AN

INVESTIGATION OF THE

CREEP OF THICK TUBES

SUBJECTED TO INTERNAL PRESSURE

by

E. M. Smith, B.Sc.

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to the University of Glasgow.

April, 1962

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343	7	$\frac{\sqrt{2}}{3}$	$\frac{1}{\sqrt{2}(1+\eta)}$

E.M.S. 22nd May, 1962.

"........ I often say that when you can measure what you are speaking about and express it in numbers, you know something about it; but when you cannot measure it in numbers your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science whatever the matter may be" ....LORD KELVIN.

CONTENTS

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Without these people, this document would not exist.

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INTRODUCTION

The incentive for more accurate design of tubes to withstand internal pressure under creep conditions springs from both economic and technological considerations.

In design, the bore of a pipe or tube is generally fixed by the allowable pressure drop in the fluid to be carried. The pipe wall may therefore be considered as being built round the required bore, and the necessary wall thickness is then determined taking into account stresses in the pipe material, corrosion and oxidation allowances, thinning allowances for bending, metallurgical considerations of fabrication and service, etc. "Over design" in any direction results in the pipe being thicker than necessary, and will introduce unnecessary stiffness into the piping system. It may also cause undesirably high thermal stresses across the wall of the tube during temperature fluctuations.

The additional material involved can only be considered as added at the outer circumference of the tube where a relatively small increase in radius represents a much greater proportional increase in volume; here also the material is most lightly stressed and is therefore used least efficiently. The C.E.G.B. spend an estimated £2M annually on high pressure high temperature pipework, and to industry as a whole the economic advantage of an accurate method of design is immediately apparent.

The day to day design of tubes in industry is made possible by the existence of recommended standards and codes of practice which are based on simple empirical formulae supported by tables of allowable stresses. Such methods of design can be accurate, and their simplicity is essential to industry where the time factor is paramount. The disadvantage of these techniques is that they can only be shown to be accurate when supported by adequate amounts of ad hoc testing.

Indeally, an accurate 'reference method' of design is required, based on a full understanding of what is actually happening in the wall of a cylinder under internal pressure subjected to creep conditions. Advanced conditions of temperature and pressure beyond the jurisdiction of the codes and standards may then be met with confidence, and the empirical design methods given in the codes and standards can themselves be checked directly and their accuracy improved. Under Professor J. Small a research programme was initiated in the Engineering Laboratories of the University of Glasgow in the autumn of 1954 as a contribution to the search for a reliable and accurate reference method of design.

The practical application of creep data to design had not previously been the subject of an investigation in these laboratories, and it was essential first to establish a proper background against which new results could be evaluated. In the first two chapters of this report, the development of the theory of creep as applied to thick tubes, and the correlation and extrapolation of creep data, are reviewed from an engineering viewpoint, conversance with both these aspects being indispensable to logical design.

The theory of creep in complex stress systems is a time-dependent extension of the theory of plastic deformation, and is still in the process of development. Because of the additional parameter (time) the mathematical representation of creep can become very complicated, particularly if refinements to the basic theory are incorporated to represent the behaviour of specific materials more accurately. For clarity this report, therefore, only deals with the basic 'framework' of the theory.

Another difficulty encountered in the application of creep data to design is that a 'creep surface' for a material, describing the stress and temperature dependence of creep deformation, does not truly exist. Thus it is not possible to represent mathematically the exact behaviour of a material under all conditions of carying temperature and varying loads. Nevertheless research workers have found it necessary to assume the existence of a creep surface in order to obtain solutions to specific problems, but these solutions are then restricted to defined loading and temperature programmes. Engineers must beware of unconsciously attributing any universal validity to the empirical analytical expressions used in these solutions.

The review of the basic theory of creep in Chapter 1 and of the correlation and extrapolation of data in Chapter 2, raised several points worthy of investigation, but since a new project generally necessitates considerable development of experimental equipment and testing techniques, it was resolved that the first experimental investigation of the programme on thick tubes should have a limited aim.

In consideration of the theory of thick tubes subjected to internal pressure the substantial simplification occurring when the assumption of zero axial creep is made was particularly apparent. The present work was therefore given the aim of establishing whether or not the assumption of zero axial creep was acceptable in design.

A literature search for previous experimental work on the creep of thick tubes under internal pressure was made. Chapter 3 gives a review of the papers located. Much background information on general techniques was acquired in the search and this formed a basis for the specification of the equipment used in the present tests.

The first essential was to select suitable test materials which would be isotropic and metallurgically stable to ensure reproducibility and permit interpretation of results. Structural changes during creep are discussed in Section 2.3 and the reasons for selecting a simple solid solution as the test material are given in Section 4.1.

The general features aimed for in the tests on tubes were:-

(a) to make the loading of the cylinders as nearly mathematically exact as possible,

(b) to ensure accurate temperature and pressure control, measurement and,

(c) to obtain the axial and diametral deformation curves of the specimen using precise techniques.

The final arrangement of the apparatus used in the tests is given in Figs. 9.1 and 9.2.

For item (a) provision was made for aligning each specimen vertically, and an end loading system for each specimen was arranged to counterbalance the weight of the bottom end closure plus half the weight of the test piece. The specimen end loading mechanism is clearly seen in Fig. 9.3. The top plate of the rig framework carried the two adjustable height fulcrums for the loading beams of the specimen counterbalance systems.

For item (b) a constant temperature jacket surrounded the specimens and a constant pressure hydraulic accumulator provided the pressure control. The constant temperature jacket was counterbalanced by a system of pulleys and weights and arranged to slide vertically on guides attached to a three column framework. Liquid from a reservoir fitted with immersion heaters was circulated rapidly at controlled temperature through the jacket surrounding the specimens. An air fan in the bottom closure of the jacket helped to maintain an even temperature distribution and a flexible heating tape fitted at the top closure compensated for the heat conducted through the specimen support bars, pressurising pipes and counterbalancing tie rods.

The constant fluid pressure to the system was provided by a sensitive dead weight hydraulic accumulator illustrated in

Figs. 9.1 and 9.4. A Morrison seal was used and the piston and dead weights continuously rotated to minimise friction. A hand pump was used to raise the accumulator initially.

For item (c) Marten's type axial and Cook type diametral extensometers operating on the mechanical and optical lever principle were fitted to the lower (and longer) specimen seen in Fig. 9.2. The extensometers were totally within the constant temperature jacket and the mirrors were viewed through ports provided in the walls. Resetting of the diametral extensometers was possible while a test was in progress, thus permitting the measurement of large diametral extensions.

An attempt was also made to develop a method of observing bore extensions by measuring the amount of oil delivered to the specimen during testing. Two tubular specimens of differing gauge length are necessary to eliminate end effects, and these were placed one above the other in the constant temperature jacket (Fig. 9.2) to ensure identical test conditions of temperature and pressure.

The amount of oil delivered to each specimen was measured by a dilation manometer (Figs. 9.1 and 9.5). Two smooth bore vertical stainless steel tubes of known dimensions were fitted into a common base block which connected with the bottom of a mercury reservoir. The top of each steel tube ended in a separate steel block, and the level of the mercury in the steel tubes could be observed by Pyrex capillary sight glasses fitted in parallel with each tube. The oil pressurising the specimens was separated from the oil supporting the piston of the hydraulic accumulator by a mercury lute and any expansion of a test specimen was thus indicated by movement of the mercury column in the appropriate sight glass. An oil injector was provided for adjustment of the amount of oil above the mercury level. The columns were isolated from each other at the top by closing the appropriate valves before a test commenced. Traps were fitted initially to prevent discharge of mercury into a ruptured specimen, but were later removed. Discussion of the philosophy behind the design of the above equipment is given in Chapter 4.

A lead (1% tin) alloy and a magnesium (2% aluminium) alloy were used in the experiments. The lead was in the form of an extruded billet $2\frac{7}{8}$ inch diameter, with the final bore of the test cylinders formed as a $\frac{3}{8}$ inch diameter hole during extrusion (Fig.9.7, Plate 10.6). The material was air cooled from the extrusion temperature and tested in this condition. The extruded billet was found to be anisotropic. The magnesium was obtained as two continuously case billets 12 inches in diameter and 12 inches long (Fig. 9.8) and heat treated in an atmosphere of sulphur dioxide to produce an isotropic structure. The preparation and testing of specimens from the above materials is described in Chapter 5.

The results of the tests are discussed in Chapter 6. Axial creep was found in every tube tested, but the amount seems to depend both on the k value of the tube* and the material of the tube. In design the assumption of zero axial creep for certain materials may therefore be good to a first approximation only. Further tests on thick cylinders are required and the next stage in the test programme has been considered briefly. Possible improvements to the experimental equipment are also included in Section 6 and the conclusions to the present test programme are given in Section 7.

* k is the ratio $\frac{\text{outside diameter}}{\text{inside diameter}}$. It is generally assumed in design that a scale effect does not exist for tubes of the sizes commonly in use.

CHAPTER I

1. AN OUTLINE OF THE DEVELOPMENT OF THE THEORY OF CREEP FOR COMPLEX STRESS SYSTEMS WITH SPECIAL REFERENCE TO TUBES.

1.1 GENERAL REMARKS

The analytical theory of creep is currently concerned with ideal materials which are metallurgically stable and behave isotropically during creep. Papers published recently have shown that it is possible by suitable modification of isotropic theory to take anisotropy into account and this must eventually be the goal when attempting to predict the performance of tubes produced commercially by extrusion. However, in the past, mainly isotropic behaviour has been considered in order to establish the major features of the theory before tackling the more obscure anisotropic conditions.

At any moment, total strain in an ideal material may be taken as a function of time, temperature, stress, strain (or strain rate) and the physical properties of the material (e.g. Young's modulus, Poisson's ratio, coefficient of thermal expansion, previous history, etc.). Since it is not possible at present to combine the effects of all these variables together in a single factor, the principle of superposition is often used and the total strain assumed to be composed of the sum of five independent parts, namely: elastic strain, plastic strain, thermal strain, transient or primary creep strain and minimum or secondary creep strain. Metallurgically it may be that the concept of separate stages of creep has to be discarded, but for analytical purposes this concept is convenient and is retained here to simplify presentation. As the onset of tertiary creep usually marks the end of the useful life of the material tertiary creep strain is not normally considered.

Elastic, plastic and thermal strain are generally taken as time independent and the theories dealing with these components have been long established. Primary creep strain and secondary creep strain are time dependent. However, since secondary creep strain is, by definition, a linear function of time, it can be treated in a pseudo time independent fashion by correlating the variables, stress, time and temperature with creep rate instead of with creep strain.

For long service life, secondary creep strain may swamp the effect of the other four strains and because it could be considered in a pseudo time-independent manner, it received initially more attention for complex stress systems than primary creep strain. Some workers, however, contend that steady state secondary creep does not really exist, and that primary creep is all important. This belief is supported by experimental results from constant time stress tensile tests from which only decreasing creep rates are observed. In a number of problems it is unsatisfactory to neglect primary creep and recent papers show that the analytical difficulties are being attacked more vigorously. Nevertheless, it is convenient however, to follow through the development of the theory of creep in complex stress systems via secondary creep and to show later how the concept may require to be modified.

Before reviewing the development of creep theory and its application to the design of thick tubes under internal pressure, a brief examination of empirical methods of design is warranted. Such methods use simple formulae which make little claim to represent the true physical situation accurately, but which nevertheless give satisfactory results for design purposes by virtue of the use of empirical constants determined by ad hoc testing.

Subsequent sections review the development of an acceptable theory of creep in complex stress systems as applied to the design of thick tubes. The mathematical models used sometimes necessitate approximations which take the results rather far from reality, but the theoretical work does provide a framework against which practical results may be evaluated.

As the theory of secondary creep was developed first it will be considered before the more complex theories of primary creep and rupture. The various published solutions for thick tubes will be discussed in their appropriate sections.

1.2 EMPIRICAL DESIGN METHODS

The relevant British Standards and the ASME Codes for Power Boilers and Unfired Pressure Vessels are examples of empirical design methods and these Rules perform the useful task of minimising computation in well charted regions of design. However, their blind use is uninspired engineering, since initial assumptions are not clearly stated and the safety factors allowed remain unknown. Papers included in this category are those of Buxton and Burrows (1951), Burrows, Michael and Rankine (1954) and Blair (1955, 1957).

Buxton and Burrows discussed, amongst other formulae, the so-called Bailey-Nadai relations for time independent secondary creep (discussed in sub-section 1.3.1) and took the illogical step of recommending the use of a modified Lamé equation for design under creep conditions because both the Bailey-Nadai and the Lamé design methods gave the same result.

In a later paper sponsored by the American Standards Association, Burrows et al, examined thirty-one formulae of various types the majority of which were not applicable to creep conditions - or even strictly valid for plastic conditions, with the object of selecting for a future Code of Practice the most suitable formula for calculating the wall thickness required for high pressure steam piping. The formula selected has the advantage of simplicity but because it is empirical it is obviously not suited for any extrapolation beyond proven design regions. Blair has criticised the previously mentioned American equations for not being rational, and recommends alternative empirical equations in an attempted simplification. However, he makes no real attempt in this paper to explain how the three principal stresses vary during the life of a tube, and his suggestion of modifying both the American equation and the permissible stresses (as recommended in the ASME Code) is simply exchanging one empirical design method for another. Blair included some discussion of safety factors. This aspect will, however, be examined later in sub-section 1.9.4.

1.3 SECONDARY CREEP UNDER COMPLEX STRESS

1.3.1 General remarks

The present theoretical approach to the secondary or pseudo-time independent creep of complex stress systems may be regarded as an extension to the original theory of plastic flow of metals suggested by St. Venant in 1870. A brief historical outline of plasticity has been given by Hill (1950) and indicates clearly, landmarks in development. Texts by Prager and Hodge (1951), Hoffman and Sachs (1953) and Nadai (1950) also discuss major contributions.

The theory of plasticity was not directly applied to the problem of secondary creep until after the classical experiments on the plastic flow of metals by Taylor and Quinney (1931). By 1933 "......a theory had been constructed, reproducing the main plastic and elastic properties of an isotropic metal at ordinary temperatures, and substantially in accord with observations" (Hill, 1950).

Theoretical treatments of the problem of secondary or time independent creep followed later.

Several papers concerning creep had appeared prior to 1932 and two books had been published - Norton (1929) and Tapsell (1931). Bailey, however, may properly be regarded as a pioneer in the application of creep test data to engineering design. He developed by hypothesis and critical experiment a theory of creep in complex stress systems which later was shown by Johnson (1951) to be a special case of the more general classical relations due to Prager (1945) and Reiner(1943) originating with St. Venamt. Bailey's theory has been simplified by later workers, but the form of the expressions given in his paper (1935) has been retained, although modified, and since extended to include primary creep as well (notably by Johnson).

Bailey's first paper concerning creep in complex stress systems appeared in 1927. It was, however, 1929 before he correctly formulated his hypothesis - some four years before the theoretical papers of Nadai, Soderberg and others, and even later, (1935) before his theory of complex stress systems was expressed in algebraic form. (Remarkably, White and Clark published the results of their first experiments on steel tubes subjected to internal pressure under creep conditions in 1926). Bailey's 1929 paper to the World Power Conference, Tokyo Sectional Meeting, may be regarded as an early landmark in the theory of creep in complex stress systems. A clear argument demolishing for practical purposes the concept of limiting creep stress was presented and the effect of metallurgical changes during service was also considered [Orowan (1947)] suggests that a limiting creep stress may exist, although it is probably too low to be of significance in design.]

Bailey correctly assumed that creep was associated with shear. Based on the fact that tests on a single crystal showed the maximum resolved shear stress to be the controlling factor in deformation he first of all postulated incorrectly (1927) "..... that creep might be assumed to result from shear only upon the planes of maximum shear stress".

Theoretical work by Sachs (1928) and by Dehlinger (1943) has indicated that the yield criterion of maximum shear stress for a single face-centred cubic crystal corresponds to the Maxwell criterion of shear strain energy (Eqn. 1.2) for a polycrystalline aggregate. This work, summarised by Hoffman and Sachs (loc.cit) has been recently quoted by Johnson (1951, 1960) in support of the use of the shear strain energy or octahedral shear stress criterion in the analysis of complex stress creep behaviour, although not all Johnson's materials were face centred cubic. More cautious comment on the apparent relationship is made by Hill (loc.cit); however, with appropriate reservations, the Maxwell criterion is probably the most suitable for universal application since it is the simplest type which most nearly fits the data.

Apparently, Bailey was unaware of Sachs' work, and when his own predictions were not confirmed by subsequent tests, he modified his original hypothesis, and in his Tokyo paper concluded that "..... creep occurred on planes where there is shear stress and is not confined to planes of maximum shear stress".

The tests* crucial to his hypothesis were carried out on thin walled lead cylinders (k = 1.083) under combined internal pressure and externally applied axial loading.

They showed that any change in axial stress caused by axial loading was accompanied by axial creep and also that circumferential creep rates were affected. Useful conclusions may also be drawn from the results of Bailey's pure torsion creep tests, and these are perhaps best expressed in the 1930 paper, namely, "..... that the normal stress acting in any plane has negligible influence upon the creep due to shear in that plane". This established that a hydrostatic system of tensile or compressive stress may be superimposed upon any system of stress without altering the creep due to the latter. (Johnson confirmed the ineffectiveness of hydrostatic stress experimentally in 1951). Thus for the yield and flow of material subjected to complex stress the known possible and simple expressions were effectively reduced to two in number, symmetrical in terms of the differences of the principal stresses, namely (a) the shear stress theory (Tresca)

* An undesirable feature of this early work is that for economy reasons, the same specimens were used for a number of tests. Each observation is dependent on the previous history of the material which should be identical for each test. The experimental work may also be criticised in other respects but results were sufficiently clear-cut to permit the formulation of a working hypothesis.

$$[(\sigma_1 - \sigma_2)^2 - 4q^2][(\sigma_2 - \sigma_3)^2 - 4q^2][(\sigma_3 - \sigma_1)^2 - 4q^2] = 0 \qquad \ldots.(1.1)$$

(b) the shear strain energy theory or octahedral shear stress theory (Maxwell)

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2q^2 \qquad \ldots\ldots\ldots(1.2)$$

Prager and Hodge (loc.cit) have pointed out that the Tresca criterion is mathematically clumsy when the largest of the three principal shearing stresses is not known, and the Maxwell criterion is thus preferred for simplicity in general analysis.

Following the conclusions on the ineffectiveness of hydrostatic stress, it is evident that creep in complex stress systems may be conveniently investigated by one of two bi-axial tests; (a) a thin tube subjected to combined tension and torsion, or (b) a thin tube subjected to combined internal pressure and external load. However, in (a) the axes of stress and strain may not remain coincident for large strains; and in (b) there is always a radial stress which has to be neglected. Tests using flat plates are possible (Johnson 1951) but the technique is probably not suitable for accurate high temperature investigations.

Bailey's classic 1935 paper included experimental results for both methods of testing thin walled cylinders. The paper, supported by experimental work, provided the first rational basis for dealing with the general case of stress distribution and creep under any system of stress. It gave the analysis for secondary creep of most of the basic components of high temperature plant.

The proposed general expressions for creep rate were:-

$$\dot{\mathcal{E}}_1 = \frac{A}{2}\left[\frac{1}{2}(\sigma_1 - \sigma_2)^2 + \frac{1}{2}(\sigma_2 - \sigma_3)^2 + \frac{1}{2}(\sigma_3 - \sigma_1)^2\right]^m \left[(\sigma_1 - \sigma_2)^v - (\sigma_3 - \sigma_1)^v\right] \qquad \ldots.(1.3)$$

$$\dot{\mathcal{E}}_2 = \ldots\ldots\ldots\ldots$$

$$\dot{\mathcal{E}}_3 = \ldots\ldots\ldots\ldots \text{ etc.}$$

and the inclusion of the Maxwell criterion is apparent.

In the discussion to the paper, Cook pointed out that for isotropy v = 1 or 3, the equation is readily seen to be of the simple form favoured by recent workers*

$$\dot{\mathcal{E}}_1 = B. \, \tau_{oct}^{\;n} \, . \, S_1 \qquad \ldots.(1.4)$$

where the octahedral shear stress is

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2} \qquad \ldots(1.5)$$

and the stress deviator in the first principal direction is

$$S_1 = (\sigma_1 - \frac{1}{3}\sum\sigma_1) \qquad \ldots.(1.6)$$

* Later theoretical work by Prager indicates however, that the form of Bailey's original equation is acceptable in certain circumstances and Johnson has indicated that a modification of this type of relation may be necessary in order to represent non-isotropic materials.

Independently of Bailey, Odqvist in Sweden developed a theory for the creep of metals under compound stress. Odqvist's original (1934) papers were not available to the writer, but later papers (1936 and discussion to Bailey (1935) indicate that a general theory had been developed which included elastic strain, and of which Bailey's relations were a special case. Odqvist appears to have been interested in turbine discs and the problem of the thick cylinder was not treated.

An abridgement of Bailey's 1935 I.Mech.E. paper (prepared by McCullough at the request of an ASME committee) was published in 1936. In the discussion Soderberg indicated the relationship between the experimental results and the long established theory for plastic flow. This may be regarded as the meeting point of classical theory and experimental research on complex stress systems under creep conditions.

Soderberg further elaborated his comments in a paper (1936) in which he indicated that his previous theoretical work (1933) based on strain hardening was untenable, and in which he developed more fully the approach outlined in his contribution to Bailey's paper. (An interesting point to note is his statement that ".... all of the creep curves for any one group of specimens with a common thermal history are geometrically similar", an observation which Johnson found of considerable value in later work on primary creep). Several possible relations between minimum creep rate and stress were considered. Also secondary creep in a thin cylinder subjected to internal pressure was treated concisely. Thick cylinders were also discussed and a method of attack indicated. A review of the work of Soderberg, Marin and Bailey is given by Nadai (1936) who discussed the validity of the various theories advanced.

A major contribution to the understanding of the problem of creep in complex stress systems has been made by Johnson (sometimes in association with others) from 1940 to the present day. A systematic experimental investigation of time independent plastic strain, primary creep and complex changing stress systems was carried out using four different materials each of which exhibited isotropic behaviour to some degree. The results obtained were used to compare existing theories by Bailey, Nadai, Kanter, Odqvist, Marin, Soderberg and later the more advanced concept of Reiner and Prager.

It is convenient to consider only the major papers to 1951 at this stage, as that date may reasonably be regarded as marking the establishment of satisfactory empirical relationships for secondary creep, and of this early work six papers may be taken as representative, viz. Tapsell and Johnson (1940), Johnson (1946 and 1949), (1948 and 1949), (1949, 1949 and 1949), (1949 and 1950) and (1951). These papers covered tests on cast steel, aluminium alloy, magnesium alloy and Nimonic 75.

Johnson's experimental technique was of a very high order. The basic test used was high temperature creep in combined tension and torsion of a thin walled tube, and specially sensitive machines were developed for this purpose.

It should be clearly understood however, and Johnson himself is at some pains to point this out, that his experimental work is in fact almost whooly restricted to investigation of the primary creep region. Nevertheless as indicated in more detail in section 1.4.1 on primary creep, the form of the relationships obtained by him are undoubtedly valid for secondary creep.

Johnson's results and conclusions are better assimilated when seen against the framework of the simple theory of creep in isotropic complex stress systems, which seems to have originated with Maxwell and St. Venant (Hill, loc.cit) and was developed for creep in various forms by Bailey, Odqvist, Soderberg and Marin (Johnson 1951). The theory is based on four assumptions:-

1. The principle axes of stress and of strain are coincident and remain so during plastic strain. (This indicates that the combined internal pressure and axial load test on a thin walled tube would permit simpler analysis for large strain than the test under combined tension and torsion).
2. The volume of the material remains unchanged during plastic strain.

3. The principal shear strain <u>rates</u> are proportional to the principal shear stresses. (This is emphasised in the texts by Hill and by Hoffman and Sachs, although not by Johnson in his earlier work).
4. The plastic strain or yielding follows the criterion of shear strain energy or octahedral shear stress.

From the above assumptions there results the basic pseudo time independent relation between creep rate and stress, which for the principal plane 1 is:-

$$\dot{\varepsilon}_1 = f(\text{inv}) \cdot S_1 \qquad \text{......(1.7)}$$

where $\dot{\varepsilon}_1$ is the creep rate in direction 1,

S_1 is the stress deviator, defined in Equation 1.6

and $f(\text{inv})$ is some invariant of the complex stress systems.

The invariant is often found to be some function of the octahedral shear stress which is given by equation 1.5 and thus equation 1.7 gives:-

$$\left.\begin{array}{ll} \text{for the principal plane} & \dot{\varepsilon}_1 = f(\tau_{oct}).S_1 \\ \text{for the octahedral plane} & \dot{\varepsilon}_{oct} = 2f(\tau_{oct})\tau_{oct} \\ \text{for the simple tension} & \dot{\varepsilon}_{ten} = 2f(\tau_{oct})\sigma_{ten} \end{array}\right\} \qquad \text{....(1.8)}$$

The function $f(\tau_{oct})$ must be determined experimentally for each material and several empirical relations have been suggested by different authors to express the results in analytical form. For convenience in processing the experimental data it is desirable that the selected function should be reducible to a straight line plot by suitable manipulation. The simplest relations proposed are:-

$f(\tau_{oct}) = A.\ \tau_{oct}^{\ n}$ (power stress law) ....(1.9)

$f(\tau_{oct}) = A\ a^{n.\tau_{oct}}$ (general exponential stress law ..(1.10)

$f(\tau_{oct}) = A.\ \sinh\ (n.\ \tau_{oct})$ (hyperbolic sine stress law .. (1.11)

Theoretical analysis of creep in complex stress systems requires a relation that is easily integrated and differentiated and thus the power law has gained favour. For the power law equations 1.8 become:-

$\dot{\varepsilon}_1 = A.\ \tau_{oct}^{\ n}\ S_1$ .....(1.12)

$\dot{\varepsilon}_{oct} = 2.A.\ \tau_{oct}^{\ n}\ .\ \tau_{oct}$ .....(1.13)

$\dot{\varepsilon}_{ten} = 2A.\ \tau_{oct}^{\ n}\ .\ \sigma_{ten}$ (equivalent to $\dot{\varepsilon} = B\sigma^m$) ...1.14)

and for the exponential law:-

$\dot{\varepsilon}_1 = Ae^{n\ \tau_{oct}}\ .\ S_1$ .....(1.15)

$\dot{\varepsilon}_{oct} = 2Ae^{n\ \tau_{oct}}\ .\tau_{oct}$ .....(1.16)

$\dot{\varepsilon}_{ten} = 2Ae^{n\ \tau_{oct}}\ .\ \sigma_{ten}$ (equivalent to $\dot{\varepsilon} = Be^{m\sigma}.\ \sigma$...(1.17)

τ_{oct} is always taken positive and the direction of creep is determined by the sign of S_1 or σ_{ten}. The above expressions are not always given their true form however, and confusion concerning sign may then easily arise.

In Appendix I of his 1951 paper, Johnson pointed out that equations of the form 1.12 and equations 1.3 previously advanced by Bailey (1935), were both special cases of a more general from proposed by Prager and Reiner, and which (for plastic deformation) do not assume proportionality of principal shear strain (rate) and principal shear stress. However, Hill (loc.cit) considers that these equations represent a reasonably good first approximation and discussion is therefore restricted to the type

$$\dot{\varepsilon}_1 = f(J_2) \cdot S_1 \qquad \text{......(1.18)}$$

where J_2 is the second invariant of the deviator stress tensor

$$J_2 = \tfrac{1}{2}(\Sigma S_1^{\,2}) = \tfrac{1}{6}\left[\Sigma(\sigma_1 - \sigma_2)^2\right] = \tfrac{3}{2}\tau_{oct}^{\,2} \qquad \text{...(1.19)}$$

Equation (1.18) above is time independent and is normally used for correlating minimum creep rates and stresses.

1.3.2 <u>Temperature dependence of secondary creep</u>

Only isothermal conditions have been considered to this stage, and the temperature dependence of secondary creep will now be discussed. A measure of agreement exists concerning the best analytical representation of the temperature dependence of stress during secondary creep. Early workers have based their relations, by analogy, on the Arrhenius rate equation (1889) which expressed the influence of temperature on chemical reaction velocity by

$$V_k = Ae^{-\frac{\Delta H}{RT}}$$

[Maxwell (1860) previously obtained a similar expression to describe the distribution of molecular velocities]. Eyring (1936) developed a theory of plastic flow based on such an expression. Kauzmann (1941) followed by Dushman, Dunbar and Huthsteiner (1944) applied the same form of equation to the theory of secondary creep and later experimental work by MacGregor and Fisher (1946) indicated the validity of that type of expression. Mott, Dorn, Feltham, Kauzmann, Nowick and Machlin and others have all applied dislocation theory to secondary creep and, with the exception of Feltham, the above named investigators have attempted to use the theory of rate processes in one form or another to describe the temperature dependence of secondary creep. The theories are discussed by Sully (1949, 1956) and by Johnson and Frost (1952).

A major contribution to the understanding of the fundamental processes occurring during creep has been made by Dorn and his co-workers, namely Sherby, Orr and Dorn (1954) and Laks, Wiseman, Sherby and Dorn (1957). Their experimental results on pure metals and solid solution alloys above 0.5 T_m strongly suggested that creep was to a large extent controlled by self-diffusion, and not the thermal activation of dislocations over free energy barriers, as the activation energy found for high temperature creep was very close to the activation energy for self-diffusion in the metal. It was stated probable that high temperature creep occurred by a dislocation-climb process where-in the rate of self-diffusion determined in part, the rate of climb*. Dorn et al's work, however, does not alter the form of the temperature dependence equation previously assumed, although their alternative explanation of the underlying mechanism may be nearer the truth.

Important additional conclusions were that the observed relations for the temperature and stress dependence of creep rate did not obtain $\frac{\sigma}{T}$ as an appropriate value for correlation thus leading to an expression of the form

$$\dot{\varepsilon} = Ae^{\frac{-\Delta H}{RT}} \cdot \phi(\sigma) \qquad \ldots.(1.20)$$

$\phi(\sigma)$ was found to be either a power or an exponential function but not a hyperbolic function. Conrad (1959) noted that dislocation theory required a power stress law.

To include temperature dependence, the creep rate equation (1.18) may thus be written

$$\dot{\varepsilon}_1 = g(T).f(J_2).S_1 \qquad \ldots.(1.21)$$

where $g(T) = Ae^{\frac{B}{T}}$

An alternative to the representation of temperature dependence of creep rate by $Ae^{\frac{B}{T}}$ is the form Ae^{BT} used by

* In addition Lake, Wiseman, Sherby and Dorn (1957) have shown that the activation energy for the high temperature creep of aluminium and some of its dilute alloys is insensitive to stress and strain.

Bailey (1935). Although not consistent with the theory of rate processes discussed previously, this form also fits test data reasonably well and, as pointed out by Traexler (1960) has the advantage that it leads to considerable mathematical simplification in analysis.

In passing it is interesting to note that in 1926 Bailey suggested that the secondary stage of creep could be regarded as a balance between strain hardening and thermal softening. Possible objections to this suggestion were given by Sully in 1949, but in a subsequent review of progress in dislocation theory Sully (1956) indicated that modern theory is in fact based on such a balance between 'work hardening' and 'recovery' processes.

1.3.3 Summary of Relationships

Stress dependence is best represented by $f(\tau_{oct})S_1$ where $f(\tau_{oct})$ may be a power stress law $A\tau_{oct}^{\ n}$, or an exponential stress law $Ae^{n\tau_{oct}}$, the power law being preferred for mathematical simplicity. The indices and constants may be determined from experimental data and for tensile tests by isothermal plots of $\log\frac{\dot{\varepsilon}}{\sigma}$ versus $\log \sigma$ for the power law, or $\log\frac{\dot{\varepsilon}}{\sigma}$ versus σ for the exponential law. Temperature dependence is best represented by Ae^{BT} or $Ae^{\frac{B}{T}}$, where the former may be preferred for mathematical simplicity. Again, the indices and constants may be determined from experimental data, and for tensile tests by isostress plots of $\log \dot{\varepsilon}$ versus T or $\log \dot{\varepsilon}$ versus $\frac{1}{T}$.

The above relations are, of course, for an ideal material and may only apply in practice over a very limited range of temperature and stress, or even not at all.

The basic theoretical relations discussed above are sufficient to permit the analysis of the problem of the creep of a thick tube subjected to internal pressure and permit a closed form solution for secondary (pseudo time independent) creep, when the important additional condition of constant axial deformation (zero axial creep) is included.

1.4 SECONDARY CREEP APPLIED TO TUBES

1.4.1 General remarks.

The first theoretical analysis of the creep of a thick walled cylinder under internal pressure is probably that due to Bailey (1930). The basic features of this analysis are retained by later workers although they make use of more recently developed complex stress-secondary creep rate relationships.

Bailey deduced the ineffectiveness of hydrostatic stress in producing creep from the results of torsion tests and of internal pressure tests on thin tubes. Zero change in length found in a thin tube subjected to internal pressure suggested that a thick tube might behave in a similar manner. A single experiment carried out on a lead tube gave no appreciable change in length, and permitted treatment of the stress analysis as a problem in plane strain. This simplifying assumption of zero axial creep occurs throughout the literature on thick tubes under internal pressure, and its experimental confirmation is therefore of considerable importance.

Generalised secondary creep rate relationships for complex stress systems were not known in 1930, and Bailey used design data obtained from the results of torsion tests. Theoretical developments have now established a suitable form in which tensile data or combined tension and torsion data may be applied to creep problems. However, the application of torsion creep data to the creep of a thick tube may have a possible parallel with the case of plastic deformation in a thick tube, as it was found by Crossland and Bones (1958) that torsion data gave the most accurate results. Experimental work is necessary to test this interesting possibility.

In describing his test material analytically, Bailey used exponential and power law expressions for the stress creep rate relationships, similar to the expressions discussed on previous pages. He also treated the case of heat transmission through the walls of his tubes but did not express the temperature dependence mathematically. Instead he reduced the problem to the isothermal case by graphical manipulation of separate exponential or power law relationships obtained for his material at different temperature levels. In 1933 MacCullough discussed the published solution by Bailey and clearly indicated all his assumptions - those made either implicity or explicitly. MacCullough's paper may usefully be read in conjunction with the original paper.

Bailey's second treatment of the problem of thick tubes (1935) is similar to his earlier solution, but the use of generalised complex stress creep rate relationships for secondary creep permitted a more general and mathematically elegant closed form solution. For an isothermal tube solutions were obtained for the three principal stresses and for the corresponding creep rates using stress-creep rate laws (eqns. 1.3) which are now considered more complicated than necessary (c.f. eqns. 1.4). Heat transmission through the wall was also treated using the empirical exponential relation for the variation of creep rate with temperature Ae^{BT}. In 1936 MacCullogh prepared a summarising abstract of this paper for the American Society of Mechanical Engineers, which was published under Bailey's name.

Nadai in 1937 gave an analysis of the isotropic creep of a thick tube using somewhat different complex stress-creep rate relations from Bailey. The relations were essentially those later developed by Johnson, i.e. $\dot{\varepsilon}_1 = f(\tau_{oct})S_1$ but a power law was used in place of $f(\tau_{oct})$. A closed form solution was obtained and in the course of his analysis Nadai showed that the assumption of zero axial creep gave an axial stress distribution which just balanced the axial tensions produced in a tube with closed ends.

Primary, secondary and tertiary creep of a thick cylinder were treated by Bailey in 1951. The secondary creep relations he used previously were simplified to give the now generally accepted form $\dot{\varepsilon}_1 = f(\tau_{oct})S_1$ and a power function was again used for $f(\tau_{oct})$. Only the isothermal case was treated and closed form solutions were obtained. Johnson also gave an identical analysis for secondary creep in 1951 using the same form of creep rate stress relationship.

It is convenient to consider the papers by Weir (1957) and by Traexler (1960) together since these are essentially identical. Both authors considered plane strain and neglected elastic and primary creep strains in their analysis, although both also stated that their approach could be extended to include transient conditions (e.g. primary creep) using a graphical or numerical approach. Only the secondary creep closed form solutions were given however. The creep rate-stress-temperature relationship used was of the form $\dot{\varepsilon}_1 = \phi\,(\tau_{oct}, T).S_1$ and general expressions were obtained for σ_r and σ_t. Considerable reduction was possible by splitting stress and temperature in the accepted way, namely $\dot{\varepsilon}_1 = f(\tau_{oct}).S_1.g(T)$ and finally a closed form solution was obtained substituting the power law $f(\tau_{oct}) = A\tau_{oct}^{\ n}$ for stress and the exponential relationship $g(T) = Ae^{BT}$ for temperature. In the closed form the result is seen to be identical to that obtained by Bailey in 1935 if in Bailey's notation we write $n = 2m - 1$ and replace n by m in the expressions for the stresses.

Rimrott (1959) gave an analysis for secondary creep in which

deformation was described in terms of logarithmic or natural strains. The use of strains defined in this way eliminates the mathematical approximation made in all previous analyses when assuming that the volume of the material remains constant.
For conventional strains

$$\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0 \qquad \ldots.(1.22)$$

is an approximation, while for logarithmic or natural strains

$$\bar{\varepsilon}_1 + \bar{\varepsilon}_2 + \bar{\varepsilon}_3 = 0 \qquad \ldots.(1.23)$$

is exact providing there is no rotation of the principal strain axes.

The equations for compatibility of strains and equilibrium of stresses are modified when using logarithmic strains and the subsequent mathematical complications require graphical or numerical treatment in the final stages. However, if small strains are used with Rimrott's method of attack (which is different from that of Weir and Traexler) a closed form solution is obtained. That this should be so is obvious since the problem is then identical with that given 24 years earlier by Bailey, but it is interesting to note the form of the equation for internal pressure which occurs, namely:

$$P = \sqrt{\frac{3}{2}} \int_{\dot{\varepsilon}_{oct_a}}^{\dot{\varepsilon}_{oct_b}} \frac{\tau_{oct}}{\tau_{oct}} \cdot d\,\acute{\varepsilon}_{oct} \qquad \ldots.(1.24)$$

The form is similar to that obtained by other workers tackling the problem of plastic straining of thick tubes, e.g.,

$$P = \sqrt{6} \int_{r_a + u_a}^{r_b + u_b} \frac{\tau_{oct}}{(r + u)} .d(r + u) \qquad \text{......(1.25)}$$

Rimrott however appears to have been the first to apply this expression in the analysis of creep.

1.4.2 <u>Summary of Basic Equations</u>

In the analytical solutions considered so far, three simultaneous independent conditions have to be met, namely:

1. The equation of <u>equilibrium of forces</u> which requires that any element not accelerated must be acted on by forces in equilibrium.
2. The equation of compatibility of strains which requires that if the structure is assumed to be composed of small elements before deformation then after deformation all the small elements should fit together perfectly.
3. The <u>stress-creep rate relationship</u> which conforms to the four assumptions stated previously in Section 1.3.1 when discussing the work of Johnson.

Two possible definitions of strain have been used, conventional strain (suitable for small deformations),

$$\varepsilon = \frac{l - l_o}{l_o} \qquad \text{...(1.26)}$$

And natural strain (necessary for large deformations)

$$\bar{\mathcal{E}} = \log_e \left(\frac{l}{l_o} \right) \qquad ....(1.27)$$

Applied to the problem of a thick tube

$$\mathcal{E}_r = \frac{\partial u}{\partial r} \qquad ....(1.28)$$

$$\bar{\mathcal{E}}_r = \log_e \left(1 + \frac{\partial u}{\partial r}\right) \qquad ....(1.29)$$

$$\mathcal{E}_t = \frac{u}{r} \qquad ....(1.30)$$

$$\bar{\mathcal{E}}_t = \log_e \left(1 + \frac{u}{r}\right) \qquad ....(1.31)$$

The equilibrium equations become:

$$r \frac{d\sigma_r}{dr} = \sigma_t - \sigma_r \qquad ....(1.32)$$

$$r \frac{d\sigma_r}{dr} = (\sigma_t - \sigma_r)^{(\bar{\mathcal{E}}_r - \bar{\mathcal{E}}_t)} \qquad ....(1.33)$$

and the compatibility equations

$$r \frac{d\mathcal{E}_t}{dr} = \mathcal{E}_r - \mathcal{E}_t \qquad ....(1.34)$$

$$d \frac{d\bar{\mathcal{E}}_t}{dr} = e^{(\bar{\mathcal{E}}_r - \bar{\mathcal{E}}_t)} -1 \qquad ....(1.35)$$

or in terms of deformation

$$ur = \text{const} \qquad ....(1.36)$$

$$2ur + u^2 = \text{const} \qquad ....(1.37)$$

1.5 PRIMARY CREEP UNDER COMPLEX STRESS

1.5.1 General Remarks

The physical understanding and the mathematical representation of primary creep, even for uniaxial creep, is as yet in an unsatisfactory state. The physical metallurgists have advanced several theories based on dislocations to describe the transient creep deformation of a poly-crystalline aggregate, but none of these theories can be said to explain the phenomenon completely at present. The theories are nevertheless of interest to the design engineer in that they indicate the form of equations which will probably give a good empirical correlation for his experimental results. These basic physical concepts advanced by Mott and Nabarro, Orowan, Smith and others will however, not be discussed here since they have been adequately summarised elsewhere by Sully (1949, 1956), by Stanford (1949) and very fully by Johnson and Frost (1952).

The engineer is concerned most often with the phenomenological behaviour of his materials, i.e., with macroscopic stresses and strains occurring under creep conditions. In design, however, he must be wary of assuming that, with co-ordinates of stress, strain and time, it is possible to construct an isothermal "creep surface" which is single valued. A unique "condition surface" with co-ordinates of pressure, volume and temperature exists for a gas, with as pointed out by Orowan (1947) the creep of a metal is influenced by its entire past history and in general a unique "creep surface" does not exist.

Nevertheless a restricted concept of a "creep surface" is found to be the most useful method of describing the behaviour of a metal under a complex stress system for design purposes provided initial temperature and load conditions do not vary excessively, and provided deformation is not large the predictions do not depart too much from the truth. This is always subject to the further proviso that the data used has not required extensive exreapolation. (Extrapolation methods are discussed in section 2).

Section 1.3 on secondary creep dealt with selected time independent creep-stress relations. A total of eight possible expressions was tabulated by Marin (1954) but only the three simplest and most convenient analytically were discussed previously namely the power stress law, the exponential stress law and the hyperbolic sine stress law. Also indicated was the most likely temperature dependence law based on the theory of rate processes.

The representation of time-dependent primary creep includes time as a factor in addition to the factors required for secondary creep. Seven proposed creep time-dependent relationships were tabulated by Marin (1954) and these were discussed in greater detail by G.V. Smith (1950) and by Stanford (1949). As with secondary creep, in the analysis of complex stress systems only those relationships which are convenient to handle mathematically have found favour, namely the exponential time law involving expressions of the type e^t and the power time law involving expressions of the type t^{α}.

Andrade's expression for simple tension

$$= (1 + \beta t^{1/3})\, e^{\alpha t} - 1 \qquad \ldots.(1.38)$$

appears to include both types of expression. Sully (1949) by analysing the terms separately, showed that the expression appeared to simulate the power time law only. However, Bhattacharya, Congreve and Thompson (1952) have emphasised that neither α nor β is ever zero and that Andrade's expression does not therefore belong in either of the two classifications mentioned above.

1.5.2 Exponential Time Law

The exponential time law to represent primary creep was proposed by McVetty in 1934 and has been discussed in the texts by Smith (1950) and by Stanford (1949). From an examination of creep-time plots McVetty observed that the minimum creep rate was approached asymptotically (also suggested by Ludwik (1909)) which suggested a purely empirical exponential relationship. Later workers have developed this concept further and the creep curve for an idealised material may, as one possibility, be considered as built up of three independent component parts, so that the total strain at any time may be considered as the sum of elastic strain*, transient creep strain and minimum creep strain. In the simplest case, uniaxial tension, it may be shown that

* Plastic strain has also been included by some workers, e.g. Griffith and Marin (1956).

$$\mathcal{E} = \mathcal{E}_{elastic} + \mathcal{E}_{transient} + \mathcal{E}_{minimum}$$

$$= \frac{\sigma}{E} + \mathcal{E}_{\substack{tran.\\max}} (1 - e^{-\alpha t}) + \dot{\mathcal{E}}_{min} \cdot t \qquad ...(1.39)$$

The maximum transient creep strain is often found to be proportional to some power of the stress so that $\mathcal{E}_{\substack{tran.\\max}} = A\sigma^{m}$. The simplest law for the minimum creep rate (discussed in Section 1.3.1) is given by the power stress law $\dot{\mathcal{E}}_1 = A.\tau_{oct}^{\ n} \cdot S_1$, which for uniaxial tension reduces to $\dot{\mathcal{E}}_{min} = B\sigma^{n}$ thus

$$\mathcal{E} = \frac{\sigma}{E} + A\sigma^{m} (1 - e^{-\alpha t}) + B\sigma^{n}. t. \qquad ....(1.40)$$

Pao and Marin (1953) have extended the above analysis to complex stress systems and have represented graphically the make-up of the composite creep curve. In this connection, it is worth while noting in passing a parallel with the more fundamental work of McLean as quoted by Sully (1956). Here a diagram of the creep curve for an aluminium specimen depicts the curve as made up of contributions from several different metallurgical mechanisms, namely creep due to grain boundaries, creep due to coarse slip and creep due to fine slip (also known as "missing" creep). McLean's analysis may offer scope for alternative and more soundly based mathematical representation of the creep curve.

Popov (1947) mentions the use of McVetty's relationship for creep relaxation in tension and the above form of expression for total creep has also been applied to creep relaxation in complex stress systems by Griffith and Marin (1956), the transient component of creep strain being regarded as fully recoverable.

1.5.3 Power Time Law

The power time law to represent creep was suggested by Sturm, Dumond and Howell (1936). By subtracting "initial instantaneous strain" from the total strain in the tensile test, a linear relationship was found when the logarithm of creep strain was plotted against the logarithm of time, giving the empirical relationship $\mathcal{E}_{creep} = At^m$. The method of determining the instantaneous strain may either be graphical or algebraic and has been described by Sturm et al, and by Bhattacharya, Congreve and Thompson (1952). Johnson in his work on creep in complex stress systems also subtracts initial plastic strain, see e.g. his work on steel (1946 and 1949), on magnesium (1949 and 1950) or the work of Johnson and Frost on aluminium (1952). An accurate study of the instantaneous strain on loading a tensile creep specimen has been made by Hazlett and Parker (1953). They used special high speed recording equipment and maintained a constant true stress in their specimens by a mechanical lever system which compensated for the reduction in area of their specimens during extension (this is further discussed in Section 5.2.1) and obtained the expression $\mathcal{E} - \mathcal{E}_0 = At^m$ which confirms the equation of Sturm et al.

Soderberg in 1936 noted for primary creep strain in geometrical similarity of creep curves for different stresses at the same temperature, and this was again confirmed by Johnson

(1948, 1949, 1951) for aluminium, steel, magnesium and nimonic 75. The function representing time dependence of creep rate may then be ecplicitly separated from the stress function representing stress dependence, and both authors employed an expression for creep strain $\mathcal{E} = f(\sigma) . g(t)$.

It follows as one possibility that primary creep strain may be represented by a power time law associated with a power stress law, a relationship sometimes known as the Nutting-Scott Blair relationship,

$$\mathcal{E} = A\sigma^{n}t^{m} \qquad \ldots(1.41)$$

for simple tension (Scott Blair, 1949). Note, however, that to fulfil the requirements for representation of creep in complex stress systems this relationship has really the true form $\mathcal{E} = A\sigma^{n-1} . \sigma . t^{m}$ (see equation 1.4). A more general equation involving temperature was proposed by Graham in 1952. Bhattacharya et al (1952) and Johnson and Frost (1952) have discussed the representation of creep by these purely phenomenological relationships and by other more fundamental relationships, and noted that the power time law gave the best representation to all temperatures for the materials tested.

Following Bailey (1951), Finnie and Heller (1959) have shown that the above power stress-power time law expression $\mathcal{E} = A\sigma^{n}t^{m}$ can be made to represent either time hardening or strain hardening creep after suitable manipulation, for if the expression is differentiated with respect to time we obtain directly,

$$\dot{\varepsilon} = mA\sigma^{n}t^{m-1} \qquad \ldots.(1.42)$$

which of the form $\dot{\varepsilon} = B\sigma^{x}t^{y}$ i.e., time hardening, but if the m^{th} root of both sides of $\dot{\varepsilon} = A\sigma^{n}t^{m}$ is taken before differentiation we obtain

$$\dot{\varepsilon} = mA^{\frac{1}{m}}\sigma^{\frac{n}{m}}\varepsilon^{\frac{m-1}{m}} \qquad \ldots.(1.43)$$

which is the form $\dot{\varepsilon} = B\sigma^{x}\varepsilon^{y}$ i.e., strain hardening. The same procedure can of course be applied to the exponential stress power time law

$$\varepsilon = Ae^{n\sigma}.\sigma t^{m} \qquad \ldots.(1.44)$$

with less useful results.

Time hardening $\dot{\varepsilon} = mAe^{n\sigma}.\sigma.\,t^{m-1}$ ....(1.45)

Strain hardening $\dot{\varepsilon} = m\left[Ae^{n\sigma}.\sigma\right]^{\frac{1}{m}}.\varepsilon^{\frac{m}{m-1}}$ ....(1.46)

Both types of expression have been used in the past, Bailey (1951) has favoured the strain hardening relationship $\dot{\varepsilon} = f(\sigma)g(t)$ while Johnson (1951) has used the strain hardening form $\dot{\varepsilon} = f(\sigma)g(t)$. Johnson's earlier work was mainly concerned with determining experimentally the form of $f(\sigma)$ for complex stress creep in expressions of the type $\dot{\varepsilon} = f(\sigma)g(t)$. In the Appendix to a unique paper which described the analysis of experiments carried out on creep under changing complex stress, Johnson, Henderson and Mathur (1958) have shown how the more general form of expression for creep strain (similar to that taken as the starting point by Finnie and Heller above) may be obtained, and subsequently discussed several theories of creep derived from this general expression - namely time hardening, strain hardening, combined time and strain hardening and superposition theories.

Since Johnson's relationship can be conveniently manipulated to represent other theories it is perhaps useful at this stage to consider in more detail his experimental work in determining $f(\sigma)$.

His tests, reported in 1948, 1949, 1951 lasted for a maximum of 150 hours only, and the results are restricted to primary creep. (The relationships obtained are therefore not directly applicable to the secondary creep conditions discussed in section 1.3.1, but nevertheless it is certain that the same experimental technique could be applied, and highly probable that expressions of identical form in $f(\sigma)$ to those obtained for primary creep would also hold for secondary creep). Johnson first determined the isotropy of his test material by tensile tests on solid and thin tubular specimens [see also Hill (1950) and Hoffman and Sachs (1953)] and later confirmed these results by a special plot of the data from his main programme of tests on thin tubes subject to combined tension and torsion.

He tested the applicability of the St. Venant-von Mises relationship by Lode plots of μ versus ν and found for the principle directions that the primary creep relationships for isotropic conditions were well represented by

$$\left.\begin{aligned} \dot{\varepsilon}_1 &= f(\tau_{oct})\, S_1 \cdot g(t) \\ \dot{\varepsilon}_2 &= \ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{\varepsilon}_3 &= \ldots\ldots\ldots\ldots\ldots\ldots \text{ etc.} \end{aligned}\right\} \quad \ldots.(1.47)$$

and for the octahedral plane

$$\dot{\varepsilon}_{oct} = 2f(\tau_{oct}) \cdot \tau_{oct} \cdot g(t) \qquad \text{....(1.48)}$$

the above expressions being similar to those previously discussed for secondary creep. The best form of $f(\tau_{oct})$ was established by plotting octahedral creep rate $\dot{\varepsilon}_{oct}$ and octahedral shear stress τ_{oct} in various combinations (e.g., power law, exponential law, etc.) the power stress law being preferred for analytical convenience. It was found that for $f(\tau_{oct})$ the materials tested could be represented by the sum of several power functions e.g.,

$$f(\tau_{oct}) = A.\tau_{oct}^{\ m} + B.\tau_{oct}^{\ n} + \ .... \qquad \text{...(1.49)}$$

so that

$$\left.\begin{aligned} \dot{\varepsilon}_1 &= (A.\tau_{oct}^{\ m} + B\tau_{oct}^{\ n} + \ .....)\ S_1 \cdot g(t) \\ \dot{\varepsilon}_2 &= \ .... \\ \dot{\varepsilon}_3 &= \ ....\ \text{etc.} \end{aligned}\right\} \qquad \text{...(1.50)}$$

which for the octahedral plane may be written

$$\dot{\varepsilon}_{oct} = 2(A.\tau_{oct}^{\ m} + B\tau_{oct}^{\ n} + \ ...)\tau_{oct} \cdot g(t) \qquad \text{....(1.51)}$$

Johnson noted slight anisotropy in some of his tests and he was able to allow for this by including anisotropy coefficients in the stress deviator portions of his expression for creep rate as suggested by Hill (1948). Anisotropic creep will, however, not be discussed here*.

* Only isotropic secondary creep was discussed previously in section 1.3.1 since no experimental data was available for anisotropic secondary creep. However this could be dealt with in an identical manner as for anisotropic primary creep.

Finally, Johnson (1960) in a comprehensive summary paper has reviewed much of the work associated with representation of complex creep by relationships of the form

$$\left.\begin{aligned} \dot{\varepsilon}_1 &= A.f(\tau_{oct}).\ S_1\phi(t) \\ \dot{\varepsilon}_2 &= \ldots\ldots\ldots\ldots\ldots\ldots \\ \dot{\varepsilon}_3 &= \ldots\ldots\ldots\ldots\ldots\ldots \text{ etc.} \end{aligned}\right\} \quad \ldots.(1.52)$$

1.5.4 <u>Temperature dependence of primary creep.</u>

Only isothermal conditions have been considered to this stage and the temperature dependence of primary creep will now be discussed.

For expressions involving the <u>exponential time</u> law (eqn.1.39) Pao and Marin (1953) have pointed out that the equations could be developed to include the effect of variation in temperature; both transient (ε_{trans}) and steady state (ε_{min}) creep strains would be affected by a term

$$e^{\frac{-\Delta H}{RT}}$$

as discussed previously (section 1.3.1). However, these authors made no attempt to evolve the theory at that time and little interest in its further development is evident probably because of the analytical difficulties encountered.

Greater success has been attained with the <u>power time</u> law for the representation of isothermal primary creep strain i.e., $\varepsilon = \phi(\sigma).t^m$ previously discussed as equation 1.41. Hazlett and Parker (1953) confirmed this expression for constant true stress tensile creep tests, and over a considerable range of temperature they further showed that the exponential temperature relationship

$$\varepsilon = Bt^{m} \, . \, e^{\frac{-\Delta H}{RT}} \qquad \ldots.(1.53)$$

accurately represented the creep curve.

The above two expressions when combined result in

$$\varepsilon = \phi(\sigma) \; t^{m} \, . \, e^{\frac{-\Delta H}{RT}} \qquad \ldots.(1.54)$$

which is a general expression for time, temperature and stress dependent primary creep strain. Caution in its application is desirable however, for the complete expression is not adequately supported by tests under varying stress and temperature conditions, and the comments at the beginning of section 1.4.1 concerning the concept of a single valued "creep surface" apply.

1.5.5 Summary of relationships

For forward creep the exponential time relationships put forward by Marin et al have not been exploited to any great extent, although for relaxation Griffith and Marin(1956) have dealt with the case of combined tension and torsion in a thin walled tube. Recent theoretical work by Blackburn and others (1960, 1961) on the creep of uranium made use of expressions of this type, but allowance for the effect of variation in temperature in the primary creep term was made by modifying the appropriate constants.

Of the possible power time law theories mentioned previously, Roberts (1951) discussed both time hardening and strain hardening expressions and showed that the latter gave the better representation of creep. Johnson, Henderson and Mathur (1958) considered four possible theories and concluded that the combined time and strain hardening expression gave predictions nearest the truth. In the practical analysis of complex stress systems where steady loads are applied and stresses do not vary excessively, the time hardening relationship is generally used because of its analytical convenience and because it gives roughly the correct result. The indices and constants in the time hardening expression may be determined from a linear plot of log $\dot{\varepsilon}$ versus log t, and for the strain hardening expression from a linear plot of log $\dot{\varepsilon}$ versus log ε.

A possible disadvantage of the use of the power time law is that steady state creep is not conveniently represented. In general, the approach due to Bailey (1951) may have to be adopted where, in the typical expression $\varepsilon = A\sigma^n t^m$, the index m is assumed to vary throughout creep, being < 1 for primary creep, unity for secondary creep and (if the Maxwell relationship applies) > 1 for tertiary creep. This is analytically inconvenient and perhaps explains the interest maintained in the exponential time law, which does not have this particular defect. However, although the power stress-exponential time law does include a term for secondary creep, unless the transient creep term dies away very rapidly, the final effect in the representation of the creep curve is almost the same as that found for the power stress-power time law, since actual steady state creep is never truly achieved. This, however, may not be such a serious disadvantage as might first appear and due attention should be paid to the suggestion of Lubahn that constant rate secondary creep does not truly exist anyway.

This contention is supported by the previously mentioned experimental work (sub-section 1.5.3) of Haslett and Parker (1953) who made isothermal constant true stress tensile creep tests on nickel and nickel alloys. They concluded that the creep rate for structurally stable metals tested under conditions of constant true stress decreased continuously until the initiation of failure, and that there was no region of constant creep rate. In addition, these authors demonstrated the danger of estimating long time secondary creep rates for extrapolated data. They showed that when the data for one creep curve were replotted on a vastly expanded time scale a new apparent constant secondary creep rate was obtained for short times. The recent work of Glen (discussed in Section 2.3) who found numerous creep rate deviations during the course of testing complex low alloy steels seems also to deny the existance of steady state creep and possibly also any fundamental analytical representation of the creep curve.

To sum up the exponential law and power time law relationships are probably the most useful types in the representation and analysis of creep in complex stress systems. However, they are by no means the only forms possible, and discussion of other alternative but less useful relationships may be found in the texts by Sully (1949), Stanford (1949), Finnie and Heller (1959) and Dorn (1961).

1.5.6 Creep Relaxation

Before leaving primary creep, some brief comments on the representation of creep relaxation may be in order. Popov (1947) has discussed the use of the power time law with an associated stress function, e.g., $\varepsilon = f(\sigma)g(t)$, and concluded that a strain hardening method which does not ignore primary creep offers the most satisfactory solution. Johnson and Frost (1952) have suggested expressing the total forward creep as the sum of two terms, one representing the non-recoverable creep, and the other, recoverable creep which becomes virtually a constant at the beginning of secondary creep, and latterly Johnson, Henderson and Mathur (1959) found for an aluminium and a magnesium alloy that the simple time hardening theory gave the closest predictions for the materials tested.

1.6 PRIMARY CREEP APPLIED TO TUBES

Consideration will first be given to solutions obtained in closed form, and then to numerical solutions.

Bailey (1951) presented an analysis for creep in thick tubes using the basic assumption of a power stress-power time law c.f. equation (2) of his paper. Equation (1) of his paper indicates that the analysis will be based on the strain hardening form $\dot{\varepsilon} = A\sigma^{n} \cdot \varepsilon^{-p}$ but in the course of generalisation Bailey reverts to the time hardening relationships favoured by Johnson. Bailey's earlier (1935) form of expression for creep in complex stress systems:-

$$\dot{\varepsilon}_1 = A\,\tau_{oct}^{\,n}\left[(\sigma_1-\sigma_2)^m-(\sigma_3-\sigma_1)^m\right]t^p \quad \ldots(1.55)$$

is discarded and the simpler expression

$$\dot{\varepsilon}_1 = A\,\tau_{oct}^{\,n}\left[(\sigma_1-\sigma_2)-(\sigma_3-\sigma_1)\right]t^p = A\,\tau_{oct}^{\,n}.S_1.t^p \quad \ldots(1.56)$$

which is supported by the theory of plasticity, and favoured by Soderberg, Johnson and others is adopted. The corresponding <u>strain hardening</u> expressions are used by Bailey in his analysis of the stress distribution in a thick tube in which however, he neglects the initial elastic strain.

The formulae obtained for radial, tangential and axial stress are of somewhat restricted application, because

(a) it is assumed that only one stage of creep is involved in the analysis at any time, although different stages of creep may exist simultaneously across the wall at certain periods during the deformation; *

(b) the analysis requires that the index p in equation 1.56 for creep rate must not be a function of time since integration with respect to time, occurs earlier in the solution.

* However, the requirement of compatibility of strains would seem to imply the existence of only one stage of creep across the wall at a time.

The solutions obtained by Bailey are, therefore, essentially discontinuous, since it is required that the index p be < 0 for primary creep, zero for secondary creep and (if the Maxwell relationship applies) > 0 for tertiary creep respectively. Moreover, his suggestion that a gradual change in p from one stage of creep to the next should be expected, only emphasises that p is time dependent and underlines the unsatisfactory nature of the solution. Nevertheless, it is interesting to note that his 1951 analysis of stress in a cylinder wall results in expressions identical in form to the 1935 solution, but with modified indices. Also, it is possible by suitable choice of indices to obtain a stress distribution corresponding to that obtained under elastic conditions, although of course creep is operating. This was also found by Johnson, Henderson and Khan (1961) for nimonic 75 and an aluminium alloy.

In order that tensile or other, e.g., torsion, data may be applied in the design of components subject to complex stress, it is necessary to have a criterion of equivalence between stress systems. The most usual assumption for conditions remote from rupture is that systems are at the same stage of creep when they possess corresponding values of the Maxwell or von Mises-Hencky functions, i.e., of octahedral shear stress and creep rate. Bailey's suggestion of the equivalence of work done in creep (1951, p.426) based on his hypothesis (1), (2) and (3)* corresponds

* see over

to the equivalence of the Maxwell or von-Mises-Hencky relationships for different stress strain systems, and he recommends that until experimental evidence suggests something different, this criterion should hold.

In his subsequent discussion of the use of tensile creep data for the design of a thick cylinder, Bailey suggested three alternative criteria for the creep strain allowable in design. Only one of these three was based on the Maxwell criterion, and the other two seem illogical in the light of his previous recommendation. The criteria used will be discussed later in sub-section 1.7.2. Bailey also considered the effect of fluctuation in temperature and pressure on design life. He proposed a method whereby pressure peaks could be replaced by equivalent temperature peaks and he then took the resulting real and equivalent temperature peaks into account to ascertain an equivalent temperature for isothermal design purposes. This, however, is concerned with the determination of rupture life and will be discussed in sub-section 1.7.3.

* (See over)

The hypotheses proposed by Bailey were

(1) that the material is isotropic;

(2) that creep behaviour is comparable for stress systems possessing the same specific shear strain energy;

(3) that the work done in creep for comparable stress systems as defined according to hypothesis (2) is the same for the same stage of creep.

Johnson, Henderson and Khan (1961) have analysed the behaviour of a thick cylinder under primary creep conditions, taking the initial elastic strains into account. Three basic assumptions were made which are universally applicable, namely:-

1. that the end closures of the pressure vessel are too remote from the centre to cause non-uniformity of axial stress;
2. that the total strain in the direction of the tube axis is constant across the section of the tube;
3. that the displacement perpendicular to the axis is radial and depends only upon the radius.

Bailey, (1930) and Johnson et al (1951) have shown that the hydrostatic stress of a triaxial stress system is ineffective in causing creep. Thus, Johnson et al (1961) were able to show that the stresses which occurred in the wall of the thick cylinder were equivalent to a biaxial stress system which, for small strains, could be represented by pure torsion of a thin cylinder. Such a procedure had previously been suggested by Bailey in 1930.

In his work on combined tension and torsion of thin tubes (1946-51) Johnson had noted little or no creep in the direction of the wall thickness. This suggested that in a thick cylinder the axial <u>creep</u> strain should remain zero, and that if axial strain did occur, it would be due to the change of axial elastic stress with time, but not with radius. In Appendix II to the 1961 paper, Johnson discussed three possible assumptions concerning axial strain, viz:-

1. that the axial creep strain is zero at all times, and is accompanied by constant elastic axial strain,
2. that axial creep strain is zero at all times and is allied with changing elastic axial strain,
3. that both axial creep strain and elastic strain may change, but that the total axial strain remains constant in value at all times.

These hypotheses will not be discussed here since a clear exposition has been given by Johnson et al, but it may be noted that none fulfil all the necessary mathematical conditions perfectly. Relationship (1) has been used by Johnson et al in the main theoretical analysis of their 1961 paper, and is the assumption generally made by other workers. It involves what appears to be only a second order discrepancy. The assumption of zero axial creep strain and changing elastic axial strain (Relationship (2)) is the one assumption made which includes time dependence of axial strain. Johnson, however, shows that the relationship does not satisfy the equation of compatibility of strains and it seems unlikely that such a condition would exist since both compatability of strain and equilibrium of stress equations must be satisfied. Assumption (3) is shown to be one of relaxation which degenerates into relationship (1).

In general, the analysis advanced by Johnson et al is a valuable contribution to the theory of the creep of thick tubes for small strains, since it takes into account the important factor of changing stress distribution with time from the initial elastic state. It is important to note, however, that a function of time $\phi(t)$ has been introduced into the analysis and that in his numerical examples Johnson takes $\phi(t)$ to be a power time law of the form t^{-p} where p is a numerical constant. Unless p is time dependent (as suggested by Bailey in 1951) it is theoretically not possible to represent so called secondary creep, and the analysis is restricted to primary creep. In addition, the actual values of the basic constants used in the analyses were obtained from the results of tests which were in general only carried to 150 hours. The comparison of creep rates computed by Johnson et al's primary creep theory at 10^3, 10^4 and 10^5 hours is therefore suspect because the data has been subject to excessive extrapolation. The <u>method</u> of computation used is however valuable.

Coffin, Shepler and Cherniak presented a numerical solution to the design of thick cylinders for primary creep in 1949. Since they did not have a background of thorough complex stress creep testing as built up by Johnson, their analysis was based on tensile data only and assumed isotropy. For the purposes of analysis they assumed, like other investigators, that a unique temperature-stress-strain creep surface existed for the material. This assumption is not strictly valid but is necessary in order to obtain a solution.

The method used by Coffin et al was to write down and solve numerically the basic equations for the creep of a thick cylinder without attempting a complete formal analytical solution. The necessary basic equations used were:-

1. the equilibrium of stress,
2. the compatibility of strain, and
3. the generalised stress-strain relationship.

An additional relationship is required to permit the use of tensile or other data, viz:-

4. the basis for equivalence of complex stress systems,

and for the particular problem of a thick cylinder two further conditions have to be satisfied;

5. the axial strain condition at all radii,

and

6. the balanced end load condition.

Essentially, the method depends on the validity of converting tensile creep curves to fictitious curves of stress versus strain with time as parameter. The problem of the thick cylinder is then treated as a series of problems in time independent elasto-plastic deformation. The stress and strain distributions at stations across the wall are obtained for each iso-time stress-strain curve.

An iterative procedure is used to obtain solutions which satisfy relationships (1) to (6) and the solution is commenced by guessing a distribution of $E\varepsilon^*$ versus $\frac{r}{r_a}$ across the tube wall, where E is Young's Modulus, ε^* is the maximum shear strain, r_a is the bore radius and r is a station radius. The procedure used for correcting guesses ensures convergence within two or three trials and sufficient time parameters are taken to build up a picture of how the stresses and strains vary throughout the tube as a function of time.

Although convergence is fairly rapid as measured by the number of trials necessary, the labour involved is considerable. However, the method has the advantage that it may be used when the analytical representation of creep data is impracticable.

In addition, the problem of steady heat flow through the tube can be treated simply since it only requires additional iso-time curves for the material at different temperatures. The solution then follows exactly the same procedure except that each station across the wall of the tube has a different temperature. Coffin et al noted that the method might also be applied to cyclic loading at some future date.

A solution was obtained for a cylinder of 12% Cr steel of K value 2, subject to an internal pressure of 12,000 p.s.i. at 850°F. Valuable graphs are presented showing the variation in stress during the life of the tube and a comparison with the stresses obtaining during secondary creep is made. A full analytical solution of the problem for secondary creep with steady heat flow is also given in the body of the paper, and it is shown that the results are identical to those of Bailey.

In their work, Coffin et al stated that they had encountered "formidable analytical difficulties" in attempting to use the Maxwell-von Mises-Hencky relationship, and were forced to adopt the analytically more simple maximum shear stress relationship to solve the equilibrium and compatibility equations. Other workers, e.g., Bailey, Johnson et al, Weir etc., have used the Maxwell-von Mises-Hencky relationship successfully in the past, but this is entirely due to the fact that they adopted an analytical expression for creep rate which incorporated the stress deviator, and also assumed zero axial creep strain. If the axial creep rate equation is written

$$\dot{\varepsilon}_z = f(\tau_{oct}).S_z.\phi(t) = 0 \qquad \text{........(1.57)}$$

then $S_z = 0$ and it follows that $\sigma_z = \dfrac{\sigma_t + \sigma_r}{2}$, whence

$$\tau_{oct} = \sqrt{6}\,(\sigma_t - \sigma_r) \qquad \text{.........(1.58)}$$

Thus, in the case of a thick cylinder with zero axial creep strain the Maxwell relationship degenerates into the maximum shear stress relationship criterion used by Coffin et al, but modified by a numerical constant. This permits an analytical solution.

Since the Maxwell-von Mises-Hencky relationship has been applied widely to creep with fair success, analytical convenience in the solution of the problem of a thick cylinder makes the assumption of zero axial creep highly desirable, and it is therefore useful to determine experimentally to what extent the assumption is justified.

Voorhees, Sliepcevich and Freeman (1956) also have proposed a numerical method for obtaining the variation of the three principal stresses throughout the life of the tube. The cylinder is imagined to be divided into a series of concentric shells in which, over short time intervals, the stresses and strain rates are assumed constant. Providing the internal pressure is not sufficient to cause plastic deformation, and assuming instantaneous application of load, the initial elastic stress distribution in the wall of the cylinder is given by Lames equations.

In the tangential and radial directions these equations predict stress gradients which are steep near the bore and which flatten with increase in radius. At the bore, therefore, the width of an imaginary shell must be taken as extremely small, whereas at greater radii a wider layer is acceptable. Voorhees et al found that the assumption of a minimum of about six layers was satisfactory and that it was convenient when each layer had twice the annular cross-sectional area of the adjacent layer at a smaller radius.

The analysis depends on the assumption that it is permissible to replace the three principal stresses and principal strains by an equivalent tensile stress and tensile strain based upon the Maxwell criterion. Isotropy is therefore implicit in the theory. All calculations are then based on equivalent tensile stress, tensile strain and tensile creep rates, the differential straining of imaginary tensile specimens is considered, and no further direct reference is made to the annuli of the tube.*

The mean stress and creep rate for each shell is assumed to act at the radius of gyration of each shell. This seems a reasonable assumption. As a first step the initial elastic equivalent tensile stress is computed for the radius of gyration of each shell using Lame's equations. Over a small time interval, the mean equivalent creep rates are obtained from tensile (or other) data for the corresponding initial elastic equivalent tensile stresses**.

* An assumption of such magnitude seems worthy of verification for validity as is discussed later.

** Here the example given by Voorhees et al appears to include an unnecessary simplification. Their analysis is based on the estimation of rates of creep before and after the minimum creep rate period, and uses three plots of creep rate versus stress, namely for the first 1% of rupture life, for the minimum creep rate, and at 80% of the expired life. The use of the original creep strain versus time data would have given the appropriate values directly.

After a short time interval at constant equivalent stress and creep rate the strains are determined for each equivalent tensile specimen (annulus). Since the tensile specimens (annuli) are to fit together after creep, the strains must be the same on each side of an interface. Voorhees et al achieve this by superimposing a compressive load on the side of the interface nearest the bore and a tensile load on the other side. This load is then evaluated and the corresponding stress reduction in the first tensile specimen (first annulus, nearest the bore) and stress increase in the second tensile specimen (second annulus) is calculated*.

For a shell not next to the bore, or to the outside diameter of the tube there will be a stress increase at its inner diameter and a stress decrease at its outer diameter. The new stress at the radius of gyration of a shell after each time interval was taken by Voorhhes et al to be the algebraic sum of the original stress at its radius of gyration, the increase at its inner diameter and the decrease at its outer diameter. The complete stress distribution across the wall of the cylinder after time interval $\Delta t'$ is thus calculable, and this distribution is then taken as the initial condition for the second time interval $\Delta t''$, and so on. The above procedure provides a complete history of effective stress and strain throughout the wall of the cylinder.

* Because Voorhees et al have worked in terms of the difference in strains for tensile specimens they seem to have failed to take into account the fact that it is not permissible to equate strains at different radii in the cylinder wall directly during this stress relaxation process.

The technique used by Voorhees et al is attractive since it provides an understanding of what is happening in the wall of a thick cylinder during creep. Their method of calculation is however unsatisfactory in several ways, and in an attempt to improve the accuracy of the technique the writer had modified their method in Appendix 8.3 to retain the concept of annuli throughout. The new relationships are significantly different from those of Voorhees et al. The analysis is limited to small strains.

The foregoing discussion takes the review of the design method of Voorhees et al to the same point reached in the papers of Bailey, of Johnson, Henderson and Kahn, and of Coffin, Shepler and Cherniak; i.e. to a knowledge of how the stresses in the wall vary throughout the life of a cylinder. Discussion of the criteria of design for all methods previously reviewed follows in Section 1.9.

Mention was made in a previous footnote of the possibility of checking whether the assumptions used in the method proposed by Voorhees et al were acceptable. It would be straighforward, although perhaps a little laborious, to compare the stress strain histories predicted by the methods of Johnson, Henderson and Kahn, of Coffin, Shepler and Cherniak, of Voorhees, Sliepcevich and Freeman and of the modification to the method of the last group of authors given in Appendix 8.3. The same creep data could be used for each method, possibly the data already used by Coffin et al for a 12% Cr steel which was obtained originally by McVetty and subsequently applied by Soderberg (1936), or the data of Johnson (1946 and 1949) for a 0.17% carbon cast steel.

1.7 TERTIARY CREEP AND RUPTURE

1.7.1 General Remarks.

Before design may be considered adequate test data on the material of interest is required. Within industry such data are generally obtained from simple tensile tests carried out at constant load and under constant temperature*. Two distinctly separate forms of tensile test are used, namely the stress rupture test and the creep strength test.
The stress rupture test is very simple to make, since it does not require the use of extensometers, but it only provides the time to rupture of the material, and possibly some measure of the ductility of the metal. The creep strength test requires the use of high temperature extensometers and is generally not taken to rupture in order to preserve the extensometers.

The total strain thus obtained (i.e. elastic and plastic strain on loading plus the creep strain), when plotted against time, provides the conventional creep curve. From such curves

*Methods of carrying out tensile tests at constant stress are discussed in sub-section 4.2.2, and other tests are considered in sub-sections 4.2.3 and 4.2.4.

it is possible to obtain the times required to reach total strains of 0.1, 0.2, 0.5 and 1.0% for each stress level tested. For a single temperature the results of the stress rupture tests and the creep strength tests may then be conveniently presented together on a plot of stress versus log time, such as that given as Fig. 2 in the paper by Freeman and Voorhees (1956).

In the past controversy has existed as to whether stress rupture data or creep strength data should be used for design. For example in the discussion to Bailey's 1954 paper Margen and Harris both attempted to draw a distinction between the use of creep strain data for precision parts and stress rupture data for non-precision parts. A steam pipe was classed as a precision part since its permitted expansion was accurately specified in order to prevent the onset of instability, and a superheater tube was considered to be a non-precision part, since although rupture would be embarrassing, its consequences would not in general be catastrophic. However, although rupture may be the final chapter in the life of a superheater tube, it is obviously unrealistic to ignore the previous creep strain history of such components. Bailey (1954) stated, and in the writer's opinion, rightly so, that stress rupture data and creep strain data were complementary to one another. This is clearly evident from Fig. 2 of the paper by Freeman and Voorhees. Criteria of design are discussed more fully in sub-section 1.9.2.

It has been assumed till now that adequate long time high temperature test data has existed whenever required; in practice this is very seldom the case and recourse has often to be made to extrapolation of data. The correct extrapolation of data is a complete study on its own, and only a brief summary of the more commonly used methods is possible in this report; the subject is considered separately in section 2.

1.7.2 Equivalence of Complex Stress Systems and Theories of Failure

The equivalence of complex stress systems under secondary and primary creep conditions was discussed in sections 1.3 and 1.5. Because of the ineffectiveness of hydrostatic stress in causing creep, the possible, and simple, expressions which were permissible, were reduced to the Tresca condition (maximum shear stress) and the Maxwell criterion (octahedral shear stress).

For primary and secondary creep, the most commonly accepted criterion is that stress systems are at the same stage of creep if the value of the Maxwell relationship (equation 1.2) is the same for both systems. The work of Johnson (1948, 1949, 1951), and Johnson and Frost (1952) supports this hypothesis.

It might have been expected therefore that the Maxwell criterion would also apply to rupture following creep in a complex stress system. However, the results of complex stress creep fracture tests on thin walled tubular specimens of 0.5% molybdenum steel at 550°C. (Johnson and Frost, 1951) and of commercially pure copper at 250°C. (Johnson, Henderson and Mathur (1956), unexpectedly showed that rupture agreed most closely with the maximum principal stress theory of failure (generally attributed to Mohr). Mohr's criterion of failure is most often associated with materials in the brittle state, e.g. cast iron at room temperature.

A possible explanation for the observations of Johnson et al may be that structural changes induced by creep can move certain initially ductile material towards the brittle state and this gains support from a later paper by Johnson, Henderson and Mathur (1960) on the behaviour of an aluminium aloy at 200°C. under complex stress *.

* Similar results were reported for an 0.2% carbon steel at 450°C. by Johnson (1960) in his summary paper on the complex stress creep of metals.

This material exhibits pronounced tertiary creep of long duration without discernable general cracking and the criterion of failure was found to correspond most closely to the Maxwell hypothesis or octahedral stress criterion, which also controls normal primary creep strain.

Micrographs taken from a tensile creep test at 200°C. twenty four hours before rupture was predicted, gave no indication of cracking for the aluminium alloy. However, similar observations for copper at 250°C showed that much cracking had taken place, even at a period only somewhat greater than half the predicted life of a tensile test. The conclusion of Johnson et al was that for materials such as the copper at 250°C and the 0.5% molybdenum steel at 550°C, cracking is gradually propagated during the test, while in the case of the aluminium alloy tested, the final crack leading to fracture only appears in a short period immediately before fracture and is apparently very localised. The isotropy of the aluminium alloy in tertiary creep was also checked and it was found that isotropy was sensibly maintained up to fracture.

Other investigations of the criterion of rupture have been made. Voorhees, Sliepcevich and Freeman (1956) tested three thin walled cylinders of annealed carbon steel at 1050°F under internal pressure and axial load, and found that the experimental criterion for rupture for two of the results lay approximately mid-way between that of maximum principal stress and maximum octahedral stress, with a slight bias towards the latter criterion. Their third test supported the maximum octahedral stress criterion more closely.

Freeman and Voorhees (1956) however, have also remarked on the much longer rupture life of notched tensile specimens compared with unnotched specimens at the same temperature and nominal stress. The stress pattern in a notched tensile specimen at the minimum cross-sectional area is complex, and an observed strengthening effect is difficult to explain unless the criterion of failure is controlled by some combination of principal stresses. This appears to refute the maximum principal stress criterion, but increase in life was not obtained with all materials. The criterion of rupture, and indeed of creep strength, to be used in design is evidently dependent on the creep characteristics of the particular material concerned. In the case of materials susceptible to cracking like the commercially pure copper and the 0.5% molybdenum steel discussed previously, the criterion of equivalence of stress systems may be slowly changing as the material alters due to creep.

Undoubtedly the aim in design will be to specify a material which does not crack during or prior to tertiary creep, since this material then offers less opportunity for further damage due to such factors as fatigue, thermal stressing, stress corrosion cracking etc. As suggested by Johnson et al, the existence of a tertiary creep region of very long duration may be indicative of a material to which the Maxwell criterion will apply.

The engineer must consult the metallurgist to determine the best material for the particular application in mind, and if a 'non-cracking' material is not available, then the criterion of failure must be modified accordingly. In the discussion of the design of tubes in section 1.9 the use of both the Maxwell and Mohr criteria of failure is considered.

1.7.3 Addibility of rupture life during stress and temperature variations.

The prediction of the life of a complex stress system subject to creep conditions where the stress and temperature levels remain constant (or nearly so) may be handled by the technique of the equivalent stress referred to tensile data, as discussed in sub-section 1.7.2. Where stress or temperature levels change significantly, or vary systematically during the life of the component, the accurate prediction of behaviour from isothermal and isostress data becomes almost impracticable.

Ideally one would like to have a simple analytical expression which would describe the behaviour of the material under a complex stress system for any combination of changing load and temperature. It is the fact that a true 'creep surface' does not exist which makes this impossible (see sub-section 1.5.1). Nevertheless Johnson, Henderson and Mathur (1958) have attempted to generalise creep rate/complex stress/time relationships to cover the case where the applied stress system is increasing only, during the course of creep (see sub-section 1.5.3). Increasing stress is involved in the problem of a thick walled cylinder creeping isothermally under internal pressure which was discussed by Johnson, Henderson and Khan (1961). For general design purposes, however, the technique involves the use of mathematical expressions of restricted validity which may not appeal to designers. The writer understands that Johnson's team at the National Engineering Laboratory is now examining the problem of the effect of cyclic temperature and stress on the creep behaviour of metals which should provide further 'special case' solutions for specified creep paths.

While the problem is being more fully investigated it is nevertheless important for designers to be able to make allowance for such changes in stress and temperature as occur in practice. The empirical procedure suggested by Robinson (1952) "..... provides a simple means for making engineering estimates with some greater validity than always to assume that the worst conditions are present all the time".

Robinson proposed that the fraction of total life used up at any stress and temperature condition should equal the ratio

$$\lambda = \frac{\text{actual time at given stress and temperature level}}{\text{rupture life at that equivalent stress and temperature level in a conventional tensile test.}}$$

For a component where the stress and temperature levels are varying continuously it is then necessary to take time intervals small enough so that the stress and temperature conditions in that time interval may be considered constant. The end of the life, of the component will be reached when the sum of the ratios λ', λ'', λ''' etc., total unity. In doing this, it is assumed that the stress changes are slow enough not to introduce impact conditions, and that the temperature changes are slow enough not to introduce thermal stressing.

Robinson gave examples of the application of his theory, but did not support the hypothesis with test data. However, the experimental results of a number of other investigators were later published in a Symposium on the Effect of Cyclic Heating and Stressing Metals at Elevated Temperatures (A.S.T.M., S.T.P. 165, 1954) Miller (1954) carried out cyclic temperature tensile rupture tests on four 'super alloys' at constant load and with three different types of temperature cycle, and presented his results as a plot of log $\left(\frac{\text{actual life}}{\text{calculated life}}\right)$ versus severity of cycling which he defined as $(T_{max} - T_{min})$ log (no. of cycles). None of his specimens had lives substantially longer than that calculated, and only four specimens had rupture lives less than half the calculated value.

Duplicates of three of these four tests had lives at least equal to half the calculated value. Miller found no apparent correlation with severity of cycling, which indicated that thermal stressing had been avoided. The mean value of $\frac{\text{actual life}}{\text{calculated life}}$ found was approximately 0.75. This may seem to throw doubt on the usefulness of Robinson's proposal, but Miller pointed out that ratios $\frac{\text{actual life}}{\text{calculated life}}$ of 1.00, 0.75, 0.50 correspond to ratios $\frac{\text{actual strength}}{\text{calculated strength}}$ of 1.00, 0.95, 0.90 respectively, when the correlation is seen to be extremely good. The above data permits the selection of allowable stresses (see sub-section 1.9.4).

Discrepancies between experimental and calculated lives obtained by Miller were in part attributed to metallurgical changes induced by the thermal cycling. This conclusion was supported by the work, amongst others of Guarnieri (1954) who tested six aircraft sheet alloys, a stainless steel, a cobalt based alloy, a nickel based alloy, a titanium alloy, an aluminium alloy and a cobalt alloy at various mean temperature levels with uniaxial stressing. The behaviour of these metals under constant temperature-constant load conditions, was compared with intermittent temperature-constant load tests, intermittent load-constant temperature tests and intermittent temperature-intermittent load tests. The results showed that the stability of microstructure was a major factor in the response of the metal to cyclic conditions.

Guarnieri found that "..... acceleration of creep and rupture was induced by intermittent loading where such processes as over-ageing, relaxation, recrystallisation, and loss of ductility occurred. Retarding of creep rupture occurred in those alloys where increase in ductility and creep recovery developed because of the intermittent load cycle while intermittent heating produced acceleration of creep and rupture in a number of cases, particularly where susceptibility to intergranular oxidation and cracking was aggravated by thermal stresses".

In conclusion, it has been shown by other workers (sub-section 1.5.1) that the previous history of a material must affect its current creep behaviour, thus Robinson's proposal cannot have any fundamental validity - a fact recognised by the originator. However, providing metallurgical changes do not occur during the life of a component, Robinson's technique gives a useful empirical way of estimating the life of a structure, once the stress pattern existing in that structure has been established.

1.8 TERTIARY CREEP AND RUPTURE APPLIED TO TUBES

A large number of papers exist which propose the use of empirical expressions for the design of tubes, generally using tensile rupture data, and some of these have been discussed briefly in section 1.2. Very few papers have been published which describe a theory of tertiary creep applied to tubes. The writer has located only two such documents (which will be discussed here), one by Rimrott, Mills and Marin (1960) and the other by Voorhees, Sliepcevich and Freeman (1956).

Hoff (1953) gave a theory for repdicting the rupture time of rods subjected to constant tensile loads, using a creep law based on secondary creep only. He ignored primary creep and then assumed that the true stress/natural strain creep rate law obtained for secondary creep could be written in terms of instantaneous values of stress and strain. This law could then be used for geometries subjected to constant load where the load bearing cross-section was reduced under the action of creep causing the stress to increase correspondingly, and thus continuously accelerating the deformation. The theory assumes that cracking of the material does not occur during tertiary creep, thus implying that the Maxwell criterion holds.

To ensure that necking of his specimen would occur, Hoff was forced to assume that the original profile of his specimen was a sine wave with a very small amplitude, thus making the original cross-sectional area of the bar 1% greater at the ends than at the centre. From subsequent theoretical analysis of the problem Hoff found that the time required to reach infinite strain was a finite quantity and was inversely proportional to the steady creep rate $\dot{\varepsilon}_0$ at the initial stress caused by the load, and to the exponent of the stress in the power stress law assumed (c.f. this result with the observations of Monkman and Grant discussed subsequently in section 2.6.)

Rimrott, Mills and Marin applied the same technique to the case of a thick walled cylinder, using the theory for creep with large strains developed by Rimrott (1959) and discussed previously in sub-section 1.4.1. If a pressure vessel is expanding under internal pressure the diameter will increase and the thickness of the wall will decrease simultaneously. The result is that the deformation will accelerate until the tube is so large and thin that the strength of the material is insufficint to contain the pressure. Rimrott et al obtained an expression relating internal pressure and equivalent creep rate for the tube based on a simple secondary creep rate/stress relationship for uniaxial tension. This expression was integrated to give creep strain as a function of time from which the time to reach infinite strain with the tube was determined.

The tube will, of course, rupture before infinite theoretical strain is reached. However, since the expansion of the vessel proceeds very slowly at first and very rapidly towards the end of the life of the vessel, Rimrott et al claimed that the actual fracture time and the time for infinite theoretical strain would occur almost together.

The mathematical solutions obtained for a thick walled pressure vessel are quite complex and Rimrott et al found it convenient to represent the results graphically. The special cases of a thin walled cylinder and a very thick walled cylinder were much simpler and it was possible to use these as asymptotes to the curves for the general case. An interesting prediction of the results given by Rimrott et al was that for constant pressure, the creep rate always increases with increasing strain, i.e., it is never possible to obtain steady state creep with a tube using steady state creep data for uniaxial tension.

The case solved by Rimrott et al (1960) corresponds to the tensile specimen which does not neck down, but which extends uniformly into a thin wire until it ruptures. In practice a tube will usually show instability such as localised bulging, before the failure time predicted by the method of Rimrott et al is reached. If the technique is to match that used by Hoff for tensile specimens, then the initial tube should be assumed to have a slight geometrical defect such as a small eccentricity. The resulting mathematical expressions may of course prove intractable.

Much the same reasoning and procedure concerning tertiary creep in tubes was used earlier by Voorhees, Sliepcevich and Freeman (1956). Their numerical method of obtaining the equivalent stress distribution across a tube wall is discussed in section 1.6. For continuously accelerating creep which occurs once the stress gradients in the tube wall have levelled out, these authors recommended the use of a factor to correct the effective stress in each annular shell for increase in diameter and thinning of the wall of the tube. The factor appears to be a simple approximation only. As with Rimrott et al, Voorhees, Sliepcevich and Freeman assume that the tube is perfectly round, concentric and free from extraneous stresses, for the purpose of calculating the maximum possible rupture life under ideal conditions. As explained in sub-sections 1.7.3 and 1.9.4 this is attained for the method proposed by Voorhees et al when the equivalent stress life fractions total unity (or unity corrected by an empirical factor). However, as soon as the initial elastic stresses present in the tube have levelled out under the action of creep, the tube is susceptible to instability which may be initiated by some minor defect. The occurrence of such instability would reduce the life of the tube considerably.

1.9 DESIGN OF TUBES

1.9.1 Behaviour during creep.

Before the design of a tube may be undertaken it is essential to understand its behaviour under creep conditions and the recent papers by Coffin, Shapler and Cherniak (1949), Voorhees, Sliepcevich and Freeman (1956), Johnson, Henderson and Khan (1961) and Rimrott, Mills and Marin (1960) describe important features of the deformation. Coffin et al and Johnson et al deal with elastic and primary creep strain; Rimrott et al with secondary and tertiary creep strain; while Voorhees and his co-workers have considered the complete behaviour of the tube.

During creep, continuous deformation of the tube takes place, and due to the simultaneous decrease in tube wall thickness and increase in tube diameter, the stresses in the tube wall caused by the internal pressure must increase. However, providing the applied pressure does not cause initial time independent plastic straining, the stresses in the tube wall immediately on loading will be elastic, and distributed according to Lamé's equations. The action of creep causes the elastic stresses to redistribute and there is a gradual levelling of the initial stress gradients, as shown in the papers mentioned above, the distribution tending towards that predicted by Bailey (1935, 1951) for secondary creep.

While the reduction in the general stress level due to redistribution is greater than the increase in the stress level caused by the continuous deformation of the tube, the diametral creep-rate will decrease steadily. As soon as the 'balance point' between stress reduction due to redistribution and stress increase due to deformation is passed, the deformation will accelerate as described by Rimrott et al, because of the increase in stress caused by simultaneous decrease in tube wall thickness and increase in tube diameter. Ideally this deformation will continue until instability of the type described by Manning (1945) and by Crossland and Bones (1958) for fully plastic thick walled cylinders occurs.

From the above remarks it can be seen that no tube made of a material which exhibits steady state creep in the tensile test, can show steady state creep under constant internal pressure, and this is confirmed by the present experimental results illustrated in Figures 9.15 and 9.16. The 'balance point' is clearly seen as the point of inflexion in the curves of diametral strain given in Figures 9.15 and 9.16 and it is interesting to note the sudden acceleration in axial creep once this point has been passed. Any design based on solutions of the problem for secondary creep must therefore be invalid.

1.9.2. Criteria of design

Discussion of the criteria of design for thick tubes must take into consideration whether the deformation of the tube has passed the point of inflexion or not, and whether the material employed follows the Maxwell or the Mohr criterion of failure discussed in sub-section 1.7.2. Whichever criterion of failure is to be used, it is first necessary to know the variation of the three principal stresses - radial, tangential and axial - across the tube wall for the complete deformation of the vessel.

Johnson et al (1961) have presented graphs which show the distribution of radial and tangential stresses across the wall, and their variation with time, for thick cylinders of $K = 2$ made from a 0.2% carbon steel, an aluminium alloy, nimonic 75, a magnesium alloy, copper and lead. These results are for small strains and indicate how the fully plastic condition at the point of inflexion is reached. Beyond the point of inflexion large strains are encountered. Providing the cylinder is perfectly round, concentric and free from defects and providing the criteria of rupture (discussed in sub-section 1.7.2) do not require the vessel to fail beforehand, the cylinder will finally burst according to the mechanism suggested by Rimrott et al (1960). Rimrott et al however did not present graphs showing the distribution of stresses across the cylinder wall during large strain deformation, but these graphs will be similar in nature to those given by Manning in 1945 for the plastic deformation and subsequent rupture of a thick mild steel cylinder at room temperature conditions.

For practical purposes the numerical method of design suggested by Voorhees et al and modified as indicated in Appendix 8.3 is probably the most convenient.

Under creep conditions, cracking of the material and rupture of the cylinder may occur before the vessel reaches large strains and, depending on the material, either the Maxwell or the Mohr criterion can be used to determine when this cracking will occur.

The criterion of failure for a cylinder used by Voorhees et al, is to calculate the rupture life of the material at each radius using the assumption of addibility of rupture life given by Robinson (1952) and discussed in sub-section 1.7.3. Rupture of the complete tube is considered to be imminent when the material at any one radius has attained its rupture life, since that material can then no longer contribute to the strength of the vessel.

For the Maxwell theory of failure, the variation with time of the maximum equivalent stress present in the tube wall must be obtained. The equivalent tensile stress is defined by

$$\sigma_{eff} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

and at any instant of time the separate distributions of radial, tangential and axial stresses across the wall of the tube may be replaced by a single distribution of equivalent stress.

For a tube subjected to internal pressure only, a plot of the variation with time of the equivalent stress at each radius indicates that the equivalent stress at the bore is normally higher than at all other radii, right up to rupture by deformation. The Maxwell criterion of design thus indicates that failure will initiate at the bore. The rupture life of the tube for cracking as opposed to deformation is thus determined by calculating the rupture life of the equivalent stress versus time curve obtained for the material at the bore.

The smaller of rupture life for cracking and rupture life for deformation, is the design life of the tube. However if the rupture life for deformation is found to be the criterion it is important to consider the possibility that premature instability of the tube may occur after the cylinder has become fully plastic, due to initial out of roundness or eccentricity of the tube. In this case it is advisable to limit the deformation to a safe value. The design life of a tube is thus the smaller of the rupture life for cracking and the life for a fixed deformation which prevents premature instability.

For the Mohr theory of failure exactly the same procedure is adopted. The variation with time of the principal stress which first attains its rupture life, mustbe obtained. Unlike the Maxwell criterion it is not always immediately obvious from the graphs of stress distribution, and their variation with time, which stress and which radius will be critical.

The initial elastic stress distribution gives the maximum principal stress as the tangential stress at the bore, As the deformation proceeds not only the location, but perhaps even the direction, of the maximum principal stress may change. This is clearly indicated in Figures 10, 28 and 31 of the paper by Johnson et al. The highest stress was initially the tangential stress at the bore, but after 100 hours the maximum tangential stress shifts to about midway in the wall of the cylinder and at this point the radial stress at the bore may perhaps be slightly greater than the maximum tangential stress. However since the radial stress is compressive it is difficult to see how this can affect creep rupture, and it is therefore probably sufficient only to trace the history of the <u>tangential</u> stress. Beyond 1000 hours, the maximum tangential stress reaches the outside of the cylinder wall as the cylinder becomes fully plastic, and remains there as large strain deformation develops.

Since the tangential stress seems not to vary much with time at a point somewhere midway in the wall (c.f. Figures 7 and 10 of the paper by Johnson et al) it is probably sufficient to consider the behaviour of the material at only three radii, namely the bore, the outside diameter, and the point of least variation * in tangential stress. The rupture life for the variation in stress with time at each of these stations is calculated as indicated previously, and the least value is taken as the design life of the tube for cracking. With the Mohr criterion it is thus possible that rupture could initiate either at the bore or at the outside diameter. The remarks concerning instability due to large deformations made during discussion of the Maxwell criterion will apply here also.

Some of the results of Johnson et al (1961), given in Figures 7, 16, 22 and 25 of their paper, may seem to indicate that the maximum tangential stress in a tube will not always shift to the outside of the tube during deformation, because the tangential stress tends asymptotically to a single steady state value across the wall of the tube. However inspection of the total strain curves corresponding to these graphs indicate that the creep rate is still decreasing. The subsequent occurrence of large scale deformations would undoubtedly raise the general stress levels as discussed previously if the creep properties of the material gave a life sufficiently long for this to happen.

Possible alternative criteria of design were suggested by Bailey (1951) based on limiting the permissible strains at

* i.e. least variation before large deformations occur.

the bore of a tube. In his theory, reviewed briefly in section 1.6, he derived expressions for the variation of stress in the primary creep, secondary creep and tertiary creep stages of deformation, which required the use of different values of his parameter $\frac{n+1}{n}$ for each stage. To permit a design however, he found it necessary to assume one value of $\frac{n+1}{n}$ which would give values of pipe wall thickness on the safe side, since he did not find it possible to allow for the variation in stress levels during deformation. The design was then based on simple tensile data at constant load and temperature.

The two criteria of design suggested by Bailey which seem the most reasonable to the present writer were

(a) to make the diametral (i.e. circumferential) creep strain at the pipe bore the same as that allowed for simple tension, and

(b) to make the octahedral creep strain at the bore the same as that allowed for simple tension.

Bailey's third proposal, namely to make the diametral strain at the outside diameter the same as that allowed for simple tension seems to the writer too dangerous a criterion to be suggested.

If the maximum allowable strain is known, Bailey's criteria could then be adapted to a stress system changing with time, by assuming that the addibility of strains for life fractions is permissible. This corresponds to the assumption of constant creep rates in small time intervals. Proposals (a) and (b) above will then be analogous to the Mohr and the Maxwell criteria respectively, but applied to strains.

1.9.3. <u>Approximate but rapid method of design</u>

From discussion in sub-section 1.9.2 of the method of design proposed by Voorhees et al it is seen that if a high creep strength material is used, the initial elastic stresses in the tube will take longer to level out, and the material has therefore a smaller proportion of its life remaining for use at lower stresses than a material which relaxes quickly. With low creep strength materials the stresses level out quickly, but the large strains accompanying the use of such materials may lead to instability and early rupture of the tube by deformation.

Where a high strength material is used, a quick and conservative estimate of the life of the tube may be made using either the Mohr or the Maxwell criterion of failure. Since large deformations will not be encountered, the highest stress levels will occur during the initial elastic stress distribution, and the rupture life of the constant load tensile test with a stress level equal to either the maximum principal stress or the maximum equivalent stress existing at zero time will provide a safe estimate for the life of the tube. The thickness of the tube will of course be greater than the optimum thickness determined by the much more laborious procedure outlined in sub-section 1.9.2

and this may introduce unnecessarily high thermal stresses during temperature changes.

1.9.4 Allowable stresses and safety factors

From the foregoing remarks concerning high and low creep strength materials the ideal material for the construction of a tube would therefore appear to possess poor resistance to primary creep in order to relax initially high elastic stresses quickly, and a good resistance in the secondary creep region to delay the onset of instability. To meet this specification Voorhees et al (1956) made the intriguing suggestion that a duplex tube of two different alloys would give the required behaviour. The weaker alloy, capable of extended creep before rupture, would be the inner shell, while the stronger alloy would form the outer shell. For a simple tube however the selection of a material will inevitably involve compromise.

For any given service conditions it is not possible to select the optimum material for a simple tube without first calculating the different thicknesses of tube required using alternative materials. The total annual cost (made up of fixed charges, running costs and overheads) for tubing in each material must then be estimated and the optimum material selected as that giving the lowest total annual cost over the life of the plant.

Using the method of design proposed by Voorhees et al, the calculation of the minimum value of a tube for a specified life under given internal pressure and temperature conditions will require several guesses of the value of K for each material considered, involving a complete calculation of the deformation of the tube each time. The labour required is considerable and the use of a digital computer thus becomes essential.

From the nature of the problem outlined above it is not possible to go further than recommending that the 'specified life' of the tube should be made, say, 80% of the life to rupture predicted in the design calculations by the technique of addibility of rupture life. This allows for the scatter in the experimental results on addibility of rupture life as observed by Miller (sub-section 1.7.3).

If 'factors of safety' are desired the writer suggests that these should be incorporated by increasing the K value of the tube, and determined by recalculating the design to obtain

(a) the increase in temperature associated with the specified pressure to give the specified life, and

(b) the increase in pressure associated with the specified temperature to give the specified life.

To have any meaning the factors of safety must then be expressed as a percentage increase in the absolute operating temperature and as a percentage increase in the absolute operating internal pressure.

Combined safety factors could also be obtained by considering simultaneous increases in temperature and pressure, but it is suggested that the two 'basic' values (a) and (b) above will provide a clear enough picture of the effect of each variable. For non-isothermal conditions heat flux would require to be considered in a similar manner.

As the life of a tube progresses at the specified temperature and pressure conditions the safety factors initially calculated will obviously increase - on the basis of addibility of rupture life. An infinite number of safety factors, as defined above, thus exist during the life of a tube. As before it is suggested that the simplest procedure is to state the minimum percentage increase permissible for each separate variable, the others remaining fixed to give the specified life of the tube. The minimum factors of safety exist at zero time.

This section would be incomplete without some quantitative comments on stress and strain levels allowable in practice. The stresses permitted by the A.S.M.E. Boiler and Pressure Vessel Code for Power Boilers are given in table 11.8. These are for use in the empirical formulae specified by the Code and are included for comparison purposes only.

Voorhees et al suggested that for a pressure vessel deformation in service should be limited to 1 or 2% to avoid instabilities. When preparing tensile data for use in the design of tubes the following extract from a paper by Freeman and Voorhees (1956) is a useful guide. 'In practice both creep and rupture data should be considered. The stress for a creep rate of 0.0001% per hour ought to be no more than 70% to 80% of that for rupture in 10,000 hours. The ratio for 0.00001% per hour creep and the 100,000 hour rupture strength should be no more than 60%. Higher ratios indicate that one or the other strength value is in error, or that failure occurs with very little deformation. The latter is indicative of stress concentration sensitivity. Lower values of the ratio usually indicate that minimum creep rates were not established in the tests or that the rupture strengths were incorrectly extrapolated'.

CHAPTER 2.

2. CORRELATION AND EXTRAPOLATION OF DATA

2.1 General Remarks

Discussion of the design of thick tubes for creep conditions would be incomplete without a brief survey of the methods of determining working stresses from short time creep data. Many of the earlier methods, e.g., the Hadfield time yield test, the DVM test (German Standard), the Rohn test and the Barr-Bardgett test have now passed into history, although some may still be used as a form of commercial acceptance test. Descriptions of these tests are available in the texts by Sully (1949), Clark (1953), Finnie and Heller (1959) etc.

Only extrapolative methods which have recently been in fairly general use will be discussed. Five primary variables exist excluding material structure, namely creep strain ε, creep rate $\dot{\varepsilon}$, stress σ, time t and temperature T. These may be used as individual parameters in correlations of results or combined to give additional single parameters e.g., $\frac{\dot{\varepsilon}}{\sigma}$, which may then be plotted with linear, semi-logarithmic or double logarithmic scales. The possible number of combinations is obviously very large, and hence it is practicable to discuss only a few. Techniques which do not depend on the use of parameters also exist and will be discussed in sections 2.7 and 2.8.

2.2 Necessity for Extrapolation

For steam plant, the required life of a tube under creep conditions may be of the order of 100,000 or 200,000 hours and it is of course possible to test a tube over such a period of time.

However, apart from the fact that the data obtained for a given tube is then restricted in application to the set of changing complex stresses present in the particular geometry tested, the testing time required is uneconomic since it is not practicable, or indeed good engineering, to wait 10 or 20 years before specifying the design.

For economic purposes the knowledge required is a proven logical method of design for each complex stress system of interest e.g., tubes under internal pressure, rotating discs etc. It should take into account the changing stress system existing in the component during its life, together with some common form of basic data for the material i.e., involving stress, time, temperature, etc., this data being readily available or quickly obtainable by a simple and reliable test procedure, e.g., uniaxial tension. To permit design, the basic data must extend over the required operating life of the plant in question, and since, generally, the designer can only permit himself a limited testing period, he is forced to adopt some method of extrapolating his data. Extrapolation is completed before the designer proceeds to the methods evolved for handling the changing complex stress systems of interest.

2.3 Structural changes during creep

Before examining several of the more common methods of extrapolation it is important to note that the principal factor which disturbs the prediction of creep resistance in a material is the possibility of a change in its structure, as a result of prolonged exposure to stress and temperature.

Glen investigated this phenomenon in detail in a series of papers (1948, 1953, 1955, 1957) culminating in a classical paper (1958) which summarised the effect of alloying elements on the creep behaviour of ferritic steels. His work was directed towards the understanding of structural changes during creep as a step in the logical development of better alloys and as a necessary preliminary to the accurate extrapolation of creep data.

In his 1958 paper, Glen first discussed the creep behaviour of 'simple' metals - i.e., pure metals, simple solid solutions, and metals with an initial precipitate in fine dispersion. With pure metals, the only structural change which can take place during creep testing is strain hardening (i.e., the multiplication and mutual interference of dislocations) which increases during primary creep and becomes constant during secondary creep*. Simple solid solutions, and metals with an initial precipitate which does not change during creep, show an initial creep resistance greater than that of a pure metal, but investigations by Dorn**

* For secondary creep, Glen considered that it was probably more correct to say that the annihilation and generation of dislocations became equal; this definition of secondary creep is perhaps the best the writer has seen.

** Dorn's work is of fundamental importance and is considered briefly in sub-section 2.5.3 in connection with extrapolation techniques.

and his co-workers demonstrated that the subsequent creep behaviour appeared to be governed by the same mechanism as that for the pure metal. This suggested that for all three cases discussed above, creep at constant stress could be correlated by a simple functional relationship. Most practical materials are not 'simple' metals, however, and changes in structure which are more complex than that caused by simple strain hardening, occur during creep testing of most alloys.

Glen observed that creep curves for the commercial alloys of interest, were highly structure dependent, and that numerous transitions in creep rate occurred during testing; this led him to suspect that the high creep strength of commercial alloys was due to some form of internal hardening process during creep.

The major effect was thought to be due to <u>strain-age hardening</u> and Glen (1955) using the true stress-natural strain short time tensile test to compare ferritic steels of different ductilities right up to rupture, showed that both carbon and nitrogen in iron caused a maximum in strength about 200°C. This was attributed to the formation and precipitation of carbides and nitrides. The addition of suitable quantities of other alloying elements which formed carbides, such as manganese, chromium, molybdenum, vanadium, titanium and niobium, produced similar beneficial strain hardening effects at higher temperatures, e.g. 300 - 350°C for manganese and 500 - 550°C for molybdenum in similar short time tests.

Tensile creep tests of these alloy combinations were made, and careful plotting of creep strain versus time revealed a sequence of transitions in creep rate each of which was identified with the presence of a separate alloying element. This determined that the action of each alloying element was to stiffen the material at different stages during creep. It was found that the materials which produced beneficial strain hardening effects at higher temperatures in the short time tensile tests, also took longer to develop their corresponding stiffening effect in the constant temperature creep tests.

Glen also considered the possibility that a strain induced precipitation hardening mechanism would operate with suitable alloys and found that the behaviour of such alloys was similar to that of strain-age hardening alloys during high temperature creep tests. In addition, the effect of prior solution treatment and of tempering was considered. The possibility of spheroidisation of the carbide due to prolonged holding at the testing temperature was also taken into account.

For alloy steel, each of the above effects was shown to modify, radically, the basic shape of the creep curve found for a 'simple' metal, and Glen therefore concluded that no prediction of the long time properties of a complex alloy steel could be made from short duration tests unless a fairly complete picture of the effects of the metallurgical changes occurring in the metal could be allowed for. This observation is fundamental to the accurate extrapolation of creep data.

Very few of the extrapolative methods to be discussed subsequently, adequately consider the metallurgical changes occurring during creep. Most analytical methods are based on theoretical considerations which may be valid for a 'simple' metal only, and therefore may only have restricted validity over a limited range of temperature and stress. The engineer must therefore be careful to extend his data only in consultation with the metallurgist who is able to interpret the metallurgical changes taking place.

2.4 SUMMARY OF BAILEY'S REVIEW OF EARLY METHODS

2.4.1 General Remarks.

The simplest technique of extrapolation is to make a suitable plot of the appropriate data generally against time, or a variable including time, as a base and extrapolate the resulting curve linearly to obtain long time working stresses. Several such procedures were critically examined by Bailey (1954) who made use of analytical expressions to represent creep in an attempt to demonstrate when extrapolation was unsound.

In the introduction to his paper Bailey made much of the effect of temperature, applied either as prior heating or during testing, on the creep behaviour of the material, and coined the phrase 'thermal action' to describe the modification of the metal structure caused by temperature effects. He attempted to extend the analytical expression for the power stress-power time relationship $\mathcal{E} = A\sigma^{n}t^{m}$ (discussed in sub-section 1.5.3) to include an exponential factor for 'thermal action' which he considered to affect only the constant A. However, in this he encountered opposition from several authorities including Allen, Glen and Orowan who considered the concept too simplified. Undoubtedly, Glen's approach, in which each metallurgical change taking place is evaluated separately, is the correct one, for as shown by Glen, changes brought about by prior heating can easily be caused by mechanisms different from those associated with the presence of strain. For clarity, therefore, the present discussion of Bailey's paper will omit the analytical expression used therein for thermal action.

Of other analytical expressions for creep considered by Bailey, the exponential stress power time relationship (of true form $\varepsilon = Ae^{n\sigma}.\sigma.t^{m}$) was not used, almost certainly because of the analytical difficulties entailed. Semi-log plots indicative of expression of the type $\varepsilon = Ae^{n\sigma}.t^{m}$ were examined however, but were criticised in conjection with the power stress power time relationships. This is untidy analytically. The expondential time relationship (see sub-section 1.5.2) was used not at all and considering its mathematical inconvenience this is not surprising.

As discussed in sub-section 1.7.1 Bailey regarded stress rupture data merely as an additional check on design. Thus, taking stress rupture tests as complementary rather than alternative to creep strain tests he considered three main extrapolative techniques. These were classified by Orowan as 'abridged', 'mechanically accelerated' and 'thermally accelerated', 'stress rupture' testing providing a fourth classification. All four techniques are discussed in turn.

2.4.2 Abridged Tests

In the 'abridged' tensile test, made at the service temperature, the creep rate is measured at the end of a fixed period of time (generally 1,000 or 10,000 hours), for several stresses close to the estimated service stress. This mainly American practice corresponds roughly to measuring the secondary or minimum creep rate during a test. The original creep curves are seldom available with the published data however, and there is therefore uncertainty as to whether at the lower stresses the creep deformation has left the primary creep stage, and at the higher stresses whether it may not have entered the tertiary creep region.

Two plots previously discussed in sub-section 1.3.1 may be used to represent secondary or minimum creep data. These are (a) the power stress law obtained by plotting log σ versus log $\frac{\dot{\varepsilon}}{\sigma}$ and (b) the exponential stress law obtained by plotting σ versus log $\frac{\dot{\varepsilon}}{\sigma}$. Bailey only considered the power stress law, which is of course, equally well represented for tensile data by the simpler plot of log σ versus log $\dot{\varepsilon}$.

His technique was to determine the analytical expression for the slope of the curve of interest and to determine if this was constant for a given material. The double log plot log σ / log $\dot{\varepsilon}$ for parameter T gives the slope of the curve as

$$i = \frac{d(\log \sigma)}{d(\log \dot{\varepsilon})} = \frac{\dot{\varepsilon}.\ d\sigma}{\sigma.\ d\dot{\varepsilon}} \qquad \ldots.(2.1)$$

In its time hardening form, the basic power stress power time law discussed in sub-section 1.5.3 provides $\dot{\mathcal{E}} = m.A\sigma^{n}t^{m-1}$. For constant time, differentiation of this expression gives $\frac{d\sigma}{d\dot{\mathcal{E}}} = \frac{\sigma}{n\dot{\mathcal{E}}}$ which when substituted in equation (2.1) gives the slope of the curve as $\frac{1}{n}$.

Providing the basic relationship $\dot{\mathcal{E}} = A\sigma^{n}t^{m}$ holds, the value of n must be invariant with time to ensure a linear plog of $\log\sigma/\log\dot{\mathcal{E}}$. To check this, Bailey suggested plotting the value of n for the steel under consideration against t. Diminishing slope with time indicates that linear extrapolation from the original plot of $\log\sigma/\log\dot{\mathcal{E}}$ at the longest times will give an overestimate of the safe working stress. It would seem that the same conclusions could have been reached merely by straightforward inspection of the original $\log\sigma/\log\dot{\mathcal{E}}$ plot, and the particular merit of Bailey's roundabout method of criticism of the extrapolative technique is not evident to the writer.

On the validity of extrapolation, linear extrapolation of $\log\sigma/\log\dot{\mathcal{E}}$ curves can lead either to over or under estimates of stress, dependent on the stage of creep in which the creep rates are measured. Bailey, Kirkby and others concluded that the 'abridged' technique was not to be recommended for extrapolation.

Providing, however, that sufficient tests are mde to demonstrate that the $\log\sigma/\log\dot{\mathcal{E}}$ curve is linear for secondary creep, it is possible to write $\dot{\mathcal{E}} = m\sigma^{n}$ which is an expression used in the Monkman-Grant correlation of data discussed later in section 2.6.

2.4.3 Mechanically Accelerated Tests

Such tests are made with the test temperature equal to the required working temperature and deformation is accelerated by increasing the stresses. Stress is plotted as a function of the time required to reach the critical service strain, and the curve obtained is then extrapolated to the required service life. Two methods of representing data were examined.

(a) For the double log plot $\log\sigma/\log t$ for parameter T, the slope of the curve is

$$i = \frac{d(\log\sigma)}{d(\log t)} = \frac{t.d\sigma}{\sigma.dt} \qquad .....(2.2)$$

The basic power stress power time law $\varepsilon = A\sigma^{n}t^{m}$, when differentiated w.r.t. time for constant critical service strain, gave $\frac{d\sigma}{dt} = -\frac{m\sigma}{nt}$, which in equation (2.2) gives the slope of the curve as $-\frac{m}{n}$. Providing the basic relationship $\dot{\varepsilon} = A\sigma^{n}t^{m}$ holds, the value of $\frac{m}{n}$ must be invariant with time to ensure a linear plot of $\log\sigma/\log t$.

As before Bailey suggested the construction of a plot of $\frac{m}{n}$ versus t. Diminishing slope with time would indicate that linear extrapolation from the original plot of $\text{Log}\,\sigma/\log t$ at the longest times gave an overestimate of the safe working stress, and Bailey found by this method that the $\log\sigma/\log t$ curve in general tended to be concave to the log t axis so that linear extrapolation would be unreliable. Again it is not clear to the present writer why the same conclusion could not have been reached merely by inspection of the trend of the original plot of $\log\sigma$ versus log t.

(b) Much the same analysis was adopted for the semi-log plot $\sigma/\log t$. The slope of the curve is

$$i = \frac{d(\sigma)}{d(\log t)} = t\,\frac{d\sigma}{dt} \qquad .....(2.3)$$

and using the power stress power time law there resulted $\frac{d\sigma}{dt} = -\frac{m}{n}\frac{\sigma}{t}$ whence $i = -\frac{m\sigma}{n}$. When the variation in i with time was graphed it was found at lower stresses and longer times that the curvature of σ/log t would be convex to the log t axis. Bailey concluded in general that 'normal' linear extrapolation of the semi-log plot would not overestimate the safe working stress. However, Kirkby in the discussion pointed out that while the extrapolation might be on the safe side, it could well predict an uneconomically low stress level, and he observed also that the stress-time plots discussed by Bailey did not always result in straight line plots in practice. Thorough metallurgical knowledge of the material is required to interpret the results correctly.

2.4.4 Stress Rupture Tests

These tests are also made with the test temperature equal to the required working temperature, deformation being accelerated mechanically by increasing the stress. They are, however, regarded as complementary rather than alternative to the creep strain tests in that they indicate an extreme beyond which stressing may not go. The stress may be plotted as a function of the time to rupture either as a double log plot of log σ/log t_{rup} or as a semi-log plot of σ/log t_{rup} and the curve then extrapolated to give the stress at the required service life.

Only the double log plot was considered by Bailey and the previously described technique for examining the validity of linear extrapolation was again used. A linear plot of $\log\sigma/\log t_{rup}$ results in $t_{rup} = m\sigma^{n}$, a second relationship used by Monkman and Grant which will be discussed in section 2.6. together with the expression mentioned previously in sub-section 2.4.2.

The slope of the $\log\sigma/\log t_{rup}$ curve was obtained as

$$i = \frac{d(\log \sigma)}{d(\log t_{rup})} = \frac{t_{rup}}{\sigma} \cdot \frac{d\sigma}{dt_{rup}} \qquad \ldots.(2.4)$$

and evaluated in terms of the material indices as before. To do this Bailey required an analytical relationship characterising rupture which he obtained by plotting $\log \varepsilon_{rup}/\log t_{rup}$ to provide (fortuitously) a linear curve (which incidentally he shows extrapolated in Fig. 22 of his paper without further explanation). This enabled him to write $\varepsilon_{rup} = ct_{rup}^{\ d}$. This expression when used with the power stress power time law previously assumed ($\varepsilon = A\sigma^{n}t^{m}$) results in $ct_{rup}^{\ d} = A\sigma^{n}t_{rup}^{\ m}$ from which it was concluded that rupture would be characterised by $\sigma^{n}t_{rup}^{\ m-d}$ = constant. The slope of the $\log\sigma/\log t_{rup}$ curve becomes $i = \frac{d - m}{n}$ and examination of the indices as before resulted in the conclusion that in general the $\log\sigma/\log t_{rup}$ curve would be concave to the $\log t_{rup}$ axis and therefore that linear extrapolation would cause over-estimation of the rupture stress at service life. This again should be evident from the original $\log\sigma/\log t_{rup}$ curves without resort to involved analysis.

The form of criticism used by Bailey for the three methods of extrapolation described above, seems to be supported by its own bootstraps in that it makes used of analytical expressions involving stress, time, strain etc., whose fundamental validity have not first been demonstrated. Despite this, Bailey does reach valid conclusions which are however probably self evident on inspection of the original plots.

Leaving aside further discussion of Bailey's method of criticism of the three experimentally supported techniques of extrapolation noted so far, the main defect of these techniques is the fact that creep was accelerated solely by an increase in stress, so that the critical strain would be reached in a short test period. Temperature dependent structural changes which would normally occur during the service life of the material would therefore not have had time to develop fully. The technique recommended by Bailey to overcome this difficulty was to carry out the tests at some suitably increased temperature to accelerate both constitutional changes and deformation.

Robertson in the discussion to Bailey's paper, pointed out that this procedure would be sound, provided that the increased temperature

1. did accelerate a structural or constitutional change that took place more slowly at the service temperature;
2. did not promote any changes that would not take place at the service temperature; and
3. did not lead to surface reactions such as oxidation or decarburisation that reduced creep resistance.

2.4.5 Thermally Accelerated Tests.

Such tests are made with the test stress equal to the working stress, and deformation and structural changes are accelerated by increasing the test temperatures. The absolute temperature is plotted as a function of the time required to reach the critical service strain, and the curve was obtained extrapolated to the required service life.

For the technique to be accurate it must be assumed that massive phase changes, e.g., from ferrite to austenite in steels, do not occur at the increased temperatures of testing. Slow modification of the structure of the material occurring at an atomic or molecular level from the operation of the Maxwell-Boltzmann distribution law of energies is generally assumed. This relationship predicts that the time t for a particular change is related to the absolute temperature of testing T by the relationship $t = Ae^{-\frac{\Delta H}{RT}}$

where H is the activation energy characterising the change,

R is the Boltzmann constant and

A is a constant.

The form of this expression has already been met in sub-section 1.3.2 where the Arrhenius rate equation for chemical reaction velocity and the Dorn expression for creep based on self diffusion of dislocations were discussed, and there can be little doubt that the relationship is of fundamental significance for creep. For complex alloy steels however, Glen's work indicates that its application would not be straightforward.

Bailey attempted to examine the validity of the above expression by evaluating $\frac{\Delta H}{R}$ for the same visual structural change induced by heating at different temperatures T_1 and T_2 over times t_1 and t_2 respectively, whence

$$\frac{\Delta H}{R} = \frac{\log t_1 - \log t_2}{0.434\left(\frac{1}{T_1} - \frac{1}{T_2}\right)} \qquad \ldots(2.5)$$

On an equivalent structure basis for carbon steel he found the values of $\frac{\Delta H}{R}$ to be of the order of 33,000 for the observed thermal action change of carbide spheroidisation. Plotting creep results directly as $\frac{1}{T}$ versus log t, however, a value of $\frac{\Delta H}{R}$
results directly as $\frac{1}{T}$ versus log t, however, a value of $\frac{\Delta H}{R}$
o[illegible]

Bailey considered the possibility that the visible changes of comparatively massive carbides (on which the observations were based) merely served as an indicator of a change associated with the invisible carbide in solution in the ferrite upon which the creep resistance most likely depended (c.f. the work of Glen where the <u>action of precipitation</u> itself seems to be important). To determine whether the value of $\frac{\Delta H}{R}$ for massive carbide spheroidisation was different from that associated with the carbide in solution Bailey prepared two samples of carbon steel, each of which had been initially preheated for different time and temperature combinations based on $\frac{\Delta H}{R} = 33,000$, to produce equivalent degrees of spheroidisation. The subsequent creep tests gave equal rates of creep for the preheated samples (and an increase as expected over that for the non pre-heated material) thus confirming that the acceleration of creep tests based on the criterion of thermal action alone (i.e. the rate of visible structural change caused solely by temperature) cannot adequately represent long time creep data since values for $\frac{\Delta H}{R}$ from 50,000 to 60,000 are then obtained.

The explanation of this phenomenon is straightforward in terms of Glen's observations discussed in section 2.3. Structural changes occurring in alloy metals during creep were often accelerated by the combined action of strain and temperature to a greater extent than changes induced during prior heating by temperature alone. The metallurgical changes in the two cases might even be of a totally different kind. This opinion was put forward by Allen in the discussion to Bailey's paper and is supported by Grant * in a contribution to an earlier paper by Larson and Miller (1952).

Bailey, however, did not accept that this was the complete explanation, and to overcome the difficulty in his own hypothesis, he described an involved technique whereby a "correct" extrapolation might be obtained. The unstressed test specimen would be first preheated at a higher temperature than the test temperature in order to accelerate structural changes, and subsequent creep testing would then be carried out either at the design stress or with a higher stress - at a test temperature suitably elevated over the working temperature.

From what has been said previously it is evident that such a procedure would be unlikely to give a completely accurate prediction. In discussion, Allen and Margen separately pointed out that it also

* Grant also suggested that although oxidation and decarburisation were rate controlled processes which could affect creep behaviour, since they were essentially surface effects they would not be strain sensitive.

presupposed a detailed knowledge of the behaviour of the material, which was not generally available until the creep tests had in fact been made. The work of Sherby and Dorn et al (1952, 1953) strongly suggests that Allen's contention of a strain-temperature dependent structure is the correct one and Glen's approach is undoubtedly the most satisfactory.

To conclude this section, the question of empirical linear extrapolation of the original creep data without resort to the calculation of slopes from analytical expressions is considered. Bailey in 1951 had noted that although the plot of $\frac{1}{T}$ versus log t gave a roughly linear relationship for ferritic steel, curvature was unquestionable, and in 1954 he emphasised that for austenitic steels curves concave to the log t axis were common making linear extrapolation of the $\frac{1}{T}$ versus log t plot unsafe. This was confirmed by Kirby in the discussion to Bailey (1954).

In 1951 Bailey actually recommended plotting T versus log t to obtain a more linear plot for ferritic steels. These steels were found to give a curve which was usually slightly convex to the log t axis, thus allowing safe temperatures to be obtained by linear extrapolation. The comments of Traexler discussed in sub-section 1.3.2 concerning the representation of temperature in analysis are perhaps an additional reason for preferring the T versus log t plot.

2.5 TIME-TEMPERATURE PARAMETERS

2.5.1 General Remarks

Bailey was not the first to apply the Maxwell-Boltzmann (1860) relationship to the extrapolation of creep data, although he made use of such an exponential relationship in 1929 to describe temperature dependent structural changes in carbon steel. The concept of trading time for temperature had been recognised by other workers. Based on the chemical rate theory of Arrhenius (1889) Eyring in 1936 developed a theory for the velocity of plastic flow as a function of temperature and stress, and on the same basis Kauzmann (1941) developed an expression for creep rate during secondary creep which was later discussed by Dushman, Dunbar and Huthsteiner (1944).

These early papers were summarised briefly by MacGregor and Fisher (1945, 1946) who developed a velocity modified temperature parameter based on the theory of such rate equations and applied this to constant natural strain rate tension tests. Glen was later to make extensive use of these new techniques developed by MacGregor and Fisher in determining the effect of alloying additions on creep behaviour of steels, and discussion of this work has already been given in section 2.3.

MacGregor and Fisher provided several clues to the possible method of correlating rupture data in their discussion of the representation of secondary creep results. These clues were subsequently noted by Larson and Miller (1952) and their new method of presentation of rupture data inaugurated a surge of interest in the art of extrapolation based on the so-called time temperature parameters. According to their bases the different time temperature parameters may be classified under three main headings,

1. Chemical rate theory,
2. Dislocation climbing theory,
3. Empirical curve fitting.

The three best known examples of the above methods are the Larson-Miller, the Sherby-Dorn and the Manson-Haferd parameters respectively. The derivation and the validity of each type is discussed in turn and their advantages subsequently compared. Two forms of each of these relationships have generally been obtained, one for the creep rate, temperature and stress relationship in <u>secondary creep</u> and one for the time, temperature and stress relationship for <u>constant creep strain</u>. The second form necessitates the assumption of an average rate of straining, and by making a further assumption concerning the constancy of strain at rupture, <u>creep rupture</u> data have been correlated.

2.5.2 <u>Chemical Rate Theory</u>

Larson and Miller (1952) took the equation relating creep rate, temperature and stress in secondary creep to be of the form

$$\dot{\varepsilon} = A e^{\frac{-\Delta H}{RT}} \qquad \text{......(2.6)}$$

where the activation energy ΔH was assumed to be a function of the applied stress. (This does not correspond exactly with the types of expression previously suggested by Kaunmann, but by plotting the results of Dushman et al for secondary creep of constantan and of aluminium at constant stress, MacGregor and Fisher showed that the two expressions gave results which were nearly coincident and which were well within the existing precision of creep results).

Equation (2.6) for secondary creep may be written as

$$f(\sigma) = T\left[C - \log_e \dot{\varepsilon}\right]$$

or in another form as

$$\sigma = f\left\{T\left[c - \log_e \dot{\varepsilon}\right]\right\} = f\left\{T_{cm}\right\} \qquad \ldots.(2.7)$$

where T_{cm} is the creep rate modified temperature. This equation predicts that, if equation (2.6) is valid, then smooth curves will be obtained by plotting stress σ against the creep rate modified temperature T_{cm}. Smooth curves were in fact obtained by MacGregor and Fisher and by Larson and Miller, and examples are to be found in their respective papers.

In the latter part of their 1946 paper MacGregor and Fisher suggested the application of the creep rate modified temperature concept to give an approximate correlation of primary creep tests and also suggested it application to tests extending into the tertiary creep range. For this purpose it was necessary to relate results at a given strain ε, and the average creep rate to this strain was defined as $\frac{\varepsilon}{t}$ where t was the time required to reach that strain. This, of course, is a considerable approximation. Larson and Miller (1952) noted that equation 2.6 for constant strain could be rewritten as

$$\frac{\varepsilon}{t} = Ae^{\frac{-\Delta H}{RT}} \qquad \ldots.(2.8)$$

and making the (unstated) rather dubious further assumption* that strain at rupture was constant in independent of temperature, they obtained an expression in which the reciprocal of the time to fracture was a measure of the average strain rate throughout the test, namely,

$$\frac{1}{t} = Ae^{\frac{-\Delta H}{RT}}$$

From this there results

$$f(\sigma) = T\left[C + \log_e t\right]$$

or alternatively

$$\sigma = f\left\{T\left[C + \log_e t\right]\right\} = f\left\{T_{rm}\right\} \quad ....(2.9)$$

where T_{rm} is the rupture time modified temperature. The expression $T\left[C + \log_e t\right]$ is known as the Larson-Miller parameter for correlating creep rupture, correspondingly, the expression $T\left[C - \log \dot{\varepsilon}\right]$ is the Larson-Miller parameter for correlating secondary creep rates.

In support of their expression the authors did not attempt to show that the strain at rupture would be constant, but merely quoted the results of many correlations which their expression fitted remarkably well. Such correlation is, of course, likely to be invalidated if major phase changed occur, but the authors considered that metallurgical changes involving diffusion, such as tempering

* Heimerl and McEvily (1957) state that Machlin (1956) has shown theoretically that the product of rupture life and the steady creep rate is invariant, and that it may also be obtained as a special case from the Monkman-Grant relationship.

and over-ageing would be accommodated, and that it was possible that oxidation was allowed for in a similar manner.

Fisher and Holloman suggested in discussion that the correlation might fail when a brittle alloy is tested. This would occur if the strains at rupture were not constant and the reciprocal of rupture time was not then a measure of the average creep rate. The authors however thought that the somewhat sparse data which they had for brittle cermets gave support to their parameter.

Larson and Miller suggested the use of C = 20 in their parameter as a best average value for a number of alloys; Finnie and Heller (1959) state that values of C varying between 15 and 27 have been found for different metals. Goldhoff (1959) showed that the optimisation of the constant C led to vastly improved results for almost all materials examined, and Mendelson and Manson (1960) confirmed this fact and recommended an optimisation procedure (1959). Betteridge (1958) found that optimisation of C was desirable for the nimonics.

In 1956, Garofolo, Smith and Royle published an excellent critical paper on the validity of the three proposals for time temperature parameter mentioned in sub-section 2.5.1. They noticed the fundamental implication in equation (2.9) that during creep the activation energy (ΔH) is a <u>single valued</u> function of the <u>initial</u> stress. They pointed out firstly that the stress varies markedly in the constant load tensile test (this is discussed further in sub-section 4.2.2). Secondly, by careful plotting of results, they demonstrated that the Larson-Miller parameters were not single valued. For example, when plotting initial stress against $T(C + \log t_{rup})$, although the scatter was sufficiently small to permit a reasonable curve to be drawn through all the points, discrete curves were distinctly obtained for tests at different temperatures.

The difficulties encountered in evaluating the constant C, and the criticisms of Garofolo et al, leads to the consideration of the equation relating creep rate, temperature and stress in secondary creep which was discussed previously in section 1.3.1. This is of the form

$$\dot{\varepsilon} = Ae^{\frac{-\Delta H}{RT}} \cdot \phi(\sigma) \qquad \ldots(2.10)$$

and is supported by the work of Sherby, Orr and Dorn, who showed (1954) that stress divided by temperature did not enter the creep equation for high temperatures. The new expression is also favoured by Johnson et al.

Treating this equation in exactly the same manner as equation 2.6 we obtain

$$\frac{\Delta H}{R} = T\left\{ \log_e \left[A.\phi(\sigma)\right) - \log_e \dot{\varepsilon} \right]\right\} \qquad \ldots.(2.11)$$

and it is seen that the constant C of equation (2.7) has been replaced by the stress dependent factor $\log_e \left[A.\phi(\sigma)\right]$. This may explain the difficulties encountered in evaluating C for the Larson-Miller parameters, and it seems to agree with the findings of Garofolo et al that the parameters $T\left[C - \log_e \dot{\varepsilon}\right]$ and $T\left[C + \log_e t\right]$ are not single valued.

Writing $f(\sigma) = \log_e \left[A.\phi(\sigma)\right]$ we obtain

$$f(\sigma) = \frac{\Delta H}{RT} + \log_e \dot{\varepsilon}$$

i.e.
$$\left.\begin{aligned} \sigma &= f\left[\frac{\Delta H}{RT} + \log_e \dot{\varepsilon}\right] \\ \text{or} \quad \sigma &= f\left[\dot{\varepsilon}\, e^{\frac{\Delta H}{RT}}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(2.12)$$

Similarly for creep rupture data, making the same assumptions concerning constant strain as before, there results

$$\left.\begin{aligned} \sigma &= f\left[\frac{\Delta H}{RT} - \log_e t\right] \\ \text{or} \quad \sigma &= f\left[t\, e^{-\frac{\Delta H}{RT}}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots(2.13)$$

If the two parameters $\left[\frac{C}{T} + \log_e \dot{\varepsilon}\right]$ and $\left[\frac{C}{T} - \log_e t\right]$ can be shown to correlate creep rate and creep rupture data respectively at least as well as the Larson-Miller expressions, their adoption in preference to the Larson-Miller formulations is to be recommended because of the commensurability with the theories of creep developed by Johnson and others. The parameters given in equations (2.12) and (2.13) have in fact been used by Sherby and Dorn (1953).

2.5.3 <u>Dislocation Climbing Theory</u>

From 1952 onwards the Californian school of Physical Metallurgy led by Dorn has made substantial progress in elucidating the relationship between the creep behaviour, the structure and the more fundamental properties of simple metals. Previous workers (listed in Sherby and Dorn's 1953 paper) demonstrated that subgrain structures were functions of creep stress, creep strain, rate of creep and temperature. Zener and Hollomon in 1944, apparently without supporting theory, suggested that the tensile properties of soft metals could be correlated by the parameter

$$\sigma = f\left(\dot{\varepsilon}\, e^{\frac{\Delta H}{RT}} \right)$$

Sherby and Dorn (1953) applied this parameter to high temperature creep (above 0.5 Tm) of pure aluminium and simple solid solutions of aluminium and found excellent correspondence for tensile secondary creep data including that

$$\sigma = f\left(\dot{\varepsilon}\, e^{\frac{\Delta H}{RT}} \right)$$

was a single valued function. It followed that the same metallurgical structure, representing the same state of metal should be obtained for all equal values of $\dot{\varepsilon}\, e^{\frac{\Delta H}{RT}}$, and where the parameter applied this was confirmed within the precision of the experimental techniques by x-ray diffraction and metallographic examinations of the structure of the specimens.

From the theory given previously in sub-section 2.5.2 it is seen that the parameter $\dot{\varepsilon}\, e^{\frac{\Delta H}{RT}}$ corresponds to a secondary creep relationship of the form

$$\dot{\varepsilon} = A e^{\frac{\Delta H}{RT}} \cdot \phi(\sigma) \qquad \text{....(2.10)}$$

Yet this must not be taken as an indication of the existence of a mechanical equation of state for creep, for the above equation is in general only valid for describing separate isothermal and constant load tensile tests carried out independently, or perhaps isothermal and constant stress complex stress tests, e.g. a thin tube subject to combined torsion and tension.

Sherby and Dorn (1953) in discussing the confirmation of the parameter $\varepsilon e^{\frac{\Delta H}{RT}}$ for equivalent structure, explained why the equation of state must fail, and the cardinal paragraph of their 1953 paper is given below.

.... "If at some stage in a low stress creep test the stress is suddenly increased, the additional tensile deformation that results will introduce changes in the structure of the specimen, bringing the structure into closer agreement with that obtained at some stage of the higher creep test. But inasmuch as the structural changes that are introduced upon increasing the load are superimposed on the substructure obtained during creep at the originally lower stress, it is improbable that the resulting structure will coincide exactly with that obtained at any stage of creep in a specimen subjected exclusively to the higher stress. Thus the past stress history modifies the structure of metals in such a way as to modify their subsequent mechanical properties".

In creep design, the use of equation (2.10) should therefore be restricted to complex stress systems where temperatures and stresses do not vary excessively with time.

As described in sub-section 2.5.2 following the method of approximating to creep rate by writing $\dot{\varepsilon} = \frac{\varepsilon}{t}$ and proceeding from equation (2.10), Dorn et al also obtained the parameter $te^{\frac{-\Delta H}{RT}}$ for correlating creep rupture tests. This, of course, involves the additional assumption that the strain at fracture remains constant with temperature, which has already been refuted by Garofalo et al.

The conclusions of Sherby, Orr and Dorn (1954) so far may be summarised.

1. The correlations suggested by the Sherby-Dorn parameters $\dot{\varepsilon} e^{\frac{\Delta H}{RT}}$ and $te^{\frac{\Delta H}{RT}}$ are valid for aluminium and its dilute alloys in the temperature range above 0.45 Tm where reasonably rapid recovery can occur.

2. The activation energy ΔH is practically a universal constant for aluminium, being independent of temperature over wide ranges (above 0.45 Tm) and insensitive to creep stress, creep strain, grain size, sub-structures developed during creep, small alloying additions, as well as cold work and dispersions of $CuAl_2$.

3. The experimentally established validity of the equations proves that the stress and not the stress divided by the absolute temperature enters the creep equation for high temperatures. It follows that the stress cannot enter the free energy of activation term.

Since the activation energy ΔH appears to be a fundamental property, Sherby, Orr and Dorn (1954) considered that it should be correlatable with other fundamental properties. Reported values of the activation energy ΔH for a number of pure metals - aluminium, iron, nickel, copper, zinc, platinum, gold and lead - were therefore studied in an attempt to uncover additional correlations. A close correspondence between the activation energy for creep and the activation energy for self diffusion was found, consistent with the hypothesis that high temperature creep occurs when barriers to the motion of dislocations are removed by means of some recovery mechanism controlled by an atomic diffusion process.

It was also anticipated that the activation energy for the creep of the elements might be a periodic function of atomic number. The data available indicated that the intermediate elements in each period had the highest activation energies for creep and self-diffusion, but Dorn et al considered that their data was yet too incomplete to select the most creep resistant elements on this basis. A close association between the activation energy for creep and the absolute melting temperature was also noted for eighteen metal elements.

In a later paper (1957) Lake, Wiseman, Sherby and Dorn investigated the effect of stress on secondary creep at high temperatures and found that a relationship of the form

$$\dot{\varepsilon} = e^{\frac{-\Delta H}{RT}} \cdot \phi(\sigma) \qquad \ldots.(2.14)$$

approximated the creep rate equation for constant structure, where $\phi(\sigma)$ was of the form $A\sigma^{n}$ for low stresses and $Be^{m\sigma}$ for high stresses. This corresponds to the equations discussed in the section on secondary creep. The observations also strongly suggested that high temperature creep did not take place by thermal activation of dislocations over a free energy barrier as assumed in most classical theories of physical metallurgy, and an attempt was made to relate the experimental results to a consistent theory based on a dislocation climb process controlled by the rate of self diffusion. The approach was not completely successful, but offered promise and scope for development.

Before leaving the Sherby-Dorn parameters which give support to the secondary creep relationships

$$\dot{\mathcal{E}} = e^{\frac{-\Delta H}{RT}} \cdot \phi(\sigma) \qquad \ldots\ldots(2.14)$$

it is perhaps of interest to consider the corresponding expression for primary creep, equation 1.54, given previously in section 1.5.4 namely

$$\mathcal{E} = e^{\frac{-\Delta H}{RT}} \cdot \phi(\sigma)\, t^{m} \qquad \ldots\ldots(2.15)$$

If this expression holds, then by suitable manipulation it may be shown that

$$\sigma = f\left[\frac{\Delta H}{RT} + \log_e \mathcal{E} - \log_e t^{m}\right] \qquad \ldots\ldots(2.16)$$

a relationship which might prove of use for correlating primary creep data.

2.5.4 <u>Empirical Curve Fitting</u>

In the course of a critical examination of the Larson-Miller parameter for creep rupture data, Manson and Haferd (1953) and Manson and Brown (1953) found that the parameter was not single valued and that discrete curves were distinctly obtained for tests at different temperatures. They also noted that a better correlation was obtained if the value of C was optimised for each material. These findings were later confirmed by Garofalo et al (1956) as discussed before in sub-section 2.5.2.

In an attempt to improve the correlation, Manson and Haferd plotted stress σ, versus temperature T, with the logarithm of rupture time, log t_{rup}, as parameter. For this plot they noted that equal increases in log t_{rup} gave equal increases in T in the range of log t_{rup} greater than unity, which suggested that if log t was plotted against T at constant stress an approximately straight line would be obtained for times greater than 10 hours. This was found to be so, and in addition, the lines representing different stresses were observed to converge to a common intersection, of which the co-ordinates were (T_a, log t_a).

From these results, the purely empirical relationship

$$\sigma = f\left[\frac{T - T_a}{\log t - t_a}\right] \qquad \ldots\ldots(2.17)$$

was obtained which Manson and Haferd named the linear time-temperature parameter since in arose from the assumption that constant stress lines were linear on the log t/T plot. (The Larson-Miller and Sherby-Dorn expressions are linear time-reciprocal temperature parameters).

No physical significance was claimed for the point (T_a, log t_a). The above parameter was also applied to the correlation of creep strain data at a given stress where t is the time to produce a given total elongation. For minimum creep rate data a linear parameter of the form

$$\frac{T - T_a}{\log \dot{\varepsilon} + \log \dot{\varepsilon}_a}$$

was used where $\dot{\varepsilon}$ is the minimum creep rate and $\dot{\varepsilon}_a$ is a material constant. Manson et al did not attempt to justify their parameters theoretically but in support they presented numerous graphs illustrating the excellence of the correlations obtained.

Garofalo et al critically reviewed the linear time-temperature parameter amongst others in 1956, and noted the disturbing feature that the proper selection of the point of intersection (T_a, $\log t_a$) seemed to require considerable experience. This criticism however, was subsequently removed in a later paper by Mendelson and Manson (1959) in which a least squares method was developed for determining the optimum constants for all three time temperature parameters, namely the Larson-Miller, Sherby-Dorn and Manson-Haferd relationships, without plotting and cross plotting the data and with the use of a minimum of judgement on the part of the analyst.

Garofalo and his co-workers also pointed out that, in common with the other two parameters, the Manson-Haferd expression was valid only within a relatively limited range of initial stress. Manson and Haferd (1953) Heimerl (1954) Manson and Succop (1956), Betteridge (1958) and Goldhoff (1959) compared all three major parameters, and although much improved results could be obtained by the optimisation of the relevant constants C, ΔH and (T_a, log t_a) respectively, in every case extrapolation with the Manson-Haferd parameter gave predictions with the smallest average deviation from the observed results. This is perhaps not surprising since the parameter has two constants for allocation.

In 1959 Conrad presented for the stress rupture data of the Nimonics 80A and 90 a correlation of which the Manson-Haferd parameter was stated to be a special form Conrad proceeded from the basic equation

$$f(\sigma, T) = t\, e^{\frac{-\Delta H}{RT}} \qquad \ldots.(2.18)$$

in which the Sherby-Dorn parameter can be recognised, and assumed an analytical expression for $f(\sigma, T)$ based on experimental results. The assumed expression does not fit in with the preferred relationships given in sections 1.3 or 1.5, but the method of attack used by Conrad suggested the possibility that a relationship might exist between the purely empirical Manson-Haferd parameter and the fundamental physical theories of creep as proposed by Dorn et al.

If such a relationship does exist, it might have far reaching effects, for the Manson-Haferd parameter and the Larson-Miller and Sherby-Dorn parameters cannot in general be analytically valid simultaneously. The basic plot for the Manson-Haferd parameter is linear in log t versus T, which if translated to a log t versus $\frac{1}{T}$ plot results in a non-linear curve with gradient proportional to T^2, making co-existance with the other two parameters impossible.

The theory of creep in complex stress systems discussed in sections 1.3 and 1.5 is in accordance with the Sherby-Dorn parameter. Theoretical support for the Manson-Haferd parameter, or some modification of it may thus necessitate complete rethinking on the theory established to date unless the Manson-Haferd and Sherby-Dorn parameters can be reconciled, or a superior version of the Sherby-Dorn parameter can be evolved. However, to keep a proper perspective, it should be remembered that the analytical expressions for creep discussed in sections 1.3 and 1.5 have themselves no universal and little fundamental validity.

2.5.5 Conclusions on time-temperature parameters

The three main parameter methods have been compared by Goldhoff (1959) for the extrapolation of high temperature data. The criterion used, namely that the creep behaviour in say 10,000 hours should be predicted from the observed behaviour during say the first 1,000 hours, and subsequently compared with the actual behaviour in 10,000 hours, was also advocated by Allen (discussion to Bailey 1954).

Goldhoff, in accordance with a number of other authorities concluded that in general any of the parameter techniques studied gave better results than those obtained with long extrapolations of say one and one half cycles on logarithmic plots. Where metallurgical transformations prevented adequate data from being obtained, and necessitated short extrapolations of the parameters themselves, the results were no more questionable than with extrapolation of conventional double or semi-logarithmic plots.

The Manson-Haferd parameter was generally superior to the Larson-Miller and Sherby-Dorn techniques for correlation and extrapolation, although considerable improvement in the Larson-Miller parameter method could be obtained by the use of an optimised value for the constant C. The mainadvantage of the parameter methods was that in theory only a few tests were required to establish the master curve.

Only three techniques have been discussed above. Conrad (1959) has given a more complete review of various alternative methods proposed, and has attempted to relate these parameters to the theories of the physical mechanism of creep.

In general, the time-temperature parameter methods substitute higher temperatures for longer times, and they have proved extremely useful in reducing the amount of testing time required in investigating new material. However, as stressed by Garofalo et al; it should be kept in mind that the parameters provide good correlations only over limited ranges of stress and temperature. Through lack of suitable data the engineer may be forced to use such parameters to assist in describing a 'creep surface' analytically for the solution of a complex stress problem. It should then be remembered that this might involve the application of greatly extrapolated correlations within a concept which is fundamentally invalid (see sub-section 1.5.1).

2.6 CREEP RATE - RUPTURE TIME PARAMETERS

In sub-sections 2.4.2 and 2.4.4 two empirical relationships which may be used to describe creep were discussed, namely

$$\dot{\varepsilon}_{min} = m\sigma^{n} \quad \text{and} \quad t_{rup} = c\sigma^{-d}$$

where m, n, c, d are constants.

Combining these two expressions there results

$$t_{rup} = q \cdot \dot{\varepsilon}_{min}^{\,p}$$

where q and p are constants

$$\text{or} \quad \log t_{rup} = w + p \log \dot{\varepsilon}_{min} \qquad \text{....(2.19)}$$

where $w = \log q$.

Equation (2.19) is known as the Monkman-Grant relationship and was proposed on an intuitive basis by these authors in 1956 as a check on the reliability of individual creep ruture data and as a useful means of estimating long time creep-rupture data from short time creep data. In this latter connection however, the difficulty of correctly estimating secondary creep rates from creep strain curves should be remembered. This has been discussed previously in sub-section 1.5.5 in connection with the work of Haslett and Parker. The Monkman-Grant relationship may therefore have greater application in determining minimum creep rates from creep rupture data. It was stated by the authors in their conclusions that the relationship was not primarily intended for extrapolative purposes.

In their paper Monkman and Grant noted that if the slope p of equation (2.19) above was unity, then the relationship $\dot{\varepsilon}_{min} \times t_{rup}$ = constant, was obtained and they further recognised that the constant was then an approximate measure of the elongation taking place during second stage creep. Goldhoff (1960) has discussed the papers of the Russian workers Ivanova and Oding who used an expression of this type and considered the above constant to represent 'usable elongation', ε_{rup}. The approximation $\dot{\varepsilon}_{min} \times t_{rup} = \varepsilon_{rup}$ however, only applies to the material which has a primary creep stage of very short duration and a secondary (constant) creep stage which extends almost over the full life of the specimens. In the discussion to the paper of Monkman and Grant it was noted that Machlin (1956) had derived equation (2.19) theoretically. The work of Machlin was also mentioned briefly in sub-section 1.3.2.

Commenting on the claim that the Monkman-Grant parameter was independent of stress, temperature and many structural factors, Manson and Brown (discussion to Monkman and Grant) stated that since the correlation was derived from a consideration of broad scatter bands on linear plots of log t_{rup} versus log $\dot{\varepsilon}_{min}$ the relationship should be taken as a first approximation only. The width of the scatter bands varied from ± 0.50 to ± 0.55 cycles of rupture time, so that for a given creep rate, the corresponding rupture time might vary by a factor of 12 and the accuracy obtained was therefore quite limited.

Underwood in the discussion pointed out that if the Larson-Miller parameters given by equations (2.7) and (2.9) could be equated at constant temperature, the simplified Monkman-Grant relationship $t_{rup}.\ \dot{\varepsilon}_{min}$ = const. was obtained providing the constants C in the respective parameters were not equal. The same applies to the

Sherby-Dorn parameters given as equation 2.12 and 2.13.

Manson and Brown in a somewhat different approach showed that a combined creep and rupture parameter could be derived from the Manson-Haferd time-temperature and creep rate temperature parameters discussed previously in sub-section 2.5.4. If the value of the constant T_a is the same in both parameters then, for constant temperature, if the parameter for creep rupture

$$\frac{T - T_a}{\log t - \log t_a} = f(\sigma)$$

is divided by the parameter for minimum creep rate

$$\frac{T - T_a}{\log \varepsilon + \log \varepsilon_a} = g(\sigma)$$

there results a creep rate-rupture time parameter

$$\frac{\log \dot{\varepsilon} + \log \varepsilon_a}{\log t - \log t_a} = \phi(\sigma) \qquad \text{.....(2.20)}$$

If $\phi(\sigma)$ is constant equation (2.20) reduces to the Monkman-Grant relationship.

Similarly the Larson-Miller creep rate-rupture time parameter is

$$\frac{C - \log_e}{C + \log_e t} = \psi(\sigma)$$

and the Sherby-Dorn creep rate rupture time parameter is

$$\frac{\log_e + \frac{A}{T}}{\log_e t - \frac{A}{T}} = \chi(\sigma)$$

Again the similarity between the expressions suggests the possibility that there may be a more general expression which embraces them all.

Although the Monkman-Grant relationship was specifically excluded by the originators from extrapolative use, Goldhoff (1960) compared it with the Manson-Haferd creep rate-rupture time parameter in this context for predicting the high temperature strength of ferritic steel. He concluded that extrapolation by the Monkman-Grant relationship was invalidated at long times by the possible dependence of the relationship on stress and temperature, and the probable dependence on material instability. The use of the Manson-Haferd methods were preferred, based either on the time-temperature parameter, or on the creep rate-rupture time parameter, although insufficient evidence exists at present to support the latter parameter unequivocally.

2.7 CREEP STRAIN-CREEP RATE DEVIATION CURVES

Glen's work on the effect of alloying elements on creep behaviour was mentioned in section 2.3. His second major contribution to the understanding of creep behaviour was his proposal of a method for the extrapolation of creep data put forward in 1958. This method, perhaps, comes nearer than any to a truly logical and scientific method of extrapolation.

The technique, which is graphical, is based on the observation that creep curves for commercial alloys of interest are highly structure dependent and that numerous transitions in creep rate occur during testing. Glen found it convenient to follow the metallurgical behaviour of his materials by plotting log strain versus log creep rate; this has the advantage that any transitions in creep rate which occur are emphasised as bumps in the curves.

To obtain instantaneous values of creep rate for these curves, the original strain-time data are plotted to very extended scales as described in the appendix to his paper.

When investigating the properties of any particular material, creep or rupture tests are generally made either at constant stress and various temperatures, or at constant temperature and various stresses. Thus either temperature or stress may be used as a parameter in plotting a family or strain/creep rate curves. Glen found it more convenient to use temperature as a parameter because the initial plastic extension for constant stress did not vary appreciably, and it was then easier to estimate the trend of the strain/creep rate curves since they were approximately parallel. This approach also fits in with Bailey's contention that in evaluating a material it is better to test at the working stress and vary the environmental temperature systematically (sub-section 2.4.5.)

For extrapolation purposes Glen found that if the creep test results were used to construct a family of strain/creep rate curves then it was possible, at least approximately, to predict the <u>shape</u> and position of the strain/creep rate curves of tests at lower temperatures. No other method of extrapolation known to the writer allows metallurgical transitions occurring during creep to be taken into account in such a positive way.

From the estimated strain/creep rate curves the ordinary strain/time creep curves may be obtained by straightforward graphical integration. Iso-strain curves of temperature versus time may then be constructed in the usual way.

Glen's technique was favourably reviewed by Goldhoff, 1960, who concluded that the method was basically sound, but was perhaps too complex for practical use. The main disadvantage appears to be the considerable amount of accurate creep data required. However, the method will undoubtedly become established as a 'reference' technique and may yet replace current empirical methods if the labour involved in computation can be reduced by the use of the digital computer. In this connection the work of Mendelson and Manson discussed in the next section is of interest.

2.8 RECURRENCE RELATIONSHIPS

The time-temperature parameter methods discussed in section 2.5 have two disadvantages. Firstly, they presuppose a knowledge of the existence of some form of functional relationship between the variables, and as pointed out in their discussion the relationship for each is different. Secondly, the normal form of the parameters give equal weight to all data, whether they be short time or long time data. For extrapolating to long times more weight should be given to long time data. The above considerations were pointed out by Mendelson and Manson in their 1960 paper on the direct extrapolation of creep data using a method of finite difference recurrence relationships. The technique is best explained in the words of the authors ......"The method is based on the assumption that a family of curves such as a set of isothermal curves can be represented approximately by a finite difference recurrence relation.

Once the coefficients of this recurrence relation are determined, it is simple matter to extrapolate each member of the family individually. The method has the advantage that it does not require an explicit knowledge or assumption as to the analytic character of the curves or as to what parametric form to use to correlate the data, although implicitly it assumes any of the broad class of functions which satisfy linear finite difference equations. It is thus much more general in nature than the parametric methods previously discussed, and can be used by itself to extrapolate creep-rupture data or as an independent check on any of the parameter methods. In addition, the method treats the data in such a way as to give more weight to the higher time data than to the lower time data."

Mendelson and Manson compared their method with the Larson-Miller, Sherby-Dorn and Manson-Haferd techniques and in every case found that the recurrence method gave better results. The only rather mild restriction on the method was that sufficient short time data are required for adequate definition of the basic curves to permit proper fairing.

This paper is an important contribution to the extrapolation of creep data and if the technique can be applied to Glen's method of using strain/creep rate curves, then a considerable step towards the reduction of time consuming creep tests will have been taken.

3. REVIEW OF EXPERIMENTAL WORK ON CREEP OF TUBES UNDER INTERNAL PRESSURE

3.1 General Remarks

Upwards of twenty-three papers describing experimental work on the creep of tubes under internal pressure. from 1926 to the present day, have been located by the writer. Criticism of these papers in this section has been slanted towards discussion of the suitability of the experimental techniques used and of the results obtained, for the development of a general theory of creep of thick cylinders.

For most of the authors concerned, however, the development of a general theory of creep was not their primary aim in the presentation of their papers - e.g. for a specific problem, creep data on a particular tube configuration subjected to fixed environmental conditions might have been urgently required. In some cases, especially for the early workers, it may therefore be that the writer is found to criticise aspects of the work with which the original authors were not primarily concerned, or which perhaps were at that time obscure. However, since the review of previous experimental work must be tackled from some standpoint, the writer asks that the intention stated in the first paragraph should be kept in mind when the reader is considering the discussion of previous work given here.

Although the final layout of the section would tend to the appearance of a 'catalogue', it was thought best to review previous work in chronological order. A list of authors is to be found inthe contents list. Each worker or group of workers is treated individually and discussion of the work of a new author or authors is indicated by underlining in the text, the first reference made to that paper.

3.2 Previous Work

White and Clark (1926, 28, 29) appear to have been the first to test tubes under internal pressure at temperatures where creep would occur. The investigations were an attempt to define safe working pressures for tubing at certain elevated temperatures by determining the time taken for a tube to 'expand' at different pressures. Short time tensile tests were also made a temperature "to determine the relationship, if any, between proportional limit as measured by the short time tensile test and the maximum working load as determined by the 'expansion test'."

The 1926 tests were carried out on 0.13% carbon steel obtained in the form of seamless tubing and tested in the 'as-received' condition which was normally a 700°C anneal after cold drawing.

Short time hot tensile tests were also made on thin tubular specimens machined from the tube, to determine the proportional limit at temperature. These results were incorporated in the 1928 paper which included new results on 0.37% and 0.38% carbon steel tubes and on 16.7% chromium 0.09% carbon steel tubes.

All internal pressure expansion tests were made on seamless tubes of 1 inch bore and 30 inches length having a reduced gauge length of 18 inches with a k value of 1.235. Steel end caps were brazed to the tubes.

The test temperature levels selected were 900, 1000, 1100, 1250 and 1500°F and internal pressures up to 1400 lb/in² were employed; the maximum duration of testing was 2,238 hours.

A constant nitrogen pressure was maintained in the tubular expansion specimen by means of an intensifier, the low pressure side of which was acted upon by oil pressurised by a dead weight accumulator. Additional nitrogen could be added to the high pressure gas system to raise the dead weight accumulator and intensifier pistons during testing if required. Leather gaskets were used as seals and it was necessary to water jacket the piston and cylinder assembly exposed to hot nitrogen and to lubricate the leather with glycerine. Three sizes of intensifier were constructed permitting nitrogen pressures of 36, 16 and 6.25 times the dead weight load.

The test specimens were placed in a horizontal electric furnace mounted on wheels and diametral expansion of the tubes was measured at regular intervals by calipers during brief periods when the furnace was run away from the tube. The tests reported in 1926 were only continued until 'expansion' was found, but later experiments were carried to rupture in most cases.

Tubular tensile specimens identical in cross section with the expansion test specimens were also tested. These had a parallel gauge length of 2¼ inches and were machined from the same tube stock as the expansion test specimens.

The tube stock material was used in the 'as received' condition and some variation in properties in the circumferential and longitudinal directions must be expected due to manufacture. The tensile results reported for the material would therefore be applicable to deformation in the axial direction but not to the circumferential direction for a tube under internal pressure.

In the discussion to the 1926 paper, the accuracy of determining proportional limits from tubular tensile specimens was questioned. Tests were therefore made in the case of 0.37% carbon steel on standard 0.505 inch diameter tensile specimens from both tubular and bar stock of almost identical chemical composition to determine the variation. However, it was stated that the tubular material was used in the 'as received' i.e., annealed condition, while the bar stock was normalised before testing, so that the complete validity of the comparison may be questioned.

White and Clark concluded that the expansion limit as obtained from tubular tests under internal pressure was considerably below the proportional limit as determined by short time tensile tests. White and Clark's tests therefore demonstrated that creep was a phenomenon requiring new design concepts and new criteria of design.

It is unfair to criticise this early pioneering work too heavily, but for the internal pressure tests it seems desirable that at least some arbitrary value of diametral strain should have been selected at which the tube would be considered to have 'expanded' so that different materials could have been more accurately compared. For 0.13% carbon steel tubing at 1,250°F the strains at 'expansion' for various pressure loads ranged from 0.10 to 0.01 as measured by calipers. No allowance in strain measurement was made for thermal expansion of the specimen. Continuous strain measurements of known accuracy and sensitivity would have enhanced the value of White and Clark's work, for then the time to reach the same value of diametral strain could have been accurately determined for different materials.

No mention was made of calibration of the pressure loading system to determine its sensitivity and accuracy. The leather gaskets would have had a measurable effect on the sensitivity of the system and no provision was made to reduce friction by continuously rotating the piston. Later workers perhaps cannot be excused for such omissions.

Bailey (1930) reported very briefly the results of a test carried out on a thick lead tube ($k = 3$) under internal fluid pressure of 1,200 lb/in.^{2} at a temperature of 19.5°C. The overall length of the specimen was 64 inches with a gauge length of 48 inches and an outside diameter of 3 inches; the duration of the test was 125 minutes and the extension readings were taken to ± 0.0001 strain. The results were presented in the form of a graph and show substantially zero axial creep up to the point where localised bulging began. No details of the apparatus used or testing technique was given, but it is probable that the method employed was substantially the same as that reported in 1929 for tests on thin walled lead tubing subjected to combined internal pressure and axial loading. No isotropy tests or supporting tensile, torsion or complex stress tests were carried out which would have provided data on the tube material.

Bailey was primarily interested in the development of a theory of creep discussed in sub-section 1.3.1 and his perceptive experimental work is characterised by its directness and relevance to the fundamental points of information being sought, without omission of essential precautions, but with little refinement in experimental technique. The fundamentals of a consistent theory of creep were confirmed by his various tests, the pattern was set for more accurate and controlled experiments and further development and refinement of his original theory. The above test on a thick tube does not form one of his number of 'critical' experiments in the development of a general theory of creep, but it is however critical in this particular case to the development of creep theory as applied to thick tubes subjected to internal pressure. If zero axial creep can be shown to be an invariant characteristic of a thick tube creeping under internal pressure, then the problem may be reduced to one of plane strain, resulting in substantial simplification of analytical treatments. The theoretical analysis is discussed in sections 1.4 and 1.6.

Moore and Alleman (1932) and Moore, Betty and Dollins (1938) reported tubular creep tests carried out on a variety of lead alloys (e.g. antimony, tin, calcium, copper) on specimens of k values from 1.05 to 1.25. Most of the material used had been aged for 1 year before testing and as the intention was to test commercially produced sheathing, no attempt was made to obtain material in an isotropic condition. The authors themselves considered that previous history of the specimen would affect the results to some extent in any case.

The tests on tubes were made at an uncontrolled 'room temperature' (which varied between 73 and 84°F over a period of 4,740 hours) and some tests were carried to 20,000 hours. The apparatus consisted of a rack holding 6 foot samples of lead cable sheath between 0.656 inch and 4.25 inch outside diameter, sealed at both ends with wiped closures. The specimens were filled with oil and a constant internal pressure of 25 lb/in² was maintained by means of a mercury column which pressurised the oil reservoir thus avoiding amalgamation of mercury and lead. Circumferential stresses up to 250 lb/in² were used, calculated on the basis of thin cylinder theory.

Specimen outside diameters were measured periodically by means of a 'dial gauge micrometer' sensitive to 0.0001 inch. Small steel studs were soldered to the tube at stations 12 inches apart and across two diameters at right angles at each station. This provided bearing points for the dial gauge micrometer anvils. The average circumferential creep of a specimen was taken as the average of these twelve measurements. To allow for changes in ambient temperature a short length of unstressed tube, kept close to the test 'reference bar' was also used for calibration of the dial gauge micrometer so that suitable allowance for temperature variations could be made.

The results for diametral creep showed considerable random differences in strains at various stations along the length of the specimen for all tests, the average strain at one station being as much as twice that at another throughout the experiment. This may have been due to variation in wall thickness of the specimen, or anisotropy produced during manipulation.

Also Moore, Betty and Dollins (1935) in another publication, themselves pointed out that the effect of soldering material to the surface of a test piece was to reduce the circumferential length which could stretch, thus introducing an additional element of uncertainty into the creep strain measurements.

Tensile tests were also reported in both papers. In the 1932 paper, flat sheets of lead were prepared by splitting a 5 inch length of sheath longitudinally at its thickest section and flattening under a 2,000 lb. load. Tensile specimens in the form of strips $4\frac{3}{4}$ inches in overall length with a width over the gauge length of $\frac{1}{4}$ inch were cut from the longitudinal and circumferential directions of the original tube. Because of the small circumference of the sheath a gauge length of only 1 inch was possible with these specimens. Tests were carried out (a) at 32°F in a triple glazed refrigerated box, (b) at room temperature, and (c) at 150°F in a double glazed insulated box. Short time tensile tests were made at stresses up to 3,850 lb/in.^{2} and long time creep tests at stresses up to 1,400 lb/in.^{2} and durations up to 5,000 hours. Creep was measured to 0.0001 strain by cathetometer and mechanical lever extensometer. The loading of the specimens was by dead weights.

The authors stated that no systematic difference had been found between the creep of transverse specimens and the creep of longitudinal specimens cut from flattened sheath. Tensile tests described in the 1938 paper were therefore made only on specimens cut from sheather in the longitudinal direction. A 10 inch gauge length was used, and experiments were carried out at 110°F and 150°F in the double glazed insulated box. Stresses up to 400 lb/in² were used and most tests were carried to 2,000 hours and in some cases to 10,000 hours.

The validity of tensile results obtained from flattened material was subsequently questioned and experimental results on the effect of flattening cable sheath before cutting circumferential and longitudinal tensile specimens, were obtained by McKeown (1937) who found an appreciable effect on the creep properties. This indicated that the hardness test used by Moore Betty and Dollins was not sufficient guarantee that the material was unaffected after being flattened, and allowed to recover. However, Moore et al subsequently agreed that the effect of flattening on a 10 inch tensile specimen was appreciable, but considered that for their purpose it was not of major importance and not predictable. A possible solution would be to warm work the split sheath to flatness and then heat treat to a known condition. For comparison of results however, the same heat treatment would then be required on the tube which would then alter its initial condition, and therefore its subsequent behaviour.

An attempt was made in the 1938 paper to correlate creep behaviour with tensile behaviour using Bailey's theory, but close agreement was not found. This is not surprising, considering the anisotropy of the material due to handling, lack of close temperature control on the tubes, possible eccentricity of the tubular sheath, effect of flattening tensile specimen, etc.

Nakahara (1939) following Bailey, tested thick walled lead cylinders of $k = 1.56$ and 1.39 under internal pressures up to 450 lb/in^2, apparently at room temperature. The tubes appear to have been strained at one pressure until a steady creep rate was obtained, and then the pressure increased to a new value and the observations on creep rate repeated. Steady state circumferential creep rates up to 5×10^{-4} strain per minute were attained at the outside surface of the tube. One tensile creep specimen from lead bar material was tested similarly in that the minimum creep rate at one stress was recorded and then the load increased to a new value for new observations on creep rate. Tensile stresses up to 1000 lb/in^2 were used and steady state creep rates of up to 5×10^{-4} strain per minute were attained.

The results of the tensile test provided experimental constants for the material, which when used in Bailey's theory for steady state creep of a tube, permitted the comparison of predicted and experimental creep rates for a thick tube. Reasonable agreement in support of the theory was obtained. It should be noted, however, that a circumferential strain of 5×10^{-4} at the outside of the large tube corresponds to a bore circumferential strain of 12.8×10^{-4}

and the tensile tests should have been carried to this creep rate for determination of the material constants.

No details of the experimental procedure or of the accuracy of the measurements were given in the English summary to the paper, and no supporting isotropy tests were carried out. Providing the lead was allowed time to recover and anneal after each observation at a given load, the tests might give reasonably true values for creep rates for the different loads, however the k value of the tubes, and the cross-sectional area of the tensile specimen would decrease for each succeeding observation and it is not clear if allowance for this has been made. The errors thus introduced may not have been serious, but for future work on other materials especially at large strains, each loading condition will require a minimum of one specimen to obviate the possibility of influence of previous strain history and geometrical modification on the specimen.

The most refined series of tests on the creep of tubes of which the writer is aware, the technique and testing conditions of which have certainly not been together surpassed, are those of <u>Norton</u> (1939, 41, 42*) The investigation was carried out under the auspices of the ASTM-ASME Joint High Temperature Committee to determine the creep of thin walled cylindrical pressure vessels as compared with the creep properties of tensile specimens of the same steel at the same temperature, and under a stress equal to the circumferential stress in the cylinder.

* In association with Soderberg.

The majority of tests were carried out on a nominal carbon ½% molybdenum steel prepared from a single heat by rolling 5 inch rounds and then piercing to form 4 inch o.d. x ¾ inch minimum wall seamless tubing. Tests were also carried out on a nominal 4-6% chromium ½% molybdenum steel.

Tubular specimens were machined both inside and outside and carefully annealed before testing (this information was only given in the discussion to the 1941 paper) and an outside diameter of 4 inches selected. Tubes of k value 1.07 and 1.23 were tested. The overall length including welded hemispherical steel end caps was 28¾ inches given a parallel gauge length of approximately 6D. Norton himself expressed doubts as to whether the length was sufficient to eliminate end effects entirely. Since, however, axial creep was measured over a gauge length of 10 inches (i.e. a distance of 2.5D) then, providing the temperature distribution over the parallel gauge length was acceptable, there is good reason to believe that the allowed length was sufficient for reliable results. Norton took several weeks to adjust the end heaters on his oven for each specimen, so there is no reason to doubt that the temperature distribution was satisfactory. Each specimen is provided with a filler piece placed inside allowing only 1/16 inch free space for the gas providing the pressure.

The test temperature levels selected were 800, 900 and 1050°F for the carbon - ½% molybdenum material and 1,200°F for the 4-6% chrome -½% molybdenum material. Internal pressures up to 9,240 lb/in.^{2} were used and tubular tests were carried to 9,000 hours in some cases.

The specimens were connected to a larger cylinder below by capillary tubing. This cylinder could be charged with nitrogen from bottles up to a pressure of 2,000 lb/in^2 and higher pressures were attained by forcing oil into the bottom of the accumulator by a small hand pump to further compress the gas above the oil. The rate of expansion of the specimens was low and it was found possible to keep the pressure constant to within ± 1.0% except for two cases, by pumping up every morning and every night. The specimens were placed in a vertical electrically heated furnace, the construction of which was carefully explained in the paper, giving a temperature control from day to day of ± 1 F° with a distribution over the gauge length of the specimen of ± 3 F°.

As far as the writer is aware, Norton's experiments on tubular specimens are the only tests carried out to the present date in which continuous and accurate diametral and longitudinal creep measurements have been made during such advanced testing conditions. The sensitivity would be reduced proportionately by the use of smaller gauge length specimens but would still be very acceptable for present day work.

Diametral extensions were measured in two mutually perpendicular directions at the centre of the gauge length of each specimen. Four small fused quartz tubes, each 3 inches in length, were used as probes, a pair being set endwise, normal to the circumference of the specimen, across one diameter and passing through the furnace wall. The inner end of a fused quartz tube rested on a platinum fixture welded to the surface of the specimen and the outer end was supported by a cantilever spring which held the tube firmly on the specimen. A pair of telescopes of 100 X magnification mounted on a common invar block (the one end of the block incorporating a micrometer adjustment) were employed to measure the distance between the outer ends of the probes). The variation in length was measured to ± 0.000025 inch which with a 4 inch gauge diameter was good to 6×10^{-6} strain.

Axial strain was determined on diametrally opposite sides by measuring the distance between two reference marks 10 inches apart consisting of fine V marks in platinum wires spot welded to the surface of the specimen. The same pair of telescopes employed for the diametral measurements was used for axial observations which were measured to 2.5×10^{-6} strain.

This method of reading extensions does not give as great a sensitivity as many of the mirror extensometers, but it is believed to have fully as much precision. This was borne out since micrometer readings of the diameter in the centre of the specimens made before and after each run gave a permanent increase which agreed in every case with the telescope readings.

The method has the advantage for diametral creep that the magnification or extensometer constant is unchanged by the temperature of testing as is the case for a diametral extensometer totally enclosed within the oven. For axial creep however, the Marten's type of extensometer as modified by the N.P.L. is to be preferred as this does not involve the welding on of reference marks which may prove troublesome with some specimen materials.

Norton also carried out tensile tests on specimens having a

10 inch gauge length and cut from the wall of the same batch of tubing with their length parallel to the axis of the tube. The tests were made in a standard tensile creep testing machine. Since all specimens were carefully annealed before testing, it was considered that any axial anisotropy due to forming the original tube would be small. This may be true but the assertion cannot be accepted until demonstrated by isotropy tests in three mutually perpendicular directions in the material.

It was stated initially that the purpose of the tests was to determine the creep properties of cylindrical pressure vessels as compared with the creep properties of tensile specimens of the same steel. Although the major deformation in a tube takes place in the circumferential direction, Norton's work was an attempt to correlate tensile data obtained for the longitudinal direction of the tube material, with the complex stress creep behaviour of the tube itself. This approach may not be considered valid until isotropy has been proved.

Norton did not analyse his own results. This was undertaken by Soderberg (1941) also under the auspices of the ASTM-ASME Joint High Temperature Committee. Soderberg evaluated four of the results and found that the test cylinders gave higher steady state creep rates than those predicted by theory from the results of tensile creep tests. This may have been due to inadequate theory, or anisotropy of the material, or both, but the lack of experimental data left this unresolved.

The defence was put up that all specimens were annealed, but if the intention was to investigate the validity of complex stress theory for design, it would have been much more satisfactory if Norton had started from an isotropic forging or continuously cast material which would permit the manufacture of tensile, torsion or complex stress specimens having any desired orientation. Tubular creep tests carried out on material produced in the form of tubing should be accompanied by supporting isotropy, axial, circumferential and radial tensile creep tests.

Despite the above criticism of the organisation of the overall test programme, Norton's experimental technique can be commended and, in the writer's opinion, has yet to be surpassed. It is to be regretted, however, that Robinson did not prevail with his suggestion to test isotropic forgings when the test programme was first mooted (see discussion to 1941 paper).

In the discussion to Norton's 1939 paper, brief mention was made by Rhys and Northrup of tests they had carried out on thin walled lead tubes of k = 1.17 under internal pressure which indicated that axial creep was negligible. No details of the tests were given.

The early work of Moore et al (1932, 1938) on lead cable sheathing was later continued by Phelps, Gates and Kahn (1940) who carried out tests on tubing under more controlled conditions. These tests were supported by tensile rupture tests made on the same material and under the same controlled conditions by Moore, Dollins and Craig (1940). The tensile results were also reported by Moore and Dollins (1943). These three papers will be considered as a group.

Phelps et al mentioned in their paper a co-operative experimental programme for tubular stress rupture tests on lead tubes initiated in 1935, which had been arranged between another electric utility company laboratory and their own, and a brief summary of the results were given without details of apparatus or testing conditions.

Their currently reported work on tubular bursting tests was carried out on a variety of alloys for lead cable sheathing tested in the commercial 'as-received' condition. The intention was to investigate the effect of flaws produced during normal manufacture. Experiments were made on tubes of k value 1.11 at pressures up to 197 lb/in^2 and specimens were suspended in thermally insulated boxes which were kept at the controlled temperatures of 50, 110 and 150°F. The maximum circumferential stress produced was 1600 lb/in^2 and tests were continued in some cases to 8,800 hours.

The specimens were approximately 12 inches in length and of 2.84 inches outside diameter. The end closures consisted of solid steel plugs 2 inches long, of slightly smaller diameter than the bore of the tube and built up with gummed wrapping paper on their circumference so as to just slide into the bore of the test piece. Malleable iron hose clamps were used on the outside of the specimens to compress the lead onto the end plugs.

Since it was only possible to measure the thickness of a specimen at its ends, Phelps et al considered it advisable to use as short a length of test specimen as possible in order to reduce the effect of possible unknown variations in thickness or eccentricity on the time-to-burst*. Phelps et al made preliminary tests which indicated that a 12 inch length of tube would be satisfactory for specimens up to 3 inch diameter if the total expansion did not exceed 25%. With end closures 2 inches long in position, this would give a free parallel gauge length to diameter ratio of only 2.66**.

No details of the method or accuracy of measuring diametral creep strain during testing were given by Phelps et al, although graphs showing extension were presented.

Water pressure was applied to the specimen from a vertical steel accumulator 3 inches in diameter and 4 feet long, having an air cushion over a column of water, the compressibility of the air keeping the pressure reasonably constant during expansion of the specimen. Adjustments were made from time to time by means of a hand pump and the pressure was maintained within ± 1 lb/in². Precautions were

* The importance of this consideration may be seen from the variation in diametral creep strain along a 6 foot specimen reported earlier by Moore, Betty and Dollins (1938).

** Cook and Robertson (1911), later confirmed by Crossland and Bones (1958), showed that a minimum gauge length of 4D was necessary to ensure results unaffected by end closures for thick tubes. The writer considers that the most satisfactory procedure for Phelps et al would have been to obtain concentric tubes of constant thickness and to carry out tests with a parallel gauge length to diameter ratio of not less than 4, the problem of measuring wall thickness along a tube 16 inches in length not being insuperable.

also taken to limit the release of high pressure fluid when the specimen ruptured.

Phelps et al were probably the first to develop and commission an apparatus which enabled cyclic pressure creep tests to be carried out. Careful experimentation with cyclic pressure equipment, when carried out in conjunction with constant pressure tests would permit the evaluation of a theory for predicting creep life under pressure variation similar to that proposed by Robinson (see sub-section 1.7.3) for temperature variattion. The apparatus is also suitable for automatically controlled constant pressure tests when used with a sensitive pressure transducer.

An external heating coil was placed around the accumulator which is partly filled with water. During an "on" cycle, the heating coil generated steam and raised the pressure to the desired value, when a pressure sensitive relay took over control of the heater and maintained the pressure at 65 $lb/in.^2 \pm 1\ lb/in.^2$ An automatic timer controlled the frequency of the "on" cycles and adjustments to the heater current determined the rate of increase of pressure. Decrease in pressure was obtained by allowing the reservoir to cool when a vacuum of approximately 28 inches of mercury inside the specimen could be obtained.

Only a single cyclic pressure test was made. A tubular specimen of 99.87% lead with a k value of 1.11 was subjected to two complete cycles per day until it ruptured after about 1 week. The rate of heating was adjusted so that the pressure built up in about 1 hour at an approximately uniform rate, when it was maintained at a constant pressure of 65 ± 1 lb/in^2 for a two hour period. Heating was then discontinued and the reservoir permitted to cool, the pressure decreasing to zero gauge in about 1½ hours and reaching a maximum of 20 inches of vacuum in approximately 4 hours.

Phelps et al, appreciating that a tube creeping under constant internal pressure was essentially a problem of increasing stress due to decrease in wall thickness and additionally increase in diameter, also tried to carry out tests with constant stress in the wall but were forced to abandon the attempt. The technique was used by Latin (1948) who later (1952) also made tension tests at approximately constant stress using an ingenious cam device to vary the applied load.

Phelps et al carried out no tests for isotropy, or complex stress creep tests, but tensile stress rupture tests on portions of the same material were made by Moore, Dollins and Craig (1940). The object was to correlate the bursting tests with tension tests on specimens with their axes in the circumferential stresses in the thin tubular specimens. (In a footnote in the 1943 paper by Moore and Dollins, it was stated that Phelps et al used a formula taking into account the effect of longitudinal stress on the circumferential strain, but no details were given). Moore et al found that the rupture time of their tensile specimens gave close agreement with the bursting time of the corresponding tubular specimen, although a thin tube under internal pressure forms a complex stress system and no allowance for this was made by the authors.

This would seem to suggest that the maximum principal stress criterion for creep rupture might apply in the case of lead (see sub-section 1.7.2) but since no attempt was made to carry out tensile tests at equivalent octahedral shear stresses, a firm conclusion on this point is not possible. However, the initial anisotropic condition of the material and the deliberate attempt to include cable sheathing defects in gauge lengths must have influenced the results. Since all bursting test specimens failed through a weld and the majority of tensile specimens also failed in a weld (reported in 1943 paper) this suggests that close agreement of results is probably due to the characteristics of tubing with defects present. The test material was not sufficiently controlled, either before or during the tests, to permit the obtaining of results which could be used to establish whether the maximum principal stress or the maximum equivalent stress criteria of failure was applicable.

<u>Basset and Snyder</u> (1940) carried out bursting tests on thin walled lead tubes similar to the tests made by Phelps et al (1940) with the stated object of comparing the quality of commercially produced materials. Ten different samples of extruded tube were obtained with a minimum lead content of 99.66% but with up to seven additional trace elements present which would be sufficient to ensure wide variation in behaviour.

Tests were made on extruded tube, apparently in the 'as-received' condition, at room temperature without any temperature control for durations up to 7,000 hours. The tubular specimens had k values between 1.11 and 1.15 and outside diameters varying from 1.81 to 2.96 inches. Two series of nominally constant-internal-pressure bursting tests were made on samples of all the tubing, the first being relatively short time (7.5 to 12,500 minutes) at high pressures, (131 to 325 lb/in.^{2}) and the second series long time (32,000 to 420,000 minutes) at lower pressures (55 to 194 lb/in.^{2})

The apparatus used was similar to that of Phelps et al except that it lacked proper temperature control. It was stated that pressures were held constant well within ± 2% for 6 days each week, but no indication of what occurred on every seventh day was given. No details of extension measurements were furnished. The length of a specimen was 15 inches overall, including end closures, giving a ratio of free parallel gauge length to outside diameter of 3.66 in the worst case - a single improvement in an experimental technique which the writer considers to be generally inferior to that of Phelps et al. No supporting tensile creep tests were made and only brief mention of short time tensile results given, with no indication as to the method used.

As mentioned previously Latin (1948) reported the results of tubular creep tests carried out on continuously extruded thin walled lead tubes of k values 1.09 and 1.11 under both constant internal pressure and constant hoop stress in the tube wall. A pure lead and a lead (0.4% tin, 0.2% antimony) alloy designated alloy 'B' were used and tests were carried out apparently at room temperature and at pressures up to 150 lb/in^2 for durations up to 10,000 hours.

The specimens were from 90 to 180 inches in length and of 2 to 3 inches outside diameter with plumbed brass end caps and taped for a few inches near each pumb for reinforcement. The pure lead specimens were tested with hoop stresses of from 400 - 800 lb/in.^{2} and the 'alloy' E specimens with hoop stresses from 800 to 1,400 lb/in.^{2} Nitrogen gas was used for pressurising at low stresses and for the high pressure tests nitrogen from a bottle was fed via a reducing valve to an accumulator containing water which was used as the pressurising medium. For constant pressure tests the pressure was kept within ± 0.25 lb/in.^{2} by manual adjustment as required, and observations of diametral strain were made at several points along the length of the tube using a tape reading to 0.002 strain.

A greater number of constant hoop stress tests were subsequently made and a theoretical expression obtained relating diametral strain and internal pressure. Considerable variation in expansion along the 180 inch lengths was found and in the constant stress tests, strain measurements were therefore taken only at the centre of each tube so that the adjustments to pressure were correct for that section. It should be noted that the wall thicknesses of 0.10 and 0.12 inches were subject to a manufacturing variation of ± 0.005 inches, i.e. approximately ½%. Pressure was adjusted as soon as alteration in the level of ¼ to ½ lb/in.^{2} became necessary. Latin carried out checks on the variation in hoop stress due to experimental error by measuring the final wall thickness of several specimens. In no case was the final stress in error by more than 5% and in many cases the error was less than 2%.

In discussing his results Latin employed an Andrade type relation to determine if it would successfully express the deformation of a tube, and found reasonable agreement.

It is to be regretted that no tensile tests were carried out in support of this interesting programme and that no attempt to prove isotropy of the material was made. A fuller description of the apparatus used would also have been useful.

Dollins (1948, 1950) summarised much of the work carried out at the University of Illinois by Moore et al on lead cable sheathing and described additional results. The experimental technique appears to have been influenced by that of Phelps Gates and Kahn (1940) in that proper temperature control of tubular specimens was installed and cyclic pressure tests were commenced. In all, about 44 different samples of extruded sheathing, as distinct from tubing, were tested, the sheathing being aged, generally for 1 year, before use.

The constant pressure tests on tubes 72 inches in length were carried out in much the same manner as reported in 1938 except that in some tests the pressure was 40 lb/in^2 instead of 25 lb/in^2, and the specimens were maintained at a controlled temperature of 150°F. presumably in a double glazed box similar to that used previously for tensile tests. Tests were also made at room temperature (about 78°F.)

The graphs of results show considerable scatter in diametral extensions along stations 12 inches apart and across two diameters at each station as mentioned previously (review of 1938 paper). The author considered that this might be due to initial out of roundness, dents, extrusion marks, charge welds, duplex grain structure, etc., which were caused or accentuated by the use of sheathing normally subjected to much more handling than tubing, having been extruded onto cable. Such a large number of uncontrolled variables only permits conclusions as to general trends and rough sorting of the performance of different alloys. This is supported by the fact that when a diameter tape was used at slightly different locations to check dial micrometer readings, in every case the tape gave a higher percentage increase in diameter, indicating that sweated on micrometer ball points restrained the specimen to some extent. Cyclic loading tests under internal pressure and vacuum were also carried out for durations up to 660 days on specimens of four lead alloys at a temperature of 110°F. An internal pressure of 28.84 $lb/in.^2$ was maintained for 8 hours a day for four consecutive days weekly, each such period being followed by a rest period of 4 hours at zero pressure and by 12 hours of vacuum at 15 inches of mercury. On the fifth day, the pressure period was left on for 24 hours and after a 4 hour rest period at zero pressure the vacuum cycle was on the sheath for 44 hours after which the cycle was repeated.

The method of measuring extension was novel in that the 60 inch long specimens were placed inside horizontal steel pipes whose ends were wiped to the sheaths, the steel pipes being large enough in internal diameter to allow for 5% expansion of the sheaths. The unrestricted length of the test samples was stated to be 4 feet. A graduated glass tube 0.414 inches in internal diameter extended upwards from a hole in the steel pipe and the volume between the sheath and the pipe was filled with degassed oil. Expansion or contraction of the sheath altered the level of the oil in the glass tube.

The author could have eliminated end effects in the measurement of extension by the use of two specimens of differing length but the presence of uncontrolled variables in his specimens did not justify this refinement. As with most experiments on lead, the tubular specimens were horizontal during testing and had lengths of the order of 5 feet. Most experimenters with such long test pieces must have had some bending of their specimens, which is undesirable. The precision of Dollins' technique probably did not warrant taking precautions which would be regarded as essential in more careful work.

Tensile creep tests were made on specimens having a 10 inch gauge length and cut from the longitudinal direction of a sheath as described in the 1938 paper. The majority of tests were taken to 2,000 hours while some passed 40,000 hours. Tensile stresses were chosen so as to be the same as the hoop stress in the cylinder as calculated by the thin-walled cylinder formula, this corresponds roughly to the assumption of the maximum principal stress criteria of failure. However, in most cases the strip specimen had a greater total creep than the corresponding tubular specimen which suggests that the maximum equivalent stress criterion might apply. On the basis of octahedral shear stress and ignoring the radial stress in the tube wall, the equivalent octahedral shear stress chosen by

Dollins for his tensile specimens is too high by a factor of $\frac{2}{\sqrt{3}}$.

Long time tensile rupture tests were also made with smaller specimens of 2 inch parallel section, as reported in 1943, cut from both longitudinal and transverse directions in the sheath. It was stated that little difference was found in the strength test results between longitudinal and transverse tests, but that the longitudinal tests gave higher values for ductility.

Further results of tensile and constant internal pressure tests were given in 1950.

Romer and Newell (1952) discussed creep and stress rupture testing of steam boiler materials and mentioned the development of tubular stress rupture equipment reported by Kooistra, Blaser and Tucker in 1952.

The object of these tests was to determine experimentally the rupture lives of tubular specimens under loading conditions as closely analogous to actual service conditions as possible, and up to 10,000 hours which was considered sufficiently long for further extrapolation. 0.2 to 0.3% carbon steel tubing in 'as-received' normalised and annealed conditions was tested at pressures up to 9,600 lb/in^2 at 850 and 950°F. No machining of any kind was carried out on the specimens, end plugs being welded directly to the tube. Each specimen was 2 inches in outside diameter with a k value of 1.175, the overall length being 20 inches with a free parallel length of approximately 9D. Each specimen was provided with a close fitting steel core taking up nearly all of the internal volume so as to minimise the explosive force due to rupture.

The vertical furnace was reinforced externally and internally to withstand explosive rupture, being provided with a liner tube of 18/8/Cb, 4½ inches outside diameter and 0.10 inches thick extending over the length of the oven. For furture tests it was recommended that the liner should be 2 inches shorter at each end in order to minimise end losses due to conduction. Temperature variations at any one point along the specimens were controlled within ± 3 F° and over a 10 inch gauge length the gradient was within ± 10 F°. Thermocouples were peened to the surface of the specimen at 5 inch intervals.

Pressure was supplied to the top of the specimen through a stainless steel connecting tube from a high pressure reservoir capable of withstanding 10,000 lb/in². The reservoir was placed in a constant temperature water jacket heated by an immersion heater operated from a pressure switch, the temperature of the jacket being maintained at 220°F by the pressure controller set at 2½ lb/in² g. The relative volume of reservoir and specimen were chosen so that the compressibility of the small volume of steam in the specimen was less than the compressibility of the large quantity of water in the reservoir. When the internal volume of the specimen increased due to plastic deformation a minute quantity of water was delivered to the specimen and immediately flashed to superheated steam thus automatically adjusting the pressure. Manual corrections to the system pressure were required at weekly intervals only, and the internal pressure was maintained within ± 25 lb/in².

Preliminary short time high temperature tensile tests were carried out on equivalent bar material to decide upon stress levels for the first few tubular specimens for the relatively short time tests, and the stress levels were progressively reduced for succeeding tests.

Creep strains during testing were not measured and correlation of results was on the basis of rupture life. No tests to prove isotropy were made and no supporting tensile or complex stress tests were carried out.

In the introduction to the paper it was observed that due to fabrication, tensile test data would not be suitable for comparison with data from internal pressure tests on thin tubes. Inclusions in the metal would be aligned in the form of stringers in the longitudinal direction of both tubes and bars, and the greatest (hoop) stress in a thin tube would be transverse to the longitudinal direction while the stress in a tensile specimen would be parallel to the longitudinal direction. The grain structure in tube wall and bar material could also show differences. The wall of the present tubular specimens was too thin to permit machining of normal tensile specimens and it was stated that sub-size specimens would be too small to yield representative data.

One possibility of investigating the anisotropy of their tubular material, which was not considered by the authors, is to make combined tension and torsion creep tests on thin tubes cut from the original tubular stock. The technique of testing has been fully developed by Johnson et al (see sub-sections 1.5.3 and 4.2.4.) However, it would still not be possible to determine the properties of the material in the radial direction.

The paper provided references to earlier work on tubes carried out by the Babcock & Wilcox Tube Co., in 1933, 1935 and 1943-44 the results of which remain unpublished.

In 1954 Foster reported a series of tubular creep rupture tests carried out at pressure up to 1,200°F. on eleven steels ranging from MS to 9 Cr 1 Mo alloy steel. Tests were continued in some instances to 10,000 hours. The experiments were essentially rough evaluation tests for the determination of safe working stresses on an ad hoc basis.

Specimens were thin walled tubes, generally in the as-rolled condition; but having a 12 inch length machined internally and externally to the required wall thickness. No details of the diametral dimensions were given.

A nest of six specimens, connected in series and located vertically in a single electrically heated furnace, were stressed by steam generated in a small electrically heated boiler. Since the generated steam was static, heated palladium plugs were incorporated at the tops of the specimens to permit the escape of hydrogen generated due to corrosion of the steel. No strain measurements were carried out and the accuracy of maintenance of the testing conditions was not quoted.

Again supporting isotropy tests and data gathering tests on the tubular material were not carried out, thus limiting further the value of results obtained with a relatively unrefined testing technique.

In 1956, Voorhees, Sliepcevich and Freeman presented a paper on the analysis of creep in thick walled pressure vessels which included results of tests on annealed carbon steel at 900 and 1050°F and on Cr-Mo-V steel at 1,100°F. The data was sufficiently complete to permit evaluation of a design method only for the annealed carbon steel at 1050°F. The results at 900°F for the carbon steel and at 1,100°F. for the Cr-Mo-V steel were restricted to tensile tests; they did, however, support conclusions drawn from the tests at 1050°F. to which discussion will be confined here.

Four types of experiment were made, constant load tensile stress rupture tests, variable load tensile multiple stress rupture tests, internal pressure tests on cylinders, and internal pressure tests on cylinders with superimposed axial loading. Details of the history of the material, of the apparatus and of the experimental techniques were not given, and only diameters of test specimens were quoted, so that appraisal of the accuracy attained is not possible.

Of the constant load tensile stress rupture tests at 1050°F. ten miniature 0.160 inch diameter specimens were tested to check initial isotropy. Three were stated to be sampled in the longitudinal direction, three in the tangential direction and four in the radial direction, presumably taken from a tube. These tests were supported by data obtained using larger, 0.350 and 0.505 inch diameter specimens sampled in the longitudinal direction. The stress range was between 5,000 and 13,000 lb/in^2 and the longest test was carried to 800 hours. It was claimed that no large or consistent variation was found with sampling direction and that no effect was noted due to specimen size, however, the plot of log rupture life versus log stress shows differences in rupture life of up to 20 hours in 90 for constant stress. The scatter is reasonable for commercial testing. Results for creep rates quoted later in the paper indicated that some form of extensometer had been used.

The tensile multiple-stress rupture tests were designed to investigate the principle of addibility of rupture life proposed by Robinson in 1952 (see sub-section 1.7.3), namely that the fraction of total life used up at any stress should equal the ratio

$$\frac{\text{actual time spent at a given stress level}}{\text{rupture life at that stress in a conventional constant load test}}$$

This criterion obviously simplifies the true state of affairs since for the time interval under consideration, the effect of the previous history of the specimens is neglected, however, the results of the tests carried out by Voorhees et al gave, on the average, life fractions which when added gave somewhat less than unity. In the case of carbon steel at 1050°F the life fraction total was 87%.

Five tests were made at 1050°F. In one case the stress was raised after an initial period, and in three others it was lowered; the fifth experiment involved first a stress increase and then a decrease. Four of the tests were made on specimens sampled in the tangential direction which is the direction of greatest interest for a thick cylinder, and the fifth was taken in the radial direction.

An important part of the test programme was concerned with the criterion of rupture in complex stress systems. The work of Johnson and Frost (1951) and Johnson, Henderson and Mathur (1956) indicated quite unexpectedly that the criterion for creep rupture might be that of maximum principle stress, which was disturbing in that it upset what had been a fairly tidy theory. Voorhees et al thereupon investigated the criterion independently using so called 'thin walled' cylinders (k = 1.25) under combined internal pressure and axial load. Three tests were made at 1050°F and indicated rather inconclusively that the Maxwell criterion might still be valid. However, the excellence of the experimental technique of Johnson and his co-workers was not matched by Voorhees et al and it cannot be said that assumption of the applicability of the Maxwell criterion is fully justified on the basis of these results. Later work by Johnson et al showed that the Maxwell criterion applied to certain materials. This was discussed in sub-section 1.7.2.

The internal pressure tests on cylinders at 1050°F were made on four tubes with k values of 2.0, one tube with k value of 1.33 and three tubes with k values of 1.25. The rupture lives obtained ranged from 7 to 150 hours and no details of the testing technique were given. As was indicated in section 1.6, the work of Voorhees et al would have been greatly enhanced by the measurement of strains throughout the life of the tube.

Further results to those of Kooistra, Blaser and Tucker on tubular stress rupture have been reported by Tucker, Coulter and Kooistra (1960)

The object of the work was to determine the effect of wall thickness on stress rupture life of three typical superheater materials and to compare data obtained from standard tension-bar specimens from the same heats of material.

The apparatus was similar to that described in 1952, but utilised horizontal furnaces. The specimen design was improved, the thickest specimen being 20 inches overall and 2 inches outside diameter with a free parallel length of approximately 8D, twice the necessary length suggested by Cook and Robertson (1911) and Crossland and Bones (1958). The electric furnace was provided with a 4 inch bore alloy liner and the temperature gradient over the central 10 inches of the specimen was improved from ± 10 F° in the previous work, to ± 4 F° with a variation at any one point of ± 3 F°. Internal pressure was maintained within ± 25 lb/in² by periodic additions of make-up water using an air driven water pump.

For most tests, standard commercial tubing with no machined surfaces was used, however, for tests to study the effect of varying the k value, specimens were prepared machined both internally and externally. Various heat treatments were carried out on all specimens and details are given in the paper.

The materials tested, the tube wall ratios used, and the maximum testing conditions (not attained simultaneously) are given in the table below.

Material	Tubular Ratios k	Maximum Pressure lb/in²	Temperature °F	Maximum Rupture Life hours
Carbon steel	1.2 to 2.0	17,590	850	3,551
		10, 030	950	4.472
Low alloy steel (2¼ Cr. 1 Mo.)	1.25 to 1.85	7,670	1,100	7,928
Austenitic steel (18 Cr. 12 Ni, 1 Ti).	1.2 to 2.0	15,000	1,200	8,972

The diameter of the specimens was measured before pressurising but no extensions were measured during or after testing.

Uniaxial tension stress-rupture tests carried out in support of the programme of tubular testing were made within the range of test conditions noted below.

Material	Maximum Stress lb/in²	Temperature °F	Maximum Rupture Time hours
Carbon steel	28,000	850	12,801
	17,000	950	6,228
Low alloy steel (2¼ Cr. 1 Mo)	17,000	1,100	7,788
Austenitic steel (18 Cr. 12 Ni. 1 Ti).	20,000	1,200	7,999

No measurements of strain were reported, taken during or subsequent to any of the tests.

In the introduction to the paper the authors noted that normally stress rupture data was based on tensile tests using specimens taken from bar material. It was pointed out that such specimens would not account for metallurgical changes in material which might occur during say the fabrication of a tube, and further that the uniaxial stress system in a tensile specimen did not duplicate the complex stress systems found in a tube. The work was therefore to determine a suitable correlation between stress rupture data for tensile specimens and for tubular specimens.

Tucker et al first attempted a correlation between the rupture stress obtained for their tensile specimens and the average hoop stress in their cylindrical specimens which was calculated using the formula for thin-walled tubes. This was unsatisfactory, and to obtain better agreement, they were forced to use a maximum hoop stress calculated on the basis of Lamés equations. The correlation thus obtained is of a purely empirical nature (which the authors realised) since stress rupture is essentially a plastic phenomenon to which Lamés equations do not apply.

The authors have evidently favoured the failure criterion of maximum principal stress first suggested by Johnson et al for creep, as discussed in sub-section 1.7.2, but they do not appear to have confirmed that the maximum principal stress criterion was valid for their materials. However, they did find in nearly every case that failure originated at the outside surface of their tubes, where plastic theory predicts the highest principal stress. The Maxwell criterion gives the greatest equivalent stress at the bore. Their test programme was not sufficiently comprehensive to establish beyond doubt which criteria of failure was applicable.

Davis (1960) carried out a series of creep tests involving rupture under uniaxial tension, pure internal pressure, and combined internal pressure/axial tension on specimens of type 316 stainless steel (nominally 16 Cr 13 Ni 3 Mo). The tests were part of an evaluation programme on pipework for high pressure steam equipment and the test temperature was 1,200°F while internal pressures up to 24,000 lb/in.^{2} were used.

Specimens were cut from a 12 inch length of sleeve forging 9¾ inches outside diameter and $4\frac{5}{8}$ inches inside diameter, reheated and held at 1,950°F for 2 hours before quenching in water. No description as to how specimens were cut from the forging was given but from the dimensions quoted it would appear that all were taken parallel to the longitudinal axis of the forging.

Six uniaxial tension creep tests were made using 0.505 inch diameter specimens with 3 inch gauge lengths at varying stresses up to 28,300 lb/in^2, with durations up to 10,200 hours. No description of the extensometers was given, but conventional Marten's type equipment appeared to have been used, and minimum creep rates as low as 5.2×10^{-6} were quoted. No tests to prove isotropy in the billet appear to have been carried out.

The tubular specimens had a 6 inch cylindrical gauge length with an outside diameter of 1⅛ inch giving an l to D ratio of 5.32 which is satisfactory. The inside diameters were 9/16 inches and ⅜ inches giving k values of 2 and 3 respectively. No details of the end closures were given and the accuracy of machining was not quoted.

Five tubular specimens were tested under pure internal pressure, three with k values of 2 and two with k values of 3. Pressures up to 24,000 lb/in^2 were used and rupture times up to 1,590 hours were recorded. The specimens were mounted vertically in an electrically heated oven constructed to withstand explosive rupture and internal pressure was supplied by an air operated pump. Friction of the pump piston was minimised by providing an intermediate vessel filled with oil between pump and specimen and arranging a calibrated leak which consisted of a length of capillary tubing open to atmosphere from the system. The specimen was connected to the intermediate vessel by small diameter heavy wall tubing and pressurised by water which formed an interface with the oil in the vessel. High pressure water at room temperature in the pumping system became high pressure steam in the specimens. No details of the accuracy with which pressure was maintained was given and the method of adjusting to different pressure levels was not described although it was probably by regulation of the air pressure supply to the pump. No details of the accuracy of temperature control for any of the tests were reported.

The diametral extensions were measured using a lateral extensometer with a magnification of 10, but no details of the construction of the extensometer or of its accuracy were given. No axial strain measurements were made during testing, instead the lengths of the specimens were checked before and after each test. The method of estimating strain at rupture was however described.

Five tubular specimens were tested under combined internal pressure/axial tension, four with a k value of 2 and one with a k value of 3. Internal pressures up to 23,000 lb/in^2 were used and average axial stresses of up to 16,100 lb/in^2, rupture times up to 1,018 hours were recorded, with one test still in progress at 1,828 hours. The tests were carried out in a fabricated portal frame machine, axial loads being applied to the specimens to make the average axial stress equal to the average tangential stress.

Axial strain was continuously indicated using standard Marten's type extensometers attached to the shoulders of the specimens. Diametral strains were obtained only by stopping some of the tests from time to time, removing the specimens and measuring the increase in diameter. The desirability of developing extensometers capable of measuring both axial strain and the increase in diameter was mentioned by the author in his concluding remarks. The classical testing techniques developed for tubes by Norton (1939, 41) appear to have been overlooked.

It is not clear why David chose to test thick tubes under combined internal pressure/axial tension, the value of the results obtained being small due to the uncertainty introduced with radial stress variation across the wall.

An attempt was made to correlate the results on the basis of effective stress and effective strain. Davis used true stress-natural strain relations in the initial portion of his theoretical analysis this being necessary for thick cylinders subjected to large strains, since it ensures that tensile and compressive stress/strain curves coincide. This obviates the otherwise careful consideration which would require to be given to the meaning of each value of strain encountered. Later, however, he was forced to consider conventional strains owing to analytical difficulties. Theoretical work by other authors, notably Rimrott (see sub-section 1.4.1) indicates that the problem may soon be overcome completely. The analysis by Davis considered only secondary creep.

This experimental work is interesting and provides increasing confidence in the use of empirical methods of design and in the use of materials at extreme conditions, but the testing techniques have not been sufficiently controlled and the specimens have not been sufficiently extensometered to allow valid comparisons of existing theories of creep on the basis of the results.

Duval (1960) has given the first progress report on experiments being carried out by the British and Allied Industries Electrical Research Association on the creep rupture of thick tubes.

Preliminary results are available on two materials, namely 2¼ Cr. 1 Mo steel and 18/12/1 Nb stainless steel both of which were tested in a heat treated condition. In all cases material was obtained in the form of tube. This decision was taken so that the results would be representative of actual conditions prevailing in tubular products subject to creep under internal pressure in service. The same criticism of Norton's work, that of Kooistra et al, Tucker et al and Davis applies here, namely that results obtained from initially anisotropic material are only satisfactory for the derivation of empirical formulae at this instant in time.

Tubular specimens were machined inside and outside before welding on end caps. The outside diameter of all tubes tested was $2\frac{7}{16}$ inches, the k value for $2\frac{1}{4}$ Cr 1 Mo material being 1.63 and for 18/8/1 Nb material being 1.51. Tests on $2\frac{1}{4}$ Cr 1 Mo were carried out at temperatures up to 1,150°F at pressures up to 11,200 lb/in.^{2} and tests have been carried to 7,600 hours. The apparatus was similar to that of Kooistra et al, and a level temperature zone of 10 inch length was maintained in the furnace.

No diametral strain measurements were made during testing but an attempt was made to estimate strain at rupture by measurements taken on completion of the test. The lack of diametral strain measurements is particularly glaring when the technique due to Norton is available.

To determine the directional properties of the tube material sections of the original tube were flattened and re-heat treated. Results of tensile creep tests on specimens from the original longitudinal and circumferential directions of the flattened tube sections were compared with the results of tensile creep tests on specimens taken from the longitudinal direction of non-flattened tube material and some discrepancy was found, although insufficient results are at present available to determine the effect of possible scatter. This technique is similar to the tests on lead by Moore and Alleman and by Moore, Betty and Dollins, but includes the necessary heat treatment.

For $2\frac{1}{4}$ Cr. 1 Mo material, tensile rupture tests were made at stresses up to 24,000 lb/in^2 at temperatures up to 1150°F. for durations up to 6,200 hours and continuing.

A fuller criticism of the results will be possible when the final report is published.

CHAPTER 4

4. DESIGN OF APPARATUS

4.1 THE CHOICE OF A TEST MATERIAL

4.1.1 General Remarks

The present work was limited in scope to the investigation of axial creep in tubes, i.e., to the investigation of the behaviour of a geometry subjected to creep, and not to the determination of the behaviour of a particular material under creep conditions. The material required was therefore a 'simple' metal, as discussed in section 2.3, which would be selected to eliminate the possible occurrence of undesirable metallurgical effects - e.g., recrystallisation during testing.

The initial experience gained with such materials would be valuable on extending the present test programme to include the evaluation of stresses in the cylinder walls, since the creep behaviour of a 'simple' metal is easier to represent analytically than that of a more complex creep resisting commercial alloy.

Variables affecting the test material which must be taken into consideration are

(a) the method of manufacture,

(b) heat treatment,

(c) grain size,

(d) effect of alloying constituents,

(e) prior strain

(f) structural changes during testing.

Isotropy is the desired characteristic and maximum creep resistance is not required.

Of the three possible types of 'simple' metal, namely a pure metal, a simple solid solution, and a metal with an initial finely distributed precipitate, the pure metal was not considered because of the difficulty of obtaining it sufficiently pure to make the effect of trace elements negligible. With a metal containing a precipitate there is the possibility that the precipitate would be modified by the action of temperature, and therefore attention was confined to simple solid solutions stable at and above the test temperatures envisaged. The advantage of a solid solution is that the addition of a single alloying element assists in masking the effect on creep of minor impurities present in the nominally 'pure' major constituent.

The specimens used could not be very large if isotropy was to be assured, and the experimental difficulties associated with accurate diametral strain measurements required the selection of testing temperatures which would not be greatly above room temperature. Rough sorting of the possible elements on the basis of testing at 0.5 of their absolute melting temperature indicated that lead, magnesium and perhaps aluminium were possibilities.

Previous work on the creep of lead tubes under internal pressure had been reported by Bailey (1930), Nakahara (1939), Moore, Dollins and Craig (1940), Phelps, Gates and Kahn (1940) and Latin (1948), and a considerable literature exists on the creep of lead and lead alloys. No papers describing internal pressure creep tests on magnesium or aluminium tubes were found by the writer although tensile and complex stress creep tests have been reported by Dorn et al and by Johnson et al.

In view of the fact that this was an entirely new project in the laboratories lead was selected for initial tests.

4.1.2 Lead

Sully (1956), Moore et al (1938), Hopkins and Thwaites (1953) have stated that lead or lead base materials at temperatures as low as 32°C appear (superficially) to behave in the same general way as steel at high temperatures. This is a generalisation which would have to be examined for each alloy in the light of Glen's work on structure, discussed in section 2.3, but it may be valid for 'simple' metals.

Small amounts of alloying elements in lead can cause considerable variation in creep properties. Over alloying results in an unstable super-saturated solution, and may cause precipitation and segregation at the grain boundaries. Elements either completely or negligibly soluble at all temperatures however, have little effect on 'precipitation hardening' properties. It was therefore decided to make sufficient alloy addition to form a simple solid solution in order to refine the grain and retard recrystallisation (providing phase changes were avoided), and to mask the effects of minor impurities. A lead 1% tin alloy was selected since this was stable at the test temperatures envisaged (Metals Reference Book, Smithells, Butterworths 1955) and because experiments by Moore, Betty and Dollins (1935, 38) indicated that this material gave a straight line secondary creep curve. The test temperature selected was 0.51 Tmelt.

Machining the bore of a thick lead tube was considered impracticable and it was decided to extrude a suitable tube whose bore would be finished inside diameter of the specimen. Extrusion tends to produce a striated structure giving grains elongated in the direction of extrusion, and to overcome this, the extrusion temperature was made high enough to prevent precipitation at the grain boundaries, and to promote recrystallisation of the alloy, so that the striated structure would no longer be present when the billet had cooled to room temperature.

The extrusion process was planned to be continuous since otherwise the heat from the press might have caused grain growth in the partly extruded portion during pauses. In addition subsequent forcing of over-cooled metal through the die-block would probably have given a small grain structure in the metal under the press stop mark.

The billets were cut into short convenient lengths and stored vertically in a cool room in the 'as received' condition for periods of the order of eight weeks before machining.

Bailey (1929) steamed his lead tubes at 100°C for eight hours before testing. It was felt however, that with this treatment the weight of the present specimens might have caused distortion of the bore. No heat treatment was therefore given to the alloy before testing other than soaking the specimens at the test temperature for about two hours before applying the loads.

4.1.3 Magnesium

Experience with lead showed that machining was a long and delicate operation in the course of which the slightest inattention, even during so called 'roughing' cuts, was sufficient to necessitate scrapping the specimen. In addition, the storage of a material which could deform under its own weight at room temperature introduced possible additional variables. When, in the course of the work, it was found that one of the two batches of extruded lead contained a flaw, it was decided to go to an alternative material and repeat the work using the same experimental equipment as far as possible.

Much work of a fundamental nature has been carried out on aluminium and its simple alloys by Dorn (1954), Sherby and Dorn (1952, 1953, 1954), Sherby Orr and Dorn (1954) and Sherby, Frenkel, Nadeau and Dorn (1954). This abundance of test data has demonstrated that the behaviour of aluminium is unambiguous and markes the material attractive for tests on the behaviour of the complex stress systems in general.

Johnson at the N.E.L., East Kilbride, has been following an experimental programme investigating stress/creep-rate relationships over a number of years. The work which is still in progress, has been concentrated on several materials which showed reasonable isotropy including a magnesium (2% aluminium) alloy, and an aluminium (RR59) alloy. The work has been published extensively, e.g., Johnson (1950, 1951), Johnson and Frost (1952), Johnson, Frost and Henderson (1955), Johnson, Henderson and Mathur (1958). The main tests on magnesium had been carried out at 20°C and 50°C isotropy having been checked at 150°C. Aluminium was investigated at 150°C and 200°C and isotropy tests made at 200°C. In the present work, the existing equipment was not capable of reaching a temperature greater than 140°C and therefore the magnesium (2% aluminium) alloy was selected.

Since the results obtained by Johnson et al were so satisfactory the magnesium alloy was manufactured, and the billets given the same heat treatment as Johnson specified for his materials. Details are given in sub-section 5.5.1.

4.2 TESTS TO DETERMINE CREEP PROPERTIES

4.2.1 General remarks

Before discussing in section 4.3. the choice of test used in the present work to determine isotropy, it is worth considering briefly the somewhat wider aspect of the data necessary for a fundamental approach to the design of a thick tube expanding under creep conditions.

In general, creep data is required for slowly changing complex stress systems. Tests providing time, creep strain and stress relationship under such conditions have been devised and have been carried out for small strains in combined tension and torsion on thin walled tubes, but with stepped increments in tensile load, by Johnson Henderson and Mathur (1958). For practical purposes, however, it is almost certain that such tests will not find favour and that resort will be made to the concept of a 'creep surface' discussed in sub-section 1.5.1.

The great bulk of creep data at present available has been obtained using creep strain or stress rupture tensile tests, and this practice will undoubtedly continue. These tests however, have certain disadvantages and refinement in experimental technique, and alternative tests are possible. Papers describing the development of the most useful short time testing techniques for use under creep conditions are indicated briefly.

4.2.2 <u>Tensile Test</u>

Studies of the tensile test have been carried out by MacGregor (1940, 1944) and MacGregor and Fisher (1945). The effect of varying the strain rate was investigated by Manjoine, Wessel and Pryle (1957) and the necking and rupture of specimens has received the attention of Hoff (1953) and Parker, David and Flenigan (1946).

Under time dependent conditions the tensile creep test is usually carried out at constant load. This is of little consequence if the strains are small since the reduction in cross-sectional area due to extension does not affect the applied stress to any measurable extent. The variation of stress may be conveniently graphed by plotting the 'time/total length' relationship for the test specimen, the ordinate of the <u>curve</u> representing the instantaneous applied stress at any instant when multiplied by the factor

$$\frac{\text{applied load}}{\text{volume of gauge length}} .$$

Tapsell and Prosser (1934) have described high sensitivity creep testing equipment at the N.P.L. and many of the available texts on creep e.g., Betteridge (1959), Clark (1953), Finnie and Heller (1959), Rotherham (1951), Smith (1950), Stanford (1949), Sully (1949), Tapsell (1931) include descriptions of equipment and provide sources of references.

For large strains several workers have attempted to maintain <u>constant stress</u> conditions during testing. Andrade (1910, 1914, 1948), Andrade and Chalmers (1932), Pearson (1934), Mellor and Jenkinson (1940), Ward and Marriott (1948), Fisher and Carrker (1949), Hopkin (1950) and Latin (1952) have all suggested different devices to maintain constant stress in the gauge length. All these devices require that no deformation takes place elsewhere in the system and require a relatively long gauge length specimen. Kinsey (1951) has attempted an evaluation of the equivalent gauge length of the fillet and shoulders of a tensile test specimen for creep conditions, but the theory is restricted to secondary creep.

Cook and Robertson (1911) and Morrison (1934, 1940) have examined the problem of axiality of loading in the tensile test, but the specimen they used could not completely ensure axiability of loading and it was concluded that the specimen screwed ends were the major cause of the misalignment.

Jones and Brown (1956) have made a further careful investigation of non-axiability in tensile tests recommending the elimination of screwed ends and the use of split adaptors and button headed specimens.

Concerning reproducibility of results, tensile tests by seven different laboratories on the same steel were reported in the Proceedings of the A.S.T.M. (1938) and the results showed considerable scattering. Bardgett and Clark (1954) have compared the results of tensile creep tests involving an exchange of specimens between a British and American laboratory and found an improved measure of agreement. Johnson and Frost, in a contribution to the last paper, stressed the advantages of the torsion test and suggested that the uniformity in results would have been more marked if pure torsion had been used.

4.2.3 Torsion Test

For predicting the strength of thick walled tubes subjected to internal pressure, Manning (1945) suggested that the relation between maximum shear stress and maximum shear strain in a thick walled cylinder was the same as that for a torsion test on a solid cylindrical specimen. (It is implied that the speed of testing must be slow enough to make the stress/strain curve unique). The theory has been confirmed by extensive and careful testing by Crossland and Bones (1955, 1958) on thick tubes and on torsion and tensile specimens.

Torsion tests on solid specimens are not subject to the instability inherent in the tension test (necking down) and may be continued to much greater strains. The analysis of torsion data, however, requires that the shear stress/strain diagram be drived from the moment/twist diagram by a numerical treatment due to Nadai (1931, 1950). The technique of torsion testing has been developed by Morrison (1940, 1948) and Swift (1947) and torsion tests under hydrostatic pressure have been carried out by Crossland (1954) and Crossland and Dearden (1958).

Jamieson (1933) has made tension and torsion creep tests on lead at 25°C and Everett and Clark (1939) have carried out torsion tests on thin tubes and tension tests on solid specimens for a 0.5% Mo steel. On the basis of modified St. Venant theory only approximate agreement was found by Jamieson, but the results of Everett and Clark showed better correlation. Professor Lea has reported torsion creep tests on solid and tubular specimens in a contribution to Bailey's (1935) paper and points out that the torsion test gives more consistent results since changes in temperature make little difference to stress and have very little effect on strain readings. These conclusions are confirmed by the results of Tapsell and Johnson (1940) who found that creep readings in torsion were less affected by temperature variations than those in other tests.

Latterly, Johnson (1950) has brought the technique of torsion testing of thin tubular specimens to a very high standard by designing a sensitive torsion creep unit capable of detecting rates of strain down to 10^{-9} per hour, although care is required to avoid buckling of the thin walls.

4.2.4 Combined Stress Test

Taylor and Quinney (1931) in their classical paper on the plastic distortion of metals described their use of the combined tension/torsion test on a thin walled tube, to investigate the relationship between distortion and stress distribution. Morrison (1940) has used the combined tension/torsion test on solid specimens.

The earliest work on creep under combined stress conditions appears to be that of Bailey (1929, 1935) who subjected thin walled tubular specimens of steel to simple tension combined with simple torsion and lead specimens to combined internal pressure and axial load. Tapsell and Johnson (1940) have pointed out that the general problem of creep of complex stress systems, is most simply studied by tests on thin walled tubes of the above two types, and describe their experiments using combined tension and torsion machines. This work has been developed considerably by Johnson and in 1954 in association with Frost, he further reported the design of a combined tension and torsion machine for relaxation tests.

4.3 ISOTROPY TESTS

4.3.1 Choice of Test

The limited objective of the present work was to test experimentally the important simplifying theoretical assumption that there is no axial creep in a thick tube with closed ends, subjected to internal pressure.

Tests to determine the creep properties of the material used were therefore not required and the test programme was limited to proving isotropy under creep conditions for the material from which the tubes were to be made.

The demonstration of isotropy requires the testing of specimens taken from three mutually perpendicular directions in the material. The kind of test used should not affect the result. If each specimen is tested under identical conditions the same result should be obtained for each direction of the material providing isotropy exists. The type of test to be used is discussed with this in mind. If, however, the creep properties of the material are required, or anisotropy is present and knowledge of the extent to which it exists is required, then a different appraisal of the test to be used would be necessary.

4.3.2 Present Equipment

The design of the present equipment for testing isotropy was influenced by the initial choice of lead as a test material.

A horizontal torsion test was not considered practical because of possible bending of the specimens. A vertical arrangement reduced the possibility of bending but due to the low mechanical strength of the lead the weight of the lower end grips would have applied an additional tensile load to the specimen.

Crossland (1954) has described torsion grips which will transmit torque only to the specimens. For vertical specimens under pure torsion, the problem of supporting the lower coupling (and, if necessary, half the weight of the specimen) remains. Taylor and Quinney (1931) and Tapsell and Johnson (1940) have designed equipment for combined tension torsion tests on thin tubes which avoid the difficulty by deliberately introducing tensile loads. Taylor and Quinney also solved the problem of gripping thin lead tubes.

Lead, however, does not lend itself to accurate machining of thin tubes, and although the torsion test appears to show less variation in reproducibility, the problem of gripping lead specimens satisfactorily seems difficult. Thus it was decided to use simple tension with dead loading.

Johnson (1949, 1950, 1951) has himself used solid tensile creep specimens to demonstrate isotropy, although the main bulk of his testing to determine creep properties was carried out in combined tension and torsion. Hill (1950) and Hoffman and Sachs (1953) also describe the use of the tensile test for proving isotropy.

The apparatus used in the present tests is shown in Fig. 9.6 and the dimensions of the small specimens and their original positions in the billet, shown in Figs. 9.7 and 9.8.

4.3.3 Axiality of Loading

The design of the specimen and grips followed the suggestions of Jones and Brown (1956) in that split adaptors, held together by an external ring were used, but conical ends were provided for the specimens in place of button heads because of the relative softness of the lead.

Split adaptors were manufactured by sweating together prepared flat faces on two pieces of metal before machining internal cones and outside diameters. The adaptors were split by gentle heating after machining.

To secure axiality of loading, accurate machining of the component connecting the end grips to the axial loading device, and of the axial loading device itself, is required. The possibilities of conventional axial loading devices, e.g., crossed knife edges and ball loading were examined, but since the designs were relatively massive and required an appreciable amount of machining, consideration was given to suspension by a flexible steel wire approximately 1/16 inch diameter and with a free length of approximately 15 inches.

For a small angular misalignment from the vertical at the capstan of the tensile testing apparatus, the initial load required to ensure that the cable is vertical at the top specimen adaptor is given by

$$W = \frac{2\ EI}{\ell^2}$$

When $I = \frac{\pi D^4}{64}$ for a solid cylinder, and when the free length of the cable is $\ell = 15$ inches with a diameter $D = 1/16$ inch, and $E = 30 \times 10^6$ lb/in.^{2} the value of W is <u>0.2 lb</u>. which ensures that axial loading is obtained almost immediately and at very low stresses in the specimen. To minimise misalignment, the top plate of the tensile testing assembly was levelled before each test. Slack in the wire was taken up by the capstan at the top of the arrangement before support was taken from below the dead weight.

4.4 TUBULAR TESTS

4.4.1 Specimens

To enable continuous bore measurements to be made, two specimens identical in all respects except in length, were required for each test, as described briefly in the introduction. These specimens were placed one above the other in the oven, to be subjected to the same temperature conditions, and connected to the same hydraulic accumulator, to withstand the same pressure.

Cook and Robertson (1911) and Crossland and Bones (1958) demonstrated that the ultimate pressure of a thick cylinder under internal pressure was independent of the length of the specimen providing the gauge length/outside diameter ratio was 4/1 or greater. The gauge length of the shorter specimen was therefore made 4D and that of the larger specimen 4D plus two inches.

Results of measurements (made after the tests at room temperature) of the diameter extensions of points along the length of the tubular specimens justified these selections of length, the end effects not extending further than a distance approximately D from the ends of the gauge length.

4.4.2 End Closures

Previous workers, Bailey (1929), Moore, Dollins, Craig (1940) Phelps, Gates, Hahn (1940), Latin (1948) have used a variety of end closures for thick walled lead tubes.

'Plumbed' joints require the application of heat which would affect the creep properties of the material and probably distort the gauge length of the relatively short specimen; a mechanical joint was therefore required. Screwed ends were to be avoided because of the low strength of lead and the closure finally adopted was a development of the type of end grips used for tests in combined tensions and torsion of thin walled lead tubes by Taylor and Quinney (1931).

The machining of specimens was much simplified by the use of a symmetrical profile which also reduced the possibility of localised regions of very high stress.

The form of the seal used is shown in the right hand diagram of Fig. 9.9. A diametrally split cone ring grips on the tapered outside diameter of the tube end and compresses the tube on to a core spigot, initially a close fit in the bore of the tube. The annular knife edge assists in ensuring a tight fit as the joint is drawn up. It was thought that the lead would creep after initial cold tightening, relax, and allow leakage when the pressure was applied. This, however, was not found to be the case even if the test was delayed for some hours after initial tightening.

Later work on magnesium alloy tubes necessitated modification of the closure since larger bore tubes were required and the knife edge dis not appear to 'bite' into the magnesium so well. The design adopted is shown as the left hand diagram of Fig. 9.9. At the end of all tests it was extremely difficult to force the 'spigot' portion out of the bore of the specimen due to the large compression effect produced by the tapered end.

In the joint used for magnesium it may appear that the rubber 'O' ring would be extruded immediately on the application of hydraulic pressure consequent on the spigot of the end closure being forced out. This however, did not occur, the differential expansion between magnesium and steel, when raised from room temperature to 130°C, being sufficient to ensure a tight joint face. In the design used the stresses set up in the end closure bolts were higher than in the end closures used for lead because of the increased diameter over which pressure was sealed. In addition the forces set up by differential expansion were considerable and before the application of pressure were sufficient on the first test to rupture five out of the total of twelve ¼ inch diameter high tensile cap screws fitted to the four end closures. An estimate of the thermal stresses involved indicated that six cap screws were required for each end closure to withstand the load. No further difficulty was experienced after this modification.

Since it was evident from tests carried out that the tube was forced into close contact with the spigot by the tapered end closure an alternative pressure seal could be produced by sitting the 'O' ring inside the bore of the tube, in a groove machined in the end closure spigot. This might provide more accurate differential manometer readings by allowing less possibility for error in allowing for end effects, and would reduce the load to be carried by the cap screws.

4.4.3 Alignment and End Loading

Specimens of low mechanical strength were used and a vertical oven was adopted to avoid bending the tubes under test.

The spherical seating arrangement of the top closure, and the provision of a levelling plate with three screws bearing on the top closure, permitted vertical alignment of specimens using a Vee block on the specimen gauge length to which was attached a horizontal spirit level. This device was moved round each specimen in turn and the levelling screws adjusted until the same reading was obtained at 120° stations, thus eliminating possible errors in the spirit level.

The massive end closures and the weight of the tubes themselves place axial loads on the specimens in addition to those produced by internal pressure. A lever system was provided to counterbalance the weight of the bottom closures plus half the weight of the specimen.

Only the middle section of each tube is 'mathematically' correct stress-wise, there being increasing axial tension at sections above and increasing axial compression at sections below the central portion, due to the weight per unit length of the specimen itself.

Diametral extensions were measured at the middle section of the gauge length of the long specimens and axial extensions were taken over the central two inches due to the weight of the lead specimens was + 0.40 lb/in.^{2} (tensile) at the top, to - 0.40 lb/in.^{2} (compressive) at the bottom.

An unbalanced end closure would contribute 3.07 lb/in.^{2} (tensile) at the middle section. The initial axial stress due to internal pressure was 187.6 lb/in.^{2} (tensile).

When preparing an experiment, the two tubular specimens were first accurately sized - externally while mounted horizontally between centres on an CMT workshop travelling microscope (capable of measuring to 0.00001 inch) and internally by Micro-Maags (capable of measurement to 0.00005 inch).

Externally, the gauge length of each tube was measured across two diameters at right angles at stations spaced at one inch intervals along the gauge length. 14 readings for the long tube and 10 readings for the short tube respectively were averaged.

Internally, each tube was measured at three diameters at each of four stations spaced along the gauge length and the readings averaged. The difference in gauge lengths of the specimens was determined on the CMT travelling microscope.

The weight of each specimen and the weight of each complete end closure assembly was determined to the nearest 0.1 gm., (although as will be shown later, this accuracy was not necessary), and the required end load calculated.

Typical approximate weights are:-

Long tubular specimen	-	Lead	1900 gms.
		Magnesium	200 gms.
Short tubular specimen	-	Lead	1570 gms.
		Magnesium	170 gms.
Bottom end closure assembly			1400 gms.

The bottom end closure assembly was fitted to each specimen and the specimen filled with oil before the top closure was placed in position.

The specimen assemblies were mounted vertically one above the other in the constant temperature jacket as shown in the diagram, and the pressure supply oil pipes flushed through by means of the oil injector so that no air bubbles were trapped before the coupling was made.

Each specimen was aligned vertically and the counterbalance systems placed in position. The balance levers were to be as sensitive as practicable and the general features were based on the design of a simple laboratory beam balance (Handbook of Experimental Physics).

Each lever beam was provided with a spirit level mounted above the central fulcrum, to determine when the beam was in balance, and the fulcrum support block was adjustable for height.

To ensure that the correct end load was applied to each specimen, each lever beam was carefully balanced in turn. The fulcrum support block was lowered as far as possible to ensure that the yoke, and the ball which applied the load to the bottom end closure of the specimen, would not bear on the bottom end closure when the lever beam was horizontal. A weight pan and masses equivalent to the weight of the bottom end closure plus half the weight of the specimen, was suspended from the yoke and allowed to hang freely. Weights were added to the outer counterbalance mass until the lever beam was in horizontal equilibrium. The inner pan was then removed and the fulcrum block raised until the beam was again horizontal with the steel ball bearing on the bottom end closure of the specimen.

The sensitivity of the lever system was dependent on friction in the ball seatings and a change in mass of 5 gm. on the weight pan was required to overcome this friction. The nearest estimate of the correct load was found by averaging the weights required just to swing the beam first one way and then the other.

The weight of half the longest lead specimen plus the end closure was approximately 2350 gms. and the maximum error in end load was therefore less than 5 gms. or 0.21%.

It has been shown previously that for a lead specimen an unbalanced end closure would add 3.07 lb/in.^{2} to an initial axial stress of 187.6 lb/in.^{2} due to internal pressure.

The maximum error in axial stress during any test was therefore

$$\frac{3.07 \times 0.21 \times 100}{187.6} = 0.0034\%$$

4.5 TEMPERATURE CONTROL SYSTEM

4.5.1 General remarks

The experimental work was to be carried out at relatively low temperatures so that the conventional types of extensometer could be used.

At high temperatures the electric resistance furnace is common, having a variety of possible compensating winding systems and alternative types of temperature sensing and control equipment. The available texts on creep - Sully (1949), Smith (1950), Stanford (1949), Tapsell (1931) and Clark (1953) give references which describe the principles of operation of the more popular types of control. Lomas, Jepson and Rait (1951) have given details of more sophisticated electronic proportional controllers as has Roberts (1951).

At lower temperatures, direct circulation of externally heated air round the specimen has been adopted by Curran and Morehead (1936).

For the present work near room temperature, it was decided to jacket the specimens in an air space and circulate oil or water through the jacket at high speed, the air inside the jacket being agitated by a small fan placed in the bottom end closure.

Immersion heaters were used in the liquid reservoir where rapid mixing occurred. High-low control by toluene, or all mercury, regulator and electronic relay was considered satisfactory because of the large thermal capacity of the liquid bulk.

4.5.2 Circulating Fluid

For temperatures above 100°F. Shell Voluta Oil 27 was used. This oil has a higher flash point than technical white oil and is less viscous than B.P. liquid paraffin oil. At lower temperatures water was used because of the unacceptably high viscosity of oil.

Data was supplied by Scottish Oils and Shell Mex and comparative figures are given in table 11.1.

4.5.3 Constant Temperature Tank

The tank used was of all welded construction, 20 inches long, 15 inches wide, 12 inches deep, with a close fitting lid, and carried five immersion heaters. Ten separate elements were incorporated, 1 x 1,000 watts, 3 x 500 watts and 6 x 125 watts, and any heater element could be used on boost heat, load or control circuits.

A toluene-mercury thermo-regulator was fitted for the lower temperature tests at 30°C and an all mercury design for the higher range at 125°C (B.Pt. toluene 110.5°C). The thermo-regulator was suspended in the tank liquid independently of the tank since it was found that vibration from the pumps, transmitted through the tank structure, caused the mercury contacts to bounce. A Sunvic electronic relay type EA3T with a toluene regulator proportioning head was used for high-low control. The circuit diagram is given in Fig. 9.11.

The two circulating pumps took the liquid from one end of the tank and returned it to a diagonally opposite point; both pumps circulated liquid through the test jacket. No pump failures were experienced during tests.

4.5.4 Large Constant Temperature Jacket

This is shown in Figs. 9.2 and 9.3. Three concentric copper tubes 8 inches, 7 inches and 6 inches internal diameter formed the walls of an annular U-bend type of heat exchanger 36 inches long. Hot liquid entered the outer annulus at three spray points set at

120° in the base of the jacket, travelled to the top with a swirling motion, spilled over into the inner annulus and returned to the base where it was led away via three similar spray points. An air bleed was provided at the top of the jacket for filling purposes.

Four openings, plugged with cork stoppers, were provided through the jacket wall for extensometer mirrors, and two retractable fork drives incorporated for adjusting the diametral extensometer setting during testing.

The jacket travelled vertically on three half inch diameter guides, and was counterbalanced by three weights each sliding on a leg of the stand. A neoprene seal was provided at the top which butted on to the underside of the half inch plate carrying the specimen counterbalance fulcrums.

The bottom end of the jacket was kept permanently sealed with 3 inches of insulating fibre in which a small variable speed electric fan was embedded. The fan ensured that the temperature of the specimen was not affected by the temporary removal of the cork from a mirror port while taking a reading.

The top jacket seal was permanently fitted to the stand and consisted of 3 inches of insulating fibre through which passed six counterbalance rods, two pressure pipes, and three specimen support rods. It was found necessary to install a small electrothermal heating tape inside the oven at this point to compensate for conduction losses through this closure.

4.5.5 Small Constant Temperature Jacket

This is illustrated in Fig. 9.6.

Two concentric tubes 4 inches and 2 inchesin diameter with silver soldered copper end plates form an annulus through which hot liquid is circulated rapidly. An elongated port 2 inches long is provided in the wall of the jacket. The ends of the jacket are closed by tight fitting cork stoppers. The jacket is held between two ¼ inch thick Sindanyo plates and supported by these on collars fitted on the stand pillars. A satisfactory temperature distribution was obtained with this oven and no compensating devices were found necessary.

4.5.6 Temperature Measurement

Since the testing temperatures were low (30 - 125°C) base metal thermocouples were used on the specimens throughout, to give maximum sensitivity. Iron-constantan provided approximately 56 μV per C° and appeared compatible with all envisaged contact materials (Ref. ASM Metals Handbook, 1948).

Nine couples with silver soldered hot junctions were made up and compared with each other, firstly in groups of three, by being placed close together and in contact with the interior of an aluminium block immersed in a hot fluid, as indicated in Fig. 9.12. One couple from each group was then taken to form a fourth group and these also compared. An oil bath was used for the higher temperature range. Standard thermometers were used to measure the temperature of the block and emergent stem corrections were made (Reilly and Rae, 1954).

The cold junction was at the temperature of melting ice and contacts were made in mercury contained in a narrow bore tube closed at one end. Copper leads were taken to a Cambridge vernier potentiometer with matching galvanometer which could detect 1×10^{-6} volts with ease.

The accurate calibration of the thermocouples, one against another was considered of more importance than a knowledge of the exact test temperature since adjustments to give isothermal conditions over gauge lengths on two relatively widely spaced tubular specimens were required. Table 11.2 shows the calibration obtained for the lower temperature range.

For the tests with the large oven, one thermocouple was placed neat the auxiliary heater at the top of the large jacket to assist with control adjustments. Four couples were equally spaced round the smaller tubular specimen and along its gauge length, the junctions falling on an imaginary helix. Four couples were also attached to the larger tubular specimens, but were restricted to being placed down two diametrically opposite imaginary axial lines because of the extensometers. These couples were also staggered so that the temperature was again obtained at four equally spaced cross sections along the gauge length.

The thermocouples were attached after completion of counter-balancing of the end loads on the tubular specimens, being bound on with find twine and broad elastic bands which ensured that the thermocouple bead maintained close contact with the specimen surface during expansion without resort to peening or welding which would have damaged the finished surface. The optical extensometers could then be mounted and set.

An analysis of the results of the thermocouple readings for the test carried out on the tubular specimens obtained from billet 1A is given in Table 11.3. The best estimate of the standard deviation for the test using Bessels correction for small samples is $\pm$ 6.4 x 10^{-6} volts and this is equivalent to approximately $\pm$ 0.114°C. It will be noted that the results for thermocouple B4 have been discounted. This thermocouple read consistently low and it was subsequently discovered that it had not been in contact with the specimen.

For the small oven, thermocouple trials on the tensile specimen showed that thermocouples placed on the gauge length gave almost identical readings to thermocouples attached to the upper and lower end grips of the specimen.

The large strains developed during testing caused the gauge length thermocouples to lose close contact with the specimens and significantly lower temperatures were indicated later in the test. Consequently, it was decided to average the recorded temperatures of the top and bottom grips and take this as the temperature of the specimens. If specimen temperatures are slightly different from this mean temperature, they should be consistently so for all tests, which should not affect the validity of the isotropy results.

4.6 PRESSURE CONTROL SYSTEM

4.6.1 General Remarks

A primary requirement of creep tests by internal pressure on tubular specimens is the maintenance of a constant pressure throughout the test, regardless of expansion of the specimen.

White and Clark (1926) used a simple accumulator with leather gasket seals which fed a pressure intensifier also with leather gasket seals. Norton (1939) carried out long time tests pressurising by nitrogen and adjusting to the correct value every morning and every evening. Kooistra, Blaser and Tucker (1952) used an ingenious system of pressurisation by water so that steam was generated inside the hot specimen. Any increase in volume of the specimens caused additional water to be delivered to the specimen and this water immediately flashed to steam. Pressure corrections were made once per week.

One of the most precise devices for pressure measurement is the dead weight piston gauge, and for the present work it was decided to construct a sensitive hydraulic accumulator (i.e., a large dead weight piston gauge) as it appeared capable of providing the necessary accuracy, with reliability over a long period of continuous operation.

A considerable literature exists on the dead weight piston gauge and recent papers of interest which also include references to earlier work are those by Singh (1958), Johnson and Newhall (1953), Bett, Hayes and Newitt (1945-55), Dadson (1957), Reamer and Sage (1955), Newhall (1957), anon (1956), and Pearce (1952).

4.6.2 Accumulator Gland

Since the accumulator was to operate at different pressures and the amount of oil leakage past the piston was of importance (as it determined the length of test possible), consideration was given to two types of controlled clearance gland.

Johnson and Newhall (1953) and Newhall (1957) have described the continuously variable type in which the clearance at any pressure is adjustable. While this type is probably the most satisfactory for minimum leakage at all pressures, it requires an additional source of pressure which would also have to be kept fairly constant during a long test.

Crossland (1954), Morrison and Crossland (1956), and Morrison et al (1960) have used the Morrison gland, which is designed for constant clearance at all pressures and does not require an external pressure source for its operation; while not so versatile as the continuously variable type it is superior to the simple clearance gland is very suitable for apparatus required to operate over long periods without attention. The Morrison gland was selected for use in the apparatus. A simplified design procedure for this gland has been given by the writer in a contribution to the paper by Crossland and Dearden (1958).

4.6.3 Cylinder and Piston

The cylinder and piston are both ground and honed to fine limits to minimise oil leakage past the piston. The cylinder was manufactured first and sized on an S.I.P. comparator in the Inspection Department of the local firm of Barr and Stroud and the piston (1 inch in diameter and 15 inches long) matched to the cylinder using a Magna Gauge electronic comparator and slip blocks capable of measurements to 0.00001 inch.

Table 11.4 shows the dimensions of the piston and cylinder pair used in the tests. Previously a piston and cylinder pair with an average radial clearance of the order of 0.00004 inch had been tried, but was found to be subject to sticking at certain positions along the length of the piston unless the oil film was maintained by continuous rotation. Initially an attempt was made to increase the clearance by reducing the depth of the gland packing in the Morrison seal, but it was found that this tended to increase the stickiness. When the packing was increased to its initial thickness again, the friction was correspondingly reduced and it was concluded that the phenomenon might be partly due to hydraulic lock shown by Manhajm and Sweeney (1955) to be caused by a divergent clearance in the piston and cylinder.

In order to reduce friction it was decided to increase the clearance as far as practicable by honing the piston, the determining factor being the amount of leakage permissible. Tests were carried out on the original piston and cylinder pair and the required clearance estimated from the results. The effective length of the piston is 12 inches, 4 inches being required for the expansion of the specimens and 8 inches being allowed for leakage.

The radial clearance finally adopted was about 0.00014 inches and this gave a leakage rate with castor oil such that the continuously rotating piston would require approximately 1,750 hours to drop the 8 inches allowed.

4.6.4 Arrangement of Deadweight.

For maximum sensitivity it was decided to rotate the piston continuously during each test.

Two possible arrangements were considered with the weights slung directly below the accumulator cylinder. The first utilised a stepped piston with a gland at the top and a gland at the bottom of the cylinder to permit the rotation of the weights but this design is subject to alignment errors and would introduce twice the leakage into the system. The second arrangement used a conventional piston and cylinder, with a yoke for the dead weights but required that the piston be rotated independently and the weights held stationary. This has been achieved by Johnson and Newhall (1953) by providing continuous forced lubrication to the axial loading ball at the top of the piston.

The above arrangements did not appear to satisfy the requirements of continuous operation, and the layout finally adopted was to suspend weights from a spider, above the cylinder support. This is shown in Fig. 9.4. The design is not so convenient for large load adjustments but has the merit of simplicity, the weights rotating with the piston. The spider sits on a $\frac{3}{8}$ inch diameter steel ball on top of the piston, thus ensuring axiality of load, and the driving fork engages the ground steel bar carried on the spindle, so avoiding any direct side pull on the piston. The torque required to keep the 1 ton mass continuously rotating against windage and oil friction at $1\frac{1}{4}$ r.p.m. was of the order of 1 lb.in. at 3,000 lb/in.^{2} pressure. The friction in the Morrison gland was considerably less (by a factor of about 50) than the friction between the axial loading ball and the piston, so that the dead weights and piston rotated together.

4.6.5 Sensitivity of the Accumulator

A Budenberg dead weight tester was coupled into the system and set for 1633 lb/in^2 (115 Kg/cm^2.) The pistons of the dead weight tester and of the accumulator were rotated simultaneously and the weights on the accumulator were adjusted until the addition of 2.2 lb. (1 kgm) just caused the Budenberg piston to rise slowly from the bottom of its travel. The time taken to rise 0.1 inch was about half a minute. The mass on the accumulator was then reduced by 2.2 lb. (1 kgm) and the Budenberg piston was seen to fall slowly from the top of its travel, the time taken again being of the order of half a minute. The addition of 1.1 lb. (0.5 kgm) to the weights permitted the Budenberg piston to remain floating centrally for as long as it would rotate without being touched. The area of the asuumulator piston is 0.7817 in^2 and the sensitivity of the accumulator is better than $\pm$ 1.41 lb/in^2 at 1633 lb/in^2 Approximately six feet of 1/16 inch bore tubing connected the dead weight tester and the accumulator so that the apparent sensitivity is probably rather less than the true value.

4.7 EXTENSOMETERS

4.7.1 General Remarks

The measurement of axial (i.e. tensile) strains under creep conditions has received considerable attention over the years, and it is not proposed to describe the various extensometer designs here. Brief descriptions of the available types, and references to the papers of the workers in this field, may be found in the published texts by Siebel and Ludwig (1955), Tapsell (1931), Clark (1953), Smith (1950), Stanford (1949) Sully (1949) and Finnie and Heller (1959).

The problem of measurement of external diametral strains on tubular specimens subject to internal pressure at room temperature has been satisfactorily solved for small extensions by Cook (1934) who designed a pair of diametral extensometers to measure across two diameters at right angles. Crossland and Bones (1958) used similar extensometers in their work up to diametral strains of 0.007 and thereafter employed a micrometer sensitive to 0.0001 strain.

Steele and Young (1952) have used electrical resistance gauges again at room temperature, to measure small axial and circumferential strain outside their specimens and have also applied these to the bore where they were subject to direct fluid pressure. This technique required the use of relatively large bore specimens.

Norton (1939) tested tubular steel specimens ($k = 1.23$) under creep conditions at high temperatures (1,200°F.) The diametral extension of the hot specimen was transmitted through the oven wall by means of small fused quartz probes which were held in place across a diameter of the specimens by cantilever springs. The displacement of the outer end of each quartz tube was measured by travelling telescopes.

Taylor and Quinney (1931) have observed the change in the volume enclosed by a thin tubular specimen which was subjected to torsion. The specimen was closed top and bottom by grips and filled with water. A capillary tube led from the top closure and the level of the liquid was adjusted by running it off through the bottom closure until the meniscus in the capilliary was at a suitable level on the scale provided. Any change in the volume enclosed by the specimen was recorded as a change in level of the meniscus in the capilliary since the volume of water remained constant.

4.7.2 Axial Extensometers

For the present work it was decided that all axial extensions would be measured directly on the parallel gauge length where the extension of the test piece and the extensometer design permitted, and between annular knife edges machined integrally with the specimen and included in the gauge length, where extensions were large and precluded the use of more sensitive extensometer designs.

A pair of Martens type extensometers by Amsler were employed for the measurement of axial strains of small order expected in the tests on the thick tubular specimens. After re-designing the spring clip fixture and shortening the rhomb-to-mirror arms it was possible to house the complete assembly totally within the constant temperature jacket so that ambient temperature variations did not affect the readings. These extensometers are illustrated in Fig. 9.10 and Plates 10.2 and 10.3. Deflection of the mirrors was observed through ports in the wall of the jacket. Resetting of the rhombs for the 2 inch gauge length comparison bars was not possible during test and in fact was found to be unnecessary since the axial extension was small. The extensometers permitted the detection of changes in axial strain 1×10^{-6} (equivalent to $\frac{1}{4}$ to 1 mm. division on the scale).

Isotropy tests were carried out on all material tested and it was decided that the order of the strains would be made that of the bore extension of the tubular specimens tested, since if isotropy was demonstrated for these large strains, then it would presumably also hold for smaller strains. Large strains of themselves can introduce anisotropic effects, but since the small specimens were all machined to be identical initially, and were similarly stressed, it was considered reasonable to assume that the same effects would be produced in each specimen if in fact the material was isotropic to start with.

The 2$\frac{3}{8}$ inch diameter of the extruded lead billet permitted a gauge length of only 1 inch on the small tensile specimens, and it was felt inadvisable to fit any mechanical attachment to the relatively delicate specimen. Measurement of extension had therefore to be by travelling microscope, using suitable gauge marks on the specimen. It was not desirable to scribe marks on the specimen because of the small cross sectional area and low material strength involved and consequently a method due to Gehman (1948) was used. The ends of a line in white pigmented rubber cement ruled parallel to the length of the test piece, sharpened during stretching, thus offering reasonably satisfactory gauge marks. (The composition of Gehman's cement was not divulged but a substitute was manufactured from a mixture of artist's colour (titanium white), rubber solution (from a cycle repair kit) and lighter fuel, the proper consistency being a matter for trial and error). The travelling microscope used was provided with a scale divided to 0.0002 inch over a travel of 1.2 inches. It was not found possible to measure with this accuracy, however, due to the thickness of the hair line in the telescope and the difficulty of determining consistently the exact position of the painted gauge marks. For the lead specimens the accuracy of measurement was not better than 0.002 strain.

The size of the magnesium tensile specimens was determined by the geometry of the continuously cast billet and a gauge length of 2 inches was chosen. At the large strains used, a considerable roughening of the surface of the specimens occurred and this precluded the scribing of gauge marks on the specimen itself, since fine scratches were difficult to see later in the test. Magnesium alloy is much stronger than lead and knife edges machined integrally with the gauge length were adopted. Since only isotropy was being tested the effects of the increased diameter at the knife edges would be the same for all specimens. Comparison bars permit the extension of a specimen of comparatively large gauge length to be measured conveniently by travelling microscope, gauge marks being scratched on flats machined at the ends of the comparison bars. Stanford (1949) has commented on the disadvantages of this type of extensometer. The comparison bars were attached to stainless steel fittings which gripped the side faces of the annular knife edges and were kept in position by springs. This arrangement was adopted for these small specimens in preference to supporting the comparison bars from the 'point' of the knife edge. Thus the comparison bar was in effect a cantilever and its support was required to be as rigid as practicable, but still capable of accommodating changes in dimensions of the annular knife edge during straining. The arrangement, illustrated in Fig. 9.6 was reasonably satisfactory and the accuracy of measurement was not better than 0.0005 strain.

4.7.3 Diametral Extensometers

Several methods of measuring diametral extensions on tubular specimens were examined and papers by Coker (1904,) Morrow (1905), Stromeyer (1894), Powell et al (1955) and articles in Engineering, Anon (1925 - 26) were consulted.

The design finally adopted (Fig. 9.10 and Plates 10.2 and 10.3) closely followed the double diametral pattern set by Cook in 1934 using knife edges described in Engineering 1926. The present extensometers incorporate a 're-set' device which permits their use over an extended range (up to 0.250 inch if required). The maximum extension recorded was of the order of 0.090 inch.

The pair of extensometers were placed across two diameters at right angles at the same cross-section. Each extensometer measured the specimen between a vee anvil (giving two line-contacts) at one side and by a pad having a spigot recess (giving two short line-contacts) at the opposite end. Resetting was achieved by increasing the distance between anvil and pad enabling the mirror rhomb to be returned to its original position at intervals. A retractable fork drive (shown in Fig. 9.3) was used to rotate the screw bearing on the pad, the nut being split to permit adjustment of backlash on the screw. The spring tension on the extensometer anvils was also adjustable.

Amsler mirrors and rhombs were used, the standard pattern being modified by shortening the rhomb to mirror arms. The assembly extensometers could be enclosed by a 6 inch diameter tube.

The extensometers were calibrated at 68°F. by measurement on an

C.M.T. measuring microscope and the extensometer constant calculated by geometry. The characteristics over the ranges of extension encountered are sensibly linear and a sensitivity of 8×10^{-6} strain was possible (equivalent to ½ mm. division on scale).

Resetting took less than 30 seconds and readings on the other diametral extensometer were taken immediately before and after resetting so that the extrapolation of the small portion of the creep curve could be more accurately determined. Readings before and after were often taken so close that it was not possible to show separate values on the deformation/time graph and any extrapolation required was very small. The maximum extension at one setting was approximately 0.015 inch, although in practice it was found convenient to limit this to 0.012 inch to avoid overshooting the scale on resetting.

Since the tests on magnesium were carried out at 270°F., the strain magnification constant was recomputed for one diametral extensometer, allowance being made for expansion of the lever arms. An increase in the magnification constant of 0.4% was found.

4.7.4 Dilation Manometer

Where several different strains in one piece of material are related by geometry, the strain to be selected for measurement should be, if possible, at the same time

(a) the largest strain present, and

(b) the most easily measured strain.

The strains at the bore of a thick tube during an internal pressure test are the largest strains but are not easily measured.

Steele and Young (1952) have used strain gauges to measure relatively small elastic strains at the bore of a cylinder subjected to internal pressure. In the present work, an attempt was made to develop an apparatus which would measure large bore strains during a test, the method being suggested by the experimental work of Taylor and Quinney (1931).

As the specimen expands, additional oil is delivered to provide for the increase in internal volume. Reference points for measurement of the movement of the oil were obtained by providing a mercury/oil interface in the pressure system and the amount of oil delivered to the specimen was noted by recording the rise in level of this interface in a vertical steel tube of known dimension. The steel tube was provided with a sight glass in the form of a capillary tube and level measurements were taken with a cathetometer.

Two tubular specimens, identical in all respects except length, were provided to eliminate end effects, and the difference in the mercury level in the two steel limbs of the dilation manometer illustrated in Fig. 9.5 gave a measure of the increase in volume of the centre portion of the longer specimen. Corrections to the readings were required for the compressibility of the oil and the temperature difference between the oil in the specimen and the oil in the columns (see Fig. 9.13). Parts of the manometer in contact with mercury were constructed from 18/12/1 stainless steel and screwed fittings of the pattern used by Bett, Hayes and Newitt (1954) were used for securing the steel tubes to the end blocks.

The sight glasses were of standard Pyrex capillary tubing 1.25 mm. bore and 6 mm. o.d. The lengths of tubing were selected for straightness and concentricity, the ends being cut and ground square before annealing at 580°C. Four circular neoprene discs $\frac{1}{8}$ inch thick were used as packing to seal the glass tube to the metal end blocks. Bursting tests on glass carried out by Onnes and Braak and mentioned by Hewitt (1936) indicated that the unsupported tubing would withstand the pressure.

Due to unavoidable eccentricities in mounting and to the distortion of the metal components under stress the deflection of the glass capillary at the middle of an unsupported 24 inch length was $\frac{1}{4}$ inch, from the initial position when subjected to a pressure of 2,000 lb/in.^{2} It was decided to provide cushioned restraint. This took the form of a rigidly supported $\frac{1}{2}$ inch bore copper tube having a lengthwise slot to permit observation of the level of the mercury, with a 20 inch length of laboratory tubing, slit along its length, placed between the copper tube and the glass capillary. The dimensions of the rubber tube were chosen so that after slitting and fitting to the capillary, little clearance was left between copper tube and rubber, the lengthwise gap permitting observation of the mercury level. The glass capillary was thus supported for $\frac{7}{8}$ths of its circumference along 9/10ths of its length and this effectively reduced the amount of bowing.

The $\frac{5}{8}$ inch diameter bore of both stainless steel (bright drawn) tubes was measured at three positions on each end, using Micro-Maags capable of measuring to 0.00005 inch, the averaged result being taken as the mean diameter of the tube.

The mercury level could be determined to 0.001 inch representing a volume of 0.001 cu. in. of oil. The difference in heights of the columns was therefore known to be 0.002 inch and this is equivalent to an extension of 0.001 inch on 0.375 inch bore diameter, i.e. a strain sensitivity of 0.003. It is possible to increase this sensitivity by using smaller bore and much longer manometer tubes and also by increasing the difference in the length of the two specimens.

Shell Vitrea 69 oil was used to pressurise the specimens. The dependence of density on pressure was computed from Grunberg's (1953) equation 6 and the dependence of density on temperature was obtained from the ASTM-IP Petroleum Measurement Tables - Sectional Volume C. The specific gravity of the oil supplied was quoted as 0.884 at 60°F. A sample was taken and a specific gravity of 0.887 obtained from two measurements using the method suggested by Reilly and Rae (1954). The ASTM-IP tables were entered at 0.885.

This method of strain measurement is limited by temperature to conditions where the straining medium remains liquid in the specimens, and where its properties are known. At higher pressures it may be necessary to adopt indirect methods of measuring the height of the mercury column, e.g. resistance of a platinum wire or detection of a floating steel ball by differential transformer. The sensitivity of the device may be improved by reducing the bore of the tubes and increasing their lengths, also by increasing the length of the long specimen relative to the short, and providing core cylinders.

Sealing of glass capillary tubes into steel end blocks of the high pressure manometer and filling with oil was a major difficulty, the manometer having to be dismantled almost completely each time, the parts being degreased and the mercury cleaned before re-assembly.

The end seal took the form of a simple gland using thin neoprene discs as packing. Dimensions were chosen so that the glass was supported entirely by neoprene and did not contact metal at any point.

To fill the capillaries with oil so that no air was trapped, mercury from the reservoir was forced into the steel manometer tubes and glass capillaries mounted and sealed in the common bottom block. The top ends of the steel tubes were plugged and the air in the capillaries displaced by mercury until a meniscus just appeared at the top of the tube. A glass funnel was attached to the capillary by plastic tube, and warm oil was run into the funnel slowly to prevent entrainment of air bubbles. The mercury level was allowed to drop drawing the oil into the capillary.

All adjustments of the level in the manometer had to be carried out slowly as sudden raising of the mercury level could result in a substantial film of oil remaining on a capillary wall which could form a pellet, breaking the mercury column.

Tightening of the seal gland nuts required careful judgement since overtightening shattered the glass and undertightening resulted in leakage.

Further work using high pressure manometers will be greatly simplified due to the availability of commercial instruments with steel tubes and with electronic means for detecting the position of a metal float on top of the mercury surface, e.g. instruments by Bopp and Reuther GmbH, Mannheim Waldhof, Germany.

5. EXPERIMENTAL PROGRAMME

5.1 PREPARATION FOR TESTS WITH LEAD

5.1.1 Production of Lead Billets

The billets were manufactured to the required composition by the firm of Alexander Ferguson & Co. of Glasgow. Seven hundred pounds of purified lead ingot and seven pounds of high quality tin bars, of the typical analyses given in Table 11.5 were melted in a specially cleaned pot held at 430°C. All tests on the lead (1% tin) alloy subsequently reported here were carried out on material from this one melt.

The pipe press used to extrude the alloy was similar to that described in the Metals Handbook 1948 (page 948), but since the pipe press cylinder only held about 450 lb. per charge, extrusion was carried out in two batches.

The press cylinder was carefully cleaned out before the initial charge was poured and extrusion of the solid but plastic alloy was carried out at around 200°C. The second batch was not poured until later and the pipe press was kept hot by gas jets during this period, but was not cleaned out to receive the second charge. This omission was of considerable significance as will be seen later.

Care was taken to avoid bending of the tube during extrusion and support was given by a long wooden box subsequently used for transporting the extrusion to the circular saw where it was cut into convenient lengths. The extruded tube was 2¾ inches outside diameter and ⅝ inch bore. To produce the finished bore an accurate mandrel was manufactured in the research laboratory workshops, the size being

0.3751 inch. Some shrinkage of the lead took place after extrusion and average bore sizes of 0.3740 inch were measured later. The bore had a smooth mirror like finish.

5.1.2 Metallographic Examination and Chemical Analysis

The successful preparation of metallographic specimens was developed after considerable experiment. The method is basically that recommended in BS.602, 1085:1956 with a modified polishing technique which was found essential. This is given in the Appendix, section 8.2.1.

Billet 1 was selected for initial tests on the development of machining and testing techniques. Samples were taken and examined metallographically in three mutually perpendicular directions and found to be free from defects. There was little evidence of elongated structure in the direction of extrusion, the grain size being of the same order as in the transverse directions.

Chemical analysis of the material was carried out at the Research Laboratories of the Associated Lead Manufacturers Ltd., by the courtesy of Dr. D.S. Laidler, and the results are given in Table 11.6.

5.1.3 Machining of Specimens

The dimensions of the specimens and their relative positions in the original billet are illustrated in Fig. 9.7. As may be seen it was not possible to obtain suitable small tensile specimens out from the radial direction in the billet.

The workshop had not previously machined lead and suitable techniques had to be developed before the relatively high initial scrap rate of about 50% on small tensile specimens was reduced to an acceptable level. These specimens were initially cut from the billet by band saw, formed into rough blocks of square cross section and turned to cylindrical form. Centres were drilled at each end and the ½ inch diameter cylinder mounted between centres; thereafter all machining was carried out at one setting. Due to the mechanical delicacy of the finished gauge length, cuts of the order of 0.002 inch deep were necessary and considerable patience on the part of the operator was required.

Machining of the tubes was not so hazardous and the specimens were turned between centres on accurately ground steel mandrels which were a close fit in the extruded bore of the billet. The resulting eccentricity gave a bore never more than 0.001 inch out of true position. The ends of the billet were first machined square and the billet then clamped firmly but gently in the axial direction using washers and nuts provided on the mandrel.

No screwed fittings had been provided on the lead specimens for reasons detailed under sub-section 4.4.2 and the external profile was relatively straightforward machining on a Hardinge precision lathe at 70 ft/min., with a feed of 0.0045 inch per rev. and a depth of cut of 0.020 inch. A vee tool was used on the rear tool port so that the chips fell away.

The mandrels were left in the finished tubes during storage and were not removed until pre-test inspection. The time elapsing between finish machining and testing was kept as short as practicable to minimise possible distortion effects during storage.

Plate 10-6 illustrates a finished short tubular specimen and a corresponding part of the billet from which it was produced. A steel mandrel is in the foreground. Small tensile isotropy specimens in various stages of manufacture are also shown.

5.1.4 Development of Apparatus and Testing Techniques

The original design of hydraulic accumulator did not permit rotation of the piston. Preliminary tests with a non-rotating piston, suitably loaded to produce a pressure of 109.5 kg/cm^2, showed that when a small quantity of oil was drawn from the system, the pressure rose to about 113 kg/cm^2. On closing the valve the pressure remained at 113 kg/cm^2 for a short time and then fell slowly to about 109.5 kg/cm^2. The temporary increase in pressure on opening the valve was evidently due to the reduction in friction obtained with a moving piston. The supporting framework was therefore redesigned to permit continuous rotation of the piston and deadweights and the arrangement finally adopted is shown in Fig. 9.4.

Temperature distribution tests were carried out in both the large and small ovens shown in Figs. 9.3 and 9.6 respectively. It was found that adjustments with specimens in place were necessary to ensure correct distributions. Tests were also carried out to determine the temperature lags during heating between the outer and inner surfaces of a lead tubular specimen and between the inner surface of the specimens and the pressurising oil. A portion of the heating curves obtained is given in Fig. 9.13.

Difficulties were experienced with the high pressure mercury manometer. Failure of the capillaries under pressure were frequent during development and the arrangement finally adopted is described in Section 4.7.4. Careful assembly to ensure alignment was found essential. On occasion, three attempts have been required to obtain an assembly which would withstand pressure. Shattering of a glass capillary did not necessarily occur under the first application of pressure and bursts during the 'check' period after stabilising temperatures and before pressurising the specimens have prevented bore measurements from being recorded. In other tests, rupture of the glass has occurred on pressurising invalidating all results from that test.

The first tests on lead tubes were carried through to rupture. Later work indicated that for measurement of bore strains using the differential manometer, tubes should not be taken beyond the secondary creep range into the region of localised bulging and the mercury traps shown in Figs. 9.1 and 9.5 were therefore removed because they were unnecessary.

Several preliminary tests on tubes were made to determine the most suitable test conditions and a further three full scale tests with extensometers were necessary to gain operating experience with the apparatus before acceptable results were obtained.

Preliminary work was carried out on material taken from billet 1 and the observations made were sufficient to indicate a general trend which was confirmed during later experiments, i.e. for all lead tubes axial contraction was observed.

Tensile tests on small isotropy specimens from billet 1 were also carried out to prove the apparatus and gain experience while selecting the most convenient stress level to be used. No serious difficulties were experienced with these tests although it was evident that the material was anisotropic.

5.2 TESTS ON LEAD BILLET 2

5.2.1 Tube Tests.

The preliminary tests on tubular specimens appeared promising and only sufficient material remained of billet 1 to make one further full scale test.

It was believed that all major experimental difficulties had been eliminated and the decision to take subsequent specimens from billet 2 was engendered by the hope that further tests would be without incident and that successful runs might form part of the reported results.

It was decided to carry out tubular tests first, since if they were unsuccessful, valuable time would not have been spent in abortive testing of small isotropy specimens. This was felt to be an acceptable risk since test data on tubes were of prime concern and the experimental technique more difficult. Tests on isotropy specimens for each tubular test would have delayed the programme by one week per set of tensile specimens, for every tubular test that was unsuccessful, and from previous tests it was anticipated that the material would show anisotropy in any event. Metallographic tests on billet 2 had not been carried out at this stage and later events were to show that this was a serious omission.

Two series of tubular tests - designated 2A and 2B - were carried out at (33°C (91°F) with an internal pressure of 106 kg/cm^2 (1510 lb/in^2) on the lead (1% tin) alloy. In both tests slight leakage from the top of the high pressure manometers invalidated measurements of bore strain and both tests were therefore taken to rupture. The creep curve form was however, very evident from the manometer readings.

Specimen 2A had an initial k-value of 3.00, the smaller tube rupturing in 319 minutes and the larger (extensometered) tube taking 342 minutes. Specimen 2B had a lesser initial k-value of 2.91 and, as might be expected, rupture took place earlier - 266 minutes for the larger specimen, when the test was stopped. As in all the tests on lead tubes, axial strains were negative. The resulting curves of total deformation on specified gauge lengths are given in Fig. 9.15.

A photograph of a ruptured lead specimen is given in Plate 10.11. The characteristics of the fracture were the first indication of abnormal behaviour in the lead.

5.2.2 Tensile Isotropy Tests

Initially three isotropy tests per set of tubular specimens were envisaged these being taken from the billet as shown in Fig. 9.7.

Tensile tests at 32°C (90°F.) were carried out for billet 2A in the tangential and axial directions using a load which gave an initial stress of 110 kg/cm.^{2} (1,564 lb/in.^{2}). The axial specimen ruptured in 141 minutes with a ductile fracture which drew down to a point, similar to that seen in the centre of Plate 10.12. The two tangential specimens ruptured suddenly in a brittle manner at 134 and 188 minutes respectively, without any apparent necking and with the unusual fracture faces seen in Plate 10.12. The faces had curvature in one plane passing through the major axis of the rupture face and the axis of the specimen, and were straight in the plane at right angles which passed through the minor axis of the face and the axis of the specimen.

For billet 2B, tensile tests at 32°C and an initial stress of 110 kg/cm.^{2} showed similar behaviour, with the exception that the tangential specimens ruptured instantaneously on loading with a similar characteristic clean fracture, while the axial specimen gave a typical creep curve and necked down, rupturing at 154 minutes.

Further tests on tangential specimens on spare material from billet 2A gave a similar fracture at 95.6 kg/cm^2 and slow creep without rupture at 85.8 kg/cm^2

5.2.3 Metallographic Examination

The results of the tensile tests and the appearance of the ruptured tubes and tensile specimens indicated that the material, in addition to being anisotropic, contained some hidden flaw. Remaining portions of billet 2 were cut up and examined metallographically in three mutually perpendicular directions, with the result that "piping" was discovered in the billet. The effect may be clearly seen in Plate 10.7, a photograph of a lightly machined cross-section of the billet.

The flaw is very apparent in Plate 10.9 which is the same cross-section, heavily etched to deepen the groove, and then lightly polished on 00 emery wetted with paraffin. Plate 10.8 shows the same surface polished and lightly etched to bring out the grain structure. The circular groove may still be seen, and the variation in grain size near the bore of the tube is apparent.

One of the ruptured tubes was sectioned transversely at the centre of a crack and the heavily etched and lightly polished section shown in Plate 10.10 clearly explained the effects noted earlier during external examination - the brittle rupturing of the "outer" cylinder. Excessive etching has caused small particles of material at the "interface" to fall out and to some extent masked originally well defined edges at the rupture. The peculiar ruptures obtained with the transverse tensile specimens become obvious when taking piping into account.

Additional transverse "discs" from billet 2 showed the same flaw. Immediate further metallographic examination of discs taken from billet 1 failed to show piping defects and the difficulties encountered with billet 2 were ascribed to the failure to clean out the pipe press before the second charge was poured, although no attempt to prove this point was made. No further tests were made using material from billet 2.

5.3 TESTS ON LEAD BILLET 1

5.3.1 Metallographic Examination

Experience with billet 2 determined that further tests would be carried out in a logical order, and sufficient material remained of billet 1 to carry out one further series of experiments.

Metallographic examination in three mutually perpendicular directions was repeated for off cuts from billet 1A indicated that the grain sizes in each direction were comparable and no evidence of striation due to extrusion was found.

5.3.2 Tensile Isotropy Tests

Three specimens were taken from billet 1 as indicated in Fig. 9.7. Unfortunately, one of the tangential specimens was severely damaged during machining and the results of only one axial and one tangential specimen are reported.

The initial tensile stress was 100 kg/cm^2 (1422 lb/in^2) and the test temperature 33°C (92°F). Natural longitudinal strains of about 15% were produced in 300 minutes (these have to be compared with natural tangential strains of about 30% at the bore of the tubes tested and of 2.5% at the outside surface), the tests were continued to over 500 minutes.

Further tensile tests under the same conditions were carried out on material taken from another portion of billet 1. The results for two tangential specimens fell together and only slightly above the result given in Fig. 9.14, for the tangential specimen reported. An axial specimen gave a creep curve approximately 0.02 strain below the curve for the axial specimen given in Fig. 9.14, a substantial discrepancy which may possibly have been due to error in the measurement of the initial gauge length since the general shape of the curve followed the reported curve closely.

These additional results, however, are not shown since they apply to a distant portion of the original extrusion which was some 15 feet in length and mention is made only to indicate that the curves given are reasonably representative of the material.

5.3.3 Tube Tests

This was the most successful test of the series in that no difficulties were experienced with the high pressure mercury manometer. The complete test results are given in Fig. 9.16 for an internal pressure of 105 kg/cm^2 (1500 lb/in^2) and a test temperature of 33°C (92°F). This test was not taken to rupture and was stopped once the accelerating portion of the curve was reached and while localised bulging had not commenced. No creep recovery after unloading was noted, and during the test axial creep was again found to be negative.

The point marked A on Fig. 9.16 gives the value of bore strain at 323 minutes, and the point B has been computed from this value assuming incompressibility of material. The point C is the estimated final outside diameter of the specimen at the end of the test. This does not correspond with the value predicted by the extensometers due to the "dig-in" of the extensometers. The average amount of "dig-in" denoted by "x" was measured by micrometer when the specimens were inspected after the test.

With the vee anvil and the pad of one diametral extensometer in place (sub-section 4.7.3, Fig. 9.10) the distance was measured across the outer faces of these components. The length of the vee anvil and of the pad were then found separately and the diameter of the tube itself determined at a point slightly removed from where the anvil and pad had been placed.

In Fig. 9.15 and 9.16 the measurements of diametral extension are not corrected for dig-in.

On completion of the test the external profiles of the specimens were measured in an O.M.T. workshop microscope. The original steel mandrels were passed through the bore of the tubes which were still a very close fit at the ends due to the restraining effects of the end closures during testing. (The end closures after all tests had to be forced off using light internal pressure, the grip on the spigot due to the wedging effect being so great).

Each specimen was in turn mounted horizontally between centres in the microscope and a 0.0001 inch clock fitted in the telescope position of the O.M.T. microscope was used to determine increase in radius.

Four rotational positions (at 0°, 90°, 180° and 270° round the specimens) were taken at various "stations" along the length of a specimen and the four results averaged for each station. The results are plotted Fig. 9.17 with the curve for the shorter specimen separated into two portions and moved to match up axially with the original fillet radii of the longer specimen.

This was done firstly to determine if the behaviour of the shorter tube could be considered independent of end effects and secondly to determine if end effects obtained with the long and short tubes were the same, as required by the principle of measurement of bore strains using the high pressure manometer.

From the data, it appears that reasonably close geometrical correspondence between the ends of the specimen pairs has been obtained. It should be pointed out, however, that small discrepancies at the o.d. will be magnified at the bore of a thick tube and the accuracy of the bore measurements should be examined in this light. The necessity for having exactly similar ends on specimens suggests the use of a copy lathe.

It was also suspected from Fig. 9.17 that the strength of the diametral extensometer springs was having a restraining effect on the expansion of the lead tubes, a reasonable tension being required to maintain the extensometer position on the specimen while resetting took place during testing.

5.4 CONCLUSIONS ON LEAD AS A TEST MATERIAL

The experience gained intesting lead was sufficient to permit a conclusion on the suitability of lead as a test material.

The advantages of lead were discussed previously in section 4.1. Testing near ambient conditions eliminates the problems of high temperature extensometry, high pressure equipment is not required and stresses are generally lower, while with a properly chosen alloy the characteristic creep curve is well defined.

The disadvantages, however, balance the advantages. The temperature of storage is important, deformation due to body forces may affect a carefully machined but carelessly stored specimen, and recrystallisation of the grain structure may occur. An accurate and smooth bore for tubular specimens requires the use of an extrusion process for manufacture, which may result in inherent anisotropy. In addition the use of an extrusion fixes the bore of all specimens tested, thus changes in diameter ratio have to be made by altering the outside diameter which may require the recalibration of the diametral extensometers. Radial isotropy specimens are not obtainable unless the original extrusion is very large. The machining of lead requires skill and patience, as small specimens in particular are very delicate. The material is soft and extensometers tend to "dig-in".

Any marginal advantages that lead may have had as a test material were considerably outweighed for the present tests on the discovery of the flaw in billet 2. Immediately on completion of the tests on billet 2, and appraisal of the material situation concerning billet 1, it was obvious that further tests on new material would be necessary. Moreover, lead was suspected of anisotropy and since there was no guarantee that piping would not occur, the decision to use a different material was taken. A continuously cast magnesium (2% aluminium) alloy was selected for reasons given in section 4.1.3.

5.5 PREPARATIONS FOR TESTS WITH MAGNESIUM

5.5.1 Production of Magnesium Billets.

Two continuously cast billets of magnesium (2% aluminium) alloy, from the same melt and machined all over, were obtained from Magnesium Elektron Ltd., Clifton Junction, Manchester, to the same specification as used by Johnson et al.

The material was in the form of 12 inch diameter billets, 12 inches long and had been held at 420 ± 5°C for 4 hours and furnace cooled to room temperature in about 10 hours. Heat treatment was carried out in an atmosphere of sulphur dioxide.

5.5.2 Metallographic Examination and Chemical Analysis

Only a portion of one billet of magnesium was used in the present programme, and this was cut up as indicated in Fig. 9.8. The second billet was held as a reserve should further tests be found necessary.

Samples were taken from three mutually perpendicular directions in the billet and metallographic specimens were prepared at the laboratories using standard techniques with nital as an etch. The billet was found to be free from visible defects, the grain size being of the same order in all directions.

A more complete metallographic examination was subsequently made by an independent laboratory and the report is given in the Appendix, section 8.2.2. Spectrographic and chemical analysis of the material was undertaken at the Research Laboratories of Magnesium Elektron Ltd., by the courtesy of Mr. W.J. Price and the results are given in Table 11.7.

5.5.3 Machining of Specimens

The magnesium alloy proved much easier to machine than lead. Fig. 918 gives the dimensions of the specimens and illustrates schematically how these were cut from the billet.

A circular disc 2 inches wide was taken from the top of the billet, from which all radial and tangential tensile specimens were cut. The remaining cylinder 10 inches high and 12 inches in diameter, was cut into quadrants and axial tensile specimens and tubular specimens were taken from three of the quadrants in a manner similar to that shown. Off-cuts of material obtained during cutting up were used for the metallogrphic examination described in section 5.5.2.

Thick walled tubes were produced by first shaping a square section block, which was then clamped to the saddle of a Holbrook lathe for drilling and boring at the one setting. The boring bar was supported at both ends and successive floating cutters used, with increments of about 0.010 inch on diameter. A ground mandrel was placed in the finished bore and the external profile turned between centres. The tubes were inspected after machining for eccentricity of bore with outside diameter, and the maximum displacement of the two centres did not exceed 0.00075 inch at any time.

5.5.4 Modifications to Apparatus

Testing of magnesium (2% aluminium) alloy, required either increased temperature or increased pressure conditions, or a combination of both, in order to produce significant deformation in short time.

Since the apparatus had been developed to a reasonable degree of reliability the decision was taken to use it in its existing form and only to make modifications to suit the new material where these were essential.

The limits of temperature and pressure attainable with the equipment were 150°C (300°F) and 280 kg/cm^2 (4,000 lb/in^2). The pressure was restricted by virtue of the glass in the differential manometer illustrated in Fig. 9.5, and the temperature by the fuming of the circulating oil from the constant temperature tank illustrated in Fig. 9.2.

To determine a suitable wall ratio for the magnesium tubes the k value was estimated using a modification of Mannings theory for the prediction of the bursting of thick walled tubes. Tensile data supplied by Magnesium Elektron Ltd., was employed and the method of design indicated by the writer in a contribution to Crossland and Bones (1958) was used. A value of k = 1.5 was found for magnesium.

The outside dimensions of the tubular lead specimens were retained to avoid redesign of the diametral extensometers. The end closures were modified to suit the enlarged bore as shown in Figs. 9.3 and 9.9. Initial difficulties due to differential expansion between magnesium and steel were successfully overcome as reported in Section 4.4.2.

The thermocouples were calibrated for use at the higher temperatures and tests were carried out to determine the time required to stabilise temperature on the outside of the magnesium specimens. The thermal conductivity of magnesium is approximately 4 times that of lead and further tests for temperature lag, similar to those for lead reported in section 5.1.4 were not required.

5.6 TESTS ON MAGNESIUM BILLET

5.6.1 Metallographic Examination

This was reported in section 5.5.2.

5.6.2 Tensile Isotropy Tests

All isotropy tests were completed before proceeding to the tests on tubes. Several preliminary tests were carried out to determine suitable testing conditions, which were eventually fixed at 124°C (258°F) with an initial applied stress of 907 kg/cm.^{2} (12,900 lb/in.^{2})

The results of ten tests on three axial, four radial and three tangential specimens, are reported in Fig. 9.19. The radial test denoted by the open triangle (uppermost curve) was carried out at 126° in error. A substantial scatter band exists and although the correspondence of curves is better than that obtained in commercial testing (c.f. typical scatter results quoted by Smith (1950) a higher degree of reproducibility is desirable. However, from the evenness of the scatter, it is evident that there was no preferred direction in the billet.

In the test natural tensile strains of the order of 15% were obtained in 300 minutes, and these have to be compared with natural tangential strains of only 7% at the bore and of 3% at the outside surface of the tubes tested after 1,100 minutes. Future work should aim to keep the maximum natural octahedral strain developed in isotropy proving tests as near as possible to the maximum natural octahedral strain occurring in the tubes being tested. This may necessitate testing some tubes before carrying out the isotropy tests.

5.6.3 Tube Tests

Four tests were carried out but only sets of results for the second and fourth tests are reported. The graphs in Fig. 9.20 show close correspondence and verify that the testing technique was capable of reproducible results.

All tests showed initial elastic extension and unlike lead, all tests gave positive axial creep. Axial deformation after initial extension is very evidently creep dependent and does not remain constant.

The results for test one were excluded because of difficulties with thermal expansion in the end closures when the retaining screws ruptured as described in Section 4.4.2.

Results for diametral extensions on the third test do not exist due to the fact that a thinner specimen ($k = 1.42$) was being tested and readings of the diametral extensometers overshot the ends of the scales on initial application of pressure before resetting could be carried out. This was particularly unfortunate as the mercury manometer appeared to be satisfactory for this test. Observations of axial extension were taken for 140 minutes and deformation almost exactly twice that of the tests given in Fig. 9.20. It appears that thinner magnesium alloy tubes will have greater axial extension under creep conditions.

Measurements of bore strain by mercury manometer requires that the test be discontinued, the pressure reduced to zero, and the recovery at the outside diameter noted before the tube passes into a state where localised bulging commences. This is necessary to allow for the compressibility of the oil, and the method of calculating bore strains is given in Appendix 8.1. Unfortunately mercury manometer readings were invalidated in the first and second tests, due to slight leakage, and one glass capillary ruptured during the pre-test 'check-over' before the start of the fourth test, the experiment being continued without the manometer. No reliable results are therefore available on bore extensions for the magnesium alloy.

Fig. 9.21 shows for the specimens reported in the upper graphs of Fig. 9.20, the final external profiles (measured at room temperature) which were obtained as described for lead previously in section 5.3.3. The results confirm that the tubes were sufficiently long for representative deformation to be obtained by the external extensometers, and the alloy sufficiently hard to eliminate "dig-in" effects. No corrections for "dig-in" were found to be necessary.

The upper graphs in Fig. 9.20 show the effect of resuming the test after a short pause - little discontinuity is apparent and the creep, after a very short portion of rapidly decreasing creep rate, takes up the form of the curve immediately before unloading.

5.7 CONCLUSIONS ON MAGNESIUM AS A TEST MATERIAL

Magnesium (2% altminium) alloy, proved very suitable for testing, and few problems were encountered.

Difficulties which did arise were due to the use of apparatus designed with lower temperatures and pressures in mind (e.g. the constant temperature jacket which leaked, and the glass seals of the differential manometer), but improved equipment for the moderate conditions required, will be easily developed as a result of experience.

Oxidation of the magnesium during testing was noted, but was relatively light and the wall thickness of the tubes ensured that the effect on stress distribution could be neglected.

Wrinkling of the surface of both tensile and tubular specimens was observed and appeared more pronounced than for tests on lead to greater strains. It seems unavoidable when testing to large strains.

Magnesium alloy is light, machines easily, stores well, is relatively hard and strong, but creeps at low temperatures and relatively low stresses. Continuously cast billets show considerable isotropy. A fire hazard exists with fine swarf dust, but negligence or deliberate attempts were required to start combustion.

CHAPTER 6

6. DISCUSSION OF RESULTS

6.1 Presentation of Results

Data for creep tests have often been published showing smoothed curves of computed strain versus time. Sometimes the statement is made that initial elastic plus plastic strain has been subtracted, but no indication is given as to what length is used in the computation of strains or what part of the extension has been taken*. In the present work, unsmoothed experimental points, giving actual total extensions (creep plus initial plastic and elastic) observed for specified initial gauge lengths measured at room temperature are presented in all cases, thus eliminating possible misinterpretation as to which strains were computed. No allowance was made for the initial thermal expansion of the specimens on heating to test temperature.

6.2 Isotropy Tests.

The results for lead were indicated previously in sub-sections 5.2.2 for billet 2 and in sub-section 5.3.2 and Fig. 9.14 for billet 1. As explained in section 5.1.3 it was not possible to obtain radial specimens from the lead billet and a complete evaluation was thus impracticable.

The piping flaw (illustrated in Plates 10.7 to 10.13) invalidated the tensile isotropy tests made for billet 2 and the results of these

* A notable exception to this is to be found in the work of Bhattacharya, Congreve and Thompson (1952) where the method used with tensile testing is clearly explained.

tests are therefore not included in this report. For billet 1A, which did not have this flaw, the only tensile isotropy tests available are those given in Fig 9.14. Supporting data for these results were obtained from another portion of billet 1 as indicated in sub-section 5.3.2. These results suggest that the lead was anisotropic in all tests.

For magnesium a much more exhaustive investigation was possible. The results were described in sub-section 5.6.2 and are given in Fig. 9.19. Although a substantial scatter band exists, the results suggest that there is no preferred direction in the billet since each sampling direction is represented across the scatter band. The method of applying the load described in sub-section 4.3.3 may have been a contributory cause of the scatter since it did not ensure that each specimen was loaded at exactly the same rate*. Future isotropy tests would aim at greater reproducibility of results.

6.3 Eccentricity and Bending of Tubes

For the cylinders tested, the results obtained using the two diametral extensometers and the two axial extensometers are presented in Figs. 9.15, 9.16 and 9.20. The axial deformation of the lead tubes of Figs. 9.15 and 9.16 is also presented in Fig. 9.18, but to an

* A hydraulic jack to support the load initially, with a calibrated bleed-off to apply the load at a known rate, would eliminate that variation. This is an adaptation of the technique used by Andrade (1910) who employed a hyperbolic weight floating in mercury to maintain the stress in his specimen constant during deformation.

enlarged scale. Variation between the deformation curves obtained using the twin diametral extensometers would be an indication of the development of eccentricity in the cylinder being tested. Variation between the deformation curves obtained using the twin axial extensometers would be an indication of bending. The results obtained indicate that the care taken in the machining of the specimens and in the setting up of the cylinders reduced these undesirable effects to the point where they could safely be ignored.

6.4 End Effects in Tubes

Before discussing the deformation time curves obtained for the thick tubes tested, it is first necessary to show that the observations are free from end effects. As indicated in sub-section 4.4.1, a minimum parallel length of four times the outside diameter was taken in order to make the behaviour of the short tubular specimens independent of end effects. Plate 10.13 illustrates the ruptured specimen pairs 2A and 2B. The shorter specimens show almost spherical deformation of the mid-portion of the gauge length at rupture, while the longer gauge length specimens remain more parallel, suggesting that, for specimens with a gauge length of four times the diameter the axial location of rupture may be defined too closely by the geometry of the specimens.

Fig. 9.17 gives the external profiles of the long and short tubular lead specimens of billet 1A after testing. These results indicate that the profiles of specimens 1A were also tending to those depicted in Plate 10.13. Fig. 9.21 gives the external profiles for the magnesium alloy specimens 'C' after testing. These curves show that at small strains, the end effects indicated by deformation die out within about half a diameter for the thinner magnesium tubes. For the thicker lead tube, however, the effect may extend to over a diameter.

The amount of deformation which the tube suffers may also affect the range of influence of end effects. For the initial elastic stress distribution, in a thin tube subjected to internal pressure and restrained at the ends, Phennah (1960) has calculated the length over which end effects are noticeable and found that the strengthening effect was restricted to a length of about half a diameter at each end.

In the present tests, the shorter specimen is only of interest up to the point where localised bulging commences, since its function is to compensate for end effects in the measurement of the deformation of the bore of the larger tube. For moderate strains, such as those of interest in the design of thick tubes for creep conditions, the above results confirm that a minimum parallel gauge length of four times the outside diameter may be sufficient. For investigation of rupture behaviour, during creep, it is desirable to use an increased parallel gauge length, and from the long specimen illustrated in Plate 10.13 it is evident that a length of six times the diameter would be sufficient.

6.5 Axial Creep in Tubes

6.5.1 Results for lead and magnesium

The aim of the experimental work undertaken was limited to the determination of whether axial creep occurred in tubes subjected to internal pressure, and in this respect the experiments are conclusive -

axial creep was found in every tube tested. This bare statement however, does not place the results in proper perspective.

The results concerning axial strains were rather unexpected. Six tests in all were carried out on pairs of lead alloy tubes (of nominal diameter ratio 3/1) when axial strains were recorded, and longitudinal contraction was observed with every experiment. The results of tests on three tube pairs are reported in Figs. 9.15 and 9.16. The axial contraction observed for these three tube tests is given to a much enlarged scale in Fig. 9.18. The plots show clearly the basic form of a curve of creep deformation. No steady state creep region was found in any of the tests made on tubes and this is in accord with the theoretical reasoning of sub-section 1.9.2.

The curves of Fig. 9.18 suggest that an effect due to diameter ratio may exist. However, it should be remembered that the axial curve of Fig. 9.18 corresponding to Fig. 9.15 were obtained for tubes which had a piping flaw. Further tests on lead tubes of more widely differing diameter ratios are desirable to confirm in general, that a diameter ratio effect exists and in particular the tentative observation that thinner lead tubes contract axially during creep more than thicker tubes.

Four tests in all were made on pairs of magnesium alloy tubes (of nominal diameter ratio 1.5/1. Axial strains were recorded and longitudinal extension was observed with every experiment. The results for two tests only are given in Fig. 9.20. As reported in sub-section 5.6.3 a third test with a tube of k = 1.43 gave almost double the axial extension of the first two tests.

Although axial deformation increases during creep with both lead alloy and magnesium alloy tubes, a contraction is found for lead and an extension is found for magnesium respectively. These results are to some extent confirmed by previous workers. Bailey (1930) gave a graph showing the deformation of a thick (k = 3) lead tube in which axial contraction was observed after the point of inflexion, (sub-section 1.9.1) was passed. For thin (k = 1.23) steel tubes Norton (1939) obtained only very small amounts of axial deformation, but these were predominately extensions.

6.5.2 Interpretation of Results

The explanation of this anomalous behaviour may be in the nature of the materials used. Johnson, Henderson and Kahn (1961) have shown that the triaxial stress system in an elastic thick cylinder is identical to that of a pure torsion system. Crossland and Bones (1958) have demonstrated experimentally that torsion data gives a very close prediction of the behaviour of a tube which is ruptured under plastic conditions, and Bailey (1930) has proposed the use of torsion creep data for the design of thick tubes subjected to creep conditions. Following the argument of Bailey (1930), an increase in length of a thin tube subjected to internal pressure would thus appear to correspond to an increase in thickness of a thin tube subjected to pure torsion, similarly a decrease in length would correspond to a decrease in thickness. Although exact equivalence between the torsion system and the internal pressure system disappears as soon as deformation takes place (since the first system represents simple shear, i.e., pure shear with a rotation, and the second represents pure shear alone), the examination of the results of torsion tests may reasonably be expected to throw some light on the phenomenon.

Swift (1947) has reported on length changes in metals under torsional overstrain. His apparatus was designed to ensure that his specimens were free to extend or contract axially while subjected to pure torsion only, and special extensometers were constructed to permit simultaneous measurements of twist and extension. Eight metals were tested, namely brass, stainless steel, aluminium, cupro-nickel, copper, mild steel, 0.5% carbon steel and lead. The first seven metals gave axial extension during torsional overstrain, brass and stainless steel giving the greatest, and mild and carbon steel the least extensions. Lead gave axial contraction under severe overstrain. Swift concluded that in an annealed material, there was no measurable change in length under torsional strain, but that with work hardening material, torsional strain was accompanied by axial stretch. It is interesting to note that with the mild and carbon steels, the effect was very small, since this has a parallel with the small axial deformations observed by Norton (1939) in his tests on thin steel tubes under internal pressure.

A direct comparison between axial creep in tubes under internal pressure, and axial deformation in round specimens subjected to torsion is of course not valid since it is the change in thickness of a thin tube subjected to torsion which is of interest. Nevertheless, the results obtained by Swift strongly suggest that the type of material used for the test specimens may be responsible for the axial deformation observed in the present tests on thick tubes.

Preliminary tests by Swift on hollow torsion specimens of mild steel indicated that the bore of the tube closed in during torsion, unfortunately no measurements of the change in thickness of the tube were reported.

Tapsell and Johnson (1940) and Johnson (1949) have found no change in the thickness of initially isotropic thin tubular specimens of 0.17% C steel subjected to creep under pure torsion at relatively small strains.

6.5.3 Axial creep observed

For practical purposes of design the important thing is to be able to assume that axial creep deformation is zero (see section 1.6). In the present experimental programme no attempt was made to establish the elastic behaviour of the materials used at test temperatures. Nevertheless from the discussion in sub-section 1.9.2 on the design of tubes, elastic behaviour during creep is seen to be of considerable importance and, as indicated in sub-section 1.9.2. future work will have to consider this aspect carefully.

For the lead alloy material, the test results reported in Fig. 9.16 and 9.18 for billet 1A indicate that little or no initial elastic strain is present. In addition, no elastic recovery was found on pressure unloading as mentioned previously in sub-section 5.3.3. The tangential and axial creep strains may therefore be calculated, since the elastic strains may be neglected for the present tests on lead.

For tube 1A at 200 minutes:- tangential natural creep strain at bore

$$\bar{\varepsilon}_{db} = \log_e\left(\frac{l_o + \delta_l}{l_o}\right) = \log_e\left(\frac{0.374 + 0.0944}{0.37}\right) = \log_e 1.252$$

$$= 0.225$$

tangential natural creep strain at outside diameter

$$\bar{\varepsilon}_{do} = \log_e\left(\frac{1.135 + 0.0256}{1.135}\right) = \log_e 1.0226 = 0.0222$$

axial natural creep strain at outside of tube

$$\bar{\varepsilon}_z = \log_e\left(\frac{2.000 + 0.000678}{2.000}\right) = \log_e 1.000339 = 0.000322$$

The tangential creep strain at the bore is thus about 690 times greater, and the tangential creep strain at the outside diameter about 69 times greater, than the axial creep strain for the lead alloy tube of k = 3 which was tested. The assumption of zero axial creep strain is thus a good approximation for lead alloy tubes.

For the magnesium alloy tested it is evident from Fig. 9.20 that significant axial deformation occurred, and that the total axial strain increased throughout creep. Since Lames equations for a fully elastic tube subjected to internal pressure predict that the tangential strain at the outside diameter of the tube is $\left(\frac{2-\nu}{1-2\nu}\right)$ times the axial strain*, it is also clear that considerable time-independent plastic strain was present on loading. An investigation of the time-independent stress-strain properties of the magnesium alloy at the test-temperature was not carried out in this experimental programme

* This is a convenient check to determine if the initial deformation was in fact elastic.

and it is therefore not possible to attempt to separate initial elastic and time independent plastic strains for the magnesium alloy tubes. Such analysis is required to establish the origin of zero time for axial creep strain.

For creep under uniaxial tension it is generally assumed that if time-independent plastic straining does not occur on initial loading, then the initial elastic strain is instantaneously and fully recoverable on unloading after creep has taken place. Studies of creep recovery have been made by Tapsell and Prosser (1934), Bailey (1935) and Johnson (1941), but the paper by Lequear and Lubahn (1952) on Cr-Mo-V steel illustrates this recovery of elastic strain most clearly.

For a thick cylinder which is strained only elastically on loading, Coffin, Shepler and Cherniak (1949) have presented graphs showing how the three principal stresses and the three principal strains vary across the wall during primary creep. By assuming that all initial elastic strain was recoverable for the cylinder, as for uniaxial tension, Coffin et al were also able to present graphs showing the distribution of permanent plastic strain (and residual stresses) in the cylinder wall of instantaneous unloading after creep.

For such a case the evaluation of axial creep strain at any time for experimental results is then straight-forward since the zero axis for all creep strains can be established. The relative magnitudes of axial and tangential creep strains may then be compared to determine if the assumption of zero axial creep strain is a reasonable approximation which may be made in the theory.

The main defect of such a procedure, however, is that the initial elastic strain distribution may alter during creep (especially if large deformations are involved), and if non-compatible amounts of elastic strain exist after creep, then the initial elastic strain will not be fully recovered. An experimental means of checking the stress distribution in the wall of such a cylinder at different stages during creep is desirable. Some of the difficulties involved in making such a check are discussed in sub-section 6.6.3.

The results for magnesium alloy presented in Fig. 9.20 indicated that time-independent plastic strain was present on initial pressurising of the cylinders. The initial stress-strain distribution in the walls must therefore have been that for either a partially or a completely plastic tube, and could have been determined by the method given by Manning (1945) and by Crossland and Bones (1958). The analysis of creep starting from either a partially or a completely plastic initial condition does not seem to have been attempted. However, a similarity exists between the stress distribution in the walls of plastic cylinders presented by Manning, and the stress distribution given by Coffin et al and by Johnson et al for cylinders in the primary creep region. The main features of the deformation of an initially elastic tube would thus be retained since deformation of the plastic tube would appear to start from an initial stress distribution corresponding roughly to that only attained by the initially elastic tube after some deformation.

If sufficient experimental data for the time-independent properties of the magnesium alloy at the test temperature had been obtained, then the initial strain distribution in the tube could have been evaluated. Assuming that initial plastic and elastic strains were unaffected by creep the zero axis for creep strains would then have been determined, making a comparison of the magnitudes of axial and tangential creep strains possible. However, a rough estimate of the initial elastic and plastic strains can be made from Fig. 9.20 and on this basis it is possible to compare the axial and tangential creep strains for specimens 'C' at say 1,000 minutes:

tangential natural creep strain at outside diameter

$$\bar{\varepsilon}_{do} = \log_e\left(\frac{\ell_o + \delta_\ell}{\ell_o}\right) = \log_e\left(\frac{1.125 + 0.0219}{1.125}\right) = \log_e 1.0195$$

$$= 0.0190$$

axial natural creep strain at outside of tube

$$\bar{\varepsilon}_z = \log_e\left(\frac{2.000 + 0.00398}{2.000}\right) = \log_e 1.00199 = 0.0019$$

For the magnesium alloy tube of k = 1.5 at 1,000 minutes, the tangential creep strain at the outside diameter is about 10 times greater than the axial creep strain, and the assumption that the axial creep strain is zero does not appear to be such a good approximation in this case.

6.5.4 Theoretical assumptions concerning axial creep

In Appendix II of the paper by Johnson, Henderson and Khan (1961) three possible assumptions concerning axial strain during the creep of a thick walled cylinder were examined.

1. That axial creep strain is zero at all times and is accompanied by constant elastic axial strain.
2. That axial creep strain is zero at all times and is allied with changing elastic axial strain.
3. That both axial creep strain and elastic strain may change, but that the total axial strain remains constant in value at all times.

The present tests on lead alloy tubes would seem to support assumption (1) where the constant elastic strain is in fact zero. Assumption (2) does not seem likely to the writer. If the initial elastic stresses relax out elastic strain must be exchanged for plastic strain and the assumption then requires that the plastic strain be annihilated. There is however a slight indication of such an occurrence in Fig. 4 of the paper by Norton (1939). A tentative suggestion of the way in which such an apparent effect might be produced would be if the elastic strain gave an initial extension and the subsequent creep strain a contraction, but this seems very unlikely.

The tests on magnesium alloy tubes reported here suggest that the assumption which may apply to most materials of interest to the designer is

4. That the initial axial elastic strain may be recoverable in full or in part, but that the total axial strain does not necessarily remain constant during creep.

6.6 GENERAL BEHAVIOUR OF TUBES

6.6.1 Lead alloy tubes

It is evident from Plate 10.8 that the mechanical properties of the lead on either side of the piping flaw of billet 2 would probably be different, although no attempt was made to establish this point. Comparing the three tests on lead tubes presented in Figs. 9.15 and 9.16, the thinnest tube had the greatest deformation and the thickest tube the least deformation as would be expected. Taking the above facts into consideration, the existence of the piping flaw present in specimens 2A and 2B does not seem to have had any significant effect on the behaviour of the tubes for creep under internal pressure. This indicates that sufficient mechanical locking between inner and outer annuli was present at the end closures to ensure that the inner annulus carried its share of the axial load.

For a duplex tube such as that suggested by Voorhees et al, and discussed in sub-section 1.9.4 prediction of its behaviour would have to take into account the initial fit between the tubes and the difference between the coefficients of thermal expansion of the materials, in order to determine the stress system created on initial heating. Subsequent creep behaviour would then depend on how the end load due to internal pressure was distributed between the components. For a duplex assembly of identical materials, or a tube having a very narrow but rotationally symmetric flaw such as that in the lead of billet 2, the creep behaviour should be identical with that of a monobloc cylinder providing the axial load due to internal pressure acting on the end covers is distributed as for the monobloc cylinder.

6.6.2 Dilation manometer technique

The measurement of the diametral extension at the bore of the test cylinders during creep was undertaken:

(a) to develop a technique which would be of value in situations where external diametral extensometers might not be practicable, e.g. under irradiation, and

(b) to demonstrate the validity of the assumption of incompressible material during plastic flow.

The dilation manometer technique has the disadvantage that two specimens identical in all respects other than in length are required, and that the axial extension of these specimens must be determined by separate means. The advantages are that the dilation manometer may operate at room temperature and the diametral extension measured at the bore of the cylinder is the average over a specific length. Details of the technique of using the manometer were given in sub-section 4.7.4 and in Appendix 8.1.

Since this test programme was initially conceived for lead, which does not exhibit elasticity at the test conditions, the determination of the elastic properties of the test materials was not included. To demonstrate (b) above it is, in general necessary to know the elastic properties of the cylinder material at the test temperature so that allowance can be made for the elastic change in volume. However, since the only experimental result available for the dilation manometer is that given in Fig. 9.16 for lead alloy specimen 1A, the problem with the magnesium specimens does not arise, but future work will have to include an evaluation of the time-independent properties of the test materials at the creep temperatures employed.

The significance of the points A, B and C in Fig. 9.16 has been discussed in section 5.3.3.; for complete incompressibility the points B and C should co-incide. The discrepancy will be partly due to the difference in deformation between the long and short specimens, although inspection of Fig. 9.17 suggests that the greater deformation of the short specimen would have led to a low prediction of the point B and not high as shown in Fig. 9.16.

Dig-in of the anvils of the diametral extensometers had been allowed for as described in sub-section 5.3.3, but the restraining effect on the lead tube due to the pressure of the extensometers probably had an additional effect. Taking such discrepancies into account the assumption of incompressible material does not seem to lead to serious error. Possible refinements to both the dilation manometer and to the external extensometry discussed in section 6.7 should give improved results in future.

6.6.3 Magnesium alloy tubes.

For the investigation of the creep of complex stress systems, the use of a stable metal for experimental investigations is essential. The advantages of a simple solid solution have been stressed in sections 2.3 and 4.1 and the present choice of magnesium (2% aluminium) alloy is attested by the excellent reproducibility of the results for two pairs of thick cylinders presented in Fig. 9.20. The magnesium alloy exhibits elastic strain on initial pressurising as discussed in sub-section 6.5.3.

To compare the several methods of design put forward by Coffin et al, Voorhees et al and Johnson et al (sections 1.6 and 1.9) it would be valuable if the stress distribution in the wall of a thick cylinder could be established experimentally at various times during the life of a tube. Several identical tubes would require to be tested under the same conditions of temperature and pressure, the tests being discontinued at different times when an attempt to determine the stress distribution would be made. The problems to be faced in such a programme are illustrated with reference to the magnesium tubes of Fig. 9.20.

The possibility of recovering the initial elastic strain in full or in part has been discussed in sub-section 6.5.3. For complete elastic recovery the stress system remaining in the tube on instantaneous unloading would resemble that presented by Coffin, Shepler and Cherniak (1949) as Fig. 12 of their paper. An experimental programme would probably aim at such a comparison assuming firstly complete elastic recovery. It is assumed that sufficient short-time and uniaxial data will have been obtained for the material being tested. This is a substantial task; however the main difficulty lies in the experimental determination of the stress distribution in the cylinder after a certain amount of creep.

For uniaxial tension instantaneous elastic recovery after creep has been discussed in sub-section 6.5.3. The workers mentioned previously in this connection were however primarily concerned with <u>time dependent creep</u> recovery after unloading which will also occur here in a complex stress system.

For a thick cylinder immediately after pressure unloading, the instantaneous stress system will resemble that of Fig. 12 of Coffin et al, mentioned previously. If the cylinder is kept at the creep test temperature after unloading two events will occur simultaneously, (a) creep relaxation of the residual stress system and (b) creep recovery of the plastically strained material. The resulting deformation will be similar to that shown in Fig. 9.20 for magnesium and the separation of these two effects does not appear practicable.

An alternative solution would be to prevent further creep in the specimen immediately after unloading, and then examine the frozen residual stress system using the technique employed by Attia, Fitzgeorge and Pope (1954), by Faupel and Furbeck (1953), and by earlier workers listed by Timoshenko (1956). The specimen would however, require to be cooled almost instantaneously, and it is difficult to see how this could be done isothermally in order to avoid the introduction of thermal stresses.

The direct experimental determination of the stress system existing in a tube during creep thus appears to be impracticable. The alternative, though somewhat unsatisfactory, is to obtain by experiment the axial and diametral deformations of a tube subject to creep conditions under internal pressure and compare the observed results with predicted results, calculated for the existing theories using tensile or other data for the material. The present test programme has developed the necessary experimental equipment for such a project which is the logical next step. The following section discusses possible improvements to the present equipment in the light of the writer's experience.

6.7 IMPROVEMENTS TO EXPERIMENTAL TECHNIQUES

6.7.1 General remarks

Some of the improvements listed here have been discussed previously. Where this is the case discussion is limited and the relevant sections are indicated. The general layout adopted for this section corresponds to that of Section 4.

6.7.2 Test material

Preferably the test material should be a simple solid solution, and for temperatures and stresses which are easily handled in the laboratory the major constituent of the solid solution should be either magnesium or aluminium (see 2.3, 4.1, 6.5). Such materials are sufficiently hard to resist indentation by the extensometer anvils.

6.7.3 Creep data

Since tensile data is generally used in the design of tubes it would be useful to establish a valid method of design using data for uniaxial tension. However, the experience of Crossland and Bones and of others, concerning the applicability of torsion data to elastic conditions should not be overlooked and a parallel development of a torsion creep testing unit might well be considered (see 1.4.1, 4.2.3).

Suitable equipment for tests under uniaxial tension may require constant stress devices to obtain creep data at large strains corresponding to those encountered at the bore of thick cylinders (see 4.2.2.) For each specimen a means of applying the load at a fixed rate at the beginning of a test is desirable. (see 6.2).

6.7.4 Isotropy tests

The remarks under sub-section 6.7.3 apply.

6.7.5 Tube tests

The O-ring pressure seal illustrated in Fig. 9.9 should be moved to the spigot to reduce the load on the end closure bolts (see 4.4.2).

6.7.6 Temperature control

Electric resistance furnaces with suitable compensation for end effects should be used, and a positive means of circulating the air in the test section by forced convection through an annulus close to the windings with return through the test section would prove advantageous. A method of controlling the rate of heating of the test specimens is desirable so that each specimen encounters identical conditions prior to testing.

A multipoint potentiometric recorder would greatly reduce the labour involved in observing temperatures.

6.7.7 Pressure control

A low mechanical effort, large swept volume, oil injector (Fig. 9.1) would reduce the setting up time of the dilation manometer considerably. This injector is seldom used at high pressures.

A new weight carrier for the hydraulic accumulator to accommodate lead bricks would allow the pressure range of the accumulator to be covered more easily.

6.7.8 Extensometers

The sensitivity of the dilation manometer can be increased considerably by using narrower bore tubes of greater length. Suitable commercial high pressure manometers are now available (see 4.7.4). On first pressurising the specimens, in order to minimise the zero shift of the mercury level in such a sensitive manometer, it is useful to reduce the oil volume trapped above the mercury level by the use of bore bars in the specimens. These can be made integral with the top closures so that no extra weight has to be carried by the end load compensators.

Improved furnace performance is obtained if windows are not required for extensometer mirrors. For measurement of the axial extension of thick cylinders, Martens type extensometers of the NPL pattern for uniaxial tension may be attached directly to the tubular specimens. For measurement of the diametral extension, probe type diametral extensometers requiring only small diameter holes through the furnace wall are recommended. The reference block for such probes would be an Invar ring surrounding the furnace. For large extensions 0.0001 inch clock gauges would suffice, for small extensions the differential motion between each probe and the Invar reference block would be determined by the use of a Martens type rhomb and mirror assembly. The deflection of the mirror would be observed with the conventional telescope and scale system. The diametral extension of the test cylinder would thus be the sum of the readings obtained on opposite sides of the cylinder.

For the probe and rhomb system, radial micrometer travel of the knife edges mounted on the reference block would permit resetting of the extensometers during testing. The probes themselves would be horizontally supported external to the furnace by a suitable arrangement of low friction Vee rollers, thus making attachments to the test specimens unnecessary.

Because of the small space taken up within the furnace, the probe type diametral extensometer allows an increase in the number of diametral measurements which may be made on any one specimen. It would be possible to arrange to measure the extensions at two or more diameters at one station, and at two or more stations along the length of a single specimen.

Finally, to permit accurate evaluation of creep strain in the tests on uniaxial tension, or in the tests on cylinders, the development of equipment to record the time-independent stress-strain curves at the instant of loading is required. The development of probe-type diametral extensometers and the use of NPL type axial extensometers should ease this problem considerably, since transducers can then operate at room temperature by sensing the movement of the extensometer limbs external to the furnace.

CHAPTER 7

7. CONCLUSIONS

1. The basic theory of creep applied to thick cylinders has been reviewed, the important papers have been indicated and assumptions made in the theory brought out (Sections 1.1 to 1.7).

2. The theory underlying the simplified technique used by Voorhees et al to obtain the variation with time of the stress distribution in a cylinder wall has been discussed ~~given~~, and their solution improved ~~to take into account the effect of initial elastic strain~~ (Sections 1.5 and 8.3).

3. A tentative design philosophy for thick cylinders has been indicated, employing the concept of addibility of rupture life and applying both the Mohr and the Maxwell criteria of failure (Sections 1.8 and 1.9).

4. Methods for the correlation and extrapolation of data have been reviewed and the importance of correct metallurgical interpretation has been emphasised (Chapter 2).

5. Previous experimental work on the creep of tubes subjected to internal pressure has been discussed. The accuracy of each experimental technique is evaluated in terms of the applicability of its results to proposed theories of design. (Chapter 3).

6. Equipment has been designed and commissioned to test cylinders under conditions as nearly mathematically exact as practicable, and extensometers have been constructed to permit measurement of tube deformation during creep (Introduction and Chapter 4).

7. Tests have been carried out on lead alloy tubes of k = 3 and on magnesium alloy tubes of k = 1.5 subjected to constant internal pressure under creep conditions. Axial creep was observed in every test, but the amount of axial creep may depend on:-

(a) the material of the tube

and (b) the k value of the tube. (Chapters 5 and 6).

8. The results obtained indicate that, as a first approximation in design, the axial creep in a thick cylinder may be considered to be zero. For some materials, however, the axial creep strain may be almost within an order of magnitude of the diametral creep strain at the outside diameter of the tube for certain values of k. (Chapter 6).

9. Because of the nature of the deformation, true secondary (minimum) creep behaviour cannot exist in a thick cylinder. Any reference method of design must be based on elastic, time-independent plastic, primary creep and tertiary creep deformation, and take into account the fact that different materials have different criteria of rupture in creep (Section 1.8 and Chapter 6).

10. The further development of the test programme has been discussed and recommendations have been made concerning improvements to the present apparatus (Section 6.7).

CHAPTER 8

8. APPENDIX

8.1 DILATION MANOMETER

The determination of bore strain by dilation manometer is not an absolute technique since additional axial strain measurement is always required (although for a closed end cylinder, the axial strain may be negligible).

Allowance has also to be made for the compressibility of the straining medium when the pressure is first applied. Either the exact volume of the oil above each mercury surface must be known directly, or some method employed for determining the compressibility of the unknown volume.

In the present work, accurate measurement of the volume of oil was impracticable and the compressibility of the oil is allowed for by observing the drop in level of each mercury column on pressure unloading the specimens. If the material being tested shows creep recovery on pressure unloading it is necessary in addition to record simultaneously the recovery indicated by the external diametral extensometers, the unloading being carried out before localised bulging has taken place.

Two specimens of differing gauge length are used to eliminate end effects. The smaller specimen is to be considered cut in the centre of its gauge length, perpendicular to its axis, to give two equal portions. These halves areimagined to be subtracted one from each end of the larger specimen, leaving the centre portion of the longer tube as the test section.

At time t, after the application of pressure, the mercury level for the larger specimen will have risen by a height h_t which gives a measure of the sum of the compressibility of the trapped oil (V_{comp}), the instantaneous internal volume expansion on pressure loading ($V_{\substack{inst\\expan}}$) and the subsequent creep extension at time t ($V_{t\,\substack{creep\\expan}}$):

i.e.

$$V_{comp} + V_{\substack{inst\\expan}} + V_{t\,\substack{creep\\expan}} = C.\,h_t \qquad .....(1)$$

where C is the constant of proportionality relating unit length of mercury column to the volume of oil inside the specimen, allowance being made for the difference in the temperature of the oil in the mercury column and in the specimen.

The test is continued to a point short of where localised bulging occurs.

At time T when the manometer level has risen to a height h_T above the initial position, the pressure is unloaded and an interval allowed for complete creep recovery until no further alteration in mercury level is noted, the new level being taken as h_R above the initial position. The drop in level from h_T to h_R is given by the sum of the initial compressibility of the oil (V_{comp}), the instantaneous internal recovery on unloading ($V_{T\,\substack{inst\\recov}}$) and the subsequent creep recovery ($V_{T\,\substack{creep\\recov}}$)

$$V_{comp} + V_{T\,\substack{inst\\recov}} + V_{T\,\substack{creep\\recov}} = C(h_T - h_R) \qquad ...(2)$$

Since h_t, h_T and h_R are all measured values, and C is known, we may subtract equation (2) from equation (1):

$$(V_{\substack{inst\\expan}} + V_{\substack{t\ creep\\expan}}) - (V_{\substack{T\ inst\\recov}} + V_{\substack{T\ creep\\recov}}) = C\,(h_t - h_T + h_R) \quad .. (3)$$

Denoting the shorter specimen by * we have, similarly

$$(V_{\substack{inst\\expan}}{}^{*} + V_{\substack{t\ creep\\expan}}{}^{*}) - (V_{\substack{T\ inst\\recov}}{}^{*} + V_{\substack{T\ creep\\recov}}{}^{*}) = C^{*}(h_t{}^{*} - h_T{}^{*} + h_R{}^{*}) \quad ....(4)$$

Subtracting equation (4) from equation (3) eliminates the end effects for the longer specimen and gives a volume change for the test section equal to:

$$(V_{\substack{inst\\expan}} - V_{\substack{inst\\expan}}{}^{*}) + (V_{\substack{t\ creep\\expan}} - V_{\substack{t\ creep\\expan}}{}^{*}) - (V_{\substack{T\ inst\\recov}} - V_{\substack{T\ inst\\recov}}{}^{*}) - (V_{\substack{T\ creep\\recov}} - V_{\substack{T\ creep\\recov}}{}^{*})$$

where

$(V_{\substack{inst\\expan}} - V_{\substack{inst\\expan}}{}^{*})$ is instantaneous internal (volume) extension on loading,

$(V_{\substack{t\ creep\\expan}} - V_{\substack{t\ creep\\expan}}{}^{*})$ is subsequent (volume) creep extension to time t during loading,

$(V_{\substack{T\ inst\\recov}} - V_{\substack{T\ inst\\recov}}{}^{*})$ is instantaneous internal (volume recovery on unloading at time T,

$(V_{\substack{T\ creep\\recov}} - V_{\substack{T\ creep\\recov}}{}^{*})$ is subsequent (volume) creep recovery after unloading at time T.

If no creep recovery of the metal occurs on unloading then the terms in $V_{\substack{T\ inst\\recov}}$ and $V_{\substack{T\ creep\\recov}}$ disappear and the manometer gives a direct measure of the bore extension if the axial movement is known. If creep recovery occurs then allowance for this has to be made from readings taken by the external diametral extensometer and the bore strain subsequently computed.

8.2 METALLOGRAPHIC EXAMINATION

8.2.1. Lead alloy

This work was carried out by the writer at the Mechanical Engineering Research Annexe of the University of Glasgow, with the advice and assistance of Mr. W.W. Mackie.

Specimen preparation. Whenever possible, the surface to be examined was machined to a flat face on a lathe using successively lighter and lighter cuts to ensure a minimum of flowed surface. The finely machined face was polished with the lightest of pressures on No. 0 Emery paper, backed by plate glass and wetted with paraffin. It was found essential to replace the paper very frequently to ensure even cutting and eliminate clogging - which could cause deep scratching. The emery paper replacement process, considered extravagent when polishing harder metals, was necessary for lead. The above process was repeated using No. 00 paper, and even lighter pressures, again changing the paper before it even appeared worn.

The above procedure, after much experiment, appeared to offer the best chance of success and it may be noted, was associated with the least removal of metal by polishing. It was found inadvisable to use metal polish on Selvyt since considerable smearing resulted.

The etching agent used was Russell's solution - Ammonium Molybdate in Nitric Acid - given in BS.602. The remaining procedure was similar to that suggested in the above standard.

Microstructure. This is discussed in Sections 5.1.2, 5.2.3 and 5.3.1.

8.2.2 Magnesium alloy

This examination was carried out at the Nuclear Research Centre of C.A. Parsons & Company Limited by a member of the staff of the Metallographic Section, by courtesy of Mr. J.K. O'Hanlon, head of the Metallographic Section.

Specimen preparation. Transverse, longitudinal and tangential sections were cut from the original billet and from the central portion of a cylinder after testing. The sections were mounted in polyester resin, ground on silicon carbide papers and polished on microcloth with a 6μ and finally 1μ diamond paste. The following etchants were used:-

(1) for photomicrography — 80 ccs of 2% nital
20 ccs of 5% nitric acid

(2) for grain size determination — 100 ccs of 5% picric acid in alcohol
5 ccs of distilled water
5 ccs of glacial acetic acid.

Microstructure. Photomicrographs of the billet and the tube (x 150 plates 10 - 14 and 10 - 15) showed both intergranular and transgranular particles. Intergranular particles in the tube were, however, smaller and more numerous than those in the billet. Both samples contained small amounts of non-metallic inclusions. The twinning evident for the tube sections is characteristic of deformed magnesium when the deformation and/or temperature are not sufficient to bring about recrystallisation of the alloy.

The grain size photographs (x 50 plates, 10-16 to 10-21 inclusive) show that there was no significant difference between grain sizes in the billet and in the tube, and that the amount of deformation was not sufficient to cause measurable elongation of the grains.

8.3 Modified Theory of Voorhees, Sliepcevich and Freeman

The basis of the method has previously been described in sections 1.6 and 1.7. The present analysis however retains the concept of the tube throughout, whereas Voorhees et al attempted to simplify the problem by replacing the annuli of the cylinder with imaginary tensile specimens held at equivalent uniaxial stresses and strains based on Maxwell's criterion. They then considered only the differential straining of these tensile specimens. A substantial discrepancy exists between the results of the technique used by Voorhees et al and the method presented here.

The cylinder is assumed to be divided into annuli, as described in section 1.6. The annuli are denoted α, β, γ, δ, etc. from the bore outwards. The radius of gyration of each annulus is r_α, r_β, r_γ etc. and the k value of each annulus is k_α, k_β, k_γ etc. The radius of the bore (outside) of the cylinder is r_a (r_b) and the radii of contact between annuli are $r_{\alpha\beta}$, $r_{\beta\gamma}$, $r_{\gamma\delta}$ etc.

Creep data for the material of the cylinder are required from the results of tensile (or other) creep tests. First curves of ε_{oct} versus t (time) are constructed with τ_{oct} as parameter, and then curves of $\dot{\varepsilon}_{oct}$ versus t, again with τ_{oct} as parameter.

The life of the tube is divided into a series of small time intervals Δt denoted by superscripts ', ", "', etc. From inspection of the $_{oct}$ versus t curves mentioned above the first time interval $\Delta t'$ is selected such that the octahedral creep rates at any given τ_{oct} are sensibly constant during this interval.

On pressurising the cylinder the initial stress distribution in the wall is elastic (providing the applied pressure is not too great) and the elastic octahedral shear stress at the radius of gyration of each annulus may be found using Lame's equations, giving τ_{oct_α}', τ_{oct_β}' etc. The elastic strains may also be determined and this is the initial condition for zero-misfit of the annuli.

By using the $\dot{\varepsilon}_{oct}$ versus t graph mentioned above, in the first time interval $\Delta t'$ the octahedral shear creep rates $\dot{\varepsilon}_{oct_\alpha}'$, $\dot{\varepsilon}_{oct_\beta}'$ etc. corresponding to τ_{oct_α}', τ_{oct_β}' etc. may be determined.

For a thick cylinder under creep conditions, for small strains $\varepsilon_t + \varepsilon_r + \varepsilon_z = 0$. Assuming no axial creep $\varepsilon_z = 0$ and it follows that $\varepsilon_{oct} = 2\sqrt{\frac{2}{3}}\ \varepsilon_t$

Hence $$\dot{\varepsilon}_{oct} = 2\sqrt{\frac{2}{3}}\ \dot{\varepsilon}_{tan} \qquad \text{...........(1)}$$

so that the tangential creep rates $\dot{\varepsilon}_{t_\alpha}'$, $\dot{\varepsilon}_{t_\beta}'$ etc. may be found for each radius of gyration.

If each annulus was free to deform under creep then the unrestrained radial deformation at the radius of gyration of each annulus would be $(\dot{\varepsilon}_t' . \Delta t')\, r\, (1 + \varepsilon_{te})$ where ε_{te} is the elastic tangential strain on loading. Neglecting second order effects for small strains this becomes $(\dot{\varepsilon}_t' . \Delta t')\, r$.

For small strains the compatibility equation for creep of a cylinder with no axial deformation is ur = const. Thus at the $\alpha\beta$ interface the radial creep deformation at the outside of the α annulus is

$$(\dot{\mathcal{E}} t_{\alpha}' \cdot \Delta t') \frac{r_{\alpha}^2}{r_{\alpha\beta}} \qquad ......(2)$$

and at the inside of the β annulus is

$$(\dot{\mathcal{E}} t_{\beta}' \cdot \Delta t') \frac{r_{\beta}^2}{r_{\alpha\beta}} \qquad ......(3)$$

In general the strain at the outside of the α annulus will not be the same as the strain at the inside of the β annulus. To ensure that the α annulus and the β annulus will fit together at the end of time interval $\Delta t'$ it is therefore necessary to assume a contact pressure $P_{\alpha\beta}'$ between them. This imaginary pressure is assumed to act elastically and to reduce the elastic strain for annulus α while increasing the elastic strain for annulus β.

Using Lamé's equations for a thick elastic cylinder the tangential strain at $r_{\alpha\beta}$ for annulus α caused by the enternal pressure $P_{\alpha\beta}'$ acting at the outside radius $r_{\alpha\beta}$ is given by

$$\frac{1}{E}\left(\frac{-P_{\alpha\beta}'}{K_{\alpha}^2 - 1}\right)\left[K_{\alpha}^2(1 - 2\eta) + K_{\alpha}^2(1 + \eta)\right] \qquad ......(4)$$

Similarly the tangential strain at $r_{\alpha\beta}$ for annulus β caused by the internal pressure $P_{\alpha\beta}'$ acting at the inside radius $r_{\alpha\beta}$ is given by

$$\frac{1}{E}\left(\frac{P_{\alpha\beta}'}{K_{\beta}^2 - 1}\right)\left[(1 - 2\eta) + K_{\beta}^2(1 + \eta)\right] \qquad .....(5)$$

Since at the $\alpha\beta$ interface we have a common radius $r_{\alpha\beta}$ we may equate strains, using equations (2),(3),(4) and (5) to find $P_{\alpha\beta}'$, thus

$$(\dot{\varepsilon}t_{\alpha}' \cdot \Delta t')\frac{r_{\alpha}^2}{r_{\alpha\beta}} - \frac{1}{E}\left(\frac{P_{\alpha\beta}'}{K_{\alpha}^2 - 1}\right)\left[K_{\alpha}^2(1-2\gamma) + (1+\gamma)\right]$$

$$= (\dot{\varepsilon}t_{\beta}' \cdot \Delta t')\frac{r_{\beta}^2}{r_{\alpha\beta}} + \frac{1}{E}\left(\frac{P_{\alpha\beta}'}{K_{\alpha}^2 - 1}\right)\left[(1-2\gamma) + K_{\beta}^2(1+\gamma)\right]$$

After some reduction this gives

$$P_{\alpha\beta}' = \frac{E}{(2-\gamma)}\frac{(K_{\beta}^2-1)(K_{\alpha}^2-1)}{(K_{\beta}^2 . K_{\alpha}^2-1)}\frac{1}{r_{\alpha\beta}^2} . \Delta t'\left[\dot{\varepsilon}t_{\alpha}' . r_{\alpha}^2 - \dot{\varepsilon}t_{\beta}' . r_{\beta}^2\right] \quad \ldots(6)$$

The effect of the pressure $P_{\alpha\beta}'$ is now determined at the radius of gyration of the α annulus and of the β annulus in turn.

For the α annulus with external pressure $P_{\alpha\beta}'$, Lamé's equations give the reduction in octahedral shear stress at the radius of gyration r_{α} as

$$\Delta\tau_{oct_{\alpha}',\alpha\beta} = \sqrt{\frac{2}{3}}\frac{1}{(K_{\alpha}^2-1)}\quad\frac{r_{\alpha\beta}^2}{r_{\alpha}^2} \cdot P_{\alpha\beta}'$$

Substituting for $P_{\alpha\beta}'$ from equation (6) and using equation (1)

$$\Delta\tau_{oct_{\alpha}'\alpha\beta} = \frac{E}{2(2-\gamma)} \cdot \frac{(K_{\beta}^2-1)}{(K_{\beta}^2 . K_{\alpha}^2-1)} \cdot \frac{1}{r_{\alpha}^2} . \Delta t'\left[\dot{\varepsilon}_{oct_{\alpha}}' . r_{\alpha}^2 - \dot{\varepsilon}_{oct_{\beta}}' . r_{\beta}^2\right] \quad \ldots\ldots(7)$$

The octahedral shear stress at r_{α} at the beginning of the second time interval $\Delta t''$ is thus

$$\tau_{oct_{\alpha}}'' = \tau_{oct_{\alpha}}' - \Delta\tau_{oct\,\alpha',\alpha\beta} \quad \ldots\ldots\ldots(8)$$

For the β annulus with internal pressure $P_{\alpha\beta}'$, Lamé's equations give the increase in octahedral shear stress at the radius of gyration r_{β} as

$$\Delta\tau_{oct_{\beta}',\alpha\beta} = \sqrt{\frac{2}{3}}\;\frac{1}{(K_{\beta}^2-1)}\frac{K_{\beta}^2}{r_{\beta}^2} \cdot r_{\alpha\beta}^2 \cdot P_{\alpha\beta}'$$

Substituting for $P_{\alpha\beta}'$ from equation (6) and using equation (1)

$$\Delta\tau_{oct_{\beta,\alpha\beta}}' = \frac{E}{2(2-\gamma)} \frac{(K_\alpha^2-1)}{(K_\beta^2 . K_\alpha^2 - 1)} \cdot \frac{K_\beta^2}{r_\beta^2} . \Delta t' \left[\dot{\varepsilon}_{oct_\alpha}' . r_\alpha^2 - \dot{\varepsilon}_{oct_\beta}' . r_\beta^2\right] \quad ....(9)$$

Similarly for the β annulus at the $\beta\gamma$ interface, the reduction in octahedral shear stress at the radius of gyration r_β is given by an equation similar to equation (7)

$$\Delta\tau_{oct_{\beta,\beta\gamma}}' = \frac{E}{2(2-\gamma)} \frac{(K_\gamma^2-1)}{(K_\gamma^2 . K_\beta^2 - 1)} \frac{1}{r_\beta^2} . \Delta t' \left[\dot{\varepsilon}_{oct_\beta}' \, r_\beta^2 - \dot{\varepsilon}_{oct_\gamma}' \cdot r_\gamma^2\right] \quad .....(10)$$

The octahedral shear stress at r_β at the beginning of the second time interval $\Delta t''$ is thus

$$\tau_{oct_\beta}'' = \tau_{oct_\beta}' - \Delta\tau_{oct_{\beta,\alpha\beta}}' + \Delta\tau_{oct_{\beta,\beta\gamma}}' \quad .....(11)$$

Similar expressions apply for all other annuli. The above relationships apply for small strains only and are approximate. The complete stress distribution across the wall of the cylinder after time interval $\Delta t'$ is thus calculable, and this distribution is then taken as the initial condition for the second time interval $\Delta t''$ and so on.

Although equations of the type (7) and (9) may appear complex, their manipulation only involves variables Δt and $\dot{\varepsilon}_{oct_r}'$, once the coefficients for each annulus have been worked out.

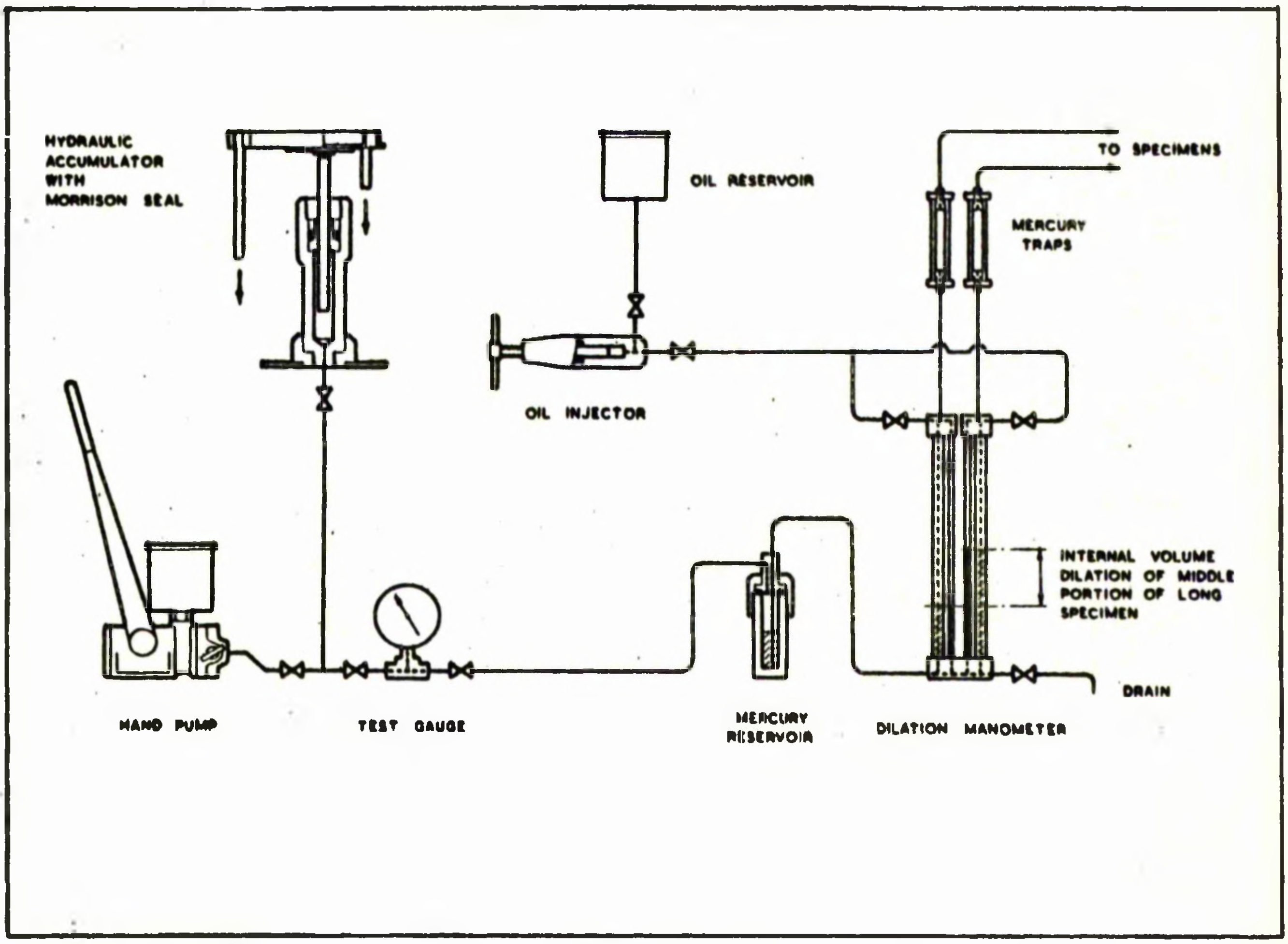
The equations of Voorhees et al corresponding to equations (7) and (9) respectively, but for effective tensile stress are

$$\Delta\sigma_{eff_{\alpha,\alpha\beta}}' = \frac{E}{(1+\gamma)} \cdot \frac{A_\beta}{A_\alpha + A_\beta} . \Delta t' \left[\dot{\varepsilon}_{eff_\alpha}' - \dot{\varepsilon}_{eff_\beta}'\right] \quad ....(12)$$

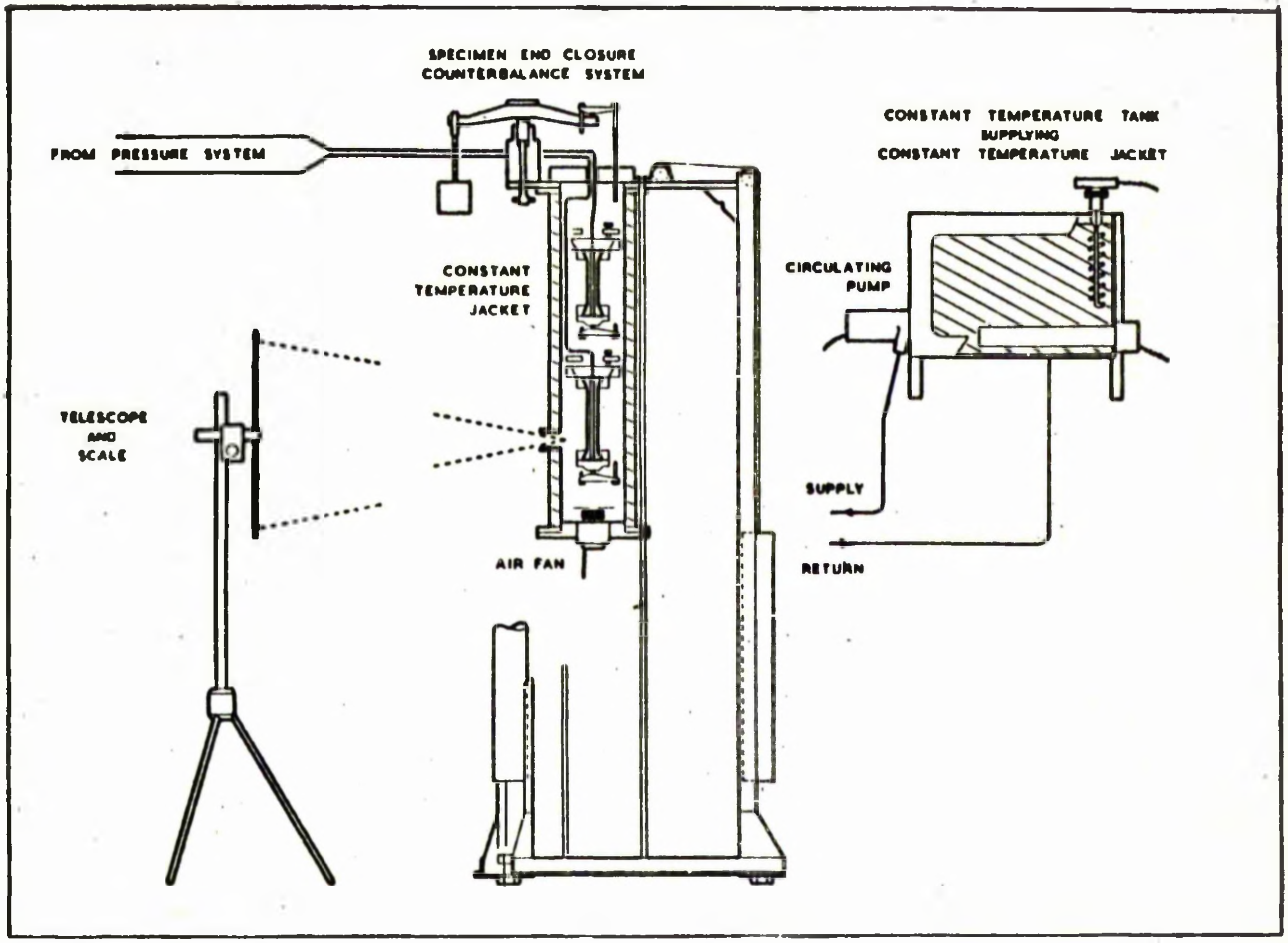
and

$$\Delta\sigma_{eff_{\beta,\alpha\beta}}' = \frac{E}{(1+\gamma)} \cdot \frac{A_\alpha}{A_\alpha + A_\beta} \cdot \Delta t' \left[\dot{\varepsilon}_{eff_\alpha}' - \dot{\varepsilon}_{eff_\beta}'\right] \quad ....(13)$$

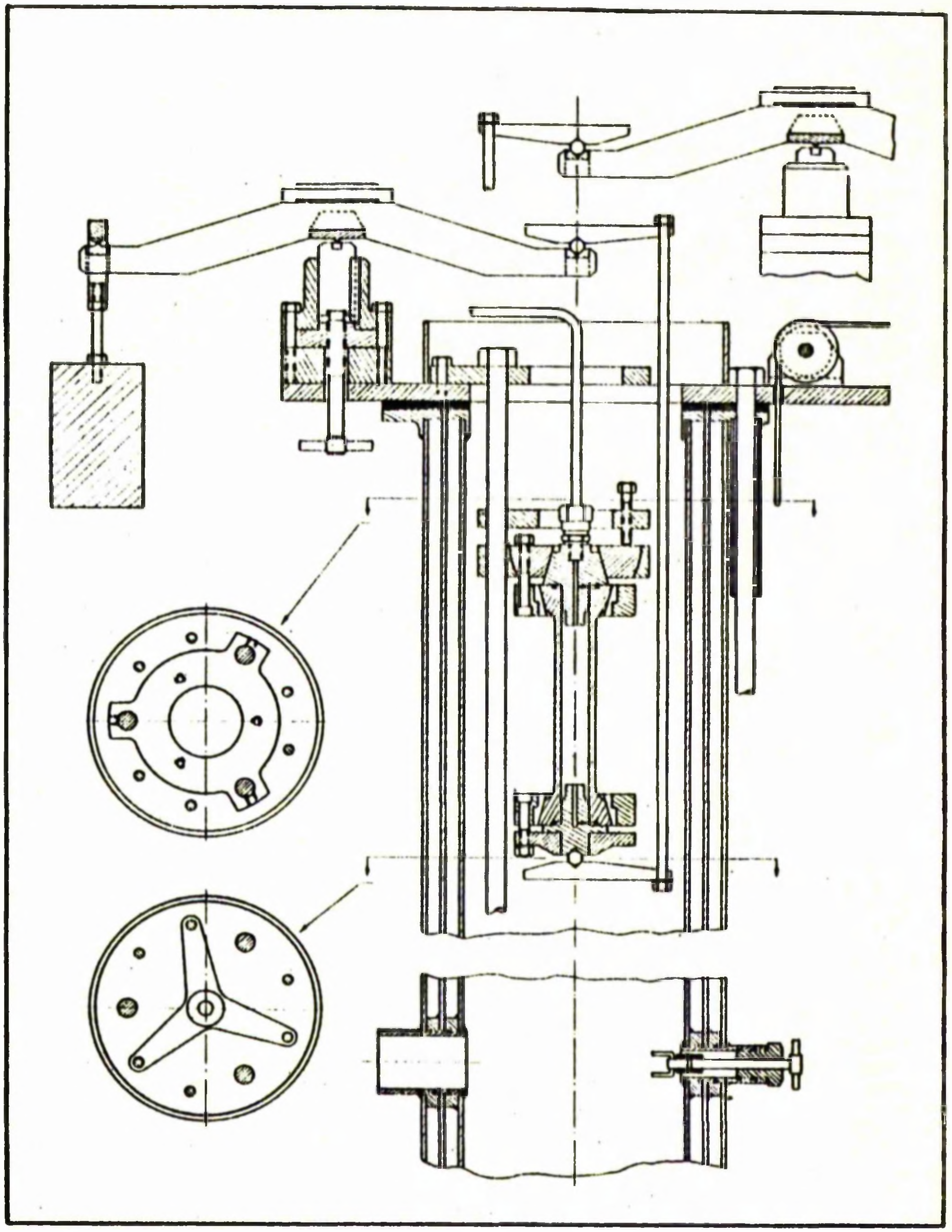
The most striking difference is the omission in equations (12) and (13) of the radius terms associated with the creep rates. This would seem to indicate that, by thinking in terms of effective tensile stress, <u>strains</u> have been equated at different radii in the cylinder which is not permissible.



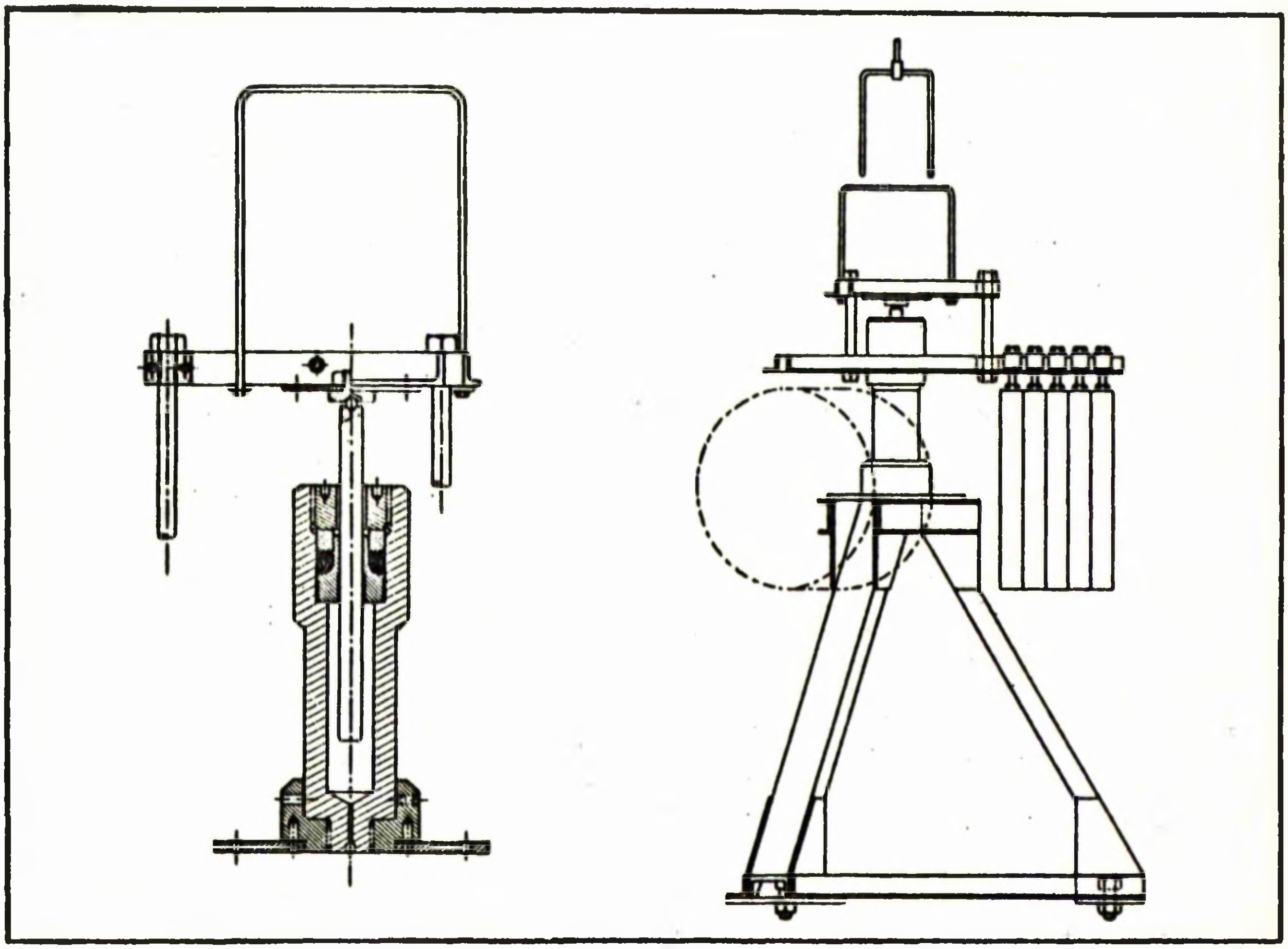
9.1 Simplified general arrangement of pressure system.



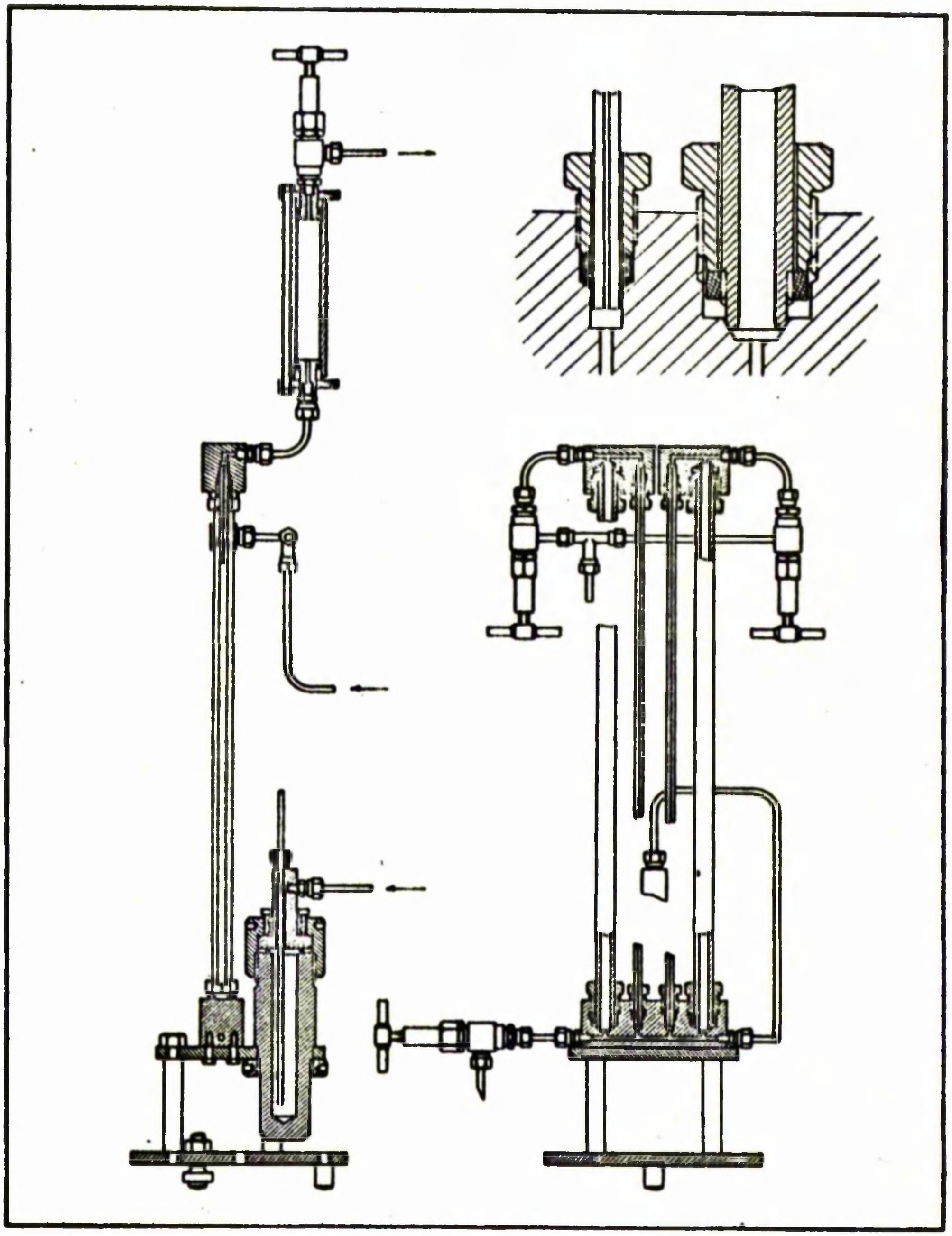
9.2 Simplified general arrangement of constant temperature system and specimen assemblies.



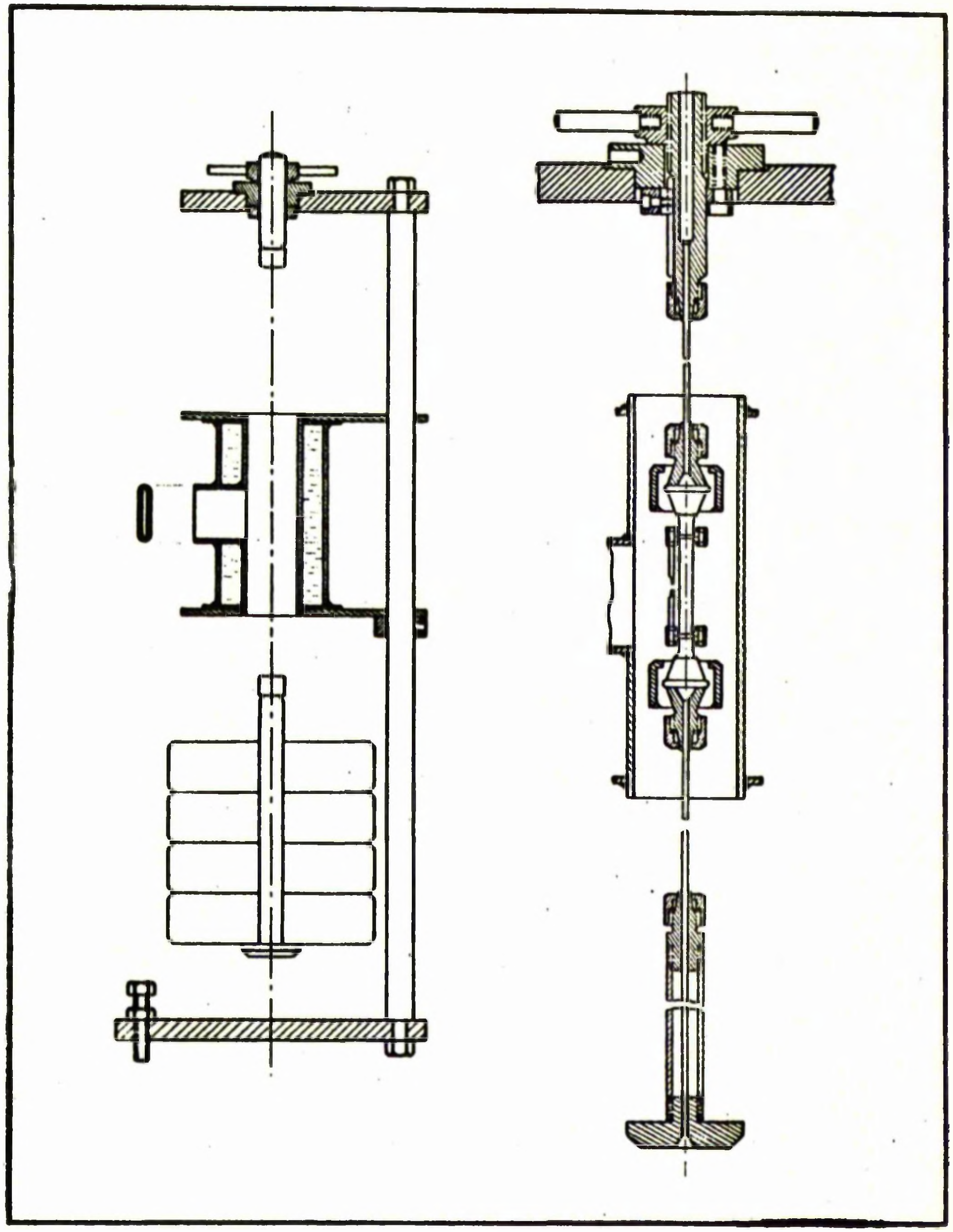
9.3 General arrangement of large constant temperature jacket showing supports and end loading mechanism for a short magnesium alloy cylinder.



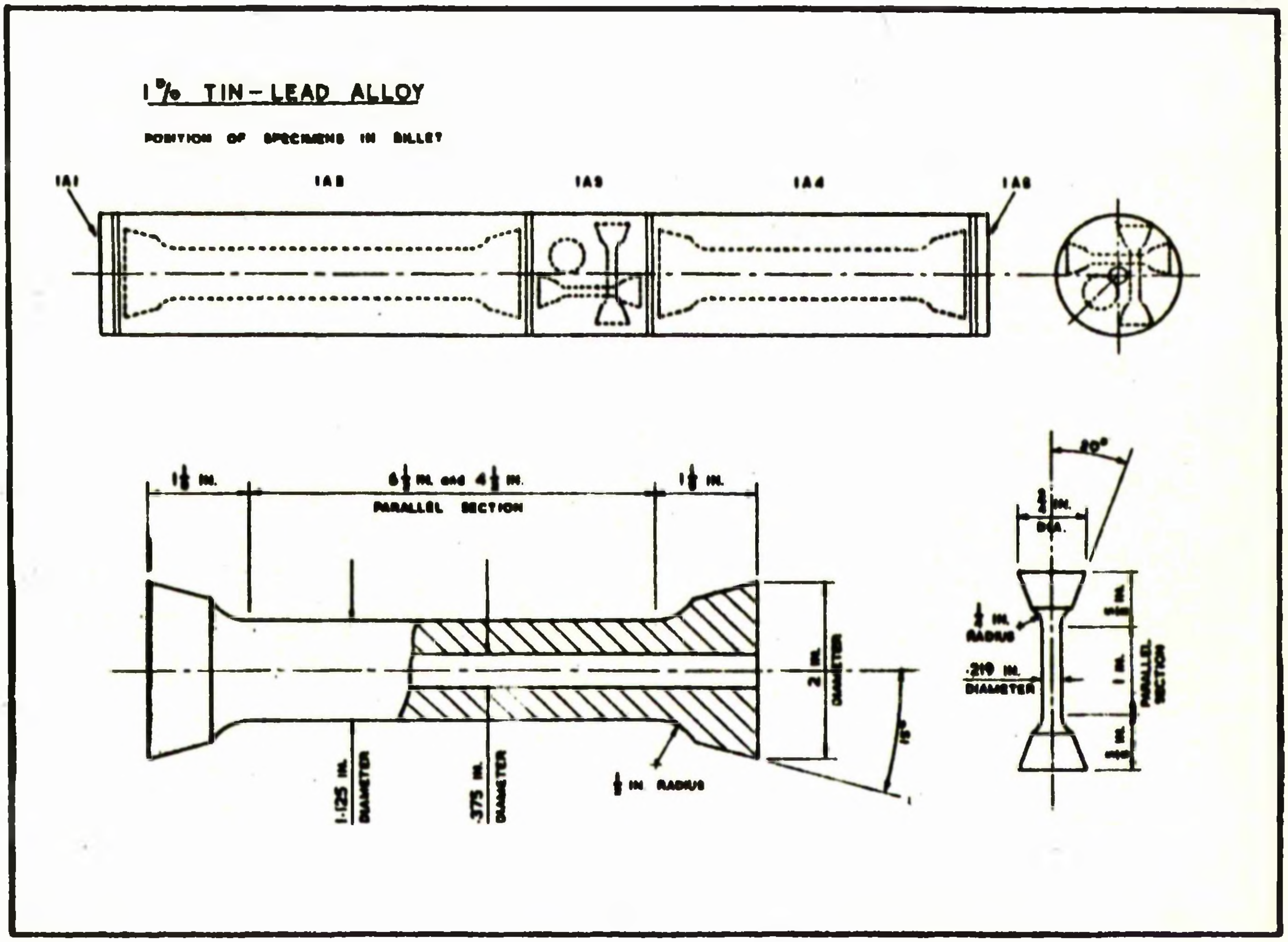
9.4 General arrangement of hydraulic accumulator.



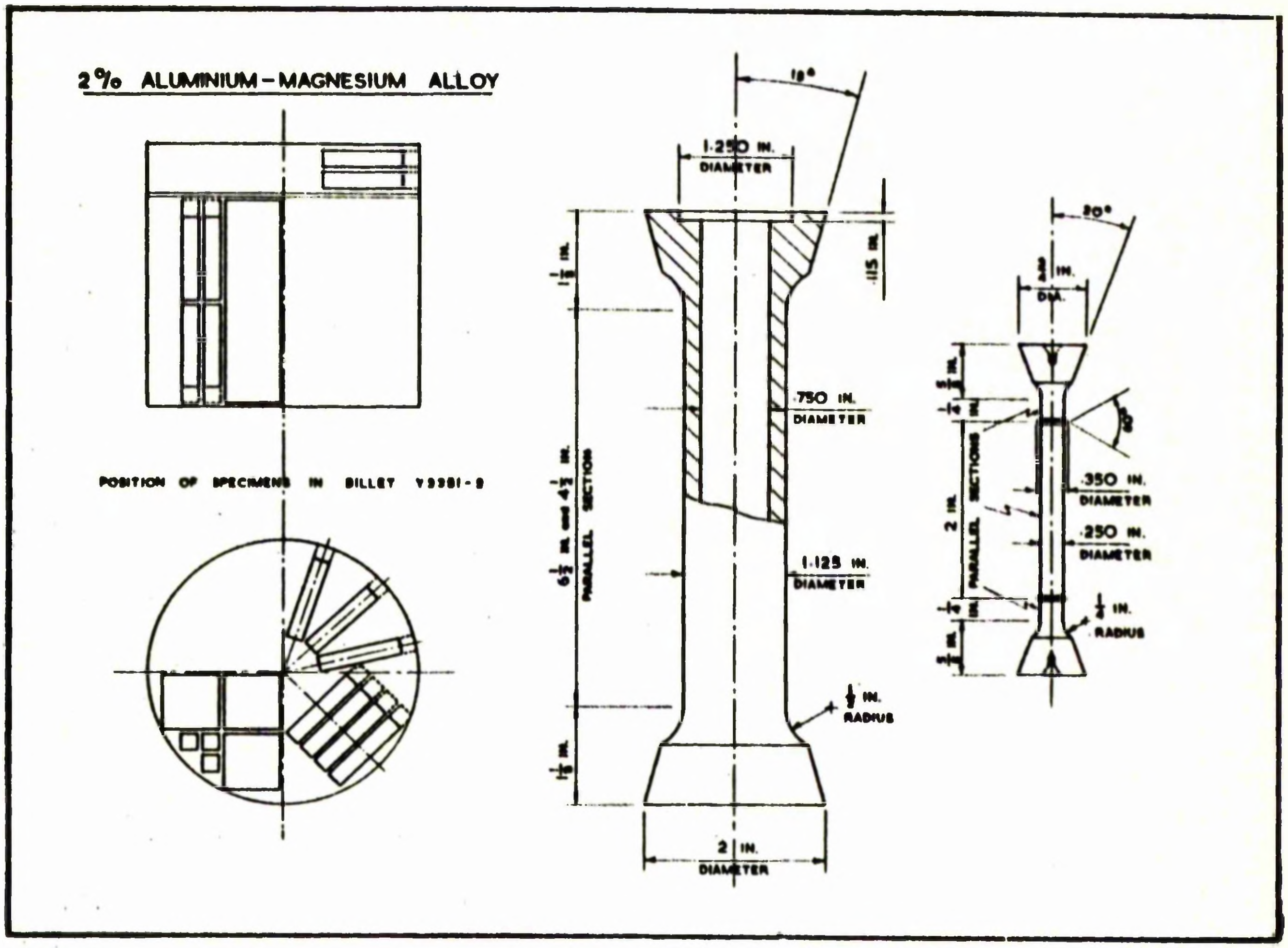
9.5 General arrangement of high pressure mercury manometer.



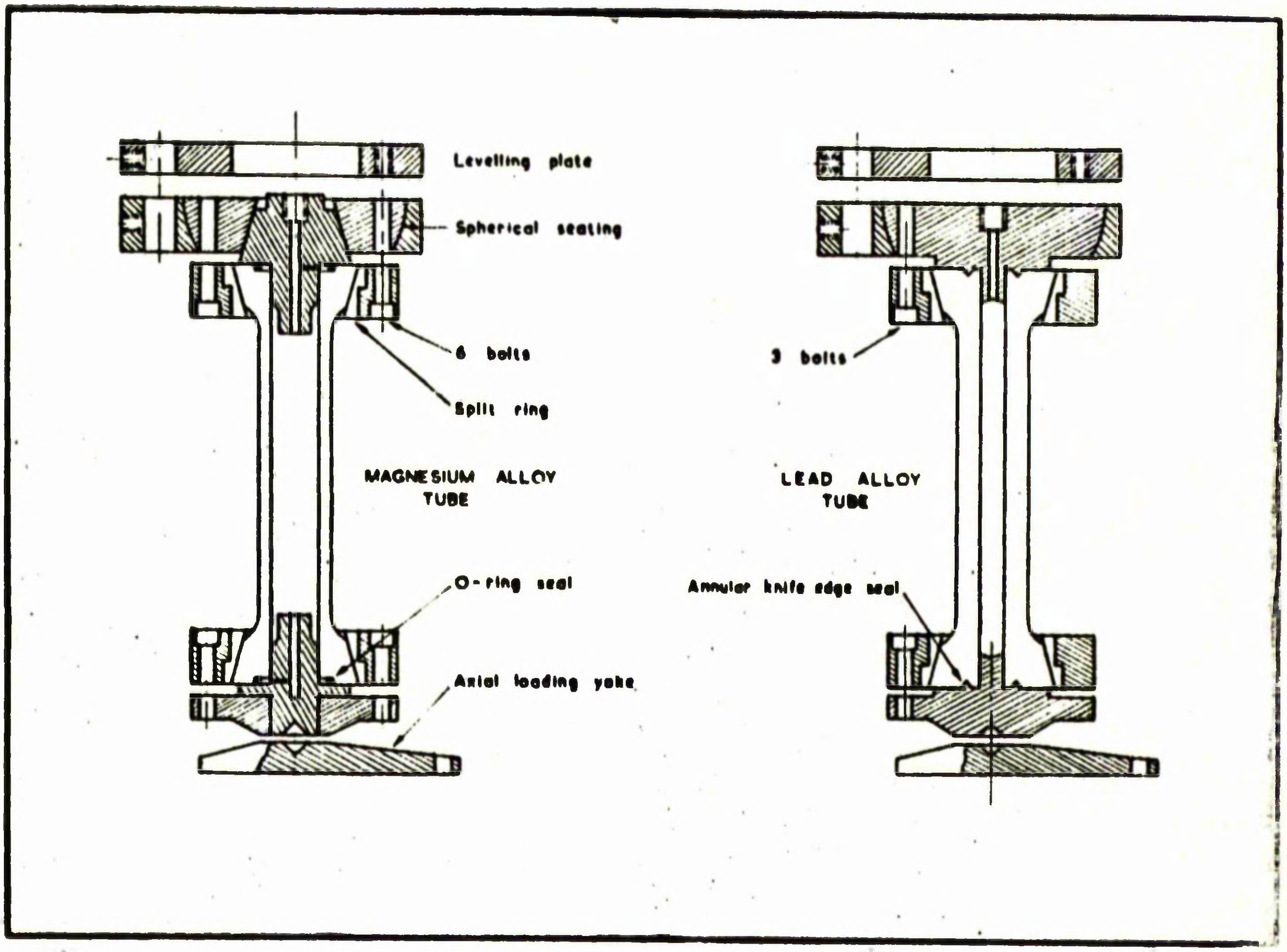
9.6 General arrangement of tensile test rig for investigation of isotropy.



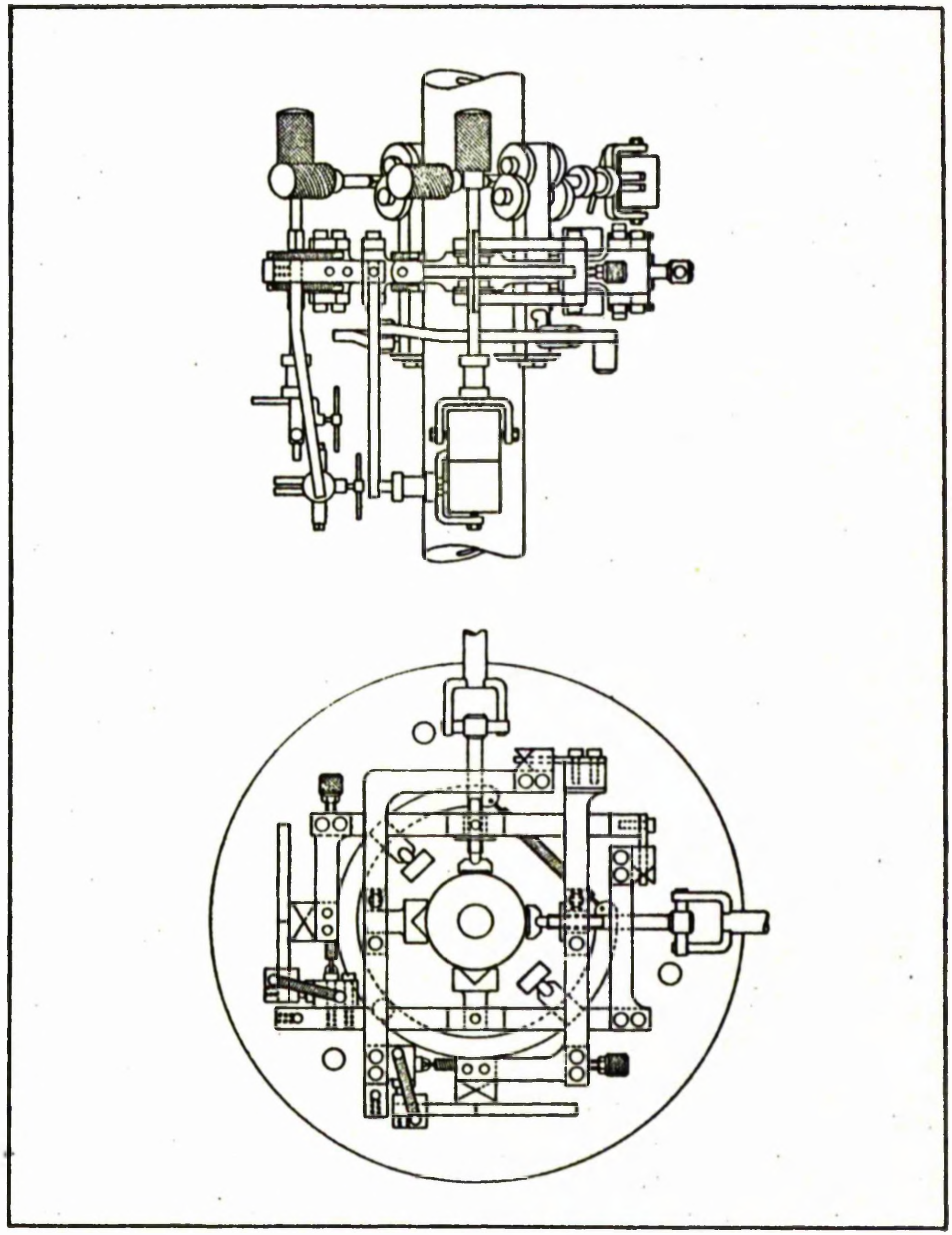
9.7 Details of lead specimens.



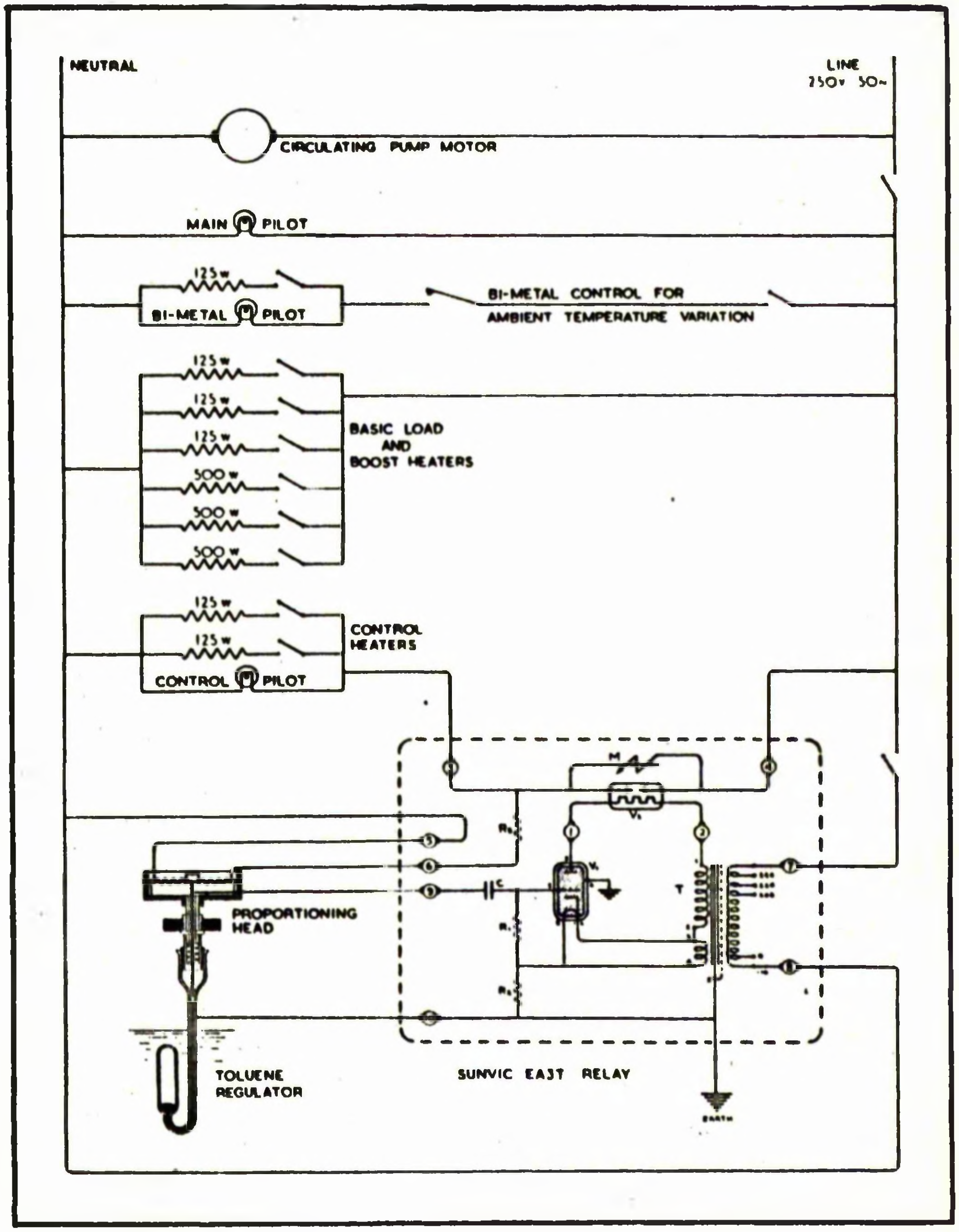
9.8 Details of magnesium specimens.



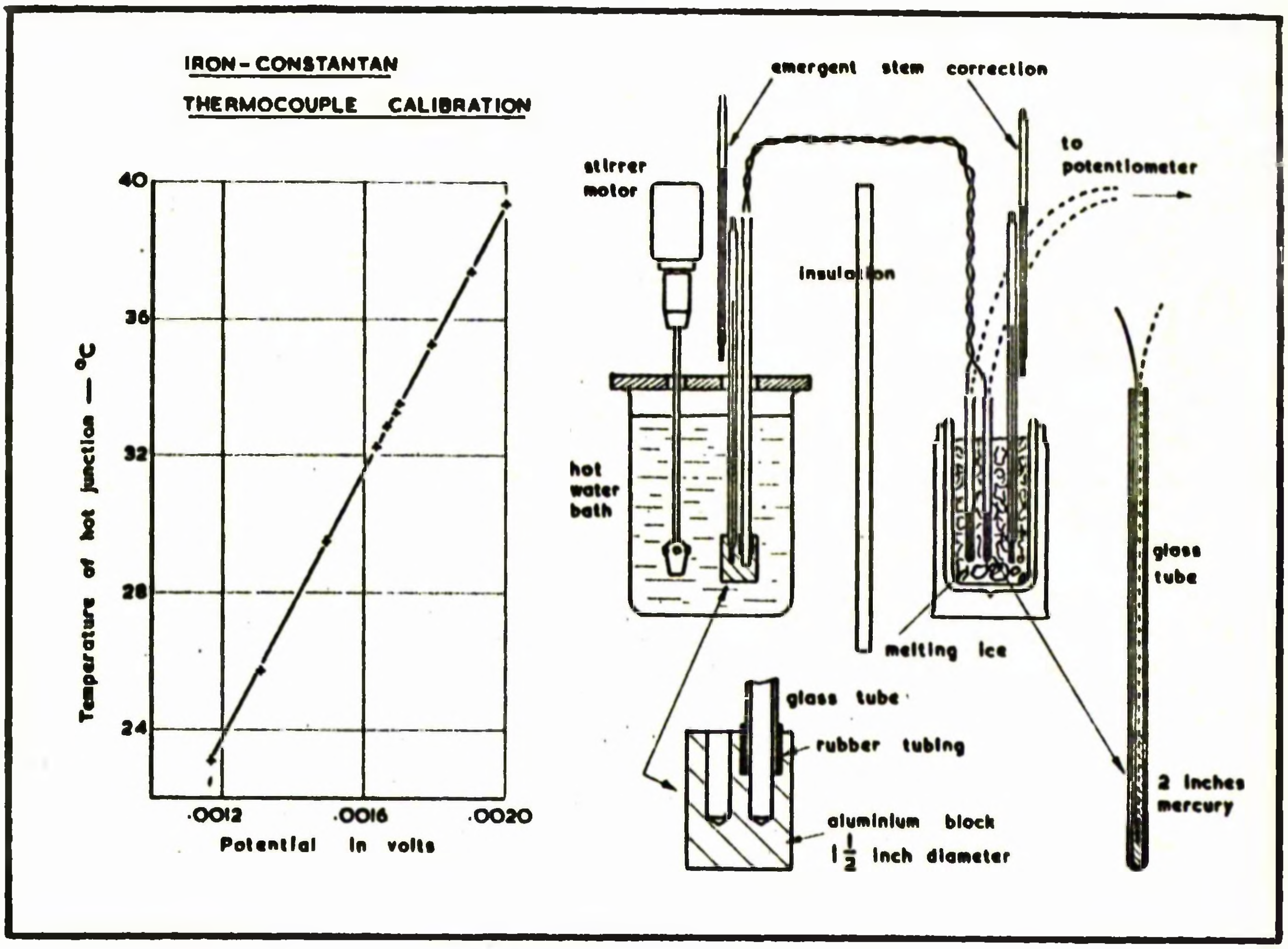
9.9 Details of end closures for lead alloy and magnesium alloy cylinders.



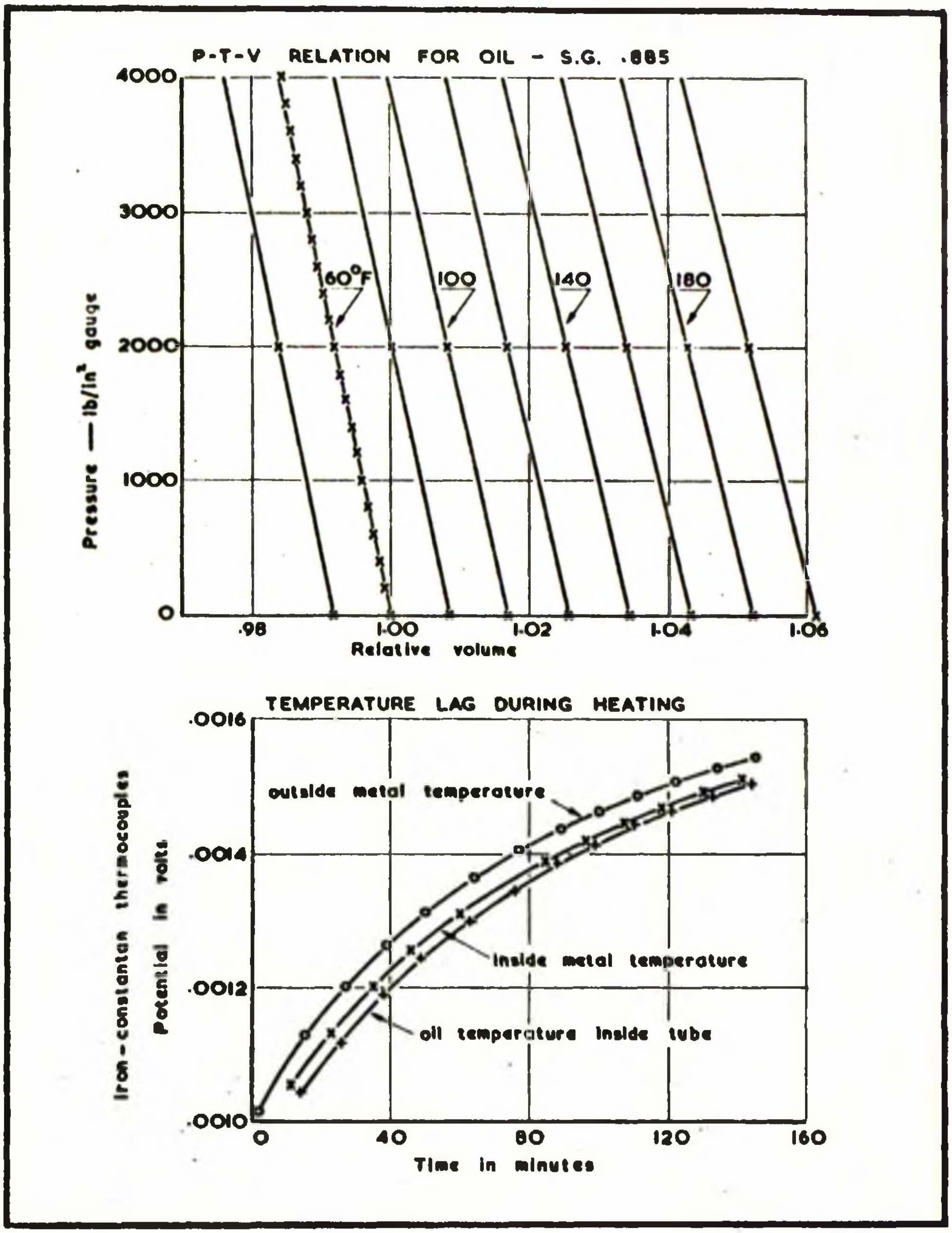
9.10 General arrangement of assembled diametral and axial extensometers.



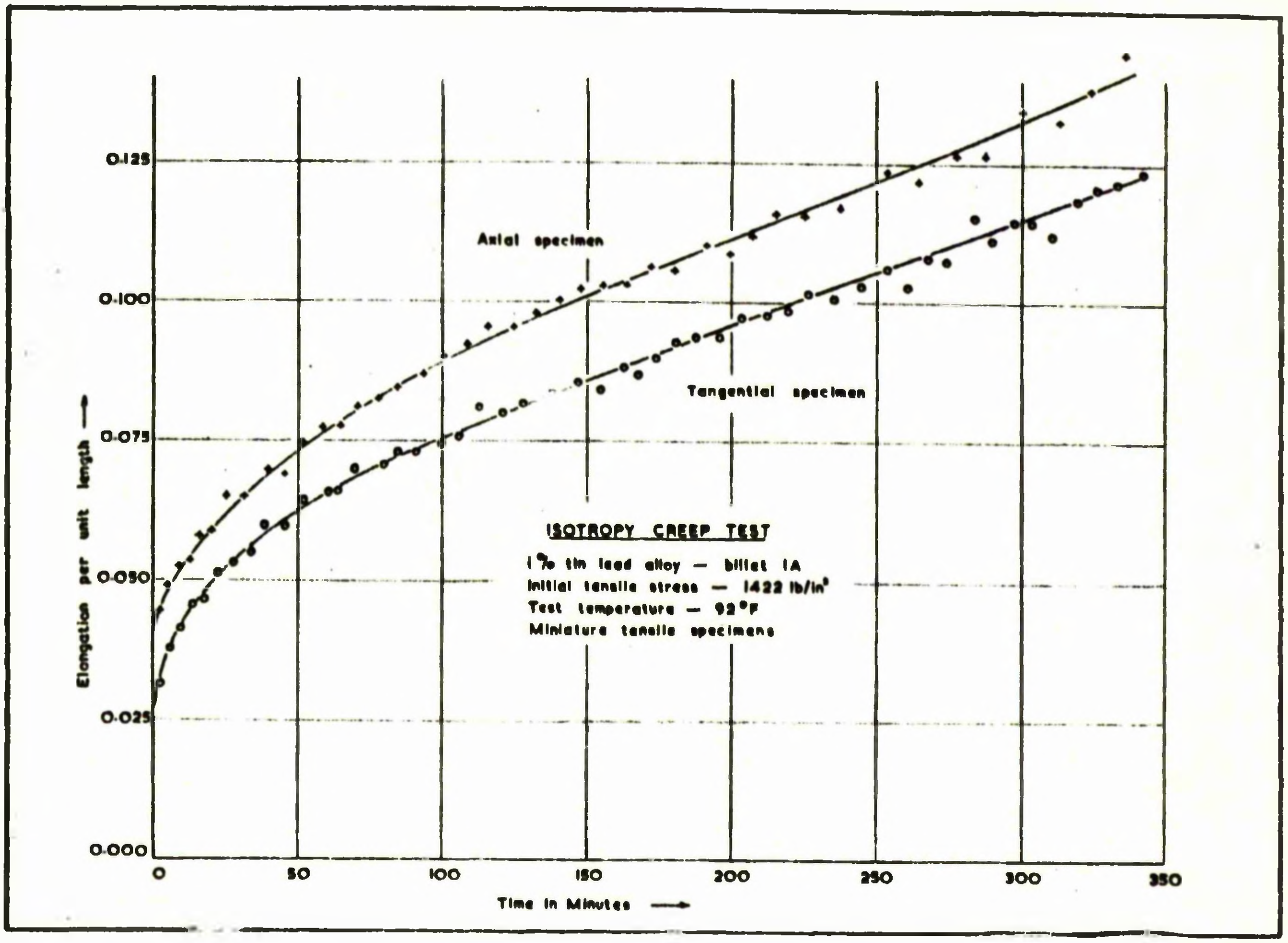
9.11 **Circuit diagram for control of heaters in constant temperature tank.**



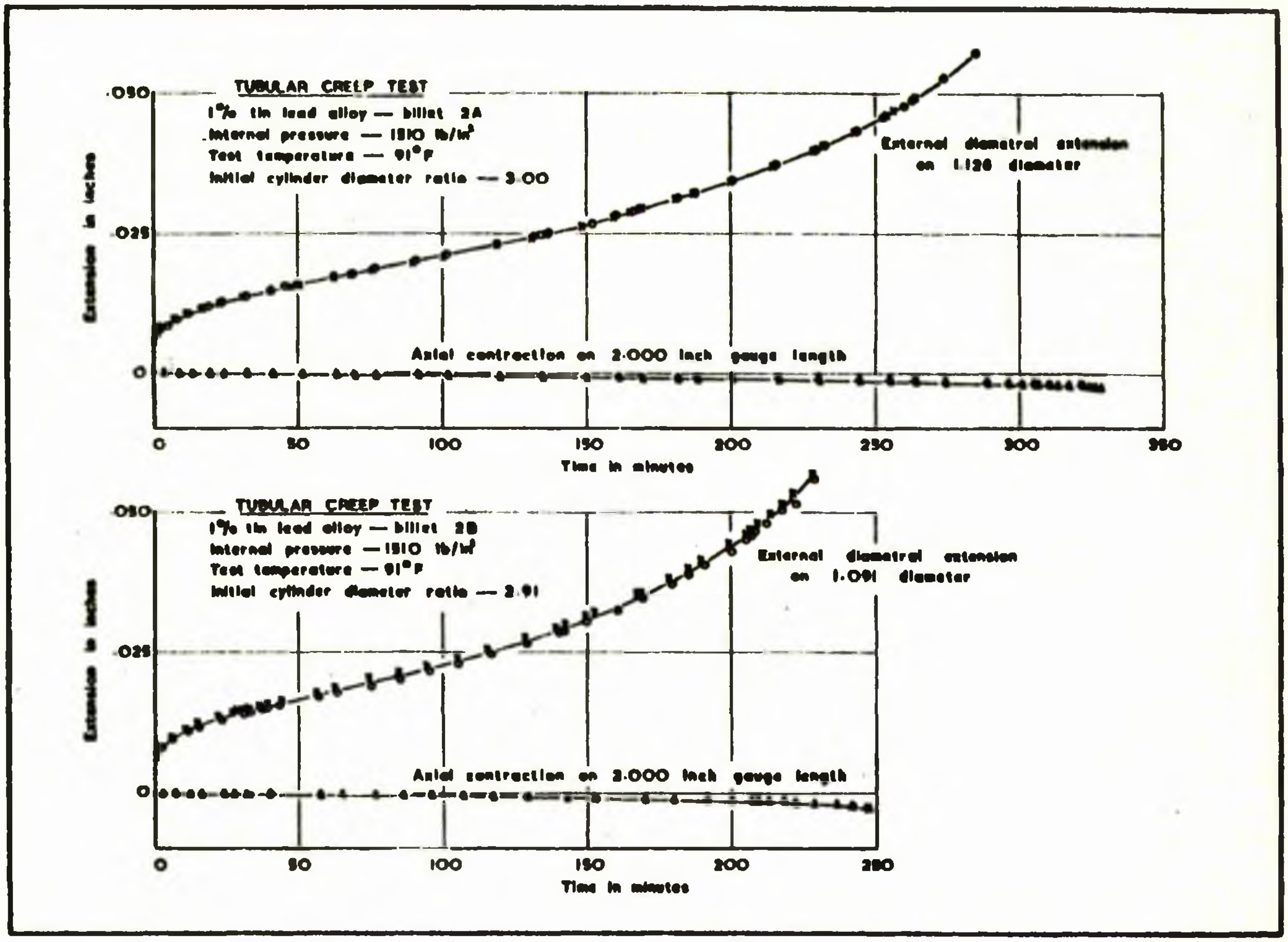
9.12 Calibration of thermocouples.



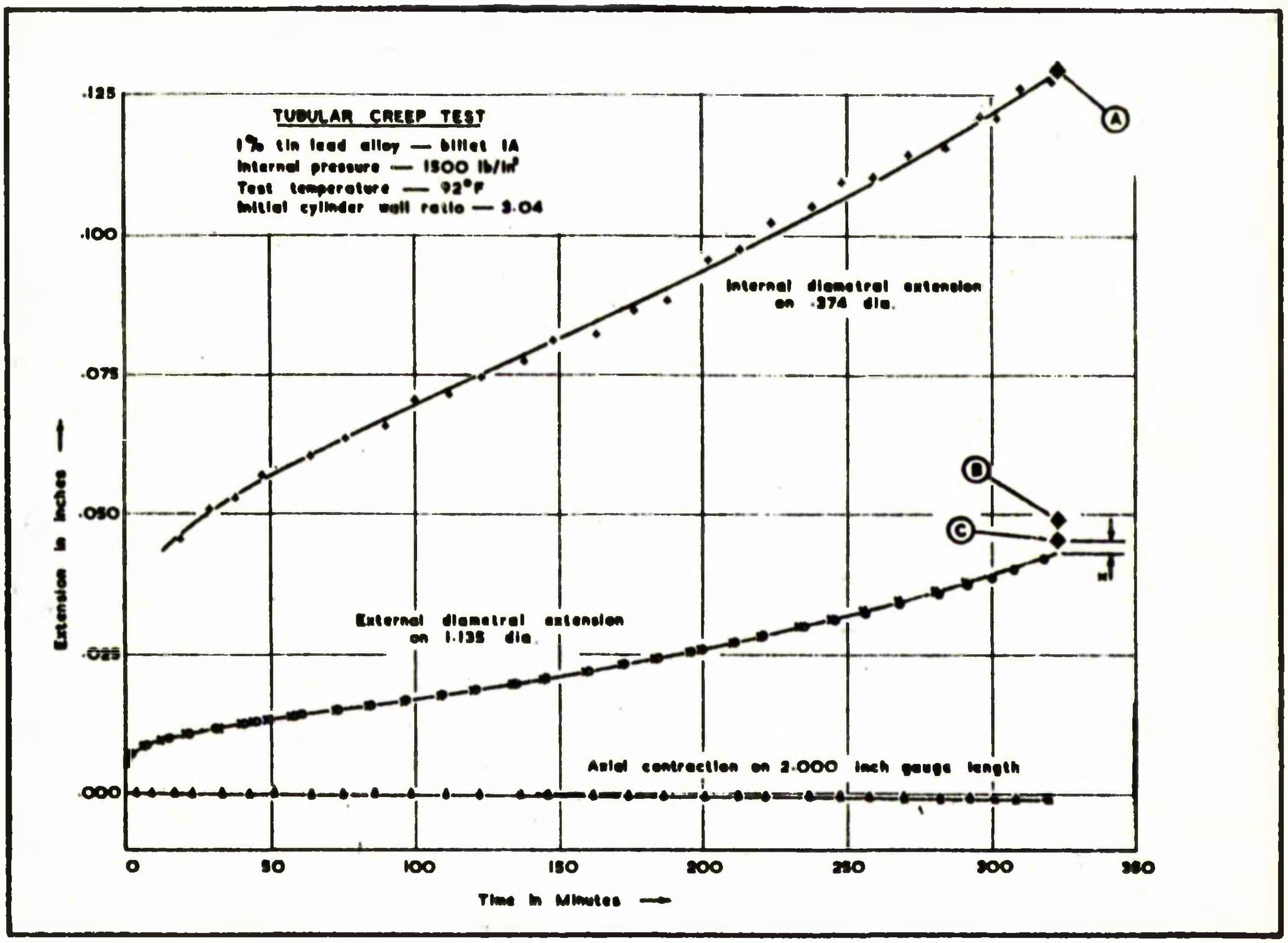
9.13 Chart for the correction of measured oil volumes inside tubular specimens. Temperature time lag across wall of lead tube during heating.



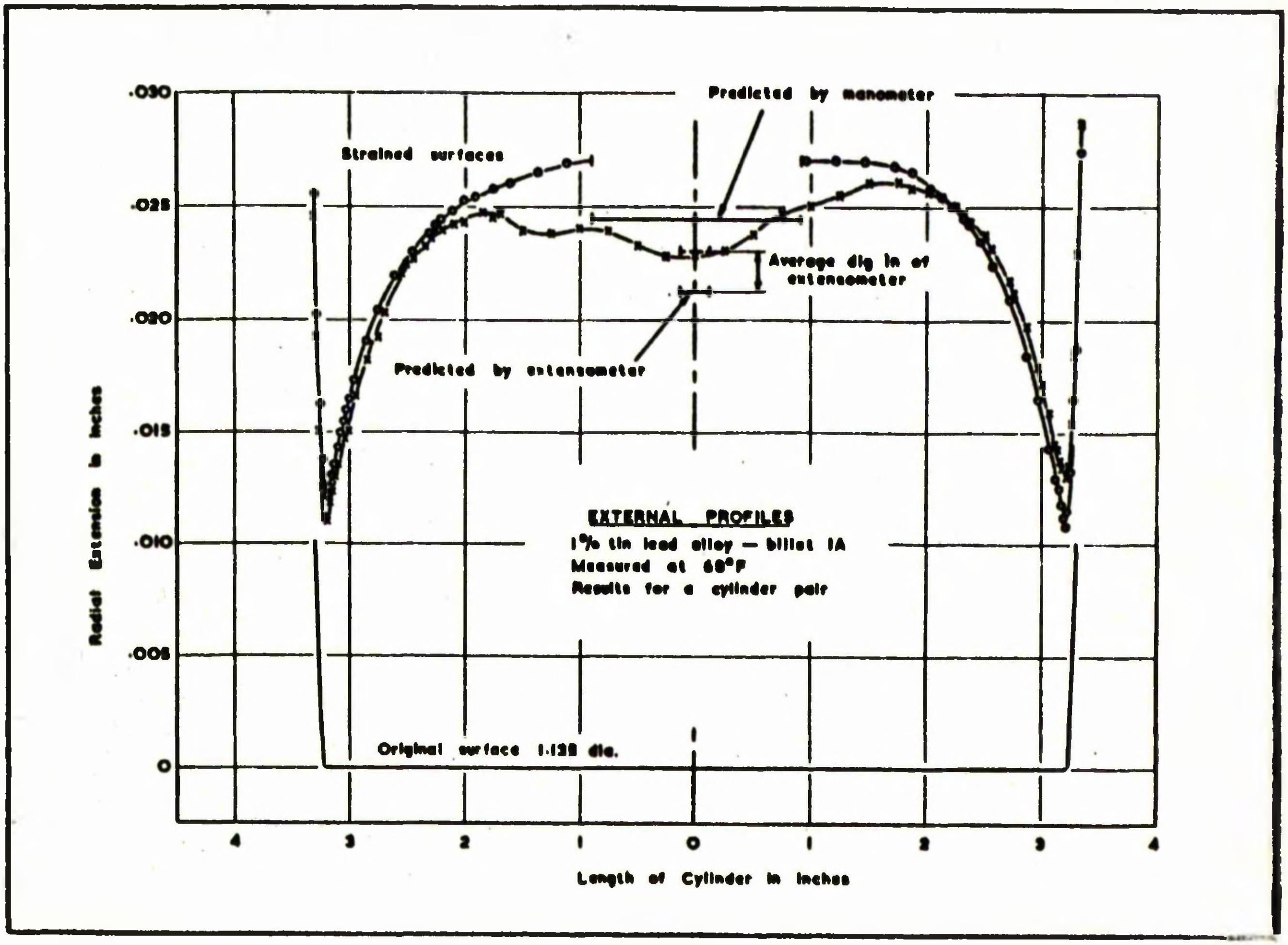
9.14 Isotropy creep test results for lead alloy tensile specimens - billet 1A.



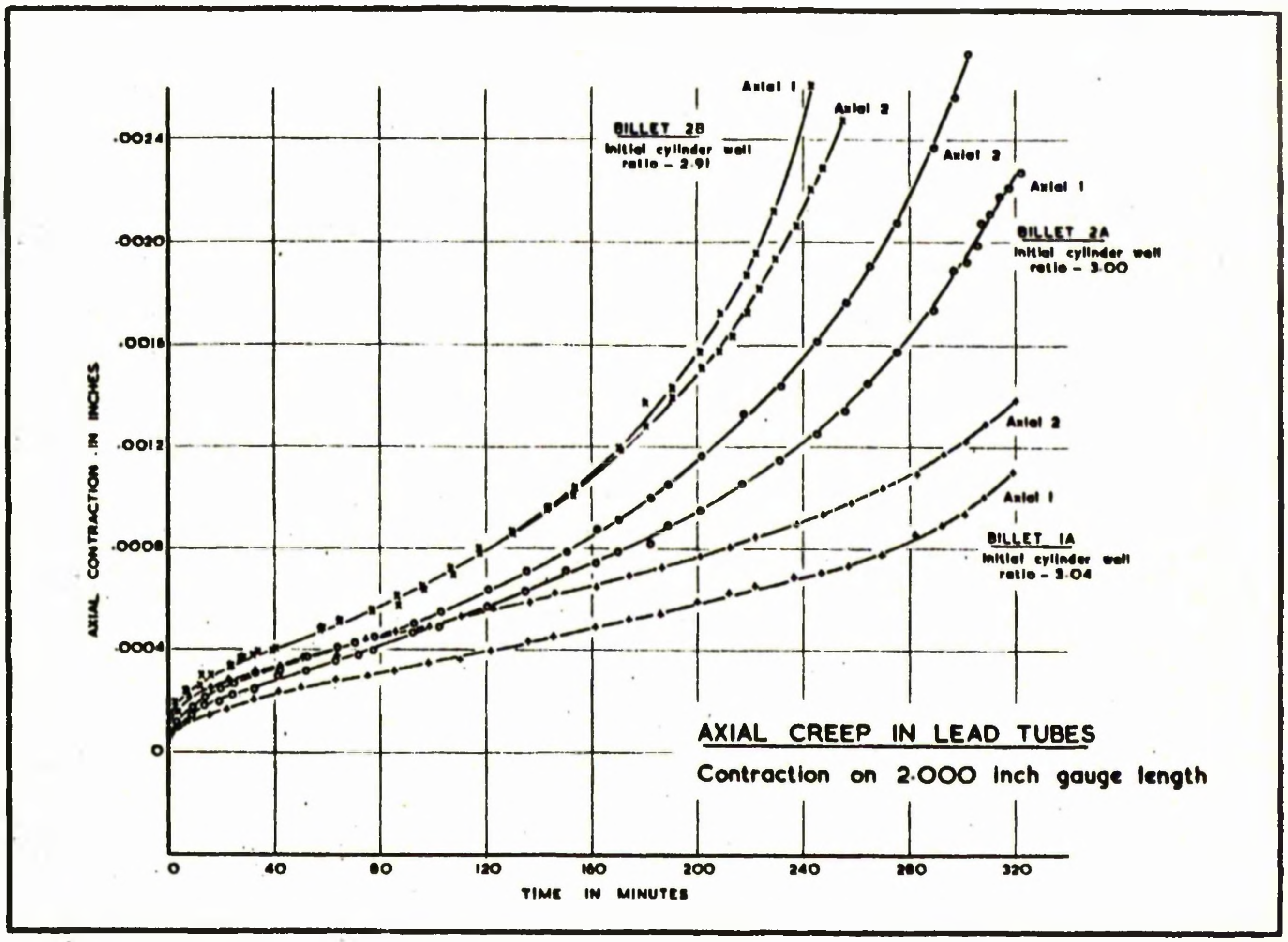
9.15 Internal pressure creep test results for lead alloy cylinders - billets 2A and 2B.



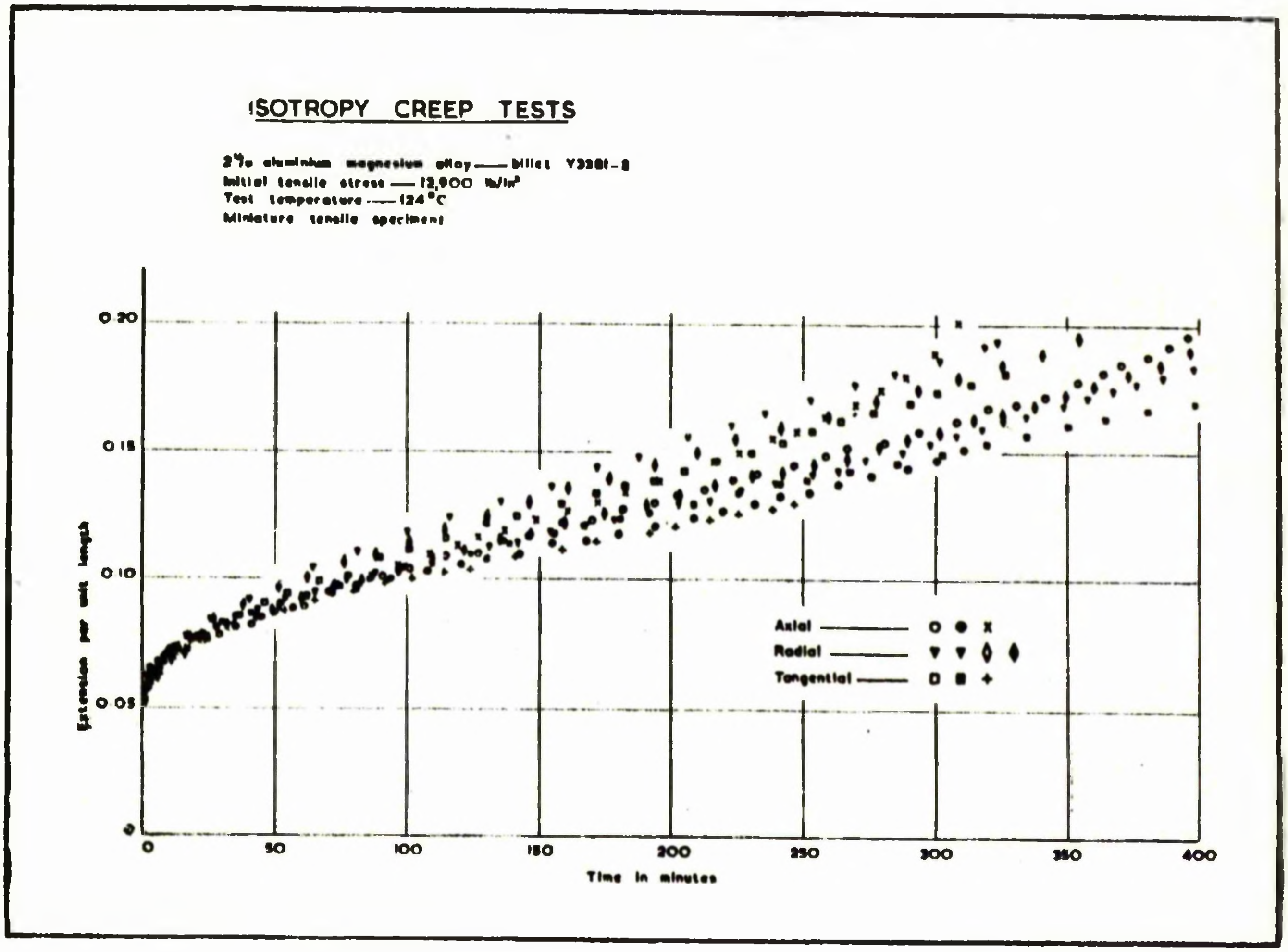
9.16 Internal pressure creep test result for lead alloy cylinders - billet 1A.



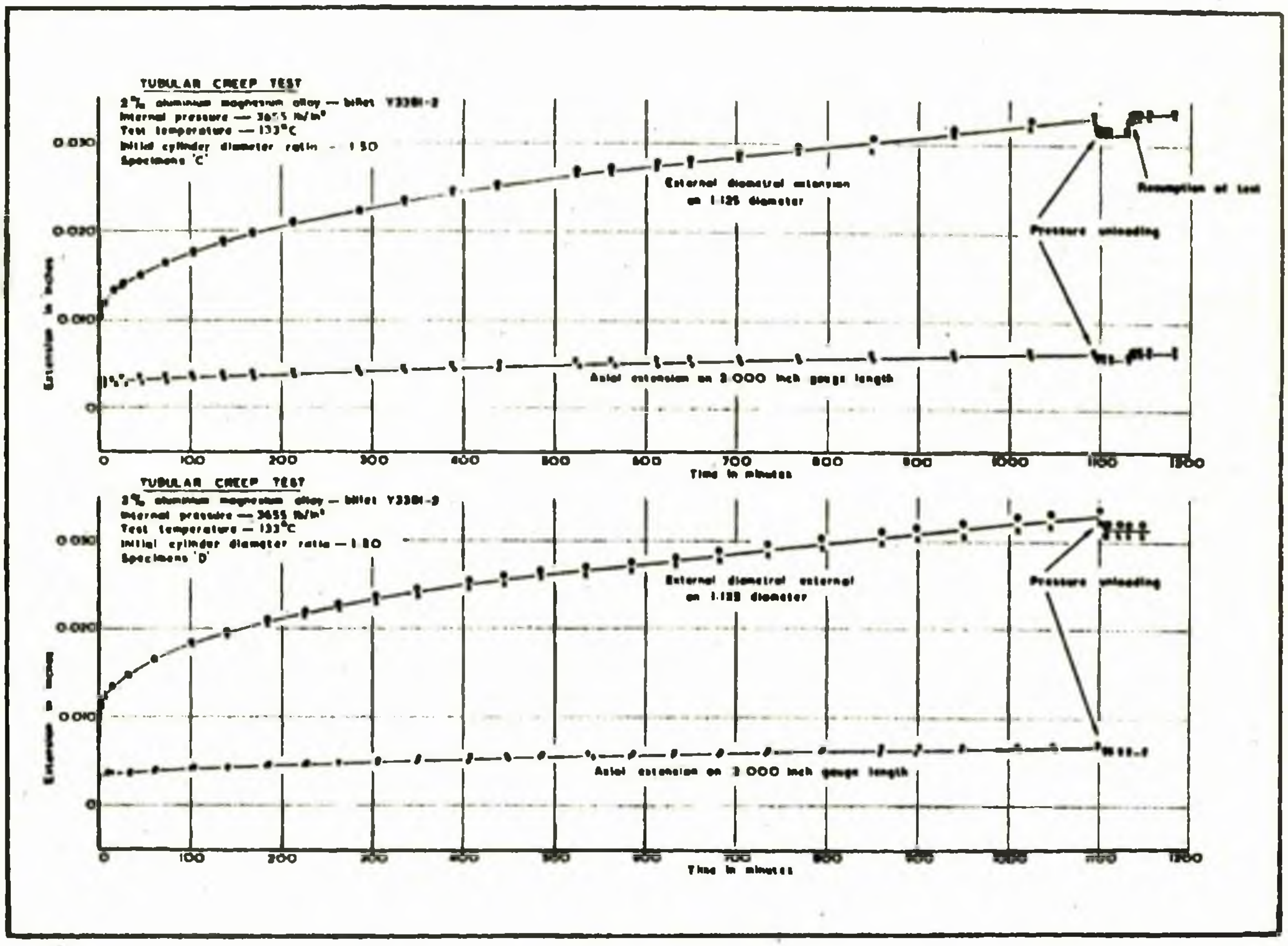
9.17 External profiles of lead alloy cylinders after testing - billet 1A.



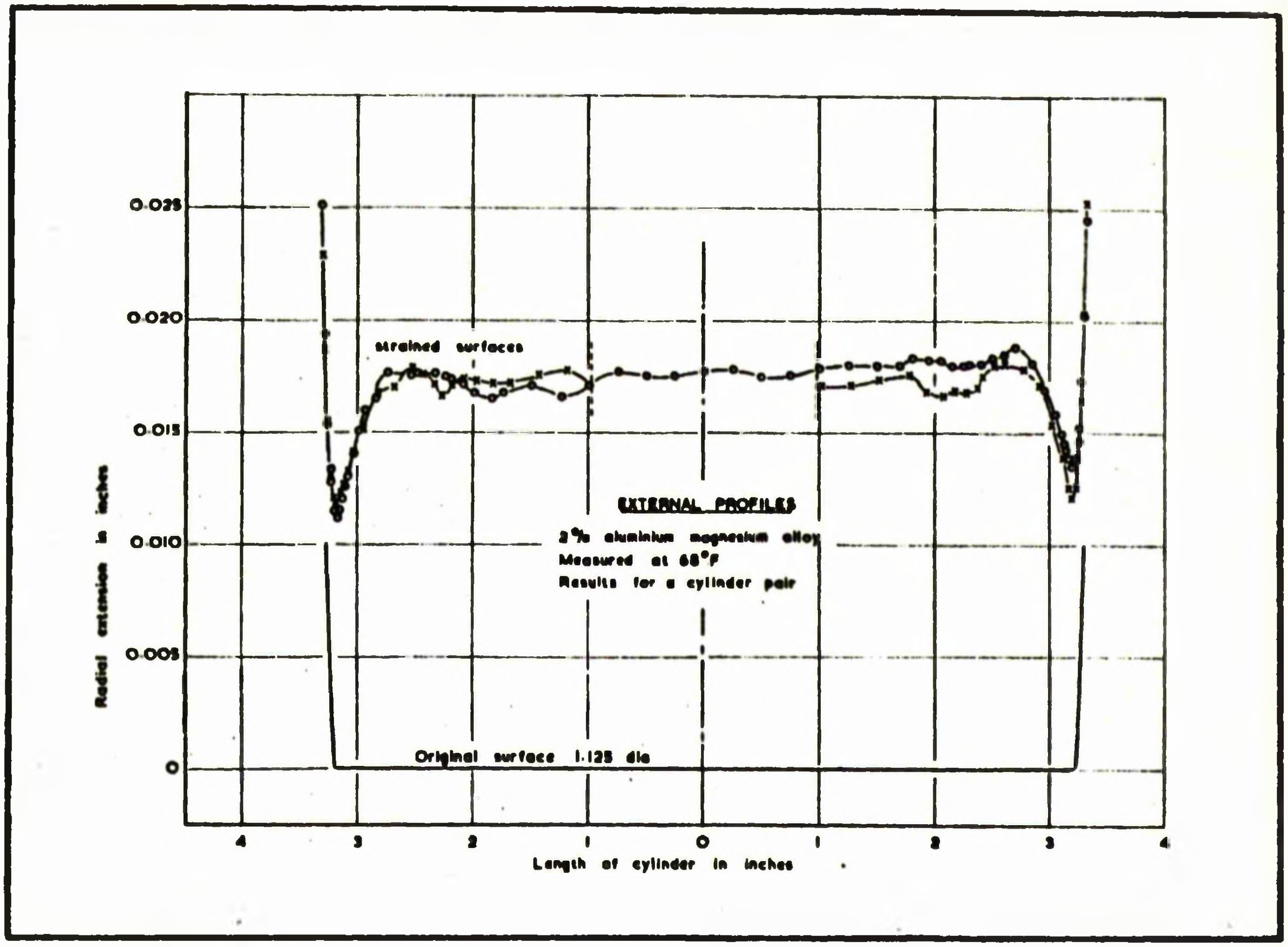
9.18 Axial creep in lead cylinders.



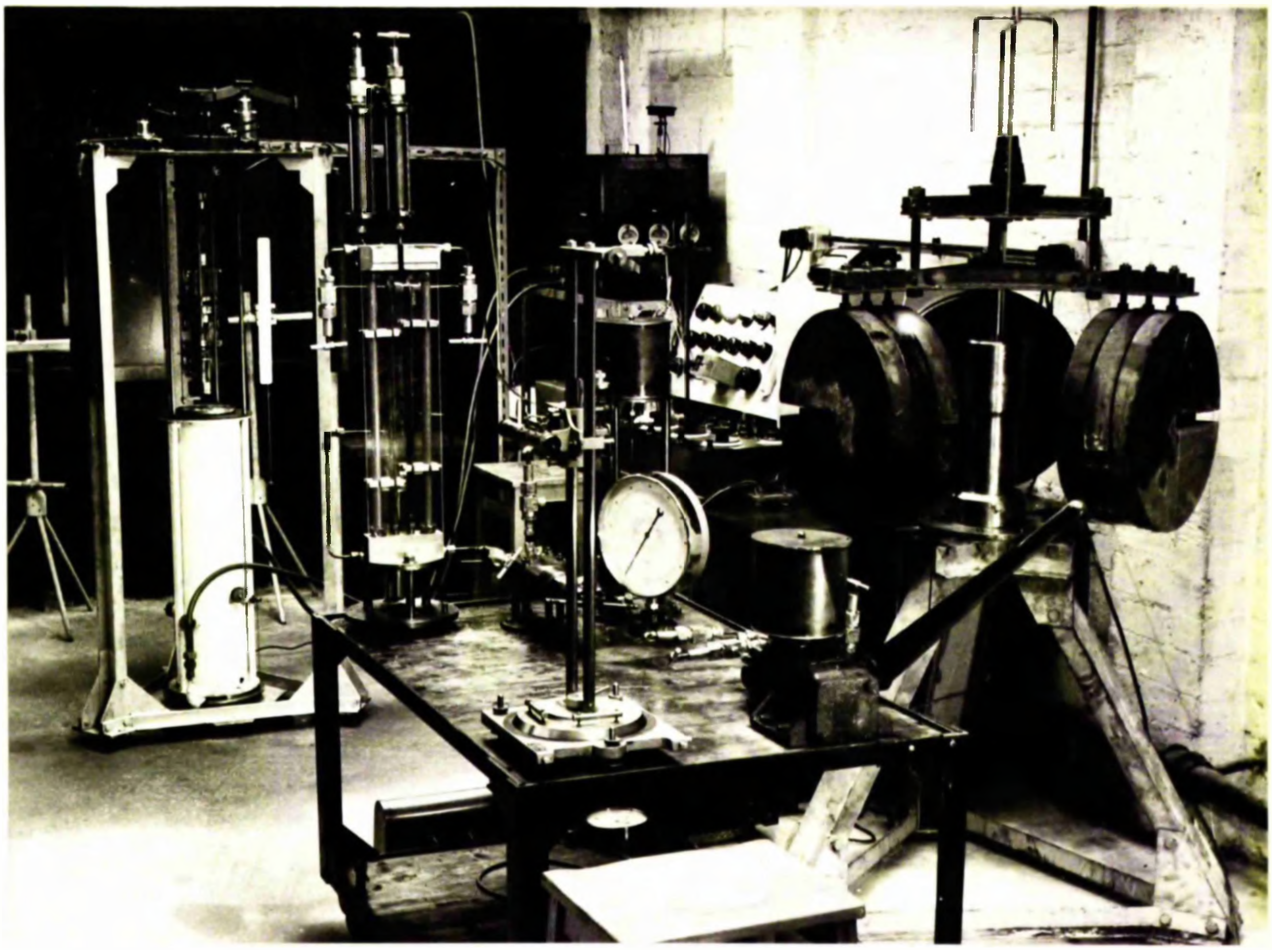
9.19 Isotropy creep test results for magnesium alloy tensile specimens. The test denoted by a hollow triangle was at a temperature 2.5°C too high.



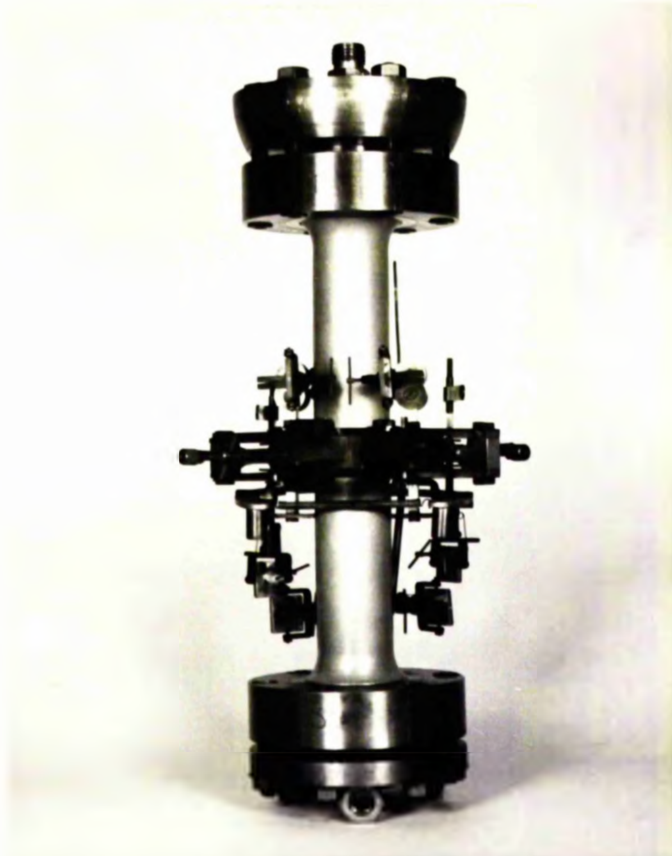
9.20 Internal pressure creep test results for magnesium alloy cylinders.



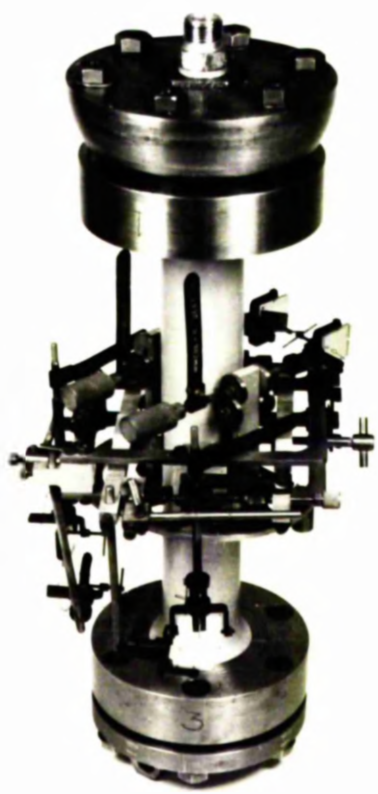
9.21 External profiles of magnesium alloy cylinders after testing.



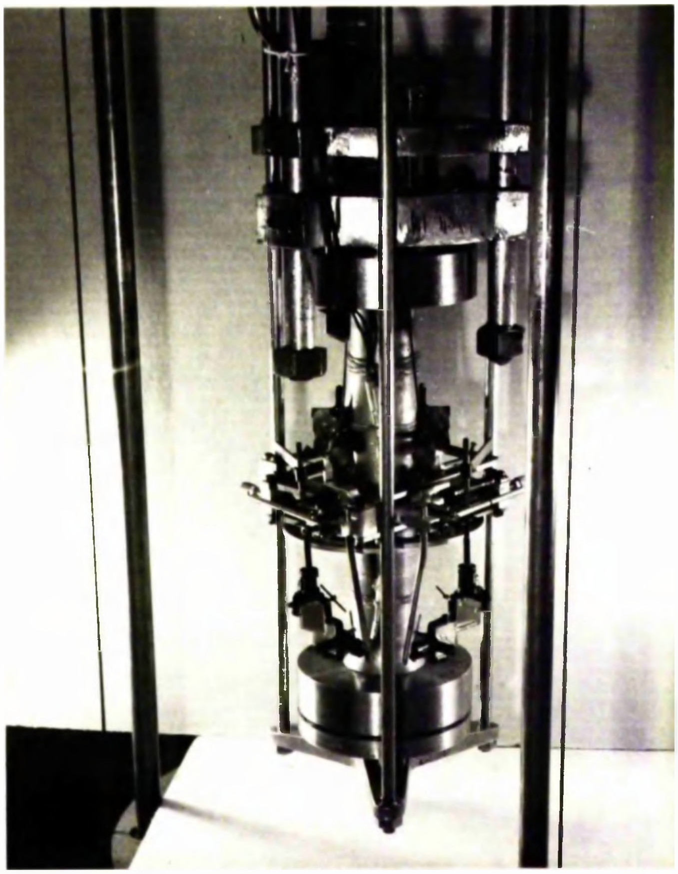
10.1 General view of apparatus.



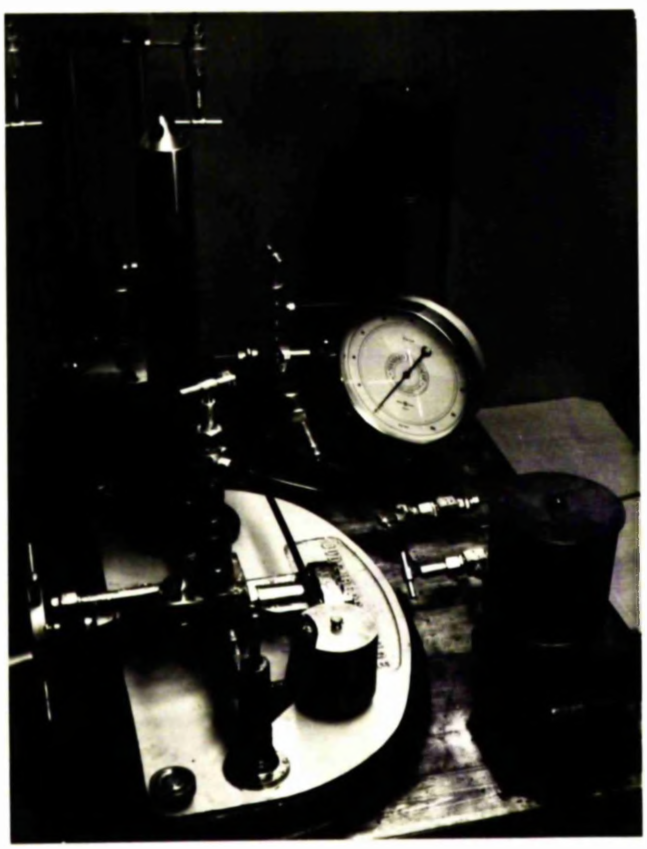
10.2 View of assembled axial and diametral extensometers.



10.3 Another view of assembled axial and diametral extensometers.



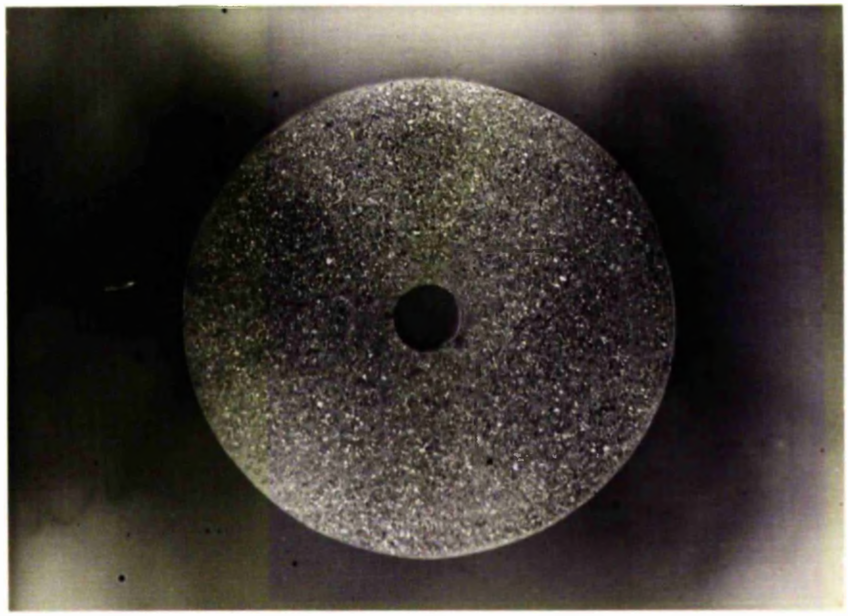
10.4 View of extensometers and thermocouples in position after a test on a lead cylinder. The bulged specimen can be seen.



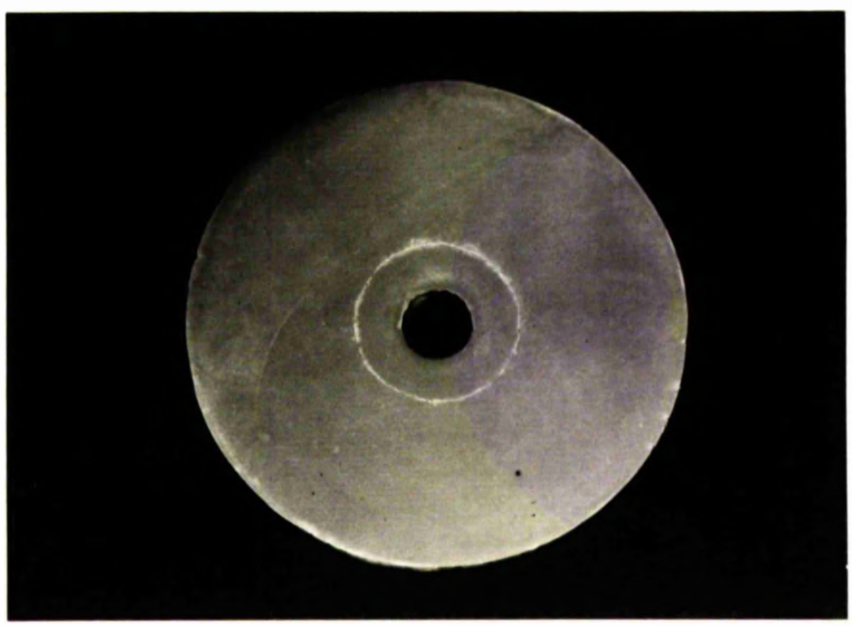
10.5 Calibration equipment for hydraulic accumulator.

10.6 Short tubular lead specimen and small tensile lead specimen standing beside portions of the original billet. A steel mandrel used in turning the lead cylinders is in the foreground.

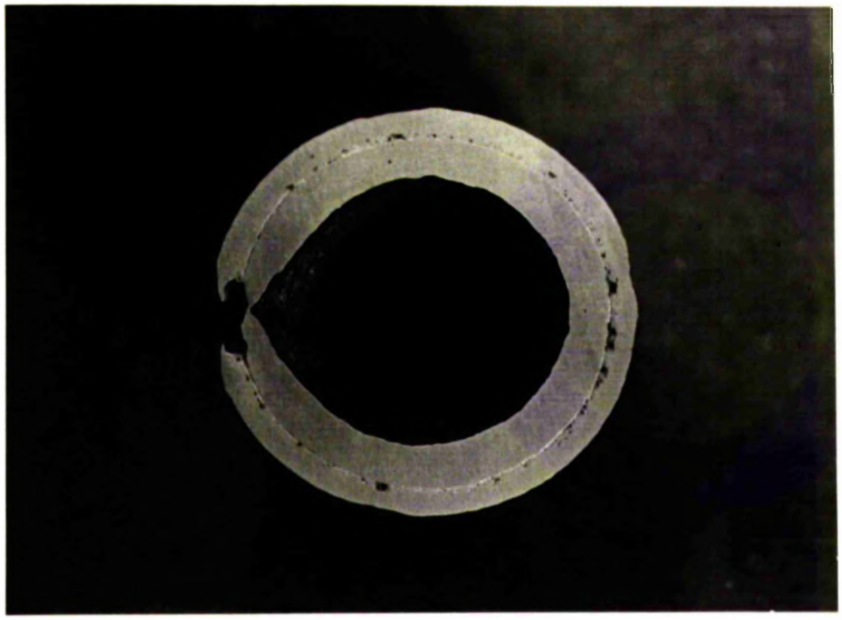
10.7 Cross section of lead billet with piping defect - as turned.



10.8 Cross section of lead billet with piping defect - etched to show grain size.

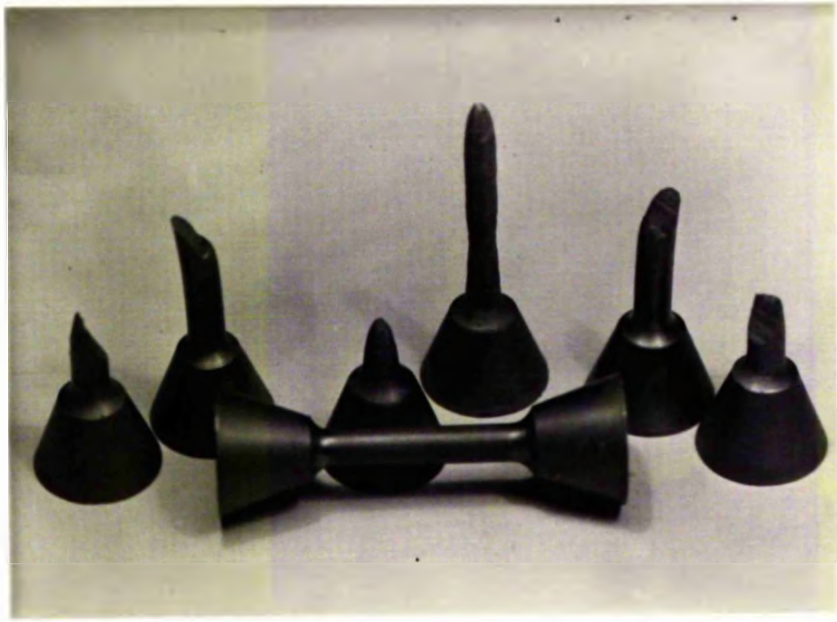


10.9 Cross section of lead billet with piping defect - deeply etched, then polished.



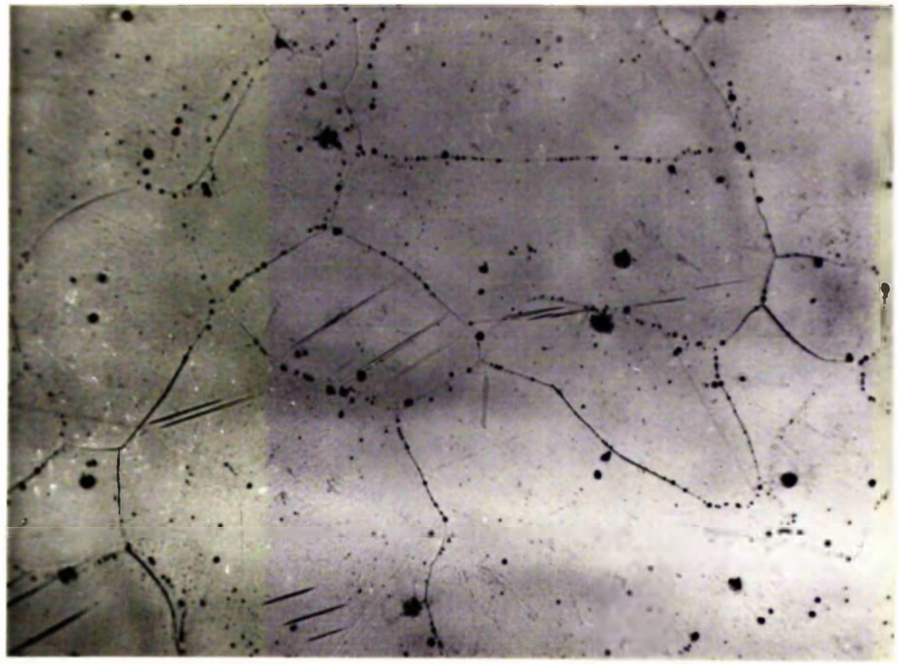
10.10 Cross section of lead cylinder (k = 3) after rupture, showing piping defect - deeply etched, then polished.

10.11 External view of long lead cylinder (k = 3) after rupture, showing piping defect.

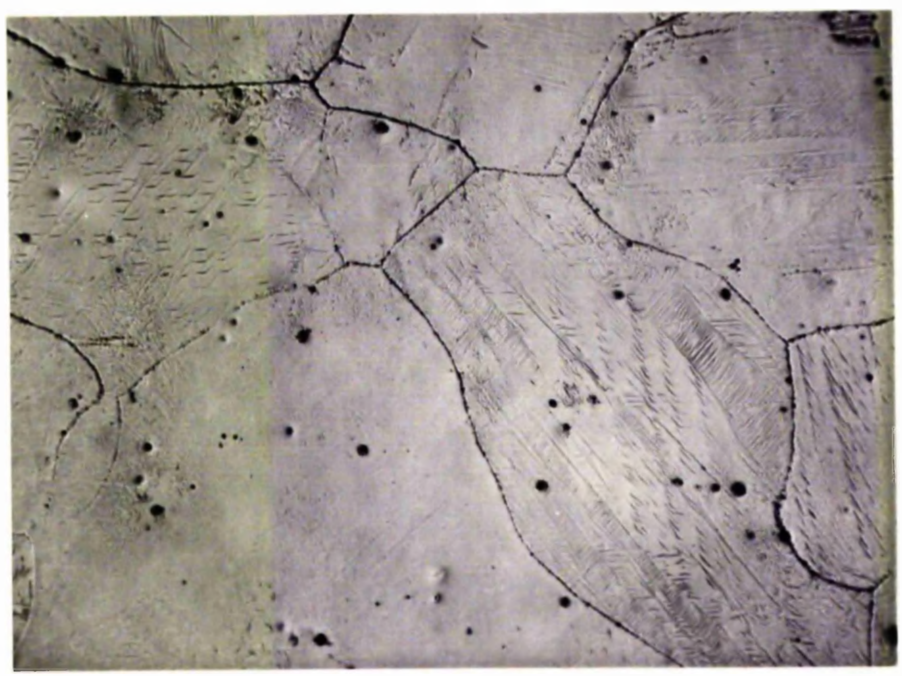


10.12 Miniature tensile specimens from lead billet with piping defect. Two tangential specimens and one longitudinal specimen after rupture, with an untested specimen in the foreground.

10.13 Two sets of lead cylinders (k = 3) after testing, showing piping defect.

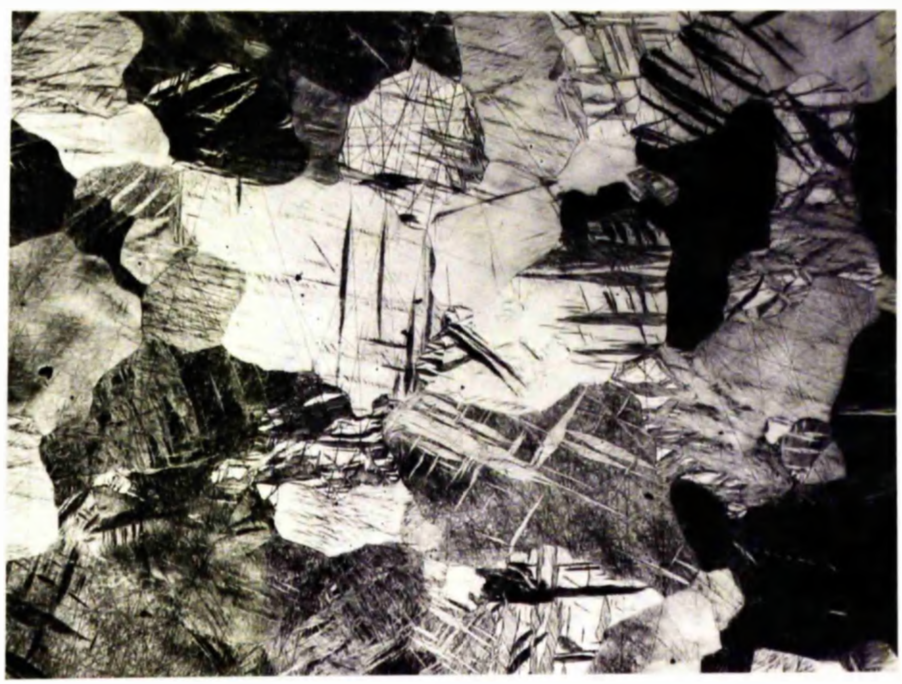


10.14 Photomicrograph of structure of magnesium alloy (× 150) before testing, transverse section through billet.

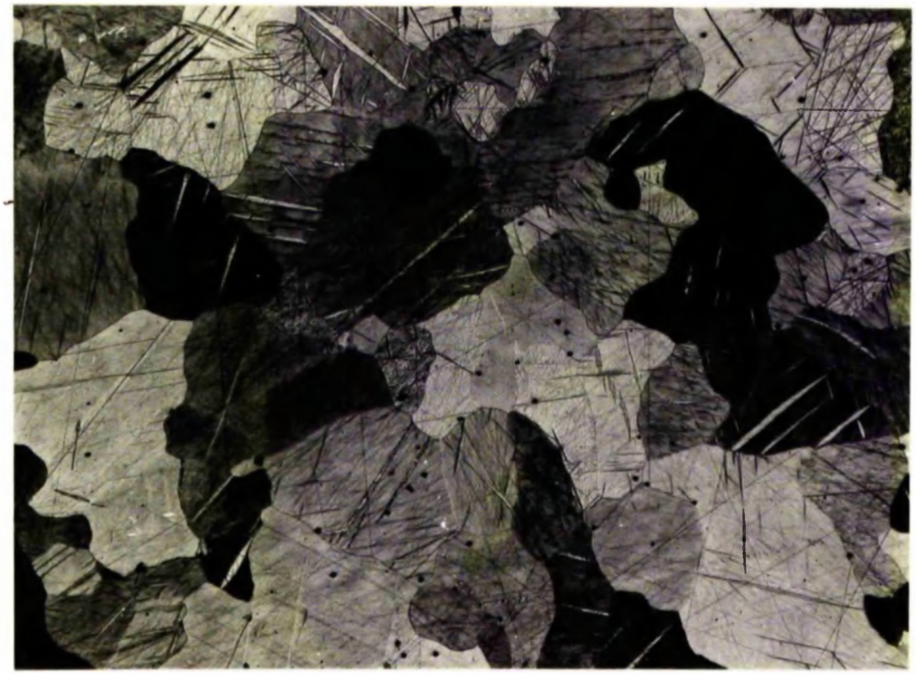


10.15 Photomicrograph of structure of magnesium alloy (× 150) after testing, transverse section through tube.

10.16 Metallographic structure of magnesium alloy (× 50) before testing, longitudinal section through billet.

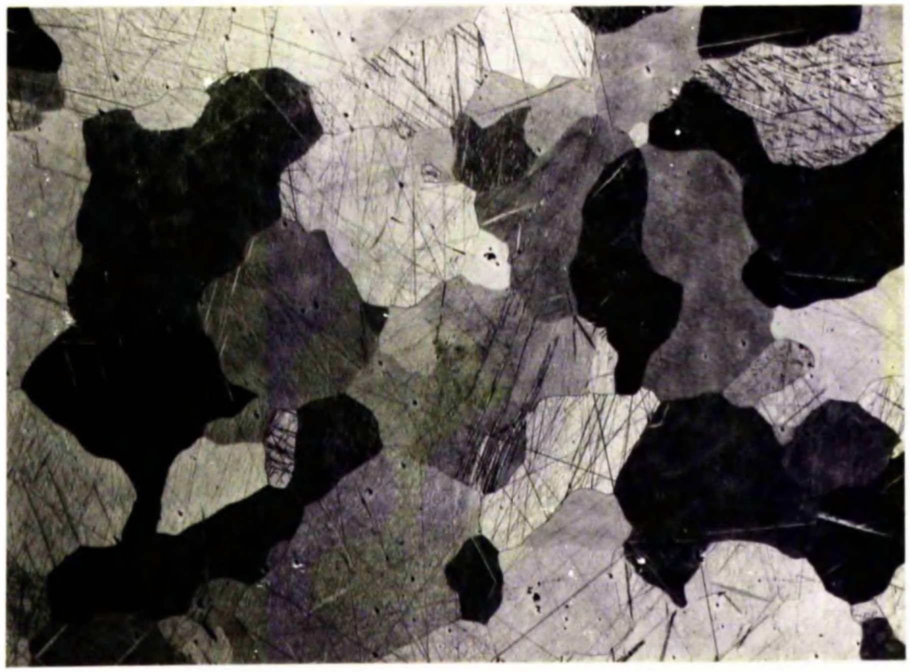


10.17 Metallographic structure of magnesium alloy (× 50) after testing, longitudinal section through cylinder.



10.18 Metallographic structure of magnesium alloy (x 50) before testing, transverse section through billet.

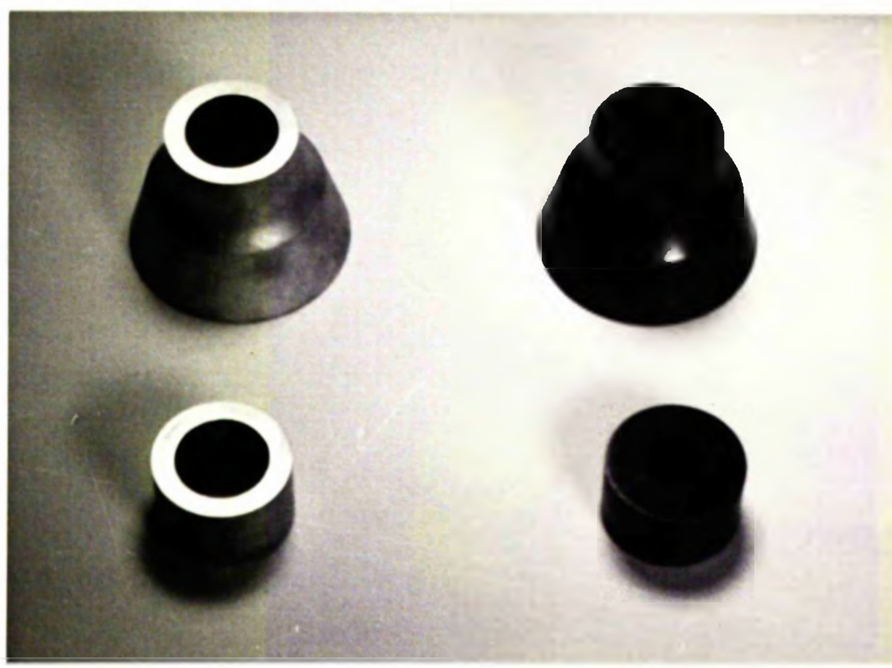
10.19 Metallographic structure of magnesium alloy (x 50) after testing, transverse section through cylinder.



10.20 Metallographic structure of magnesium alloy (x 50) before testing, tangential section through billet.

10.21 Metallographic structure of magnesium alloy (x 50) after testing, tangential section through cylinder.

10.22 Pair of magnesium alloy cylinders after testing, standing beside three miniature tensile specimens. One of the small specimens has ruptured, the centre miniature specimen is untested.



10.23 End and central transverse sections of lead alloy (k = 3) and magnesium alloy (k = 1.5) cylinders without defects, after testing. The perfect concentricity of wall at large strains is evident.

TABLE 11.1

COMPARISON OF CIRCULATING FLUIDS

Circulating Fluid	Temp °F	Viscosity Redwood 1 secs	Kinematic Viscosity Centistokes	Open Flash Point
	70	400		
	100	160		
	140	70	16.00	400°F
Shell Voluta 27	200	42	6.80	
	250	35	4.50	
	300	32.3	3.25	
	350	30.75	3.00	
Technical White Oil	70	130		330°F
B.P. Liquid Paraffin Oil	140 200	115 51		445°F
Water	68		1.00	-

TABLE 11.2

THERMOCOUPLE CALIBRATION - LOWER TEMPERATURE RANGE

Thermocouple No.	Potential Volts	Corrected Temperatures	
		Hot Junction °C	Cold Junction °C
A_1	0.0016646		
A_2	0.0016660	32.84	0.08
A_3	0.0016623		
A_4	0.0017042		
A_5	0.0017042	33.50	0.08
B_5	0.0017042		
B_4	0.0016579		
B_3	0.0016684	32.87	0.08
B_2	0.0016711		
A_1	0.0016379		
A_5	0.0016374	32.22	0.08
B_3	0.0016264		

TABLE 11.3

THERMOCOUPLE READINGS ON TUBULAR SPECIMENS FROM BILLET 1A DURING TEST

Time from application of pressure	Short specimen: thermocouple voltage x 10^7				Long specimen: thermocouple voltage x 10^7			
MINS.	B_3	A_1	B_4	B_2	A_4	B_5	A_5	A_3
9	16488	16502	15806	16542	16326	16328	16300	16365
34	16427	16447	15791	16500	16288	16278	16263	16316
51	16405	16426	15752	16468	16263	16260	16246	16305
78	16355	16382	15757	16431	16249	16250	16233	16293
105	16324	16347	15742	16401	16226	16223	16223	16274
139	16312	16324	15734	16383	16204	16205	16204	16357
164	16314	16319	15732	16374	16277	16308	16299	16343
190	16311	16329	15758	16379	16287	16312	16298	16343
215	16308	16322	15761	16382	16276	16291	16276	16326
240	16302	16319	15757	16378	16283	16311	16282	16332
273	16248	16305	15751	16366	16272	16296	16257	16326
304	16284	16305	15742	16368	16259	16296	16252	16315
325	16274	16268	15733	16349	16233	16272	16222	16284
Mean	16335	16353	--	16409	16265	16279	16258	16314
Best estimate standard deviation	67	67	--	58	29	37	15	30

TABLE 11.4

DIMENSIONS OF PISTON AND CYLINDER ASSEMBLY AT 68°F

PISTON

Distance from top of piston in inches	Angular position round circumference in degrees	Diameter in inches
2.5	0	0.99778
4	0	0.99795
6	0	0.99797
8	0	0.99793
8	45	0.99790
8	90	0.99795
8	135	0.99795
10	0	0.99790
13	0	0.99788

CYLINDER

Distance from top of lip in inches	Diameter of bore in inches
0.250	0.99822
1.125	0.99819
1.500	0.99819

TABLE 11.5

TYPICAL ANALYSIS OF CONSTITUENTS FOR LEAD ALLOY BEFORE MELTING

Element	Tin Bar	Lead Ingot
Ag	Nil	0.0005
As	0.001	0.0001
Bi	Nil	0.0006
Cd	-	0.00005
Co	-	0.00001
Cu	0.0002	0.0006
Fe	0.002	-
Ni	-	0.00001
Pb	0.001	99.997
S	0.004	-
Sb	0.02	0.0002
Sn	99.975	0.001

TABLE 11.6

ANALYSIS OF COMPOSITION OF LEAD ALLOY

Spectrographic analysis:-

Thallium		0.0012%
Silver		0.00013%
Copper		0.0004%
Bismuth		0.0004%
Antimony		0.0003%
Arsenic		0.0001%
Cadmium	less than	0.0001%

No other impurities were detected.

Chemical analysis:-

Tin	0.98%
Lead	Remainder

TABLE 11.7

ANALYSIS OF COMPOSITION OF MAGNESIUM ALLOY

Spectrographic analysis:-

Aluminium		2.06%
Zinc		0.007%
Silicon		0.003%
Copper	less than	0.001%
Manganese		0.003%
Tin	less than	0.001%
Nickel	less than	0.001%
Iron		0.004%
Magnesium		Remainder

Chemical analysis:-

Aluminium	2.06%
Zinc	0.005%

TABLE 11.8

A.S.M.E. BOILER AND PRESSURE VESSEL CODE - 1959

SECTION 1 - POWER BOILERS

Allowable stress for use in the empirical formulae specified by the code is the lowest of

(a) 25% of the minimum specified ultimate tensile strength at room temperature

(b) 25% of the tensile strength at temperature as reported by test data

(c) 62½% of the 0.2% yield strength at temperature as reported by test data

(d) a conservative average of the 100% stress to give a creep rate of 0.01% in 1000 hours as reported by test data

(e) 60% of the average or 80% of the minimum stress to produce rupture in 100,000 hours as reported by test data.

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NOMENCLATURE

Roman alphabet

a	numerical constant
A	numerical constant
B	numerical constant
c	numerical constant
C	numerical constant
d	numerical constant
D	diameter
e	base of natural logarithms
E	Young's Modulus
f( )	function of ( )
g( )	function of ( )
h	height
i	gradient of curve
inv	any invariant of the complex stress system.
I	moment of inertia
J_2	second invariant of the deviator stress tensor $J_2 = \frac{1}{2}(\Sigma\, S_1^2)$
k	ratio $\frac{\text{outside diameter of tube}}{\text{inside diameter of tube}}$
ℓ	length
m	numerical constant
n	numerical constant
p	numerical constant
P	internal pressure

q	numerical constant
r	any radius of tube
R	gas constant or Boltzman constant
S	stress deviator $S_1 = \frac{1}{3}(2\sigma_1 - \sigma_2 - \sigma_3) = (\sigma_1 - \frac{1}{3}\Sigma\sigma_1)$
t	time
t_a	fixed time (material constant)
t_{rup}	rupture time
T	absolute temperature
T_a	fixed temperature (material constant)
T_{cm}	creep rate modified temperature
T_m	absolute melting temperature
T_{rm}	rupture time modified temperature
u	radial displacement at radius r
v	numerical constant
V	volume
V_k	velocity coefficient
w	numerical constant
W	load
x	numerical constant
y	numerical constant

Greek alphabet

α	numerical constant in Andrades equation
β	numerical constant in Andrades equation
ΔH	energy of activation
Δt	time interval
ε	conventional strain $\varepsilon = \frac{\ell - \ell_o}{\ell_o}$
ε_{eff}	effective strain $\varepsilon_{eff} = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$
ε_{oct}	octahedral shear strain $\varepsilon_{oct} = \frac{2}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$
ε_{rup}	strain at rupture
ε^*	maximum shear strain
$\bar{\varepsilon}$	natural strain $\bar{\varepsilon} = \log_e \left(\frac{\ell}{\ell_o}\right)$
$\dot{\varepsilon}$	conventional strain rate $\dot{\varepsilon} = \frac{d\varepsilon}{dt}$
ε_s	fixed strain rate (material constant)
$\dot{\varepsilon}_{min}$	Minimum creep rate
$\dot{\varepsilon}_{oct}$	octahedral shear creep rate $\dot{\varepsilon}_{oct} = \frac{d\varepsilon_{oct}}{dt}$
η	Poissons ratio
λ	ratio: $\frac{\text{actual time at given stress and temperature level}}{\text{rupture life at that equivalent stress and temperature level in a conventional tensile test}}$
μ	Lode plot, $\mu = \frac{2(\sigma_3-\sigma_2)}{(\sigma_1-\sigma_2)} - 1$ for $\sigma_1 > \sigma_3 > \sigma_2$

ν	Lode plot, $\nu = \dfrac{2(\varepsilon_3 - \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)} - 1$
σ	stress
σ_{eff}	effective stress

$$\sigma_{eff} = \frac{\sqrt{2}}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

τ_{oct}	octahedral shear stress

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$\phi(\ \ )$	function of ( )
$\chi(\ \ )$	function of ( )
$\psi(\ \ )$	function of ( )

Subscripts

1,2,3	Principal directions of complex stress system
z,r,t	axial, radial and tangential directions in a cylinder
a,b	inner and outer radii of a cylinder
o	denotes zero time
α, β, γ	denotes annular shells of a thick cylinder

Superscripts

', ", '''	denotes particular time intervals.