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THE ROYAL COLLEGE OF SCIENCE AND TECHNOLOGY, GLASGOW.

DEPARTMENT OF MINING.

SHAFT RESISTANCE - THE EFFECT OF THE CAGES.

(S U M M A R Y).

by

GEORGE BAIRD B.Sc.

September, 1962.

SUMMARY

A critical review of the literature is made in which attention is focussed on the efforts of previous workers to produce theoretical analyses of some related problems. Their results are briefly discussed.

The model shaft and some stationary cage tests are then described. The accuracy of the airflow measurements was ascertained and the validity of using the Pressure Drop Coefficient (P.D.C.) was checked.

Interference effects between the cage and buntons had a profound influence on the value of the cage P.D.C., this effect varying for different types of bunton arrangement. It was shown also, that the cage P.D.C. may be assumed constant, whatever its position in the (unlined) duct.

In Section 4, the Pressure Drop Theory of Dr. Rynearz (in conjunction with whom some of this work was carried out) is presented.

Formulae are proposed which define the P.D.C. of both a single cage and two cages in the side-by-side position at their

passing place in the shaft. Both stationary and moving cages are considered.

Those equations defining the P.D.C.'s for stationary cages were verified using the results of three previous workers and a correlation coefficient of 0.94 was obtained for the 28 types of model cage considered.

Section 5, after a description of the pressure recording equipment, deals with some model tests on moving cages.

Recordings were made which showed the effect of the bunton spacing on a test length pressure drop with a cage moving in it. These indicated that the effect on the accuracy of the recording diminished as the cage speed to air velocity ratio increase

The reduction in airflow as the cages passed was shown to be more accurately detectable by means of a centrally positioned pitot-static tube than by the flowmeter itself, whose readings fluctuated greatly.

An attempt was also made to evaluate the theoretically predicted increase in P.D.C. of the two cages passing, when they were in the side-by-side position, compared with the stationary value, but no detectable difference could be found for the range of cage speeds available.

Finally, the influence of the direction of the cage movement, relative to the airflow, on the value of the cage P.D.C. was illustrated. Some measure of success was achieved in correlating some moving cage test results with the theoretical cage P.D.C. values predicted from the Pressure Drop Theory.

In the last part of this work, an attempt has been made to predict certain conditions which might arise in the upper horizon of a two horizon mine using single cage winding in one of its shafts.

An equation is put forward which defines, for time steady conditions, the circumstances necessary for stopping the airflow in the upper level in terms of the circuit resistances and the cage and fan pressure characteristics.

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CHAPTER 1

INTRODUCTION

In recent years, mine shafts have become increasingly deep and the mine workings more concentrated. Climatic conditions under these circumstances would have deteriorated rapidly if it had not been for the general raising of ventilation standards to their present level. That this has been accomplished is due in no small measure to the efforts of many workers, both in the field and in the laboratory.

Whereas the ventilation capacity used to be considered relatively unimportant, the tendency now is to design the shaft primarily for this purpose. Since all the mine air has to pass through the shaft and the quantities flowing are on the increase, the cost of the power absorbed is becoming higher. It is therefore of prime importance to reduce the shaft energy losses. To this end, much work has already been done, particularly as regards the smoothness of the shaft walls and the streamlining of the buntons.

However, it is well known that appreciable disturbances to the ventilation of a mine can be caused by the movement of cages in the shaft. Hence the effects on the ventilation must be taken into account when cage sizes and winding speeds are being planned.

This particular aspect of the problem has also been the subject of extensive investigation. Most of the work has been carried out at the Mining Department of the Royal College of Science and Technology, Glasgow, but there is still a great deal to be done. Three works have already been presented on various facets of the problem and this investigation represents a further step towards the fuller understanding of the aerodynamical resistance of a mine cage and the factors influencing it.

The scope of the chapters is as follows :-

Chapter 2

In this chapter, a critical review of the literature closely connected with the problem under investigation is presented. Some of the results of these previous investigations have been used later and the relevant data is given here. Since the problem is related to that of trains and tubs in restricted airstreams, attention has also been focussed on the works of Bryan, Blaho and Butler. In particular, great emphasis has been placed on their various theoretical considerations, for comparison with that presented in Chapter 4 of this work. In addition, mention is made of several other paperswhich have a more general bearing on the investigation.

Chapter 3

Section (A) describes the equipment used for a series of model tests using stationary cages in a wind tunnel built up to represent a mine shaft.

The purpose of the tests of Section (B) was to determine the factors which influence the cage Pressure Drop Coefficient. As well as checking some of the previous results, tests were conducted to show the influence of the cage position in the shaft, the presence of buntons, the bunton spacing and the presence of a ladder compartment, on the cage P.D.C.

In addition, emphasis was laid on the accuracy of the determination of quantity of air flowing in the duct.

Chapter 4

In this chapter, which is entirely the work of Dr. Ryncarz and without which this work would be incomplete, an attempt has been made to establish a theory of mechanical energy changes due to moving cages in the air flowing in the shaft which would generalise all the results so far obtained.

On the basis of some simplifying assumptions, a general formula has been derived which expresses the P.D.C. due to a moving cage in terms of all the factors taken into account. This formula enabled the theoretical prediction of the P.D.C. due to two cages moving in opposite directions for two extreme cases, viz.

(a) two cages side-by-side

(b) two cages outside the interaction zone.

In order to verify the derived formulae, the P.D.C.'s were calculated for those cages where P.D.C.'s had been experimentally determined by Stevenson and Wilkie. The discrepancy between the corresponding calculated and measured values was less than 10 per cent in all but a few cases.

Chapter 5

Here, some further model tests are described, this time using moving cages.

Section (A) gives details of the equipment it was necessary to use to obtain a continuous recording of the pressure fluctuations in the duct plus some practical notes on its operation.

Section (B) contains a description of the tests which were carried out. For purposes of correlation with the stationary cage tests, recordings were made, with the cage moving, to illustrate the effect of the bunton spacing and to show the increase in pressure drop, as the cages passed, of the central test length. As well as these a test recording was made to show the effect of the cages passing on the air quantity flowing in the duct.

Another test was done to indicate the effect of the direction of motion of the cage (i.e. with or against the airstream) on the pressure drop of the test length containing it. In connection with this phenomenon an attempt was also made to correlate some test results of moving cage P.D.C.'s with those predicted by the Pressure Drop Theory advanced in Chapter 4.

Chapter 6

Here, an attempt was made to predict, from a theoretical point of view the conditions necessary for the stopping and the reversal of airflow in the upper horizon of a two horizon mine using single cage winding. On the basis of the simple network theory, for time steady conditions, a tentative effort was made to determine a general equation.

It was found that a completely general solution would have involved the fourth order of the air quantities flowing. However, for certain conditions, it was possible to produce a more specific equation though difficulty was found here also since the formula necessarily involved such quantities as the fan characteristic.

Chapter 7.

The results of the tests and theoretical schemes are fully discussed where they occur in their appropriate sections. In this chapter, the various conclusions are summarised in chronological order and their limitations defined.

CHAPTER 2

A CRITICAL REVIEW OF THE LITERATURE

Introduction

Any attempt to discuss all the literature, both directly connected with and related to the subject of this work would be a task of tremendous scope. Although it may be said that much of the existing work in connection with conveyances moving in mine shafts has been carried out at this Department, there are many related publications, some with a direct, others with a marginal bearing on these investigations.

In this chapter it is intended to present the conclusions of some of these investigations and to at least mention the more important works which are of particular interest and use here. These various works will, of course, be listed in the reference but in the meantime it is felt that attention should be drawn to their findings without the necessity of reading the works themselves.

Much experimental data has been collected and some of it will be used in subsequent chapters of this work. That used will be given in this section.

Research at the Mining Department of the Royal College of Science and Technology, Glasgow.

The first paper published from this Department, related to the general field of the current investigation, was "The Effect of Standing Tubs on the Resistance of Mine Airways by Tests on Models" by W. Miller and Professor A.M. Bryan (1). This work contains much experimental data as well as a theoretical section in which an attempt is made to deduce a general equation for the pressure losses due to a tub. All the tests were carried out on model roadways, using two different tub designs.

Various conclusions were reached in the eight sections of the paper and the following are of interest here.

It was seen that the effect of a tub was fairly local and that there was a critical interval beyond which the loss due to a number of tubs was that of a single one times the number. It was noted also that the pressure loss incurred in a given length of roadway by a train of tubs was less than for the same number of tubs spaced out along that length. These two statements illustrate the presence and effects of the physical phenomenon of interference between the tubs.

In addition, it was opined that the pressure losses due to a tub could be divided roughly into two parts, viz.,

(a) Wake Losses - occurring at the downstream end of the tub arising from shock and eddying due to the higher velocity stream behind the tub.

(b) Surface Losses — which occur in the restricted area around the tub and arise from the increased velocity along the walls of the airway and from the friction and turbulent effects due to the surfaces of the tub.

For the two tub sizes and the two different types of roadway used it was found that for a train of up to 6 tubs, the loss

$$P = a + N b$$

where P = pressure loss a = wake loss

N = number of tubs b = surface loss

a and b depending on the size and construction of the tub and its lining.

It was found that smooth linings reduce the surface loss, while the wake loss remains fairly constant for the same tub.

In the next section of the paper, a general equation for tub loss was deduced in terms of the dimensions of tub and roadway. This equation consisted of two parts, one representing the wake loss and the other the surface loss.

The theoretical wake loss is given by

$$P = \frac{w\,(V^1 - V)^2}{2g} = C\,(V^1 - V)^2$$

where V^1 = mean velocity of air passing tub

V = " " " " downstream of tub

C = experimental coefficient for a specified tub and airway at stated air density w.

This may be written

$$P = C\left(\frac{a}{A - a}\right)^2 \quad \frac{Q^2}{A^2}$$

where A = cross section of airway

a = end area of tub

Q = quantity of air flowing.

The surface loss, due to increased velocity on roadway walls and turbulent effects of the tub, can be covered by an Atkinson type equation

$$P^1 = \frac{C^1 L (M + M^1) Q^2}{(A - a)^3}$$

where C^1 = experimental coefficient

L = length of tub

M = perimeter of airway

M^1 = end perimeter of tub

The total loss for a single tub then becomes

$$P = C\left(\frac{a}{A - a}\right)^2 \quad \frac{Q^2}{A^2} + \frac{C^1 L (M + M^1) Q^2}{(A - a)^3}$$

Coefficients C and C^1 depending on design of tub, nature of walls of airway and possibly on shape of cross section of roadway.

It was also stated that an expression for the wake loss could be deduced from consideration of the force on a flat plate normal to an air stream. In the above notation

$$p = \frac{k\ w\ V^2\ a}{2\ g\ A}$$

where k is an experimental coefficient.

Tests were also carried out on the pressure drop due to tubs standing side by side, the results showing a marked increase in both wake and surface losses.

This investigation was about the first in this particular field and was the forerunner of many others written in a similar vein. Not only did it provide much experimental data and emphasise the advantages of systematic model testing for this type of mine ventilation problem, it also attempted to base its findings on a sound theoretical basis. As such, it represents a first step to the fuller understanding of the phenomena.

It is interesting to see that the difficulty of carrying out tests on moving cages was appreciated even at that stage " The problem of tests on moving tubs is difficult with models, as the speed requires to be multiplied in accordance with the scale of the models ". The effect of the direction of tub movement, relative to the air stream was also noted.

Professor Bryan, in summarising the work, said that the model tests showed the importance of the tubs as a factor in airway resistance and that the subject was one which merited more consideration and further investigation. In fact, after the war, a great deal of work was done on the Continent into methods of reducing shaft resistance by experiments on models but this was confined to the shaft furnishings; the influence of the cages was ignored.

The next important work on this aspect was : "Mine Ventilation Investigations. Section C; Shaft Pressure Losses due to Cages" by A. Stevenson (2).

Here, stationary cages were used in a model shaft and the investigations covered single cage rope guide, double cage rope or rigid guide, and quadruple cage rigid guide installations as well as an examination of the effectiveness of reducing the resistances of cages by attaching fairings. Some of the preliminary work included the installation of the wind tunnel and fan unit which were used in the present work. In this section also, the Pressure Drop Coefficient (P.D.C.) of a cage was defined as the ratio of the pressure loss produced by the cage to the mean velocity head in the shaft.

The conclusions reached were, briefly, as follows :

(i) Relationship between P.D.C. and number of decks N.

This was found to be of the form

$$P.D.C. = a + b N$$

in complete agreement with Miller and Bryan for trains of tubs. For each series, the loss due to shock (frontal and wake losses) was constant, again in agreement with Miller and Bryan, and the loss due to friction and eddying was directly proportional to the number of decks.

(ii) Effect of loading cage with tubs – reduces the energy loss due to eddying in the space between decks and affects the shock losses. This effect was also noted by Miller and Bryan in the case of tubs full to the top.

(iii) Effect of Coefficient of Fill C_f – the graph of P.D.C. to a base of C_f was a type of parabola.

(iv) Effect of cage shape – scope of this section limited but a slight reduction in P.D.C. was evident as the cage frontal shape alters from square to rectangular, probably due to velocity profile in the duct.

(v) Effect of Streamling.

Joukouski aerofoil nose and tail pieces were fitted and it was found that the shock losses were considerably reduced.

Tests were carried out on all the installations using straight sided fairings on both nose and tail and their benfits noted.

In addition, when the change was made from the rope guide to the rigid guide installation it was found that the buntons produced a noticeable influence on the P.D.C.

The cage P.D.C.'s were measured with the cages apart and in the side-by-side position for a large number of types of cage. This data, together with the table giving details of the model dimensions proved ideal for the purpose of verifying the theory proposed in Chapter 5 and the relevent information is given in Table 2.1.

Also stated is the fact that for all the many types of cage tested, the region of duct over which the models exerted mutual influence was, at worst, twice the wind tunnel diameter. Hence the resistance due to the cages can be assumed to be the sum of the individual resistances.

In his general conclusion, Stevenson draws attention, inter alia, to the need for investigation into the effect of the buntons when cages are moving relative to them in a rigid guide installation and in his "suggestions for future work" points out the necessity for tests with moving cages and the development of a pressure recording device.

This next stage was carried out by A. Wilkie and the results and conclusions are reported in his "Study of the Effects of Moving Cages in Shafts" (3).

TABLE 2.1 Data extracted from results of previous investigations for use in subsequent chapters.

STEVENSON

Cage Serial	A1/1	A2/1	A3/1	A4/1	A5/1	A6/1	A1/2
Fill coefft	0.435	0.393	0.310	0.240	0.152	0.088	0.435
C_c	2.05	1.70	0.926	0.574	0.270	0.138	2.46
C_c11C_c	-	-	-	-	-	-	-

A1/3	A1/4	B1/1	C1/1	D1/1	E1/1	F1/1	G1/1
0.435	0.435	0.461	0.464	0.449	0.463	0.400	0.396
2.84	3.16	2.66	2.64	2.66	2.53	1.9	1.85
-	-	-	-	-	-	-	-

H1/1	I1/1	J1/1	K1/1	L1/1	L1/4	M1/1	M1/4
0.397	0.378	0.167	0.181	0.214	0.214	0.252	0.252
1.81	1.70	0.360	0.380	0.495	0.545	0.810	0.940
-	-	1.42	1.64	1.85	2.18	3.94	5.89

WILKIE

Cage Serial	A/2	C/2	C/4		
Fill coefft.	0.113	0.226	0.226		
C_c	0.2	0.56	0.70	0.39	0.43
C_c11C_c	0.60	2.74	3.20	1.53	1.60

STEWART

Cage Serial	A	B	C	D
Fill Coefft	0.427	0.427	0.427	0.427
C_c	2.92	3.02	3.20	3.50

E	F	G	H
0.500	0.357	0.43	0.427
4.59	1.75	2.97	3.20.

Much of Wilkie's work involved the construction of a suitable recording manometer and an investigation into its response to oscillating pressures. A great amount of investigation and modification was necessary before the equipment, full details of which are given in Chapter 6, was finally ready for use. This included the testing of various types of transducers - a variable capacitance type being finally chosen - and, also worthy of note, the installation of a three-quarter radius pitot tube flowmeter. Only then could Stevenson's work be extended.

After some preliminary investigations into conditions in the wind tunnel, tests were then carried out on both stationary and moving cages. Some of the more relevant (to the present investigation) conclusions are given in the following :

(a) Stationary Cage Tests

These were conducted mainly for purposes of comparison with the subsequent moving cage tests and the conclusions reached were substantially the same as those of Stevenson. Some of the data collected will be used later in Chapter 5 and that used is given in Table 2.1.

(b) Moving Cage Tests

(i) Influence of passing cages on shaft resistance.

Here, the variations in test length pressure drop were plotted as a percentage of normal. In spite of the advantages enumerated for this method it is felt that it might have been better to record the test length pressure drop with the cages stationary as well as that produced by the moving cages. The recorded valueswould thus allow a more direct comparison between the pressure drops. The limitation of Wilkie's method, as he was at pains to point out, was that when comparing the stationary curves, plotted as a percentage of normal, with those for moving cages, no quantitative account could be taken of the reduction in air flow caused by the increased resistance when the cages were passing. An attempt was made to record the variation in flowmeter head while the cages were passing but its natural

fluctuations precluded any reliability being placed on the results. Further reference will be made to these aspects of the problem in Chapter 6.

It was found that the addition offairings lowered the maximum pressure drop with very little change in the length of the zone of interaction. From an examination of the many records it can be seen that, in the range of cage speeds available, the effect of increase of cage speed on the maximum value of the pressure drop is completely random.

(ii) Influence of passing cages on conditions upstream and downstream from the passing place. These records illustrated the fall and rise in static pressure respectively, more particularly with the larger cages.

(iii) Influence of a cage passing on conditions at that point.

These tests showed the increase in pressure (below atmosphere) corresponding to the higher velocity due to the reduced area available for flow. Also indicated was the rapidity of the return to steady conditions after the cage had passed.

In his conclusion, Wilkie opines that the temporary decrease in quantity flowing is probably the most serious effect produced by the passing of two cages, but depends on (a) the shape of the fan characteristic, (b) the proportional increase in total circuit resistance and (c) the inertia of the air in the circuit; factors (b) and (c) probably causing the reduction in quantity to be less in a full scale shaft than in the model.

Following this work, the next investigation to be published by this Department was "Fluctuating Pressure, in the Mine Ventilating Circuit, Caused by Single Cage Winding" by R. Stewart (4) which, as can be seen, is concerned with certain other phenomena in the mine ventilating circuit. However, one section of this work, that concerning the pressure drop produced by a moving cage, is of particular interest here.

The work is concerned with investigating the behaviour of the ventilating air under the unsteady conditions produced by the movement of a single large cage in one of the shafts. In tests using one of the cages, the pressure drop due to the cage was plotted against $(u \pm v)$, where u is the air velocity and v the cage speed, for five air velocities at each of six cage speeds. These graphs are given in Fig. 2.1 and the results will be used later in Chapter 6. Knowing the mean air velocities, the cage P.D.C.'s could be calculated and also the coefficient of velocity $C_v = \frac{v}{u}$ and the results used to check the theory proposed in Chapter 5. The range of air and cage velocities and the cage dimensions are given on Fig. 2.1.

It was thought that the individual lines of Fig. 2.1 would form one line when plotted on a single graph, but such was not the case. The reason for this is possibly tied up with the correction factor $\frac{h}{1.79}$ which was used to compensate the effect of the moving cage on the total resistance of the fan circuit so that the true air velocity could be calculated.

Research at Other Institutions

In addition to the extensive research which has been conducted at this Department, much work has also been done and many problems are in the process of being investigated at other institutions,

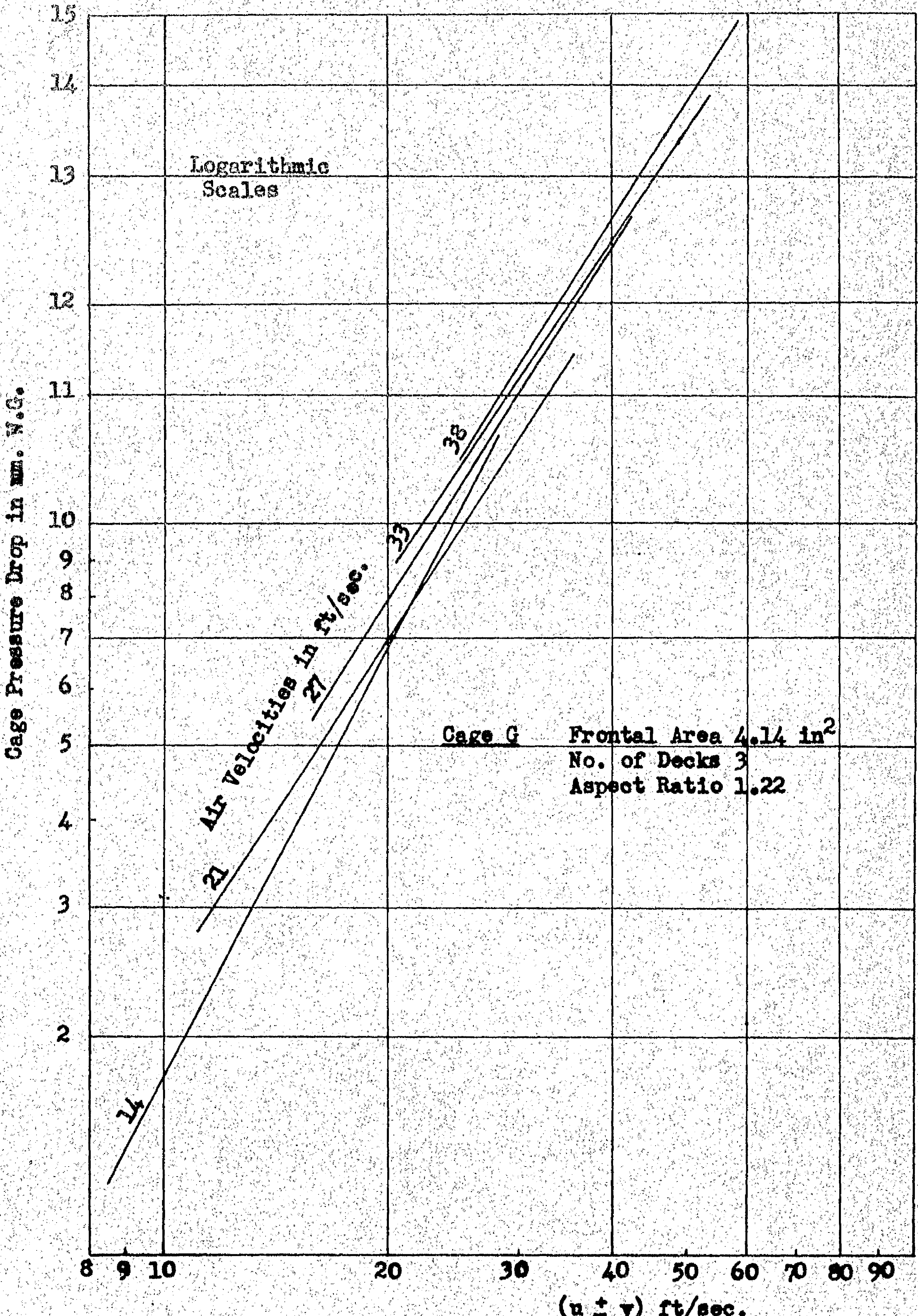


Fig. 2.1 Pressure Drop Produced by a Moving Cage in an Airstream.

both in the United Kingdom and abroad, in this and in other closely related fields.

An extensive programme of shaft research is being conducted at the National Mechanical Engineering Research Institute of South Africa (5). The current work includes a study of the air flow resistance in shafts but so far has been confined to the aerodynamic design of the shaft's internal structure. No tests have been carried out using moving cages, only "A Theoretical Analysis of the Dynamic Behaviour of Rope Guided Mine Shaft Conveyances". It is intended that the work of the Institute and of some other independent workers in this field will shortly be published as a Symposium on Shaft Resistance and the Design of Shaft Equipment.

As stated earlier, much work in the same vein, i.e. shaft furnishings, has been done by investigators in the U.S.S.R. and in Germany. In fact, two Russian mine ventilating engineers, Khokhlov and Ouchakov, did some work on moving cages at this Department during 1960-61. It is on the shaft lining and cages installed by them that the present work has been based.

Another useful paper is "The Estimation and Reduction of Aerodynamic Resistance in Mine Shafts" by J.G. Bromilow (6) which summarises much of the work in this field and quotes some of Stevenson's results. In addition, it indicates the practical means by which these results can be applied both in the design of a mine shaft and the calculation of its resistance.

It will have been gathered that most of the work so far has been conducted on scale models and the reliability of the results, when applied to full scale installations, has also been the subject of detailed investigation by several groups.

It will be appreciated that one of the principal difficulties arising from the use of air models is that they cannot generally be used at the same Reynolds numbers as the prototype. While Reynolds number is generally accepted as a good correlation factor for turbulent flow, it is usually necessary to extrapolate the curves to the higher ranges.

It has been suggested by Jones and Hinsley (7), on the basis of tests with two different sizes of models, that a better correlation is obtained on the basis of mean velocity. In other words, the mean velocity can be used as an approximate similarity parameter for turbulent flow in this type of model. Thus, the model can be operated at the same mean velocity as the prototype, and in this way the necessity for extrapolation is avoided.

This possibility was also noted by Chasteau (8) who obtained much better correlation on a basis of mean speed than on Reynolds number. His tests were conducted on 12 in. and 78 in.

diameter models of a 24 ft diameter shaft and on the full scale shaft itself.

An interesting series of tests has been carried out conjointly at University College, Cardiff and in the field, into the problem of "Induced Road Tunnel Ventilation", and this work has been reported by the National Engineering Laboratory, East Kilbride (9) (10). The investigation concerned the prediction of likely traffic induced velocities in tunnels of various dimensions with various traffic conditions. The experimental work consisted of tests on a laboratory water tunnel analogue, full scale experiments in the London Airport Tunnel, and tests on a laboratory model road tunnel.

While these investigations were primarily concerned with tunnel ventilation for the control of toxic gas contamination, an attempt was made to supplement the experimental results by a simple theoretical consideration. On the basis that, in still external atmospheric conditions with steady traffic flow in the tunnel, the forward drag of the vehicles upon the air in the tunnel is balanced by the wall friction drag plus the retarding forces due to entry and exit losses, the following equation if applicable.

$$C_v \, A_v \, n \; \tfrac{1}{2} \rho (u - v)^2 = C_t \, A_w \, \tfrac{1}{2} \rho v^2 + k \, A_t \; \tfrac{1}{2} \rho v^2$$

where C_v = Vehicle drag coefficient

A_v = Vehicle frontal area

n = Number of vehicles in tunnel

u = Vehicle speed

v = Induced air speed

C_t = Tunnel wall drag coefficient

A_w = Wetted area of tunnel

k = Tunnel end loss factor

A_t = Cross sectional area of tunnel

ρ = Air density

This equation, since it was the induced draught v with which the author was concerned, reduced to

$$v = \frac{u}{1 + \sqrt{S} \sqrt{\dfrac{k A_t + C_t P_t \ell}{C_v A_v \ell}}}$$

where ℓ = Tunnel length

P_t = Tunnel perimeter

S = Vehicle spacing ($= \frac{\ell}{n}$)

and denoted a straight line passing through the origin. This did not agree precisely with the experimental lines and it was found necessary to subtract a factor C_1, dependent on the type of traffic, to provide a better fit. After manipulation to establish suitable values of k, C_t and C_v the formula was proposed as a working basis for prediction of induced air velocities. It was conceded that the analysis was an over simplification but it was considered adequate for the practical purposes of tunnel design.

Although it has not been stated by the author, it is felt that one of the main discrepancies is due to his assumption of a constant induced draught, i.e. flow everywhere in the direction of motion of the vehicles. In fact, this is not so; it is known that a high pressure is set up at the front of a moving vehicle and a low pressure in its wake. Hence the flow in the vicinity of the vehicle will be in the opposite direction to the main induced draught.

Another fact, allegedly shown by the test results was the effect of the interference drag of the vehicles on each other at various spacings. These values, obtained by manipulation of the equation, are shown in Table 2.2.

Models	Model Spacing	Drag Coefficient C_v
Riley Saloon Card ($2\frac{3}{4}$ in. long)	1 ft 2 ft 3 ft	0.500 0.605 0.798
Bedford Truck Models ($3\frac{1}{2}$ in. long)	2 ft 4 ft	1.18 1.48

Table 2.2

However, it is felt that this effect must be due to some other influence since, according to many results given by Hoerner (11), the interference should be negligible even at the lowest lift spacing, for this size of model. This is also supported by the results of the present investigation and in fact, by all the workers in this field at the Department. The general order of the ratio of spacing distance to vehicle size at which interference effects cease being not more than four or five.

Possibly, in the case considered, it might have been more advantageous to have calculated the drag coefficient for one vehicle and introduced a separate interference factor based on the vehicle spacing.

Yet another series of tests have been carried out this time at the Technical University, Budapest, the subject of the experiments being the "Drag of Trains in Tube Tunnels" (12). The author, M. Blaho, undertook model tests on trains in a duct, the bulk of the experimental work consisting of measurements of the pressure distribution along the duct and the drag force acting on the train for different lengths of trains and fill coefficients.

This work differed from that of Butler and his co-workers, in that the fill coefficient was of much greater importance, being, in this case in the range 0.36 to 0.63 compared with values of

0.035 and 0.065 in the road tunnel tests. Also, Blaho was principally interested in evaluating the drag of the train rather than the air flow caused by its passage. It is worthy of note that while Butler's test method is the more correct in principle in that the vehicles are moved relative to the tunnel and the air speed measured, Blaho's procedure, that of exhausting the air and, with the train held stationary, measuring its resistance, is the commonly accepted routine test method. For cages in a mine shaft, of course, it is necessary to move the cages and exhaust the air simultaneously.

From the experimental work, the conclusions are briefly :

(a) There is a sudden pressure drop at the fore part of the train.

(b) The pressure drops linearly alongside the train, but much more rapidly than in the empty tunnel.

(c) The pressure increases in the wake of the train due to the widening cross section. Also, from the previous works it is clear that there is an energy loss in the wake which is independent of the length of the train.

An attempt was made to derive an equation for the train's drag coefficient. It is felt that although the formula proposed may be of use in certain specific cases, its general application is limited by the fact that it does not include certain variables. The more important of these are the shape of the front and rear of the train and the roughness of the side walls.

It is stated that when the train is running in the open, i.e. when the fill coefficient is zero, the equation gives a drag coefficient of unity. This is correct as the formula stands, but, contrary to the statement that this is in fair agreement with experience it must be pointed out that Hoerner quotes values varying from 0.38 upwards, showing quite clearly the dependence of the drag coefficient on the shape of the train, its length and the nature of its surfaces.

Although there are many text books worthy of mention in connection with this work and indeed, several are given in the References, it is felt, with all due respect to the others, that one in particular deserves special recognition. This is "Fluid Dynamic Drag" by S.F. Hoerner and it is true to say it is an outstanding treatment of the subject, of great use to engineers who have any concern whatsoever with aerodynamic or hydrodynamic drag.

Reference has already been made to it in this Chapter in connection with vehicle and interference drag. Suffice it to say that great use was made of Dr. Hoerner's treatise throughout this project.

C H A P T E R 3

STATIONARY CAGE TESTS

(A) EQUIPMENT

Introduction

Having thoroughly studied the background of the problem, it was now proposed to conduct stationary cage tests using the equipment at our disposal, with only minor modifications. This was necessary not only to check qualitatively the results of the previous workers but also to investigate some of the phenomena which were brought out by an analysis of their results, and still remained unexplained. All the experimental work was carried out using the one type of cage and shaft lining which is described in this Chapter. Although this might appear a limiting factor it was felt that it would be sufficient for the scope of this work and that the selective use of the known data would provide ample material for correlation of the theory advanced in Chapter 5.

The Model Shaft

The experimental wind tunnel was constructed using circular steel ducting, and equipped with model buntons, guides, cages and ladder compartment so that it was an exact replica of a full scale shaft with, as near as possible, complete geometrical similarity.

The tunnel itself was built up from sections of circular steel ducting, each 6 feet long and 11¼ inches nominal internal diameter, the sections being bolted together with rubber gaskets at the joints. Five 4 foot long sections, each made up of two half cylinders bolted together along longditudinal flanges were also used. The upper half of each of these sections was made of thick perspex with strong steel flanges, the sections being placed at convenient points along the length of the assembled wind tunnel. These provided very useful observation and access points, being disposed strategically along the duct. Removal of the perspex section was very simple so that dismantling was unnecessary when it was required to examine the cages or lining.

Tapped holes, designed to allow the insertion of a pitot-static tube, were provided at various points along the duct and when not in use, air tight metal plugs were screwed into these holes. As a final precaution against air leakage into the ducting, plasticine was used to seal all joints and tapping holes.

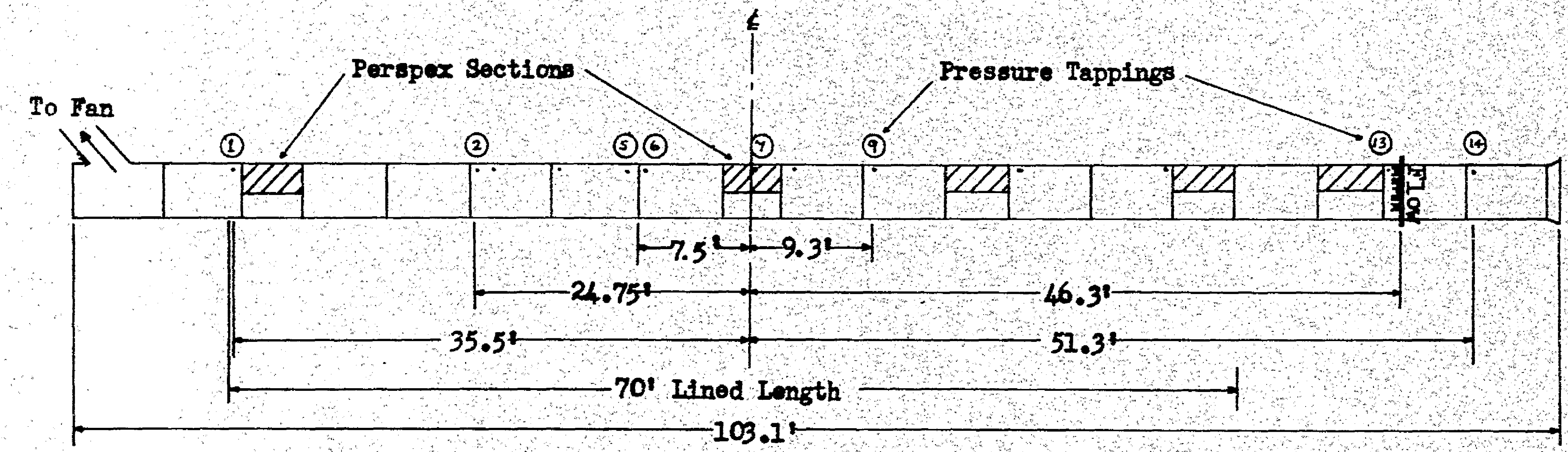


Fig. 3.1 Duct Layout

The wind tunnel was 103 feet long and the layout, indicating the position of the perspex sections, tapped holes etc. is shown in Fig. 3.1.

Shaft Lining

The pattern of guides and buntons used here is shown, half size, in Fig. 3.2, with one of the cages in position.

The centre buntons, made of $\frac{1}{4}$" x $\frac{1}{8}$" I section, were situated at $8\frac{1}{3}$" centres along the duct and the side buntons similarly, while the guides were constructed of $\frac{1}{4}$" x $\frac{1}{4}$" channel section brass. In addition, strips of $\frac{1}{8}$" x $\frac{3}{8}$" brass were fitted along the ends of the buntons to give the necessary rigidity. These strips were fixed to the duct walls by self tapping screws. As well as these features, the shaft was also equipped with a ladder compartment made of lengths of cardboard and supported by struts of the same size and section as the buntons.

The lining was made up in lengths, all the components being soldered together making up a total lined length of 70 feet. Thus the various test lengths, as described in part B of this Chapter could be so situated that a reasonable settling length was provided to help improve similarity between the model and full scale. At the same time, sufficient empty ducting remained at the inlet end to allow insertion of the flowmeter where the airflow

Scale: Half Size

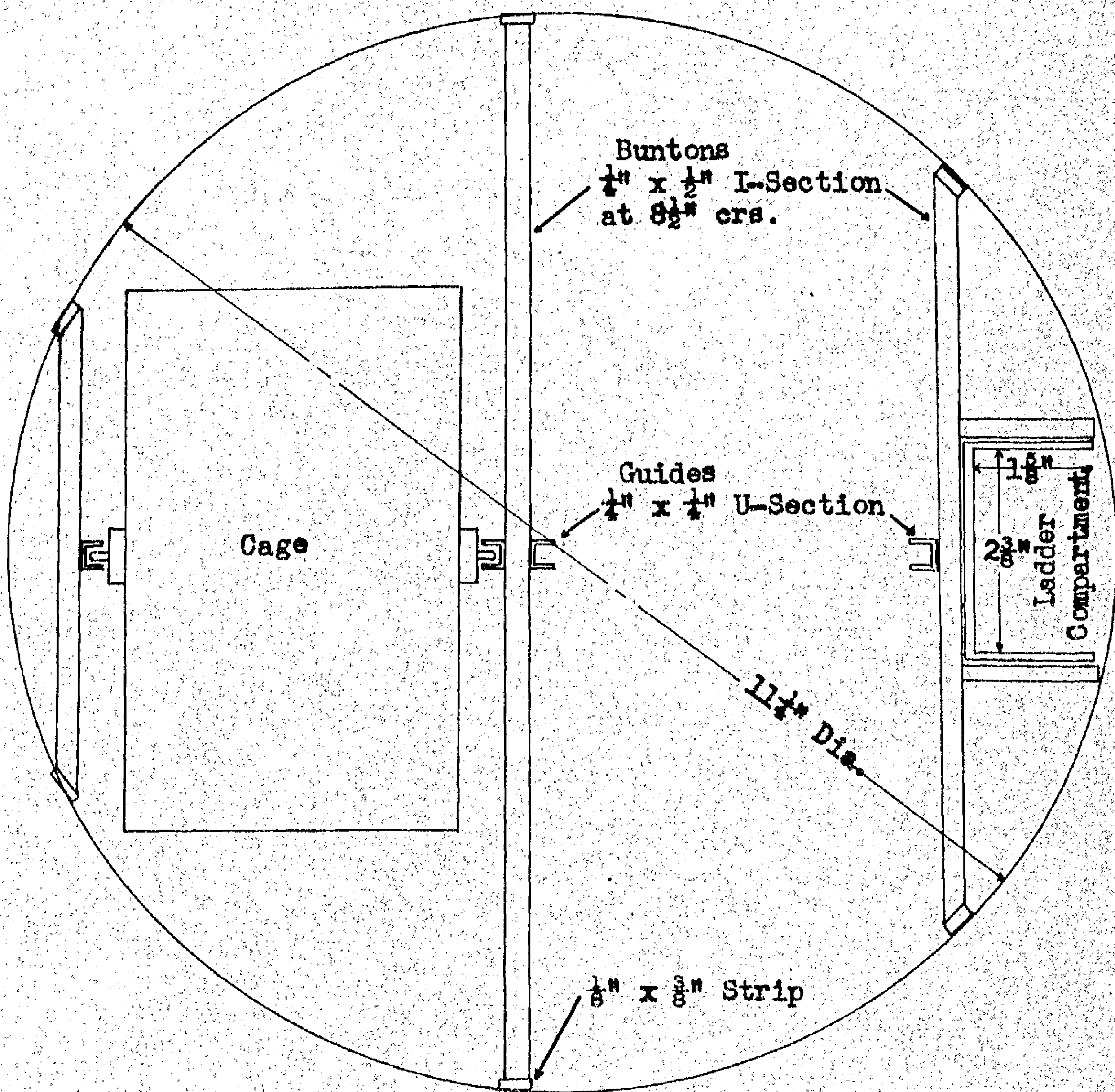


Fig. 3.2 Shaft Furnishings

would be subjected to the minimum of disturbance.

The Cage : Its Drive and Control

The cages were made of wood and were of a very simple two deck design as shown in Fig. 3.3 which gives the leading dimensions. The cage shoes were mounted on spring loaded strips attached to the sides of the cages and ran inside the channel section of the guides. Thanks to the care taken in their construction and in the soldering together of the guide lengths very little trouble was experienced with this arrangement.

The nominal breaking strength of the rope was 538 lb so, as an additional safety precaution, it was fixed to the cages, as illustrated, by easily broken links of soft wire. Thus the breaking of these links would prevent any serious damage to the cages and lining. In addition, it was found in practice that the rope tension could be adjusted such that the rope would slip on the driving pulley if the cages stuck for any reason.

The cages were driven by a $1\frac{1}{2}$ h.p. 3 phase a.c. motor fitted with a continuously variable hydraulic reduction gear, the power being transmitted to the cages by friction drive on the $\frac{1}{4}$" circumference galvanised steel rope passing $1\frac{1}{2}$ times round a 10" diameter driving pulley. With a motor output torque of 510 in.lb. at all speeds from 5 to 124 r.p.m. in either direction, Scale : Half Size

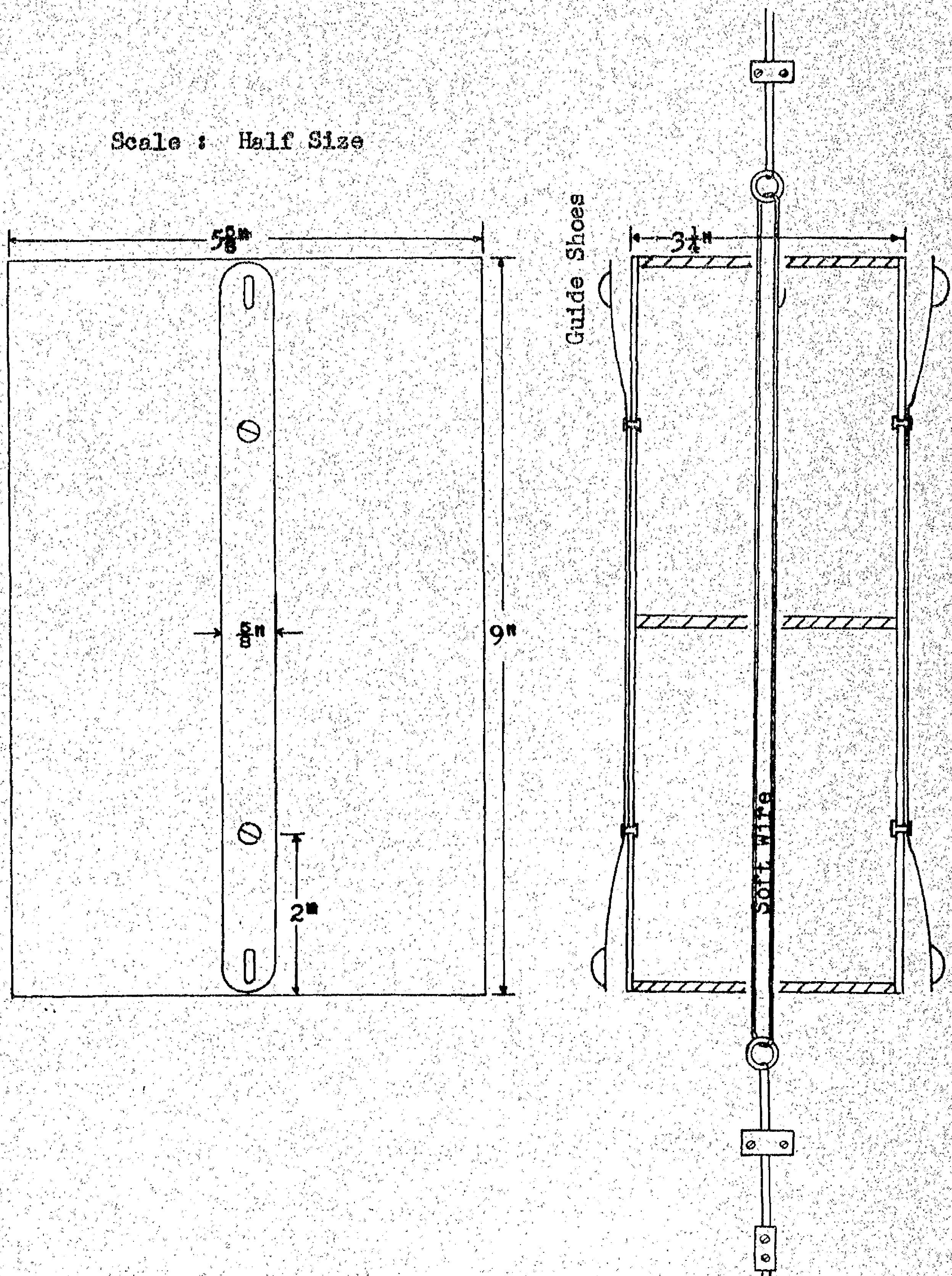


Fig. 3.3 Details of Cage

this meant that a rope pull of 102 lb. was available to drive the cage at any speed from $12\frac{1}{2}$ to 325 ft/min. Tension was maintained on the driving rope by the tail rope which passed from the cages round a tensioning pulley on which cheese weights were hung.

The motor was fitted with a reversing starter, and, for convenient operation, a set of remote control buttons for this starter was connected to the end of a long wander lead. This enabled the motor to be started from any desired position along the duct. In addition, the starter could be operated automatically by the cages themselves by means of micro-switches with specially designed actuators fitted inside the duct at the end of the lining. These switches stopped the motor before restarting it in the opposite direction, thus preventing accidental over-run of the lining by the cages, and allowing completely automatic operation if required.

The Fan

The air was drawn through the wind tunnel by a $14\frac{1}{2}$ inch Howden centrifugal fan driven directly by a 20 h.p. a.c. motor running at 2,850 r.p.m. No speed control was available so that a throttling device was necessary to control the air quantity flowing in the duct. A butterfly type shutter, situated at the fan inlet, was used for this purpose.

The fan inlet was connected to a 45° Y-junction piece at the duct outlet by a length of 12 in. diameter Spiratube, this arrangement being suitable for many reasons.

One important consequence was that it left a blank flange on the main line of the duct through which the driving rope could pass to the cages without interfering with the fan. It helped damp pulsations from the fan at their source and allowed easy alignment of the fan motor to the ducting. At the same time, it prevented vibrations being transmitted from the fan motor unit. These vibrations would have an adverse effect not only on air flow conditions but also, quite conceivably, on the relatively weak perspex sections.

The Flowmeter

A three-quarter radius pitot tube flowmeter was used for measuring air flow in the duct.

This instrument was made up using four pitot tubes and four equally spaced static holes in the wall of the duct. The pitot tubes were positioned at 90° intervals on the three-quarter radius. The static holes, at least 10 pitot tube diameters upstream of the radial part of the pitot tube, were in line with the pitot tube mouths, but lay on diameters offset 45° from those on which the pitot tubes lay in order to avoid interference. The

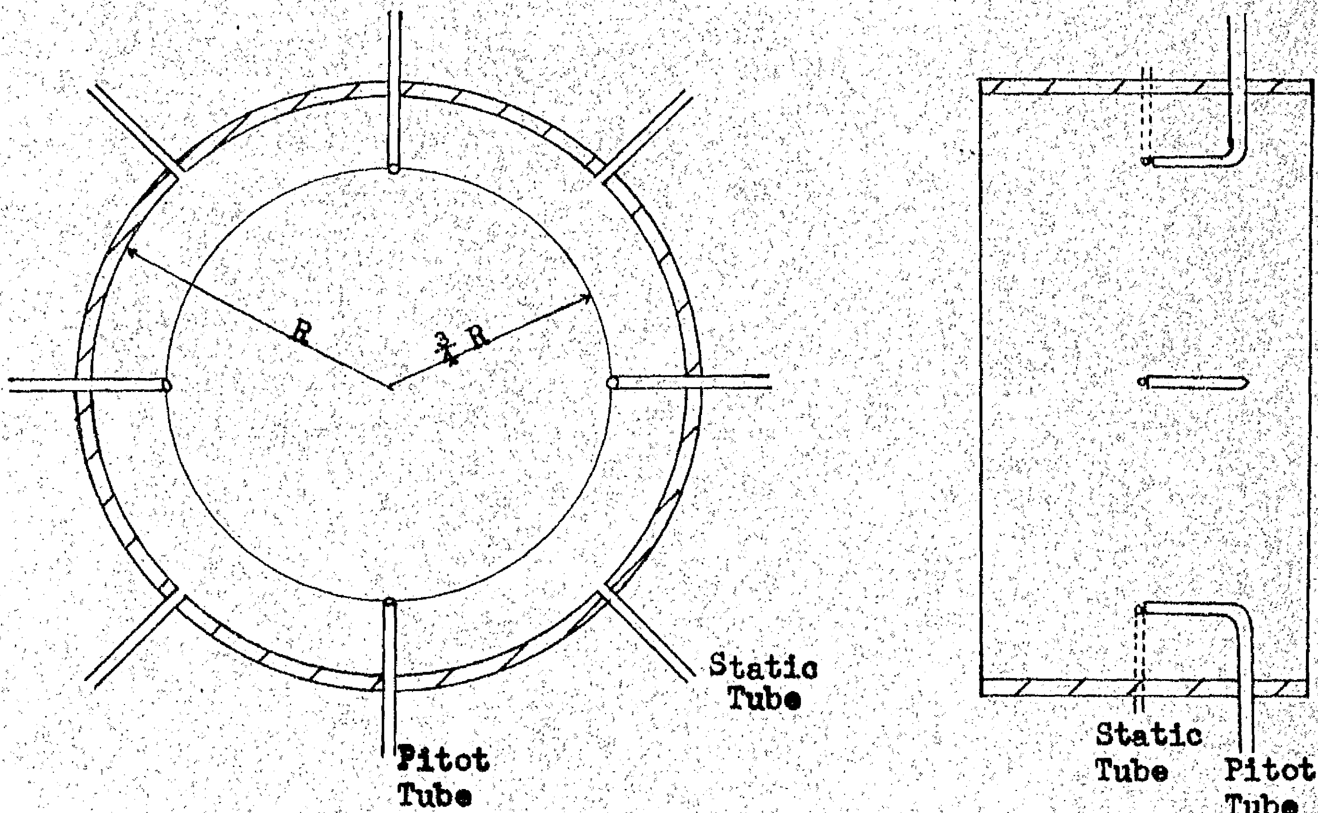


Fig. 3.5 The Flowmeter

arrangement is shown in Fig. 3.5 which clearly illustrates its compact and simple design.

Although several devices had been used by past investigators, it had been found that this instrument had many advantages. It had negligible loss and a fairly constant calibration coefficient of about unity for varying velocity distributions arising from change of Reynolds number or other causes. Tests had been successfully carried out to show this, and a theoretical basis advanced (13). In addition, it could take into account asymmetry of flow and, since the three-quarter radius position of the pitot tubes was close to the point where the contribution to the flux was greatest, better accuracy could be obtained than with a centrally placed pitot tube if irregular velocity distributions were encountered. On the other hand, the velocity fluctuations were greater in this position, making the manometer more difficult to read.

The settling length upstream of the measuring section varied, in different standards, from 3 to 20 duct diameters. In general, in the absence of any upstream disturbance causing flow asymmetry, a settling length of about 10 diameters ensures a velocity distribution which resembles the final one fairly closely. Some authorities in fact, give 3 or 4 diameters as sufficient provided a shaped inlet is fitted to reduce inlet disturbances.

The shaped inlet largely reduces the effect of the end of the pipe acting as a sharp edged orifice.

In this case, the flowmeter was situated some 12 diameters from the inlet and a bell mouthed entry was provided.

Instruments

In addition to the flowmeter described above, for the measurement of static air pressure use was made of pitot-static tubes in conjunction with two water manometers.

The former were standard N.P.L. type round nosed pitot-static tubes while the latter were both Betz Projection Manometers. The application of a differential pressure caused a change in water level inside the stem of the instrument. This change was observed, through an optical system, on a long, finely divided scale which was suspended in the water from a float. The total range of the manometer was 0 to 400 mm. W.G. and the smallest scale division was 0.1 mm. W.G.

The fact that two manometers were in use made simultaneous measurements easy and T-piece manifolds were available which allowed connections to be rapidly switched without interfering with the instrument.

A typical test set up is illustrated in Fig. 3.7 which shows the whole layout of the duct and its associated equipment.

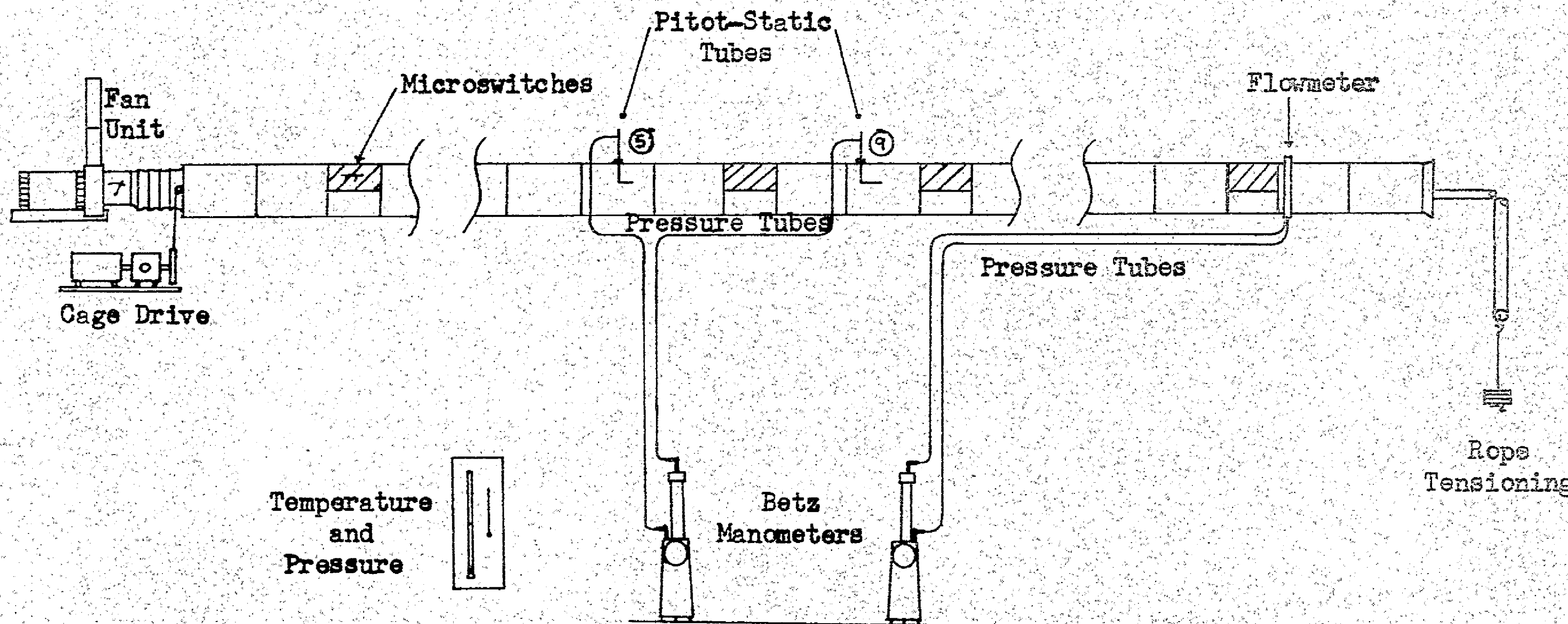


Fig. 3.7 A Typical Test Set-Up

(B) TESTS

Introduction

Having acquired complete familiarity with the equipment, all was now ready to conduct a series of stationary cage experiments into the several phenomena which it was felt necessary to investigate. The principal factors influencing the scope of these tests were

(a) The limitations of the equipment.
(b) The necessity for correlation with the moving cage tests (Chapter 5).
(c) The necessity for correlation with the theory (Chapter 4).
(d) The avoidance of needless repetition of the tests of previous workers.

In addition, emphasis was laid on the determination of the accuracy of the tests and any unforeseen effects were immediately investigated.

The tests were subdivided as follows :

(a) Calibration of the Flowmeter, including an investigation into the flow conditions immediately upstream and downstream of it.

(b) Note on the use of the Pressure Drop Coefficient (P.D.C.) together with tests relevant to its use.

(c) The determination of the P.D.C. of the stationary cages at the point where they pass and their zone of interaction.

(d) The investigation of the influence of the bunton spacing on the cage P.D.C. This was done for two different types of bunton arrangement: in addition, the influence of the ladder compartment was shown.

(e) Tests to find the P.D.C. of the cage in an unlined length of ducting and the influence of its position.

Calibration of the Flowmeter

The section of ducting relevant to this part of the work is shown in Fig. 3.8.

The initial attempts to calibrate the flowmeter were carried out by means of velocity head traverses at tap (12), immediately behind it. The traversing method was judged to be the most accurate possible in the circumstances. Another means, the log linear method, was considered and rejected on the basis of the experience of previous workers who found the traversing method just as good, for the same number of measuring points, without the attendant difficulties of positioning the pitot-static tube at a predetermined position.

Only vertical traverses were possible but it was hoped that these would be sufficient. The method of traversing and calculating the mean speed is set out in The British Standard

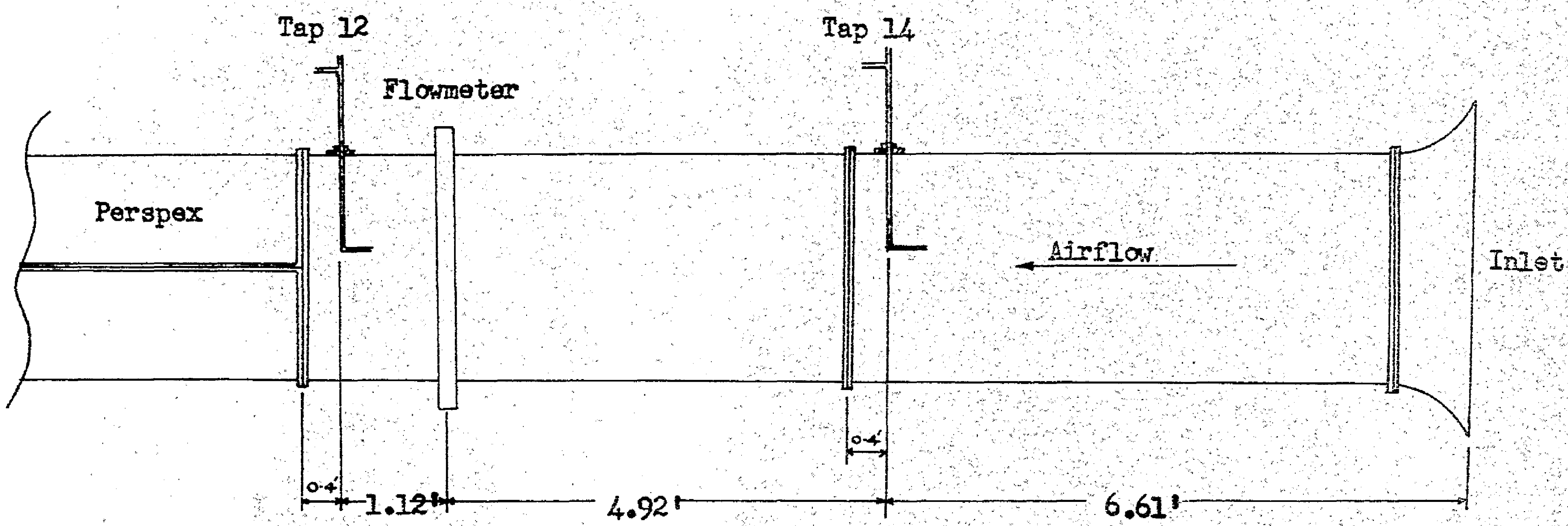


Fig. 3.8 Flowmeter Section

Code (14) and it might be appropriate to give a brief summary here.

The pitot-static tube was moved from one side of the duct to the other by equal increments, measurements of velocity pressure being taken at each station. A plot was then made of $\sqrt{h"\ W.G.}$ to duct diameter. Ten points on the traverse, each specified as a certain fraction of the duct diameter were then marked on the graph and the corresponding values of $\sqrt{h}$ noted. These were arranged, giving $\sqrt{h_{AV.}}$ from which the mean velocity head h can be found.

The mean velocity is got from the relation

$$V_{mean} = \sqrt{\frac{2g}{W}}\sqrt{h_{AV.}} = \frac{18.29}{\sqrt{W}}\sqrt{h_{AV.}}$$

where W = density in lb/ft^3

and h = velocity head in inches W.G.

Traverses were done at various air speeds, under similar conditions of temperature and pressure. At each air speed, the flowmeter reading was noted together with the limits of its fluctuations and the true mean velocity and mean velocity head were calculated from the traverses.

Some typical traverses are given in Figs. 3.9 and 3.10 plotted on a base of duct diameter. In addition to the mean velocity

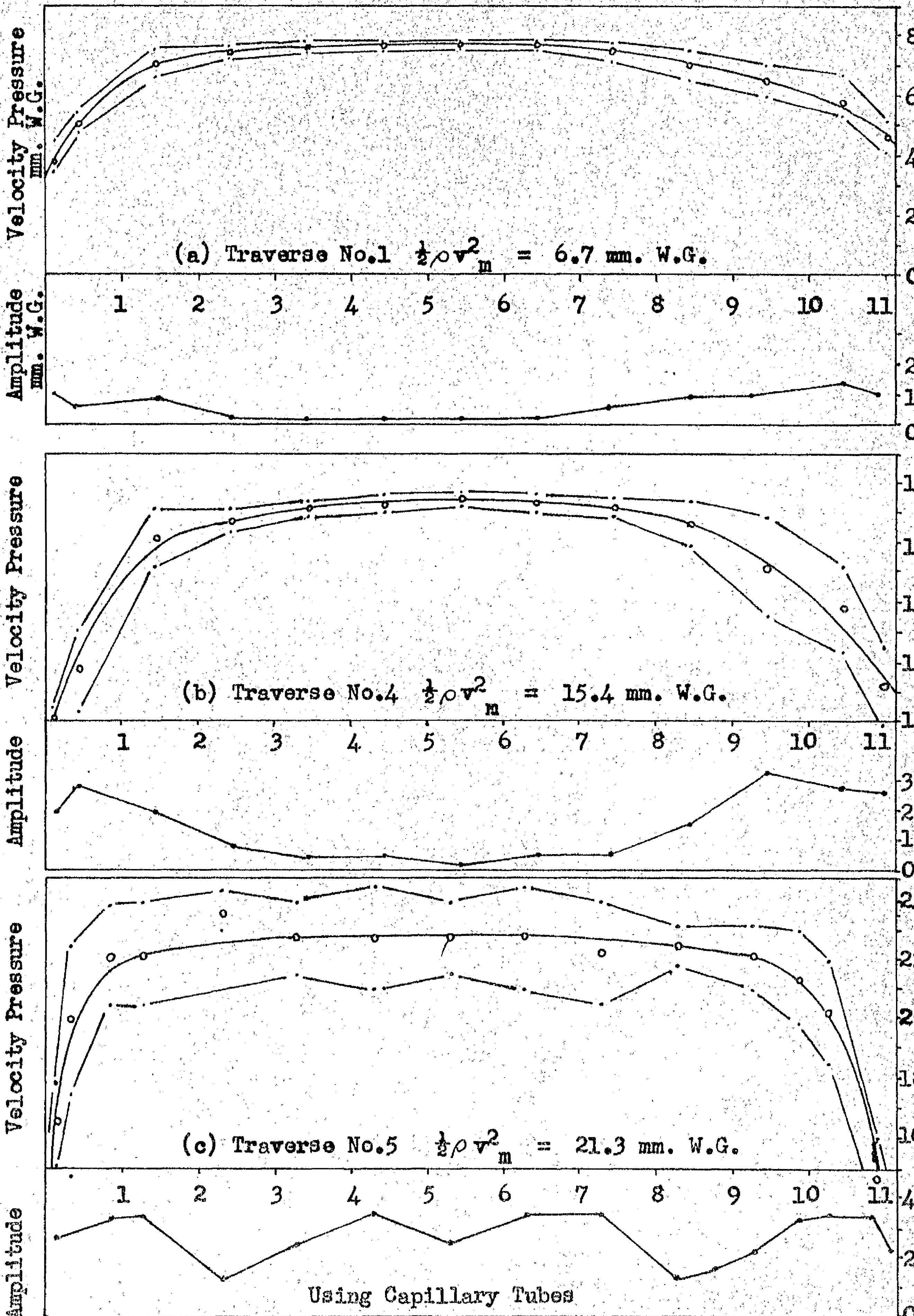


Fig. 3.9 Velocity Traverses and Amplitude of Fluctuation at Tap 12

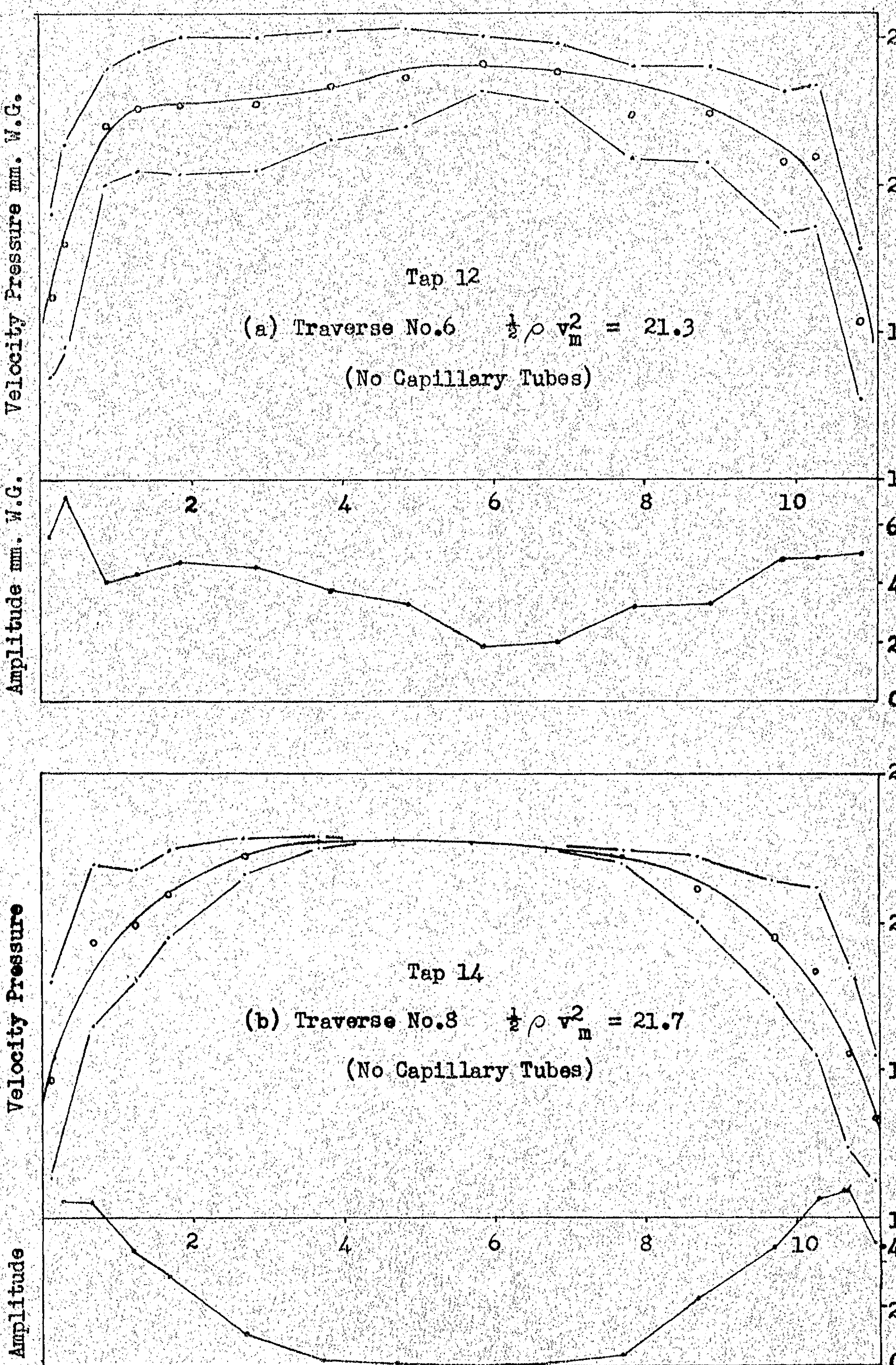


Fig. 3.10 Comparison of Velocity Traverses at Taps 12 and 14

pressure readings it was decided, in view of their magnitude, to plot the upper and lower limits of the fluctuations, as seen on the manometer. It soon became apparent that the traverse measurements were subject to fairly large fluctuations and the amplitude of these has also been plotted. In general, it can be said that the amplitude at the duct centre is usually less than at the sides while the fluctuations at the lower air speeds is much less than at the higher. This does not, of course, mean that the calculated velocities are more accurate at the lower speeds since the percentage variation from normal remains about the same.

An attempt was also made to find the effect of damping the pitot-static tube readings by using short lengths of capillary tubing inserted near the instrument. Figs. 3.9(c) and 3.10(a) show traverses at the same mean air velocity — the former utilising capillary tubes, the latter without. It can be seen that no improvement was obtained.

Also, more particularly at the lower mean air speeds, it was found that a pronounced asymmetry existed in the vertical velocity profiles, these being more smoothly rounded at the upper part of the duct. It was thought that this effect was probably due to the proximity of the perspex half sections which allow, due to the smooth nature of their surface, the more rapid achievement of a fully developed velocity profile.

It was becoming apparent that the large fluctuations at tap (12) were most likely due to the air flow disturbance caused by the presence of the flowmeter only 1.12 feet upstream of it. Although it had been thought that the disturbance could be ignored it was evident that it was, in fact, sufficient to spoil the accuracy of the traverse.

Therefore, in an attempt to improve the accuracy it was then decided to conduct traverses at tap (14), approximately 5 diameters upstream of the flowmeter and 7 diameters downstream from the duct inlet.

A specimen traverse is shown in Fig. 3.10(b). It is immediately seen that, especially at the duct centre, the pressure fluctuations have become very small even at this, the maximum air speed. Nearing the side walls, the amplitude of the fluctuations again increases.

This pointed to the interesting possibility, described in Chapter 5, of using a central pitot-static tube at this position in the duct. Since the flowmeter reading was subject to fairly large fluctuations, measurement of the variations in air quantity flowing with the movement of the cages in the shaft was well nigh impossible. A central pitot static tube would show this, qualitatively at least. A preliminary test of this stage showed a 10 per cent reduction in the reading at tap (14) when the cages were together at the centre of the duct compared with when they were far apart.

It was noted, also, that some asymmetry in the velocity head profile still existed. One fairly likely reason for this could be the presence of the pitot static tube shaft but unfortunately it was not possible to check this by traversing from the opposite direction since no tapping was available. Bad fitting of the gaskets could conceivably have an adverse effect but it is improbable that both joints – one at tap 12, the other at tap 14 – were the same.

As stated, traverses were conducted at both tappings and the results of these are given in Table 3.1.

1	2	3	4	5	6	7	8	9	10
Tap	No.	Flowmeter			Conditions		Calc.	$\frac{h_{AV.}}{h_m}$	$\sqrt{\frac{h_{AV.}}{h_m}}$
		h_m	Range	Ampl'd	Temp. ^oF	Press. "Hg	$h_{AV.}$		
12	1	6.7	6.3 – 7.1	0.8	71.0	30.15	6.5	0.970	0.986
	2	9.2	8.6 – 9.7	1.2	65.5	29.84	8.7	0.892	0.941
	3	11.9			66.0	30.44	11.6	0.976	0.987
	4	15.3	14.6 – 15.9	1.3	71.0	30.15	15.0	0.955	0.977
	5	21.3	19.5 – 23.0	3.5	65.5	29.84	21.7	1.018	1.010
	6	21.3	19.8 – 22.8	3.0	69.5	30.29	21.3	1.000	1.000
	7	21.3			71.0	29.71	21.6	1.011	1.005
14	8	21.7	21.3 – 22.2	0.9	67.0	30.47	19.7	0.910	0.955
	9	21.3	20.7 – 22.0	1.3	69.0	30.36	20.0	0.940	0.975

Table 3.1 Flowmeter Constants

Columns 1 and 2 give the tapping and the traverse number; 3, 4 and 5, the mean velocity pressure reading of the flowmeter, its range and its amplitude respectively; 6 and 7, the temperature and pressure obtaining; 8, the average velocity pressure calculated from the traverse. Column 9 lists the correction coefficients for the mean velocity head; Column 10, those for the mean velocity.

It can be seen that these coefficients fell mainly in the range 0.9 to 1.0 , though only in one case was a value of exact unity obtained. At the maximum air flow, it can be seen that for tap (12), the coefficients are all very close to 1. At tap (14) they are anything up to 9 per cent less. However, it must be remembered that the flowmeter reading itself is subject to wide fluctuations (Columns 4 and 5) and does not easily lend itself to precise measurement in spite of the care exercised in reading the manometer.

With these facts in mind it was decided to use a correction factor of unity in all subsequent computations.

The Pressure Drop Coefficient

In common with Stevenson and Wilkie, all resistances were determined as a Pressure Drop Coefficient (P.D.C.). This was decided, not only to give ease of comparison with the data of these workers but also because it was simple to evaluate and was a dimensionless ratio and could thus be compared with the results of other investigators, whatever system of units they used.

The P.D.C. was defined by Stevenson as the loss in pressure due to any particular part of the arrangement divided by the mean velocity pressure of the air. It was used since energy losses in the air were being investigated. In addition, it could be applied directly to a full scale arrangement if geometrical similarity was maintained and change in Reynolds number is ignored.

When calculating the P.D.C. of a particular component in the test length, a cage say, it was necessary to assume that the overall P.D.C. was the algebraic sum of the $P.D.C._d$ caused by the duct resistance and the $P.D.C._c$ due to the cage

$$\text{i.e.} \quad P.D.C._c = P.D.C. - P.D.C._d$$

It has been shown that the resistance to flow in a duct is measured by the difference in the static pressure at the ends of the duct (15).

Briefly, for a straight parallel pipe, with flow parallel to the sides, the static pressure is constant at any cross-section and, at a sufficient distance from the inlet, the velocity distribution is the same at all cross-sections. Now the mean total head is equal to the static pressure p plus the velocity head corresponding to the mean kinetic energy of unit volume of air passing the section. Let this velocity be $\overline{v}$, hence the velocity head is $\frac{1}{2}\rho\overline{v}^2$.

Consider a length of pipe AB. The mean total head at A will be greater than that at B by a pressure P, which is equal to the work done on unit volume of the fluid in moving from A to B, i.e. to the resistance of the length AB of the pipe. Hence,

$$\frac{1}{2}\rho_A\overline{v}_A^2 + p_A = \frac{1}{2}\rho_B\overline{v}_B^2 + p_B + P$$

It was assumed that the velocity distribution at A and B are identical, so that $\overline{v}_A = \overline{v}_B$. Further, if the static pressure difference $p_A - p_B$ is small compared with p_A or p_B it can be assumed that $\rho_A = \rho_B$. The equation then becomes

$$P = p_A - p_B$$

The physical significance of this equation is that (in the special case of flow in a straight length of parallel pipe in which the velocity distributions at the two ends are the same) the resistance is measured by the difference in static pressure

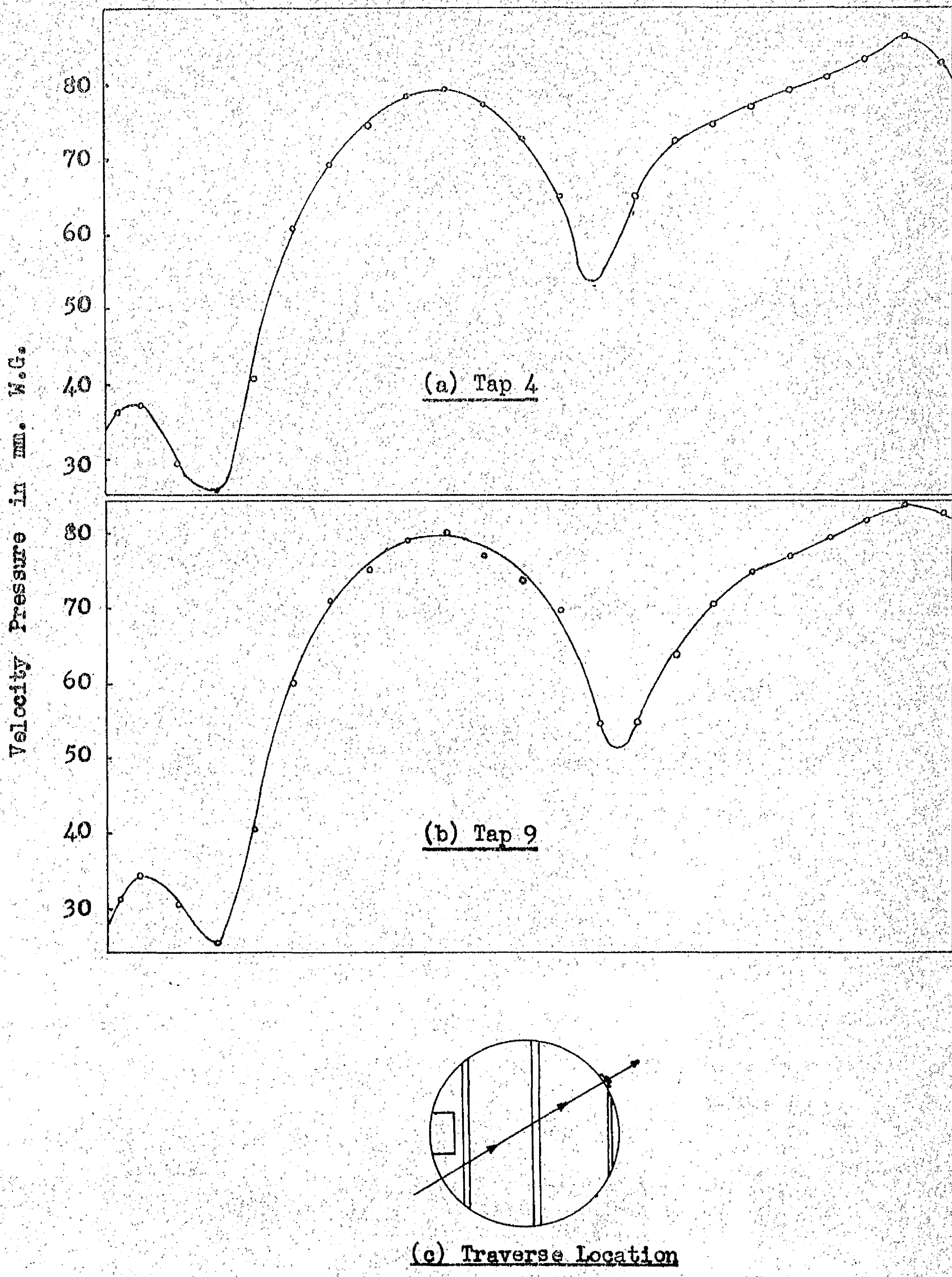


Fig. 3.11 Velocity Traverses in Lined Ducting

at the two ends.

This gives the P.D.C. $= \dfrac{p_A - p_B}{\frac{1}{2}\rho \bar{v}^2}$

As a check, it was decided to conduct velocity traverses at tappings ④ and ⑨ where they could be done on almost identical duct diameters. The graphs of velocity to duct diameter are shown in Fig. 3.11 (a) and (b), and it can be seen that they are practically identical. Any differences were probably due to slight variations in the trajectory of the traverse.

The approximate traverse diameter is shown in Fig. 3.11 (c).

Interaction of Two Stationary Cages

The next stage was to investigate the effect of the interaction of the cages, as they passed, on the P.D.C. at the centre of the duct length. Test length ⑤ — ⑨ was chosen for this purpose since it provided a fairly short test length in which any abnormal effects would quickly be noticed. At the same time it was long enough to allow the cages to be moved over a considerable distance without interfering with the pitot-static tubes at each end of the test length. The relevant section of ducting is shown in Fig. 3.12.

The method used was to position the cages side by side

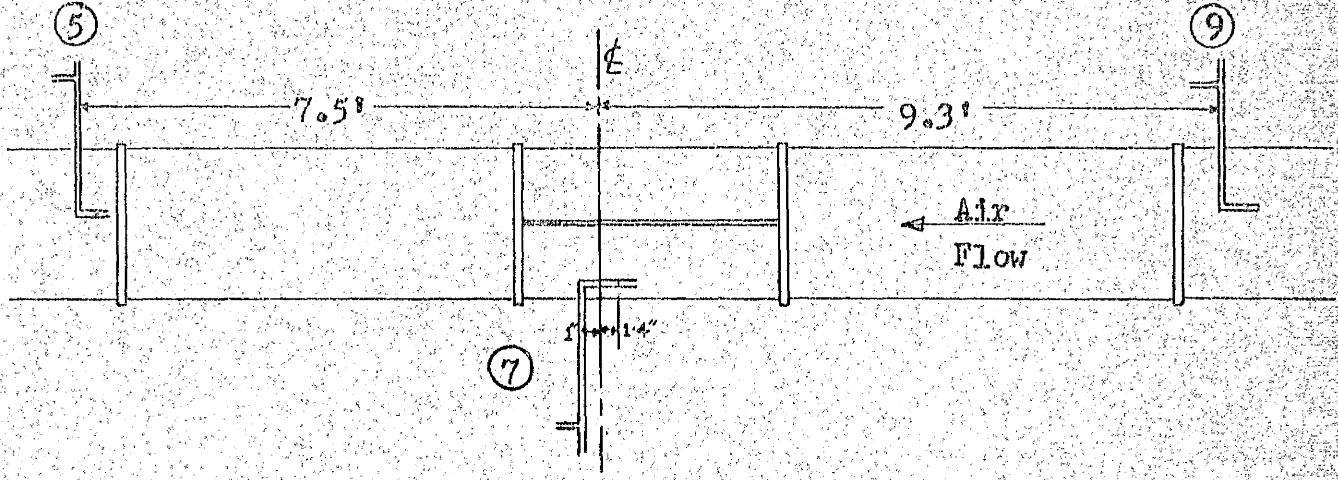
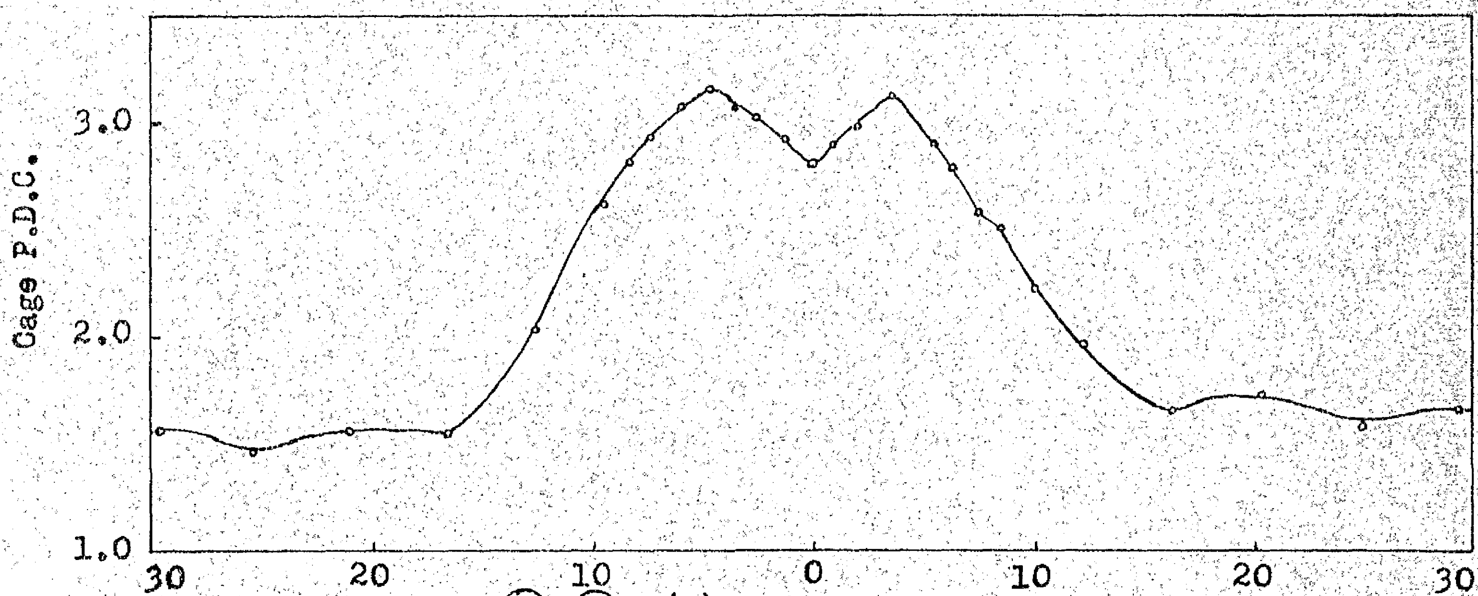
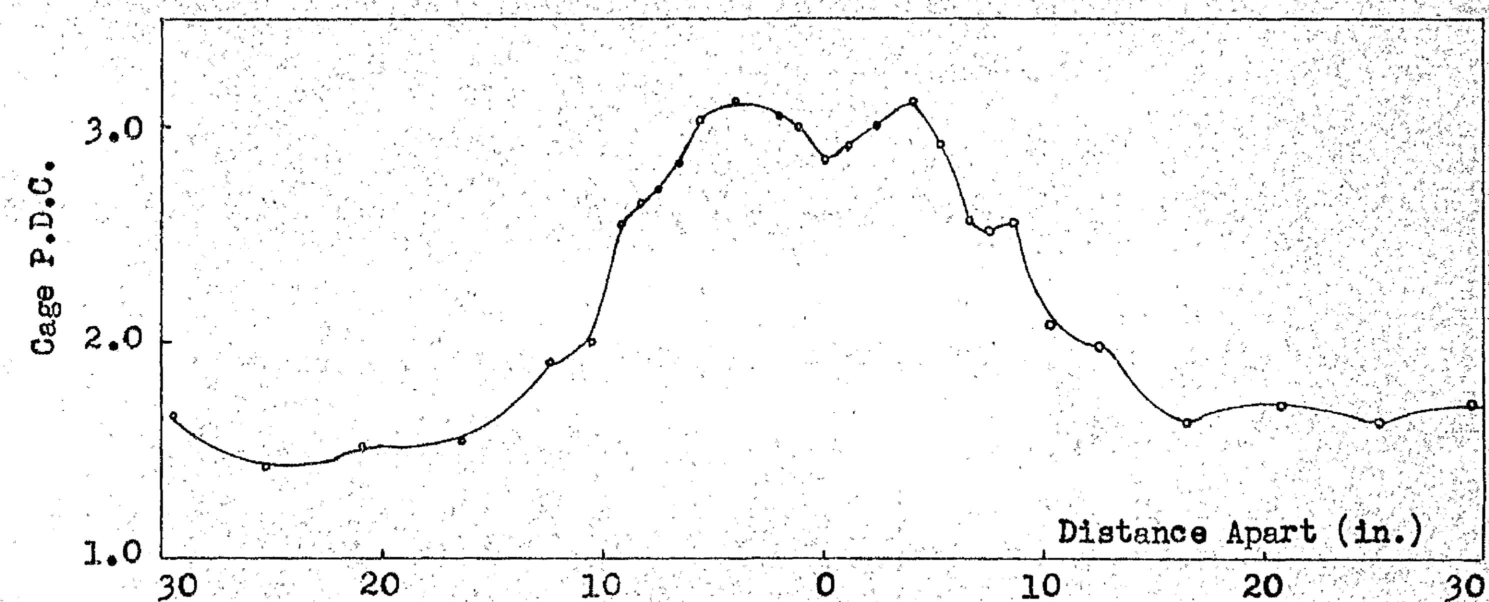


Fig. 3.12 Centre Compartment



P.D.C. (5)-(9) (a) High Speed



P.D.C. (5) - (9) (b) Low Speed

Fig. 3.13 Graphs of P.D.C. to Distances Apart of Cage Centre Lines and then to gradually move them apart, first in one direction, then in the other, till no further significant pressure changes took place. At the same time, measurements were taken of the differences in static pressure between taps ⑤ and ⑨ , ⑦ and ⑨ and ⑦ and atmospheric. In addition, readings were taken of the mean velocity pressure on the flowmeter and the distance apart of the cage centre lines at each cage position.

This was done at each of two mean air speeds, one the maximum 61.7 f.p.s., the other much lower 32 f.p.s. and the overall P.D.C.'s were calculated. It might be mentioned also, that the static pressure differences were very steady, thus affording accurate measurement of their values on the manometer.

At the end of each test, the various readings were taken with the test length empty of cages. Thus, the P.D.C. of the empty duct could be calculated and by subtracting this from the total P.D.C., the cage P.D.C.'s could be determined for the various positions, from outside the zone of interaction to the side by side position.

Figs. 3.13 (a) and (b) show the graphs of the cages' P.D.C., as measured in test length ⑤ to ⑨ , plotted to a base of distance apart of the cage centre lines, for the two air speeds. It is at once apparent that these graphs are almost identical showing that the P.D.C. is constant for this range of Reynolds numbers. Also, the zone of interaction of the two cages, each 9" long, extended till the cage centre lines were 18" approximately apart: or, till the end faces of the cages were 9" apart.

The maximum P.D.C. of 3.16 did not occur when the cages were exactly side by side but when their centre lines were about 4 inches apart. Also, it can be seen that the cage P.D.C. does not settle down to a constant value outside the zone of direct interaction of the two cages. It was soon recognised that both these effects were due to the influence of the buntons. Stevenson had noted this when changing from rope guide to rigid guide models and earmarked it for future investigation.

This effect was also shown by Hoerner who demonstrated the extent to which the drag of a streamline body was increased through the addition or presence of a comparatively small obstacle. Even without touching the main body, the small one had an appreciable effect upon the flow pattern (separation from the rear)and drag coefficient of that shape. Of course, the effect would be less for a main body with some separation to begin with and a correspondingly higher drag coefficient.

At this stage it was evident that the buntons had quite an appreciable effect on the cage P.D.C. and a fuller investigation would be appropriate. This was especially desirable in view of the assumption, made earlier, that the total P.D.C. equalled the sum of the duct (including lining) and cage P.D.C.'s. Even at this stage it could be seen that these interference effects were exerting a relatively great influence on the accuracy of the tests.

However, apart from the anomalies associated with the buntons, the graphs were much as expected from the results of previous workers, showing, as they did, the considerable increase in P.D.C. as the cages passed.

The Influence of Bunton Spacing on the Cage P.D.C.

Having recognised that the presence and spacing of the buntons had an appreciable influence on the value of the cage P.D.C. it was now necessary to investigate this effect in greater detail. The results previously obtained for test lengths ⑤ to ⑨ and ⑦ to ⑨ were used for this purpose. In addition, tests were carried out in test length ① to ② for reasons to be explained later.

As depicted in Fig. 3.14, the buntons were arranged in an even fashion, with all three of each set in the same cross section. From the previous results, the P.D.C. of both cages (length ⑤ to ⑨) and of a single cage (test length ⑦ – ⑨) were known and

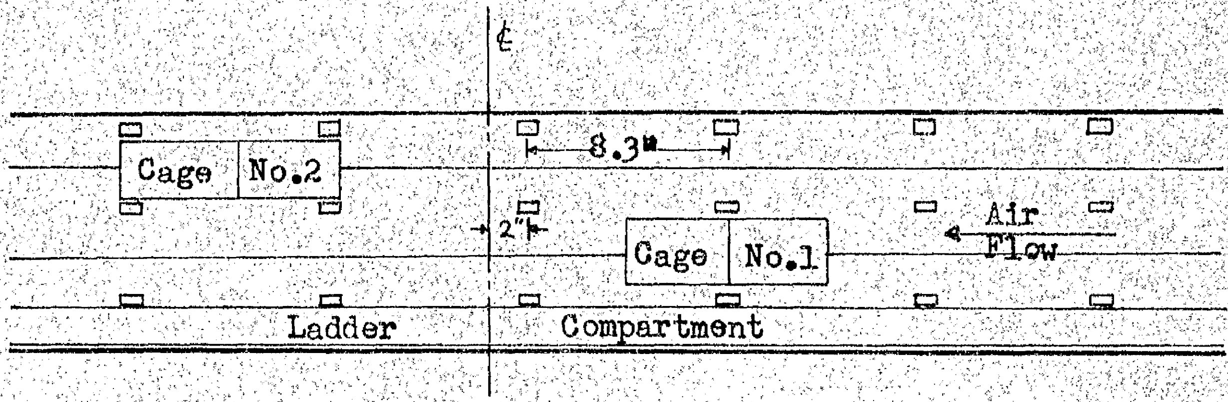


Fig. 3.14 Bunton Arrangements at Duct Centre and Relative Positions of Cages

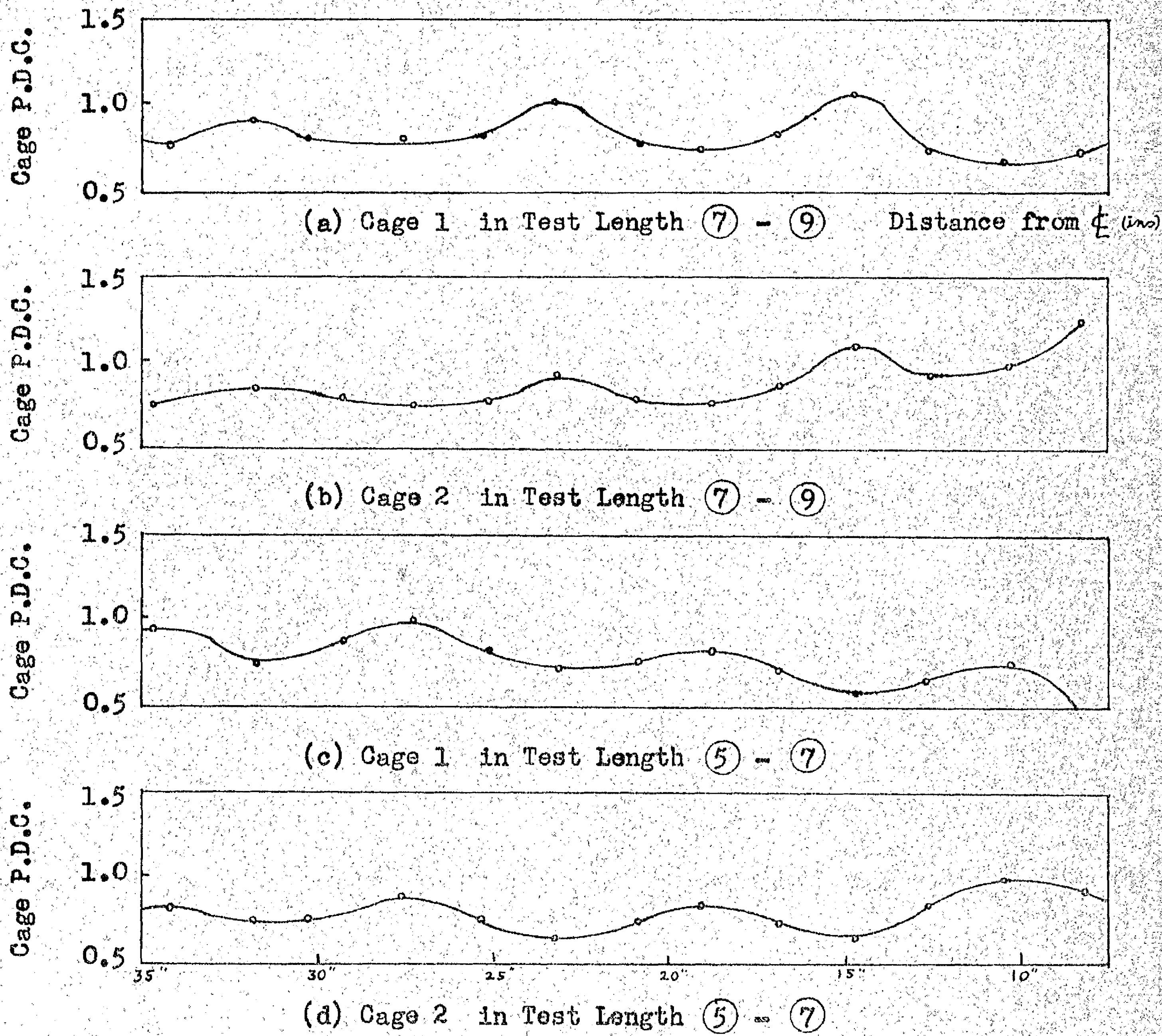


Fig. 3.15 P.D.C. of a Cage at Duct Centre

by subtraction of these two, the P.D.C. of the cage in test length (5) to (7) could be obtained. These values are plotted to a base of cage distance from the centre line (starting just outside the zone of direct interaction of the two cages) in Fig. 3.15.

It is immediately apparent that the relative positions of buntons and cage had a great effect on the P.D.C. The peaks corresponded to a cage position where there was a bunton at each corner — the bunton spacing being equal to the cage length; the troughs to the position where there were two buntons, one on each side of the cage and halfway along it. Intermediate arrangements gave P.D.C. values between these two extremes and with movement of the cage along the duct, the P.D.C. took the form of a wave whose wave length was equal to the bunton spacing.

Another interesting feature which emerged from a study of the graphs of Fig. 3.15 was that, outside the zone of direct interaction of the two cages, the mean P.D.C. value and the mean wave amplitude of cage 1 were greater than the corresponding figures for cage 2. These values are given in Table 3.2, columns 1 and 2.

	1	2	3	4	5
	Test Length ⑤-⑨				Test Length ①-②
	Cage 1	Cage 2	$\sum$1&2	Both Cages	Cage 1
Mean P.D.C.	0.87	0.79	1.66	1.65	0.77
Mean Amplitude	0.29	0.21		0.13	0.16

Table 3.2

It can be seen that the values obtained differ by more than a negligible amount. The reason for this difference must be the result of the presence of the ladder compartment. Not only does this cause an unsymmetrical flow pattern (see Fig. 3.11), but also the justaposition of cage wall and ladder compartment hinders the airflow and increases the resistance.

Fig. 3.16 illustrate the staggered bunton arrangement in test length ①-② . Cage 1 only was tested and the resulting plot of P.D.C. to distance moved is presented in Fig. 3.17. The mean P.D.C. and wave amplitude are given in Table 3.2, column 5 and it can be seen that there is a significant decrease from the corresponding figures for Cage 1 (column 1) at the centre compartment. The wave length also has decreased to half the bunton spacing distance, the maximum and minimum values corresponding to the cage

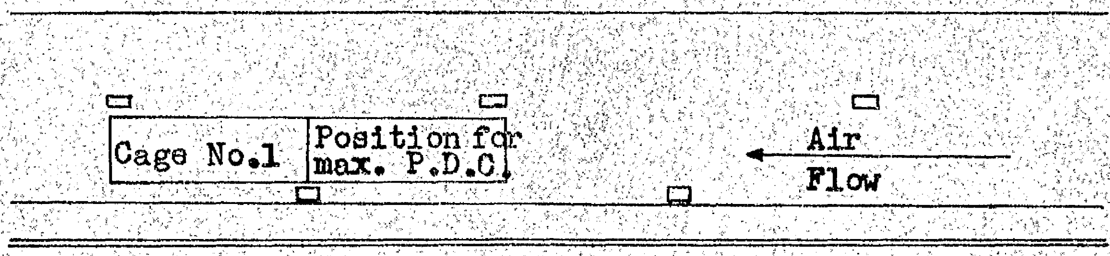


Fig. 3.16 Bunton Arrangement at Test Length ① - ②

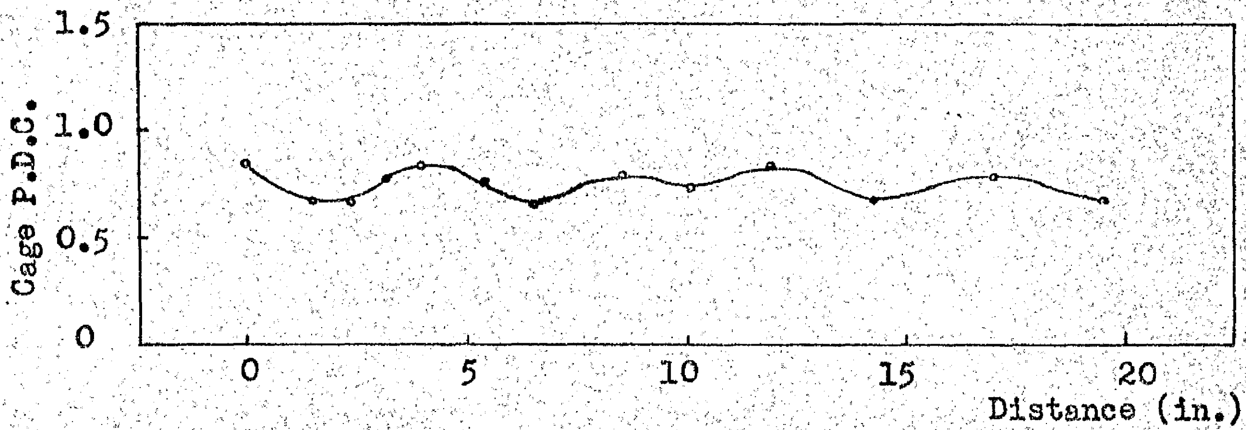


Fig. 3.17 Cage P.D.C. at Test Length ① - ②

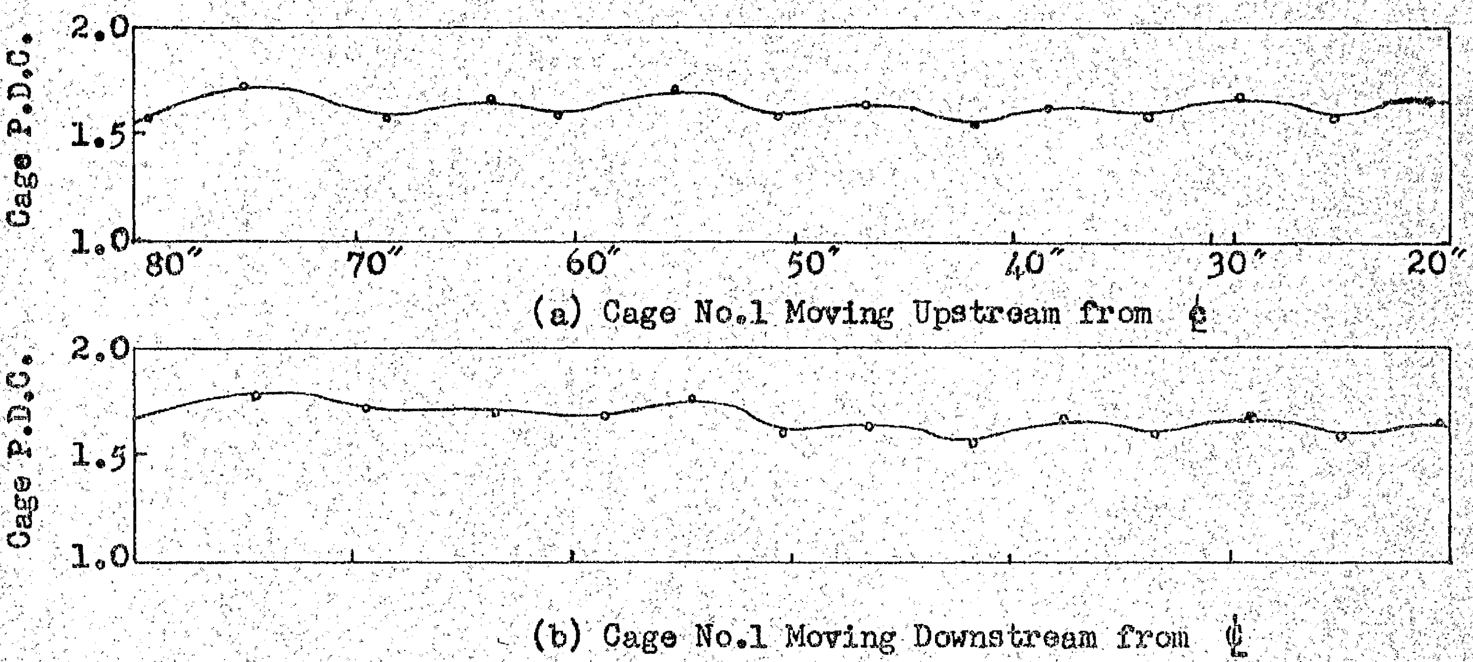


Fig. 3.18 P.D.C. of Both Cages – Test Length ⑤ - ⑨

positions shown.

Fig. 3.18 illustrates the plots of the P.D.C. of both cages, outside the zone of interaction but within test length ⑤ –⑨ . Reference to Fig. 3.14 shows that when Cage 1 is in the position corresponding to maximum P.D.C., Cage 2 is in the position corresponding to minimum P.D.C. and vice versa. This position, in fact, is the one corresponding to maximum P.D.C. for the two cages together. Thus, interference of the two waves occurs, resulting in the graphs shown. The average value of P.D.C. for the two cages is given in Table 3.2, column 4 together with the mean amplitude of the wave.

Graphs 3.18 (a) and (b), representing readings taken on each side of the centre line are almost identical, as expected. It may be said that although all the points on the graphs are taken from the high speed tests, the results from the low speed tests were identical.

It was also noticed that there was a slight, but perceptable increase in P.D.C. as the distance between the cage centres increased. It is thought that this is probably due to the fact that both cages were wholly in the rougher metal ducting of the wind tunnel when they were further apart.

The P.D.C. of the Cage in an Unlined Duct

From a comarison with previous works it was seen that the P.D.C. value obtained for the cage (approximately 0.8) in the furnished shaft was very much higher than the figures quoted by Stevenson for cages of a similar size and fill coefficient. It was therefore thought that it would be of use to measure the cage P.D.C. in an unlined section of the ducting.

As the model stood, the only possible section was that at the inlet end of the wind tunnel, Fig. 3.8. However, to avoid taking measurements too near the inlet it was necessary to modify the ducting by removing a perspex section, shifting the inlet sections and inserting a 6 foot section between the bell mouthed entry and the test length. A $\frac{1}{4}$ inch rod was fitted to the cage, and holes bored in the top and bottom of the ducting so that the cage position could be varied, in the vertical plane, right across the duct section. The arrangement is shown in Fig. 3.191

Initially, the P.D.C. of the empty test length was found. Also, an estimate was made of the P.D.C. of the length of rod necessary to support the cage by inserting the appropriate length into the ducting and measuring the additional pressure drop. The pressure drop of test length ⑫ — ⑮ was then measured, as the cage traversed the wind tunnel.

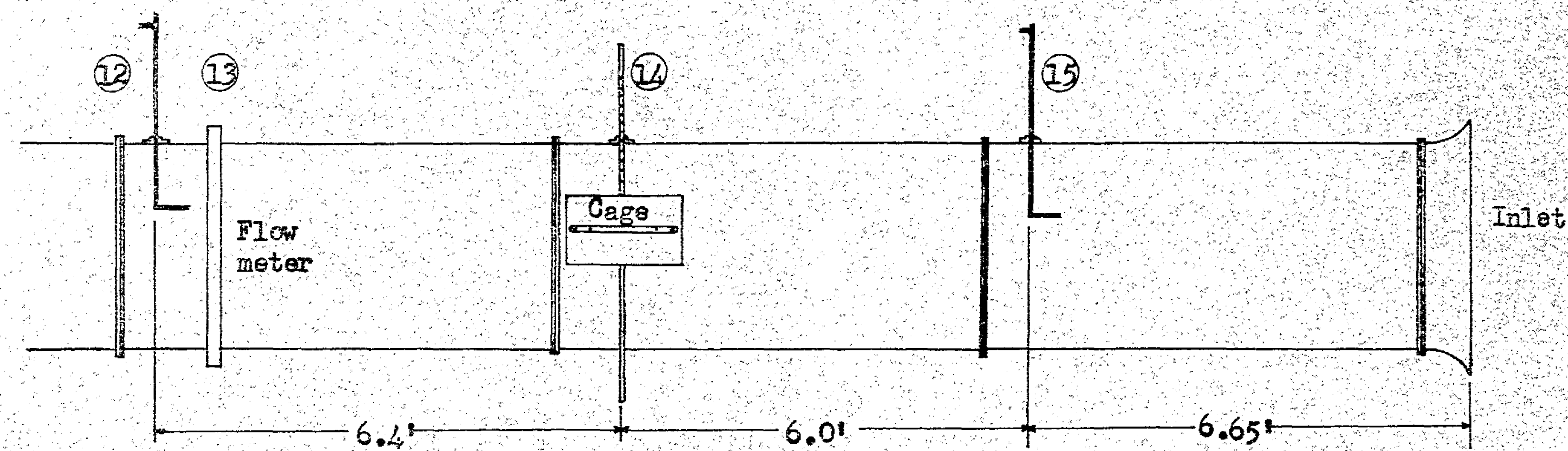


Fig. 3.19 Duct Layout for Test on Cage P.D.C. in Unlined Shaft

It was found that even very slight yawing of the cage produced a considerable increase in the test length pressure drop. Actually, this provided a simple means of ascertaining that the cage was, in fact, facing directly upstream since in this position, the pressure drop was at its lowest value.

The mean velocity pressure was also measured, and from these results, the cage P.D.C. was calculated for each cage position.

In the first instance, the cage was in the position shown in Fig. 3.19. The next stage was to repeat the procedure, this time with the cage turned 90 degrees, thus simulating the effect of the cage moving across the duct at right angles to the first diameter.

The results of these traverses are shown in Fig. 3.20 together with sketches showing the cage positions.

Figs. 3.20 (a) and (b) show the cage position and P.D.C. values at each station in the first traverse. It was found that the arithmetic mean of the P.D.C. values was 0.355, all the traverse values being within 4 per cent of this. The second traverse, Figs. 3.20 (c) and (d), gave a mean of 0.348, the individual traverse values again being within 4 per cent of this value.

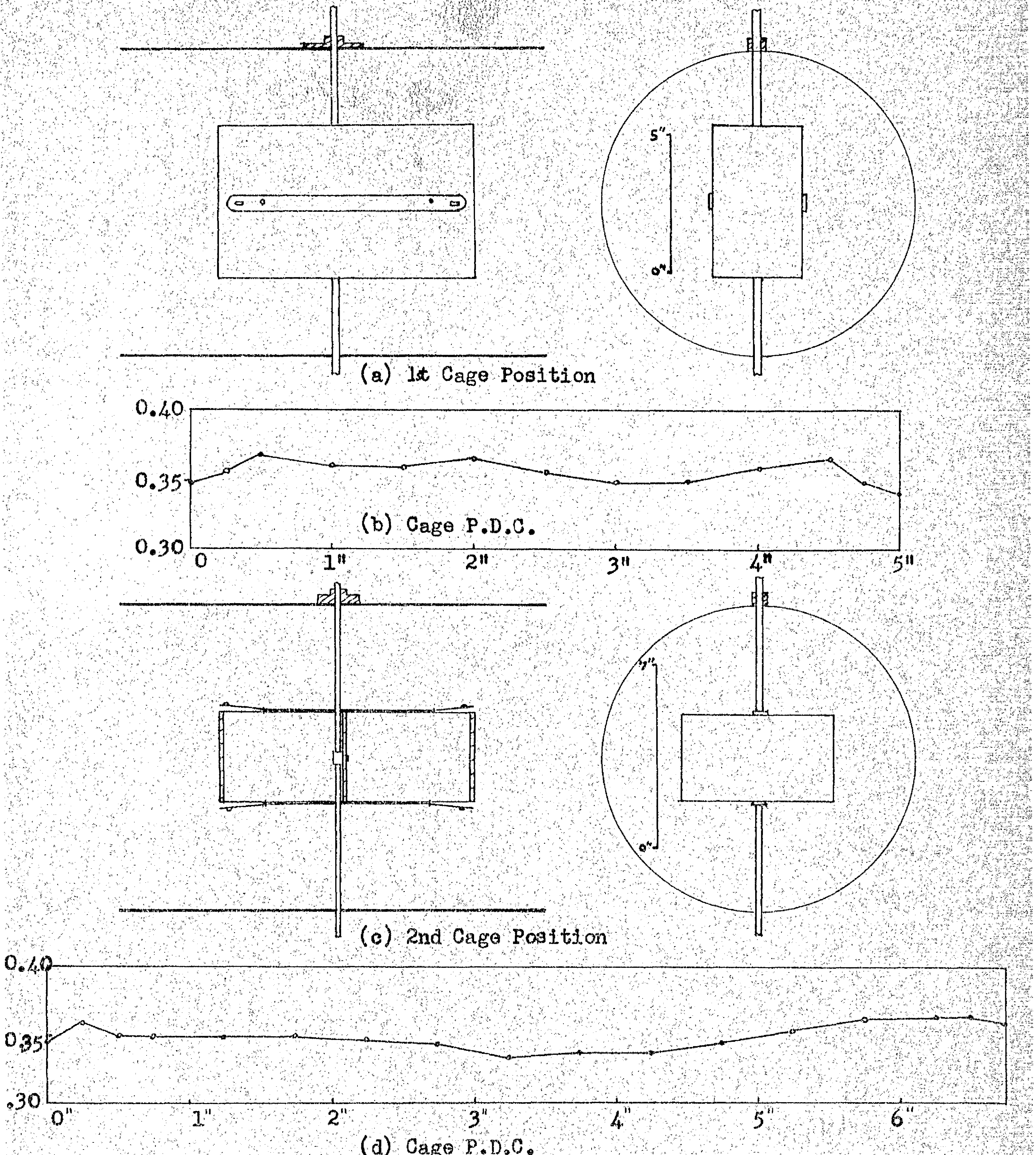


Fig. 3.20 P.D.C. Traverse Measurements

The immediate conclusion which can be reached from these results is that, in general, the P.D.C. of a stationary cage is practically the same for all positions in a duct. However, this does require some qualification since the graphs of Fig. 3.20 (b) and (d) are by no means straight and constant.

It can be seen that there is a tendency for the P.D.C. to be slightly lower in value towards the upper half of the duct— this effect may be due to the slight asymmetry of the vertical velocity profile which was noted in a previous section. Also, as the cage approaches the duct walls, there appears to be a slight increase in the P.D.C. value. However, these effects are small and for practical purposes it would appear that there is only a slight error in assuming that the P.D.C. is constant.

Of much greater interest here is the fact that the cage P.D.C., at a mean value of approximately 0.35 is less than half that obtained with the cage situated in the shaft lining. This clearly shows the considerable influence of interference between the cage and the shaft furnishings, notably the buntons and the ladder compartment.

As stated previously, the effect was noted by Stevenson when changing from rope guides to rigid guides. Unfortunately, different types of cages were used in these installations so that the interference effects for the types of shaft furnishing used cannot be estimated.

CHAPTER 4

APPROXIMATE THEORY OF PRESSURE DROP DUE TO CAGES MOVING IN A SHAFT [16]

The Formulation of the Preliminary Equation

The problem of the determination of the pressure drop due to cages moving in mine shafts involves so many variables, even when some simplifying assumptions have been made, that it is quite impossible to solve it by means of model investigations alone.

It has been shown (16) that the general relationship determining the dimensionless pressure drop coefficient takes the form

$$\frac{p}{\frac{1}{2}\rho V^2} = f\left[M, F_r, R_e, \frac{V_c}{V}, S\right]$$

where M = Mach No. R_e = Reynolds No.

F_r = Froude No. S = Scale Factor

$\frac{V_c}{V}$ = Ratio of cage speed to mean air velocity

Constancy of the Scale Factor is required for geometrical similarity;

the ratio $\frac{V_c}{V}$ on account of kinematic similarity; Mach, Froude and Reynolds numbers on account of dynamical similarity.

In this chapter an attempt will be made to determine the form of the function $f\left[\quad\right]$ on the basis of some simple theoretical considerations.

The following introductory assumptions are made.

(a) The compressibility of air is neglected since the mean air velocities in mine shafts are far below the velocity of propagation of sound waves in air.

(b) The influence of the gravitational forces of air is neglected. This is a normal assumption made in most aerodynamical considerations on air resistance.

(c) The friction factors do not depend on the value of Reynolds number. This assumption has been found to be true in almost all measurements which have been carried out in mine shafts and shaft models. This was also proved in the model tests described in Chapter 3.

(d) The pressure drop, as a result of interaction between the cages and the buntons is not considered separately and the presence of buntons in the shaft is taken into account in the overall friction coefficient of the shaft. Considering the great influence of the buntons and their spacing on the cage P.D.C., which was shown very clearly in the model tests, this assumption is recognised as the weakest point of the constructed theory. Although the extension of the following theory to include the bunton spacing seems to be possible, it would require a great amount of additional investigation. Of course, in shafts with rope guides instead of rigid guides and buntons, this restricting assumption loses its importance.

In order to calculate the pressure drop caused by the presence of a cage in the shaft, the sum of the energy losses and eventual energy gains due to the moving cages has to be established.

Usually, the energy of the streams flowing in ducts and pipes is expressed in terms of total head which is the energy of one unit of weight of fluid flowing. The difference in total heads in two cross sections of the same area is equal to the difference in pressure heads.

The P.D.C. of a cage C_c is defined as the ratio of pressure head losses ΔH_p caused by the cage, to the velocity head calculated for the mean shaft air velocity.

$$C_c = \frac{-\Sigma \Delta H_t}{H_v} = \frac{-\Sigma \Delta H_p}{H_v} = \frac{-\Sigma \Delta p}{\frac{1}{2} \rho V^2} \quad \ldots\ldots\ldots \quad 4.1$$

The following sources of energy change due to the cage will be considered.

(a) Facing wall of cage :

$\pm \Delta H_1'$ Energy gained or lost due to facing wall of moving cage.

$- \Delta H_1''$ Energy lost in contraction or expansion of the air stream in the cross-section of the facing wall.

(b) Space along the length of the cage :

$\pm \Delta H_2'$ Energy lost or gained along the side walls of the cage.

$- \Delta H_2''$ Additional energy loss along the shaft wall due to the increased air velocity in the space between the shaft wall and the cage sides.

(c) Rear wall of the cage :

$\pm \Delta H_3'$ Energy gained or lost in the wake behind the cage.

$- \Delta H_3''$ Energy lost in contraction or expansion of the air stream in the cross-section of the rear wall of the cage.

It will be assumed that all the above energy changes are independent of each other and are therefore additive. The larger the cage, the better this assumption holds. However, some results obtained by Stevenson in model tests on the influence of nose and tail pieces on the cage state that this assumption is valid, to a certain extent, for relatively short cages, with one or two decks.

The energy change quantities will now be calculated.

$\pm \Delta H_1'$ The drag force acting on the facing wall of the cage is defined by

$$F_{D_1} = C_{D_1} \cdot A_c \frac{\rho (V - V_c)^2}{2} \text{ sign } (V - V_c) \quad \ldots\ldots\ldots\ldots \quad 4.2$$

where C_{D_1} = Drag coefficient of facing wall dependent on its shape.

A_c = Area of facing wall of cage.

V = Mean shaft air velocity.

V_c = Cage speed

ρ = Air density.

If the cage moves in the opposite direction to the air flow, the cage speed has to be taken with a negative sign. When the cage moves in the same direction as the air, but at a greater speed than the air velocity, the drag force does not resist the air flow but even helps it. This means that the drag force changes its direction and the cage thus contributes energy to the air stream. To denote this change of direction of the drag force, the function sign $(V - V_c)$ has been introduced in equation 2 and takes the following values

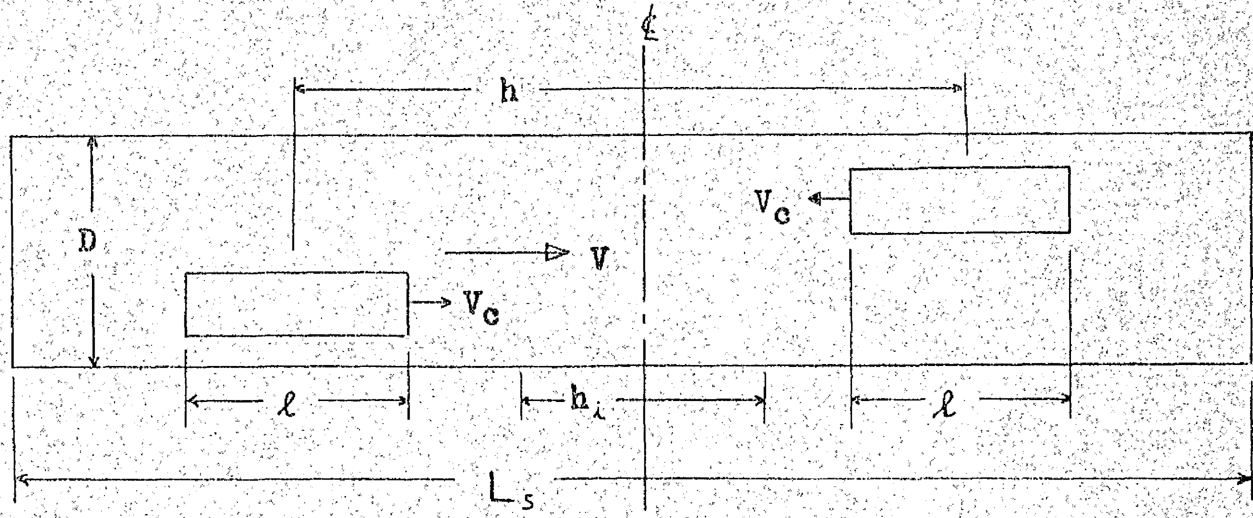


Fig. 4.1

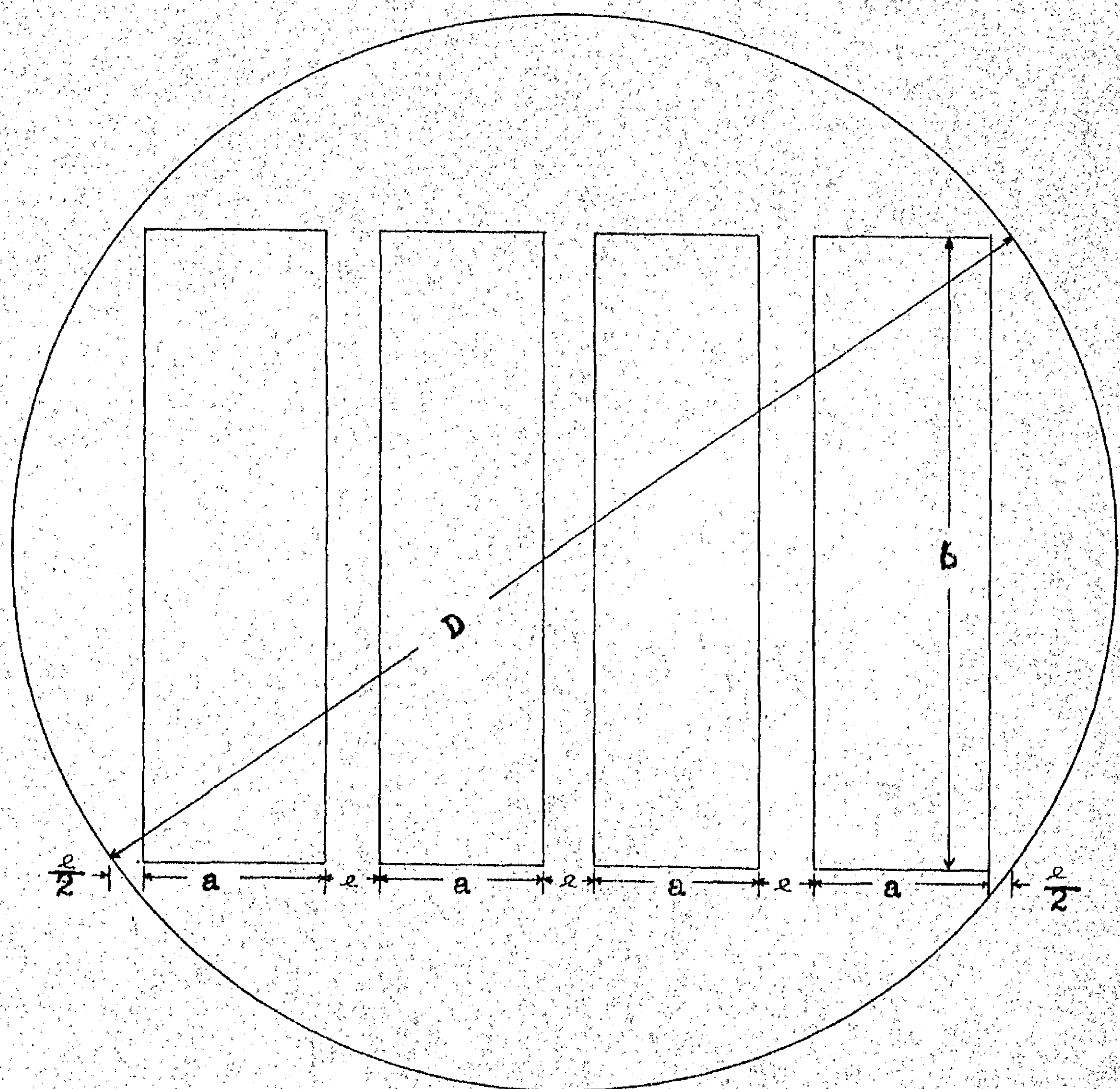


Fig. 4.2

$$\text{sign}\,(V - V_c) = 1 \qquad \text{when} \quad V \geq V_c$$

$$\text{sign}\,(V - V_c) = -1 \qquad \text{when} \quad V < V_c$$

The work done by the drag force F_{D_1} over a distance L_s is

$$W_{D_1} = F_{D_1} \cdot L_s = \frac{C_{D_1} \; A_c \; \rho (V - V_c)^2 \; \text{sign}\,(V - V_c) \; L_s}{2} \quad \ldots\ldots 4.3$$

The total weight of air in the length L_s is

$$G_a = \rho \, g \left[(L_s - \ell) \, A + \ell \, (A - A_c) \right] \quad \ldots\ldots 4.4$$

where g = gravitational acceleration

A = cross sectional area of the shaft

ℓ = length of the cage

Thus

$$-\Delta H_1' = \frac{W_{D_1}}{G_a} = \frac{C_{D_1} \; A_c \; \rho (V - V_c)^2 \; \text{sign}\,(V - V_c) \; L_s}{2 \rho \, g \left[L_s A - \ell A_c \right]} \quad \ldots\ldots 4.5$$

By introducing the following dimensionless ratios

$$C_f = \frac{A_c}{A} = \text{fill coefficient} \quad \ldots\ldots 4.6$$

$$C_v = \frac{V_c}{V} = \text{ratio } \frac{\text{cage speed}}{\text{mean air speed}} \quad \ldots\ldots 4.7$$

$$C_\ell = \frac{\ell}{L_s} = \text{ratio } \frac{\text{cage length}}{\text{shaft length along which pressure drop is measured.}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.8$$

equation 5 can be written

$$-\Delta H_1' = \frac{C_{D_1} \; C_f \, (1 - C_v)^2 \; \text{sign} \, (1 - C_v)}{1 - C_\ell \; C_f} \cdot \frac{V^2}{2g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.9$$

$\underline{-\Delta H_1''}$ The energy losses due to contraction or expansion of the air stream can be calculated using the Carnot Borde hypothesis, as follows

$$-\Delta H_1'' = \frac{k_c \, (V_1 - V)^2}{2g} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.10$$

where k_c = dimensionless factor

V_1 = velocity of air passing the cage.

On the basis of the equation of continuity, the relationship between the mean shaft air velocity V and the air velocity in the restricted space between the shaft wall and the cage sides V_1 can be expressed in the form

$$VA = V_1 (A - A_c) \quad \ldots\ldots\ldots \quad 4.11$$

Hence $$V_1 = V \frac{A}{A - A_c} = V \frac{1}{1 - C_f} \quad \ldots\ldots\ldots \quad 4.12$$

Inserting relation 12 into equation 10 one obtains

$$-\Delta H_1'' = \frac{k_c \; C_f^2}{(1 - C_f)^2} \cdot \frac{V^2}{2g} \quad \ldots\ldots\ldots \quad 4.13$$

$\underline{\pm \Delta H_2'}$ In order to determine the energy losses or gains in the space between the shaft walls and the sides of the cage, the well known Darcy formula will be applied. Thus, the energy lost or gained along the cage sides is given by

$$-\Delta H_2' = \frac{f_c \, \ell \, (V_1 - V_c)^2 \; \text{sign} \, (V_1 - V_c)}{\frac{4 \, (A - A_c)}{2 \, (a + b)} \; 2g} \quad \ldots\ldots\ldots \quad 4.14$$

$$\text{Hence} \; -\Delta H_2' = \frac{f_c \, \ell \, (\frac{1}{1 - C_f} - C_v) \; \text{sign} \, (\frac{1}{1 - C_f} - C_v)}{\frac{4 \, A \, (1 - C_f)}{2 \, (a + b)}} \cdot \frac{V^2}{2g} \quad \ldots\ldots\ldots \quad 4.15$$

and finally

$$-\Delta H_2' = \frac{C_o\ f_c\ \ell \left[1-(1-C_f)\ C_v\right]^2\ \text{sign}\ \left[1-\ (1-C_f)\ C_v\right]}{D\ (1-C_f)^3} \cdot \frac{V^2}{2g}$$

.... 4.16

where f_c = dimensionless friction coefficient of the cage walls.

$$C_o = \frac{2\ (a+b)}{D} = \text{ratio}\ \frac{\text{perimeter of cage}}{\text{circumference of shaft}}$$

$\underline{-\Delta H_2''}$ The additional energy loss along the shaft wall caused by the presence of the cage is equal to the difference between the energy loss occurring due to the increased air velocity V_1 and the energy loss in normal flow with mean air velocity. V.

$$-\Delta H_2'' = \frac{f_d\ \ell\ V_1^{\ 2}}{2g\ \frac{4(A-A_c)}{\pi\ D}} - \frac{f_d\ \ell\ V^2}{\frac{4\ A}{\pi\ D}\ 2g} \qquad \text{............ 4.17}$$

Hence

$$-\Delta H_2'' = \frac{f_d\ \ell}{\frac{4A}{\pi\ D}\ 2g} \left[\frac{V_1^{\ 2}}{(1-C_f)} - \frac{V^2}{1}\right] \qquad \text{............ 4.18}$$

and finally

$$-\Delta H_2'' = \frac{f_d\ \ell\ C_f\ (3-3\ C_f + C_f^{\ 2})}{D\ (1-C_f)^3} \cdot \frac{V^2}{2g} \qquad \text{........ 4.19}$$

where f_d = dimensionless friction coefficient of the empty shaft.

$\underline{\pm \Delta H_3'}$ The energy lost or gained in the wake behind the rear wall of the cage is calculated in the same way as $\Delta H_1'$ due to the facing wall. However, a different drag coefficient C_{D_3}, depending on the shape of the rear wall of the cage, will be applied.

Hence,

$$-\Delta H_3' = \frac{C_{D_3} \quad C_f \ (1 - C_v)^2 \ \ \text{sign} \ (1 - C_v)}{1 - C_\ell \quad C_f} \ \frac{V^2}{2g} \qquad \ldots\ldots \ 4.20$$

$\underline{-\Delta H_3''}$ The energy loss due to expansion of the air stream behind the cage is calculated in the same way as $\Delta H_1''$ giving

$$-\Delta H_3'' = \frac{k_e \quad C_f^{\ 2}}{(1 \quad C_f)^2} \cdot \frac{V^2}{2g} \qquad \ldots\ldots\ldots\ldots \ 4.21$$

where k_e = dimensionless energy loss coefficient due to expansion of the airstream.

Inserting all six energy constituents into equation 1, the following general expression for the P.D.C. of the cage C_c is obtained.

$$C_c = \frac{(C_{D_1} + C_{D_2})\ C_f\ (1 - C_v)^2\ \text{sign}\ (1 - C_v)}{1 - C_\ell\ C_f} + \frac{(k_c + k_e)\ C_f^{\ 2}}{(1 - C_f)^2}$$

$$+ \frac{\ell}{D}\left[\frac{f_d\ C_f\ (3 - 3\ C_f + C_f^{\ 2})}{(1 - C_f)^3} + \frac{f_c\ C_o\left[1 - (1 - C_f)C_v\right]^2 \text{sign}\left[1 - (1 - C_f)C_v\right]}{(1\ \ C_f^{\ 3})}\right]$$

.... 4.22

In spite of the many assumptions that have been made, equation 22, determining the P.D.C. of a cage in a shaft, is by no means simple. Not all the quantities occurring in the formula are independent of each other, viz., the cage length ℓ is involved in the lengths coefficient C_ℓ and a relationship exists between the fill coefficient C_f and the ratio of the perimeters of cage and shaft C_o. However, despite these complications, equation 22 does provide a means of analysing the influence of the individual quantities involved in it on the overall pressure coefficient C_c. Of course, there is no difficulty in evaluating the P.D.C. if the quantities occurring in equation 22 are known.

When the case of a stationary cage is considered, equation 22 takes a simpler form since in this instance V_c is zero and hence C_v is also zero giving

$$C_{cs} = \frac{C_D\ C_f}{1 - C_f\ C_\ell} + \frac{k\ C_f^{\ 2}}{(1 - C_f)^2} + \frac{\ell}{D}\left[\frac{f_d\ \ C_f(3 - 3C_f + C_f^{\ 2})}{(1 - C_f)^3} + \frac{f_c\ C_o}{(1 - C_f)^3}\right]$$

.... 4.23

where $C_D = C_{D_1} + C_{D_3}$ and $k = k_e + k_c$

If the influence of C_ℓ is neglected, and this may be done in most cases, the relation between C_c and the length of the cage takes a very simple form

$$C_c = S + t\ell \qquad \text{.............. } 4.24$$

where S and t are quantities which are independent of ℓ.

This form of the relationship between the P.D.C. and the length of a train of tubs in a roadway or the length of a cage in a shaft has already been found experimentally in the model tests of Miller and Bryan and of Stevenson as stated in Chapter 2.

The Fill Coefficient C_f of Rectangular Cages in Circular Shafts

Most of the present mine shafts, and all new ones which are being sunk have a circular cross section. If the assumption is made that in a given circular shaft there is a winding system with "n" equal rectangular cages, then the fill coefficient C_f of one cage may be expressed as a function of the number "n" and the cross sectional shape factor "x" of the cage.

The following relationships exist

$$x = \frac{a}{b} = \text{cross sectional shape factor of a cage}$$

$$\text{............... } 4.25$$

$$C_f = \frac{a\,b}{\frac{\pi}{4}\,D^2} = \text{fill coefficient of a cage} \qquad \ldots\ldots\ 4.26$$

and according to Pythagoras' theorem (see Fig. 4.2)

$$D^2 = b^2 + n^2\,(a + e)^2 \qquad \ldots\ldots\ 4.27$$

where e = distance between walls of two passing cages.
The distance "e" will be expressed as a dimensionless ratio to the cage width "a".

$$\frac{e}{a} = m\,, \quad \text{hence} \quad e = m\,a \qquad \ldots\ldots\ 4.28$$

By means of equations 25, 27 and 28, the fill coefficient of a cage, defined by equation 26, can be written in the form

$$C_f = \frac{4x}{\pi\left[1 + n^2\,(1 + m)^2\;x^2\right]} \qquad \ldots\ldots\ 4.29$$

On the basis of equation 29, the maximum value of the fill coefficient can be determined by differentiation.

$$\frac{d\,C_f}{dx} = \frac{4}{\pi}\cdot\left[\frac{1 - n^2\,(1 + m)^2\;x^2}{\left[1 + n^2\,(1 + m)^2\;x^2\right]^2}\right] \qquad \ldots\ldots\ 4.30$$

$$\frac{dC_f}{dx} = 0 \text{ when } 1 - n^2 (1 + m)^2 \; x^2 = 0 \qquad \text{............ } 4.31$$

$$\text{hence, } x_o = \frac{1}{n (1 + m)} \qquad \text{............ } 4.32$$

x_o is the value of the cross sectional shape factor of a cage at which the fill coefficient reaches its maximum value. To calculate this maximum, the value of x_o has to be inserted in equation 29.

$$\text{i.e. } C_{f\ max.} = \frac{2}{\pi} \cdot \frac{1}{n (1 + m)} \qquad \text{............ } 4.33$$

The total maximum fill coefficient of "n" cages side by side is

$$n \; C_{f_{max}} = \frac{2}{\pi} \cdot \frac{1}{1 + m} \qquad \text{............ } 4.34$$

It can be seen that the maximum value of total fill coefficient of rectangular cages in a circular shaft does not depend on the number of cages "n" in the winding system, and if "m" is negligible it takes the value of $\frac{2}{\pi}$ or 0.64. The maximum value of the C_f of one cage is, of course, "n" times smaller.

The Relationship between C_o and C_f in a Circular Shaft

As has already been mentioned, a relationship exists between the fill coefficient of a shaft and the dimensionless ratio of the perimeters of cage and shaft. In this section, an attempt will be made to determine this relation for rectangular cages and circular shafts and to find the range of possible changes of the value of C_o.

The ratio of the circumferences of cage and shaft is defined as follows :

$$C_o = \frac{2\,(a + b)}{\pi D} \qquad \ldots\ldots\ldots\ldots \quad 4.34$$

By means of equations 26, 27 and 28, the ratio C_o can be expressed in terms of the number of cages "n" in the winding system and the fill coefficient C_f.

$$C_o = \frac{\pi C_f + 2 + \sqrt{4 - \pi^2 \; n^2 \,(1 + m)^2 \; C_f^{\,2}}}{\pi \sqrt{2 + \sqrt{4 - \pi^2 \; n^2\,(1 + m)^2 \; C_f^{\,2}}}} \qquad \ldots\ldots\ldots\ldots \quad 4.35$$

Substituting the maximum possible value of fill coefficient $C_{f_{max.}}$ determined by equation 33 into equation 35 we obtain

$$C_{o_{max.}} = \frac{\sqrt{2} + \sqrt{2}\ n(1 + m)}{\pi n\ (1 + m)} \qquad \ldots\ldots\ldots\ldots \quad 4.36$$

The possible range of value of the ratio of perimeters can be determined using equations 35 and 36. From equation 35, the lower limit of this range, for $C_f = o$, is $C_o = \frac{2}{\pi} = 0.64$. Assuming $m = o$, equation 36 provides the values of the upper limits of C_o which are :

For a 2 cage winding system $\quad n = 2, \quad C_{o_{max.}} = 0.68$

For a 4 cage winding system $\quad n = 4, \quad C_{o_{max.}} = 0.58$

From these calculations, one can conclude that, for the most common winding systems in circular shafts, the range of variation in C_o is very small. This leads to the assumption that, for the case considered, the fill coefficient and the ratio of perimeters are independent.

Pressure Drop Coefficient of a Cage as a Function of Fill Coefficient

Now that it has been established that C_o can be C_f

assumed independent of C_f, equation 22, determining the P.D.C. C_c of a cage may be written in a shorthand form.

$$C_c = C_D B_1(C_f)\ Y_1(C_v) + k B_2(C_f) + \frac{\ell f_d}{D} B_3(C_f)$$

$$+ \frac{\ell f_c C_o}{D} B_4(C_f)\ Y_2(C_f C_v) \qquad \ldots\ldots\ldots\ldots\ 4.37$$

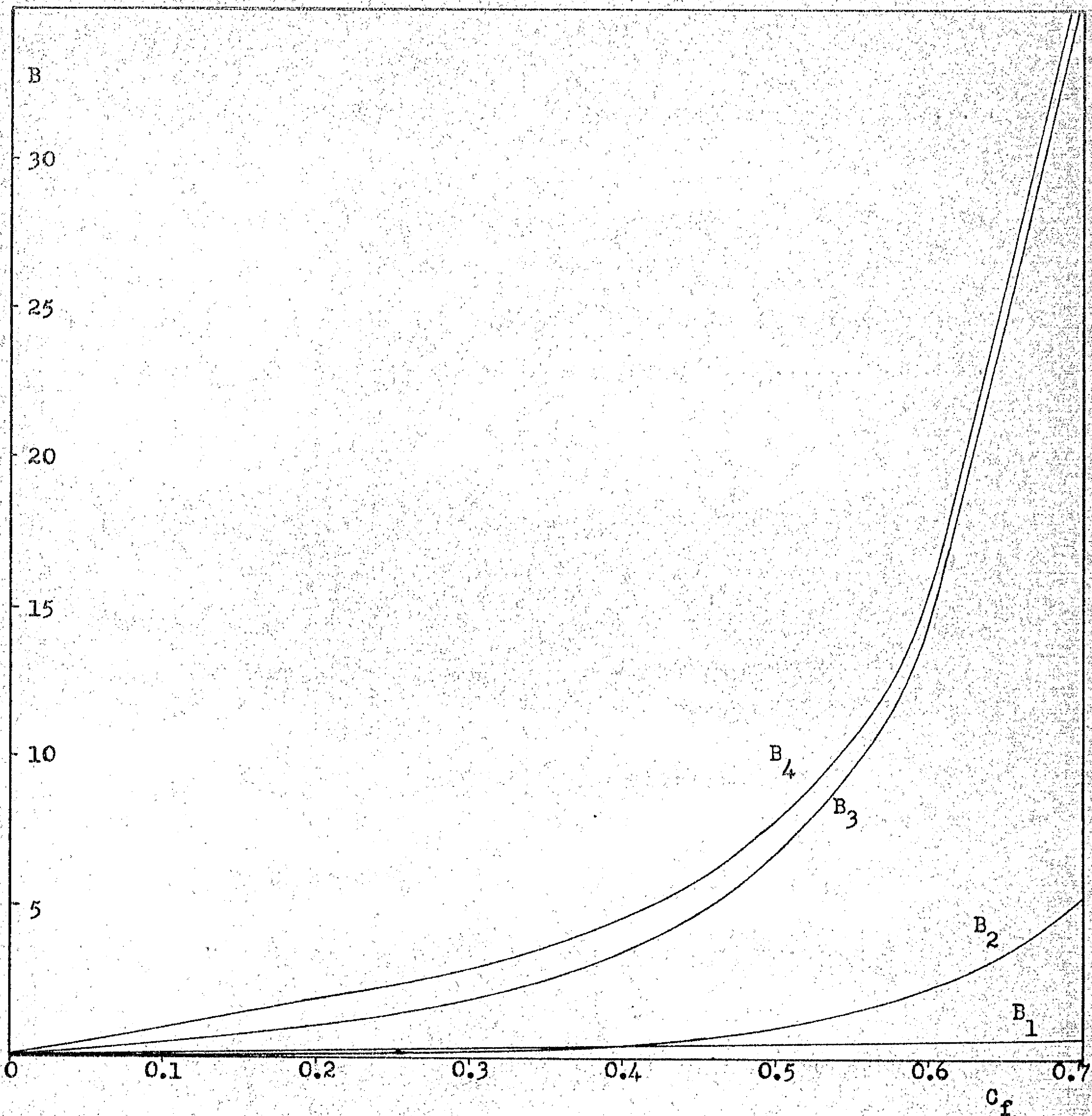
where $B_1(C_f) = \dfrac{C_f}{1 - C_\ell C_f}$ $\qquad B_2 = \dfrac{C_f^{\,2}}{(1 - C_f)^2}$

$$B_3 = \frac{C_f\ (3 - 3C_f + C_f^{\,2})}{(1 - C_f)^3} \qquad B_4 = \frac{1}{(1 - C_f)^3}$$

and $Y_1(C_v) = (1 - C_v)^2 \ \text{sign}\,(1 - C_v)$

$$Y_2(C_f C_v) = \left[1 - (1 - C_f)\,C_v\right]^2 \ \text{sign}\left[1 - (1 - C_f)\,C_v\right]$$

For the stationary cage, the functional coefficients Y_1 and Y_2 both take the value unity. The forms of the coefficients B_1, B_2, B_3 and B_4 as functions of fill coefficient C_f are given in Fig. 4.3 In the case of B_1, C_ℓ is assumed zero . It can be seen that all four functions rise with increase



$$C_c = C_p\ B_1\ (C_f) + k\ B_2\ (C_f) + \frac{\ell\ f_d}{D}\ B_3\ (Cf) + \frac{\ell\ f_c\ C_o}{D}\ B_4\ (C_f)$$

Fig. 4.3

of C_f, but at different rates which are determined by their derivatives.

Since the determination of the values of functions B_1, B_2, B_3 and B_4 for a given C_f value requires some laborious calculations, the curves of Fig. 4.3 can be used to ease their determination for practical purposes.

The Pressure Drop Coefficient of a Cage as a Function of Cage Speed

Equation 37, determining the P.D.C. C_c of a cage is the sum of four constituents, two of which contain C_v, the ratio of cage speed to mean air velocity. In order to illustrate the change of C_c dependent on C_v, four curves have been given on Fig. 4.4 (a). Two of those are straight lines parallel to the C_v axis and illustrate the two constituents of C_c which are independent of C_v. The remaining two are parabolas whose equations are

$$P_1 = C_D B_1 \; Y_1 (C_v) = C_D B_1 (1 - C_v)^2 \; \text{sign} \, (1 - C_v) \quad \ldots\ldots \quad 4.44$$

and $$P_2 = \frac{\ell f_c C_o}{D} Y_2 (C_v) = \frac{\ell f_c C_o}{D} \left[1 - (1 - C_f) C_v\right]^2 \; \text{sign} \left[1 - (1 - C_f) C_v\right]$$

$$\ldots\ldots \quad 4.45$$

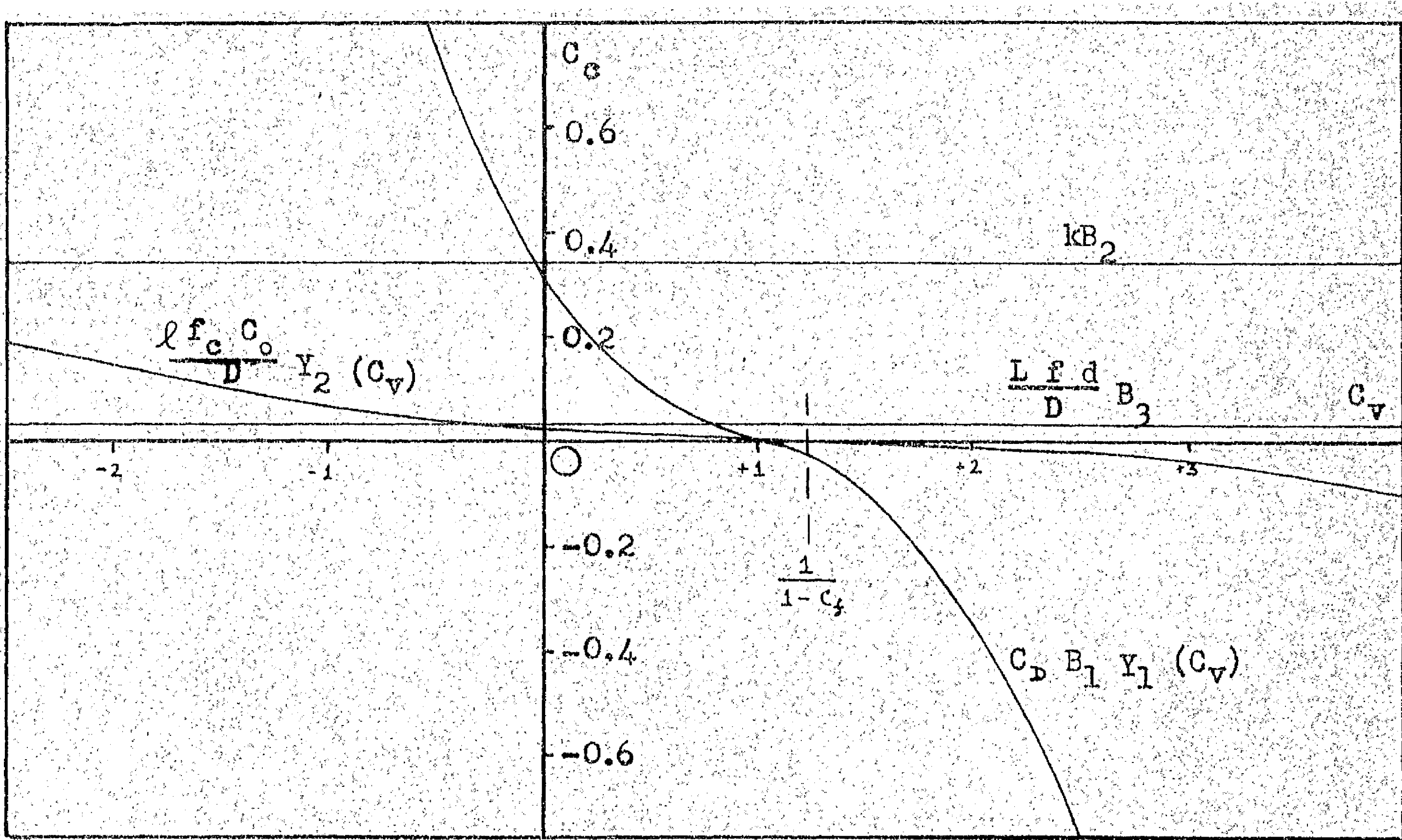


Fig. 4.4(a)

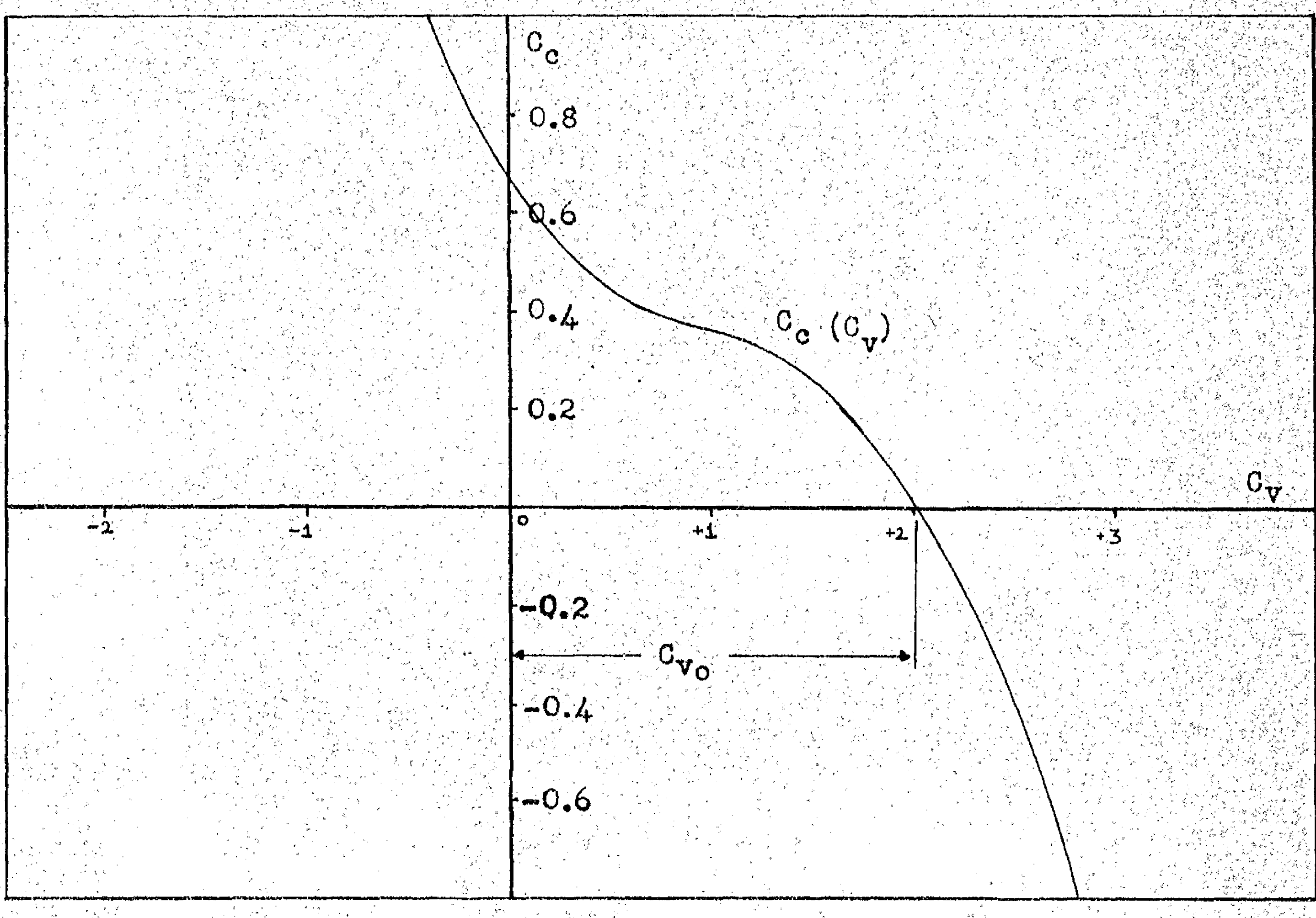


Fig. 4.4(b)

Parabola 1 crosses the C_v axis at the point where $C_v = 1$

" 2 " " " " " " " " $C_v = \frac{1}{1-C_f}$

The total curve illustrating the dependence of C_c on the cage speed involved in C_v is given in Fig. 4.4 (b). It can be seen that the P.D.C. C_c decreases with increase in the positive value of C_v. At a certain point, $C_v = C_{v_o}$, the curve crosses the C_v axis and the value of C_c is equal to zero. This means that the energy added to the air by the moving cage is equal to the energy lost. If $C_v > C_{v_o}$, the energy added is greater than the energy lost and the P.D.C. becomes negative. The value of C_{v_o} can be calculated from equation 37 if C_c is put equal to zero. Then the result is :

$$C_{v_o} = \frac{C_D B_1 D - \ell f_c C_o (1-C_f)}{C_D B_1 D - \ell f_c C_o (1-C_f)^2} + \sqrt{\left[\frac{C_D B_1 D - \ell f_c C_o (1-C_f)}{C_D B_1 D - \ell f_c C_o (1-C_f)^2}\right]^2 + \frac{C_{os} D - 2 C_D B_1 D}{C_D B_1 D - \ell f_c C_o (1-C_f)^2}}$$

.... 4.46

and for $C_v > \frac{1}{1-C_f}$

$$C_{v_o} = \frac{C_D B_1 D + \ell f_c C_o (1-C_f)}{C_D B_1 D + \ell f_c C_o (1-C_f)^2} + \sqrt{\left[\frac{C_D B_1 D + \ell f_c C_o (1-C_f)}{C_D B_1 D + \ell f_c C_o (1-C_f)^2}\right]^2 + \frac{C_{os} D - 2 C_D B_1 D}{C_D B_1 D + \ell f_c C_o (1-C_f)^2}}$$

.....4.47

where $$C_{os} = C_D B_1 + k B_2 + \frac{\ell f_d}{D} B_3 + \frac{\ell f_c C_o}{D} B_4 \quad \text{............ 4.48}$$

= the P.D.C. for a stationary cage when $C_v = 0$.

The Pressure Drop Coefficient of Two Cages moving in a Shaft

Since most existing winding systems consist of two cages, it is essential to investigate the problem of the P.D.C. due to two cages moving in opposite directions in a shaft.

All the model tests which had been carried out in connection with this problem had shown that a limited zone of aerodynamical interaction h_i (see Fig. 4.1) exists between the two cages when moving in a shaft. For cages outside this zone, the total P.D.C. is simply the sum of the two coefficients due to the two cages. If the cages are moving inside the zone of interaction, the overall P.D.C. increases as the distance h between the central points of the cage decreases, reaching its maximum value when the cages are side by side i.e., when $h = 0$.

In the following sections, the overall P.D.C. for two extreme positions of the two cages in the shaft will be considered, viz. (a) outside the zone of interaction and (b) in the side-by-side position.

It is worthy of note that the same method of consideration may be applied to winding systems consisting of more than two cages.

A. Two cages outside the zone of interaction

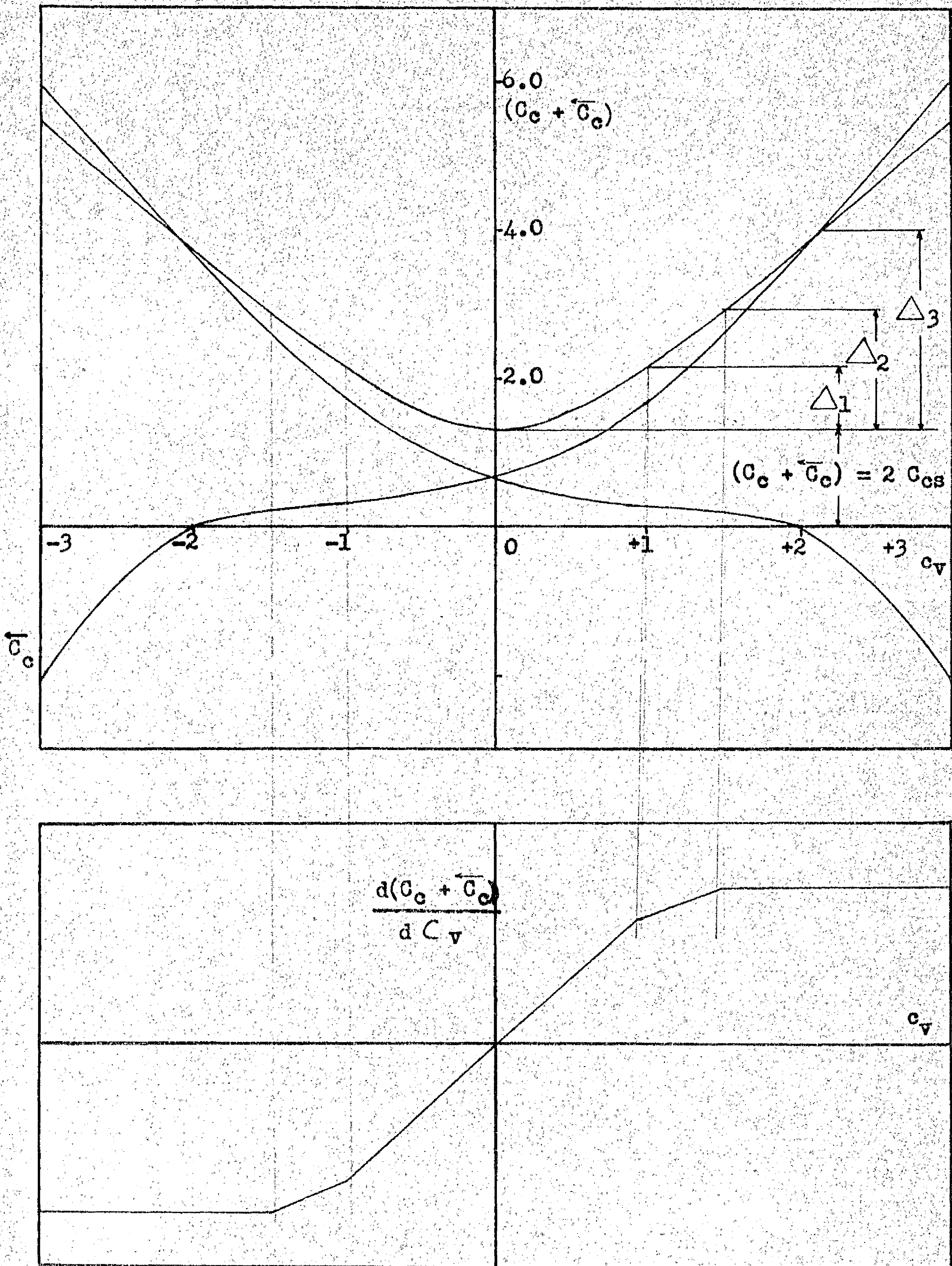
As shown previously, the P.D.C. of a cage moving in a shaft is determined by equation 4.37.

$$C_c = C_D B_1 (1 - C_v)^2 \text{ sign } (1 - C_v) + k B_2 + \frac{\ell f_d}{D} B_3 + \frac{\ell f_c C_o}{D} B_4 \left[1 - (1 - C_f) C_v\right]^2 \text{ sign } \left[1 - (1 - C_f) C_v\right] \qquad \ldots\ldots\ldots\ldots 4.37$$

The P.D.C. $\overleftarrow{C_c}$ of the second cage moving in the opposite direction is also determined by equation 4.37 with the sign of the ratio C_v changed.

$$\overleftarrow{C_c} = C_D B_1 (1 + C_v)^2 \text{ sign } (1 + C_v) + k B_2 + \frac{\ell f_d}{D} B_3 + \frac{\ell f_c C_o}{D} B_4 \left[1 + (1 - C_f) C_v\right]^2 \text{ sign } \left[1 + (1 - C_f) C_v\right] \qquad \ldots\ldots\ldots\ldots 4.49$$

The graphs of functions 4.37 and 4.49 are given in Fig. 4.5.



6.0
$(C_c + \overleftarrow{C}_c)$
4.0
2.0
Δ_1
Δ_2
Δ_3
$(C_c + \overleftarrow{C}_c) = 2\ C_{cs}$
-3
-2
-1
0
+1
+2
+3
c_v
$\overleftarrow{C}_c$
C_c
$\frac{d(C_c + \overleftarrow{C}_c)}{d\ C_v}$
c_v

Fig. 4.5

It is obvious that the latter is the mirror image of the former, reflected in the C_c axis.

To determine the overall P.D.C. of two cages moving outside the interaction zone, the two functions 4.37 and 4.49 have to be added and this gives

$$
\begin{aligned}
(C_c + \overleftarrow{C_c}) = C_D B_1 \Big[(1-C_v)^2 \text{ sign } (1-C_v) + (1+C_v)^2 \text{ sign } (1+C_v) \Big] \\
+ 2 k B_2 + \frac{2 \ell f_d}{D} B_3 \\
+ \frac{\ell f_c C_o}{D} B_4 \Big\{ \big[1-(1-C_f)C_v\big]^2 \text{ sign } \big[1-(1-C_f)C_v\big] \\
+ \big[1+(1-C_f)C_v\big]^2 \text{ sign } \big[1+(1-C_f)C_v\big] \Big\}
\end{aligned}
\qquad \text{............ } 4.50
$$

The graph of function 4.50 is also given in Fig. 4.5. It is a kind of parabola with a minimum value at $C_v = 0$. That is, the overall P.D.C. is a minimum if the cages are stationary. This minimum value can be calculated from equation 4.50 with $C_v = 0$.

$$
\begin{aligned}
(C_c + \overleftarrow{C_c}) &= 2 C_D B_1 + 2 k B_2 + \frac{2 \ell f_d}{D} B_3 + \frac{2 \ell f_c C_o}{D} B_4 \\
&= 2 C_{c_s}
\end{aligned}
\qquad \text{............. } 4.51
$$

It is also possible to determine the increase in overall P.D.C. due to the movement of the cages.

i.e. $$\Delta(C_c + \overleftarrow{C}_c) = (C_c + \overleftarrow{C}_c) - (C_c + \overleftarrow{C}_c)_{ST} \qquad \ldots\ldots\ldots\ldots\ldots\ 4.52$$

The following formulae are the final results of calculations based on relation 4.52.

In the range $|C_v| \leqslant 1$

$$\Delta_1 = 2\left[C_D B_1 + \frac{\ell f_c C_o}{D} B_4 (1 - C_f)^2\right] C_v^{\,2} \qquad \ldots\ldots\ldots\ 4.53$$

In the range $1 < |C_v| < \frac{1}{1 - C_f}$

$$\Delta_2 = 2\left[C_D B_1 (2C_v - 1) + \frac{\ell f_c C_o}{D} B_4 (1 - C_f)^2 C_v^{\,2}\right]$$

$$\ldots\ldots\ldots\ 4.54$$

In the range $|C_v| > \frac{1}{1 - C_f}$

$$\Delta_3 = 2\left\{C_D B_1 (2C_v - 1) + \frac{\ell f_c C_o}{D} B_4 \left[2(1 - C_f)C_v - 1\right]\right\}$$

$$\ldots\ldots\ldots\ 4.55$$

On the basis of those formulae, it can be seen that in the first and second ranges, the increase in overall P.D.C. due to the movement of the cages is proportional to the second order of C_v, while in the third range, only to the first order of C_v.

B. Two Cages in the Side by Side Position

In the general case, where "n" equal cages are in the side by side position, the P.D.C. due to these "n" cages may also be determined by means of equation 4.22 if the quantities "nC_f" and "nC_o" are substituted for C_f and C_o.

$$(C_c \| C_c \quad \ldots\ldots\ldots \quad \| n) = C_D M_1 (C_f) Y_1 (C_v) + k M_2(C_f) + \frac{\ell f_d}{D} M_3 (C_f)$$

$$+ \frac{\ell f_c C_o}{D} n M_4 (C_f) Y_2 (n C_f \cdot C_v) \qquad \ldots\ldots\ldots\ldots \quad 4.56$$

$$\text{where } M_1(C_f) = \frac{n C_f}{1 - n C_f C_\ell} = B_1 (n C_f) \qquad \ldots\ldots\ldots\ldots \quad 4.57$$

$$M_2(C_f) = \frac{n^2 C_f^2}{(1 - n C_f)^2} = B_2 (n C_f) \qquad \ldots\ldots\ldots\ldots \quad 4.58$$

$$M_3(C_f) = \frac{n C_f (3 - 3n C_f + n^2 C_f^2)}{(1 - n C_f)^3} = B_3 (n C_f) \quad \ldots\ldots \quad 4.59$$

$$M_4(C_f) = \frac{1}{(1-n\,C_f)^3} = B_4\,(n\,C_f) \qquad \ldots\ldots\ldots\ldots\ldots\ 4.60$$

It can be seen from relations 4.57 to 4.60 that the values of functions M_1, M_2, M_3 and M_4 can be determined by means of the same graphs as functions B (Fig. 4.3) with the abscissa taken equal to $n\,C_f$.

It is natural to assume that the P.D.C. of one cage, positioned parallel to other $(n-1)$ cages is equal to one n-th of the overall P.D.C. given by equation 4.56.

$$\uparrow C_c = \frac{C_D}{n} M_1 Y_1 + \frac{k}{n} M_2 + \frac{\ell f_d}{n\,D} M_3 + \frac{\ell f_c\,C_o}{D} M_4 Y_4\ (n\,C_f\ .\ C_v)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ 4.61$$

The form of $\uparrow C_c$ as a function of C_v is, of course, exactly analogous to that of equation 4.33 but it has other numerical values at its intersections with the axes. These values are dependent on the number of cages positioned side by side.

Equation 4.61 will now be applied to a winding system consisting of two cages $n = 2$. The overall P.D.C. due to two cages moving at the same speed in opposite directions, at the moment when they are in the side by side position will be the sum of two expressions of the equation 4.61 type, taken with different signs of the ratio C_v.

$$\uparrow C_c = \frac{C_D}{2} M_1 (1 - C_v)^2 \text{ sign } (1 - C_v) + \frac{k}{2} M_2 + \frac{\ell f_d}{2D} M_3 + \frac{\ell f_c C_o}{D} M_4 \left[1 - (1 - 2C_f)C_v\right]^2 \text{ sign } \left[1 - (1 - 2C_f)C_v\right] \quad \ldots\ldots 4.62$$

and for the 2nd cage

$$\downarrow C_c = \frac{C_D}{2} M_1 (1 + C_v)^2 \text{ sign } (1 + C_v) + \frac{k}{2} M_2 + \frac{\ell f_d}{2D} M_3 + \frac{\ell f_c C_o}{D} M_4 \left[1 + (1 - 2C_f)C_v\right]^2 \text{ sign } \left[1 + (1 - 2C_f)C_v\right] \quad \ldots\ldots 4.63$$

The overall P.D.C. for two cages side by side is thus :

$$\left[C_c \uparrow \downarrow C_c\right] = \frac{C_D}{2} M_1 \left[(1 - C_v)^2 \text{ sign } (1 - C_v) + (1 + C_v)^2 \text{ sign } (1 + C_v)\right] + k M_2 + \frac{\ell f_d}{D} M_3 + \frac{\ell f_c C_o}{D} M_4 \left\{\left[1 - (1 - 2C_f)C_v\right]^2\right.$$

$$\text{sign}\left[1-(1-2\ C_f)C_v\right] + \left[1+(1-2\ C_f)C_v\right]^2 \ \text{sign}\left[1+(1\ 2C_f)C_v\right]\Bigg\}$$

.... 4.64

The graphs of the P.D.C.'s given by expressions 4.62 to 4.64 are drawn as functions of C_v on Fig. 4.6. If there are two stationary cages in the side by side position, $C_v = 0$ and equation 4.64 takes the following simple form :

$$\left[C_c \uparrow \downarrow C_c\right]_s = C_D M_1 + k\, M_2 + \frac{\ell f_d}{D} M_3 + \frac{2\ell f_c\ C_c}{D} M_4$$

.... 4.65

It can be proved by mathematical analysis and also seen on Fig. 4.6 that a minimum P.D.C. value occurs if the cages are stationary. This value is determined by means of equation 4.65.

The increase in P.D.C. caused by the movement of the cages, which is the difference between the actual value of P.D.C. and the value for the stationary cages i.e. $\Delta\left[C_c \uparrow \downarrow C_c\right] = \left[C_c \uparrow \downarrow C_c\right]$

$$- \left[C_c \uparrow \downarrow C_c\right]_s$$ 4.66

can be easily calculated on the basis of equations 4.64 and 4.65 and

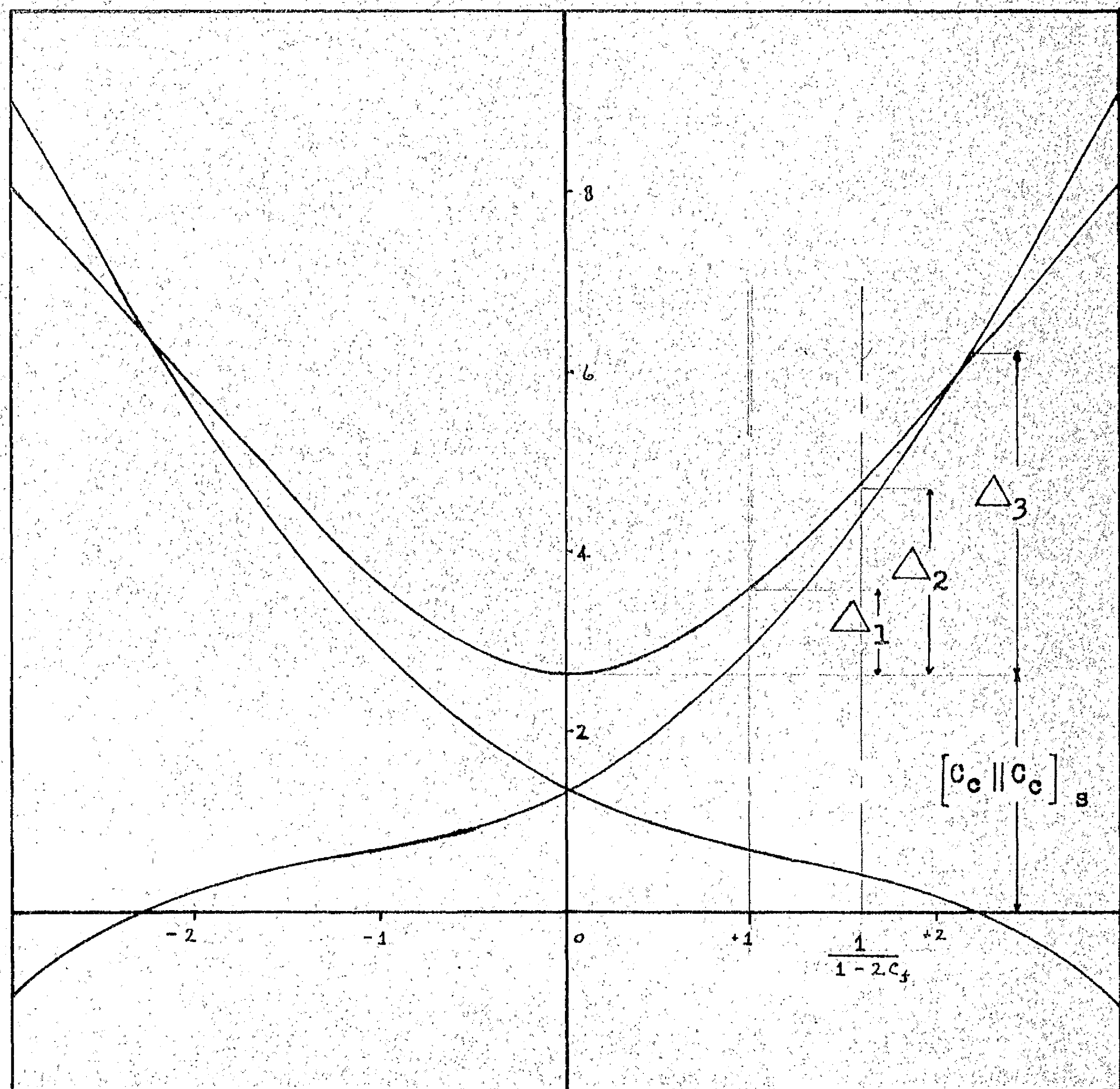


Fig. 4.6

the results are as follows :

In the range $\left| C_v \right| \leqslant 1$

$$\Delta_1 = 2\left[\frac{C_D}{2} M_1 + \frac{\ell f_c\ C_o}{D} M_4\ (1-2\ C_f)^2\right] C_v^{\ 2} \qquad \ldots\ldots\ldots\ldots\ 4.67$$

In the range $1 < \left| C_v \right| \leqslant \frac{1}{1-2\ C_f}$

$$\Delta_2 = 2\left[\frac{C_D}{2} M_1\ (2\ C_v - 1) + \frac{\ell f_c\ C_o}{D} M_4\ (1-2\ C_f)^2\ C_v^{\ 2}\right] \qquad \ldots\ldots\ldots\ldots\ 4.68$$

In the range $\left| C_v \right| > \frac{1}{1-2\ C_f}$

$$\Delta_3 = 2\left\{\frac{C_D}{2} M_1\ (2\ C_v - 1) + \frac{\ell f_c\ C_o}{D} M_4 \left[2\ (1-2\ C_f) C_v - 1\right]\right\} \qquad \ldots\ldots\ldots\ldots\ 4.69$$

Verification of the Theory

In order to verify the validity of the assumptions made in the course of the establishment of the theory, some experimental results have to be compared with the corresponding values calculated on the basis of the theory.

Results of model investigations on stationary cages are contained in the works of both Stevenson and Wilkie. Stevenson's results, in particular, are very suitable for checking the theory since most of the tests were carried out in an unlined shaft.

Table 4.1 contains some results of P.D.C., due to stationary cages, measured in model tests together with the corresponding results calculated from the formulae derived in previous sections of this chapter. Columns 1 to 4 contain the numerical values of the coefficients used in the calculations. The values of P.D.C. calculated from equations 4.48 and 4.51 for a single cage and for two cages side by side are given in columns 5 and 6. In columns 7 and 9, the corresponding measured values are given. Finally, columns 8 and 10 give the relative differences expressed as a percentage of the calculated value.

These results are also presented in graphical form in Fig. 4.7 and from both this and Table 4.1 it can be seen that most of the results are in fair agreement, most of the relative differences being within the $\pm$ 10 per cent mark. The overall Coefficient of Correlation between the calculated and measured results given in Table 4.1 is 0.94.

It must be admitted, however, that among the results of Stevenson and Wilkie, discrepancies sometimes occur which cannot

	k	f_d	f_c	Calculated C_c	Calculated $C_c 110 C_c$	Measured C_c	Measured %	Measured $C_c 110 C_c$	%	Type	Source
	2	3	4	5	6	7	8	9	10	11	12
.8	2.00	0.014	0.150	0.209	0.570	0.20	-4.3	0.60	+5.3	A2	[illegible]
	"	"	0.220	0.556	2.374	0.56	+0.7	2.74	+15.5	C/2	
	"	"	0.220	0.701	3.118	0.70	-0.1	3.20	+2.6	C/4	
	"	"	0.100	0.386	1.490	0.39	+1.0	1.53	+2.7		
	"	"	0.100	0.456	1.778	0.43	-5.7	1.60	-10.0		
.8	2.00	"	0.180	2.045	-	2.050	+0.2	-	-	A1/1	
	"	"	"	1.652	-	1.700	+2.9	-	-	A2/1	
	"	"	"	0.908	-	0.926	+2.0	-	-	A3/1	
	"	"	"	0.595	-	0.574	-3.5	-	-	A4/1	
	"	"	"	0.254	-	0.270	+6.3	-	-	A5/1	
	"	"	"	0.145	-	0.138	-4.8	-	-	A6/1	
.8	2.00	"	"	2.580	-	2.46	-4.6	-	-	A1/2	
	"	"	"	3.115	-	2.84	-9.1	-	-	A1/3	
	"	0.014	0.180	3.650	-	3.16	-13.4	-	-	A1/4	
.8	2.00	0.016	0.250	2.755	-	2.66	-3.4	-	-	B1/1	STEVENSON
	"	"	0.210	2.632	-	2.67	+0.1	-	-	C1/1	
	"	"	0.190	2.547	-	2.66	+0.4	-	-	D1/1	
	"	"	0.180	2.515	-	2.53	+0.6	-	-	E1/1	
.8	2.00	"	0.250	1.788	-	1.90	+0.3	-	-	F1/1	
	"	"	0.215	1.708	-	1.85	+8.3	-	-	G1/1	
	"	"	.195	1.658	-	1.81	+9.3	-	-	H1/1	
	"	"	0.180	1.632	-	1.70	+4.2	-	-	I1/1	
.8	2.00	"	0.280	0.384	1.283	0.36	-6.3	1.42	+11.1	J1/1	
	"	"	0.200	0.377	1.350	0.38	+0.8	1.64	+21.5	K1/1	
	"	"	0.110	0.417	1.620	0.495	+18.6	1.85	+14.0	L1/1	
	"	0.016	0.110	0.582	2.364	0.595	+2.2	2.18	-7.8	L1/4	
.8	2.00	0.027	0.110	0.732	2.976	0.810	+106	3.94	+35	M1/1	
	"	0.027	0.110	1.002	4.614	0.940	-6.2	5.89	+27.4	M1/4	

TABLE 4.1

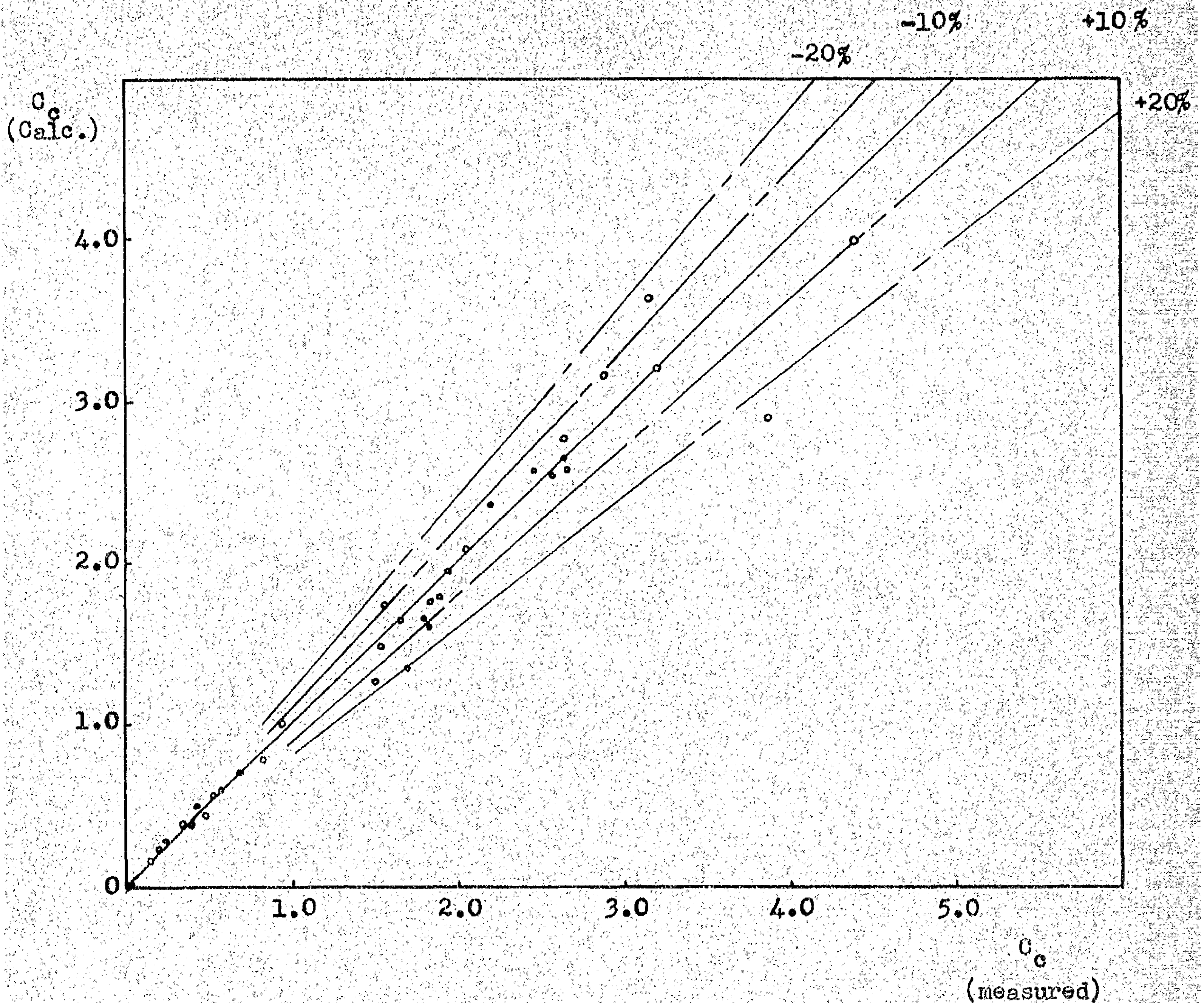


Fig. 4.7

be explained, at the moment at least, on the basis of the theory. An example of this is given in Table 4.2.

Cage Dimensions	C_f	C_c	$C_c \| \| C_c$	Source
2.00" x 5.38" x 6.38"	0.108	0.200	0.600	Wilkie A2
1.90" x 5.45" x 7.40"	0.100	0.255	0.775	Stevenson N1/2

Table 4.2

This table contains the dimensions of quite similar types of two deck cage, set up in similar 4 cage rigid guide installations by the two investigators. The measured P.D.C. values show a discrepancy of 20 – 25%.

Apart from some discrepancies it may be generally assumed that the results of model tests on stationary cages give good proof of the derived formulae. In addition, there is no objection to assuming that these formulae are valid for moving cages since, in the course of their derivation, the only factor related to the movement of the cages was the square law of aerodynamical resistance, and this has been proved many times.

Equations 4.53 and 4.67, which determine the increase in

P.D.C. due to the movement of the cages, give the explanation for the failure of the moving cage tests as already mentioned in Chapter 2. For example, for cages of series C/4 of Wilkie's tests, the increase in P.D.C. for two cages passing at the maximum speed of 5.50 D/sec can be calculated from equation 4.67 as follows.

$$\Delta_1 = 2\left[\frac{1.18}{2}.2.0{\cdot}215 + \frac{12.38 \times 0.22 \times 0.53}{11.25}5.3(1-2\times 0.255)^2\right]\left(\frac{62}{740}\right)^2$$

$$= 2\left[0.253 + 0.710 \times 0.570^2\right](0.084)^2 = 2 \times 0.487 \times 0.007$$

$$= \underline{0.007}$$

But increases of this order are impossible to measure with the present equipment.

CHAPTER 5

MOVING CAGE TESTS

(A) Equipment

Introduction

Up to now, the tests described have been concerned with stationary cages only. In this Chapter it is intended to describe some tests carried out using moving cages. The first section will comprise a detailed description of the additional equipment necessary for these tests; the second section, the tests themselves.

The wind tunnel arrangements and cages were the same as before, but with the moving cage tests it was necessary to make a continuous recording of the various test length pressure fluctuations. For this purpose, the equipment developed by Wilkie in his previous work on moving cages was used.

Although this recording apparatus had already been described it was felt that it would be advantageous to reiterate the details with more emphasis on the practical side of its operation. Without these hints, much time could be needlessly lost if one were unfamiliar with the idiosyncracies of the equipment. It is hoped that these notes will not go unheeded and will help towards the rapid establishment of an accurate technique without the repetitions and frustrations of trial and error processes.

In this preliminary section, the details of the various components will be first of all given. This will be followed by a description of the apparatus as a whole and some notes on its operation. Finally, a typical test will be described to illustrate the experimental procedure.

The purpose of the equipment was to transform the pressures into mechanical displacement by means of a diaphragm which was part of the a.c. bridge of a Proximity Meter. The meter had a galvanometer recorder in circuit which made a continuous record of the pressure changes on photographic paper. The details of the various components are as follows :-

The Fielden Proximity Meter

This instrument provides an electronic means of measuring small mechanical displacements, in this case the distance between two brass plates. In the type used here, the P.M.4, a capacitance system is used; this has the advantage that the sensitivity of the device is infinite and small mechanical displacements can be faithfully indicated. This follows from the fact that by making the condenser gap progressively smaller, the sensitivity is increased, the limitations being the structural dimensional stability of the specimen and the condenser head arrangement and the flatness and parallelism of the surfaces.

The electrical arrangement is shown in Fig. 5.1. An oscillator, operating at 500 kc/s supplied a small R.F. voltage to a phase balancing network (an R.F. bridge), which was so designed that its output consisted of two earth-free, equal and antiphase voltages joined in series at point P. One side of this circuit was connected to earth via the internal variable condenser C_1 (balance control), and the other side was fed to earth via C_2, this being the capacity at the termination of the R.F. cable. When C_1 was adjusted to equal C_2, the voltage at the centre point P was zero and likewise the instrument output. Any asymmetry in the values of C_1 and C_2 resulted in an "out of balance" voltage at the input to the amplifier and a consequent deflection of the meter.

The R.F. cable was constructed with an intermediate screen which was connected such that the stray capacitances of the cable were eliminated from the measuring circuit, thus making it

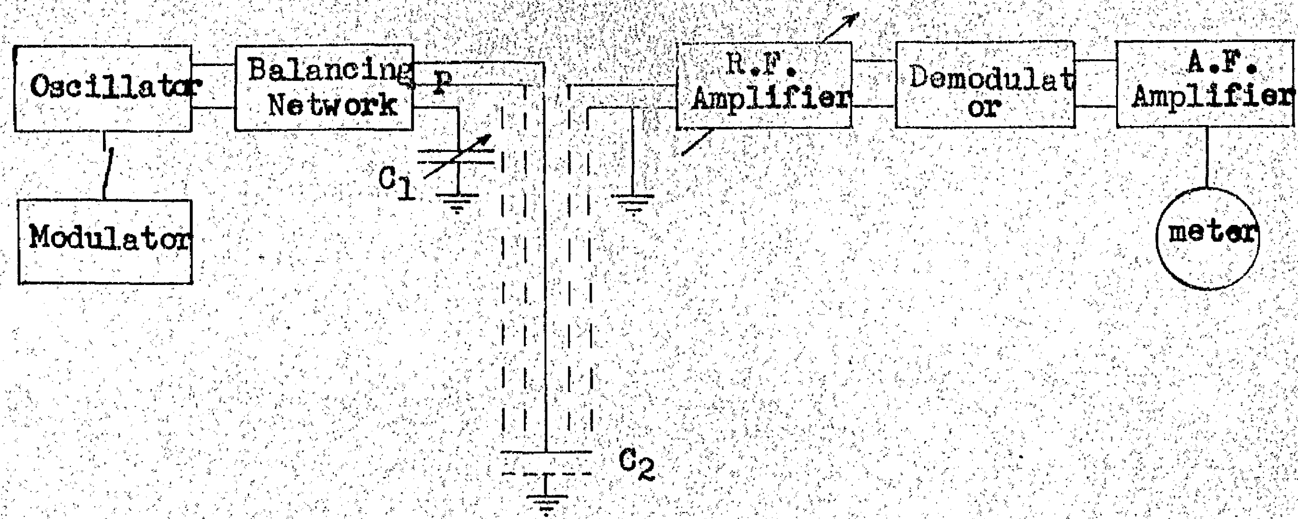


Fig. 5.1 Schematic Arrangement of Proximity Meter

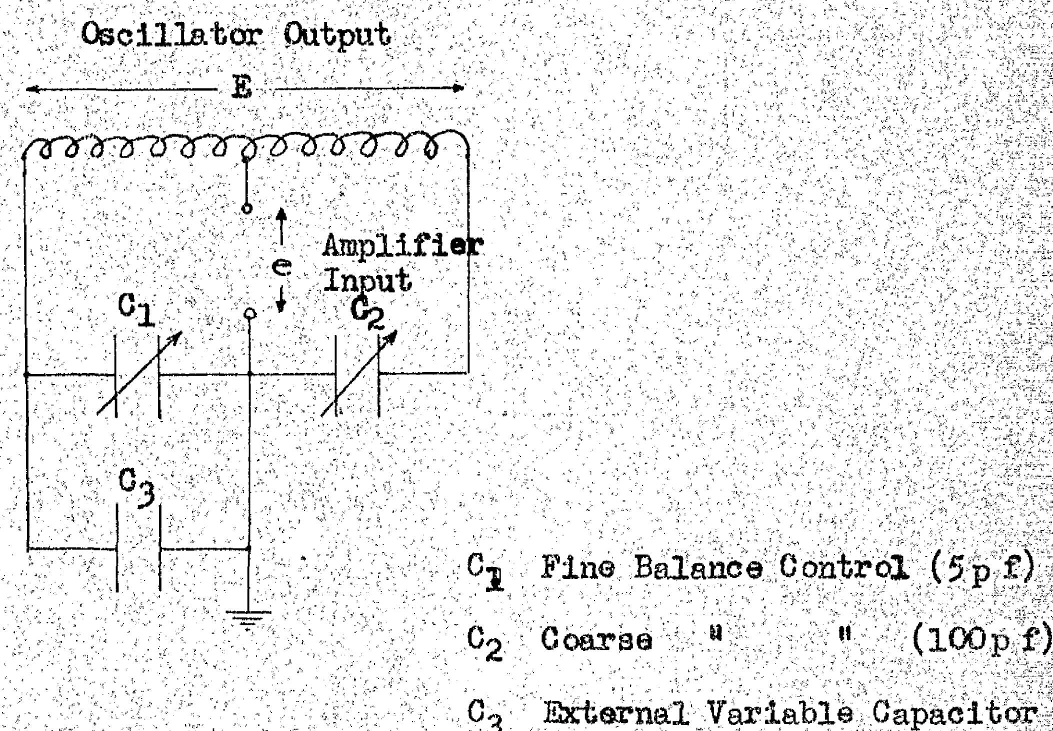


Fig. 5.2 Basic R.F. Bridge

unnecessary for the instrument to be at the actual measuring point.

In addition to the capacitances shown in the schematic diagram, a further variable condenser was incorporated in the bridge circuit as a fine balance control. A resistive balance was also provided so that any necessary power factor corrections could be applied, in order to balance the instrument correctly to zero.

The basic R.F. bridge is shown again in Fig. 5.2 where C_1 is the electrode capacitance to earth and C_3 is the effective capacitance of the double screened cable plus stray capacitances inside the instrument.

It can be shown that the change in "out of balance" voltage, V, for a given fractional change in C_1, is given by

$$\frac{dV}{d\ C_1/C_1} = -\frac{C_1\ C_2}{(C_1 + C_2 + C_3)^3} \times E$$

where E is the applied voltage.

For near balance conditions, which always exist when operating at high sensitivities (as in the present series of tests), C_1 and C_2 may be taken as approximately equal to C_2 and therefore

$$\frac{dV}{d\ C_1/C_1} \simeq -\frac{C_1}{4\ (C_1 + C_3)} \times E$$

Now if C_1 /

Now if C_1 is large compared with C_3

$$\frac{dV}{d\,C_1/C_1} \simeq -\frac{E}{4}$$

i.e. the change in output voltage for a given fractional change in C_1 is constant and is independent of the initial value of C_1.

The reading obtained on the meter, which is shown in Fig. 5.3, varied in a substantially linear manner with change of capacitance at the electrode, especially at the lower sensitivities. Even at the maximum sensitivity, a non-linearity of less than ½% was possible over the range of meter readings from 5 to 85 scale divisions.

The external bridge unit was connected to the main instrument via three co-axial cables and a further co-axial cable from the bridge unit was used to connect to the electrode. The controls provided very fine balance adjustment, and the external mounting of the bridge unit reduced temperature drift considerably.

The Variable Capacitance Diaphragm Gauge

In order to operate the P.M.4 it was necessary to use a condenser whose capacitance could be varied by the pressures which were required to be measured. This was done by varying the

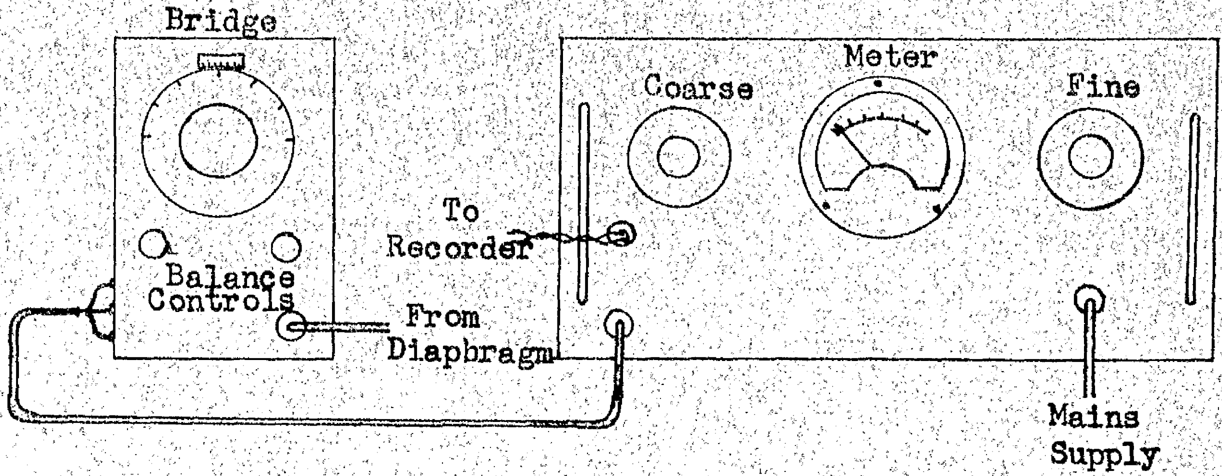


Fig. 5.3 Fielden Proximity Meter Type P.M.4.

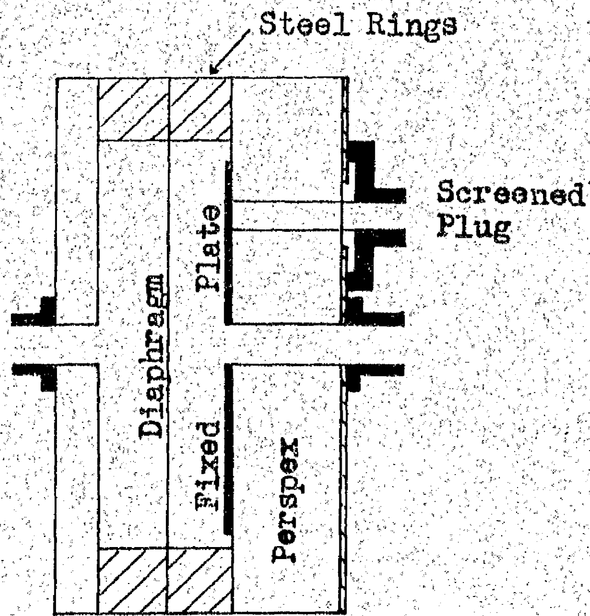


Fig. 5.4 The Diaphragm

distance between two plates, one earthed, the other connected to the bridge through a short double-screened cable. A jack socket in the circuit made it possible to connect a galvanometer recorder in series with this meter as a means of continuously measuring the output.

Several diaphragms were available and the most sensitive of these, with a range of 0 to 5 ins. W.G., was used throughout this series of tests. It was built from 0.005 in. thick brass, clamped tightly and flat between steel rings and perspex cover plates were used to make the diaphragm sensitive to differential air pressures. The fixed plate was bolted on the inside of the perspex cover plate and was thus separated from the diaphragm by the thickness of the metal clamping ring. Since it was cut slightly smaller than the inside diameter of the ring, it was insulated from it by the perspex. The central core of the proximity meter cable was connected to the plate by a screened plug passing through the perspex. The diaphragm was enclosed in a screened box to minimise interference from outside sources. A condenser input filter was connected in the output circuit to remove any ripple in the output. The arrangement is shown in Fig. 5.4.

In operation, the diaphragm was earthed and the flat brass disc concentric with the diaphragm was rigidly fixed a short distance away. This formed the variable condenser of the Proximity Meter bridge circuit. Under pressure, the diaphragm was deflected, thus producing the necessary change in capacitance.

This method has been used successfully by both Wilkie and Stewart and is very convenient as the sensitivity can be varied by changing the design constants. For instance, the sensitivity increases as the area of the diaphragm is increased, as the distance between the plates is decreased and as the deflection of the plate increases (for a given pressure change). However, the linearity is better if the deflection is small compared with the distance apart of the plates.

The Galvanometer Recorder

As stated previously, a galvanometer recorder was connected in series with the proximity meter in order to continuously measure the pressure change impressed on the diaphragm.

This was done by feeding the meter output into one of the recorder's twelve insulated channels. The light from a lamp is focussed onto the galvanometer mirror which, in turn reflects the light onto the recording paper through an optical system. The paper is then developed, thus giving a permanent record of the pressure fluctuations during a test. The meter was operated using a 24 volt D.C. supply which gave recording speeds up to 5 feet/sec.

Ilford N.L.6, which is an extremely fast non-colour-sensitive paper made particularly for recording fast moving light spots was used here and gave good results provided the necessary precautions were taken during the processing.

For the present purpose, three of the channels were generally used simultaneously. One for the pressure, one of the time and the third to indicate where the cages were passing.

The Operation of the Recording Equipment

The whole set up is illustrated in Figs. 5.5 and 5.6. In the former, the course of the pressures produced in the duct test length can be traced as they are transformed, first into mechanical displacements in the diaphragm and then into electrical energy at the proximity meter. After passing through the smoothing circuit, this impulse energises the appropriate galvanometer,

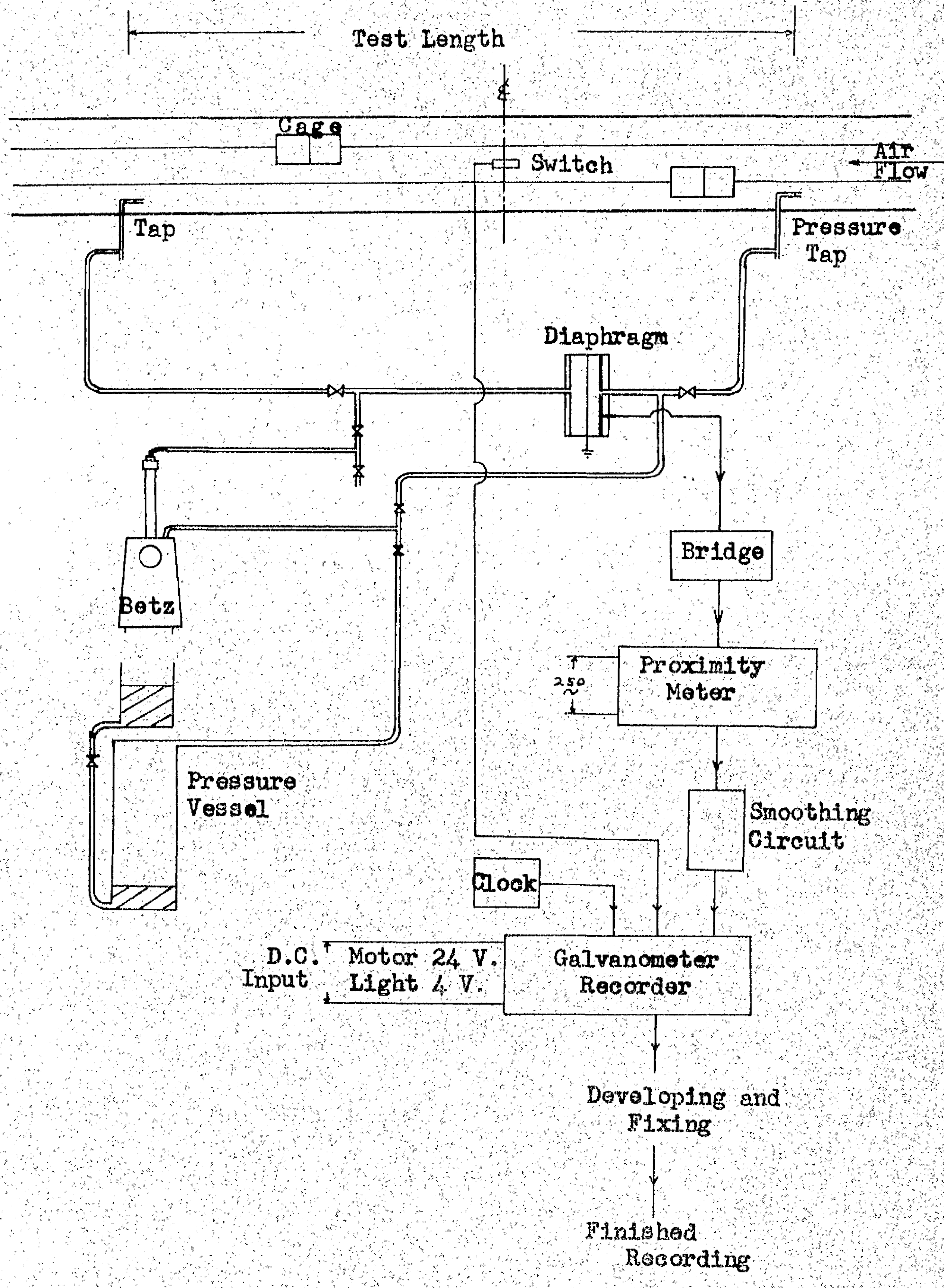


Fig. 5.5 Schematic Arrangement of Recording Equipment

whose mirror reflects a light spot onto the sensitised paper which is finally developed and fixed to give a permanent record of the pressure fluctuations.

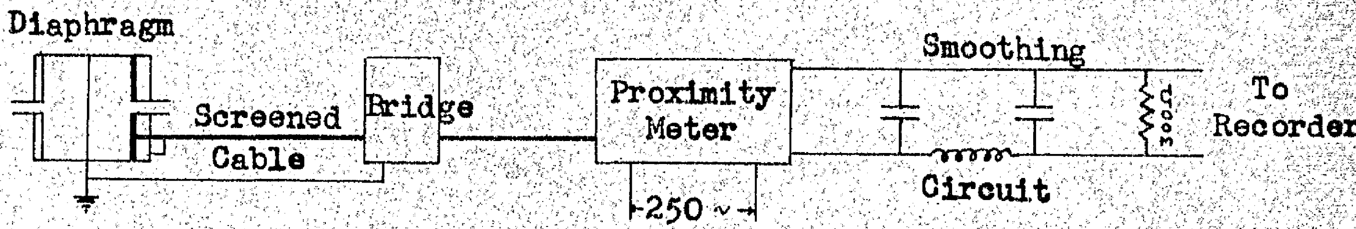
In addition, the manometer is shown connected in circuit, together with a pressure vessel. This was required so that known pressures could be fed into the diaphragm for calibrating purposes.

Fig. 5.6 shows the external wiring of the equipment in more detail.

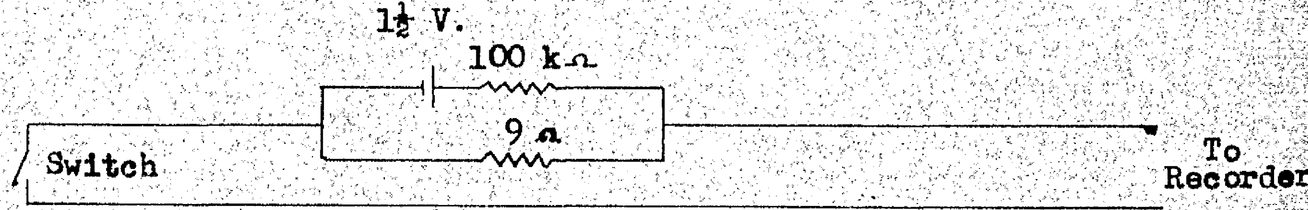
What follows now is a description of the practical details of the use of the equipment and some notes on its operation and limitations.

The first step was to zero the Proximity Meter accurately and find its range for the various sensitivies available.

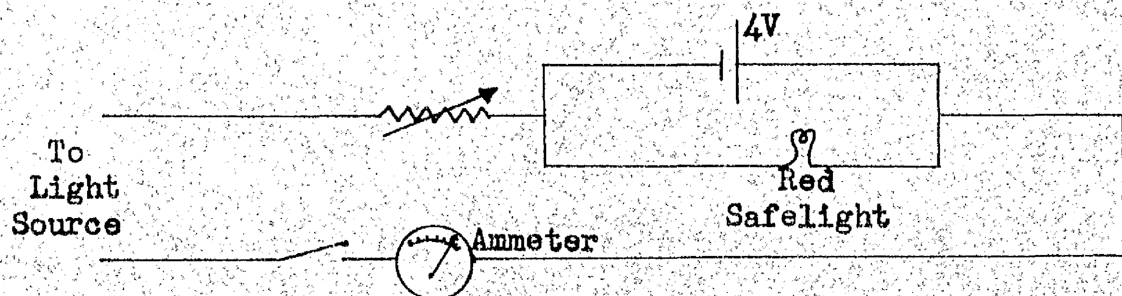
Accordingly, with the mains supply switched on, and the controls set at a fairly low sensitivity, the instrument was allowed to warm up for about half an hour. The "coarse sensitivity" was then increased, at the same time keeping the pointer at zero by adjustment of the "balance" controls on the bridge unit. The next step was to increase the "fine sensitivity", keeping the



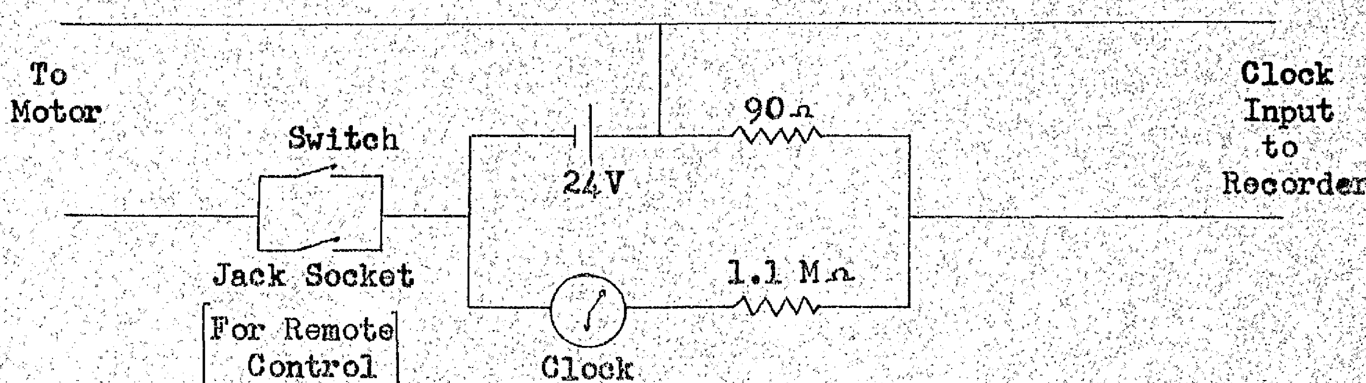
(a) Pressure Input Circuit



(b) Shaft ℄ Switch Circuit



(c) Light Source Supply Circuit



(d) Motor Supply and Clock Input Circuit

Fig. 5.6 External Circuit of Recording Equipment instrument zeroed as before. At each sensitivity, the range of pressure indicated by full scale deflection of the meter was determined. This was used for future reference so that the best setting could be chosen for any particular test. Some of these values are given in Table 5.1.

Table 5.1

Sensitivity	Coarse	12	12	12	12	12	12	12	12	11
	Fine	12	11	10	9	8	7	6	5	12
Range in mm. W.G.	on Meter	21	29	39	52	135				135
	on Width of Recording Paper	3.3	5	7	8	18	28	37	57	
Drift ins./min.		0.1				0.0035			0.000	

In addition, calibration curves were drawn for some of the settings and were found to conform to the standards already described.

After locating the pressure leads in the appropriate channel in the galvanometer recorder, another, similar test was carried out. This time, the purpose was to find, for each sensitivity setting, the range of pressures indicated by the light spot on the width of the recording paper. Some of these figures are given in Table 5.1 as a rough guide. It was noted here that more especially at the highest sensitivities, the light spot tended to drift with time.

From a knowledge of the static cage tests, the figures given in Table 5.1, and the projected moving cage test programme it was desired to use a fairly high sensitivity to manifest the rather small pressure difference. Thus, it was necessary to investigate the magnitude of this drift in order to see just how this would affect the accuracy of the recorded pressure trace. This was done simply by allowing the equipment to warm up and, with no pressure difference across the diaphragm and the instrument zeroed, recording the trace at fixed time intervals. Some of these results have been inserted in Table 5.1 and illustrate the drift effects clearly. It may be said here that it was later found that the drift was, if anything, greater, when there was a pressure difference on the diaphragm.

Accordingly, from a scrutiny of these drift records and from a knowledge of the approximate time it would take to run a complete test it was decided that the most convenient sensitivity setting would be Coarse 12, Fine 8 (C12 F8). This setting had the best combination of sensitivity and drift for the tests envisaged.

Before any given test it was found desirable to take readings of the expected extreme values of the pressures to be recorded. This was necessary so that the proximity meter would not be off the scale (at 0 – 135 mm.W.G. f.s.d. at a setting of C12 F8 this was unlikely for the range of pressures expected here) and, more important, so that the pressure trace could be accurately set within the confines of the recorder aperture. This was done by feeding into the diaphragm the expected middle pressure value (say) and setting the light spot, by means of the galvo adjusting key, to the centre of the aperture.

It must be emphasised that extreme care was necessary during this preliminary exercise if the recording was to be successful first time. It is obvious that if due care was not taken while setting the reflected light spot the pressure fluctuations could quite easily be off the paper.

In addition to these precautions, it was also necessary to make a calibration for each test run. The procedure, after setting the light spot, was to feed known pressure differences into the diaphragm by means of the pressure vessel. At each pressure value, the recording paper was run for a short distance thus giving the calibration at the start of every test. This was also done at the end of every test to provide a check on the drift and in addition, if the test was lasting overlong, check calibrations of say two values only were made at appropriate points in the test. Thus, a constant check could be kept on the actual values of the recorded pressures, the calibration procedure being simple to perform.

When recording fluctuating pressures it is imperative that the Betz manometer should be cut out of the pressure circuit (see Fig. 5.5). If left connected, the manometer causes extreme damping of the pressures and tends to level out all but the severest fluctuations. However, it is quite permissable to keep the Betz in circuit under certain special circumstances where steady pressures are being recorded, e.g. when calibration is in progress.

There are one or two additional points which, although they may seem obvious to the experienced investigator, are worthy of mention. Under this heading is the necessity of having enough recording paper in the spool, not only for the envisaged test, but also for a repeat if something goes wrong with the first attempt. Also included here are such necessary precautions as keeping the recorder motor and light source batteries in good condition. In the former case, running down of the cells quickly leads to a fall off in recording speed. On the subject of light spot intensity, the optimum can only be determined by trial and error but one fairly evident rule is that the brilliance must be increased, the faster the recording speed.

Last, but by no means least, care is also necessary during the processing of the records — the developing and fixing solutions should be kept up to scratch and their containers clean.

Thus, a complete test would be conducted on the following lines:

(a) Static test using Betz manometer to determine the approximate limits of the pressures to be recorded. At the same time, the recorder is loaded and the equipment allowed to warm up.

(b) Feed in appropriate pressure to the diaphragm and set light spot in aperture.

(c) Calibration over expected range.

(d) First part of test e.g. with cages stationary in test length.

(e) Check calibration — say two values.

(f) Second part of test e.g. with cages moving in test length.

(g) Calibration over expected range.

(h) End of test or, more usually, repeat test.

It will be appreciated that a good procedure can only be learned through experience. It is hoped that these few notes will help any subsequent workers to avoid needless delays and frustrations when using this equipment.

(B) Tests

Introduction

Tests were now carried out to investigate some of the phenomena associated with moving cages in order to correlate the results with those of Chapter 3, and to check the theory of Chapter 4 where possible.

They may be conveniently subdivided as follows :

Section 1 : Recording of the effect of the bunton spacing on the cage pressure drop (a) staggered arrangement, (b) even arrangement.

Section 2 : Recording of the effect of a cage passing in both directions on the pressure drop of a test length.

Section 3 : Recording to show the influence of two cages passing on the quantity flowing in the shaft.

Section 4 : Recording to show the influence of two cages passing on the pressure drop of the test length containing the passing place.

Section 5 : The correlation of some moving cage tests with the pressure drop theory.

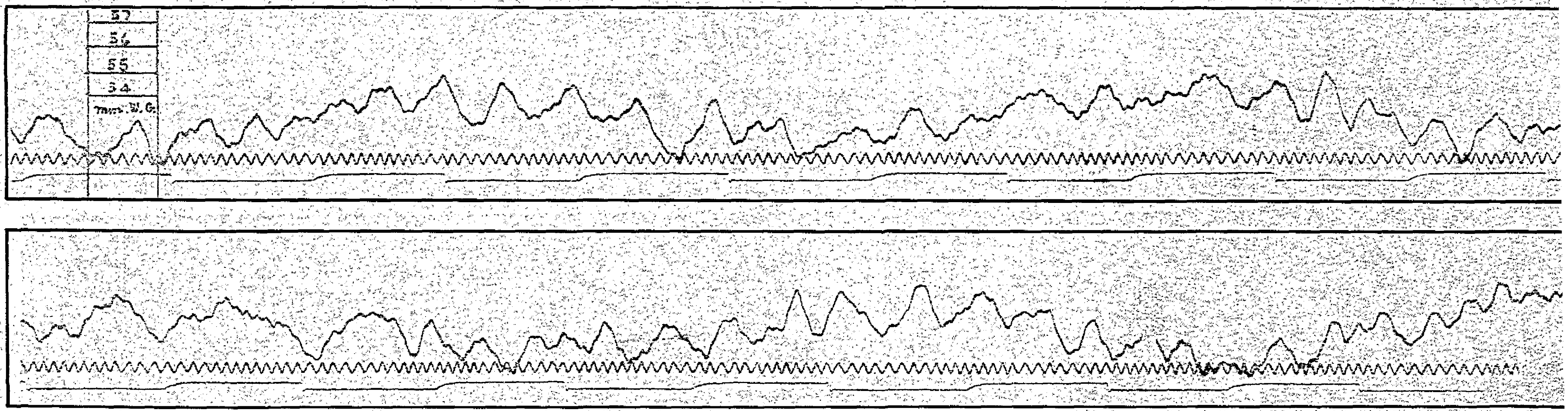
Section 1

The object of this first section was to attempt to record the effect of different bunton arrangements on the pressure drop due to a cage. In addition, it was desired to compare the results of this continuous recording with the results from the stationary cage tests. As before, the same two bunton arrangements and test lengths were used. Also, in order to allow the effects to fully manifest themselves, the air speed was the maximum possible (just over 60 f.p.s.) and the cage speed was very low, approximately 0.27 f.p.s. A recording speed of 3 ins. per sec. was utilised.

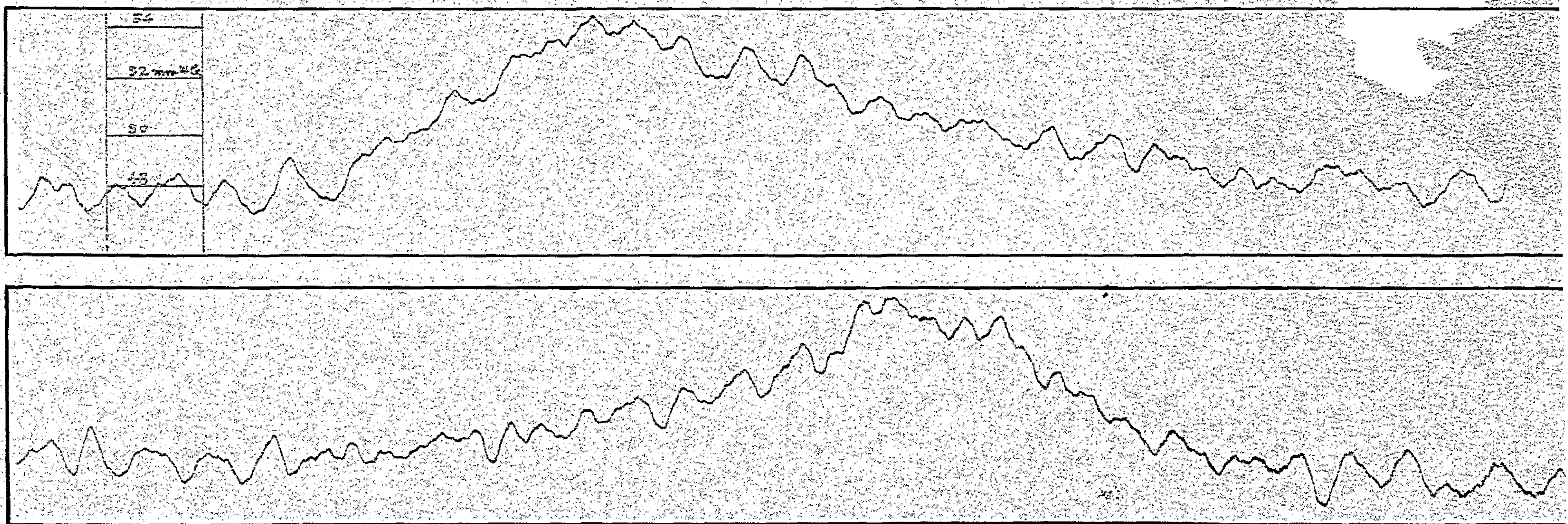
(a) As with the stationary cage tests, test length ①-② was used to determine the effects of a staggered bunton arrangement and No.1 cage was used.

First of all, a preliminary stationary test was done to check that conditions were unchanged and to obtain the range for the recorder. Recording was then carried out, the cage moving with and against the airstream in succession and several repetitions were made. A typical part of the record is shown in Fig. 5.7(a).

(b) A similar procedure was carried out for test length ⑦-⑨. In this test length, exactly the same section was used, containing



(a) Staggered Bunton Arrangement



(b) Even Bunton Arrangement

Fig. 5.7 Recordings of Cage Moving in Different Bunton Arrangements evenly spaced buntons, as in the stationery tests. No.1 cage was used and recording made for a cage movement of about two feet. Fig. 5.7(b) depicts a typical part of the record.

The relevant calibrating figures have been given and since the two test lengths were about the same size, their pressure drops were roughly the same.

The first thing noticed was the fact that the pressure drop fluctuations could be easily seen, with the higher frequency fluctuations, due to turbulence in the airstream, superimposed. As well as this, the records were the same whether the cage moved upstream or downstream. With the air and cage speeds used here this is not surprising.

The records were correlated with the appropriate positions of cage relative to buntons and it was found that the peaks and troughs corresponded to cage/bunton positions in exactly the same way as in the stationary tests.

As stated, preliminary stationary tests had been carried out and it was found that there was no detectable difference in the pressure drop values for corresponding cage positions. Similarly, the P.D.C. values were the same.

Once again it was found that the "waves" produced by a staggered bunton arrangement were half the amplitude and duration of those of an even arrangement.

With the high air speed and low cage velocity used in these tests this correlation with the stationary tests is as might be expected. However, with higher cage velocities and lower air speeds, the relative importance of these fluctuations decreases. It can be seen that the cage P.D.C. depends much more on the quantity of the buntons, their arrangement having a much smaller influence.

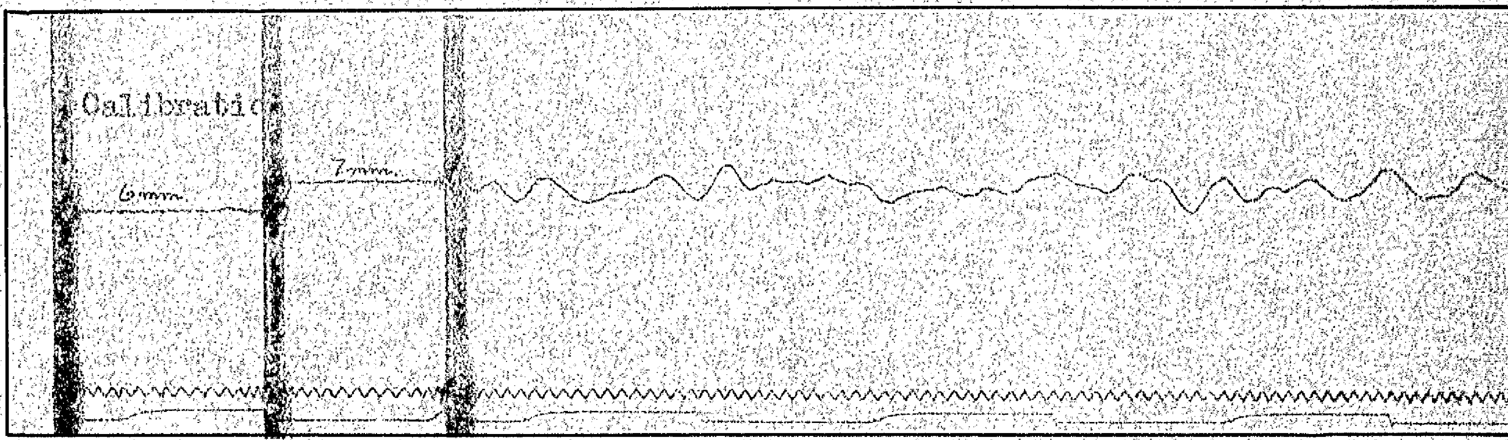
While this might not matter in a full scale shaft, it can be appreciated that the bunton configuration could exert an undesirable influence in a model investigation, where precise measurements are desired, if little thought was given to the spacing. The best arrangement would be that which gives the least deviation from a mean line. In the case considered here, the staggered arrangement is the better from the point of view of accuracy.

Section 2

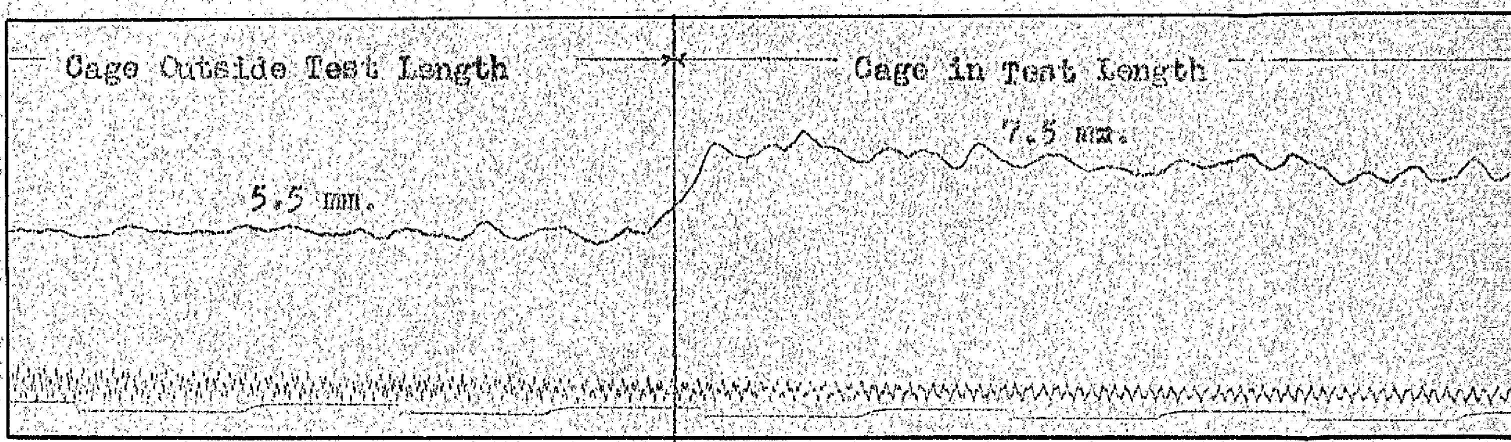
In this section, an attempt was made to show the influence of the direction of motion of a single cage relative to the air flow direction on the pressure drop and P.D.C. For this purpose, an 18 foot test length ②–⑥ was used with cage 1 only. The maximum cage speed of 5.5 f.p.s. was constant and the air speed varied.

Apart from those tests conducted at very low air speeds where the fluctuations were too small to be picked up by the recorder, even at its maximum sensitivity, all the records showed the influence of the direction of the cage's movement on the pressure drop.

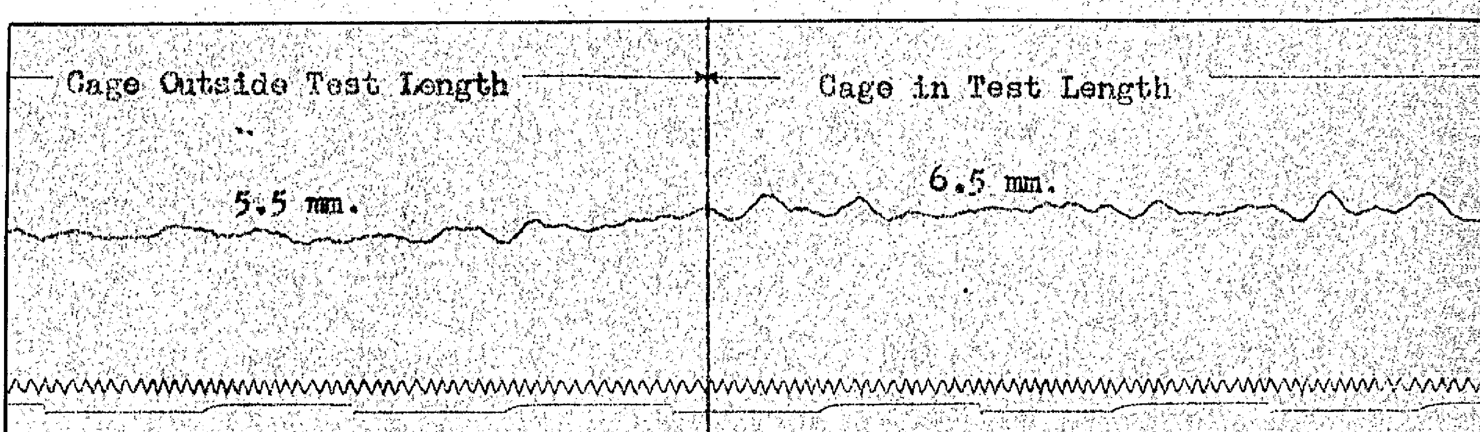
Fig. 5.8 illustrates a typical record, made with an air speed of 19.5 f.p.s. in the duct. Part (a) is a record of the pressure drop with the cage stationary; part (b) shows the pressure rise as the cage enters the test length, going against the airstream — the pressure drop rising to a value greater than the stationary; part (c) similarly, but with the cage moving in the same direction as the air — the pressure drop being less than the stationary value in this case. The relevant values are given on the records.



(a) Cage Stationary



(b) Cage Moving Against the Air Stream



(c) Cage Moving with the Airstream

Fig. 5.8 Records of Cage in Test Length (2) - (6)

At the higher air speeds, up to 60 f.p.s., the fluctuations were rather troublesome, probably due to the increased turbulence, and the relatively greater influence of the buntons. However, the same effects were noted.

The only general conclusions which can be reached from this limited series of tests apart from recognising the effect, is that at the higher air speeds the motion of the cage results in a larger increase or decrease in the test length pressure drop compared with the stationary value.

A larger series of tests were carried out by R. Stewart. Reference will be made to these results in Section 5.

Section 3

This series of tests was concerned with showing what reduction in air flow and air velocity was caused when the two cages passed each other at the centre of the shaft. It was well known, from our own and from Wilkie's experience, that the flowmeter reading fluctuated considerably. It was therefore decided to record the velocity pressure at the centre of the duct at Tap 14, as well as the mean velocity pressure of the flowmeter, while the cages passed each other. It will be remembered that in Chapter 3 it was found that the readings at the Tap 14 position were subject to very small variations, especially at the duct centre, compared with flowmeter readings. It was hoped therefore that any abnormal changes would be shown up more clearly.

A calibration of the flowmeter and Tap 14 readings might have been useful but it was not thought necessary since a qualitative result only was wanted. No theory had been advanced to be experimentally verified.

While a broad relationship seems to exist between the effects described in this section and in Section 4 (static pressure drop of ⑥-⑨ as cages pass) for which a theory has been advanced, many different quantities are involved. For instance, instead of

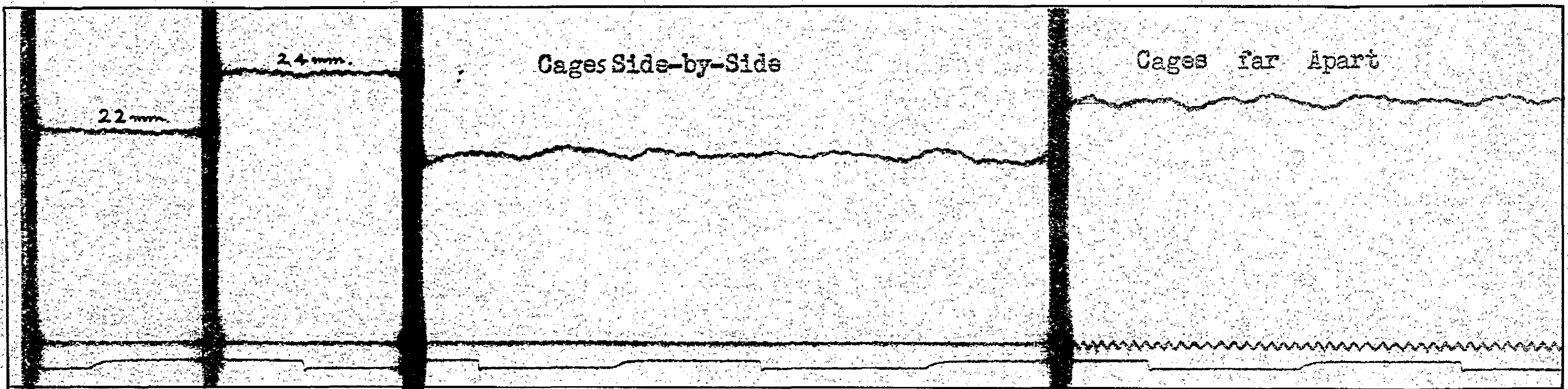
Section 3

simply a static pressure drop, which is independent of the fan characteristic, here one is concerned with a velocity pressure and air quantity which brings the fan characteristic into any consideration.

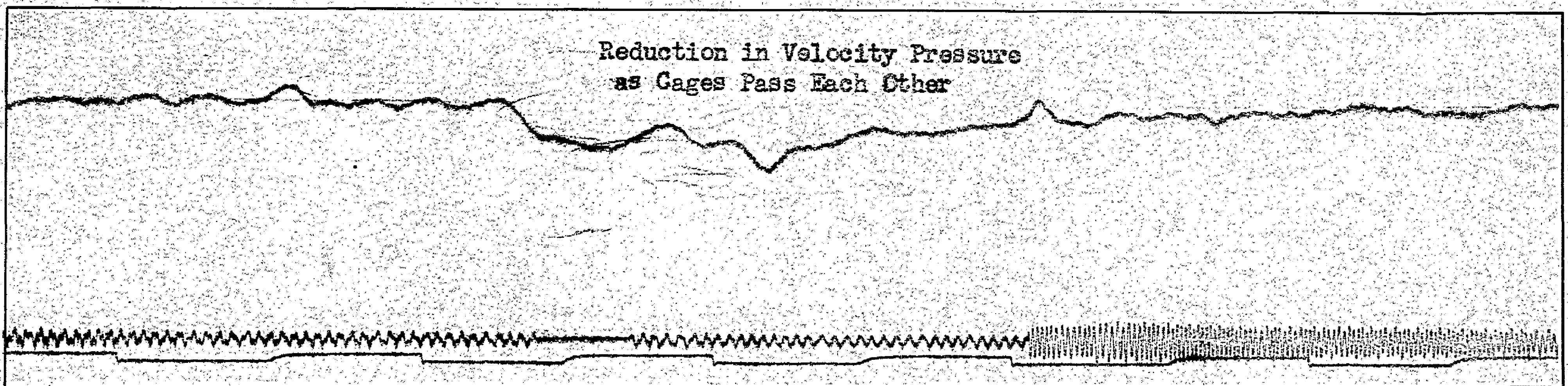
The tests using Tap 14, a typical recording of which is shown in Fig. 5.9, will be described first of all.

The first step was to record the pressure with the cages stationary and in the two extreme positions, i.e. side-by-side and apart, as shown in Fig. 5.9 (a) to ascertain that the difference was in fact measurable. Since this was so, tests were then carried out at the maximum air velocity ($\approx$ 60 f.p.s) and cage speed ($\approx$ 5 f.p.s) and a specimen record is depicted. For those conditions it was found that, in general, the maximum drop in pressure was about the same as for the stationary cages. The shape of the recording seemed to indicate that the reduction was characterised by a sudden drop at first followed by a more gradual return to normal. The reduction amounted to approximately 6 per cent.

In the next stage, see Fig 5.10, the flowmeter reading was recorded. Since the fluctuations were so large, recording of the pressure, with the cages stationary, was done at a low recording speed for a longer time. This gave a clearer and more compact picture and Figs. 5.10 (a) & (b) illustrate these stationary values. Again, the reduction in pressure when the cages are side-by-side is evident, but not nearly so clear cut as those of Fig. 5.9. Under the same conditions as previously, records were then taken with the cages moving, see Fig.5.10 (c). As expected, the reduction shown by these recordings was not nearly so clear as those of Tap 14.

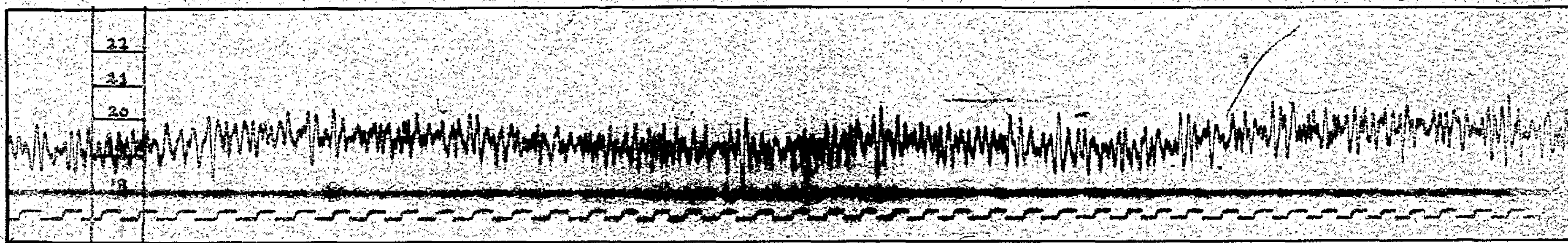


(a) Cages Stationary

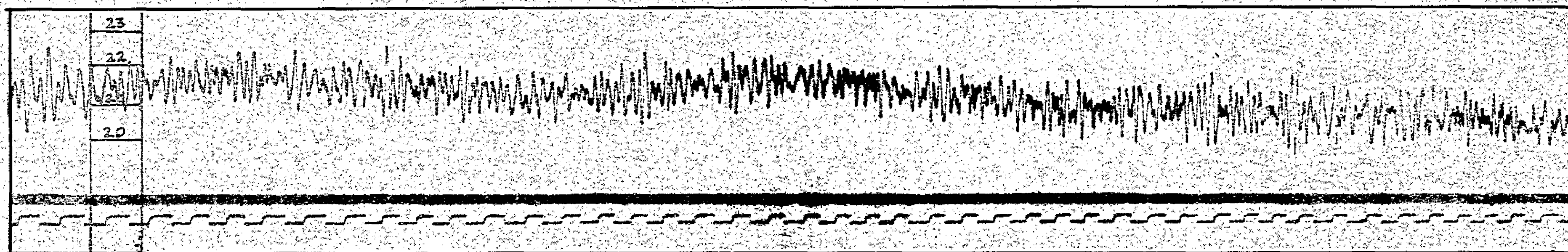


(b) Cages Moving

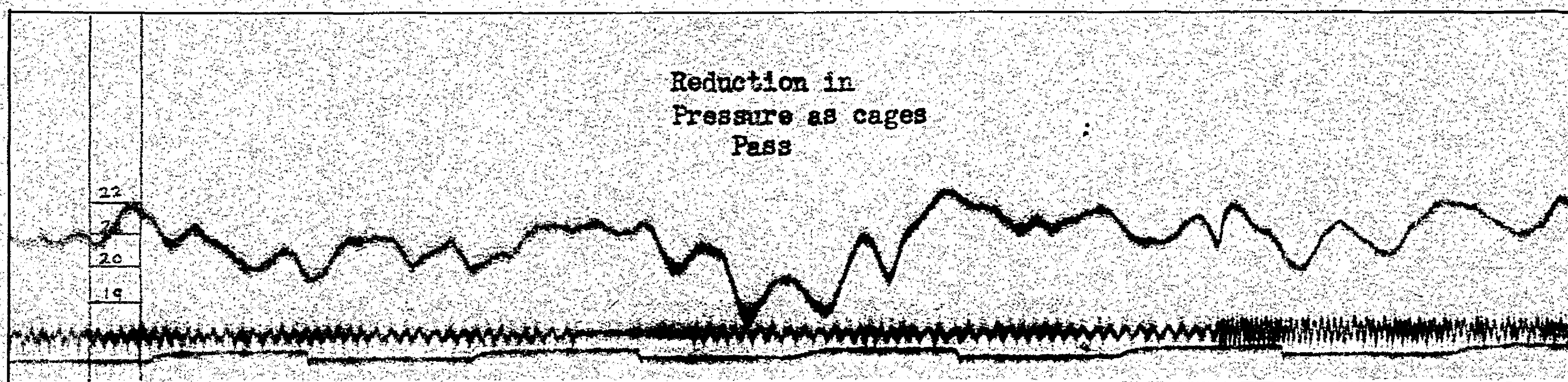
Fig. 5.9 Recording of Velocity Pressure at Tap 14



(a) Cages Stationary, Side-By-Side



(b) Cages Stationary, Far Apart



(c) Cages Moving

Fig. 5.10 Recording of Flowmeter Pressure

Section 3

However, the tendency of the mean velocity pressure to be significantly reduced in value can be seen without any doubt as in Fig. 5.10 (c).

It is also worth remarking here that in Figs 5.10 (a & b) there seems to be an underlying wave form with a period of about 6 seconds possibly due to pulsating air flow in the circuit.

Section 4.

This section constituted an attempt to show the effect of the cages passing on the shaft resistance. This was done by recording the static pressure difference in test-length ⑥-⑨ as the cages approached, passed, and receded from the zone of interaction.

Similar tests had already been carried out by Wilkie. However, the results of these tests had not shown any relation between cage P.D.C. and speed for the range available. Since the cage speed could not be increased and, from the previous Chapter the P.D.C. increment was theoretically extremely small, it was hoped only to show the same effects again.

This time however, a higher measuring sensitivity could be used and, by recording the pressure drop with the cages stationary (side-by-side and outside the zone of interaction) as well as when moving, a better comparison could be achieved. From Section 3 it is known that there is a lowering of the mean velocity pressure when the cages pass. Also, this reduction is roughly the same whether the cages are moving or stationary. Thus, the test length pressure drop would be affected similarly whether the cages were moving or stationary thus giving a good basis for comparison by the above method.

From a consideration of the results of the stationary cage tests and the theoretical analysis it was decided that the purpose of this section would best be suited by lowering the air velocity as much as possible while keeping the cage speed at its maximum value. Thus it

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would be possible to record the full range of pressure fluctuations on the full width of the paper at the chosen sensitivity.

It was hoped that the air speed could be lowered to that of the cages or less but at that level, variations in test length pressure drop were too small to be picked up by the recording equipment. In addition, it was suspected that Reynolds number effects would have to be considered at these low air speeds. A test was made and a graph drawn of the P.D.C. of test length ⑥-⑨ to air velocity. This is shown in Fig. 5.11 where it is clear that below an air speed of about 30 to 40 f.p.s. corresponding to a mean velocity pressure of about 7.5 mm W.G. the results are not strictly comparable.

Thus the lower range of air speeds was on the sloping part of the graph. However, since the purpose here was qualitative rather than quantitative it was decided to utilise an air speed of about 20 f.p.s. (corresponding to 3 mm W.G. mean velocity pressure). This value gave the most convenient range of test-length pressure drops for the various cage positions, easily accommodated on the width of the recording paper.

As usual recordings were made with the stationary cages side-by-side and outside the zone of interaction, a typical specimen being shown in Fig. 5.12.

Recordings were then made with the cages moving. Many of these were done and in all cases it was found that, within the limits of the measurements, the peak pressure difference, corresponding to the position

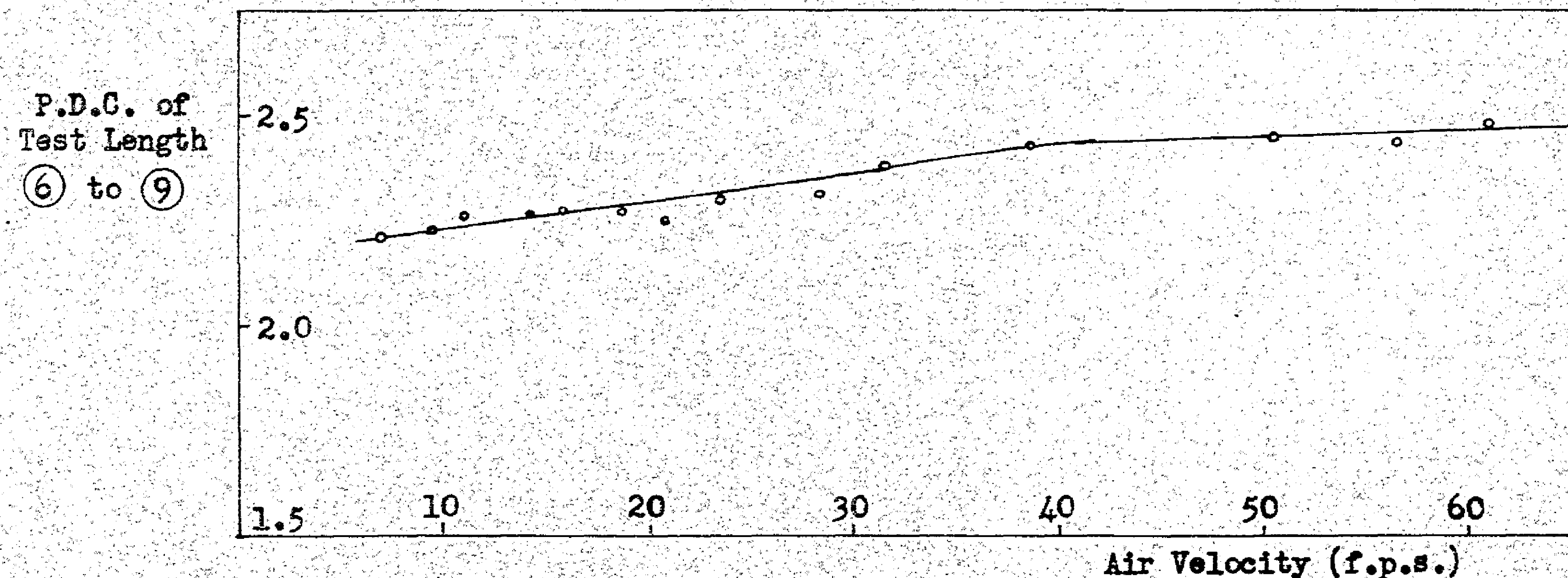


Fig. 5.11

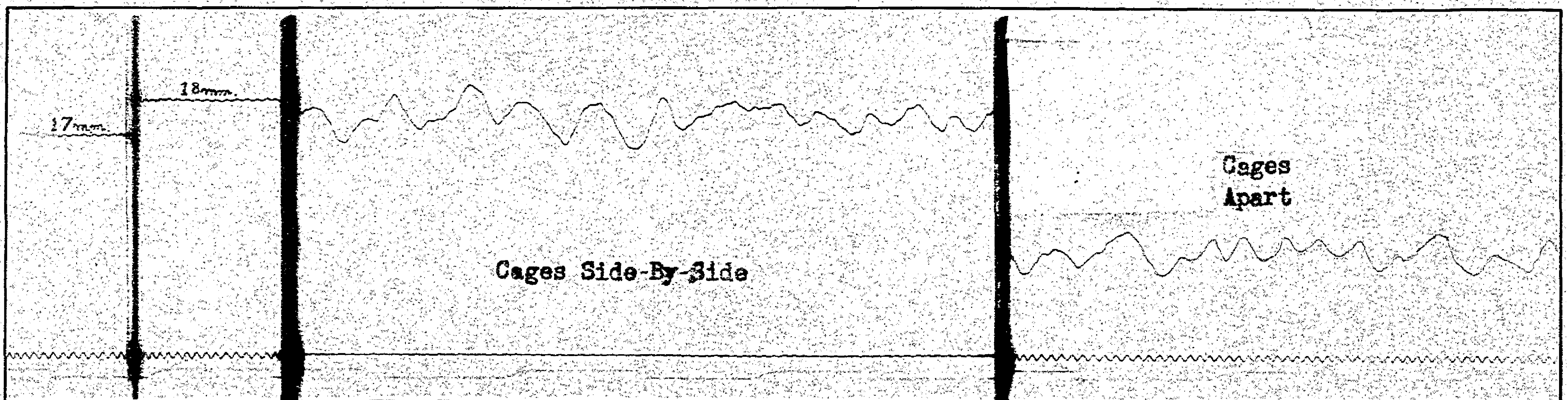
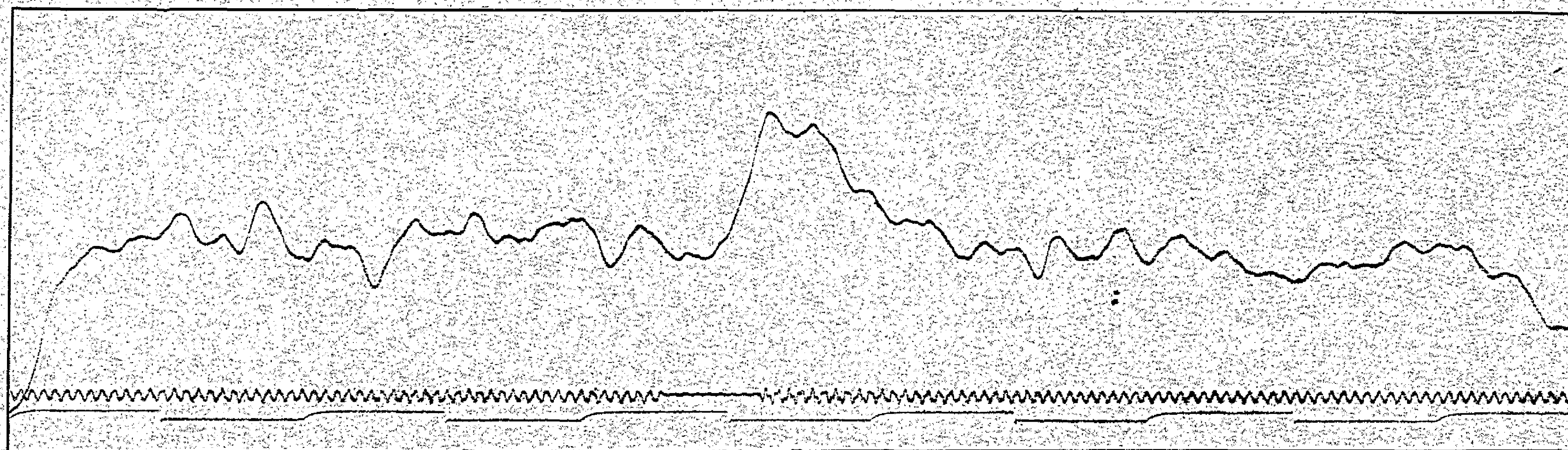


Fig. 5.12 Cages Stationary

where the cages passed, was substantially the same as when the cages were in the stationery side-by-side position. For purposes of comparison, Figs. 5.13 (a) & (b), showing the peaks as the cages pass, are taken from the same record as the stationary cage recording of Fig. 5.12.

These results were much as expected and it would appear that in order to attempt to verify the theory it will be necessary to increase the cage speed considerably. This of course poses the difficulty of constructing a lining, robust enough to withstand the increased stresses, keeping in mind what has already been stated about the interference between cage and buntons. Another factor would be the short time the cages would be in the test length though a faster recording speed might help.

(a) No.1 Cage Going Downstream, No.2 Upstream



(b) No.1 Cage Going Upstream, No.2 Downstream

Fig. 5.13 Recording of Pressure Drop of Test Length ⑥ - ⑨ Cages Moving at Maximum Velocity

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The tests described in Section 2 serve to illustrate the effect of the direction of movement, relative to the air flow, of the cage. In this section it is proposed to compare the results of the larger test series carried out by Stewart with the theoretical values calculated from the Pressure Drop Theory.

The experimental results have already been noted in Chapter 2 where the pressure drop due to a moving cage was plotted versus $(V \pm V_c)$, in the present notation, the cage speed being varied at five different air speeds. Knowing the various air speeds, the pressure drop values were converted to P.D.C.'s and the corresponding velocity coefficients C_v calculated.

Examination of Stewart's work showed that cage G only had been used throughout the tests. The following factors could then be evaluated.

CAGE G

Cage leading dimensions = 6 in x 2.25 in x 1.84 in.

Cage frontal area A_c = 4.14 sqr.in. f_d = 0.0222

Duct area A = 9.63 sq.in. f_c = 0.22

Fill coefficient C_f = 0.43 C_d = 1.18
C_o = 0.744.

B_1 = 0.43; B_2 = 0.569; B_3 = 4.44; B_4 = 5.40.

Knowing these values, the P.D.C.'s could then be calculated using Equation 4.37

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$$C_c = C_D^2 B1 (C_f) Y_1 (C_v) + k B_2 (C_f) + \frac{\ell f d}{D} B_3 (C_f)$$
$$+ \frac{\ell f_c C_c}{D} B_4 (C_f) Y_2 (C_f, C_v)$$

for the range of C_v's used in the tests and a theoretical C_c/C_v graph drawn.

Both the theoretical and experimental graphs are given in Fig. 5.14. While it would have been possible to plot an infinite number of test points, only representative values, for each of the five air speeds were selected.

It is at once evident that there is a fairly wide scatter of the measured values, but despite the large scale used for this comparatively small C_v range, an approximate line can be drawn. The actual results would appear to diverge from the theoretical line as the C_v value increases, become greater as the cage moves relatively faster against the air; less as the cage moves faster in the same direction as the airflow.

This discrepancy could be attributed to several factors. For example, to the assumptions made in the derivation of the theory, notably that neglecting the influence of the guides. However, the stationary cage results gave good proof of the theory and the only additional factor involved here relating to the movement of the cages, was the square law of aerodynamical resistance and this has been proved many times.

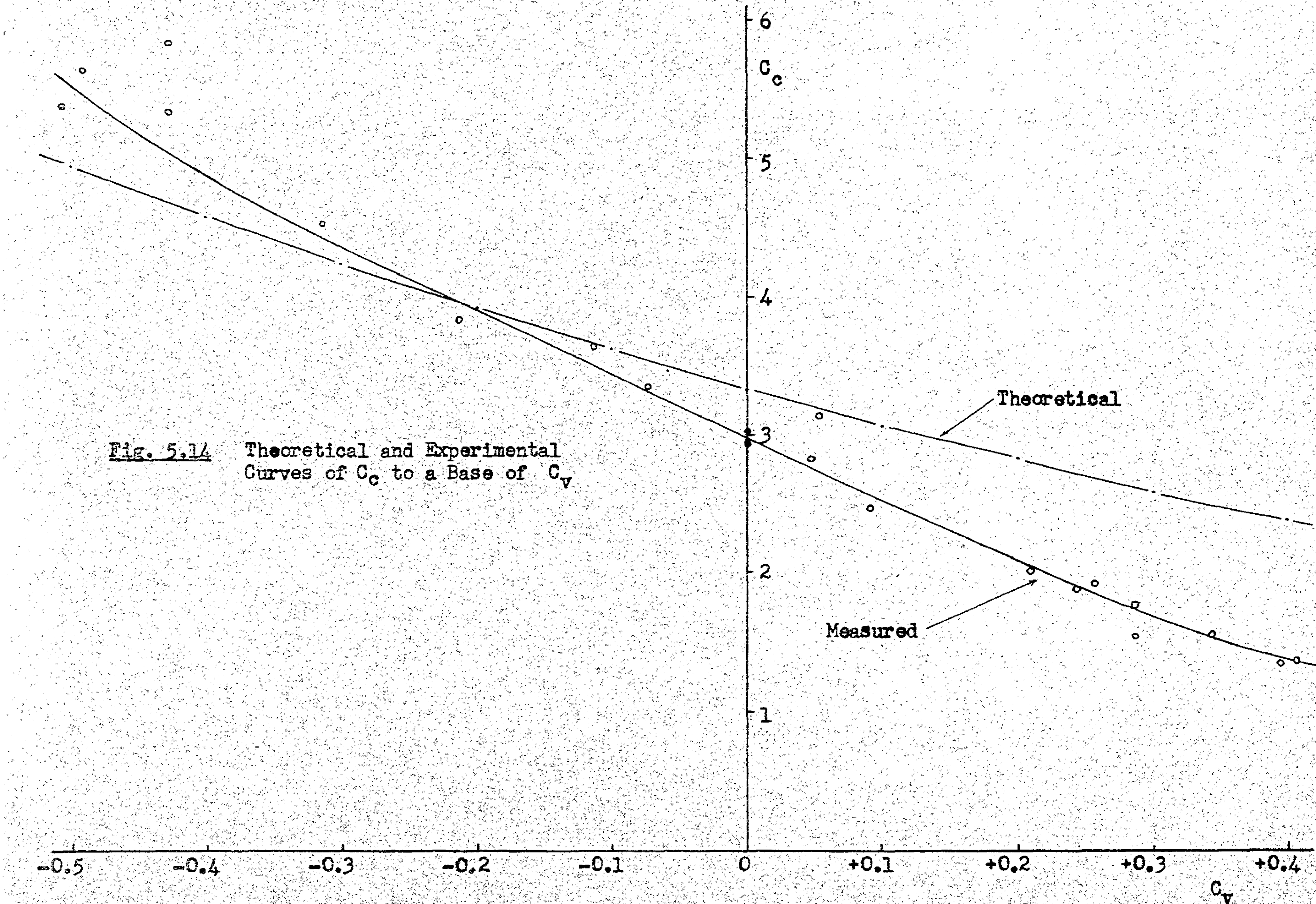

Fig. 5.14 Theoretical and Experimental Curves of C_c to a Base of C_v

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In spite of the precautions taken to allow for its effects, it is felt that the main error if probably due to the difficulty of measuring the true mean velocity pressure when the cage is moving in the duct.

It is possible that the influence of the moving cage on the resistance of the duct may have been over-estimated. This would result in the measured results being lower than their true value at the higher positive C_v values; higher at the negative C_v values. Both these effects would tend to increase as the numerical value of C_v increased.

However, the difficulties involved in allowing for these effects are appreciated, and in no way detract from the value of these test results for purposes of checking the theory.

The graphs of Fig 5.14 do serve to further verify the validity of the assumptions made in the derivation of the Pressure Drop Theory. More tests will be necessary if the correlation is to be extended over a wider range of C_v values.

C H A P T E R 6

REVERSAL OF AIRFLOW DUE TO SINGLE CAGE WINDING

INTRODUCTION

An investigation has already been carried out into disturbances in the mine ventilation circuit caused by single cage winding, this research being prompted by report from collieries in the North of England and the East Midlands. In these collieries it was found that winding with a single cage instead of two smaller balanced cages had a very noticeable effect on the ventilation.

In this Chapter it is intended to present a short appraisal of the problem from a different point of view to that used by Stewart. The consideration is restricted to the case considered here. However the method is widely applicable. As will be seen, it is not altogether easy to give a general conclusion, but it is possible to state the conditions for certain specific cases.

In addition, certain other general conclusions can be reached on the basis of this section of the work, and that of the previous Chapters. These will be stated in due course.

THEORETICAL CONSIDERATION

Consider the simple mine circuit shown in Fig. 6.1

a.b.c. is the downcast shaft with single cage winding.

d.e.f. is the upcast shaft - for the moment assume no winding arrangements.

b.e. represents the mine workings at an upper horizon.

c.d. represents the mine workings at a lower horizon.

Also, let R = aerodynamical resistance of a branch of the circuit.

and $h_f(Q)$ = fan pressure characteristic

and $h_c(Q)$ = cage pressure characteristic.

In the previous chapters the pressure drop coefficient of a cage C_c has normally been used. A convenient conversion is given by

$$h_c = C_c \frac{V^2}{2g} = \frac{C_c}{2gA^2} Q^2$$

The circuit may be conveniently redrawn and labelled as in Fig. 6.2

Where
- R_{D1} = Resistance of upper part of D/C shaft.
- R_{D2} = Resistance of lower part of D/C shaft.
- R_1 = Resistance of upper horizon.
- R_2 = Resistance of lower horizon including the lower part of the U/C shaft)
- R_u = Resistance of upper part of U/C shaft.
- Q = Total airflow in the mine.
- Q_1 = Airflow in the upper horizon.
- Q_2 = Airflow in the lower horizon.

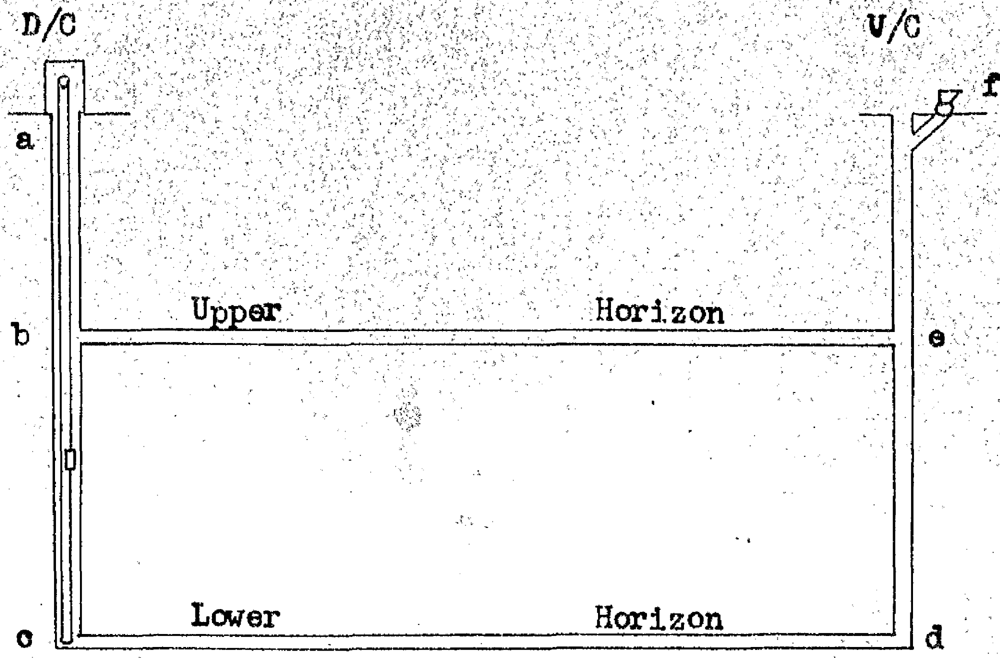

Fig. 6.1 Sketch of Mine Circuit Considered

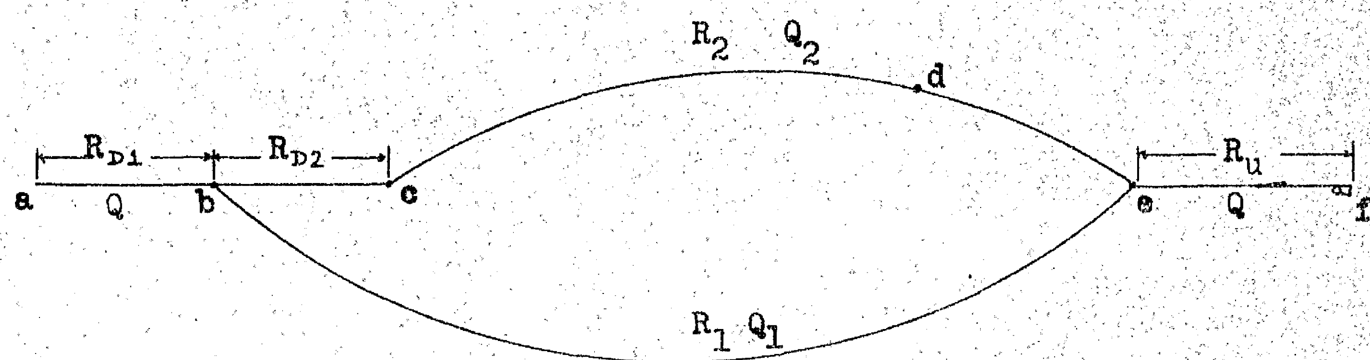

Fig. 6.2 Labelled Sketch of Circuit

It is assumed that the air in mine roadways is incompressible and obeys Atkinson's Laws. In addition, Kirchoff's Laws, on which the whole of the circuit theory of steady airflow is based, are used. From the previous work it is evident that the cage acts as a source of pressure loss or gain dependent on its velocity and the direction of its motion.

In the first instance, the cage will be considered moving in the upper part of the downcast shaft only, between a and b

$$\text{Then} \quad h_f + h_c = \left[(R_{D1} + R_u + \frac{R_1 (R_2 + R_{D2})}{(\sqrt{R_1} + \sqrt{R_2 + R_{D2}})^2} \right] Q^2 \ldots\ldots\ 6.1$$

$$R_1 Q_1^{\,2} = \left[R_2 + R_{D2} \right] Q_2^2 \ldots\ldots\ 6.2$$

$$\text{and} \quad Q = Q_1 + Q_2 \ldots\ldots\ 6.3$$

From equations 6.2 and 6.3

$$Q_1 = Q_2 \sqrt{\frac{R_2 + R_{D2}}{R_1}}$$

$$= (Q - Q_1) \sqrt{\frac{R_2 + R_{D2}}{R_1}}$$

$$\text{i.e.} \quad Q_1 \left[1 + \sqrt{\frac{R_2 + R_{D2}}{R_1}} \right] = Q \sqrt{\frac{R_2 + R_{D2}}{R_1}}$$

$$\text{Hence,} \quad Q_1 = Q \left[\frac{\sqrt{R_2 + R_{D2}}}{\sqrt{R_1} + \sqrt{R_2 + R_{D2}}} \right] \ldots\ldots\ 6.4$$

$$\text{Similarly,} \quad Q_2 = Q \left[\frac{\sqrt{R_1}}{\sqrt{R_1} + \sqrt{R_2 + R_{D2}}} \right] \ldots\ldots\ 6.5$$

Therefore, on the basis of the simple network theory, for time steady conditions, the quantity of air flowing in each horizon depends on the resistances of the various branches and the characteristics of the various sources. The sources, in this case, are the fan and the cage, their pressure drops both being involved in the Q term of equations 6.4 and 6.5 (see equation 6.1).

The only condition when the pressure drop of the cage can cause a reversal of airflow is when the cage is moving against the airflow in the shaft. In these circumstances the circuit has two fans acting in opposite directions. Only in the normally highly improbable condition when the pressure heads produced by fan and cage are equal will the airflow in the mine be stopped and in this case, both horizons will be affected equally.

The case will now be considered where the cage has passed the inset at the upper horizon and is moving in the lower part of the downcast shaft between b and c. Under these circumstances

$$R_{D1} Q^2 + R_1 Q_1^2 + R_u Q^2 = h_f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.6$$

and $$(R_{D2} + R_2)\, Q_2^2 + h_c = R_1 Q_1^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 6.7$$

As before $$Q_1 + Q_2 = Q$$

Equations 6.6 and 6.7 are both second order of Q and a general solution of these for Q_1 and Q_2 would involve the fourth order.

However, the main concern here is with find the conditions such that the air flow in the upper horizon would be stopped, i.e., the special case of $Q_1 = 0$.

Suppose $Q_1 = 0$, then $Q_2 = Q$ and equations 6.6 and 6.7 then become,

$$R_{D1}\ Q^2 + R_u Q^2 = h_f$$

and $$(R_{D2} + R_2)Q^2 + h_c = 0$$ respectively,

giving $$h_f = (R_{D1} + R_u)Q^2$$

and $$- h_c = (R_{D2} + R_2)Q^2$$ respectively,

hence, $$h_f = \left[\frac{R_{D1} + R_u}{R_{D2} + R_2}\right](-h_c) \qquad \text{.................................} \quad 6.8$$

DISCUSSION

Equation 6.8 thus defines the conditions necessary to cause the cessation of air-flow in the upper horizon.

The negative sign of h_c implies that the effect of the cage must be to give a negative pressure loss, i.e. a pressure gain. In other words, the cage acts as an additional fan in the lower part of the down-cast shaft, causing flow in the same sense as the main one. Consequently, only when the cage is moving in the same direction as the air will these circumstances be possible.

As stated above, equation 6.8 defines the conditions, but some qualification is necessary since the quantities flowing, the fan and cage pressure drops and the resistances are all interdependent. Its solution must therefore be based on some system of continuous approximation.

As a first step it would be necessary to find, from the fan and mine characteristics, the fan pressure and quantity with the cage stationary. Knowing these values, and the resistances of the various branches the pressure drop which the cage must produce can be calculated. Hence, knowing the shaft air velocity, the cage velocity, to give the required conditions, could be found. At each subsequent step, the pressures and speeds would be adjusted to maintain these airflow conditions.

Conversely, knowing the branch resistances and the cage dimensions and design speed, the minimum fan pressure to give conditions of no flow in the top horizon could be calculated.

Hence, the required fan duty could be estimated such that these conditions would not arise; alternatively the cage dimensions or its speed could be suitably modified.

It must be said that this applies to single cage winding systems only. With a two cage system, the pressure drops, as shown in the previous chapters, cancel each other out, resulting in no net pressure gain or loss. In addition of course, in a two cage system, the individual cage P.D.C's are usually much less than for a single cage.

Equation 6.8 can equally well apply to the case of a single cage ascending the lower part of the upcast shaft from points d to e. For a combination of both cages, the effects would obviously be additive, giving $2\ h_g$ for the case of the two cages moving in the same sense.

The consideration is of course limited to two horizons but it is hoped that the approach will help to shed some more light on the problem.

On the more practical side it may be said that to produce such an effect as the stopping of the airflow in the upper horizon, the fill coefficient, and consequently the P.D.C., of the cage would have to be quite large. From the point of view of model investigations it may be as well to mention also that when conducting tests it would be best to measure the airflow at a point as far as possible from the inset. Much turbulence is caused by the passage of the cage which could quite easily result in erroneous readings.

CHAPTER 7

CONCLUSIONS

The test results and theoretical schemes have been fully discussed in their appropriate sections. The conclusions, both general and specific, which can be drawn from this work, will be presented here.

From the Stationary Cage Tests of Chapter 3 it can be said that

(a) Allowing for its fluctuations, the Flowmeter reading can be taken as the true mean velocity pressure of the air flowing in the duct. Use of a correction factor of unity for the mean velocity results in an error of not more than 5 per cent, for the range of air speeds involved.

(b) The use of the P.D.C. $= \dfrac{\Delta P_s}{\frac{1}{2}\rho \bar{v}^2}$ is valid provided the P.D.C. is that of all the components of the section of ducting in use.

(c) The assumption that the overall P.D.C. of a test length is the algebraic sum of the P.D.C.'s of its various components, measured separately is only very approximate. It has been shown that interference between cage and shaft furnishings causes the cage P.D.C., measured in the lined test length and calculated as described, to be more than twice the value obtained in the unlined ducting.

(d) The value of cage P.D.C. may be assumed constant for any position in the duct. The error incurred in doing so in the present case amounts to less than 4 per cent.

(e) Interaction between the two cages causes their combined P.D.C., in the side-by-side position, to be approximately twice that when they are far apart.

(f) The zone of direct interaction is about twice the cage length.

(g) The relative position of cage and buntons exerts a great influence on the measured value of cage P.D.C. For a single cage, a staggered bunton arrangement results in a smaller amplitude of P.D.C. wave than an even arrangement. This is due to the fact that with the former configuration, the same number of buntons (3) are nearly always in contact with the cage; with the latter, the number of buntons producing interference effects varies from 2 to 4.

In short, the bunton arrangement affects the accuracy of any measurements, while the number of them obviously affects the total combined P.D.C. of cage and buntons.

In Chapter 4, Dr. Ryncarz's Pressure Drop Theory has been presented. The following are of importance.

(a) On the basis of some simplifying assumptions, the relationship between the P.D.C. C_c of a cage and its length ℓ in the form

$$C_c = s + t\ell$$

where s and t are constants, independent of ℓ, has been re-verified.

(b) For a stationary cage it has been shown that

$$C_c = C_D B_1 + k B_2 + \frac{\ell f_d}{D} B_3 + \frac{\ell f_c C_o}{D} B_4$$

and for two stationary cages, side-by-side,

$$\left[C_c \,||\, C_c\right] = C_D M_1 + k M_2 + \frac{\ell f_d}{D} M_3 + \frac{2 \ell f_c C_o}{D} M_4$$

the various symbols being defined in the text.

The results of three previous investigations have been used to cheke these equations and good correlation has been obtained between theoretical and measured results.

(c) Formulae have also been put forward for finding the P.D.C.'s of moving cages. They are similar to the above, plus terms involving the cage speed.

(d) The minimum P.D.C.'s occur when the cages are stationary.

(e) For the case of rectangular cages in a circular shaft, it can be assumed that the fill coefficient is independent of the ratio of the perimeters of cage qnd shaft.

Chapter 5, describing the Moving Cage Tests, indicates the following general conclusions.

(a) As with the stationary cages, the type of bunton arrangement exerts a very great influence on the P.D.C. of a moving cage. This has the greatest effect on the recording when the cage speed/air velocity ratio is low. As this ratio increases, the effect on the accuracy of the recorded trace becomes less noticeable.

(b) The direction of motion of the cage exerts a great influence on the cage P.D.C. When the cage moves against the air, its P.D.C. increases; moving with the air, the P.D.C. decreases. The difference from the stationary value increases as the cage speed/air velocity ratio increases.

(c) A velocity pressure reading at the centre of the duct cross-section provides a more accurate method of estimating the reduction in airflow when the cages pass one another, than a direct recording of the flowmeter pressure.

(d) For the cage speeds and air velocities used here, the reduction in airflow due to the cages passing each other may be assumed the same for both moving and stationary cages.

(e) For the range of cage speeds and air velocities available, no increase in the P.D.C. of two cages passing, compared with the stationary side-by-side value, could be detected. If this effect is to be shown, some modification will be necessary so that much higher cage speeds can be used.

(f) Comparison of some moving cage test results with the theoretically predicted values of cage P.D.C. showed a fair degree of correlation. The difficulty of measuring the true mean velocity pressure when the cages are moving in the duct, especially at the higher C_v values, appears to be one of the main causes of discrepancy.

In Chapter 6 an attempt was made to analyse the conditions necessary to cause the reversal of airflow in the upper level of a two horizon mine with single cage winding in one of its shafts.

The following have emerged from the analysis.

(a) The conditions necessary for stopping the airflow in the upper horizon are given by

$$h_f = \left[\frac{R_1 + R_u}{R_2 + R_2} \right] (-h_c)$$

where the various quantities are defined in Chapter 6.

(b) To produce these conditions, the cage must be travelling in the same direction as the airflow in the lower part of either the downcast or the upcast shaft, and producing a pressure gain.

(c) When conducting tests, the air quantities should be measured as far as possible from the shaft insets where local disturbances might affect the instrument readings.

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