams on injection. An indication of the injection pressures may be obtained from the pressures measured at the nearest tappings. These are indicated in Figure 41 for each nozzle where it may be seen that neither of them are exactly in the plane of injection. The tapping indicating the primary injection pressure is one eighth of an inch inside the mixing duct and that indicating the secondary injection pressure is within the secondary nozzle. The ratio of the indicated injection pressure to the reservoir total pressure was found to be of the order of 2 to 3 percent too high when the nozzle is fully choked. This is not surprising since, not only were the total pressure measurements only accurate to one percent, but in transonic flow the static pressure is very sensitive to small geometric variations. However, it is throught that the indicated injection pressures will be much closer to the true values when the nozzles are not choked.

4.5.3. Ejector Flow Experiments.

As mentioned above approximately one hundred ejector flows were recorded during this programme. These were grouped in four major experiments which will be described below. Throughout the entire programme the ejector geometry was kept constant with an injection area ratio of unity and mixing duct dimensions of 20 inches in length and one by one and a half inches in cross-section. Thus the only parameters that remained free to be varied were the two total pressures and the back pressure. The desire to be able to study the complete set of flows corresponding to any fixed total pressure ratio limits the range of possible total pressure ratios to values below 2.0. The four main experiments undertaken were as follows :-

1. Complete sets of flows corresponding to the five total pressure ratios, 1.1, 1.3, 1.5, 1.7 and 1.9 were recorded to try and establish an overall picture of the behaviour of the flows that occur in the region of investigation.

2. A more detailed study was made of the flows corresponding to the total pressure ratio of 1.5. The main idea of this experiment was to find out where and how the flow becomes choked. However, since it involved repeating several flows already recorded in the first experiment it also provides a check on the repeatability of the flows.

3. A study was made of the flows in the Supersonic Regime, that is those that were fully choked, over the total pressure range 1.0 to 2.0. These were compared with the predictions of the one-dimensional theory.

4. The results from four flows in the Supersonic Regime with the same total pressure ratio but different actual total pressures were compared to investigate any dimensional effects.

4.5.4. The Complete Back Pressure Dependent Flows.

Experiments 1 and 2, outlined above, may be considered together here, as experiment 2 is just a more detailed repeat of part of experiment 1. In these experiments the flows corresponding to the complete range of back pressures acting on ejectors with fixed total pressure ratios were studied. The secondary total pressure was maintained at two atmospheres throughout the two experiments and the primary total pressure was adjusted to give the desired total pressure ratios. For a fixed total pressure ratio the following experimental procedures was adopted :-

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Starting at the Supersonic Regime end of the solution, an ejector run was made with the blocking ~~co~~ntre body completely removed from the exist of the mixing duct. This was the flow corresponding to the lowest back pressure available with the ejector in that particular configuration. To check whether the flow was tru~~e~~ly supersonic, that is that it was independent of the back pressure, it was noted whether the static pressure just before the duct exit was greater or less than atmospheric If it was greater it was then assumed that the fully supersonic condition had been reached, and that there would have been no point in trying to obtain lower back pressures. This was found to be the case with the majority of the flows recorded. Once the supersonic condition had been established for any particular total pressure ratio, flows were recorded corresponding to a series of increments in the back pressure simulated by progressively blocking the mixing duct exit with the movable centre body. The simulated back pressure rises as the available exit area is gradually reduced. In this way the total mass flow through the ejector is also gradually reduced until eventually the secondary injection pressure becomes equal to the secondary total pressure and the Base Pressure condition is reached. Since only one flow could be recorded at one time the ejector had to be shut down while the centre body was adjusted to simulate the next increment in back pressure and the instrumentation was reset to record the next flow.

In the first experiment about ten flows were recorded for each total pressure ratio covering the full back pressure range in each case. For the purposes of illustration the complete results from the runs in experiment 1 for a total pressure ratio of 1.5 are given in Appendix 4.

In the second experiment the series of flows corresponding to a total pressure ratio of 1.5 were re-run only this time with smaller increments between each recording. Thus a more gradual rise in back pressure was simulated. Twenty flows in all were recorded in this experiment.

(i) The Results.

There appear from the results of these experiments, when considered as a whole, two major points which must be appreciated before entering into any detailed discussion. Firstly the mixing is by no means complete by the end of the mixing duct. Secondly the ejector appears to become choked at a much smaller secondary mass flow than predicted by any of the Two Stream Theories. This second effect is so marked that, for all flows with total pressure ratios of 1.5 or less, the system chokes while the primary injection nozzle is still unchoked. Needless to say, the choking is found to take place in a very different way to that predicted by the theory. It appears that the choking takes place during the mixing process itself.

The results will now be described and discussed in more detail. Three single flows representing the three flow regimes found to occur will be discussed first. Then the complete series of flows corresponding to the total pressure ratio of 1.5 will be discussed and lastly the complete results from experiment 1.

(ii) The Single Flows.

A flow in the Supersonic Regime.

Ejector Test 21, the results of which are given in Appendix 4, consisted of operating the ejector, at a total pressure ratio of 1.5, with the centre body completely removed from the mixing duct exit. The static pressure at the end of the mixing duct was higher than the atmospheric; it was, therefore assumed that the flow was in the fully choked condition. Consider the results as shown in Appendix 4. The ratio of the secondary injection pressure to the secondary total pressure, according to the predictions of the Two Stream Theories should be of the order of 0.68 for the fully choked flow. In Test 21 this ratio was 0.80. Since the injection pressures are felt to be reasonably accurately indicated in this region, it can only be concluded that the secondary mass flows was appreciably less than predicted. In fact, in this case, the indicated value of the secondary injection pressure is greater than the value (0.793) at which the primary injection nozzle theoretically becomes choked. It appears, therefore, that the flow becomes choked straight from the Subsonic Regime, without even choking the primary injection nozzle. Looking at the total pressure distribution across the end of the mixing duct it appears that the mixing region has only just filled the whole duct and the stream, as a whole, is certainly not completely mixed at this stage. The mixing region may be seen as a light area in the Schlieren picture gradually spreading as the streams move downstream. It may be seen from the static pressure distributions that the indicated injection pressures are not equal, in fact the primary injection pressure is substantially higher than that of the secondary. There are two factors that may contribute to this effect, firstly the positions of the relevant pressure tappings and secondly the fact that the two streams meet at a slight angle instead of being injected parallel to one another as theoretically supposed.

There does not appear to be any evidence of a primary expansion of the type assumed in the Two Stream Theories. This again points to the conclusion that the primary nozzle is not choked in this case. On looking at the static pressure distributions down the mixing chamber it is readily seen that the static pressure is uniform across any cross-section normal to the duct axis, except maybe those very immediately downstream of the injection plane. The static pressure seems to fall only very slightly for the first two thirds of the duct, the major part of the total static pressure drop taking place in the last third of the duct. This might suggest that the choking process was taking place at a cross-section about two thirds of the way down the duct, but that is about all that may be said on this evidence alone. The Pitot tubes at the end of the mixing tube, being positioned at the nearest at least a quarter of an inch away from the nearest mixing duct wall, were not in a position to give any information on boundary layer effects.

A flow in the Subsonic Regime.

Ejector Test 25, the details of which are also given in Appendix 4, represents a typical flow in what has been defined as the Subsonic Regime. Again it has a total pressure ratio of 1.5. Both the injection pressures and the simulated back pressure are substantially higher than in the Supersonic flow just described. The exhaust static pressure in this case has been affected by the intrusion of the blocking centre body and is, therefore, not representative of the simulated back pressure. The pressure at the penultimate tapping should, therefore, be used in this connection. Considering the total pressure distribution across the end of the mixing duct, the flow appears to be better mixed than in the last case but is still by no means fully mixed. The mixing region is not very easily distinguishable in the particular photograph corresponding to this flow, but it may be seen from pictures of other similar flows that the mixing region tends to spread more quickly (in other words the angle between the nearly straight mixing region boundaries is greater) than in the Supersonic flows. The indicated injection pressures are again not quite equal. This is thought to be due again to the positions of the relevant tappings and the angle of injection of the streams. The static pressure is almost constant throughout the entire ejector, there being a very slight rise as the mixing progresses.

Other flows in the Subsonic Regime may be seen in Tests 23 to 26 in Appendix 4. On looking through this series of tests the change in the angle occupied by the mixing region may be fairly easily observed. It may also be noticed that a fair degree of turbulence is developed on the mixing region-secondary boundary as the Base Pressure condition is approached.

A Base Pressure Flow.

A typical example of a Base Pressure flow is that recorded in Test 28 in Appendix 4. Here the simulated back pressure was very high and the secondary injection pressure was equal to the secondary total pressure. The total pressure distribution across the end of the mixing duct was fairly uniform, as might be expected with the duct being entirely filled with one stream. The static pressure distributions down the mixing duct, however, are slightly more interesting. For the initial part of the duct the static pressure remains constant at the injection value, it then quite suddenly starts to rise and rises fairly quickly to the value corresponding to the back pressure. It then remains at this value until the duct exit. This, together with the evidence of considerable turbulence in the Schlieren picture, tends to suggest that a bubble type region of vorticity is formed in the space between the primary stream and the secondary wall, before the primary stream fills the whole duct.

(iii) The Complete Set of Flows Corresponding to a Total Pressure Ratio of 1.5.

A comprehensive picture of the performance of this ejector operating at a total pressure ratio of 1.5 may be obtained by combining the relevant results from experiments 1 and 2. In the theoretical calculations made earlier in this paper the performance of an ejector was described in terms of the relation between the exhaust pressure and the secondary injection pressure. The theoretical ejector performance according to the Polytropic Two Stream Theory is shown in this form in Figure 50. The supersonic solution is included in this graph, for although it has been stated that these solutions are unlikely to exist physically if the choking takes place as predicted by the one-dimensional theory, this supersonic solution will still provide an extreme solution if the choking is found to take place in some other way.

Also plotted on Figure 50 are two sets of experimental points. The light ring spots represent plots of the static pressure at the last pressure tapping in the primary wall of the mixing duct against the secondary injection pressure. As can be seen from looking at ejector test results in Appendix 4, this static pressure is not always truely representative of the actual exhaust pressure. In the flows with low simulated back pressures there is a strongly negative pressure gradient towards the end of the mixing duct and, therefore, to obtain the true pressure at the duct exit it would seem necessary to extrapolate the static pressure distribution to the end of the duct. Also in the flows with high simulated back pressures the centre body interferes with the last pressure tapping and so the penultimate tapping would give a more correct indication of the appropriate exhaust pressure. The full dark spots represent a replot of the exhaust pressure taking into account the two factors just mentioned.

On looking at the plots in Figure 50 it is fairly obvious that the ejector is choking at a much higher secondary injection pressure than predicted by the one-dimensional theory. Now, speaking theoretically, even if the flow does choke at the secondary injection pressure indicated by the experiments, the exhaust pressure for a fully mixed stream would still be given by a curve of the form ABC in Figure 50. But since the exhaust stream never becomes fully mixed in these experiments the exhaust pressure is obviously not going to be as predicted for a fully mixed stream. Since the static pressure gradient during mixing is negative for flows with low back pressures and positive for flows with high back pressures it would be expected that the static pressure for a particularly mixed stream would be higher than that for a fully mixed stream in flows with low back pressures, and vice versa for flows with high back pressures. The experimental points in Figure 50 appear to agree with this prediction.

As mentioned above, the main object in collecting together all these flows corresponding to the same total pressure ratio was to try to investigate the mechanism of choking. The first question that must be answered is "where is the choking taking place" ? Once this is known we will be in a much better position to investigate the actual mechanism responsible. According to the definition given earlier in this report a flow becomes choked when pressure communication in the upstream direction is no longer possible. Thus it would appear that the best way to find where the choking is taking place would be to find, for each point in the ejector, the back pressure at which the static pressure at this point first becomes influenced by the raising of the back pressure. Since it was not always possible in these experiments to determine the correct back pressure corresponding to a particular flow, it was decided to use the setting of the blocking centre body in place of the back pressure in this context. In Figures 51 and 52 the static pressures at nine points throughout the ejector are plotted against A_3 , the centre body setting. When A_3 is large the mixing duct exit is unblocked and when A_3 is small the exit area is restricted. Plotted at the top of Figure 51 is a small graph showing the scatter in the total pressure ratios in the flow used to provide the static pressures for the following graphs. Looking at the behaviour of these pressures as A_3 varies it is seen that at high values of A_3 all the pressures are independent of A_3 , but as A_3 drops they all become dependent on it at some stage or another. It is not possible, with such a wide scatter in the total pressure ratios, to obtain an accurate estimate of the value of A_3 at which each static pressure becomes back pressure dependent, but a rough attempt to locate these values may be made by drawing in curves through the points in the neighbourhood of the suspected values. Values of A_3 obtained in this way are plotted against the positions in the ejector to which they correspond, in a graph at the bottom of Figure 52. It can be seen that the static pressures in that part of the ejector up stream of the section $x/D = 9.0$, including the two injection pressures, all become dependent on the back pressure at roughly the same value of A_3 . The three stations downstream of this section, however, become back pressure dependent at much higher values of A_3 . It is, therefore, suggested that the choking occurs somewhere between the stations $x/D = 9.0$ and $x/D = 11.0$. It is not felt justifiable to try to tie it down any closer in view of the limited data available.

With regard to finding out how the choking mechanism works the obvious approach is to try to build up a picture of the flow in the region in which the choking is thought to take place. It has been established with a fair degree of certainty that the static pressure distributions across the cross-sections normal to the duct axis are uniform. The static pressure distributions along the duct walls are known and, therefore, it may be assumed that the complete static pressure distribution throughout the downstream half of the mixing duct is also known. It is the total pressure distribution throughout this region that is not yet known. In view of the limited data at present available all that we are able to do is to assume an approximate distribution, based on the available data together with several assumptions. An example of such a distribution will now be given and from it an approximate form for the sonic line for this particular case will be derived.

The flow chosen for this exercise was that of ejector Test 21 in Appendix 4. Looking at the total pressure distribution at the mixing duct exit it may be fairly well approximated by a straight line intersection on either side of the two boundary values of 1.0 and 1.5. Such an idealised end distribution is shown in Figure 57. Now these intersection points may be taken to represent the boundaries of the mixing region. Then if the mixing region is assumed to spread in a linear fashion the boundaries may be drawn in as in Figure 57. Making a final assumption that the total pressure within the mixing region always varies linearly between the two unmixed values across the region, we have fixed the total pressure distribution throughout the entire mixing duct. Using this assumed total pressure distribution together with the known static pressure distribution an approximation for the flow in the suspected region of choking may be derived. Using this technique the sonic line shown in Figure 57 was computed. This indicates that the choking process probably takes place at a cross-section containing both supersonic and subsonic flow. It is felt that not much more can be deduced from these results without the support of further experiments.

(iv) The Complete Sets of Flows Investigated in Experiment 1.

Of the four other tests of flow investigated in experiment 1 only those corresponding to the total pressure ratios 1.3 and 1.7 had a small enough total pressure ratio scatter to be treated in a similar manner to that corresponding to the ratio 1.5 above. Graphs of the static pressure variations against A^{*}_{3} etc., are given in Figures 53 to 56. In both cases it is again indicated that the choking takes place somewhere between the stations $x/d = 9.0$ and $x/D = 11.0$. Again the sonic lines may be computed using assumed total pressure distributions based on the experimental findings. The assumed distributions and the resulting sonic lines are shown in Figure 58. The details of the flows upon which these calculations are based are given in Ejector Tests 11 and 31 in Appendix 4. According to these estimates only a very small part of the mixing duct contains supersonic flow in the case of the total pressure ratio of 1.3 and only a small part of this region coincides with the region within which the choking is suspected of taking place. It would, therefore, appear that the choking, if it is going to take place in some generalised manner suggested in Section 2.3, will occur in the neighbourhood of the cross-section $x/D = 11$. In the flow with the total pressure ratio of 1.7, however, the primary stream is injected in a slightly under-expanded state and a supersonic primary expansion takes place as assumed in the one-dimensional theory. Part of the flow remains supersonic throughout the whole length of the mixing duct;

hence the different shape of the corresponding sonic line in Figure 58. The cross-section of choking in this case must definitely contain both supersonic and subsonic flow.

With regard to the suspected position of the choking, it should be noted that in the experimental ejector used there was a joint between the end of the Schlieren windows and the side plates at a cross-section corresponding to a value of 10 for x/D. This is within the region in which the choking is suspected of taking place and it is, therefore, just possible that this joint could have disturbed the flow in such a way as to precipitate the choking process. Although this possibility cannot be ruled out on the strength of the data at present available it is thought that the effect of this disturbance, if it is occurring at all, is only very slight.

4.5.5. Experiment 3, A Survey of the Fully Choked Flows.

In this third experiment the set of fully choked flows corresponding to the total pressure range 1.1. to 2.0 was investigated. The blocking centre body was completely withdrawn from the mixing duct exit and positioned so that the Pitot tubes were just in the exit plane. The secondary total pressure was again kept at two atmospheres and the primary total pressure was adjusted to give the desired ratios. Ten flows in all were recorded and some of the results may be seen in Figure 59.

The behaviour of these flows is probably most meaningfully indicated by the behaviour of the secondary injection pressure. For this pressure gives an indication of both the primary and secondary mass flows. The experimentally determined secondary injection pressures are compared in Figure 59 with those predicted by the one-dimensional theory. Also plotted in this graph is the secondary injection pressure corresponding to the choking of the primary injection nozzle (double chain dotted line). It is clearly seen that the ejector chokes at a much higher secondary injection pressure thanpredicted by any of the one-dimensional theories, and therefore also at lower mass flow rates. At low total pressure ratios the experimental secondary injection pressure is even in excess of that corresponding to the choking of the primary nozzle. So at total pressure ratios less than about 1.5 the ejector chokes while it is still, as it were, in the Subsonic Regime, and the phenomenon referred to as the primary expansion never in fact takes place. Only when the total pressure ratio becomes greater than 1.5 does the primary nozzle choke and the expansion occur, but even then the secondary mass flow is still too low for the flow to choke at the end of the expansion. However, the gap between the theoretical and experimental results does seem to close slightly as the total pressure ratio is increased and it is possible that they would agree better at higher total pressure ratios when the choking process is more likely to take place further upstream.

Since the total mass flows are lower than predicted it would be expected that the exhaust pressures corresponding to a fully mixed stream would also be lower than predicted, the flow having the room to expand to a faster supersonic speed. This is found to be so for the low total pressure ratio flows in which the mixing is relatively complete. But, as can be seen from Figure 60, the degree of mixing reached by the end of the mixing duct decreases with increasing the total pressure ratio. Now in all these fully supersonic flows there is always a negative static pressure gradient during mixing and, therefore, if the mixing does not proceed to completion the exhaust pressure will be higher than it ought to be. This would explain the rise in exhaust pressure shown in Figure 59. At the top of Figure 59 the theoretical static pressure at the end of the primary expansion is compared with experimental static pressure measured at the second tapping in the secondary wall of the mixing duct ($x/D = 1.6$). As would be expected in view of the findings above these results do not compare.

4.5.6. Experiment 4, Investigation of Scale Effect.

In this experiment four fully choked flows were recorded each with a total pressure ratio of 1.5 but with the actual values of total pressures different in each case. The primary total pressure was varied between three and five atmospheres. No perceptable differences could be found between the normalised results.

5. CONCLUSIONS.

To obtain a better understanding of the phenomena involved in ejector flows and to find the best methods of predicting them have been the major objectives of this work. As mentioned before the four basic phenomena occurring in ejector flows are mixing, boundary layer growth, injection pressure adjustment and choking. Of these all but the choking are fairly well understood and have well established theories connected with them. Probably the most worthwhile addition that could be made to high pressure ejector theory at present is a deeper understanding of the phenomenon of choking, together with the establishment of a satisfactory theory of choking.

The work has been limited in the main to the constant area mixing of two streams of the same perfect gas. Both sonic and supersonic injection have been considered and the resulting ejector flows have been classified into Regimes. This classification is summarised in the following table.

Regime.	Primary Stream	Ejector Flow as a Whole	Injection Pressures
Subsonic	not choked	not choked	primary = secondary
Supersaturated Mixed	choked	" "	primary $<$ secondary
Saturated Mixed	"	" "	primary = secondary
Supersonic Mixed	"	" "	primary $>$ secondary
Supersaturated Supersonic	"	choked	primary $<$ secondary
Saturated Supersonice	"	"	primary $\geqslant$ secondary
Supersonic	"	"	primary $\gg$ secondary

The phenomena of mixing, pressure adjustment and boundary layer formation can very often be said to take place in well defined regions of the flow. For example, in a short region just downstream of the injection nozzles it may be that the effects of mixing and boundary layer growth are negligible. It is in this region that the dissipation of the difference in injection pressures takes place, through the expansion or contraction of the primary stream. This process may be considered to end at the cross-section at which the static pressure first becomes uniform. The normal static pressure gradient is zero throughout the remainder of the mixing duct and it is here that the major portion of the mixing takes place. The boundary layer build up takes place over the entire surface of the mixing duct walls but again may only become significant towards the end of the duct where it will probably become merged with the mixing region.

The two contexts in which theories on ejector flows are likely to be used must be distinguished as their requirements may be somewhat different. Firstly, it may be desired to assess the likely behaviour of a definite ejector system of known geometry. In this case it may be worthwhile using some fairly exact but complex theory to obtain a detailed idea of the flows likely to occur. Such measures might include two-dimensional analysis of the primary expansion and a Navier-Stokes solution of the mixing region. Alternatively, it may be desired to obtain an overall picture of possible ejector performance which would entail assessing the flows corresponding to many sets of configurations. In this latter case it is obviously necessary to use as simple a theory as possible and it is here that the application of one-dimensional theories becomes particularly attractive. It is on this simple approach that the emphasis in this paper has been laid. One-dimensional theories, however, can only supply a limited amount of information. All they are able to do is to supply certain sets of relations between states at two cross-sections of the flow at an arbitrary distance apart. The states at such cross-sections are usually assumed to be uniform. This is not entirely necessary; the states can be assumed to take any suitable distribution provided that they can be expressed uniquely in the same number of parameters as the equivalent uniform state. Thus the rates of growth of the mixing region and of the boundary layer are also factors that must be assumed when using any one-dimensional theory. The way in which this type of theory may be applied to ejector flows by splitting them up into distinct regions was shown in Section 2.

One of the most important predictions required of any theory on compressible flow in ejectors is the prediction of the choked condition. In general it is quite possible for choking to take place in a variety of different ways and places within any ejector flows. The physical phenomenon of choking will occur as a result of the interactions of the other three ejector phenomena, mixing, pressure adjustment and boundary layer growth, and, therefore, the prediction of choking will be dependent upon the theories used to describe these other phenomena. Also if it is accepted that choking may take place in a generalised form, such as that suggested in Section 2.3, it will be seen that it would be necessary to monitor every cross-section of any ejector flow as the mass flow is increased in order to ascertain where and when the choking takes place. The one-dimensional theories of the simpler kind do not go this far, but assume that the choking occurs only at the cross-sections in terms of which they describe the flows. More complex one-dimensional methods by which a tendency for a flow to choke during the mixing process might be detected are suggested in Section 2.4.

Before coming to any definite conclusions as to the relative merits or limitations of the various one-dimensional theories it is necessary to discuss in detail this phenomenon of choking. Choking in this paper has been defined as the loss of streamwise pressure communication in the upstream direction. The cross-section at which such a phenomenon first occurs on increasing the mass flow is said to be the section at which the systems chokes. It has been shown that if a stream tube has the freedom to react in an isentropic manner to small streamwise pressure disturbance then the disturbance velocity at the point in question will be stationary. A general theory of choking has been put forward based on this concept. It states that a flow will become choked over any cross-section which as a whole has the freedom to accommodate a small pressure disturbance without affecting the total mass flow. It has been shown that any such section must, in general, contain both subsonic and supersonic flow. The main defect in the theory is that in the derivation of the wave speeds in a stream tube the inertia effects in the direction normal to the stream tube have been neglected. However, this is not thought to be a serious defect as it is small disturbances that are being considered.

If such a theory of choking is accepted then some of the traditional modes of choking such as the sonic choking of the secondary stream at the end of the primary expansion can no longer be regarded as choked cross-sections. They in fact become "supersonic" cross-sections, and the actual choking will occur upstream of them.

Let us now return to the problem of describing the flow within an ejector. Starting at the upstream end of the ejector and assuming that the injection conditions are reasonably uniform and well behaved the first phenomenon which has to be dealt with is the primary expansion. The theories describing the primary expansion have been termed the Two Stream Theories and in the main they have two objectives. The first is simply to describe the expansion and the second is to detect or predict any tendency for the system to choke at the end of that expansion. In the context of the one dimensional theories these two aims may not always be compatible. The cross-section at which the expansion is assumed to end may not be that at which the choking takes place. Assumptions also have to be made as to whether the primary becomes over-expanded or not, and as to whether it can really be represented by a uniform state at the end of the expansion.

Three Two Stream Theories were presented and discussed in Section 2.1. Two of these assumed that the primary stream did not become over-expanded and that the choking took place in the generalised form. They were the Polytropic and Isentropic Two Stream theories. Fabri's theory on the other hand assumed that the primary stream did become over-expanded and also that the system choked by the secondary stream becoming sonic at the section of maximum primary expansion.

Of these theories the Polytropic Two Stream theory appears to be the best founded, allowing both for an irreversible primary expansion and for the generalised mode of choking. With regard to the relative success of these theories in predicting experimental flows it is not possible at this stage to come to any definite conclusions without the support of further experimental results. The experimental work completed so far seems to indicate that for sonic ejectors with medium to long mixing ducts the choking takes place within the mixing process towards the end of the duct. These experiments, however, were all carried out with total pressure ratios of 2.0 or less and there was some indication that the gap between the experimental results and the theoretical results predicted by the Two Stream theories tended to become smaller as the total pressure ratio rose. It may be possible then that the choking process moves further upstream towards the end of the primary expansion as the total pressure ratio is increased, and thus gradually tends towards the solutions predicted by the Two Stream theories. Such behaviour would be compatible with the theory that those flows, for which the theory predicts a nearly choked fully mixed state, are the most susceptable to the phenomenon of choking within the mixing process. This concept is supported by limited results obtained for supersonic injection. Here the total pressure ratios were within the range 4.0 to 7.0 and the predicted fully mixed states were well supersonic and well subsonic. The predictions of the Polytropic Two Stream theory give remarkably good results in the Supersonic Regime. The predictions according to Fabri's theory are relatively poor. However, these results only cover a very small part of the total set of solutions covered by these theories, and so one cannot really draw any sweeping conclusions without the support of further results.

Further experimental work is also needed to investigate the exact mechanism and location of the choking processes within these supersonic injection flows. Messrs. Chow and Addi (Ref. 3) in their two-dimensional treatment of the primary expansion assume that the choking takes place at the cross-section of maximum primary expansion with secondary stream becoming sonic, but according to the general theory of choking such a cross-section is "supersonic" . Messrs Chow and Addi support their theory with a static pressure distribution along the mixing duct wall which indicates that the secondary stream does become sonic. This would also be the case if the choking took place in the general form and the flow then expanded as it mixed. An experiment that would help to resolve this question would be one in which the effect of back pressure on a flow is investigated; may be with the help of some interferometric technique to observe detailed changes in the flow.

A better unstanding of the flows in the Saturated Supersonic Regime is also needed before any useful theory can be evolved. Here again the actual mechanism and situation of the choking is not really known.

Returning again to the problem of describing the flow within an ejector, once a method of describing the primary expansion and its associated mode of choking has been chosen the problems of the mixing and the boundary layer effect must be considered. The boundary layer effect is obviously heavily dependent on the geometry of the ejector and thus is not easy to include in an all embracing one-dimensional theory. In fact in most one-dimensional theories the boundary layer effect is entirely neglected. Very occasionally it is included in some empirical way. One-dimensional theories of mixing on the whole have two main objectives. The first is to attempt to predict the state of the mixed stream when it reaches the end of the mixing duct given fixed conditions upstream. The second is to detect those flows that are likely to choke at the end of the mixing process. The simplest theory, the Straight Mixing Theory, assumes that the mixing is complete by the end of the duct and that the state of the exhausting stream is uniform. This uniform state is simply determined by equating the mass, momentum and energy fluxes at the end of the primary expansion to those in the exhaust plane. In general two solutions emerge one indicating a subsonic flow and the other a supersonic flow. The two solutions are related to one another through the normal shock relations. They converge into the sonic condition when the two solutions become coincident indicating a choked condition in the fully mixed flow. There are several major weaknesses in this theory. Firstly, in most physical ejector flows the mixing is no where near complete by the end of the mixing duct and if the duct is lengthened to allow further mixing the boundary layer effect becomes quite considerable and may no longer be neglected. No doubt an ejector could be devised in which the mixing becomes complete without any boundary layer effects, but this is hardly the point. Another weakness is that it does not provide any means of telling whether any choking is likely to occur within the mixing process. The simple theory does, however, prove useful in that it provides an envelope solution; that is the experimental exhaust static pressure is likely to lie somewhere between the two extreme solutions theoretically predicted. For in fully supersonic flows there is usually a negative streamwise pressure gradient during the mixing and in flows with relatively high back pressures, that is those with subsonic exhaust conditions, the streamwise pressure gradient is usually positive during mixing (in the absence of large boundary layer effects). The Straight Mixing Theory of course suffers from a weakness common to all one-dimensional theory in that it provides no indication of the steamwise scale of events. It cannot predict the effect of length ening or shortening the mixing duct. In order to obtain any information on these matters it is necessary to resort to a Navier-Stokes form of analysis.

Several improvements on the simple one-dimensional theory are possible, however. Some of these were discussed in Section 2.4. Basically they consist of assuming the likely form the total pressure distribution at the cross-section under investigation and solving the one-dimensional equations to give the static pressure and a parameter finally fixing the total pressure distribution. It was shown how such an approach could be used to follow through the mixing and check whether the flow would be likely to choke during the mixing. Obviously the same idea of detecting choking could be used when applying the more complex but more exact approach of using a Navier-Stokes solution, or even when going one step further and including a boundary layer effect.

It must also be noted that since the mixing does not usually reach completion in actual ejector flows those flows which the Straight Mixing Theory predicts will become choked at the end of the mixing process will in fact probably choke in some general form at the end of the duct. It would thus appear that ejector systems in which a primary expansion occurs will all choke in some generalised mode. This choking appears to take place well downstream of the end of the primary expansion in ejectors with low total pressure ratios, high total enthalpy ratios and relatively long mixing ducts. These are the types of ejector configurations for which the one-dimensional theory predicts choked or nearly choked fully mixed conditions. For ejectors with short ducts or ejectors with low total enthalpy ratios, high total pressure ratios and high primary injection Mach numbers the choking tends to take place at or near the end of the primary expansion. These are the flows for which the simple one-dimensional theories predict well supersonic fully mixed conditions. They are also the type of flows, therefore, for which the Two Stream Theories might be expected to give reasonable results as to the maximum mass flows.

The extent to which the boundary layer build up affected modes of choking observed in the experimental programme is not really known. It is expected, however, that even a fairly small boundary layer might have a significant effect. For, not only are flows near the critical condition very sensitive to small changes in cross-sectional area such as would be caused by the boundary layer displacement effect, but the plane flow system used in these experiments, with its rectangular sectioned mixing duct, would be fairly susceptable to boundary layer effects. However, a thorough investigation of the extent of the boundary layer in the experimental ejector must be undertaken before this question can be answered. If the boundary layer effects were in fact very significant, it is quite possible that ejectors with less boundary layer prone geometries may behave in a manner more like that predicted by the Two Stream Theories

Suggestions for Further Work.

Further investigation is suggested in three particular directions; these are briefly :-

1. The completion of the experimental programme as set out in Section 4, including a thorough investigation of the boundary layer effects.

2. The design and execution of some experiments to test the validity of the suggested general theory of choking.

3. A numerical investigation using some of the one-dimensional theories suggested in Section 2.4. In particular an assessment of the behaviour of the parameter B (a parameter that is positive in "supersonic" flow and negative in "subsonic" flow) as the mixing proceeds and the mass flow is increased in an ejector of fixed geometry and stagnation conditions.

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APPENDIX 1.
DERIVATION OF THE MASS FLOW DENSITY AND MOMENTUM FLOW FUNCTIONS.

The mass flow, $\dot{m}$, of a perfect gas flowing steadily past a certain cross-section in a duct, is given one-dimensionally by the following expression :-

$$\dot{m} = A\rho v$$

where A is the duct cross-section,
ρ is the density of the gas,
and v the velocity of the gas.

Introducing the perfect gas law,

$$p = \rho R\,T$$

where p is the pressure,
R the gas constant,
and T the temperature,

and using the normal isentropic relations the expression for the mass flow may be rewritten,

$$\dot{m} = A\rho\, v$$

$$= A\omega P_t\, a\, M/(R\,T)$$

$$= \frac{A\,P_t}{\sqrt{R\,T}}\sqrt{\frac{2\gamma\ \left(1 - \omega^{(\gamma - 1)/\gamma}\right)}{\gamma - 1}}\ .\ \omega^{1/\gamma}$$

where M is the Mach number,
T_t the total temperature,
P_t the total pressure
γ the ratio of the specific heats,
ω the isentropic pressure ratio $\frac{p}{P_t}$

Now if we define the mass flow density function y as the function $\left[1 - \omega^{\frac{\gamma - 1}{\gamma}}\right]^{\frac{1}{2}}.\ \omega$ of ω, we see that the mass flow may be written :-

$$\dot{m} = \frac{A\,P_t}{\sqrt{T_t}}\ .\sqrt{\frac{2\gamma}{R\,(\gamma - 1)}}\ .\ y\,(\omega)$$

Similarly the energy flow may be written,

$$\dot{m}\,C_p\,T_t = A\,P_t\,T_t^{\frac{1}{2}}\ .\ C_p\sqrt{\frac{2\gamma}{R\,(\gamma - 1)}}\ .\ y\ (\omega)$$

The momentum flow, $\dot{M}$, is given by,

$$\dot{M} = A(p + \rho v^2).$$

This may also be written in terms of the stagnation conditions and a single property which in this case we shall call the momentum flow function f (ω).

$$M = A(p + \rho v^2),$$

$$= A(P_t\omega + P_t \omega M^2),$$

$$= A P_t \cdot \frac{\gamma + 1}{\gamma - 1} \omega\left[\frac{2}{\gamma + 1} \omega^{(1-\gamma)/\gamma} - 1\right],$$

$$= A P_t \frac{\gamma + 1}{\gamma - 1} f(\omega).$$

where $f(\omega)$, the momentum function, is defined as :

$$f(\omega) = \omega\left[\frac{2\gamma}{\gamma + 1} \omega^{(1-\gamma)/\gamma} - 1\right].$$

APPENDIX 2.
EXTENSION OF THEORY TO THE CASE OF MIXING TWO DIFFERENT GASES.

The theory developed above is also applicable when the compositions of the two streams are not the same. However, the equations presented with this theory have to be rewritten in a more general form in order to take into account the differences in properties. Each stream will have its own values of gas constant, R, and ratio of specific heats, γ . The values of these constants for the mixed stream will depend upon the ratio of the primary and secondary mass flows and thus may only be determined when these are already known. The mass flow density and momentum flow functions must be redefined as functions the two variables ω and γ .

The mass flow past a cross-section now becomes

$$\dot{m} = A\,P_t\,(R\,T_t)^{-\frac{1}{2}}\,y(\omega,\ \gamma).$$

where $$y(\omega,\ \gamma) = \left[\frac{2\gamma}{\gamma - 1}\left(1 - \omega^{(\gamma - 1)/\gamma}\right)\right]^{\frac{1}{2}}\omega^{1/\gamma}\ .$$

Similarly the energy flow,

$$\dot{e} = A\,P_t\,C_p(T_t/R)^{\frac{1}{2}}\ y(\omega,\ \gamma)$$

and the momentum flow,

$$\dot{M} = A\,P_t\ f(\omega,\ \gamma)$$

where f is redefined as

$$f(\omega,\ \gamma) = \omega\left[\frac{2\gamma}{\gamma - 1}\,\omega^{(1 - \gamma)/\gamma} - \frac{\gamma + 1}{\gamma - 1}\right]$$

If the mass flows of the primary and secondary streams are m_1 and m_2 respectively, the gas constant R_3 for the mixed stream will be given by :-

$$R_3 = \frac{m_1R_1 + m_2R_2}{m_1 + m_2}$$

The ratio of the specific heats γ for the mixed stream will be given by

$$\gamma_3 = \frac{m_1Cp_1 + m_2Cp_2}{m_1Cv_1 + m_2Cv_2}$$

APPENDIX 3.

In the following appendix the outlines of three Algol procedures are given. These procedures may be used to solve the equations set up by the one-dimensional theory of section 2.1. Two of them determine the charecteristics of the primary expansion for the case where the choking takes place at the end of the expansion and the other determines the state of the fully mixed stream given known conditions at the start of the mixing. The procedures are written for the mixing of two streams of the same gas with a specific heats ratio of 1.4. Each procedure uses sub-procedures,determining functions such as those derived in appendix 1,and these are given below.

1. The necessary sub-procedures.

```
a.  real procedure phy(p); value p; real p;

                   phy := (1-p↑0.286)↑0.5×p↑0.714;

b.  real procedure f(p); value p; real p;

                   f := p×(7×p↑(-0.286)-6);

c.  real procedure F(p); value p; real p;

             begin real ee,ff;

                   ee := (p↑0.429-1.2×p↑0.714)/(1.4×phy(p));

                   ff := 5×p↑(-0.286)-6;

                   F  := (phy(p)×ff-f(p)×ee)/phy(p)↑2;

             end;

d.  real procedure P(p); value p; real p;

             begin real ee;

                   ee := (p↑0.429-1.2×p↑0.714)/(1.4×phy(p));

                   P  := (phy(p)-p×ee)/phy(p)↑2;

             end;
```

2. The procedure finding the choked solution according to Fabri s Theory.

Variables. in program, in text.

Known	z	z	Primary throat/mixing tube area
	pie	π	Total pressure ratio
	p11	w11	Primary injection pressure ratio
	p22	w22	Secondary critical pressure ratio
To be determined	p21	w21	Secondary injection pressure ratio
	p12	w12	Static/total pressure ratio defining the state of the primary stream at the end of the expansion.

Non-local
```
real f11,y11,z1,z,a1,a,p1,pie,p21,y21,y12,
   p12,f12,F12,p22;

integer n,m;
```

PROCEDURE:-

```
procedure Z2(p); value p; real p;
      begin f11:=f(p);y11:=phy(p);z1:=z×0.2588/y11;a1:=1-z1;
            p1:=(p×pie-0.5283)/2;p21:=0.5283+p1;
            a:=a1/(z×pie×0.2588);for n:=1 step 1 until 14 do
      begin y21:=phy(p21);y12:=z1×0.2588×y11/(0.2588-a1×y21);
            p12:=p/2;for m:=1 step 1 until 14 do p12:=
            p12-sign(phy(p12)-y12)×p/2↑(m+1); f12:=f(p12);
            F12:=y12×(z1×pie×f11+a1×(f(p21)-y21×4.9))/(z1×
            pie×y11); p21:=p21+sign(f12-F12)×p1/2↑n;
      end;
            p22:=0.5283;
      end;
```

PROCEDURE CALL:-   Z2(p11);

3. An algol procedure finding the state of the fully mixed stream according to the Straight Mixing Theory.

Variables.	in program,	in text.	
Known	alpha2	-	Area ratio primary/secondary at start of mixing.
	pie22	π'	Total pressure ratio at start of mixing.
	p12	w12	Static/total pressure ratio of primary at start of mixing.
	p22	w22	Static/total pressure ratio of secondary at start of mixing.
	h2	h	Total enthalpy ratio.
To be determined	mew	-	Mass flow ratio secondary/primary.
	pie3	π''	Total pressure ratio mixed/secondary.
	p3	-	Mixed static pressure/ Secondary total pressure.
	h3	h3	Total enthalpy ratio, mixed/ secondary.
Non-local	real f1,f2,y1,y2,alpha3,alpha2,C,Q,R,V,aa,bb, dd,k3,k31,k32,f3,y3,pie22,p12,p22,p3,mew, pie3,h2,h3;		

PROCEDURE:-

```
procedure ZA(p); value p; real p;
begin f1:=f(p12); f2:=f(p22); y1:=phy(p12); y2:=phy(p22);
      alpha3:=1+alpha2; C:=pie22×p×y1×y2×(h2↑0.5+h2↑(-0.5));
      R:=f2+p×pie22×f1; Q:=(p×pie22×y1)↑2; V:=(y2↑2+C+Q)/R↑2;
      aa:=49×V-1; bb:=14×V-1; dd:=bb↑2-4×aa×V;
      if dd<0 then begin k31:=(-bb+sqrt(dd))/(2×aa);
                         k32:=(-bb-sqrt(dd))/(2×aa);
                         mew:=y2/(y1×p×pie22×h2↑(-0.5));
                         for k3:=k31,k32 do
                   begin f3:=(1+7×k3)/(1+k3)↑3.5;
                         y3:=k3↑0.5/(1+k3)↑3.0;
                         pie3:=R/(alpha3×f3);
                         p3:=pie3/(1+k3)↑3.5;
                   end;
                         h3:=((y2+p×pie22×y1×h2↑0.5)/
                              (alpha3×pie3×y3))↑2;
                   end;
end;
```

PROCEDURE CALL:-   ZA(alpha2);

4. An algol procedure finding the choked solution according to the Polytropic Two Stream Theory.

Variables. in program, in text.

Known	z	z	Primary throat area/mixing duct area.
	pie	π	Total pressure ratio.
	p11	w11	Primary injection pressure ratio.
To be determined	p21	w21	Secondary injection pressure ratio.
	p12	w12	Static/total pressure ratios
	p22	w22	defining states of streams at the end of the primary expansion.

Non-local

```
real f11,y11,a,a1,z,pie,p1,p21,f21,y21,p2,W,p22,
     F12,aa,p121,p12,P12,q;
integer n,l,m;
```

PROCEDURE:-

```
procedure Z1(p); value p; real p;
begin f11:=f(p);y11:=phy(p);a1:=1-z×0.2588/y11;
    a:=a1/(z×pie×0.2588);p1:=(p×pie-0.5283)/2;
    p21:=0.5283+p1; for n:=1 step 1 until 14 do
begin f21:=f(p21);y21:=phy(p21);p2:=0.23585;W:=f11/y11+a×f21;
    p22:=0.76415; for m:=1 step 1 until 14 do
begin F12:=W-a×y21×f(p22)/phy(p22);aa:=p121:=p12:=0.26415;
    for l:=1 step 1 until 14 do p12:=p12+sign(f(p12)/phy(p12)
                                        -F12)×aa/2↑l;
    P12:=a×p22×(1/a1-y21/phy(p22)); for l:=1 step 1 until
    14 do p121:=p121+sign(P12-p121/phy(p121))×aa/2↑l;
    q:=(y21×P(p22)-1/a1)/P(p121)-y21×F(p22)/F(p12);
    p22:=p22-sign(q)×p2/2↑m;
end;
    p21:=p21+sign(p12-p121)×p1/2↑n;
end;
end;
```

PROCEDURE CALL:- Z1(p11);

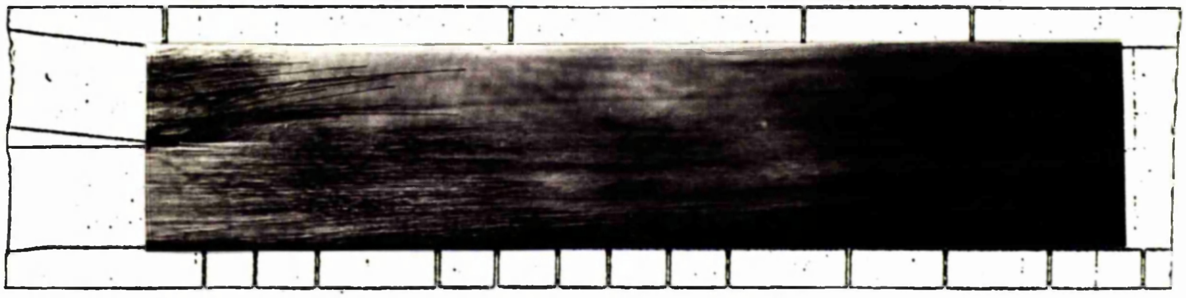
APPENDIX 4.
SPECIMEN EXPERIMENTAL RESULTS

This appendix contains the full normalised results from twelve experimental ejector flows recorded during the pilot experimental programme. Each page represents the results from a different flow. Below the particulars of each flow a Schlieren picture of the initial mixing region is given. The positions of the first few pressure tappings are marked along the edge of the picture to help to establish the relation between the picture and the pressure distributions. The static pressure distribution along the primary and secondary walls of the mixing duct are given in the graph below the picture. The pressures on the primary wall are marked by solid triangles and those on the secondary by hollow triangles. At the bottom of each page the total pressure distribution across the end of the mixing duct is plotted on a small graph.

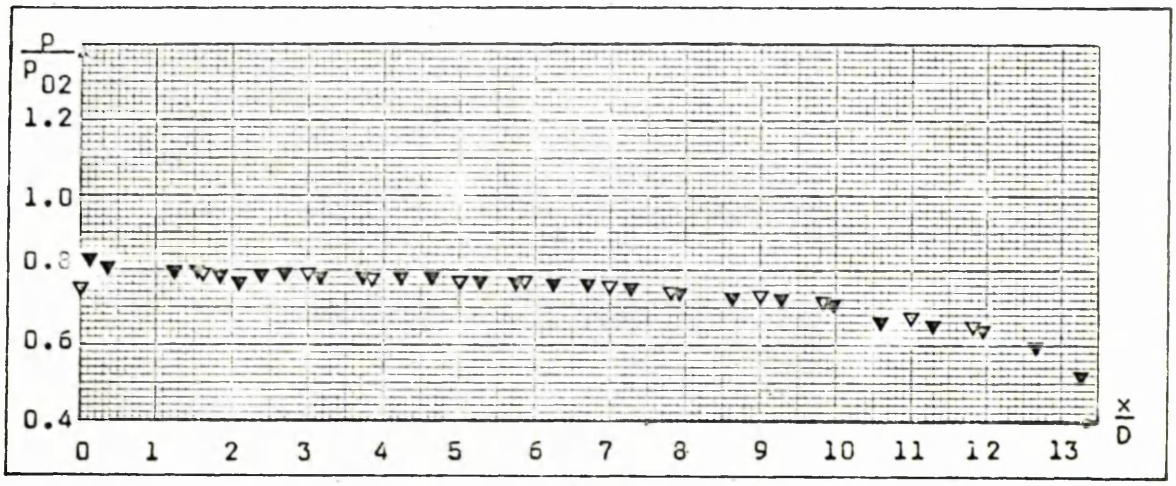
Ejector Test Number 11

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	$A3$	=	1.4912
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.2727
Primary Injection Pressure	ω_{11}	=	0.6519
Secondary Injection Pressure	ω_{21}	=	0.7521
Exhaust Static Pressure	p_3	=	0.524
Average Exhaust Total Pressure	π''	=	1.1028

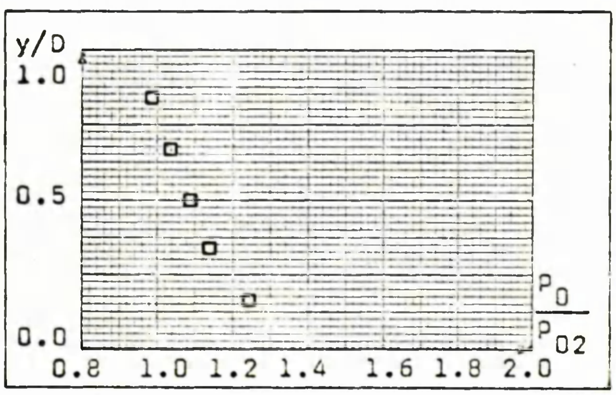
Schlieren Picture of Initial mixing Region



Static Pressure Distributions along Top and Bottom Walls



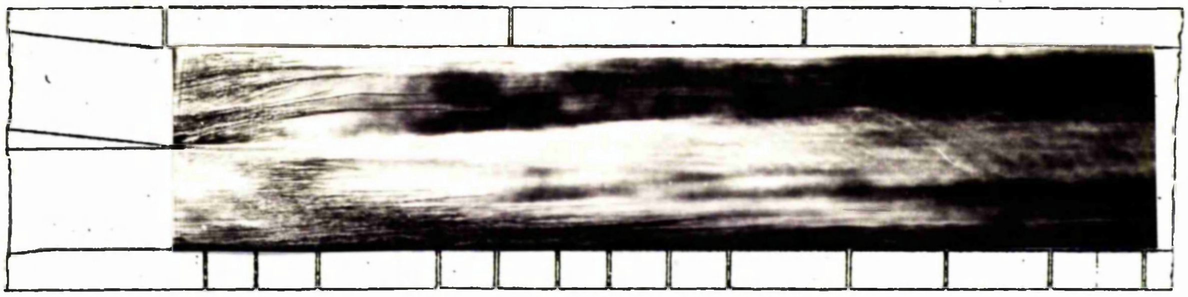
Total Pressure Distribution across End of Mixing Tube



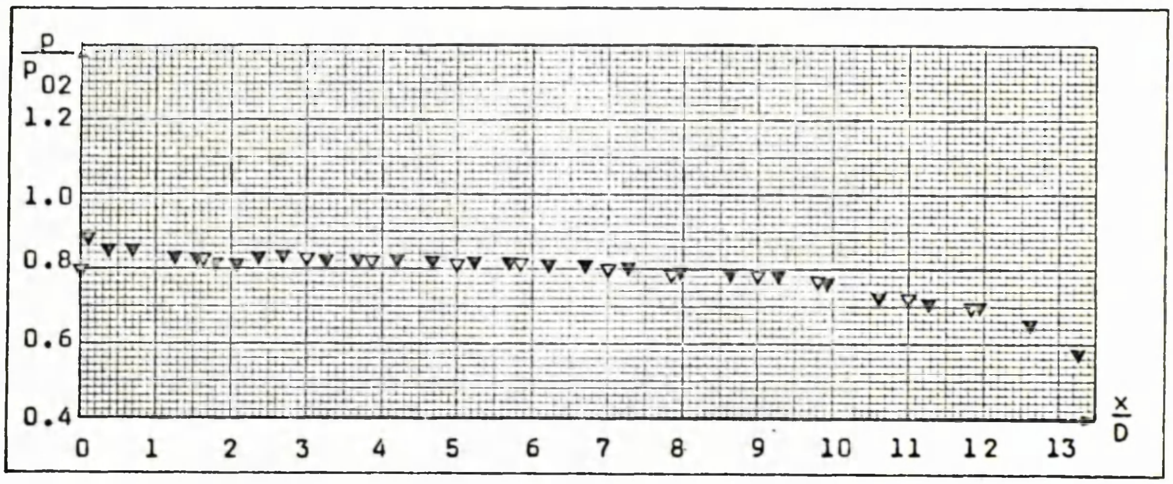
Ejector Test Number 21

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	1.4912
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.5008
Primary Injection Pressure	w_{11}	=	0.5874
Secondary Injection Pressure	w_{21}	=	0.8003
Exhaust Static Pressure	p_3	=	0.5753
Average Exhaust Total Pressure	π''	=	1.2183

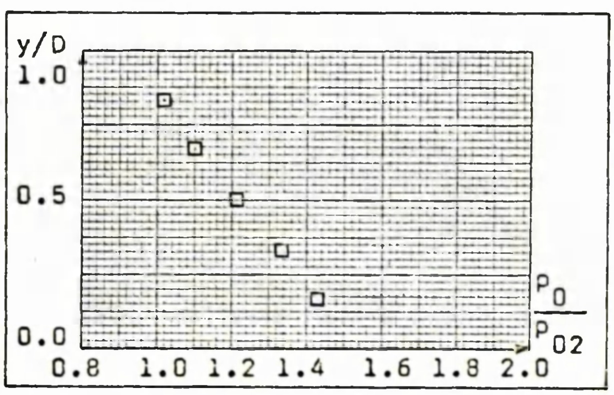
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



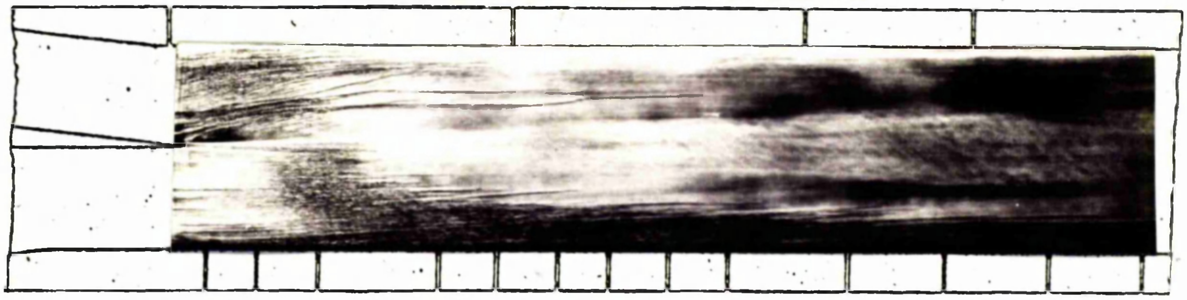
Total Pressure Distribution across End of Mixing Tube



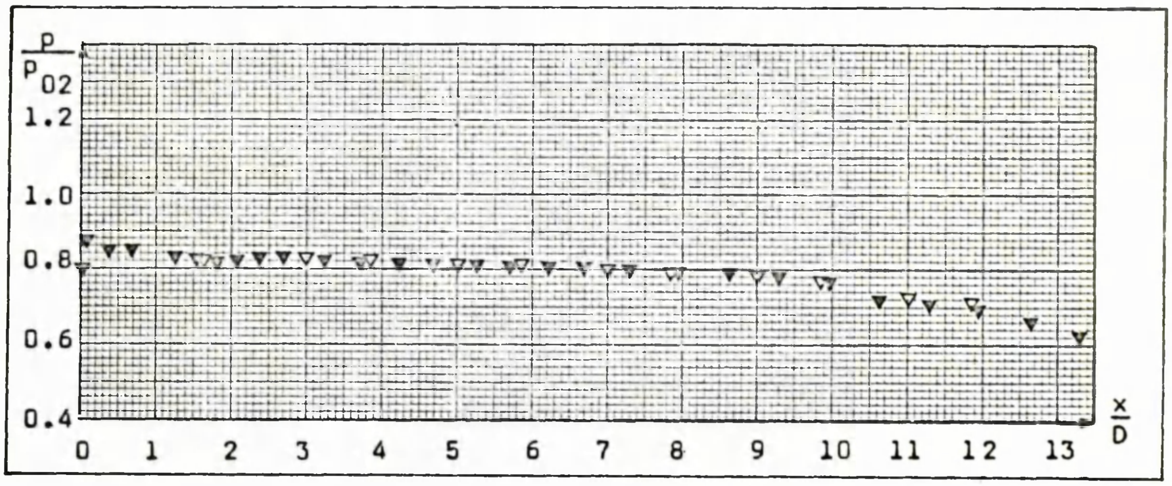
Ejector Test Number 22

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	$A3$	=	1.1184
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4958
Primary Injection Pressure	w_{11}	=	0.5882
Secondary Injection Pressure	w_{21}	=	0.7986
Exhaust Static Pressure	p_3	=	0.6244
Average Exhaust Total Pressure	π''	=	1.2284

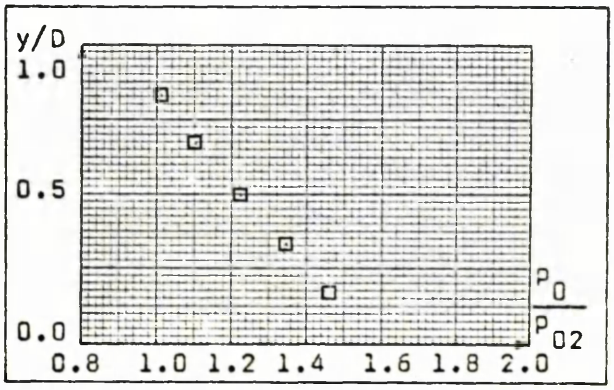
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



Total Pressure Distribution across End of Mixing Tube

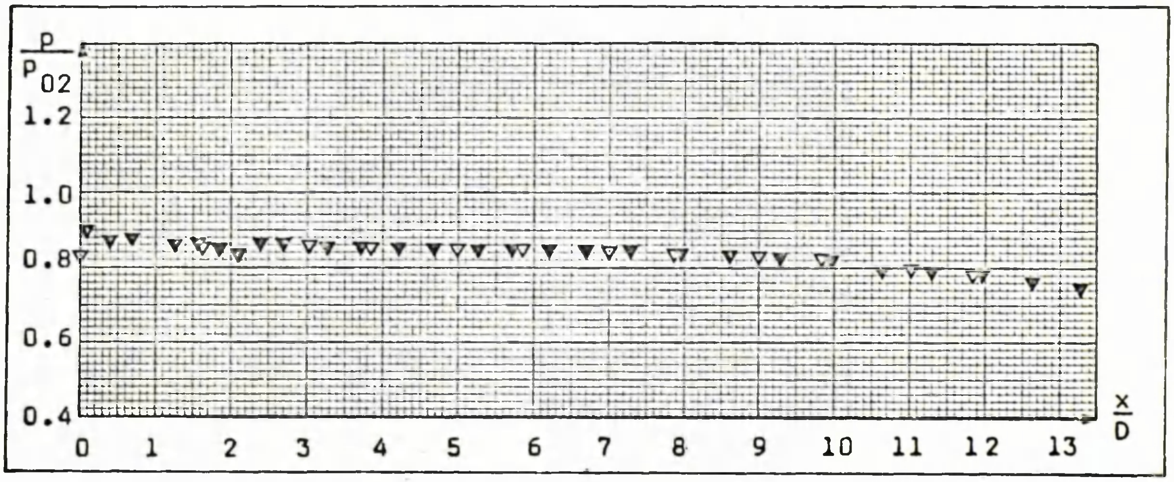


Ejector Test Number 23

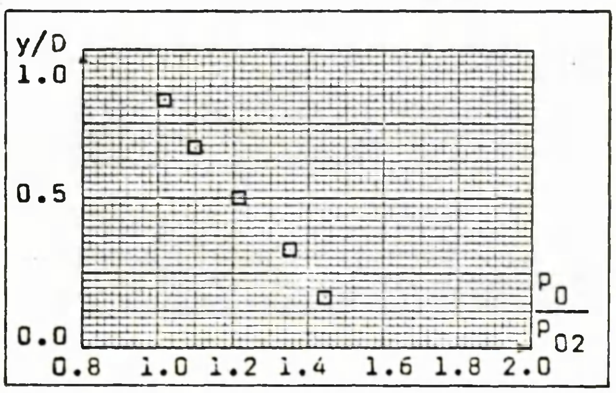
Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.932
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4726
Primary Injection Pressure	w_{11}	=	0.6115
Secondary Injection Pressure	w_{21}	=	0.8308
Exhaust Static Pressure	p_3	=	0.7529
Average Exhaust Total Pressure	π''	=	1.2235

Schlieren Picture of Initial Mixing Region

Static Pressure Distributions along Top and Bottom Walls



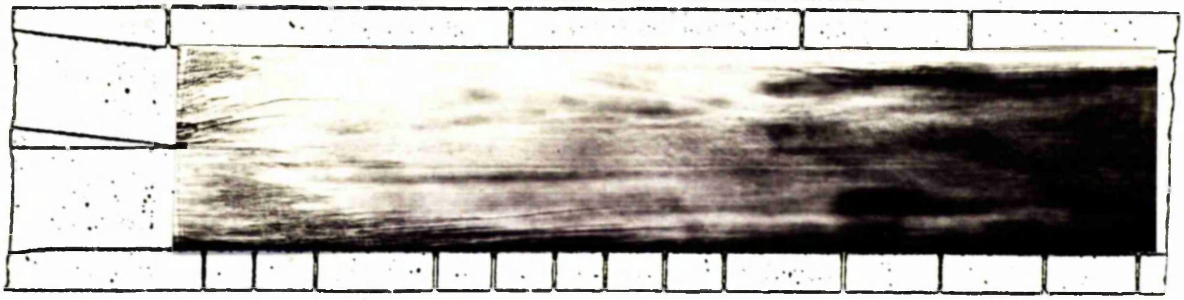
Total Pressure Distribution across End of Mixing Tube



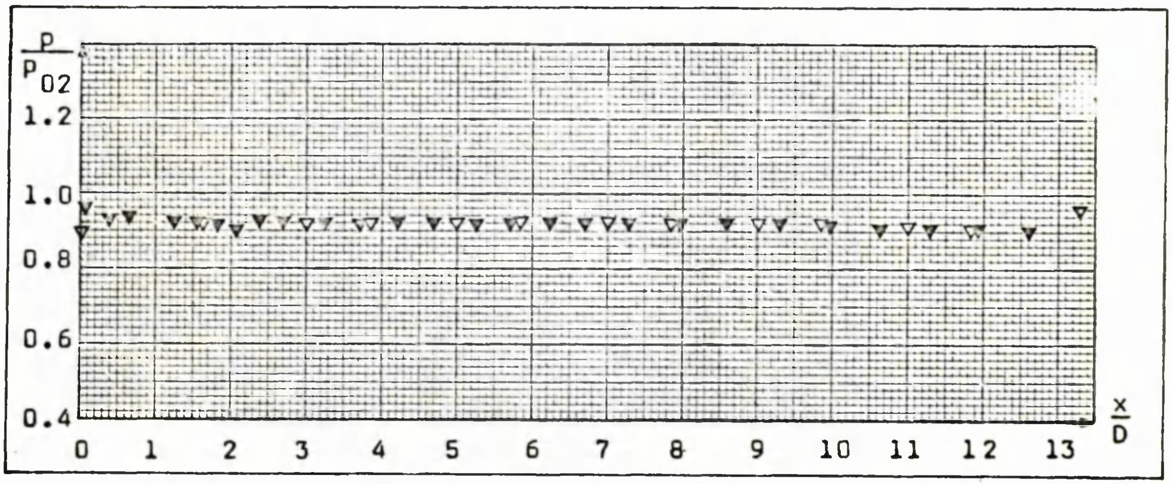
Ejector Test Number 24

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.8202
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4842
Primary Injection Pressure	w_{11}	=	0.638
Secondary Injection Pressure	w_{21}	=	0.8972
Exhaust Static Pressure	p_3	=	0.9652
Average Exhaust Total Pressure	π''	=	1.2438

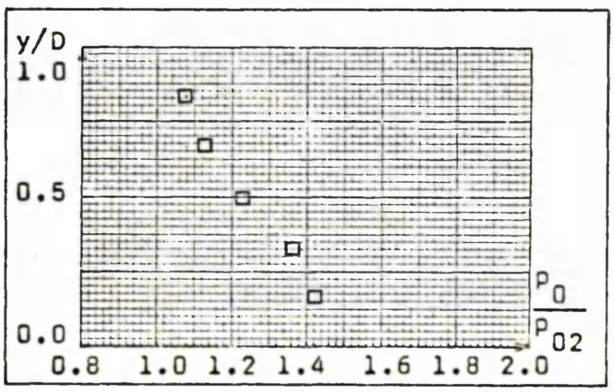
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



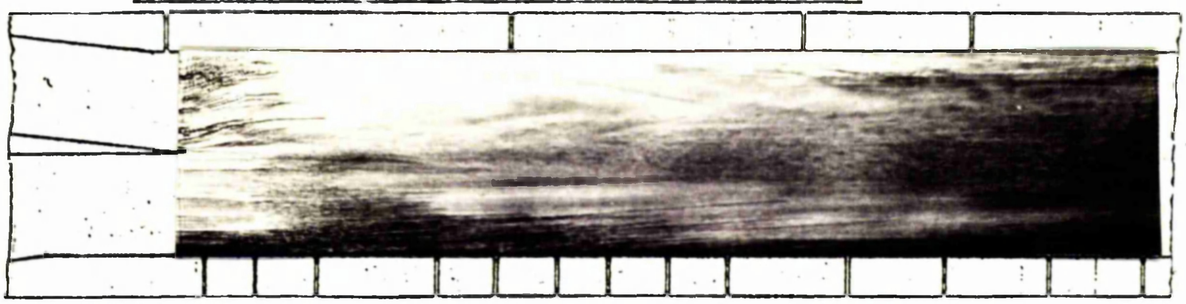
Total Pressure Distribution across End of Mixing Tube



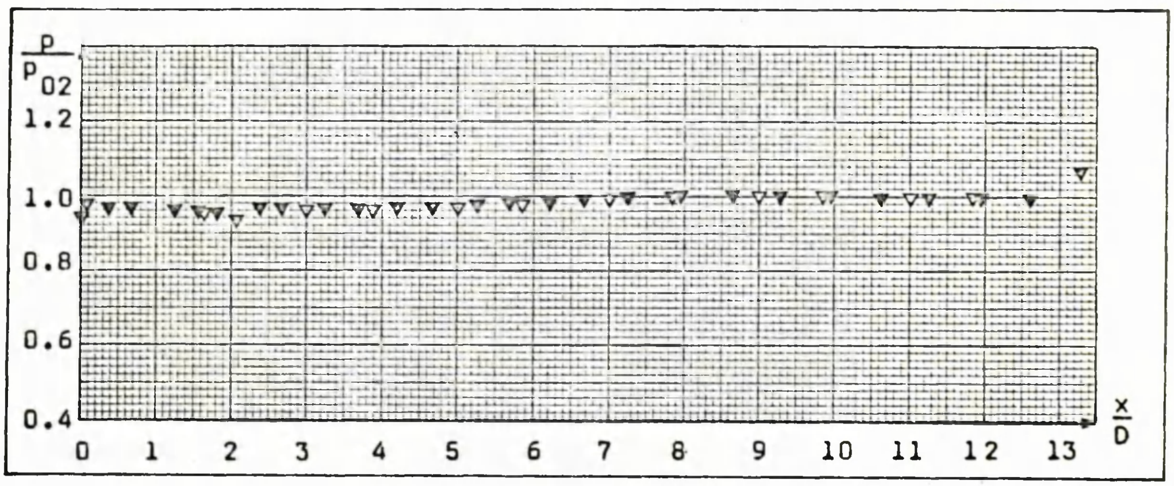
Ejector Test Number 25

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	$A3$	=	0.7456
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.5187
Primary Injection Pressure	ω_{11}	=	0.6461
Secondary Injection Pressure	ω_{21}	=	0.9388
Exhaust Static Pressure	p_3	=	1.0663
Average Exhaust Total Pressure	π''	=	1.2864

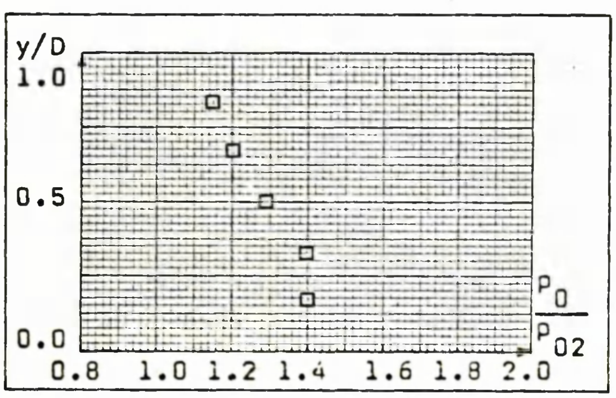
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



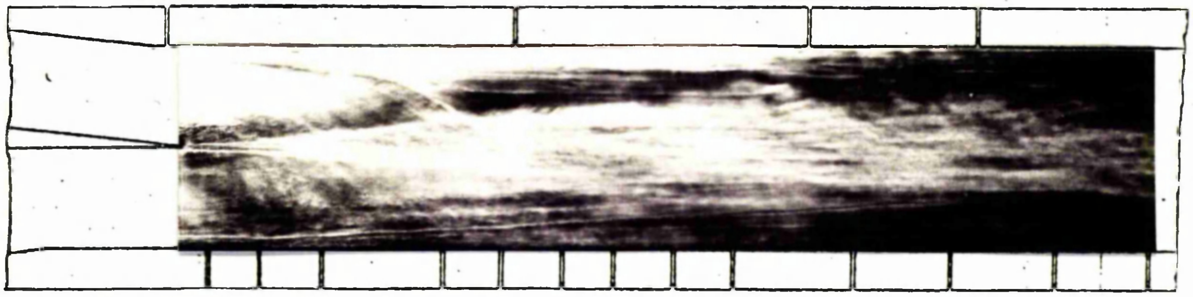
Total Pressure Distribution across End of Mixing Tube



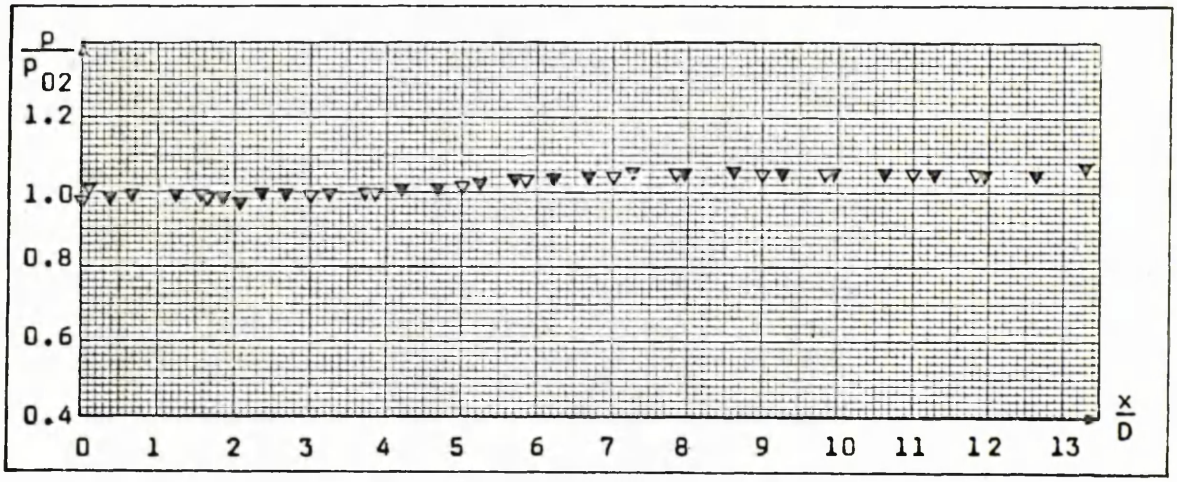
Ejector Test Number 26

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.671
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4773
Primary Injection Pressure	w_{11}	=	0.6791
Secondary Injection Pressure	w_{21}	=	0.9725
Exhaust Static Pressure	p_3	=	1.0534
Average Exhaust Total Pressure	π''	=	1.2783

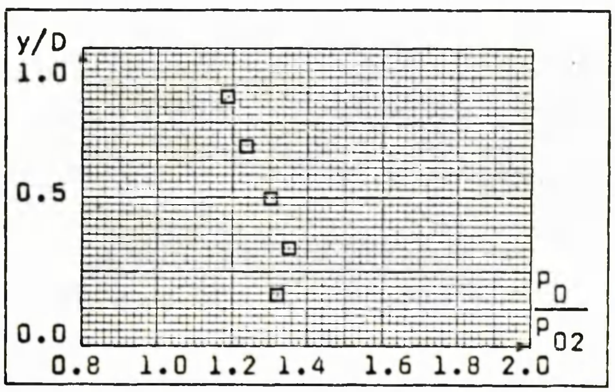
Schlieren Picture of Initial mixing Region



Static Pressure Distributions along Top and Bottom Walls



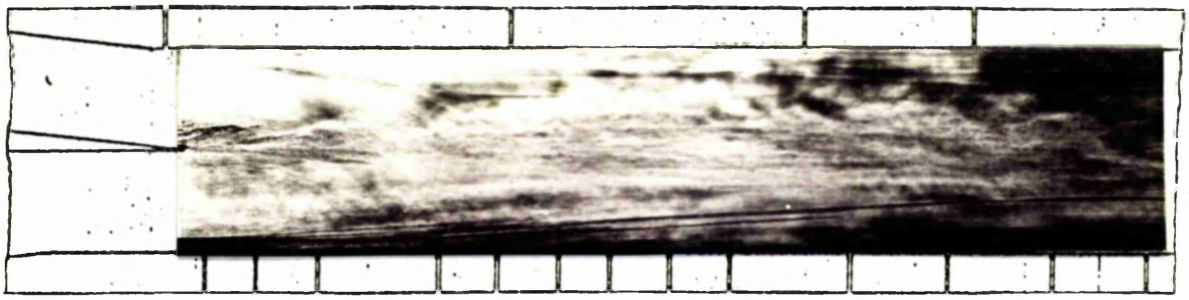
Total Pressure Distribution across End of Mixing Tube



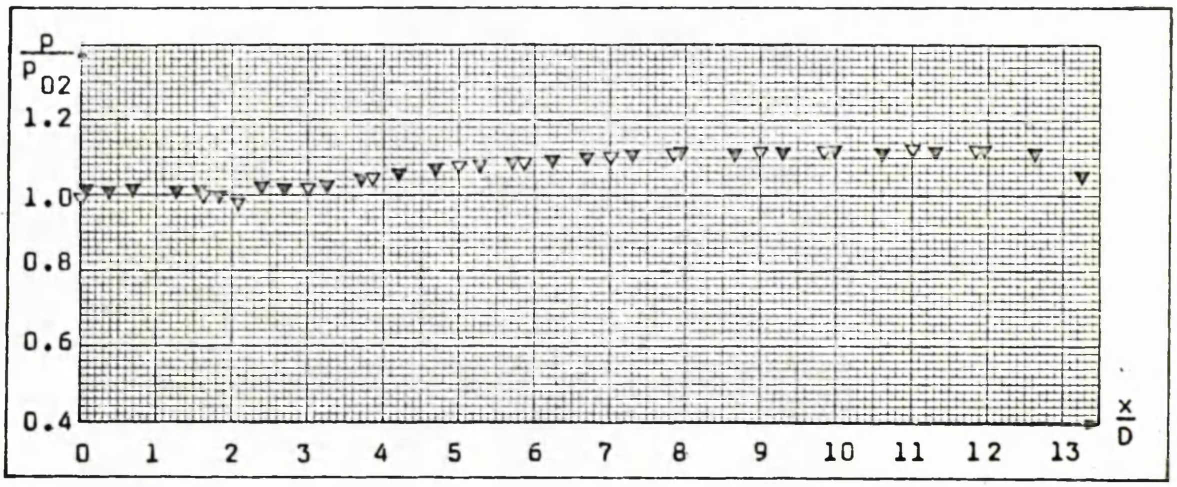
Ejector Test Number 27

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.5965
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4773
Primary Injection Pressure	w_{11}	=	0.6879
Secondary Injection Pressure	w_{21}	=	0.9919
Exhaust Static Pressure	p_3	=	1.1185
Average Exhaust Total Pressure	π''	=	1.2964

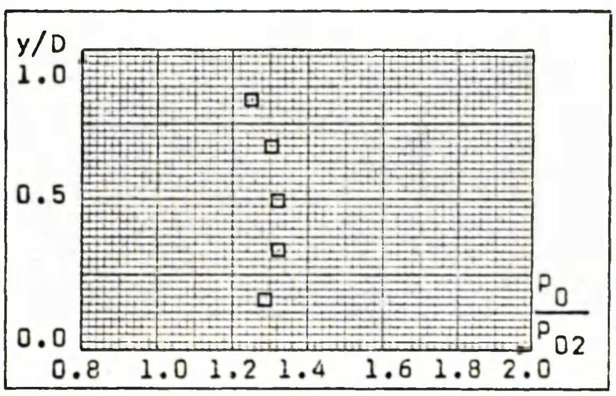
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



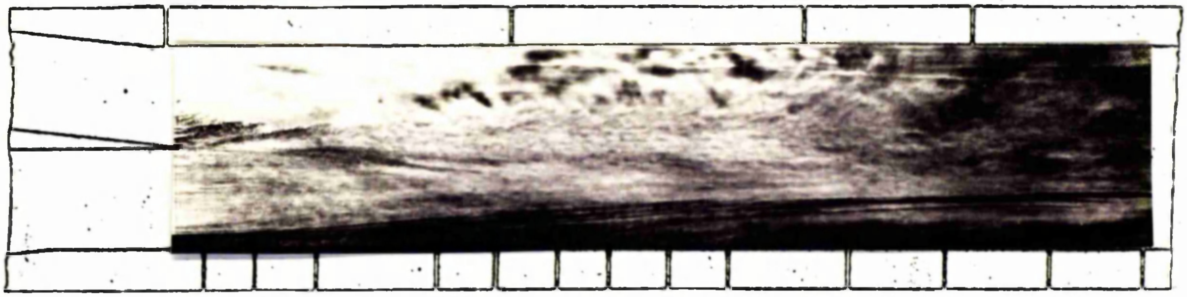
Total Pressure Distribution across End of Mixing Tube



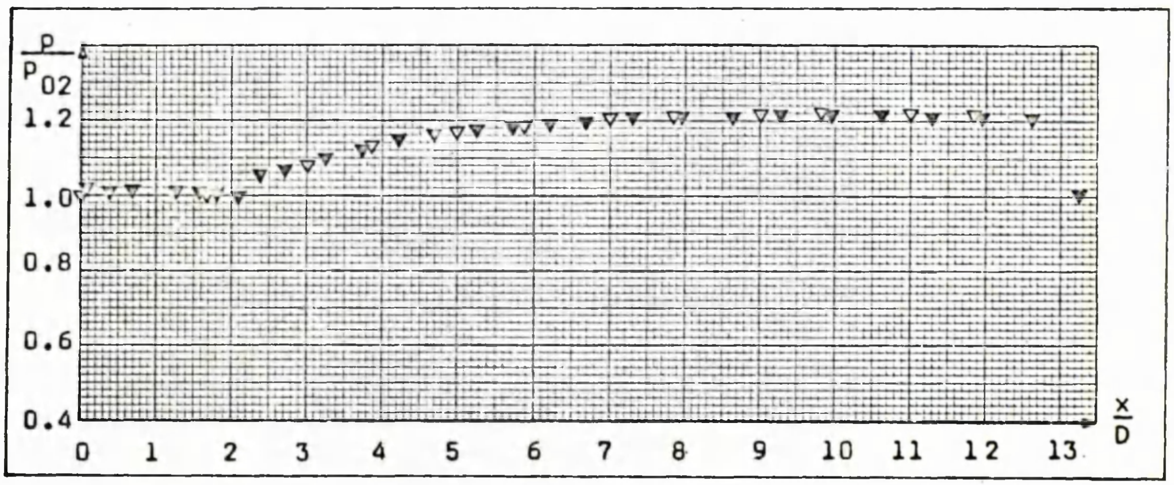
Ejector Test Number 28

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	$A3$	=	0.5219
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.5398
Primary Injection Pressure	w_{11}	=	0.6637
Secondary Injection Pressure	w_{21}	=	1.0
Exhaust Static Pressure	p_3	=	1.2115
Average Exhaust Total Pressure	π''	=	1.3408

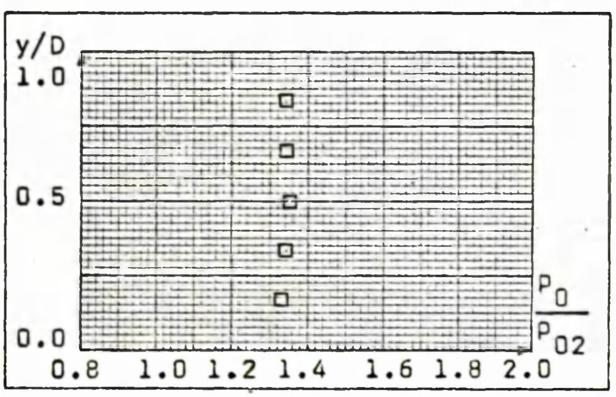
Schlieren Picture of Initial mixing Region



Static Pressure Distributions along Top and Bottom Walls



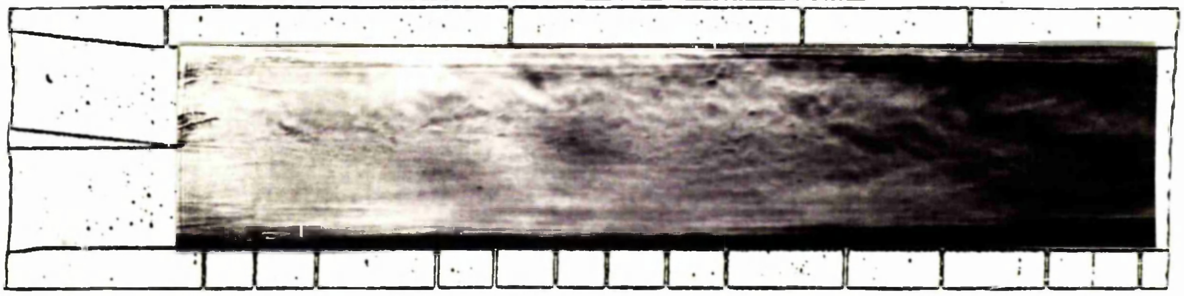
Total Pressure Distribution across End of Mixing Tube



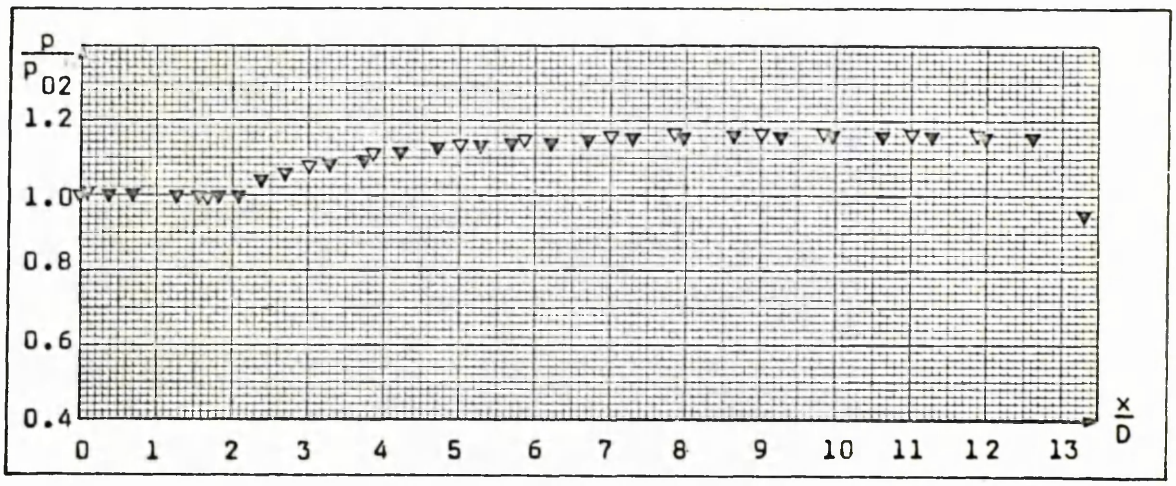
Ejector Test Number 29

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.4474
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.3615
Primary Injection Pressure	w_{11}	=	0.7431
Secondary Injection Pressure	w_{21}	=	0.993
Exhaust Static Pressure	p_3	=	1.1596
Average Exhaust Total Pressure	π''	=	1.2404

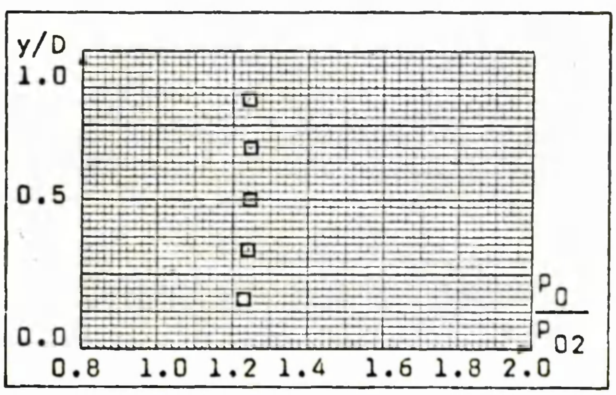
Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



Total Pressure Distribution across End of Mixing Tube

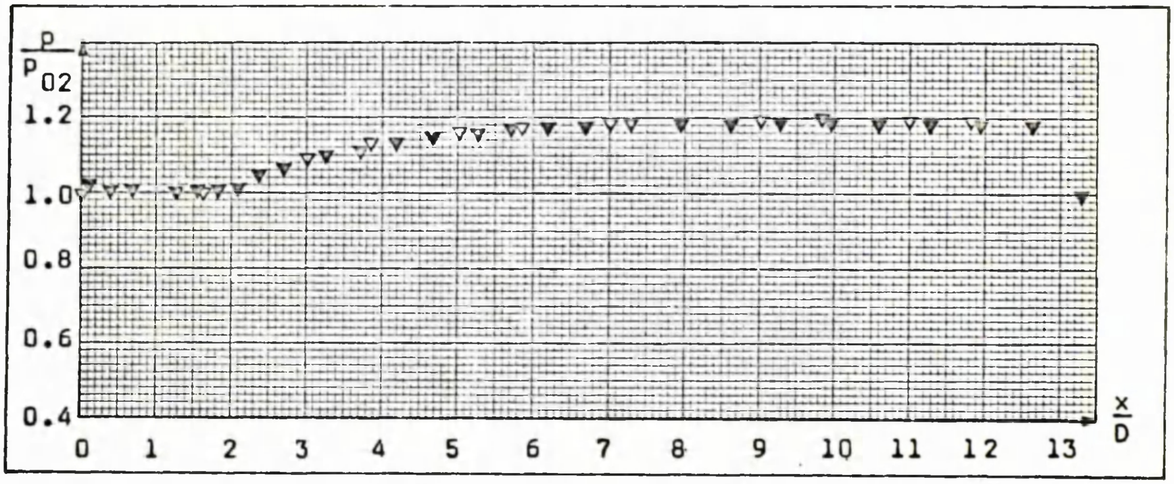


Ejector Test Number 30

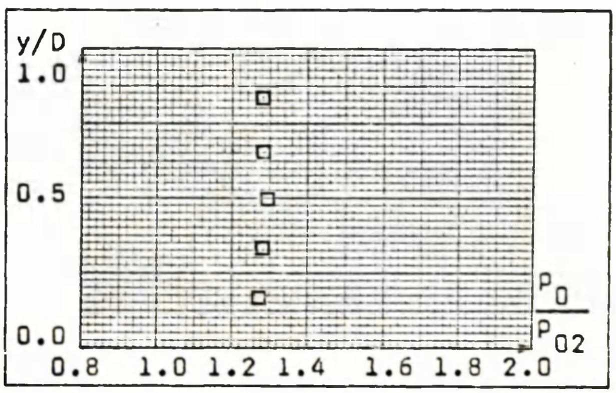
Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	A3	=	0.4846
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.4289
Primary Injection Pressure	w_{11}	=	0.7136
Secondary Injection Pressure	w_{21}	=	0.9951
Exhaust Static Pressure	p_3	=	1.1838
Average Exhaust Total Pressure	π "	=	1.3299

Schlieren Picture of Initial Mixing Region

Static Pressure Distributions along Top and Bottom Walls



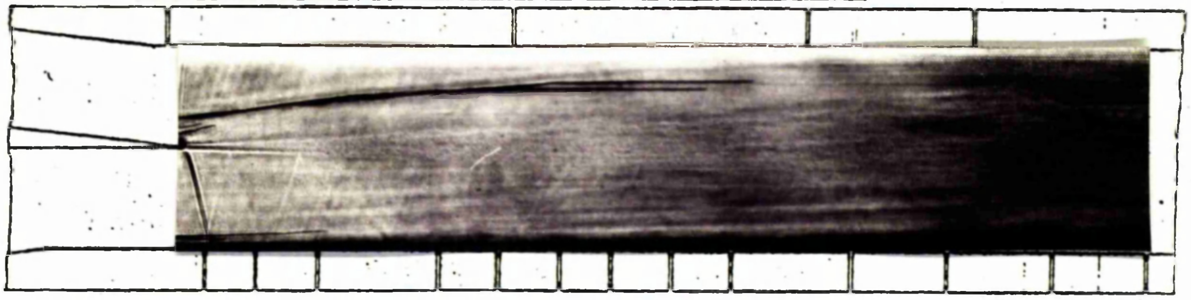
Total Pressure Distribution across End of Mixing Tube



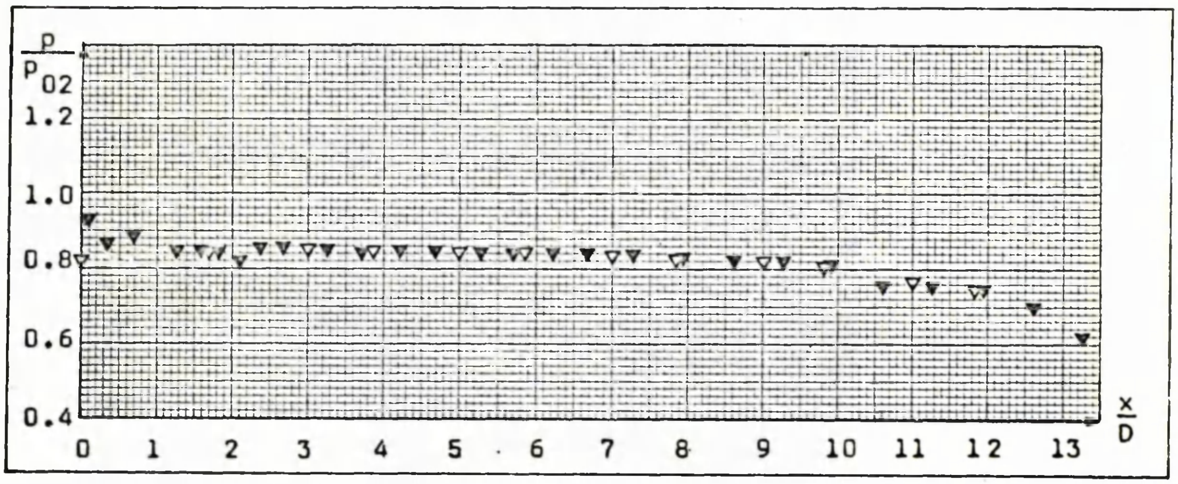
Ejector Test Number 31

Injection Nozzle Area Ratio	z	=	0.5
Exhaust Area	$A3$	=	1.4912
Total Enthalpy Ratio	h	=	1.0
Total Pressure Ratio	π	=	1.7066
Primary Injection Pressure	w_{11}	=	0.5455
Secondary Injection Pressure	w_{21}	=	0.8196
Exhaust Static Pressure	p_3	=	0.6189
Average Exhaust Total Pressure	π''	=	1.312

Schlieren Picture of Initial Mixing Region



Static Pressure Distributions along Top and Bottom Walls



Total Pressure Distribution across End of Mixing Tube

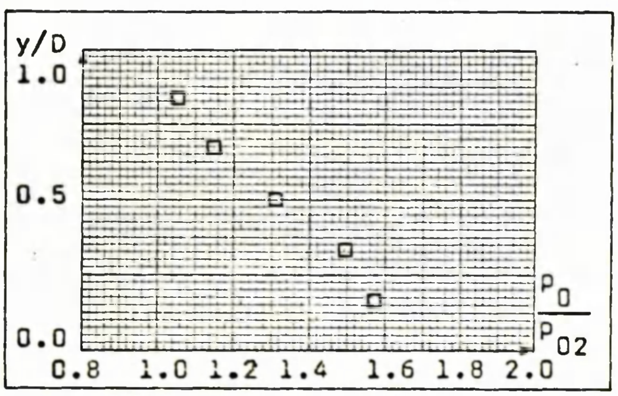


Figure 1.

Sonic Ejector with Constant Area Mixing Duct.

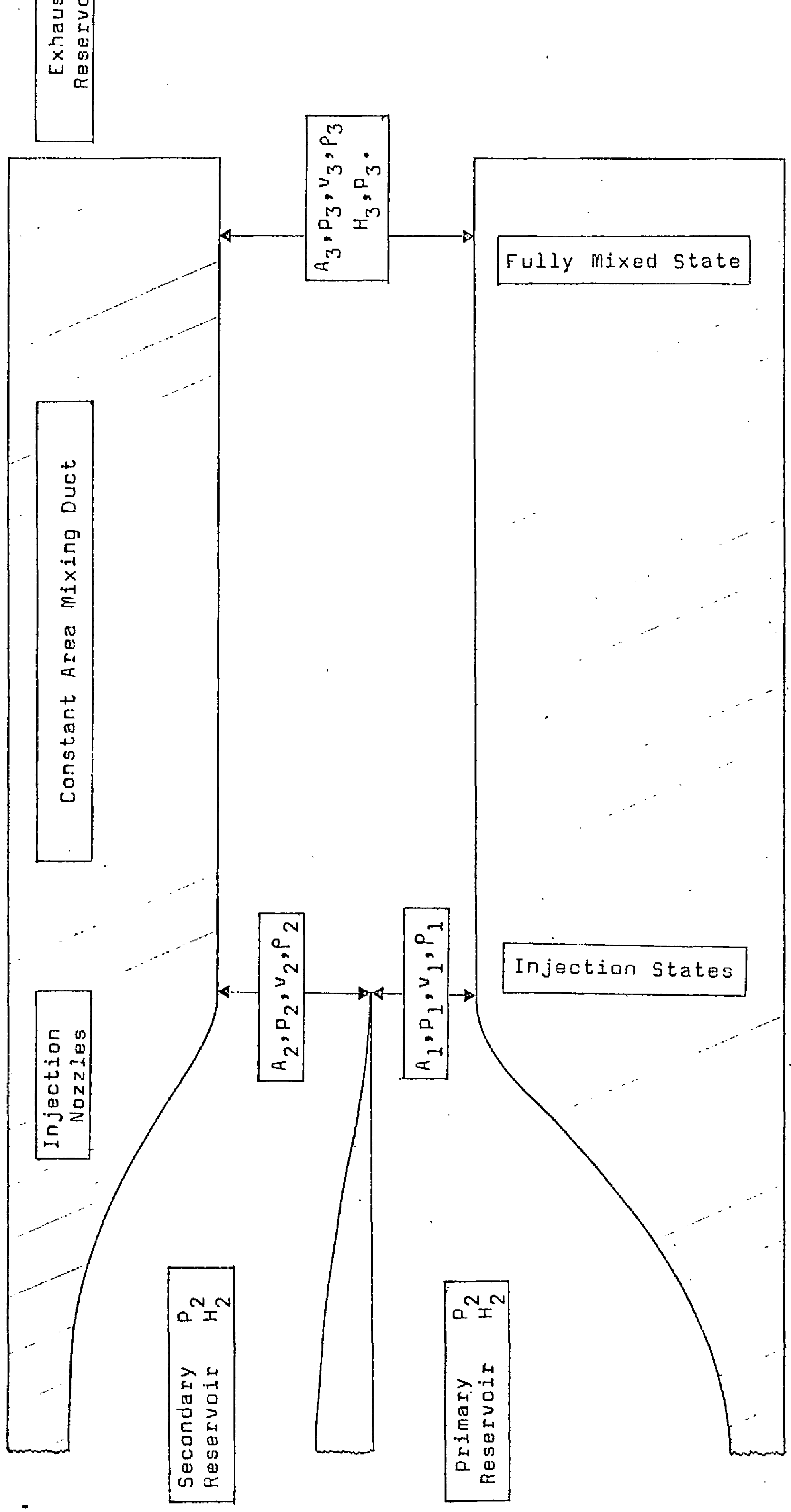
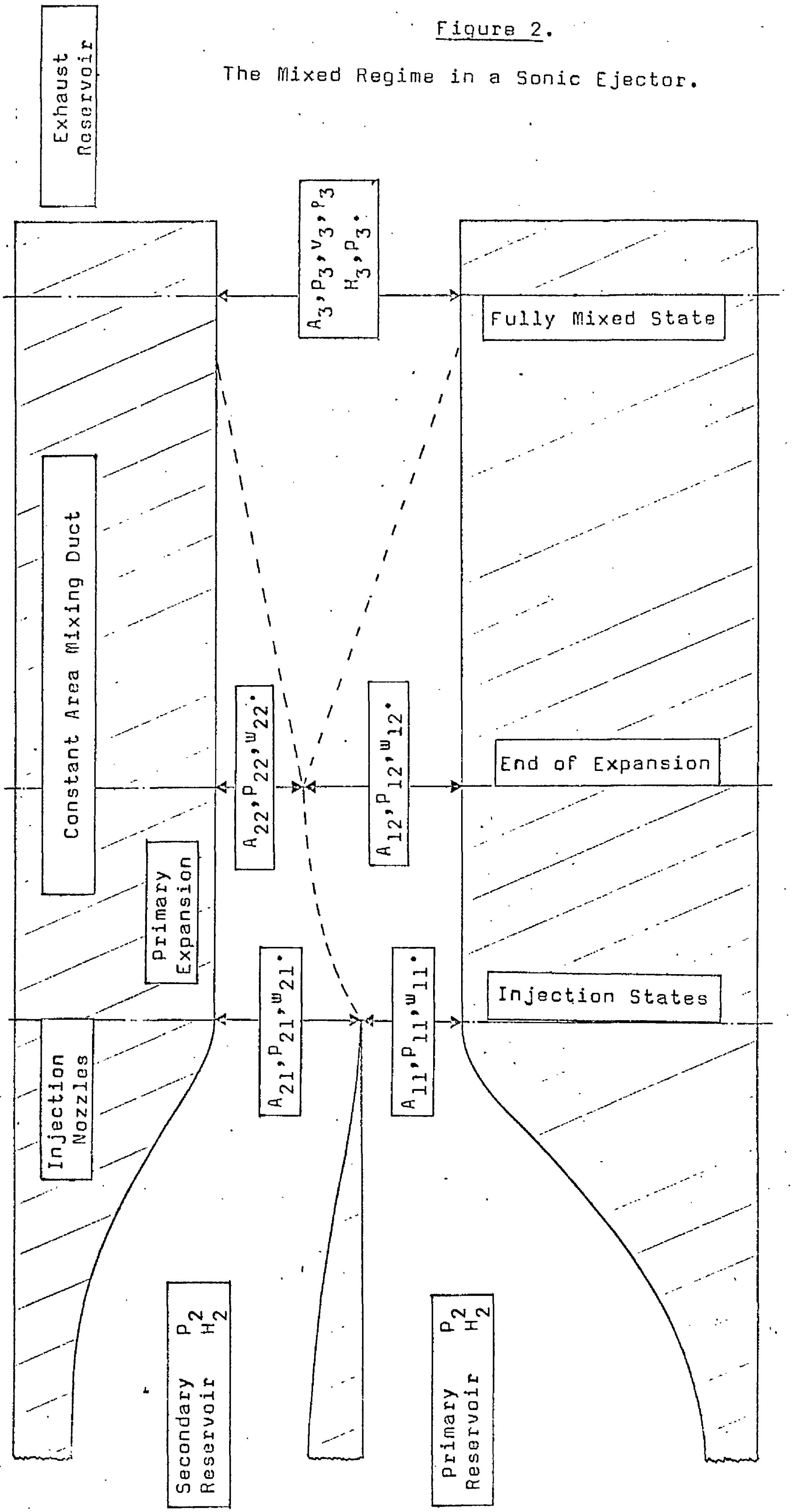


Figure 2.

The Mixed Regime in a Sonic Ejector.



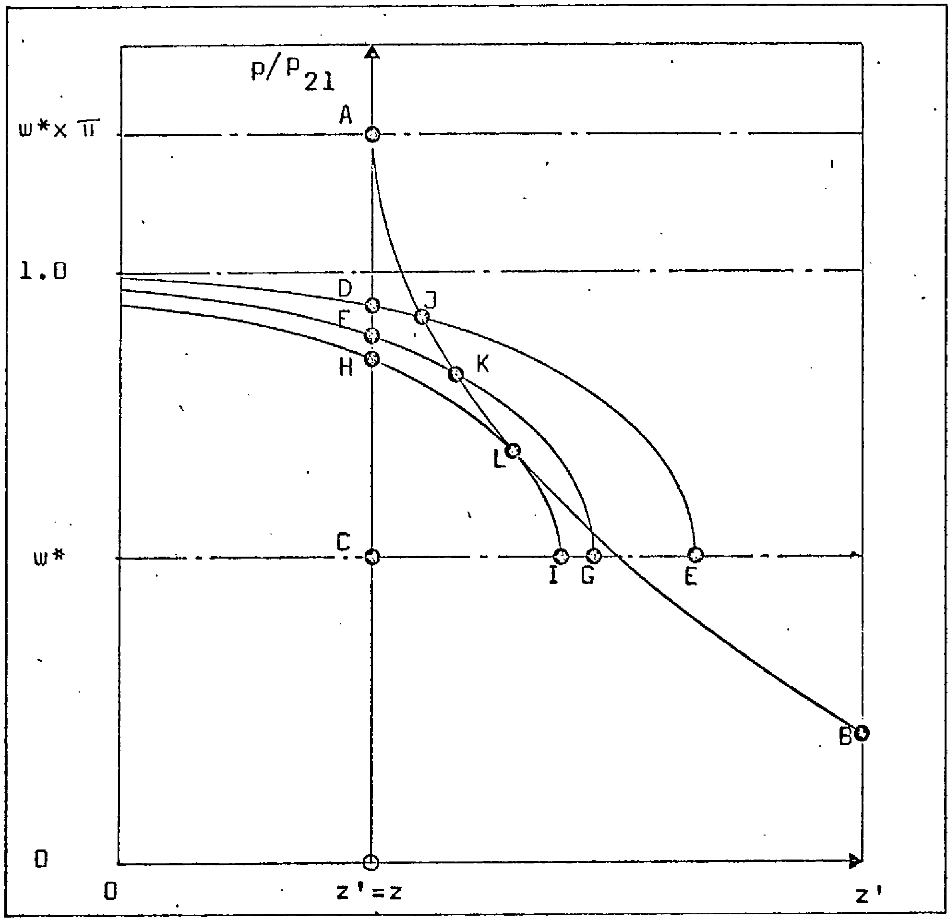


Figure 3

Diagramatic Representation of the Isentropic Two Stream Theory.

A   Choked primary injection state.
AB   Isentropic primary expanssion, supersonic.
C   Choked secondary injection state.
D, F&H Secondary injection states.
DE,
FG
& HI   Isentropic secondary expansion, subsonic.
J, K&L States at end of expansion.

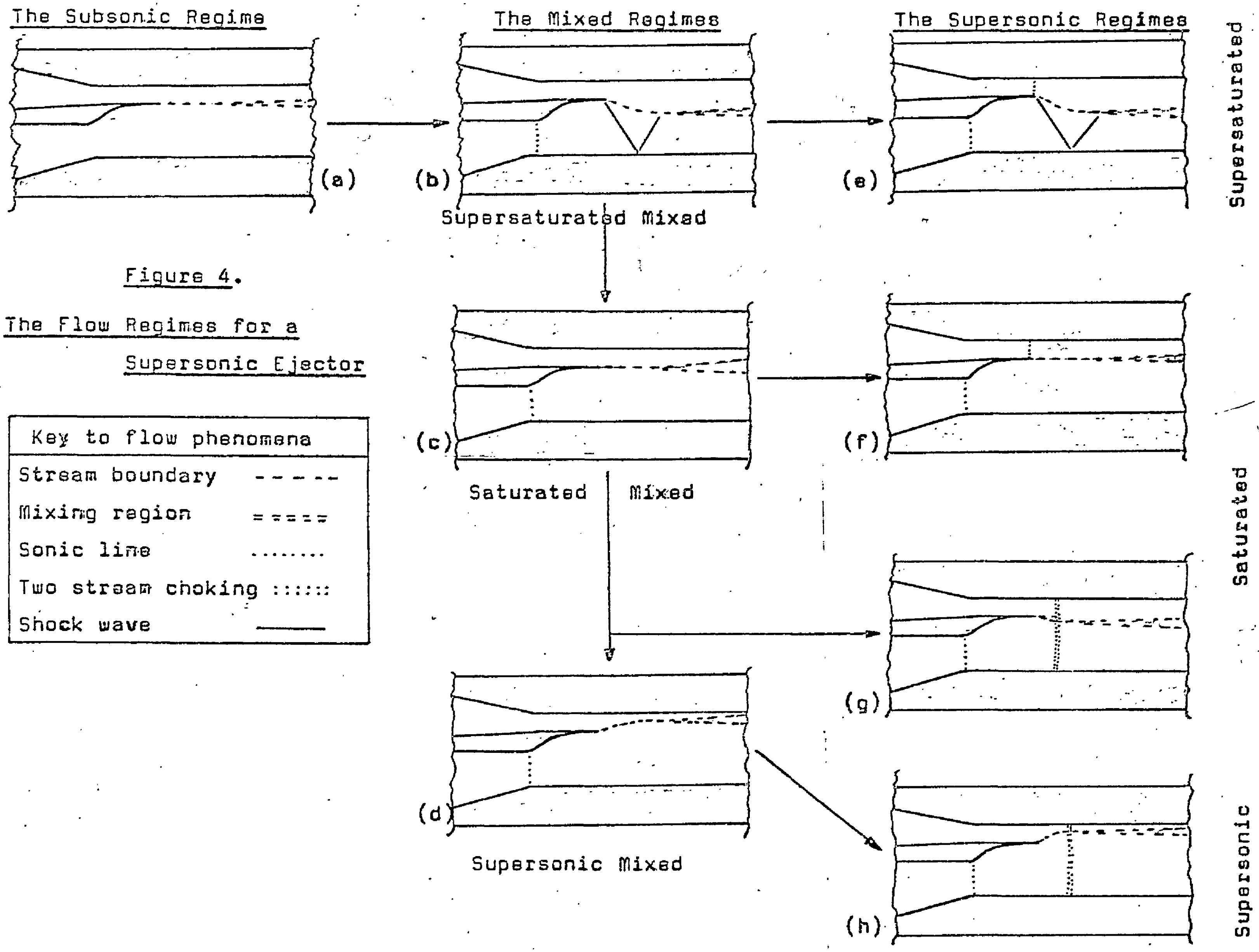


Figure 4.

The Flow Regimes for a Supersonic Ejector

Key to flow phenomena	
Stream boundary	- - - - -
Mixing region	= = = = =
Sonic line	....
Two stream choking	::::::
Shock wave	———

Figure 5

The Supersonic Regimes for a Supersonic Ejector of fixed Geometry

A comparison of the predictions of the one- and two-dimensional theories.

Secondary injection Mach number versus
the ratio of the secondary injection pressure
over the primary total pressure.

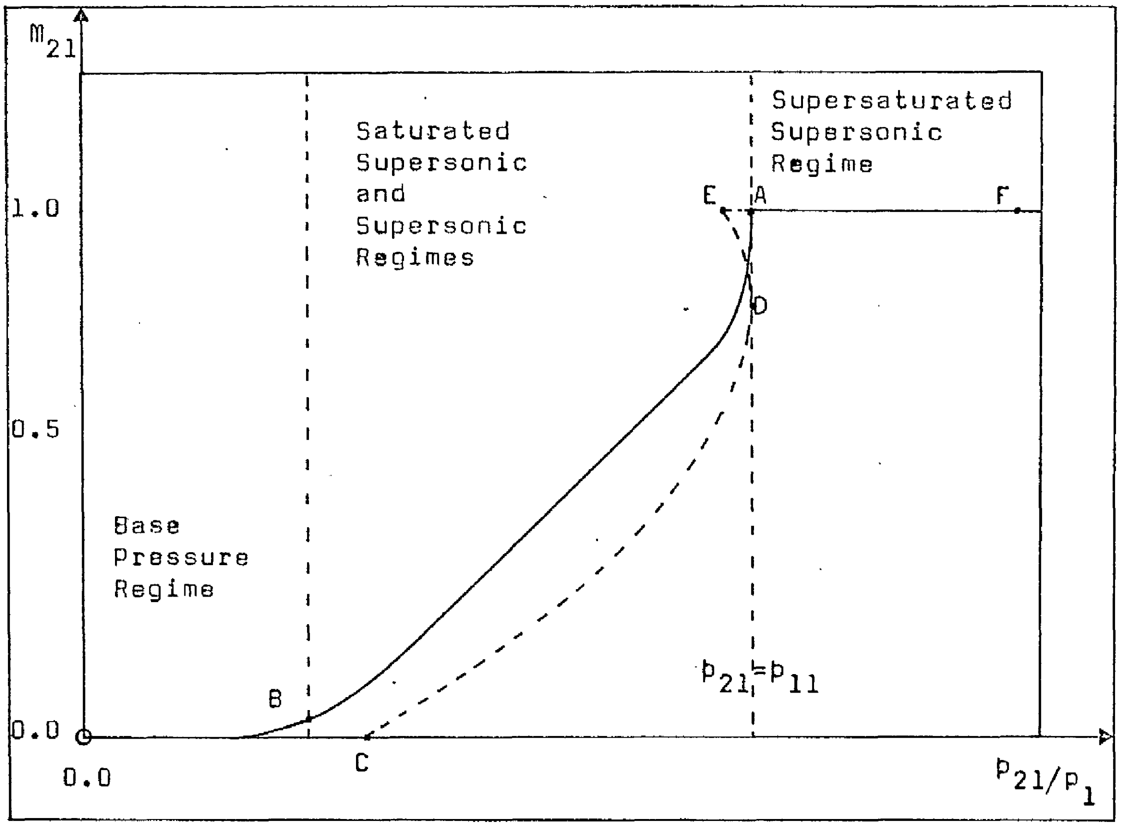


Figure 5 illustrates the forms of the solutions for the performance of a supersonic ejector exhausting into a vacuum;that is operating in the Supersonic Regimes. The continuous curve represents the form of the solution according to the two-dimensional theory of Messrs.Chow and Addi (ref 3) and the dotted curve indicates the form of the one-dimensional solutions provided by the Isentropic and Polytropic theories. The line AF,representing the Supersaturated Supersonic Regime,may be considered common to both types of solution. The total pressure ratio π is high at the Base Pressure ends of the curves and falls as the solution points move along the curves towards the point F.

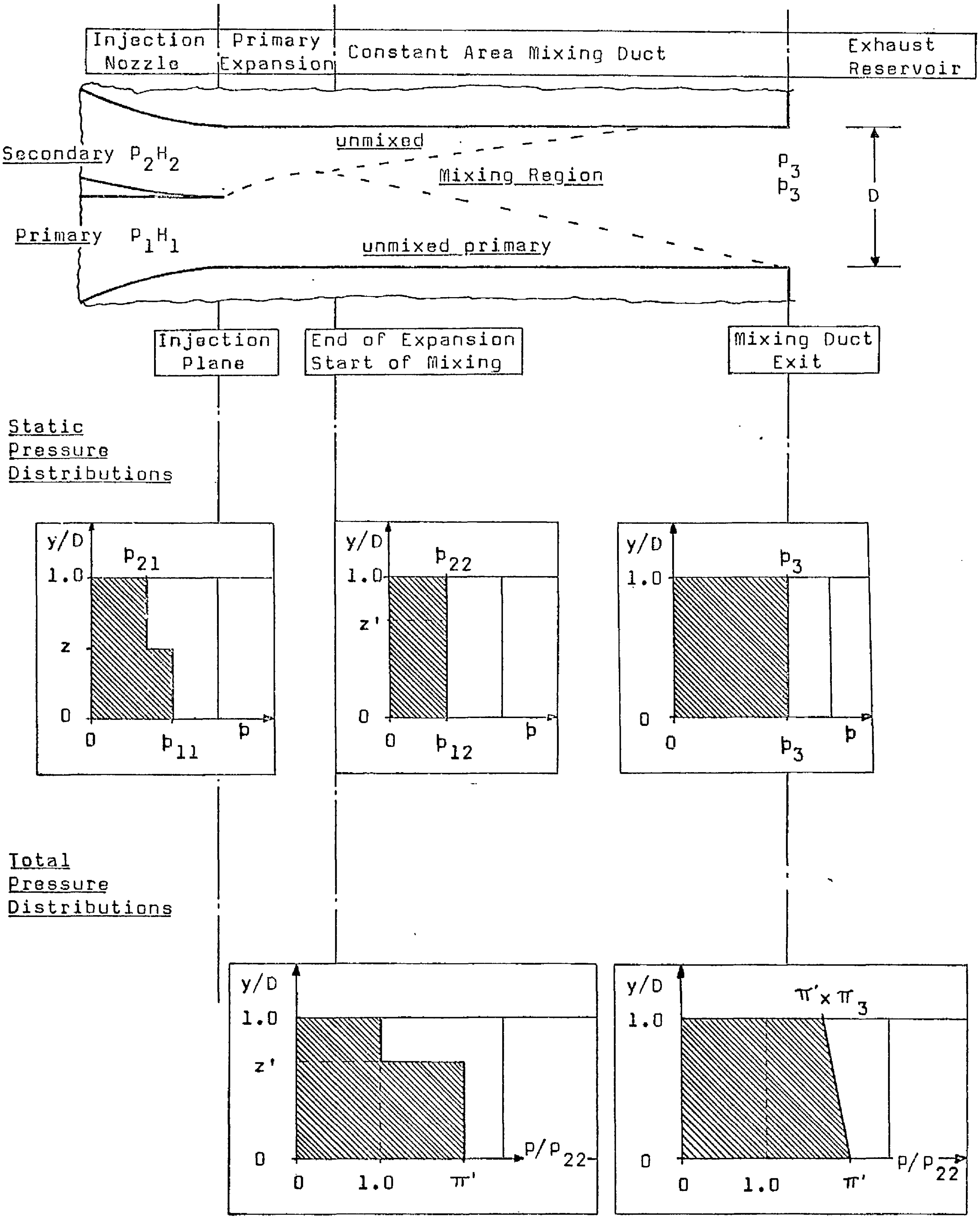


Figure 31.

Pressure distributions for the linear mixing theory.

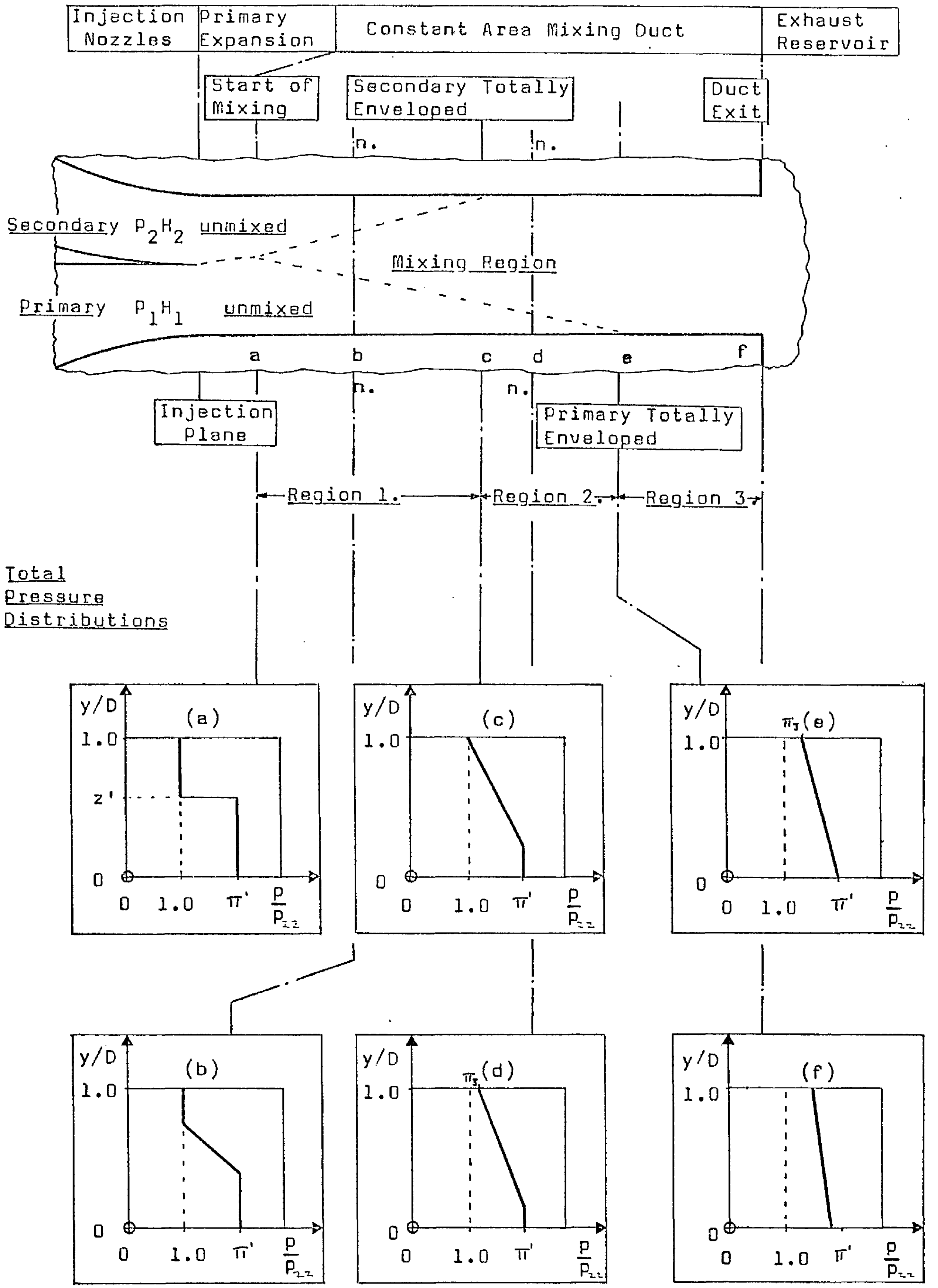


Figure 32

A One-dimensional Description of the Mixing Process.

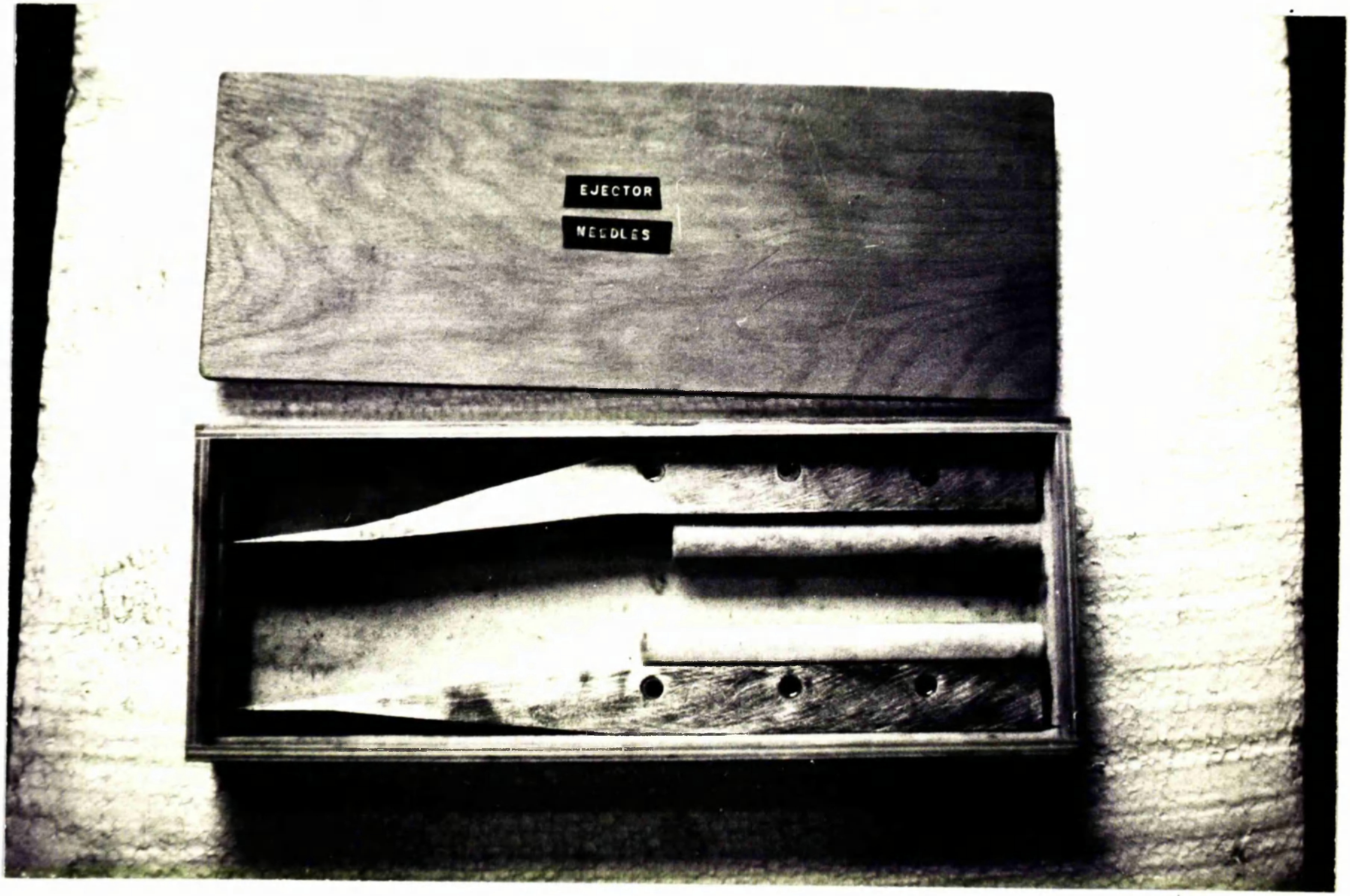


Figure 33. Needles Forming the various Injection Nozzles.

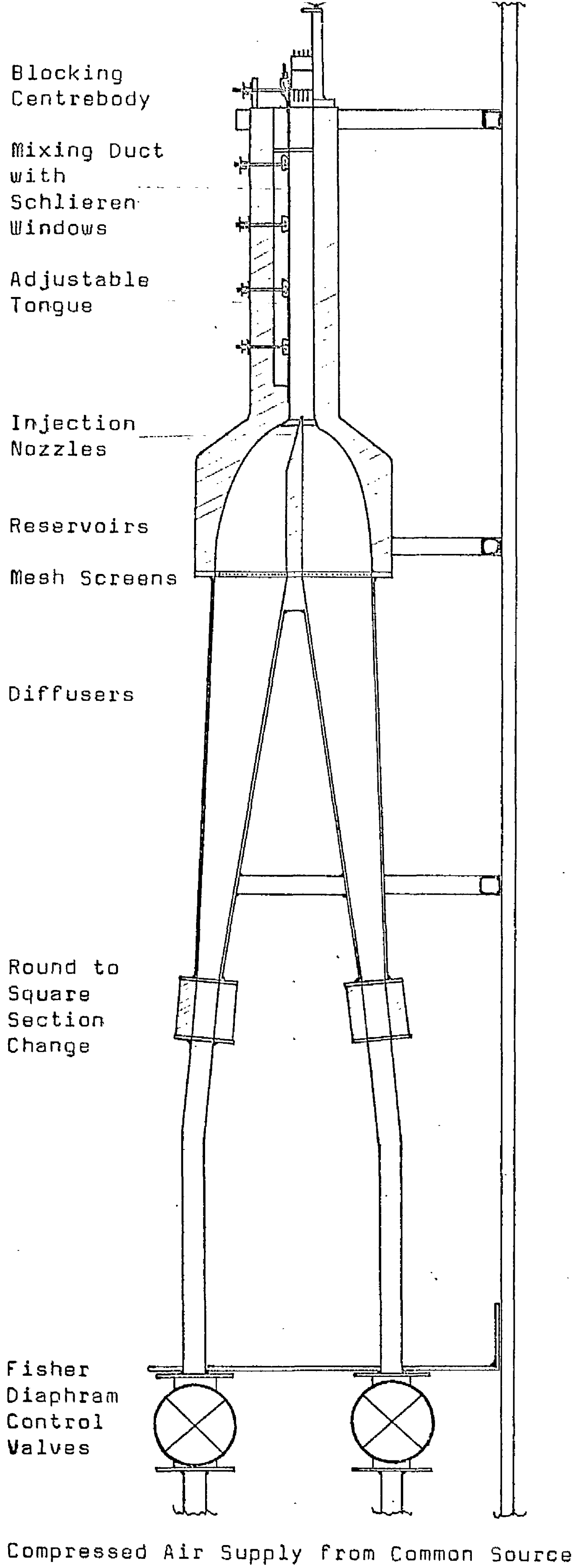


Figure 34.

A Diagram showing the layout of the Experimental Ejector.

The compressed air comes from the storage tanks through a two inch line, it passes through a manual stop valve and then is split up into two one and a half inch lines which go directly to the Fisher valves. A section of one and a half inch line then connects each valve to a diffuser. The upstream part of the diffuser consists of a moulded duct which changes the cross-section of the flow from round to $1\frac{1}{2}$ inches square. The second half of the diffuser is a welded duct ending with a cross-section one by five inches. Each flow then passes through a wire mesh screen into its respective reservoir. From these reservoirs the injection nozzles lead into the mixing duct. At the end of the mixing duct the flow exhausts into the atmosphere.

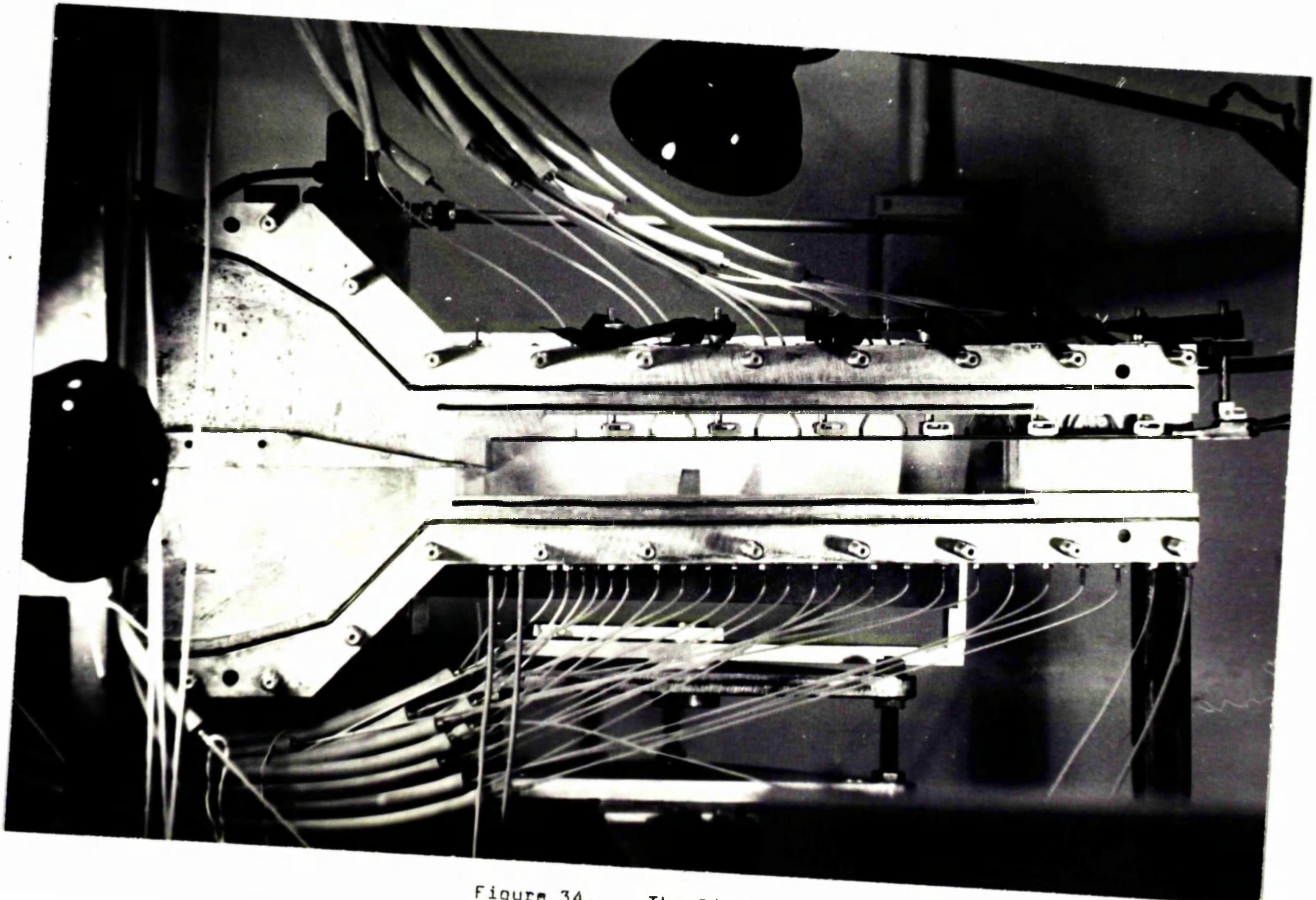
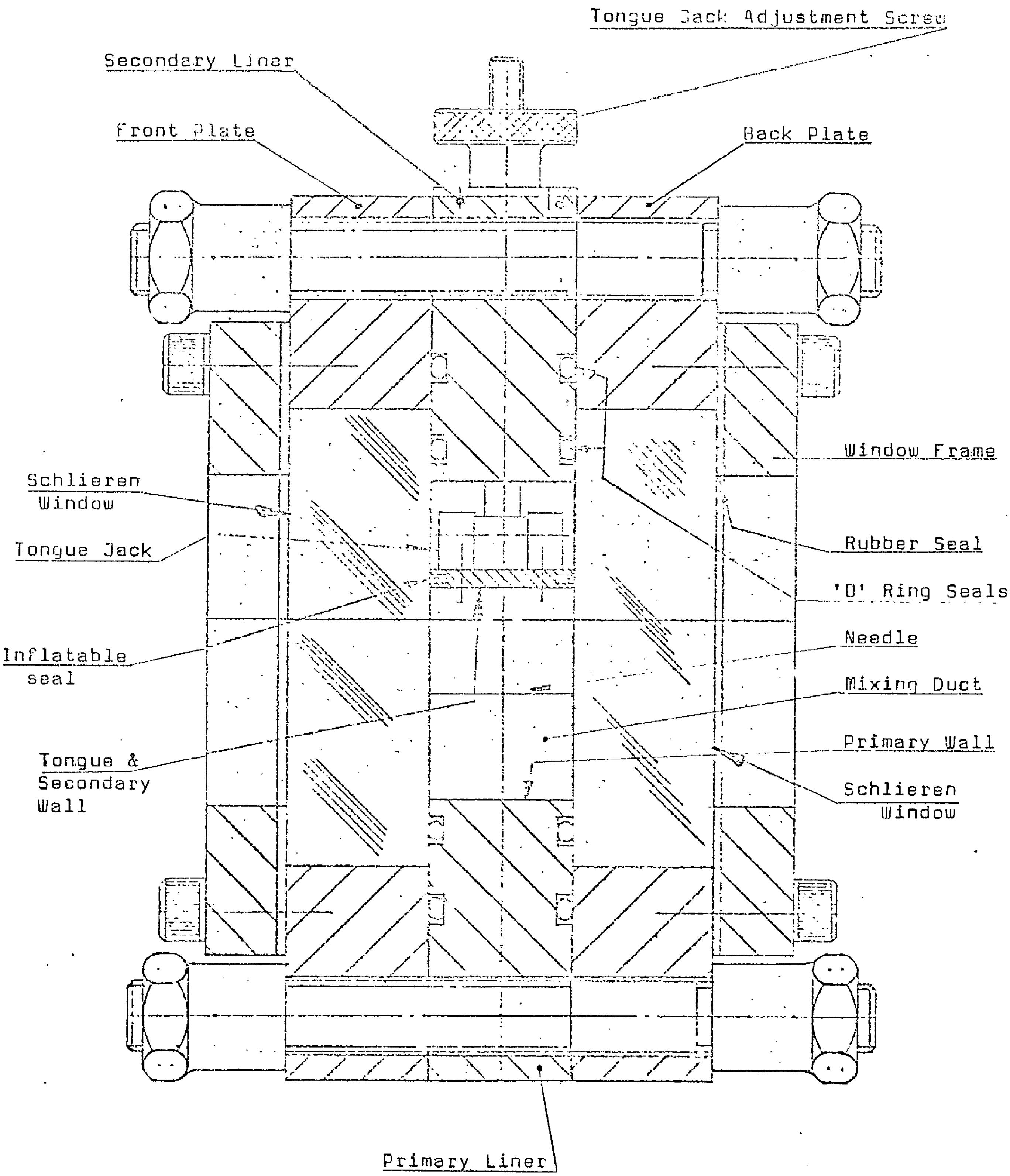


Figure 34. The Ejector.

Cross-section through the Mixing Duct.



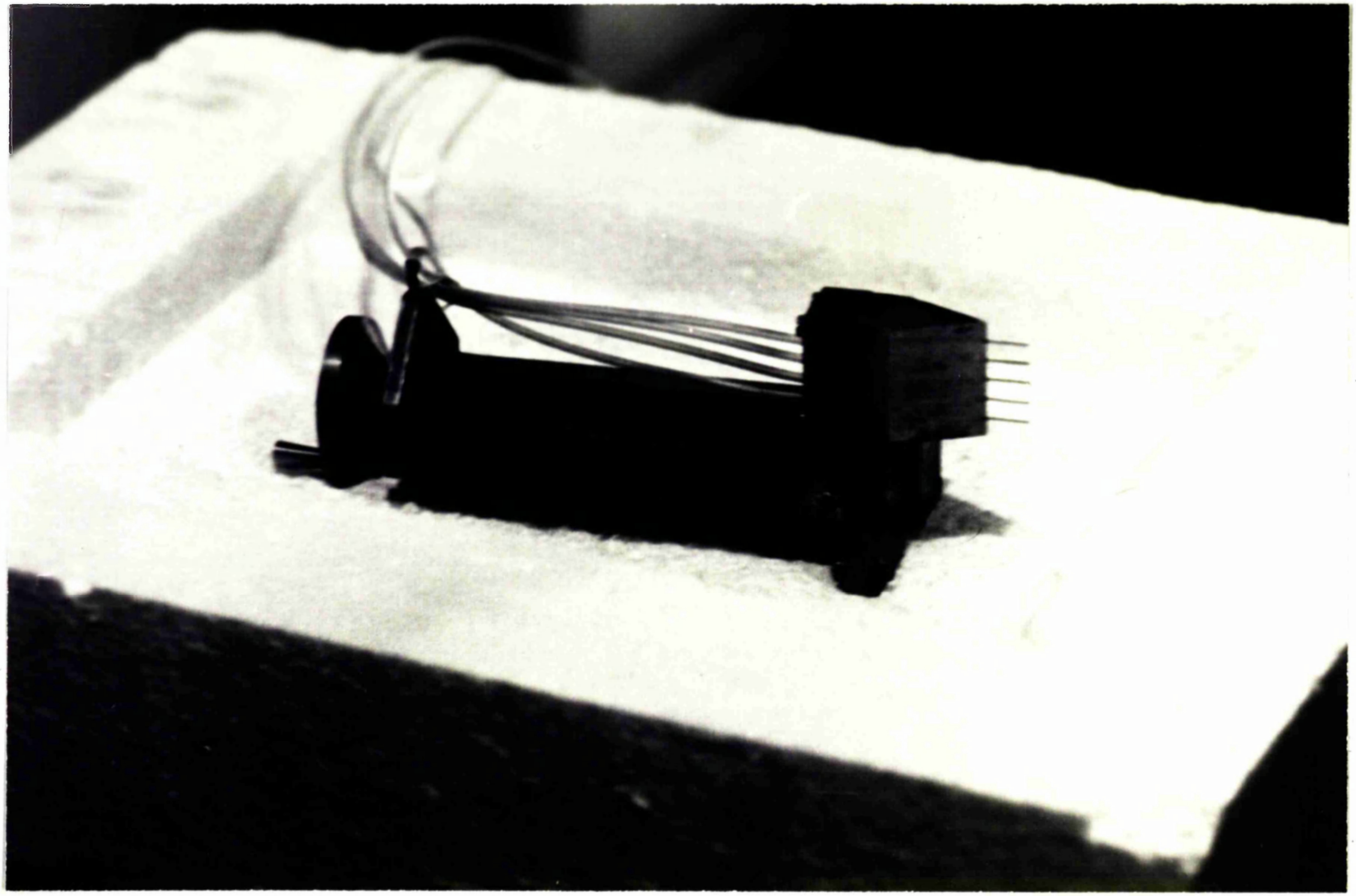


Figure 36. The Blocker.

Figure 37. The Manometers.

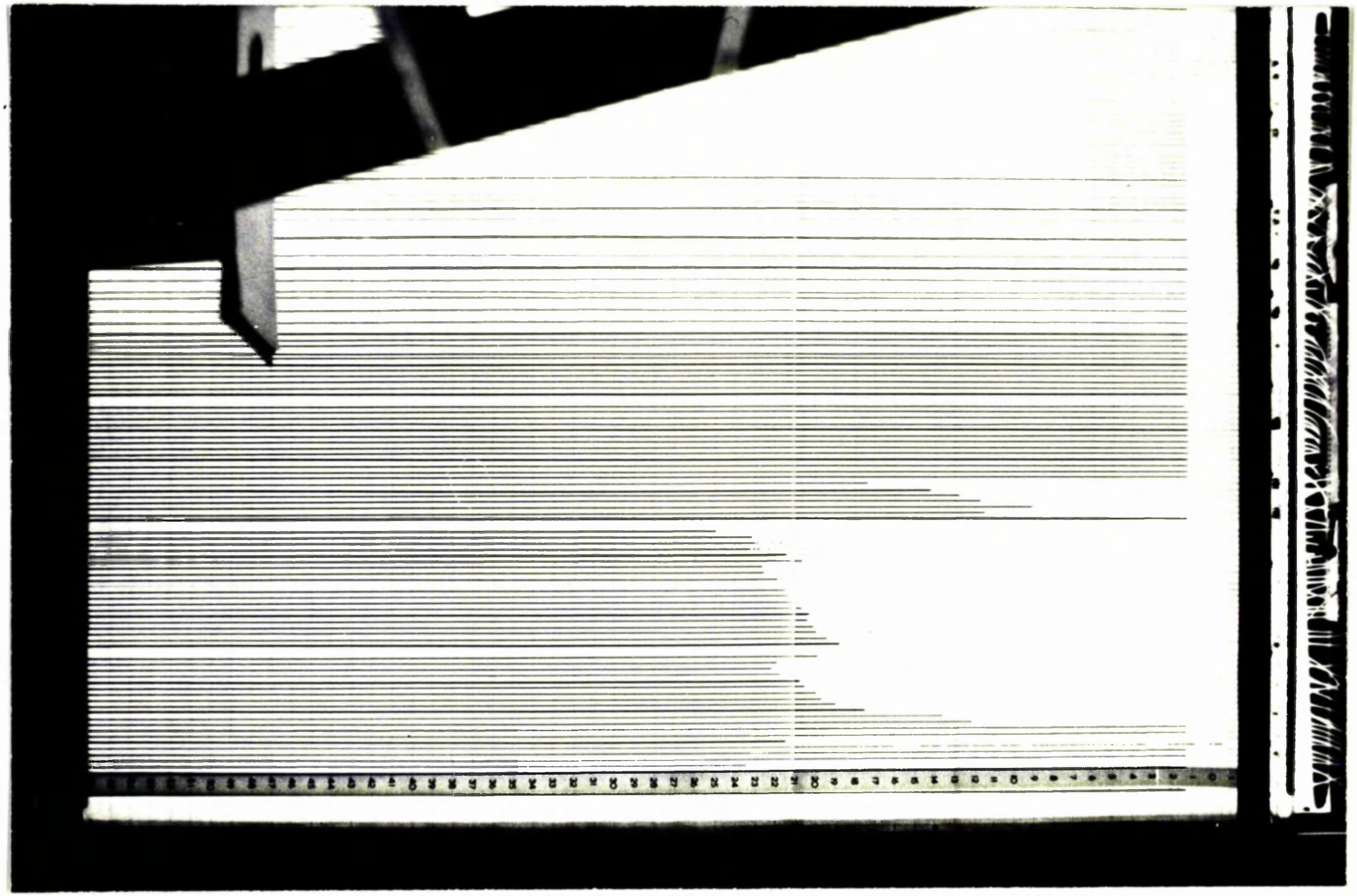
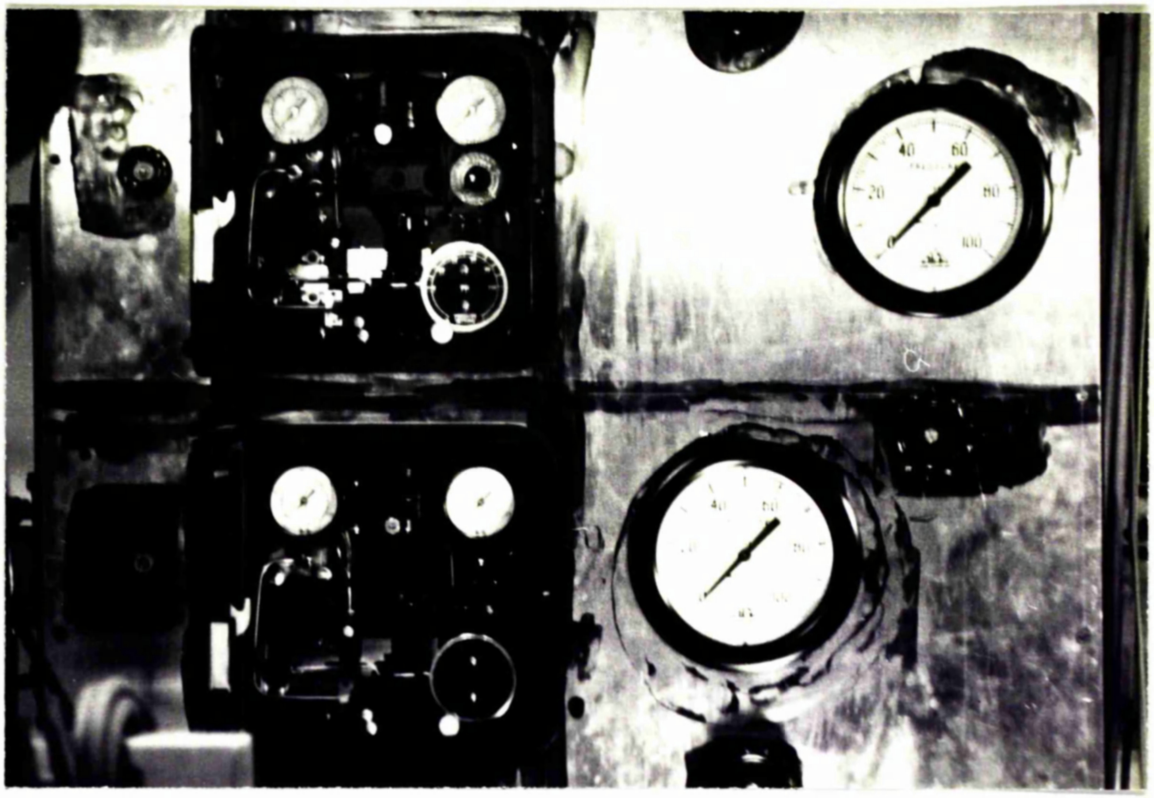


Figure 38. Manometer Recording.

Figure 39.



The Control Panel.

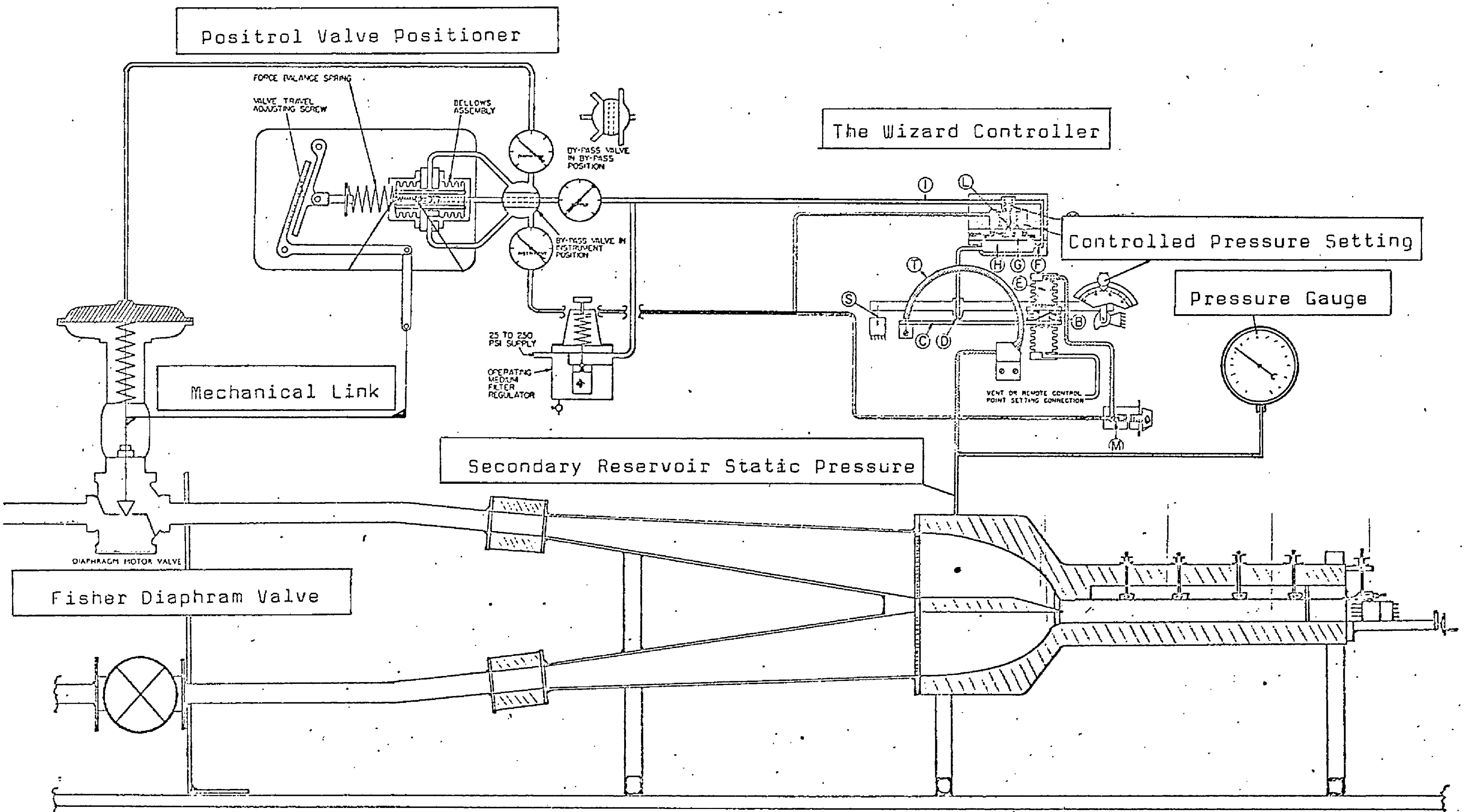
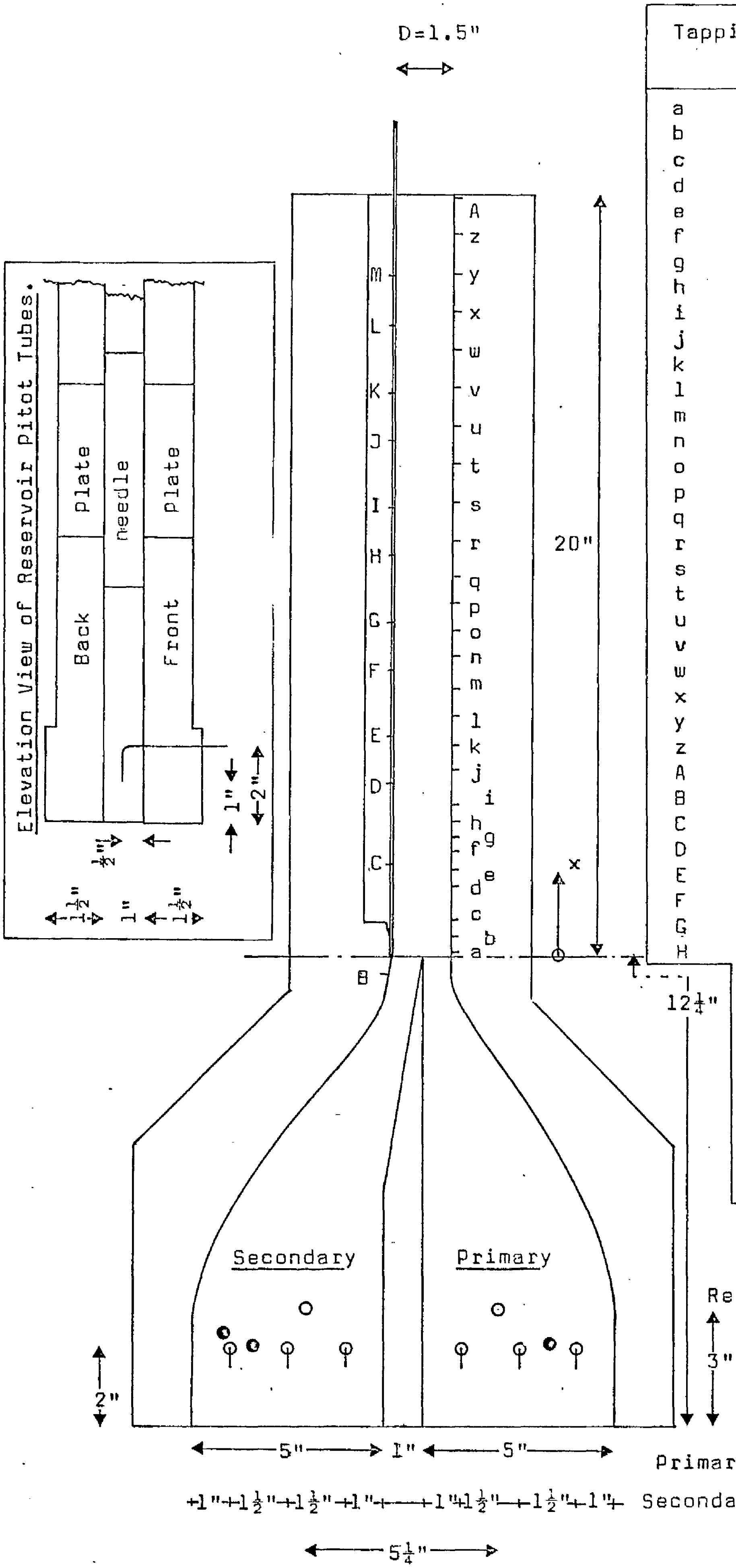


Figure 40. The Control System. A diagram of the system controlling one stream.

Figure 41.

Diagram Showing Positions of Pressure Tappings and Pitot Tubes.



Tapping.	Distance $\frac{x}{D}$ from injection plane.
a	0.0833
b	0.3750
c	0.6667
d	1.2500
e	1.5417
f	1.8333
g	2.0833
h	2.3750
i	2.6667
j	3.2500
k	3.7083
l	4.2083
m	4.6667
n	5.2500
o	5.7083
p	6.2083
q	6.6667
r	7.2917
s	7.9583
t	8.6250
u	9.2917
v	9.9583
w	10.6250
x	11.2917
y	11.9583
z	12.6250
A	13.2917
B	
C	1.6042
D	3.0208
E	3.8542
F	5.0208
G	5.8542
H	7.0208

Tapping	x/D
I	7.8542
J	9.0208
k	9.8542
L	11.0208
M	11.8542

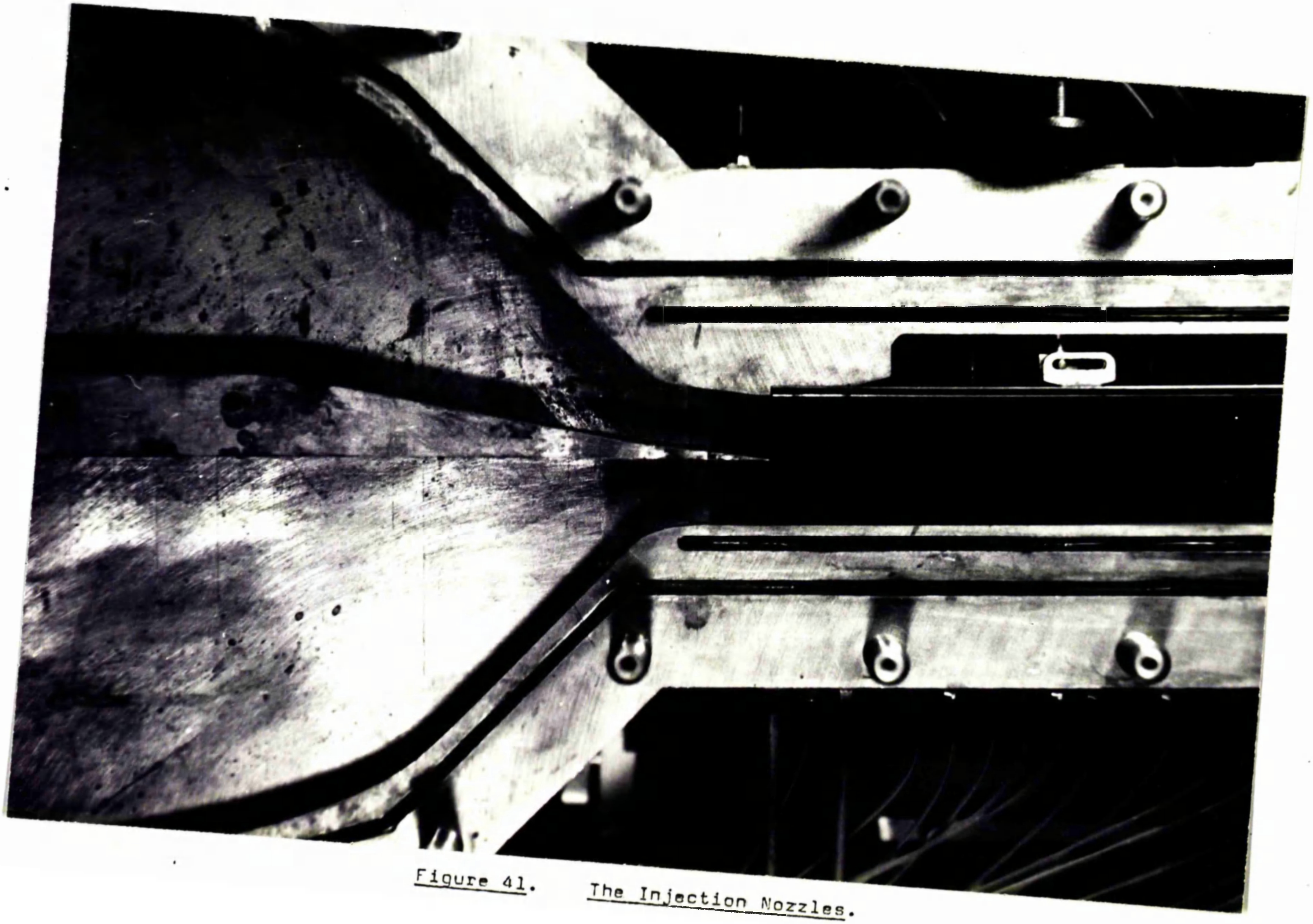


Figure 41. The Injection Nozzles.

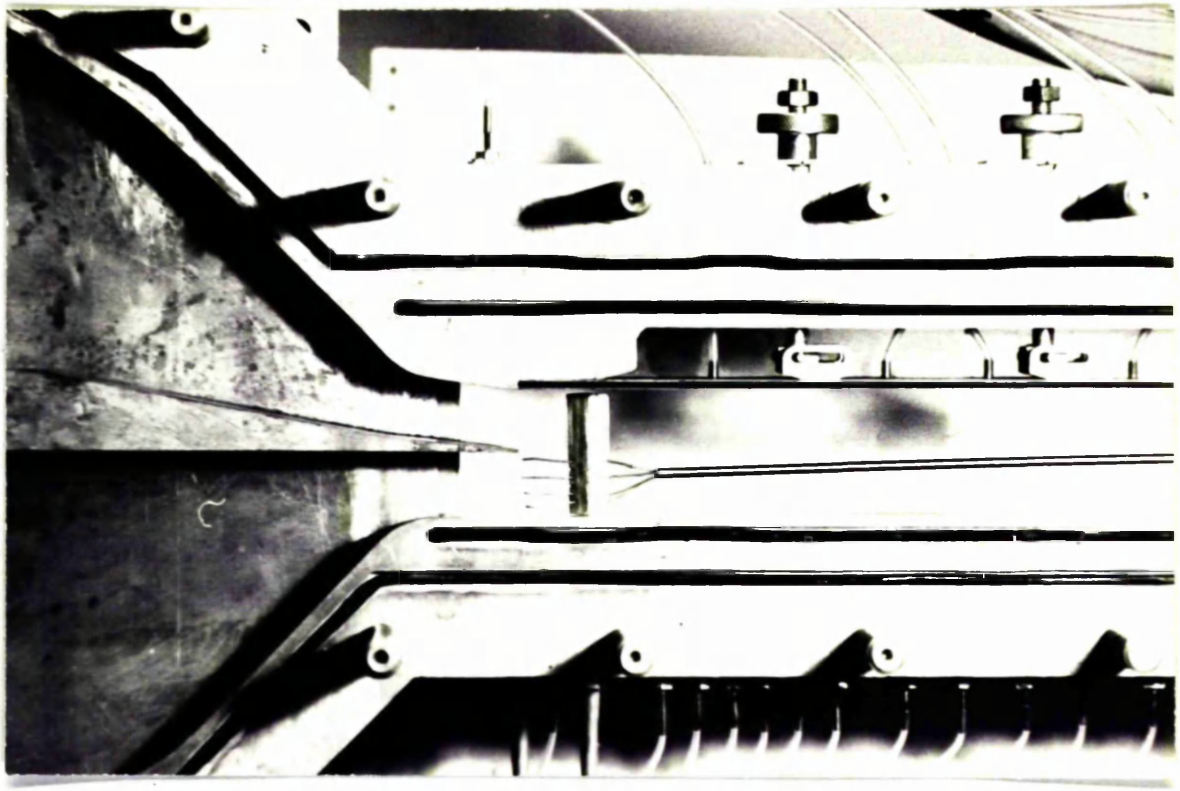
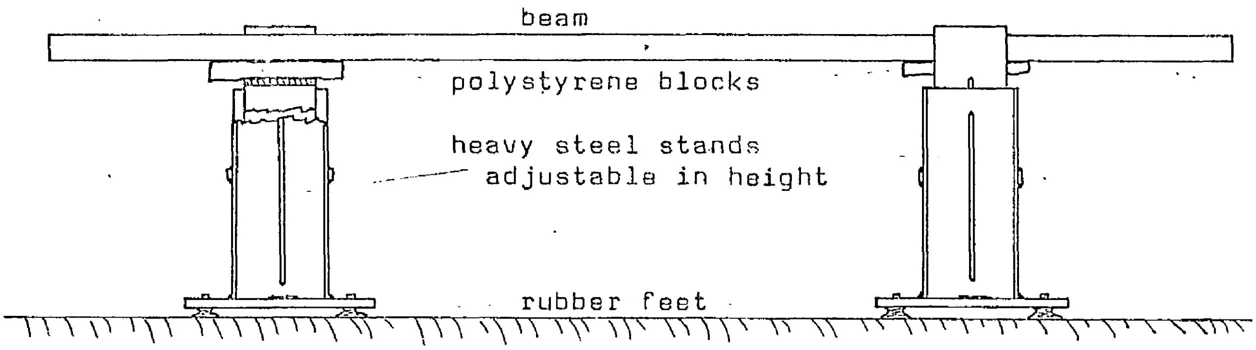


Figure 42. Pitot rake mounted in the injection plane.

Figure 43. Thermocouple mounting.

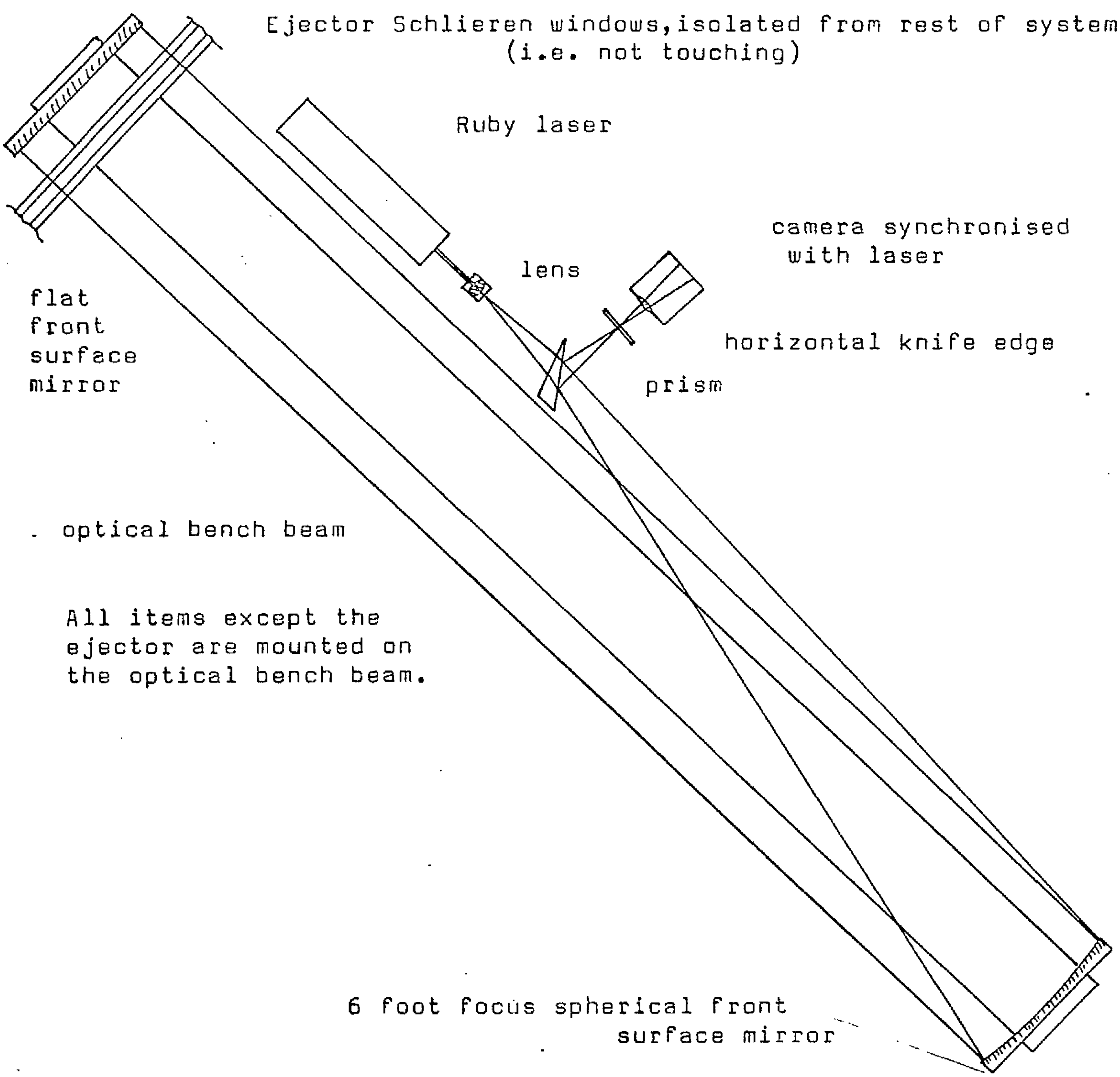
The Optical Bench Assembly



The optical bench consisted of a sixteen foot box beam of 4 by 8 inch cross-section mounted on blocks of expanded polystyrene which were in turn mounted on heavy stands. These stands were adjustable in height and were themselves mounted on rubber feet. Entire optical systems can be mounted on the beam free from vibrations.

Figure 45.

The Double Pass Schlieren System.



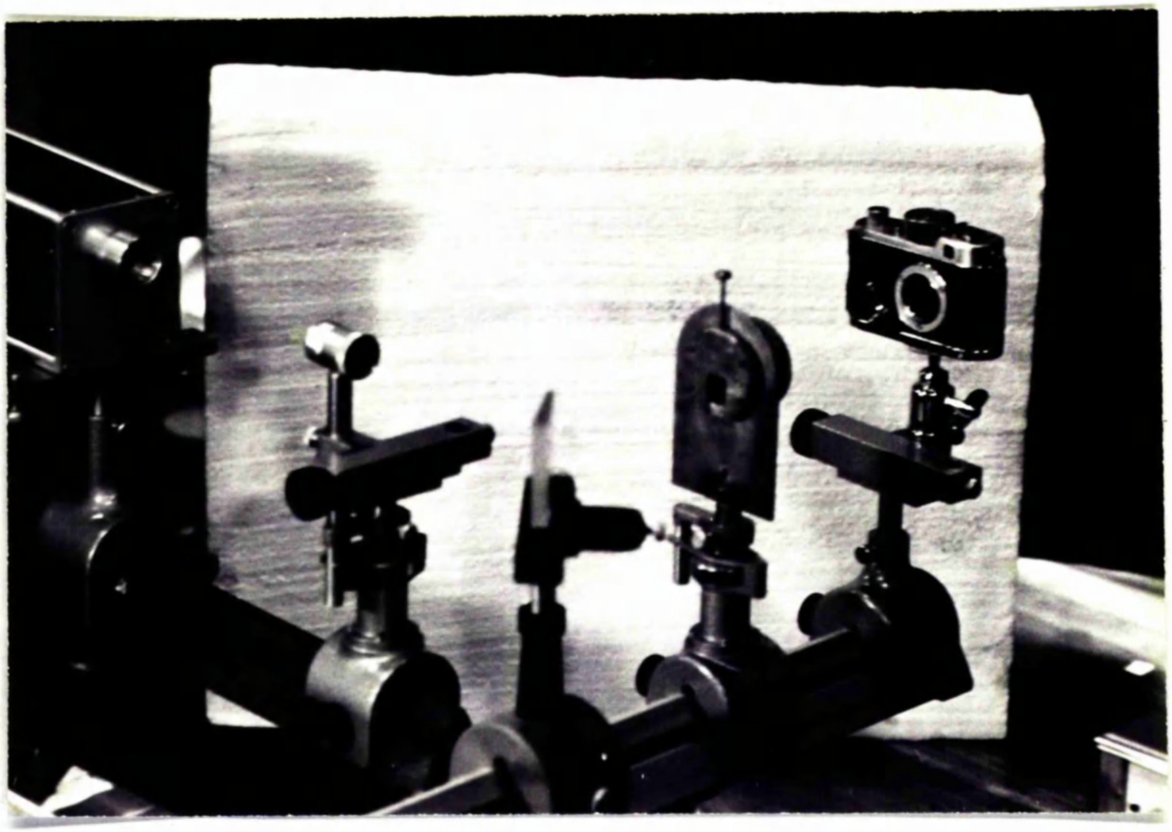


Figure 45. The Schlieren System.

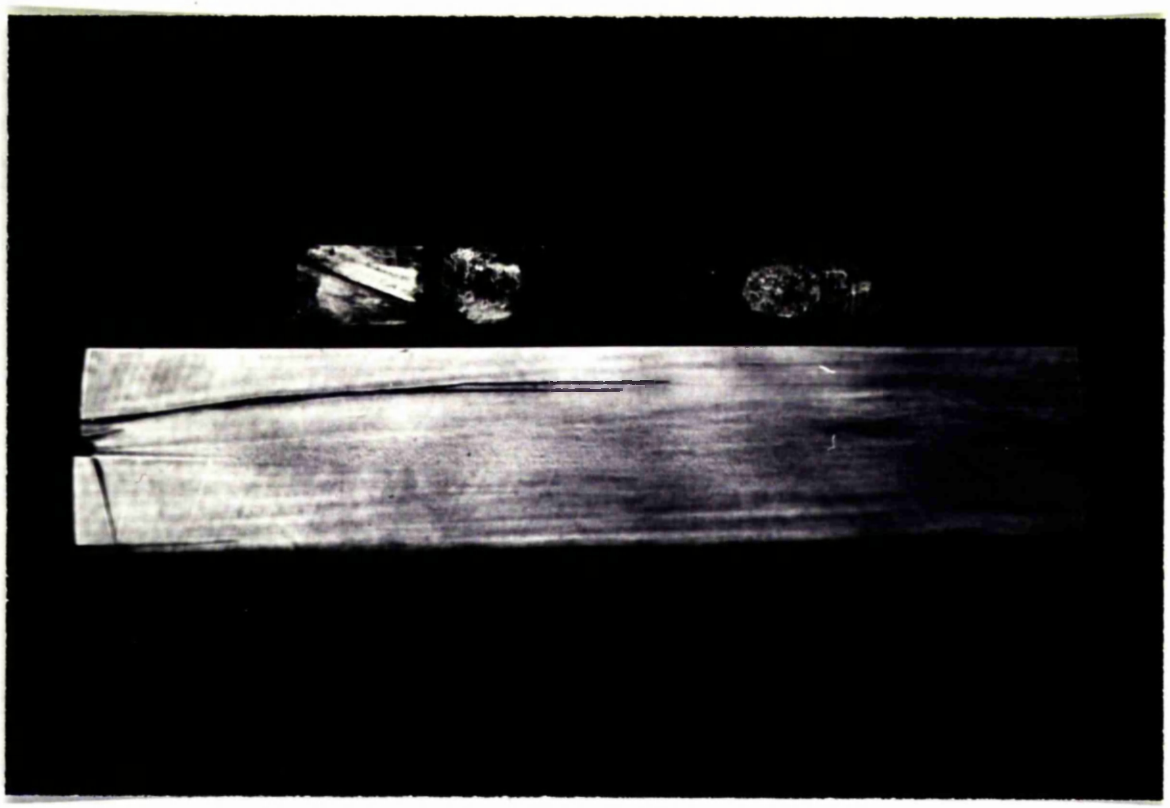


Figure 46. A Specimen Schlieren Photograph.

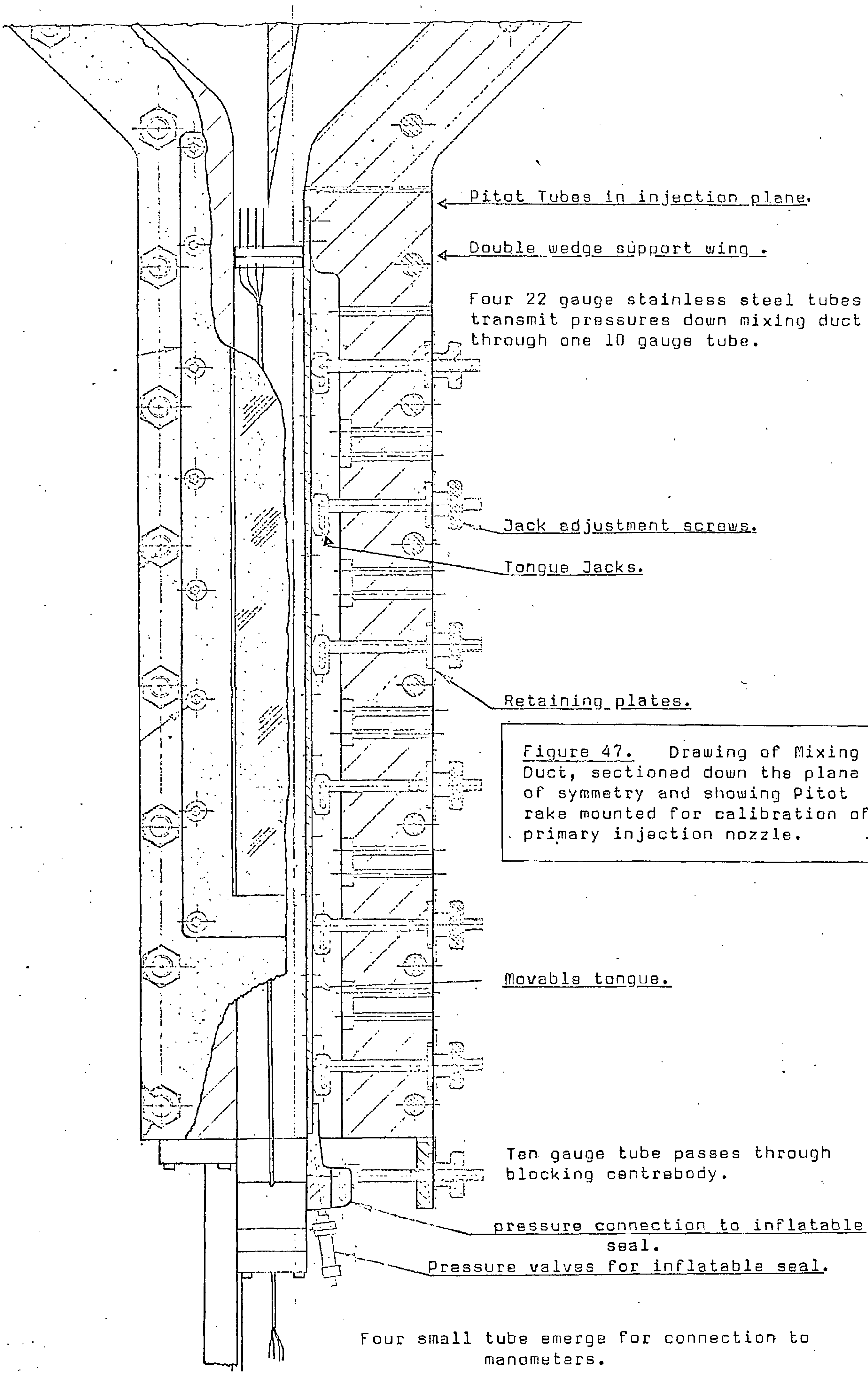
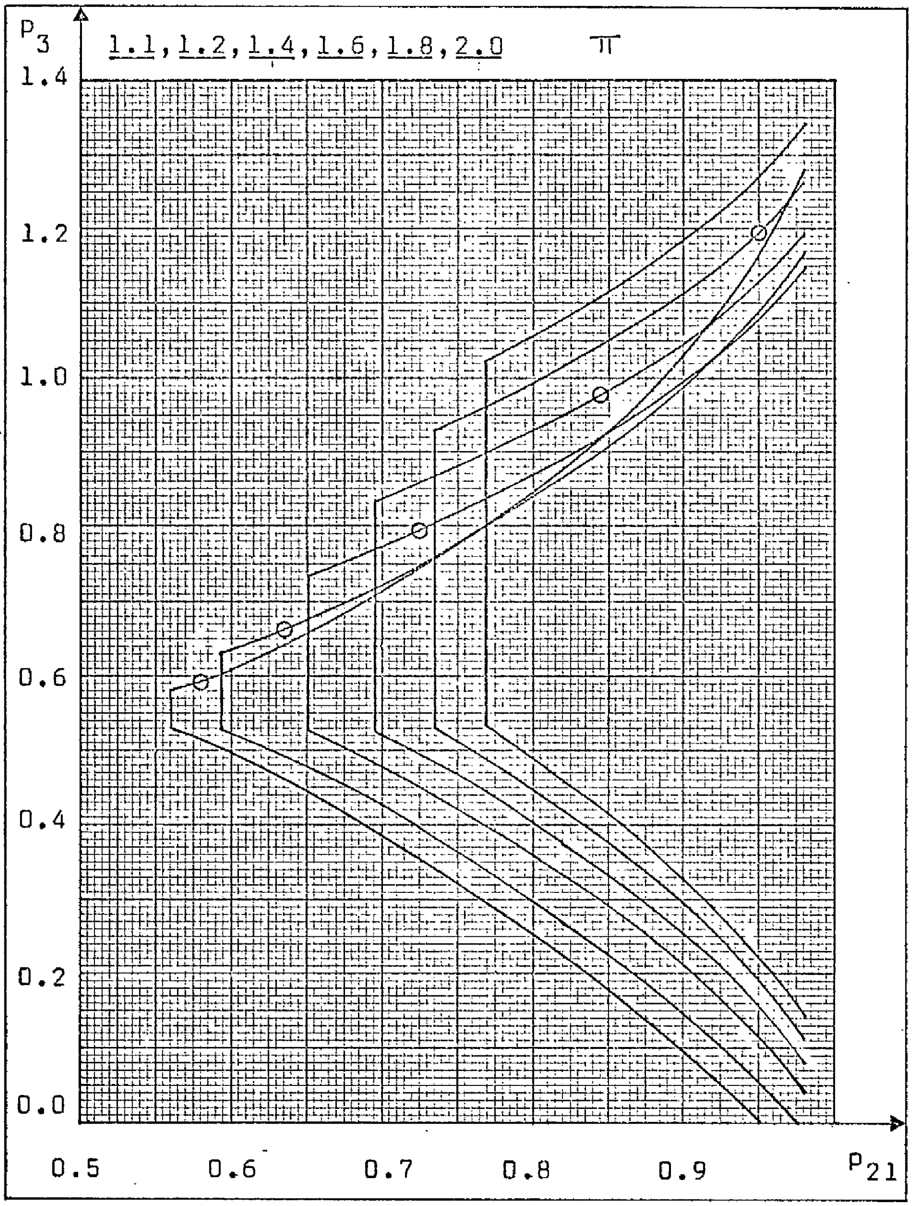


Figure 47. Drawing of Mixing Duct, sectioned down the plane of symmetry and showing Pitot rake mounted for calibration of primary injection nozzle.

The Full Back Pressure Dependent Solutions according to the Polytropic Two Stream Theory for an Ejector with the configuration:-

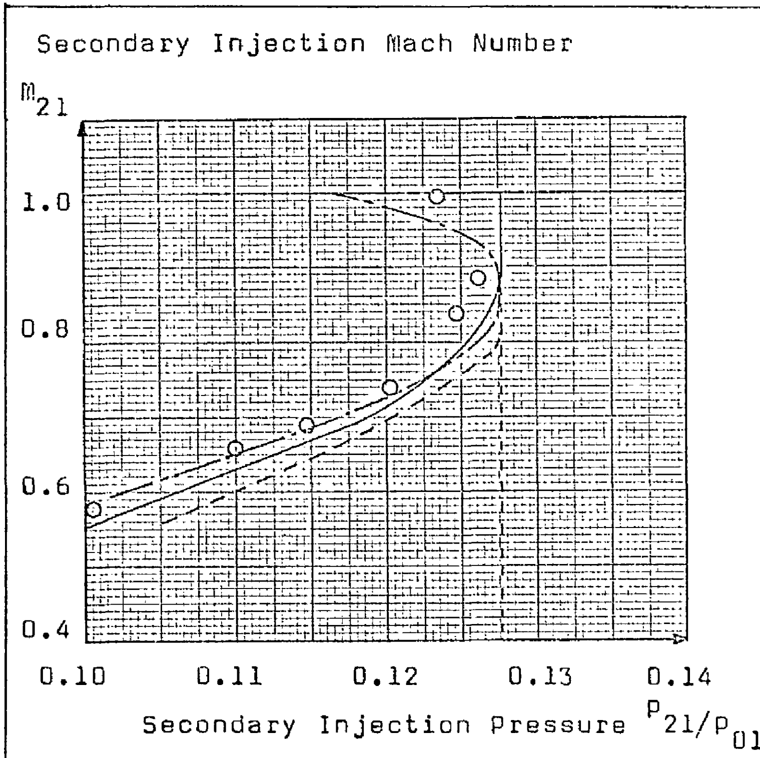
Sonic injection, z = 0.5, h = 1.0, within the range 1.0 to 2.0 .

The Exhaust Pressure versus the Secondary Injection Pressure for a series of Total Pressure Ratios.



Maximum Flow Solutions for a Supersonic Ejector

In the following figures results from the two stream choking theory are compared with those from the two dimensional theory of Messrs. Chow and Addy and also with those from Fabris one dimensional theory. The two figures represent two different ejector systems, both with Mach 2 injection nozzles, but with different nozzle area ratios. The experimental points shown are due to Messrs. Chow and Addi.



Ejector Configuration

$z = 0.1937$

$h = 1.0$

$M_{11} = 2.0$

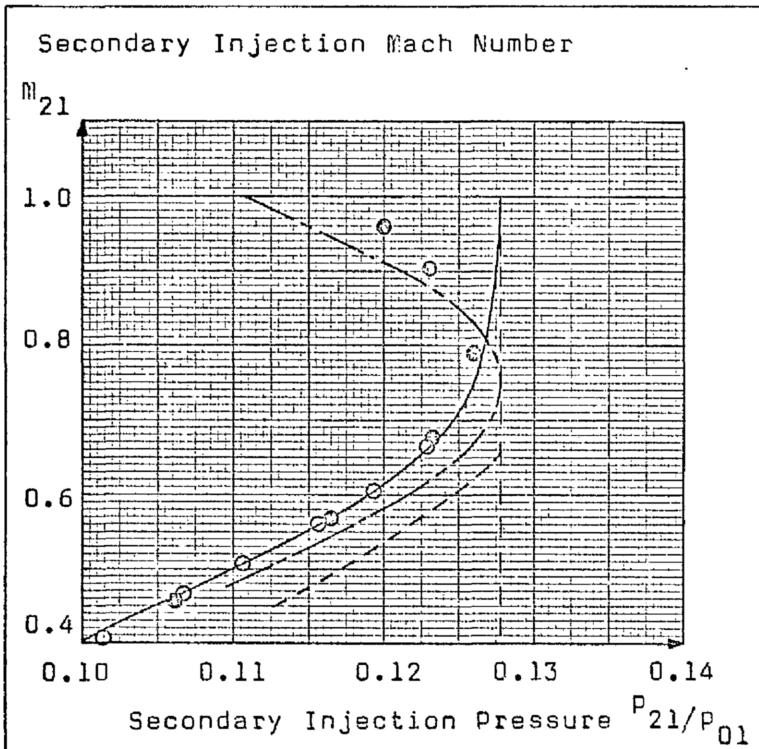
in range 3.5 to 8

T.S.C. —·—·—

Chow & Addy ———

Fabri – – – – – –

Experimental ʘ & O



Ejector Configuration

$z = 0.3025$

$h = 1.0$

$M_{11} = 2.0$

in range 3.5 to 8

T.S.C. —·—·—

Chow & Addy ———

Fabri – – – – – –

Experimental ʘ & O

Figure 50.

A Comparison of the full back pressure dependent solution with Experimental results for an ejector with the configuration:-

Sonic injection, z = 0.5, h = 1.0,   = 1.5.

Exhaust pressure v. Secondary injection pressure.

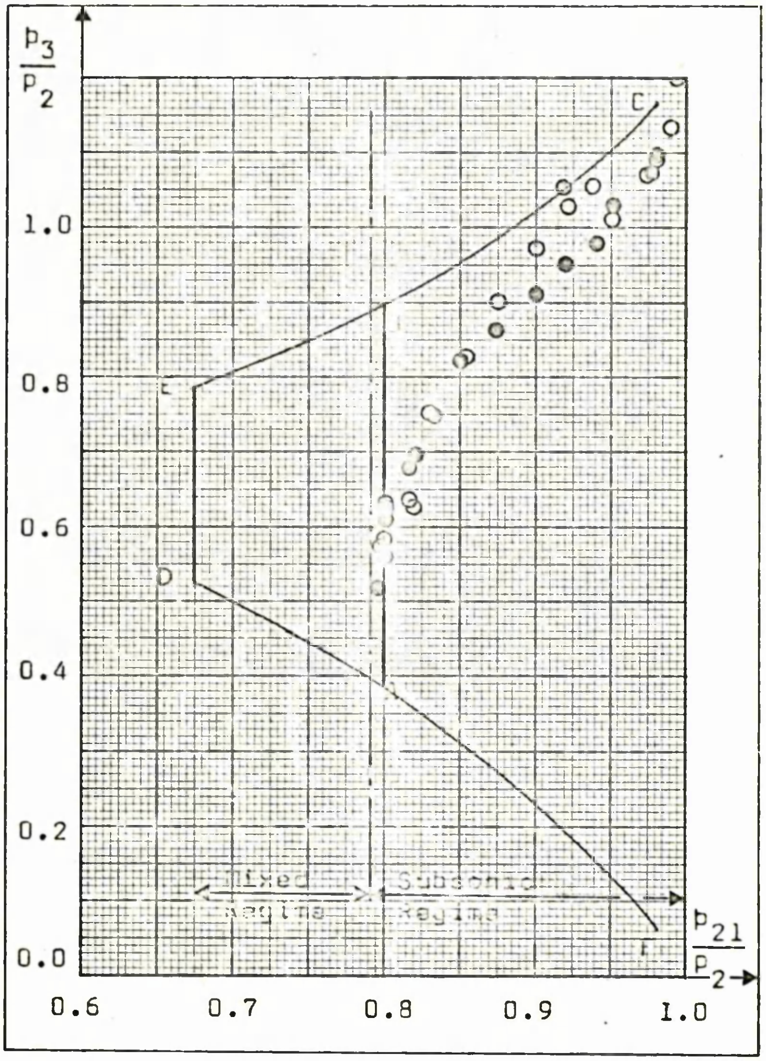
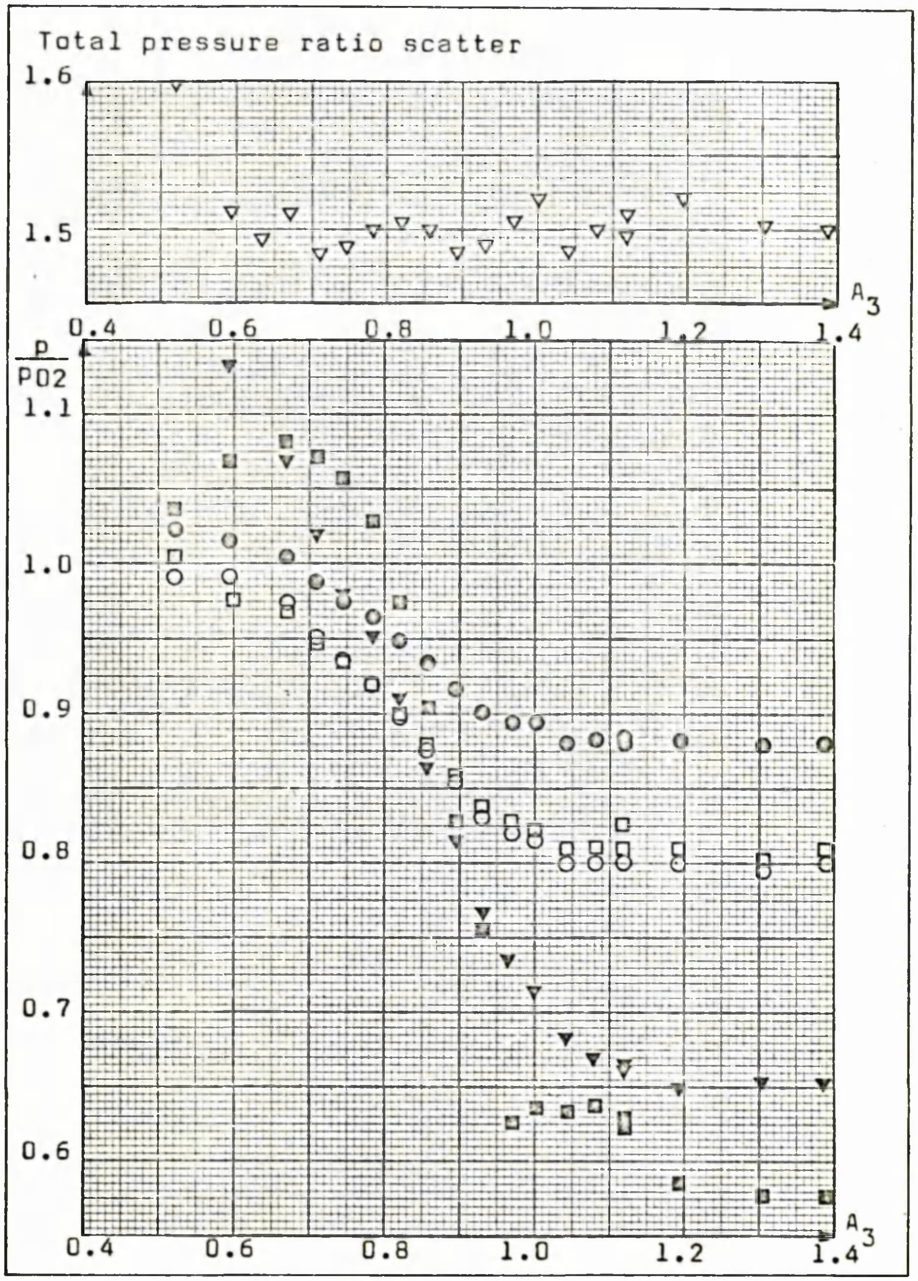


Figure 51.



The Variation of Static Pressure at various points in the ejector as the end of the mixing tube is gradually blocked.

Ejector Configuration:- z = 0.5, h = 1.0, = 1.5.

Experimental points:-

● represents p_{11}

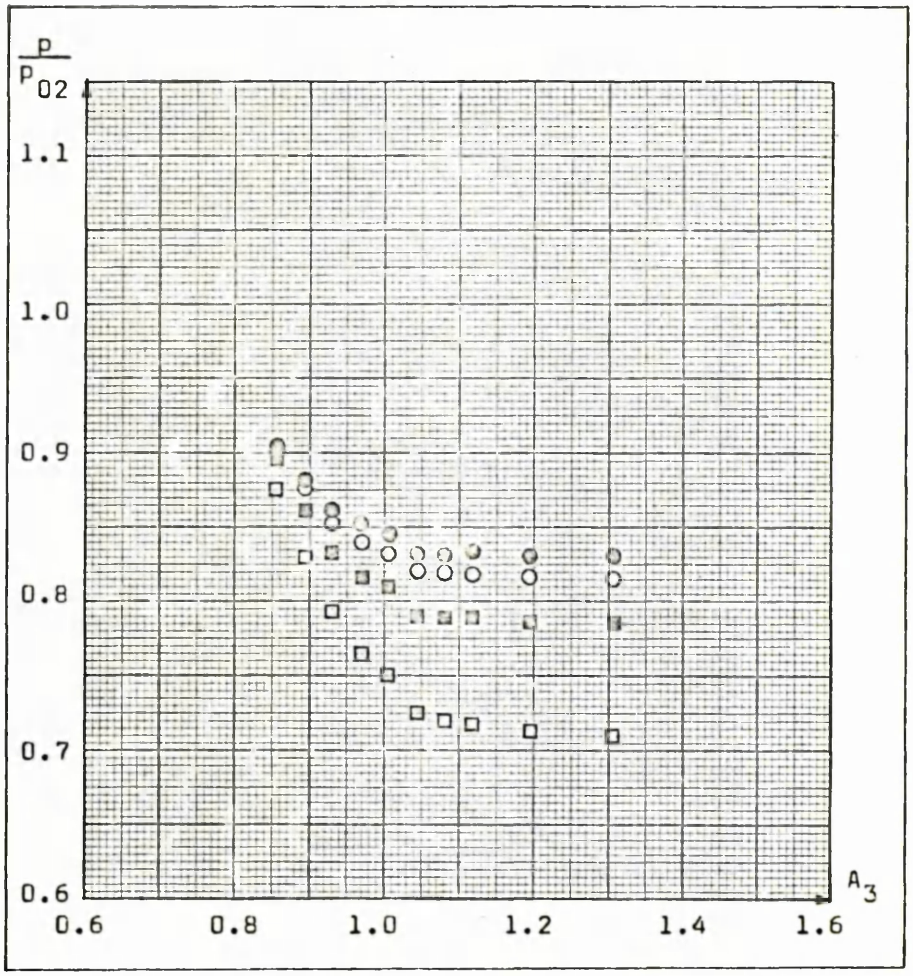
○ represents p_{21}

□ represents the static pressure at x/D = 2.08

▼ " " " " " " = 12.6

■ " " " " " " = 13.3

Figure 52.



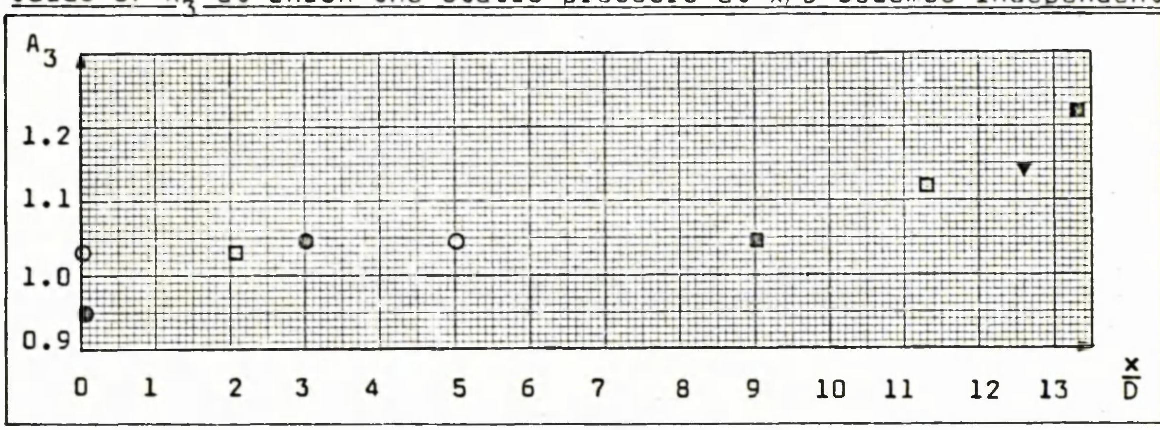
Variation of static pressure v. Exhaust area A_3

Ejector Configuration: z = 0.5, h = 1.0, π = 1.5

Experimental points:

◐	represents	the	static	pressure	at	x/D	=	3.0
○	"	"	"	"	"	"	=	5.0
■	"	"	"	"	"	"	=	9.0
□	"	"	"	"	"	"	=	11.3

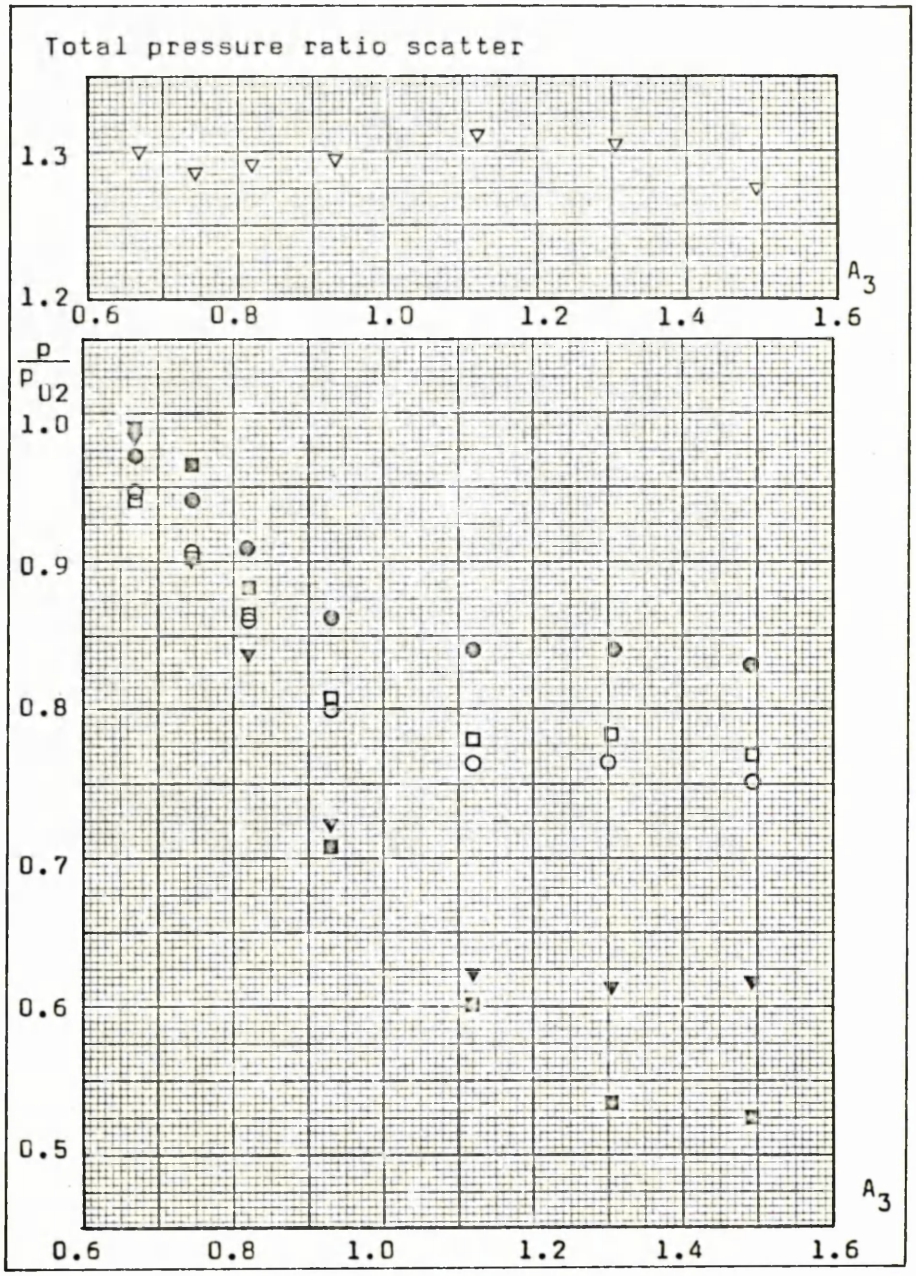
Value of A_3 at which the static pressure at x/D becomes independent of A_3



Flow is entirely subsonic over cross section x/D = 9.0

Flow is supersonic over $\frac{1}{3}$ of cross section x/D = 11.0

figure 53.



Variation of Static Pressure at various points in the ejector as the end of the mixing tube is gradually blocked.

Ejector Configuration: $z = 0.5$, $h = 1.0$, $= 1.3$

Experimental points:

● represents p_{11}

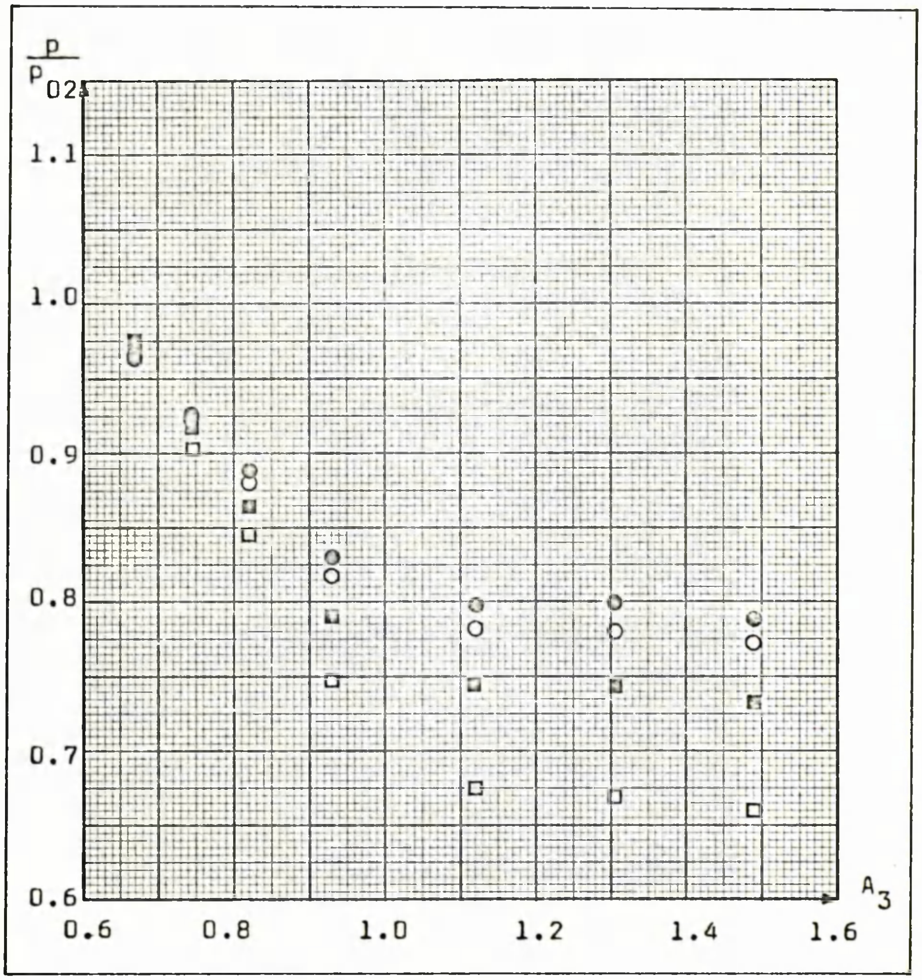
○ represents p_{21}

□ represents the static pressure at $x/D = 2.08$

▼ " " " " " " $= 12.6$

■ " " " " " " $= 13.3$

Figure 54.



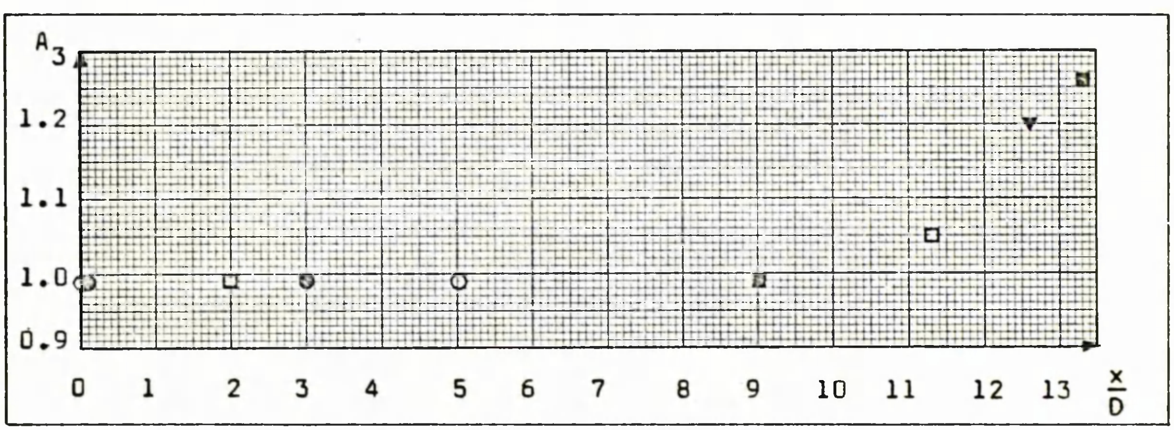
Variation of static pressure v. Exhaust area A_3

Ejector configuration: $z = 0.5$, $h = 1.0$, $\pi = 1.3$

Experimental points:

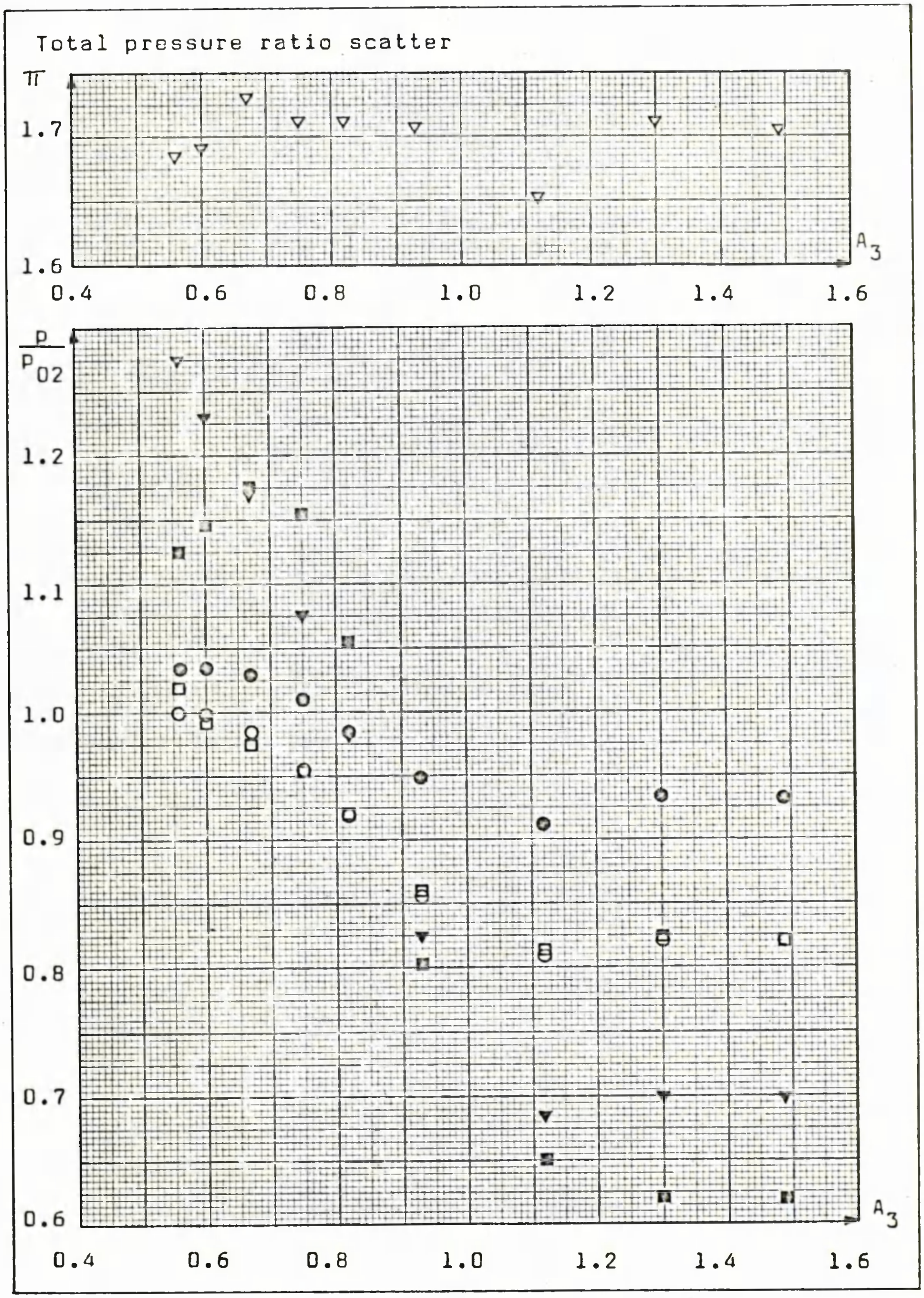
●	represents the static pressure at x/D	= 3.0
○	" " " " " "	= 5.0
■	" " " " " "	= 9.0
□	" " " " " "	= 11.3

Value of A_3 at which the static pressure at $\frac{x}{D}$ becomes independent of A_3



Flow is entirely subsonic over cross section x/D = 9.0
Flow is entirely subsonic over cross section x/D = 11.0

Figure 55.



Variation of Static Pressure at various points in the ejector as the end of the mixing tube is gradually blocked.

Ejector configuration; $z = 0.5$, $h = 1.0$, $\pi = 1.7$.

Experimental points:

● represents p_{11}

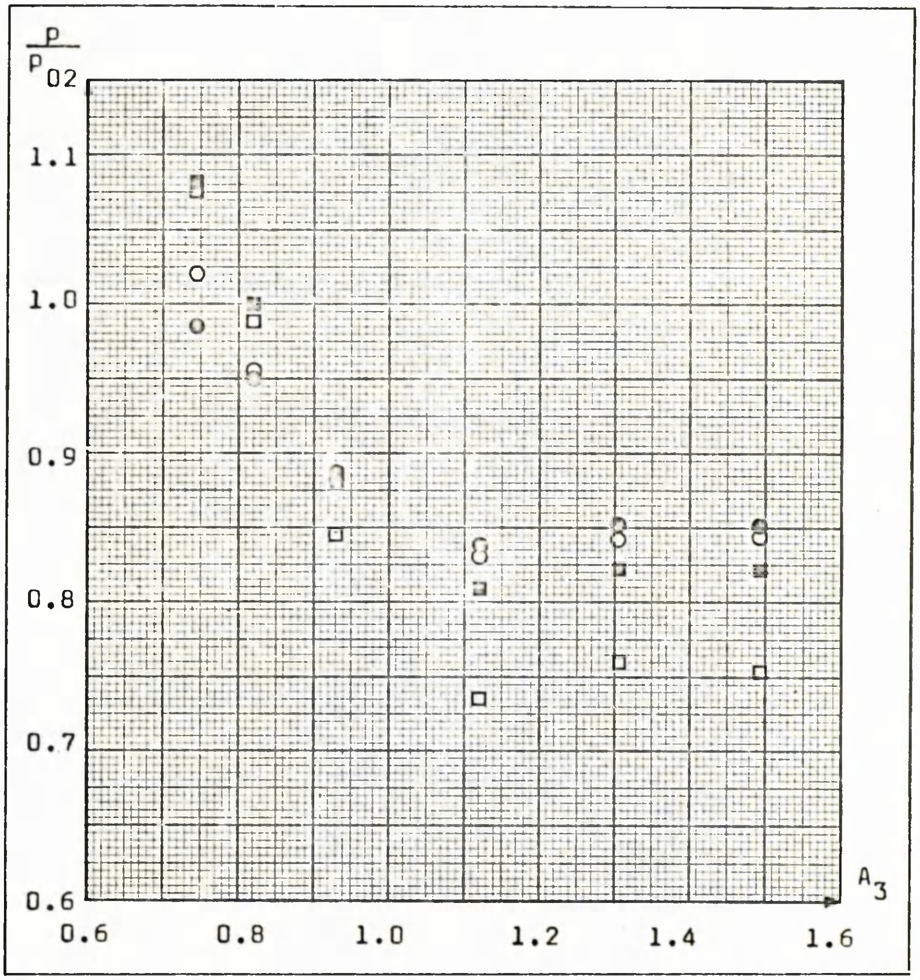
○ represents p_{21}

□ represents the static pressure at $\frac{x}{D} = 2.08$

▼ " " " " " " $= 12.6$

■ " " " " " " $= 13.3$

Figure 56.

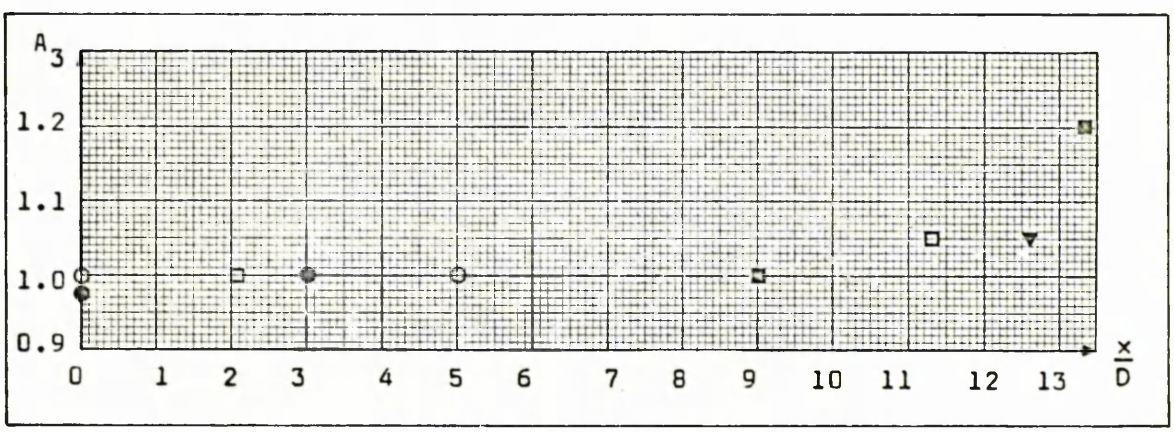


Variation of static pressure v. Exhaust area A_3

Ejector Configuration : $z = 0.5$, $h = 1.0$, $\pi = 1.7$

Experimental points:

represents the static pressure at $\frac{x}{D} = 3.0$
" " " " " " = 5.0
" " " " " " = 9.0
" " " " " " = 11.3



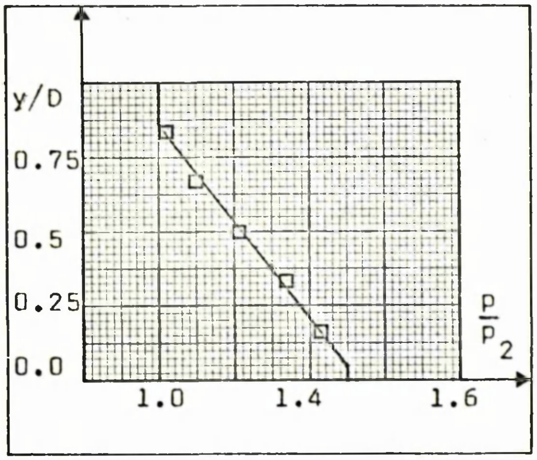
Value of A_3 at which the static pressure at $\frac{x}{D}$ becomes independent of A_3

Figure 57.

The Total Pressure Distribution Assumed for the Flow in Ejector Test 21.

The idealised distribution across the end of the mixing duct.

In the following graph a straight line has been drawn through the experimental points depicting the total pressure distribution across the end of the mixing duct. This line intersects with the two lines $P/P_2 = 1.0$ and $P/P_2 = 1.5$ at the points B and C. The idealised total pressure distribution is then taken to be that given by the line ABCD. The points B and C are also assumed to represent the boundaries of the mixing region at the duct exit.



The total pressure distribution assumed throughout the mixing duct.

Assuming that the mixing region spreads linearly with distance down the duct, its boundaries may be drawn as in the graph below. If it is also assumed that the total pressure varies linearly across the mixing region at all cross-sections, the position of the sonic line may then be computed, since the static pressure distribution is already known. The sonic line is shown in the form of a series of dots connected by a dotted line.

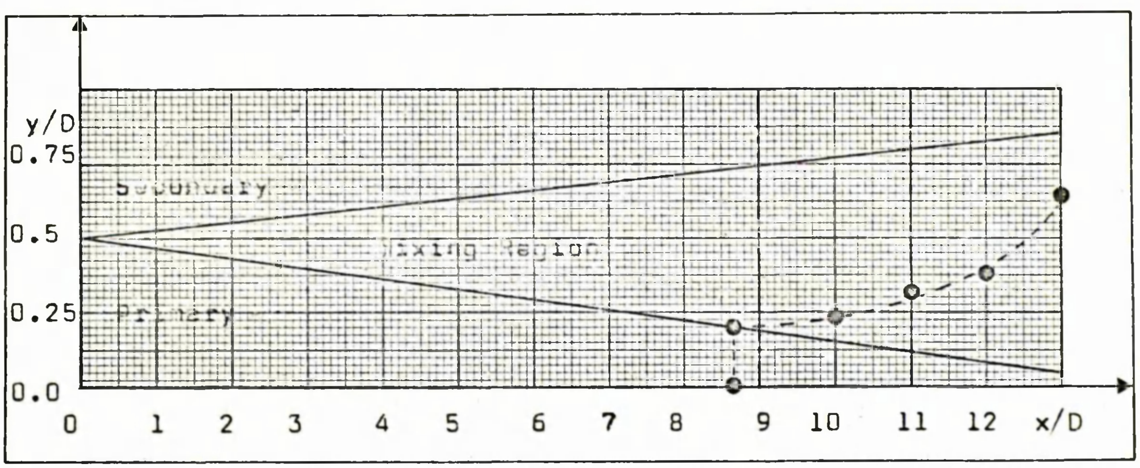
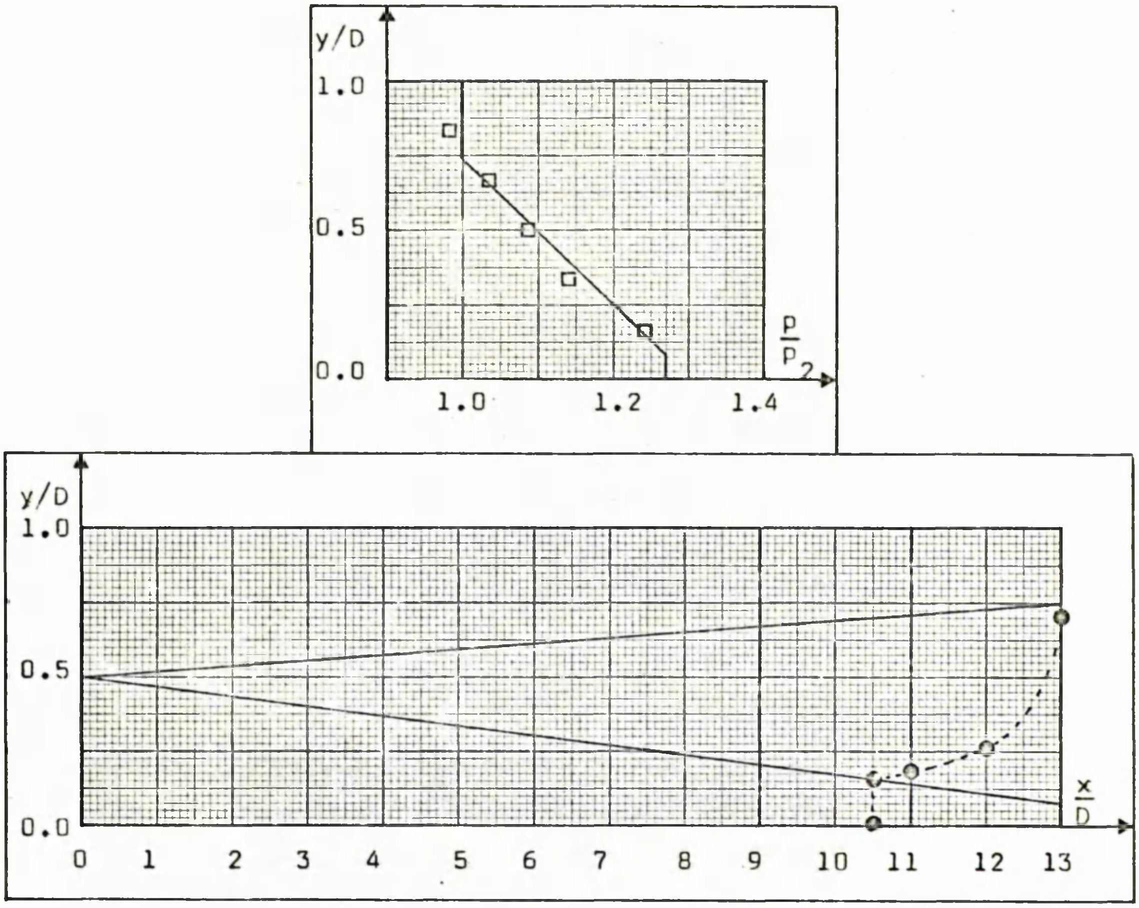


Figure 58.

The Total Pressure Distributions assumed for the flows in Ejector Tests 11 and 31.

Test 11. Total pressure ratio = 1.3 (nominal)



Test 31. Total pressure ratio = 1.7 (nominal)

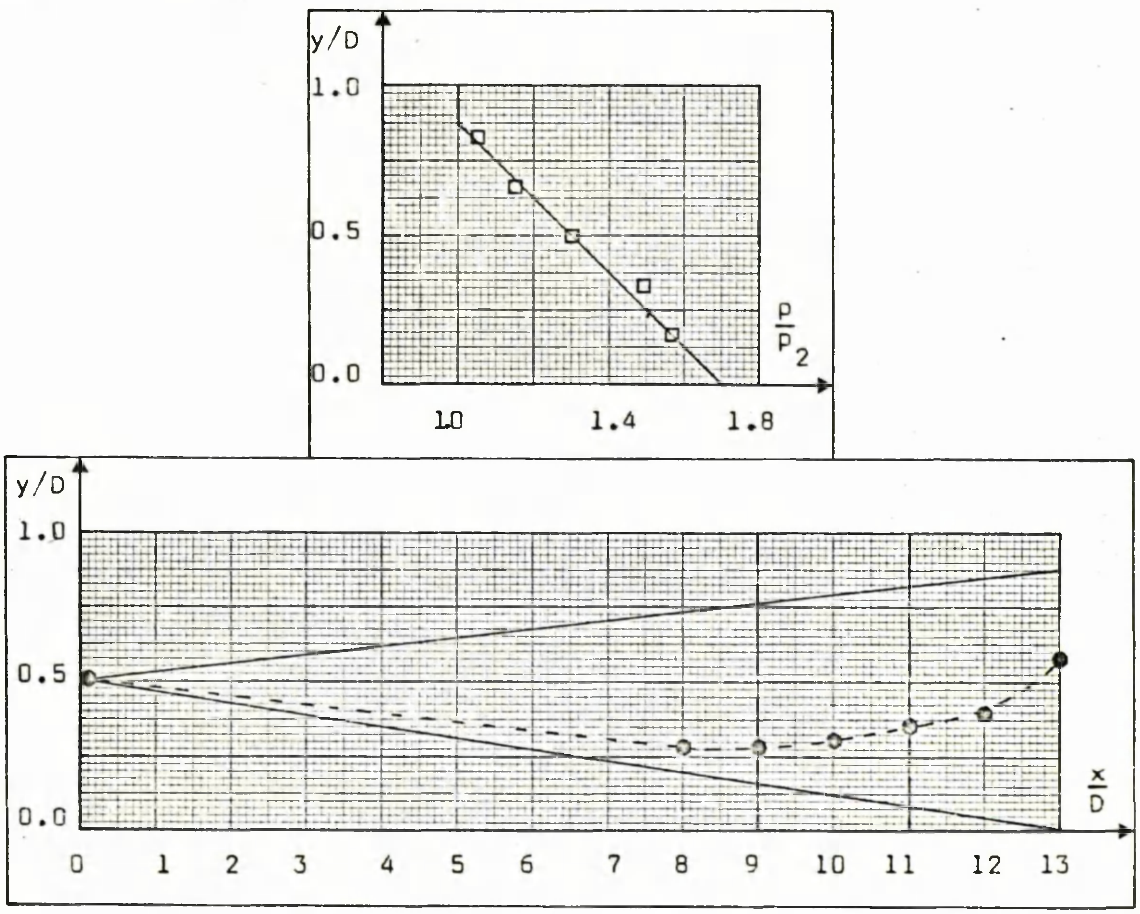
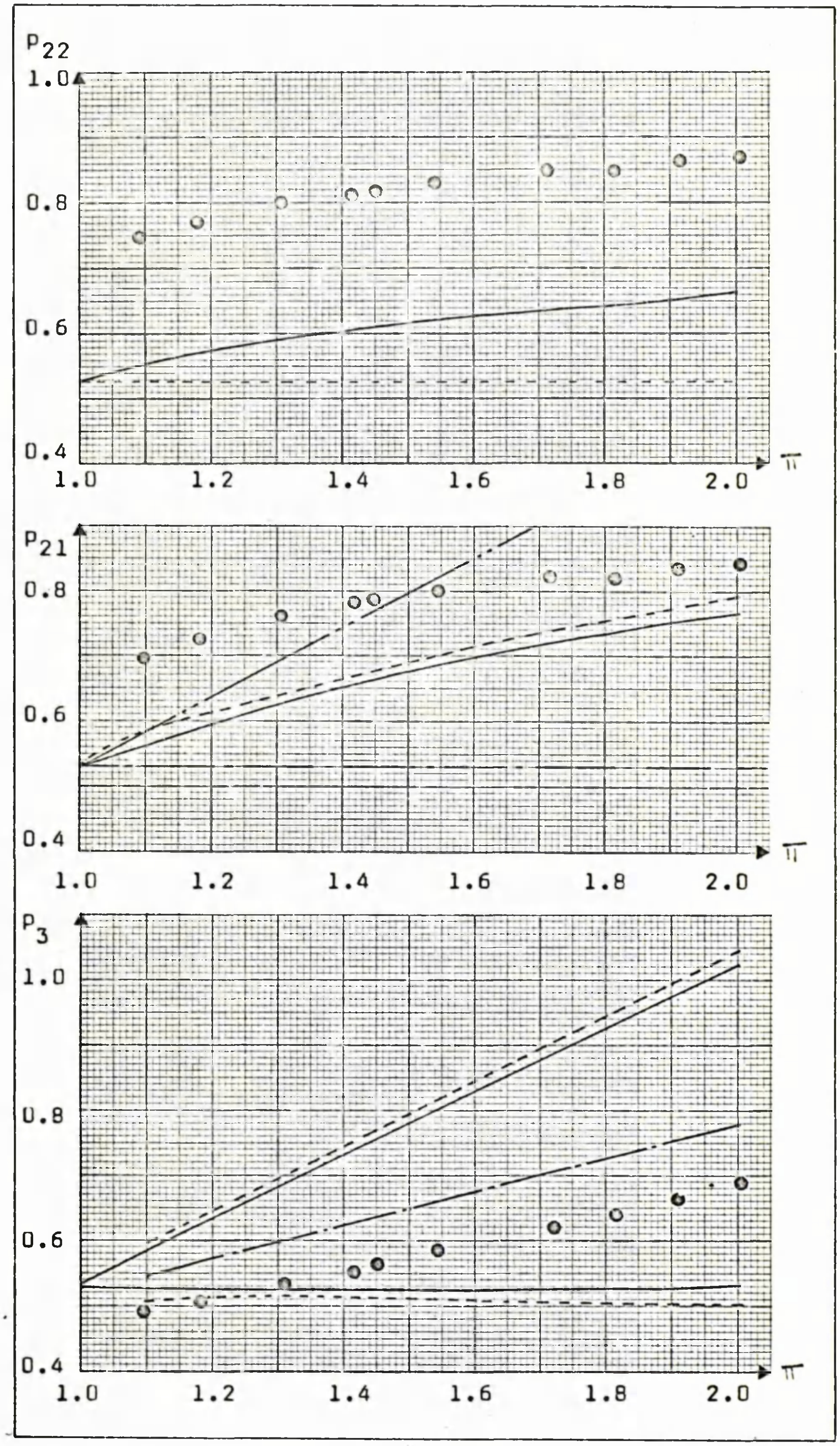


Figure 59.



Comparison of theory and experiment for fully choked flows.

Top:- Theoretical static pressure at end of primary expansion compared against static pressure at x/D = 1.6 on secondary wall.

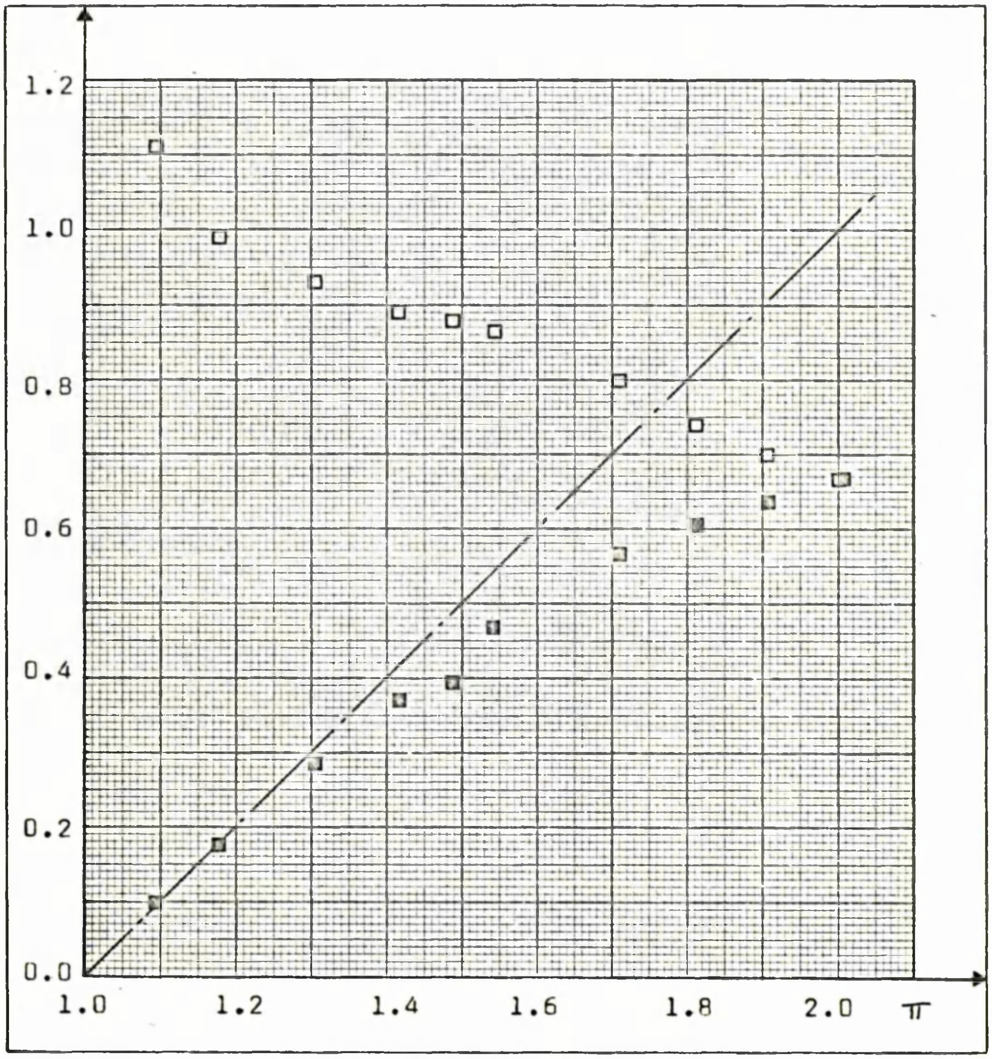
Middle:- Secondary injection pressures.

Bottom:- Experimental exhaust pressure and theoretical exhaust pressures for fully mixed stream.

Key:- Polytropic two stream theory ——— Fabri's method - - - - - -
Sonic choking at duct exit ——-—— Experimental ●

Figure 60.

The Degree of Mixing reached by the End of the Mixing Duct for the Fully Choked Flows.

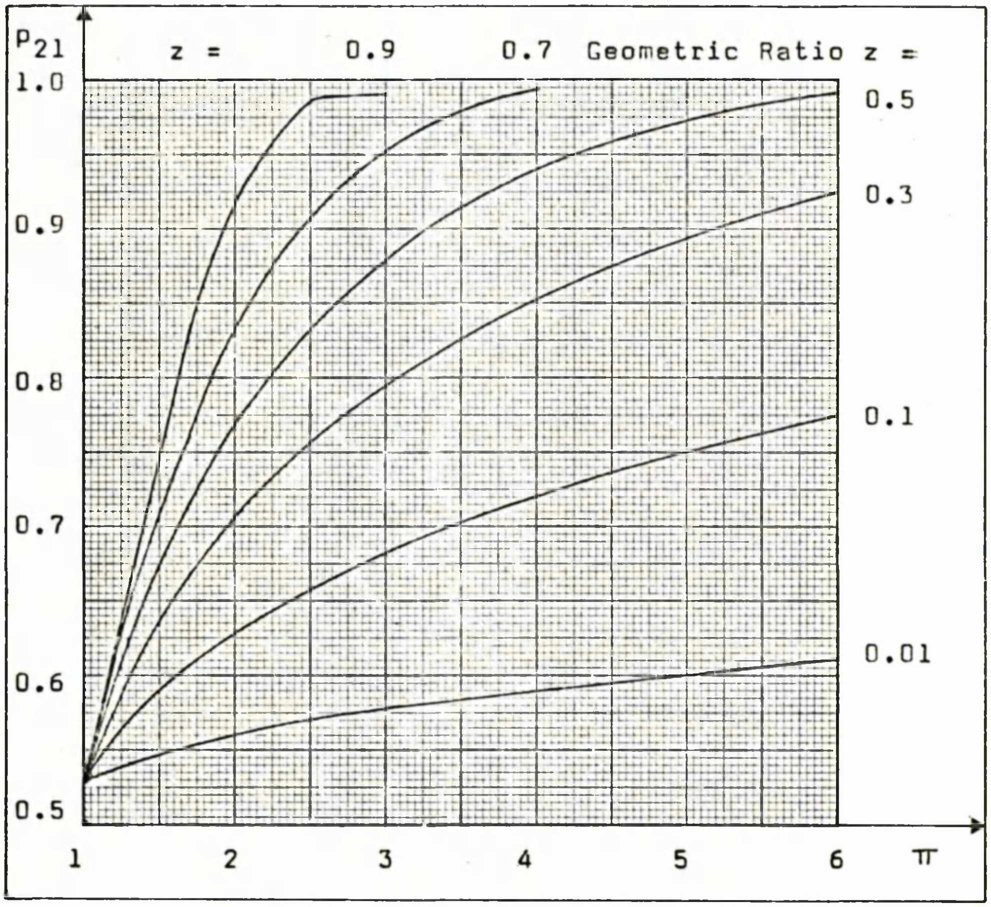


For the purpose of this comparison the degree of mixing is gauged by the difference in the total pressures indicated by the top and bottom Pitot tubes mounted in the blocking centrebody. Their positions are given by $y/D = 0.167$ and $y/D = 0.833$. If no mixing takes place then this difference would take the value $\pi - 1$. The chain dotted line in the graph above represents this situation. The solid squares represent the experimentally measured differences. The hollow squares represent these differences as fractions of initial differences, that is $\pi - 1$.

Figure 61

The Secondary Injection Pressures For Flows Choking in the Manner Predicted by the Polytropic Two Stream Theory.

Secondary Injection Pressure v. Total Pressure Ratio
Lines of constant geometric ratio,z.

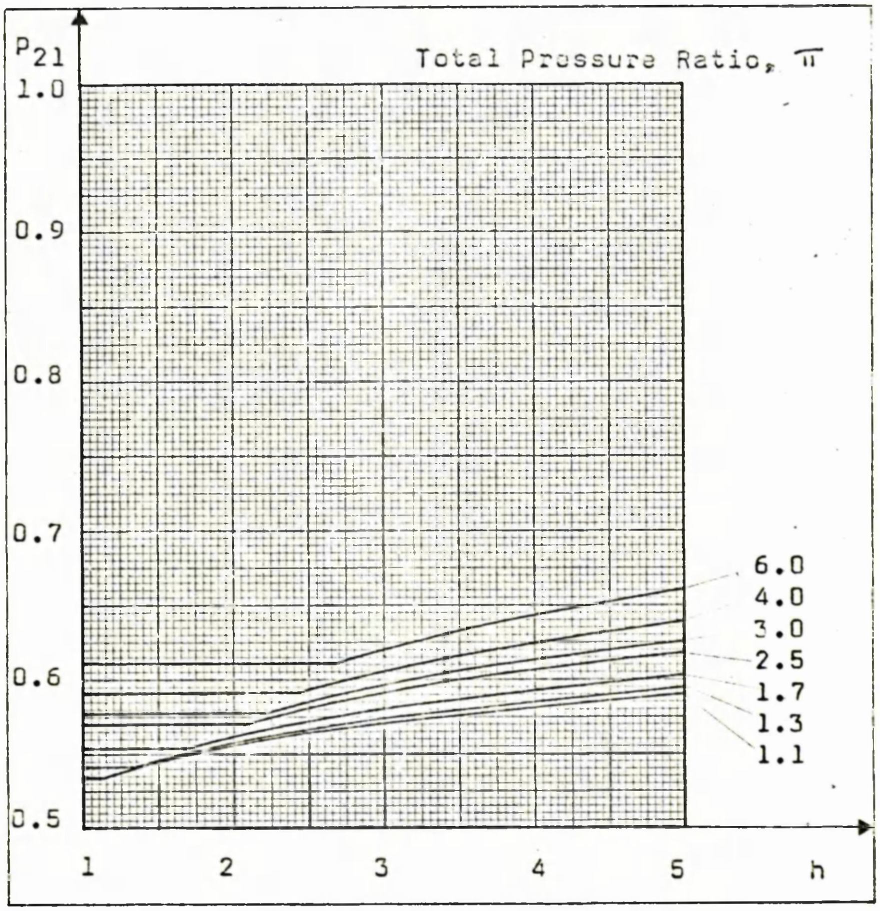


The Secondary Injection Pressure at which the two stream choking process takes place is independent of the Total Enthalpy ratio,h,but if the total enthalpy ratio is too high the flow will choke at the end of the mixing process instead of due to the two stream effect.

Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 62 $z = 0.01$ sonic injection.

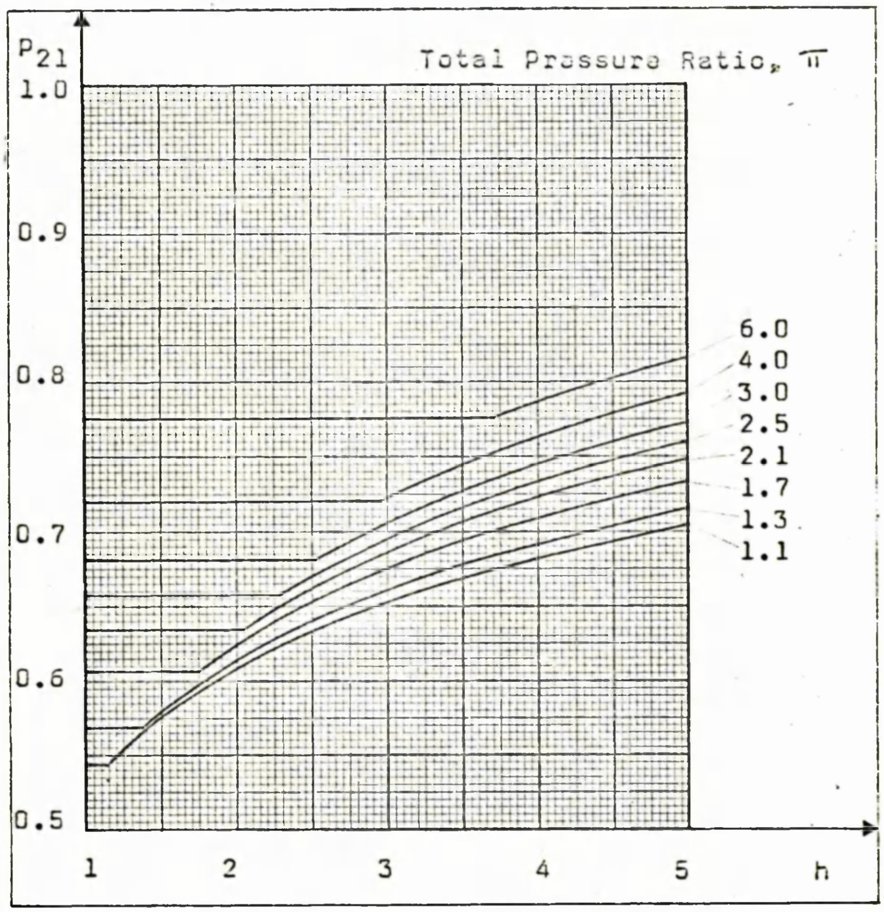
Secondary Injection Pressure v. Total Enthalpy Ratio.



Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 63. $z = 0.1$, sonic injection.

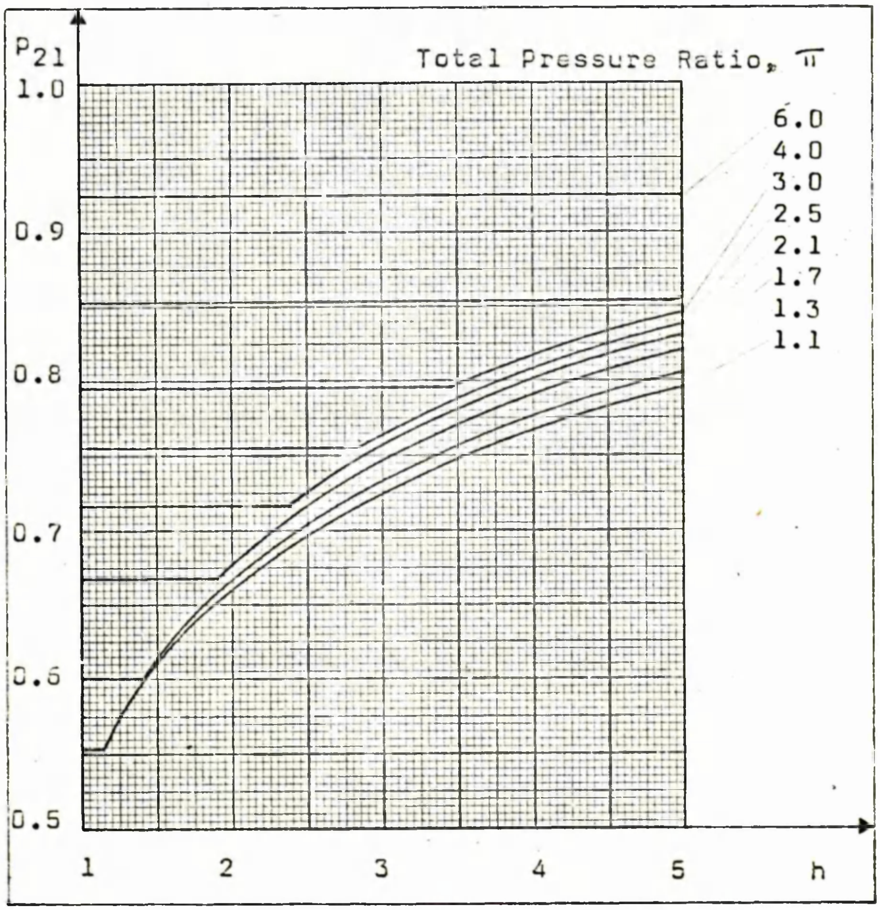
Secondary Injection Pressure v. Total Enthalpy Ratio.



Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 64. $z = 0.3$, sonic injection.

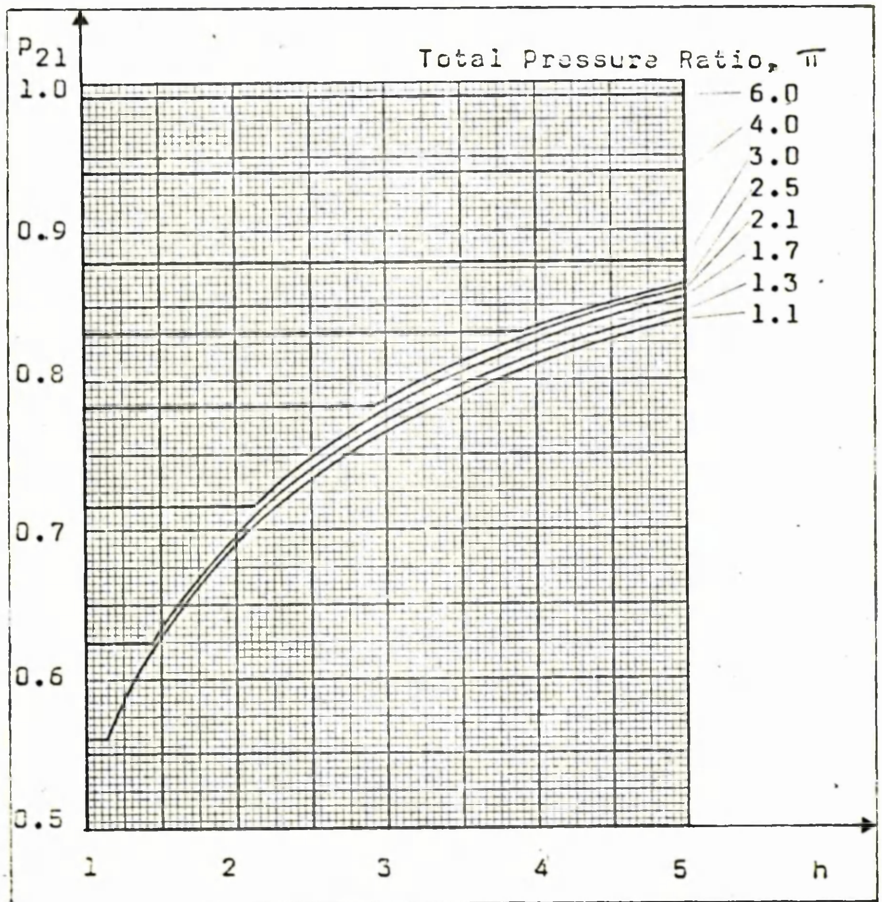
Secondary Injection Pressure v. Total Enthalpy Ratio.



Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 65. z = 0.5 , sonic injection.

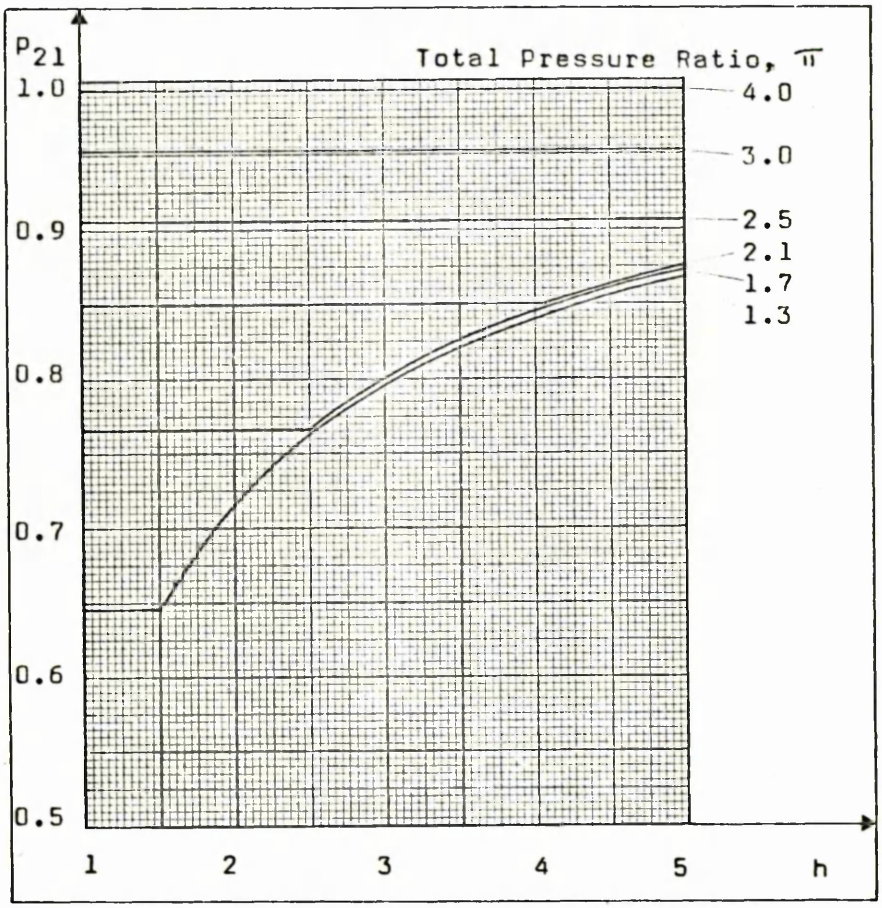
Secondary Injection Pressure v. Total Enthalpy Ratio.



Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 66. $z = 0.7$, sonic injection.

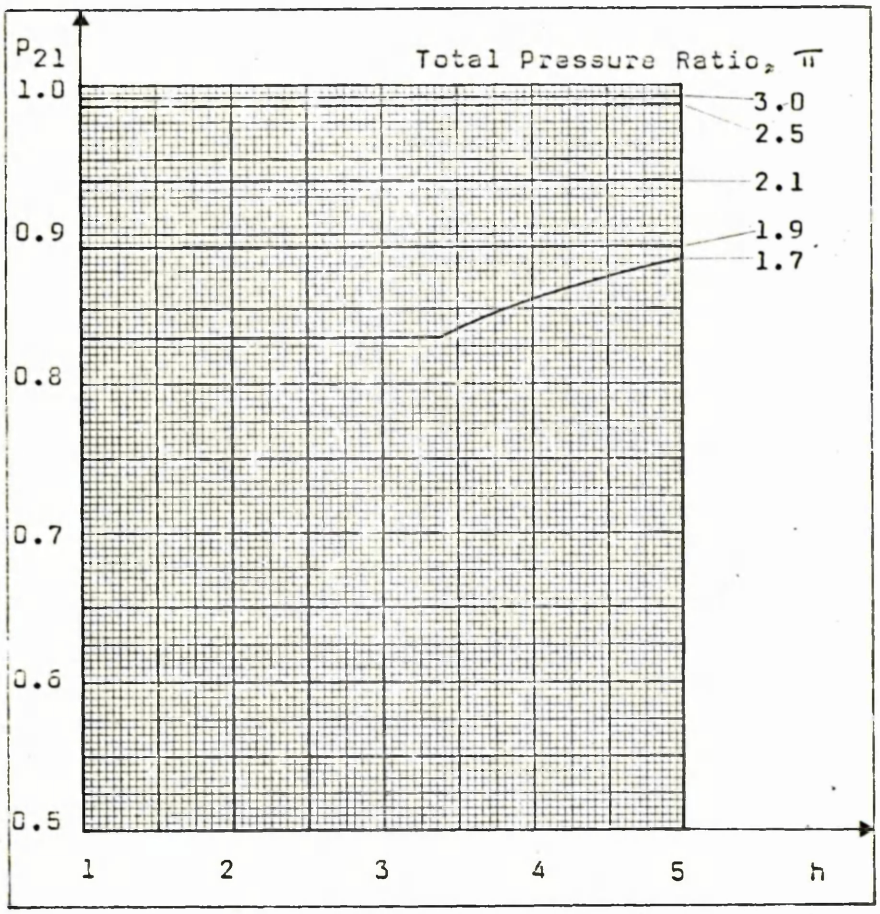
Secondary Injection Pressure v. Total Enthalpy Ratio.



Secondary Injection Pressures for the fully choked flows according to the Polytropic Two Stream Theory.

Figure 67. $z = 0.9$, sonic injection.

Secondary Injection Pressure v. Total Enthalpy Ratio.



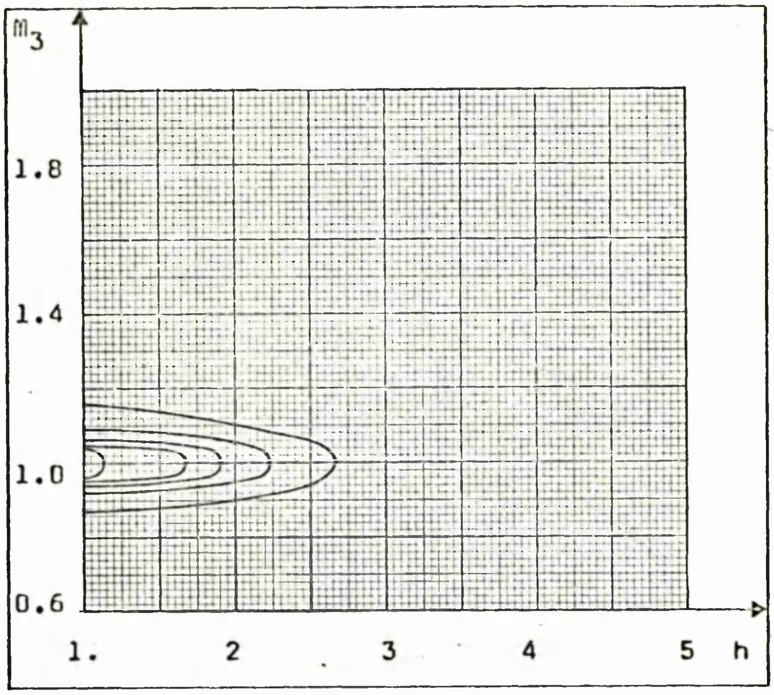
Figures 68 to 73 , The Exhaust Mach Numbers.

These figures depict the Exhaust Mach Number solution surfaces as constant total pressure ratio contours in the Mach Number - Total Enthalpy ratio field according to the Polytropic Two Stream Theory.

Figure 68.

z = 0.01

Exhaust Mach No.



Total Pressure ratios from the inside outwards.

1.3
1.7
2.1
3.0
6.0

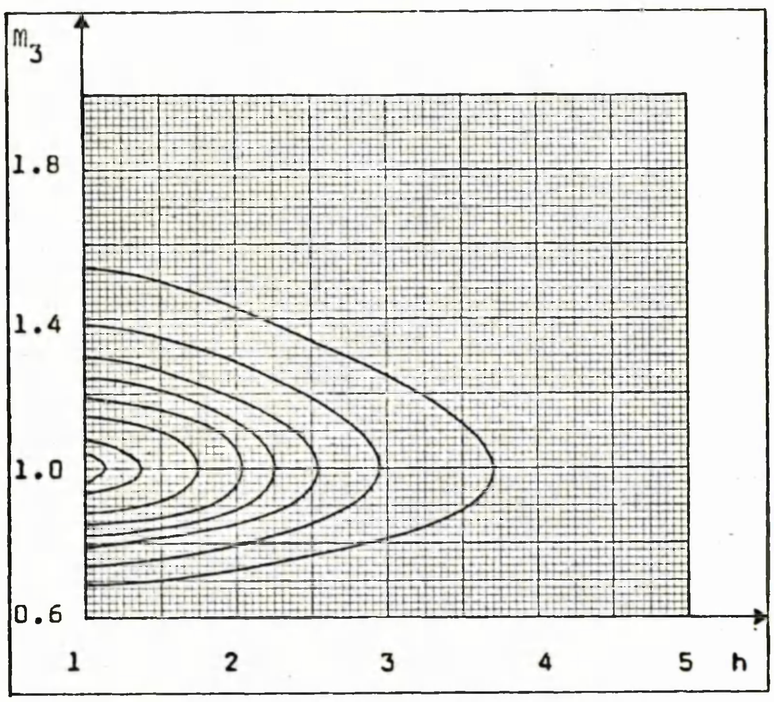
Total Enthalpy Ratio

*   *   *

Figure 69

z = 0.1

Exhaust Mach No.



Total Pressure ratios from the inside outwards.

1.1
1.3
1.7
2.1
2.5
3.0
4.0
6.0

Total Enthalpy Ratio

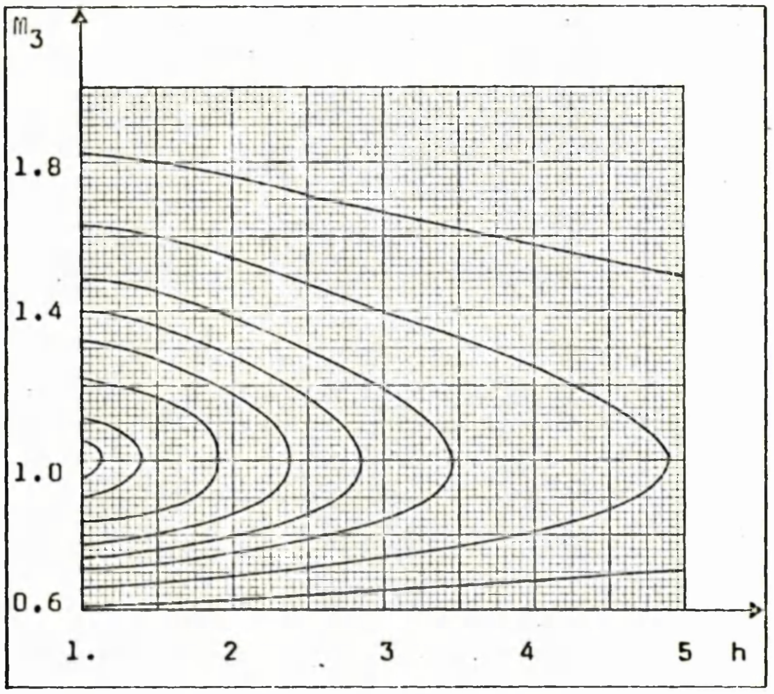
Figures 68 to 73 , The Exhaust Mach Numbers.

These figures depict the Exhaust Mach Number solution surfaces as constant total pressure ratio contours in the Mach Number - Total Enthalpy ratio field according to the Polytropic Two Stream Theory.

Figure 70

z = 0.3

Exhaust Mach No.



Total Enthalpy Ratio

Total Pressure ratios from the inside outwards.

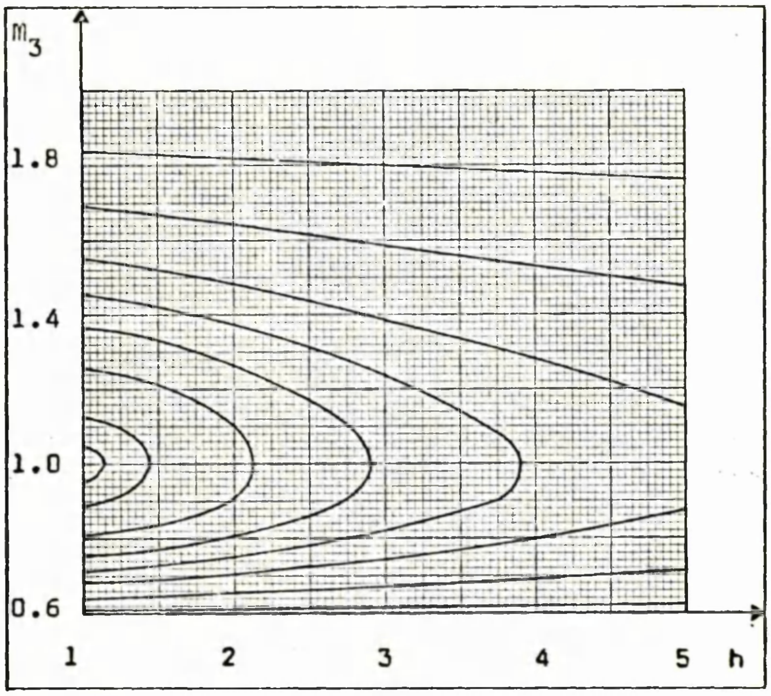
1.1
1.3
1.7
2.1
2.5
3.0
4.0
6.0

* * *

Figure 71

z = 0.5

Exhaust Mach No.



Total Enthalpy Ratio

Total Pressure ratios from the inside outwards.

1.1
1.3
1.7
2.1
2.5
3.0
4.0
6.0

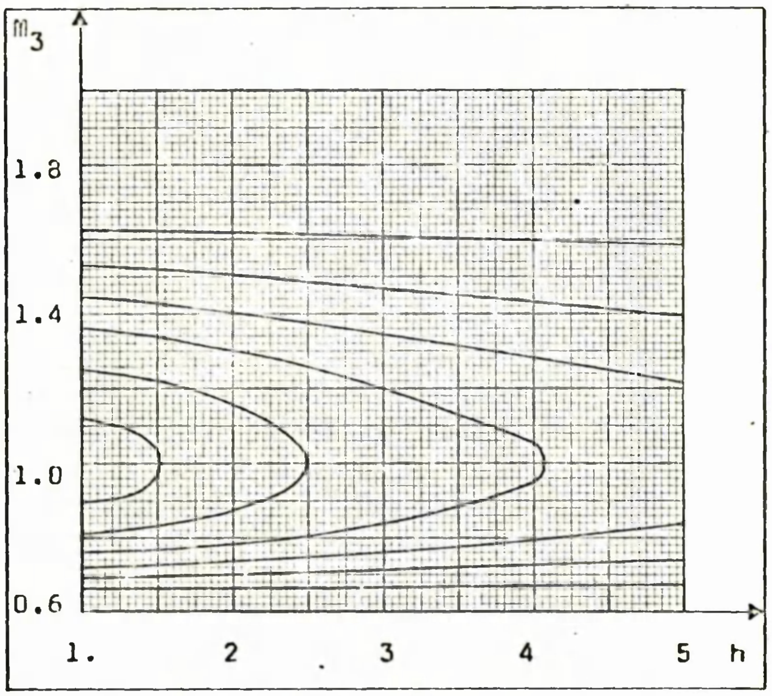
Figures 68 to 73 , The Exhaust Mach Numbers.

These figures depict the Exhaust Mach Number solution surfaces as constant total pressure ratio contours in the Mach Number - Total Enthalpy ratio field according to the Polytropic Two Stream Theory.

Figure 72

$z = 0.7$

Exhaust Mach No.



Total Enthalpy Ratio

Total Pressure ratios from the inside outwards.

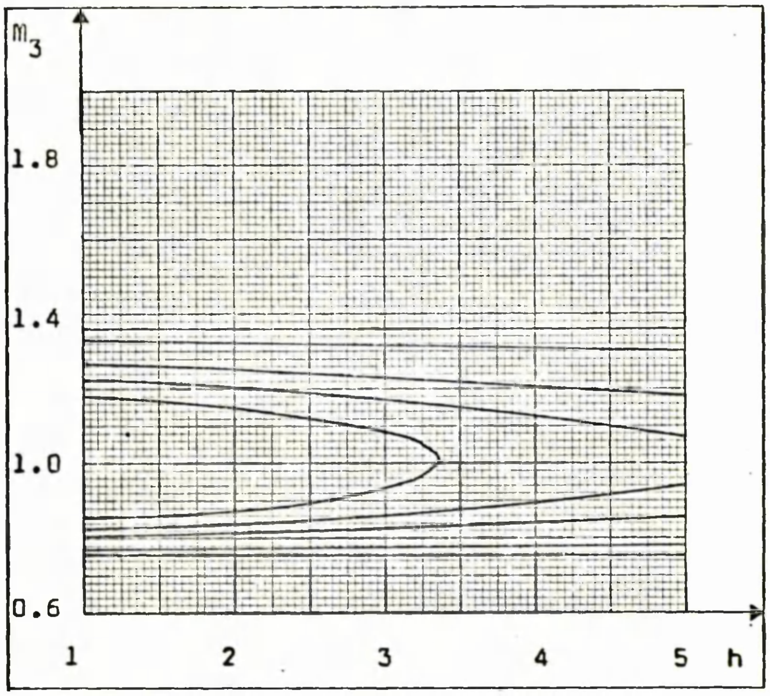
1.3
1.7
2.1
2.5
3.0
4.0

* * *

Figure 73

$z = 0.9$

Exhaust Mach No.



Total Enthalpy Ratio

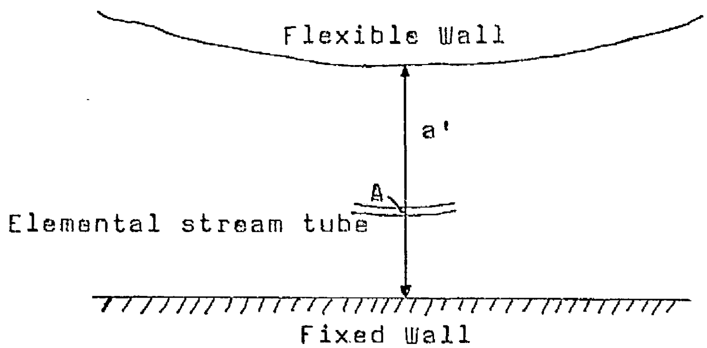
Total Pressure ratios from the inside outwards.

1.7
1.9
2.1
2.5
3.0

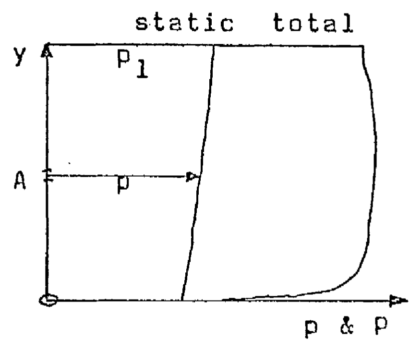
Figure 74.

The Application of a General Theory of Choking to a General Shear Flow.

Flow.



Static and Total Pressure Distributions.



NOMENCLATURE.

A	=	Cross-sectional Area.
a	=	Speed of sound.
a'	=	Cross-sectional area of flow.
B	=	Parameter that is positive for "supersonic" flows and negative for "subsonic" flows.
b	=	Parameter positive for "supersonic" streamtubes and negative for "subsonic" streamtubes.
D	=	Width of mixing duct.
f	=	Momentum function (See Appendix 1).
H	=	Total enthalpy.
h	=	Total enthalpy ratio H_1/H_2.
h	=	Total enthalpy ratio H_3/H_2.
M	=	Mach number.
$\dot{m}$	=	Mass flow.
p	=	Total pressure.
p	=	Static pressure.
q	=	Wave velocity.
r	=	Streamtube elasticity.
s	=	Distance normal to flow.
v	=	Gas velocity.
x	=	Distance streamwise.
x_j	=	Mass fraction of ith stream to have entered the mixing zone by the jth plane.
y	=	Mass flow function (See Appendix 1).
z	=	Area ratio, primary throat over mixing duct.
z'	=	Area ratio, primary injection over mixing duct.
z''	=	Area ratio, primary over mixing duct after expansion.
γ	=	Ratio of specific heats Cp/Cv.
π	=	Total pressure ratio, primary over secondary.
π'	=	Total pressure ratio, primary/secondary after expansion.
π''	=	Total pressure ratio, mixed/secondary.
ω	=	Isentropic pressure ratio, static/total.
ρ	=	Density.

Suffices.

1.	Primary stream.
2.	Secondary stream.
3.	Mixed or exhaust condition.
ij.	Refers to the ith stream at the jth plane.