A THEORETICAL AND EXPERIMENTAL INVESTIGATION INTO THE COMPONENT LOSSES IN THE WORKING PASSAGES OF TURBO MACHINERY.

SUMMARY

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Introduction.

It is the function of the elements in a turbine or compressor stage to effect the transformation of energy from one form to anot with the minimum loss. In the past losses and efficiencies in th elements and stages of turbo-machinery have been defined and measu in a variety of different ways each associated with the designer c research worker in a particular field. The steam turbine designe has in the main been concerned with high speed flow in impulse or fifty degree reaction stages. In the gas turbine plant the degre of reaction varies not only from stage to stage but with blade hei within a particular stage and in axial flow compressors the designe has been concerned with low speed flow, where for many application the working fluid could be regarded as incompressible.

Present day trends are towards steam turbine stages with vari amounts of reaction in the moving blade and to compressor stages w with rising operating Mach numbers, the working fluid can no longe be regarded as incompressible.

Summary.

The enthalpy loss in a blade element, which is the net loss o high grade energy caused by friction forces, is shown to consist o a basic friction loss plus an auxiliary loss which accounts for th "heating" effect of the friction work. The auxiliary loss is positive for compressions and negative for expansions. Relations are derived between the enthalpy loss and other methods commonly u to express the irreversibilities in the blade elements. The equa involved are applicable to static or moving elements in axial flow

turbines or compressors with high or low speed flow.

Factors affecting turbine or compressor stage efficiencies ar accounted for by (a) defining an efficiency for the flow process i the stage and by (b) using the concept of blading (diagram) effici It is shown that, while the "total to static" stage efficiency is directly proportional to the blading efficiency, the "total to tot stage efficiency is almost independent of blading efficiency if th process efficiency and blading efficiency are high.

Expressions are developed for the blading efficiency and load factor in a stage for any given reaction effect in the rotor blade It was found that the conditions for maximum blading efficiency co best be obtained in terms of a criterion which is called here the "reaction coefficient". This analysis will fill the gap between existing theories applying to the impulse blade and the fifty degr reaction blade.

In most of the previous work in this field of blading researc use is made of "atmospheric" air in a wind tunnel. It was apprec that a steam circuit could readily be adapted as a variable densit tunnel. Hence, using superheated steam as the working fluid, it decided to explore the flow and loss characteristics for a small rectangular nozzle and impulse blade pair over a range of Reynolds number and Mach number. An impact tube and traversing gear were designed to operate in a relatively high pressure and temperature stream and were so arranged that the entire exit area of either th nozzle or blade could be completely traversed.

A considerable amount of calculation is however involved in relating the local impact tube readings to local values of efficien and in converting these local values into mean effective values. Hence a procedure is given here which will ease this transformation and will take account of compressibility effects, of variable densi flow and of supersonic flow with the possibility of a shock wave formation upstream of the impact tube.

The nozzle loss is confined to a thin wall boundary layer with isentropic flow elsewhere in the nozzle. The larger blade loss, which is very sensitive to quite small amounts of reaction or compression in the blade passage, is greatest on the convex side of th blade. For this small nozzle, the stream attains a constant efflu angle in only a small central core of the flow with large variation in angle as the nozzle boundaries are approached. This efflux angle pattern remains unchanged with varying Reynolds number and subsonic Mach number. When the Mach number increases beyond unity however, the pattern changes so that the stream conforms more to th geometric outlet angle.

The observed variation in blade loss coefficient at high Reynolds number is compared with other similar work and confirms that a critical Reynolds number exists at 2×10^5 below which value there is a substantial increase in blade loss.

THE UNIVERSITY OF GLASGOW

A THEORETICAL AND EXPERIMENTAL INVESTIGATION INTO THE COMPONENT LOSSES IN THE WORKING PASSAGES OF TURBO MACHINERY

THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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Summary.

Introduction.

In the thermodynamic design of turbines and compressors it is essential firstly to decide on the basic features of the machine. In turbines, for example, one must determine whether high specific mechanical energy output is the aim or whether one may sacrifice some of this high specific output for an increase in gross stage efficiency. It is then necessary to decide on such factors as blade speeds, stream velocities and degree of reaction or compression. Formerly in steam turbine practice the blading was either of pure impulse design or a 50° reaction design, and for these cases the principles on which the rotor speeds and steam velocities were determined were widely known and practiced. With the advent of the gas turbine plant however, more use is being made of blading where the degree of reaction is other than 50° or zero, both in the turbine and compressor, and this trend is finding its way into present day steam turbine practice. Once the obove mentioned design features have been established, it is necessary to have available data on friction coefficients, efflux angles etc, and these are normally obtained from experiments on the proposed blading. With the development of the gas turbine plant and especially with the axial flow compressor, this work has usually been done in atmospheric wind tunnels, where air, passing into the atmosphere through the proposed blading, is examined with impact or pitot tubes and yawmeters.

Purpose of the Present Work.

In the turbine or compressor stage the stage loss is a loss of mechanical work energy which is affected by the loss of high grade energy in the elements of the stage. These element losses are due to the effect of friction forces and it is necessary to understand fully the nature of a friction loss and to be able to correlate the various ways in which the irreversibilities in the elements may be expressed. Part of the loss of mechanical work in the stage can however be dissociated from the friction losses in the elements and depends on the form of the stage velocity triangles. In turn the form of the velocity triangles is dependant on the initial choice of such design parameters as the degree of reaction and the loading factor for the stage.

When dealing with efficiencies in the turbine stage it is desirable to separate friction effects from effects associated with the form of the velocity triangles. It is the object of the present work to redefine the efficiencies associated with the stage so as to separate these effects and to do so in such a manner that the concepts may be applied to either the compressor or the turbine stage.

Due to the increasing interest in stages which are not either of an impulse or a fifty degree reaction design it is proposed to analyse the influence on the stage efficiency of degree of reaction, blade speed to jet speed ratio and loading factor. The analysis should be comprehensive enough to embrace turbine stages designed for any positive or negative degree of reaction and the theory evolved should be applicable to the compressor stage.

In most of the previous work in this field of blading research use has been made of "atmospheric" air in a wind tunnel. It was appreciated that a steam circuit could readily be adapted as a variable density tunnel. Hence it was decided to explore the flow and loss characteristics for a small rectangular nozzle and blade pair using superheated steam as the working fluid. This entailed more attention to the design of the instumentation compared with the "atmospheric" tunnel but the scheme allowed for testing over a wider range of conditions.

Part 1.

Using the concept of high and low grade energy the net useful high grade energy released by unit mass of working fluid in any steady flow process is shown to be $-\int VdP - \int fv_r$, where V is the specific volume of the working fluid, dP the elemental change in pressure, f the friction force per unit mass and v_r the relative velocity of the working fluid, the term $\int fv_r$ being the friction work done in the process. Comparing an adiabatic steady flow process with a similar isentropic process the net loss of useful high grade energy, which is often referred to as the enthalpy loss is shown to be $\int fv_r + \int dP\ (V - V_s)$ where V_s is the isentropic specific volume. Thus the enthalpy loss is divided into a basic friction work loss term plus an auxiliary loss term which accounts for the heating effect of the friction work. The auxiliary term is positive for compressions and negative for expansions.

The enthalpy loss is related to other methods used to express the irreversibilities in blade elements. Relationships are derived between the enthalpy loss, the total head pressure loss, the increase in entropy, the enthalpy loss coefficient, the total head pressure loss coefficient, the blade velocity coefficient, the blade drag coefficient and the efficiency of the expansion or compression process in the blade. The equations involved are applicable to static or moving elements in axial flow turbines and compressors with high and low speed flow.

Two stage efficiency definitions are used in turbine work, the total to static stage efficiency is the criterion of performance where the exhaust kinetic energy is degenerated by friction and the total to total stage efficiency is used where the exhaust kinetic energy is not degenerated. These stage efficiencies are derived as functions of (a) a blading or diagram efficiency and (b) an efficiency for the steady flow process in the stage. The diagram efficiency depends on the form of the stage velocity triangles which in turn depends on the choice of design parameters for the stage. These design parameters are the blade speed to jet speed ratio, the degree of reaction based on the inlet total head condition of the working fluid and the loading factor. The efficiency of the expansion process in the stage depends on the friction losses in the elements and is derived in terms of the loss coefficients in the static and moving blade rows.

It is shown that the total to static stage efficiency is directly proportional to the blading efficiency while the total to total efficiency is sensibly independant of blading efficiency provided that both the blading efficiency and process efficiency are high.

The axial flow compressor stage efficiency is also derived in terms of a blading efficiency and in terms of an efficiency for the compression process in the stage.

Having shown that the turbine stage efficiencies are dependant on the blading efficiency, an analysis is given of the way in which the blading efficiency varies with speed ratio and degree of reactio It was found that the blading efficiency and loading factor could best be expressed in terms of (a) the blade speed to tangential jet speed ratio and (b) a reaction coefficient for the stage, define as the ratio of the reaction effect in the moving blade to the kinetic energy available at inlet to the moving blade. The relationships between this reaction coefficient, the total head degree of reaction and other ways in which degree of reaction is sometimes defined are derived and the relative merits of the different definitions discussed. The variation of loading factor and blading efficiency is presented in graphical form. The optimum speed ratio for any given reaction coefficient is obtained along with the value of the maximum blading efficiency. The analysis is developed for stages where the degree of reaction is positive, zero, or negative and it is shown that when a stage is designed for maximum blading efficiency the value of the blading efficiency increases and the loading factor decreases as the degree of reaction is increased.

The way in which the blading efficiency of a compressor stage depends on the design parameters of the stage can be seen by looking upon the compressor stage as a similar turbine stage in which all of the stage velocities are reversed in direction.

Part 2.

The Determination in a static test rig of Local Total Head Efficiency of Expansion and of Stream Condition by means of an Impact Tube and the Conversion of Local Values into Mean Effective Values.

The material herein is associated with the experimental methods of obtaining efficiency and efflux angle. Even when dealing with the "incompressible" flow of air through blading there is a considerable amount of calculation involved in relating the local impact tube or pitot tube readings to local values of efficiency and further in converting these local values into mean effective values. The calculations become even more complex when account must be taken of compressibility effects, of variable density flow and of supersonic flow with the possibility of a shock wave formation upstream of the impact tube. Hence in this section a procedure is given which will ease the transformation of the local impact tube readings into local efficiency and local stream conditio and finally will allow these local values to be correctly converted into mean effective values.

Part 3.

An Experimental Investigation into the Flow of Superheated Steam through a Small Nozzle and Impulse (zero pressure drop) Blade yielding results on Flow Pattern and Efficiency with Varying Reynolds Number and Mach Number.

In this section experiments are described in which the flow and loss characteristics of a small nozzle and impulse blade pair were investigated using superheated steam as the working fluid. The nozzle is of rectangular cross-section and was made to suit a standard set of impulse blading. An impact tube and traversing gear were designed so that the entire exit area of either the nozzle or blade could be completely traversed. By this means local values of total head pressure and efflux angle were obtained at nozzle and blade outlet for a range of Reynold's number covering subsonic and supersonic Mach numbers. The results were converted to mean effective values of efficiency and efflux angle by the methods described in Part 2, and serve partly as an example of the application of these methods to blading research. It was found that the greater part of the total loss across the nozzle and blade pair occurs in the "parallel" passage of the blade and that this loss is very sensitive to quite small amounts of reaction or compression in the blade passage. The nozzle loss is confined to a thin wall boundary layer with isentropic flow elsewhere in the nozzle. The boundary layer in the blade passage is more pronounce than in the nozzle and is thickest on the convex side of the blade. The efficiency of both the nozzle and of the nozzle and blade pair rise with increasing Mach number as the value of unity is approache At low Mach numbers there is a rise in nozzle efficiency as the Mach number decreases but the efficiency of the nozzle and blade pair, where the losses are greater, does not show this characteristi For this small nozzle, the stream attains a constant efflux angle in only a small central core of the flow, with large variations in angle as the nozzle boundaries are approached. This efflux angle pattern remains unchanged with varying Reynolds number and subsonic Mach number. The efflux angles in the central core however rise slightly with increasing Mach number up to unity. As the Mach number progressively increases beyond unity the efflux angle pattern changes so that there is less variation in angle across the nozzle and the stream conforms more and more to the geometric outlet angle.

The tests on the impulse blade indicate a sharp rise in blade loss as the chord Reynolds number is reduced to 2×10^5. In appendix 3 the variation of loss coefficient at high Reynolds number is compared with similar work by Armstrong [40] and helps to confirm his conclusion that a critical Reynolds number exists at 2×10^5, below which value there is a substantial increase in blade loss.

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Nomenclature.

b	------	Breadth.
C_D	------	Drag coefficient.
C_L	------	Lift coefficient.
c_p	------	Specific heat at constant pressure.
D	------	Mean diameter.
da	------	Elemental area.
DH	------	Heat drop.
e	------	Specific internal energy.
f	------	Force of friction.
fda	------	Fraction of the elemental area da.
fV	------	Ratio of actual to isentropic specific volume.
F	------	Force normal to vane.
G	------	Reaction coefficient.
G_c	------	Compression coefficient.
g	------	Acceleration due to gravity.
H	------	Specific enthalpy.
H.M.D.	----	Hydraulic mean diameter.
h	------	Blade height.
i	------	Incidence.
J	------	Mechanical equivalent of heat.
K	------	Blade velocity coefficient.
M	------	Mass or Mach number.
m	------	Mass flow rate.
P/c	------	Pitch to chord ratio.
P	------	Pressure.
P_1	------	Inlet total head pressure static tests.
P_2	------	Exit static pressure ------ " "
P_3	------	Exit total head pressure -- " "
P_4	------	Static pressure after a normal shock jump at exit --- static tests.
P_5	------	Total head pressure after a normal shock jump at exit ---- static tests.
Q	------	Heat quantity.
R	------	Degree of reaction or gas constant.

Ro	------	Total head degree of reaction.
Rc	------	Degree of compression.
Roc	------	Total head degree of compression.
Rn	------	Reynolds number.
r	------	Radius.
s	------	Blade velocity.
t/c	------	Maximum thickness to chord ratio.
T	------	Temperature.
t	------	time.
u	------	Radial velocity of the working fluid.
V	------	Specific volume.
v	------	Absolute velocity of the working fluid.
v_a	------	Axial velocity or accoustic velocity.
v_r	------	Relative velocity of the working fluid.
v_W	------	Absolute whirl or tangential velocity of the working fluid.
WI	------	Specific work input.
WD	------	Specific work output.
Y	------	Total head pressure loss coefficient.
Z	------	Work done or loading factor.
ε	------	Enthalpy loss coefficient.
φ or s	---	Entropy.
γ	------	Isentropic index.
ω_V	------	Angular velocity of blade or vane.
ω	------	Absolute angular velocity of the working fluid.
α	------	Angle between the absolute velocity vector leaving the nozzle blade and the plane of rotation, in a turbine stage and angle between the absolute velocity vector entering the stator blade and the plane of rotation, in a compressor stage. Referred to as the nozzle outlet angle and stator blade inlet angle respectively. --- also local efflux angle in static tests.
β	------	Angles between the relative velocity vectors associated with the moving blade and the plane of rotation. Referred to as the "blade" angles.

φ ——— Angle between tangent to vane and the radius.
θ ——— Gas deflection.
μ ——— Coefficient of viscosity.
γ ——— Isentropic index.
ρ ——— Blade speed to jet speed ratio or density.
σ ——— Blade speed to tangential jet speed ratio.
ρ' ——— Blade speed to jet speed corresponding to the stage total head heat drop.
σ' ——— Blade speed to tangential jet speed corresponding to the stage total head heat drop.
h ——— An imperical coefficient.
η ——— Efficiency.
η_{ts} ——— Total to static stage efficiency.
η_{tt} ——— Total to total stage efficiency.
η_B ——— Blading or diagram efficiency.
η_b ——— Total head efficiency of expansion in nozzle and blade pair. —— static tests.
η_n ——— Total head efficiency of expansion in nozzle —— static tests.
Σ ——— Prefix designating summation.
$\bar{x}$ ——— Denotes a mean value of x.

Subscripts.

$(\quad)_{SY}$	------	denotes "system".
$(\quad)_{SU}$	------	denotes "surroundings".
$(\quad)_{f}$	------	denotes "with friction".
$(\quad)_{c}$	------	denotes compression.
$(\quad)_{o}$	------	denotes total head condition.
$(\quad)_{r}$	------	denotes velocity or condition relative to the moving blade.
$(\quad)_{s}$	------	denotes condition after an isentropic process in general.
$(\quad)_{ss}$	------	denotes condition after an isentropic process in the stage.
$(\quad)_{N}$	------	denotes "for the nozzle blade" in a turbine stage.
$(\quad)_{D}$	------	denotes "for the diffuser or stator blade" in a compressor stage.
$(\quad)_{R}$	------	denotes "for the rotor blade" in turbine or compressor stages.
$(\quad)_{S}$	------	denotes "for the stage" in turbines or compressors.
$(\quad)_{p}$ or $\mathrm{C\!\!\!\!L}$	----	denotes "profile or centre line" coefficient.
$(\quad)_{t}$	------	denotes "total or three dimensional" coefficient.
$(\quad)_{n}$	------	denotes nozzle outlet -- static tests.
$(\quad)_{b}$	------	denotes blade outlet -- static tests.

Introduction.

Since the impetus given to blading research by the advent of the gas turbine plant, and especially by the compressor unit there has been an increasing interest in turbines and compressors of widely differing types. These types include the radial and axial flow turbine or compressor and the use of the shock wave in supersonic compressors. Against this varied background it is necessary to be clear as to the nature and effect of losses in the working passages in turbo machinery and as to how the losses may best be determined experimentally.

The principal parts in turbines and compressors are the static elements which act as nozzles or diffusers and the moving elements which as well as being nozzles or diffusers are agents for the transfer of mechanical energy. The function of the static elements is the interchange of enthalpy and kinetic energy while that of the moving elements is the interchange of enthalpy on the one hand with kinetic and mechanical energy on the other. The principal factors affecting the efficiency of these energy interchanges, may be grouped under two headings.
These are:-

(1) Those which cause the degeneration of high grade kinetic energy into low grade energy.

(2) Those involved in the design and operation of the turbine or compressor and which influence the conversion of high grade energy to mechanical work energy.

The degeneration of high grade energy affects both the moving and static elements and the losses to which the elements are liable under this heading are:-

(a) The boundary layer or skin friction loss:-

This loss is characterized by a velocity gradient in the flow in the vicinity of the solid boundaries.

(b) Turbulent flow loss:-

This type of loss can effect the region within the boundary layer as well as the main stream and is due to random velocity variations with time, which are superimposed on the main velocity of flow. These random velocities cause an interchange of mass between adjacent streamlines with the result that shear stresses are set up within the body of the fluid.

(c) Eddy Motion or vorticity losses:-

This is caused by small variations in static pressure normal to the principal direction of the flow resulting in secondary circulatory motions being imposed on the main flow. The kinetic energy for these secondary motions reduces the energy of the main stream.

(d) Losses due to the separation of the boundary layer:-

Under certain circumstances especially in decelerating flow the low energy boundary layer separates from the wall and is energised by the main flow. This results in momentum mixing with consequent loss.

(e) Shock losses at the entrance to the elements:-

These losses are due to misalignment of the fluid stream and blade inlets causing eddy formations.

(f) Discontinuity losses:-

These losses are inherent in shock discontinuities or sudden changes in pressure, specific volume, temperature and velocity and occur only in supersonic flow. These losses (a) to (f) are known as reheat losses due to the fact that as the kinetic energy disappears, the stream is "heated" by an energy quantity equal to that of the lost kinetic energy.

Losses under the second heading are:-

(g) Deviation losses:-

This loss is due to the variation in the direction of the fluid flow from the direction dictated by the solid boundaries of the machine elements and affects the transfer of mechanical energy only.

and

(h) Leaving losses:-

These are losses in the amount of mechanical energy transferred because of the retention in the fluid stream of kinetic energy.

Other parasitic losses are those due to disc friction, windage and leakage of the working fluid past the machine elements.

The parameters which influence losses may be classified as:-

(1) Those due to the geometry of the machine elements.

For example, the shape of the cross sections normal to the flow, the curvature of the axis of the elements, and the surface finish of the boundaries.

(2) Those due to the properties of the working fluid. For example:- pressure, temperature, density, velocity, and viscosity.

The object of the investigation is to analyse the effect of these losses on the efficiency of energy transformation, to determine the conditions for maximum mechanical energy transfer from or to a working fluid, to establish a technique for measuring losses in machine elements with variable density flow and to apply the technique to determine losses in a nozzle and blade pair.

With regard to the effect of friction, an attempt has been made to show the basic nature of a frictional process, and to distinguish the friction work from the net loss of useful energy which is usually loosely called the loss or friction loss. An analysis is made for the general case of the flow of the working fluid through a radial flow machine and the equations developed, with suitable madification, may then readily be applied to any type of stationary or moving element. Particular reference is made to the effect of friction on the energy transfer terms in turbo machinery and to the different effects of friction in expansion and compression processes.

A section of theory is devoted to the efficient transfer of mechanical energy. In it a criterion is given which determines the best operating speed of the rotor blade for any given amount of reaction or compression in the blade passage. In the theory it is found to be convenient to define a term called "Reaction

Coefficient" and the criterion mentioned above is given as a function of this Reaction Coefficient. The conflicting factors of high specific mechanical energy output and blading efficiency are discussed for turbine blades with reaction or compression in the blade passage and for compressor blading.

In testing for the effect of friction in turbine or compressor elements, if the total head condition of the working fluid entering the machine element is known, a traverse of the outlet area with an impact tube will allow local values of the loss in total head pressure to be obtained. In the experimental work here the machine elements used are a small rectangular nozzle and blade pair with superheated steam as the working fluid and the arrangement is such that the density of the steam in the test section may be readily controlled and varied. The object is to test the nozzle and blade pair for friction losses and efflux angles under various operating conditions and, taking a broader view, to establish a method of translating the results of the traverse readings into mean values of efficiency and efflux angle which would be applicable to any type of machine element. To this end a section is devoted to the determination of local efficiency and local state point of the fluid from the observed impact tube readings. These local values over the cross-section of the flow are then reduced to mean effective values. The method used applies generally in that the effect of large density change within the element and the effect of supersonic speeds may be accounted for.

Review.

The problems presented by the flow of the working fluid through the elements of turbo machinery have been tackled from many viewpoints and with increasing intensity since the turn of the century. On the theoretical side the classical theories of potential flow, based on Euler's equations of motion for a non viscous fluid, represented a major step forward in an attempt to describe by mathematics the nature of fluid flow past and around solid boundaries. These equations were used, for example, to calculate the pressure distribution around cylinders, spheres, and aerofoils and gave results which were found to be in substantial agreement with observations. Various anomalies were however apparent and these were suspected to be due to the neglect of fluid friction. The Navier Stokes equations of motion, which account for fluid friction, were found to be capable of solution in only a few simple cases.

Prandtl was the first to give a practical approach to the problem of fluid friction and to its effect on fluid flow. He divided the flow around bodies into regions where friction could be neglected and where the classical theories of potential flow could be applied and into regions where friction had to be accounted for, these he referred to as boundary layers. Even at this stage Prandtl's theories were the results of mathematical treatment confirmed by observation.

The initial experimental work was of course associated with steam turbines. In 1910 Briling[1] investigated loss characteristics for plate type impulse turbine blades. Using a static test rig he determined the effect of different blade inlet forms and of steam deflection on the blade velocity coefficient. The coefficient was shown to decrease with increasing steam deflection for a given inlet form.

The most systematic approach to finding experimental coefficients in the early days was due to the work of the Steam Nozzle Research Committee of the Institution of Mechanical Engineers. This Committee was formed in 1914 with the initial aim of providing much needed information on the efficiency of typical steam turbine nozzle forms. The bulk of the experimental work took place between 1921 and 1930 and is summarized in the sixth and final report[2]. In the Committee's apparatus the impulse principle was used to obtain a nozzle velocity coefficient. The momentum of the steam jet issuing from the nozzle was measured by the force exerted by it on a flat plate which was placed at right angles to the direction of the jet. Great care was taken to ensure that all of the jet momentum was absorbed at the plate and, since the plate had to be at right angles to the mean direction of the jet, preliminary experiments to determine the efflux angle of the jet were necessary. This efflux angle was initially measured by a metal vane placed in the jet. Tests were made on "cast in" impulse nozzles, "built up" impulse nozzles, Parsons blade type nozzles and on elementary nozzles with a straight axis. In the bulk of the experiments the efflux angle was always less than the nominal outlet angle of the nozzle. In addition it was found that as the pressure ratio across the nozzle was decreased the efflux angle increased and so approached a value nearer the geometric outlet angle. The difference between the efflux angle and the geometric angle was found to depend on the nozzle plate thickness and it was thought that the proximity of the flat surface, inclined at a small angle to the stream at outlet, caused a suction effect on the jet. Thus with the curved type of outlet in the Parsons nozzles or with a chamfered outlet the deviation of the jet from the geometric angle was not so great. In general the nozzle velocity coefficient was found to decrease with increasing steam velocity from low velocities up to approximately 800 ft/sec, to remain fairly constant to about 1600 ft/sec and to increase there after. The minimum values of the coefficient were found to be sensitive to the length of the parallel portion, a large parallel portion being detrimental. It was also found that a roughness on the nozzle profile, produced by machining a screw thread on the nozzle had an adverse effect on the coefficient. In an interesting series of experiments on straight elementary nozzles with different inlet forms, it was found that at high velocity (about 2,000 ft/sec) a nozzle with sharp inlet had a velocity coefficient almost as high as a similar nozzle with a rounded inlet.

The impulse plate method of the Committee's apparatus was used in an improved form by Giffen and Crang[3] in work preceeding 1939 but not published until 1946. The object of this was to investigate the rising characteristic of nozzle velocity coefficient with decreasing steam velocity at low speeds as obtained by the Nozzle Research Committee and the results were in apparent disagreement with those obtained by the Committee.

In a Parsons Memorial Lecture in 1939 Guy[4] describes experiments with which he was associated wherein the vane method of measuring efflux angle was dispensed with and a yawmeter used to find local values of efflux angle at the exit of typical steam nozzles. He again found that the mean efflux angle increased with increasing steam speed and found that at higher velocities the mean efflux angle is given very closely by $\sin^{-1}$ opening/pitch.

In 1940 Dollin[5] used the reaction principle to obtain steam nozzle efficiency. He supported the nozzles in a static testing machine in such a way that they had limited freedom of torsional and axial movement and by means of balances measured the forces necessary to restore the nozzles to their original position. This was claimed to be superior to the impulse plate method because, in addition to giving efficiency, it also gave mean efflux angle. The results of tests on a group of reaction nozzles showed that, as the pressure ratio was decreased, the nozzle efficiency continuously increased up to the critical pressure ratio. Also the mean efflux angle was always greater than that given by $\sin^{-1}$ opening/pitch and indeed decreased slightly with decreasing pressure ratio.

Stodola[6] also gives the results of efflux angle tests but in this case on single convergent parallel nozzles with overcritical expansion. The nozzles were mounted in a nozzle holder with the axis of the nozzle pointing vertically upwards and the outlet face of the nozzle machined off obliquely to the axis. The efflux angle distribution was apparently obtained by photographing the steam jet and making measurements from the photographs. The results show that for small pressure ratios, in the region of 0.2, the nozzle angle increased by as much as 44 degrees at the inner or shorter boundary with a small positive or negative deviation at the outer boundary.

A considerable impetus to blading research came however with the advent of the gas turbine plant. The need for a compressor which would be efficient at wide operating conditions led to a great deal of work on compressor blading. The principal contribution is that of Howell[7] in his paper "Fluid Dynamics of Axial Compressors" in which he pays particular attention to the variation of blade loss with incidence of the gas stream. Air was used as the working fluid in wind tunnels with cascades of wooden blades, and detailed traverses were made of the outlet flow areas of the cascade with pitot or impact tubes and yawmeters. Thus for a given compressor cascade average values, usually the arithmetic mean values, of total head loss and of stream deflection were obtained for various tunnel settings or incidence angles of the inlet stream. This in turn gave the permissable range of efficient operation of the cascade, since points of positive or negative stall were obtained by the above methods. In addition the total loss at a given incidence was divided into a profile loss or skin friction loss which could be measured at the mid height of the cascade, a secondary loss due to trailing vortices formed by unequal static pressure distribution within the blade passage and an annulus loss due to skin friction at the annulus walls. Fluid speed effects on the loss characteristics in compressor cascades were allowed for by making use of criteria known as critical and maximum Mach number. Critical Mach number is reached at inlet to the blading when, at a given incidence, there is a distinct increase in loss as the inlet speed is raised. Maximum Mach number is obtained when the cascade chokes and the losses are so great that there is no pressure rise in the blading. Maximum Mach number was found to increase continuously from large negative incidence to large positive incidence. Critical Mach number on the other hand increases from large negative incidence to approximately zero incidence and decreases thereafter. The progressive increase in loss between critical and maximum Mach number is expressed in a simple graphical form in terms of the actual inlet Mach number and the efficiency at critical Mach number. Proceeding from the results of his research Howell evolved rules of procedure which are the present day basis of subsonic compressor design. Details of these are given in papers by Howell[8] and by Howell and Bonham[9].

Todd[10] in his paper "Practical Aspects of Cascade Wind Tunnel Research" gives details of low speed and high speed (up to Mach No. of one) wind tunnels and of the methods and instruments used in observing flow pattern and loss. The methods given in this paper are substantially the same as those used by other investigators including Howell. The success of the cascade work on compressor blading led to an appreciation of the need for more information on turbine blading. Ainley[11] in his paper "The Performance of Axial Flow Turbines" gives results of similar cascade wind tunnel testing on turbine blade sections. The work on turbine blading is however not as comprehensive as that on compressor blading and is often limited to cascade testing and subsequent modification of a proposed blade section until a suitable profile is obtained. Attempts have been made, notably by Weske[12] and Carter[13] to correlate the results of cascade testing with the classical theories, mentioned previously, of potential flow in order to obtain a logical procedure of calculation which would relate the known basic laws to the performance of a turbine or compressor stage. These methods have still to obtain widespread recognition and experimental work is still the basis of the design.

A feature of all cascade wind tunnel work is the use of the pitot tube or impact tube to measure local total energy leaving the blade. The tube itself has been the subject of much investigation notably by Todd [10 & 14] and Hodkinson[15]. With suitable precautions it gives an accurate method of obtaining local values of blade loss. In a supersonic stream, however, modification must be made to the observed impact tube pressure to obtain the stream total head pressure, due to the formation of a detached shock wave in front of the impact tube hole. Details of these modifications are given by Bairstow[16] and Durand[17].

The volume of present day knowledge on the behaviour of turbine and compressor elements is considerable. The future needs of industry will nevertheless demand an increasing research programme on blading elements. The effects of high and low deflection and of varying degrees of reaction in turbine blades, the effects of blade form and pitch especially in supersonic compressors, the effects of compressibility, Reynolds number, secondary flows and of radial equilibrium are some of the subjects requiring further investigation. One disadvantage of cascade wind tunnels is that the test sections normally operate at one pressure and usually the exhaust from the cascade is to the atmosphere. This atmospheric exhaust has many obvious advantages, but it does preclude testing to find the effects of large density change. In addition the design of a variable density wind tunnel with its associated compressor is relatively difficult and is expensive. The apparatus used here, while not giving the flexibility of variable change inherent in the atmospheric cascade tunnel, does give a simple way of obtaining a variable density test section using a steam boiler plant in place of a compressor circuit.

PART 1 (a).

Methods of obtaining the high grade energy exchanged between the working fluid and its surroundings in flow and non flow processes with particular reference to the effect of friction in the steady flow process.

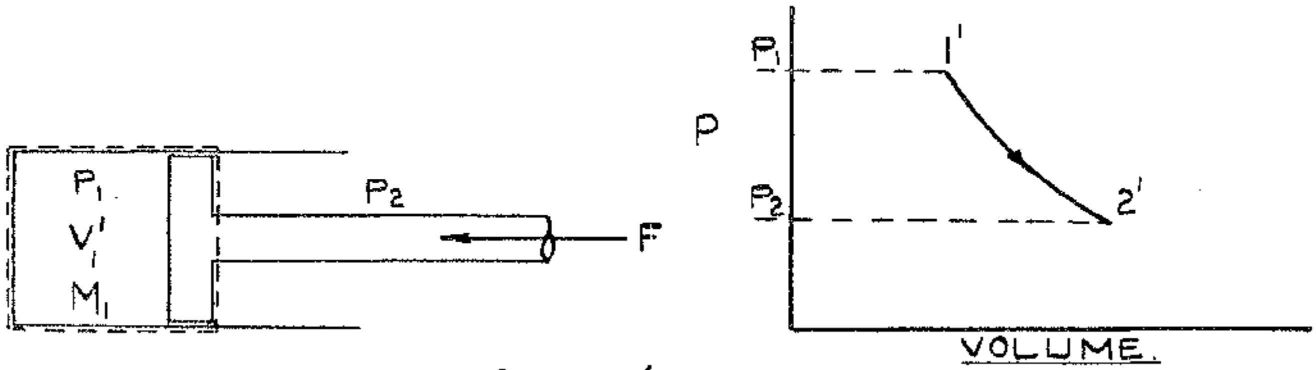


FIGURE 1a.

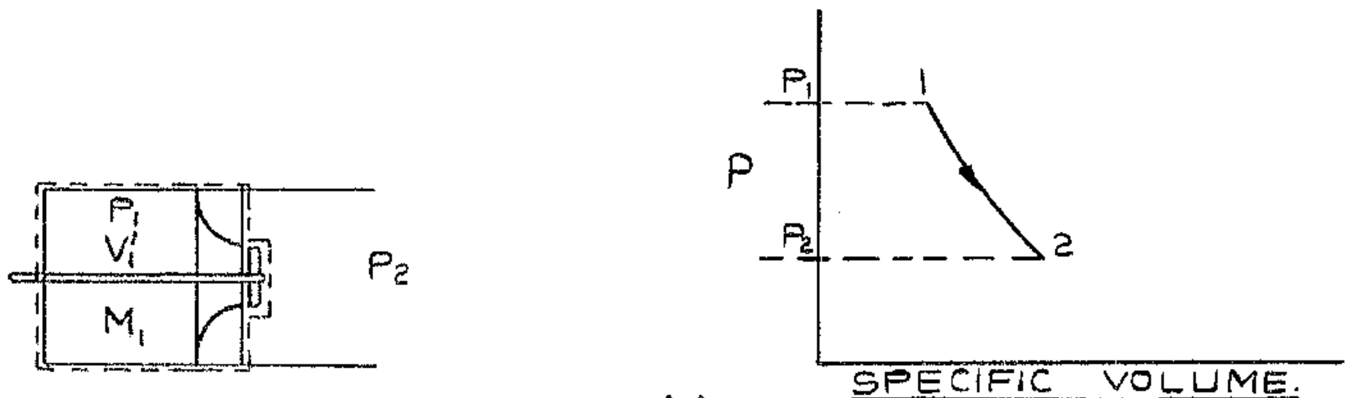


FIGURE 1b.

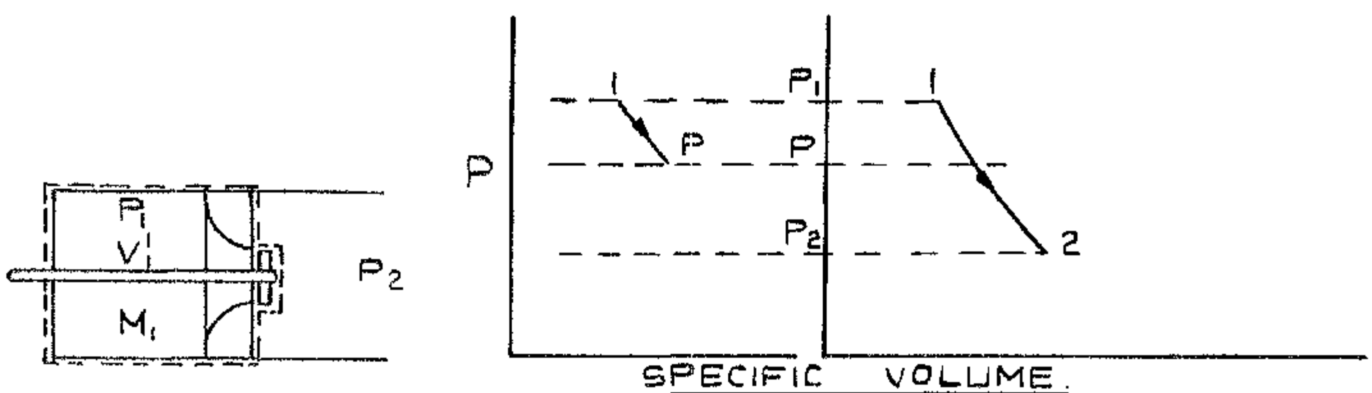


FIGURE 1c.

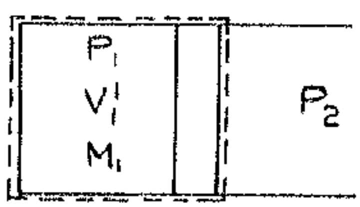


FIGURE 1d.

FIG. 1.

Part 1 (a).

Methods of Obtaining the high grade energy exchanged between the Working Fluid and its surroundings in Flow and Non-flow processes with particular reference to the effect of friction in a Steady Flow Process.

In turbines and compressors we are concerned in the main with the steady flow process and in particular with the transfer of mechanical work energy from or to the working fluid. Since the relationships between kinetic energy changes, mechanical work transfer terms and the changes in the properties of state of the working fluids in steady flow processes, are but one particular (though extremely important) application of more general laws, it is fruitful to consider the methods available for obtaining energy transfer in different processes.

Section 1 - Processes without friction.

Consider a cylinder fitted with a "weightless" piston where the space between the piston and the cylinder head is filled with M_1 lbs of a perfect gas at pressure P_1 and specific volume V_1. (Figs 1a, b, c, and d). The volume of gas involved is then $V_1' = M_1V_1$. The original boundary of the system is shown by the dotted lines and the pressure of the fluid in the surroundings of the system is P_2 ($P_2 < P_1$) In considering the subsequent behaviour of the gas in the system we shall presume that no heat crosses the system boundary and that effects of friction may be neglected.

In case (a) (fig. 1a) suppose that the gas expands slowly to P_2 so that at all times equilibrium is established between the gas and its surroundings. This means that a variable restraining force (F) must be maintained at the piston rod, of such a magnitude that this force, together with the force exerted on the piston by the fluid surroundings, is only slightly less than the gas force on the piston. The total work done by the gas which appears entirely as displacement work at the piston (the fluid boundary) is given by

$$\frac{P_1\ V_1' - P_2\ V_2'}{\gamma - 1} = M_1\ \left(\frac{P_1\ V_1 - P_2\ V_2}{\gamma - 1}\right) \qquad \text{——— (1).}$$

This total work may be divided into useful external work done on the force, F, and into work which is absorbed by the surrounding fluid at the fluid boundary. The useful external work is then

$$M_1\ \left[\left(\frac{P_1\ V_1 - P_2\ V_2}{\gamma - 1}\right) - P_2\ (V_2 - V_1)\right] \qquad \text{——— (2).}$$

and this equation may be rearranged to give

$$M_1\ \left(\frac{P_1\ V_1 - P_2 V_2}{\gamma - 1}\right) - M_1\ \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right)\ P_2\ V_2 \qquad \text{——— (3).}$$

In case b and c (fig 1b and c) a stop is fitted to the piston and the gas in the system is initially contained in a rigid closed vessel. We shall presume that the piston is fitted with a suitably shaped nozzle, which may be opened or closed instantaneously, and that at the nozzle exit an ideal impulse rotor is fitted, which is capable of absorbing all of the kinetic energy supplied to it. Suppose that the nozzle is opened allowing the pressure inside the vessel to fall. In practice, if the vessel is large compared with the throat area of the nozzle, and if the nozzle is suitably designed we may assume isentropic expansion in the vessel and in the nozzle.

Let M, P and V be the instantaneous mass, pressure and specific volume of the gas remaining in the vessel sometime after the nozzle is first opened.

Then $\frac{M}{M_1} = \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}$ and $\frac{dM}{dP} = \frac{1}{\gamma}\ \frac{M_1}{(P_1)^{\frac{1}{\gamma}}}\ P^{\frac{1-\gamma}{\gamma}}$ ——— (4).

By applying the steady flow energy equation to the instantaneous ejection of mass δM through the nozzle, the elemental kinetic energy generated at the nozzle exit, when the pressure inside the vessel is P, is given by

$$\delta M \quad \frac{\gamma}{\gamma - 1} \quad (PV - P_2 V_2) \qquad \text{\_\_\_\_\_ (5).}$$

Hence the total kinetic energy generated at nozzle exit, as the pressure inside the vessel falls from P_1 to P, is given by

$$\int_{P_1}^{P} - \frac{1}{\gamma - 1} \frac{M_1}{(P_1)^{\frac{1}{\gamma}}} \quad (P)^{\frac{1-\gamma}{\gamma}} \left(PV_1 \left(\frac{P_1}{P}\right)^{\frac{1}{\gamma}} - P_2 V_2\right) dP \qquad \text{\_\_ (6).}$$

this gives

$$\frac{M_1 \ V_1}{\gamma - 1} \ (P_1 - P) \ - M_1 \frac{\gamma}{\gamma - 1} \ \left(1 - \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}\right) P_2 V_2 \qquad \text{\_\_\_\_\_ (7).}$$

and is identical with the work obtained at the turbine.

<u>Case b.</u> If the pressure inside the vessel is allowed to fall to P_2, then from equation 7 it can be shown that the total kinetic energy is given by

$$M_1 \left(\frac{P_1 \ V_1 - P_2 \ V_2}{\gamma - 1}\right) \ - \ M_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right) P_2 \ V_2 \qquad \text{\_\_\_\_\_ (8).}$$

which is identical with equation 3.

In this event all of the original M_1 lbs of gas whether finally inside or outside the vessel, has expanded from 1 - 2.

$M_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right)$ is the mass which has discharged from the vessel and has been exhausted against the back pressure P_2.

Rogers and Mayhew[18] approach the problem of case (b) by applying the unsteady flow energy equation and they show that the kinetic energy (or turbine work) is equal to the decrease of internal energy of the gas in the vessel minus the enthalpy of the fluid which has left the vessel. It can of course be shown that this gives the same result as that derived in equation 8.

<u>Case c.</u> If the process is stopped by closing the nozzle when the pressure inside the vessel reaches P, then the total kinetic energy generated can be shown from equation 7

to be

$$M_1 \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}} \left(\frac{P_1 V_1 - PV}{\gamma - 1}\right) + M_1 \left(1 - \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}\right) \left(\frac{P_1 V_1 - P_2 V_2}{\gamma - 1}\right)$$

$$- M_1 \left(1 - \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}} \right) P_2 V_2 \qquad \text{——— (9).}$$

Here $M_1 \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}$ is the mass which remains inside the vessel and has expanded from 1 - P (fig 1c). $M_1 \left(1 - \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}\right)$ is the mass which has expanded from 1 - 2 and has been discharged against the back pressure P_2.

In cases a and b all of the original mass of gas expands from the initial state (1) to the final pressure P_2. The total high grade energy released appears entirely as displacement work in (a) and as a combination of displacement work and kinetic energy in (b). For either case the total high grade energy released is given by $\int_{SY} PdV$, where P is the pressure of the gas in the system (SY) and dV is the elemental change in total volume. In (a) and (b) the displacement work done, by the gas in the system, on the fluid surrounding the system is $M_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right) P_2 V_2$.

Here $M_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\right) V_2$ is the total volume of surrounding fluid which is displaced and P_2 is the constant pressure of the fluid in the surroundings. This displacement work at the fluid boundary can be written as $\int_{SU} PdV$, where P is the pressure of the surrounding fluid and dV the total elemental change in volume at the surroundings. This boundary displacement work, which is absorbed by the surrounding fluid, is not in general useful work and the useful high grade energy released, (equations 3 or 8), may be written as

$$\int_{SY} PdV - \int_{SU} PdV \qquad \text{——— (10).}$$

In case (c) where the process ceases when the pressure in the vessel is P, the system may be divided into two parts. The first part being that which contains the mass, $M_1 \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}$,

which is finally left in the vessel, and has expanded from 1 - P. The second is that containing the mass, $M_1 \left(1 - \left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}\right)$ which

is finally outside the vessel having expanded from 1 - 2. Although no single pair of properties can be used to describe the final state of the gas in the system, the total high grade energy released is obtained by finding $\int PdV$ for each of the parts of the system (equation 9). Thus for case (c) the useful high grade energy released is given by

$$\underbrace{\int PdV}_{\text{Mass remaining in the vessel.}} + \underbrace{\int PdV}_{\text{mass finally outside the vessel.}} - \int_{SU} PdV$$

In the general case a system may be divided into a number of parts and in addition the system may be in contact with surrounding fluid at a number of points, at each of which the fluid pressure may be different, and indeed at each point the fluid pressure may vary during the process. For this general case the useful high grade energy released will be given by

$$\sum_{SY} \left[\int PdV \right] - \sum_{SU} \left[\int PdV \right] \qquad \text{———— (11).}$$

In case (d), (fig 1d) the weightless piston is initially fitted with a stop. If the stop is suddenly removed the gas in the system is subjected to a free unrestrained expansion and directional kinetic energy will be released in various amounts throughout the system. Referring again to cases (b) and (c), the rate at which the process proceeds depends on the size of the nozzle in relationship to the vessel, and in thermodynamics the energy relationships are independant of the time taken for a process to be completed. Thus in case

(d) removal of the stop at the weightless piston is equivalent to replacing the piston with a nozzle of the same area. The process proceeds at a rapid rate and approach velocity will be generated in the vessel as well as at the nozzle exit. In practice the directional kinetic energy will be reabsorbed as random molecular energy but, if one postulates that viscous drag forces will be absent, then case (d) is the same thermodynamic problem as cases a, b, or c. The system in case (d) cannot be conveniently divided into two parts as in case (c), and to obtain the high grade energy released, at any given time, (by using equation 11) the pressure-volume history of all the elements of the system would have to be accounted for.

In a steady flow process a mass of gas, contained within a system boundary, already possesses kinetic energy as the system moves towards a control volume. During this approach to the control volume the properties of state of the gas remain constant and there is no deformation of the system boundary. Within the control volume all of the elements of mass in the system suffer the same series of pressure volume changes. Again in the exhaust the properties of state remain constant. The total high grade energy released is given by

$$\int_{SY} PdV.$$

To obtain the useful high grade energy released, one must consider the displacement work done on the system by the fluid boundary at inlet, where the pressure is constant at P_1, as well as the work done by the system on the fluid boundary at exhaust, where the pressure is constant at P_2.

Thus from equation 11 the net displacement work absorbed by the fluid surroundings is

$$\sum_{SU} \left[\int PdV\right] = -P_1 \int dV_1 + P_2 \int dV_2 = -P_1 V_1 + P_2 V_2$$

and the useful high grade energy released is

$$\int_{SY} PdV + P_1 V_1 - P_2 V_2 = - \int_{SY} VdP \qquad \text{——— (12).}$$

Section 1 - Summary.

There are a number of ways for obtaining work done and kinetic energy changes in closed systems and in open systems with steady and unsteady flow. These include the non-flow energy equation, the steady and unsteady flow energy equations. These relationships have been applied here, to processes where friction may be neglected, to illustrate the meaning that may be attached to areas on the pressure volume diagram in all of these systems.

It is shown that, when any of the elements of mass, which together comprise the total mass within a system, suffer a pressure-volume change the summation of $\int PdV$ for all such elements gives the total high grade energy released in the process. This high grade energy may appear as a combination of mechanical shaft work, displacement mechanical work, displacement work done at the fluid boundary on the surrounding fluid, and in increase of directional kinetic energy, which is retained by some or all of the elements of mass.

A distinction is made between the total high grade energy released in the process and the useful high grade energy released. The useful high grade energy released being the total high grade energy less the displacement work done on the fluid surrounding the system. This useful high grade energy for any flow or non-flow process is given by equation 11 and for the particular case of a steady flow process equation 11 reduces to $- \int_{SY} VdP$.

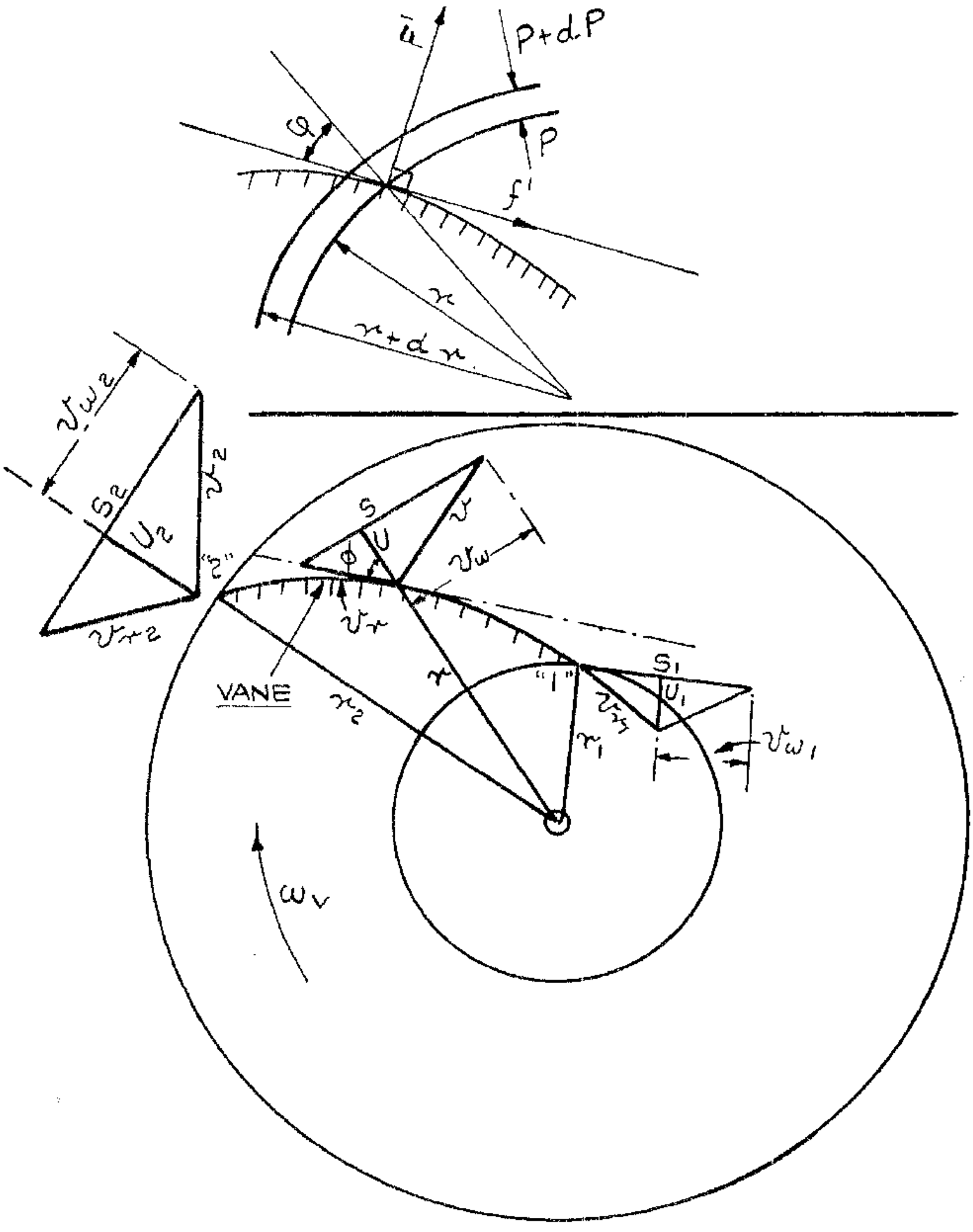


FIG. 2.

Section 2. - The steady flow process with friction in the elements of Turbo Machinery.

In all actual turbo-machinery elements there is some degeneration of high grade energy to low grade molecular energy by friction forces. In the previous section we have seen how the high grade energy involved in a process may be obtained. In this section the influence of friction on the relationships derived in section 1a will be considered by studying a steady flow process through the rotor blades of a centrifugal machine (Fig. 2.).

In the flow of any real fluid through the blades of turbine or compressor elements the properties and velocity of the fluid are not constant over any given cross-section in the flow. As the fluid molecules pass through the blade passage, those molecules in immediate contact with the blade surface are at rest relative to the blade. Thus there is relative motion between layers of molecules resulting in momentum transfer from one layer to another. This involves resistance to the flow and results in a drag force on the blade. Hence in any real flow, friction results in an uneven distribution of properties and velocity normal to the mean direction of the flow. In addition, it is the difference in static pressure at a given cross-section between the concave and convex sides of the blades, that results in blade lift.

In an account of blading performance for axial flow turbines and compressors by Shepherd [19], the lift and drag forces on the blade are related to the change of whirl velocity between inlet and outlet, to give the work energy transferred. Since friction is involved it follows that the whirl components of velocity are not uniform over the inlet or outlet cross-section and the use of average values of these velocity components is implied in Shepherd's analysis. Thus Shepherd combines the effects of a non uniform distribution of fluid properties with the essentials of a one dimensional flow treatment.

As a general example of a steady flow process with friction consider the motion of a working fluid through the rotor blades of a centrifugal compressor. The arrangement is shown in fig 2. The fluid enters at radius r_1 at the rate of m lb/sec and leaves at radius r_2. The rotor vane is defined by the path 1 - 2, lying between the inner and outer radii and ω_V is the angular velocity of the vane. At radius r let

u = the radial velocity of the fluid.

s = the tangential velocity of the vane.

ω = the absolute angular velocity of the fluid.

v_W = the absolute tangential velocity of the fluid.

v_r = the velocity of the fluid relative to the vane.

v = the absolute velocity of the fluid.

V = the specific volume of the fluid.

φ = the angle between the tangent to the vane and the radius r.

and b = the breadth of the passage.

We shall presume that the properties and velocities are constant with angular position at a given radius, a state which is approached as the number of blade passages is increased. In such a one dimensional treatment it is then necessary to presume that the force of friction is concentrated at the boundary (see Rogers and Mayhew [20]).

Thus at radius r

let f^1 = the total friction force tangential to the vane.

and F^1 = the total force normal to the vane.

As each element of mass arrives at radius r its acceleration along and perpendicular to the radius may be shown to be

$$\frac{du}{dt} - \omega^2 r \text{ and } r\frac{d\omega}{dt} + 2u\omega \text{ respectively.}$$

Thus for a continuum of particles passing through radius r the radial and tangential external forces on the fluid are

$$m\,(du - \omega^2 r dt) = F^1 \sin\varphi - f^1 \cos\varphi - dP2\pi rb$$

and m $(rd\omega + 2u\omega dt) = F^1 \cos\varphi + f^1 \sin\varphi$ respectively.
This gives the radial force per unit mass flow as

$$du - \omega^2 r dt = F \sin\varphi - f\cos\varphi - \frac{dP2\pi rb}{m} \qquad \text{\_\_\_\_\_ (13).}$$

and the tangential force per unit mass flow as

$$rd\omega + 2u\omega dt = F\cos\varphi + f\sin\varphi \qquad \text{\_\_\_\_\_ (14).}$$

Where F and f are forces per unit mass flow.
The elemental work input at radius r per unit mass is obtained from equation 14 as

$$dWI = (rd\omega + 2u\,\omega dt)\, r\, \omega_V \qquad \text{\_\_\_\_\_ (15).}$$

$$\text{now } v_W = \omega r,\ dv_W = \omega dr + rd\omega \text{ and } udt = dr \qquad \text{\_\_\_\_\_ (16).}$$

$$\therefore dWI = (rdv_W + v_W dr)\,\omega_V \qquad \text{\_\_\_\_\_ (17).}$$

This well known relationship is given by Hunsaker and Rightmire[21]. In their generalized approach to the problem the external forces are obtained by considering the momentum of particles entering and leaving a control volume and the rate of change of momentum is referred to two arbitrary directions mutually at right angles. In the analysis shown here some simplification is obtained by choosing at the outset reference directions along and at right angles to the radius. Since however this involves a moving coordinate system, account must be taken in equations 13, and 14 of centrifugal and coriolis accelerations. (Shapiro[22]). For turbines and compressors the control volume is the space within an annulus concentric with the axis of rotation. In Hunsaker's analysis it is assumed that the momentum within the control volume does not vary with time and that the velocity and direction of the fluid are uniform entering and leaving the control surfaces, that is at the inner and outer radii of the annulus. These assumptions are in accord with the provisions stated earlier when dealing with a one dimensional flow with friction . From the geometry of the velocity triangle at r, $u^2 + (s - v_W)^2 = v_r^2$ or in differential form

$$\therefore s dv_W + v_W ds = \omega_v\,(rdv_W + v_W dr) = udu + sds + v_W dv_W - v_r dv_r \qquad \text{\_\_\_\_\_ (18).}$$

Hence from 17

$$dWI = \omega_V\ (r dv_W + v_W dr) = v dv + s ds - v_r dv_r \qquad (19).$$

Equation 17 is an exact differential

$$\therefore\ WI = (s_2 v_{W_2} - s_1 v_{W_1}) = \frac{v_2^2 - v_1^2}{2} + \frac{s_2^2 - s_1^2}{2} + \frac{v_{r_1}^{\ 2} - v_{r_2}^{\ 2}}{2} \qquad (20).$$

From 16, and using the mass flow equation, 13 and 14 may be expressed as

$$du - \frac{v_W^{\ 2}}{u}\ \frac{dr}{r} = F \sin\varphi - f \cos\varphi - \frac{dPV}{u} \qquad (21).$$

and

$$dv_W + v_W \frac{dr}{r} = F \cos\varphi + f \sin\varphi \qquad (22).$$

Eliminating F from 21 and 22 gives

$$u du - v_W^{\ 2}\ \frac{dr}{r} = u \tan\varphi\ dv_W + u v_W \tan\varphi \frac{dr}{r} - dPV - \frac{fu}{\cos\varphi} \qquad (23).$$

now $u \tan\varphi = s - v_W$, $u = v_r \cos\varphi$ and $\frac{dr}{r} = \frac{ds}{s}$.

Hence from 23

$$u du + v_W dv_W = s dv_W + v_W ds - dPV - f v_r \qquad (24).$$

Hence from 19

$$dPV + f v_r = dWI - v dv \qquad (25).$$

and

$$dPV + f v_r = s ds - v_r dv_r \qquad (26).$$

Equations 25 and 26 apply to any steady flow process with friction. There is no restriction in the proof to adiabatic flow and the equations may be used where there is a combination of friction and heating from a reservoir external to the system. The steady flow energy equation gives

$$dH - dQ = dWI - v dv \qquad (27).$$

$$\text{or } de + PdV + VdP - dQ = dWI - v dv \qquad (28).$$

where dH and de are the changes in enthalpy and internal energy in the process and dQ is the heat supplied from an external reservoir.

Interpretation of equation 25 for steady flow compression and expansion processes.

For a compression process from 1 - 2, equation 25 gives

$$\int_1^2 dPV + \int_1^2 fv_r = WI + \frac{v_1^2 - v_2^2}{2} \qquad (29).$$

$WI + \frac{v_1^2 - v_2^2}{2}$ is the total high grade energy absorbed in the compression and $\int_1^2 fv_r$ is the friction work. Thus of the total high grade energy utilized in the process a proportion of it, the friction work, is degenerated and $\int_1^2 dPV$ gives that part of the high grade energy used, which is usefully employed in creating the pressure rise. For an expansion process equation 25 is best written as

$$- dPV = dWD + vdv + fv_r \qquad (30.$$

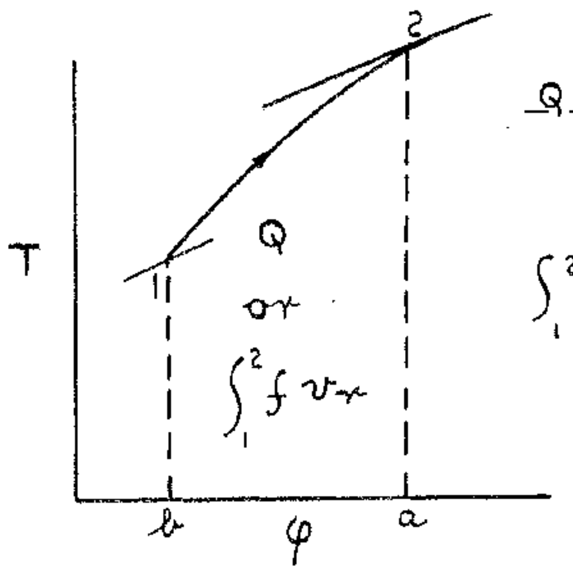
or for an expansion from 1 - 2

$$\int_2^1 dPV = WD + \frac{v_2^2 - v_1^2}{2} + \int_1^2 f\, v_r \qquad (31).$$

where $\int_2^1 dPV$ is a positive area on the P- V diagram.

In section 1A it was shown that for an isentropic steady flow process $- \int dPV$, (equation 12), gave the useful high grade energy released. The total high grade energy released being the sum of the useful high grade energy and the work absorbed by the fluid surroundings. In considering equations 30 or 31 a further distinction must be made. The term $\int_2^1 dPV$ may again be regarded as the total useful high grade energy which is ever released during the expansion process. However during the process a part of this energy is degenerated by friction to low grade energy so that the <u>net</u> useful high grade energy released is $\int_2^1 dPV - \int_1^2 fv_r$ or $WD + \frac{v_2^2 - v_1^2}{2}$

Comparison of an adiabatic steady flow process with friction and a similar diabatic process without friction.



Q = HEAT SUPPLIED IN DIABATIC PROCESS.

$\int_1^2 f\, \underline{vr}$ = FRICTION WORK IN ADIABATIC PROCESS.

FIG. 3.

Suppose that in the diabatic process the heat supplied from an external reservoir is such that the path traced out by the state point of the working fluid is the same as that in the adiabatic process.

Hence from 25

for the adiabatic, $\int_1^2 dPV + \int_1^2 fv_r = \int_1^2 (dWI - vdv)_f$ ____(32).

and for the diabatic $\int_1^2 dPV = \int_1^2 (dWI - vdv)_{f=o}$ ____(33).

For a compression process since $\int dPV$ is common 32–33, or $\int_1^2 fv_r$, gives the extra high grade energy required to effect the pressure rise.

For an expansion $\int_1^2 fv_r$ is the loss of useful high grade energy released.

From 27 for the adiabatic

$$dH = (dWI - vdv)_f \qquad \text{\_\_\_\_(34).}$$

and for the diabatic

$$dH - dQ = (dWI - vdv)_{f=o} \qquad \text{\_\_\_\_(35).}$$

Since dH is common 34 – 35 = 32 – 33 gives

$$\int_1^2 dQ = \int_1^2 fv_r \qquad \text{\_\_\_\_(36).}$$

i.e. the friction work in the adiabatic process is equal to the low grade heat energy supplied in the diabatic process. Hence referring to figure 3, the area a 2 1 b on the T/φ diagram represents the heat supplied in the diabatic process or the friction work in the adiabatic process.

<u>Interpretation of the "non-flow" energy equation applied to a system which is subjected to an adiabatic one dimensional steady flow process with friction.</u>

For an adiabatic steady flow process with friction equations 28 and 25 give

$$o = de + PdV - fv_r \qquad \text{\_\_\_\_(37).}$$

In the non-flow adiabatic process in a closed system this equation is usually written, $o = de + dWD$. dWD is then the net total high grade energy released. Equation 37 is in accord with the conclusions reached for isentropic processes in section 1a. The $\int_1^2 PdV$ again gives the total high grade energy released and includes the work absorbed by the surrounding fluid. The net useful total high grade energy released is obtained by subtracting the friction work from $\int_1^2 PdV$, and this quantity of high grade energy is obtained by depleating the store of internal energy of the system. Equation 37 may be written as

$$o + fv_r = de + PdV \qquad \text{(38)}.$$

In the form of equation 38 the friction term appears along with the external heat term suggesting that in its effect friction is of the same nature as heat. In equation 37 on the other hand friction appears as a work term. In fact friction is the mechanism whereby high grade work energy is degenerated to low grade energy and as is shown in the comparison of the diabatic and adiabatic process the effect on the working fluid, of the low grade energy created, is the same as that of supplying the same amount of heat from an external reservoir. (see equation 36).

In present day concepts heat is considered as energy in transfer across the boundary of a system. With this definition there should be no need to refer to "external" heat since all of the heat entering the system must come from an external source. In a one dimensional treatment of fluid friction the friction force occurs at the boundary and the low grade energy created could be considered as crossing into the system at the boundary. Since in the actual process however, friction occurs within the body of the fluid there would appear to be room for the terms internal heating or reheating.

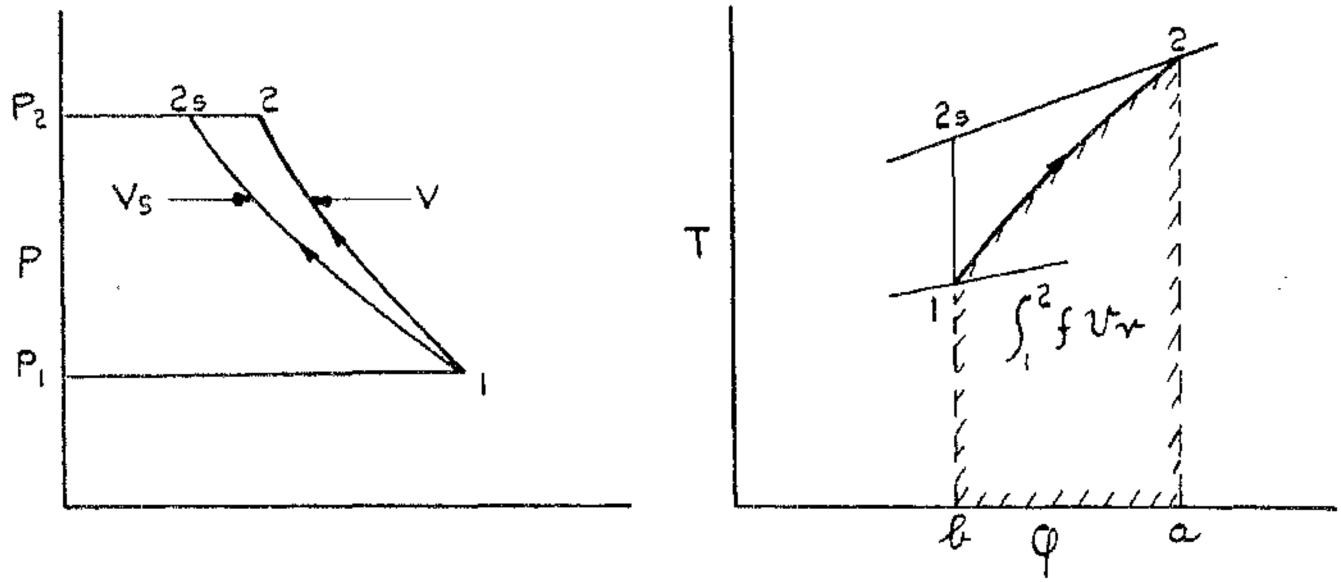


FIGURE 4 a.

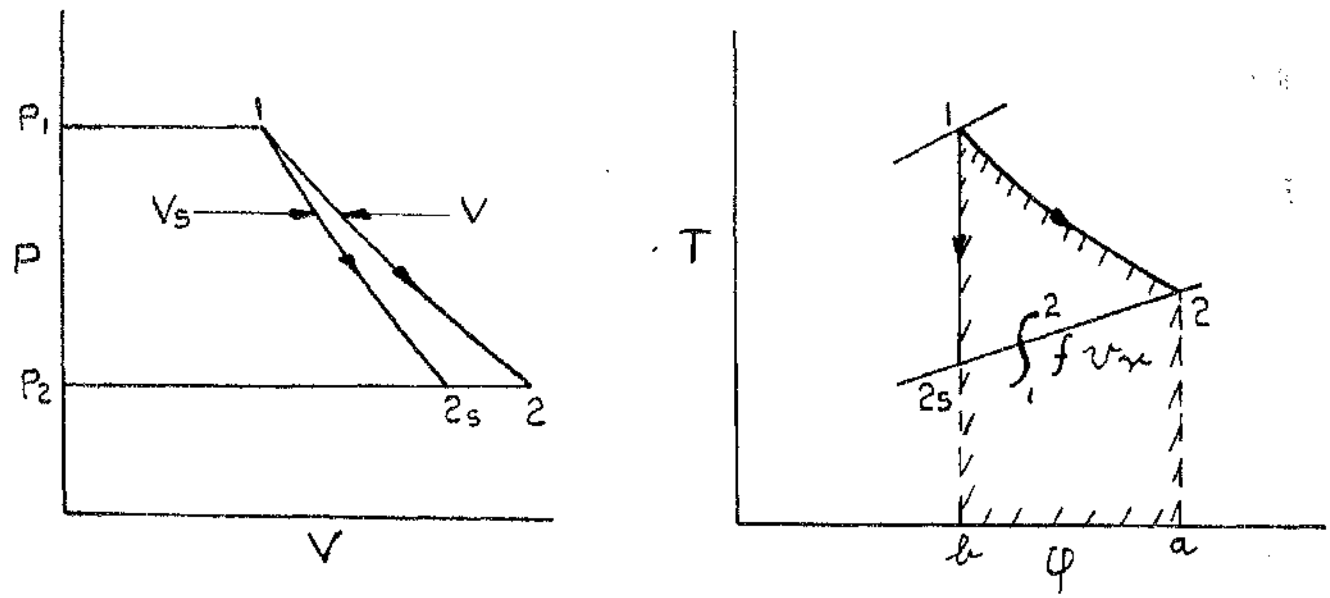


FIGURE 4 b

FIG. 4.

<u>The adiabatic steady flow process in axial flow turbo-machinery elements.</u>

Equation 27 gives

$$dH + vdv = dWI \qquad \text{\_\_\_\_(39).}$$

or

$$H_{o_2} - H_{o_1} = WI \qquad \text{\_\_\_\_(40).}$$

i.e. the change in total head enthalpy is equal to the work input.

For axial flow elements where there is no appreciable radial shift equations 25, 26, and 27, which are alternative expressions for the net useful high grade energy, give

$$dH + v_r dv_r = o \qquad \text{\_\_\_\_(41).}$$

or

$$H_{o_{2r}} - H_{o_{1r}} = o \qquad \text{\_\_\_\_(42).}$$

i.e. the relative stagnation enthalpy remains constant across the moving blade row, the relative stagnation enthalpy being that obtained by complete diffusion of the relative velocity.

From 25, 26 and 27 the total high grade energy involved may be expressed as :-

For a compression:-

the total high grade energy absorbed is

$$WI + \frac{v_1^2 - v_2^2}{2} = \int_1^2 dPV + \int_1^2 fv_r = \frac{v_{r_1}^2 - v_{r_2}^2}{2} = H_2 - H_1 \qquad \text{\_\_\_(43)}$$

and for an expansion :-

the net useful high grade energy released is

$$WD + \frac{v_2^2 - v_1^2}{2} = \int_2^1 dPV - \int_1^2 fv_r = \frac{v_{r_2}^2 - v_{r_1}^2}{2} = H_1 - H_2 \qquad \text{\_\_\_(44)}$$

<u>Comparison of an adiabatic flow process and a similar isentropic process for axial flow elements.</u>

Consider an adiabatic steady flow process (1 - 2) and an isentropic process from the same inlet state point to the same final pressure. (see figures 4A and 4B).

For the isentropic compression process the high grade energy absorbed is

$$WI_s + \frac{v_1^2 - v_2^2}{2} = \int_1^{2s} dPV_s = \frac{v_{r_1}^2 - v_{r_{2s}}^2}{2} = H_{2s} - H_1 \quad (45).$$

and for the isentropic expansion the useful high grade energy released is

$$WD_s + \frac{v_{2s}^2 - v_1^2}{2} = \int_{2s}^{1} dPV_s = \frac{v_{r_{2s}}^2 - v_{r_1}^2}{2} = H_1 - H_{2s} \quad (46).$$

From 43 and 45 the extra high grade energy required in adiabatic compression compared with the isentropic compression is

$$43 - 45 = \int_1^2 fv_r + \int_1^2 dP(V - V_s) = \frac{v_{r_{2s}}^2 - v_{r_2}^2}{2} = H_2 - H_{2s} \quad (47)$$

and this is a loss of initial high grade energy.
For the expansions the loss of useful high grade energy released is

$$46 - 44 = \int_1^2 fv_r - \int_2^1 dP(V - V_s) = \frac{v_{r_{2s}}^2 - v_{r_2}^2}{2} = H_2 - H_{2s} \quad (48)$$

The loss of useful high grade energy in 47 and 48 is known as the enthalpy loss.

Referring to figures 4a and 4b the enthalpy loss may be divided into the basic friction work loss in the process, ($\int_1^2 fv_r$ = area a21b on the T/φ diagram), plus an auxiliary loss term, ($\int_1^2 dP(V - V_s)$ = area 122_s on the P/V or T/φ diagrams), which is the result of the "heating" effect of the friction work. Kearton[23] refers briefly to this division of the enthalpy loss for compressions. For the expansion process, fig 4b, the auxiliary term is negative showing that the increase in specific volume causes some of the basic friction work loss to be returned as useful high grade energy. In similar expansion and compression processes therefore, where the basic friction work is the same, the loss of high grade energy will be greatest in the compression process.

Section 2 - Summary.

In a steady flow process in the elements of turbo-machinery the friction work is given by $\int f\, v_r$ where f is the friction force per unit mass and v_r the velocity of the working fluid relative to the moving blade. Friction is the mechanism by which high grade energy is degenerated to low grade energy. The low grade energy produced is equal in magnitude to the friction work and causes an increase in the internal energy and in the specific volume of the working fluid. This "heating" effect of friction is the same as that which would occur in a diabatic process without friction where the heat supplied from an external reservoir is equal in magnitude to the friction work. In a steady flow compression process, where one is interested in the efficient utilization of available high grade energy to create a pressure rise, the friction work is that proportion of the total high grade energy used, which is degenerated and does not produce pressure rise. The high grade energy which is usefully absorbed is given by $\int dPV$.

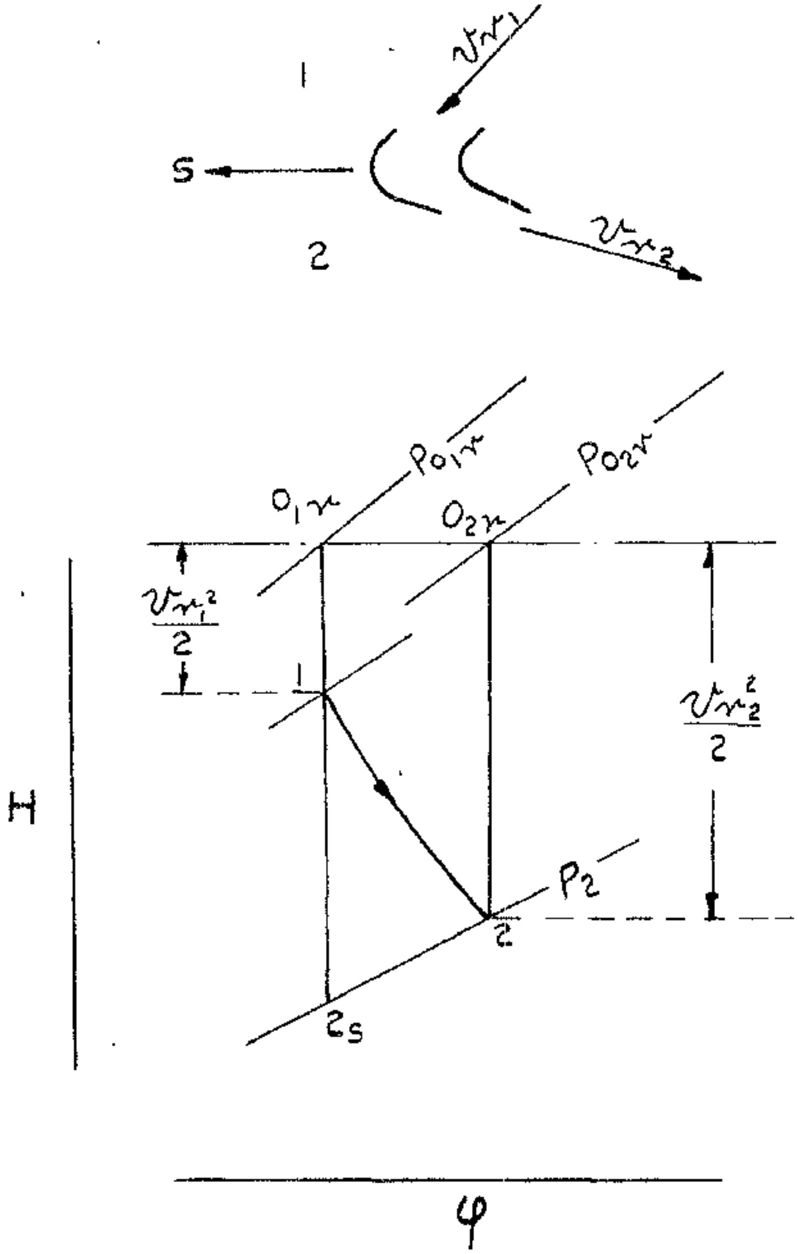
For the steady flow expansion process the term $-\int dPV$ gives the total useful high grade energy ever produced during the process. During the process however, part of this energy, the friction work, is degenerated and the difference $(-\int VdP - \int fv_r)$ is the net useful high grade energy released. Comparing a steady adiabatic flow process with a similar isentropic process, the loss of useful high grade energy is given by

$$\int fv_r + \int dP(V - V_s).$$

The "heating" effects of the basic friction work loss causes an increase in the specific volume of the working fluid so that the actual specific volume (V) is greater than the isentropic specific volume (V_s). Hence the loss of high grade energy may be divided into a friction work term plus an auxiliary loss term reflecting this heating effect of the friction work. In compressions the auxiliary loss term is positive so that friction results in more high grade energy being required. In expansions the auxiliary loss term is negative and there is a return of some of the friction work as useful high grade energy.

PART 1 (b).

Losses and loss coefficients in axial flow turbo-machinery elements - a review of the methods used to express losses in blade elements and their interrelationships.



ADIABATIC EXPANSION (1-2) IN THE ROTOR BLADE OF A TURBINE STAGE.

FIG. 5.

Part 1 (b).

Losses and Loss Coefficients in Axial Flow Turbo-Machinery Elements. - A review of the methods used to express losses in blade elements and their interrelationships.

Section 1.-The axial flow turbine blade element.

Consider a steady adiabatic expansion in the rotor blade of an axial flow turbine stage from inlet state point 1 to the state point 2. (fig 5).

The loss of useful high grade energy known as the enthalpy loss is given by equation 48 as

$$\int_1^2 f v_r - \int_2^1 dP(V-V_s) = \frac{{v_{r_{2s}}}^2 - {v_{r_2}}^2}{2} = H_2 - H_{2s}$$

dividing by $\frac{{v_{r_2}}^2}{2}$ this gives

$$\frac{H_2 - H_{2s}}{\frac{{v_{r_2}}^2}{2}} = \frac{1}{\frac{{v_{r_2}}^2}{{v_{r_{2s}}}^2}} - 1 \qquad \text{\_\_\_\_\_ (49).}$$

The ratio $\frac{H_2 - H_{2s}}{{v_{r_2}}^2/2} = \mathcal{E}$ is known as the enthalpy loss coefficient. The ratio $\frac{v_{r_2}}{v_{r_{2s}}} = K$, is the blade velocity coefficient and the ratio $\frac{{v_{r_2}}^2}{{v_{r_{2s}}}^2} = \eta_{or}$ is the relative total head efficiency of expansion in the moving blade.

Equation 49 gives the relationship between these methods of expressing the loss as

$$\mathcal{E} = \frac{H_2 - H_{2s}}{{v_{r_2}}^2/2} = \frac{1}{K^2} - 1 = \frac{1}{\eta_{or}} - 1 \qquad \text{\_\_\_\_\_ (50).}$$

The irreversibility in the expansion may also be expressed as the increase in entropy during expansion i.e. as

$\varphi_2 - \varphi_{2s}$ or $\varphi_2 - \varphi_1$. For this adiabatic process the the change in entropy is entirely due to internal friction effects and is sometimes called the internal entropy change. It can be shown that, where such an irreversible increase in entropy occurs in a cycle, the loss of cycle work is given by the product of the change in entropy and the lowest temperature at which heat is rejected in the cycle.

From figure 5 assuming the relationships for a perfect gas the entropy increase is given by

$$\varphi_2 - \varphi_{2s} = c_p \log_e \frac{T_2}{T_{2s}} \qquad \text{——— (51).}$$

Equation 42 gives $H_{o_{2r}} = H_{o_{1r}}$ or $T_{o_{2r}} = T_{o_{1r}}$ and, while there is no change in relative total head temperature across the moving blade row, the drop in relative total head pressure affords another way of expressing the loss.

From 51

$$\varphi_2 - \varphi_{2s} = c_p \log_e \frac{T_2}{T_{o_{2r}}} \times \frac{T_{o_{1r}}}{T_{2s}} = c_p \log_e \left(\frac{p_{o_{1r}}}{p_{o_{2r}}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \varphi_2 - \varphi_{2s} = -c_p \log_e \left(1 - \frac{H_2 - H_{2s}}{H_2}\right)$$

$$= c_p \log_e \left(1 + \frac{p_{o_{1r}} - p_{o_{2r}}}{p_{o_{2r}}}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{——— (52).}$$

This relates the entropy increase with the enthalpy loss and with the loss in relative total head pressure. This loss in relative total head pressure furnishes a useful method of determining losses in practice and is usually expressed as a total head pressure loss coefficient defined as

$$Y = \frac{p_{o_{1r}} - p_{o_{2r}}}{\frac{1}{2} \rho_2 \; v_{r_2}^2} \qquad \text{——— (53).}$$

Substituting in 52 for the enthalpy loss and total head pressure loss from 50 and 53 respectively gives

$$\varphi_2 - \varphi_{2s} = -c_p \log_e \left(1 - \frac{\mathcal{E}\, {v_{r_2}}^2/2}{c_p\, T_2}\right)$$

$$= c_p \log_e \left(1 + \frac{Y\, \tfrac{1}{2}\rho_2\, {v_{r_2}}^2}{p_{o_{2r}}}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \text{———} \quad (54).$$

This gives

$$\varphi_2 - \varphi_{2s} = -c_p \log_e \left(1 - \mathcal{E}\, \frac{\gamma - 1}{2}\, {M_{2_r}}^2\right)$$

$$= c_p \log_e \left(1 + \frac{Y\, \frac{\gamma}{2}\, M_{2r}^2}{\left(1 + \frac{\gamma - 1}{2} M_{2r}^2\right)^{\gamma/\gamma-1}}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \text{———} \quad (55).$$

Where M_{2r} is the relative outlet Mach number. Equation 52 relates the losses while equation 55 relates the corresponding loss coefficients. Useful approximate relationships may be obtained from these equations using the series expression for log (1 + x) where x is small. Hence approximately from 52 and 55

$$\therefore \varphi_2 - \varphi_{2s} = \frac{H_2 - H_{2s}}{T_2} \doteq R\left(\frac{p_{o_{1r}} - p_{o_{2r}}}{p_{o_{2r}}}\right) = \mathcal{E}\, R\, \frac{\gamma}{2}\, M_{2r}^{\ 2}$$

$$= \frac{YR\, \frac{\gamma}{2}\, M_{2r}^{\ 2}}{\left(1 + \frac{\gamma - 1}{2}\, M_{2r}^2\right)^{\gamma/\gamma-1}} \qquad \text{———} \quad (56).$$

Hence approximately

$$Y = \mathcal{E}\left(1 + \frac{\gamma - 1}{2}\, M_{2r}^2\right)^{\frac{\gamma}{\gamma - 1}} \qquad \text{———} \quad (57).$$

and when $M_{2r} < 1$

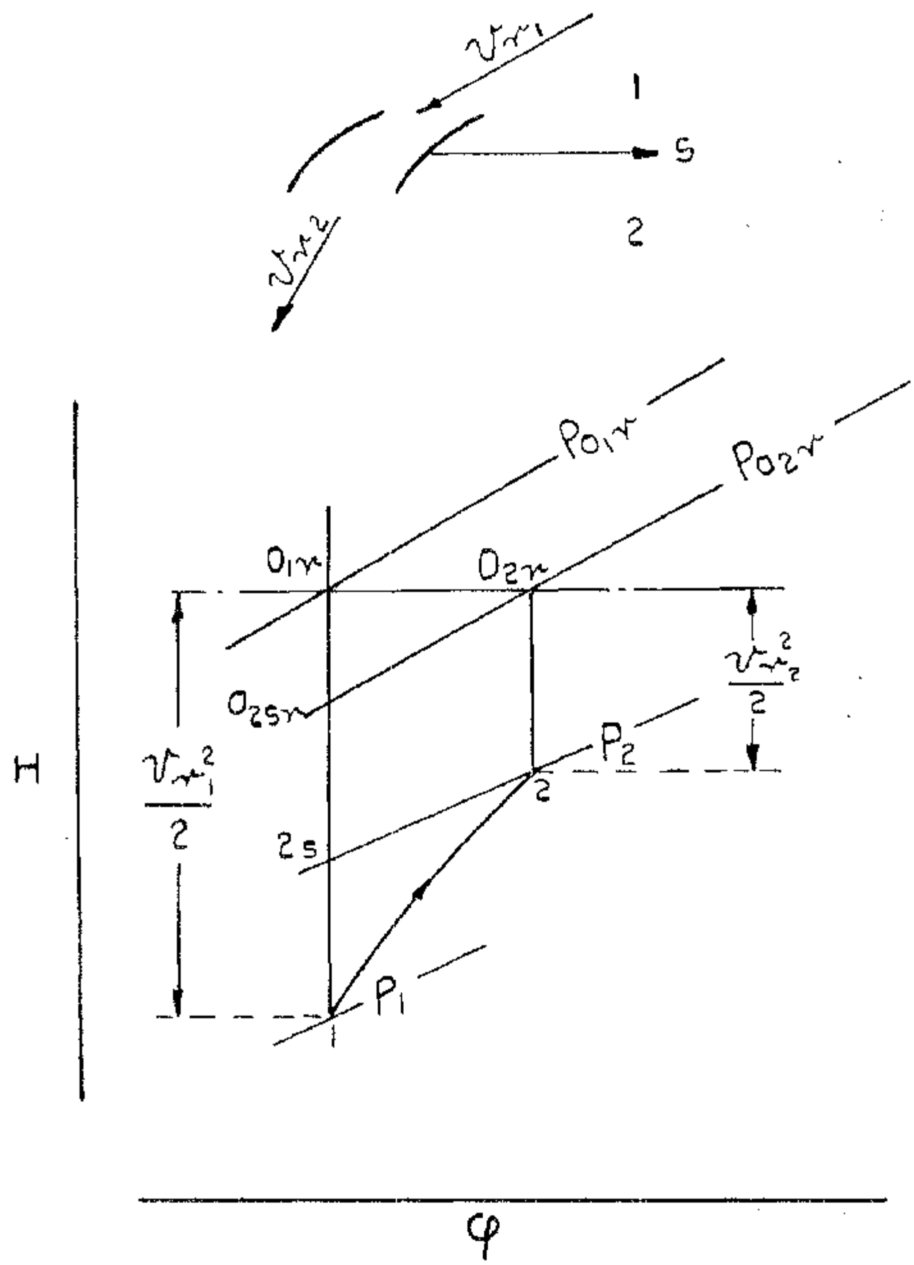
$$Y = \mathcal{E}\left(1 + \frac{\gamma}{2}\, M_{2r}^2\right) \qquad \text{———} \quad (58).$$

and for incompressible flow

$$Y = \mathcal{E} \qquad \text{———} \quad (59).$$

A further method of expressing the irreversibility in the blade is the use of a blade drag coefficient (C_D). This is given in terms of the pressure loss coefficient (Y) as

$$C_D = Y\, p/c\ \frac{\cos^3 \alpha_m}{\cos^2 \alpha_2} \qquad \text{———} \quad (60).$$



ADIABATIC COMPRESSION (1-2) IN THE ROTOR BLADE OF A COMPRESSOR STAGE.

FIG. 6.

where $p/_c$ is the pitch to chord ratio for the blade, α_m the mean direction of the fluid as it passes through the blade and α_2 the fluid efflux angle, the angles being measured from the axial direction.

The enthalpy and total head pressure loss coefficients are sometimes defined as

$$\mathcal{E}_s = \frac{H_2 - H_{2s}}{\frac{1}{2}\, v_{r_{2s}}^{\,2}} \qquad \text{———} (61).$$

$$\text{and } Y_s = \frac{p_{o_{1r}} - p_{o_{2r}}}{\frac{1}{2}\,\rho_{2s}\, v_{r_{2s}}^{\,2}} \qquad \text{———} (62).$$

If these definitions are used relationships similar to those above are obtained but in terms of the relative isentropic outlet Mach number $M_{2_{sr}}$.

<u>Section 2 - The axial flow compressor element.</u>

Consider an adiabatic steady flow compression process in the rotor blades of an axial flow compressor stage from the inlet state point 1 to state point 2 (fig 6).

In the compression 1 - 2 the loss of high grade energy compared with an isentropic compression from state point 1 to the same outlet static pressure is given by equation 47 as

$$\text{Enthalpy loss} = H_2 - H_{2s} \qquad \text{———} (63).$$

In compressions however one is often concerned with the energy absorbed in the actual adiabatic process in reaching the final outlet total head pressure. Compared with an isentropic compression from state point 1 to the outlet total head pressure the loss is given by

$$\text{"total head" enthalpy loss} = H_{o_{2r}} - H_{o_{2sr}} \qquad \text{———} (64).$$

The general expression for enthalpy loss is given in equation 47 as

$$\int f\, v_r + \int dP\,(V - V_s)$$

In equations 63 and 65 the friction work term is common (as is the entropy increase) but because the specific volume V_2 is greater than the specific volume V_{2s} the auxiliary loss term is greater in 64. Hence $H_{o_{2r}} - H_{o_{2sr}} > H_2 - H_{2s}$

In compressions either of these definitions of enthalpy loss may be used and they are related approximately by the expression

$$H_{o_{2r}} - H_{o_{2sr}} = \frac{T_{or}}{T_2} (H_2 - H_{2s}) \qquad \text{\_\_\_\_ (65).}$$

For the compression the enthalpy loss coefficient is defined

as $$\xi_c = \frac{H_{o_{2r}} - H_{o_{2sr}}}{\frac{1}{2} \; v_{r_1}^{\;2}}$$

Thus $$\xi_c = \frac{H_{o_{2r}} - H_{o_{2sr}}}{\frac{1}{2} \; v_{r_1}^{\;2}} = 1 - \frac{H_{o_{2sr}} - H_1}{H_{o_{1r}} - H_1} = 1 - \eta_{orc} \qquad \text{--- (50c}$$

where η_{orc} is the relative total head efficiency of compression for the moving blade.

The total head pressure loss coefficient for a compression is defined as

$$Y_c = \frac{p_{o_{1r}} - p_{o_{2r}}}{\frac{1}{2} \rho_1 \; v_{r_1}^{\;2}} \qquad \text{\_\_\_\_ (53c).}$$

By methods similar to that for an expansion process it may be shown that the entropy increase is related to the enthalpy and total head pressure losses by the relationship

$$\varphi_2 - \varphi_{2s} = - c_p \log_e \left(1 - \frac{H_{o_{1r}} - H_{o_{2sr}}}{H_{o_{1r}}}\right)$$

$$= - c_p \log_e \left(1 - \frac{p_{o_{1r}} - p_{o_{2r}}}{p_{o_{1r}}}\right)^{\frac{\gamma - 1}{\gamma}} \qquad \text{\_\_\_\_ (52c).}$$

and substituting in 52c for the enthalpy and pressure losses from 50c and 53c gives

Part 1(b) - Summary.

The loss of useful high grade energy in a rotating turbine blade passage is variously expressed as an increase in entropy, a blade velocity coefficient, an enthalpy loss coefficient or as a total head pressure loss coefficient. The relationships between these various methods for expressing the loss are given in equations 50 and 55. They may be applied to a static nozzle or blade row in which case the relative velocities, Mach numbers etc become absolute values. Approximate relationships between the coefficients which suffice in many applications are given in equations 56 to 59.

The method by which the loss in the blade row is expressed has historical associations with the techniques of loss measurement available at the time to the research worker. The earliest method was that used by the steam turbine designer in which he "worked back" from overall test figures to determine the efficiencies of the elements. Later the impulse plate method was used to determine the total force of the jet issuing from a nozzle or blade. Here the loss was expressed as a nozzle or blade velocity coefficient. With the advent of the gas turbine, with its associated pitot tube technique for loss determination, the losses are given as a total head pressure loss coefficient. When comparing data from these sources it should be borne in mind that the steam turbine data may be obtained from high speed tests whereas the gas turbine cascade tests are usually for Mach numbers less than 0.5. Before making a comparison therefore, Mach number effects should be accounted for by applying equation 57. In addition, results from tests using the impulse plate method give three dimensional loss in the blade. Horlock[24], who has previously summarized for the turbine nozzle or moving blade the relationships given in this section, points out that the results apply to an imaginary two dimensional flow having the same flow rate and overall losses as the actual stream. In a three dimensional flow there is a small discrepancy between the square of the velocity coefficient and the total head efficiency. This discrepancy increases with decrease in nozzle efficiency. Values for this discrepancy are given by Kearton [25].

With regard to the total head efficiency of the nozzle or blade row Kearton[26] uses a different method to express the loss. He splits the total loss into two parts. The first part is the loss of kinetic energy between the outlet of the previous row and the immediate inlet to the row under consideration. This loss is accounted for by using a carry over coefficient. The remainder of the total loss is then attributed to effects within the blade passage. Horlock points out that this division of the total loss is artificial since the boundary layer behaviour within the blade is dependant on the entry flow. In addition it has been shown here that the bulk of the blade passage loss is given by $\int f v_r$ and the average value of v_r within the passage is affected by the entry velocity.

Loss coefficients and efficiencies for axial flow compressor blading are defined in this section and their inter-relationship is given in equations 50c and 55c, the corresponding approximate forms are given in equations 56c and 59c. Most of the present data on compressor blade loss is for low speed flow (Mach number <0.5) where the loss is expressed as a total head pressure loss coefficient. However there is a tendency at present for the operating Mach numbers to greatly exceed 0.5. This in turn necessitates high speed cascade testing where it may be convenient to express the loss as an enthalpy loss coefficient.

In turbine and compressor rotor blade elements (figs 5 and 6) the high grade energy released (or absorbed) in the process is given by the enthalpy difference $H_1 - H_2$ (or H_2-H_1) and for each case the loss of high grade energy is given by

$H_2 - H_{2s}$. On the other hand the quantities $H_{o_{1r}} - H_2$ (and $H_{o_{1r}} - H_1$) which appear in the definitions of relative total head efficiency cannot be expressed in terms of high grade energy associated with the moving blade. The loss in the rotor blade however depends largely on the term $\int fv_r$ which suggest that the blade be examined in a static test rig, where the pressure and temperature at inlet are the same as those for the moving blade, but where the inlet velocity is made equal to the actual inlet relative velocity. In this case the quantities $H_{o_{1r}} - H_2$ and $H_{o_{1r}} - H_1$ are the high grade energy released or absorbed respectively in the static test rig, in expansion or compression from or to the total head pressure. Again in the compressor element (figure 6) two definitions for enthalpy loss are given. These are the enthalpy loss $H_2 - H_{2s}$ and the "total head" enthalpy loss $H_{o_{2r}} - H_{o_{2rs}}$. The relationship between them is given in equation 65. The total head loss has a meaning in terms of high grade energy only for a simulated static test on the element. These points are referred to again in the definitions of efficiency given in the next section.

PART 1 (c).

Efficiencies in axial flow turbine and compressor stages. ---- A method of expressing the efficiency of the flow process in the stage and the interrelationships between the process efficiency, the stage efficiency and the blading efficiency.

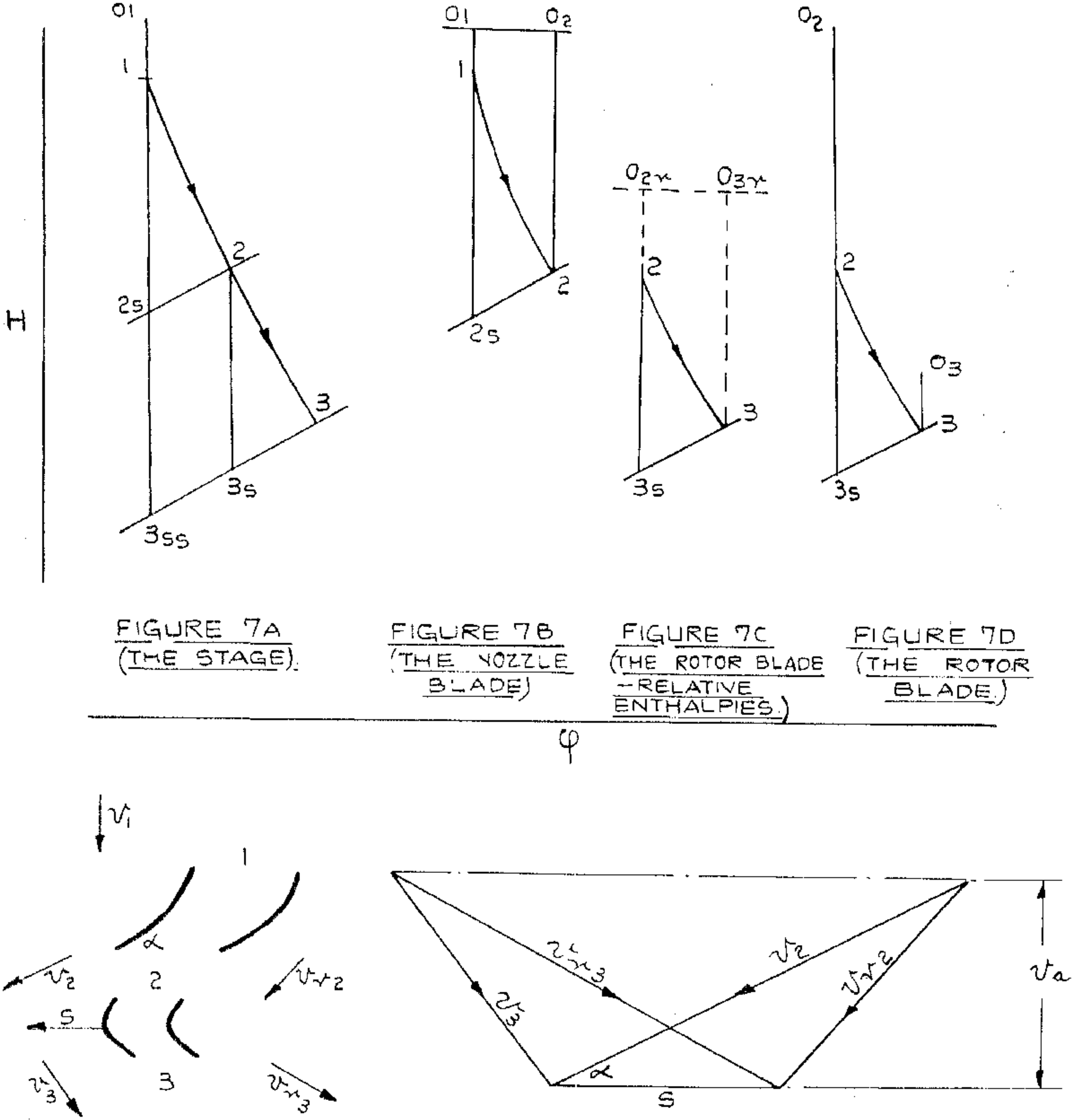


FIGURE 7A (THE STAGE).

FIGURE 7B (THE NOZZLE BLADE)

FIGURE 7C (THE ROTOR BLADE – RELATIVE ENTHALPIES.)

FIGURE 7D (THE ROTOR BLADE.)

FIGURE 7E.

THE AXIAL FLOW TURBINE STAGE.

FIG. 7.

Section 1. - Efficiencies in the Turbine Stage.

The function of the following section is to correlate the various ways in which the efficiency of a turbine stage may be defined and to relate these efficiencies to the losses in the individual elements which comprise the stage. Definitions of the efficiency of a turbine stage depend on the application for which the stage is intended. The function of most turbine stages, one possible exception being the turbine of a turbo-jet engine, is the efficient production of mechanical work energy and stage efficiency definitions reflect this fact. In all turbine stages however one is concerned with a steady flow process in the stage and in the elements which comprise the stage. At the outset here an efficiency definition for the steady flow process is given which is applicable either to the nozzle row, the moving blade row or to the stage, and which is independant of the use made of the stage.

Consider the turbine stage shown in figure 7. The working fluid enters the turbine nozzle at condition 1, total head condition 01, and expands adiabatically to the state point 3 at outlet from the moving row.

(1) The efficiency of expansion.

The efficiency of the expansion process in the nozzle (N), or in the rotor blade (R) or in the stage (s) may be defined as

$$\eta = \frac{\text{Net useful high grade energy released in adiabatic expansion from the inlet condition to the exhaust pressure.}}{\text{Useful high grade energy released in isentropic expansion from the inlet condition to the exhaust pressure.}}$$

Hence from equation 44

for the nozzle (fig 7B)

$$\eta_N = \frac{H_1 - H_2}{H_1 - H_{2s}} = \frac{1}{1 + \dfrac{H_2 - H_{2s}}{H_1 - H_2}} \qquad (66).$$

For the rotor blade (fig 7D)

$$\eta_R = \frac{H_2 - H_3}{H_2 - H_{3s}} = \frac{1}{1 + \frac{H_3 - H_{3s}}{H_2 - H_3}} \qquad \text{(67).}$$

and for the stage (fig 7A)

$$\eta_s = \frac{H_1 - H_3}{H_1 - H_{3ss}} = \frac{1}{1 + \frac{H_3 - H_{3ss}}{H_1 - H_3}} \qquad \text{(68).}$$

Expression for the stage efficiency of expansion in terms of the losses in the elements.

The increase in entropy for the stage is

$$\varphi_3 - \varphi_{3ss} = \varphi_3 - \varphi_{3s} + \varphi_2 - \varphi_{2s}$$

Hence from equation 56

$$H_3 - H_{3ss} = H_3 - H_{3s} + \frac{T_3}{T_2} (H_2 - H_{2s}) \qquad \text{(69).}$$

Hence from 68 and 69

$$\eta_s = \frac{1}{1 + \frac{\frac{T_3}{T_2} (H_2 - H_{2s}) + H_3 - H_{3s}}{H_1 - H_3}} \qquad \text{(70).}$$

and from 67, 68, and 70

$$\eta_s = \frac{1}{\frac{T_3}{T_2} \left(\frac{1}{\eta_N} - 1\right)(1 - R) + \left(\frac{1}{\eta_R} - 1\right) R} \qquad \text{(71).}$$

where R is the degree of reaction for the stage

$$\text{defined as } R = \frac{H_2 - H_3}{H_1 - H_3} \qquad \text{(72).}$$

alternatively from 70 and 50

$$\eta_s = \frac{1}{\frac{T_3}{T_2} \frac{\left(\mathcal{E}_N \frac{v_2^2}{2} \right)}{H_1 - H_3} + \frac{\mathcal{E}_R \frac{v_{r_3}^{2}}{2}}{H_1 - H_3}} \qquad \text{(73).}$$

(2) The total head efficiency of expansion.

In many instances one is more concerned with the final high grade energy available at the end of a process rather than with the energy released during the process.

The final available high grade energy is obtained by rearranging equation 44 as

$$H_{o_1} - H_2 = WD + \frac{v_2^2}{2} \qquad \text{——— (74).}$$

H_{o_1} being the enthalpy corresponding to the state point reached after isentropic diffusion of the inlet absolute velocity.

The total head efficiency of expansion for the nozzle rotor blade, or stage, may be defined as

$$\eta_o = \frac{\text{Final available high grade energy obtained in adiabatic expansion from the inlet total head state to the exhaust pressure.}}{\text{Final available high grade energy obtained in isentropic expansion from the inlet total head state to the exhaust pressure.}}$$

This gives

for the nozzle (fig 7B).

$$\eta_{oN} = \frac{H_{o_1} - H_2}{H_{o_1} - H_{2s}} = \frac{1}{1 + \dfrac{H_2 - H_{2s}}{H_{o_1} - H_2}} \qquad \text{——— (75).}$$

for the rotor blade (fig 7D).

$$\eta_{oR} = \frac{H_{o_2} - H_3}{H_{o_2} - H_{3s}} = \frac{1}{1 + \dfrac{H_3 - H_{3s}}{H_{o_2} - H_3}} \qquad \text{——— (76).}$$

and for the stage (fig 7A).

$$\eta_{os} = \frac{H_{o_1} - H_3}{H_{o_1} - H_{3ss}} = \frac{1}{1 + \dfrac{H_3 - H_{3ss}}{H_{o_1} - H_3}} \qquad \text{——— (77).}$$

Hence from 69

$$\eta_{os} = \frac{1}{1 + \dfrac{\frac{T_3}{T_2}(H_2 - H_{2s}) + H_3 - H_{3s}}{H_{o_1} - H_3}} \quad \text{——— (78).}$$

from 75 this gives

$$\eta_{os} \doteq \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{1}{\eta_{oN}} - 1\right)(1 - Ro) + \frac{H_3 - H_{3s}}{H_{o_1} - H_3}} \quad \text{——— (79).}$$

Where Ro, the "total head" degree of reaction is defined as

$$Ro = \frac{H_2 - H_3}{H_{o_1} - H_3}, \quad \text{——— (80).}$$

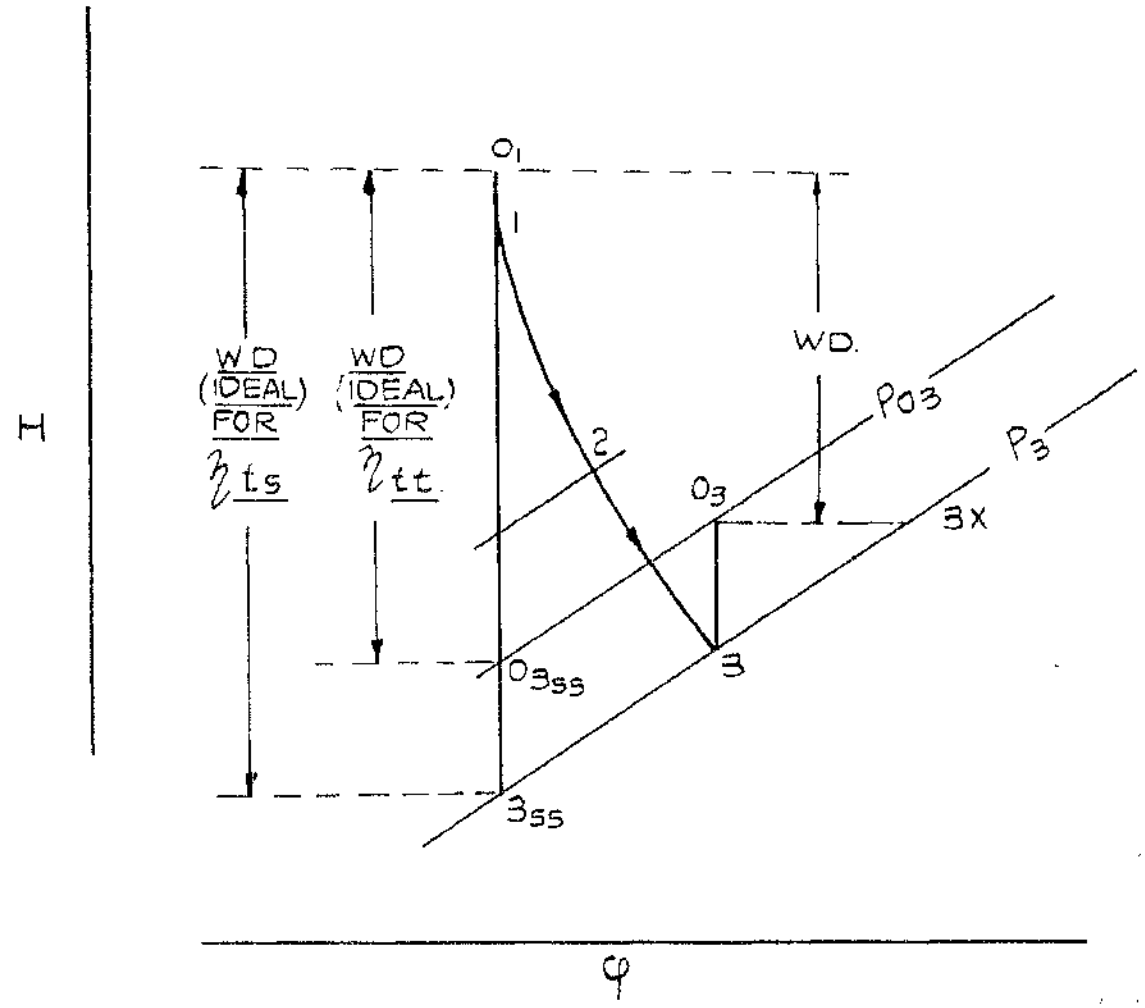
From 67

$$\eta_{os} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{1}{\eta_{oN}} - 1\right)(1 - Ro) + \left(\frac{1}{\eta_R} - 1\right) Ro} \quad \text{(81).}$$

Alternatively from 78 and 50

$$\eta_{os} = \frac{1}{\frac{T_3}{T_2}\left(\frac{\mathcal{E}_N \frac{v_2^2}{2}}{H_{o_1} - H_3}\right) + \frac{\mathcal{E}_R \frac{v_{r_3}^2}{2}}{H_{o_1} - H_3}} \quad \text{——— (82).}$$

In figure 70 the appropriate relative total head enthalpies for the expansion process in the rotor blade are shown. It will be noted that the definition of total head efficiency of expansion for the moving blade, given by equation 76, is not the same as the relative total head efficiency (η_{or}) which is related to the loss coefficients for the blade in equation 50. As has been pointed out previously the enthalpy drop $H_{o_{1r}} - H_2$ cannot be expressed in terms of the release of high grade energy and the relative total head efficiency should be regarded as the total head efficiency of expansion obtained in a simulated static test rig.



IDEAL WORK IN "TOTAL TO TOTAL" AND "TOTAL TO STATIC" STAGE EFFICIENCY DEFINITIONS.

FIG. 8.

<u>(3) Stage efficiency - the turbine as a producer of mechanical work energy.</u>

The efficiencies of expansion in the stage, which are related above to the irreversibilities in the flow through the elements of the stage, are concerned with the production of high grade energy. This high grade energy is a combination of mechanical work energy and kinetic energy and in the design of the stage one tries to ensure that as much as possible of the high grade energy appears as mechanical work.

The stage efficiency (distinct from the efficiency of expansion in the stage) is defined as

$$\eta = \frac{\text{Actual mechanical work output}}{\text{Ideal mechanical work output}}$$

The actual mechanical work output is given from equation 40 as $H_{o_1} - H_{o_3} = WD$. The ideal mechanical work output depends on the history of the exhaust kinetic energy after the working fluid leaves the moving row. This kinetic energy $\left(\frac{V_3^2}{2}\right)$ is known as the leaving kinetic energy, or carry over kinetic energy.

<u>The total to static stage efficiency.</u>

For single stage turbines which are not fitted with an exhaust diffuser, or for partial admission stages of steam turbines, where in each case the exhaust kinetic energy is degraded by friction, the final condition of the working fluid is given by the state point 3x (figure 8). The stage enthalpy loss is increased by $\frac{V_3^2}{2}$ and the ideal work output is $H_{o_1} - H_{3ss}$.

For this case the stage efficiency is known as the "total to static" efficiency (η_{ts}) and is given by

$$\eta_{ts} = \frac{WD}{WD_{(ideal)}} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_{3ss}} = \frac{1}{1 + \dfrac{H_3 - H_{3ss} + \dfrac{V_3^2}{2}}{H_{o_1} - H_{o_3}}} \qquad (83).$$

or

$$\eta_{ts} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{\mathcal{E}_N \frac{V_2^2}{2}}{H_{o_1} - H_{o_3}}\right) + \frac{\mathcal{E}_R \frac{V_{r_3}^2}{2}}{H_{o_1} - H_{o_3}} + \frac{\frac{V_3^2}{2}}{H_{o_1} - H_{o_3}}} \qquad (84).$$

The total to total stage efficiency.

For cases where the exhaust kinetic energy is not degraded, e.g. in a single stage turbine fitted with an exhaust diffuser, or in a multistage turbine where the nozzle blades are designed to absorb the leaving kinetic energy, the effective stage exhaust pressure is P_{o_3}. (fig. 8). The ideal work output is then given by an isentropic expansion from state point 01 to the total head exhaust pressure P_{o_3}. This is given approximately by $H_{o_1} - H_{3ss} - \frac{V_3^2}{2}$ and the stage efficiency which is known as the "total to total" efficiency is given by

$$\eta_{tt} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_{3ss} - \frac{V_3^2}{2}} = \frac{1}{1 + \frac{H_3 - H_{3ss}}{H_{o_1} - H_{o_3}}} \qquad (85).$$

or

$$\eta_{tt} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{\mathcal{E}_N \frac{V_2^2}{2}}{H_{o_1} - H_{o_3}}\right) + \frac{\mathcal{E}_R \frac{V_{r_3}^2}{2}}{H_{o_1} - H_{o_3}}} \qquad (86).$$

This definition of total to total stage efficiency in terms of the loss coefficients of the elements is due to Hawthorne[27].

Special application of the total to total efficiency.

For the particular case of a typical stage in a multistage turbine, which is designed for a constant carry over velocity, the total to total efficiency given by 85 reduces to

$$\eta_{tt} = \frac{H_1 - H_3}{H_1 - H_{3ss}} = \eta_s \qquad (87).$$

i.e. the total to total efficiency is then identical with the efficiency of expansion in the stage.

(4) The blading or diagram efficiency.

For a given ideal stage isentropic heat drop, $H_{o_1} - H_{3ss}$, the total head efficiency of expansion in the stage determines the high grade energy made available. It is the function of the rotor blade to convert some of this high grade energy into mechanical work energy. The blading or diagram efficiency is defined here as

$$\eta_B = \frac{\text{Mechanical work energy produced}}{\text{Total high grade energy available to the rotor blade.}}$$

i.e.

$$\eta_B = \frac{WD}{H_{o_1} - H_3} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_3} = 1 - \frac{\frac{v_3^2}{2}}{H_{o_1} - H_3} \qquad (88).$$

This definition of blading efficiency differs somewhat from that normally used for impulse stages, but is considered by the writer to have a wider application for both turbine and compressor stages. A comparison with the usual impulse definition is given in subsection 6.

(5) Relationships between the expansion efficiency, the blading efficiency and the stage efficiencies.

From 83, 87 and 77

$$\eta_{ts} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_{3ss}} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_3} \times \frac{H_{o_1} - H_3}{H_{o_1} - H_{3ss}}$$

$$\therefore \eta_{ts} = \eta_B \times \eta_{os} \qquad (89).$$

From 85 and 77

$$\eta_{tt} = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_{3ss} - \frac{v_2^2}{2}} = \frac{\eta_{os} - \frac{v_3^2/2}{H_{o_1} - H_{3ss}}}{1 - \frac{v_3^2/2}{H_{o_1} - H_{3ss}}} \qquad (90).$$

and from 88 and 77

$$(1 - \eta_B)\ \eta_{os} = \frac{v_3^2/2}{H_{o_1} - H_{3ss}} \qquad \text{\_\_\_\_ (91).}$$

Hence from 90 and 91

$$\eta_{tt} = \frac{\eta_{os} - (1 - \eta_B)\ \eta_{os}}{1 - (1 - \eta_B)\ \eta_{os}} \qquad \text{\_\_\_\_ (92).}$$

or from 89

$$\eta_{tt} = \frac{\eta_{ts}}{1 - \eta_{os} + \eta_{ts}} = \frac{\eta_{os}}{\frac{1}{\eta_B}(1 - \eta_{os}) + \eta_{os}} \qquad (93).$$

(6) Design parameters and the "impulse" stage.

Consider an axial flow turbine stage where the axial velocity is constant. (figure 7).

Design Parameters. Important design parameters for the stage are :-

The total head degree of reaction, $Ro = \dfrac{H_2 - H_3}{H_{o_1} - H_3}$

The blade speed to jet speed ratio, $\rho = \dfrac{s}{v_2}$

The work done factor or loading factor, $Z = \dfrac{WD}{s^2/2}$

The blading or diagram efficiency, $\eta_B = \dfrac{WD}{H_{o_1} - H_3}$

If values of Ro and ρ are chosen for the stage then the stage velocity triangles may be drawn to an unknown scale. Thus the initial choice of Ro and ρ determines all the velocity ratios in the stage, from which the loading factor and blading efficiency may also be obtained. The total head degree of reaction may be negative, zero or positive corresponding to the velocity ratio v_{r_3}/v_{r_2} being < 1, $= 1$ or > 1 respectively.

Enthalpy changes in the stage. If any one of the stage velocities or any one of the energy quantities involved in the design parameters (e.g. the blade speed or the specific

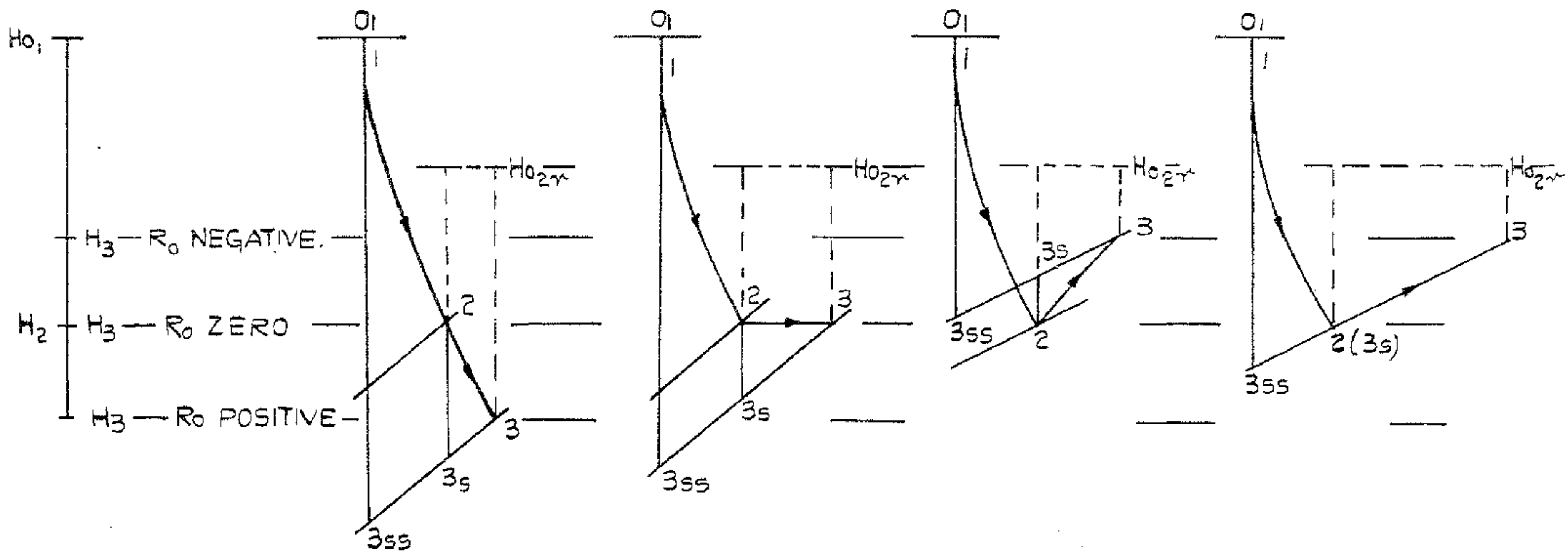


FIGURE 9A. FIGURE 9B. FIGURE 9C. FIGURE 9D. FIGURE 9E.

POSSIBLE PRESSURE CHANGES IN THE TURBINE STAGE CORRESPONDING TO POSITIVE, NEGATIVE AND ZERO DEGREES OF REACTION.

FIG. 9.

work output) is chosen, then the magnitude of all velocities and energy quantities for the stage are determined. This being so the enthalpy changes across the nozzle row and moving blade row are known. In figure 9A enthalpy levels corresponding to positive, negative and zero degrees of reaction are shown. When the degree of reaction is positive the quantity $H_2 - H_3$ is known as the reaction effect. For negative degrees of reaction the quantity $H_3 - H_2$ is called here the compression effect.

Pressure changes in the stage. The pressure changes across the nozzle and blade row depend in the first instance on the location of the process in the H / φ field. When however the initial state point is known the enthalpy changes and element losses are the controlling factors. Possible pressure changes corresponding to positive and zero degree of reaction are shown in figures 9B and 9C respectively. For negative degrees of reaction possible pressure changes are shown in figures 9D and 9E. To encompass these possibilities the total head efficiency of expansion for the stage is best retained in the form of equation 79. i.e. as

$$\eta_{os} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{1}{\eta_{oN}} - 1\right)(1 - Ro) + \frac{H_3 - H_{3s}}{H_{o_1} - H_3}} \qquad \text{——— (79).}$$

In this equation the expression

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = Ro\left(\frac{H_2 - H_{3s}}{H_2 - H_3} - 1\right) \qquad \text{——— (94).}$$

is the fractional stage loss occuring in the moving blade.

Ro positive figure 9B. For positive values of Ro equation 94 gives

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = Ro\left(\frac{1}{\eta_R} - 1\right) \qquad \text{——— (95).}$$

where η_R is the efficiency of expansion in the moving blade.

<u>Ro = 0, figure 9C.</u> For zero degree of reaction equation 94 gives

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = \frac{H_2 - H_{3s}}{H_{o_1} - H_3} \qquad \text{——— (96).}$$

where $H_2 - H_{3s}$ is the isentropic enthalpy drop in the moving blade. This enthalpy drop corresponds to the pressure drop required to maintain a constant relative velocity in the moving blade passage. The zero reaction stage may be regarded as a special case of a stage with a pressure drop in the moving blade where the efficiency of expansion in equation 95 is zero.

<u>Ro negative, figure 9D.</u> Where Ro is negative there is a pressure rise in the moving blade passage and equation 94 may be written

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = -\,Ro\left(1 - \frac{H_{3s} - H_2}{H_3 - H_2}\right) = -\,Ro\,(1 - \eta_{cR}) \qquad \text{——— (97).}$$

where η_{cR} is the efficiency of compression in the rotor blade. When the degree of reaction is negative it is better to work in terms of a degree of compression defined as

$$R_{oc} = \frac{H_3 - H_2}{H_{o_1} - H_3} \qquad \text{——— (98).}$$

in which case equation 97 reduces to

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = R_{oc}\,(1 - \eta_{cR}) \qquad \text{——— (99).}$$

If the efficiency of compression in the blade passage is zero then there is no pressure rise in the blade passage and we are dealing with the zero pressure drop impulse stage. (figure 9E). Equation 99 then gives

$$\frac{H_3 - H_{3s}}{H_{o_1} - H_3} = \frac{H_3 - H_2}{H_{o_1} - H_3} = R_{oc} \qquad \text{——— (100).}$$

The zero pressure drop impulse stage should therefore be regarded as a special case of a stage designed to have a compression effect in the blade passage, where the efficiency of compression is zero.

In such an impulse stage the blade passage loss is often expressed in terms of a blade velocity coefficient defined as $K = {v_{r_3}}/{v_{r_2}}$.

It will be noted that this definition for the zero pressure drop blade is identical with $K = {v_{r_3}}/{v_{r_{3s}}}$ which is the general definition used in equation 50. For the zero pressure drop impulse stage the usual definition of blading efficiency is

$$\eta'_B = \frac{H_{o_1} - H_{o_3}}{H_{o_1} - H_2} \qquad \text{——— (101).(Kearton[28]).}$$

$H_{o_1} - H_2$ being the kinetic energy available to the moving blade.

For the total to static efficiency in the stage this gives

$$\eta_{ts} = \eta_{oN} \times \eta'_B \qquad \text{——— (102)}$$

This relationship is similar in form to the expression given here for total to static efficiency (equation 89) which may also be applied to the zero pressure drop impulse stage. This is

$$\eta_{ts} = \eta_{os} \times \eta_B \qquad \text{———(89).}$$

Equation 101 may be shown to give

$$\eta'_B = \cfrac{1}{1 + \cfrac{\mathcal{E}_R \cfrac{v_{r_3}^{\,2}}{2} + \cfrac{v_3^{\,2}}{2}}{H_{o_1} - H_{o_3}}} \qquad \text{——— (103). (Horlock[24]).}$$

while from equation 88

$$\eta_B = \cfrac{1}{1 + \cfrac{v_3^2/2}{H_{o_1} - H_{o_3}}} \qquad \text{——— (104).}$$

It will be seen from equation 103 that the usual definition of blading efficiency for the impulse stage includes a term to account for the blade passage loss. Thus when using this definition it is assumed at the outset that it will not be possible to recover, as useful pressure rise, some of the energy associated with the compressor effect available to the rotor blade, so that all of the available compression effect becomes irreversible enthalpy loss.

Section 1 - Summary.

Definitions of efficiency of expansion (equation 68) and of total head efficiency of expansion (equation 77) for a turbine stage are given, which account for the irreversibilities of the flow process in the stage and which are independant of the application for which the turbine is intended. Since the definitions are for the flow process they may be applied to the individual elements of the stage. The efficiency of expansion for the stage is given in terms of the element efficiencies and degree of reaction in equation 71 and is related to the loss coefficients of the elements in equation 73. For the total head efficiency of expansion in the stage the corresponding relationships are given in equations 81 and 82, where equation 81 involves the total head degree of reaction.

Thus for a given total head stage isentropic heat drop ($H_{o_1} - H_{3ss}$, figure 7) the total head efficiency of expansion determines the total amount of high grade energy made available to the rotor blade. The blading or diagram efficiency, defined in equation 88, then determines the proportion of this high grade energy which appears as mechanical work energy, the remainder being retained by the working fluid as leaving kinetic energy.

Two definitions of stage efficiency (as distinct from efficiency of expansion in the stage) are given. Here one is concerned with the ratio of actual to ideal work obtained in the stage. In applications where the leaving kinetic energy is degraded the total to static stage efficiency is the criterion of performance and where the exhaust kinetic energy is not degraded the total to total stage efficiency is used. The relationship between the total to total stage efficiency and the loss coefficients in the elements is due to Hawthorne[27] and is given here in equation 86.

The total to static and total to total stage efficiencies are related to the blading efficiency and total head stage

VARIATION IN TOTAL TO TOTAL AND TOTAL TO STATIC STAGE EFFICIENCY WITH BLADING EFFICIENCY FOR DIFFERENT VALUES OF THE TOTAL HEAD EFFICIENCY OF EXPANSION IN THE STAGE.

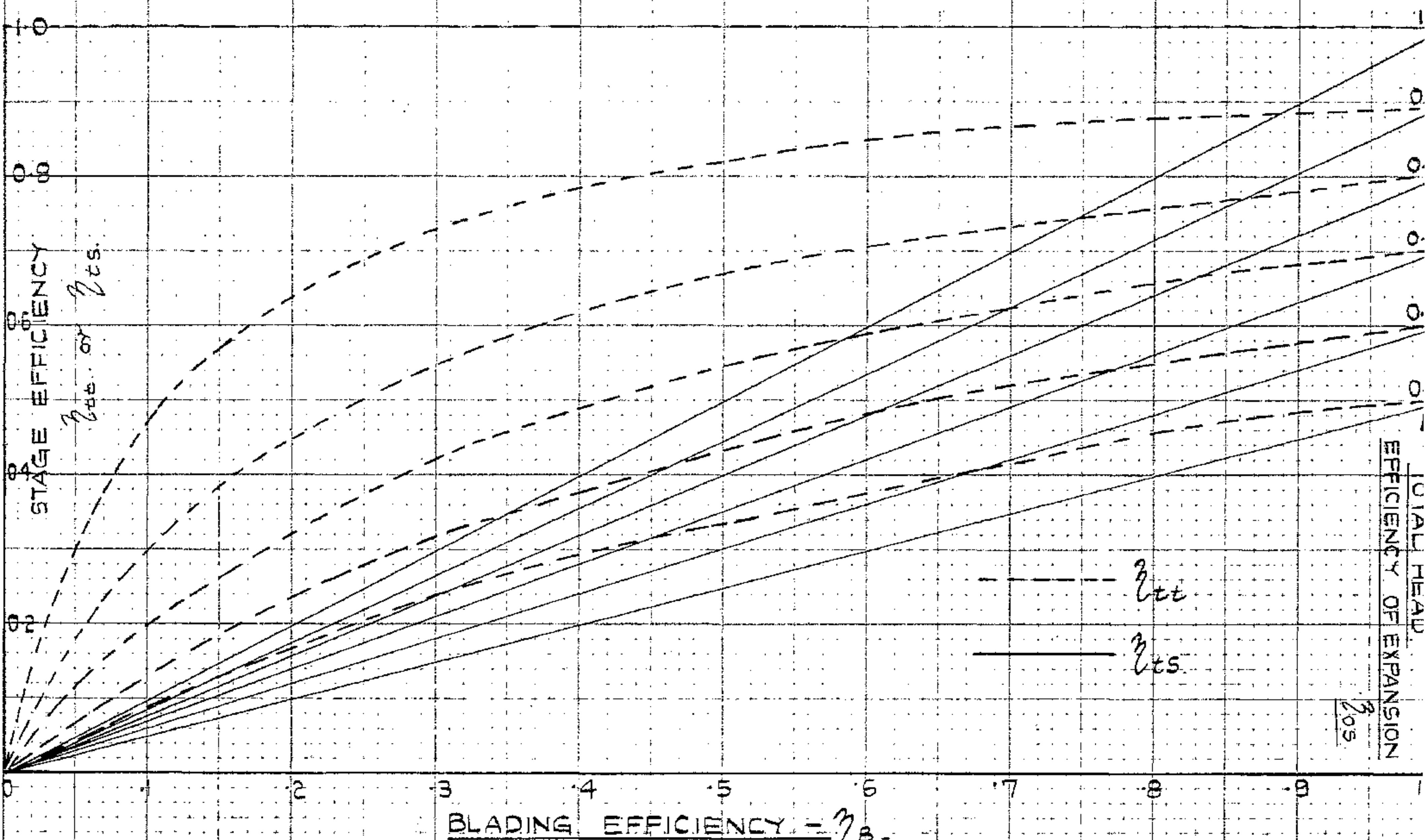


FIG. 10.

efficiency of expansion in equations 89 and 93. It will be seen that if the total head efficiency of expansion is assumed constant then as the blading efficiency increases both stage efficiencies also increase. Hence from the point of view of stage efficiency it is desirable to design the turbine stage for maximum blading efficiency.

The actual variation in the stage efficiencies with blading efficiency, for different values of the total head efficiency of expansion, is shown in figure 10. It will be noted that, whereas the total to static stage efficiency varies directly with the blading efficiency, the rate of change of total to total stage efficiency depends on the blading efficiency range. For high values of blading efficiency (above say 0.6) the variation in total to total efficiency is small. This variation does however increase as the efficiency of expansion decreases.

It has been shown that a choice of total head degree of reaction and of blade speed to jet speed ratio determine the form of the stage velocity triangles and the blading efficiency. It follows from this that as the blading efficiency varies, the deflection suffered by the gas in the blading elements will change. The effect of this variation in gas deflection on the loss coefficients in the elements, should be taken account of in assesing the variation in stage efficiencies with blading efficiency.

There are two possible definitions of an impulse stage. The "pure" impulse stage should be taken as one where the degree of reaction is zero. In practice this means that a pressure drop across the blade passage is necessary to maintain the relative velocity constant within the passage. The traditional impulse stage, where there is no pressure drop across the moving blade, should be regarded as a special case of a blade which is designed to utilize an increase in enthalpy (or compression effect) in the blade passage, but where the efficiency of the compression process is zero.

Section 2. - The Axial Flow Compressor Stage.

The axial flow compressor stage as a "reversed" turbine stage.

Consider the axial flow turbine stage shown in figure 11A. As discussed in the previous section a choice of total head degree of reaction, blade speed to jet speed ratio and specific work done determines all the velocities and enthalpy changes in the stage. If all the stage velocities are somehow maintained in magnitude but reversed in direction the stage will operate as a compressor stage in which the enthalpy increases in the rotor and stator blades will be equal in magnitude to the corresponding enthalpy drops in the turbine stage. Using the nomenclature of the turbine stage the velocities of the corresponding compressor stage are shown in figure 11B. It will be noted that the enthalpy increases in the elements of the compressor stage are determined by the choice of total head degree of reaction, blade speed to jet speed ratio and specific work output for the turbine stage. The pressure changes in the elements of the stages depend on the element losses as well as on the enthalpy changes in the elements. Thus, while we may consider that the velocities of the compressor stage are the reverse of those in the turbine stage, neither stage is reversible in the thermodynamic sense.

A complete comparison between the stages is obtained if the turbine is fitted with an exhaust diffuser to utilize the leaving kinetic energy ${V_3}^2/2$. The exhaust diffuser of the turbine stage is equivalent to the inlet guide or nozzle blades, which preceed the rotor blade, in the compressor stage.

It will be noted that (a) the velocity (V_3) entering the compressor stage rotor blade is equivalent to the velocity entering the turbine stage diffuser and (b) the velocity (V_1) leaving the compressor stage diffuser is equivalent to the velocity of approach to the turbine stage nozzles. This

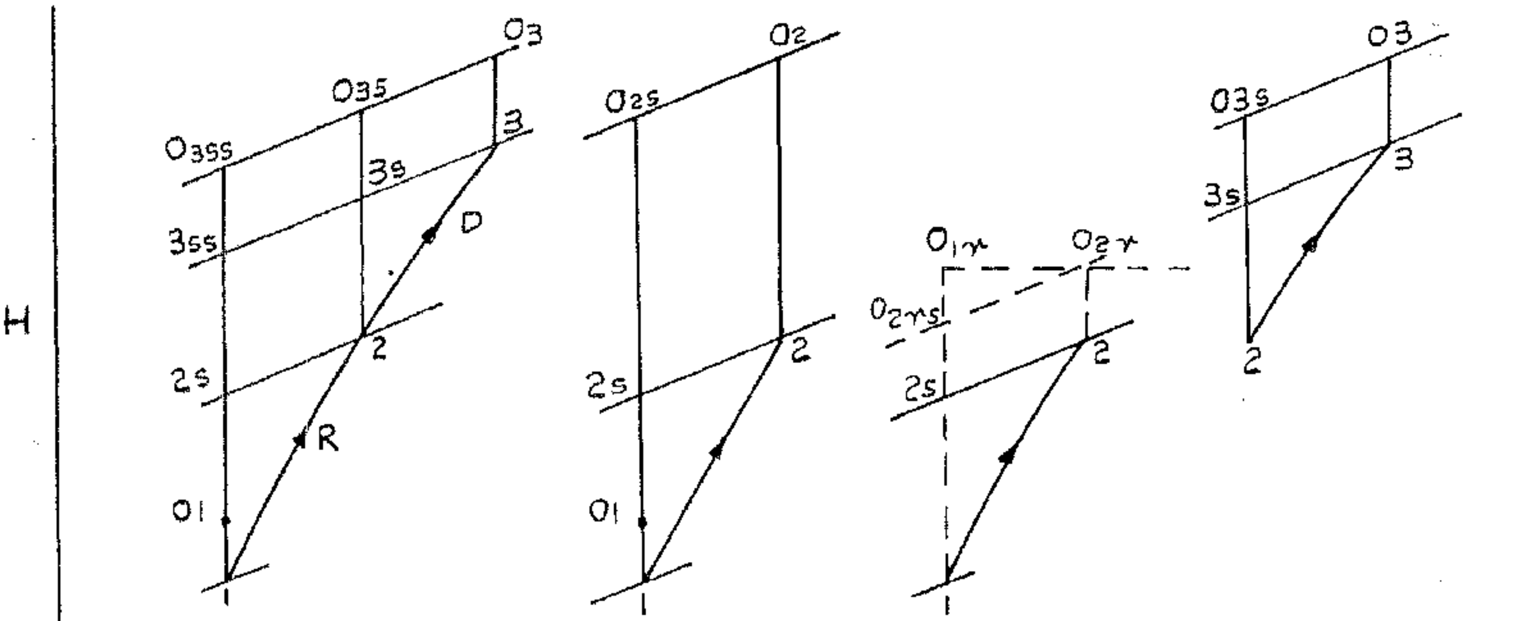


FIGURE 12A (THE STAGE.)

FIGURE 12B. (THE ROTOR BLADE)

FIGURE 12C. (THE ROTOR BLADE RELATIVE ENTHALPIES)

FIGURE 12D. (THE STATOR OR DIFFUSER BLADE.)

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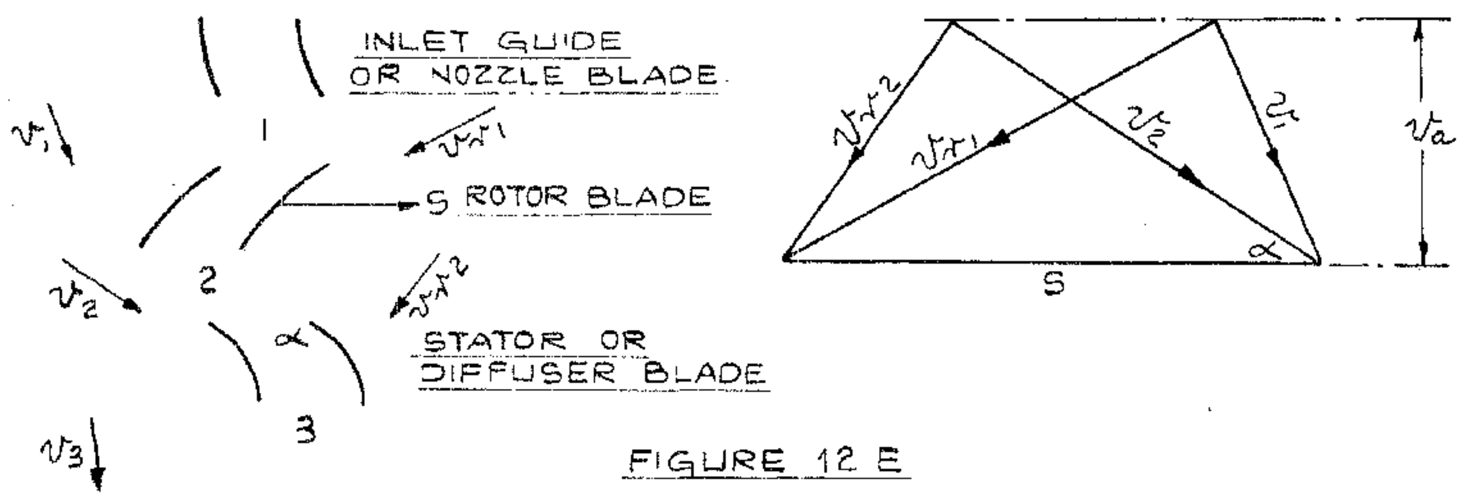


FIGURE 12 E

THE AXIAL FLOW COMPRESSOR STAGE.

FIG. 12.

velocity does not appear in the velocity triangles.

Efficiencies in the axial flow compressor stage.

Consider the axial flow compressor stage shown in figure 12. The working fluid enters the rotor blade (R) at condition 1, and leaves at condition 2. The fluid passes through the diffuser blade (D) leaving this blade at condition 3. In the rotor blade, stator blade or in the stage we are interested in the efficient utilization of available high grade energy. The high grade energy available to the rotor blade or to the stage is the kinetic energy at 1 plus the work input to the rotor blade. The high grade energy available to the diffuser blade is the kinetic energy leaving the rotor blade. In either of the elements or in the stage part of the total high grade energy initially available remains at outlet as directional kinetic energy when the compression process ceases.

(1) Efficiency of compression.

For any one element or for the stage the efficiency of the compression process may be defined as

$$\eta_c = \frac{\text{Total high grade energy absorbed during isentropic compression from the inlet condition to the exhaust pressure.}}{\text{Actual total high grade energy absorbed in adiabatic compression from the inlet condition to the exhaust pressure.}}$$

Hence from 43

for the diffuser blade (figure 12D)

$$\eta_{cD} = \frac{H_{3s} - H_2}{H_3 - H_2} = 1 - \frac{H_3 - H_{3s}}{H_3 - H_2} \qquad \text{——— (66c).}$$

for the rotor blade (figure 12B)

$$\eta_{cR} = \frac{H_{2s} - H_1}{H_2 - H_1} = 1 - \frac{H_2 - H_{2s}}{H_2 - H_1} \qquad \text{——— (67c).}$$

and for the stage (figure 12A)

$$\eta_{cs} = \frac{H_{3ss} - H_1}{H_3 - H_1} = 1 - \frac{H_3 - H_{3ss}}{H_3 - H_1} \qquad \text{——— (68c).}$$

2. Total head efficiency of compression.

If at outlet of either of the elements or of the stage the remaining kinetic energy is absorbed in a reversible manner the pressure attained is the maximum actual pressure possible from the process. The actual exhaust total head pressure is thus the maximum pressure which can be reached in the actual process when all of the available high grade energy at inlet is absorbed. In the total head efficiency of compression a comparison is made between the actual available high grade energy at inlet and the ideal high grade energy, which should be available at inlet, to attain the same total head pressure.

Thus the total head efficiency of compression for either element or for the stage is defined as

$$\eta_{oc} = \frac{\text{Total high grade energy required at inlet to attain in isentropic compression the total head exhaust pressure of the actual process.}}{\text{Actual total high grade energy required at inlet to attain in adiabatic compression the actual exhaust total head pressure.}}$$

Thus for the diffuser blade (figure 12D)

$$\eta_{ocD} = \frac{H_{o_{3s}} - H_2}{H_{o_3} - H_2} = 1 - \frac{H_{o_3} - H_{o_{3s}}}{H_{o_3} - H_2} \qquad \text{——— (75c).}$$

for the rotor blade (figure 12B)

$$\eta_{ocR} = \frac{H_{o_{2s}} - H_1}{H_{o_2} - H_1} = 1 - \frac{H_{o_2} - H_{o_{2s}}}{H_{o_2} - H_1} \qquad \text{——— (76c).}$$

and for the stage (figure 12A)

$$\eta_{ocs} = \frac{H_{o_{3ss}} - H_1}{H_{o_3} - H_1} = 1 - \frac{H_{o_3} - H_{o_{3ss}}}{H_{o_3} - H_1} \qquad \text{——— (77c).}$$

Relationships between the element efficiencies of compression and the stage efficiencies of compression.

Using equations 65 and 56c it may be shown that approximately

$$H_3 - H_{3ss} = H_3 - H_{3s} + \frac{T_3}{T_2}(H_2 - H_{2s})$$

and

$$H_{o_3} - H_{o_{3ss}} = H_{o_3} - H_{o_{3s}} + \frac{T_{o_3}}{T_2}(H_2 - H_{2s}) \qquad \text{(69c).}$$

Hence for the efficiency of compression in the stage 68c and 69c give

$$\eta_{cs} = 1 - \frac{H_3 - H_{3s} + \frac{T_3}{T_2}(H_2 - H_{2s})}{H_3 - H_1} \qquad \text{(70c).}$$

This may be expressed in terms of the element efficiencies given in equations 66c and 67c as

$$\eta_{cs} = 1 - ((1 - \eta_{cD})(1 - Rc) + \frac{T_3}{T_2}(1 - \eta_{cR})\,Rc) \qquad \text{(71c).}$$

where Rc is the degree of compression for the stage defined as

$$Rc = \frac{H_2 - H_1}{H_3 - H_1} \qquad \text{(72c).}$$

alternatively η_{cs} may be expressed in terms of the total head enthalpy loss coefficients (equation 50c). It is shown in equation 65 that approximately

$$H_2 - H_{2s} = \frac{T_2}{T_{o_{2r}}}(H_{o_{2r}} - H_{o_{2sr}}) = \frac{T_2}{T_{o_{2r}}}\,\mathcal{E}_{cR}\,\frac{{v_{r_1}}^2}{2}$$

and

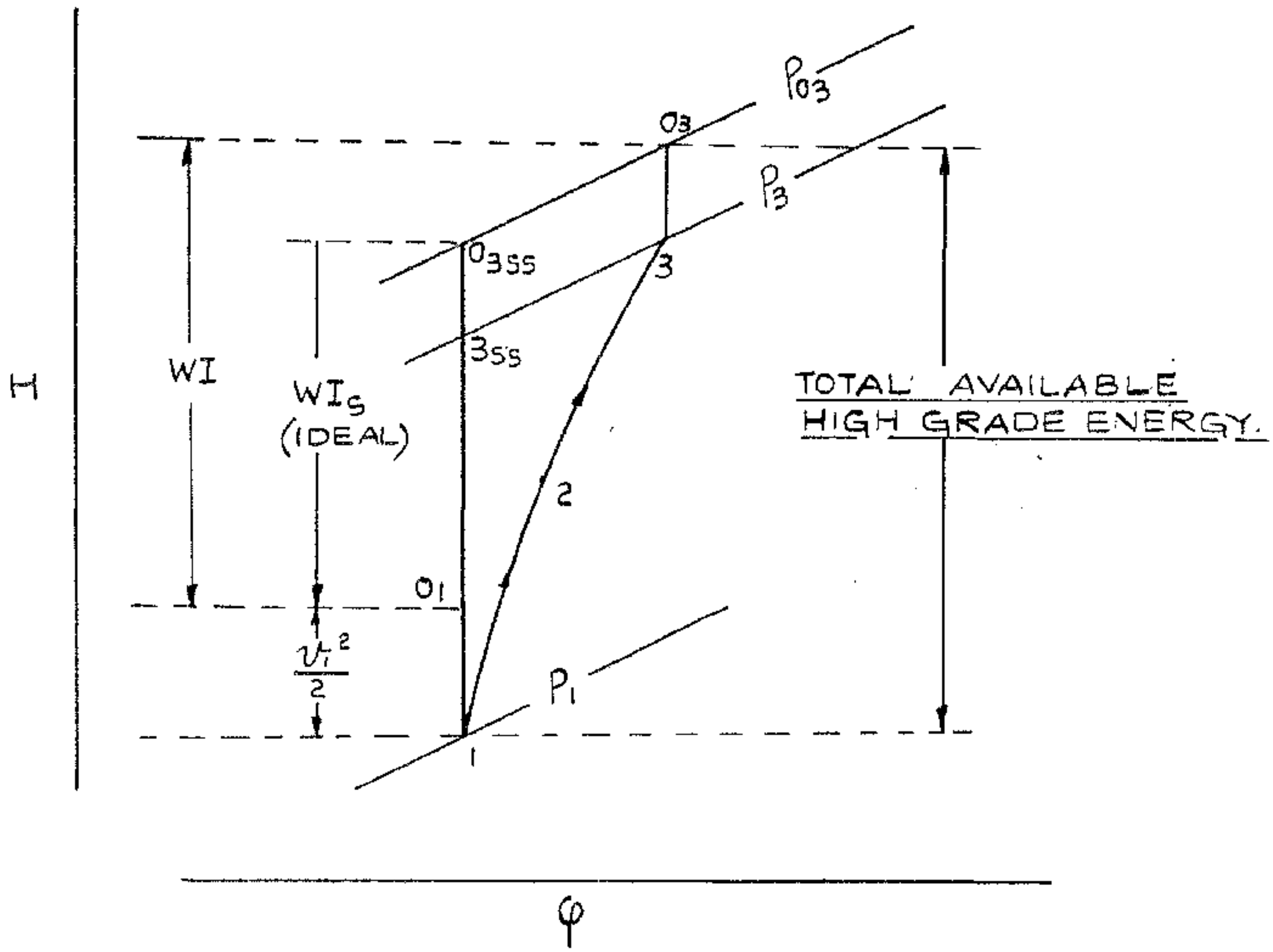
$$H_3 - H_{3s} = \frac{T_3}{T_{o_3}}(H_{o_3} - H_{o_{3s}}) = \frac{T_3}{T_{o_3}}\,\mathcal{E}_{cD}\,\frac{{v_2}^2}{2}$$

Hence from 70c

$$\eta_{cs} = 1 - \left[\frac{T_3}{T_{o_3}}\left(\frac{\mathcal{E}_{cD}\;v_2^2/2}{H_3 - H_1}\right) + \frac{T_3}{T_2}\left(\frac{T_2}{T_{o_2}}\left(\frac{\mathcal{E}_{cR}\;v_{r_1}^2/2}{H_3 - H_1}\right)\right)\right] \qquad \text{(73c).}$$

By similar methods the corresponding relationships may be obtained for the total head efficiency of compression. These are

$$\eta_{ocs} = 1 - \frac{H_{o_3} - H_{o_{3s}} + \frac{T_{o_3}}{T_2}(H_2 - H_{2s})}{H_{o_3} - H_1} \qquad \text{(78c).}$$



IDEAL WORK INPUT IN "TOTAL TO TOTAL"
STAGE EFFICIENCY DEFINITION FOR A COMPRESSOR.

FIG. 13.

or

$$\eta_{ocs} = 1 - \left[(1 - \eta_{ocD}) (1 - Roc) + \frac{T_{o_3}}{T_2} (1 - \eta_{cR}) Roc \right] \quad \text{(81c)}$$

where the total head degree of compression is defined as

$$Roc = \frac{H_2 - H_1}{H_{o_3} - H_1} \quad \text{(80c)}$$

and

$$\eta_{ocs} = 1 - \left[\left(\frac{\mathcal{E}_{oD} \frac{v_2^2}{2}}{H_{o_3} - H_1} \right) + \frac{T_{o_3}}{T_2} \left(\frac{T_2}{T_{o_{2r}}} \left(\frac{\mathcal{E}_{oR} \frac{v_{r_1}^2}{2}}{H_{o_3} - H_1} \right) \right) \right] \quad \text{(82c)}$$

<u>3. Blading efficiency, total to total and total to static efficiency in the compressor stage.</u>

<u>Blading efficiency.</u> In the design of a turbine stage it is desirable to arrange for a high value of the blading efficiency. The velocity triangles for the stage and the blading efficiency are fixed by a choice of total head degree of reaction, blade speed to jet speed ratio, and specific work done, the blading efficiency being a function of degree of reaction and blade speed to jet speed ratio only. Thus for a given specific work output a choice of blading efficiency for the stage in effect means choosing a value for the stage leaving velocity. If this same design is used as a compressor stage, by maintaining the magnitude of the velocities but reversing their directions, the work input in the compressor stage will be equal in magnitude to the work output in the turbine stage, and the inlet absolute velocity (v_1 figure 13) to the compressor stage rotor blade will be determined by the initial choice of blading efficiency.

The blading or diagram efficiency for the compressor stage is defined as

$$\eta_B = \frac{WI}{WI + \frac{v_1^2}{2}} = \frac{H_{o_3} - H_{o_1}}{H_{o_3} - H_1} \quad \text{(88c).}$$

Thus the blading efficiency for the compression stage gives the proportion of the total high grade energy, available at inlet to the stage, which is mechanical work energy.

The total to total efficiency for the compressor stage.

While the efficiencies of the compression process in the stage have involved the total high grade energy available to the stage, it is with the efficient use of mechanical work energy that we are mainly concerned.

The total to total efficiency for the compressor stage is defined as

$$\eta_{ctt} = \frac{\text{Minimum work input required to attain the actual outlet total head pressure of the stage.}}{\text{Actual work input required to reach the same total head pressure.}}$$

Thus $\eta_{ctt} = \frac{WIs}{WI}$ (see figure 13).

$$\text{or} \quad \eta_{ctt} = \frac{H_{o_{3ss}} - H_{o_1}}{H_{o_3} - H_{o_1}} = 1 - \frac{H_{o_3} - H_{o_{3ss}}}{H_{o_3} - H_{o_1}} \qquad \text{\_\_\_\_\_}(85c).$$

and the efficiency may be expressed in terms of the loss coefficients in the elements as

$$\eta_{ctt} = 1 - \left(\frac{\mathcal{E}_{cD}\frac{V_2^2}{2}}{H_{o_3} - H_{o_1}} + \frac{T_{o_3}}{T_2}\left(\frac{T_2}{T_{o_{2r}}} \quad \frac{\mathcal{E}_{cR}\frac{V_{r_1}^2}{2}}{H_{o_3} - H_{o_1}}\right)\right) \qquad \text{\_\_\_\_}(86c).$$

If the total to total stage efficiency is applied to a typical stage in a multistage compressor where the "carry over" velocity is constant then

$$\eta_{ctt} = \frac{H_{o_{3ss}} - H_{o_1}}{H_{o_3} - H_{o_1}} = \frac{H_{3ss} - H_1}{H_3 - H_1} = \eta_{cs} \qquad \text{\_\_\_\_\_}(87c).$$

i.e. the total to total stage efficiency is the same as the efficiency of compression in the stage.

The total to static efficiency in the compressor stage.

In the case of a single stage turbine one has a choice of fitting or not fitting an exhaust diffuser. For the single stage compressor however the inlet nozzle or guide vanes are an essential part of the stage. The roll of these nozzle vanes for a typical stage in a multistage compressor is assumed by the preceeding stage. Hence there is no parallel in the compressor stage for the total to static efficiency of the turbine stage.

<u>Relationship between blading efficiency, total to total stage efficiency and total head efficiency of compression in the compressor stage.</u>

From 85c and 77c

$$\eta_{ott} = \frac{1 - \dfrac{v_1{}^2/2}{H_{o_{3ss}} - H_1}}{\dfrac{1}{\eta_{ocs}} - \dfrac{v_1{}^2/2}{H_{o_{3ss}} - H_1}} \qquad \text{\_\_\_\_(90c).}$$

and from 88c and 77c

$$\therefore \quad \frac{1 - \eta_B}{\eta_{ocs}} = \frac{v_1{}^2/2}{H_{o_{3ss}} - H_1} \qquad \text{\_\_\_\_(91c).}$$

This gives

$$\eta_{ctt} = 1 - \frac{(1 - \eta_{ocs})}{\eta_B} \qquad \text{\_\_\_\_(93c).}$$

Hence from the stage efficiency view-point it is desirable to work with high values of blading efficiency, which means in effect operating the stage with a low approach velocity at inlet to the rotor blade.

Section 2 - Summary.

An analogy between the axial flow turbine and compressor stage is given, which is a useful tool with which to study the effect in the compressor stage of the stage design parameters. The compressor stage is considered as a similar turbine stage where the velocity vectors of the turbine stage are reversed in direction. If the two design parameters, the total head degree of reaction and the blade speed to jet speed ratio are chosen for the turbine stage then the form of the velocity triangles, the blading efficiency and the loading factor are determined for both the turbine stage and its compressor counterpart. This approach facilitates the interpretation of the role of the compressor stage elements. For example, it is evident that the inlet guide or nozzle blades of the compressor stage are equivalent to the exhaust diffuser of the turbine stage. The analogy would be complete for a turbine and similar compressor stage where there were no losses in the elements, so that each stage was also reversible in the thermodynamic sense. Then the pressure drops associated with the turbine stage would be equal in magnitude to the corresponding pressure increases in the compressor stage.

In dealing with efficiencies in the compressor stage, the proceedure adopted is the same as that used in the turbine stage. Definitions are given of the efficiencies of the compression process in the elements and in the stage, and the interrelationships between the stage and element efficiencies are developed. The total head efficiency of compression in the stage is given in equation 97c and is related to the element losses in equations 81c and 82c.

In the total head efficiency of compression we are concerned with the high grade energy available at inlet to the stage to effect a pressure rise. This high grade energy is the sum of the work input to the stage and the kinetic energy generated in the inlet nozzle blades, which preceed the stage. The proportion of kinetic energy to work energy in the available high grade energy depends on the initial choice for the blading efficiency of the stage. Since however we are particularly concerned with the work energy absorbed, a definition of total to total stage efficiency is given which is the ratio of the minimum to the actual work required to attain the maximum possible stage pressure. (equation 85c). A relationship between the total to total stage efficiency, the blading efficiency and the total head efficiency of compression in the stage is given in equation 93c. It will be seen that, for a constant total head efficiency of compression, the stage efficiency increases as the blading efficiency increases.

With regard to the total to total stage efficiency it will be noted that the definition given here is similar to one which is sometimes called the "total head efficiency of the stage". This latter efficiency is however usually applied from the state point at inlet to the stationary nozzle blades and would therefore take account of losses in these blades.

In a turbine stage the total to static efficiency is the criterion of performance where there is no exhaust diffuser. In a compressor stage however, kinetic energy must be generated at inlet to the rotor blade and hence the equivalent in the compressor stage of the turbine exhaust diffuser must always be present. There is thus no parallel in the compressor stage for the total to static efficiency of the turbine stage.

PART 1 (d).

Conditions for maximum blading efficiency in axial flow turbines and compressors - factors affecting the choice of design parameters for the stage.

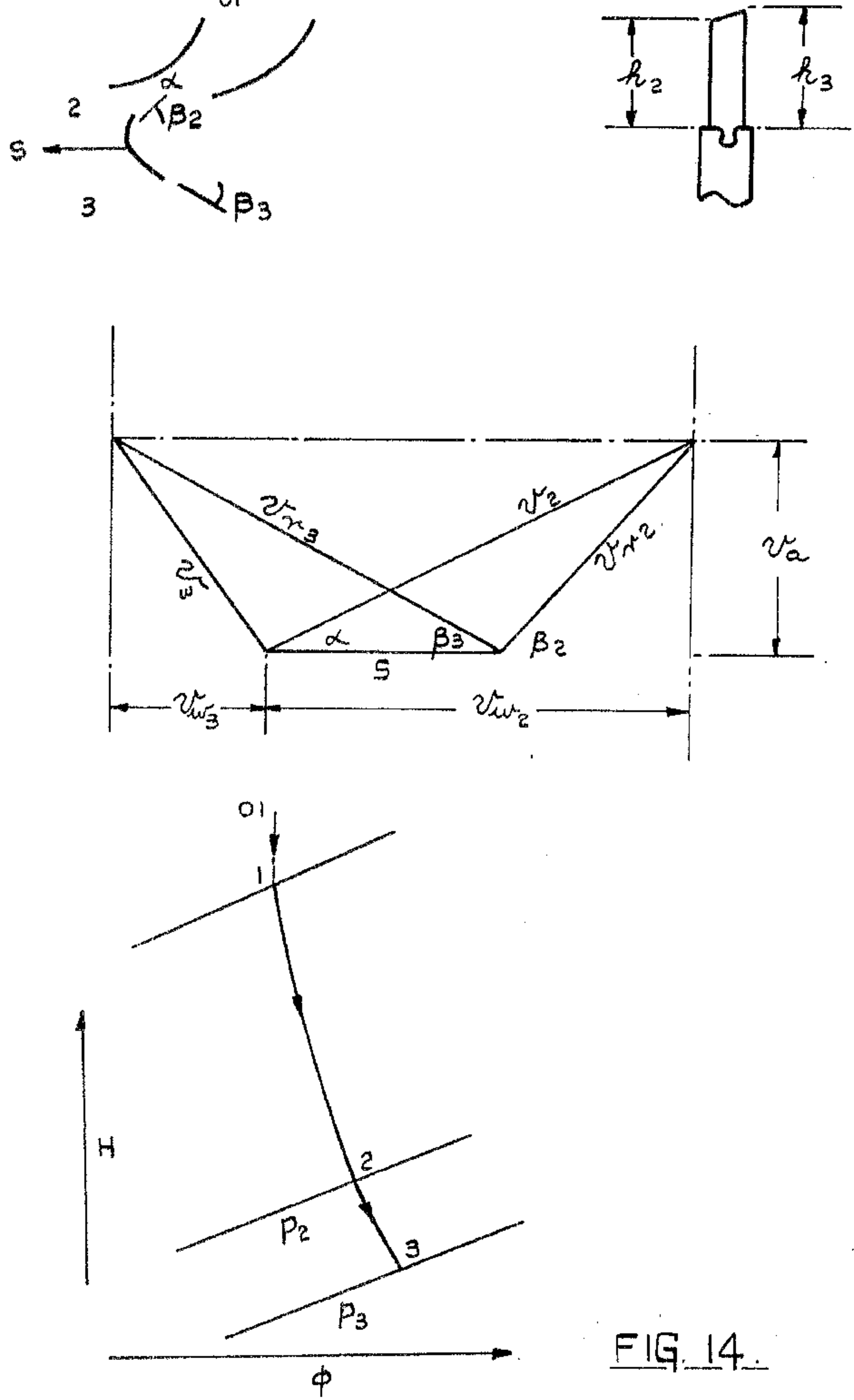


FIG. 14.

Part 1 (d).

Conditions for maximum blading efficiency in axial flow turbines and compressors - factors affecting the choice of design parameters for the stage.

(1). The blading efficiency. Consider the axial flow turbine stage shown in figure 14. We shall presume that the stage axial velocity is constant, which means that the blade heights at inlet and outlet are adjusted to nullify the effect of the change in specific volume across the moving blade. The reaction effect in the moving blade is

$H_2 - H_3 = \dfrac{v_{r_3}^2 - v_{r_2}^2}{2}$ and we shall define a reaction coefficient (G) given by

$$G = \frac{H_2 - H_3}{H_{o_1} - H_2} = \frac{\dfrac{v_{r_3}^2 - v_{r_2}^2}{2}}{\dfrac{v_2^2}{2}} \qquad \text{\_\_\_\_(105).}$$

The work done is given by

$$WD = s\,(v_{r_3} \cos \beta_3 + v_{W_2} - s) \qquad \text{\_\_\_\_(106).}$$

From 105

$$v_a^2 + (v_{r_3} \cos \beta_3)^2 - (v_a^2 + (v_{W_2} - s)^2) = G\, v_2^2$$

$$\therefore v_3 \cos \beta_3 = \sqrt{\frac{G\, v_{W_2}^2}{\cos^2 \alpha} + (v_{W_2} - s)^2} \qquad \text{\_\_\_\_(107).}$$

Hence from 106

$$WD = s\left(\sqrt{\frac{G\, v_{W_2}^2}{\cos^2 \alpha} + (v_{W_2} - s)^2} + v_{W_2} - s\right) \qquad \text{\_\_\_\_(108).}$$

using the blade speed to tangential jet speed ratio $\Gamma = s/v_{W_2}$ this gives

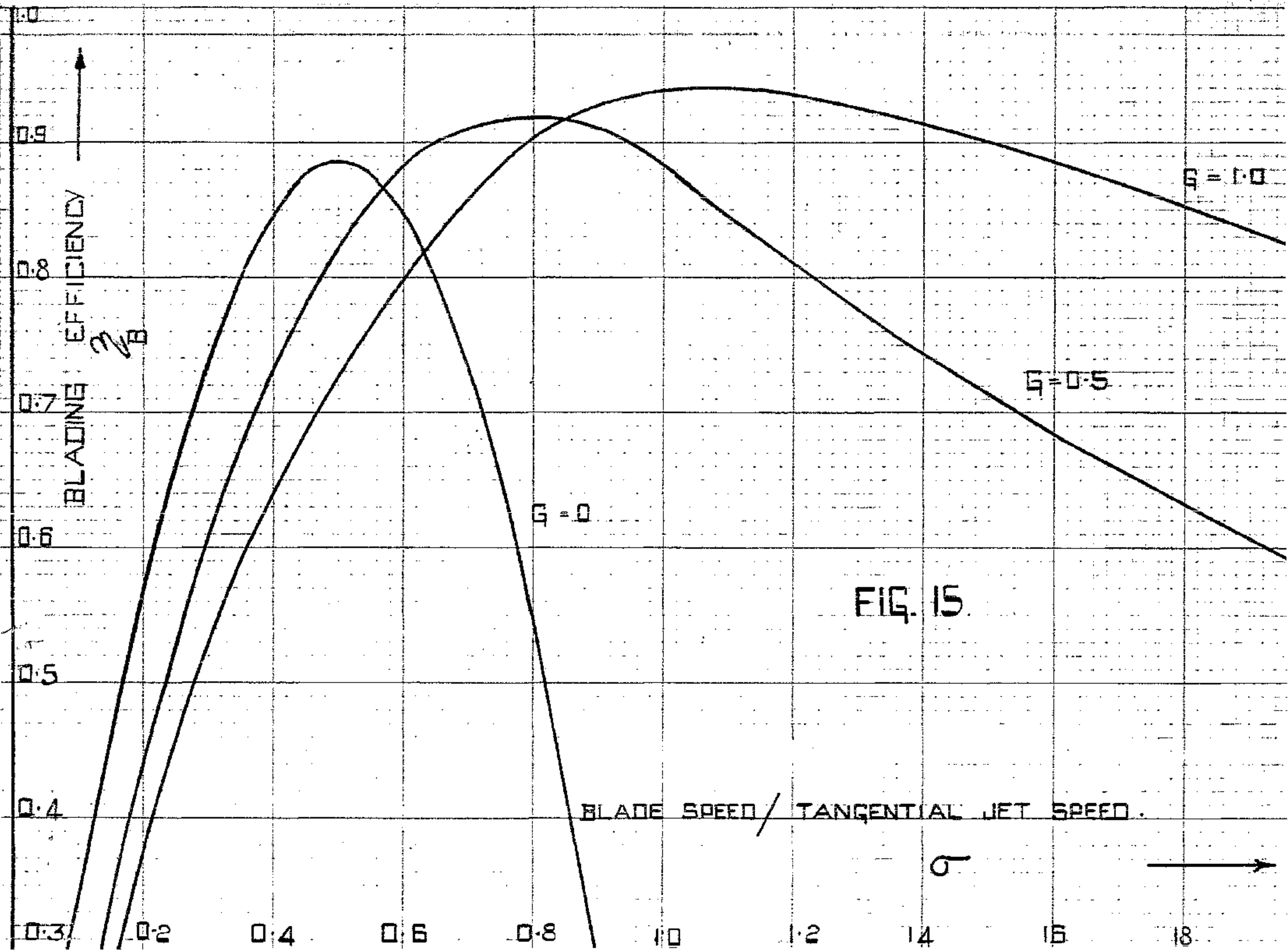
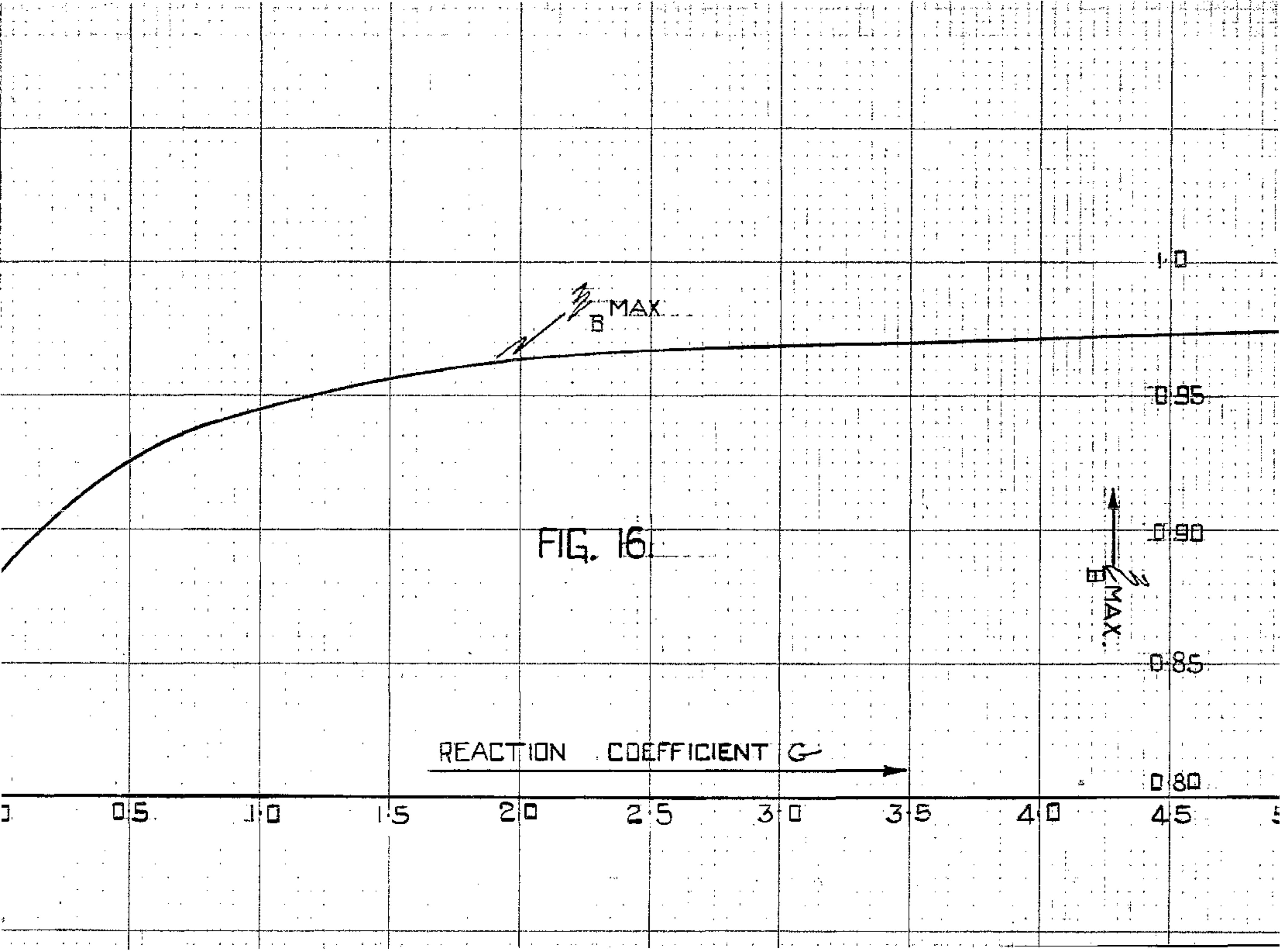

1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
BLADING EFFICIENCY η_B
G = 1·0
G = 0·5
G = 0
BLADE SPEED / TANGENTIAL JET SPEED.
σ
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8

FIG. 15.



η_B MAX
1.0
0.95
0.90
0.85
0.80
η_B MAX.
REACTION COEFFICIENT σ
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5

FIG. 16.

$$WD = \sigma v_{W_2}^2 \left(\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma \right) \qquad \_\_\_(109).$$

$$= \frac{s^2}{\sigma} \left(\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma \right)$$

The high grade energy available to the moving blade is

$$H_{o_1} - H_3 = \frac{v_2^2}{2}(1 + G) = \frac{v_{W_2}^2}{2\cos^2\alpha}(1 + G) \qquad \_\_\_(110).$$

Hence from 109 and 110 the blading or diagram efficiency is given by

$$\eta_B = \frac{2\sigma\cos^2\alpha}{1 + G} \left(\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma \right) \qquad \_\_(111).$$

Thus the blading efficiency depends on σ, α and G and we shall henceforth treat the nozzle angle α as a constant, (taken as 20^o throughout). Hence for a given value of G, η_B is a function of σ only and we can find the speed ratio for maximum blading efficiency ($\eta_B max$) by differentiating η_B with respect to σ and equating to zero. Hence for maximum blading efficiency for a given reaction coefficient

$$\sigma \ (\text{for max } \eta_B) = \frac{1 + \frac{G}{\cos^2\alpha}}{2} \qquad \_\_\_(112).$$

The maximum value of the blading efficiency is obtained by substituting 112 for σ in 111, this gives

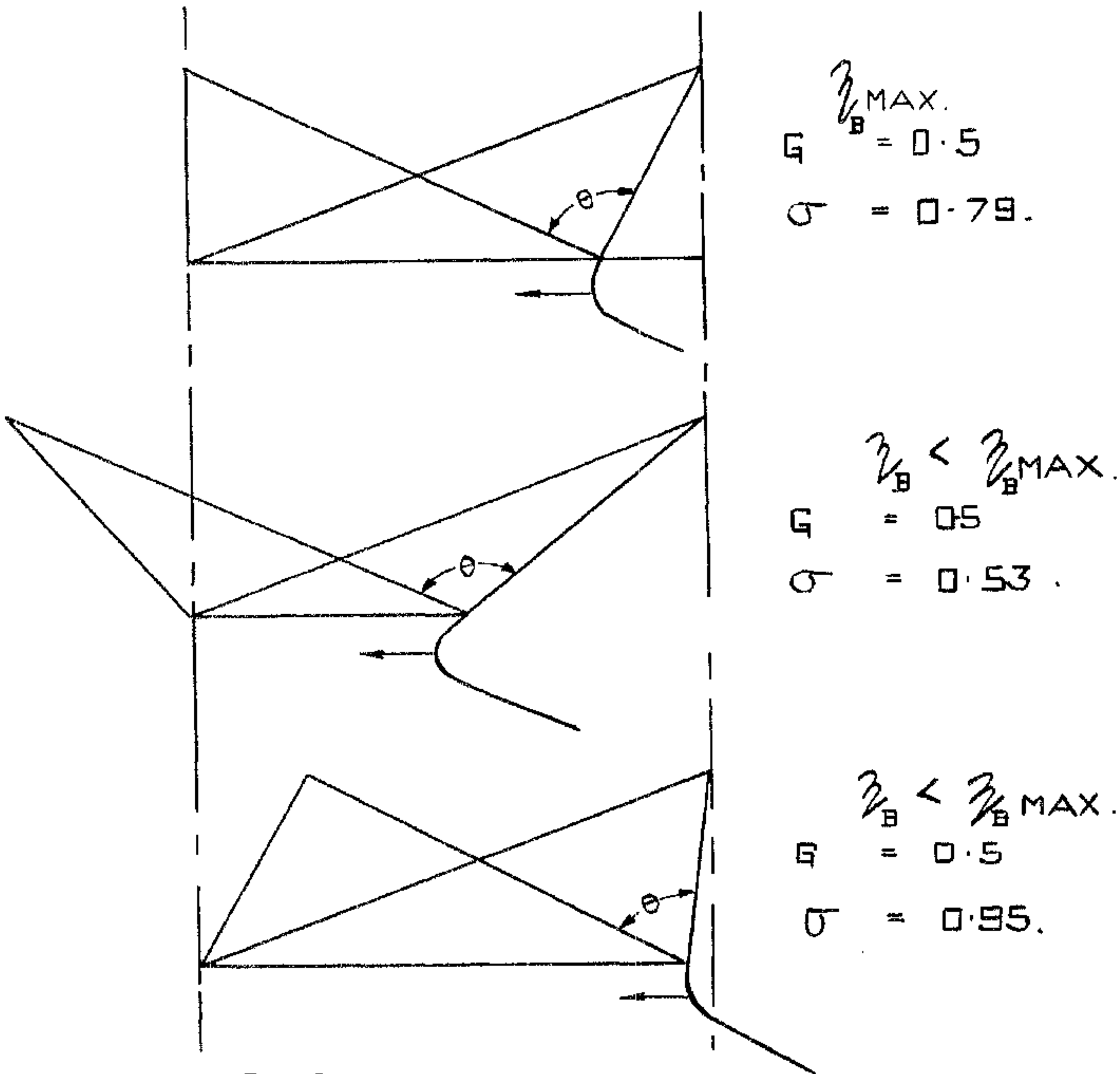
$$\eta_{Bmax} = \frac{G + \cos^2\alpha}{1 + G} \qquad \_\_\_(113).$$

The variation of η_B and σ for various values of G is shown in figure 15. The maximum value of the blading efficiency increases with increasing values of the reaction coefficient and the variation is shown in figure 16.

<u>(2). Form of the velocity triangles when the stage is designed for maximum blading efficiency.</u>

From equation 107,

$$v_{r_3}\cos\beta_3 = v_{W_2}\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} \qquad \_\_\_(114).$$



θ = GAS DEFLECTION.

FIG. 17.

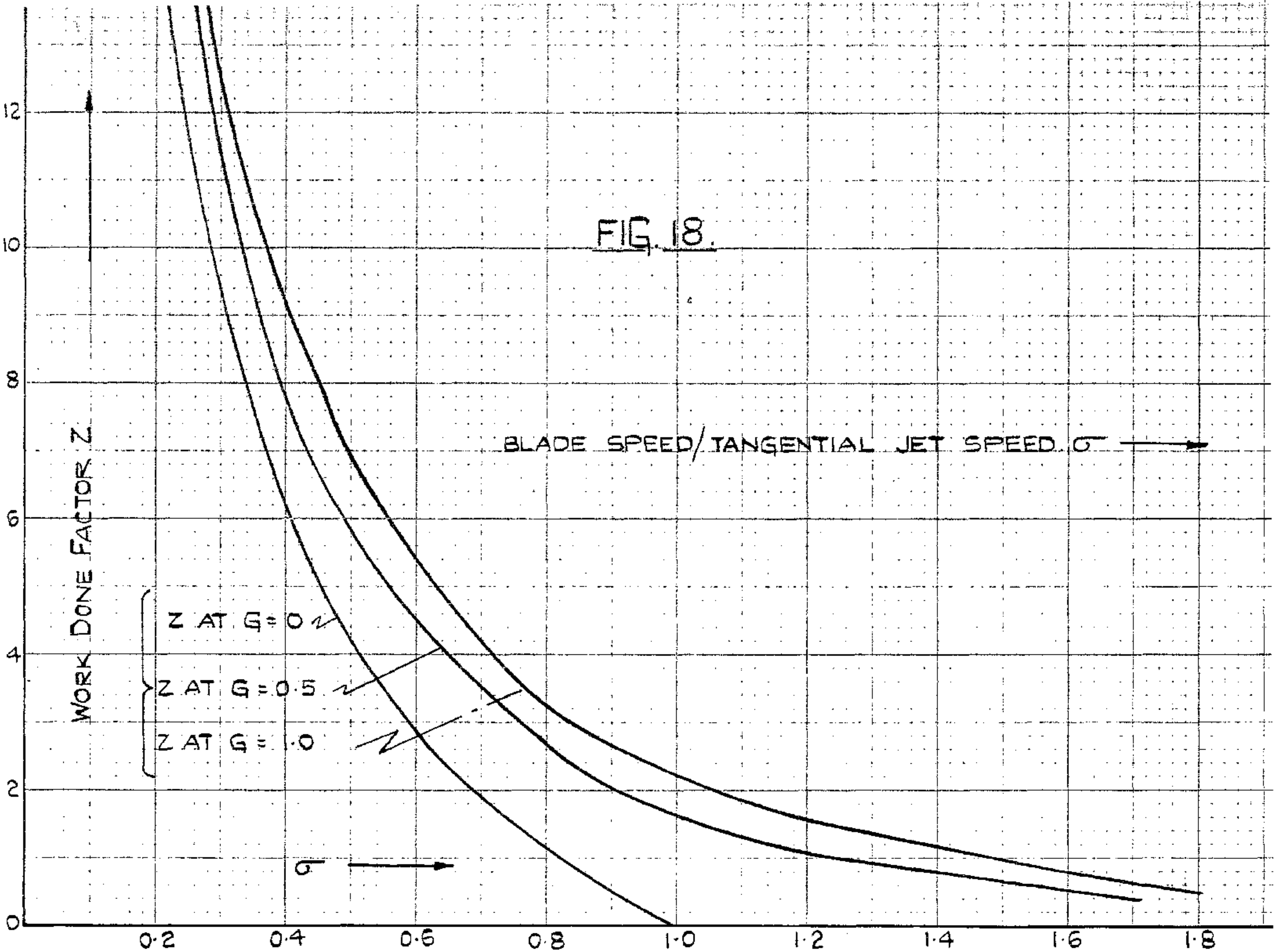

WORK DONE FACTOR Z
BLADE SPEED/TANGENTIAL JET SPEED σ
Z AT G = 0
Z AT G = 0·5
Z AT G = 1·0
σ
0
2
4
6
8
10
12
0·2
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8

FIG. 18.

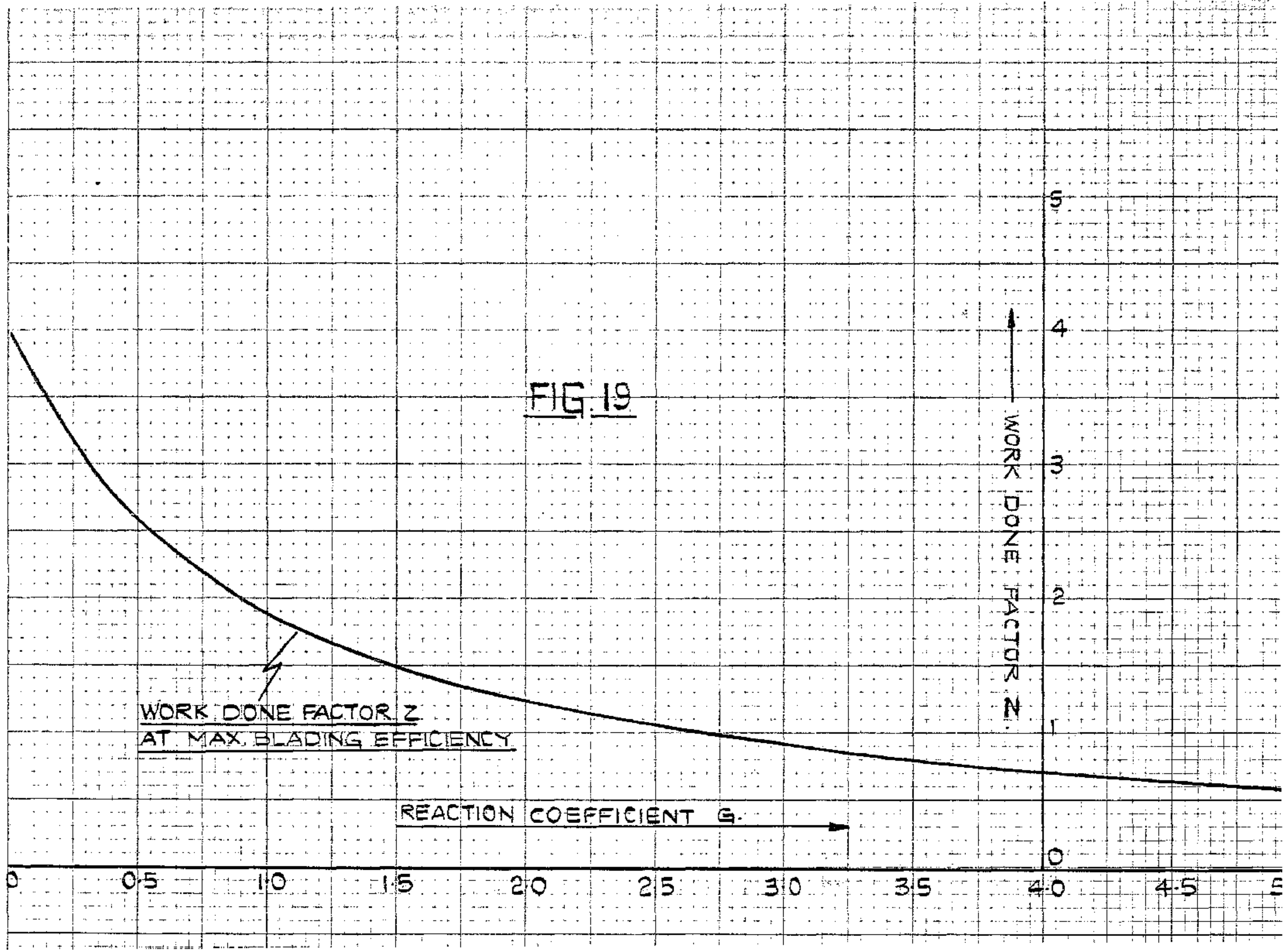


FIG. 19
WORK DONE FACTOR Z
WORK DONE FACTOR Z
AT MAX. BLADING EFFICIENCY
REACTION COEFFICIENT G.
0
1
2
3
4
5
0
0·5
1·0
1·5
2·0
2·5
3·0
3·5
4·0
4·5
5

and at maximum blading efficiency σ is given in 112 as a function of G. Substituting in 114 for σ gives

$$v_{r_3} \cos\beta_3 = v_{W_2} \sqrt{\frac{G}{\cos^2\alpha} + \left(\frac{1 - \frac{G}{\cos^2\alpha}}{2}\right)^2} = s. \qquad (115).$$

Thus at maximum η_B the outlet whirl velocity v_{W_3} is zero and only the inlet whirl component v_{W_2} is removed. The form of the velocity triangles when the stage is designed to operate above and below $\sigma = \frac{1 + \frac{G}{\cos^2\alpha}}{2}$ is shown in figure 17 for a given reaction coefficient.

<u>(3). The work done factor or loading factor for the stage.</u>

The work done or loading factor for the stage is defined as $Z = WD \Big/ \frac{s^2}{2}$

from 109 this gives

$$Z = \frac{2}{\sigma}\left(\sqrt{\frac{G}{\cos^2\alpha} + (1 - \sigma)^2} + 1 - \sigma\right) \qquad (116).$$

i.e. Z is a function of G and σ. The variation in Z with σ is shown in figure 18 for different reaction coefficients. When the stage is designed for maximum blading efficiency the value of the work done factor is obtained by substituting 112 for σ in 116, this gives,

$$Z_{\text{at max } \eta_B} = \frac{2}{\sigma}\left(\frac{1 + \frac{G}{\cos^2\alpha}}{2} + 1 - \sigma\right) = \frac{2}{\sigma(\text{for max } \eta_B)}$$

$$= \frac{2}{\left(\frac{1 + \frac{G}{\cos^2\alpha}}{2}\right)} \qquad (117).$$

The work done factor at maximum blading efficiency is a function of G only and is twice the reciprocal of the blade speed to tangential jet speed ratio for maximum blading efficiency. The relationship of equation 117 is given in graphical form in figure 19.

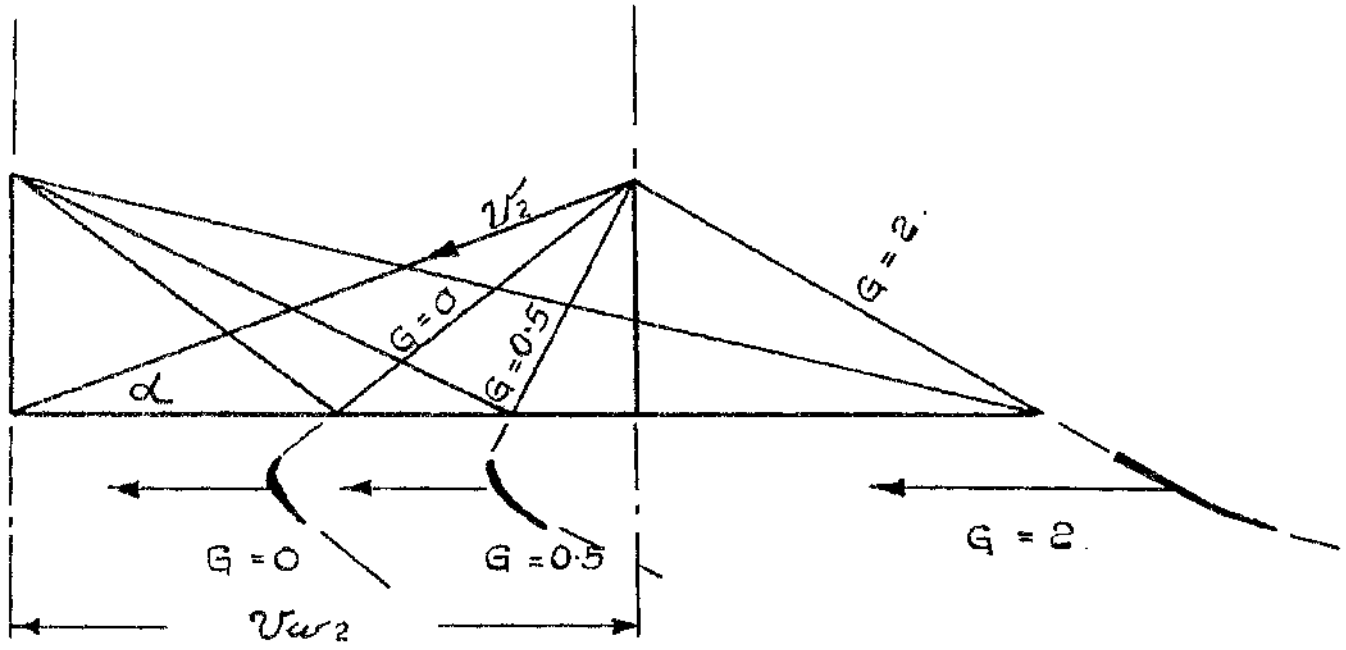


FIG. 20 .

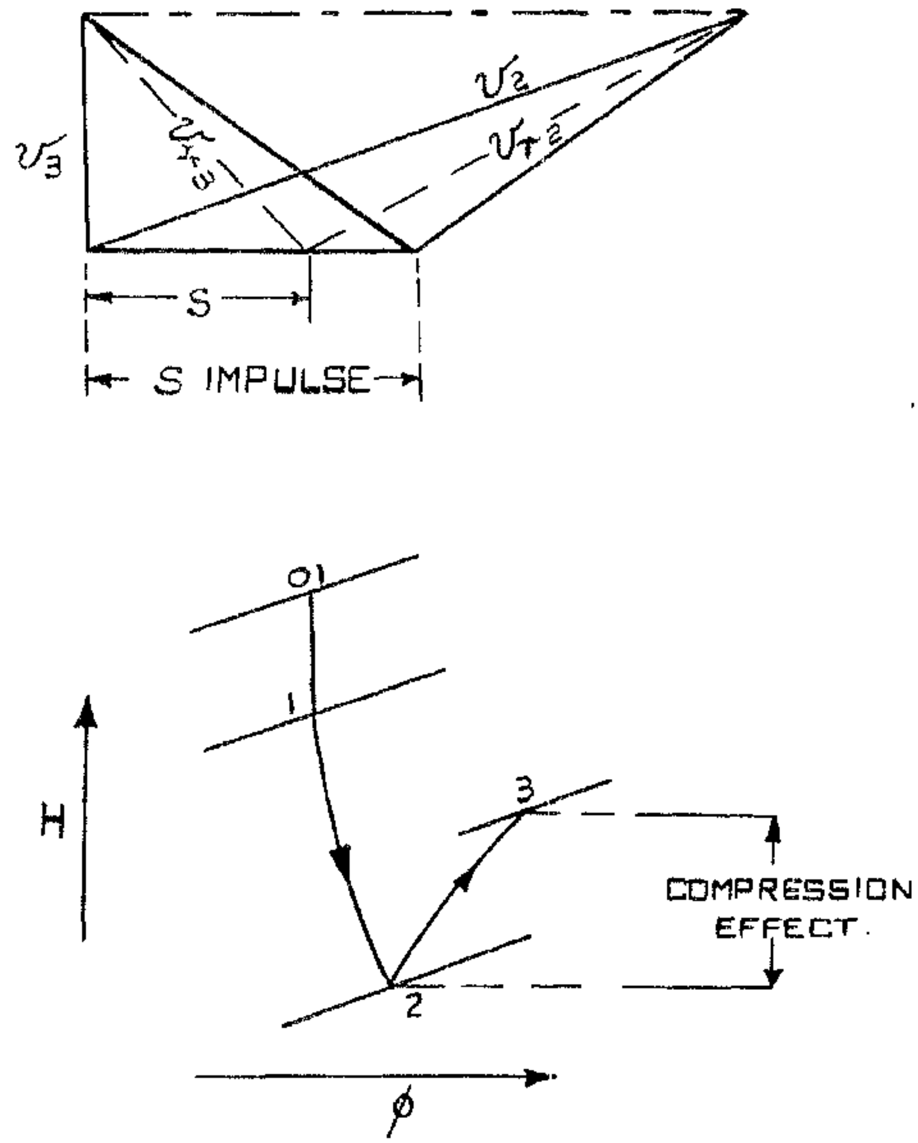


FIG. 21.

Thus when the stage is designed for maximum blading efficiency for a given reaction coefficient, the work done factor decreases with increasing reaction coefficient, and hence when efficiently utilizing a given blade speed the specific work output decreases as the reaction coefficient rises. An impulse blade therefore gives a large specific work output with a low blading efficiency, and a reaction blade gives a small specific work output with a higher blading efficiency, when each is operating at the same blade speed.

At maximum blading efficiency the work output given in equation 109 reduces to

$$WD = s\, v_{W_2} = v_{W_2}^{\;2} \left(\frac{1 + \frac{G}{\cos^2\alpha}}{2} \right) \qquad \text{\_\_\_\_(118).}$$

If the whirl velocity at inlet to themoving blade is maintained constant, then as the reaction coefficient increases we get more output work per pound but this also means a progressively increasing blade speed. (see figure 20).

<u>(4). Compression in the rotor blade passages of a turbine stage.</u>

The effect of various amounts of compression in the rotor blade passage can be studied from figure 21, where the blade speed is reduced below that required for zero degree of reaction. For negative degrees of reaction, where $v_{r_3} < v_{r_2}$, a compression coefficient (G_c) may be defined as

$$G_c = \frac{v_{r_2}^{\;2} - v_{r_3}^{\;2}}{v_2^2/2} \qquad \text{\_\_\_\_(119).}$$

As before it may be shown that the blading efficiency is given by

$$\eta_B = \frac{2\sigma \cos^2\alpha}{1 - G_c} \left(\sqrt{(1-\sigma)^2 - \frac{G_c}{\cos^2\alpha}} + 1 - \sigma \right) \text{—(120).}$$

FIGS. 22 & 23.

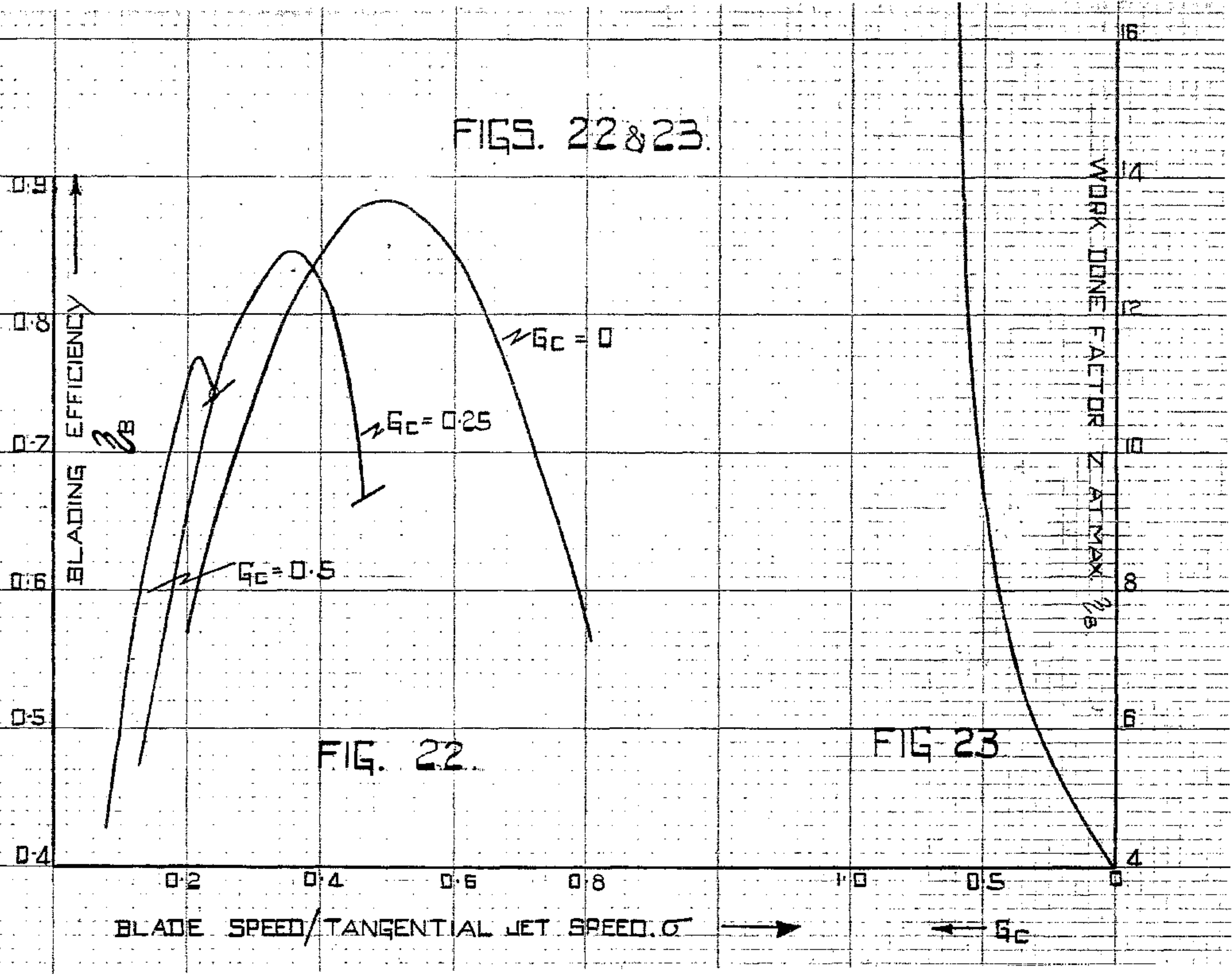


FIG. 22.

FIG 23

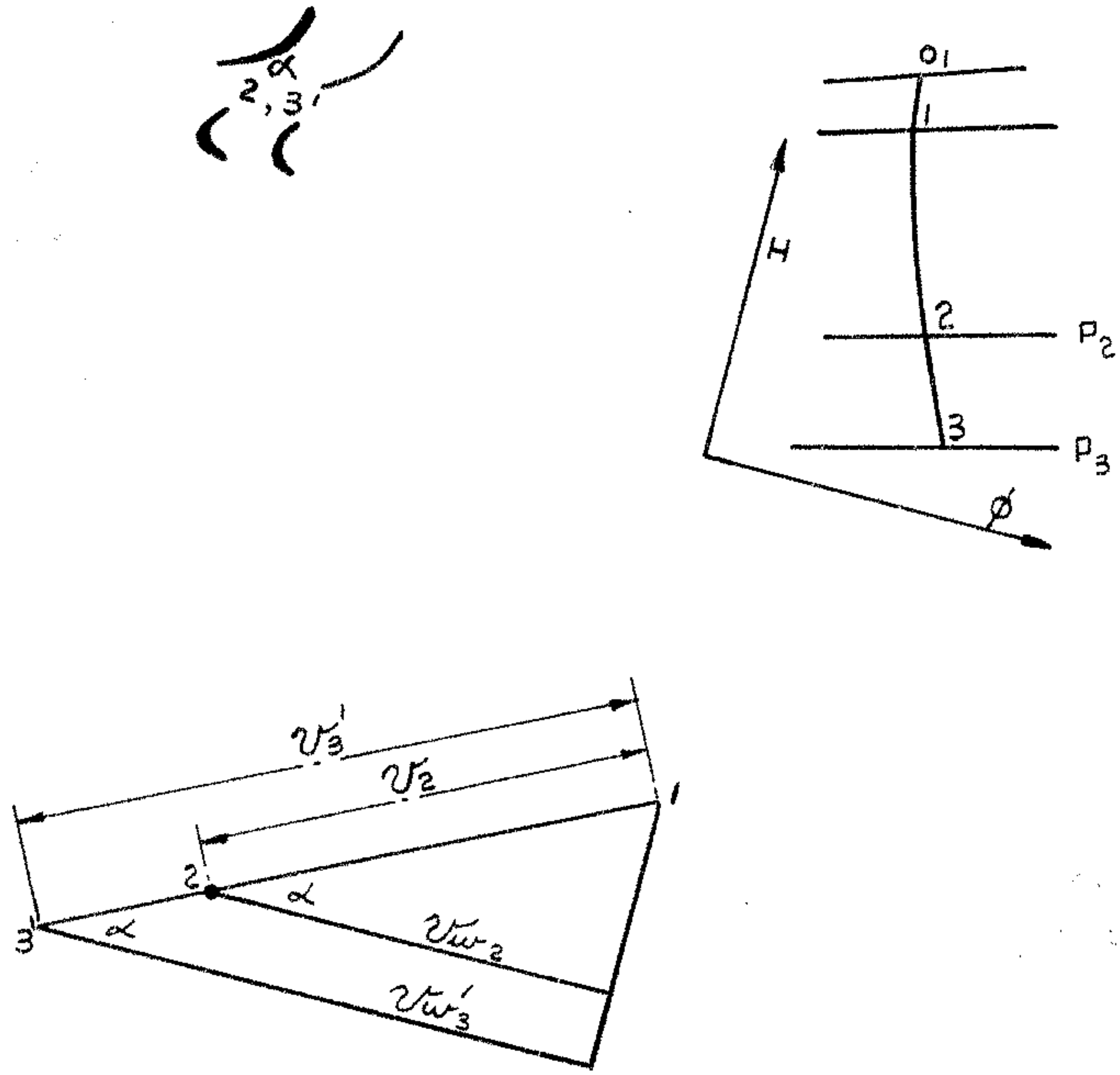


FIG. 24.

The condition for maximum blading efficiency is

$$\sigma = \frac{\frac{G_c}{1 - \cos^2\alpha}}{2} \qquad \text{\_\_\_\_\_(121).}$$

and the work done factor at maximum blading efficiency is given by

$$Z = \frac{\mu \cos^2\alpha}{\cos^2\alpha - G_c} \qquad \text{\_\_\_\_\_(122).}$$

The variation of η_B with σ for various values of G_c is shown in figure 22 and it will be observed that the maximum blading efficiency reduces substantially as G_c increases. In addition, as G_c is increased, the permissable variation in σ, about σ (for max η_B), to avoid a serious fall in blading efficiency is considerably reduced. The work done factor at maximum blading efficiency is shown in figure 23 and increases as the compression coefficient increases.

<u>(5). Relationship between the reaction coefficient (G), the total head degree of reaction (Ro) and the degree of reaction (R).</u>

Consider the stage enthalpy changes shown in figure 24. The total head degree of reaction is defined as,

$$R_o = \frac{H_2 - H_3}{H_{o_1} - H_3} \qquad \text{\_\_\_\_\_(80).}$$

and the reaction coefficient as

$$G = \frac{H_2 - H_3}{H_{o_1} - H_2} \qquad \text{\_\_\_\_\_(105).}$$

Thus the reaction coefficient and total head degree of reaction are related by the expression

$$G = \frac{Ro}{1 - Ro} \text{ or } Ro = \frac{G}{1 + G} \qquad \text{\_\_\_\_\_(123).}$$

The usual definition of degree of reaction is

$$R = \frac{H_2 - H_3}{H_1 - H_3} \qquad \text{\_\_\_\_\_(72).}$$

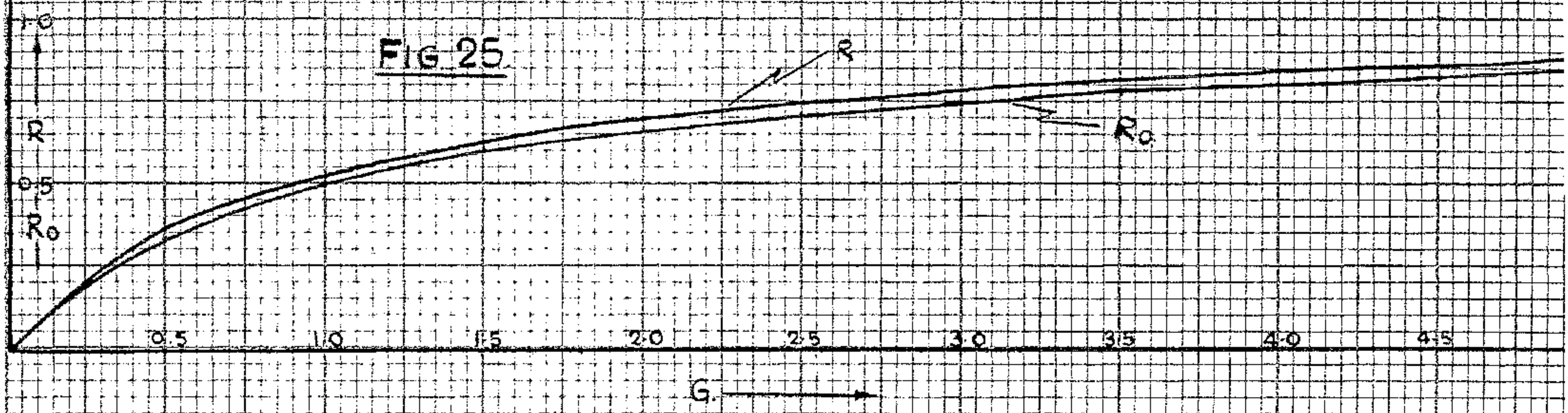

R
1.0
0.5
R
R_o
R
R_o
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
G

FIG 25

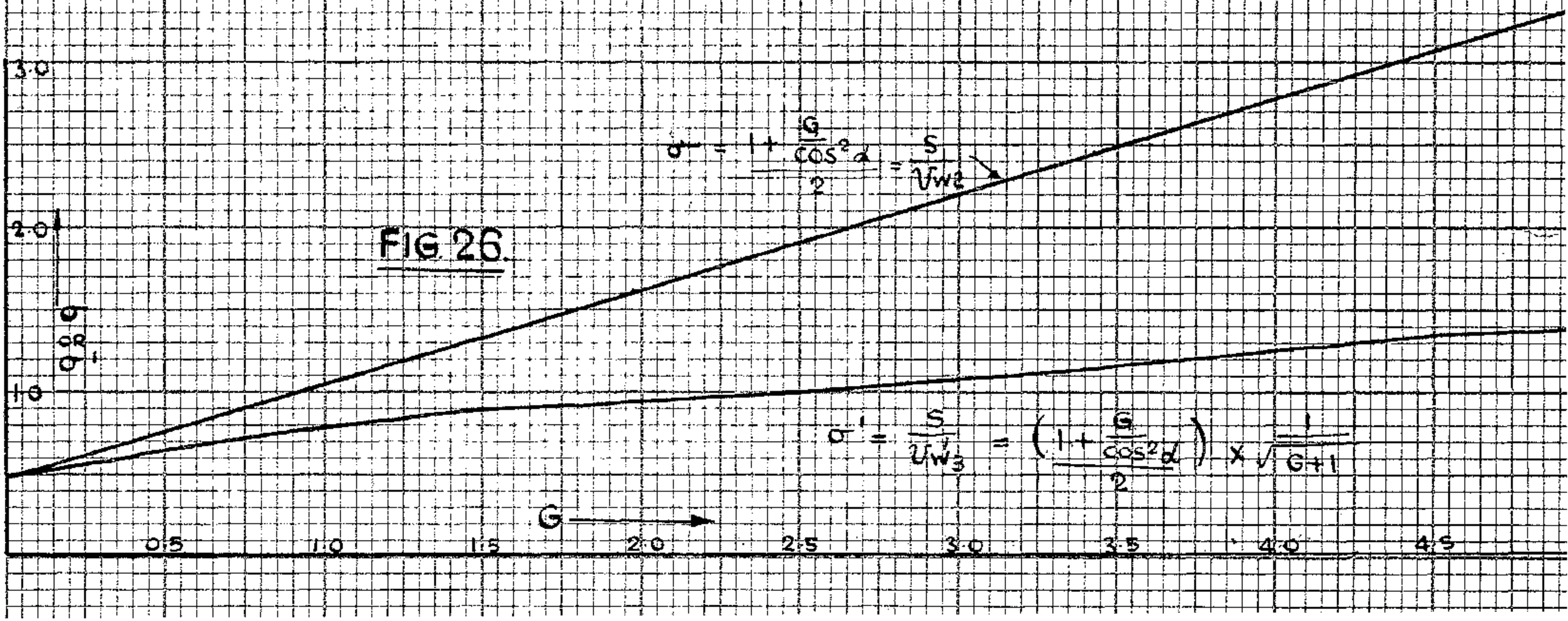

3.0
2.0
1.0
σ or σ'
$\sigma = \frac{1 + \frac{G}{\cos^2 \alpha}}{2} = \frac{S}{Vw_2}$
$\sigma' = \frac{S}{Uw'_3} = \left(\frac{1 + \frac{G}{\cos^2 \alpha}}{2} \right) \times \sqrt{\frac{1}{G+1}}$
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
G

FIG 26

This definition is useful only when applied to a typical stage in a multistage turbine where the carry over velocity is constant. In such a case the stage enthalpy change $H_1 - H_3$ is identical with the stage work output. Thus from equations 105 and 109,

$$R = \frac{H_2 - H_3}{WD} = \frac{G \quad v_2^2/2}{\sigma \cos^2\alpha \;\; v_2^2 \left(\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma\right)} \qquad (124).$$

Thus R is in general a function of G and σ.

When designing at maximum blading efficiency however equation 124 gives

$$\underset{(\text{for max } \eta_B)}{R} = \frac{G}{\cos^2\alpha + G} \qquad (125).$$

For a "50 degree" reaction blade, R = 0.5, G = 0.884 and hence

$$\sigma\,(\text{for Max } \eta_B) = \frac{1 + \frac{G}{\cos^2\alpha}}{2} = 1$$

The relationships between Ro, G and R given in equations 123 and 125 are shown in graphical form in figure 25.

<u>(6). The blade speed to jet speed ratio.</u>

The blade speed to jet speed ratio, $\rho = \frac{s}{v_2}$ is related to the blade speed to tangential jet speed ratio by

$$\rho = \sigma \cos\alpha \qquad (126).$$

A speed ratio defined by

$$\rho' = \frac{s}{v_3'} \qquad (127).$$

is however sometimes used, where the velocity v_3' is the velocity which would be attained in expansion in a nozzle, in which the nozzle heat drop is equal to the stage total head heat drop $H_{o_1} - H_3$. i.e. $v_3' = 223.8 \sqrt{H_{o_1} - H_3}$.

The equivalent blade speed to tangential jet speed ratio would be defined as

$$\sigma' = \frac{s}{v_{W_3}'} \qquad (128).$$

where v_{w_3}' is the tangential component of the velocity v_3' assuming that the nozzle outlet angle is the same as that for the nozzle of the stage under consideration (see figure 24). The speed ratios given in equations 126, 127 and 128 may be related to the total head degree of reaction for the stage. Thus

$$\frac{\rho'}{\rho} = \frac{\sigma'}{\sigma} = \frac{v_2}{v_3'} = \sqrt{1 - Ro} \qquad \text{\_\_\_\_\_(129).}$$

Hence when the stage is designed for maximum blading efficiency equation 129 gives

$$\sigma' = \left(\frac{1 + \frac{G}{\cos^2\alpha}}{2}\right)\sqrt{1 - Ro} = \left(\frac{1 + \frac{G}{\cos^2\alpha}}{2}\right)\frac{1}{\sqrt{1 + G}} \qquad \text{\_\_\_\_\_(130).}$$

This form of the speed ratio is useful in a number of applications. For example, if the stage heat drop and blade speed are determined, equation 130 gives the total head degree of reaction for maximum blading efficiency and so determines the best way to split the available stage heat drop between the static and moving rows. The relationships between σ, σ' and G for maximum blading efficiency are shown in figure 26.

(7). Conditions for maximum blading efficiency in the axial flow compressor stage.

For the compressor stage shown in figure 12, a compression coefficient may be defined as

$$G_c = \frac{H_2 - H_1}{H_{o_3} - H_2} = \frac{\frac{v_{r_1}^2 - v_{r_2}^2}{2}}{v_2^2/2} \qquad \text{\_\_\_\_\_(105c).}$$

and the blade speed to tangential jet speed ratio as

$\sigma = \dfrac{s}{v_{w_2}}$ where v_{w_2} is the whirl component of the absolute velocity entering the stator blade.

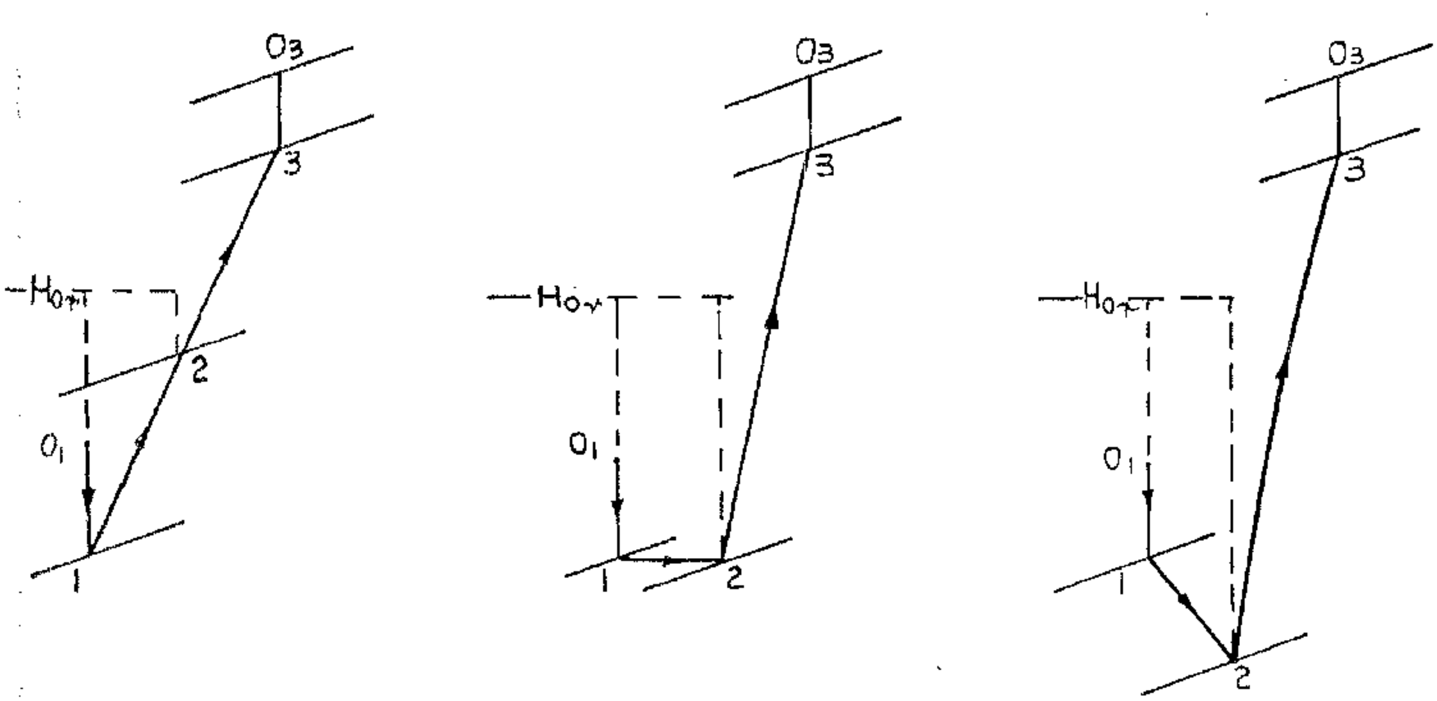


FIGURE 27A

FIGURE 27B.

FIGURE 27C.

φ

COMPRESSOR STAGES WITH POSITIVE, ZERO AND NEGATIVE DEGREES OF COMPRESSION.

FIG. 27.

In a similar manner to that of the turbine stage it may be shown that the work input is given by

$$WI = \sigma v_{W_2}^2 \left(\sqrt{\frac{G}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma \right) \qquad \text{(109c).}$$

and the blading efficiency is

$$\eta_B = \frac{WI}{H_{o_3} - H_1} = \frac{2\,\sigma\cos^2\alpha}{1 + G_c} \left(\sqrt{\frac{G_c}{\cos^2\alpha} + (1-\sigma)^2} + 1 - \sigma \right) \qquad \text{(111c).}$$

where α is the stator blade inlet angle.

Thus for a given stator blade inlet angle maximum blading efficiency is obtained when

$$\sigma = \frac{1 + \frac{G_c}{\cos^2\alpha}}{2} \qquad \text{(112c).}$$

The analogy between the compressor stage and the turbine stage, discussed in Part 1(c), may be used to determine the effect in the compressor stage of varying the stage design parameters. Thus the conclusions reached regarding the variation of blading efficiency and loading factor, with reaction coefficient and speed ratio for the turbine stage, may be applied to the compressor stage. These conclusions are summarized in figures 15, 16, 18, 19, 22 and 23 and when they are applied to the compressor stage it is only necessary to replace the reaction coefficient G with the compression coefficient G_c.

The H/φ diagrams for compressor stages corresponding to turbine stages with positive, zero and negative degrees of reaction are shown in figure 27. They are

(a) a conventional compressor with compression effect in the rotor blade.

(b) a zero degree of compression stage.

and (c) a stage with expansion effect in the rotor blade.

While from the point of view of stage efficiency a turbine with compression in the rotor blade is liable to be inefficient, because of the greater losses associated with a diffusion process, a compressor with expansion in the rotor blade could give a more efficient compressor with a large loading factor. Shepherd[29] indicates that this type of compressor stage is only possible where the stage velocities are low, otherwise the operating Mach number will be such that flow separation may occur in the stator blade.

Part 1 (d) - Summary.

For an axial flow turbine stage the blading efficiency and loading factor are derived as functions of the blade speed to tangential jet speed ratio (σ), and of the reaction coefficient (G). (Equations 111 and 116). The reaction coefficient is defined as the reaction effect in the moving blade divided by the kinetic energy at inlet to the blade and is given in equation 123 as a function of the total head degree of reaction for the stage. For a given reaction coefficient and nozzle outlet angle the blading efficiency is a maximum when

$$\sigma = \frac{1 + \cos^{\frac{G}{2}} \alpha}{2}$$

where α is the nozzle outlet angle.

The variation of blading efficiency with σ and G is shown in figure 15. It will be noted that there is a limited range of σ , about that for maximum blading efficiency, within which σ may be chosen without incurring a serious fall in blading efficiency. This range of σ , along with the value of the maximum blading efficiency increase with increase in reaction coefficient. For a given reaction coefficient, the loading factor increases with decreasing values of σ , (figure 18), but the choice of loading factor is in general limited to the range of σ within which there is no serious fall in blading efficiency. When the stage is designed for maximum blading efficiency, the value of the maximum blading efficiency and the loading factor are functions of reaction coefficient only, the value of the loading factor being twice the reciprocal of the blade speed to tangential jet speed ratio. As the reaction coefficient rises the maximum blading efficiency increases (figure 16) while the loading factor decreases (figure 19). In choosing a reaction coefficient for the stage therefore, a compromise must be made between these conflicting factors.

If the stage is designed with a small compression effect in the rotor blade a considerable rise in loading factor can be obtained. This is however accompanied by a serious fall in blading efficiency. In addition, the position with regard to blading efficiency is further aggravated if the blade speed to tangential jet speed ratio is only slightly removed from that which gives maximum blading efficiency.

The variation in the stage efficiencies with speed ratio and reaction coefficient may be studied using the results derived in this section. The total to static, and total to total stage efficiencies are related to the blading efficiency and total head efficiency of expansion in the stage by the expressions

$$\eta_{ts} = \eta_B \times \eta_{os} \qquad (89).$$

and

$$\eta_{tt} = \frac{\eta_{os}}{\frac{1}{\eta_B}(1 - \eta_{os}) + \eta_{os}} \qquad (93).$$

where η_{os} is given by

$$\eta_{os} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{1}{\eta_{oN}} - 1\right)(1 - Ro) + \left(\frac{1}{\eta_R} - 1\right)Ro} \qquad (81).$$

or

$$\eta_{os} = \frac{1}{1 + \frac{T_3}{T_2}\left(\frac{\mathcal{E}_N \frac{V_2^2}{2}}{H_{o_1} - H_3}\right) + \frac{\mathcal{E}_R \frac{V_{r_3}^2}{2}}{H_{o_1} - H_3}} \qquad (82).$$

It will be seen from equation 89 that the plots of blading efficiency, given in figure 15, may be regarded as design plots of total to static efficiency against σ where η_{os} is unity.

It has been shown that the blading efficiency and loading factor depend primarily on the choice of reaction coefficient for the stage. (figures 16 and 19). In making this choice of reaction coefficient, the effect on the stage efficiencies may be obtained by assuming constant element efficiencies and using η_{os} in the form of equation 81. When the reaction coefficient is thus determined, a comparison of a number of stage designs may be made, each corresponding to a blade speed to jet speed ratio within the range where the blading efficiency is sensibly constant. By this means the optimum design from the loading factor and stage efficiency viewpoint may be determined. In assesing the stage efficiencies at this point, account should be taken of the variation in the loss coefficients of the elements as the gas deflection varies. An example of the way in which gas deflection varies with ∇ at constant reaction coefficient is given in figure 17. It may be shown that equation 82 can be expressed as

$$\eta_{os} = \frac{1}{1 + \frac{T_3}{T_2}\frac{\mathcal{E}_N}{1+G} + \frac{\mathcal{E}_R}{1+G}\left(G + \sin^2\alpha + (1-\nabla)^2\cos^2\alpha\right)}$$

and to determine η_{tt} and η_{ts} it only remains to substitute experimental figures for $\mathcal{E}_N$ and $\mathcal{E}_R$.

Typical such results for the variation in loss coefficients with gas deflection, for blading which is designed for optimum pitch to chord ratio, are given by Soderburg[30]. The gas deflection and loss coefficient decrease as the blade speed to jet speed ratio rises. Thus the total head efficiency of expansion in the stage will rise continuously as the speed ratio is increased. The variation of stage efficiency with speed ratio may then be deduced from the blading efficiency graphs in figure 15 using the relationships given in equations 89 and 93. For a given reaction coefficient the total to static stage efficiency will exhibit a steeper characteristic at speed ratios below that for maximum blading efficiency and flatter characteristic above, compared with the corresponding blading efficiency curve. Rogers and Mayhew[31] have considered the effect of friction losses on the stage efficiency characteristic and have suggested that the speed ratio for maximum stage efficiency may differ somewhat from that for maximum blading efficiency.

The total to total stage efficiency has been shown to be much less dependant on the blading efficiency, especially at high values of the blading efficiency. (figure 10). At values of the blade speed about that for maximum blading efficiency the total to total stage efficiency will exhibit a much flatter characteristic than the total to static efficiency.

Equation 93 may be shown to give

$$\eta_{tt} \doteq 1 - \frac{1 - \eta_{os}}{\eta_{ts}}$$

from which the total to total stage efficiency curve may be determined once the total to static characteristic has been established.

With regard to the definitions of degree of reaction given in this section, Shepherd[32] used a different approach. He defines the degree of reaction as the reaction effect in the moving blade divided by the work output in the stage, thus :-

$$R_{sh} = \frac{H_2 - H_3}{WD}$$

This is a general definition which is not restricted to stages where the velocity entering and leaving the stage is the same. The relationship between R_{sh} and G is obtained by substituting R_{sh} for R in equation 124. This gives R_{sh} as a function of speed ratio and reaction coefficient. At maximum blading efficiency R_{sh} is a function of reaction coefficient only and is given by

$$R_{sh} = \frac{G}{\cos^2\alpha + G}$$

Shepherd gives the blade speed to jet speed ratio for maximum blading efficiency as

$$\rho = \frac{\cos\alpha}{2(1 - R_{sh})}$$

and the value of the maximum blading efficiency as

$$\eta_{B_{max}} = \frac{\cos^2\alpha}{1 - R_{sh}\sin^2\alpha}$$

Using the relationship between R_{sh} and G for maximum blading efficiency it may be shown that these latter expressions are identical with the corresponding expressions given here in equations 112 and 113.

For the axial flow compressor stage maximum blading efficiency is obtained when

$$\sigma = \frac{1 + \frac{G_c}{\cos^2\alpha}}{2}$$

Here σ is the ratio of blade speed to the tangential component of the absolute velocity leaving the rotor blade, α is the stator blade inlet angle and G_c, the compression coefficient, is the ratio of the compression effect in the rotor blade to the kinetic energy at inlet to the stator blade. The way in which the blading efficiency and loading factor for the compressor stage vary with the compression coefficient and speed ratio, can be obtained by using the analogy, between the compressor stage and similar turbine stage, discussed in the previous section.

Part 2.

(a) The Determination in a static test rig of Local Total Head Efficiency of Expansion and of Stream Condition by means of an Impact Tube.

The measurement in a static test rig of the local efficiency of expansion and of stream condition in a subsonic or supersonic stream by means of an impact tube.

The loss of high grade energy in a turbine or compressor element may vary from point to point in the exit plane of the element. The impact tube when set at zero incidence to a stream records the local value of the total head or stagnation pressure. Thus if the total head condition of the working fluid entering a nozzle or diffuser is known the tube may be used to give local losses in total head pressure at the exit plane of the element. The function of the following section is to relate the recorded value of total head pressure to the local total head efficiency of expansion in a nozzle or blade element and to the local state of the fluid when the element is examined in a static test rig used to simulate the actual operating conditions. The equations developed allow for the case where the fluid stream approaching the impact tube is supersonic, in which case the recorded stagnation pressure is less than the actual stagnation pressure of the stream due to the loss in the total head pressure across the detached shock front up-stream of the tube. The relationships allow one to determine whether the local flow is subsonic or supersonic and in either case the local total head efficiency of expansion and local specific volume is then given in graphical form as a function of the local outlet stagnation pressure of the stream and of the theoretical Mach number after expansion. The graphs are drawn for the theoretical Mach numbers at outlet from the nozzle or blade which were used in the experimental work.

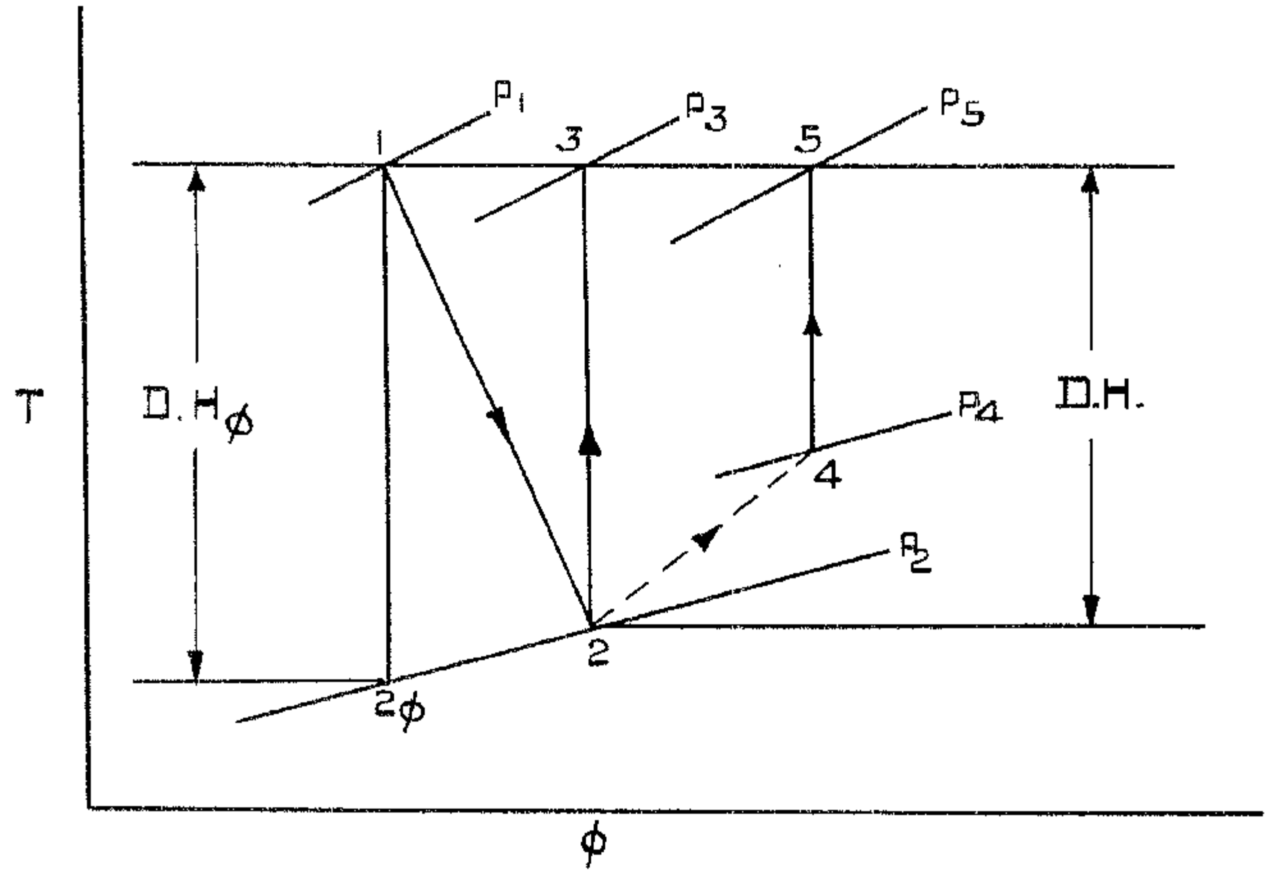


FIG. 28.

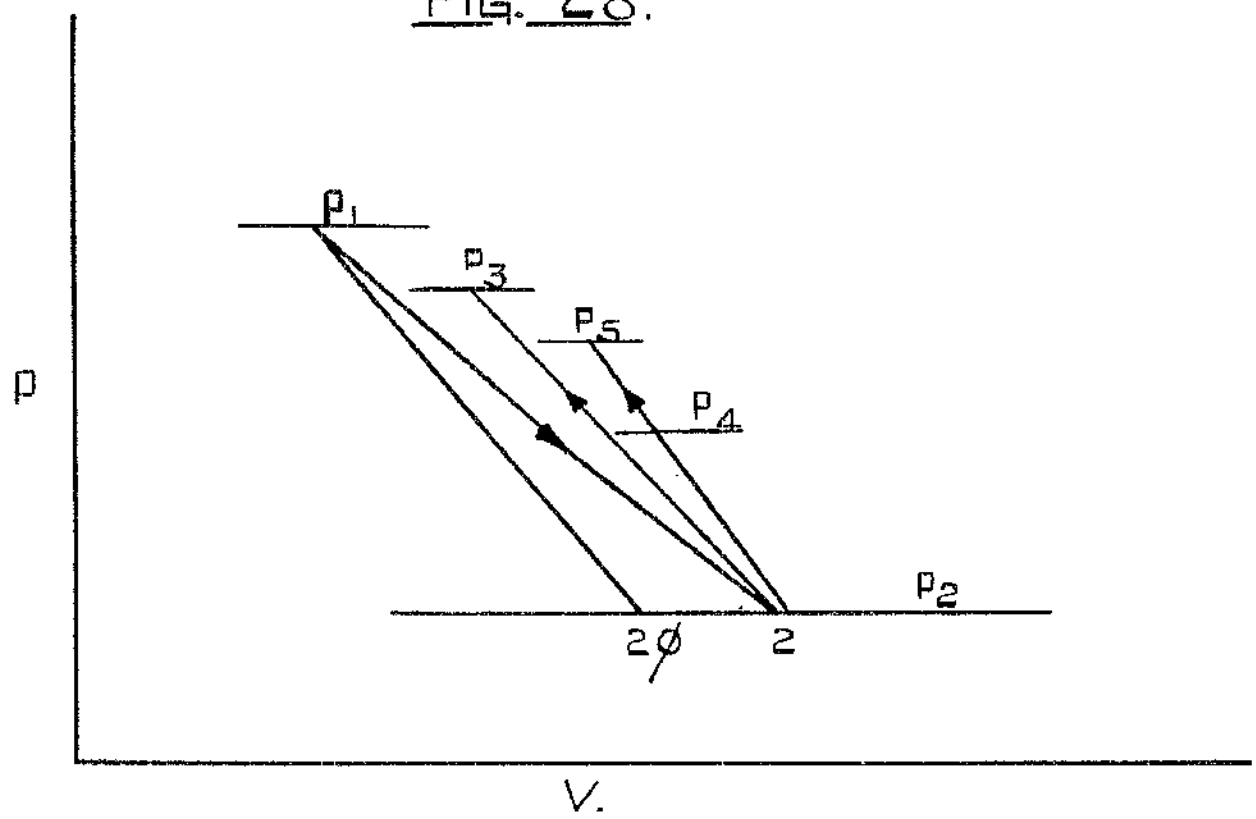


FIG. 29.

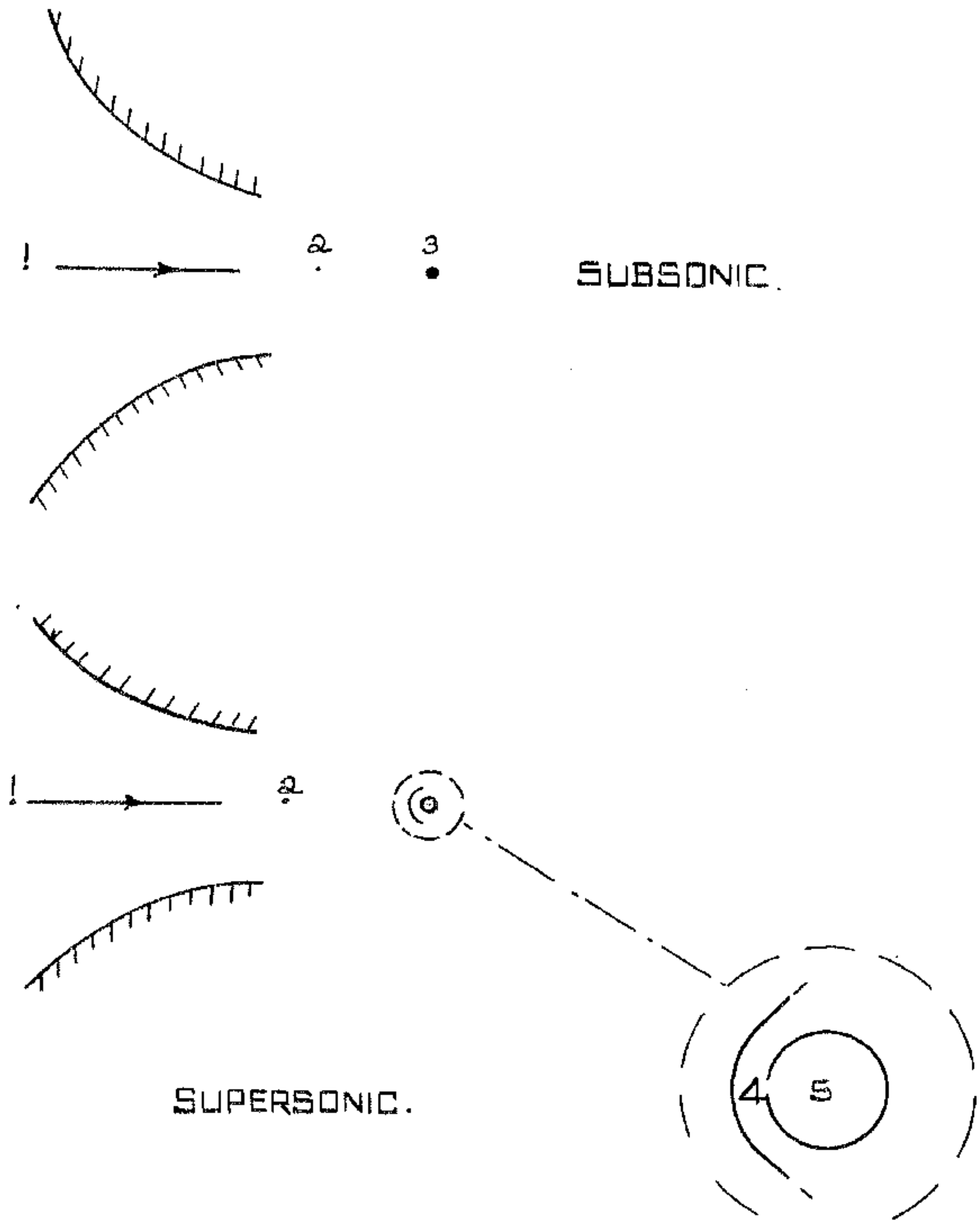


FIG. 30.

The measurement, in a static test rig, of the local efficiency of expansion and of steam condition in a subsonic or supersonic stream by means of an impact tube.

Let M denote Mach No.

"	v	"	velocity ft/sec.
"	V	"	specific volume ft^3/lb.
"	P	"	pressure lb/ft^2.
"	v_a	"	accoustic velocity.
"	γ	"	isentropic index.
"	η	"	local total head efficiency of expansion.
"	DH	"	heat drop B.T.U./lb.
"	g	"	acceleration due to gravity.

Referring to figures 28, 29 & 30, consider an expansion from the total head condition (1) (Total head pressure P_1) to the static pressure P_2. Thus an isentropic expansion from (1) gives the state point 2ϕ after expansion and an adiabatic expansion gives the point 2 with corresponding total head pressure P_3.

Hence $$\frac{v_{2\phi}^2}{2g} = \frac{\gamma}{\gamma - 1} (P_1V_1 - P_2V_{2\phi})$$

$$\therefore \; v_{2\phi}^2 = \frac{2g\gamma}{\gamma - 1} P_1V_1 \left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}\right)$$

$$= \frac{2g\gamma}{\gamma - 1} P_2V_{2\phi} \left(\left(\frac{P_1}{P_2} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right) \qquad \text{———(1).}$$

$$\because \; v_{a_2}^2 = g\,\gamma\, P_2V_{2\phi}$$

$$\therefore \quad M^2_{2\phi} = \frac{v^2_{2\phi}}{v^2_{a\,2\phi}} = \frac{2}{\gamma - 1}\left(\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \qquad \text{———— (2).}$$

$$\text{also } v^2_2 = \frac{2\,g\,\gamma}{\gamma - 1}\; P_3 V_3 \left(1 - \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$= \frac{2\,g\,\gamma}{\gamma - 1}\; P_2 V_2 \left(\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \qquad \text{———— (3).}$$

$$\because \quad v^2_{a_2} = g\,\gamma\,P_2 V_2$$

$$\therefore \quad M^2_2 = \frac{2}{\gamma - 1}\left(\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right) \qquad \text{———— (4).}$$

Since $P_1\ V_1 = P_3\ V_3$ then from (1) and (3)

$$\eta = \frac{DH}{DH_\phi} = \frac{v^2_2}{v^2_{2\phi}} = \frac{1 - \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \text{———— (5).}$$

$$\text{from (2) } \frac{P_2}{P_1} = \left(\frac{\gamma - 1}{2}\,M^2_{2\phi} + 1\right)^{\frac{-\gamma}{\gamma-1}}$$

$\therefore$ from (5)

$$\eta = \frac{1 - \left(\frac{P_2}{P_3}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{\gamma-1}{2}\,M^2_{2\phi} + 1\right)^{-1}} \qquad \text{———— (6).}$$

also from (1) and (3)

$$\eta = \frac{V_2}{V_{2\phi}}\;\frac{\left(\left(\frac{P_3}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}{\left(\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} \qquad \text{———— (7).}$$

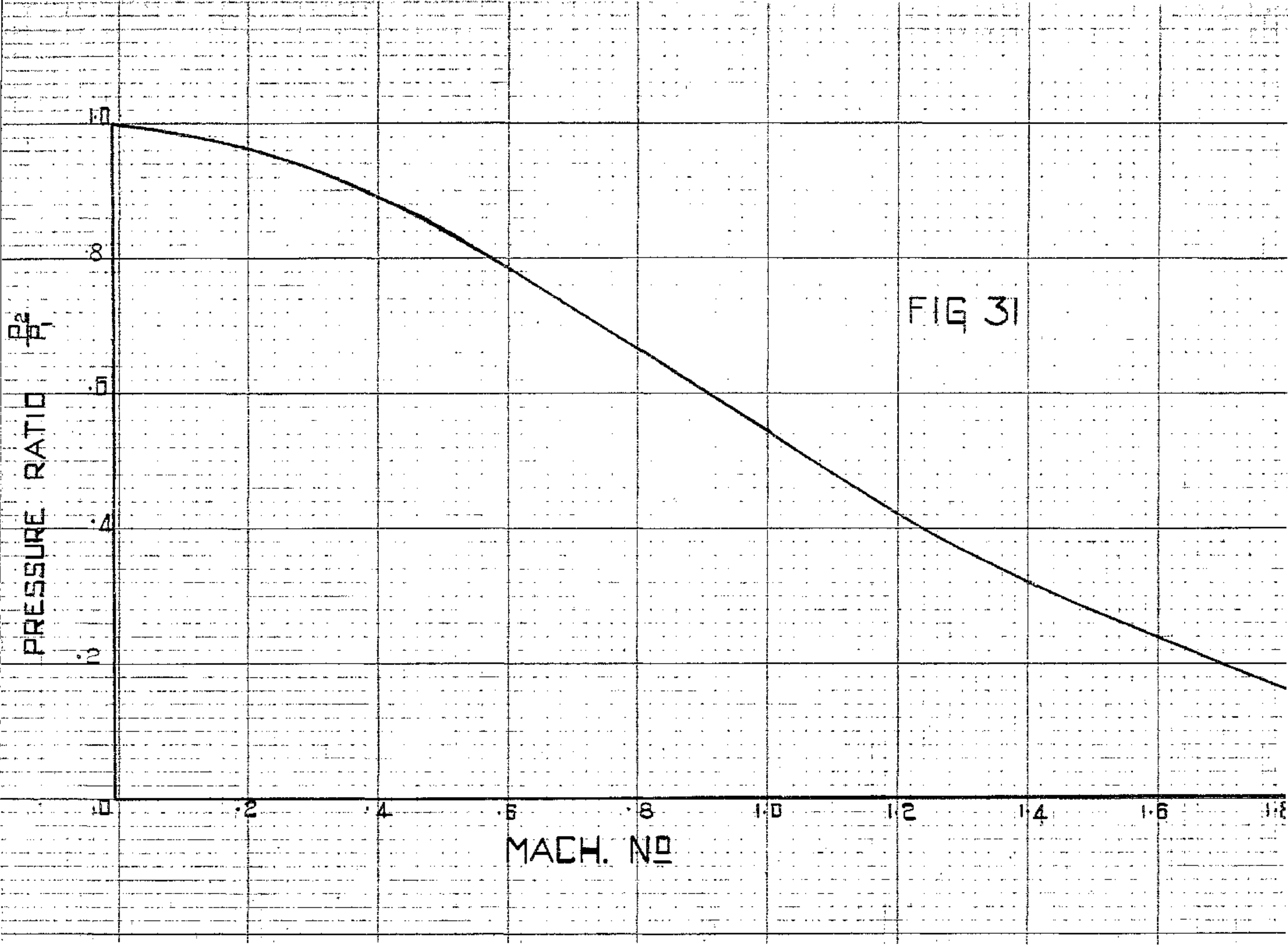


FIG 31
PRESSURE RATIO $\frac{P_2}{P_1}$
MACH. No
1·0
·8
·6
·4
·2
0
·2
·4
·6
·8
1·0
1·2
1·4
1·6
1·8

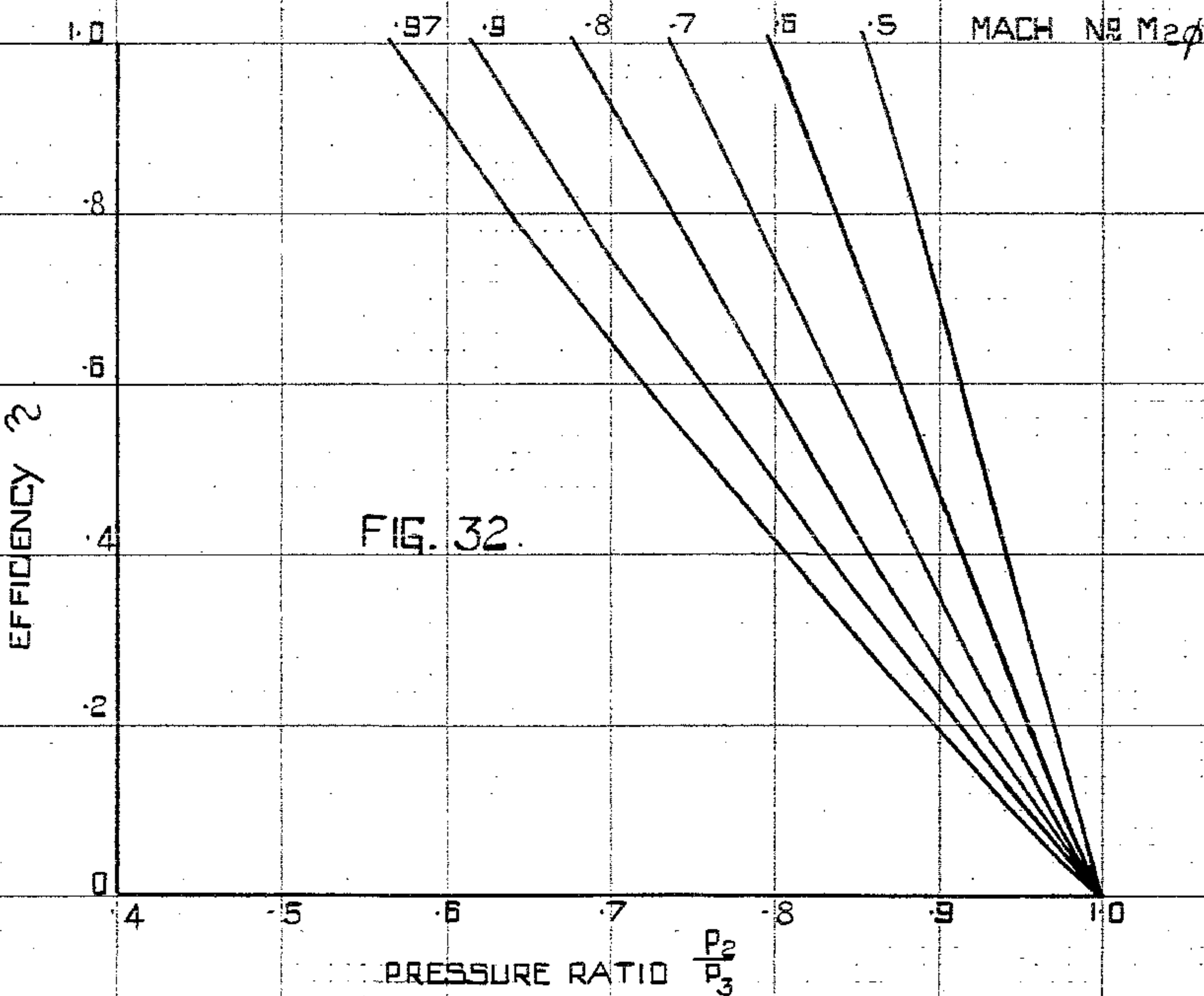
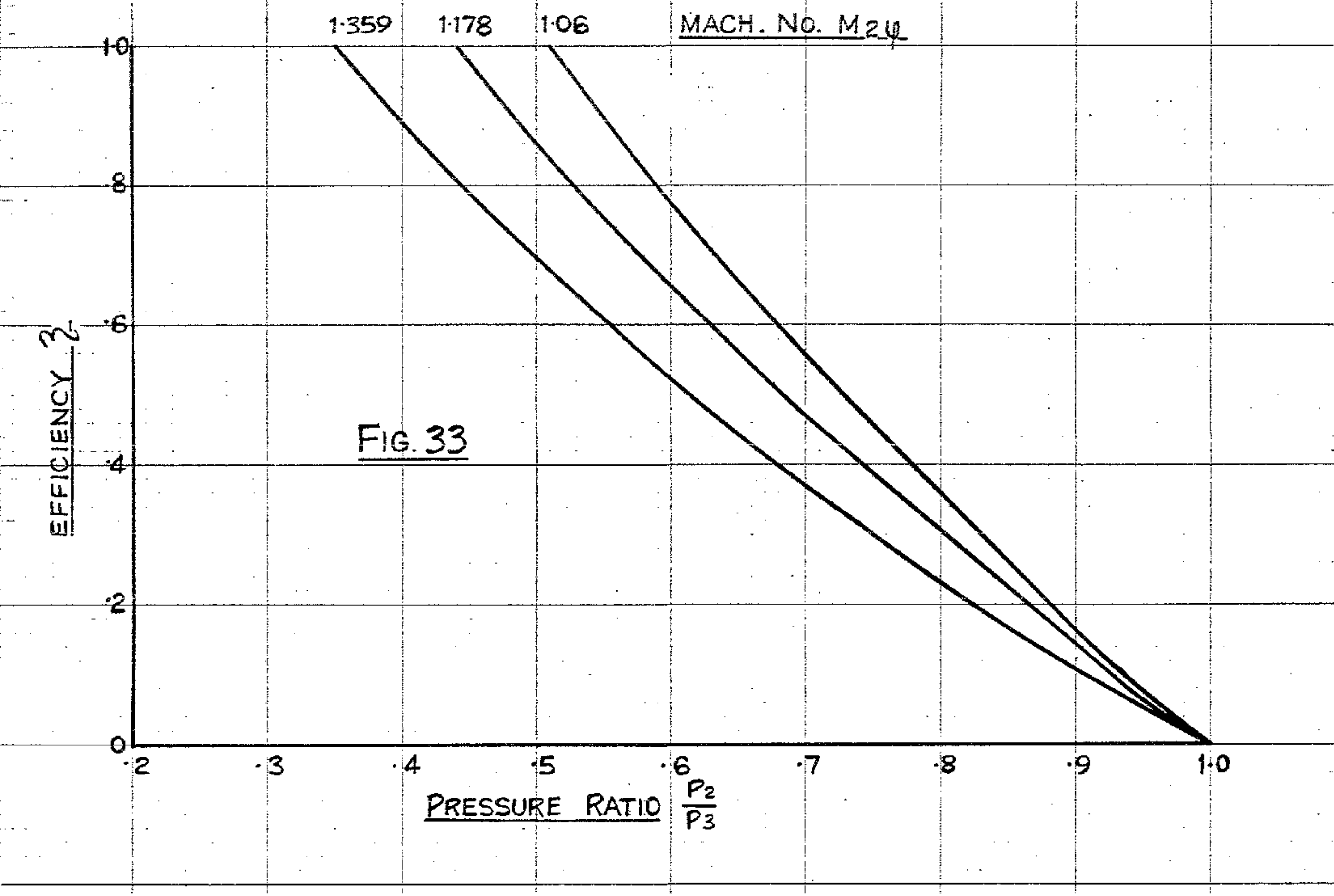

MACH № $M_{2\phi}$
·97
·9
·8
·7
·6
·5
EFFICIENCY η
1·0
·8
·6
·4
·2
0
·4
·5
·6
·7
·8
·9
1·0
PRESSURE RATIO $\frac{P_2}{P_3}$

FIG. 32.



1·359
1·178
1·06
MACH. No. $M_{2\psi}$
EFFICIENCY η
1·0
·8
·6
·4
·2
0
·2
·3
·4
·5
·6
·7
·8
·9
1·0
PRESSURE RATIO $\frac{P_2}{P_3}$

FIG. 33

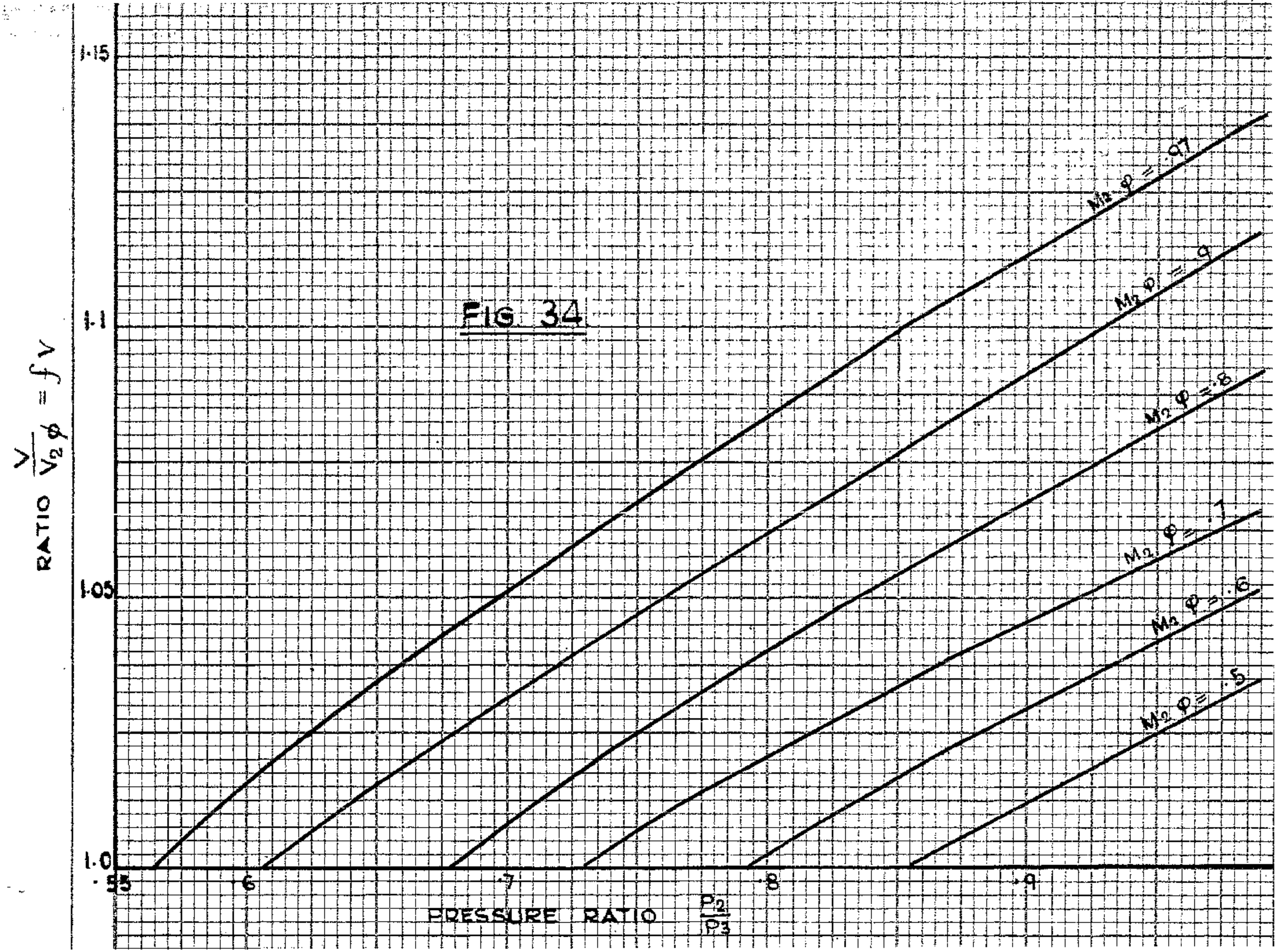
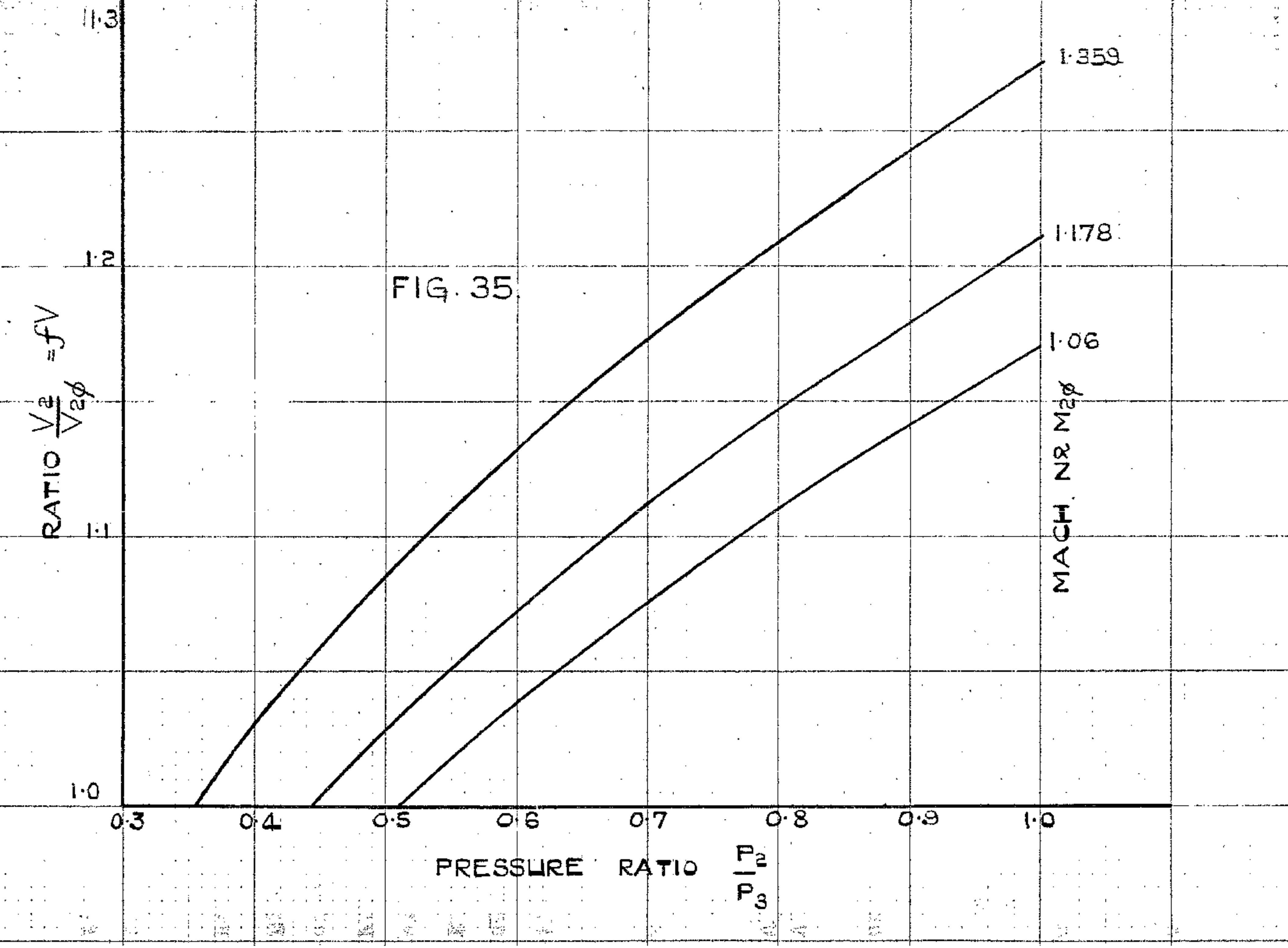


FIG 34
RATIO $\frac{V}{V_2 \phi} = f V$
PRESSURE RATIO $\frac{P_2}{P_3}$
1·15
1·1
1·05
1·0
·55
·6
·7
·8
·9
$M_2\,\phi = .97$
$M_2\,\phi = .9$
$M_2\,\phi = .8$
$M_2\,\phi = .7$
$M_2\,\phi = .6$
$M_2\,\phi = .5$



1·3
1·2
1·1
1·0
RATIO $\frac{V_2}{V_{2\phi}} = fV$
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
PRESSURE RATIO $\frac{P_2}{P_3}$
1·359
1·178
1·06
MACH Nº $M_{2\phi}$

FIG. 35

$$\therefore \frac{V_2}{V_{2\varphi}} = \zeta \left(\frac{\left(\frac{P_1}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1}{\left(\frac{P_3}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1} \right)$$

$\therefore$ from 5

$$\frac{V_2}{V_{2\varphi}} = \frac{1 - \left(\frac{P_2}{P_3}\right)^{\frac{\gamma - 1}{\gamma}}}{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}} \times \frac{\left(\frac{P_1}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1}{\left(\frac{P_3}{P_2}\right)^{\frac{\gamma - 1}{\gamma}} - 1}$$

on simplifying this gives

$$\frac{V_2}{V_{2\varphi}} = \frac{\left(\frac{P_2}{P_3}\right)^{\frac{\gamma - 1}{\gamma}}}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}}$$

$$\therefore \frac{V_2}{V_{2\varphi}} = \left(\frac{P_2}{P_3}\right)^{\frac{\gamma - 1}{\gamma}} \left(\frac{\gamma - 1}{2} M_{2\varphi}^2 + 1\right) \qquad \text{———} \quad (8).$$

The above relationships apply with subsonic or supersonic flow at point 2. Equation (2) is plotted in Fig. 31 as $\frac{P_2}{P_1}$ against Mach number.

Equation (6) relates ζ, $\frac{P_2}{P_3}$ and theoretical Mach No. after expansion $M_{2\varphi}$.

Figs. 32 and 33 give ζ to a base of $\frac{P_2}{P_3}$ for various values of $M_{2\varphi}$.

Equation (8) relates $\frac{V_2}{V_{2\varphi}}$, $\frac{P_2}{P_3}$ and $M_{2\varphi}$ and these are shown in Figs 34 and 35 for various values of $M_{2\varphi}$.

Hence by obtaining the total head pressure P_3 the local total head efficiency and local specific volume can be obtained from the above relationships.

In subsonic flow the total head pressure is equal to the observed impact tube pressure, in supersonic flow however this is not so and a correction has to be made to the impact tube pressure to obtain the actual total head pressure of the stream.

Referring to Figs. 28, 29 and 30, for supersonic flow at 2 the impact tube acts like a large angled wedge so that a detached shock front forms up-stream of the impact tube. The total shock front is made up of a series of oblique shocks at various angles to the stream. However when the impact tube hole is aligned to be in the direction of the fluid stream, the changes of state experienced by the fluid approaching the hole correspond to those of a normal shock front.

For supersonic flow then P_4 and P_5 are the static pressure and total head pressure respectively after a normal shock jump from a stream Mach no. M_2, and P_5 is equal to the observed impact tube pressure.

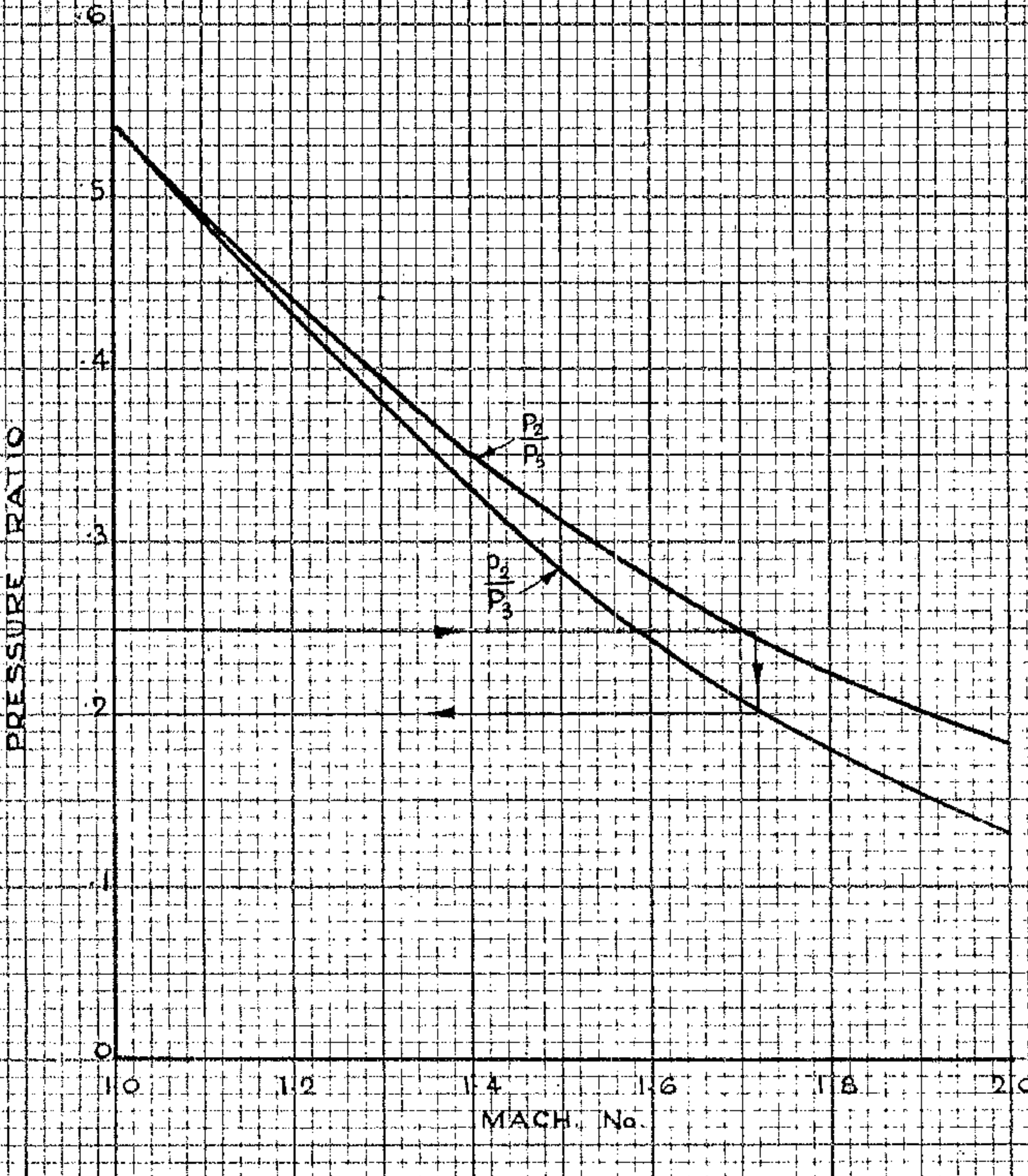
Hence from the theory of the normal shock wave the Rayleigh Supersonic Pitot - Tube relationship is

$$\frac{P_2}{P_5} = \frac{\left(\frac{2\gamma}{\gamma+1} M_2^2 - \frac{\gamma-1}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}{\left(\frac{\gamma+1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}} \qquad \text{——— (9).}$$

(Shapiro[33]).

also we have that $$\frac{P_2}{P_3} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{-\gamma}{\gamma-1}} \qquad \text{——— (10).}$$

Equation 9 relates the static pressure at outlet with the impact tube pressure for supersonic flow and is shown in Fig. 36. Superimposed on this graph is the relationship given by equation



PRESSURE RATIO
·6
·5
·4
·3
·2
·1
0
1·0
1·2
1·4
1·6
1·8
2·0
MACH. No.
$\frac{P_2}{P_5}$
$\frac{P_2}{P_3}$

FIG No. 36.

10 which is general for subsonic or supersonic flow.

Fig. 36 enables one to determine if the flow is supersonic or subsonic and if supersonic to obtain from the observed impact tube pressure the actual total head pressure of the stream.

Fig. 36 is drawn for superheated steam taking $\gamma = 1.3$, and has been reduced in size from its more accurate and larger original for presentation here. Thus if the ratio stream static pressure/ Impact tube pressure is less than 0.545 the stream Mach number is greater than unity and the ratio P_2/P_3 can be obtained as indicated. This ratio can then be used in Figs. 32 - 35 to obtain local efficiency and specific volume.

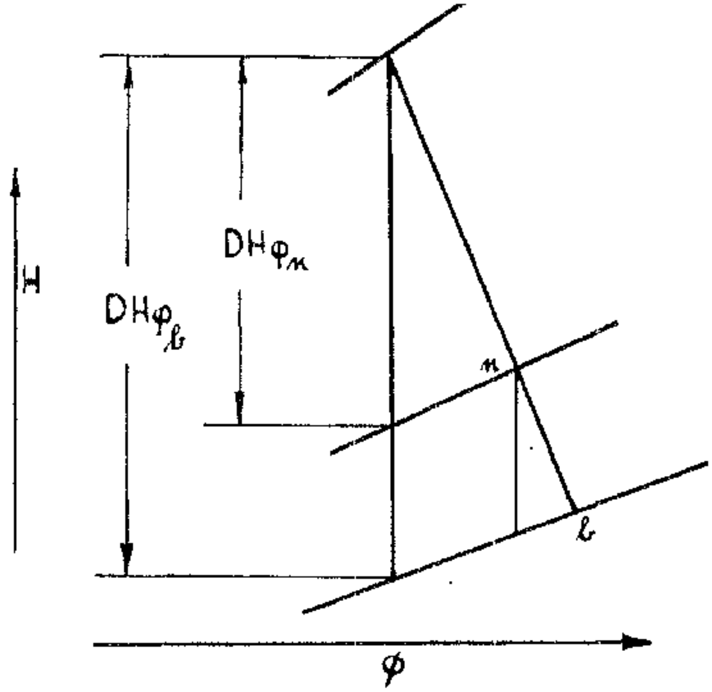
When plotting the relationships given in this section use may be made of Keenan and Kayes Gas Tables.[34] Table 34 for isentropic compressible flow with $\gamma = 1.3$ gives values of total head to static temperature ratio and total head to static pressure ratio for stream Mach numbers ranging from 0 to 10. and may be used to obtain the plots of pressure ratio, total head efficiency, and volume ratio in figures 31 to 35. Table 52 gives normal shock functions from which the plot of the Ragleigh pitot tube relationship in fig 36 may be made.

Summary of Part 2(a)

In subsonic flow the impact tube, if placed correctly in the stream at the exit plane of a nozzle or of a nozzle and blade pair, will record local values of the total head pressure of the stream. If locally the flow is supersonic a correction must be made to the observed impact tube pressure to obtain the actual total head pressure of the stream. The determination of whether or not the local flow is supersonic and the subsequent correction should this prove necessary are obtained using figure 36. The local total head efficiency of expansion and the ratio of the actual to the theoretical specific volume after expansion are shown to be functions of the pressure ratio, stream static pressure/ stream total head pressure, and of the theoretical Mach number after expansion. Hence if one plots the local efficiency and volume ratio to a base of the above pressure ratio, for the various theoretical outlet Mach numbers at which an investigation is to take place, the local total head efficiency and state point of the fluid may be readily and expressly obtained. These relationships are shown in figures 31 - 35.

Part 2.

(b) The Conversion of Local Values of Efficiency and of Efflux Angle into Mean Effective Values.



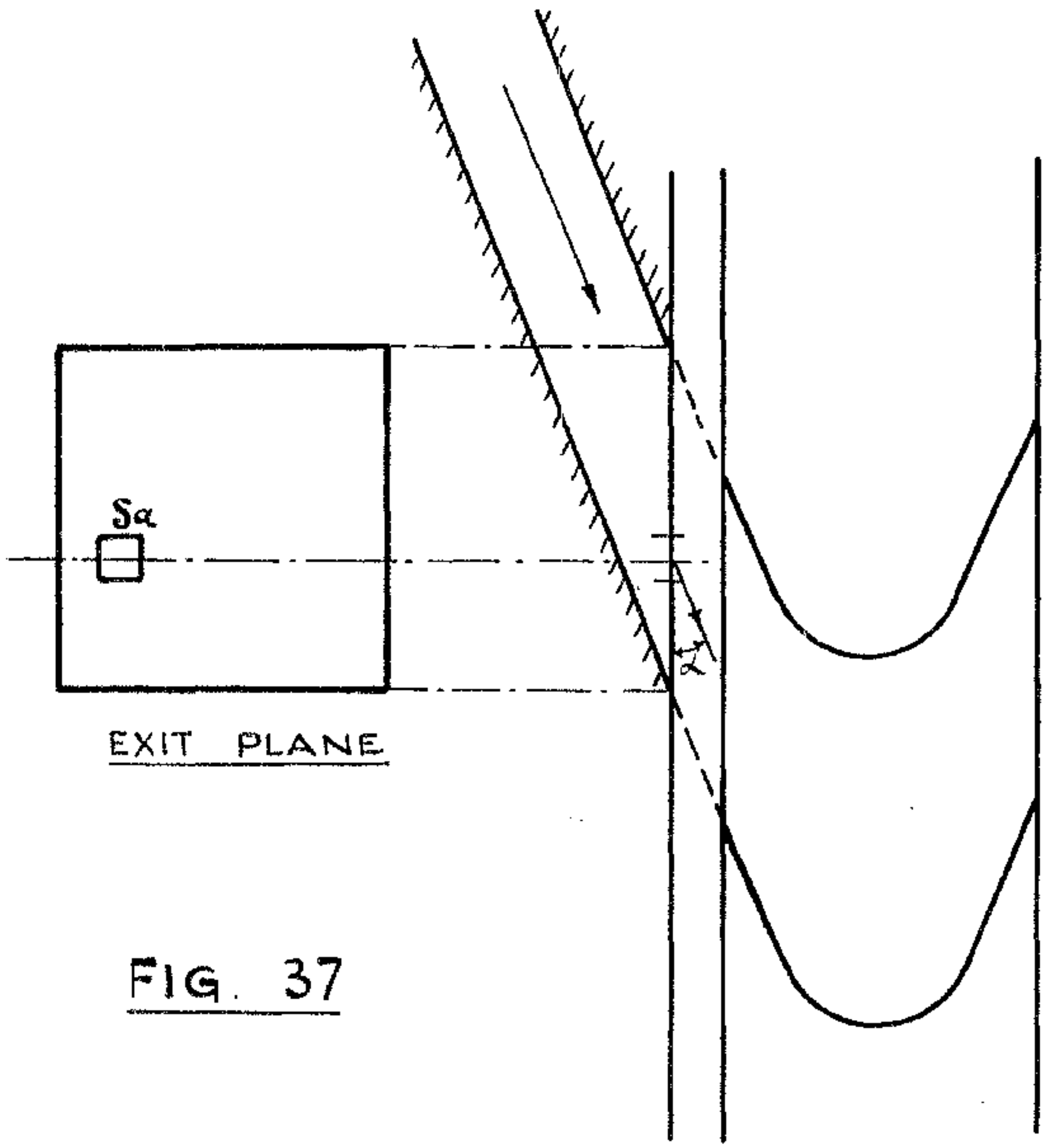


FIG. 37

Determination of the Mean or Effective Values of Efficiency of Expansion etc. from Traverses over the Exit Planes of a Nozzle and Blade Pair.

It will be appreciated that spot centre line readings of efficiency and efflux angle have to be modified to take account of two dimensional effects obtained from centre line traverses. The work of Howell[7] and Ainley[11] further emphasised the need for three dimensional traverses to account for secondary losses. Having obtained the local values of efficiency, specific volume and efflux angle from the recorded impact tube readings at any cross-section of the flow in a nozzle or diffuser it is necessary to reduce these local values to a mean effective value which will apply to the whole cross-section.

This is done below with particular reference to the nozzle and blade pair used in the experimental work but the same general method would be used in any turbine or compressor element. In terms of local values, expressions are developed for the mean total head efficiency of expansion in the nozzle or in the nozzle and blade pair. The mean efficiency in the blade itself may then be obtained and for the particular case where there is no reaction in the blading a mean blade velocity coefficient is derived. The theory is further extended to give mean efflux angles for the nozzle and blade.

Referring to figure 37, consider a nozzle and blade pair in which the isentropic heat drop in the nozzle is $DH_{\phi n}$ and in the nozzle and blade pair $DH_{\phi b}$. Suppose that traverses of the nozzle and blade exit planes have been made and the values of local total head efficiency, specific volume and angle obtained over the areas.

Let DH = heat drop BTU./lb.

" m = mass flow lb. per sec.

" v = local velocity ft. per sec.

" α = local flow angle degrees.

" V = local specific volume $ft^3/lb.$

" η = local total head efficiency of expansion.

" da = local elemental area ft^2.

" fV = the volume ratio V/V_φ .

" Σ = summation.

" K = blade velocity coefficient.

" $\bar{x}$ = a mean value of x.

also let subscripts n and b denote the nozzle and blade outlet respectively.

Then at either plane the elemental mass flow at area da is given by $dm = \dfrac{v \sin\alpha \, da}{V}$

The total mass flow is then $m = \Sigma \dfrac{v \sin\alpha \, da}{V}$

But $\dfrac{v^2}{v_\varphi^2} = \eta$ and $\dfrac{V}{V_\varphi} = fV$.

$$\therefore m = \frac{v_\varphi}{V_\varphi} \quad \Sigma \, \eta^{\frac{1}{2}} \sin\alpha \frac{da}{fV}$$

$$= \frac{223.8 \quad DH}{V} \qquad {}^{\frac{1}{2}} \sin \quad \frac{da}{fV} \qquad \text{------} \quad (1).$$

Hence at the nozzle exit

$$m = \frac{223.8 \sqrt{DH \, \varphi n}}{V_{\varphi n}} \; \Sigma \left(\eta^{\frac{1}{2}} \sin\alpha \frac{da}{fV} \right)_n \text{ lb/sec.} \qquad \text{------} \quad (2).$$

and at the blade exit

$$m = \frac{223.8\ \sqrt{DH\,\varphi b}}{V\varphi b}\ \Sigma\left(\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}\right)_b \text{ lb/sec.} \qquad \text{———— (3).}$$

Kinetic energy at the exit planes.

At any plane the total flux of kinetic energy is

$$\Sigma\ \frac{v^2}{2g}\ \frac{v\sin\alpha\,da}{V}\ \text{ ft lb/sec.}$$

∴ Total flux of kinetic energy

$$= \frac{223.8^3\ DH_\varphi^{3/2}}{2gV_\varphi}\ \Sigma\left(\eta^{3/2}\sin\alpha\frac{da}{fV}\right) \text{ ft lbs/sec.} \qquad \text{———— (4).}$$

Hence for one lb,

$$K.E. = \frac{223.8^2\ DH_\varphi}{2g}\ \frac{\Sigma\eta^{3/2}\sin\alpha\frac{da}{fV}}{\Sigma\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\ \text{ ft lbs/lb.}$$

$$\therefore DH = DH_\varphi\frac{\Sigma\eta^{3/2}\sin\alpha\frac{da}{fV}}{\Sigma\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\ \text{ B.T.U./lb.}$$

∴ The mean total head efficiency of expansion is given by

$$\bar{\eta} = \frac{\Sigma\eta^{3/2}\sin\alpha\frac{da}{fV}}{\Sigma\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}} \qquad \text{———— (5).}$$

Thus $\bar{\eta}_n = \left(\frac{\Sigma\eta^{3/2}\sin\alpha\frac{da}{fV}}{\Sigma\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_n$ ———— (6).

and $\bar{\eta}_b = \left(\frac{\Sigma\eta^{3/2}\sin\alpha\frac{da}{fV}}{\Sigma\eta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_b$ ———— (7).

The mean exit velocities for the nozzle and blade are given by

$\bar{v}_n = 223.8\ \sqrt{\bar{\eta}_n DH\varphi_n}$ and $\bar{v}_b = 223.8\ \sqrt{\bar{\eta}_b DH\varphi_b}$

It should be noted that $\bar{\eta}_b$ is the mean total head efficiency of expansion for the nozzle and blade pair. The efficiency of the expansion process in the blade itself is given by

$$\bar{\bar{\zeta}}_b = \frac{H_n - H_b}{DH\varphi_b - DH\varphi_n}$$

$$\bar{\bar{\zeta}}_b = \frac{DH\varphi_b\left(\dfrac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_b - DH\varphi_n\left(\dfrac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_n}{DH\varphi_b - DH\varphi_n} \quad \text{——— (8).}$$

and the loss of high grade energy within the blade passage is

$DH\varphi_b - DH\varphi_n - (H_n - H_b)$ B.T.U.

∴ blade passage reheat loss is

$$DH\varphi_b - DH\varphi_n - \left[DH\varphi_b\left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_b - DH\varphi_n\left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_n\right] \quad \text{—(9)}$$

For an impulse (zero pressure drop) blade this becomes

$$DH\varphi_n\left[\left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_n - \left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_b\right] \quad \text{——— (10)}$$

For the impulse blade the loss in the blade passage may be expressed as

$$\text{Loss} = (1 - K^2)\frac{\bar{V}_n^2}{2gJ} = (1 - K^2)\ \bar{\zeta}_n\ DH\varphi_n \quad \text{——— (11)}$$

where K is the blade velocity coefficient.

Substituting for $\bar{\zeta}_n$ from (6) in (11) and equating to (10) gives

$$K = \sqrt{\left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_b \div \left(\frac{\Sigma\zeta^{\frac{3}{2}}\sin\alpha\frac{da}{fV}}{\Sigma\zeta^{\frac{1}{2}}\sin\alpha\frac{da}{fV}}\right)_n} \quad \text{——— (12).}$$

Effective mean exit angles.

Let the mean exit angles for the nozzle and blade be $\bar{\alpha}_n$ and $\bar{\alpha}_b$ respectively.

Tangential force on the blade = $\frac{m}{g}$ x (change of tangential velocity) lbs.

= $\frac{m}{g}$ (inlet tangential velocity + outlet tangential velocity) lbs.

Hence the inlet tangential momentum flux is

$$= \frac{1}{g}\Sigma(\frac{v_n}{V_n} \sin \ _n \, da \times v_n \cos \ _n) \text{lbs.}$$

$$= \frac{223.8^2 DH \varphi n}{gV_{\varphi n}} \Sigma \quad (\zeta_n \sin\alpha_n \cos\alpha_n \frac{da}{fV}) \text{ lbs.}$$

$$= \frac{m}{g} \bar{v}_n \cos \bar{\alpha}_n \text{ lbs.}$$

$$\therefore \cos\bar{\alpha}_n = \left(\frac{\Sigma \zeta \sin\alpha \cos\alpha \frac{da}{fV}}{(\Sigma \zeta^{\frac{1}{2}} \sin\alpha \frac{da}{fV})\sqrt{\zeta}}\right)_n \qquad \text{———— (13).}$$

$$\text{similarly } \cos\bar{\alpha}_b = \left(\frac{\Sigma \zeta \sin\alpha \cos\alpha \frac{da}{fV}}{(\Sigma \zeta^{\frac{1}{2}} \sin\alpha \frac{da}{fV})\sqrt{\zeta}}\right)_b \qquad \text{———— (14).}$$

Summary of Part 2 (b).

In Part 2 (b) local values of efficiency and efflux angle are converted to mean effective values taking account of variations in mass flow per unit area throughout the exit plane element. Equation 8 gives the mean efficiency of the expansion process based on the static state of the fluid entering the element. Equation 5 may be applied generally to obtain the mean total head efficiency of expansion in a static test on a stator or rotor blade element. In the derivation of equation 5, it is assumed that the inlet total head state of working fluid is constant over the entire inlet flow area and the ideal final kinetic energy is that which would be obtained after isentropic expansion from this total head state to the static pressure at exhaust from the element. When testing cascades of blades in a wind tunnel, the total head state at inlet to the cascade is usually uniform and the reference total head pressure is often taken as the static pressure in the settling chamber upstream of the tunnel or nozzle.

In equations 6 and 7 the relationship for mean total head efficiency is applied at the exit plane of the tunnel or nozzle ($\bar{\eta}_n$) or at the exit plane of the blade cascade ($\bar{\eta}_b$). If there is a tunnel loss $\bar{\eta}_n$ will be less than unity and to calculate $\bar{\eta}_b$ the total head pressure at the immediate inlet to the cascade should be used. If the reference total head pressure is however taken as the static pressure in the settling chamber then the mean efficiency will include an allowance for the tunnel loss.

Expressions for the mean efflux angle at the nozzle or at the blade exit are given in equations 13 and 14. These angles may be used in conjunction with the mean efflux velocities at the nozzle and blade exit to obtain the tangential blade force, the mean velocities being obtained using the appropriate mean total head efficiency.

In the experimental work described here, loss characteristics over a range of Reynolds and Mach numbers were obtained for a small nozzle and blade pair. The nozzle acted as a "wind" tunnel for a blade section which was essentially of impulse design and the test arrangement was such that the blade was forced to operate with zero pressure drop across the blade passage. Pitot traverses were made at the nozzle and blade exit planes and for the total head efficiency in each case the reference total head pressure was that at inlet to the nozzle. Since there was a small nozzle loss, this loss is included in the mean total head efficiency of the expansion for the blade. For this type of impulse blade the mean blade velocity coefficient, is given in equation 12, in terms of the mean total head nozzle and blade efficiencies.

Part 3.

An experimental investigation into the flow of superheated steam through a small nozzle and impulse blade pair yielding results on flow pattern and efficiency with varying Reynolds number and Mach number.

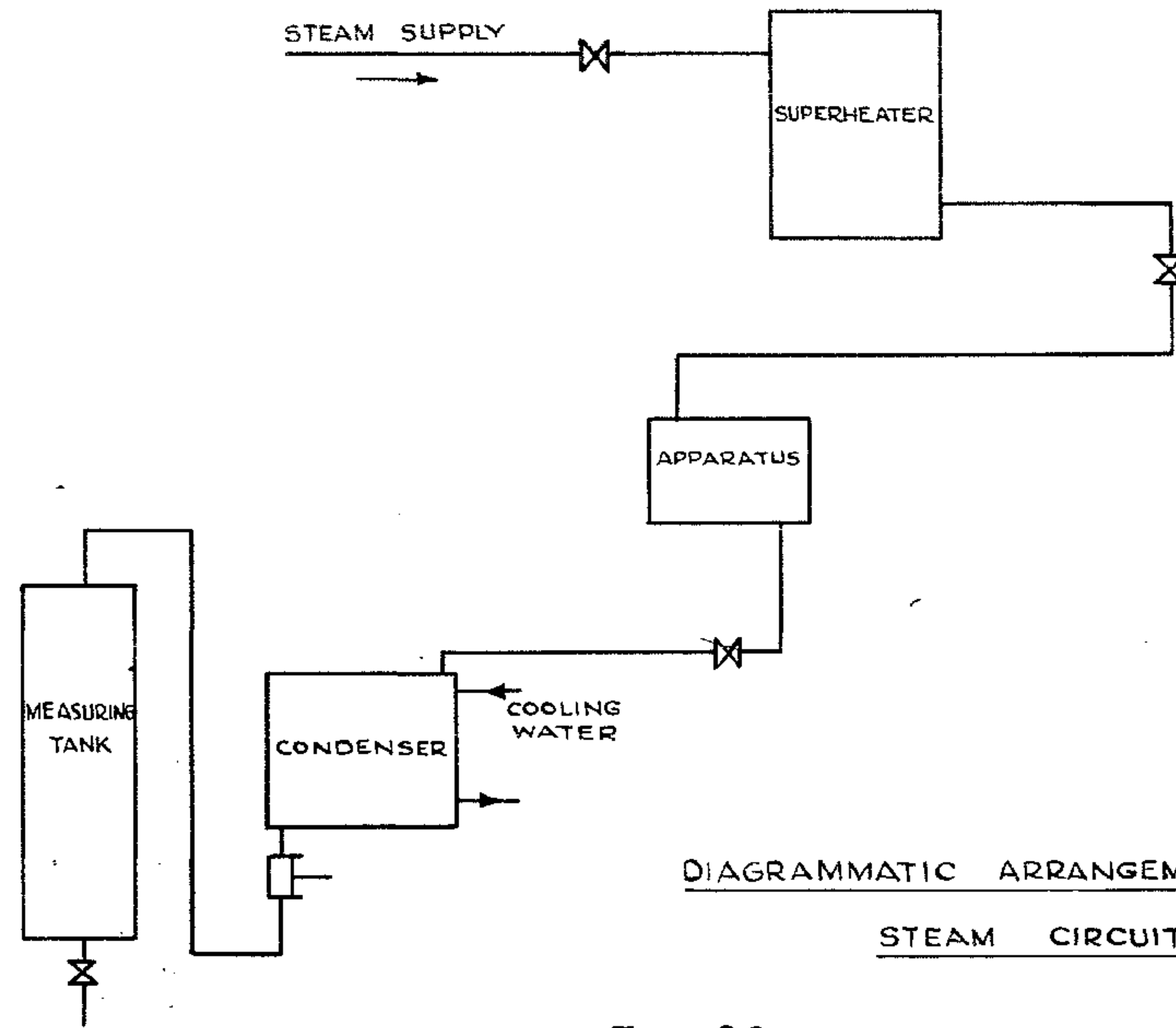
Object of the Experimental Work.

The underlying object is to develop an apparatus in which the methods of previous sections may be applied to typical machine elements and of course to examine these elements for flow patterns and friction factors.

It was decided to use superheated steam as the working fluid because of its availability and also due to the ease with which its density in a test section may be controlled and varied.

A nozzle of rectangular cross-section was manufactured to suit a standard set of blading which were essentially of impulse design. An impact tube and its associated traversing gear was then developed by means of which the flow patterns at either the nozzle or blade exit could be examined as near to true working conditions as possible. For this reason the tube and the traversing gear had to be of fairly robust construction so that they would operate efficiently in an atmosphere of relatively high pressure and temperature.

Description of the Apparatus.



DIAGRAMMATIC ARRANGEMENT OF THE STEAM CIRCUIT.

FIG. 38.

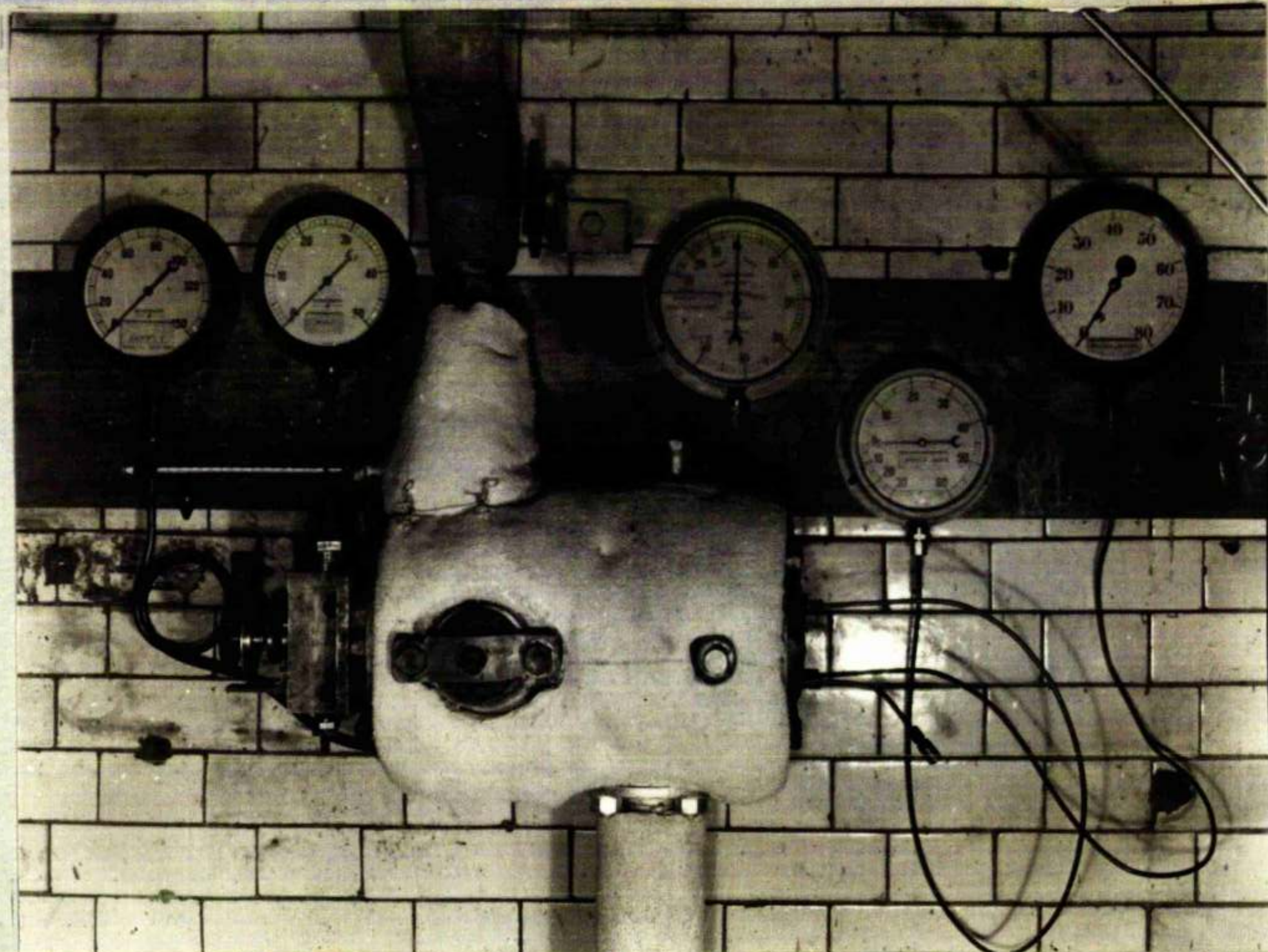


Plate 1.

General View of the Apparatus.

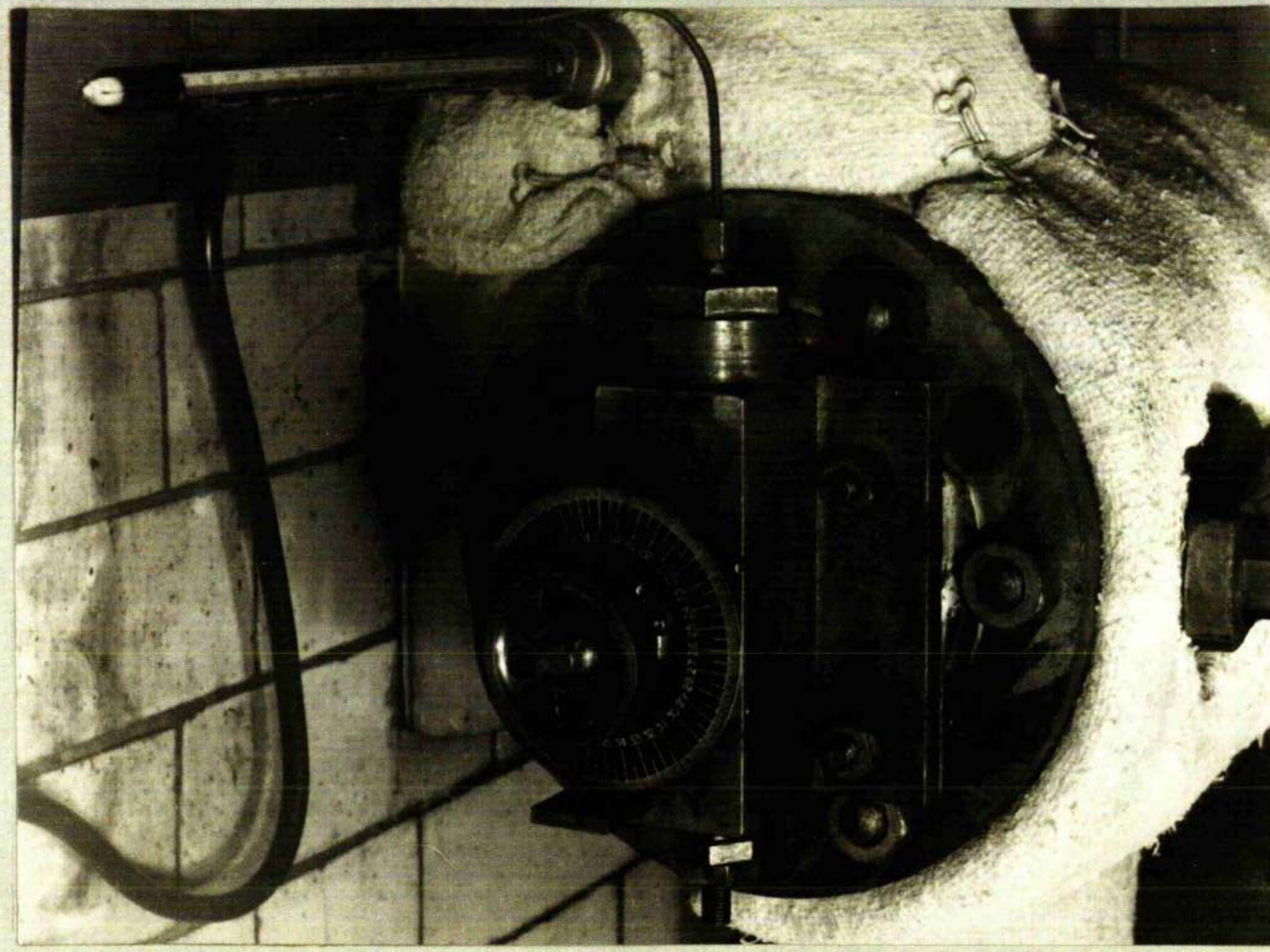
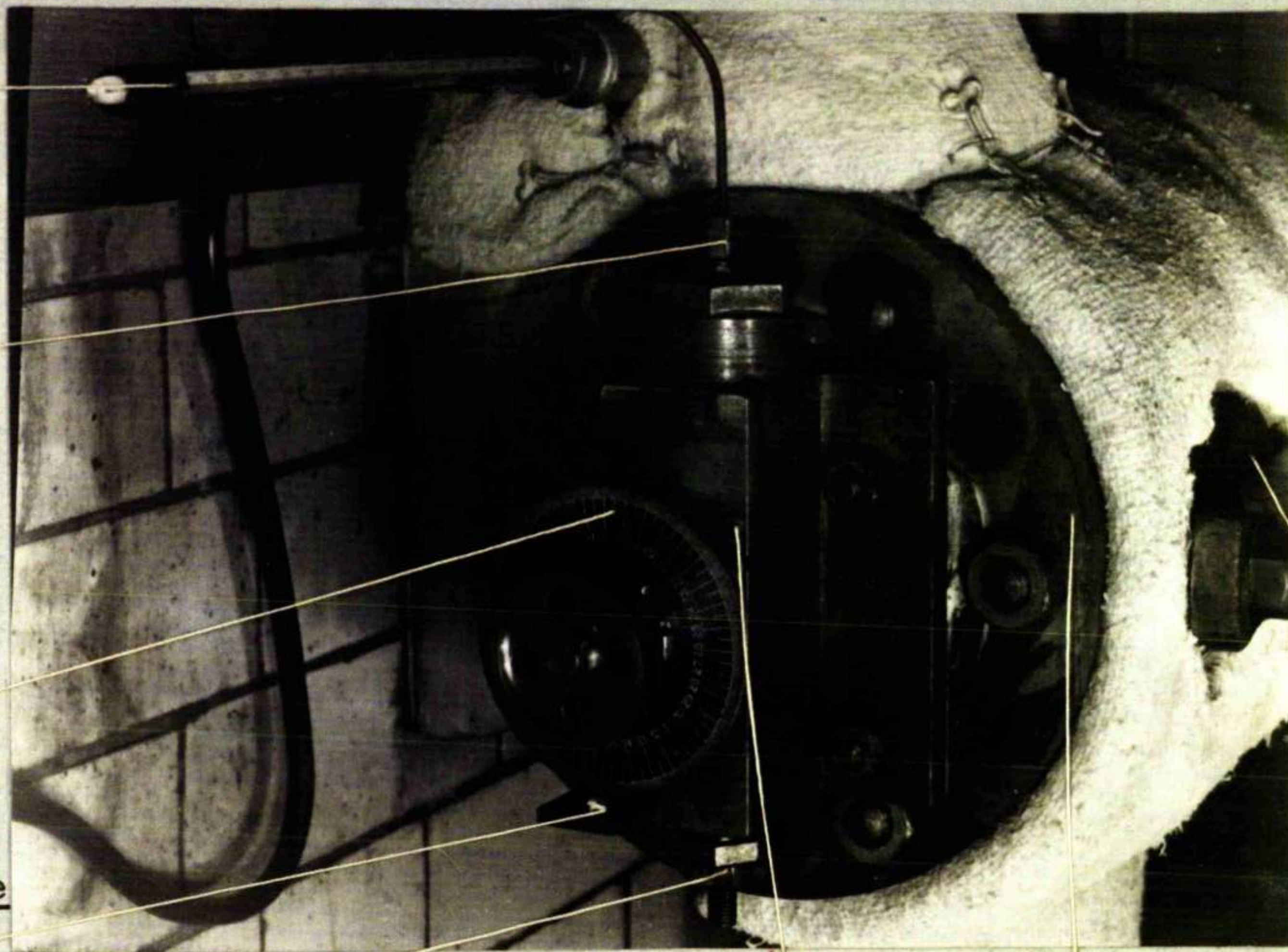


Plate 2.

Traversing Gear Mounted on End Cover.



Thermometer.

Impact Tube Pressure Connection.

Protractor.

Horizontal Scale Bracket.

Nut for Vertical Movement.

Main Block.

End Cover.

Inspection Port.

Plate 3.

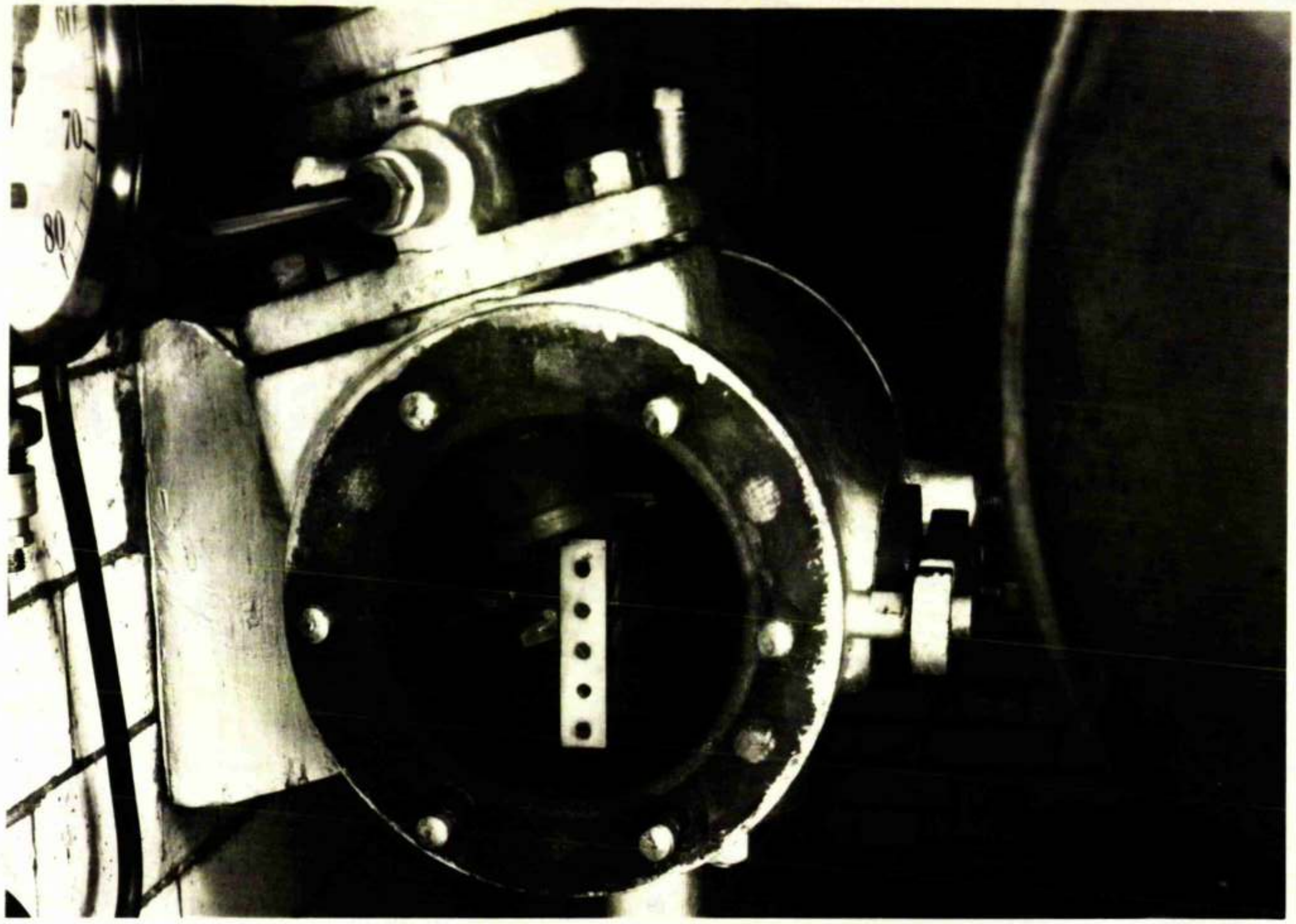


Plate 4.

Casing Interior.

Description of the Apparatus.

A diagramatic arrangement of the steam circuit is shown in Fig. 38 (Page. 99). Wet steam is passed from a boiler through a gas fired superheater to a casing which contains the nozzle and blade elements. After passing through this test section the steam is exhausted to a surface condenser and the condensate may be measured in a graduated vessel. Throttle control valves are fitted before and after the superheater and between the casing and the condenser. By this arrangement considerable variation of the inlet state of the steam and of the exhaust pressure may be obtained.

The principal parts are:-

(1) The casing.

(2) The nozzle and blades.

(3) The impact tube traversing gear.

The casing is shown on Plates 1, 2, 3, & 4, and is a cast iron pipe eight inches internal diameter by fourteen inches long fitted with a nozzle box, two end cover plates, a side inspection port, an exhaust opening and an arrangement for mounting the blades known as the blade carrier.

The nozzle box attaches to the steam supply pipe and is offset from the vertical at an angle of 15^{0}. It is fixed to the casing by four studs which, passing through elongated holes, allow lateral movement of the nozzle box and hence of the nozzle position within the casing. The nozzle box is fitted with a thermometer.

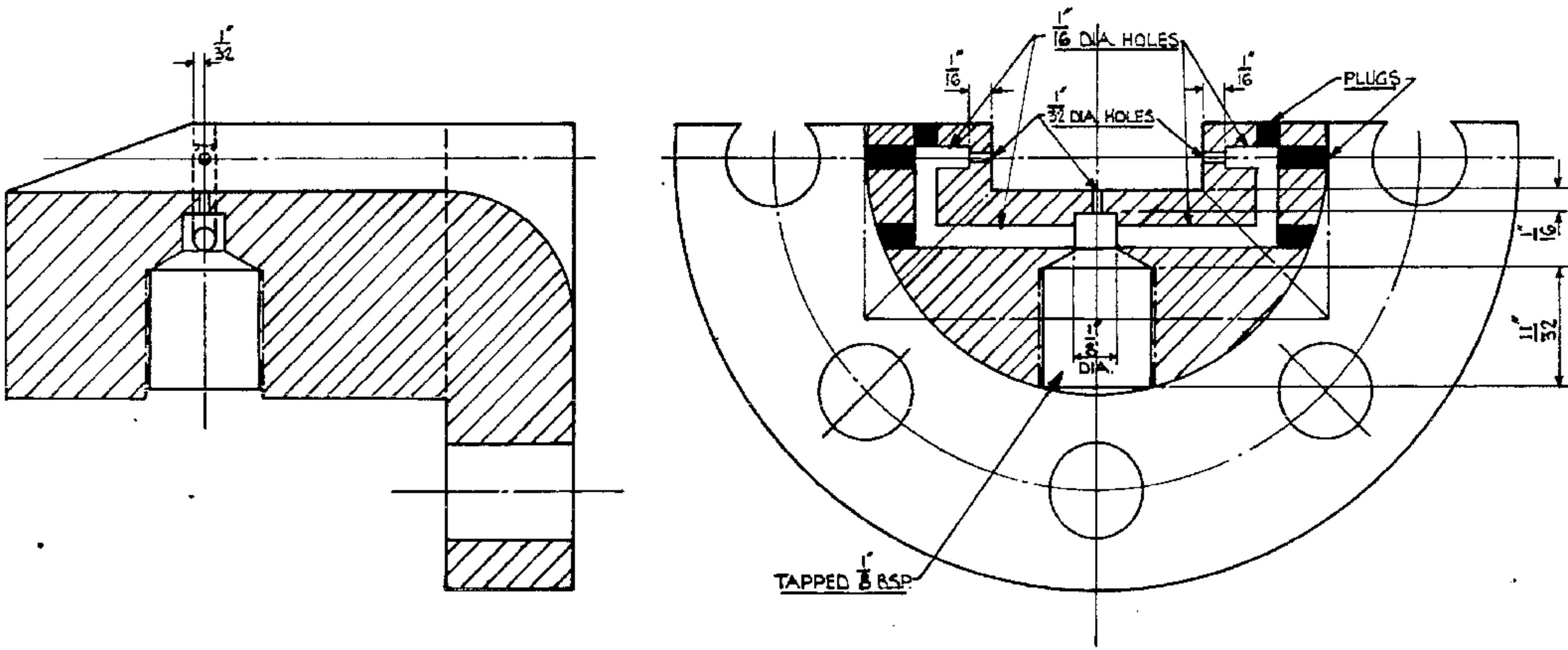
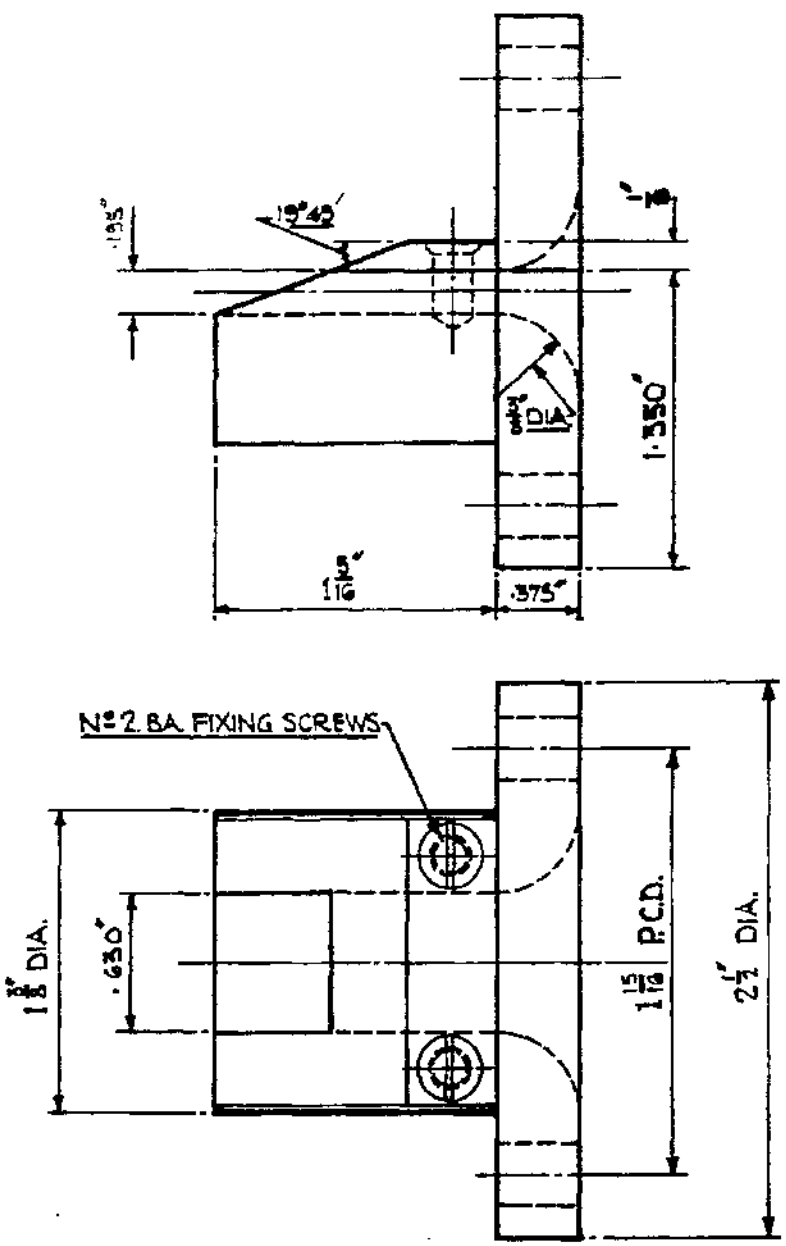


FIG 39 DETAIL OF PRESSURE TAPPING

SCALE:- TWICE FULL SIZE



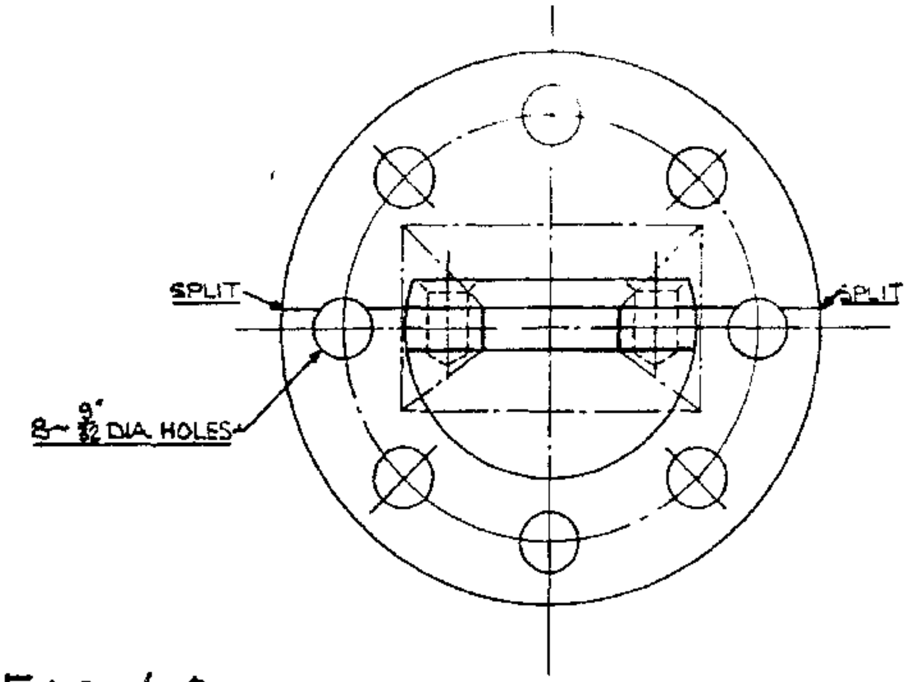


FIG 40

ARRANGEMENT OF RECTANGULAR NOZZLE

CONVERGENT PARALLEL

SCALE :- FULL SIZE

NOTE :- FOR DETAIL OF PRESSURE TAPPING SEE SEPARATE DETAIL

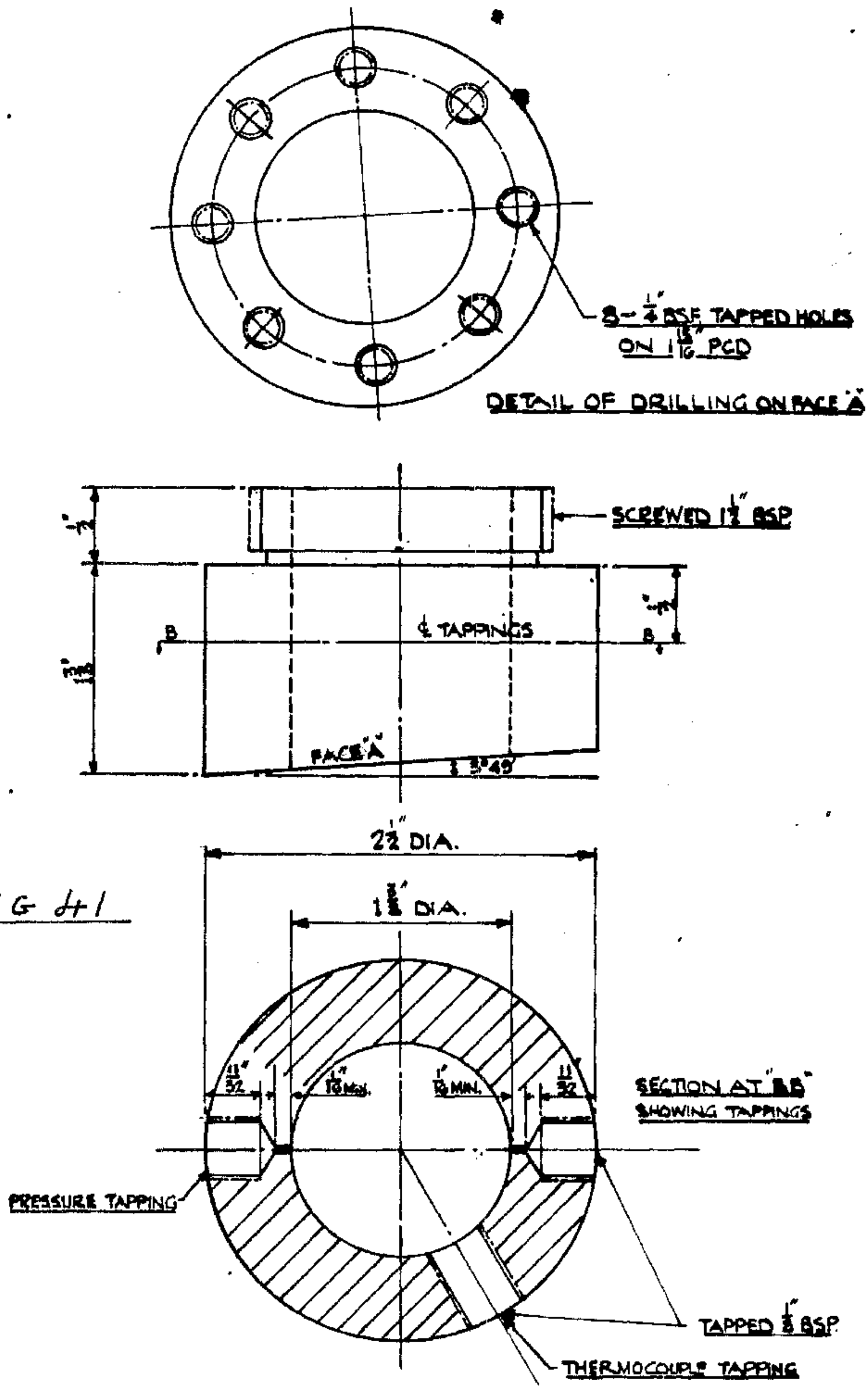


FIG 41

DETAIL OF ADAPTER PIECE

SCALE:- FULL SIZE

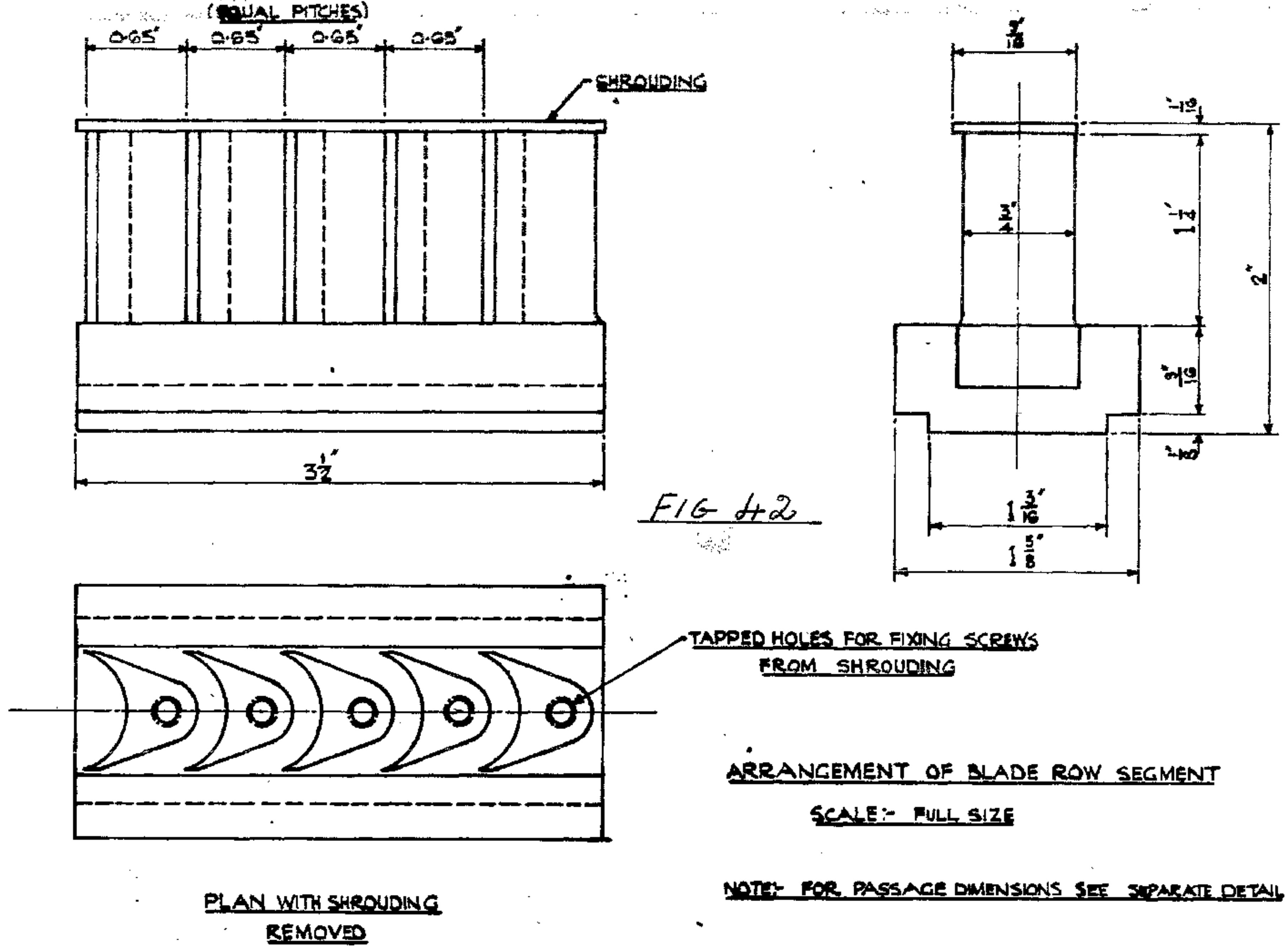


FIG 42

ARRANGEMENT OF BLADE ROW SEGMENT

SCALE:- FULL SIZE

NOTE:- FOR PASSAGE DIMENSIONS SEE SEPARATE DETAIL

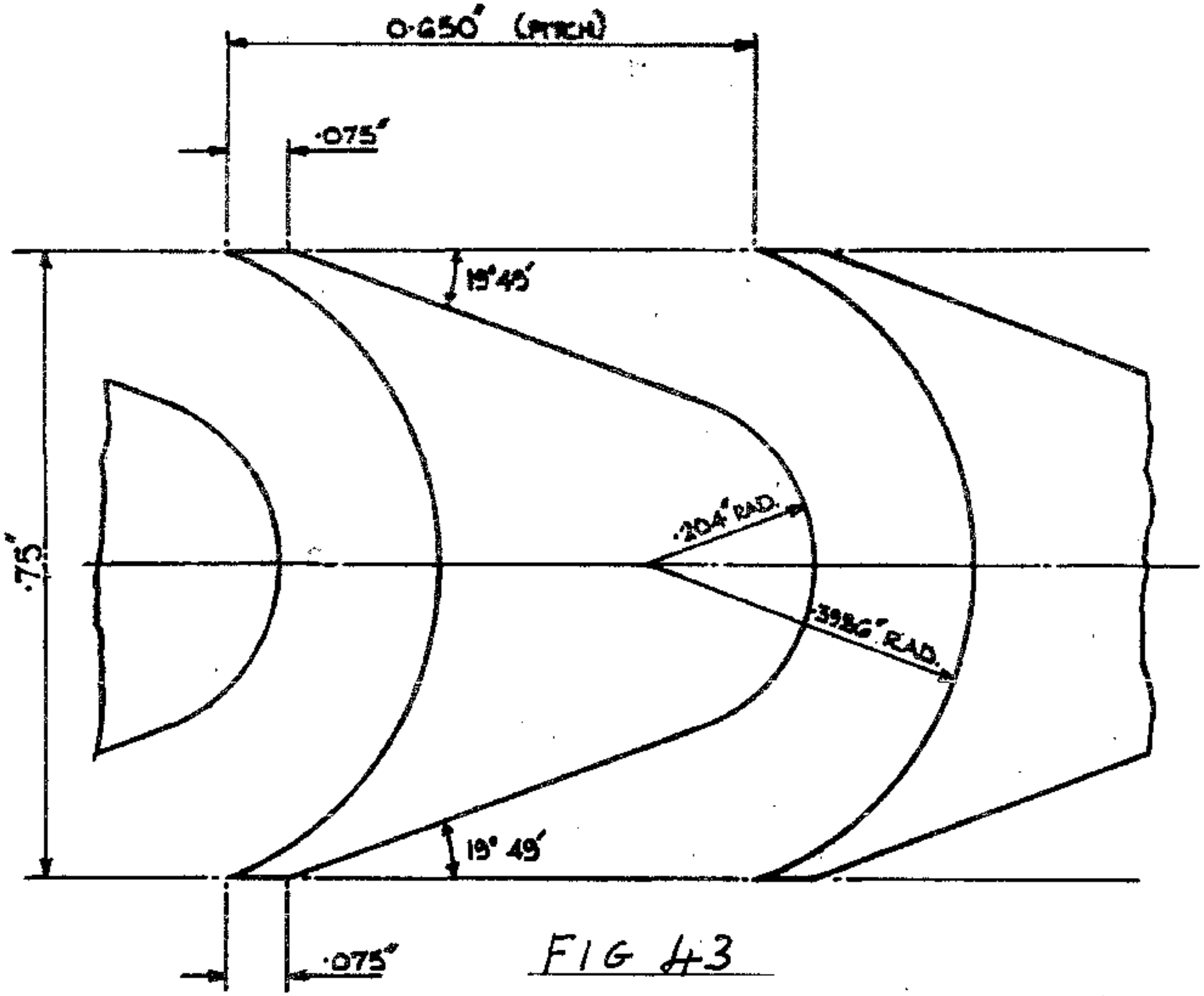


FIG 43

DETAIL OF BLADE PASSAGE (PARSONS S630)

SCALE:- 4 TIMES FULL SIZE

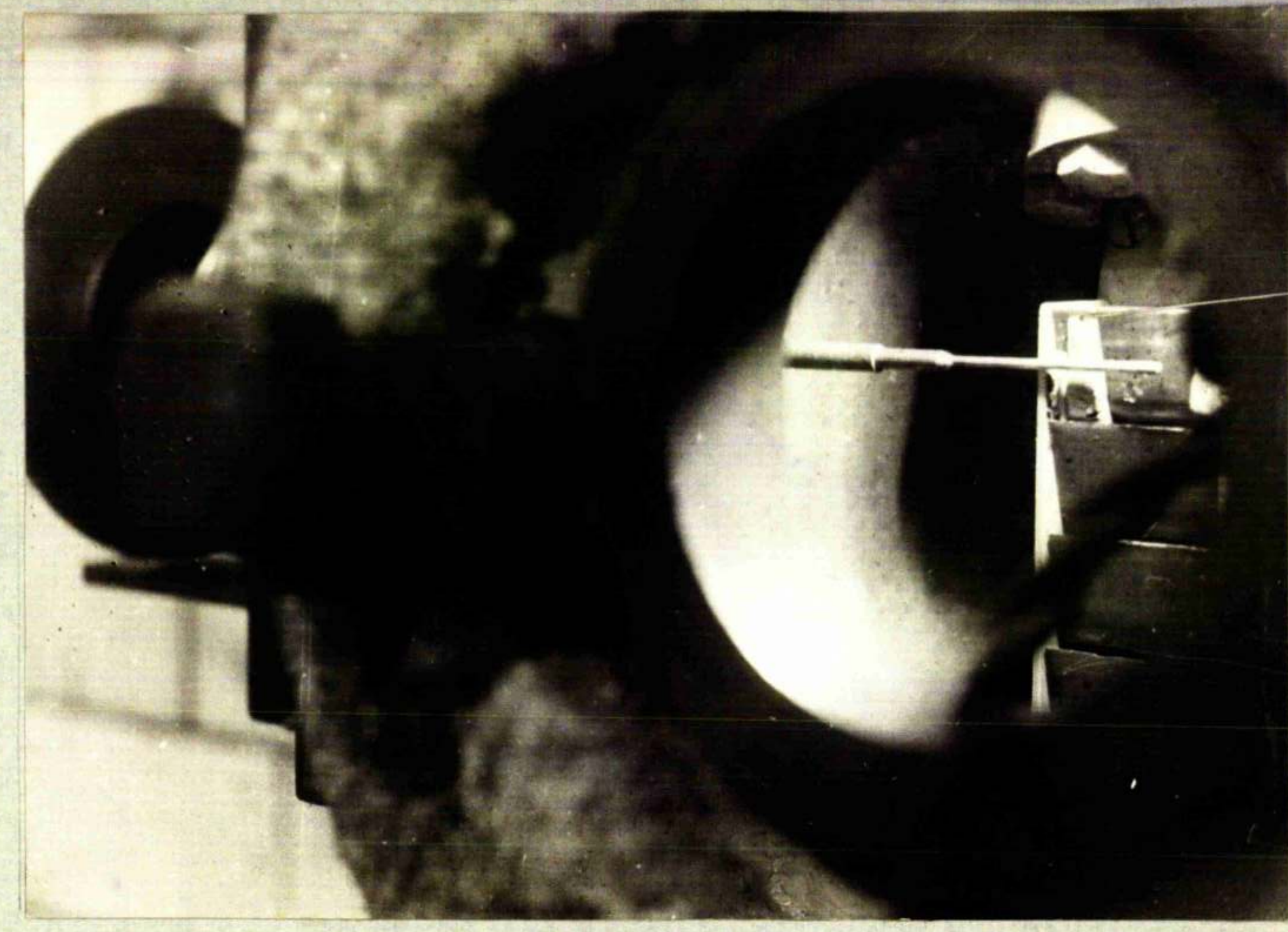


Plate 5.

View Through Inspection Port.

One end cover has a rectangular opening and four studs for mounting the traversing gear. The other cover has pipe connections for pressure tappings from the nozzle.

The blade carrier, Plate 4, is a lever arrangement mounted within the casing and pivoted on screwed trunnions, which pass through the wall of the casing.

The other end of the lever has a vertical face on which the blading is fixed by friction plates fastened by four screws. Lateral movement of the blading is obtained by varying the relative position of the two trunnions while the vertical position depends on where the blades are clamped to the carrier face.

The nozzle is convergent parallel of rectangular section and is shown on Plate 4, and on Figs. 39 & 40, (Pages 105 & 106). It was manufactured in two parts to facilitate machining. The nozzle is mounted onto the nozzle box through an adapter piece so that the nozzle outlet angle is equal to the blade inlet angle of $19^{\circ} - 49'$. The adapter piece Fig. 41, (Page 107) is fitted with a static wall pressure tapping and with a tube which measures the total head pressure at inlet to the nozzle. Tappings of pressure are taken from three of the nozzle passage walls at exit and lead to one chamber. Thus the average nozzle exit pressure is obtained.

The blading used was a Parsons set of essentially impulse design and is shown on Plates 4 & 5 and on Figs. 42 & 43.

Packing pieces were introduced into the blade passages

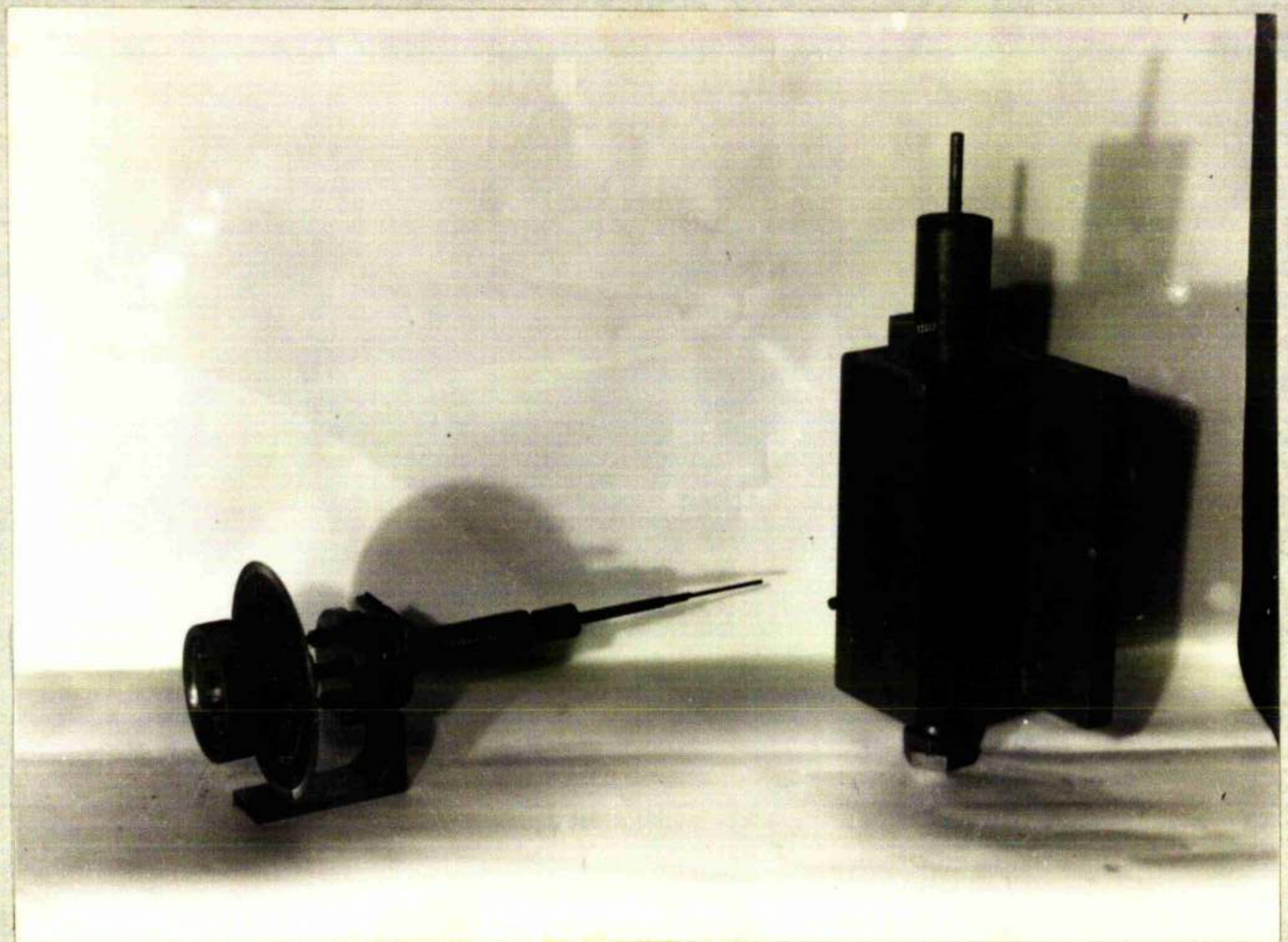
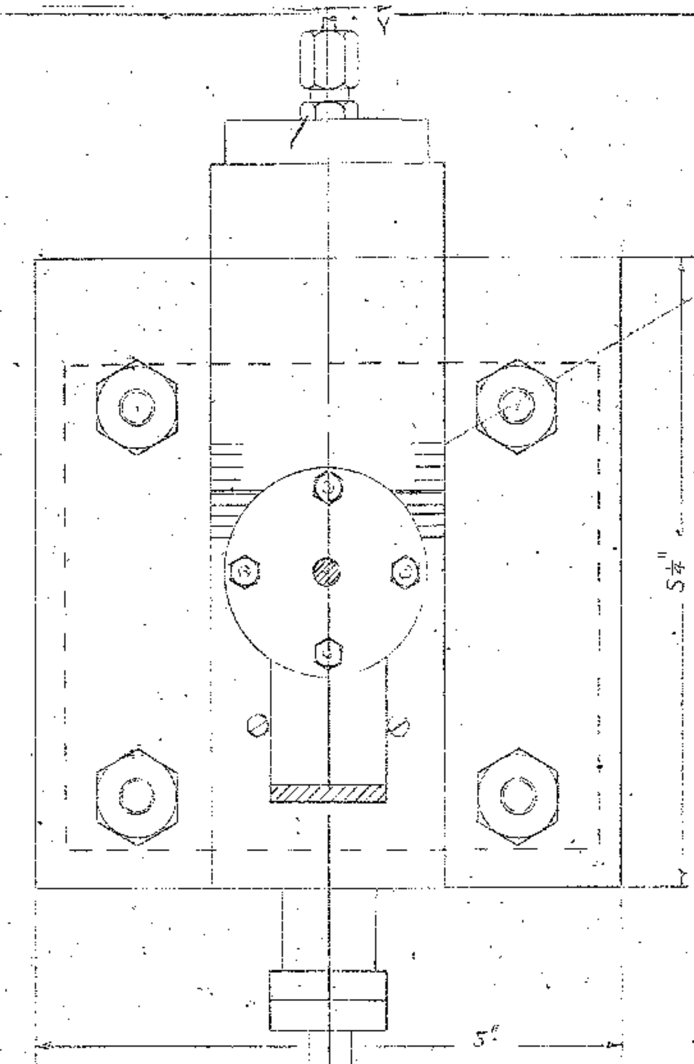
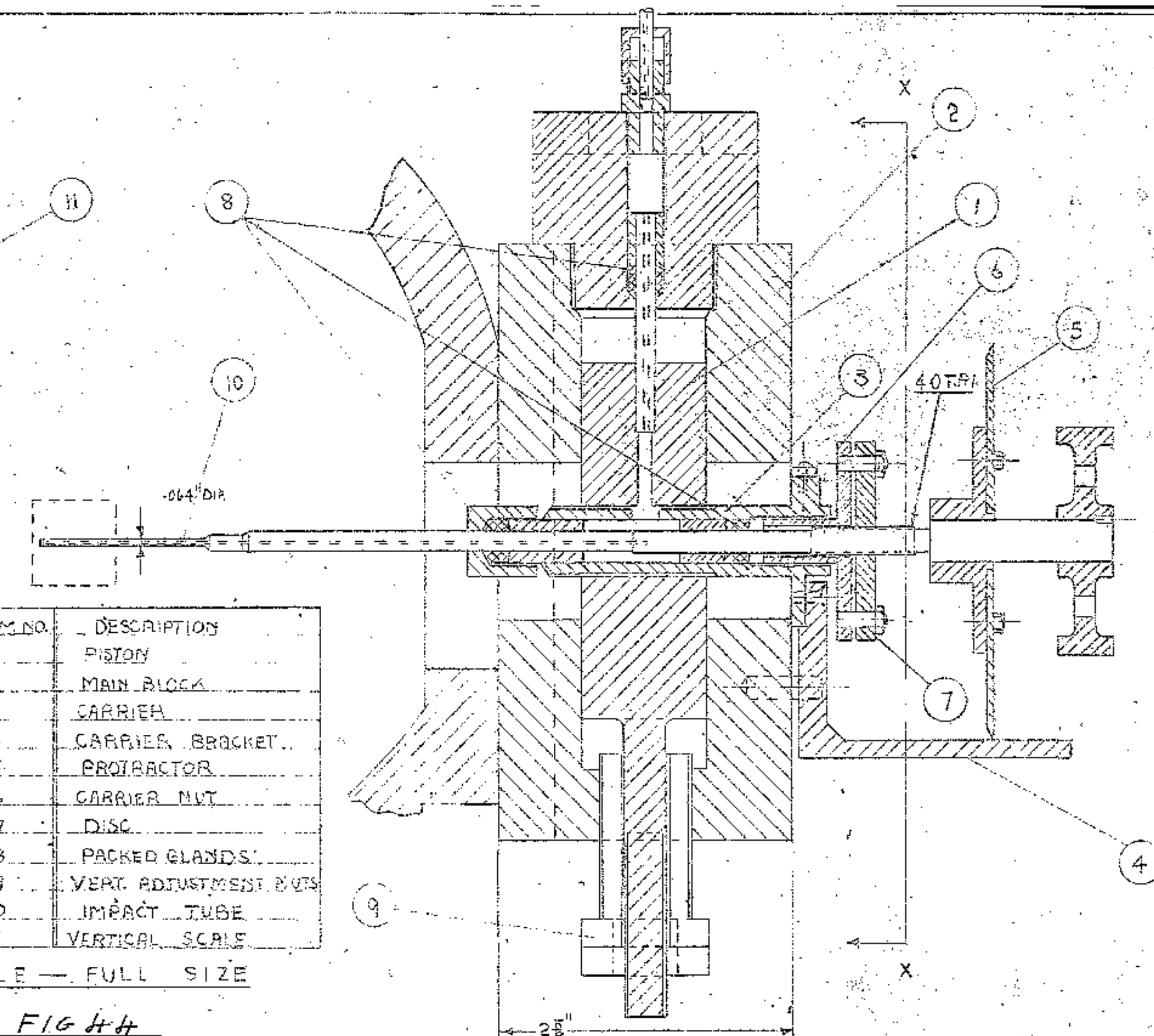


Plate 6.

Traversing Gear.



VIEW ON XX



ITEM NO.	DESCRIPTION
1	PISTON
2	MAIN BLOCK
3	CARRIER
4	CARRIER BRACKET
5	PROTRACTOR
6	CARRIER NUT
7	DISC
8	PACKED GLANDS
9	VERT. ADJUSTMENT NUTS
10	IMPACT TUBE
11	VERTICAL SCALE

SCALE — FULL SIZE

FIG 44

SECTION ON YY

IMPACT TUBE TRAVERSING GEAR

so that the effective blde height could be varied. The packing pieces are shown in position on Plate. 5. The impact tube traversing gear is shown on Plate 6 (Page 112) and in the drawing on Fig. 44. The traversing gear is designed so that the impact tube hole can be located at any point in the exit plane of the nozzles or blades. Thus it must be capable of vertical and horizontal movement as well as being able to rotate.

The main block (item 2) is clamped to the end cover facing the nozzle and has a vertical hole machined in it into which a piston (item 1) is lapped. At right angles to the axis of the piston near the middle of its length there is a screwed hole. This hole takes the impact tube carrier (item 3). The impact tube runs through the carrier and is located centrally in the carrier by sleeves. Pressure packing glands (item 8) are fitted at both ends of the carrier. Item 6, the carrier nut is screwed into the carrier and controls the pressure on the gland packing. Bolted to the carrier nut is a disc (item 7) which is screwed internally with 40T.P.I. and which engages with a screwed portion of the impact tube. Thus rotation of the impact tube causes a horizontal movement of the tube. This movement is measured on a scale scribed on the face of the carrier bracket (item 4).

A protractor is fitted to measure the angular position of the impact tube hole and a knurled nut is used to rotate the tube.

Vertical movement is obtained by two nuts (item 9) which can be arranged to push up or pull down the piston. The piston, carrier, impact tube and carrier bracket move as one unit.

The impact tube (item 10) consists of four lengths of tubing of increasing diameterbrazed together. The largest is a length of steel bar screwed for a part of its length with a 40 T.P.I. thread. The portion of the tube within the fluid stream is made of hypodermic tubing 0.0625" external diameter and with an impact hole 0.010" diameter at $\frac{3}{8}$" from its end.

The pressure at the impact hole is communicated through the tubing to approximately the centre of impact tube and is released into a chamber within the carrier. This chamber is pressure sealed by packing glands mentioned previously. The chamber is connected through a vertical hole in the piston to a tube brazed to the top of the piston. The tube is pressure sealed in a large nut which screws into the top of the main block. Hence the impact tube pressure can be taken off from a simplex coupling at the top of the large nut. By this means the impact tube pressure may be observed without any direct pipe coupling onto the impact tube itself. In addition the parts of the tube used in the operation remain cool for handling.

Considerable attention was paid to the alignment of the protractor relative to the impact tube hole. The protractor was set so that when the impact tube hole was at T.D.C. the zero mark on the protractor was at B.D.C. The impact tube was then clamped in a V-block and placed in a Hilger projector with the hole at approximately 90° from T.D.C. The outline of the tube was magnified on the screen 50 times and the impact hole showed up clearly. The tube was then rotated until the reflection on the screen showed the hole to be equidistant from the edges of the tube. Thus the hole was accurately positioned at 90° to T.D.C. The V-block was then placed on a surface table and by means of a height gauge the protractor was set so that the 0° and 180° marks were at 90° to T.D.C. The protractor was then clamped to the tube by the three screws provided.

The pressures recorded were taken from wall tappings at the nozzle inlet and at the nozzle outlet, and also from the impact tube and from a tapping in the casing. In addition a fixed impact tube was placed upstream of the nozzle inlet but it was found that the velocity head here could be neglected and readings from the inlet nozzle wall tappings were taken to give the inlet total head pressure.

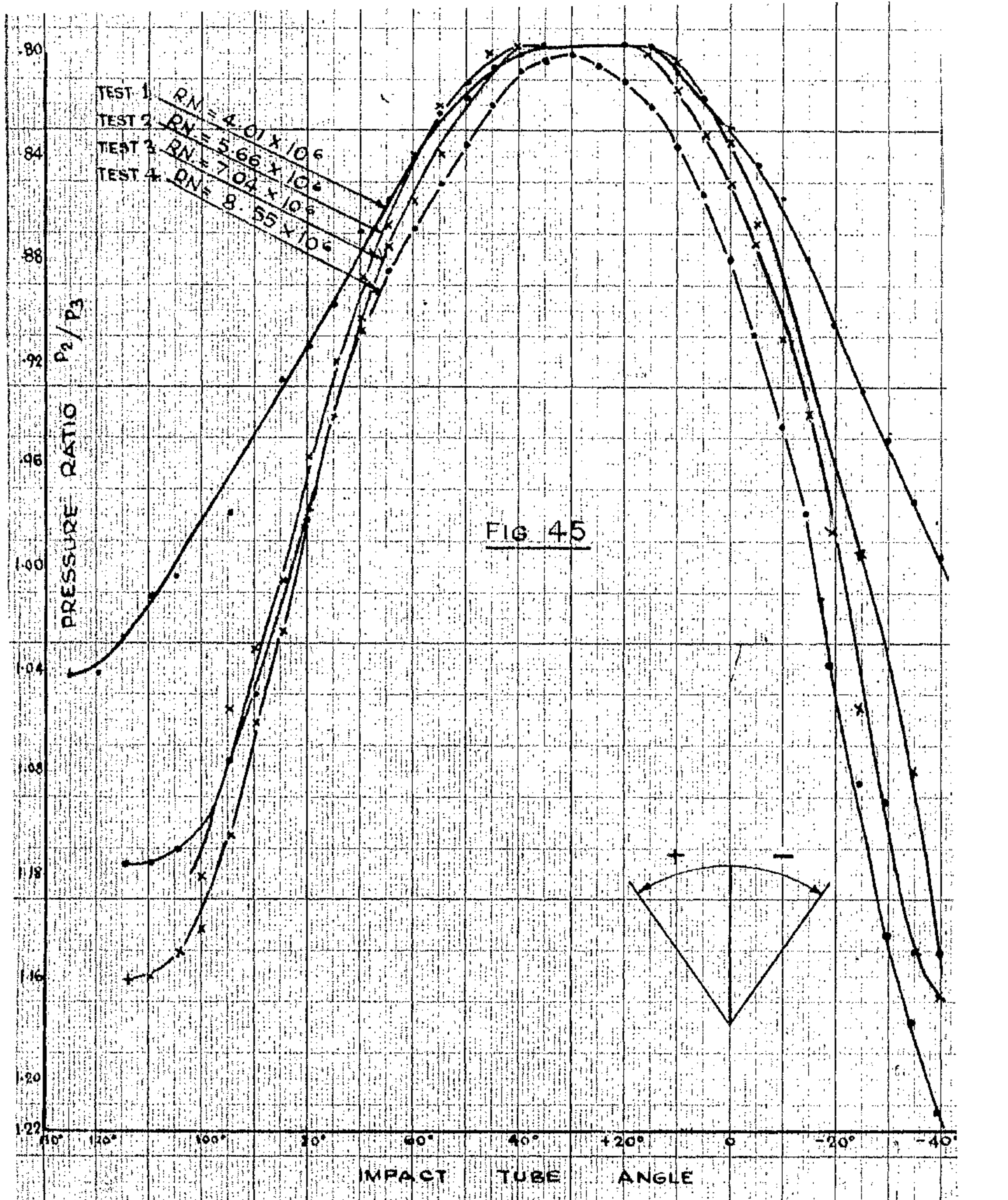
Preparation of the apparatus for testing.

The nozzle was screwed into position so that the exit plane of the nozzle was vertical and parallel to the axis of the casing. Horizontal and vertical lines 0.10" apart were scribed on the nozzle face to divide the exit area regularly. Allowance was made for the fact that the actual flow area to be traversed would be lower than the exit face of the nozzle due to the clearance between the face and the hypodermic tubing. The traversing gear was then bolted to the end cover and corresponding horizontal and vertical lines were scribed on the carrier bracket and on the main block. The pressure gauges were accurately calibrated and a special large scale test gauge was used for the impact tube pressure. The procedure for examining the flow at the blade exit was similar to that at the nozzle exit.

<u>Limitations set by the experimental rig on the applicability of the experimental results.</u>

(a). It should be noted that the packing pieces which control the effective blade height were adjusted at each blade test so that the static pressure at the nozzle outlet was equal to that at the blade outlet. Thus the blade passage is forced to operate as a zero pressure drop impulse blade (figure 9E). The observed variation of loss with Reynolds number and Mach number will therefore apply to such a zero pressure drop impulse stage, one example of which is the partial admission stage in a steam turbine. Here the turbine disc runs in a constant pressure medium, the static pressure on either side of the disc being maintained the same by drilling balance holes through the disc.

(b). The nozzle outlet pitch was made equal to the blade pitch so that the steam flow covers one blade pitch only and the whole blading cascade does not therefore run full in the pitchwise direction. This is particularly important when considering the observed variation in efflux angle in the pitchwise direction, as the variation would be influenced by neighbouring blades running full of steam.



ONE DIMENSIONAL TRAVERSE AT CENTRE OF NOZZLE EX

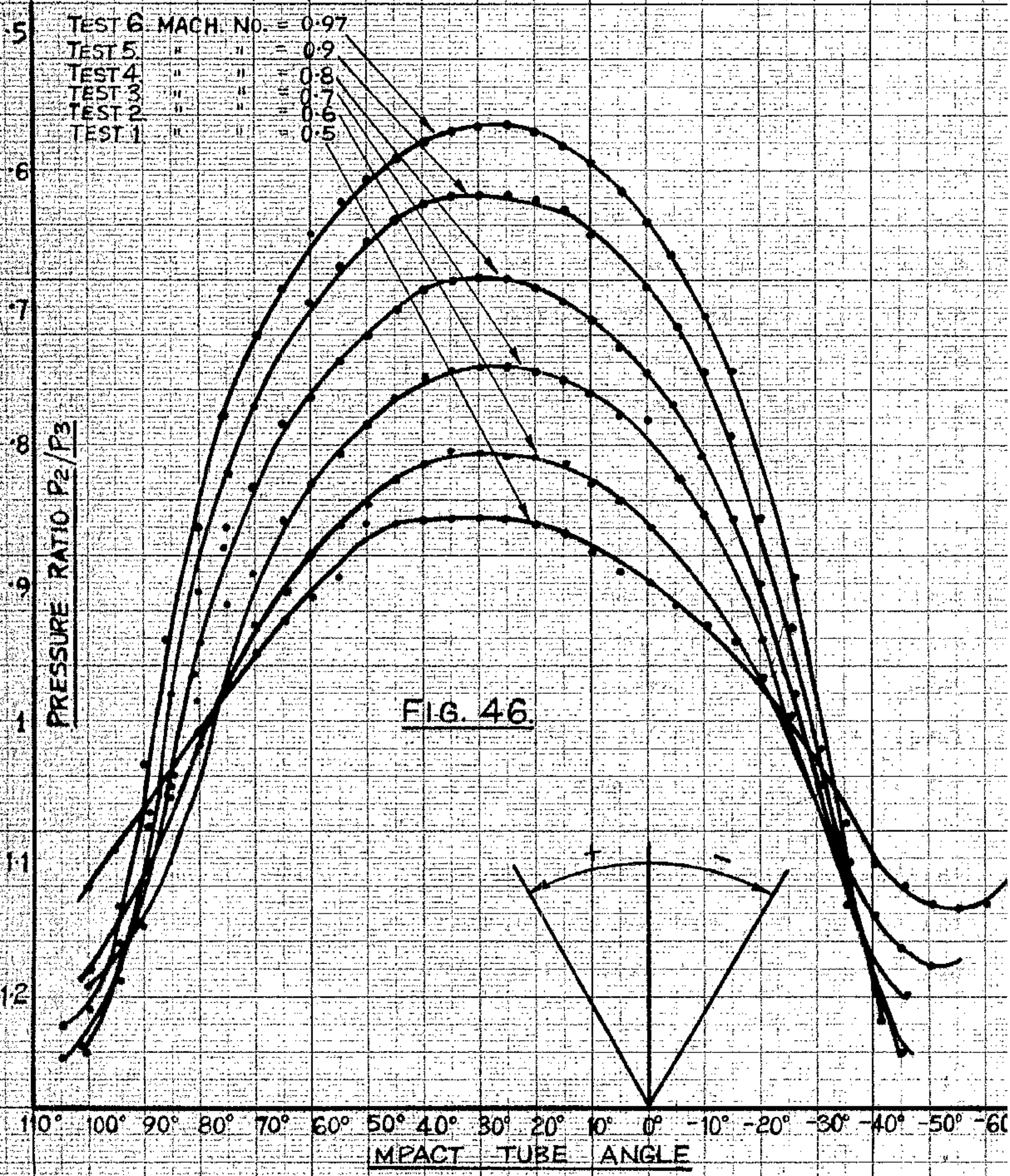


FIG. 46.

ONE DIMEMSIONAL TRAVERSE AT CENTRE OF NOZZLE EXIT

Test Programme

A preliminary investigation of the characteristics of the impact tube was first made with the impact tube hole situated at the centre of the nozzle exit. Figures 45 and 46 show experimental readings of the variation of pressure round the impact tube, for various Reynolds numbers and Mach numbers, as the position of the hole is varied by rotating the tube. The correct alignment of the impact tube hole with the absolute direction of the stream is essential if the maximum impact tube pressure is to be observed.

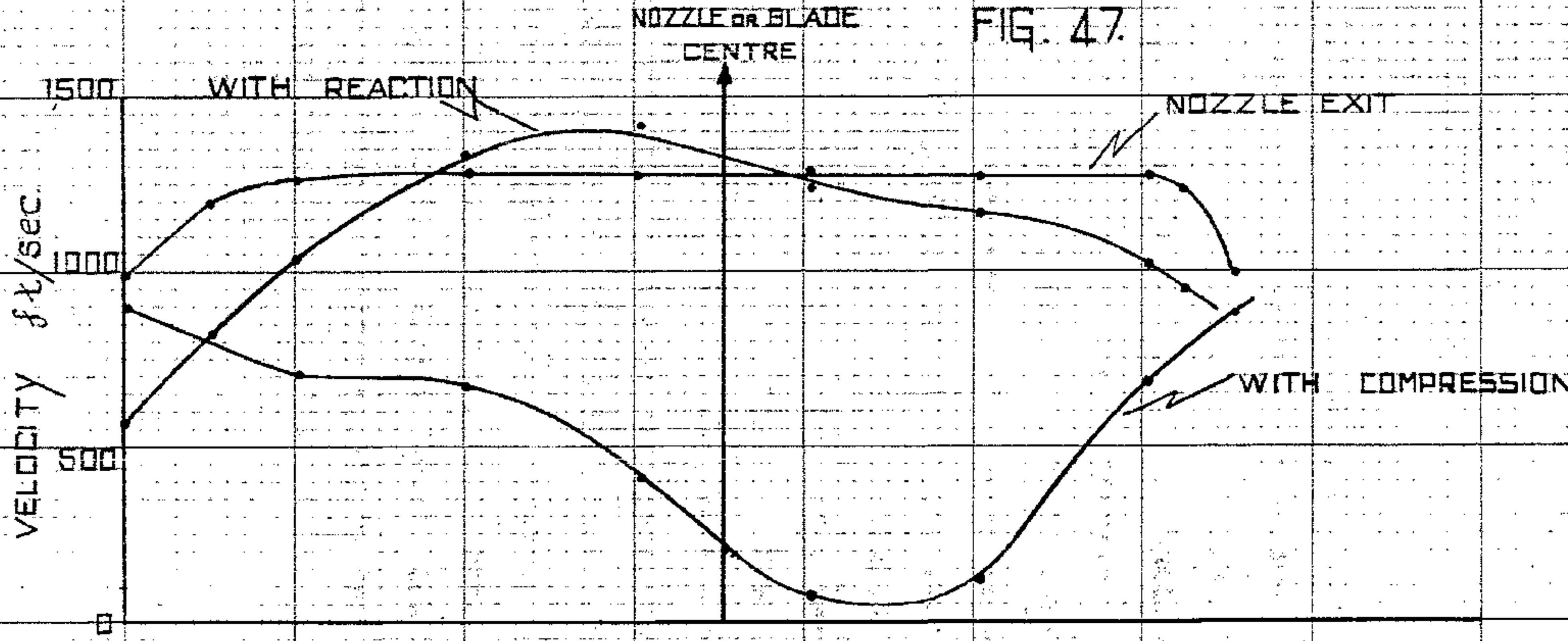
The local direction of the stream was found by rotating the tube in one direction and observing an angle at some suitable arbitrary pressure. The tube was then rotated in the opposite direction until the same pressure was obtained, the new angle being noted. The direction of the stream is then the mean of these two angles and by positioning the tube at this angle the correct total head pressure is obtained.

This is more accurate than rotating the tube until the maximum pressure is observed due to the relative flatness of the top of the curve and the time necessary for the fluid pressure to build up inside the tube.

The dependance of loss on Reynolds number and Mach number indicates a main test series which would show the variation of loss with these parameters. Since the blades were essentially impulse in design, it was decided to test as near to pure impulse conditions as possible. Thus a preliminary test was

COMPARISON OF BLADE EXIT VELOCITY DISTRIBUTION FOR SMALL AMOUNTS OF COMPRESSION AND REACTION ACROSS THE BLADE PASSAGE.

FIG. 47.



TEST CONDITIONS

	NOZZLE ALONE	WITH REACTION	WITH COMPRESSION
NOZZLE INLET PRESSURE.	34·7 LB/IN^2	34·7 LB/IN.^{2}	34·7 LB/IN.^{2}
NOZZLE EXIT PRESSURE	22·7 " "	22·7 "	21·7 " "
BLADE EXIT PRESSURE	—	20·7 "	22·7 " "

arranged during which the blade packing pieces which control the effective height of the blades were adjusted so that there was no pressure drop within the blade passage. During this operation the flow pattern at the blade outlet was observed with firstly a small amount of reaction in the blade passage and secondly with a small amount of compression in the passage. The test conditions along with graphs of exit velocity distribution along a horizontal centre line for the nozzle and blade are shown on Fig. 47 for this preliminary test.

The main tests are grouped into three series and for simplicity test conditions, results and calculations for each series are given separately.

Series 1 on page 124.

Series 2 on page 151.

Series 3 on page 164.

In all the series the steam remains superheated during expansion to avoid the complication of either supersaturation[35] or wetness. In each series specimen experimental observations, calculations and derived tables only are given here. A complete set of experimental and derived talbes is given in appendix 2.

The symbolism used in the presentation of the test results is the same as that in Part 2 (a) (page 75) and in Part 2 (b) (page 90).

Series 1.

In this series the exit Mach number was kept constant at 0.6 and the Reynolds number varied by changing the inlet nozzle pressure. There are in total eight three dimensional tests, four at the nozzle and four at the blade exit flow area. The nozzle tests are numbered 1 - 4 and the blade tests 1A - 4A.

The nozzle test conditions are given in the following table. The blade test conditions vary only slightly from these due to daily variations in barometric pressure.

Test 1	Test 2	Test 3	Test 4
P_1 = 24.3 lb/in^2	P_1 =35.15 lb/in^2	P_1 =44.15 lb/in^2	P_1 =54.32 lb/in
P_2 = 19.3 lb/in^2	P_2 =28.0 lb/in^2	P_2 =35.15 lb/in^2	P_2 =43.25 lb/in
H_1 =1229.2BTU/lb	H_1 =1229.2BTU/lb	H_1 =1229.2BTU/lb	H_1 =1229.2BTU/l
T_1 = 380^{o}F	T_1 = 383.6^{o}F	T_1 = 386.7^{o}F	T_1 = 389.8^{o}F
M_{2_ϕ} = 0.6	M_{2_ϕ} = 0.6	M_{2_ϕ} = 0.6	M_{2_ϕ} = 0.6
Reynolds number 4.01×10^6	Reynolds number 5.66×10^6	Reynolds number 7.04×10^6	Reynolds number 8.55×10^6

On the following pages the results and calculations for tests 1 and 1A are given in detail. The remaining tests results and tables of calculations are given in appendix 2 (pages 2-32).

The calculations are concerned with the determination of the effective values as described in Part 2.

Specimen Results and Calculations ------ Series 1.

Test 1

Barometric pressure 29.22 ins Hg = 14.3 lb/in^2

$P_1 = 24.3\ lb/in^2$

$P_2 = 19.3\ lb/in^2$ $\qquad \frac{P_2}{P_1} = 0.796 \therefore M_{2\phi} = 0.6$ from Fig 31.

$H_1 = 1229.2$ BTU/lb $\qquad DH_\phi = 20.74$ BTU/lb

$T_1 = 380^oF$ $\qquad T_{2\phi} = 334.5^oF$

$$V_{2\phi} = \frac{1.253\ (H_1 - DH_\phi - 835)}{P_2} = 24.25\ ft^3/lb$$

$$v_{2\phi} = 223.8\sqrt{DH_\phi} = 1020\ ft/sec.$$

viscosity $\mu_{2\phi} = 3.26 \times 10^{-7}\ lb\ sec/ft^2$ from appendix 1 Fig. 66 (also ref. 36).

Reynolds number $R_n = \frac{1 \times 1020 \times 10^7}{24.25 \times 32.2 \times 3.26} = 4.01 \times 10^6$

Test 1A

Barometric pressure 29.32 ins Hg = 14.37 lb/in^2

$P_1 = 24.37\ lb/in^2$

$P_2 = 19.39\ lb/in^2$ $\qquad \frac{P_2}{P_1} = 0.796 \quad \therefore M_{2\phi} = 0.6$

$H_1 = 1229.25$ BTU/lb $\qquad DH_\phi = 20.58$ BTU/lb

$T_1 = 380^oF$ $\qquad T_{2\phi} = 334.9^oF$

$V_{2\phi} = 24.15\ ft^3/lb$ $\qquad v_{2\phi} = 1015$ ft/sec

$\mu_{2\phi} = 3.27 \times 10^7$ lb. sec/ft^2 $\qquad R_n = 3.99 \times 10^6$

Measured flow 14.1 gallons in 1¼ hours.

The observed readings of impact tube pressure P_3 and efflux angle are given on pages126and127for tests 1 and 1A, in Table 1.

TABLE 1 TEST 1. HORIZONTAL STATIONS.

VERTICAL STATIONS.

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°
3	21.5	51	22.3	56	23.3	58	23.6	50	23.6	53	23.7	50	23.7	50	22.4	55	21.3	52.5
$2\frac{1}{2}$	20.5	37	22.7	36.5	24.1	40	24.3	35	24.0	38	23.9	37	23.8	37.5	23.5	38	22.6	37.5
$1\frac{1}{2}$	21.0	31	22.8	29	24.0	30.5	24.3	32	24.3	32	24.3	31	24.3	30.5	23.8	29.5	22.9	29.5
$\frac{1}{2}$	21.0	23	24.1	26	24.3	27	24.3	28	24.3	29	24.1	28.5	24.1	26	24.0	25.5	22.9	25
$-\frac{1}{2}$	21.8	23	23.0	26	24.1	26	24.3	27	24.3	29	24.1	29	24.1	27	23.4	25.5	22.9	24
$-1\frac{1}{2}$	21.8	19	22.9	20	24.2	20	24.3	21	24.1	21.5	24.1	20.5	23.4	19	23.1	18	22.1	18
$-2\frac{1}{2}$	20.1	10	21.3	10	24.1	11.5	24.1	10.5	24.1	12	24.2	10	23.8	11	22.3	12	21	18.5
-3	20.0	3	20.5	3	24.1	9.5	23.8	6.5	23.5	4.5	23.8	5	23.8	8.5	21.6	9	20.3	8.5

TABLE 1 TEST 1A.

HORIZONTAL STATIONS →

↓ VERTICAL STATIONS

STATION	−3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		−2		−1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°
2							20.47	10	20.57	3	20.17	5								
$1\frac{1}{2}$	19.77	10	19.87	10	20.27	10	22.37	10	21.87	6	21.37	2.5	20.67	2.5	20.27	3	20.57	3	20.97	24.5
1	20.07	7	20.37	7	21.07	7	23.17	12.5	23.17	13	23.07	12	22.47	10.5	20.97	7	20.77	10	21.97	24
$\frac{1}{2}$	20.47	7	21.27	7.5	23.07	19	24.07	21	24.07	24	24.17	24	23.87	24	23.57	18	23.37	20	23.27	24
$-\frac{1}{2}$	21.07	33	22.77	33	23.57	35	24.07	37	24.17	37	23.97	36.5	23.97	36	24.07	37.5	22.97	37.5	22.87	40
$-1\frac{1}{2}$	20.27	64	21.07	64	22.57	61	22.87	61	23.87	55	23.97	54	23.67	54	23.47	54	23.37	53.5	22.07	52
−2							19.67	82	20.17	82	20.97	82.5	21.27	81	21.47	80	20.57	81	19.77	81

Measured Flow = 14.1 galls./$1\frac{1}{4}$ hrs.

Table 2 is shown for tests 1 and 1A on pages129and130.

In this table the observed readings are converted to pressure ratio $\frac{P_2}{P_3}$ and to local values of efficiency of expansion.

The values of efficiency are obtained from the pressure ratio using the graphsfor $M_{2\phi} = 0.6$ in Fig 32 (page 78).

Flow area covered by each impact tube reading.

Typical diagrams or maps of the nozzle and blade exit flow areas are given in Figs 48 and 49 (pages134 & 135). In a locality where the rate of change of impact tube pressure was large it was necessary to take more readings. Hence the flow area is split up into smaller areas each represented by one impact tube reading. Where the stations are close together an area factor f(da) is introduced. These area factors are shown in the diagrams. Since the basic stations are 0.1 inches apart the basic elemental area is da = 0.01 in^2 and any smaller elemental area is given by f(da) x 0.01 in^2.

On pages 131 and 132, table 3 for tests 1 and 1A are given.

This table gives the figures for the calculations of $\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{f da}{fV}$ which is proportional to the mass flow given by equation 1 page90.

The table shows local values of

(1) $\eta^{\frac{1}{2}}$ --------- from table 2

(2) sinα--------- from table 1

(3) f(da) -------- from the area factor diagram.

(4) fV --------- from Fig. 34. (page 80).

TEST 1 TABLE 2

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
3	·898	·478	·865	·643	·828	·830	·818	·883	·818	·883	·815	·899	·815	·899	·862	·656	·906	·440
$2\frac{1}{2}$	·941	·272	·850	·716	·801	·973	·796	1·0	·804	·960	·807	·940	·811	·920	·821	·867	·854	·696
$1\frac{1}{2}$	·919	·377	·847	·732	·804	·960	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·811	·920	·843	·752
$\frac{1}{2}$	·919	·377	·801	·973	·796	1·0	·796	1·0	·796	1·0	·801	·973	·801	·973	·804	·960	·843	·752
$-\frac{1}{2}$	·885	·544	·839	·774	·801	·973	·796	1·0	·796	1·0	·801	·973	·801	·973	·825	·845	·843	·752
$-1\frac{1}{2}$	·885	·544	·843	·752	·798	·988	·796	1·0	·801	·973	·801	·973	·825	·845	·835	·794	·874	·598
$-2\frac{1}{2}$	·960	·183	·966	·440	·801	·973	·801	·973	·801	·973	·798	·988	·811	·920	·865	·643	·919	·377
-3	·965	·160	·941	·272	·801	·973	·811	·920	·821	·867	·811	·920	·811	·920	·894	·500	·960	·230

TEST 1a TABLE 2

STATION	−3		−2¾		−2½		−2		−1		0		1		2		3		3¼	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
2							·848	·240	·944	·259	·962	·174								
1½	·981	·087	·976	·110	·956	·202	·867	·632	·887	·535	·908	·430	·939	·283	·956	·202	·944	·259	·926	·343
1	·966	·155	·953	·216	·921	·367	·837	·783	·837	·783	·841	·762	·864	·648	·926	·343	·935	·300	·883	·555
½	·968	·240	·912	·411	·841	·762	·806	·946	·806	·946	·803	·961	·813	·909	·823	·857	·830	·820	·834	·800
−½	·921	·367	·852	·706	·823	·857	·806	·946	·803	·961	·810	·924	·810	·924	·806	·946	·844	·748	·868	·727
−1½	·956	·202	·921	·367	·860	·667	·848	·727	·813	·909	·810	·924	·820	·871	·827	·835	·830	·820	·879	·574
−2							·986	·065	·962	·174	·926	·343	·912	·411	·904	·450	·944	·259	·981	·087

TEST 1 TABLE 3

STATION	−3¼			−3			−2			−1			0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da	$\eta^{\frac{1}{2}}$	Sin α	f da
	f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \sin\alpha\, f da}{fV}$	
3	·692	·777	¼	·803	·829	¼	·911	·848	½	·941	·766	½	·941	·799	½	·949	·766	½	·949	·766	½	·810	·819	¼	·664	·793	¼
	1·028	·131		1·019	·1635		1·009	·383		1·006	·358		1·006	·374		1·005	·362		1·005	·362		1·018	·163		1·03	·128	
2½	·522	·602	⅜	·866	·595	⅜	·987	·663	¾	1	·574	¾	·980	·616	¾	·970	·602	¾	·960	·609	¾	·931	·616	⅜	·835	·609	⅜
	1·039	·1135		1·015	·186		1·001	·475		1	·4305		1·002	·452		1·003	·437		1·004	·437		1·007	·2135		1·016	·188	
1½	·614	·515	½	·856	·685	½	·980	·508	1	1	·530	1	1	·530	1	1	·515	1	1	·508	1	·950	·492	½	·868	·492	½
	1·033	·153		1·014	·205		1·002	·497		1	·530		1	·530		1	·515		1	·508		1·004	·235		1·013	·211	
½	·614	·391	½	·987	·438	½	1	·454	1	1	·470	1	1	·485	1	·987	·477	1	·987	·438	1	·950	·430	½	·868	·423	½
	1·033	·116		1·001	·216		1	·454		1	·470		1	·485		1·001	·471		1·001	·432		1·002	·210		1·013	·1815	
−½	·738	·391	½	·880	·438	½	·987	·438	1	1	·454	1	1	·485	1	·987	·485	1	·987	·450	1	·920	·430	½	·868	·407	½
	1·024	·141		1·012	·1905		1·008	·432		1	·454		1	·485		1·001	·478		1·001	·445		1·008	·196		1·013	·174	
−1½	·738	·326	½	·868	·342	½	·995	·342	1	1	·358	1	·987	·367	1	·987	·350	1	·920	·336	1	·892	·369	½	·773	·309	½
	1·024	·1175		1·013	·1465		1·001	·340		1	·358		1·001	·362		1·001	·345		1·005	·293		1·011	·136		1·021	·117	
−2½	·428	·174	⅜	·664	·174	⅜	·987	·199	¾	·987	·182	¾	·987	·208	¾	·995	·174	¾	·960	·191	¾	·803	·208	⅜	·414	·317	⅜
	1·043	·027		1·03	·042		1·001	·147		1·001	·135		1·001	·154		1·001	·130		1·004	·137		1·019	·0615		1·033	·071	
−3	·400	·052	¼	·522	·052	¼	·987	·165	½	·960	·113	½	·931	·079	½	·960	·087	½	·960	·148	¼	·707	·156	¼	·430	·148	¼
	1·045	·005		1·039	·0065		1·001	·081		1·004	·054		1·007	·0365		1·004	·042		1·004	·071		1·027	·027		1·041	·017	
Σ	0·804			1·156			2·809			2·7895			2·8785			2·780			2·693			1·242			1·0875		

$$\Sigma \frac{\eta^{\frac{1}{2}} \sin\alpha .\, f da}{fV} = 18{\cdot}2395$$

TEST 1a TABLE 3

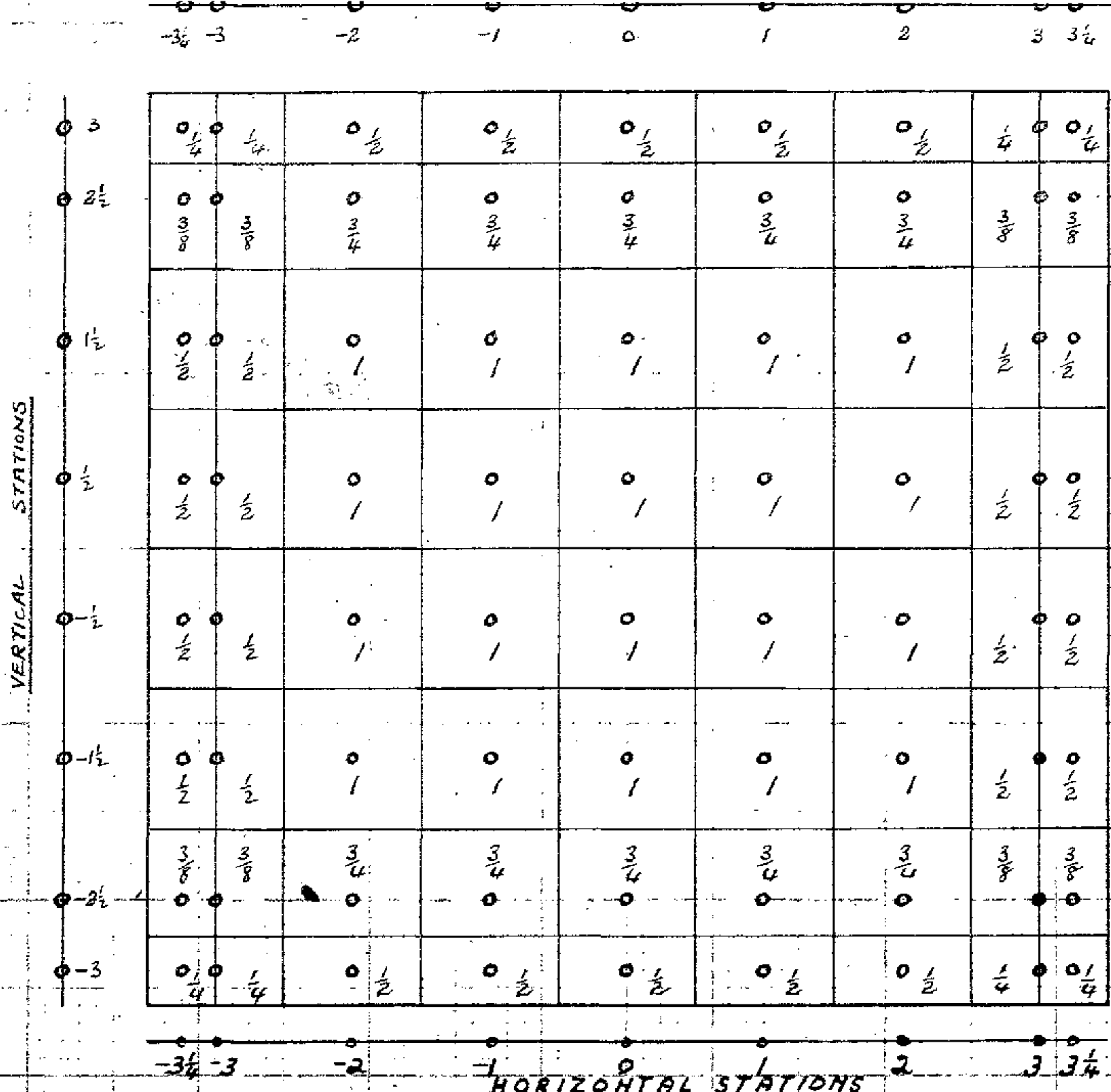
Station	-3			$-2\frac{3}{4}$			$-2\frac{1}{2}$			-2			-1			0			1			2			3			$3\frac{1}{2}$		
	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda
	fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	
2										.490	.174	3/8	.509	.052	1/2	.417	.087	1/2												
										1.04	.031		1.0395	.013		1.044	.017													
$1\frac{1}{2}$	.295	.174	1/8	.332	.174	1/8	.450	.170	3/16	.795	.174	3/8	.731	.105	1/2	.656	.0144	1/2	.532	.0144	1/2	.450	.052	1/2	.509	.056	5/16	.585	.415	3/16
	1.048	.006		1.047	.007		1.0425	.014		1.0195	.051		1.025	.0375		1.03	.014		1.038	.011		1.0425	.011		1.0395	.008		1.035	.0464	
1	.394	.122	1/8	.466	.122	1/8	.606	.122	3/16	.886	.216	3/8	.886	.225	1/2	.875	.208	1/2	.805	.152	1/2	.585	.122	1/2	.547	.174	5/16	.765	.407	3/16
	1.045	.006		1.042	.007		1.034	.013		1.011	.071		1.011	.0895		1.013	.0895		1.019	.072		1.035	.0365		1.037	.029		1.024	.0555	
$\frac{1}{2}$	.490	.122	1/4	.642	.131	1/4	.873	.326	3/8	.972	.358	3/4	.972	.407	1	.980	.407	1	.954	.407	1	.875	.309	1	.906	.342	5/8	.895	.407	3/8
	1.04	.014		1.031	.020		1.013	.105		1.003	.260		1.003	.393		1.002	.398		1.005	.386		1.008	.284		1.009	.1915		1.01	.135	
$-\frac{1}{2}$	.606	.545	1/4	.840	.545	1/4	.925	.574	3/8	.972	.602	3/4	.980	.602	1	.962	.595	1	.962	.588	1	.972	.604	1	.864	.609	5/8	.853	.663	3/8
	1.034	.080		1.016	.113		1.008	.197		1.003	.436		1.002	.588		1.004	.571		1.004	.564		1.003	.590		1.013	.324		1.014	.203	
$-1\frac{1}{2}$	.450	.899	1/8	.606	.899	1/8	.817	.875	3/16	.853	.875	3/8	.954	.819	1/2	.962	.809	1/2	.954	.809	1/2	.914	.809	1/2	.906	.804	5/16	.756	.788	3/16
	1.062	.048		1.036	.066		1.018	.132		1.014	.276		1.005	.388		1.004	.387		1.007	.375		1.009	.366		1.009	.227		1.023	.109	
-2										.255	.990	3/8	.417	.990	1/2	.586	.991	1/2	.642	.988	1/2	.570	.985	1/2	.509	.988	5/16	.295	.988	3/16
										1.05	.090		1.044	.198		1.035	.280		1.031	.307		1.029	.322		1.035	.151		1.048	.052	
Σ	0.154			0.213			0.461			1.215			1.716			1.7565			1.715			1.6075			0.9305			0.5985		

$$\Sigma\, \eta^{\frac{1}{2}} \sin\alpha \,\frac{fda}{fV} = 10.367$$

For each station the products $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$ are obtained and the products are summated.

<u>Test 1</u> $\sum \eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV} = 18.239$

<u>Test 1A</u> $\sum \eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV} = 10.367$



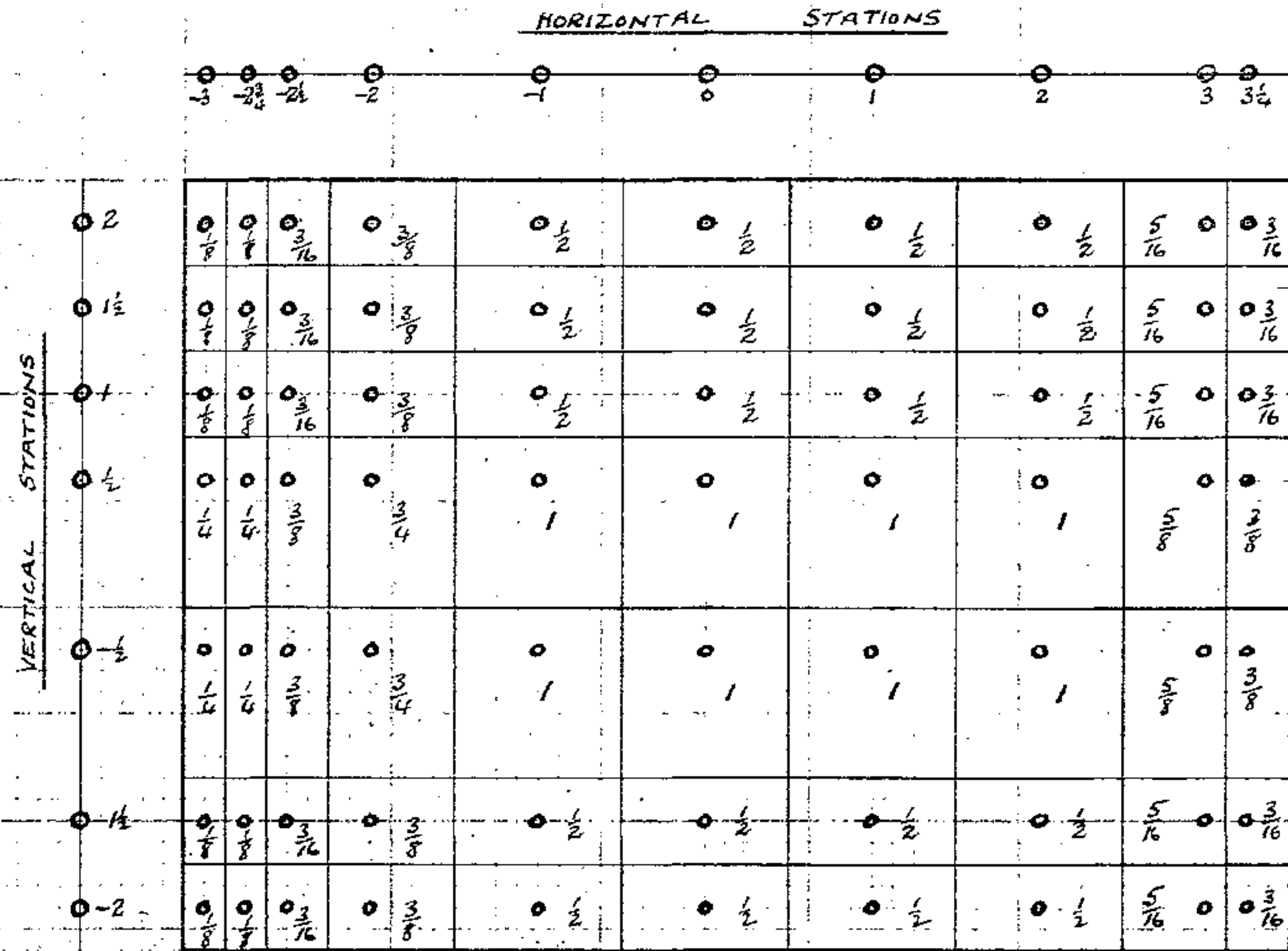
SCALE – 1" ≡ 0·125"

NOZZLE EXIT FLOW AREA

IMPACT READINGS TAKEN AT POINTS MARKED o

SERIES I FIG. 48.

NUMBERS IN RECTANGLES ARE AREA FACTORS (f_{da}) FOR SMALL AREAS SHOWN.



SERIES I FIG. 49.

SCALE:- 1" = 0·125"

BLADE EXIT FLOW AREA

IMPACT READINGS TAKEN AT POINTS MARKED o

NUMBERS IN RECTANGLES ARE AREA FACTORS (f.da) FOR THE SMALL AREAS SHOWN.

On pages 137 and 138 table 4 for tests 1 and 1A are given. These enable one to calculate the mean values of nozzle efficiency, efficiency of expansion for the pair, blade velocity coefficient and blade loss.

The table gives local values of

(1) η from table 2.

(2) $\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$ from table 3.

For each station the product $\eta^{3/2} \sin\alpha \frac{fda}{fV}$ is given and the products are summated.

Test 1 $\quad \Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV} = 16.407$

From equation 6 page 91.

$$\bar{\eta}_n = \left(\frac{\Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV}}{\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}}\right)_n = \frac{16.407}{18.239} = 0.900$$

Similar calculations taking account only of variations along the vertical centre line give

$$\bar{\eta}_{n \, ₵} = \frac{2.798}{2.878} = 0.972$$

Test 1A $\quad \Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV} = 8.060$

From equation 7 page 91 the mean efficiency of the pair $\bar{\eta}_b$ is

$$\bar{\eta}_b = \left(\frac{\Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV}}{\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}}\right)_b = \frac{8.060}{10.367} = 0.778$$

and $\bar{\eta}_{b \, ₵} = \frac{1.441}{1.756} = 0.820$

TEST 1 TABLE 4.

STATION	$-3\frac{1}{4}$	-3	-2	-1	0	1	2	3	$3\frac{1}{4}$
	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$
3	·478 ·131	·643 ·1635	·830 ·383	·883 ·358	·883 ·374	·999 ·362	·899 ·362	·656 ·163	·440 ·128
	·0625	·105	·318	·316	·330	·325	·325	·107	·056
$2\frac{1}{2}$	·272 ·1135	·716 ·186	·973 ·475	1 ·4305	·960 ·452	·960 ·437	·920 ·437	·867 ·2135	·695 ·188
	·031	·133	·462	·4305	·434	·411	·402	·185	·131
$1\frac{1}{2}$	·377 ·153	·732 ·205	·960 ·497	1 ·530	1 ·530	1 ·515	1 ·508	·920 ·235	·752 ·211
	·058	·150	·477	·530	·530	·515	·508	·216	·1585
$\frac{1}{2}$	·377 ·116	·973 ·216	1 ·454	1 ·470	1 ·485	·973 ·471	·973 ·432	·960 ·210	·752 ·1815
	·044	·210	·454	·470	·485	·459	·420	·2015	·1365
$-\frac{1}{2}$	·544 ·141	·774 ·1905	·973 ·432	1 ·454	1 ·485	·973 ·478	·973 ·448	·845 ·196	·752 ·174
	·077	·1475	·420	·454	·485	·465	·436	·1655	·131
$-1\frac{1}{2}$	·544 ·1175	·752 ·1465	·988 ·340	1 ·358	·973 ·362	·973 ·345	·845 ·298	·794 ·136	·598 ·117
	·064	·110	·336	·358	·352	·336	·252	·108	·070
$-2\frac{1}{2}$	·183 ·027	·440 ·042	·973 ·147	·973 ·135	·973 ·154	·988 ·130	·920 ·137	·643 ·0615	·377 ·071
	·005	·0185	·143	·131	·150	·1285	·126	·0395	·027
-3	·160 ·005	·272 ·0065	·973 ·081	·920 ·054	·867 ·0365	·920 ·042	·920 ·071	·580 ·027	·230 ·017
	·001	·002	·079	·080	·032	·039	·065	·0135	·004
Σ	0·3425	0·876	2·689	2·7395	2·798	2·6785	2·534	1·036	0·714

$$\Sigma\, \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 16{\cdot}4075$$

TEST 1a TABLE 4

Station	−3	−2¾	−2½	−2	−1	0	1	2	3	3¼
	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$
2				·240 ·031	·259 ·013	·174 ·017				
				·008	·003	·003				
1½	·087 ·006	·110 ·007	·202 ·014	·632 ·051	·535 ·0375	·430 ·014	·283 ·011	·202 ·011	·259 ·008	·343 ·04[illegible]
	·0005	·000	·003	·032	·020	·006	·003	·002	·002	·015
1	·155 ·006	·216 ·007	·367 ·013	·783 ·071	·783 ·0985	·762 ·0895	·648 ·072	·343 ·0345	·300 ·029	·555 ·05[illegible]
	·001	·0015	·005	·0555	·077	·068	·047	·012	·009	·031
½	·240 ·014	·411 ·020	·762 ·105	·946 ·260	·946 ·393	·961 ·398	·909 ·386	·857 ·284	·820 ·1915	·800 ·13[illegible]
	·003	·008	·080	·246	·372	·383	·351	·244	·157	·108
−½	·367 ·080	·706 ·113	·857 ·197	·946 ·436	·961 ·588	·924 ·571	·924 ·564	·946 ·590	·748 ·324	·727 ·20[illegible]
	·029	·080	·169	·414	·565	·527	·521	·558	·242	·147
−1½	·202 ·048	·367 ·066	·667 ·132	·727 ·276	·909 ·388	·924 ·387	·871 ·375	·835 ·366	·820 ·227	·574 ·109
	·097	·024	·088	·201	·352	·358	·326	·306	·186	·0625
−2				·065 ·090	·174 ·198	·343 ·280	·411 ·307	·450 ·322	·259 ·151	·087 ·05[illegible]
				·006	·034	·096	·1265	·165	·039	·0045
Σ	0·1305	0·1135	0·345	0·9625	1·423	1·441	1·3745	1·267	0·635	0·368

$$\Sigma\, \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 8{\cdot}060$$

From equation 12 page 92 the mean blade velocity coefficient.

$$K = \sqrt{\left(\frac{\Sigma\, \eta^{3/2} \sin\alpha \frac{fda}{fV}}{\Sigma\, \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}}\right)_b \div \left(\frac{\Sigma\, \eta^{3/2} \sin\alpha \frac{fda}{fV}}{\Sigma\, \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}}\right)_n}$$

$$\therefore K = \sqrt{\frac{0.778}{0.900}} = 0.920$$

Similarly $K_{\text{℄}} = \sqrt{\frac{0.820}{0.972}} = 0.920$

The blade loss $= (1 - K^2)\ \bar{\eta}_n\ DH_{\varphi}$

$$= (1 - 0.865) \times 0.900 \times 20.58 = 2.50 \text{ BTU/lb}$$

and the blade loss ℄ $= (1 - 0.844) \times 0.972 \times 20.58 = 3.12$ BTU/lb .

Table 5 for tests 1 and 1A are given on pages 141 and 142 . These tables enable one to calculate mean efflux angles. The tables show the local values of

(1) $\eta^{\frac{1}{2}}$ from table 2

(2) $\cos\alpha$ from table 1

(3) $\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$ from table 3

for each station the product $\eta\cos\alpha\sin\alpha\frac{fda}{fV}$ is given and the products are summated.

Test 1

$$\Sigma\,\eta\cos\alpha\sin\alpha\frac{fda}{fV} = 14.512,\quad \Sigma\,\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV} = 18.239.$$

$$\bar{\eta}_n = 0.900.$$

From equation 13 page 93.

$$\cos\bar{\alpha}_n = \left(\frac{\Sigma\,\eta\sin\alpha\cos\alpha\frac{fda}{fV}}{\Sigma\,\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}\sqrt{\bar{\eta}_n}}\right)_n$$

$$= \frac{14.512}{0.95 \times 18.239} = 0.838.$$

$$\therefore \bar{\alpha}_n = 33^\circ.$$

Test 1A

$$\Sigma\,\eta\cos\alpha\sin\alpha\frac{fda}{fV} = 6.505,\quad \Sigma\,\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV} = 10.367$$

$$\bar{\eta}_b = 0.788$$

From equation 14 page 93.

$$\cos\bar{\alpha}_b = \frac{6.505}{0.883 \times 10.367} = 0.71.$$

$$\therefore \bar{\alpha}_b = 44.7^\circ$$

TEST 1 TABLE 5

STATION	$-3\frac{1}{4}$	-3	-2	-1	0	1	2	3	$3\frac{1}{4}$
	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$ Cos α $\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$
	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$	η Cos α Sin α $\frac{fda}{fV}$
3	·692 ·629 ·131 ·057	·803 ·559 ·1635 ·073	·911 ·530 ·383 ·180	·941 ·643 ·358 ·206	·941 ·602 ·374 ·202	·949 ·663 ·362 ·221	·949 ·643 ·362 ·221	·810 ·574 ·163 ·076	·664 ·609 ·128 ·052
$2\frac{1}{2}$	·522 ·799 ·1135 ·047	·846 ·804 ·186 ·1265	·987 ·761 ·475 ·356	1 ·819 ·4305 ·352	·980 ·788 ·452 ·348	·970 ·799 ·437 ·338	·960 ·793 ·437 ·332	·931 ·788 ·2135 ·156	·835 ·793 ·188 ·124
$1\frac{1}{2}$	·614 ·857 ·153 ·081	·856 ·875 ·205 ·154	·980 ·862 ·497 ·420	1 ·848 ·530 ·450	1 ·848 ·530 ·450	1 ·857 ·515 ·442	1 ·862 ·508 ·437	·960 ·870 ·235 ·195	·868 ·870 ·211 ·159
$\frac{1}{2}$	·614 ·921 ·116 ·0655	·987 ·899 ·216 ·191	1 ·891 ·454 ·404	1 ·883 ·470 ·415	1 ·875 ·485 ·424	·987 ·879 ·471 ·408	·987 ·899 ·432 ·382	·980 ·963 ·210 ·186	·868 ·906 ·1815 ·143
$-\frac{1}{2}$	·738 ·921 ·141 ·096	·880 ·899 ·1905 ·1505	·987 ·899 ·432 ·382	1 ·891 ·454 ·404	1 ·875 ·485 ·424	·987 ·875 ·478 ·413	·987 ·791 ·448 ·393	·920 ·903 ·196 ·163	·868 ·914 ·174 ·138
$-1\frac{1}{2}$	·738 ·946 ·1175 ·0815	·868 ·960 ·1465 ·120	·995 ·960 ·360 ·318	1 ·934 ·358 ·334	·987 ·930 ·362 ·333	·987 ·937 ·345 ·318	·920 ·946 ·298 ·260	·892 ·951 ·136 ·115	·773 ·951 ·117 ·086
$-2\frac{1}{2}$	·428 ·985 ·027 ·011	·664 ·985 ·042 ·027	·987 ·980 ·147 ·142	·987 ·983 ·135 ·131	·987 ·978 ·154 ·149	·995 ·985 ·130 ·128	·960 ·982 ·137 ·129	·803 ·978 ·0615 ·048	·614 ·968 ·071 ·041
-3	·400 ·999 ·005 ·002	·522 ·999 ·0065 ·003	·987 ·986 ·081 ·079	·960 ·994 ·054 ·0515	·931 ·997 ·0365 ·034	·960 ·996 ·042 ·040	·960 ·989 ·071 ·067	·707 ·988 ·027 ·019	·480 ·989 ·017 ·008
Σ	0·441	0·845	2·281	2·3435	2·364	2·308	2·221	0·958	0·751

$$\Sigma\ \eta \ \text{Cos}\,\alpha \ \text{Sin}\,\alpha \ \frac{fda}{fV} = 14{\cdot}5125$$

TEST 1a TABLE 5

Station	−3			−2¾			−2½			−2			−1		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$
		η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$	
2										.490	.985	.031	.509	.998	.013
											.015			.007	
1½	.295	.986	.006	.332	.985	.007	.450	.985	.014	.795	.985	.051	.731	.995	.0375
		.002			.002			.006			.040			.027	
1	.394	.993	.006	.466	.993	.007	.606	.993	.013	.886	.976	.071	.886	.974	.0985
		.002			.003			.008			.061			.085	
½	.490	.993	.014	.642	.991	.020	.873	.966	.105	.972	.934	.260	.972	.914	.393
		.007			.013			.087			.236			.349	
−½	.606	.839	.080	.840	.839	.113	.925	.819	.197	.972	.799	.436	.980	.799	.588
		.041			.0795			.149			.338			.465	
−1½	.480	.438	.048	.606	.438	.066	.817	.485	.132	.853	.485	.276	.954	.576	.388
		.009			.0175			.052			.114			.211	
−2										.256	.139	.090	.417	.139	.198
											.003			.0115	
Σ		0.061			0.115			0.302			0.807			1.1555	

Station	0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}$ Sin α $\frac{f d\alpha}{f\psi}$
		η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α $\frac{f d\alpha}{f\psi}$			η Cos α Sin α	
2	.417	.996	.017												
		.007													
1½	.656	.999	.014	.532	.999	.011	.450	.998	.011	.509	.997	.008	.585	.910	.0
		.009			.006			.005			.004			.023	
1	.873	.978	.0895	.805	.983	.072	.585	.993	.0345	.547	.986	.029	.745	.994	.
		.076			.057			.020			.016			.038	
½	.980	.914	.398	.954	.914	.386	.925	.951	.284	.906	.900	.195	.895	.914	.13
		.356			.337			.2495			.1645			.110	
−½	.962	.804	.571	.962	.809	.564	.972	.793	.590	.864	.793	.324	.863	.766	.2
		.442			.438			.455			.221			.1325	
−1½	.962	.588	.387	.934	.588	.375	.914	.588	.366	.906	.595	.227	.786	.616	.1
		.219			.205			.197			.1225			.061	
−2	.585	.131	.280	.642	.156	.307	.670	.174	.322	.509	.156	.151	.295	.156	.0
		.021			.031			.038			.012			.002	
Σ		1.130			1.074			0.9645			0.540			0.3566	

$$\Sigma\ \eta \operatorname{Cos}\alpha \operatorname{Sin}\alpha \frac{f d\alpha}{f\psi} = 6.5055$$

<u>Mass flow calculations and results for tests 1A to 4A.</u>

The table below gives experimental mass flow readings for tests 1A to 4A along with the function $\sum \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$ for each test from table 3.

Also shown are the calculated mass flow figures obtained as follows:-

<u>For test 4A.</u>

$$DH_{\varphi} = 20.55 \text{ BTU/lb.} \quad H_1 = 1229.25 \text{ BTU/lb.}$$

$$V_{2\varphi} = \frac{1.253(H_1 - DH_{\varphi} - 835)}{43.25} = 10.81 \text{ ft}^3/\text{lb.}$$

$$da = \frac{0.01}{144} \text{ ft}^2$$

From equation 3 page 91.

$$m = \frac{223.8\sqrt{DH}\varphi}{V_{2\varphi}} \times \sum \eta^{\frac{1}{2}} \sin\alpha \frac{da}{fV}$$

$$m = \frac{223.8\sqrt{DH_{\varphi}}}{V_{2\varphi}} \times da \times \sum \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$$

$$= \frac{223.8\sqrt{20.55}}{V_{2\varphi}} \times \frac{.01}{144} \times 10.6425 = .0692 \text{ lb/sec.}$$

The actual measured flow is $\frac{31.5 \times 10}{1.25 \times 3600}$ = .0700 lb/sec.

<u>Series 1 Mass flow measurements.</u>

Test No.	1A	2A	3A	4A
Measured flow per $1\frac{1}{4}$ hrs.(Galls)	14.1	20.1	25.2	31.5
$\sum \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	10.367	10.906	10.591	10.6425
Measured flow lb/sec.	0.0313	0.0449	0.0560	0.0700
Calculated flow lb/sec.	0.0303	0.0464	0.0561	0.0692
% age difference	-3.20	+3.34	+0.18	-1.14

Series 1 -------- Final Results.

The results for the series 1 tests are shown in the table on page 145. From these results the following graphs have been drawn and are shown in subsequent figures.

(1) Mean nozzle efficiency and efficiency of expansion for the pair to a base of Reynolds number. Fig. 50 (page 146).

(2) The blade velocity coefficient to a base of Reynolds number. Fig. 51 (page 147).

(3) Blade passage loss to a base of Reynolds number. Fig. 52 (page 148).

(4) Typical graphs showing velocity distribution for the nozzle and blade exit along a vertical centre line. Figs. 53 and 54 (pages 149 & 150).

(5) Typical graphs shown variations of efflux angle at nozzle and blade exit along a vertical centre line. Figs. 53 and 54 (pages 149 &150).

Series 1-Table of Final Results.

Test	1	1A	2	2A	3	3A	4	4A
Mach No. M_{2_ϕ}	←			0.6				→
Reynolds No. $\times 10^6$	4.01	3.99	5.66	5.72	7.04	7.04	8.55	8.55
$\bar{\eta}_n$	.900		.926		.908		.872	
$\bar{\eta}_b$		.778		.762		.762		.778
K		.930		.909		.916		.946
$\bar{\alpha}$	33°	44.7°	33°	44.4°	32.5°	44.6°	38°	44.1
Blade loss B.T.U./lb		2.50		3.38		3.00		1.19
$\bar{\eta}_n$ ℄	.972		.981		.971		.935	
$\bar{\eta}_b$ ℄		.820		.826		.805		.817
K ℄		.920		.919		.911		.936
Blade loss ℄		3.12		3.20		3.41		2.41

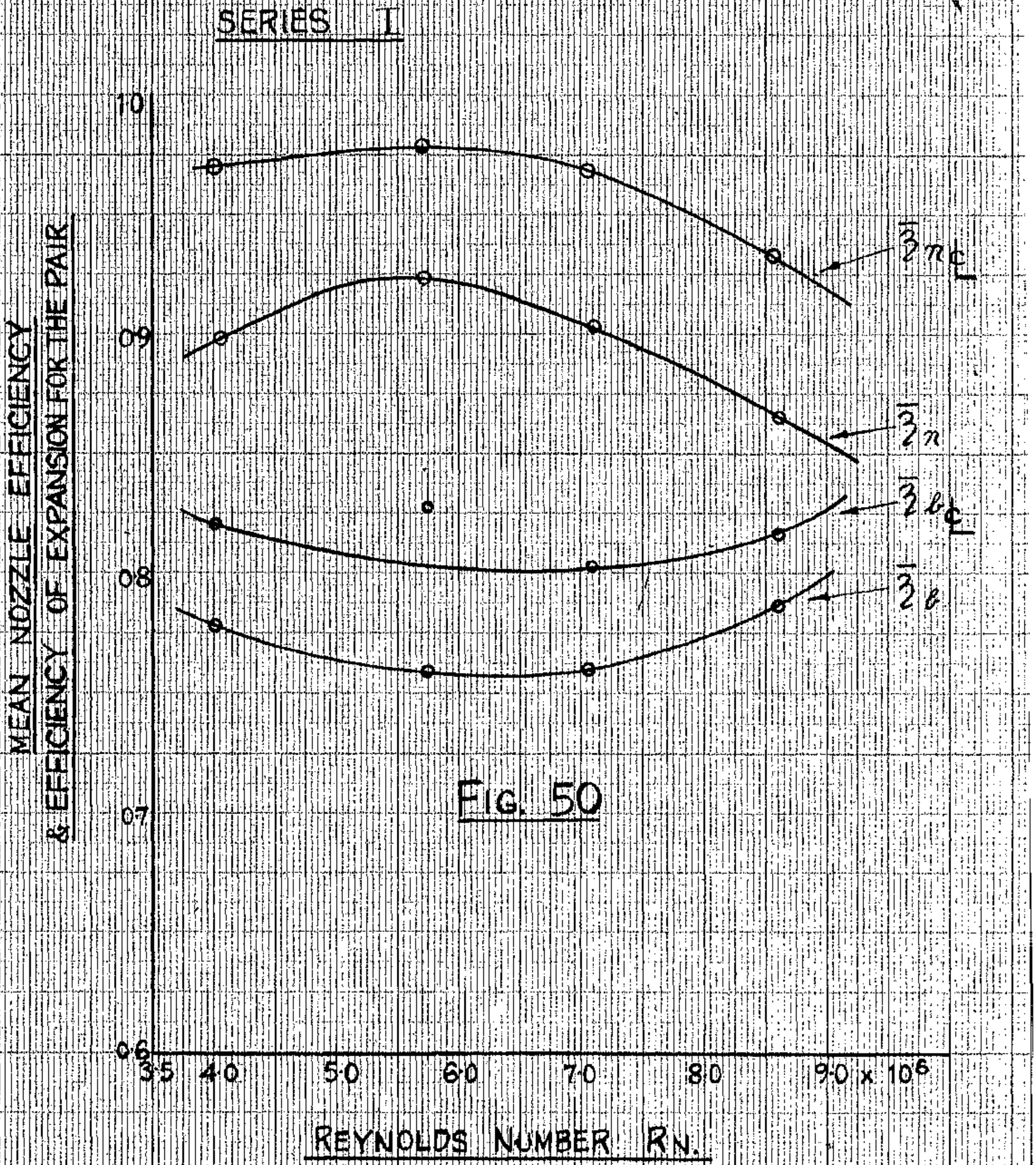

SERIES I
MEAN NOZZLE EFFICIENCY
& EFFICIENCY OF EXPANSION FOR THE PAIR
1·0
0·9
0·8
0·7
0·6
3·5
4·0
5·0
6·0
7·0
8·0
9·0 x 10⁶
REYNOLDS NUMBER RN.
$\bar{\eta}_{nc}$
$\bar{\eta}_n$
$\bar{\eta}_{bc}$
$\bar{\eta}_b$

Fig. 50

BLADE VELOCITY COEFFICIENT

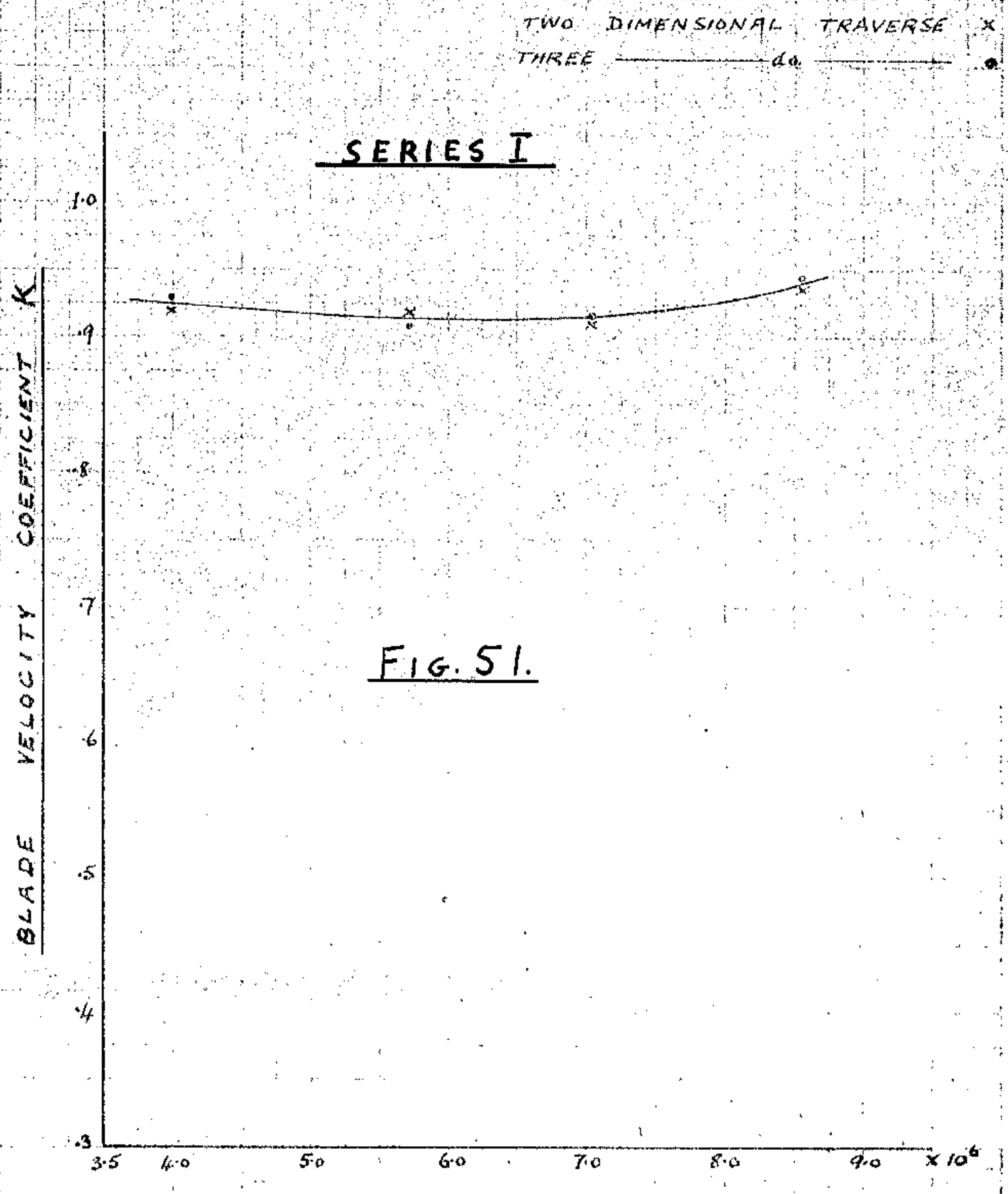
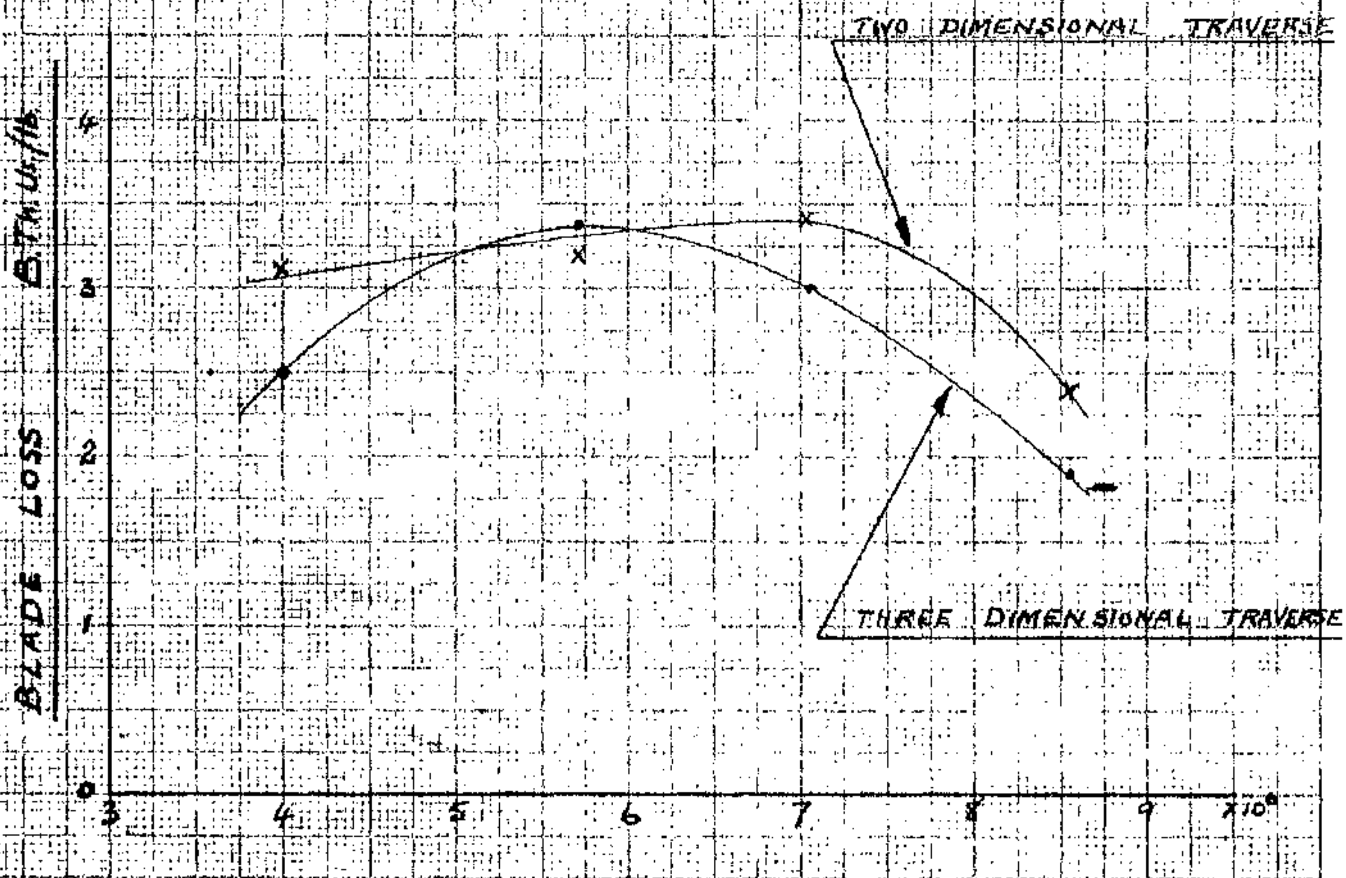


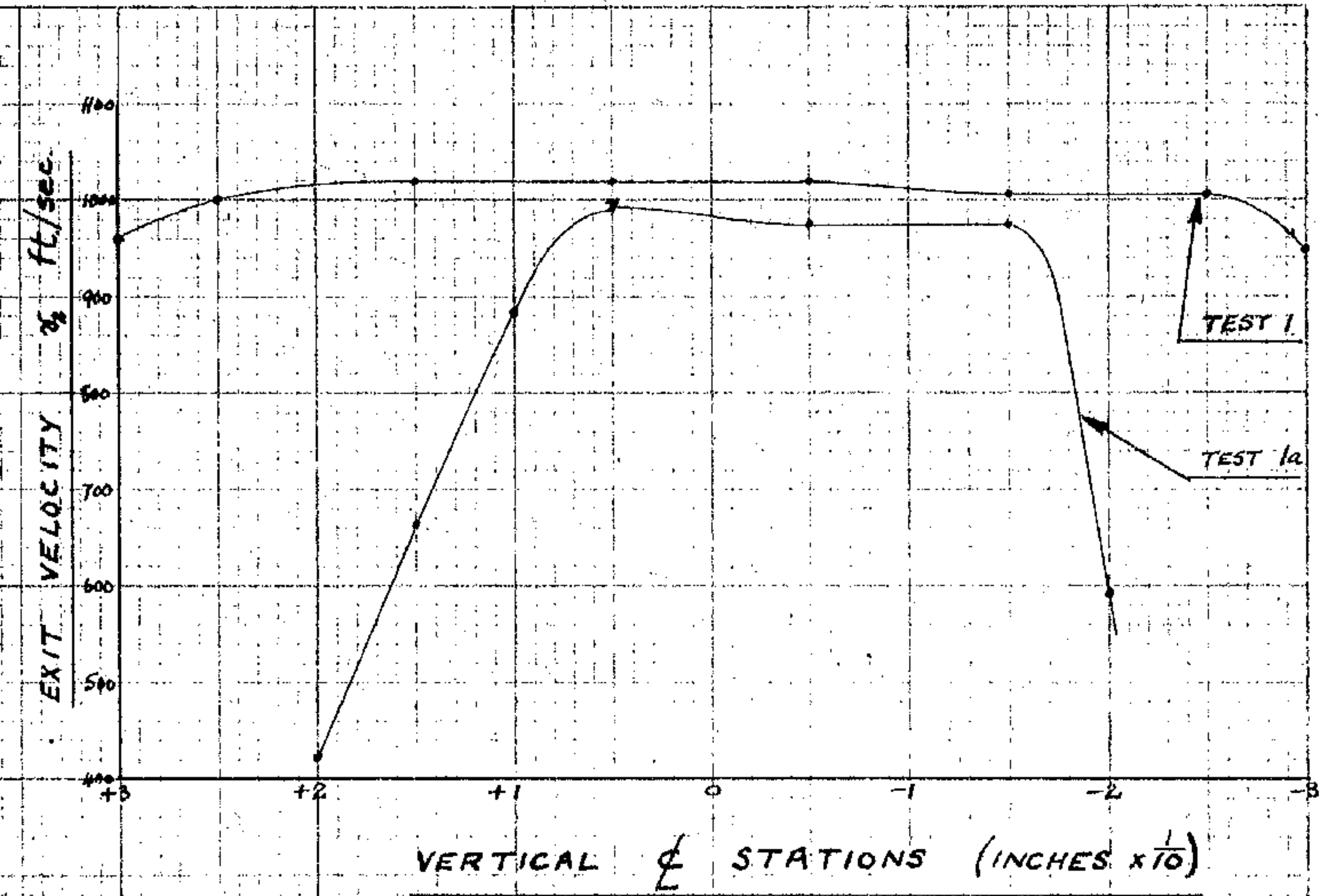
FIG. 51.

SERIES I

BLADE LOSS

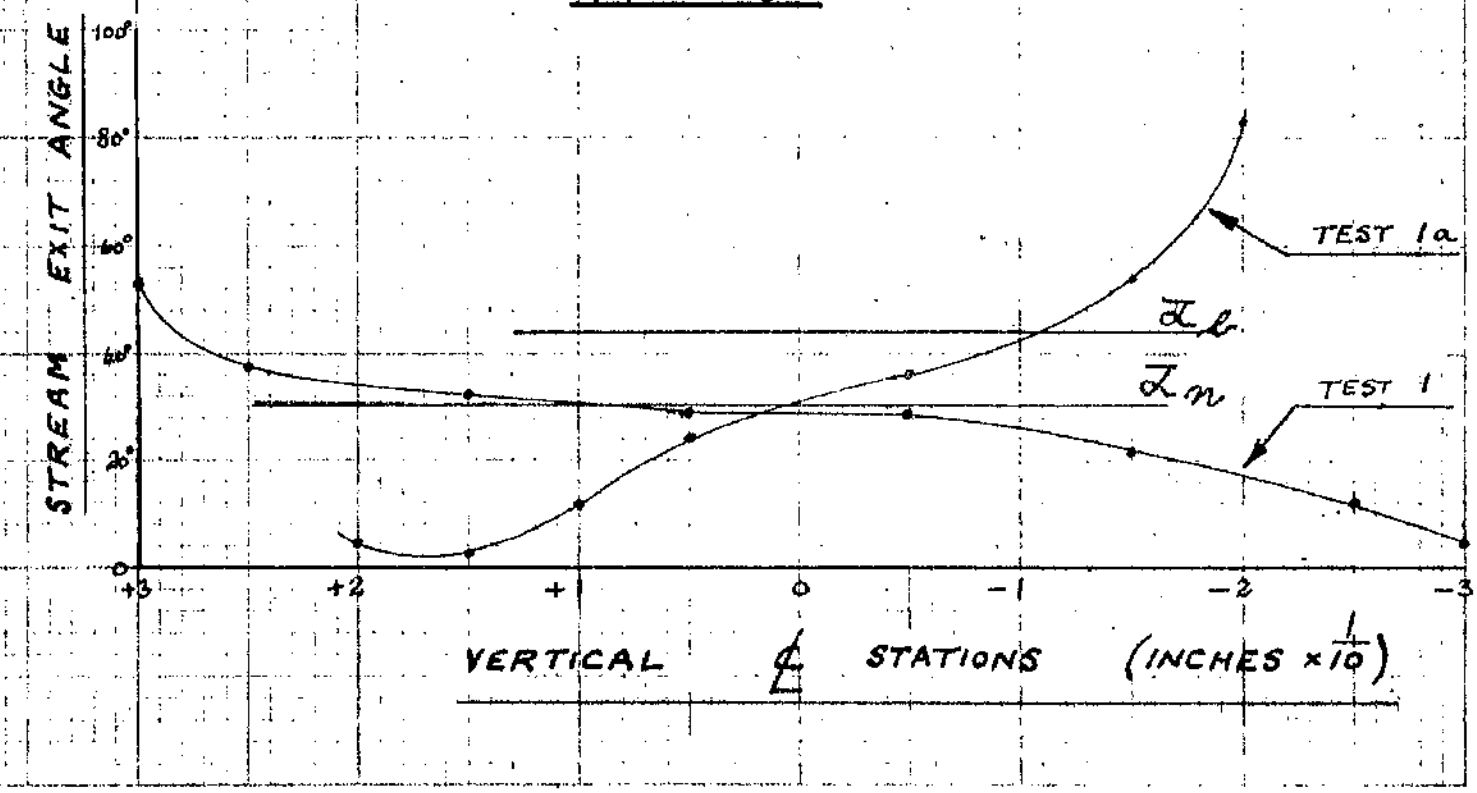
FIG. 52.

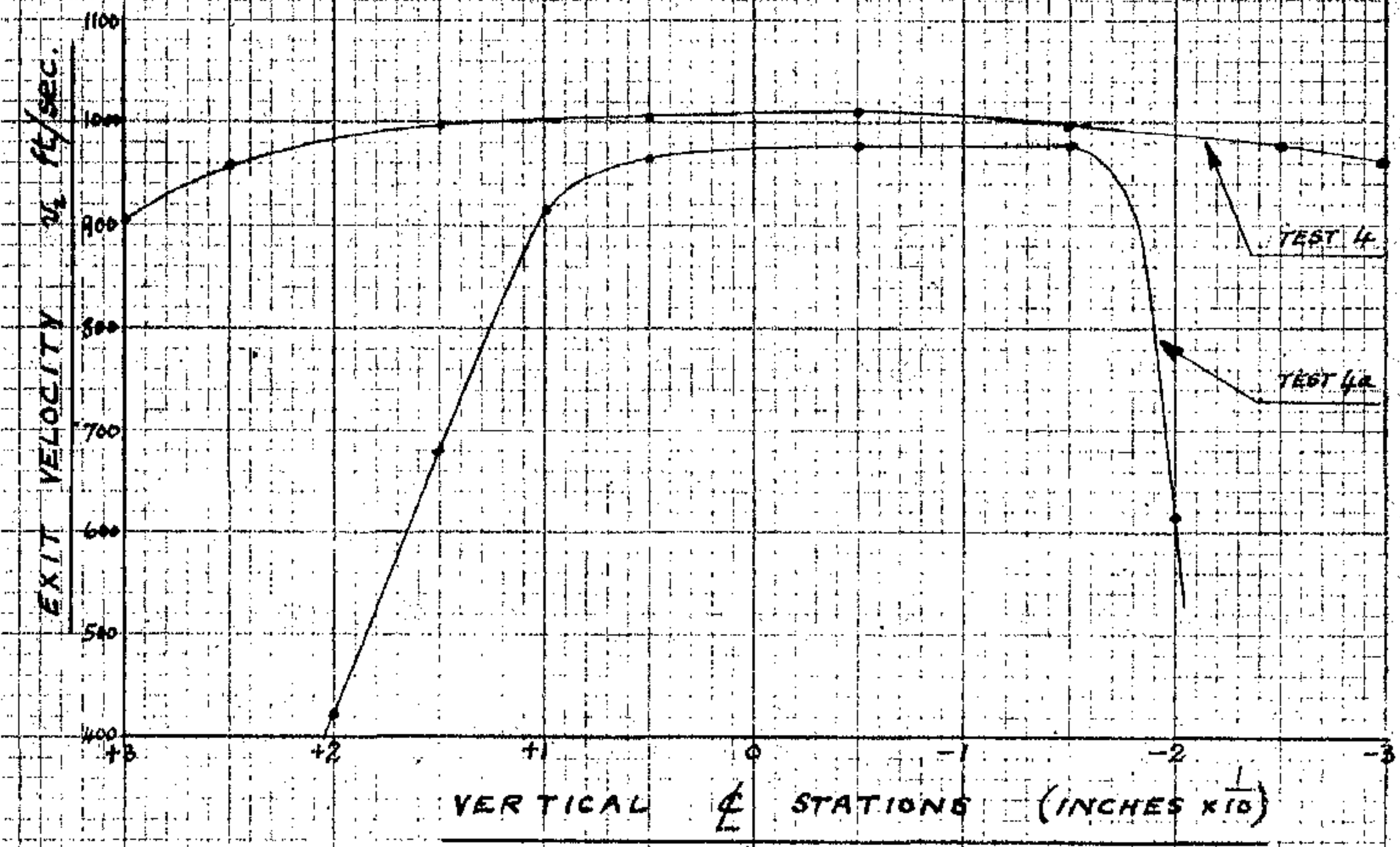




SERIES. I.

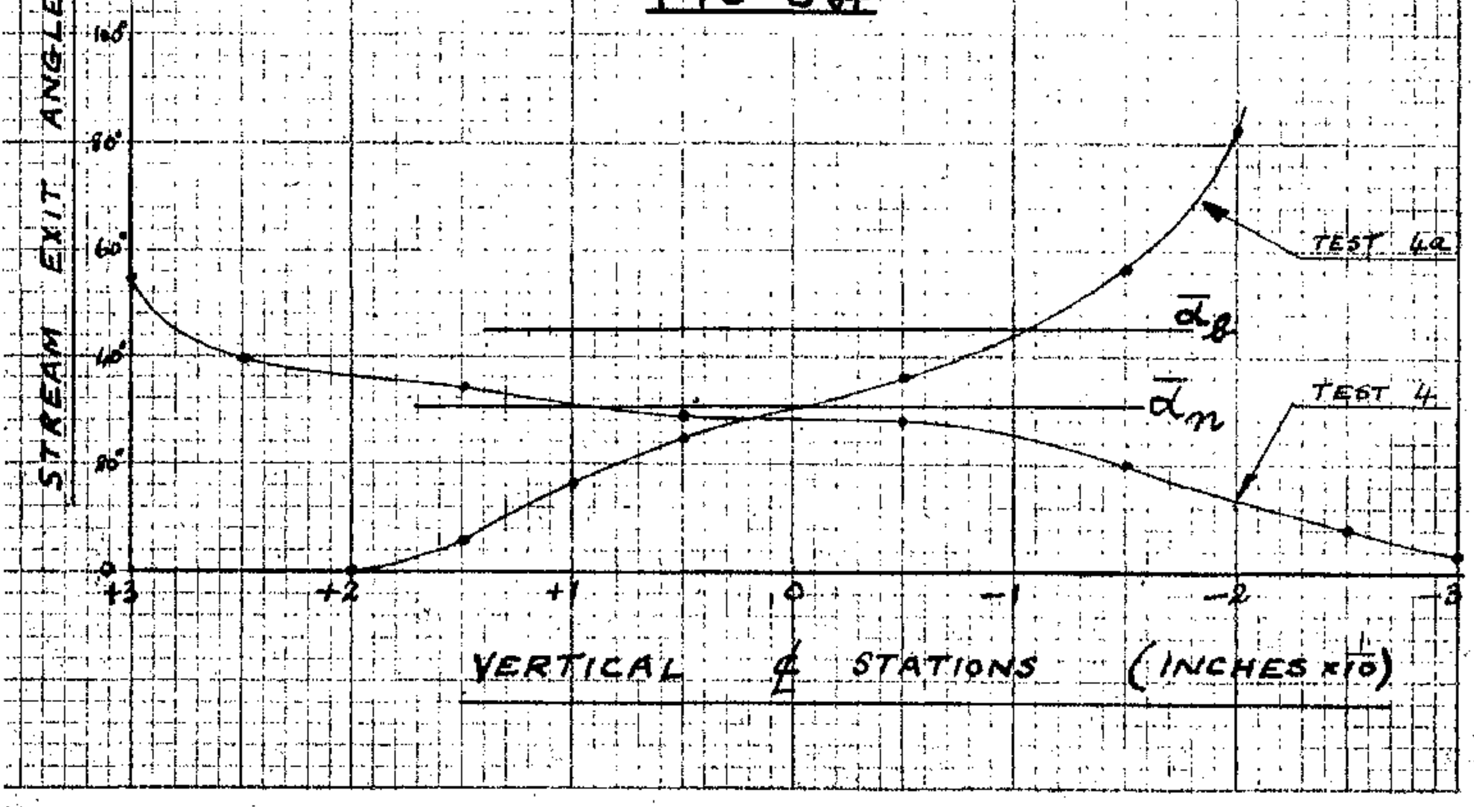
FIG. S3.





SERIES I.

FIG 5d



Series 2

Series 2.

In this test series the exit Mach number is varied within the subsonic region. There are twelve three dimensional tests, six at the nozzle and six at the blade exit. The nozzle tests are numbered 1 - 6 and the blade tests 1A - 6A. The exit Mach numbers taken are 0.5, 0.6, 0.7, 0.8, 0.9, and 0.97. For the tests in the range $M_{2\varphi_2}$ equal to 0.5 to 0.8 the inlet nozzle pressure is 24.3 lb/in^2 which is the same as in test 1 series 1. The back pressures are chosen to give the correct exit Mach numbers from Fig. 31 (page 77). In the tests at $M_{2\varphi} = 0.9$ and 0.97 the inlet pressure was raised above 24.3 lb/in^2 so as to prevent the expansion entering the wet field.

The test conditions are shown in the table on page 153.

The calculation of the effective values takes the same form as in series 1 except that for the values of local efficiency the graphs in Fig. 32 (page 78) must be used.

Specimen calculations for tests 1 and 1A are given in appendix 1 (page 197), and tables 1 to 5 for tests 1 and 1A are given in appendix 1 pages 199to210 . The remaining tables of observed and derived results for tests 2 to 6 and 2A to 6A for series 2 are given in appendix 2 (pages 33 to 83).

The final results for series 2 are described on page 155.

Series 2 ——— Test Conditions.

Test No.	1	1A	2	2A	3	3A	4	4A	5	5A	6	6A
Inlet pressure P_1 lb/in^2.	24.3	24.3	24.3	24.3	24.3	24.3	24.3	24.3	30.0	30.0	32.5	32.6
Back pressure P_2 lb/in^2.	20.7	20.7	19.32	19.32	17.88	17.88	16.35	16.35	18.38	18.38	18.30	18.30
Inlet temperature T_1 ^oF.	←					380^{o}F						→
Exit Mach No. $M_{2\phi}$	0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	0.97	0.97

Mass flows for tests 1A to 6A.

The table below gives a comparison between the actual measured flow and the flow figures calculated from the traverse readings for tests 1A to 6A. The calculations follows the same pattern as those for the mass flow figures in the series 1 tests.

Series 2 Mass flow results.

Test No.	1A	2A	3A	4A	5A	6A
Measured flow lb/sec.	0.0288	0.0343	0.0375	0.0384	0.0494	0.0539
Calculated flow lb/sec.	0.0284	0.0331	0.0352	0.0380	0.0501	0.0528
%age difference	-1.39	-3.50	-6.13	-1.04	+1.42	-2.04

Series 2 ———————— Final Results.

The final results for the series 2 tests are given in the table on page 156. From these results the following graphs have been drawn and these appear in subsequent figures.

(1) The mean nozzle efficiency and efficiency of expansion for the pair to a base of Mach number Fig. 55 (page 157).

(2) Blade velocity coefficient to a base of Mach number Fig. 56 (page 158).

(3) Blade loss to a base of Mach number Fig. 57 (page 159).

(4) Typical graphs of velocity distribution along a vertical centre line for the nozzle and blades Figs 58 and 59 (pages 160 and 161).

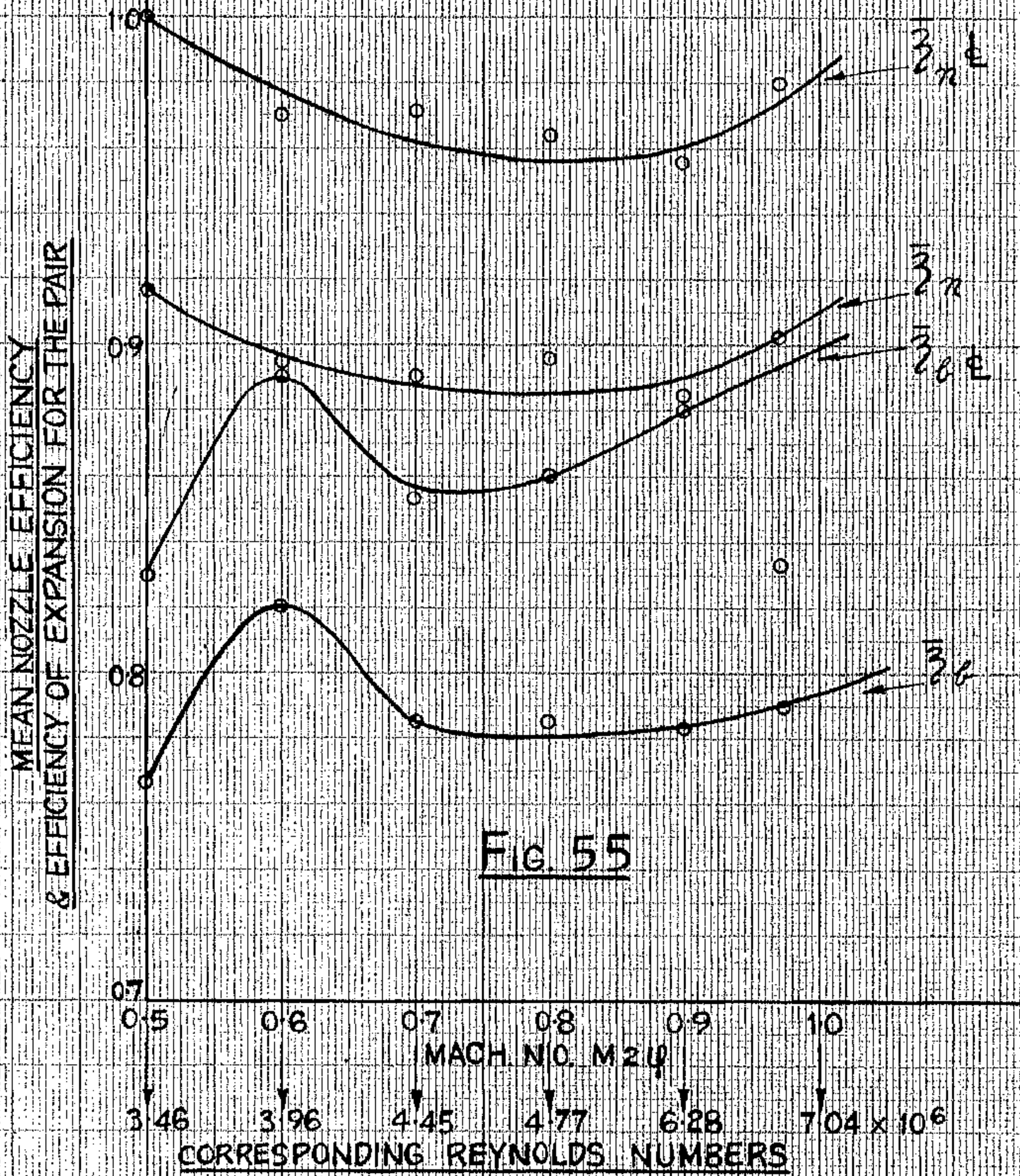
(5) Typical graphs of efflux angle along a vertical centre line for nozzle and blades Fig. 60 (page 162).

(6) Table 1 for test 1A to 6A show that as the impact tube traverses in a horizontal position, there is a considerable decrease in impact tube pressure on either side of the centre line. Velocity distributions for a horizontal traverse have been calculated for these tests and typical results appear in graphical form in Fig. 61 (page 163).

Series 2 —— Table of final results.

Series 2.

Test No.	1	1A	2	2A	3	3A	4	4A	5	5A	6	6A
Mach No. $M_{2\varphi}$	0.5	0.5	0.6	0.6	0.7	0.7	0.8	0.8	0.9	0.9	0.97	0.97
Reynolds No ($\times 10^6$)	3.46	3.46	3.96	3.96	4.45	4.45	4.77	4.77	6.28	6.28	7.04	7.04
$\bar{\eta}_n$	0.917		0.894		0.891		0.897		0.884		0.902	
$\bar{\eta}_b$		0.766		0.821		0.785		0.785		0.783		0.790
K		0.914		0.959		0.940		0.935		0.941		0.935
Blade loss BTU/lb.		2.185		1.455		2.860		3.900		4.310		5.570
$\bar{\alpha}$	33–11°	44–24°	34–0°	45–0°	35–6°	45–15°	35–24°	46–46°	36–58°	47–51°	35–6°	45–46°
$\bar{\eta}_n$ ℄	1.0		0.97		0.973		0.965		0.957		0.98	
$\bar{\eta}_b$ ℄		0.83		0.894		0.854		0.86		0.881		0.832
K ℄		0.911		0.96		0.935		0.944		0.928		0.922
Blade loss BTU/lb. ℄		2.46		1.585		3.33		3.70		5.68		7.26



SERIES II
MEAN NOZZLE EFFICIENCY
& EFFICIENCY OF EXPANSION FOR THE PAIR
1·0
0·9
0·8
0·7
0·5
0·6
0·7
0·8
0·9
1·0
MACH. NO. M2ψ
3·46
3·96
4·45
4·77
6·28
7·04 × 10⁶
CORRESPONDING REYNOLDS NUMBERS

FIG. 55

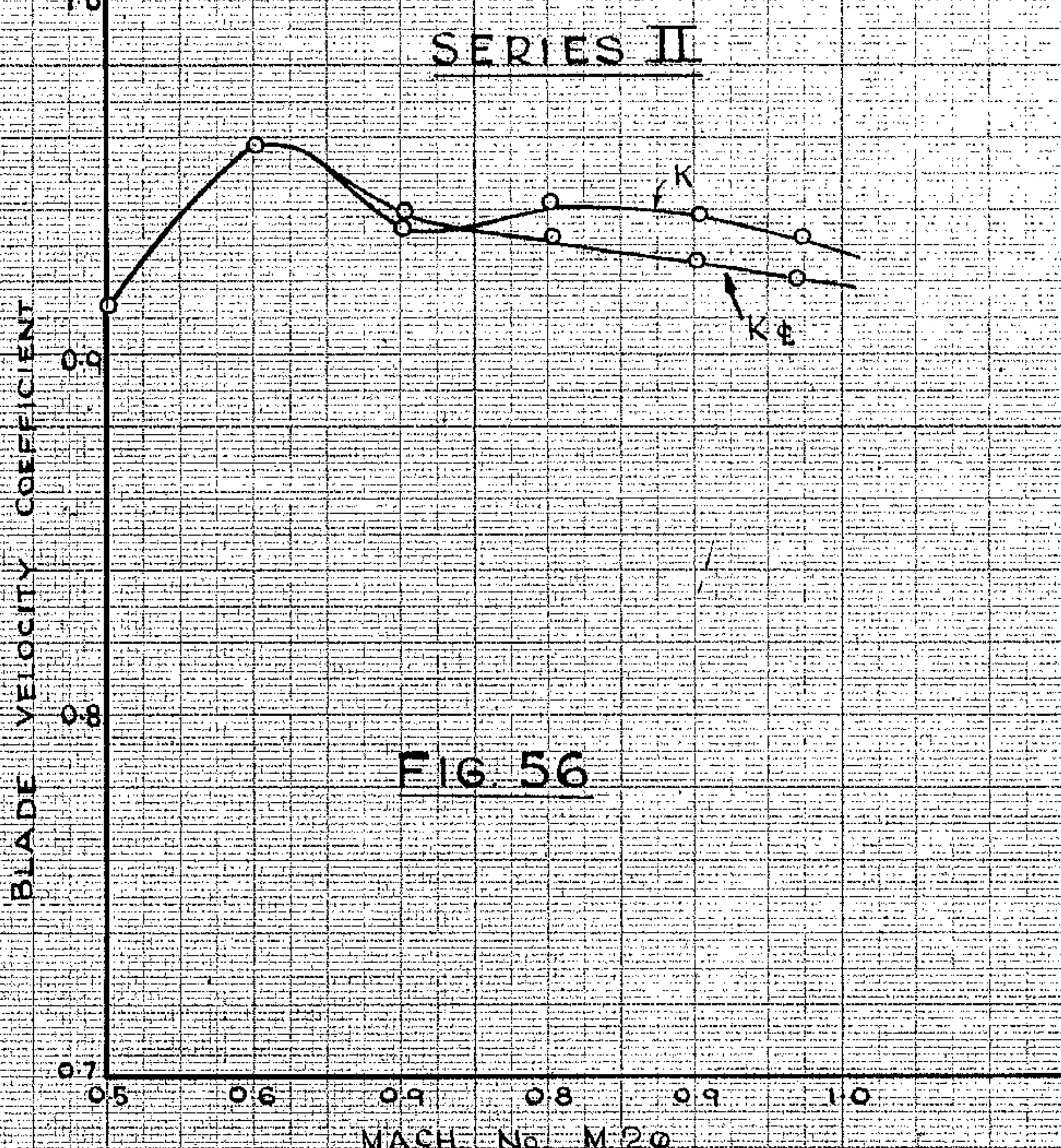

SERIES II
BLADE VELOCITY COEFFICIENT
1.0
0.9
0.8
0.7
0.5
0.6
0.9
0.8
0.9
1.0
MACH. No M 2φ
K
K φ

FIG. 56

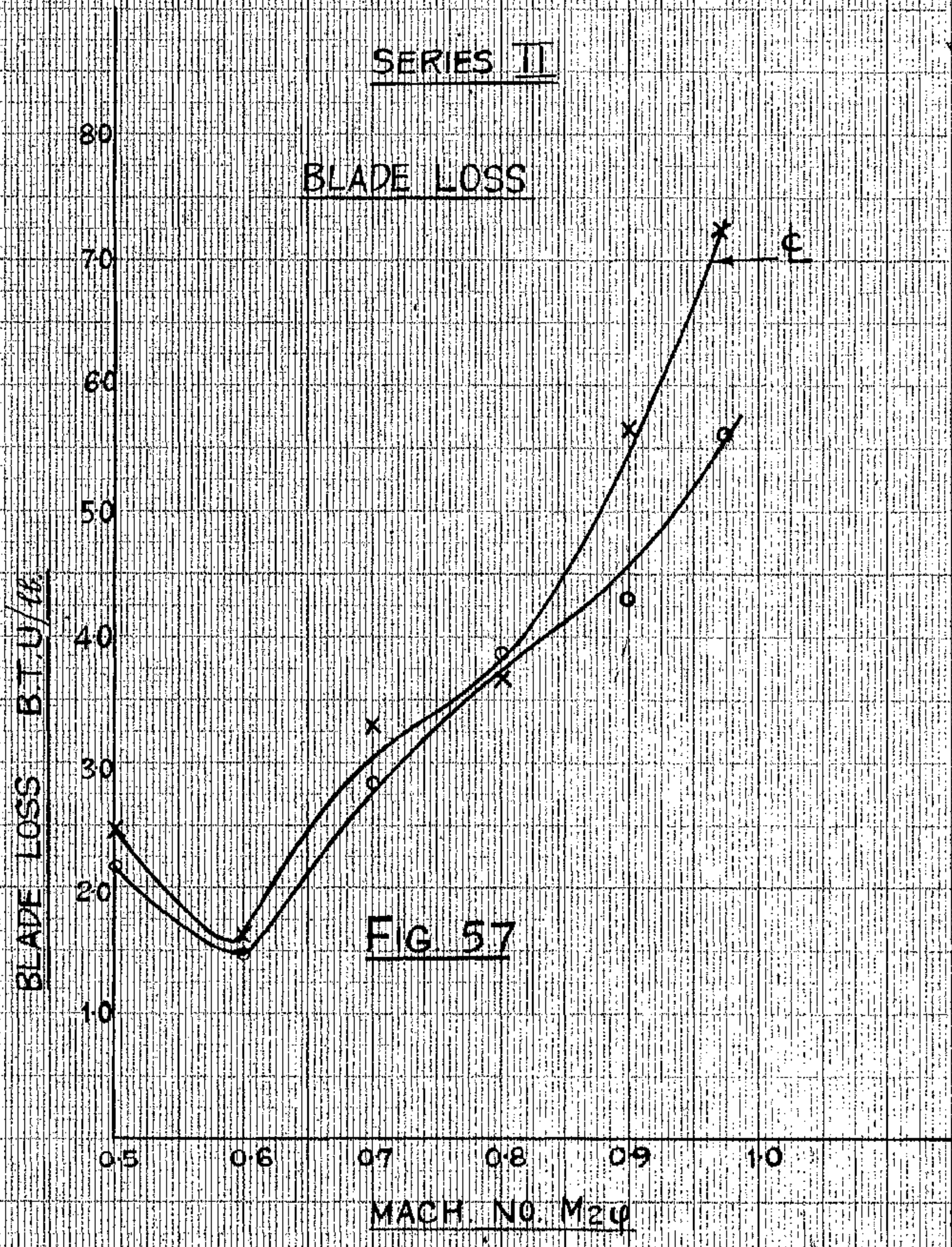
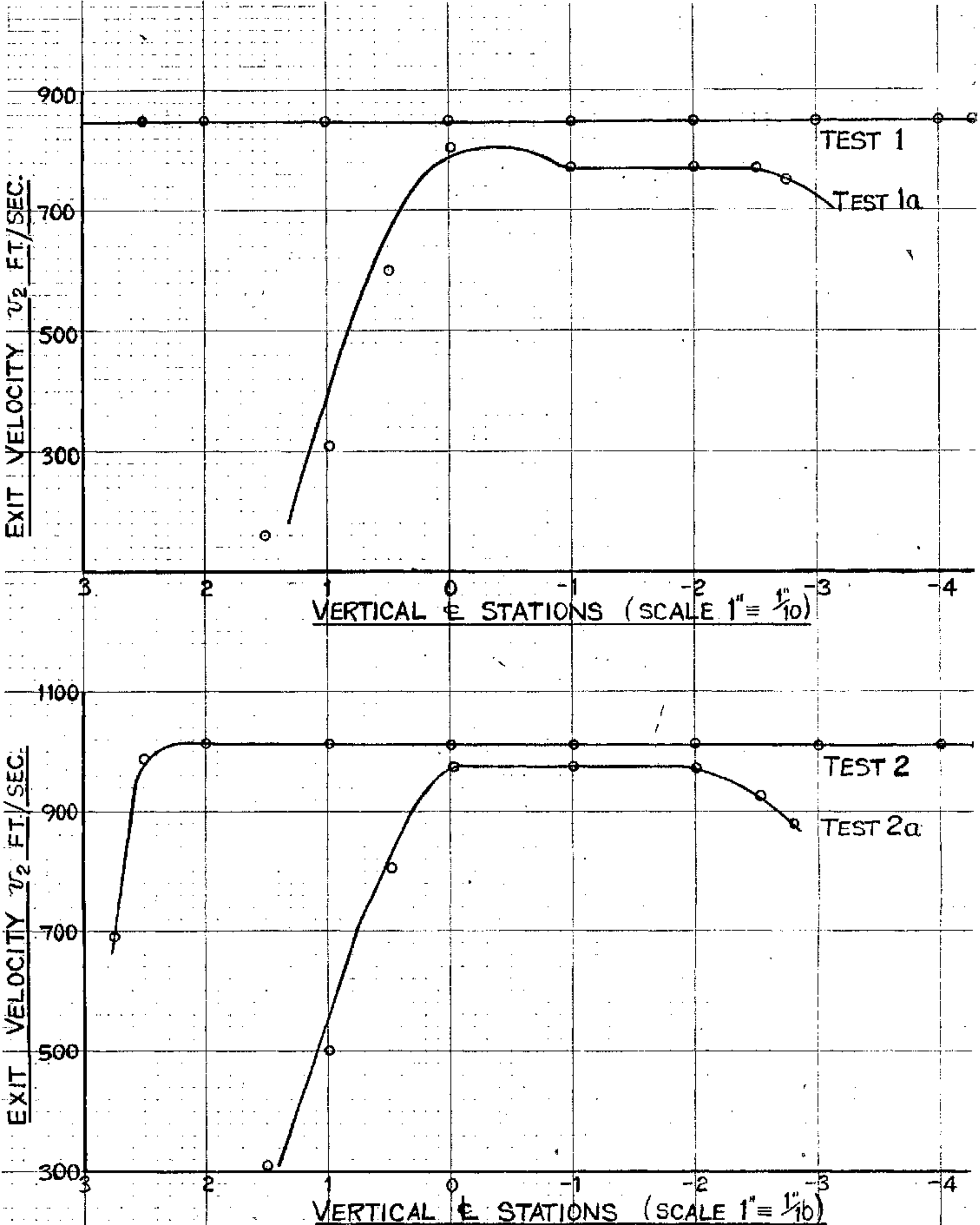
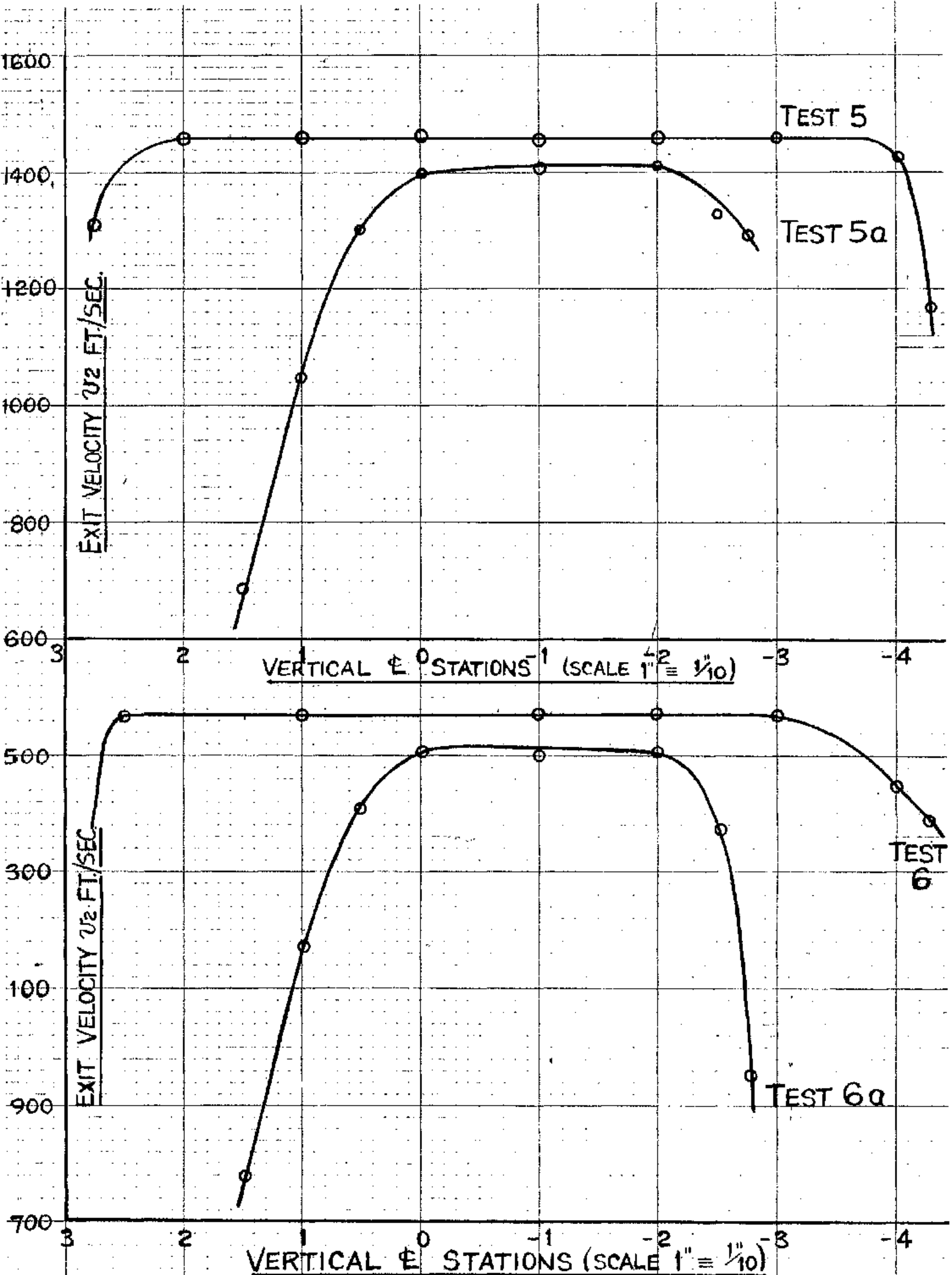

SERIES II
BLADE LOSS
BLADE LOSS B.T.U./lb.
80
70
60
50
40
30
20
10
0·5
0·6
0·7
0·8
0·9
1·0
MACH. NO. M2φ
℄

FIG. 57



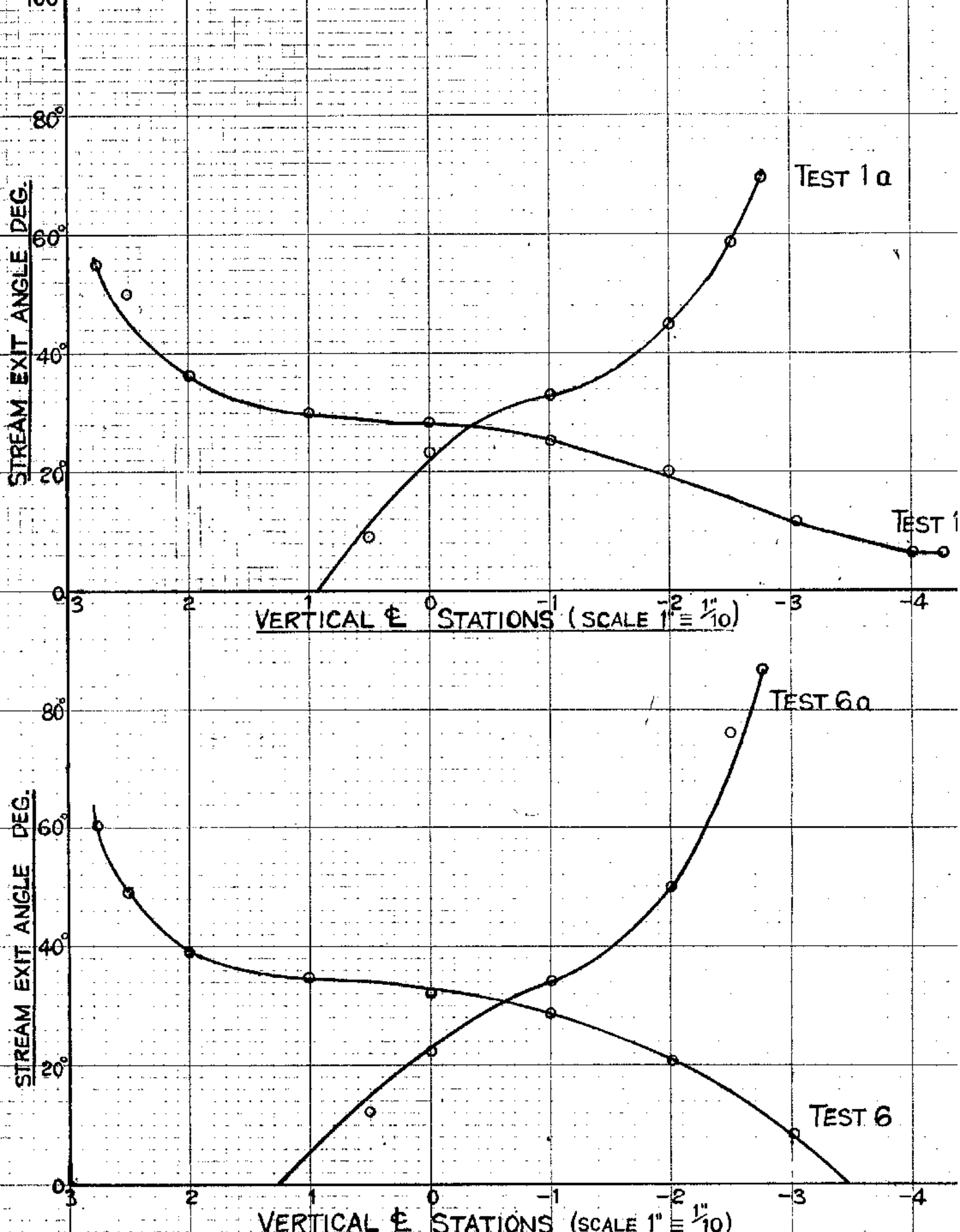
SERIES III

FIG. 58



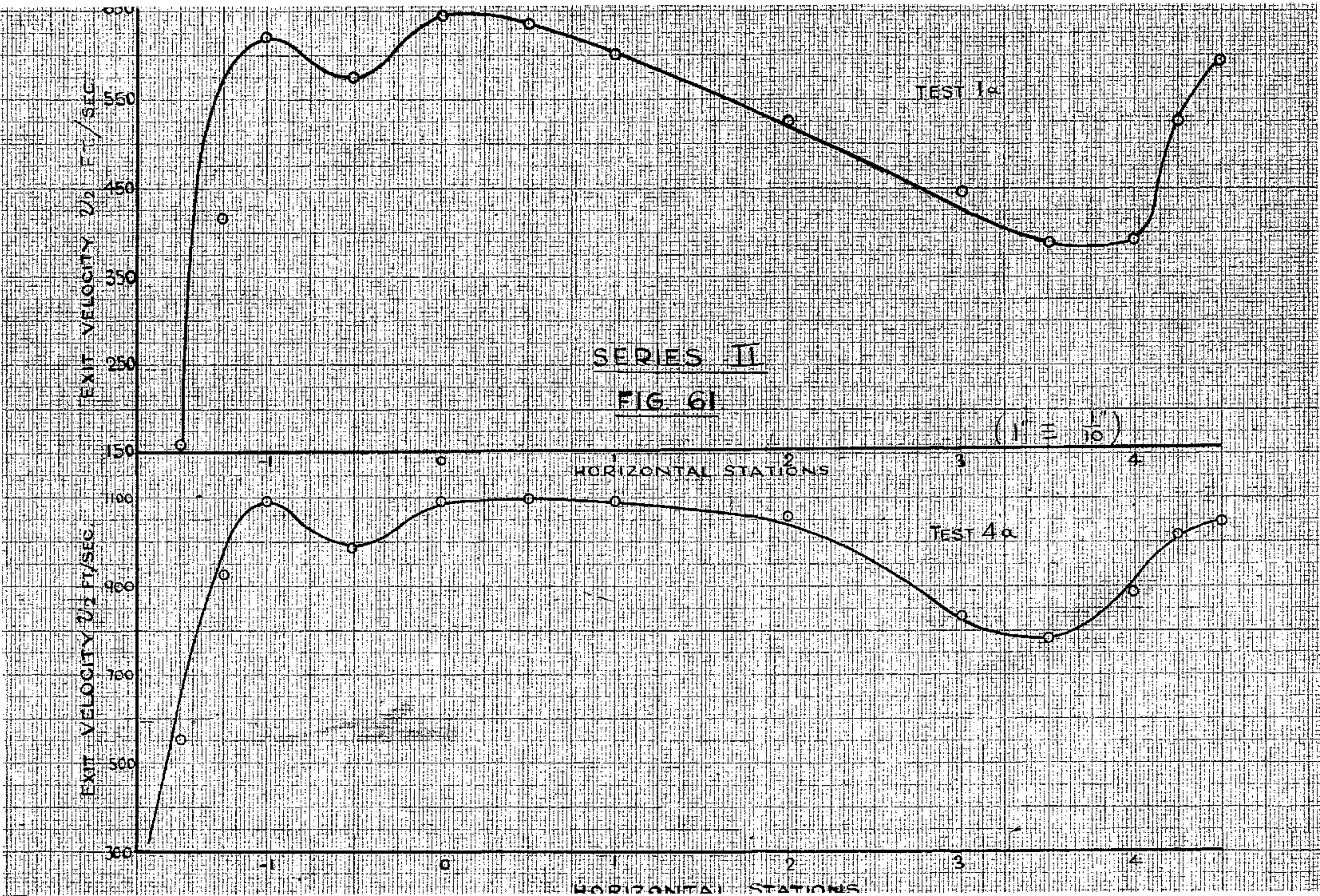
SERIES II

FIG. 59



SERIES II

FIG. 60



SERIES II

FIG 61

Series 3.

Series 3.

In this test series the range of outlet Mach number was extended into the supersonic field. The tests were carried out only at the blade exit and along a vertical centre line. There are three groups of tests with inlet pressures of 64.5, 53.37, and 44.45 lb/in^2. In each group the maximum Mach number is obtained with a back pressure of 1 lb/in^2G. and the inlet temperature is chosen to avoid expansion into the wet field at maximum Mach number. In all tests, including those in series 1 and 2 where the Mach numbers is less than unity, the back pressure in the casing and the nozzle outlet pressure were as expected equal. For this test series, in the cases where the outlet Mach number was greater than unity the nozzle outlet pressure corresponded to the critical pressure.

The test conditions for series 3 are shown in the tables on pages 166,167,168 along with the final results. Specimen calculations are given on page 169 and the graphs of the final results are described on page 170.

Effect of supersonic Mach number on efflux angle.

An additional test in this series wherein the variation of nozzle efflux angle with supersonic Mach number was investigated is described on page 174.

Series 3 ----- Test conditions and final results.

Group 1

Inlet pressure P_1 lb/in^2	← 64.5 →			
Back pressure P_2 lb/in^2	32.5	24.5	19.5	15.5
Mach No. $M_{2\varphi}$	1.075	1.298	1.46	1.6
Inlet Temp. T_1 °F	← 490 →			
Reynolds No. (x 10^6)	11.4	11.75	11.55	11.5
$\bar{\eta}_{b\,¢}$	0.849	0.815	0.88	0.868
K ¢	0.921	0.903	0.938	0.931
Blade loss BTU/lb. ¢	9.85	16.6	12.8	17.1

Series 3 ——— Test conditions and final results.

Group 2

Inlet pressure P_1 lb/in.^{2}	←			54.37	→				
Back pressure P_2 lb/in.^{2}	46.25	43.2	40.0	36.56	33.0	26.5	22.5	18.5	15.
Mach No. $M_{2\varphi}$	0.5	0.6	0.7	0.8	0.9	1.08	1.228	1.37	1.5
Inlet Temp. T_1 °F	←			450	→				
Reynolds No. (x 10^6)	6.66	7.61	8.55	9.3	10.1	10.66	10.65	10.2	10.
$\bar{\eta}_b$ ℄	0.733	.771	.794	.828	.81	.76	.744	.791	.78
K ℄	0.856	.879	.891	.910	.90	.872	.864	.891	.88
Blade loss BTU/lb. ℄	4.16	5.01	6.04	6.3	8.8	15.4	20.0	19.6	23.

Series 3 —— Test conditions and final results.

Group 3

Inlet pressure P_1 lb/in.^{2}	←				44.45				→
Back pressure P_2 lb/in.^{2}	37.6	35.2	32.43	29.7	26.84	24.84	22.45	19.45	15.45
Mach No. M_2 φ	0.5	0.6	0.7	0.8	0.9	0.97	1.06	1.178	1.359
Inlet Temp. T_1 °F	←				408				→
Reynolds No. (x 10^6)	5.91	6.76	7.55	8.14	8.7	8.89	9.24	9.36	9.32
$\bar{\eta}_b$ ¢	.7125	.762	.830	.871	.851	.841	.806	.755	.797
K ¢	.845	.875	.911	.935	.924	.916	.896	.869	.893
Blade loss BTU/lb. ¢	4.32	5.02	4.80	4.56	6.66		11.5	17.25	17.75

Calculations. The calculations differ slightly from those in series 1 and 2 due to the possibility of the local outlet Mach number being greater than unity. The method described in detail on pages 83 and 85 is used for dealing with this. The specimen calculations given below are for the test in group 3, with $M_{2\varphi} = 1.359$. The necessary tables for group 3 are given in appendix 1 pages 211 to 217. Those for group 1 and 2 are given in appendix 2 pages 84 to 103.

Group 3, $M_{2\varphi} = 1.359$.

$P_1 = 44.45 \text{ lb/in}^2$
$P_2 = 15.45 \text{ lb/in}^2$
$\therefore \frac{P_2}{P_1} = 0.347 \quad \therefore M_{2\varphi} = 1.359$

$T_1 = 408^\circ F$
$H_1 = 1240$ BTU/lb.
$\therefore T_{2\varphi} = 214.64^\circ F, \quad DH_\varphi = 87.55$ BTU/lb,
$v_{2\varphi} = 2094$ ft/sec, $V_{2\varphi} = 25.7 \text{ ft}^3/\text{lb}$,
$\mu_{2\varphi} = 2.71 \times 10^{-7}$, $R_n = 9.32 \times 10^6$.

At station 0, $\frac{P_2}{P_5} = 0.415$.

$$\therefore \frac{P_2}{P_3} = 0.4065 \text{ ——— Fig. 36 (page 84).}$$

$$\therefore \eta = 0.88 \text{ ——— Fig. 33 (page 79).}$$

From table 3 $\sum_{\text{¢}} \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 1.8227$ ——— appendix 1 (page 216).

From table 4 $\sum_{\text{¢}} \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 1.4545$ ——— appendix 1 (page 217).

$$\therefore \bar{\eta}_{b\,\text{¢}} = \frac{1.4545}{1.8227} = 0.797$$

Taking $\bar{\eta}_n = 1 \quad \therefore K_{\text{¢}} = \sqrt{.797} = 0.893$.

Blade loss $\text{¢} = (1 - K^2)\, DH_\varphi = (1 - .797) \times 87.55 = 17.75$ BTU/lb.

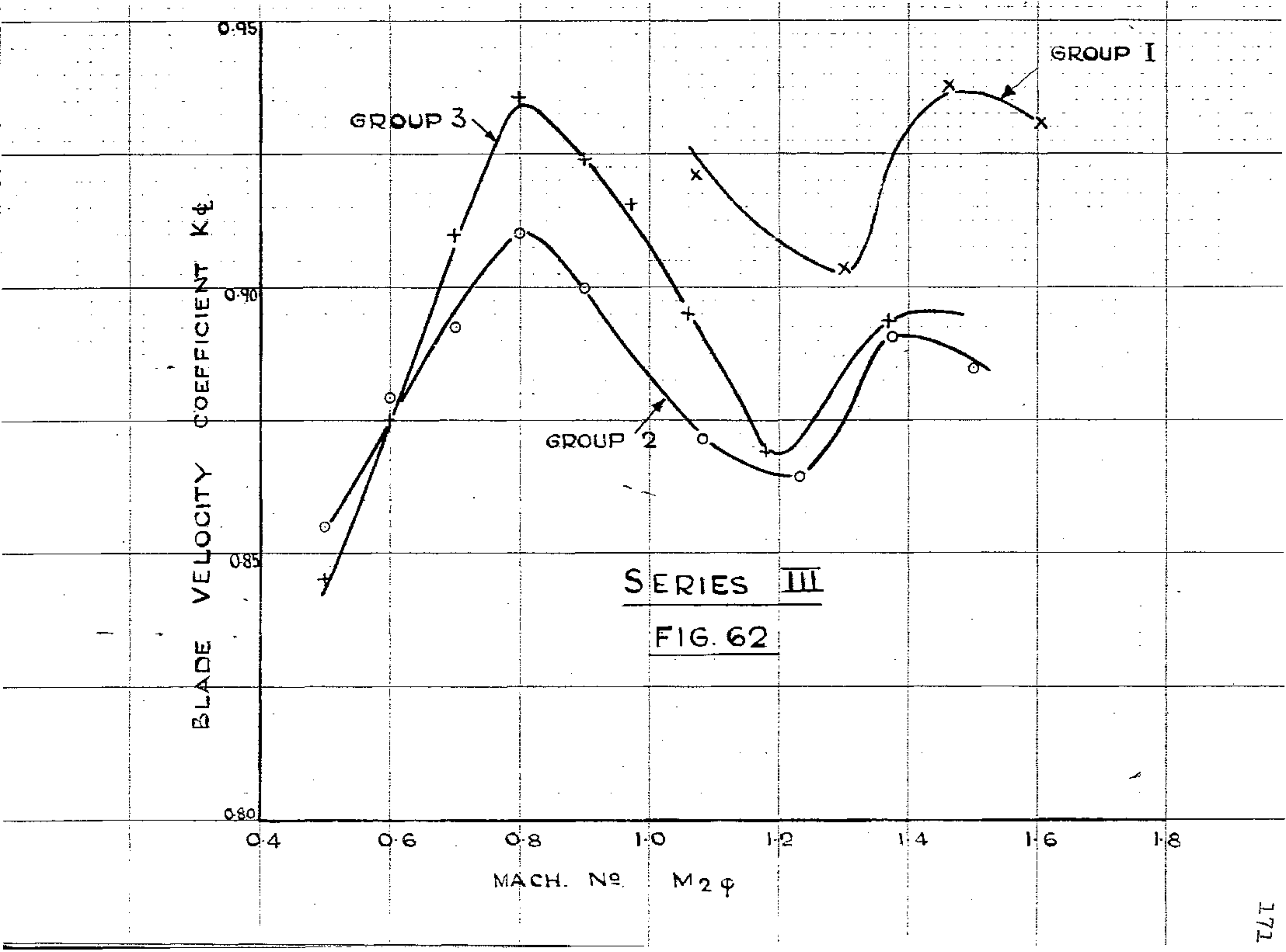
Series 3 —— Final Results.

From the final average results the following graphs have been drawn.

(1) The efficiency of expansion as $K_{¢}$ $\left(\sqrt{\eta_{b_{¢}}}\right)$ to a base of Mach number Fig. 62 (page 171).

(2) Blade velocity coefficient $K_{¢}$ to a base of Reynolds number. Fig. 63 (page 172).

(3) Blade loss to a base of Mach number Fig. 64 (page 174).


GROUP I
GROUP 3
GROUP 2
SERIES III
FIG. 62
BLADE VELOCITY COEFFICIENT K_ϕ
0.95
0.90
0.85
0.80
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
MACH. No. $M_{2\phi}$

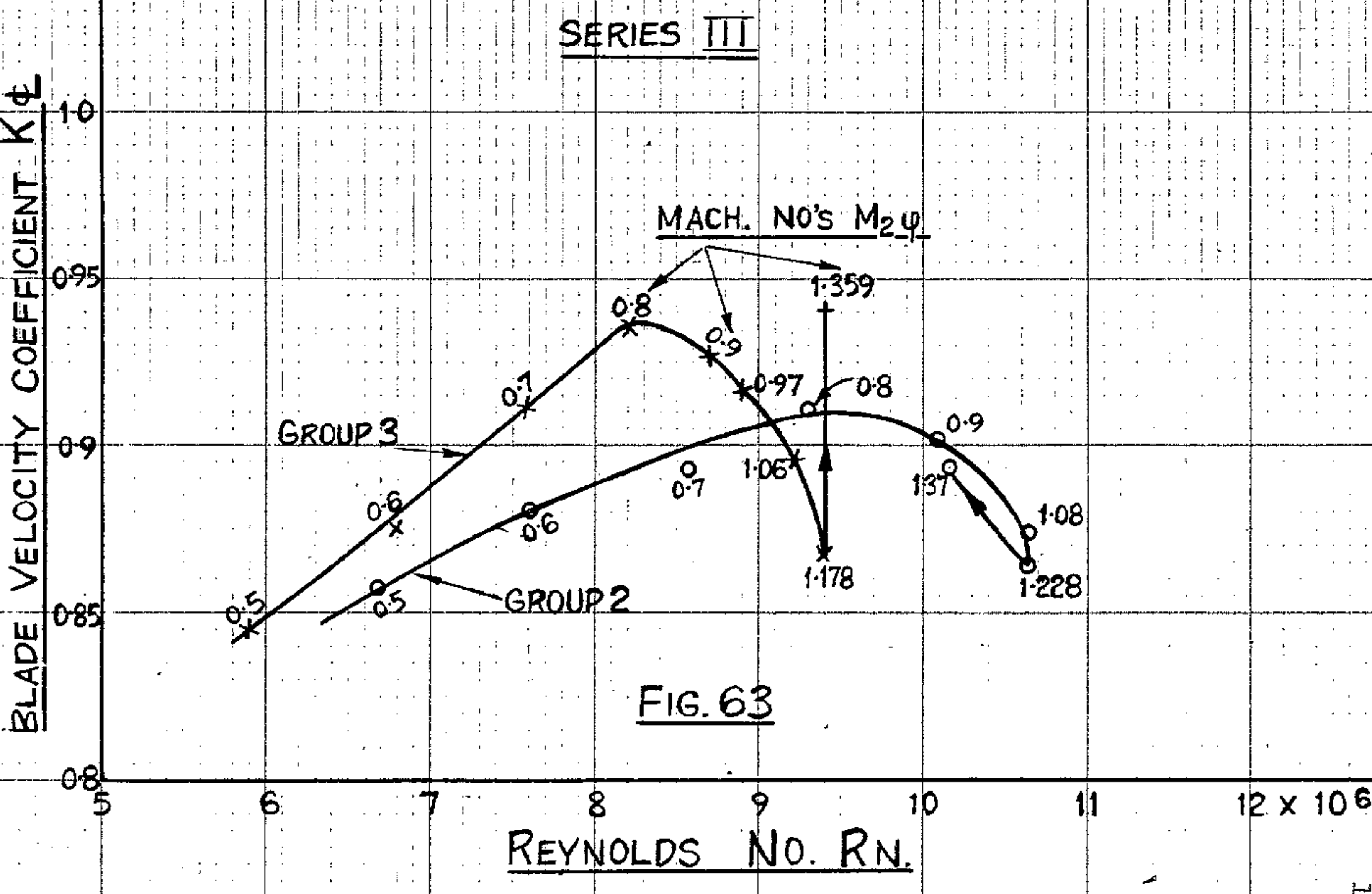

SERIES III
BLADE VELOCITY COEFFICIENT Kψ
1·0
0·95
0·9
0·85
0·8
5
6
7
8
9
10
11
12 x 10^6
REYNOLDS NO. RN.
MACH. NO'S M2ψ
GROUP 3
GROUP 2
0·5
0·6
0·7
0·8
0·9
0·97
1·06
1·178
1·359
0·5
0·6
0·7
0·8
0·9
1·08
1·228
1·37

FIG. 63

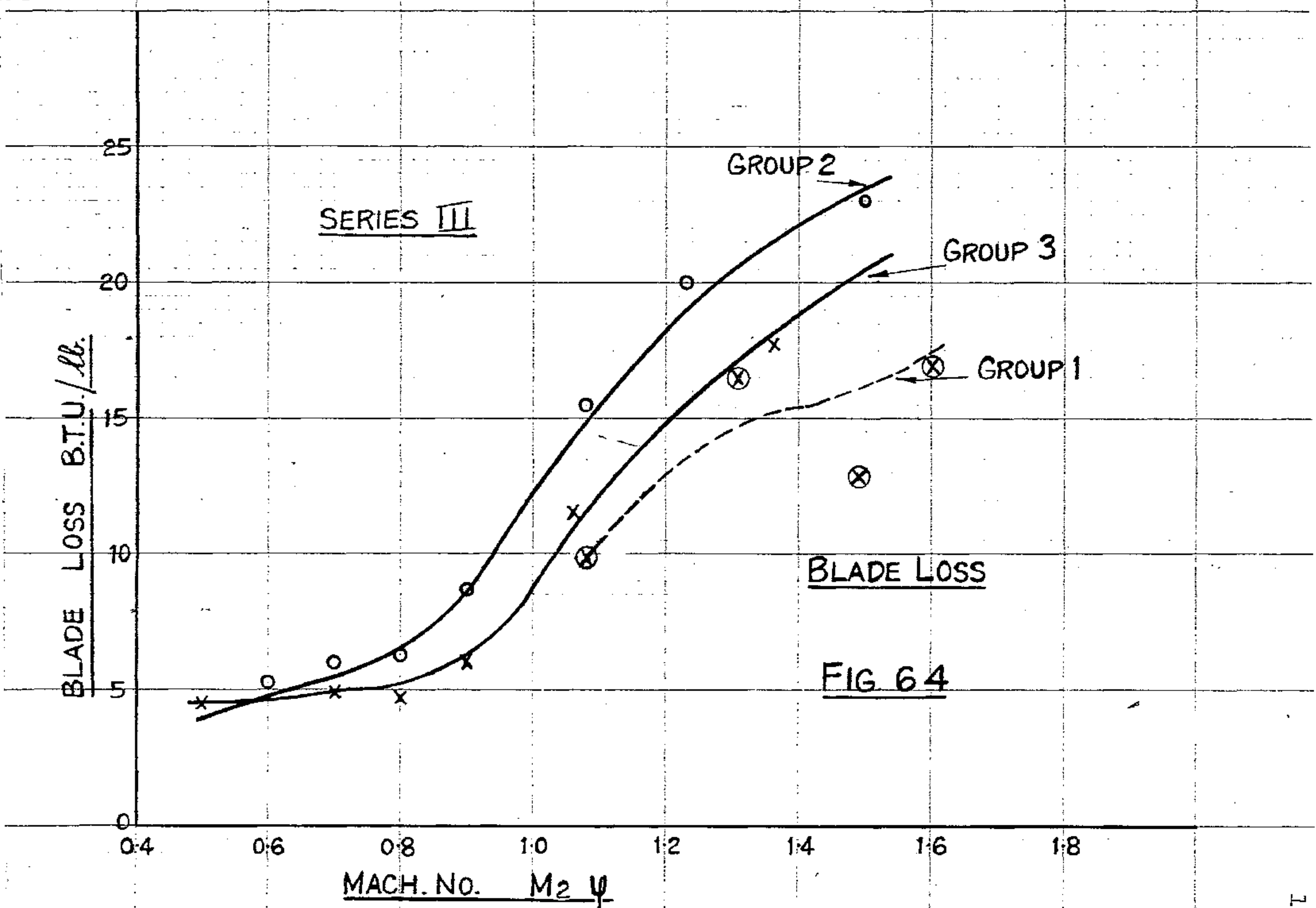

SERIES III
GROUP 2
GROUP 3
GROUP 1
BLADE LOSS
BLADE LOSS B.T.U./lb.
MACH. NO. M2 ψ
25
20
15
10
5
0
0·4
0·6
0·8
1·0
1·2
1·4
1·6
1·8

FIG 6 4

Effect of Supersonic Mach Number on Efflux Angle.

It was observed in series 3 that the efflux angle at any given station varied considerably as the Mach number was increased into the supersonic field. To investigate this further a traverse was made along the vertical centre line of the nozzle outlet at values of Mach number greater than unity.

The results are given in Fig. 65 (page 175) and show the change of efflux angle with Mach number for various vertical stations.

Discussion of Results.

Discussion of Results.

Part 1.

The stage efficiency of a turbine or compressor has been shown to depend on two broad considerations. The first of these is the effect of friction in the stage elements which causes the degeneration of high grade energy and the second is the choice of such design parameters as the degree of reaction or loading factor for the stage. The enthalpy loss or loss of high grade energy in a static or moving element is shown to be a composite quantity comprising a basic friction work term plus an auxiliary loss term. The friction work is equal in magnitude to the high grade energy which is degenerated to low grade energy and the auxiliary loss term is due to the "heating" effect of the low grade energy thus created. For a compression process the auxiliary term is positive while for an expansion it is negative. Comparing similar expansion and compression processes therefore, where the friction work is the same, the compression process will be less efficient than the expansion process. The enthalpy loss is related to other ways of expressing the irreversibility in the blade element and the expressions developed may be applied to static or moving elements in axial flow turbines or compressors with high and low speed flow.

The stage efficiencies in turbines or compressors are shown to be dependant on the blading or diagram efficiency as well as on the friction losses in the elements. A comprehensive theory is developed giving the variation in blading efficiency with such stage design parameters as the degree of reaction, the blade speed to jet speed ratio and the loading factor. The analysis is developed for positive, zero, or negative degrees of reaction and the speed ratio for maximum blading efficiency is obtained for any given degree of reaction. It is shown that the value of the maximum blading efficiency increases as the degree of reaction increases. However the loading factor at maximum blading efficiency reduces as the degree of reaction increases and in choosing design parameters for the stage account must be taken of these conflicting factors. For a turbine stage the total to static stage efficiency is used where the exhaust kinetic energy is degraded by friction and it is shown that this stage efficiency is directly proportional to the blading efficiency. The total to total stage efficiency, used where the exhaust kinetic energy is not degraded, also increases as the blading efficiency increases. It is shown however that where the friction losses in the elements are low and the blading efficiency high, the total to total stage efficiency is almost independant of blading efficiency. The characteristic variation of the two stage efficiencies with blade speed to jet speed ratio at a given degree of reaction may be deduced from the corresponding variation of blading efficiency and from the way in which the process efficiency varies as the velocity triangles change.

Part 2.

Having decided on the broad features of a design by determining the amount of reaction or compression, blade speeds and stream velocities to be used, it is then necessary to obtain experimentally the net friction loss in the proposed element and also the path taken by the working fluid as it passes through the element. In the section on the measurement of local efficiency by means of an impact tube, it has been shown how the tube may be used in either subsonic or supersonic flow to obtain local values of efficiency, specific volume, and efflux angle. The observed local total head pressures across a section must be converted to mean effective values at that section. To ease this transformation a method has been formulated using the theory, graphs and tables provided here. The calculations given for mean effective values represent a maximum requirement andthey may be modified to lessen the work involved in individual applications. For example, in many cases using cascades of blades in wind tunnels the variation of density across the traverse plane may be neglected and in calculating mean efficiency the local efficiency need not be weighted for mass flow.

Part 3.

The technique evolved for the measurement and calculation of loss represents a method which is applicable to all types of turbo machine elements with either compression or expansion in the elements and proves effective here in its application to a nozzle and blade pair. The inherent disadvantage in the method is the length of time required both for obtaining the impact tube readings and for completing the calculations. Some considerable amount of work has been done towards reducing the time factor by introducing automatic traversing gears and calculators. (See references 37 and 38).

The advantages of using steam as a working fluid in this type of testing lie in the ease with which the inlet condition and exhaust pressure can be varied giving a very flexible variable density tunnel and also in the simplicity of accurate flow measurement by condensation. The accuracy of the calculated mean values is difficult to assess due to the number of variables involved, but a check on the calculated mass flow rates is obtained by comparison with the observed flow figures and show a good agreement.

With regard to the experimental results it should be emphasized that these apply to the type of machine element tested that is to a single nozzle and impulse blade pair and that the losses measured exclude incidence effects. This limits the variables making the experimental work an attempt to correlate loss and flow characteristics with Reynolds number and Mach number for a constant geometry system.

The general impressions from all the subsonic test results are that, while the losses change, the general flow pattern at the nozzle or blade exit remains substantially the same with varying Reynolds number and Mach number. For the nozzle we find isentropic flow throughout the major part of the outlet area with the region of low flow and high loss confined to a small wall boundary layer, while the variation of efflux angle in the vertical direction is considerable.

At the blade exit plane the isentropic condition is nowhere realized. On the vertical centre line the velocity (Figs 53 and 54 pages 149 and 150) reaches a constant maximum for only 40% of the outlet height and this region is flanked by large unsymetrical boundary layers, the maximum loss being on the convex side of the blade passage. Whether these loss regions can be described as boundary layers is doubtful because they are probably caused by detachment of the true boundary layer due to the curvature of the blade passage walls. Centrifugal effects within the blade passage will result in the lowest static pressure normal to the flow occuring at the convex surface of the blade. Hence at some subsonic Mach number a local Mach number of unity will appear firstly at the convex surface. This can result in a shock discontinuity stemming from the convex surface with consequent detachment or thickening of the boundary layer downstream from the point of origin of the shock.

In the horizontal direction the blade velocity distribution (Fig. 61 page 163) as well as showing the wall loss displays regions of low efficiency on either side of the vertical centre line. This is explained by the internal distribution of static pressure formed by the curvature of the blade passage which causes circulatory motion or vortices to be imposed on the main flow. Again vertically we find the same form of efflux angle distribution as for the nozzle.

The variation of mean nozzle efficiency with Reynolds number (Fig. 50 page 146) follows the same pattern for the two and three dimensional tests and show a maximum value at a Reynolds number of 5.7×10^6. The variation of the mean pair efficiency with Reynolds number is however in the reverse direction to the nozzle efficiency. One would expect that in a region of high nozzle efficiency, that is with a good quality jet entering the blades passage, that the loss in the blade passage would be small. Remembering however that the loss regions in the nozzle are confined to the boundary layer, with good nozzle efficiency we then have a high energy stream passing from the vicinity of the nozzle wall to the wall of the blading. A slower layer of fluid would be expected to conform more to the path dictated by the blade passage wall with consequently less disturbance to the rest of the flow. Thus we assume that regions of high velocity approaching a blade boundary will cause greater subsequent loss than regions of low velocity. The reverse would apply to the jet quality away from the boundaries. The blade velocity coefficient as would be expected follows the same pattern as the mean pair efficiency.

In the graphs of mean nozzle and mean pair efficiency with Mach number (Fig. 55 page 157) we see that the nozzle efficiency reduces with increase of Mach number up to 0.9 and then increases from 0.9 to 1.0. In the mean pair efficiency graphs, where the losses are greater, there is a similar tendency for the efficiency figures to increase as a Mach number of unity is approached. Ainley[11] and Todd[10] have reported increase in efficiency as the Mach number approaches unity. They say that this is due to a local shock wave appearing first at some point on the convex surface of the blade and some way inside the passage and that as the free stream Mach number is increased the shock front moves towards the blade outlet leaving less surface in contact with the thickened boundary layer. For circular nozzles Guy[4] and Giffen[3] have each indicated a rise in efficiency at high steam speeds although they disagreed as to the form of the curve at low steam speeds.

The graphs of mean pair efficiency also indicate a rise of efficiency with increasing Reynolds number below a Mach number of 0.6 and again a rise of efficiency with increased Mach number above 0.7 with the range 0.6 to 0.7 as a transition zone. This to some extent confirms the observations of a number of observers to the effect that Reynolds number is the controlling parameter below a Mach number of 0.66 and Mach number the controlling parameter thereafter. This is again supported by the graphs of loss in Fig. 57 (page 159).

The series 3 test results plotted in Fig. 62 (page 171) show a regular pattern for the variation of centre line blade velocity coefficient with Mach number and indicate maximum values at 0.8 in the subsonic field and 1.4 in the supersonic field. To show the influence of Reynolds number the coefficients are again plotted to a base of Reynolds number in Fig. 63 (page 172). In the Mach number range 0.9 to 1.2 it will be observed that the variation in Reynolds number is small so that the reduction in blade velocity coefficient here must be solely due to Mach number effects. The effect of Reynolds number on the coefficient in this same range of Mach number can be seen by comparing the group 3 tests with the group 2 tests where the increase of Reynolds number from group 3 to group 2 reduces the coefficient. The variation of centre line loss with Mach number (Fig64 page 173) shows a progressive increase of loss with Mach number.

As stated previously the pattern of efflux angles for the nozzles and blades varies very little with Reynolds number and subsonic Mach number. In the efflux angle graphs in Figs. 53 and 54 (pages 149 and 150), the nozzle efflux angle steadies to a value of 28^{o} over a large proportion of the vertical traverse and is almost constant with Reynolds number. In the graphs in Fig. 60 (page 162) the corresponding steady values are 28^{o} at Mach number 0.5 rising to 32^{o} at Mach number 0.97, and the mean calculated values of nozzle efflux angle agree very closely with these central steady values. This small variation in efflux angle with Mach number is consistent with the results obtained by Ainley[11]. Ainley quotes variations in efflux angle with subsonic Mach number for various theoretical outlet angles. His results show that the jet is deflected towards the axial direction as the Mach number is increased from low subsonic values up to unity. The variation is about 6^{o} for a theoretical outlet angle of 40^{o} (measured from the axial direction) and the variation decreases as the theoretical axial outlet angle is increased. There is however a considerable difference of about 8^{o} between the mean values quoted above and the geometric outlet angle and in addition the efflux angles of the nozzle or blade differ from those quoted by Guy where the formula $\sin^{-1}$ opening/pitch was said to give the mean outlet angle. It should be remembered however that in Guy's experiments the nozzles being tested were supported on either side by other nozzles running full of steam.

The change in the pattern of the nozzle efflux angle distribution in the supersonic field (Fig. 65 page 175) is in contrast to the small changes in the subsonic region. Considering the vertical centre line at the mid point of the nozzle, as the Mach number approaches unity the efflux angle is 32^{o} with angles varying up to 56^{o} and down to 10^{o} at the boundaries. As the Mach number increases to 1.5 the flow is deflected away from the boundaries towards the main stream and with increasing Mach number the vertical variation of efflux angle is reduced. Hence with increasing supersonic Mach number the efflux angle tends to follow more closely the geometric outlet angle.

In all the tests the pressure at exhaust from nozzle or blades has been taken as equal to the pressure observed in the casing and the assumption is then made that this pressure is constant along the line of the traverse. This is justified provided the variation in pressure is small due to the relative difficulty of making accurate static pressure measurements in the free stream, reliance normally being placed on wall tappings. In the subsonic tests the casing pressure and nozzle wall tappings were found to coincide. In the supersonic tests the nozzle wall tappings, as would be expected, gave the critical back pressure and the casing pressure was again taken as the static pressure along the traverse line. Stodola[37] has shown that the variation in static pressure at the exhaust from a convergent parallel nozzle with super critical expansion is small and in any case the impact tube measures the total energy of the stream so that a slight departure of the local static pressure from the final back pressure means only that a little of the total energy is retained as potential energy. The transformation to kinetic energy will be complete a little further downstream with only a slight effect on efficiency.

We have seen in the section on the analysis of losses that the effective loss in turbo machine elements can be divided into two parts. The first part is the friction work or the basic loss of high grade energy which is dissipated as low grade energy. The second part, which is positive or negative depending on the pressure gradient, represents the "heating" effect of the low grade energy on the specific volume. The internal heating will be equivalent to external heating of the same amount and is therefore distinct and separate from the basic friction work loss. One feels that it is this basic friction work loss and not the effective loss which should be related to Reynolds number and Mach number. Unfortunately the measurement of the basic loss is much more difficult than that of the effective loss, since it would entail traverse measurements through the whole expansion or compression process.

In the section on efficient mechanical energy transfer we saw, for a turbine stage, that the maximum blading or diagram efficiency rises as the degree of reaction increases. The comparative effects of a small amount of reaction and of a small amount of compression have shown in Fig. 47 (page 122) where the flow with compression appears to break down completely in the centre of the blade. Ainley [11] has stated for the turbine blades with which he was dealing that the loss decreased with increased reaction. Hence from the point of view of increased efficiency both in expansion in the blading and in diagram efficiency high reaction is beneficial. The upper limit on reaction will be due only to the decrease in work done factor with increased reaction.

Conclusions.

It is well known that where friction is involved care must be exercised in applying thermodynamic principles. It has been shown here how the expressions for the energy inter-changed between the rotor blades and the working fluid in turbines and compressors are modified in a friction process. The loss of useful energy caused by friction is shown to comprise of two parts, one of which accounts for the degeneration by friction work of high grade energy into low grade energy and the other for the effect of this low grade energy on the specific volume of the fluid. Thus the nature of any friction process may be more readily examined and it is shown here how friction affects the efficiency of similar expansion and compression processes.

Some work has already been done on the choice of design parameters for the efficient operation of both the impulse blade and 50° reaction blade. The section on maximum blading efficiency, with its concept of Reaction Coefficient, forms a comprehensive theory which will fill the gap between these two as well as embracing the operation of the axial flow compressor. The flexibility of the theory will be apparent to those who are contemplating unorthodox designs. For example it is well known that in axial flow compressors the growth of the boundary layer from stage to stage reduces the efficiency. If occasionally a stage is introduced with a little reaction in say the moving blade, an improvement in the axial velocity distribution would result.

The determination of local efficiency and efflux angle and the conversion of these local values into mean effective values is a necessary step in the analysis of the performance of machine elements. It is suggested that the methods herein developed will ease and simplify this transformation. These methods take account of variable density and supersonic flow and may therefore be used, in individual applications, to assess the errors involved in applying quicker but less rigorous procedures.

With regard to the flow pattern in the nozzle and blade pair, there was an unexpected variation in nozzle efflux angle about the centre of the nozzle, the stream apparently paying little heed to the geometric outlet angle except in a small central core. This flow pattern remained comparatively unchanged with varying Reynolds number and subsonic Mach number. When however the Mach number was increased to values greater than unity, there was a marked straightening of the stream lines, so that with progressively increasing Mach number the stream conformed more and more to the geometric outlet angle at all sections of the nozzle.

The variation of profile and total loss coefficients with Reynolds and Mach number for the writers zero pressure drop impulse blade is compared in appendix 3 with similar research by other workers, notably Armstrong[40], Ainley[41] and Kearton[42]. Armstrong examined the profile loss characteristics for an impulse blade at high Reynolds number and found that above a Reynolds number of 2×10^5, based on blade chord, the loss coefficient remains substantially constant. When however the Reynolds number is reduced from high values and approaches the 2×10^5 region there is a sudden increase in the loss coefficient. The writers work, which covers a similar high Reynolds number range, helps to confirm that a chord Reynolds number of 2×10^5 is in fact a critical value of this parameter. The following conclusions regarding the variation of loss coefficients were found to be consistent with most of the experimental data given in appendix 3.

Below a Mach number of 0.66 the controlling parameter is the Reynolds number. As the Reynolds number is reduced below 2×10^5 both the profile and the total loss coefficient show a considerable increase and above this critical value they remain substantially constant. Where the Reynolds number is high (above 2×10^5) and the Mach number is above 0.66 then (a) the total enthalpy loss coefficient remains constant while the total stagnation pressure loss coefficient increases as the Mach number increases, and (b) the profile enthalpy and stagnation pressure loss coefficients are both Mach No. dependant and increase with increasing values of the Mach number.

The impact tube and traversing gear have been shown to be capable of use in relatively high pressure, temperature and density streams. Hence the arrangement may be used to test steam or gas turbine elements near to their actual working conditions, or indeed the gear may be adapted for use on actual turbines or compressors.

In general however the experimental and analytical procedures involved in testing turbo machinery elements are time consuming and repetitive. In future work it would be desirable to intoduce more labour saving devices, such as automatic traversers and integrators, and certain suggestions along these lines by other workers have already been referred to.

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Appendix 1.

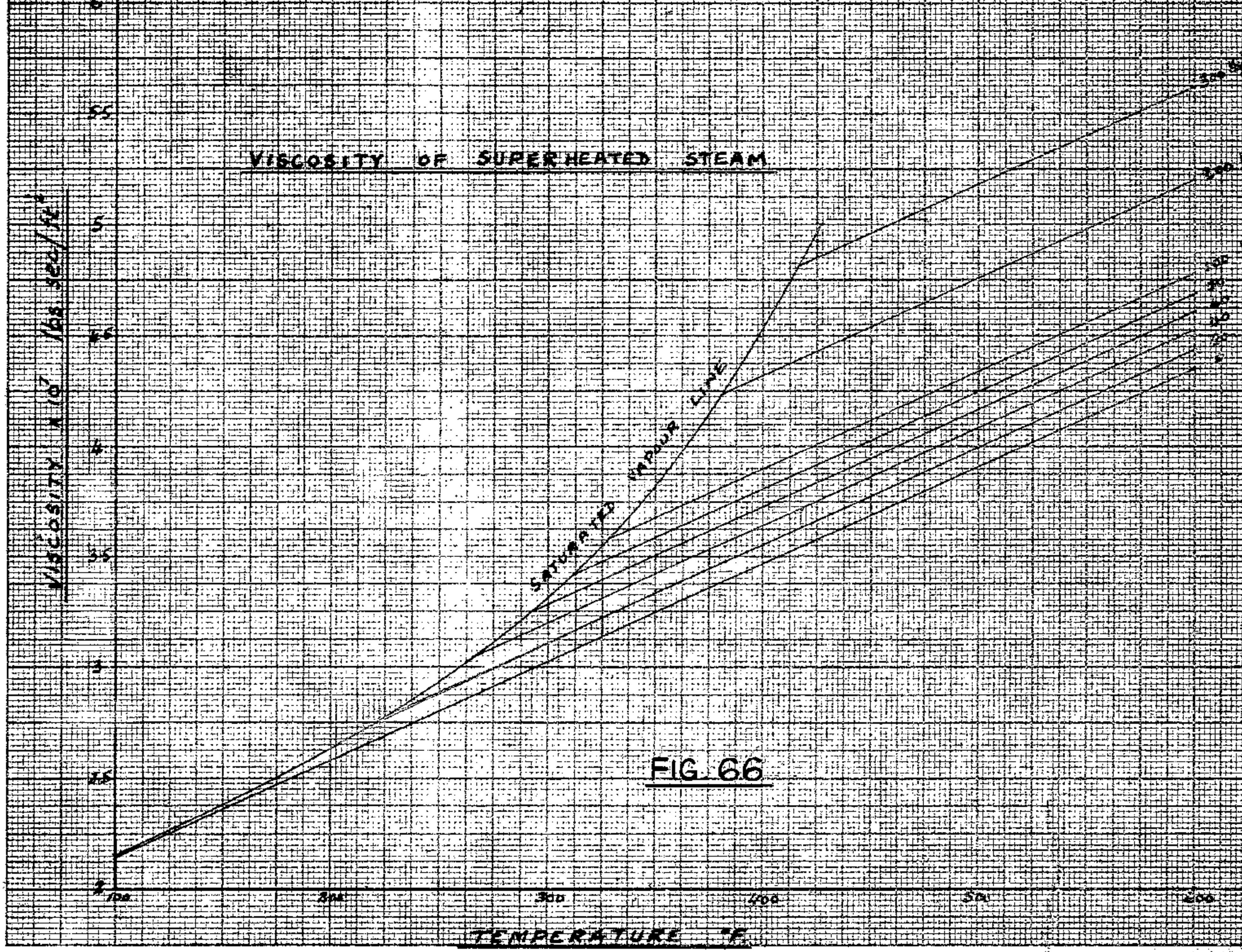
VISCOSITY OF SUPERHEATED STEAM
VISCOSITY × 10^7 lbs sec/ft^2
TEMPERATURE °F
SATURATED VAPOUR LINE
2
2.5
3
3.5
4
4.5
5
5.5
100
200
300
400
500
600

FIG. 66

Specimen tables of observed and derived results Series 2.

Tests 1 and 1A.

Specimen Calculations for test 1 and 1A Series 2.

$P_1 = 24.3 \text{ lb/in}^2$

$P_2 = 20.7 \text{ lb/in}^2 \quad \therefore \frac{P_2}{P_1} = 0.852 \quad \therefore M_{2\varphi} = 0.5.$

$H_1 = 1229.25$ BTU/lb. $\quad DH_\varphi = 14.45$ BTU/lb.

$T_1 = 380^\circ F \quad T_{2\varphi} = 348.5^\circ F$

$v_{2\varphi} = 223.8 \sqrt{DH_\varphi} = 851$ ft/sec. $\quad V_{2\varphi} = 23 \text{ft}^3/\text{lb}.$

$\mu_{2\varphi} = 3.325 \times 10^{-7} \text{ lb sec/ft}^2$

$R_n = 3.46 \times 10^6$

From Test 1 Table 3.

$\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 19.129$, and $\Sigma_{\mathbb{C}} \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 2.992$ (for vertical ℄)

From Test 1A Table 3

$\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 11.083$ and $\Sigma_{\mathbb{C}} \eta^{1/2} \sin\alpha \frac{fda}{fV} = 1.358$ (for vertical ℄)

From Test 1 Table 4.

$\Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV} = 17.541$ and $\Sigma_{\mathbb{C}} = 2.992.$

From Test 1A Table 4.

$\Sigma \eta^{3/2} \sin\alpha \frac{fda}{fV} = 8.467$ and $\Sigma_{\mathbb{C}} = 1.126.$

Hence $\quad \bar{\eta}_n = \frac{17.541}{19.129} = 0.917. \quad \bar{\eta}_{n\mathbb{C}} = \frac{2.992}{2.992} = 1.0.$

$\bar{\eta}_\ell = \frac{8.467}{11.083} = 0.766. \quad \bar{\eta}_{\ell\mathbb{C}} = \frac{1.126}{1.358} = 0.83$

Blade velocity coefficient $K = \sqrt{\frac{0.766}{0.917}} = 0.914.$

and $K_{\mathbb{C}} = \sqrt{\frac{0.83}{1}} = 0.911.$

Blade loss $= (1 - K^2)\, \bar{\eta}_n \, DH_\varphi = 2.185$ BTU/lb.

$$\text{Blade loss}_{\mathbb{C}} = (1 - K_{\mathbb{C}}^2)\ \bar{\zeta}_{n\,\mathbb{C}}\ DH_{\varphi} = 2.46 \text{ BTU/lb.}$$

From test 1 table 5.

$$\Sigma \zeta \cos \alpha \sin \alpha \frac{fda}{fV} = 15.331.$$

From test 1A table 5.

$$\Sigma \zeta \cos \alpha \sin \alpha \frac{fda}{fV} = 6.902.$$

Hence for the mean nozzle exit angle.

$$\cos \bar{\alpha}_n = \frac{\Sigma \zeta \cos\alpha \sin\alpha \times \frac{fda}{fV}}{\sqrt{\bar{\zeta}_n}\, \Sigma \zeta^{\frac{1}{2}} \sin \alpha \frac{fda}{fV}} = \frac{15.331}{0.957 \times 19.129} = 0.837.$$

$$\therefore \bar{\alpha}_n = 33^\circ - 11'$$

and for the mean blade exit angle.

$$\cos \bar{\alpha}_b = \frac{6.902}{0.875 \times 11.083} = 0.714 \quad \therefore \bar{\alpha}_b = 44^\circ - 24'$$

TABLE I

Test No :- 1
SERIES II

Mach No. ·5

Barometric Press = 14·3 LB/IN²

p_1 = 24·3 LB/IN² Ab.

p_2 = 20·7 LB/IN² Ab.

Temp. 380°F.

HORIZONTAL STATIONS →

VERTICAL STATIONS →

STATIONS	$-3\frac{3}{4}$		$-3\frac{1}{2}$		$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
$2\frac{3}{4}$	20·8	55	20·9	55	21·5	55	22·3	58	23·4	56	23·8	54	24·3	55	23·8	56·5	22·8	58	21·1	57	SS	
$2\frac{1}{2}$	20·8	51	21·1	51	22·4	51·5	23·3	55	24·3	52	24·3	49	24·3	50	24·3	50	24·3	51	24·8	52·5	SS	[illegible]
2	20·9	34	21·8	38	23·5	40	24·3	40·5	24·3	39·5	24·3	37	24·3	36·5	24·3	38	24·3	39	22·7	41·5	21·0	44
1	21·4	25	22·3	29	23·6	29	24·3	31	24·3	30	24·3	30	24·3	30	24·3	31·5	24·3	30·5	23·1	30	21·8	29
0	21·5	21	22·5	23	23·5	26	24·3	26·5	24·3	27	24·3	28	24·3	28	24·3	27	24·3	27	23·1	25	21·7	25
-1	21·3	21	22·4	23	23·6	24·5	24·3	23	24·3	23	24·3	26	24·3	25	24·3	25	24·3	26	22·8	23	21·4	22
-2	21·2	15·5	22·3	19	23·3	18	24·3	18	24·3	19	24·3	19	24·3	20	24·3	20	24·3	19·5	22·1	17	21·2	17
-3	21	5	21·4	6	22·2	9	23·4	10	24·3	11	24·3	11	24·3	12	24·3	11	24·3	12	21·2	11	20·9	
-4			20·9	-3	21·1	-3	21·7	4	24·3	7·5	24·3	6·5	24·3	6·5	24·3	6·5	24	7	20·8	7	SS	
$-4\frac{1}{4}$			20·9	-4	21	-4	21·5	0	24·3	5·5	24·3	6	24·3	6·5	24·3	6·5	23·6	6	20·7			

TABLE I

TEST No:- IA.
SERIES II

MACH. No. ·5

P_1 = 24·3 LB/IN² Ab

BAROMETRIC PRESS = 14·42 LB/in² A

P_2 = 20·7 LB/IN² Ab

TEMP. 380 °F

HORIZONTAL STATIONS →

VERTICAL STATIONS →

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
1½	–	–	–	–	20·52	2	20·82	2	21·42	2	21·32	4	21·22	4	20·82	–	–	–	–	–	–	–	20·82	18	20·62	18	20·82	8
1	20·62	4	20·82	7	21·02	10	21·72	9	21·42	6	21·52	2	21·52	1	21·22	0	20·82	0	20·62	–	20·42	–	20·52	–	21·32	20	21·42	15
½	20·42	5	20·82	10	21·52	13	22·52	10	22·22	8	22·72	10	22·62	10	22·42	9	22·02	8	21·62	7	21·42	–	21·42	19	22·02	20	22·92	21
0	21·22	17	21·92	11	23·32	17	23·42	17	23·32	14	23·52	20	23·62	22·5	23·92	23	23·72	24	23·42	21	23·52	20	23·32	20	23·22	19	23·02	19
-1	22·32	21·5	23·32	27	23·62	29	23·62	30	23·62	31	23·82	31	23·72	32	23·72	33	23·72	34	23·62	34	23·52	33	23·42	32	23·42	31	23·22	29
-2	22·42	48	23·42	44·5	23·72	44·5	23·62	46	23·72	46	23·72	47	23·62	46	23·72	45	23·92	47	23·82	46	23·62	46	23·52	45	23·42	46	22·82	47
-2½	21·22	69	22·02	67	23·22	64	22·82	66	23·52	62	23·42	61	23·72	60	23·72	59	23·72	58	23·92	57	23·72	57	23·52	60	22·22	66	21·12	65
-2¾	–	–	21·22	–	21·92	78	21·52	79	22·52	79	22·92	76	23·12	75	23·52	69	23·52	67	23·72	67	23·42	67	23·32	73	20·92	–	–	–

TEST No :- 1
SERIES II

MACH No. 5   TABLE 2.   BAROMETRIC PRESS = 14.3 LB/in²

p_1 = 243 LB/IN² Ab. p_2 = [illegible] LB/IN² Ab.   TEMP. 380°F.

STATIONS	−3 3/4		−3 1/2		−3 1/4		−3		−2		−1		0		1		2		3		3 1/4	
	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η	p_2/p_3	η
2 3/4	·995	·027	·99	·055	·963	·232	·928	·462	·855	·76	·869	·872	·852	·995 1	[illegible]	·862	·908	·6	·962	·31	~~·985~~	
2 1/2	·995	·027	·982	·105	·929	·49	·867	·747	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	·95	·32	·95	[illegible]
2	·99	·055	·949	·33	·88	·792	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	·912	·575	·985	·09
1	·967	·207	·929	·462	·877	·817	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	[illegible]	~~·995~~ 1	·896	·685	·95	·32
0	·963	·24	·919	·52	·88	·792	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	[illegible]	~~·995~~ 1	·896	·685	·954	·292
−1	·971	·182	·924	·49	·877	·817	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	[illegible]	~~·995~~ 1	·958	·6	·965	·2
−2	·977	·14	·938	·442	·885	·74	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	[illegible]	~~·995~~ 1	·937	·407	·977	·14
−3	·986	·082	·968	·2	·9325	·43	·854	·765	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	·977	·14	·99	·055
−4			·99	·055	·981	·112	·954	·292	·852	~~·995~~ 1	·852	·995 1	[illegible]	~~·995~~ 1	·852	~~·995~~ 1	·863	~~·915~~	·995	·027		
−4 1/4			·99	·055	·985	·09	·962	·24	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	·852	~~·995~~ 1	·877	·815	1	1		

TEST No:- 1A
SERIES II

MACH. No. 5 TABLE 2
P_1 = 24.3 LB/IN² Ab.

BAROMETRIC PRESS = 14.42
P_2 = 20.7 LB/IN² Ab.

TEMP. 380. °F

STATIONS	−1¾		−1½		−1¼		−1		½		0	
	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η
1½	S	S	S	S	S	S	.994	.035	.966	.215	.973	.168
1	S	S	.994	.035	[illegible]	[illegible]	.954	.295	.957	.275	.962	.24
½	S	S	.994	.035	[illegible]	[illegible]	.919	.525	.931	.45	.912	.575
0	.974	.16	.944	.36	[illegible]	[illegible]	.884	.765	.889	.735	.881	.788
−1	.928	.469	.881	.735	[illegible]	.817	.877	.817 ~~.75~~	.876	.823	.87	.865
−2	.924	.49	.934	.765	[illegible]		.877	.817 ~~.85~~	.866	.894	.874	.835
−2½	.975	.155	.94	.385	[illegible]	.72	.906	.617	.881	.788	.871	.817 ~~.75~~
−2¾	S	S	.975	.155	[illegible]	.355	.963	.235	.92	.52	.904	.63

STATIONS	½		1		2		3		3½		4		4¼		4½	
	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η	P_2/P_3	η
1½	.975	.155	.994	.035	S	S	S	S	S	S	.994	.035	.994	.035	.994	.035
1	.962	.24	.978	.132	.994	.035	S	S	S	S	S	S	.971	.285	.966	.222
½	.914	.562	.923	.50	.94	.385	.936	.28	.966	.215	[illegible]	.215	.94	.385	.925	[illegible]
0	.876	.823	.866	.894	.874	.84	.883	.775	.881	.76	.887	.755	.972	.714	.90	.96
−1	.874	.892	.874	.834	.874	.84	.877	.817	.881	.75	.884	.755	.884	.705	.922	.715
−2	.874	.823	.874	.834	.866	.874	.87	.865	.877	.917	.888	.74	.884	.765	.776	.617
−2½	.874	.836	.874	.836	.874	.84	.87	.865	.873	.845	.888	.74	.733	.435	.78	.12
−2¾	.895	.75	.881	.788	.88	.775	.873	.846	.884	.765	.927	.40	.994	.035	S	S

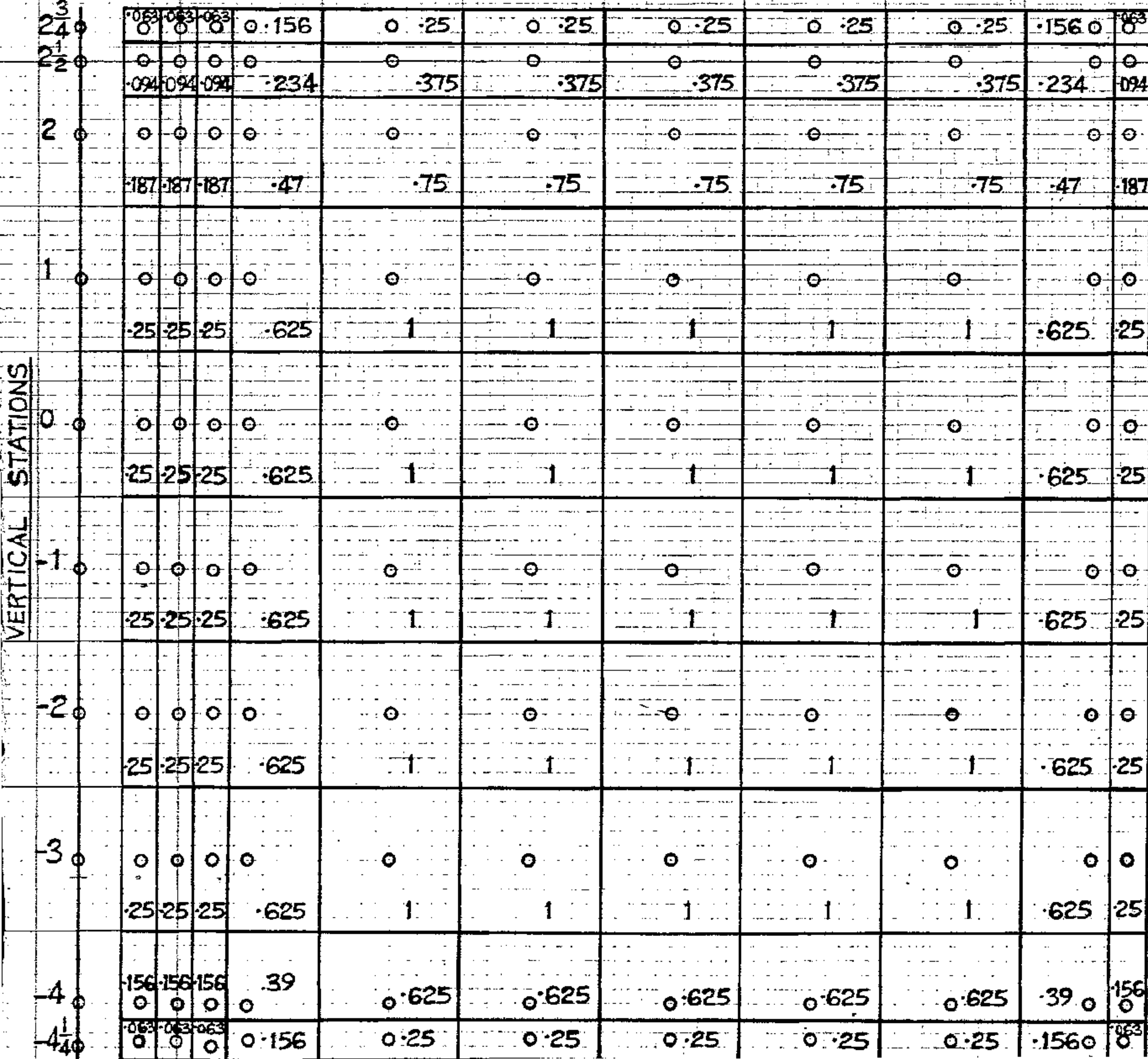

FIG. 67

SERIES II

IMPACT READINGS MARKED o

AREA FACTORS (fda) ARE ENCLOSED IN THE RECTANGLE

HORIZONTAL STATIONS

VERTICAL STATIONS	$-1\frac{3}{4}$	$-1\frac{1}{2}$	$-1\frac{1}{4}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	$3\frac{1}{2}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$
$1\frac{1}{2}$	·094	·094	·094	·141	·187	·187	·187	·281	·375	·281	·187	·141	·094	·141
1	·125	·125	·125	·187	·25	·25	·25	·375	·5	·375	·25	·187	·125	·187
$\frac{1}{2}$	·125	·125	·125	·187	·25	·25	·25	·375	·5	·375	·25	·187	·125	·187
0	·187	·187	·187	·281	·375	·375	·375	·563	·75	·563	·375	·281	·187	·281
-1	·25	·25	·25	·375	·5	·5	·5	·75	1	·75	·5	·375	·25	·375
-2	·187	·187	·187	·281	·375	·375	·375	·563	·75	·563	·375	·281	·187	·281
$-2\frac{1}{2}$	·094	·094	·094	·141	·187	·187	·187	·281	·375	·281	·187	·141	·094	·141
$-2\frac{3}{4}$	·063	·063	·063	·094	·125	·125	·125	·187	·25	·187	·125	·094	·063	·094

SERIES II

EXIT FLOW MAP FOR BLADE

FIG. 68.

IMPACT READINGS MARKED O

AREA FACTORS ($\int da$) ARE ENCLOSED IN THE RECTANGLES

SERIES II TEST:-1a TABLE :-3

TION	−1¾			−1½			−1¼			−1			−½			0			½			1			2			3			3½			4			4¼			4½		
	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da	$\eta^{\frac{1}{2}}$	Sinα	∫da
	fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{\int da}{fV}$	
										182 / 209	·035	141	464	·035	187	41	·07	167	394	·07	187	187	·07	281			375							187	·31	141	187	242	094	187	139	141
:		—			—			—		1·036	·001		1·029	·003		1·031	·005		1·032	·050		1·03	·004			—			—			—		1·036	·008		1·036	·004		1·036	·004	
				187	122	125	398	174	125	544 / 47	156	167	525	405	25	49	·035	25	49	·018	25	364	0	375 / 25	187	0	·5			375			25			187	534	342	125	471	259	167
		—		1·036	·003		1·034	·006		1·026	·015		1·027	·013		1·028	·004		1·028	·002		1·033	·000		1·036	·000											1·03	·022		1·029	·022	
				187	·174	125	49	225	125	725	·174	187	67	139	25	758	·174	25	75	174	25	704	·156	375	621	139	5	53	122	375	444	·122	25	444	326	187	621	342	125	696	358	187
2		—		1·036	·004		1·028	·013		1·017	·023		1·02	·023		1·016	·033		1·016	·032		1·018	·041		1·023	·042		1·027	·024		1·029	·014		1·029	·027		1·023	·026		1·019	·046	
	·4	292	187	·6	191	187	86	292	187	875	292	281	869	242	375	887	342	375	908	383	375	946	391	563	914	407	75	88	358	563	866	342	375	857	342	281	846	326	187	812	326	281
)	1·031	·021		1·024	·021		1·01	·047		1·009	·071		1·010	·078		1·008	·113		1·006	·130		1·004	·208		1·006	·278		1·008	·176		1·009	·110		1·01	·081		1·01	·051		1·012	·073	
	682	367	25	856	454	25	904	485	25	904	·5	375	907	515	5	93	515	5	916	·53	5	914	545	75	910	559	1	904	559	75	844	545	5	875	53	375	875	515	25	844	485	375
	1·02	·061		1·01	·096		1·006	·109		1·006	·168		1·006	·232		1·005	·238		1·006	·241		1·006	·371		1·006	·510		1·006	·377		1·009	·234		1·008	·172		1·008	·112		1·01	·152	
	·7	743	187	874	701	187	914	701	187	904	719	281	946	719	375	914	731	375	91	719	375	914	707	563	946	731	75	93	719	563	904	719	375	86	707	281	875	719	187	785	731	281
l	1·019	·095		1·008	·115		1·006	·120		1·006	·181		1·024	·254		1·005	·249		1·005	·244		1·005	·362		1·004	·516		1·005	·374		1·005	·242		1·009	·170		1·008	·117		1·004	·159	
	394	·934	094	62	921	094	85	899	094	785	914	141	896	883	187	904	875	187	1·5	·866	187	914	857	281	946	898	375	93	839	281	92	839	187	86	866	141	66	914	094	34	906	141
½	1·032	·034		1·023	·052		1·01	·071		1·024	·100		1·008	·145		1·026	·147		1·025	·147		1·026	·219		1·006	·290		1·005	·218		1·006	·143		1·009	·104		1·021	·055		1·033	·043	
				394	·978	063	596	·978	063	483	982	094	721	982	125	794	·97	125	835	966	125	886	934	187	892	·921	25	92	921	187	875	921	125	678	956	094	187	956	063			
¾		—		1·032	·023		1·024	·036		1·025	·044		1·02	·087		1·014	·095		1·01	·100		1·008	·153		1·039	·204		1·006	·158		1·008	·100		1·02	·061		1·037	·011			—	
Σ	0·211			0·314			0·402			0·603			0·835			0·884			0·946			1·358			1·840			1·327			0·843			0·623			0·398			0·499		

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SERIES II TEST: 4 TABLE :- 4

-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼							
η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta^{3/2}\sin\alpha$	$\frac{fd\alpha}{fV}$
·7	·008	055	·012	232	·024	462	·088	·76	·179	872	·189	1	·205	862	·192	·6	·161	·11	·042								
·000		·000		·006		·040		·136		·164		·205		·165		·096		·005									
27	·012	105	·023	49	·051	747	·164	1	·296	1	·283	1	·288	1	·288	1	·292	32	·102								
·000		·002		·025		·122		·296		·283		·288		·288		·292		·033									
55	·024	33	·065	792	·105	1	·302	1	·477	1	·452	1	·446	1	·463	1	·472	575	·232	·09	·038						
·001		·021		·083		·302		·477		·452		·446		·463		·472		·133		·003							
07	·047	462	·081	817	·109	1	·322	1	·500	1	·500	1	·500	1	·523	1	·508	685	·256	32	·067						
·010		·037		·089		·322		·500		·500		·500		·523		·508		·175		·021							
24	·043	52	·069	·719	·077	1	·279	1	·454	1	·470	1	·470	1	·454	1	·454	685	·216	292	·056						
·010		·036		·070		·279		·454		·470		·470		·454		·454		·148		·016 ~~·021~~							
182	·037	49	·067	817	·073	1	·242	1	·391	1	·438	1	·423	1	·423	1	·438	·6	·186	·2	·041						
·007		·033		·075		·242		·391		·438		·423		·423		·438		·111		·008							
14	·024	462	·064	74	·066	1	·193	1	·326	1	·326	1	·342	1	·342	1	·334	407	·114	·49	·026						
·003		·025		·049		·193		·326		·326		·342		·342		·334		·047		·004							
082	·006	·2	·011	43	·025	·765	·109	1	·191	1	·191	1	·208	1	·191	[illegible]	·208	14	·043	055	·011						
·000		·002		·011		·084		·191		·191		·208		·191		·208		·006		·001							
		055	·002	42	·003	·292	·014	1	·082	1	·082	1	·082	1	·082	915	·072	027	·008								
		·000		·000		·004		·082		·082		·082		·082		·066		·000									
		·065	·001	·09	·001	·24	·000	1	·024	1	·026	1	·028	1	·028	815	·024										
		·000		·000		·000		·024		·026		·028		·028		·019											
0·031		0·156		0·408		1·588		2·817		2·932		2·992		2·959		2·887		0·658		0·053							

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f d\alpha}{f V}$$

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SERIES II TEST :- 1A. TABLE :- 4

N	$-1\frac{3}{4}$		$-1\frac{1}{2}$		$-1\frac{1}{4}$		-1		$\frac{1}{2}$		0		$\frac{1}{2}$		1		2		3		$3\frac{1}{2}$		4		$4\frac{1}{4}$		$4\frac{1}{2}$	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}} \sin\alpha$	$\frac{fda}{fV}$
							·035	·001	·215	·003	·168	·005	·155	·050	·035	·004							·035	·008	·035	·004	·035	·004
							·000		·001		·001		·008		·000								·000		·000		·000	
		·	·035	·003	·095	·006	·295	·015	·275	·013	·24	·004	·24	·002	·132	·000	·035	·000							·285	·022	·222	·022
			·000		·001		·004		·004		·001		·000		·000		·000								·006		·005	
			·035	·004	·24	·013	·525	·023	·45	·023	·575	·033	·562	·032	·50	·041	·385	·042	·28	·024	·215	·014	·215	·027	·385	·026	·480	·046
			·000		·003		·012		·010		·019		·018		·021		·016		·007		·003		·006		·010		·022	
	·16	·021 ~~·10~~	·36	·021	·74	·047	·765	·071	·735	·078	·788	·113	·823	·130	·894	·208	·84	·278	·775	·176	·75	·110	·735	·081	·714	·051	·66	·073
	·003		·008		·035		·054		·057		·089		·107		·186		·234		·136		·082		·060		·036		·048	
	·465	·061 ~~·105~~	·735	·096	·817	·109	·817	·168	·823	·232	·805	·238	·836	·241	·830	·371	·84	·510	·817	·377	·75	·234	·765	·172	·765	·112	·715	·152
	·028		·071		·089		·137		·191		·206		·202		·310		·428		·308		·175		·131		·086		·109	
	·49	·095	·765	·115	·837	·120	·817	·181	·894	·254	·835	·249	·823	·244	·830	·362	·894	·516	·865	·374	·817	·242	·74	·170	·755	·117	·617	·159
	·047		·088		·101		·148		·227		·208		·201		·304		·461		·324		·198		·126		·089		·098	
	·155	·034	·385	·052	·72	·071	·617	·100	·788	·145	·817	·147	·836	·147	·830	·219	·84	·290	·865	·218	[illegible]	·143	·74	·104	·435	·055	·12	·043
	·005		·020		·051		·062		·114		·120		·123		·184		·244		·189		·121		·077		·024		·005	
			·155	·023	·355	·036	·235	·044	·52	·087	·63	·095	·695	·100	·736	·153	·795	·204	·845	·158	·765	·100	·40	·061	·235	·011		
			·004		·013		·010		·045		·060		·070		·121		·162		·134		·077		·028		·000			
	0·083		0·191		0·293		0·427		0·649		0·704		0·729		1·126		1545		1·098		0·656		0·428		0·251		0·287	

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{\int da}{\int V}$$

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SERIES II TEST:- I TABLE:- 5

-3¾	-3½	-3¼	-3	-2	-1	0	1	2	3	3¼			
$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A	$\eta^{\frac{1}{2}}$ $\cos\alpha$ A
$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$	$\eta\cos\alpha$ $\sin\alpha$ $\frac{fda}{fV}$
164 574 008	234 574 012	461 574 024	68 53 088	87 559 179	935 588 189	1 574 205	928 552 192	775 53 161	332 545 042				
·001	·002	·007	·032	·087	·104	·118	·098	·066	·008				
164 629 012	234 629 023	·7 623 051	864 574 164	1 616 296	1 656 283	1 643 288	1 643 288	1 629 292	565 609 102				
·001	·005	·022	·081	·183	·186	·185	·185	·184	·035				
134 829 026	575 768 066	89 766 105	1 76 302	1 772 477	1 799 452	1 804 446	1 788 463	1 777 472	758 749 232	·3 719 038			
·005	·029	·072	·229	·368	·360	·358	·364	·367	·132	·008			
182 906 047	68 875 081	904 875 109	1 857 322	1 866 500	1 866 500	1 866 500	1 853 523	1 862 508	626 866 256	366 875 067			
·019	·048	·086	·276	·433	·433	·433	·445	·437	·183	·033			
49 934 043	721 921 069	89 899 097	1 895 279	1 891 454	1 883 470	1 883 470	1 891 454	1 891 454	626 906 216	54 906 056			
·020	·046	·078	·250	·404	·415	·415	·405	·405	·162	·027			
126 934 037	·7 921 067	904 91 093	1 921 242	1 921 391	1 899 438	1 906 423	1 906 423	1 899 438	774 921 186	446 927 041			
·015	·043	·077	·223	·360	·394	384	·383	·394	·133	·017			
374 964 024	68 946 054	86 951 066	1 951 193	1 946 326	1 946 326	1 94 342	1 94 342	1 943 334	636 956 114	374 956 026			
·009	·035	·054	·184	·309	·308	·322	·322	·315	·069	·009			
136 996 006	446 995 011	655 985 025	575 985 109	1 983 191	1 982 191	1 978 208	1 982 191	1 978 208	379 982 043	234 982 011			
·002	·005	·016	·094	·187	·187	·204	·187	·204	·016	·002			
	234 997 002	335 997 003	54 998 014	1 992 082	1 994 082	1 994 082	1 998 080	955 993 072	169 993 008				
	·000	·001	·008	·081	·082	·082	·082	·068	·001				
	234 998 001	·3 998 001	49 1 000	1 995 024	1 995 026	1 994 028	1 994 028	904 995 022	995 -				
	·000	·000	·000	·024	·026	·028	·028	·022					
0·072	0·213	0·413	1·377	2·436	2·495	2·529	2·499	2·462	0·739	0·096			

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$

15·331

SERIES II TEST:- 1A TABLE:- 5

	-1¾	-1½	-1¼	-1	-½	0	½	1	2	3	3½	4	4¼	4½
	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A
	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$	η cosα sinα $\frac{fd\alpha}{fV}$
				187 999 001	464 999 003	41 998 005	394 998 050	187 998 004				187 ·951 008	187 ·97 004	187 99 004
				·000	·001	·002	·022	·001				·001	·001	·001
		187 993 003	308 985 006	544 988 015	525 995 013	49 999 004	49 999 002	364 1 000	187 1 000				534 94 022	471 [illegible] 022
		·001	·002	008	007	·002	·001	·000	·000				·011	·010
		187 985 004	49 974 013	725 985 023	67 99 023	758 985 033	75 985 032	706 986 041	621 99 042	53 993 024	464 993 014	464 996 027	621 94 026	696 934 046
		·001	·006	·019	·015	·025	·024	·029	·026	·013	·006	·012	·015	·030
	4 986 021	6 982 021	86 956 047	875 956 071	869 97 078	887 94 113	908 924 130	946 921 208	916 914 278	88 934 176	866 94 110	857 94 081	846 946 051	812 946 073
	·008	·012	·039	·059	·066	·094	·109	·181	·234	·145	·090	·065	·041	·056
	682 93 061	856 891 096	904 875 109	904 866 168	907 857 232	93 657 238	915 898 241	914 839 371	916 829 510	904 829 377	866 839 848 234	875 848 172	875 857 112	84 865 152
	·039	·073	·086	·132	·181	·190	·187	·284	·387	·282	·170	·128	·084	·111
	7 669 095	874 713 115	914 713 120	904 695 181	946 695 254	914 682 249	91 695 244	914 707 362	946 682 516	93 695 374	904 695 242	86 707 170	875 695 117	785 682 159
	·044	·081	·078	·114	·167	·156	·155	·234	·334	·242	·152	·104	·071	·085
	394 358 034	62 391 052	85 438 071	785 407 100	886 47 145	904 485 147	915 5 147	914 515 219	916 63 290	93 545 218	92 545 143	86 ·5 104	66 407 055	34 423 043
	·005	·013	·030	·032	·061	·064	·076	·103	·141	·111	·072	·045	·015	·006
		394 208 023	596 208 036	485 191 044	721 191 087	794 242 095	835 259 268 100	886 358 153	892 391 204	92 391 158	875 391 100	678 292 061	187 292 011	
		·002	·004	·004	·012	·018	·022	·049	·071	·057	·034	·012	·001	
	0·096	0·183	0·245	0·368	0.510	0.551	0.596	0.881	1193	0.850	0.534	0.367	0.239	0.299

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fd\alpha}{fV}$.

6.902

Specimen Tables of Observed and Derived Results

Series 3 (Group 3).

TABLE OF OBSERVED READINGS OF PRESSURE + ANGLE.

GROUP :- 3
SERIES III

p_1 = 44.15 LB/IN² Ab.

BAROMETRIC PRESS = 14.45 lb/in².
p_2 = LB/IN² Ab. TEMP. °F

STATIONS	-2¾		-2½		-2¼		-2		-1		0		½		¾		1		1¼		1½		→ VERTICAL STATIONS					
p_2 (lb/in²) ↓	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
23.15	27.5	111	28.1	124	28	131	27.9	135	27.7	146	27.6	159	25.9	168	24.7	174	24	177	23.2	181	–	–						
20.75	27.1	108	28.1	122	27.9	131	27.9	135	27.6	147	27.4	160	25	169	23.5	172	22.4	178	21.3	181	20.9	184						
18.08	24.2	101	27.9	115	28.1	126	28.1	134	28	148	28.3	159	26	167	24	172	21.9	176	20	184	19.3	183						
15.25	23.9	101	27.4	117	27.9	126	27.7	133	27.3	147	27.3	160	24.1	169	21.8	173	19	177	18	182	16.1	186						
12.39	21.7	100	26	112	27.4	124	27.3	132	26.9	146	27	169	23.9	169	21.4	173	18	177	15.2	180	14	182						
10.39	18	95	24	106	26.9	123	27	131	26.5	146	26.7	159	23.1	169	20.8	173	17.3	178	13.8	183	12.7	183						
8	13	94	18.9	102	24.2	115	26.3	127	26	147	26.3	159	23.2	169	21	173	18.3	177	15	181	12.8	183						
5	7.1	95	12	103	17.1	111	21.6	119	25.2	145	25.1	158	24.2	168	22	173	20.4	175	17.9	178	15.6	180						
1	8	93	14.7	106	20.5	122	22.9	132	21.7	152	22.9	156	22.8	162	22	165	21.8	167	20.7	173	19.1	176						

RATIO OF $\frac{\text{BACK PRESSURE}}{\text{PRESSURE READING}}$ AND CONVERTED ANGLE

GROUP:- 3
SERIES III

TABLE.I.

P_1 = 14.45 LB/IN² Ab.

BAROMETRIC PRESS = 14.45 lb/in²
P_2 = LB/IN² Ab.
TEMP.

M_{2f} ↓	STATIONS	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½		13 →		VERTICAL STATIONS.			
		$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α	$\frac{p_2}{p_5}$	α
·5		·898	69	·883	56	·888	49	·889	45	·892	34	·894	21	·911	12	·936	6	·955	3	·972	-ve -1	—							
·6		·848	72	·828	58	·832	49	·823	45	·838	33	·842	20	·894	11	·928	8	·957	2	·985	-ve -1	·995	-ve -4						
·7		·839	79	·768	65	·763	54	·763	46	·766	32	·769	21	·805	13	·847	8	·881	4	·945	-ve -4	·961	-ve -2						
·8		·75	79	·688	63	·679	54	·681	47	·689	33	·689	20	·743	11	·792	7	·86	3	·888	-ve -2	·939	-ve -6						
·9		·741	80	·663	68	·641	56	·642	48	·65	39	·648	21	·7	11	·729	7	·827	3	·962	0	·944	-ve -2						
·97		·766	85	·697	74	·6	57	·599	49	·607	34	·602	21	·66	11	·709	7	·781	2	·878	-ve -3	·912	-ve -3						
1·06		·82	86	·674	78	·58	65	·55	53	·555	33	·56	21	·596	11	·633	7	·685	3	·762	-ve -1	·822	-ve -3						
1·178		·9	85	·738	77	·615	69	·539	61	·49	35	·492	22	·503	12	·535	7	·56	5	·602	2	·696	0						
1·359		·688	87	·53	74	·448	58	·415	48	·428	28	·415	24	·415	18	·425	15	·426	13	·438	7	·46	4						

READINGS OF CONVERTED PRESSURE RATIO AND LOCAL EFFICIENCY

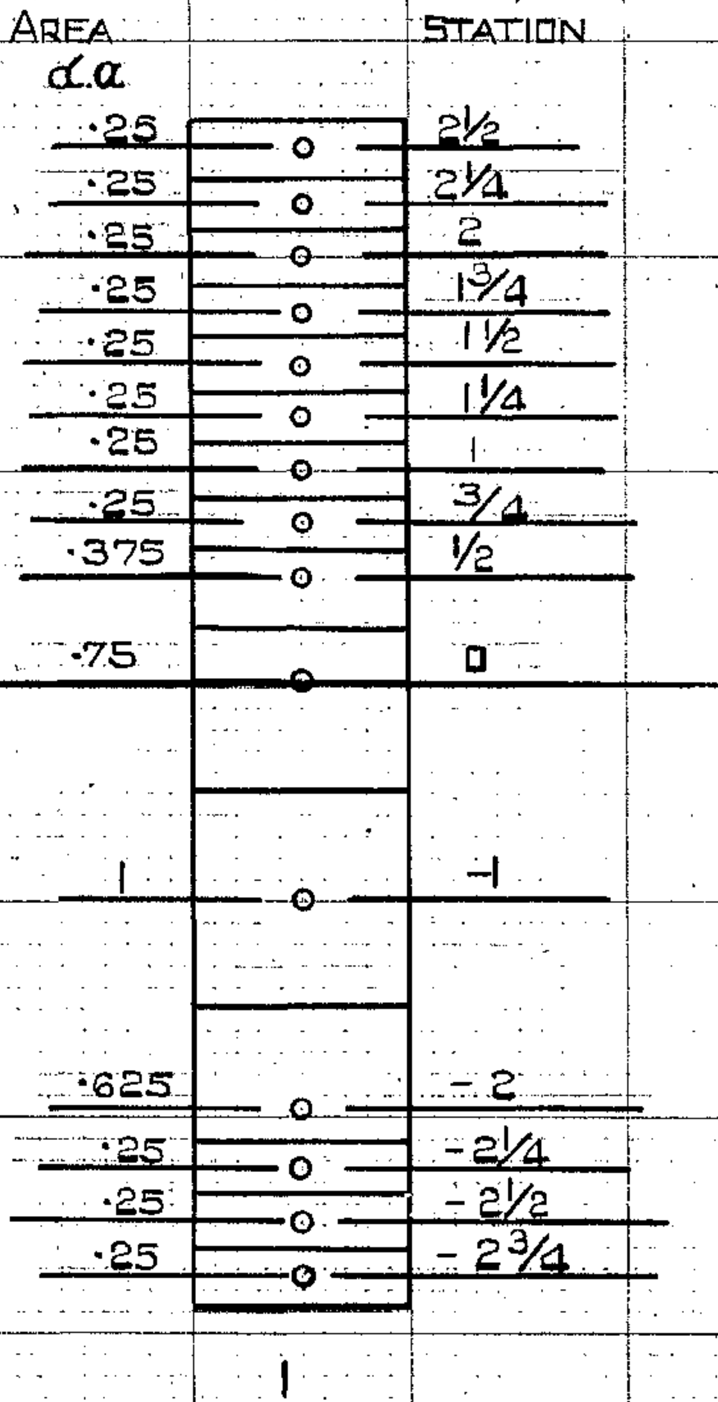
GROUP:- 3
SERIES III

TABLE II

P_1 = 44.45 LB/IN² AD.

BAROMETRIC PRESS 14.45 lb/in².
P_2 = LB/IN² AD
TEMP °F

	STATIONS	-2¾		-2½		-2¼		-2		-1		0		½		¾		1		1¼		1½.		→ VERTICAL STATIONS					
		$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
	$M_{2\phi}$																												
.5	.5	.898	.665	.883	.766	.888	.759	.889	.733	.892	.718	.894	.697	.911	.58	.934	.42	.955	.29	.972	.175	—	—						
.6	.6	.848	.72	.828	.82	.833	.795	.833	.775	.838	.77	.842	.752	.899	.5	.828	.335	.957	.2	.985	.06	.995	.02						
.7	.7	.839	.58	.768	.863	.763	.884	.763	.884	.766	.872	.759	.9	.805	.717	.847	.552	.881	.42	.945	.18	.961	.126						
.8	.8	.75	.74	.688	.947	.679	.977	.681	.972	.689	.942	.689	.942	.743	.764	.792	.604	.86	.388	.888	.297	.939	.161						
.9	.9	.741	.629	.663	.84	.641	.91	.642	.9	.65	.88	.698	.885	.7	.736	.729	.66	.827	.412	.902	.228	.944	.12						
.97	.97	.766	.484	.697	.77	.6	.895	.599	.898	.607	.894	.602	.895	.66	.738	.704	.63	.781	.45	.878	.235	.912	.166						
1.06	1.06	.82	.311	.674	.612	.58	.81	.55	.89	.555	.883	.55	.89	.595	.78	.633	.711	.685	.585	.762	.422	.822	.307						
1.178	1.178	.9	.131	.738	.396	.615	.616	.539	.77	.4818	.89	.4899	.88	.5016	.87	.5317	.785	.56	.722	.602	.696	.696	.562						
1.39	1.39	.688	.386	.5217	.64	.4426	.8	.4065	.88	.421	.848	.4069	.88	.4064	.88	.4176	.86	.4186	.855	.432	.826	.4554	.776						



FLOW AREA DIAGRAM FOR CENTRE-LINE TRAVERSE.

SERIES III

FIG. 69

TABLE SHOWING CALCULATION OF $\Sigma \eta^{1/2} \sin\alpha \frac{f\,da}{fV}$

SERIES III GROUP :- 3   TABLE :- 3   → VERTICAL STATIONS.

STATION	-2¾			-2½			-2¼			-2		
	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da
$M_{\alpha f}$	fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$	
·5	·815	·9336	·25	·875	·8290	·25	·856	·7547	·25	·856	·7071	·625
	1·012	·1875		1·007	·1802		1·0086	·1604		1·009	·3785	
·6	·848	·9511	·25	·905	·8481	·25	·871	·7647	·25	·891	·7071	·625
	1·016	·1987		1·011	·1895		1·0123	·1656		1·0123	·3885	
·7	·761	·9816	·25	·93	·9063	·25	·94	·8090	·25	·94	·7193	·625
	1·03	·1815		1·022	·20795		1·01	·1880		1·01	·4180	
·8	·86	·9816	·25	·973	·8910	·25	·986	·8090	·25	·986	·7314	·625
	1·025	·2060		1·004	·2159		1·0	·2000		1·001	·4500	
·9	·793	·9848	·25	·916	·9272	·25	·951	·8290	·25	·999	·7431	·625
	1·018	·1865		1·02	·2082		1·0128	·1950		1·0126	·4345	
·97	·696	·9962	·25	·876	·9613	·25	·946	·8387	·25	·947	·7547	·625
	1·0726	·1620		1·0325	·2038		1·015	·1951		1·015	·4395	
1·06	·557	·9976	·25	·784	·9702	·25	·9	·9063	·25	·998	·7986	·625
	1·116	·1243		1·066	·1798		1·03	·198		1·018	·462	
1·178	·3725	·9962	·25	·629	·9744	·25	·785	·9334	·25	·877	·8746	·625
	1·178	·0789		1·125	·1494		1·077	·170		1·047	·4586	
1·359	·621	·9986	·25	·8	·9613	·25	·795	·8481	·25	·939	·7431	·625
	1·466	·133		1·1	·1748		1·066	·1597		1·039	·42	
Σ												

STATION	-1			0			½			¾		
	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da
$M_{\alpha f}$	fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$	
·5	·846	·5592	1	·834	·3584	·75	·761	·2079	·375	·649	·1095	·25
	1·01	·4685		1·011	·2215		1·015	·0584		1·021	·01625	
·6	·878	·5446	1	·868	·3420	·75	·706	·1908	·375	·579	·1392	·25
	1·013	·4710		1·014	·2194		1·0275	·0493		1·0365	·01943	
·7	·935	·5299	1	·949	·3584	·75	·846	·225	·375	·744	·1392	·25
	1·011	·4895		1·009	·2530		1·022	·06975		1·0325	·02508	
·8	·971	·5446	1	·971	·3420	·75	·875	·1908	·375	·778	·1219	·25
	1·004	·5260		1·004	·2480		1·0225	·0612		1·0325	·02286	
·9	·939	·5592	1	·94	·3584	·75	·858	·1908	·375	·812	·1219	·25
	1·015	·5165		1·0149	·2490		1·021	·0594		1·04	·0238	
·97	·946	·5592	1	·946	·3584	·75	·86	·1908	·375	·794	·1219	·25
	1·0175	·5210		1·016	·2505		1·0375	·0593		1·0526	·02292	
1·06	·991	·5446	1	·991	·3584	·75	·889	·1908	·375	·899	·1219	·25
	1·02	·5015		1·018	·2498		1·037	·061		1·05	·0245	
1·178	·944	·5736	1	·938	·3746	·75	·933	·2079	·375	·886	·1219	·25
	1·024	·528		1·026	·2572		1·03	·0785		1·095	·0258	
1·359	·92	·4695	1	·938	·4067	·75	·938	·3090	·375	·927	·2588	·25
	1·043	·414		1·039	·276		1·037	·1047		1·091	·0575	
Σ												

STATION	1			1¼			1½			Σ								
	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da	$\eta^{1/2}$	Sin α	f da
$M_{\alpha f}$	fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$		fV	$\eta^{1/2}$ Sin α $\frac{f da}{fV}$	
·5	·539	·0523	·25	·419	-ve ·0175	·25	0	0	·25									
	1·0265	·00686		1·03	-ve ·001779		0	0			1·6763							
·6	·447	·0349	·25	·245	-ve ·0175	·25	·141	-ve ·0698	·25									
	1·043	·003725		1·05	-ve ·001021		1·0626	-ve ·002335			1·7018							
·7	·648	·0698	·25	·425	-ve ·0698	·25	·3596	-ve ·0349	·25									
	1·041	·01084		1·066	-ve ·00703		1·059	-ve ·002925			1·8337							
·8	·629	·0523	·25	·595	-ve ·0349	·25	·3985	-ve ·1045	·25									
	1·0564	·00773		1·064	-ve ·00446		1·0775	-ve ·00944			1·9238							
·9	·692	·0523	·25	·477	0	·25	·396	-ve ·0349	·25									
	1·069	·00785		1·091	0		1·104	-ve ·002735			1·8780							
·97	·671	·0349	·25	·485	-ve ·0523	·25	·407	-ve ·0523	·25									
	1·077	·00543		1·107	-ve ·00574		1·116	-ve ·00477			1·8490							
1·06	·765	·0872	·25	·648	-ve ·0175	·25	·555	-ve ·0523	·25									
	1·07	·00935		1·097	-ve ·00258		1·17	-ve ·00065			1·80117							
1·178	·85	·0872	·25	·804	·0349	·25	·75	0	·25									
	1·056	·01788		1·073	·00652		1·09	0			1·7625							
1·359	·925	·2250	·25	·91	·1219	·25	·881	·0698	·25									
	1·042	·0498		1·098	·0187		1·062	·01448			1·82268							
Σ																		

SERIES III GROUP:-3 TABLE η, K and M_{B} → VERTICAL STATIONS.

-2¾		-2½		-2¼		-2		-1		0		½		¾		1		1¼		1½		Σ		K_t		M_{Bt}	
	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{3/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{1/2}$ Sin α	$\frac{fda}{fV}$	$\eta^{1/2}$ Sin α	$\frac{fda}{fV}$
·5	·1875	·766	·1802	·724	·1604	·733	·3785	·718	·4685	·697	·2215	·58	·0589	·42	·01625	·29	·00686	·175	-ve ·00178	0							
·1248		·1382		·1178		·2778		·3365		·1642		·0338		·00684		·00199		-ve ·000312		0		1·1916		·845		·7125	
2	·1987	·82	·1895	·795	·1666	·795	·3885	·77	·4710	·752	·2194	·5	·0493	·335	·01943	·2	·00374	·06	-ve ·00102	·02	-ve ·00234						
·1430		·1555		·1318		·3092		·3630		·1651		·02465		·00651		·000748		-ve ·000612		-ve ·0000468		1·2994		·8745		·762	
58	·1815	·863	·2078	·884	·1880	·884	·4180	·872	·4815	·9	·2530	·717	·06975	·552	·02508	·42	·01084	·18	-ve ·00703	·126	-ve ·00242						
·1052		·1791		·1660		·3695		·4260		·2277		·0501		·01386		·00455		-ve ·001265		-ve ·000368		1·5404		·911		·83	
74	·2060	·947	·2159	·977	·2	·972	·4600	·942	·5260	·942	·2480	·769	·0612	·604	·02286	·388	·00773	·297	-ve ·00496	·151	-ve ·00944						
·1525		·2041		·1954		·4370		·4955		·2339		·04675		·01380		·0030		-ve ·001329		-ve ·001425		1·6792		·935		·871	
29	·1865	·84	·2082	·91	·1950	·9	·4346	·88	·5166	·885	·2490	·736	·0594	·66	·0238	·412	·00785	·228	0	·12	-ve ·00274						
·1171		·1751		·1773		·3910		·4550		·2205		·0437		·01570		·003235		0		-ve ·0003288		1·5983		·924		·851	
984	·1620	·77	·2038	·895	·1951	·898	·4376	·894	·5210	·895	·2505	·738	·0593	·63	·02292	·45	·00693	·235	-ve ·00874	·166	-ve ·00477						
·0784		·1569		·1749		·3945		·4655		·2243		·0437		·01442		·002441		-ve ·00135		-ve ·000792		1·5629		·916		·841	
311	·1243	·612	·1798	·81	·198	·89	·462	·883	·5015	·89	·2498	·78	·061	·711	·0245	·585	·00935	·422	-ve ·00258	·307	-ve ·0065						
·03875		·1099		·1605		·411		·444		·222		·04755		·01742		·00546		-ve ·00109		-ve ·001995		1·4535		·896		·806	
139	·0789	·396	·1494	·616	·170	·77	·4586	·89	·528	·88	·2572	·87	·0705	·785	·0258	·722	·01758	·646	·00652	·562	0						
·01098		·0592		·105		·3608		·47		·2265		·0613		·02025		·0127		·00422		0		1·33096		·869		·755	
386	·133	·64	·1748	·8	·1597	·88	·42	·848	·414	·88	·276	·88	·1047	·86	·0576	·855	·0498	·826	·0187	·776	·01498						
·0513		·1119		·1276		·3695		·3508		·243		·0816		·0495		·0426		·01547		·01123		1·4545		·893.		·797	

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$$

Appendix 3.

An examination of the variation of loss coefficients with Mach number and Reynolds number with particular reference to the existance of critical values of these parameters.

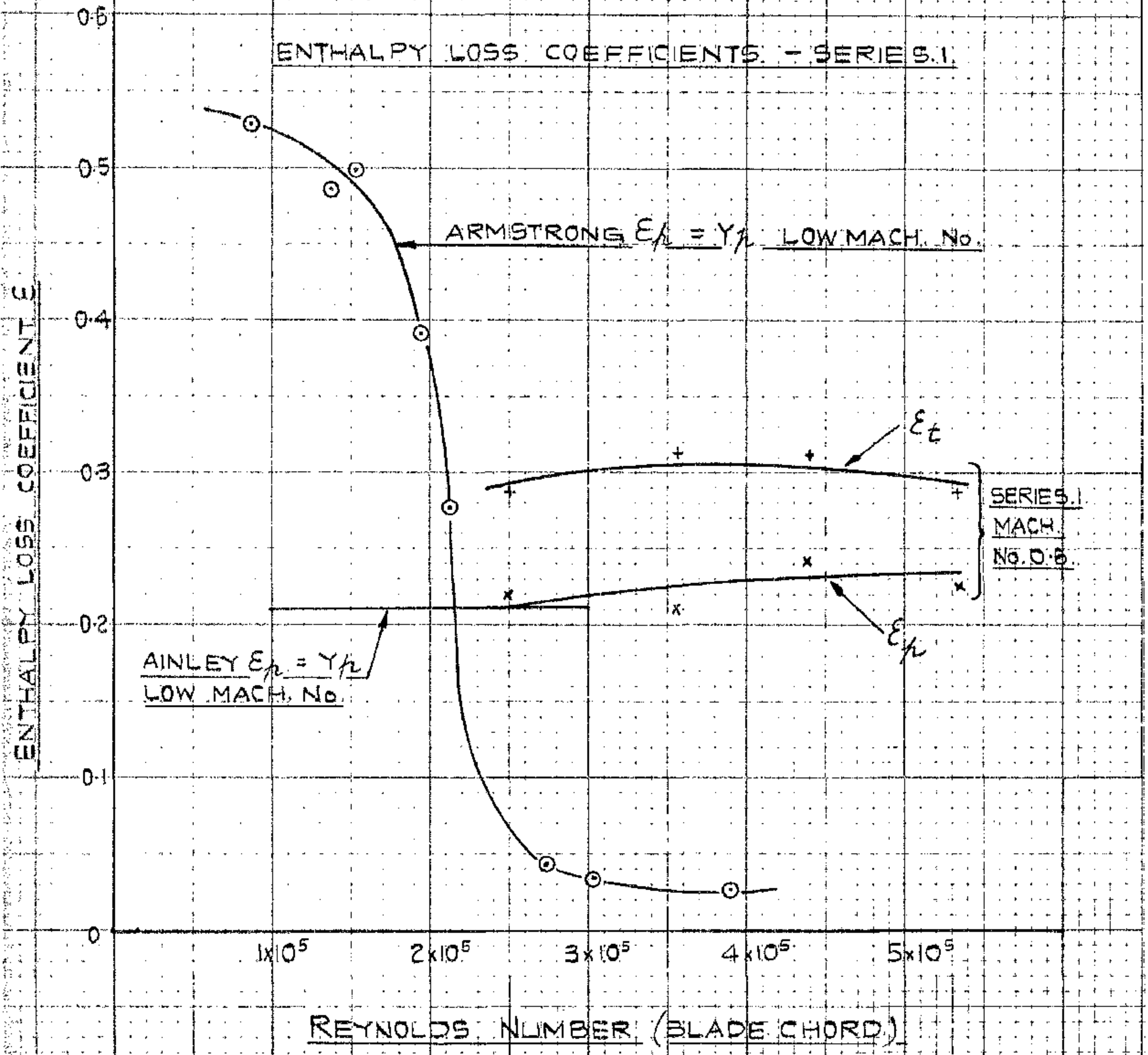
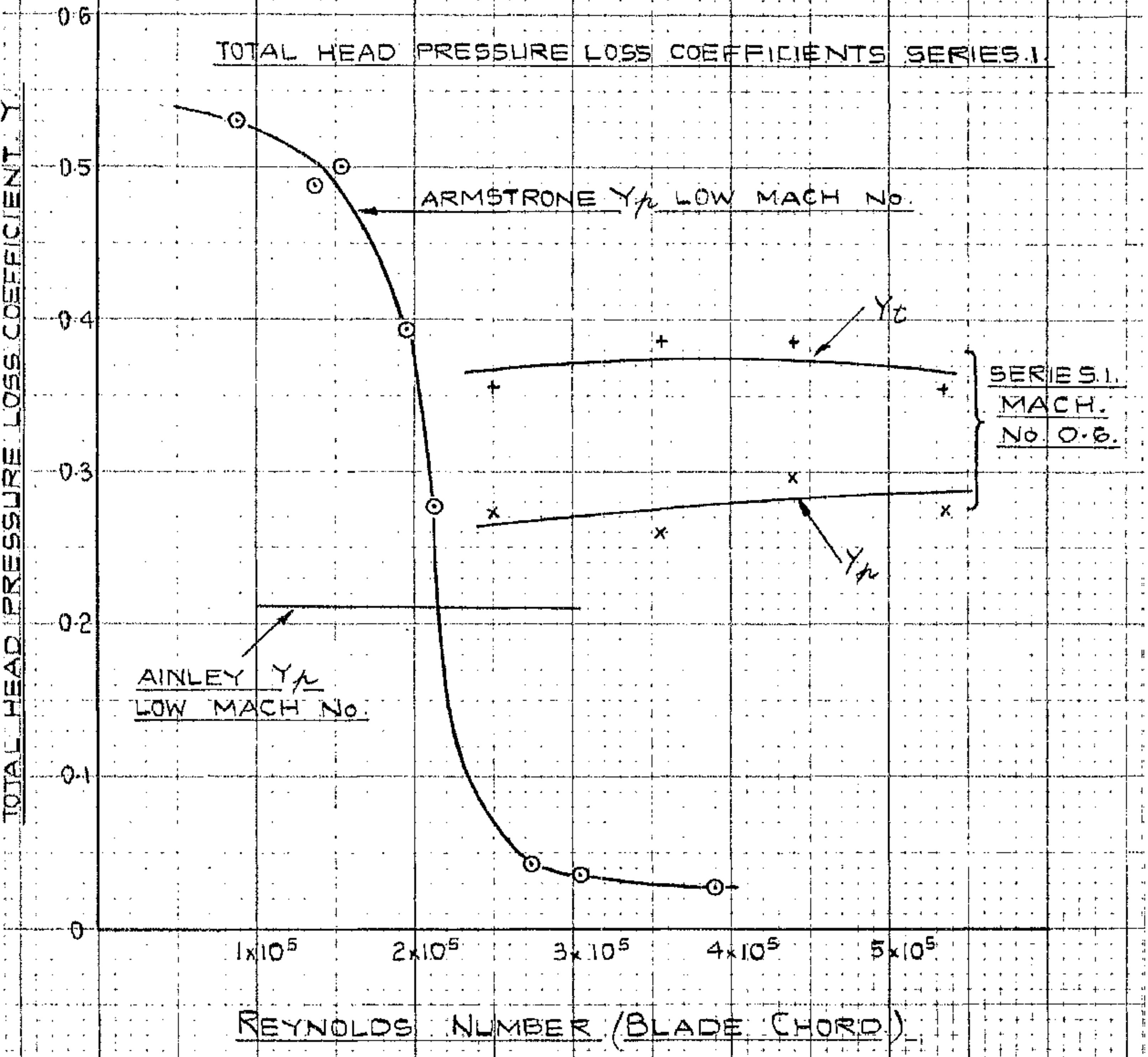
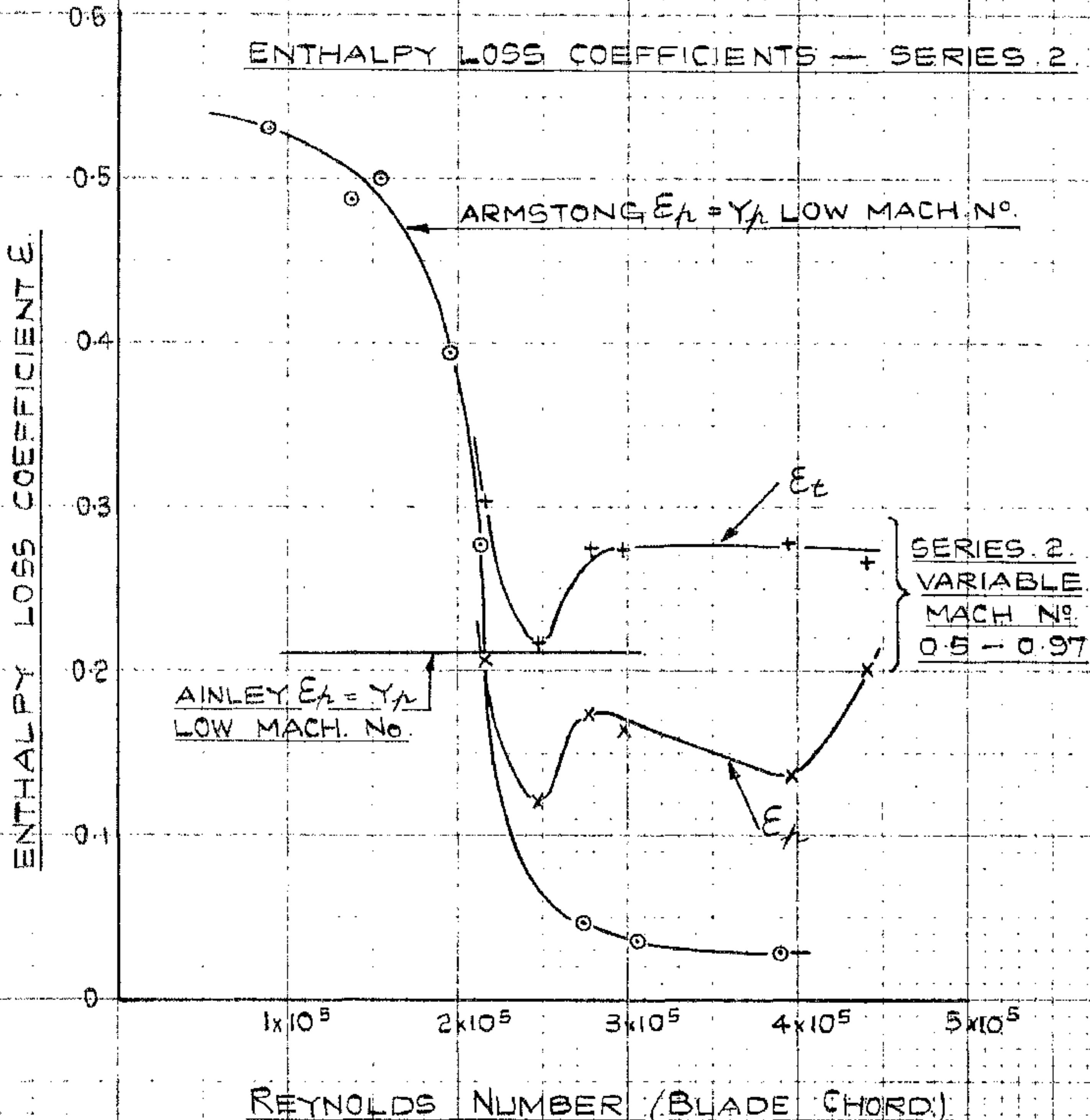

ENTHALPY LOSS COEFFICIENTS - SERIES 5.1.
ARMSTRONG $\mathcal{E}_p = Y_p$ LOW MACH. No.
$\mathcal{E}_t$
SERIES 5.1
MACH. No. 0.6.
$\mathcal{E}_p$
AINLEY $\mathcal{E}_p = Y_p$ LOW MACH. No.
ENTHALPY LOSS COEFFICIENT $\mathcal{E}$
0
0.1
0.2
0.3
0.4
0.5
0.6
1x10^5
2x10^5
3x10^5
4x10^5
5x10^5
REYNOLDS NUMBER (BLADE CHORD)

FIG. 70



TOTAL HEAD PRESSURE LOSS COEFFICIENTS SERIES.1.
ARMSTRONE Yp LOW MACH No.
Yt
Yp
SERIES.1. MACH. No. 0·6.
AINLEY Yp LOW MACH No.
TOTAL HEAD PRESSURE LOSS COEFFICIENT. Y.
0·6
0·5
0·4
0·3
0·2
0·1
0
1x10^5
2x10^5
3x10^5
4x10^5
5x10^5
REYNOLDS NUMBER (BLADE CHORD)

FIG. 71



ENTHALPY LOSS COEFFICIENTS — SERIES 2.
ARMSTONG $\mathcal{E}_p = Y_p$ LOW MACH No.
$\mathcal{E}_t$
SERIES 2.
VARIABLE
MACH No.
0·5 – 0·97
AINLEY $\mathcal{E}_p = Y_p$
LOW MACH. No.
$\mathcal{E}_p$
ENTHALPY LOSS COEFFICIENT $\mathcal{E}$.
0
0·1
0·2
0·3
0·4
0·5
0·6
1x10⁵
2x10⁵
3x10⁵
4x10⁵
5x10⁵
REYNOLDS NUMBER (BLADE CHORD)

FIG. 72

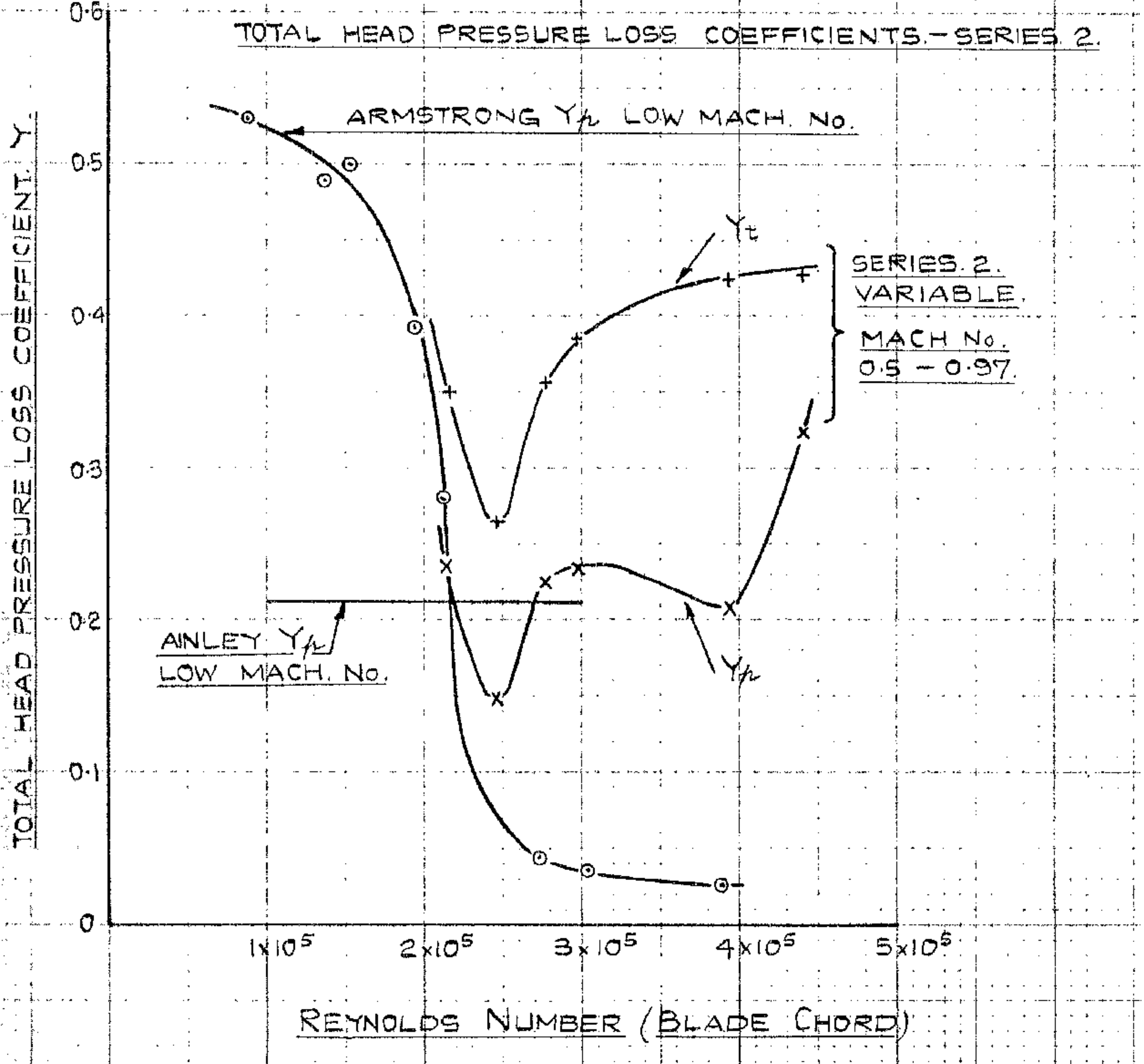

TOTAL HEAD PRESSURE LOSS COEFFICIENTS.—SERIES 2.
ARMSTRONG Yp LOW MACH. No.
Yt
SERIES. 2.
VARIABLE.
MACH No.
0·5 – 0·97.
AINLEY Yp
LOW MACH. No.
Yp
TOTAL HEAD PRESSURE LOSS COEFFICIENT. Y.
0·6
0·5
0·4
0·3
0·2
0·1
0
1×10^5
2×10^5
3×10^5
4×10^5
5×10^5
REYNOLDS NUMBER (BLADE CHORD)

FIG. 73

Appendix 3.

An examination of the variation of loss coefficients with Mach number and Reynolds number with particular reference to the existance of critical values of these parameters.

A particular study of the variation of loss coefficients at high Reynolds number has been made by Armstrong[40], Ainley[41] and Kearton[42]. To compare the writer's work with theirs, the results from test series 1 and 2 are given in the accompanying tables along with details of the geometry of the impulse blade used in the tests. The efficiencies of expansion for the blade have been converted to enthalpy loss coefficients ($\mathcal{E}$) and to total head pressure loss coefficients (Y) by the relationships derived in Part 1. The coefficients are given as profile or centre line coefficients ($\mathcal{E}_p$ and Y_p) and as total loss coefficients ($\mathcal{E}_t$ and Y_t), where the total loss coefficients include the profile and secondary losses. The test Reynolds numbers (previously based on a length dimension of one foot) have been calculated on a basis of blade chord and on a characteristic dimension equal to four times the hydraulic mean depth at the outlet throat section of the blade passage.

For test series 1, where the outlet Mach number is constant at 0.6, $\mathcal{E}_p$ and $\mathcal{E}_t$, and Y_p and Y_t are plotted in figures 70 and 71 respectively to a base of chord Reynolds number. The corresponding results for series 2, where the Mach number varies from 0.5 to 0.97 are given in figures 72 and 73.

Comparison with Armstrong's work.

In Armstrong's work air tests at low velocities were carried out on a cascade of large impulse blades. The blades were designed for good incidence characteristics with a well rounded inlet edge.

In the tests the air inlet angle was maintained constant at 35 degrees (from the axial direction) and total head profile pressure loss coefficients were obtained over a range

Geometry of the impulse blade used in test series 1 & 2 (see fig 43) pitch 0.65 ins; height 0.65 ins; blade chord = axial chord, 0.75 ins; hydraulic mean depth at the outlet throat section of the blade, 0.075 ins; hydraulic mean diameter 0.30 ins; aspect ratio 0.866; pitch/chord ratio 0.866; thickness to chord ratio 0.615; inlet and outlet angle 20°; nominal fluid deflection 140°; opening to pitch at blade outlet = 0.30.

Table 1.

Test results for the impulse blade from series 1 Constant Mach number 0.6.

Test No.	1A.	2A.	3A.	4A.
R_n (L = 1ft) x 10^6	3.99.	5.27.	7.041.	8.55.
R_n (chord) x 10^5	2.50.	3.58.	4.40.	5.35.
R_n (H.M.D.) x 10^5	1.00.	1.43.	1.76.	2.14.
η_p profile	0.820.	0.826.	0.805.	0.817
ε_p profile	0.220.	0.210.	0.242.	0.223
Y_p profile	0.272.	0.259.	0.298.	0.275
Y_p profile (Ainley)	0.212			
η_t TOTAL	0.778.	0.762.	0.762.	0.778
ε_t TOTAL	0.287.	0.312.	0.312.	0.287
Y_t TOTAL	0.354.	0.385.	0.385.	0.354
Y_t TOTAL (Ainley)	0.628			

Table 2.

Test results for the impulse blade from series 2 - variable Mach number.

Test no.	1A.	2A.	3A.	4A.	5A.	6A.
Mach No.	0.5	0.6	0.7	0.8	0.9	0.97.
R_N (L = 1ft) x 10^6	3.46	3.96	4.45	4.77	6.28	7.04
R_N (chord) x 10^5	2.16	2.48	2.78	2.98	3.94	4.40
R_N (H.M.D.) x 10^5	0.86	0.99	1.12	1.20	1.57	1.76
ζ_p profile.	0.83	0.894	0.854	0.860	0.881	0.832
$\bar{\varepsilon}_p$ profile.	0.204	0.120	0.171	0.164	0.133	0.200
Y_p profile.	0.236	0.148	0.225	0.232	0.203	0.322
Y_p profile. (Ainley)	0.212					
ζ_t TOTAL	0.766	0.821	0.785	0.785	0.783	0.790
ε_t TOTAL	0.302	0.215	0.271	0.271	0.277	0.266
Y_t TOTAL	0.350	0.265	0.356	0.383	0.422	0.429
Y_t TOTAL (Ainley)	0.628					

of chord Reynolds number varying from 0.8×10^5 to 4.0×10^5. The average air deflection in the tests was approximately 68^o, and since the tests were performed at low Mach numbers the enthalpy and pressure loss coefficients may be assumed identical.

The results are shown in figures 70 to 73 and indicate an abrupt rise in loss coefficients as the Reynolds number is reduced. The critical region lies between 1.8 and 2.3×10^5, an average value of approximately 2.0×10^5. In test series 1 the writer's Reynolds numbers are greater than 2.5×10^5 (figures 70 and 71) but in series 2 (figures 72 and 73) where the lower Reynolds numbers are approaching Armstrong's critical value, the loss coefficients rise sharply confirming Armstrong's result. Below the critical Reynolds number Armstrong also found that the boundary layer growth on the convex side of the blade surface was of such a magnitude that the blade channel acted like a convergent nozzle.

<u>Ainley's loss correlation.</u>

Ainley and Mathieson[41] give the results of air cascade tests at low Mach numbers. For nozzle blades (fluid inlet angle α_1 zero) and for impulse blades (fluid outlet angle α_2 = - fluid inlet angle α_1), they correlate the profile loss coefficient at zero incidence with the pitch to chord ratio ($P/_c$) in the form of a series of graphs each corresponding to a different fluid outlet angle. The results are for a thickness to chord ratio ($t/_c$) of 0.2.

A relationship is given between the opening to pitch ratio at blade outlet and the fluid outlet angle. For the writer's blade this gives a fluid outlet angle very close to the nominal inlet angle of 20^o. Hence for comparison with Ainley's data, the impulse blade of series 1 and 2 has been assumed to have fluid inlet and outlet angles of 20^o from the tangential direction, giving a fluid deflection of 140^o.

Hence at $t/_c = 0.2$, $P/_c = 0.866$ and $\alpha_2 = 70^o$ (axial direction), Ainley gives $Y_p = 0.170$

and for this impulse blade where $t/_c = 0.25$

$$Y_p = 0.170 \left(\frac{0.25}{0.2}\right)^{-1} = 0.212.$$

For the secondary loss in the blade passage Ainley gives

$$Y_s = \lambda \left(\frac{C_L}{P/_C}\right)^2 \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m}$$

where $\frac{C_L}{P/_C} = 2 (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m$

$$\operatorname{Tan} \alpha_m = \frac{\tan \alpha_1 + \tan \alpha_2}{2}$$

and λ; an empirical coefficient is given in graphical form as a function of

$$\left(\frac{A_2}{A_1}\right)^2 \times \left(1 + \frac{D - h}{D + h}\right)^{-1}$$

Here A_1 is the inlet annulus area x cos α_1

A_2 " " outlet " " x cos α_2

D is the mean diameter of the blade and h is the blade height.

Hence for the writer's blade :- $\alpha_m = 0$.

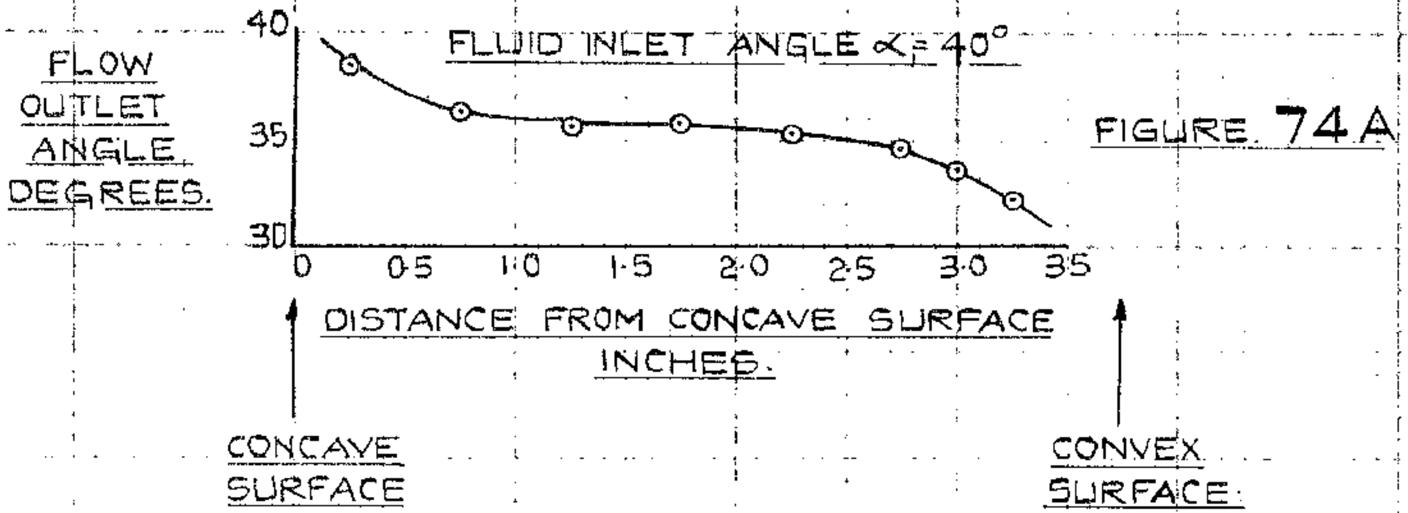
$$\left(\frac{C_L}{P/_C}\right)^2 \times \frac{\cos^2 \alpha_2}{\cos^3 \alpha_m} = \left[2 \left(2 \tan 70^0\right)\right]^2 \cos^2 70^0 = 14.10$$

and λ is a function of $\left(1 + \frac{D - h}{D + h}\right)^{-1}$

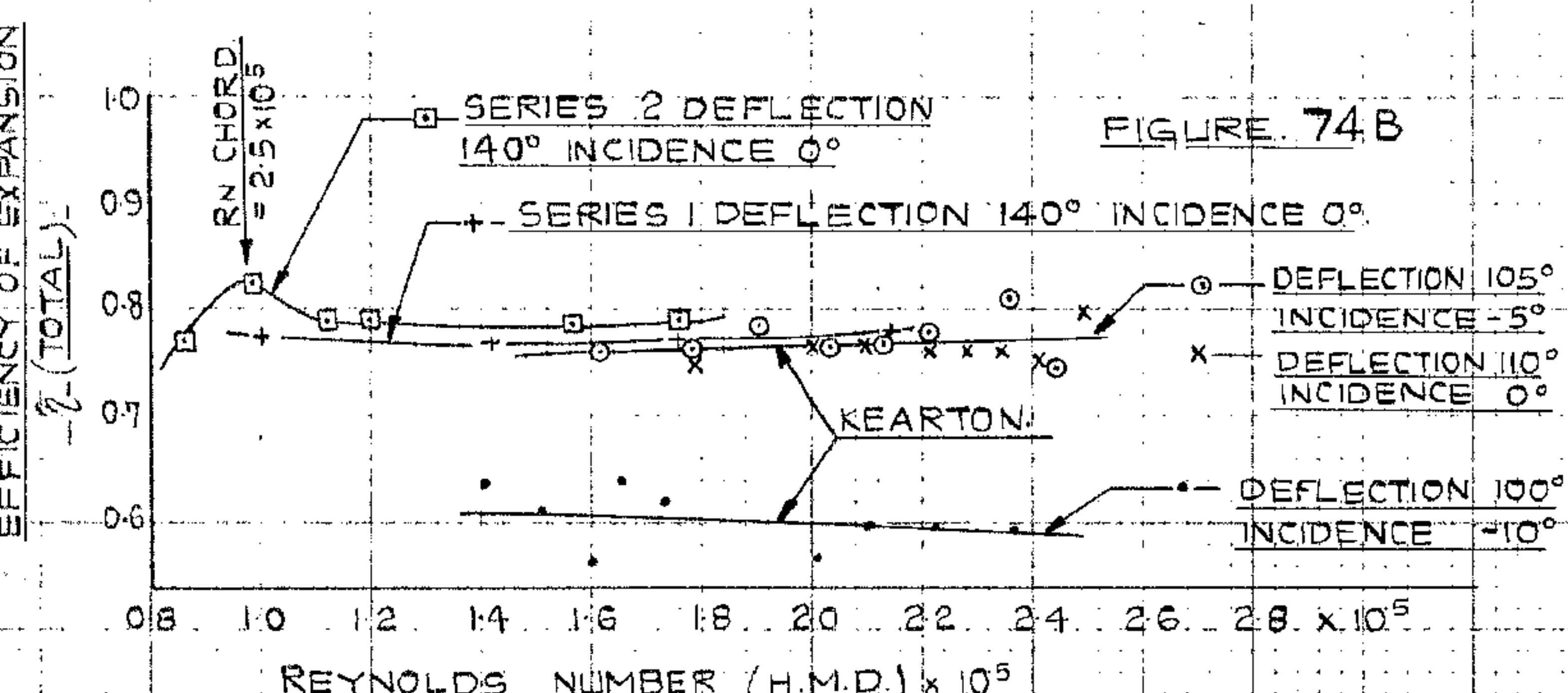
Using values of mean diameter from 10 in. to 40 in. and the blade height of 0.65 in. , the quantity $\left(1 + \frac{D - h}{D + h}\right)^{-1}$ varies from 0.534 to 0.506. An average value of 0.52 has been used here, for which Ainley gives $\lambda = 0.0295$.

Hence $Y_s = 0.0295 \times 14.1 = 0.416$ and therefore the estimated value of the total loss using Ainley's correlation is

$$Y_t = 0.416 + 0.212 = 0.628.$$



FLUID DIRECTION AT IMPULSE BLADE OUTLET KEARTON.



EFFICIENCIES OF PROFILE TYPE IMPULSE BLADES.

FIG. 74

In Ainley's cascade tests the operating Mach numbers were less than 0.5 and the chord Reynolds number varied from 1×10^5 to 3×10^5. Ainley suggests that in this Reynolds number range there is little variation in loss coefficient but proposes however that, below his average Reynolds number of 2.0×10^5, a correction should be applied to the overall efficiency of the turbine by assuming that the stage losses are proportional to $R_N^{-0.2}$.

Ainley's estimates of profile and total loss coefficient are shown in tables 1 and 2 and in figures 70 to 73. The prediction for profile loss corresponds favourably with the writer's value but for the total loss coefficient Ainley's method greatly overestimates the loss in this "profile type" of steam turbine impulse blade.

Comparison with Kearton's work.

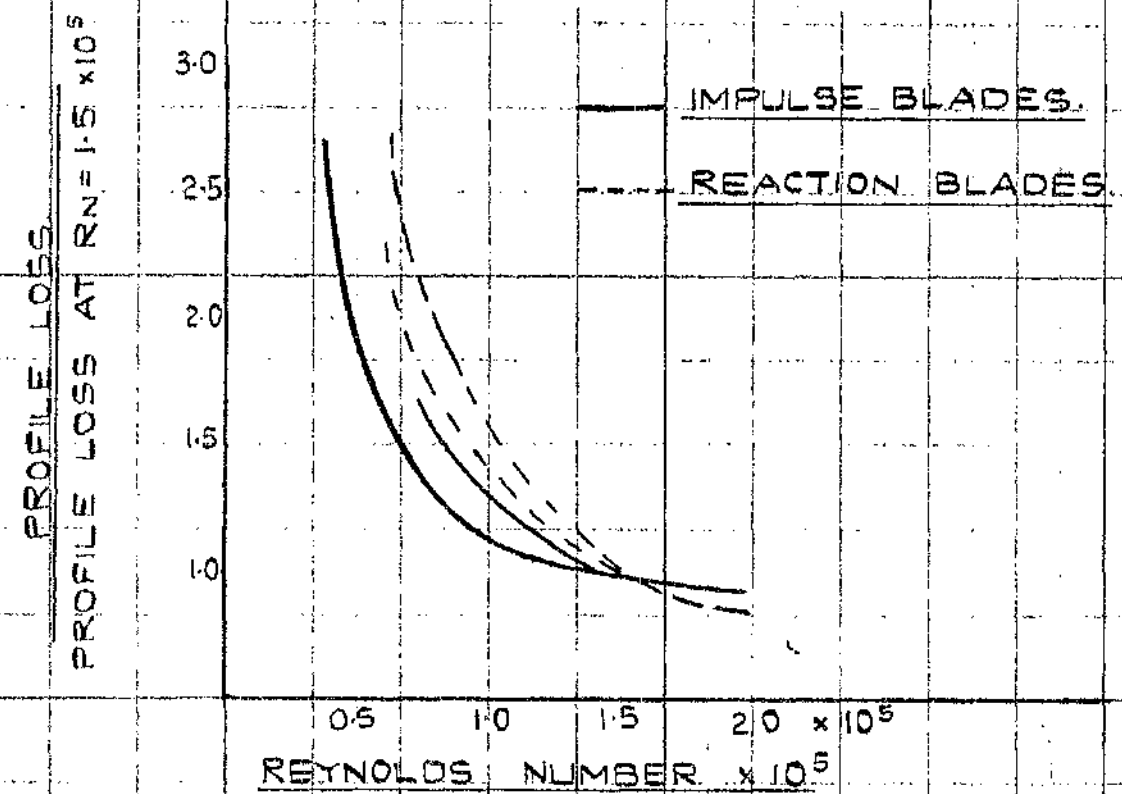
Kearton[42] has examined the flow and loss characteristics for model profile type impulse blades at high Reynolds numbers using air as the working fluid. The blade had inlet and outlet angles of 35° (from the tangential direction) and the tunnel was arranged so that tests could be made with air inlet angles of 30°, 35°, 40° and 45°. Direct measurements were made of the tangential blade force, by connecting the blade cascade to weights suspended from a pulley. Total head pressure and fluid direction measurements were made at a number of stations along the blade pitch at the mid height of the blade. Kearton found that, at an inlet angle of 45°, the average outlet angle was equal to the nominal outlet angle of 35°. For the other three tunnel settings the average outlet angle was slightly greater than 35° showing a positive deviation. A typical result is shown in figure 74 A. In the pitchwise direction the outlet angle is greatest at the concave surface (positive deviation) and is least at the convex surface (negative deviation).

Like Armstrong, Kearton also observed the greatest loss in total head pressure at the convex side of the blade passage.

The variation in fluid outlet angle observed by Kearton is very similar in form to that obtained by the writer and shown in figures 53 and 54 (test 1a and 4a). The maximum positive or negative deviation about the mean efflux angle for the writer's blade is however greater than that observed in Kearton's tests. This is because in Kearton's experimental rig the blades adjacent to the blade passage under test were also passing air, whereas in the writer's apparatus the adjacent blades do not run full of steam. (see page 117). From figures 53 and 54 it will also be observed that the biggest loss region is on the convex side of the blade.

From the measured blade force, the air flow and assuming an average fluid outlet angle of 35°, blade velocity coefficients were calculated over a range of Reynolds number, based on the hydraulic mean diameter at the blade outlet. Kearton's results are shown in figure 74B. The velocity coefficients have been converted to efficiencies for comparison with the results from test series 1 and 2 which are also shown in the figure. When calculating the blade velocity coefficients Kearton made a correction for a small reaction effect obtained during the tests. In the writer's experiments the arrangement was such that no pressure drop occurs across the blade passage. Kearton's results apply therefore to the zero reaction stage while the writer's pertain to the zero pressure drop impulse stage.

Kearton's figures are for total loss in the blade passage at low air velocities. The lower curve corresponds to a fluid deflection of 100° and the upper curve to deflections of 105° - 110°. One would have expected the efficiency to decrease with increase in fluid deflection but the main factor here is probably incidence effects and not fluid deflection. Both Kearton's impulse blade and that



VARIATION OF PROFILE LOSS
WITH REYNOLDS No. - CHESHIRE

FIG. 75

used in series 1 and 2 are the same form of "profile" blade, where the sharp inlet edge results in poor incidence characteristics. It is Kearton's zero incidence results therefore which should be compared with those of series 1 and 2 where the incidence is also zero. It will be noted from figure 74B that these results do compare favourably, bearing in mind that Kearton's figures are for low Mach numbers and for Reynolds numbers well above Armstrong's critical Reynolds number, while those of series 1 and 2 are for high Mach numbers and that some of the figures are in the region of the critical Reynolds number.

It is worthwhile at this point to summarize the limits within which the above results are applicable.

(1). Armstrong's data applies to profile pressure loss coefficients. Low Mach number. Chord Reynolds number range $0.8 \times 10^5 - 4 \times 10^5$. Exhibits a critical Reynolds number at approximately 2×10^5.

(2). Ainley's data applies to profile and total loss coefficients. Mach numbers less than 0.5. No critical Reynolds number in the range $1 \times 10^5 - 3.5 \times 10^5$ (chord). Suggests that below a Reynolds number of 2×10^5 the turbine loss is proportional to $R_n^{-0.2}$.

(3). Kearton's data applies to total loss coefficients only. Low Mach number. Reynolds number based on hydraulic mean diameter but above Armstrong's critical Reynolds number.

(4). Series 1. For profile and total loss coefficients. Mach number constant at 0.6. Reynolds number range $2.5 \times 10^5 - 5.35 \times 10^5$ (chord).

(5). Series 2. For profile and total loss coefficients. Mach number range 0.5 - 0.97. Reynolds number range $2.16 \times 10^5 - 4.4 \times 10^5$ (chord).

In addition to the work quoted above some results on the variation of losses with Reynolds number are given by Cheshire[43] and by Sodorburg[30]. These are:-

(6). Cheshire. Cheshire's results are shown in figure 75.

They are for profile loss coefficients in the Reynolds number range $0.5 \times 10^5 - 2 \times 10^5$.

<u>(7). Soderburg.</u> Soderburg gives the following relationship for the variation in total loss coefficient with Reynolds number.

$$\mathcal{E} = \left(\frac{10^5}{R_n}\right)^{0.25} \times \mathcal{E}'$$

where $\mathcal{E}'$ is the total enthalpy loss coefficient at $R_n = 10^5$.

Appendix 3 - Summary.

Below a Reynolds number of 1×10^5 there seems to be general agreement that the loss coefficients rise. Above 1×10^5 on the other hand Ainley's results suggest that the coefficients remain constant, while Soderburg's indicate a continuous decrease. Cheshire's figures also exhibit a considerable decrease in coefficient as the Reynolds number rises above 1×10^5. Armstrong's critical Reynolds number at 2×10^5 is confirmed by the results of series 2 for both profile and total loss coefficients at low Mach numbers. If this figure is used as the controlling Reynolds number parameter then Ainleys data, Cheshire's results and Soderburg's relationship would all confirm an increase in losses at Reynolds numbers below the critical value. Above 2×10^5 it would appear that, at low velocities, the losses are substantially constant. For example Ainley's data gives a constant loss coefficient, while Cheshire's results, in which the loss ratio curve flattens out as the Reynolds number approaches Armstrong's critical value, are not inconsistant with the losses remaining constant above 2×10^5.

It will be noted that most of the loss coefficient results at high Reynolds numbers are obtained from tests at low Mach numbers. It has been suggested previously that a Mach number of 0.66 is a controlling parameter. Using a critical Reynolds number of 2×10^5 and a critical Mach number of 0.66 the following conclusions are consistant with most of the above data.

1. Below a Mach number of 0.66 the controlling parameter is the Reynolds number.
   - (a) below $R_n = 2 \times 10^5$ both the profile and total loss coefficients rise. (Ainley, Armstrong, Soderburg, Cheshire, Series 2).
   - (b) above $R_n = 2 \times 10^5$ both the profile and total loss coefficients remain substantially constant. (Ainley, Armstrong, Series 1, Kearton, Cheshire).

2. Above a Mach number of 0.66 and above a Reynolds number of 2×10^5.

(a) the total enthalpy loss coefficient $\mathcal{E}_t$ remains constant while the total head pressure loss coefficient (Y_t) varies with Mach number. (Series 2).

For a constant enthalpy loss coefficient this variation in pressure loss coefficient is expected from the relationship.

$$Y = \mathcal{E}\left(1 + \frac{\gamma}{2} M^2\right)$$

(b) The profile enthalpy and total head pressure loss coefficients are both Mach number dependant.

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Appendix 2.

Series 1 ------------------ Page 2.

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Series 3 (Groups 1 & 2)---- Page 84.

Series 1

Tests 2 - 4

and 2A - 4A

TABLE 1   TEST 2

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	
3	28.35	55	30.25	55.5	33.45	56	34.55	53	34.15	58	34.75	52	34.25	52	32.95	53	31.05	53
$2\frac{1}{2}$	24.65	42.5	32.15	41.5	35.15	40	35.15	38.5	35.15	39.5	35.15	39	35.15	39	34.25	41	32.65	[illegible]
$1\frac{1}{2}$	33.45	28	34.95	30	35.15	30.5	35.15	31	35.15	32	35.15	32	35.15	30.5	34.75	30	33.55	[illegible]
$\frac{1}{2}$	33.65	26	34.35	26	35.15	26	35.15	27	35.15	30	35.15	27.5	35.15	26.5	34.45	24.5	33.55	[illegible]
$-\frac{1}{2}$	31.75	24	33.75	25.5	35.15	25	35.15	27	35.15	27	35.15	26	35.15	25	34.65	23	33.35	[illegible]
$-1\frac{1}{2}$	31.05	19.5	33.25	19	35.15	19	35.15	20	35.15	20	35.15	19.5	35.15	19	34.15	18.5	32.55	[illegible]
$-2\frac{1}{2}$	24.25	7	31.35	9.5	35.15	10.5	35.05	9.5	35.15	9	35.15	9	34.85	10	32.65	8	34.25	[illegible]
-3	28.55	-3	29.55	$+0.5$	35.15	7.5	34.85	6	34.85	4.5	34.85	4	34.65	6.5	31.05	3.5	29.45	[illegible]

<u>TABLE 1</u> TEST 2A

STATION	−3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		−2		−1		0		1		2		3		3
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3
	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2	°	lbs/in^2
2	28.6	4	29.0	4	29.0	4	30.6	4	30.6	8.5	29.8	8.5	29.3	5	28.8	5	29.6	24.5	29.7
$1\frac{1}{2}$	29.0	2	29.3	2	29.9	2	32.5	11	32.3	7	31.5	6	30.5	3	28.6	5.5	30.2	23	30.9
1	29.4	0	30.2	0	31.4	9.5	33.9	16	34.3	17	34.3	16.5	33.6	14.5	31.0	9	31.1	20	32.0
$\frac{1}{2}$	30.2	3.5	31.7	9	33.6	21	34.8	22	34.8	24	35.0	25	34.7	23.5	33.7	18	33.6	19.5	53.7
$-\frac{1}{2}$	30.9	32	33.2	33.5	34.4	35.5	34.8	36.5	34.9	35.5	34.9	36.5	34.6	37.5	34.5	42.5	34.3	36	34.0
$-1\frac{1}{2}$	29.3	63	31.2	63.5	33.3	59	33.4	59.5	34.6	56.5	35.0	53	35.0	52.5	34.9	52	34.4	53	33.4
−2	28.6	84	28.6	84	28.6	84	28.6	84	29.2	84	29.3	82.5	30.7	83	31.4	81.5	29.8	81	28.9

Measured Flow = 20.1 galls./$1\frac{1}{4}$ hrs.

TABLE 1 TEST 3

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	
3	35.45	53	38.15	54	42.35	53	42.95	50	42.85	53	43.45	48.5	43.35	48.5	41.35	48	38.55	[illegible]
$2\frac{1}{2}$	36.65	39	40.45	36.5	44.05	39.5	44.15	38	44.15	39	44.15	38	44.05	38.5	42.55	40	40.15	[illegible]
$1\frac{1}{2}$	38.15	26	41.65	28	44.15	31	44.15	30.5	44.15	31.5	44.15	31.5	44.15	31	42.85	30	39.95	[illegible]
$\frac{1}{2}$	38.45	23	41.85	24	44.15	27	44.15	28.5	44.15	29	44.15	28	44.15	27	43.05	25	41.25	[illegible]
$-\frac{1}{2}$	39.35	23.5	41.95	24.5	44.15	26	44.15	27	44.15	27.5	44.15	27	44.15	26.5	43.05	24	41.15	[illegible]
$-1\frac{1}{2}$	38.75	17	[illegible]	19	43.95	23.5	43.95	20	43.85	19.5	43.95	19.5	43.85	18.5	42.45	18	40.15	[illegible]
$-2\frac{1}{2}$	36.55	5	38.75	7	43.85	10.5	43.75	9	43.25	8	43.65	8	43.45	8.5	40.35	8.5	37.35	[illegible]
-3	35.85	-2	36.85	-2	43.65	+6.5	43.45	5	43.15	3	43.36	2	43.15	5	38.75	2	35.95	[illegible]

TABLE 1 TEST 3A

STATION	−3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		−2		−1		0		1		2		3		$3\frac{1}{4}$
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3
	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²
2			35.4	−4.5	35.6	−4.5	37.4	+4.5	37.6	+2.5	36.5	−2.5	35.5	−6			36.6	+23	36.7
$1\frac{1}{2}$	35.7	.0	36.8	0	36.9	0	40.3	10	39.9	7.5	38.9	6	37.8	5	36.0	3.5	37.2	27.5	37.9
1	36.4	−1.5	37.3	−1.5	39.0	+7	42.1	16	42.3	15.5	41.8	15	41.1	14.5	38.0	8.5	38.0	22	39.4
$\frac{1}{2}$	37.4	4	39.1	9	41.9	19	43.0	21	43.2	23.5	43.3	24	42.9	22.5	41.3	17	41.2	19	41.7
$-\frac{1}{2}$	38.8	33.5	41.7	33	42.8	35	43.2	36	43.3	36	43.2	37	43.2	37	43.2	37	42.7	35	42.3
$-1\frac{1}{2}$	35.9	75	38.9	64.5	41.5	58.5	41.4	59	43.3	55.5	43.4	53.5	43.4	53	43.4	52	42.6	51	41.7
−2					36.4	80	36.3	80	36.6	80	38.5	80.5	39.3	80	40.5	78.5	37.9	79	35.7

Measured Flow = 25.2 galls./$1\frac{1}{4}$ hrs.

TABLE 1 TEST 4

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	
3			46·52	55	51·12	53	52·12	50	51·82	55	52·32	51	52·32	52·5	50·12	50	46·62	52
$2\frac{1}{2}$	46·82	40	49·32	38·5	53·32	40	53·92	38·5	53·02	40	53·32	40·5	53·62	39	51·62	39	48·52	40
$1\frac{1}{2}$	47·02	27	51·12	33·5	54·32	30·5	54·22	30·5	53·92	34	54·02	32	53·72	31	52·42	29	50·12	28
$\frac{1}{2}$	47·62	23·5	51·12	26	54·32	21	54·32	29	54·12	29	54·12	27·5	54·02	28	52·12	25	49·42	25
$-\frac{1}{2}$	47·92	26	51·02	25	54·12	26	54·22	28·5	54·22	28	54·12	27·5	53·92	26	52·02	24·5	49·32	23·5
$-1\frac{1}{2}$	47·22	20	50·52	20	53·92	19·5	53·92	20·5	53·92	20	54·02	20	53·62	18·5	51·82	17·5	48·82	17
$-2\frac{1}{2}$	46·82	7	47·62	6·5	53·62	10	53·52	9	53·42	8·5	53·92	8·5	53·52	8	49·92	9	45·82	7
-3	43·92	0	45·52	6·5	53·52	7·5	53·62	4	53·12	3	53·22	3·5	52·92	5	47·52	+2	46·02	−1

TABLE 1 TEST 4A

STATION	−3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		−2		−1		0		1		2		3		3	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	
	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	°	lbs/in²	
2			43·37	0	43·67	0	46·17	+6	46·37	+3	44·97	0	43·37	−8	43·25	−8	45·27	+24	45·37	+
$1\frac{1}{2}$	43·67	−8	44·17	−8·5	44·87	+0·5	49·27	10	49·27	7	47·87	5·5	46·67	4	43·87	1	45·87	26	46·87	2
1	45·07	−5	46·57	+1·5	49·17	11·5	51·97	16	52·67	17·5	52·07	17	51·37	16	47·57	9·5	47·97	18	49·37	
$\frac{1}{2}$	46·47	7	48·87	12	51·97	21	53·07	22	53·47	24·5	53·17	25	53·37	25·5	51·47	18	51·77	20	51·87	
$-\frac{1}{2}$	47·47	33·5	51·37	34	52·77	35	53·37	36	53·47	36·5	53·47	36	53·37	36·5	53·27	36	52·67	36	52·27	
$-1\frac{1}{2}$	43·47	67	46·37	67	49·47	63	49·87	62·5	53·07	59	53·57	56	53·67	55·5	53·67	55	52·27	54	50·77	
−2									44·57	84	46·97	82·5	47·37	82	48·87	80	45·47	78	43·57	

Measured Flow = 31·5 galls./1¼ hrs.

TEST 2 TABLE 2

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
3	·986	·065	·926	·344	·837	·783	·810	·924	·820	·871	·805	·951	·818	·883	·850	·716	·902	·460
$2\frac{1}{2}$	·965	·253	·870	·616	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·818	·883	·857	·682
$1\frac{1}{2}$	·837	·783	·801	·973	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·805	·951	·834	·799
$\frac{1}{2}$	·832	·810	·815	·899	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·813	·908	·834	·799
$-\frac{1}{2}$	·882	·560	·830	·820	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·807	·940	·839	·773
$-1\frac{1}{2}$	·902	·460	·842	·758	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·820	·871	·860	·667
$-2\frac{1}{2}$	·967	·198	·893	·505	·796	1·0	·799	·983	·796	1·0	·796	1·0	·804	·955	·857	·682	·926	·344
-3	·981	·088	·947	·246	·796	1·0	·804	·955	·804	·955	·804	·956	·807	·940	·902	·460	·960	·183

TEST 2a TABLE 2

STATION	-3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
2	.986	.065	.973	.124	.973	.124	.922	.361	.922	.361	.946	.250	.963	.169	.980	.091	.953	.216	.949	.235
$1\frac{1}{2}$	.973	.124	.963	.169	.943	.262	.868	.626	.873	.602	.895	.494	.925	.348	.986	.065	.934	.305	.913	.405
1	.959	.188	.934	.305	.898	.478	.832	.810	.823	.857	.823	.857	.839	.774	.910	.420	.907	.435	.882	.560
$\frac{1}{2}$	.934	.305	.890	.520	.839	.774	.810	.924	.810	.924	.806	.946	.813	.909	.837	.783	.839	.774	.837	.783
$-\frac{1}{2}$	.913	.405	.849	.722	.820	.871	.810	.924	.808	.935	.808	.935	.815	.899	.818	.882	.823	.857	.829	.825
$-1\frac{1}{2}$	.963	.169	.904	.450	.847	.732	.844	.768	.815	.899	.806	.946	.806	.946	.808	.935	.820	.871	.844	.768
-2	.986	.065	.986	.065	.986	.065	.986	.065	.966	.155	.963	.169	.919	.375	.898	.478	.946	.250	.976	.110

TEST 3 TABLE 2

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
3	·991	·042	·921	·367	·829	·825	·818	·882	·820	·871	·809	·930	·811	·920	·850	·716	·912	·411
$2\frac{1}{2}$	·960	·183	·869	·624	·798	·988	·796	1·0	·796	1·0	·796	1·0	·798	·988	·826	·840	·876	·587
$1\frac{1}{2}$	·921	·367	·845	·743	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·820	·871	·880	·567
$\frac{1}{2}$	·903	·455	·840	·767	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·817	·888	·853	·702
$-\frac{1}{2}$	·893	·505	·838	·778	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·796	1·0	·817	·888	·855	·692
$-1\frac{1}{2}$	·907	·435	·848	·727	·800	·977	·800	·977	·802	·966	·800	·977	·802	·966	·827	·835	·876	·587
$-2\frac{1}{2}$	·962	·174	·907	·435	·802	·966	·804	·956	·813	·909	·807	·940	·809	·930	·871	·612	·941	·273
-3	·980	·091	·954	·211	·807	·940	·809	·930	·815	·899	·811	·920	·815	·899	·907	·435	·978	·101

TEST 3a TABLE 2

STATION	-3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
2			·993	·033	·988	·055	·940	·275	·935	·300	·964	·165	·990	·045			·961	·177	·958	·193
$1\frac{1}{2}$	·985	·069	·974	·120	·953	·216	·872	·606	·881	·564	·904	·450	·930	·325	·976	·110	·945	·253	·928	·335
1	·966	·155	·943	·262	·902	·460	·835	·794	·830	·820	·841	·762	·857	·682	·925	·348	·925	·348	·892	·510
$\frac{1}{2}$	·940	·275	·900	·469	·839	·774	·818	·882	·814	·903	·812	·915	·820	·871	·852	·706	·855	·692	·844	·748
$-\frac{1}{2}$	·906	·440	·844	·748	·822	·860	·814	·903	·812	·915	·814	·903	·814	·903	·814	·903	·824	·850	·830	·820
$-1\frac{1}{2}$	·979	·098	·904	·450	·847	·732	·849	·722	·812	·915	·810	·924	·810	·924	·810	·924	·826	·840	·844	·748
-2					·993	·033	·996	·018	·961	·177	·913	·405	·895	·494	·868	·626	·928	·335	·985	·069

TEST 4 TABLE 2

STATION	−3¼		−3		−2		−1		0		1		2		3		3¼	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
3			·930	·325	·846	·736	·830	·820	·835	·794	·827	·835	·827	·835	·864	·668	·928	·335
2½	·983	·078	·877	·584	·812	·915	·802	·966	·816	·894	·812	·915	·807	·940	·839	·774	·892	·510
1½	·920	·371	·846	·736	·796	1·0	·798	·988	·802	·966	·801	·970	·805	·950	·825	·846	·864	·648
½	·909	·425	·846	·736	·796	1·0	·796	1·0	·800	·978	·800	·978	·801	·970	·830	·820	·875	·593
−½	·902	·460	·848	·727	·800	·978	·798	·988	·798	·988	·800	·978	·802	·966	·832	·810	·877	·584
−1½	·916	·390	·856	·687	·802	·966	·802	·966	·802	·966	·801	·970	·807	·940	·835	·794	·886	·540
−2½	·983	·078	·909	·425	·807	·940	·809	·930	·810	·924	·802	·966	·809	·930	·866	·636	·944	·259
−3	·985	·069	·950	·230	·809	·930	·807	·940	·815	·899	·814	·903	·817	·888	·911	·415	·983	·078

TEST 4a TABLE 2

STATION	-3		$-2\frac{3}{4}$		$-2\frac{1}{2}$		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
2			·997	·014	·991	·041	·937	·291	·933	·310	·962	·174	·997	·014			·956	·202	·953	·216
$1\frac{1}{2}$	·991	·041	·980	·091	·964	·165	·878	·578	·878	·578	·904	·450	·927	·338	·985	·069	·943	·262	·923	·357
1	·960	·183	·929	·329	·880	·567	·833	·864	·822	·861	·831	·814	·842	·759	·910	·419	·902	·459	·876	·587
$\frac{1}{2}$	·931	·320	·885	·544	·833	·804	·816	·896	·809	·930	·814	·904	·810	·924	·840	·767	·836	·789	·834	·800
$-\frac{1}{2}$	·912	·411	·842	·759	·820	·871	·810	·924	·809	·930	·809	·930	·810	·924	·812	·915	·822	·861	·828	·830
$-1\frac{1}{2}$	·995	·024	·933	·310	·874	·598	·867	·632	·816	·896	·808	·935	·806	·946	·806	·946	·828	·830	·853	·720
-2									·971	·132	·921	·367	·914	·400	·885	·544	·951	·225	·993	·032

TEST 2 TABLE 3

STATION	−3¼			−3			−2			−1			0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da	$\eta^{\frac{1}{2}}$	Sin α	f.da
	f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$		f.V	$\frac{\eta^{\frac{1}{2}} \text{Sin}\alpha \, f da}{fV}$	
3	·255	·819	¼	·587	·824	¼	·886	·829	½	·962	·799	½	·934	·848	½	·976	·788	½	·941	·788	½	·846	·799	¼	·679	·804	¼
	1·050	·050		1·035	·117		1·0115	·363		1·004	·383		1·007	·394		1·003	·384		1·006	·369		1·015	·167		1·029	·133	
2½	·503	·676	⅜	·785	·663	⅜	1	·643	¾	1	·623	¾	1	·636	¾	1	·629	¾	1	·629	¾	·941	·656	⅜	·826	·669	⅜
	1·040	·123		1·0205	·191		1	·482		1	·468		1	·477		1	·472		1	·472		1·006	·230		1·0175	·204	
1½	·886	·470	½	·987	·500	½	1	·508	1	1	·515	1	1	·530	1	1	·530	1	1	·508	1	·976	·500	½	·894	·485	½
	1·0115	·206		1·0015	·246		1	·508		1	·515		1	·530		1	·530		1	·508		1·003	·244		1·011	·214	
½	·900	·438	½	·969	·438	½	1	·438	1	1	·454	1	1	·500	1	1	·462	1	1	·446	1	·954	·418	½	·894	·407	½
	1·01	·195		1·0055	·207		1	·438		1	·454		1	·500		1	·462		1	·446		1·005	·198		1·011	·180	
−½	·749	·407	½	·906	·430	½	1	·423	1	1	·454	1	1	·454	1	1	·438	1	1	·423	1	·970	·391	½	·880	·375	½
	1·0235	·149		1·0095	·193		1	·423		1	·454		1	·454		1	·438		1	·423		1·003	·189		1·012	·163	
−1½	·679	·334	½	·871	·326	½	1	·326	1	1	·342	1	1	·342	1	1	·334	1	1	·326	1	·934	·317	½	·817	·276	½
	1·029	·110		1·013	·140		1	·326		1	·342		1	·342		1	·334		1	·326		1·007	·147		1·018	·111	
−2½	·445	·122	⅜	·711	·165	⅜	1	·182	¾	·992	·165	¾	1	·156	¾	1	·156	¾	·978	·174	¾	·826	·139	⅜	·587	·130	⅜
	1·0425	·0195		1·022	·043		1	·1365		1·001	·123		1	·117		1	·117		1·002	·1275		1·0175	·042		1·035	·028	
−3	·297	·052	¼	·495	·009	¼	1	·130	½	·978	·105	½	·978	·079	½	·978	·070	½	·970	·113	½	·679	·061	¼	·428	·061	¼
	1·0485	·004		1·04	·001		1	·065		1·002	·051		1·002	·039		1·002	·036		1·003	·055		1·029	·010		1·0435	·006	
Σ	0·8565			1·138			2·7415			2·790			2·853			2·771			2·7265			1·227			1·039		

$$\sum \frac{\eta^{\frac{1}{2}} \, \text{Sin}\alpha \, . \, f da}{fV} = 18{\cdot}1425$$

TEST 2a TABLE 3

Station	−3			−2¾			−2½			−2			−1			0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda	$\eta^{\frac{1}{2}}$	Sin α	fda
	fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sin α $\frac{fda}{fV}$	
2	·255	·070	1/8	·352	·070	1/8	·352	·070	3/16	·601	·070	3/8	·601	·148	1/2	·500	·148	1/2	·411	·087	1/2	·302	·087	1/2	·466	·445	5/16	·485	·366	3/16
	1·05	·002		1·046	·003		1·046	·004		1·034	·015		1·034	·043		1·04	·036		1·044	·017		1·048	·0125		1·042	·058		1·04	·032	
1½	·352	·035	1/8	·411	·035	1/8	·512	·035	3/16	·791	·191	3/8	·776	·122	1/2	·702	·105	1/2	·590	·052	1/2	·255	·096	1/2	·552	·391	5/16	·636	·438	3/16
	1·046	·0015		1·044	·002		1·039	·003		1·02	·0555		1·021	·046		1·027	·036		1·035	·015		1·05	·012		1·037	·065		1·032	·0505	
1	·434	0	1/8	·552	0	1/8	·692	·165	3/16	·900	·284	3/8	·925	·299	1/2	·925	·284	1/2	·880	·250	1/2	·648	·156	1/2	·660	·362	5/16	·749	·423	3/16
	1·043	·000		1·037	·000		1·028	·021		1·01	·095		1·008	·138		1·008	·131		1·012	·109		1·031	·049		1·03	·070		1·023	·058	
½	·552	·061	1/4	·721	·156	1/4	·880	·358	3/8	·962	·375	3/4	·962	·407	1	·972	·423	1	·954	·399	1	·886	·309	1	·880	·334	5/8	·886	·375	3/8
	1·037	·008		1·025	·0275		1·012	·117		1·004	·271		1·004	·390		1·003	·410		1·005	·380		1·011	·270		1·012	·182		1·011	·123	
−½	·636	·530	1/4	·850	·552	1/4	·934	·581	3/8	·962	·595	3/4	·966	·581	1	·966	·595	1	·949	·609	1	·960	·676	1	·925	·588	5/8	·909	·574	3/8
	1·032	·0815		1·015	·115		1·007	·202		1·004	·426		1·003	·560		1·003	·574		1·005	·574		1·006	·630		1·008	·368		1·009	·194	
−1½	·411	·891	1/8	·670	·895	1/8	·857	·857	3/16	·864	·862	3/8	·949	·824	1/2	·972	·799	1/2	·972	·793	1/2	·966	·788	1/2	·934	·799	5/16	·864	·829	3/16
	1·044	·044		1·029	·073		1·014	·136		1·013	·275		1·005	·388		1·003	·386		1·003	·383		1·003	·379		1·007	·231		1·013	·132	
−2	·255	·995	1/8	·255	·995	1/8	·255	·995	3/16	·255	·995	3/8	·394	·995	1/2	·411	·991	1/2	·613	·993	1/2	·692	·989	1/2	·500	·988	5/16	·332	·988	3/16
	1·05	·030		1·05	·030		1·05	·045		1·05	·091		1·045	·188		1·044	·195		1·033	·295		1·028	·334		1·04	·149		1·047	·059	
Σ	0·167			0·2505			0·528			1·2285			1·753			1·768			1·773			1·6865			1·103			0·6485		

$$\sum \eta^{\frac{1}{2}} \sin\alpha \frac{f da}{f V} = 10{\cdot}906$$

TEST 3 TABLE 3

STATION	$-3\frac{1}{4}$	-3	-2	-1	0	1	2	3	$3\frac{1}{4}$
	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da	η^2 Sinα f.da
	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$	f.V $\frac{\eta^2 \text{Sin}\alpha \text{ f.da}}{\text{fV}}$
3	.205 .799 ¼	.606 .869 ¼	.909 .799 ½	.940 .766 ½	.934 .799 ½	.965 .749 ½	.960 .749 ½	.846 .743 ¼	.642 .777 ¼
	1.051 .039	1.034 .1185	1.009 .360	1.006 .358	1.007 .371	1.004 .360	1.004 .358	1.015 .155	1.0315 .121
$2\frac{1}{2}$	.428 .629 ⅜	.791 .595 ⅜	.995 .636 ¾	1 .616 ¾	1 .629 ¾	1 .616 ¾	.995 .623 ¾	.917 .643 ⅜	.767 .663 ⅜
	1.0435 .097	1.02 .173	1.001 .474	1 .462	1 .472	1 .462	1.001 .464	1.0085 .219	1.022 .1865
$1\frac{1}{2}$	.606 .638 ½	.863 .470 ½	1 .515 1	1 .508 1	1 .523 1	1 .523 1	1 .515 1	.934 .500 ½	.753 .492 ½
	1.034 .1285	1.014 .200	1 .515	1 .508	1 .523	1 .523	1 .515	1.007 .232	1.023 .181
$\frac{1}{2}$	.675 .391 ½	.876 .407 ½	1 .454 1	1 .476 1	1 .485 1	1 .470 1	1 .454 1	.943 .423 ½	.839 .415 ½
	1.029 .128	1.012 .176	1 .454	1 .476	1 .485	1 .470	1 .454	1.006 .198	1.016 .1715
$-\frac{1}{2}$	.711 .399 ½	.882 .415 ½	1 .438 1	1 .454 1	1 .462 1	1 .454 1	1 .446 1	.943 .407 ½	.833 .399 ½
	1.026 .138	1.012 .181	1 .438	1 .454	1 .462	1 .454	1 .446	1.006 .191	1.016 .164
$-1\frac{1}{2}$	.660 .292 ½	.853 .326 ½	.989 .399 1	.989 .342 1	.983 .334 1	.989 .334 1	.983 .317 1	.914 .309 ½	.767 .284 ½
	1.03 .0935	1.0165 .137	1.001 .394	1.001 .338	1.002 .328	1.001 .330	1.002 .311	1.009 .140	1.022 .1065
$-2\frac{1}{2}$	.417 .087 ⅜	.660 .122 ⅜	.983 .182 ¾	.978 .156 ¾	.956 .139 ¾	.970 .139 ¾	.965 .148 ¾	.783 .148 ⅜	.523 .130 ⅜
	1.044 .013	1.03 .029	1.002 .134	1.002 .114	1.005 .099	1.003 .101	1.004 .107	1.021 .0425	1.039 .025
-3	.302 .035 ¼	.460 .035 ¼	.970 .113 ½	.965 .087 ½	.949 .052 ½	.960 .035 ½	.949 .087 ½	.660 .035 ¼	.318 .035 ¼
	1.048 .0025	1.042 .004	1.003 .055	1.004 .042	1.0055 .025	1.004 .017	1.0055 .041	1.03 .0055	1.048 .003
Σ	0.6395	1.0185	2.824	2.752	2.765	2.717	2.696	1.183	0.9585

$= \eta^2$ Sinα fda

TEST 3a TABLE 3

STATION	-3			$-2\frac{3}{4}$			$-2\frac{1}{2}$			-2			-1		
	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda
	fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	
2				·182	·078	$\frac{1}{8}$	·234	·078	$\frac{3}{16}$	·525	·078	$\frac{3}{8}$	·547	·044	$\frac{1}{2}$
				1·051	·002		1·05	·003		1·038	·015		1·037	·012	
$1\frac{1}{2}$	·263	0	$\frac{1}{8}$	·346	0	$\frac{1}{8}$	·466	0	$\frac{3}{16}$	·778	·174	$\frac{3}{8}$	·751	·131	$\frac{1}{2}$
	1·049	·000		1·047	·000		1·042	·000		1·021	·050		1·023	·048	
1	·394	·026	$\frac{1}{8}$	·512	·026	$\frac{1}{8}$	·679	·122	$\frac{3}{16}$	·892	·276	$\frac{3}{8}$	·906	·267	$\frac{1}{2}$
	1·045	·001		1·039	·002		1·029	·015		1·011	·091		1·009	·1195	
$\frac{1}{2}$	·525	·070	$\frac{1}{4}$	·685	·156	$\frac{1}{4}$	·880	·326	$\frac{3}{8}$	·940	·358	$\frac{3}{4}$	·951	·399	1
	1·038	·009		1·028	·026		1·012	·106		1·006	·252		1·005	·378	
$-\frac{1}{2}$	·664	·552	$\frac{1}{4}$	·864	·545	$\frac{1}{4}$	·926	·574	$\frac{3}{8}$	·951	·588	$\frac{3}{4}$	·957	·588	1
	1·03	·089		1·013	·116		1·007	·198		1·005	·416		1·004	·561	
$-1\frac{1}{2}$	·313	·966	$\frac{1}{8}$	·670	·903	$\frac{1}{8}$	·856	·853	$\frac{3}{16}$	·850	·862	$\frac{3}{8}$	·957	·824	$\frac{1}{2}$
	1·048	·036		1·029	·074		1·014	·135		1·015	·271		1·004	·392	
-2							·182	·985	$\frac{3}{16}$	·134	·985	$\frac{3}{8}$	·421	·985	$\frac{1}{2}$
							1·051	·032		1·052	·047		1·044	·199	
Σ	0·135			0·220			0·489			1·142			1·7095		

STATION	0			1			2			3			$3\frac{1}{4}$		
	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda	$\eta^{\frac{1}{2}}$	$\sin\alpha$	fda
	fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	
2	·406	·044	$\frac{1}{2}$	·212	·105	$\frac{1}{2}$				·421	·391	$\frac{5}{16}$	·439	·358	$\frac{3}{16}$
	1·044	·009		1·051	·011					1·044	·049		1·043	·028	
$1\frac{1}{2}$	·670	·105	$\frac{1}{2}$	·570	·087	$\frac{1}{2}$	·332	·061	$\frac{1}{2}$	·503	·462	$\frac{5}{16}$	·579	·399	$\frac{3}{16}$
	1·029	·034		1·036	·024		1·047	·010		1·04	·070		1·035	·042	
1	·873	·259	$\frac{1}{2}$	·826	·250	$\frac{1}{2}$	·590	·168	$\frac{1}{2}$	·590	·395	$\frac{5}{16}$	·715	·431	$\frac{3}{16}$
	1·013	·111		1·017	·102		1·035	·042		1·035	·067		1·026	·0565	
$\frac{1}{2}$	·957	·407	1	·934	·383	1	·840	·301	1	·833	·326	$\frac{5}{8}$	·864	·367	$\frac{3}{8}$
	1·004	·388		1·007	·355		1·016	·249		1·016	·167		1·013	·117	
$-\frac{1}{2}$	·951	·602	1	·951	·602	1	·951	·602	1	·922	·574	$\frac{5}{8}$	·906	·566	$\frac{3}{8}$
	1·005	·570		1·005	·570		1·005	·570		1·008	·329		1·009	·190	
$-1\frac{1}{2}$	·962	·804	$\frac{1}{2}$	·962	·799	$\frac{1}{2}$	·962	·788	$\frac{1}{2}$	·917	·777	$\frac{5}{16}$	·864	·799	$\frac{3}{16}$
	1·004	·385		1·004	·383		1·004	·378		1·008	·222		1·013	·127	
-2	·636	·986	$\frac{1}{2}$	·702	·985	$\frac{1}{2}$	·791	·980	$\frac{1}{2}$	·579	·982	$\frac{5}{16}$	·263	·976	$\frac{3}{16}$
	1·032	·304		1·027	·338		1·02	·380		1·035	·172		1·049	·046	
Σ	1·801			1·783			1·629			1·076			0·6065		

$$\sum \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 10{\cdot}591$$

TEST 4 TABLE 3

STATION	$-3\frac{1}{4}$			-3			-2			-1			0			1			2			3			$3\frac{1}{4}$		
	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$f.da$
	$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$		$f.V$	$\frac{\eta^{\frac{1}{2}}\sin\alpha\, fda}{fV}$	
3			$\frac{1}{4}$	·570	·819	$\frac{1}{4}$	·858	·799	$\frac{1}{2}$	·906	·766	$\frac{1}{2}$	·892	·819	$\frac{1}{2}$	·914	·717	$\frac{1}{2}$	·914	·793	$\frac{1}{2}$	·805	·766	$\frac{1}{2}$	·579	·788	$\frac{1}{4}$
		·000		1·036	·113		1·014	·337		1·0095	·345		1·011	·360		1·009	·352		1·009	·358		1·019	·1515		1·035	·110	
$2\frac{1}{2}$	·279	·663	$\frac{3}{8}$	·765	·623	$\frac{3}{8}$	·957	·663	$\frac{3}{4}$	·983	·623	$\frac{3}{4}$	·946	·663	$\frac{3}{4}$	·957	·649	$\frac{3}{4}$	·970	·629	$\frac{3}{4}$	·880	·629	$\frac{3}{8}$	·715	·663	$\frac{3}{8}$
	1·049	·064		1·022	·181		1·0065	·460		1·002	·458		1·006	·454		1·0045	·464		1·003	·456		1·012	·2055		1·026	·168	
$1\frac{1}{2}$	·610	·454	$\frac{1}{2}$	·858	·552	$\frac{1}{2}$	1	·508	1	·995	·508	1	·983	·559	1	·985	·530	1	·975	·515	1	·920	·485	$\frac{1}{2}$	·805	·477	$\frac{1}{2}$
	1·0335	·134		1·014	·233		1	·508		1·001	·505		1·002	·547		1·002	·520		1·003	·502		1·008	·222		1·019	·189	
$\frac{1}{2}$	·652	·399	$\frac{1}{2}$	·858	·438	$\frac{1}{2}$	1	·358	1	1	·485	1	·989	·485	1	·989	·462	1	·985	·470	1	·906	·423	$\frac{1}{2}$	·771	·423	$\frac{1}{2}$
	1·0305	·1265		1·014	·192		1	·358		1	·485		1·001	·478		1·001	·456		1·002	·462		1·0095	·190		1·022	·165	
$-\frac{1}{2}$	·679	·438	$\frac{1}{2}$	·863	·423	$\frac{1}{2}$	·989	·438	1	·995	·477	1	·995	·470	1	·989	·462	1	·983	·438	1	·900	·415	$\frac{1}{2}$	·765	·399	$\frac{1}{2}$
	1·029	·145		1·0165	·178		1·001	·434		1·001	·470		1·001	·466		1·001	·455		1·002	·432		1·01	·185		1·022	·150	
$-1\frac{1}{2}$	·625	·342	$\frac{1}{2}$	·829	·342	$\frac{1}{2}$	·983	·334	1	·983	·350	1	·983	·342	1	·985	·342	1	·970	·317	1	·892	·301	$\frac{1}{2}$	·735	·292	$\frac{1}{2}$
	1·0325	·1035		1·017	·1395		1·002	·327		1·002	·363		1·002	·335		1·0015	·336		1·003	·309		1·011	·134		1·0245	·105	
$-2\frac{1}{2}$	·279	·122	$\frac{3}{8}$	·652	·113	$\frac{3}{8}$	·972	·174	$\frac{3}{4}$	·965	·156	$\frac{3}{4}$	·962	·168	$\frac{3}{4}$	·983	·168	$\frac{3}{4}$	·965	·139	$\frac{3}{4}$	·798	·156	$\frac{3}{8}$	·509	·122	$\frac{3}{8}$
	1·049	·012		1·0305	·027		1·003	·126		1·004	·1125		1·004	·106		1·002	·109		1·004	·100		1·0195	·046		1·0395	·022	
-3	·263	0	$\frac{1}{4}$	·480	·113	$\frac{1}{4}$	·965	·130	$\frac{1}{2}$	·970	·070	$\frac{1}{2}$	·949	·052	$\frac{1}{2}$	·951	·061	$\frac{1}{2}$	·943	·087	$\frac{1}{2}$	·645	·040	$\frac{1}{4}$	·279	·018	$\frac{1}{4}$
	1·0495	·000		1·041	·013		1·004	·0625		1·003	·034		1·0065	·0245		1·005	·029		1·006	·041		1·031	·006		1·049	·001	
Σ		0·585			1·0765			2·6125			2·7525			2·7705			2·721			2·660			1·140			0·910	

$$\sum \frac{\eta^{\frac{1}{2}} \sin\alpha . fda}{f.V} = 17{\cdot}228$$

TEST 4a TABLE 3

STATION	−3			−2¾			−2½			−2			−1		
	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda
	fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$	
2				·118	0	1/8	·202	0	3/16	·560	·105	3/8	·556	·052	1/2
				1·052	·000		1·051	·000		1·038	·0205		1·037	·014	
1½	·202	·139	1/8	·302	·148	1/8	·606	·009	3/16	·760	·174	3/8	·760	·122	1/2
	1·051	·003		1·048	·005		1·044	·001		1·022	·0485		1·022	·045	
1	·428	·087	1/8	·574	·026	1/8	·753	·199	3/16	·896	·276	3/8	·928	·301	1/2
	1·043	·004		1·036	·002		1·023	·0275		1·01	·092		1·007	·139	
½	·566	·122	1/4	·738	·208	1/4	·896	·358	3/8	·946	·375	3/4	·965	·415	1
	1·036	·017		1·024	·0375		1·01	·119		1·006	·265		1·004	·398	
−½	·642	·552	1/4	·871	·559	1/4	·934	·574	3/8	·962	·588	3/4	·965	·595	1
	1·031	·086		1·013	·120		1·007	·199		1·004	·422		1·004	·572	
−1½	·155	·921	1/8	·556	·921	1/8	·773	·891	3/16	·795	·887	3/8	·946	·857	1/2
	1·052	·017		1·037	·062		1·021	·126		1·019	·260		1·006	·404	
−2													·363	·995	1/2
													1·046	·173	
Σ	0·127			0·2265			0·4725			1·108			1·745		

STATION	0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda	$\eta^{\frac{1}{2}}$	Sinα	fda
	fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$		fV	$\eta^{\frac{1}{2}}$ Sinα $\frac{fda}{fV}$	
2	·417	0	1/2	·118	·139	1/2				·450	·407	5/16	·466	·366	3/16
	1·044	·000		1·052	·008					1·042	·055		1·042	·031	
1½	·670	·096	1/2	·581	·070	1/2	·263	·018	1/2	·512	·438	5/16	·598	·423	3/16
	1·029	·031		1·035	·020		1·049	·002		1·039	·068		1·034	·046	
1	·902	·292	1/2	·871	·276	1/2	·647	·165	1/2	·676	·309	5/16	·767	·391	3/16
	1·01	·131		1·013	·118		1·031	·052		1·029	·0635		1·022	·055	
½	·951	·423	1	·962	·431	1	·876	·309	1	·888	·342	5/8	·895	·395	3/8
	1·005	·401		1·004	·414		1·012	·268		1·011	·187		1·01	·124	
−½	·965	·588	1	·962	·595	1	·957	·588	1	·928	·588	5/8	·911	·552	5/8
	1·004	·564		1·004	·570		1·004	·558		1·007	·340		1·009	·187	
−1½	·966	·829	1/2	·972	·824	1/2	·972	·819	1/2	·911	·809	5/16	·839	·839	3/16
	1·003	·398		1·003	·398		1·003	·397		1·009	·229		1·016	·130	
−2	·606	·991	1/2	·632	·990	1/2	·738	·995	1/2	·474	·978	6/16	·179	·978	3/16
	1·034	·290		1·032	·302		1·024	·356		1·041	·139		1·051	·031	
Σ	1·815			1·830			1·633			1·0815			0·604		

$$\sum \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV} = 10{\cdot}6425$$

TEST 2 TABLE 4

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{V}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	
3	·065	·050	·344	·117	·783	·363	·924	·383	·871	·394	·951	·384	·883	·369	·716	·167	·460	·133
	·003		·040		·284		·353		·343		·365		·326		·120		·061	
$2\frac{1}{2}$	·253	·123	·616	·191	1	·482	1	·468	1	·477	1	·472	1	·472	·883	·230	·682	·204
	·031		·118		·482		·468		·477		·472		·472		·203		·139	
$1\frac{1}{2}$	·783	·206	·973	·246	1	·508	1	·515	1	·530	1	·530	1	·508	·951	·244	·799	·214
	·161		·239		·508		·515		·530		·530		·508		·232		·171	
$\frac{1}{2}$	·810	·195	·899	·207	1	·438	1	·454	1	·500	1	·462	1	·446	·908	·198	·799	·180
	·158		·186		·438		·454		·500		·462		·446		·180		·144	
$-\frac{1}{2}$	·560	·149	·820	·193	1	·423	1	·454	1	·454	1	·438	1	·423	·940	·189	·773	·163
	·0835		·158		·423		·454		·454		·438		·423		·178		·126	
$-1\frac{1}{2}$	·460	·110	·758	·140	1	·326	1	·342	1	·342	1	·334	1	·326	·871	·147	·667	·111
	·0505		·106		·326		·342		·342		·334		·326		·128		·074	
$-2\frac{1}{2}$	·198	·0195	·505	·043	1	·1365	·983	·123	1	·117	1	·117	·955	·1275	·682	·042	·364	·028
	·004		·022		·1365		·121		·117		·117		·122		·029		·010	
-3	·088	·004	·245	·001	1	·065	·955	·051	·955	·039	·955	·034	·940	·055	·460	·010	·183	·006
	·000		·000		·065		·049		·037		·0325		·052		·005		·001	
Σ	0·491		0·869		2·6625		2·756		2·800		2·7505		2·675		1·075		0·726	

$$\Sigma\ \eta^{\frac{3}{2}}\sin\alpha\,\frac{fda}{fV} = 16{\cdot}805$$

TEST 2a TABLE 4

STATION	−3		−2¾		−2½		−2		−1		0		1		2		3		3¼	
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	
2	·065	·002	·124	·003	·124	·004	·361	·015	·361	·043	·250	·036	·169	·017	·091	·0125	·216	·058	·235	·032
	·000		·000		·0005		·005		·016		·009		·003		·001		·013		·0075	
1½	·124	·0015	·169	·002	·262	·003	·626	·0555	·602	·046	·494	·036	·348	·015	·065	·012	·305	·065	·405	·0505
	·000		·000		·001		·035		·028		·018		·005		·001		·020		·0205	
1	·188	·000	·305	·000	·478	·021	·810	·095	·857	·138	·857	·131	·774	·109	·420	·049	·435	·070	·560	·058
	·000		·000		·010		·077		·118		·112		·084		·021		·0305		·0325	
½	·305	·008	·520	·0275	·774	·117	·924	·271	·924	·390	·946	·410	·909	·380	·783	·270	·774	·182	·783	·123
	·002		·014		·0905		·250		·360		·388		·345		·212		·161		·096	
−½	·405	·0815	·722	·115	·871	·202	·924	·426	·935	·560	·935	·574	·899	·574	·882	·630	·857	·348	·825	·194
	·033		·083		·176		·393		·524		·536		·515		·564		·299		·160	
−1½	·169	·044	·450	·073	·732	·136	·748	·275	·899	·388	·946	·386	·946	·383	·935	·379	·871	·231	·748	·132
	·007		·033		·100		·205		·348		·365		·362		·354		·202		·099	
−2	·065	·030	·065	·030	·065	·045	·065	·091	·155	·188	·169	·195	·375	·295	·478	·334	·250	·149	·110	·059
	·002		·002		·003		·006		·029		·033		·111		·160		·037		·0065	
Σ	0·044		0·132		0·381		0·971		1·423		1·461		1·425		1·313		0·7425		0·422	

$$\Sigma\, \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 8·3145$$

TEST 3   TABLE 4

STATION	$-3\frac{1}{2}$		-3		-2		-1		0		1		2		3		$3\frac{1}{2}$	
	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$	h	$h^{\frac{1}{2}}\sin\alpha\frac{tda}{fV}$
	$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$		$h^{\frac{3}{2}}\sin\alpha\frac{tda}{fV}$	
3	·042	·039	·367	·1185	·825	·360	·882	·358	·871	·371	·930	·360	·920	·358	·716	·155	·411	·121
	·002		·0435		·297		·316		·323		·335		·330		·111		·050	
$2\frac{1}{2}$	·183	·097	·624	·173	·988	·474	1	·462	1	·472	1	·462	·988	·464	·840	·219	·587	·1865
	·018		·108		·469		·462		·472		·462		·458		·184		·1095	
$1\frac{1}{2}$	·367	·1285	·743	·200	1	·515	1	·508	1	·523	1	·523	1	·515	·871	·232	·567	·181
	·047		·149		·515		·508		·523		·523		·515		·202		·1025	
$\frac{1}{2}$	·455	·128	·767	·176	1	·454	1	·476	1	·485	1	·470	1	·454	·888	·198	·702	·1715
	·058		·135		·454		·476		·485		·470		·454		·176		·1205	
$-\frac{1}{2}$	·505	·138	·778	·181	1	·438	1	·454	1	·462	1	·454	1	·446	·888	·191	·692	·164
	·070		·141		·438		·454		·462		·454		·446		·170		·1135	
$-1\frac{1}{2}$	·435	·0935	·727	·137	·977	·394	·977	·338	·966	·328	·977	·330	·966	·311	·835	·140	·587	·1065
	·041		·0995		·385		·330		·317		·322		·301		·117		·0625	
$-2\frac{1}{2}$	·174	·013	·435	·029	·966	·134	·956	·114	·809	·099	·940	·101	·930	·107	·612	·0425	·273	·025
	·002		·013		·1295		·110		·081		·095		·0995		·026		·007	
-3	·091	·0025	·211	·004	·940	·055	·930	·042	·899	·025	·920	·017	·899	·041	·435	·0055	·101	·003
	·000		·001		·052		·039		·0225		·016		·037		·0025		·000	
Σ	0·238		0·690		2·7395		2·695		2·6855		2·677		2·6405		0·9885		0·5655	

$$\sum h^{\frac{3}{2}} \sin\alpha \frac{tda}{\ldots} = 15.9195$$

TEST 3a TABLE 4

STATION	−3		−2¾		−2½		−2		−1		0		1		2		3		3¼	
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	
2			·033	·002	·055	·003	·275	·015	·300	·012	·165	·009	·045	·011			·177	·049	·193	·028
			·000		·000		·004		·004		·0015		·0005				·009		·005	
1½	·069	·000	·120	·000	·216	·000	·606	·050	·564	·048	·650	·034	·325	·024	·110	·010	·253	·070	·335	·042
	·000		·000		·000		·030		·027		·015		·008		·001		·018		·014	
1	·155	·001	·262	·002	·460	·015	·794	·091	·820	·1195	·762	·111	·682	·102	·348	·042	·348	·067	·510	·0565
	·000		·0005		·007		·072		·098		·0845		·0695		·015		·023		·030	
½	·275	·009	·469	·026	·774	·106	·882	·252	·903	·378	·915	·388	·871	·355	·706	·249	·692	·167	·748	·117
	·0025		·012		·082		·222		·342		·355		·309		·175		·116		·0875	
−½	·440	·089	·748	·116	·860	·198	·903	·416	·915	·561	·903	·570	·903	·570	·903	·570	·850	·329	·820	·190
	·039		·0865		·170		·376		·512		·515		·515		·515		·280		·156	
−1½	·098	·036	·450	·074	·732	·135	·722	·271	·915	·392	·924	·385	·924	·383	·924	·378	·840	·222	·748	·127
	·0035		·033		·099		·195		·369		·355		·354		·349		·189		·095	
−2					·033	·032	·018	·047	·177	·199	·405	·304	·494	·338	·626	·380	·335	·172	·069	·046
					·001		·001		·036		·123		·167		·238		·0575		·003	
Σ	0·045		0·132		0·359		0·900		1·388		1·449		1·423		1·293		0·6925		0·3905	

$$\sum \eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV} = 8{\cdot}072$$

TEST 4 TABLE 4

STATION	$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$		$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	
3			·325	·113	·736	·337	·820	·345	·794	·360	·835	·352	·835	·358	·648	·1515	·335	·110
			·037		·248		·282		·286		·294		·298		·098		·037	
$2\frac{1}{2}$	·078	·064	·584	·181	·915	·460	·966	·458	·894	·454	·915	·464	·940	·456	·774	·2055	·510	·168
	·005		·1055		·421		·443		·405		·424		·428		·159		·086	
$1\frac{1}{2}$	·371	·134	·736	·233	1	·508	·988	·505	·966	·547	·970	·520	·950	·502	·846	·220	·648	·189
	·050		·171		·508		·499		·528		·505		·476		·186		·1225	
$\frac{1}{2}$	·425	·1265	·736	·192	1	·358	1	·485	·978	·478	·978	·456	·970	·462	·820	·190	·593	·165
	·054		·141		·358		·485		·468		·446		·446		·156		·098	
$-\frac{1}{2}$	·460	·145	·727	·178	·978	·434	·988	·470	·988	·466	·978	·455	·966	·432	·810	·185	·584	·150
	·067		·1295		·424		·465		·460		·445		·417		·150		·088	
$-1\frac{1}{2}$	·390	·1035	·687	·1395	·966	·320	·966	·343	·966	·335	·970	·336	·940	·309	·794	·134	·540	·105
	·040		·096		·309		·331		·324		·326		·290		·106		·057	
$-2\frac{1}{2}$	·078	·012	·425	·027	·940	·126	·930	·1125	·924	·106	·966	·109	·930	·100	·636	·046	·259	·022
	·001		·0115		·118		·1045		·098		·105		·093		·029		·006	
-3	·069	·000	·230	·013	·930	·0625	·940	·034	·899	·0245	·903	·029	·888	·041	·415	·006	·078	·001
	·000		·003		·058		·032		·022		·026		·036		·0025		·000	
Σ	0·217		0·6945		2·444		2·6415		2·591		2·571		2·484		0·8865		0·4945	

$$\Sigma\, \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 15.024$$

TEST 4a TABLE 4.

STATION	−3			−2¾			−2½			−2			−1		
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$
2				·014	0	·000	·041	0	·000	·291	·0205	·006	·310	·014	·004
1½	·041	·003	·000	·091	·005	·0005	·165	·001	·000	·578	·0485	·028	·578	·045	·026
1	·183	·004	·001	·329	·002	·001	·567	·0275	·016	·864	·092	·074	·861	·139	·120
½	·320	·017	·005	·544	·0375	·020	·804	·119	·0955	·896	·265	·238	·930	·398	·370
−½	·411	·086	·035	·759	·120	·091	·871	·199	·173	·924	·422	·389	·930	·572	·532
−1½	·024	·017	·000	·310	·062	·019	·598	·126	·075	·632	·260	·164	·896	·404	·362
−2													·132	·173	·023
Σ			0·041			0·1315			0·3595			0·899			1·437

STATION	0			1			2			3			3¼		
	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha\frac{fda}{fV}$	η	$\eta^{\frac{1}{2}}\sin\alpha$	$\eta^{\frac{3}{2}}\sin\alpha$
2	·174	0	·000	·014	·008	·000				·202	·055	·011	·216	·031	·007
1½	·450	·031	·014	·338	·020	·007	·069	·002	·000	·262	·068	·018	·357	·046	·016
1	·814	·131	·105	·759	·118	·089	·419	·052	·022	·459	·0635	·029	·585	·05	·032
½	·904	·401	·362	·924	·414	·382	·767	·268	·206	·789	·187	·147	·800	·12	·099
−½	·930	·564	·524	·924	·570	·526	·915	·558	·510	·861	·340	·293	·830	·18	·155
−1½	·935	·398	·372	·946	·398	·376	·946	·397	·376	·830	·229	·190	·702	·13	·091
−2	·367	·290	·106	·400	·302	·121	·544	·356	·193	·225	·139	·031	·032	·03	·001
Σ			1·483			1·501			1·307			0·719			0·401

$$\Sigma\, \eta^{\frac{3}{2}} \sin\alpha \frac{fda}{fV} = 8{\cdot}279$$

TEST 2 TABLE 5

STATION	$-3\frac{1}{4}$			-3			-2			-1			0			1			2			3			$3\frac{1}{4}$		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
3	·255	·574	·050	·587	·566	·117	·886	·559	·363	·962	·602	·383	·934	·530	·394	·976	·616	·354	·941	·616	·367	·846	·602	·167	·679	·595	·133
	·007			·039			·180			·222			·195			·231			·213			·085			·054		
$2\frac{1}{2}$	·503	·737	·123	·785	·749	·191	1	·766	·482	1	·783	·468	1	·772	·477	1	·777	·472	1	·777	·472	·941	·755	·230	·826	·743	·204
	·046			·112			·370			·366			·368			·367			·367			·163			·125		
$1\frac{1}{2}$	·886	·883	·206	·987	·866	·246	1	·862	·508	1	·857	·516	1	·848	·530	1	·848	·530	1	·862	·508	·976	·866	·244	·894	·875	·214
	·161			·210			·438			·442			·450			·450			·438			·206			·167		
$\frac{1}{2}$	·900	·899	·195	·949	·899	·207	1	·899	·438	1	·891	·454	1	·866	·500	1	·887	·462	1	·895	·446	·954	·910	·198	·894	·914	·810
	·158			·1765			·394			·405			·433			·410			·399			·172			·147		
$-\frac{1}{2}$	·749	·914	·149	·906	·903	·193	1	·906	·423	1	·891	·454	1	·891	·454	1	·899	·438	1	·906	·423	·970	·921	·189	·880	·927	·163
	·102			·158			·384			·405			·405			·394			·384			·169			·133		
$-1\frac{1}{2}$	·679	·963	·110	·871	·946	·140	1	·946	·326	1	·940	·342	1	·940	·342	1	·943	·334	1	·946	·326	·934	·948	·147	·817	·981	·110
	·0705			·1155			·3085			·322			·322			·315			·3085			·130			·0865		
$-2\frac{1}{2}$	·445	·993	·0195	·711	·986	·043	1	·983	·1365	·992	·986	·123	1	·988	·117	1	·988	·117	·978	·985	·1275	·862	·990	·042	·587	·991	·028
	·0085			·030			·134			·120			·116			·116			·123			·036			·016		
-3	·297	·999	·004	·495	1	·001	1	·991	·065	·978	·995	·051	·978	·997	·039	·978	·998	·034	·970	·996	·065	·679	·998	·010	·428	·998	·006
	·001			·0005			·064			·050			·038			·033			·053			·007			·0025		
Σ	0·554			0·8415			2·2725			2·332			2·327			2·316			2·2855			0·968			0·731		

$$\Sigma\, \eta \cos\alpha \sin\alpha \frac{fda}{fV} = 14{\cdot}6275$$

TEST 2a TABLE 5

Station	−3			−2¾			−2½			−2			−1			0		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$
	$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$		
2	·255	·998	·002	·352	·998	·003	·352	·998	·004	·601	·998	·015	·601	·989	·043	·500	·989	·036
	·0005			·001			·001			·009			·026			·018		
1½	·352	·999	·0015	·411	·999	·002	·512	·999	·003	·791	·982	·0556	·776	·993	·046	·702	·995	·036
	·0005			·001			·0015			·043			·035			·025		
1	·634	1	·000	·552	1	·000	·692	·986	·021	·900	·961	·095	·925	·956	·138	·925	·959	·131
	·000			·000			·014			·082			·122			·116		
½	·552	·998	·008	·721	·988	·0275	·880	·934	·117	·962	·927	·271	·962	·914	·390	·972	·906	·410
	·004			·020			·096			·241			·343			·361		
−½	·636	·848	·0815	·850	·834	·115	·934	·844	·202	·962	·804	·426	·966	·814	·560	·966	·804	·574
	·044			·0815			·1535			·330			·440			·446		
−1½	·411	·454	·044	·670	·466	·073	·856	·515	·136	·864	·508	·275	·949	·566	·388	·972	·602	·386
	·008			·020			·060			·121			·2085			·226		
−2	·255	·105	·030	·256	·105	·030	·255	·105	·045	·255	·105	·091	·394	·105	·188	·411	·131	·195
	·001			·001			·001			·002			·008			·0105		
Σ	0·058			0·1245			0·327			0·828			1·1825			1·2025		

Station	1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	[illegible]
	$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$			$\eta \cos\alpha \sin\alpha \frac{fda}{fV}$		
2	·411	·996	·017	·302	·996	·0125	·466	·910	·058	·465	·930	[illegible]
	·007			·004			·025			·014		
1½	·590	·999	·015	·255	·995	·012	·552	·921	·065	·636	·899	[illegible]
	·009			·003			·033			·029		
1	·880	·968	·109	·648	·988	·049	·660	·940	·070	·769	·966	[illegible]
	·093			·031			·043			·039		
½	·954	·917	·380	·886	·951	·270	·880	·943	·182	·886	·927	[illegible]
	·332			·228			·151			·101		
−½	·969	·793	·574	·960	·737	·630	·925	·804	·348	·909	·814	[illegible]
	·386			·436			·259			·1435		
−1½	·972	·609	·383	·966	·616	·379	·934	·602	·231	·864	·559	[illegible]
	·227			·2255			·130			·066		
−2	·613	·122	·295	·692	·148	·334	·500	·156	·149	·332	·156	[illegible]
	·022			·034			·012			·003		
Σ	1·076			0·9615			0·663			0·3935		

$$\Sigma\, \eta \cos\alpha \sin\alpha \frac{fda}{fV} = 6{\cdot}8065$$

TEST 3 TABLE 5

STATION	−3¼			−3			−2			−1			0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{t\,d\alpha}{tV}$
	$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$			$\eta\cos\alpha\sin\alpha\frac{t\,d\alpha}{tV}$		
3	·205	·602	·039	·606	·588	·1185	·909	·602	·360	·960	·643	·358	·934	·602	·371	·965	·663	·360	·960	·663	·358	·866	·669	·156	·662	·629	·121
	·005			·042			·197			·216			·208			·230			·228			·0875			·049		
2½	·428	·777	·097	·791	·804	·173	·995	·772	·474	1	·788	·462	1	·777	·472	1	·788	·462	·995	·783	·464	·917	·764	·219	·767	·749	·1865
	·032			·110			·364			·364			·366			·364			·361			·154			·107		
1½	·606	·899	·1285	·863	·883	·200	1	·857	·515	1	·862	·508	1	·853	·523	1	·853	·523	1	·857	·515	·934	·866	·232	·753	·870	·181
	·070			·152			·442			·438			·446			·446			·442			·188			·1185		
½	·675	·921	·128	·876	·914	·176	1	·891	·454	1	·879	·476	1	·875	·486	1	·883	·470	1	·891	·454	·943	·906	·198	·839	·910	·1715
	·0795			·141			·405			·418			·424			·415			·405			·169			·131		
−½	·711	·917	·138	·882	·910	·181	1	·899	·438	1	·891	·454	1	·887	·462	1	·891	·454	1	·895	·446	·943	·914	·191	·833	·917	·164
	·090			·1455			·394			·404			·410			·404			·400			·1645			·1255		
−1½	·660	·956	·0955	·853	·946	·137	·989	·917	·384	·989	·940	·338	·983	·943	·328	·989	·943	·330	·983	·948	·311	·914	·951	·140	·767	·959	·1065
	·059			·110			·357			·314			·304			·307			·289			·122			·0785		
−2½	·417	·996	·013	·660	·993	·029	·983	·983	·134	·978	·988	·114	·954	·990	·099	·970	·990	·101	·965	·989	·107	·783	·989	·0625	·523	·991	·025
	·005			·019			·1295			·110			·0935			·097			·102			·033			·013		
−3	·302	·999	·0025	·460	·999	·004	·970	·994	·055	·965	·996	·042	·949	·999	·025	·960	·999	·017	·949	·996	·041	·660	·999	·0055	·318	·999	·003
	·0075			·002			·053			·040			·024			·016			·039			·004			·001		
Σ	0·348			0·7215			2·3415			2·304			2·2755			2·279			2·266			0·922			0·6235		

$$\Sigma\, \eta \cos\alpha \sin\alpha \frac{t\,d\alpha}{tV} = 14{\cdot}081$$

TEST 3a TABLE 5

STATION	−3			$-2\frac{3}{4}$			$-2\frac{1}{2}$			−2			−1			0			1			2			3			$3\frac{1}{4}$		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	$\cos\alpha$	$\eta^{\frac{1}{2}}$ [illegible]
		$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha$ [illegible]	
2				·182	·997	·002	·234	·997	·003	·525	·997	·015	·547	·999	·012	·406	·999	·009	·212	·995	·011				·621	·921	·049	·439	·936	·0[illegible]
					·000			·001			·008			·0065			·004			·002						·019			·011	
$1\frac{1}{2}$	·263	1	·000	·346	1	·000	·466	1	·000	·778	·985	·050	·751	·991	·048	·670	·995	·034	·570	·996	·024	·332	·998	·010	·583	·887	·070	·579	·917	·0[illegible]
		·000			·000			·000			·038			·036			·023			·014			·003			·031			·022	
1	·394	1	·001	·512	1	·002	·679	·993	·015	·892	·961	·091	·906	·964	·1195	·873	·966	·111	·826	·968	·102	·590	·989	·042	·590	·927	·067	·715	·903	·05
		·000			·001			·010			·078			·1045			·0935			·0815			·0245			·037			·036	
$\frac{1}{2}$	·525	·998	·009	·685	·988	·026	·880	·946	·106	·940	·934	·252	·951	·917	·378	·957	·914	·388	·934	·924	·355	·840	·956	·249	·833	·946	·167	·864	·930	·11
		·005			·018			·088			·221			·329			·339			·306			·200			·1315			·094	
$-\frac{1}{2}$	·666	·834	·089	·864	·839	·116	·926	·819	·198	·951	·809	·416	·957	·809	·561	·951	·799	·570	·951	·799	·570	·951	·799	·570	·922	·819	·329	·966	·824	·19
		·049			·084			·149			·320			·435			·432			·432			·432			·249			·1415	
$-1\frac{1}{2}$	·313	·259	·036	·670	·431	·074	·856	·523	·135	·850	·515	·271	·957	·566	·392	·962	·595	·385	·962	·602	·383	·962	·616	·378	·917	·629	·222	·864	·602	·12
		·003			·021			·060			·119			·212			·220			·221			·224			·128			·066	
−2							·182	·174	·032	·134	·174	·047	·424	·174	·199	·636	·165	·304	·702	·174	·338	·791	·199	·380	·579	·191	·172	·263	·216	·04
								·001			·001			·015			·032			·041			·060			·019			·003	
Σ		0·057			0·124			0·309			0·785			1·138			1·1435			1·0975			0·9435			0·6145			0·3735	

$$\sum \eta \cos\alpha \sin\alpha \frac{fda}{fV} = 6\cdot5855$$

TEST 4 TABLE 5

STATION	−3¼			−3			−2			−1		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
3				.570	.574	.113	.858	.602	.337	.906	.643	.345
					.037			.174			.201	
2½	.279	.766	.064	.765	.783	.181	.957	.766	.460	.983	.783	.458
		.014			.1085			.337			.353	
1½	.610	.891	.134	.858	.834	.233	1	.862	.508	.995	.862	.505
		.073			.1665			.438			.433	
½	.652	.917	.1265	.858	.899	.192	1	.934	.358	1	.875	.485
		.072			.148			.334			.424	
−½	.679	.899	.145	.853	.906	.178	.989	.899	.434	.995	.879	.470
		.0885			.138			.386			.411	
−1½	.625	.940	.1035	.829	.940	.1395	.983	.943	.327	.983	.937	.343
		.061			.109			.303			.316	
−2½	.279	.993	.012	.652	.994	.027	.970	.985	.126	.965	.988	.1125
		.003			.0175			.1205			.107	
−3	.263	1	.000	.480	.994	.013	.965	.991	.0625	.970	.998	.034
		.000			.006			.060			.033	
Σ		0.3115			0.7305			2.1525			2.278	

STATION	0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$	$\eta^{\frac{1}{2}}$	Cos α	$\eta^{\frac{1}{2}}\sin\alpha\frac{fda}{fV}$
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
3	.892	.574	.360	.914	.629	.352	.914	.609	.358	.805	.643	.1515	.579	.616	.110
		.184			.202			.199			.078			.039	
2½	.946	.766	.454	.957	.760	.464	.970	.777	.456	.880	.777	.2055	.715	.766	.168
		.329			.337			.344			.1405			.092	
1½	.983	.829	.547	.985	.848	.520	.975	.857	.502	.920	.875	.222	.805	.879	.189
		.446			.434			.420			.179			.134	
½	.989	.875	.478	.989	.887	.456	.985	.883	.462	.906	.906	.190	.771	.906	.165
		.414			.400			.402			.156			.115	
−½	.995	.883	.466	.989	.887	.455	.983	.899	.432	.900	.910	.185	.765	.917	.150
		.410			.399			.382			.1515			.105	
−1½	.983	.940	.335	.985	.940	.336	.970	.968	.309	.892	.954	.134	.735	.956	.105
		.309			.3115			.284			.114			.074	
−2½	.962	.989	.106	.983	.989	.109	.965	.990	.100	.798	.998	.046	.509	.993	.022
		.101			.106			.0955			.036			.011	
−3	.949	.999	.0245	.951	.998	.029	.943	.996	.041	.645	.999	.006	.279	1	.001
		.023			.0275			.0385			.004			.000	
Σ		2.216			2.217			2.165			0.859			0.570	

$$\sum \eta \cos\alpha \sin\alpha \frac{fda}{fV} = 13.4995$$

TEST 4a TABLE 5.

STATION	-3			$-2\frac{3}{4}$			$-2\frac{1}{2}$			-2			-1		
	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$
	η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$		
2				·118	1	·000	·202	1	·000	·560	·995	·0205	·556	·999	·014
					·000			·000			·011			·008	
$1\frac{1}{2}$	·202	·990	·003	·302	·989	·005	·406	1	·001	·760	·985	·0485	·760	·993	·045
		·005			·0015			·0005			·036			·034	
1	·428	·996	·004	·574	1	·002	·753	[illegible]	·0275	·846	·961	·092	·928	·954	·139
		·002			·001			·020			·079			·123	
$\frac{1}{2}$	·566	·993	·017	·738	·978	·0375	·886	·934	·119	·946	·927	·265	·965	·910	·398
		·0095			·027			·099			·233			·349	
$-\frac{1}{2}$	·662	·834	·086	·871	·829	·120	·934	·819	·199	·962	·809	·422	·965	·804	·572
		·066			·087			·152			·328			·444	
$-1\frac{1}{2}$	·155	·391	·017	·556	·391	·062	·773	·454	·126	·795	·462	·260	·966	·515	·404
		·001			·013			·044			·0956			·197	
-2													·363	·105	·173
														·007	
Σ	0·059			0·1295			0·3155			0·7825			1·162		

STATION	0			1			2			3			$3\frac{1}{2}$		
	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$	η^{ϵ}	Cos α	$\sin\alpha \frac{fd\alpha}{fV}$
	η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$			η Cos α Sin α $\frac{fd\alpha}{fV}$		
2	·417	1	·000	·118	·990	·008				·650	·94	·055	·666	·930	·021
		·000			·001						·023			·013	
$1\frac{1}{2}$	·670	·995	·031	·581	·98	·020	·263	1	·002	·512	·899	·068	·598	·906	[illegible]
		·021			·0115			·0005			·031			·025	
1	·902	·956	·131	·871	·961	·118	·647	·986	·052	·676	·951	·0636	·767	·921	[illegible]
		·113			·099			·033			·041			·039	
$\frac{1}{2}$	·951	·906	·401	·982	·903	·404	·876	·957	·268	[illegible]	·940	·187	·845	·927	·124
		·345			·360			·224			·156			·103	
$-\frac{1}{2}$	·965	·809	·564	·962	·806	·570	·957	·809	·558	·928	·809	·340	·911	·834	·187
		·440			·442			·433			·255			·142	
$-1\frac{1}{2}$	·966	·559	·398	·972	·566	·398	·972	·574	·397	·911	·588	·229	·839	·545	·130
		·215			·219			·221			·123			·0595	
-2	·606	·131	·290	·632	·139	·302	·738	·174	·356	·474	·208	·139	·179	·208	·031
		·023			·0265			·046			·014			·001	
Σ	1·157			1·159			0·9575			0·643			0·3825		

$$\Sigma\, \eta \cos\alpha \sin\alpha \frac{fd\alpha}{fV} = 6{\cdot}7475$$

<u>Series 2</u>

<u>Tests 2 - 6</u>

<u>and 2A - 6A</u>

TEST No :- 2 MACH No. ·6 BAROMETRIC PRESS = 14·5 LB/IN²

P_1 = 24·3 LB/IN² Ab. P_2 = 19·3 LB/IN² Ab. TEMP. 380°F.

STATIONS	-3 3/4		-3 1/2		-3 1/4		-3		-2		-1		0		1		2		3		3 1/4	
	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α
2 3/4							19·9	70	20·4	69	21·2	65	21·4	67	20·8	78	19·8	85				
2 1/2			20	55	21·3	55	22·1	55	23·7	52·5	23·9	46	24	46	24	48	23·5	51	20·7	57	19·5	56
2	19·7	45	20·5	45	22·5	45	23·6	45	24·3	43	24·3	41	24·3	41	24·3	42	23·9	43	21·2	48	19·6	48
1	19·9	21	21·5	28	23·7	32	24·3	33	24·3	32	24·3	30	24·3	31	24·3	33·5	24·3	33	24·2	32·5	21·3	24·5
0	20	18·5	21·5	25	23	25	24·3	26	24·3	28·5	24·3	29·5	24·3	30·5	24·3	30	24·3	24	21·5	30	20·1	17
-1	20·3	22	21·5	22	23·3	23·5	24·2	24·5	24·3	25·5	24·3	25·5	24·3	26·5	24·3	25·5	24·2	24·5	21·5	25·5	20·5	25·5
-2	20	18	21	18	22·6	18·5	23·7	19	24·3	19	24·3	20	24·3	20	24·3	21	24	19	21·3	19	20·3	18
-3			20·5	10	21·4	10	23	12	24·3	11	24·3	11	24·3	12	24·3	12	24	13	20·3	7	19·8	7
-4			19·5	-5	20·1	-5	21	0	24·3	7	24·3	6	24·3	7	24·3	9	24·3	8	20·5	8	19·7	8
-4 1/4					19·7	8·5	20·6	-3	24·3	4·5	24·3	4	24·3	6	24·3	6	23·7	5·5	19·7			

Test No:-2A MACH. No. ·6 p_1=24·3LB/IN² Ab. BAROMETRIC PRESS = 14·3LB/IN² p_2=19·3LB/IN² Ab. TEMP. °F

STATIONS	$-1\frac{3}{4}$		$-1\frac{1}{2}$		$-1\frac{1}{4}$		-1		$-\frac{1}{2}$		0		$\frac{1}{2}$		1		2		3		$3\frac{1}{2}$		4		$4\frac{1}{4}$		$4\frac{1}{2}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
$1\frac{1}{2}$	–	–	19·5	–	19·5	–	20·1	5	20·6	3	20·3	1	20·1	-2	19·7	-2	19·3	–	19·3	–	19·3	–	19·4	15	19·8	17	19·8	12
1		–	19·3	–	20·2	7	21·5	10	20·9	2	21	5	20·8	3	20·4	2	19·7	-1	19·3	–	19·4	8	19·5	16	20·6	21	20·7	19
$\frac{1}{2}$	19·3	–	19·6	7	20·9	9·5	22·3	10	21·9	7	22·2	9	22·3	10	22·3	10	21·7	8	20·8	9	20·4	10	20·7	17	21·7	20	22·2	21
0	[illegible]	11	20·8	9·5	23·2	16·5	23·4	15	23·3	13	23·3	16	23·6	21	23·9	23	23·8	24	23·4	20	23·1	19	22·9	18	22·8	19	22·8	19·5
-1	21·8	23	23	18	23·6	31	23·8	31	23·9	32	23·9	31	23·7	30	23·9	34	23·8	34	23·6	33	23·5	34	23·5	33	23·3	31	23	31
-2	21·5	48	23·1	45	23·5	45	23·7	46	23·9	46	23·9	47	23·6	46	23·9	46	23·7	46	23·6	46	23·6	46	23·4	46	23	47	22·4	48
$-2\frac{1}{2}$	19·62	73	21·42	70	22·42	66	21·72	66	23·42	64	23·42	62	23·42	62	23·42	52	23·62	60	23·82	59	23·62	59	22·92	62	21·72	69	19·62	70
$-2\frac{3}{4}$	19·42	8	19·82	80	20·42	79	20·42	79	21·42	78	22·12	76	22·22	77	22·92	74·5	23·92	70·5	23·52	66·5	23·42	68	21·42	75	19·52	75	–	–

TEST No :- 2

MACH No. ·6

BAROMETRIC PRESS = 14·5 LB/IN²

P_1 = 24·3 LB/IN² Ab.

P_2 = 19·3 LB/IN² Ab.

TEMP. -380°F.

STATIONS	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼	
	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]	$\frac{P}{P_3}$	[illegible]
2¾							·969	·162	·946	·25	·91	·42	·901	·465	·927	·34	·974	·12	~~·897~~			
2½		·965	·965	·162	·905	·625	·873	·6	·814	·901	·807	·937	·804	·955	·804	·955	·821	·865	931 ~~·81~~	·322	·99	·06
2	·98	·09	·942	·27	·857	·682	·817	·885	·794	1	·794	1	·794	1	·794	1	·807	·937	91 ~~·797~~	·42	·985	·067
1	·97	·14	·897	·482	·814	·901	·794	1	·794	1	·794	1	·794	1	·794	1	·794	1	797 ~~·897~~	·992	·905	·445
0	·965	·162	·897	·482	·839	·775	·794	1	·794	1	·794	1	·794	1	·794	1	·794	1	·897 ~~·897~~	·482	·959	·19
-1	·951	·225	·897	·482	·828	·83	·797	·992	·794	1	·794	1	·794	1	·794	1	·798	·992	·897 ·905	·482	·941	·272
-2	·965	·162	·919	·377	·854	·695	·814	·901	·794	1	·794	1	·794	1	·794	1	·804	·955	905 ~~·96~~	·445	·949	·17
-3			·942	·27	·902	·46	·839	·775	·794	1	·794	1	·794	1	·794	1	·804	·955	95 ~~·944~~	·23	·974	·12
-4			·989	·097	·96	·172	·918	·38	·794	1	·794	1	·794	1	·794	1	·794	1	941 0	·272	·98	·09
-4¼					·98	·09	·937	·292	·794	1	·794	1	·794	1	·794	1	·814	·901	·96	·09		

Test No:- 2a. MACH. No. 6 BAROMETRIC PRESS = 14·3

P_1=24·3 LB/IN² Ab. P_2=19·3 LB/IN² Ab. TEMP. °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]	P_2/P_3	[illegible]
1½	s	s	s	s	s	s	·96	181	·935	·30	·951	·225	·96	·18	·98	·09	s	s	s	s	s	s	·995	·017	·975	·115	·974	·117
1	s	s	s	s	·955	·21	·897	·488	·922	·364	·92	·37	·927	·34	·947	·245	·98	·09	s	s	·985	·017	·995	·017	·927	·337	·932	·315
½	s.	s.	·985	·067	·924	353	·866	·635	·88	·565	·869	·62	·865	·64	·865	·64	·89	·589	·929	·33	·966	·25	·932	·345	·89	·52	·87	·617
0	·97	·64	·928	·334	·833	·807	·825	·845	·829	·825	·828	·83	·818	·88	·806	·962	·81	·922	·825	·845	·836	·79	·844	·75	·848	·727	·868	·727
-1	·886	·536	·84	·77	·816	·891	·811	·915	·808	·93	·807	·934	·815	·895	·806	·962	·817	·887	·817	·887	·821	·867	·821	·867	·828	·83	·84	·77
-2	·898	·478	·835	·995	·824	·865	·815	·895	·808	·93	·807	·936	·818	·88	·806	·962	·815	·895	·817	·887	·818	·88	·825	·846	·838	·78	·863	·65
-2½	·984	·073	·90	·466	·86	·666	·889	·525	·823	·855	·824	·85	·824	·85	·824	·85	·816	·891	·81	·922	·817	·887	·841	·765	·91	·42	·984	·072
-2¾	·994	·025	·973	·125	·92	·371	·943	·262	·90	·466	·874	·596	·868	·625	·862	·76	·807	·936	·82	·87	·825	·845	·904	·46	·987	·066	S	S

TEST No :- 3

MACH No. ·7

P_1 = 24·3 LB/IN^2 Ab.

BAROMETRIC PRESS = 14·38 LB/IN^2

P_2 = 17·88 LB/IN^2 Ab.

TEMP. 300°F.

STATIONS	−3 ¾		−3 ½		−3 ¼		−3		−2		−1		0		1		2		3		3 ¼	
	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α
2 ¾			18·18	57	19·28	57	19·68	63	20·98	64·5	22·38	60	21·98	60·5	21·38	62	19·58	62	18·78	62	17·78	
2 ½			18·88	59	20·28	59	21·38	59	22·78	55·5	23·98	51·5	23·78	50·5	23·78	51·5	22·68	56	19·68	55	17·88	4·0
2	18·18	45	18·98	45	21·88	45	23·18	45	24·3	42	24·3	40	24·3	40	24·3	41	23·88	42	21·28	44	18·58	45
1	18·58	25	20·58	29	22·78	29	23·78	31	24·3	31·5	24·3	32	24·3	33·5	24·3	33·5	24·3	33·5	21·98	33	19·68	33
0	18·88	29	20·88	29	22·78	28	23·58	30	24·3	29	24·3	32	24·3	28	24·3	29	24·3	27·5	21·78	29	20·18	29
−1	19·38	24·5	20·88	25	22·58	27	23·48	25	24·3	26	24·3	27	24·3	27	24·3	27	23·78	24	21·68	24·5	19·68	23
−2	18·68	13	20·18	18	23·08	18·5	24·29	17·5	24·3	18·5	24·3	18·5	24·3	18	24·3	19	23·88	18	20·78	18	19·08	15
−3	18·18	5	18·7	7	20·2	9	22	11	24·3	10·5	24·3	12·5	24·3	11	24·3	11	24·3	10·5	19·3	5	18	5
−4			18	−2	18·4	−2	19·5	0	24·3	4	24·3	2	24·3	2	24·3	3	23·9	2	19·1	1	17·9	1
−4 ¼					18·3	−1	19·3	−1	24·3	6	24·3	2·5	24·3	4	24·3	3 ½	23·4	6	18·1	6		

TEST No:-3A- MACH. No. ·7 BAROMETRIC PRESS = 14·4 LB/IN²

P_1 = 24·3 LB/IN² Ab. P_2 = 17·88 LB/IN² Ab. TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½	
	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α
1½	17·7	-	17·8	-	18	-	18·6	4	19·4	1	19	-5	18·7	-4
1	18·3	3	19·4	3	19·4	8	20·4	10	19·6	4	19·8	0	19·4	1
½	18·4	4	19·5	9	20·5	15	21·9	11	21·1	8	21·8	9	21·9	8·5
0	19·2	7	19·7	11	22·5	18	23·1	16	22·9	14	23·2	18	23·4	21
-1	20·8	21	22·6	29	23·2	30	23·5	33	23·6	32	23·6	32	23·7	34
-2	20·5	50	22·2	48	23	46	23·2	48	23·5	47	23·8	48	23·7	47
-2½	18·5	75	19·7	74	21·4	69	20·6	70	23	66	23·2	67	23·2	66
-2¾	-	-	18·3	78	19·4	79	18·9	82	20·2	80	21·2	79	21·3	79

STATIONS	1		2		3		3½		4		4¼		4½	
	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α
1½	18·4	-5	17·9	-5	17·8	-5	17·6	4	18·1	[illegible]	18·8	11	18·6	10
1	19·3	1	18·4	0	17·9	2	17·9	2	18·5	2	19·2	-2	19·4	2
½	21·6	8·5	20·9	9	19·5	7	19·5	10	20·4	17	21·4	20	21·5	19
0	23·4	19	23·4	18	22·9	19	22·5	19	22·6	19	22·6	21	22·4	19
-1	23·7	34	23·4	33	23·3	33	23·2	33	23	33	22·8	32	22·4	31
-2	23·4	47	23·4	46	23·4	46	23·3	45	23·1	45	22·6	48	21·5	46
-2½	23·3	63	23·2	62	23·3	59	23·3	60	22·1	63	19·6	71	18·3	70
-2¾	22	73	22·4	75	22·8	72	22·6	70	19·6	78	18·2	-	17·8	-

TEST No :- 3

MACH No. ·7

P_1 = 24·3 LB/IN²Ab.

BAROMETRIC PRESS =

P_2 = 17·88 LB/IN²Ab.

TEMP. 380°F.

STATIONS	-3 3/4		-3 1/2		-3 1/4		-3		-2		-1		0		·1		2		3		3 1/4	
	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]	p/p_3	[illegible]
2 3/4			·984	·05	·926	·255	·909	·315	·853	·525	·799	·74	·814	·678	·837	·586	·918	·282	·958 ·988	·157	·55 ·963	
2 1/2			·947	·547	·88	·42	·837	·557	·785	·792	·745	·957	·753	·925	·753	·925	·787	·786	·91	·311	X	
2	·984	·05	·942	·2	·816	·675	·792	·845	·735	1	·735	1	·735	1	·735	1	·748	·945	·839	·58	·962	·125
1	·963	·122	·87	·457	·784	·797	·752	·928	·735	1	·735	1	·735	1	·735	1	·735	1	·814	·678	·908	·318
0	·947	·547	·854	·52	·784	·797	·758	·902	·735	1	·735	1	·735	1	·735	1	·735	1	·821	·65	·886	·4
-1	·923	·265	·854	·52	·792	·766	·762	·89	·735	1	·735	1	·735	1	·735	1	·735	1	·823	·642	·908	·318
-2	·957	·142	·885	·405	·809	·697	·735	1	·735	1	·735	1	·735	1	·735	1	·735	1	·821	·65	·938	·212
-3	·984	·05	·955	·15	·885	·405	·812	·685	·735	1	·735	1	·735	1	·735	1	·735	1	·926	·255	·992	·025
-4			·992	·025	·973	·086	·917	·288	·735	1	·735	1	·735	1	·735	1	·747	·95	·988	·035	·998	·005
-4 1/4					·977	·072	·925	·26	·735	1	·735	1	·735	1	·735	[illegible]	·754	·92	[illegible]	[illegible]	–	

<u>TEST No:-3A.</u> <u>MACH. No. 0·7</u> <u>BAROMETRIC PRESS = 14·4</u>

P_1=24·3LB/IN² Ab. P_2=17·8LB/IN² Ab. TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ	$\frac{P_2}{P_3}$	ζ
1½	S	S	S	S	·994	·017	961	·127	921	·272	·94	·205	95	..	·11	[illegible]	998	0	S	S	S	S	·985	0	95	·168	96	·182
1	·977	·071	971	·092	921	·272	875	·437	91	·31	·904	·352	922	267	925	·26	97	·097	·996	·015	996	·015	·965	·125	93	·24	921	·272
½	·977	·092	966	·11	871	·455	816	·671	848	562	82	·655	816	·67	827	·626	854	·52	915	295	916	·29	876	·435	835	595	83	·615
0	931.	·237	906	324	794	·757	774	·837	781	·81	·77	·852	764	877	764	877	764	877	·78	812	794	757	79	775	79	775	778 787	·742
-1	859	·50	·79	·774	·77	·852	·76	894	756	·911	756	·911	755	915	755	·915	764	877	766	87	77	·851	775	835	784	795	798	762
-2	87	·457	804	·717	777	·825	77	·852	·76	·895	751	·931	·755	915	764	877	764	877	764	877	766	87	773	842	79	775	831	·612
-2½	965	·115	906	·324	836	59	866	·472	·779	·817	772	·845	772	·845	766	87	772	845	766	87	766	87	808	·70	91	·31	976	·075
-2¾	S	S	977	·071	921	272	946	·182	884	·407	843	562	84	575	811	·69	797	747	784	·795	79	·775	902	34	·981	·06	S	S.

TEST No :- 4

MACH No. ·8

BAROMETRIC PRESS = 14·45 LB/IN²

p_1 = 24·3 LB/IN² Ab.

p_2 = 16·35 LB/IN² Ab.

TEMP. 380°F.

STATIONS	$-3\frac{3}{4}$		$-3\frac{1}{2}$		$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
$2\frac{3}{4}$	SS		16·78		17·48	66	18·38	66	19·88	65	21·08	62	20·98	60	19·98	61	18·58	65	17·18	65	SS	
$2\frac{1}{2}$	SS		16·88		19·08	59	20·48	59	22·58	56·5	23·98	52·5	23·98	51	23·88	52·5	22·38	59	18·88	58·5	SS	[illegible]
2	SS		16·98	46	19·08	46	23·18	46	24·18	42	24·3	41·5	24·3	41·5	24·3	42·5	23·98	44	20·78	47·5	16·38	
1	SS		19·45	29	22·05	30	23·65	32	24·3	33·5	24·3	33	24·3	35	24·3	34	24·05	35	21·75	33	17·55	33·5
0	16·45	26·5	19·75	26·5	22·65	27	23·55	28·5	24·3	30	24·3	29	24·3	27·5	24·3	30	23·95	32	21·45	30·5	19·25	27
−1	SS		20·15	25	22·25	27	23·25	27	24·3	27	24·3	28	24·3	29	24·3	27	24·15	26	21·45	26	19·45	26
−2	17·18	19	19·38	19	21·78	23	23·28	23	24·3	21	24·3	19·5	24·3	20	24·3	20	24·08	19·5	20·58	20	18·28	22
−3	SS		17·28	12	19·28	10	21·78	12	24·3	8	24·3	10	24·3	10	24·3	10	24·3	11	19·38	11	SS	
−4	SS		16·48		17·18	3	18·38	−5	24·3	1·5	24·3	−1	23·98	−1	24·08	1	23·58	5	16·58	5		
$-4\frac{1}{4}$			16·38		16·68		18·38	−8	23·98	6	23·98	6·5	24·3	−2	24·3	1·5	23·08	4	SS			

TEST No:-4a

MACH. No. ·8

BAROMETRIC PRESS = 14·38 LB/IN²

p_1=24·3 LB/IN² Ab.

p_2=16·35 LB/IN² Ab.

TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
1½	-	-	-	-	16·98	1	18·38	3	18·38	0	17·68	-1	17·38	0	17·08	0	-	-	-	-	-	-	17·28	17	17·78	14	17·38	12
1	-	-	-	-	18·48	9	20·28	10	19·08	4	18·88	6	18·88	5	18·08	4	17·28	4	17·08	2	16·78	3	18·08	21	19·28	20	18·98	19
½	16·88	11	17·58	9	19·78	13	21·38	8	20·28	4	21·28	9	21·38	11	21·28	10	20·98	11	19·08	8	18·68	10	19·38	17	20·48	21	20·78	18
0.	17·88	10	19·28	13	22·28	18	22·58	16	22·38	13	22·98	17	23·28	20	23·38	22	23·83	23	22·48	21	22·08	19	22·08	19	22·08	21	21·68	22
-1	19·68	18	22·08	29	22·98	28	23·18	30	23·38	30	23·38	31	23·38	33	23·18	33	22·98	32	22·78	34	22·78	33	22·48	34	22·18	32	21·58	31
-2	19·08	55	21·78	51	22·78	49	22·68	51	23·28	50	23·38	49	23·18	49	23·28	49	23·28	49	23·18	48	23·08	48	22·58	48	21·68	50	19·88	66
-2½	17·48	80	19·58	72	21·18	65	20·18	68	22·68	67	22·28	68	22·38	68	22·78	65	22·88	62	23·08	61	22·98	60	21·38	64	18·18	63	-	-
-2¾	16·78	82	17·38	82	18·48	77	17·88	78	19·78	74	21·08	79	21·08	78	22·38	75	22·98	74	23·04	71	22·68	70	19·28	78	17·78	80	-	-

Test No :- 4

Mach No. 0·8

P_1 = 24·3 LB/IN^2 Ab.

Barometric Press =

P_2 = 16·35 LB/IN^2 Ab.

Temp. - °F.

STATIONS	−3¾		−3½		−3¼		−3		−2		−1		0		1		2		3		3¼	
	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]	$\frac{P_2}{P_3}$	[illegible]
2¾	S	S	·976	·045	·935	·16	·892	·295	·824	·50	·776	·652	·781	·635	·82	·511	·881	·327	·954	·101	S	S
2½	S	S	·97	·06	·858	·398	·797	·585	·726	·816	·684	·986	·684	·986	·685	·96	·734	·79	·866	·374	S	S
2	S	S	·965	·072	·858	·398	·707	·882	·678	·983	·673	1·0	·673	1·0	·673	1·0	·684	·962	·788	·615	S	S
1	S	S	·84	·45	·742	·765	·691	·94	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·68	·976	·752	·731	·931	·175
0	S	S	·827	·49	·721	·832	·695	·925	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·684	·962	·761	·70	·849	·425
−1	S	S	·812	·536	·735	·786	·704	·894	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·678	·952 (·972)	·741	·70	·84	·451
−2	·953	·105	·844	·44	·753	·727	·701	·905	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·68	·976	·795	·572	·895	·286
−3	S	S	·946	·126	·849	·425	·752	·731	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·673	1·0	·865	·437	S	S
−4	S	S	·922	·012	·952	·109	·89	−·30	·673	1·0	·673	1·0	·684	·986	·68	·976	·695	·925	·986	·026	S	S
−4¼	S	S	·999	0	·981	·035	·89	·30	·684	·962	·684	·962	·673	1·0	·673	1·0	·709	·875	S	S	S	S

Test No:- 4a   Mach. No. ·8   Barometric Press = 14·38

P_1=24·3 LB/IN² Ab. P_2=16·35 LB/IN² Ab.   Temp. °F

STATIONS	−1¾		−1½		−1¼		−1		−½		0		½		1		2		3		3½		4		4¼		4½	
	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]	$\frac{p}{P_3}$	[illegible]
1½	-	-	-	-	·964	·075	·89	·30	·89	·30	·925	·192	·742	·765	·958	·091	-	-	-	-	-	-	·987	·122	·92	·207	·94	·146
1	-	-	-	-	·886	·312	·807	·552	·858	·397	·867	·371	·867	·371	·905	·256	·947	·125	·958	·091	·975	·048	·905	·366	·845	·436	·871	·357
½	·97	·06	·93	·175	·827	·49	·766	·685	·807	·552	·768	·677	·765	·681	·768	·677	·78	·64	·858	·397	·876	·342	·844	·44	·787	·585	·788	·614
0	·915	·229	·849	·425	·733	·792	·725	·82	·731	·80	·713	·86	·702	·901	·7	·907	·703	·897	·727	·812	·74	·77	·74	·77	·74	·77	·753	·727
−1	·832	·476	·74	·772	·712	·865	·707	·882	·7	·907	·7	·907	·7	·907	·706	·886	·712	·865	·719	·84	·719	·84	·727	·811	·737	·777	·756	·71
−2	·858	·397	·752	·732	·718	·844	·72	·836	·702	·901	·7	·907	·704	·884	·708	·886	·702	·901	·706	·886	·707	·882	·725	·82	·753	·727	·823	·505
−2½	·937	·155	·836	·465	·773	·662	·811	·54	·721	·832	·734	·79	·731	·80	·732	·84	·714	·857	·707	·882	·713	·86	·765	·687	·9	·27	-	-
−2¾	·976	·049	·942	·14	·889	·318	·915	·225	·821	·49	·775	·656	·775	·656	·732	·795	·727	·812	·707	·882	·718	·845	·918	·43	·919	·211	-	-

TEST No :- 5

MACH No. ·9

BAROMETRIC PRESS = 14·38 LB/IN²

p_1 = 30 LB/IN² Ab.

p_2 = 18·38 LB/IN² Ab.

TEMP. -380°F.

STATIONS	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
2¾	SS		18·88		21·18	60	22·18	66	24·28	67	27·18	62·5	27·18	62	24·28	67·5	21·78	67	19·78	60	SS	
2½	18·78		19·38	61	23·08	61	25·58	66	26·98	64·5	28·88	56	29·48	61·5	27·68	61·5	25·48	64	21·68	62	18·88	[illegible]
2	18·88		20·78	48	26·58	47	29·08	47	30	44	30	42	30	42	30	43·5	29·88	47	25·88	48	20·48	48
1	21·08		23·68	31	27·58	35·5	29·28	33	29·88	34·5	30	34·5	30	34	29·48	35·5	29·38	35·5	25·98	33·5	21·88	32·5
0	20·68		24·48	27	27·58	27	29·28	28·5	30	30	30	31·5	30	31·5	30	31	30	29	26·68	30	23·38	27·5
-1	20·78	23	24·68	25	27·38	25	28·88	26	30	27·5	30	28	29·88	29	30	29	29·28	27	26·18	26	22·68	24
-2	20·88	19	23·88	19	27·48	20	29·28	19	30	19	30	19·5	30	19·5	30	21	29·88	20	25·38	21	21·98	21
-3	20·28	9	20·88	6	24·28	10	27·58	11	30	10	30	9·5	30	9	30	9	29·88	10	21·48	8	19·28	8
-4			19·28		19·38	-5	21·18	-1	29·88	-1·5	29·58	-2	29·38	-6·5	29·78	-3	29·28	4	19·18	-2	SS	
-4¼							19·18		25·78	3	25·78	-3	24·98	-5½	25·98	-2	25·68	3	19·48			

TEST No:- 5a   MACH. No. ·9 p_1=30 LB/IN² Ab.   BAROMETRIC PRESS = 14·38 LB/IN² p_2= 18·38 LB/IN² Ab.   TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		+½		1		2		3		3½		4		4¼		4½	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
1½	18·45	0	19·05	0	20·55	6	23·35	[illegible]	21·95	0	21·15	-1	20·55	-1	20·35	-1	18·75	-2	18·55	-2	18·55	·1	20·75	15	21·65	13·5	20·65	11
1	18·85	0	19·65	4	24·25	14	26·35	9	23·85	1	24·05	15	24·15	6	23·45	5	21·05	2	19·45	2	19·65	7	22·45	14	24·65	18	23·65	15
½	19·25	-1	20·75	9	26·05	14	27·65	10	26·15	6	27·25	11	27·55	12	27·15	12	26·65	13	22·65	9	[illegible]	11	25·15	15	26·25	19	24·35	18
0.	20·65	0	23·35	15	28·05	18	28·75	16	28·45	14	26·75	18	28·85	21	28·75	22	28·75	23	27·55	19	27·15	18	27·65	20	27·85	20	27·25	21
-1	29·15	25	27·15	29	28·55	32	28·85	32	29·15	33	29·15	33	28·95	34	28·85	33	28·75	34	28·55	35	28·25	35	27·65	33	27·25	33	26·25	33
-2	21·35	55	26·55	51	27·45	50	27·65	51	28·85	50	28·95	50	28·95	49	28·95	49	28·95	49	28·65	48	28·25	47	27·55	47	26·05	48	23·45	60
-2¼	19·25	74	20·85	76	24·15	72	22·35	72	24·45	74	27·05	73	27·05	71	27·85	69	28·25	65	28·35	66	28·15	63	25·05	66	21·05	70	18·65	73
-2¾	18·45	81	19·35	80	21·25	78	20·65	78·5	22·95	82	24·15	82	24·65	81	26·45	79	26·45	77	27·45	75	27·05	72	28·25	78	[illegible]	80	—	—

TEST No :- 5 MACH No. 0·9 BAROMETRIC PRESS = 14·38 LB/IN²

p_1 = 30 LB/IN² Ab. p_2 = 18·38 LB/IN² Ab. TEMP. - °F.

STATIONS	−3¾		−3½		−3¼		−3		−2		−1		0		1		2		3		3¼	
	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]
2¾	S	S	·772	·06	·868	·306	·83	·40	·755	·592	·676	·802	·676	·802	·755	·592	·845	·362	·927	·162	S	S
2½	·977	·05	·948	·112	·795	·488	·72	·685	·681	·787	·635	·93	·623	·675	·664	·84	·72	·684	·847	·357	·973	·057
2	·974	·055	·885	·265	·692	·756	·632	·94	·612	1·0	·612	1·0	·612	1·0	·612	1·0	·614	·997	·709	·712	·896	·237
1	·872	·296	·775	·541	·667	·83	·631	·944	·615	·995	·612	1·0	·612	1·0	·622	·97	·626	·96	·708	·714	·838	·38
0	·886	·262	·75	·605	·667	·83	·631	·944	·612	1·0	·612	1·0	·612	1·0	·612	1·0	·612	1·0	·688	·766	·786	·511
−1	·885	·265	·745	·617	·672	·815	·636	·926	·612	1·0	·612	1·0	·615	·995	·612	1·0	·627	·955	·704	·725	·809	·452
−2	·88	·277	·77	·555	·668	·827	·631	·944	·612	1·0	·612	1·0	·612	1·0	·612	1·0	·615	·995	·724	·674	·837	·381
−3	·905	·216	·88	·277	·756	·59	·666	·833	·612	1·0	·612	1·0	·612	1·0	·612	1·0	·615	·995	·855	·337	·952	·105
−4	S	S	·953	·101	·946	·116	·868	·306	·615	·995	·621	·975	·625	·962	·618	·985	·626	·96	·957	·092	S	S
−4¼	S	S	S	S	S	S	·957	·093	·714	·70	·714	·70	·736	·644	·708	·714	·715	·696	·957	·092	S	S

TEST No :- 5a MACH. No. 9

P_1=30 LB/IN² Ab

BAROMETRIC PRESS = 14·38 LB/IN²

P_2=18·38 LB/IN² Ab. TEMP. 380 °F

STATIONS	−1¾		−1½		−1¼		−1		−½		0		½		1		2		3		3½		4		4¼		4½	
	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η	$\frac{p}{p_3}$	η
1½	·995	·008	·964	·075	·894	·241	·787	·508	·837	·38	·868	·305	·881	275 ~~[illegible]~~	·903	·22	·98	·207	·991	·016	·991	·016	·885	·265	·848	·3[illegible]	[illegible]5	·361
1	·975	·05	·935	·141	·757	·590	·697	·742	·77	·552	·763	·572	·761	·576	·783	·518	·872	·296	·945	·116	·935	·141	·817	·431	·744	·62	·[illegible]	·52
½	·954	·10	·885	·265	·705	·762	·664	·838	·702	·73	·673	·812	·667	·83	·677	·798	·686	·766	·804	·466	·804	·466	·73	·658	·699	·736	·697	·741
0	·889	·255	·786	·51	·654	·87	·639	·92	·645	·898	·639	·92	·636	·926	·639	·92	·639	·92	·667	·83	·677	·798	·664	·839	·659	·854	·673	·812
−1	·761	·576	·676	·80	·644	·902	·636	·926	·63	·946	·63	·946	·635	·93	·636	·926	·639	·92	·636	·926	·65	·882	·659	·854	·673	·812	·699	·737
−2	·84	·372	·692	·756	·669	·825	·664	·838	·636	·926	·635	·93	·635	·93	·635	·93	·635	·93	·636	·926	·65	·882	·667	·83	·705	·721	·783	·519
−2½	·954	·10	·88	·275	·761	·576	·922	·47	·702	·73	·678	·796	·679	·795	·667	·83	·65	·882	·648	·89	·653	·872	·732	·652	·873	·293	·989	·02
−2¾	·995	·008	·949	·11	·865	·313	·889	·255	·8	·475	·761	·576	·745	·617	·681	·788	·694	·75	·677	·798	·679	·795	·824	·414	·974	·052	S	S

TEST No:- 6

MACH No. ·97

P_1 = 32·5 LB/IN² Ab.

BAROMETRIC PRESS = 14·3

P_2 = 18·3 LB/IN² Ab.

TEMP. - °F.

STATIONS	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼	
	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α	P_3	α
2¾	-	-	18·7	-	22·3	62	23·5	65	25·4	64·5	28·7	60·5	29	60	25·3	67	22·5	67	19·6	57	-	-
2½	-	-	19·4	60	26	53	29·1	55·5	31·5	53	32·5	47	32·5	48·5	32·1	51	[illegible]	57	22·5	60	-	[illegible]
2	18·8	-	22·9	40·5	29·2	39	31·5	40	32·5	40	32·5	39	32·5	39	32·5	40	32·1	41	27·5	42	20·5	43
1	19·8	31	25	32	30	31	31·6	33	32·5	35	32·5	35	32·5	35	32·5	35	32·3	35	28·2	33·5	22·5	34
0	20·8	28	26·7	28	20·6	28	32	27	32·5	30·5	32·5	31	32·5	32	32·5	31	32·3	30	26·4	27·5	24·3	28·5
-1	22·8	28	26·2	26	29·2	27	30·7	28	32·3	27	32·5	27	32·8	28·5	32·3	27·5	31·5	27·5	26·8	27	22·7	27
-2	19·8	15	23·6	18	27·9	19	30·4	19·5	32·3	19	32·5	19	32·5	20	32·4	21	31·4	22	24·7	18	21	18
-3	19·35		20·7	4	25	8·5	29·6	10	32·5	8	32·5	10	32·5	8·5	32·5	10	32·2	8·5	32·5	9	19·6	7
-4			18·8	-	19·7	-9·5	22·4	-2	32·3	1	31·2	-8	29·7	-15·5	31·7	-5·5	31·6	-[illegible]	18·9	-4	-	-
-4¼			5	5	19·3	-13	21·6	-6	32·2	1	31·4	-7	28·5	-15	31·7	-6·5	31·4	7·5	18·9	-	-	-

<u>TEST No:-6a.</u> <u>MACH. No. ·97</u> <u>BAROMETRIC PRESS = 14·25 LB/IN²</u>

p_1=32·5 LB/IN² Ab. p_2=18·3 LB/IN² Ab. TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		+½		1		2		3		3½		4		4¼		4½	
	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
1½	18·25	-2	19·25	5	22·75	7	25·95	6	23·15	-1	21·65	-1	21·95	-1	20·95	-3	19·25	-5	19·05	-6	19·25	5	22·85	13	23·55	9	21·95	7
1	19·25	-1	20·45	8	25·95	14	29·05	10	25·75	2	25·55	4	26·25	7	24·85	5	21·75	0	19·85	-1	21·25	9	25·35	17	27·55	18	25·55	12
½	19·45	-3	21·35	10	27·75	16	28·65	13	27·55	6	29·15	11	29·45	14	28·85	12	30·45	13	24·25	11	28·65	10	27·45	17	28·25	17	27·75	15
0.	20·95	7	24·25	15	29·65	20	30·55	17	30·45	15	31·05	18	31·05	20	31·05	22	30·95	21	27·35	18	29·45	18	30·15	21	29·95	22·5	29·05	22
-1	24·65	24	28·75	29	30·25	32	30·55	33	31·05	33	31·35	33	31·05	33	30·75	34	30·75	35	30·45	34	30·25	34	29·75	33	29·05	33	27·65	33
-2	21·75	58	27·25	52	29·25	49	28·75	51	31·05	50	31·25	50	31·05	50	30·45	50	30·95	50	30·55	49	30·45	49	29·35	50	27·15	53	23·45	65
-2½	18·95	80	20·35	78	22·95	74	21·55	74	24·65	80	26·15	79	26·65	78	29·05	76	28·35	74	28·75	74	29·15	70	28·15	75	19·25	77	–	–
-2¾	–	–	19·45	93	19·35	91	19·05	89	19·75	88	20·25	87	21·05	87	22·35	87	23·15	88	23·45	88	24·25	83	19·65	84	–	–	–	–

TEST No :- 6 MACH No. ·97 BAROMETRIC PRESS = 14·3

$p_1 = 32·5$ LB/IN² Ab. $p_2 = 18·3$ LB/IN² Ab. TEMP. - °F.

STATIONS	-3¾		-3½		-3¼		-3		2		-1		0		1		2		3		3¼	
	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]	p_2/p_3	[illegible]
2¾	S	S	·978	·04	·82	·361	·778	·453	·72	·59	·637	·792	·631	·807	·724	·58	·814	·375	·933	·125	S	S
2½	S	S	·942	·107	·704	·628	·629	·812	·581	·947	·564	1	·564	1	·57	·98	·606	·876	·813	·377	S	S
2	·974	·046	·80	·405	·626	·73	·581	·947	·564	1	·564	1	·564	1	·564	~~·971~~ 1	·57	·98	·665	·722	·892	·207
1	·923	·142	·731	·564	·61	·866	·579	·955	·564	1	·564	1	·564	1	·564	1	·566	·991	·641	·762	·814	·375
0	·88	·235	·685	·674	·598	·897	·572	·975	·564	1	·564	1	·564	1	·564	1	·566	·991	·645	·77	·754	·57
-1	·839	·32	·677	·642	·626	·82	·596	·905	·566	·991	·564	1	·564	1	·566	·991	·581	·948	·682	·681	·806	·39
-2	·933	·125	·775	·46	·656	·744	·602	·887	·566	·991	·564	1	·564	1	·565	·995	·582	·945	·741	·54	·872	·25
-3	·945	·101	·884	·225	·733	·557	·617	·845	·564	1	·564	1	·564	1	·564	1	·563	·986	·814	·375	·934	·125
-4	S	S	·974	·046	·93	·131	·816	·37	·566	·991	·586	·935	·616	·848	·577	·96	·58	·951	·967	·06	S	S
-4¼	S	S	S	S	·948	·095	·849	·30	·568	·985	·584	·94	·642	·798	·577	·96	·582	·945	·967	·06	S	S

TEST No:-6a MACH. No ·97 BAROMETRIC PRESS = 14·25 LB/IN²

p_1=32·5 LB/IN² Ab. p_2=18·3 LB/IN² Ab. TEMP. 380 °F

STATIONS	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ	p/p_3	η/θ
1½			·95	·09	·805	·392	·705	·625	·79	·425	·844	·311	·834	·33	·874	·245	·95	·09	·961	·068	·951	·068	·8	·405	·777	·456	·834	·33
1	·951	·887	·895	·205	·705	·625	·83	·809	·724	·58	·716	·599	·697	·642	·735	·552	·842	·314	·922	·147	·861	·772	·722	·584	·664	·724	·716	·597
½	·941	·107	·858	·28	·66	·735	·838	·787	·664	·725	·628	·815	·621	·832	·634	·799	·642	·777	·754	·507	·736	·55	·666	·72	·647	·765	·66	·735
0	·874	·245	·754	·507	·617	·845	·609	·867	·6	·892	·588	·927	·588	·927	·588	·927	·591	·92	·624	·824	·621	·832	·607	·873	·611	·861	·63	·807
-1	·736	·55	·637	·791	·604	·871	·609	·867	·585	·927	·584	·94	·585	·927	·595	·907	·595	·907	·601	·89	·604	·881	·615	·85	·63	·807	·657	·742
-2	·843	·312	·67	·71	·625	·822	·637	·791	·588	·927	·585	·937	·588	·927	·593	·912	·591	·92	·599	·897	·601	·89	·624	·825	·674	·7	·78	·447
-2½	·917	·157	·9	·191	·897	·197	·85	·296	·735	·552	·7	·636	·69	·661	·652	·755	·645	·77	·637	·791	·628	·813	·768	·5	·951	·87	–	–
-2¾			·992	·015	·946	·098	·96	·07	·925	·142	·904	·184	·869	·255	·819	·362	·891	·211	·78	·448	·754	·509	·922	·147	–	–	–	–

TEST :- 2 TABLE :- 3

STATION	-3¾			-3½			-3¼			-3			-2		
	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$
	fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$	
2¾										·377	·94	156	·5	934	25
										1·047	·0528		1·041	·112	
2½				·403	819	·094	·791	819	094	·775	819	234	·95	793	375
				1·046	·0296		1·03	·0591		1·022	·145		1·006	·281	
2	·3	707	187	·519	707	187	·826	707	187	·94	707	47	1	682	75
	1·05	·0378		1·04	·066		1·018	·1072		1·007	·31		1·002	·511	
1	·374	358	25	·695	470	25	·95	530	25	1	545	625	1	530	1
	1·047	·032		1·028	·0796		1·006	·1253		1·002	·339		1·002	520	
0	·403	317	25	·695	431	25	·88	431	25	1	438	625	1	477	1
	1·046	·0305		1·028	·073		1·013	·0935		1·002	·273		1·002	·477	
-1	·474	375	25	·695	375	25	·911	383	25	·996	415	625	1	431	1
	1·042	·0426		1·028	·0635		1·01	·0864		1·002	·2575		1·00	·431	
-2	·403	309	25	·615	309	25	·834	317	25	·96	326	625	1	326	1
	1·046	·0298		1·034	·0459		1·017	·065		1·006	·1925		1·00	·326	
-3				·519	174	25	·678	174	25	·88	208	625	1	191	1
				1·04	·0217		1·028	·0287		1·013	·113		1·00	·191	
-4				·312	$\bar{0}87$	156	·414	087	156	·616	0	39	1	122	625
				1·052	·00401		1·045	·00537		1·234	0		1·00	·0764	
-4¼					~~[illegible]~~	063	·3	148 ~~052~~	063	·54	·062 ~~077~~	156	1	·079 ~~07~~	25
							1·05	·00266		1·036	·00424		1·00	·01975	
Σ	·174			·385			·573			1·688			2·955		

STATION	-1			0			1			2			3			3¼		
	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$	$\eta^{1/2}$	$\sin\alpha$	$\int d\alpha$
	fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$		fV	$\frac{\eta^{1/2}\sin\alpha\int d\alpha}{fV}$	
2¾	·647 ~~877~~	906	25	·681	931	25	·584	978	25	·346	996	25						
	1·031	·142		1·029	·1523		1·036	·1379		1·048	·0823							
2½	·966	719	375	·978	719	375	·978	743	375	·93	777	375	·567	839	234	·2	819	094
	1·004	·256		1·004	·2595		1·004	·268		1·008	·269		1·037	·1071		1·052	·01462	
2	1	656	75	1	656	75	1	669	75	·968	682	75	·649	743	47	·259	743	187
	1·002	·491		1·002	491		1·002	·501		1·005	·492		1·031	·2199		1·251	·0342	
1	1	5	1	1	515	1	1	552	1	1	545	1	·996	537	625	·666	415	25
	1·002	·5		1·002	515		1·002	552		1·002	·545		1·002	·3335		1·03	·066	
0	1	492	1	1	508	1	1	5	1	1	407	1	·795	5	625	·436	292	25
	1·002	·492		1·002	·508		1·002	·5		1·002	·407		·039 1·028	·2118		1·044	·0304	
-1	1	431	1	1	446	1	1	431	1	·996	415	1	·695	431	625	·521	431	25
	1·00	431		1·00	·446		1·00	·431		1·002	·412		1·028	·1822		1·04	·0539	
-2	1	342	1	1	342	1	1	358	1	·978	326	1	·667	326	625	·412	309	25
	1·00	·342		1·00	·342		1·00	·358		1·004	·319		1·03	·132		1·042	·0304	
-3	1	191	1	1	208	1	1	208	1	·978	225	1	·48	122	625	·347	122	25
	1·00	·191		1·00	·208		1·00	·208		1·004	·219		1·063	·03518		1·048	·0101	
-4	1	105	625	1	122	625	1	156	625	1	139	625	·522	139	39	·3	139	156
	1·00	·0656		1·00	·0764		1·00	·0974		1·00	·087		1·04	·0272		1·05	·0062	
-4¼	1	·07 ~~105~~	25	1	106 ~~105~~	25	1	105 ~~096~~	25	·96	096	25	·3	~~096~~	156			
	1·00	·0175		1·00	·02625		1·00	·02625		1·006	·02265		1·05	·00427				
Σ	2·929			3·022			3·079			2·855			1·263			·245		

Σ 19·158

TEST:- 2a TABLE :- 8

STATION	$-1\frac{3}{4}$			$-1\frac{1}{2}$			$-1\frac{1}{4}$			-1			$-\frac{1}{2}$			0		
	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$
	fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$	
$1\frac{1}{2}$										·125	·067	·141	·495	·0052	·187	·475	·015	·187
										1·046	·00499		1·038	·00514		1·0[illegible]	·00153	
1							·496	·123	·125	·596	·174	·187	·603	·035	·25	·608	·067	·25
							1·048	·0067		1·028	·02205		1·034	·0051		1·036	·0128	
$\frac{1}{2}$				·289	·123	·125	·594	·165	·125	·796	·174	·187	·751	·123	·25	·787	·156	·25
				1·041	·00376		1·035	·0118		1·02	·0254		1·024	·02225		1·02	·03	
0	·375	·191	·187	·578	·165	·187	·897	·259	·187	·92	·259	·281	·908	·225	·375	·911	·226	·375
	1·047	·01278		1·036	·0172		1·011	·0471		1·009	·0664		1·01	·076		1·01	·0935	
-1	·731	·391	·25	·876	·309	·25	·946	·515	·25	·956	·515	·375	·964	·53	·5	·967	·515	·5
	1·005	·0696		1·018	·0666		1·007	·1209		1·005	·1835		1·005	·254		1·005	·248	
-2	·691	·743	·187	·891	·707	·187	·93	·707	·187	·945	·719	·281	·914	·719	·375	·967	·731	·375
	1·0[illegible]	·0935		1·002	·1162		1·008	·1222		1·006	·19		1·005	·2584		1·005	·264	
$-2\frac{1}{2}$	·265	·956	·094	·682	·94	·094	·815	·914	·094	·725	·914	·141	·925	·9	·187	·922	·863	·187
	1·041	·02283		1·029	·0586		1·018	·0689		1·026	·091		1·009	·1548		1·007	·151	
$-2\frac{3}{4}$	·158	·985	·063	·254	·965	·063	·309	·982	·063	·512	·982	·094	·682	·978	·125	·771	·97	·125
	1·064	·0093		1·048	·021		1·034	·0364		1·04	·0455		1·027	·0812		1·022	·0914	
		·20801			·28336			·4140			·62884			·85689			·89223	
Σ																		

STATION	$\frac{1}{2}$			1			2			3			$3\frac{1}{2}$			4			$4\frac{1}{4}$			$4\frac{1}{2}$		
	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$	$\eta^{\frac{1}{2}}$	$\sin\alpha$	$\int d\alpha$
	fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$		fV	$\eta^{\frac{1}{2}}\sin\alpha\frac{\int d\alpha}{fV}$	
$1\frac{1}{2}$	·424	$\overline{\cdot035}$	·187	·3	$\overline{\cdot035}$	·281	0									·13	·259	·141	·339	·292	·094	·3	·208	·141
	1·045	·00266		1·05	·00281											1·04	·0045		1·048	·00855		1·0[illegible]	·0036[illegible]	
1	·583	·052	·25	·495	$\overline{\cdot035}$	·375	·3	$\overline{\cdot018}$	·50				·13	·139	·25	·13	·07	·187	[illegible]	·355	·125	[illegible]	·326	·187
	1·036	·00731		1·04	·00624		1·05	·00257					1·054	·00429		1·048	·001612		1·026	·0251		1·028	·03215	
$\frac{1}{2}$	·8	·174	·25	·8	·174	·375	·72	·139	·50	·574	·156	·375	·5	·174	·25	·561	·292	·187	·73	·292	·125	·785	·268	·187
	1·02	·0341		1·02	·0511		1·026	·0488		1·037	·0324		1·041	·0209		1·037	·0296		1·026	·03		1·021	·0515	
0	·936	·358	·375	·971	·391	·563	·96	·407	·75	·93	·342	·563	·889	·326	·375	·865	·309	·281	·852	·326	·187	·862	[illegible]	·281
	1·007	·125		1·006	·212		1·005	·292		1·009	·176		1·012	·1081		1·014	·074		1·015	·0511		1·015	·0789	
-1	·946	·5	·5	·971	·559	·75	·981	·559	1·0	·981	·545	·75	·93	·559	·5	·93	·545	·375	·91	·53	·25	·877	·53	·375
	1·006	·2355		1·004	·405		1·007	·524		1·007	·382		1·008	·258		1·008	·189		1·01	·1192		1·02	·172	
-2	·938	·719	·375	·971	·719	·563	·946	·719	·75	·941	·719	·563	·938	·719	·375	·92	·719	·281	[illegible]	·731	·187	·946	·743	·281
	1·007	·2444		1·004	·391		1·006	·506		1·007	·3785		1·007	·2515		1·009	·236		1·018	·119		1·019	·1653	
$-2\frac{1}{2}$	·921	·883	·187	·921	·788	·281	·945	·866	·375	·96	·857	·281	·941	·857	·187	·575	·883	·141	·648	·934	·094	·278	·94	·141
	1·009	·151		1·009	·202		1·006	·306		1·006	·23		1·007	·15		1·013	·1075		1·031	·0551		1·041	·035	
$-2\frac{3}{4}$	·771	·974	·125	·871	·964	·187	·946	·943	·25	·932	·917	·187	·92	·927	·125	·471	·966	·094	·319	·966	·063			·094
	1·02	·0941		1·013	·154		1·004	·226		1·008	·159		1·009	·1059		1·03	·0593		1·032	·01268				
		·89407			1·42415			1·40537			1·3579			·89769			·701512			·42106			·53929	
Σ																								

Σ 11·42437

TEST :- 2 TABLE :- 4

STATION	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼							
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$
							142	·053	·25	·112	42	·142	465	·152	34	·138	·12	·082										
2¾							·008		·028		·06		·071		·047		·01											
			·162	·03	625	·059	6	·145	·901	·281	937	·256	955	·26	955	·268	865	·269	322	·107	04	·015						
2½			·005		·037		·087		·244		24		·248		256		233		·034		·006							
	·09	·038	·27	·066	682	·107	885	·31	1	·511	1	·491	1	·491	1	·501	·937	492	42	·22	067	·034						
2	·003		·018		·073		·27		·51		·491		·491		501		·461		·092		·002							
	·14	·032	482	·08	901	·125	1	·339	1	·53	1	·5	1	·515	1	·552	1	545	·492	·334	445	·066						
1	·004		·039		·113		·339		·53		·5		·515		552		·545		·331		·029							
	·162	·031	482	·073	775	·094	1	·273	1	477	1	492	1	508	1	·5	1	407	482	·212	·19	·03						
0	·005		·035		·073		·273		·477		492		508		·5		407		·102		·006							
	225	·043	482	·064	83	·086	992	·258	1	431	1	431	1	·446	1	431	·992	412	482	·182	272	·054						
-1	·01		031		·071		·256		·431		·431		·446		·431		·404		·088		·015							
	·162	·03	377	·046	695	·065	901	·193	1	·326	1	·342	1	·342	1	·358	955	319	445	·132	17	·03						
-2	·005		·017		·045		·174		·326		·342		·342		·358		·304		·06		005							
			27	·022	46	·029	775	·113	1	·191	1	·191	1	·208	1	·208	955	·218	23	·055	12	·01						
-3			006		·013		·087		·191		·191		·208		·208		·207		·008		·001							
			·097	·004	172	·005	38	0	1	·076	1	·066	1	·076	1	·097	1	·087	272	·027	·09	·006						
-4			000		·001		0		·076		·066		·076		·097		·087		·007		000							
					·09	·003	·292	·004	1	·02	1	·018	1	·026	1	·026	901	·023	·09	·004								
-4¼					·008		·001		·02		·018		·026		·026		·021		·000									
Σ	·027		·151		·426		1·495		2·833		2·831		2·931		2·976		2·686		·722		·064							

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{\int da}{\int V}$$

Σ 17·142

TEST :- 2A. TABLE :- 4

ATION	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$
1½							181	·005	30	·005	225	·002	[illegible]	·003	[illegible]	·003							·017	·005	·115	·009	117	·004
							·0009		·0015		·0005		·0005		·0003								·000		·001		·000	
1					21	·007	485	·022	364	·005	37	·013	34	·007	245	·006	69	·003			·017	·004	·017	·002	337	·025	315	·033
					·001		·011		·002		·005		·002		·001		·000				·000		·000		·008		·01	
½			267	·004	353	·012	635	·025	565	·022	62	·03	34	·034	[illegible]	·051	[illegible]	·049	33	·032	25	·021	315	·03	52	·03	617	·052
			·000		·004		·016 ~~·002~~		·012		·019		·022		·033		·025		·011		·005		·009		·015		·032	
0	14	·013	334	·017	307	·047	845	·066	725	·076	83	·094	83	·125	142	·212	122	·292	345	·176	79	·107	75	·074	727	·051	727	·079
	·002.		·006		·038		·056		·063		·078		·11		·2		·269		·149		·085		·055		·037		·058	
-1	·536	·07	77	·067	811	·121	915	·184	73	·254	736	·248	395	·236	942	·405	887	·524	887	·382	867	·158	867	·184	83	·119	77	·172
	·037		·052		·105		·168		·236		·232		·211		·382		·464		·338		·224		·164		·099		·182	
-2	·478	·094	715	·116	805	·122	895	·19	73	·258	736	·264	[illegible]	·244	742	·391	895	·506	887	·379	88	·252	345	·236	78	·119	65	·165
	·045		·092		·106		·17		·24		·247		·214		·368		·452		·336		·222		·2		·093		·107	
	·072	·023	406	·059	[illegible]	·068	525	·091	355	·155	85	·151	85	·151	85	·202	891	·306	722	·23	887	·15	765	·108	42	·055	072	·035
-2½	·002		·027		·045		·048		·133		·128		·128		·172		·272		·212		·133		·083		·023		·003	
-2¾	025	·009	25	·021	37	·036	[illegible]	·046	[illegible]	·081	[illegible]	·091	025	·094	[illegible]	·154	[illegible]	·226	47	·159	345	·106	45	·059	043	·013		
	0		·003		·013		·012		·038		·054		·059		·117		·212		·138		·089		·027		·006			
Σ	·086		·18		·315		·482		·726		·764		·747		1·273		1·694		1·184		·758		·538		·282		·342	

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f d\alpha}{f V}$$

Σ 9 371

TEST:- 2 TABLE:- 6

Station	-3¾	-3½	-3⅓	-3	2	1	0	1	2	3	3½			
	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A	$\eta^{\frac{1}{2}}$ cos α A
	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$	$\eta^{\frac{1}{2}}\cos\alpha\sin\alpha\frac{f d\alpha}{\Sigma V}$
2¾				377 342 .053	.5 358 .112	647 403 .142	651 391 .152	584 208 .138	346 087 .002					
				.007	.02	.027	.04	.017	.002					
2½		403 574 .03	791 574 .059	776 574 .145	95 609 .281	966 695 .256	978 695 .26	978 669 .268	93 629 .269	567 565 .107	2 574 .015			
		.007	.027	.065	.163	.172	.177	.175	.157	.033	.002			
2	.3 707 .038	519 707 .066	826 707 .107	94 707 .31	1 731 .511	1 755 .491	1 755 .491	1 743 .501	966 731 .492	649 669 .22	259 669 .034			
	.008	.024	.062	.206	.374	.37	.37	.372	.348	.095	.006			
1	574 934 .032	895 883 .06	95 848 .125	1 839 .339	1 848 .53	1 866 .5	1 857 .515	1 834 .552	1 839 .546	996 863 .334	666 91 .066			
	.811 .008	.049	.101	.284	.45	.434	.441	.46	.457	.288	.04			
0	403 948 .031	675 906 .073	88 906 .094	1 899 .273	1 899 .477	1 890 .492	1 862 .506	1 866 .5	1 914 .407	686 866 .212	436 982 .03			
	.012	.046	.075	.245	.42	.428	.438	.433	.372	.128	.013			
-1	574 927 .043	695 927 .064	911 917 .086	996 914 .258	1 903 .431	1 903 .431	1 898 .446	1 903 .431	996 91 .412	695 903 .182	521 908 .054			
	.019	.041	.072	.234	.39	.389	.398	.398	.374	.114	.025			
-2	403 967 .03	615 957 .046	834 968 .065	95 946 .193	1 946 .326	1 94 .342	1 94 .342	1 934 .358	978 946 .319	667 946 .132	412 961 .03			
	.011	.027	.051	.173	.308	.322	.322	.334	.294	.083	.012			
-3		519 985 .022	678 985 .029	88 978 .113	1 982 .191	1 982 .191	1 978 .208	1 978 .208	974 974 .219	48 993 .035	347 993 .01			
		.011	.019	.097	.187	.187	.204	.204	.208	.017	.003			
-4		312 996 .004	414 996 .005	616 1 0	1 993 .076	1 995 .066	1 993 .076	1 988 .098	1 99 .088	522 99 .027	.3 99 .006			
		.001	.002	0	.075	.065	.075	.096	.086	.014	.002			
-4¼			3 989 .003	54 99 .004	1 997 .02	1 998 .018	1 995 .026	1 995 .026	95 995 .023	3 995 .04				
			.001	.002	.02	.018	.026	.026	.022	.012				
Σ	.061	.20	.41	1.313	2.407	2.412	2.491	2.515	2.330	.776	.103			

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f d\alpha}{\Sigma V}$

$\Sigma\, 15.008$

TEST :- 2a TABLE:- 5

STATION	-1¾			1½			-1¼			-1			-½			0			½		
	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A
	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$
										475	996	·005	547	999	·005	475	999	·005	424	999	·003
1½										·002			·003			·002			·001		
							458	993	·007	696	985	·022	603	999	·005	608	996	·013	583	999	·007
1							·003			·015			·003			·008			·004		
				259	993	·004	594	986	·012	796	985	·025	751	993	·022	787	988	·03	·8	985	·034
½				·001			·007			·019			·016			·023			·027		
	374	982	·013	578	986	·017	897	959	·047	92	966	·066	908	·97	·076	911	961	·094	938	934	·125
0	·000			·001			·04			·059			·067			·082			·11		
	731	921	·07	876	951	·067	945	557	·121	956	857	·184	964	848	254	767	857	248	946	866	236
-1	·047			·056			·098			·151			·207			·206			193		
	691	669	·094	891	707	116	93	707	122	945	695	·19	964	695	258	767	682	264	938	695	244
-2	·043			·073			·08			125			·173			·174			·159		
	268	292	·023	682	342	·059	815	407	·068	725	407	·091	725	438	·155	722	470	151	721	47	151
-2½	·002			·014			·023			·027			·063			·065			065		
	158	174	009	354	174	·021	609	191	·036	512	191	·046	682	208	·084	771	242	·091	791	225	·094
-2¾	0			·001			·004			·005			·012			·017			·017		
Σ	·092			·146			·255			·403			·544			·577			·576		

STATION	1			2			3			3½			4			4¼			4½		
	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A	$\eta^{\frac12}$	$\cos\alpha$	A
	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{fV}$
	·3	999	·003										13	966	·005	339	956	·009	13	978	·004
1½	·001												·001			·003			·001		
	495	999	·006	·3	999	·003				13	99	·004	13	961	·002	581	934	·025	561	946	·033
1	·003			·001						·005			0			·014			·018		
	·8	985	·051	72	99	·049	574	968	·032	·5	985	·021	561	956	·03	72	94	·03	785	934	·052
½	·04			·035			·018			·01			·016			·02			·038		
	971	921	212	96	914	282	92	·94	176	889	946	·107	865	951	·074	882	946	·054	852	943	·078
0	·19			·256			·152			·09			·061			·041			·063		
	771	829	405	941	829	524	941	839	362	93	829	258	93	839	189	91	857	·192	877	857	172
-1	·326			·408			·301			·198			·147			·093			·128 (129)		
	771	695	391	946	695	506	941	695	379	938	695	252	92	695	236	882	682	119	806	669	165
-2	·263			·332			·248			164			·151			·072			·089		
	921	616	202	945	5	306	96	515	23	941	515	·13	875	47	·108	648	358	·065	278	342	·035
-2½	·115			·145			·114			·073			·0444			·012			·033		
	871	267	154	966	334	·226	932	399	169	92	375	·106	671	259	·059	219	259	·013			
-2¾	·036			·073			·059			037			·01			·001					
Σ	·974			1·250			·892			·577			·430			·256			·371		

$A = \eta^{\frac12} \sin\alpha \frac{fd\alpha}{fV}$

Σ 7·343

TEST :- 3 TABLE :- 3

STATION	-3¾			-3½			-3¼			-3		
	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα
	∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$	
2¾				·724	·839	·063	·905	·839	·063	·562	·891	·156
				1·069	·011		1·052	·025		1·05	·074	
2½				·74	·857	·094	·846	·857	·094	·765	·857	·234
				1·033	·058		1·063	·050		1·03	·149	
2	·324	·705	·187	·446	·707	·187	·621	·707	·187	·92	·707	·47
	1·069	·028		1·058	·056		1·024	·106		1·012	·302	
1	·346	·423	·25	·676	·485	·25	·893	·485	·25	·963	·515	·625
	1·064	·034		1·039	·079		1·016	·106		1·009	·307	
0	·74	·485	·25	·721	·485	·25	·893	·47	·25	·95	·50	·625
	1·033	·087		[illegible]	·084		1·016	·103		1·011	·294	
-1	·515	·415	·25	·721	·423	·25	·675	·454	·25	·944	·423	·625
	1·053	·051		1·035	·073		1·018	·097		1·012	·246	
-2	·376	·225	·25	·436	·309	·25	·835	·317	·25	1	·307	·625
	1·062	·020		1·063	·047		1·023	·065		1·00	·192	
-3	·324	·087	·25	·368	·122	·25	·636	·156	·25	·828	·191	·625
	1·069	·005		1·062	·011		1·043	·024		1·023	·096	
-4				·158	·035	·156	·293	·035	·156	·536	·00	·39
				1·072	·001		1·067	·002		1·052	·000	
-4¼							·268	·018	·063	·51	·018	·156
							1·068	·000		1·054	·001	
Σ	0·225			0·420			0·578			1·661		

STATION	-2			-1			0			1		
	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα
	∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$	
2¾	·724	·903	·25	·86	·866	·25	·824	·883	·25	·765	·883	·25
	1·034	·157		1·02	·183		1·024	·177		1·03	·164	
2½	·89	·824	·375	·977	·783	·375	·961	·772	·375	·961	·783	·375
	1·016	·270		1·004	·285		1·006	·278		1·006	·282	
2	1	·669	·75	1	·643	·75	1	·643	·75	1	·656	·75
	1·00	·501		1·00	·482		1·00	·482		1·00	·494	
1	1	·523	1	1	·53	1	1	·552	1	1	·552	1
	1·00	·523		1·00	·530		1·00	·552		1·00	·552	
0	1	·485	1	1	·53	1	1	·47	1	1	·485	1
	1·00	·485		1·00	·530		1·00	·470		1·00	·485	
-1	1	·438	1	1	·454	1	1	·454	1	1	·454	1
	1·00	·438		1·00	·454		1·00	·454		1·00	·454	
-2	1	·317	1	1	·317	1	1	·309	1	1	·326	1
	1·00	·317		1·00	·317		1·00	·309		1·00	·326	
-3	1	·182	1	1	·216	1	1	·191	1	1	·191	1
	1·00	·182		1·00	·216		1·00	·191		1·00	·191	
-4	1	·07	·625	1	·035	·625	1	·035	·625	1	·052	·625
	1·00	·044		1·00	·022		1·00	·022		1·00	·033	
-4¼	1	·105	·25	1	·064	·25	1	·07	·25	1	·061	·25
	1·00	·026		1·00	·011		1·00	·018		1·00	·015	
Σ	2·943			3·030			2·953			2·996		

STATION	2			3			3¼		
	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα	$\eta^{1/2}$	Sin α	∫dα
	∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$		∫V	$\frac{\eta^{1/2} \sin\alpha \int d\alpha}{\int V}$	
2¾	·531	·883	·25	·396	·883	·156			
	1·052	·111		1·041	·051				
2½	·866	·829	·375	·558	·819	·234			
	1·017	·271		1·05	·102				
2	·971	·669	·75	·761	·694	·47	·359	·707	·187
	1·005	·484		1·031	·240		1·066	·043	
1	1	·552	1	·824	·545	·625	·565	·545	·25
	1·00	·552		1·006	·278		1·049	·074	
0	1	·462	1	·806	·485	·625	·333	·485	·25
	1·00	·462		1·008	·242		1·063	·073	
-1	1	·407	1	·802	·415	·625	·566	·371	·25
	1·00	·407		1·009	·206		1·065	·053	
-2	1	·309	1	·805	·309	·625	·46	·257	·25
	1·00	·309		1·008	·154		1·057	·028	
-3	1	·182	1	·505	[illegible]	·625	·168	·087	·25
	1·00	·182		[illegible]	·026		1·072	·003	
-4	·974	·035	·625	·167	·018	·39	·071	·018	·156
	1·006	·021		1·071	·001		1·073	·000	
-4¼	·96	·105	·25	·167	·105	·156			
	1·007	·025		1·071	·003				
Σ	2·824			1·303			0·374		

19·207

TEST :- 3a   TABLE :- 3

STATION	-1¾			-1½			-1¼			-1		
	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½							·13	·07	·094	·356	·07	·141
							1·072	·00797		1·068	·03296	
1	·246	·052	·125	·302	·139	·125	·521	·139	·125	·662	·174	·187
	1·068	·00162		1·066	·00594		1·058	·0086		1·04	·0207	
½	·303	·07	·125	·332	·156	·125	·675	·259	·125	·82	·191	·187
	1·06	·007485		1·046	·00607		1·037	·0211		1·025	·0286	
0	·488	·122	·187	·566	·191	·187	·87	·309	·187	·915	·276	·281
	1·055	·01051		1·04	·01935		1·019	·0494		1·013	·07	
-1	·707	·358	·25	·88	·485	·25	·921	·5	·25	·945	·545	·375
	1·036	·0612		1·017	·105		1·012	·1135		1·009	·1915	
-2	·676	·766	·187	·846	·743	·187	·909	·719	·187	·924	·743	·281
	1·037	·0932		1·01	·115		1·014	·1202		1·012	·189	
-2½	·339	·946	·094	·569	·961	·094	·768	·934	·094	·686	·94	·141
	1·064	·0288		1·04	·049		1·03	·0655		1·038	·0876	
-2¾				·268	·978	·063	·521	·982	·063	·427	·99	·094
				1·068	·015		1·053	·0306		1·059	·0376	
Σ	·198			·315			·418			·630		

STATION	-½			0			½			1		
	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½	·521	·018	·187	·452	·067	·187	·384	·07	·167	·303	·067	·281
	1·053	·001662		1·058	·00922		1·062	·00476		1·066	·00695	
1	·556	·07	·25	·576	0	·25	·516	·018	·25	·51	·018	·375
	1·05	·00926		1·068	0		1·053	·0022		1·054	·00327	
½	·736	·139	·25	·81	·156	·25	·817	·148	·25	·79	·148	·375
	1·033	·02465		1·020	·0308		1·025	·0294		1·028	·0427	
0	·9	·242	·375	·924	·309	·375	·936	·358	·375	·936	·326	·563
	1·015	·0805		1·012	·1059		1·01	·1245		1·01	·17	
-1	·955	·53	·5	·956	·53	·5	·956	·559	·5	·956	·559	·75
	1·007	·251		1·007	·251		1·007	·264		1·007	·396	
-2	·946	·731	·375	·966	·743	·375	·956	·731	·375	·936	·731	·563
	1·009	·257		1·006	·264		1·007	·2605		1·01	·382	
-2½	·904	·914	·187	·92	·921	·187	·92	·914	·187	·934	·891	·281
	1·014	·152		1·012	·1565		1·01	·155		1·01	·232	
-2¾	·638	·985	·125	·75	·982	·125	·759	·982	·125	·831	·956	·187
	1·043	·0752		1·032	·0891		1·031	·0904		1·023	·145	
Σ	·852			·907			·931			1·378		

STATION	2			3			3½		
	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½	0	·067	·375						
	1·073	0							
1	·311	0	·5	·122	·035	·375	·122	·035	·25
	1·066	0		1·073	·00149		1·073	·000994	
½	·721	·156	·5	·544	·122	·375	·538	·174	·25
	1·035	·0542		1·031	·0237		1·037	·0222	
0	·936	·309	·75	·901	·326	·563	·87	·326	·375
	1·01	·215		1·015	·1627		1·019	·1044	
-1	·936	·545	1	·934	·545	·75	·923	·545	·5
	1·01	·504		1·011	·378		1·012	·248	
-2	·936	·719	·75	·936	·719	·563	·932	·707	·375
	1·01	·5		1·01	·374		1·011	·248	
-2½	·92	·883	·375	·934	·857	·281	·934	·866	·167
	1·01	·3		1·02	·222		1·011	·15	
-2¾	·864	·966	·25	·892	·951	·187	·88	·94	·125
	1·019	·205		1·016	·156		1·017	·1015	
Σ	1·778			1·319			·873		

STATION	4			4¼			4½		
	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da	$\eta^{\frac{1}{2}}$	Sin α	∫ da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½	0	·139	·141	[illegible]	·191	·094	·363	·174	·141
	1·07	0		1·06	·00695		1·063	·00837	
1	·354	·035	·187	[illegible]	·035	·125	·521	·035	·187
	1·064	·00218		1·065	·00204		1·063	·00323	
½	·66	·292	·187	[illegible]	·312	·125	·785	·326	·187
	1·04	·0396		1·03	·03205		1·022	·0465	
0	·88	·326	·281	[illegible]	·356	·187	·861	·326	·281
	1·017	·0793		1·017	·058		1·02	·0774	
-1	·914	·545	·375	[illegible]	·53	·25	·861	·515	·375
	1·013	·184		1·016	·116		1·02	·1625	
-2	·918	·707	·281	[illegible]	·743	·187	·782	·719	·281
	1·013	·18		1·017	·1203		1·02	·1535	
-2½	·836	·891	·141	·550	·946	·094	[illegible]	·94	·141
	1·022	·1027		1·05	·047		1·067	·034	
-2¾	·584	·978	·094	·265	·978	·063			
	1·048	·0513		1·067	·0141				
Σ	·634			·396			·486		

Σ 11·115

TEST :- 3 TABLE :- 4

STATION	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼							
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{f\,da}{fV}$
			·05	·011	255	·025	·315	·074	525	·157	74	·183	678	·177	586	·164	282	·111	157	·051								
2¾			·001		·006		·023		·083		·135		·120		·096		·031		·008									
			547	·058	42	·050	·587	·149	792	·270	957	·285	925	·278	925	·282	786	·271	311	·102								
2½			·031		·021		·088		·214		·273		·256		·261		·213		·032									
	·05	·028	·2	·056	675	·106	·845	·302	1	·501	1	·482	1	·482	1	·494	945	·484	58	·240	125	·043						
2	·001		·011		·072		·255		·501		·482		·482		·494		·456		·139		·006							
	·122	·034	457	·079	797	·106	·928	·307	1	·523	1	·530	1	·552	1	·552	1	·552	678	·278	318	·074						
1	·004		·036		·085		·284		·523		·530		·552		·552		·552		·189		·023							
	547	·087	52	·084	797	·103	·902	·294	1	·485	1	·530	1	·470	1	·485	1	·462	65	·242	·4	·073						
0	·047		·044		·082		·264		·485		·530		·470		·485		·462		·157		·029							
	265	·051	52	·073	766	·097	89	·246	1	·438	1	·454	1	·454	1	·454	1	·407	642	·206	318	·053						
-1	·013		·038		·075		·219		·438		·454		·454		·454		·407		·132		·017							
	142	·020	405	·047	697	·065	1	·192	1	·317	1	·317	1	·309	1	·326	1	·309	65	·154	212	·028						
-2	003		·019		·045		·192		·317		·317		·309		·326		·309		·101		·006							
	·05	005	15	·011	405	·024	·685	·096	1	·182	1	·216	1	·191	1	·191	1	·182	255	·026	025	·003						
-3	·000		·002		·010		·066		·182		·216		·191		·191		·182		·007		·000							
			·025	·001	086	·002	·288	·000	1	·044	1	·022	1	·022	1	·033	95	·021	·035	·001	·005	·000						
-4			·000		·000		·000		·044		·022		·022		·033		·020		·000		·000							
					072	·000	·26	·001	1	·026	1	·011	1	·018	1	·015	92	·025	·035	·003								
-4¼					·000		·000		·026		·011		·018		·015		·023		·000									
Σ	0·068		0·182		0·396		1·391		2·813		2·970		2·874		2907		2·665		0·765		0·081							

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f\,da}{fV}$

17·102

TEST :- 3A. TABLE :- 4

ATION	$-1\frac{3}{4}$		$-1\frac{1}{2}$		$-1\frac{1}{4}$		-1		$-\frac{1}{2}$		0		$\frac{1}{2}$		1		2		3		$3\frac{1}{2}$		4		$4\frac{1}{2}$		$4\frac{3}{4}$	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{\frac{1}{2}}\sin\alpha$	$\frac{\int d\alpha}{\int V}$
$1\frac{1}{2}$					·017	·008	127	·003	272	·002	·205	·009	147	·005	·092	·007	0	0			~~·015~~		0	0	168	·007	132	·008
					·000		000		·001		·002		·001		·001		0						0		·001		·001	
1	·071	·002	·092	·006	272	·009	437	·021	31	·009	332	0	267	·002	26	·003	·097	0	·015	·002	·015	·001	125	·003	242	·002	272	·003
	·000		·001		·002		·009		·003		0		·001		·001		0		·000		000		·000		000		·001	
$\frac{1}{2}$	·092	·002	·11	·006	455	·021	671	·029	542	·025	655	·031	67	·029	625	·043	52	·054	295	·024	29	·022	435	·035	595	·032	615	·047
	·000		·001		·01		·019		·014		·02		·019		·027		·028		·007		·006		·015		·019		·029	
0	·237	·011	·324	·019	·757	·049	837	·07	·81	·061	852	·106	877	·125	877	·17	877	·216	812	·163	757	·104	776	·079	775	·058	742	·077
	003		·006		·037		·059		·066		·09		·109		149		188		132		·079		·061		·045		·057	
-1	·50	·061	774	105	852	·114	894	·192	911	251	911	251	915	264	915	396	877	504	87	378	851	248	835	184	795	116	742	163
	·03		·081		·097		·172		·229		·229		242		·362		·443		329		211		154		·092		·121	
-2	·457	093	717	115	825	·12	852	189	895	257	931	264	915	261	877	382	877	5	877	374	87	246	842	·18	775	12	612	154
	·042		·082		·099		161		·23		244		239		·326		44		328		214		152		·093		094	
$-2\frac{1}{2}$	·115	·029	324	049	59	·066	·472	·088	·817	162	845	157	845	155	87	232	845	·3	87	222	87	·15	70	103	31	047	075	·034
	·003		·016		·039		·041		·124		·133		131		·202		·254		193		·13		·072		·015		·003	
$-2\frac{3}{4}$			071	015	272	·031	182	·038	·407	·075	562	·089	575	·09	69	145	747	205	795	156	775	·102	34	051	·06	·014		
			·001		·008		·007		·03		·05		·052		·1		·153		124		·079		·017		·001			
Σ	·078		·188		·292		·368		·697		·768		·784		1·178		1·506		1·113		·719		·471		·266		·306	

$A = \eta^{\frac{1}{2}}\sin\alpha\,\frac{\int d\alpha}{\int V}$

Σ 8·744

TEST :- 3 TABLE :- 5

STATION	-3¾	-3½	-3¼	-3	-2	-1	0	1	2	3	3¾			
	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A	$\eta^{\frac{1}{2}}$ cos A
	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$	η cos sin $\frac{fda}{\Sigma V}$
		229 545 011	506 545 025	862 454 074	729 43 157	86 ·5 183	829 492 177	765 47 164	531 47 111	396 47 051	47			
2¾		·001	·007	·019	·049	·079	·072	·059	·028	·010				
		74 515 058	148 515 050	765 515 149	89 566 270	977 623 286	961 636 278	961 623 282	886 559 271	558 574 102				
2½		·022	017	059	·136	173	170	·169	·134	·033				
	224 707 028	446 707 056	821 707 106	92 707/743 302	1 743/766 501	1 766 482	1 766 482	1 765 496	971 743 484	761 719 260	354 707 063			
2	·004	·018	·062	·196	373	370	·370	·372	·350	·131	·011			
	396 906 034	676 875 079	893 825 106	963 857 307	1 653 523	1 848 530	1 834 652	1 834 552	1 834 652	829 639 278	565 839 076			
1	·011	·047	·083	·253	·445	·450	·460	·461	·460	·192	·035			
	74 875 087	721 875 084	893 883 103	95 866 294	1 875 485	1 848 530	1 863 470	1 875 485	1 887 462	806 875 242	·33 875 073			
0	·057	·053	·081	·242	·425	·450	·415	·425	·410	·171	·040			
	515 ·91 051	721 ·906 073	875 891 097	944 906 246	1 899 438	1 891 454	1 891 454	1 891 454	1 914 407	802 91 206	565 921 053			
-1	·024	048	·076	·210	394	·404	·405	·405	·372	·151	·028			
	376 974 020	636 951 047	835 948 066	1 954 192	1 948 317	1 948 317	1 951 309	1 946 326	1 951 309	805 951 156	46 966 028			
-2	·007	·029	·052	·183	·300	·300	·294	·309	·294	·118	·012			
	126 996 005	363 993 011	636 988 024	828 982 096	1 983 182	1 976 216	1 982 191	1 982 191	1 983 182	505 996 026	158 996 003			
-3	·001	·004	·015	078	·179	·211	·188	187	179	·013	·000			
		158 $\bar{999}$ 001	293 $\bar{999}$ 002	536 1 000	1 998 044	1 999 022	1 999 022	1 999 033	979 999 021	167 999 001	071 999 000			
-4		·000	·001	·000	·046	·022	·022	·033	·020	·000	·000			
			268 $\bar{999}$ ·000	51 $\bar{999}$ 001	1 995 026	1 999 011	1 998 018	1 998 015	96 995 025	167 995 003				
-4¼			·000	·001	·026	·011	·018	·015	·024	·001				
Σ	0·104	0·222	0·394	1241	2371	2470	2·414	2·435	2371	0·820	0·126			

$A = \eta^{\frac{1}{2}} \sin\angle \frac{fda}{\Sigma V}$.

14868

TEST :- 3A TABLE:- 5

STATION	$-1\frac{3}{4}$			$-1\frac{1}{2}$			$-1\frac{1}{4}$			-1			$-\frac{1}{2}$			0			$\frac{1}{2}$		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
						.	13	998	008	356	998	003	521	999	002	452	996	004	384	998	005
$1\frac{1}{2}$							·001			·001			·001			·004			·002		
	[illegible]	999	002	322	99	006	521	99	009	567	985	021	552	998	009	574	1	0	516	999	002
1	·000			·002			·005			·014			·005			0			·001		
	303	998	002	332	988	006	315	966	021	82	982	029	732	99	025	71	988	031	917	959	029
$\frac{1}{2}$	·001			·002			·014			·023			·018			·025			·023		
	436	993	011	568	982	019	81	951	049	915	961	07	9	97	081	924	951	106	136	924	125
0	·005			·01			·04			·021			·071			·094			·109		
	107	934	061	88	875	105	121	866	114	76	839	192	155	848	251	150	848	251	[illegible]	829	264
-1	·04			·081			·091			·152			204 ~~·244~~			·204			·209		
	016	643	093	846	669	115	709	695	12	724	669	189	74	682	257	74	669	264	750	652	261
-2	·04			·065			·076			·117			·166			·164			·17		
	637	259	029	569	276	049	708	358	066	786	342	088	72	407	152	92	391	157	12	407	155
$-2\frac{1}{2}$	·003			·008			·018			·03			·056			·056			·058		
				209	208	015	621	191	031	42	139	038	638	174	075	75	191	089	751	191	09
$-2\frac{3}{4}$				·001			003			002			·008			·013			·013		
Σ	·089			·169			·248			·4			·529			·56			·585		

STATION	1			2			3			$3\frac{1}{2}$			4			$4\frac{1}{4}$			$4\frac{1}{2}$		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
	393	996	007	0	996	0							0	99	0	411	982	007	363	985	008
$1\frac{1}{2}$	·002			0									0			·003			·003		
	[illegible]	999	003	311	1	0	122	999	002	122	999	001	354	999	002	412	999	002	521	999	003
1	·002			0			·000			000			00[illegible]			·001			·002		
	77	989	043	14	988	054	544	993	024	538	985	022	66	956	035	112	94	032	783	946	047
$\frac{1}{2}$	·034			·038			·013			·012			·022			·023			·035		
	13	946	17	126	951	215	109	946	163	87	946	104	881	946	079	88	934	058	861	946	077
0	·15			·191			·139			·085			·066			·048			·063		
	750	829	396	73	839	504	774	839	378	723	839	248	714	839	184	811	848	116	861	857	163
-1	·314			·396			·296			·192			·141			·087			·12		
	7	652	382	72	695	5	732	695	374	732	707	246	718	707	18	8	669	12	782	695	154
-2	·244			·326			·244			·162			·119			·071			·084		
	734	454	232	72	47	3	734	515	222	734	5	15	836	454	103	550	326	047	274	342	034
$-2\frac{1}{2}$	·098			·13			107			·07			·039			·009			·003		
	331	292	145	7	259	205	892	309	156	88	342	102	584	208	051	245	208	014			
$-2\frac{3}{4}$	·035			·048			·043			·031			·006			·001					
Σ	·879			1·127			·842			·552			·392			·243			·31		

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$.

Σ 6925

TEST:- 4 TABLE:- 3

STATION	-3¾			-3½			-3¼			-3			-2			-1			0			1			2			3			3¼		
	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα	$\eta^{1/2}$	Sinα	∫dα
	fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$		fV	$\frac{\eta^{1/2} Sin\alpha \int d\alpha}{fV}$	
				·212	914	063	4	914	063	544	914	156	706	906	25	807	883	25	796	866	25	716	875	25	572	906	25	318	906	156			
2¾				1.089	.011		1.078	.021		1.067	.073		1.047	.153		1.033	.173		1.035	.163		1.047	.150		1.064	.122		1.083	.042				
				245	857	094	631	857	094	765	857	234	902	834	375	992	793	375	992	777	375	·98	793	375	·89	857	375	611	853	234			
2½				1.087	.018		1.087	.048		1.04	.147		1.016	.278		1.002	.294		1.002	.288		1.002	.290		1.05	.272		1.06	.115				
				268	719	187	631	719	187	·94	719	47	992	669	75	1	663	75	1	663	75	1	676	75	981	695	75	784	737	47			
2				1.086	.033		1.087	.080		1.01	.314		1.001	.497		1.00	.497		1.00	.496		1.00	.506		1.002	.512		1.037	.262				
				671	485	25	875	5	25	·97	53	625	1	552	1	1	545	1	1	574	1	1	559	1	988	574	1	856	545	625	419	552	25
1				1.052	.077		1.022	.107		1.005	.320		1.00	.552		1.00	.545		1.00	.574		1.00	.559		1.001	.566		1.025	.284		1.077	.054	
				·7	446	25	912	454	25	962	477	625	1	·5	1	1	485	1	1	462	1	1	·5	1	981	53	1	836	508	625	653	454	25
0				1.048	.075		1.015	.102		1.006	.285		1.00	.500		1.00	.485		1.00	.462		1.00	.500		1.002	.519		1.028	.259		1.055	.070	
				732	423	25	886	454	25	946	454	625	1	454	1	1	470	1	1	485	1	1	454	1	985	438	1	936	438	625	672	438	25
-1				1.044	.074		1.02	.099		1.009	.268		1.00	.454		1.00	.470		1.00	.485		1.00	.454		1.001	.432		1.028	.223		1.052	.070	
	·324	326	25	·64	326	25	851	391	25	951	391	625	1	358	1	1	335	1	1	342	1	1	342	1	988	334	1	768	342	625	535	375	25
-2	1.183	.024		1.058	.051		1.025	.081		1.008	.230		1.00	.358		1.00	.335		1.00	.342		1.00	.342		1.001	.330		1.039	.158		1.067	.047	
				354	208	25	652	174	25	855	208	625	1	139	1	1	174	1	1	174	1	1	174	1	1	191	1	661	191	625			2·
-3				1.081	.017		1.033	.027		1.025	.109		1.00	.139		1.00	.174		1.00	.174		1.00	.174		1.00	.191		1.054	.075				
				·11	052	156	·33	052	156	547	$\overline{087}$	39	1	026	625	1	$\overline{017}$	625	994	$\overline{017}$	625	987	017	625	962	087	625	158	087	39			
-4				1.093	.001		1.083	.002		1.066	.018		1.00	.016		1.00	.011		1.002	.011		1.001	.011		1.006	.052		1.072	.005				
				0	$\overline{139}$	063	187	$\overline{139}$	063	547	$\overline{139}$	156	982	105	25	981	113	25	1	$\overline{035}$	25	1	026	25	936	07	25			156			
-4¼				1.095	.000		1.09	.002		1.066	.011		1.002	.026		1.002	.028		1.00	.009		1.00	.007		1.011	.016							
Σ	0.024			0.357			0.569			1.175			2.973			3.012			3.004			2.993			3.012			1.423			0.241		

19.383

TEST:- 4a TABLE :- 8

STATION	−1¾ η½	−1¾ Sin α	−1¾ ∫da	−1¾ fV	−1¾ η½ Sin α ∫da/fV	−1½ η½	−1½ Sin α	−1½ ∫da	−1½ fV	−1½ η½ Sin α ∫da/fV
1½										
1										
½	245	191	·125	1·067	·00539	418	156	·125	1·077	·00758
0	474	174	·187	1·073	·01435	651	225	·187	1·055	·026
−1	·69	304	·25	1·05	·0507	·88	965	·25	1·022	·1045
−2	63	819	·187	1·063	·0907	856	777	·187	1·025	·1213
−2½	394	985	·094	1·08	·0338	682	951	·094	1·051	·0582
−2¾	212	990	·063	1·089	·01213	374	99	·063	1·08	0216
					·20707					·339
Σ					·207					·339

STATION	−1¼ η½	−1¼ Sin α	−1¼ ∫da	−1¼ fV	−1¼ η½ Sin α ∫da/fV	−1 η½	−1 Sin α	−1 ∫da	−1 fV	−1 η½ Sin α ∫da/fV
1½	274	·018	·094	1·086	·000427	547	·052	·141	1·066	·00377
1	558	156	·125	1·065	·01022	744	174	·187	1·043	·0232
½	·70	·225	·125	1·019	·0188	828	139	·187	1·03	·02088
0	89	309	·187	1·019	·0505	905	276	·281	1·016	·0691
−1	93	47	·25	1·012	·108	94	5	·375	1·01	·174
−2	919	·756	·187	1·014	·128	915	777	·231	1·015	·1477
−2½	814	906	·094	1·032	·0671	735	927	[illegible]	1·044	·092
−2¾	564	974	·063	1·065	·03245	475	978	·094	1·073	·0406
					·41547					
Σ					·415					·622

STATION	−½ η½	−½ Sin α	−½ ∫da	−½ fV	−½ η½ Sin α ∫da/fV	0 η½	0 Sin α	0 ∫da	0 fV	0 η½ Sin α ∫da/fV
1½	547	0	·187	1·066	0	437	·018	·187	1·026	·00137
1	·63	·07	·25	1·057	·01045	·609	105	·25	1·06	·0151
½	745	·07	·25	1·043	·0125	822	·156	·25	1·031	·0314
0	894	225	·375	1·018	·0742	926	292	·375	1·013	·1
−1	952	5	·5	1·006	·2362	958	515	·5	1·006	·2438
−2	·95	·766	·375	1·008	·271	952	755	·375	1·008	·268
−2½	916	921	·187	1·015	·155	·89	927	·187	1·02	·152
−2¾	·7	961	·125	1·049	·0802	81	982	·125	1·033	·0962
Σ					·839					·907

STATION	½ η½	½ Sin α	½ ∫da	½ fV	½ η½ Sin α ∫da/fV	1 η½	1 Sin α	1 ∫da	1 fV	1 η½ Sin α ∫da/fV
1½	875	0	·187	1·022	0	302	0	·281	1·084	0
1	61	·087	·25	1·06	·0125	506	·07	·375	1·07	·0124
½	828	·191	·25	1·03	·0384	821	174	·375	1·031	·052
0	·95	342	·375	1·009	·1219	·95	375	·563	1·008	·199
−1	953	545	·5	1·004	·258	941	545	·75	1·01	·38
−2	941	755	·375	1·01	·264	946	755	·563	1·01	·396
−2½	·895	927	·187	1·018	·1525	916	·906	·281	1·014	·23
−2¾	81	978	·125	1·033	·0958	891	966	·187	1·018	·158
Σ					·99					1·427

STATION	2 η½	2 Sin α	2 ∫da	2 fV	2 η½ Sin α ∫da/fV	3 η½	3 Sin α	3 ∫da	3 fV	3 η½ Sin α ∫da/fV
1½										
1	354	·07	·50	1·062	·01148	302	·035	·375	1·084	·00366
½	·80	191	·50	1·034	·0738	·63	139	·375	1·057	·0312
0	946	391	·75	1·009	·275	·90	358	·563	1·017	·1788
−1	93	53	1	1·012	·487	916	569	·75	1·014	·378
−2	·95	755	·75	1·008	·534	941	743	·563	1·01	·388
−2½	925	863	·375	1·018	·302	·94	875	·281	1·01	·228
−2¾	901	961	·25	1·017	·213	·94	946	·187	1·01	·165
Σ					1·897					1·373

STATION	3½ η½	3½ Sin α	3½ ∫da	3½ fV	3½ η½ Sin α ∫da/fV	4 η½	4 Sin α	4 ∫da	4 fV	4 η½ Sin α ∫da/fV
1½						349	·292	·141	1·062	·01325
1	219	·052	·25	1·089	·00254	502	358	·187	1·07	·0314
½	585	·179	·25	1·062	·024	664	292	·187	1·053	·0344
0	876	326	·375	1·022	·1045	876	326	·281	1·022	·0785
−1	915	545	·5	1·014	·246	901	559	·375	1·017	·186
−2	·94	743	·375	1·01	·2599	904	743	·281	1·016	·1855
−2½	926	866	·187	1·018	·1478	829	·899	·141	1·03	·102
−2¾	·92	940	·125	1·014	·1065	656	978	·094	1·055	·0571
Σ					·893					·688

STATION	4¼ η½	4¼ Sin α	4¼ ∫da	4¼ fV	4¼ η½ Sin α ∫da/fV	4½ η½	4½ Sin α	4½ ∫da	4½ fV	4½ η½ Sin α ∫da/fV
1½	454	242	·094	1·074	·0096	381	209	·141	1·08	·01029
1	661	342	·125	1·054	·0268	598	309	·187	1·061	·0326
½	765	358	·125	1·04	·0329	784	309	·187	1·037	·0436
0	876	358	·187	1·022	·0574	851	325	·281	1·026	·0844
−1	881	53	·25	1·02	·1142	842	515	·375	1·017	·1584
−2	852	766	·187	1·026	·1191	711	914	·281	1·047	·1742
−2½	·52	841	·094	1·069	·0408					
−2¾	·46	985	·063	1·074	·0266					
Σ					·428					·503

Σ 11·628

TEST :- 4 TABLE :- 4

STATION	-3¾		-3½		-3¼		-3		-2		-1		0		1		2		3		3¼							
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{f V}$
			045	·011	·16	·021	295	·073	·5	·153	·652	·173	635	·163	511	·150	327	·122	101	·042								
2¾			·001		·003		·021		·076		·113		·104		·077		·040		·004									
			·06	·018	398	·048	585	·147	816	·278	986	·294	986	·288	·96	·290	79	·272	374	·115								
2½			·001		·019		086		·227		·290		280		·278		·215		·043									
			·072	·033	398	·080	882	·314	983	·497	1	·497	1	·496	1	·506	·962	·512	615	·262								
2			·002		·032		·277		·490		·497		·496		·506		·493		·161									
			·45	·079	765	·107	94	·320	1	·552	1	·545	1	·574	1	·559	·976	·566	731	·284	195	·054						
1			035		·082		300		·552		545		·574		·559		·552		·208		·009							
			·49	·075	832	·102	925	·285	1	·500	1	·485	1	·462	1	·500	·962	·519	·7	·259	425	·070						
0			·036		085		·264		·500		·485		·462		·500		·499		·181		·030							
	~~105~~		536	·074	786	·099	894	·268	1	·454	1	·470	1	·485	1	·454	·972	·432	·7	·223	451	·070						
-1			·040		·078		·238		·454		·470		·485		·454		·419		·156		032							
	105	·024	44	·051	727	·081	905	·230	1	·358	1	·335	1	·342	1	·342	·976	·330	·592	·158	286	·047						
-2	·003		·022		·059		·208		·358		·335		·342		·342		322		·094		·014							
			126	·017	425	·027	731	·109	1	·139	1	·174	1	·174	1	·174	1	·191	·437	·075								
-3			·002		·012		·079		·139		·174		·174		·174		·191		·033									
			·012	·001	109	·002	·3	·018	1	·016	1	·011	986	·011	976	·011	925	·052	·025	·005								
-4			·000		·000		·005		·016		·011		·010		·010		·048		·000									
			·000 999		·035	·002	·3	·011	962	·026	962	·028	1	·009	1	·007	876	·016										
-4¼			000		·000		·004		·025		·027		·009		·007		·014											
Σ	0·003		0·139		0·370		1·482		2·837		2·947		2·936		2·907		2·793		0·880		0·085							

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f d\alpha}{f V}$$

17·379

TEST :- 4A TABLE :- 4

ATION	-1¾		-1½		-1¼		-1		-½		0		½		1		2		3		3½		4		4¼		4½	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{3/2}\sin\alpha$	$\frac{f da}{f V}$
½					075	000	30	004	·3	0	192	001	765	0	091	0							·122	013	207	·01	195	·01
					·000		·001		·0		·000		0		0								·002		·002		·001	
1					312	·01	552	023	397	·01	371	015	371	013	256	·012	125	·012	·091	·004	048	·003	256	·031	436	·027	357	·033
					·003		·013		·004		·005		·005		·003		·002		000		·000		·008		·012		·012	
½	·08	005	175	·008	49	019	685	021	552	013	677	031	687	·084	677	052	64	074	397	031	342	024	44	034	585	·033	614	·044
	·000		·001		·009		·014		·007		·021		·058		·035		·047		·012		·008		·015		·019		·027	
0	225	014	425	026	792	051	82	069	·8	074	86	·1	901	122	907	199	897	225	812	179	77	105	77	079	77	057	727	084
	·003		·011		·04		·057		·059		·086		·11		·181		·247		145		·081		·061		·044		·061	
-1	·476	051	772	105	865	108	882	174	907	236	907	244	907	258	886	38	865	467	84	378	84	246	·811	186	777	114	·71	158
	·024		·081		·093		·153		·214		·222		234		·336		·421		·317		·206		151		·088		·0118	
-2	397	091	732	121	844	128	836	198	901	271	907	268	886	264	886	396	901	534	886	388	882	26	82	186	727	119	·505	174
	·036		·089		108		·166		·244		·243		234		351		481		343		·229		·151		·086		·088	
-2½	·155	034	465	058	662	067	54	092	832	155	79	152	8	153	84	23	857	302	882	228	·86	148	687	102	27	041		
	0058		·027		044		·05		·129		·12		122		193		258		·201		127		·07		·011			
-2¾	·045	012	14	022	318	032	225	041	49	·08	656	096	656	096	795	158	812	213	882	165	845	107	43	057	21	027		
	0008		·003		·01		·0098		·039		·063		·063		·126		173		145		·09		024		·006			
Σ	·068		·212		·307		·463		·696		·76		·826		1·225		1·629		1·163		·741		·482		·268		·2	

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{f da}{f V}$$

Σ 9·040

TEST :- 4   TABLE :- 5.

STATION	−3¾	−3½	−3¼	−3	−2	−1	0	1	2	3	3¼			
	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A	$\eta^{1/2}$ cos α A
	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$	η cos α sin α $\frac{fd\alpha}{\Sigma V}$
		212 407 011	·4 407 021	594 407 073	706 423 153	807 47 173	796 5 163	716 485 150	372 423 122	318 423 042				
2¾		·001	·003	·016	·046	·066	·065	·052	·029	·006				
		245 515 018	631 515 048	765 515 147	902 552 278	992 609 294	992 629 288	96 609 290	89 515 272	611 523 115				
2½		·002	·016	·058	·139	·177	·180	·173	·124	·037				
		266 695 033	631 695 080	94 695 314	992 743 497	1 749 477	1 749 496	1 737 506	981 719 512	764 676 262				
2		·006	·035	·205	·367	·372	·371	·373	·362	·139				
		671 875 077	875 866 107	97 848 320	1 834 552	1 839 545	1 819 574	1 829 559	968 819 566	856 839 284	419 834 054			
1		·045	·081	·263	·460	·457	·468	·464	·458	·204	·019			
		7 895 075	912 891 102	942 879 285	1 866 500	1 875 485	1 857 462	1 866 500	981 848 519	836 862 259	653 891 070			
0		·047	·083	·241	·433	·425	·410	·433	·431	·187	·041			
	[illegible]	732 906 074	966 891 099	946 891 268	1 891 454	1 883 470	1 875 485	1 891 454	985 899 432	836 899 223	672 899 070			
−1		·049	·078	·226	·405	·415	·425	·405	·382	·168	·042			
	538 946 020	664 946 051	851 921 081	951 921 230	1 934 358	1 943 335	1 94 342	1 94 342	988 943 330	768 94 158	535 927 047			
−2	·007	·032	·064	·202	·334	·316	·322	·321	·308	·114	·023			
		354 978 017	652 985 027	855 978 109	1 99 137	1 985 174	1 985 174	1 985 174	1 982 191	661 982 075				
−3		·006	·017	·091	·138	·171	·172	·171	·188	·049				
		·11 999 001	33 999 002	597 996 018	1 1 016	1 $\bar{1}$ 011	994 $\bar{1}$ 011	967 1 011	962 996 052	156 996 005				
−4		·000	·001	·010	·016	·011	·011	·011	·050	·001				
		0 $\bar{99}$ 000	167 $\bar{99}$ 002	547 $\bar{99}$ 011	982 995 026	961 994 028	1 $\bar{999}$ 009	1 1 007	936 998 016					
−4¼		·000	·000	·006	·025	·027	·009	·007	·015					
Σ	0·007	0·188	0·378	1·318	2·363	2·437	2·433	2·410	2·347	0·905	0·125			

$A = \eta^{1/2} \sin\alpha \frac{fd\alpha}{\Sigma V}$.

14·911

TEST :- 4A.   TABLE:- 5

STATION	-1¾	-1½	-1¼	-1	-½	0	½	1	2	3	3½	4	4½	4½
	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A	$\eta^{\frac{1}{2}}$ cosα A
	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$	η cosα sinα $\frac{fda}{\Sigma V}$
1½			274 999 .000	547 999 .004	547 1 0	437 999 .001	875 1 0	312 1 0				349 956 .013	454 97 .01	381 978 .01
			.000	.002	0	.000	0	0				.004	.004	.004
1			558 988 .01	744 985 .023	63 998 .01	609 995 .015	61 996 .013	506 998 .012	354 998 .012	302 999 .004	219 999 .003	502 934 .031	661 94 .027	598 946 .033
			.006	.002	.006	.008	.008	.006	.004	.001	.001	.018	.017	.019
½	295 982 .005	416 988 .008	.7 974 .019	826 99 .021	746 998 .013	822 988 .031	826 982 .084	821 985 .052	.8 982 .024	63 99 .031	565 985 .024	664 956 .034	765 934 .033	764 95 .044
	.001	.003	.013	.017	.01	.025	.068	.042	.058	.019	.014	.022	.024	.033
0	474 985 .014	651 974 .026	89 95 .051	905 961 .069	894 974 .074	926 956 .1	95 94 .122	95 927 .199	946 921 .275	9 934 .179	576 946 .105	576 946 .079	876 934 .057	851 927 .084
	.007	.016	.043	.06	.064	.088	.109	.176	.24	.151	.057	.066	.047	.066
-1	69 951 .051	68 875 .105	93 883 .108	94 866 .174	952 866 .236	952 857 .244	952 839 .258	941 839 .38	93 848 .487	916 829 .378	915 839 .246	901 829 .186	881 848 .114	892 857 .158
	.033	.081	.089	.142	.195	.2	.206	.3	.384	.287	.189	.139	.085	.114
-2	63 574 .091	856 629 .121	919 656 .128	915 627 .198	95 643 .271	952 656 .248	941 656 .264	941 656 .396	95 656 .534	991 669 .388	94 669 .26	904 669 .186	852 643 .188	711 609 .174
	.033	.065	.077	.114	.166	.167	.163	.244	.332	.244	.164	.112	.065	.05
-2½	394 174 .034	682 309 .058	814 423 .067	735 375 .092	912 391 .155	89 375 .152	895 375 .153	916 423 .23	925 47 .302	94 485 .228	926 .5 .148	829 438 .102	52 454 .081	
	.002	.012	.023	.025	.055	.051	.051	.089	.131	.104	.068	.037	.01	
2¾	212 139 .012	374 139 .022	569 225 .032	475 208 .041	7 276 .08	81 191 .096	81 208 .096	891 259 .158	901 276 .213	99 326 .165	92 342 .107	656 308 .057	46 174 .027	
	.000	.001	.004	.004	.015	.015	.016	.036	.053	.051	.034	.008	.002	
			3	2	3	3	4	3	3 2	3	3 3	4	3	
Σ	.076	.178	.255	.366	.511	.555	.621	.893	1.202	.857	.527	.403	.254	.286

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{\Sigma V}$.

Σ 6.984

TEST :- 5 TABLE :- 3

STATION	-3 3/4			-3 1/2			-3 1/4			-3			-2			-1			0			1			2			3			3 1/4		
	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da	$\eta^{1/2}$	Sin α	∫da
	∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$		∫V	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\int da}{\int V}$	
			·063	245	866	·063	554	866	·063	634	914	·156	·77	921	·25	896	887	·25	896	883	25	·77	924	·25	·917	921	·25	·403	866	·156			·063
2 3/4				1·112	·012		1·084	028		1·072	·084		1·049	·169		1·024	194		1·024	193		[illegible]	·170		[illegible]	·129		1·12	·050				
	·224	875	·094	334	875	·094	·698	875	·094	·826	914	·234	·886	903	·375	[illegible]	829	·375	821	879	·375	[illegible]	879	·375	·821	899	·375	·598	983	·234	·237	883	·094
2 1/2	1·113	017		1·106	·025		1·061	·054		1·038	171		1·025	294		1·009	298		1·004	·210		1·02	·296		1·035	·268		1·07	·115		1·12	·018	
	·234	743	·187	·515	743	·187	·87	731	·187	·97	731	·47	1	695	·75		669	·75	1	669	·75	1	688	·75	·998	631	·75	·845	743	·47	·498	743	·187
2	1·113	·029		1·081	·066		1·029	·116		1·008	330		1·00	·521		1·00	·502		1·00	·502		[illegible]	516		1·001	·474		1·034	·234		1·091	·052	
	·545	515	·25	·736	515	·25	·911	581	·25	·972	545	·625	·997	566	1		560	1	1	559	1	·965	581	1	·98	581	1	·846	552	·625	·616	537	·25
1	1·085	065		1·055	090		1·02	·130		1·007	330		1·00	·565		1·00	566		1·00	559		[illegible]	·572		1·005	570		1·034	282		1·074	·077	
	·512	454	·25	·777	454	·25	·911	454	·25	·972	477	·625	1	·50	1		523	1	1	523	1	1	515	1	1	485	1	·876	·50	·625	·710	462	·25
0	1·089	054		1·047	084		1·02	·101		1·007	·288		1·00	·500		1·00	523		1·00	523		1·00	515		1·00	·485		1·28	·267		1·058	077	
	·515	391	·25	·785	423	·25	·902	423	·25	·96	438	·625	·	401	1	·	47	1	·998	485	1		485	1	·976	454	1	·851	438	·625	·672	407	·25
-1	1·089	·046		1·046	·079		1·22	·093		·99	·261		1·00	·461		1·00	·410		1·001	·484		[illegible]	·435		1·006	·440		1·232	·226		1·266	064	
	·526	326	·25	·745	326	·25	·909	342	·25	·972	326	·625	·	326	1		334	1	1	334	1		358	1	·993	342	1	·921	358	·625	·516	358	·25
-2	1·087	·040		1·054	·058		1·021	·076		1·007	·197		1·00	·326		1·00	·334		1·00	334		[illegible]	·358		1·0	·341		1·039	·177		1·074	·051	
	·465	156	·25	526	105	·25	·768	174	·25	·914	171	·625	1	174	1	·	156	1	1	156	1	·	150	1	·91	174	1	·581	137	·625	·324	137	·25
-3	1·094	·017		1·087	·012		1·042	·032		1·02	·107		1·00	·174		1·00	·165		1·00	·156		1·00	·156		1·00	·172		1·08	·047		1·107	·010	
				·318	$\bar{0}87$	·156	·341	$\bar{0}87$	·156	·554	$\bar{0}18$	·39	·497	$\bar{0}26$	·625	·978	$\bar{0}35$	·625	·982	$\bar{1}13$	·625	·992	$\bar{0}52$	·625	·93	·07	·625	·303	$\bar{0}35$	·39			
-4				1·08	·004		1·035	·004		1·084	·004		1·00	·016		1·004	·022		1·005	·069		1·003	·032		1·005	·043		1·08	·004				
										·304	$\bar{0}52$	·156	·836	$\bar{0}52$	·25	·830	$\bar{0}52$	·25	·804	$\bar{0}90$	·25	·845	$\bar{0}35$	·25	·934	052	·25	·303	~~035~~ 052	·156			
-4 1/4										1·108	·002		1·036	·011		1·036	·011		1·043	·013		1·04	·007		1·036	·010		1·08	·002				
Σ	0·268			0·430			0·634			1·774			3·037			3·085			3·108			3·107			2·932			1·454			0·361		

20·190

TEST:-5a TABLE :-8

STATION	-1¾			-1½			-1¼			-1			-½			0		
	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½	·09	0	·094	·274	0	·094	·491	·105	·094	·712	·139	·141	·617	[illegible]	·187	·551	·017	·187
	1·118	0		1·11	0		1·097	·00445		1·059	·0132		1·074	0		1·083	·001615	
1	·223	0	·125	·376	·07	·125	·769	·242	·125	·862	·156	·187	·744	·017	·25	·758	·087	·25
	1·113	0		1·102	·00298		1·09	·0222		1·03	·0244		1·054	·003		1·051	·0157	
½	·316	$\overline{·017}$	·125	·575	·156	·125	·874	·242	·125	·915	·174	·187	·855	·105	·25	·904	·191	·25
	1·08	·000606		1·048	·00925		1·033	·0256		1·019	·0292		1·032	·0217		1·022	·0421	
0	·505	0	·187	·715	·259	·187	·932	·309	·187	·959	·276	·281	·947	·242	·375	·957	·309	·375
	1·07	0		1·059	·0327		1·016	·0531		1·01	·0737		1·013	·0847		1·01	·11	
-1	·759	·423	·25	·894	·465	·25	·95	·530	·25	·961	·530	·375	·972	·545	·5	·972	·545	·5
	1·051	·0762		1·034	·106		1·012	·1242		1·009	·1895		1·007	·264		1·007	·264	
-2	·61	·819	·187	·867	·777	·187	·91	·766	·187	·944	·777	·281	·961	·766	·375	·965	·766	·375
	1·075	·0869		1·039	·123		1·021	·1275		1·019	·196		1·009	·274		1·009	·275	
-2½	·316	·961	·094	·526	·970	·094	·758	·951	·094	·685	·951	·141	·854	·961	·187	·89	·956	·187
	1·109	·0258		1·087	·0441		1·051	·0644		1·07	·0857		1·032	·1486		1·024	·1555	
-2¾	·09	·988	·063	·331	·985	·063	·56	·978	·063	·505	·98	·094	·689	·99	·125	·759	·99	·125
	1·118	·00501		1·116	·0184		1·083	·0318		1·09	·0426		1·063	·0801		1·051	·0895	
Σ	·195			·336			·453			·654			·877			955		

STATION	½			1			2			3			3½			4			4¼			4½		
	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da	$\eta^{\frac{1}{2}}$	Sin α	∫da
	∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$		∫V	$\eta^{\frac{1}{2}}$ Sin α $\frac{\int da}{\int V}$	
1½	·525	·017	·187	·469	$\overline{·035}$	·281	·455	$\overline{·035}$	·375	·126 ·445	·035	·281	·126 ·445	[illegible]	·187	·515	·259	·141	·524	·233	·094	·601	·191	·141
	1·087	·00154		1·094	·00205		1·114	·00536		1·117	·00111		1·117	·000358		1·085	·01735		1·077	·0121		1·077	·01503	
1	·76	·105	·25	·72	·087	·375	·544	·035	·5	·341	·035	·375	·375	·122	·25	·656	·242	·187	·787	·309	·125	·745	·259	·187
	1·05	·019		1·058	·0222		1·086	·00877		1·105	·00406		1·102	·0104		1·068	·0278		1·045	·0291		1·054	·0342	
½	·906	·208	·25	[illegible]	·208	·375	·875	·225	·5	·682	·156	·375	·682	·191	·25	·81	·259	·187	·856	·326	·125	·861	·309	·187
	1·02	·0461		1·024	·0679		1·028	·096		1·064	·0375		1·064	·0306		1·04	·0376		1·031	·0338		1·03	·0484	
0	·961	·358	·375	·96	·375	·563	·96	·391	·75	·911	·326	·563	·892	·309	·375	·915	·392	·281	·925	·342	·187	·901	·358	·281
	1·009	·128		1·01	·205		1·01	·279		1·02	·164		1·024	·101		1·019	·0864		1·017	·0582		1·022	·0885	
-1	·965	·559	·5	·962	·545	·75	·96	·559	1·0	·962	·574	·75	·94	·574	·5	·924	·545	·375	·901	·595	·25	·858	·545	·375
	1·009	·268		1·009	·39		1·01	·531		1·009	·411		1·04	·266		1·017	·186		1·022	·12		1·031	·17	
-2	·965	·755	·375	·962	·755	·563	·965	·755	·75	·962	·743	·563	·14	·731	·375	·91	·731	·281	·85	·793	·187	·72	·866	·281
	1·009	·271		1·009	·407		1·009	·542		1·009	·398		1·04	·254		1·02	·1832		1·033	·1142		1·057	·166	
-2½	·891	·946	·187	·91	·934	·281	·944	·906	·375	·944	·914	·281	·934	·891	·187	·807	·914	·141	·54	·940	·094	·444	·956	·141
	1·024	·1538		1·02	·234		1·04	·315		1·018	·2384		1·015	·1535		1·04	·1		1·086	·044		1·116	·01705	
-2¾	·785	·988	·125	·887	·962	·187	·865	·974	·25	·892	·966	·187	·891	·951	·125	·644	·978	·094	·228	·985	·063			·094
	1·044	·0927		1·025	·159		1·03	·2024		1·024	·1572		1·025	·1032		1·07	·0552		1·03	·0128				
Σ	·981			1487			1979			1·411			·918			·693			·424			489		

Σ 11·852

TEST :- 5 TABLE :- 4

STATION	$-3\frac{3}{4}$		$-3\frac{1}{2}$		$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$							
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int d\alpha}{\int V}$
			·06	·012	306	·028	40	·084	592	·169	802	·194	802	·193	592	·170	362	·129	162	·050								
$2\frac{3}{4}$			·001		009		·034		·100		155		·155		·101		·047		·008									
	·05	·017	·112	·025	488	·054	·685	·171	787	·294	93	·298	675	·270	84	·296	684	·268	357	·115	·057	·018						
$2\frac{1}{2}$	001		003		·026		117		234		276		182		248		184		·041		·001							
	·053	·029	·265	·066	756	·116	·94	330	1	·521	1	·502	1	·502	1	·516	997	·474	712	·284	237	·062						
2	·0024		·017		088		310		·521		·502		·502		·516		·472		·202		·015							
	·295	·065	541	·090	83	·130	944	·330	995	·565	1	·566	1	·559	97	·572	·96	·570	714	·282	38	·077						
1	·019		·049		108		312		·563		·566		·559		·556		·546		·201		·029							
	·262	·054	·601	·084	83	·101	·944	·288	1	·500	1	·523	1	·523	1	·515	1	·485	766	·267	511	·079						
0	·014		·051		·084		·274		·500		·523		·523		·515		·485		·204		·040							
	·265	·046	·617	·079	815	·093	926	·261	1	·461	1	·47	995	·484	1	·485	·955	·440	725	·226	452	·064						
-1	·012		·049		·076		·242		·461		·470		·482		·485		·420		·164		·029							
	·277	·040	·555	·058	827	·076	·944	·197	1	·326	1	·334	1	·334	1	·358	·995	·341	674	·177	381	·051						
-2	011		·032		·063		·186		326		·334		·334		358		339		119		·020							
	·216	·017	·277	·012	59	·032	833	·107	1	·174	1	·165	1	·156	1	·156	995	·172	337	·047	105	·010						
-3	·004		003		·019		·089		·174		·165		·156		·156		·171		·016		·001							
			·101	·004	·116	·004	306	·004	995	·016	975	·022	962	·069	985	·032	·96	·043	·092	·004								
-4			000		000		·001		·016		·021		·066		·032		·041		·000									
							·093	·002	·7	·011	7	·011	644	·018	714	·007	696	·010	·092	·002								
$-4\frac{1}{4}$							·000		·007		·007		·012		·005		·007		·000									
Σ	0·063		0·205		0·473		1·565		2·902		3·019		2·971		2·972		2·712		0·955		0·135							

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{\int d\alpha}{\int V}$$

17·972

TEST :- 5A. TABLE :- 4

ATION	$-1\frac{3}{4}$		$-1\frac{1}{2}$		$-1\frac{1}{4}$		-1		$-\frac{1}{2}$		0		$\frac{1}{2}$		1		2		3		$3\frac{1}{2}$		4		$4\frac{1}{4}$		$4\frac{1}{2}$	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$
	·008	0	075	0	241	·004	508	·013	38	0	305	·002	275	·002	22	·002	207	·005	·016	·001	016	0	265	·017	353	·012	361	·015
$1\frac{1}{2}$	0		0		·001		·007		0		·001		·001		000		·001		000		0		·005		·004		·005	
	·05	0	141	·003	59	022	742	024	552	·003	572	016	576	·019	518	·022	296	·009	116	·004	141	·01	431	·028	62	·029	552	·034
1	0		·000		·013		·018		·002		·009		·011		·011		·003		·005		·001		·012		·018		·019	
	·1	·001	265	·009	762	·026	838	029	73	022	812	·042	83	·046	798	068	766	096	466	·038	466	·031	658	·038	736	·034	741	·048
$\frac{1}{2}$	·000		·002		·02		·0248		·016		·034		·038		·054		·073		·018		·014		·025		·025		·036	
	255	0	51	033	87	·053	92	·074	898	·085	92	·11	926	·128	92	·205	92	·279	83	·164	798	·101	839	·086	854	·058	812	·089
0	0		·017		·046		·068		·075		·101		·118		·188		·257		·136		·08		·072		·049		·072	
	576	·076	·8	106	902	·124	926	·189	946	264	946	264	93	·268	926	·39	92	531	926	411	882	266	854	186	812	12	737	17
-1	·044		·085		·112		·175		·25		·25		·25		·361		·488		·38		·235		·159		·097		·013	
	372	·087	756	·123	825	·128	838	196	926	274	93	275	93	271	93	407	93	542	926	398	882	254	83	183	721	·114	519	·116
-2	·032		·093		·103		·164		·254		·256		·252		·378		·505		·368		·224		·152		·082		·06	
	·1	026	275	·044	576	·064	47	·086	73	·149	796	·156	795	·154	83	·234	882	315	89	238	872	153	852	·1	293	·044	·02	·017
$-2\frac{1}{2}$	·003		·012		·037		·04		·11		·124		·122		·195		·278		·212		·133		·065		·013		·003	
	·008	005	·11	·018	313	032	255	043	475	08	576	·09	617	·093	788	159	75	·202	798	·157	795	103	414	·055	·052	·013		
$-2\frac{3}{4}$	·000		·002		·01		·011		·038		·052		·057		·125		·152		125		·082		·023		·007.			
Σ	·079		·101		·142		·507		·745		·827		·849		1·312		1·757		1·244		·769		·513		·295		·108	

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$$

Σ 9·248

TEST :- 5 TABLE :- 5

STATION	-3¾			-3½			-3¼			-3			-2			-1			0			1			2			3			3¼											
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A
	$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$			$\eta\cos\alpha\sin\alpha\frac{fda}{fV}$		
				245	.5	.012	554	.5	.028	634	407	.084	77	391	.169	896	462	.194	896	47	.193	77	383	.170	602	391	.129	403	.5	.050												
2¾				.001			.008			.022			.051			.080			.081			.050			.030			.010														
	224	485	.017	334	485	.025	698	485	.054	826	407	.171	886	431	.294	965	559	.298	821	477	.270	916	477	.296	827	438	.268	598	47	.115	239	47	.018									
2½	.002			.004			.018			.058			.112			.160			.106			.130			.097			.032			.002											
	234	669	.029	515	669	.066	87	682	.116	97	682	.330	1	719	.521	1	743	.502	1	743	.502	1	725	.516	998	682	.474	845	669	.284	488	669	.062									
2	.005			.023			.069			.218			.375			.373			.373			.375			.322			.161			.020											
	545	857	.065	736	857	.090	911	814	.130	972	839	.330	997	824	.565	1	824	.566	1	829	.559	985	819	.572	98	814	.570	896	834	.282	616	843	.077									
1	.030			.057			.096			.269			.465			.466			.464			.456			.455			.199			.040											
	512	891	.054	777	891	.084	911	891	.101	972	879	.288	1	866	.500	1	853	.523	1	853	.523	1	857	.515	1	875	.485	876	866	.267	716	887	.079									
0	.025			.058			.082			.246			.433			.446			.446			.441			.425			.202			.050											
	515	921	.044	785	906	.079	903	906	.097	961	.899	.261	1	887	.461	1	883	.470	998	875	.486	1	875	.485	976	841	.460	851	899	.226	672	914	.064									
-1	.022			.056			.076			.225			.410			.415			.422			.425			.384			.173			.039											
	526	946	.040	745	946	.058	909	94	.076	972	946	.197	1	946	.326	1	943	.334	1	943	.334	1	934	.358	998	94	.341	821	934	.177	616	939	.051									
-2	.020			.041			.065			.181			.308			.315			.315			.334			.320			.136			.029											
	465	988	.017	526	995	.012	768	985	.032	914	982	.107	1	985	.174	1	986	.165	1	988	.156	1	988	.156	998	985	.172	581	99	.047	324	99	.010									
-3	.008			.006			.024			.096			.172			.163			.154			.154			.169			.027			.003.											
				318	$\bar{9}96$	.004	341	$\bar{9}96$	.004	554	$\bar{1}$	.004	997	$\bar{1}$	.016	988	$\bar{9}99$	.022	982	$\bar{9}94$	.069	992	$\bar{9}99$	.032	98	998	.043	303	$\bar{9}99$	.004												
-4				.001			.001			.002			.016			.022			.067			.032			.042			.001														
										304	999	.002	836	999	.011	836	$\bar{9}99$	.011	804	$\bar{9}95$	.018	845	$\bar{9}99$	.007	834	999	.010	303	999	.002												
-4¼										.001			.009			.009			.014			.006			.008			.001														
Σ	0.112			0.247			0.439			1.318			2.351			2.449			2.442			2.403			2.252			0.942			0.183											

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}.$$

15.138

TEST :- 5A TABLE :- 5

STATION	$-1\frac{3}{4}$	$-1\frac{1}{2}$	$-1\frac{1}{4}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	$3\frac{1}{2}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$
	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A	$\eta^{\frac{1}{2}}$ bad A
	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$	η bad $\sin\alpha \frac{fda}{fV}$
	09 1 0	274 1 0	491 995 004	712 99 013	617 1 0	551 999 002	525 999 002	469 999 002	455 999 005	126 999 001	126 999 0	515 966 017	594 973 012	·601 982 015
$1\frac{1}{2}$	0	0	·002	·009	0	·001	·001	·001	·002	·000	0	·008	·007	·009
	223 1 0	376 998 003	769 97 022	·862 988 024	744 999 003	758 996 016	76 995 019	72 996 022	544 999 009	341 999 004	375 993 01	656 97 ·028	767 951 029	745 966 034
1	0	·001	·016	·02	·002	·012	·014	·016	·005	·001	·004	·018	·022	·024
	316 999 001	515 987 009	874 97 026	915 985 029	855 995 022	901 982 042	906 978 046	893 978 068	875 974 096	682 988 038	682 982 031	81 966 038	856 946 034	861 951 048
$\frac{1}{2}$	·000	·005	·022	·026	·019	037	·041	·058	·082	·026	·021	·03	·028	·039
	505 1 0	715 966 033	932 951 053	959 961 074	947 97 085	959 951 ·11	961 934 128	96 927 205	·96 921 279	911 946 164	892 951 101	915 94 086	925 94 058	901 934 089
0	0	023 ·029	·047	·068	·078	·1	·115	·181	·246	·141	·086	·074	·05	·075
	759 906 076	894 875 106	95 848 124	961 848 189	972 839 264	972 839 264	965 829 268	962 839 39	96 829 531	962 819 411	94 819 266	924 839 186	901 839 12	858 839 17
-1	·052	·083	·1	·154	·216	·216	·214	·316	·422	·324	·204	·144	·091	·122
	·61 574 087	869 629 123	·91 643 128	914 629 196	961 643 274	965 643 275	965 656 271	965 656 407	965 656 542	962 669 398	94 682 254	91 682 183	85 669 114	72 ·5 116
-2	·03	·067	·075	·112	·169	·171	·171	·257	·343	·256	·163	·114	·065	·042
	·316 276 026	525 242 044	758 304 064	665 309 086	854 276 149	891 292 156	891 326 154	91 358 234	94 423 315	944 407 238	934 454 153	807 407 ·1	541 342 044	191 292 017
$-2\frac{1}{2}$	·002	·006	·015	·018	·035	·04	·045	·076	·125	·092	·065	·033	·008	0
	·09 156 005	331 174 018	56 208 032	505 199 043	689 139 ·08	759 139 ·09	785 156 ·093	887 191 159	865 225 202	892 259 157	891 309 103	644 208 055	228 174 013	
$-2\frac{3}{4}$	0	·001	·004	·004	·008	·009	·011	·027	·039	·036	·028	·007	·000	
Σ	·084	·186	·281	·411	·527	·586	·612	·832	1·264	·876	·371	·408	·271	·311

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$.

Σ 7·020

TEST :- 6 TABLE :- 3

STATION	−3¾			−3½			−3¼			−3		
	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda
	fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$	
2¾				·2 ~~·987~~	·883	063	·601 ~~·905~~	883	063	·73	906	156
				1·136	·010		1·04	·031		[illegible]	·089	
2½				·328 ~~·97~~	866	094	·792 ~~·857~~	799	094	·901	824	234
				1·124	·024		1·058	·056		1·025	·169	
2	·214 ~~·987~~	643	187	·636 ~~·994~~	649	187	·855 ~~·79~~	629	187	·971	643	47
	1·135	·023		1·064	·071		1·024	·098		1·007	·292	
1	·378 ~~·961~~	515	25	·751 ~~·855~~	53	25	·93 ~~·81~~	515	25	·977	545	625
	1·12	·044		1·062	·094		1·018	·118		1·006	·332	
0	·485 ~~·939~~	·47	25	·821 ~~·827~~	·47	25	·946 ~~·943~~	47	25	·987	454	625
	1·108	·051		1·04	·092		1·04	·109		1·004	·278	
−1	·565 ~~·946~~	47	25	·801 ~~·834~~	438	25	·903 ~~·77~~	454	25	·951	47	625
	1·09	·060		1·05	·084		1·04	·100		1·013	·276	
−2	·354 ~~·980~~	259	25	·678 ~~·66~~	309	25	·862 ~~·87~~	326	25	·941	334	625
	1·123	·020		1·07	·049		[illegible]	·068		[illegible]	·194	
−3	·318 ~~·771~~	069	25	·474 ~~·94~~	069	25	·746 ~~·856~~	148	25	·92	174	625
	1·127	·005		1·09	·007		[illegible]	·026		1·02	·098	
−4				·214 ~~·987~~	$\overline{165}$	156	·363 ~~·965~~	$\overline{165}$	156	·608	$\overline{035}$	39
				1·35	·005		1·2	·008		1·089	·008	
−4¼							·308 ~~·974~~	$\overline{225}$	063	·547	$\overline{105}$	156
							1·128	·004		1·099	·008	
Σ	0·203			0·436			0·618			1·744		

STATION	−2			−1			0			1		
	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda
	fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$	
2¾	·768	903	25	·89	87	25	·898	866	25	·761	921	25
	1·059	·164		1·28	·189		1·26	·190		1·059	·166	
2½	·972	799	375	1	731	375	1	749	375	·99	777	375
	1·007	·290		1·00	·274		1·00	·281		1·003 ~~1·00~~	·288	
2	1	643	75	1	629	75	1	629	75	1	643	75
	1·00	·481		1·00	·471		1·00	·471		1·00	·482	
1	1	574	1	1	574	1	1	574	1	1	574	1
	1·00	·574		1·00	·574		1·00	·574		1·00	·574	
0	1	508	1	1	515	1	1	53	1	1	515	1
	1·00	·508		1·00	·515		1·00	·53		1·00	·515	
−1	·996	454	1	1	454	1	1	477	1	·995	462	1
	1·001	·452		1·00	·454		1·00	·477		1·009	·460	
−2	·996	326	1	1	326	1	1	342	1	·997	358	1
	1·001	·324		1·00	·326		1·00	·342		1·001	·357	
−3	1	139	1	1	174	1	1	148	1	1	174	1
	1·00	·139		1·00	·174		1·00	·148		1·00	·174	
−4	·996	·018	625	·986	·139	625	·92	$\overline{263}$	625	·98	$\overline{096}$	625
	1·001	·011		1·009	·083		1·02	·150		1·005	·059	
−4¼	·992	·018	25	·97	$\overline{122}$	25	·882	$\overline{259}$	25	·98	$\overline{13}$	25
	1·002	·004		1·008	·029		1·03	·055		1·05	·032	
Σ	2·947			3·089			3·218			3·107		

STATION	2			3			3¼		
	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda	$\eta^{1/2}$	Sin α	fda
	fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$		fV	$\frac{\eta^{1/2}\,\mathrm{Sin}\,\alpha\,\mathrm{fda}}{\mathrm{fV}}$	
2¾	·612	921	25	·354	·839	156			
	1·088	·130		1·23	·041				
2½	·935	839	375	·619	·866	234			
	1·017	·290		1·088	·123				
2	·99	656	75	·851	669	47	·455	682	187
	1·013	·485		1·038	·258		1·12	·052	
1	·995	574	1	·874	552	625	·613	559	·25
	1·001	·570		1·033	·292		1·088	·079	
0	·995	5	1	·877	462	625	·755	477	25
	1·00	·497		1·031	·246		1·07	·084	
−1	·973	462	1	·826	454	625	·624	454	25
	1·007	·448		1·04	·224		1·086	·065	
−2	·972	375	1	·735	309	625	·5	309	25
	1·007	·363		1·065	·133		1·106	·035	
−3	·992	148	1	·612	156	625	·354	122	25
	1·002	·146		1·088	·055		1·124	·010	
−4	·976	$\overline{035}$	625	·245	$\overline{07}$	39			
	1·007	·022		1·183	·006				
−4¼	·972	·044	25	·245	·044	156			
	1·006	·011		1·133	·001				
Σ	2·962			1·379			0·325		

Σ 20·028

20·028

TEST:- 6A   TABLE :- 3

STATION	−1¾			−1½			−1¼			−1			−½		
	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$
	$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$	
1½			·094	·3	·087	·094	·126	·123	·094	·79	·105	·141	·651	·018	·187
				1·128	·002		1·086	·007		1·053	·011		1·081	·002	
1	·295	·018	·125	·452	·139	·125	·79	·242	·125	·899	·174	·187	·761	·035	·25
	1·128	·001		1·113	·007		1·058	·023		1·026	·028		1·059	·006	
½	·327	·052	·125	·579	·174	·125	·856	·276	·125	·886	·225	·187	·851	·105	·25
	1·126	·002		1·105	·010		1·037	·029		1·028	·036		1·038	·022	
0	·495	·122	·187	·71	·259	·187	·919	·342	·187	·93	·292	·281	·995	·259	·375
	1·106	·010		1·07	·032		1·022	·057		1·018	·075		1·014	·090	
−1	·741	·407	·25	·89	·485	·25	·935	·53	·25	·93	·545	·375	·961	·545	·5
	1·064	·071		1·028	·105		1·016	·122		1·018	·187		1·01	·260	
−2	·559	·848	·187	·842	·788	·187	·908	·755	·187	·89	·777	·281	·961	·766	·375
	1·097	·081		1·04	·120		1·024	·125		1·028	·189		1·01	·274	
−2½	·397	·985	·094	·457	·978	·094	·144	·961	·094	·543	·961	·141	·744	·985	·187
	1·119	·033		1·144	·036		1·13	·012		1·099	·067		1·063	·128	
−2¾			·063	·122	·999	·063	·314	1·00	·063	·34	1·00	·094	·377	·999	·125
				1·14	·007		1·127	·018		1·131	·022		1·121	·042	
Σ	0·198			0·319			0·393			0·615			0·824		

STATION	0			½			1			2			3		
	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$
	$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$	
1½	·508	·018	·187	·574	·018	·187	·495	·052	·281	·3	·087	·375	·26	·105	·281
	1·098	·002		1·094	·002		1·106	·007		1·128	·009		1·131	·007	
1	·774	·07	·25	·802	[illegible]	·25	·744	·087	·375	·56	·00	·50	·364	·017	·375
	1·057	·013		1·05	·023		1·063	·023		1·097	·000		1·12	·002	
½	·902	·191	·25	·912	·242	·25	·892	·208	·375	·881	·225	·50	·711	·191	·325
	1·025	·042		1·022	·054		1·027	·068		1·03	·096		1·07	·048	
0	·961	·209	·375	·961	·342	·375	·961	·375	·563	·959	·358	·75	·906	·309	·563
	1·01	·110		1·01	·122		1·01	·201		1·01	·255		1·023	·154	
−1	·97	·545	·5	·961	·565	·5	·952	·559	·75	·952	·574	1·0	·942	·559	·75
	1·008	·262		1·01	·260		1·013	·395		1·013	·536		1·015	·388	
−2	·968	·766	·375	·961	·766	·375	·955	·766	·563	·959	·766	·75	·946	·755	·563
	1·008	·276		1·01	·274		1·012	·405		1·011	[illegible]		1·014	·408	
−2½	·796	·982	·187	·814	·978	·187	·87	·970	·281	·876	·961	·375	·89	·961	·281
	1·05	·139		1·047	·142		1·034	·229		1·031	·306		1·028	·234	
−2¾	·428	·998	·125	·505	·999	·125	·401	·999	·187	·459	·999	·25	·67	·999	·187
	1·15	·048		1·105	·057		1·09	·103		1·11	·103		1·078	·116	
Σ	0·892			0·934			1·431			1·849			1·357		

STATION	3½			4			4¼			4½		
	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$	$\eta_{\frac{1}{2}}$	$\sin\alpha$	$\int da$
	$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$		$\int V$	$\eta_{\frac{1}{2}}\sin\alpha\frac{\int da}{\int V}$	
1½	·296	·087	·187	·636	·225	·141	·676	·156	·094	·575	·122	·141
	1·128	·004		1·084	·019		1·077	·009		1·094	·009	
1	·521	·156	·25	·765	·292	·187	·851	·309	·125	·772	·208	·187
	1·103	·018		1·059	·040		1·038	·032		1·056	·028	
½	·742	·174	·25	·848	·292	·187	·875	·292	·125	·856	·259	·187
	1·063	·030		1·039	·045		1·022	·031		1·037	·040	
0	·912	·309	·375	·935	·358	·281	·928	·383	·187	·896	·375	·281
	1·022	·103		1·017	·093		1·019	·065		1·026	·092	
−1	·94	·539	·5	·921	·545	·375	·9	·545	·25	·961	·545	·375
	1·016	·259		1·02	·185		1·026	·120		1·036	·171	
−2	·944	·753	·375	·91	·766	·281	·836	·799	·187	·678	·806	·281
	1·014	·264		1·023	·192		1·041	·120		1·078	·160	
−2½	·902	·94	·187	·707	·966	·141	·931	·972	·094			·141
	1·025	·155		1·071	·090		1·128	·076				
−2¾	·713	·993	·125	·384	·995	·094			·063			·094
	1·09	·082		1·12	·032							
Σ	0·915			0·696			0·453			0·500		

Σ 11·376

TEST :- 6 TABLE :- 4

STATION	$-3\frac{3}{4}$		$-3\frac{1}{2}$		$-3\frac{1}{4}$		-3		-2		-1		0		1		2		3		$3\frac{1}{4}$							
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{fda}{fV}$
			04	·040	·361	·031	453	089	59	164	792	189	807	190	58	166	375	130	125	041								
$2\frac{3}{4}$			·000		·011		040		·097		·150		·153		096		·049		·005									
			107	024	628	056	812	169	947	290	1	274	1	281	98	288	876	290	377	123								
$2\frac{1}{2}$			003		035		·137		274		·274		·281		·282		·254		046									
	046	·023	405	071	·73	098	947	292	1	481	1	471	1	471	1	482	98	485	722	258	207	052						
2	·001		029		·071		·276		·481		·471		·471		482		·476		·186		·011							
	142	·044	564	094	866	·118	955	332	1	574	1	574	1	574	1	574	991	570	762	292	375	079						
1	·006		·053		·102		·317		574		·574		·574		574		·565		·222		·030							
	235	051	674	092	·897	109	975	278	1	508	1	515	1	530	1	515	991	497	77	246	57	084						
0	·012		·062		·098		·271		·508		·515		·530		515		494.		·189		048							
	32	·060	642	084	·82	·100	905	276	991	452	1	454	1	477	991	460	948	448	681	224	39	065						
1	019		054		·082		·250		448		454		·477		456		·425		·152		·025							
	125	·020	46	049	·744	068	887	194	991	324	1	326	1	342	995	357	945	363	54	133	25	035						
2	·003		023		·051		·172		321		326		·342		356		·343		072		·009							
	·101	·005	226	007	·557	026	845	098	1	139	1	174	1	148	1	174	986	146	375	055	125	010						
3	·001		·002		·015		·083		139		174		·148		·174		·144		·021		·001.							
			046	005	·131	008	37	008	991	011	935	083	848	150	96	059	951	022	·06	006								
4			·000		·001		·003		·011		·078		·127		057		·021		·000									
					095	004	3	008	985	004	94	029	778	055	96	032	945	011	·06	001								
$4\frac{1}{4}$					·000		·002		·004		·027		·043		·031		·010		·000									
Σ	0·042		0·226		0·466		1·551		2·857		3·043		3·146		3·023		2·781		0·893		0·124							

$$A = \eta^{\frac{1}{2}} \sin\alpha \frac{\int da}{\int V}$$

$\Sigma = 18 \cdot 152$

18·152

TEST :- 6A. TABLE :- 4

STATION	$-1\frac{3}{4}$		$-1\frac{1}{2}$		$-1\frac{1}{4}$		-1		$-\frac{1}{2}$		0		$\frac{1}{2}$		1		2		3		$3\frac{1}{2}$		4		$4\frac{1}{4}$		$4\frac{1}{2}$	
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$	$\eta^{\frac{3}{2}}\sin\alpha$	$\frac{f da}{f V}$
$1\frac{1}{2}$			·09	·002	392	007	·425	·011	·425	·002	·311	·002	[illegible]	·002	245	007	·09	·009	·068	·007	·088	·004	·405	·019	·456	·009	·33	·009
			·000		003		007		001		000		001		·002		·001		·000		·000		008		·004		·003	
1	·087	·001	205	·007	·525	·023	[illegible]	·028	[illegible]	·006	571	·013	[illegible]	·023	552	023	314	·000	147	·002	272	·018	584	·040	724	032	597	·028
	·000		·001		014		023		004		008		·015		013		·000		000		005		·023		·023		·017	
$\frac{1}{2}$	·107	·002	28	·010	·735	·029	[illegible]	·036	·25	022	[illegible]	042	832	·054	777	068	777	·096	507	·048	55	·030	72	·045	765	·031	·735	·040
	·000		·003		·021		029		016		·034		045		·054		074		·024		017		·032		·024		·029	
0	·245	·010	507	·032	·845	·057	[illegible]	·075	·592	·070	[illegible]	·110	727	·122	727	·201	12	·255	824	·154	832	·103	873	·093	861	·065	807	·092
	·002		016		·048		065		·081		102		·113		186		235		·126		·086		·081		·056		·074	
-1	·55	·071	·791	·105	871	·122	[illegible]	·187	[illegible]	·260	[illegible]	·262	927	·260	[illegible]	·395	707	·536	87	·388	881	·259	85	·185	807	·120	·742	·171
	·039		083		·106		162		241		248		·241		358		·486		345		·228		·159		·097		·126	
-2	·312	·081	71	·120	822	·125	[illegible]	·187	[illegible]	·274	[illegible]	·276	927	274	·2	·405	72	·544	877	·408	89	·264	825	·192	·7	·120	447	·160
	·025		·085		103		·149		·254		258		·254		368		500		·366		234		·158		·084		·071	
$-2\frac{1}{2}$	·157	·033	[illegible]	·036	·117	·012	246	·067	552	·128	[illegible]	·139	661	·142	[illegible]	·229	77	·306	771	·234	313	·155	·5	·090	·87	·076		
	·005		007		·002		020		071		089		·094		·173		·236		·185		126		·045		·066			
$-2\frac{3}{4}$			·015	·007	[illegible]	·018	·7	·022	·42	·042	·34	·048	255	·057	·2	·103	21	·103	4·8	·116	509	082	147	·032				
			·000		002		002		·006		·009		·015		·037		·022		·046		·042		·005					
Σ	0·071		0·195		·299		0·457		0·674		0·748		0·778		1·191		1·554		1·092		0·738		0·509		0·354		0·320	

$$\dot{A} = \eta^{\frac{1}{2}} \sin\alpha \, \frac{f da}{f V}$$

Σ 8·980

8·980

TEST :- 6 TABLE :- 5

Position	-3¾			-3½			-3¼			-3			-2			-1			0			1			2			3			3¼		
	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A	$\eta^{\frac{1}{2}}$	$\cos\alpha$	A
	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$	$\eta\cos\alpha$	$\sin\alpha$	$\frac{fd\alpha}{\Sigma V}$
				·2	469	010	601	469	031	673	423	089	768	431	164	89	492	189	898	·5	190	761	391	166	612	391	130	359	545	041			
2¾				·001			009			·025			054			083			·085			·049			·031			·008					
				326	·5	026	792	602	056	901	566	169	974	602	290	1	662	276	1	663	281	99	629	288	925	595	290	417	5	123			
2½				·004			·027			097			·170			187			·186			179			148			·038					
	414	766	023	636	76	071	656	777	098	971	766	292	1	766	481	1	777	471	1	777	471	1	766	482	99	755	485	951	793	258	455	731	052
2	·004			·034			·065			·218			·369			·366			·366			·370			·362			·163			·017		
	276	857	044	751	848	094	93	857	118	977	839	332	1	819	574	1	819	574	1	819	574	1	819	574	995	819	570	874	839	292	413	879	079
1	·014			·060			·094			·272			·470			·470			·470			·470			·464			·213			·040		
	465	83	051	821	83	092	946	83	109	967	871	278	1	862	508	1	857	515	1	898	53	1	857	515	995	866	497	771	887	246	755	879	084
0	·021			·063			·086			·245			·438			·442			·450			·442			·430			·192			·056		
	526	83	060	801	899	084	905	891	100	951	83	276	996	891	452	1	891	454	1	879	477	995	887	460	973	887	446	626	891	224	624	891	066
-1	·028			·061			·081			·218			·402			·404			·420			·405			387			·165			·036		
	359	966	020	676	951	049	822	966	068	941	943	194	996	996	324	1	996	326	1	99	342	997	934	357	972	927	363	725	957	133	5	959	035
-2	007			·032			055			172			·306			308			·322			·332			·328			·093			·017		
	315	998	005	674	998	007	796	989	026	92	965	098	1	99	139	1	965	174	1	989	148	1	985	174	992	967	146	412	986	056	354	993	010
-3	·002			·003			·019			·089			·138			·172			·147			·172			·143			·033			·004		
				214	986	005	362	966	008	685	999	008	996	1	011	966	99	083	92	964	150	98	985	059	976	999	022	265	996	006			
-4				·001			003			005			011			·079			·133			·058			·022			·001					
							300	979	004	517	995	008	992	1	004	97	993	029	962	966	055	98	999	032	972	999	011	205	999	001			
-4¼							·001			·004			·004			·028			·047			·031			·011			·000					
Σ	0·076			0·259			0·440			1345			2362			2539			2626			2508			2326			0906			0170		

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fd\alpha}{\Sigma V}$

$\Sigma = 15 \cdot 557$

TEST:- 6A TABLE:- 5

STATION	$-1\frac{3}{4}$	$-1\frac{1}{2}$	$-1\frac{1}{4}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3	$3\frac{1}{2}$	4	$4\frac{1}{4}$	$4\frac{1}{2}$
	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A	$\eta^{\frac{1}{2}}$ / cos∝ / A
	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$	η cos∝ / Sin∝ / $\frac{fda}{fV}$
$1\frac{1}{2}$	999	3 / 996 / 002	[illegible] / 993 / 007	77 / 995 / 011	[illegible] / 994 / 002	[illegible] / 994 / 002	[illegible] / [illegible] / 002	445 / 99 / 007	3 / 996 / 009	26 / 995 / 007	[illegible] / 996 / 004	636 / 974 / 019	[illegible] / 988 / 009	676 / 993 / 009
		·001	·004	·009	·001	·001	·001	·003	·003	·002	·001	·012	·006	·005
1	295 / 999 / 001	452 / 990 / 007	77 / 970 / 023	711 / 985 / 028	[illegible] / 999 / 006	714 / 998 / 013	302 / 993 / 023	141 / 996 / 023	56 / 1 / 000	384 / 999 / 002	521 / 988 / 018	765 / 986 / 040	851 / 981 / 032	792 / 978 / 228
	·000	003	·018	·025	·005	·010	·018	·017	·000	·001	·009	·029	·026	·021
$\frac{1}{2}$	327 / 999 / 002	529 / 985 / 010	[illegible] / 961 / 029	[illegible] / 974 / 036	[illegible] / 985 / 024	17 / 982 / 042	[illegible] / 97 / 054	[illegible] / [illegible] / 068	[illegible] / 914 / 096	711 / 982 / 048	742 / 985 / 030	848 / 956 / 045	858 / 936 / 031	957 / 966 / 040
	·001	·005	·024	·031	·019	·037	·048	·059	·082	·034	·022	·036	·026	·033
0	485 / 993 / 010	71 / [illegible] / 032	[illegible] / [illegible] / 057	[illegible] / 95 / 075	[illegible] / [illegible] / 090	[illegible] / 937 / 110	[illegible] / [illegible] / 122	[illegible] / 927 / 201	[illegible] / 934 / 255	[illegible] / [illegible] / 154	[illegible] / [illegible] / 133	935 / 934 / 093	28 / [illegible] / 065	878 / 927 / 092
	·005	·022	·049	·067	·082	·101	·110	·179	·228	·133	·089	·081	·056	·077
-1	741 / 914 / 071	89 / 875 / 105	135 / 849 / 122	13 / 801 / 187	[illegible] / [illegible] / 260	[illegible] / 831 / 262	[illegible] / 831 / 262	152 / 829 / 375	152 / 811 / 536	[illegible] / 829 / 388	14 / 829 / 259	911 / 839 / 185	910 / 839 / 120	861 / 839 / 171
	·049	·082	·097	·146	·210	·213	·209	·312	·417	·302	·201	·143	·091	·124
-2	557 / 53 / 081	[illegible] / [illegible] / 120	[illegible] / [illegible] / 125	[illegible] / [illegible] / 187	[illegible] / [illegible] / 274	169 / [illegible] / 276	[illegible] / [illegible] / 274	155 / 643 / 405	157 / 643 / 544	[illegible] / 656 / 408	[illegible] / 656 / 264	91 / 643 / 192	886 / [illegible] / 120	678 / 423 / 160
	·024	·062	·075	·106	·169	·172	·169	·248	·334	·254	·164	·112	·061	·046
$-2\frac{1}{2}$	[illegible] / 74 / 033	437 / [illegible] / 036	[illegible] / 276 / 012	[illegible] / 276 / 067	[illegible] / [illegible] / 128	[illegible] / [illegible] / 139	[illegible] / [illegible] / 142	[illegible] / 342 / 229	[illegible] / 276 / 306	[illegible] / 276 / 234	[illegible] / 342 / 155	771 / 259 / 090	931 / 225 / 076	931
	·002	·003	·000	·010	·017	·021	·024	·048	·074	·058	·048	·017	·016	
$-2\frac{3}{4}$		122 / [illegible] / 007	214 / 18 / 018	[illegible] / [illegible] / 022	[illegible]	[illegible] / [illegible] / 048	[illegible] / [illegible] / 057	[illegible] / [illegible] / 103	45 / [illegible] / 103	[illegible] / 035 / 116	[illegible] / 122 / 082	384 / 105 / 032		
		·000	·000	·000	·001	·001	·001	·003	·002	·003	·007	·001		
Σ	0·081	0·178	0·267	0·394	0·504	0·556	0·580	0·869	1·140	0·787	0·541	0·431	0·282	0·306 -

$A = \eta^{\frac{1}{2}} \sin\alpha \frac{fda}{fV}$.

6·916

Σ = 6· 9166

Series 3

Groups 1 and 2

TABLE OF OBSERVED READINGS OF PRESSURE AND ANGLE.

GROUP :- 1

p_1 = LB/IN² Ab.

BAROMETRIC PRESS =

p_2 = LB/IN² Ab.

TEMP. °

STATIONS	$-2\frac{3}{4}$		$-2\frac{1}{2}$		$-2\frac{1}{4}$		-2		-1		0		$\frac{1}{2}$		$\frac{3}{4}$		1		$1\frac{1}{4}$		$1\frac{1}{2}$							
P_2(IN HG)	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
18	30.7	102	36.3	105	40.6	113	44	129	43	148	42.6	159	36.1	172	29.9	179	25.9	180	23.7	183	33.4	175						
10	12.2	92	23.1	105	26.7	113	32.3	123	38.2	148	38.4	157	38	165	35	173	32.5	177	30.4	180	36.9	170						
5	18.9	95	24.9	105	35	125	37.3	134	33.7	157	30	162	36	161	36	167	34	172	33.5	173	36.7	164						
1	21	95	34.6	118	32.8	135	31.9	140	24.5	157	20.3	160	19.3	167	19.1	155	19.6	150	30.3	161	19.3	157						

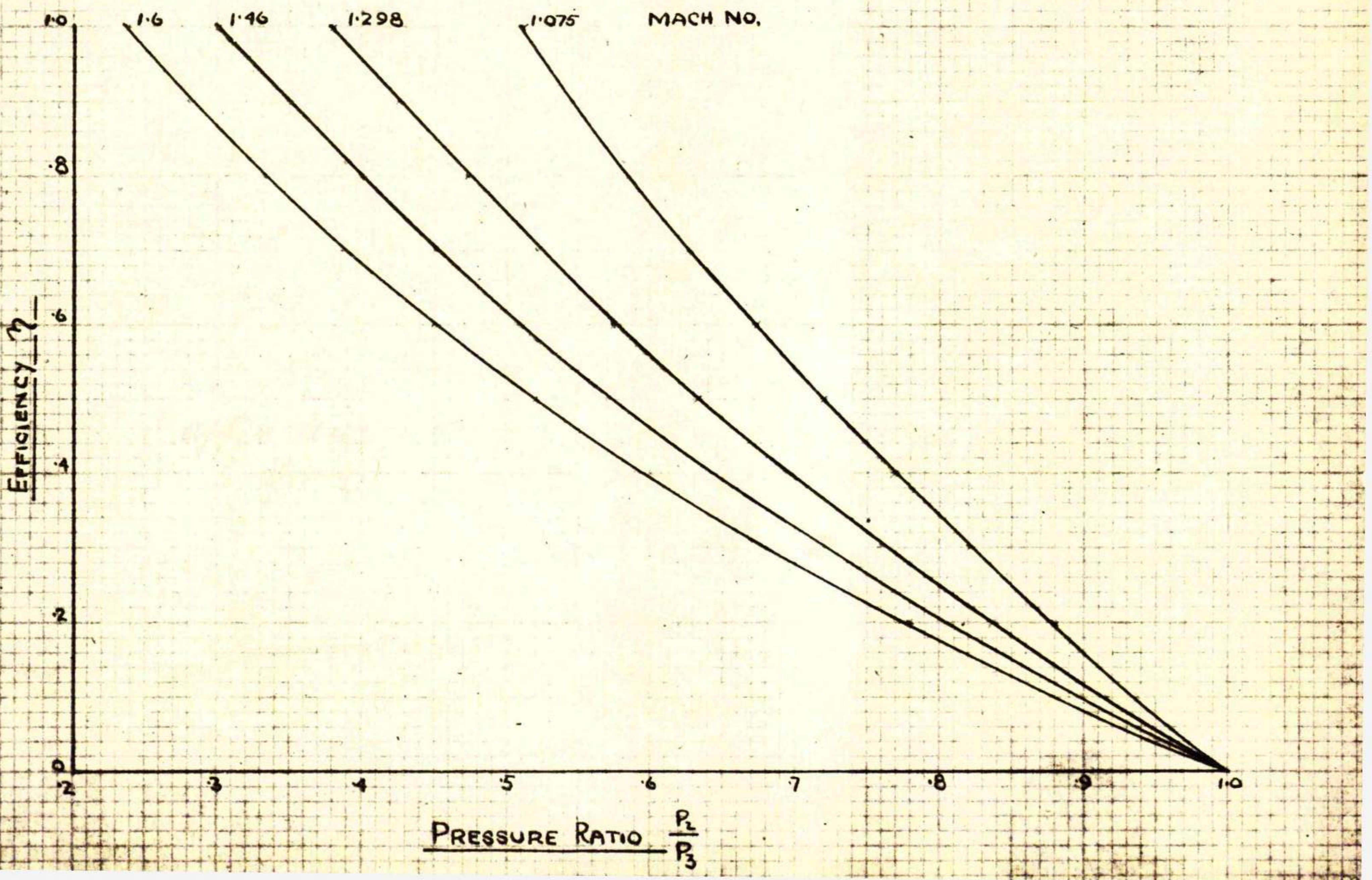
Test. No 1. Vertical Traverse Slot 1¼.

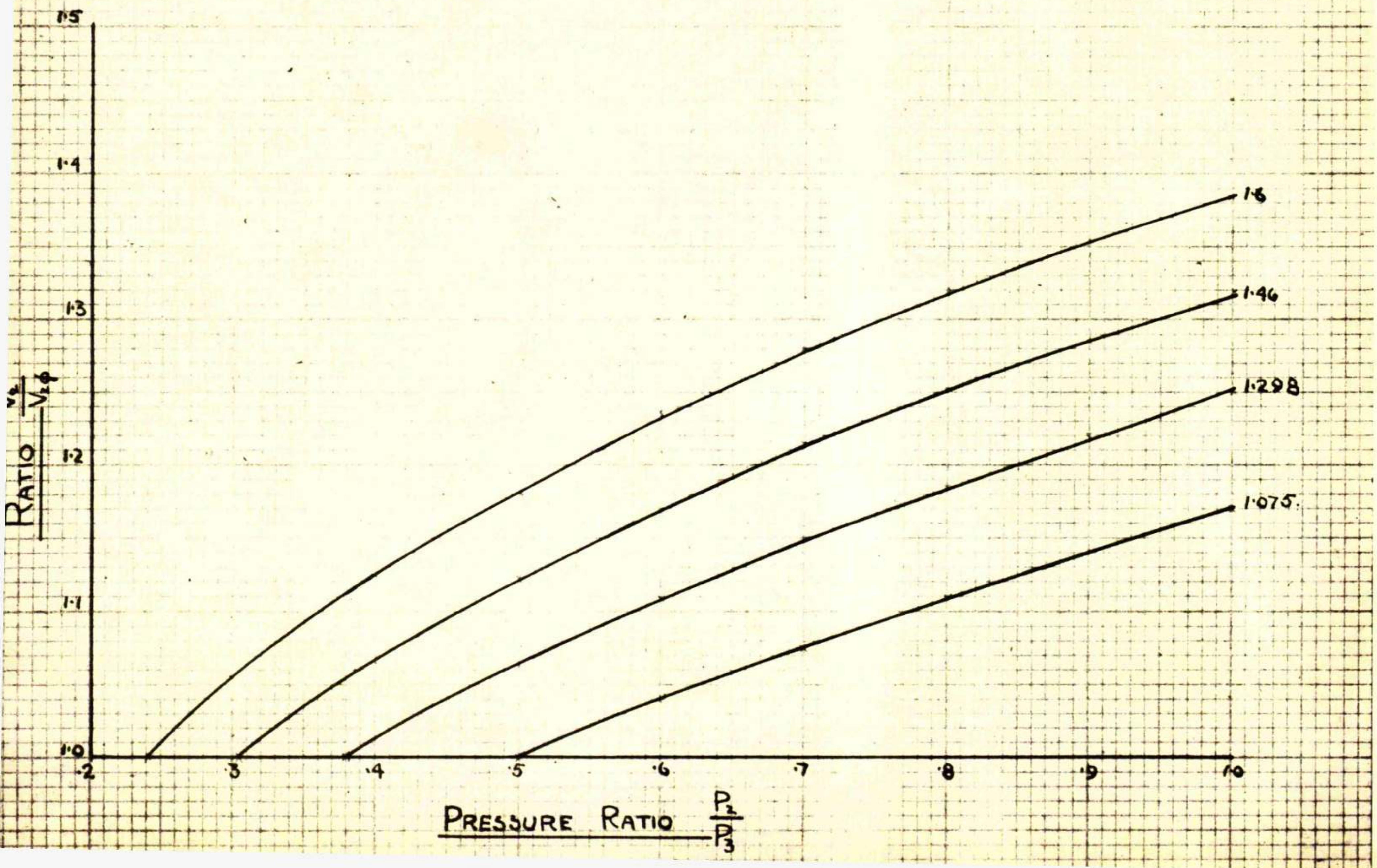
RATIO OF $\frac{\text{BACK PRESSURE}}{\text{PRESSURE READING}}$ AND CONVERTED ANGLE

GROUP :- 1

P_1= LB/IN² Ab. BAROMETRIC PRESS = P_2= LB/IN² Ab. TEMP. °F

STATIONS	-2¾		-2½		-2¼		-2		-1		0		½		¾		1		1¼		1½		1¾					
	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α
p_2=32.5	·718	85	·64	62	·591	45	·554	40	·563	23	·568	20	·641	23	·678	23	·732	25	·805	30	·851	19						
p_2=24.5	·918	85	·651	75	·595	55	·523	46	·466	23	·464	18	·467	19	·476	16	·495	13	·521	8	·546	7						
p_1=19.5	·585	88	·495	75	·394	67	·377	57	·405	32	·438	23	·386	15	·38	10	·386	7	·403	3	·407	0						
p_2=15.5	·437	78	·316	75	·328	67	·334	51	·378	32	·446	21	·458	8	·458	5	·46	1	·455	0	·316	-3	·326					


1·6
1·46
1·298
1·075
MACH NO.
1·0
·8
·6
·4
·2
0
·2
·3
·4
·5
·6
·7
·8
·9
1·0
EFFICIENCY η
PRESSURE RATIO $\frac{P_2}{P_3}$



1·5
1·4
1·3
1·2
1·1
1·0
Ratio $\frac{V_2}{V_{2\phi}}$
1·6
1·46
1·298
1·075
·2
·3
·4
·5
·6
·7
·8
·9
1·0
Pressure Ratio $\frac{P_2}{P_3}$

READINGS OF CONVERTED PRESSURE RATIO AND LOCAL EFFICIENCY

GROUP:- 1

BAROMETRIC PRESS =

P_1 = LB/IN^2 Ab P_2 = LB/IN^2 Ab TEMP = °F

STATIONS	$-2\frac{3}{4}$		$-2\frac{1}{2}$		$-2\frac{1}{4}$		-2		-1		0		$\frac{1}{2}$		$\frac{3}{4}$		1		$1\frac{1}{4}$		$1\frac{1}{2}$	
M_{2f}	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
1.075	.718	.5	.64	.67	.591	.785	.559	.894	.563	.851	.568	.845	.641	.665	.678	.59	.732	.48	.805	.33	.851	.25
1.298	.918	.095	.651	.47	.595	.565	.5223	.693	.4622	.818	.46	.821	.4632	.816	.473	.798	.4932	.75	.52	.7	.596	.65
1.46	.585	.478	.4932	.623	.3845	.825	.3639	.871	.3955	.805	.432	.74	.3743	.855	.3675	.868	.3743	.851	.3932	.812	.3976	.805
1.6	.431	.632	.2895	.875	.3048	.865	.3128	.84	.3877	.7	.4405	.616	.4535	.596	.4535	.596	.4558	.594	.45	.6	.3278	.81

VERTICAL TRAVERSE

GROUP No 1

CALCULATION OF LOCAL EFFICIENCY & VELOCITY

$u_{2\rho}$	$v_{2\rho}$	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½	
		$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v	$\eta^{1/2}$	v
·9	1492																						
·97	1590																						
1·075	1807	·707	1277	·82	1480	·886	1600	·945	1708	·924	1668	·92	1660	·815	1471	·768	1388	·693	1250	·575	1039	·5	902
1·298	2120	·031	65·6	·685	1451	·752	1591	·833	1764	·904	1916	·895	1897	·891	1888	·894	1892	·866	1838	·836	1771	·806	1710
1·46	2319	·691	1600	·79	1830	·908	2102	·934	2160	·896	2075	·861	1994	·925	2140	·931	2158	·924	2139	·901	2085	·898	2079
1·6	2545	·796	2027	·936	2385	·925	2356	·916	2335	·836	2132	·785	2000	·771	1963	·771	1963	·77	1962	·774	1970	·9	2292

GROUP N°1 TABLE :- 3.

STATION	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½				Σ			
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
M=4	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$	$\eta^{3/2}\sin\alpha$	$\frac{\int da}{\int V}$
1.075	·707	·1626	·82	·1719	·886	·148	·945	·3715	·924	·351	·92	·2292	·815	·1122	·768	·0903	·693	·0759	·575	·0696	·5	·0361						
	·115		·1408		·1312		·351		·324		·2106		·0914		·054		·0526		·03718		·01805				1·52583.			
1.29	·0308	·0063	·685	·1465	·752	·143	·833	·348	·904	·3375	·895	·1985	·891	·1041	·894	·0556	·866	·0436	·836	·0272	·806	·0227						
	·000194		·1001		·1078		·29		·3018		·1775		·093		·04915		·03785		·02275		·0183.				1·19844			
1.46	·691	·151	·79	·1708	·908	·1981	·934	·4775	·896	·459	·861	·2328	·925	·0965	·931	·03866	·924	·01957	·901	·01109	·898	0.						
	·1086		·1349		·18		·4455		·411		·2006		·0892		·03606		·0134		·0100		0				1·62926			
1.6	·796	·1743	·936	·2165	·925	·2015	·916	·4178	·836	·397	·785	·1838	·771	·0348	·771	·01455	·77	·00291	·774	0	·9	−ve ·0097						
	·1388		·203		·1863		·3837		·332		·1442		·02685		·01122		·002242		0		−ve. ·00873				1·91958			
Σ																												

$$A = \eta^{\frac{3}{2}} \sin\alpha \, \frac{\int da}{\int V}$$

TABLE SHOWING CALCULATION OF $\Sigma\, \eta_t^{\frac{1}{2}} \sin\alpha \frac{\int da}{\int V}$.

GROUP:- N°1 TABLE :- ψ, K & η_p.

STATION	$-2\frac{3}{4}$	$-2\frac{1}{2}$	$-2\frac{1}{4}$	-2	-1	0	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$		Σ	K_ϕ	$\eta_{B\phi}$
	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	$\eta_t^{\frac12}$ Sinα $\int da$	
Mach	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	$\int V$ $\eta_t^{\frac12}$ Sin$\alpha \frac{\int da}{\int V}$	
	·757 ·9962 ·25	·82 [illegible] ·25	·886 ·7071 ·25	[illegible] ·6428 ·625	·924 ·3907 1	·92 ·3420 ·75	·815 ·3907 ·375	·768 ·3907 ·25	[illegible] ·9224 ·25	·575 ·5 ·25	·5 ·3256 ·25			✓	✓
1·075	[illegible] ·1626	[illegible] ·1714	[illegible] ·148	[illegible] ·3715	[illegible] ·351	[illegible] ·2292	[illegible] ·1122	[illegible] ·07025	[illegible] ·0759	1·11 ·0646	[illegible] ·0361.		1·79275	·921	·849
	[illegible] ·9962 ·25	[illegible] ·9659 ·25	·762 ·8192 ·25	[illegible] ·7193 ·625	[illegible] ·3907 1	[illegible] ·3090 ·75	·891 ·3264 ·375	[illegible] ·2756 ·25	·866 ·225 ·25	·834 ·1392 ·25	·806 ·1219 ·25				
1·298	[illegible] ·00626	1·13 ·1465	1·105 ·143	1·074 ·348	1·077 ·3375	[illegible] ·1985	[illegible] ·1041	1·108 ·05555	[illegible] ·0436	[illegible] ·0272	1·103 ·02268		1·46889	·903.	·815.
	·691 ·9994 ·25	·79 ·9659 ·25	·908 ·9205 ·25	·934 ·8387 ·625	[illegible] ·5299 1	[illegible] ·3907 ·75	·925 ·2588 ·375	·931 ·1737 ·25	[illegible] ·1219 ·25	·901 ·0523 ·25	[illegible] 0 ·25				
1·46	[illegible] ·151	[illegible] ·1708	[illegible] ·1981	[illegible] ·4775	[illegible] ·459	[illegible] ·2328	1·05 ·0965	[illegible] ·03865	1·05 ·01451	[illegible] ·01109	1·12 0		1·84995	·938.	·88
	·796 ·9782 ·25	[illegible] ·9659 ·25	·925 ·9205 ·25	·916 ·7772 ·625	[illegible] ·5299 1	·785 ·3684 ·75	·771 ·1392 ·375	·771 ·0872 ·25	·77 ·0175 ·25	[illegible] 0 ·25	·9 ·0523 ·25				
1·6	1·12 ·1743	[illegible] ·2165	[illegible] ·2015	[illegible] ·4178	1·17 ·397	1·15 ·1838	[illegible] ·0348	[illegible] ·01555	[illegible] ·002913	[illegible] 0	[illegible] −ve. ·0097		1·63346	·931	·868
Σ															

Group 2.

TABLE OF OBSERVED READINGS OF PRESSURE AND ANGLE

GROUP - 2

BAROMETRIC PRESS = ______

p_1 = LB/IN² Ab. p_2 = LB/IN² Ab. TEMP. °F

STATIONS	$-2\frac{3}{4}$		$-2\frac{1}{2}$		$-2\frac{1}{4}$		-2		-1		0		$\frac{1}{2}$		$\frac{3}{4}$		1		$1\frac{1}{4}$		$1\frac{1}{2}$							
p_2 (lb/in²)	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α	p_3	α
31.8	37.7	110	38	125	38	131	37.9	135	37.6	146	37.3	160	34.9	167	33.4	174	32.7	178	32.2	182	–	–						
28.75	36.4	109	37.7	120	37.6	126	37.9	133	37	146	37.2	160	34.7	167	32.8	172	30.9	175	29.6	180	–	–						
25.55	34.2	104	37.1	118	37.3	128	37	134	36.8	147	36.4	160	32.2	170	29.9	174	28.4	177	26.8	182	25.7	185						
22.11	33.3	103	36.4	114	37.1	126	37	133	36.2	147	36	161	32.1	170	28.8	175	26.1	178	24.2	182	22.9	186						
18.55	28	97	33.5	107	36.1	122	36.2	131	35.3	147	35.3	161	32.1	168	28.9	173	24.7	177	22.9	182	21.2	182						
16.13	27.1	98	33	107	35.7	121	36	131	35.3	147	35.3	160	30.7	169	27.9	174	24	178	20.4	182	18.8	184						
12	16.8	96	23.3	102	29.2	108	23	121	34.8	147	35	159	30.9	171	28.7	172	25.9	170	21.4	180	19.1	183						
8	12.7	97	18.8	106	23	114	28.7	126	33	145	32.8	157	32.6	167	30.9	172	29	176	25.9	180	23.3	181						
4	10.2	91	14	104	28.5	123	30.9	137	29.6	154	31.6	158	30.4	171	30.3	165	30.1	167	28.6	174	26.3	177						

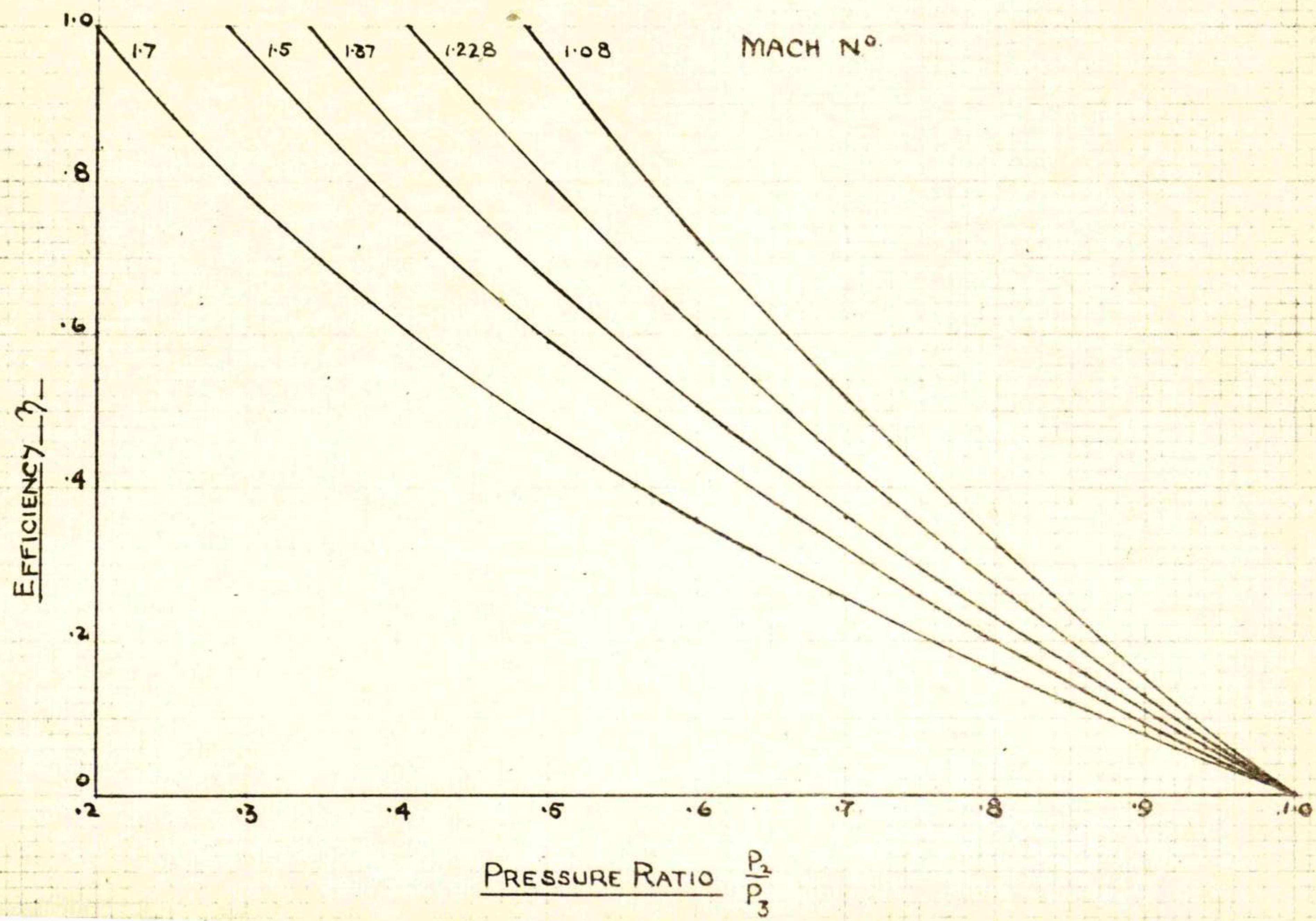
RATIO OF $\frac{\text{BACK PRESSURE}}{\text{PRESSURE READING}}$ AND CONVERTED ANGLE

GROUP :- 2

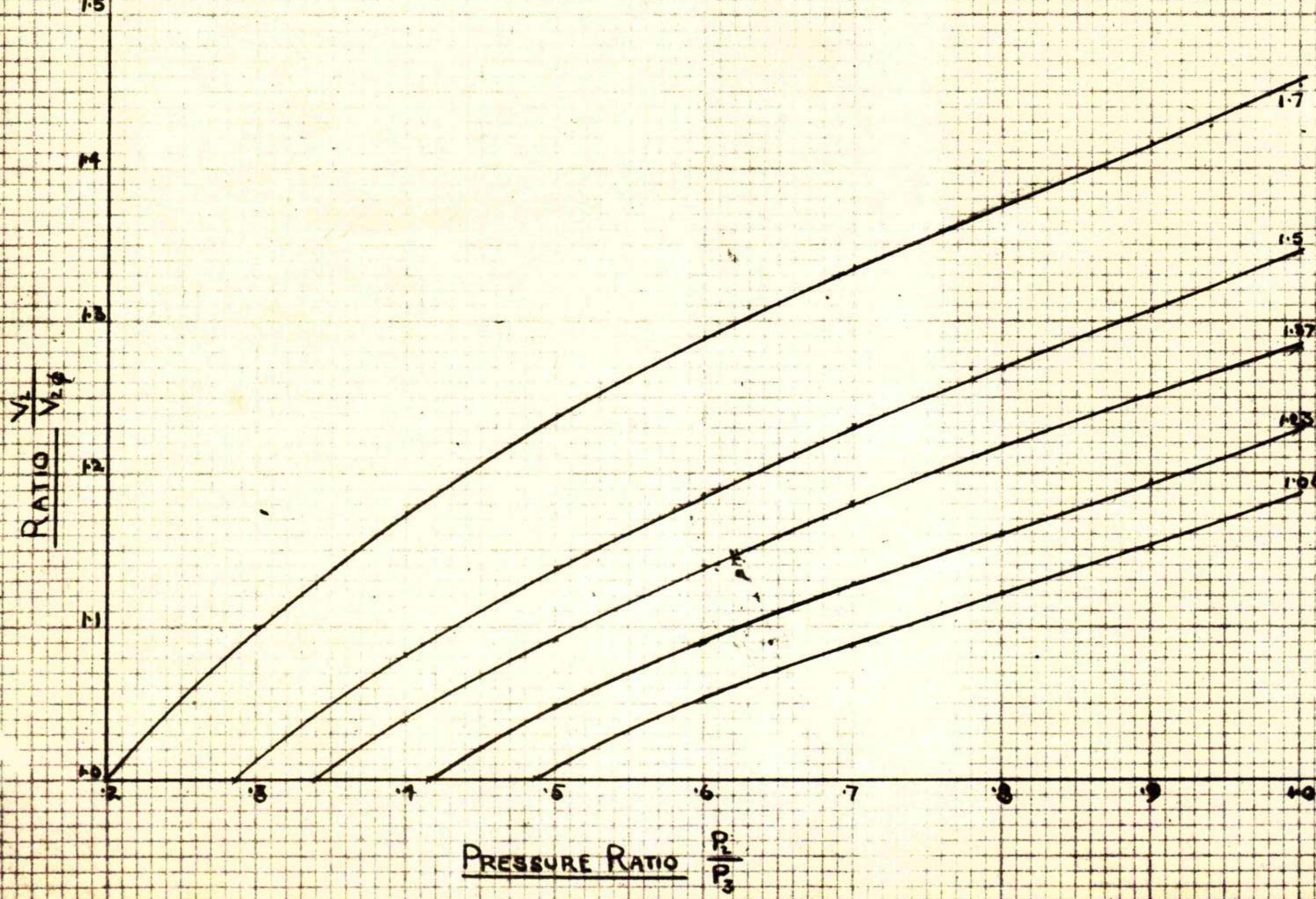
BAROMETRIC PRESS =

$p_1 =$ LB/IN² Ab $p_2 =$ LB/IN² Ab. TEMP. °F

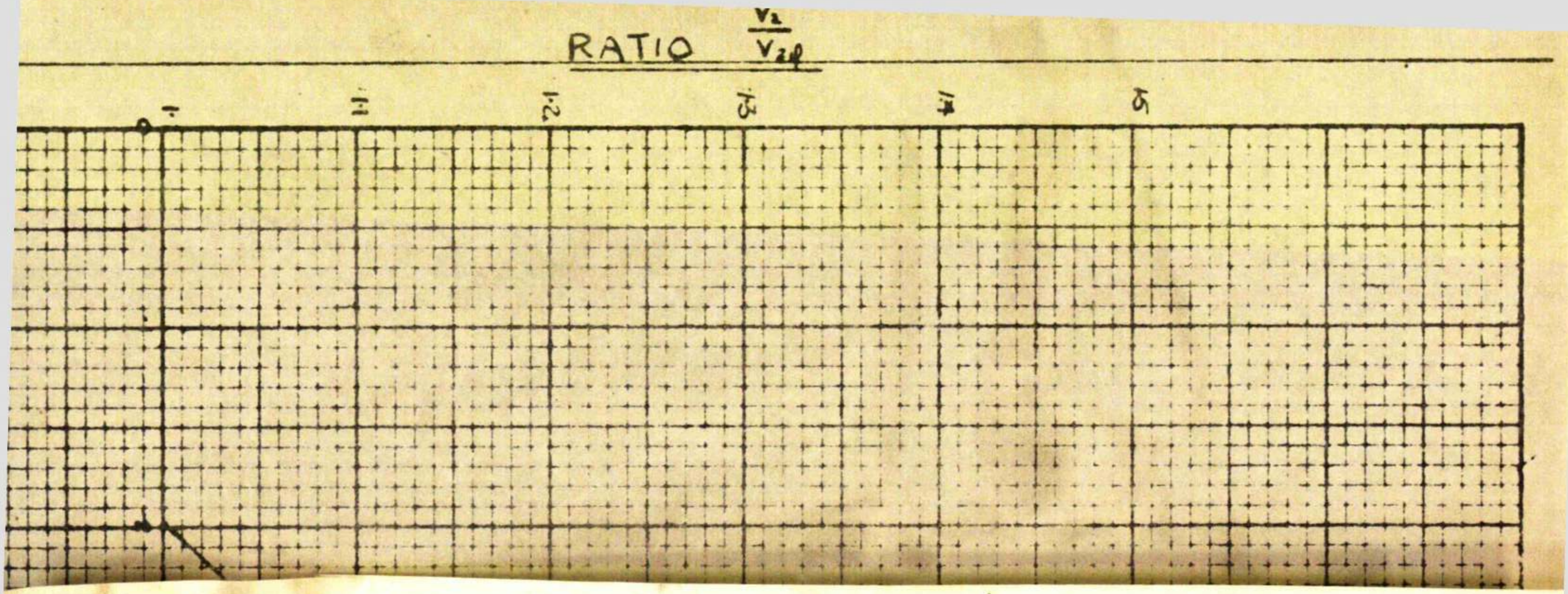
$M_{2\phi}$	STATIONS	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½							
		p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α	p_2/p_5	α
·5	p_2 = 46·25	·886	70	·882	55	·882	49	·884	45	·888	34	·894	20	·937	11	·966	6	·98	2	·99	−2	—							
·6	p_2 = 43·2	·85	71	·828	60	·831	54	·826	47	·841	34	·836	20	·885	13	·915	8	·954	5	·981	0	—	·						
·7	p_2 = 40	·821	76	·775	62	·773	52	·778	46	·78	33	·787	20	·856	10	·903	6	·934	3	·969	−2	·996	−5						
·8	p_2 = 36·56	·765	77	·719	66	·708	54	·71	47	·72	33	·724	19	·784	10	·845	5	·899	2	·945	−2	·977	−6						
·9	p_2 = 33·0	·777	83	·689	73	·651	58	·65	49	·663	33	·663	19	·708	12	·764	7	·849	3	·883	−2	·926	−2						
1·08	p_2 = 26·5	·847	84	·702	78	·606	72	·558	59	·538	33	·535	21	·584	9	·613	7	·658	4	·739	0	·788	−3						
1·228	p_2 = 22·5	·828	83	·675	74	·6	66	·521	·54	·479	35	·475	23	·477	13	·496	8	·518	4	·558	0	·596	−1						
1·37	p_2 = 18·5	·748	89	·552	76	·431	57	·407	43	·42	26	·401	22	·413	9	·414	15	·415	13	·43	6	·454	3						
1·5	p_2 = 15·5	·507	87	·384	67	·358	48	·363	41	·394	21	·44	17	·46	18	·453	22	·374	19	·358	11	·374	6						

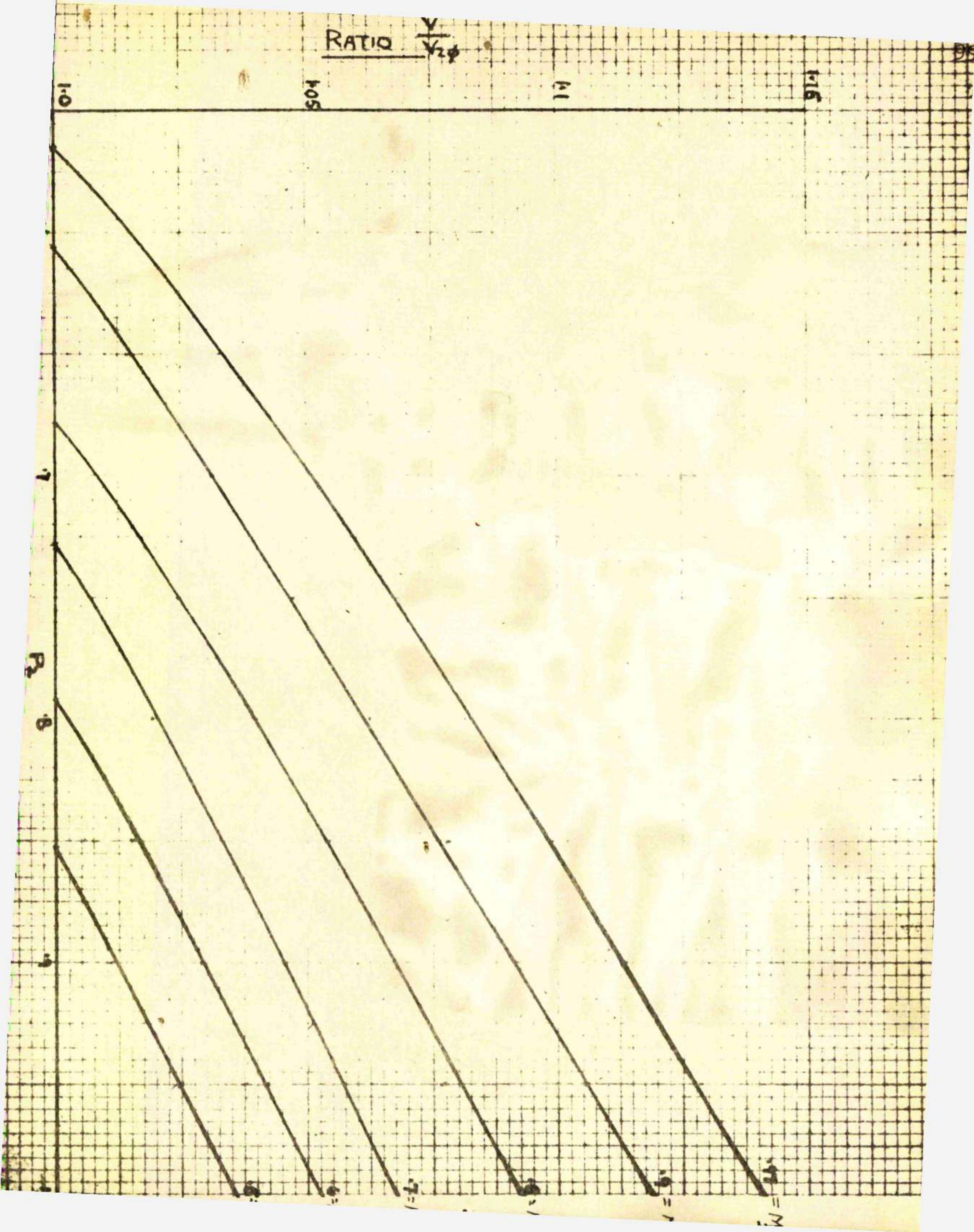


1·0
·8
·6
·4
·2
0
·2
·3
·4
·5
·6
·7
·8
·9
1·0
1·7
1·5
1·37
1·228
1·08
MACH N°
EFFICIENCY η
PRESSURE RATIO $\frac{P_2}{P_3}$



1·5
1·4
1·3
1·2
1·1
1·0
Ratio $\frac{V_1}{V_{2\phi}}$
·2
·3
·4
·5
·6
·7
·8
·9
1·0
Pressure Ratio $\frac{P_1}{P_3}$
1·7
1·5
1·37
1·23
1·08


RATIO $\frac{V_2}{V_{2\theta}}$
1·
1·1
1·2
1·3
1·4
1·5



RATIO V/V2φ
1.0
1.05
1.1
1.15
.7
.8
.9

Test N 2. Vertical traverse

READINGS OF CONVERTED PRESSURE RATIO AND LOCAL EFFICIENCY

GROUP:- 2

BAROMETRIC PRESS =

P_1 = LB/IN^2 Ab P_2 = LB/IN^2 Ab TEMP °F

STATIONS	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½	
M_{2d}	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η	$\frac{P_2}{P_3}$	η
·5	·886	·75	·882	·781	·882	·781	·884	·761	·888	·73	·894	·7	·937	·42	·966	·215	·98	·12	·99	·052	–	–
·6	·85	·713	·828	·818	·831	·81	·826	·834	·841	·76	·836	·785	·885	·545	·915	·399	·959	·211	·981	·082	–	–
·7	·821	·651	·775	·836	·773	·85	·778	·82	·78	·815	·787	·799	·856	·515	·903	·34	·934	·221	·969	·095	·996	·011
·8	·765	·69	·719	·842	·708	·876	·71	·872	·72	·84	·724	·826	·784	·629	·845	·435	·899	·265	·945	·139	·977	·057
·9	·777	·54	·689	·768	·651	·88	·65	·881	·663	·84	·663	·84	·708	·711	·764	·57	·849	·352	·883	·27	·926	·168
1·08	·847	·25	·702	·515	·606	·714	·558	·815	·538	·865	·535	·876	·584	·75	·613	·698	·658	·606	·739	·447	·788	·352
1·228	·828	·232	·675	·42	·6	·609	·520	·76	·471	·866	·472	·865	·474	·864	·494	·805	·517	·775	·558	·69	·595	·61
1·37	·748	·29	·552	·588	·424	·82	·398	·87	·412	·895	·391	·884	·404	·86	·405	·857	·407	·856	·423	·82	·449	·766
1·5	·5056	·583	·372	·82	·342	·877	·348	·868	·383	·795	·434	·704	·456	·666	·448	·678	·361	·837	·342	·879	·361	·837

TEST N°2

n₂ℓ	V₂ℓ	−2¾		−2½		−2¼		−2		−1		0		½		¾		1		1¼		1½									
		η½	v	η½	v	η½	v	η½	v	η½	v	η½	v	η½	v	η½	v	η½	v	η½	v	η½	v								
5	880	866	763	884	776	884	776	874	769	854	769	836	736	649	570	464	407	347	305	228	201	0	0								
·	1048	845	885	905	947	·9	942	914	956	871	912	886	927	739	774	632	662	46	582	286	300	0	0								
7	1208	806	972	914	1100	922	1111	905	1092	903	1090	895	1080	718	865	583	704	47	566	308	372	105	127								
8	1352	831	1125	919	1244	936	1270	935	1269	916	1240	909	1230	793	1072	66	895	515	696	373	505	239	324								
9	1520	735	1115	876	1330	938	1423	94	1429	916	1390	916	1370	844	1280	755	1146	594	900	52	790	41	623								
97	1620	748	1211	·871	1411	·934	1511	·938	1520	·925	1500	·925	1500	·835	1352	·765	1240	·648	1050	·494	800	·394	639								
08	1794	5	897	718	1289	845	1518	903	1620	93	1670	936	1681	860	1555	835	1499	778	1299	669	1200	594	1065								
228	1978	481	951	685	1354	·777	1538	871	1721	93	1840	93	1840	73	1840	896	1772	881	1741	831	1642	78	1541								
37	2165	539	1167	766	1660	905	1960	933	2019	92	1971	94	2035	926	2004	925	2004	925	2004	706	1962	875	1894								
·5	2315	765	1770	905	2093	936	2166	931	2155	872	2062	839	1940	815	1888	822	1900	914	2115	936	2168	914	2115								

GROUP:- 2 TABLE :- 3

ION	$-2\frac{3}{4}$		$-2\frac{1}{2}$		$-2\frac{1}{4}$		-2		-1		0		$\frac{1}{2}$		$\frac{3}{4}$		1		$1\frac{1}{4}$		$1\frac{1}{2}$				Σ			
	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A	η	A
	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$	$\eta^{3/2}\sin\alpha$	$\frac{f d\alpha}{\Sigma V}$
	·75	·202	·781	·1798	·781	·1652	·761	·3838	·73	·4740	·7	·2131	·42	·0955	·216	·01179	·12	·00293	·052	-ve ·00237	—	0						
5	·1515		·1403		·1291		·292		·346		·14917		·01910		·002535		·0003518		-ve ·0001231		0				1·2299			
	·713	·1967	·818	·1936	·81	·1715	·834	·4142	·76	·481	·785	·2245	·546	·0608	·399	·02132	·211	·0096	·082	0	—	0						
6	·1401		·1583		·1453		·345		·3659		·1761		·03318		·00851		·002025		0		0				1·3744			
	·651	·1907	·834	·1989	·85	·1792	·82	·40	·815	·1815	·799	·2255	·516	·0962	·34	·01456	·221	·00582	·095	-ve ·00268	·011	-ve ·0215						
7	·1241		·1665		·15225		·328		·3945		·1800		·0233		·00495		·001289		-ve ·00024		-ve ·00002365				1·3806			
	·69	·1970	·842	·2065	·876	·1869	·872	·1220	·84	·4910	·826	·2181	·629	·0498	·435	·01365	·265	·00921	·139	-ve ·00302	·057	-ve ·0056						
8	·1360		·1740		·1633		·3685		·418		·1805		·03125		·00594		·001119		-ve ·00042		-ve ·000319				1·4729			
	·54	·1730	·768	·2034	·88	·1958	·881	·4355	·84	·1877	·84	·2188	·711	·0636	·57	·02185	·352	·0072	·27	-ve ·003	·168	-ve ·0226						
9	·0934		·1562		·1723		·3841		·41		·1840		·0453		·01248		·002535		-ve ·00081		-ve ·000485				1·4590.			
	·25	·11	·515	·1615	·714	·1919	·815	·468	·865	·4936	·876	·2455	·75	·0487	·618	·0291	·606	·0127	·447	0	·362	-ve ·00695						
08	·0275		·0831		·137		·3815		·426		·2153		·03646		·01683		·0077		0		-ve ·002445				1·32896			
	·232	·102	·47	·1472	·604	·1628	·76	·4155	·866	·5155	·865	·2633	·864	[illegible]	·806	·0299	·775	·0146	·9	0	·61	-ve ·00314						
288	·02365		·0694		·0983		·316		·4465		·2278		·0655		·02405		·01132		0		-ve ·001915				1·28061			
	·29	·1145	·588	·1662	·82	·1805	·87	·3835	·845	·386	·884	·256	·86	·0535	·857	·0576	·856	·05	·82	·0226	·766	·01072						
·37	·0332		·0976		·1479		·3335		·326		·226		·046		·04945		·0428		·01854		·00823				1·32922			
	·583	·1675	·82	·1965	·877	·1662	·868	·364	·795	·2982	·704	·167	·666	·0848	·678	·0693	·837	[illegible]	·79	·0428	·837	·0226						
·5	·0976		·1603		·1458		·3164		·2372		·1175		·05655		·04695		·0589		·03765		·0189				1·29375			
Σ																												

$$A = \eta^{\frac{1}{2}}\sin\alpha\,\frac{f d\alpha}{\Sigma V}$$

TABLE SHOWING CALCULATION OF $\Sigma \eta^{\frac{1}{2}} \sin\alpha \frac{f\,da}{f\,V}$

GROUP:- 2. TABLE :- H, η_B + K.

STATION	$-2\frac{3}{4}$			$-2\frac{1}{2}$			$-2\frac{1}{4}$			-2			-1		
	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$
$M_{2\varphi}$	$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$	
	·8655	·9397	·25	·884	·8192	·25	·884	·7547	·25	·874	·7071	·625	·854	·5592	1
·5	1·008	·202		1·0075	·1798		1·0075	·1652		1·0076	·3838		1·009	·4740	
	·845	·9455	·25	·905	·866	·25	·9	·8090	·25	·914	·7314	·625	·871	·5592	1
·6	1·017	·1967		1·011	·1936		1·0115	·1795		1·01	·4142		1·014	·4810	
	·806	·9703	·25	·914	·883	·25	·922	·7880	·25	·905	·7193	·625	·903	·5446	1
·7	1·026	·1907		1·0135	·1989		1·013	·1792		1·015	·400		1·015	·4845	
	·831	·9744	·25	·919	·9136	·25	·936	·8090	·25	·935	·7314	·625	·916	·5446	1
·8	1·029	·1970		1·015	·2065		1·0115	·1864		1·012	·4220		1·015	·4910	
	·735	·9926	·25	·8755	·9563	·25	·938	·8481	·25	·94	·7597	·625	·916	·5446	1
·9	1·054	·1730		1·03	·2034		1·0151	·1958		1·015	·4355		1·02	·4877	
	·5	·9945	·25	·718	·9782	·25	·845	·9611	·25	·903	·8572	·625	·93	·5446	1
1·08	1·129	·11		1·088	·1615		1·051	·1919		1·033	·468		1·025	·4936	
	·981	·9926	·25	·685	·9613	·25	·777	·9136	·25	·871	·8090	·625	·93	·5736	1
1·228	1·17	·102		1·118	·1472		1·09	·1628		1·058	·4155		1·034	·5165	
	·539	·9999	·25	·766	·9703	·25	·905	·8387	·25	·933	·682	·625	·92	·4384	1
1·37	1·2	·1145		1·119	·1662		1·05	·1805		1·037	·3835		1·096	·386	
	·765	·9986	·25	·905	·9205	·25	·936	·7431	·25	·931	·6561	·625	·892	·3584	1
1·5	1·14	·1675		1·066	·1955		1·095	·1662		1·05	·364		1·071	·2982	
Σ															

STATION	0			$\frac{1}{2}$			$\frac{3}{4}$			1			$1\frac{1}{4}$			$1\frac{1}{2}$		
	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$
$M_{2\varphi}$	$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$	
	·836	·3420	·75	·699	·1908	·375	·469	·1045	·25	·3965	·0349	·25	·2281	-ve ·0349	·25	0	·	·25
·5	1·01	·2131		1·021	·04545		1·028	·01179		1·032	·002125		1·035	-ve ·00237		0	—	
	·886	·3420	·75	·739	·225	·375	·634	·1392	·25	·46	·0872	·25	·2861	0	·25	0		·25
·6	1·0125	·2245		1·0255	·0608		1·0325	·02132		1·0425	·0096		1·099	0		0	—	
	·8945	·3420	·75	·718	·1737	·375	·583	·1045	·25	·47	·0523	·25	·3081	-ve ·0349	·25	·105	-ve ·0872	·25
·7	1·017	·2255		1·035	·0452		1·0466	·01456		1·053	·00582		1·062	-ve ·00253		1·0675	-ve ·002145	
	·909	·3256	·75	·793	·1737	·375	·6595	·0872	·25	·515	·0349	·25	·373	-ve ·0349	·25	·2385	-ve ·1045	·25
·8	1·017	·2181		1·035	·0498		1·0525	·01365		1·0625	·00421		1·079	-ve ·003018		1·113	-ve ·0056	
	·916	·3256	·75	·844	·2079	·375	·765	·1219	·25	·594	·0523	·25	·52	-ve ·0349	·25	·41	-ve ·0349	·25
·9	1·02	·2188		1·034	·0636		1·051	·02185		1·028	·0072		1·086	-ve ·0030		1·0985	-ve ·00326	
	·936	·3584	·75	·866	·1564	·375	·835	·1219	·25	·778	·0698	·25	·669	0	·25	·594	-ve ·0523	·25
1·08	1·023	·2455		1·043	·0487		1·055	·0241		1·071	·01268		1·1	0		1·118	-ve ·00695	
	·93	·3907	·75	·93	·225	·375	·896	·1392	·25	·881	·0698	·25	·831	0	·25	·78	-ve ·0175	·25
1·228	1·035	·2633		[illegible]	·0759		1·045	·02986		1·057	·01456		1·073	0		1·088	-ve ·00335	
	·94	·3746	·75	·926	·1564	·375	·925	·2588	·25	·925	·225	·25	·906	·1045	·25	·875	·0523	·25
1·37	1·032	·266		1·04	·0535		1·04	·05755		1·091	·05		1·05	·02256		1·065	·01072	
	·839	·2924	·75	·815	·3090	·375	·822	·3746	·25	·914	·3256	·25	·936	·1908	·25	·914	·1045	·25
1·5	1·102	·167		1·112	·0848		1·11	·0693		1·058	·0704		1·095	·04275		1·058	·0226	
Σ																		

STATION				Σ			η_{B_d}			K
	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	$\eta^{\frac{1}{2}}$	Sinα	$\int da$	
$M_{2\varphi}$	$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		$\int V$	$\eta^{\frac{1}{2}}$ Sin$\alpha \frac{\int da}{\int V}$		
·5					1·6757			·733		·85
·6					1·7812			·771		·87
·7					1·7397			·794		·89
·8					1·7800			·828		·9
·9					1·8006			·81		·9
1·08					1·7489			·76		·87
1·228					1·7235			·744		·86
1·37					1·6810			·791		·89
1·5					1·6481			·7856		·88
Σ										