AIR ENTRAINMENT BY FLUID JETS AND STREAMS

by

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C O N T E N T S

Page No.

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PREFACE

During the preliminary design of the Glen Shira Project of the North of Scotland Hydro-Electric Board, the problem of air entrainment at a free surface was considered. In this original design, water from subsidiary catchment areas was led into the main tunnel through a steeply inclined pipe running down a hill side, and it was considered that air might be entrained where this subsidiary flow met the free surface in the pipe and be carried into the main tunnel. In order to investigate this possibility a model of the relevant portion of the pipe-line was made in Pyrex glass, the behaviour of the air bubbles observed and the quantity of air entrained measured by the method of Binnie and Wright.(1)* Very little relevant information on this particular problem exists and it was intended to carry out experiments using a series of models to determine scale effect. However, the visual observation of air being entrained and carried into the pipe-line so impressed the engineers responsible that major alterations were made in the proposed scheme, and the characteristics of this preliminary design ceased to be of

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*Numbers refer to list of references, Appendix I.

practical interest and no further model work was authorized. The problem of air entrainment and its prediction from model tests in such a case remained.

About the same time as these experiments were being carried out, a suggestion was made by the late Sir Edward MacColl that it might be possible to use a hydraulic compressor for compressing the air for use in gas turbines (perhaps peat burning) in certain localities. This suggestion, which was studied analytically, raised the question of the efficiency of the hydraulic compressor, a device which depends for its functioning on the entrainment of air.

These two instances brought to the writer's attention the need for fundamental information on the entrainment of air by jets or streams of water. The problem of air entrainment is also present in the design of high speed flumes, channels and spillways and this aspect has received and is receiving attention both on the Continent and in the U.S.A. In fact, most of the information available deals with such cases.

This thesis describes experiments in which jets of water of various sizes and various velocities were made to strike a free surface, thus entraining air which was collected and measured. The results are discussed with reference to the two cases mentioned above, in one of which no air is wanted while in the other (the hydraulic compressor) the greatest possible quantity of entrained air is desired. The experiments are described in chronological order so that the development of the apparatus may be explained.

Since the technique used was evolved from the original Glen Shira model experiments, these are briefly described in Section 1. Section 2 contains a short survey of available information on the subject of air entrainment and Section 3 relates the first stage in the development of the apparatus. In this, jets of water from a series of orifices entrained air in a T-piece in the glass pipe-line. The next modification in which vertical jets were directed straight into the collecting tank is recorded in Section 4, while Section 5 describes some additional experiments having special reference to the design of a hydraulic compressor. The results of the nozzle experiments are analysed and theories are discussed in Section 6. The general conclusions drawn from the experiments are stated in Section 7.

The thesis is a report of an original investigation. The sources from which information were derived are referred to in the text and listed in the bibliography, Appendix I.

SECTION 1

GLEN SHIRA HYDRO-ELECTRIC SCHEME - MODEL PIPELINE TESTS

SECTION 1

GLEN SHIRA HYDRO-ELECTRIC SCHEME - MODEL PIPE-LINE TESTS.

(a) Introduction.

The original design of the Brannie Burn Intake and Diversion Works (Glen Shira Project) is shown in Fig. 1. Flow from subsidiary catchment areas was led into the main tunnel through the inclined pipe. The pipe was not intended to run more than about half full even under maximum flow conditions, and the level of the free surface in the inclined pipe varied with the level in the reservoir, as indicated. It was desired to know whether air would be entrained where the flow down the inclined pipe met this free surface and whether any entrained air would be carried into the main tunnel. If the experiments showed that air was carried into the tunnel, it would be necessary to investigate methods of minimising or preventing this occurrence. This part of the experimental work, however, is not relevant to the main object of this thesis and is not reported here.

(b) Apparatus.

A geometrically similar model of the inclined pipe line, the main tunnel and the junction piece, was made in Pyrex glass to a scale of 1 to 60, i.e. the 5 ft. diameter inclined

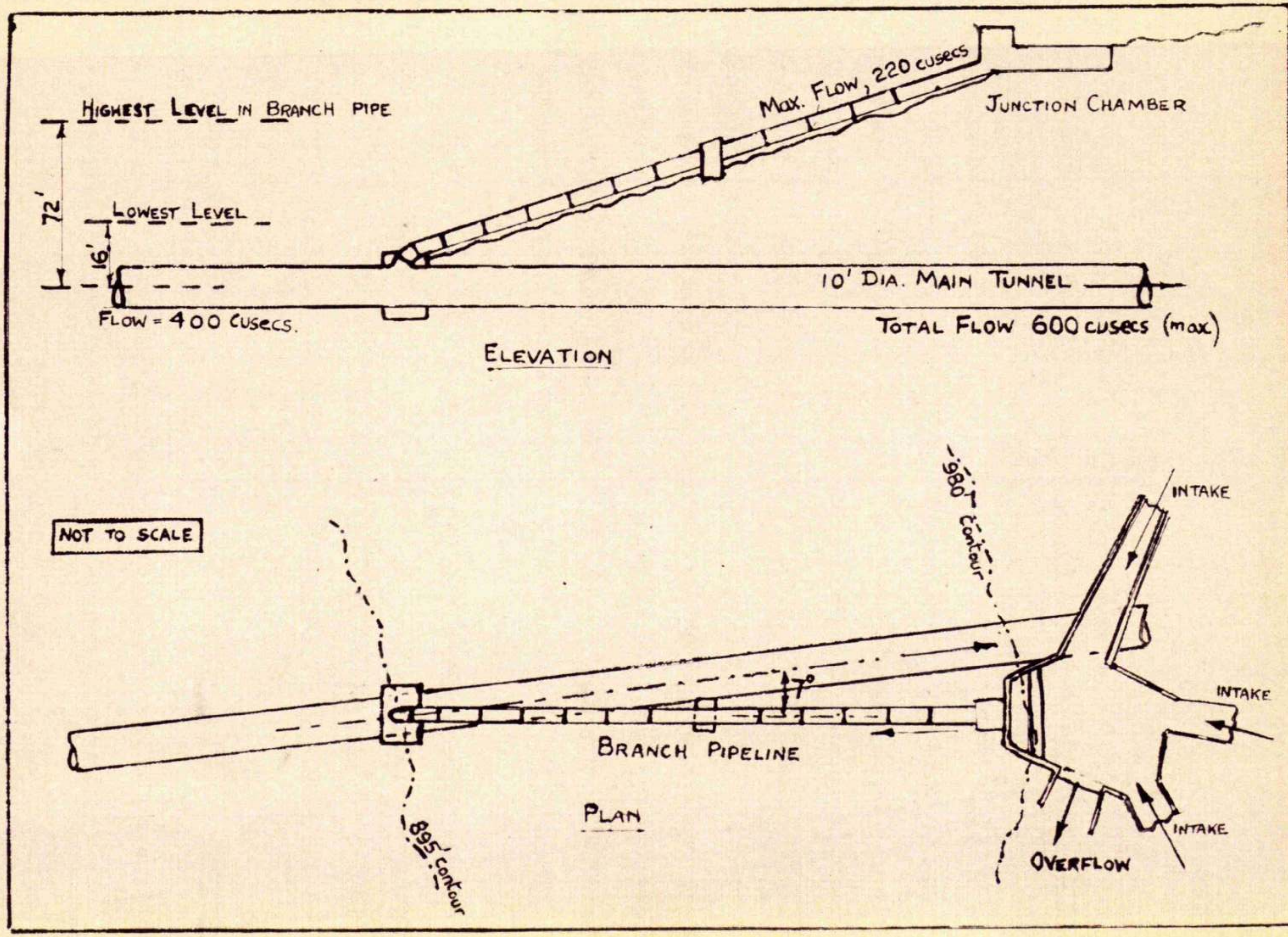

HIGHEST LEVEL IN BRANCH PIPE
72'
LOWEST LEVEL
16'
FLOW = 400 CUSECS.
MAX. FLOW, 220 CUSECS
JUNCTION CHAMBER
10' DIA. MAIN TUNNEL
TOTAL FLOW 600 CUSECS (MAX)
ELEVATION
NOT TO SCALE
980' CONTOUR
INTAKE
INTAKE
INTAKE
7°
BRANCH PIPELINE
OVERFLOW
PLAN
895' CONTOUR

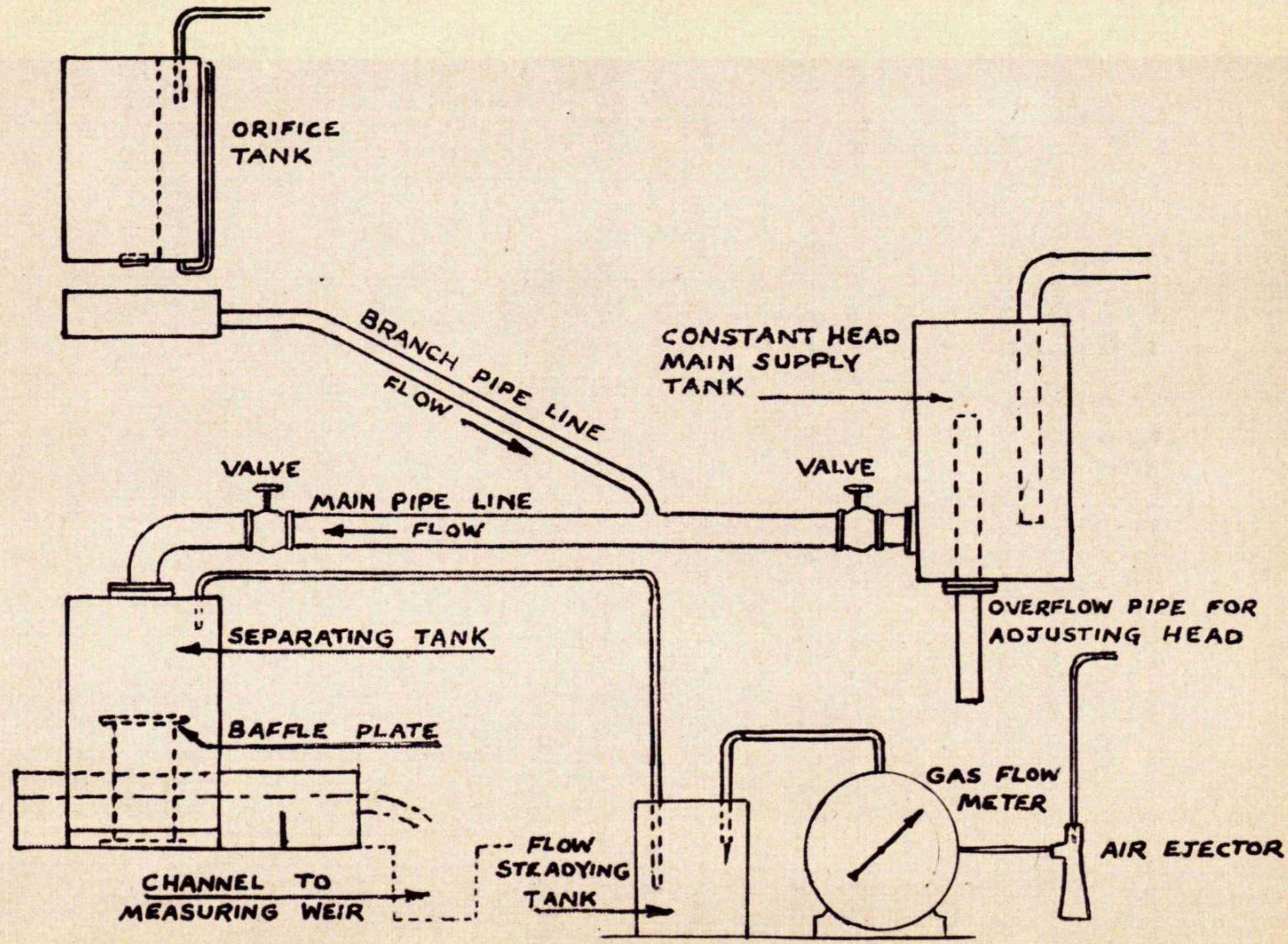
FIG. 1

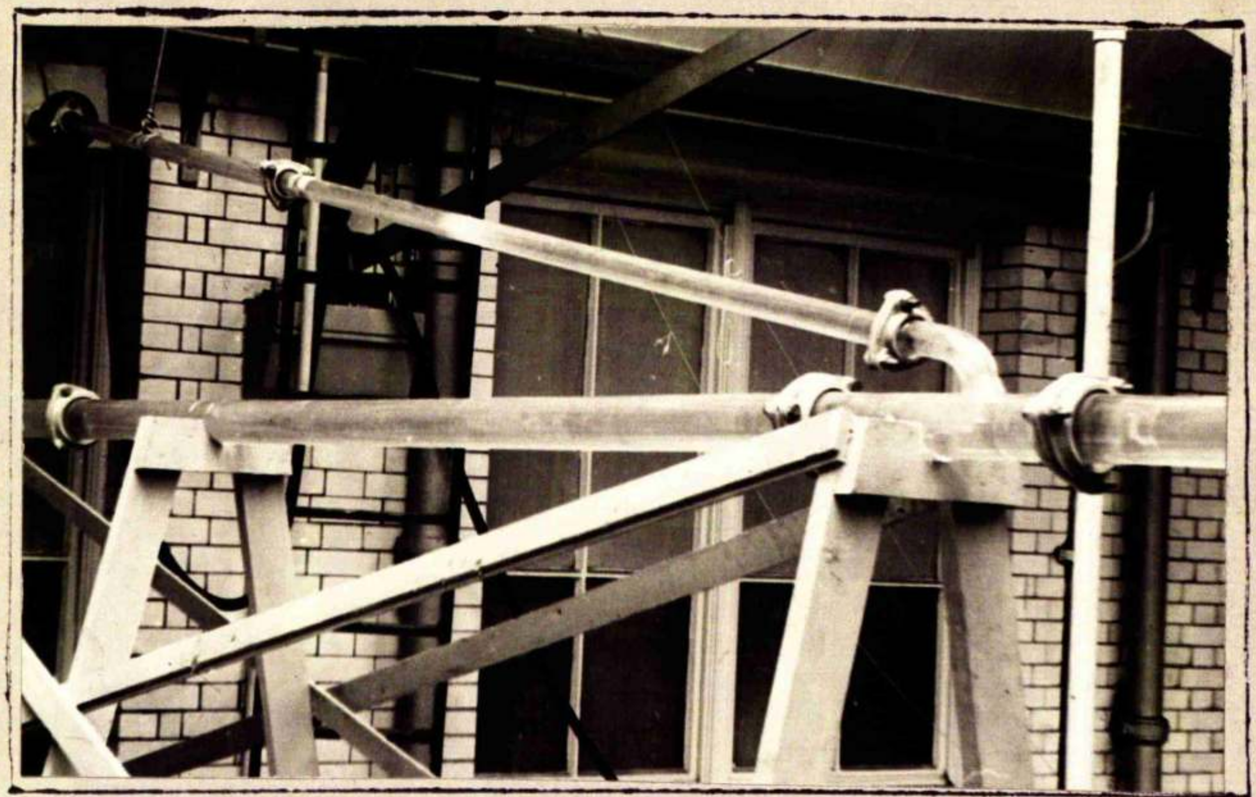
pipe and the 10 ft. diameter tunnel were represented by model pipes 1 inch and 2 inches diameter respectively. The flow into the inclined pipe line was measured by an orifice and the total flow from the main pipe was measured by a triangular notch.

The air entrained was measured by the method of Binnie and Wright[1] and the apparatus is shown diagramatically in Fig. 2 and in the photographs, Fig. 3. The water with air entrained in it was led to a separating tank containing a baffle plate. Here the air separated from the water and was drawn off through a gas flow meter by an ejector pump which was adjusted to maintain atmospheric pressure in the separating tank. This method of measuring the air proved quite satisfactory and remained essentially unchanged in the later experiments.

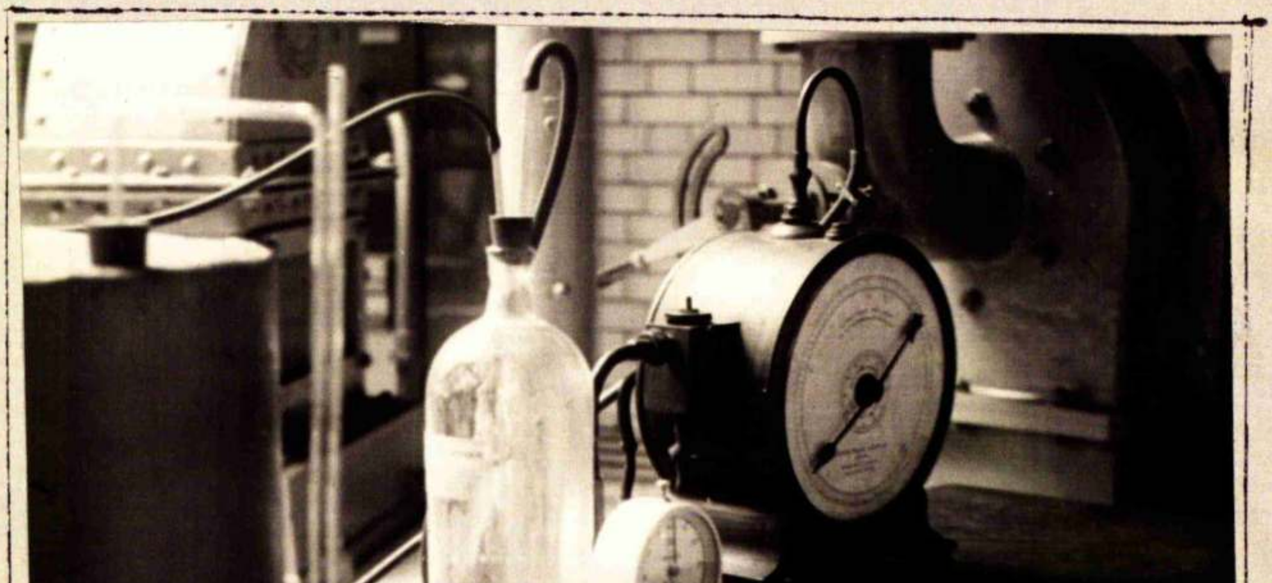
(c) Scale Effects.

On the assumption that the pipes are smooth and that the model and prototype are geometrically similar, reasoning used, for example, by Prof. J. Allen[2] leads to $\frac{V_2}{V_1} = (\frac{1}{x})^{0.714}$ where the horizontal and vertical scales are both in the ratio 1 : x, V_2 is the model speed and V_1 the full scale speed. If, however, the friction coefficient 'f' is assumed constant instead of being a function of Reynolds Number, the loss of head is proportional to the square of the speed and


ORIFICE
TANK
BRANCH PIPE LINE
FLOW
CONSTANT HEAD
MAIN SUPPLY
TANK
VALVE
VALVE
MAIN PIPE LINE
FLOW
SEPARATING TANK
OVERFLOW PIPE FOR
ADJUSTING HEAD
BAFFLE PLATE
GAS FLOW
METER
AIR EJECTOR
FLOW
STEADYING
TANK
CHANNEL TO
MEASURING WEIR



(a) Pyrex Glass Model Pipeline



(b) Air Measuring Apparatus

FIG. 3 GLEN SHIRA PROJECT (BRANNIE BURN) MODEL the velocity ratio = $(\frac{1}{x})^{0.5}$ which is the same as saying that the Froude Number is constant for model and full size.

It was decided to work with the same Froude Number for the model and prototype pipeline since the main feature under investigation concerned a surface effect (channel flow in the branch pipe line) and deceleration. This criterion led to

$$\frac{V\ \text{model}}{V\ \text{full scale}} = \left(\frac{1}{60}\right)^{0.5} = 0.0123 \text{ and } \frac{Q\ \text{model}}{Q\ \text{full scale}} = \frac{A_m V_m}{A_f V_f} =$$

$$= \left(\frac{1}{60}\right)^{2} \left(\frac{1}{60}\right)^{0.5} = \left(\frac{1}{60}\right)^{2.5} = .0000359,$$ where V = water velocity, Q = water volume flow rate and A = cross-sectional area of flow.

The behaviour of completely submerged bubbles of air should, however, probably be studied at the same Reynolds' Number for model and full scale rather than the same Froude Number. Since these conflicting requirements could not be satisfied in one model, it was originally intended to test several models of different sizes to investigate scale effects, but, as mentioned earlier, the design of the pipe-line was altered and these further tests were not required by the Hydro-Electric Board.

(d) Method of Testing.

The quantity of air entrained under various operating conditions was measured. In the first set of tests the flow in the main pipe-line was kept constant while the flow in the branch pipe was varied, the experiment being repeated for various water levels in the inclined pipe line. This water level is referred to as the "static junction head" in Fig. 4 and was measured vertically from the main pipe centre line. In the second set of tests the branch pipe flow was kept constant while the flow in the main pipe was varied. This was repeated with various junction heads. A further series of tests was undertaken to try the effect of modifications to the junction.

The technique of air measurement was developed during these tests. For small quantities of air, measurement was continued for 10 to 20 minutes for any given operating condition. It was found easier to control the pressure in the separating tank by having the ejector working with the water valve fully open all the time and adjusting a clip on the rubber tube between the steadying tank and flow-meter rather than attempting to alter the water flow to the ejector. The minimum air flow measurable was found to be approximately 0.016×10^{-3} ft^3/sec (.058 ft^3/hr.)

(e) Results.

The results of the first series of tests are shown in Figs. 4 and 5, the latter being deduced from Fig. 4 on the

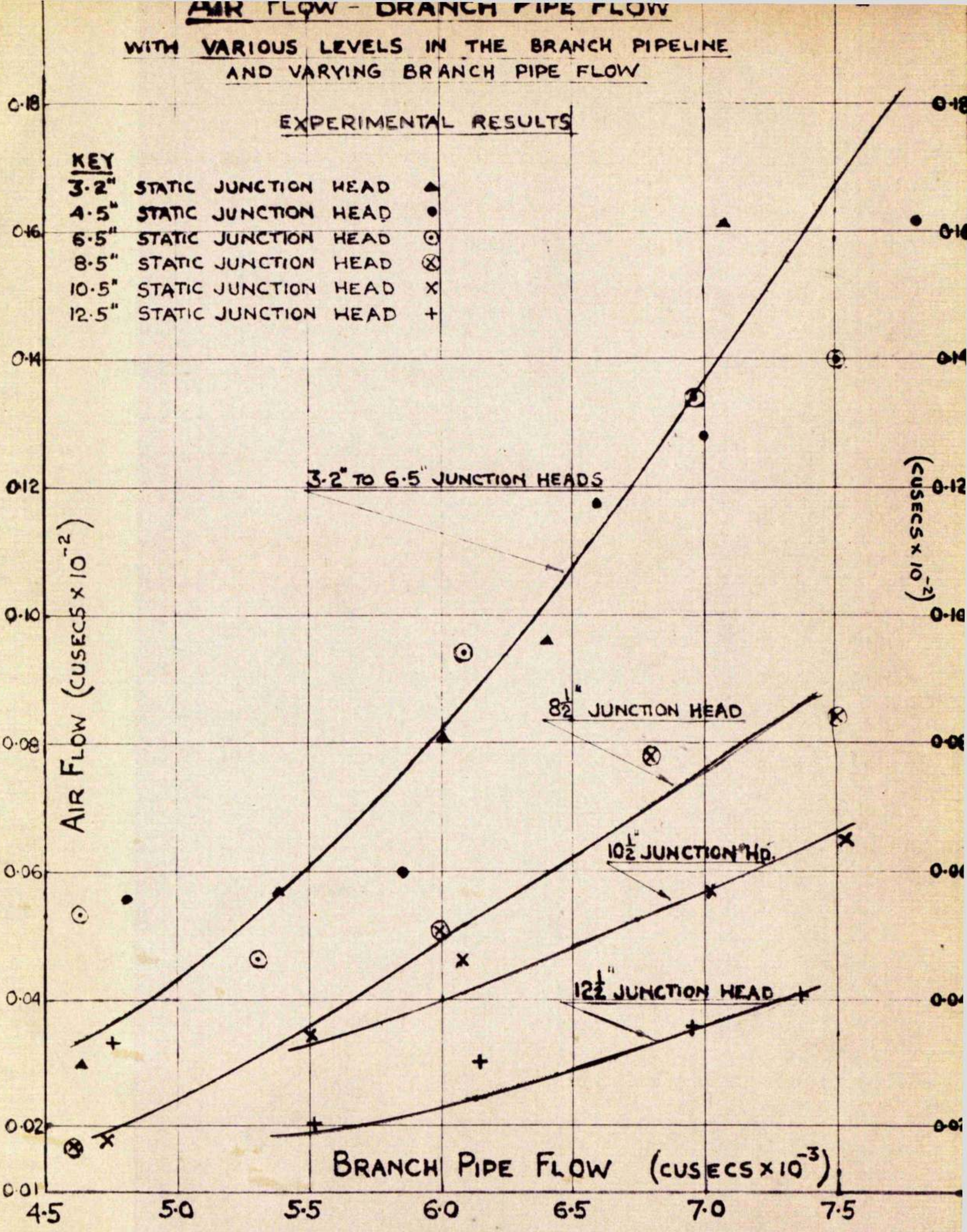


FIG. 4 AIR FLOW - BRANCH PIPE FLOW, BRANNIE BURN MODEL

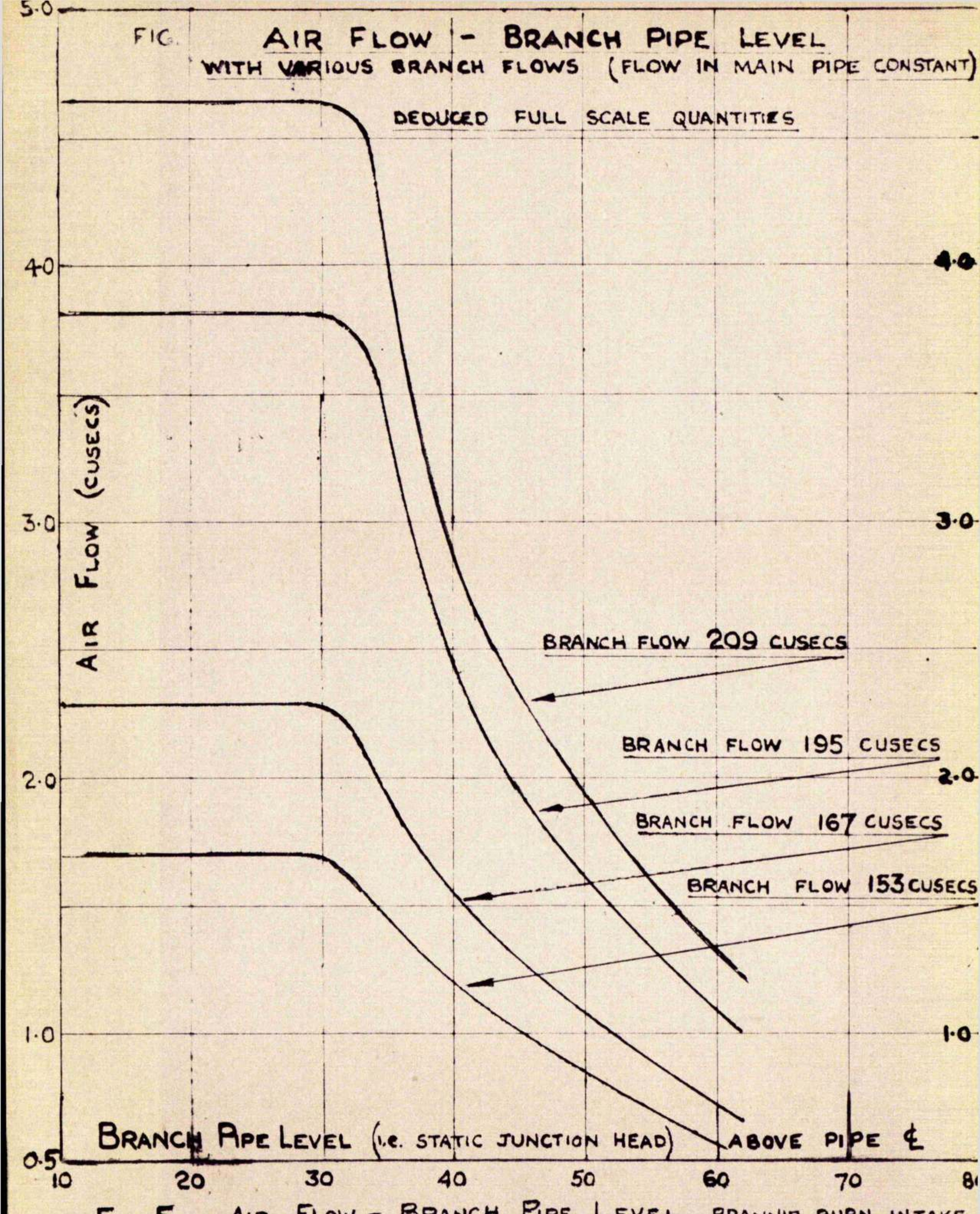


FIG. 5. AIR FLOW - BRANCH PIPE LEVEL, BRANNIE BURN INTAKE

assumption that the Froude criterion holds. Since the results of these experiments on a particular design are not of general or fundamental interest it is not proposed to detail the results obtained or to discuss them further here.

The behaviour of the air bubbles was observed visually. At first small air bubbles collected on the top of the pipe and gradually formed clusters which coalesced into one large bubble occupying almost half the pipe bore. This large bubble moved slowly down the pipe becoming trapped at the bend of the pipe. The size of this bubble remained approximately constant and small bubbles of entrained air were either swept into the main pipe past this bubble or apparently became attached to it at the upper end while a corresponding quantity of air was swept off the lower end. By modifying the junction details so that the bubbles could rise to a free surface, it proved possible to eliminate air from the main pipe line. An alternative solution, of course, was to fit a valve so that this pipe always ran full.

Once again the detailed results need not be considered here. The problem which these experiments raised was that of predicting behaviour of a system from model tests where air entrainment was concerned. The proper scaling laws to use, the mechanism whereby air is entrained by a jet or stream of water striking a free surface and the general suitability or applicability of model tests for air entrainment phenomena seemed to require further investigation. It was decided to carry out research into the air entrained by jets, varying both the size of the jets and the velocity of the water in the jets, and to study the mechanism of entrainment. The application of results obtained to the problem of model tests such as those described in this section and to the efficient design of hydraulic air compressors was the ultimate aim.

A survey of available information on the subject is given in the next section and the experimental work is then reported in subsequent sections.

SECTION 2

SHORT SURVEY OF AVAILABLE INFORMATION ON AIR ENTRAINMENT

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When the decision to carry out work on air entrainment had been taken, the literature was searched for all relevant information including not only the physics of air-water mixtures and the behaviour of air bubbles in pipe flow but also the practical application of such ideas in the hydraulic compressor and the air lift. Very little data of direct application to the particular problem which was to be investigated was found.

A comprehensive survey and annoted bibliography(3) was published by the University of Minnesota in August 1949 but this did not become available to the writer until after the jet research had been started and an independent search of the literature had been made. Although apparently interested primarily in the entrainment of air in high speed channels or flumes, the Minnesota bibliography contains a survey of all related information and must be acknowledged as a comprehensive and useful compilation. This publication contains no reference to the work of Shirley(4) which is of a later date and is closely related to the work reported in this thesis.

It is proposed to note briefly here the information which seems applicable to the problem of air entrainment by jet and to the design of a hydraulic compressor and then discuss Shirley's work separately in greater detail.

(a) Physical Properties of Air-Water Mixtures.

Several experimental and theoretical investigations of gas bubbles ascending in a liquid column and of droplets of liquid falling in a gas have been made and such features as bubble size, distribution, shape, growth by coalescence, velocity, have been studied. Miyagi[(5)] showed that a marked change in the rising velocity and motion of bubbles takes place when a critical radius of bubble is exceeded. The buoyancy force is predominant in the larger bubbles, causing a natural sorting of sizes in entrained mixtures. Allen[(6)] carried out experiments on the rate of rise of various sized bubbles and this work together with that of several other investigators is summarized by Pekeris.[(7)] Kortweg[(8)] put forward a mathematical development of the equations of motion of fluids in which the density varies rapidly from point to point and has considered a thin transition layer between the solid and vapour state of a fluid which may have a bearing on the entraining mechanism.

Ohnesorge,[(9)] Haenlein,[(10)] Weber,[(11)] Rayleigh,[(12)]

Smith and Moss[13] and others have studied the dissolution of liquid jets and the formation of drops from a jet discharging into the air. Entrainment at the lip of an overflow pipe was investigated by Kalinske.[14] Kalinske and Robertson[15] investigated a means of evacuating air pockets from pipes by means of a hydraulic jump which entrained air. A direct analysis of surface tension forces readily applicable to problems of air entrainment could not be found in the literature.

(b) The Air Lift and Hydraulic Compressor.

Papers by Purchas[16] and Owens[17] give elementary theory and operating characteristics of the air lift pump and contain bibliographies of more advanced investigations. Design studies on these machines do not seem to contribute much to a better understanding of the phenomena concerned, although the design and efficiency of such apparatus would benefit from a more thorough knowledge of the principles involved in their operation.

Several large hydraulic compressors were installed and operated in America and Germany in the years 1896-1920.[18,19,20,21 The American (Taylor) entraining heads created suction by radial flow while the early Continental types drew in air through pipes tapped into a venturi-type throat. The later plants entrained air by merely allowing a jet to strike a water surface. No proper comparison of these different types can be made from available published data. The efficiency of the earlier Taylor compressors was 50% to 62.4% but higher efficiencies - up to 85% - were claimed for later installations. The dependence of any improvement on a better understanding of the phenomenon of air entrainment is obvious.

(c) Open Channel Flows.

The subject of air entrainment in open channels is too large and complex to be discussed in detail here - various empirical relationships have been put forward by different investigators but the data do not fit any generally accepted function. The proposal of Lane[22] that the onset of entrainment depends on the state of the turbulent boundary layer is, however, of interest and may be accepted since turbulence is almost certainly the mechanism by which air is held in the flow. De Lapp[23] carried out careful measurements of air concentration in entrained channel flows. The difficulty of reconciling model and full scale results in such cases has often been discussed (e.g. by Ehrenberger[24]). A model spillway, for example, will show no indication of entrainment phenomena although the prototype may have "white water" at all operating stages. The writer feels that this discrepancy may be due, at least in some measure, to faulty model technique as will be discussed later. Éscande[25] has shown analytically that exact similarity of air entrainment phenomena is impossible in systems comprising a free surface subjected to atmospheric pressure. The condition for similarity requires the same reduced pressure at the free surface as would be adopted in model studies of cavitation. Dimensional similarity and the general form of the functions of gas-liquid mixtures are discussed by Schmidt.[26]

Research into air entrainment in open channel flows is being actively pursued on the Continent and in the U.S.A. as indicated by the fact that twelve papers on the subject were read at the recent meeting of International Association for Hydraulic Research at Minnesota.[27]

(d) Shirley's Experimental Work.

A thesis on "Entrainment of Air by Liquid Jets" was submitted to the State University of Iowa by R. W. Shirley in August 1950. The object of this research was almost identical with that reported here. As knowledge of this work was not obtained until after the main series of jet experiments had been completed, Shirley's research and that of the writer are two independent and almost simultaneous attacks on the same problem. The method of collection of the bubbles and measurement of the flow quantities used by Shirley differed completely from the writer's and a comparison is of interest.

Shirley directed his jets into a pool at an angle and collected the air as it rose to the surface in a reservoir, the flow from which could be calculated. His apparatus is shown diagrammatically in Fig. 6. The angle which his jets made with the surface could be varied from about 45° to 67° and results from these tests were extrapolated to obtain figures for vertical entry of the jets. Determination of the critical flow at which entrainment begins was made from observation of vertical jets. This critical flow was found to vary considerably when a different design of nozzle was used.

Small quantities of bubbles were observed to rise before reaching the box (maximum loss judged to be 5%) and at high rates of flow with large nozzles a tendency for bubbles to exceed the limits of the reservoir was seen and a correction was made based on visual observation of the quantity of air escaping.

Difficulty was experienced at times with vortices formed at the surface causing excessive entrainment and in some cases an obstruction in the form of a small block was inserted to prevent vortex formation. Entrainment at the critical flow was observed to be erratic and surface bubbles moving into the jet caused increased entrainment. The nozzle sizes used were 3/32", 3/16", 3/8", ¾" and 1½" diameter and the

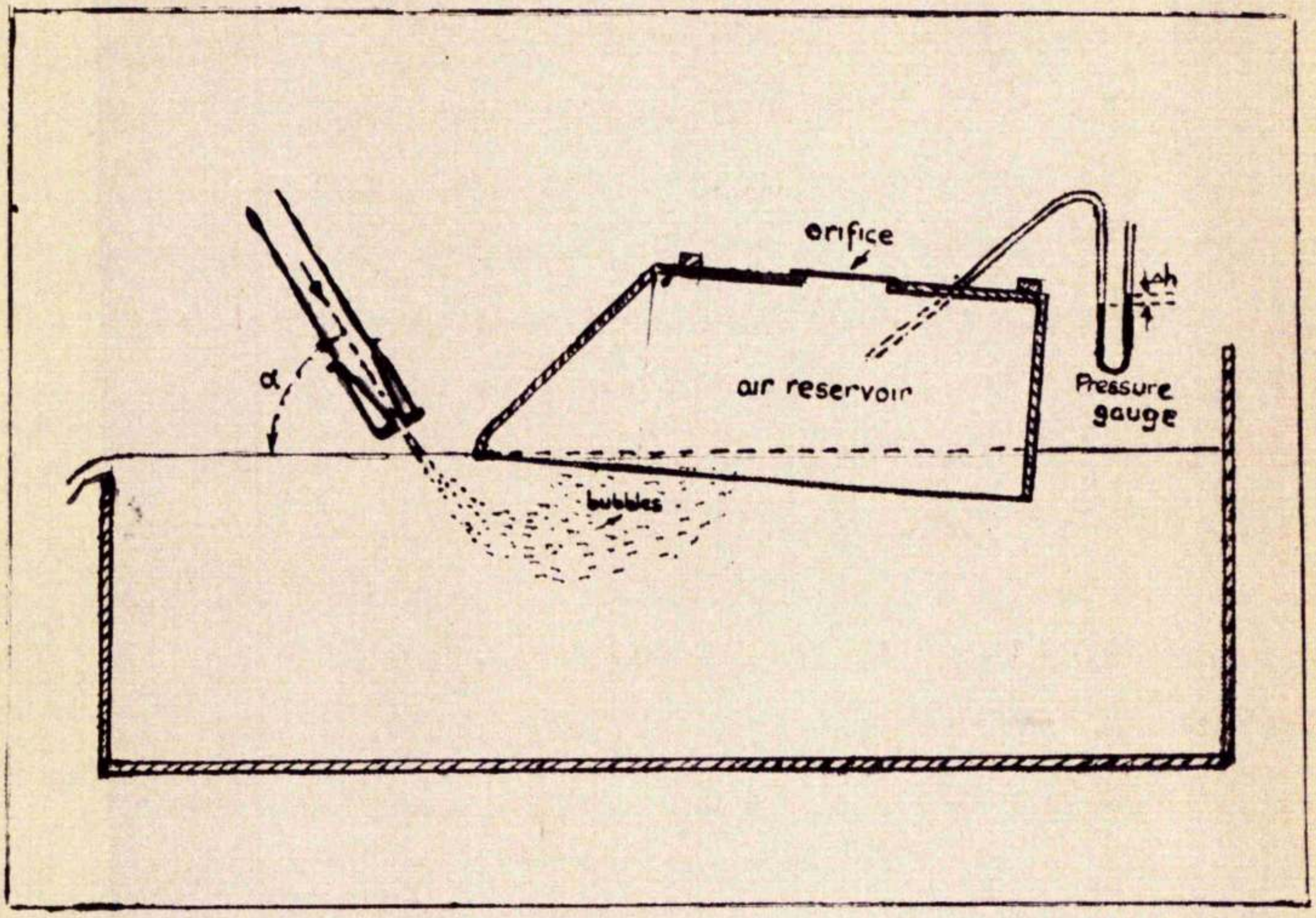


FIG.6 SHIRLEY'S APPARATUS (DIAGRAMMATIC)

distance from nozzle to centre of area of entrance of jet was 4 to 6 jet diameters.

This experimental work was, like the writer's, largely exploratory. Briefly, Shirley concluded:

(1) The entrainment of air is not a direct function of velocity and diameter of the jet alone.

(2) Roughness of the free surface of the jet and degree of turbulence in the pool have an influence on the critical flow at which entrainment starts.

(3) There was an indication that for critical flow the influence of surface tension as represented by Weber Number may become more pronounced as the Froude Number becomes larger than 20, but experimental limitations prevented verification.

(4) The data collected approached a single curve of generalized nature when entrainment tended to become independent of Froude Number and more dependent on Weber criterion.

(5) More sensitive equipment should be used to continue the investigation for smaller nozzles.

(6) Experiments should be continued with a fluid of different characteristics to allow conclusions regarding relative importance of each fluid property.

Shirley's results will be commented on when the results of the present research are considered. His method of air collection and measurement with the accompanying experimental difficulties, do not appear to be superior in any way to the technique evolved and described in the following pages, while his extrapolation of data from angled jets to vertical jets is an obvious source of error.

SECTION 3

PRELIMINARY TESTS WITH JETS FROM ORIFICES

SECTION 3

PRELIMINARY TESTS WITH JETS FROM ORIFICES

(a) Apparatus.

In order to investigate the process of air entrainment it was decided to try to collect the air entrained by jets of different sizes and velocities in a pipe-line similar to that already used in the Glen Shira model tests. The same apparatus for separation and measurement of the air, which had proved to be satisfactory in operation was used. An open Pyrex Tee-piece was fitted into a length of 2" bore Pyrex pipe (in place of the special junction and inclined pipe of the Brannie Burn model). The 2 inch bore pipe carried a flow of water from a tank into the separator, a free surface being formed in the neck of the T-piece. Into this free surface was directed a jet of water from a calibrated orifice, any entrained air being swept into the pipeline and thence into the separator. The apparatus is shown in the photographs, Fig. 7 and 8. The air measuring apparatus was the same as that used in the Glen Shira model experiments.

In order to get jets of various diameters a series of orifice plates was made, the diameters of these orifices being 1/32", 1/16", ⅛", 3/16", ¼", ⅜", and ½" respectively. The parallel portions of these orifices were given a carefully

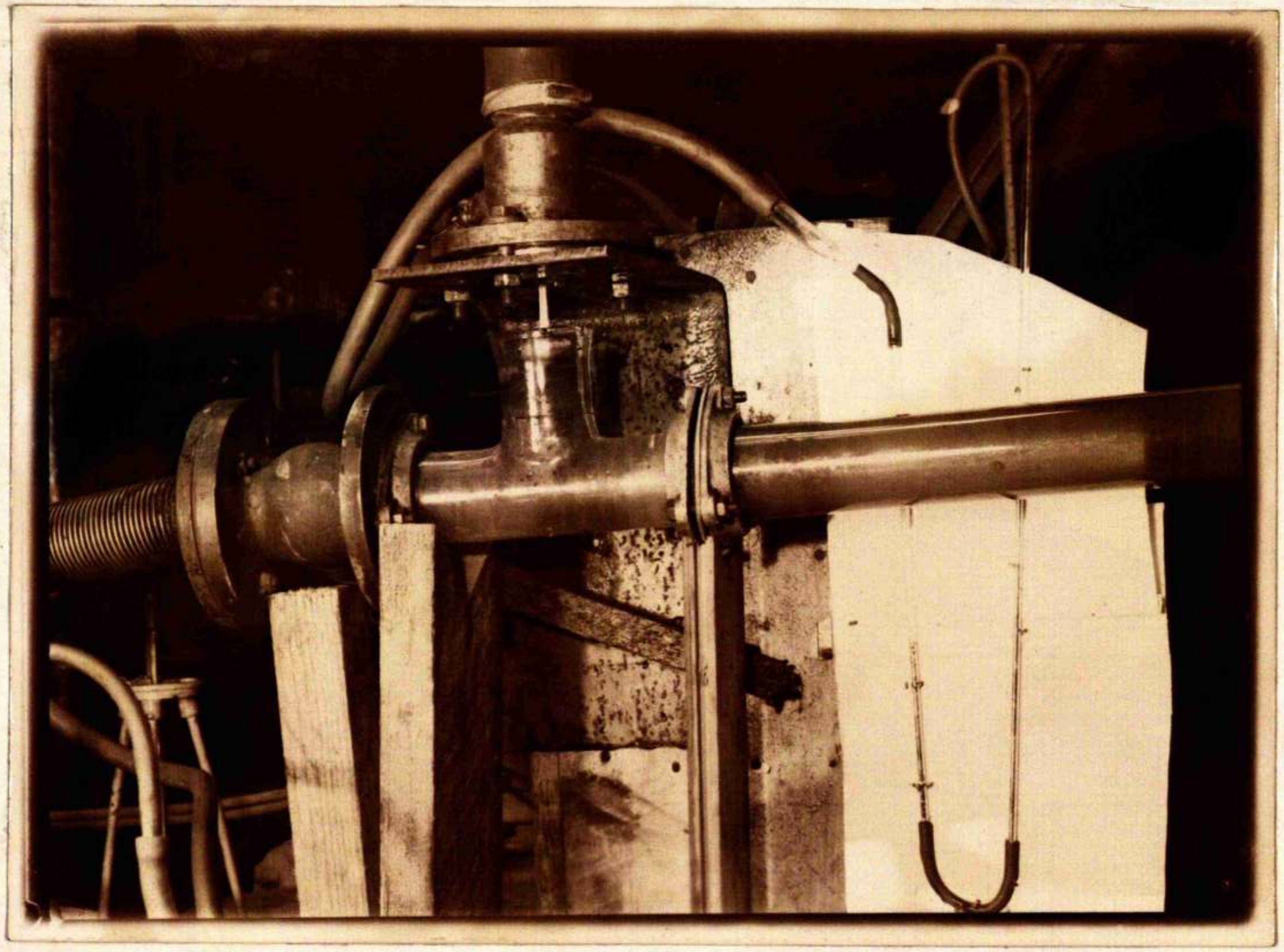


FIG. 7 ORIFICE APPARATUS

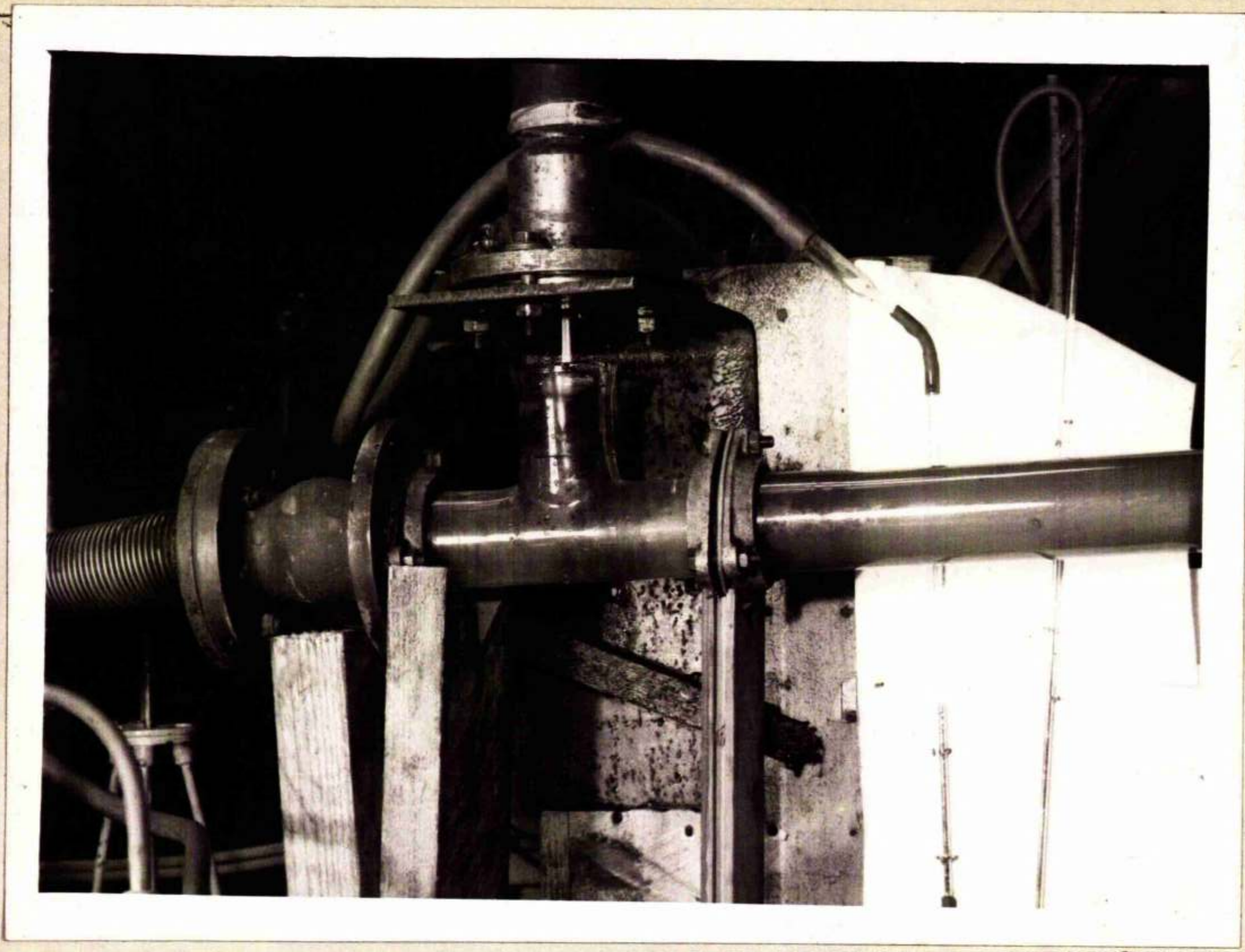


FIG. 8 CLOSE-UP OF GLASS T-PIECE, SHOWING JET

reamed finish.

The $\frac{1}{2}$ inch diameter orifice was first tested using an orifice tank, the heads above the orifice being read on a gauge glass. Later it was found that the smaller jets required higher heads than could be obtained by this means before entrainment would start. An orifice plate holder was therefore designed to hold the orifice plates, the flow of water to the orifice plates being supplied through a 2 inch diameter flexible hose from a centrifugal pump. This meant that the flow was subject to fluctuations, as will be discussed later. A pressure gauge could be fitted to the orifice plate holder.

In deciding on this experimental arrangement it was realised that several important variables, such as surface tension, viscosity, pressure and density of the fluids concerned, were not controllable and that the results must therefore be lacking in generality. The problem being investigated is the entrainment of air by jets or streams of water rather than the general case where a jet of one fluid entrains another on striking a free surface, although dimensional analysis including the fluid properties as variables will give guidance as to the interpretation of results. If it were desired to extend the investigation to embrace several different fluids the experimental complications would become very great and do not, at present, seem to be justified.

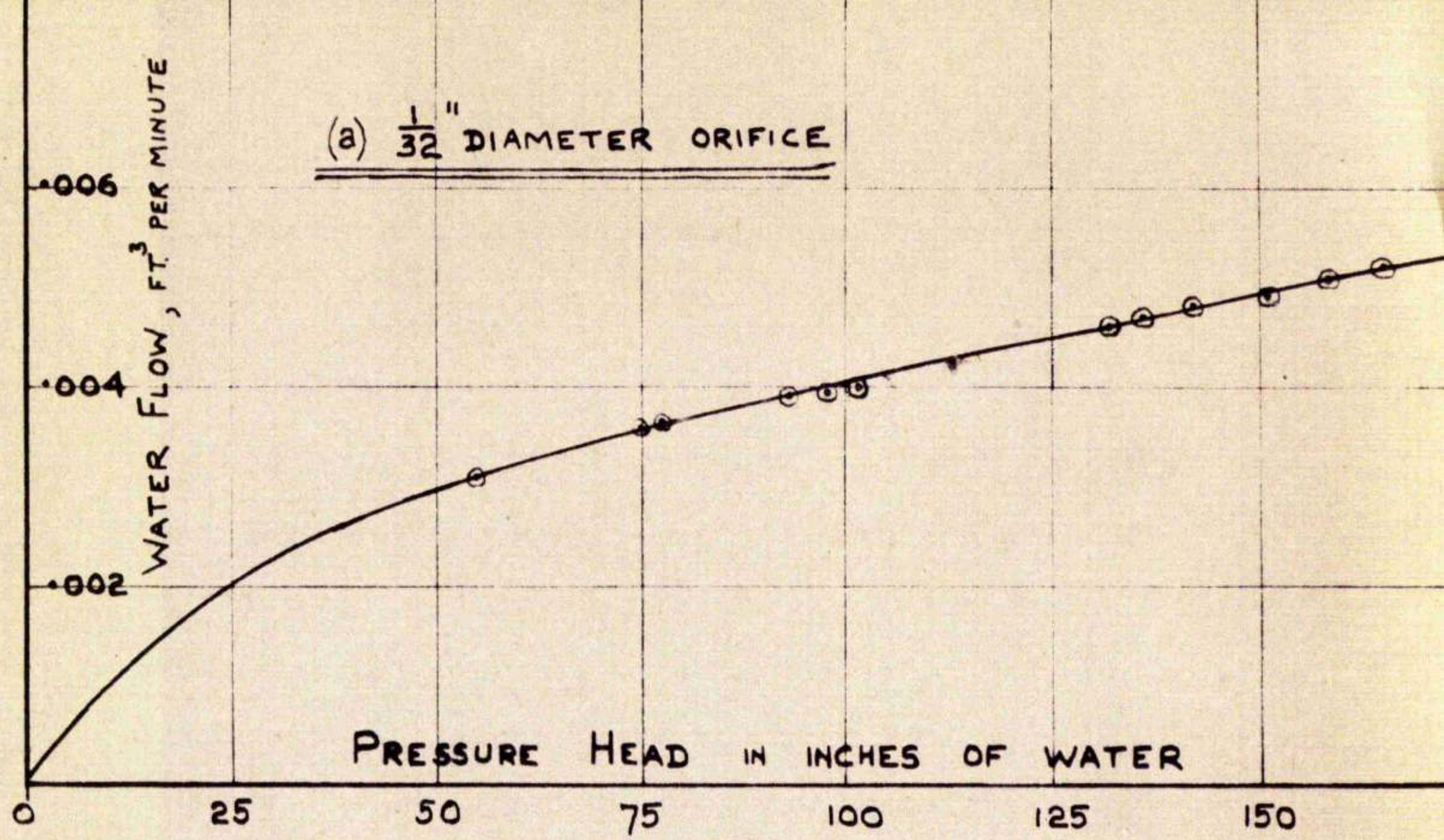
(b) Calibration of Orifices.

The orifices were all calibrated for heads up to about 16 inches of water while fitted to the orifice tank. The water was collected over a period of time, the head being maintained constant by controlling an inflow to the tank. The calibration was also checked by the following level method, i.e. noting the time for the level to fall through successive steps of one inch. When it was found that higher heads would be required in the air entrainment experiments, the calibration was extended by fitting the orifice plates to the holder described above. The calibration of the various orifices is shown in Figs. 9 to 12. Many more experimental points than are shown were obtained in the range 0 to 16 ~~miles~~ inches head.

(c) Air Entrainment Experiments: Teething Troubles.

The first experiments were made to try out the proposed technique. The ½ inch orifice was fitted to the orifice tank and the air bubbles were observed visually.

Some air was entrained with heads as low as 2" to 3", but this seemed to be due primarily to the surface disturbance caused by the jet in the confined space of the T-piece neck. There was also a tendency for a vortex to form in the T-piece. At higher heads the jet appeared to be entraining air before



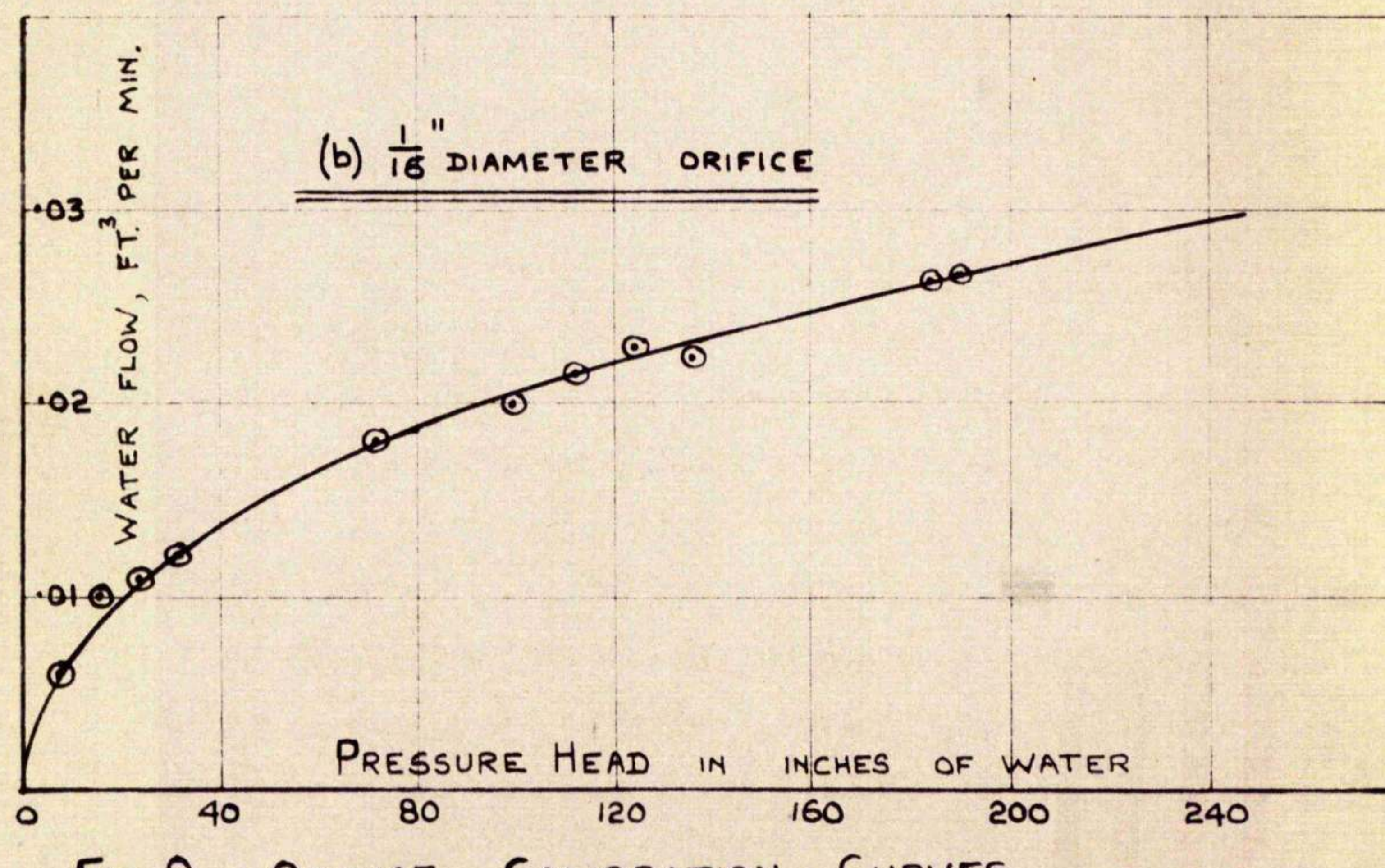
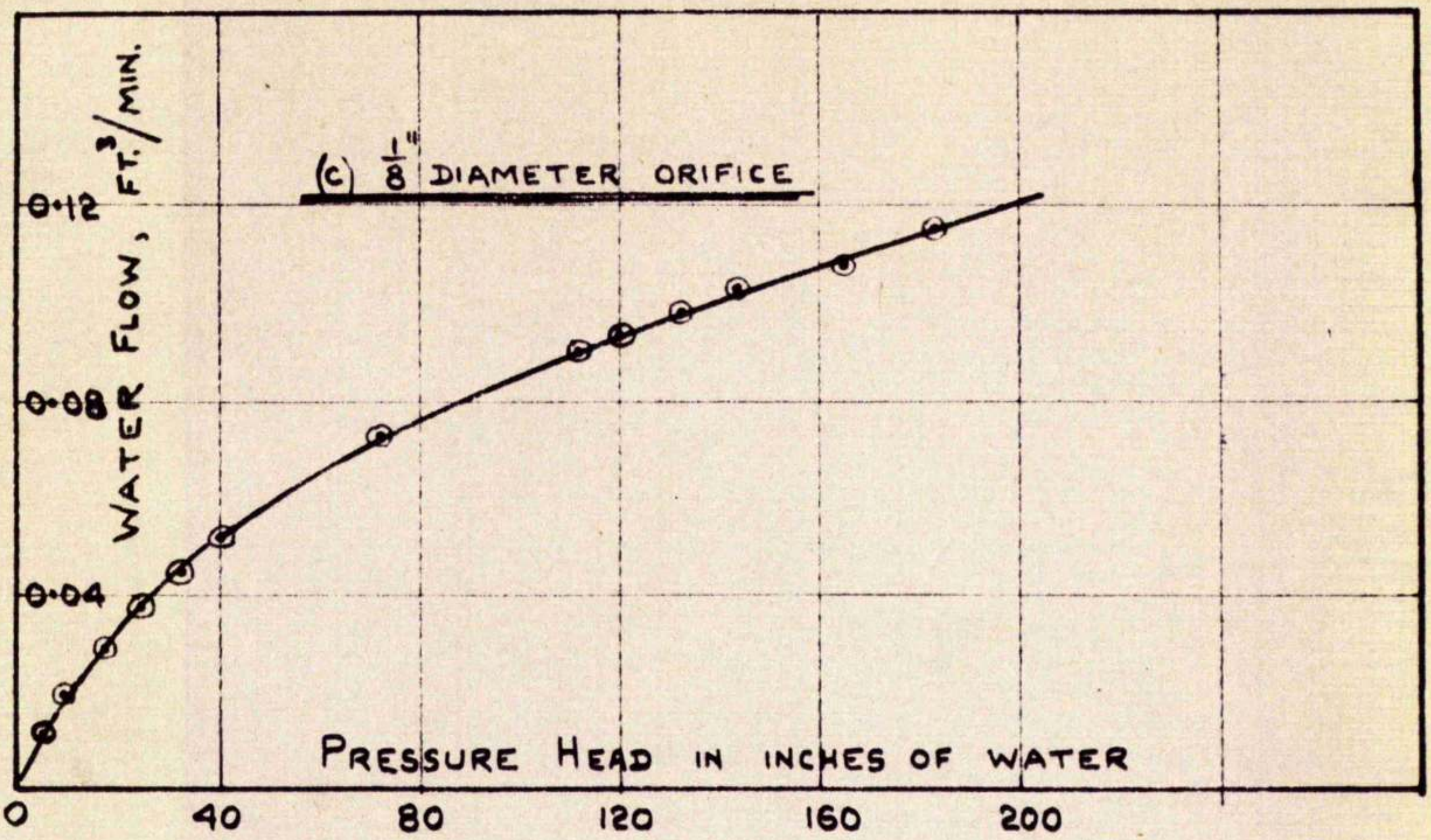


FIG. 9 ORIFICE CALIBRATION CURVES



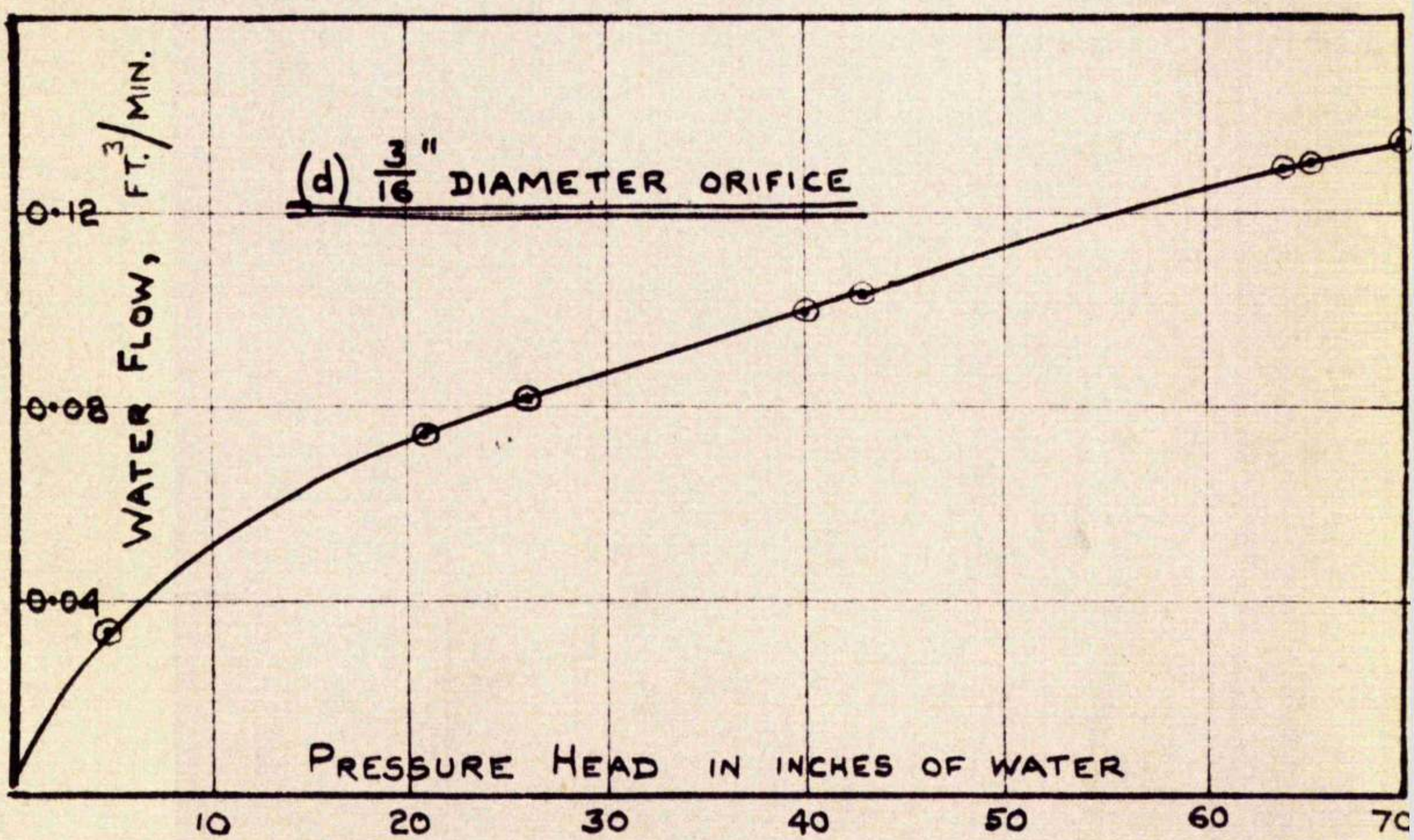
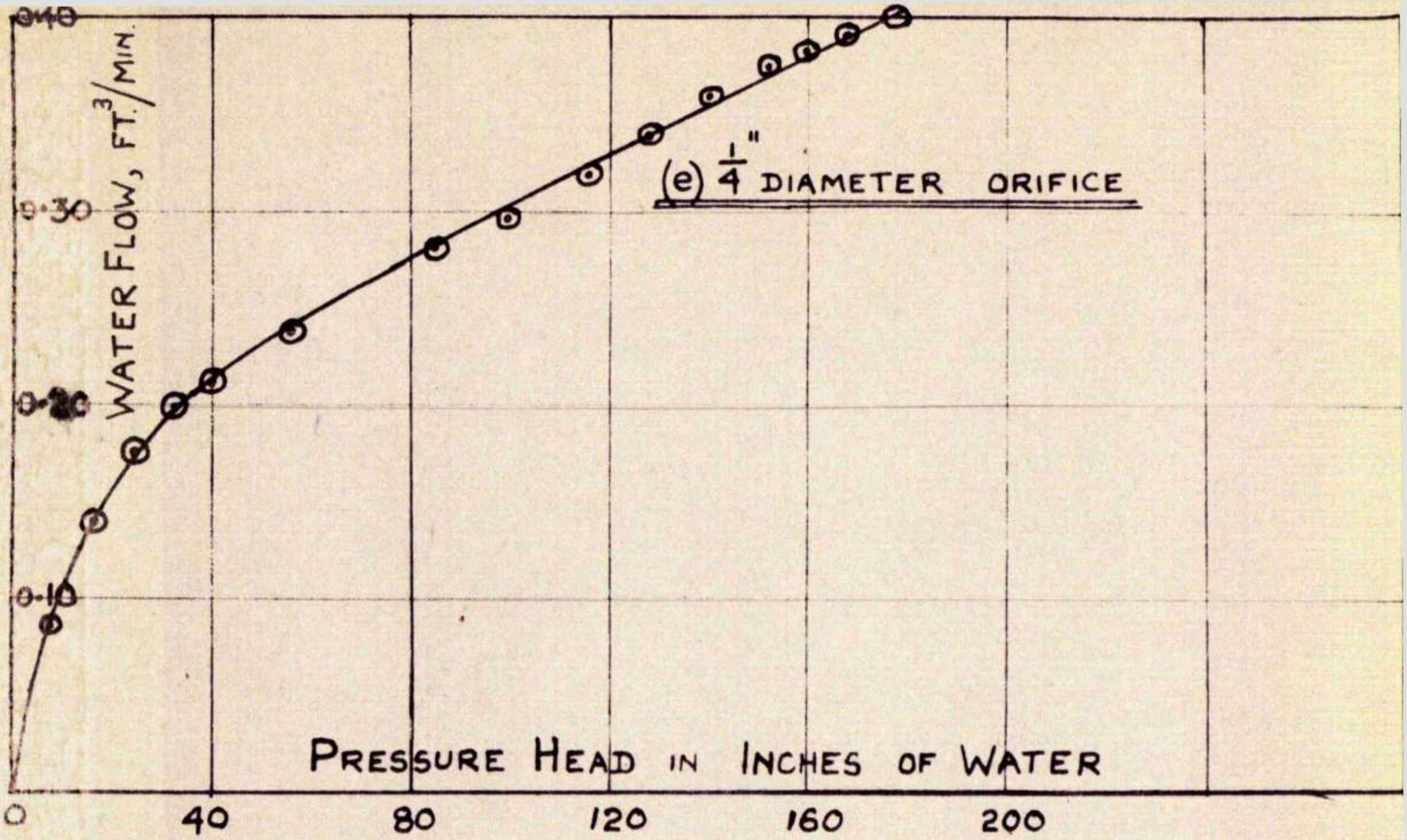


FIG.10. ORIFICE CALIBRATION CURVES



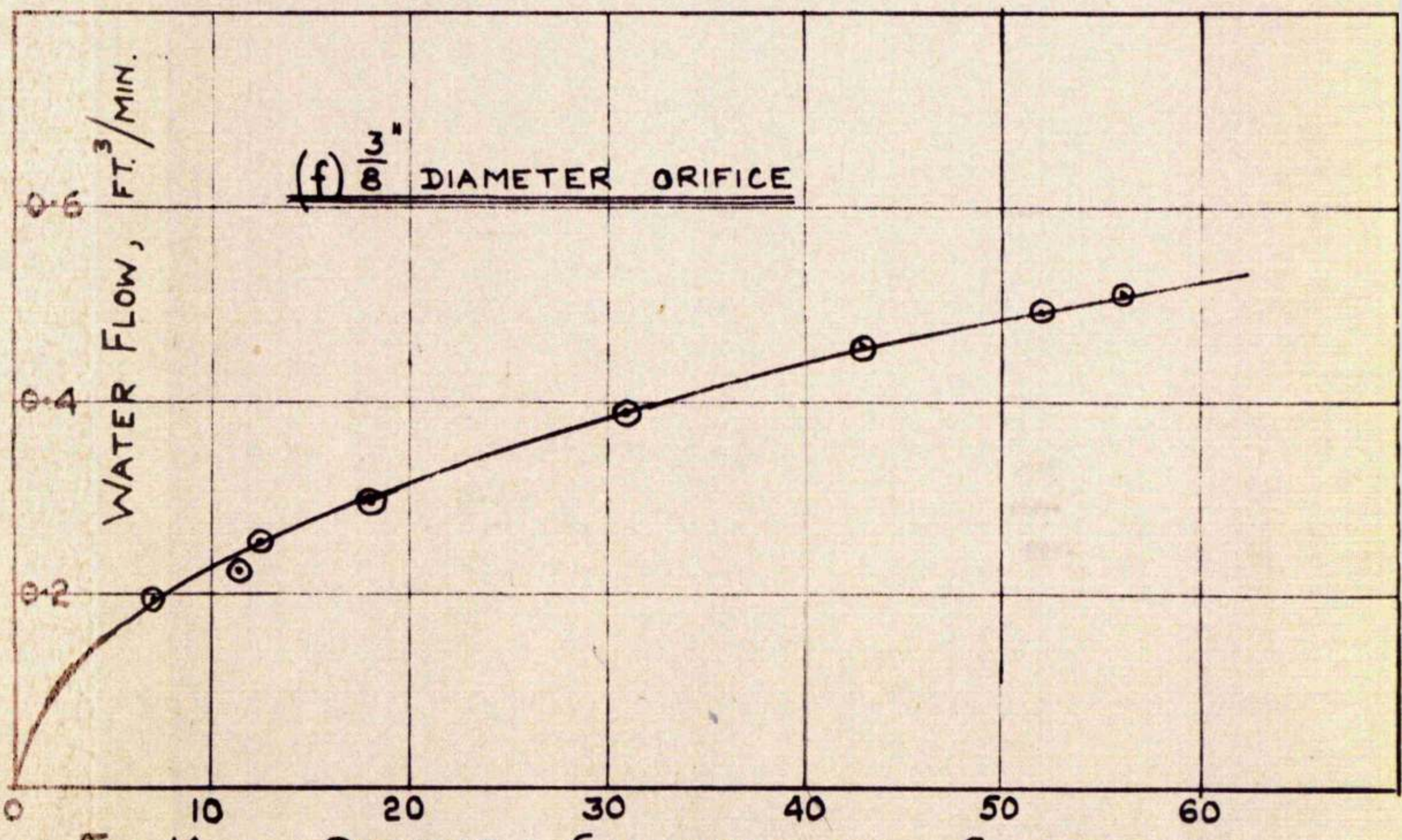


FIG. 11. ORIFICE CALIBRATION CURVES

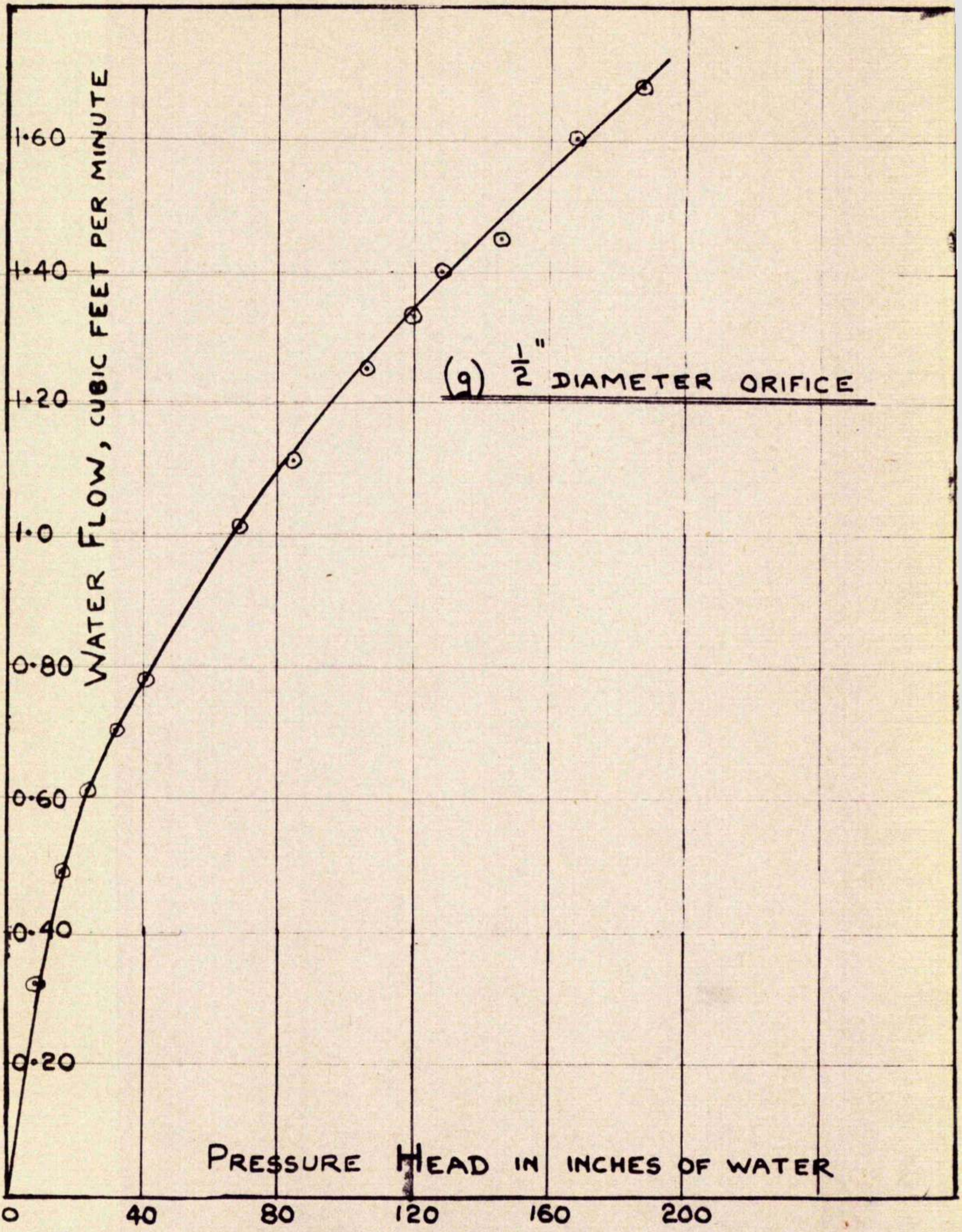


FIG. 12 ORIFICE CALIBRATION CURVE

striking the surface. Air could apparently be carried below the surface by any of three methods, viz., (1) by rupture of the surface, air being carried under the surface by viscous forces, (2) by entrainment of air by the jet itself before striking the water, (3) by impact on surface irregularities, trapping and forcing air below the surface. It was decided to attempt to eliminate this last source of inconsistency by isolating the surface from the secondary currents caused by the jet impinging on the straight-through flow in the pipe-line

A piece of rubber sheeting with a hole approximately 0.7 inch diameter was inserted in the T-piece as shown in Fig. 13(a). This was a great improvement. The surface was much calmer and no entrainment was observed until heads of about 15 inches to 18 inches were reached. There was still intermittent entrainment due to surface irregularity as the head was increased. It also became apparent that, with surface disturbances reduced as much as possible, higher heads than had been anticipated would be required and the orifice holder was therefore designed. A tendency for vortex formation persisted.

It was observed that when the water was turned off so that the orifice head fell slowly, entrainment appeared to persist down to heads of approximately 14 inches compared with about 18 inches when the head was increasing.

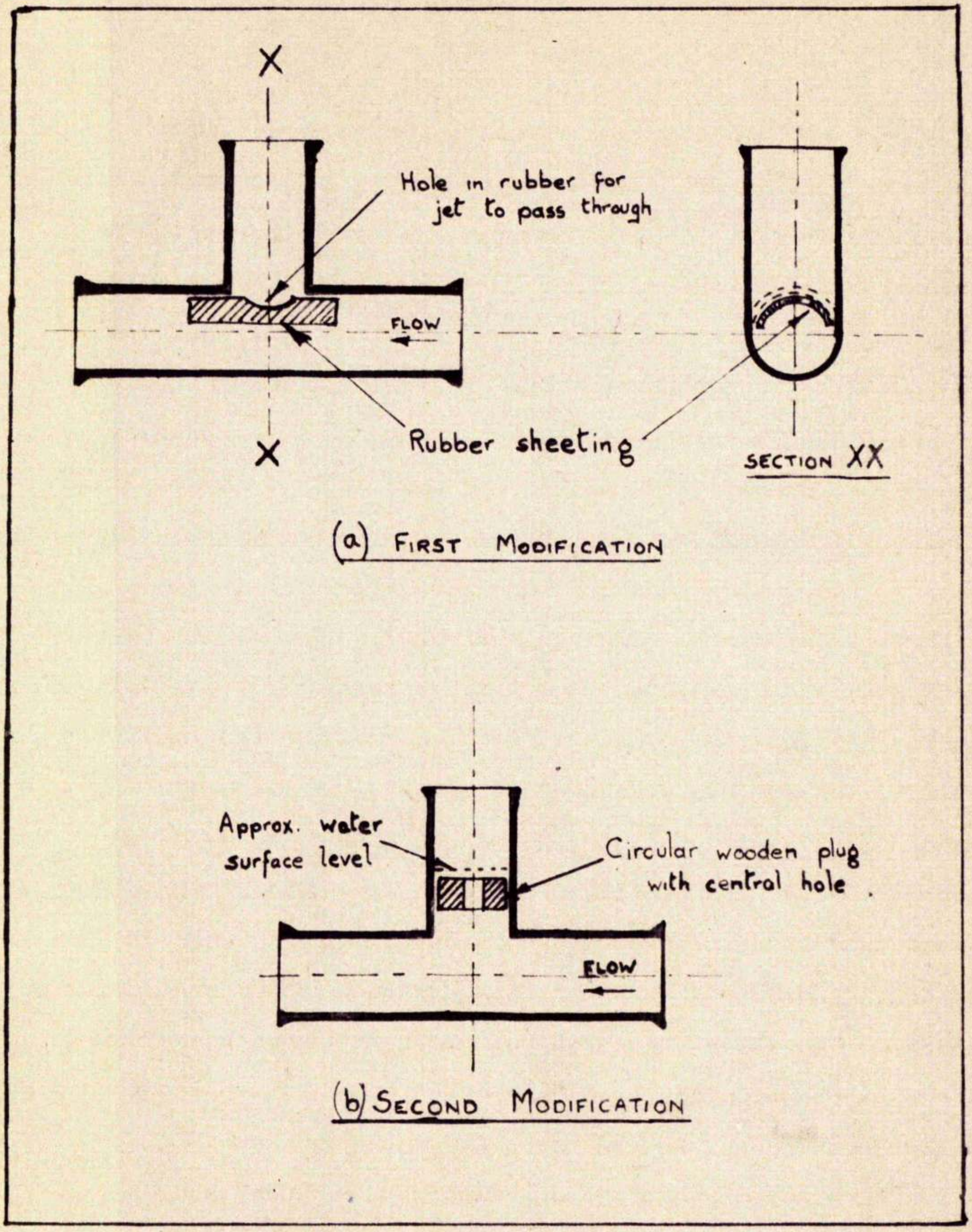


FIG. 13 METHODS OF OBTAINING A SMOOTHER WATER SURFACE IN THE NECK OF THE TEE-PIECE

The point at which air entrainment starts was determined by observation of the water surface against a plain background. The head was increased slowly and at the critical point a small white puff of very fine air bubbles was seen to accompany the jet under the surface. This critical point, because of its dependence on personal judgment, was difficult to determine exactly, particularly if the surface was not absolutely still. It was later checked by extrapolating the curve of air entrained on a base of head at the orifice to zero air. Observed and extrapolated values were found to give reasonably good agreement, as will be seen from the results - a useful check which is not obtainable with Shirley's apparatus.

At first some difficulty was experienced due to blocking of the manometer and air extraction tubes from the separator by drops of water from the splashing inside. This was cured by increasing the diameter of these pipes where they entered the separator. It was found easier to carry out the initial adjustments if one of the bungs were removed from the separator, thus ensuring atmospheric pressure inside. Other occasional sources of trouble in operating were (a) air trapped in the glass pipe-line before starting a test, (b) vortex formation in the T-piece.

The rubber sheeting was removed from the T-piece and a wooden circular plug, approximately $\frac{3}{4}$ inch thick with a 1 inch diameter central hole through which the jets flowed, was tried instead. It was intended that this should be a floating "mat" but soon after it was inserted it jammed in the neck of the T-piece. All subsequent tests were run with the surface approximately $\frac{1}{4}$ inch to $\frac{1}{2}$ inch above the upper surface of the wooden plug and a reasonably smooth surface resulted. (Fig. 13(b)). A small piece of rubber sheeting placed radially above the top of the wooden plug helped to reduce the tendency to vortex formation.

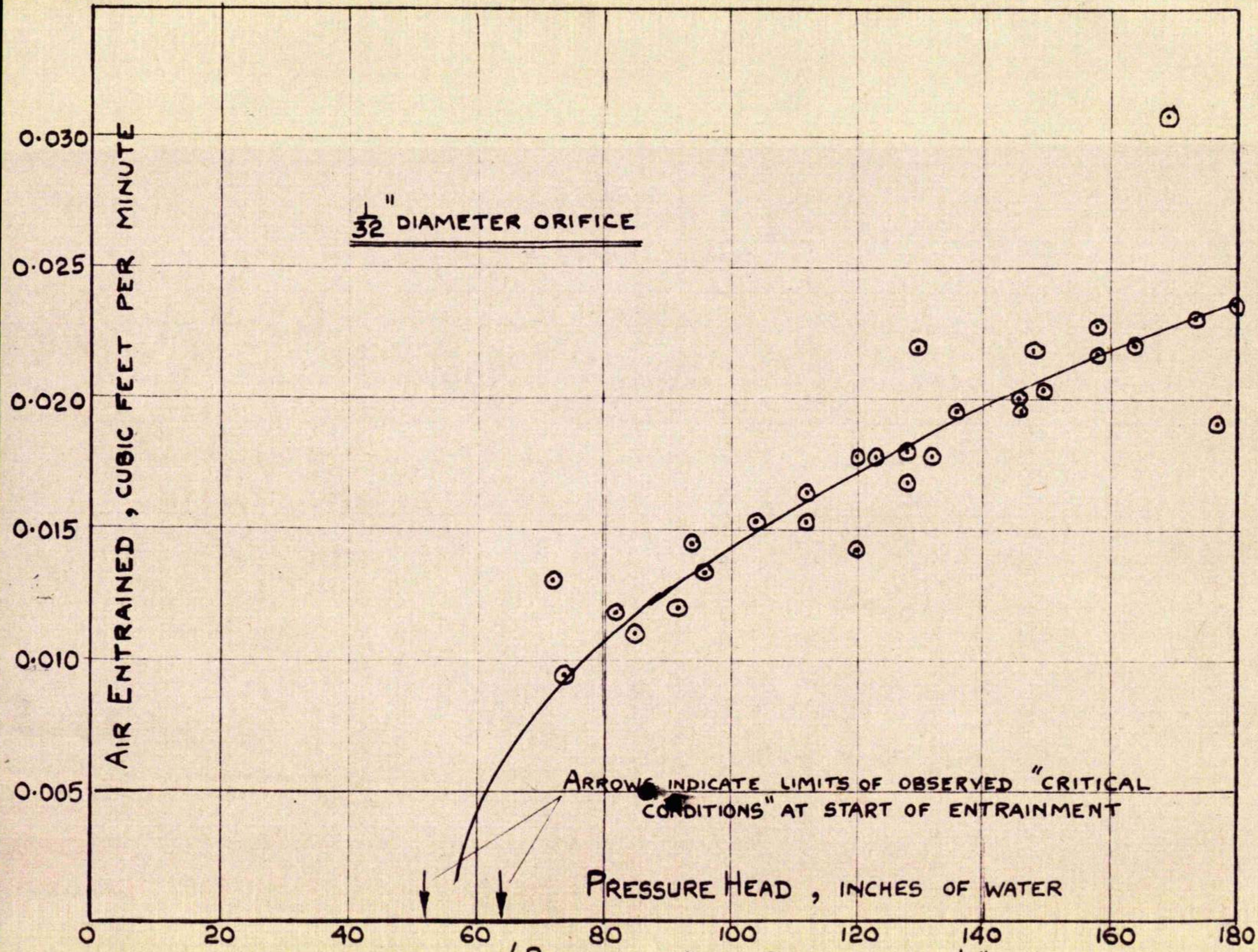
The water temperature was not taken during every test but measurements taken at intervals indicated little variation from 50° F.

(d) Results.

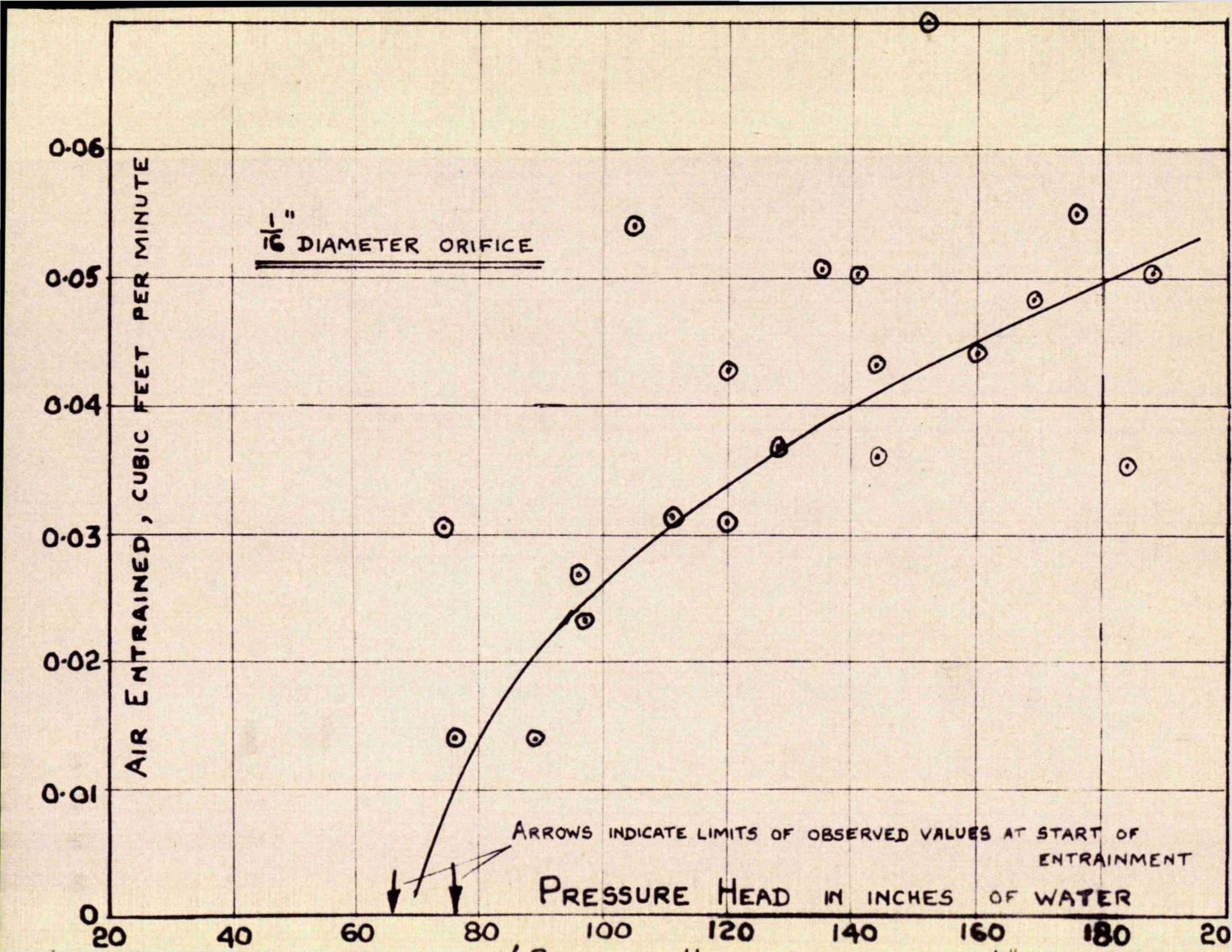
Although the results obtained in these experiments were, in certain cases, not confirmed by the later nozzle experiments they are shown graphically in Figs. 14 to 20.

It will be observed that considerable scatter of experimental readings was obtained. This scatter seemed to be due to two main causes, viz.:

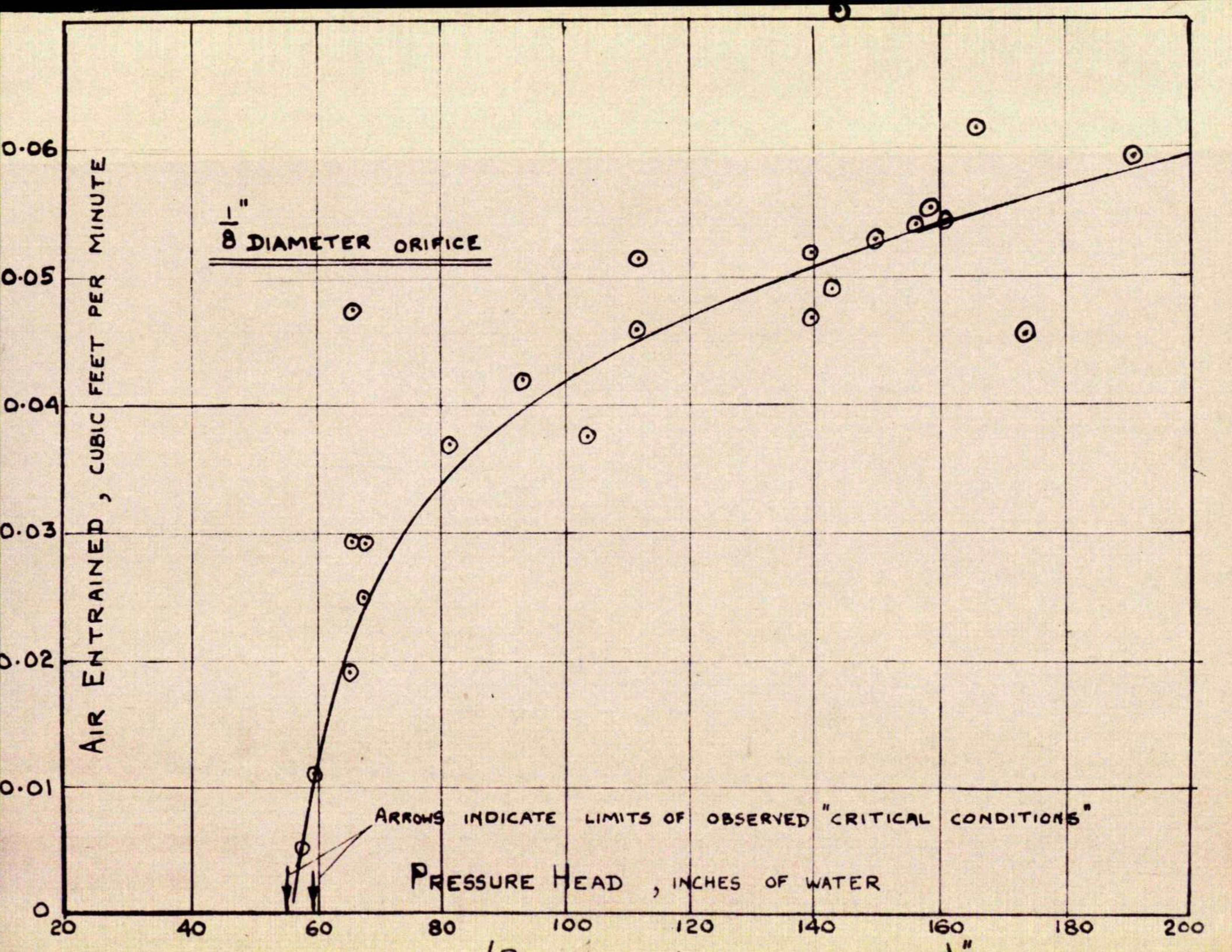
(a) the impingement of the jet on the water in the neck of the T-piece caused a certain amount of surface disturbance which was variable and uncontrollable (and as already mentioned, there was occasionally a tendency to vortex formation). This surface disturbance generally caused rather more air to be entrained than if the surface were still (see Section 3(c) above);



1/32" DIAMETER ORIFICE
AIR ENTRAINED, CUBIC FEET PER MINUTE
0·030
0·025
0·020
0·015
0·010
0·005
ARROWS INDICATE LIMITS OF OBSERVED "CRITICAL CONDITIONS" AT START OF ENTRAINMENT
PRESSURE HEAD, INCHES OF WATER
0
20
40
60
80
100
120
140
160
180



1/16" DIAMETER ORIFICE
AIR ENTRAINED, CUBIC FEET PER MINUTE
PRESSURE HEAD IN INCHES OF WATER
ARROWS INDICATE LIMITS OF OBSERVED VALUES AT START OF ENTRAINMENT
0·06
0·05
0·04
0·03
0·02
0·01
0
20
40
60
80
100
120
140
160
180



1/8" DIAMETER ORIFICE
AIR ENTRAINED, CUBIC FEET PER MINUTE
PRESSURE HEAD, INCHES OF WATER
ARROWS INDICATE LIMITS OF OBSERVED "CRITICAL CONDITIONS"
0.06
0.05
0.04
0.03
0.02
0.01
0
20
40
60
80
100
120
140
160
180
200

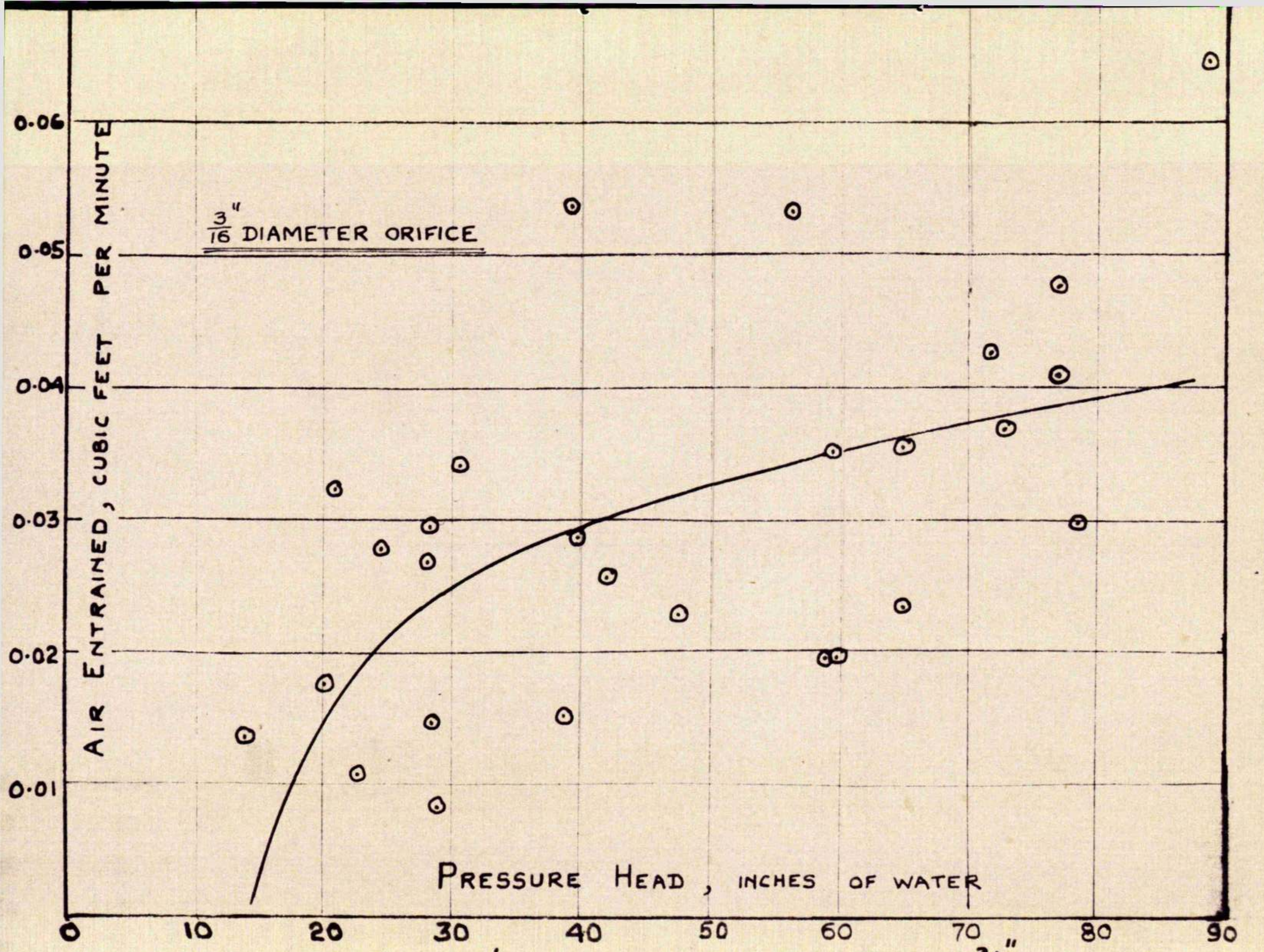


FIG 17 AIR ENTRAINED / PRESSURE HEAD CURVE FOR $\frac{3}{16}$" DIA. ORIFICE

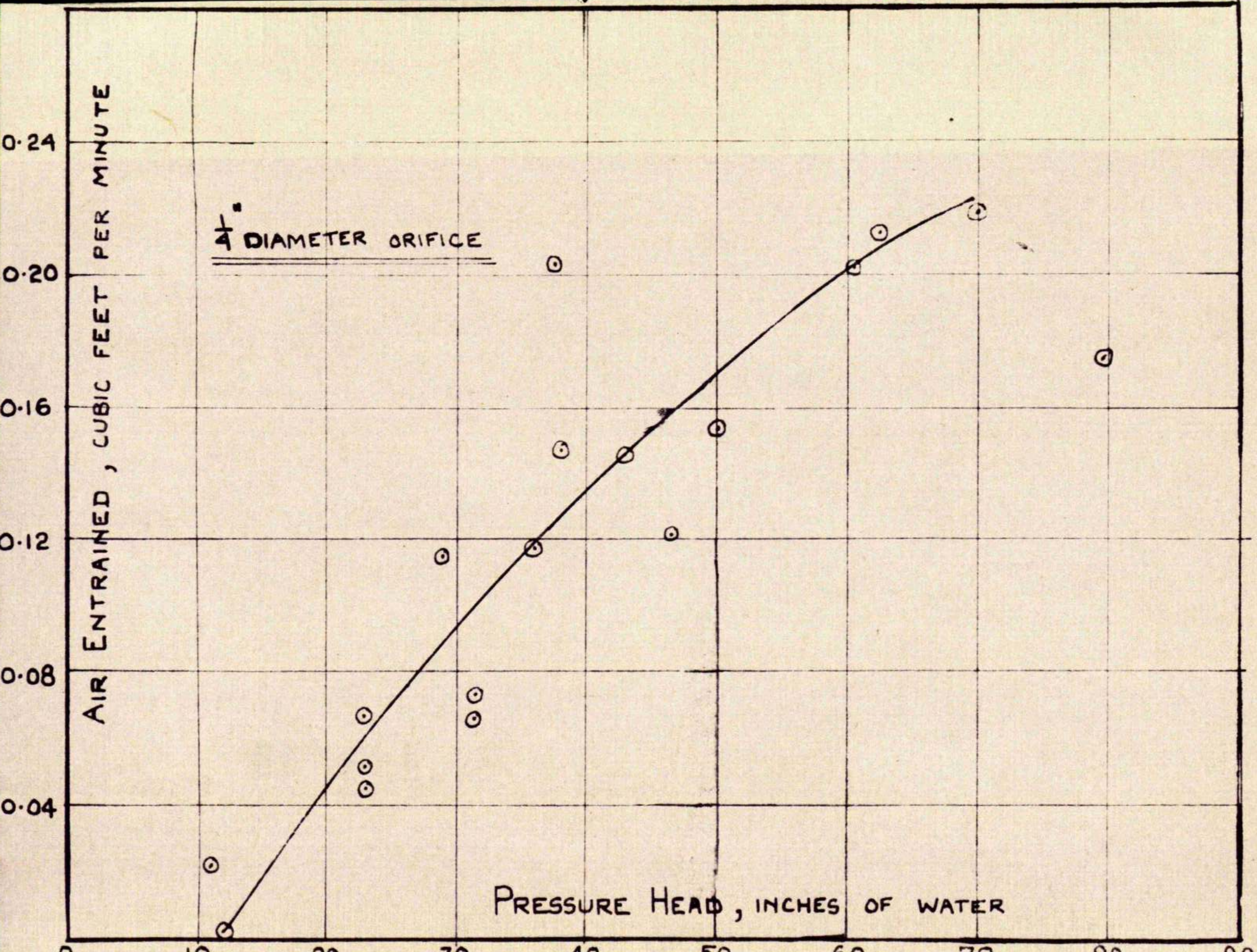


FIG 10. AIR ENTRAINED/PRESSURE HEAD CURVE FOR ¼" DIA. ORIFICE

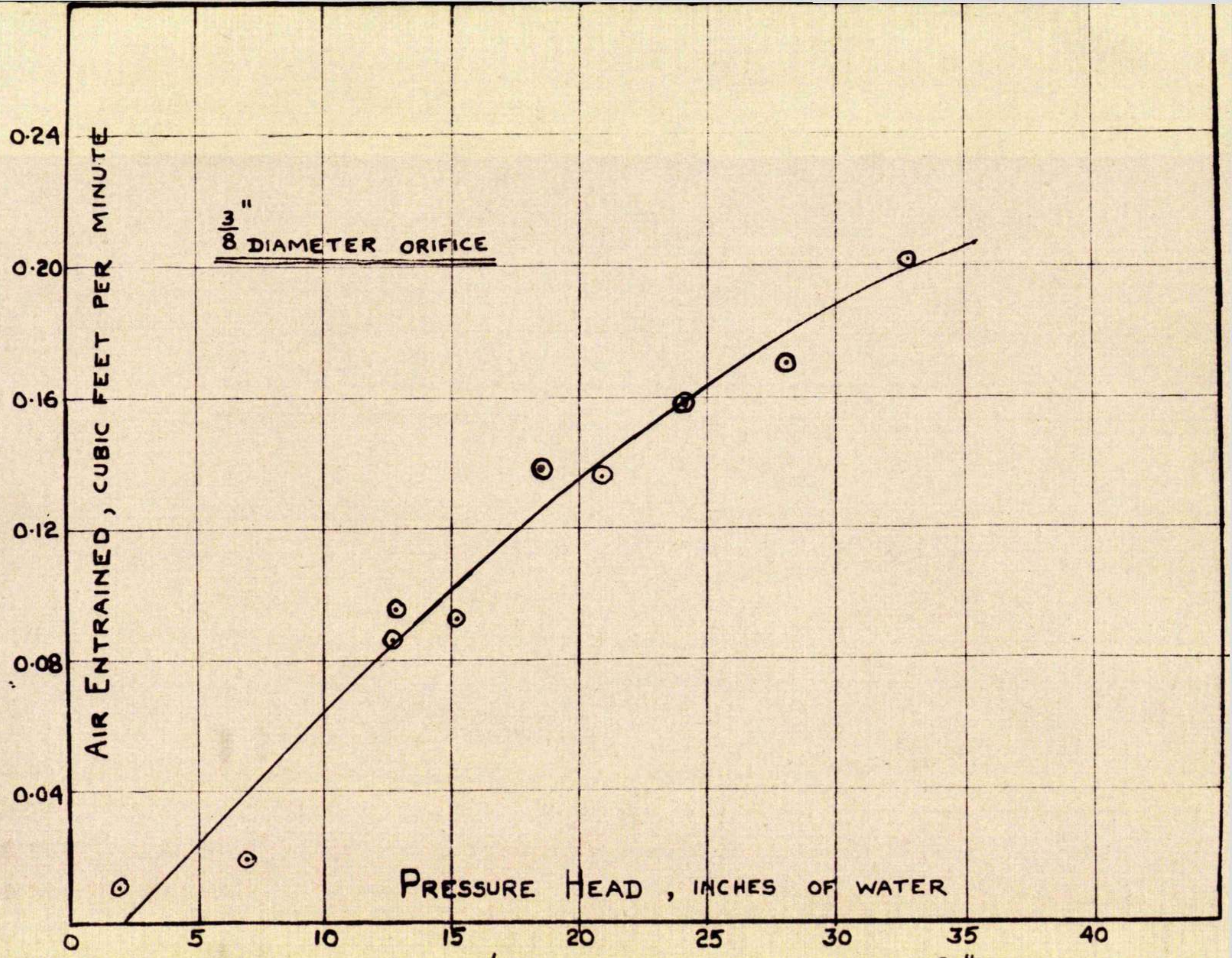


FIG. 10. AIR ENTRAINED / PRESSURE HEAD CURVE FOR 3/8" DIA. ORIFICE

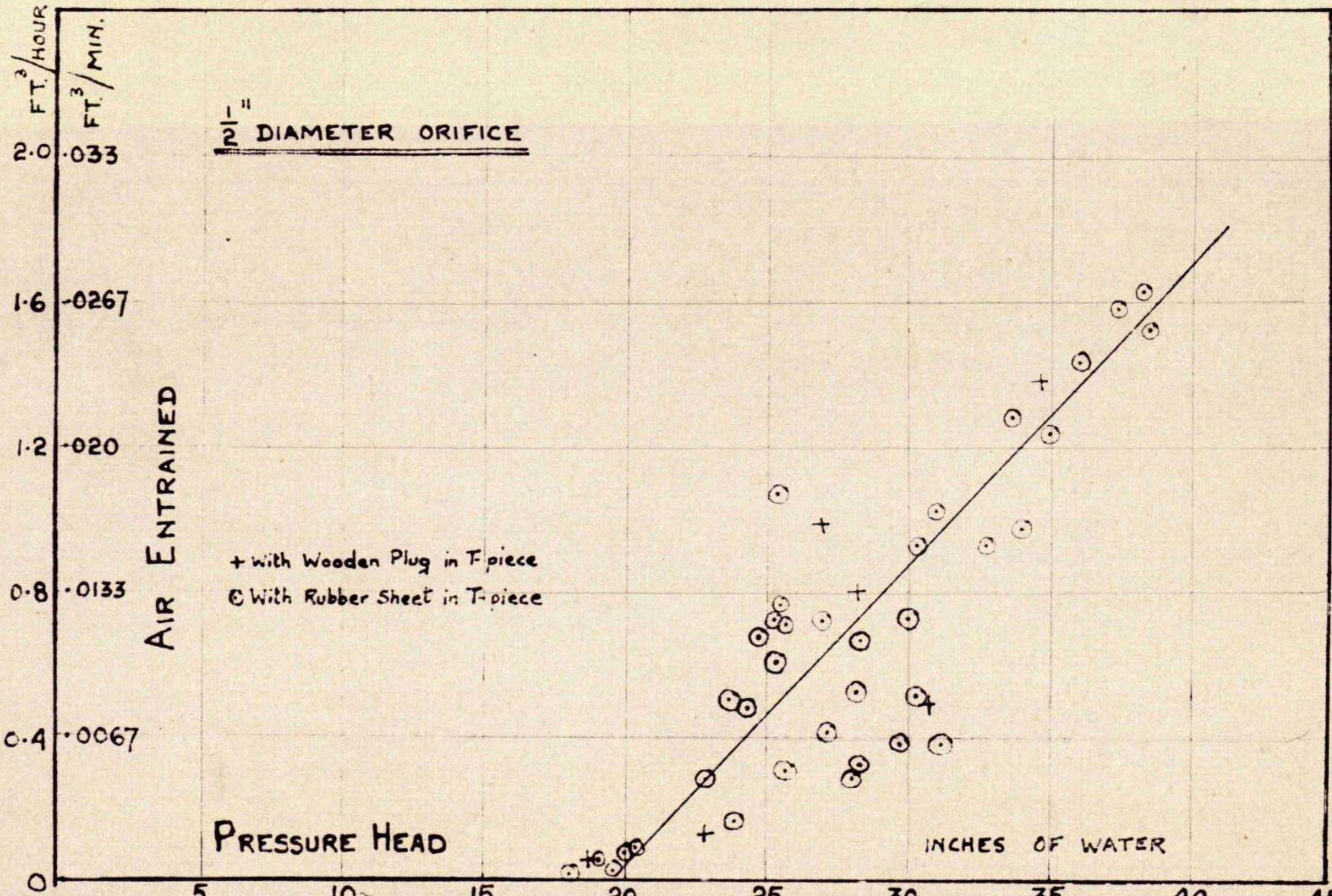


FIG. 20. AIR ENTRAINED / PRESSURE HEAD CURVE FOR ½" DIA. ORIFICE

(b) non-parallel jets issuing from the orifices. In some cases the jets appeared to have entrained air before striking the surface and the smallest jets showed signs of breaking up into droplets.

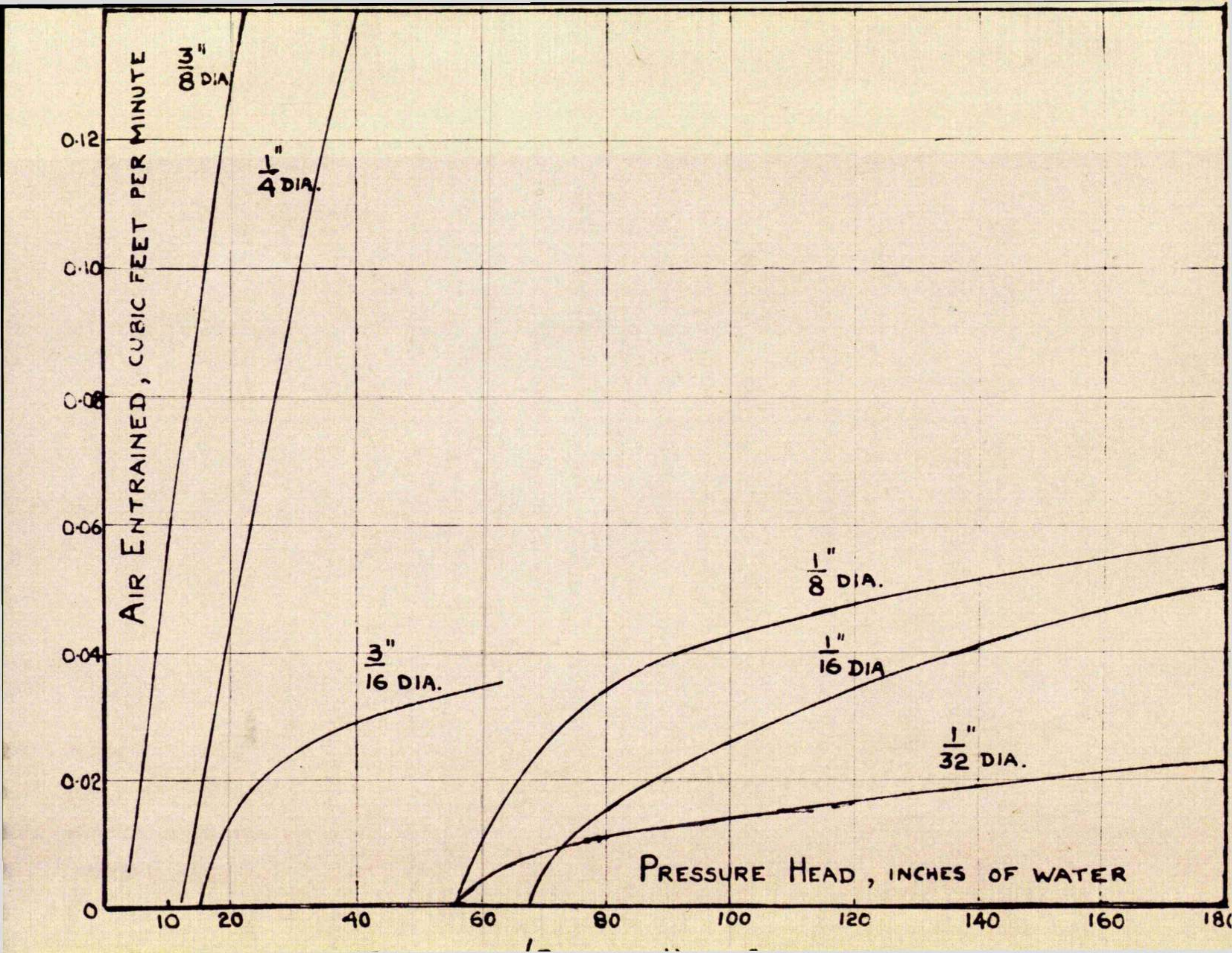
In addition to these effects, the fact that the flow of air bubbles into the separator is a discontinuous phenomenon which is averaged over a period of ten to twenty minutes may give rise to differences, particularly if conditions are not steady during the test - and these conditions (pressure head at the orifice, pressure in the separator, state of the surface) were never all absolutely constant during a test. This will be discussed later.

Despite the scatter of the experimental points a mean curve could be drawn through each set of points sufficiently accurate at this preliminary stage to indicate qualitative if not quantitative results. In drawing these curves it is borne in mind that the effect of the two sources of irregularity discussed above is to increase the air entrained compared with what might be considered the true minimum quantity.

The mean curves obtained from these air entrainment tests are shown in Fig. 21 and further interpretation and analysis is carried out on the basis of these curves.

(e) Analysis of Results.

Fig. 21 shows that the smaller jets require higher heads before entrainment starts.



AIR ENTRAINED, CUBIC FEET PER MINUTE
PRESSURE HEAD, INCHES OF WATER
0·12
0·10
0·08
0·06
0·04
0·02
0
10
20
40
60
80
100
120
140
160
180
3/8" DIA.
1/4" DIA.
3/16" DIA.
1/8" DIA.
1/16" DIA.
1/32" DIA.

The ratio of the volume of air entrained by the jet (Q_a) to the volume of water flowing in the jet (Q_w) is plotted to a base of pressure head at the orifice in Fig. 22. These curves show that the ratio $\frac{Q_a}{Q_w}$ is largest for the smallest jets. This result was not confirmed by the later nozzle experiments described in the next section and is almost certainly attributable to break up of the jets before striking the surface.

The curves of Fig. 22 also seemed to indicate that for the ⅛" diameter and 3/16" diameter orifices the ratio $^{Qa}/_{Qw}$ reached a constant maximum value and it was thought at this stage that this might be a criterion for the efficiency (as regards entrainment) of the different jets. This result also was not confirmed by later experiments - the range of jet velocities and quantities of air which could be handled by the apparatus was too small for any such limit, if it existed, to be determined - but at this stage the criterion was adopted and extrapolated values obtained for the other jets which had not apparently reached the condition in the experimental range. These are shown plotted against jet diameter in Fig. 23.

Fig. 24 shows the pressure head at the nozzle plotted against the jet diameter for two conditions, (1) start of entrainment, (2) entrainment "complete", i.e. when the ratio $\frac{Qa}{Qw}$ first reaches the constant value obtained from Fig. 22 as

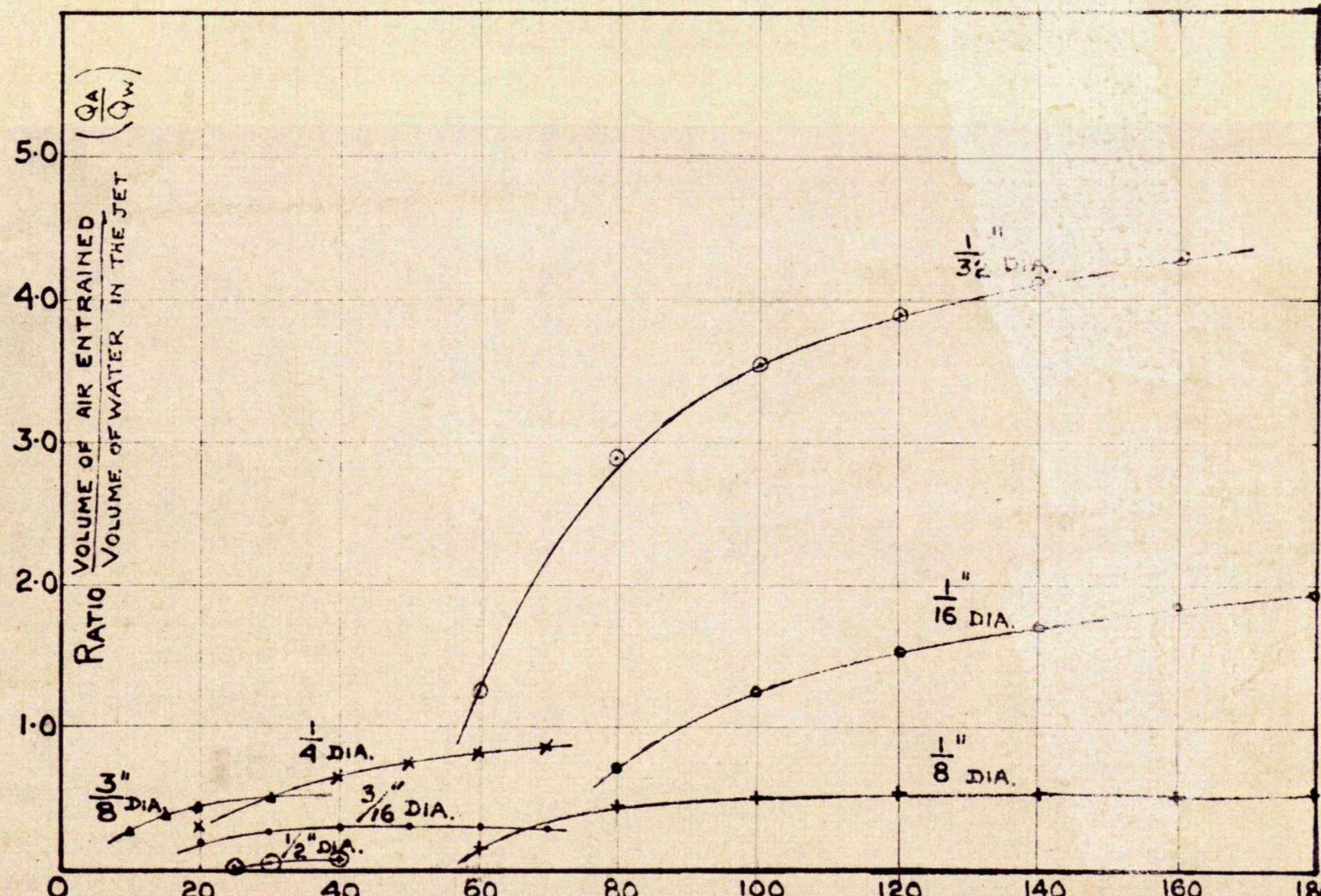


Fig. 22 Ratio $\frac{Q_A}{Q_W}$ plotted against Pressure Head (all orifices)

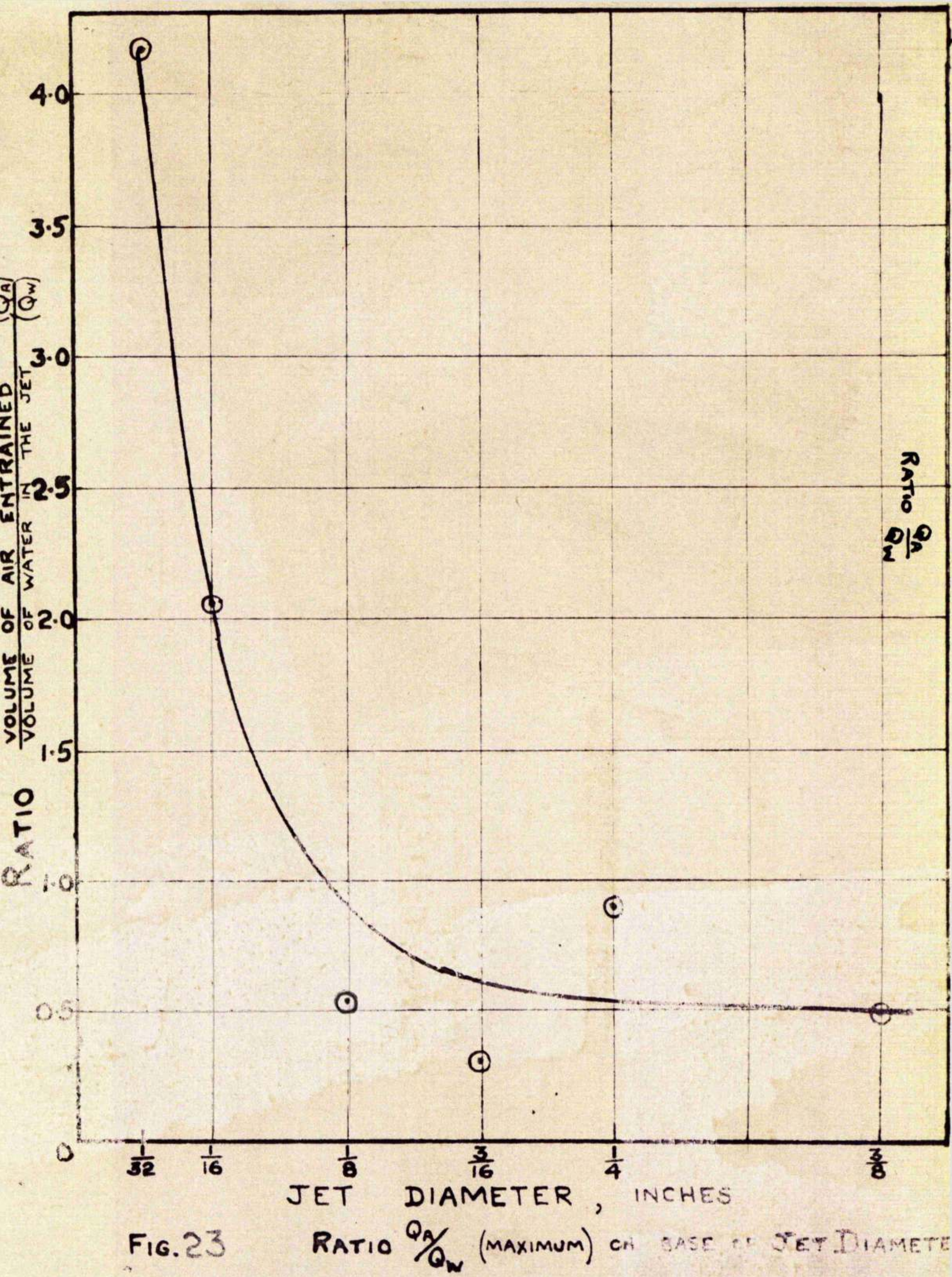


FIG. 23 RATIO Q_A/Q_W (MAXIMUM) ON BASE OF JET DIAMETE

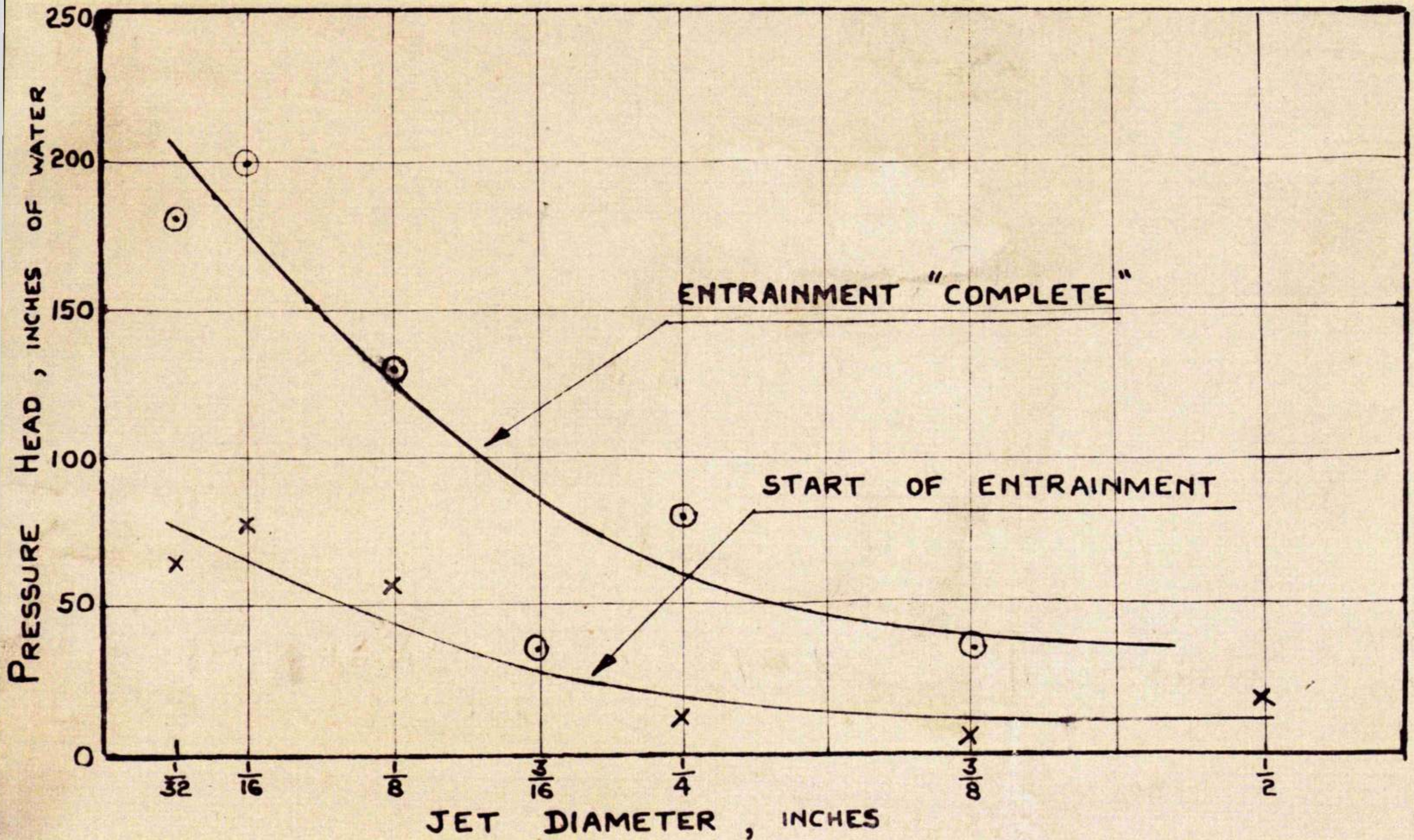


FIG. 24 PRESSURE HEAD AT NOZZLE ON BASE OF JET DIAMETER

explained. These curves show that the pressure head, and therefore the jet velocity, at these two stages decreases as the jet diameter increases, the difference between the two also decreasing as the jet diameter increases. It should be stated here that the nominal jet diameter figures used are actually the orifice diameters and not the true jet diameters. The reasons for adopting this simplification (as far as plotting results are concerned) are (i) because of the scatter in measured air quantities the obtaining of the actual jet diameter for every condition seemed an unnecessary refinement at this stage, (ii) the coefficient of discharge obtained for each orifice over a range of heads up to 14 inches was found to vary from about 0.6 to 0.7 becoming more constant as the head increased beyond this - the value differed comparatively little from orifice to orifice, (iii) the jets obtained from some of the orifices were not satisfactory (i.e. smooth and parallel), appearing to entrain air or tend to disperse in droplets, and this made the actual measurement of the jet diameter extremely difficult and subject to large error.

Since the conception of a maximum ratio of $\frac{Qa}{Qw}$ was shown to be incorrect by later work it will not be pursued further here, although maximum values of $\frac{Qa}{Qw}$ were plotted against Reynolds Number, Froude Number and Weber Number in the course of the research.

Dimensional analysis (Section 6(b)) shows that the quantity of air entrained may be expected to be some function of the Reynolds Number R, the Froude Number F and the Weber Number W. This follows from the assumption that the complex phenomenon depends on viscous, surface tension and gravity forces. In order to generalise the results, therefore, they were plotted on bases of these non-dimensional numbers. Since the same fluids, viz. air and water, were used throughout and the temperature was practically constant, the surface tension, viscosity and density of the fluids could be considered as constants, as far as these experiments were concerned, and the only true variables were the head at the orifice H (i.e. jet velocity) and jet diameter d. Thus,

Reynolds Number R, $\frac{\rho dV}{\mu}$, is proportional to $d\sqrt{H}$ since ρ (density) and μ (viscosity) are constants.

The Froude Number F, $\frac{V^2}{gd}$, is proportional to $\frac{H}{d}$, where d is considered to be the characteristic length.

The Weber Number W, $\frac{V^2}{S/\rho d}$, is proportional to Hd, since S the surface tension and ρ are constants.

Assuming a water temperature of 50° F, a value of ν $(=\frac{\mu}{\rho})$ of 0.00001407 ft²/sec (International Critical Tables) and a value of 0.00509 lb/ft. for the surface tension of water, the following values are obtained:-

$$\text{Reynolds Number} = \frac{Vd}{\nu} = \frac{d\sqrt{H} \times 8.025}{.00001407 \times 12 \times \sqrt{12}} \quad \text{if d and H are in inches}$$

$$= 13{,}720 \; d\sqrt{H}$$

$$\text{Froude Number} = \frac{2gH}{gd} = \frac{2H}{d} \quad \text{with H and d in the same units.}$$

$$\text{Weber Number} = \frac{2gH \times d}{S/\rho} = \frac{Hd \times 64.4}{.00509/1.94} = 24{,}550\ Hd \text{ if H and d in feet}$$

$$= 170.3\ Hd \text{ if H and d in inches.}$$

The forms of the dimensionless groups given above are those generally used in this country but the Froude and Weber Numbers may be written alternatively as $\frac{V}{\sqrt{gd}}$ and $\frac{V}{\sqrt{S/\rho\, d}}$ respectively. This latter form appears to be gaining popularity in America (see e.g. "Engineering Hydraulics" by H. Rouse) and was used by Shirley in presenting his results. Both forms have been used in plotting results in this thesis, the form of the dimensionless group used being stated on the graphs where necessary.

Since the viscosity varies more markedly with temperature than surface tension, the error involved in neglecting temperature variation will lead to more scatter between values calculated using the above numerical coefficients and the actual values (corresponding to conditions during the experiments) in the case of Reynolds Number than in the case of Weber Number.

The above relationships have been used in plotting the graphs, Figs. 25, 26 and 27, the apparently ludicrous use of Reynolds Numbers which do not seem to include viscosity and Weber Numbers which do not seem to include surface tension being due to the assumption of uniform temperature as explained.

From the results of Fig. 21 the following relationships at the start of entrainment are obtained:-

TABLE I

d, Jet dia. (inch)	H, head at start of entrainment (inches of water)	$d\sqrt{H}$	H/d	Hd	$R=\frac{\rho dV}{\mu}$	$F=\frac{V^2}{gd}$	$W=\frac{V^2}{S/\rho\, d}$
1/32	56	0.233	1790	1.75	3200	3580	298
1/16	68	0.515	1088	4.25	7080	2176	725
1/8	56	0.935	448	7.0	12,830	896	1192
3/16	14	0.701	74.6	2.62	9620	149.2	446
1/4	12	0.866	48	3.0	11,900	96	511
3/8	2	0.53	5.3	0.75	7280	10.6	127.7
1/2	15 or 18	1.93 or 2.12	30 or 36	7.5 or 9	26,500 or 29,100	60 or 72	1277 or 1533

Reynolds Number, Froude Number and Weber Number at start of entrainment are plotted on a base of jet diameter in Figs. 25 and 26.

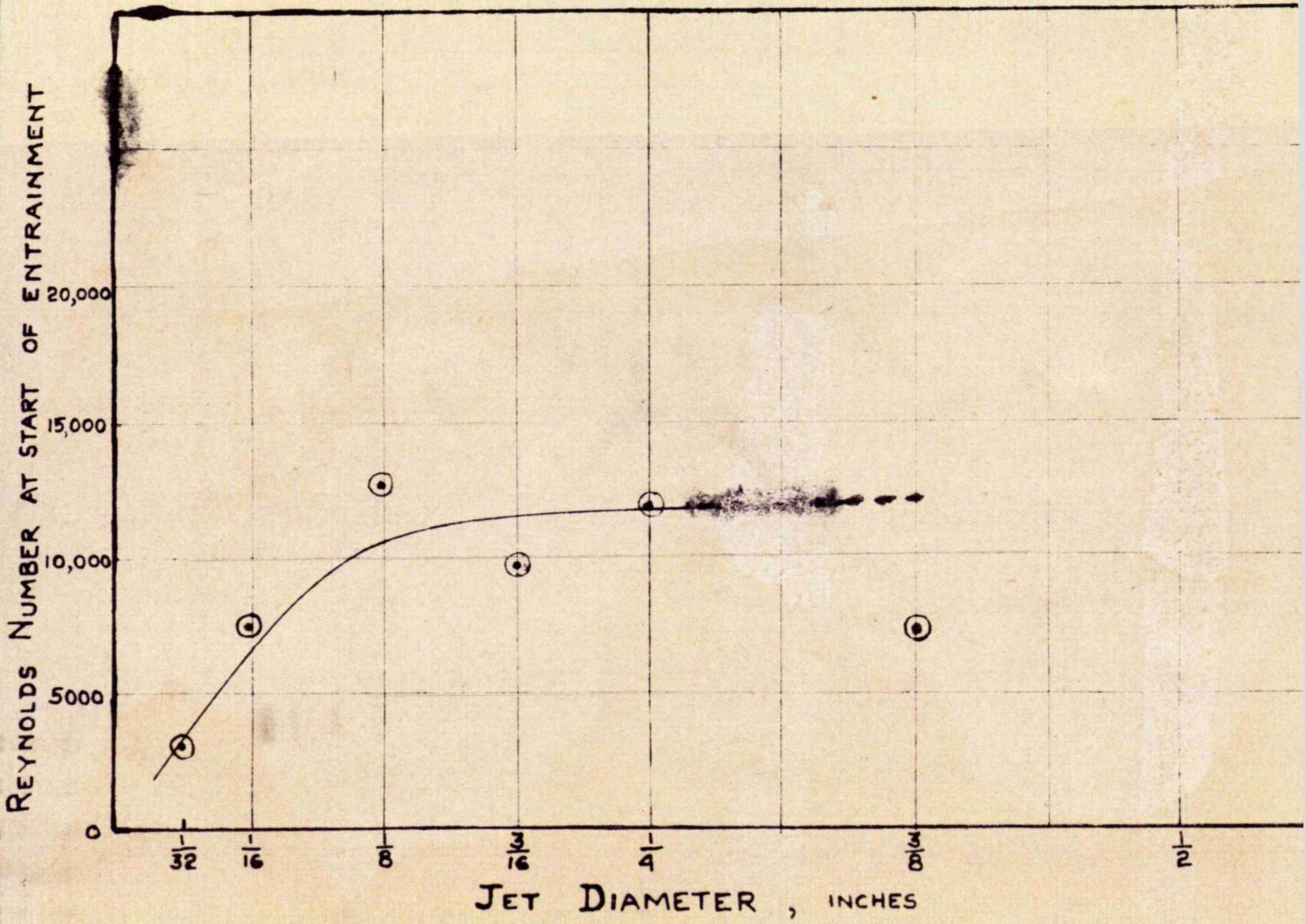


Fig. 25 Reynolds Number at Start of Entrainment

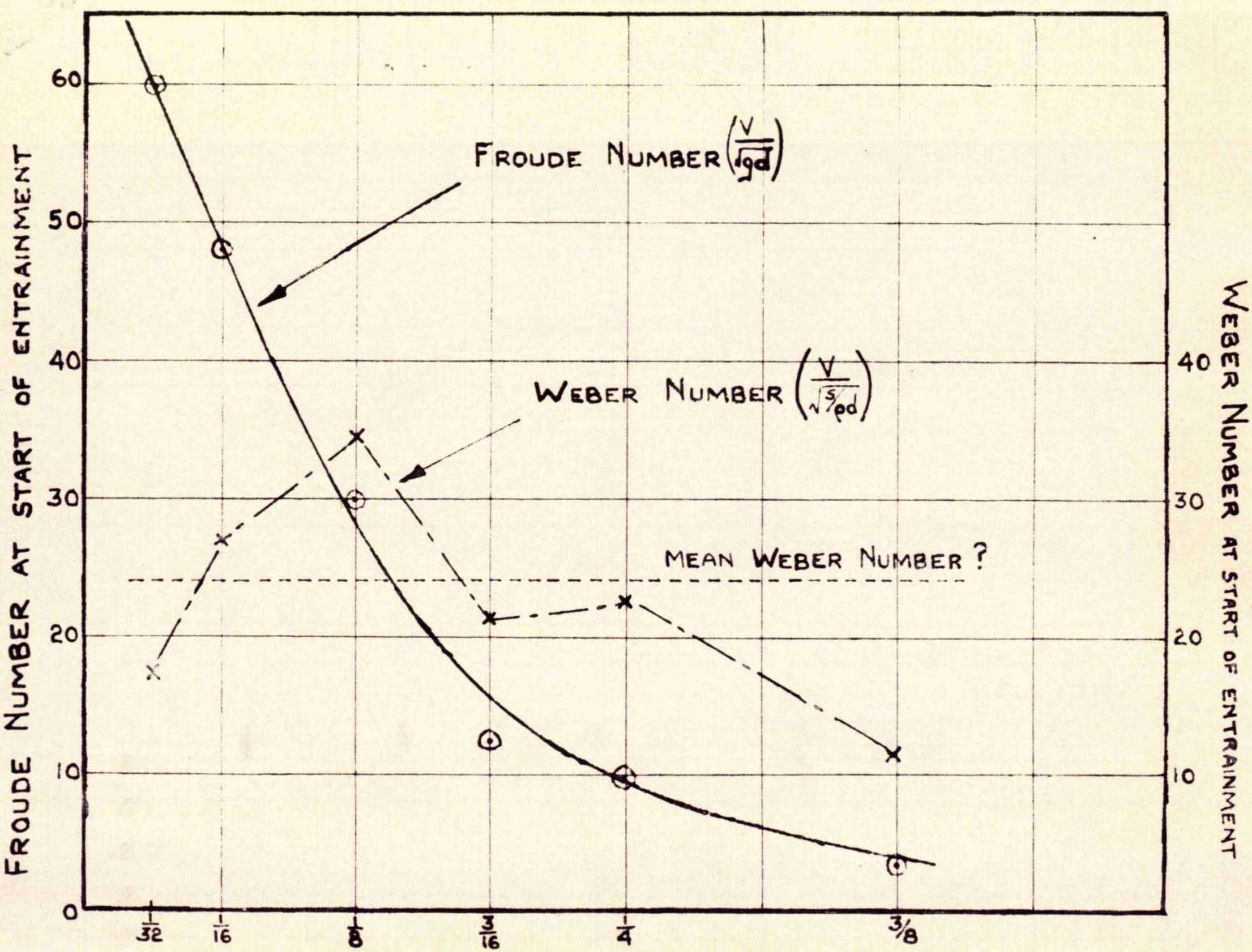


FIG. 26. FROUDE & WEBER NUMBERS AT START OF ENTRAINMENT

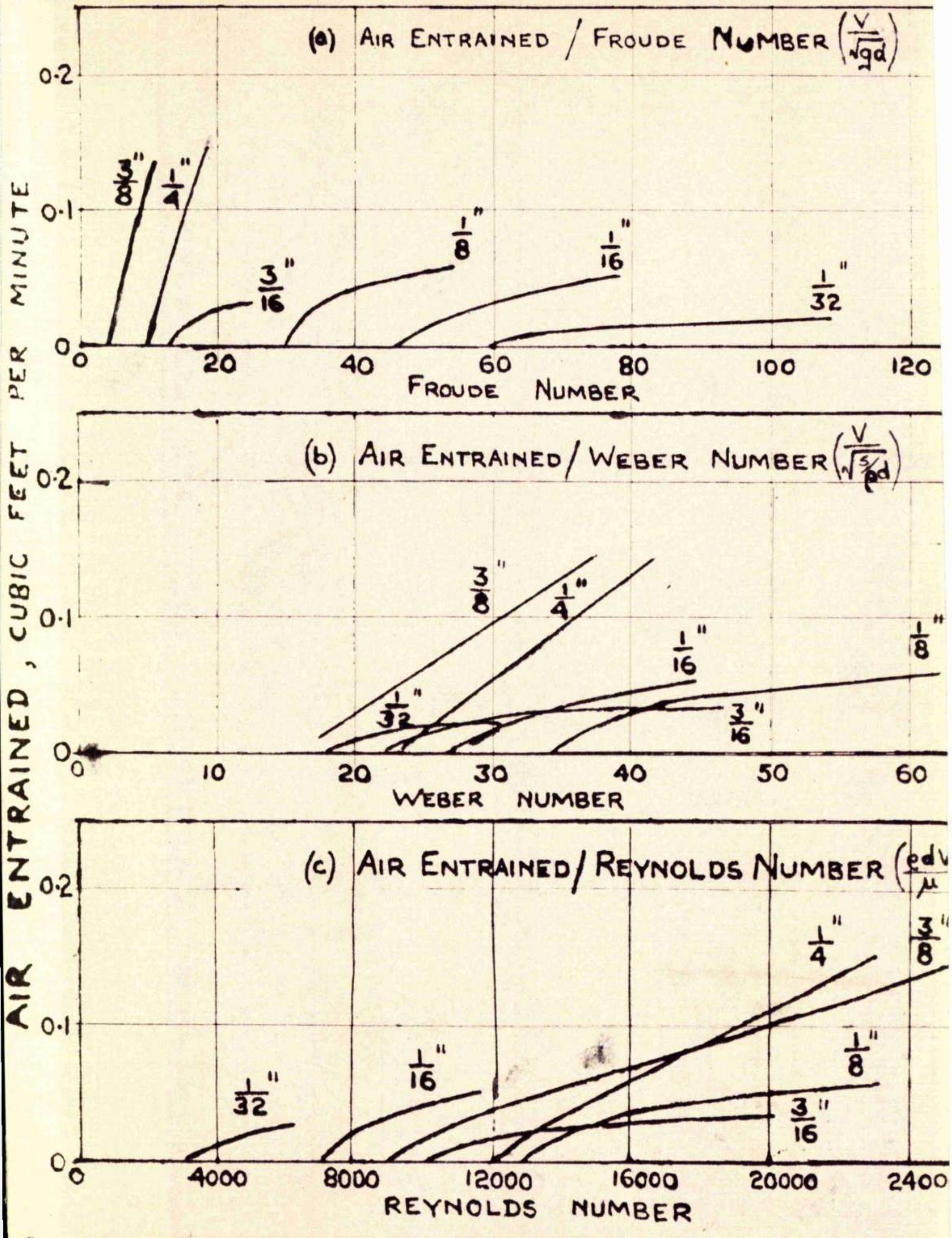


FIG. 27 AIR ENTRAINED / N_F, N_W AND N_R

It can be seen that the conditions recorded for the start of entrainment with the $\frac{1}{2}$ inch diameter orifice are almost certainly inaccurate - the jet not only appeared to be entraining air before striking the surface but it was too large for the apparatus and caused too much surface disturbance for reliable readings. It is therefore omitted from further consideration.

The Reynolds Number at start of entrainment appears to increase fairly sharply with increase in jet diameter for the smaller jets and then tend to a more constant, or slightly increasing, value. The Froude Number at start of entrainment decreases sharply with increase of jet diameter in the experimental range used. This is significant for hydraulic model testing if air entrainment is of interest - the same Froude Number for model and full scale will not give "corresponding" conditions.

The Weber Number figures show considerable irregularity. The graph might suggest either a wide scatter about some constant mean value or an increase with jet diameter up to approximately $\frac{3}{4}$ inch diameter and then a decrease with further increase in diameter. Theory suggests that a constant value of the Weber Number may give the criterion for the start of entrainment (see Section 6(c)).

Figs. 27(a), (b), and (c) show the volume of air entrained on bases of R, F and W respectively.

The results could be summarised as follows:-

(1) The heads (i.e. jet velocities) required for entrainment to commence increase with decrease in jet diameter.

(2) The Reynolds Number when entrainment starts increases rapidly with jet diameter up to $\frac{1}{8}$ inch, perhaps becoming constant or increasing more slowly as jet size is increased beyond this figure.

(3) The Froude Number at the start of entrainment decreases rapidly with increasing jet diameter becoming very small as jet size increases beyond about $\frac{3}{8}$ inch diameter.

(4) The Weber Numbers at the start of entrainment show considerable irregularity which, however, could be interpreted as scatter about some constant mean value.

(5) Little useful information about the variation of the quantity of air entrained with R, F or W has been obtained and so far it would not be possible to estimate from tests on one size of jet, the quantity of air likely to be entrained by a larger jet. Nevertheless, the results seem to indicate that the quantity of air entrained may be a function of Reynolds Number rather than Weber Number or Froude Number.

(f) Conclusions from Preliminary Orifice Tests.

The apparatus and technique used had several features which influenced the results obtained.

The right-angled bend through which the entrained air had to flow in the T-piece imposed limitations on the quantity of air which could be handled satisfactorily and the surface in the neck of the T-piece tended to become disturbed, leading to variable results. The jets from the orifices, despite care in manufacture of the orifice plates, were not sufficiently uniform to make size the only variable - the nature of the jets also differed.

These preliminary experiments showed the complexity of the problem in which viscous, surface tension and gravity forces are all present. They seemed to indicate that the Froude Number is not the correct similarity criterion where air entrainment is of interest although this is almost automatically adopted for channel models. The quantity of air entrained appeared to depend most closely on Reynolds Number, i.e. entrainment is mainly due to the action of viscous forces.

The need for further experimental work was obvious. The main points requiring modification or refinement were the means of trapping the entrained air bubbles and carrying them into the separator and the method of producing uniform jets of various diameters. The method evolved is described in the next section.

SECTION 4

JET TESTS WITH NOZZLES

SECTION 4

JET TESTS WITH NOZZLES

(a) Modifications to Apparatus.

In an endeavour to obtain more uniform parallel jets, it was decided to use nozzles instead of orifices. Several welding blowpipe nozzles, manufactured by Messrs. C. S. Milne and Co., London, S.E.8., were obtained (Fig. 28) and a nozzle plate into which they could be screwed was made to fit the existing orifice holder. The diameters of these nozzles were carefully measured in the Metrology laboratory of the Mechanical Engineering Department, Royal Technical College, and the particulars are given in Table II below.

TABLE II

Nozzle No. (Type CH)	Two diameters at right angles (inches)		Average diameter (inches)	Cross-sectional area (in^2)
3	.0359	.0347	.0353	.000979
7	.0490	.0507	.0499	.00192
10	.0601	.0601	.0601	.00284
13	.0626	.0640	.0633	.00315
25	.0870	.0869	.0870	.00593
35	.1030	.1026	.1028	.00830
55	.1303	.1274	.1289	.0130
90	.1501	.1493	.1497	.0176

The right angle bend of the T-piece was the other feature which it was desired to modify. The first arrangement tried is shown in Fig. 29(a). A Pyrex glass fitting (a

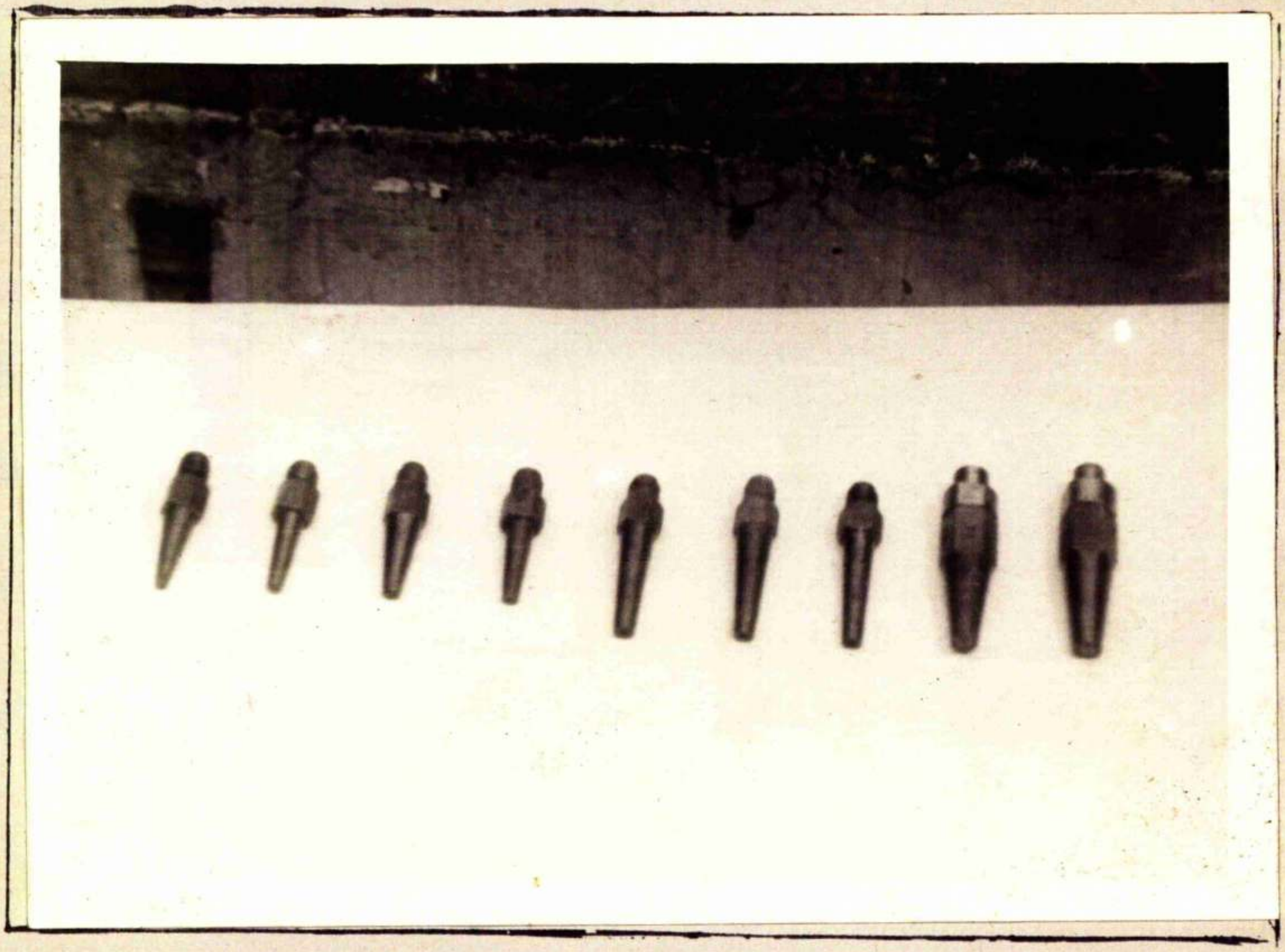


FIG. 28 NOZZLES USED IN THE EXPERIMENTS.

(The two on the right are multi-jet nozzles)

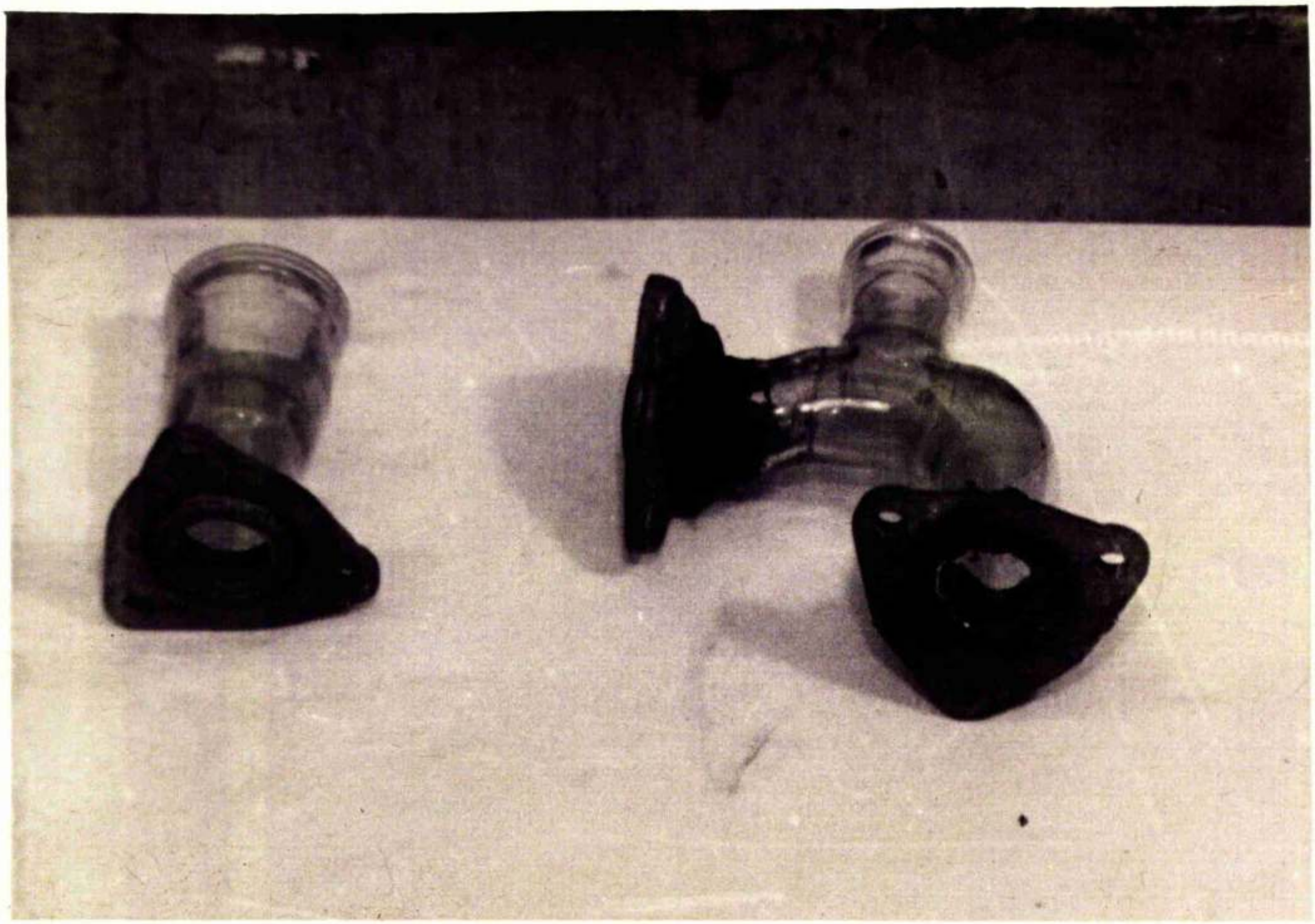


FIG. 29 PYREX GLASS PIECES USED IN EXPERIMENTS.

(a) ON THE LEFT, FIRST ARRANGEMENT FOR STRAIGHT-THROUGH FLOW
(b) ON THE RIGHT, SECOND MODIFICATION WITH ADDITIONAL FLOW

junction piece for 2" and 1" bore pipes) was used as the receiver into which the jet impinged. The level in the receiver could be controlled by a valve between the receiver and the air separator. This was found to be unsatisfactory for the range of pressure heads used because the valve, being almost closed to maintain the required level in the receiver, checked the flow and allowed the bubbles of entrained air to rise to the surface instead of being carried into the separator. At higher heads (about 23.1 inches of mercury, i.e. 314 inches of water) a violent vortex formed in the receiver. If the jet were carefully centred the air entrained was carried straight down into the separator but if it were very slightly off-centre the bubbles were released and surfaced under buoyancy force while the conditions in the receiver became very agitated and turbulent.

While experiments were being made with this apparatus the following phenomenon was observed. At low heads, say 1 inch of mercury or less, the jet did not appear to be entraining air on striking the surface, nevertheless occasional bubbles appeared one or two inches below the surface in the line of the jet. These were large-ish bubbles, up to perhaps ¼ inch in diameter, and they appeared to "dance" in a more or less stationary position for quite long periods at a time. The origin of these bubbles was not clear - they may have been surface bubbles carried down by the jet or caused by separation of air already entrained in the jet before striking the surface - but they were held in position by the jet (momentum of jet = buoyancy of bubble). The nozzle used for these experiments was No. 25 (.0870 inch diameter).

It was decided to improve the apparatus by introducing an additional flow of water below the Pyrex receiver and above the valve, so that the valve could be kept more widely open thus causing less obstruction to the flow while the additional flow, by increasing the downward velocity would help to carry the bubbles into the separator. In assembling this modification use was made of a specially shaped bend of 1 inch diameter Pyrex glass pipe which had been used in a modification of the original Brannie Burn model (Fig. 29(b)). This arrangement allowed the valve to be kept more fully open but the entrained air still escaped unless the surface were lowered into the straight neck portion. This, however, was essentially like the first T-piece apparatus but with only 1 inch diameter pipe instead of 2 inch diameter and with a smaller permissible variation of surface level. The arrangement was unsatisfactory because the added flow did not increase the downward velocity in the entraining section. Vortex formation was also observed.

The third arrangement tried, (Figs. 30, 31), was the

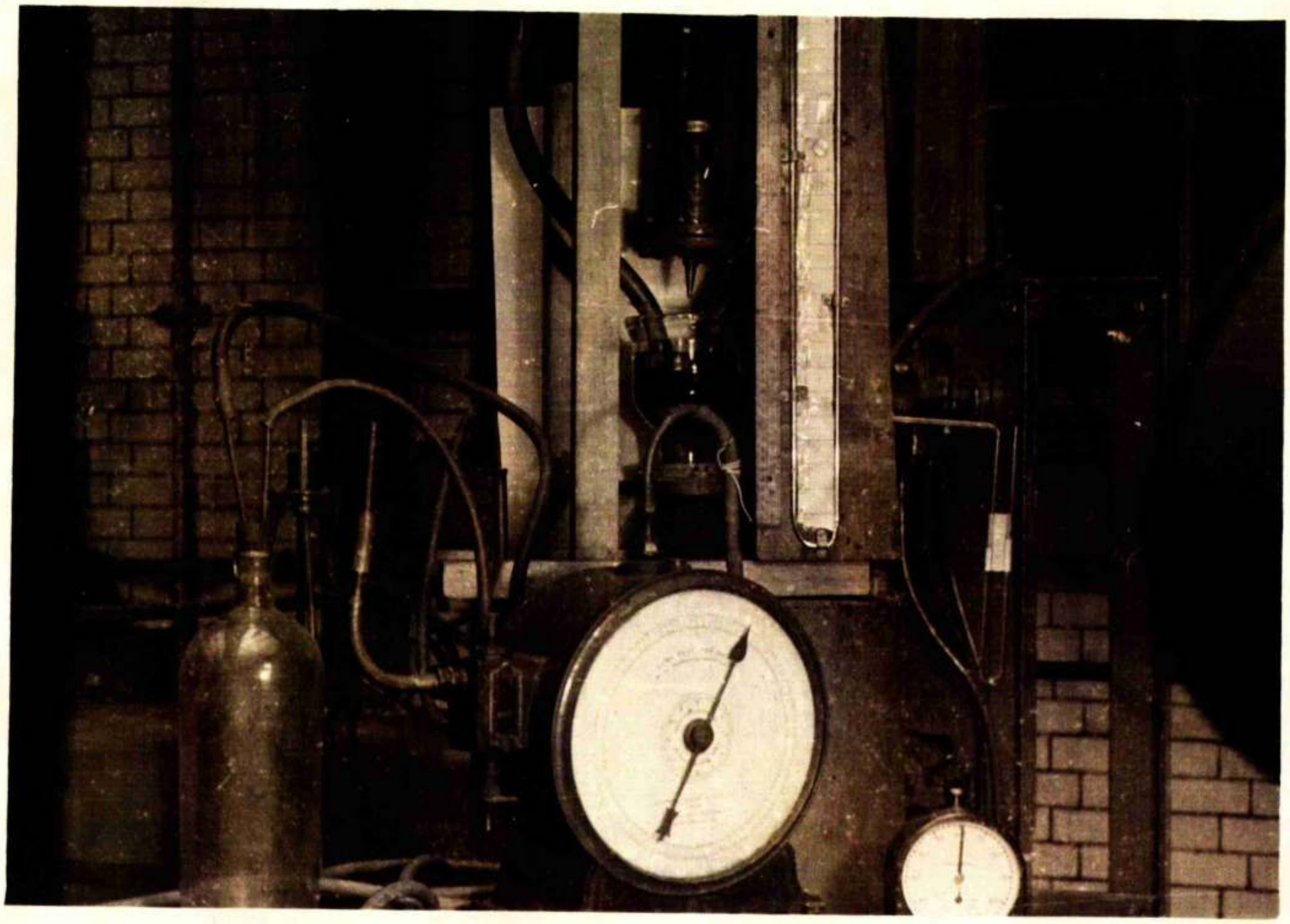


FIG. 30 APPARATUS USED FOR NOZZLE EXPERIMENTS.

(AIR SEPARATING TANK NOT SHOWN)

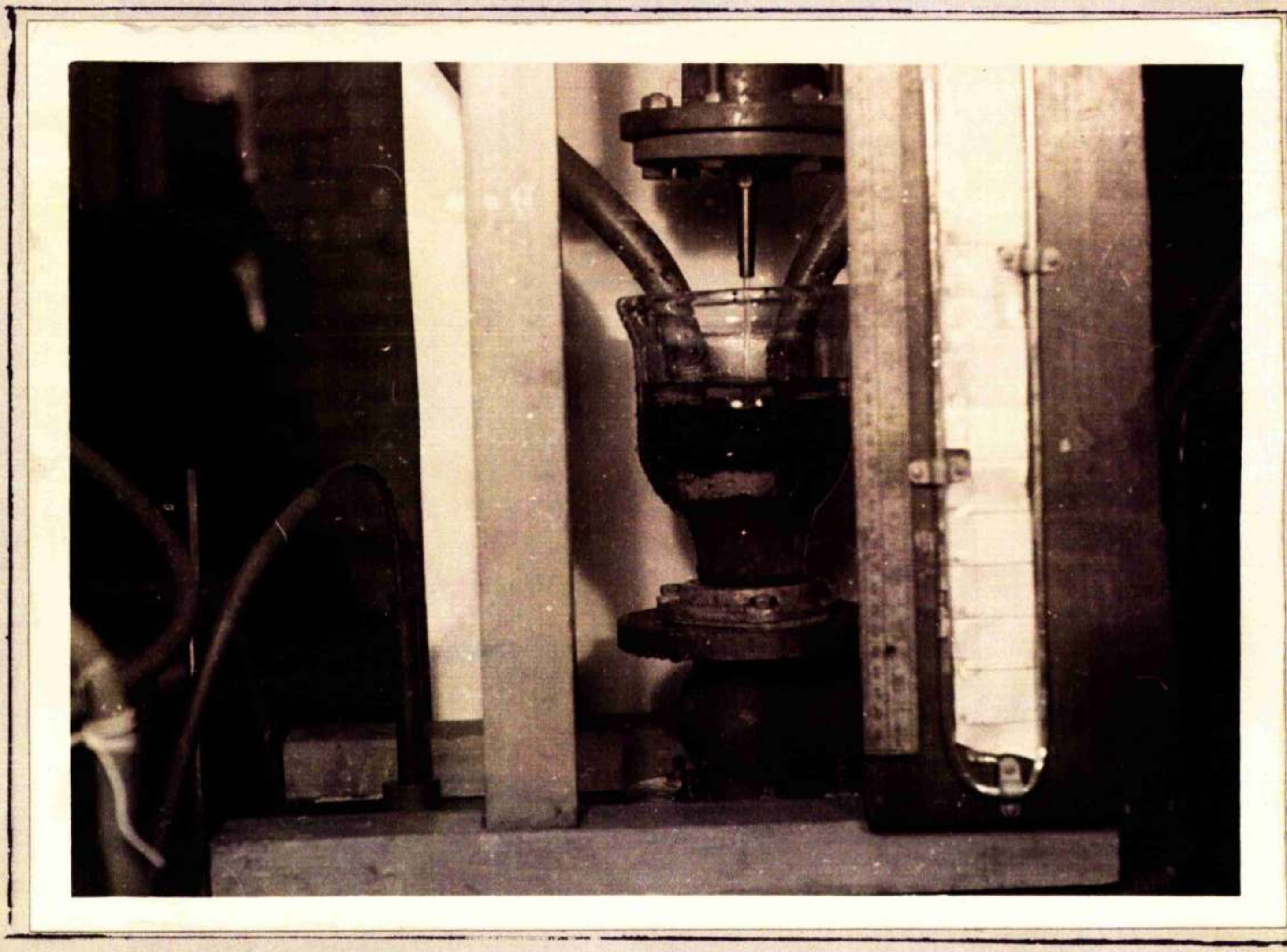


FIG. 31 CLOSE-UP SHOWING NOZZLE AND JET

(RUBBER TUBE CONNECTIONS TO THE "DIFFUSER" IN THE GLASS RECEIVER CAN BE SEEN) one which, with minor improvements, was used in all the experiments described in this section. A 4 inch diameter/2 inch diameter Pyrex adaptor was used instead of the 2inch/1 inch adaptor of the previous modifications and into this was led a secondary flow through a diffuser. The diffuser (Fig. 32) had a central hole approximately one inch in diameter through which the jet with the entrained air flowed. The diffuser was constructed of a brass top plate and sheet forming the outer diameter, the bottom plate and the central inner diameter being of wire gauze soldered to these other component parts. Two inlet pipes were soldered to the top plate diametrically opposite each other, as shown. These were first fitted at an angle of approximately 45^{o}, but this was later altered to the vertical position to allow the diffuser to be lowered further into the Pyrex receiver. It was found necessary to seal the space between the diffuser and receiver to stop any leakage of stray bubbles of air, and for this a rubber ring and Bostik was used. The holes in the central part of the diffuser were found to affect the capturing of the entrained air bubbles - the radial inward flow at right angles to the jet caused a retardation of the jet and allowed some air bubbles to rise to the surface - accordingly the central gauze portion was covered, so that only the bottom portion remained for the flow. This was satisfactory. A

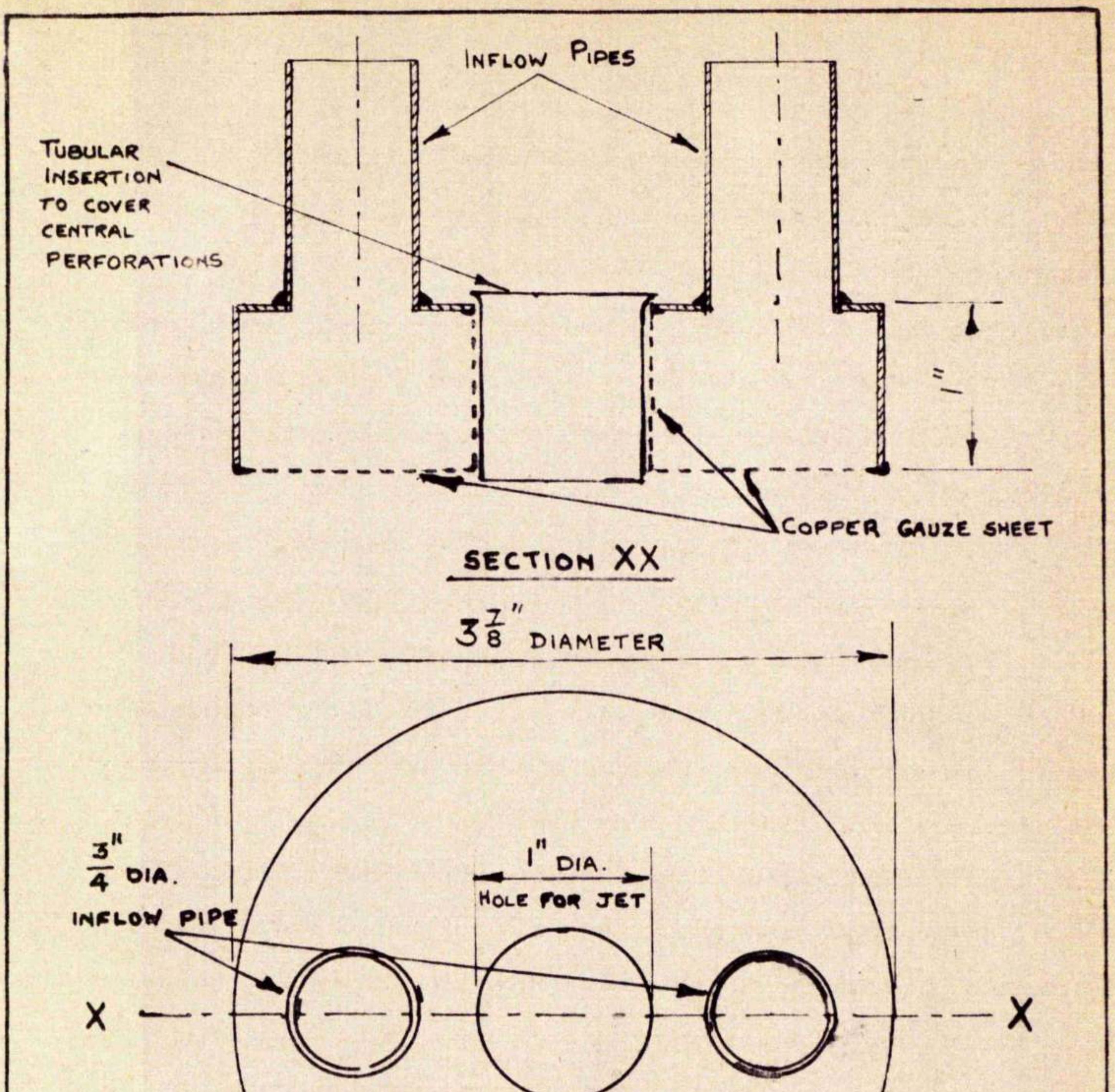


FIG. 32 SKETCH OF DIFFUSER THROUGH WHICH SECONDARY FLOW WAS INTRODUCED

smoother jet was obtained and the jets carried the entrained air through the diffuser, but the position of the surface relative to the top of the diffuser could not be varied more than about $\frac{1}{2}$" or $\frac{3}{4}$" if the air bubbles were to be carried into the separator effectively. This control of the surface level was obtained by adjustment of the valve. The height of the nozzles above the top surface of the diffuser could not be readily reduced to less than about 2 inches because of the nozzle holder fouling the rubber tubing. The apparatus therefore limited the possible variation of the distance from the nozzle to the free surface. Preliminary experiments indicated that the quantity of air entrained depended on the length of the jet, i.e. the distance from nozzle to free surface, but since the range of variation conveniently obtainable with this apparatus was comparatively small it was decided as a first step to carry out the experiments with an approximately constant distance from nozzle to surface.

(b) Method of Carrying Out Experiments; Control of Apparatus.

The method of carrying out the experiments was essentially the same as used previously with the orifices. The secondary inflow was obtained from the ordinary water supply system and the level in the Pyrex receiver was controlled by the valve between the receiver and the separator. The supply to the nozzle was obtained from a single stage centrifugal pump which also supplied the ejector pump used for withdrawing air from the separator. The pressure head at the nozzle was measured by a mercury manometer and adjusted by a throttle valve and/or altering the speed of the D.C. motor driving the pump. The duration of practically all the tests was ten minutes and it was endeavoured to maintain constant conditions during this period.

While the principle of the apparatus is simple it was found that the operation of the various controls required to maintain uniform conditions was extremely difficult for one experimenter. The various flows all tended to fluctuate during any one test and each result recorded represents a mean value. The observations and adjustments required are therefore detailed below:-

(1) The surface level in the receiver was observed to fluctuate presumably due to varying demand on the domestic supply mains causing the secondary inflow to vary. By adjusting the valve, it was attempted to maintain the surface with $\pm \frac{1}{4}$" of its nominal position, since this controlled the length of the jet.

(2) The pressure at the nozzle (and therefore the jet velocity) varied due to speed fluctuation of the pump motor. The valve controlling this supply was not within reach of the

observer during an experiment and any such variation had to be accepted. The limits between which the pressure varied in each test was recorded. For almost all tests this was less than ± 0.2 inch of mercury in the manometer, the greatest fluctuation being ± 0.7 inch of mercury on a mean reading of 11.1 inches of mercury. The mercury had to be observed carefully throughout.

(3) The air pressure in the separator had to be maintained at atmospheric pressure as closely as possible. The air pressure in the separator was indicated by a water manometer so that this gauge also had to be kept under observation during a test and the air flow regulated accordingly. Binnie and Wright[1] showed that a difference of ¼" of water caused little difference to the air quantity measured, and the fluctuations were controlled within closer limits than this.

(4) The gas meter had to be observed and the readings noted.

(5) The duration of the test had to be noted.

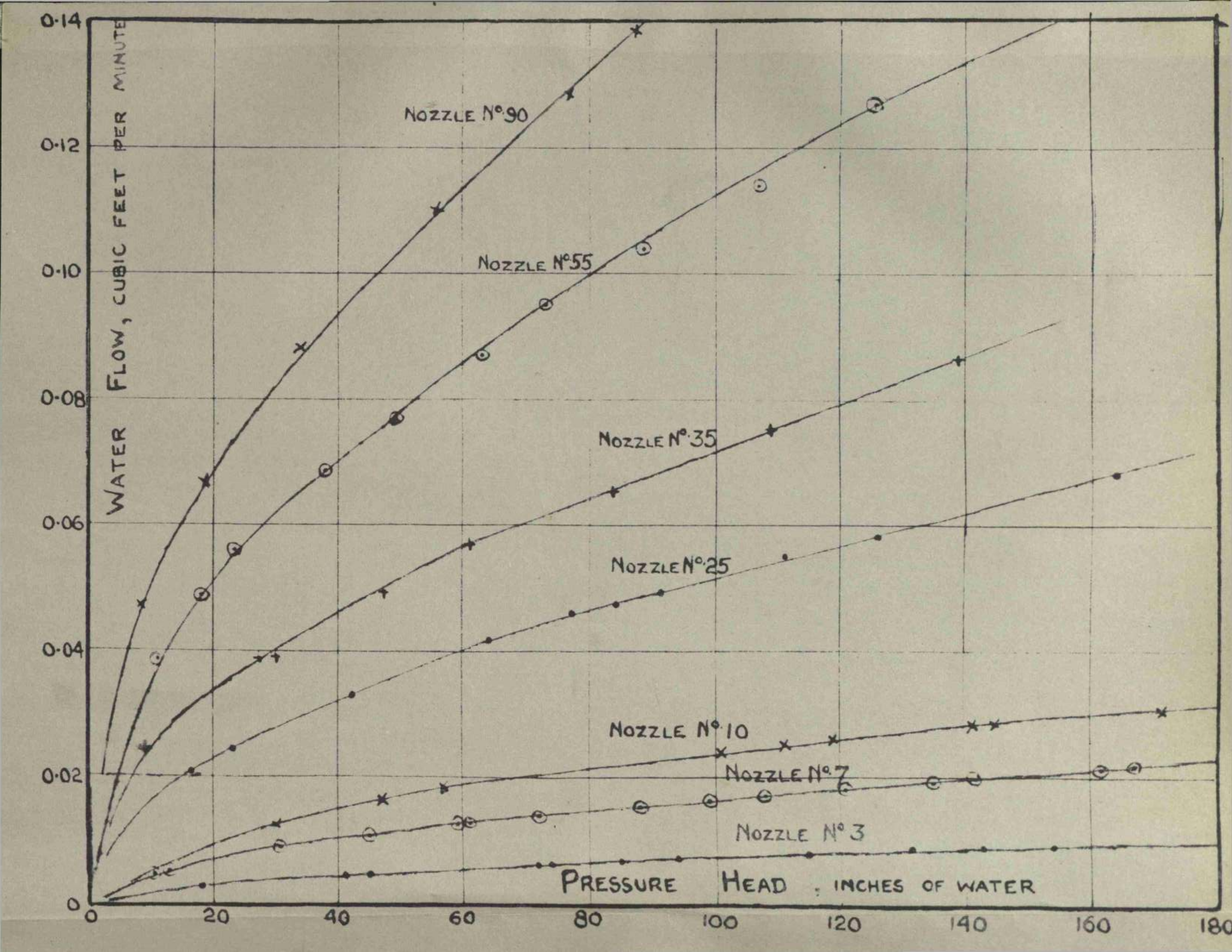
Thus five observations (surface level, pressure at nozzle, air pressure in separator, gas meter reading and time) had to be made and two controls (surface level and air flow) had to be manipulated throughout each test.

These fluctuations, together with the discontinuous nature of the phenomenon and variations in atmospheric pressure and temperature which were not observed on every occasion, account for the scatter of the experimental points. Nevertheless it is considered that the mean curves supply useful data and constitute a step towards a fuller understanding of the nature of air entrainment.

(c) First Series of Nozzle Tests.

Each nozzle was first calibrated by collecting the discharge in a given time with various pressures at the nozzle. The calibration curves are shown in Fig. 33.

The results of the air entrainment tests are shown in Figs. 34-40. The larger diameter nozzles were used first and the distance from the nozzle to the surface was maintained at 2½"-3" in every case. When nozzle No. 7 (.0499 inch diameter) was fitted it was observed that the jet seemed to be breaking up into droplets before striking the surface. The point at which entrainment started could not be determined accurately. The nozzle was therefore lowered to 1 inch-1½ inches from the surface and the tests repeated. Both sets of readings are plotted in Fig. 35. The tests with nozzle No. 3 (.0353 inch diameter) at a distance of approximately 2½"-3" from the surface also gave very scattered results. It was found that the quantity of air entrained varied


0·14
0·12
0·10
0·08
0·06
0·04
0·02
0
20
40
60
80
100
120
140
160
180
WATER FLOW, CUBIC FEET PER MINUTE
PRESSURE HEAD : INCHES OF WATER
NOZZLE Nº 90
NOZZLE Nº 55
NOZZLE Nº 35
NOZZLE Nº 25
NOZZLE Nº 10
NOZZLE Nº 7
NOZZLE Nº 3

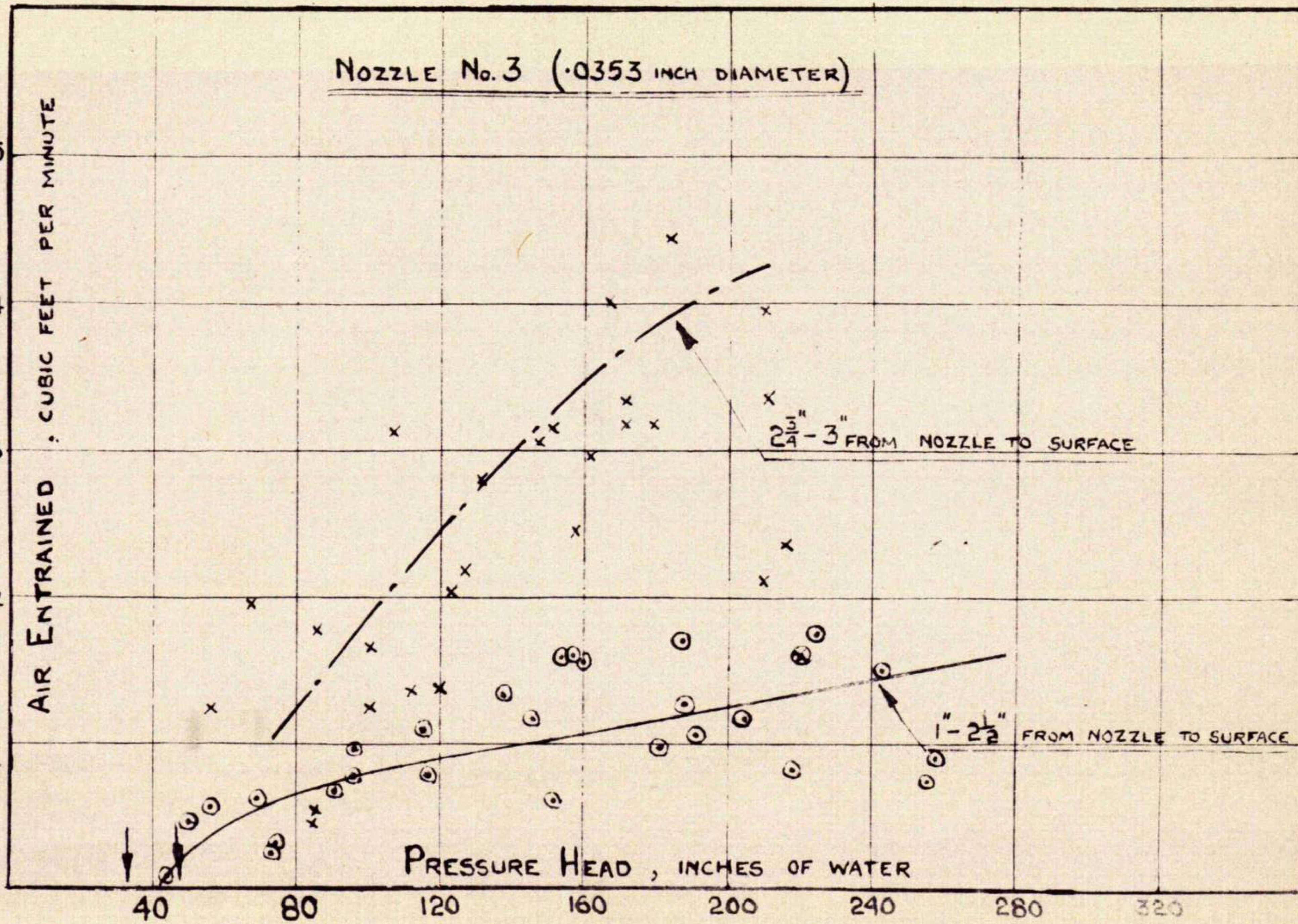


FIG. 34 AIR ENTRAINED / PRESSURE HEAD CURVES FOR 0·0353 INCH NOZZLE

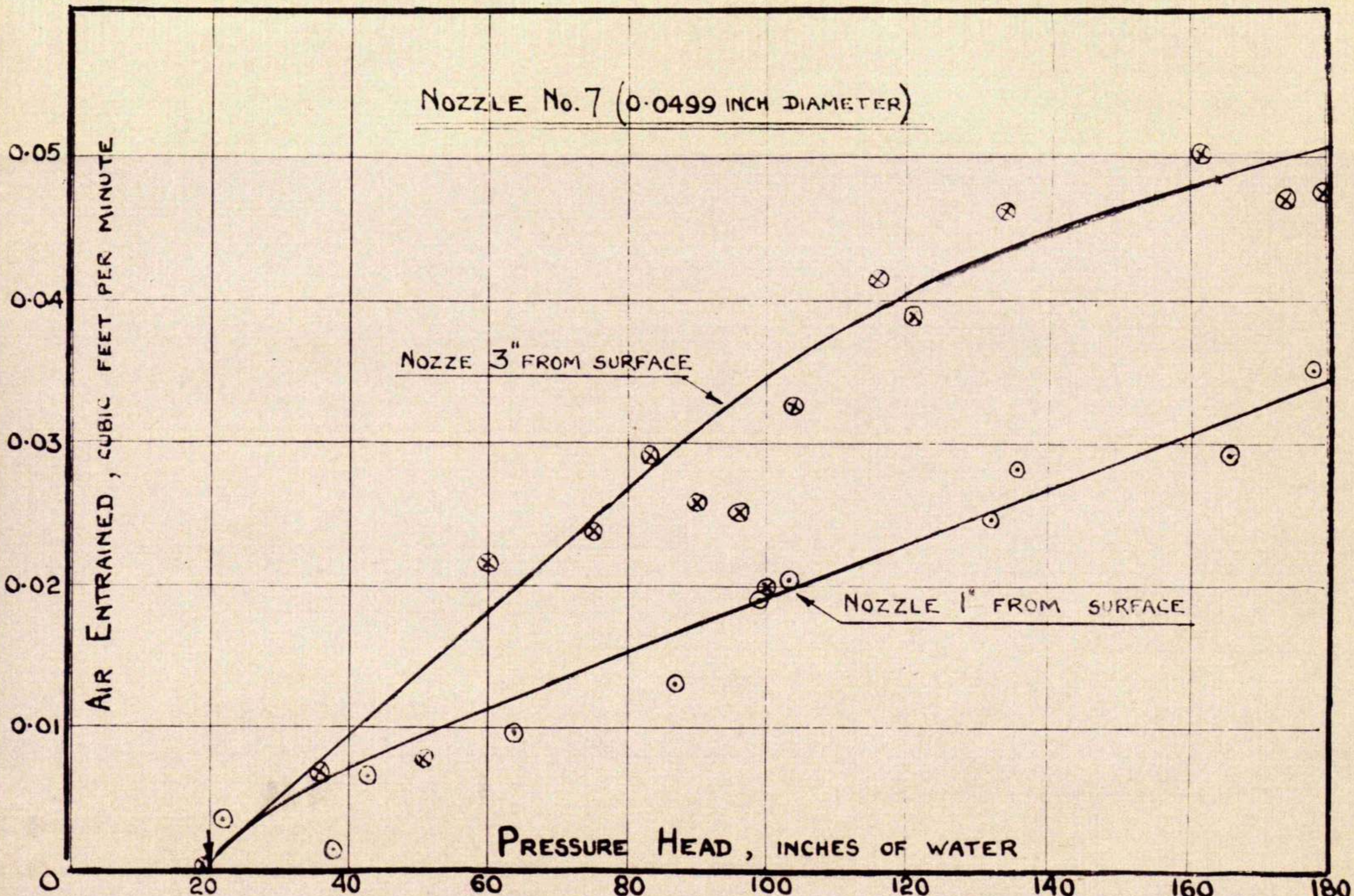


Fig. 35 Air Entrained / Pressure Head Curve for 0·0499 inch dia. nozzle

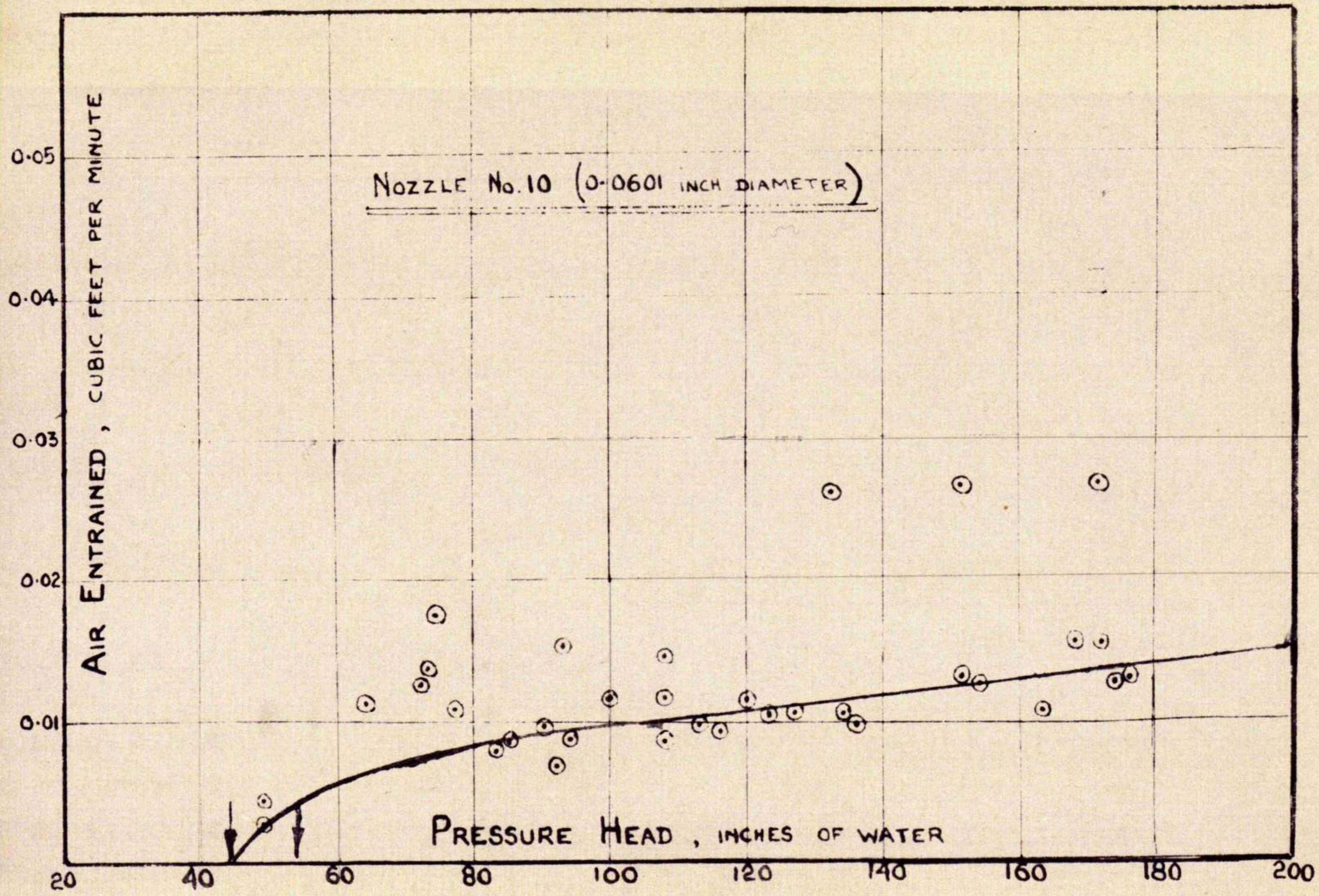


FIG. 36 AIR ENTRAINED / PRESSURE HEAD CURVE FOR 0·0601 INCH DIA. NOZZLE

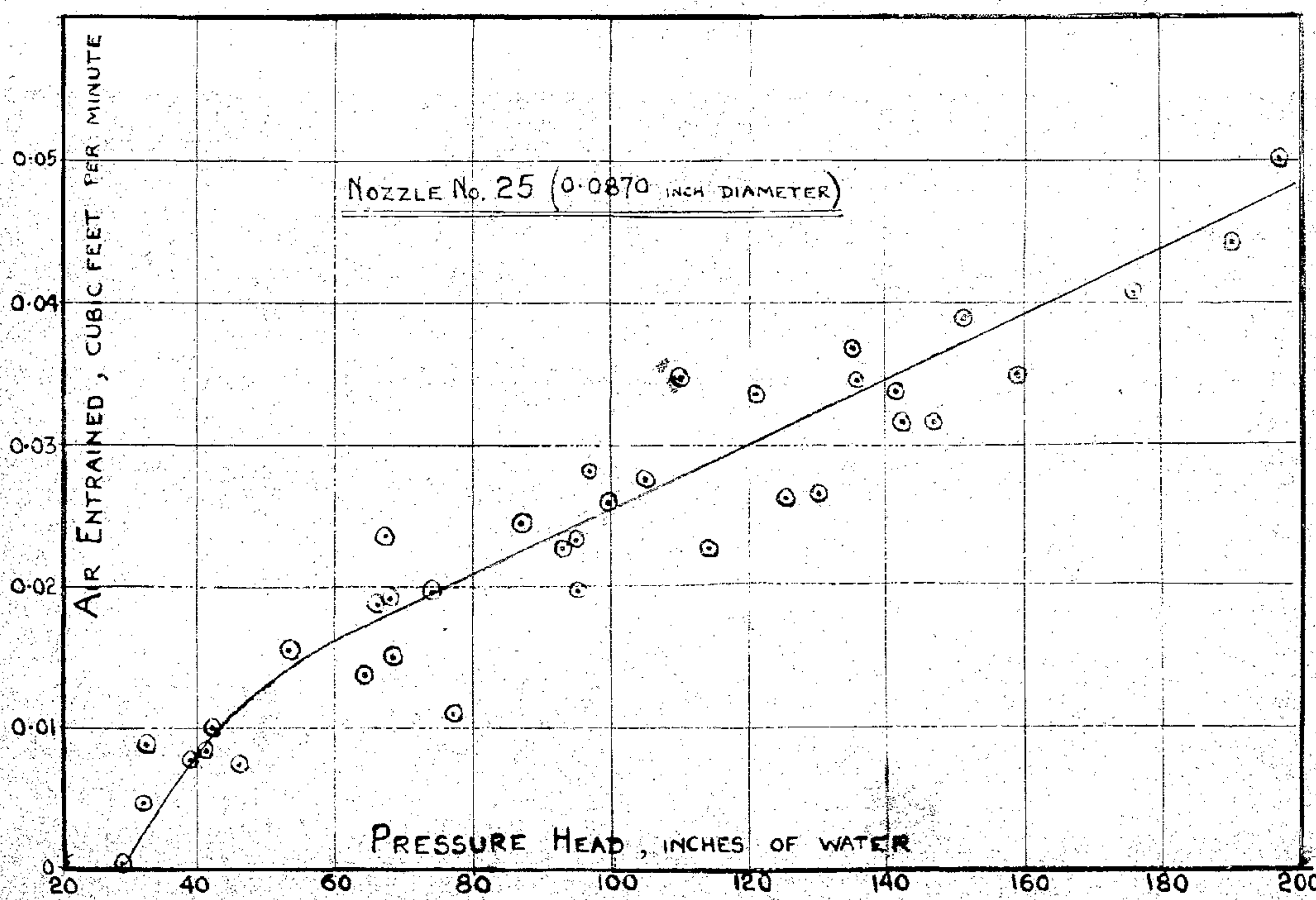


Fig. 37 Air Entrained / Pressure Head Curve for 0·0870 inch dia. nozzle

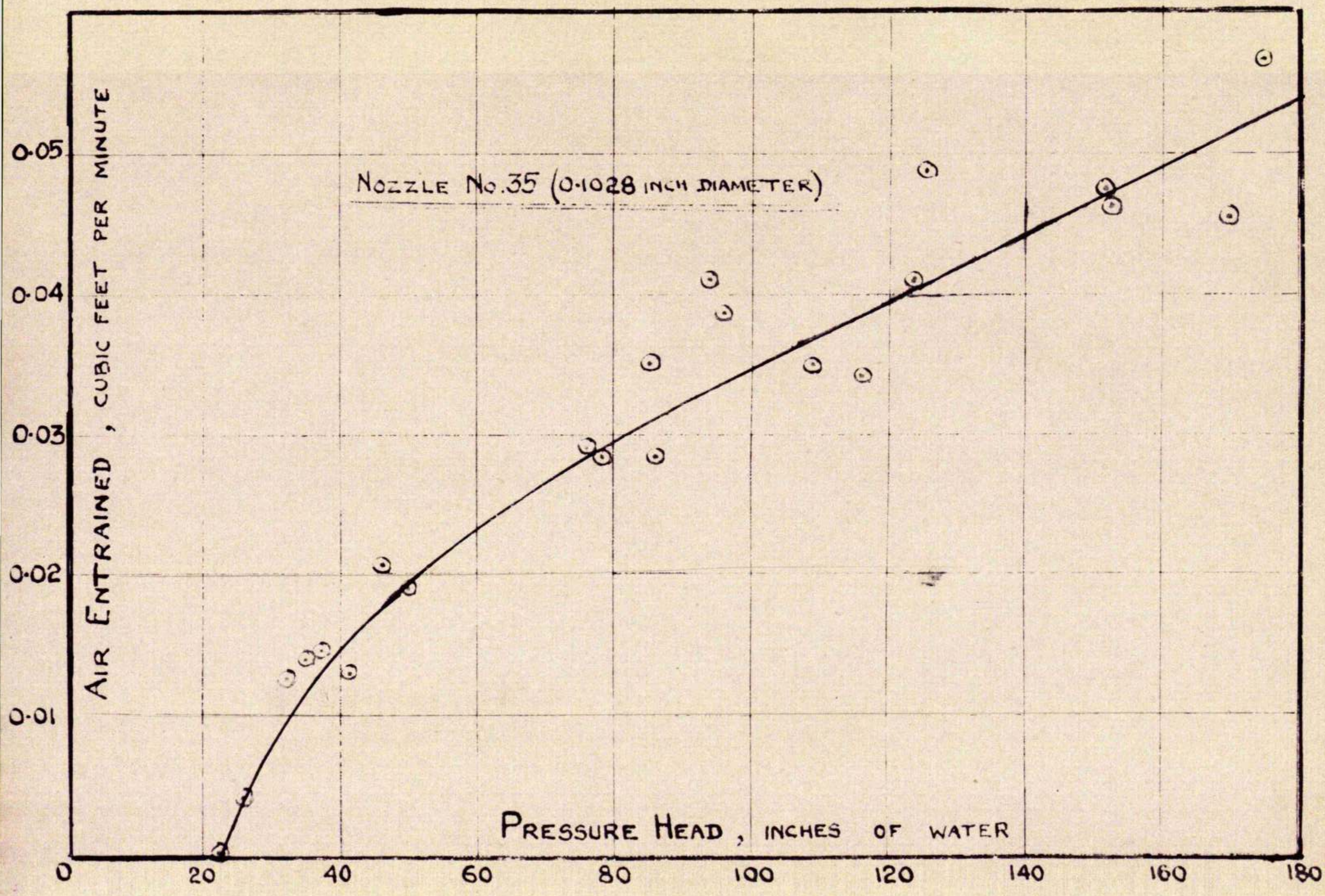
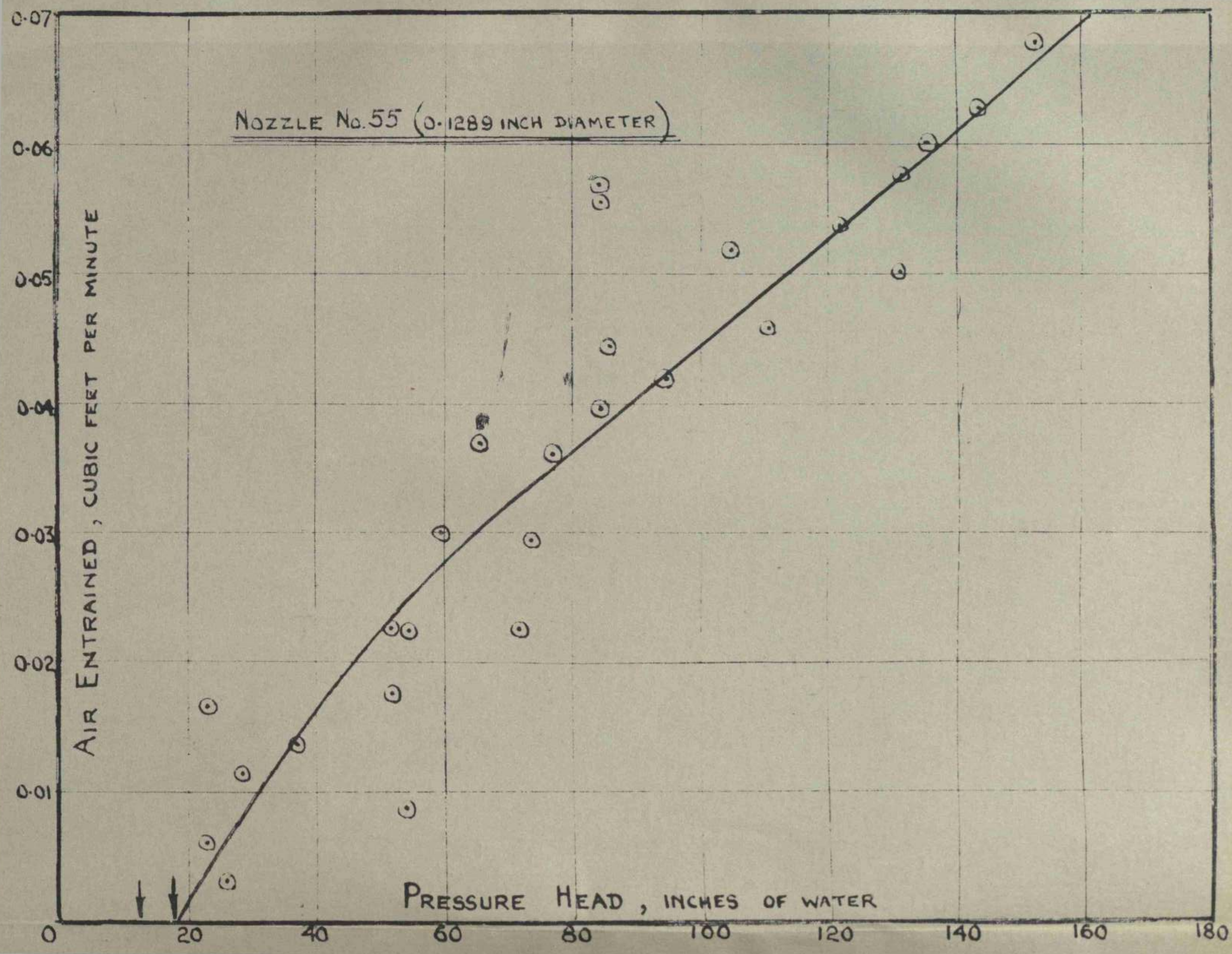


Fig. 38 Air Entrained / Pressure Head Curve for 0·1028 inch dia nozzle



Nozzle No. 55 (0·1289 inch diameter)
Air Entrained, cubic feet per minute
Pressure Head, inches of water
0·07
0·06
0·05
0·04
0·03
0·02
0·01
0
20
40
60
80
100
120
140
160
180

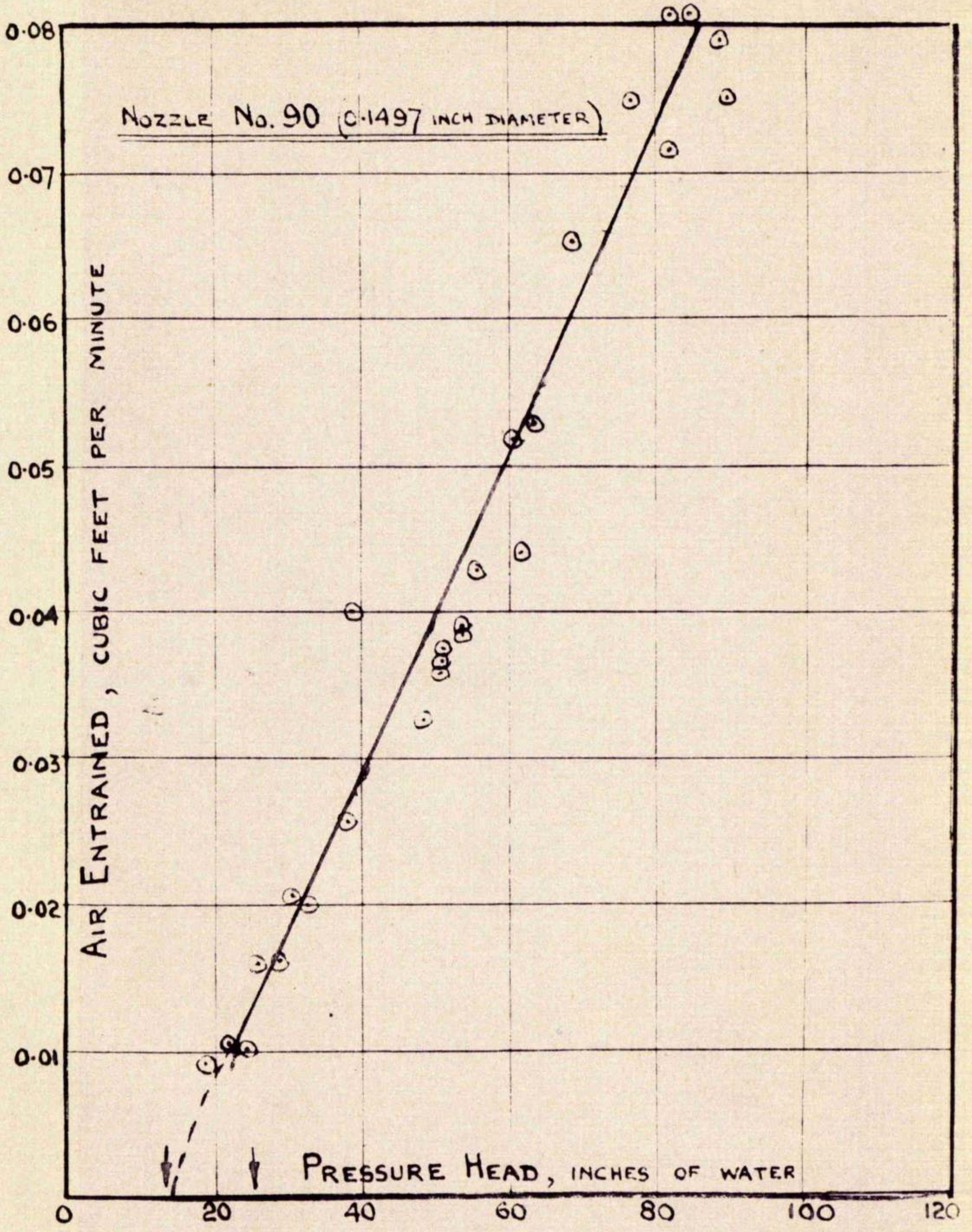


Fig. 40 Air Entrained | Pressure Head Curve for 0·1497" nozzle

considerably with a comparatively small change in surface level and observation showed that the jet appeared to break up into droplets or entrain air in the jet at a distance of about 2 inches from the nozzle with a head of about 71 inches of water, this distance increasing as the head decreased. Nozzle No. 3 was therefore also lowered to a distance of 1"-2½" from the surface and the results shown in Fig. 34 were obtained.

The length/diameter ratio of the jets used in these tests was below that at which dissolution of the jets under surface tension forces might have been anticipated[13] and so the behaviour was probably due to imperfections (e.g. surface finish) in the nozzle. The results obtained with nozzles Nos. 10, 25, 35, 55, 90 at a distance of 2½" to 3" from the surface and nozzles Nos. 3 and 7 at a distance of 1" to 2" from the surface were considered together.

Fig. 41 shows that the ratio $\frac{Qa}{Qw}$, plotted on a base of pressure head at the nozzle, is much greater for the two smallest nozzles than for the larger nozzles. Since the same result had been obtained with the orifices, it seemed at first as if this result might be significant.

The conditions at start of entrainment are shown in Figs. 42 and 43. The break in the curves caused by the results from the two smallest nozzles is obvious and further

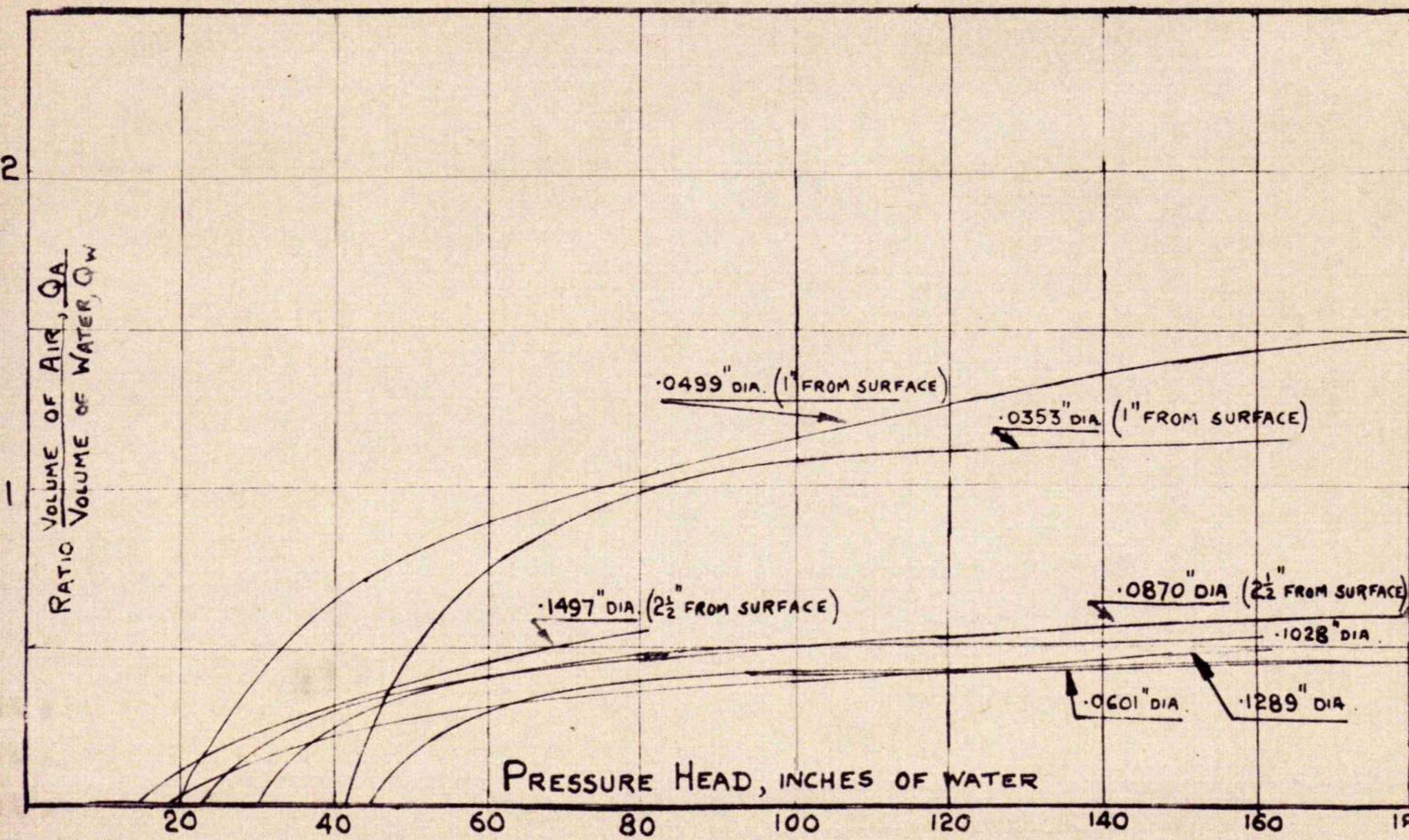


FIG. 41 RATIO $\frac{Q_A}{Q_W}$ ON BASE OF PRESSURE HEAD, ALL NOZZLES 2½" FROM SURFACE EXCEPT THE TWO SMALLEST (·0499" & ·0353" DIA) WHICH WERE 1" (APPROX.) FROM SURFACE.

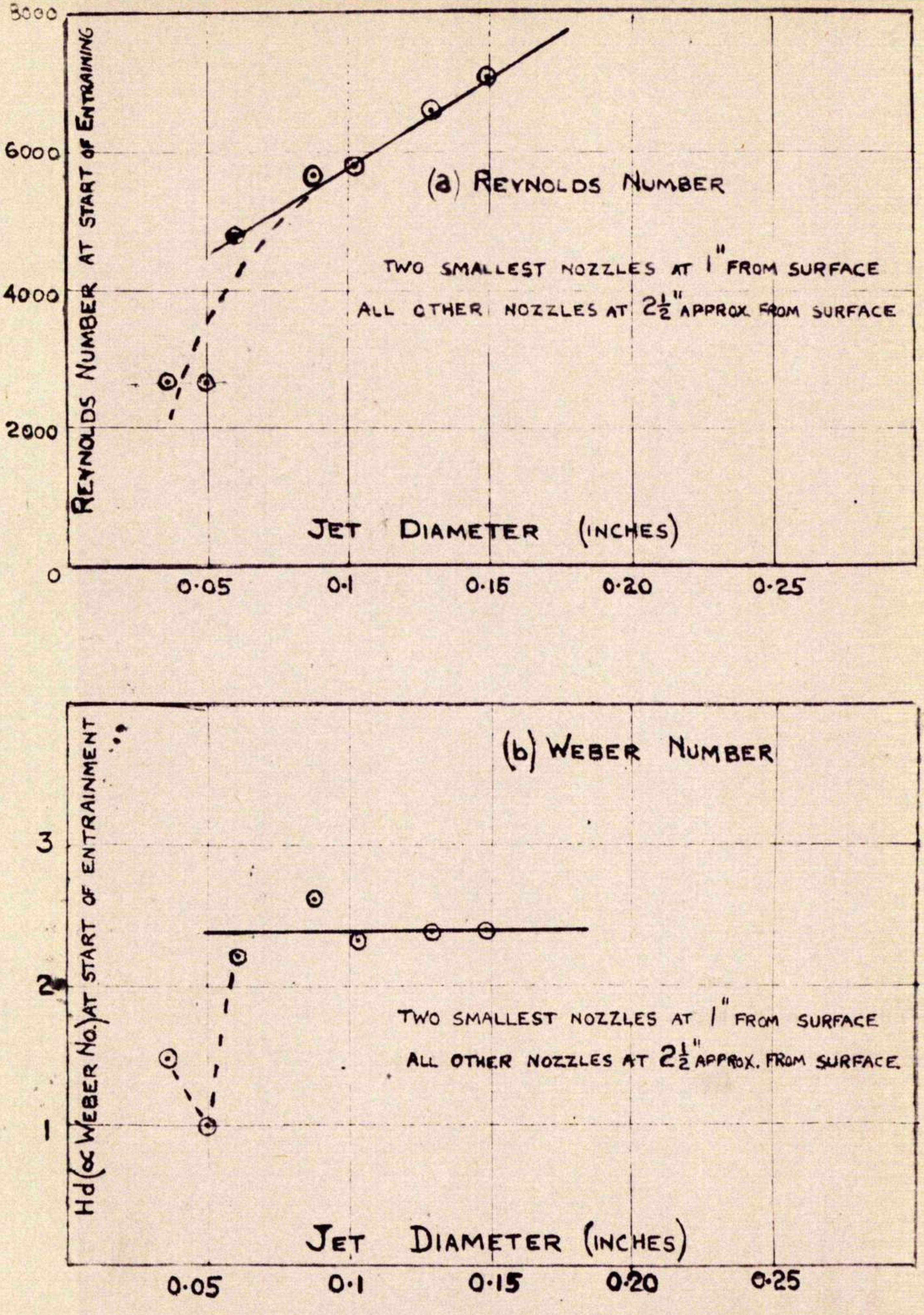


FIG. 42. CONDITIONS AT START OF ENTRAINMENT

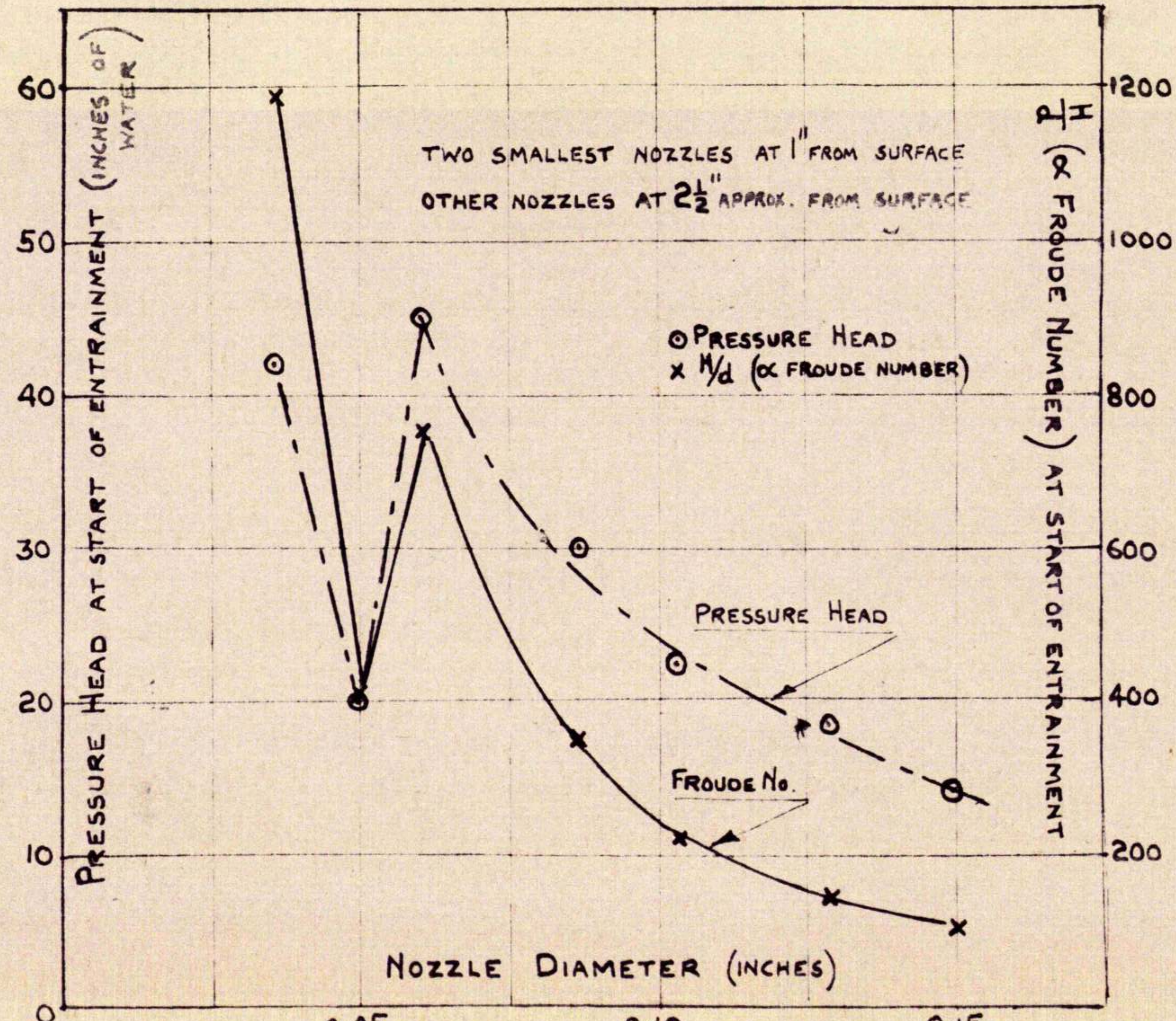


FIG. 13 ... START OF ENTRAINMENT

analysis served to emphasise the fact that the results from Nozzles No. 3 and 7 did not conform with the trend of the other results. Either there was some abrupt change in the mechanism of air entrainment when the jet size was reduced below a certain figure or the results from the two smaller results were unreliable. It was therefore decided to modify the apparatus further, so that the nozzles could be lowered much nearer the surface, thus overcoming the feature of jet disintegration.

(d) Further Tests with Nozzle Extension Piece; Jet Observations

A brass extension piece was made which screwed into the orifice holder and which enabled the nozzle to be lowered $3\frac{1}{2}$" without fouling the rubber tubes carrying the secondary flow.

A further series of tests was run, concentrating on the point when entrainment starts. Some difficulty was experienced in the control of the surface level in the receiver since, with the nozzle only about $\frac{1}{4}$" from the surface, little fluctuation was permissible. The distance of the free surface above the diffuser was also found to be somewhat critical since, if the surface were too near the diffuser observation of entrainment was difficult and a very slight fall caused the surface to fall below the top of the diffuser. If the surface were too high some of the bubbles escaped instead of being carried into the separator. The level of the surface was maintained at approximately the same level as it had been in the earlier series of tests.

The head at which entrainment starts was carefully observed for all the nozzles and air entrainment measurements were made. The critical point at which entrainment starts was found to be very sensitive to the distance from nozzle to surface, especially with the smaller jets. A jet might be entraining no air at a certain head but a slight fall in surface level (less than ¼ inch) would cause entrainment to start. This extreme sensitivity had not been expected and could not be controlled by the existing apparatus. The results are shown in Table IV below.

TABLE IV

Jet dia. (inches)	Critical head H (inches of water) (a) ¼" from surface	(b) 2½"-3" from surface	Critical Hd (a)	(b)
.0353	72	41	2.54	1.45
.0499	60	20	3.0	1.0
.0601	70	44	4.2	2.64
.0870	76	30	6.61	2.61
.1028	56	23	7.75	2.36
.1289	62	18	8.0	2.32
.1497	30	15	4.5	2.25

The sudden commencement of entrainment due to a slight fall in surface level led to a more critical observation of the jets produced by the nozzles. Each nozzle was examined under several heads and two approximate measurements were made:-

(a) the length of the jet from nozzle to the point where the jet broke up into droplets, (b) the length of the jet from the nozzle which was smooth and glossy in appearance - no entrainment by the jet. The following stages were observed:

(i) The jet was glassy-smooth - a scale could easily be read through the jet.

(ii) The jet had a dull appearance and the surface was marked parallel to the flow - a scale could be seen but not read clearly through the jet.

(iii) The jet appeared to be sparkling at the surface due to wavelets or small surface instabilities or disturbances.

(iv) There was pronounced break-up into droplets.

These stages merged into one another and were not clearly separated.

As a result of these jet observations it seemed reasonable to conclude that the two smaller jets were in a state of at least incipient break-up when entrainment started in the original tests at 1" approximately from the surface and this would account for the discrepancy with these results. At approximately ¼" to ½" from the nozzle these jets appear to be similar to the larger jets at 2½" to 3" from the nozzle, for the heads concerned. Hence it seems permissible to include the results of the two smallest nozzles at a distance of $\frac{1}{4}$" from the surface with the other nozzles at a distance of $2\frac{1}{2}$" to 3" from the surface.

In the further analysis of results, therefore, the readings obtained with the two smallest nozzles at $\frac{1}{4}$" from the surface and the other nozzles at $2\frac{1}{2}$" to 3" will be considered together (Fig. 47). This somewhat illogical step seems justified by the jet observations mentioned above for the particular nozzles used in this investigation. The obtaining of "similar" jets of different diameters was one of the unexpected difficulties of this research and further investigation into the production of smooth jets (e.g. the effect of surface roughness of the nozzle which may be expected to have an appreciable effect particularly on small diameter jets) seems required in the future.

The air entrainment measurements with the nozzles about $\frac{1}{4}$" from the surface gave less air entrained with a given head than when the nozzles were $2\frac{1}{2}$"-3" from the surface but generally similar curves were obtained (see Fig. 44).

(e) Check Tests with Specially Made Nozzles.

In order to check whether the behaviour of the jets described in the previous section was peculiar to the nozzles used, three additional nozzles were made in the workshops of the Mechanical Engineering Department, The Royal Technical College. These nozzles were of the form shown in Fig. 45 and

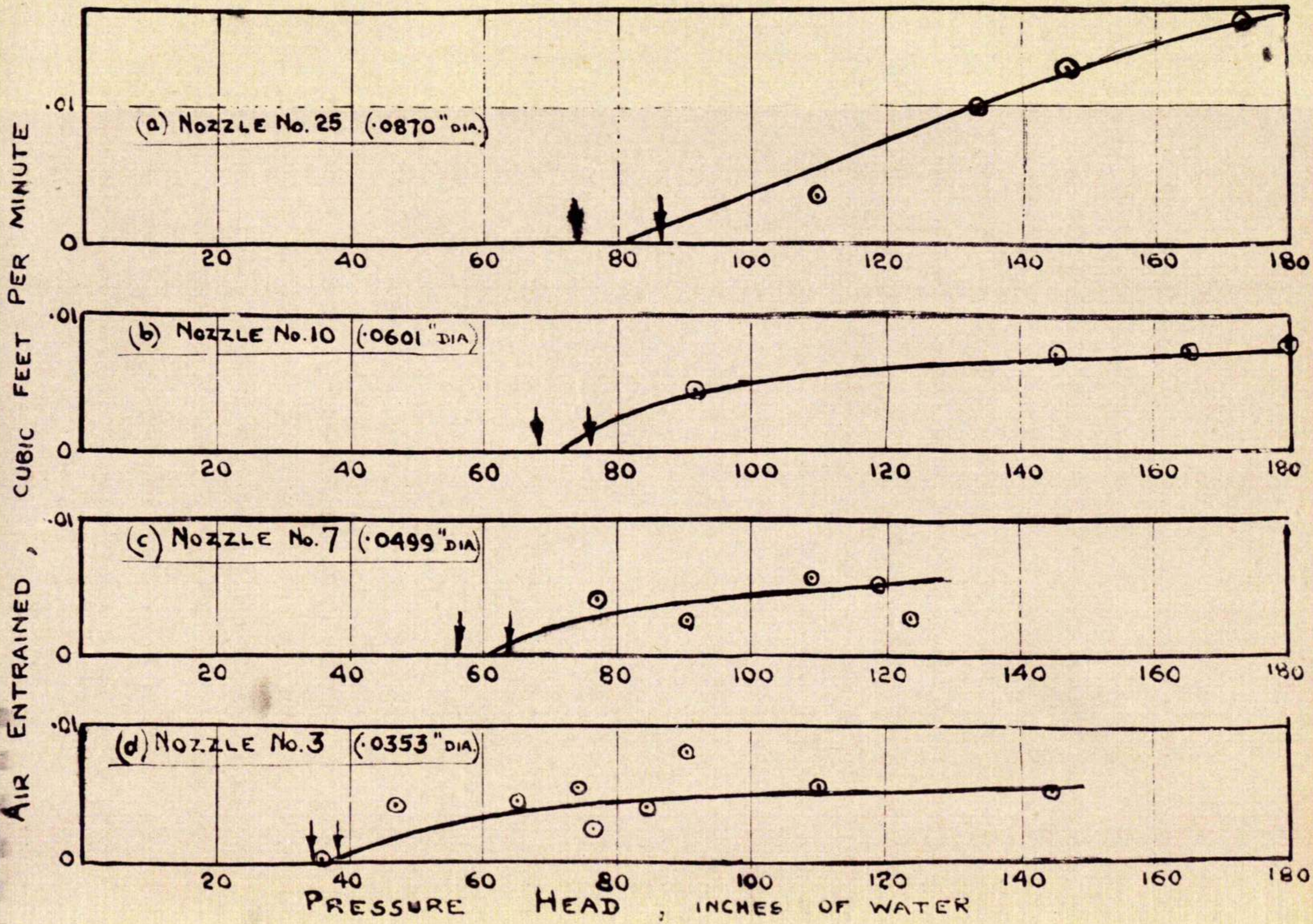
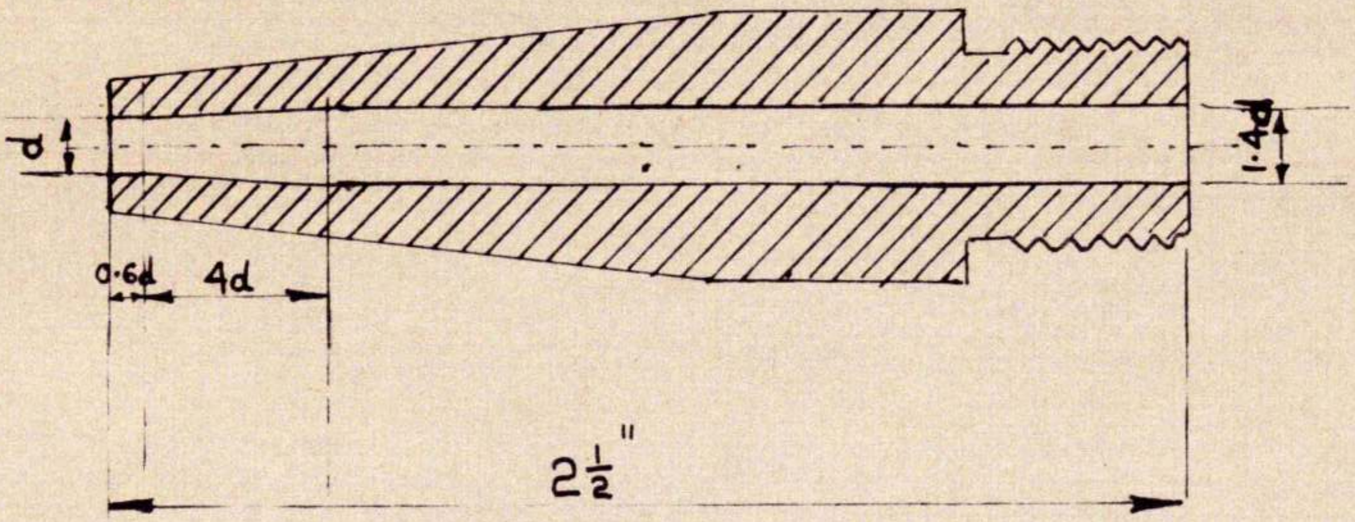


Fig 44 Air Entrainment Experiments with Nozzles $\frac{1}{4}$" from Surface



DIMENSIONS OF R.T.C. NOZZLES

NOZZLE DIAMETER d = ·03125", ·0625" & ·125" RESPECTIVELY

FIG. 45

had diameters of 0.03125, 0.0625 and 0.125 inch respectively. Although they were made as accurately as possible, it was found impossible to obtain a real "super finish" on such small nozzles.

The jets from these nozzles were found to be very similar to those from the corresponding nozzles in the main series, and the results agreed with the previous experiments within the accuracy obtainable. These results are shown in Fig. 46 and added little useful information.

(f) Results of Nozzle Experiments.

Assuming that it is permissible to include the tests with the two smaller nozzles at ¼" from the surface with the larger nozzles at approximately 2⅛" from the surface, the final results from the series of nozzle experiments are shown in Figs. 47 and 48. The results from the various nozzles now lie in proper sequence. The experimental range is too small - allowing for experimental scatter - to determine whether the curves of Qa on a base of pressure head at the nozzle are linear (apart from the initial portion) or flat parabolic curves.

Two aspects of the experiments are of interest, viz., (a) the conditions governing the start of air entrainment, (b) the quantity of air entrained under given conditions (or alternatively the ratio of the volume of air entrained to the

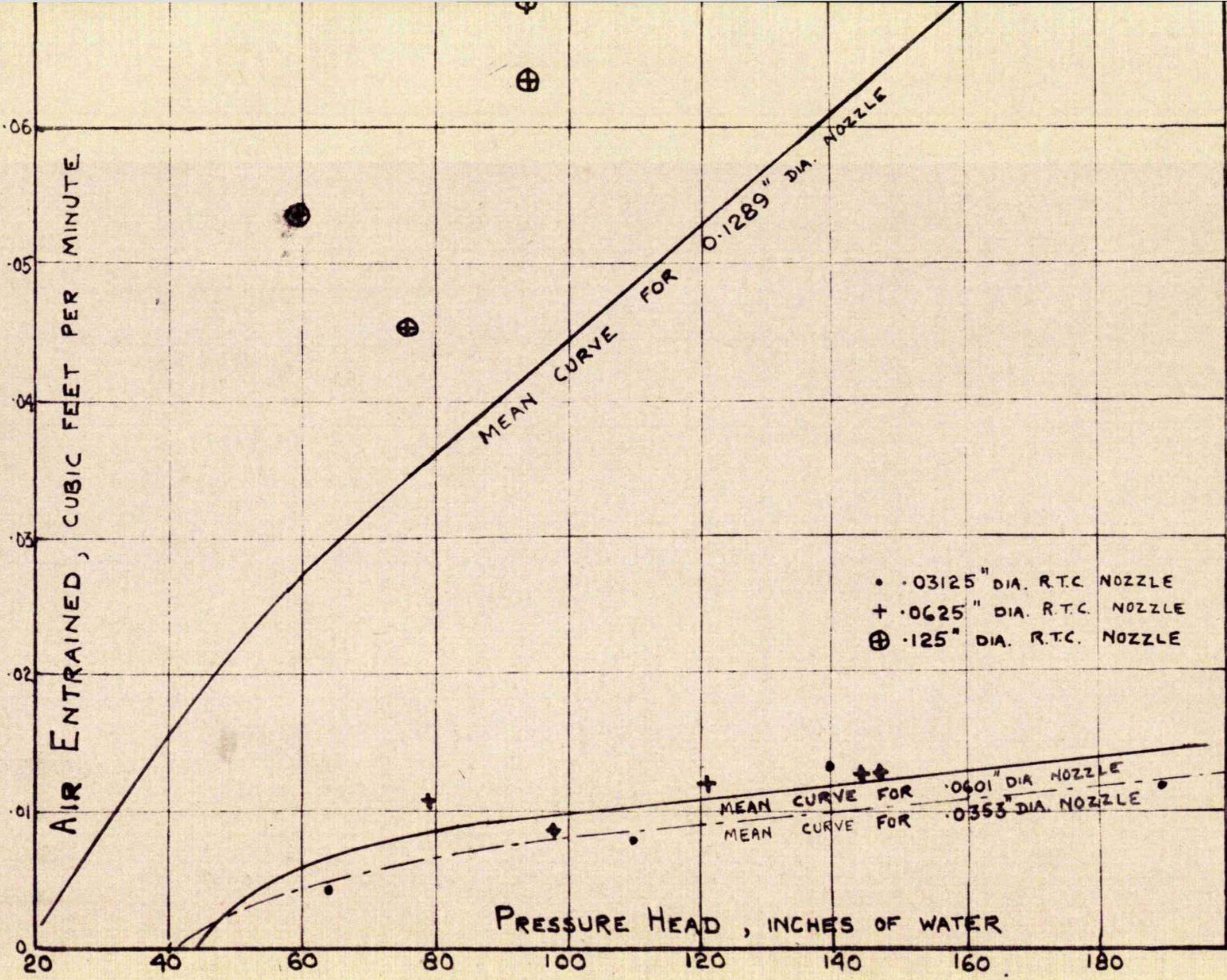


FIG 46 RESULTS OF TESTS WITH R.T.C. NOZZLES (ORIGINAL NOZZLE CURVES FOR)

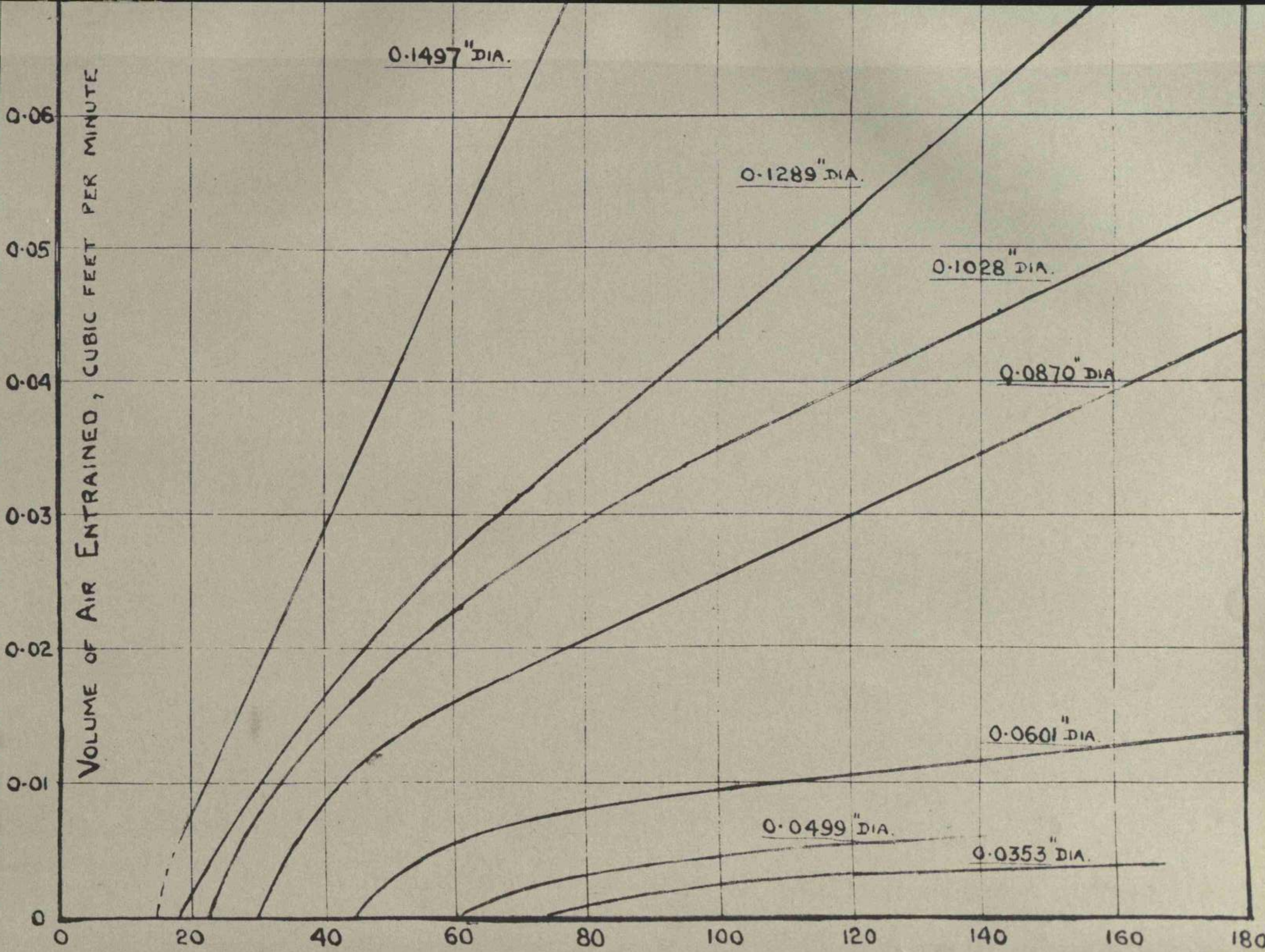

0·1497" DIA.
0·1289" DIA.
0·1028" DIA.
0·0870" DIA.
0·0601" DIA.
0·0499" DIA.
0·0353" DIA.
VOLUME OF AIR ENTRAINED, CUBIC FEET PER MINUTE
0·06
0·05
0·04
0·03
0·02
0·01
0
0
20
40
60
80
100
120
140
160
180

Fig. 47 Mean

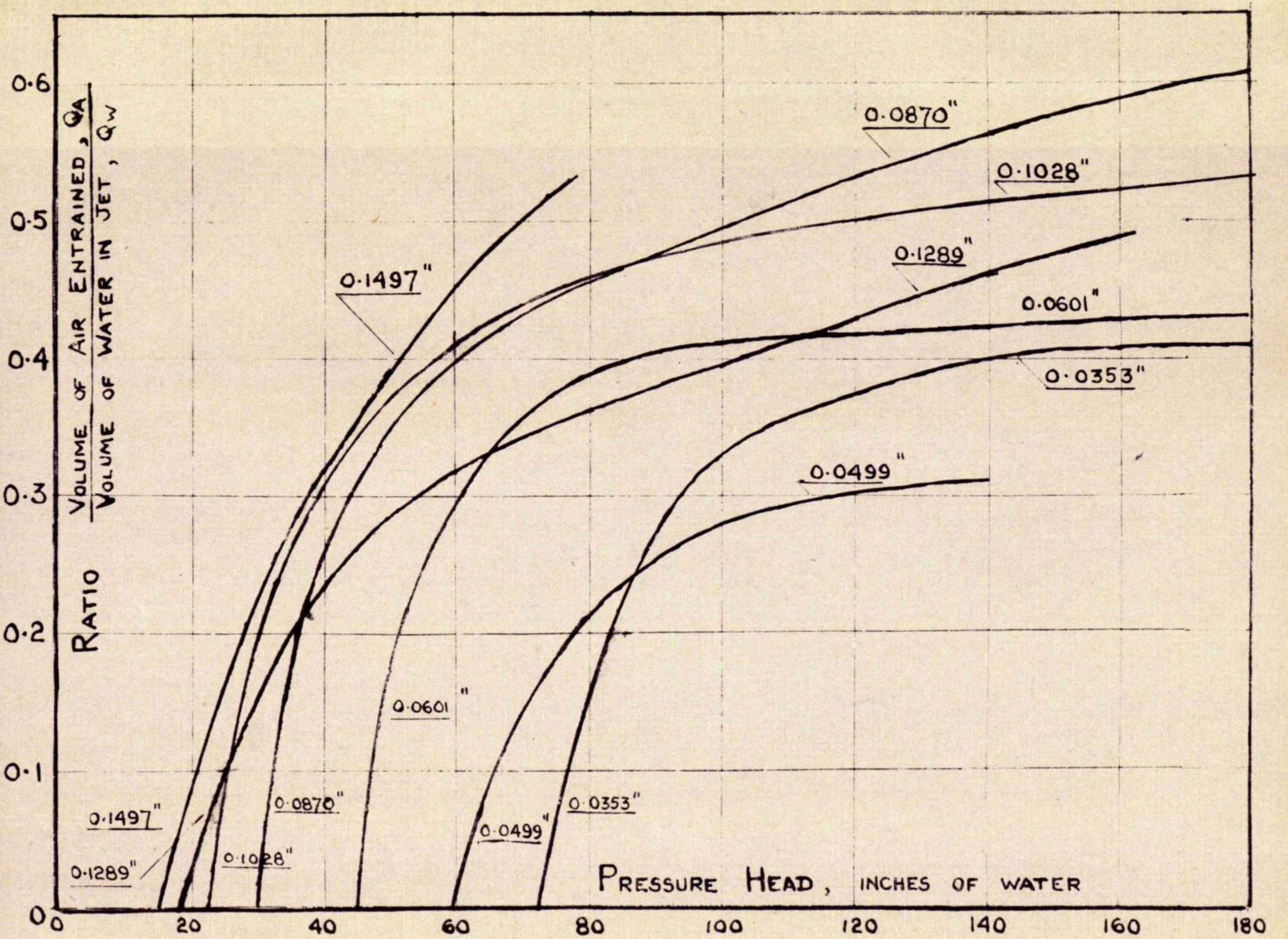


Fig. 48 Mean Curves of Q_A/Q_W (all nozzles)

volume of water, $\frac{Qa}{Qw}$). Considering first the conditions at start of entrainment, these are shown in Figs. 49 and 50. The pressure head at the start of entrainment decreases as the jet diameter increases (Fig. 49(a)). The trend indicated is similar to that of the orifice experiments although the actual values of the pressure head are considerably lower - this fact indicating a refinement in technique. The Froude Number at start of entrainment decreases rapidly with increase in jet diameter in the range of the experiments (Fig. 49(b)). The Reynolds Number at the start of entrainment increases with jet diameter (Fig. 50(a)). These three curves all show how the results obtained with the two smaller nozzles ¼" from the surface fall smoothly on the curves of the results from the other nozzles at 2½" approximately from the surface, thus giving further justification of the procedure. Values of $\sqrt{Hd}$ (proportional to Weber Number) at start of entrainment are plotted in Fig. 50(b). These points are rather scattered to indicate any definite trend - they may either be considered to be grouped about a mean constant value or to show a slight decrease in Weber Number as the jet diameter increases. However when these values of $\sqrt{Hd}$ at start of entrainment are plotted on a base of $^H/_d$ at start of entrainment (i.e. Critical Weber Number W_o on base of critical Froude Number F_o, to use Shirley's nomenclature) as in Fig. 51

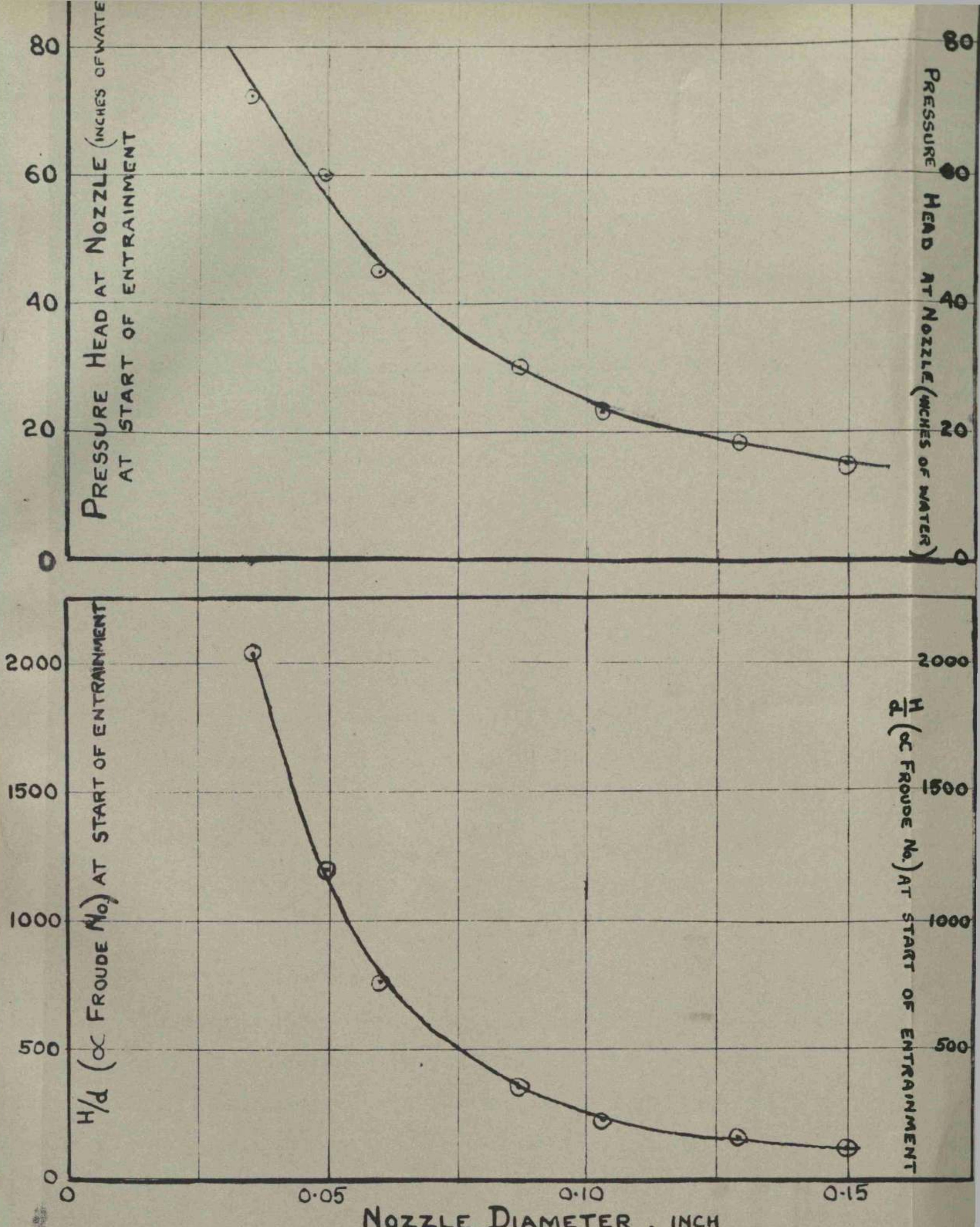


Fig. 49 Conditions at Start of Entrainment

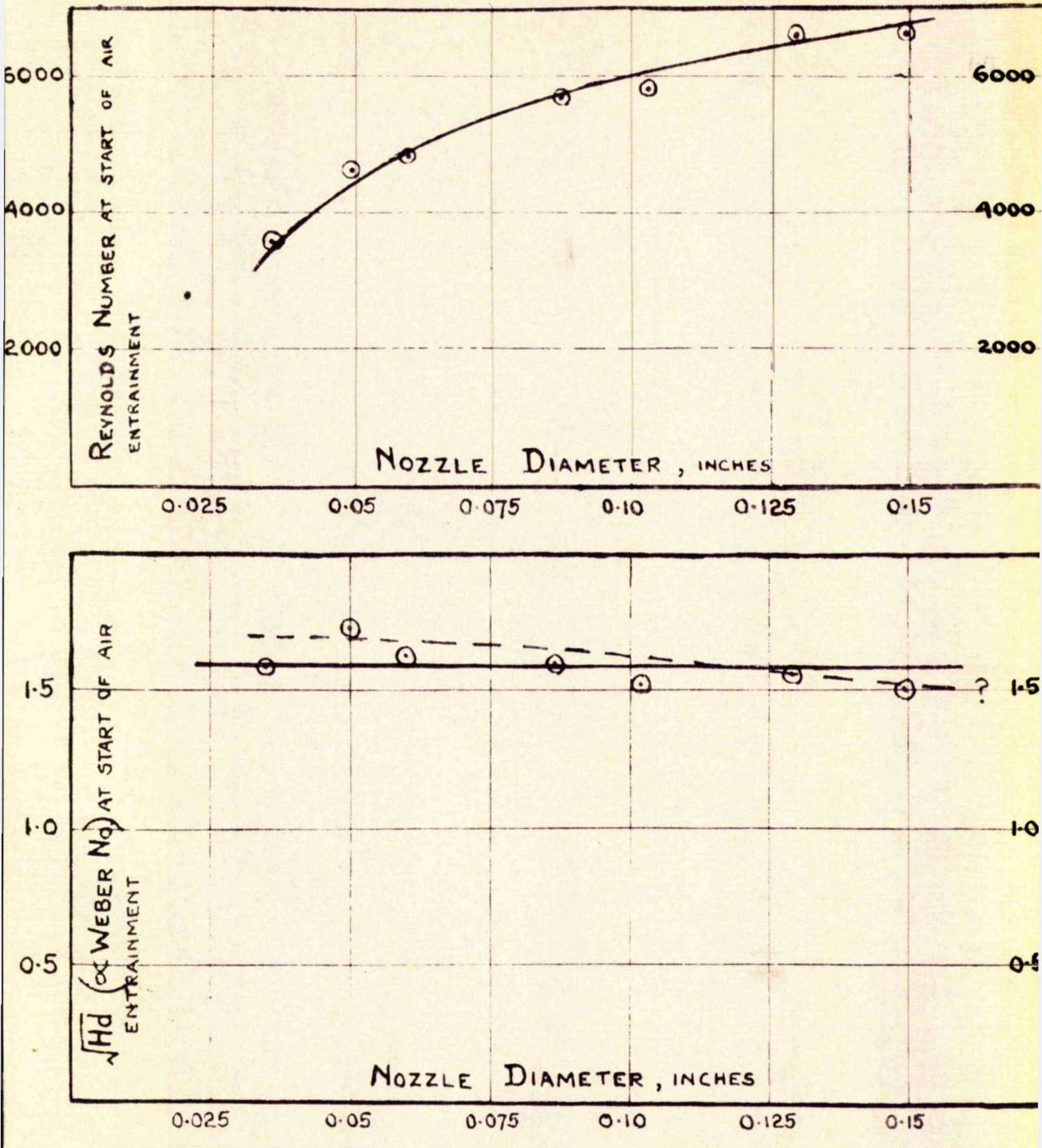


Fig. 50 Reynolds No. and Weber No. at start of air entrainment

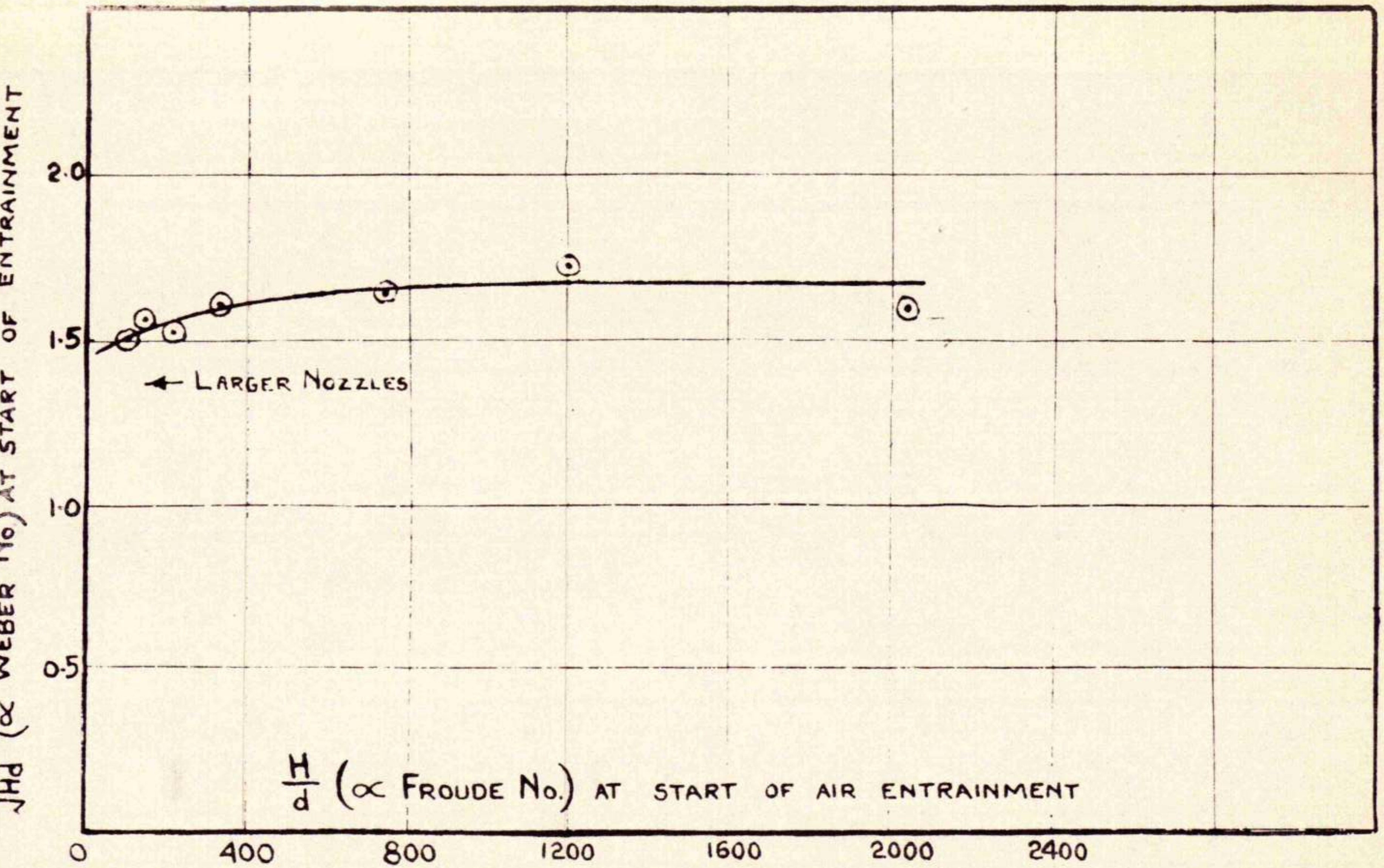


FIG. 51 WEBER No. ON BASE OF FROUDE No. (AT START OF AIR ENTRAINMENT)

the values of $\sqrt{Hd}$ which were apparently scattered about a mean value when plotted on a base of jet diameter now appear to be on a definite curve which shows a more pronounced change in Weber Number for a given change in Froude Number at the lower Froude Numbers (which correspond to the larger nozzles). At the higher Froude Numbers the Weber Numbers tend to a constant value. This seems to indicate that the Froude Number becomes of less importance for the smaller jets - it may have any value at start of entrainment while the Weber Number tends to a constant value. For the larger jets the start of entrainment depends on both the Weber Number and the Froude Number.

Considering next the quantity of air entrained, this is shown plotted on bases of Weber Number, Froude Number and Reynolds Number in Figs. 52, 53 and 54 respectively. The curves of Qa on a base of Reynolds Number give the closest approach to a general curve and strongly suggest that the quantity of air depends primarily on the Reynolds Number. These curves all seem to show a slight change in slope at some value of Reynolds Number but since this corresponds approximately with the "knee" of the Qa/H curve it is thought that it may be due to the assumed shape of these curves in this region, and is not considered to be significant.

The ratio $^{Qa}/_{Qw}$ is plotted on a base of Weber Number,

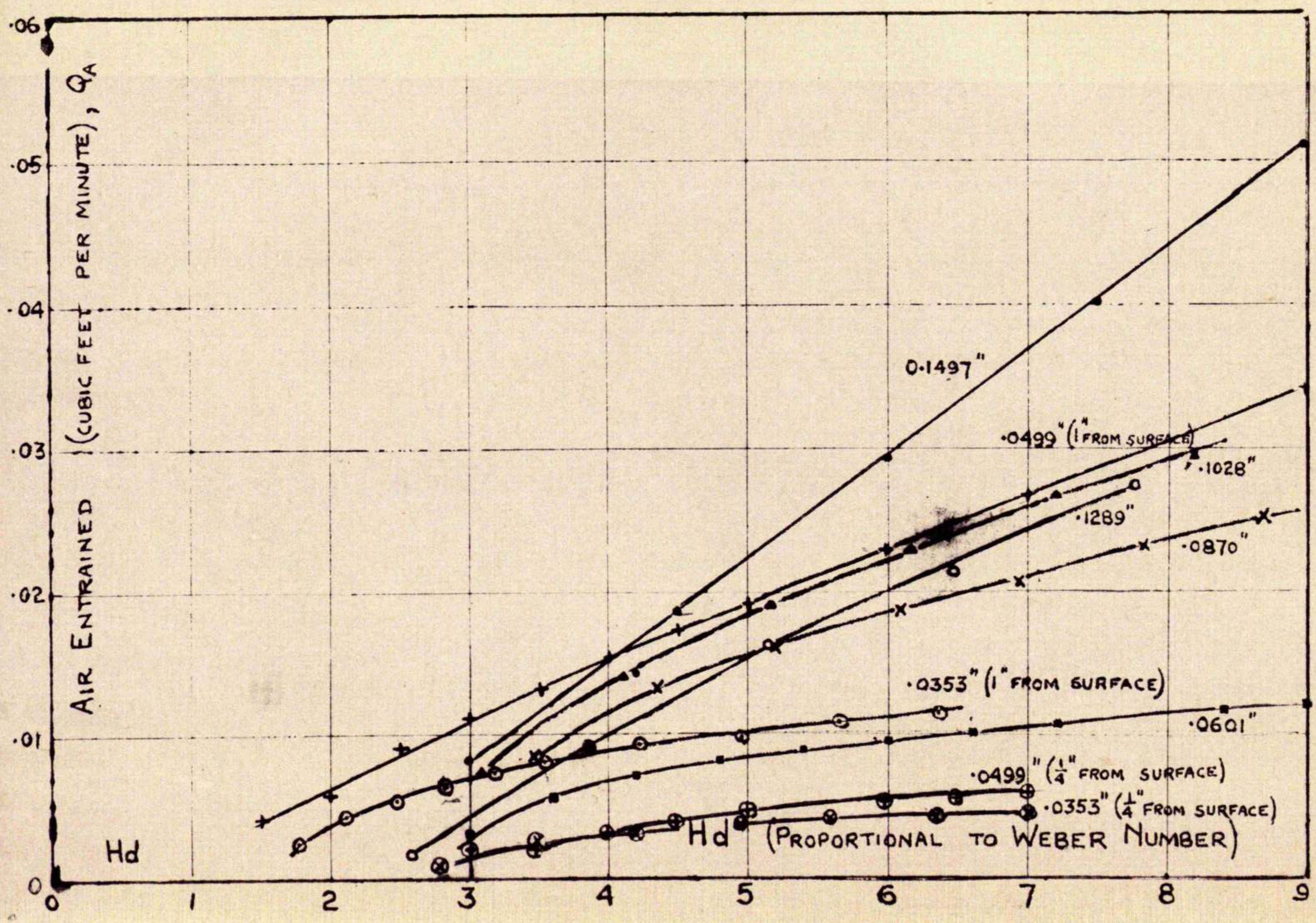


FIG. 52 CURVES OF Q_A ON BASE OF Hd ($\propto$ WEBER NUMBER)

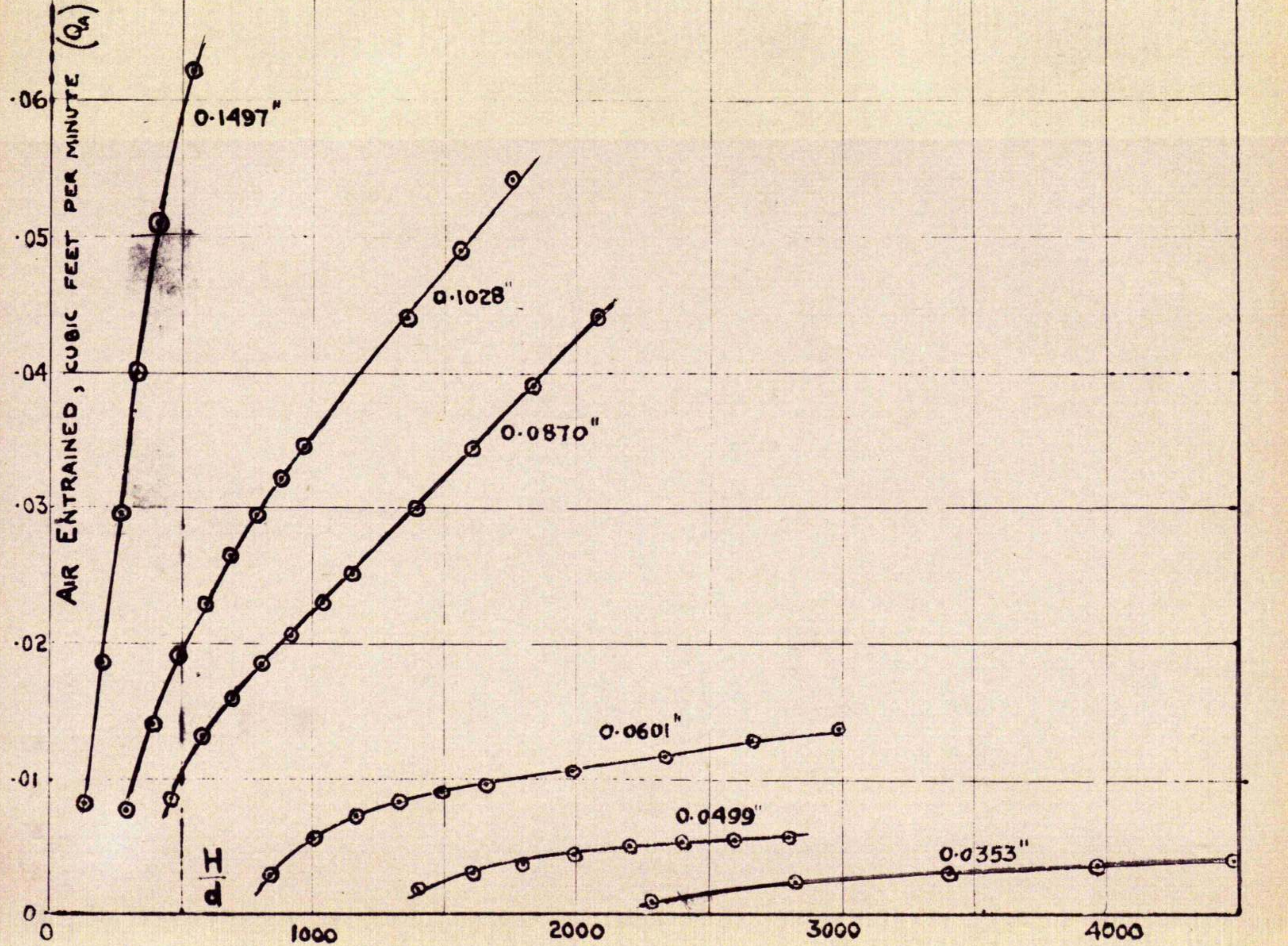
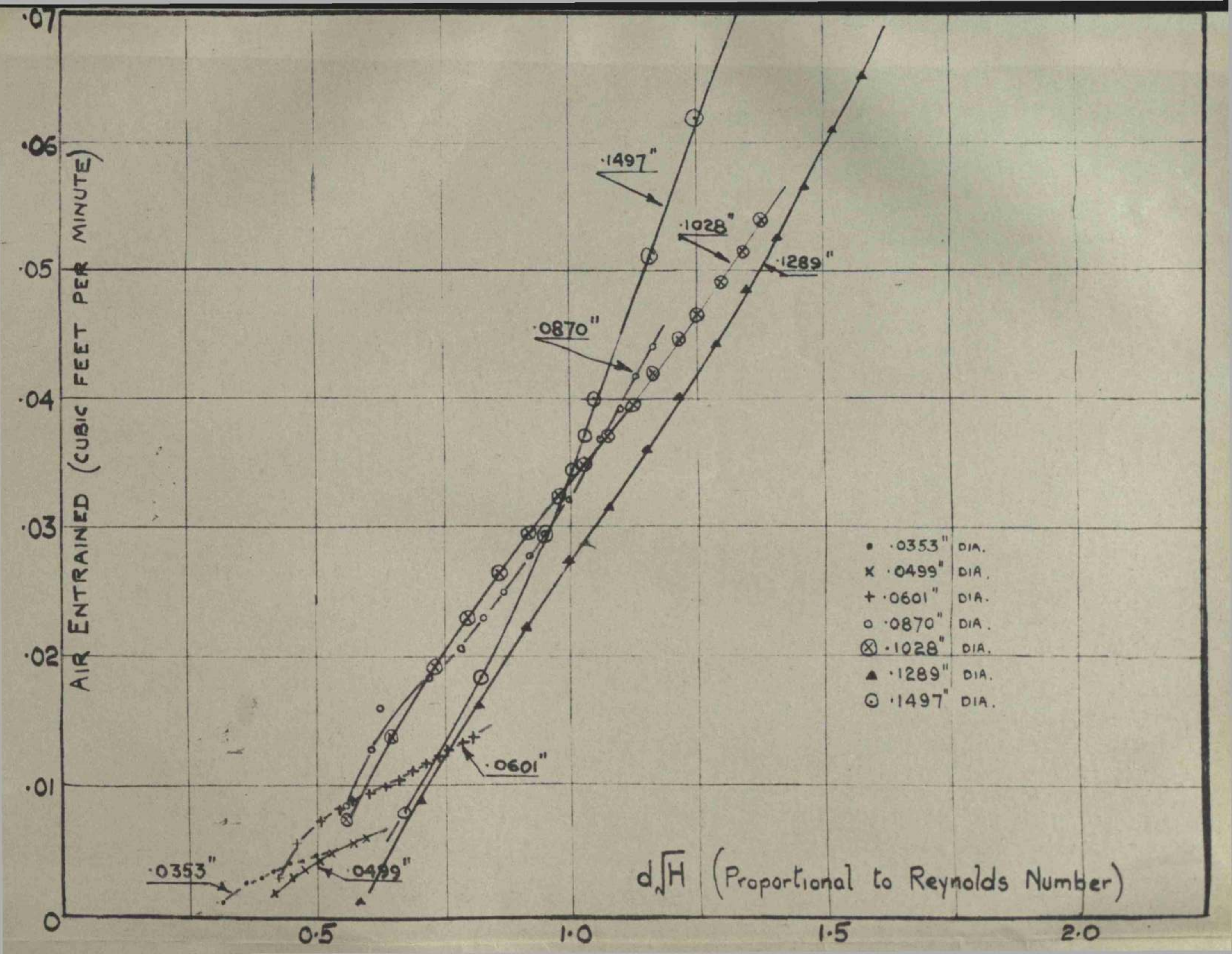


FIG. 53 CURVES OF Q_A ON BASE OF $\frac{H}{d}$ ($\propto$ FROUDE NUMBER)


·07
·06
·05
·04
·03
·02
·01
0
0·5
1·0
1·5
2·0
AIR ENTRAINED (CUBIC FEET PER MINUTE)
$d\sqrt{H}$ (Proportional to Reynolds Number)
·1497"
·1028"
·1289"
·0870"
·0601"
·0353"
·0499"
• ·0353" DIA.
x ·0499" DIA.
+ ·0601" DIA.
o ·0870" DIA.
⊗ ·1028" DIA.
▲ ·1289" DIA.
⊙ ·1497" DIA.

Froude Number and Reynolds Number in Figs. 55, 56 and 57 respectively. This ratio shows no clearly defined trend although it seems generally to increase with jet diameter. Within the limited range covered by these experiments the maximum value of $\frac{Qa}{Qw}$ lies between 0.3 and 0.6. The ratio, $Qa/_{Qw}$, seems less amenable to generalisation than the actual quantity of air entrained.

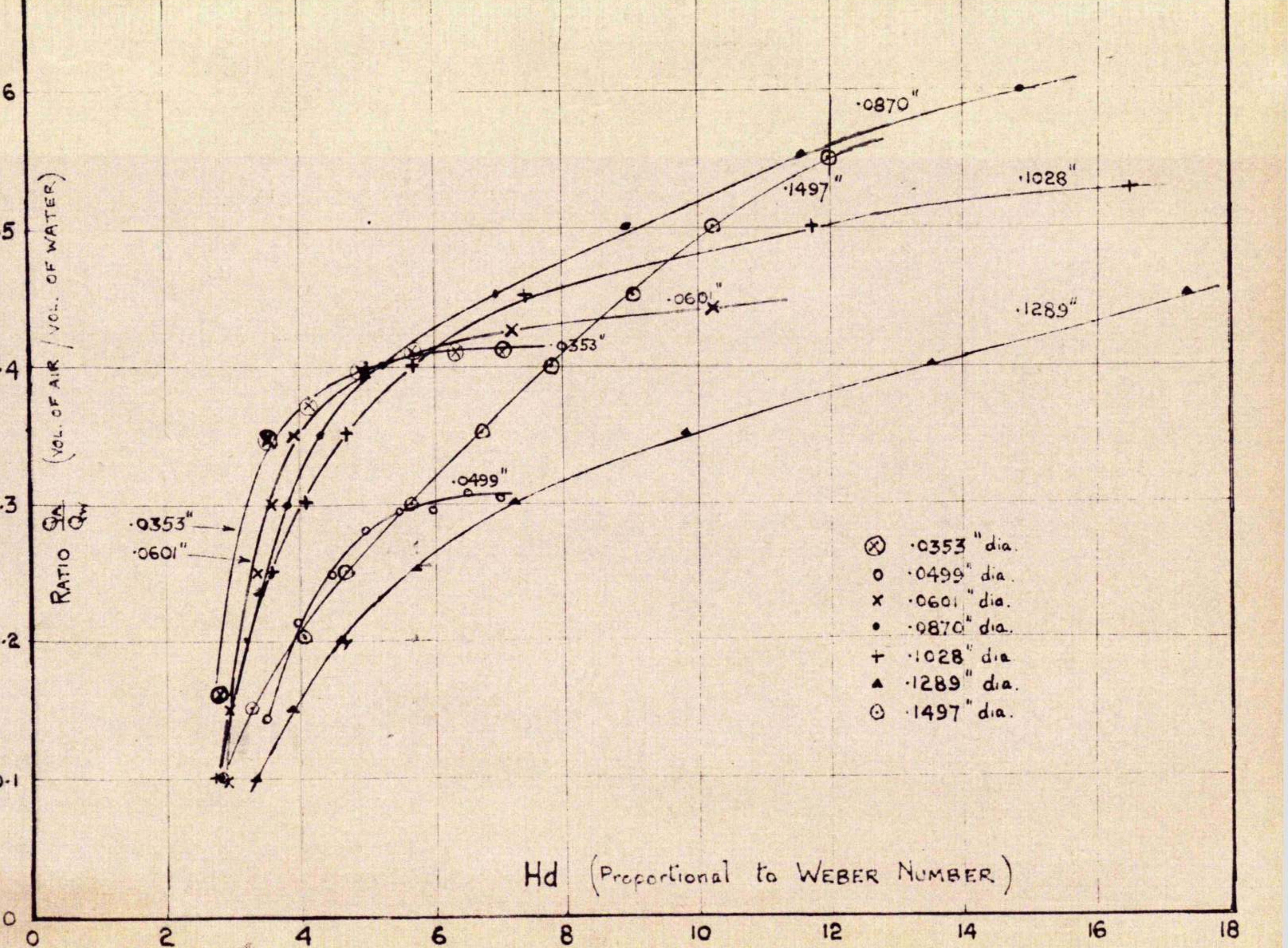


FIG. 55 Ratio $\frac{Q_A}{Q_W}$ on base of Hd ($\propto$ Weber Number)

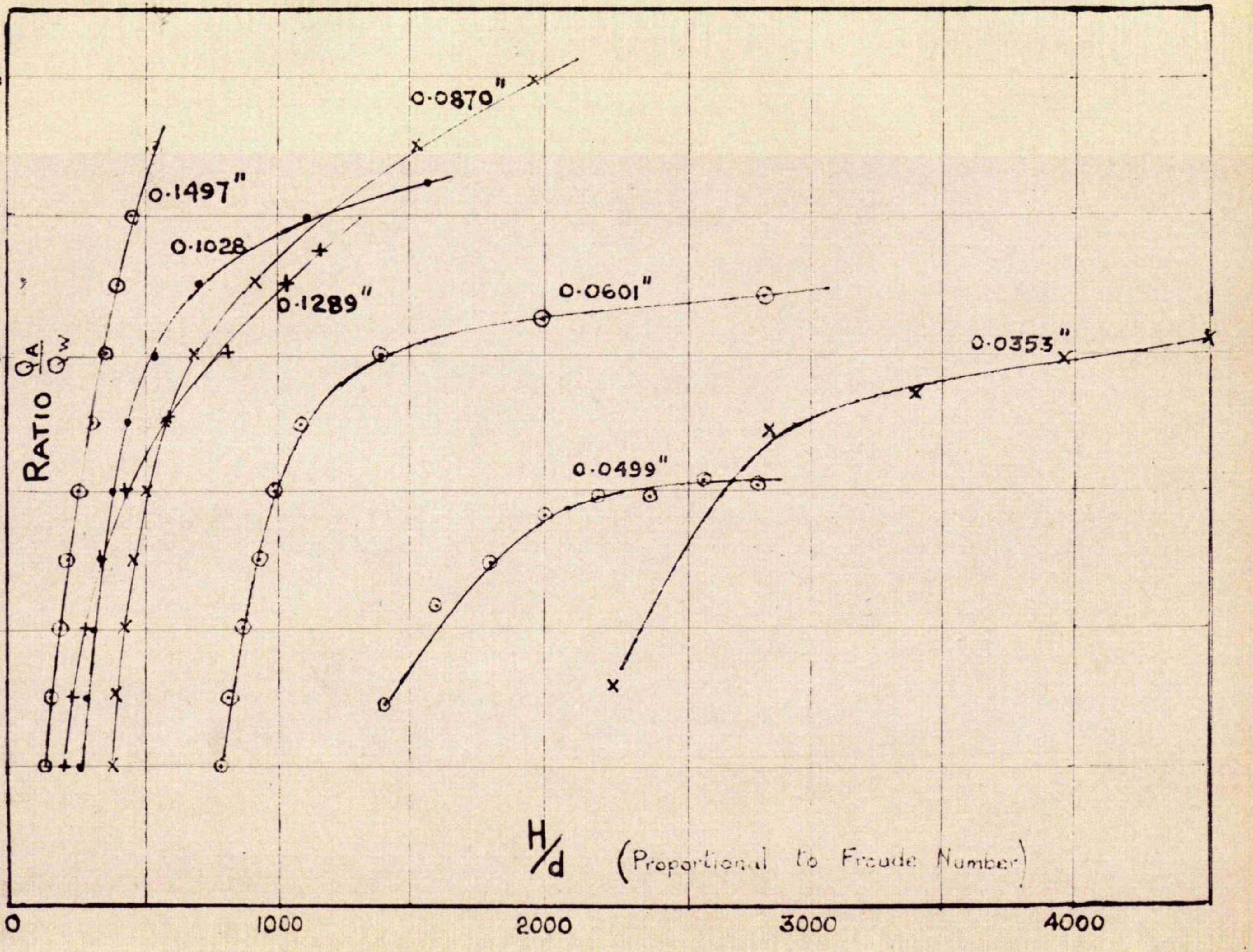


FIG. 56. RATIO $\frac{Q_A}{Q_W}$ ON BASE OF H/d ($\propto$ FROUDE NUMBER)

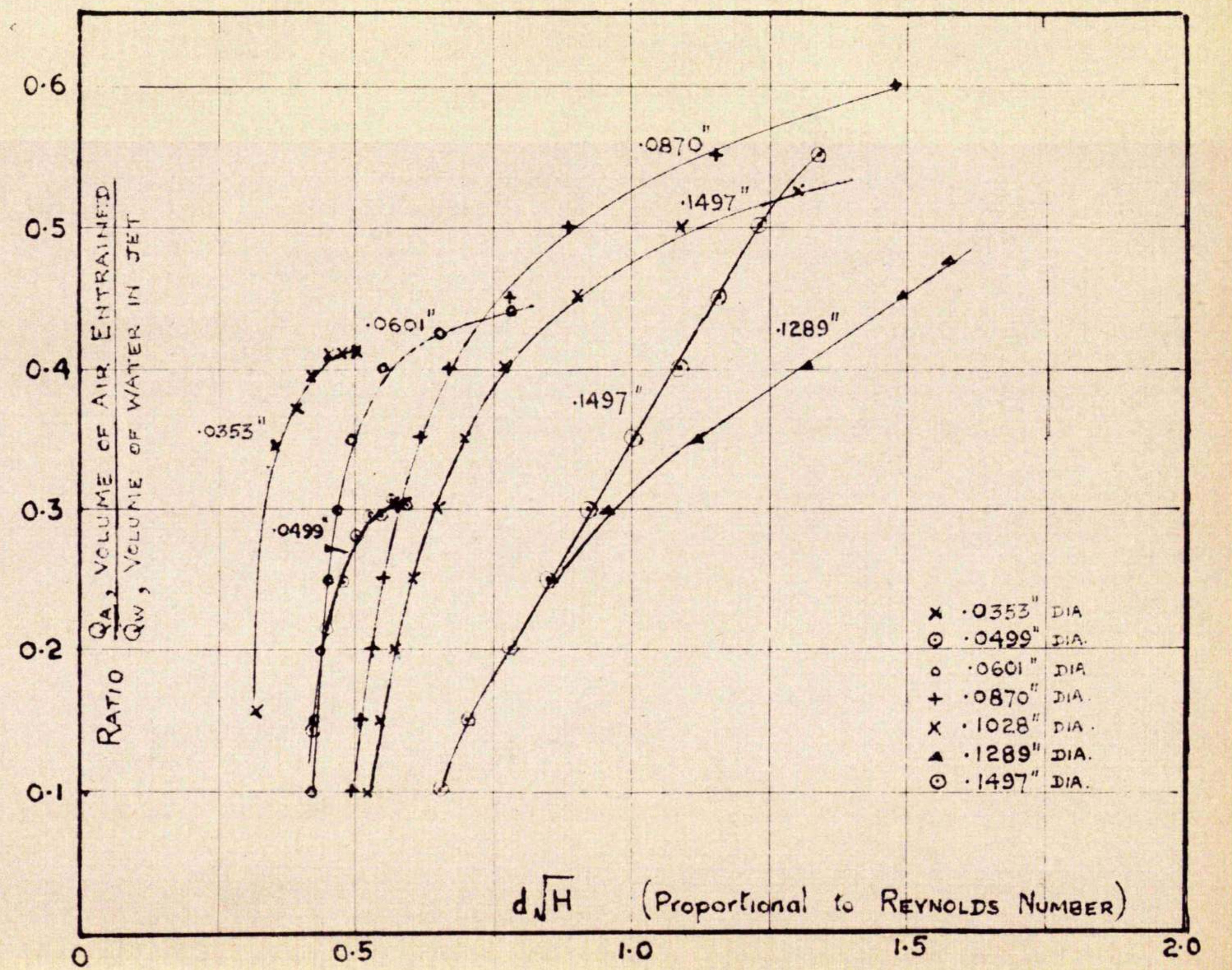


FIG. 57 RATIO $\frac{Q_A}{Q_W}$ ON BASE OF $d\sqrt{H}$ ($\propto$ REYNOLDS NUMBER)

SECTION 5

ADDITIONAL EXPERIMENTS RELEVANT TO THE DESIGN OF A HYDRAULIC COMPRESSOR

SECTION 5

ADDITIONAL EXPERIMENTS RELEVANT TO THE DESIGN OF A HYDRAULIC COMPRESSOR

The experiments described in the previous sections had two main objects, viz. (a) to find a criterion for air entrainment in such cases as the Brannie Burn Intake Works of the Glen Shira Project, or at any rate to provide guidance for model tests on such problems, (b) to collect information which would be applicable to the design of a hydraulic compressor.

In the first case, the step from experiments on small jets to entrainment in large streams is obviously such a big one that, lacking further experimental evidence, any conclusions must be tentative at this stage. It may therefore be justifiable to concentrate further on the second application. Here the object is to achieve the maximum entrainment of air or the maximum values of the ratio $\frac{Qa}{Qw}$. Attempts to increase the quantity of air were accordingly made. These experiments were not exhaustive but were merely designed to see whether such an increase might be readily obtained. Two methods were tried, viz. (a) by surrounding the jet with a glass venturi-shaped tube to see whether the increased velocity and decreased pressure at the throat could be made to improve the entrainment, (b) by experimenting with jets of different cross-section obtained by using triangular and square orifices.

(a) Experiments with Glass Venturi-Shaped Tubes.

Three glass venturi tubes were made from 0.3 inch bore glass tubing. Two of these had a throat diameter of approximately 0.1 inch and the third 0.2 inch. One of the 0.1 inch throat venturis was 1.6 inches long overall, the other two being 3.3 inches overall. The throat was approximately 0.3 inch from the bell-mouthed inlet in each case. It was decided to experiment with different nozzles, thereby altering the ratio of jet diameter to throat diameter, and with the venturis situated at different levels (with reference to nozzle and water surface) so that the venturi effect of increased velocity at the throat - on which any increased air entrainment by such devices would depend - was obtained (a) before the jet struck the water surface, (b) in the stream of entrained air just below the water surface. A sketch of the arrangement is shown in Fig. 58.

The following observations were made when the venturi tubes were placed so that the length from nozzle to surface was approximately 2½" and the venturi throat surrounded the jet above the surface. The distance from nozzle to venturi throat was approximately ½". If the jet diameter was less than the throat diameter, the jet went straight through, the

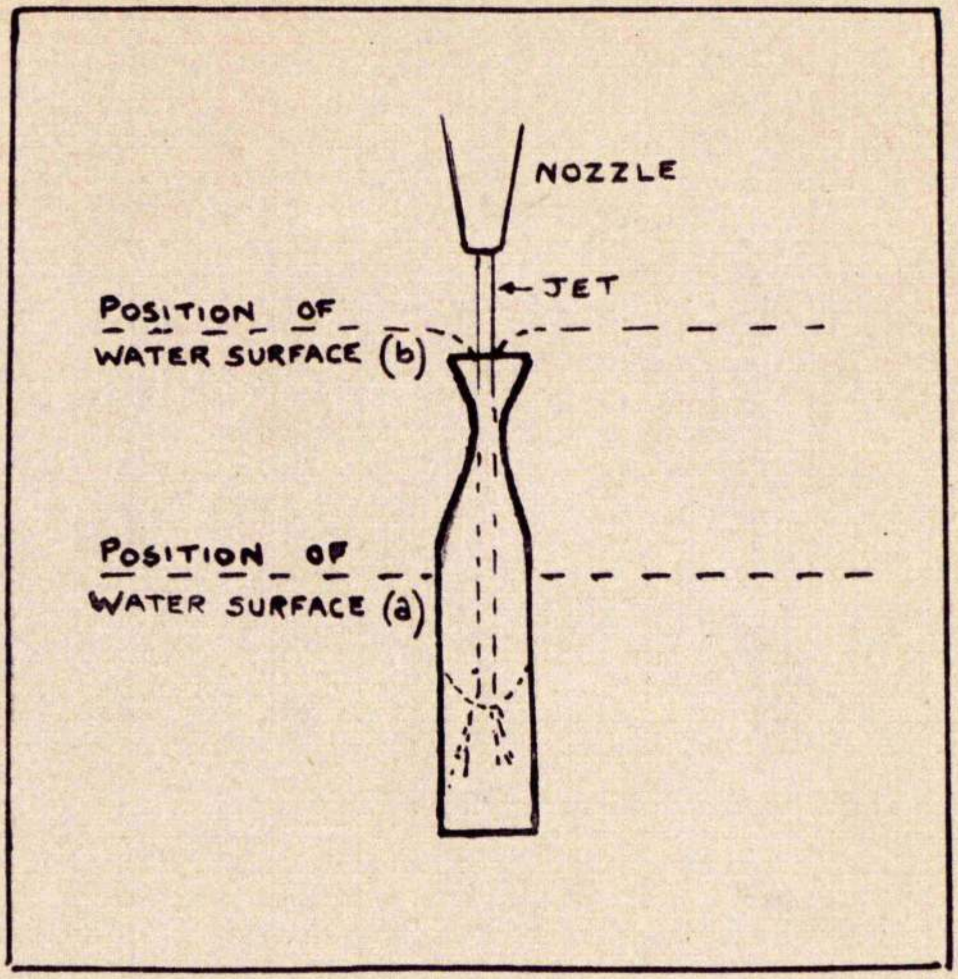


FIG. 58 SKETCH OF ARRANGEMENT FOR EXPERIMENTS WITH GLASS VENTURI TUBE SURROUNDING JET

surface level was depressed inside and air was entrained at this depressed surface in a similar manner to the original experiments. The amount by which the surface was depressed was considerable (more than ½ inch) and in some cases it disappeared below the top of the diffuser, thus preventing any further observation of the entrainment and, consequently, control of the surface level. The jet length was increased to such an extent that conditions were no longer comparable with the earlier tests. Apart from demonstrating this effect of the jet momentum on the free surface - an effect which is not apparent with the larger surface area originally used - it is difficult to see how such an arrangement could increase the quantity of air entrained and this was borne out by the results which showed that less air was, in fact, entrained.

If the jet diameter was greater than the throat diameter, the jet appeared to cling to the walls of the venturi and run down them. At some critical point, the surface appeared to be sucked up into the tube which was thenceforth filled with a mass of "boiling" white water (i.e. water with entrained air). This was accompanied by an audible hissing noise and it seemed that air entrainment had been increased. But the results showed that less air had, in fact, been entrained probably due to the fact that the velocity was not sufficient to carry the air bubbles below the surface. If some arrangemen for carrying this white mass of water and air below the surface could be devised some improvement might be obtained but the existing arrangements allowed the air to escape.

A further series of tests was run with the mouth of the venturi tubes just below the surface. Attempts were made to keep the surface at such a level that entrainment occurred at, or as near as possible to, the venturi throat. Small fluctuations in surface level, which were impossible to control, affected the surface contour and flow into the venturi tube. It appeared as if there might be a critical level at which the flow through the venturi throat induced additional air but such a critical condition could not be maintained and is not reflected in the results.

The results of these tests with three nozzles (.0499", .0870" & .1497" dia.) are shown in Figs. 59 & 60&61. The tests were not pursued further because it was obvious that the quantity of air entrained by the jets was not increased by any of the arrangements tried.

(b) Square and Triangular Orifices.

Since the entrainment of air by a jet striking a free surface occurs at the circumference of the jet, it was thought that it might be worth experimenting with jets of various cross sectional shapes. Two special sharp-edged orifices were therefore made in the College workshops in $\frac{1}{8}$" thick

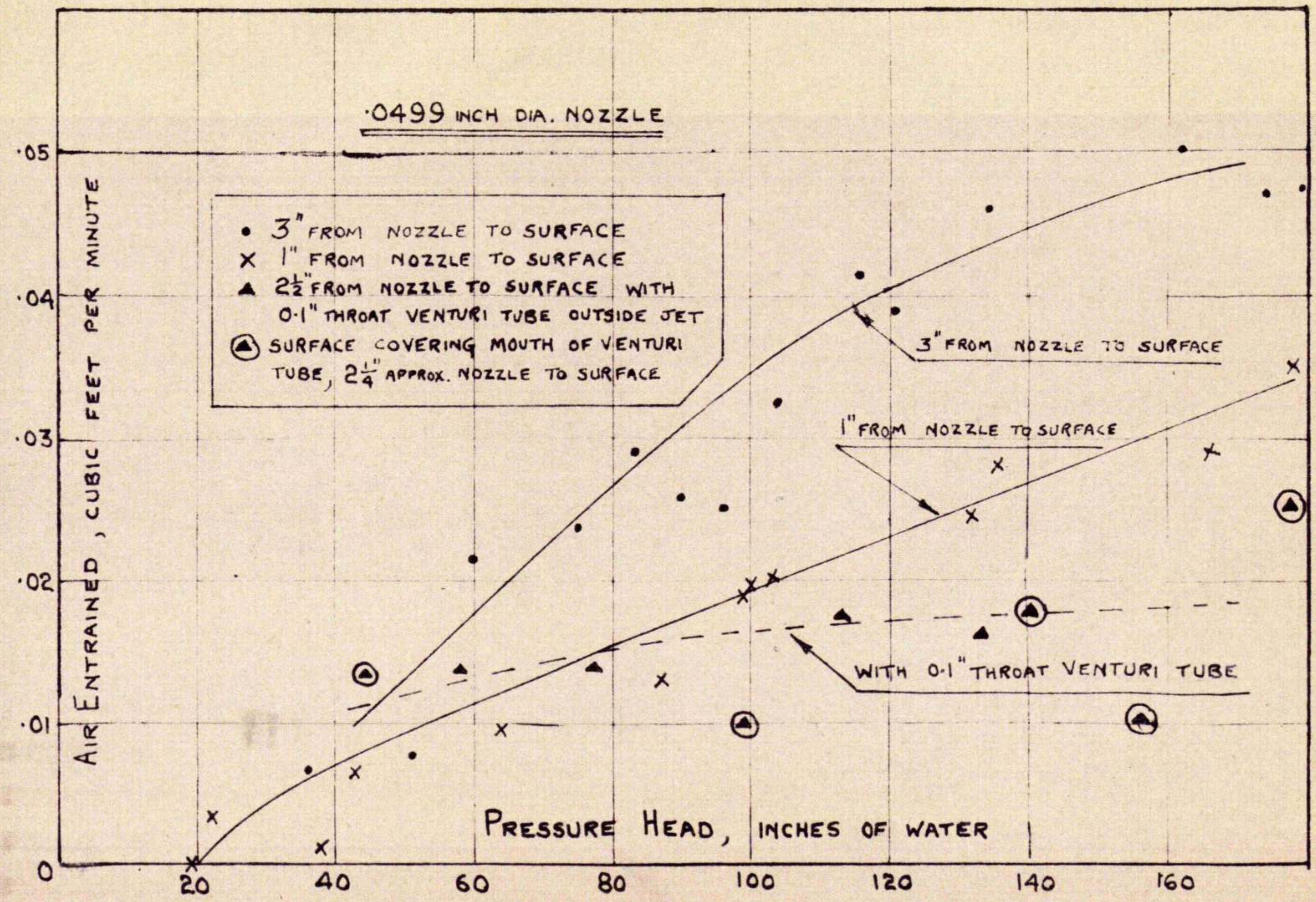


FIG. 59 EXPERIMENTS WITH ·0499" DIA. NOZZLE, SHOWING EFFECT OF VENTURI

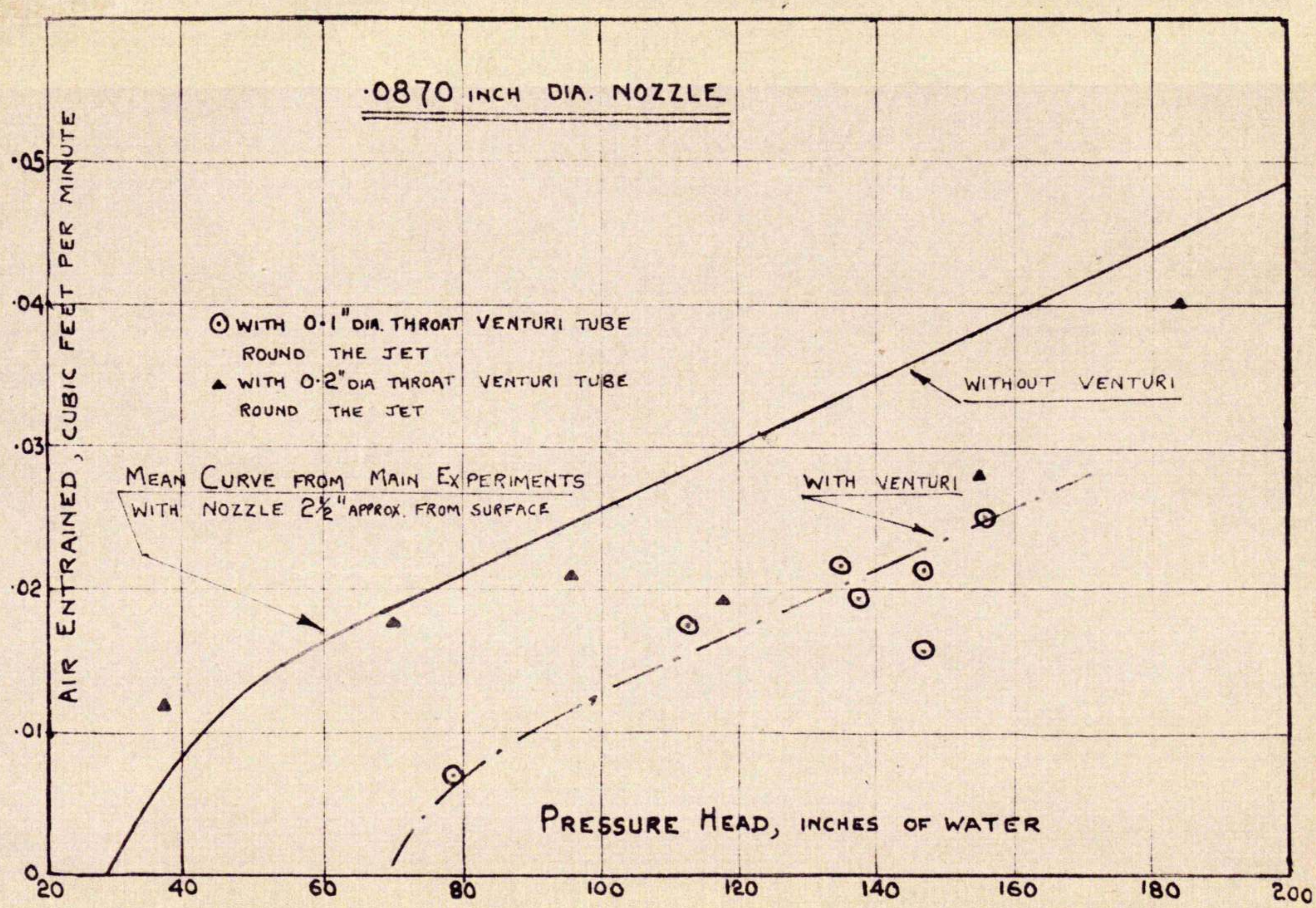


FIG. 60 EXPERIMENTS WITH ·0870" DIA. NOZZLE, SHOWING EFFECT OF FITTING VENTURI TUBES.

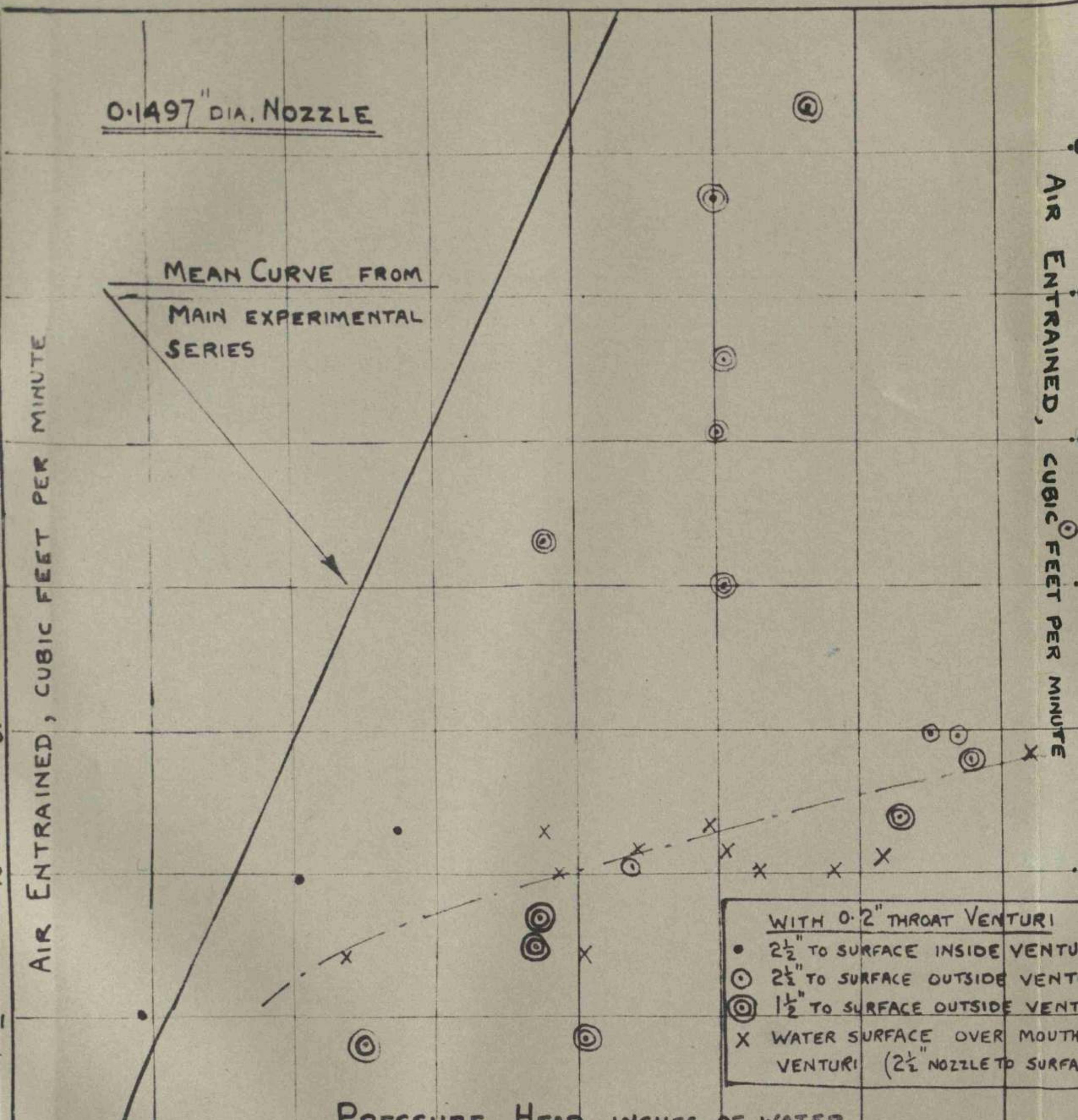


FIG. 61. EXPERIMENTS WITH 0·1497" DIA. NOZZLE SHOWING EFFECT OF US
VENTURI TUBE SURROUNDING THE JET

brass sheet, designed to fit the orifice holder of the apparatus. The first was square in shape with sides 0.1 inch long and the second of triangular shape with each side 0.1 inch long. Calibration and air entrainment tests were made with these nozzles exactly as in the main experiments.

From the calibration tests the coefficient of discharge of these orifices was determined and then assuming a value of 0.97 for the coefficient of velocity, the value of the coefficient of contraction which was obtained, allowed the jet area to be estimated. This was used to give the approximate equivalent nozzle diameter with which to compare the orifice. Thus, for the square orifice, the area is .01 in.^{2} and the approximate area of the jet was found to be .007 in.^{2} The equivalent jet dia. is $\sqrt{\frac{.007 \times 4}{\pi}}$ or .0945 in. and the results from this square orifice should therefore be compared with a nozzle intermediate between .0870 inch and .1028 inch dia. Similarly the area of the triangular orifice is .00433 in.^{2}, the approximate area of the jet is .00394 in.^{2}, and the equivalent jet diameter is .0709 inches, i.e. between .0601 inch and .0870 inch diameter.

The experimental results are shown in Figs. 62, and 63. Both these special jets apparently require higher heads to start entrainment than the equivalent circular jet. The curves of $\frac{Qa}{Qw}$ show that the square orifice is inferior in

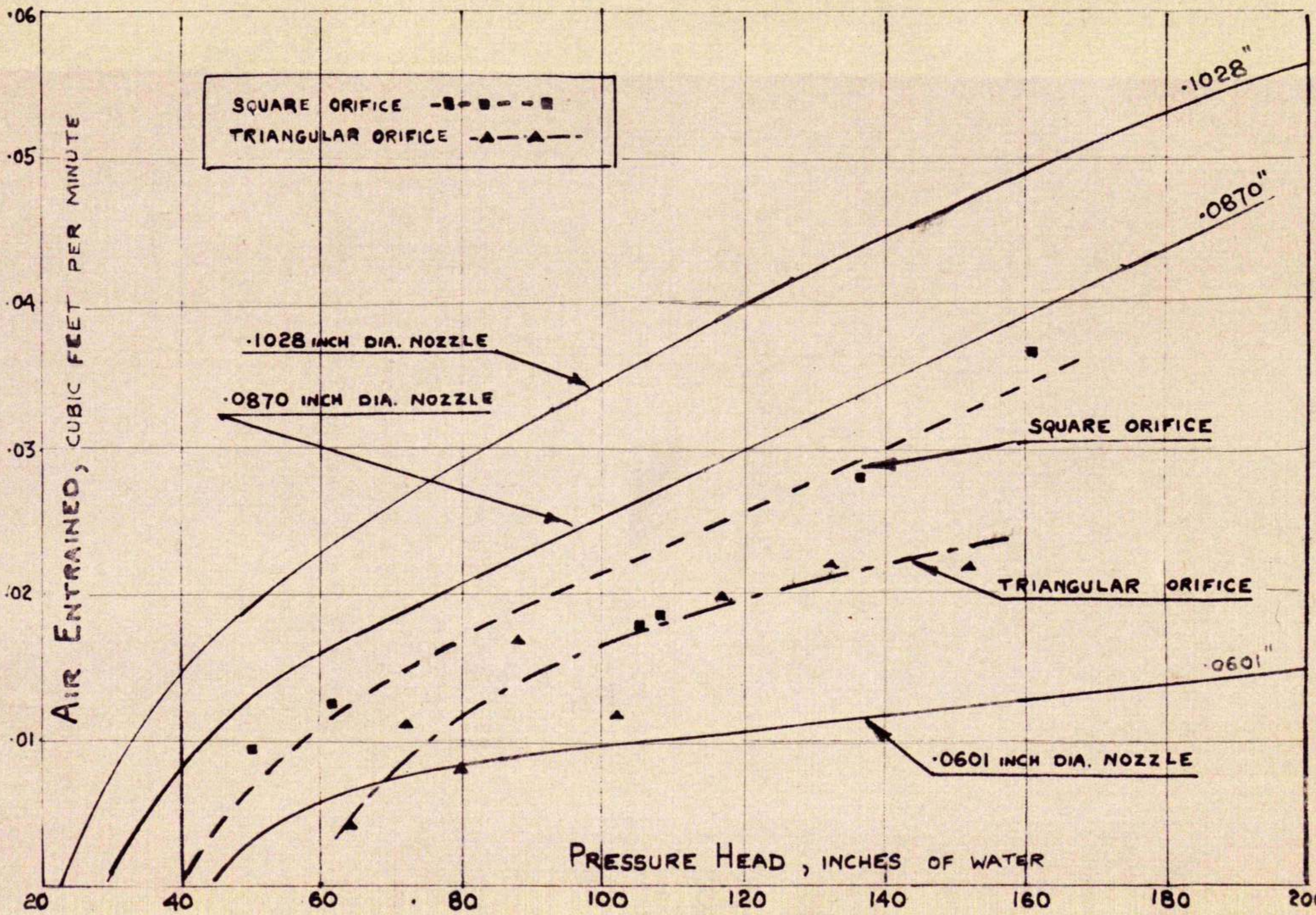


FIG. 62 EXPERIMENTAL RESULTS FROM SQUARE AND TRIANGULAR ORIFICES (NOZZLE RESULTS FOR COMPARISON)

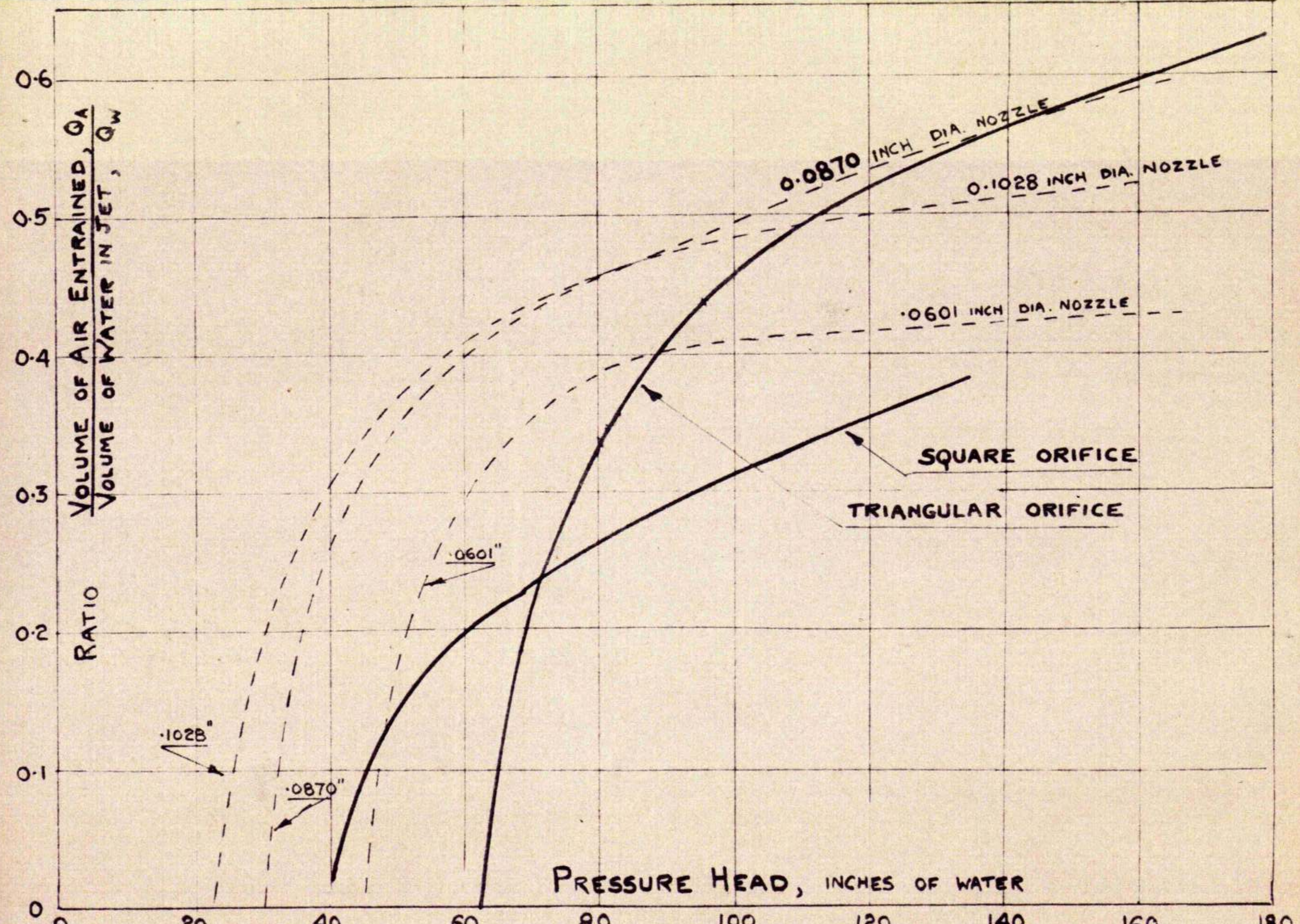


Fig. 63 Curves of $\frac{Q_A}{Q_W}$ for square and triangular orifices
(Nozzle results for comparison)

performance to the nearest equivalent nozzle but that the triangular orifice gave results which were roughly comparable with the equivalent nozzle and the trend of the curve suggests that at higher heads than those used this orifice may give higher values of $\frac{Qa}{Qw}$ than the equivalent circular jet.

An attempt to experiment with another type of nozzle was unsuccessful. The multi-jet nozzle shown in Fig. 28 gave unsatisfactory performances because the nozzle holes were not accurately parallel and the jets interfered with each other before striking the surface.

(c) Conclusions from the above Experiments.

This section has described certain attempts which were made to increase the quantity of air entrained by a jet. They were intended to indicate possibilities only and were not continued beyond this point. The conclusion to be drawn from them is, that for the purposes of air entrainment in an hydraulic compressor, the simple circular jet produced by nozzles or orifices, gives the best performance. The slightly improved performance of the triangular jet as regards the ratio $\frac{Qa}{Qw}$ is not sufficiently marked to justify the practical complications of manufacture of such nozzles or orifices or to offset the disadvantage of the higher head necessary for entrainment to start.

SECTION 6

ANALYSIS AND DISCUSSION OF RESULTS

SECTION 6

ANALYSIS AND DISCUSSION OF RESULTS

In this section the results of the nozzle experiments are analysed and discussed with the object of (a) providing a basis for a theory of air entrainment and (b) giving data which might be useful for the design of a hydraulic compressor. In order to achieve as much generalisation as possible, the results have been plotted in various ways suggested by the dimensional analysis of Section 6(b) below. A comparison is made with the results of Shirley, who, although his experimental method was different and his jets were considerably larger than the writer's, has furnished the only known similar data.

(a) Visual Observation of Phenomenon.

Before discussing the results further it may be useful to describe the phenomena as observed visually during the experiments. At first, when the jet velocity or momentum is small, the jet appears to behave as if it were striking a rigid, or, at any rate, impenetrable surface. The jet does not penetrate but spreads uniformly over the surface, with perhaps a slight mixing action at the interface. As the jet velocity is increased the critical conditions at start of entrainment are reached. The jet penetrates the surface and carries some air below the surface with it. The air appears to be carried below the surface in a discontinuous manner - small puffs of air bubbles are seen - and there may be an intermittent "make and break" effect at the surface as surface tension tends to decrease the surface area and the jet tends to increase it. As these critical conditions are exceeded, air is entrained more or less continuously at the surface. A divergent cone of small bubbles is observed surrounding the jet after it strikes the surface. It is not possible to determine visually whether this entrainment is actually continuous or whether it proceeds in an intermittent manner, but, whatever the mechanism, it seems obvious that the air is carried below the surface by viscous forces.

(b) Dimensional Analysis.

A dimensional analysis of the problem was made. To be completely general, the behaviour of two fluids with different properties should be considered, but the number of variables involved becomes unwieldy and the analysis was restricted to the case of a liquid entraining air at constant atmospheric pressure and temperature (i.e. a fluid of known, or constant, properties). By this simplification only one set of fluid properties is involved in the analysis instead of two. Assuming, therefore, that the quantity of air entrained, Q, is a function of jet diameter d, velocity V, viscosity μ, surface tension S, fluid density ρ, length from nozzle to surface l, acceleration g, the relationship may be written

$$Q = \phi \left\{ d^a V^b \mu^c s^d \rho^e l^f g^h \right\}$$

Equating indices and eliminating (say) a, b and e leads to

$$Q = d^2 V \phi \left\{ \left(\frac{\mu}{\rho d V}\right)^c, \left(\frac{s}{\rho d V^2}\right)^d, \left(\frac{gd}{V^2}\right)^h, \left(\frac{l}{d}\right)^f \right\}$$

which may be re-written

$$Q = d^2 V \phi \left\{ \left(\frac{\rho d V}{\mu}\right), \left(\frac{V^2}{s/\rho d}\right), \left(\frac{V^2}{gd}\right), \left(\frac{l}{d}\right) \right\}$$

$$\text{or} \quad Q = d^2 V \phi \left\{ \frac{l}{d}, R_l \text{ or } R_d, W, F \right\}$$

The same result can be obtained by applying the Buckingham Pi theorem.

In the particular case of jets of water entraining air at constant temperature and pressure, μ, s and ρ may be considered constants rather than variables (Section 3(e)), and since the jet velocity is proportional to $\sqrt{H}$ (where H is the head at the nozzle) the analysis may be simplified to

$$Q = d^2 \sqrt{H} \phi \left\{ \left(\frac{l}{d}\right), \left(d\sqrt{H}\right) \text{ or } \left(l\sqrt{H}\right), \left(\frac{H}{d}\right), \left(Hd\right) \right\}$$

This analysis is the basis of the plotting of the results in Figs. 64 to 68 inclusive. These curves do not show the generalisation hoped for, although the graphs could be considered to indicate scatter about mean curves. Fig. 67 suggests that better agreement might be obtained at much higher values of Reynolds Number. The best single curve is shown in Fig. 68 in which values of $Q\sqrt{H}$ are plotted on a base of $d\sqrt{H}$. These curves are discussed further in section (d).

Since the fluid properties are considered to be constant it might be thought that an analysis based on the assumption that Q is a function of V and d only would be valid, but such an analysis which leads to

$$Q = \phi(Vd^2) \quad \text{or} \quad \phi(d^2\sqrt{H})$$

omits the groupings of the more general analysis given above.

(c) Start of Entrainment.

The following rough analysis of the conditions at start of entrainment suggests that the Weber Number should be approximately constant.

Equating the jet momentum force to the surface tension force as indicated in Fig. 69, we get

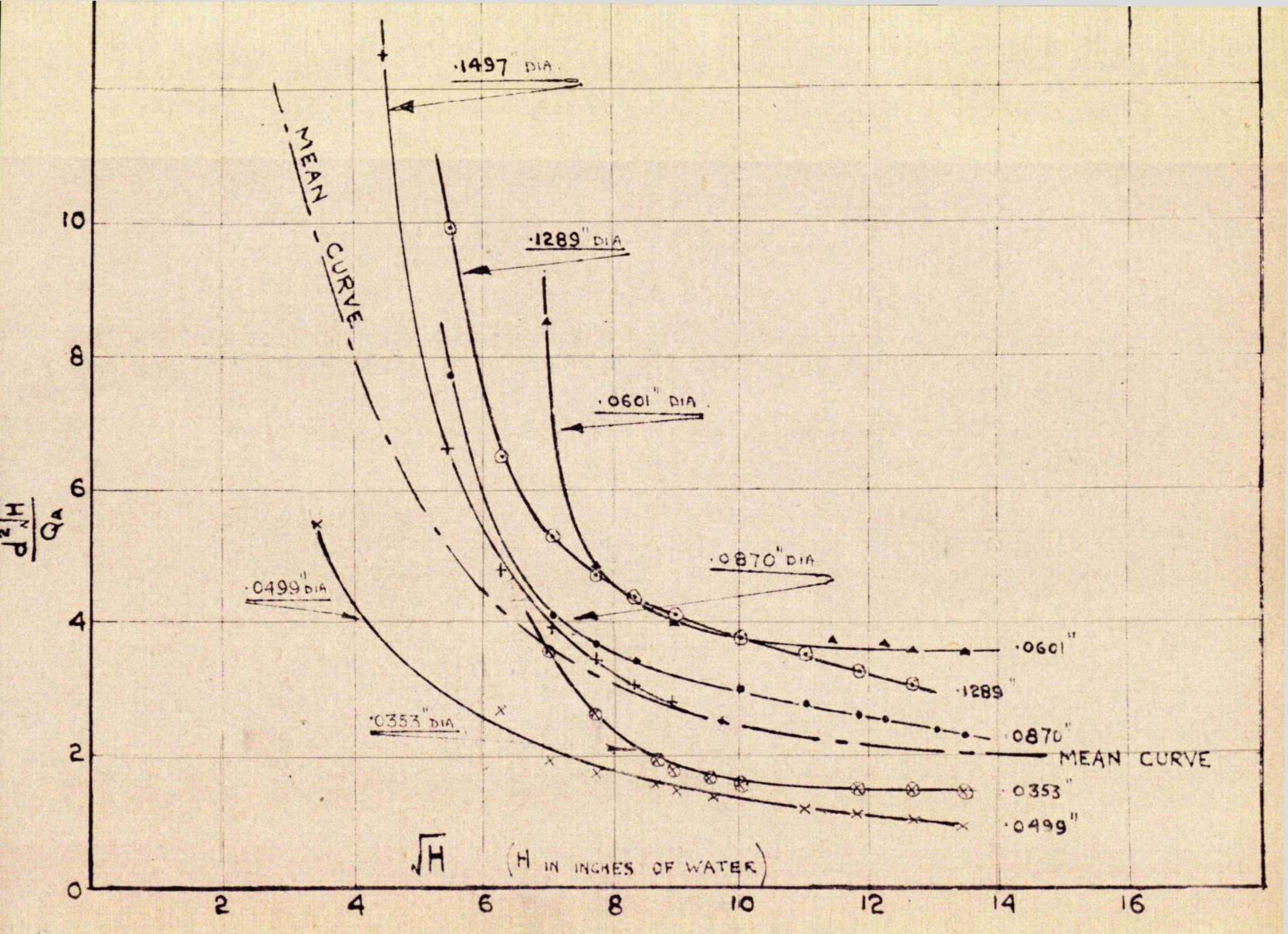


FIG. 64. $\frac{d^2\sqrt{H}}{Q}$ CURVES ON BASE OF $\sqrt{H}$

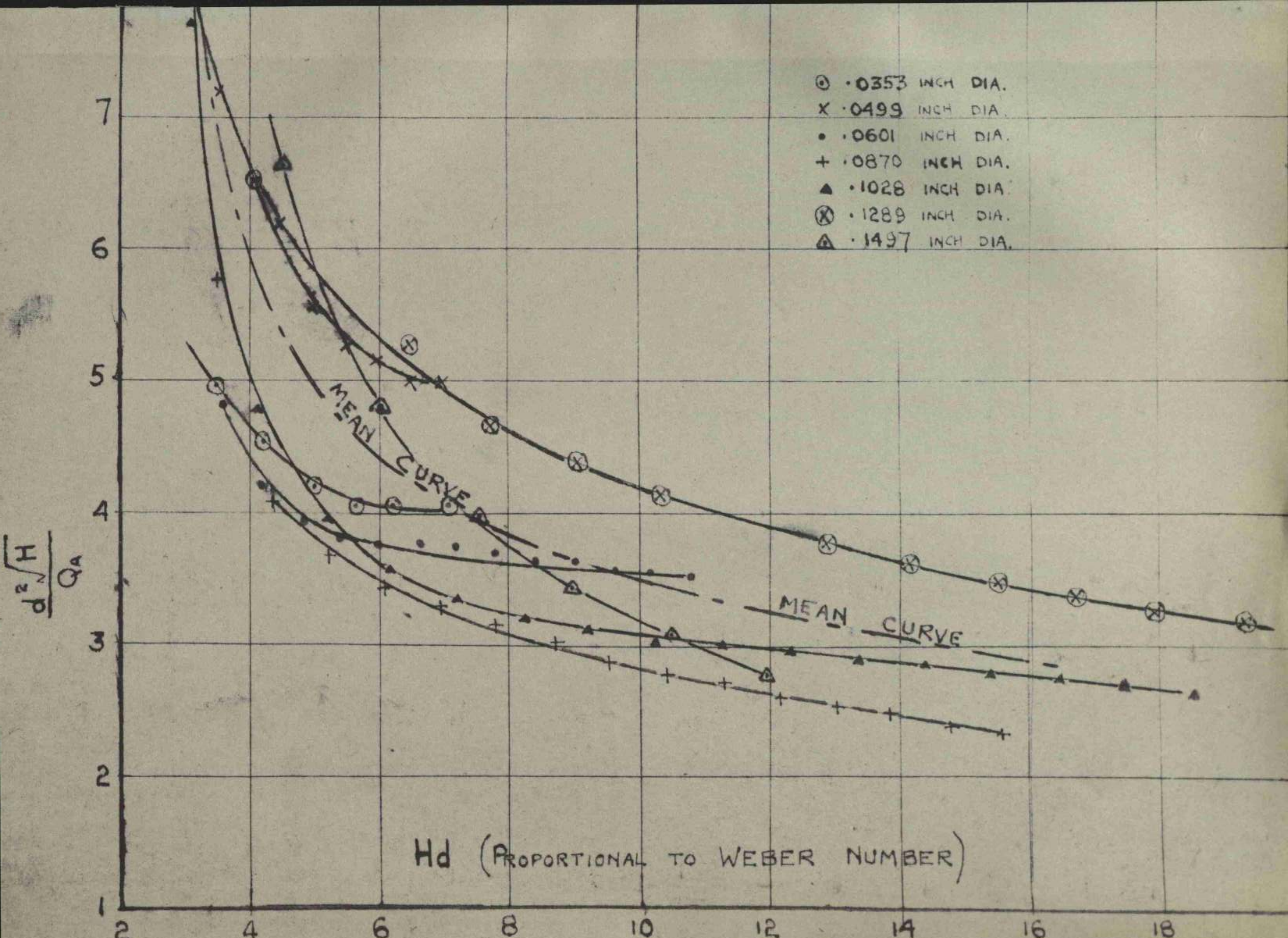


FIG 6.6 $\frac{d^2\sqrt{H}}{Q_A}$ CURVES ON BASE OF Hd ($\propto$ WEBER NUMBER)

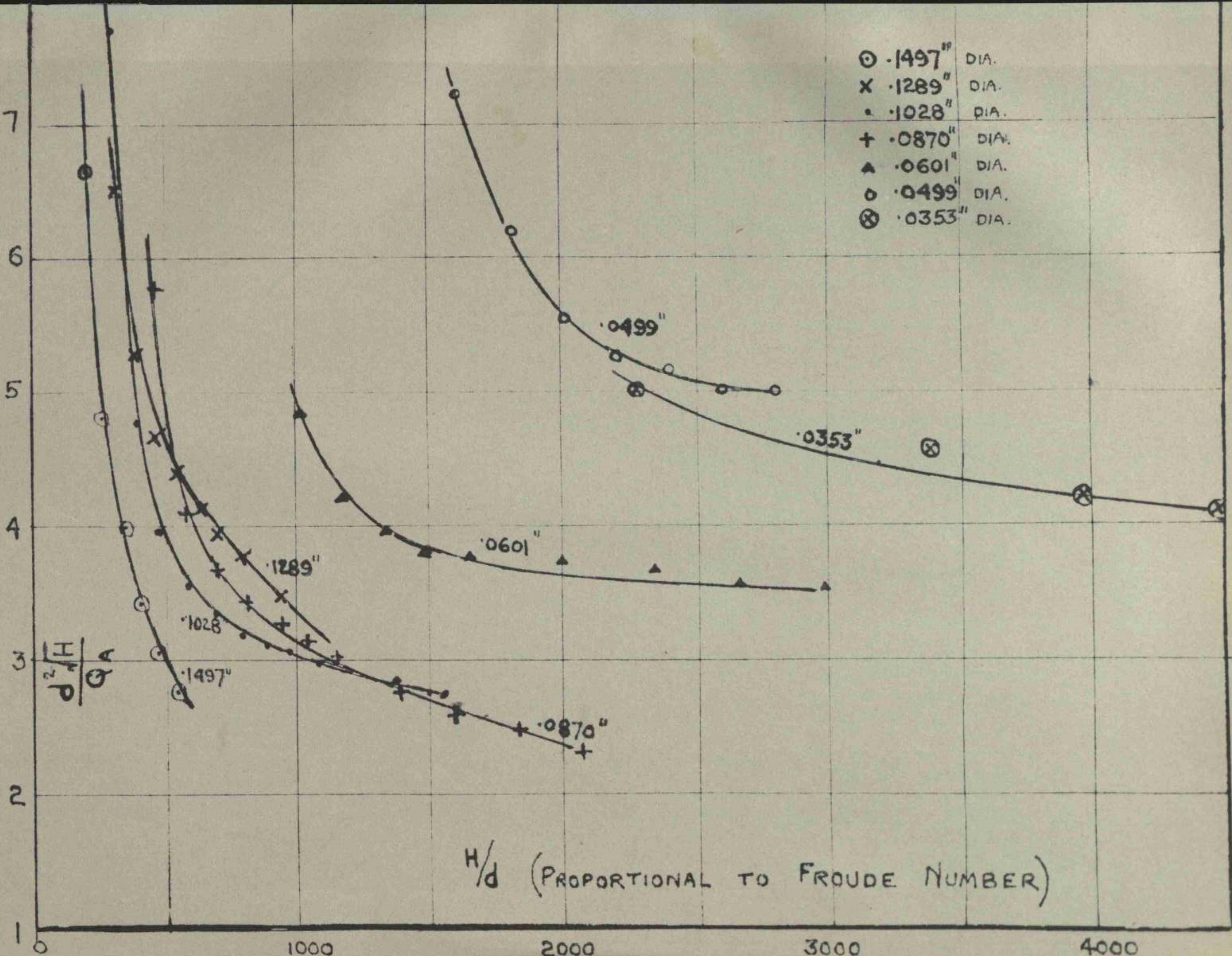


FIG. 66 $\frac{d^2\sqrt{H}}{Q}$ CURVES ON BASE OF $\frac{H}{d}$ ($\propto$ FROUDE NUMBER)

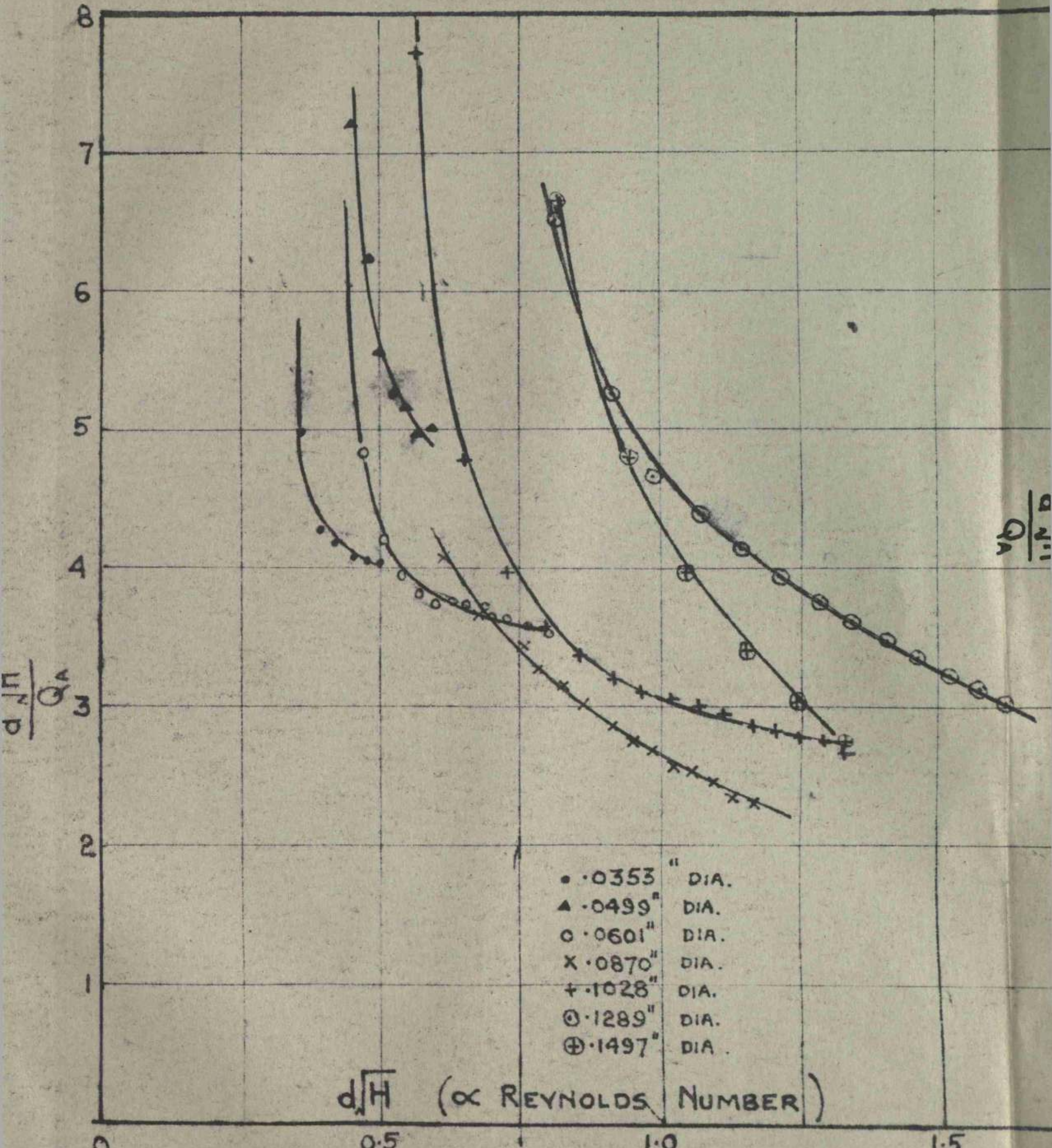


FIG. 67. CURVES OF $\frac{d^2\sqrt{H}}{Q_A}$ ON BASE OF $d\sqrt{H}$

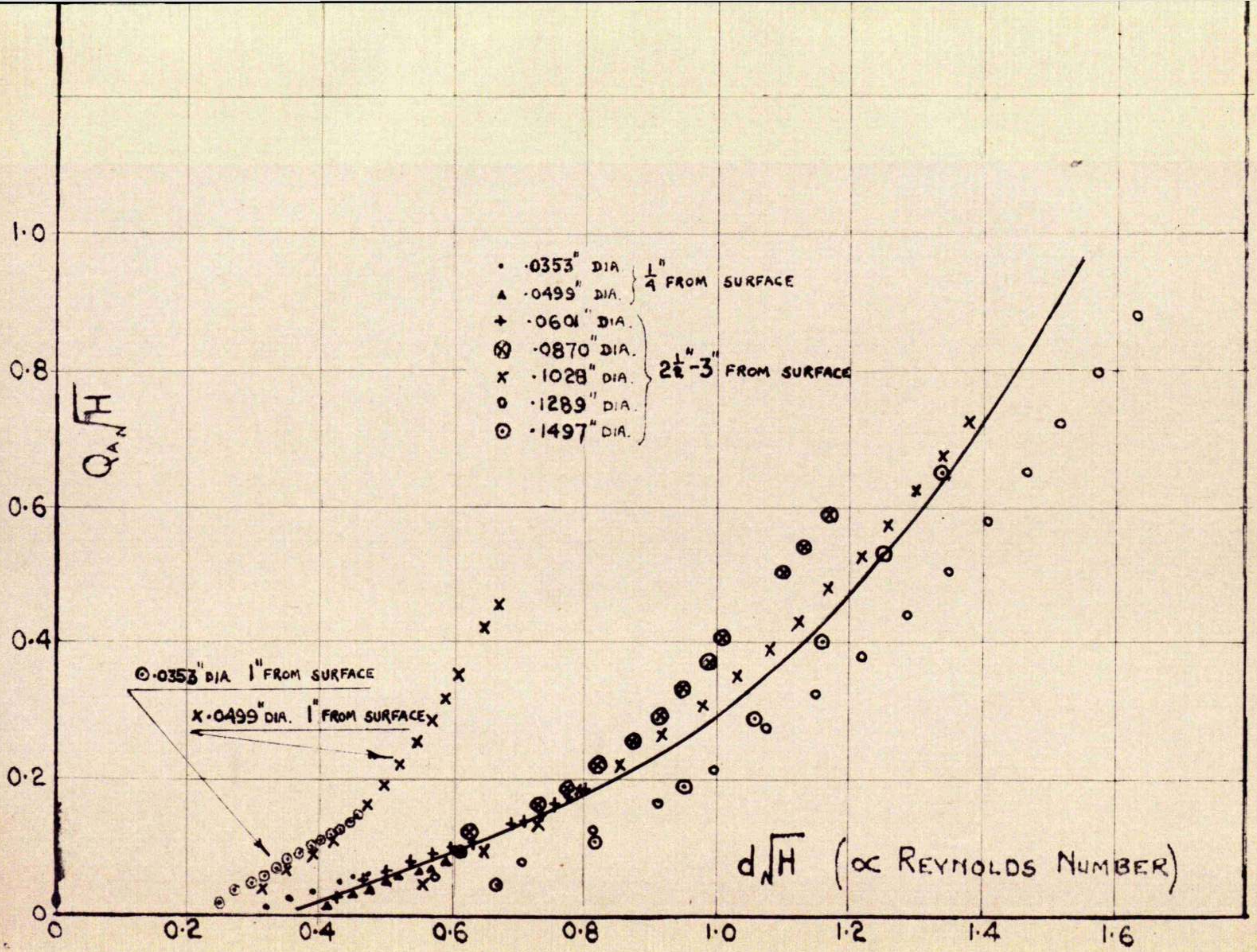


FIG. 68. $Q_A\sqrt{H}$ ON BASE $d\sqrt{H}$ (∝ REYNOLDS NUMBER)

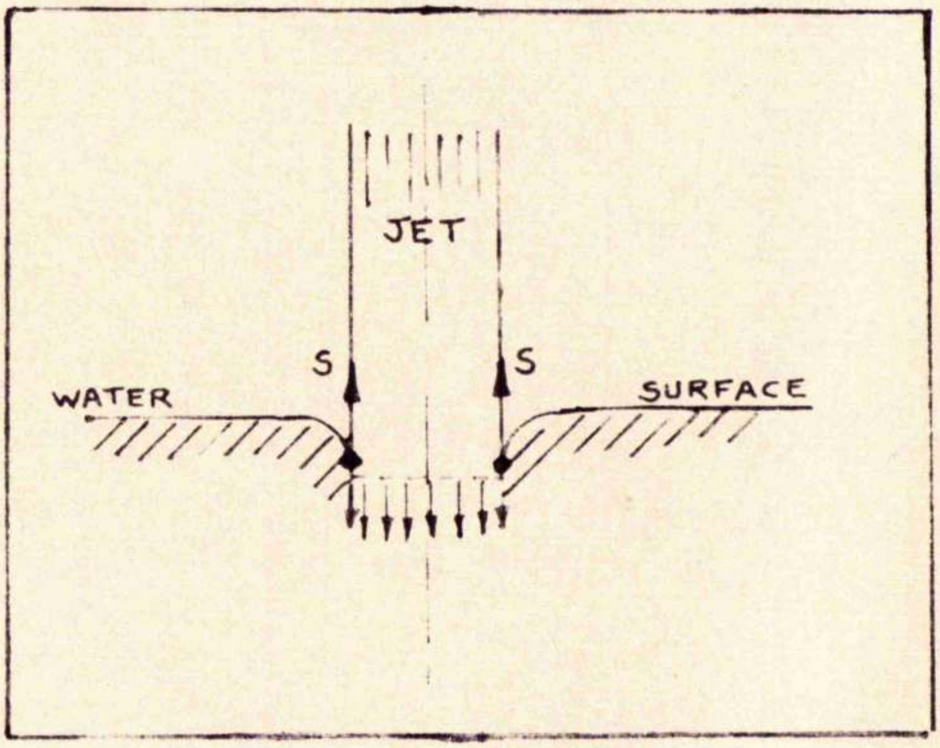


Fig. 69

for the critical condition.

Mass flow x change of velocity = surface tension x circumference

i.e. $\rho \times \frac{\pi}{4} d^2 V \times kV = S \times \pi d$, where kV is the deceleration of the jet.

$$\therefore \quad \rho k dV^2 = 4S$$

or $$\frac{\rho dV^2}{s} = \text{constant, if } k \text{ and } s \text{ are constant}$$

i.e. $\frac{V^2}{s/\rho d}$ = Weber Number = constant, if k & s are constant.

When the jet is on the point of entraining air, therefore, it seems likely that the Weber Number could be expected to have a constant value. Since deceleration effects are also involved, the Froude Number which expresses the ratio of inertia to acceleration (or gravity) forces may also be significant.

Fig. 70 shows the Froude Number at the start of entrainment plotted as a function of jet diameter and includes both the results of the present research and those of Shirley. Considering the differences in technique, the various experimental difficulties encountered, the differences in both jet sizes and water temperatures (about 23° F.), the fact that the experimental points all lie fairly closely on one curve is gratifying. This curve shows that the critical Froude Number decreases with increase in jet diameter. The decrease is rapid as diameter increases up to about 0.01 ft. (say ⅛

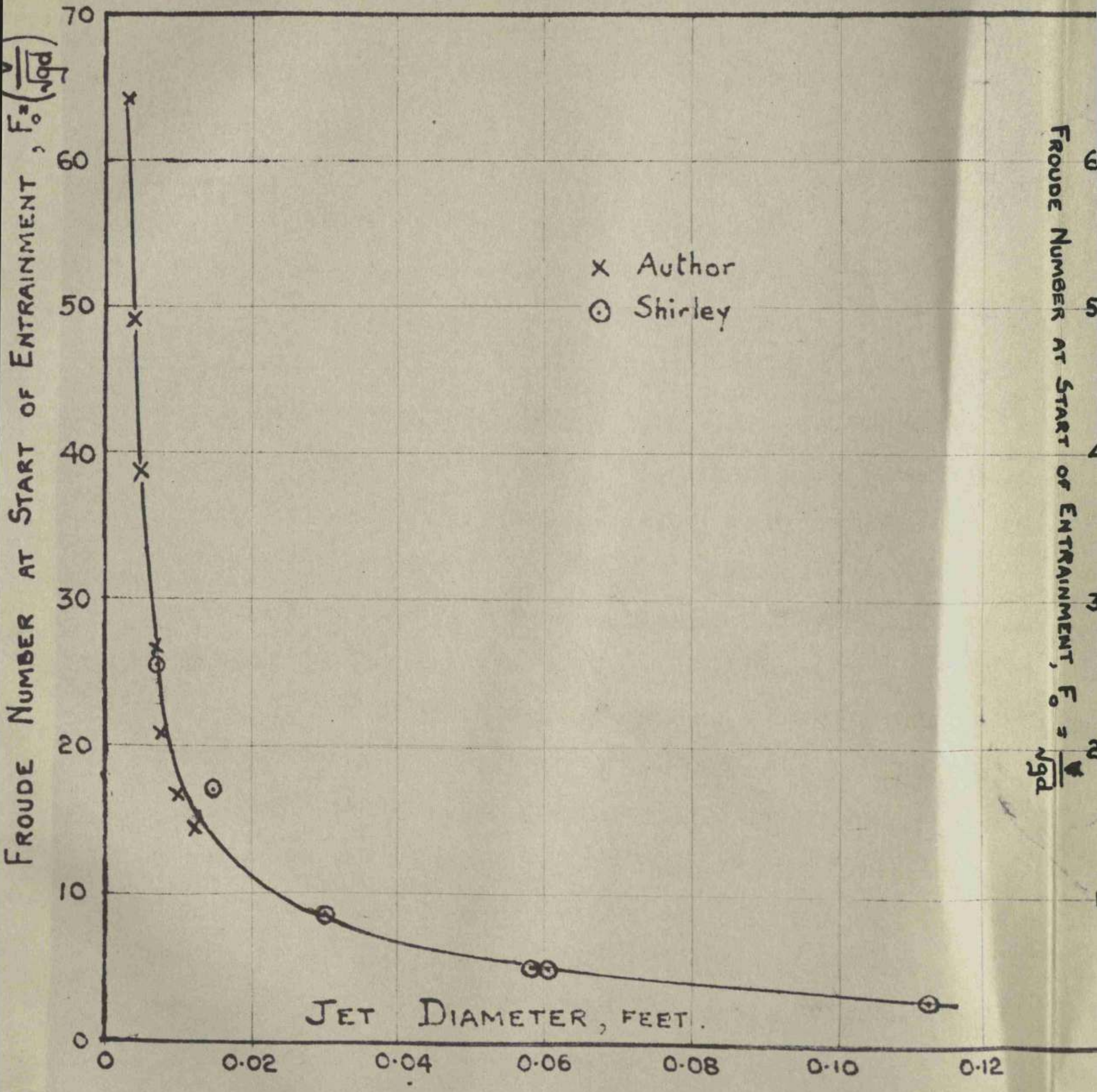


Fig. 70 Critical Froude Number (Shirley and Baxter)

inch) diameter but thereafter becomes much less pronounced, the Froude Number becoming nearly constant for jets greater than about 0.12 ft. (say 1½ inches) diameter. From this curve it may be deduced that the Froude Number alone does not give a criterion for the start of entrainment and therefore, if this phenomenon is of interest, it would be misleading to use the Froude Number to give "corresponding conditions" for model and full size prototype without further consideration. Since the Froude Number at the critical point increases as the jet size decreases, a small jet may not entrain air whereas a larger jet operating at the same Froude Number would entrain air. The writer feels that some of the difficulty experienced with model tests of air entrainment on high speed flumes and spillways in the past may be due to the use of Froude Number as the similarity criterion.

The Weber Number at start of entrainment is plotted on a base of jet diameter in Fig. 71. Here Shirley's results and those of the writer do not appear to lie on a common curve but the fact that one of Shirley's readings which overlaps the range covered by the writer agrees with these results suggests that either there is an abrupt change or discontinuity in the value of the critical Weber Number with nozzle diameters of approximately 0.01 ft. (0.12 inch) or there is considerable experimental "scatter" in the results. There seems no reason

FIG 71. W_o ON BASE OF JET DIAMETER

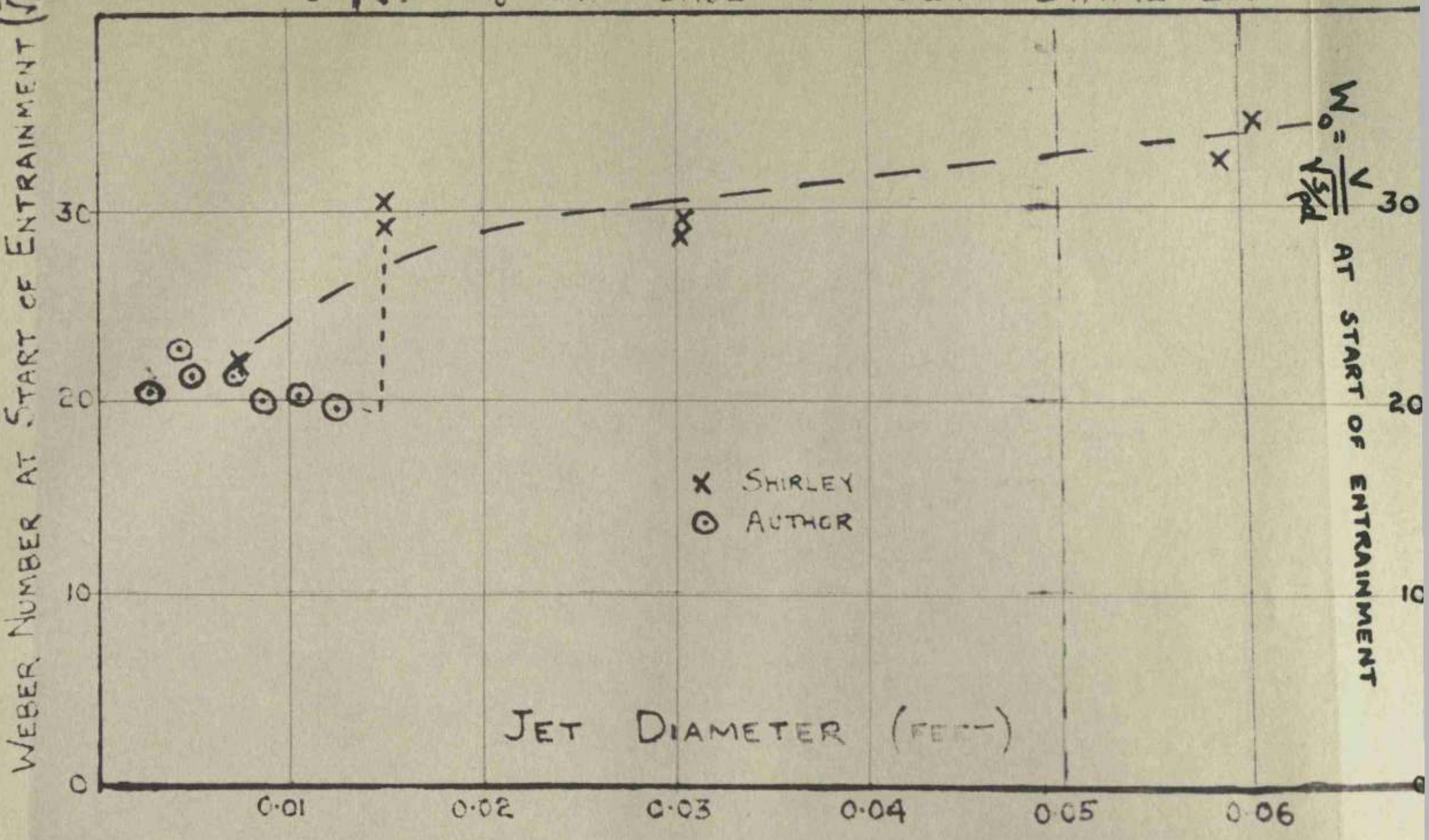
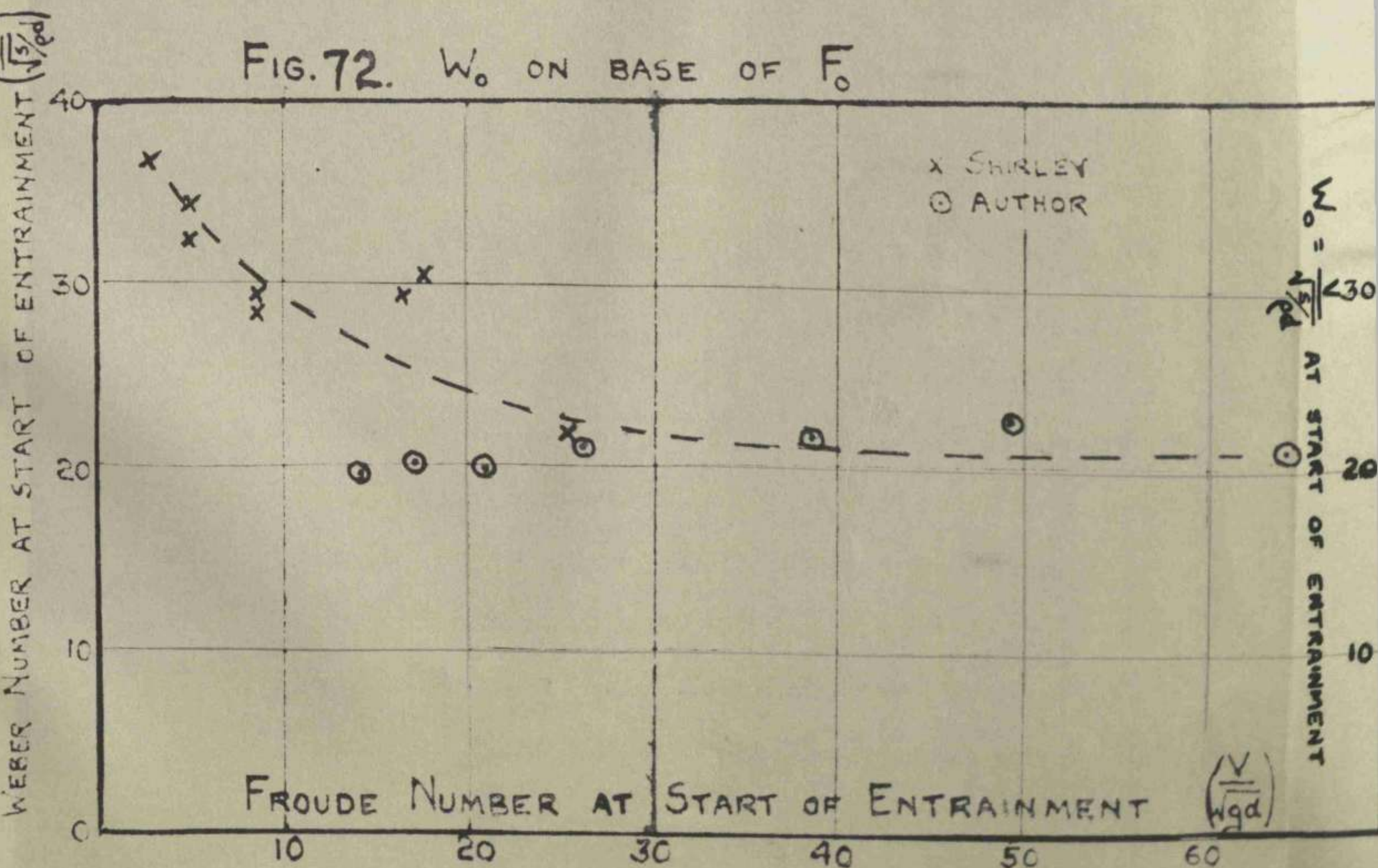


FIG. 72. W_o ON BASE OF F_o



to expect any such sudden change and the different values of the critical Weber Number (Shirley's results indicate a figure of about 30 for $\frac{V}{\sqrt{S/\rho d}}$ while the writer's suggest 21) are therefore probably due to "scatter". The different jet lengths used by the two experimenters is a possible reason for the discrepancy.

Fig. 72 shows the Weber Number at start of entrainment, W_o, plotted on a base of Froude Number at start of entrainment, F_o. The results of Shirley and the writer, considered separately exhibit opposite trends - Shirley records a <u>decrease</u> in W_o with increasing F_o while the writer's results indicate a slight <u>increase</u> - but when both are considered together on one diagram, a mean curve can be drawn through them, as shown. This figure may be said to summarise information regarding the critical conditions at start of entrainment. For the smaller jets (smaller than about 0.1 ft. diameter) the Weber Number $\left(\frac{V}{\sqrt{S/\rho d}}\right)$ at start of entrainment appears to be constant at about 21 while the Froude Number $\left(\frac{V}{\sqrt{gd}}\right)$ may have any value from 25 upwards - i.e. the Weber Number is the main criterion as to whether air entrainment will start. For the larger jets, however, both the Froude Number and the Weber Number vary together, the Froude Number probably tending to become dominant as the jet size is increased.

(d) The Volume of Air Entrained and the Ratio $\frac{Qa}{Qw}$.

It is the object of this section to discuss the results with a view to establishing a theory which would enable the quantity of air likely to be entrained under given conditions to be calculated. The available experimental data is not complete enough to enable this object to be achieved but various hypotheses are considered.

The various plots of the experimental results suggested by dimensional analysis did not give any satisfactory general curve. As stated earlier, it seemed obvious that the air is carried below the surface by viscous forces and therefore it was expected that the Reynolds Number would be the significant parameter. Fig. 54 in which the quantity of air entrained is plotted on a base of Reynolds Number did, in fact, give the best approach to a single curve but the results from the various nozzles showed rather more scatter than is desirable.

The first attempt to apply theoretical reasoning to an analysis of the results was made on the basis of boundary layer theory. In the jet experiments, the air in contact with the jet will be moving at the jet velocity and this air velocity will fall to zero as the radial distance from the jet increases. The phenomenon will be complicated by surface disturbances and increasing pressure in the direction of flow but, as a first approximation, the ideas of boundary layer theory were applied to the problem. In considering this description of the phenomenon it is convenient to think of the air, carried below the surface by viscous forces, as forming an annulus round the jet. The first step was to estimate the quantity of air which would be entrained if it were assumed that the thickness of the annulus were, in fact, that of the boundary layer.

Boundary layer theory gives the thickness of the layer δ and at a distance x from the leading edge of a flat plate as $\frac{0.38\,x}{\left(\frac{Vx}{\nu}\right)^{\frac{1}{5}}}$. This flat plate theory is applied to curved surfaces as well, but the radius of curvature should be great in comparison with δ, which is not the case with the small diameter jets used in the experiments. The validity of the theory as applied to these jets is therefore very questionable, since the thickness of the boundary layer at a distance of 2½ inches from the start is of the same order as the jet diameter. The boundary layer thickness is independent of jet diameter. If this flat plate theory is applied to the jet and it is assumed that the whole of the boundary layer is travelling with the speed of the jet, the quantity of air entrained = area of annulus x speed of jet,

i.e. $Q_a = \frac{\pi}{4}\left\{(d+2\delta)^2 - d^2\right\}V$

where d = jet diameter
V = jet speed
δ = boundary layer thickness.

$$= \pi V\left(d\delta + \delta^2\right)$$

Quantities of air calculated from this expression give curves similar in form to the experimental curves but the numerical values are of the order of ten times too great (Fig. 73). The thickness of the air annulus is therefore probably much less than the complete boundary layer. The actual thickness of the annulus is probably determined by the values of shear stress and surface tension at the boundary. The contour of the surface at the point of entrainment may alter to maintain this balance.

Applying the above boundary layer theory to the calculation of the ratio $\frac{Qa}{Qw}$ we get

$$\frac{\text{Volume of air entrained, } Qa}{\text{Volume of water in jet, } Qw} = \frac{\frac{\pi}{4}\left\{(d+2\delta)^2 - d^2\right\}V}{\frac{\pi}{4}d^2V} = \frac{(d^2+4\delta d+4\delta^2)-d^2}{d^2}$$

$$= \frac{4\delta}{d} + \frac{4\delta^2}{d^2}$$

This expression also gives values much higher than those determined experimentally (again the order of discrepancy is about ten times). Simple boundary layer theory is not sufficient to account for the results.

An alternative theoretical approach is on the basis of viscous flow between parallel plates in relative motion. In this case the air annulus is the narrow slot through which viscous flow is taking place and the jet forms the moving plate. Text book (e.g. 28) theory gives:

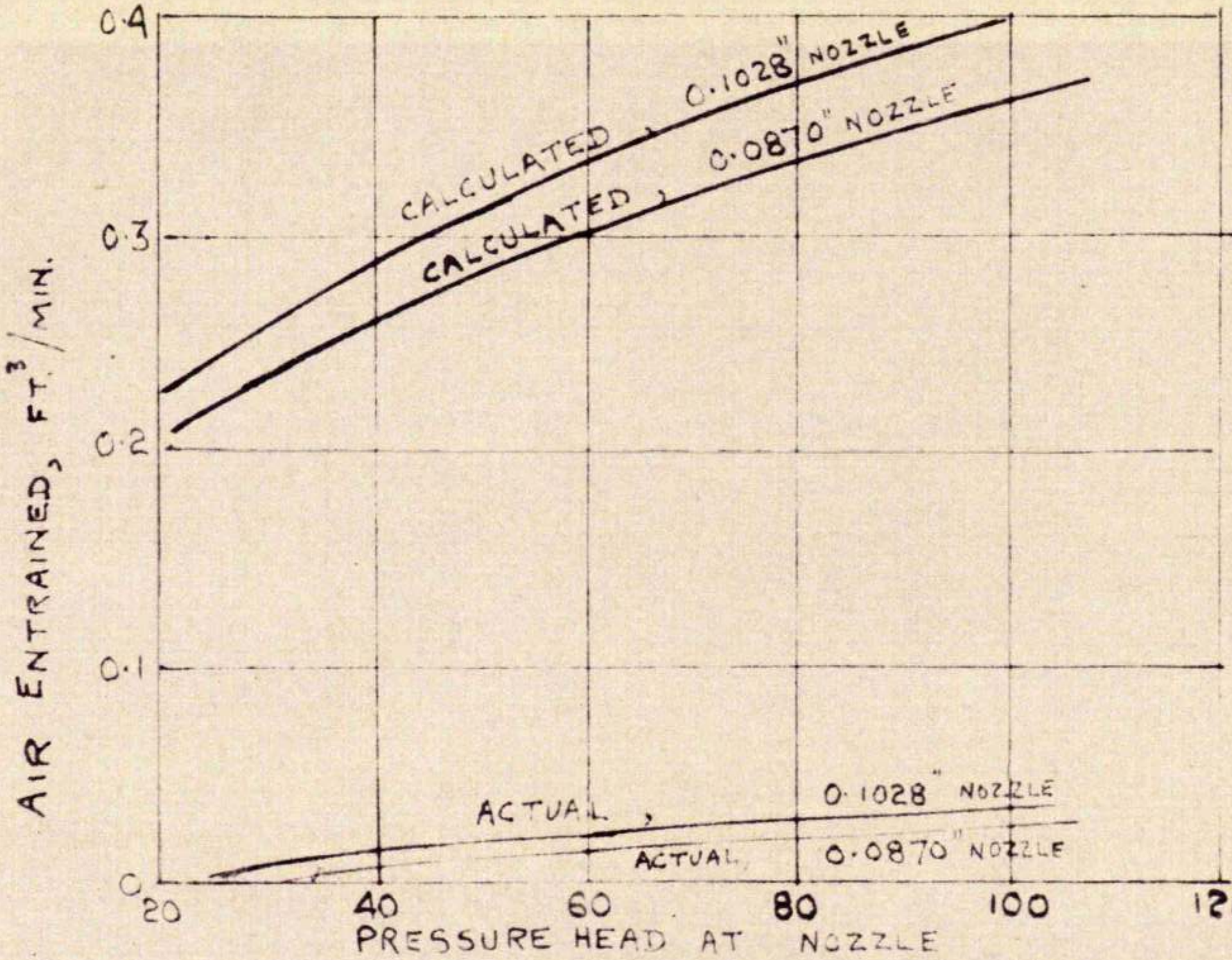


FIG. 73. CURVES OF AIR ENTRAINED CALCULATED BY BOUNDARY LAYER THEORY.

$Q = -\frac{m}{\mu}\frac{P}{l}\frac{h^3}{12} + mh\frac{u_1}{2}$ where m = width of slot (periphery
h = depth of slot (annular thickness)
l = length
P = pressure
u_1 = speed of moving plate
μ = viscosity

If, as a first approximation the pressure effect is neglected, then the flow is due to relative motion only, i.e. $Q = mh\frac{u_1}{2}$

The drag force on the stationary plate (in this case the water surrounding the air annulus) $= -P\frac{h}{2}m + \mu\frac{u_1}{h}ml$

If, as above, the pressure effect is neglected, this becomes $\mu\frac{u_1}{h}ml$.

i.e. Force $= \mu\frac{u_1}{h}ml = s \times \pi d$ for equilibrium with surface tension force.

i.e. $\mu\frac{V}{h}ml = s \times \pi d$ where V = jet velocity

$\therefore \mu\frac{V}{h}\pi d l = s\pi d$

$\therefore h = \frac{\mu V l}{s}$

If the length l is assured to be proportional to the jet diameter d (which would be the case if geometrical similarity existed) then $h \propto Vd$.

Now $Q_a = mh\frac{u_1}{2}$ (see above)

i.e. $Q_a = \pi d \times k(Vd) \times \frac{V}{2}$ i.e. proportional to d^2V^2

The quantity of water in the jet is $\frac{\pi}{4}d^2V$, therefore the ratio $\frac{Qa}{Qw} \propto V$. This approach does not allow any calculations of air quantities to be made at this stage without making further assumptions but it appears to be justified by consideration of the experimental results.

A plot of Qa on a base of d^2H ($\propto d^2V^2$) is shown in Fig. 74 and the experimental points from all nozzles lie scattered about a mean straight line. Shirley's results are shown similarly plotted in Fig. 75. Despite the spread of the results these two graphs suggest that, at least as a first approximation, the quantity of air entrained varies linearly with d^2V^2 and the mean curve obtained could be used for estimating the quantity of air likely to be entrained by a given jet. Fig. 75 therefore summarises the results of both Shirley and the writer as far as the quantity of entrained is concerned, just as Figs. 70 and 72 summarise the results regarding the conditions at start of entrainment. Some of the difference in the results of the two experimenters is probably due to the different jet lengths used by them.

None of the plots of the ratio $\frac{Qa}{Qw}$ showed a single general curve although Fig. 55 ($\frac{Qa}{Qw}$ on a base of Hd) could be considered to indicate a spread about a mean curve. Fig. 76 which shows $\frac{Qa}{Qw}$ on a base of $\sqrt{H}$ (or jet velocity) does not exhibit the agreement which the above analysis would lead one to expect, although considerable scatter about a mean line might still be the explanation. The curves suggest, however, that the approximate theoretical analysis is

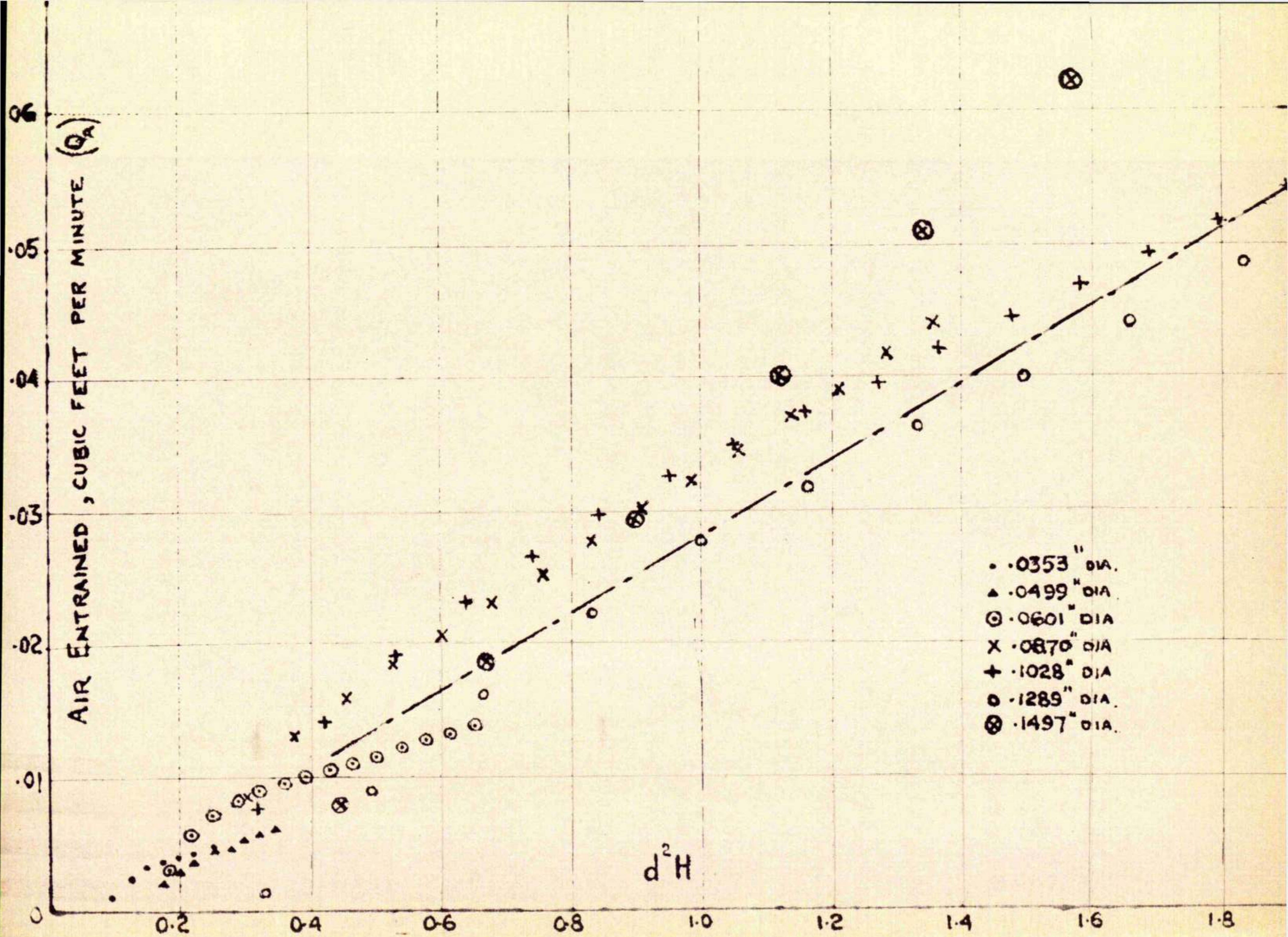


FIG 74 Q_A PLOTTED ON BASE d^2H (ALL NOZZLES)

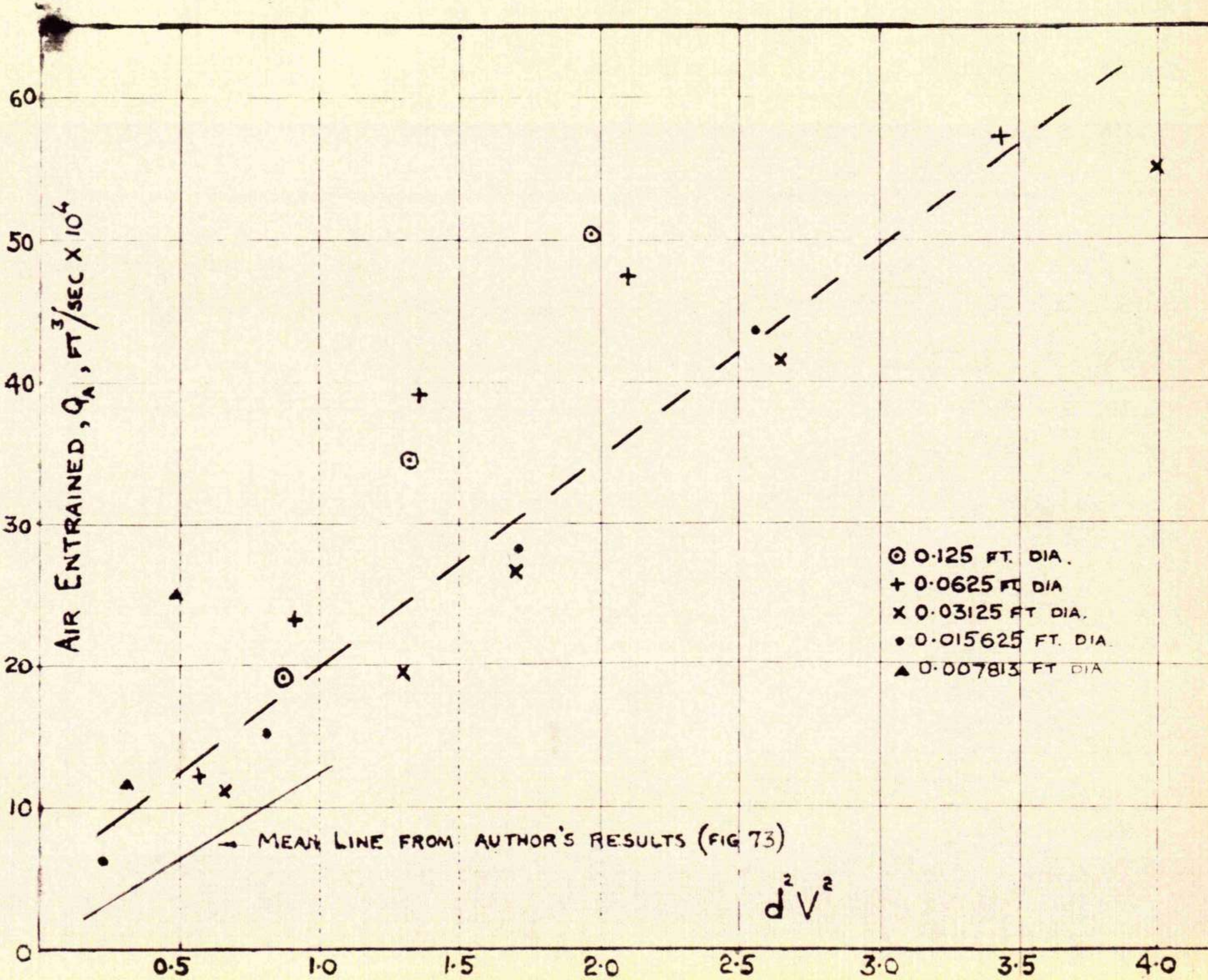


FIG 75 ...'S EXPERIMENTS Q PLOTTED ON BASE d^2V^2

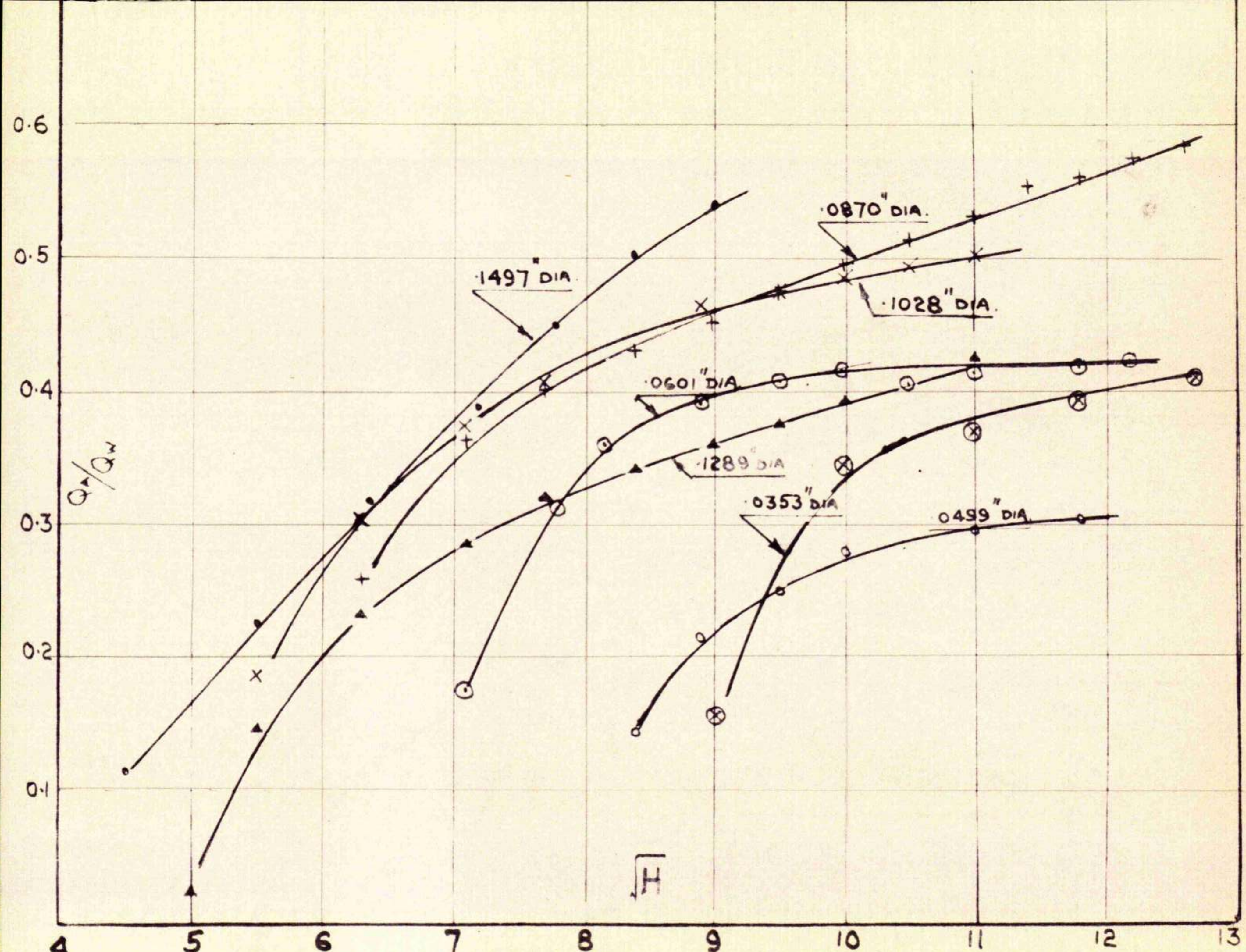


FIG. 76 RATIO $\frac{Q_A}{Q_W}$ PLOTTED ON BASE OF $\sqrt{H}$

insufficient and will require modification (e.g. by including the neglected pressure terms) before a satisfactory explanation of the results can be put forward.

From the practical point of view it seems that attention should be concentrated in the first place on the quantity of air entrained under given conditions rather than on the ratio $\frac{Qa}{Qw}$ since this ratio, for some reason which is not clear, appears to be less amenable to generalisation. The quantity of air likely to be entrained could be estimated from Figs. 68 or 75. The more general statement that the quantity of air entrained is a function of Reynolds Number may not be a justifiable conclusion from the limited range of experimental data, but it seems probable that this is the case.

SECTION 7

C O N C L U S I O N S

SECTION 7

C O N C L U S I O N S

Consideration of the experimental results leads to the following conclusions regarding the technique and the application of results to (a) model tests involving air entrainment by jets and streams, (b) design of a hydraulic compressor.

It seems possible that complications were introduced by using such small jets in which surface tension forces were not negligible compared with gravity and viscous forces. An extension to larger jets is desirable but this would increase considerably the experimental difficulties and a different technique would probably require to be evolved. The difficulty of getting "similar" jets of different diameter requires further investigation and it seems probable that the surface finish of the nozzles may be important.

The difficulty of predicting performance from models in which air is entrained by jets or streams is obvious. Where small models are concerned and the onset of entrainment is of importance, the same Froude Number alone will not give conditions corresponding to those of the full scale. Conditions at the start of entrainment appear to depend on both the Weber Number and Froude Number while the quantity of air appears to be mainly dependent on Reynolds Number. One set of conditions cannot be used to simulate both the start of entrainment and the quantity of air entrained. The model would require to be operated at two different "corresponding conditions" to investigate these effects separately. This conclusion is of great importance in all cases such as the Glen Shira Project model experiments described at the beginning of this thesis.

If the primary concern of such model tests is the start of entrainment Figs. 70, 72 summarise the relevant experimental results. The agreement shown in Fig. 70 between the results obtained by Shirley and those of the writer is significant. For small jets a critical value of Weber Number ($\frac{V}{\sqrt{S/\rho d}}$ = 21 approximately) must be exceeded before entrainment will start - the Froude Number is not the relevant criterion - but as the jet or model size increases both the Froude Number and the Weber Number are concerned. For still larger jets it seems probable that the effect of the Weber Number will become insignificant and the Froude Number will be the criterion of similarity. There is a danger that if model and prototype are operated at the same Froude Number, the small model may show no air entrainment while the full scale prototype will.

Concerning the design of a hydraulic air compressor the experiments have provided data which could be useful to such a project. Curves of Figs. 54, 68, 74 or 75 would allow estimates to be made of quantities of air likely to be entrained under given conditions. The experiments also indicate that simple circular jets give the best results - complications introduced by the design of various shaped orifices or by addition of venturi tubes are not justified. Some advantage in using long jets near the break-up stage seems to be indicated provided the air can be carried successfully into the separator.

Further experimental data are required before a satisfactory quantitative theory can be put forward, but, as already stated, the experimental difficulties are so great that it is doubtful if the same method could be used in an extension to larger jets. It seems probable that any further data must await the development of a new experimental technique.

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"AIR ENTRAINMENT BY FLUID JETS AND STREAMS"

by A. M. BAXTER, B.Sc., M.Sc.

SUMMARY.

Model experiments on the preliminary design of a sectio of pipeline for the Glen Shira hydro-electric scheme showed that under certain conditions air was entrained by a stream of water and carried into the main pipeline. Although the design was altered to overcome this, these experiments emphasised the need for further knowledge of the behaviour of such systems. Information was lacking regarding the proper scaling laws to use, the possibility of predicting behaviour of a prototype from model tests in such cases and the mechanism whereby air is entrained at a free surface. An investi tion into the more fundamental aspects of the problem was therefore made and forms the subject of this thesis. The experiments were also designed to supply data which would be useful in the design of hydraulic compressors, devices which depend for their functioning on air entrainment.

A technique had to be developed whereby air was entrain by a jet of water striking a free surface and subsequently collected and measured, and the experimental difficulties encountered are described. In the method finally evolved,

jets of water from nozzles of various sizes (.0353 inch to .1497 inch diameter) were directed vertically into water in a receiver into which was also led a secondary flow of sufficient magnitude to carry the entrained air into a separato After separation from the water, the air was measured by a gas flow-meter. A series of experiments was also made to investigate the possibility of increasing the quantity of ai entrained, this being of importance for the design of hydrau compressors.

The results of the experiments are analysed and discuss A comparison is made with the results of Shirley who, in a thesis to the State University of Iowa, has given the only known comparable data. Allowing for the experimental difficulties, the difference in method and range of jet sizes used, reasonably good agreement is recorded.

The results show that the Froude Number alone is not th correct similarity criterion if the start of air entrainment is of importance. For the smaller jets, the entrainment starts at a critical value of Weber Number. The quantity o air entrained depends most closely on Reynolds Number. Whe plotted on a base of d^2V^2 (where d = jet diameter and V = je velocity) both the results of Shirley and those of the write lie scattered about a mean straight line. This curve and others included in the thesis enable estimates to be made of the quantity of air likely to be entrained by jets working under given conditions. The supplementary experiments showed that simple circular jets are best for a hydraulic compressor. The data collected would therefore be applicable to the design of such a machine.

For model experiments, such as those mentioned at the beginning, the experiments indicated that different similarity criteria are required depending on whether conditions at the start of entrainment or the quantity of air entrained is the major concern. Both cannot be simulated simultaneously in one model.

ADDENDUM TO THESIS. "AIR ENTRAINMENT BY FLUID JETS & STREAMS"

BY A.M. BAXTER.

The object of this addendum is to clarify one or two points in the thesis and to amplify the reference to boundary layer theory on pp. 62-64.

It should be made clear that the explanation of the onset of entrainment in terms of a critical Weber Number is not conclusive at this stage but it appears both credible and consistent with the available experimental results.

The method of presenting the results successively in terms of Froude, Reynolds and Weber Numbers has been adopted in order to obtain information which would be applicable to model technique in such cases. The usual procedure is to adopt one of these dimensionless parameters as the appropriate similarity criterion and to consider the effect of the other groups as contributing to "scale effect". (For example, in the type of hydraulic model described in the earlier part of the thesis the Froude Number is usually chosen). Thus, although it is realised that all three parameters may be involved, it is assumed that the effect of one of them is predominant and, therefore, defines the scaling laws to be used. In the case of air entrainment by jets, in which gravity, viscous, inertia and surface tension forces are all involved it is not readily apparent which criterion should be used and it was to give guidance on this point that the method of presentation of results was chosen.

Fig. 48 shows that at large values of head H the ratio $\frac{Q_A}{Q_W}$

is probably independent of H, but the scatter is rather large and the experimental range small. An attempt was later made in the thesis to put forward a tentative theoretical basis to account for the experimental data. For this purpose the quantity of air entrained by a jet, Q_A, was considered rather than the ratio $\frac{Q_A}{Q_W}$, but the reasoning was extended to include this ratio and Fig. 76 was the result. Although no particular reference is made to Fig. 48 in the text it should be made clear that the conclusion that $\frac{Q_A}{Q_W}$ is probably independent of H at large values of H, (the curves being scattered about a mean) is derived from that figure and not arrived at only after consideration of Fig. 76. This latter merely illustrates an attempt to explain the result.

In view of the complexity of the phenomenon of air entrainment, the application of simple boundary layer theory, derived from consideration of flow over a flat plate, to the small diameter jets used in the experiments can hardly be expected to yield results in close agreement with the measured quantities. This attempt is, therefore, not stressed in the thesis but some amplification may be desirable. The effective length 'x' to be used in estimation of Reynolds Number on the flat plate theory should probably be related to the depth of penetration of the jet below the surface, a dimension which is unknown and probably varies with jet velocity. Assuming a value of $x = 2\frac{1}{2}$ inches for the larger jets and $x = \frac{1}{4}$ inch for the two smallest jets, the air quantities have been calculated on the basis of a jet surrounded by a laminer boundary layer of thickness $\delta = \frac{5.2\,x}{\sqrt{\frac{Vx}{\upsilon}}}$

(δ is arbitrarily taken and $3\delta^*$ where δ^*, the displacement thickness = $\dfrac{1.73x}{\sqrt{\dfrac{Vx}{\nu}}}$ see, e.g. Rouse, "Fluid Mechanics for Hydraulic Engineers") and are shown in the attached table. The magnitude of the air quantities calculated on the assumption of a laminar boundary layer are, therefore, closer to the experimental values than those shown in Fig. 73 (which were calculated on the assumption of a turbulent boundary layer) but the slopes of the curves plotted from these values differ considerably from those of the experimental curves. This is perhaps due to the use of an arbitrary length in the calculation of Reynolds Number whereas this length probably ought to vary with jet velocity as explained above. While, therefore, it seems probable that the ideas of laminar boundary layer theory may provide the basis for future work, they cannot at present be applied satisfactorily to the existing experimental data.

A. M. Baff

CALCULATED QUANTITIES OF AIR ENTRAINED, ASSUMING A LAMINER BOUNDARY LAYER.

Assuming a boundary layer thickness of δ, the quantity of air entrained = $\pi V(d\delta + \delta^2)$ where V = jet velocity and d = jet diameter (see thesis)

Jet dia. (in.)	Head (inches of water).	Q_A (ft^3/min) Calculated	Q_A (ft^3/min) Experimental.
.0353	80	.00248	.001
	90	.00255	.002
	100	.00262	.0025
	110	.00267	.0030
	120	.00273	.0032
	130	.00278	.0035
	140	.00284	.0036
	150	.00289	.0037
	160	.00293	.0039
.0499	90	.00364	.0153
	100	.00373	.016
	110	.00380	.017
	120	.00387	.018
	130	.00394	.019
	140	.00401	.020
.0601	50	.0127	.003
	60	.0133	.0058
	70	.0137	.0072
	80	.0141	.0082
	90	.0145	.0090
	100	.0149	.0096
	110	.0151	.010
	120	.0154	.0106
	130	.0157	.0111
	140	.0160	.0117
.0870	30	.0157	
	40	.0168	.0083
	50	.0177	.0131
	60	.0184	.0160

CALCULATED QUANTITIES OF AIR ENTRAINED (CONTINUED).

Jet dia. (in)	Head (inches of water)	Q_A (ft^3/min) Calculated.	Q_A (Experimental)
.0870	70	.0192	.0185
	80	.0197	.0207
	90	.0203	.023
	100	.0207	.025
.1028	30	.0184	.0075
	40	.0196	.014
	50	.0206	.019
	60	.0215	.023
	70	.0223	.027
	80	.023	.030
	90	.0236	.032
	100	.0243	.035
.1289	20	.0204	.0013
	30	.0225	.0092
	40	.0241	.0162
	50	.0254	.0222
	60	.0266	.0275
	70	.0275	.0317
	80	.0283	.0360
	90	.0292	.040
	100	.030	.044
	110	.0306	.048
.1497	20	.0235	.008
	30	.0260	.0185
	40	.0278	.0295
	50	.0293	.040
	60	.0306	.051
	70	.0318	.062
	80	.0327	.0726