THERMODYNAMIC METHOD OF MEASURING EFFICIENCY OF HYDRAULIC MACHINES

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C O N T E N T S

This Thesis should be read in conjunction with the addendum inserted in a pocket at the outside of back cover.

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1. INTRODUCTION:

Basically there are two methods available for measuring the losses in large commercial hydraulic machines. Firstly, the losses in a model are found by conventional experiments in a laboratory, and the actual losses for the prototype are then estimated, allowing for the majoration effect. Secondly, the actual flow through the machine is measured by various methods and by determining the head available together with other factors involved, the losses in the turbines or pumps are estimated. The first method cannot be accepted except as an approximation. The second method, however, gives accurate and reliable values but it entails complicated and elaborate equipment. Moreover, it necessitates closure of machines during installing and dismantling of the measuring equipment resulting in a considerable waste of time and money.

Routine checks on hydro-electric stations are usually impeded due to this rather cumbersome method. Recent advances in the design of hydraulic machines have made it increasingly important that a routine check on the performance of the machines should be made so that they may be operated in the most efficient way.

Power stations consisting of two or more machines usually present the problem of running the machines efficiently in combination, especially when there is a limited supply of water. In these conditions knowledge of the actual performance curves could ensure the most efficient way of running the machines.

Bearing in mind the problems associated with the existing methods and the importance of finding the performance of hydraulic machines, which cannot be overstressed, a new method called the thermodynamic method has been developed.

The principal conception of this method was first introduced by A. Poirson in 1914. Later in 1929 further investigations were carried out by L. Barbillon, Fontaine and Poirson in France, which was followed by Willmer and Umfenbach in Germany and Katzman in Canada.

Fundamentally/

Fundamentally the thermodynamic method involves the determination of the change in internal energy of the fluid flowing through hydraulic machines as a result of dissipation of energy losses. Practically it can be computed by observing the rise in temperature of fluid flowing through the machines. It is known that if water is dropped through 1400 ft. and brought to rest ensuring that there is no exchange of heat from the surroundings, the temperature will rise through 1 deg.C. Therefore, if a turbine operating at 1400 ft. of head is say, 90% efficient, the water at the tail race will be roughly 0.1deg.C. warmer than the water entering the turbine, assuming that the specific heat of water is unity at that temperature. Theoretically this statement is not correct, because the losses do not occur as change in enthalpy of the fluid, but as a change in internal energy of the fluid. Although a detailed theoretical investigation will be given later, here for a general argument we have assumed that losses appear as rise in temperature of fluid flowing through a machine.

In 1954 G. Willm and P. Campmas of Electricite de France published a paper on the fundamental investigation of the Poirson method which has formed the basis of all work carried out on the thermodynamic method of measuring the efficiency of hydraulic machines. They have given a detailed explanation of the theoretical relationship, followed by an analysis of possible sources of error and a description of measuring equipment suitable for industrial applications.

The present investigation is also based on the work of Willm and Campmas. It mainly consists of detailed study of the fundamental principles involved and the calculation of change in internal energy using basic laws of thermodynamics leading to the development of the thermodynamic expression for efficiency of hydraulic machines; also the development of measuring equipment in general and an adiabatic expander in particular with special reference to low head machines and its technique of operation, followed by

details/

details of a series of tests at high and low head machines spread over a period of three years.

A comparison test with current meters was carried out on a machine operating under approximately 180 ft. of head, which is the lowest ever tested successfully. An encouraging result has shown that this method can be developed for low heads as effectively as it has been done for high heads in France and Germany.

The new adiabatic expander, and a new technique of measurement have been the two main factors in the development of this method, and for low heads they have proved to be a definite improvement.

2. THEORY:

2.1 Energy equation for steady flow:-

The law of conservation of energy may be stated in several different forms. One of the forms could be written as:

Energy entering the system	=	Increase or decrease of energy stored within the system	+	Energy leaving the system

As a hydraulic machine delivers work at a constant rate while operating under steady flow conditions, the rate of flow of each mass stream entering and leaving the system is constant; so it may be analyzed as a steady flow system. Let us apply these conditions to the general form of the energy equation. Since by definition there is no change in the energy stored within a steady flow system, the statement of the law of conservation of energy reduces to,

Energy entering the system = Energy leaving the system.

For an arbitrary system as shown in Fig.1, the energy equation may be written as:

$$Z_1 + K_1 + W_{f_1} + E_1 + q = Z_2 + K_2 + W_{f_2} + E_2 + W$$

where/

where Z is potential energy.

K is kinentric energy written as $\frac{V^2}{2g}$ where V is velocity.

q is heat input

E is internal energy.

W_f is flow work and written as P/w, where P is pressure and w is specific weight.

Suffix 1 shows entry point.

Suffix 2 shows exit point.

(Each term in the above equation represents energy per unit weight, i.e. ft.lb./lb.).

Solving for q and writing in difference form, we get

$$q = \Delta E + \Delta W_f + \Delta K + \Delta Z + W \quad \ldots\ldots 2{\cdot}1{:}1$$

which is a general form of the steady flow energy equation.

2.2 Energy equation for flow of incompressible fluids.

The well-known Berroulli equation may be derived using a force analysis. Heat and internal energy are not involved for good historic reasons. Bernoulli, a contemporary of Newton, derived the equation around 1700, whereas the laws of conservation of energy were not accepted until about 1850 - 150 years later. However, the energy law throws additional light on the Bernoulli equation, forming the basis of the work published by Willm and Campmass in 1954.

Let us consider an open system of fluid flowing in a pipe between boundaries 1 and 2 as shown in Fig.2, and let us apply the steady flow energy equation 2·1:1 to this system. In this system $W = 0$ and q can be negligibly small if there is little temperature difference between the inside and outside or if the pipe is well insulated. An incompressible fluid in frictionless flow with $q = 0$ will undergo no change of internal energy i.e. $\Delta E = 0$. Between the boundaries 1 and 2, the energy equation 2·1:1 then becomes

P

$$\Delta\left(\frac{P}{W}\right) + \Delta K + \Delta Z = 0$$

or $$\frac{P_2}{w_2} - \frac{P_1}{w_1} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + Z_2 - Z_1 = 0 \text{ (ft.lb/lb)} \quad \ldots 2.2:1$$

which is a form of Bernoulli's equation for the flow of incompressible fluids without friction. But in an actual process friction prevents some of the energy at 1 from issuing as useful energy at 2, and this is termed energy loss. If 'q' is negligible during the flow of an incompressible fluid, the energy loss due to friction appears in the internal energy of the fluid. Thus using all the energy terms shown in Fig.2, we have

$$\frac{P_2}{w_2} - \frac{P_1}{w_1} + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + Z_2 - Z_1 = -(E_2 - E_1)\text{ft.lb/lb.}$$

This is a form of Bernoulli's equation with friction if we designate $E_2 - E_1$ as the friction head, which is the sum of the changes of "all heads", a net loss.

If the system of Fig.2 should lead to a hydraulic turbine at 2, some of the potential energy ΔZ would be converted into kinetic energy and then some of the kinetic energy would be converted into work in the turbine.

All the change of potential energy is not converted to work because of friction; and the energy loss due to friction appears in the internal energy of the fluid.

Willm and Campmas in 1954 extended the application of the energy equation to hydraulic machines and reached an interesting result which enabled them to calculate the efficiency.

2.3 Energy equation applied to hydraulic turbines:-

We shall apply the energy equation to the system formed by the turbine and water passing through it as shown in Fig.3. For this system we will consider a cross-section A at inlet and B at/

at outlet. The energies at A will be shown by suffix 'i' and the energies at B will be shown by suffix 'o'.

We now recall that the energy equation (2·1:1) for steady flow is given by,

$$q = \Delta E + \Delta W_f + \Delta K + \Delta Z + W$$

If the flow through the turbine is relatively large, the heat added or removed from the system per pound weight will be very small, and so it will be justifiable to take $q = 0$.

The energy relation then becomes

$$W = \Delta W_f + \Delta K + \Delta Z + \Delta E \quad \ldots\ldots\ldots \; 2{\cdot}3{:}1$$

which can also be written as

$$W = \frac{P_i}{w_i} - \frac{P_o}{w_o} + \frac{V_i^2}{2g} - \frac{V_o^2}{2g} + Z_i - Z_o + E_i - E_o \text{ ft.lb/lb}$$

or re-arranging

$$W = \left(\frac{P_i}{w_i} + \frac{V_i^2}{2g} + Z_i\right) - \left(\frac{P_o}{w_o} + \frac{V_o^2}{2g} + Z_o\right) + E_i - E_o \quad \ldots\ldots \; 2{\cdot}3{:}2$$

Putting

$\frac{P_i}{w_i}$ (the pressure head) + $\frac{V_i^2}{2g}$ (the velocity head) + Z_i (the potential head) = H_{ti} the total hydraulic head at inlet

and

$\frac{P_o}{w_o} + \frac{V_o^2}{2g} + Z_o = H_{to}$ the total hydraulic head at outlet

we get from 2·3:2

$$W = H_{ti} - H_{to} + E_i - E_o$$

or $$W = (H_{ti} + E_i) - (H_{to} + E_o) \quad \ldots\ldots\ldots \; 2{\cdot}3{:}3$$

If in a system there is no exchange of energy with the

outside/

outside, then $W = 0$; and we can write equation 2·3:3 as,

$$H_{ti} + E_i = H_{to} + E_o \quad \ldots\ldots\ldots \quad 2\cdot3:4$$

Therefore if we take some water inlet section A and bring it without any change in total energy (for example through a throttle valve) to a measuring section defined by suffix 1, we can write,

$$H_{ti} + E_i = H_{t1} + E_1 \quad \ldots\ldots\ldots \quad 2\cdot3:5$$

Similarly if we define a measuring section at outlet by suffix 2, we get

$$H_{to} + E_o = H_{t2} + E_2 \quad \ldots\ldots\ldots \quad 2\cdot3:6$$

Taking the values from equation 2·3:5 and 2·3:6 and substituting in equation 2·3:3 we can write,

$$W = (H_{t1} - H_{t2}) - (E_2 - E_1) \text{ ft. lb/lb.} \quad \ldots\ldots\ldots \quad 2\cdot3:7$$

which is a general form of the useful work supplied by the turbine per unit weight measured at two experimental sections, one at inlet and the other at outlet defined by suffixes 1 and 2 respectively.

We can also write equation 2·3:7 using all the "head" terms as,

$$W = \left(\frac{P_1}{w_1} + \frac{V_1^2}{2g} + Z_1\right) - \left(\frac{P_2}{w_2} + \frac{V_2^2}{2g} + Z_2\right) - (E_2 - E_1) \text{ ft.lb/lb.} \quad \ldots\ldots\ldots \quad 2\cdot3:8$$

2.31 Calculation of change in internal energy.

From the definition of entropy and from the first law of thermodynamics, for a reversible or irreversible transformation we have,

$$ds = \frac{dE + P.dv}{T} \quad \ldots\ldots\ldots\ldots \quad 2\cdot31:1$$

where/

where T is absolute temperature

s " entropy

E " internal energy

P " absolute pressure

v " Specif volume

Re-writing equation 2·31:1, we get

$$dE = T.ds - Pdv.$$

If we choose T and P as our independent variables, E becomes a function of these variables and we can write

$$dE = T\left(\frac{\partial s}{\partial T}\right)_P \cdot dT + T\left(\frac{\partial s}{\partial P}\right)_T \cdot dP +$$

$$- P\left(\frac{\partial v}{\partial T}\right)_P \cdot dT \div P\left(\frac{\partial v}{\partial P}\right)_T \cdot dP$$

Referring to equation 2·31:1, if we choose T and P as independent variables we can write

$$ds = \left\{\left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P\right\}\frac{dT}{T} + \left\{\left(\frac{\partial E}{\partial P}\right)_T + P\left(\frac{\partial v}{\partial P}\right)_T\right\} \times \frac{dP}{T}$$

On the other hand,

$$ds = \left(\frac{\partial s}{\partial T}\right)_P \cdot dT + \left(\frac{\partial s}{\partial P}\right)_T \cdot dP$$

Since dT and dP are independent of each other we can equate the co-efficients of dT and dP separately. This gives us

$$\left(\frac{\partial s}{\partial T}\right)_P = \frac{1}{T}\left(\frac{\partial E}{\partial T}\right)_P + \frac{P}{T}\left(\frac{\partial v}{\partial T}\right)_P \quad \text{.......... } 2\cdot31:3$$

$$\left(\frac{\partial s}{\partial P}\right)_T = \frac{1}{T}\left(\frac{\partial E}{\partial P}\right)_T + \frac{P}{T}\left(\frac{\partial v}{\partial P}\right)_T \quad \text{.......... } 2\cdot31:4$$

By/

By differentiating equation 2·31:3 with respect to P and equation 2·31:4 with respect to T we get

$$\frac{\partial^2 s}{\partial T.\partial P} = \frac{1}{T}\left(\frac{\partial^2 E}{\partial T.\partial P}\right) + \frac{P}{T}\left(\frac{\partial^2 v}{\partial T.\partial P}\right) + \frac{1}{T}\left(\frac{\partial v}{\partial T}\right)_P$$

$$\frac{\partial^2 s}{\partial P.\partial T} = \frac{1}{T}\left(\frac{\partial^2 E}{\partial T.\partial P}\right) + \frac{P}{T}\left(\frac{\partial^2 v}{\partial T.\partial P}\right) - \frac{1}{T^2}\left(\frac{\partial E}{\partial P}\right)_T - \frac{1}{T^2}\left(\frac{\partial v}{\partial P}\right)_T$$

It follows that,

$$\left(\frac{\partial E}{\partial P}\right)_T = - T\left(\frac{\partial v}{\partial T}\right)_P - P\left(\frac{\partial v}{\partial P}\right)_T$$

If we put the value of $\left(\frac{\partial E}{\partial P}\right)_T$ from above equation in equation 2·31:4 we get

$$\left(\frac{\partial s}{\partial P}\right)_T = - \left(\frac{\partial v}{\partial T}\right)_P \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2{\cdot}31{:}5$$

We can write for any heating process

$$\frac{q}{dT} = \frac{dE}{dT} + P.\frac{dv}{dT} = \text{constant (say) } C_2$$

and if the heating is under constant pressure, $dP = 0$ and $C_2 = C_p$ the specific heat at constant pressure.

Therefore we can write,

$$\left(\frac{\partial E}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial T}\right)_P = C_p$$

From the above equation and equation 2.31:3 we get

$$\left(\frac{\partial s}{\partial T}\right)_P = C_p.\,T^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 2{\cdot}31{:}6$$

Substituting the values of $\left(\frac{\partial s}{\partial P}\right)_T$ and $\left(\frac{\partial s}{\partial T}\right)_P$ from equations 2·31:5 and 2·31:6 respectively in equation 2·31:2 we can write

$$dE = C_p.\,T - T\left(\frac{\partial v}{\partial T}\right)_P . dP - P\left(\frac{\partial v}{\partial T}\right)_P .dT - P\left(\frac{\partial v}{\partial P}\right)_T . dP$$

and/

and rearranging dP and dT together, we can write

$$dE = -\left\{ T \left(\frac{\partial v}{\partial T}\right)_P + P \left(\frac{\partial v}{\partial P}\right)_T \right\} .dP + \left\{ C_p - P \left(\frac{\partial v}{\partial T}\right)_P \right\} . dT$$

We recall now that the change of internal energy depends only on the location of state points 1 and 2; that it is independent of the nature of the process connecting the points, and that therefore the change is the same between any two states no matter whatever is the path.

Therefore if we consider an isothermal expansion from P_1 to P_2 constant at T_1, followed by a heating at constant pressure P_2 from T_1 to T_2 we may write

$$\int_1^2 dE = \int_{P_1}^{P_2} -\left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_P + P \left(\frac{\partial v}{\partial P}\right)_T \right\} . dP + \int_{T_1}^{T_2} \left\{ C_p - P \left(\frac{\partial v}{\partial T}\right)_P \right\} . dT$$

or

$$E_2 - E_1 = C_{p_2} (T_2 - T_1) - P_2 \left\{ v(P_2, T_2) - v(P_2, T_1) \right\} + \int_{P_2}^{P_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_P + P \left(\frac{\partial v}{\partial P}\right)_T \right\} dP \quad \ldots\ldots\ldots\ldots \quad 2{\cdot}31{:}7$$

NOTE:-

$\int_{T_1}^{T_2} C_p \, . dT$ is taken as $C_{p_2} (T_2 - T_1)$, since the temperature difference is very small. Brandt has argued that as C_{p_2} varies with temperature by not more than 0.1% per deg C over the range 0 to 20°C, this simplification is permissible.

2.4 Thermodynamic expression for the useful work done by a hydraulic turbine.

If we put the value of $E_2 - E_1$ from equation 2·31:7 in equation/

equation 2.3:8 a general expression for the useful work done can be obtained as,

$$W = \frac{P_1}{w_1} - \frac{P_2}{w_2} + P_2\left\{v(P_2,T_2) - v(P_2,T_1)\right\} - \int_{P_2}^{P_1}\left\{T_1\left(\frac{\partial v}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial P}\right)_T\right\} dP$$

$$+ J.C_{p2}(T_1 - T_2) + \frac{1}{2g}(V_1^2 - V_2^2) + Z_1 - Z_2 \quad \text{ft lb/lb.}$$

..............2.4:1

To bring this equation to a simplified form, Willm and Campmas collected together all those quantities obtainable from the tables of the properties of water, and by introducing a reference specific weight w_s (where w_s is measured at standard temperature and pressure; for water $w_s = 62.4$ lb/ft^3 at 4°C and 14.7 lb/in^2 abs.) showed that these known quantities are conveniently represented by a dimensionless quantity $(1 - \alpha)$.

In appendix equation A1:3 it has been shown that,

$$\frac{P_1}{w_1} - \frac{P_2}{w_2} + P_2\left\{v(P_2,T_2) - v(P_2,T_1)\right\} - \int_{P_2}^{P_1}\left\{T_1\left(\frac{\partial v}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial P}\right)_T\right\} dP$$

$$= \frac{1}{w_s}(P_1 - P_2)\quad(1 - \alpha)$$

where w_s is the specific weight at S.T.P., and where

$$\alpha = \frac{w_1 - w_s}{w_1} + P_2 w_s\left\{\frac{v(P_2,T_1) - v(P_1,T_1)}{P_1 - P_2}\right\} + \frac{w_s}{P_1 - P_2}\int_{P_2}^{P_1}\left\{T_1\left(\frac{\partial v}{\partial T}\right)_P + P\left(\frac{\partial v}{\partial P}\right)_T\right\}dP$$

Therefore we can write the fundamental thermodynamic expressions for the useful work done by a hydraulic turbine as

$$W = \frac{1}{w_s}(P_1 - P_2)(1 - \alpha) + JC_{p2}(T_1 - T_2) + \frac{1}{2g}(V_1^2 - V_2^2) + Z_1 - Z_2$$

ft lb/lb.........2.4:2

2.5/

2.5 Thermodynamic expression for the work done by an ideal turbine (no losses).

We recall that even in an ideal turbine there would be a change of internal energy between inlet and outlet. Therefore if we consider that the water is being admitted at the same initial conditions as in the actual turbine, defined by head terms.

$$\frac{P_i}{w_i} \; ; \; \frac{V_i^2}{2g} \; ; \; Z_i \; ; \; E_i \; (P_i, T_i)$$

we can say that the water at the outlet of an ideal turbine will have the same conditions as at the outlet of the actual turbine except that the temperature T'_o at the outlet of The Ideal turbine will be less than the temperature T_o at the outlet of the actual turbine. Consequently, the internal energy and specific weight being functions of temperature, will be different at the outlet of the ideal turbine compared to the actual turbine. Hence we can define the energy terms for the conditions at outlet of an ideal turbine as

$$\frac{P_o}{w'_o} \; ; \; \frac{V_o^2}{2g} \; ; \; Z_o \; ; \; E'_o \; (P_o, T'_o)$$

where w'_o is the specific weight at the outlet of the ideal turbine

E'_o " " internal energy at the outlet of the ideal turbine

T'_o " " temperature " " " " " " "

and

P_o is the pressure at the outlet of actual and ideal turbine

V_o^2 " " velocity head at " " " " " "

Z_o " " position " " " " " "

If we define W' as the work done by an ideal turbine, from energy balance and equation 2.3:2 we can write the ideal work done per lb. as

$$W' = \frac{P_i}{w_i} - \frac{P_o}{w_o} + \frac{V_i^2}{2g} - \frac{V_o^2}{2g} + Z_i - Z_o + E_i - E'_o \; \ldots\ldots \quad 2.5{:}1$$

Since/

Since in an ideal flow without losses, the change in entropy $ds = o$, we can write

$$dE = - Pdv, \text{ for an ideal turbine}$$

$$\text{or} \quad \int_i^o dE = \int_i^o - P.dv$$

$$\text{or} \quad E'_o - E_i = \int_i^o - P.dv$$

$$\text{or} \quad E_i - E'_o = \int_i^o P.dv \qquad \ldots\ldots \; 2.5{:}2$$

Therefore we can express

$$W' = \frac{P_i}{w_i} - \frac{P_o}{w'_o} + \frac{V_i^2}{2g} - \frac{V_o^2}{2g} + Z_i - Z_o + \int_i^o P.dv.$$

It is shown in Appendix 2 that we can write

$$\frac{P_i}{w_i} - \frac{P_o}{w'_o} + \int_i^o P.dv = \frac{1}{w_s}\,(P_i - P_o) \quad (1 - \beta)$$

where w_s is the specific weight of the fluid at S.T.p. and

$$\beta = \frac{w_i - w_s}{w_i} - \frac{w_s}{Pi - Po} \int_{P_i}^{P_o} P.\left(\frac{\partial v}{\partial P}\right)_{T_i} .dP$$

and is a dimensionless quantity like .

Hence we can define the work done by an ideal turbine as

$$W' = \frac{1}{w_s}\,(P_i - P_o)\,(1 - \beta) + \left(\frac{V_i^2}{2g} - \frac{V_o^2}{2g}\right) + Z_i - Z_o \quad \text{ft. lb/lb.}$$

$$\ldots\ldots\ldots\ldots \; 2.5{:}3$$

2.6 Thermodynamic expression for efficiency.

We can define the efficiency of a hydraulic turbine as

$$\eta = \frac{\text{Actual work done}}{\text{Ideal work done}} = \frac{W}{W'}$$

From equations 2·4:2 and 2·5:3, we can write

$$= \frac{\frac{1}{w_s}(P_1 - P_2)(1-\alpha) + J\,C_{p_2}(T_1 - T_2) + \frac{1}{2g}(V_1^2 - V_2^2) + Z_1 - Z_2}{\frac{1}{w_s}(P_i - P_o)(1-\beta) + \frac{1}{2g}(V_i^2 - V_o^2) + Z_i - Z_o} \qquad \ldots\ldots\ldots \; 2\cdot6:1$$

Singh has argued that

(i) V_1 = 0.895 ft/sec at the point of measurement in the calorimeter, assuming a discharge of 0.833 lb/sec ($\equiv$ 0.4 litre/sec).

(ii) $V_2 = V_o$ = 4 ft/sec or less in tail race.

(iii) $\frac{V_i^2}{2g}$ is taken into account by the use of a pitot probe in the measurement of P_i.

(iv) $\frac{V_1^2 - V_2^2}{2g}$ in the numerator and $-\frac{V_o^2}{2g}$ in the denominator are of the same order of smallness compared with P_i and P_o respectively.

Then the efficiency expression 2·6:1 reduces to

$$\eta = \frac{\frac{1}{w_s}(P_1 - P_2)(1 - \;) + J\,c_p(T_1 - T_2) + Z_1 - Z_2}{\frac{1}{w_s}(P_i - P_o)(1 - \;) + Z_i - Z_o}$$

If we choose the measuring test section at outlet of the turbine (i.e. at tail race) then,

$P_2 = P_o$ = atmospheric.

Since we can more conveniently represent gauge pressure head measured above atmosphere in ft. of water, we can therefore write/

write

$$\frac{1}{w_s}\left(P_1 - P_2\right) = H_1 \text{ ft of water}$$

$$\frac{1}{w_s}\left(P_i - P_o\right) = H_i \text{ ft of water}$$

In most test cases $Z_1 = Z_i$ and $Z_2 = Z_o$,

and the efficiency can be expressed as

$$= \frac{H_1 (1 - \alpha) + J\, C_{P_2} (T_1 - T_2) + Z_1 - Z_2}{H_i (1 - \beta) + Z_1 - Z_2} \quad \ldots\ldots 2{\cdot}6{:}2$$

3. MEASUREMENT OF EFFICIENCY.

The efficiency equation leads to 4 general methods of measurement:-

(i) Method of absolute measurement:-

In this method the absolute value of T_1 and T_2 is measured by precision thermometers and the value of C_{P_2} is obtained from tables. H_1 and H_i are measured by pressure gauges and hence the efficiency of the machine can be calculated.

This method is impractical as it entails the measurement of the absolute value of temperatures - a very difficult and lengthy process especially when the desirable accuracy is $\pm$ 0.001°C, and also because it requires the calculation of specific heat with reasonable accuracy, not easily obtainable.

(ii) Method of complete expansion:-

In this method water under pressure from the penstock is expressed adiabatically to the atmospheric pressure. The difference between the temperature of the expressed water T_1 and the temperature of the tail race T_2 is measured by Thermometers. C_{P_2} is obtained from tables and H_i is measured by pressure gauge, which enables calculation of the efficiency.

This method has an advantage that the term does not come

into/

into calculation as H_1 becomes zero. Also only one pressure i.e. H_1 need be measured which simplifies the operation.

However, this method suffers from other great disadvantages. Since the temperature term $JCp_2(T_1 - T_2)$ becomes the principal term, therefore the temperature difference and the specific heat have to be very accurately measured. A slight error in the measurement of either temperature difference or specific heat, will cause an appreciable error in the efficiency calculation. It also involves the measurement of the absolute value of the difference of temperature $T_1 - T_2$. Accurate calculations of the specific heat necessitates another measurement of the absolute temperature of tail race. Moreover the chamber(into which the water is expressed) together with all the connecting pipes must be completely insulated as a small heat transfer will alter the actual value of the principal term in the efficiency equation and thus will introduce a gross error.

(iii) Method of Partical Expansion:-

In this method water under pressure from the penstock is partially expressed adiabatically in a well insulated chamber, such that the temperature T_1 of the expressed water becomes equal to the temperature T_2 of the tail race. Thus the temperature term is eliminated and the corresponding values of H1 and H_1 are then measured by pressure gauge.

This method has several advantages. The measurement of absolute temperature is not required. The measurement of difference of temperature is not necessary as the temperature term $T_1 - T_2$ becomes equal to zero. Also it does not entail the calibration of the measuring instrument because the value of the difference of temperature is not required in the efficiency equation. Therefore the method becomes simple, as only two pressures are to be measured, and it is most commonly used to measure the efficiency.

However, it has some practical disadvantages. To obtain the condition $T_1 - T_2$, several readings have to be taken by adjusting the pressure settings, which is a lengthy process and entails the assumption/

assumption that the graph of pressure versus temperature is a straight line. Also the exchange of heat between the expander and surroundings becomes a major problem.

In view of the above mentioned practical disadvantages a new adiabatic expander was developed and a new technique of measurement was employed to eliminate the disadvantages.

(iv) Method of direct measurement:-

The difference of temperature between the penstock and tail -race is measured directly by inserting one thermometer in the penstock and the other in the tail race. Now since H_1 becomes equal to H_i, only one pressure reading will be required.

However, this method necessitates the calibration of temperature in terms of pressure head for the measuring equipment, as the value of the difference of temperature is required. The calibration can also be done in the laboratory prior to the field tests.

In view of the simplicity of the apparatus and measuring technique, this method becomes very attractive and needs to be investigated and widely employed. Some preliminary experiments were carried out at a test at Bonnington Power Station which gave a very encouraging results. But the method is in an embryonical stage, and at present a comparison with other methods will not be justifiable.

4. METHOD OF MEASUREMENT.

As discussed in Article 3, the method of partical expansion is most commonly used for efficiency measurements. A small sample of water is withdrawn steadily from the penstock at a section where the total energy of the water is fairly constant. The sampling water is then led to a chamber where it is expressed adiabatically. Expansion and rate of flow is controlled by two valves one at the inlet of the chamber and the other at the outlet, so that the temperature of the expressed water is raised exactly to that of tail race. This is ascertained by means of thermometers, one/

one placed in the expansion chamber, the other in the tail race. It is most common to use two carefully matched platinum resistance thermometers requiring no absolute calibration as the adjacent arms of a wheatstone bridge. A galvanometer used with the bridge shows the equality in the temperature of the expansion chamber and the tail-race.

Sampling of the tail race water is not necessary in most cases as the tail race is easily accessible and the thermometer is directly placed in the tail race. However, the temperature gradient across the tail race presents a great problem at low heads. Therefore during tests at low heads a device has to be installed to sample water at different points on the cross section to give a mean temperature reading.

The pressure H_1 in feet of water inside of the expansion chamber is then accurately measured, and also H_1 the total head in feet of water at inlet of the turbine is noted by stopping the flow through the expansion chamber brought about by the closure of the outlet valve. The efficiency can then be calculated by the efficiency equation 2.6:2 which reduces to

$$= \frac{H_1 (1-\alpha) + Z_1 - Z_2}{H_1 (1-\beta) + Z_1 - Z_2} \quad \ldots\ldots\ldots \quad 4:1$$

where Z_1 and Z_2 are the datum levels of the two test sections; α and β are thermodynamic coefficients which correct for the variation of specific volume of water and change of its internal energy with temperature and pressure.

5. APPARATUS FOR MEASUREMENT.

For measurement of efficiency the apparatus required may be divided into five main groups:-

(i) Extracting probe (for penstock).

(ii) Tail race water mixing device.

(iii) Temperature measuring equipment.

(iv) Expander of throttling calorimeter.

(v) Accurate pressure recording gauge.

5.1 <u>Extracting Probe</u>:-

It is desirable that the probe for extraction of water should be constructed on the principle of the Pitot tube to provide for the measurement of pressure head plus velocity head. The design of the probe should be based on the following factors:-

(a) It should be strong enough to withstand stresses due to bending moments and fatigue failures due to vortex shedding.

(b) It should be stiff to resist large amplitude of vibration; as the vibration might lead to mechanical failure and the flutter might cause erroneous pressure readings.

(c) The pitot holes should be large enough to ensure a constant flow of 0.4 litres per second through the expander at all pressure levels.

(d) The general layout should be such that it may be easily erected or dismantled.

The probe used during test was a contilevered pitot tube shown in Fig. 4. It was made of thin stainless steel tube of 1.128 inches outside diameter and 1.046 inches bore, and was closed at the orifice end. The orifice was 9/16 inches in diameter and the centre of the orifice facing upstream was fixed at a distance of two inches from the rounded top of the closed end. Care was taken during insertion to avoid any intense vibration. Singh observed that the depth of immersion at which the vibration may be expected was computed by obtaining the natural frequency of the probe from charts prepared by Winternitz and calculating the approximate shedding frequency of the vortices from the Rayleigh and Taylor formula. The most favourable depth of insertion as prescribed by other investigators is 1/7th of the pipe diameter.

To find any possible temperature gradient across the pipe diameter an adjustable probe has been suggested by R. Vaucher, which eliminates/

eliminates the inconvenience of altering the depth of insertion by pushing or pulling the probe. This adjustable probe consists of two concentric tubes with holes in both at specified positions. By displacing the inner tube, appertures at various distances from the wall can be opened one by one. This type of probe has successfully been used by Escher Wyss in different plants, to measure the energy distribution across the test section more rapidly.

5.2 Tail-race water mixing device:-

Temperature gradient across the tail race becomes critical at low heads, where the variation in temperature in some extreme cases might result in ±2% variation in measured efficiency. A very extensive and useful work has been done by Thom to show the effect of temperature variation across the width and the depth of the tail race. The readings observed by Thom were plotted in the form of temperature contours across the cross-section of the flow. These contours show that the temperature variation does not obey a particular law but rather changes in a haphazard manner. Moreover the shapes of the contours change with the change of load on the Machine. During tests it is not possible to draw the temperature contours for every load to calculate the correct efficiency. Some tests were carried out at Bonnington in June and December 1959; the traverses measured at three different loads, although making it possible for a correction to be applied for the temperature gradient across the tail race, took about two hours. It is difficult to justify this expenditure of time. Therefore, if the preliminary investigation of the temperature gradient across the tail race shows that variation in temperature will result in variation in efficiency outside the required accuracy it is recommended that a device described in the following paragraph is to be adopted to find the mean temperature of the tail race, without taking a traverse for temperature variation.

Generally in installing the tail race thermometer it has been usual to fix a timber beam in a vertical plane over sections of the tail race. Twelve U-shaped brackets are attached to the beams to allow the/

the thermometer to be positioned at any of the twelve fixed points. The new device was designed to sample water from these twelve different fixed points on the cross-section of the tail race and to mix them in a chamber to give a mean temperature reading. The chamber shown in Fig. 5 was a spherical shell of 1 ft. in diameter made of $\frac{1}{2}$" thick perspex. Twelve short length pipes of 1" bore and 2" long were mounted on twelve holes of the same bore placed on the surface of the shell for the water inlets. The outlet consisted of $3\frac{1}{2}$" dia. pipe with a 90° bend, one limb of which penetrated the spherical shell from the bottom vertically and extended to $\frac{3}{4}$ of its diameter, the other limb, i.e., the exit end, faced downstream. The inlet diameters of the holes and the outlet diameter of the pipe were so designed that the total inlet was equal to the total outlet area to ensure equality of inlet and outlet velocity. The upper edge of the vertical limb of the outlet pipe was about 2" higher than the level of the inlet holes to ensure that the water inside was mixed thoroughly before entering the outlet pipe. At the top of the sphere an opening was made for insertion of the thermometer probe which was kept in a vertical position inside the outlet pipe. The twelve samples of water were taken from two different depths, six of them at one quarter of the depth and the other six at three quarters of the depth below the water level. The tubes used for sampling the water were $1\frac{1}{4}$ inch bore made of polythene and about 12 ft. long. The whole unit was submerged in water and continuous equal flow would occur in each of the twelve pipes.

5.3 Temperature measuring equipment:-

Accurate temperature measurements are not easy to make, even with accurate instruments. In the Thermodynamic method the accurate measurement of small temperature differences in the range of 0 deg.C - 20 deg. is required. The measurement becomes a problem of prime importance when the accuracy required is higher than the order of ± 0.001 deg.C. From the following table it can be surmised that with an instrument of one millidegree/

millidegree accuracy, turbines with heads of less than 140 ft. cannot be tested if the tolerance of efficiency error is restricted to ± 1%.

Table 1

Temperature Difference Equivalent to 1% of Effcy.	Net Head Across Turbine in Ft. of Water
0.02	2,800
0.01	1,400
0.001	140
0.0001	14

Willm and Campmas have discussed the possibilities of using differene types of thermometers for measuring small temperature differences. Also Leslie and Hunter have suggested various thermometers for use in the thermodynamic method. These studies and observations together with the work of other investigators lead us to the choice of the following types of the thermometers available for measurement of small temperature differences.

(a) Mercury in glass thermometers:-

These are accurate only to 1/100 deg. C and therefore not suitable for heads lower than 1400 ft. of water. Other disadvantages are that they are fragile and require easy access for observation.

(b) Differential thermometers:-

These are simple and can be very sensitive depending on the liquid and differential manometer used. Therefore further investigation becomes attractive from the point of view of measurements in the Thermodynamic method. Advantages of this type of theremometer are ruggedness, distant reading and the fact that a long bulb can be used for averaging the temperature across a large area, such as the expansion chamber.

(c) Mercury-in-Quartz thermometers:-

Moreau has recently describes tests on a mercury filled quartz thermometer which he claimed gave an error less than ± 0.001 deg.C. To achieve such accuracy the thermometer must be a totally immersed in the fluid and observed through a telescope to avoid parallax. In the thermodynamic method use of such a thermometer will not/

not be possible as the access to the tail race for observation will present a great problem and also simultaneous readings of temperatures at two stations several hundred feet apart could not be conveniently achieved.

(d) Solid-expansion thermometers:-

These types of thermometers lack published information and therefore are not in general use. However, the principle is very simple; the differential expansion of two rods of dissimilar materials could yield a very reproducible indication of temperature. The sensitive element is usually a bimetallic strip or coil, fixed at one end carrying an indicator or contact at the other. The differential expansion could be indicated remotely by a differential inductance transducer. Measurement of inductance to give an accuracy of temperature of $\pm$ 0.001 deg.C would not be very difficult and this method merits further investigation.

(e) Thermocouple thermometers:-

One of the most generally useful temperature-sensing devices is the thermocouple. Thermocouples are pairs ofwires, of dissimilar metal connected at both ends. When the two junctions are subjected to different temperatures an electrical potential is set up between them. When one junction is maintained at a fixed temperature (say that of melting ice) the other junction may be used as a thermometer and the voltage curve is highly reliable and reproducible. Table 2 gives the e.m.f generated at various temperatures in the range of $0^{o}C - 25^{o}C$ with commonly used thermocouples.

TABLE 2/

TABLE 2.

Temperature - Millivolt relations for Thermocouple

Deg.C $0^{\circ}C$ Cold Junction	Copper + Constantan -	Iron + Constantan -	Chromel + Alumel -	Platinum + 10% Rhodium -
5	0.17	0.23	0.18	0.024
10	0.39	0.51	0.40	0.055
15	0.61	0.80	0.62	0.087
20	0.83	1.09	0.84	0.120
25	1.06	1.37	1.06	0.153

This table enables us toestimate the voltage which could be generated by the most sensitive thermocouple (ie Iron and constantan) for a temperature difference of 0.001 deg.C. and it comes approximately to the order of 0.4×10^{-6} volts. The problem of measuring accurately this small voltage becomes immense because it is difficult to eliminate and even to discover the parasitic electromotive forces which arise in the lines and connections between couples; generally this method is not very suitable for measurement of temperature differences between two points distant from each other.

(f) Electrical-resistance thermometers:-

Electrical resistance thermometers are the most convenient means of measuring temperature. They are reliable and sensitive and the readings can be observed remotely. The resistance thermometer depends upon the inherent characteristics of metals to change in electrical resistance when they undergo a change in temperature. Although industrial resistance thermometers bulbs are usually made of platinum, copper or nickel the temperature resistance phenomenon also applies to certain semi conductors called thermistors. Therefore electrical resistance thermometers can be divided into two groups (i) Metal resistance thermometers (ii) Thermistor or semi-conductor Thermometers.

Since/

Since this type of thermometer seems to be the best for the measurement of temperature difference in the thermodynamic method, it will be discussed briefly below.

5.31 <u>Metal resistance thermometers.</u>

Willim and Campmas selected this type of thermometer for efficiency tests because (i) of the experience acquired over many years in high precision thermometric measurements and (ii) because of flexibility and convenience in use. Three principal metals, Nickel, Copper and Platinum are most commonly used for the windings of a resistance Thermometer bulb. These metals possess the characteristics which make them suitable for resistance windings. The required characteristics of material employed in the resistance winding should include:(a) Relative high temperature coefficient of resistance - the greater the resistance change per degree for a given value of resistance the greater the possible sensitivity of measurement.

(b) High resistivity -the higher the resistivity of the material, the more resistance there is available for a given length of wire and consequently for a given space; also the higher the resistance for a given temperature the greater is the change per degree of temperature, and hence this factor contributes to sensitivity.

(c) Stability - the material must be stable over a long period of time and over the working range of temperature without changing its electrical characteristics.

(d) Linearity of Resistance - temperature relationship - this characteristic obviously is desirable, since it results in a linear temperature scale and greatly simplifies the calculation of temperature from the measured resistance.

Table 3 gives the resistivity and temperature coefficient of three principal metals Nickel, Copper and Platinum.

TABLE 3/

TABLE 3

METAL	Resistivity in Micro Ohm - cm at $0^{o}C$.	Temperature Coefficient in $Ohm/Ohm^{o}C$ at $0^{o}C$ and Temperature Coefficient in $Ohm/Ohm^{o}C$ at $0^{o}C$ and
Nickel	6.38	0.0063 to 0.0066
Copper	1.56	0.00425
Platinum	9.83	0.00392

Nickel:- Despite some of its inherent limitations Nickel is most commonly used in industry because of its high temperature coefficient and adequate resistivity. However, from the point of view of the present investigation Nickel as a thermometer material is not suitable. The principal disadvantages of Nickel are (1) its non linear resistance-temperature relationship (2) its inherent variation in temperature coefficient over various lots of wire and (3) its lack of availability in pure form.

Platinum:- With exception of its cost it is one of the most suitable of all metals for resistance thermometry. It has been used for a long time for precision Thermometric measurements and because Platinum is obtainable in a highly purified form, the values of the temperature coefficient and resistivity are constant and dependable. The linearity of the resistance temperature relationship together with a high degree of accuracy and sensitivity, makes platinum a very attractive material for use in resistance thermometry. Most investigators interested in the Thermodynamic method have preferred Platinum resistance thermometers because of extensive published information available and of well known and definite characteristics. During the present investigation the same type of element was used as suggested by Willm and Campmas. Each element consisted of a thin platinum spiral embedded in Pyrex (manufactured by Comptoir Lyon - Alemand, Paris) had an ice resistance $R_0 = 100 \pm 0.020$ ohms and was rated for a continuous maximum power disipation/

dissipiation of 600 micro-watts. Each sensitive element was fitted by soft soldering with short flexible leads approximately 25 cm in length. These leads were made of standard copper (200 strands per lead). The element was then inserted with small mechanical play in a copper bulb with pyrotenax heads and filled with oil. The copper bulb was ½" outside diameter and about 7" long. The thermometer was completely watertight, sufficiently robust and could be easily handled.

Owing to the necessity of having almost identical thermometers it is essential that they should have almost no hysteresis and that they both should be identical within a tolerance of the order of 10^{-4}deg C. Willm and Campmas suggest that this can only be achieved by careful annealing ageing and selection from a large number of high grade elements. Singh observes that to avoid calibration hysteresis it was necessary to use the same cycle of temperature change. Willm and Campmas have described this phenomenon as a "strain gauge" effect between platinum and glass and suggest that one solution would be to replace pyrex glass by a glass with tested mechanical characteristics such as that used in making thermometers, such glass also having a coefficient of expansion of 8×10^{-6} much closer to that of platinum 9×10^{-6}. In conclusion Willm and Campmas write that gauges with free platinum can be made; the platinum spiraled like the filament of ½ watt lamp is mounted on a support made of aluminium alloy, a material which possesses in relation to glass, the very great advantage of having a coefficient of thermal conductivity about 30 times higher than than of glass. During the present efficiency tests several thermometer bulbs were made from platinum embedded in Pyrex type element. These resitor bulbs were tested in pairs over the range of 0 deg C to 20 deg C to give minimum deflection change with change of temperature after the balance was obtained at 0 deg C when two resitor bulbs were connected to two arms of a Wheat stone bridge. Good pairs were chosen and others were discarded. Willm and Campmas have theoretically analysed a way to compensate the error produced by the slight difference in matching.

It/

It has been pointed out by Leslie & Hunter that all platinum resistance thermometers of millidegree accuracy have been designed for careful handling under laboratory conditions; if they are subjected to vibration and knocks the resistance at a given temperature alters due to mechanical strains in the wire and the estimated temperature may be in error by several hundredths of degree celsius. A routine check therefore should be made every day to ensure that their characteristics were not changed during operation. This necessitates finding a thermometer more robust in field use but possessing the same degree of accuracy and sensitivity as of platinum.

Copper:-

Electrolytic copper of the highest purity is obtainable and since the temperature coefficient of the pure metal is quite consistent, it can very successfully be used in resistance thermometers. Copper has a higher temperature coefficient than platinum and is therefore relatively more sensitive. But it has not been so widely used as platinum because of its low resistivity resulting in a rather low temperature range. Since in thermodynamic measurements the working temperature range is only 0 deg C - 20 deg C which is well within the useful temperature range of Copper given as - 200 to 120 deg C, Copper will be very suitable for measuring small temperature differences.

Dauphinee has shown that properly designed copper Thermometers can maintain their ice-point calibration within 0.05 deg K even when cycled between 20 deg K and room temperature. This robustness of copper thermometers together with their relatively higher sensitivity; definitely makes their use a better proposition than platinum in the case of the thermodynamic method. Leslie & Hunter have attributed this success of Dauphinee's design to a combination of a rigid mounting for the resistance thermometer wire and complete freedom from differential expansion of the wire and its mounting. At N.E.L. Leslie & Hunter have designed a new type of copper Thermometer of millidegree accuracy which is very robust in construction for field use. They employed the method of mounting the resistance/

resistance Thermometer element on a former of the same material with the minimum of high quality insulating adhesive; and used 800 turns of 48 S.W.G. Vinyl-acetate insulated copper wire to wind on a similarly insulated 11 S.W.G. Copper wire, to construct resistor bulbs. The results obtained were very encouraging but lack of publication of observed data deters from drawing any definite conclusion. Dr. T.R. Foord has also constructed Copper Thermometers of a new design at Glasgow University (details of which are not available at present) and has successfully used them in measuring small temperature differences across inlet and outlet of a hydraulic machine used for laboratory, demonstrations. He has obtained accurate and consistent results and has contributed a "very useful development" in resistance thermometry for small temperature differences. It can therefore, be concluded that copper resistance thermometers could be one of the most suitable temperature measuring devices in testing hydraulic machines by the thermodynamic method.

5.32 Semiconductor thermometer or Thermistor:-

A thermistor is an electrical device made of a solid semi-conductor with a high temperature coefficient of resistivity which would exhibit a linear voltage-current characteristic if its temperature were held strictly constant. When a thermistor is used as a temperature sensing element the relationship between its electrical resistance and the temperature is of primary concern. The approximate relationship applying to most thermistors is $R = Ro\, e^{B\left(\frac{1}{T} - \frac{1}{T_o}\right)}$

where Ro = resistance value at reference temperature T_o

R = resistance at any other temperature $T, {}^{o}K$

B is approximately a constant over moderate temperature range and depends upon composition and process of manufacture.

This equation shows that thermistors have an exponential relationship between resistance and temperature and resistance decreases with rise of temperature/

temperature. The temperature coefficient at room temperature is approximately ten times that of the metals of which resistance thermometers are constructed. The high temperature coefficient of thermistors results in their having greater available sensitivity as temperature sensing elements than metal resistance thermometers or common thermometers. As for all resistance thermometers the measuring current should be kept small enough to avoid significant heating in order that element resistance shall be dependent upon the temperature of the surrounding alone. Thermistors are readily designed for sufficiently high resistance valves to make lead-resistance errors negligible. Hence Therimstor primary elements unlike metallic resistance thermometers and thermocouples can be located remotely from their associated measuring circuit without introducing any appreciable error - a very desirable property from the point of view of the thermodynamic method. The availability of thermistors in small sizes and their mechanical simplicity gives them advantages over sensing elements such as conventional resistance thermometers. Harrington and Handley also conclude that excepting reproducibility, thermistors are generally superior in having other basic qualities such as sensitivity, ruggedness, small size and small lag coefficient.

However thermistors suffer from the disadvantage of calibration stability. To Friedberg also, reproducibility appears to be the principal problem with thermistors which seem to undergo a systematic resistance change with time. It has been found by other investigators that if thermistors are held at a fixed temperature with negligible current flow, resistance changes of the order of 1% per year are typical. Green mentions that in certain glass-enclosed type smaller aging results are observed of the order of a few tenths of a percent in resistance per year. This corresponds to a calibration change of 0.1 deg.C or less per year. In the thermodynamic method we are not concerned with absolute measurement of temperature. What we require is an extremely sensitive temperature sensing/

sensing element, available in pairs having identical characteristics without hysteresis. Moreover the thermometers could easily be calibrated for measuring difference of temperature almost every day before starting the actual test. Therefore, a calibration change of 0.1 deg.C per year will not introduce any appreciable error. Katzman has successfully used the thermistors calibrated with standard platinum thermometers in experiments on the thermodynamic method. Singh carried out tests on Glenlee Power Station in which a "Stantel" type F.23 Thermistor was used and the results obtained were reasonable.

It will not be wrong to conclude that thermistors can be successfully used in thermodynamic tests because of their high sensitivity, ruggedness and high resistance valve. The only disadvantage of having poor calibration stability has been overcome to a greater extent by modern developments and also by the fact that precision type stability is not desired in thermodynamic method.

Several thermistors suitable for the thermometry of the thermodynamic method are described in Table 4. and are illustrative of various shapes, sizes and resistance valves available. Use of these thermistors is strongly recommended for measuring temperature differences.

TABLE 4.

Properties of Typical Thermistors

Description	Temp. Coeff. at 25°C. %/°C approx.	Unheated Resistance in Ohms At 0°C approx.	At 25°C approx.	Dissipi-ation factor mw/°C	Thermal Time Constant in Seconds
0.4 in. dia disk, radial leads	- 4.4	3270	1000±10%	7	160
0.11 x 1$\frac{5}{8}$ in.rod, axial leads	- 3.8	28500	10000±10%	6	95
Glass coated bead, 0.015 in.dia; 0.001 in leads,opposite	- 3.4	5000	2000±25%	0.1	1
Glass coated bead, 0.045 in.dia;0.0 4 in leads,adjacent	- 3.8	5800	2000±20%	0.7	2
Bead on 0.07 x 2 in. glass probe	- 3.8	5800	2000±20%	1	25
0.4 in. dia.-disk, radial leads	- 3.8	283	100±10%	7	160

5.33 BRIDGE CIRCUITS:-

Since, basically, a resistance thermometer is an instrument for measuring electrical resistance, a bridge has to be employed for the measurement of temperature. A d.c bridge has commonly been used by most investigators interested in the thermodynamic method. Important disadvantages of a d.c bridge are the presence of thermo electric and stray e.m.f's and the fragility and relatively slow response of the sensitive galvanometer, together with sensitivity to vibration. An A.c bridge is definitely superior to a d.c. bridge and merits a thorough investigation for use in thermodynamic method. Leslie and Hunter have described in considerable detail transformer ratio arm type A.c bridges, and have successfully used these bridges in measurement of temperature by copper resistance thermometer. They have also suggested various types of circuits for multiple thermometers - a very desirable arrangement for simultaneous measurement of temperature variation across a cross-section of tail race. Dr. T. R. Foord and Mr. Langlands of Glasgow University have also investigated the use of A.c bridge methods in measurement of temperature. The results obtained have been extremely successful. The advantages of an A.c bridge in thermodynamic measurements cannot be over-emphasized and it is proposed that this type of bridge shall be employed in further investigation of the method.

However, a d.c. bridge is much more simple to use and has the advantage of long tradition and experience behind it. Several types of bridge network are commonly employed in high precision measurement of temperature with platinum thermometers.

1. Wheatstone bridge.
2. Callander-Griffiths bridge.
3. Mueller bridge.
4. Smith bridge type III.

In the thermodynamic method we are interested only in detection of small differences of temperature, and are not concerned with absolute measurement of temperature. Therefore, a simple Wheatstone/

Wheatstone bridge designed for high degree of precision is all that is required.

High precision of a bridge network depends on its accuracy and sensitivity. For the given resistances of a network the relationship between accuracy and sensitivity could be defined as "the greater the sensitivity the smaller the accuracy". Since the thermodynamic method does not involve direct measurements of temperatures or resistances, we can dispense with the accuracy of the network and can concentrate on increasing its sensitivity up to practical limits. By choosing the best ratio of the arms of the bridge and a galvanometer of high sensitivity with resistance equal to the required galvanometer resistance for maximum sensitivity, a bridge network can be designed to observe a greater deflection in the galvanometer for very small differences between resistances of two thermometers, caused by the difference of temperatures.

We shall first investigate theoretically the conditions for maximum sensitiveness of a Wheatstone bridge. A sensitivity equation can then be developed and by imposing the practical and circumstantial limits on that equation the condition for permissible sensitivity could be known.

5.331. <u>Theoretical sensitivity of Wheatstone bridge:-</u>

The Wheatstone bridge is a network of six conductors and is usually represented as an arrangement of conductors in trilateral symmetry about the point D (Fig.6a & Fig. 6b). In this system a battery is inserted in the branch AB and a galvanometer in the branch CD. Some adjustments of one or more of the branches are made until the current through the galvanometer is zero.

A. Gray has given a detailed theoretical analysis of sensitivity of Wheatstone bridge. He concludes that when everything except R is a matter of choice and arrangement, we should make $R_1 = R_2 = R_3 = R_4 = R_5 = R_6$.

This, however, is almost never a practical arrangement and is impossible to achieve these theoretical conditions. Since the sensitivity of a higher order is not easily obtainable, the only solution left to this problem is to find out how sensitivity depends on the relative values of the various resistances of the bridge. And this being known the sensitivity of the bridge could be increased to the optimum value allowed by various limitations and practical restrictions.

5.332. Practical sensitivity of Wheatstone bridge:-

Let us consider now the Wheatstone bridge as a network of five conductors (Fig.7); the resistance of battery being small we will consider it equal to zero. It has been shown in Appendix III that i_g the current through the galvanometer can be expressed as

$$i_g = \frac{E\,(R_2 R_3 - R_1 R_4)}{Y + R_g (R_1 + R_3)(R_2 + R_4)} \qquad \text{.......... } 5\cdot332{:}1$$

where E is the e.m.f. of the battery applied to the circuit

& $Y = R_1 R_2 R_3 + R_2 R_3 R_4 + R_3 R_4 R_1 + R_4 R_1 R_2$

In order to discover the best arrangement of the bridge, let us suppose for example that balance is obtained by adjustment of R_3. Since the precision of adjustment is greater, only when for a given fractional change of R_3 at the point of balance, there is a larger galvanometer deflection; therefore the bridge sensitivity could be defined as S, where

$$S = S_g \, R_3 \, \frac{i_g}{R_3}$$

where S_g is galvanometer deflection per unit current.

By referring to eqn. 5·332:1 we can write the partial derivations of i_g with respect to R_3 as

$$\frac{\partial i_g}{\partial R_3} = E\, \frac{R_2 \left\{Y + R_g(R_1+R_3)(R_2+R_4)\right\} - (R_2R_3 - R_1R_4)\cdot\frac{\partial}{\partial R_3}\left\{Y + R_g(R_1+R_3)(R_2+R_4)\right\}}{\left\{Y + R_g\,(R_1 + R_3)\,(R_2 + R_4)\right\}^2}$$

and/

and at balance $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

$$R_2 \; R_3 - R_1 \; R_4 = 0$$

$$\frac{\partial i_g}{\partial R_3} = \frac{E R_2}{Y + R_g (R_1 + R_3)(R_2 + R_4)} \qquad \text{........ } 5\cdot332{:}2$$

$$S = \frac{S_g \; R_2 \, R_3 \; E}{Y + R_g (R_1 + R_3)(R_2 + R_4)} \qquad \text{........ } 5\cdot332{:}3$$

as $R_2 = \frac{R_1 \; R_4}{R_3}$

$$S = \frac{S_g \, R_1 \, R_4 \; E}{Y + R_g (R_1 + R_3)(R_2 + R_4)} \qquad \text{........ } 5\cdot332{:}4$$

Evidently 'S' has exactly the same form if R_2 or R_4 were adjusted and therefore for a given set of resistances, the bridge is equally sensitive to a variation of any arm.

As in practice it is not always possible to obtain a galvanometer made to specific requirements namely the sensitivity and the resistance, therefore we will first investigate the condition for a maximum value of 'S' the sensitivity of the bridge, with a given battery and galvanometer. That is E and R_g are constants but R_2, R_3 & R_4 may be varied within the limitation of the relation

$$R_2 \, R_3 = R_1 \; R_4$$

In addition to this mathematical restriction there are certain physical limitations which must be observed, because each resistance element has a definite current capacity or maximum safe current, the magnitude of which depends on the construction of the element. Therefore with the given e.m.f. there is a minimum possible value for $R_1 + R_2$ and also $R_2 + R_4$. The minimum values may be different in the/

the two cases. Here it could be mentioned that the current capacity of a given type of resistor element is usually greater for low resistances than for high. By supposing that E is small enough so that the current through R_2 & R_4 will never be excessive, we may avoid any restriction on R_2 & R_4. We cannot however avoid any restriction on R_1 & R_3, because in thermodynamic method the two resistance thermometers constitute a part of the resistances R_1 & R_3. And as the current capacity of the elements of the resistance thermometers commonly used are very low, so to avoid any overheating of thermometers, we cannot escape the condition

$$R_1 + R_3 = 2R_T + \delta \; ; (\delta \geqslant 0).$$

where R_T is the minimum safe value of R_1 & R_3

and is subject to another relation

$$i\,max \times 2R_T = E$$

where i_{max} is the maximum permissible current through the resistance thermometer & E is the e.m.f. of the battery used in the network.

The sensitivity of the bridge is given by

$$S = \frac{S_g \cdot E \cdot R_1 R_4}{Y + R_g (R_1 + R_3)(R_2 + R_4)}$$

$$\text{or } S = \frac{Sg.\ E}{\frac{R_1R_2R_3}{R_1R_4} + \frac{R_2R_3R_4}{R_1R_4} + \frac{R_3R_4R_1}{R_1R_4} + \frac{R_4R_1R_2}{R_1R_4} + Rg\left(\frac{R_1R_2}{R_1R_4} + \frac{R_1R_4}{R_1R_4} + \frac{R_2R_3}{R_1R_4} + \frac{R_3R_4}{R_1R_4}\right)}$$

$$\text{or } S = \frac{Sg.\ E}{\frac{R_2R_3}{R_4} + \frac{R_2R_3}{R_1} + R_3 + R_2 + Rg\left(\frac{R_2}{R_4} + 1 + \frac{R_2R_3}{R_1R_4} + \frac{R_3}{R_1}\right)}$$

Since $R_2R_3 = R_1R_4$

$$\text{or } S = \frac{}{R_4 + R_3 + R_2) + Rg\left(\frac{R_1}{R_3} + 1 + 1 + \frac{R_3}{R_1}\right)}$$

$$\text{or } S = \frac{Sg.\ E}{(R_1 + R_3 \quad 2R_T) + Rg\left(\frac{R_1}{R_3} + 2 + \frac{R_3}{R_1}\right)}$$

and 'S' will be maximum when denominator (say D) will be minimum, but D will be minimum subject to conditions.

$$R_2 R_3 = R_1 R_4$$

$$R_1 + R_3 = 2R_T + \delta \quad ; (\delta \geqslant 0)$$

Therefore for minimum 'D'

$$\delta = 0, \text{ and } \frac{\partial D}{\partial R_3} = 0$$

where $D = (R_1 + R_4 + 2R_T) + Rg \left(\frac{R_1}{R_3} + 2 + \frac{R_3}{R_1} \right)$

$$\therefore \frac{\partial D}{\partial R_3} = 1 + Rg \left(\frac{-R_1}{R_3^2} + \frac{1}{R_1} \right) = 0$$

$$\text{or } R_3^2 = \frac{R_1^2 . Rg}{R_1 + Rg}$$

$$\text{or } R_3 = R_1 \times \sqrt{\frac{Rg}{R_1 + Rg}} \qquad \text{-------- } 5.332 : 5$$

$$R_2 = \frac{R_1 R_4}{R_3} = \frac{R_4}{\frac{R_3}{R_1}} = \frac{R_2 R_4}{\frac{R_3}{R_1} + 1}$$

$$\therefore R_2 = \frac{2 R_T}{1 + \sqrt{\frac{Rg}{R_1 + Rg}}} \qquad \text{-------- } 5.332 : 6$$

$$R_4 = \frac{2 R_T . \sqrt{\frac{Rg}{R_1 + Rg}}}{1 + \sqrt{\frac{Rg}{R_1 + Rg}}} \qquad \text{-------- } 5.332 : 7$$

Since 'S' the sensitivity of the bridge is directly proportional to Sg the current sensitivity of galvanometer which in turn is directly proportional to the square root of its resistance; the value of Rg should be of a higher order. But it should not be too large as Rg appears in the denominator of the sensitivity equation and so tends to decrease the value of 'S'. However, if Rg is large compared with R_1, the best arrangement of the bridge by inspection of equations 3, 4, 5 : 5.332 will be as follows

$$R_1 = R_3 = R_T \text{ (say)}$$

$$R_2 = R_4 = R \text{ (say)}$$

Here it will be useful to find out the best resistance of a galvanometer for a given bridge network. We know that for a galvanator

$$Sg = h \times \sqrt{Rg} \quad \text{where} \quad h \text{ is a constant}$$

Substituting $Sg = h \times \sqrt{Rg}$ in the sensitivity equation 5.332 : 4 we get

$$S = \frac{h.\sqrt{Rg} \times R_1R_4 E}{Y + Rg (R_1 + R_3) (R_2 + R_4)}$$

Differentiating S with respect to Rg we get

$$\frac{\partial S}{\partial Rg} = h \times R_1R_4 \times E \left\{ \frac{\frac{1}{2.\sqrt{Rg}} \left\{ Y + Rg(R_1+R_3)(R_2+R_3) - \sqrt{Rg}(R_1+R_3)(R_2+R_3) \right.}{\left\{ Y + Rg (R_1+R_3) (R_2+R_4) \right\}^2} \right.$$

For maximum S , $\frac{\partial S}{\partial Rg} = 0$

i.e. $\frac{1}{2.\sqrt{Rg}} \left\{ Y + Rg (R_1+R_3)(R_2+R_4) \right\} = \sqrt{Rg} (R_1+R_3)(R_2+R_4)$

or $Rg (R_1 + R_3) (R_2 + R_4) = Y$

or $Rg = \frac{R_1R_2R_3 + R_1R_2R_4 + R_1R_3R_4 + R_2R_3R_4}{(R_1 + R_3) (R_2 + R_4)}$

$$= \frac{R_2R_4 (R_1 + R_3) + R_1R_3 (R_2 + R_4)}{(R_1 + R_3) (R_2 + R_4)}$$

$$Rg = \frac{R_2R_4}{R_2R_4} + \frac{R_1R_3}{R_1R_3}$$

At balance $\frac{R_1}{R_3} = \frac{R_2}{R_4}$

which gives $\frac{R_1+R_3}{R_3} = \frac{R_2+R_4}{R_4} = \frac{R_1+R_2+R_3+R_4}{R_3+R_4}$

$$\therefore Rg = \frac{R_2R_4}{\frac{R_4}{R_3}(R_1+R_3)} + \frac{R_1R_3}{R_1+R_3} = \frac{R_2R_3 + R_1R_3}{R_1+R_3}$$

or $Rg = \frac{(R_3 + R_4) \times R_3 (R_1 + R_4)}{R_3(R_1 + R_2 + R_3 + R_4)}$

or $Rg = \frac{(R_1 + R_2) (R_3 + R_4)}{(R_1 + R_2 + R_3 + R_4)}$ ---------- 5.332 : 8

or $\frac{1}{Rg} = \frac{1}{R_1+R_2} + \frac{1}{R_3 + R_4}$ ---------- 5.332 : 9

Thus the resistance of the galvanometer for maximum sensitivity should/

should be equal to the parallel resistance of the bridge.

These discussions and mathematical treatments lead us to the general conditions for the most sensible arrangement of the wheatstone bridge, as applied to Thermodynamic method to ensure a high degree of sensitivity.

Referring to Fg. 7 we should have for the most sensible arrangement

(a) $R_2 = R_4 = R$ (say) as small as possible

(b) $R_1 = R_3 = R_T$(say) { This should be the Arm containing resistance thermometer and should also be of lower order

(c) $\frac{1}{Rg} = \frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}$

(d) $E = i_{max} \quad (R_1 + R_3)$ where i_{max} is the maximum allowable current through resistance thermometers.

5.333 Calculation of deflection with change of resistance :-

Let Δi_g amps be the small change in current through galvanometer and Δn mm be the corresponding movement of spotlight. If the current sensitivity of the galvanometer is defined as k mm per micro ampere. Then

we can write $\Delta i_g = \frac{k^{-1} x 10^{-6}}{(\Delta n)^{-1}}$ amperes ---------- 5.333 : 1

Re writing equation 5.332 : 2 in difference form, we can express Δi_g also as

$$\Delta ig = \Delta R_3 \quad x \quad \frac{E R_2}{Y + Rg \ (R_1+R_3)(R_2+R_4)} \quad \text{---------- 5.333 : 2}$$

From 5.333 : 1 and 5.333 : 2

we can write

$$\Delta n = \quad R_3 \ x \ \frac{E R_2 \ x \ k \ x \ 10^6}{Y + Rg \ (R_1+R_3)(R_2+R_4)}$$

In article 5.332 it has been shown that for the most sensible arrangement of the bridge

$$R_2 = R_4 = R$$
$$R_1 = R_3 = R_T$$

and/

and since $E = i_{max} \times (R_1 + R_3)$

therefore $E = i_{max} \times 2 R_T$

equation of deflection can therefore be written as

$$\Delta n = \Delta R_3 \times \frac{i_{max} \times R_T \times R \times k \times}{R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_qR_1R_2 + R_g (R_1 + R_3)(R_2 + R_4)}$$

$$= \Delta R_3 \times \frac{i_{max} \times k \times 10^6 \times 2R}{R R_T^2 + R^2 R_T + R R_T^2 + R^2 R_T + Rg\, 2 R_T \cdot 2^R}$$

$$= \Delta R_3 \times \frac{i_{max} \times k \times 10^6 \times 2 R R_T}{2 R R_T (R + R_T) + 2 R_g}$$

$$\Delta n = \Delta R_3 \times \frac{i_{max} \times k \times 10^6}{(R + R_T) + 2 R_g} \text{ mm} \quad \ldots\ldots\ldots 5{\cdot}333{:}3$$

With the help of this equation we can calculate the deflection Δn in mm for a small change of resistance ΔR_3 ohm for a given galvanometer whose resistance is R_g ohms and sensitivity is defined as k mm per micro amperes; and i_{max} is the current flowing through resistance thermometers, R is the resistance of equal ratio arms and R_T is the resistance of equal thermometer arms.

5.334. <u>Comparison of different networks for sensitivity:-</u>

It would be interesting to compare the sensitivity of various bridge networks for a given pair of thermometers and a galvanometer of fixed resistance.

The following assumptions will be made

(i) The thermometers used have each resistance of approximately 100 ohms.

(ii) The fundamental interval of thermometers is given as $R_{100} - R_o = 40$ ohms per 100^oC.

(iii) /

(iii) The current capacity of thermometer i.e. $i_{max} = 2.0 \times 10^{-3}$ amperes.

(iv) The galvanometer used has a fixed resistance $R_g = 478.6$ ohms and a current sensitivity of 525 mm per micro amperes.

(v) The above conditions will remain similar for the arrangements of various networks.

Basis:- The comparison will be made on the basis of a change of temperature of 0.001°C.

Since the fundamental interval of the resistance thermometer used is given as 40 ohms per 100°C, a temperature change of 0.001°C $\equiv$ a resistance change of 0.0004 ohms

Therefore we will calculate Δn the deflection in mm for a change in resistance $\Delta R_3 = 0.0004$ ohms.

Re-writing equation 5·333:3

$$\Delta n = \Delta R_3 \times \frac{i_{max} \times k \times 10^6}{(R + R_T) + 2 R_g}$$

we get

$$n = \frac{0.0004 \times 2.0 \times 10^{-3} \times 525}{(R + R_T) + 2 \times 478.6}$$

$$\text{or} \qquad n = \frac{420}{(R + R_T) + 957.2} \text{ mm} \quad \ldots\ldots \ 5{\cdot}334{:}1$$

The equation 5·334:1 will enable us to calculate the galvanometer light spot deflection in mm for a change of temperature of 0.001°C.

We shall now examine the various networks.

(a) Simplified Smith bridge:-

For certain preliminary investigations it was decided to use the Smith bridge in a simplified form, with its four arms each equal to 1000 ohm. The arrangement used is shown in Fig. 8

here/

here R_T = 1000 ohm

and R = 1000 ohm

$$\Delta n = \frac{420}{(1000 + 1000) + 957.2} = 0.142 \text{ mm}$$

(b) Modified Smith bridge:- (Used by Dr. A. S. Thom).

Since the simplified Smith bridge was not sufficiently sensitive the arrangement shown in Fig.9 was used to obtain maximum sensitivity;

here R_T = 300 ohm and R = 40 ohm

$$\Delta n = \frac{420}{(40 + 300) + 957.2} = 0.322 \text{ mm}$$

(which shows that the arrangement of Fig.9 is 125% more sensitive than the arrangement of Fig.8).

(c) Modified Callander and Griffith bridge:- (Used by Dr. Singh)

The arrangement is shown in Fig.10

here R_T = 120 ohm

R = 111 ohm

$$\Delta n = \frac{420}{(111 + 120) + 957.2} = 0.354 \text{ mm}$$

(d) Willm bridge:- (Fig.11)

here R_T = 100 ohm

R = 100 ohm

$$\Delta n = \frac{420}{(100 + 100) + 957.2} = 0.363 \text{ mm}$$

It is evident that of all the four arrangements discussed, the bridge used by Willm is by far the most sensitive. However it is not ideal and the sensitivity of this arrangement can be much improved.

5.335. <u>Design of Wheatstone bridge for thermodynamic method (for given galvanometer and resistance thermometers):-</u>

For the most sensible arrangement of a bridge network for thermodynamic method as discussed in 5.332, the conditions relating to the values of resistance of arms are:-

(a) $R_2 = R_4 = R$ (should be as small as possible)

(b) $R_1 = R_3 = R_T$ (should be of a lower order)

In practice however we cannot assign any desired values to the resistances R_1, R_2, R_3 & R_4. Since the contact resistances add to the resistance in the arm, and any variation in the contact resistance contributes to the uncertainty in ratio which in turn affects the sensitivity of the bridge, it is therefore desirable that the resistances of the arms be large. However, high resistances are affected by humidity to a greater extent than low resistances and are less stable, so that they are accurately known for a shorter time. It is therefore better to use intermediate values of resistance i.e. between 10 ohms and 100 ohms.

In choosing the values of the resistances R_1 and R_3, the branches which contain thermometers, we are left with a very narrow choice. For the sensible arrangement $R_1 = R_3 = R_T$ (say) and with R_T appearing in the denominator of the deflection equation 5·333:3, it is necessary that these resistances should be of lower order. Therefore arms R_1 and R_3 should comprise only resistance thermometers having resistances of intermediate value, tending towards the higher order so that the lead resistances should be small compared with that of the thermometers themselves.

Platinum thermometers have already been used by most workers for the thermodynamic method and also used for the present investigation. Therefore all design calculations will be made on the assumption that platinum resistance thermometers are to be used in the bridge. Platinum resistance thermometers having values 100 ohms at $0^{o}C$ were used by Willm and Campmas, Thom and Singh and appear to make a good/

good compromise between high and low order of resistances.

Therefore,

$$R_1 = R_3 = R_T = 100 \text{ ohm approximately}$$

shall be chosen for a sensitive arrangement.

In article 5.334 it has been shown that Willm's bridge network is the most sensitive compared with the arrangements used by other investigators. It should be noted that the calculation of deflections for comparison was based on a galvanometer different from the type used originally by Willm and was less sensitive. For further improvement of sensitivity of a bridge network therefore, the Willm type bridge together with the galvanometer originally used by Willm and Campmas will be taken as the basis of all design calculations.

Willm used a Schlumberger S.P.E. type SP SD galvanometer, whose internal resistance R_g = 120 ohms and sensitivity is given as 10^{-9} ampere per milli radian and the length of ray of light is 750 mm.

Therefore the spotlight deflection per mm will be equivalent to $\frac{1}{750}$ radian,

or 1 mm deflection $\equiv \frac{1}{0.75}$ milli radian

$\therefore$ current sensitivity $\equiv$ 0.75 mm per 10^{-9} amperes

or current sensitivity can be defined as

750 mm per micro amperes.

Using equation 5.333:3 we can calculate the deflection of a galvanometer in a Willm type bridge for a change of 0.001°C in temperature. And it gives

$$\Delta n = 0.0004 \times \frac{2.0 \times 10^{-3} \times 750 \times 10^{6}}{(100 + 100) + 2 \times 120}$$

$$\text{or} \quad \Delta n = \frac{600}{640} = 1.36 \text{ mm}$$

Let us now investigate how to improve the sensitivity of a Willm type bridge network.

It should be noted that we are restricted to the

following/

following values of the various factors involved.

(i) $R_1 = R_3 = R_T = 100$ ohm approx.

(ii) $_{max} = 2.0 \times 10^{-3}$ amperes.

(iii) $R_g = 120$ ohms.

and current sensitivity of galvanometer = 750 mm/micro ampere

(iv) $R_3 = 0.0004$ ohms $\equiv$ a change of 0.001°C of temperature

and therefore eqn. 5·333:3 can be written as

$$\Delta n = \frac{0.0004 \times 2 \times 10^{-3} \times 750 \times 10^{6}}{(R + 100) + 2 \times 120}$$

or

$$\Delta n = \frac{600}{R + 340} \quad \ldots\ldots\ldots 5{\cdot}335{:}1$$

From equation 5·335:1 it is evident that the deflectional sensitivity can be improved only by using lower values of R, where $R = R_2 = R_4$.

In choosing the values of the ratio arm R_2 and R_4 we have a considerable choice and so we will calculate the percentage increase in deflection by taking different values of R_2 and R_4 with one restriction that $R_2 = R_4 = R$ which is desired for maximum sensitivity.

(a) let $R_2 = R_4 = R = 50$ ohms.

From 5·335:1 we get

$$= \frac{600}{50 + 340} = 1.54 \text{ mm},$$ which corresponds to an increase of 13% in sensitivity compared with Willm type bridge arrangement.

(b) let $R_2 = R_4 = R = 10$ ohms.

From 5·335:1 we get

$$= \frac{600}{10 + 340} = 1.71 \text{ mm.}$$

Compared to the Willm bridge this corresponds to an increase in sensitivity of 26%.

(c) let $R_2 = R_4 = R = 1$ ohm

From/

From 5·335:1 we get

$$\Delta n = \frac{600}{1 + 340} = 1.76 \text{ mm}$$

which, compared with the Willm bridge, corresponds to an increase of 29% in sensitivity.

These computations show that the most sensible value to be chosen for R_2 and R_4 is 10 ohms. It may be argued that when R_2 & R_4 have value of 1 ohm, the percentage increase in sensitivity is 29% compared with 26% when R_2 & R_4 are 10 ohms. An increment of only 3% is not justifiable at the expense of the risk of having a very low resistance in the ratio arm, as the change in contact resistance will considerably alter the balance of bridge. With a view of making the ratio arm as small as possible to achieve a higher degree of sensitivity, the best arrangement would be to insert a small slide wire at the junction of the two resistances R_2 & R_4; and hence the equality of the arms would be ensured - a necessary condition from the point of view of sensitivity. Such an arrangement is expensive and cumbersome and therefore the most sensible arrangement is to use a ratio arm of 10 : 10 ohm.

In conclusion, the best possible arrangement of a bridge network to ensure a high degree of sensitivity can be given as:-

(i) $R_1 = R_3 = R_T'$ (Resistance of thermometers) = 100 ohm approx.

(ii) $R_2 = R_4 = R$ (Resistance of ratio arm) = 10 ohm

5.336. <u>Ideal arrangement of Bridge network:-</u>

So far, in designing the bridge we had assumed that the best choice was restricted to a type having R_g = 120 ohm and a current sensitivity of 750 mm per micro ampere, because of its availability in the market. However it will be interesting to compute the resistance and sensitivity of the galvanometer which should be used in the arrangement summarised in article 5.335 to ensure maximum sensitivity.

It/

It has been shown in equation 5.332 : 9 that the best resistance of a galvanometer for maximum sensitivity is given by

$$\frac{1}{Rg} = \frac{1}{R_1+R_2} + \frac{1}{R_3+R_4}$$

Since For best arrangement, $R_1=R_3=R_T=100$ ohms and

$R_2=R_4=R = 10$ ohms

$$\therefore \frac{1}{Rg} = \frac{1}{110} + \frac{1}{110}$$

ie $Rg = 55$ ohms

Now if we assume that the new galvanometer is made of the same type of sensitive element as SP2SD type, whose current sensitivity is given as 750 mm per micro ampere $R_g = 120$ ohms, and knowing that current sensitivity is proportional to square root of R_g, we can calculate the current sensitivity S_g' of the new galvanometer. It is given by

$$S_g' = 750 \times \frac{55}{120} = 507 \text{ mm per micro ampere}$$

From eqn. 5.333:3 we can calculate deflection for a change of 0.001°C of temperature equivalent to a change of 0.0004 of resistance, which gives

$$\Delta n = \frac{0.0004 \times 2.0 \times 10^{-3} \times 507 \times 10^6}{(100 + 10) + 2 \times 55}$$

$$\text{or} \quad \Delta n = \frac{405.6}{220} = 1.85 \text{ mm}$$

which corresponds to an increase of 36% in sensitivity compared with the Willm bridge arrangement.

Therefore the ideal arrangement is the most sensitive arrangement, and gives an increase of 7% in sensitivity compared with the circuit when SP2SD type galvanometer was used. Another advantage of this arrangement is that this new galvanometer will be more robust because of its lower current sensitivity and the response will be quicker.

It can be concluded therefore that in a properly designed bridge, the galvanometer has also to be made to specific requirements to achieve a higher degree of bridge sensitivity without using a very sensitive galvanometer.

5.4. Expander or Throttling Calorimeter.

In the partial expansion method the expander plays an important part, and should be carefully designed. Basically an expander consists of a chamber where the pressure is reduced without any loss of energy, and is provided with a pair of throttle valves one at the inlet and the other at the outlet. Reduction in pressure results in rise of temperature, which enables calculation of the loss of head in the machine; therefore the expander must be well insulated to prevent any transfer of heat to or from the surroundings.

Most investigators have used basically the same expander as designed by Willm and Campmas. It consists of a vertical steel cylinder, in which water is admitted at the bottom through an interchangeable throttle valve into the measuring space. The water then passes through another valve at the top into Dewar vacuum flask containing expanded water by means of a divergent cone for pressure recovery. The measuring space is shielded by a removable cover consisting of a shock proof Dewar container. Willm himself points out that heat transfer is the main practical problem with this type of expander. In spite of the fact that up to a certain degree, the use of vacuum flasks, foam rubber, vinyl chloride and the immersion of calorimeter pipes and controls in the fully expanded water does reduce the error due to heat transfer, this cannot be taken as a satisfactory measure of eliminating the effect of heat flow.

Vaucher has suggested a method for applying a correction due to errors caused by heat transfer in the expander. He says that complete thermal insulation would only be possible theoretically, with an infinitely large flow of sampling water q'; in the other extreme case if $q' = o$ the measurement would indicate ambient temperature. By plotting pressure as a function of the reciprocal $\frac{1}{q'}$ of the flow of sampling water and by extrapolating to $\frac{1}{q'} = o$ the value of the pressure can be determined which would be obtained with an infinitely large quantity of sampling water, that is with perfect heat insulation.

This/

This method has two great disadvantages. Theoretically it needs the assumption that the function of $\frac{1}{q'}$ varies lineally with heat transfer effect which is caused by the difference of temperature brought about by changing the pressure inside the expander. Practically it necessitates observing a series of readings of pressure against galvanometer deflection for at least two rates of flow through the expander. It is therefore difficult to justify this method of correction for heat flow during field tests.

Brandt has developed a new expander called "Reducing probe type". He has managed to avoid all the elaborations of insulated pipes and Dewar type containers. In Brandt's reducing probe type expander the pressure reduction takes place in a fixed throttle at the extraction aperture; the measuring space lies within the probe itself, into which the resistance thermometer is introduced through a long sheath, and a thin tube is provided for the pressure measurement. The second adjustable throttle is outside of the probe at a convenient point, and also an auxiliary outlet by means of which flow quantity can be varied so that the heat exchange with the surroundings can be measured, is provided at that point.

The reducing probe type expander is definitely an improvement but still suffers from great disadvantages. The fixed throttle at the extraction aperture prevents maintenance of a constant flow at all pressure levels. Measurements of heat exchanged from surroundings are not easily obtainable, and corrections for heat transfer effects as suggested by Vaucher and followed by Brandt require certain assumptions which have no theoretical justifications and necessitate a long expenditure of time.

A new adiabatic expander was developed for present investigations and this is described in the following article; it was successfully used for testing machines with low heads and has proved to be superior to other existing expanders.

5.41 /

5.41. Adiabatic expander:-

Studies of various existing expanders described in the preceding article and the tests at Bonnington (8·2) called for the development of an expander in which expansion could take place adiabatically without having any elaborate insulations, and whose operation should not involve tedious experimentation or correction for heat transfer. Consideration was given to many designs of expanders from theoretical and practical points of view. The following conclusions were reached.

(a) The expander should be so designed that it could be assembled directly on the probe, thus eliminating the connecting hose pipe and its insulating tubes and in turn reducing the effect of heat transfer to a minimum.

(b) A method has to be developed of maintaining the immediate vicinity of the inner part of the expander at a constant temperature equal to that of the tail-race or, failing that, at a temperature as near as possible to that of the tail-race.

(c) As the amount of heat transferred to the water flowing through the expander from the surrounding atmosphere is inversely proportional to the flow, the flow should remain constant at all pressures throughout the experiment to ensure a fair degree of repeatability and accuracy.

Bearing in mind these three basic requirements a new adiabatic expander was developed (Fig.12). It consisted of a main expansion chamber of 2 inches inside diameter. Flow was controlled by two valves, one at each end, which controlled the pressure as required. This chamber was enclosed in a jacket of $4\frac{1}{2}$ inches inside diameter through which water was led from the penstock, as was done for the main expansion chamber. The jacket was also provided with an inlet and outlet valve for controlling the pressure and hence the temperature of the flow.

In an expander not properly insulated, although the

temperature/

temperature of the water could be raised to the temperature of the tail-race, the corresponding pressure inside the expander could not be taken as correct because of heat transfer from the atmosphere; part of the temperature effect observed on the galvanometer would not be the result of destroyed pressure but would be caused by the heat transferred from or into the surrounding air. To eliminate this source of error, fitting of an outer concentric expander was considered to be the best method. Since the new jacket enveloped the whole inner expander it followed that, on bringing the jacket temperature and the inner expander temperature to that of the tail-race, the corresponding inner pressure would be correct, and the temperature effect observed on the galvanometer would be a function of destroyed pressure only.

In the original design of Willm and Campmas the expander was fitted with a divergent cone after the outlet valve in which the final part of the expansion took place. In the new adiabatic expander the outlet valve was coupled to a vertical variable-area-type flowmeter (Fischer & Porter) by a 3 ft. long high pressure hose. At the outlet of the flowmeter, final total expansion took place, and the expanded water was then led to waste. This design enabled the rate of flow to be kept constant at all pressures. The outside jacket was provided with the same arrangement as the main inner expansion chamber, complete with flowmeter.

To minimize the effect of heat transfer, the main expansion chamber was made of stainless steel and it was covered with glass fibre moulded to the cylindrical surface. The jacket was made of mild steel tube about 1 ft. long. The outside of the jacket was covered with cylindrical cork pads shaped to suit the surface irregularities; plastic tubes of about $\frac{1}{4}$ inch bore, through which pressurized water from the penstock was kept flowing at a constant rate, was wrapped over the cork pads. The whole unit was wrapped with strips of 1 inch thick foam rubber to ensure good thermol insulation.

Two/

Two Bourdon gauges were provided to give an estimate of the pressure in the expander and jacket. As in the original design the piston manometer was connected to the main expansion chamber to record the exact pressure.

That portion of the probe exposed between the outside wall of the pen-stock and the inlet valve of the expander was wrapped in 1+ inch thick plastic foam covered with $\frac{3}{8}$ inch bore high-pressure hose, through which pressurized water from the penstock was allowed to run. By this means it was hoped to make the temperature of that portion of the probe outside of the penstock as nearly as possible equal to that of the penstock water.

A procedure of measurement for the adiabatic expander is given in the article 5.1. The results obtained by using this expander are given in article 8.6, which shows that this type of expander is almost indispensible for testing low head machines.

5.42. Valves for Expander:-

In designing an expander choice of valves plays an important part. Most of the commercial valves available are the conventional pinch type which tend to give a high friction loss, thus preventing measurements at high efficiency points; it also gives a rather steep rise of flow with small percentage displacement of stroke near the opening, which impedes accurate settings of different pressure levels at a given constant flow. Various designs of valves were studied and a valve of Swedish design called "Sala Slurry Valve" produced by Neldco Process Limited is considered to be the most suitable for adiabatic expanders. The Sala valve is a non plugging valve and is used for continuous process control. It is unique in having round holes at all apertures. It is operated by hydraulic or pneumatic pressure applied to a central muscle which brings the change in aperture and hence the change in flow. The round aperture is infinitely variable from full bore to full closure and the venturi shape of the aperture produces a low pressure loss so that original flow pressure is maintained - a very desirable feature especially for the/

the inlet valve of the expander. Apart from the flexing of muscles there are no moving parts, and since the passage of the flow is always circular, it eliminates the possibility of recording any false measurement of pressure caused by the presence of foreign bodies in suspension within the flowing medium. The hydraulic or pneumatic operator can be mounted at any convenient point and can be controlled manually or by power, and thus gives the advantages of remote control - an attractive feature where access to penstock for observation is not easily available. Unlike conventional valves, the percentage flow in the Sala valves increases gradually with the percentage of opening throughout the whole range of apertures - this unusual performance ensures accurate settings of different pressures for a given constant flow, thus rendering it possible to repeat the readings at exactly the same conditions during an efficiency test. Fig.13 shows the performance curves of conventional pinch valve and Sala valve at a constant inlet pressure. These curves illustrate that in a conventional pinch valve the distribution of the flow does not change gradually with the opening of the aperture as it does in a Sala valve.

Due to long delivery dates, the Sala valves in spite of their inherent qualities were not used during present investigations but it is recommended for use in all future tests by thermodynamic method. It should be noted that use of Sala valves requires a special design of expander to get the best results.

However, whatever type of valve is used, careful choice is necessary and the working range for the pair of valves has to be found by experiment. During present investigations a ½ inch inside diameter conventional pinch valve was found to be quite suitable for a constant flow rate of 0.4 litres/sec - a flow rate accepted by most investigators. If the valves at inlet and outlet are opened to full capacity, and then the outlet valve is gradually closed until the flow-meter reads 0.4 l/s, it will give the maximum pressure which can be maintained at that flow. Similarly, after fully opening the inlet and outlet valves, if the inlet valve is gradually closed until the flow is

0.4/

0.4 l/s it will give the minimum pressure which can be maintained at that flow. Any pressure level between those defined limits can be maintained for a constant flow rate of 0.4 litres/sec. Following the same procedure the upper and lower limits of pressure can be determined for any particular rate of flow.

This experiment makes it possible to find out whether a pair of valves will be suitable for successful readings at high-efficiency loads. A quick computation can be made. Suppose that the total head is 180 ft. and assume that the highest efficiency point attained by the machine will be 95 per cent; the maximum pressure in the expander should not be less than 0.95 x 180 - 171 ft. of water, when the outlet valve is gradually closed to restrict the flow to 0.4 l/s. A similar computation can be made for the lower range of working pressure assuming any low value of efficiency.

5.5 <u>Pressure gauges</u>:-

Ordinary Bourdon pressure gauges are not accurate enough for the measurement of pressure in thermodynamic tests. In view of this fact Willm developed a manometric piston gauge, which in principle is a combination of a deadweight gauge and a water manometer. Singh following Willm and Campmas constructed a gauge, shown in Fig.14, 14a, which permits measurements of the pressures to be made to $\pm$ 0.1 ft. for all measurements above 100 ft. of water. During present investigations Singh's gauge was used to record all pressures and excellent results were obtained. Singh has given a theoretical analysis of the tolerances to be allowed in the measurements of heads and levels; he concludes that manometric piston gauges can conveniently measure pressure heads within the given tolerance for the efficiency tests of machines operating under 100 ft. of water and above.

For low pressure heads Singh used a $\frac{7}{8}$" diameter piston made up of leather washers. This was very unsatisfactory and gave erroneous results. A modification of this piston became necessary.

A/

A solid piston was not desirable as it would reduce the range; therefore it was decided to use a thin steel tube closed at both ends and approximately $\frac{7}{8}$ inch outside diameter as a piston for low pressure ranges. The piston was calibrated and the results obtained were very accurate.

6.0 TECHNIQUE OF MEASUREMENT:-

It has been the general practise to note the galvanometer deflection at different pressure settings of the expander to determine the pressure corresponding to an expander temperature equal to the tail-race temperature. A graph of pressure against deflection has usually been plotted and a straight line drawn passing through these points, the point of intersection of the straight line and the zero deflection line giving the desired pressure. It was observed that the graph of pressure against deflection did not always give a straight line, although the resistance thermometers had linear resistance/temperature characteristics. Moreover there is no theoretical justification to assume that deflection plotted against temperature should give a straight line. From equation 2.6:2 we can write efficiency as

$$= \frac{H_1(1-\alpha) + JC_{P2}(T_1 - T_2) + Z_1 - Z_2}{H_1(1-\beta) + Z_1 - Z_2}$$

Since the expander has zero efficiency, it follows that

$$H_1(1-\alpha) + Z_1 - Z_2 - JC_{P2}(T_1 - T_2) = 0 \text{ --------6:1}$$

where H_1 is the pressure in the expander

T_1 is the temperature in the expander

α is a non-dimensional coefficient and is a function of H_1 and T_1

JC_{P2}, $Z_1 - Z_2$ and T_2 (being the temperature of the tail-race) are constants.

Rewriting/

Rewriting equation 6:1 we get

$$E_1 (1 - \alpha) + \text{constant} + \text{constant} \times T_1 = 0 \quad\text{------------}\quad 6:2$$

This will represent a straight line only if $(1 - \alpha)$ is constant; but as varies with temperature as well as pressure, it is not justifiable to say that equation 6:2 represents a straight line. However, if for small change of pressure in the expander it is assumed that change of α is so small that it can be taken as a constant, then it will no longer be necessary to take many different readings of pressure in the expander and the corresponding deflection to compute the condition when $T_1 = T_2$.

As the characteristics of bridge, galvanometer and resistance thermometers are known, a theoretical relationship between the deflection and change of temperature and hence pressure can be determined. This can be verified easily in the laboratory. It has been shown in equation 5.333:3 that Δn the deflection of galvanometer is given by

$$\Delta n = \Delta R_3 \times \frac{i_{max} \times k \times 10^6}{R + R_T + 2Rg}$$

As the fundamental interval of the resistance thermometer is usually denoted by

$$\frac{R_{100} - R_o}{T_{100} - T_o} \quad \text{ohm per deg C}$$

we can write

$$\frac{\Delta R_3}{\Delta T} = \frac{R_{100} - R_o}{T_{100} - T_o}$$

where ΔT is the change of temperature experienced by the resistance thermometer.

Therefore we can write the deflection equation as

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$$\Delta n = \Delta T \cdot \frac{R_{100} - R_o}{T_{100} - T_o} \cdot \frac{i_{max} \times k \times 10^6}{R + R_T + 2Rg} \quad \text{------------ } 6{:}3$$

In equation 6:1, if the change of α is assumed to be negligible for small pressure changes, the pressure-head change ΔH inside the expander can be expressed as

$$\Delta H = JC_{p2} \cdot \Delta T$$

and if C_{p2} is assumed to be unity, the value for pure water at 4^oC - which is justifiable because of the small rate of change of C_p with temperature in the range $0-15^oC$; we can write

$$\Delta H = J.\Delta T$$

and hence equation 3:6 transform into

$$\Delta n = \Delta H \cdot \frac{1}{J} \cdot \frac{R_{100} - R_o}{T_{100} - T_o} \cdot \frac{i_{max} K.10^6}{R + R_T + 2Rg} \quad \text{--------- } 6{:}4$$

Equation 6:4 thus makes it possible to calculate theoretically the galvanometer deflection for unit change of head in the expander, i.e., the gradient of the deflection/pressure-head graph. It should be noted, however, that this value of the gradient will be true only for small pressure cha ges.

Therefore, if readings are taken close to the condition where $T_1 = T_2$ a state point will be obtained on the deflection/pressure graph. Knowing from equation 6:4 the value of the gradient, a line can be drawn passing through the state point and from the intersection of this straight line with the straight line representing $T_1 = T_2$, the pressure in the main expansion chamber for the condition when $T_1 = T_2$ can be determined.

This method reduces the necessity of taking a large number of readings at different pressure settings, and time will be/

be saved. Moreover, by taking the mean of four or five readings at one pressure setting which closely corresponds to the condition of $T_1 = T_2$, it can be ensured that conditions in the expander at the time of measuring are steady and that the results obtained are reliable.

This method also enables approximate values of the efficiency to be obtained at loads when the efficiency is very high and it is not possible to maintain the required high pressure in the expander and to keep the desired rate of flow due to the losses occuring at the inlet valve. Under such conditions, when it is not possible to make $T_1 = T_2$, a reading as close to this required condition as possible can be taken and, using equation 6:4, an approximate though not accurate val e of the efficiency can be calculated.

6.1 Procedure for Measurement:-

Three resistance thermometers suitably matched were used to measure the temperature differences between the expander and the tail-race. One thermometer (T_1) was placed inside the main expansion chamber; the second thermometer (T_2) was placed in the tail-race section and the third thermometer (T_3) was placed inside the jacket. It was not found necessary to match (T_1) and (T_3) because the only desired combinations were (T_2) with (T_1) and (T_2) with (T_3). Carefully selected thermometers were tested in the range 0^o to 15^oC. The combinations (T_2) with (T_1) and (T_2) with (T_3) were placed in a bath at say 0^oC and bridge settings were so adjusted that the deflection of the galvanometer was zero. Theoretically, if the thermometer elements are identical, the deflection should remain zero after raising the temperature of the bath to, say 15^oC, but this does not happen in practise, and a choice was made, from ten thermometers, of those two pairs which gave the minimum variation of deflection.

Before/

Before any measurements are made, the required rate of flow through the expander has to be determined. Theoretically it is desirable to make this flow as large as possible, because when the flow tends to infinity the heat transfer effect tends to zero. However, it is impractical to make the flow very large. A quantity has to be chosen within limits, the upper of which must not be so large that high pressures at the desired fixed flow cannot be maintained, whilst the smaller limit must be large enough to keep the heat transfer effect neglibible. For this type of apparatus a flow of 0.4 litres/second has normally been used by Willm and Campmas and by Thom and Singh. It has been observed experimentally that it is difficult to obtain pairs of valves which can maintain 0.5 l/s at high pressure settings; readings at high efficiency loads might then be unobtainable. At 0.3 l/s, however, it is possible to maintain high pressures, but the heat transfer effect becomes appreciable. Even with the adiabatic expander, for a difference of only 5 deg C between water temperature and the surrounding atmosphere, an adverse effect was observed. It could therefore be concluded that 0.4 l/s is the most suitable flow through the expander.

Some preliminary experiments using the adiabatic expander were carried out with 0.04 l/s flowing through the jacket and 0.4 l/s flowing through the main expansion chamber. As the heat transfer is a function of the flow, it is desirable that the flow through the outer jacket as well as the inner expansion chamber should be the same, so that if any heat transfer did take place caused by inequality of temperature between the two chambers, due to experimental error, its effect would be a minimum.

For a flow of 0.4 l/s a ½ inch inside diameter 'Ermeto' valve was found to be most suitable. Two pairs of valves were chosen, one pair for the expander and the other for the jacket. The/

The working range for the pair of valves was found by experiment following the method outlined in the last paragraph of article 5.42. The valves were found to be suitable for a maximum efficiency of 92% for a machine operating under 180 ft of water approximately.

The three thermometers (T_1), (T_2) and (T_3) were connected by four-way cable to the bridge through a low resistance 4-pole two-way switch. The switch connections were so arranged that combination (T_2) with (T_1) and (T_2) with (T_3), each had two of the four wires of the cables for compensation leads. The cable soldered to (T_2) was joined to the input side of the 4-pole two-way switch, and the cables soldered to (T_1) and (T_3) were joined to the two output sides respectively.

The three thermometers were placed in a flask of water taken from the penstock. The bridge was then adjusted so that there was minimum deflection for both pairs, (T_2) with (T_1) and (T_2) with (T_3). The valve of the deflection for both pairs should be the same for one particular setting, because of the fact that the lengths and types of the cables and the resistances of the elements were identical. However, due to the different resistances of the soldered joints the total resistances were not the same - thus resulting in inequality of deflection; hence a compromise had to be made between the two deflections when the bridge settings were adjusted. Deflections for each combination were noted and it was taken that these corresponded to conditions when $T_1 = T_2$ and $T_3 = T_2$.

Thermometer (T_2) was then placed, by means of a probe, in the tail-race mixing chamber into which water from twelve different positions was led to give a mean temperature. Thermometer (T_1) was placed in the main expansion chamber through a gland. Care was taken that the gland was sufficiently tight to prevent any pressure leakages between the main expansion chamber and the outer jacket. Thermometer (T_3) was placed in the annulus formed by the inside of the jacket and the outside of the main expansion chamber.

The/

The outer cover flange for the jacket, through which the two cables passed by means of a gland, was then bolted to the jacket and made pressure-tight. The complete adiabatic expander was then well wrapped in strips of 1 inch thick and 3 inch broad polythene foam.

The inlet valve for the main expansion chamber was kept closed and the inlet and outlet valves of the jacket were opened slowly to maximum opening. After allowing the water to flow for about two minutes, the outlet valve of the jacket was gradually closed until the discharge was 0.4 l/s. Flow was then allowed for about fifteen minutes to ensure steady conditions in the jacket expander. The inlet and outlet valves of the jacket were then manipulated, maintaining a flow of 0.4 l/s, to bring conditions in the jacket as nearly equal as possible to conditions defined by $T_2 = T_3$. Deflection of the galvanometer was recorded and the variation from the deflection for $T_2 = T_3$ was noted. Any variation in deflection of more than 1 mm was corrected by remanipulating the valves. The expander was then allowed to run for some time, and a constant watch was kept on the flowmeter to ensure steady conditions.

After conditions in both chambers had become steady, the inlet and outlet valves of the main expansion chamber were adjusted, maintaining a flow of 0.4 l/s to bring the conditions inside as nearly equal as possible to the conditions defined by $T_2 = T_1$. Galvanometer deflection was then noted, and if the variation in the deflection from the deflection required for $T_1 = T_2$ was more than 1 mm, another valve setting was made and the corresponding deflection noted. This variation may be either positive or negative. The current was reversed at every reading of the galvanometer, and a mean of the two readings was recorded to eliminate stray current effects. Several readings of the galvanometer deflection at the same pressure setting were made and/

and an arithmetic mean of all the readings was taken as the corresponding deflection. The pressure inside the main expansion chamber was then measured accurately by the manometric piston gauge. With the help of equation 6:4, the exact value of the pressure corresponding to the condition $T_1 = T_2$ was calculated. Readings of total head and measurement of datum levels at the two test sections of the machine were also taken. The efficiency was finally calculated by the equation given in 4:1 as

$$\eta = \frac{H_1 (1 - \alpha) + Z_1 - Z_2}{H_1 (1 - \beta) + Z_1 - Z_2}$$

7.0 SOURCES OF ERRORS AND PRECAUTIONS

Errors can be introduced either by (a) the inaccuracy of experimental observations, or by (b) the occurence of various physical and chemical phenomena.

Experimental errors can be introduced during the measurements of temperature difference, pressure and of datum levels, or in the determination of and the non-dimensional coefficients. Therefore a limit of tolerance for each of these factors has to be defined for a particular machine operating under certain head. Serpaud and Coffin have shown under the heading of theoretical precision that these limits can be easily calculated by making some simple and valid assumptions and have supplied illustrative examples.

Various physical and chemical phenomena likely to introduce errors have been outlined by Willm and Campmas, which can be summarised as follows:-

(a) Impurities of water

(b) Heat exchanges with outside

(c) Changes of state of the water due to canitation or vaporization

(d)/

(d) Changes in the gas content of water

(e) Variation of temperature of water with time.

It has been observed that excess impurities in water occur only in exceptional occasions and in general cases, this factor is relatively unimportant.

Errors caused by the effect of heat transfer between the system and surroundings are difficult to estimate and can only be approximately determined. Wilm points out that this source of error can entirely be neglected for the machines of large capacity - he bases his argument on the fact that the thermodynamic method agrees with other methods of measurement. However, Brendt observes that specially high atmospheric humidity can raise the heat transfer coefficient between the walls of the system and air, and consequently increased the heat flow, by a factor of more than ten, if water from the atmosphere condenses on the housing. For machines of relatively small capacity, the error introduced by the heat transferred from the surroundings to the flowing medium can be of high value, and therefore corrections must be applied to obtain accurate results.

Errors similar to the above may occur due to the heat transfer between the surroundings and the sampling flow passing through the measuring expander. Vaucher followed by Brandt has given a method of evaluating the amount of heat transferred by varying the flow. This evaluation is tedious in experimentation and has got little theoretical justification. The necessity of heat transfer corrections which are difficult to estimate can be avoided by thermal insulation, using suitable materials such as stainless steel, and by appropriate design of the expander. It can be claimed that use of the adiabatic expander (5.41) ensures almost perfect insulation and does not necessitate heat transfer correction; and hence it is strongly recommended to eliminate/

eliminate this type of error.

Willm and Campmas point out that measurement at the exit of the system should be taken at a considerable distance from the runner of the machine to avoid any error due to change of state of water produced by canitation. They observed that at Val Beneyte plant, samples of water taken from immediately under the wheel led to the deduction of a turbine efficiency near unity, whereas the measurements at the exit of the draft tube were realistic.

The presence of dissolved gases in the water can introduce a gross error, but only in special cases; an assessment of the quantity of gas released could then become essential. Willm and Campmas have described a simple apparatus for this purpose; but they suggest that if in exceptional cases the effect of dissolved gases were responsible for too great a correction term, it would be preferable to abandon the thermodynamic method.

A further cause of error in many cases could be the instability of the temperature of the flowing medium. As a matter of fact, when the temperature water does vary with time the expression for efficiency (2.6:2) is no longer theoretically valid. Therefore the stability of the temperature is essential to an accurate measurement of efficiency. Willm and Campmas emphasize that the temperature of the water be constantly observed and if the variation of temperature exceeds 0.3 deg. C/hr, it is preferable to postpone the measurement to a more favourable time. However, it is noted with considerable interest that Serpaud and Coffin have developed a simple gradient simulator, and have been able to reproduce consistently the effect of temperature gradient by this device. They claim to obtain correct results even in the difficult cases.

An additional source of error in low head machines could be the etrogeneity of the temperature across the cross-section of the tail-race. In article 5.2 a device for mixing the tail-race water has been described, which ensures compensation of the effect of temperature gradient. Umpfenbach suggests that the tail-race measuring section sould be taken further downstream to reduce this error.

8.0 FIELD TESTS

8.1 Tests at Bonnington, June, 1959:

At Bonnington two Francis turbines operate under a head of 189 ft. and each develop about 5.2 megawatts. The tail-race is divided into two sections for each machine, and is approximately 23 ft. wide. Thom and Singh noted that at this station the bearing cooling water is discharged into the draft tube which affects the tail-race temperature, and so the efficiency measured by the thermodynamic method was not a true hydraulic efficiency.

All tests were carried out by the Thermodynamic method, and on Machine No. 1

(a) The expander was similar to that developed by Willm and Campmas and used by Thom and Singh.

(b) The bridge circuit used was as shown in Fig. 8. The total current was 4 mA

(c) The two platinum resistance Thermometers of French make were the same as used by Thom and Singh.

(d) A mixing chamber described in article 5.2 was used for taking the average temperature of the tail-race.

Because of shortage of water it was not possible to run the machine at or near full power. The range of tests extended from 1.5 to 2.8 MW. The results obtained are given in Table 5. In this low range the efficiencies calculated were not the same as those measured by Thom and Singh in April 1958. To quote a typical example, the efficiency at 1.58 MW found on 8th April, 1958 was 77.4 per cent and at the same load in June 1959 it was 69.8 per cent.

A six-point traverse as a preliminary investigation was also carried out, and the observations are given in Table 7. One Thermometer was put into the mixing chamber defined by position M and the other thermometer was placed successively at each of six different positions equally spaced across a cross-section of the tail-race,

at/

at a depth of 2 ft. below the water level. (Fig. 15). These traverse tests were performed at three different loads in the range 1.5 to 1.8 MW. In an extreme case a variation of temperature resulting in a 3.7 per cent variation in measured efficiency was found to exist between the temperature of the mixing chamber and the mean temperature of the water at the six different positions of the tail-race cross-sections. It shows that the effect of temperature gradient across the tail-race could be critical in some cases, and further shows that selection of six point is inadequate to measure the mean temperature.

Another series of traverses by choosing twelve points across the tail-race was taken and it was carried out by putting one Thermometer in the expander and the other thermometer successively at each of the thirteen different points in the tail-race cross-section, (Fig. 16). The first position was in the mixing chamber defined by M, while the other twelve were defined by the entrance of the twelve different pipes leading to the mixing chamber. This arrangement made it possible for the difference to be found between the mean temperature of the tail-race across the twelve different positions and the mean temperature of the tail-race obtained by the mixing chamber, assuming that there was no heat flow between the mixing chamber and its surrounding water. Table 8 shows the observed readings; and the maximum deviation inefficiency, between the measured mean obtained from traverse observations and the experimental mean obtained by mixing chamber, appears to be only 0.92 per cent.

During the traverse it was seen that stray currents were affecting the galvanometer which showed considerable fluctuations in the readings. At some points it was not even possible to record the reading, as the galvanometer spot moved off the scale. These effects remained after switching off the battery current but it stopped/

stopped when the thermometer was taken out of the tail-race water.

8.11 Discussions of Bonnington results, June 1959

Study of the results obtained leads to the following conclusions.

(a) The efficiency at the same load differed by about 7.6 per cent from the April 1958 test. It was noted that the temperature difference between the surrounding air at the expander and the water flowing through it was considerably higher in April 1958 than in June 1959.

(b) The mixing chamber gives a sample at a reasonably accurate average temperature and its use saves the work of measuring the temperature gradient across the tail-race.

(c) At Bonnington random deflections were observed on the galvanometer. These may be due to the presence of electrically charged particles whose effect can be eliminated by reversing the current in the circuit and taking the mean of the two readings. However, when the current is so strong that it is not feasible to record the readings, the position of the Thermometer in the tail-race should be changed. It has been observed that these currents only exist at certain positions in the tail-race, whereas at other positions either they are not noticed or their effect is not noticeable.

8.2 Tests at Bonnington, December '59 to February '60

A further series of tests was carried out at Bonnington from December 1959 to February 1960. The apparatus was practically the same as in the June 1959; and the same machine, namely Machine No. 1, was tested. At the beginning of the tests it was observed that it was no longer possible to obtain a reading at high-efficiency loads. The effect of heat transfer was so great that the temperature in the expander could not be brought to the temperature of the tail-race by manipulating the valves of the expander./

expander.

This problem was solved by altering the design of the expander developed by Willm and Campmas. In this original design the thermometer was placed inside a copper pocket, with externally fixed fins, placed inside of the expander facing upstream (Fig.17a). In the altered design the thermometer with copper shield suitably lengthened, was placed directly inside the expander, through a gland arrangement (Fig. 17b). Heat transfer to the thermometer from the warmer air in the vacuum flask would now be reduced.

The above arrangement made it possible for the efficiency at every load to be measured with a fair degree of repeatability. Readings are shown in Table 9 for the two arrangements. In Fig. 18 the readings are plotted showing the difference obtained between the old copper pocket with fins and the new gland arrangement. Also some efficiency results obtained by using the new gland arrangement at different loads are given in Table 6. It should be noted that this was a preliminary investigation and therefore results are not accurate.

A series of traverses was taken at three different loads with one thermometer in the expander and the other successively at twelve different positions across the tail-race. The positions defined were the same as in June 1959 test (Fig. 16). These traverses followed the efficiency test carried out with thermometer (T_2) at a position in the tail-race where the velocity was a maximum as judged by visual observation; it should be noted that in all three cases, the chosen position came to be No. 6. Following the normal procedure of plotting deflection against pressure (Fig. 20), efficiencies were computed. Efficiencies were also calculated by using the average of the traverse deflections at a measured expander pressure kept constant throughout the experiment, and extra polating it by knowing the theoretical gradient of deflection/ pressure/

pressure (Fig.20); theoretical gradient was calculated from equation 1:5.334 - for the bridge circuit used (Fig. 8) it was found to be as 1 mm = 10 ft. of water. Table 11 shows the observed readings and the calculations involved for three different loads. Corrections were then estimated by comparing the values of efficiency obtained from the conventional test procedure, with that obtained from the average of the twelve different traverse readings. The corrections applied at three different loads were:

(i) at 4.92 MW, - 1.7 per cent efficiency

(ii) at 4.0 MW, - 0.5 per cent efficiency

(iii) at 3.0 MW, - 0.8 per cent efficiency.

An experiment was also carried out to find the effect of lagging the exposed parts of the measuring circuit between the expander and the wall of penstock, by wrapping it with flexible plastic tubings and allowing the water from penstock to flow through the tubings. Table 10 and Fig. 19 shows two sets of readings, one taken without lagging i.e. no water of conditioning through the tubes and the other with lagging i.e. water of conditioning flowing through the tubes.

8.21 Discussion of Bonnington results, December '59 to February '60

From Fig. 3 calculations showed that with the old arrangement of expander the efficiency obtained was greater than 100 per cent, whereas the efficiency obtained using new gland arrangement was 81 per cent, which is quite realistic. It should be noted that in the June 1959 test, it was possible to take the readings with the old arrangement, whereas a few months later the same apparatus failed to give realistic readings. The only logical explanation is, that heat was being transferred from the atmosphere to the expander due to the increased temperature gradient. An examination of the temperature of the atmosphere surrounding the expander, and of the temperature of the water flowing through the expander shows that in the June 1959 test the difference was about

4.5/

4.5 deg. C, whereas in the December '59 to February '60 test the difference was about 15 deg. C. It is therefore concluded that during the December '59 to February '60 tests, the amount of heat transferred from the atmosphere to the expander was so much that the efficiency values obtained were unrealistic. This emphasises the importance of using an adiabatic expander.

The traverses carried out at three different loads, although making it possible for a correction to be applied for the temperature gradient across the tail-race, took about two hours. It is difficult to justify this expenditure of time. Observations showed that the correction to be applied was within ± 1.5 per cent for the majority of the load range of the machine. Therefore, if the accuracy required is within ± 1 per cent, some method has to be adopted to find the mean temperature of the tail-race. All later tests were carried out by means of mixing chamber described in article 5.2

An interesting result was obtained by taking consecutive readings with and without lagging the portions of measuring circuit between the penstock and the expander. The error in efficiency resulting from removal of the lagging brought about by stopping the flow of conditioning water, is as high as 6.7 per cent. This shows the importance of using a well-insulated expander and also leads towards the design having the expander built on the probe, thus completely eliminating the corrections between the probe and the expander - successfully incorporated in the design of adiabatic expander.

8.3 Tests at Finlarig, June 1960

The alternator was driven by two overhung Pelton wheels, each being supplied by a 3 ft. diameter pipe bifurcated from a main pipe of about 5 ft diameter. The two turbines had a common tail/

tail-race, and the upstream and downstream test sections were chosen in the main pipe and common tail-race respectively. Net was 1275 ft rated output was 30 MW and maximum flow was approximately 360 cu. secs.

Preliminary tests were carried out at this station with the new adiabatic expander for two reasons.

(a) The head was high and the expander could be tested under high pressure.

(b) Previous tests with the ordinary expander has shown relatively higher valves of efficiency, and therefore any improvement due to using adiabatic expander could be observed easily.

No measurement of flow through either of the expansion chambers was made. A mixing chamber was provided for measurement of the mean temperature across the tail-race section. The arrangement was similar to that used at Bonnington (section 8.1) and the bridge circuit used was the same as in Fig. 8.

A series of tests showed encouraging results and it was observed that the pressure in the outer jacket had to be maintained at a higher value than the pressure inside the main expansion chamber, to bring the temperature of these two compartments equal to that of the tail-race. This was exactly as expected. The reason was quite obvious: the outer jacket was receiving heat from the warm surroundings and had to be kept at a lower temperature than the inner chamber, and an equilibrium temperature equal to the inner chamber temperature was reached. Efficiency values determined were lower than those obtained at previous tests in July and September 1958 by Dr. Thom. No flow meters were used at this stage, so no control could be kept over the amount of flow at each setting. Consequently the different rates of heat flow resulted in some scatter of the values of efficiencies obtained.

Three/

Three traverses at low, medium and high load were made to determine the temperature distribution across the tail-race. Two methods were employed.

(a) The conventional method, of placing one thermometer inside the main expansion chamber, where a constant pressure was maintained, and the other thermometer successively at thirteen different positions across the tail-race.

(b) The two thermometer methods, where one thermometer was placed inside the mixing chamber and the other was placed successively at twelve different positions across the tail-race.

The variations of deflections were observed. The arithmetic mean of the deflections at different positions was taken as reference and each valve of deflection at a particular position was then represented as a variation in percentage of efficiency - shown in Fig. 21 and Fig. 22. As seen in the graphs, the variation was within the limit of ± 0.6 per cent. In this particular station, therefore, use of the mixing chamber can be dispensed with, if the accuracy required is less than ± 0.6 per cent.

The effect of air intake into the turbine casing on efficiency measurement was investigated. A large air duct had been fitted to each casing, allowing air from the station building to be drawn into the casing and entrained in the spray of water from the buckets. Efficiency measurements were made at two loads, each time with and without the air flow. A simple cover was arranged to stop the air flow when required. Results are shown in Table 12. The presence of the air from the station appears to have the effect of cooling the tail-race and of showing a rise in efficiency of about 0.8 per cent. No measurements of relative humidity were made at the time but a later request to the meteorological office for an approximate valve of humidity was/

was fruitful and calculations of the effect, with the help of a monograph constructed by Willm, gave a value of approximately 0.4 per cent change of efficiency in the same sense.

It is possible that the effects of humidity on condensation on the outside of the turbine casing are also important and it is evident that in future this phenomenon should be considered more carefully.

At one time during the period of testing part of one of the two overhung Pelton turbines was under repair. With the co-operation of the station staff the unit was used with hydraulic power from only one turbine. For outputs of between 3 and 12 MW thermodynamic tests gave the expected shape of efficiency curve. It was evident that each runner could easily be tested in this way. The experiment was not continued.

Calculations were commenced to determine the effect of heat transfer along the shaft to the wheels from the alternator. The effect was, however, considered to be small and work on this question was discontinued.

8.4 <u>Tests at Bonnington, December 1960</u>

A detailed study of the sensitivity of the wheatstone bridge network revealed that the circuit used by Dr. Thom (Fig. 9) was more sensitive than the network used on the previous tests at Bonnington (article 5.334). In further tests therefore the circuit shown in Fig. 9 was employed, with an increase in galvanometer deflection per unit of head.

The adiabatic expander was used, with flow meters to control the flow in its two compartments. Flow in the outer chamber was kept steady at 0.04 l/s. The repeatable results obtained were encouraging and showed the practical validity of this new technique. At certain loads, because of the very low flow/

flow in the outer chamber, it was not possible to make the temperature of the outer jacket equal to that of the tail-race. However, the deflection obtained due to the inequality of temperature was noted and later a correction was applied. The results were satisfactory, but this procedure was undesirable. It was therefore decided that in future a flow of 0.4 l/s instead of 0.04 l/s would be maintained through the outer jacket.

Several efficiency tests were performed but later they were abandoned due to inaccurate and scattered results caused by the instability of the temperature of flowing water.

8.5 Tests at Stonebyres, January 1961

The layout of Stonebyres is similar to Bonnington. Two vertical Francis turbines were separately supplied by pipes of 6 ft. 6 inches diameter from a surge chamber at the end of a tunnel. The net turbine head was 92.5 ft. for rated output of 2.9 MW.

Previous tests carried out by Thom and Singh at this station had been unsuccessful. A test was therefore made at this station to examine the suitability of the new technique and the successful working of the new adiabatic expander.

The apparatus and the bridge circuit used were the same as described in article 8.4. No mixing chamber was employed. Thermometer (T_2) was placed in the tail-race where the velocity was judged to be maximum.

The efficiency results obtained from a test carried out on machine No. 2 are given in Table 13 and a graph of the efficiency curve is shown in Fig. 23. The gap in the curve is explained by the fact that the machine is not operated between 1.5 and 2.0 MW due to excessive vibration.

No accuracy is claimed for the results. However, it shows that at a head as low as 95 ft. the new technique is valid and the new adiabatic expander is workable. A definite improvement can therefore be claimed in the development of the thermodynamic method.

8.6 Comparative tests at Bonnington, April 1961

It was desirable that a comparison of methods should be undertaken and in April 1961 such tests were carried out at Bonnington. The comparison was made between results calculated from a series of current meter and power output measurement, and measurements taken simultaneously by the thermodynamic method.

8.61 Estimation of hydraulic efficiency

Measurements of flow using current meters, measurements of power output and the estimation of generator efficiency are described in appendices of Reference 16. Total head measurement was by the piston manometer, used also for the thermodynamic method. Results are given in Tables 4, 5 and 6 of Reference 16.

8.62 Measurement by the thermodynamic method

Previous tests at Bonnington (article 8.4) showed that a flow of 0.4 l/s through the outer jacket was most desirable, so two similar probes were provided, one for the outer jacket and one for the inner chamber, to extract water from the penstock. Depth of insertion was the same and the complete apparatus was lagged as before with high-pressure hose, plastic tubings etc. (article 5.41).

The expander arrangements were the same as described in article 5.41 and the technique of measurement is described in article 6.1. The electrical circuit used is shown in Fig. 9. A mixing chamber was used as described in 5.2 for measuring the mean temperature across the tail-race section.

Five or six tests were made per day and the results on two consecutive days are given in Table 15 and plotted in Fig.24.

8.63 Discussion of Comparative tests

From Fig. 24 it is evident that the envelope or shape of the thermodynamic curve is similar to that of the overall efficiency/

efficiency curve. Moreover, the repeatability of thermodynamic readings is firmly established, the curve being smooth and regular, in spite of the fact that observations were spread over two consecutive days.

It is therefore evident that the method may also be employed successfully at stations of relatively low head and will give consistent results.

Exact evaluation of the generator efficiency is, however, not practically possible, involving as it does such unknown quantities as change of bearing friction with load, windage of turbine runner in air, unknown stray losses in the generator operating on load, etc. It follows, therefore, that a comparison of thermodynamic efficiency and conventional efficiency is difficult since there may be indeterminate differences.

It is of interest to note that during the comparison test, experiments were abandoned due to flood in the river, which caused a great amount of weeds later verified as Nitella to flow through the pipe line. It was not possible to find the efficiency by conventional method; but even in such adverse circumstances, thermodynamic method enabled us to calculate several efficiency points without any appreciable error.

9.0 TRIAL TESTS AT BONNINGTON BY DIRECT MEASUREMENT

The method of direct measurement is described in paragraph (iv) of article 4. Some preliminary investigations using this method were carried out at Bonnington in April 1961, which immediately followed the comparative tests (article 8.0). The thermometer (T_1) was inserted in the stainless steel probe used for sampling water and was placed in the pen stock; the other thermometer (T_2) was placed in the mixing chamber of the tail-race. The bridge circuit employed is shown in Fig. 9.

At each load the pressure in the pen stock i.e. H_1 was measured and the temperature difference $T_1 - T_2$ between the penstock and the tail-race was computed by noting the galvanometer deflection/

deflection. At each point of loading several readings were taken to ensure steady conditions, and the arithmetic mean of these readings was taken as the true observed deflection.

Since the expander is not used in this method and the measurements are made inside the penstock, therefore H_1 in the efficiency equation 2.6 : 2 from theoretical consideration will be equal to H_i, and this equation can be written as:-

$$\eta = \frac{H_i(1-\alpha) + J\,C_{p_2}(T_1 - T_2) + Z_1 - Z_2}{H_i(1-\beta) + Z_1 - Z_2} \qquad \text{------- } 9 : 1$$

As α and β are small, therefore without causing any appreciable error; we/write H_n the net effective head as

$$H_n = H_i + Z_1 - Z_2, \text{ and}$$

we can write efficiency equation 9 : 1 as

$$\eta = 1 - \frac{J\,C_{p_2}(T_2 - T_1)}{H_n} \qquad \text{------- } 9 : 2$$

C_{p_2} can be taken as unity the value of water at 4 deg. C - Justifiable because of small rate of change of C_p with temperature in the range 0 - 15°c, and equation 9 : 2 simplifies to

$$\eta = 1 - \frac{J(T_2 - T_1)}{H_n} \qquad \text{------- } 9 : 3$$

Assuming that temperature varies linearly with pressure, we can represent $T_2 - T_1$ the difference of temperature in terms of galvanometer deflection.

From equation 6 : 3 the value of $\frac{\Delta T}{\Delta n}$ can be calculated and it will remain constant for a given pair of thermometers and a bridge network.

Therefore if we write $\frac{\Delta T}{\Delta n} = C_1$, we can express equation 9 : 2 as,

$$\eta = 1 - \frac{J.C_1.\text{ deflection}}{Hn}$$

$$\text{or } \eta = 1 - \frac{C\text{ . deflection}}{Hn} \qquad \text{------- } 9 : 4$$

where $C = C_1 . J$ (J = 1400 in C.H.U.)

For/

For the network arrangement (Fig. 9) and thermometers used during present investigations shown in Fig.9 we can calculate C, which is given by

$$C = J \times \frac{T_{100} - T_o}{R_{100} - R_o} \times \frac{R + R_T + 2\ Rg}{C_{max} \times K \times 10^6}$$

$$\text{or} \quad C = 1400 \times \frac{100}{40} \times \frac{40 + 300 + 2 \times 478.6}{10^3 \times 2.15 \times 525 \times 10^6}$$

$$\text{or} \quad C = 4.0$$

And hence the efficiency was calculated by equation given as

$$\eta = 1 - \frac{4 \times \text{deflection}}{H_n} \qquad \text{-------- 9 : 6}$$

where deflection is in

and H_n is in ft of water.

Table 14 shows the results at different loads in the range 1.0 MW to 5.0 M W.

The results obtained are by no means accurate, and they tend to be of a lower order than obtained by comparative tests (Article 8). This error can be explained as the effect of stagnation temperature rise at the penstock thermometer causing a decrement in the measured efficiency.

It is therefore desirable that some method has to be employed applying suitable correction for stagnation temperature rise, if the method of direct measurement is being developed.

10. CONCLUSIONS.

The theoretical investigation of the sensitivity of the wheat stone bridge led to a very simple design of bridge, which can be successfully employed in thermodynamic tests for increased accuracy.

Development of the adiabatic expander proved to be successful/

successful. The technique of using three thermometers, one each for the inner expander and tail-race and one for the outer expander, showed much better results than were previously obtained. Insulation of the whole unit by wrapping in high-pressure hose was successful. Use of the mixing chamber, to obtain the average temperature in the tail-race saved time and gave consistent and fairly accurate results. It may be concluded from the smooth efficiency curve finally obtained at a low-head station that the method can be successfully employed in such power stations for routine checks on performance.

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APPENDIX I

CALCULATION OF α

We can write $\frac{p_1}{w_1} - \frac{p_2}{w_2}$ in the form

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} = \frac{p_1}{w_1} - \frac{p_2}{w_1} + \frac{p_2}{w_1} - \frac{p_2}{w_2}$$

$$= \frac{1}{w_1} (p_1 - p_2) + p_2 \left(\frac{1}{w_1} - \frac{1}{w_2}\right) \text{ ---------- A1:1}$$

If we introduce a reference specific weight w_s, where w_s is the specific weight of the fluid under standard conditions of temperature and pressure, (for water at $4^{o}C$ and 14.7 lb/in $w_s = 62.43\ ft^3/lb$), we can write

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} = \frac{1}{w_1} (p_1 - p_2) + p_2 \left(\frac{1}{w_1} - \frac{1}{w_2}\right) + (p_1 - p_2)\frac{1}{w_s} +$$

$$-(p_1 - p_2) \frac{1}{w_s}$$

or
$$\frac{p_1}{w_1} - \frac{p_2}{w_2} = \frac{1}{w_s} (p_1 - p_2)\left(\frac{w_s}{w_1} + 1 - 1\right) + p_2 \left(\frac{1}{w_1} - \frac{1}{w_2}\right)$$

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} = \frac{1}{w_s} (p_1 - p_2) \left(1 + \frac{w_s - w_1}{w_1}\right) + p_2 \left(\frac{1}{w_1} - \frac{1}{w_2}\right)$$

------------A1:1a

As w_1 and w_2 are specific weights at 1 and 2 respectively we can represent

$$\frac{1}{w_1} - \frac{1}{w_2} = v(p_1, T_1) - v(p_2, T_2) \text{ .}$$

where v is the specific volume.

Hence/

APPENDIX I (continued)

Hence

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} = \frac{1}{w_s} \; (p_1 - p_2) \; \left(1 + \frac{w_s - w_1}{w_1}\right) + p_2 \left\{ v(p_1, T_1) - v(p_2, T_2) \right\}$$

——————A1:2

Therefore we can write

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} + p_2 \left\{ v(p_2, T_2) - v(p_2, T_1) \right\} - \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} dp$$

$$= \frac{1}{w_s} (p_1 - p_2) \left[1 + \frac{w_s - w_1}{w_1} \right] + p_2 \left\{ v(p_1, T_1) - v(p_2, T_2) \right\} +$$

$$+ p_2 \left\{ v(p_2, T_2) - v(p_2, T_1) \right\} - \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} dp$$

$$= \frac{1}{w_s} (p_1 - p_2) \left(1 + \frac{w_s - w_1}{w_1}\right) - p_2 \left\{ v(p_2, T_1) - v(p_1, T_1) \right\} +$$

$$- \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} dp.$$

$$= \frac{1}{w_s} (p_1 - p_2) \left[1 + \frac{w_s - w_1}{w_1} - p_2 w_s \left\{ v(p_2, T_1) - v(p_1, T_1) \right\} + - \frac{w_s}{p_1 - p_2} \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} dp \right]$$

$$= \frac{1}{w_s} (p_1 - p_2) \left[1 - \left[\frac{w_1 - w_s}{w_1} + p_2 \cdot w_s \left\{ \frac{v(p_2, T_1) - v(p_1, T_1)}{(p_1 - p_2)} \right\} + + \frac{w_s}{(p_1 - p_2)} \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} \cdot dp \right] \right]$$

Consequently we can write

$$\frac{p_1}{w_1} - \frac{p_2}{w_2} + p_2 \cdot \left[v(p_2, T_2) - v(p_2, T_1) \right] - \int_{p_2}^{p_1} \left\{ T_1 \left(\frac{\partial v}{\partial T}\right)_p + p \left(\frac{\partial v}{\partial p}\right)_T \right\} \cdot dp$$

$$= \frac{1}{w_s} \cdot (p_1 - p_2) \cdot (1 - \alpha)$$

——————A1:3

where/

APPENDIX I (continued)

where

$$\alpha = \frac{w_1 - w_s}{w_1} + p_2 . w_s \left[\frac{v(p_2, T_1) - v(p_1, T_1)}{p_1 - p_2} \right]$$

$$+ \frac{w_s}{(p_1 - p_2)} \int_{p_2}^{p_1} \left[T_1 \left(\frac{\partial v}{\partial T} \right)_p + p \left(\frac{\partial v}{\partial p} \right)_T \right] dp \text{ ---------- A1:4}$$

Under normal industrial conditions p_2 is in the region of atmospheric pressure and temperature at Section 1 lies between 0° and $20^\circ C$, hence in equation A1:4

$$p_2 w_s \cdot \left[\frac{v(p_2, T_1) - v(p_1, T_1)}{p_1 - p_2} \right] < p_2 \left[\frac{1}{v} \cdot \frac{\partial v}{\partial p} \right] \text{ max.}$$

which is of the order of 5×10^{-5}. This term can therefore be neglected, and the expression for α is reduced to

$$\alpha = \frac{w_1 - w_s}{w_1} + \frac{w_s}{p_1 - p_2} \cdot \int_{p_2}^{p_1} \left[T_1 \left(\frac{v}{T} \right)_p + p \left(\frac{v}{p} \right)_T \right] . dp$$

------------ A1:5

The value of α can be obtained from the tabulated properties of the fluid, since

w_1 and w_s are specif wts at specified temperature and pressures,

$\left(\frac{\partial v}{\partial T} \right)_p$ is called the Isopiestic thermal expansion and

$\left(\frac{\partial v}{\partial p} \right)_T$ is called the Isothermal Compression of water.

Figure 25 shows the value of α against upstream temperature in deg. C for four different pressure levels obtained by Willm and Campmas.

[Note: All P's are capitals representing absolute pressure.]

APPENDIX II

Calculation of β

Following the same procedure as for the calculation of α, from equation A1:2 appendix I where w_s is the specific weight of fluid at S.T.P we can write,

$$\frac{p_i}{w_i} - \frac{p_o}{w_o'} = \frac{1}{w_s}(p_i - p_o)\left(1 + \frac{w_s - w_i}{w_i}\right) + p_o\left(\frac{1}{w_i} - \frac{1}{w_o'}\right)$$

and where v is the specific volume it follows that

$$\frac{p_i}{w_i} - \frac{p_o}{w_o'} + \int_i^o p.dv = \frac{1}{w_s}(p_i - p_o)\left(1 + \frac{w_s - w_i}{w_i}\right) + p_o\left(\frac{1}{w_i} - \frac{1}{w_o'}\right) + \int_i^o p.dv$$

$$= \frac{1}{w_s}(p_i - p_o)\left[1 + \frac{w_s - w_i}{w_i} + \frac{p_o\, w_s}{p_i - p_o} \cdot \left(\frac{1}{w_i} - \frac{1}{w_o'}\right) + \frac{w_s}{p_i - p_o}\int_i^o p.dv\right]$$

$$= \frac{1}{w_s}(p_i - p_o)\left[1 - \left[\frac{w_i - w_s}{w_i} + \frac{p_o\, w_s}{p_i - p_o}\left(\frac{1}{w_o'} - \frac{1}{w_i}\right) - \frac{w_s}{p_i - p_o} \cdot \int_i^o p.dv\right]\right]$$

Therefore we can write

$$\frac{p_i}{w_i} - \frac{p_o}{w_o'} + \int_i^o p.dv = \frac{1}{w_s} \cdot (p_i - p_o) \cdot (1 - \beta)$$

where/

Note: All P's are capitals representing absolute pressure.

APPENDIX II (continued)

where

$$\beta = \frac{w_i - w_s}{w_i} + \frac{p_o\, w_s}{p_i - p_o} \left(\frac{1}{w_o'} - \frac{1}{w_s} \right) - \frac{w_s}{p_i - p_o} \cdot \int_i^o p.dv$$

Under normal industrial conditions the temperature of water entering the turbine lies between 0 deg C and 20 deg C and pressure is less than than 4940 ft. of water (150 kg/cm^2 or 2139 lb/in^2). Willm and Campmas have argued that under these conditions the factor

$$\frac{p_o . w_s}{p_i - p_o} \cdot \left(\frac{1}{w_o'} - \frac{1}{w_i} \right) \text{ is less than } 5 \times 10^{-5}$$

and hence may be neglected.

They have further argued that, since

$$\frac{w_s}{p_i - p_o} \cdot \int_i^o p.dv = \int_i^o \left[p\left(\frac{\partial v}{\partial T}\right)_p dT + p \left(\frac{\partial v}{\partial p}\right)_T \right] dp$$

does not differ from

$$\frac{w_s}{p_i - p_o} \cdot \int_{p_i}^{po} \left[p \left(\frac{\partial v}{\partial p}\right)_{T_i} . dp \right]$$ by a quantity of the order of 10^{-4}, the expression for β can be written as

$$\beta = \frac{w_i - w_s}{w_i} - \frac{w_s}{p_i - p_o} \int_{p_i}^{p_o} \left[p \left(\frac{\partial v}{\partial P} \right)_{T_i} \right] .d P$$

The values of β can be obtained from tabulated properties of fluid, since

w_s and w_i are specific weights at specified temperature and pressure, and

$\left(\frac{\partial v}{\partial p}\right)_T$ is the compressibility.

Figure 25 shows the values of β against upstream temperatures in deg. C for four different pressure levels obtained by Willm and Campmas.

[Note: All P's are capitals representing absolute pressure.]

APPENDIX III

Calculation of ig the current through galvanometer for Fig.7

To find the current ig we will apply Kirchhoff's 1st law at points A, C and D and we get

$$i = i_1 + i_3 \quad \text{------} \quad (i)$$

$$i_1 = i_2 + i_g \quad \text{------} \quad (ii)$$

$$i_4 = i_g + i_3 \quad \text{------} \quad (iii)$$

and applying Kirchhoff's second law to the loop A C B A we get

$$i_1\, R_1 + i_3\, R_3 = E$$

$$\text{or} \quad i_1\, R_1 + (i_1 - i_g)\, R_3 = E \qquad \text{(from (ii))}$$

$$\text{or} \quad i_1\, R_1 + i_1\, R_3 - i_g\, R_3 = E \quad \text{---------}(iv)$$

similarly from loop A C D A, we get

$$i_1\, R_1 + i_g\, Rg - i_2\, R_2 = 0 \quad \text{---------}(v)$$

and from loop C B D A, we get

$$R_3\, i_3 - R_4\, i_4 - Rg\, i_g = 0$$

$$\text{or} \quad i_1\, R_3 - i_2\, R_4 - i_g\, (R_3 + R_4 + Rg) = 0 \quad \text{----------}(vi)$$

Multiplying (vi) by R_2 and (v) by R_4 and subtracting the letter from the former, we get

$$i_1\, R_2\, R_3 - i_2\, R_2\, R_4 - i_g\, (R_2R_3 + R_2R_4 + R\ R\ ) = 0$$

$$i_1\, R_1\, R_4 - i_2\, R_2\, R_4 + i_g\, R_4\, R_5 \qquad = 0$$

$$- \qquad + \qquad -$$

$$\text{or} \quad i_1\, (R_2R_3 - R_1R_4) - ig(R_2R_3 + R_2R_4 + R_2R_5 + R_4R_5) = 0$$

$$\text{or} \quad i_1 = \frac{i_g(R_2R_3 + R_2R_4 + R_2R_5 + R_4R_5)}{R_2R_3 - R_1R_4}$$

and substituting the value of i_1 in equation (iv), we get

$(R_1$ /

APPENDIX III (continued)

$$\frac{(R_1+R_3)\ ig \times (R_2R_3+R_2R_4+R_2Rg+R_4Rg)}{R_2R_3 - R_1R_4} = i_5 R_3 = E$$

$$\text{or } ig \frac{R_1R_2R_3+R_1R_2R_4+R_1R_2Rg+R_1R_4Rg+R_2R_3^2 + R_2R_3R_4+R_2R_3Rg + R_3R_4Rg-R_2R_3^2 + R_1R_3R_4}{R_2R_3 - R_1R_4} = E$$

$$\text{or } ig = \frac{E(R_2R_3 - R_1R_4)}{(R_1R_2R_3+R_1R_2R_4+R_2R_3R_4+R_1R_3R_4) + Rg\ (R_1R_2+R_1R_4+R_2R_3+R_3R_4)}$$

$$\text{or } ig = \frac{E(R_2R_3 - R_1R_4)}{Y + Rg\ (R_1+R_3)\ \ (R_2+R_4)} \qquad \text{----------------(vii)}$$

where $Y = R_1R_2R_3 + R_2R_3R_4 + R_3R_4R_1 + R_4R_1R_2$

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LIST OF SYMBOLS

Symbol	Definition
C	Constant of measuring apparatus.
C_p	Specific heat of working fluid at constant pressure.
E (article 2)	Internal energy of working fluid.
E	Applied electromotive force.
g	Acceleration due to gravity.
h	Constant for galvanometer sensitivity.
H	Head i.e. Pressure in Ft. of water.
H_t	Total head.
H_n	Net effective head ($= H_1 + Z_1 - Z_2$).
H_a	Available head ($= H_n - V^2/2g$).
i	Current in arms of Wheatstone bridge.
i_{max}	Maximum permissible current through thermometer.
i_g	Current through galvanometer.
J	Mechanical equivalent of heat.
k	Current sensitivity of galvanometer.
K	Kinetic energy of working fluid.
n	Deflection of galvanometer spotlight.
p	Pressure.
P	Absolute pressure.
q	Heat input to the turbine.
R_1 & R_3	Resistances of ratio arm of bridge.
R_2 & R_4	Resistances of bridge arms containing thermometers.
R	$= R_1 = R_3$
R_T	$= R_2 = R_4$
R_g	Resistance of galvanometer.
s	Entropy of working fluid.
S	Sensitivity of Wheatstone bridge.

Symbol	Definition
S_g	Sensitivity of galvanometer.
T	Absolute temperature of working fluid.
(T)	Thermometer measuring temperature T.
v	Specific volume of working fluid.
V	Velocity of flow
w	Specific weight of working fluid.
w_s	Specific weight of working fluid at S.T.P. (14.7 lb/in^2 & 4°C for water).
W	Useful work supplied by turbine.
W_f	Flow work of working fluid ($= P/_w$)
Z	Potential energy i.e. Datum head.
α, β	Thermodynamic coefficients which correct for the variation in specific volume of the working fluid with temperature and pressure.
Y	$= R_1 R_2 R_3 + R_1 R_2 R_4 + R_1 R_3 R_4 + R_2 R_3 R_4$
Δ(Function)	Shows change in function.
η	Efficiency of turbine.

Suffices

i	Inlet test section of turbine.
o	Outlet test section of turbine.
1	Measuring point at inlet test section.
2	Measuring point at outlet test section.
3	Measuring point in the outer jacket of adiabatic expander.
'	When applied to any symbol refers to an ideal turbine having no losses.

LIST OF TABLES.

LIST OF FIGURES

TABLE 5

Observations at Bonnington, June 1959

Date	Load in M.W	Efficiency
17.6.59	2.80	0.804
17.6.59	2.72	0.784
17.6.59	2.70	0.764
16.6.59	2.30	0.751
22.6.59	1.58	0.698
22.6.59	1.50	0.650

TABLE 6

Observations at Bonnington, Dec. '59 to Feb. '60:

Date	Load in M.W	Efficiency
30.1.60	5.2	0.818
16.12.59	5.1	0.806
9.12.59	5.0	0.824
30.1.60	4.0	0.856
30.1.60	3.0	0.801
30.1.60	2.0	0.723
30.1.60	1.0	0.586

TABLE 7

Six-point traverse at Bonnington, June '59.
(See Fig. 15)

Time	Gate (in)	Load (mw)	Position see Fig.15	Deflection (mm)		Mean deflection	Remarks
				Direct current	Reversed current		
10.45	4.13	1.8	1	-7.5	4.5	6.0	Deflection at M = 6.0
10.55	"	"	2	-6.5	4.5	5.5	Average deflection = 5.6
11.10	"	"	3	-7.0	4.75	5.87	Difference = 0.4mm 0.4 = 4.4 ft.head
11.20	"	"	4	-5.0	4.0	4.75	Hi = 178 ft. Deviation in effcy
11.30	"	"	5	-6.0	5.0	5.5	$= \frac{4.4 \times 100}{178} = 2.4\%$
11.40	"	"	6	-3.0	9.0	6.0	
12.10	4.0	1.7	1	-4.0	7.0	5.5	Deflection at M = 6.0
12.20	"	"	2	-4.5	6.5	5.5	Average deflection = 5.4
13.30	"	"	3	-5.5	6.5	6.0	Difference = 0.6 m 0.6 = 6.6 ft. head
13.40	"	"	4	-5.0	5.25	5.12	Hi = 178 ft. Deviation in effcy
13.50	"	"	5	-7.0	7.0	7.0	$= \frac{6.6 \times 100}{178}$
14.00	"	"	6	-7.5	7.5	7.5	= 3.7 %
14.25	4.0	1.7	1	Fluctuations			Deviation in efficiency not calculable.
14.35	"	"	2	Fluctuations			
14.45	"	"	3	Fluctuations			
14.55	"	"	4	-7.0	6.0	6.5	
15.10	"	"	5	-5.25	5.75	5.5	
15.20	"	"	6	Fluctuations			
15.35	3.88	1.5	1	-4.0	8.0	6.0	Deflection at M = 6.0
15.45	"	"	2	-5.0	7.0	6.0	Average deflection = 5.9
15.55	"	"	3	-5.0	7.0	6.0	Difference = 0.1 0.1 = 1.1 ft. head
16.05	"	"	4	-5.75	5.75	5.75	H_i = 178 ft.
16.15	"	"	5	-5.5	-6.0	5.75	Deviation in effcy. $= \frac{1.1 \times 100}{178}$
16.25	"	"	6	Fluctuations			= 0.62%

TABLE 8

Twelve-point traverse at Bonnington, June '59.
(See Fig. 16)

Time	Gate (in)	Load (mw)	Position see Fig.15	Deflection (mm) Direct current	Deflection (mm) Reversed current	Mean deflection	Remarks
11.40	5.38	2.6	M	-6.1	4.5	5.3	
12.15	"	"	1	-7.75	2.25	5.0	Average deflection for 1 to 12 = 5.2
12.26	"	"	2	-6.5	4.0	5.25	
12.33	"	"	3	-8.4	3.1	5.75	Difference = M-5.2 =0.1mm
12.40	"	"	4	-4.75	6.25	5.5	0.1mm = 1.1 ft. head
12.47	"	"	5	-8.5	1.5	5.0	
12.55	"	"	6	-2.0	8.5	5.25	Hi = 178 ft.
13.08	"	"	7	Fluctuations			Deviation in efficiency
13.15	"	"	8	-4.0	5.5	4.75	
13.21	"	"	9	-3.5	6.5	5.0	$= \frac{1.1 \times 100}{178}$
13.27	"	"	10	-4.5	5.5	5.0	= 0.62%
13.33	"	"	11	0	10.5	5.25	
13.39	"	"	12	-4.0	6.0	5.0	
14.40	5.5	2.7	M	-5.1	6.0	5.55	
14.47	"	"	1	-6.5	4.25	5.38	Average deflection = 5.4
14.53	"	"	2	-4.75	5.75	5.25	
14.59	"	"	3	Fluctuations			Difference = M-5.4 = 0.15mm
15.05	"	"	4	-3.0	7.5	5.25	
15.11	"	"	5	Fluctuations			0.15mm=1.85 ft.head
15.17	"	"	6	-3.8	7.2	5.5	
15.31	"	"	7	-2.5	8.5	5.5	H_1 = 178 ft.
15.37	"	"	8	-3.1	7.2	5.5	Deviation in efficiency
15.44	"	"	9	-3.0	8.5	5.75	$= \frac{1.65 \times 100}{178}$
15.50	"	"	10	-3.5	8.0	5.75	= 0.92%
15.56	"	"	11	-2.5	9.0	5.75	
16.02	"	"	12	-3.0	8.0	5.5	

TABLE 9

Galvanometer deflection, thermometer with and without pocket.

Time	Load MW	Deflection		Mean deflection	Wt. Lb.	Remarks
		Direct current	Revised Current			
2.40	5.2	+3.0	-0.5	1.75	28	Gland arrangement used
2.50	"	+3.5	-1.1	2.3	26	
2.58	"	+4.0	-2.0	3.0	24	
3.05	"	+5.0	-3.0	4.0	22	
3.55	"	+6.0	-4.0	5.0	28	Copper pocket arrangement used
4.05	"	+6.8	-5.0	5.9	26	
4.13	"	+6.0	-6.0	6.0	32	
4.25	"	+5.5	-4.0	4.75	30	
4.32	"	+6.0	-4.5	5.25	28	

TABLE 10

Galvanometer deflection, with and without lagging of flexible tube.

Time	Load MW	Deflection		Mean deflection	Wt. lb.	Remarks
		Direct current	Revised current			
2.42	5.1	+4.0	-0.2	2.1	30	Without lagging
2.48	"	+4.1	-0.8	2.45	28	
2.55	"	+4.85	-1.05	2.95	26	
3.02	"	+5.0	-1.8	3.4	24	
4.37	"	+3.0	+0.6	1.2	30	With lagging
4.45	"	+3.2	0	1.6	28	
4.51	"	+4.0	-0.5	2.25	26	
4.56	"	+4.2	-1.0	2.6	24	
5.03	"	+5.0	-2.0	3.5	22	

TABLE 11

Efficiency test with traverse correction

(See Fig. 20 and Fig. 16)

(a) LOAD = 4.92 MW

Time	Position of probe in tail-race	Deflection (mm) Direct current	Reversed current	Mean deflection	H_1 the press in expander in Ft. of water	Remarks
2.0.37	6	2.2	-2.1	2.15	151.6	For $T_1 = T_2$
20.47	"	2.8	-2.9	2.85	143.9	Deflection = 2.9 and H_1 (From Fig. 20)
20.57	"	3.4	-3.8	3.60	136.0	= 143.75 ft.

Traverse observations

21.10	6	3.0	-2.8	2.9	143.9	Average deflection = 2.6 mm
21.25	5	1.8	-3.0	2.4	"	
21.41	4	3.5	-3.0	3.25	"	For $T_1 = T_2$
21.53	3	4.0	-2.0	3.0	"	Deflection = 2.9
22.08	2	3.0	-3.0	3.0	"	and H_1 (From Fig. 20)
22.24	1	3.5	-2.0	2.75	"	= 140.5 ft.
22.38	8	2.0	-3.0	2.5	"	
22.50	7	1.0	-3.0	2.0	"	
23.03	10	3.5	-1.1	2.3	"	
23.14	9	3.0	-2.0	2.5	"	
23.25	12	3.0	-1.6	2.3	"	
23.35	11	3.0	-2.0	2.8	"	

H_i = 165.3 ft: $Z_1 = Z_2$ = 12.06 ft.

observed efficiency with T_2 at position 6 = $\frac{143.75}{165.3+12.06}$ = 0.877

calculated efficiency using traverse = $\frac{140.5}{165.3+12.06}$ = 0.860

Applied correction = -1.7%

TABLE 11 (continued)

(b) Load = 4.0 MW

Time	Position of Probes in tail-race	Deflection (mm) Direct current	Reversed current	Mean deflection	H_1 the press. in expander in ft. of water	Remarks
00.50	6	4.0	-2.2	3.1	143.9	For $T_1 = T_2$
1.00	"	3.0	-1.8	2.4	151.6	Deflection=2.9
1.11	"	3.4	-2.0	2.7	147.9	and H_1 (from Fig. 20) = 145.9 ft.

Traverse observations

1.20	6	4.0	-2.2	3.1	143.9	Average deflect-
1.34	5	3.6	-2.6	3.1	"	ion = 3.0 mm
1.50	4	3.8	-2.2	3.0	"	For $T_1 = T_2$
2.00	3	3.4	-2.6	3.0	"	Deflection=2.9
2.12	2	3.8	-2.0	2.9	"	and
2.22	1	3.35	-2.35	2.85	"	H_1 (from Fig.20)
2.36	8	3.9	-2.2	3.05	"	= 145.0 ft.
2.46	7	3.6	-2.6	2.8	"	
2.58	10	3.8	-2.2	3.0	"	
3.09	9	3.6	-2.6	3.1	"	
3.22	12	4.0	-2.2	3.1	"	
3.33	11	3.5	-2.7	3.1	"	

H_i = 168.2 ft; $Z_1 - Z_2$ = 12.06 ft.

observed efficiency with T_2 at position 6 = $\frac{145.9}{168.2+12.06}$ = 0.876

Calculated efficiency using traverse = $\frac{145.0}{168.2+12.06}$ = 0.871

Applied Correction = 0.5%

TABLE 11 (continued)

(c) Load = 3.0 MW

Time	Position of Probes in tail-race	Deflection (mm) Direct Current	Deflection (mm) Reversed Current	Mean deflection	H_1 the expander pressure in Ft. of water	Remarks
4.58	6	3.0	-1.4	2.2	143.9	For $T_1 = T_2$
5.09	"	3.5	-2.3	2.9	136.0	Deflection=2.9
5.19	"	4.2	-3.0	3.6	128.1	and H_1 (from Fig. 20) = 136 ft.
Traverse observations						
5.28	6	3.5	-2.3	2.9	136.0	Average deflection = 2.75 mm
5.38	5	3.1	-2.5	2.8	"	
5.50	4	3.4	-2.2	2.8	"	For $T_1 = T_2$
6.00	3	3.1	-2.7	2.9	"	Deflection=2.9
6.12	2	3.3	-2.5	2.9	"	and
6.35	1	3.1	-2.7	2.9	"	H_1 (from Fig.20)
6.47	8	3.1	-2.1	2.6	"	= 134.3 ft.
7.00	7	3.0	-2.4	2.7	"	
7.12	10	3.4	-2.0	2.7	"	
7.23	9	3.1	-2.1	2.6	"	
7.34	12	3.0	-2.1	2.55	"	
7.43	11	3.0	-2.2	2.6	"	

H_1 = 171 ft; $Z_1 = Z_2$ = 12.06 ft.

observed efficiency with T_2 at position 6 = $\frac{136}{171+12.06}$ = 0.808

Calculated efficiency using traverse = $\frac{134.3}{171+12.06}$ = 0.800

Applied Correction = -0.8 %

TABLE 12

Effect of air inlets, Finlarig

Date	Time	Load mm	Deflection mm	Remarks	
25th June 1960	21.27	12.7	7.5	Room temperature 19°C	
	21.43	12.7	6.7	Air flowing through ducts	
	21.57	12.7	7.0	Mean 7.06	By closing ducts an increase in deflection of 1.05mm=0.73 per cent decrease in efficiency
	21.35	12.7	8.5	Air ducts covered	
	21.49	12.7	7.5		
	22.05	12.7	8.3	Mean 8.10	
30th June 1960	20.39	10.0	7.0	Room temperature 20.5°C	
	20.54	10.0	7.6	Air flowing through ducts	
	21.08	10.0	7.5	Mean 7.36	By closing ducts an increase in deflection of 1.34mm=0.85 per cent decrease in efficiency
	20.47	10.0	9.4	Air ducts covered	
	21.01	10.0	8.2		
	21.15	10.0	8.5	Mean 8.70	

TABLE 13

Observations at Stonebyres

Load MW	Temperature of expanded water °C	H_1 ft.	H_i ft.	Z_1-Z_2	Efficiency η	Remarks
2.1	2.5	55.4	82.1	15.4	0.726	
1.46	2.5	54.6	82.8	15.4	0.723	$\eta = \frac{H_1(1-\alpha)+Z_1-Z_2}{H_i(1-\beta)+Z_1-Z_2}$
1.00	2.5	48.0	83.0	15.4	0.653	
2.9	2.5	59.2	81.8	15.4	0.768	
2.7	2.4	60.0	81.9	15.4	0.774	
2.4	2.4	51.9	82.0	15.4	0.753	
2.0	2.4	53.5	82.2	15.4	0.705	

TABLE 14

Observations of Direct measurement at Bonnington April 61.

Load MW	Mean deflection (mm)	H_i (Ft.)	Hn (Ft)	Efficiency η	Remarks
1.0	25.90	176.29	185.54	0.420	$\eta = \frac{1 - 4 \times \text{deflection}}{H_n}$
2.0	12.25	175.04	184.19	0.739	
3.0	9.30	172.73	181.68	0.795	
4.0	7.40	169.9	178.68	0.834	
5.0	8.70	167.45	176.4	0.794	

T A B L E 15

Efficiencies - Machine No. 1 Bonnington

Date	Test No.	MW	Surface level tail-race	Velocity head tail-race	Available head H_a ft.	Pipe °C	Q cusecs	W lb/ft³	η_{oa}	Approx. η_{gen}	η_h	$\eta_{ther.}$	Notes
10th April, 1961	1	zero	376.9	0.004	186.56	6.87	60.22	62.3555	-	-	-	0.350	Tail race bed level 371.00
	2	1.020 09	376.9	0.017	185.74	7.37	121.87	62.3542	0.5327	-	-	0.602	H_a = (Head at gauge corrected for pipe temperature at 20°C at which gauge was calibrated)
	3	1.599 92	376.9	0.027	185.03	7.47	155.15	62.3539	0.6424	-	-	0.726	
	4	2.021 86	377.0	0.037	184.81	7.87	185.60	62.3527	0.6967	0.936	0.744	0.760	$+(Z_{piston} - Z_{tail\ race\ surface})$
	5	2.499 29	377.0	0.052	184.13	8.17	218.62	62.3515	0.7340	0.945	0.777	0.806	$- V_T^2/2g$; $Z_{piston} = 386.15$
11th April, 1961	6	3.007 04	376.9	0.070	183.34	8.37	249.79	62.3505	0.7763	0.952	0.807	0.842	
	7	3.479 68	376.9	0.088	182.56	8.78	280.45	62.3485	0.8032	0.956	0.840	0.873	$\eta_{oa} = 1340 \times MW \times 550/wQH_a$
	8	3.970 26	376.8	0.113	182.14	8.88	312.15	62.3480	0.8256	0.958	0.862	0.905	$\eta_h = \eta_{oa}/\eta_{gen}$
	9	4.461 89	376.8	0.141	181.15	9.47	348.09	62.3459	0.8362	0.960	0.871	0.902	
	10	5.015 92	376.8	0.190	179.05	10.08	404.70	62.3454	0.8200	0.962	0.852	0.892	
	11	1.001 01	376.8	0.017	186.27	10.17	120.44	62.3429	0.5274	-	-	0.620	

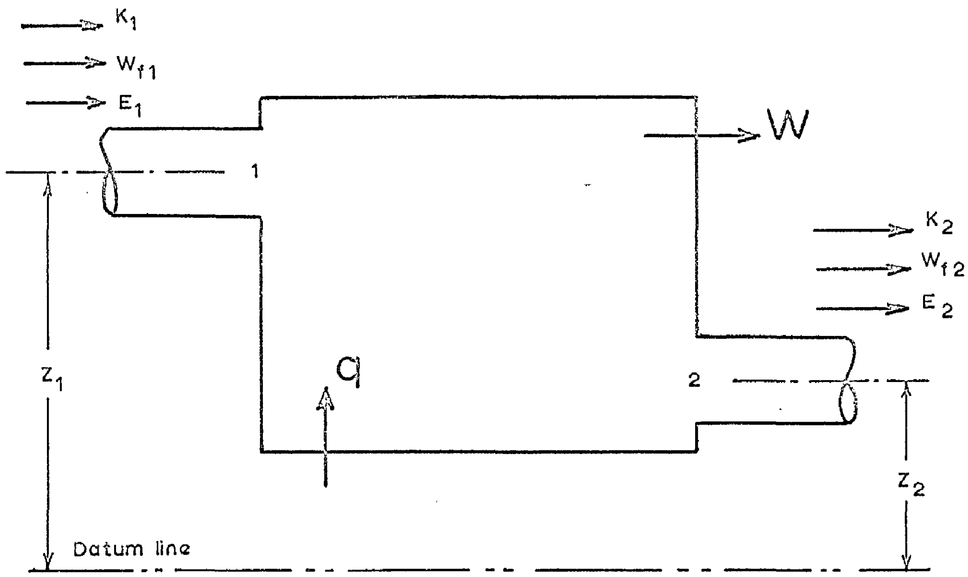

FIG. 1 ARBITRARY SYSTEM

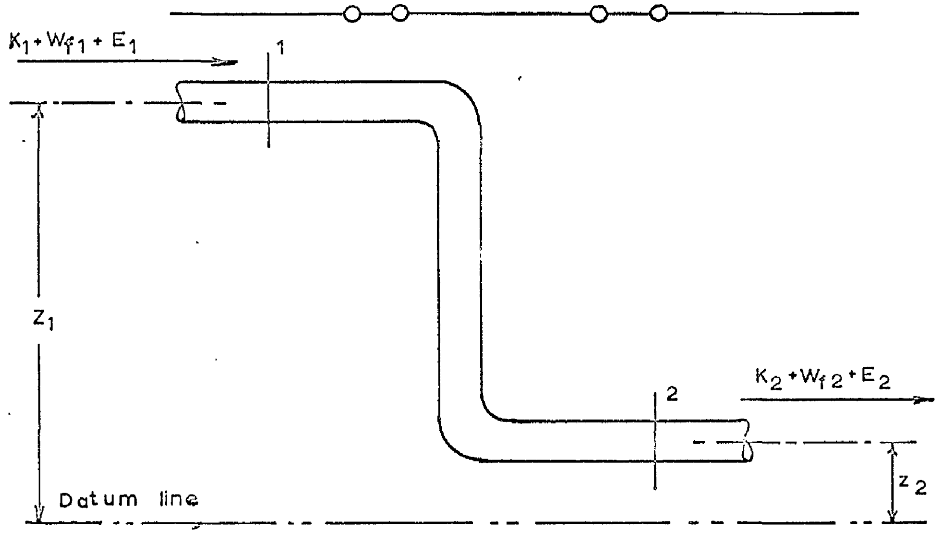

FIG. 2 PIPE SYSTEM

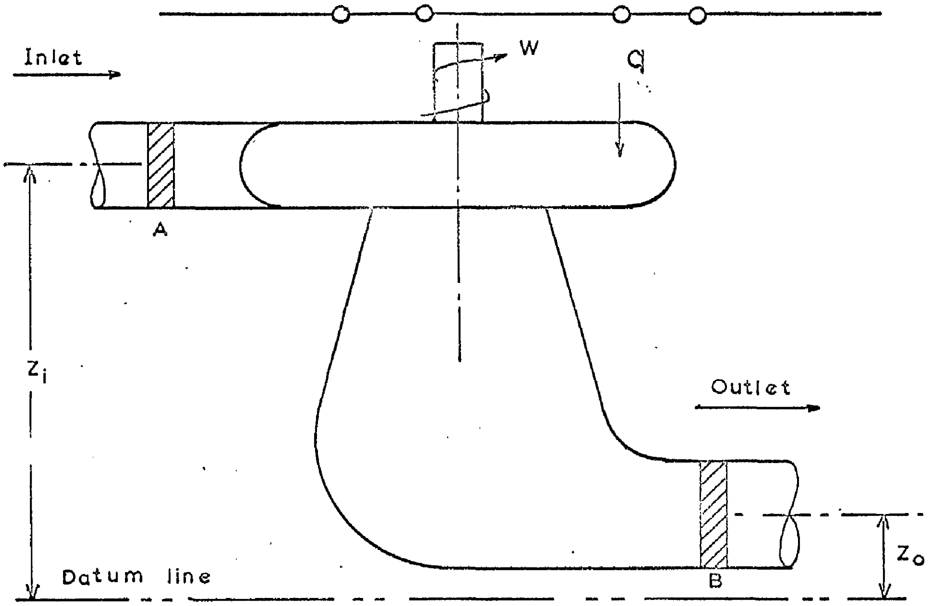

FIG. 3 TURBINE SYSTEM

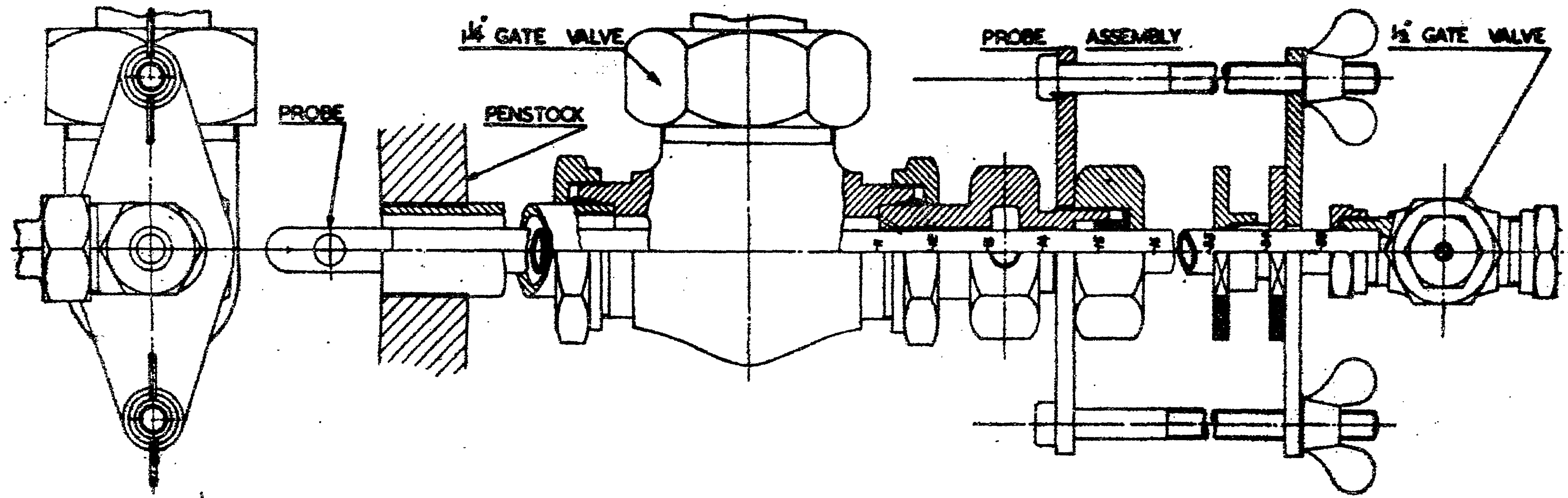

FIG. 4 SAMPLING PROBE

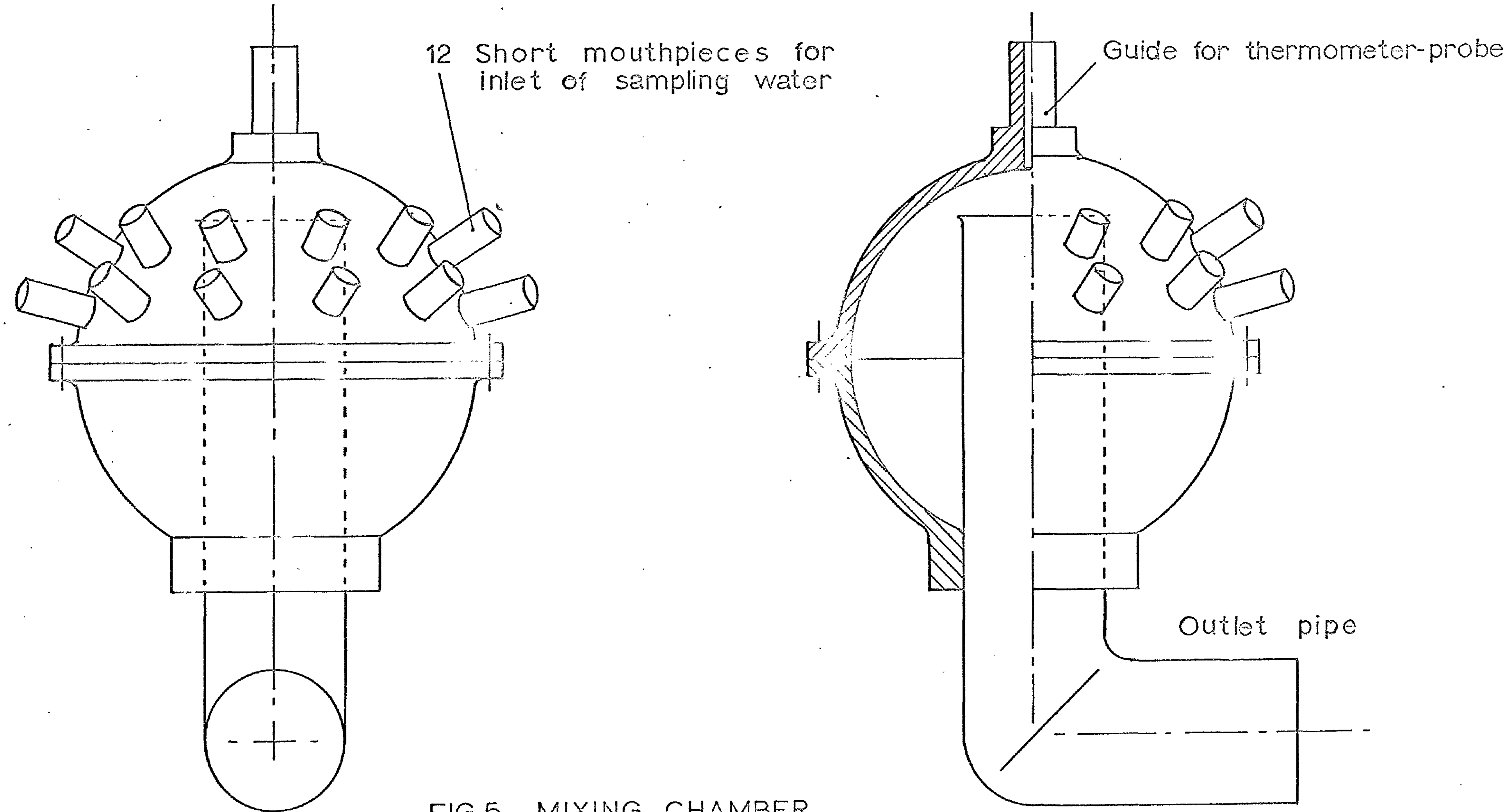

FIG.5 MIXING CHAMBER

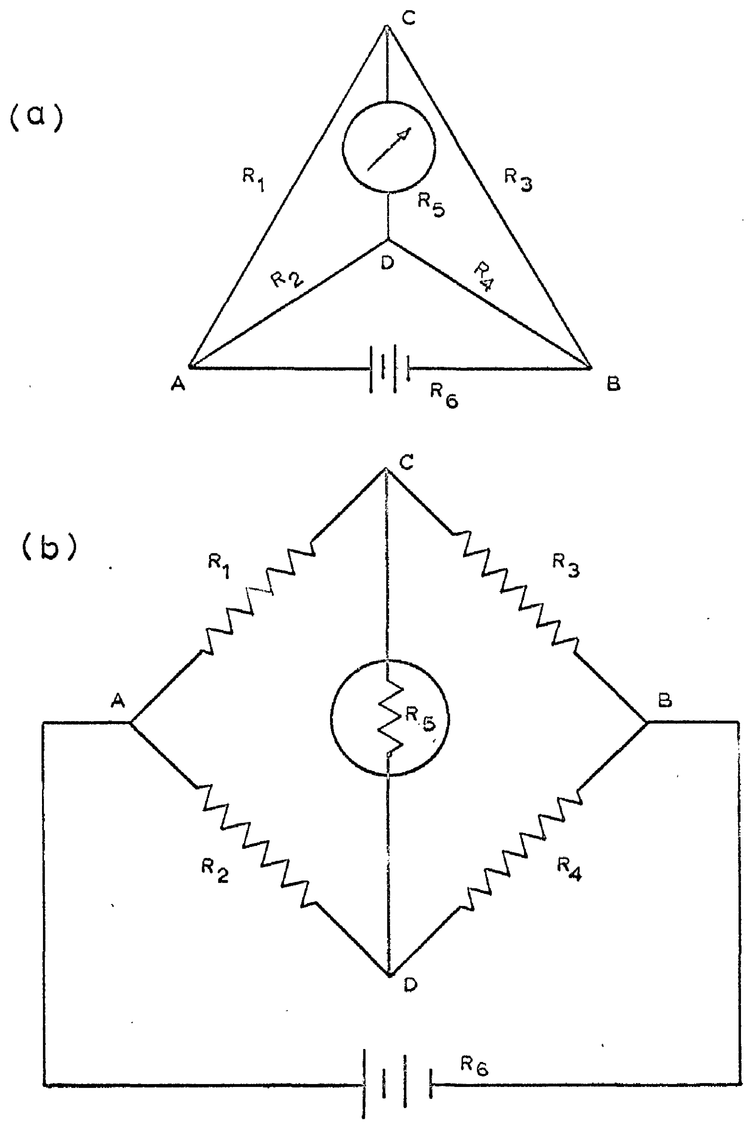

FIG. 6 THEORETICAL WHEATSTONE BRIDGE

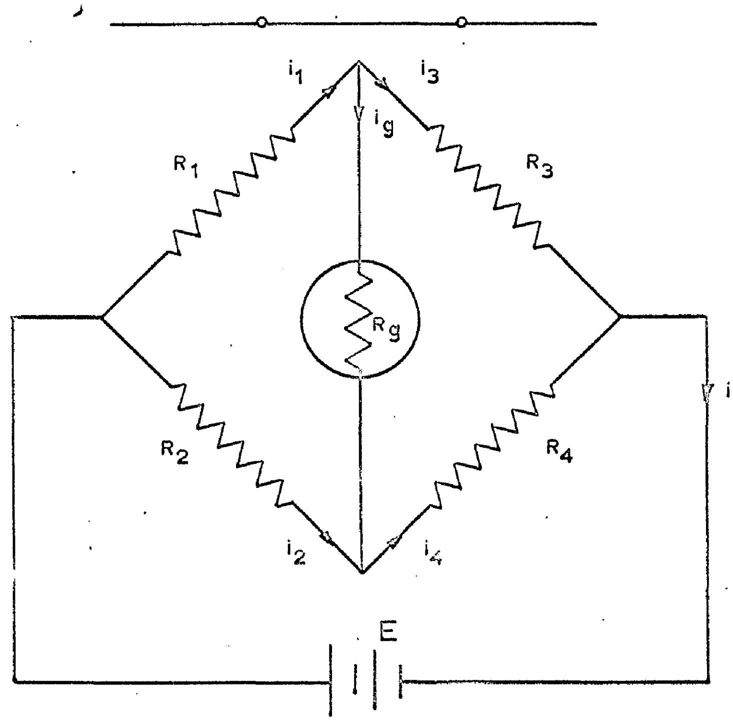

FIG. 7 WHEATSTONE BRIDGE

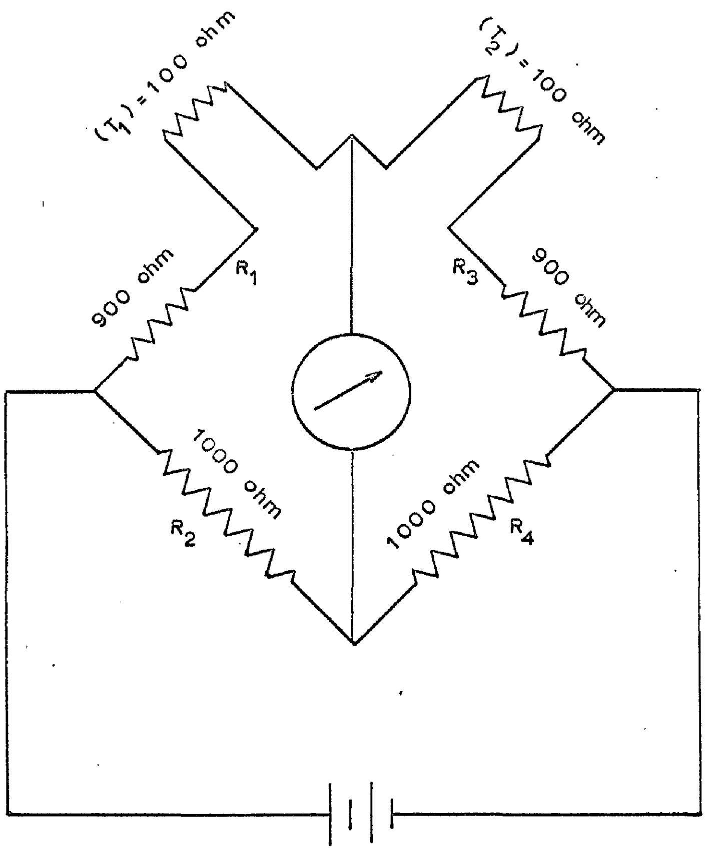

FIG.8 SIMPLIFIED SMITH BRIDGE

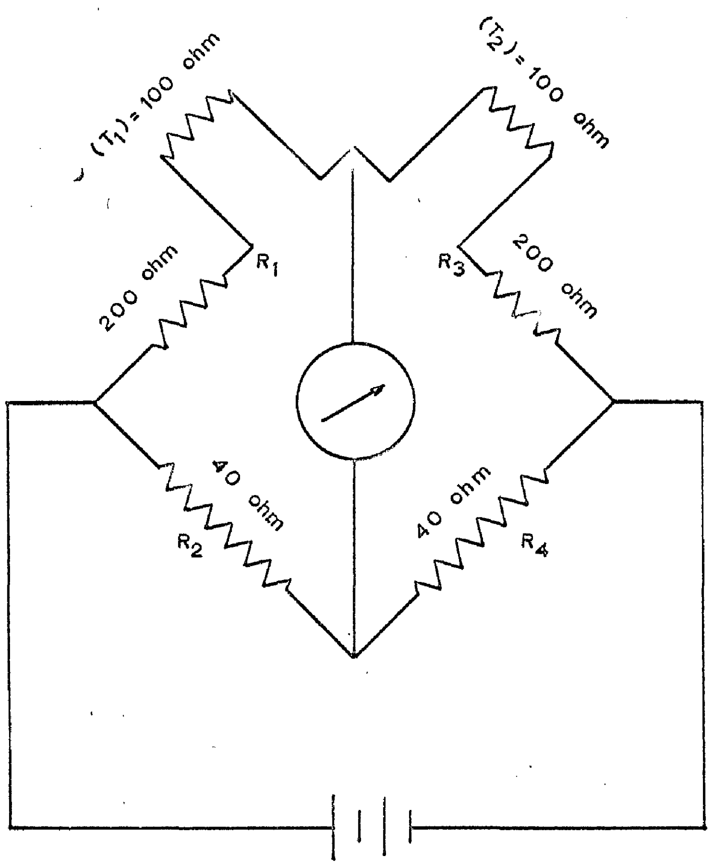

FIG.9 MODIFIED SMITH BRIDGE

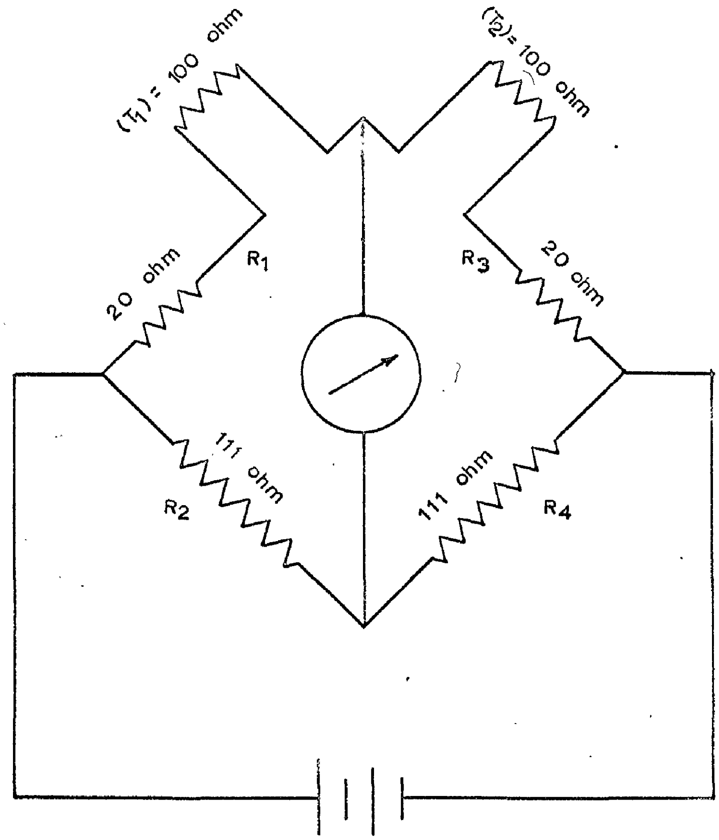

FIG.10 CALLANDER & GRIFFITH BRIDGE

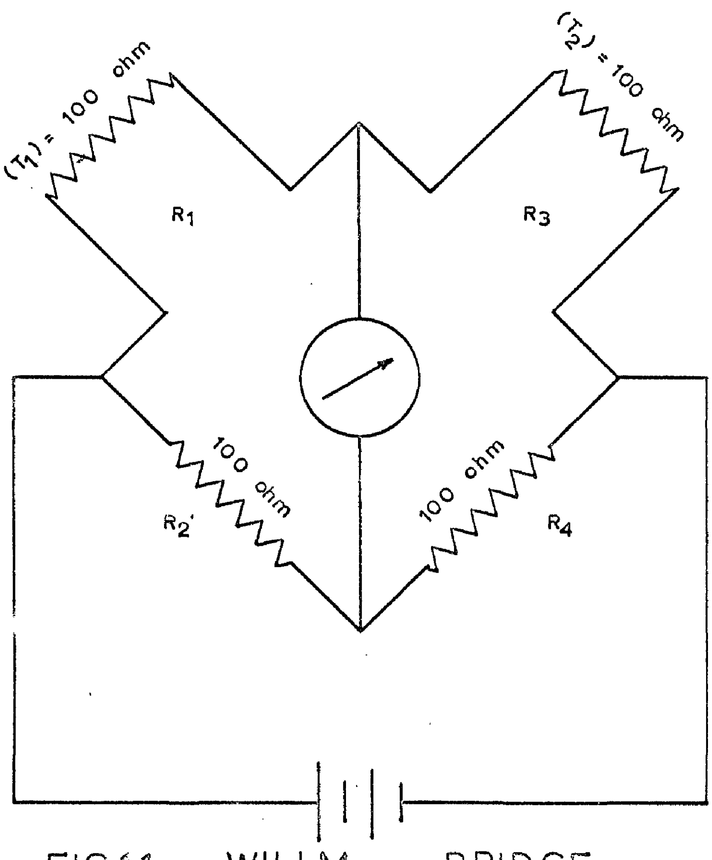

FIG.11 WILLM BRIDGE

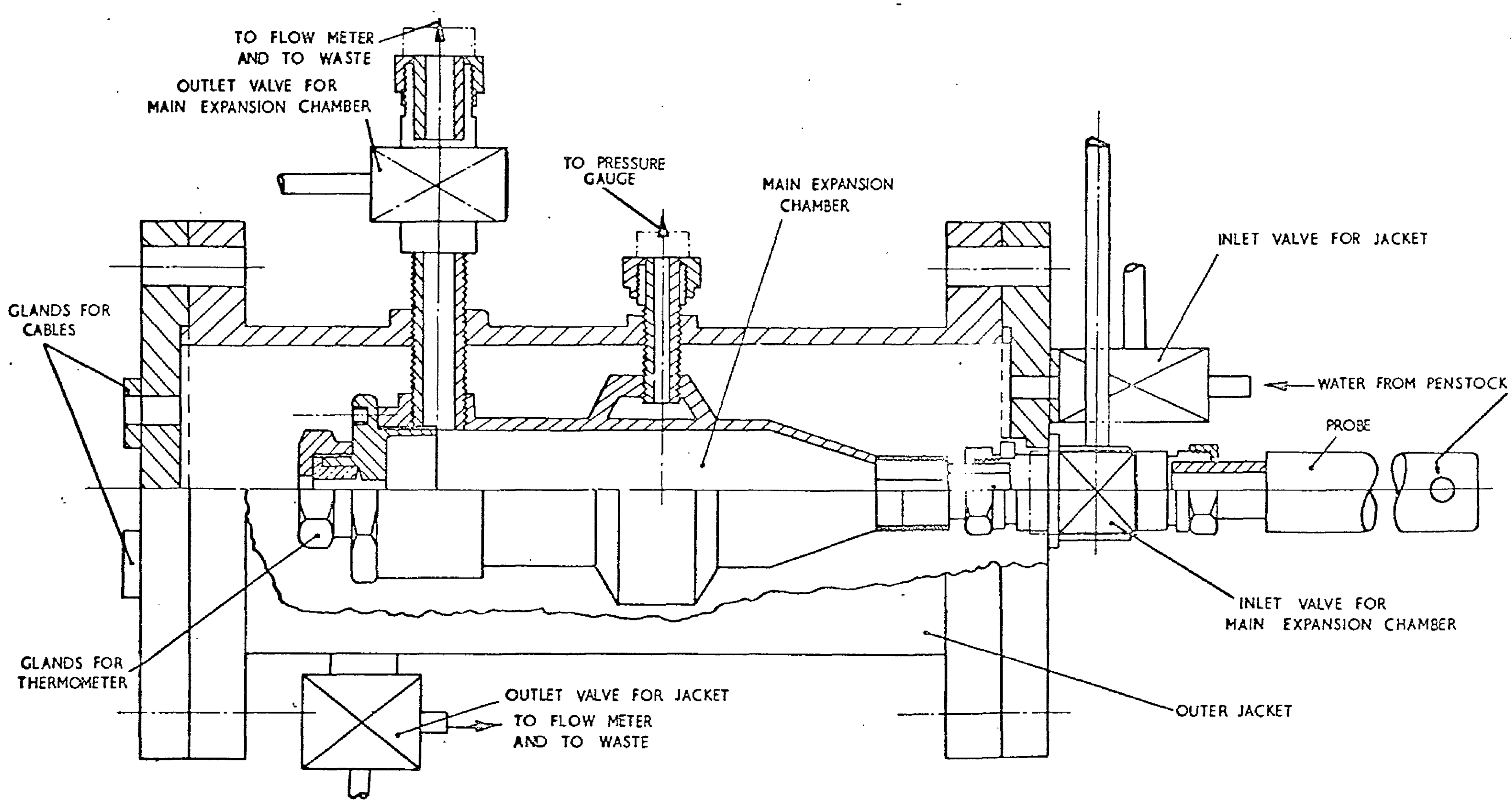

FIG. 12 ADIABATIC EXPANDER

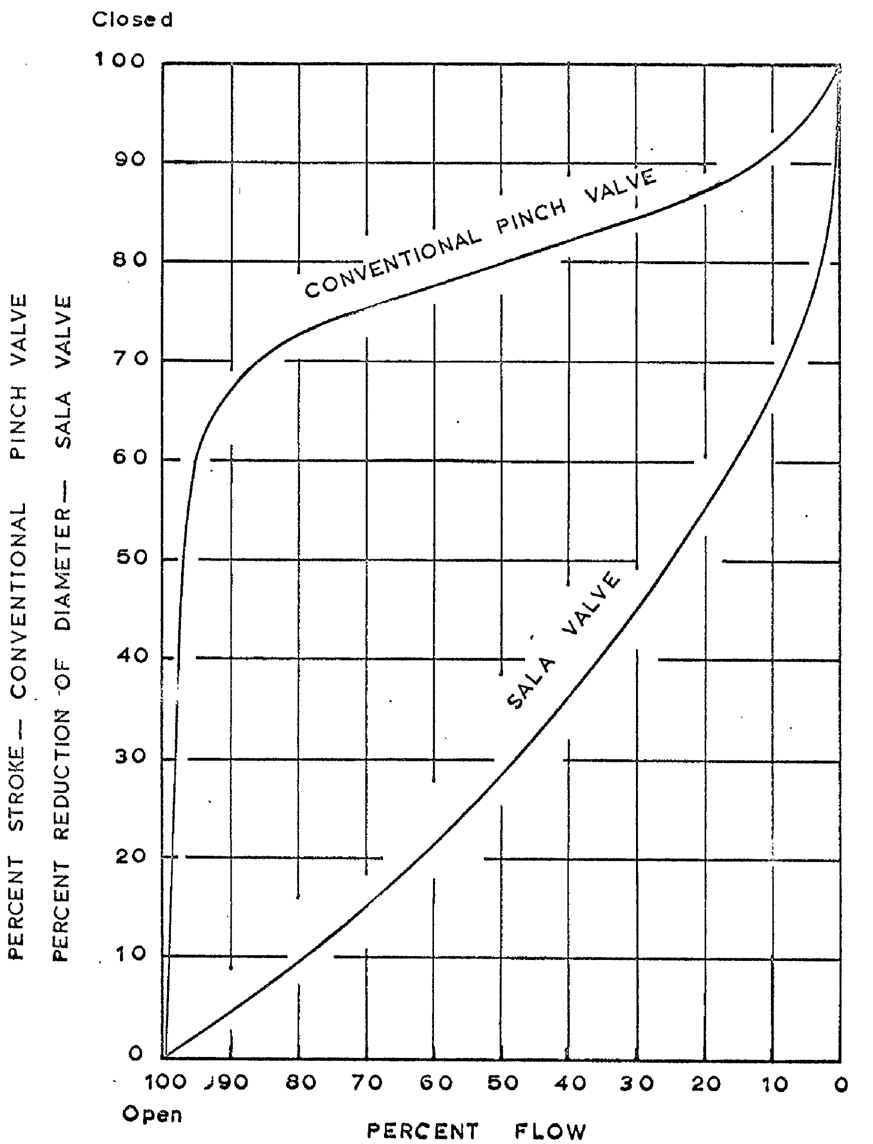

FIG.13 PERFORMANCE CURVES OF SALA VALVE & CONVENTIONAL PINCH VALVE at constant inlet pressure

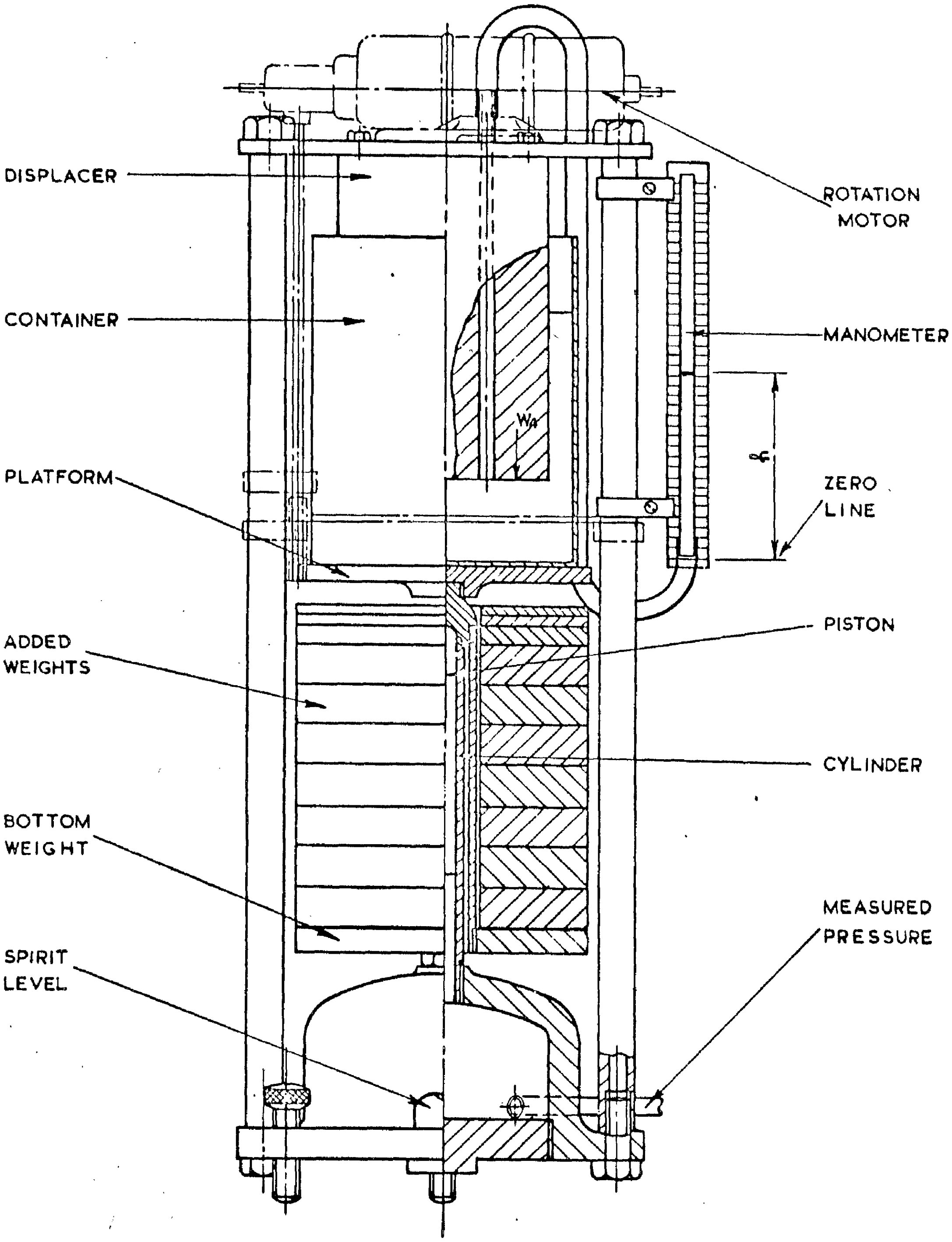

FIG.14 MANOMETRIC PRESSURE GAUGE

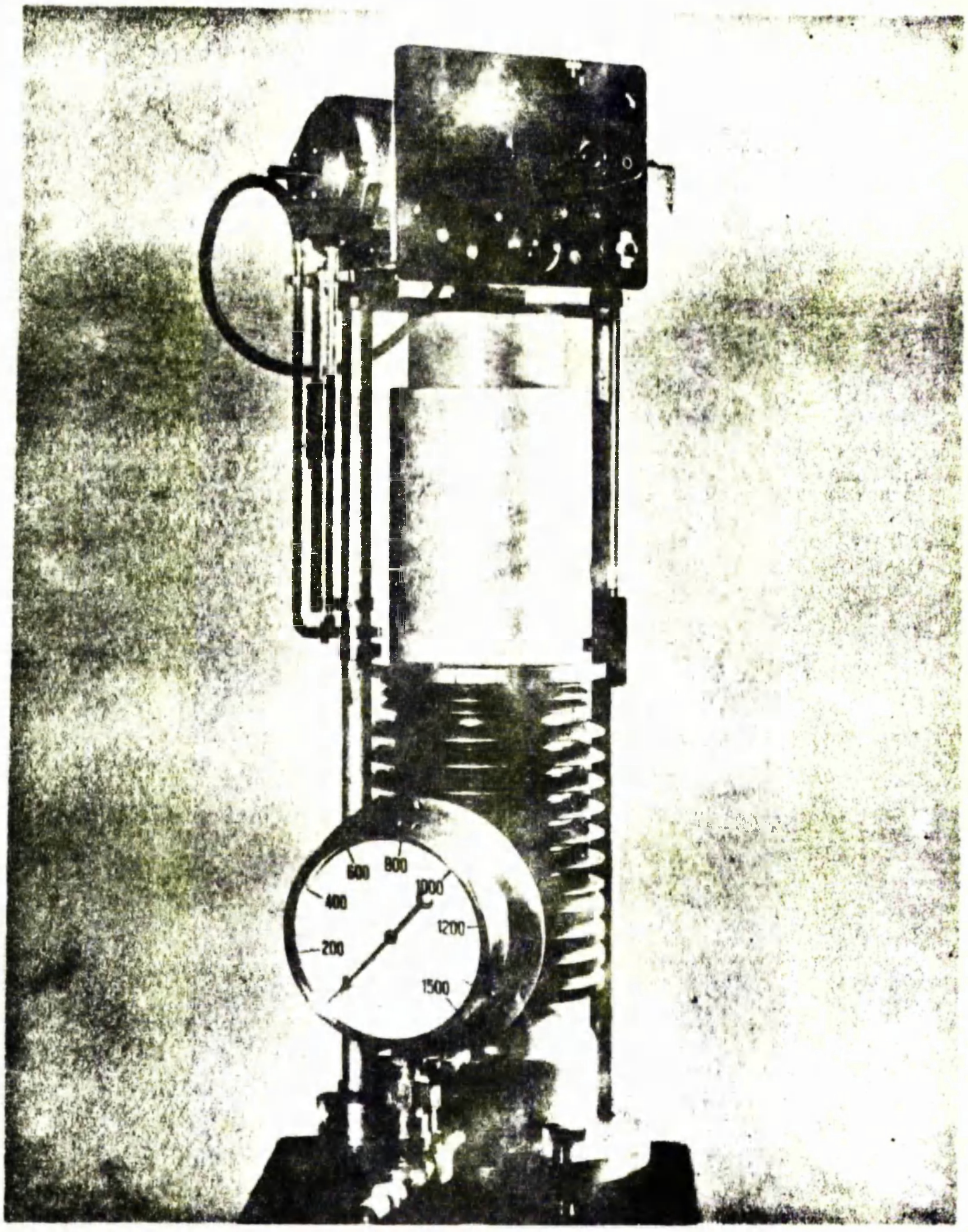

FIG.14a MANOMETRIC PRESSURE GAUGE

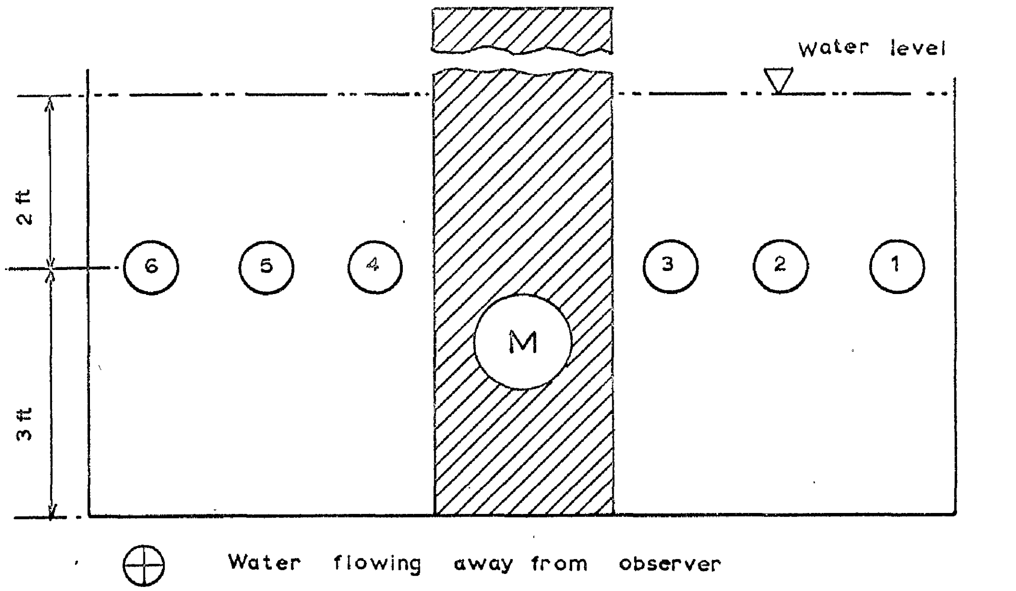

FIG. 15 CROSS-SECT'N OF BONNINGTON TAIL-RACE SHOWING POSITIONS OF 6-POINT TRAVERSE

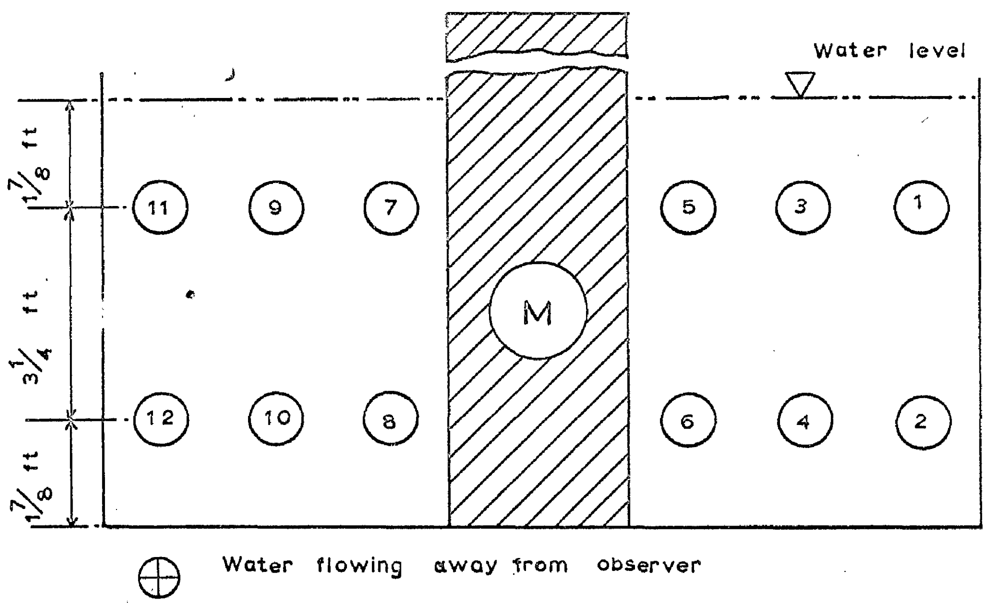

FIG. 16 CROSS-SECTION OF BONNINGTON TAIL-RACE SHOWING POSITIONS OF 12-POINT TRAVERSE

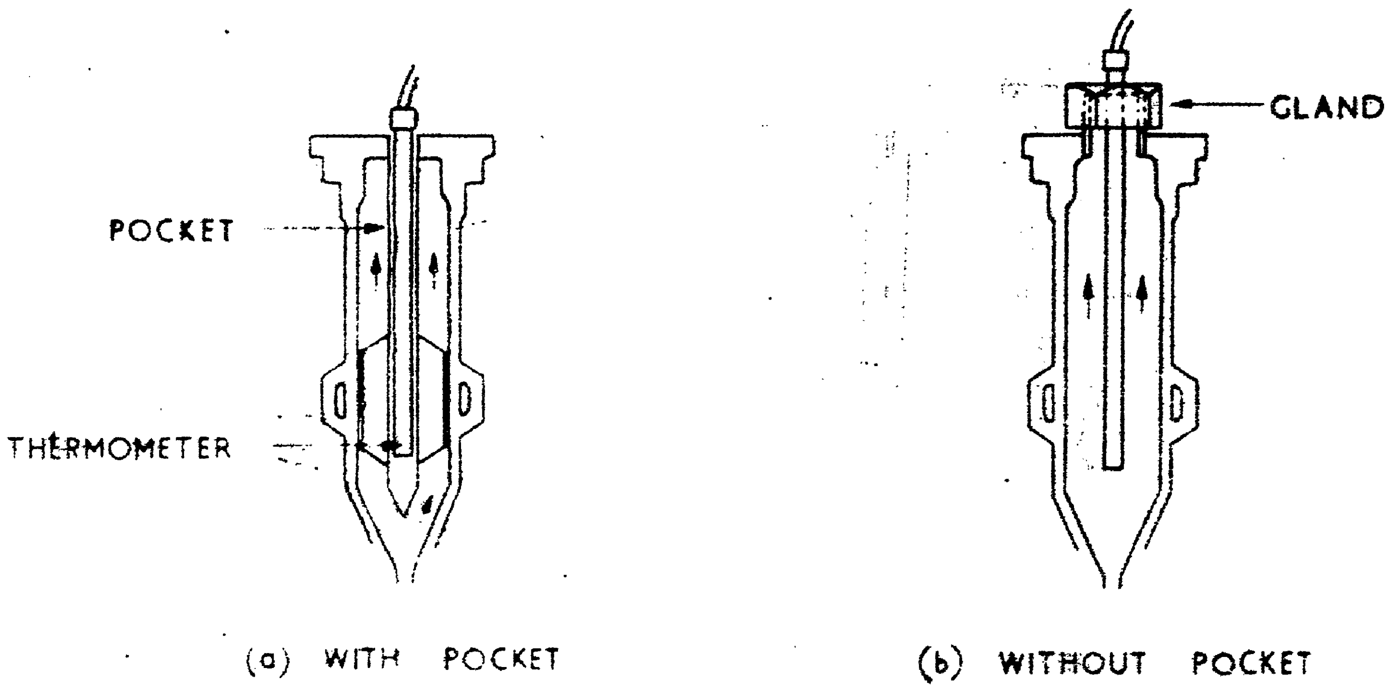

FIG.17 THERMOMETER

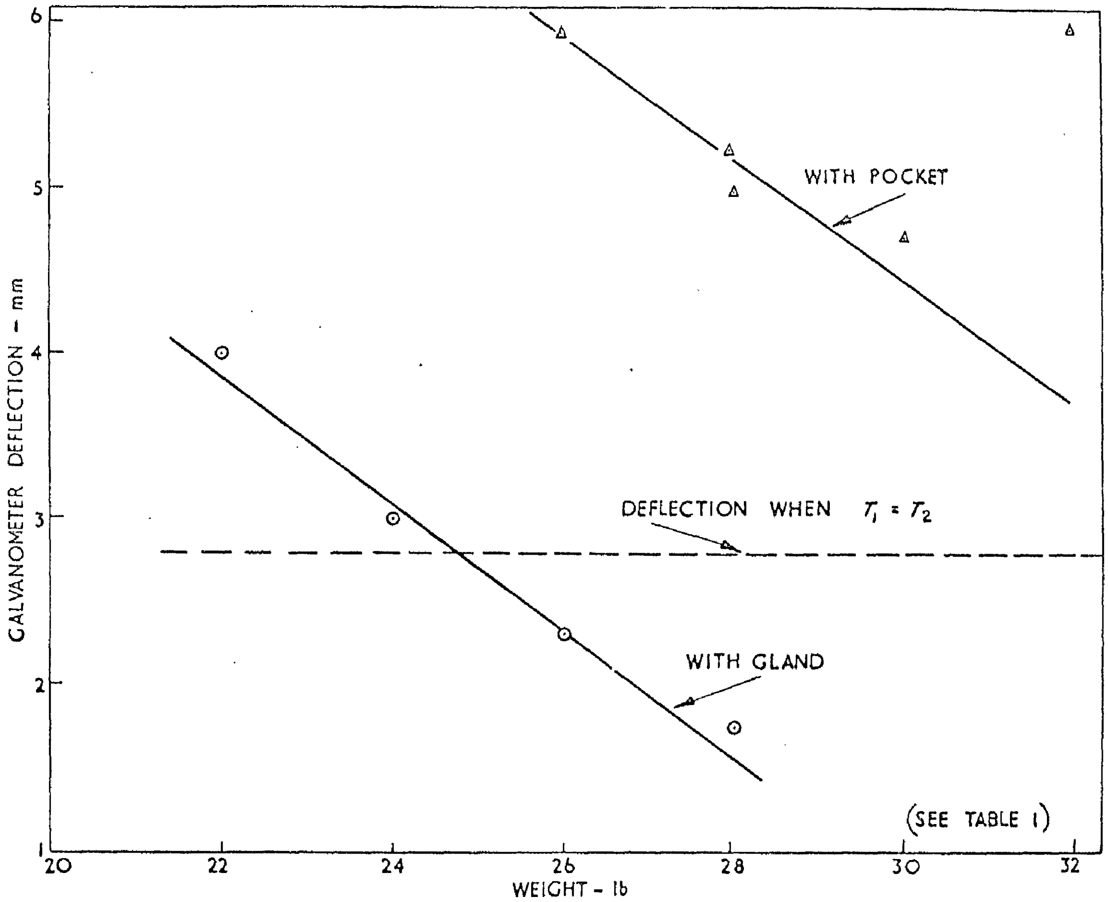

FIG. 18 GALVANOMETER DEFLECTIONS, THERMOMETER WITH AND WITHOUT POCKET

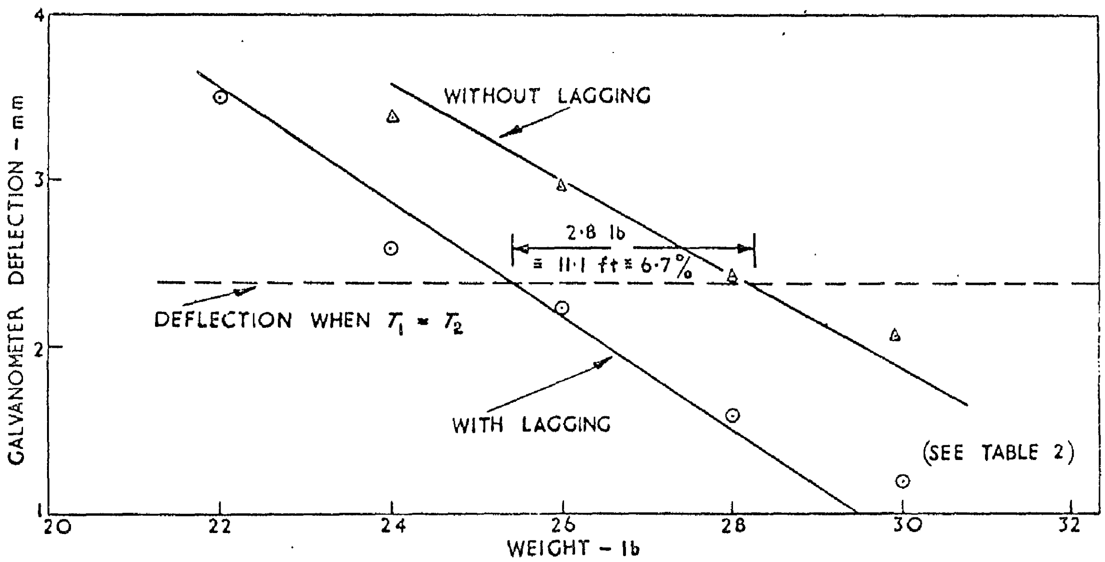

FIG. 19 GALVANOMETER DEFLECTIONS, WITH AND WITHOUT LAGGING ON FLEXIBLE TUBE

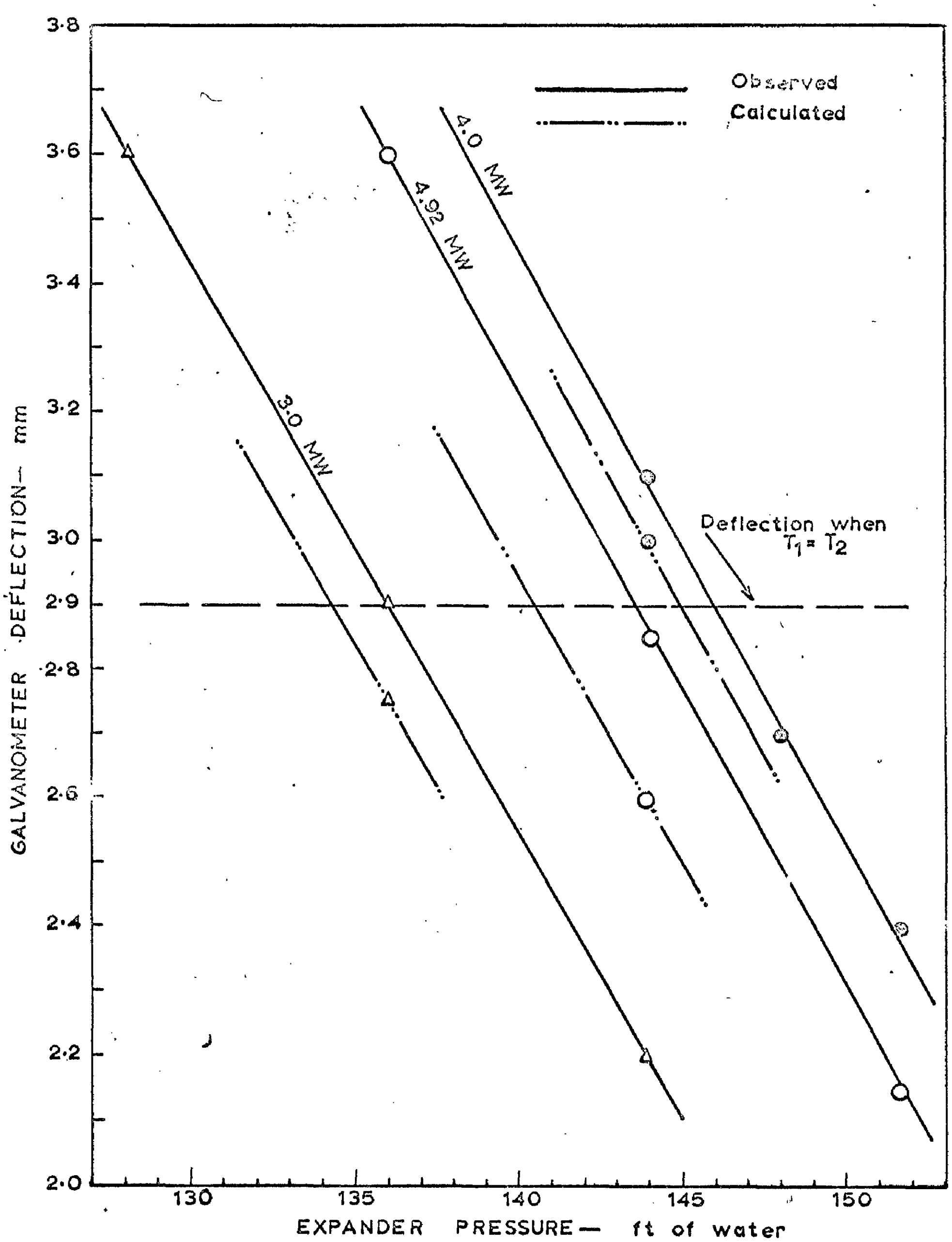

FIG. 20 PRESSURE vs. DEFLECTION for TABLE 1[illegible]

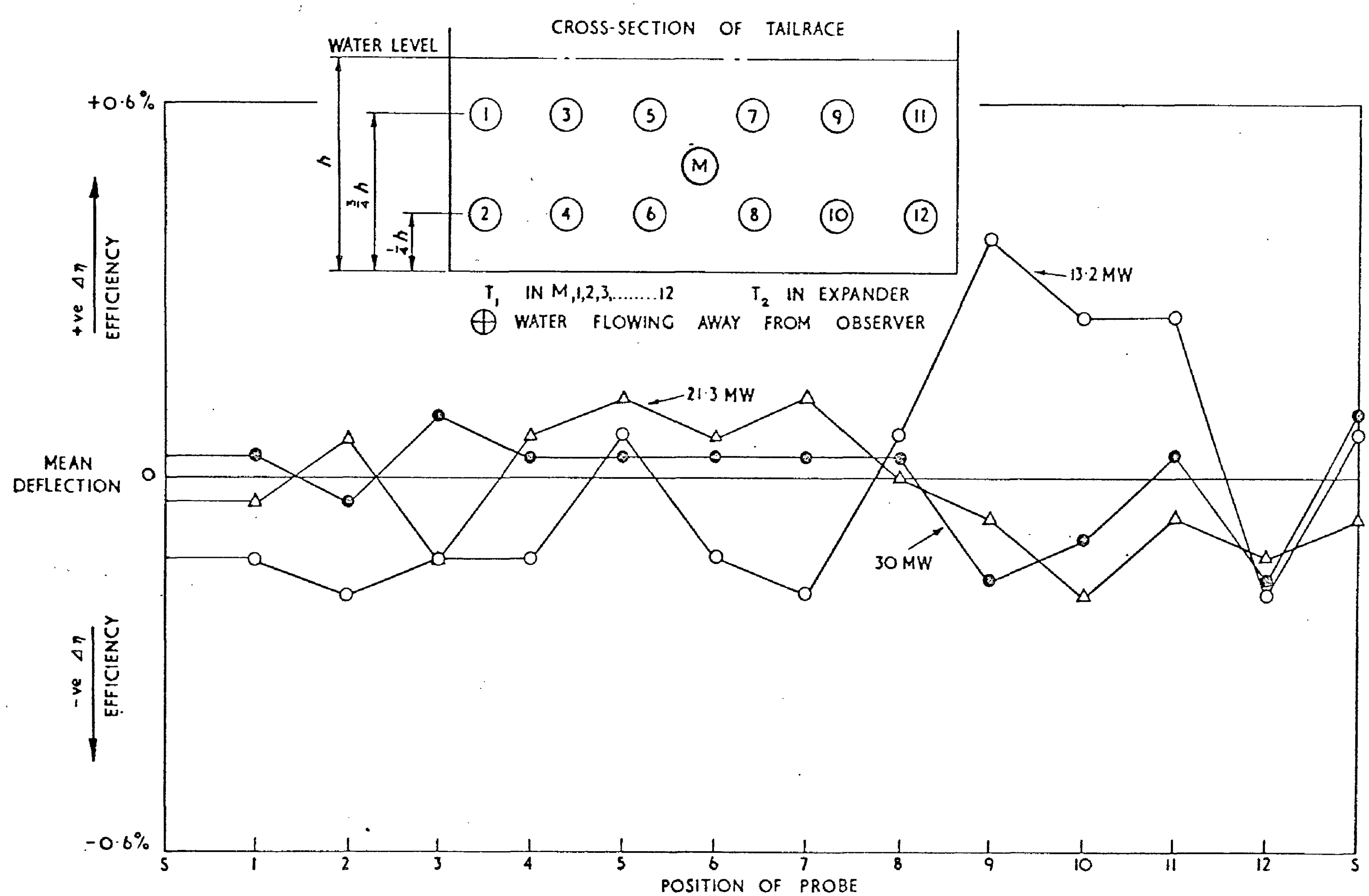

FIG. 21 TRAVERSE BY CONVENTIONAL METHOD

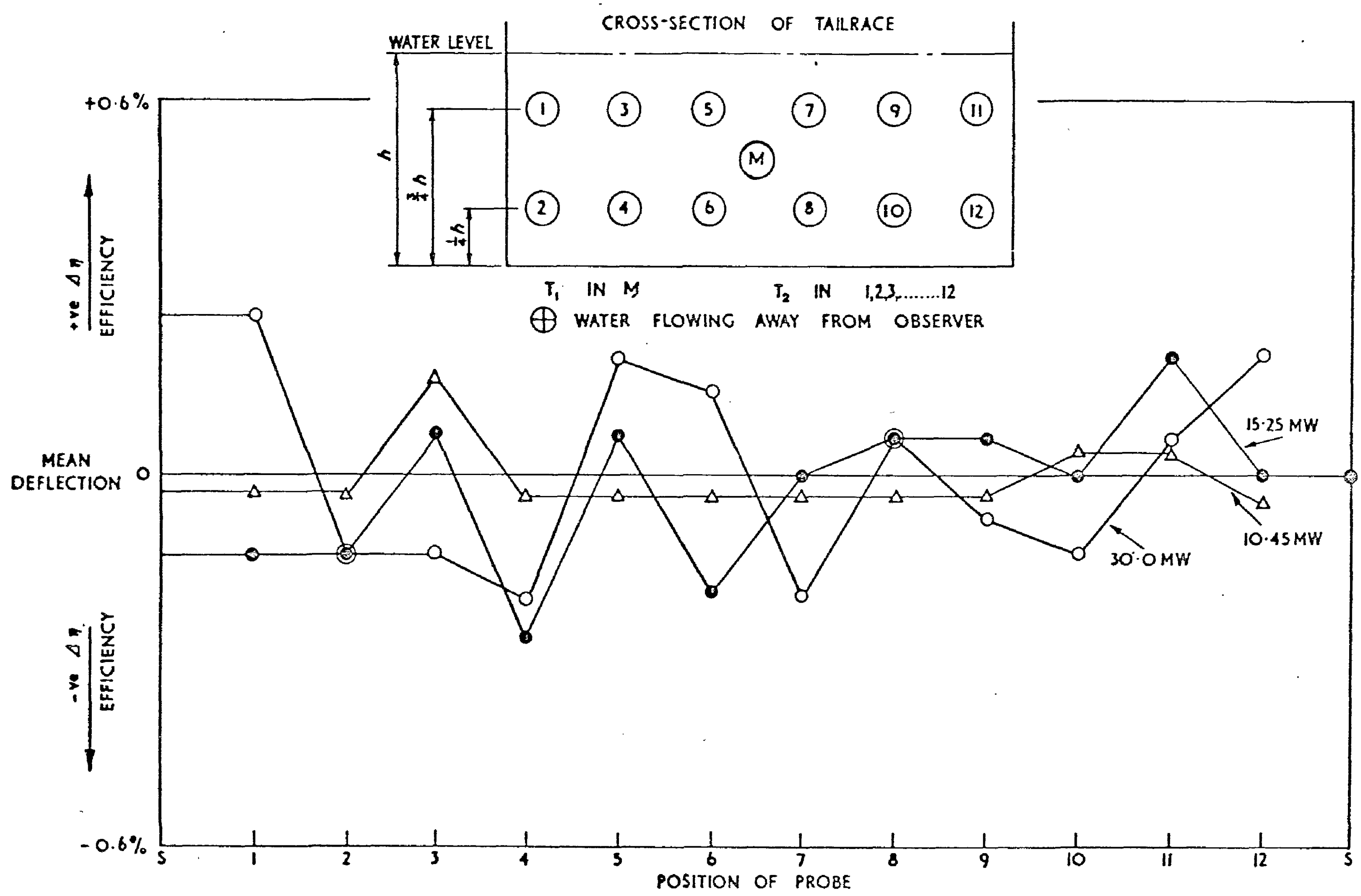

FIG. 22 TRAVERSE BY TWO THERMOMETER METHOD

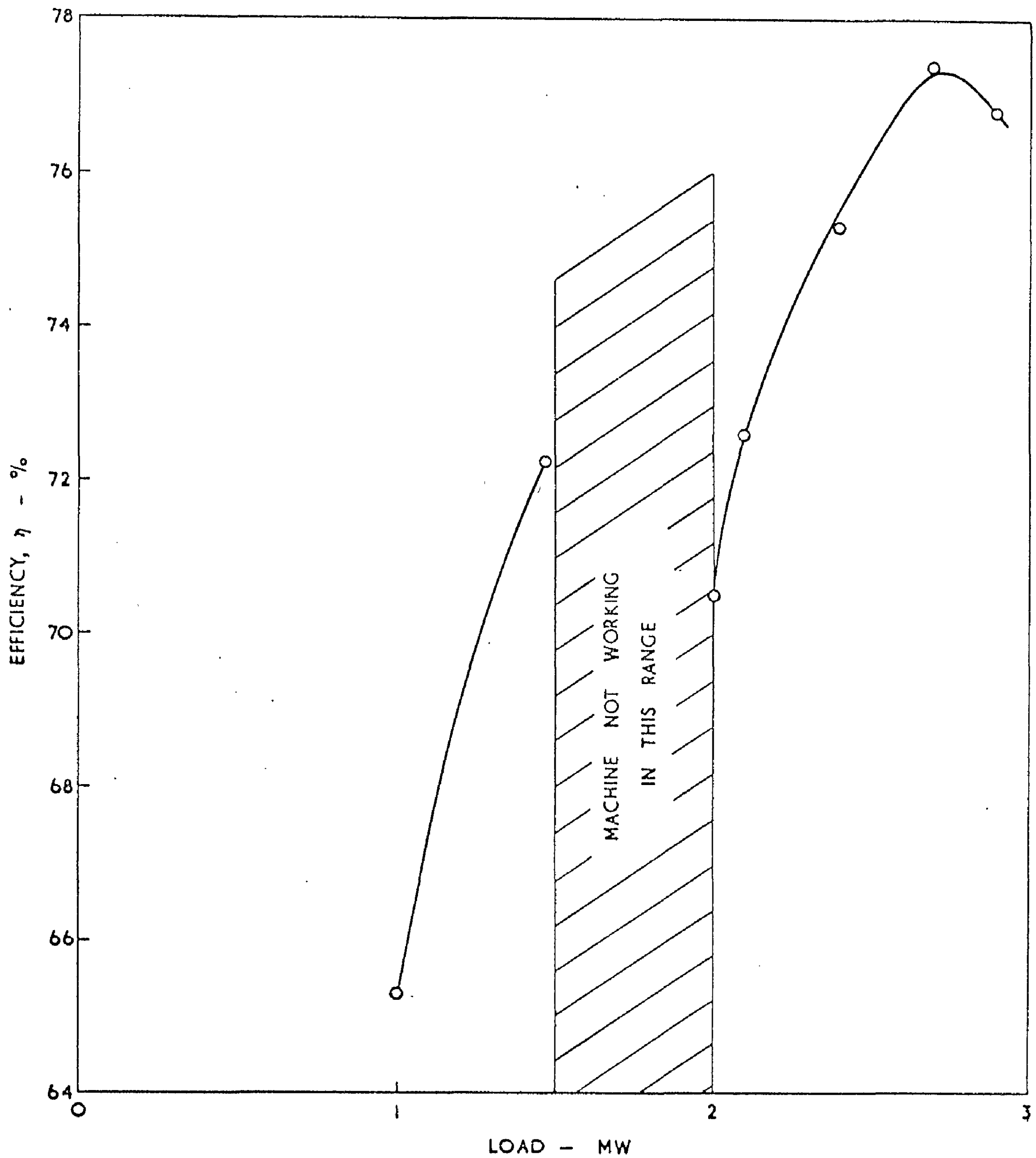

FIG. 23 EFFICIENCY, STONEBYRES MACHINE No 2

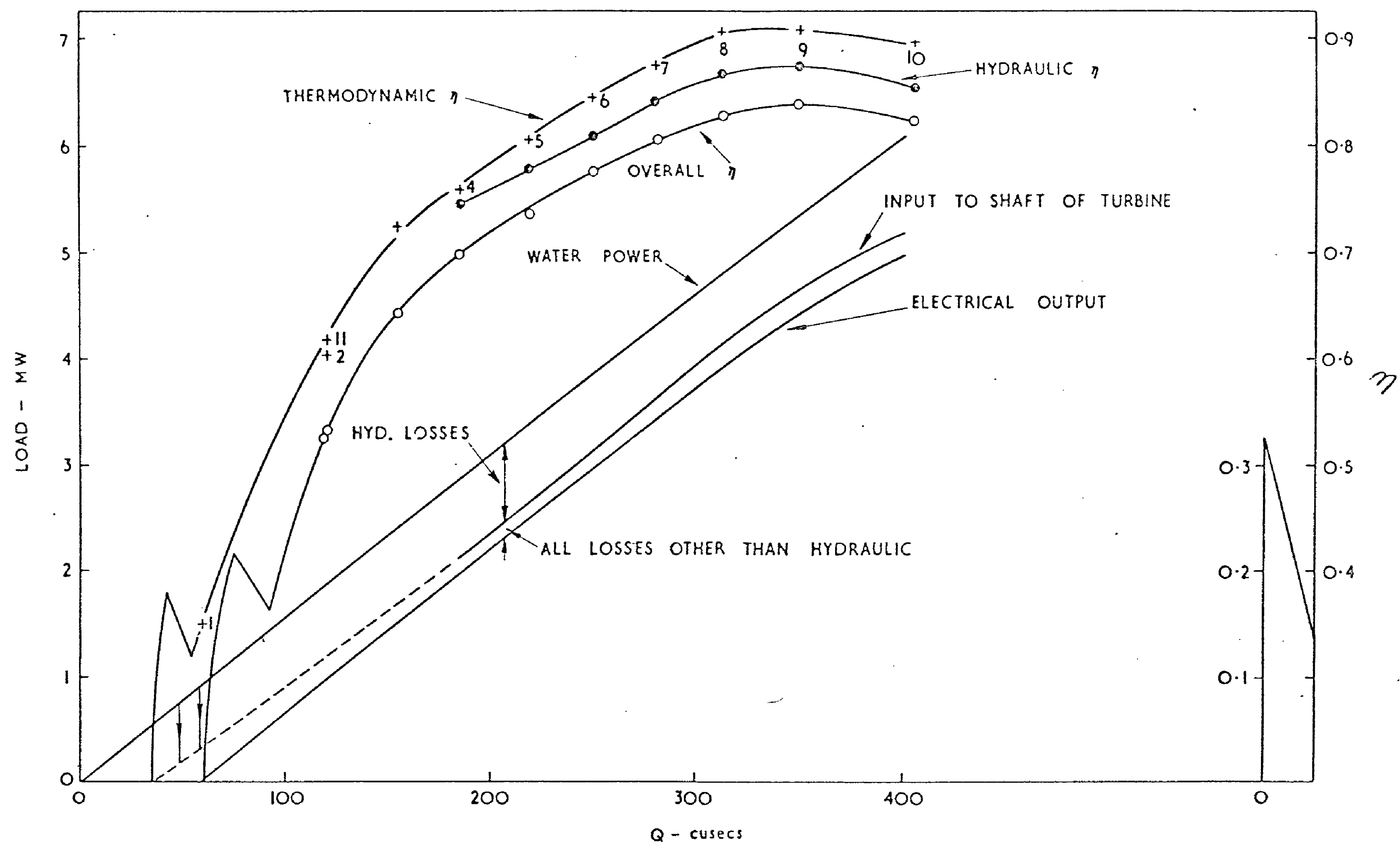

FIG. 24 EFFICIENCY AND POWER CURVES BONNINGTON, MACHINE N°1

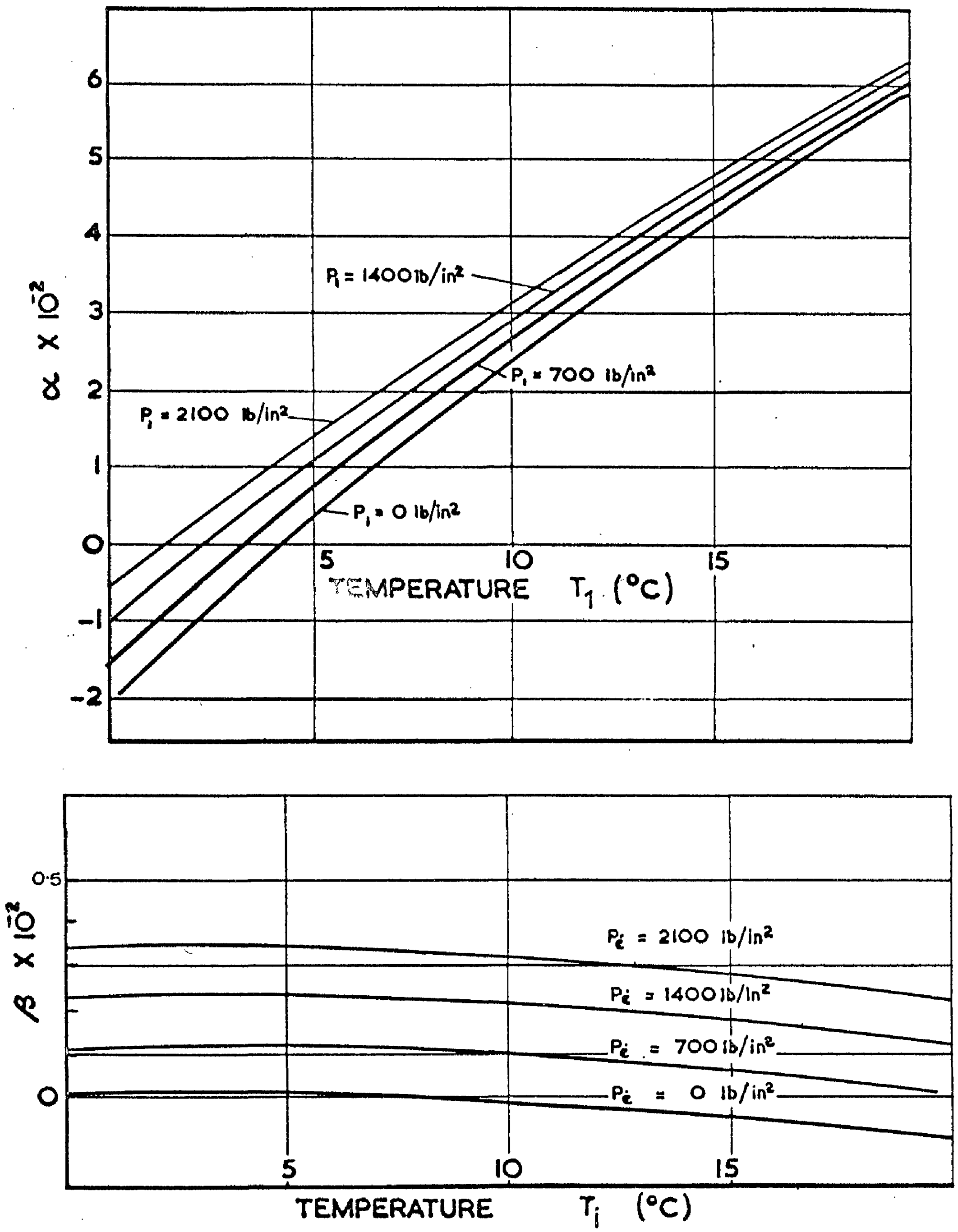

FIG. 25 · VARIATION OF THERMODYNAMIC COEFFICIENTS α AND β WITH TEMPERATURE AND PRESSURE

ADDENDUM TO THESIS

on

Thermodynamic Method of Measuring Efficiency of Hydraulic Machines

by

SYED YOUSUF AHMAD

(University of Glasgow)

C O N T E N T S

5.1A <u>Extracting probe</u>

The purpose of an extraction probe is to provide a sample of water from the penstock which fairly represents the total energy (i.e. hydraulic and internal) of the cross-section at inlet. The expression for efficiency has been established on the assumption that the total energy was uniformly distributed across the section. In practice however, especially in large pipes, the total energy is not uniformly distributed, and therefore the probe should be inserted to such a position where the total energy is nearly equal to the average total energy.

This necessitates determination of the distribution of different types of energy across the measuring section. In the thermodynamic method as required by the efficiency equation 2.6 : 1, we are only interested in kinetic energy, pressure energy and the internal energy of the flowing fluid at the inlet section of the machine. Fortunately, in most cases, the turbine entrance is a large circular pipe, which makes it easier to determine the distribution of the various types of energy. During the present investigation, since all the tests performed were on hydraulic machines of relatively large capacity, it can be safely assumed that the flow was turbulent under all tests and at all gate openings, and the Reynolds number involved was of the order of 10^6 in most cases.

To determine the average total hydraulic head in a turbulent stream at a particular section, it is necessary to know the distribution of static pressure and the distribution of velocity. Goldstein has shown theoretically that the measured static pressure at a single point is always greater than the true mean static pressure. Fage has observed experimentally that the static pressure varies across a section and the measured static pressure in a turbulent stream does not give a true value of static pressure. He explains this variation as an effect due to fluctuating cross velocities. Addison however comments that these variations of pressure are of small magnitude and concludes that for all engineering purposes the distribution of static pressure across a diameter of a uniform circular straight pipe could be taken as uniform. Willm and Campmas, Brandt, Thom and Singh and all other investigators interested in thermo-dynamic method did not find any appreciable variation of static pressure across the measuring section. Numachi measured the velocity distribution for a 6 ft. dia. water duct by a stream-lined pitot tube. He used two methods of measurement, one on the assumption that static pressure was constant throughout the whole cross section and the other on the basis of actual measurement of static pressure at each velocity point. The resulting error in the velocity head amounted to a range of $\pm$ 1 per cent. This shows that the variation of static pressure was of small magnitude.

In the discussion on Numachi's paper, Hooper quoting Cole's and Hubbard's paper raised an interesting point that the presence of a pitot rod might disturb the pressure distribution appreciably and hence might give erroneous readings of static pressure. Numachi explained however that the experimental work done by Cole and Hubbard was related to pipes of small diameter (e.g. 4 - 12 in. dia. by E.S. Cole) with small pitot tube bar. He concludes on the basis of experimental evidence that the magnitude of variation in pressure due to the presence of the pitot rod is almost negligible.

It can therefore be concluded that the distribution of static pressure can be taken as uniform across a circular cross-section. Hence it follows that if the extraction probe is placed at some radius which will give the average velocity, the sample of water extracted at that point will also give the average total hydraulic head. It should however be noted that the velocity head determined from the mean velocity (or average velocity) does not represent the true mean kinetic energy per pound of fluid unless multiplied by a correction factor k. Streeter has calculated these correction factors for turbulent flow in pipes for a large range of Reynolds number. The correction factor decreases with the increase of Reynolds number, and at Reynolds number 10^6, the correction factor has a value of about 1.023. As we are concerned with flows, where Reynolds number is of the order of 10^6, the correction factor can therefore be taken as unity without causing any appreciable error.

To evaluate the position of average velocity, it is imperative that the velocity distribution across the cross-section should be known.

5.1A.1 Velocity distribution.

Velocity distributions for turbulent flow in closed conduits have been studied very thoroughly by different investigators. Numerous velocity profiles have been determined experimentally and attempts have been made to establish a universal relationship to express the velocity distribution for flow in circular pipes. Most of the work on velocity profiles was performed on small diameter pipes and under laboratory conditions, therefore they may not agree with the conditions in large pipes. However, these works will provide a good basis for estimating the position of the average velocity point for turbulent flow in pipes.

The most extensive work on the study of turbulent velocity profiles was done by Prandtl. He derived a preliminary relationship for velocity profiles in turbulent flow, which is known as the power law for the velocity distribution and it is expressed as

$$\frac{u}{u_{max}} = c \left(\frac{y}{r_o} \right)^{1/n} \quad \ldots\ldots\ldots\ldots \quad 5.1A:1$$

where u is the velocity at any radius r,

u_{max} is the maximum velocity,

r_o is the outer radius of the pipe,

$y = r_o - r$ measured from the wall,

c is a constant, and

n is an integer whose value depends on the Reynolds number of the flow.

If we define v as the average velocity in the pipe we can write

$$v \pi r_o^2 = \int_0^{r_o} 2 \pi r u . dr .$$

Substituting the value of u from 5.1A:1, we get

$$v \pi r_o^2 = \int_o^{r_o} 2 \pi c \, u_{max} \left(\frac{y}{r_o}\right)^n . (r_o - y) . dr$$

On integration we get,

$$v = 2 \, c \, u_{max} \left[\frac{1}{n+1} - \frac{1}{n+2} \right]$$

$$\text{or} \quad v = \frac{2 \, c \, u_{max}}{(n+1)(n+2)}$$

From 5.1A:1 we can represent u_{max} in terms of u and y, and therefore we can express the above equation as

$$\frac{u}{v} = \frac{(n+1)(n+2)}{2} \left(\frac{y}{r_o}\right)^n .$$

To find the position of average velocity point, we should determine the value of y for $\frac{u}{v} = 1$, and therefore we can write

$$\left(\frac{y}{r_o}\right)^n = \frac{2}{(n+1)(n+2)}$$

$$\text{or} \quad \frac{y}{r_o} = \left[\frac{2}{(n+1)(n+2)} \right]^{1/n} .$$

In the velocity distribution n varies with Reynolds number. At Reynolds number 10^6, the value of $n = 10^{-1}$ (according to Nikuradse's data), therefore

$$\frac{y}{r_o} = (0.8608)^{10} = 0.244,$$

$$\text{or} \quad y = 0.122 \, D \quad \text{where } D = 2 \, r_o .$$

This calculation shows that the average velocity can be obtained at a point equal to 0.122 D measured from the wall of the pipe.

Another law of velocity distribution known as the logarithmic velocity-distribution, derived by Prandtl, is

$$\frac{u}{u_*} = \frac{1}{k} \log_e y + \text{constant} \quad \ldots\ldots\ 5.1A:2$$

where $u_* = \sqrt{\frac{\tau_0}{\rho}}$ and is known as shearing velocity,

u is the velocity at any radius r,

y is the distance from the outer wall $= r_0 - r$,

r_0 is the outer radius of the pipe, and

k is a constant determined by experiment.

This equation agrees well with experiment and is quite satisfactory to apply to turbulent flow in pipes.

However, to obtain a more general equation for the velocity distribution in circular pipes, Von Karman analysed the experimental data and suggested that the flow field should be divided into three definite regions; the laminar sublayer, the buffer layer and the turbulent core. Thus the velocity distribution for the whole cross-section of the circular tube is expressed by three logarithmic type equations (5.1A:2) and is known as the universal velocity distribution.

In determination of the mean velocity point however, it can be assumed that the velocity distribution for the whole pipe is represented by the equation for the turbulent core. This simplification will not cause any appreciable error as the thickness of the laminar sublayer and buffer layer in large pipes at high Reynolds number is almost negligible.

Therefore from equation 5.1A:2 we can derive that,

when $y = r_o$, $u = u_{max}$

and we can write the equation 5.1A:2 as

$$\frac{u}{u_*} = \frac{u_{max}}{u_*} + \frac{1}{k} \log_e \frac{y}{r_o} \quad \ldots\ldots\ldots 5.1A:3.$$

As the discharge is obtained by integrating the velocity distribution, therefore we can write

$$v \pi r_o^2 = 2 \int_o^{r_o - \delta} ur.dr = 2\pi \int_{\delta}^{r_o} (u_{max} + \frac{u_*}{k} \log \frac{y}{r_o})(r_o - y).dy$$

where δ is the thickness of boundary layer.

The integration cannot be carried out to $y = 0$, since the equation holds in the turbulent zone only. The volume per second flowing in the laminar zone is so small that it may be neglected.

Then,

$$v = 2 \int_{\delta/r_o}^{1} (u_{max} + \frac{u_*}{k} \log_e \frac{y}{r_o}) (1 - \frac{y}{r_o}). \ d(\frac{y}{r_o})$$

in which variable of integration is y/r_o .

By performing the integration we get

$$v = 2 \left[u_{max} \left\{ \frac{y}{r_o} - \frac{1}{2} \left(\frac{y}{r_o} \right)^2 + \frac{u_*}{k} \quad \frac{y}{r_o} \log_e \frac{y}{r_o} - \frac{y}{r_o} + - \frac{1}{2} \left(\frac{y}{r_o} \right)^2 \log_e \frac{y}{r_o} + \frac{1}{4} \left(\frac{y}{r_o} \right)^2 \right\} \right]_{\delta/r_o}^{1} .$$

As δ/r_o is very small, terms including δ/r_o are negligible, so

$$v = u_{max} - \frac{3}{2} \frac{u_*}{k} \qquad \ldots\ldots\ldots\ldots \quad 5.1A:4$$

Re-writing equation 5.1A:3 we get

$$u = u_{max} + \frac{u_*}{k} \log_e \frac{y}{r_o} \qquad \ldots\ldots\ldots\ldots \quad 5.1A:5$$

If we substitute the value of u_{max} from equation 5.1A:4 we can write

$$u = v + \frac{3}{2} \frac{u_*}{k} + \frac{u_*}{k} \log_e \frac{y}{r_o} \qquad \ldots\ldots \quad 5.1A:6 .$$

It can be shown that $u_* = v \sqrt{\frac{f}{2}}$ where v is the average velocity and f is the friction factor of the pipe.

Thus,

$$u = v + \frac{3}{2k} v \sqrt{\frac{f}{2}} + \frac{v}{k} \sqrt{\frac{f}{2}} \log_e \frac{y}{r_o}$$

or $$\frac{u}{v} = 1 + \frac{1}{k} \left(\frac{3}{2} + \log_e \frac{y}{r_o} \right) \sqrt{\frac{f}{2}} \qquad \ldots\ldots \quad 5.1A:7$$

And if $\frac{u}{v} = 1$, then from 5.1A:7

$$\frac{3}{2} + \log_e \frac{y}{r_o} = 0$$

or $$\log_e \frac{r_o}{y} = 1.5$$

or $$y = 0.234\ r_o$$

or $$y = 0.117\ D \qquad (D = 2\ r_o)$$

Thus the universal velocity profile enables us to determine the average velocity at a point situated at a distance of 0.117 D from the wall of the pipe.

The equation 5.1A:7 is universally true, both for rough and smooth pipes. Examination of equation shows that the velocity profiles will be different for different Reynolds numbers and for different friction factors, but the position of the mean velocity point will be the same in all cases. This equation was first derived by Rouse and has been used by various investigators interested in turbulent flow. Streeter has also used this equation in calculating the kinetic energy and momentum correction factors.

The logarithmic velocity distribution equation no doubt is in good agreement with experimental velocity data. The relationships, however, are not consistent with physical reality. The change from laminar to turbulent motion is gradual; it is therefore not correct to have three different curves representing the complete velocity distribution when a single curve should be used. This however is not very serious from the point of view of the present investigation, as we neglect the laminar and buffer layers which are very small at high Reynolds numbers in large pipes. A second more serious inconsistency is the fact that at the axis of the tube it does not give a zero velocity gradient. For this reason a velocity profile calculated from equation 5.1A:7 gives an average velocity somewhat higher than the true average velocity.

Aware of these inconsistencies mentioned above, a number of investigators have developed relationships to avoid them if possible. Most of these relationships are of complex nature and beyond the scope of present investigation. Pai however, has developed a comparatively simpler equation, which agrees with Nikuradse's data for water in a tube at Reynolds number of 3.24×10^6, and it is expressed as

$$\frac{u}{u_{max}} = 1 - 0.204\left(\frac{r}{r_o}\right)^2 - 0.25\left(\frac{r}{r_o}\right)^{32} \quad \ldots\ldots \; 5.1A{:}8$$

In view of the fact that Nikuradse's data in the vicinity of the wall is questionable, the integration of the equation 5.1A:8 shall not be carried to y = 0.

Following the same procedure as before, we can express the average velocity v as

$$v \pi r_o^2 = 2\pi \int_0^{r_o - \delta} r . u_{max} \left[1 - 0.204\left(\frac{r}{r_o}\right)^2 - 0.25\left(\frac{r}{r_o}\right)^{32} \right] dr \;,$$

$$\text{or} \quad v = \frac{2\,u_{max}}{r_o} \int_{\delta}^{r_o} \left(1 - \frac{y}{r_o}\right) \left[1 - 0.204\left(1 - \frac{y}{r_o}\right)^2 - 0.25\left(1 - \frac{y}{r_o}\right)^{32} \right] . \, dr$$

By substituting $\frac{y}{r_o} = t$, we get

$$v = 2\,u_{max} \int_{\delta/r_o}^{t} (1 - t) \left[1 - 0.204(1 - t)^2 - 0.25(1 - t)^{32} \right] . \, dt$$

$$v = 2\,u_{max} \left[\frac{-(1-t)^2}{2} + 0.204\frac{(1-t)^4}{4} + \frac{0.25}{34}(1-t)^{34} \right]_{\delta/r_o}^{1} .$$

Neglecting δ/r_o and higher powers of δ/r_o, as it is very small, we get

$$v = 2\,u_{max}\left(\frac{1}{2} - 0.051 - \frac{0.25}{34} \right),$$

$$\text{or} \; \frac{v}{u_{max}} = 0.8833 \;,$$

Therefore equation 5.1A:8 can be expressed as

$$\frac{u}{v} \times 0.8833 = 1 - 0.204(1 - \frac{y}{r_o})^2 - 0.25(1 - \frac{y}{r_o})^{32} .$$

For the determination of mean velocity measuring point, $\frac{u}{v} = 1$, therefore,

$$0.204\ (1 - \frac{y}{r_o})^2 + 0.25(1 - \frac{y}{r_o})^{32} = 0.1167,$$

$$\text{or} \quad (1 - \frac{y}{r_o})^2 \left[0.204 + 0.25(1 - \frac{y}{r_o})^{30}\right] = \left[0.1167.\right]$$

Solving we get $\frac{y}{r_o} = 0.244$

or $y = 0.122\ D.$

Therefore Pai's equation gives the distance of the single reading, measuring position as 0.122 D measured from the wall.

Several other investigators also have tried to establish a rule for single measuring position for mean velocity. Winternitz and Fischl have recently developed a new technique of velocity integration called the log linear rule. In this paper they report that the determination of mean flow from a single reading was proposed by Aichelen, and similar rules had previously been mentioned by Ott and by Colebrook and White. They further add that the position specified by Aichelen as a result of empirical analysis is 0.119 D; and they mention that by comparison, the log linear rule with two points per diameter yields 0.112 D, the close agreement being noteworthy. Aichelen used Nikuradse's data to check on the accuracy of his results which was within one per cent for flow in smooth pipes. Winternitz and Fischl also applied Aichelen's technique on Nikuradse's data and obtained results with a mean error of less than one per cent, with a scatter comparable to both that of the ten-point tangential and the four-point log linear rule.

Professor Jakob also has reached somewhat the same conclusion about the position of mean measuring point. He writes in the discussion of Bate's paper that he has found empirically that measuring velocity at a distance y = 0.12 D from the wall yields very closely the mean velocity in the wide range of Reynolds number from 3000 to 4.5×10^6.

The results for the determination of a single reading gauging position for mean velocity obtained by different calculations in this paper and also by other investigators are summarised in the table below.

Table 16 <u>Single reading gauging positions for the determination of mean velocity.</u>

Rules	Distance from wall in pipe diameters
Prandtl power law	0.122
Universal logarithmic law	0.117
Pai's polynomial	0.122
Aichelen's empirical	0.119
Winternitz log-linear	0.112
Jakob's empirical	0.120

All these rules seem to be in quite good agreement with each other. The mean of these six rules can be taken as 0.119 D. Hence a point at a distance of 0.119 D from the wall can be taken as a single reading gauging position representing the mean velocity. In his book on flow measurement and meters Linford observes that for straight pipes the velocity might be measured at a point 0.762 times the pipe radius from the centre (= 0.119 D from the wall). He adds that here it was found that the velocity was within $\frac{1}{2}$ per cent of the mean velocity over a wide range of Reynolds number and pipe-wall roughness.

It is to be remembered that most of the equations of velocity distribution were based on the data collected under controlled laboratory experiments. Addison points out that these simple formulas are limited in application in the following ways:-

(i) They only apply to those sections of uniform circular pipes, at some distance from the inlet, at which a permanent regime of flow has been established - Winternitz suggests that for the velocity profile to be fully developed a settling length of the order of fifty times the diameter is ideally required.

(ii) They do not apply to points very close to the wall.

(iii) They may not agree with the conditions in large pipes.

It is therefore necessary to verify the position of mean velocity obtained analytically from the observed velocity distributions in large pipes. For comparison some calculations were made on the velocity distributions obtained from pitot tubes in large pipes published in

Cole's paper. The calculations were confined to those velocity distributions which were approximately symmetrical. Fig. 26 shows three velocity traverses in a 78" dia. pipe at three different gate openings (taken from Cole's paper). The position of mean velocity expressed as fraction of diameter as calculated from Fig. 26 is given in the table below.

Table 17. Positions of mean velocity in 78" dia. pipe. (calculated from Fig. 26)

Gate opening	Position of mean velocity measured from wall in pipe diameters.
0.5	0.123
0.6	0.122
0.8	0.124

Fig. 27 shows ten tests of velocity traverses in a 7.5 ft. dia. penstock. The positions of mean velocity calculated from these traverses are shown in the following table.

Table 18. Positions of mean velocity in 7.5 ft. dia. penstock (calculated from Fig. 27)

Tests	Positions of mean velocity measured from wall in pipe diameter.
IV	0.112
I	0.12
VI	0.12
II	0.12
III	0.12
V	0.118
VIII	0.102
IX	0.115
X	0.112

Tables 17 and 18 show that the experimental results are in good agreement with the formula obtained analytically and hence provided experimental verification of the formula.

In conclusion it can therefore be stated that at a distance of 0.12 D from the wall, a single measurement by pitot probe can obtain the mean velocity of the flow without causing appreciable error. In the thermodynamic expression of efficiency (2.6), the velocity head appears as a secondary term, hence very accurate measurement of velocity is not required. However, it was desirable to determine the position which gives the mean velocity with least error in most general cases. A rule of the insertion of a probe equal to $^{D}/_{8}$ (= 0.12 D approx.) has been established (as explained above) to represent the mean velocity of flow. It should be remembered that this rule can only be applied to fully developed flows having symmetrical velocity profiles.

5.1A.2 Temperature distribution.

It has been shown in 2.31 that the determination of the internal energy primarily concerns the measurement of temperature of the flowing medium. Unlike velocity distribution, adequate published work is not available for turbulent flow temperature distribution. Some investigators in problems of heat transfer by forced convection have presented some empirical formulas for temperature distribution in circular pipes. According to Professor Hsu, the motion of fluid particles in turbulent flow is complicated, and because we lack theoretical knowledge on the velocity and temperature distribution in turbulent flow, difficulties are encountered in analytical treatment. In solving problems of this sort, principles of the analogy between momentum and heat transfer have been found useful and invariably used by different investigators. Reynolds' original work was based on the same principle, which was later extended by Prandtl and further improved upon by Von Karman and Boelter. Among the later workers Martinelli was first to present a thorough analysis of turbulent-flow heat transfer in circular pipes, in his detailed studies of temperature distribution. Seban and Shimazaki made an anslysis similar to that of Martinelli for turbulent flow in circular pipes. They did not, however, assume a linear radial distribution of heat flux, as Martinelli did, and they considered the case of constant wall temperature. They used Martinelli's equation to calculate the first approximation of temperature distribution, which was subsequently put into the equation developed by them. An iterative process was continued until the calculated temperature distribution did not change. Lyon, adopting Martinelli's main concept modified the theory by using a much easier procedure, but the actual calculation of velocity profile is rather tedious. The most recent work on temperature profiles for turbulent flow in pipes has been done by Deisseler. The theoretical analyses have covered both high and low-Prandtl-number fluids and have also taken into consideration the effect of variable fluid properties. To discuss the work of all investigators and to compare the different temperature profiles obtained is beyond the scope of the present investigation. Martinelli's work can be taken as the basis of all other investigations. Deissler's work however is a further development but it is not important from the point of view of the present investigation owing to the fact that the temperature across a section of a penstock does not change sufficiently to make any appreciable change in the fluid properties. Therefore it will be sufficient to deal only with Martinelli's equation for the determination of temperature distribution for turbulent flow in circular pipes.

In Martinelli's work the general emphasis is being given on calculation of Nussett number for liquid metals, but the laws of temperature distributions are also presented in detail which apply to all fluids at all Reynolds numbers. For the purpose of analysis Martinelli followed Von Karman and sub-divided the flow field into three definite regions; the laminar sublayer, the buffer layer and the turbulent core. He also used the velocity distribution equations for the three different regions given by Von Karman. Martinelli obtains temperature distribution as a function of 'y' the distance measured from the wall and gives three equations for the different regions. These equations are very involved but for clarity, only the equation for turbulent core is given below.

$$\frac{t_w - t}{t_w - t_c} = \frac{Pr + \ln(1 + 5\alpha Pr) + 0.5 \ln \frac{Re}{60}\sqrt{\frac{f}{8}} \cdot \frac{y}{r_o}}{Pr + \ln(1 + 5\alpha Pr) + 0.5 \ln \frac{Re}{60}\sqrt{\frac{f}{8}}}$$

where t_w is the temperature of the pipe wall
t_c is " " " fluid at r=0 in deg.F
is " " at any radius r
$y = r_o - r$
Pr is Prandtl number of the fluid.
Re is Reynolds " " " "
α is the ratio of eddy diffusivity for heat and eddy diffusivity of momentum, and
f is friction factor.

The temperature distribution for turbulent flow in pipes is a function of the Reynolds number and the Prandtl number, as indicated by Martinelli's equations. The temperature profiles calculated from Martinelli's equations at Reynolds number of 10^4 and 10^6 for different Prandtl numbers are given in Fig. 28 and Fig. 29 taken from Martinelli's paper. From Fig. 28 it can be noted with interest that in turbulent flow for the fluids at Pr = 1, the velocity and temperature profiles are quite close to each other when plotted on a dimensionless basis. Another important point can be observed from Fig. 29 that the thickness of the laminar sublayer and buffer layer at Reynolds number 10^6 is almost negligible and therefore not shown in the diagram, as it was shown in Fig. 28. The effect of Prandtl number on the temperature distribution is clearly indicated in these figures.

Once the law of temperature distribution has been established, the average value of the temperature and its position can be calculated with the help of the temperature and the velocity profiles. The mean-mixed temperature of the fluid t_m is defined as the temperature that would be measured if all liquids flowing through a section were thoroughly mixed. It is also known as cup-mixing or mixing box temperatures, but is generally referred to as bulk temperature.

The mean temperature therefore can be expressed as

$$t_m = \frac{\int_0^r t.u.2\pi r.dr}{\int_0^r u.2\pi r.dr}$$

To express the mean temperature in the dimensionless form we write the above equation as

$$\frac{t_w - t_m}{t_w - t_c} = \frac{\int_0^{r_o} \frac{u}{u_{max}} \left(\frac{t_w - t}{t_w - t_c} \right) r.\ dr}{\int_0^{r_o} \frac{u}{u_{max}}\ r.\ dr} \qquad \ldots\ldots \ 5.1A:9$$

Martinelli calculated the ratio of temperature difference from equation 5.1A:9 by actually performing the integration by numerical method. He used for temperature distribution the profiles shown in Fig. 28 and Fig. 29, and for velocity distribution, used equations given by Von Karman. The results obtained by Martinelli are shown in Fig. 30 for a number of Prandtl moduli. For convenience Martinelli has associated a number of fluids with certain magnitudes of Prandtl modulus. He observes that this figure brings out clearly the importance of having mixing chambers during tests on water, air and other fluids, since the measured temperature difference may be in appreciable error if a single thermometer is immersed at the centre of the unmixed fluid stream; the error is appreciable even for Reynolds modulus as large as 10^6.

A similar error is to be expected if attempts are made to measure the mean velocity by placing a pitot tube in the axis of the tube. In section 5.1A.1, it has been shown that for circular pipes, measuring the velocity at a distance $y_{um} = D/8$ (or = 0.24 R) from the wall yields very closely the mean velocity in the wide range of Reynolds number, from 3×10^3 to 4.5×10^6. Due to the similarity between the distribution of velocities and temperatures, it is likely that also a fixed distance y_{tm}/R exists, where y_{tm} is that distance from the wall at which the temperature t is equal to the mean temperature t_m of the fluid. This however is only true in ranges where the influential properties of the fluid are independent of temperature.

The present investigation is concerned only with flow of water at ordinary temperatures, representing a fluid at Prandtl number 8 to 10; therefore the value of y_{tm} was calculated by means of Figs. 28, 29 and 30 for fluids of Pr = 10 at two different Reynolds number. For Re = 10^4, the distance ratio $y_{tm}/R = 0.275$, and for Re = 10^6, the distance ratio $y_{tm}/R = 0.28$. According to these results it seems that the mean temperature of water (at ordinary temperatures) flowing through a circular pipe can be measured by placing a thermometer at a distance of $D/7$ (= 0.28 R approx.) from the wall.

Professor Jakob in the discussion on Martinelli's paper has given a table of distance ratio for two different Reynolds numbers and various Prandtl numbers, associated with fluids like air, water and mercury etc. For the sake of interest the table as prepared by Jakob from Fig. 28, 29 and 30 is given below.

Table 19. Distance ratio y_{tm}/R

Re \ Pr	0.001	0.01	0.1	1	10	Average
10^4	0.305	0.335	0.3	0.3	0.275	0.309
10^6	0.31	0.295	0.3	0.335	0.28	0.304

Jakob comments that according to this table it seems that the mean temperature of all fluids flowing in circular pipes can be measured by placing a thermometer at a distance $y_{tm} = 0.3$ R from the wall at all Reynolds numbers.

Experiments from which y_{tm} could be checked are rare. Some experimental work was done on air by Pannell and by Seban and Shimazaki. Recently Isakoff and Drew have obtained experimental data for velocity and temperature distribution for flow of mercury. It is rather unfortunate that experimental work for water has not yet been published by any investigator. The velocity and temperature profiles drawn from Pannel's data for air shown in Fig. 31, are nearly coincidental. A numerical integration of these distribution curves leads to $y_{um} = 0.247$ R and $y_{tm} = 0.274$ R. The agreement with the foregoing values is satisfactory.

Recently in Rosemount Engineering Co's bulletin a paper has been published by Leo, Werner & Hagen on measurement of mean temperature in a duct. They have prescribed for fluids at Pr = 1.0 and Reynolds number greater than 6000, a distance of y_{tm} = 0.28 for single thermometers, which is in good agreement with Pannel's data. They do not however, give details of their calculations nor have they given any experimental verification of the results. About liquids they comment that since Prandtl number varies considerably with temperature, this greatly complicates evaluating average temperature, because temperature distribution is much more complex even for round pipes and they suggest that an actual survey of temperature and velocity distribution should be made. In the present investigation an attempt has been made successfully to evaluate the complicated equations of average temperature.

In the absence of experimental verification of temperature distribution for flow of water, it is difficult to draw any definite conclusions. It has however been established analytically in the present paper, that the mean velocity can be measured at D/8 from the wall and the mean temperature at D/7 from the wall. An important point has been demonstrated analytically by Jakob that y_{tm} must indeed be larger than y_{um} if the excess of the temperature t over the surface temperature t_w can be assumed to be proportional to u for any radius r. He has shown from the equations of definition of the mean velocity and the excess of mean mixing temperature that y_{tm} necessarily will exceed y_{um}. This deduction leads us to the fact that the aperture of the extraction tube should be inserted to a distance equal to D/7 from the wall to draw a sample of water representing average total energy, as temperature is a primary term in the efficiency equation. The kinetic energy at this position no doubt will be in error by a few per cent but it will not make any appreciable difference as the velocity head is a secondary term in the efficiency equation.

It should be remembered that the rule of D/7 insertion was obtained analytically for small diameter pipes. The conditions in large pipes (with which the present investigation is concerned) may not agree with the analytical temperature and velocity distributions. In this case it is necessary to resort to a detailed survey of temperature and velocity distribution of the actual flow and to use numerical integration.

It will be of interest to note the observations of different investigators interested in the thermodynamic method about the distribution of temperature in the penstock. Katzman had put one of his thermometers in the head race water at the emergency gate opening and the other into the tail race. The thermistor at the head race was moved vertically up and down for a distance of 20 ft. to determine whether the temperature of the water was uniform with depth. He found no change in temperature with depth. Willm and Campmas have assumed it to be known that a probe situated at a distance of 1/7 of diameter from the wall gave a sample of water, the energy of which is extremely close to the mean hydraulic energy in the cross section. They report that they had no evidence of any variation of temperature in such sections. From the argument of the present investigation it seems that in the absence of any temperature variation, a sample of water drawn off at a distance of D/8 from the wall will represent more closely the mean value of the total energy of the flowing medium.

Singh carried some probe insertion tests up to 20 inches in a penstock of 5 ft. 8 in dia. (max. static head = 510 ft.). He obtained variation of temperature which resulted in variation of efficiency of the order of ± 0.3%. He carried out some further tests in a penstock of 6 ft. dia. (max. static head = 410 ft.) at three different loads. He observed in one of the worst cases a variation of efficiency of the order of ± 3%, which is rather high. No explanation has been furnished by Singh for such large variation in efficiency. He suggests, however, that the maximum possible insertion of the probe, without undue vibration would give a fairly good representative sample at all loads. Singh further remarks that suitable values of insertion obtained by trials i.e. where the curves tend to flatten out and become parallel, do not differ much from the $^1/_7$th of the diameter given by Willm and Campmas. Brandt also does not report any evidence of temperature gradient across the penstock and has simply followed Willm and Campmas in adopting the D/7 insertion rate. Vaucher has recommended an adjustable probe (5.1) for rapidly measuring the energy distribution across the test section. He does not furnish any experimental results, nor does he discuss the energy distribution.

In conclusion it can be deduced that probe insertion in most general cases should be made to a depth of D/7, as has been established analytically. If, however, there is evidence that the probe traverse affects the measured values of efficiency by more than 1 per cent, the most suitable insertion should be determined by actually measuring the velocity and temperature at each point.

5.1A.3 <u>Geometry of the aperture of the extracting probe.</u>

In article 5.1 it has been stated that the diameter of the aperture for the pitot probe was quite arbitrarily chosen as 9/16". The hole was simply made large enough to ensure a volume of 0.4 l/s at all pressure levels. Nevertheless, a relationship between the diameter of the cantilevered pitot cylinder and the diameter of aperture can be established theoretically on the basis of pressure distribution.

For a cylinder placed with its axis normal to the flow, theoretical and practical studies show that the distribution of the pressure difference $(p - p_o)$ at each point on the surface is somewhat as indicated by the radial ordinates shown in Fig. 32. At a certain critical angle the pressure at the cylinder surface p equals the approach static pressure p_o. According to Binder, the theory based on incompressible flow indicates an angle of 30^o, whereas experimental work with real incompressible fluids show the critical angle as 39^o for turbulent flow.

It evidently follows that if the diameter of the aperture subtended an angle more than twice the critical angle, the sample of water would not represent the true hydraulic energy (kinetic and pressure energy) of the flowing fluid. Therefore the critical angle can be taken as the basis for the determination of the optimum value of the diameter of aperture.

A relationship between the diameter of the aperture and diameter of the probe can be established from the geometry of the pressure distribution.

Let D be the diameter of the pitot probe

d " " " " " aperture, and

θ " " critical angle.

From the geometry of triangle OAB of Fig. 32, it can be shown that for optimum value,

$$d = \frac{D}{2} \cdot \frac{\sin 2\theta}{\cos \theta} = D \sin \theta .$$

Experiments have shown that $\theta = 39^{o}$ for incompressible fluids in turbulent flow, therefore

$$d = D \sin 39 = 0.629 \, D$$

or $$d = \frac{5}{8} D \quad \text{(approximately)}$$

The above calculations show that the diameter of the aperture should not exceed $\frac{5}{8}$th of the diameter of probe; therefore, it can be taken as a good basis for designing the aperture. During the present investigations an aperture of 9/16" diameter on a 1" diameter probe was used, which gave a flow of 0.4 litres/sec and also was within the optimum limit of design.

It may be argued that design of aperture should also be based on the temperature distribution. Unfortunately, enough in formation is not available to establish any design formula, but it can be safely assumed that the pressure and temperature distribution will be similar. It may be further argued that the error caused by large apertures will be almost negligible as the velocity head is usually very small compared to the static head in the machines under test during the present investigation. These arguments are quite valid, but instead of choosing the diameters arbitrarily, it is more desirable to choose the diameter within the optimum design limit, to reduce the magnitude of error, however small it may be.

5.2A Mixing Chamber.

For tests of high head machines, it was assumed that both velocity and temperature of the fluid passing through the tail-race stayed unchanged across the measuring section. However, the situation is quite different for machines of low heads where small differences in temperatures might result in gross error of efficiency. In the inlet measuring section the position of the measuring point was calculated on the basis of temperature and velocity profiles obtained analytically. As the outlet measuring section mostly consists of open channel of rectangular cross-section, it is not easy to calculate analytically the position of mean temperature. A relationship which gives universal velocity profiles in channels has been established by Vanoni, but no published information is available on temperature profiles in channels, therefore any analytical calculation is not possible. Moreover, extensive work done by Thom on temperature profiles conclusively shows that it is not practical to establish any empirical relationship for temperature distribution in the tail-race. Therefore it is necessary to resort to experimental determination of temperature and velocity profile in order to obtain average temperature. It should be noted however, that the determination of mean temperature by means of graphical integration of experimental temperature and velocity profiles will be many times more tedious and time consuming than the determination of discharge per unit time in the conventional way. It is evident that this will defeat the whole object of the present investigation, as the thermodynamic method has been developed with a view of eliminating the tedious methods of measurement of discharge for calculation of efficiency.

To solve this problem a mixing chamber described in article 5.2 was developed to measure the average temperature of fluid passing through a cross-section of the tail-race. It is desirable to determine analytically whether the temperature measured in the mixing chamber will represent the true average temperature. An attempt has been made to establish a relationship between the true mean temperature and the measured mean temperature.

Before any relationship is established it is necessary to define the reference mean temperature. In thermodynamics the reference temperature has been defined in many ways, e.g.:-

(a) the undisturbed temperature.

(b) the energy mean temperature.

(c) the arithmetic mean temperature over the cross-sectional area.

(d) the mixing box, or cup-mixing or bulk temperature.

(e) the probe mean temperature.

(f) the film temperature.

The choice for the reference temperature depends upon the conditions under which the efficiency equation is derived. We recall that the efficiency equation 2.6:2

was established from energy balance. The energy mean temperature which is used conceptually to describe the enthalpy of the moving fluid, would therefore be employed as the reference temperature. It is defined as the temperature representing an average energy per unit volume which, multiplied by the total volume of fluid passing a section in a period of time, equals the energy of that fluid as it passed the section.

The overall change in temperature across the section suffered by the fluid is so small that the density and the specific heat of the fluid can be assumed constant. Hence, in accordance with definition, the energy mean temperature has been expressed by Trefethen as,

$$T_{energy} = \frac{\int_{\Delta t} \int_{A} (\bar{\bar{T}}\, \bar{\bar{v}}.dA)\; dt}{\int_{\Delta t} \int_{A} (\bar{v}.dA)\; dt} \qquad \ldots\ldots\ldots\; 5.2A:1$$

where $\bar{\bar{T}}$ is the instantaneous value of temperature
$\bar{\bar{v}}$ " " " " " velocity
A " " area of cross-section
Δt " " time taken by the volume of fluid in passing the section

$\bar{T}$ and $\bar{v}$ are instantaneous values, it would be difficult to measure them directly and Trefethen remarks that they are never measured in turbulent flow. The evaluation of this integral therefore is not possible. The mean temperature is usually determined experimentally by using velocity and temperature probes. It is to be noted that the probe mean temperature at a section is defined as

$$T_{probe} = \frac{\int_A T.U.d\,A}{\int_A U.d\,A} \quad \ldots\ldots\ldots\ldots 5.2A:2$$

where T and U are the mean temperature and mean velocity of the element and is measured from traverses obtained by velocity and temperature probes. The probe mean temperature however is not the same as the energy mean temperature. Trefethen has provided a relationship between probe and energy mean temperature obtained analytically, but he remarks that for turbulent flow, present knowledge is too meagre to permit evaluation of the relationship. He suggests however that for practical purposes in turbulent non-conducting fluids such as air and water, the difference in probe mean temperature and energy mean temperature can be assumed negligible. We can then define the reference temperature as,

$$T_{energy} = T_{probe} = \frac{\int_A T.U.d\,A}{\int_A U.d\,A} \quad \ldots\ldots\ldots 5.2A:4$$

A relationship can be established between the reference energy mean temperature and the mean temperature observed in a theoretical mixing box. The mixing box mean temperature or cup mixing temperature has been defined as the temperature of the fluid mass passing through the tail race cross sectional area per unit time thoroughly mixed in a box inserted at the section. Trefethen concludes on the basis of experimental evidence that in practice the discrepancy between the probe and mixing-box mean temperature would be important only for $R_e \times P_r$ values less than 30; therefore for the purpose of the present investigation where $R_e \times P_r$ is far greater than 30, the mixing box mean temperature can be taken as probe mean temperature which is equal to the reference temperature defined in equation 5.2A:4.

It is apparent that introducing a mixing box at the section of the tail-race where the mean temperature is to be measured, would involve a number of physical and practical problems. Among these would be velocity and temperature changes similar to entry effects. To a degree such complications could be avoided by careful design, or a correction factor could be introduced. One major problem would remain however; the insertion of a theoretical mixing box at a cross-section of the tail-race for large volumes of fluid would present an immense practical problem, and considering the size of the mixing box required, it would be practically impossible to introduce such a measuring device. Even for relatively small flows, the physical and practical problems are complex, so much so that Professor Hsu points out that the mixing box temperature is not determined by actual measurement as the definition might imply, but by calculation from the temperature and velocity profile across the section, and is usually evaluated by the graphical integration of equation 5.2A:4.

The above discussions show that the measurement of mean temperature either by mixing box or by velocity and temperature probes is not feasible during thermodynamic tests. What is required in essence is a device for providing, through sampling and mixing, an isothermal volume in which the average temperature of the fluid can be readily measured. Such a device is described in detail in article 5.2. A relationship shall now be established between the reference mean temperature and that obtained from the mixing chamber used.

It will be assumed that the cross-section of the tail-race is divided into a number of imaginary areas. Each area shall be defined as a control area, and the volume passing through this area per unit time shall be referred to as the control volume.

If a is the control area, v the velocity of the fluid passing through the area, q(= va) the control volume, T the mean temperature of the control volume and c_p the mean specific heat of the fluid, we can write that,

the heat flux of control volume = $q\ c_p\ T$.

Therefore the total heat flux over the whole cross section of tail-race

$$= c_p\ (q_1\ T_1 + q_2\ T_2 + q_3\ T_3 + \ldots\ldots\ldots)$$

$$= \sum c_p\ q\ T \qquad \ldots\ldots\ldots\ldots\ldots\ 5.2A:6.$$

The total heat flux can also be expressed as

$$= Q\ c_p\ T_m \qquad \ldots\ldots\ldots\ldots\ldots\ 5.2A:7$$

where Q is total volume flowing per unit time

T_m is reference mean temperature of fluid.

Hence from 5.2A:6 and 5.2A:7, we can write

$$T_m = \frac{c_p\ \sum q\ T}{c_p\ Q} = \frac{\sum q\ T}{Q} \qquad \ldots\ldots\ 5.2A:8a.$$

or
$$T_m = \frac{\sum q\ T}{\sum q} \qquad \ldots\ldots\ldots\ldots\ldots\ 5.2A:8b$$

It should be noted that the above equation is simply another form of equation 5.2A:4 which defines reference mean temperature.

To calculate the mean temperature given by the mixing chamber, it will be assumed that each control area is provided with a sampling tube which leads to the mixing chamber and the sampling tube runs parallel to the direction of fluid flow as shown in Fig. 33 where A B C D is the control area.

If a' is the area, ℓ the length of the sampling tube, v' the velocity of fluid inside the tube and q' the volume through the tube flowing per unit time, we can obtain by applying Bernoulli's equation to the entrance and exit of the sampling tube, that

$$\frac{v^2}{2g} + i_w \ell = \frac{v'^2}{2g} + i_p \ell + \frac{(v - v')^2}{2g} \text{ ft lbs/lb} \quad \ldots.5.2A:9$$

where i_w and i_p are hydraulic gradients of tail race channel and sampling tube respectively, and

$$i_w = \frac{fc}{m} \cdot \frac{v^2}{2g} , \quad i_p = \frac{4 f \ell v'^2}{2 gd}$$

where fc is the friction factor for channel

m is the hydraulic mean depth for channel

f is the friction factor for tube

d is the diameter of tube.

If we write

$$i_w \ell = c \frac{v^2}{2g} \quad \text{where} \quad c = \frac{fc \, \ell}{m}$$

and $$i_p \ell = P \frac{v'^2}{2g} \quad \text{where} \quad P = \frac{4 f \ell}{d}$$

we can write equation 5.2A:9, as

$$\frac{v^2}{2g} + c \frac{v^2}{2g} = \frac{v'^2}{2g} + P \frac{v'^2}{2g} + \frac{(v - v')^2}{2g}$$

$$\text{or} \quad C\,\frac{v^2}{2} = (2 + P)\,\frac{v'^2}{2} - v\,v' \qquad \text{............ 5.2A:10}$$

$$\text{Let } v' = k\,v \qquad \text{......................... 5.2A:11}$$

then we can write 5.2A:10 as

$$k^2\,(2 + P) - 2\,k - c = 0$$

$$\text{or} \quad k = \frac{1 \pm \sqrt{1 + c(2 + P)}}{(2 + P)} \qquad \text{........... 5.2A:12}$$

Equation 5.2A:12 gives us the ratio of the velocity of sampling volume and of the control volume, which depends on pipe constant P and channel constant c.

An experiment in the laboratory was carried out to verify the value of k obtained from the equation 5.2A:12. A ten ft. length of $1\frac{1}{8}$" dia. tube was laid in a channel 4" wide, and the velocity ratio was calculated by the actual measurement of velocity before the entrance and at the exit of the tube. Readings were observed for three different rates of flows and they are tabulated in the following table.

Table 20. Experimental determination of k

Rate of flow in channel in ft^3/sec.	v the velocity in channel in ft/sec.	v' the velocity at the exit of pipe in ft/sec.	$k = \frac{v'}{v}$	k from equation 5.2A:12
0.198	1.14	0.78	0.685	0.644
0.247	1.26	0.85	0.675	0.656
0.305	1.35	0.925	0.685	0.650
		MEAN	0.68	0.65

In conclusion it can be said that theoretical and experimental results are in good agreement.

It is known that for water flowing in tubes, since no external work is done, the energy lost in friction will partly reheat the water causing an increase in internal energy at the outlet end of the pipe, which is theoretically equal to the loss of energy due to friction. The energy balance for fluid flowing through a pipe is given in equation 2.2:2, but there it was assumed that the exchange of heat from the surroundings was negligible. In practice however, some heat will be lost by the sampling volume to the control volume in the course of flow. Therefore in calculation of the temperature of the sampling volume the heat exchange term will be added to the energy equation. It is to be remembered that the theory of the thermodynamic method assumes that the heat exchange between the system itself and its surroundings is negligible (2.3).

An energy balance between a point in the control volume immediately before entering the sampling tube and a point at the inlet of sampling tube (Fig. 33), enables us to write,

$$H + \frac{v^2}{2gJ} = H_1 + \frac{v'^2}{2gJ} + \frac{(v - v')^2}{2gJ}$$

where H is the sum of internal energy and flow work and is known as enthalpy of fluid.
By writing $v' = kv$ from 5.2A:1, we can re-write the above equation after simplifying and rearranging,

$$H_1 = H + \frac{2k(1 - k)}{J} \cdot \frac{v^2}{2g}$$

Since variations of specific volume and pressure are very small, we can write enthalpy $H = c_p T$, where T is the absolute temperature. The above equation then transforms to,

$$T_i = T + \frac{2k(1 - k)}{J\ c_p} \cdot \frac{v^2}{2g} \qquad \ldots\ldots\ldots\ 5.2A:13$$

The fluid during the course of flow through the sampling tube will be re-heated due to friction and will also be cooled by the fluid flowing outside the tube surface. The energy balance for the inlet and outlet of the tube can be expressed as,

$$H_i + \frac{v'^2}{2g} = H' + \frac{v'^2}{2gJ} - \frac{Pv'^2}{2gJ} + q_h$$

where q_h is the amount of heat transferred from the sampling volume to the control volume per unit time.

If the enthalpy H is written as $c_p T$ and the value of T_i is substituted in the above equation from 5.2A:13, then by rearranging we can write,

$$T' = T + \left[\frac{2k(1 - k) + Pk^2}{J\ c_p} \right] \cdot \frac{v^2}{2g} - \frac{q_h}{c_p} \qquad \ldots\ldots\ 5.2A:14$$

To calculate q_h it shall be assumed that the sampling tube lies parallel to and within the path of the control volume and the effect of other sections on a control volume is negligible. In an arbitrary length of the tube let fluid be flowing at a rate of

w_o lb per unit time, outside the tube

w_i " " " " inside " "

If T_i and T' are the bulk temperatures at inlet and outlet for the fluid flowing inside of the tube and T and T_o are the bulk temperatures at inlet and outlet for the fluid flowing outside of the tube, by following the usual procedure of thermodynamics of heat transfer for parallel flow, the temperature difference at the end of the heating surface can be written as,

$$T' - T_o = e^{-\mu A_1 Y} (T_i - T) \quad \ldots\ldots\ldots \; 5.2A{:}15a$$

where $\mu = \frac{1}{c_p} \left(\frac{1}{w_i} + \frac{1}{w_o} \right)$

$A_1 = \pi d \ell$, where d is diameter and ℓ the length of the tube

Y is overall heat transfer coefficient.

Let the ratio of the sampling area a' and the control area a be expressed as,

$$\frac{a'}{a} = c'$$

then it follows that the ratio of sam pling volume and control volume can be written as

$$\frac{q'}{q} = \frac{a'v'}{av} = c'k = m \text{ (say)}$$

or $q' = m\, q$ 5.2A:15b

If w is the specific wt of the fluid flowing through the cross-section, we can write

$$w_i = wq' = w\, m\, q \quad \ldots\ldots\ldots\ldots \; 5.2A{:}15b2$$

$$w_o = wq \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; 5.2A{:}15b3$$

and hence $\mu = \frac{1}{w\, c_p\, q} \left(\frac{m+1}{m} \right)$ 5.2A:15c

Let n be the number of sampling points, and A be the area of cross-section of the tail race, the control area can then be written as

$$a = \frac{A}{n}, \quad \text{and we can express}$$

$$m = k\frac{a'}{a} = \frac{n}{\frac{A}{a'k}}$$

$$\text{or} \quad m = \frac{n}{b} \qquad \dots\dots\dots\dots\dots\dots \quad 5.2A{:}15d$$

$$\text{where} \quad b = \frac{A}{a'k} = \frac{4A}{\pi d^2 k}$$

From 5.2A:15c and 5.2A:15d we obtain that

$$\mu = \frac{n + b}{w\ c_p\ nq} .$$

Without causing any appreciable error in heat transfer calculation we can write that

$$Q \doteq nq,$$

therefore

$$\mu = \frac{n + b}{w\ c_p\ Q} \qquad \dots\dots\dots\dots \quad 5.2A{:}15e$$

The overall heat transfer coefficient Y is the sum of the quotient of thermal conductivity and film coefficient at the inside and outside of the tube. To calculate the film coefficient at the outside of the tube we shall assume that the tube behaves as a thin flat plate - this is justifiable as the total surface area of the control volume is considerably larger than the surface area of the tube. The inside film coefficient will be determined in the usual manner for flow in circular tubes.

If h_1 is the film heat transfer coefficient for inside of the tube, it can be calculated by the formula

$$h_1 = Nu \cdot \frac{k'}{\ell}$$

where Nu is Nusselt number
k' " thermal conductivity of the fluid
ℓ " the length of the tube.

Nusselt number can be calculated by the empirical equation given by Colburn for turbulent flow heat transfer in circular tubes for moderate temperature differences, and is expressed as

$$Nu = \frac{f}{2} \, Re \, (P_r)^{1/3} \; ;$$

hence we can write

$$h_1 = \frac{k'}{\ell} \cdot \frac{f}{2} \, Re.(P_r)^{1/3} \; .$$

Similarly if h_2 is the film heat transfer coefficient for outside of the tube and we calculate Nusselt number from Colburn's equation for turbulent heat transfer over a flat plate, we obtain

$$h_2 = \frac{k'}{\ell} \times 0.0365 \times Re^{4/5} \cdot P_r^{1/3} \; .$$

If the thickness of the tube wall is given by t and its thermal conductivity by k_w, the overall heat transfer coefficient Y can be expressed as

$$\frac{1}{Y} = \frac{1}{h_1} + \frac{t}{k_w} + \frac{1}{h_2} \quad \ldots\ldots\ldots \; 5.2A:15f$$

Therefore from equation 5.2A:15e and 5.2A:15f we can write that

$$e^{-\mu A_1 Y} = e^{-\frac{n+b}{wc_pQ}\frac{\pi d\ell}{\left(\frac{1}{h_1}+\frac{t}{k_w}+\frac{1}{h_2}\right)}}$$

or $$e^{-\mu A_1 Y} = e^{-F(n+b)} = B \text{ (say)} \quad \ldots\ldots\ldots \quad 5.2A:15e$$

where $$F = \frac{\pi d\ell}{wc_pQ\left(\frac{1}{h_1}+\frac{t}{k_w}+\frac{1}{h_2}\right)} .$$

And the temperature difference equation 5.2A:15a can be expressed in a simpler form as

$$T' - T_o = B\,(T_i - T) \quad \ldots\ldots\ldots\ldots\ldots \quad 5.2A:16$$

Assuming that there are no heat losses, the heat delivery of the sampling water must be equal to the heat absorbed by the water flowing in the control tube. The heat balance then gives that

$$w_i\, c_p\, (T_i - T') = w_o\, c_p\, (T_o - T) .$$

Substituting the value of w_i and w_o from 5.2A:15b2 and 5.2A:15b3 and re-arranging we obtain

$$T_o - T = m\,(T_i - T') \quad \ldots\ldots\ldots\ldots\ldots \quad 5.2A:17$$

It can be shown by adding equations 5.2A:16 and 5.2A:17 and by re-arranging that

$$T' = T\,\frac{1-B}{1+m} + T_i\,\frac{B+m}{1+m} \quad \ldots\ldots\ldots\ldots \quad 5.2A:18$$

Since the heat transferred from the sampling volume per lb of flow is given by

$$q_h = c_p\,(T_i - T')$$

we can show on substituting the value of T' from 5.2A:18 in the above equations, and T_i from 5.2A:13 that

$$\frac{q_h}{c_p} = \frac{1 - B}{1 + m} \quad \frac{2(1 - k)}{J\ c_p} \cdot \frac{v^2}{2g} \qquad \ldots\ldots\ 5.2A{:}19$$

The above equation 5.2A:19 enables us to re-write the equation 5.2A:14, giving the relationship between the temperature T of the control volume and temperature T' of the sampling volume at exit of the sampling tube, as

$$T' = T + \left[\frac{2k(1-k)}{J\ c_p} \cdot \frac{m+B}{1-m} + \frac{Pk^2}{J\ c_p} \right] \cdot \frac{v^2}{2g} \ .$$

Substituting the value of m and B from 5.2A:15d and 5.2A:15e respectively, we obtain,

$$T' = T + \left[\frac{2k(1-k)}{J\ c_p} \cdot \frac{n+be^{-F(n+b)}}{n + b} + \frac{Pk^2}{J\ c_p} \right] \cdot \frac{v^2}{2g}$$

$$\text{or} \qquad T' = T + \left[c_1 \cdot \frac{n+be^{-F(n+b)}}{n + b} + c_2 \right] \cdot \frac{v^2}{2g} \ \ldots\ldots\ 5.2A{:}20$$

$$\text{where} \quad c_1 = \frac{2k(1-k)}{J\ c_p} \quad \text{and} \quad c_2 = \frac{Pk^2}{J\ c_p} \ .$$

Water after flowing through the sampling tube enters the mixing chamber, therefore the heat flux entering the mixing chamber through each sampling tube per unit time can be written as

$$\text{heat flux through each tube} = c_p\ q'\ T' \ .$$

Hence total heat flux entering the mixing chamber

$$= c_p (q_1' T_1' + q_2' T_2' + \ldots)$$

$$= c_p \sum q' T'$$

If the total volume leaving the mixing chamber is Q', then,

$$Q' = q_1' + q_2' + \ldots\ldots$$

$$\text{or} \quad Q' = \sum q'$$

Let T_m' be the temperature measured in the mixing chamber, then the total heat flux leaving the mixing chamber can be expressed in terms of T_m' and Q', i.e.

$$\text{total heat flux leaving the mixing chamber} = c_p\, Q'\, T_m' \;.$$

Assuming that the heat exchange in the mixing chamber itself is negligible because of small surface area, we can state that the heat flux entering the mixing chamber must be equal to the heat flux leaving the mixing chamber, hence it follows that

$$T_m' = \frac{c_p \sum q'T'}{c_p\, Q'} = \frac{\sum q'T'}{q'} \;.$$

From equation 5.2A:15b we get $q' = mq$, therefore

$$T_m' = \frac{m \sum qT'}{m\; q} = \frac{\sum qT'}{\sum q}$$

Writing the value of T' from equation 5.2A:20, we obtain,

$$T_m' = \frac{\sum qT}{\sum q} + \left[c_1 \cdot \frac{n + be^{-F(n+b)}}{n+b} + c_2 \right] \frac{\sum (q \cdot \frac{v^2}{2g})}{\sum q} \;.$$

It has been shown in equation 5.2A:8 that the energy mean temperature $T_m = \frac{q\,T}{q}$, therefore

$$T_m' = T_m + \left[c_1 \cdot \frac{n + be^{-F(n+b)}}{n+b} + c_2 \right] \cdot \frac{\sum (q \cdot \frac{v^2}{2g})}{\sum q} \;.$$

At high Reynolds number, on the basis of Streeter's calculation of kinetic energy correction factor, we can write without causing any appreciable error that

$$\frac{\sum (q \cdot \frac{v^2}{2g})}{\sum q} = \frac{V_m^{\,2}}{2g}$$

where V_m is the mean velocity in the tail race.

Hence the relationship between the measured mean temperature T_m' and the reference energy mean temperature T_m can be expressed as,

$$T_m = T_m' - \left[c_1 \cdot \frac{n + be^{-F(n+b)}}{n + b} + c_2 \right] \cdot \frac{V_m^{\,2}}{2g} \quad \ldots\ldots \ 5.2A:21$$

where

T_m = Mean energy temperature in deg. R

T_m' = Temperature measured inside the mixing chamber in deg. R

V_m = Mean velocity at measuring section of tail race in ft/sec.

n = number of sampling points

b = $\frac{4A}{\pi d^2 k}$

A = Cross-sectional area of tail-race in ft^2.

k = $\frac{1 \pm \sqrt{1 + c(2 + P)}}{2 + P}$

c = $\frac{f_c \ell}{m'}$

f_c = frictional factor for tail race channel.

m' = hydraulic mean depth of tail race cross-section in ft.

ℓ = length of sampling tube in ft.

P = $\frac{4 f \ell}{d}$

f = frictional factor for sampling tube.

$c_1 = \frac{2k\,(1 - k)}{J\ c_p}$

J = Mechanical equivalent of heat in B.Th.U.

c_p = Specific heat of fluid in BTU/lb. deg.F

$c_2 = \frac{P\ k^2}{J\ c_p}$

$$F = \frac{\pi\, d\, \ell}{3600\ w_s\ c_p\ Q\left(\frac{1}{h_1} + \frac{t}{k_w} + \frac{1}{h_2}\right)}$$

w_s = Specific weight of fluid at S.T.P. in lb/ft^3

Q = $v_m A$ = Volume rate of flow in tail race in ft^3/sec.

t = Thickness of sampling tube wall in ft.

k_w = Thermal conductivity of the tube wall in BTU/ft.hr. deg.F.

$h_1 = \frac{k'}{d} \cdot \frac{f}{2}\ Re\ (P_r)^{1/3}$

k' = Thermal conductivity of fluid in BTU/ft.hr.deg.F.

$h_2 = \frac{k'}{\ell} \times 0.0365 \times Re^{4/5} \cdot P_r^{1/3}$

Re = Reynolds number = $\frac{k\ v_m\ d}{\nu}$ (for h_1)

and = $\frac{k\ v_m\, \ell}{\nu}$ (for h_2)

ν = Kinematic viscosity of fluid in ft^2/sec.

Pr = Prandtl number = $\frac{c_p \mu}{k}$

μ = Viscosity of fluid in $lb.sec/ft^2$.

The above equation 5.2A:21 enables us to determine the mean energy temperature T_m by making a single observation of temperature T_m' inside the mixing chamber and by subsequent application of a correction term.

To bring the equation to a standard form we write c_f as correction factor where

$$c_f = c_1 . \frac{n + be^{-F(n+b)}}{n + b} + c_2 \quad \ldots\ldots\ldots \; 5.2A:22.$$

and hence equation 5.2A:21 reduces to

$$T_m = T_m' - c_f \frac{V_m^2}{2g} \quad \ldots\ldots\ldots\ldots\ldots \; 5.2A:23$$

It is of interest to note that the above equation is similar to the equation used in aerodynamics to represent the relationship between the static temperature and the recovery temperature for fluids flowing at high velocities.

Examination of equation 5.2A:23 shows that the ideal conditions of $T_m = T_m'$ can never be achieved, therefore the mixing chamber should be designed in such a way that the correction factor is as small as possible in order to reduce the magnitude of error. The correction factor depends on many variables, such as length, diameter, roughness number etc. of the sampling tube. Nevertheless, it can most conveniently be decreased by increasing the number of sampling tubes. However the number cannot be increased indefinitely. By differentiating c_f with respect to n (equation 5.2A:22) and equating it to zero we can find the required value of n which will determine the turning point of c_f, after which the correction term will start increasing with increase in value of number of sampling tubes. Such a calculation was carried out for test no. 10 (of Table 15) and it was found that the optimum value of n was nearly 150. It is obvious that to incorporate such a large number of sampling tubes will not be a practical proposition. It can therefore be concluded that the number of sampling tubes should be as large as practical limitations will allow.

Another pertinent question can be asked whether equal sampling is desirable or even possible in the design of a mixing chamber. To obtain equal sampling, the velocity at each sampling station must be known, and then the sampling area can be designed in that ratio. Obviously it will be practically impossible, as the velocity in the tail race changes with variation of load on the machine. Furthermore this procedure is undesirable, because according to the definition of energy mean temperature, the value of heat flux of a certain volume per unit time at a particular section, is what we require, and this is obtainable in the constant area sampling.

It is evident from equation 5.2A:23 that the correction term $c_f \frac{V_m^2}{2g}$ will increase with increase in V_m. Therefore the correction term will be relatively of less importance for low head machines where the velocity in the tail-race is of a lower order. For the present investigation, which is concerned with low head machines, the value of c_f was calculated for test no. 10 (Table 13), during which maximum velocity occurred and it was found to be less than 10^{-3} deg.F per ft. head. It follows that the value of the correction term will be less than 0.2×10^{-3} deg.F, (since $\frac{V_m^2}{2g} = 0.24$) or roughly 10^{-4} deg.C, which will cause an error in efficiency of less than 0.08%. It can therefore be safely assumed that for low and medium head machines, the temperature observed in the mixing chamber will represent the true energy mean temperature of the fluid flowing through the tail race.

Eleven different tests were successfully performed to determine the effect of measurement of mean temperature by mixing chamber. The results of these experiments have been shown in tables 7, 8, 11a, 11b, 11c and figures 21 and 22, and have been discussed in article 5.2 and 8. The difference between the readings of mean temperature observed by traverse and that obtained by the mixing chamber has been caused because of the assumption that the arithmetic mean temperature is equal to the energy mean temperature. A true comparison can only be made if the energy mean temperature is measured by graphical integration of velocity and temperature profiles obtained by measuring probes. All investigators obtained temperature traverses and calculated the mean temperature by assuming that the velocity remained constant throughout the section. Unfortunately the true comparison could not be determined during the present investigation due to shortage of time, and the fact that such procedure of measurement requires a full scale investigation of its own.

6.1A Matching of Thermometers.

When the two platinum gauges are at the same temperature, theoretically the bridge must remain in equilibrium whatever the value of temperature may be. This however, is not obtainable in practice. In spite of manufacturers' claims that the gauges are made of identical elements, the fundamental interval remains different for each element which is more noticeable when used for the measurement of small temperature differences. Therefore it is necessary to test the platinum gauges in pairs over the working range of temperature (0 deg.C - 20 deg.C); and that pair should be chosen which gives minimum deflection change with change in temperature. Table 21 and Fig. 34 show a test carried out on certain pairs of thermometers. Since we required three thermometers, (T_1), (T_2) and (T_3) to give us two pairs (T_1) and (T_2), and (T_2) and (T_3), thermometers 2 and 7 and 2 and 9 were therefore chosen. These appear to be the best combination available as shown by Fig. 34.

Willm and Campmas have theoretically analysed a particular method of compensating the effect of temperature variation and have concluded that the procedure outlined by them eliminates any error due to variation of temperature. The method includes a particular type of bridge which incorporates two potentiometers inserted between the junctions of two different arms of the bridge network. In essence they have provided a design of a bridge which remains in equilibrium, whatever the value of temperature sensed by the gauges may be. However, they have not published any experimental data to support this analysis.

An assessment of, or comparison with Willm's method was not possible as the bridge used during the investigation was a Smith type bridge which does not incorporate two potentiometers at the junctions of the two arms of the bridge. Furthermore, Willm's method is restricted to one pair of gauges with their wires, therefore it would not be possible at all to use Willm's method for the adiabatic expander which requires three thermometers simultaneously for measurement.

8.0A. Tests at Bonnington Dec. '60 and April '61.

It has been stated in article 8.4 that because of the low flow of 0.04 l/s in the outer chamber, it was not possible to make the temperature of the outer jacket equal to that of tail race. However, the deflection due to the inequality of temperature was noted and later a correction was applied. Such a series of tests is given in Table 22 for six different loads. The values of efficiencies obtained after correction, cannot be taken as accurate, because the temperature of the flowing water was changing and the average drop in temperature was observed to be 0.4 deg. C/hr. The method of correction enables us however to calculate values of efficiencies, but this procedure is undesirable and the results obtained are not accurate.

To determine the effect of different rates of flow through the jacket on the values of efficiency, another series of tests was carried out for four different loads each at four rates of flow. The observations are shown in Table 23. In Fig. 35, the efficiencies obtained are plotted against flow time ($= \frac{1}{q}$, where q is the rate of flow) to show the effect of increasing the rate of flow in the jacket.

It is apparent that the most desirable condition for determining the efficiency will be when the flow time is zero, or in other words when the flow is infinitely large which eliminates the heat transfer effect - but to achieve this theoretical condition will not be possible. Examination of Fig. 35 however shows that the values of efficiencies do not appreciably change between the flow time of 2.5 sec/litre and 2.0 s/l and it can be inferred from the shape of the four curves that there will be almost no change in the value of efficiency when the flow time is further decreased. It can therefore be concluded that flow time of 2.5 sec/litre or a flow of 0.4 litre/sec. will be the most suitable value. The flow time of 2.0 s/l no doubt will give slightly more accurate values of efficiency, but at this flow time it will not be possible to determine the values of efficiency at high efficiency points because of larger pressure drop between the valves. It is therefore not justifiable to use a flow of 0.5 litres/sec. instead of 0.4 l/s for the given pair of valves. It could however be possible to use higher rates of flow with better designed valves, such as described in article 5.42.

It should be noted that these experiments were carried out for moderate temperature differences between the surrounding air and the flowing water. For large differences of temperature, either the flow rate must be increased or, if this is not possible, the test should be postponed till some suitable time.

Another series of efficiency tests was carried out for fifteen different loads. The observed values are given in Table 24 and plotted in Fig. 36. The results obtained are scattered and their accuracy is unreliable. This could be explained as the effect of instability of the temperature of the flowing medium caused by the formation of ice. The average fall of temperature during the test was recorded as 0.36 deg. C/hr. According to Willm, in such cases the tests should be abandoned as the efficiency equation was derived on the assumption that the temperature of the flowing medium was constant. The efficiency tests were therefore discontinued and were not reported in the main thesis.

Tests at Bonnington April '61.

In article 8.63 it was claimed that, where the conventional method failed to determine the efficiency readings due to the influx of a great amount of weed in the pipe line brought over by flood, the thermodynamic method enabled us to calculate efficiencies in such adverse circumstances. The values obtained under such circumstances are given in Table 25 measured over two consecutive days. These readings are plotted in Fig. 37 and are represented by plain circles. For comparison the values of efficiencies obtained on two previous days (Table 15) are also plotted on the same graph shown by dots. The two results are in very good agreement and the scatter is less than ± 1%. In conclusion it can be said that the thermodynamic method has definite advantages over conventional methods.

AI/AII.A **Determination of α and β**

The derivation of the formulas for α and β are given in Appendix I and Appendix II. Examination of equation AI:5 shows that calculation of α and β basically requires the determination of variation of specific volume of water with temperature and pressure. Several investigators have published the P, v, T (Pressure, volume and temperature) data for pure water for different pressures and temperatures. Most of them have taken large intervals of pressure or temperature; since the calculation of α and β involves determination of $\frac{\partial v}{\partial t}$ and $\frac{\partial v}{\partial p}$, it is therefore necessary to know the values of the specific volume of water at small intervals of temperature and pressure.

Willm used Amagat's P, v, T data given at intervals of 25 atm pressure from 1 atm to 200 atm and 5 deg.C. interval of temperature from 0 deg.C. to 20 deg.C. The values of α and β obtained by Willm and Campmas derived from Amagat's data are shown in Fig. 25. Smith and Keyes have provided an empirical equation as a satisfactory representation of their observations of specific volume of water. The advantages of this type of representation are various but the most important is that small intervals of pressure and temperature increase the accuracy of the calculation for isopiestic thermal expansion of water $(\frac{\partial v}{\partial t})$ and isothermal compression $(\frac{\partial v}{\partial p})$ of water. A comparison of Amagat's data and those obtained by Smith and Keyes is given in Table 16, in the form of $\Delta = (1 - v)10^4$, where v is the specific volume. In the analysis, Amagat's observations were taken from the tables published by Dorsey and the data of Smith and Keyes were calculated from the empirical equation given in the original paper of Smith and Keyes. Table 16 includes the values of specific volume for the temperature range of 0°C. to 50°C. and for the pressure range of 1 atm to 200 atm. Comparison shows that the maximum difference in two observed values occurs at 0 deg.C. and 200 atm. pressure and is of the order of 13 parts in 10,000. This difference diminishes gradually with increase in temperature, so much so that at 50°C. and at all pressures the difference is never greater than 2 parts in 10,000. It can be said that the values are in good agreement.

α and β were calculated using Smith and Keyes data. The values of $(1 - \alpha)$ and $(1 - \beta)$ were plotted against pressure for various temperatures and are shown in Fig. 38 and Fig. 40 respectively. The summary of the method of calculation employed is given below.

Method of Calculation:

(1) Specific volume of water was calculated from the empirical equation set up by Smith and Keyes.

(2) Isothermal compression of water $(-\frac{\partial v}{\partial p})$ was calculated by directly differentiating the empirical equation relating specific volume and pressure. (Given by Smith and Keyes).

(3) Isopiestic thermal expansion of water $(-\frac{\partial v}{\partial t})$ was calculated by means of central difference interpolation formula using 1 deg.C. interval of temperature for five values of specific volume.

(4) The integrals of the functions were calculated by using Simpson's rule employing eleven ordinates for every pressure range.

(5) The value of v_s was taken at 4 deg.C. at 1 atm. and it was equal to 1.0002 cc/gm.

(6) All computations were performed by SiriUs computer and the results were printed up to five significant figures.

The results plotted in Fig. 38 show that the rate of change of $(1 - \alpha)$ with pressure remains constant at all temperatures and hence it can almost be represented by straight lines of the same gradient. The values of $(1 - \alpha)$ were calculated at 1 deg.C. interval between 5 deg.C. and 20 deg.C. which was the working range of the present investigation; however it is possible to calculate the value of $(1 - \alpha)$ at any temperature and pressure by means of the method outlined above. For comparison the values of $(1 - \alpha)$ were compiled from Willm's data (Fig. 25) and plotted on the same scale as Fig. 38 and are shown in Fig. 39. Within the working range at a temperature of 5 deg.C. the maximum difference in the values of $(1 - \alpha)$ was observed; this was equivalent to 0.5% of efficiency. This variation however is not insignificant, in spite of the fact that the difference in the values of specific volume as shown by Table 26 is not greater than 12 parts in 10,000. This difference in the values of $(1 - \alpha)$ can be explained by the difference in the methods of calculation. Unfortunately Willm and Campmas do not furnish the method of their calculation and therefore no definite conclusions can be drawn. However, since they have used Amagat's data they most likely have used mean values of isiopiestic thermal expansion and isothermal compression instead of accurately calculating $\frac{\partial v}{\partial t}$ and $\frac{\partial v}{\partial p}$ at each state point. To elucidate this argument two values of $(1 - \alpha)$, one at 10 deg.C. and the other at 15 deg.C. between 1 atm and 25 atm, were calculated by using mean values of isiopiestic thermal expansion and isothermal compression, but still using P,v,T data of Smith and Keyes. The values obtained differed from that of Fig. 38, equivalent to 0.4% of efficiency - this explains the difference in the values of the thermodynamic coefficients obtained compared to that of Willm.

These values of $(1 - \alpha)$ which have been calculated from the P,v,T data given by various investigators are known as the static values, and in view of the fact that the results are not in very close agreement, it shall be useful to determine the dynamic values of the thermodynamic coefficients α and β. In conclusion it can be said that the values of Fig. 38 can be taken as more reliable than those obtained by Willm (Fig. 25 and Fig. 39), because his values are based on the approximation of $\frac{\partial v}{\partial t}$ and $\frac{\partial v}{\partial p}$. The values of Fig. 38 however can not be taken as accurate, because the empirical equation set up by Smith and Keyes is itself an approximation in strict sense of the term. Values of $(1 - \alpha)$ obtained experimentally can be taken as really the only accurate values, which unfortunately has not been determined yet. Brand has mentioned in his paper that the data of Willm and Campmas was checked experimentally and was in good agreement; but he has neither published his experimental results nor has he given any details about the experiment.

It is therefore difficult to deduce any conclusions from Brand's findings.

The values of $(1 - \beta)$ compiled from Fig. 25 were also plotted on the same scale as Fig. 40 and are shown in Fig. 41. Like $(1 - \alpha)$ these values also differ from each other, but the variation expressed as a percentage of efficiency is nearly .05% which can be taken as negligible. It can be concluded that variation of $(1 - \beta)$ due to different values of specific volumes observed is insignificant.

Effect of impurities on α and β

The values of α and β given in Fig. 38 and Fig. 40 are only for pure water. These values will be affected by presence of impurities, because impurities change the specific volume of water which in turn will change the value of the thermodynamic coefficients. The most common cause of any significant change in the specific volume of water is the presence of salt. To determine the effect of salt content, the values of α and β were calculated for water containing 3.1% salt (by wt.). The results obtained are plotted as $(1 - \alpha)$ and $(1 - \beta)$ against pressure in Fig. 42 and Fig. 43 respectively. Also the values of α and β were calculated for water containing 3.9% salt (by wt.). The results are plotted in Fig. 43 and Fig. 44. (The values of specific volumes were taken from the Dorsey's book).

The results show that within the working range of temperature of water (5 - 20 deg.C.), one per cent (by wt.) of salt in water will change the value of $(1 - \alpha)$ sufficiently to give 1.2% change in efficiency, and this change in efficiency reduces with the rise of temperature of water.

The value of $(1 - \alpha)$ for sea water can be interpolated by means of Fig. 42 and Fig. 44 - sea water usually contains 3.5% salt (by wt.). The values of $(1 - \beta)$ also changes with change in concentration of salt but the effect of change on efficiency is almost negligible.

In conclusion it can be said that a correction factor should be applied if there are reasons to believe that the salt concentration is higher than the average for river water. It is to be remembered that the values of thermodynamic coefficients given in Fig. 42, Fig. 43 and Fig. 44 are purely hypothetical, and these conditions may never arise in industrial applications.

Dissolved air increases the specific volume of water, and hence it will affect the values of α and β. However the maximum change observed in specific volume due to dissolved air as reported by Emeleus is 2 parts in 10^6, which is very small. Its effect on thermodynamic coefficients will be insignificant. It can therefore be concluded that the value of α and β does not change with saturation of water with air.

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TABLE 21

Deflection for pair of thermometers with change of temperature.

(See Fig. 34)

PAIRS	TEM' IN DEG.C.	DEF' IN MM.	TEM' IN DEG.C.	DEF' IN MM.	TEM' IN DEG.C.	DEF' IN MM.	TEM' IN DEG.C.	DEF. IN MM.	REMARKS
2 & 1	2.4	4.4	6.3	4.25	13.5	3.97	20.1	3.73	
2 & 3	1.5	4.3	7.6	4.1	13.6	3.85	19.8	3.63	
2 & 4	3.2	4.1	8.3	4.2	14.1	4.35	20.2	4.5	
2 & 5	2.1	3.64	7.9	3.4	15.8	3.1	21.0	2.92	
2 & 6	3.0	4.08	9.2	3.75	15.1	3.45	18.9	3.21	
2 & 7	2.5	3.39	6.3	3.38	12.8	3.34	19.4	3.32	Good pair
2 & 8	2.9	3.8	8.4	3.92	14.4	4.1	19.1	4.2	
2 & 9	1.8	4.0	9.7	4.04	15.2	4.06	18.8	4.08	Good pair
2 & 10	2.3	3.2	8.8	3.05	13.4	2.95	20.4	2.78	

TABLE 22

Efficiency Observations at Bonnington, Dec.'60.

DATE	LOAD IN MW	EFF'CY OBSERVED	GALVANOMETER DEVIATION FROM ZERO IN MM. (Outer jacket)	CORRECTED EFF'CY	REMARKS
19.12.60	5.2	83.6	6	80.8	1. Temperature of water dropped from 4.5°C to 1.2°C in eight hours.
"	4.28	85.7	9	82.4	
"	3.7	84.2	8	81.3	2. Therefore average temp. drop was about 0.4 deg.C/hr.
"	3.15	78.1	6	75.7	
"	2.5	72.2	3	71.3	3. Flow in outer jacket was 0.04 litre/sec.
"	1.5	69.8	2	69.1	

TABLE 23

Efficiency variation with flow time, Bonnington, Dec.'60.

(See Fig. 35)

LOAD	Flow 'q' in outer jacket in l/s.	Flow time $= \frac{1}{q}$	EFFICIENCY	REMARKS
1.01 MW	0.2	5	56.7	1. Average difference between the temp. of surrounding air and the flowing water was about 10 deg.C.
	0.3	3.33	58.2	
	0.4	2.5	58.6	
	0.5	2.0	58.7	
2.0 MW	0.2	5.0	71.75	
	0.3	3.33	73.7	
	0.4	2.5	74.1	2. Average fall of temperature of flowing water was about 0.35 deg.C/hr.
	0.5	2.0	74.22	
3.02 MW	0.2	5.0	79.8	
	0.3	3.33	81.7	
	0.4	2.5	82.2	
	0.5	2.0	82.32	
4.04 MW	0.2	5.0	84.5	
	0.3	3.33	86.4	
	0.4	2.5	86.9	
	0.5	2.0	Unobtainable	

TABLE 24

Efficiency, Bonnington, Dec.'60
(See Fig. 36)

DATE	LOAD IN MW	EFFICIENCY	REMARKS
29.12.60	1.0	58.0	1. Average fall of temperature of flowing water was about 0.36 deg.C. per hour.
"	1.52	71.2	
"	2.05	72.0	
"	2.5	78.2	
"	2.71	79.1	2. Flow in outer jacket was 0.4 l/s.
"	3.04	84.0	
"	3.25	82.1	
"	3.5	85.0	
"	3.75	87.3	
"	4.0	86.9	
"	4.25	87.6	
"	4.5	88.5	
"	4.75	86.1	
"	5.02	87.1	
"	5.2	84.6	

TABLE 25

Efficiency, Bonnington, April 1961
(See Fig. 37)

DATE	LOAD IN MW	EFFICIENCY	REMARKS
13.4.61	0.6	59.0	(Flowing water full of weeds, later identified as Nitella)
	1.0	64.8	
	1.5	72.6	
	2.0	74.6	
	2.5	79.0	
	3.0	85.0	
14.4.61	3.5	87.4	
	4.0	88.0	
	4.5	90.7	
	5.0	89.4	
	1.0	64.4	
	0	32.3	

TABLE 26

Comparison of observed specific volume of water

$\Delta = (1 - v)\, 10^4$

$t^\circ C$ / P in atm	0 deg.C.	$5^\circ C.$	$10^\circ C.$	$15^\circ C.$	$20^\circ C.$	$30^\circ C.$	$40^\circ C.$	$50^\circ C.$	OBSERVER
1	-1.3	-0.1	-2.7	-8.7	-17.7	-43.4	-77.0	-119.5	Amagat
	-1.9	-2.0	-5.2	-11.2	-20.0	-44.9	-78.7	-120.6	Smith & Keyes
25	11.3	12.2	9.3	3.2	-5.9	x	x	x	Amagat
	9.1	8.9	5.7	-0.4	-9.2	-34.1	-67.8	-109.6	Smith & Keyes
50	24.2	24.6	21.6	15.2	6.0	-19.8	-54.5	-96.7	Amagat
	20.4	20.2	16.9	10.7	1.9	-22.9	-56.6	-98.2	Smith & Keyes
75	36.9	36.7	33.4	26.8	17.4	x	x	x	Amagat
	31.6	31.7	27.9	21.7	13.0	-11.9	-45.5	-87.0	Smith & Keyes
100	49.4	48.7	45.1	38.2	28.7	2.3	-32.2	-74.5	Amagat
	42.7	42.3	38.9	62.7	23.9	-1.0	-34.5	-75.9	Smith & Keyes
125	61.7	60.6	56.7	49.5	39.9	x	x	x	Amagat
	53.6	53.2	49.7	43.5	34.6	9.8	-23.7	-64.9	Smith & Keyes
150	73.9	72.5	68.2	60.8	51.0	24.0	-10.3	-53.0	Amagat
	64.5	64.0	60.4	54.2	45.3	20.5	-12.9	-54.0	Smith & Keyes
175	86.1	84.3	79.7	72.0	62.0	x	x	x	Amagat
	75.2	74.6	71.1	64.8	55.9	31.1	-2.3	-43.3	Smith & Keyes
200	98.2	96.0	91.1	83.1	72.9	45.5	10.8	-31.7	Amagat
	85.8	85.2	81.6	75.2	66.3	41.6	8.7	-32.6	Smith & Keyes

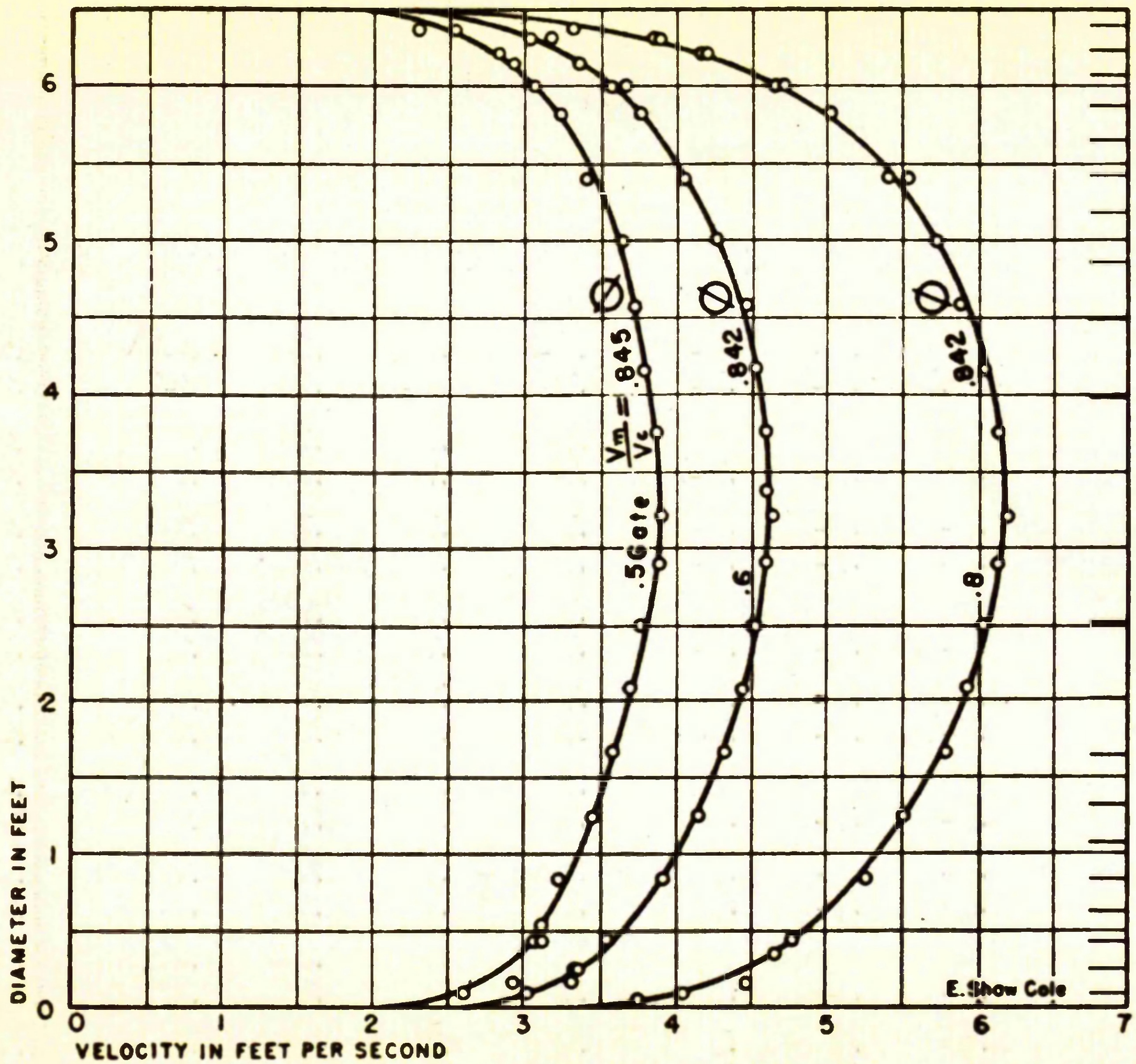

FIG. 26 VELOCITY TRAVERSES IN 78 in. WELDED STEEL PENSTOCK. (Taken from E.S.Cole.)

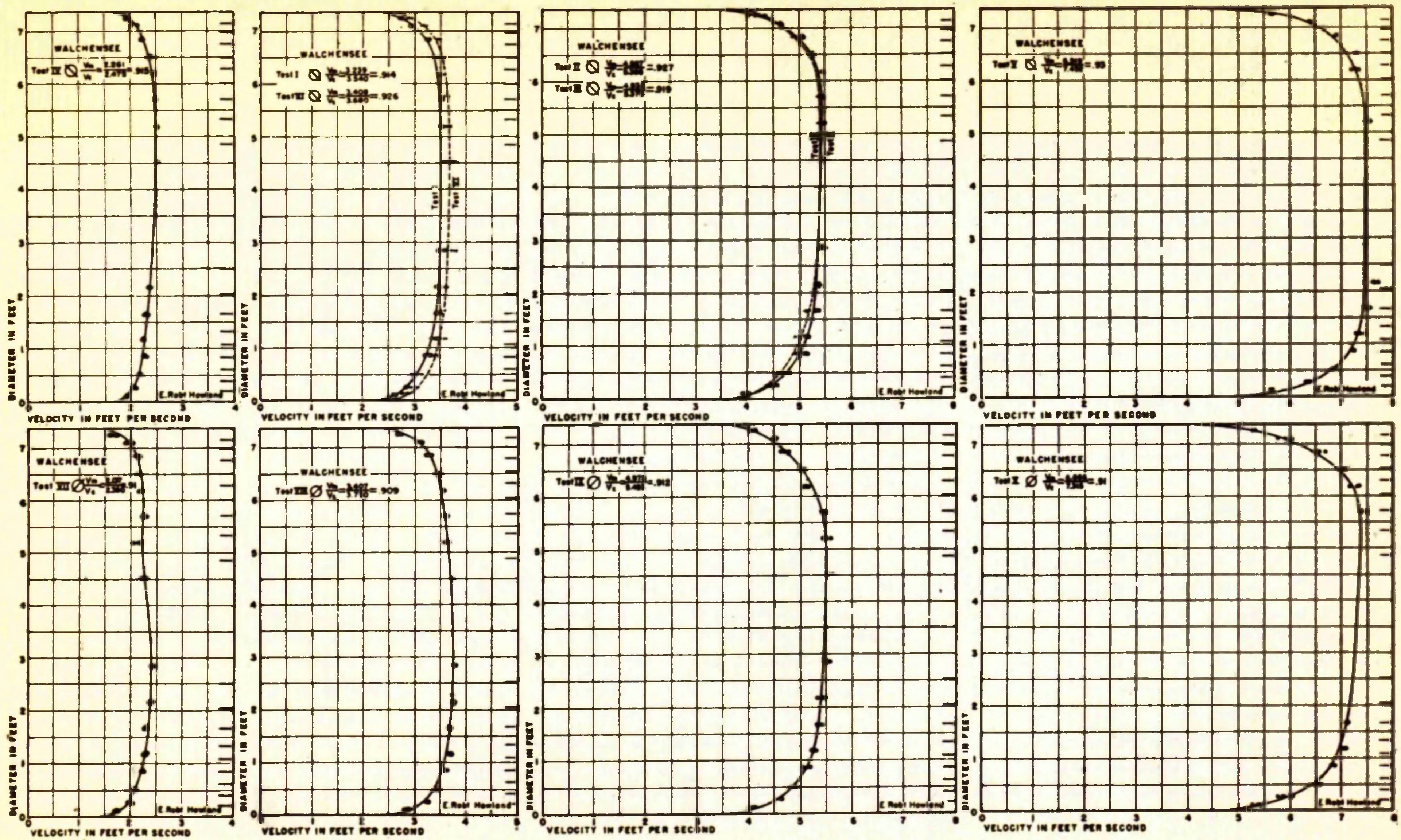

FIG.27 VELOCITY TRAVERSES IN 7.5 ft dia. PENSTOCK.(From E.S.Cole.)

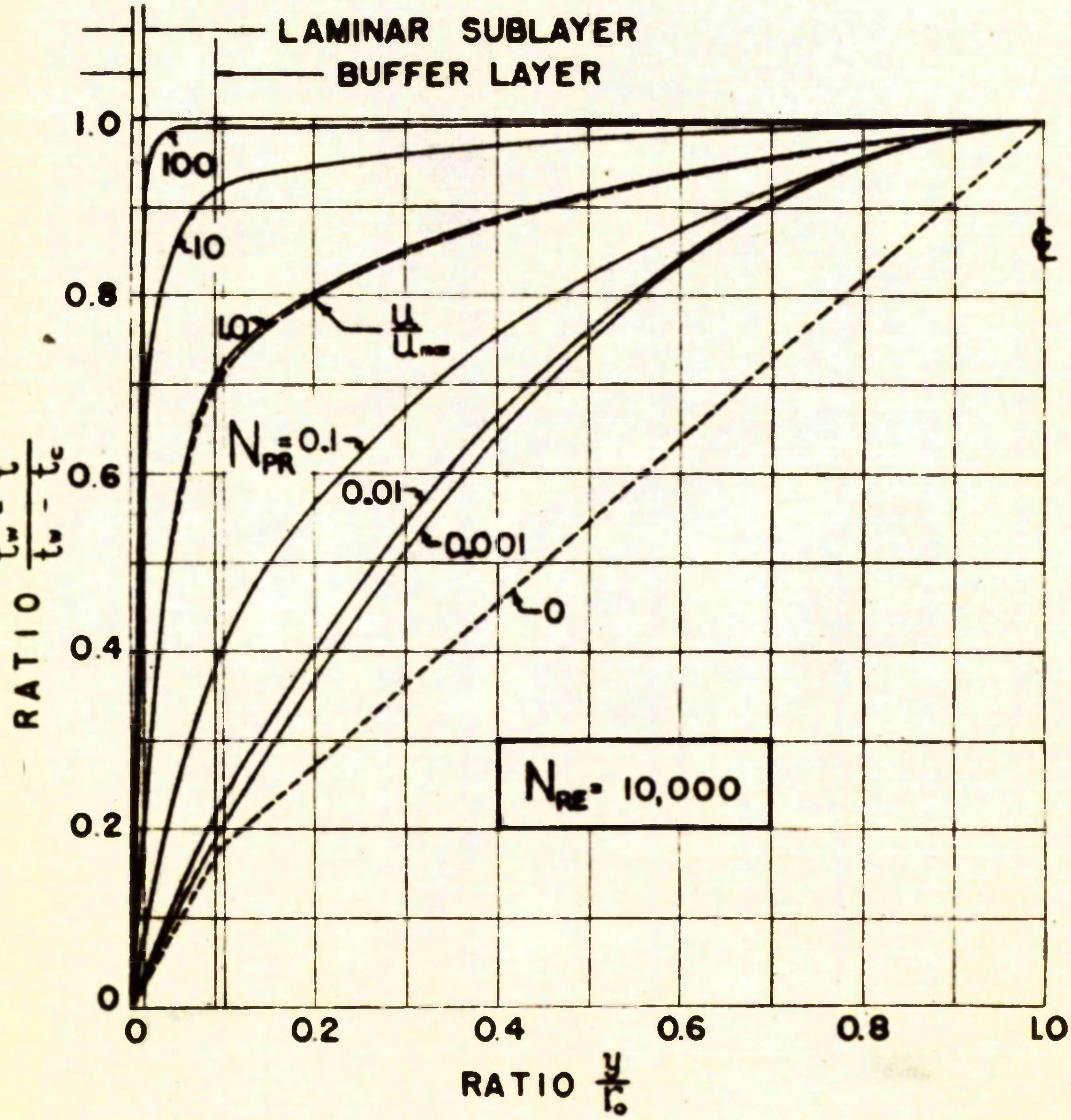

FIG. 28 FULLY DEVELOPED TEMPERATURE DISTRIBUTION DURING TURBULENT HEAT TRANSFER IN A CIRCULAR PIPE FOR VARIOUS MAGNITUDES OF PRANDTL MODULUS. ($Re = 10^4$)

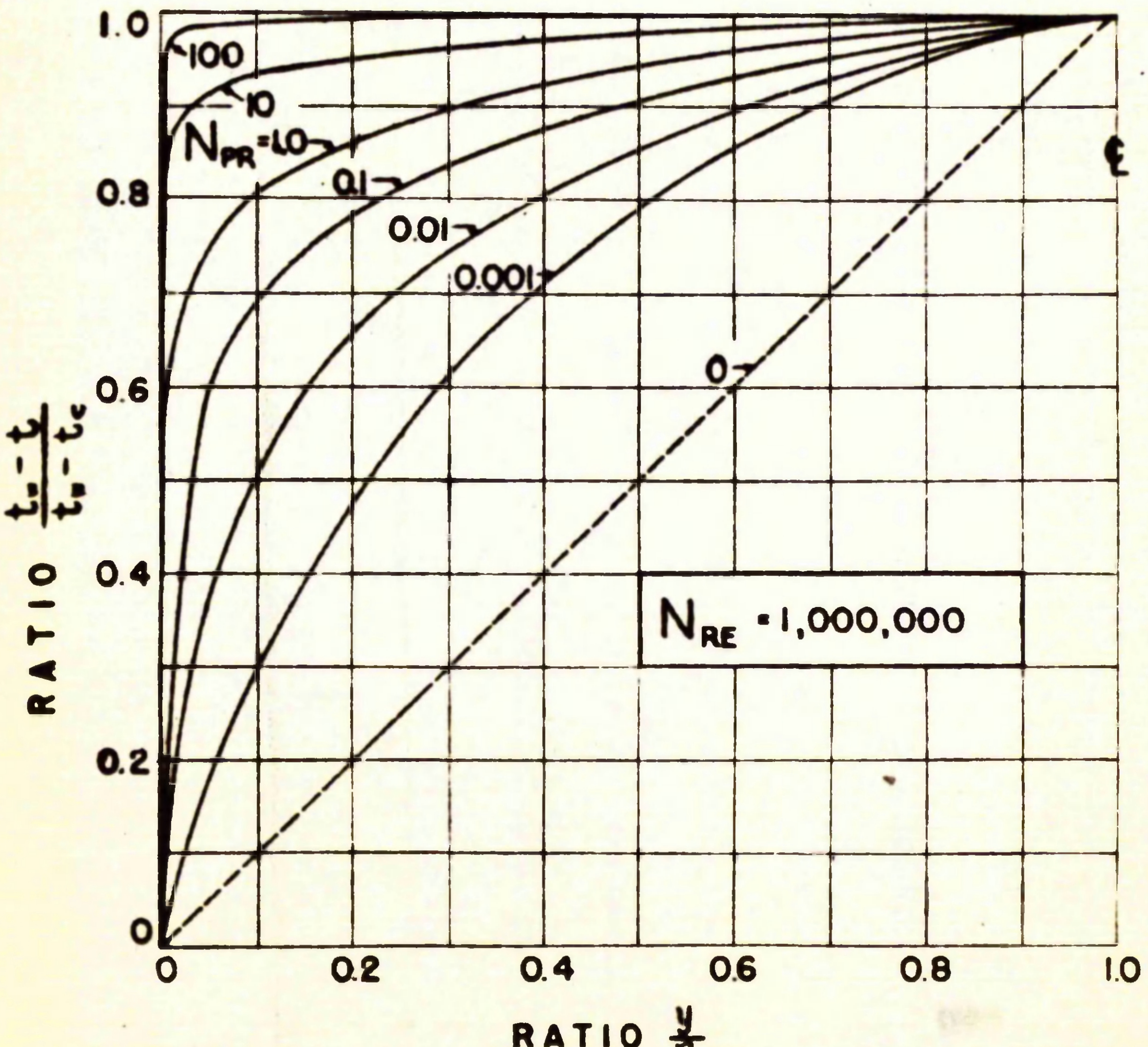

FIG. 29 FULLY DEVELOPED TEMPERATURE DISTRIBUTION DURING TURBULENT HEAT TRANSFER IN A CIRCULAR PIPE FOR VARIOUS MAGNITUDES OF PRANDTL MODULUS. (RE=10^6) (According to Martinelli.)

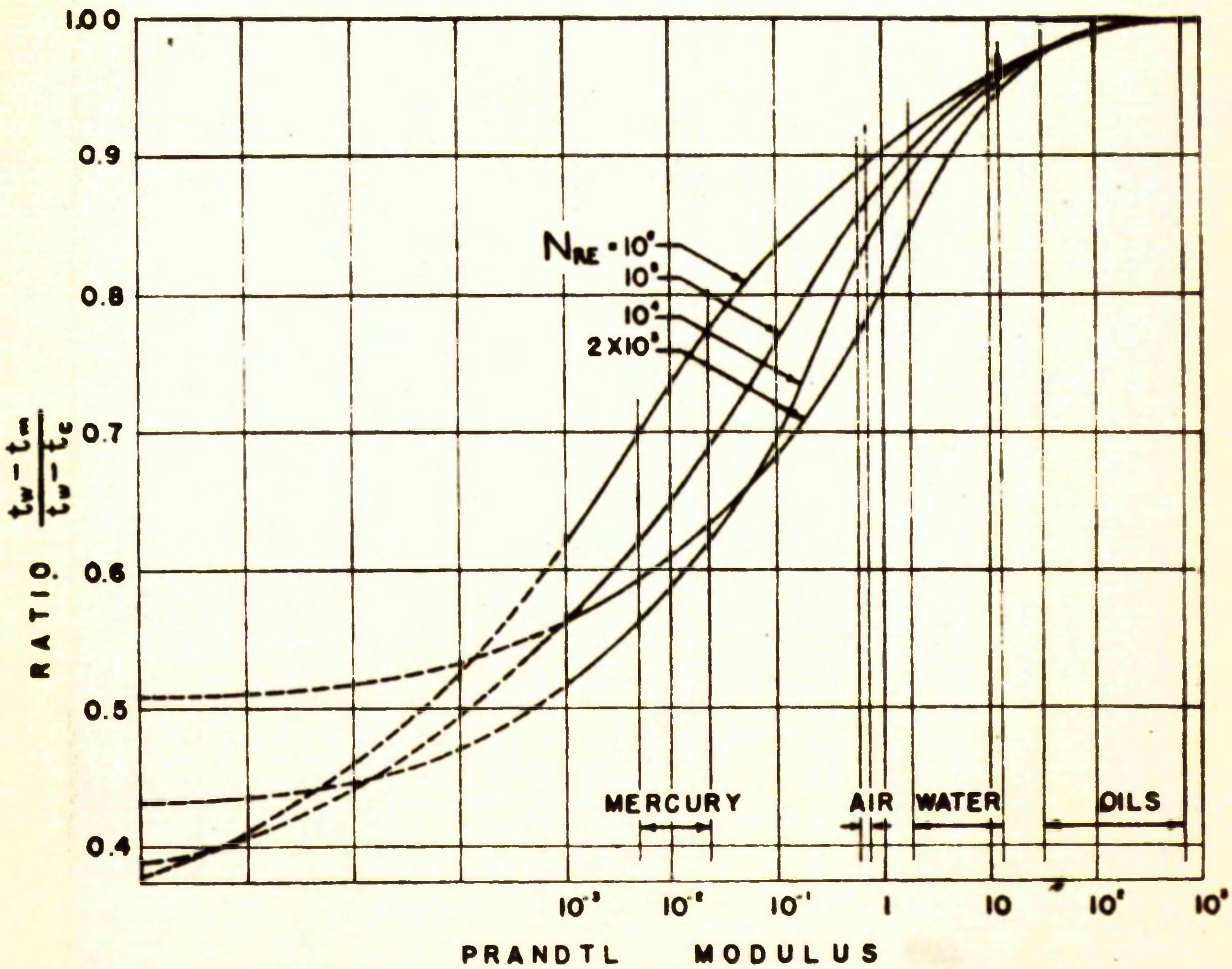

FIG. 30 RATIO OF MEAN TO MAXIMUM TEMPERATURE DIFFERENCE AS FUNCTION OF REYNOLDS AND PRANDTL MODULUS FOR TURBULENT HEAT TRANSFER IN CIRCULAR PIPE. (According to Martinelli.)

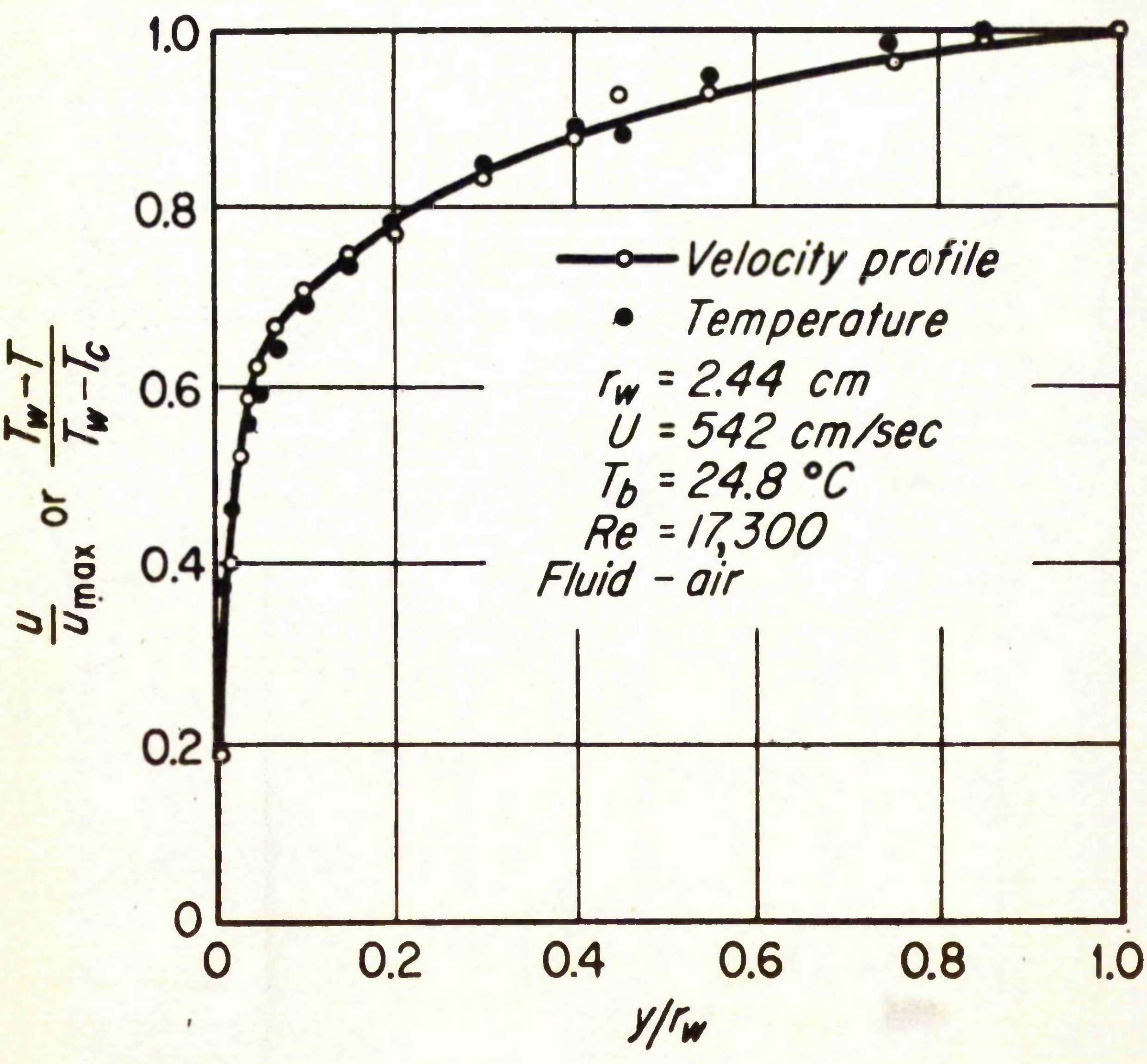

FIG. 31 VELOCITY AND TEMPERATURE PROFILES FOR TURBULENT FLOW IN A CIRCULAR TUBE. (Data from J.R.Pannel, Brit.Aer.Res.Comm. R and M.243ii, 1916.)

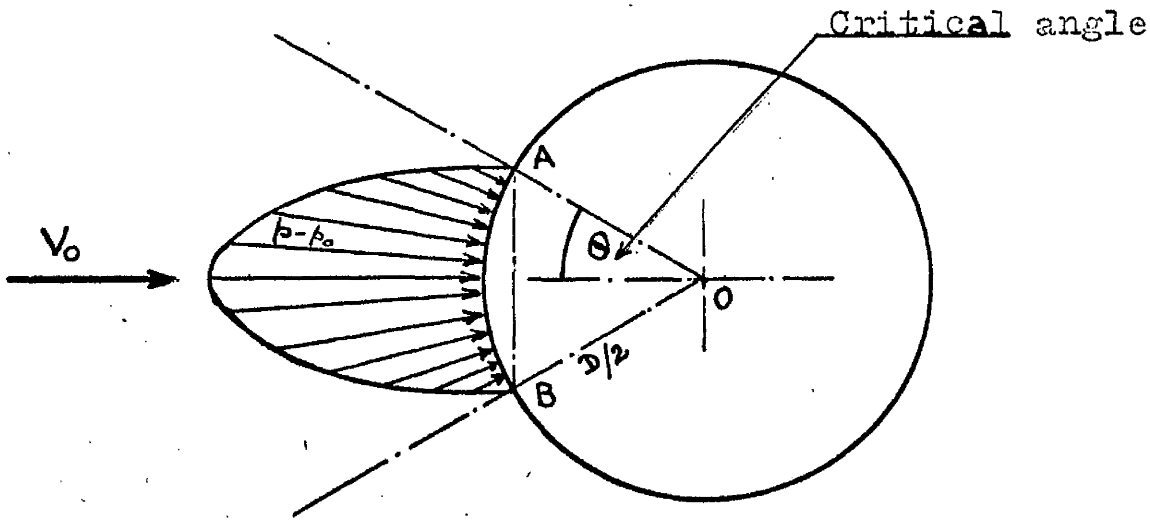

FIG.32 PRESSURE AROUND A CYLINDER.

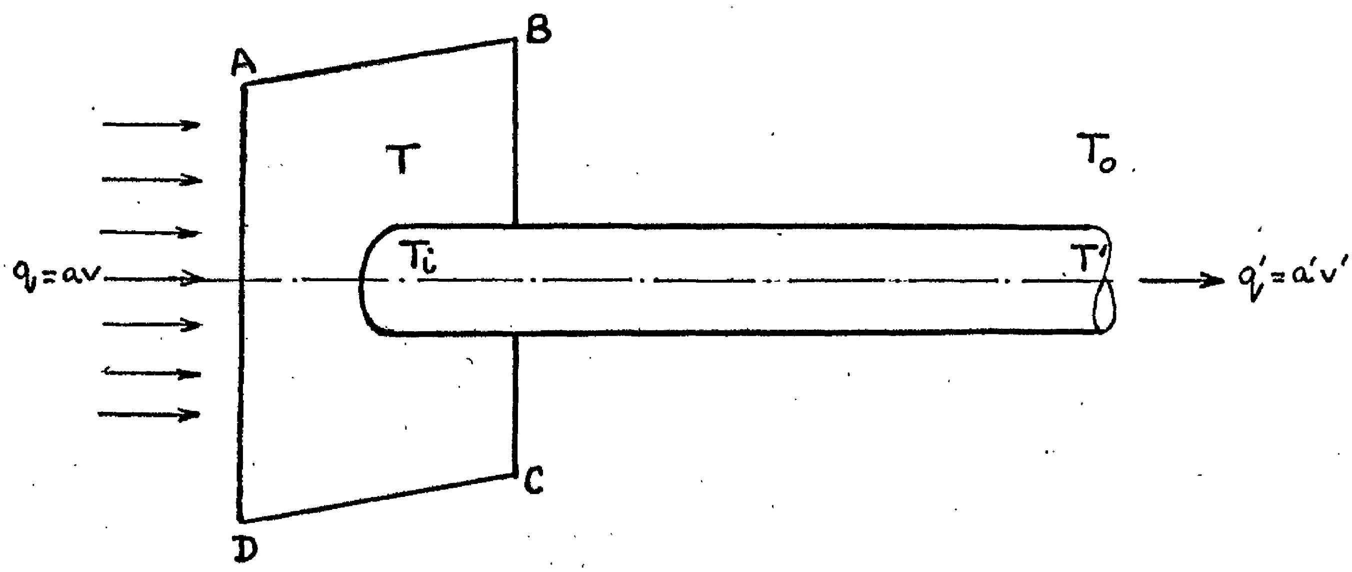

FIG.33 CONTROL AREA WITH SAMPLING TUBE.

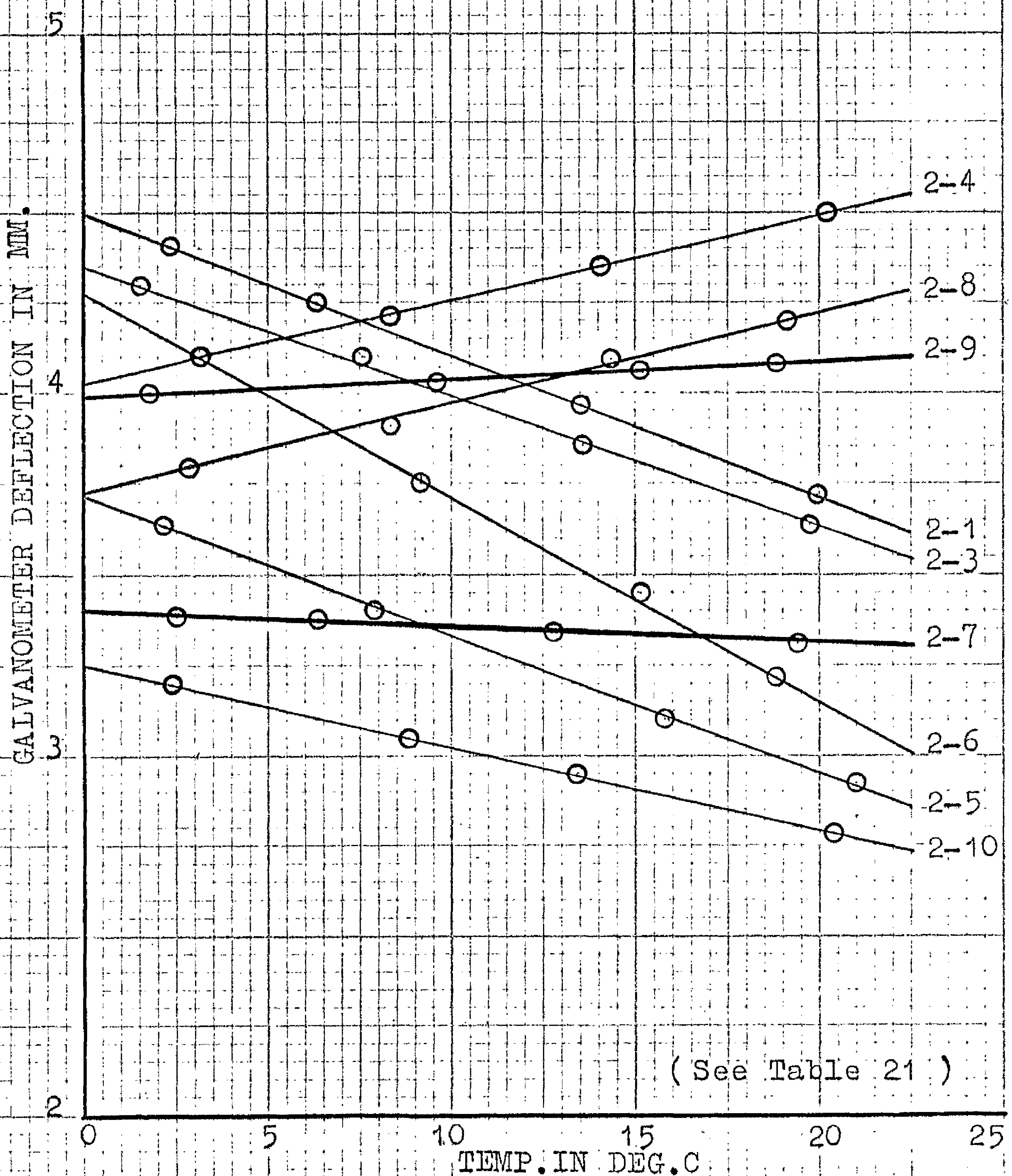

FIG. 34 DEFLECTION vs TEMP. FOR VARIOUS PAIRS OF THERMOMETERS.

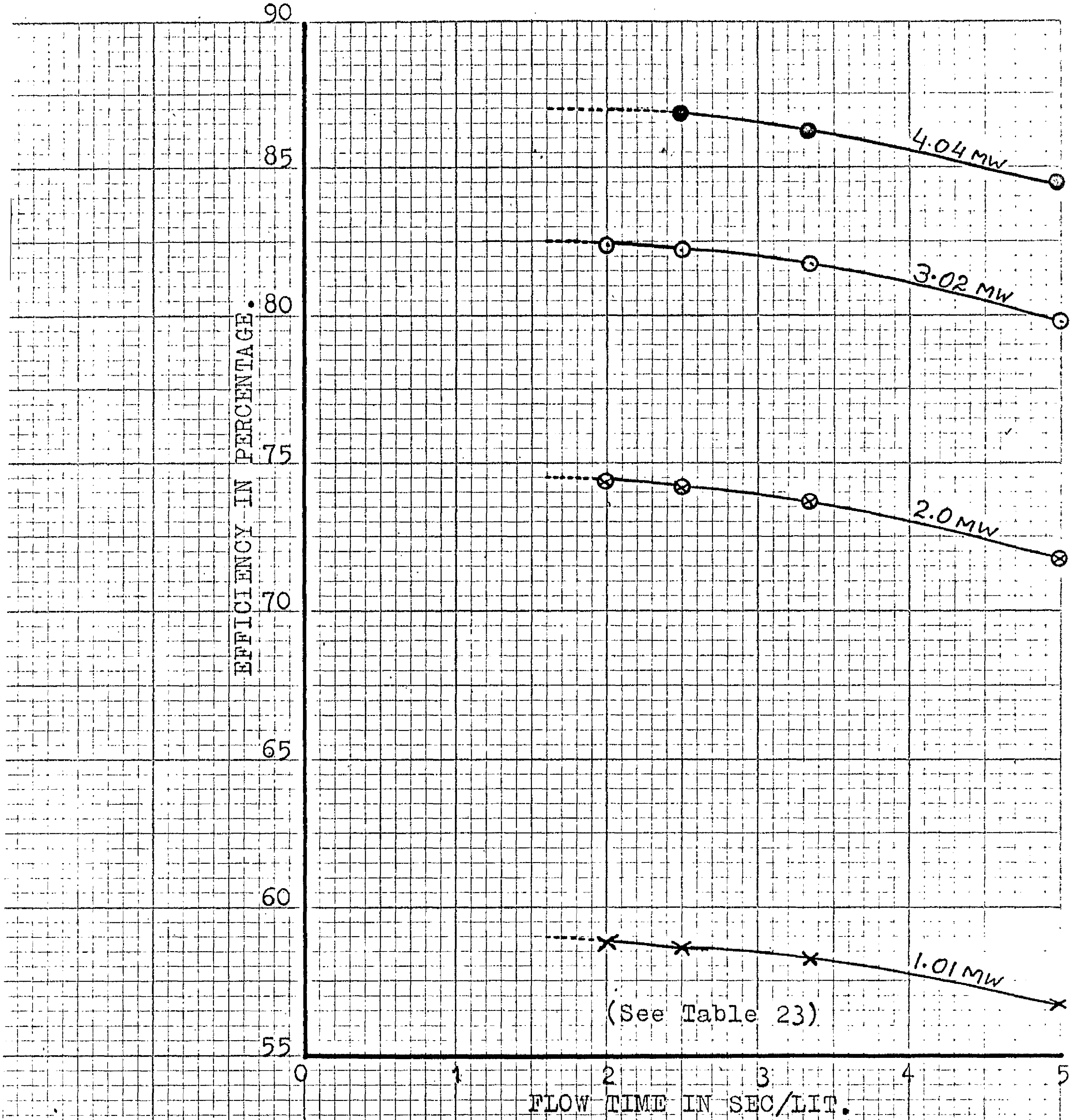

FIG. 35 EFFICIENCY VARIATION WITH FLOW TIME.

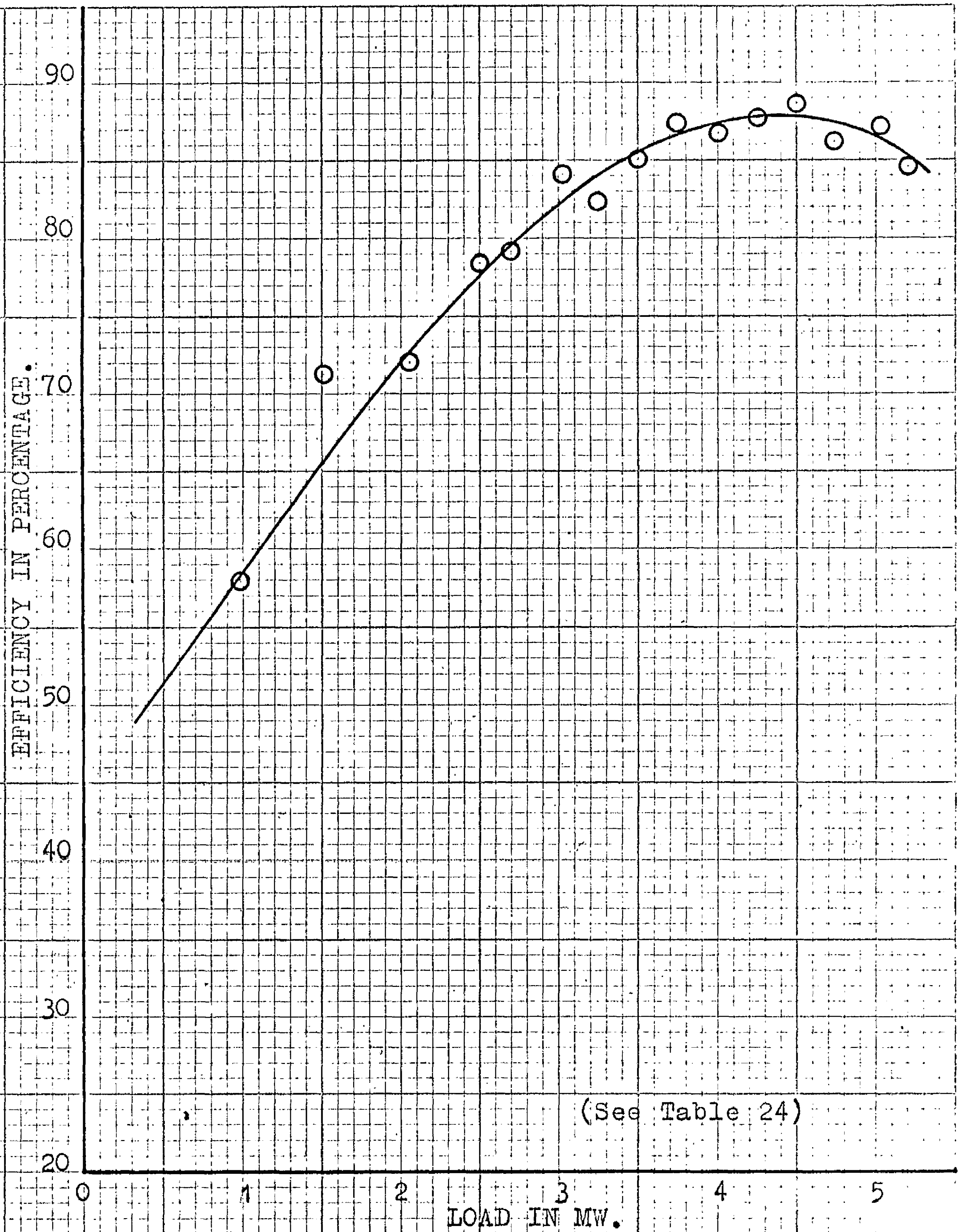

FIG.36 EFFICIENCY BONNINGTON DEC.60.

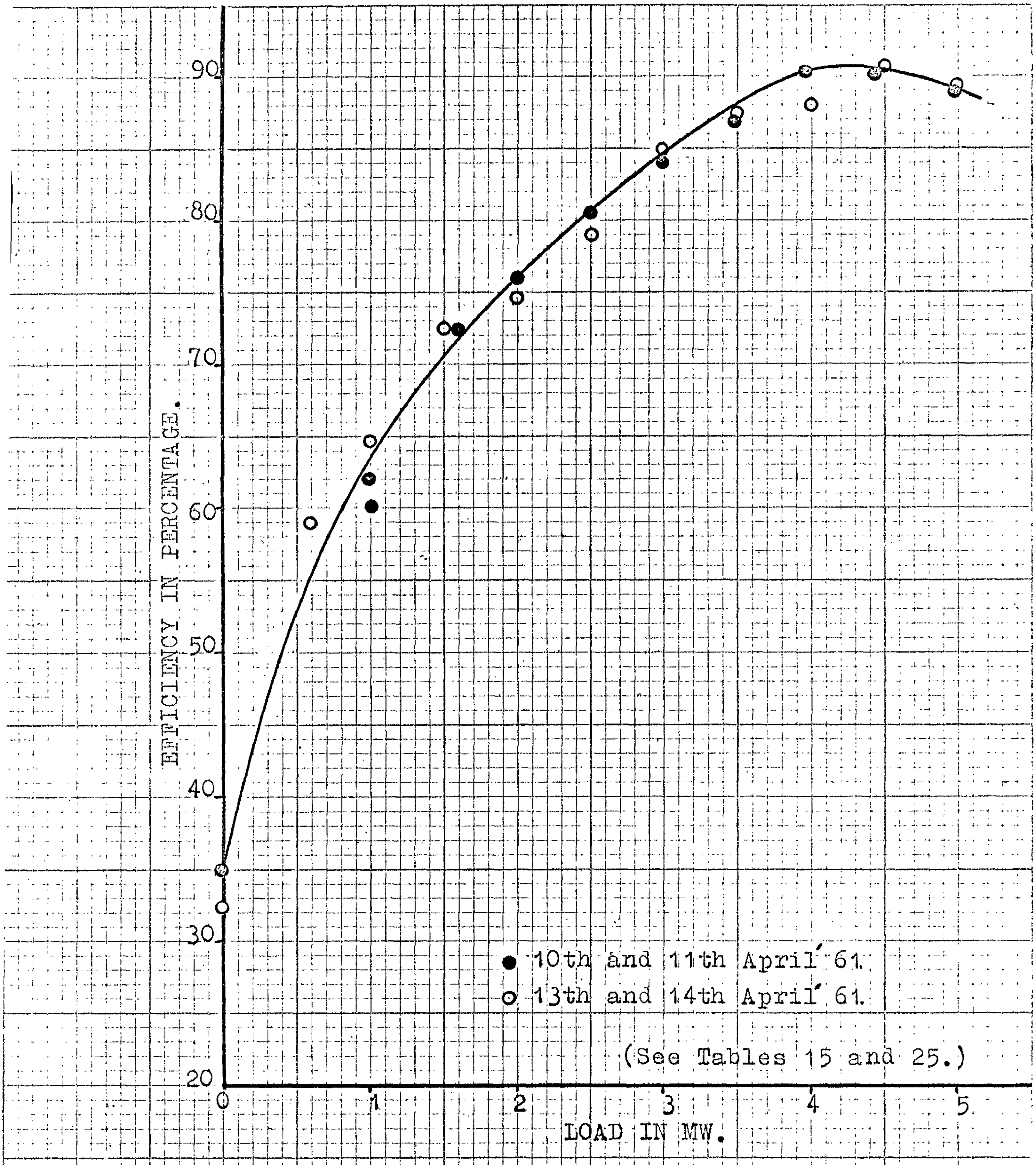

FIG. 37. EFFICIENCY BONNINGTON APRIL' 61.

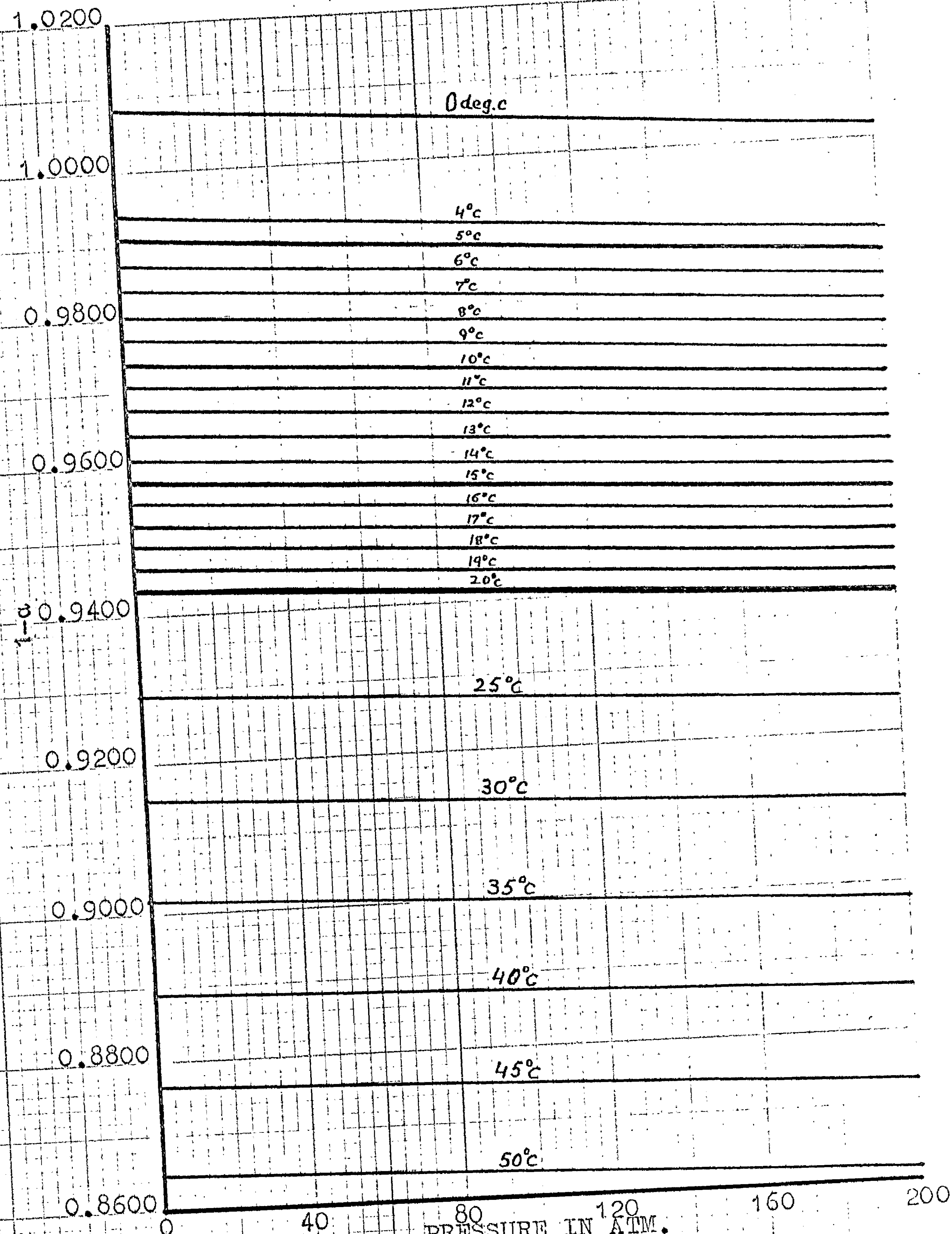

FIG. 38 THERMODYNAMIC COEFFICIENT (1-α) FOR PURE WATER.

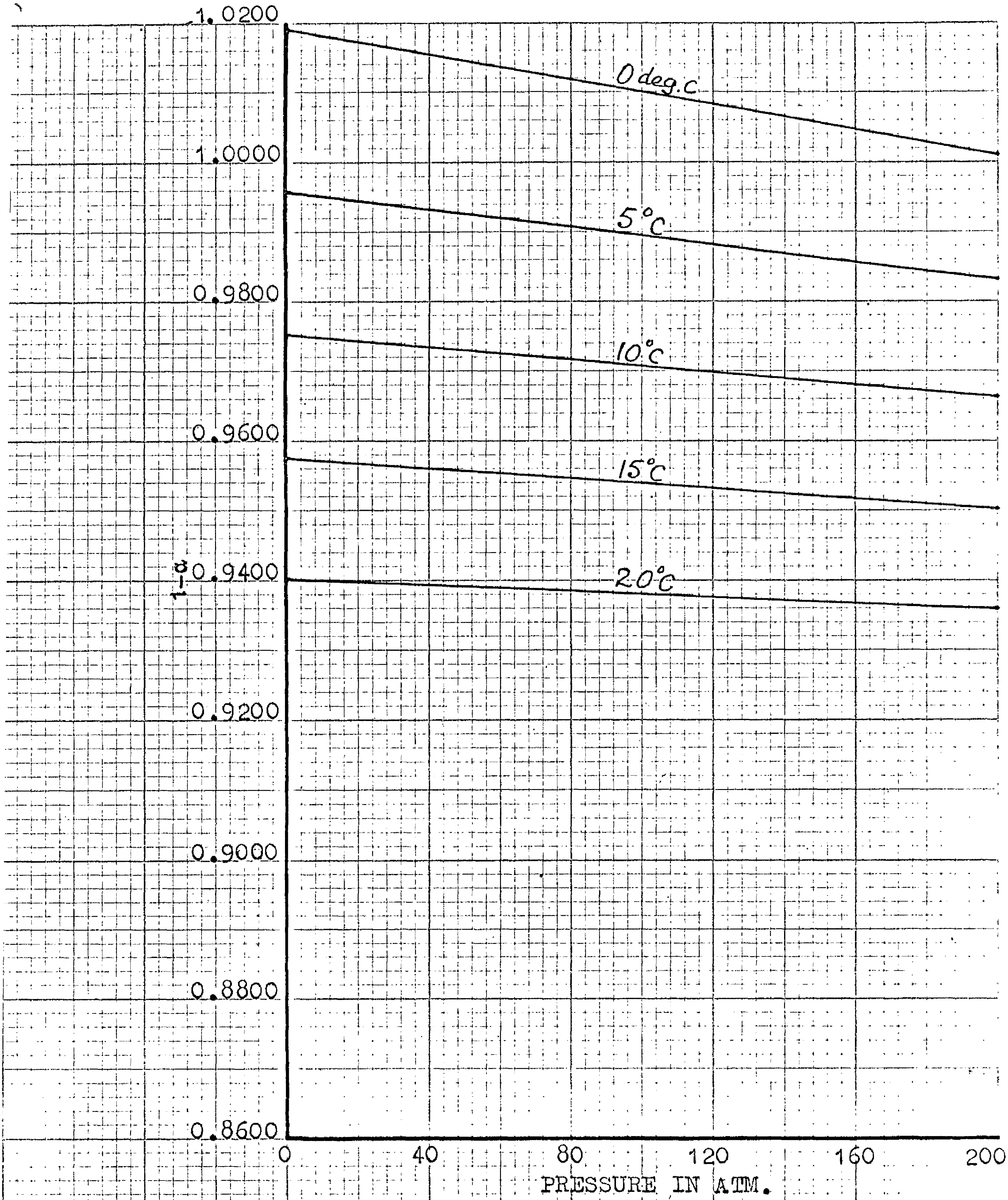

FIG. 39 THERMODYNAMIC COEFFICIENT (1-α) FOR PURE WATER. (According to WILLM)

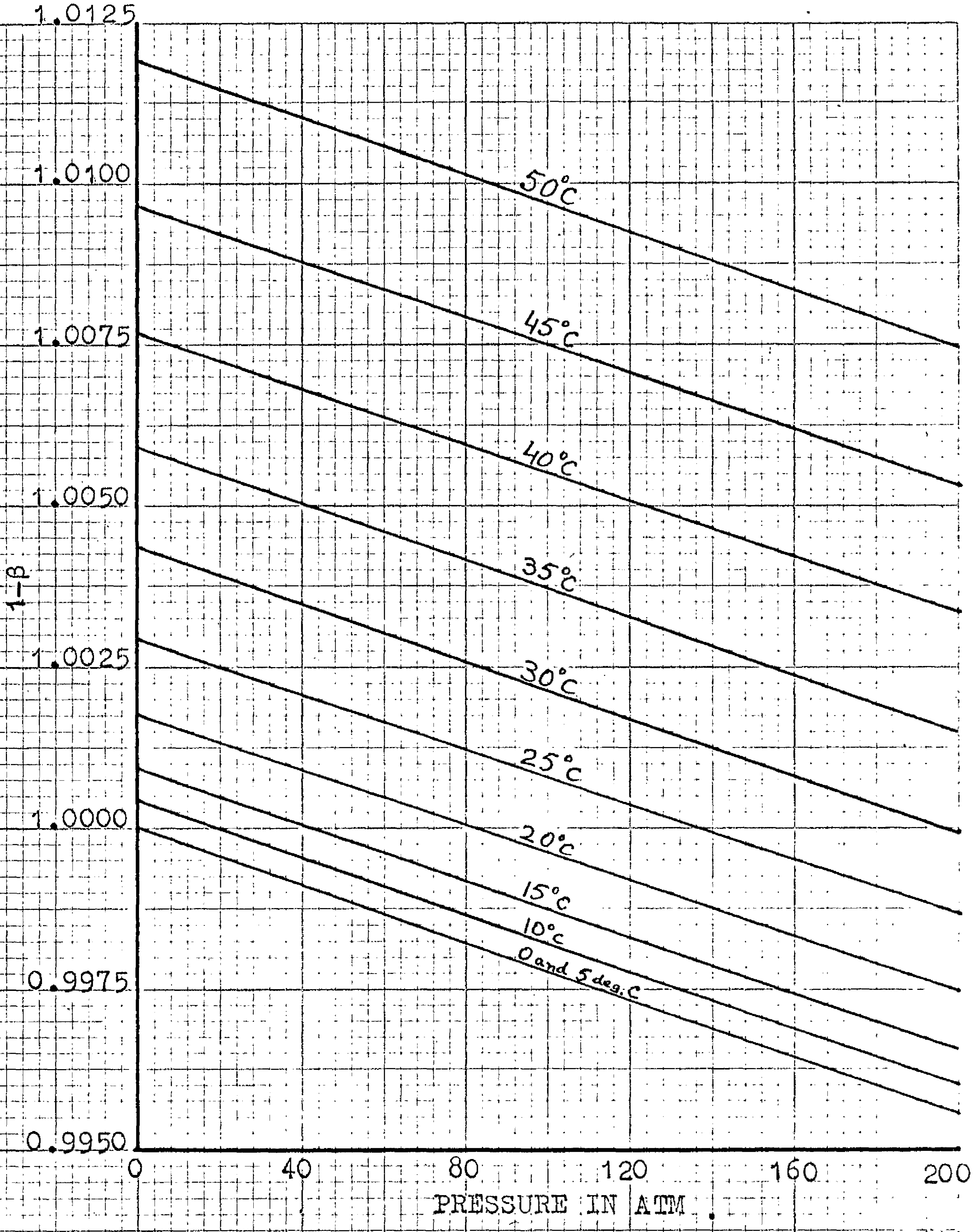

FIG. 40 THERMODYNAMIC COEFFICIENT (1-β) FOR PURE WATER.

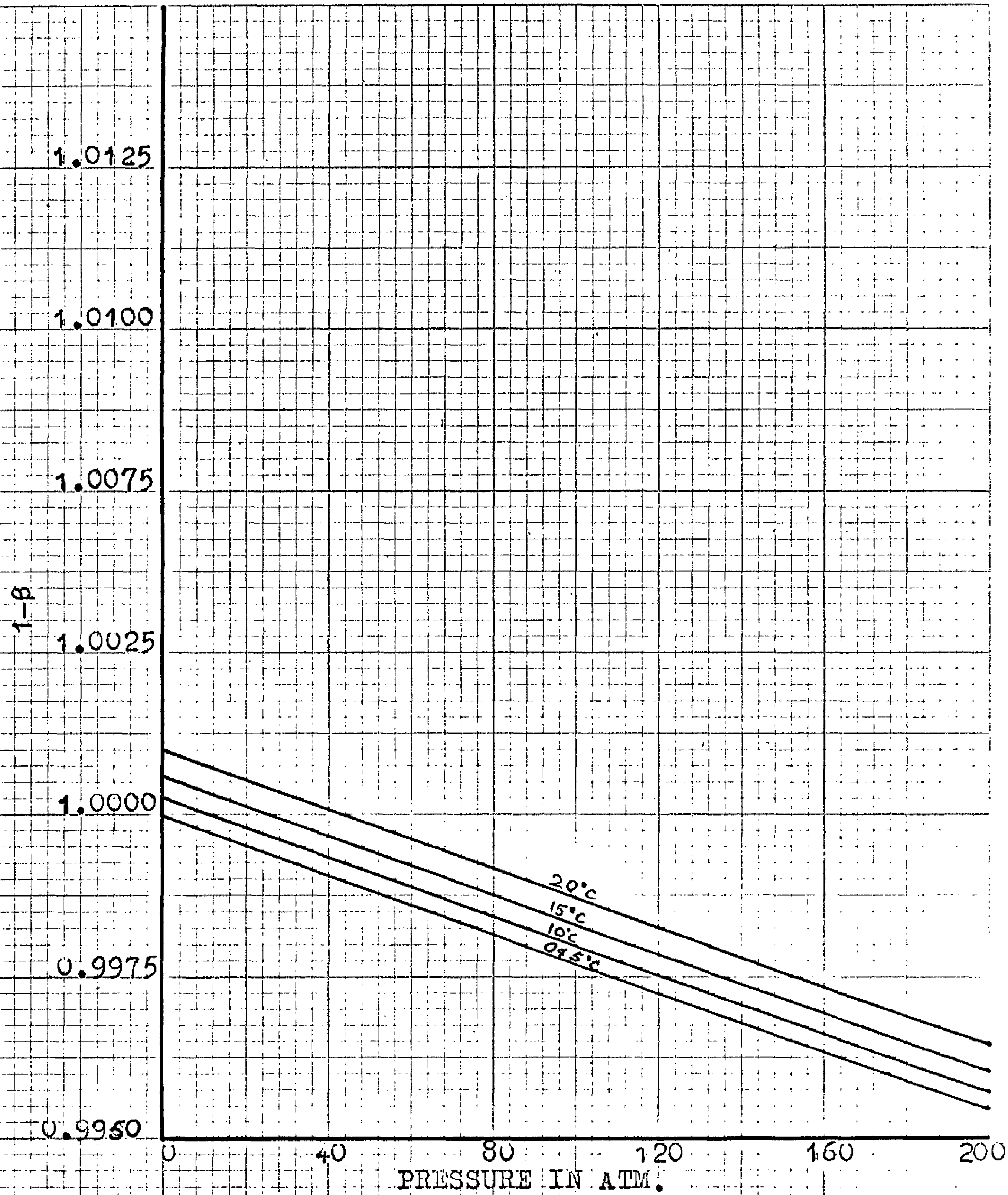

FIG. 41 THERMODYNAMIC COEFFICIENT (1 — β) FOR PURE WATER. (According to WILIM)

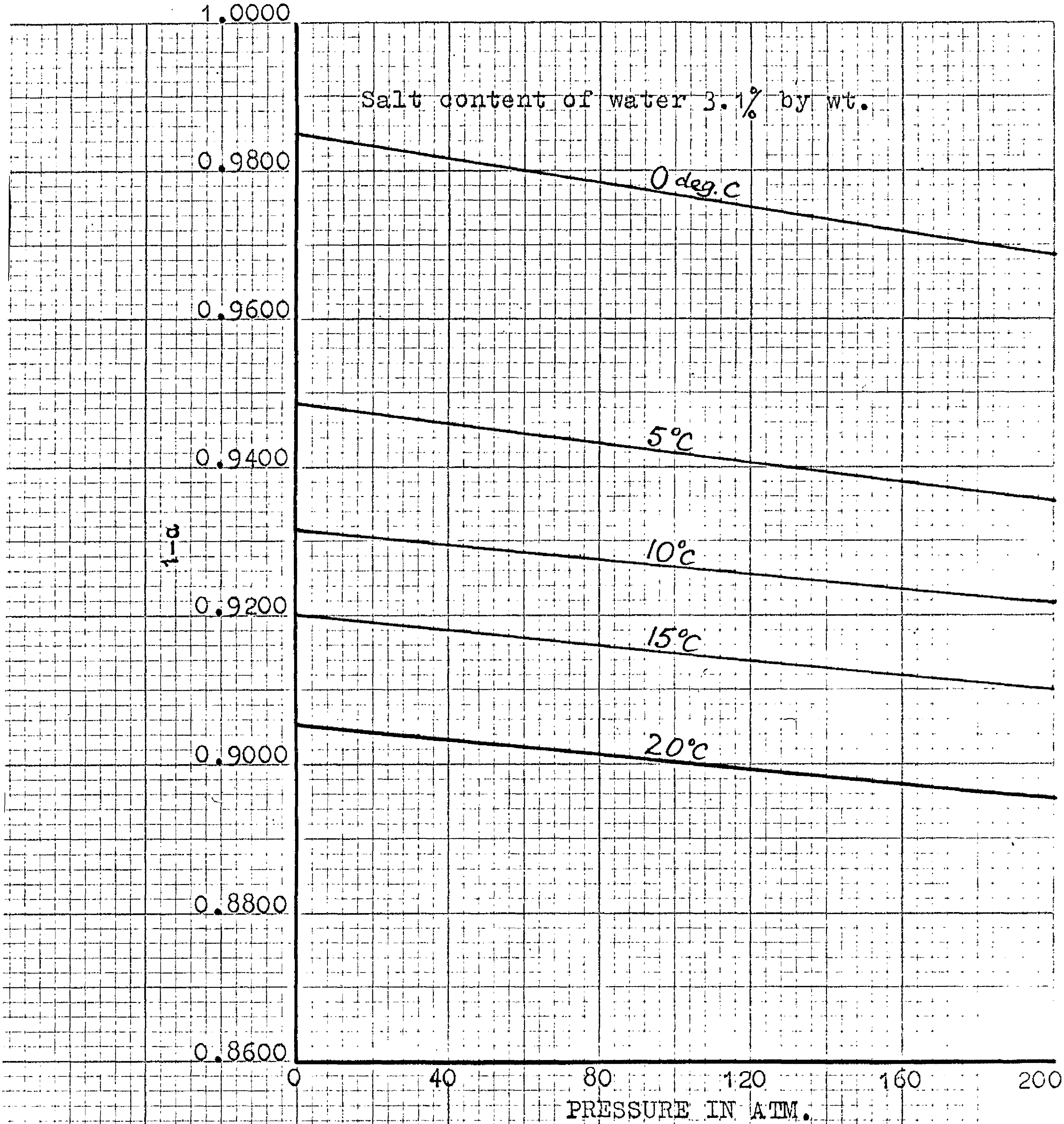

FIG.42 THERMODYNAMIC COEFFICIENT (1-a) FOR SALINE WATER containing 3.1% salt.

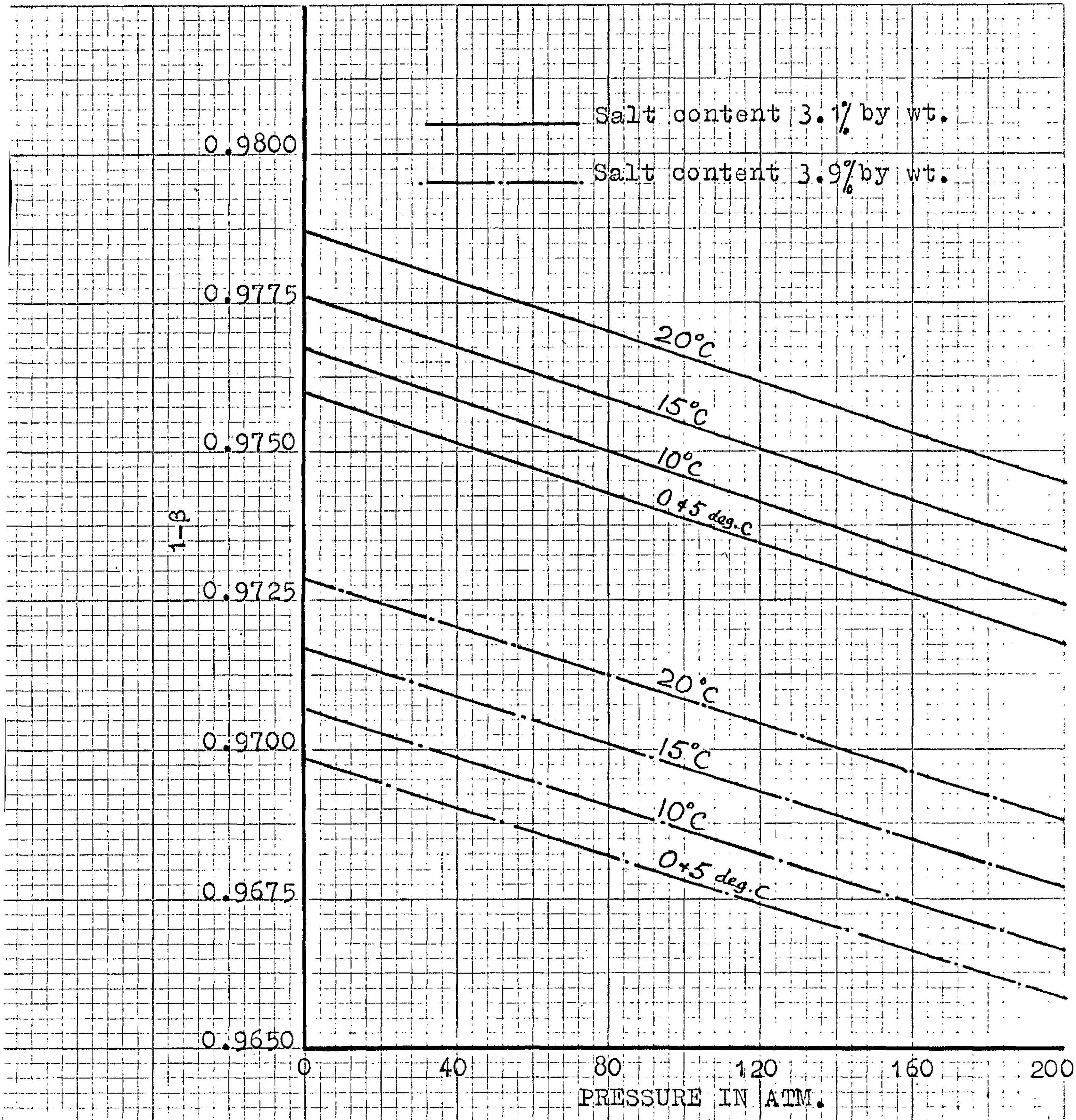

FIG. 43 THERMODYNAMIC COEFFICIENT (1-β) FOR SALINE WATER.

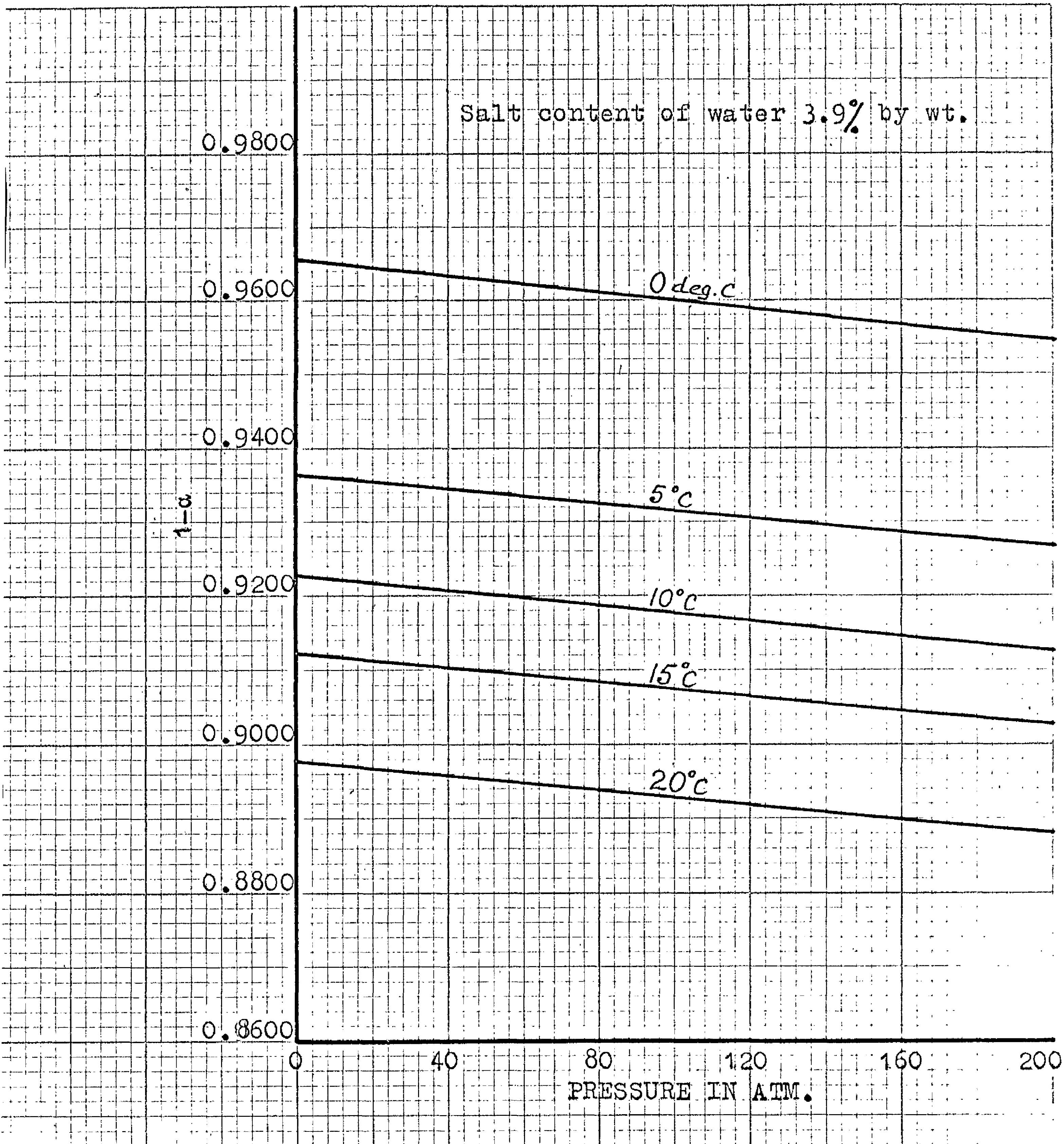

FIG. 44 THERMODYNAMIC COEFFICIENT (1-a) FOR SALINE WATER containing 3.9% salt.