THESIS

FACTORS AFFECTING THE DESIGN

OF

MARINE REDUCTION GEARING

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The study of loading on the teeth of large marine reduction gears has been greatly retarded over a long period by inability to produce units in which an adequate analysis of performance was not rendered impossible by the consequences of inaccuracies of manufacture.

Serious failures with early double reduction gears in the years following the first World War set the seal on a conservative attitude to design without widely inspiring an urge for more accurate manufacture and as a consequence the limited advances made in Naval design met only with mixed success and the position in the early days of the last war was revealed as unsatisfactory.

The writer has taken some part in developments which followed and progressive stages are represented by papers he prepared on the subject, copies of which accompany this thesis as additional papers and which may be briefly described as follows :-

No.1. Fairfield Shipbuilding & Engineering Company's report "Loading on Gear Teeth" 1941, circulated to the marine industry in this country and referred to by Joughin in his 1951 paper to the Institution of Mechanical Engineers "Naval Gearing - War experience and Present Development". This was an early attempt to assess the amplification of tooth loading due to inaccuracies of manufacture and to distortion, and to provide a yardstick for the seemingly fortuitous incidence of "scuffing" of tooth surfaces.

No. 2. "Report on visit to U.S.A. to investigate problems associated with Design, Production and Maintenance of Main Gearing" 1944. This was prepared for the Admiralty and the marine engineering industry in this country and showed that the redevelopment of the double reduction gear in America, which so greatly benefited their naval performance, was supported by gearcutting practice much in advance of general British practice at that time, although there was no evidence that their appreciation of gear tooth design was superior to contemporary knowledge here.

No. 3. "Corrections and Measurements carried out on Pinion Hobbing Machine A111 at Fairfield" 1945. This is not a published report but is a record of work carried out under the author's instigation, guidance and instructions and represents the kernel of developments based on some novel methods of fine measurement which established a world wide reputation for the quality of gears cut on this and adjacent machines that had subsequently been treated in the same manner.

No. 4. "Current practice in Marine Gear Cutting" 1945, published in the transactions of the Institution of Engineers and Shipbuilders in Scotland, vol. 88, provided a more general resume of American practice and gave details of developments in this country that were rapidly closing the gap in technique.

No. 5. "Trends in the development of Marine Reduction Gearing" was the title of a paper given to the Institute of Marine Engineers in 1949, published in volume 61 of their transactions. Its purpose was to trace the development of marine gearing, to show how the best practice in this country had by that time overtaken American wartime technique, and to forecast the lines which developments might take. In this latter respect the writer was on delicate ground and subsequent experience showed that progress in the employment of higher duty through hardened steels was to be fraught with more complications than were envisaged.

No.6. The Twenty-eighth Thomas Lowe Gray Lecture "Marine Reduction Gearing" given in 1956 to the Institution of Mechanical Engineers and reproduced by International Shipbuilding Progress. After giving particulars of some of the most significant features in the present day production of hobbed and shaved marine gears, the lecture proceeded to a detailed examination of gear tooth design, making hitherto unestablished distinctions pertaining to modes of failure and providing a logical basis for design in relation to the materials used.

It/

It is this latter section of the last mentioned lecture which forms the basis of this thesis, and in fact the following text with its associated Appendices 1-12 are abstracted substantially from the final printed record of the lecture. Appendices 13-16 are added providing explanation or supporting evidence for material given in the text and in Appendices 2, 3 and 4, which it was not appropriate to include in the lecture. Where additional symbols are used in any of these latter appendices their significance is given in that same appendix. A tabular list of all other symbols is given by Appendix 1.

Appendix No.17 comprises notes on approximations made in certain of the Appendices 5 to 12 and the Appendix, No.18, illustrates the types of gears to which the subject applies. Appendix 19 presents a bibliography of works to which reference is made in the present text.

In general outline the text commences with a consideration of the amplifying effects on tooth loading of pinion distortion and malalignment as modified by tooth deflexion, and the slew of pinion journals in their oil films. It then proceeds to a new presentation of tooth loading criteria cast so as to provide clear guidance in the choice of tooth form for any particular duty,and having particular reference to the effects of employing different qualities of steels.

When the additional paper No.1 was written in 1941, the general errors of gearcutting were so serious /

serious as to justify inclusion of their effects in the calculation of tooth loading; the degree of compensation by tooth deflexion could only be roughly estimated in the absence of knowledge of the relevant characteristics of the teeth of broad helical gears.

Additional paper No.3 illustrates the transformation which the gear hobbing process underwent, commencing seriously in this country in 1944 (actually with the machine exemplified), and which together with the advent of the post hobbing process known as "Selective Shaving", resulted in the production of gears of an entirely different quality. Shaving is an American process by which a cutter in the form of a narrow faced, serrated toothed wheel is run in solid mesh and crossed axes with the gear to be treated, the face of which is slowly traversed by a feeding mechanism. The process is not suited to the improvement of badly hobbed gears but corrects minor irregularities of tooth surface and also of tooth profile. Tooth spacing is entirely a function of hobbing, but minor errors of helical angle can be corrected by shaving selectively in a manner devised by the author whereby the cutting pressure is varied as the process continues. This ensures uniformity of undeformed contact along the meshing teeth and provides a method for helix correction to which subsequent reference is made. The advantages of shaving are amplified when applied to gears hobbed on machines fitted with a creep mechanism to the master wheel driving the table and the merits of such a drive have been critically examined by Tuplin (1948 and 1951).

Therefore when the additional paper No.6 came to be prepared in 1955, the general character /

character of marine gears had so improved that it was no longer necessary to consider the effects of gear cutting errors on tooth loading, other than errors of malalignment over which the manufacturer might have little control. The vital effects of tooth deflexion were by this time the subject of research by the author and the conclusions reached in this matter are embraced in the text and amplified by Appendices 2, 13, 14 and 15.

The conclusions reached in the text regarding the merits of tooth form (d) reflect the uniform sucess achieved with this form of tooth during 10 years experience. The design was originally submitted to the Admiralty in 1944 by the author, the form being as adopted except for the preference for a flank angle of 17^{o} rather than 16^{o}.

CONSIDERATIONS ARISING FROM PINION DISTORTION AND MALALIGNMENT MODIFIED BY TOOTH DEFLEXION.

It has been customary to design gears allowing for a maximum circumferential tooth separation of 0.00075 inch due to combined bending and torsion of the pinion on the assumption of a uniformly distributed load along the teeth. In fact the teeth deflect differentially along their length in sympathy with an altered distribution of load brought about by the distortion. The problem is capable of elaborate treatment but by the approach described in Appendix 3 it is shown that approximately the maximum as compared with the average tooth load for double-helical pinions is given by :

$$\frac{P_m}{P} = 1 + \frac{\sec.\psi_N}{16\,\delta_T \times 10^6}\left(\frac{W + \frac{1}{2}G - 6s}{D + (2m-1)s}\right)^{2.5}$$

or in the case of a double-helical pinion engaging a pair of divided train gears at 180 deg. spacing

$$\frac{P_m}{P} = 1 + \frac{1}{19{\cdot}5\,\delta_T \times 10^6}\left(\frac{W - 6s}{D + (2m-1)s}\right)^2$$

Values of δ_T, the tooth flexibility factor (in terms of inches tangential deflexion in a circumferential plane per lb. of tangential load per axial inch of gear face), have been determined from repeated full-size deflexion tests on equipment illustrated by Fig. 1 and by the method described in Appendix 2, the results of which are graphed in terms of $\delta_T \times 10^6 / Q$ on Fig.2 where $Q = \frac{h_e}{t} \Big/ 1{\cdot}25\frac{s}{p}$.

Fig.1. Equipment for Measuring Tooth Flexibility.

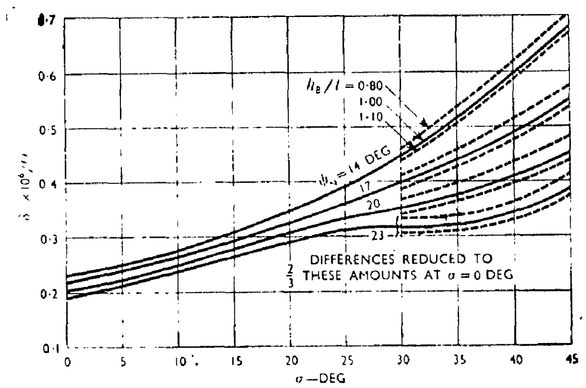


Fig.2. Tooth Flexibility Chart

The values bear additive quantities representing Hertzian compression derived from calculations by Niemann (1949).

For ease of comparison, values of $\delta_T \times 10^6$ for the tooth forms in more common use are given by Table 1. It may be noted that a deep form of tooth does not show up as more flexible than a

Table 1. $\delta_T \times 10^6$ *and Y Values for Particular Tooth Forms*

	Tooth form		$\delta_T \times 10^6$					Bending stress factor, Y
	ψ_N, deg.	s/p	Helical angle, deg.					
			0	10	20	30	40	
a	$14\frac{1}{2}$	$2/\pi$	0·25	0·30	0·375	0·48	0·64	0·79
b	20	$2/\pi$	0·185	0·225	0·275	0·315	0·38	0·915
c	$22\frac{1}{2}$	$2/\pi$	0·22	0·265	0·33	0·35	0·38	1·0
d	16	$2/\pi \times 1{\cdot}2$	0·245	0·295	0·36	0·445	0·565	1·0
e	$14\frac{1}{2}$	$2/\pi \times 1{\cdot}44$	0·215	0·255	0·32	0·415	0·55	1·0

shallower tooth of the same flank angle, because actual load per unit length along the teeth is reduced on account of the longer zone of contact by the same factor as the flexibility is increased, so that no further advantage accrues to a deep tooth so far as flexibility is concerned.

In Appendix 3 it is also shown, as a derivation from the above equations, that with a double-helical pinion in /

in single engagement having a central gap of about 3 inches, the optimum ratio of face width to pinion diameter to give the least localized tooth load as based on this theory is given by

$$\frac{W'-2}{D} = 2.6\left[1+(2m-1)\frac{s}{D}\right]\left(\frac{\delta_T \times 10^6}{\sec.\psi_N}\right)^{1/2.5}$$

and for double engagement at 180 deg. spacing

$$\frac{W'-2}{D} = 4.4\left[1+(2m-1)\frac{s}{D}\right]\left(\delta_T \times 10^6\right)^{1/2}.$$

Appropriate to these particular ratios P_m/P has the values 1.67 and 2.0 respectively.

In considering the theoretical value of W'/D it is important to recognize that:

(i) the conditions determining this turning point of low loading are extremely sensitive to changes in tooth alignment that might be brought about (a) deliberately by relief of the tooth flanks or (b) accidentally by errors of bearings, gear-case distortion, or helical angle;

(ii) the characteristic is not sharply defined and some deviation from whatever may be the theoretical value is not important; and

(iii) whatever theory is employed, the nature of the problem demands resort to approximations, the accuracy of which can only be verified by a co-ordinated examination of failures.

The reflection of this last point is seen in that the results of the theory presented pass the test that they indicate W'/D values substantially in accordance with what is believed to be the best current practice and which has in effect evolved over years through trial and error.

The difficulty of obtaining precision must not be allowed to obscure the importance of having a sound theoretical basis for W'/D as a nucleus about which certain vital influences may be crystallized, influences which have sometimes evaded proper attention with dire consequences.

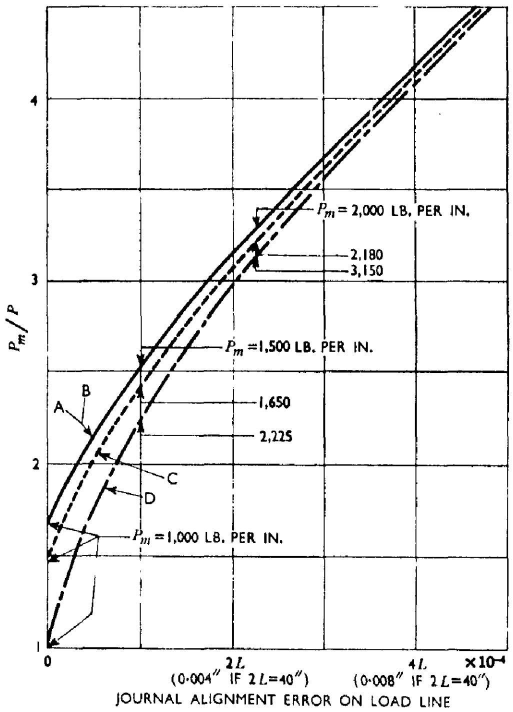
First/

First, however, the effect of gross alignment errors requires consideration and repeated reference follows to Fig. 3, which illustrates some general principles with particular examples. With a centre distance between effective bearing supports of 40 inches, a journal alignment error of about 0·007 inch in the direction of the load line at the untorqued end of a double-helical pinion in single engagement will, with tooth flexibility $\delta_T \times 10^6$ of 0.4, give $P_m/P \doteqdot 4$. If the pinion having diameter 17 inches is designed in accordance with the above theory to have a minimum P_m/P when in true alignment, it will have face width of 32 inches and the variation of P_m/P with alignment is shown by the curve marked A, being 1·67 in true alignment.

If the tooth flanks of this same pinion are now relieved 0·00025 inch at the torqued end, in Appendix 3 it is shown that $\frac{P_m}{P} - 1$ is reduced by the factor

$1 + \frac{250}{P \cos\sigma . \delta_T \times 10^6}$, and if P = 1,000, P_m/P is reduced to 1·48, the condition being illustrated by curve C on the same diagram. This would enable the gear to carry 14 per cent greater load without increasing the maximum loading in true alignment. But a journal alignment error of 0·007 inch would still give approximately the same P_m/P as the unrelieved pinion A, and under these abnormal but not infrequently possible conditions gear C is not suitable for carrying any heavier load than gear A.

From Appendix 3 it will be seen that the optimum face width when such relief is carried out increases in the /



A - Uncorrected gear.

W = 32 inches (theoretical optimum).

Relative horsepower = 100 as basis of comparison.

B - Gear with helix end relieved 0·00025 inch.

W = 40 inches (theoretical optimum).

Relative horsepower = 126 for same maximum load under any similar condition of alignment.

C - Gear with helix end relieved 0·00025 inch.

W retained at 32 inches.

Relative horsepower = 114 for same maximum load only when in alignment.

D - Corrected gear.

If W = 32 inches, relative h.p. = 167) for same maximum
If W = 40 inches, relative h.p. = 210) load only when in alignment.

2L = effective length between pinion bearing centres.

Fig. 3. Tooth Load Concentration.

the ratio $\left(1+\frac{250}{P\cos\sigma.\delta_T\times10^6}\right)^{1/2\cdot5}$ to about 40 inches in this example, the P_m/P at true alignment reverting to 1·67. Such a gear, referred to as B, would carry 26 per cent more load than A for the same maximum tooth pressure, and as its characteristic with increasing malalignment is almost identical to gear A, the curves representing the two gears are virtually coincident. The bearing-supporting centres would now increase to 50 inches and it would take a proportionately greater journal displacement of about 0·0085 inch to make P_m/P equal to about 4. Thus under all conditions gear B is suitable to carry 26 per cent greater load than gear A. Gears bearing the particular combined characteristic as described here for gear B are now being referred to as "broad relieved". The relief should be approximately in the proportion to P shown above and clearly becomes too small for practical consideration when P is much less than 1,000. A relief of 0·00025 inch has the merit of corresponding to a definite witness when checking gear marking in the best practice.

This comparison justifies careful reflection. Part of its import is seen with greater effect in referring to the "corrected" gear (Joughin 1951), of which gear D on Fig. 3 is an example, that is a gear of the type which has been considered advantageous to meet severe conditions of loading, and in which the helical angle of the pinion has been adjusted along the face width so as to conform with the reflected deflexion of the pinion under full load when evenly loaded along its length so that $P_m/P = 1$. Under true alignment gear D, with face width 32 inches, would carry 67 per cent more load than gear A, but with 0·007 inch journal alignment error

P_m/P /

P_m/P again approximates to 4 and the gear is not suitable for any greater load than A. Thus the conception of a "corrected" gear fails in that such a gear is extremely sensitive to alignment errors.

From the viewpoint of gear-load assessment the significant finding of this investigation is that a gear having P_m/P less than the optimum value (1·67 on basis presented) is not suitable for advantage to be taken of the fact by increasing load. One conclusion to be drawn from this would be that the existing practice of ignoring P_m/P in defining gear loading is fully supported. But this does have the serious disadvantages that:

(a) There is no recognised method of assessing the effect of gear loading when P_m/P is more than the optimum value. Perhaps a lack of information regarding permissible limits of Hertzian stress and root stress has obscured the significance of the omission. But more is known of the incidence of scuffing, and a subsequent section illustrates the vital use of the P_m/P factor.

(b) There is no recognizable warning sign to prevent serious mistakes being made in taking seemingly innocent advantage of low P_m/P values to overload malaligned gears.

The view is submitted that P_m/P should be incorporated as a factor in all formulae for the assessment of gear loading but that by the method of calculation shown here, it should never be less than 1·67 even although by calculation the figuring shows a lesser value. To simplify procedure the distortion coefficient c_d should be employed in lieu of P_m/P and should equal $\frac{1}{1{\cdot}67}\cdot\frac{P_m}{P}$ or $\frac{3}{5}\cdot\frac{P_m}{P}$. Thus c_d could never be entered as less than 1·0 and for the bulk of well-designed gears c_d would be 1·0. If for any reason a slender gear were required /

required, due regard would automatically be given to its shortcomings in determining permissible loading.

In a divided train gear, however, as the optimum face-width ratio occurs when $P_m/P = 2.0$ c_d must never be less than 1.2.

None of the particulars given above is applicable without modification to the older type pinions with centre bearings.

It would be unfitting to close this section where such urgent reference has been made to malalignment, without a word on the correcting influence of slew of the journals in the bearings due to differential loading of the oil film. It has often been hopefully imagined that such a process would occur to a useful degree in compensating for shortcomings in alignment, but doubts as to the useful proportion of the slew to the error are confirmed from recent measurements reported by Newman (1956) on the attitude of journals in bearings, and from which it can be estimated, as shown in Appendix 4, that, when the bearing load is maldistributed $\pm$ 50 per cent, the total slew correction is $\dfrac{N.D_b.d}{10,000 P_b}$ thousandths of an inch; from this it can be calculated that slew will not usually correct more than 10 per cent of the malalignment. To take the effect into account on the basis of the assumptions made in Appendix 4, C_D is substituted for c_d in all the above remarks, the relationship being approximately

$$C_D = 1 + \frac{c_d - 1}{2.35/K^{1/6}}$$

for gears in single engagement.

No slew can occur for gears in double engagement.

It will be seen that the correction is quite small and for most practical purposes with well-designed gears $C_D = c_d$.

One trend which might not be altogether of advantage to the performance of gear teeth is the tendency to reduce diametral clearance. This has resulted from suspected vibration due to whirl of the bearing oil film under light loads (Shawki 1955), but reference to Appendix 4 shows that the ability of a journal to correct maldistribution of loading by slew is in proportion to the bearing clearance. A gear should be so designed as to make correction unnecessary and indeed it has been shown that with a properly designed gear the slew correction is very small, but in the event of a disturbance to the bearings causing malalignment, the effect can be quite substantial in minimizing load concentration. Reduction in pinion-bearing clearance should therefore only be carried out after the most careful consideration of all the circumstances of the service.

TOOTH LOADING CRITERIA AND CHOICE OF TOOTH FORM.

In its approach to gear design, B.S.436 takes the reader immediately into an empirical treatment based on long experience in general operation, focusing on wear and fatigue life. But the marine gear must be provided with such a margin of safety to meet exceptional conditions of storm, that these factors have no primary application, and the best treatment for /

for design is therefore by recourse to more fundamental principles rather than general conventional formulae.

Appendices 5 to 9 inclusive show the development of three criterion coefficients C_P, C_R, and C_S relating respectively to tooth failure by pitting, root stress, and scuffing. The numerical coefficients have been chosen so that when the undernoted conditions apply, a thoroughly sound merchant gear design is, in the author's view, achieved when none of the criterion numbers exceeds 100. This is conditional upon the use of what have been for many years regarded as standard materials (Dorey 1942), viz. 0·3 per cent carbon steel 31/35 tons ultimate tensile strength (U.T.S.) for the rims and 3·0 per cent nickel steel, oil quenched to 40-45 tons U.T.S. for the pinions, lubricated with a standard undoped oil and having normal high accuracy tooth finish by, say, shaving, aligned according to proper shop and fitting-out practice and submitted to conditions of transmission which do not involve exceptional cyclic irregularity.

With the numerical coefficients chosen it is believed that under normal conditions of service at sea there is a factor of safety of about 1·5 on tooth loading against pitting fatigue, and 2·5 against bending fatigue, /

fatigue, although as records are based on gears of older construction the safety margin is probably now greater. Against scuffing the factor of safety is about 1·25 under full-power trial conditions with proper tooth profiles, but this rapidly and indefinitely increases with improvement of surface and profile and by work hardening under running conditions. The reasons for the variation in the respective factors of safety are obvious, but it is to be observed that a gear which scuffs on trial and is subsequently "dressed up", or a gear which has excessive tip relief in an effort to avoid such trouble, carries a lesser margin of safety against subsequent pitting fatigue by reason of the lesser useful tooth surface that remains.

The expressions for the criterion numbers are

$$C_P = K.k_P \quad ; \quad C_R = K.k_R \; ; \quad \text{and} \quad C_S = K.k_S$$

where K = P/De conventionally and k_P, k_R and k_S are respectively given by expressions in Appendices 7, 8, and 9.

General use may be made of these expressions for the analysis of existing designs, and they also form the basis upon which to examine the character of patterns produced by altering variables with the object of defining the most suitable proportions of teeth in terms of variables that are relevant, for which reference should be made to Appendix 10.

The cardinal points emerging from this analysis are:

(1) Referring to Fig. 4, tooth forms, e, d, and c form a series meeting all requirements, having certain consistent characteristics. From the point of view of the master expression representing surface pressure, the forms are to be preferred in the above order.

(2) Form e should not be used for a pitch less than $\frac{7}{10}$ inch.

(3) /

(3) The liability to scuffing with these tooth forms is in the reverse order to that preferred for surface pressure and this entails an overriding restriction which will follow as an outcome of this analysis.

(4) Existing practice is confirmed in that tooth forms e and d should be associated with a helical angle of 30 deg. Reference to Table 3 will show why, for a flank angle of 22½ deg. the helical angle for minimum root stress should be 40 deg., and why, for the sake of consistency in the present examination it is temporarily assumed to be associated with a 20 deg. helical angle.

(5) The proportion of pinion addendum to active depth should only be increased above 0·5 if a slight increase enables a tooth of smaller pitch to be used. The particular case of an all-addendum tooth is referred to in Appendix 10.

Before this information can be channelled to lead to any broad conclusion it is necessary to establish a

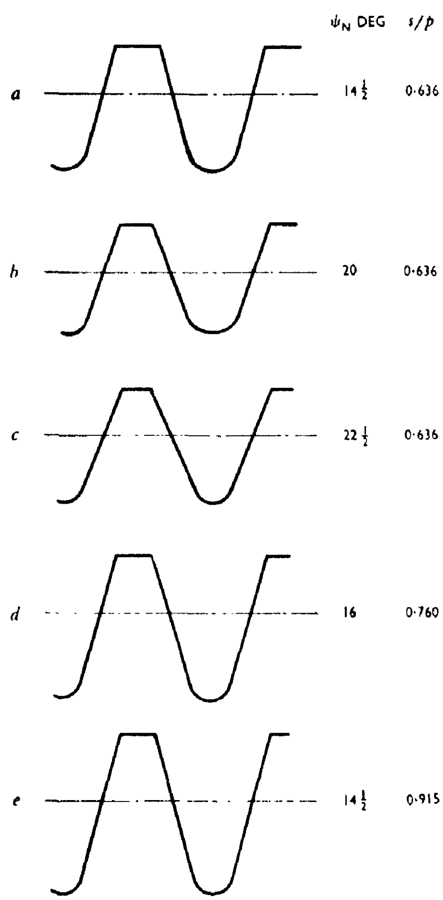


Fig.4. Tooth Forms

suitable expression for loading, and this will be in the form of a C factor which will be developed from the basic relationship

$$C = C_P\left[\frac{C_R}{C_P} \text{ or } \frac{C_S}{C_P} \text{ or } 1{\cdot}00\right] \text{ whichever is greatest}$$

$$= K.k_P\left[\frac{k_R}{k_P} \text{ or } \frac{k_S}{k_P} \text{ or } 1{\cdot}00\right] \text{ whichever is greatest}$$

In this regard C_P is in the nature of a master while C_R and C_S are, in a manner, slaves; it will be seen subsequently that C_R/C_P has a major influence on tooth pitch and C_S/C_P on depth/pitch ratio. additionally.

It is shown in Appendix 11 how, on this basis of presentation, the information derived from Appendix 10 may be crystallized about a consistent series of teeth approximating closely to the forms represented by e, d, and c in Fig. 4 , flank angle being subjugated to depth/ pitch ratio, involving an error of up to no more than 1¼ deg. in any of the forms. The method adopted enables the C factor introduced above to be recast in a way to show quite clearly the effect of the vital variables, unclouded by other variables legitimately subjugated and surviving only as factors in numerical coefficients or as indices of the vital variables. In its amended general form the C factor is

$$C = K.\frac{C_D}{\left(\frac{s}{p}\right)^{0{\cdot}83}\left(\frac{N}{1000}\right)^{0{\cdot}17}}\left[\left(\frac{p}{p_f}\right)^{0{\cdot}83} \text{ or } \frac{\left(\frac{s}{p}\right)^{3{\cdot}4} . D_e^{0{\cdot}5} . \frac{p_f}{p}}{2{\cdot}2\left(N/1000\right)^{0{\cdot}125}} \text{ or } 1{\cdot}00\right]$$

whichever is greatest

where the theoretically correct pitch, as distinct from pf the pitch fitted, is

$$p = \frac{D_e}{19{\cdot}5}\left(\frac{s}{p}\right)^{0{\cdot}67}\left(\frac{N}{1000}\right)^{0{\cdot}20}$$

p/p_f in preliminary design being 1·0.

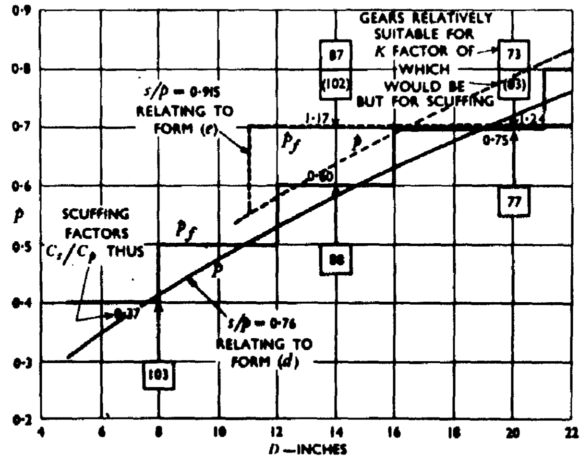
In the expression $s/_p$, p represents the pitch actually fitted and this is not to be confused with the distinction between p and p_f in another term. The possibility of confusion on this point is unavoidable whatever /

whatever system of symbols is employed in the derivation, and particular reference to the point is warranted.

The quantities within the square bracket only contribute to the product if the conditions they represent, respectively root stress and propensity to scuffing, are vital in indicating the limitation of permissible loading. The full significance of the $s/_p$ ratio on scuffing is clearly shown.

The next step provides in itself an indication of the significance of the equations to all marine gear design. It is convenient temporarily to adopt the general relationship of diameter to revolutions per minute referred to in Appendix 7, enabling a chart to be drawn showing, for normal materials, p, p_f, and C_S/C_P on a base of pinion diameter, taking $D_e = 0{\cdot}875D$. This is shown by Fig. 5, a study of which will reveal the restriction which considerations of scuffing place upon the use of the deep tooth form e supporting the view that its satisfactory employment is often to be associated either with limitation of loading or excessive tip relief.

The C_S/C_P and K values relating to the tooth form d on the diagram ($s/p = 0{\cdot}76$) show respectively that, for standard materials, there is a good margin for the avoidance of scuffing and that this tooth is suitable for carrying a greater load than the deeper tooth when proper restriction is placed on the latter against scuffing.



'Standard' materials. D_e^2 (N/1,000) = 250, D_e = 0·875D. Helical angle, 30 deg. m = 0·5.

Fig.5 Typical Load Criteria with Different Tooth Forms.

The fact that, but for this restriction, the deeper tooth would carry a greater load and also that the shallower tooth embraces such a substantial margin against scuffing, leads to an examination of the load-carrying capacity of a tooth form intermediate between them having s/p equal to, say, 0·84. Such a form would, on the average conditions portrayed by the diagram, run up to the safe scuffing limit and carry a load represented by K values midway between those given for form d and those in brackets for form e which would hold good but for scuffing. This represents, for a 20-inch pitch circle diameter (P.C.D.) pinion, an increased load capacity of 4 per cent over form d and 10 per cent over form e, the sharp characteristic indicating the intermediate form being brought about by the intersection of converging allowable load curves rather than the turning point of a single curve.

Attractive though this may be, the author would not recommend the establishment of such an additional standard, at least at this stage, on account of the following /

following considerations. It is believed that with modern gear-cutting and alignment standards, the factors of safety against ultimate fatigue by both pitting and root failure are in fact greater than have been suggested, although the margin against scuffing on a new gear is fairly well established. The form d, by reason of its generous margin against scuffing, would probably permit an increase of nominal loading of at least 20 per cent over the limits that are suggested here and as such an increase would not be possible with the intermediate form, the justification for its establishment disappears. Conversely, form d probably provides a more uniform additional margin of safety with the limits proposed. Furthermore, the adoption of higher tensile materials may, by reason of scuffing characteristics, to which subsequent reference is made, favour a reduction rather than an increase in depth ratio.

For these reasons, and having regard to the suitability of form d through the full range of gear sizes, it is appropriate to regard this existing tooth form as a standard upon which to base a specialized statement of the C factor, namely,

$$C = K \frac{1{\cdot}3\, C_D}{\left(\frac{N}{1000}\right)^{0{\cdot}17}} \left[\left(\frac{p}{p_f}\right)^{0{\cdot}83} \text{ or } \frac{D_e^{\,0{\cdot}5} \cdot p_f/p}{5{\cdot}65\left(\frac{N}{1000}\right)^{0{\cdot}125}} \text{ or } 1{\cdot}00 \right]$$

whichever is greatest

where $$p = \frac{D_e}{23{\cdot}4}\left(\frac{N}{1000}\right)^{0{\cdot}20}$$

With this form of tooth, when normal materials are used, the slave factors within the square brackets are unlikely ever to be significant, that is, to exceed 1.00, particularly as, normally, the pitch chosen would be the standard pitch next above the calculated figure. These calculated pitches are slightly smaller than conventionally/

Gear	Naval vessels quoted by Joughin (1951)								Merchant vessels					Alternative designs for S or T	
	Egret, Pelican, and *Sutlej*	'Hunt' class des-troyer	Italian cruisers	'J' and 'K' class des-troyers	*Dido* (cruis-ing) turbine)	*Bittern*	*Erne* and *Ibis*	*For-midable*	P	Q	R	S	T	U	V
D, inches	5·14	8·78	14·3	10·93	6·0	5·14	5·44	18·2	5·99	9·21	13·86	16·71	16·71	16·71	16·71
D_e, inches	4·77	7·87	12·95	9·84	4·4	4·77	5·06	16·31	5·41	7·9	12·18	14·84	14·84	14·84	14·84
$W+G$, inches	12+3	24½+3	45½+3	32+3	8+3	12+3	15·7+2	52+3	16½+3½	20+3½	27+3	38+3	38+3	38+3	38+3
N	3,700	3,300	2,631	3,070	6,500	3,700	4,030	2,000	3,000	5,000	800	829	829	829	829
P	616	918	1,070	1,098	540	564	462	1,281	459	606	1,437	1,010	1,010	1,010	1,010
p, inches	$\frac{7}{12}$	$\frac{7}{12}$	1	$\frac{7}{12}$	$\frac{7}{12}$	$\frac{7}{12}$	$\frac{5}{16}$	$\frac{7}{12}$	$\frac{7}{12}$	$\frac{5}{16}$	1·047	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{7}{16}$	$\frac{5}{16}$
ψ_N, deg.	22½	22½	22½	22½	22½	14½	22½	22½	14½	16	20	22½	14½	16	16
s/p	0·636	0·636	0·636	0·636	0·636	0·636	0·636	0·636	0·636	0·760	0·636	0·636	0·915	0·760	0·760
σ, deg	30	30	30	30	30	30	30	30	36·4	42	30	30	30	30	30
m	1	1	1	1	1	0·5	1	1	0·5	0·6	0·5	1·0	0·5	0·6	0·6
$K = P/D_e$	129	116	83	111	123	118	91	78	85	77	118	68	68	68	68
C_D	1·29	1·82	2·07	2·01	1·00	1·16	1·88	1·93	1·34	1·00	1·10	1·26	1·16	1·15	1·16
C_P	222	287	244	310	149	163	228	224	147**	78	201**†	149**	90	107	108
C_S	135 **	138 **	172 **	135 *	70	112	88	87	97	46	178*‡	98	109*	100	86
C_B	77	158	126	208	53	63	120	228 *	64	64	87†	100	83	91	105
C	193	250	214	268	130	159	196	231	137	74	—	129	118	101	102

Gears which have pitted, scuffed, or suffered from broken teeth shown ** Varied experience with different gears of the class *

† To correct for 65 tons U.T.S. pinion steel $C_P' = C_P/C_{MP} = 130$, $C_B' = C_B/C_{MB} = 60$.
‡ History unknown in relation to C_{MS} factor, but scuffing probably led to pitting on a reduced surface.

C has been calculated from the general expression derived for the consistent series of tooth forms and shows, by its variation from the maximum of individual C_P, C_B, or C_S values, typical effects of using other tooth forms, or addenda distribution other than 50/50.

Table 2.

conventionally adopted, reflecting the author's view that pitch is frequently made excessive.

Table 2 shows criterion numbers and the C factor for a number of naval and merchant gears. The naval gears show the high C_P values which fast-running all-addendum teeth seem able to sustain and to which reference is made in App.10. Gear S represents slow-running all-addendum gears which fail by pitting, the sizes being chosen for direct comparison with gear T, and to which the designs U and V also relate.

The use of material constants is introduced in Appendix 12, whereby

$$C_P' = K.\,k_P/C_{MP}$$
$$C_R' = K.\,k_R/C_{MR}$$

and

$$C_S' = K.\,k_S/C_{MS}$$

C_{MP}, C_{MR}, and C_{MS} are unity for the materials currently regarded as standard. To avoid complication these constants have been excluded from the equations leading to the C factor which is now repeated in its specialized form to show the characteristic of their employment and termed C'.

$$C' = K\,\frac{1{\cdot}3\,\frac{C_D}{C_{MP}}}{\left(\frac{N}{1000}\right)^{0{\cdot}17}}\left[\left(\frac{p}{p_f}\right)^{0{\cdot}83} \text{ or } \frac{D_e^{0{\cdot}5}\,.\,p_f/p}{5{\cdot}65\,(N/1000)^{0{\cdot}125}}\,.\,\frac{C_{MP}^{2{\cdot}25}}{C_{MS}\,.\,C_{MR}^{1{\cdot}25}} \text{ or } 1{\cdot}00\right]$$

whichever is greatest

and

$$p = \frac{D_e}{23{\cdot}4}\left(\frac{N}{1000}\right)^{0{\cdot}20}\left(\frac{C_{MP}}{C_{MR}}\right)^{1{\cdot}25}$$

The latter equation, taken in conjunction with the expressions for C_{MP} and C_{MR} in terms of U assumed in Appendix 12, shows that for an established $s/_p$ ratio, desirable pitch varies as $\sqrt[4]{U/45}$.

If the material constants are applied to the previous and more general statement of the C factor, which approximately covers the e, d, c tooth series, it is seen that the scuffing or depth/pitch slave factor embraces /

embraces the multiple

$$\left(\frac{s}{p}\right)^{3\cdot4}\cdot\frac{p_f}{p}\cdot\frac{C_{MP}^{2\cdot25}}{C_{MS}\cdot C_{MR}^{1\cdot25}} \quad \text{or} \quad \left(\frac{s}{p}\right)^{3\cdot4}\frac{p_f}{p}\cdot\left(\frac{U}{45}\right)^{1\cdot45}\Big/C_{MS}$$

Thus, if $\left(\frac{U}{45}\right)^{1\cdot45}\Big/C_{MS} > 1\cdot0$, as is quite likely in the employment of advanced materials to which reference is made later, immunity from scuffing is reduced and when the safety limit is reached, K ceases to benefit further in proportion to C_{MP} and unless C_{MS} is artificially raised by the use of an E.P. lubricant either s/p must be reduced, possibly rendering desirable a new tooth form intermediate between d and c, or p_f must be reduced. Within the scuffing zone, reference to the same statement of the C factor will show that reduction in s/p will permit further load increase inversely proportional to the ratio of reduction raised to the power (3·4 – 0·83) or about 2½. On the other hand, unless p_f is larger than necessary (in which case K may be increased inversely proportional to a reduction in p_f) the allowable gain in K due to reduction in p_f is negligible since the root stress slave factor suffers to almost the extent that the scuffing slave factor benefits.

These considerations are essential to research on gears with advanced materials to ensure that comparison is being made between optimum conditions.

While the desirable values of the foregoing criterion numbers have been assessed from practical experience of gears on trial and in service, based on a good quality of tooth finish, certain refinements may justify the raising of some limits and the question of the best tip relief to be provided for the teeth is silhouetted against any such consideration.

Before referring in detail to the extent of tip relief required it is essential to refer to two features that have been responsible for confusion.

First, tip relief on a spur gear (Merritt 1954) is necessary to provide "sweetness" of engagement in compensating for the deflexion of the load-carrying teeth. The comparable feature in a helical gear is the relief that must be provided at the ends of the pinion teeth; no such consideration arises affecting tooth profile. Secondly, because gears have sometimes been designed in a manner that has made them susceptible to scuffing, an erroneous impression has arisen that heavy tip relief is a basic necessity of normal engagement of helical gear teeth, in disregard of the palliative nature of the measure.

In extreme cases the tip relief has been progressively increased to amputate tooth surface and when the measure has not been carried sufficiently far to overcome scuffing it has been described as inadequate with reference to the thickness reduction rather than the radial dimension of mutilation. With a gear tooth properly designed to meet the requirements of the transmission, the factors dictating the need for tip relief are entirely different.

Attention/

Attention is now directed to the measurements described in Appendix 2 in which the values of factors a, b, c, d, e, and f represent the variation in loading intensity along the line of contact. Typical values of the factors are plotted in Fig. 6 as applying to a pair of teeth of $\frac{8}{10}$ inch pitch having $\psi_N = 16$ deg. and s/p = 0·76 with a length of contact corresponding to a helical

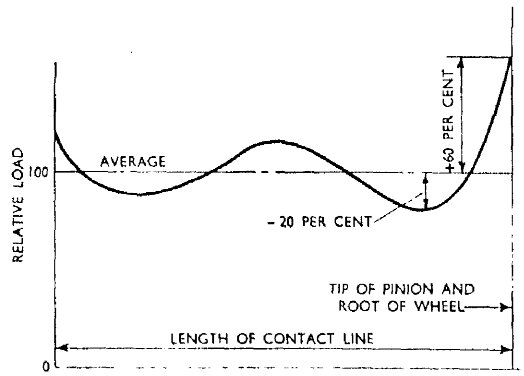


Fig.6. Typical Load Variation on Contact Line

angle of 30 deg. The nature of the variation correspond to a forecast envisaged by Walker (1946) but appears to be of lesser total variation than he anticipated, being for this form of tooth plus 60 per cent and minus 20 pe cent of the average load, totalling 80 per cent. Corresponding variations and percentages measured for ε tooth of the same pitch having $\psi_N = 22\frac{1}{2}$ deg. and s/p = 0·636, total 95 per cent. For the $\frac{7}{10}$ inch pitch deep tooth with $\psi_N = 14\frac{1}{2}$ deg. and s/p = 0·915, the tot variation in load is 40 per cent.

In Appendix 2 reference was also made to the corollary to be drawn from the deflexion calculations by assuming a, b, c, d, e, and f to be equal, giving differing local deflexions that might be regarded as the mirror reflection of an initial profile correction to give uniform loading. But the total correction to give the /

the required effect is only 0·00005 inch. * The importance of this finding is that the whole conception of correcting the involute to compensate for the not inconsiderable variation in loading that nominally occurs by reason of tooth deflexion due to bending and shear, is, with a helical gear, purely theoretical and incapable of practical interpretation. It nevertheless emphasizes the value of plasticity in the running-in of a new gear.

The sole legitimate reason for profile correction is to provide for the generation of a hydrodynamic oil film at the regions of initial and final contact and this is effected by a tip relief of, say, 0·00025 inch plus an allowance to compensate for Hertzian depression (which, unlike bending and shear deflexion, tends to absorb tip relief effect). The total relief to be provided varies then from, say, 0·0003 inch for a 0·4-inch pitch to 0·0005 inch for 0·8-inch pitch.

The problem of producing these small corrections naturally requires patient attention. In shaving, the requirement is met by a swelling on the root of the cutter profile of about twice the desired relief, the ratio having been established by comparing the routine measurement of a shaving cutter profile with the profile of a tooth finished with the same tool, and obtained laboriously in the interest of research.

Finishing /

* See Appendix 14.

Finishing by hobbing does not provide assurance that the correction will be properly achieved. Two fundamental difficulties obtrude. First, there is the difficulty of a slightly eccentrically-running hob producing errors even under most careful control greater than the required correction. Secondly, the generating process is such that the position of the swelling on the hob tooth to give proper tip relief varies with the diameter of the gear to be cut, and unless a much greater relief is to be accepted for large diameters with loss of effective tooth surface, a considerable stock of only slightly differing hobs must be carried.

END RELIEF *

End relief, to be greater than the total tooth deflexion and depression, should be, say, 0.0008 inch for 0.4-inch pitch to 0.0015 inch for 0.8-inch pitch, normal to the teeth on both flanks. While relief is largely to ease the rate of deflexion on engagement, it also eases an end peak of surface pressure on the teeth. Root strength is adequately safeguarded by the conventional chamfer and the fact that the enhanced flexibility of the tooth end does not permit that part to carry greater root stress than the adjacent body of the tooth, similar deflexions being reflected by similar root stresses as experiment has shown. The provision of this slight relief which should wash away in about ¼ inch, can form part of the selective shaving process. Excessive end relief is detrimental in just the/

* See Appendix 15.

the same way as excessive tip relief. The author does not consider that there is any useful purpose to be served in attempting to make this small relief taper off in line with the line of contact in a manner which has been suggested, and which entails costly work on every wheel tooth.

(W - 6s) is employed rather than W in the calculations for tooth loading, not because the teeth are relieved for 1·5s inches at each end, but because this allowance represents the reduction in load. carried by the ends of the teeth regardless of relief.

MATERIALS FOR GEAR-WHEEL RIMS AND PINIONS

Development in the production of highly-finished teeth led naturally to the desire to adopt higher tooth loadings by taking advantage of higher tensile steels with greater surface hardness and root strength, it being considered that loss of plasticity for the accommodation of irregularities of tooth contour would axiomatically be compensated by the higher finish. (Such development is represented by Fig. 7 reproduced from Paper No. 5.)

Factually this is an oversimplification of the problem, and the relative merits of retaining certain qualities of materials to the sacrifice of other qualities will emerge only as the results of further research materialize. The next ten years are likely to see a concentration on this problem matched by the intensity of the quest for greater accuracy of tooth production in the previous ten years.

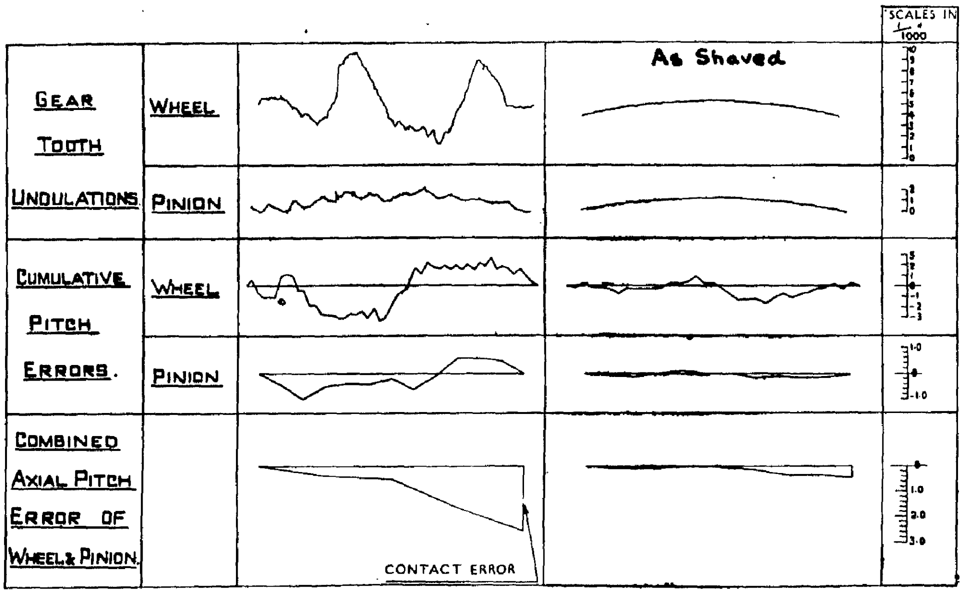


FIG.7 - Comparative error characteristics of gears of twenty-five years ago and modern shaved gears.

Basically, a higher-duty material is required in order to obtain a greater fatigue resistance to pitting, and the material conditions which supply this quality also provide for a greater resistance to fatigue in relation to root stress. Toughness, that is, the ability to deform without inter-crystalline cracking, is provided in greater measure by an alloy steel than by a high-carbon steel and this consideration has led to the demonstration of one viewpoint that higher duty can best be obtained by making both pinions and rims of alloy steel having suitable U.T.S. differing by, say, 15 tons per sq. in. to minimize mutual weldability.

Unfortunately, such a difference in U.T.S. between the mating elements is not effective in providing an increase in resistance to scuffing (as represented by the/

the C_{MS} factor) proportional to the increase in fatigue resistance (as represented by the C_{MP} and C_{MR} factors). Indeed, there is some practical evidence, both from service results and disk testing, to show that the C_{MS} factor is reduced with a combination of alloy steels as compared with standard materials as the following examples demonstrate:

Secondary gears having teeth of $\frac{7}{10}$ inch pitch deep form ($s/p = 0{\cdot}915, \psi_N = 14\frac{1}{2}$ deg.) scuffed when running with a K value of about 62 and at 95 r.p.m. The severity of the scuffing was such that C_S' was assessed at 130 on the temporary assumption that normal steels were involved, this corresponding with a C_{MS} value of 0·78.

In the second example a similar set of machinery incorporating secondary gears having a pitch of $\frac{6}{10}$ inch with $s/p = 0{\cdot}76$ and $\psi_N = 16$ deg. scuffed in very mild form at the same power condition. C_S' was assessed at 120, giving a C_{MS} value of 0·64.

Both these wessels have primary gears of the same combination of alloy steels, and operate completely satisfactorily with a K value of 91 and 5,000 r.p.m. Assessing C_{MS} at the worse determined value of 0·64, this gives C_S' for the primary gears as only 84, thus demonstrating why they have been free from scuffing.

The reason for the scuffing of the alloy steel combination may not be completely determinable, and the assessment of different combinations must be the subject of research.

One factor that might aggravate the condition is that the difference between the two alloy steels is largely in their heat treatment which locally becomes purely historical when an area, however small, has reached /

reached welding temperature. On a subsequent contact there may be a local molecular affinity between wheel and pinion which is not foreshadowed by study of the respective U.T.S. values of the test-pieces.

More definitely it can be added that as an alloy steel has a relatively high yield point with poor work-hardening qualities the tooth asperities which are so small as to be of no consequence when a carbon gear is employed, do actually assume a significance that cannot be ignored. The difficulty can be satisfactorily overcome by employing one of the new E.P. lubricants suitable for turbine machinery; the function of the additive material in these oils is to form a sulphide or an oxide locally on the tooth surface where the temperature is raised by metallic contact, thereby not only preventing molecular cohesion but corroding the local high spot and affecting a so-called chemical polishing. After running for a few hours with such an oil the gears are seen to have achieved a particularly fine polish. Fig. 8 shows Talysurf

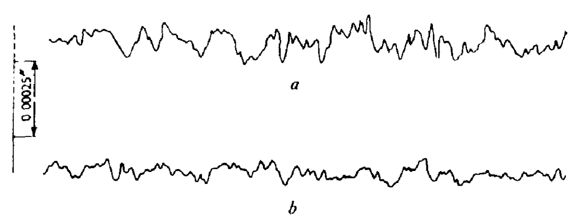


Fig.8. Talysurf Records

a. Before running with an E.P. lubricant.
b. After running with an E.P. lubricant.

records of tooth pinion surface before and after such a run with E.P. lubricant. It is to be noted that the Talysurf undulations before running, being on a highly magnified scale, correspond with Tomlinson undulation records in the "as shaved" condition shown in Fig. 7. Subsequent trial-and-service records on two vessels show that such a combination of alloy steels when running /

running with an E.P. lubricant have a C_{MS} value of at least 1·15, there being no evidence of scuffing under any conditions to indicate what the actual top limit may be,. although disk testing might suggest the value to be between 1·40 and 1·90.

An alternative to the use of a combination of alloy steels for high duty is the adoption of an alloy steel of high U.T.S. for the pinions running with a carbon-steel rim having a relatively high carbon content. Opinions have been divided on the point but it does appear that with suitable heat treatment an increase from 0·3 to 0·4 per cent carbon results in a steel having higher fatigue resistance proportional to the increase in U.T.S.

The adoption of case hardened gears is slow in the marine industry due to the present uneconomical character of its application to large units and the mathematical treatment which has been described applies specifically to carbon steel gears or through hardened alloy steel gears running with through hardened alloy steel pinions.

Nevertheless the system of design criteria propounded will have equivalent application to case hardened gears with the use of material constants determinable from current researches on such gears.

APPENDIX 1.

Symbols

Symbol: Having application in final conclusions	Symbol: Having permanent application in detail	Symbol: Used only for convenience of expression	Meaning (lb. in. units unless otherwise stated)
Gear proportions			
		B	Breadth of one helix
D			Pinion P.C.D.
D_e			D (gear ratio/(1+gear ratio))
G			Gap between helices
		I	Moment of inertia of pinion section
		L	½ (effective length between pinion bearing centres)
W			2 x B
W'			Theoretical optimum value of W for minimum load concentration
Bearing dimensions			
D_b			Journal diameter
d			Diametral bearing clearance in $\frac{1}{1000}$ inch
Loads, forces, and pressures			
		F	Reaction on one pinion bearing
P			Average tangential load per inch gear face (based on (W-2) inches
		P_b	Bearing pressure on projected area
		P_f	Pressure in oil film between teeth
	P_m		Maximum value of P
		P_N	P converted to force normal to tooth
Stresses/			

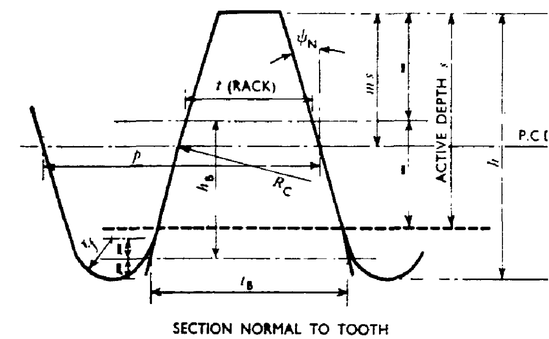


Fig. 9. *Symbols for Tooth Dimensions*

p_f Pitch fitted (if differing from desirable or theoretical pitch)
σ Helical angle. ψ_T Pressure angle in transverse plane.
z Tangential length of contact zone in transverse plane.

Symbols

Symbol: Having application in final conclusions	Symbol: Having permanent application in detail	Symbol: Used only for convenience of expression	Meaning (lb. in. units unless otherwise stated)
Stresses			
	S_H		Hertzian stress (conventional)
	S_P		Hertzian stress (see Appendix 12)
	S_R		Root bending stress
	U		U.T.S. of pinion material, tons per sq. in.
Speeds			
N			Pinion speed, r.p.m.
		V_S	Tooth sliding speed (relative)
		V_R	Tooth rolling speed (relative)
Criterion numbers and factors			
C, C'			Gear loading C factor, ′ indicates incorporation of material constants
C_D	c_d		Pinion distortion multiplying factor (about 1·0 for well-designed gear)
		C_F, k_F	Criterion of Martin oil film pressure
		C_H, k_H	Criterion of Hertzian stress
C_P, C_P', k_P	k_P''		Criteria of pitting, root stress, and scuffing respectively
C_R, C_R', k_R	k_R''		($C_P = K \cdot k_P$, etc.) ″ indicates use in particular comparison
C_S, C_S', k_S	k_S''	C_S	and $C_P = K \cdot k_P''$, etc.)
C_{MP}/			

Symbols

Symbol: Having application in final conclusions	Symbol: Having permanent application in detail	Symbol: Used only for convenience of expression	Meaning (lb. in. units unless otherwise stated)
Criterion numbers and factors (contd.)			
C_{MP} C_{MR} C_{MS}			Material constants representing effect on resistance to pitting, root fatigue and scuffing respectively for materials other than current standard materials
K			Conventional K factor P/D_e
Common symbols used in mechanics			
		E	Modulus of elasticity
		e	2·718
		λ	Poisson's ratio
		μ	Viscosity (relative)
Sundry functions and coefficients			
		A	Local significance (Appendix 3)
		a	Local significance (Appendix 3)
		b	Local significance (Appendix 3)
	δ_{BSN}, δ_{BST}		Normal and tangential deflexions of a pair of teeth due to bending and shear
	δ_{HN}, δ_{HT}		Normal and tangential deflexions of a pair of teeth due to Hertzian compression
δ_T			Combined tangential tooth deflexion, in. per lb. per in. load
		f	Thickness of oil film between teeth
		H /	

Symbols

Symbol			Meaning
Having application in final conclusions	Having permanent application in detail	Used only for convenience of expression	(lb. in. units unless otherwise stated)
Sundry functions and coefficients (contd.)			
		H	Local significance (Appendix 2)
		J_s	Local significance (Appendix 4)
		l_f	Effective length of loaded oil film between teeth
Q			Tooth proportion factor for estimation of δ_T
		t_A	Local significance (Appendix 3)
		x	Local significance (Appendix 3)
	X_β		Root fillet stress raiser
	X_δ		Stress modifier for helical teeth
	X_σ		Function of helical angle that affects root stress
	Y		Function of tooth form (equal to 1·0 for consistent series)
	y		Function of angularity of line of contact as it affects tooth deflexion
		y_A	Local significance (Appendix 3)

APPENDIX 2

FLEXIBILITY OF GEAR TEETH

Full-size samples of long, single teeth have been machined integral with supporting flanges and subjected to loading with short, blunt knife-edges as shown by Fig. 1a and b, deflexions being measured with a see-saw toe registering on a clock gauge with $\frac{1}{10000}$-inch calibrations.

The line of contact of typical helical gears was drawn on the tooth and equally loaded successively at six equidistant stations into which it was divided, the deflexion being recorded at each of the stations for every load, giving thirty-six readings for each test. Corresponding readings were taken for replica rack and involuted pinion teeth and for every two such sets of thirty-six readings the following process of evaluation was adopted. The constant load was imagined to be multiplied by respective factors a, b, c, d, e, or f depending upon its station, the reverse sequence being adopted for rack and pinion teeth when counting from the tip of the teeth in each instance. The deflexions were multiplied by the same appropriate factors so that at each station the summation of six deflexions became an algebraic expression involving the six letters. Such expressions for stations of the same letter on rack and pinion teeth were then added to give six further expressions of a similar character, and these represented the combined deflexion of rack and pinion teeth at each of the six stations, and these of course must be equal and were represent by a single constant.

Solution /

Solution of the six simultaneous equations gave the combined deflexion of the teeth for the chosen line of contact so far as bending and shear were concerned. Any irregularities due to the inability of locating the recording toe at the point of load, which applied to a sixth of the readings, were evened out by graphing each of the seventy-two readings of a combination before forming the equations.

This method of loading the teeth gave a measure of the distribution of load along the line of contact and overcame the vital uncertainty of minute contact attending the use of a loading edge the full length of the line of contact.

Furthermore, in the nature of a corollary to this method of calculation, the six unknown factors in the final equations can be equated so that each equation gives a different total deflexion, the differences being the mirror reflection of the correction required to involute form to secure uniform loading from tip to root of the teeth for any chosen magnitude of loading.

The satisfactory nature of the equipment having been demonstrated there were the effects of many variables to be examined, viz.:

Influence of involuting to represent different pinion diameters.

Different rack tooth forms.

Influence of altered angles of line of contact to represent different helical angles.

Influence of length of tooth in giving greater flexibility at ends of helices.

A very large number of readings would be required to cover this investigation fully and this has not been possible; indeed it would be all too easy to over-elaborate the problem at this stage. The following approximations have been evolved from the results of some twenty tests.

Involuting has the opposite effect on flexibility to that which might be imagined, because of the

intensification /

intensification of loading nearer the root of the pinion teeth, thus offsetting the lesser beam strength. Repeated readings showed pinion tooth forms, for average diameter relative to pitch, in mesh with a rack tooth giving lesser flexibility in the ratio $0{\cdot}82/\cos^2\psi_N$ as compared to rack tooth with rack tooth having the same diagonal length of line of contact (albeit an imaginary condition).

It was not possible to define the varying effects of tooth depth, flank angle, and helical angle by any simple relationship, or indeed by any relationship at all until another unexpected feature was recognized. As the helical angle is reduced the increased angularity of the line of contact results in the deflexion normal to the tooth (in terms of in. per lb. per in. of true load on the load line) increasing, but this effect is reversed with further reduction of helical angle, because of the relatively greater buttressing effect of the adjacent unloaded portions of the tooth as the contact line shortens further. The effect is modified by the factor required to convert the deflexions to a tangential direction to suit the calculations for pinion distortion given in Appendix 3.

The expression derived for the normal bending and shear deflexion of the loaded portion of a long tooth, except at the ends, expressed in inches x 10^6 per lb. per in. of load along the load line, is:

$$\delta_{BSN} \times 10^6 = \frac{\left(\dfrac{0{\cdot}065}{\tan^{1\cdot5}\psi_N} + \dfrac{0{\cdot}85}{y}\right)\dfrac{1}{\tan^{1/4}\psi_N}\cdot\dfrac{S}{p}\cdot Q}{1 + \dfrac{150\tan^2\psi_N}{e^y + 1}}$$

There is a noteworthy significance in the components of this expression. The first term in the numerator is due to parallel loading at zero helical angle and was readily assessed from measurements on short lengths /

lengths of teeth. The second term in which

$y = 0.655 / \sin\phi_N \cos^2\psi_N \tan\sigma$ (being proportional to the ratio of the longitudinal to radial lengths of the contact line)

represents the increase in deflexion relative to a more radially loaded tooth as the helical angle increases.

The exponential function in the denominator where $e = 2.718$ represents the stiffening effect of the long unloaded portions of teeth.

The term $Q = \dfrac{h_B/t}{1.25\, s/p}$ introduces for closer reflection of bending and shear effects, functions related to full depth and tooth thickness as distinct from active depth and pitch, with numerical constant arranged so that the term approximates to 1·0. The expression satisfies experimental results within ± 10 per cent.

The factor required to convert this bending and shear deflexion into a tangential direction per lb. per in. of face width of gear is shown:

$$\delta_{BST} = \delta_{BSN}\left[1.11\,\frac{p}{s}\cdot\frac{\sin\psi_N}{\cos^3\sigma}\right]$$

To obtain the total tooth deflexion, there must be added to this an amount representing the Hertzian depression and compression of the mating teeth. Substituting in equation (6) of Niemann's D.S.I.R. Sponsored Research Report No.3 (1949) the appropriate expression in a direction normal to the teeth is given by

$$\delta_{HN} \times 10^6 = 0.0887 \log_{10} 2550 \frac{P\sqrt{\dfrac{\cos\sigma}{\sin\psi_T}}}{\sqrt{P_N . D_e}} - 0.0166$$

The log term in δ_{HN} is relatively insensitive and hence it is reasonable to simplify the expression and to eliminate p, D_e, and P_N, which is a function of P (see Appendix 5), by taking $p/D_e = 0.050$ (normally varies 0·030- 0·075) and $P/D_e = 70$, whence and converting to

the /

the tangential direction again as above $\left(\frac{\delta_{HT}}{\delta_{HN}} = \frac{\delta_{BST}}{\delta_{BSN}}\right)$

$$\delta_{HT} \times 10^6 = \frac{H}{\sin\psi_N \cos\sigma}\left[\log_{10}\left\{\frac{4.72\cos\sigma}{\sqrt{H}}\right\} - 0.187\right]$$

where $$H = 0.0985\,\frac{p}{S}\cdot\frac{\sin^2\psi_N}{\cos^2\sigma}$$

This can, in fact, be substituted by the wholly empirical expression

$$\delta_{HT} \times 10^6 = \frac{(1 + 1.2\tan^2\sigma)\sin^{2/3}\psi_N}{10.25\ S/p}$$

Finally, the total tangential deflexion of the teeth in a circumferential plane in in. per lb. of tangential load per in. of face width of gear is given by

$$\delta_T = Q\left[\frac{\delta_{BST}}{Q} + \frac{\delta_{HT}}{Q}\right]$$

The expression within the brackets from which certain terms cancel is graphed in Fig.2.

It was originally thought to be something of a disadvantage that, by the method described, only the bending and shear effects could be actually measured and that resort had to be made to calculation for the Hertzian effect, but reference to the factor X_δ, which is introduced in Appendix 8, shows that there has in fact been considerable advantage in obtaining such a separation of the deflexions.

APPENDIX 3

MALDISTRIBUTION OF TOOTH LOADING WITH PINION DISTORTION UNDER COMBINED BENDING AND TORSION MODIFIED BY TOOTH DEFLEXION

The helix of a double helical gear with the significant maldistribution of load is the one adjacent to the driving end. A method of assessment is introduced by Fig. 10, it being assumed that the

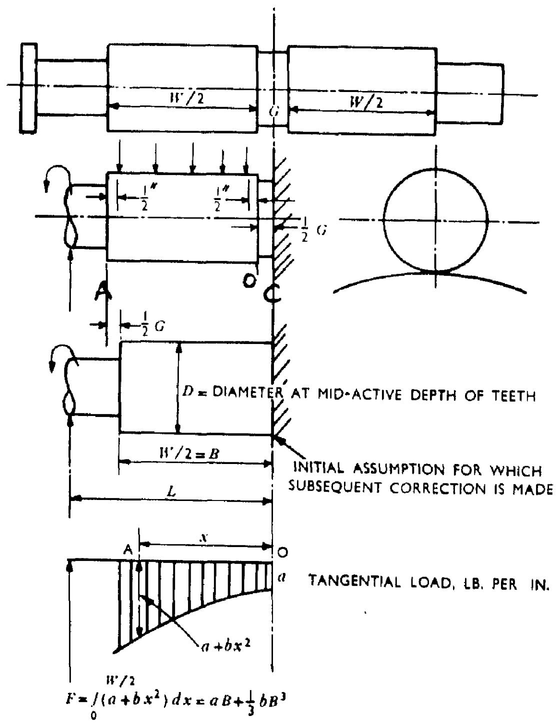


Fig.10. Loading on Pinion Helix.

loading along the helix takes a parabolic form represented by $a + bx^2$, x being distance axially along the helix from the inner edge of the helix, a correction being made subsequently for the gap.

It can readily be shown that if I = transverse moment of inertia and y_A = equivalent tangential bending/

/bending deflexion over the helix of a gear in single engagement

$$EI\cos^2\psi_T\, y_A = bB^6\left(\frac{1}{6}\frac{L}{B} - \frac{23}{180}\right) + aB^4\left(\frac{1}{2}\frac{L}{B} - \frac{7}{24}\right)$$

Likewise the twist over the helix, expressed as a tangential movement t_A at the mid-active tooth depth radius, is given by

$$EI\, t_A = \frac{25}{192}\, bB^6\left(\frac{D}{B}\right)^2 + \frac{15}{32}\, aB^4\left(\frac{D}{B}\right)^2$$

Taking δ_T = tangential flexibility of teeth per unit tangential load per in., from Appendix 2, the differential tangential load between the two ends of the helices bB^2 produces a movement $bB^2.\delta_T$ which must equal $(y_A + t_A)$. Substituting and transposing

$$\frac{bB^2}{a} = \frac{\frac{15}{32}\left(\frac{D}{B}\right)^2 + \left(\frac{1}{2}\frac{L}{B} - \frac{7}{24}\right)\sec^2\psi_T}{\frac{\pi}{64}E\left(\frac{D}{B}\right)^4\delta_T - \left[\frac{25}{192}\left(\frac{D}{B}\right)^2 + \left(\frac{1}{6}\frac{L}{B} - \frac{23}{180}\right)\sec^2\psi_T\right]}$$

the maximum load/inch $\quad P_m = a + bB^2$

the average load/inch $\quad P = \frac{F}{B} = a + \frac{1}{3}bB^2$

whence $\quad \frac{P_m}{P} = \frac{1 + bB^2/a}{1 + \frac{1}{3}bB^2/a}$

The cumbersome solution of the above equations may be graphed and an empirical relationship derived to give an expression for Pm/P in terms of the proportions of the gears and the tooth flexibility, it being reasonable and in the interests of simplicity to take

$$L = B + \tfrac{1}{4}D$$

In so deriving the final expression, consideration must be given to the effect, significant at the highly loaded end only, of the greater flexibility of the ends of the teeth due to absence of the full buttressing of continuous teeth. It is found from the experimental results of tooth flexibility that quite a close approximation/

/approximation is given by taking a corrected width of helix as (B - 3s). To this must be added a correction for the unloaded gap between the helices, a good approximation being half its axial width in relation to the whole width of the two helices.

The diameter of the pinion is confirmed by experiment to be correctly taken as the diameter at mid-active depth of the teeth, namely, D + (2m - 1)s. Taking $\cos\psi_N = 1{\cdot}015\cos\psi_T$ the final expression for the distortion coefficient for a double-helical pinion in single engagement without centre bearing is

$$\frac{P_m}{P} = 1 + \frac{\sec\psi_N}{16\,\delta_T \times 10^6}\left(\frac{W + \frac{1}{2}G - 6s}{D + (2m-1)s}\right)^{2\cdot5}$$

For a locked train pinion in double engagement at 180 deg. spacing, there is no bending term and the gap between the helices has no bearing on the distortion. The quantity bB^2/a is as above but without the bending terms (identifiable by the factor $\sec^2\psi_T$), and the final similarly derived expression is

$$\frac{P_m}{P} = 1 + \frac{1}{19{\cdot}5\,\delta_T \times 10^6}\left(\frac{W - 6s}{D + (2m-1)s}\right)^2$$

Important derivations from these values are theoretical expressions for the ratio of face width to pinion diameter to give the least intensity of local loading on the teeth.

Taking the case of a gear in single engagement and letting (W + ½G - 6s) = (W - 2), which conveniently is the same reduction in width customarily used to assess average tooth loading, it follows that

/it follows that

$$P_m = \frac{2F}{W-2} \cdot \frac{P_m}{P}$$

$$= 2F\left[\frac{1}{W-2} + A(W-2)^{1.5}\right]$$

where $$A = \frac{\sec \psi_N}{16\,\delta_T \times 10^6}\left(\frac{1}{D+(2m-1)s}\right)^{2.5}$$

Differentiating

$$\frac{dP_m}{d(W-2)} = 2F\left[-\frac{1}{(W-2)^2} + 1.5A(W-2)^{0.5}\right] = 0$$

when (W - 2) is at its most advantageous value of, say, (W' - 2), whence

$$\frac{W'-2}{D} = 2.6\left[1+(2m-1)\frac{s}{D}\right]\left(\frac{\delta_T \times 10^6}{\sec \psi_N}\right)^{1/2.5}$$

By a similar process for a pinion in double engagement at 180 deg. spacing

$$\frac{W'-2}{D} = 4.4\left[1+(2m-1)\frac{s}{D}\right]\left(\delta_T \times 10^6\right)^{1/2}$$

<u>Broad Relief of Helix.</u> The tooth surface towards the loaded end of the helix at the driving end of the gear can be shaved to reduce thickness about 0.00025 inch on the ahead flank just without loss of witness when marking the gears in mesh according to the best practice, the effect being tapered off to cover one-third of the helix width.

When such procedure is adopted

$$bB^2\delta_T = y_A + t_A - \frac{0.00025}{\cos \sigma}$$

For single engagement in the expression for Pm/P and W'/D, $\delta_T \times 10^6$ then becomes replaced by $\left(\delta_T \times 10^6 + \frac{250}{P \cos \sigma}\right)$ the transposition being subject to the condition that as a first approximation to the optimum face width ratio, P_m/P remains equal to 1.67. The inclusion of a loading term illustrates the fact that a static correction can only give effect as a function of the load.

For/

For double engagement the substitution of $\delta_T \times 10^6$ in the expression for Pm/P and W'/D is by $\left(\delta_T \times 10^6 + \dfrac{250}{1.5\,P\cos\sigma}\right)$ this being based on $\dfrac{bB^2}{a} = 3$ appropriate to the optimum face width ratio at $\dfrac{P_m}{P} = 2.0$ From the description in text $C_d = \dfrac{3}{5}\,\dfrac{P_m}{P}$ but $C_d \not< 1.0$ or 1.2 for single and double engagement respectively.

APPENDIX 4

MALDISTRIBUTION OF TOOTH LOADING AS FURTHER MODIFIED BY SLEW OF JOURNALS IN BEARING OIL FILM.

It may be deduced very approximately from work carried out by Pametrada on the attitude of journals in bearings that with a constant viscosity of lubricant corresponding to average pinion bearing conditions, an inequality of bearing load on a two-bearing pinion of ± 1 per cent produces a combined slew of the journals in the plane of the load of

$$\frac{N.D_b^2\,d}{0.5\times 10^6\,P_b} \quad \text{in } 1/1000''$$

the diametral bearing clearance also being expressed in these units.

For a double-helical pinion in single engagement, taking account of the different loading characteristic on the helix adjacent to the untorqued end it can be shown, taking moments, that the percentage inequality of load on the two bearings is $\pm\,0.57\left(\frac{P_m}{P}\right)^{1.5}$

Slew is similar in its effect to broad relief of the helix in reducing P_m/P, and hence from the reasoning given in Appendix 3, it follows that $\left(\frac{P_m}{P}-1\right)$ is reduced to, say, $\left[\left(\frac{P_m}{P}\right)'-1\right]$ by the factor

$$1+\frac{N.D_b^2\,d\times 0.57\left(P_m/P\right)^{1.5}\times 1000}{0.5\times 10^6\,P_b .\,P\cos\sigma .\,\delta_T\times 10^6}$$

The effect of the correction is usually less than 10 per cent and in this circumstance simplification by approximation is desirable and accordingly the following relationships are adopted:

$$d = D_b = \tfrac{2}{3}D\ ;\ \frac{D}{D_e}=1.1\ ;\ P_b = 150\ ;\ \sigma = 30^\circ\ ;\ \delta_T\times 10^6 = 0.42\,.$$

It /

It then follows that

$$\left(\frac{P_m}{P}\right)' = 1 + \frac{\frac{P_m}{P} - 1}{1 + J_s\left(P_m/P\right)^{1\cdot5}}$$

where

$$J_s = \frac{6\cdot85\ ND^2}{\frac{P}{D_e} \times 10^6}$$

Taking $D_e^2 N$ as a constant (see Appendix 7), giving $D^2N = 325$ (as adopted for the general case shown by Fig.5), and using C_D for the distortion coefficient in lieu of C_d corresponding to the effect of $\left(P_m/P\right)'$ instead of P_m/P, then

$$C_D = 1 + \frac{C_d - 1}{2\cdot35/K^{1/6}}$$

APPENDIX 5

HERTZIAN STRESS COEFFICIENT - C_H

Hertzian stress $$S_H = \sqrt{\frac{P_N . E . \cos\sigma}{D_e . \pi . \sin\psi_T (1-\lambda^2)}}$$

$$\therefore S_H^2 = \frac{P_N}{D_e} . \frac{E}{2\cdot85} . \frac{\cos\sigma}{\sin\psi_T}$$

$$= \frac{P}{D_e} . k_H \times 12\cdot8 \times 10^6$$

Hertzian stress coefficient $$C_H = \frac{P}{D_e} . k_H$$

$$= S_H^2 / 12\cdot8 \times 10^6$$

For the determination of k_H

$$P_N = P . \frac{p}{z} . \frac{\cos^2\psi_T}{\cos\sigma . \cos\psi_N} . C_D$$

$$z = \frac{(1-m)s}{\tan\psi_T} + \frac{1}{2} D_e . \cos\psi_T \left[\sqrt{\left(1+\frac{2ms}{D_e}\right)^2 - \cos^2\psi_T} - \sin\psi_T \right]$$

$$\doteqdot \frac{0\cdot81 . s . \cos\sigma}{m^{1/6} . \sin\psi_N}$$

and $\cos\psi_T \doteqdot 0\cdot985 \cos\psi_N$

whence $$k_H = \frac{C_D . m^{1/6} . \cos\psi_N}{s/p}$$

APPENDIX 6

OIL FILM PRESSURE COEFFICIENT - C_F

Martin (1916) has shown that between two loaded teeth:

Maximum oil film pressure $P_f \propto \mu . V_R . R_c^{0.5} / f^{1.5}$

Load carried per inch of tooth $P_N \propto \mu . V_R . R_c / f$

In expanding P_f in terms of P_N the following relationships are used :

P_N (see Appendix 5)

$V_R \propto D.N. \sin \psi_T$

$R_c \propto D_e . \sin \psi_T / \cos \sigma$

whence $$P_f^{2/3} \propto \frac{P}{D_e} . \frac{C_D \, m^{1/6}}{S/p} . \frac{\cos \psi_N}{N^{1/3}} \left[\left(\frac{D_e}{D} \right) \Big/ \cos \sigma \right]^{1/3}$$

Neglecting the near unity term in the square bracket, the oil film pressure coefficient may now be expressed as

$$C_F = \frac{P}{D_e} . k_F$$

$$\propto P_f^{2/3}$$

where $$k_F = \frac{C_D . m^{1/6} . \cos \psi_N}{\frac{S}{p} . N^{1/3}}$$

APPENDIX 7

TOOTH SURFACE PRESSURE COEFFICIENT - C_P

The true maximum shear stress below the working surface of the tooth must be reflected by a combination of C_H and C_F. An approximation to such a combination, at least so far as the relationship between the maximum stress and the applied load to the tooth is concerned, is made feasible without recourse to indefinite assumptions by the chance similarity of C_H and C_F, the latter containing the denominator term $N^{1/3}$ which the former lacks.

The simplest solution would be to take $N^{1/6}$ as representing the combined condition, and indeed no definite information is available to suggest any other course. General practice, in so far as it is presumed to reflect a consistent factor of safety, does, however, lend support to making this convenient assumption, as the following will show.

For many years (Douglas 1940) it has proved satisfactory to design for the widely differing conditions of primary and secondary gears by taking $P/D_e^{2/3}$ = constant, and despite general recognition of the theoretical value of P/D_e (Merritt 1954) as representing the Hertzian stress, it will be found that, comparing the successful highest loaded small and large gears, the $\frac{2}{3}$ index represents practice.

If a large number of gears of all sizes are plotted on logarithmic paper on co-ordinates of D_e and N, it will be found that, by and large, D_e varies inversely as $N^{1/2}$. There is nothing fundamental in this fact, any more than there would appear to be in the $P/D_e^{2/3}$ rule/

/rule, but taken together, when representing the same group of successful gears, they reflect the constancy of $\frac{P}{D_e^{2/3}} \cdot \frac{N^{-1/6}}{D_e^{1/3}}$ or $P/D_e . N^{1/6}$

It is proposed here to adopt this quantity as the basis for representing tooth surface stress, but the subject requires more investigation. Actually it is difficult to see how this can be done other than by a long-term examination of the service performance of gears designed on this basis, having good initial tooth contours unblemished by initial scuffing or any preventative measures taken in that connexion.

It does, however, seem possible that as, say, $D_e^2 . N$ increases relative to the cube of the K factor, reflecting an increase in the effectivelength of oil film relative to the 'Hertzian flat', that the oil-film pressure criterion will play a greater part in the combined effect, and that the index of N in the load formula will increase numerically. It is under such circumstances that the advantages claimed for the all-addendum tooth at high speeds, and known not to apply at low speeds, could be rendered effective, perhaps owing to a reduction in the peak of the oil film pressure which in fact reaches its maximum where it is being translated in its position relative to the meshing centre by a change-over in the direction of sliding.

Accordingly/

Accordingly, surface pressure coefficient C_p

s given by

$$C_p = \frac{P}{D_e} \cdot k_p = K \cdot k_p$$

here

$$k_p = \frac{1.15\, C_D \cdot m^{1/6} \cdot \cos\psi_N}{\frac{S}{P} \cdot \left(N/1000\right)^{1/6}}$$

The numerical coefficients for k_p and subsequently or k_R and k_S have been chosen to meet the equirements discussed in the text.

APPENDIX 8

ROOT STRESS COEFFICIENT - C_R

Root bending stress at approximately mid-height of root fillet of pinion teeth

$$S_R = \frac{P_N . \cos\psi_N . h_B . X_\beta . X_\delta}{\frac{1}{6} t_B^2}$$

$$= \frac{P}{D_e} . k_R \times \cancel{145}\ 130$$

Root stress coefficient

$$C_R = \frac{P}{D_e} . k_R = K . k_R$$

For the determination of k_R

P_N (see Appendix 5)

$$h_B = h - \tfrac{1}{2}(s + r_f)$$

$$t_B = (0{\cdot}48p + 2h_B \tan\psi_N)\left[1 - 3\left(\frac{s}{p}\right)^{1{\cdot}5}\frac{\cos^2\sigma\,(1-m)^{1{\cdot}5}}{\tan\psi_N . D/p}\right]m^{1/12}$$

The expression in the square bracket and following thereafter gives an approximation to reduction in thickness from rack to involute form.

$X_\beta = 1 + \frac{0{\cdot}15}{r_f}\left(0{\cdot}48p + 2h_B\tan\psi_N\right)$ is a stress raiser for the rack tooth fillet as adopted by Timoshenko and Baud (1926). The correction is not susceptible to more elaborate treatment, the fillet curve varying as it does with different diameters of gear cut by the same hob.

X_δ is a stress modifier having regard to a helical tooth as distinct from a spur tooth to which the stress would otherwise apply. Measurements have been taken from a strain gauge recessed into the supporting flange in way of the root fillet to obtain relative values from one of the specimen teeth described in Appendix 2 and it has been determined that, as compared with a spur tooth, the root stress varies very/

/very closely as the measured normal deflexion of the teeth (excluding Hertzian depression) as given by δ_{BSN}. It is therefore convenient to take X_δ as the ratio of this deflexion at the required helical angle to the deflexion at zero helical angle. It is a function of flank angle and helical angle and is graphed in Fig. 11

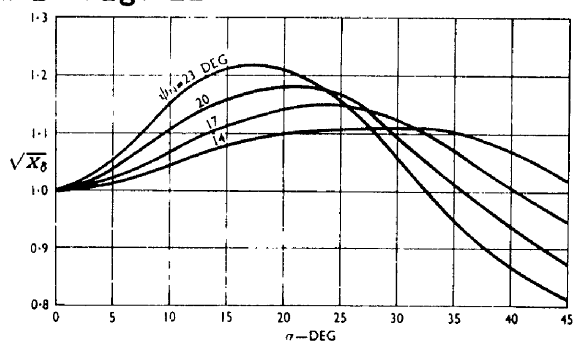


Fig. 11. Relative Deflexion of Helical to Rack Teeth

the square root of the function being taken for convenience in plotting. (The position of maximum stress lies about one-sixth the length of the line of contact from and within the normal section through the outer end of the line.)

Values of $X_\delta / 1{\cdot}67 \cos^2\sigma$ are given in Table 3, from which it will be seen that for σ = 30 deg. both tooth forms d and e have the value 1.0.

Table 3. Values of $X_\delta/1{\cdot}67 \cos^2\sigma$

Flank angle, ψ_N, deg.	$X_\delta/1{\cdot}67 \cos^2\sigma$				
	Helical angle, deg.				
	0	10	20	30	40
14½	0·60	0·68	0·83	1·00	1·16
16	0·60	0·70	0·86	1·00	1·09
20	0·60	0·76	0·95	0·96	0·90
22½	0·60	0·80	1·00	0·92	0·80

It is convenient to take a function of tooth form

$$Y = \frac{2{\cdot}375\, S/p}{\cos\psi_N} \cdot \frac{h_B}{S} \cdot X_\beta \cdot \cos^2\psi_N \cdot \sin\psi_N \Big/ \left[0{\cdot}48 + \frac{2h_B}{p}\tan\psi_N\right]^2$$

Particular/

Particular values of Y are given in Table 1, from which it is seen that each of the tooth forms c, d, and e has value 1.0.

It can now be stated that

$$k_R = \frac{D_e/p}{\underset{25.5}{\cancel{28.5}}\left[1 - 3\left(\frac{s}{p}\right)^{1.5} \frac{\cos^2\sigma\,(1-m)^{1.5}}{\tan\psi_N \; D/p}\right]^2} C_D \cdot \frac{Y\cos\psi_N}{s/p} \cdot \frac{X_\delta}{1.67\cos^2\sigma}$$

APPENDIX 9

SCUFFING CRITERION - C_S

In addition to the proportionalities given in Appendix 6 Martin (1916) has also shown that the effective length of loaded oil film between two teeth

$$l_f \propto (f.R_c)^{0.5}$$

The author has previously suggested (Davis 1941) that a criterion of failure of the oil film at the incidence of scuffing may be taken as an expression of work done on the film at the end of recess, viz.,

$$P_f.\frac{l_f}{f}.\frac{V_s}{V_R}$$

viscosity and coefficient of friction being constant at the conditions pertaining to failure.

For convenience the square root of this expression is taken and transposed becomes

$$C_s = \frac{P_N}{(V_R.R_c)^{0.5}}\left(\frac{V_s}{V_R}\right)^{0.5} \times \text{a constant}$$

In expanding the terms, the following additional relationships are used:

$$\frac{V_s}{V_R} \doteqdot \frac{2.67\ m.S.\cos^{5/3}\sigma}{D_e(D_e/D)^{0.5}\tan^{5/3}\psi_N}$$

$$\frac{\sin^{5/6}\psi_N}{\cos^2\psi_N} \mathrel{\dot{\propto}} \tan\psi_N$$

There was an earlier impression that for an equal addendum gear (m = 0.5) the conditions appertaining to the beginning of approach were more onerous and should supplant consideration of those for the end of recess, and this conception influenced the adoption of m = 0.6 for many gears. Recent observations have shown this to be misleading, and also that conditions promoting scuffing in approach and recess are approximately equal when m = 0.5.

This/

This theoretical criterion for scuffing may then be expressed as

$$\tau_s = \frac{P}{D_e}\sqrt{\frac{p}{D.N}} \cdot C_D \frac{\sqrt{p/s}}{\tan\psi_N} \cdot m^{2/3}$$

omitting the relatively insignificant product term

$$\left(\frac{D}{D_e}\right)^{1/4} \cos^{1/3}\sigma \quad .$$

From the considerable amount of evidence, largely accumulated during 1939-45 on classes of vessels rather than individual examples, it is apparent that this formula is at fault in practice in that it assumes an even bearing down the depth of the teeth, whereas at least in the case of a new vessel, this often does not apply.

It may be considered that for a new gear the breadth of marking bears no relationship to the depth of the teeth, or possibly that it increases somewhat with increase of pitch. A good reflection of practical results is obtained if, in fact, the depth of contact on a new gear is taken as proportional to $\sqrt{p}$ regardless of s instead of being theoretically s.

Thus the above criterion requires revision for new gears by inclusion of the factor $s/\sqrt{p}$, when the scuffing criterion may be expressed:

$$C_s = \frac{P}{D_e} \cdot k_s = K \cdot k_s$$

where

$$k_s = 100\, p\sqrt{\frac{s/p}{D.N}} \cdot \frac{m^{2/3}}{\tan\psi_N} \cdot C_D$$

(To avoid confusion it is to be noted it is fortuitous that this particular numerical coefficient is 100.)

APPENDIX 10

CARDINAL RELATIONSHIPS OF DESIGN

Flank Angle and Tooth Depth. An examination of the expressions for k_p and k_s reveals that while a large depth ratio s/p is beneficial to the former, it may have to be sacrificed to avoid scuffing. It is an unhappy fact that the amelioration is only to the square root power of the sacrifice, but it is mitigated by the maximum opportunity being taken when reducing depth ratio of increasing flank angle which benefits in proportion to its tangent in reduction of liability to scuffing.

Referring to Fig. 10, the tooth forms e, d, and c form a consistent series in meeting these requirements progressively and from the points of view referred to, the best tooth form for a particular example will be the one of these three nearest to the bottom of the list that is immune from scuffing.

But another consideration arises. The least tooth pitch for satisfactory root strength will be employed to ensure the best conditions of operation as reflected by quietness. The deep tooth form e is at the limit for practical hobbing with a tooth pitch of $\frac{7}{10}$ inch and it is considered that any smaller pitch of this form would lead to severe trouble with hob breakages. For pitches smaller than $\frac{7}{10}$ inch tooth form e should therefore be avoided.

Now, if the scuffing factor is low and the design is not being influenced by this consideration and, in addition, the tooth pitch is going to be less than $\frac{7}{10}$ inch, there is best advantage in matching the otherwise/

/otherwise unwanted reduction in s/p with the greatest possible increase in strength to keep pitch to a minimum. The consistent series d, c stemming from e in practice meets this requirement, but it is of interest to consider the minor divergence that is involved relative to the theoretical optimum condition. The attainment of maximum s/p combined with maximum flank angle for each tooth of the series has resulted in a relatively small fillet, which, by Appendix 8, is shown to give a maximum Y value to the series, reflecting a fillet stress raiser overbalancing the greater root thickness associated with a big flank angle. This tends to be an over-theoretical consideration because localized yielding would relieve this highly localized stress were the factor of safety to be severely reduced by, say, malalignment, but if taken at its face value, the conditions under immediate review would be best met by a reduction in s/p retaining 14½ deg. flank angle, towards a form intermediate to those represented by e and a in Fig. 10. No such intermediate form is, in fact, in existence and its variation from d would be so small as to make its inception facetious. As tooth form d is suitable for hobbing to the smallest desired pitch, no occasion can arise to justify the use of form a. Thus the consistent series, e, d, c meets all possible requirements.

Helical Angle. The common multiplier C_D of each of the expressions C_P, C_R, and C_S is affected by helical angle, but reference to Appendix 11 will show the significance of the statement that it is only in its application to the master expression C_P that it is of consequence in that the effect cancels out on the slave expressions/

/expressions C_R and C_S. The effective component of C_D is seen in the differential value dPm appearing in the expression for optimum width ratio in Appendix 3, namely, $\left(\cos\psi_N \cdot \delta_T \times 10^6\right)^{1/2.5}$. The effect of this factor is approximately to reduce or increase the allowable loading on 14½ and 16 deg. teeth of the consistent series by about 3 per cent depending upon whether the helical angle is reduced from 30 to 20 deg. or increased to 40 deg. respectively. In the case, however, of the 22½ deg. tooth of the series, the idea of consistency breaks down in that such an alteration in helical angle has virtually no effect on the load-carrying factor C_P as a study of δ_T values in Table 1 will show.

An inspection of the expressions C_P, C_R, and C_S will show that the only other respect in which helical angle affects the design is in its effect upon root stress. The effect is best shown by plotting component factors of C_R on a basis of σ, namely, the function

$$X_\sigma = \frac{\cos\psi_N}{S/p} \cdot \frac{X_\delta}{\cos^2\sigma} \Big/ \left[1 - 3\left(\frac{S}{p}\right)^{1.5} \frac{\cos^2\sigma\,(1-m)^{1.5}}{\tan\psi_N \cdot D/p}\right]^2$$

The character of X_σ in relation to σ does not alter within the normal limits of the variables concerned and the typical example given by Fig. 12 applies to m = 0.5 and D/p = 1.8. An examination of this graph reveals

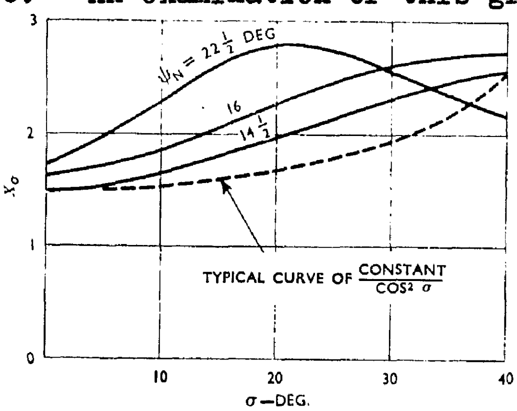


Fig. 12. Relative Stress at Root of Teeth

the further inconsistency of the series in that, from the/

/the point of view of minimum tooth stress, a flank angle of 22½ deg. is favoured by an increase to a helical angle of 40 deg. whereas the 14½ and 16 deg. teeth of the series are favoured by a small helical angle. The rate at which the load on these teeth increases with increase in helical angle is greater than the simpler theory based on $\cos^2\sigma$ (excluding tooth flexibility effects) would suggest, and for the purpose of comparison a curve of this simple trigonometrical ratio is shown on the same graph.

The conclusion to be drawn from this examination is that a reduction or an increase in tooth pitch of about 0.1 inch is brought about respectively by a reduction from 30 to 20 deg. or an increase to 40 deg. in helical angle for the 14½ and 16 deg. teeth, the advantage of a lesser pitch being offset by the disadvantage of a lower load-carrying capacity as previously described. It is therefore the intention to strike a compromise and base subsequent expression on a 30 deg. helix for these two tooth forms. In the case of the 22½ deg. tooth, subsequent analysis will show that its use could only be justified to take advantage of any amelioration of surface loading which the all-addendum characteristic might convey at high speeds.

<u>Addendum Distribution.</u> An examination of the expressions for both C_p and C_s shows that a disadvantage accrues in raising the proportion of pinion addendum to active depth above the minimum of 0.5 particularly in the case of C_s. From such a viewpoint the only minor exception which could be acknowledged would be that, when a design is not being influenced by the consideration of scuffing/

/scuffing, a slight sacrifice might be accepted in C_p by accepting a small increase in m to enable a tooth pitch to be employed that might otherwise have been regarded as just too small.

However, some provision must be made in the general assessment of this position to take account of the peculiar conditions arising in the case of the all-addendum pinion tooth and to which reference has been made in Appendix 7. There is evidence to support the fact that under the high-speed conditions therein referred to, the lubrication conditions dictating the limit that otherwise applies to C_p are upset, enabling a somewhat higher tooth pressure to be sustained. Values of C_p applying to high-speed naval gears listed in Table 2 support this view, but can be misleading to the extent that extremely limited time of operation at or near full power minimizes the demonstration of fatigue failure. First-hand experience available to the author is contradictory on the subject, but the impression is gained that the loss of s/p in adopting the large flank angle (that is necessary to depress C_s when an all addendum ratio is employed) and which reflects with disadvantage on the C_p factor is fully compensated by the effect of the all-addendum characteristic when the peripheral speed is sufficiently high. No account is taken of this possible amelioration in the derived formulae, but on the basis of satisfactory evidence, C_{MP} as described in Appendix 11 might be increased from 1.0 for high-speed all-addendum gears as a reflection of a peculiar lubricating condition; in such a way no confusion of the scuffing slave factor C_s/C_p is introduced.

APPENDIX 11

DERIVATION OF A LOADING FORMULA

It is the intention to express a loading formula as a C factor, where, K being the well-known P/D_e ratio,

$$C = K . k_P \left[\frac{k_R}{k_P} \text{ or } \frac{k_S}{k_P} \text{ or } 1{\cdot}00 \right]$$

whichever is greatest.

This method of presentation has certain advantages which will become apparent. The terms outside the bracket represent the master equation, the bracketed quantities being slaves which only supply their quota to the final product if the conditions they represent, respectively root stress and propensity to scuffing, are vital in indicating the limitation of permissible loading.

There is an essential difference in the values to be desired of the two slave functions. For a well-designed gear with minimum pitch of teeth, for full loading $C_P = C_R = 100$ and therefore $k_P = k_R$ and $\frac{k_R}{k_P} = 1{\cdot}0$. Natural limitations in available pitches will usually result in k_R/k_P being different from and preferably less than 1.0. On the other hand, for full loading C_S will only equal 100 if the conditions of running make consideration of scuffing necessary, and under other circumstances to make $C_S = 100$ would be to produce a grotesque design.

The best pitch can most readily be determined by equating k_R and k_P , but to enable this to be done to embrace the tooth forms covered by the series c, d, e, already established as relevant, it is necessary for simplicity of calculation and comparison to postulate a series of tooth forms differing slightly from the actual/

/actual standards and which are defined by the following relationships:

$$\cos \psi_N = (s/p)^{0.17}$$
$$\tan \psi_N = 1/4.75\,(s/p)^{1.42}$$
$$Y = 1.00$$
$$\frac{X_\varepsilon}{1.67 \cos^2 \sigma} = 1.00$$ (which presumes $\sigma = 30^\circ$ when $s/p = 0.915$ or 0.760 and $\sigma = 20^\circ$ when $s/p = 0.636$)
$$D_e/D = 0.875$$
$$m = 0.5$$

The involuting expression forming the squared term in the denominator of k_R is substituted by indices for the terms D_e/p and s/p together with an appropriate numerical coefficient to reproduce the thickness reduction without significant error, whence using " to differentiate from the previous expressions.

$$k_p'' = 1.0 \times C_D \Big/ \left(\frac{s}{p}\right)^{0.83} \left(\frac{N}{1000}\right)^{0.17}$$
$$k_R'' = \left(\frac{D_e}{p}\right)^{0.83} C_D \Big/ 11.75 \left(\frac{s}{p}\right)^{0.27}$$
$$k_s'' = 8.8\, p \left(\frac{s}{p}\right)^{1.92} C_D \Big/ D_e^{0.5} \left(\frac{N}{1000}\right)^{0.5}$$

Now equating k_R'' and k_p''

$$p = \frac{D_e}{19.5} (s/p)^{0.67} (N/1000)^{0.20}$$

If pf is the pitch actually to be fitted then

$$\frac{k_R''}{k_p''} = \left(\frac{p}{p_f}\right)^{0.83}$$

In preliminary design this is taken as equalling 1.0.

An expression for k_s''/k_p'' is obtained from the formulae quoted but this contains p which obviously must be replaced by p_f, but it is convenient to follow this by multiplying by p/p, the numerator being in the expanded form already given above which neatly combines with other terms in the equation and the denominator being bracketed with p_f to give p_f/p, which would be taken as 1.0 in the initial stages of design when neither p nor p_f were known, whence/

/whence

$$\frac{k_s''}{k_p''} = \frac{\left(\frac{s}{p}\right)^{3.4} D_e^{0.5} \frac{p_f}{p}}{2.2\left(N/1000\right)^{0.125}}$$

The full significance of depth ratio in its effect upon scuffing is clearly seen from this equation.

The C factor may now be written

$$C = K \cdot \frac{C_D}{\left(\frac{s}{p}\right)^{0.83}\left(\frac{N}{1000}\right)^{0.17}}\left[\left(\frac{p}{p_f}\right)^{0.83} \text{ or } \frac{\left(\frac{s}{p}\right)^{3.4} D_e^{0.5} \frac{p_f}{p}}{2.2\left(N/1000\right)^{0.125}} \text{ or } 1.00\right]$$

whichever is greatest.

APPENDIX 12.

MATERIAL CONSTANTS

All the foregoing remarks have assumed the use of materials and lubricants referred to in the second paragraph of the text under 'Tooth Loading Criteria'. A later section of the lecture refers to the use of other materials and as the effect of such variants is not the same on the different criteria, it is desirable to make suitable provision in the formulae when presenting a complete system of design.

A fundamental property of a material used for gear teeth is its fatigue strength, and in general, if not in particular, this may be taken as proportional to the U.T.S.

Assuming a chosen pinion steel to have U.T.S. equal to U as distinct from the normal 45 tons per sq. in. then the material constant C_{MR} which affects root stress, being a factor in the denominator of the root stress criterion C_R, is equal to U/45.

The effect on the pitting criterion C_P is more complex.

Appendix 5 shows Hertzian stress to vary as $K^{1/2}$.

Appendix 6 shows oil film pressure to vary as $K^{3/2}/N^{1/2}$.

Appendix 7 adopts the relationship that the stress below the surface of the tooth is a function of $K/N^{1/6}$

This might imply that this actual stress is regarded as being proportional to

$$\left(K^{1/2}\right)^{2/3}\left(\frac{K^{3/2}}{N^{1/2}}\right)^{1/3} \quad \text{or} \quad S_P \propto \frac{K^{5/6}}{N^{1/6}}$$

Then the corresponding material constant appearing in the denominator of the pitting criterion is $C_{MP} = \left(\frac{U}{45}\right)^{6/5}$ but this requires experimental verification and possible qualification with relation to pitch line speed, for which/

/which a small gear tester would be admirably suited.

It is assumed in each of these cases that no less than a corresponding increase is made in the U.T.S. of the wheel rim material.

The U.T.S. of the materials employed have no bearing on the propensity of a well-finished gear to scuff, this being some function of the relation between the molecular structures of the materials themselves and the properties of the lubricant. Disk testing in a laboratory can be a guide to the development of new materials and lubricants in this connexion, and the running of test gears under carefully controlled conditions should eliminate bitter experience afloat. The corresponding material constant in the denominator of the scuffing criterion is C_{MS}, a value lower than 1.0 indicating that by experience a particular combination of materials is liable to scuff at a proportionately lesser load than the normal materials referred to in the first paragraph of this appendix. The reference made in the text to factors of safety shows that C_{MS} must be cast to correspond with the fringe of scuffing when $C_S = 120$.

APPENDIX 13

SOME NOTES ON THE PROCEDURE REFERRED TO IN APPENDIX 2, ESTABLISHING VALUES FOR TOOTH FLEXIBILITY

Computation of Measurements

If two large helical toothed wheels are in mesh, the length of the line of contact between two teeth is a maximum; this is referred to as the rack tooth contact line and may conveniently be divided into 100 equal sections, commencing 0 at the tip to 100 at the root of the tooth.

If one of the wheels now be substituted by a pinion of average diameter relative to the pitch of the teeth, rack contact will commence at 0 but will terminate at less than 100. The contact line on the pinion tooth may be similarly equally divided into 100 equal sections commencing external to the line at point $0'$ corresponding to the full nominal active tooth depth, lying on the circle which is tangential to the tip circle of the wheel and finishing at $100'$ at the tip of the tooth. On account of the curvature of the pinion its divisions are longer than the rack divisions.

The determination of the reference stations referred to in Appendix 2 is carried out geometrically giving a result of which the following is an example relating particularly to a 14" P.C.D. pinion and rack with 0·7" pitch teeth, tooth form (e), addendum distribution $m = 0{\cdot}5$:-

Rack contact commences 0, finishes 92, length 4·05" (100≡4·4")
Pinion " " $15'$, " $100'$

Reference stations, being centre points of six equally divided loading sections on full contact line on rack:- 8, 25, 42, 58, 75, 92

Ditto contracted to occupy shortened rack contact line distance of 92 :-
7, 23, 39, 53, 69, 85

By/

By geometry these points are in respective contact with the following pinion points :- 17′, 27′, 39′, 53′, 70′, 91′

The actual measurements were based on straight sample teeth of rack and appropriate involute profile respectively, the six stations referred to in Appendix 2 being 8,--- 92 as above. Interpolated readings were calculated for 7,--- 85 on the rack and for 17′,----91′ on the involuted form. To demonstrate their character these values are given in Table 4 for individual loadings of 1800 lbs.

Loading Position	Deflexion in $^1/10,000"$ at					
	A	B	C	D	E	F
	Rack profile					
7 = A	12.5	4.0	1.2	0.6	0.4	–
23 = B	4.0	9.0	2.9	0.9	0.6	0.1
39 = C	1.0	3.0	6.0	2.1	0.7	0.2
53 = D	0.1	0.7	2.2	4.2	1.6	0.5
69 = E	–	–	0.5	1.6	2.8	1.1
85 = F	–	–	–	0.3	1.2	2.0
	Involute profile					
17′ = A	2.8	1.0	0.1	–	–	–
27′ = B	1.4	3.8	1.1	0.2	–	–
39′ = C	0.5	1.6	4.7	1.3	0.3	–
53′ = D	0.5	0.6	1.9	6.0	1.5	0.4
70′ = E	0.2	0.5	0.8	2.3	8.2	2.0
91′ = F	–	0.1	0.3	1.0	3.1	12.5

Table 4

Taking the loads at A, B,--- as (a, b, ----) 1800 and the common combined deflexion of rack and involuted forms as $100 \times \frac{1}{10,000}$ ins., then six equations can be established /

established, viz :-

$$15{\cdot}3a + 5{\cdot}4b + 1{\cdot}5c + 0{\cdot}6d + 0{\cdot}2e = 100$$
$$5{\cdot}0a + 12{\cdot}8b + 4{\cdot}6c + 1{\cdot}3d + 0{\cdot}5e + 0{\cdot}1f = 100$$
$$1{\cdot}3a + 4{\cdot}0b + 10{\cdot}7c + 4{\cdot}1d + 1{\cdot}3e + 0{\cdot}3f = 100$$
$$0{\cdot}6a + 1{\cdot}1b + 3{\cdot}4c + 10{\cdot}2d + 3{\cdot}9e + 1{\cdot}3f = 100$$
$$0{\cdot}4a + 0{\cdot}6b + 1{\cdot}0c + 3{\cdot}1d + 11{\cdot}0e + 4{\cdot}3f = 100$$
$$0{\cdot}1b + 0{\cdot}2c + 0{\cdot}9d + 3{\cdot}1e + 14{\cdot}5f = 100$$

From which $a = 4{\cdot}54$, $b = 3{\cdot}63$, $c = 4{\cdot}78$, $d = 5{\cdot}05$, $e = 4{\cdot}73$, $f = 5{\cdot}51$. (The significance of the character of this solution is shown by Fig. 6, drawn for a different tooth form.)

The total load to give the assumed deflexion is

$$(a + b + c + d + e + f)\,1800 = 28{\cdot}24 \times 1800 \text{ lbs.}$$

Whence the combined deflexion $= \dfrac{100 \times 10^{-4}}{1800 \times 28{\cdot}24}/4{\cdot}05$

$= 0{\cdot}795 \times 10^{-6}$ ins/lb. per in. of contact length

No allowance is made for a small but indeterminate addition to stiffness consequent upon pinion tooth curvature along the line of contact

The expression for δ_{BSN} given in Appendix 2 has in fact been formulated on the basis of addendum distribution $m = 0{\cdot}6$. This alteration of standard has a quite negligible effect on the foregoing loading equations but the length of the contact line is reduced from 4·05" to 3·95" and hence the combined deflexion (which may now be termed δ_{BSN}) becomes

$$\delta_{BSN} = \frac{100 \times 10^{-4}}{1800 \times 28{\cdot}24}/3{\cdot}95$$

$= 0{\cdot}775 \times 10^{-6}$ ins/lb. per in. of contact length.

A simpler approximation can be made on the basis of measurements on the rack form only, assuming, initially, large wheels to be in mesh and taking the aforementioned reference stations 8, 25, 42, 58, 75, 92 and their mirror reflections. The deflexion readings are those obtained from the rack profile prior to the interpolations of Table 4, and after evening up are given in Table 5 for the same individual loadings.

Loading Position	Deflexion in $^{1}/_{10,000}$" at					
	A	B	C	D	E	F
8 = A	12·2	3·9	1·1	0·5	0·3	-
25 = B	3·9	8·7	2·8	0·8	0·4	0·1
42 = C	0·9	2·8	5·5	2·0	0·6	0·2
58 = D	0·1	0·6	2·0	3·8	1·5	0·5
75 = E	-	-	0·4	1·4	2·5	0·9
92 = F	-	-	-	0·3	1·1	1·9

Table 5

Taking the loads at A & F, B & E, C & D as (a, b, c) 1800 respectively and the combined deflexion as before, then :-

$$14{\cdot}1a + 4{\cdot}9b + 1{\cdot}7c = 100$$

$$5{\cdot}3a + 11{\cdot}6b + 5{\cdot}5c = 100$$

$$1{\cdot}9a + 5{\cdot}4b + 13{\cdot}3c = 100$$

From which $a = 5{\cdot}02$, $b = 4{\cdot}90$, $c = 3{\cdot}00$

The total load to give the assumed deflexion is

$$(a + b + c)\ 2 \times 1800 = 25{\cdot}84 \times 1800 \text{ lb.}$$

At this stage the length of the contact line may be taken as 4·4" (100) consistent with the nature of the assumption, or as being compatible with an average ratio of pinion diameter/tooth pitch, in this case 4·05" (92).

The first method gives a combined deflexion of

$$\frac{100 \times 10^{-4}}{1800 \times 25{\cdot}84/_{4{\cdot}4}} = 0{\cdot}945 \times 10^{-6} \text{ ins/lb. per inch}$$

and the second method

$$\frac{100 \times 10^{-4}}{1800 \times 25{\cdot}84/_{4{\cdot}05}} = 0{\cdot}87 \times 10^{-6} \text{ ins/lb. per inch.}$$

Comparison with these of the more correctly determined value of $0{\cdot}795 \times 10^{-6}$ shows the effect (in substituting a pinion for a wheel) of the redistribution of loading, firstly along the shortened line of contact and secondly on equalized short lines of contact.

The /

The second method , (converting at the same time to the basis of m = 0·6) gives a ratio

$\frac{\cdot 775}{\cdot 87} = \cdot 89\cdot$

Similar measurements and calculations made with pairs of teeth of forms (d) and (c) gave corresponding ratios ·90 and ·95, which may be fairly represented by the expression $0\cdot 82/\cos^2\psi_N$ (giving ·88, ·89 and ·96 for the three cases respectively).

The use of this factor enables estimates to be made of combined wheel and pinion tooth deflexions on the basis of wheel tooth measurements only and this has curtailed the number of measurements required to give a broad impression of the significance of other variables involved in their effect upon tooth flexibility.

Co-ordination of Results

The method by which the results have been co-ordinated is shown by the equation for δ_{BSN} in Appendix 2 and in which the first term in the numerator represents the combined deflexion of wheel and pinion teeth at zero helical angle, namely (when $y = \infty$) $\frac{0\cdot 052\, h_B/t}{\tan^{1\cdot 75}\psi_N}$

As the line of contact for this condition is parallel with the teeth (as for spur gears), deflexion measurements were able to be made on conveniently short teeth, loaded their full length and using the same equipment already described. Pinion and rack teeth were loaded at their mid active depth (50) and the deflexions were recorded at this same depth.

The second term in the numerator of the equation for δ_{BSN} and the term forming the denominator, take reasoned form about the parameter y, which by a trial and error examination of the experimental evidence, is found to be of particular significance, representing as it does the ratio of the length of the contact line along the tooth to the differential

radial /

radial distance between its ends. While the second numerator term nominally represents the increase in deflexion relative to a more radially loaded tooth as the helical angle increases, and the denominator term the stiffening effect of long unloaded portions of teeth, in fact the only distinguishing feature is that the latter is an exponential function which is appropriate to its designation, but there is no experimental evidence to delineate the terms.

Experimental values of δ_{BSN} obtained by the methods here described are shown plotted on a base of helical angle in Fig.13 which also shows the character of the derived expression and percentage discrepancies.

The derivation of the factor $\delta_{BST}/\delta_{BSN}$ for the conversion of the flexibility expression from representing conditions normal to a single pair of teeth to deflexions in a tangential direction per unit load per inch width of gear face is as follows :-

Total length of teeth in contact over 1" width of gear face = No. of teeth in contact x length of full tooth in contact over 1" axial band

$$= \frac{\gamma/\cos\psi_T}{p\cos\psi_T/\cos\sigma} \times 1/\cos\sigma$$

$$= \gamma/p.\cos^2\psi_T$$

$\therefore$ Tangential load per inch length of tooth $= P.p.\cos^2\psi_T/\gamma$

Normal load " " $= P_N = \dfrac{P.p.\cos^2\psi_T}{\gamma} \cdot \dfrac{1}{\cos\sigma.\cos\psi_N}$

$$= P.\frac{p}{\gamma}.\frac{\cos^2\psi_T}{\cos\sigma.\cos\psi_N}$$

Let x_N = deflexion normal to teeth = $P_N.\delta_{BSN}$

x_T = " tangential to gear = $P.\delta_{BST}$

$x_N = x_T.\cos\sigma.\cos\psi_N$

$$\therefore \delta_{BST}/\delta_{BSN} = P_N/P.\cos\sigma.\cos\psi_N$$

$$= \frac{p}{\gamma}.\frac{\cos^2\psi_T}{\cos^2\sigma.\cos^2\psi_N}$$

By /

By approximation in Appendix 5, $\zeta = \dfrac{0{\cdot}81\ s.\cos\sigma}{m^{1/6}.\sin\psi_N}$.

Take $\dfrac{\cos\psi_T}{\cos\psi_N} = \cdot 985$ (corresponding to $\sigma = 30^\circ$)

The value of m must be taken as for the calculation of the δ_{BSN} quantities, namely 0·6, the effect cancelling out so that δ_{BST} <u>does not alter with variation of m</u>.

Whence $\delta_{BST}/\delta_{BSN} = 1{\cdot}11\ p.\sin\psi_N / s.\cos^3\sigma$

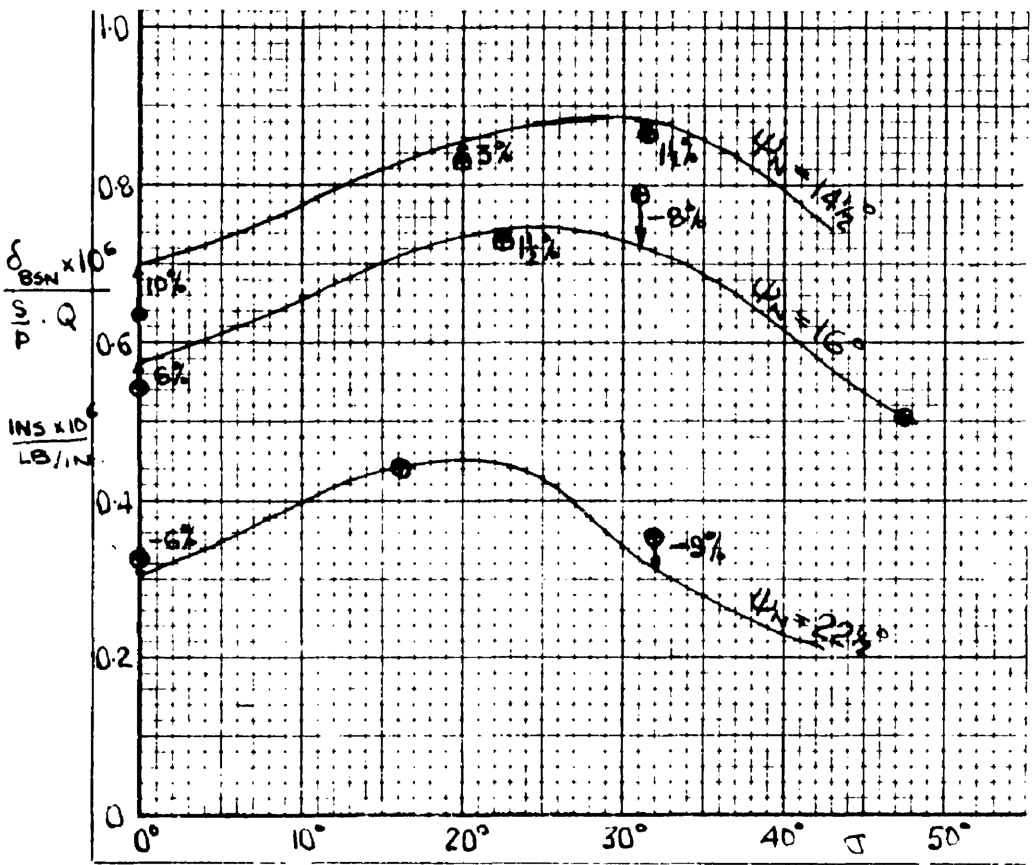


FIG. 13.

For End Effect see Appendix 15.

APPENDIX 14

ADDITIONAL NOTE ON TIP RELIEF

As referred to in Appendix 2, there is a corollary to the solution of the equations for variation in load along the contact line shown in detail in Appendix 13. This lies in determining the virtual variation in deflexion for uniform loading as reflecting an initial profile modification.

Taking the example following the loadings given by Table 4, as it is now stipulated that $a = b = c = d = e = f$, the combined tooth deflexions in $^1/_{10,000}$ inch along the load line must vary from station to station as defined by the L.H.S. of the six equations following the table namely 23·0 a, 24·3 a, 21·7 a, 20·5 a, 20·4 a, 18·8 a.

The total profile correction that would be required to effect this condition as between the two teeth is then 5·5 a for a normal load per inch of contact line of

$$6 \times 1800\ a/4.05 \quad \text{or} \quad 2660\ a \ \text{lbs/inch.}$$

The full load figure to be expected on a gear of this size may correspond with $a = \frac{1}{6}$, whence the required profile correction in 0·00009" total for the two teeth (compare with 0·00005" given in text under heading of "Tip Relief" which applied to teeth of $\frac{8}{10}$" pitch having $\psi_N = 16^\circ$ and $\frac{s}{p} = 0\cdot76$).

APPENDIX 15

SOME NOTES ON THE PROCEDURE REFERRED TO IN APPENDIX 3, ESTABLISHING DISTRIBUTION OF TOOTH LOADING ACROSS THE GEAR FACE.

Loading Characteristic

The algebraical form to be assigned to the loading characteristic can best be considered by calculating, for a few significant cases, the pinion deflexion at the mid point along the helix due to such loading and comparing this with the tooth deflexion to which it should be equal but for any inaccuracy of the assumption.

The form taken in Appendix 3, $(a + bx^2)$, was first adopted for simplicity of expression and although the following examination shows the function $(a + bx^{1\cdot 5})$ to be slightly more accurate, the empirical solution of the dependent equation for P_m/P in Appendix 3 in fact more nearly satisfies the initial assumption of $(a + bx^{1\cdot 5})$ than $(a + bx^2)$. While the descriptive use of $(a + bx^2)$ is retained in Appendix 3, subsequent calculations on the corrections for the gap between helices and end effects in this Appendix 15 are based on $(a + bx^{1\cdot 5})$.

Reverting to the comparison between $(a + bx^2)$ and $(a + bx^{1\cdot 5})$ it can first of all be shown on a basis of $(a + bx^2)$, taking $L = B + \frac{1}{4}D$, that if y_B and t_B be bending and torsional deflexions at the mid point of the helix corresponding to y_A and t_A respectively at the end :-

$$EI\cos^2\psi_T \cdot y_B = bB^6\left(\frac{239}{23040} + \frac{1}{96}\frac{D}{B}\right) + aB^4\left(\frac{23}{384} + \frac{1}{32}\frac{D}{B}\right)$$

and $$EI \cdot t_B = \frac{165}{3072}bB^6\left(\frac{D}{B}\right)^2 + \frac{25}{128}aB^4\left(\frac{D}{B}\right)^2$$

Taking a particular value of $\cos^2\psi_T$, say 0·9, values can be obtained for the ratio $\dfrac{y_B + t_B}{y_A + t_A}$;

from /

from a consideration of tooth deflexion these should, if the assumption made for the loading characteristic is good, be equal to $\frac{b(\frac{1}{2}B)^2.\delta_T}{b.B^2.\delta_T} = 0{\cdot}25$ or $\frac{b(\frac{1}{2}B)^{1.5}.\delta_T}{b.B^{1.5}.\delta_T} = 0{\cdot}354$ for $(a + bx^2)$ and $(a + bx^{1\cdot5})$ characteristics respectively.

On the alternative basis of $(a + bx^{1\cdot5})$, the expressions corresponding to those given in Appendix 3 for y_A and t_A are :-

$$EI\cos^2\psi_T . y_A = bB^{5\cdot5}\left(0{\cdot}200\frac{L}{B} - 0{\cdot}148\right) + aB^4\left(\frac{1}{2}\frac{L}{B} - \frac{7}{24}\right)$$

$$EI . t_A = 0{\cdot}161\, bB^{5\cdot5}(D/B)^2 + \frac{15}{32}aB^4(D/B)^2$$

and for $L + B + \frac{1}{4}D$

$$EI\cos^2\psi_T . y_B = bB^{5\cdot5}\left(0{\cdot}0143 + 0{\cdot}0125\frac{D}{B}\right) + aB^4\left(\frac{23}{384} + \frac{1}{32}\frac{D}{B}\right)$$

$$EI . t_B = 0{\cdot}0657\, bB^{5\cdot5}\left(\frac{D}{B}\right)^2 + \frac{25}{128}aB^4\left(\frac{D}{B}\right)^2$$

For this case $\frac{P_m}{P} = \frac{1 + bB^{1\cdot5}/a}{1 + \frac{1}{2\cdot5}bB^{1\cdot5}/a}$

where $$\frac{bB^{1\cdot5}}{a} = \frac{\frac{15}{32}\left(\frac{D}{B}\right)^2 + \left(\frac{1}{2}\frac{L}{B} - \frac{7}{24}\right)\sec^2\psi_T}{\frac{\pi}{64}E\left(\frac{D}{B}\right)^4\delta_T - \left[0{\cdot}161\left(\frac{D}{B}\right)^2 + \left(0{\cdot}200\frac{L}{B} - 0{\cdot}148\right)\sec^2\psi_T\right]}$$

The following examples, taken with alternative values of $\delta_T \times 10^6 = {\cdot}30$, ${\cdot}45$ and ${\cdot}60$, provide a suitable matrix for comparison.

I. Gears having 33% less width in relation to pinion diameter than the "optimum" as defined by :-

II. Gears having D/B ratio to give $\frac{P_m}{P} = \frac{5}{3}$ by the expression for single engagement given in Appendix 3, $(W + \frac{1}{2}G - 6s)$ being taken as $1{\cdot}85B$, $m = 0{\cdot}5$, $\cos\psi_M = 0{\cdot}96$.

III. Gears having 33% greater width than for II in relation to pinion diameter.

The /

The comparison is given by Table 6 which also vitally includes $\frac{Pm}{P}$ values as calculated by the expression referred to in II above and distinguished by the heading "Derivation adopted". The close agreement between actual and "correct" values for $(y_B + t_B)/(y_A + t_A)$ under the a + bx$^{1\cdot5}$ heading shows how well the characteristic is represented by this expression, while the correspondingly close agreement between the Pm/p values obtained on the same basis as compared with the "derivation adopted", well supports an empirical expression based on this form.

Basis of calculation		a + bx ^{2}			a + bx$^{1\cdot5}$			Derivation adopted		
$\delta_T \times 10^6$		·3	·45	·6	·3	·45	·6	·3	·45	·6
	Gears									
Matrix for comparison	I				1·76		1·33			
D/B	II				1·175	1·0	·89			
	III				·88		·665			
$\frac{y_B + t_B}{y_A + t_A}$	I	·378	–	·368	·379	–	·367	–	–	–
	II	–	·353	–	–	·352	–	–	–	–
	III	·343	–	·328	·342	–	·328	–	–	–
"Correct" value	All		·250			·354			–	
P_m/p	I	1·29	–	1·29	1·26	–	1·26	1·24	–	1·24
	II	–	1·77	–	–	1·70	–	–	1·67	–
	III	2·50	–	2·63	2·34	–	2·46	2·38	–	2·38

Table 6

A similar examination of the empirical expression derived for locked train pinions in double engagement at 180° spacing provides corresponding support for the treatment adopted, as shown by Table 7 for the same D/B ratios.

Basis of calculation		a + bx^2			a + bx$^{1\cdot5}$			Derivation adopted		
$\delta_T \times 10^6$		·3	·45	·6	·3	·45	·6	·3	·45	·6
$\frac{y_B + t_B}{y_A + t_A}$	I	·415	–	·416	·415	–	·415	–	–	–
	II	–	·415	–	–	·415	–	–	–	–
	III	·414	–	·414	·412	–	·414	–	–	–
"Correct" value	All		·250			·354			–	
P_m/p	I	1·22	–	1·20	1·20	–	1·18	1·17	–	1·15
	II	–	1·45	–	–	1·40	–	–	1·35	–
	III	1·85	–	1·75	1·76	–	1·67	1·67	–	1·59

Table 7

Correction for Gap between Helices

Referring to the second and third diagrams in Fig.10, the basic calculations for pinion deflexion are based on the moment of the bending moment diagram between O and A giving $y_A - y_o + B i_o^*$, the differential deflexion $(y_A - y_o)$ being the quantity required regardless of the fact that y_o is assumed to be zero; but the fact that the inclination at O, i_o, is taken as zero, represents in its full consequence the temporary neglect of the central gap between the helices.

The area of the bending moment diagram between O and C gives $EI i_o = -\frac{1}{2} G \left[bB^{3.5}\left(0.114 + \frac{1}{10}\frac{D}{B}\right) + aB^2\left(\frac{1}{2} + \frac{1}{4}\frac{D}{B}\right)\right]$ (again taking L = B + ¼D).

The expression for EIy_A already referred to in this appendix, being in reality the quantity for $EI(y_A - y_o) + EIBi_o$, therefore requires correction in the ratio

$$1 + \frac{\frac{1}{2} \cdot \frac{G}{B}\left[bB^{5.5}\left(0.114 + \frac{1}{10}\frac{D}{B}\right) + aB^4\left(\frac{1}{2} + \frac{1}{4}\frac{D}{B}\right)\right]}{bB^{5.5}\left(0.052 + 0.050\frac{D}{B}\right) + aB^4\left(\frac{5}{24} + \frac{1}{8}\frac{D}{B}\right)}$$

which by inspection closely equals $1 + G/B$

In the derivation of the fundamental relationship for $bB^{1.5}/a$, that which is really $\left(\frac{y_A}{1 + G/B} + t_A\right)$ is substituted by a function of δ_T. As δ_T but not $(y_A + t_A)$ appears in the derivation adopted for P_m/P it is convenient to regard the gap correction as applying to δ_T and to state that δ_T should be replaced by $\delta_T\left(\frac{y_A}{1 + G/B} + t_A\right)\Big/(y_A + t_A)$, or that $\left(\frac{P_m}{P} - 1\right)$ requires correction by the factor

$$\left(\frac{y_A}{t_A} + 1\right)\Big/\left[\frac{y_A}{t_A(1 + G/B)} + 1\right]$$

An /

* i = inclination.

An examination of the bending and torsional terms already quoted (taking $\cos^2\psi_T = 0.9$) gives

$$\frac{y_A}{t_A\left(1+\frac{G}{B}\right)} = \frac{\frac{bB^{1.5}}{a}\left(0.058+0.0555\frac{D}{B}\right)+\left(0.231+0.139\frac{D}{B}\right)}{\frac{bB^{1.5}}{a}\left(0.161\left(\frac{D}{B}\right)^2\right)+0.469\left(\frac{D}{B}\right)^2}$$

$= \text{say } \gamma$

The correction made in the derivation adopted for $\left(\frac{P_m}{P} - 1\right)$ is $\left(\frac{2B + gG}{2B}\right)^{2.5}$, g being a constant,

whence $$g = \frac{2}{G/B}\left[\left\{\frac{(1+G/B)\gamma+1}{\gamma+1}\right\}^{0.4} - 1\right]$$

Taking the same broad range of sample gears chosen for Table 5 and with alternative values of G/B, 0.1 and 0.5, the appropriate values of g are given by Table 8.

$\delta_T \times 10^6$		·3		·45		·6	
G/B		·1	·5	·1	·5	·1	·5
	Gears						
D/B	I	1·76		–		1·33	
	II	–		1·00		–	
	III	·88		–		·665	
g	I	·200	·192	–	–	·280	·260
	II	–	–	·320	·328	–	–
	III	·360	·360	–	–	·460	·420

Table 8

The variation in g is small enough to justify establishing a constant value of $\frac{1}{3}$ although $\frac{1}{2}$ has in fact been adopted to make suitable allowance for the 60^o chamfer which it is customary to apply to the ends of the teeth.

Correction for End Effect

Tooth flexibility measurements were extended to the ends of three specimen rack teeth to determine the relative increase of flexibility due to partial loss of end buttressing effect. Measurements obtained with tooth form (e) having 0·7" pitch and length of contact corresponding to $\sigma = 30^\circ$ are after evening up, given by Table 9 for 1800 lbs loads, deflexions being at the loading position in each case.

Loading and Deflexion Position	Deflexion in 1/10,000" with outer end of line of contact distanced from end of tooth by			line of contact central on long tooth as Table 5	inner end of line of contact distanced from end of tooth by	
	0	¼"	¾"		¾"	0
8 = A	21·0	17·5	15·2	12·2	12·2	12·4
25 = B	15·0	13·0	11·0	8·7	8·8	9·7
42 = C	10·5	9·3	7·7	5·5	5·8	7·8
58 = D	7·1	6·5	5·2	3·8	4·4	6·5
75 = E	4·5	4·0	3·2	2·5	3·1	5·0
92 = F	2·6	2·3	2·0	1·9	2·3	3·9

Table 9.

Considering the line of contact away from the ends of the teeth and excluding the effect of any load producing deflexion at another position on the contact line, the combined deflexion of A in contact with F is 14·1, B with E 11·2, and C with D 9·3, averaging 11·5. But an examination of the solution on a more correct basis in conjunction with Table 5 shows the average combined deflexion for the same loading to be

$$\frac{14{\cdot}1\,a + 11{\cdot}6\,b + 13{\cdot}3\,c}{a+b+c} = 13{\cdot}0$$

Considering /

Considering now the very commencement or the very end of contact, F on the driving tooth is in contact with A on the driven while B, C, D and E are unloaded and this provides a deflexion for the same intensity of loading of 3·9 + 21·0 = 24·9.

Consider now the position when the line of contact has just assumed its full length; A is in contact with F giving deflexion 21·0 + 3·9 = 24·9, B with E giving 15·0 + 5·0 = 20·0, C with D giving 10·5 + 6·5 = 17·0, averaging 20·6; the examination of the spreading effect of the deflexions in a central position suggests that this latter deflexion should be raised to 20·6 x 13·0/11·5 = 23·2.

When the contact line has moved $\frac{3}{4}$" further along the tooth or about ·65" across the face of the gear, the corresponding combined deflexion as so raised is 16·5.

An examination of the deflexions shows that by the time the full contact line has travelled 3" along the teeth the loss of end effect is complete.

The co-ordination of these estimates can best be achieved graphically in relation to a particular example, say a 14" P.C.D. pinion, m = 0·5, face width 27", gap 3" with average loading P = 1000lbs/in.

$\delta_T = \cdot 415$. Taking $\frac{P_m}{P} = 1 + \frac{\sec.\psi_n}{16\,\delta_T \times 10^6}\left(\frac{W + \frac{1}{3}G}{D}\right)^{2.5}$ (no chamfer)

$$P_m/_P = 1{\cdot}885 = \left(1 + bB^{1.5}/a\right)/\left(1 + \tfrac{1}{2.5}\,bB^{1.5}/a\right)$$

$$P_m = 1885 = a + bB^{1.5}$$

being the loading at the outer end of the torqued helix without allowance for any end effect

whence a = 410, being the corresponding loading at the central end.

The /

The tangential Hertzian flexibility is given by

$$\delta_{HT} \times 10^6 = \frac{(1 + 1\cdot2 \tan^2 30^\circ) \sin^{2/3} 14\tfrac{1}{2}^\circ}{10\cdot25 \times 0\cdot915} = 0\cdot06$$

$$\therefore \delta_{BST} \times 10^6 = 0\cdot415 - 0\cdot06 = 0\cdot355$$

δ_{BST} values are proportional to the combined deflexions deduced and when the δ_{HT} is added, the load intensity which the teeth carry must locally be reduced in inverse ratio to the summations as compared with the central value (·415 in this case).

This further step in the calculation is shown in tabular form in Table10 and a first approximation to the resulting loading on the torqued helix of the gear exemplified by the graph in Fig.14.

Axial distance from middle of line of contact to central end of helix inches	Relative combined deflexions as deduced	Corresponding δ_{BST} $\times 10^6$	δ_{HT} $\times 10^6$	δ_T $\times 10^6$	Local value of P: To give average P = 1000 without end effects	Local value of P: To give reduced average P to take account of end effects
0	24·9	·68	·06	·74	410	305
1·75	23·2	·635	"	·695	480	285
2·4	16·5	·45	"	·51	520	425
4·3	13·0	·355	"	·415	675	675
6·75	"	"	"	"	930	930
9·2	"	"	"	"	1240	1240
11·1	16·5	·45	"	·51	1510	1230
11·75	23·2	·635	"	·695	1610	960
13·5	24·9	·68	"	·74	1885	1395

Table 10

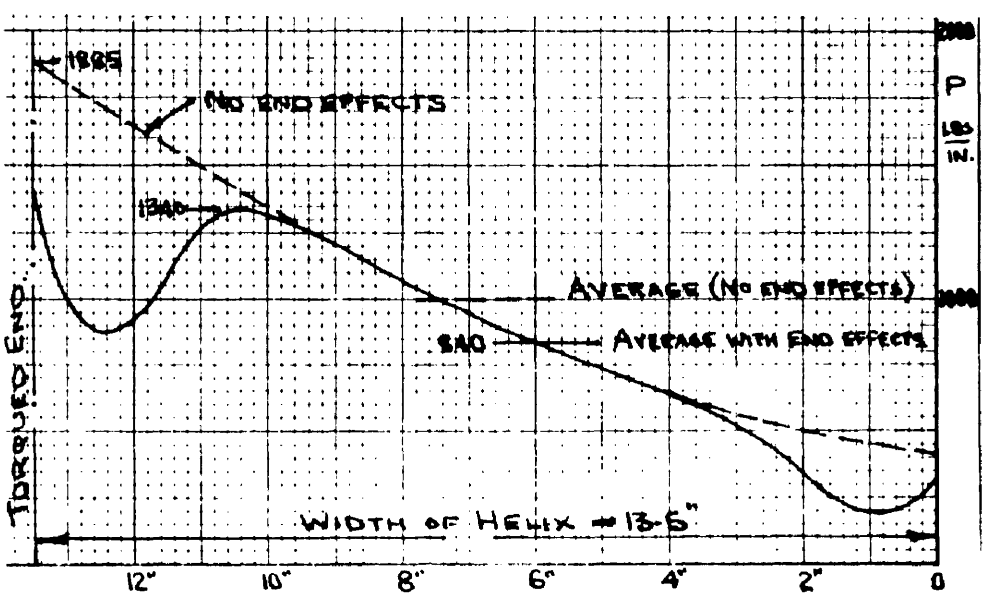


FIG. 14.

The graph shows that the loss of end loading reduces the average load to P = 840 lbs/inch (it is more convenient for the present purpose to stop short of stepping up the whole of the loading to restore the original average figure). The reduced maximum load is P_m = 1340 lbs/in (excluding the peak of 1395 lbs/in. at the very end of the teeth which could never apply because of relief, and which varies slightly in the passage of contact from one tooth to another).

Thus the modified value of P_m/p = 1·60.

If the correction is to be provided in the basic formula for P_m/p by an equivalent reduction in face width of the gear, the reduction to give the desired effect in this case is 4·0".

Corresponding results with other two racks suggest that this figure can best be taken as a function of active tooth depth which in this case is 6·25s. For simplicity 6s has been adopted and the corrected formula for P_m/p is as given by Appendix 3.

APPENDIX 16

SOME NOTES ON THE PROCEDURE REFERRED TO IN APPENDIX 4 GIVING AN APPROXIMATION TO THE EFFECT OF SLEW OF JOURNALS IN BEARING OIL FILM.

Based on the loading characteristic $(a + bx^2)$ as being sufficiently accurate for the purpose of this small correction, the system on the untorqued helix is given by

$$\frac{bB^2}{a} = \frac{-\frac{5}{32}\left(\frac{D}{B}\right)^2 + \frac{1}{8}\left(\frac{D}{B}\right) + \frac{5}{24}}{\frac{\pi}{64}E\left(\frac{D}{B}\right)^4\delta_T - \left[-\frac{5}{64}\left(\frac{D}{B}\right)^2 + \frac{1}{24}\left(\frac{D}{B}\right) + \frac{7}{180}\right]}$$

Taking $\delta_T = 0{\cdot}4 \times 10^{-6}$ the loading on both helices was evaluated for $\frac{D}{B} = 0{\cdot}8$, $1{\cdot}00$ and $1{\cdot}25$ respectively giving relative values as shown by Table 11.

D/B	Torqued Helix			Untorqued Helix		
	Maximum increment of load $\frac{1}{3}bB^2/a$	Uniform Load	Total	Uniform Load	Maximum increment of load $\propto \frac{1}{3}bB^2/a$	Total
0·8	1·94	1·00	2·94	2·22	0·72	2·94
1·00	0·63	1·00	1·63	1·47	0·16	1·63
1·25	0·29	1·00	1·29	1·25	0·04	1·29

Table 11.

The system of loads is shown diagramatically on Fig.15.

Taking moments about the bearing at the free end the percentage inequality of loading on the two bearings is found to be $\pm$ 1·0, 1·35 and 2·0 respectively for the three examples, corresponding with $\pm 0{\cdot}57\left(P_m/P\right)^{1{\cdot}5}$.

The further development of a working formula for the correction is described in Appendix 4.

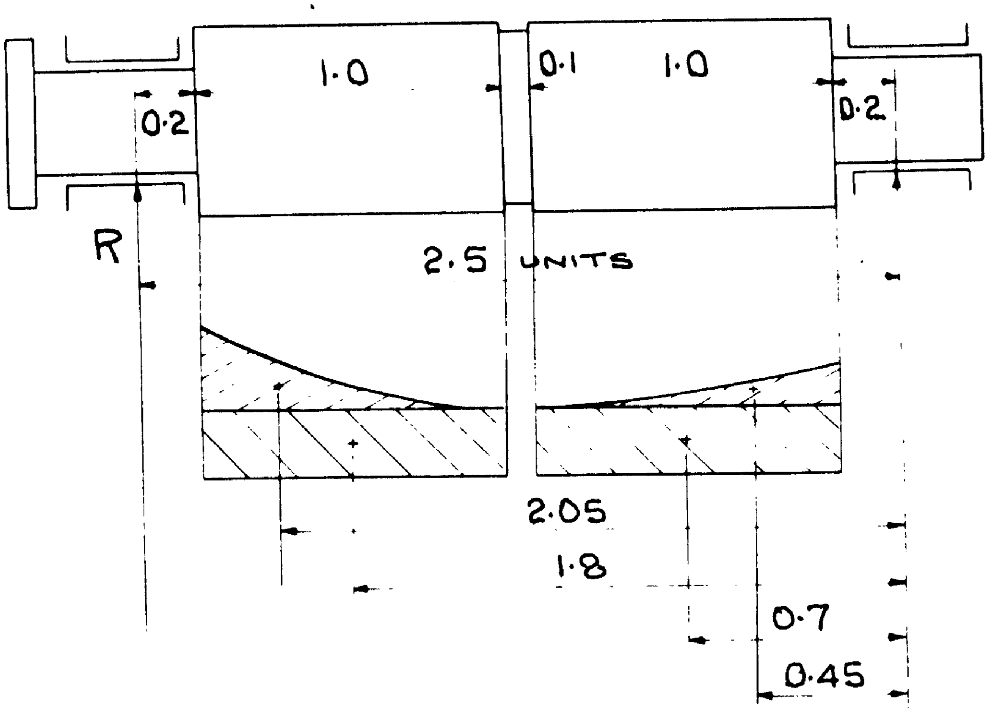


Fig. 15.

APPENDIX 17

NOTES ON APPROXIMATIONS IN APPENDICES 5-12

Empirical expressions are introduced for γ (Appendix 5), t_B (Appendix 8) and V_s/V_R (Appendix 9), each of which are accurate to ± 3% within the range of normal gear design.

The correct expression for γ is given in Appendix 5 itself, the first term of the summation being appropriate to the zone of approach and the second to the zone of recess.

The function for t_B is obtained from a graphical analysis.

The correct value of V_s/V_R as applying to recess is given by the expression $\sec.\psi_T \Big/ \left(\frac{D_e \sin\psi_T}{2\gamma_R} - \frac{(1 - D_e/D)}{\cos\psi_T}\right)$ where γ_R is the portion of γ referred to above as applying to recess.

In the assessment of stress at the root of the teeth, bending has been taken as the criterion; direct compressive stress varies with quite a different function of tooth proportions but as it averages only about 6% of the bending stress its neglect is justified by the complication and indeed the confusion avoided; shear stress gives a combined stress with bending which varies in relation to the pure bending stress approximately in proportion to $\tan^{1/6}\psi_r$, giving an extreme variation of ± 4%. With some additional complication this aspect can be incorporated into the calculations but without material effect on the final conclusions.

In Appendix 11, the conversion of the square of the involuting expression affecting tooth thickness /

thickness to the product $\dfrac{\cancel{0.41}\;0.47\,(D_o/p)^{0.17}}{(S/p)^{0.56}}$ is carried out with an error less than ± ~~3%~~. 5%

In each of these cases the relevant coefficients have been so arranged that the error is a minimum for gears of average proportions.

APPENDIX 18

TYPES OF MARINE REDUCTION GEARS TO WHICH THE FOREGOING ANALYSIS APPLIES

The three principal types of gear to which the criteria and various formulae apply are shown by the following three figures relating respectively to single reduction, double reduction articulated (or semi-articulated, depending upon whether solid or flexible type couplings are employed) and locked or divided train gears.

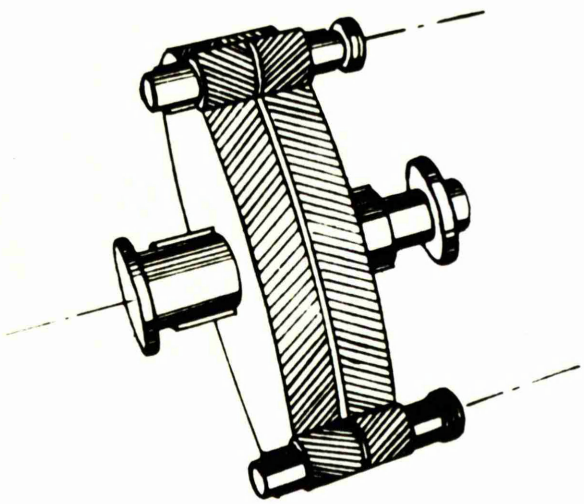


Fig. 16 - Single Reduction Gear.

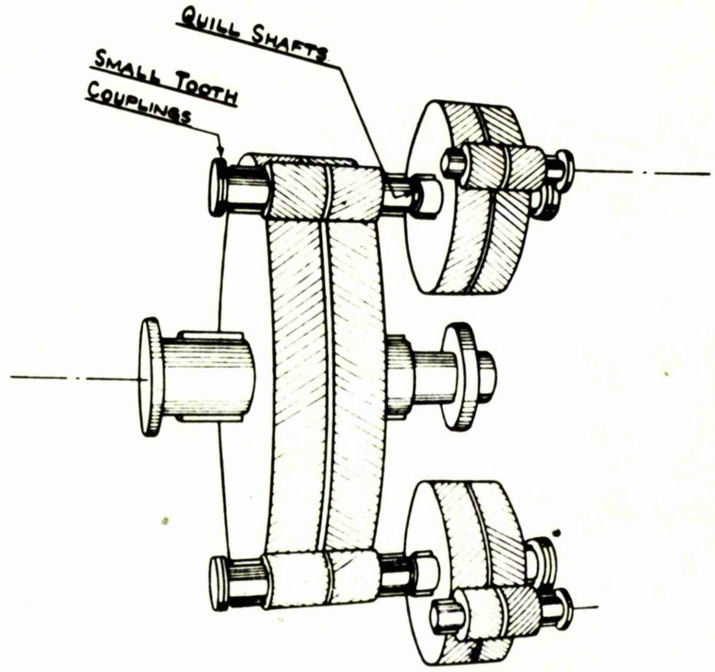


Fig. 17 - Double Reduction Articulated Gear.

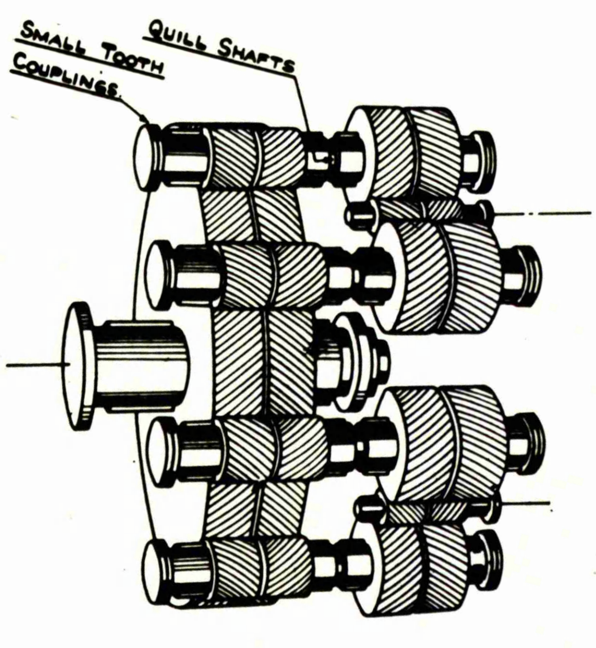


Fig. 18 - Double Reduction Divided or Locked Train Gear.

APPENDIX 19

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THE FAIRFIELD SHIPBUILDING
AND
ENGINEERING C^{o} L^{TD}

REPORT ON LOADING ON GEAR TEETH

OCTOBER 1941

A.W.DAVIS

R E P O R T

on

L O A D I N G on G E A R T E E T H

1941

LOADING on GEAR TEETH

FORWARD - Recent cases of "scuffing" of gear teeth have led to a general investigation and reconsideration of the knowledge relating to the loading and lubrication of gears, results of which are presented in this Report.

With particular reference to scuffing, the analysis has been greatly helped by the Admiralty Report E.N.24/250/41 dealing extensively with their experience in this connection. The final conclusions detailed in the present Report however give no evidence in contradiction to the essence of the theory as derived from the limited information formerly in our possession, and as stated in our letters to Messrs. Parsons dated 10th and 25th June 1941.

Upon more detailed investigation we find that the incidence of scuffing appears to be closely related to a simple expression, representing the work done per unit volume of oil film by the frictional forces arising from sliding action of the teeth, a function which is a measure of the temperature rise, and consequently the reduction in viscosity of the oil. The application of this criterion value (Cw) is illustrated with respect to the gears typified in your own Memorandum (Appendix L); and also in Appendix M to gears of other Classes of Vessels. Expressing the results of this analysis in relative numbers, we have found that of all the examples of Fairfield experience and of cases recently brought to our notice, scuffing has been in evidence between 238 and 121, the former being an extremely bad example, and the latter a borderline case. Gears which have not been prone to this defect have varied between 116 and 20.

The/

The expression is of a similar nature to the criterion proposed in the Admiralty Report and consists of the factors

$\frac{P}{D}$ $\frac{1}{\sqrt{N}}$ and $\sqrt{V_s/V_r}$ together with a refinement denoted by the pressure increase factor C_d associated with contact errors of the mesh and a term of fundamental importance incorporating length of contact zone, pitch and flank angle of teeth, by which "pressure per inch width of face" is converted into true normal pressure on the tooth flanks. The omission of this latter term has led to confusion of the main issue in several instances where different types of teeth have been compared on a supposedly common basis.

The occurrence of scuffing at low speeds is considered to be due to its inception when manoeuvring, and particular attention is drawn in the Report to the severe loading to which the gears may be unwittingly subjected under such circumstances, before the teeth have attained their ultimate polish. This conception provides a variable factor which could well account for the capricious nature of the defect in Vessels of the same Class.

The occurrence of this scuffing defect in such an unexpected manner has demonstrated that the basis of gearing design is not altogether consistent with present-day requirements; and such simple factors as $\frac{P}{D}$; $\frac{P}{\sqrt{D}}$; and $P\,(W/D)^2$, while they have their proper place in preliminary design, cannot be expected to reveal the true conditions under which the gears function such as to enable the safety margin to be reduced to a known minimum with sufficient certainty. The opportunity has therefore been taken in the accompanying Report to present the problem of gearing design/

/design in a rather more analytical form, although, for convenience in working on this proposed basis, the functions involved have been simplified to a maximum extent by graphical representation. The criteria which have been developed are -

PINION DISTORTION - C_d, the criterion being the maximum ratio by which it is estimated that tooth pressure is increased due to the contact errors. arising from bending and torsional distortion of the pinion, together with contact error of manufacture. The latter varies in its effect over a certain range, and while its nominal minimum value is included in C_d, its nominal maximum value is included in the associated criterion C_d'. Successive criteria each include this pressure increase ratio as an integral factor, although its inclusion is not inimical to the consistency of the C_w criterion with respect to scuffing.

ROOT STRENGTH, Cr - The chart associated with this criterion (Appendix H) clearly shows the relative strength of different types of standard teeth on a basis of pinion diameter. The root stress as indicated by C_r represents the maximum stress induced, having complete regard to contact error effects.

WORK DONE on OIL FILM, C_w - The significance of this term has been mentioned with regard to scuffing in recess. When considered with respect to approach in the manner later described, a value is obtained with small pinions which is greatly in excess of that associated with scuffing in recess. There is no reason to suspect the validity of the criterion on this account since the actions occurring in approach and/

/and recess are so different, particularly when pinion teeth tips are not relieved, that no reason exists why a given local drop in oil viscosity should produce the same effect in each respective case. Comparison between the two conditions is rendered less positive by the uncertainty attaching to the definition of sliding/rolling ratio in approach; excessively high values of C_w would not have occurred in approach if this ratio had been taken on the same basis as indicated in Admiralty Memorandum E.N.24/250/41, but it is considered that the basis chosen in this Report (taking $Vr = YO_p$ instead of YO_w as described in Appendix C) gives a fairer estimate of the true condition of the oil film; it also demonstrates the unsuitability of contact in the earlier stages of the approach zone in which experience has shown failure of oil film evidenced not in scuffing but in ridging and grooving.

MAXIMUM PRESSURE IN OIL FILM, C_p - If sliding/rolling is not excessive, C_w might, in consequence, be reduced to a sufficiently small degree to conceal the fact that the actual minimum pressure in the oil film may be excessive, leading to pitting in the manner which will be described. This maximum pressure as estimated by the criterion C_p may therefore require individual attention in certain circumstances, particularly in the case of non all-addendum gears.

This criterion is taken from H.M.Martin's article on gear lubrication in Engineering 1916, and its approximate truth is verified by the fact that it forms part of the basic structure of the sliding work criterion C_w which in turn has been shown to be a reasonable representation of true conditions by the consistency of its relationship to scuffing.

In/

In designing gears to carry a maximum load, it is agreed that the recent policy of utilising the minimum tooth pitch consistent with root strength is most desirable. Further, there is no doubt that the gear is assisted in succesfully overcoming the excessive pressures associated with the early stages of running in, by the limited polishing of the pinion teeth.

A proposal to relieve pinion tips was included in the Report, prior to receipt of Commander Given's letter of 6th October requesting our views on the subject. Although Fairfield practice has so far not been to provide any such relief, I agree that it is most desirable, and although it could be, and for the present would have to be, provided by hand filing, there is no doubt that special hobs would make a more uniform job, although it does not follow that filing would be less satisfactory.

It is considered desirable that relief at the tip of the tooth should be provided for a depth of 1/16" towards the root, and the Parsons proposal of two standards of pinion hobs would be able to provide this within limits of ± 18% each group being suitable for the ranges 5" - 9" p.c.d., and 9" - 20" p.c.d. respectively. No trouble has of course been experienced with scuffing of pinions of the latter range having 7/12" pitch teeth, but it is considered that the provision of relief is an aid to the successful lubrication of the teeth, and as such is desirable in all instances.

By judicious choice, having regard to flank angle errors, "mating" pairs of hobs for cutting wheels could probably be chosen from existing groups in the possession of individual firms. This proposal is considered to be preferable to cutting the relief with a special hob in an independent operation.

There/

There seems little doubt that the cures of scuffing through the medium of hand work have in fact been effected by relief of the tips of the pinion teeth. It is the tips which are always most affected by the complaint, and, whether intentional or otherwise, it is a natural result that a greater proportion of the hand work should be applied to this region.

We are still of the opinion that the lubrication and working conditions of gears would be improved by the adoption of a pinion addendum equal to 75 per cent of the meshing depth. Although comparison between C_W or K_W values in approach and recess may not be an entirely reliable guide for reasons already indicated, the fact remains that comparison of the respective values for equal (50%) addendum $14\frac{1}{2}^{\circ}$ gears on this basis does emphatically indicate the lesser suitability of the commencement of the approach zone as compared with the end of the recess zone, conforming at least in principle with the known characteristics of this type of gear. Such a distinction would be less apparent with equal (50%) addendum gears having a $22\frac{1}{2}^{\circ}$ flank angle, and would be more than counteracted if the pinion addendum were increased to 75%. It is considered that this proposal, if put into effect, would not be accompanied by any risk of failure and it is to be borne in mind that the present lack of practical evidence in this direction is due to the simultaneous conversion made between extreme limits of both flank angle and addendum, without regard to the finer selection of intermediate conditions.

It is possible, however, that the analysis which follows may tend to clarify the comparison between these two extreme types of teeth, and show more precisely to what/

/what extent the variation of each of the factors, flank angle and addendum, is responsible for the difference in characteristics of the gears. Reference is made in Section I (e) to the fallacy of the theory of "the point of reversal of sliding", a factor conceived to account for pitting and grooving troubles, which being associated with oil film failure had escaped discrimination due to the limited scope of the simple criteria utilised. The all-addendum gear was a product of this conception, which probably would never have arisen had arrangements been made to observe the effects of progressive reduction of the pinion dedendum.

The effects of alteration of tooth pitch and addendum on the various criteria is indicated for gears of various classes in Appendix N.

Troubles to which gear teeth are prone when prudent limits of loading are exceeded may be listed as follows -

(1) Excessive smooth abrasion

(2) Flaking and pitting

(3) Scuffing or seizing

(4) Ridging and grooving

(5) Fatigue fracture of teeth in extreme cases.

Fundamental causes of these troubles, assuming that the cutting of the gears, the material, and the lubricant leave nothing to be desired, are -

1. Smooth abrasion: Excessive relative sliding of the tooth surfaces in conjunction with a high pressure in the oil film, is liable to cause undue smooth abrasion. The relatively great amount of work done on the oil film causes it to rise in temperature locally with consequent reduction of viscosity, so that in way of microscopic high spots, failure of the oil film may occur from time to time resulting in mild abrasion of the surface. The effect is likely to commence and be most prevalent where the sliding is greatest, i.e., remote from the pitch line, but it may spread to an extent on account of the convection of heat to adjacent oil and the distribution of small metallic particles.

2. Flaking and Pitting: Flaking and pitting may be caused by excessive pressure in the oil film (and consequently on the tooth surface) in the absence of any considerable sliding. The heavy local pressure on the tooth surface causes high fatigue stresses at the surface of the metal, which may result in the flaking of microscopic high spots/

/spots or the pitting of the surface of the tooth. Pressures which cause this trouble would also be associated with a degree of abrasion in zones subjected to sliding, and it should be noted that the latter action, as wear occurs, has the effect of transferring pressure to, and causing pitting at, the portion of the teeth not subjected to sliding, namely, the pitch line zone. It has been observed that it is in the region of the pitch line that pitting is most prevalent.

3. <u>Scuffing or Seizing:</u> Scuffing is considered to be a product of the same factors which result in smooth abrasion when encountered to a lesser degree. When the combination of pressure of oil film and sliding become altogether excessive, local failures in the oil film become more extensive, and the rupturing of high spots from the parent metal produces sufficient heat to involve seizure, with the consequence of an increase rather than a reduction in the surface irregularity - a process which rapidly becomes cumulative. The first evidence of the trouble occurs at a region where the sliding speed is great, and the oil film is weakest, namely, at the root of the wheel teeth where the pinion is terminating its contact.

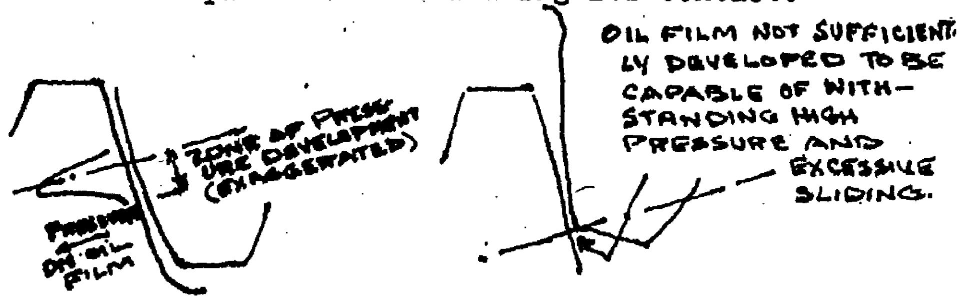

FIG. 1A.   FIG. 1B.

Fig. 1A shows the normal distribution of oil pressure over the effective length of contact, while Fig. 1B shows how at the final point of contact/

/contact the development of the oil film is seriously handicapped. The wheel tooth is affected prior to the pinion tooth probably because the sliding is more localised on the former, due to the advance of the contact point in contradirection to the rolling, the conditions on the pinion tooth being vice versa and consequently less severe. In addition the comparatively rough surface presented by the unpolished teeth of the wheel invites incipient scuffing rather than does the smooth surface of pinions polished in accordance with general practice.

4. <u>Ridging and grooving:</u> Ridging and grooving of the tooth surfaces is another product of the factors which cause smooth abrasion and scuffing, but in this instance it is plastic flow of the surface metal of the teeth brought about by the combination of excessive pressure in conjunction with sliding, subsequent to partial failure of the oil film, a failure which occurs as a result of the same excesses. The wheel tooth ridges in the region of the pitch line, the sliding being towards the pitch line both in approach and recess; the pinion tooth grooves at the pitch line, the sliding being in the reverse direction.

5. <u>Fatigue Fracture of Teeth</u>: Incidence of this trouble in the past was nearly always associated with material or bad gear cutting. Attention should, however, be paid to the root strength of pinion teeth in the determination of suitable pitch, and in considerations of undercutting with gears other than those of all addendum type.

Prevention/

Prevention of excessive loading in the design, construction and operation of high duty gears -

To be considered under four headings -

I. Design variables

II. Design Criteria

III. Manufacture of gears

IV. Operation of machinery

I. In the design of a pinion to transmit a certain horse power at known revolutions, the factors which may be varied are -

(a) Pitch circle diameter

(b) Width of face

(c) Flank angle

(d) Pitch and depth of teeth

(e) Distribution of addendum

(a) and (b). It is the obvious desire that the diameter and width of face be kept as small as possible, consistent with a performance that is to be free from any of the aforementioned defects. The two factors must be considered in close relationship in determining the working pressure per unit width of tooth, having regard both to conditions of lubrication and errors in contact, brought about by bending and torsional deflections of the pinion.

(c) Existing practice determines that the choice of flank angle must be $14\frac{1}{2}^{o}$ or $22\frac{1}{2}^{o}$ (measured axially on the hob), these angles representing almost extreme limits of general engineering practice. The $22\frac{1}{2}^{o}$ angle is undoubtedly superior to the $14\frac{1}{2}^{o}$/

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/$14\frac{1}{2}^{\circ}$ on account of the stronger pinion tooth which results therefrom, the lesser sliding which is involved, and the reduction in curvature of the pinion tooth flank, factors which more than compensate for reduction in the length of the zone of contact and increase of the normal pressure between the tooth faces due to the greater obliquity. It is unfortunate that in the simultaneous change made in favour of both the $22\frac{1}{2}^{\circ}$ flank angle and all addendum tooth, no opportunity arose to demonstrate to how a great extent the improvement in performance was due to the altered flank angle.

(d) Existing practice again determines that the choice of tooth pitch must be 1", 7/12" or 4/10" (measured axially on the hob), the depth of cut being in accordance with normal procedure. Sliding velocities are reduced with the 4/10" tooth, a feature which makes its use most desirable in the case of small diameter all addendum gears; the shortening of the zone of contact is precisely compensated by the increased number of teeth in contact at any instant. With bigger diameters and consequent greater loading per inch length, teeth of greater pitch must be employed to provide the necessary root strength.

(e)/

(e) Other factors having been determined the distribution of addendum remains to be chosen. An equal addendum and dedendum gear is highly unsatisfactory on account of the excessive rate of sliding/rate of rolling ratio which occurs at the commencement of contact and which may be five times as great with $14\frac{1}{2}^{\circ}$ gears as the value corresponding to the final point of contact at the end of recess; this leads to excessive smooth abrasion of the teeth and probable ridging and grooving unless the loading is very light. Furthermore, the pinion tooth is unnecessarily weakened at the root due to the involuting of the flanks. It should be noted that it was in conjunction with the $14\frac{1}{2}^{\circ}$ flank angles that these defects were manifest.

In direct contrast is the "All Addendum" gear in which more than half of the most satisfactory range of the possible zone of contact is sacrificed merely to avoid contact at the pitch point. In consequence the rate of sliding is excessive at the end of recession and instances occur in which the oil film breaks down due to the resulting high temperature.

The only argument in favour of the "All Addendum" gear (as distinct from a corrected gear having greater pinion addendum than dedendum) is this avoidance of pitch point contact; the evidence against the desirability of such contact is taken entirely from equal addendum $14\frac{1}{2}^{\circ}$ gears which advertised their unsuitability by pitting, grooving and ridging at the pitch/

/pitch line, but the possibility that these defects were manifestations of grossly excessive sliding at the commencement of contact has been ignored.

Reference is made to the pitch point as the point of "reversal of sliding" implying that a crosshead and its guide might be a suitable analogy. In actual fact the rate of sliding in the region of the pitch point is so small, positive or negative, in comparison to the rolling velocity that such reference is entirely academic and the points at which sliding becomes of practical importance in each direction are widely separated on the tooth flanks.

If equal sliding/rolling ratios were to be allowed in approach and recess with $22\frac{1}{2}^{\circ}$ gears, the pinion addendum would be about 55%-60% of the meshing depth, but as a conservative proposal it is suggested that gears with a 75% pinion addendum should be adopted. The sliding characteristics of gears of this type as compared with equal addendum and all addendum gears are illustrated on an accompanying graph.

II. Design Criteria - (for derivation see Appendix)

The following criteria are suggested for determination of the maximum allowable tooth pressures having regard to pinion distortion, tooth strength and lubrication.

Pinion distortion - Criterion of the distortion of the pinion due to bending and torsion as compared with the amount by which the teeth bend to correct the resulting contact error. The criterion is intended to represent directly the ratio of the maximum resulting increase in tooth pressure as compared with the mean value.

Cd/

$Cd = (k_d + {}^B/P)\ A$ ... Lower limit of static contact error of manufacture,

$C_d' = (k_d + {}^{2B}/P)A$ ... Upper - do -

$A,\ k_d\ \&\ B$ = coefft. from accompanying chart (See Appendix G)

P = 126000 SHP/ND (w-2)

N = pinion RPM

D = pinion diameter - inches

w = gross width of helices - inches.

Root Stress -

$Cr = P.C_d.k_r$ ... k_r = coefft. from accompanying chart (Appendix H)

Max. pressure of oil film -

Criterion of pitting and flaking -

$$Cp = \frac{P}{D} \cdot \frac{1}{\sqrt[3]{N}} \cdot Cd.k_p$$ k_p = coefft. from chart (Appendix J)

Work done on oil film - Criterion of condition of oil film to be considered with respect to commencement of contact (if not all-addendum) and end of contact. In each case the figure obtained should be related to actual instances of excessive smooth abrasion, ridging, grooving and scuffing in order of ascending severity. High values which might indicate possibility of scuffing at the end of recession would not infer similar conditions if applying to the commencement of approach, since in the latter instance the breakdown of the oil film is not aggravated as described in paragraph (3). Pitting may occur as a secondary effect if this criterion is excessive.

$$Cw = \frac{P}{D} \cdot \frac{1}{\sqrt{N}} \cdot Cd.k_w$$ k_w = coefft. from chart (different values for approach and recess) (Appendix K)

The/

The use of charts has been necessitated in order to avoid excessively complicated formulae on the one hand, or undue approximation on the other. The derivation of the formulae is indicated in the Appendix to this memorandum, together with examples of their application to several types of gear including those listed as A-M in Admiralty report E.N.24/250/41. A typical comparison is also given of all-addendum, 75% addendum, and equal addendum gears.

III. Manufacture of Gears:

It is not necessary here to draw attention to the necessity of accuracy in gear cutting machines and procedure to ensure minimum errors in respect of cumulative circular pitch, spiral angle, creep marking and form of tooth, excesses in any of which would involve the gears being subjected locally to loading in excess of the design allowance.

Local errors in the spiral angle call for limited hand-work to give contact over the full width of tooth face but the obvious objection to such a process being employed in the correction of excessive errors is the impossibility of maintaining the true contour of the teeth, particularly in the case of small diameter pinions where the curvature of the flanks is appreciable.

As the result of much experimental work in the application of the axial pitch gauge which has recently been developed by the Admiralty in conjunction with the N.P.L., satisfactory measurements have been made of several typical wheels and pinions, showing the magnitude of contact error (measured normal to the teeth) which/

/which is likely to arise in gears which are accepted as being of a high standard of accuracy. The results of the investigation are the subject of a separate report but it may here be mentioned that, irrespective of the width of the element, the contact error appears to vary between .0005" and .0015", so that in the worst instance, mating gears could have a combined contact error of .003" prior to hard points being relieved by hand. In considering the relative importance of these figures it is useful to recall that the contact error that has been permitted at full power over the width of the forward helix, due to the bending and torsion of the pinion, has varied from approximately .0005" to .0017", depending on the width of face, and with such allowances no evidence of pressure at the forward end of the helix in excess of that at the aft has been shown, indicating that the flexibility of the teeth is sufficient to take care of a contact error of this magnitude when evenly distributed. Incidently, there seems to be no justification for allowing a lesser contact error on a narrow than on a broad gear since as the teeth are helical, each individual tooth must bend in the region of the instantaneous band of contact, in order to correct the contact error in its path, and in so doing it is in no way affected by the width of face such as would be the case if the gear were of the spur type. It is unfortunate that it so happens that in the case of pinions, the involuting of the flank removes all trace of the radius which in the case of a rack or wheel is formed by the hob at the tip of the teeth, since it is the pinion rather than the wheel which is in need of the relief. In extreme cases there seems little doubt that scuffing is partly a consequence of this abrupt termination/

/termination of contact as described in paragraph (3), and if relief is provided at the tips of the pinion teeth the production of an adequate oil film is ensured up to the instant of final contact, thus more than compensating for the consequent reduction in the length of contact zone. In the case of small All Adendum pinions such relief also has the effect of reducing the sliding velocity where this is excessive. This provides adequate reason why the hand relief of the tips of pinion teeth should effect a cure for scuffing. It is considered that if it were made general practice that the tips of pinion teeth be slightly relieved by file, tapering the teeth for a depth of say 1/16" towards the root, the region subjected to the most severe condition would be benefitted, even although extreme evidence of oil film failure as exhibited by scuffing was not anticipated.

It is agreed that the polishing of the pinions provides them with a surface which is highly beneficial to the running of the gear in its early stages. The polishing process however, is only intended to remove high spots and if adequate witness of the hob marking is not retained, there is a severe risk of spoiling the contour of the tooth, should the grinding wheel be at all worn.

1V. Operation of Machinery -

At no time are gears subjected to such severe conditions as during the very early running life of the Vessel before the hard spots on thetooth surfaces have been worn down and the bearing has fully spread over the width of each helix. Gears which have scuffed even prior to the preliminary trial indicate that not only at full power is the effective loading heavy, but that the gears can be damaged by undue "acceleration" at low speeds.

With/

With particular regard to scuffing (even although it may not become manifest in sufficient severity to warrant comment), it is certain that during running in the portion of the teeth subjected to the greatest sliding action, will be relieved of minor surface irregularities more rapidly than those portions where the sliding is less severe. If this initial process is performed without excessive pressure, the "lumps" will be reduced gently, and, in consequence, the loading on the gears will be thrown more heavily on to the portion of the teeth subjected to less sliding and a permanent state of balance will be set up in which the highly sliding portions of the tooth surfaces will never again be so heavily loaded. Alternatively, if the initial running in process is too severe, local pressure will cause more extensive breakdown of the oil film, and seizure or scuffing may be liable to occur.

With regard to acceleration at low speeds it has been stated that the work done on the oil film is a function of $P/\sqrt{N}$ at full power, or say $p/\sqrt{n}$ at a reduced power corresponding to revolutions per minute. If the Vessel is steaming steadily at this lower speed $p' = P(n/N)^2$ or $p'/\sqrt{n} = P/\sqrt{N} \times (n/N)^{1.5}$ which is obviously less than $P/\sqrt{N}$ indicating lighter loading on the gear.

If the extreme though hypothetical case then be imagined that full steam be put on the turbines when steaming at n revolutions per minute, considerations of turbine efficiency show $p' = P(N/n)^{.5}$ or $p'/\sqrt{n} = \frac{P}{\sqrt{N}} \times N/n$ which is obviously greater than $P/\sqrt{N}$ showing the heavier loading on the gear as compared with full power.

The following table shows relative loads on H.P. pinions assuming full power revolutions 360 and the corres - ponding control pressure to be 250 lbs. per square inch gauge/

/gauge, the full power nozzles being open under all conditions. The full power loading is represented by 1.00 and relative loadings are indicated for other speeds appropriate to steady steaming and also to conditions represented by a sudden increase in control pressure. The analysis is of particular interest considered with regard to manoeuvring Ahead and Astern.

Revs. per min.	Control Pressure Steaming Steadily lbs/sq." G.	Relative Gear Loading (C_w) when control Pressure is sharply increased to lbs/sq.in.G.							
		0	25	50	75	100	150	200	250
120	0	0.16	0.51	0.79	1.11	1.41			
180	25	-	0.34	0.53	0.74	0.95	1.30		
240	75	-	-	-	0.55	0.71	0.97	1.37	
300	150	-	-	-	-	-	0.78	1.01	1.20
360	250	-	-	-	-	-	-	-	1.00

In conclusion it is considered that if more analytical attention is paid to the design of gears, possibly on lines similar to those suggested, avoiding the least satisfactory zones of contact in approach and recess, it should be possible by working up to allowable limits with more definite precision, to permit generally of greater loading on the gears and consequent reduction in weight and space. Such a policy would be assisted if the general practice were adopted of relieving pinion teeth tips as described. Furthermore, it must be recognised that the successful after-life of a heavily loaded gear is largely dependent on the care with which it is treated when new, and the greatest respect is demanded until the major surface irregularities of the teeth have been given adequate opportunity of being eased down.

In/

In the production of this memorandum, valuable reference has been made to the following -

Admiralty report on Damage of Turbine Reduction Gears by scuffing, E.N.24/250/41.

The Lubrication of Gear Teeth, H.M. Martin - Engineering 1916, vol.C11, p.120.

The Lubrication of Gear Teeth, H.E. Merritt - Transactions I.M.E., 1937.

Mechanical Gearing for Large Power Transmission L.M.Douglas - Transactions N.E.C.1940-41.

An accompanying table shows specific instances of how typical gears could, according to the views expressed herein, be reduced in general proportions without exceeding reasonable limits of loading.

APPENDIX -

The following items are included -

A. Derivation of contact error due to bending and twisting of pinion; compensating bending of wheel and pinion teeth; pinion distortion criterion C_d denoting ratio of increase of pressure on teeth.

B. Derivation of criterion of root stress of teeth, Cr.

C. Derivation of criterion of maximum pressure in oil film, Cp.

D. Derivation of criterion of maximum work done on unit volume of oil film, C_w.

E. Chart of Rate of Sliding/Rate of Rolling ratios for equal addendum $14\frac{1}{2}^{\circ}$ gears in comparison with other types.

F. Ditto to larger scale for gears other than equal addendum $14\frac{1}{2}^{\circ}$.

G. Chart of pinion distortion coefft. kd for different types of gears.

H/

H. Chart of root stress coefft. k_r for different types of gears.

J. Chart of maximum oil pressure coefft. k_r for different types of gears.

K. Chart of maximum work done on oil film coefft. k_w for different types of gears.

L. Tabular analysis and comparison of gears given in Admiralty report E.N.24/250/41.

M. Tabular analysis of other gears, Naval and Merchant.

N. Tabular comparison of typical gears incorporating different types of teeth, according to the proposed basis of design.

O. List of symbols used.

APPENDIX A -

Derivation of contact error due to bending and twisting of pinion in conjunction with errors of cutting; compensating bending of wheel and pinion teeth; pinion distortion criterion Cs -

In gears without centre pinion bearings the Forward helix of the pinion is subject to the greater distortion contact error since on the one hand the torque is geater than in the Aft helix, and on the other hand the bending deflection aggravates the torsional deflection in the Forward, while rather more than counterbalancing it in the Aft helix. Considering only primary effects, this contact error may be expressed in relative terms as follows -

Forward helix	Contact Error	(Torsion plus Bending=Total)	39+61=100
Aft helix	-do-	-do-	13-61=-48

Two secondary effects, however, play an important part. In the first place the pinion journals are slightly accommodated by the bearing oil film, thus permitting the unequal radial reaction forces, brought about by the uneven contact errors, to slew the pinion out of line, thus tending to equalise the errors on the two helices. Secondly, the bending errors are minimised on account of the redistribution of loading which occurs with the pinion in the bent condition, resulting in excess loading towards the ends of the pinion with a resultant reduction in the amount of bend, which may be shown to be approximately 1/3 in amount.

The/

The resulting relative torsion and bending errors are thus -

Forward helix	Contact Error	(Torsion plus bending plus slew = Total)	39 + 40 - 26= 53
Aft helix	- do -	- do -	13 - 40 - 26= -53

In assessing this contact error let -

P = Tangential load per inch net width of face = $\frac{126000 \text{ SHP}}{ND\ (w-2)}$

N = pinion RPM

D = pinion PCD

w = gross width of faces - inches

g = gap between faces - inches

α = circumferential pressure angle of teeth

= 25° - 33' for nominal $22\frac{1}{2}^{\circ}$ flank

= 16° - 35' for nominal $14\frac{1}{2}^{\circ}$ flank

Then the uncorrected bending error over one helix (proportional to 61 in relative figures quoted) can be shown to be approximately -

$$e_b = \frac{P}{1000}\left(\frac{w-2}{D}\right)^4\left(\frac{w+g}{w-2}\right)^3 \times \frac{7.65}{10^6}\left(1+\sin\alpha\tan\alpha\right) \text{ ins} \text{---}(1)$$

The corresponding uncorrected torsional deflection of the Forward helix (proportional to 39 in relative figures quoted) is approximately -

$$e_t = \frac{P}{1000}\left(\frac{w-2}{D}\right)^2 \times \frac{69}{10^6} \text{ ins.} \text{ - - - - - - - - - - }(2)$$

The corrected bending and torsional error over either helix (proportional to 53 in relative figures quoted) incorporating the slew correction as a proportion of the torsional error is -

$$e = \frac{2}{3}e_b + \frac{1}{3}e_t$$
$$= \cdot 187\left(\frac{P}{1000}\right)\frac{K_\alpha - 1}{B} \text{ inches} \text{ - - - - - - }(3)$$

where

$$K_\alpha = \frac{1}{1000\,w}\left[3\cdot4\left(\frac{w-2}{D}\right)^4\left(\frac{w+g}{w-2}\right)^3\left(1+\sin\alpha\tan\alpha\right)+15\cdot3\left(\frac{w-2}{D}\right)^2\right]+1$$
$$\text{- - - }(4)$$

This/

This function is plotted for different types of teeth on an accompanying chart (Appendix G) which also gives values of B. The significance of B and u are described hereafter.

The relative effect of such errors in increasing tooth pressures in various gears cannot be considered without reference to superimposed contact errors, due to irregularities of gear cutting which even in high class work are of a magnitude comparable with the distortion errors. After limited handwork in reducing the hardest bearing areas of the teeth, it may be taken that in good practice this static error is reduced to .001" on each helix, which on the average will be reduced to .0005" by natural slewing of the pinion in the journals in the manner already described, although in the least favourable case, the higher value would apply.

Under running conditions the total contact error, if not excessive, is absorbed by bending of individual wheel and pinion teeth. It is impossible to make a precise estimate of the amount by which they will yield for a given load since within the zone of contact of each pair of teeth, the point of application varies from tip to root of wheel tooth and root to tip of the pinion tooth respectively, while the adjacent unloaded portion of each provides indeterminate additional strength. Calculations indicate that the combined deflection of a pair of $22\frac{1}{2}^0$, 7/12" pitch teeth with pinion of medium diameter (8"), would vary from $\cdot 30\,P\times 10^{-6}$ ins at commencement of contact to $\cdot 15\,P\times 10^{-6}$ ins at mid-contact, and $\cdot 40\,P\times 10^{-6}$ ins at end of contact, so that if some allowance is made for the rigidity supplied by the adjacent lengths of unloaded teeth, the average deflection may be taken as -

$$\delta = 0\cdot 2\,P\times 10^{-6} \text{ inches for } 22\tfrac{1}{2}^{\circ},\ \tfrac{7}{12}'' \text{ pitch teeth.}$$

(It/

(It will be noticed that the precise value is not of great importance so long as the different flexibilities of each type of tooth are linked up on a similar basis, since the deflections are to be used in comparison with contact errors which may vary considerably with respect to their assumed magnitude).

It will be seen that the effective deflection has thus been taken as 2/3 of the wheel tooth deflection, when the load line is at its tip, and no support is offered by adjacent lengths of unloaded tooth. The expression for the deflection is thus -

$$\delta = \frac{1}{26{,}000}\left(\frac{P}{1000}\right)\frac{1}{\tan^2\psi . \sin\psi} . \frac{p}{z}\left[\log_e \frac{t_R}{t_T} - .5\left(\frac{t_T}{t_R}\right)^2 + 2\left(\frac{t_T}{t_R}\right) - 1.5\right] \text{ins} \quad \text{----}(5)$$

where p = nominal tooth pitch

z = tangential length of contact zone in circumferential plane (See Appendix B)

t_R = root thickness of rack tooth measured normally

t_T = tip - do - - do -

ψ = flank angle of hob

or in general terms $\quad \delta = uP \times 10^{-6}$ - - - - - - (6)

On a similar basis it is shown that u equals 0.2 in the case of 4/10" pitch, $22\frac{1}{2}^{\circ}$ teeth also.

If the local increase in tooth pressure be considered due to bending and torsional contact errors alone, it will be found that on account of the error being approximately parabolic in form, the maximum ratio of local pressure increase is - $\left(1 + {}^{2e}/_{3\delta}\right)$

The static contact error is likely to be of such a form that initial contact between pairs of teeth, is over quite a broad area except for microscopic creep marking on/

/on the wheel. Considering all factors it seems reasonable to assume that due to static contact error alone, the maximum ratio of pressure increase would be $\left(1 + {\cdot0005}/{4\delta}\right)$ or $\left(1 + {\cdot001}/{4\delta}\right)$ depending upon whether the maximum effective value of the error be taken as .0005" or .001".

In considering the combined effect of the two errors, and assuming one aggravates the other, it is approximately correct to take the maximum ratio of pressure increase as -

$$C_d = \left(1 + 2e/3\delta + {\cdot0005}/4\delta\right) \text{ - - - - - } (7)$$

or the worse case -

$$C_d' = \left(1 + 2e/3\delta + {\cdot001}/4\delta\right) \text{ - - - - - } (8)$$

In practice the simultaneous use of these two factors in conjunction with the design criteria, briefly described in the body of the report, gives an immediate impression of the relative effect of a known deterioration in static tooth contact in any specific case.

The factors may be expressed more conveniently in the form -

$$C_d = \left(K_d + B/p\right)A \text{ - - - - - } (9)$$

$$C_d' = \left(K_d + 2B/p\right)A \text{ - - - - - } (10)$$

where A = factor dependent on graphing of function (see below)

$$B = p\left({\cdot0005}/4\delta\right) = 125/\mu \text{ - - - } (11)$$

In the case of gears having pinions with centre bearings it may be shown that the contact error for the same pinion diameter, face width and loading per inch width, expressed in the same relative terms as those already used are -

Forward/

/Forward 55% of Forward helix (torsion plus bending = total)

	27 + 3 = 30	39
Aft 45% - do - - do -	do - 12 - 3 = 9	

Forward 45% of Aft helix	- do - 10 + 3 = 13	13
Aft 55% - do -	- do - 3 - 3 = 0	

It will be seen that a slewing correction of the pinion journals would even up the error to a relative figure of ± 13 on the respective helices. The bending error is so small in proportion to the torsion error that the latter may be credited with the total relative error of 13 per helix.

This may be shown to be -

$$e = \left(\frac{P}{1000}\right)\left(\frac{w-2}{D}\right)^2 \times 23/10^6$$

$$= \cdot 127\left(\frac{P}{1000}\right)\frac{(K_d-1)}{B} \quad \text{---} \quad (12)$$

$$\text{where} \quad K_d = \frac{15\cdot 3}{1000\,\mu}\left(\frac{w-2}{D}\right)^2 + 1 \quad \text{---} \quad (13)$$

and $\mu = 0\cdot 32$

The pressure increase factors C_d and C_d' are obtained by the aforementioned formulae (9) and (10), B again equalling $125/\mu$. K_d is plotted for this instance also on the accompanying chart (Appendix G).

The functions $\left(K_d + B/P\right)$ and $\left(K_d + \frac{2B}{P}\right)$ are obtained from the K_d chart drawn with respect to 14½° centre bearing pinions, and 22½° non-centre bearing pinions, in which instances the factor A is taken as 1.0. To suit the following cases A has the value as stated.

14½° flank without centre bearings $A = \cdot 625$

22½° flank with centre bearings $A = 1\cdot 60$

APPENDIX B -

Derivation of criterion of root stress of teeth, C_r -

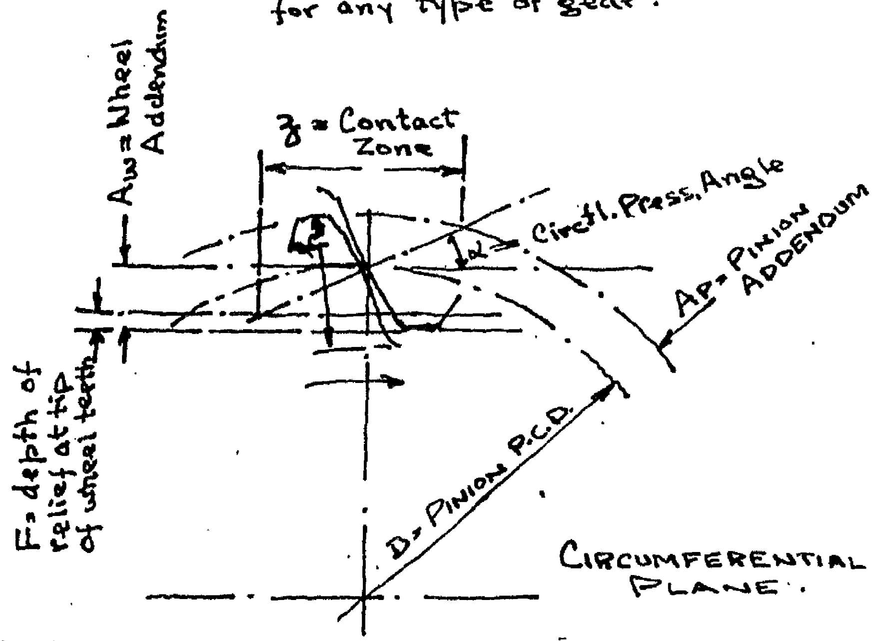
Using the symbols already defined, the actual normal load acting per inch length of tooth flank, when the helical angle is 30° is -

$$P_o = P.\ C_\alpha.\ \frac{p}{z}.\ \frac{1}{\cos 30^\circ.\cos\psi} \quad ---- (14)$$

For the determination of it may be shown that approximately -

$$z \doteq \frac{A_w - F}{\tan\alpha} + D\left[-\tfrac{1}{2}\tan\alpha + \sqrt{\frac{A_p}{D} + \left(\tfrac{1}{2}\tan\alpha\right)^2}\,\right] \text{ ins} \quad ---(15)$$

for any type of gear.



With an equal addendum and dedendum gear the variation of with diameter is small, being in the case of 7/12" standard 14½° tooth, 1.00" for a 5" p.c.d. pinion and 1.07" for a 12" p.c.d. pinion. The % variation is greater for an all-addendum gear, z in the case of the 7/12" pitch 22½° tooth, being .575" for a 5" p.c.d. and .64" for 12" p.c.d. pinion. The maximum bending moment per inch of tooth = $P_o.h$ where h = tooth depth.

The/

The maximum stress in the pinion tooth neglecting the support afforded by its adjacent unloaded length is -

$$6 P_o h / t_r^2 \quad \text{where } t_r = \text{root thickness of pinion teeth (normal)}$$

This stress will be halved if the support of the adjacent unloaded length of tooth is considered to have the restraining effect as assumed in Appendix A, when mean δ was taken as $\cdot 2 P \times 10^{-6}$ instead of $\cdot 4 P \times 10^{-6}$ the value estimated as being appropriate to the load line at the pinion tip (corresponding to the condition now under consideration).

∴ Maximum stress in pinion tooth, which is a criterion is:-

$$C_r = P.C_d.\frac{ph}{z.\cos\psi} \times \frac{3.46}{t_r^2} \text{ lbs}/\square'' \quad \text{---} (16)$$

$$\text{or } C_r = P.C_d.K_r \quad \text{---} (17)$$

$$\text{where } K_r = \frac{3.46}{t_r^2}.\frac{ph}{z.\cos\psi} \quad \text{---} (18)$$

this coefficient being plotted on a base of pinion diameter for various tooth forms on an accompanying chart (Appendix H).

APPENDIX C -

Derivation of criterion of maximum pressure in oil film, Cp -

In his detailed analysis of the rolling action of gear teeth given in Engineering Vol.102, p.119, H.M. Martin showed the following relationships to exist -

Maximum pressure of oil film $P_m \propto \frac{\mu V_R}{t^{1.5}} \sqrt{R}$ ---(19)

Load carried/inch of tooth $P \propto \mu V_R . R/t$ ---(20)

Effective length of loaded film $l \propto \sqrt{t.R}$ ---- (21)

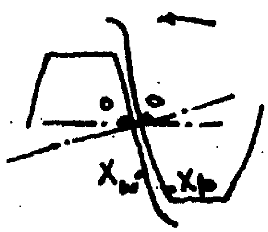
where -

μ = viscosity which may be taken as a constant for this purpose

V_R = rolling velocity of tooth flanks

R = radius of curvature of pinion tooth flank

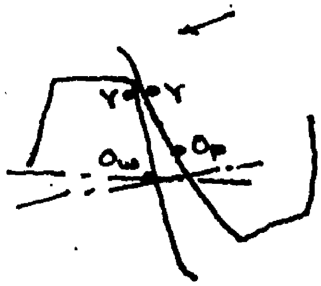
t = minimum thickness of oil film



Recess

The evaluation of V_R entails consideration of true rolling as distinct from sliding. In recess, as the point of contact moves from O to X there is no question but that X_pX_w represents the amount of sliding. Meanwhile, rolling action has occurred over the depth OX_p of pinion flank, and OX_w of wheel flank. The effective oil wedge forming rolling movement is the lesser value OX_w

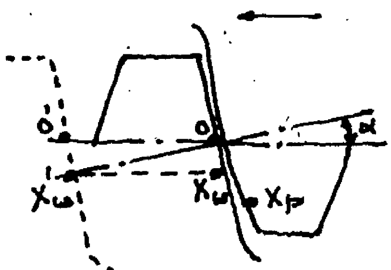
In/



Approach

In approach, as the point of contact moves from Y to O, O_pO_w is the sliding movement. Simultaneous rolling action has occurred over the depth YO_p of pinion flank and YO_w of wheel flank. The effective oil wedge forming rolling movement is again the lesser value YO_p but it will be noted that in contra-distinction to the action in recess, the effective rolling is measured by the advance of the point of contact on the pinion instead of on the wheel.

In later considerations of rate of sliding/rate of rolling (Appendix D), the value is in **consequence** taken as $\frac{X_pX_w}{OX_w}$ in recess and $\frac{O_pO_w}{YO_p}$ in approach.



Circumferential Plane.

To evaluate V_R in recess, consider the point of contact advancing from O to X during which time the peripheral movement at the pitch line is given by

$$OO' \propto D.N$$

The point of contact rolls

$$OX_w = O'X_w' = OO' \sin \alpha$$

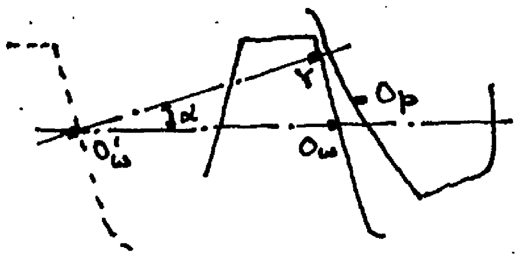
∴ In recess

$$V_R \propto D.N \sin\alpha \quad \text{---} (22)$$

To evaluate V_R in approach consider point of contact advancing from Y to O during which time the peripheral movement at the pitch line is given by

$$O_wO_w' \propto D.N$$

The/



Circumferential Plane

The point of contact rolls YO_p

$$= YO_w \cdot \frac{YO_p}{YO_p + O_pO_w}$$

$$= O_wO_w' \sin\alpha \, \frac{1}{1 + V_s/V_R}$$ where V_s = sliding velocity

In approach

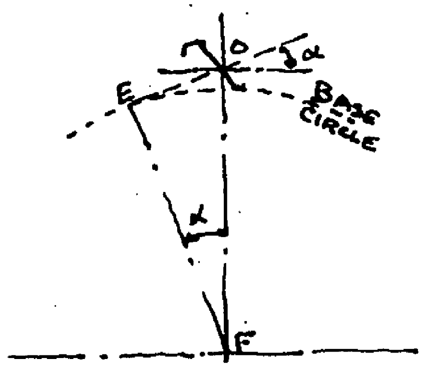
$$V_R \propto D.N.\sin\alpha \Big/ \left(1 + V_s/V_R\right) \quad \text{---(23)}$$

Radius of curvature of pinion tooth flank = R = OE = EF $\tan\alpha$.

But $EF \doteqdot \frac{1}{2} D$ where D = P.C.D

Approximately

$$R \propto D \sin\alpha \quad \text{- - - - (24)}$$



Circumferential Plane.

From equations (19) and (20)

$$P_m \propto P_0^{1.5} \cdot \frac{1}{R} \cdot \frac{1}{V_R^{.5}}$$

In substituting for from equation (14) in the following step, and also where it occurs in Appendix D, pressure angle α has been substituted for flank angle ψ for convenience, the relative difference introduced thereby for the various types of teeth being negligible.

For reasons described in the body of the report, particular consideration is given to conditions at the pitch point with regard to maximum oil pressure and consequently V_R is the same whether equation (22) or (23) is used, V_s/V_R being zero. The result will also be applicable to conditions obtaining in recess.

From/

From equations (14) (22) and (24)

$$P_m \propto \left(\frac{P}{D}\right)^{1.5} \cdot C_d^{1.5} \cdot \frac{1}{\sqrt{N}} \left(\frac{p}{y}\right)^{1.5} \frac{1}{\sin^{1.5}\alpha \cdot \cos^{1.5}\alpha} \quad ---- (25)$$

As a criterion this expression can be more easily used when reduced to 2/3 power, i.e. -

$$P_m^{2/3} \propto \frac{P}{D} \cdot C_d \frac{1}{\sqrt[3]{N}} \cdot \frac{p}{y} \cdot \frac{1}{\sin\alpha \cdot \cos\alpha} = C_p \quad \cdots (26)$$

or Pressure criterion

$$C_p = \frac{P}{D} \cdot C_d \cdot \frac{1}{\sqrt[3]{N}} \cdot K_p \quad -- \;\; -- \;\; -- (27)$$

$$\text{where } K_p = \frac{p}{y} \cdot \frac{1}{\sin\alpha \cdot \cos\alpha} \quad ----- (28)$$

the coefficient being graphed on a base of pinion diameter for different forms of teeth on an accompanying chart (Appendix J)

(It might here be noted that L.M. Douglas of Messrs. Parsons Marine Steam Turbine Company, in his paper "Mechanical Gearing for large power transmission", suggests the use of pressure criterion $\frac{P}{D^{2/3}} \cdot \frac{1}{(DN)^{1/3}}$ which is identical to (27) for any one tooth form).

APPENDIX D -

Derivation of criterion of maximum work done on unit volume of oil film, C_W -

At the high oil film pressures obtaining between gear teeth of the class under consideration, it is approximately correct to assume a constant coefficient of friction. Thus the work done on the film per second is

$$\propto P_m . l' . b . V_s \quad \text{ft.lbs/sec} \text{ ---- } (29)$$

where l' is proportional to the effective length of the oil film say

and b = breadth of oil film

V_s = sliding velocity

But during each second the volume of oil passing under the influence of the maximum pressure is

$$V_R . t . b. \quad \text{ft}^3\text{/sec} \text{ ------- } (30)$$

where V_R = rolling velocity of tooth flanks

t = minimum thickness of oil film

From equations (29) and (30), the work done on each cubic unit of oil is -

$$W \propto \frac{P_m . l' b V_s}{t . b . V_R} \propto P_m . \frac{l}{t} . \frac{V_s}{V_R}$$

or from equations (21) and (24)

$$W \propto P_m . \frac{V_s}{V_R} \sqrt{\frac{D \tan \alpha}{t}} \text{ ------ } (31)$$

For/

For recess -

From equations (14) (20) (22) and (24)

$$\frac{1}{t} \propto \frac{P}{D^2} \cdot C_d \cdot \frac{1}{N} \left(\frac{P}{\gamma}\right) \frac{1}{\sin^2\alpha} \text{ - - - - - - } (32)$$

Hence from equations (25) (31) and (32)

$$W \propto \left(\frac{P}{D}\right)^2 \cdot C_d^2 \cdot \frac{1}{N} \left(\frac{P}{\gamma}\right)^2 \cdot \frac{V_s}{V_R} \cdot \frac{1}{\sin^2\alpha \cos\alpha} \text{ - - - } (33)$$

This is the criterion which as stated in the body of the report, might be considered as a guide to the possible occurrence of "scuffing" from which it will be observed that the term in P is squared.

For simplicity in use, the square root of this expression may be used as the criterion written thus -

$$C_w = \frac{P}{D} \cdot C_d \cdot \frac{10}{\sqrt{N}} \cdot K_w \text{ - - - - - - - } (34)$$

where for recess

$$K_w = \frac{P}{\gamma} \sqrt{\frac{V_s}{V_R}} \cdot \frac{10}{\sin\alpha \sqrt{\cos\alpha}} \text{ - - - - - } (35)$$

and is plotted on an accompanying chart (Appendix K) to a base of pinion P.C.D. for various tooth forms.

For approach -

Due to the modified value of V in approach, equation (32) becomes -

$$\frac{1}{t} \propto \frac{P}{D^2} \cdot C_d \cdot \frac{1}{N} \left(\frac{P}{\gamma}\right) \left(1 + \frac{V_s}{V_R}\right) \frac{1}{\sin^2\alpha} \text{ - - - - } (36)$$

Equation (25) becomes -

$$P_m \propto \left(\frac{P}{D}\right)^{1.5} \cdot C_d^{1.5} \cdot \frac{1}{\sqrt{N}} \cdot \left(\frac{P}{\gamma}\right)^{1.5} \sqrt{1 + \frac{V_s}{V_R}} \cdot \frac{1}{\sin\alpha \cos\alpha \sqrt{\tan\alpha}} \text{ - - - } (37)$$

Whence equation (33) becomes -

$$W \propto \left(\frac{P}{D}\right)^2 \cdot C_d^2 \cdot \frac{1}{N} \left(\frac{P}{\gamma}\right)^2 \frac{V_s}{V_R} \left(1 + \frac{V_s}{V_R}\right) \frac{1}{\sin^2\alpha \cdot \cos\alpha} \text{ - - } (38)$$

As/

As before the criterion may be written -

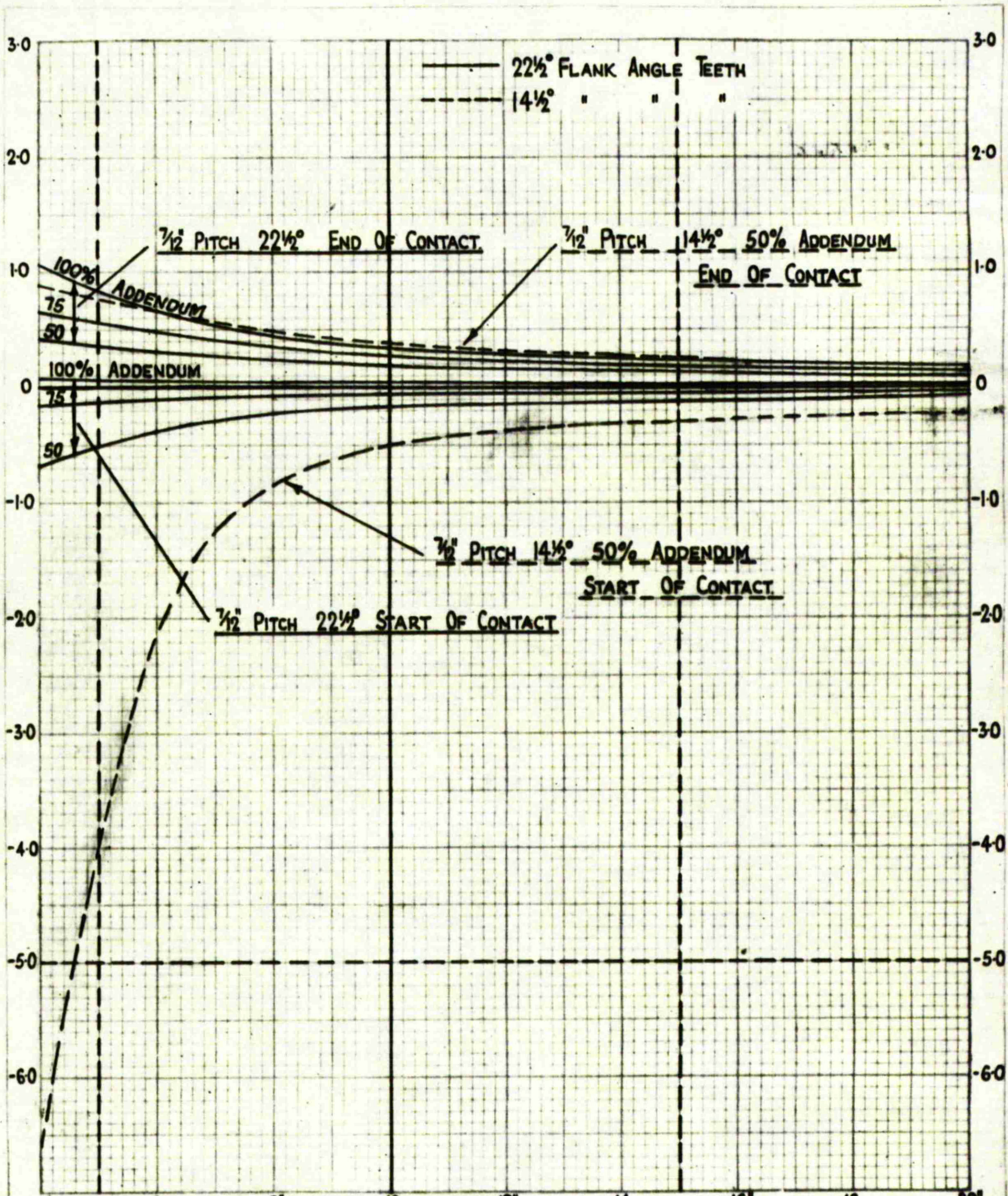
$$C_\omega = \frac{P}{D} \cdot C_d \cdot \frac{10}{\sqrt{N}} \cdot K_\omega$$

but for approach -

$$K_\omega = \frac{P}{\gamma} \cdot \sqrt{\frac{V_s}{V_R}\left(1+\frac{V_s}{V_R}\right)} \cdot \frac{10}{\sin\alpha\sqrt{\cos\alpha}} \quad \text{---} \; (39)$$

values of which are plotted on the aforementioned chart (Appendix K).

In the use of equations (17) (27) and (34) a more severe condition is represented by the substitution of C_d' for C_d for reasons already described.



GEARING DESIGN

Vs/Vr CHART (1) SLIDING/ROLLING RATIOS.

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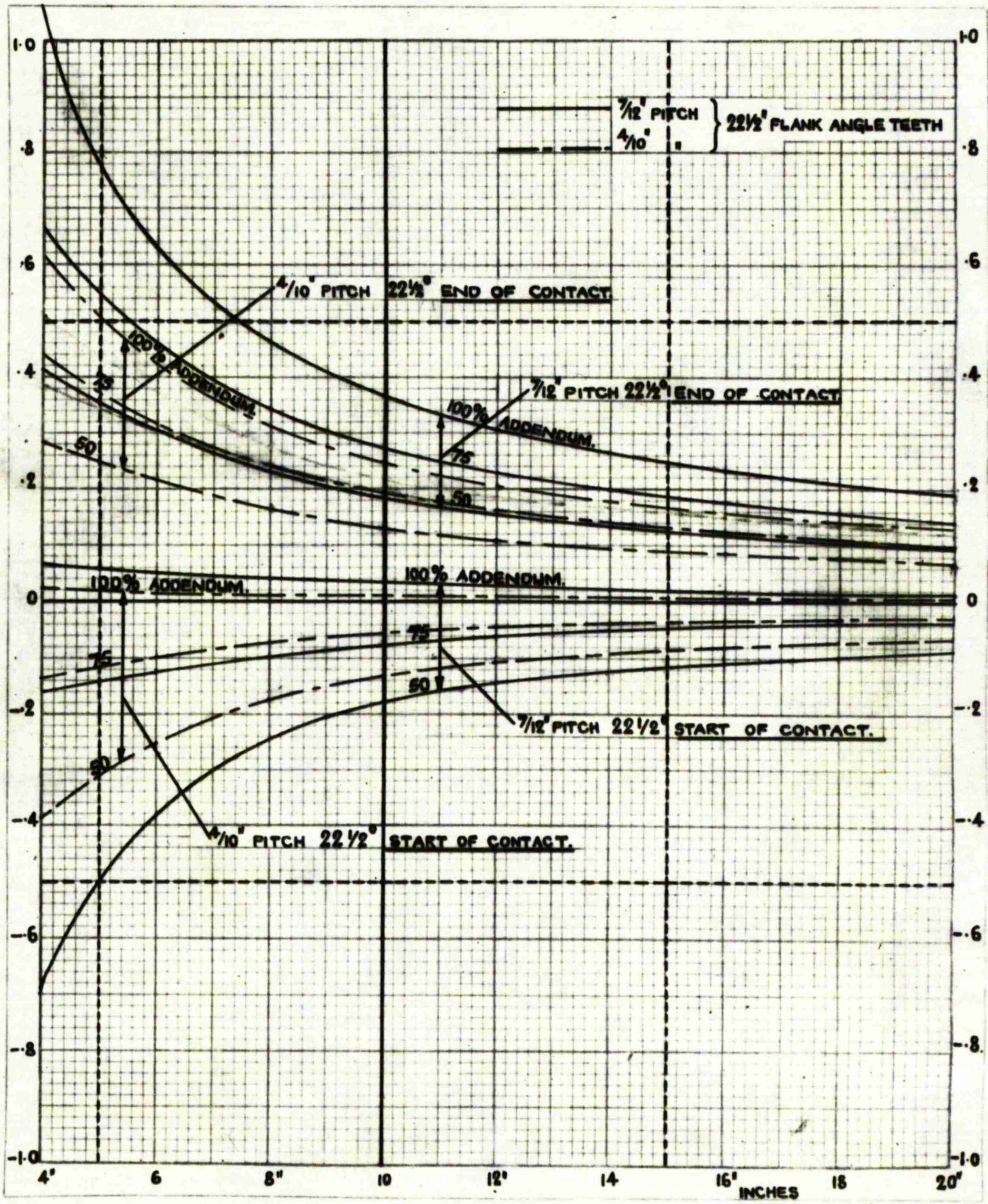
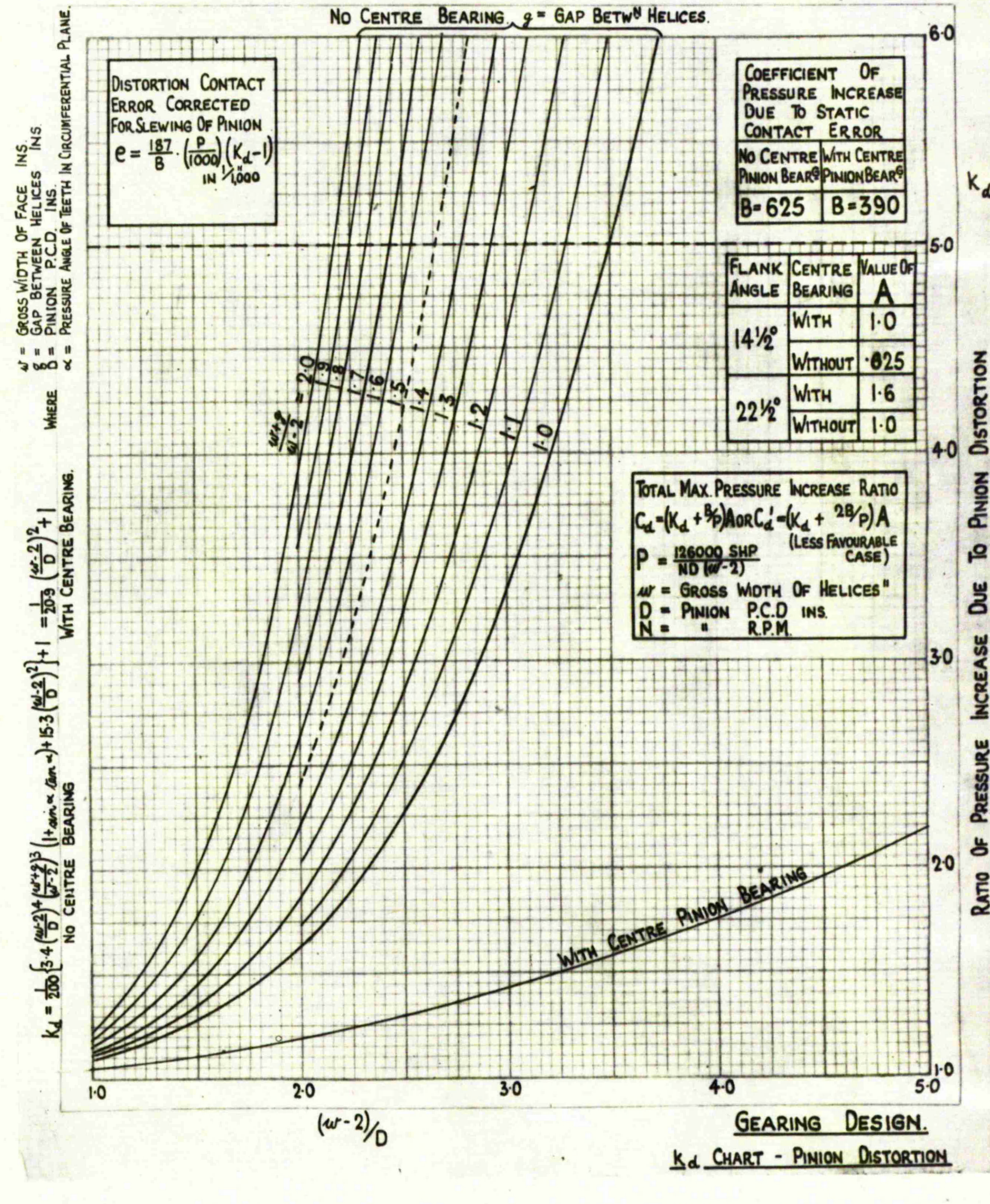
APPENDIX F.



GEARING DESIGN.

Vs/VR. CHART(2) SLIDING / ROLLING RATIO.

APPENDIX G

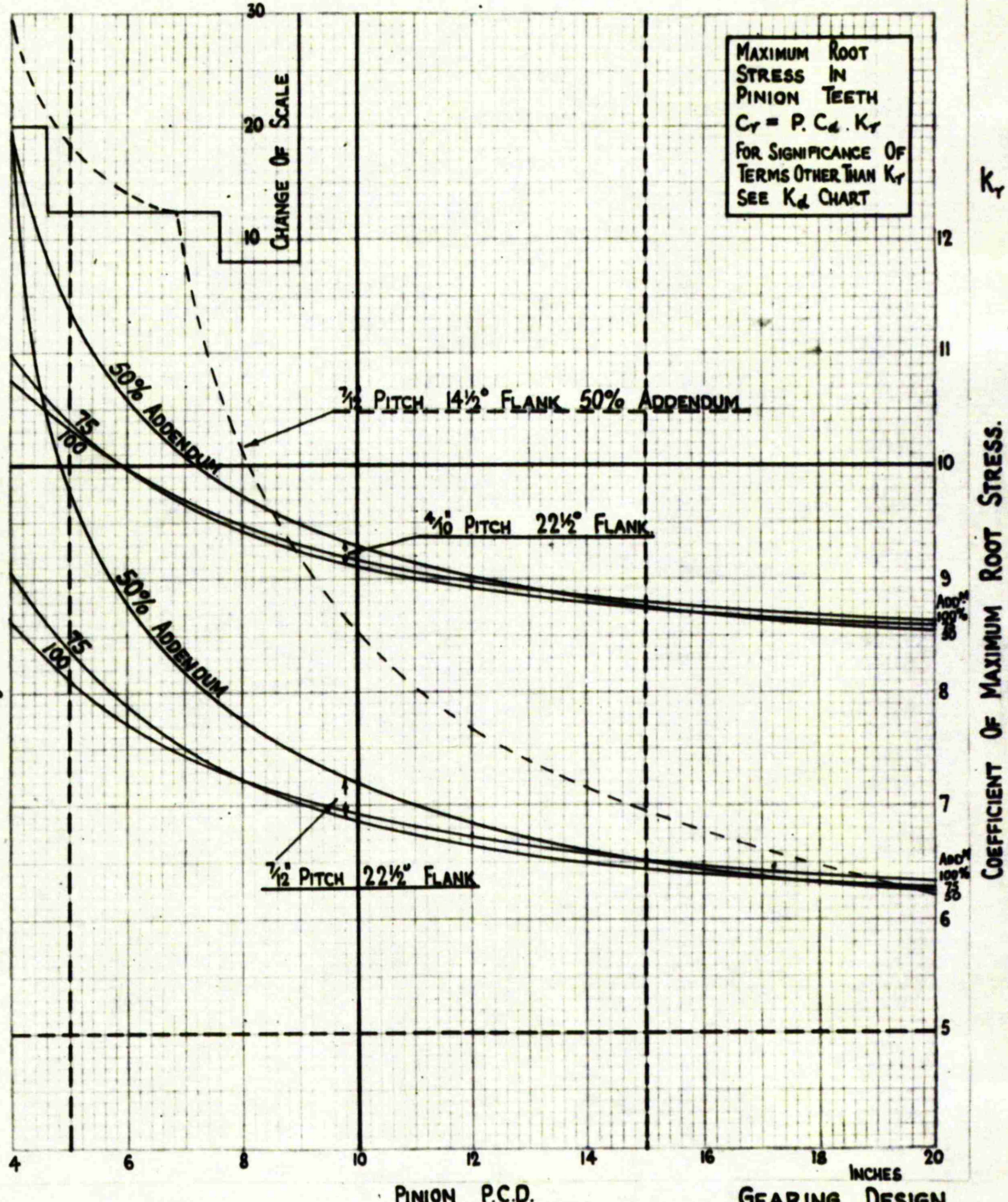


GEARING DESIGN.

Kd CHART - PINION DISTORTION

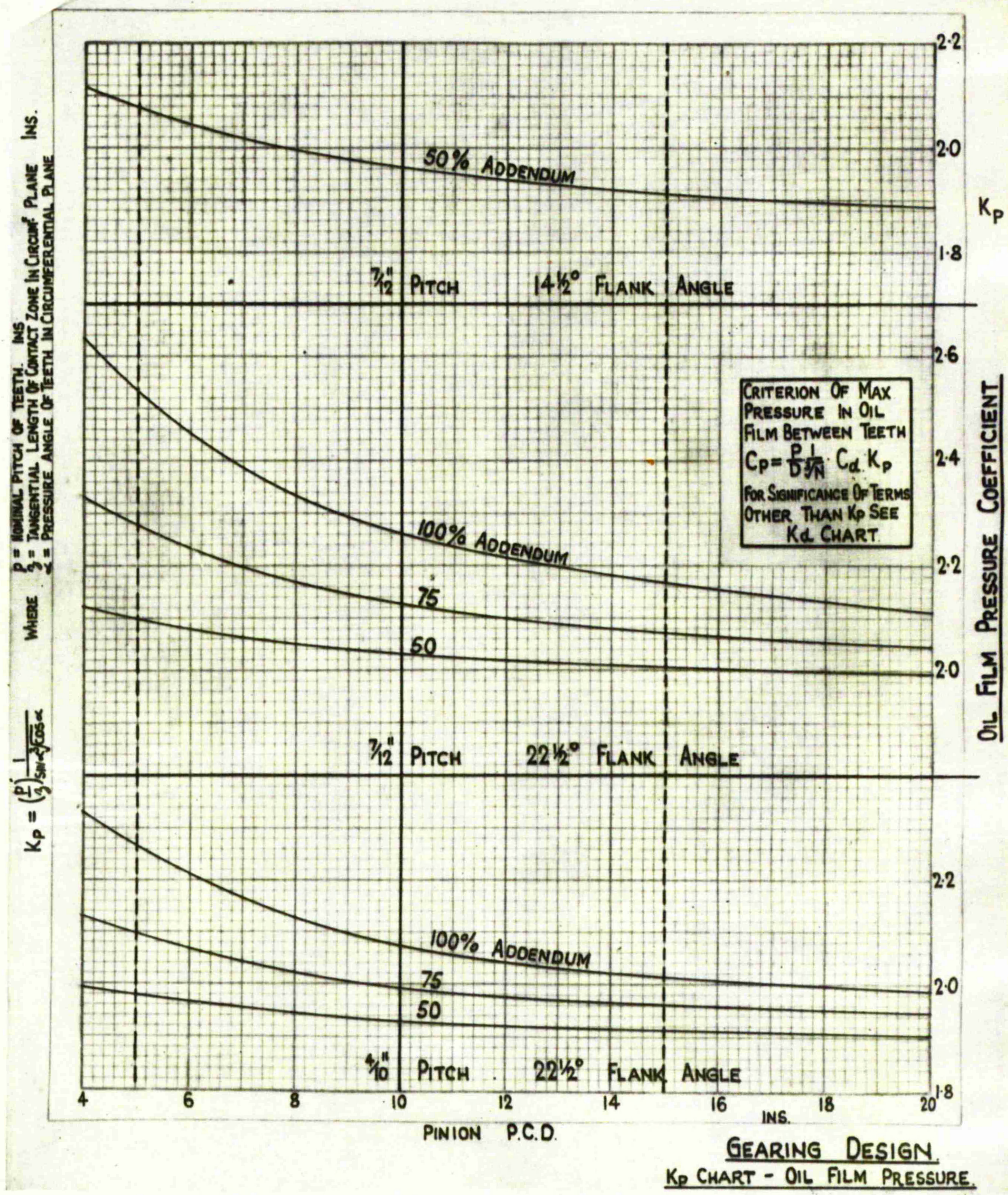
$$K_r = \frac{3{\cdot}46}{t_r^2} \cdot \frac{P\,h}{z \cos\psi}$$

t_r = Root Thickness of Pinion Teeth Ins. h = Depth of Pinion Teeth
P = Nominal Pitch of Teeth Ins. ψ = Flank Angle of Hob.
z = Tangential Length of Contact Zone in Circumferential Plane.



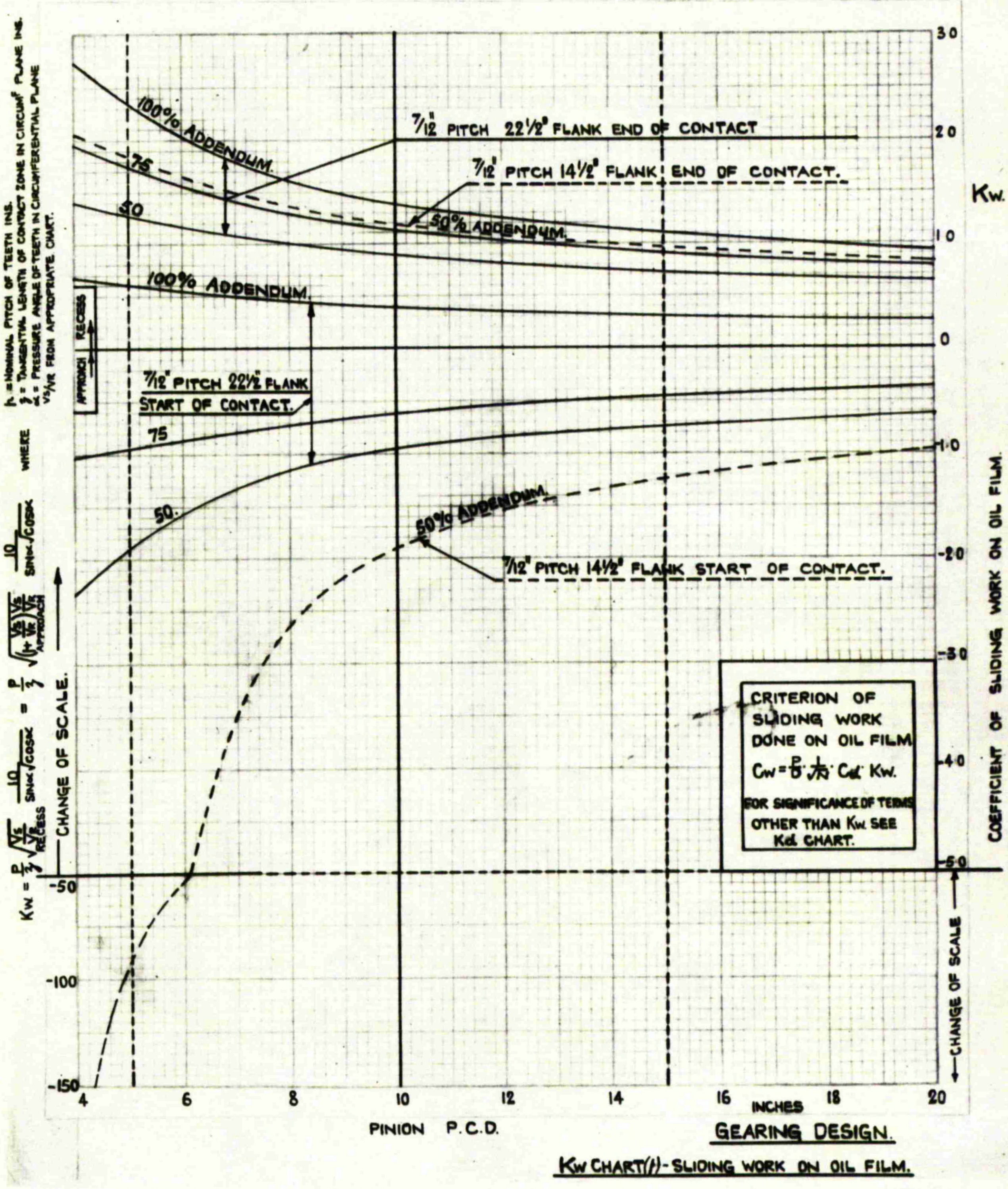
GEARING DESIGN

K_r CHART - ROOT STRESS



GEARING DESIGN.

Kp CHART - OIL FILM PRESSURE.



GEARING DESIGN.

Kw CHART(1)- SLIDING WORK ON OIL FILM.

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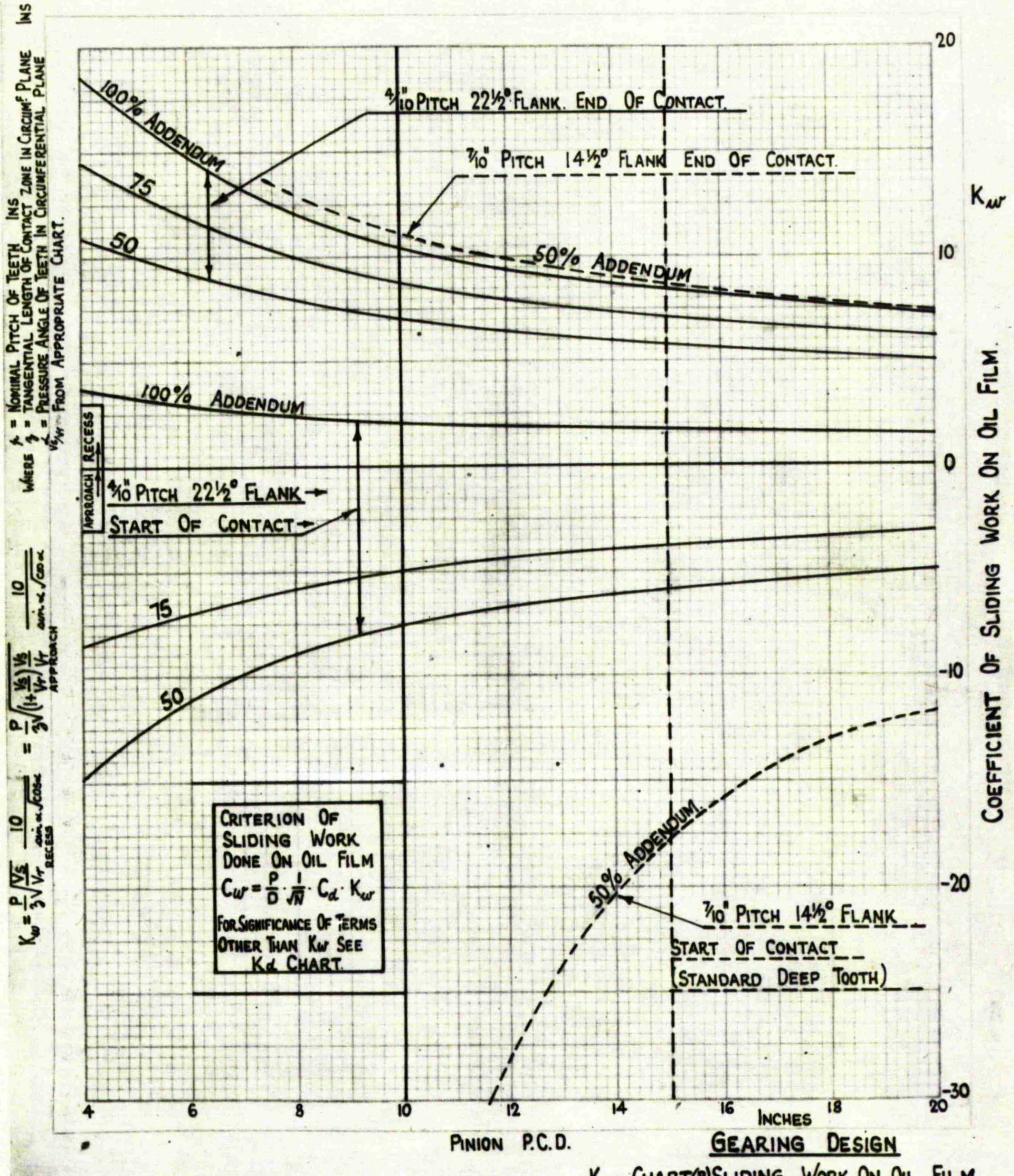
APPENDIX K.



GEARING DESIGN

K_w CHART(2) SLIDING WORK ON OIL FILM.

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APPENDIX L.

ANALYSIS OF GEARS TABLED IN ADMIRALTY REPORT EN 24/250/41

REFERENCE TO GEAR	A	B	C	D	E	F	G	H		J	K	L	M
NORMAL PITCH OF TEETH ins p	7/12	7/12	7/12	7/12	7/12	1	7/12	7/12		8/10	7/12	4/10	4/10
FLANK ANGLE ψ	22½°	22½°	22½°	22½°	22½°	22½°	22½°	22½°		22½°	14½°	22½°	22½°
ADDENDUM OF PINION AS % MESHING DEPTH	100	100	100	100	100	100	100	100		100	50	100	100
WIDTH OF FACE ins w	12	12	12	11½	24½	45¼	8	32		50	12	15·7	12·2
GAP BETWEEN HELICES ins g (ASSUMED IN SOME INSTANCES)	3	3	3	3	3	3	3	3		3	3	2	2
R.P.M. OF WHEEL AT FULL AHEAD POWER.	275	300	350	275	380	270	2300	340		330	275	300	320
PINION	L.P.	L.P.	L.P.	L.P.	L.P.	L.P.	CRUIS[g]	H.P.	L.P.	L.P.	L.P.	L.P.	L.P.
PINION P.C.D." D	5·14	5·14	5·57	5·36	8·78	14·3	6·0	10·93	15·21	17·57	5·14	5·44	5·44
AHEAD													
FULL POWER S.H.P.	930	1100	930	530	4750	13750	1000	8750	11250	18000	850	1100	540
" " R.P.M. N	3700	4030	3810	3420	3300	2631	6500	3070	2200	2063	3700	4030	3240
PRESSURE/INCH NET WIDTH OF FACE $(w-2)$ P LBS	616	666	553	384	918	1070	540	1098	1413	1305	564	462	379
PINION DISTORTION – COEFFICIENT K_d	2·30	2·30	1·93	1·93	3·08	4·07	1·21	3·39	1·77	3·08	2·30	3·20	1·97
CORRECTED CONTACT ERROR e 1/1000"	·24	·26	·32	·22	·85	·98	·20	1·11	·75	1·20	·39	·44	·22
INCLUDING STATIC CONTACT ERROR EFFECTS: MIN. PRESSURE INCREASE RATIO C_d	3·31	3·24	3·06	3·56	3·76	4·66	2·37	3·96	2·21	3·56	2·13	4·55	3·62
MAX. " " " C_d'	4·32	4·18	4·19	5·19	4·44	5·25	3·53	4·53	2·65	4·04	2·82	5·90	5·27
ROOT STRENGTH – COEFFICIENT K_r	8·05	8·05	7·89	7·95	7·10	1·40	7·71	6·82	6·50		17·6	10·17	10·17
MAX. STRESS (BASED ON C_d) C_r LBS/□"	16400	17400	18200	10800	24500	6900	9800	19700	20300		21100	21300	14000
OIL FILM PRESS[e] – COEFFICIENT K_p	2·52	2·52	2·49	2·50	2·30	2·36	2·45	2·24	2·17		2·07	2·24	2·24
CRITERION BASED ON C_d C_p	65	66	49	42	115	60	28	61	34		31	55	38
" " " C_d' C_p'	85	86	66	62	136	67	42	70	41		41	71	56
	← GEARS WHICH HAVE SCUFFED →						← GEARS WHICH HAVE NOT SCUFFED →						
WORK ON OIL FILM – MAX. SLIDING/ROLLING RATE V_s/V_r	·75	·75	·69	·71	·42	·44	·63	·33	·24		·74	·47	·47
RECESS: COEFFICIENT K_w	23·0	23·0	21·7	22·3	15·0	15·8	20·3	13·0	10·9		18·1	15·7	15·7
RECESS: CRITERION EX. C_d C_w/C_d	45·5	47·0	35·0	27·3	27·3	23·1	22·7	23·5	21·6		32·6	21·0	19·2
RECESS: " BASED ON C_d C_w	151	152	107	97	103	108	54	93	48		69	95	70
RECESS: " " " C_d' C_w'	196	197	147	142	121	121	80	116	57		92	124	101
APPROACH: MAX. SLIDING/ROLLING RATE V_s/V_r	–	–	–	–	–	–	–	–	–	–	-3·67	–	–
APPROACH: COEFFICIENT K_w	–	–	–	–	–	–	–	–	–	–	-84	–	–
APPROACH: CRITERION EX. C_d C_w/C_d	–	–	–	–	–	–	–	–	–	–	-151	–	–
APPROACH: " BASED ON C_d C_w	–	–	–	–	–	–	–	–	–	–	-321	–	–
APPROACH: " " " C_d' C_w'	–	–	–	–	–	–	–	–	–	–	-425	–	–
ASTERN (BASED ON ⅓ AHEAD POWER AT ⅔ AHEAD REVS)													
FULL ASTERN S.H.P.	600	717	600	333	3166	9166	–	–	6667	12000	550	717	333
" " R.P.M. N	2467	2687	2540	2280	2200	1754	–	–	1467	1375	2467	2687	2160
PRESSURE/INCH NET WIDTH OF FACE $(w-2)$ P LBS	596	655	535	362	918	1070	–	–	1256	1305	547	452	351
PINION DISTORTION – CORRECTED CONTACT ERROR e 1/1000"	·23	·26	·31	·21	·85	·98	–	–	·67	1·20	·38	·43	·20
INCLUDING STATIC CONTACT ERROR EFFECTS: MIN. PRESSURE INCREASE RATIO C_d	3·35	3·25	3·10	3·66	3·76	4·66	–	–	2·27	3·56	2·15	4·58	3·75
MAX. " " " C_d'	4·40	4·20	4·27	5·39	4·44	5·25	–	–	2·77	4·04	2·86	5·96	5·53
ROOT STRENGTH – MAX. STRESS (BASED ON C_d) C_r LBS/□"	16100	17100	13100	10500	24500	6900	–	–	18500		21700	21000	13400
OIL FILM PRESSURE – CRITERION BASED ON C_d C_p	73	75	55	47	70	68	–	–	36		35	61	42
" " " C_d' C_p'	95	97	75	69	82	77	–	–	44		47	80	62
WORK ON OIL FILM – RECESS: CRITERION BASED ON C_d C_w	180	184	129	115	126	132	–	–	53		84	116	82
RECESS: " " " C_d' C_w'	236	238	177	170	148	148	–	–	65		111	150	120
APPROACH: CRITERION BASED ON C_d C_w	–	–	–	–	–	–	–	–	–	–	-388	–	–
APPROACH: " " " C_d' C_w'	–	–	–	–	–	–	–	–	–	–	-518	–	–

Appendix M.

Analysis of Some Other Naval and Merchant Gears.

Reference to Gear	Sloop	Destroyers						Cruisers						Battleship	Aircraft Carrier	Merchant		
	Repeat Black Swan	"D" Class		"G" Class		"M" Class		"Norfolk"		"Liverpool"		"Phoebe"		"Howe"	Implacable	Liner	Cross Channel 43	Cross Channel 56
Nominal Pitch of Teeth ins p	4/10	7/12		7/12		7/12		7/12		7/12		7/12		7/12	7/12	7/12	7/12	7/12
Flank Angle ψ	22½°	14½°		22½°		22½°		14½°		22½°		22½°		22½°	22½°	14½°	14½°	22½°
Addendum of Pinion as % Meshing Depth	100	50		100		100		50		100		100		100	100	50	50	100
Width of Face ins w	15	42		29½		37		45		32		27		43	53	59	16½	9
Gap Between Helices ins g	2	Centre Bearing		3		3		Centre Bearing		3		3		3	3	Centre Bearing	3½	3
R.P.M. of Wheel at Full Ahead Power	300	356		343		340		310		329		340		236	235	124	312	410
Pinion	L.P.	H.P.	L.P.	H.P.	L.P.	H.P.	L.P.	H.P.	L.P.	H.P.	L.P.	H.P.	L.P.	L.P.	L.P.	L.P.	L.P.	L.P.
Pinion P.C.D. D	5·44	10·07	15·64	9·85	14·57	12·21	16·93	10·71	15·42	10·93	15·42	9·85	12·43	15·21	18·21	13·07	5·99	5·14
AHEAD.																		
Full Power S.H.P.	1100	8600	9600	7360	10080	10700	13300	8310	12100	9970	10790	6810	8400	12500	19000	5620	950	430
" " R.P.M. N	4046	3390	2180	3380	2285	2965	2140	3120	2165	3310	2340	3425	2715	2200	2040	1540	2920	4130
Pressure/Inch Net Width of Face $(w-2)$ P Lbs	485	794	885	1012	1387	1061	1320	662	1060	1155	1250	1018	1250	1145	1260	617	471	365
Pinion Distortion - Coefficient K_d	2·94	1·75	1·32	3·62	1·68	3·62	1·86	1·76	1·37	3·41	1·74	2·96	1·89	3·04	3·27	1·90	3·29	1·50
Corrected Contact Error e 1/1000	·28	·29	·14	·79	·28	·83	·34	·24	·19	·83	·28	·60	·33	·70	·86	·27	·32	·05
Including Static Contact Error Effects: Min. Press. Increase Ratio C_d	4·23	2·24	1·76	4·24	2·13	4·21	2·33	2·36	1·74	3·95	2·24	3·57	2·39	3·58	3·76	2·53	2·88	3·21
Including Static Contact Error Effects: Max. " " C_d'	5·52	2·73	2·20	4·86	2·58	4·80	2·80	2·96	2·11	4·49	2·74	4·18	2·89	4·12	4·25	3·16	3·71	4·92
Root Strength - Coefficient K_r	10·17	8·50	6·82	6·95	6·54	6·70	6·43	8·17	6·87	6·82	6·50	6·95	6·69	6·50	6·39	7·35	14·0	8·05
Max Stress (Based on C_d) C_r Lbs/□"	20900	15100	10600	29800	19300	29900	19800	12500	12700	31100	18200	25200	20000	26700	30300	11500	19000	9400
Oil Film Pressure - Coefficient K_p	2·24	1·96	1·91	2·27	2·17	2·21	2·14	1·95	1·91	2·24	2·16	2·27	2·21	2·16	2·13	1·93	2·04	2·52
Criterion Based on C_d C_p	53	23	15	66	33	56	30	19	18	63	30	56	38	45	44	20	32	36
" " " C_d' C_p'	69	28	18	76	40	64	36	24	21	71	36	65	46	51	50	25	42	55
	← None of these gears have scuffed. →																	
Work on Oil Film - Max Sliding/Rolling Rate W_s/W_r	·46	·38	·25	·37	·25	·30	·22	·35	·25	·33	·24	·37	·29	·24	·21	·30	·67	·75
Recess: Coefficient K_w	15·7	11·8	9·4	14·0	11·1	12·2	10·3	11·4	9·5	13·0	10·9	14·0	12·1	11·0	9·9	10·2	16·5	23·0
Recess: Criterion ex C_d C_w/C_d	22·1	16·0	11·4	24·8	22·1	19·5	17·4	12·4	14·0	23·8	18·3	24·7	23·3	17·7	15·2	12·3	24·0	25·4
Recess: " Based on C_d C_w	94	36	20	105	47	82	41	29	24	94	41	88	56	63	57	31	69	82
Recess: " " C_d' C_w'	122	44	25	120	57	94	49	37	30	107	50	104	67	73	65	39	89	125
Approach: Max Sliding/Rolling Rate W_s/W_r	–	-·50	-·29	–	–	–	–	-·45	-·30	–	–	–	–	–	–	-·33	-2·15	–
Approach: Coefficient K_w	–	-18·9	-12·0	–	–	–	–	-17·5	-12·0	–	–	–	–	–	–	-14·1	-53·0	–
Approach: Criterion ex C_d C_w/C_d	–	-25·6	-14·5	–	–	–	–	-19·1	-17·7	–	–	–	–	–	–	-17·0	-77·3	–
Approach: " Base on C_d C_w	–	-57	-25	–	–	–	–	-45	-31	–	–	–	–	–	–	-43	-223	–
Approach: " " C_d' C_w'	–	-70	-32	–	–	–	–	-56	-37	–	–	–	–	–	–	-54	-287	–
ASTERN (Based on ⅓ Ahead Power at ⅔ Ahead R.P.M.)																		
Full Astern S.H.P.	717	–	6067	–	5813	–	8000	–	6803	–	6920	–	5070	8333	12667	4560	440	430
" " R.P.M. N	2698	–	1454	–	1524	–	1426	–	1444	–	1560	–	1810	1466	1360	1370	1950	3320
Pressure/Inch Net Width of Face $(w-2)$ P Lbs	475	–	840	–	1200	–	1193	–	893	–	1207	–	1135	1145	1260	564	327	454
Pinion Distortion - Corrected Contact Error e 1/1000	·28	–	·13	–	·24	–	·31	–	·16	–	·20	–	·30	·70	·86	·24	·22	·07
Including Static Contact Error Effects: Min. Press. Increase Ratio C_d	4·25	–	1·78	–	2·20	–	2·38	–	1·81	–	2·26	–	2·44	3·58	3·76	2·59	3·25	2·88
Including Static Contact Error Effects: Max. " " C_d'	5·56	–	2·24	–	2·72	–	2·90	–	2·25	–	2·78	–	2·99	4·12	4·25	3·28	4·45	4·26
Root Strength - Max Stress (Based on C_d) C_r Lbs/□"	20200	–	10200	–	17300	–	18300	–	11100	–	17700	–	18500	26700	30300	10800	14900	10500
Oil Film Pressure Criterion Based on C_d C_p	60	–	16	–	34	–	32	–	18	–	33	–	40	51	50	20	29	43
" " " C_d' C_p'	78	–	20	–	42	–	39	–	22	–	40	–	50	59	57	25	40	64
Work on Oil Film - Recess: Criterion Based on C_d C_w	112	–	24	–	52	–	46	–	26	–	49	–	63	77	70	31	66	102
Recess: " " C_d' C_w'	147	–	30	–	64	–	56	–	33	–	60	–	78	89	79	39	91	150
Approach: Criterion Based on C_d C_w	–	–	-30	–	–	–	–	–	-33	–	–	–	–	–	–	-43	-212	–
Approach: " " C_d' C_w'	–	–	-38	–	–	–	–	–	-41	–	–	–	–	–	–	-54	-291	–

APPENDIX N -

The analysis presented in tabular form in Appendices L and M illustrates several features principal of which are -

1. The value of (C_W, C_W') in recess for gears which have scuffed varies between 238 - 121 in the upper limit and 184-103 in the lower limit (which corresponds with a lesser contact error). Gears which have not scuffed vary between 116 - 20 in the lower limit and 150 - 25 in the upper limit, the latter corresponding with a greater contact error. The consistency of these values is such that every example would fit in with the rule that "gears with C_W greater than 120 will scuff (unless very carefully handled when new or unless relieved at pinion tips), gears with C_W' less than 120 will not scuff (with reasonable lubrication) and gears in which 120 is intermediate between the values of C_W and C_W' are likely to be very dependent for their behaviour on such variables as contact errors of manufacture, polish of pinion teeth, manoeuvring of engines at sea.

2. It appears to be permissible to work up to a root stress (based on C_d) of 30,000 lbs/square inch. If in designing a gear C_W is low but the face width is determined by the root stress, it is a clear case for teeth of larger pitch, enabling the face width to be reduced thus hinging C_r and C_W each towards their safe limit. The gears of the Aircraft Carrier typified are an example of where it is considered teeth of greater pitch might have been more satisfactory, in that the width of gear could have been reduced.

3/

3. The Hunt Class gears are a good example of where it is considered a change to 75% pinion addendum would have particularly beneficial results. For ahead running the (C_W, C_W') range would be reduced from 103 - 121 to 83 - 97 in recess, while the approach value brought into existence would be only minus 47 - 55. The stress at the root of the teeth would be increased only ½%.

4. Analysis on the basis proposed indicates in many instances that excessive face width is introducing higher local stresses and loads on account of pinion distortion, than would apply were the width to be reduced. Outstanding among these is the modified Black Swan Sloops, in which it would appear that the local loading and stresses would be reduced 14% were the face width to be reduced from 15" to 10". This may indicate that distortion correction criterion bears excessive influence on the results, but on the other hand Fairfield experience of failure of pinions of a similar type showed that scuffing was restricted to narrow bands showing that the load was being carried by a comparatively narrow width and that contact errors were far from insignificant although the cutting of the gears was up to standard and the alignment was good.

5. The general impression conveyed by the results substantiates the views regarding the excessively empirical nature of design as expressed in the body of the report.

APPENDIX O -

List of Symbols -

A Correction factor of C_d and C_d'

A_p Addendum of pinion, inches

A_w Addendum of wheel, inches

B Coefficient of pressure increase due to static contact error

b Breadth of oil film

C_d Criterion - ratio of pressure increase due to pinion distortion and errors.

C_d' Criterion - ratio of pressure increase due to pinion distortion and errors (less favourable case)

C_p Criterion of maximum pressure of oil film

C_r Criterion stress at root of pinion teeth

C_w Criterion of work done per unit volume of oil film

D Pinion p.c.d., inches

e Contact error due to bending and twisting of pinion, (corrected) inches

e_b Contact error due to bending of pinion (uncorrected)

e_t Contact error due to twisting of pinion (uncorrected)

F Depth of relief at tip of wheel teeth

g Gap between helices, inches

h Tooth depth, inches

k_d Coefficient related to C_d - See chart Appendix G

k_p Coefficient related to C_p - See chart Appendix H

k_r Coefficient related to C_r - See chart Appendix J

k_w Coefficient related to C_w - See chart Appendix K

l Length of oil film

l' Proportion of l, being length subjected to maximum pressure

N Pinion R.P.M. at full ahead power

n Pinion R.P.M. at reduced power

P/

P	Tangential load per inch net width of face at full ahead power, lbs.
P_m	Maximum pressure of oil film at full ahead power, lbs.
P_o	Actual normal load per inch of tooth flank at full ahead power, lbs.
p	Nominal pitch of teeth
p'	Tangential load per inch net width of face at reduced revs. n lbs.
R	Radius of pinion tooth flanks, inches
t	Minimum thickness of oil film
t_r	Normal thickness at root of pinion tooth, inches
t_R	- do - - do - rack tooth, inches
t_T	- do - at tip of rack tooth, inches
u	Coefft. of flexibility of rack tooth
V_r	Rolling velocity of teeth, ft/sec.
V_s	Sliding velocity of teeth, ft/sec.
W	Work done per cubic unit of oil film
w	Gross width of tooth face, inches
γ	Tangential length of contact zone in circumferential plane, inches.
α	Pressure angle of teeth in circumferential plane
δ	Tooth deflection under load
ψ	Flank angle of hob
μ	Viscosity of oil

2

COMMITTEE
ON
STEAM TURBINE INSTALLATIONS

REPORT ON VISIT TO

UNITED STATES OF AMERICA

TO INVESTIGATE PROBLEMS ASSOCIATED WITH
DESIGN, PRODUCTION & MAINTENANCE OF

MAIN GEARING

FEBRUARY 1944

A.W. DAVIS FAIRFIELD S. & E. Co. LTD

CONTENTS.

The following itinerary was followed -

February 1944		
Monday,	14th	Disembarked New York 6 p.m.
Tuesday,	15th	Arrived Washington 2.30 p.m. and visited E. in C. (W), ADMIRAL BURT of the BRITISH ADMIRALTY DELEGATION.
Wednesday,	16th	Meeting at BUREAU OF SHIPS, Navy Dept. 9.30 a.m. to 1.p.m. Depart Washington 5.30 p.m.
Thursday,	17th	Arrived Milwaukee 10.30 a.m. Visited FALK CORPORATION, Milwaukee. 12 noon to 6.30 p.m.
Friday,	18th	Visited FALK CORPORATION, Milwaukee. 9.30 a.m. to 5 p.m. Depart Milwaukee 6.15 p.m.
Saturday,	19th	Arrived Boston 11.30 p.m.
Monday,	21st	Visited GENERAL ELECTRIC CO., Lynn, Boston, 9.30 a.m. to 6 p.m.
Tuesday,	22nd	Visited GENERAL ELECTRIC CO., Lynn, Boston 9.30 a.m. to 5 p.m. Discussion with Mr. R.H. TINGEY, CHIEF ENGINEER, BETHLEHEM STEEL CO. Fore River Yard, Quincy, Mass. at Statler Hotel, Boston, 8 to 11 p.m. Depart Boston 11.30 p.m.
Wednesday,	23rd	Arrived Trenton 9.30 a.m. (indisposed).
Thursday,	24th	Visited DE LAVAL STEAM TURBINE CO. Trenton, 9 a.m. to 3 p.m. Depart Trenton 3.30 p.m. Arrived Philadelphia 5 p.m.
Friday,	25th	Visited WESTINGHOUSE ELECTRIC MANFG. CO., Essington, 9.30 a.m. to 5.30 p.m.
Saturday,	26th	Visited WESTINGHOUSE ELECTRIC MANFG. CO., Essington, 9.30 a.m. to 3 p.m. Left for Washington 3.30 p.m.
Monday,	28th	Meeting at BUREAU OF SHIPS, Navy Dept. 9 to 11 a.m. Obtaining Exit Permit 11 a.m. to 5 p.m. Depart for New York 6 p.m.
Tuesday,	29th	Visited ADMIRAL IRISH, U.S.N. at his Office, Broadway, and thereafter with him visited MR. ALFRED MYER at MESSRS. GIBBS & COX 9.30 to 11 a.m. Passage Arrangements 11 a.m. to 1 p.m. Visited MR. J.W. ATKINSON of PARSONS MARINE TURBINE CORPORATION, West Street, 4 p.m. to 6 p.m. Embarked 8 p.m.

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The principal personnel met at the Bureau of Ships and at each of the four firms visited were as follows:-

Bureau of Ships

	Admiral Brand:	
	Captain Sharp:	
	" Ward:	
*	Mr. Ball:	

Falk Corporation

	Messrs. Hermann Falk:	President
	Harold Falk Senr:	Vice President
	Carpenter:	-do-
*	Louis Falk:	Plant Superintendent Engineers.
	Harold Falk:	
*	Richardson:	
	Green (Left David Brown about 1910):	Works' Manager.
	Lieutenant Frasier:	U.S.N. Supervisor.

G.E.C.

	Messrs. K.M. Holt:	Turbine Engineer
*	E.N. Twogood:	Gearing Engineer
*	J.J. Zrodowski:	-do-
	L.E. Whitescarver:	Assistant Sales Manager.
	Young:	Superintendent Gear Shop
	R.J. Trenholm:	Asst. -do-
	J.J. Davey:	Application Engineer, Marine Dept., Schenectady.
	H. Papazian:	International G.E.C. Schenectady.
	Commander Sampson:	U.S.N. Supervisor.

De Laval

*	Messrs. C.R. Waller:	Vice President and Chief Engineer.
	A. Peterson:	Chief Engineer, Pump and Compression Department.
	Bauer.	
	Navy Dept. Supervisor:	

Westinghouse

*	Messrs. J.R. Davies:	Plant Engineer.
*	Seymour:	Works' Manager.
	Navy Dept. Supervisor:	

* Denotes Personnel with whom principal discussions took place.

On the visits to the Navy Department and the four principal firms, I was accompanied by -

X Commander (E) L.E. Rebbeck, R.N. - Asst. Naval Attaché
Lieutenant Commander (E) G. Miles R.N.V.R.
Mr. G.W. Richmond, E. in C's Dept.
Lieutenant A.J. Kroog, U.S.N.R.

X Except at De Laval and Westinghouse.

GENERAL IMPRESSIONS.

The four firms which were visited differ very much in character.

G.E.C. and Westinghouse, particularly the former, are large highly organised plants which might be said to typify my pre-formed notions of American Industry as based on popular comment. On the other hand, De Laval and Falk are strongly reminiscent of British firms. The latter, however, had an atmosphere of its own, which could only be described as being born of the outstanding character of the Management.

The President, Mr. Hermann Falk, is now an elderly man, but their shops show much evidence of his practical engineering instinct, and this tradition is being worthily maintained by his nephew, Mr. Louis Falk and his colleagues. No doubt the extreme success of the firm as Steel Founders, has been a direct result of this applied engineering intelligence. Incidentally, it may be remarked that the Plant, which employs about 3000, is manned entirely by Non-Union Labour. Walking through the shops with the President, it was apparent that he knows most of the men personally, and, furthermore, that he was well respected by them.

As an extreme contrast G.E.C. could be quoted, where, having regard to the tremendous size of this firm (in all their various Plants they employ 230,000), there is an impersonal atmosphere, and the high degree of organisation that would be expected in an undertaking of the magnitude. Questions of man power and expense appear to be of little consequence in the endeavours of the Executive of each Department to ensure that their products are of the highest quality, and that no smudge shall besmirch the name of their firm before the eyes of the engineering world.

The attitude of the Navy Department to all these firms is interesting. Where questions of interchangeability and general policy are involved, they specify their requirements, but so far as details /

details are concerned, even large details, the respective firms are given an entirely free hand, and the responsibility is theirs for producing a unit that functions satisfactorily; thus for example while gearcase seatings and gear dimensions are standardised from the point of view of interchangeability, the details of the construction of the gearcases and running parts vary between extreme limits according to the opinions and special facilities of the Manufacturers. All this goes to emphasise that there is more than one satisfactory way of fulfilling specific requirements in the manufacture of any unit, and it is misleading to imagine that various American Manufacturers holding different view points, have been brought together and have reached an absolutely agreed decision regarding a design, subsequent to open discussion. The perfect design which appears to glitter on its pinnacle is an idealism for which all are striving and while the nearest approaches thereto are as many as they are various, any distinction between the best of them tends to become more a matter of prejudice than fact.

So far as gearing is concerned, the Manufacturers in the States have been forced into a common policy to suit the requirements of the U.S. Navy as regards type of design, and the firms which have had to bow to the inevitable in this respect and have been overruled in their individual opinions still hold their original views, even after experiencing the merits emphasised in reaching the decision for the adoption of a rival firm's design.

Even on a question of theoretical design policy, the same situation exists.. The Gearing Manufacturers in the States were brought together to agree upon a loading formula. Each firm had its own particular ideas in this connection, varying in degrees of complexity, but in reducing their ideas to a common basis, the gross product of their deliberations was a recapitulation of the well known formula P/D corrected for the finite diameter of the main wheel by incorporating a simple expression of the gear ratio. It was made clear that this arrangement was brought about to satisfy/

satisfy the Navy Department, but that firms which have thought more deeply in the matter are not impressed with the usefulness of this oversimplified coefficient.

The opening remarks made by Mr. Davies of Westinghouse during our discussion are worth particular reference. He said that he was willing to give us all the information we wanted, but that it surprised him that we had come so far in connection with any problem of gearing construction. He questioned whether all concerned were not becoming too academic in this connection, particularly with regard to tooth surface, and contrasted the immunity from serious trouble of gears of all types to the difficulties which have been associated with the turbines of high pressure installations. He would have been much less surprised to have met a mission associated with Turbine design. Incidentally his remarks regarding gearing itself might find corroboration in the service experience of British gears in both Naval and Merchant practice.

In concluding these general remarks, it would be appropriate to express acknowledgement of the care which must have been expended on the involved arrangements for the tour by Admiral Burt and his staff at the British Admiralty Delegation, Washington, and also to Commander (E) Rebbeck and Lieutenant Kroog U.S.N.R. who accompanied us on the visits; also to the firms themselves for the extreme hospitality and attitude of helpfulness which was shown in all instances, although the latter are unlikely to read these words.

Permission to visit the four plants and also Messrs. Gibbs & Cox, Consulting Engineers in New York, was granted officially by the U.S. Navy Department. With regard to the latter firm it had been the Committee's request that I should, if possible, meet Mr. Gibbs personally but at the last minute circumstances, as far as Mr. Gibbs was concerned, made this impossible and his place was taken by his Chief Assistant, Mr. Alfred Myer.

To meet the wishes of the Committee and further the investigation to its utmost, Commander Rebbeck endeavoured to make a private arrangement for me to meet Mr. Burkhardt of Bethlehem Steel Corporation but owing to his immediate departure from Boston on other business Mr. Burkhardt arranged for his very able assistant Mr. Tingey to meet me privately at the Statler Hotel.

A. COMPARISON OF NESTED AND LOCKED TRAIN GEARS.

1. General.

Bureau of Ships. 16/2/44: On the one hand Falk and Westinghouse are in favour of the nested type while, on the other hand, G.E.C., Farel Birmingham and De Laval (the originators) favour the locked train type. For the purpose of standardisation, locked train gears have been chosen for all classes of vessel although no trouble has been experienced with the nested design in general. Nested gears are in fact being employed for powers as high as 32,500 S.H.P. per shaft at 185 R.P.M. on battleships now fitting out. The Navy Department have had no more difficulty with alignment in one type than the other.

Falk: Have supplied nested gears for 40 destroyers of 25,000 S.H.P. per shaft and all have been entirely successful although one batch of them has been rather noisier than it should have been. De Laval developed the locked train which particularly suits the small hobbing machines, of which they have an abundance. G.E.C. have refused to have anything to do with the nested design. Falk were strongly opposed to the locked train on account of its complication and the added hobbing times, having regard to the fact that their machines were laid out for the production of nested gears. When standardisation was brought about the Bureau decided on locked train, the preference of the main gearcutting firms being three to two in its favour. Behind the scenes Mr. Gibbs has had a large influence with the Bureau and as he fathered the locked train type in its infancy, his opinions have presumably carried weight in that direction. In their earlier designs of nested gears, Westinghouse ran into some trouble with their so-called floating frame which did not maintain alignment. They also had helical angle troubles which landed them into difficulties with both the nested and locked train designs.

G.E.C.: Have never built any nested type gears and in view of this fact they are not in a good position to offer criticism of its /

its merits. Their facilities are suited to the locked train design which they first employed in cargo boats during the last War and which they claim to have developed into its present form in 1933. Apart from considerations of possible alignment difficulties, they have no technical objections to the nested type, but from a manufacturing standpoint, they prefer to have separate primary and secondary units so that production can proceed independently on the various components until such time as they are ready for assembly. Damage to a secondary pinion does not entail dismantling the built up unit and recutting the primary wheels after reassembly. They state that troubles have been experienced due to lack of rigidity in the nested type (see possibly De Laval). G.E.C. merchant and minesweeper gears are of articulated type.

Mr. Tingey of Bethlehem Steel Corporation emphasised that from the experience which he had had with repair work, there was no marked distinction between the two types of gear in performance.

When the Navy Department decided to standardise gears, the two largest firms in the States wished to adopt different designs. G.E.C. were wedded to the locked train design and Westinghouse were in favour of the nested design but at that time the latter firm was having considerable trouble with some turbines and possibly on account of this fact they were not able to shout with quite such a loud voice at Washington, and in consequence G.E.C. won their point.

Mr. Tingey was not asked for a categorical opinion of the two designs, although he had been advised by Commander Rebbeck, our Assistant Naval Attaché in Washington, of the purpose of my visit, and he volunteered the statement that in his opinion the locked train design was an abortion. Under ideal conditions in the shop, it is possible to set these gears, but, in the event of a replacement being required on board ship, the difficulties are such that it is their practice to scrap a complete train to replace one pinion. The difficulty is magnified by the fact that any variation in tooth thickness between any unit in one train and the other /

other renders it impossible to obtain a balanced load astern with the same setting as for ahead. Further difficulties are associated with variations in lengths of quill shafts as these are usually finished to place to suit individual gearcase set ups.

From the point of view of overhaul, the accessibility of the thrust block and forward main bearing is abominable.

De Laval: A photograph was produced showing an early De Laval design of nested gear with which the Navy Department had had trouble in alignment. This showed the centre bearings (4 bearing type) of the primary pinion to be of extremely light construction. Mr. Waller is of the opinion that there is greater chance of distortion with the nested type of gear than with the locked train type and they are fully in agreement with the Navy Department's decision to standardise on the locked train.

Westinghouse: 1938 programme destroyers DD397-420 all incorporated nested gears which so far as Mr. Davies knows have been entirely satisfactory. At the present time, the Battle Ships "ALABAMA" and "INDIANA" are being fitted out with 32,500 S.H.P. units of the nested type.

In one particular design, the locked train gear has been shorter than the nested type but this is an exception. The locked train is a problem to line up but when this job is done carefully the gear is entirely satisfactory in operation. If the power is unevenly distributed between the trains the bearings of the primary pinion take a load for which they were never designed, and gears have failed from this cause..

Westinghouse very much dislike the lack of accessibility to the main thrust and have, in fact, modified their designs to provide the oiltight plate, flush with the foremost plate of the gearcase, thus completely housing in the thrust and forward main bearing. When this oiltight plate is removed the access to the thrust and main gearing is possibly somewhat better than in other designs but the drawbacks of such a design are obvious.

Westinghouse /

Westinghouse prefer the simplicity of the nested type of gear and Mr. Davies cited the well-known engineering criterion that simplicity usually goes hand in hand with the best product.

Bureau of Ships. 28/2/44: Mr. Bell was asked regarding tooth thickness requirements, in view of difficulties of interchangeability with the locked train design of gears. He stated that there were no requirements in this respect, and that he was not aware of any difficulties having been met with.

Admiral Irish & Messrs. Gibbs & Cox: Admiral Irish emphasised that he had not been intimately associated with gearing production during the last few years but he was satisfied in his own mind that the nested type of gear was easier to produce.

In operation, he was of the opinion that the two designs were six and half a dozen.

When it came to a question of overhaul, Admiral Irish stated that the locked train design presented difficulties. Conditions on board ship were not suitable to the concentration of care and thought which the adjustment of these gears demanded and which they receive under ideal conditions in the shop.

Mr. Myer stated that G.E.C. had developed the locked train gear and that no serious troubles had occurred with these designs.

The Navy Department desired to introduce a policy of standardisation of interchangeability of running parts and gearcases and speaking for Messrs. Gibbs & Cox he felt they were fully justified in adopting the locked train design.

With regard to alignment difficulties, there had been instances of residual stresses in castings producing unfortunate effects in this direction in both nested and locked train designs.

So far as space requirements of the two designs of gear were concerned, there was very little to choose between.

He thought that one of the main advantages of the locked train type of gear was that the bearing thermometers were placed prominently /

prominently for observation.

2. Production Time.

Bureau of Ships. Mr. Ball has gathered that the locked train can be set up quicker than the nested type of gear but states that comparison is difficult with different firms.

Falk: State there is 25% more gearcutting in the locked train design, this being largely brought about by additional setting time for the greater number of units (Falk's speeds and feeds are high). They agree that 60% is a reasonable statement of additional fitting time for locked train as compared with nested gears but when they first went into production they were even more badly delayed due to such items as missing couplings, muddled quill shafts, etc.

G.E.C.: For the locked train design the elements of production are not interdependent and they consider that this speeds up production.

De Laval: 20% saving in gearcutting time for locked train type.

Westinghouse: State there is no difference in gearcutting times for the two designs but that the total production time for the locked train type of gear is twice that for the nested type.

3. Weight.

Falk: Gave the weight of a 30,000 S.H.P. set of locked train destroyer gears (dry) as 2 lbs/S.H.P. Their corresponding nested gear would be 10-15% lighter than this although the ratio would probably cross over and favour the locked train design for main wheel diameters greater than 10 feet.

Mr. Tinney: With regard to weight, this depends more upon the manufacturer than upon the design of gear.

De Laval /

De Laval: The locked train gear for a destroyer is 10,000 lbs lighter per shaft than the corresponding nested design.

Westinghouse: No difference in weight for nested and locked train gears.

4. Noise.

Falk: Noise level of nested and locked train designs not markedly different but possibly the locked train is slightly quieter due to the lighter parts but there have been both relatively quiet and noisy gears of both types.

Mr. Tingey: As regards the noise level, his opinion is that it depends upon the individual gears rather than the type of gear. Even gears of similar design made by the same manufacturer may have widely different noise characteristics.

De Laval: Do not consider that there is any difference in noise for the nested and locked train designs.

Westinghouse: The noise levels of the two types of gear are the same, and are the same also as a single reduction unit at the same power. The adoption of double reduction gears does not involve added noise.

B. DESIGN & CONSTRUCTION OF NESTED GEARS.

1. Primary Pinion Bearing Arrangement.

Falk: Until recently Falk have preferred primary pinions of the three bearing type, the shaft being tapered from the tooth root diameter to the journal diameter, in contrast to the four bearing type developed by Westinghouse. In their latest designs, however, they have now developed the two bearing pinion, the portion between the helices being swelled in diameter, in some instances to as great a diameter as that of the coupling. They consider that this eliminates any possibility of the primary pinion snaking and their limiting provisional rule in this respect is

$$F \text{ max.} = \frac{5.5D}{\sqrt[4]{G}} \qquad \left.\begin{array}{l} F = \text{face width} \\ D = \text{pinion p.c.d.} \\ G = \text{gap} \end{array}\right\} \text{See Fig.1.}$$

Where the shaft diameter is swelled in excess of the helix diameter, the hobs may inevitably run in to the ends of the swelled portion.

Westinghouse: Westinghouse are now also developing the nested type of gear with the two bearing primary pinions in which they limit the actual deflection over one face to .6/1000", the pinion being swelled to the coupling diameter, if necessary, between the helices and the end of the helix teeth washing into this swelled portion if as may be unavoidable.

2. Permissible Deflection of Primary Wheel - Secondary Pinion Unit

Westinghouse: The maximum bending deflection which Westinghouse permit for a secondary pinion is .001" over one face.

3. Torsional Vibration.

Falk: Second order torsional oscillation frequency of nested gears is very high - well above working revolutions, even in Destroyer designs.

Westinghouse: The nested type of gear is a stiff system and the critical torsional approximates to twice the running speed. The articulated type of gear (of which the locked train is an example) can not have such a high critical in view of the flexibility of the quill drive.

4. Hobbing Arrangements for Primary Wheels.

Falk: In the hobbing of nested type primary wheels it is Falk's standard practice to rough cut the wheels on mandrels with wheels, say, 3" apart then to shrink them on to the pinion with their standard shrinking rings and thereafter to finish hob the wheels, turning the element over in order to cut the second helix or otherwise depending on machine available and the length of the unit. It was stated, however, that for repair jobs they have finish cut the wheels on mandrels and these have been sent to the repair shipyard, fitted to the pinion and put into service without ill effects.

5. Typical Proportions of Gears for Naval Vessels.

Falk: The type of gears proposed by Falk for the nested design in the latest Class of Destroyers (which have actually all been fitted with locked train type) is indicated on Fig. 2. The particulars of the proposed design for these gears are as follows:-

	Primary			Secondary	
	H.P. Pinion	L.P. Pinion	H.P. & L.P. Wheels	H.P.& L.P. Pinion	Wheel
Nominal D.P.		7		5	
Actual "		6.190		4.419	
No. of teeth	76	90	277	91	358
P.C.D.	12.3"	14.55"	44.75"	20.6"	80.6"
Total face plus gap		22½" + 33¼"		30¼" + 2¼"	
Spiral angle		40°		40°	
Hobs		.389" Pitch	15° Flank	15° Flank	
S.H.P.					
R.P.M.	5695	4805	1563	1563	397
Pressure lbs/in.	1170	1238		HP 1900 LP 2000	
P/D	121	112.7		HP-116	LP 122.1
Bearing sizes		7½" x 7½"		14" dia. x 11"	16½"x16½"

C. DESIGN AND CONSTRUCTION OF LOCKED TRAIN GEARS.

1. Quill Shaft Design, Torsional Oscillation, Steady Bearings.

Bureau of Ships: There are no torsional oscillation difficulties. After prolonged investigation, the Navy Department had ceased to require consideration of torsional characteristics, in view of the heavy damping in the system. In earlier designs which incorporated elaborate boring of pinions and quill shafts, this was done entirely with a view to saving weight. Later wartime designs have plain bores.

Falk: No torsional oscillation problem for Naval gears where the excitation is low, but with slow running Merchant gears, this is not necessarily the case.

Confirm that the elaborate bores in pre-war pinions were to save weight to avoid a 25 c/lb. penalty for exceeding the guaranteed figure.

Quill shafts are designed for a stress of 8,000 lbs/sq.in. nickel steel being used formerly but now a straight carbon.

Neo-prene (synthetic rubber) steady bushes for the quill shafts have been tried by Falk, but they are not considered necessary.

G.E.C.: The second order torsional critical has no effects. It used to be customary to design for nodal tuning, but this has now been abandoned having regard to the high damping effect of the bearings.

The combined bending and torsional stress in the quill shafts is based on a maximum of 5,000 lbs/sq. in. for carbon steel, and 10,000 lbs/sq. in. for nickel steel.

No steady bearings are fitted for the quill shafts.

De Laval: Confirm torsional vibration of no importance, having regard to the damping effect of the bearings.

Quill shafts designed for torsional stress of 6,000 lbs/sq. in. with even distribution of load between the trains, using carbon steel. If nickel steel is employed, the stress limit is raised /

raised to 8,500 lbs./sq. in.

Steady bushes are not used, and the quill shaft is provided with a radial clearance of 1/16" or 1/8".

Westinghouse: Still prefer to balance the system for a second order torsional critical, although the Navy Department have dropped any requirements in this respect.

Plain bores are now adopted, and the quills are usually of nickel steel, no steady bearings being provided for them. There is a tendency for the quills to rust up solid, and it is Westinghouse practice to paint them with a rust preventive before assembling.

2. Location of Rotating Parts.

De Laval: Secondary wheel is located at the centre of the oil pan and the gears are set up to accommodate this setting. Primary wheels have a total clearance of .015". There is about 1/16" axial play in each coupling, so that the total axial play in the secondary pinion is about .125" + .015" say .140". There is 1/4" clearance between the butting shafts of the secondary pinion and primary wheels.

3. Coupling Design.

Bureau of Ships: Star couplings are considered as completely out of date, and it is now difficult even to obtain replace components in the States.

Four firms in the States specialise in the production of small tooth couplings; included among which are Falk and G.E.C.

80% simultaneous bearing of teeth is required, and this is frequently only obtained by hand work.

Falk: Small tooth couplings are much preferred.

In designing the couplings, the tooth pressure is kept as low as accommodation for the unit will permit, although they have no /

no knowledge of any trouble having arisen with any firm due to excess loading.

It is Falk practice to shave the coupling teeth after cutting and 80% contact is normally obtained without hand work. The ends of the teeth are relieved during the shaving process.

The claws are hobbed and the sleeve teeth are machined on a Fellows planer.

No difficulty is found in lubricating the teeth, but the end plates are now omitted so as to allow a circulation of oil without risk of collection of mush.

<u>G.E.C.</u>: Couplings are designed on a basis of 400 lbs/in. loading on the teeth based on 80% teeth in contact. Both claw and sleeve teeth are machined on Syke's Planers, and no hand work is required to obtain the requisite contact.

<u>De Laval</u>: Designs are based on 350 lbs/in. loading on the coupling teeth, 90% of the teeth being in simultaneous contact for any position of engagement, a requirement which De Laval insist upon being observed by their Subcontractors. The designed backlash of the couplings is .015" but as little as .009" is permitted, and as much as .090".

<u>Westinghouse</u>: Design based on 350 lbs/in. on the teeth, the assumption being that they are all in contact. Some couplings have been designed on twice this loading, and of these, some have worn badly. It is essential to provide an adequate circulation of oil to the couplings to avoid the collection of mush, and it is useful to provide a byepass for oil flow from any restricted portion of the coupling to avoid this occurring. Designed backlash .015".

Westinghouse find it necessary to specify very carefully the conditions which must be satisfied to give satisfactory meshing. They permit a maximum out of balance of 8 oz. inches.

4 /

4. Boring of Gearcases.

Falk: Cast steel jigs are provided carrying all the boring bars in roller bearings, it being so arranged that the gearcases can be set up on the machine bed without disturbing the jigs. Two separate boring heads on continuous horizontal shears, provide for simultaneous boring of pinion and primary wheel housings for the H.P. and L.P. sides of the gearcase. A separate motor and specially adapted boring head, situated on the remote side of the gearcase, provides for simultaneous boring of the main wheel housings. This unit is non-adjustable for position.

Light cast steel frames are set up above this latter motor for supporting the boring bars when withdrawn.

The time taken to bore the housings of a 30,000 S.H.P. gearcase of the Destroyer type with this equipment is 40 hours. This is accomplished in 3 cuts, the first which leaves 1/16" radially is accomplished by 2 tools per bar, one working in each housing of the same size. The second cut is taken with 2 split collars on each bar, each ring carrying 6 carefully set tools. .012" is left for the finishing cut, which is accomplished by 2 one tool cutters provided with micrometer adjustment.

G.E.C.: Boring is effected by similar set up to that employed by Falk, except in that the boring bar jigs are provided with plain instead of roller bearings, that multiple cutter tools are not provided for the semifinishing cut, and that the jigs are of fabricated construction. The boring time per gearcase is about 100 hours, during 60% of which time the main boring bar is in operation. The bores in the gearcase are of uniform diameter throughout in any one axial line, and there are no recesses. In boring out, .002" is left for bedding the mandrels by hand. No importance is laid on maintaining alignment between primary and secondary gears within this limit.

De Laval: Boring is accomplished by a single head machine.

Cast /

Cast iron jigs are employed for the boring bars running in plain bores. The average time taken per gearcase is 190 hours.

Westinghouse: Boring of gearcases is carried out with a single headed machine, employing cast iron jigs with plain bearings for the boring bars. To increase the capacity, additional gear-cases are bored on a similar machine without the use of boring bar jigs. The relative times are as follows:-

	Hours With Jigs	Hours Without Jigs (bored after assembly)
Upper portions	110	
Base - Main Bearings and Thrust	120	
Vertical Milling	37	
Top and bottom portions together	30	
Total ...	297	380

5. Adjustment for load Distribution between Locked Trains and gear alignment.

Bureau of Ships: Advised that it would be found that the opinions of the various firms differ widely on this point.

Falk: Falk consider that the best distribution between the trains that can be relied upon is 62/38 although on an average they have probably now improved their technique to give say 58/42.

Their procedure for adjusting the trains is firstly to provide the pinions and wheels with zero clearance bearings all concentric except in the case of those for the main wheel which are eccentric in the direction of the ahead loading. The aft lower couplings of the H.P. and L.P. trains are left undrilled.

Backlash is removed throughout the trains in the ahead direction with suitable levers. The undrilled coupling is then marked, removed, drilled and reamed. The unit is then replaced and meshing is checked by painting a band of lamp black marking on the pinions and joggling the gears ahead and astern by hand with a lever on the turbine coupling flange. This process was witnessed and showed that the black lead marking had been lifted right across both /

both the ahead and astern faces of each train.

It was emphasised that secondary pinions or primary wheels with thin teeth must be paired off or the astern marking would be indefinite.

An error of .002" in the marking of the coupling flange represents a 10% out of balance loading. In consequence of this, there are no interchangeable spares for locked train gears. In making this statement it should be noted that Falk are not in favour of adjusting the gears by making use of prime teeth in the flexible couplings or the difference in the number of teeth between the primary and secondary gears, because of the difficulty of knowing what adjustment has been achieved when the process is not based on the manual absorption of backlash; consequently Falk provide their forward and aft couplings with a uniform number of teeth.

The process described above is the very antithesis of the procedure which Falk originally thought necessary when they started to build locked train gears. At that time they torqued up the gear in an effort to take account of gearcase distortion, but the results which they achieved were variable and unsatisfactory.

In meshing the bearing housings, allowance is made for .002" scraping to obtain alignment. Alignment in a vertical plane is obtained with the use of a spirit level having an accuracy of .0005" per inch per division.

<u>G.E.C</u>: G.E.C. consider that there is a 50/50 distribution of power between the two sides of the locked train, both in theory and practice. Adjustment of gears is effected by prime difference in number of teeth between primary wheels and secondary pinions. The small tooth couplings at the forward and aft ends are fitted with an equal number of teeth, thus providing no additional adjustment The coupling keyways are finished to drawing size and the adjustment of the train is effected entirely by the above method. It is their experience that if a single coupling is left for marking off the /

the keyway to place the errors involved in machining are sufficient to put the adjustment of the trains out to such an extent that prime tooth adjustme.t has to be resorted to in any case. At my particular request I was permitted to wait for about two hours at the test bed where they happened to be effecting an adjustment of this nature. The back to back test had shown a li:htness of marking on one side of the teeth, as evidenced by the fact that the printers' ink which they paint in axial bands on the pinion teeth prior to the back to back test had not worn off one train. Regardless of the elaboration of the back to back tests, the amount by which the trains were out of adjustment, had of course, to be measured and this was done by insertion of shims between the tooth faces in the following manner:- shims of a thickness of .005" were placed between the outer ends of all teeth in mesh. Torque was then applied by lever on the primary pinion and it was found that the shims of one train remain free. By a process of trial and error, it was found that all shims became simultaneously nipped when an additional thickness of .014" was inserted between the secondary teeth of the upper pinion and main wheel. The bearings, other than those of the main wheel, were packed up to make the journals concentric in the oil clearance. Reference to the Chart provided for the purpose showed the relative movement of secondary pinion and primary wheel that it was required to make. Unfortunately, the squad employed on this job appeared to be unfamiliar with the requirements, and after certain confusion had arisen, the job was abandoned for the next shift to tackle.

De Laval: With their original method of assembling gears under static torque with loaded arms, they concluded that the load division might be as poor as 40-60. They have now abandoned that method of setting up and consider that they have appreciably improved the balance by the following method:- The complete assembly of the gears and the fitting of all the couplings except one from which they omit the key, the keyways having all been finished to drawing dimensions /

dimensions. The backlash of the gears is then taken up by suitable levers and the amount by which the keyways of the unkeyed coupling is out of line with the keyway in its shaft is carefully measured. They then make the necessary adjustment to the relative position of the secondary pinions and primary wheels of one train, having regard both to the fact that the number of teeth in the respective elements have a prime difference and also that further adjustment can be obtained due to the fact that the forward and aft couplings are also provided with a prime difference in the number of their teeth.

A description and typical calculation of the method employed was provided by the firm, and is attached hereto, Fig.3.

It was stated that a 10% out of balance of load between the two sides of the train was represented by a key setting error of .005" for the H.P. and .003" for the L.P.

During this process, the journals are allowed to lie in the bottom of the oil clearance. A check on the setting is provided by marking the gears with blue marking and rotating against a light restraining torque.

De Laval are not particular about the division of power for astern loading and they do not take account of varying tooth thicknesses on the two trains, although they do not anticipate any undue variation arising in this way.

Westinghouse: Design for 50/50 balance of load, but they do not imagine that this is achieved in practice. They consider that the actual balance of power which is achieved is indeterminable.

It is Westinghouse practice to torque up their gears H.P. against L.P. with a system of levers and springs, so as to obtain 40% of the full power meshing torque when balancing the load between the trains.

With regard to balance of load for astern running, they would pair off any pinion with thin teeth, a process which is greatly facilitated when there is a large volume of work of the same design going /

going through the shops simultaneously.

The maximum contact error which they permit over the face of one helix is .0005" up to .001" measured statically. In event of a greater error occurring the pinion bearing housings are scraped out. This was, in fact, actually witnessed and a set which had been running on torque during the first day of the visit was dismantled and the mandrels were being rebedded in the housings when seen during the second day. Gears are chosen for the most suitable meshing from the large selection which are in simultaneous production.

Admiral Irish: Is of the opinion that the adoption of a simplified method of balancing the loading on the trains may be justifiable by a firm th t has had lengthy experience with the locked train type of gear but that for a firm commencing in its construction the very greatest care would be required to avoid the possibility of mishap.

6. Turning Gear.

General Note: The type of turning gear standardised on all locked train designs is indicated on drawings available in this Country. On account of the high reduction ratio employed the gear may be used for the dual purpose of holding the shaft at sea even with the other shaft developing full power.

D. GENERAL GEAR DESIGN OTHER THAN TOOTH FORM

1. Gearcase Construction & Rigidity

Falk: Great emphasis is laid on gearcase rigidity and they employ heavier plating than was seen in any of the other Plants visited. Weight is kept down by extensive lightening holes carefully designed so that the strength of the main structure is well distributed. This policy necessitates a great deal of flame cutting which is accomplished on large pantograph machines. The plate to be cut is placed on a steel grid from which it is insulated by copper pads which conduct the heat away and prevent burning of the grids. Scrap is directed on to a sloping grate which deposits it into a sump below the grating forming the floor, from whence it is easily collected. Plate thicknesses employed are of the order of $2^1/4$" for the inner wall and $1^3/4$" for the outer wall. Lightening holes in oiltight walls are closed by thin steel welded panels which have been previously pressed with a cruciform indentation to provide flexibility for welding.

For the locked train design, the A frames are cast in pairs Port and Starboard and afterwards parted by a flame cutter. Bearing assembly houses are centrifugally cast. In some designs the entire top half of the gearcase is of cast steel. No trouble has been experienced with distortion but it must be recognised that Falk's steel founding technique is quite exceptional and other firms in the States have attempted to follow them in some respects, without success. In the Welding Department extensive use is made of a wood lined pit about 30 ft. square and 10 ft. deep for facilitating welding, the partly fabricated structures being leaned up against the sides of the pit at angles suited to the work in hand. Two large manipulators are also in use. One welding machine is employed over the pit working on the Union Melt system.

G.E.C./

G.E.C. The general design of fabrication is similar to practice in this Country. The extent to which welding is employed in gearcase construction varies, some designs employing cast steel bearing housings and others being all fabricated. The view was expressed that the former are the easier to construct, as can well be appreciated.

De Laval: The type of fabrication employed is very similar to that of G.E.C. although possibly of rather lighter scantlings.

Westinghouse: The characteristic appearance of a Westinghouse fabricated structure is the unbroken surface of the outside plates. This is in striking contrast to Falk and accessibility to some of the welding must be difficult. After completion gearcases are tested by filling with oil up to the joint at a temperature of 140°F.

2. Main Wheel Construction.

Falk: Wheels are of cast steel construction and of design as indicated in diagram 4. The hub is split and is fitted against a shoulder on two parallel diameters on the shaft, retention being provided by two shrunk rings and location ensured by a ring fitted in halves in a groove in the shaft at one side of the wheel, this ring itself being retained in position with a shrunk-on ring. Wheels are balanced dynamically.

G.E.C. The general design of G.E.C. wheel consists of a cast steel hub, boiler plate discs and forged rim all welded. For the rims G.E.C. prefer high carbon content, say up to .50% but the Navy Department limit is .40%, Brinell being 160-190. All welds are magnafluxed. Prior to and during welding the hub is heated by gas jets and the rim by an induction coil consisting of about 15 turns of cable absorbing 40 K.w. and giving a rim temperature of about 150°C. The actual process of welding the wheel takes six men about 100 hours for a Destroyer wheel. They can only work/

work at the job for a limited period without a rest on account of the heat. The hubs are fitted on a $\frac{1}{2}^{o}$ taper and are hydraulically pressed up against a collar, the draw being 1/2". The bore of the hub is finally scraped by hand to obtain the required draw. After annealing the rims are finish turned with a carbide tipped tool at 220 ft/min. giving a highly polished finely serrated surface. Wheels are balanced dynamically.

De Laval: The De Laval design of main and primary wheels incorporates two dished side plates. These are bought in from outside, finish welded. In accordance with Navy Department requirements for metals having a carbon content of .40% or over all welds are Magnafluxed. The rims are heated to 130°F. for welding. Thereafter the wheels are annealed to a temperature of 50° less than the rolling temperature of the rim, a careful history of which is kept. The Brinell of the rims, varies between 160 and 190, 150 being regarded as the minimum permissible.

Wheels and in fact all rotating parts above 150 R.P.M. are dynamically balanced.

The main wheel hub is pressed up on a $\frac{1}{2}^{o}$ taper on the spindle by a certain definite amount until it comes hard home against the shoulder provided. The amount of the axial draw-up is calculated so as to give a ring stress of 8,000 lbs/sq.in. in the hub. A nut is fitted as a provision against slackening back.

Westinghouse: Main wheels are all fabricated with flat side plates. Intercostals between these plates are provided but the plates and the welding is kept clear of the rim so as not to cause a variation in rim stiffness and induce possible vibration (a condition of which De Laval have had experience). Rims are flame heated to 150°F. for welding. The rims are of .35% carbon and as such do not require to be Magnafluxed. Brinell 160-190. Wheels for 150 R.P.M. and above are dynamically balanced only as a Navy requirement - Westinghouse consider that static balancing is adequate/

adequate. Main wheel hubs are pressed up against a shoulder on a taper of .005" per inch, the amount of draw being such as to give a ring stress of 15,000 lbs/sq.in. in the hub, viz., about .005" per inch diameter. No nut is fitted. Primary wheels for the nested type of gears are pressed on to a double parallel fit against a shoulder and secured with a nut and six 3/4" dowels.

3. Bearings, Loading Clearances, Wear Down, Position of Joint.

Bureau of Ships: Minimum and maximum permissible bearing clearances according to U.S.N. practice are indicated in Fig.5. The Bureau is prepared to accept gearing bearing pressures up to 200 lbs/sq.in. although the maximum which is in service at the moment (on a Cruiser) is 190 lbs/sq.in. running at 154 ft/sec. The use of bridge gauges for gearing bearings has been abandoned having regard to the fact that the static reading is no measurement of wear down on the load line.

Falk: Bearing clearances are .002" per inch diameter concentric. A value of L/D about 1.0 is favoured. Bearing peripheral speeds are considered to be of small significance and Falk would not hesitate to employ 200 ft/sec. The minimum angle between the joint and the load line is regarded as 30°. Whitemetal is centrifugally spun with a small dovetail at each end. For primary wheel bearings of nested type gears Falk are now employing solid ring bearings without a split.

G.E.C. Clearances adopted are for 2" diameter - .002" per inch diameter: 6" diameter - .0015" per inch diameter: 12" and above .001" per inch diameter. The minimum permissible angle between load line and joint is regarded as 45°. Rubbing speeds up to 150 or 170 ft/sec. are employed. G.E.C. believe in conservative loading for gearing bearings and regard 150 lbs/sq.in. as a maximum.

With regard to the proportions of the bearing a ratio L/D = 1.0 is favoured although for main wheel bearings this ratio is/

is allowed to rise to 1.5.

Whitemetal is cast centrifugally into steel shells without dovetails.

De Laval: Bearing pressures up to 205 lbs/sq.in. have been used. Little attention is paid to rubbing speed. It is usually arranged that the load line be at 90° from the joint. Bridge gauges have been dispensed with and instead the bush thickness is stamped on the shell of the bearing in way of the load line and at 45° each side therefrom. Bearing shells are of steel plate cross rolled before bending, thereafter annealed and lined with whitemetal, using a 4" riser. Centrifugal casting is not employed; cast steel shells have not been found satisfactory.

Westinghouse: Bearing clearances are - .003" per inch diameter for high speed high temperature bearings; .002" per inch diameter for first and second reduction pinions and .0015" per inch diameter for slow speed wheels. Bearing loadings are conservative - H.P. 125 lbs/sq.in. and L.P. 150 lbs/sq.in. Typical rubbing speeds are 150 ft/sec. for first reduction gears and 100 ft/sec. for second reduction, but although 150 ft/sec. used to be considered as the limit it is now considered that 200 ft/sec. would not be excessive and, in fact, the nested gears for the Indiana have bearing speeds of 187 ft/sec.

4. Lubrication - Quantity, Distribution, Oil Troughs.

Bureau of Ships. The maximum oil temperature rise that is allowed is 50°F. and the maximum permissible outlet temperature is 180°F. It is only specified that sprayers be provided for the ahead side of the mesh. The capacity of the sump is for two minutes' supply of oil at full power. The oil shield that has been adopted for U.S. Destroyers permits of the gear being placed lower in the ship and yet allows the main wheel to run clear of oil. A drain hole must be provided in the shield or the shield will fill up with mush /

mush, but when the wheel is running the pressure created inside the trough is sufficient to prevent ingress of oil through the drain hole.

G.E.C. Lubrication to journals in the locked train design of gear is effected by passages cast in the housings in way of the primary gears permitting oil to flow upwards round the outside of the housing of each bearing in turn, an appropriate proportion of the feed being discharged into each bearing. In the case of the secondary pinion bearings, the connection between the lower and upper housings is effected by an internal pipe. Oil for the bearings is fed into distributing boxes at the lower extremity of the A frames. All internal pipes are of steel with welded joints.

De Laval: No astern sprayers are fitted. The following is a typical distribution of oil for a 30,000 S.H.P. set.

Bearings	...	...	240	gallons/minute
Sprayers	...	...	60	"
Couplings	...	...	40	"
Margin	...	...	30	"
		Total	370	"

They emphasise the desirability of using good grade oil particularly for initial running and recommend the use of anti-rust inhibitors.

Westinghouse: Bearings are designed on the basis of a 30° temperature rise. The journal loss is appreciably greater than the tooth losses in the gears and it is probable that the teeth have three times the quantity of oil that they require. Sprayers are provided for L.P. astern. They are not really necessary on the H.P. but it is standard U.S.N. practice and is probably desirable for Aircraft Carriers.

5. Dehumidifiers.

Bureau of Ships: This is considered as a gadget which is of no particular value since an air vent must be provided to atmosphere somewhere or trouble is experienced with the lubrication of the couplings and if an air vent is provided it cuts out the effectiveness of a dehumidifier.

G.E.C: Consider dehumidifiers a distinct asset for keeping down corrosion of the gears.

Westinghouse: Dehumidifiers are not considered necessary. They are a palliative and it is more fundamental to stop water getting into the oil by the proper provision of baffles.

E. TOOTH DESIGN.

1. Loading Factors.

Bureau of Ships: The Navy Department have no dogmatic ideas about tooth loading but it was on their instigation that the principal gearcutting firms in the States got together and agreed upon a common loading factor. To a certain extent each firm had its own relatively involved proposals but the only factor upon which common agreement could be reached was the well-known simple expression

$$\frac{P}{FD} \times \frac{R + 1}{R} = K$$

Where P/FD = lbs. loading per inch face per inch diameter

R = gear tooth ratio.

Tooth speeds up to 10,000 feet per minute are permitted but thereafter it is thought there is danger of the oil being thrown off.

All Firms visited: Are in agreement regarding the statement of the basis of design given at the Bureau of Ships. The following are values of K -

Destroyers	-	110
Cruisers	-	90
Battleships	-	85
Cargo Vessels	-	65

All firms confirmed the first of these values and the latter three were supplied by G.E.C. It will be observed that the figures for the larger ships are based on more conservative design.

A (face + gap)/P.C.D. ratio of 2.5 for pinions is generally regarded as an absolute maximum.

Pinion teeth are not relieved at the ends. The process is generally considered unnecessary and even undesirable from the point of view of the gradual formation of a ridge towards the end of the wheel tooth.

2. Tooth Depth Pitch and Addendum Distribution.

Bureau of Ships /

__Bureau of Ships:__ These factors are left entirely to the discretion of the respective gear manufacturers. A pitch of 9 D.P. (·35") is, however, considered too small for any main propulsion gear.

__Falk:__ Take addendum equal to dedendum = $\dfrac{1}{\text{normal D.P.}}$ = Parsons' normal standard

The total tooth depth including the root clearance = $\dfrac{2.25}{\text{normal D.P}}$

In assessing these depths, the basis of design has been to obtain two complete teeth in contact in one circumferential plane. From this it will be seen that an equal addendum and dedendum is worked to. A slight correction is made, however, by reason of the fact that they cut a greater number of teeth in a wheel than the nominal pitch circle would indicate. Thus, while the number of teeth corresponding to a given P.C.D. may be 356 the actual number of teeth cut in the wheel is made 358.

5 D.P. (·628") is employed for secondary teeth and 7 D.P. (·448") for primary teeth based on normal to tooth D.P.

__G.E.C:__ Were not explicit regarding tooth proportion which they employ, but the following information was obtained for a particular design of Naval Gear -

	Primary	Secondary
Pinion addendum	.170	.200
" dedendum (ex root clearance)	.112	.200
Total	.282	.400
Normal pitch	.444	.624
Given by G.E.C. as	7 D.P.	5 D.P.
Parsons' normal standard meshing depth/pitch	.636	
G.E.C. meshing depth/pitch	.636	.640
	= .636 x 1.00	= .636 x 1.01

From/

From these figures it is derived that G.E.C. are employing -

	Primary	Secondary
Pinion dedendum/meshing depth	.603	.500

They made a casual reference to an addendum correction.

De Laval: Typical tooth forms employed by De Laval are as follows -

	Primary Gears	Secondary Gears
Circumferential D.P.	5	3.85034
Normal Pitch	.4444	.5830
Circumferential pitch	.6283	.8159
Addendum: Pinion	.141	.185
Wheel	.141	.185
Meshing Depth	.282	.370
Total Depth	.349	.450
Meshing depth/pitch	.640	.636

The secondary gears obviously correspond with Parsons' Standard $^7/12"$ tooth so far as pitch and depth is concerned but 50% addendum is employed. Hobs are formed to give radius at tip of teeth. De Laval are not disturbed by pitting if this is uniform across the tooth.

Westinghouse: Employ Parsons' standard tooth proportions with 50% addendum.

3. Flank and Helical Angles.

Bureau of Ships: This matter is left to the discretion of individual gearing contractors. Falk employ 15° flank angle and 40° helix. The latter has been adopted in favour of a 30° helix as it was thought that this would tend to reduce noise but the results are inconclusive.

G.E.C: Employ 15° flank angle. Helical angles which they employ are as follows:-

	Primary	Secondary
Naval Locked Train Gears	45°	40°
Cargo Vessels Articulated Gears	35°	30°

De Laval:/

De Laval: Employ a normal flank angle of 15° and helical angle 45°. With nested type gears in order to avoid deflection of wheel a lower helical angle is employed about 18°. It was at one time considered that a large helical angle would be efficacious in minimising gear noise but De Laval now question whether noise is in any way associated with the particular helical angle chosen. In the case of the latest Destroyers De Laval were given a free hand by the Navy Department to adopt whatever angle they desired and they kept 45° merely for convenience, having regard to the fact that that had been their recent practice.

Westinghouse: Employ a 14½° flank angle in accordance with Parsons' standard as also for pitch and depth. Westinghouse maintain that helical angle has no connection with noise and they favour 18°.

4. Backlash.

Bureau of Ships: The U.S. Navy Department standard backlash is less than ours, namely, about .015" but they are not particular as to how great it may be. It has nothing to do with the satisfactory operation of the gears.

G.E.C: Minimum backlash for 1" pitch teeth .060" with other tooth forms in proportion. They do not fix any maximum limit.

Westinghouse: See no reason to be concerned about the amount of backlash so long, of course, that it is adequate.

F. GEARCUTTING AND FINISHING.

1. Hobbing Machines, Speeds and Feeds.

Falk: In general Falk are hobbing their wheels with double headed machines. Their machines incorporate a secret compensating device which in principle has something of the creep action. In appearance the wheels and pinions look as though they had been cut on fairly good creep machines, the creep markings being of rather smaller pitch than is customary in this country. Their hobbing speeds are greatly in excess of those met with in this country and indeed of any other plants visited in the States. Using a 4" - 4½" diameter hob and 7 D.P. the following are typical of pinion cutting speeds -

	R.P.M.	Feed	No. of Starts
Roughing	40	.062	5
Finishing	50	.100	1

They admit that as a result of these heavy feeds the tooth surface has deteriorated but this is corrected for in the shaving process.

Gears in excess of 50" diameter are too large for their shaving machine and these are cut with more conservative hobbing speeds.

They agree that it is desirable to finish hob a wheel with the same direction of rotation of the table for each helix but they do not lay great str ss on achieving this end, the object of which is in any case destroyed when using a double headed hobber.

Five of their new machines are of their own design which is quite unorthodox and is shown diagrammatically in Figure 6. The machine tables are not provided with brakes but the hob spindles are so fitted.

G.E.C: Have 31 Gear Hobbing Machines several of which take a 200" diameter wheel, hobbing with double heads. Machines are all of non-creep design. It is their opinion that a creep wheel is only a palliative for a poor worm wheel, and they have concentrated rather on obtaining really accurate worm wheels. Many of their machines, including their largest, are of their own manufacture following Gould & Eberhardt practice. They employ cast iron worm wheels with bronze worms, and they do not experience wear with the worm after it is initially /

initially run into mesh. They emphasise the desirability of running the machines with a light load, so as not to strain them, and so destroy their accuracy. With this in mind they only use single start hobs for roughing, although they run up to about 30 revs. the feed being of the order of .030". For finish cutting, they have a feed of .030" and hob revs. about 43.

De Laval: All large gearwheels are cut with the spindle horizontal. In their newest machines there has been some disagreement between the Managers of the Firm as to whether this principle should be continued and they admit that in so doing they have merely been adopting a conservative policy. Their gearcutting arrangements were not very impressive. Using small diameter hobs their feeds and speeds are as follows -.

Roughing	-	.045" feed	-	50 R.P.M.
Finishing	-	.035" feed	-	30 R.P.M.

using all single lead hobs.

Westinghouse: Have 29 hobbing machines, many of which are in the new Maritime Defence Shop and these latter are fully enclosed and temperature controlled. Most of the machines are of Gould and Eberhardt design, many of which have been completed and erected by Westinghouse. The machines are of non-creep type with small pitched wormwheels. With regard to creep machines, Westinghouse are of the same opinion as G.E.C. The older machines have steel wormwheels and bronze worms but the latest machines have hardened steel worms and bronze wheels. These wheels have been milled by Westinghouse on their best gear hobbing machine. The large machines (taking about 200" diameter wheel) have 720 teeth in the wormwheels, about 5/8" pitch. A separate wormwheel is provided for roughing, having a pitch of about three times this amount. The worms are about 6" diameter. The worm shaft thrusts are of the ball type but these have been specially manufactured for the purpose. Mention was made of the unfortunate effects which accompany the use of a mechanite wheel with a hard steel worm. The appearance of wheels as finished hobbed varied. Those from some machines gave only slight evidence of /

of the wormwheel spiral while in the case of other machines the effect was prominent. The general tooth surface presented the evenness associated with a non-creep table. A characteristic which was unusual in the hobbing of the gears was the ragged edge which was formed at the tips of the teeth. The new machines are of the double headed type. Hobbing speeds and feeds utilised by Westinghouse with $3^3/4$" diameter hobs are -

Roughing	-	Feed .060" at 35 R.P.M.
Finishing	-	Feed .040" at 35 R.P.M.

Westinghouse are not particularly keen on the two worm drive for the hobbing machines as they are always afraid that the roughing worm may be used for the finishing cut.

2. Hobbing Machine Accuracy.

Falk: Lead screw accuracy .5/1000" in 18".

Cumulative error readings were not witnessed but individual pitch error readings over a span of about 12" showed values varying up to ±.0005".

G.E.C.: Lead screw accuracy .6/1000" in 30". 1.2/1000" overall.

Individual pitch error readings over single pitches on elements as cut give a maximum error reading of .3/1000". The maximum cumulative error was stated to be about 1/1000" for a wheel of about 14'0" diameter, but from the appearance of the single pitch readings which were witnessed, it is considered that the cumulative error quoted is extremely optimistic.

De Laval: Lead screw accuracy .5/1000" in 18".

Westinghouse: Split test wheel cumulative error .001" maximum for small machines but develops to .002" with wear. For larger machines the latter figure may rise as high as .003". For a contact error in excess of .001" the spiral wheels on the machine are altered to provide correction. This in all probability entails using different spiral change wheels for cutting left and right hand /

hand helices. Westinghouse gave the error of their lead screws as .3/1000" to .5/1000" in 12", but if the lead screws are indeed as accurate as this, there would seem to be little need for the spiral wheel corrections. Verticallity given as being true to within .5/1000" and worm error per revolution .1/1000". No hand work is done on the worm wheels.

3. Hobs and Hob Testing.

Bureau of Ships: Have no specific requirements for hobs and there is no standard calibration corresponding to our N.P.L. measurement.

De Laval and Westinghouse: Use hobs of a considerably smaller diameter and shorter length than those employed in this country. Those seen at Westinghouse were particularly noticeable in this respect, and with a tooth of about 7/12" pitch, the outside diameter of the hobs were only $3\frac{3}{4}$", their overall length being also $3\frac{3}{4}$". After sharpening, these hobs are carefully checked for profile in a hob testing machine. Errors in excess of about .5/1000" are corrected by honing, a highly skilled operation and a lengthy one.

4. Shaving

Westinghouse: Westinghouse are now employing shaving to an increasing extent and they have recently put into operation a 90" machine in addition to their earlier 36" machine. Westinghouse do not consider that shaving is a necessary process but suggest that it may have some future. The appearance of a shaved tooth is one of considerable polish and slight close pitched undulations. It was stated that the shaving process rectifies tooth form, minimises undulation errors in the teeth and can to an extent be used to correct for errors in the helical angle, depending upon the amount and direction of feed, although results in this latter connection have tended to be erratic. The principle of shaving is that the element to be treated is set up on its own journals in bearings on the machine, truly parallel to the cross slide carrying the shaving tool, and is driven by a motor, a suitable peripheral speed being 400 ft/min. The shaving tool is really a broach and consists of a wheel 9-12" diameter and 1" wide cut with teeth at a helical angle about 10° or 15° less than that of the gear to be shaved and of opposite hand. This involves the spindle of the shaving tool being set 10°-15° off the line of the cross slide, giving an inclination of the spindle corresponding in direction to the helix of the wheel or pinion at the instant it engages the broach. This feature gives the process its name "Crossed Axes Shaving". Each tooth of the shaving tool has about 12 cutting edges formed along the length of its flanks. The wheel is set into rotation and the broach is fed into the tooth to be shaved until it is in full mesh, and is then given an additional radial feed of .001". The traverse motion is then engaged and the broach is fed axially across the face of the wheel with a feed varying between .003" and .008" per revolution of the wheel, a liberal supply of oil being provided. The broach is running free on its spindle and is driven only by the wheel with which it is in mesh. After completing /

completing the traverse of the wheel face, the broach is fed back at a similar rate but with no additional radial feed. In the choice of a broach, it is necessary to provide a hunting tooth with as great a prime difference as possible between the number of its teeth and that of the wheel to be shaved; e.g. a broach having 61 teeth is considered quite unsuitable for cutting a wheel with 123 teeth. An L.H. broach is used to shave an R.H. helix and vice versa.

Westinghouse have no information to suggest that the shaved gears are quieter in operation but they prefer the process to lapping.

Falk: Gears less than 50" diameter are shaved and this firm are taking advantage of the process to speed up their hobbing times, the rougher finish of the teeth as hobbed being corrected in the shaver.

The process is similar to that described for Westinghouse, but instead of one double pass of the broach Falk employ $4\frac{1}{2}$ double passes for each helix, 5 in one direction with a radial feed on the broach and 4 return passes driving light. Meshing elements which have been shaved are thereafter lightly lapped together.

G.E.C: Do not do any shaving.

It is their opinion that hobbing machines of inferior accuracy can be utilised if a gear is to be shaved but that the process is no substitute for a good hobbing machine. They further state that as shaving does not correct for helical angle, gears still have to be lapped, thereby spoiling the shaved surface.

<u>Falk</u>: Elements which have not been shaved are lapped with a cast iron lap in a set up indicated diagrammatically in Fig. 7. The lapping wheel being overhung on its brackets, its weight carries it into full mesh with the element to be lapped and it is this component of gravity which provides the total load between the teeth. The process is continued until marking shows along the whole length of the tooth by which time the tooth thickness has probably been reduced by about .006". While the gears are running a compound of machine oil and medium carborundum is poured by hand into the mesh. Lapping wheels are recut after lapping each element. After this process, mating gears are set up in special bearings, great importance being laid on maintaining correct centres for parallelism, and are then lapped together for a further period. Shaved gears are also subjected to this process. The sides of the wheel rims are finished to a true circumferential plane so that a special gauge applied to the pinion journals may be employed for checking parallelism in the tangential direction. As can well be imagined, confusion can arise in assessing the degree of contact by the appearance of the lapped surface since a certain amount of roughening occurs due to the presence of the grinding compound even where the tooth surfaces may not locally be in hard contact. Therefore, during the final lapping the process is periodically stopped, the elements are cleaned and lamp black marking is applied to the pinion teeth. Contact is assessed by joggling the pinion circumferentially back and fore in the backlash with a suitable lever. In appearance, a lapped gear shows a polished band in the region of the pitch line while the addendum and dedendum surfaces of the flanks are of a matt finish. Falk state that thickness reduction is uniform over the whole depth of the tooth but it is difficult to see that this should be the case and the markings which were witnessed certainly appeared to evidence heaviest contact in the region of the pitch line although it is accepted that this may be an optical illusion having /

having regard to the different nature of the tooth surfaces in that region, as already described.

G.E.C.: G.E.C. have been lapping gears for 5 or 6 years. With regard to lapping, it is their practice first of all to lap with separate cast iron laps until an overall good bearing is achieved, and then to lap mating elements together, and these processes were witnessed in the shop. Lapping, it was stated, was for the purpose of correcting helical angle and tooth form.

On being questioned as to the merit of lapping with a separate cast iron lap to correct for helical angle (which might, in fact, be making conditions worse), Mr. Twogood agreed that this was a pertinent question, and admitted that they were employing separate cast iron laps to a decreasing extent; e.g. with Destroyer secondary wheels they now only lap with the mating pinions, having regard to the fact that there are 4 pinions to be lapped.

Questioned as to the possibility of lapping destroying the tooth contour rather than improving it by reason of the fact that it would seem that a greater amount of metal would be removed from the surfaces remote from the pitch line, it was stated that particular care was taken both in modifying the flank angle and also juggling with the addendum of the lapping wheels to ensure that no such evil consequence ensued. They were not explicit as to the nature of these corrections, but, in any case, they can not be applied to instances wherein mating pinions and wheels are lapped without the use of a cast iron intermediary, and in the circumstances it would seem difficult to justify the argument; Nevertheless, it must be admitted that meshings which were witnessed were broad and uniform as a result of the lapping process; although the pitch line was quite evident on the surface of the teeth, that being the only region where the tooth surface was polished.

G.E.C. /

G.E.C. have no evidence of excessive wear with the lapped teeth and although pitting on the pitch line is not unknown, they are not unduly concerned about it. In the process of lapping, the elements to be lapped are set up in special bearings and driven through gearing from motors kept for the purpose. Great care is taken in setting spindles parallel and after initial setting, clock gauges are retained to check the maintenance of alignment. The teeth are pushed hard into mesh and rotation is maintained for 30 to 40 hours, depending upon the condition of marking of the teeth, lapping compound in the form of medium carborundum in oil being applied by hand throughout the process. As lapping continues, the pinion bearing blocks are moved radially in towards the wheel to make up for the wear of the teeth.

With regard to the marking which might be considered as satisfactory, they would not accept a single line of marking along the pitch line, but they would be prepared to accept the gear if there was a second line of marking outwith that region. When mating gears are lapped together, the process is continued for at least an hour after ceasing application of the lapping fluid in order as they say to spark out embedded carborundum. During this period, the teeth are fed with E.P. oil.

It was emphasised that lapping was not associated with the adoption of the locked train gear.

De Laval: Mating gears are meshed together in temporary bearings and run in with powdered glass and oil. They emphasise that too much lapping spoils the tooth shape and that the lapping is not to correct helical angle. Their gears are cut non creep and it would seem that the lapping process is to reduce the worm wheel wave on the teeth. Lapping with powdered glass leaves an extremely rough surface on the tooth.

Westinghouse: Naval elements which have not been shaved are lapped with cast iron laps using Garnet which is of the hardness of glass but softer than carborundum. Westinghouse dislike the practice /

and would not employ it, were it not required by the Navy Department.

6. Polishing:

Falk: This is a particular process peculiar to Falk. After mating gears have been lapped they are burnished by the action of meshing them with two hardened steel wheels, mounted as indicated diagrammatically in Fig. 9, a system of weigh bars providing a force which tends to close the centre distance between the hardened wheels. The load is adjusted so that the bearing pressure between the teeth is raised to two to three times the designed full power loading on the teeth. The mechanism is traversed slowly across the width of the helix, the wheel or pinion element being rotated meanwhile by a motor drive, and the meshes being lubricated with oil jets. It is considered by Falk that this process improves the state of the material of the teeth.

7. Gear Measurement.

Bureau of Ships: Have no specific requirements.

Falk: Cumulative pitch readings are taken using the polishing mechanism described above as a pitchometer, the weights being removed therefrom except for a light tensioning load and a clock gauge being inserted between the jaws, as indicated in Fig. 8.

G.E.C: Cumulative pitch errors are measured by what they call a pin check. In essence, this consists of placing a hardened ground cylinder in the bosom of a tooth, its diameter being such that it protrudes possibly a 1/16" above the periphery of the wheel. The wheel is supported in bearings associated with the equipment, and is rotated by an electric motor, so that the ground pin passes under a shoe attached to a lever operating a clock gauge with suitable magnificiation; thus the depth of insertion of the pin into the tooth is measured and from this the pitch of the adjacent teeth is deduced. As the process of turning the wheel to pass the pin under the shoe and clear involves a turn of possibly about 7 teeth, it is arranged that say each 7th tooth is measured during the /

the first rotation of the wheel and intermediate teeth are gauged during successive revolutions of the wheel, until every tooth has been dealt with. A similar procedure is adopted for the pinions, but in this case to facilitate the drive, the pinion to be measured is meshed into the wheel already set up in the machine, and is driven therefrom, the measuring mechanism being slid back along shears to accommodate the positioning of the pinion.

De Laval: No cumulative error measurements are made.

Westinghouse: No cumulative pitch measurements are taken, Helical angle errors are measured by a clock type gauge which registers two pins in each helix of the wheel or pinion. The pins are set with extreme precision to the true axial pitch of the teeth and two clocks on the instrument are set to zero with the four pins in line. Thus, when applied to the wheel if the angle of one helix is at fault the fact will be registered by the resulting variation in the readings of the clocks but the failing with a gauge of this type is that no indication is provided as to which of the helices, if either, is true.

G. GEAR TESTING.

1. Torque tests, reasons for and against.

Bureau of Ships: The clause in the Navy Department machinery Specification calling for back to back test has now been eliminated to speed up production but may be reinstated after the War. On occasion gears have had to be realigned after test but it was agreed that the initial alignment may not have been as good as possible.

Falk: Have now abandoned the back to back test in view of the fact that it is no longer a Navy Department Requirement. They had some experience with the test during the period that it was required but they consider that the results obtained were inconclusive, e.g. some gears have given trouble with thrusts which did not show up in the tests. During the test there is no irregularity of loading or distortion of the gearcase such as occurs in a seaway and from these points of view the test is not so severe as service operation. As regards meshing, Falk are of opinion that results only indicate what is shown by a static test. No distress was ever observed on the teeth due to starting under torque.

G.E.C.: Consider that the back to back test is absolutely necessary to check against the possibility of a gear having to be removed from a Ship. Questioned as to whether they ever found it necessary to make an adjustment after the back to back test, they stated that probably one in ten times they did. Evidence of this was witnessed in the Shop but it was apparent even then that the adjustment could have been correctly effected at an earlier stage either by more careful marking or measuring of the tooth clearance with shims in the manner to which they finally had to revert in resetting. Two sets of Cruiser gears were witnessed running full load back to back. It so happened that while walking through the gearcutting shop I saw a main wheel being recut which had scuffed. On enquiring I was told that this was a result of extensive load on back to back test with elements having too large addendum correction (possibly 70% addendum). They were emphatic that a back to back

test /

test on all addendum gear would be most undesirable. It was emphasised that the back to back test was not associated with the adoption of the locked train design and indeed it was seen being applied to articulated gears for the Maritime Commission.

De Laval: De Laval are strongly averse to the back to back test on account of the amount of equipment required, the time taken, danger of damage to the teeth and finally because the results obtained are considered to be an unreliable guide to the functioning of the gears unless indeed it proves that the design is altogether faulty.

Westinghouse: Were emphatic that the back to back tests serve no useful purpose for gears in production.

Westinghouse do, however, employ a method of checking contact under load by driving one set of gears from an electric motor through reduction gearing coupled to one of the primary pinions with full torque and at 10% of the full power revolutions, the power being absorbed by a prony brake driven by the other primary pinion. The gears are run at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and full load in each direction for periods of an hour or so, the marking being examined periodically.

2. Equipment Required and Methods Employed for Back to Back Tests.

G.E.C.: For running the back to back torque test a special extension is provided on one of the secondary pinions to provide drive from a steam turbine. It was recommended that the power available at this source should be at least 4% of the total power to be transmitted by the two gears under full load service conditions. The primary pinions of the respective gears were coupled back to back through long shafts supported in steady bearings, one of these shafts being broken in way of the final drive of a multi-reduction gearbox coupled through an air operated clutch to a 60 B.H.P. electric motor for /

for starting purposes. This is to overcome the static load which is very heavy and the air operated clutch automatically cuts out the motor when the turbine takes up the full drive. The equipment is elaborate.

Toothed coupling flanges for applying the torque were integral with the shafts connecting the primary pinions adjacent to the starting gearbox and were capable of being united by coupling bolts through slotted holes. The torque was applied by a pair of toothed jaws fitted with long levers. Each jaw engaged one of the small toothed flanges and the outer ends of the levers were connected through a link arrangement with a portable air cylinder. The requisite torque was provided by admitting compressed air to a predetermined pressure through a reducing valve. While in this condition the coupling bolts were hardened up, the jaws thereafter being removed. It is G.E.C. practice to run the gears light for a quarter of an hour then under quarter load for 15 to 20 hours, at the completion of which stage they check to verify that they are obtaining 100% marking over all faces. The gears are then run up to 75% load followed by a period at full load, the combined time of these last two trials taking 15 to 20 hours.

G.E.C. have in operation for submarine gears a torque applying device by which the torque may be increased while the gears are running. So far as can be gathered, this unit, which is hydraulically operated, functions through the medium of right and left handed splines on which work a coupling piece. G.E.C. have no intention to adopt this principle for use with larger gears.

Falk: From the description which was given of the method which they used to employ when making the test a very large motor (which was seen) was used for driving the gears coupled to one of the main shafts. The load was applied through a coupling uniting the primary shafts, somewhat similar in nature to the arrangement used by G.E.C. except that loading was applied by weights on the end of /

of a bar instead of by air pressure.

De Laval: In operating the back to back test De Laval used to employ a 900 H.P. turbine and starting up was assisted by an overhead crane pulling on a rope wound round a drum on one of the primary pinion coupling shafts.

3. Noise Levels. (for comparison of nested and locked train designs see A.4. For the effect of helical angle see E.3).

Bureau of Ships: Have standardised the measurement of sound to be averaged from readings taken 1" from the outer surface of the gearcase in decibel units. Mr. Ball's description of decibel ratings was as follows:-

108 - Loud gears necessitating shouting to make oneself heard.

122 - Painfully loud rendering it impossible to make oneself heard however loud one shouts.

The general standard of gears fitted in U.S. Navy Vessels is 108 D.B. An instance was recorded of 122 D.B. Lapping has the effect of reducing noise emission by about 2 D.B.

G.E.C.: Cruiser gears which were witnessed running on back to back test gave the impression of being average good so far as noise was concerned. There was no scream and the volume of noise emitted was not such as to make conversation with a raised voice difficult. It was stated by Mr. Zrodowski that the noise level would be about 105 D.B. and that this would probably reduce to 103 D.B. when the gears were fitted aboard the ship.

De Laval: De Laval's description of decibel ratings was as follows:- /

follows:-

95 - Quiet Gears; converse in normal voice.

100 - Loud Gears; necessary to face person to whom conversation is directed.

110 - Unpleasantly loud; necessary to shout to make oneself heard.

They gave the following figures as being typical of a set of De Laval locked train Naval gears:-

Running idle in Shop at full power revolutions	90-95 decibels
On board under full power load ...	95-100 decibels
Back-to-back test	103-105 decibels.

Westinghouse: Are of the opinion that decibel readings are of academic interest only. A gear giving 100 decibels with a high frequency would be described as bad, while another gear giving 110 decibels with a low frequency might well be considered a good quiet gear. At one time consideration was being given by the Navy Department to condemning gears which in operation gave a decibel reading above a certain limit, but, having regard to the uncertainty of decibel measurements and the fact that at the present time no one was in a position to state the exact cause of noise or to give a specific remedy for the complaint (assuming that the gear teeth are of high class finish), the Department was persuaded to drop the proposal.

Admiral Irish: Is satisfied that the lapping process renders gears less noisy in operation.

GROUP IV. PERSONAL COMMENTS ON DISCUSSIONS.

Section A. Items 1-4

I have arrived at the following conclusions:-

(a) The locked train design takes longer to produce than the nested type.- available statements vary from 160-200% of the time required for nested gears. These figures correspond with estimates made in this Country.

(b) Gear cutting time for the locked train varies with circumstances between 80%-125% of the time required for nested gears. It seems likely that the lower figure would be more generally applicable.

(c) Nested gears for 30,000 S.H.P. have been quoted as being 9,000 lbs. lighter and alternatively 10,000 lbs. heavier than the corresponding locked train design. General characteristics of design would lead one to expect the locked train gear to be lighter but, in view of the conflicting nature of the evidence, it is clear that the difference should not be great. Falk's reaction to the examination of the provisional nested design prepared by Fairfield was that it was of sound construction and could afford to be reduced in scantlings without running risk of the distortion which was under emphasis at the time the design was prepared.

(d) The space occupied by the gears is slightly in favour of the nested type but the difference is so small as to be of little consequence.

(e) The prominence of the bearing thermometers, quoted by Mr. Myor, is poor compensation for the bad accessibility of the thrust and forward main bearing of the locked train design. In any case, the bearing thermometers and indeed all the bearings of the nested design are readily visible and accessible. Nevertheless, it is the case that, with only the light covers removed, the running parts /

parts and bearings (ex main) of the locked train gears present a most pleasing appearance as a piece of engineering and accessibility to these parts is ideal.

(f) In operation the two types of gear are equally serviceable.

(g) Falk make the suggestion that the locked train gear may be slightly quieter in operation than the nested type. No other evidence was available to suggest that there is any difference between the two but it is obvious that Falk's statement must be made with some foundation and it is, therefore, probable that while in general there is little difference between the two, the locked train design might on the average be expected to run with slightly less noise than the nested type.

(h) In replacement or repair the locked train gear is clearly at a disadvantage and it is to be noted that to scrap a number of elements for a defect in one, vide American practice, does not line up with the British temperament. The question all hinges about the meshing of coupling elements, the location of coupling keyways, the length of quill shafts and the necessity of maintaining equal tooth thicknesses in the tandem trains.

(k) The question of small tooth coupling production with adequate accuracy of meshing is regarded with extreme anxiety so far as British output is concerned. Apart from the large number of these couplings per gearcase it must be recognised that for purposes of adjusting the trains, the quill shaft couplings must be remeshed in various tooth registers and in consequence the required degree of perfection in meshing must be attained with any position of the claw with respect to the sleeve unless Falk's method of setting can be realised and accepted. In this connection see also comments on Section C.5.

(l)/

(l) With regard to the present programme in Britain, it is strongly suggested that the coupling production question be fully examined in the light of the above notes before we are committed too deeply in the decision as to design. It is felt that, having regard to the closely competing claims of the two designs, the industry and all concerned in this Country should be given a chance to form their own opinions which could only be accomplished by building a number of each type with full confidence that each will be entirely satisfactory. If, however, the question of man power is to be given its proper consideration, it cannot be recommended that the locked train design be constructed at the present time in other than limited numbers. Incidentally, this policy would probably line up with the coupling production situation.

(m) It is a matter of incidental interest that only the antagonists of the locked train gear in the States refer to it by that name. G.E.C. and De Laval are respectively insistent that the titles "Divided train" and "Twin Drive" be applied. The reason for this is clearly that the adjective "locked" is descriptive in that it emphasises not just a peculiarity but an undesirable peculiarity of the gear.

<u>Section B.</u>

The nested proposal put forward in January 1944 is being examined in the light of the information received from Falk and Westinghouse.

Section C

Item 1. High damping very evidently renders torsional oscillation of small moment. Quill shaft design stresses vary 5000/8000 lbs/sq. in. for carbon steel and 8500/10000 lbs/sq.in. for nickel steel, say, 6000 and 9000 lbs/sq. in. for carbon and nickel steel respectively on a basis of equal power distribution.

Item 2: In every instance the gears are set up to suit the positioning of the main wheel. Extremely careful setting is required to enable employment of quill shafts to drawing length.

Item 3: G.E.C. and Westinghouse agree on a design basis of 320/350 lbs/inch of coupling teeth, all teeth being in simultaneous contact. Further comments are entered under Section A(k) above. See also comments on C.5 below.

Item 4: A set up such as those employed by Falk and G.E.C. would only be warranted if a very large number of gearcases of similar design were under construction by one firm, but in view of the fact that the times taken by these firms represent only about 35% and 85% of typical boring times in this Country for single reduction units, it certainly appears that a simpler set up is justified. De Laval, with a single headed boring machine, would appear to have struck a happy medium suited to the possible requirements of this Country, giving an output per machine of approximately two gearcases in three weeks.

Item 5: In observing that Falk specifically do not favour the seemingly simple prime tooth correction for load balancing, either based on wheel and pinion teeth and/or coupling teeth, one is bound to query whether the real truth of the matter is that they are prepared to take considerable pains in the method of load balancing ascribed against their name in order to ensure that their couplings /

couplings can be definitely registered in one meshing position. It is therefore, possible that Falk's method of setting will be of particular interest to firms in this Country and, of course, their statement that there are no interchangeable spares for locked train gears lines up with this reasoning and confirms the conditions in repair yards described by Mr. Tingey.

Falk's method of assessing marking by "joggling" is not nearly as accurate as methods employed in this Country with single reduction gears since the process depends rather upon "lifting" a definite thickness of marking, than of obtaining a metallic bearing. It is most probable, however, that they cannot afford to show a more accurate marking as no means of further adjustment is open to them when once the coupling flange is drilled and reamed. It will be noted in this connection that G.E.C. have actually referred to the errors peculiar to the method employed by Falk but then G.E.C. (somewhat optimistically) believe they ultimately obtain 50/50 power division, while Falk only claim 62/38.

If the couplings available are sufficiently accurate for indiscriminate meshing the De Laval procedure is preferred although it is suggested that the practice of allowing the pinions to lie in the bottom of the oil clearance may be deceptive and would require special consideration. The zero clearance bearings employed by Falk are good only if it can be guaranteed that the pinions and wheels, when fitted in their correct bushes will lie truly parallel with their earlier position in the temporary bushes - this element of uncertainty appears to be introduced unnecessarily and the employment of brass shims in the oil clearance may well be preferred.

No advantage is seen in loading the gears up with 40% full power torque for checking balance of power in the manner of Westinghouse and it will be noted that both Falk and De Laval have abandoned similar processes.

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In the construction of single train gears there is no advantage to be gained in giving very close attention to the maintenance of design tooth thickness, always provided that a specific minimum backlash is exceeded when meshing. In the construction of locked train gears, however, this item demands the closest consideration or one train may well carry the full astern loading, and not only may the teeth be punished but the primary pinion bearings will be called upon to provide reaction loading for which they were never designed and the primary pinion will deflect under a bending loading system which it was never intended to withstand, thereby providing added localised loading on the ends of the primary teeth.

The remark attributed to Admiral Irish related to Falk's method of checking their load balance but as the problem is now seen in the manner explained above, no alternative may be open to them.

<u>Item 6.</u> The disadvantages associated with the complication of the standard U.S.N. turning gear are largely offset if in consequence of its possible adaption as holding gear, separate holding gear can be dispensed with.

Section D:

Item 1: Falk's general principles of fabricated construction are impressive and it is considered that they could be followed more closely with advantage where adequate flame cutting capacity is available. Weights quoted by Falk for completed gears show an appreciable saving and this must be largely attributable to their principles of gearcase construction.

Item 2: The cast steel wheels which Falk manufacture give every appearance of a first class job but, having particular regard to the failure of other American firms to follow their procedure, it is considered undesirable to contemplate the development of this method in Britain. The method of locating and securing wheels to shafts as standardised by Falk is interesting and may on occasion find suitable application in this Country.

The policy of welding disc wheels is not impressive. For the flat disc type employed by G.E.C. and Westinghouse, the man hours taken in actual fabrication for a Cruiser second reduction wheel as stated by G.E.C. is 600. They are anxious to emphasise the large amount of machining and fitting which is avoided by the abandonment of the bolted construction, but it is interesting to note that for a Destroyer single reduction wheel of similar proportions, the average time allowed at Fairfield to set the plates, drill, ream and fit the bolts is 82 man hours. To this must be added the time required for the manufacture of the bolts and nuts for which a liberal allowance is 28 hours, giving a total of 110 man hours for the bolted type of wheel in comparison with 600 man hours for the fabricated type.

The De Laval type of fabricated wheel with dished side plates is attractive in appearance but as regards production time the situation is hardly likely to be more attractive than the flat plated wheels, and the plant required in their manufacture is more elaborate.

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The dynamic balancing of main wheels appears to be an unjustifiable requirement, a view it will be noted that is held by Westinghouse. Certainly some of the Battleship wheels which were seen at G.E.C. showed evidence of having been, by our standards, appreciably out of balance having regard to the number of blind balancing holes drilled in the strips which are welded near the periphery of the plates for that purpose. It is probable that the fabrication method of wheel construction was responsible for the poor initial balance and this factor might tend to warrant the dynamic process of correction which presents another reason in favour of bolted wheels.

<u>Item 4:</u> A contradiction will be observed between the report of the discussions at the Bureau of Ships and at Westinghouse with regard to the Bureau's requirements in respect of astern sprayers.

Section E:

Item 1: The loading coefficient might be described as the Highest Common Factor of Technical Agreement between the firms concerned and does not represent the considered views of those who have examined the problem in greatest detail.

With conservative tooth bending stresses and relatively small face width/diameter ratios, the need for thinning the ends of pinion teeth is less essential to a certain degree, and it might be on this latter account that the Americans have had no trouble with unthinned teeth. Nevertheless, the policy cannot be recommended although it is considered that it is our present practice to remove an altogether excessive amount of metal and all that should be required is a light relief to ensure that no heavy bearing occurs on the ends of the teeth.

Their policy regarding maximum face width/diameter ratio is concurred in.

Item 2: According to the information obtained all firms are working to a conventional ratio of meshing depth relative to pitch although, from information which the E. in C's Department had previously obtained, Falk would appear to have standardised on a meshing depth 16% in excess of the standard, relative to pitch.

The method of addendum correction employed by Falk requires close analysis before comment can be made thereon. The only other firm to make an addendum correction is G.E.C. who in the case of primary gears only (and then apparently only in some instances) apportion 60% of the addendum to the pinions.

No one was prepared to express any opinion regarding optimum flank angle and it is apparent that in this respect American policy has been conservative and the earlier conventional $14^1/2^o$ - 15^o has been maintained without any specific reason that could be ascertained.

It is surprising to find that the various American firms are /

are permitted by the Bureau to work to different tooth standards as it would seem to upset the position as regards interchangeable spares. No information was obtained on this point but it may be that a concession has been made with a view to easing the hob situation and utilising existing stocks. Alternatively the aforementioned difficulties with which the locked train spares question abounds may in any case render it impossible to give consideration to replace components being supplied by any firm other than the original manufacturers.

Item 3: The theory that noise is minimised by a large helical angle appears to have been brought into disfavour. Having regard to the axial independent freedom of the locked train units the need for a large helical angle is not obvious unless, indeed, it be with a view to minimising the lack of balance between the two trains consequent upon pitch errors in the units concerned. It is noted that Westinghouse and De Laval prefer an 18° helix for nested type gears in order to reduce axial loading on the independent wheels. This is presumably more desirable when the primaries are nested than when the secondaries are nested on account of the stiffer construction of the single wheels in the latter type, so that having regard to the lack of axial freedom in these designs generally, it is, on the whole, not agreed that a small helical angle is desirable when it is the secondaries that are nested.

Item 4: Clearly no one is concerned about any maximum limit for backlash.

Section F. Items 1, 2 and 3

The successful application of non creep machines is entirely dependent upon the use of worms and worm wheels of extreme accuracy.

Westinghouse quote a worm error per revoluation of .1/1000", although it is considered from inspection of wheels cutting on various machines that they have not attained this degree of perfection in all instances. This figure compares with a minimum worm rotation error of 0.3/1000" which one of the leading Machine Tool Manufacturers in Britain was prepared to guarantee in a recent quotation. Machines are, however, in operation in this Country with an appreciably greater error than this and there is clearly room for progress to be made in this direction before the finer points of preference in the choice of creep or non creep drive need be discussed and meanwhile we would be well advised to adhere to our creep ring policy.

Tooth surfaces exhibited the smooth finish associated with non creep drive and, as mentioned, the worm spiral markings varied from average good to excellent but no visual inspection of a wheel can reveal the accuracy of pitch division, and tooth surface must not be regarded as a predominent criterion of quality for this reason.

Inquiries into pitch accuracy met with disappointingly negative response. Both at Falk and G.E.C. individual pitch error readings were witnessed being taken and their magnitude gave the impression of being inferior to our best standards, but in neither case were cumulative error curves available and while the latter firm stated and confirmed that their total error for a wheel of about 14'0" diameter was just under 1.0/1000" I am satisfied that 1.0/1000" per foot diameter is nearer the truth, if indeed this was not/

Not actually the statement intended. Curves demonstrating the lesser error would I feel, have been prominent for inspection. This would suggest that the error of G.E.C's worm wheels is generally about twice that of a good spur master wheel in this country which is not surprising having regard to the impossibility of hand touching a worm wheel to improve pitch without introducing worm rotation errors, while spur wheels can be effectively hand finished.

Errors of verticallity and lead screws as quoted appear to be commensurable with good practice in this country but Westinghouse's reference to spiral wheel correction would seem to confirm, as suggested by Falk, that they have in fact been up against serious trouble with helical angles. Furthermore, reference to Group III, Section F, 5, shows that G.E.C. allow for a tooth thickness reduction of .006" during lapping - a figure which is suggestive of helical angle difficulties of a magnitude not experienced in good class work in this country. My impression is that in general the U.S. leadscrews are inferior to the class of screw produced by the best firms in this country and particularly by the N.P.L. on their cam bar corrected machine.

In respect of hobbing times both De Laval and Westinghouse line up closely with say Fairfield practice except where time is saved by double headed hobbing; their feeds, however, are less although this is compensated by increased hob revolutions; in this latter connection it is to be noted that the smaller diameter hobs which these firms employ counter balance the increased revolutions to give approximately equal shearing speeds to those which we employ.

The cutting times taken by G.E.C. appear to be some 50% greater than those quoted above in accordance with their proclaimed policy of restricting their machines to a light load. On the other hand Falk are much more rapid for their small units as was indeed apparent from inspection of their machines in operation but it /

it seems doubtful whether the feeds which they were understood to have quoted can be correct and half these values would appear more likely.

It is questioned whether the use of small diameter hobs is not likely to prejudice accuracy of finish, the cutting effect being similar to say a two-start hob operating under our normal conditions. Furthermore hobbing machines for cutting large diameter wheels with large helical angles have to be specially constructed for clearance of the hob carriage to suit the use of small diameter hobs. The merit of these diminutive hobs lies of course in their relative speed of manufacture, sharpening and gauging.

With regard to hob profile testing, as particularly witnessed at Westinghouse, the need for such a test in this Country does not arise as the calibration is fully and more clearly covered by the N.P.L. pantograph records and other gaugings of tooth profiles, Section F:

Items 4 & 5: The tooth surface of a shaved gear is attractive in appearance, and it is stated, as can be imagined if given an accurate broach, that it is a good method of correcting tooth profile. More obviously the process evens out short undulations and irregularities along the tooth surface. I agree with G.E.C., however, in their assertion that shaving should not be regarded as a substitute for good hobbing and the question which naturally follows is, what standard of tooth finish requires to be achieved by hobbing before shaving becomes a redundant requirement? There is scope for research in this connection but as the chief respect in which ultimate benefit is likely to accrue is associated with possible noise reduction (and it is to be noted that opinion is not consolidated even on this possibility), the question is essentially dissociated from wartime production.

Although the principle of the shaving machine is simple the equipment represents quite a considerable item which could not be justified unless the gain was real. The broaches themselves would cost several hundred pounds a pair, and a considerable variety would be required for each tooth form in view of the necessity for avoiding many tooth combinations. Each pair of broaches is also suited to only a limited range of helical angle.

The whole question of the advisability of shaving is further complicated by G.E.C's objection that to correct for helical angle it is necessary to destroy the shaved surface. This involves consideration of the lapping processes adopted in the States, bearing in mind that after a gear has been lapped it is impossible to shave it (as stated by Falk) without damaging the broach - an incidental admission of the abrasive condition of the tooth surface after lapping. A gear could, of course, be shaved after hand dressing for the correction of helical angle. Westinghouse are not apparently troubled by this consideration and obtain the desired degree of meshing with their shaved gears but they are not averse to /

to scraping out the bearing housings to accommodate the marking and in assessing marking they are not dealing with the stringency of a static light load test but with bearing under full torque - a much easier condition.

It is clear that Falk and G.E.C. lap essentially to correct helical angle for which purpose a much more extended process is necessary than would be required to improve tooth form or smooth out undulations. In fact, prolonged lapping is a menace to the tooth form as emphasised by De Laval. Falk stated a preference for hand dressing teeth to correct for helical angle but their trouble in this connection is an inadequacy of skilled fitters for the purpose.

In several gearcutting firms in this country, hand work on the teeth has been either eliminated or reduced to a small amount and in such cases it is doubtful whether lapping would be desirable. The limited noise reduction claimed in the States as being consequent upon lapping may be associated particularly with non creep cut gears which used to be notorious for their noise in this Country. Again, however, this is a question which is not associated with war production so far as good quality gears are concerned, although lapping might possibly be applied with success in the case of gears cut on machines which are not considered sufficiently accurate in their present state for the production of Naval gears.

Item 6: It can be visualised that for a tooth surface which has been lapped, the Falk polishing process may well be advantageous in closing the grain and reducing the abrasive character of the surface. For a tooth which has not been lapped, it is considered that the Falk polishing process would be of no avail.

Item 7: Methods of gear measurement, in so far as they exist are definitely inferior to those in use in this Country.

Section G:

Items 1 & 2: The only firm not opposed to the running of the back to back torque test was G.E.C. to whom the operation clearly appealed as a means of proving their products and demonstrating the extent of the care which they are prepared to lavish on contracts with which their organisation is entrusted. The gear which they employ for the purpose is elaborate, and the space which the set up occupies would in itself present a problem to firms not blessed with the remarkable acreage of shops which G.E.C. have at their disposal.

As a laboratory experiment to test say the overload capacity which a particular design would carry for a long period, the arrangement is clever but as a check of production technique, I am satisfied that it is misplaced and where the man power situation is critical the test would seriously impede output.

Item 3: Combining the definitions of decibel ratings given by the Navy Department and De Laval, the following is derived -

95	Quiet:	converse in normal voice
100	Fairly loud:	necessary to face person to whom conversation is directed.
105	Loud	converse with raised voice.
110	Unpleasantly loud:	necessary to shout to make oneself heard.
115	-do-	conversation almost impossible.
120	Painfully loud:	conversation impossible.

Having regard to Westinghouse's remarks, a scream from a gear probably cuts right across the above sound ratings.

The respective opinions of the Navy Department and various firms regarding average noise emitted in service under full load are-

Navy Department (Mr. Ball)	108
G.E.C.	103
De Laval	95-100

It is evident that Mr. Ball, the U.S.N. gear specialist, rates the noise emission of U.S. Naval gears in general as verging on being unpleasantly loud. Furthermore, either De Laval are optimistic in their estimate or the average is raised by the inclusion of "unpleasantly loud gears" manufactured presumable by the firms who are more reticent with information regarding their decibel estimates.

FIG. 1.

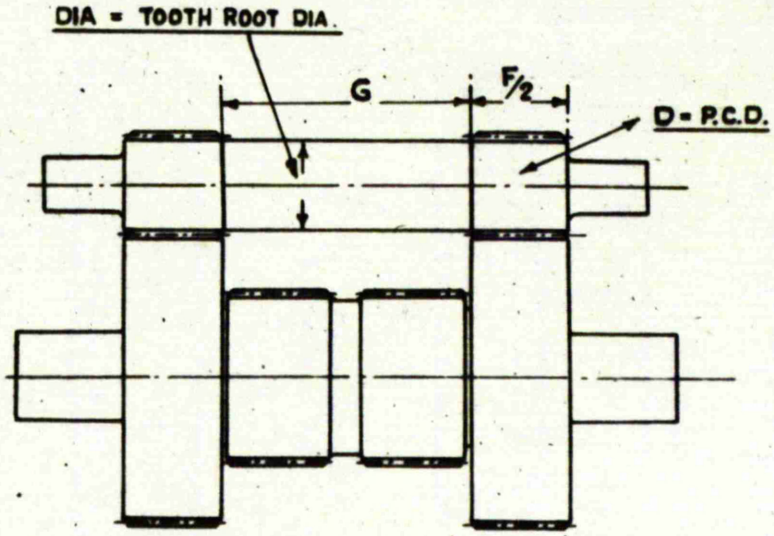


FIG. 2.

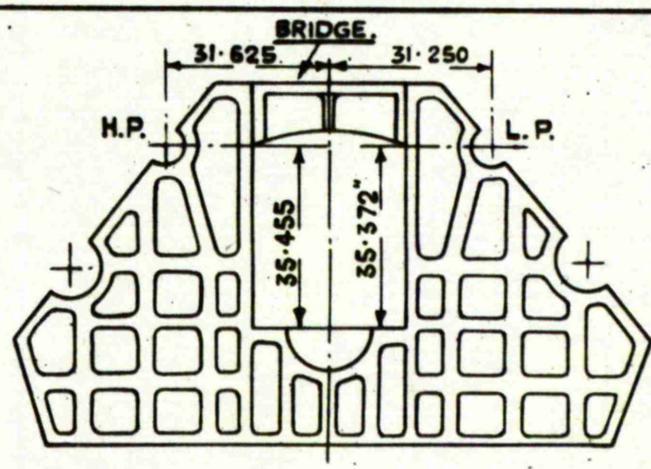


FIG. 3.

SEE NEXT SHEET.

FIG. 4.

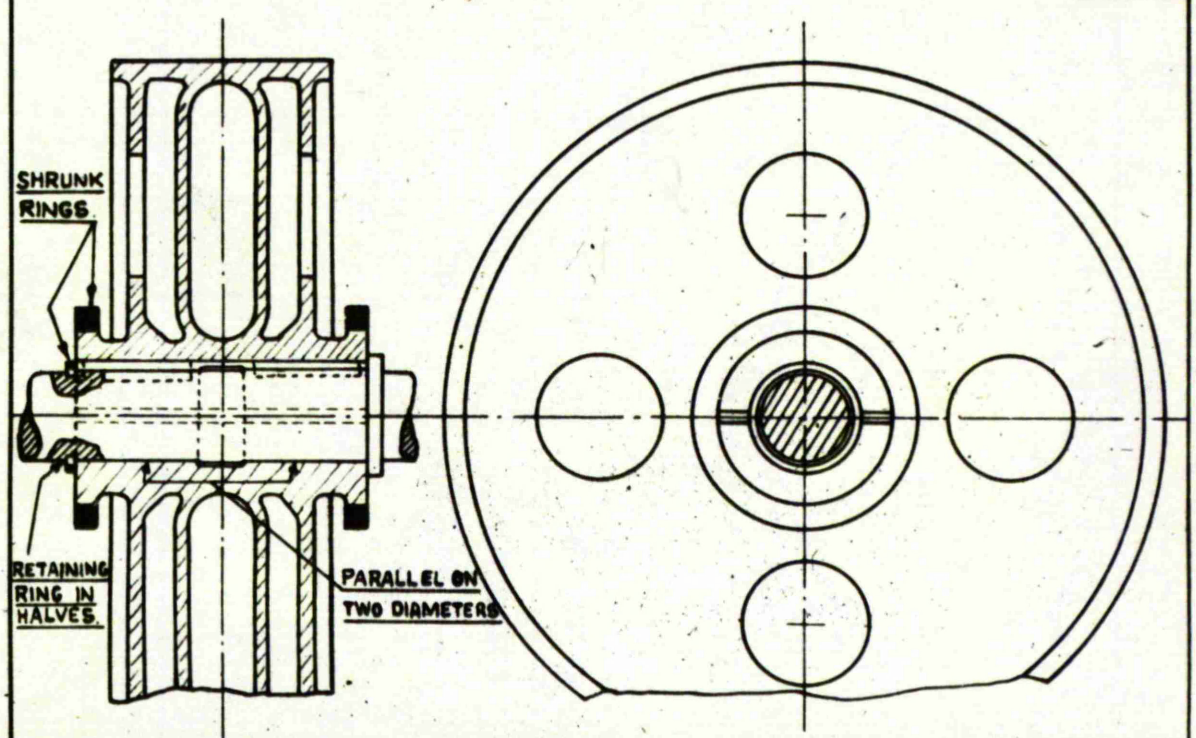


FIG. 5.

OIL CLEARANCE FOR TURBINE, REDUCTION GEAR, GENERATOR AND MOTOR BEARINGS.

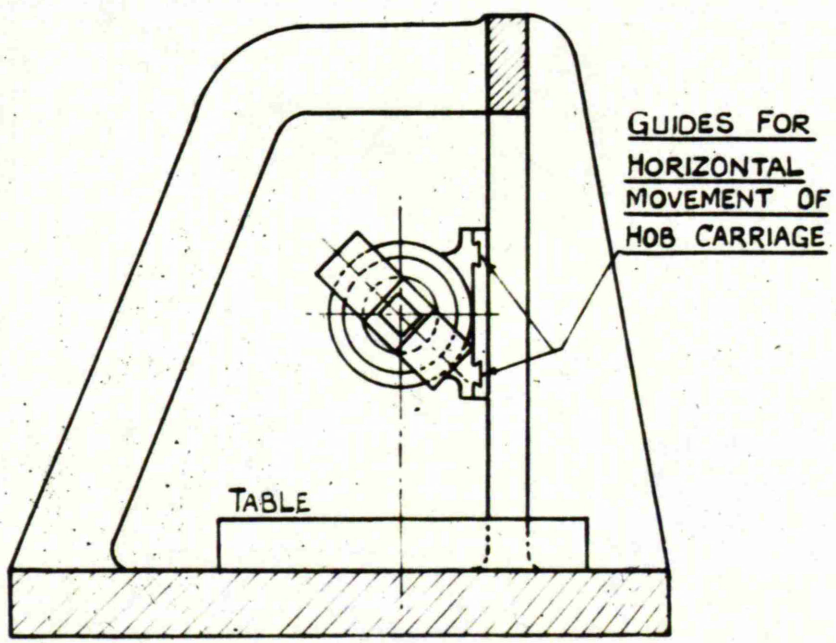
BASIC DIAMETER OF JOURNAL.	MINIMUM OIL CLEARANCE WHEN REBABBITTING BEARINGS	MAXIMUM OIL CLEARANCE WHEN REBABBITTING BEARINGS	BEARINGS SHALL BE REBABBITTED AT OR BEFORE THE FOLLOWING OIL CLEARANCES ARE REACHED.
INCHES	INCHES	INCHES	INCHES.
1	·005	·007	·015
2	·005	·007	·015
3	·005	·007	·015
4	·006	·008	·015
5	·007	·009	·015
6	·008	·010	·018
7	·009	·011	·021
8	·010	·012	·024
9	·011	·013	·027
10	·012	·014	·030
11	·012	·015	·033
12	·013	·016	·036
13	·013	·017	·039
14	·014	·018	·042
15	·015	·019	·045
16	·016	·020	·045
17	·017	·021	·045
18	·018	·022	·045
19	·019	·023	·045
20	·020	·024	·045
21	·021	·026	·045
22	·022	·027	·045
23	·023	·028	·046
24	·024	·029	·048
25	·025	·030	·050
26	·026	·031	·052
27	·027	·032	·054
28	·028	·033	·056
29	·029	·034	·058
30	·030	·035	·060

NOTES:-

1. THIS SHEET SUPERSEDES 2099-SK. & SHALL BE FOLLOWED BY ALL NAVY YARDS AND FORCES AFLOAT EXCEPT AS MODIFIED BY NOTE 2.
2. ALL BEARINGS OF A UNIT, SUCH AS ONE H.P. TURBINE OR ONE I.P. TURBINE OR ONE PINION SHAFT, SHALL HAVE THE SAME SCHEDULE OF CLEARANCES. WHEN PARTIALLY REPLACING BEARINGS OF A UNIT, USE SAME RELATIVE CLEARANCE AS EXISTS IN OTHER BEARINGS.
3. THIS SHEET SUPERSEDES TABLE 6 OF CHAPTERS 9 & 39 M.E.I. AS REVISED IN 1929 & ALL BEARING OIL CLEARANCES SPECIFIED BY CONTRACTORS PLANS.
4. THE OIL CLEARANCE SHALL NOT BE LESS THAN THE MINIMUM FIGURES LISTED ABOVE AT ANY POINT IN THE BEARING.

SCALE DATE:- JUNE 4TH 1929 REF. LET. BY MESNY O.K.

BUREAU OF ENGINEERING. 2184 - SK.



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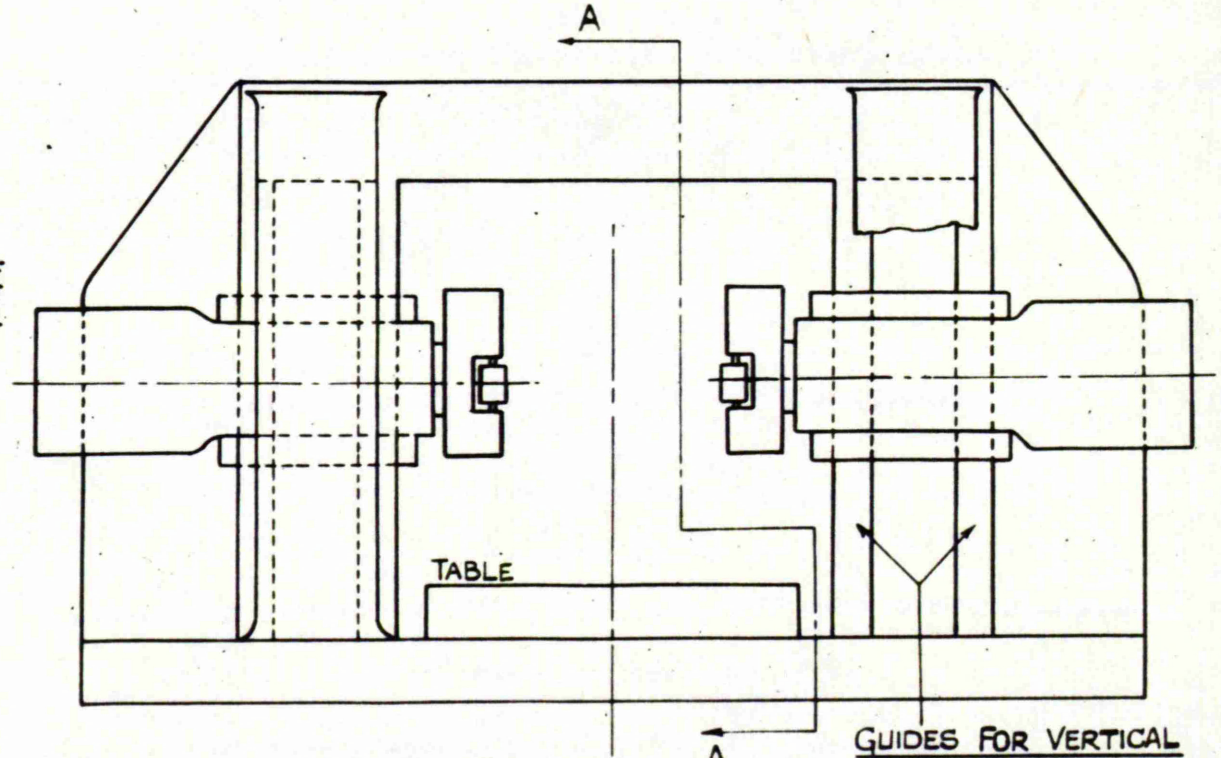


FIG. 0

Fig. 7

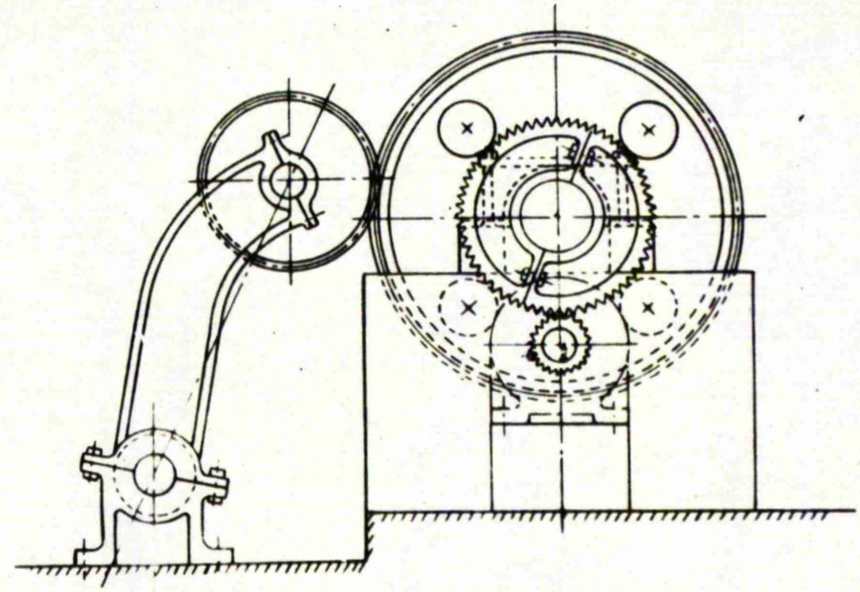
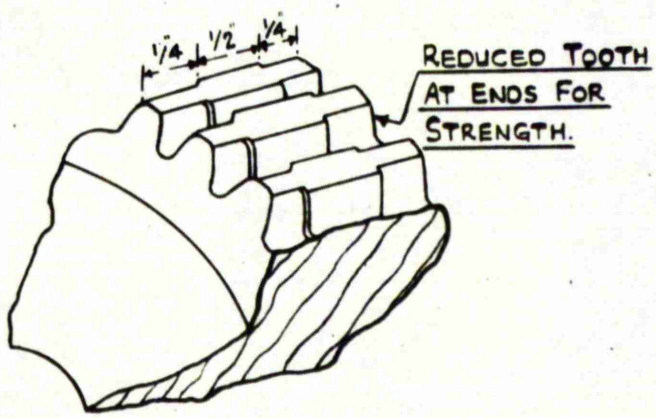


Fig 8



Tooth Form On Polishing Wheels.