In the Name of God, Most Gracious, Most Merciful

The Terrestrial/Photogrammetric (TP) Technique

for

The Detection and Compensation of Systematic Height Errors

in

Block Aerial Triangulation

by

Mohammed Mukhtar Abdel Rahim

Diploma, Land Surveying (Khartoum Polytechnic, Khartoum, Sudan)
Flight Dispatcher's Certificate (Airline Operations Training Inc., New York, U.S.A.)
Certificate of successful training in Air Navigation and Photogrammetry (Federal Aviation Agency (FAA), Philadelphia & Washington D.C., U.S.A.)
B.Sc., Photogrammetric Engineering (ITC, Delft, The Netherlands)
M.Sc., Photogrammetric Engineering (ITC, Delft, The Netherlands)

Submitted for the degree of
Doctor of Philosophy (Ph.D) in Photogrammetry

Department of Geography,
University of Glasgow (U.K)

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<u>28th, May, 1980.</u>

M.M.Abdel Rahim

<u>Geography Department</u>

<u>University of Glasgow</u>

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Summary

The thesis is concerned with a new technique (the Terrestrial/ Photogrammetric (TP) technique) for the detection and compensation of systematic height errors in block aerial triangulation. This technique improves the height accuracy of such triangulation by reducing the original bridging distance using new bands of corrected photogrammetric tie points as height control, together with the original bands of the terrestrial height control. Various photogrammetric blocks with different characteristics and configurations have been tested in a comprehensive manner.

The results show that the TP technique gives consistent results over a wide range of circumstances and that it is a most effective, simple and inexpensive method for the compensation of systematic errors. The technique shows that, in principle, a minimum of only two bands of terrestrial height control points together with an additional terrestrial height check point lying midway between them are enough to obtain the optimum height accuracy. That is to say, the terrestrial heights of only (2n + 3) points will be required to obtain the optimum height accuracy after the application of the TP technique, in any rectangular shaped photogrammetric block of parallel strips, where n is the number of these strips.

The TP method promises therefore to have a great impact on aerial triangulation practice, since the requirements for ground control are less than any other triangulation method devised up till now - with a consequent economy in the overall provision of such control.

INTRODUCTION

INTRODUCTION

Aerial triangulation in photogrammetry has been a popular subject of research for many years. Initially the geometrical and instrumental aspects were the main concern of researchers, but with the development of sophisticated computational tools attention has focussed more on the procedures for data reduction and adjustment. The objectives however were always firmly linked with devising methods or equipment which would yield higher accuracies in the results, or the same results in a shorter time, or the same results using fewer ground control points. The ultimate stage in this progression of interests, was that dealing with the detection and compensation of the systematic errors which remained after block adjustment. It represented the stage concerned with the final refinement of co-ordinates, so that remaining errors were distributed in a random fashion, and were of a magnitude consistent with that predicted from theory.

The theoretical accuracy of the results of block adjustment has been studied in the post - 1960 era. These studies, based on the classical principles of propagation of error, gave a clearer insight into the accuracies expected for different patterns and densities of control. They were, however, based on some oversimplified assumptions regarding the mathematical models used. For example, it was assumed that the observed co-ordinates (whether plate co-ordinates, or model co-ordinates) were uncorrelated and of equal weight. In other words the possibility that some of these co-ordinates might contain systematic errors arising from a common source was conveniently, and understandably, overlooked. From a scientific point of view, this state of affairs was unsatisfactory. Obviously it was essential to carry out experiments which would test whether these oversights were justified or not. It was also important in practical terms to assess the extent of the differences between practice and theory. If they were significant, the mathematical models could be improved - thus improving the results obtained in practice.

Experimental confirmation or disproval of a theory is often difficult, and it is particularly so in the case of aerial triangulation. Most triangulated areas do not have sufficient ground control or check points to allow general conclusions regarding accuracy to be made. During the period 1967/68 long discussions and preparations concerning experimental investigations took place within the OEEPE (Organisation Européenne d'Etudes Photogrammétriques Expérimentales). In the Autumn of 1968, the OEEPE decided to carry out extensive experiments concerning the accuracy of adjusted strips and blocks, and the nature of the errors involved. The main objectives were to assess the relationships between accuracy and other parameters in a block adjustment, such as overlap, block size, type of photography, distribution of control, adjustment methods, etc.

The outcome of these discussions was a set of specifications for a test field, and for the aerial photography which was to be taken of the field. The test area, given the name "Oberschwaben" (see Ackermann, 1973a), was chosen to be the area lying between the Danube and the Lake of Constance in Southern Germany. The first results of the program of investigations were presented by Stark (1973), Ackermann (1973b), and Ebner (1973), at the OEEPE Symposium in Brussels in 1973. A large part of these were the results of block triangulation based on independent models. Although they met certain expectations regarding the effectiveness of perimeter control in planimetry and bridging distance on height, the details clearly contradicted the theory based on propagation of random error (Haug, 1976). Amongst other conclusions, the results (see Wiser, Ackermann (1976)) also showed that the difference in accuracy between bundle-based and model-based adjustments was not consistent with theory. This confirmed the presence of uncompensated systematic errors, and led to consideration being given to the source and possible elimination of these errors. In recent years therefore much attention has been devoted to this aspect of aerial triangulation, and various techniques have been devised to cope with the problem.

The methods devised so far are somewhat cumbersome and elaborate in application. Each has problems associated with its successful implementation, and these tend to discourage the use of the method for practical mapping projects. The development of a new approach (the Terrestrial/Photogrammetric (TP) technique), which is both simple to apply and effective in its results, is discussed in this dissertation; in particular the investigation concerns the application of the technique to obtain an improvement in height accuracy after block adjustment.

The first two chapters review aerial triangulation - the methods of block adjustment, and the sources and effects of systematic errors in aerial triangulation. The third chapter discusses the various approaches taken to compensate for these systematic errors. Chapter 4 describes the TP technique; Chapter 5 describes the material (blocks of observed data) used to test the technique, and presents the results of these tests. Chapter 6 compares the TP technique with other methods pursuing the same objective, and Chapter 7 draws conclusions based on the tests.

CHAPTER I

A Review of Aerial Triangulation and Methods of Block Adjustment

1. A Review of Aerial Triangulation and methods of Block Adjustment.

1.1 Review of Aerial Triangulation

1.1.1 Single Photo Orientation in Space

Three elements of inner orientation and six elements of exterior orientation are required to be known in order to determine the orientation of a camera in space.

(i) The three elements of inner orientation defining the position of the photograph (focal plane) with respect to the projection centre are the focal length (f) of the camera and the position of the principal point (xp, yp) with respect to the fiducial centre. (See Fig. 1.1).

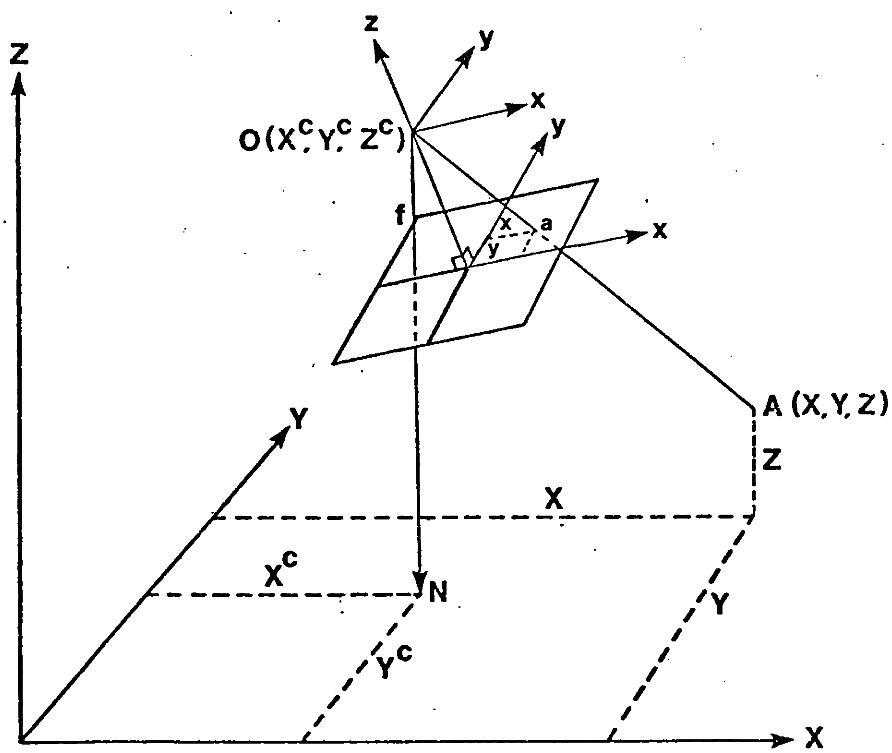


Fig. 1.1 Orientation of a photograph in Space

The principal point is the point where the camera axis pierces the photographic plane. Its location is the geometric centre of the photograph obtained by connecting the images of the fiducial marks attached to the frame of the camera. The focal length is determined by camera calibration techniques based usually on the known angular values between targets in object space.

Manufacturers of metric cameras provide these elements of inner orientation to a certain degree of accuracy. However it is necessary to re-calibrate the camera from time to time in order to re-establish these values.

Fig.1.1 shows the position of a photograph in space with a focal length f and exposure station at O. The following notation is used in Fig. 1.1:-

x, y, z = image space co-ordinate system. (In analytical aerial triangulation, all co-ordinate systems are right hand orthogonal systems).

X, Y, Z = object space co-ordinate system.

O = exposure station having co-ordinates (X^c, Y^c, Z^c) in the object space co-ordinate system.

A = an object point on the ground with co-ordinates X, Y, Z in the object space co-ordinate system.

a = image point of object A on the ground having image co-ordinates (x, y, z). z is -f for all image points.

A, a, and O are assumed to be on a straight line. This is the colinearity condition (explained later in Chapter 2).

(ii) The six elements of exterior orientation are the three translations in the object (terrain) space orthogonal co-ordinate system and the three rotations about axes which are parallel to the same object space system. These six elements may be obtained by resecting each individual photograph through the use of suitable located ground control points.

1.1.2 Orientation of Pairs of Photographs

When two overlapping photographs forming a stero-model are being considered all 18 elements of orientation must be determined in order to form a correct three-dimensional model of the terrain. The establishment of the inner orientation of each photograph accounts for three elements i.e. six for the pair of photographs. The relative orientation of a model in a stero-plotting instrument is achieved by eliminating the x and y parallaxes at five points on the photograph thus determining a further five elements of orientation. Some ground control points are necessary to achieve the correct position and altitude of the stereo-model with respect to the terrain so determining the remaining seven orientation elements. Of these, the scale and azimuth of the stereo-model can be fixed if at least two horizontal control points each with known X and Y co-ordinates (giving four co-ordinate values) appear within the model. The model can be levelled with respect to the terrain if three points with known Z co-ordinates appear within the model. Therefore as is well known, a minimum of two plan control points and three height control points are required in order to position and level an individual stereoscopic model with respect to a particular map-co-ordinate system. Such a process of scaling, levelling, and positioning of a model is termed Absolute Orientation.

Once inner, relative and absolute orientation of the model are completed the co-ordinates of any point within the model may be measured, and a topographic map of the area may be drawn.

In the early days of photogrammetry the control points, required for the absolute orientation of each model, were determined by ground survey methods. These methods were expensive and time consuming, and so photogrammetric techniques were developed to meet the control requirements for mapping by photogrammetric methods. These techniques are referred to as aerial triangulation.

1.1.3 Definition of Aerial Triangulation

Aerial triangulation is the method of establishing supplementary planimetric and height control, using the geometric relationships between successive aerial photographs.

Another definition according to ASP (1966) is as follows: "Aerial Triangulation is the process for the extension of horizontal and/or vertical control whereby measurements of angles and/or distance, on overlapping photographs are related into a spatial solution using the perspective principles of the photograph".

1.1.4 Methods of Aerial Triangulation

It is well known that aerial triangulation came into practical application around 1935 after preliminary experiments lasting about 15 years (mainly in The Netherlands, Germany, Italy, and France).

Two different methods of aerial triangulation can be distinguished as follows:-

(1) Radial Triangulation

(2) Spatial Triangulation

Radial Triangulation is limited essentially to flat terrain and provides planimetric positional information only. For all practical purposes, radial triangulation, which is excluded from any discussion in this thesis, could be considered as a special case of the three-dimensional spacial triangulation. The term "aerial triangulation" will always refer, in this thesis, to spatial triangulation which is not subject to terrain restrictions and provides both planimetric and height information. There are many different methods of aerial triangulation, and the method or methods used by different organizations will depend to a great extent on the equipment available. These different methods can be classified into three groups, depending mainly on the amount of computation necessary (see Kubik, Tait (1967)), as follows:

1.1.4.1 Aeropolygon Method

In this type of aerial triangulation procedure, the relative orientation is carried out in a plotting instrument using only the elements of the "new" projector, the "old" projector being left untouched. The X component of each air base is fixed by the scaling procedure, while the Y and Z components are determined by the relative orientation. The formation of the strip is therefore achieved in the triangulation instrument. This method is possible on a Multiplex assembly, and also on an instrument with a Zeiss parallelogram device - such as for example the Zeiss C8, Wild A7, or Wild A9 stereo-plotting instruments, commonly referred to as universal instruments.

The output from this method are the strip co-ordinates of all photogrammetric points referred to the co-ordinate system of the first model.

1.1.4.2 Independent Model Triangulation

Independent model triangulation is carried out when there are only two projectors available, and either when no parallelogram device is available (e.g. as in the Wild A8), or when no use is made of the parallelogram device on a universal instrument (e.g. the Wild A7). After determining the co-ordinates of the projection centres, it is only necessary to perform relative orientation in the instrument. The connection of the single models to form a strip (or block) is achieved by computation, and because this is rather complicated and normally requires the use of an electronic computer, other methods have been developed which transfer some of the elements of absolute orientation in the instrument. However these other methods are time-consuming, of limited accuracy, and rather messy to execute. Thus they will not be considered any further.

1.1.4.3 Analytical Triangulation Methods

In this third broad group, almost all of the triangulation procedure is carried out by computation, since not even relative orientation (and the formation of models) is carried out in the instrumental phase. This phase consists of measuring the plate co-ordinates (x, y) of all points of interest in a comparator. The co-ordinates are fed to a computer suitably programmed and the final output of the computer is a list of co-ordinates of the points of interest, either as independent model co-ordinates, or as co-ordinates in the ground system.

A more detailed classification of the methods of aerial triangulation may be found in the Literature (e.g. Mikhail (1963), ASP (1966), Kenefick (1973)). The accuracy attainable by any method of aerial triangulation depends on the ability of the method to deal realistically and effectively with the inherent errors and discrepancies in the overall photogrammetric system. This accuracy is affected by many factors; some of which are mentioned below.

1.1.5 Factors affecting the Accuracy of Aerial Triangulation

Various, quite separate procedures and instruments are involved in the data acquisition and data reduction phases in any photogrammetric system. The quality of the photogrammetric solution will therefore be affected by many factors in these two phases.

1.1.5.1 Factors associated with Data Acquisition

Some of the factors associated with the data acquisition phase are those related to the following:-

(1) Camera e.g. resolution; quality and updating of the calibration parameters; stability; lens quality; angular field of view; effectiveness and repeatability of film flattening.

(2) Photograph e.g. scale; type; overlap; filters.

(3) Film processing and handling e.g. type and quality of film base; type and quality of emulsion; development; drying, storage and handling.

(4) Diapositive printing e.g. the method used for producing contact or projection prints; compensation plates for reduction of image deformation.

(5) Identification and transfer of pass points e.g. operator accuracy and consistency of measurement; type of instrument used; form of mark used.

(6) Ground Control e.g. type, quality and density; distribution and configuration; means of identification.

(7) Measurement of image co-ordinates e.g. the calibration of the instrument; type of instrument used; accuracy and precision of the instrument; basic unit of measurement i.e. whether a photograph, a model or a strip.

1.1.5.2 Factors associated with Data Reduction

Some of the factors related to the data reduction and processing are:

(1) Data editing e.g. elimination of blunders, either automatically or manually; criteria for rejecting a measurement.

(2) Image refinement e.g. types of errors considered such as lens distortion, atmospheric refraction, film deformation, film and platen flatness, microfilm deformation, instrumental errors; mathematical models to compensate for systematic errors, residual systematic errors and self calibration. (These will be discussed later in Chapters 2 & 3).

(3) Adjustment e.g. basic unit of adjustment, i.e. whether a photograph, stereo model, strip, triplet or sub-block.

A more detailed discussion of the factors affecting the accuracy of aerial triangulation can be found in the Literature. (See, for example, ASP (1966)). In general, one can say that the three basic components interacting with one another to produce several combinations in a photogrammetric system are:

(1) instruments and materials,

(2) processes, and

(3) mathematical models.

A good understanding of the interaction and combined effect of these factors is necessary if reliable judgements are to be made regarding the required accuracy within a fixed framework of cost and time.

1.2 Review of Methods of Adjustment of Aerial Triangulation

1.2.1 Introduction

Systematic and random errors will exist in the aerial triangulation data as a result of the different factors mentioned previously in para. 1.1.5.

The primary objective of an aerial triangulation adjustment is to compensate for the propagation of systematic errors, and also to control the odd behaviour of random errors, since all of these errors are bound to have adverse effects on the accuracy of minor control point co-ordinates. The adjustment consists of estimating the most probable values of the co-ordinates of minor control points, whose images appear on at least one pair of overlapping photographs. The basis for this estimation is

(1) the known co-ordinates of available ground control,

(2) the measured co-ordinates of minor control points, and ground control points, in the system of the photographs, the model or the strip.

The next section deals with some historical aspects of the adjustment of aerial triangulation and this is followed by a short account of Strip and Block adjustment procedures. A strip may be considered as a special case of the block, occuring in mapping projects requiring a single strip coverage such as road and pipe-line surveys. However the general case of a photogrammetric block occurs when an area is covered by at least 2 strips, with suitable lateral overlap between the strips.

1.2.2 Historical Remarks

Before the serious consideration of block adjustment round about 1955, photogrammetrists were concerned mostly with the strip, as this seemed the natural unit for adjustment purposes. The application to a block was generally a simple extension of the same technique applied to each strip in turn. Finally the arithmetic means of common points in the lateral overlaps were adopted as the final adjusted values. Other approximate methods included the use of linear transformation equations for planimetry, and a method developed by Zeller which used cross strips. The latter was an extension of a geodetic idea, the principle being comparable with that of first and second order geodetic triangulation chains. The essential feature of all these simple procedures was the fact that some consideration was given to the lateral ties between strips, but in such a way that it was still possible to carry out the adjustment strip by strip. With the development of genuine block adjustment procedures, these methods lost their importance.

As early as 1935, Von Gruber had been investigating the sources of systematic error in aerial triangulation. The investigations included tests to observe what effect random errors in the measurement of co-ordinates in a photogrammetric instrument would have on a triangulated strip. Bachman proved in 1946 that the error propagation was of the third power in X for all three co-ordinates, X, Y, and Z. With the growth in knowledge of these patterns of errors, rational methods of strip adjustment developed -

in the first place based on graphical procedures and then with the development of calculators, by computation. A well-known and widely used graphical method was that introduced by Zarzycki (1949). The graphical methods are also extended into a form of block adjustment such as that developed by Brandenberger (1951), which used 3 or 4 widely spaced tie (or cross) strips. These were triangulated and adjusted first, and then the other strips were measured and made to fit their values.

Least squares strip adjustment procedures were first proposed around 1950 by Roelofs (1949), Verdin, Bjerhammer, and others. However their serious application was inhibited at that time by the lack of suitable automatic computational facilities, so that graphical methods remained popular. Computational adjustment using polynomials was not such a serious problem however, and in 1951 Roelofs proposed some rather complicated correction formulae, which were later simplified by Van der Weele (1954). Although these served a useful purpose at the time, it was inevitable that block adjustment, with its much stronger geometry, would become the focus of attention.

The first major progress in block adjustment came when analogue computers were developed in the period 1955-60. Although the early electronic computers were appearing at that time, they were expensive and rare, and numerical solutions were still a great problem. The answer lay in the application of mechanical analogue computers which performed the bulk of the computations. Two main developments took place - one at the Institut Geographique Nationale in Paris, France, and the other at the International Training Centre for Aerial Survey (I.T.C) in Delft, The Netherlands. Both gave excellent results for that period, and the methods were close approximation to a theoretically sound least squares block adjustment. The former, known as the IGN Analogue Computer, was in use until 1961 when it was abandoned in favour of electronic computation. The latter, known as the ITC-

Jerie Analogue Computer was essentially an improvement on the IGN system. There were two main improvements - the first allowed the discrepancies between models to be introduced at larger scales for each iteration cycle, and the second concerned the use of material more consistent with the theory of error propagation in aerial triangulation. Both improvements gave rise to greater accuracy in the final results of an adjustment. The ITC - Jerie computer was used for many years before being replaced by electronic computation. Indeed it is highly likely that it is still in use in the more remote and less developed parts of the world where electronic computers are rare.

The serious implementation of numerically-based simultaneous block adjustment methods took place during the period 1960-70, which coincided quite naturally with the wider availability of electronic computers. The Ordnance Survey of Great Britain had a working system as early as 1953 (see Thompson (1953)) and, by the beginning of 1970, a number of very sophisticated block adjustment programs were operational in some of the larger mapping agencies.

In the next two main sections (1.2.3 and 1.2.4) some of the current strip and block adjustment procedures are discussed in more detail.

1.2.3 Strip Adjustment

Most strip adjustment procedures are smooth interpolation procedures which use the differences between the machine co-ordinates and ground co-ordinates for some control points to derive corrections for all other triangulation points. Some strip adjustment procedures are summarised below.

1.2.3.1 Procedures based on Polynomials

The graphical method devised by Zarzycki (1949) is a second degree interpolation procedure. It derives the X, Y and Z co-ordinate corrections separately, and is based on a special distribution of control (See Fig.1.2).

Fig.1.2 Correction curves derived from contradictions at control points

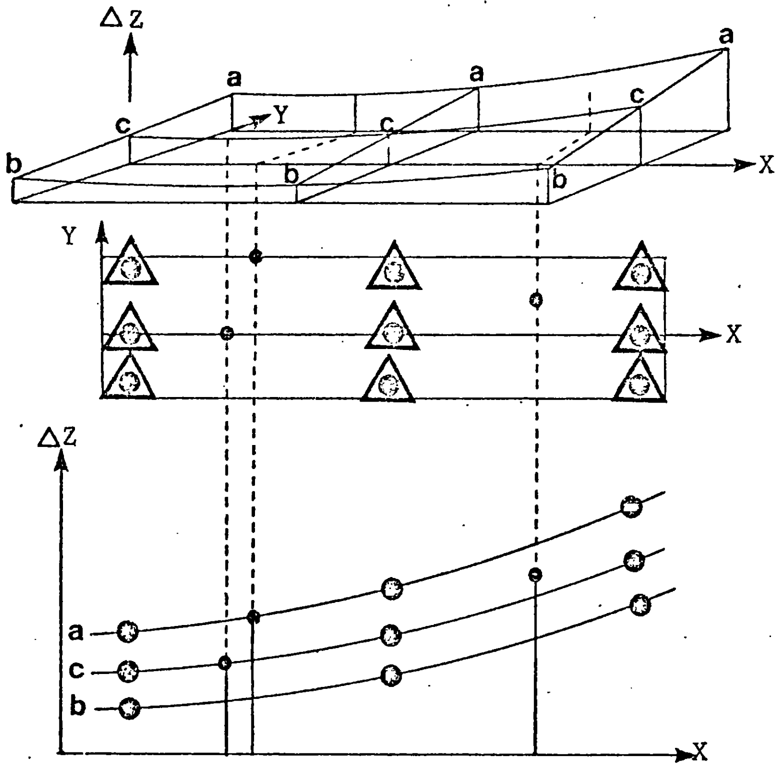


Fig.1.2(a)

Fig.1.2(b)

Fig.1.2(c)

△ —— Terrestrial Planimetric Control

◉ —— Terrestrial Height Control

• —— Arbitrary Point

Fig.1.2(a) Second degree correction surface for ΔZ. Other similar surfaces are established for ΔX and ΔY.

Fig.1.2(b) Plan of strip, showing the ideal position of control.

Fig.1.2(c) Correction curves for ΔZ.

The method has been used extensively for triangulation work in many parts of the World. Its success is largely due to its simplicity in concept and application. The mathematical formulae which are equivalent to the graphical procedure of Zarzycki are:-

$$\Delta X = a_0 + a_1X + a_2X^2 + a_3Y + a_4XY + a_5X^2Y$$
$$\Delta Y = b_0 + b_1X + b_2X^2 + b_3Y + b_4XY + b_5X^2Y$$
$$\Delta Z = c_0 + c_1X + c_2X^2 + c_3Y + c_4XY + c_5X^2Y$$

The first three terms in each equation represent the parabola for corrections to points along the strip axis. The fourth and fifth terms represent additional linear corrections for points with Y co-ordinates, and the sixth term (involving X^2Y) takes care of any additional second degree (in X) difference for these same points. (See Fig.1.3).

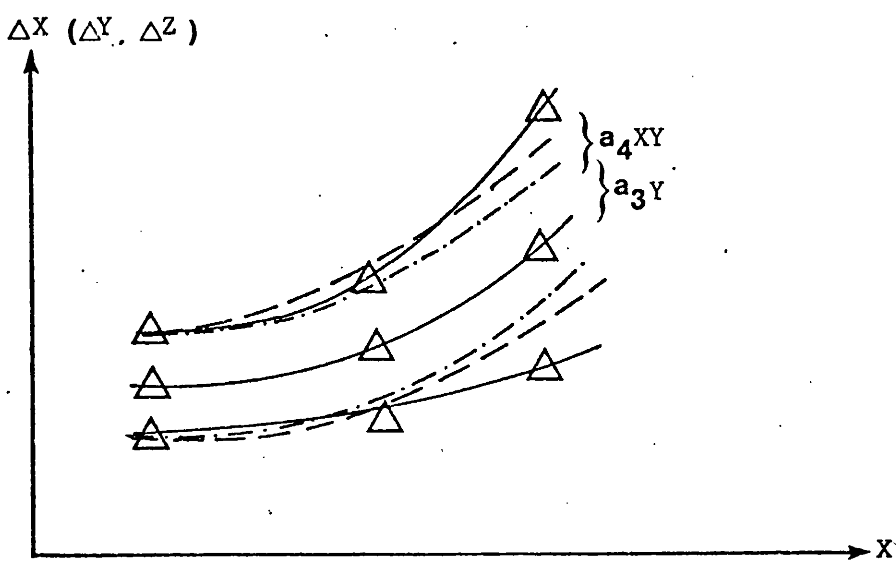


Fig.1.3 Correction Curves

In total, each correction surface has 6 degrees of freedom.

The above formulae not only compensate for all systematic strip deformations caused by constant transfer errors (Ackermann (1966)), but also deal quite efficiently with the effects of random transfer errors. Thus it can be said with some justification that Zarzycki's method of strip adjustment was a most significant contribution to the development of aerial triangulation.

Graphical methods of strip adjustment, rely heavily on control being located in the ideal positions (See Fig.1.2). With irregularly located or more numerous additional control points, computational methods involving formulae of the polynomial type are easier to apply. They include terms which are at least second degree in X. A typical example for flat terrain would be

$$\Delta X = a_0 + a_1X - b_1Y + a_2X^2 - 2b_2XY$$
$$\Delta Y = b_0 + b_1X + a_1Y + b_2X^2 + 2a_2XY$$
$$\Delta Z = c_0 + c_1X + c_2Y + c_3X^2 + c_4XY$$

In this case (in contrast to the previous set of formulae), ΔX and ΔY are interrelated, but ΔZ can still be treated independently.

The discrepancies (ΔX, ΔY, and ΔZ) are determined from the known control points and a set of equations is solved to give values for the 11 unknown transformation parameters, $a_0 \ldots\ldots c_4$. If redundant control is available, then a least squares solution is normally implemented.

The following more general formulae (which include Z terms) are appropriate for mountainous terrain (Ackermann (1966)).

$$\Delta X = a_0 + a_1X - b_1Y + c_1Z + a_2X^2 - 2b_2XY + 2c_2XZ$$
$$\Delta Y = b_0 + b_1X + a_1Y - d_1Z + b_2X^2 + 2a_2XY - 2d_2XZ$$
$$\Delta Z = c_0 - c_1X + d_1Y + a_1Z - c_2X^2 + 2d_2XY + 2a_2XZ$$

Although the formulae appear more complicated, they do in fact contain the same number of unknown transformation parameters. The difference lies in the fact that ΔX, ΔY, and ΔZ are inter-related, and it is no longer possible to consider a planimetric solution separately from that for height.

Polynomials of higher order can also be used. The choice of the formulae depends on whether the actual strip deformations are approximated better by second or third degree surfaces. This depends on:

(1) the total number and the grouping of the control points, and

(2) the distances between the control points.

Second degree formulae are used in cases where the control distribution is similar to the standard Zarzycki pattern - i.e. 3 bands, with 3 control points in each band. If 4 bands are available, the general rule is that second degree polynomials are used for bridging distances less than 5 models (Ackermann (1966)).

Polynomial adjustments are used basically because they are simpler to apply, and they produce good results for strips with standard patterns of control. When more control points are available and are arranged in a more irregular pattern, then the interpolation principle with polynomials should be abandoned in favour of more rigorous adjustment methods, such as the least squares strip adjustment procedure.

1.2.3.2 Least Squares Strip Adjustment

In this method, all the available control points are fully exploited, and any number of control points can be handled (i.e. within the computer capacity). The problem of the choice of formulae (as in polynomial adjustments) does not arise and the same type of mathematical model is used for any number and distribution of control points. This method is based on the theory of Vermeir, and assumes that the observational errors are in the transfer and setting operations, and that these are responsible for the strip deformations. A least squares adjustment is carried out which applies corrections to these quasi-observations. Strip triangulation may be regarded as the process of constructing 3-dimensional models and joining them to the proceeding ones by the transfer of absolute orientation, and that in each connection between models there are 7 transfer parameters. These parameters are associated with:

(1) The X - co-ordinate connection,

(2) The Y - co-ordinate connection,

(3) The Z - co-ordinate connection,

(4) S - The Scale transfer,

(5) a - The azimuth transfer,

(6) φ - The Longitudinal tilt transfer, and

(7) ω - The Lateral tilt transfer

Thus, according to the theory of Vermier the transfer elements are treated as "observations" in a strip.

$\Delta S_1, \Delta S_2$ ------------ ΔS_{i-1}

$\Delta a_1, \Delta a_2$ ------------ Δa_{i-1}

$\Delta \varphi_1, \Delta \varphi_2$ ------------ $\Delta \varphi_{i-1}$

$\Delta \omega_1, \Delta \omega_2$ ------------ $\Delta \omega_{i-1}$

and the setting elements Δx, Δy, and Δz.

These observations are assumed to be independent and of equal weight in setting up a least squares adjustment. From the known strip deformations at the control points, corrections are derived for the quasi-observations. Let the corrections required for these observations be as follows:

$V_{\Delta S_1}, V_{\Delta S_2}$ ------------- $V_{\Delta S_{i-1}}$ and $V_{\Delta x}$ ---------

$V_{\Delta a_1}, V_{\Delta a_2}$ ------------- $V_{\Delta a_{i-1}}$ $V_{\Delta y}$ ---------

$V_{\Delta \varphi_1}, V_{\Delta \varphi_2}$ ------------- $V_{\Delta \varphi_{i-1}}$ $V_{\Delta z}$ ---------

$V_{\Delta \omega_1}, V_{\Delta \omega_2}$ ------------- $V_{\Delta \omega_{i-1}}$

Let there be a functional relationship between the corrections to the observations and the strip deformations (i.e. the contradictions at the control points). So, for instance, in the case of flat terrain, there is for each ΔZ-contradiction (ΔZ = Z terrain -Z machine) a relationship (condition) of the type

$$\sum_{\gamma=1}^{i-1} (X - X_\gamma)\, V_{\Delta\varphi_\gamma} + Y \sum_{\gamma=1}^{i-1} V_{\Delta\omega_\gamma}$$

$$+ V_{\Delta z} + \Delta Z_0 + X \cdot \Delta\Phi_{(1)} + Y \cdot \Delta\Omega_{(1)} = \Delta Z \text{ - - - - - - - - } (1)$$

[for $i > 1$; i (nadir point number) = 0 - - - - - n; γ = 1 - - - - - - i-1]

where,

$\sum$	denotes the sum of,
X and Y	are the strip co-ordinates of any point in model (i) (with the origin of a right handed co-ordinate system in the first projection center), (i) (model number) = 1 - - - - - - - - n,
$\Delta Z_0, \Delta\Phi_{(1)}, \Delta\Omega_{(1)}$	are three unknown absolute orientation corrections for the first model.

Equation (1) can be written in matrix notation as:

$$uv + a\Delta c = t \text{ - - - - - - - - - - - - - - - } (2)$$

where,

v and Δc	are vector matrices for the unknowns,
u and a	are coefficient matrices of the unknowns contained in v and Δc, and
t	is a column matrix of constants (representing ΔZ in equation (1))

Equation (2) is the approach to be used for an adjustment utilising "Condition equations with unknowns". (Refer to the ITC publications A17 and A18, Ackermann (1966) for more details).

1.2.4 Block adjustment

1.2.4.1 Introduction

Aerial triangulation by means of single strips is fundamentally weak. The error propagation (double summation) is unfavourable and high accuracy can only be expected if control points are supplied at a sufficient density to compensate for such an unfavourable accumulation of error. This contradicts the basic idea of aerial triangulation, namely to use as few ground control points as possible. On the other hand the adjustment of blocks involving several strips improves the accuracy of aerial triangulation without the need for extra control. With the exception of some special cases (road surveys, pipe lines, etc), most mapping projects require at least two or more strips, and block adjustment is then most appropriate since it takes full account of the lateral ties between strips.

Accepting the fact that the strip may be considered as a special case of the block, a classification of current computational block adjustments may be presented according to the following criteria (Anderson 1973), Brown (1973)):

(1) The Unit of Computation or Adjustment

There are three distinct units of computation:

(a) The Single Photograph i.e. a single bundle of rays

(b) The Section i.e. a single model, or two or more from a single strip, or a group from overlapping strips.

(c) The Strip i.e. a single strip, or parallel strips, or a combination of parallel and cross or tie strips.

(2) The Computational Procedure

(a) A Sequential Procedure - where a piecemeal adjustment is carried out using linear, second degree, or higher degree polynomial transformations.

(b) A Simultaneous Procedure - where the desired parameters (for either planimetry, or height, or both) are computed through the use of one single simultaneous least squares adjustment.

(3) The Mathematical Formulation - whether condition equations, observation equations, or some variations of these may be used.

(4) The Numerical Solution - whether direct (or Gaussian) or iterative.

The direct solutions in operation today include:

(a) The Anblock Procedure (Van den Hout (1966)), which solves the transformation parameters directly, and then determines the tie point co-ordinates.

(b) The M.C.E. Geodetic Office Procedure (Smith (1967)) which applies a Cholesky reduction to the reduced normal equations and makes use of the banded form of the system.

(c) The P.A.T. - M series Hychol procedure (Ackermann (1972b)) which also applies the Cholesky reduction method to the submatrices of a hyper-matrix.

(d) The S.B.A.I.M (Simultaneous Block Adjustment of Independent Models) Program belonging to the University of Glasgow and developed by Elmaleeh (1976), which is similar in principle to the previously mentioned solutions, and deals separately and successively with the planimetric and height adjustment phases. It is the program used for processing the aerial triangulation data in this research.

The iterative solutions solve the normal equations by introducing approximate values (e.g. zero values) for some of the unknowns, and then solve for the remaining unknowns. By substituting the intermediate solutions as new approximate values, the process is repeated until the solution approaches the ultimate solution of the unknowns. The speed of convergence of iterative solutions depends on the conditioning of the equations, the distribution of ground control and the initial approximate values. The Gauss-Seidel method of solution is a typical iterative solution.

Some of the iterative solutions are:

(a) The Ordnance Survey Method (described in section 1.2.4.3).

(b) The Digital Block Adjustment of Amer (described also in section 1.2.4.3).

(c) NRC System (National Research Council; Canada) which is an iterative block adjustment of strips with sequential plan (XY) and height (Z) adjustment using polynomials of specified degree.

(d) NOS System (National Ocean Survey; USA) which is the same as the above NRC System.

The detailed descriptions of the current block adjustment methods are given below.

1.2.4.2 Analytical Block Adjustment with Strips

Block adjustment with strips can be achieved by the use of polynomials, or by applying the principles of least squares strip adjustment. The task is to deform and change the triangulated strips by simultaneously considering the values of points in the lateral overlaps and the values at ground control points. This is done in such a way that the adjusted strips fit the ground control as well as possible while, at the same time, discrepancies in the lateral overlaps are minimised. The method should be flexible enough to allow the inclusion of cross-strips or strips flown at peculiar angles.

The corrections for each strip are given by polynomial formulae of the second or third degree, and the parameters for all the strips are determined simultaneously. Thus it is an extension of the simple polynomial theory of strip adjustment mentioned previously in para. 1.2.3.1, except that in the case of the block, each strip is corrected by its own unique polynomial. The additional condition imposed on the solution is that identical points, measured in adjacent strips, should have the same final co-ordinates.

In this type of adjustment the number of unknown transformation parameters remains quite small (say 11-20 per strip), and it is therefore a suitable method for small computers. The number of unknowns to be determined simultaneously may be reduced even further if the adjustment is carried out in two phases - i.e. by separating the adjustment of planimetry from that of height. Provided the terrain is not mountainous, such an approach gives satisfactory results, particularly for small-scale work. However, from a theoretical point of view, the polynomial approach is not flexible enough, and a more correct use of the data is a least squares adjustment procedure based on independent models rather than strips.

1.2.4.3. Analytical Block Adjustment with Independent Models

In this case, stero-models are available and are treated as independent units. The task of the block adjustment with independent models is to connect the models amongst themselves and to the ground control by considering simultaneously the relative and absolute discrepancies. The models are usually changed by linear conformal transformations, and generally the triangulation and the adjustment are performed together. Strictly speaking therefore, the adjustment does not necessarily form strips in the first instance, although some adjustment procedures do start from strips as a good initial approximation. The principle of this type of block adjustment is to minimise the relative and absolute contradictions by applying 3-dimensional linear transformations to the models.

The Anblock method, developed by Van den Hout (1966), was an early application (for planimetry only) of this principle. It is virtually the same procedure as that used in the ITC Jerie Analogue Computer, but implemented by digital electronic computation rather than by mechanical analogy. The method adjusts the planimetric co-ordinates by applying linear transformations to each model. The basic units are independent models, and each model has 4 unknown transformation parameters which transform the model co-ordinate system (x, y) into the ground co-ordinate system (X, Y). The transformation equations are

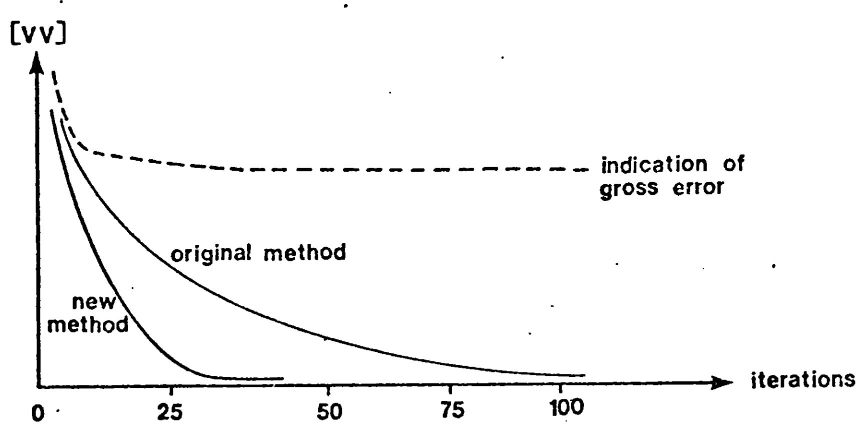
$$X = ax - by + c$$

$$Y = bx + ay + d$$

where a, b, c, d are the unknown transformation parameters. These parameters are determined by least squares such that the block fits the ground control as well as possible, and also so that relative discrepancies between models at the tie points are minimised. The total system has two groups of unknowns - the transformation parameters and the co-ordinates of the tie points. The total number of unknowns is considerable. For example, in a block of 200 models there are 800 unknown transformation parameters and about 500 unknown co-ordinates of the tie points (the exact number depends, of course, on how many tie points are used).

The Ordnance Survey Method (Proctor (1962)), which is basically the same idea as the Anblock method, has been in operation since about 1960. It does however include an adjustment of the heights, and a 3-dimensional transformation of the co-ordinates of each model. These 3-dimensional transformation formulae are not linear, and so they must be linearised before they can be used to solve for the transformation parameters. Iterative cycles are therefore an essential feature in the computational procedure of this method, and any other 3-dimensional method. In this method, accurate heights are only possible if the projection centres are also regarded as tie points.

The iterative procedure developed at the Ordnance Survey avoids the formation and solution of normal equations. Each model is transformed separately by spatial transformation (x, y, z) to fit temporary control points. The co-ordinates of a temporary control point are obtained by taking the means of the machine co-ordinates of the tie point for all models in which that particular tie point occurs. Improved terrain co-ordinates of the tie points (including the projection centres) are found from linear transformations of each model separately onto its temporary control points, and wherever terrestrial control points appear they are used directly. After all models have been fitted as well as possible to these temporary control points, there will still be gaps between adjacent models. New temporary control points are computed, and again the models are fitted onto these control points. This procedure is repeated until the gaps between models are small enough. Originally about 80 iterations were required, but by introducing so called <u>acceleration factors</u> this has now been reduced (for blocks of 100-200 models and moderate control density) to 30-35 iterations. However a fast computer is still necessary. Fig.1.4 shows the decrease of the sum of the squares of the residuals as a function of the number of iterations for different cases.



<u>Fig.1.4.</u> The decrease of the sum of the squares of the residuals as a function of the number of iterations for different cases

Similar to the Ordnance Survey method is the digital block adjustment of Amer (1962). The latter is a typical example of a block adjustment method based on the concept of analogue computers. It was developed during the same period as the Ordnance Survey method, and the planimetric adjustment phase is very similar in principle to the ITC-Jerie analogue method. The features may be summarised as follows:

(1) Sections (one or more models) are used as adjustment units.

(2) The linear transformation parameters (4 per section) are determined iteratively, and successively, for each section in the block. The parameters are thus updated within each iteration cycle.

(3) The formation of the normal equations for the whole block at the same time is not required for the determination of the transformation parameters.

(4) The iteration process of the adjustment is directed at reducing the sums of the squares of residuals at tie points (or section corners) successively through the block.

The number of iterations required for convergence is the main difference between the two solutions of Amer and Jerie. Amer's solution is equivalent to an iterative solution of the normal equations and it requires as many iterations as there are sections in the block. Jerie's solution is equivalent to a direct solution of the normal equations and it requires 2-3 iterations. The large number of iterations for convergence in Amer's solution is due to the iterative procedure which depends on the initial provisional data. This number, as reported by Boniface (1967), has been reduced and the method modified greatly since 1962.

The PAT-M43 Hychol procedure (University of Stuttgart Program) is analogous to the Gauss elimination of the unknowns but uses submatrices instead of single elements. Hence the procedure solves for groups of unknowns by inverting the respective small-submatrices and carrying out a back solution.

The S.B.A.I.M. Program (University of Glasgow) is a three-dimensional, simultaneous block adjustment procedure, similar in principle to the PAT-M43 procedure and based on the following criteria:-

(1) the use of independent models, measured in a stereo-plotter or determined from comparator observations, as the basic units of the adjustment;

(2) the concept of a spatial similarity transformation of models, in which the transformation parameters (7 per model) are determined separately and successively in groups of 4 planimetric parameters and 3 vertical parameters per model;

(3) the least squares approach and a direct solution of reduced normal equations pertaining to the transformation parameters. (The convergence requires 2-3 iterations).

The mathematical formulation of this three-dimensional simultaneous block adjustment procedure is given in Appendix B.

1.2.4.4 Analytical Block Adjustment with Independent Photographs.

This is the most general approach. It uses the single photograph (or bundle of rays) as the basic unit for measurement, computation and adjustment. Plate co-ordinates are measured in a comparator and all other operations are carried out by computation. Relative orientation (and the formation of models), the joining of models to give strips and the joining of strips to give the block are carried out by computation as part of the overall adjustment process. The relative orientation is represented therein by intersection conditions; the joining of strips is represented by so-called co-ordinate conditions, and the control points are introduced by so-called control conditions.

This approach (the bundle adjustment) is the most rigorous of all least squares block adjustment. There are no intermediate instrumental steps between the photo co-ordinate measurements and the final adjusted co-ordinates. The adjustment can cope with any auxiliary data, any distribution of control, and any variation in overlap, scale, distortion information, etc. It is based on the collinearity condition equations and solves for the intersection of all conjugate rays simultaneously. In other words, this approach involves the simultaneous solution of the relative and absolute orientation of all photographs, i.e. the determination of 6 orientation parameters (3 rotations and 3 shifts) per photograph, and the determination of the ground co-ordinates of all points. Within n photos and m points there will be 6n + 3m unknowns to be solved simultaneously. This number of unknowns is based on the assumption that the measured plate co-ordinates of all points have been previously corrected for all significant sources of systematic error. If additional appropriate parameters for minimising the effect of uncompensated systematic errors are added as unknowns and solved for within the bundle adjustment procedure, then the adjustment is referred to as bundle adjustment with self-calibration (refer to Chapter 3).

The solution by means of a general block adjustment program is an enormous computational problem, even for a large electronic computer. For example, in the case of the triangulation of a block of ten strips with 20 photos in each strip, the number of absolute orientation unknowns would be 1,200. The co-ordinates of tie points which have to be determined would amount to about another 1,000 unknowns. This means that the total number of unknowns in such a problem could be well over 2,000. This large number of unknowns has been a major factor preventing the widespread adoption of this general approach to block adjustment.

The main requirements for implementing a bundle adjustment type of solution may be summarised as follows:

(1) A comparator to measure the plate co-ordinates (x, y) of all image points participating in the adjustment.

(2) A suitable mathematical model based on one form or another of the collinearity condition equations.

(3) A large fast computer for an efficient solution of the large number of unknowns to be determined.

(4) Estimates of the exposure station positions and camera orientations, since the collinearity equations are non-linear. Depending on the approach taken, estimates for the co-ordinates of all pass points may also be needed.

It is not surprising therefore that organisations have preferred simpler approaches to meet their technical requirements.

1.3 Concluding remarks

The expected accuracy at different points in a strip or a block with different control patterns and densities may be derived theoretically, relying on the principles of the propagation of errors. It is far more difficult however to draw firm conclusions from practical tests, since most triangulated areas do not have sufficient ground control or check points to allow these conclusions to be made. In recent times, it was realised that experimental confirmation or disproval of the theory was necessary, and during the period 1967-68 long discussions took place in the OEEPE regarding the establishment of a comprehensive test area. The Oberschwaben Test Area (Ackermann (1973)) was established as a direct result of these discussions, and experiments with photography of this area showed the existence of uncompensated systematic errors after block adjustment. These systematic errors are discussed more fully in the next chapter.

CHAPTER II

Systematic errors in Aerial Triangulation

2. Systematic Errors in Aerial Triangulation

Systematic errors are one of three types of error encountered in any measuring science; the other two types are blunders (or gross errors) and random errors. Blunders are identified and removed from a set of observations by data editing or data verification techniques (refer to Davis (1967), and Wilke (1967)), and random errors are dealt with by the well-known least squares computational procedures. On the other hand systematic errors require special consideration, and this is particularly so in aerial triangulation. However before this aspect of aerial triangulation is considered in some detail it is necessary to define more exactly what is meant by the expression "systematic error".

2.1 Definition

The definition of systematic image deformation is a difficult task in itself (Kubik, Bosman, Clerici, and Eckhart (1973)). An intuitive description reported by these authors is as follows:

"Let us consider a photogrammetric block consisting of P photograms. In every photogram nine points are selected at standardised locations. (See Fig.2.1).

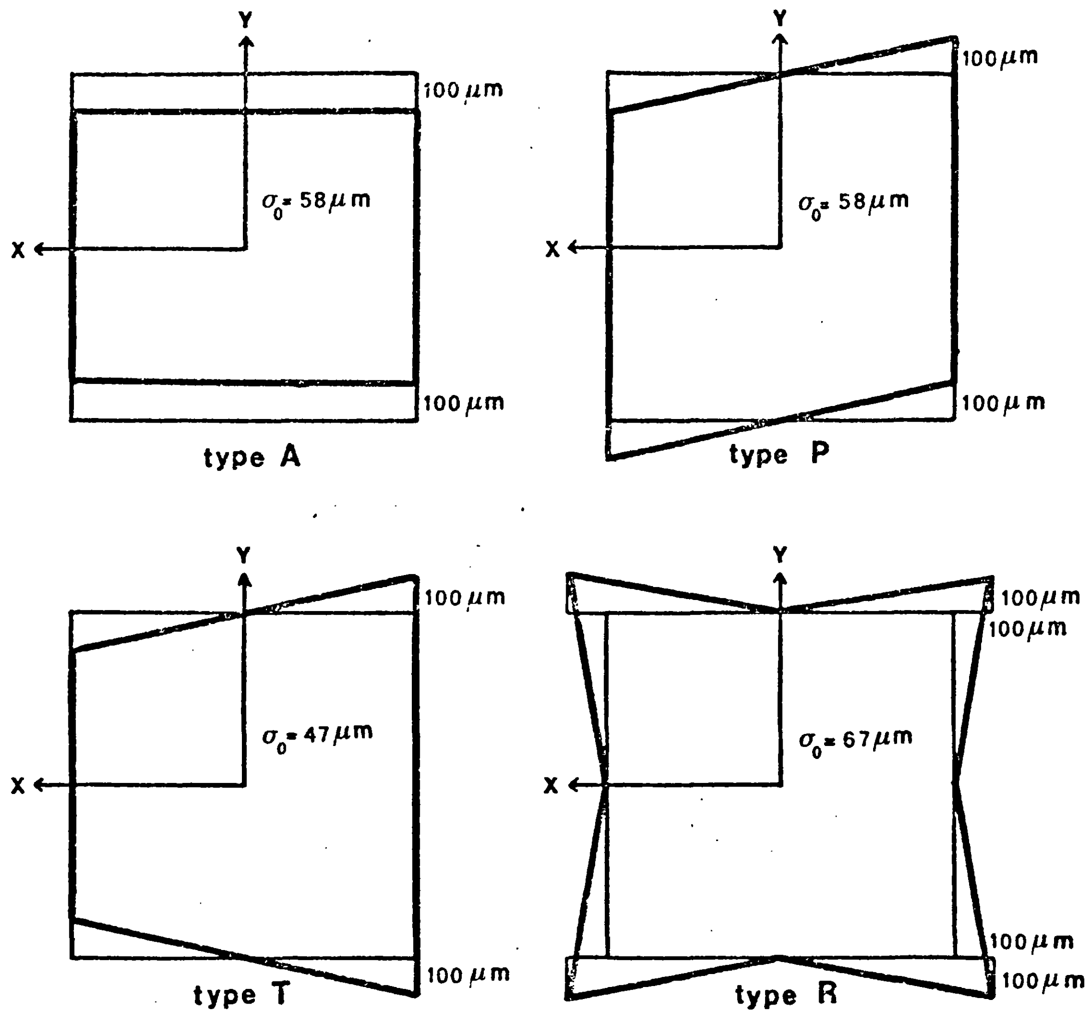


Fig.2.1 The basic pattern of image deformation

We then define the systematic image deformation by the set of 2 x 9 co-ordinate errors at each of these points, which is the same for all photograms in the block. We also agree on some (linear) interpolation rule, by which the co-ordinate errors may be obtained in any other point of the plates. In other words, we assume the image deformation to be constant for all photograms in the block".

This somewhat tortuous definition is in fact an over simplification, but it suffices for the time being.

2.2 Sources

Systematic errors originate from different sources. A light ray traversing from the object space to the image plane passes through the atmosphere and a lens system. The rays will be deflected as they pass through these media, giving rise to systematic errors termed atmospheric refraction and lens distortion. Film shrinkage and expansion will displace the positions of image points with respect to theoretical positions on the photographic plane, and because these displacements are of a similar type (i.e. of a recognisable pattern) they are classified as being systematic. Lack of film flatness during exposure or during printing are other examples of sources of systematic error. Certain mechanical or optical errors in the photogrammetric instrument result in systematic errors in the values of measured image or model co-ordinates. These systematic errors may be classified according to the sources from which they originate, in the following way:

1. Symmetrical or radial lens distortion.
2. Asymmetrical lens distortion or decentering distortion.
3. Atmospheric refraction.
4. Earth's Curvature.
5. Image Plane deformations resulting from:

   (a) film shrinkage and expansion,

(b) curvature of the camera platen, and

(c) micro undulations of the film.

6. Distortions due to instrumental errors.

A detailed presentation of these systematic image deformations is given below.

2.2.1 Symmetrical or radial lens distortion

The projection centre in photogrammetry is supposed to be a single point in space. However, in practice, this is not physically possible. A single lens produces large aberrations on the film and so photogrammetric cameras are always equipped with a lens system which is a combination of several components. Designing a lens system which behaves exactly as a single projection centre is physically impossible and, because of lens aberrations, the object space ray is bent and displaced when it passes through a lens system into the image space. The effect of this is that the angle made by the image space ray and the optical axis will differ from the angle between the object space ray and the optical axis. This effect is known as lens distortion; it is generally measurable and is usually presented as a displacement which is a function of position in the image field. It causes the tangent to the actual light ray, at the centre of projection, to deviate from a line expressing the condition of co-linearity between object point, center of projection and image point.

The lens distortion may be expressed as two components; one component along the radial direction from the principal point, and the other perpendicular to the radial direction (termed tangential distortion The major part of the radial component is symmetrical about a point at or near the principal point and so it is referred to as symmetrical lens distortion. The remaining component of the radial lens distortion and the tangential distortion form what is known as asymmetric or decentering lens distortion. The imperfect centering of lens elements during their assembly causes this decentering lens distortion, as the name implies.

Lens distortions may be compensated for in instrumental or analogue aerial triangulation by:

(1) using the same camera lens for the projection instrument, or

(2) using compensating plates located in the image plane, or

(3) changing the principal distance automatically in either an optical projection solution (e.g. the Kelsh Plotter) or in a mechanical projection solution (e.g. the Galileo Stereosimplex) as the photograph is being scanned.

In analytical aerial triangulation a mathematical model, derived from known information about the lens distortion, may be used to apply corrections to the measured image co-ordinates. A generally accepted model for symmetrical lens distortion is an odd-ordered polynomial* of the form

$$\Delta = K_1 d^3 + K_2 d^5 + K_3 d^7 + K_4 d^9 + \ldots$$

where

Δ is the radial lens distortion,

$K_1, K_2, K_3, K_4, \ldots$ are coefficients of radial lens distortion,

d is the distance from the centre of symmetry for the point under consideration, given by

$d = [(x_\gamma - x_p)^2 + (y_\gamma - y_p)^2]^{\frac{1}{2}}$ (see Fig.2.2), and

where,

x_γ , y_γ are image co-ordinates of the point, and

x_p, y_p are image co-ordinates of the point of symmetry.

* A term in d is generally not included for reasons which will become clear shortly.

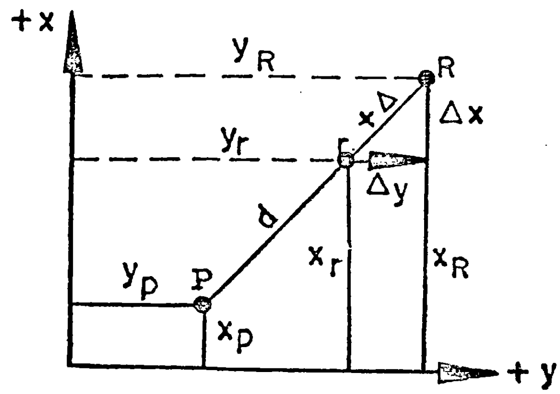


Fig.2.2 Radial lens distortion

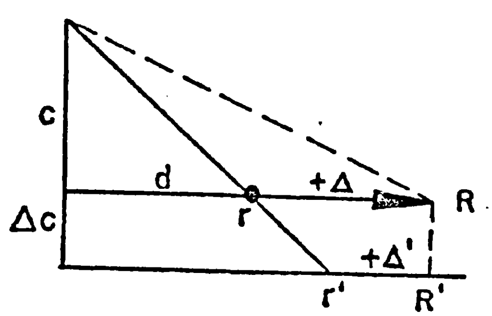


Fig.2.3 Correction to principal distance

The radial lens distortion Δ is positive if the image point is displaced away from the point of symmetry.

Due to the relatively small magnitude of lens distortion, it is immaterial if d is interpreted as the radial distance with or without distortion.

It may be seen from Fig.2.3 that:

$$\frac{\Delta - \Delta'}{\Delta c} = \frac{d}{c} \quad \text{or} \quad \Delta' = \Delta - \frac{\Delta c}{c} d$$

where

c is the principal distance,

Δc is a correction to the principal distance, and

Δ' is a new value for lens distortion, which corresponds to the corrected principal distance $(c + \Delta c)$.

The type of polynomial which best describes lens distortion varies from one lens system to another. Thus some are represented more realistically by a full polynomial such as

$$\Delta = K_1 d + K_2 d^2 + K_3 d^3 + K_4 d^4 + K_5 d^5 + \ldots$$

However the lens calibration procedure is usually based on a mathematical model which minimises Δ', and when this is the case, Δc is determined as an integral part of the procedure. This explains the omission of the term in d and the deletion of the even-ordered terms. Thus the remaining distortion may then be represented by the equation

$$\Delta' = k_1 d^3 + K_2 d^5 + K_3 d^7 + K_4 d^9 + \ldots$$

2.2.2 <u>Asymmetrical Lens Distortion (or Decentering Distortion)</u>

As mentioned previously in para. 2.2.1, the asymmetric or decentering lens distortion is the remaining component of lens distortion.

Brown (1956) has developed the following mathematical model for decentering lens distortion:

$$\Delta x = [P_1(\gamma^2 + 2\bar{x}^2) + 2P_2\bar{x}\bar{y}](1 + P_3\gamma^2 + P_4\gamma^4 + \ldots)$$
$$\Delta y = [P_2(\gamma^2 + 2\bar{y}^2) + 2P_1\bar{x}\bar{y}](1 + P_3\gamma^2 + P_4\gamma^4 + \ldots)$$

where

Δx is the x - component,

Δy is the y - component,

$\bar{x} = x - x_p$,

$\bar{y} = y - y_p$,

$r^2 = \bar{x}^2 + \bar{y}^2$,

x, y are image co-ordinates of a point with respect to the fiducial centre,

x_p, y_p are image co-ordinates of the principal point with respect to the fiducial centre and

P_1, P_2, P_3, . . . are coefficients for the decentering distortion.

Only the first two terms of the above mathematical model are normally found to be significant in practice. The amount of the decentering distortion depends, of course, on how well the centering of the lens elements has been carried out during their assembly.

2.2.3 Atmospheric Refraction (See Fig.2.4)

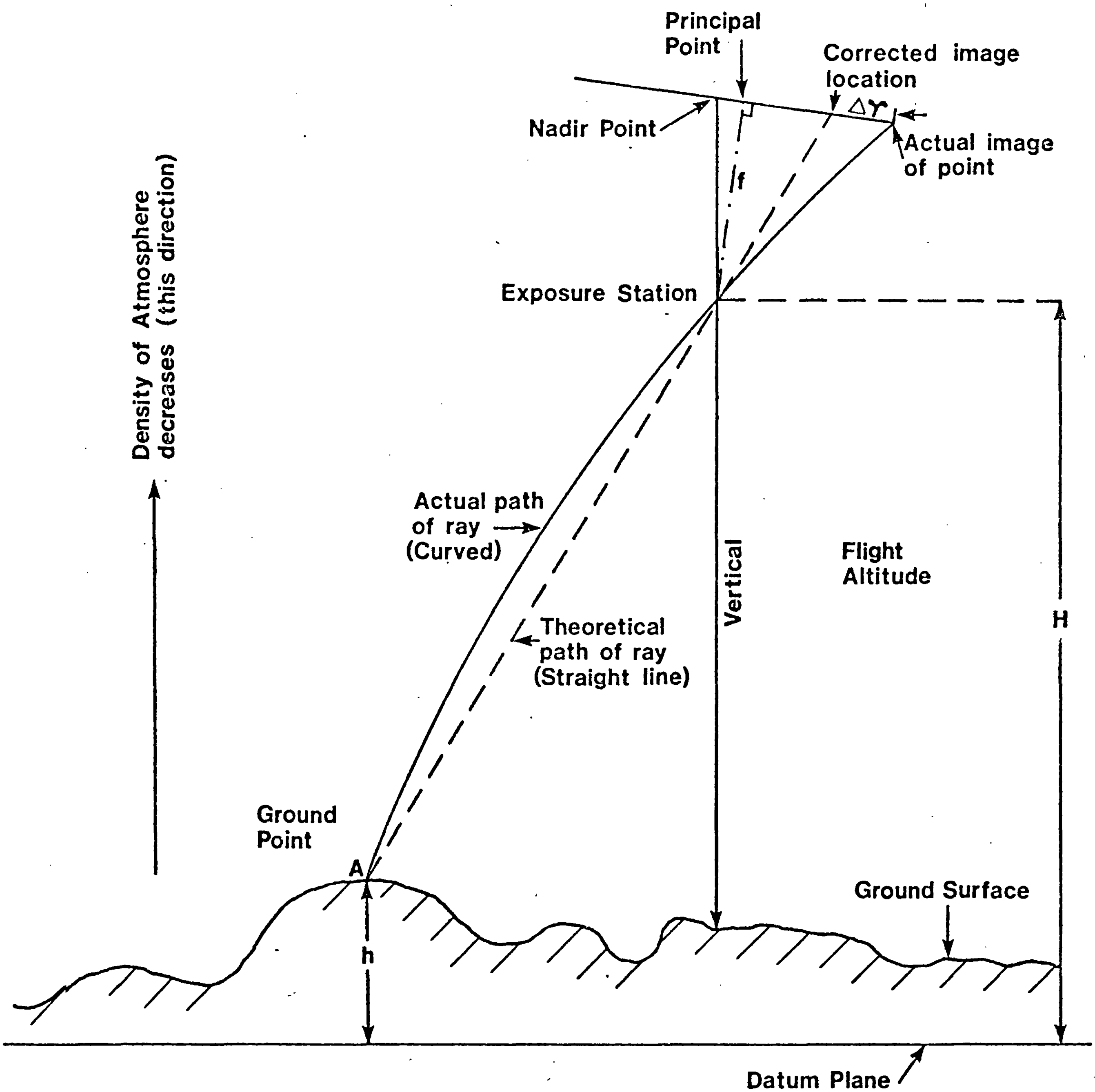


Fig.2.4. Atmospheric Refraction

A light ray passes through the atmosphere when it traverses from the object point on the ground to the camera lens, and because the density of the atmosphere decreases with height, it will be refracted from its theoretical straight line path. All image points are displaced outward from the photo nadir point. An alternative way of viewing the matter is to say that atmospheric refraction causes the tangent to the actual light ray to deviate from a line expressing the condition of co-linearity.

As reported in ASP (1966), the atmospheric refraction is represented by the following formula:

$$d\alpha = K \tan\alpha \quad \text{(see Fig.2.5)}$$

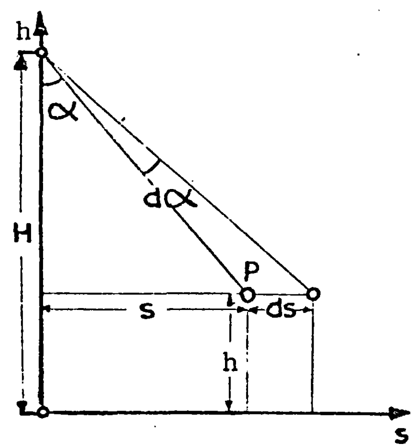


Fig.2.5 The influence of refraction

where,

α is the angle between the actual ray path and the vertical;

$d\alpha$ is the angle between the actual ray path and the theoretical (straight line path); and

K is a coefficient, calculated according to a formula such as

$$K = \left[\frac{2410\ H}{H^2 - 6H + 250} - \frac{2410h}{h^2 - 6h + 250} \cdot \frac{h}{H}\right] \cdot 10^{-6}$$

which has been determined empirically.

In this formula

H is the flight altitude above sea level in kilometres,

and

h is the ground elevation above sea level in kilometres.

The rigorous use of the above formulae will not be possible initially since the location of the nadir point is not known before the completion of the aerial triangulation. However in the normal case , where vertical or nearly vertical photographs are employed, a first assumption can be made that the nadir point coincides with the principal point. In such a case, the refraction correction becomes radial from the principal point and may be treated in a manner similar to radial lens distortion. That is to say (see ASP, 1966).

$$\Delta x = \frac{x}{\gamma} K \left(\gamma + \frac{\gamma^3}{f^2}\right)$$

$$\Delta y = \frac{y}{\gamma} K \left(\gamma + \frac{\gamma^3}{f^2}\right)$$

where,

Δx, Δy are corrections for atmospheric refraction,

γ is the radial distance from the principal point,

f is the focal length of the camera; and

x, y are image co-ordinates of the point under consideration.

In the early period of analytical photogrammetry atmospheric refraction was not given the same close attention as lens distortion, and it was only in the early sixties that it came under close scrutiny. Leyonhufvud (1952/3), Schmid (1959), Brown (1962), Faulds and Brock (1964), and Bertrom (1965) are among the few who have published on the subject in the English language. The value of the refraction coefficient becomes more reliable with increasing flight altitude, and the relationship between the percentage error (E) in the value and flying height is shown in Fig.2.6 (See Malinen (1969)).

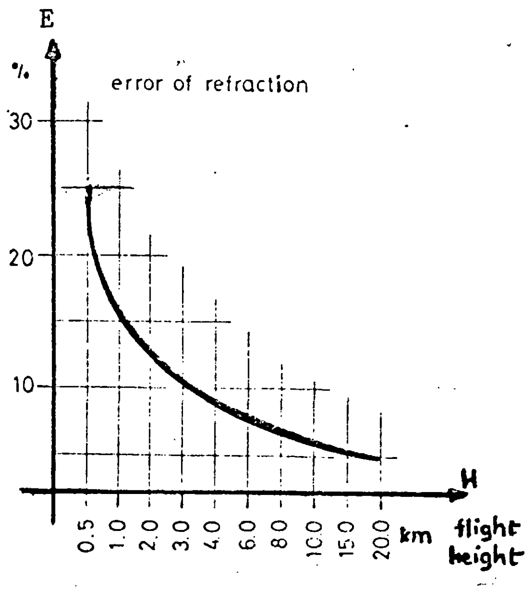


Fig.2.6 The relative error of the refraction coefficient

2.2.4 Earth Curvature

Earth's curvature has no effect on aerial triangulation if the co-ordinates of ground control points are in a 3-dimensional orthogonal co-ordinate system. However the adjustment of most triangulation projects is based usually on plane or map projection co-ordinate systems and, in these cases, the deformation due to Earth's curvature can be quite significant. Image points will be displaced inwardly towards the nadir point as shown in Fig.2.7.

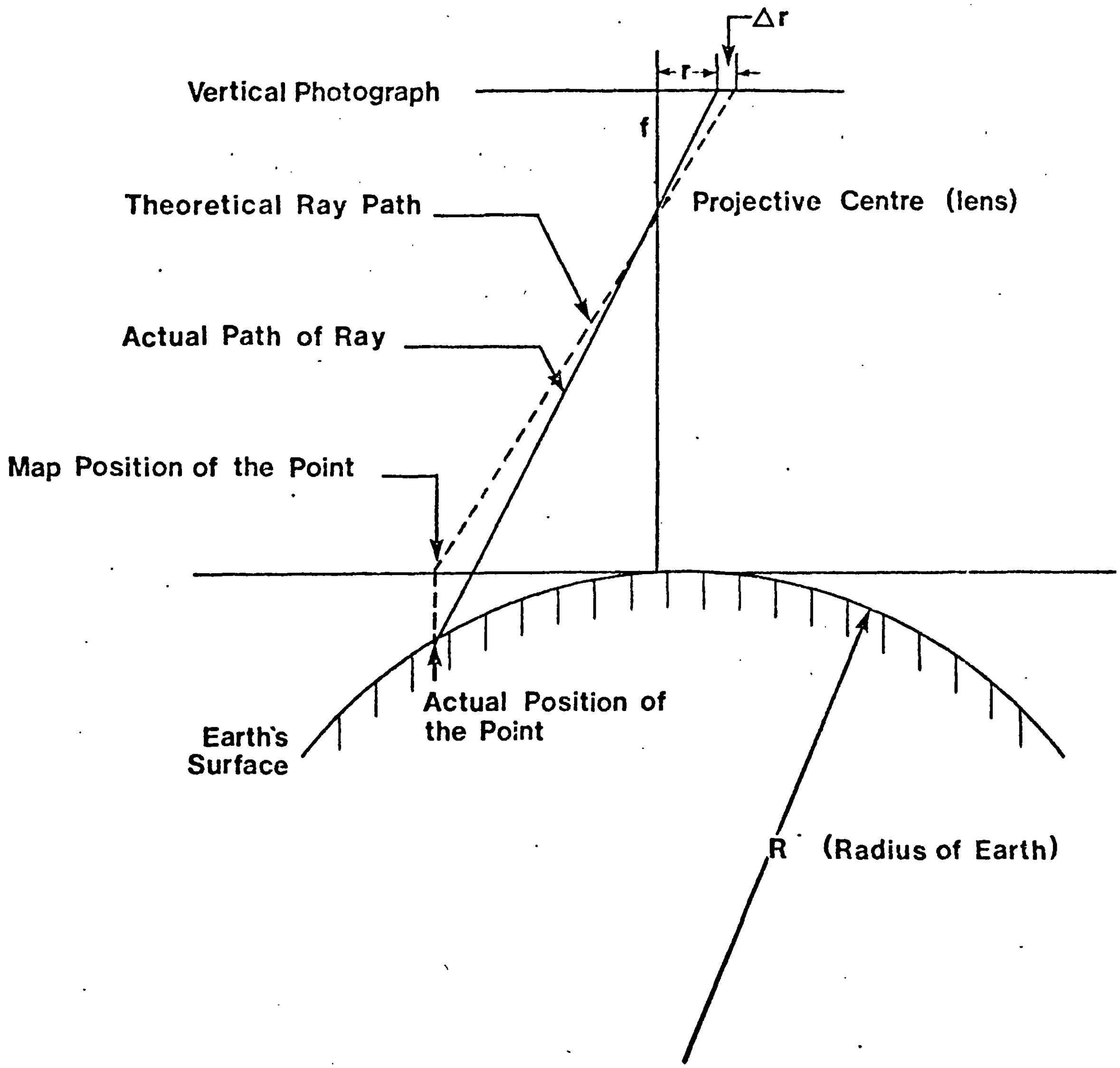


Fig.2.7 Correction for Earth Curvature

The displacement is due to the vertical distance between the map position of the point and the Earth's curved surface. Thus, for a near-vertical photograph

$$\Delta\gamma = \frac{H\,\gamma^3}{2Rf^2}$$

where

$\Delta\gamma$ is the inward displacement of the image point due to Earth's curvature,

γ is the radial distance of the image point from the principal point,

H is the flight altitude above the datum plane,

R is the mean Radius of the Earth, and

f is the focal length.

The use of a truly orthogonal co-ordinate system, such as the geocentric co-ordinate system or a local space rectangular co-ordinate system (sometimes referred to as a "secant plane" system), avoids the difficulties caused by Earth's curvature, and is to be recommended wherever possible. The secant plane system is a local 3-dimensional orthogonal system in which the co-ordinate directions are comparable to map directions, and in which the co-ordinate values are considerably less than geocentric values. What is more, the third dimension is in approximately the same direction as height values in a photogrammetric model - a matter of some importance with certain block adjustment procedures. For further details on the geometric and secant plane systems refer to ASP (1966) and Harris, Tewinkel, Whitten (1962).

2.2.5 Image Plane Deformations

Displacements in the positions of image points within the photographic plane may take place both during the exposure of the film in the aerial camera and in the interval between the time of exposure and the time of actual measurement in a photogrammetric instrument. They are due to:

(1) dimensional change of the film itself due to shrinkage or expansion of the film base material;

(2) displacements due to lack of film flatness during exposure or diapositive printing;

(3) deformation of the image plane during exposure.

The dimensional changes within the film itself are related to:

(a) the type of emulsion base;

(b) the method and conditions of film processing and developing; and

(c) the storage conditions.

Lack of film flatness gives rise to two different types of deformations

(a) macro film; and

(b) micro film deformation.

The macro film deformation is the large undulation of the photographic plane during the film flattening and is represented usually by a low-order polynomial. The micro film deformation is the result of very small undulations of high spatial frequency and cannot be represented by a low order polynomial. These small undulations introduce serious accuracy problems, particularly at the corners of the photographs.

Some experimental work on the problem of film flattening has been done by Clark (Clark (1972)) and Meier (Meier (1972)) but their methods of eliminating macro film deformation did not produce any significant improvement in accuracy. They did however prove that micro film undulations produced significant changes in the positions of the image points. The very high spatial frequency of these undulations, coupled with the fact that very often they tend to be random in occurance, makes their representation by mathematical functions extremely difficult. Brown (1973) has nevertheless stated that a practical method of overcoming these short-period undulations is desirable. The use of glass as an emulsion base is a well-known method which has been used in the past, but is considered impractical in modern photogrammetric operations due to the weight, handling problems, fragility and bulk associated with the glass plates themselves. The problem of compensation for film deformation is approached in three ways when dealing with analytical photogrammetry:

(1) By using more stable emulsion bases;

(2) By introducing a calibrated reseau plate into the camera through which a grid of reseau crosses, is imaged on each photograph at the instant of exposure. The co-ordinates of the reseau marks on the reseau plate should be known very accurately, and the co-ordinates of the photographic images of the same marks are measured together with the images of terrain points. Differences between the two sets of values for the marks close to a terrain point image then serve as a basis for a linear correction of the measured co-ordinates of the terrain point image. The method of using a reseau plate to model the deformations resulting from lack of film flatness is effective, but it is time consuming and costly.

(3) By defining, determining, and applying a mathematical correction model which is a polynomial function of the image co-ordinates.

The following is a generally agreed form of the polynomial representing film deformation. (See Brown (1973)).

$$\Delta x = a_1x + a_2y + a_3x^2 + a_4xy + a_5y^2 + a_6x^2y + a_7xy^2$$

$$\Delta y = b_1x + b_2y + b_3x^2 + b_4xy + b_5y^2 + b_6x^2y + b_7xy^2$$

where

Δx, Δy are corrections to x and y image co-ordinates,

$\left.\begin{matrix} a_1, a_2, \ldots\ldots, a_7 \\ b_1, b_2, \ldots\ldots, b_7 \end{matrix}\right\}$ are unknown co-efficients defining the film deformation.

The curvature of the camera platen which is used as the supporting surface in cameras utilizing vacuum (pneumatic) flattening techniques to flatten the film during exposure, is another source of error caused by lack of film flatness.

The following polynomial can represent the systematic deformations resulting from curvature of the platen if the same platen is used throughout a flight. (See ASP (1966), Brown (1973)).

$$\Delta x = \frac{x}{r} (C_1x^2 + C_2xy + C_3y^2 + C_4x^3 + C_5x^2y + C6xy^2 + C_7y^3)$$
$$\Delta y = \frac{y}{r} (C_1x^2 + C_2xy + C_3y^2 + C_4x^3 + C_5x^2y + C6xy^2 + C_7y^3)$$

where

$C_1, C_2, \ldots, C_7$ are coefficients defining the curvature of the camera platen, and

x, y are image co-ordinates.

2.2.6 Distortions due to Instrumental Errors.

All photogrammetric instruments, whether mechanical, optical, optical-mechanical or electronic in nature, have limitations arising from their design and manufacture.

Instrumental defects give rise to errors in the measured co-ordinates when the operator positions the measuring mark on the point of interest. Stereo-plotting machines and comparators are two main types of instrument used to measure x and y co-ordinates of image points. The errors which are more important in stereo plotters are those which will affect:

(1) a correct inner orientation;

(2) a correct relative orientation;

(3) a correct absolute orientation; and

(4) accurate measurements of the three dimensional model co-ordinates of each point.

As far as comparators are concerned, the two most important sources of error are those which produce effects similar to those of linear film deformation and lens distortion, namely

(1) periodic screw errors; and

(2) non-orthogonality of the x and y axes.

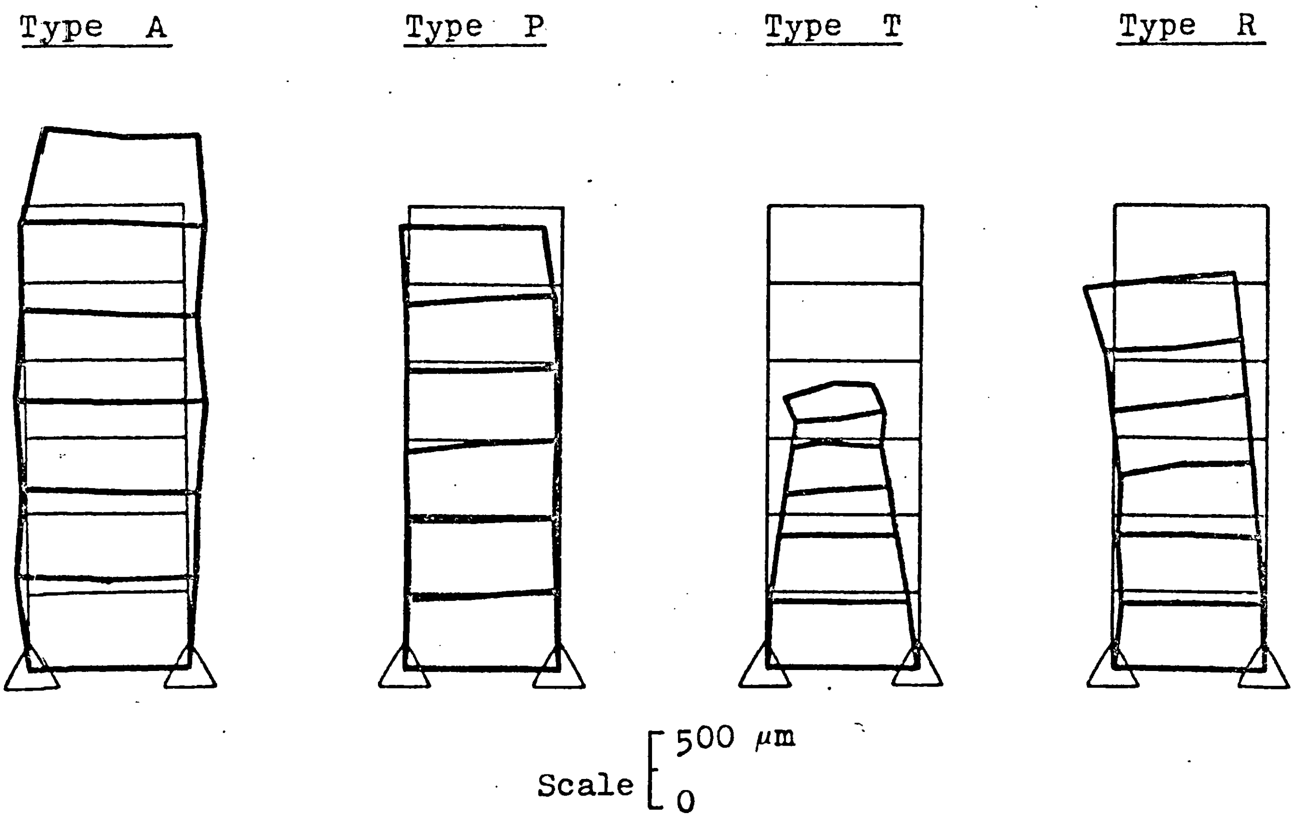
Turning to stereo-plotters, the construction of these devices is much more complicated than that of a comparator, so there are far more sources of error likely to be present which have to be taken into account. Yassa (1976) has presented a paper concerning the metric performance of precision stereo-plotters in which he describes a method of identifying and eliminating the calibration errors in stereo-plotters by the use of additional parameters.

To keep the instrumental errors within tolerable limits, photogrammetric instruments need to be checked periodically and calibrated from time to time.

2.3 Effects

Some authors such as Wiser (1973), Camps (1973), Kupfer (1973), Bauer (1973) and Haug (1976) have conducted independent studies dealing primarily with the problem of systematic image errors in the Oberschwaben material (see Ackermann (1973a)). They come to the conclusion that uncompensated systematic errors have a significant effect on the results of aerial triangulation and can be very dangerous and unpredictable. It was found also in various studies conducted by Kubik (1971), Anderson (1972), Bauer and Müller (1972), Brown (1973), Ebner (1973), and Ackermann (1973b), that there were a number of discrepancies between the accuracy models derived from the propagation of random errors and the results of practical tests. Kubik (1971) and Clerici (1972) have also studied the propogation of error in synthetic strips and blocks of wide angle photography, up to a block size of nine strips, each with eighteen models. The terrain covered by the blocks was assumed to be horizontal. Their aim was to answer certain questions concerning the effects of selected basic types of image deformation. The selected types were the four basic patterns, denoted by A, P, T, and R in Fig. 2.1, . . .

which were proposed by Kubik (1971). Clerici (1972) studied the effects of systematic image errors after Anblock Adjustment (see Van den Hout (1966)), using models as computational units. Some of the deformations for different patterns of systematic image deformation are shown below in Figs, 2.8, 2.9, and 2.10.



	Type A	Type P	Type T	Type R
σ_x	= 265 μm	= 73 μm	= 601 μm	= 276 μm
σ_y	= 64 μm	= 45 μm	= 139 μm	= 112 μm
$\hat{\sigma}_0$	= 192 μm	= 61 μm	= 436 μm	= 211 μm
$e_x max$	= 468 μm	= 148 μm	=1237 μm	= 537 μm
$e_y max$	= 128 μm	= 133 μm	= 324 μm	= 250 μm

Fig.2.8 Planimetric deformation of a strip of six models due to systematic image errors. (Practical triangulation projects, with superimposed systematic image errors)

Fig.2.9 Height deformation of a strip of six models due to systematic image errors. (Practical triangulation projects). — Terrestrial height control.

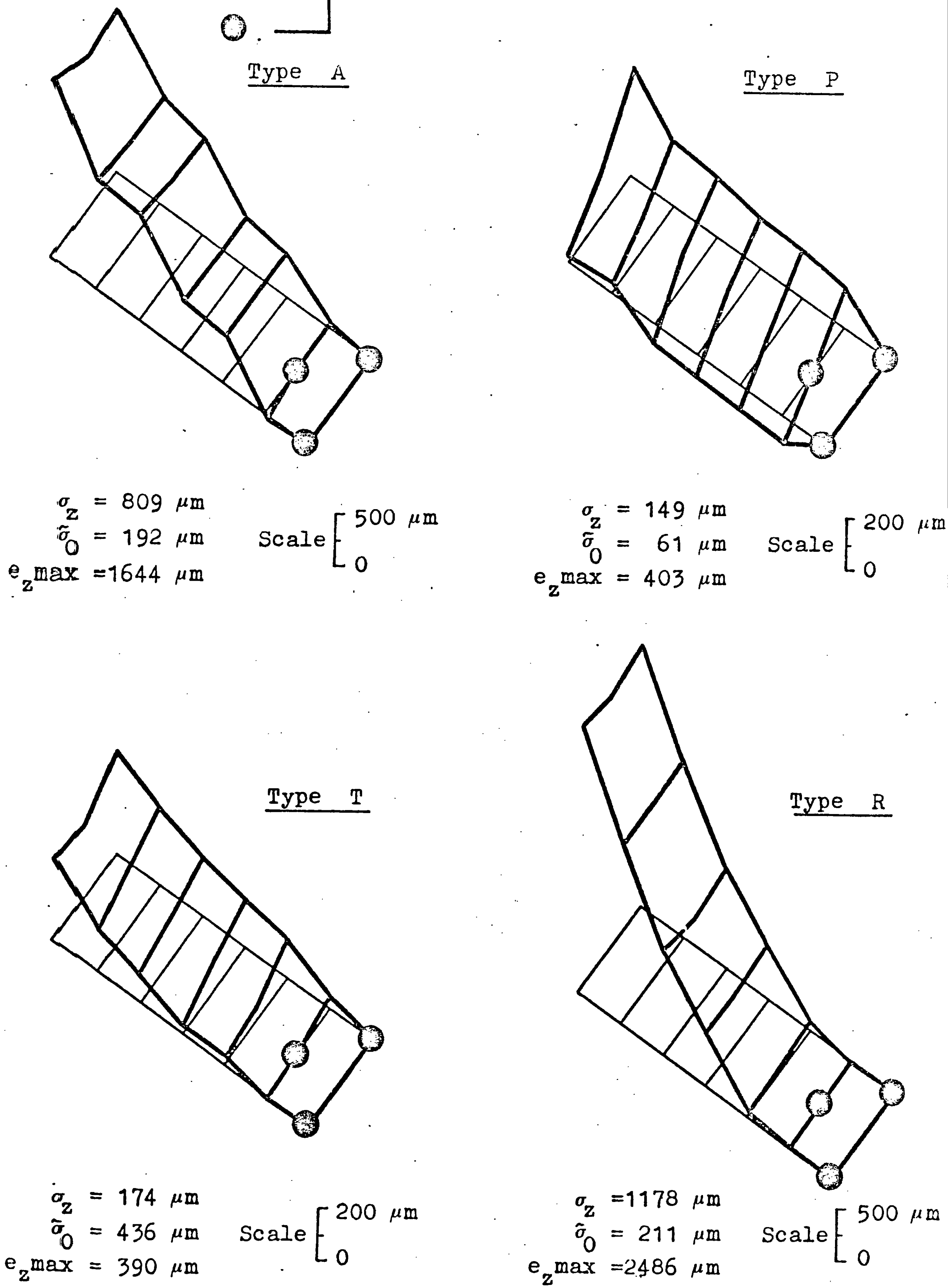
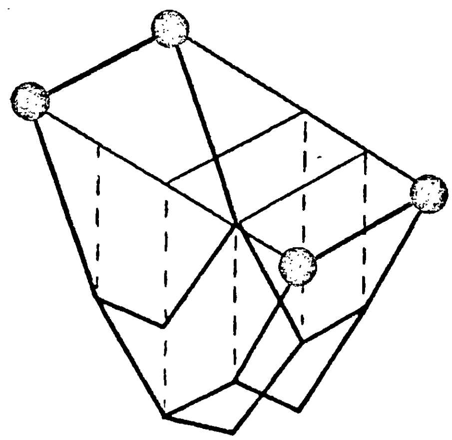


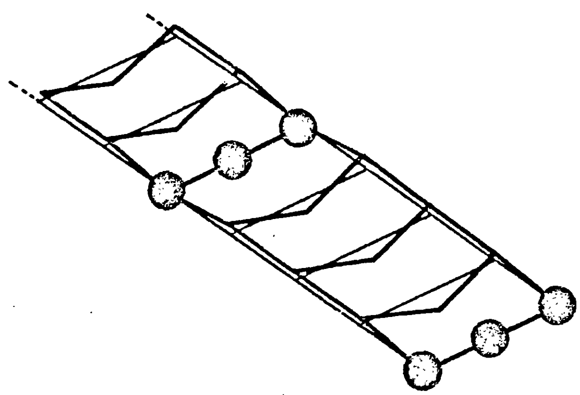
Fig.2.10. The effects of systematic height image errors after Bundle adjustment (Deformation type R)



Scale [500 μm — 0]

Deformation type R, strip with two bands of control.

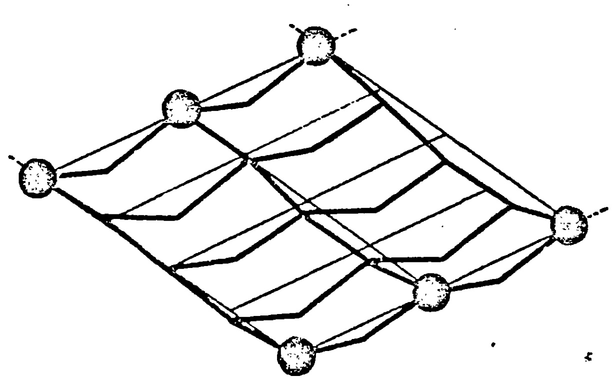
σ_z =430 μm
$\tilde{\sigma}_0$ = 27 μm
$e_z max$ =754 μm



Scale [500 μm — 0]

Deformation type R, strip with three bands of control. (The third band is not shown).

σ_z = 61 μm
$\tilde{\sigma}_0$ = 54 μm
$e_z max$ =128 μm



Scale [500 μm — 0]

Deformation type R, block with three bands of control. (The third band is not shown).

σ_z =140 μm
$\tilde{\sigma}_0$ = 45 μm
$e_z max$ =292 μm

The significent effects of uncompensated systematic errors in bundle adjustment and adjustment of independent models, as reported in the literature quoted above, may be summarised as follows:

(1) Accuracy decreases more rapidly than predicted as control is reduced.

(2) The absolute accuracy of adjusted blocks in relation to the standard errors of unit weight does not agree with theory.

(3) The theoretical expectations are not met when the results of bundle adjustment are compared with those obtained from the adjustment of independent models.

(4) Changing from 20% to 60% sidelap does not increase the accuracy to the extent that had been predicted by theory, and there is even a loss in vertical accuracy.

(5) The accuracy of the planimetric co-ordinates X and Y differs greatly, when in theory they should be approximately the same.

(6) The standard error of unit weight σ_o depends very significantly on the control distribution and the overlap, when in theory this should not be the case.

(7) With the same unit density of control per photograph, the decrease in accuracy with increasing block size is larger than expected.

2.4 Conclusion

The effects of systematic image errors in triangulation can be very serious and unpredictable. The empirical results have confirmed that there are a number of discrepancies between the accuracy models derived from theory and the results obtained from practical tests. The conclusion is that systematic errors remain in the results, and the solution to such a problem has become one of the main objectives of recent research in aerial triangulation. Various techniques for eliminating or rather minimising uncompensated systematic errors have been developed, and these techniques will be discussed in the following Chapter.

CHAPTER III

The Various Approaches to Compensation of Systematic Errors in Aerial Triangulation

3. The Various Approaches to Compensation of Systematic Errors in Aerial Triangulation

3.1 Introduction

Steady progress in improving the available materials and instruments used in photogrammetry (such as film, film base, lens and auxiliary camera equipment, comparator, stereo-plotter, etc.) has been made ever since photogrammetry became a tool of topographic mapping. Although the qualities of the materials and instruments have improved steadily, there is an ever increasing demand for photogrammetry to achieve even higher accuracies. In fact, with the modern processing tools available at present and provided that great care is taken during the flight mission, film processing and handling, the systematic image errors can be kept within reasonable limits. The remaining systematic errors should not, in most cases, be dangerous for conventional mapping projects; these components only become dangerous in special applications. However one can expect their effects to be eliminated by the various methods mentioned in this chapter.

Before proceeding any further, it should be mentioned that the simplest method to eliminate a large part of the systematic errors is to correct systematic image errors as far as possible before the commencement of the triangulation process. This has particular merit from a practical point of view, since the methods devised so far for the detection and compensation of systematic errors after triangulation are both cumbersome and elaborate.

3.2 Various approaches

3.2.1 The Methods of reducing the Effect of Systematic Errors before the Commencement of Triangulation.

Such methods are:

(1) increasing the side lap - for instance, to 60%;

(2) varying the arrangement of photos by flying in both

longitudinal and transverse directions;

(3) eliminating atmospheric influences by repeating the flight at different times;

(4) using another camera/film combination; and

(5) calibrating the camera by means of a test area at the beginning and end of the flight.

The significant disadvantage of all these methods is that their expense is greater than the expense of an ordinary flight with 60% forward overlap, and 30% side lap (Bauer and Müller (1972)). Furthermore, the increase in accuracy does not justify this increase in expenditure.

3.2.2 The Method of providing Sufficient Ground Control throughout the Interior of the Block

This is a well-known and effective method for the partial elimination of uncompensated systematic errors. The dense control constrains the error propagation and reduces the effect of systematic errors.

The disadvantage of this approach is that it involves high costs due to the large numbers of ground control points that will be required. However, as reported by Brown (1975), Satellite Doppler methods may provide dense ground control economically with sufficient accuracy in the near future.

3.2.3 The Method of Image Refinement

In this method, attention is directed towards developing a full understanding of the types and physical sources of systematic errors, so that these can be represented by some kind of empirical or mathematical model. Correction models to compensate for systematic errors in image co-ordinates before adjustment can then be formulated.

It is well known that the results of practical tests in aerial triangulation are not entirely consistent with theoretical accuracy models if residual systematic errors are present in the photogrammetric data. Tests may be made to determine whether the residuals are significant or not. If they are significant, then it may be possible to modify the correction models so as to further reduce the effect of systematic errors to a tolerable level. This procedure is no more than repeating the original process of detecting and eliminating the systematic errors before the adjustment. There are physical and practical limitations to this "image refinement" process and as a result, there will always remain some residual systematic errors in the data.

A typical example of this method is the analysis and correction of systematic errors in the Oberschwaben test area (see Haug (1976)) by the technique devised by Masson d'Autume (1972). This method starts after the corresponding block adjustment. It can be applied both to the bundle method, for an analysis and correction of the systematic errors of the image co-ordinates, and also to the method of independent models for an analysis and correction of the systematic errors of the model co-ordinates. The block of photographs (photo scale 1:28,000 in Oberschwaben test area), or models, is subdivided into sub-areas for example as shown below in Fig. 3.1 and Fig.3.2 which are equivalent in size to the area covered by a single photo and single model respectively. The tie points (lying inside the small squares) are located in each sub-area (as shown by the boundaries A B C D in the Figs.)

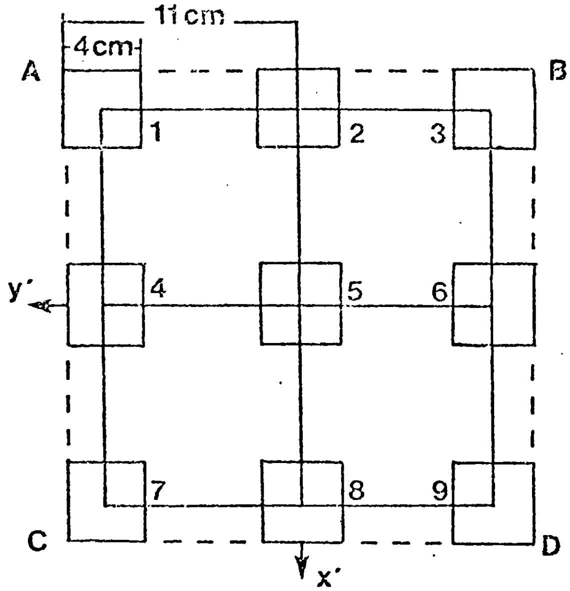


Fig.3.1 (Sub-area of a block of photos)

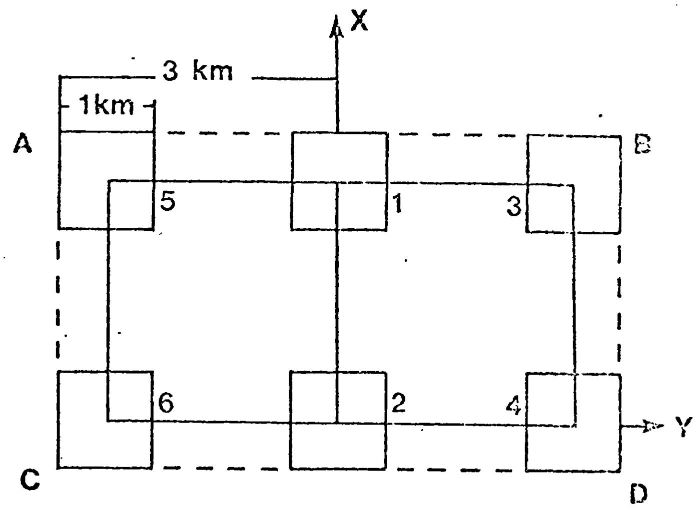


Fig.3.2 (Sub-area of a block of models)

For analysis, use is made of the corrections to the transformed image or model co-ordinates (obtained after block adjustment) of these tie points (i.e. use is made of contradictions at tie points). The average correction to the tie points in each sub-area describes the systematic errors of the corresponding group of photographs or models to a good approximation. These systematic errors are corrected with the aid of second-degree correction polynomials. These corrections are applied to the image or model co-ordinates of the tie points. The corrections for planimetry and height are separate in the case of the model co-ordinates. The corrected image or model co-ordinates are then used for another block adjustment and the results are again analyzed for residual systematic errors following the procedure outlined above. The photographs or models are corrected once more if necessary, followed by another adjustment; and as a rule, one or two correction cycles will be enough.

In his publication, Haug (1976) refers to the results of tests using observations of the "Oberschwaben" test area. The results are given for block adjustment of data which has not been corrected for systematic error, and also for the same data which has been corrected for systematic error using the method advocated by Masson d'Autume (1972). Haug reports that the results of bundle block adjustments which did not include corrections for systematic error, were almost always less accurate than the results of the corresponding model block adjustments. But after including corrections for systematic error, the results obtained from the bundle block adjustments were better than those of the model block adjustments. An improvement in accuracy by a factor of 2.1 in planimetry and 3.5 in height was obtained in the former case. Haug reports also that changing from 20% to 60% side lap had resulted in an average increase in accuracy of 20 to 30%. This was still below the expected theoretical accuracy.

3.2.4 The Method of Self-calibration or Additional Parameters.

A more general and elegant method for the treatment of systematic errors would be to extend the mathematical model of the projective relationships, and to include additional parameters in adjustments for systematic image errors, so that not only random errors but also systematic errors can be compensated for during the adjustment of photogrammetric data (Schmid (1971), Kubik (1972), Brown (1975), Ebner (1976)).

In this method (the method of self-calibration), the mathematical model is modified by adding some terms which are polynomials of image co-ordinates with unknown coefficients which are determined from the adjustment of the data. These additional unknown coefficients are called "additional parameters". They do not modify or extend the error model but rather they bring corrections to the image co-ordinates of each point. Each of these has been displaced by systematic deformations from its true colinear position depending on the location or position of this image point. In this sense, the additional parameters give the image points the flexibility of moving in the image plane in order to satisfy the colinearity condition as closely as possible.

The colinearity equations would then be as follows:

$$x - x_0 + \Delta x - f \frac{A\,(X-X_c) + B\,(Y-Y_c) + C(Z-Z_c)}{D\,(X-X_c) + E\,(Y-Y_c) + F\,(Z-Z_c)} = 0$$

$$y - y_0 + \Delta y - f \frac{A'\,(X-X_c) + B'(Y-Y_c) + C'(Z-Z_c)}{D\,(X-X_c) + E\,(Y-Y_c) + F\,(Z-Z_c)} = 0$$

where,

X, Y, Z are co-ordinates of ground points in a rectangular ground system,

X_c, Y_c, Z_c are co-ordinates of the exposure station in the ground system,

x, y are image co-ordinates referred to the fiducial centre of the photograph,

x_0, y_0 are co-ordinates of the principal point referred to the fiducial centre, and

f is the focal length of the camera.

$\left.\begin{matrix} A, B, C \\ A', B', C' \\ D, E, F \end{matrix}\right\}$ are elements of the rotation matrix which defines the rotation of the terrain system with respect to the image system.

$$\Delta x = \Sigma_{ij}\, a_{ij}\, x\, y$$

$$\Delta y = \Sigma_{ij}\, b_{ij}\, x\, y$$

where,

a_{ij} and b_{ij} are the coefficients of the polynomials for the systematic image deformations; i.e., Δx and Δy are equal to the polynomials of the x, y image co-ordinates with unknown parameters of a_{ij} and b_{ij}.

The type and number of additional parameters to be introduced into colinearity equations are important and a decision has to be made in advance as to the type of polynomials as well as to the number of unknown parameters to be used. Brown (1975) suggests that these should be a combination of physically interpretable expressions along with empirical expressions. It is desirable to have terms corresponding to well-known sources of systematic errors such as lens, film, atmosphere, etc., Another basic assumption to be followed is that the additional parameters should be block-invariant, which means that exactly the same terms are applied to all photographs in the block.

Brock (1973) and Brown (1975) proposed an error model which incorporates a total of 29 parameters, most of which are designed to account for the well-known physical sources of systematic image deformations. Also some empirical terms are included in order to compensate for anomalous distortions, as well as for any other type of otherwise unmodeled systematic errors. Bauer and Müller (1972), Bauer (1973), Bauer (1975), Schut (1975), Salmenpara, Anderson, and Savolainen (1975), and Ebner (1976), have proposed error models with fewer parameters and studied them with varying degrees of success. In fact, there is not much reported in literature about the improvement of accuracy, the numerical problems arising from the inclusion of additional parameters on the adjustment, or the effects of the weights in handling these parameters. Bauer and Müller (1972) have suggested that the following principles should be followed for the introduction of additional parameters into the adjustment (together with some valuable comments).

"(a) <u>All photos of the block should obtain the same correction</u>

This means that the conditions do not alter from exposure station to exposure station. Apart from the fact that the same camera is used, it may be doubtful whether this unqualified assumption is sufficient. On the other hand, a different treatment of different photos requires additional information for the adjustment.

(b) The number of parameters for the correction statement has to be as small as possible.

The additional unknowns require more control points. Often the questions as to how many control points should be used, and the arrangement or manner in which they should be deployed can not be answered simply. Consequently it is difficult to handle block triangulations with many correction terms.

(c) The parameters have to be chosen in such a way that their mutual algebraic correlation is small.

High correlation causes a deterioration in the accuracy of the final result and indicates that at least one correction term is superfluous. This is particularly so considering the small size of the image corrections.

(d) The parameters have to be as uncorrelated as possible with the orientation unknowns of the bundle adjustment (3 co-ordinates of the exposure station, x - tilt, y - tilt, and swing for each photo). This requires, for instance, that the image scale is not altered.

(e) Terms which are insensitive to a rotation around 180° are preferable.

In this case, it is not necessary to distinguish the direction in which the strips were flown. Furthermore, the position of the photos in the comparator (direction of triangulation) has no influence on computation."

Some tabulated results of block adjustments with and without corrections for systematic errors can be found in Bauer and Müller (1972), Bauer (1973), and Ebner (1976). These are summarised in Table 3.1 below.

Author	Test Area	Side Lap	Comments regarding the improvement when systematic errors are accounted for
Bauer & Müller (1972)	Oberschwaben	20%	The height accuracy (at negative scale) improved from 20μm to 14μm i.e. 1.4 times
Bauer (1973)	Oberschwaben	20%	The height accuracy improved slightly, namely by about 20%
		60%	An experiment to reach a height accuracy improvement greater than 20% was not successful
	Steinbergen	60%	An increase in height accuracy of about 40% was obtained.
Ebner (1976)	Oberschwaben	20%	A significant improvement in height accuracy was obtained.

Table 3.1 (Summary of comments on some recent tests)

As reported by Bauer and Müller (1972), the values for height accuracy are 20μm (in the negative scale) for bundle adjustment without corrections and 14μm (again in negative scale) for Bundle Adjustment with Additional Parameters (BAP), in which a control point distance of five models is assumed. This rise of 1.4 times in accuracy was obtained simply by an improved formulation of the error equations.

Bauer (1973) reports that the height accuracy in existing tests using a side-lap of 20% could be improved only slightly, namely by about 20%. An experiment in the Oberschwaben test to reach a greater improvement of height accuracy, using 60% side-lap, was not successful. An increase in accuracy of about 40% was obtained in the Steinbergen test (reported in the same publication) using a 60% side-lap. In his compensation of systematic errors by analytical block adjustment with common image deformation parameters, Bauer comes to the conclusion that a side-lap of 60% is necessary for good accuracy of height as it gives the stability to a block such that correction terms for height will yield a good increase in accuracy.

The test results of bundle block adjustment and block adjustment by independent models (Ebner (1976)), demonstrated that, by simultaneous self-calibration, excellent accuracies can be obtained, even when systematic errors of considerable size exist. He makes an important statement that the test results obtained with simultaneous self-calibration meet the theoretical expectations in a two-fold way. Firstly, the standard deviations of unit weight are practically independent of the control distribution patterns used in his tests. Secondly, the empirical ratios representing the error propagation in a block are in good agreement with the corresponding theoretical predictions based on random errors only. He concludes by saying that these facts indicate that the systematic deformations of the image and model co-ordinates are compensated adequately and that the remaining errors can be considered as random.

3.2.5 The Method of Bundle Adjustment with Increased Weight for the Control Point Observations.

Often residual errors remain at the control points, which cannot be explained by the block adjustment process or by the observation errors at the control points. This method accounts for these residuals by assigning a larger weight to the observations of the control points than to those of the tie points.

It has to be mentioned that proper weight allocation is not easy to achieve and moreover the obtained results after this method has been applied suggest that the method is ineffective (Bauer and Müller (1972)).

3.3 Conclusion

The various methods devised so far for the detection and compensation of systematic errors after triangulation are both cumbersome and elaborate. It is clear that each of them has problems of one type or another regarding the effectiveness, simplicity, or cost of application of the method. These difficulties discourage their practical application so that methods need to be devised which will avoid such problems. The development of a new approach (the Terrestrial/Photogrammetric (TP) technique) which is both simple and effective to apply is discussed in the next chapter.

CHAPTER IV

The TP Technique - a New Approach to the Compensation of Systematic Errors

4. The TP Technique - a New Approach to the Compensation of Systematic Errors

4.1 Introduction and Notation

It is well known that the expected accuracy of aerial triangulation can be derived theoretically for different points in a strip or a block with different control patterns and densities based on the principles of propagation of errors.

The theoretical investigations which have been carried out with simplified error models may be classified into three main groups (Kubik and Kure 1972) as follows:-

(1) Accuracy of planimetric strip and block triangulation based on the independent model approach (e.g. Ackermann (1966) and Ebner (1971));

(2) Accuracy of height strip and block triangulation based on the theory of transfer errors e.g. Jerie (1964) and Jerie (1968);

(3) Accuracy of planimetric and height strip and block triangulation based on the bundle approach e.g. Kunji (1968), Kilpela (1970), and Talts (1973).

The results of these theoretical investigations are shown together with remarks on their significance and conclusions in Appendix A. The summary results which are given are mainly those for height since that is the main concern of this research. Some conclusions from these height accuracy investigations are as follows:-

(1) The maximum standard errors in a block with bands of control across the strips will occur at the edges of the block.

(2) Maximum and mean standard errors in a block are almost independent of the size of the block, but depend mainly on the bridging distance between the bands of control (See Figs. A.2 - A.8 in Appendix A).

(3) The bridging distance between bands of control must be reduced if the overall accuracy of the block is to be improved.

When auxiliary data is not available, the standard arrangement of height control in a block is in bands across the strips. These control points should be located in (or close to) each lateral overlap, in order to control the lateral tilts of the strips.

The Terrestrial/Photogrammetric (TP) technique which will be described below in Section 4.2, makes use of these conclusions in detecting and eliminating uncompensated systematic height errors in adjusted photogrammetric blocks. The notation which is used throughout the thesis to describe the technique and the patterns of different points in the various tests that follow is given below.

△ —— Terrestrial Planimetric Control

⊙ —— Terrestrial Height Control

○ —— Uncorrected Photogrammetric Height Control

◎ —— Corrected Photogrammetric Height Control

● —— Terrestrial Height Check Point

▣ —— Terrestrial Height Check Point (for detection of maximum systematic error after first adjustment).

○ —— Photogrammetric Height Check Point.

+ —— Terrestrial Planimetric Check Point

• —— Tie Point

The technique improves the height accuracy by reducing the original bridging distance using new bands of corrected photogrammetric tie points as height control together with the original bands of terrestrial height control. The maximum height errors (which will exist midway between bands of height control) are detected and corrected. The arrangement of the terrestrial control points (which will be discussed in detail later) provides a means of finding the systematic height errors, including the

the effects of Earth's curvature, existing after the first adjustment. That is to say, the pattern of these errors can be recognized and so they can be corrected. The systematic height errors will of course also affect the planimetric accuracy and, by eliminating them, the latter can be expected to be improved also.

The above comments have focussed mainly on systematic height error. Systematic planimetric error arises also in block adjustment and again the amount will depend primarily on the pattern and distribution of the control points. However, because of the geometry of photogrammetric blocks, the problem is not as serious as for height adjustment. Accuracy requirements (particularly for topographic mapping) are met far more readily and with the use of fewer control points than in height adjustment. Although this thesis concentrates on the improvement of height accuracy, it is conceivable that a similar technique could be used to detect and compensate for systematic planimetric error.

4.2 Theory and description of the TP technique

4.2.1 Basic geometry

4.2.1.1 Least Squares Collocation

The problem of estimating a random quantity from certain available data arises in the TP technique. Least squares collocation solves this problem. (See Mikhail (1976)). It makes possible the estimation of parameters (Δ) based on observations at control points. It also makes possible the estimation of filtered values for the variables representing the observations (signals) at all points. It combines the well-established techniques of adjustment with those of interpolation and filtering. The task of interpolation is to estimate (interpolate) the values at locations other than those for which control data are given. Since measuring errors also occur at the control points, the process of estimation applies to these points as well.

For an explanation of interpolation and filtering, three functions

$l(u)$, $s(u)$, and $r(u)$ may be defined such that

$$l(u) = s(u) + r(u)$$

The observable function $l(u)$ comprises both systematic and random components. The systematic component is represented by the function $s(u)$ and the random component is represented by the function $r(u)$. The process of finding estimates of the systematic and random functions is sometimes referred to as collocation, and it involves both the processes of interpolation and filtering when a set of values $l(u_1)$, $l(u_2)$, ..., $l(u_n)$ from a given population $l(u)$ are available.

Some examples of least squares collocation are the general least squares technique of adjustment of observations and functionally independent parameters (see Mikhail (1976)), and the two adjustment procedures referred to by J.M. Tienstra (1956) as Standard Problem I (Adjustment of Conditional Observations) and Standard Problem II (Adjustment of indirect observations). (See also M. Tienstra (1966) and Kure (1970)). It must be clearly understood that a least squares adjustment gives only the best distribution of residual errors, and the reliability of the results after its application is strongly correlated with both the accuracy of the observations and the reliability of the formulae adopted in the mathematical model. Once a general mathematical model is specified, the model remains in the background and consideration is then given to the practical and computational aspects of selecting a particular least squares technique. The mathematical formulation varies from one adjustment method to another and consequently the adjustment unknowns will also vary. However, whatever the type of equations or the number of unknowns, the latter must include the ground co-ordinates of the pass points which are required for the application of the TP technique. In this research work, the S.B.A.I.M (Simultaneous Block Adjustment of Independent Models) Program developed by El Maleeh (El Maleeh (1976)) has been used to provide these ground co-ordinates. (See Appendix B).

The collocation procedure provides adjusted heights which may include systematic errors arising from other sources, explained previously in Chapter 2. The TP technique detects and reduces these systematic errors, and, in the next section, four typical examples are taken to help explain the principle and the practical implementation of the technique. The two basic procedures have been termed Procedures A and B. Each of these is described with two different control patterns (Patterns 1 and 2), so comprising the four examples.

4.2.1.2 Application of the technique - Specific cases.

Procedure A (Control Pattern 1)

Fig.4.1 (a) represents a block of models with a particular height control pattern. The horizontal lines define strips of photography, while the vertical lines represent sections across the strips at specific intervals along the strips. Individual models are not shown on the diagram. The pattern of height control used in this example consists of three lines of height control points located at the beginning, middle and end of each strip.

Step 1 The procedure starts with the formation of the block and its initial adjustment using the height control points available in the block, i.e. lying along Sections 1, 3 and 5. From this first stage of the adjustment, the maximum height errors may be assumed to exist midway between the bands of height control, i.e. along the vertical sections marked 2 and 4.

Step 2 This involves a repetition of the adjustment procedure, using as height control the photogrammetrically-determined values lying along Sections 2 and 4 derived from Step 1. This produces new values for the points lying along Section 3, which can be compared with the known values for these points. The difference between the two sets of values for the points lying along Section 3 is the basis for correcting the photogrammetric values of the points lying along Sections 2 and 4.

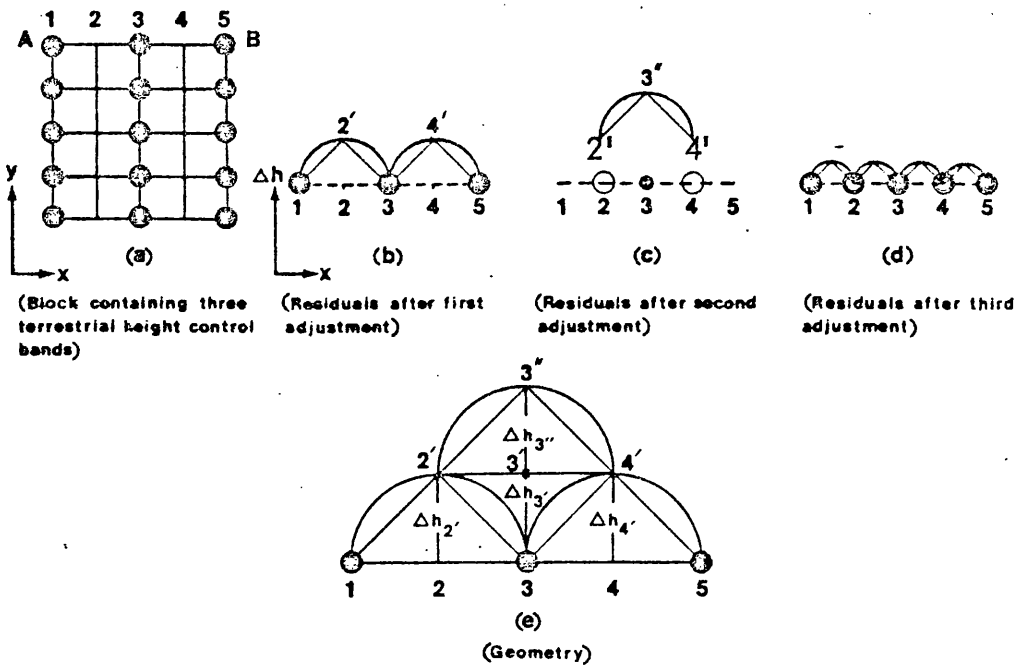

1 2 3 4 5
A
B
y
x
(a)
(Block containing three terrestrial height control bands)
2′ 4′
Δh
1 2 3 4 5
x
(b)
(Residuals after first adjustment)
3″
2′ 4′
1 2 3 4 5
(c)
(Residuals after second adjustment)
1 2 3 4 5
(d)
(Residuals after third adjustment)
3″
Δh3″
2′ 3′ 4′
Δh3′
Δh2′
Δh4′
1 2 3 4 5
(e)
(Geometry)

Fig. 4.1

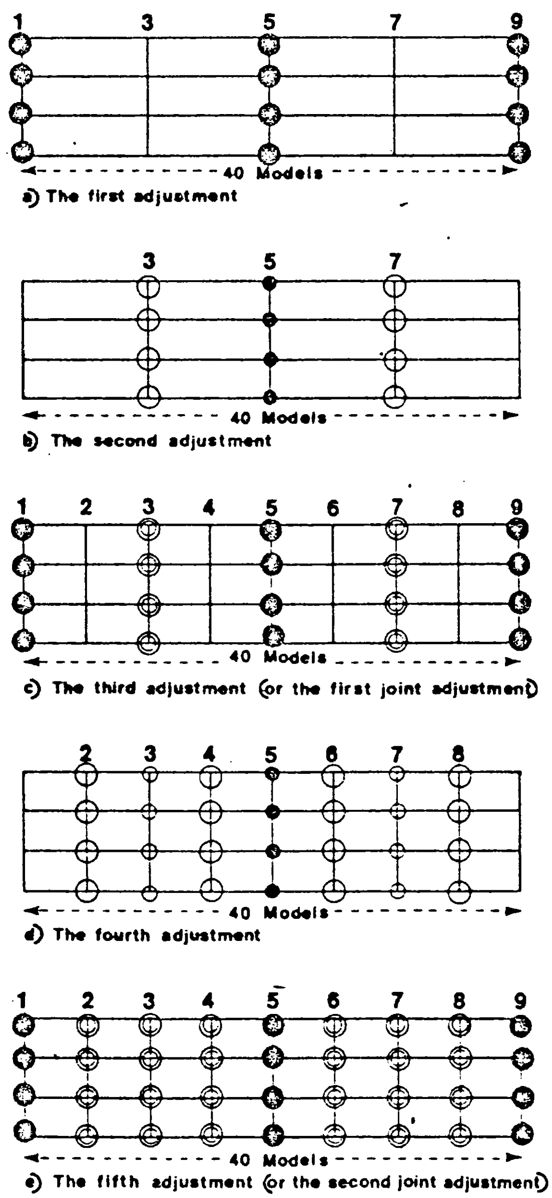

1 3 5 7 9
40 Models
a) The first adjustment
3 5 7
40 Models
b) The second adjustment
1 2 3 4 5 6 7 8 9
40 Models
c) The third adjustment (or the first joint adjustment)
2 3 4 5 6 7 8
40 Models
d) The fourth adjustment
1 2 3 4 5 6 7 8 9
40 Models
e) The fifth adjustment (or the second joint adjustment)

Fig.4.2

Step 3 The final step in the procedure is to repeat the adjustment process using all five bands of control points lying along Sections 1 to 5, i.e. the known (terrestrial) values for Sections 1, 3 and 5 and the corrected photogrammetric values for Sections 2 and 4. The final results of the procedure will be an improvement in the absolute accuracy of the height points throughout the block.

The Basis for and Explanation of the Procedure.

Consider a horizontal section AB in the block. Points lying along the section AB will have systematic height errors after adjustment as shown by the curved lines in Fig.4.1(b). The maximum error will occur at points 2 and 4, in this case shown by points 2' and 4'. The line 1, 2, 3, 4, 5 is the surface to which all systematic errors are referred, and the two curved lines 1, 2', 3 and 3, 4', 5 represent the error surfaces produced from Step 1 of the adjustment. The level 2', 4' becomes the reference surface for the second step of the adjustment.

The curved surface 2'3"4' represents the error surface which is produced from the adjustment carried out in Step 2 with a maximum value 3" (Fig.4.1c).

After Step 3 in which all the control points have been used for the adjustment, the residual systematic errors will have the pattern and magnitude given in Fig.4.1(d).

The final diagram (Fig.4.1(e)) represents an amalgam of the previous diagrams Fig.4.1(b) & (c) to which numerical values have been assigned. The maximum errors at points 2' and 4', after Step 1 are given the values $\Delta h_2'$ and $\Delta h_4'$ (corresponding to the positions 2' and 4'). In Step 2, since the level 2', 4' is the reference level, the maximum error over the bridging distance 2, 4 will be at point 3" and this error is assigned the value $\Delta h_3''$. It will be noted that this represents the height above the point 3' which lies on the same level as the new reference surface 2', 4'. Since the bridging distance 2, 4 in Step 2 is equal to the bridging distances 1, 3 and 3, 5 used in

Step 1, the magnitude of the systematic height errors will be equal, in which case,

$$\Delta h_2' = \Delta h_4' = \Delta h_3' = \Delta h_3''.$$

Extension of the Procedure

It will be obvious that the basic method is capable of being extended so that the strip or block of aerial traingulation can be sub-divided into still smaller sections and the basic process repeated a greater number of times. This would be possible both for a strip or block of the same length as has been discussed in Fig.4.1 above - in which case, the magnitude of the residual systematic errors will be still smaller - or for a strip or block of still greater length - in which case the errors will be reduced to the same order as outlined above.

Taking the latter case, the procedure would then be to use, for example, 9 sections over a block of say 40 models in length so cutting the maximum bridging distance to 5 models. The actual steps can be seen by inspection in Fig.4.2. It will be noted that there will then be five steps instead of those as discussed in the first example of the procedure in Fig.4.1.

Procedure A (Control Pattern 2)

In this, the basic procedure is applied to a control pattern which consists of two lines of height control points located at the beginning and end of each strip with only a single height check point located in any position in Section 3(Fig.4.3).

Step 1 This first utilises all the height control points which are available for the adjustment procedure (Fig.4.3a). The bands of known points along Sections 1 and 5 are held fixed, the maximum systematic height error being produced along Section 3. Here the known value of the single height check point is compared with the value given by the adjustment to produce a height difference $\Delta h_3'''$

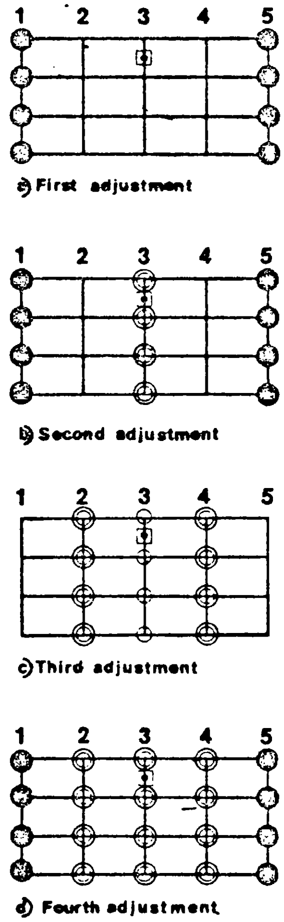

1 2 3 4 5
a) First adjustment
1 2 3 4 5
b) Second adjustment
1 2 3 4 5
c) Third adjustment
1 2 3 4 5
d) Fourth adjustment

Fig.4.3

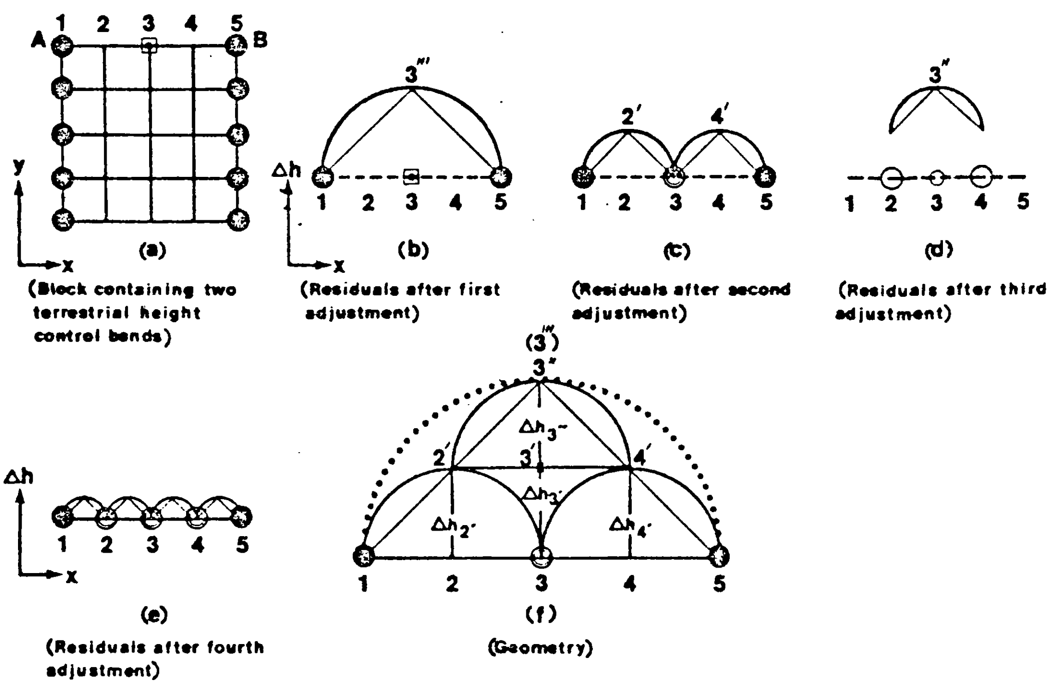

1 2 3 4 5
A
B
y
x
(a)
(Block containing two terrestrial height control bends)
3'''
Δh
1 2 3 4 5
x
(b)
(Residuals after first adjustment)
2' 4'
1 2 3 4 5
(c)
(Residuals after second adjustment)
3''
1 2 3 4 5
(d)
(Residuals after third adjustment)
Δh
1 2 3 4 5
x
(e)
(Residuals after fourth adjustment)
(3''')
3''
Δh3''
2'
3'
4'
Δh3'
Δh2'
Δh4'
1 2 3 4 5
(f)
(Geometry)

Fig. 4.4

This value is used to correct all the photogrammetrically-determined (i.e. provisional height) values lying along Section 3.

Step 2 These corrected values for Section 3, together with the known values of the height points lying along Sections 1 and 5 are used as control values for the second stage of the procedure which determines the photogrammetric height values (again termed provisional values) of all points lying along Sections 2 and 4. (Fig.4.3b).

Step 3 The provisional heights along Sections 2 and 4 are now used as control for the next stage of the adjustment (Fig.4.3c). Along Section 3, the corrected photogrammetric values determined in Step 1 will be compared with the newly determined photogrammetric values of the same points to give differences which will form the basis for correcting the provisional values of the points located along Sections 2 and 4.

Step 4 The corrected values for the points lying along Sections 2, 3 and 4 are then used, together with the given terrestrial values lying along Sections 1 and 5 as control for the last stage of the procedure. (Fig.4.3d).

The diagram Fig.4.4 represents the height errors in the same manner as has been done for the previous example in Fig.4.1. The quantities $\Delta h_2'$, $\Delta h_3'$, $\Delta h_3''$ and $\Delta h_4'$ are all equal in magnitude and are the same terms as those determined in the previous example. Of course they have been determined in a slightly different manner and with a four step procedure rather than the three steps used in the previous example.

Procedure B (Control Pattern 1)

A variant of the basic procedure A can also be employed. This will be discussed for the same control pattern 1 and for long strips as discussed in the extension case of Procedure 1, Control Pattern 1.

Step 1 As before, the block is formed and adjusted to the three bands of given height control lying along Sections 1, 5 and 9

(Fig.4. 5a). This produces photogrammetric heights for all the points lying along Sections 2, 3, 4, 6, 7 and 8.

Step 2 The adjustment procedure is once again repeated using the photogrammetrically-determined height values lying along Sections 3 and 7, located mid-way between the bands of given height control. For all the points lying along Section 5, there will be two values, the given values and the photogrammetrically-derived values. The mean value of the differences between these two values is calculated and half of this difference is applied as a correction to all the photogrammetrically-derived values lying along Sections 3 and 7. (Fig.4.5b).

In addition, ¾ of the correction value applied to the tie points in Sections 3 and 7 is also applied to all the tie points lying in the intermediate Sections 2, 4, 6 and 8.

Step 3 The third and final step is to repeat the adjustment procedure using as height control both the bands of terrestrial control points lying along Sections 1, 5 and 9 and the bands of corrected photogrammetric points. (Fig.4.5c).

Basis for and Explanation of the Procedure

It will be obvious that the procedure is an alternative method of adjusting blocks of long strips to that already discussed in Procedure A with the same control pattern. The photogrammetric heights (i.e. The provisional height values) will have been determined for all of the intermediate sections 2 to 8 in Step 1. Fig.4.6b shows the errors after Step 1 with maximum values of the height errors at 3' and 7'.

The provisional height values of the points lying in Sections 3 and 7 are then used as control for Step 2 and the resulting error pattern is shown in Fig. 4.6c in which the maximum error occurs at 5". Points lying along Section 5 will have two sets of values, the known and the photogrammetrically-derived values. These differences derived along Section 5 form the basis for correcting the photogrammetric values in all the intermediate sections.

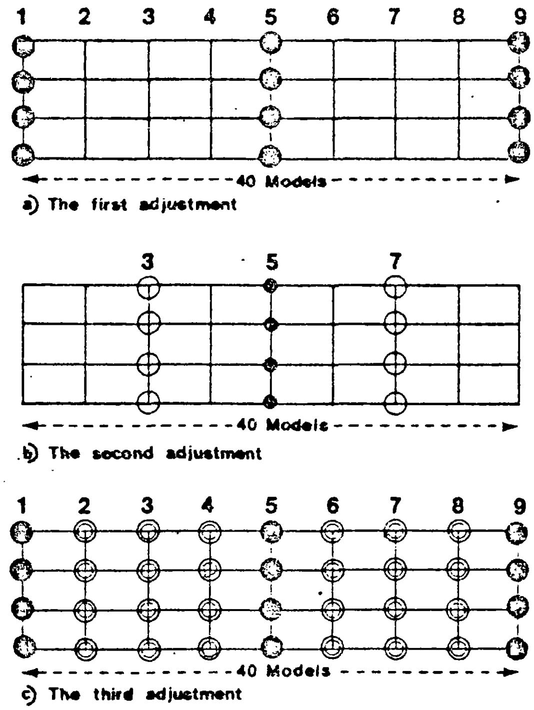

1 2 3 4 5 6 7 8 9
40 Models
a) The first adjustment
3 5 7
40 Models
b) The second adjustment
1 2 3 4 5 6 7 8 9
40 Models
c) The third adjustment

Fig. 4.5

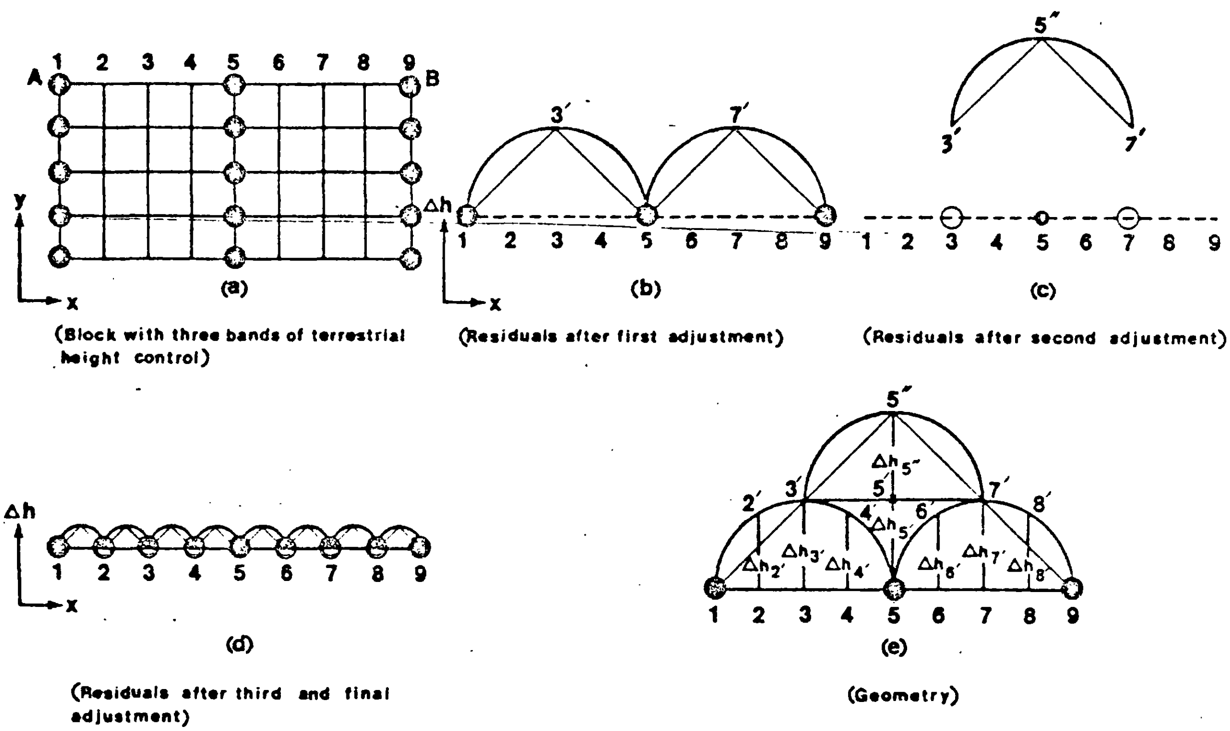

A
B
1 2 3 4 5 6 7 8 9
y
x
(a)
(Block with three bands of terrestrial height control)
3′
7′
Δh
(b)
(Residuals after first adjustment)
5″
(c)
(Residuals after second adjustment)
Δh
(d)
(Residuals after third and final adjustment)
$\Delta h_{5''}$
$\Delta h_{5'}$
$\Delta h_{2'}$
$\Delta h_{3'}$
$\Delta h_{4'}$
$\Delta h_{6'}$
$\Delta h_{7'}$
$\Delta h_{8'}$
(e)
(Geometry)

Fig. 4.6

It will be clear from the explanations given for the previous cases that the corrections to points lying in Sections 3 and 7 will be half the magnitude of those occurring along Section 5. Because of the parabolic nature of the error curves, the values of the corrections to be applied along Sections 2, 4, 6 and 8 will be approximately ¾ of the correction value applied to the points lying in Sections 3 and 7.

A summary of the situation is given in Fig.4.6e, in which

$\Delta h_5' = \Delta h_5'' = \Delta h_3' = \Delta h_7'$; and

$\Delta h_2' = \Delta h_4' = \Delta h_6' = \Delta h_8'$ $(= \frac{3}{4}\Delta h_5')$

A simple formula for the interpolation of the corrections would be the following: $\Delta Z = a + b\,X^2$.

where ΔZ = the required correction

a = the maximum error $(\Delta h_5' = \Delta h_5'' = \Delta h_3' = \Delta h_7')$

$b = -\dfrac{a}{(D/2)^2}$ (a constant)

D = the bridging distance between the bands of terrestrial height control (i.e. between Sections 1 and 5 and Sections 5 and 9).

X = the horizontal distance between the Section where maximum errors exist after Step 1 (i.e. in Sections 3 or 7) and the particular Section for which a correction is required. (X and D must be in the same units).

Procedure B (Control Pattern 2)

The Procedure B is then shown in Fig.4.7 for the second control pattern 2 already used as an example for Procedure A. The steps in the procedure will by now be quite obvious since they are analogous to those already described.

Step 1 This involves the adjustment of the block using two bands of height control located along Sections 1 and 9 and the determination of photogrammetric height values at the intermediate sections 2 to 8. The single terrestrial point lying in Section 5 is used to derive an error value (and correction value) for all the points lying in this Section. (Fig.4.7a).

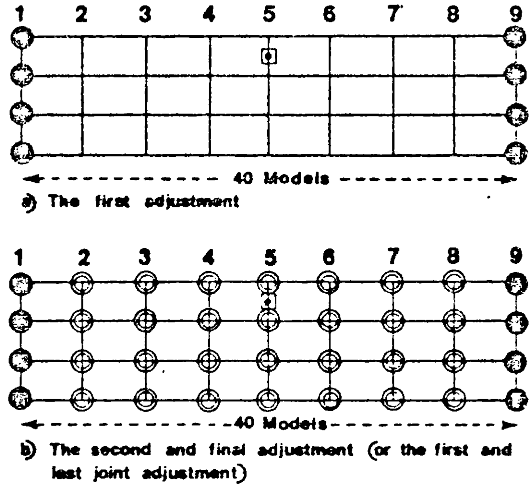

1 2 3 4 5 6 7 8 9
40 Models
a) The first adjustment
1 2 3 4 5 6 7 8 9
40 Models
b) The second and final adjustment (or the first and last joint adjustment)

Fig. 4.7

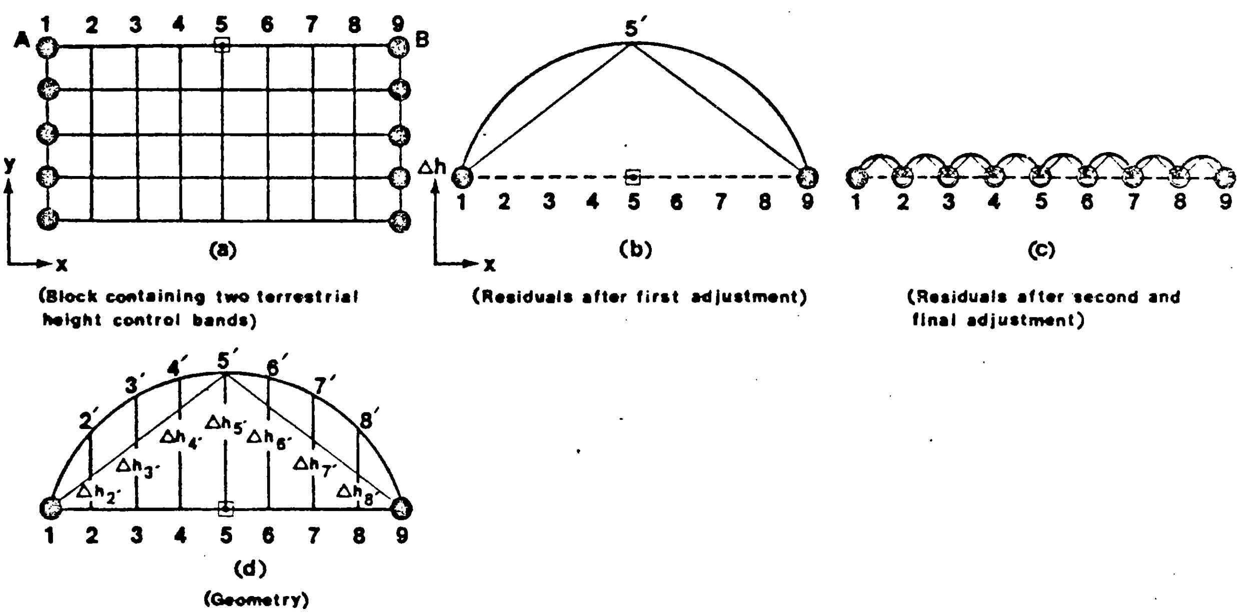

1 2 3 4 5 6 7 8 9
A
B
y
x
(a)
(Block containing two terrestrial height control bands)
5′
Δh
1 2 3 4 5 6 7 8 9
x
(b)
(Residuals after first adjustment)
1 2 3 4 5 6 7 8 9
(c)
(Residuals after second and final adjustment)
2′ 3′ 4′ 5′ 6′ 7′ 8′
Δh2′ Δh3′ Δh4′ Δh5′ Δh6′ Δh7′ Δh8′
1 2 3 4 5 6 7 8 9
(d)
(Geometry)

Fig. 4.8

In addition, correction values are interpolated for all the intermediate sections 2, 3, 4, 6, 7 and 8 in a manner similar to that already described.

Step 2 This utilises both the known terrestrial values for Sections 1 and 9 and the corrected photogrammetric values for Sections 2, 3, 4, 5, 6, 7 and 8, in a single combined final adjustment. (Fig.4.7b).

The errors present in each of these two steps are presented in a manner similar to that of previously described cases in Fig.4.8. The maximum systematic errors at point 5' after Step 1 are shown in Fig. 4.8(b) and the extent of the systematic errors after the second step is given in Fig.4.8(c). Finally Fig.4.8(d) is a composite diagram showing the errors present at all the Sections 1 to 9, in which the intermediate values are interpolated from the value derived from Section 5 ($\Delta h_5'$).

4.3. Predictions for the Accuracy of the TP method

Returning to the matter of the theoretical investigations into the accuracy of aerial triangulation discussed previously in para. 4.1, these studies have shown that accuracy can be expressed in the form

$$\sigma_{max} = c.\sigma_o$$

where

σ_{max} = the standard error after adjustment associated with the point which is the weakest in terms of the amount and distribution of control;

c = a constant which corresponds to n, the number of models bridged. For height, this is the number of models between bands of height control. For planimetry, it is the number of models between planimetric control points located around the perimeter of the block.

σ_o = the standard error of observation of unit weight.

Values for c against n are given below (see Table 4.1) for the three cases of:-

(i) Strip adjustment (height and planimetry)

(ii) Block adjustment (height)

(iii) Block adjustment (planimetry)

Bridging Distance (n models)	$c = \frac{\sigma_{max}}{\sigma_o}$		
	(i) Strip Adjustment (height and planimetry)	(ii) Block Adjustment (height)	(iii) Block Adjustment (planimetry)
2	1.5		1.0
4	2.0	1.80	1.4
6	2.5	2.15	1.8
8	3.5		2.3
10	4.4	3.10	2.6
12	5.4		3.1
14	6.4	4.15	3.6
16			
18			
20		5.80	

Table 4.1 Theoretical accuracy of aerial triangulation

Note: Blank spaces above indicate that values are not available for these conditions.

It will be seen that the decisive point which emerges is that if the bridging distance between control points could be reduced, then the accuracy of the aerial triangulation process will be increased. Obviously however, to employ more control points with all the consequent increase in cost which this would entail, would be to defeat the whole object of the aerial triangulation process. What the TP technique offers as an alternative is a method of attaining an accuracy equivalent to that which would result from a densification of the control network, without having to actually provide this control. This results from the technique itself whereby the provisional photogrammetrically-determined points are used to act as a form of control by which the systematic errors are determined and eliminated in an intermediate stage before the final stage of the block adjustment is carried out.

As far as the TP technique is concerned, it was again stated earlier in para. 4.1 that control must be located in the ideal positions. When this is the case, the following predictions may be made regarding the accuracy of the results after application of the technique to improve height accuracy.

(1) The maximum residual for <u>height</u> will approach the value of σo.

(2) The maximum residual for <u>planimetry</u> will be c.σo.

It should be noted that the value of σo varies with the method of triangulation, the type and condition of the measuring equipment used, the type of photography used, etc., In general however, the following values are representative of modern methods and conditions.

σo = 0.20‰ H for height,

where H = flying height over ground.

σo = 16-20µm in the negative scale for planimetry.

σo = 20-30µm in the negative scale for planimetry when point identification is difficult.

To test and verify the TP technique and the procedures which have been outlined in this chapter, it has of course been necessary to carry out experimental work on a variety of photographs of different scale with blocks of different size and having different patterns and distributions of control points.

A detailed account of this extensive test work and the results which have been produced from it are discussed in the next chapter.

CHAPTER V

Test Data and Results

5. Test Data and Results

5.1 Test Data

Testing of the TP technique was carried out using the procedures described in the previous Chap. 4. In all the tests data from practical blocks of photography have been used. The data was obtained by Mr. B.D. F. Methley from the following sources:-

(i) The Durban Test Block and the Pietermaritzburg - Durban Test Strip were both obtained from the Survey Department of the University of Natal in Durban.

(ii) The DOS Test Block was obtained from the Directorate of Overseas Surveys, Tolworth, England.

(iii) The Oberschwaben Test Block, which has been used extensively in international tests organised by the O.E.E.P.E, was obtained from the I.T.C. Enschede, The Netherlands.

Extensive use was made of the University of Glasgow mainframe computer (an ICL 2976 machine) in testing these blocks in a variety of different ways - with different block sizes and different control patterns. Throughout the tests, use was made of the S.B.A.I.M Program already mentioned.

5.1.A. The Durban Test Block

This block comprises 4 strips with 12 models each, and has the following characteristics:-

Type of Photography: Wide Angle (f = 152mm)

Format : 23 cm x 23 cm.

Scale of Photography : 1/8,000

Number of Models : 48

Measuring Instrument : Hilger & Watts Stereo comparator

Camera : Wild RC8R film camera fitted with an Aviogon lens and a reseau plate.

Focal length : 152mm approximately.

Tie or Transfer Points : Mostly artificial but natural points sometimes used.

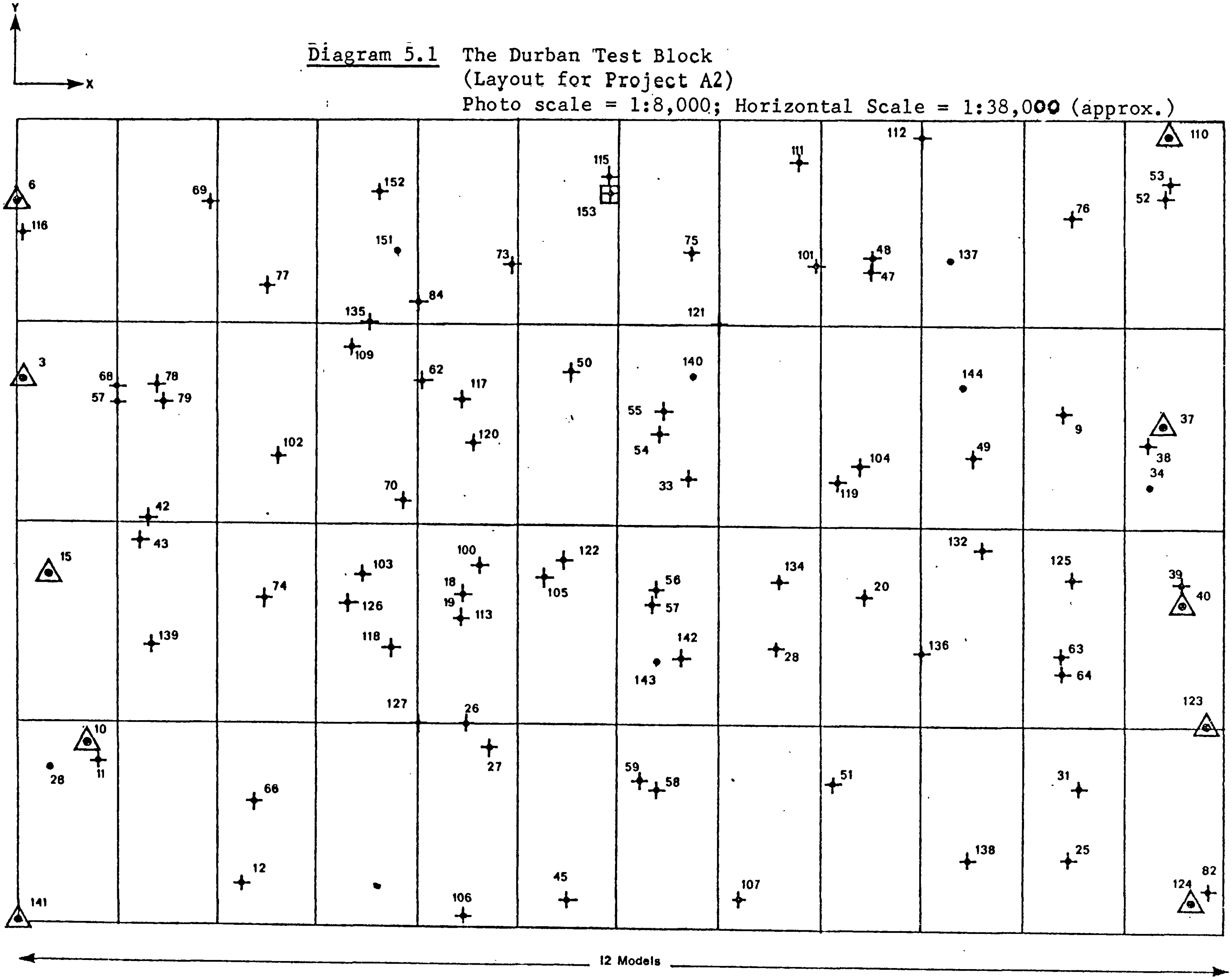
Point Transfer Device : Wild PUG3

Control and Check Points :

The whole Durban Block area includes about 80 pre-marked points, usually located in pairs, and fixed in X, Y and Z. Since very few of these points were suitable for use as transfer points, the entire block was based on artificial transfer points.

The Durban Test Block layout for a particular control point pattern is shown schematically in Diagram 5.1.

<u>Diagram 5.1</u> The Durban Test Block
(Layout for Project A2)
Photo scale = 1:8,000; Horizontal Scale = 1:38,000 (approx.)



5.1.B. <u>The Pietermaritzburg to Durban Test Strip</u>

This strip is at a much smaller scale and has a much greater length (i.e. no. of models) compared with the previous Durban Block.

The important characteristics of this test strip are as follows:-

<u>Type of Photography</u> : Wide Angle (f = 152mm)

Format : 23 cm x 23 cm

<u>Scale of Photography</u> : 1/30,000

<u>Number of Models</u> : 31

<u>Measuring Instrument</u> : Hilger & Watts Stereo comparator.

<u>Camera</u> : Wild RC8R film camera fitted with an Aviogon lens and a reseau plate.

<u>Tie or Transfer Points</u> : Unknown

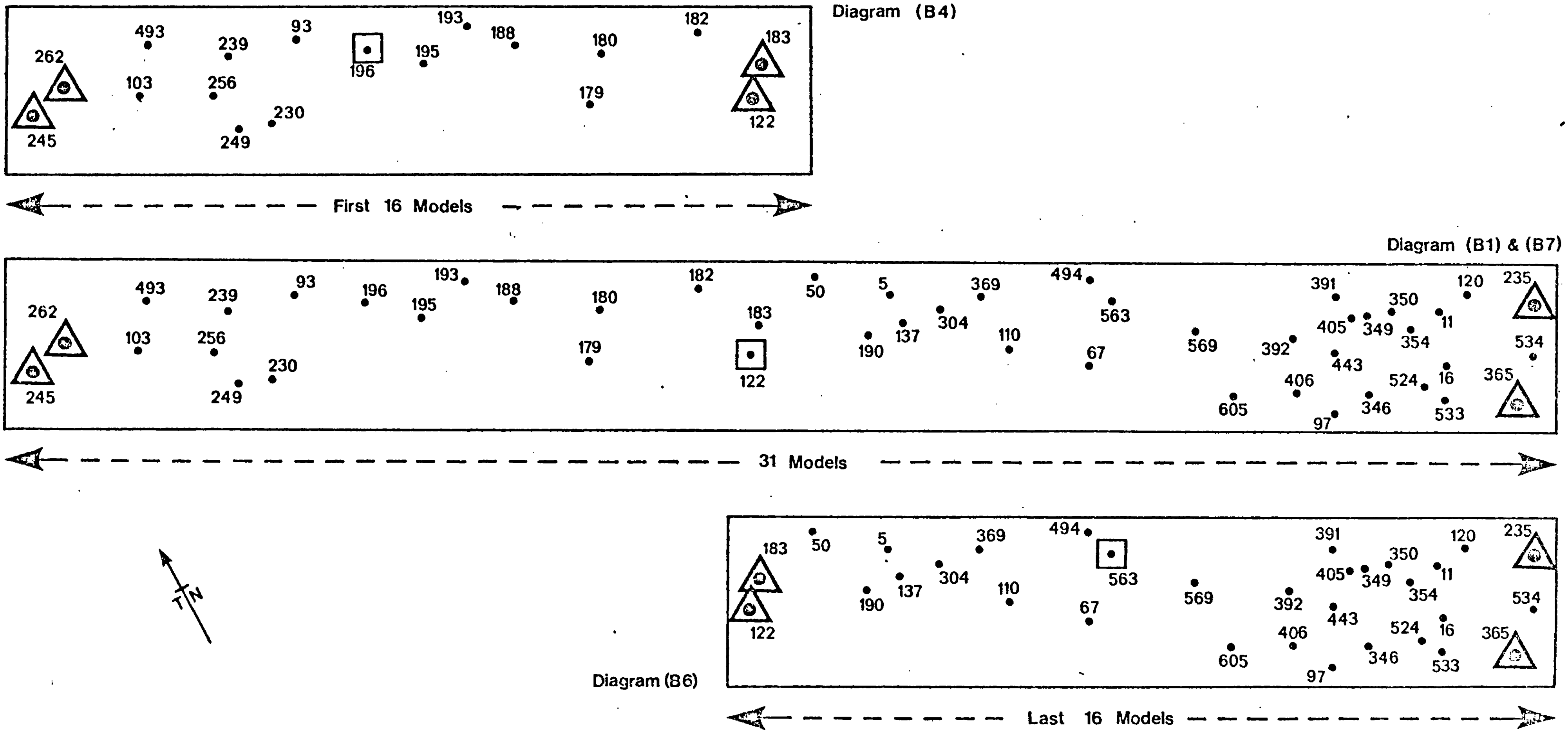
<u>Control and Check Points</u> : Identified (but not premarked) trig. beacons.

A layout of the Pietermaritzburg to Durban Test Strip is shown in Diagram 5.2 which includes layouts for particular control point patterns.

Diagram 5.2 The Pietermaritzburg to Durban Test Strip (Layout for Projects B1, B4, B6 and B7)

Photo Scale = 1:30,000
Horizontal Scale = 1:312,500 (approx)

△ —— Terrestrial Planimetric Control.
⊙ —— Terrestrial Height Control
• —— Terrestrial Height Check Point
⊡ —— Terrestrial Height Check Point for detection of maximum systematic error after first adjustment.



5.1.C. <u>The D.O.S. Test Block</u>

This block comprises part of a larger block of 5 strips. Tests were carried out using only two of the strips of this block (Nos. 9 & 10), since the control point locations were not appropriate for the application of the TP technique throughout the rest of the block. The important characteristics of the tested block are as follows:-

<u>Type of Photography</u>: Wide Angle (f = 152mm)

Format : 23cm x 23cm

<u>Scale of Photography</u> : 1/12,500

<u>Number of Models</u> : 34

<u>Measuring Instrument</u> : Wild A8

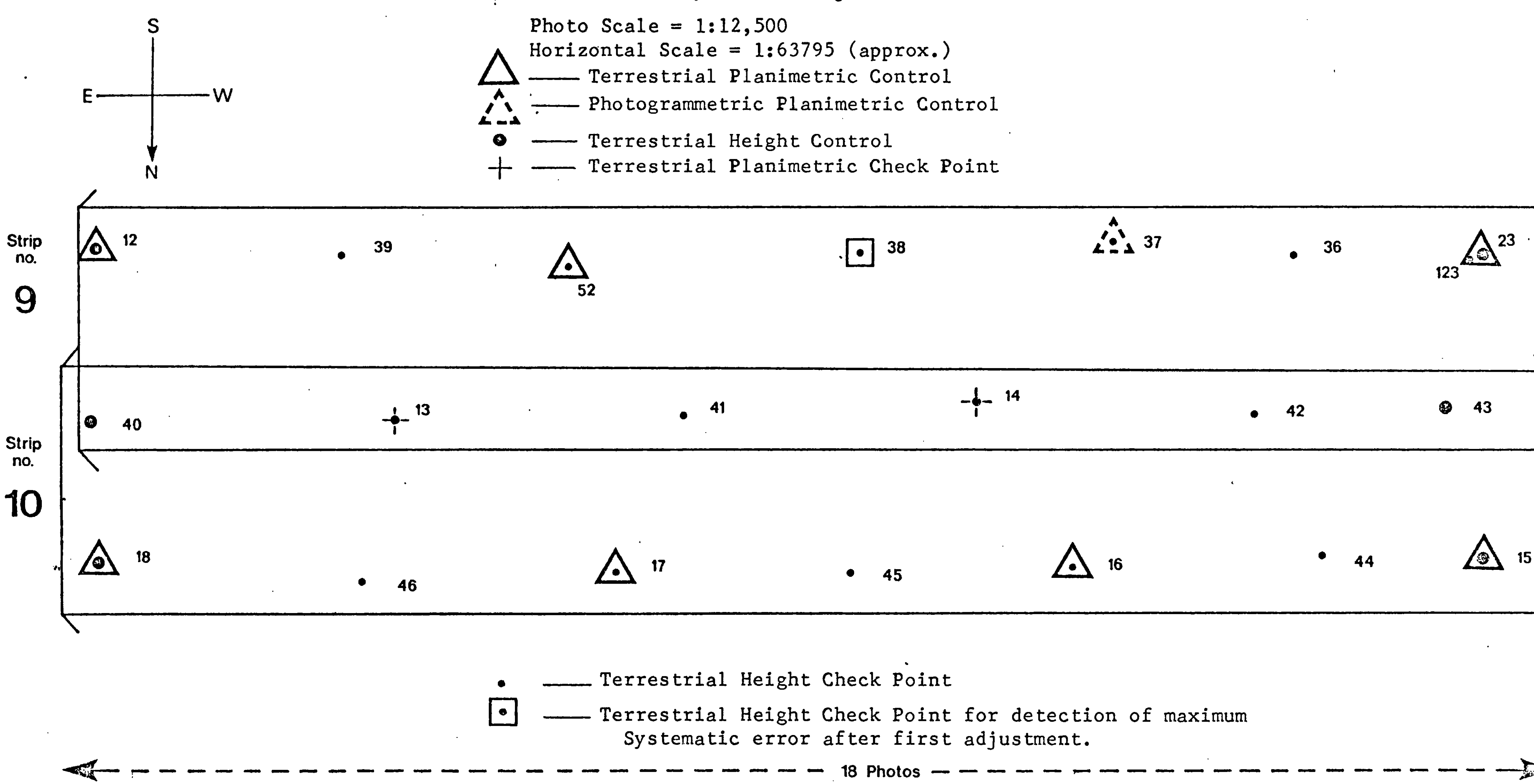
<u>Camera</u> : Wild RC10 film camera

<u>Tie or Transfer Points</u> : Unknown

<u>Control and Check Points</u> :

The tested block layout for a particular control pattern is shown in Diagram 5.3.

Diagram 5.3 The D.O.S. Test Block
(Layout for Project C2)



5.1.D. The Oberschwaben Test Block

A section of the Oberschwaben Test Block was used. It comprises 4 strips with 8 models each. These models lie in the strips numbered 1, 3, 5 and 7 of the test area "Oberschwaben" of the O.E.E.P.E. The important characteristics are as follows:-

Type of Photography : Wide Angle

Format : 23 cm x 23 cm

Scale of Photography : 1/28,000

Number of Models : 32

Measuring Instrument : Zeiss PSK Stereo comparator.

Camera : The Zeiss Oberkochen RMK A 15/23 Wide-Angle Camera.

Tie or Transfer Points : All standard tie points (6 per model) were premarked in the terrain using double signals.

Control and Check Points :

Diagram 5.4 shows the layout of the block for a particular control point pattern.

Diagram 5.4 The Oberschwaben Test Block(layout for project D2)

Photo Scale = 1:28,000; Horizontal Scale = 1:121,840 (approx.)

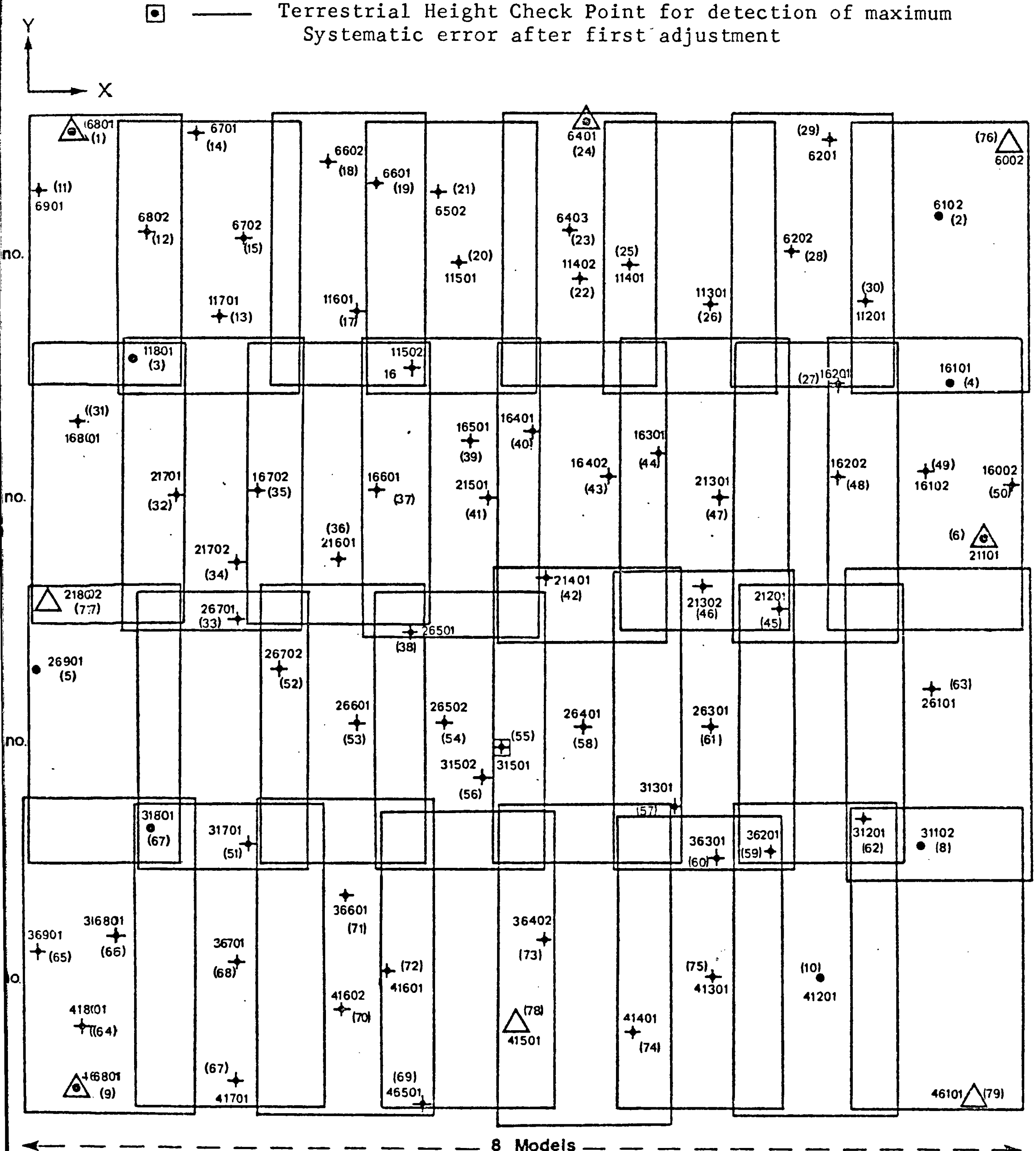
△ —— Terrestrial Planimetric Control

● —— Terrestrial Height Control

+ —— Terrestrial Planimetric Check Point

• —— Terrestrial Height Check Point

⊡ —— Terrestrial Height Check Point for detection of maximum Systematic error after first adjustment



5.2 Results and Analysis

5.2.1 Introduction

To present all the detailed results from the great number of tests carried out would require two or three large-sized books of print-out paper. Since it is not practicable to present them in this way and in any case it would be confusing to the readers, the results have been presented in summary form in a series of 14 tables. The four tables 5.1, 5.3, 5.8, 5.10 give the height accuracies using Procedure A. The results from each of the four blocks is presented in a separate table. The seven tables 5.2, 5.4, 5.5, 5.6, 5.7, 5.9 and 5.11 comprise detailed results of certain projects from all four blocks. In the case of two of the three remaining tables, i.e. Table 5.12 (Planimetric Accuracy, Procedure A)), and Table 5.13 (Height Accuracy, Procedure B)), each table summarizes the accuracy of the block adjustments carried out on all four blocks in a single table. The remaining table 5.14 shows a comparison between the accuracies, the number of times of height accuracy improvement, and the number of adjustments carried out in certain projects common to Procedures A & B discussed previously in Chapter 4.

The tables show the accuracy of the results at check points in terms of

(i) the maximum residual error before and after accuracy improvement;

(ii) the estimated standard error of unit weight σ_o before and after accuracy improvement;

(iii) the standard deviation of the residuals before and after accuracy improvement;

The standard deviation gives the absolute accuracy and is computed as follows:-

$$\sigma_z = \sqrt{\frac{\sum \Delta Z . \Delta Z}{n_z}}$$

$$\sigma_p = \sqrt{\frac{\sum \Delta P . \Delta P}{n_p}}$$

where,

ΔP = $\sqrt{((\Delta X)^2 + (\Delta Y)^2)}$;

σ_z = The standard deviation of the height residuals detected at all the terrestrial check points;

σ_p = The standard deviation of the planimetric residuals detected at all the terrestrial check points;

ΔZ = The height residual detected at a terrestrial check point;

ΔX = The planimetric residual (in the X direction of the terrain system) detected at a terrestrial check point;

ΔY = The planimetric residual (in the Y direction of the terrain system) detected at a terrestrial check point;

n_z = Number of terrestrial height check points; and

n_p = Number of terrestrial planimetric check points.

In the summary tables mentioned above, the expected accuracies for height and planimetry are expressed in terms of the maximum errors derived from theoretical considerations and previously presented in Table 4.1 of para. 4.3, Chapter 4. In the computations of the expected maximum error (σ_{max}) σ_o (the standard error of unit weight) is taken as 0.20‰H for height, where H is the flying height above ground, and as 20µm in the negative scale for planimetry. The empirical values for σ_o obtained from the results before and after height accuracy improvement are tabulated in these summary tables and are computed as follows:

$$\sigma_o = \frac{|\text{Maximum Residual}|}{c}$$

where,

|Maximum Residual| = the absolute value of the maximum residual (in height or planimetry) detected at the

terrestrial check points (before or after the height accuracy improvement). This value is assumed to represent the estimated σ_{max}.

$c = \frac{\sigma_{max}}{\sigma_o} =$ a constant which corresponds to n, the number of models bridged. For height this is the number of models between bands of height control, and for planimetry it is the number of models between planimetric control around the perimeter of the block.

DURBAN BLOCK

RESULTS AND ANALYSIS

5.2.2 Analysis of the Results of the Durban Test Block

Referring to Table 5.1 (Height accuracy(Procedure A)), it is apparent from the tabulated results of Project A2 that the expected accuracy of 0.72‰H for a bridging distance of 12 models could not be reached before improvement. The maximum height residual detected at the 62 terrestrial height check points was -1.42‰H before improvement and it became -0.42‰H after improvement using the TP technique. The standard deviation improved 5.3 times (from 0.83‰H to 0.16‰H). The maximum residual after improvement in Project A2 is even better than the expected accuracy of Project A1 which has a bridging distance of 6 models. This significant improvement of the height accuracy in Project A2 is reflected also in a planimetric accuracy improvement as shown in the tabulated results of Project A2 in Table 5.12 (Planimetric Accuracy (Procedure A)). The maximum planimetric residual detected at the 62 terrestrial planimetric check points was found to improve from 62μm to 56μm, and the standard deviation improved from 28μm to 20μm. This means that a planimetric accuracy improvement of 1.4 times occurred as a direct result of height accuracy improvement.

Referring again to Table 5.1, it will be noticed (from the number of the control points used in projects A1, A2, and A3, and from their tabulated accuracies) that the accuracy of an adjusted block depends on the density (i.e. the number) of control points. This is a conclusion which agrees with that of the theoretical investigations.

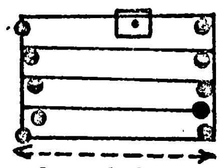
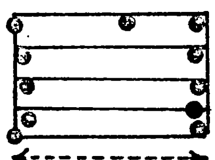
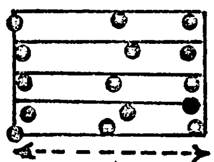
A better accuracy than expected was reached in Project A1 using a bridging distance of 6 models. The maximum height residual in this project was 0.31‰H before improvement, while the expected σ_{max} was 0.43‰H. This was not the case with project A2 in which the maximum height residual detected at the check points before the accuracy improvement was -1.42‰H while the expected σ_{max} was 0.72‰H. It could be noticed from Table 5.1 that the estimated σ_o obtained before height accuracy improvement in project A1 was 0.15%oH while it was 0.39%oH in project A2. That is to say, σ_o used in computing the maximum standard error (σ_{max}) expected in project A1 is <u>more</u> than the estimated value σ_o obtained before the height accuracy improvement of this project, while this was not the case with project A2 in which σ_o is <u>less</u> than the estimated σ_o obtained before height accuracy improvement and is more than the estimated σ_o (0.12%oH) obtained <u>after</u> height accuracy improvement. This illustrates the effect of systematic errors on the value of σ_o. The effect is even more evident in the tabulated results in Table 5.2 and Diagram 5.5 shown below.

Referring to Table 5.14 (Comparison of Procedures A & B), it will be seen from the tabulated information of Project A2 that Procedure A (Control Pattern 2) yields better results than Procedure B (Control Pattern 2) and the number of adjustments carried out in the former case was 4 while it was 2 in the latter case. The accuracy improved 5.3 times (from 0.83%oH to 0.16%oH) in Procedure A (Control Pattern 2) and it improved 4.5 times (from 0.83%oH to 0.19%oH) in Procedure B (Control Pattern 2). The reason that the former case yields better results than the latter may be explained as follows.

Corrections to photogrammetrically determined control points in the former case are based more on the photogrammetric values of points lying along several intermediate sections, whereas in the latter case, they depend wholly on the values of those sections lying midway between the banks of terrestrial control, which are interpolated over longer distances between bands of terrestrial control points.

Table 5.1 Table 5.1.1

(Tables 5.1.1 to 5.1.4) - Height Accuracy, Procedure A, Durban Block

Project Photo Scale = 1 : 8000 Flying Height (H) = 1218 meters 4 strips; 48 models total		Bridging Distance (Models)	Number of Terrestrial Height Control Points
No.	Details		
A. 1	3 Ground Height Control Bands (15 Points) 	6	15
A. 2	2 Ground Height Control Bands (10 Points) plus one Ground Height Check Point. 	12	10
A. 3	Height Control Bands (10 Points) plus one extra Ground Height Control Point 	12	11

⊙ ——— Terrestrial height control.

⊡ ——— Terrestrial height check point (for detection of maximum systematic error after first adjustment).

Table 5.1.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error,* (%oH)	Maximum height residuals detected at check points. (%oH)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
A. 1	57	±0.43	0.314	
A. 2	62	±0.725	-1.420	-0.417
A. 3	63	±0.725	-1.053	

* The value of the expected maximum error is in fact the value of the expected Maximum Standard Error (σ_{max}) which is assumed to represent the expected maximum error.

Note:- Blank spaces above indicate that values are not available for these conditions.

Table 5.1.3

Project No.	σ_o = the standard error of unit weight (‰H)		
	σ_o used in computing the expected maximum Error	σ_o obtained	
		Before height accuracy improvement	After height accuracy improvement
A.1	0.20	0.15	
A.2	0.20	0.39	0.12
A.3	0.20	0.29	

Table 5.1.4

Project No.	The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Points (%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
A.1	0.144		
A.2	0.832	0.156	5.3
A.3	0.425		

Table 5.2

Table 5.2.1

(Tables 5.2.1 to 5.2.4) - Comparison Tables, Procedures A & B, Project A2

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in Meters). Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
116	-0.258	-0.236	-0.280
77	-0.967	0.049	0.219
152	-1.472	-0.044	-0.452
153	-1.536	0.075	-0.086
75	-1.378	0.172	0.043
111	-1.013	0.391	0.168
112	-0.749	0.388	0.116
47	-1.065	0.179	-0.034
48	-1.158	0.086	-0.128
76	-0.301	0.234	0.123
53	0.056	0.211	0.203
52	-0.018	0.141	0.132
7	-0.177	-0.078	-0.117
68	-0.554	-0.062	-0.210
102	-1.027	0.114	-0.189

Table 5.2.2

Terrestrial Check Point No.	Height (ΔZ) Residuals detected at the Terrestrial Check Points (in Meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A&B	Procedure A	Procedure B
109	-1.319	0.002	-0.255
117	-1.537	-0.063	-0.297
120	-1.042	0.436	0.185
33	-1.381	0.064	-0.261
54	-1.468	0.062	-0.152
55	-1.419	0.124	-0.072
140	-1.365	0.154	-0.027
104	-0.943	0.154	-0.323
49	-0.688	0.057	-0.363
9	-0.297	0.193	-0.028
38	-0.113	0.003	-0.096
43	-0.427	0.130	-0.091
139	-0.220	0.135	0.034
74	-0.837	0.054	-0.191
118	-1.210	0.089	-0.134
103	-1.156	0.090	-0.238

Table 5.2.3

Terrestrial Check Point No.	Height (ΔZ) Residuals detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
126	-0.824	0.323	0.052
100	-1.060	0.396	0.129
113	-0.874	0.535	0.324
19	-1.129	0.270	0.019
122	-1.384	0.099	-0.088
105	-1.317	0.155	-0.027
56	-1.323	0.141	-0.093
57	-1.130	0.332	0.097
142	-1.129	0.298	0.128
143	-1.194	0.256	0.097
128	-0.995	0.284	0.002
134	-1.034	0.270	-0.089
20	-0.984	0.128	-0.316
132	-0.290	0.437	-0.174
63	-0.530	-0.036	-0.246
64	-0.479	0.017	-0.191

Table 5.2.4

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
125	-0.308	0.109	-0.232
11	-0.085	0.024	0.012
12	-1.171	-0.177	-0.508
66	-0.778	0.088	-0.147
106	-1.729	-0.132	-0.439
27	-1.251	0.146	0.019
45	-1.660	-0.020	-0.203
59	-1.407	0.117	0.013
58	-1.277	0.246	0.137
107	-1.082	0.480	0.238
51	-0.902	0.388	0.102
138	-0.644	0.366	0.017
31	-0.314	0.226	0.041
25	-0.220	0.426	0.197
82	0.099	0.061	0.072

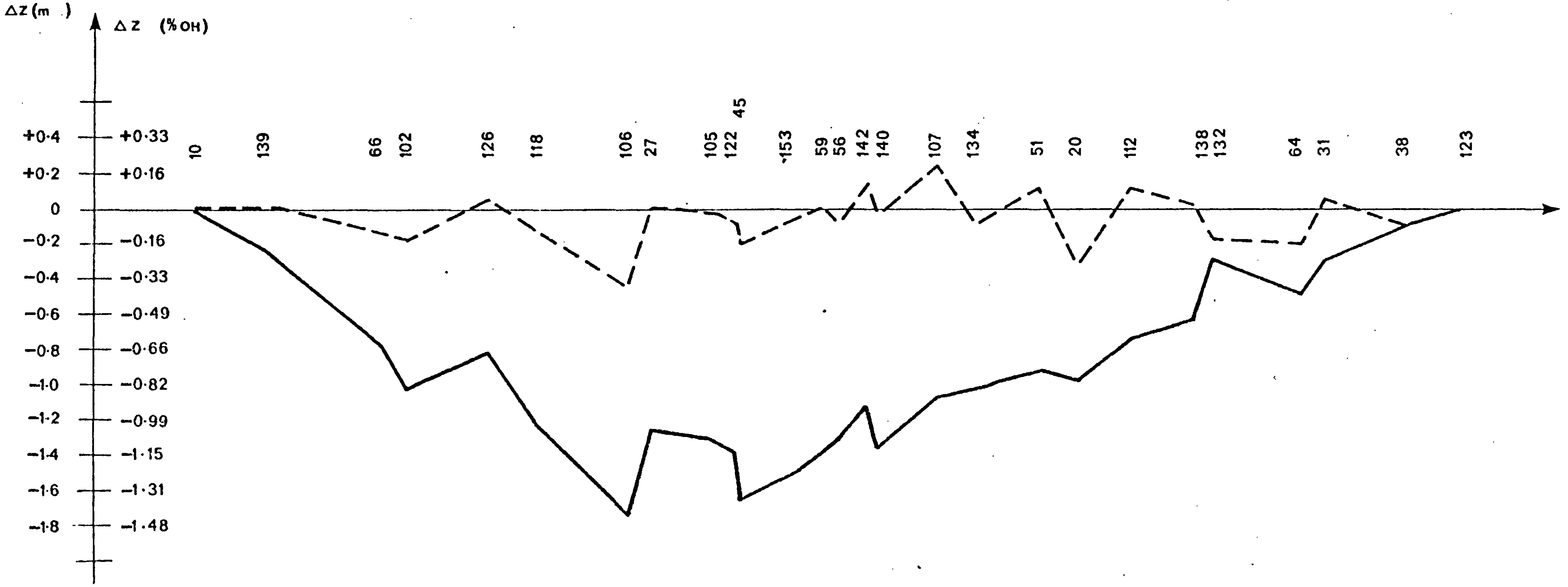
Diagram 5.5 The Durban Test Block
(Height errors in project A2. Procedure A (Control pattern 2))

—— for height residuals (ΔZ) before improvement

– – – for height residuals (ΔZ) after improvement

ΔZ Scale = 1:33

Horizontal Scale = 1:35,175 (approx.)



PIETERMARITZBURG - DURBAN STRIP

RÉSULTS AND ANALYSIS

5.2.3. Analysis of the Results of the Pietermaritzburg to Durban Test Strip.

The series of Tables 5.3.1 to 5.3.4 gives the summary results for the Pietermaritzburg - Durban strip for Procedure A. It will be noted that the improvement factors are 2.2, 1.9, 5.6, 2.3, 12.9, 6.4, and 1.3 for Projects B2, B3, B4, B5, B6, B7 and B9 respectively. The maximum height residuals were all larger than the expected σ_{max} values, due again to the presence of systematic error. Tables 5.4, 5.5, 5.6, and 5.7 and diagrams b1, b4, and b6 (see Diagram 5.6) illustrate the systematic errors more clearly. A comparison of the results of B3 with B4, B5 with B6 and B7 with B9, support the theory that accuracy depends on the number of models between control rather than the length of the strip.

Consider now the results of projects B2, B3, and B5. All have the same number of models between control, and therefore the results should be similar. However this is clearly not the case. A possible explanation is the location of the control and check points; some of the control points are not in the ideal positions and they could have an adverse effect on the results. Such effect can be very clearly noticed in the results of projects B7 and B9 in which the estimated value σ_o obtained (even after height accuracy improvement) is more than 3 times larger than σ_o used in computing the maximum standard error σ_{max}. It will be noticed that this was not the case with projects B2, B3, B4, B5, and B6 in which the estimated σ_o obtained after height accuracy improvement has an average value of 0.16‰H which is less than the value of 0.20‰H given for σ_o.

It may be concluded from the tabulated information of projects B7 and B8 in Table 5.3 that the accuracy of an adjusted block depends on the density of control points.

Referring to Table 5.14 and the tabulated information of projects B5, B6 and B7 the following points can be made:-

(i) In Project B5, Procedure A (Control Pattern 1) yields slightly better accuracy than Procedure B (Control Pattern 1). The accuracy improved 2.3 times (from 0.93‰H to 0.40‰H) in the former case and 2.2 times (from 0.93‰H to 0.43‰H) in the latter case.

(ii) In project B6, Procedure A (Control Pattern 2) yields significantly better accuracy than Procedure B (Control Pattern 2). The accuracy improved 12.9 times (from 5.40‰H to 0.42‰H) in the former case and 3.0 times (from 5.40‰H to 1.81‰H) in the latter case. It will be noticed that the number of adjustments carried out was 4 in Procedure A (Control Pattern 2) and 2 in Procedure B (Control Pattern 2).

(iii) In Project B7, Procedure A (Control Pattern 2) yields the same accuracy as Procedure B (Control Pattern 2). The accuracy improved 6.4 times (from 11.87‰H to 1.84‰H) in both cases, but the number of carried out adjustments was 4 in the former case and 2 in the latter.

It has to be noted that the bridging distance (16 models) in project B6 is longer than that (8 models) of Project B5 and shorter than that (31 models) of Project B7. It will also be noted that the distribution of control and check points in Project B6 is better than that in Projects B5 and B7; and the distribution of control and check points in Project B5 is better than that in Project B7. (See Diagram 5.2). So, one can conclude firstly that the two control patterns (1 & 2) of Procedure A yield better results than those of the two control patterns of Procedure B. The reasons for this have been discussed in the previous section. Secondly, it may be concluded that a less than ideal distribution of terrestrial control will yield inferior results.

Table 5.3 Table 5.3.1

(Tables 5.3.1 to 5.3.4) - Height Accuracy, Procedure A, Pietermaritzburg - Durban Strip

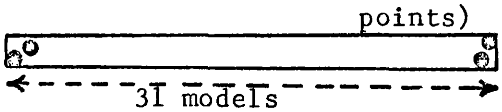
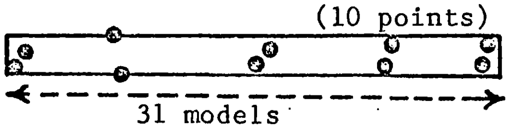
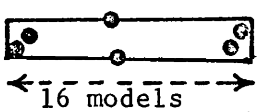
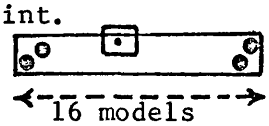
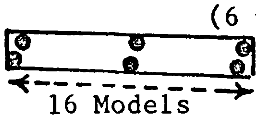
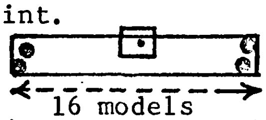
Project Photo Scale = 1:30,000 Flying Height (H) = 5000 metres		Bridging Distance (models)	Number of Terrestrial Height Control Points
No.	Details		
B.1	31 Models; 2 Ground Height Control Bands (4 points)  31 models	31	4
B.2	31 Models; 5 Ground Height Control Bands (10 points)  31 models	8	10
B.3	First 16 Models; 3 Ground Height Control Bands (6 points)  16 models	8	6
B.4	First 16 Models; 2 Ground Height Control Bands (4 points) plus one Ground Height check point.  16 models	16	4
B.5	Last 16 Models; 3 Ground Height Control Bands (6 points)  16 Models	8	6
B.6	Last 16 Models; 2 Ground Height Control Bands (4 points) plus one Ground Height check point.  16 models	16	4

Table 5.3.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error, (%oH)	Maximum height residuals detected at check points. (%oH)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
B.1	44	±1.28	-17.946	
B.2	38	±0.70	- 1.491	-0.820
B.3	10	±0.70	- 0.755	-0.366
B.4	10	±1.28	- 2.876	-0.990
B.5	26	±0.70	- 1.385	- 0.624
B.6	28	±1.28	- 7.378	- 0.797

Table 5.3.3

Project No.	σ_o = the standard error of unit weight (%oH)		
	σ_o used in computing the expected maximum Error	σ_o obtained	
		Before Height Accuracy Improvement	After Height Accuracy Improvement
B.1	0.20	2.8	
B.2	0.20	0.43	0.23
B.3	0.20	0.22	0.10
B.4	0.20	0.45	0.15
B.5	0.20	0.40	0.18
B.6	0.20	1.15	0.12

Table 5.3.4

Project No.	The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Points(%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
B.1	11.874		
B.2	0.831	0.380	2.2
B.3	0.363	0.191	1.9
B.4	2.250	0.405	5.6
B.5	0.929	0.405	2.3
B.6	5.398	0.418	12.9

Table 5.3 Table 5.3.1

(Tables 5.3.1 to 5.3.4) - Height Accuracy, Procedure A, Pietermaritzburg - Durban Strip

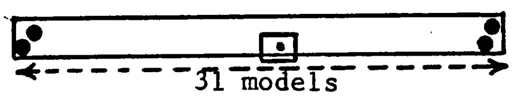
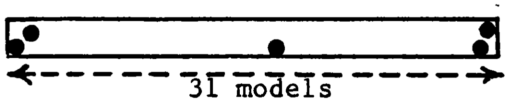
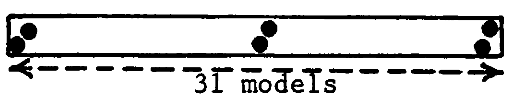
Project Photo Scale = 1:30,000 Flying Height (H) = 5000 mts. 1 Strip; 31 models		Bridging Distance (models)	Number of Terrestrial Height Control Points
No.	Details		
B.7	2 Ground Height Control Bands (4 points) plus One Ground Height Check Point 	31	4
B.8	2 Ground Height Control Bands (4 points) plus One extra Ground Height Control Point 	31	5
B.9	3 Ground Height Control Bands (6 Points) 	16	6

Table 5.3.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error, (%oH)	Haximum Height Residuals detected at Check Points (%oH)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
B.7	44	±1.28	-17.946	4.726
B.8	44	±1.28	3.690	
B.8	42	±1.28	3.739	4.430

Table 5.3.3

Project No.	σ_o = the standard error of unit weight (%oH)		
	σ_o used in computing the expected maximum error	σ_o Obtained	
		Before Height Accuracy Improvement	After Height Accuracy Improvement
B.7	0.20	2.8	0.74
B.8			
B.9	0.20	0.58	0.69

Table 5.3.4

Project No.	The standard deviation of the Height Residuals detected at the Terrestrial Height Check Points (%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
B.7	11.874	1.844	6.4
B.8	2.167		
B.9	2.274	1.804	1.3

Table 5.4

(Tables 5.4.1 & 5.4.2) - Detailed Results, Procedure A, Project Bl

Table 5.4.1

Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)	Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)
	After Block Adjustment of Project B.1		After Block Adjustment of Project B.1
1493	-9.777	1304	-88.337
1103	-15.497	1369	-86.388
1256	-29.892	1110	-87.206
1249	-36.576	1494	-77.469
1239	-26.028	1563	-77.228
1230	-41.032	1067	-81.283
1093	-33.645	1569	-69.144
1196	-42.899	1605	-64.586
1195	-52.035	1406	-54.839
1193	-55.121	1392	-54.132
1188	-62.312	1405	-45.329
1180	-72.193	1391	-48.499
1179	-77.365	1443	-48.288
1182	-77.322	1097	-49.226
1122	-87.197	1349	-43.105
1183	-85.241	1350	-37.757
1050	-84.334	1354	-32.933
1190	-89.728	1346	-40.543
1005	-87.914	1011	-25.533
1137	-89.050	1016	-25.171

Table 5.4.2

Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)	Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)
	After Block Adjustment of Project B.1		After Block Adjustment of Project B.1
1524	-28.932		
1533	-23.683		
10120	-19.826		
1534	-10.059		

Table 5.5 - Detailed Results, Procedure A, Project B4

Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)	
	Before Height Accuracy Improvement	After Height Accuracy Improvement
1103	-6.833	-1.163
1256	-12.769	-3.523
1239	-9.677	-0.433
1093	-10.446	0.037
1196	-11.451	-0.316
1195	-12.943	-1.408
1193	-11.482	-0.136
1188	-11.484	-0.427
1180	-9.167	-0.484
1179	-14.379	-4.948

Table 5.6

(Tables 5.6.I & 5.6.2) - Detailed Results, Procedure A, Project B6

Table 5.6.1

Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)	
	Before Height Accuracy Improvement	After Height Accuracy Improvement
1050	-10.315	3.255
1190	-19.304	2.948
1005	-22.546	3.505
1137	-23.726	3.983
1304	-28.117	3.423
1369	-31.869	2.822
1110	-33.540	1.625
1494	-34.998	2.713
1563	-36.675	0.748
1067	-36.888	-0.023
1569	-36.459	0.570
1605	-34.946	0.480
1406	-32.018	-0.137
1392	-32.365	0.092
1405	-28.549	-0.638
1391	-29.884	0.595
1443	-29.275	0.308
1097	-29.865	-1.215
1349	-27.588	-0.974
1350	-24.328	-1.000

Table 5.6.2

Terrestrial Check Point No.	Height Residuals Detected at the Terrestrial Check Points (in meters)	
	Before Height Accuracy Improvement	After Height Accuracy Improvement
1354	-21.643	-1.572
1346	-25.131	-0.717
1011	-17.221	-1.956
1016	-16.728	-1.886
1524	-19.381	-3.294
1533	-15.958	-3.001
10120	-13.788	-1.892
1534	-7.338	-2.479

Table 5.7

Table 5.7.1

(Tables 5.7.1 to 5.7.3) - Comparison Tables, Procedures A & B, Project B7

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
1493	-9.777	11.433	15.605
1103	-15.497	7.281	8.046
1256	-29.892	12.071	11.154
1249	-36.576	11.855	8.308
1239	-26.028	15.780	17.415
1230	-41.032	13.594	9.189
1093	-33.645	21.239	20.076
1196	-42.899	23.974	19.303
1195	-52.035	23.304	16.758
1193	-55.121	23.387	18.070
1188	-62.312	20.860	15.169
1180	-72.193	14.740	10.937
1179	-77.365	11.078	5.486
1182	-77.322	9.638	8.998
1122	-87.197	1.004	0.141
1183	-85.241	2.627	1.929

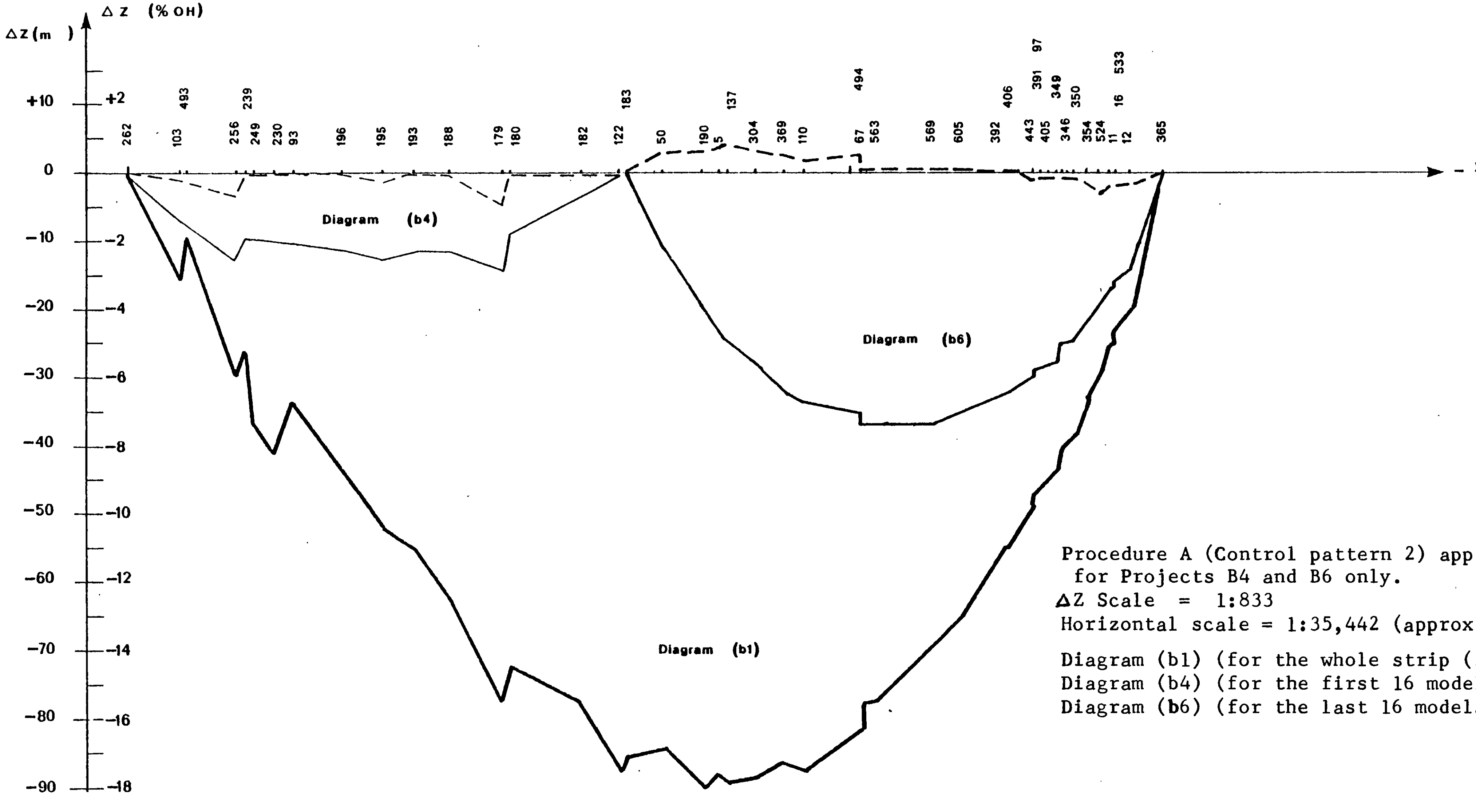
Table.5.7.2

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in metres). Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
1050	-84.334	4.546	2.250
1190	-89.728	1.029	-4.753
1005	-87.914	2.986	-3.507
1137	-89.050	2.413	-5.273
1304	-88.337	3.102	-6.090
1369	-86.388	4.265	-6.614
1110	-87.206	2.212	-9.861
1494	-77.469	7.300	-5.583
1563	-77.228	5.051	-7.901
1067	-81.283	2.375	-10.811
1569	-69.144	2.904	-8.017
1605	-64.586	0.541	-8.408
1406	-54.839	-1.151	-7.024
1392	-54.132	-0.050	-6.139
1405	-45.329	-1.282	-5.198
1391	-48.499	0.244	-4.641

Table 5.7.3

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial check points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
1443	-48.288	-0.595	-5.075
1097	-49.226	-2.666	-6.729
1349	-43.105	-1.623	-5.070
1350	-37.757	-1.603	-4.483
1354	-32.933	-2.235	-4.394
1346	-40.543	-1.917	-4.876
1011	-25.533	-2.386	-3.979
1016	-25.171	-2.523	-3.973
1524	-28.932	-4.152	-5.710
1533	-23.683	-3.839	-4.922
10120	-19.826	-2.128	-3.276
1534	-10.059	-2.781	-3.114

Diagram 5.6 The Pietermaritzburg to Durban Test Strip
(Height residuals in projects B1, B4, and B6)



Procedure A (Control pattern 2) applied
for Projects B4 and B6 only.
ΔZ Scale = 1:833
Horizontal scale = 1:35,442 (approx.)

Diagram (b1) (for the whole strip (31 mo
Diagram (b4) (for the first 16 models)
Diagram (b6) (for the last 16 models)

——— for height residuals (ΔZ) before improvement
– – – for height residuals (ΔZ) after improvement

D.O.S. BLOCK

RESULTS AND ANALYSIS

5.2.4 Analysis of the Results of the D.O.S. Test Block

Tables 5.8.1 to 5.8.4 and Tables 5.12.1 to 5.12.5 give the results for the D.O.S. Block. Improvement factors for height and planimetry are 2.1, 14.8, 1.3 and 1.1, 1.5, 1.0 respectively for Projects C1, C2 and C4. Although the planimetric control was the same for all these projects, the planimetric accuracy improves with the improvement in height accuracy - as is to be expected.

It will be noted in the results in Table 5.8 that the maximum height residuals after accuracy improvement are significantly larger than the expected σmax. Also the standard deviations after improvement are larger than might be expected. It will be noted also that the estimated σo obtained before and after application of the TP technique to Projects C1, C2 and C4 has improved as follows in these projects in which σo used in computing the maximum standard error (σmax) was 0.20‰H :

(i) from 0.74‰H to 0.39‰H in Project C1;

(ii) from 3.17‰H to 0.27‰H in Project C2;

(iii) from 1.39‰H to 1.07‰H in Project C4.

It is very clear from these results that the largest improvement of the estimated value of σo is in Project C2 which has a better distribution of height control than either Project C1 or Project C4. However the estimated σo (0.27‰H) obtained after the height accuracy improvement of Project C2 is still more than σo (0.20‰H) used in computing σmax. This agrees with the results of Projects B7 and B9 (mentioned previously in para. 5.2.3) and is contrary to the results of all other projects in the other test areas. A possible explanation would seem to be that the DOS Block is one which was measured for normal production mapping purposes, and not one designed specifically for test purposes which the other three blocks were. Hence there are quite a number of factors which could have caused the value of σo to be larger for this particular block. Systematic error is also larger than expected in this block. Although this is compensated for quite effectively by the TP technique (as shown in Table 5.9 and Diagram 5.7), there appears to be some residual systematic error still unaccounted for.

The results for planimetry (shown in Table 5.12) improved with improved height accuracy, and the final standard deviations were of an acceptable order of magnitude. However it is again noticeable that some of the maximum residuals are slightly larger than the expected σmax values. The reason for this is probably the fact that 14 of the 16 check points were determined photogrammetrically, and the co-ordinates are those determined from an adjustment of the DOS Test Block of 2 strips using all the available control. Thus the check points for this particular test cannot be regarded as being truly terrestrial.

Referring to Table 5.14 the following specific points can be seen from the tabulated information of projects C1 and C2.

(i) In project C1, Procedure A (Control Pattern 1) yields the same accuracy as Procedure B (Control Pattern 1). The accuracy improved 2.1 times in both cases, the number of adjustments carried out being the same in each case.

(ii) In Project C2, Procedure A (Control Pattern 2) yeilds significantly better accuracy than Procedure B (Control Pattern 2). The accuracy improved 14.8 times (from 11.08‰H to 0.75‰H) in the former case and 8.8 times (from 11.08‰H to 1.25‰H) in the latter case. It will be noted that the number of adjustments carried out was four in Procedure A (Control Pattern 2) and two in Procedure B (Control Pattern 2).

It will also be noted that the bridging distance is 8 models in Project C1 and 16 models in Project C2. Furthermore, the arrangement and distribution of control is better in Project C2 than in Project

C1. So, it is possible to conclude that the significant improvement of Procedure A (Control Pattern 2) over Procedure B (Control Pattern 2) lies in its application in projects with a good distribution of control and with long bridging distances.

Referring once more to Table 5.8, one can also conclude from the tabulated information of Projects C1, C2, C3 and C4 that the accuracy of an adjusted block depends on the density of control points.

Table 5.8

(Tables 5.8.1to 5.8.4) - Height Accuracy, Procedure A,D.O.S. Block

Table 5.8.1

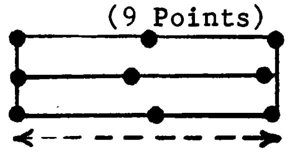
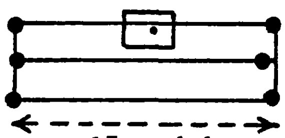
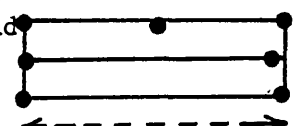
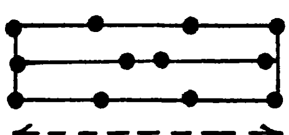
Project Photo Scale = 1:12500 Flying Height (H) = 1915 metres 2 strips; 34 models total		Bridging Distance (models)	Number of Terrestrial Height Control Points
No.	Details		
C.1	3 Ground Height Control Bands (9 Points)  17 models	8	9
C.2	2 Ground Height Control Bands (6 Points) plus one Ground Height Check Point  17 models	16	6
C.3	2 Ground Height Control Bands (6 Points) plus One Extra Ground Height Control Point,  17 models	16	7
C.4	4 Ground Height Control Bands (12 Points)  17 models	6	12

Table 5.8.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error, (%oH)	Maximum height Residuals detected at check points. (%oH)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
C.1	13	±0.53	-1.950	1.036
C.2	16	±0.94	-14.920	1.273
C.3	15	±0.94	3.481	
C.4	10	±0.43	-2.995	-2.301

Table 5.8.3

Project No.	σ_o = The Standard Error of Unit Weight (%oH)		
	σ_o used in computing the expected maximum Error	σ_o Obtained	
		Before Height Accuracy Improvement	After Height Accuracy Improvement
C.1	0.20	0.74	0.39
C.2	0.20	3.17	0.27
C.3	0.20	0.74	
C.4	0.20	1.39	1.07

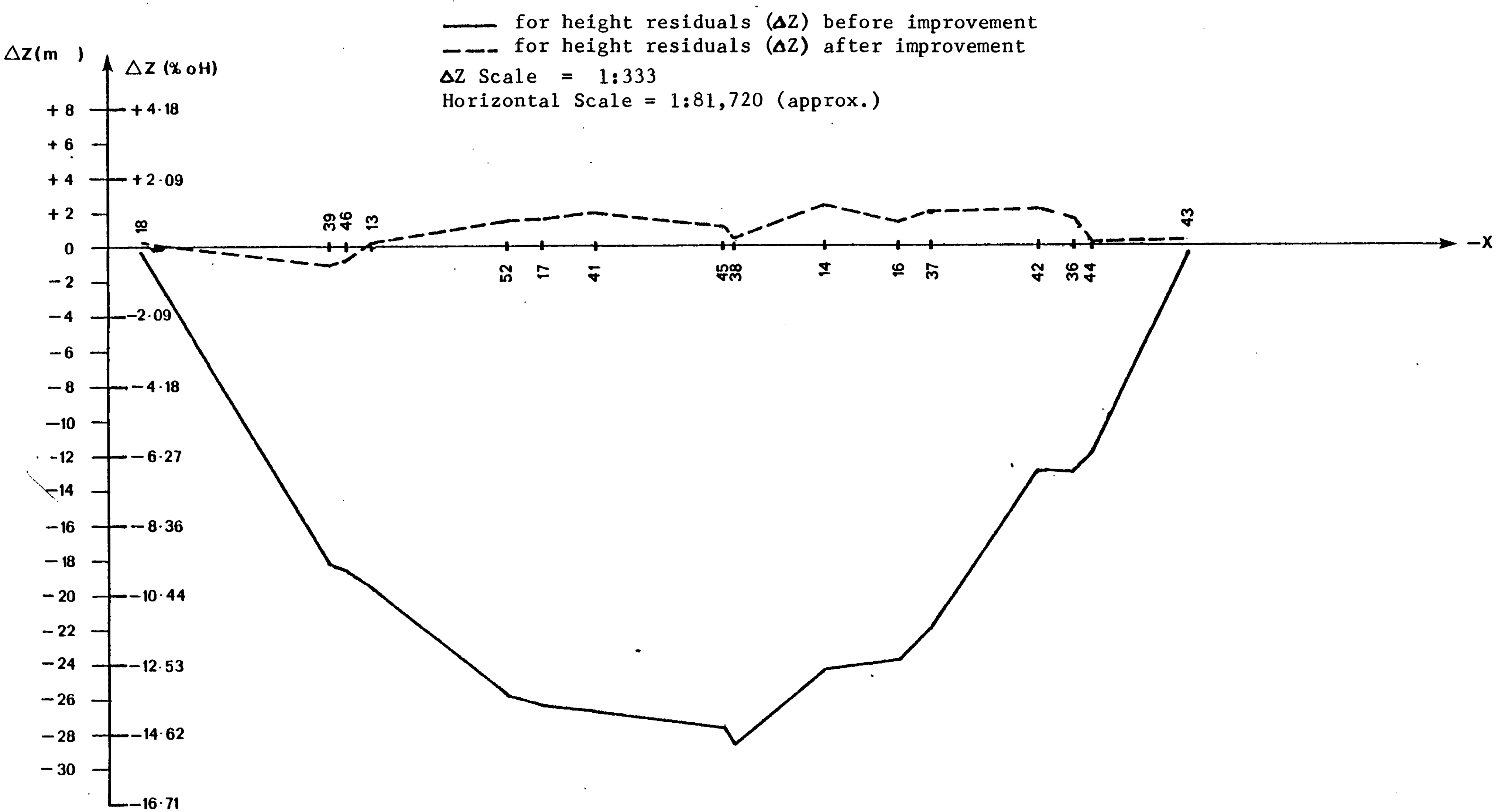
Table 5.8.4

Project No.	The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Points (%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
C.1	1.149	0.546	2.1
C.2	11.077	0.747	14.8
C.3	2.298		
C.4	0.927	0.713	1.3

Table 5.9 - Comparison Table, Procedures A & B, Project C2

Terrestrial Check Point No.	Height (ΔZ) Residuals detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
39	-18.221	-1.096	-5.096
13	-19.539	0.382	-3.646
52	-25.772	1.574	-3.021
41	-26.727	1.999	-0.852
38	-28.572	0.587	-0.202
14	-24.336	2.437	2.022
37	-21.844	2.057	0.495
42	-12.775	2.145	1.182
36	-12.842	0.205	0.336
123	0.032	0.342	0.178
46	-18.502	-0.712	-4.282
13	-19.653	0.266	-3.755
17	-26.421	1.572	-1.580
45	-27.650	1.201	0.782
16	-23.713	1.539	1.203
44	-11.818	0.363	-0.054

Diagram 5.7 The D.O.S. Test Block
(Height residuals in project C2. Procedure A (Control pattern 2))



OBERSCHWABEN BLOCK

Results and Analysis

5.2.5 Analysis of the Results of the Oberschwaben Test Block

The results of the Oberschwaben Test Block are given in Tables 5.10.1 to 5.10.4. The results from this particular block have a special importance, since it is one of the best designed and executed blocks ever devised for testing purposes. Thus the results have been used by several authors in recent years for both theoretical and practical studies in aerial triangulation. It is therefore a particularly important yardstick for the effectiveness of the TP Technique.

Improvement factors of 1.2 and 2.4 were obtained for Projects D1 and D2 respectively. The accuracies for all of the Oberschwaben projects are better than expected, even before height accuracy improvement which contrasts markedly with the results for the DOS Block. This is reflected also by the estimated values σ_o obtained for Projects D1 and D2 before height accuracy improvement. The estimated value σ_o obtained in both projects was 0.13‰H, which is less than the value (0.20‰H) of σ_o used in computing the expected maximum standard error (σ_{max}). However, as noted above, great care has been taken to establish the test area at Oberschwaben, and it has been designed specifically for test purposes rather than for mapping purposes which is the case with the D.O.S block. Also greater care has undoubtedly been taken with the observations of the Oberschwaben block than the D.O.S block. Nevertheless, a systematic error component is still present in the results before improvement by the TP technique (see Table 5.11 and Diagram 5.8). This residual component is compensated for quite readily by the TP technique. The final result of 0.086‰ standard deviation for Project D2 is indeed most gratifying, and it is a reflection not only on the quality of the observations but also on the value of the TP technique when such a well-observed block with an optimised pattern of control can still derive benefit from the method.

It could be concluded from the tabulated information (number of control points and the standard deviations before height accuracy improvement) of Projects D1, D2, and D3 in Table 5.10 that, if the pattern of Control is irregular, then the accuracy of an adjusted block depends on the density of control points.

As far as planimetric accuracy is concerned, Table 5.12 shows that there is no improvement in planimetry resulting from height improvement. This is due to the fact that in this case, the improvement in height accuracy is insufficient to affect the planimetry, and indeed one could say that the limit for planimetric accuracy had already been reached before the height accuracies were improved. This is confirmed by the value of the estimated value σ_o which was 19μm both before and after the height accuracy improvement of Projects D1 and D2 while σ_o used in computing the maximum standard error (σ_{max}) expected was 20μm. The fact that there is virtually no difference in the values for planimetric accuracy between Projects D1 and D2 would seem to verify this.

Referring to Table 5.14, it can be seen from the tabulated information for Project D2 that Procedure A (Control Pattern 2) yields slightly better results than Procedure B (Control Pattern 2). The height accuracy improved 2.4 times (from 0.204‰H to 0.086‰H) in the former case (carrying out 4 adjustments) and 2.3 times (from 0.204‰H to 0.090‰H) in the latter case (carrying out 2 adjustments). However, it must be remembered that accuracies for the Oberschwaben projects are all better than expected, even before height accuracy improvement. The effectiveness of Procedure A (Control Pattern 2) over Procedure B (Control Pattern 2) is seen more clearly from Projects A2, B6 and C2 where the accuracy before improvement is of a much lower order.

Table 5.10

(Tables 5.10.1 to 5.10.4) - Height Accuracy, Procedure A, Oberschwaben Block

Table 5.10.1

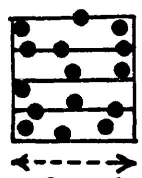
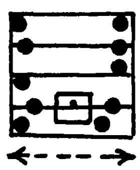
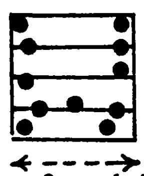
Project Photo Scale = 1:28,000 Flying Height (H) = 4290 metres. 4 Strips; 32 models total.		Bridging Distance	Number of Terrestrial Height Control Points
No.	Details	(models)	
D.1	3 Ground Height Control Bands (15 points)  8 models	4	15
D.2	2 Ground Height Control Bands (10 points) plus one Ground Height Check Point.  8 models	8	10
D.3	2 Ground Height Control Bands (10 points) plus one extra Ground Height Control Point  8 models	8	11

Table 5.10.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error, (%oH)	Maximum height residuals detected at check points. (%oH)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
D.1	60	±0.36	0.233	0.245
D.2	65	±0.525	-0.342	0.304
D.3	64	±0.525	0.261	

Table 5.10.3

Project No.	σ_o = the Standard Error of Unit Weight (%oH)		
	σ_o used in computing the expected maximum Error	σ_o obtained	
		Before Height Accuracy improvement	After Height Accuracy improvement
D.1	0.20	0.13	0.14
D.2	0.20	0.13	0.12
D.3	0.20	0.10	

Table 5.10.4

Project No.	The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Point (%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
D.1	0.116	0.100	1.2
D.2	0.204	0.086	2.4
D.3	0.136		

Table 5.11

(Tables 5.11.1 to 5.11.4) - Comparison Tables, Procedures A & B, Project D2

Table 5.11.1

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
6901 (11)	1.053	0.943	0.960
6802 (12)	-0.002	0.246	0.169
11701 (13)	-0.621	-0.190	-0.312
6701 (14)	-1.148	-0.510	-0.709
6702 (15)	-0.856	-0.167	-0.369
11502 (16)	-1.317	-0.371	-0.526
11601 (17)	-1.072	-0.200	-0.376
6602 (18)	-1.362	-0.449	-0.659
6601 (19)	-1.136	-0.144	-0.344
11501 (20)	-1.273	-0.261	-0.377
6502 (21)	-1.102	-0.073	-0.208
11402 (22)	-1.245	-0.240	-0.369
6403 (23)	-0.943	0.080	-0.060
6401 (24)	-0.696	0.341	0.169
11401 (25)	-0.708	0.258	0.115
11301 (26)	-0.679	0.147	-0.031

Table 5.11.2

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
16201 (27)	-0.442	0.007	-0.083
6202 (28)	-0.668	-0.093	-0.241
6201 (29)	0.176	0.646	0.488
11201 (30)	-0.039	0.348	0.245
16801 (31)	-0.337	-0.247	-0.313
21701 (32)	-1.207	-0.522	-0.847
26701 (33)	-0.947	0.163	-0.425
21702 (34)	-0.918	0.081	-0.415
16702 (35)	-0.994	-0.100	-0.466
21601 (36)	-1.302	-0.100	-0.526
16601 (37)	-1.281	-0.151	-0.470
26501 (38)	-1.065	0.415	0.006
16501 (39)	-1.244	-0.135	-0.309
16401 (40)	-0.735	0.384	0.272
21501 (41)	-0.725	0.529	0.298
21401 (42)	-1.119	0.244	-0.139

Table 5.11.3

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters) Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
16402 (43)	-1.341	-0.210	-0.456
16301 (44)	-1.313	-0.279	-0.500
21201 (45)	0.298	1.305	0.774
21302 (46)	-1.466	-0.324	-0.811
21301 (47)	-0.882	0.098	-0.243
16202 (48)	-0.284	0.293	0.062
16102 (49)	-0.717	-0.456	-0.578
16002 (50)	-0.299	-0.348	-0.396
31701 (51)	0.096	0.626	0.528
26702 (52)	-0.217	0.854	0.413
26601 (53)	-0.927	0.164	-0.146
26502 (54)	-1.075	0.085	-0.177
31501 (55)	-1.089	0.041	-0.179
31502 (56)	-0.714	0.309	0.152
31301 (57)	-0.739	0.100	-0.038
26401 (58)	-0.943	0.175	-0.096

Table 5.11.4

Terrestrial Check Point No.	Height (ΔZ) Residuals Detected at the Terrestrial Check Points (in meters). Control Pattern 2		
	Before Accuracy Improvement	After Accuracy Improvement	
	Procedures A & B	Procedure A	Procedure B
36201 (59)	-0.563	-0.078	-0.110
36301 (60)	-0.323	0.253	0.224
26301 (61)	-0.735	0.170	-0.108
31201 (62)	-0.826	-0.580	-0.633
26101 (63)	0.213	0.532	0.323
41801 (64)	0.192	0.245	0.226
36901 (65)	-0.080	-0.205	-0.155
36801 (66)	0.332	0.456	0.444
41701 (67)	-0.760	-0.133	-0.331
36701 (68)	-0.700	-0.060	-0.224
46501 (69)	-0.621	0.566	0.326
41602 (70)	-0.837	0.088	-0.100
36601 (71)	-0.739	0.053	-0.069
41601 (72)	-1.226	-0.243	-0.407
36402 (73)	-0.976	0.020	-0.050
41401 (74)	-0.708	0.291	0.175
41301 (75)	-0.509	0.223	0.121

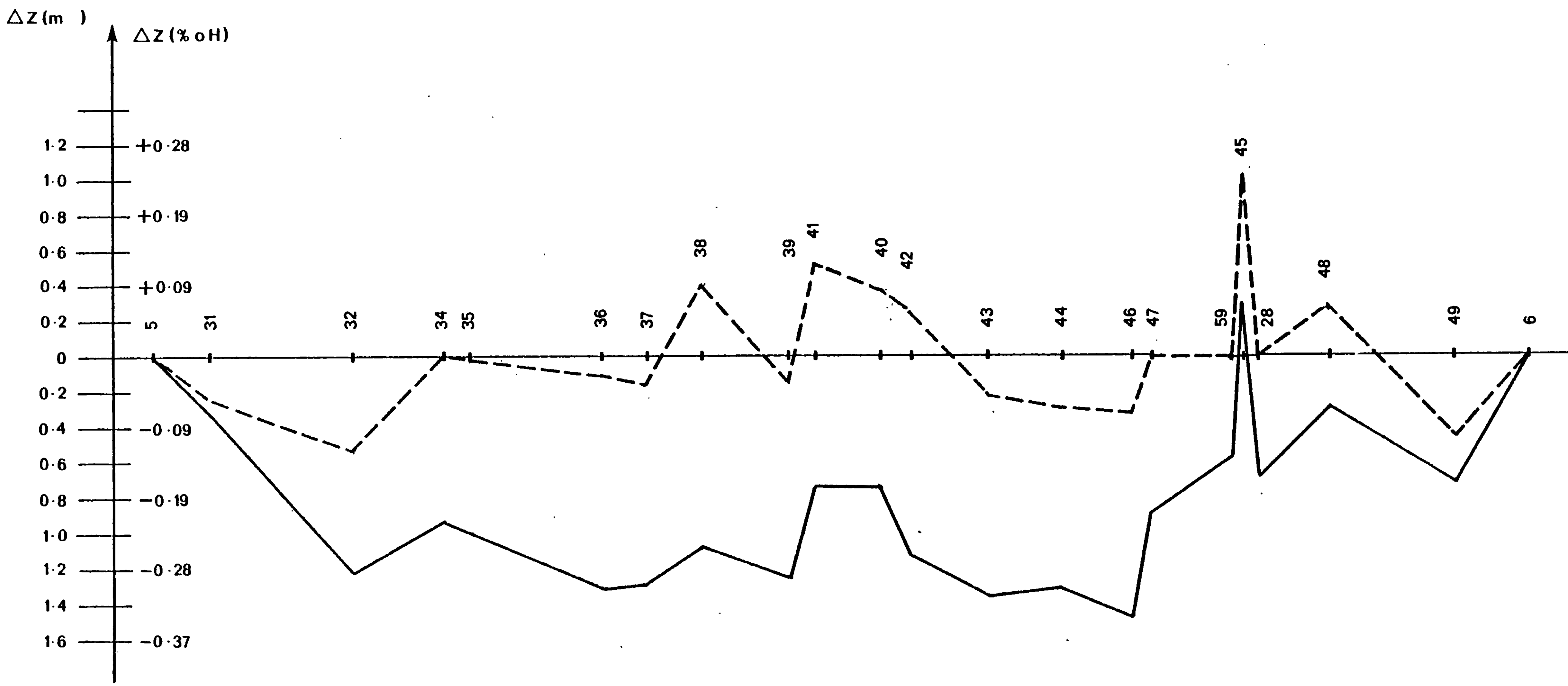
Diagram 5.8 The Oberschwaben Test Block
(Height residuals in project D2.Procedure A (Control Pattern 2))

—— for height residuals (ΔZ) before improvement

- - - for height residuals (ΔZ) after improvement

ΔZ Scale = 1:33

Horizontal scale = 1:81,170 (approx.)



<u>SUMMARY AND</u>

<u>COMPARISON TABLES</u>

<u>ALL BLOCKS</u>

5.2.6 Summary and Comparison Tables

In the previous sections, detailed analysis was given of the contents of tables 5.12.1 to 5.12.5, tables 5.13.1 to 5.13.4 and tables 5.14.1 to 5.14.2 shown below. In general, one can say the following when inspecting these summary and comparison tables:

(i) The planimetric accuracy is improved by improving the height accuracy. (See tables 5.12.1 to 5.12.5).

(ii) Procedure B, discussed previously in Chapter 4, succeeds in improving the height accuracy. (See tables 5.13.1 to 5.13.4).

(iii) Procedure A yields better results than Procedure B in Control Pattern 2 (See tables 5.14.1 to 5.14.2). More explanation is given in the next section.

Table 5.12

(Tables 5.12.1 to 5.12.5) - Planimetric Accuracy, Procedure A, All Blocks

Table 5.12.1

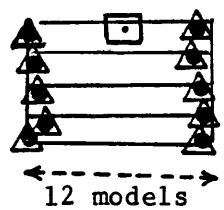
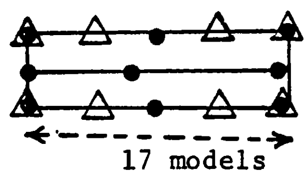
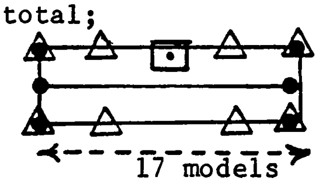
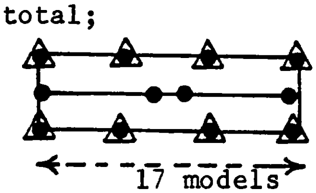
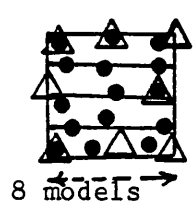
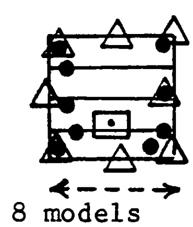
<table>
<tr><td colspan="2">Project
△ = Terrestrial Planimetric Control; ● = Terrestrial Height Control; ⊡ = Terrestrial Height Check Point (for detection of maximum systematic error after first adjustment)</td><td rowspan="2">Photo Scale</td></tr>
<tr><td>No.</td><td>Details</td></tr>
<tr><td>A.2</td><td>Durban Block; 4 Strips; 48 models total; 2 Height Control Bands (10 points) plus one Height Check Point.
</td><td>1:8000</td></tr>
<tr><td>C.1</td><td>D.O.S. Block; 2 Strips; 34 models total; 3 Height Control Bands (9 points)
</td><td>1:12500</td></tr>
<tr><td>C.2</td><td>D.O.S. Block; 2 Strips; 34 models total; 2 Height Control Bands (6 points) plus one Height Check Point.
</td><td>1:12500</td></tr>
<tr><td>C.4</td><td>D.O.S. Block; 2 Strips; 34 models total; 4 Height Control Bands (12 points)
</td><td>1:12500</td></tr>
<tr><td>D.1</td><td>Oberschwaben Block; 4 Strips, 32 models total; 3 Height Control Bands (15 points)
</td><td>1:28000</td></tr>
<tr><td>D.2</td><td>Oberschwaben Block; 4 strips; 32 models total; 2 Height Control Bands (10 points) plus one Height check point.
</td><td>1:28000</td></tr>
</table>

Table 5.12.2

Project No.	Bridging Distance (models) for Planimetry (P) In Direction X	Y	Number of Terrestrial Control Points for Planimetry (P)	Bridging Distance (models) for Height (Z)	Number of Terrestrial Control Points for Height (Z)
A.2	12	2	10	12	10
C.1	6	4	8	8	9
C.2	6	4	8	16	6
C.4	6	4	8	6	12
D.1	4	4	8	4	15
D.2	4	4	8	8	10

Table 5.12.3

Project No.	Number of Terrestrial Check Points	Expected Maximum Error, (μm in negative scale)	Maximum Planimetric residuals detected at check points. (μm in negative scale)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
A.2	62	62	62	56
C.1	13	36	49	48
C.2	15	36	43	48
C.4	10	36	28	27
D.1	60	28	27	27
D.2	65	28	27	27

Table 5.12.4

Project No.	σ_o = the Standard Error of Unit Weight (µm in negative scale)		
	σ_o used in computing the expected maximum Error	σ_o obtained	
		Before Height Accuracy Improvement	After Height Accuracy Improvement
A.2	20	20	18
C.1	20	27	26
C.2	20	24	27
C.4	20	16	15
D.1	20	19	19
D.2	20	19	19

Table 5.12.5

Project No.	The standard deviation of the planimetric residuals (ΔP) detected at the terrestrial check points. (μm in negative scale)		Planimetric Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
A.2	28	20	1.4
C.1	18	17	1.1
C.2	27	18	1.5
C.4	14	14	1.0
D.1	18	18	1.0
D.2	18	18	1.0

Table 5.13

(Tables 5.13.1 to 5.13.4) - Height Accuracy, Procedure B, All Blocks

Table 5.13.1

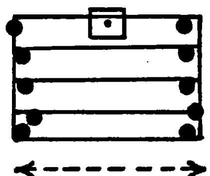
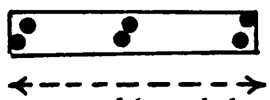
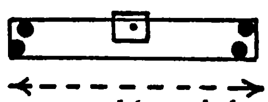
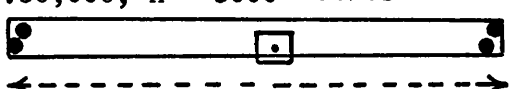
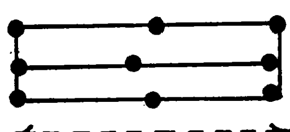
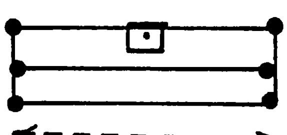
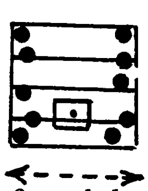
Project No.	Project Details (S = Photo Scale, H = Flying Height)	Bridging Distance (Models)	Number of Terrestrial Height Control Points
A.2	S = 1:8000 H = 1218 metres  12 models	12	10
B.5	S = 1:30,000 H = 5000 metres  16 models	8	6
B.6	S = 1:30,000 H = 5000 metres  16 models	16	4
B.7	S = 1:30,000; H = 5000 metres  31 models	31	4
C.1	S = 1:12,500 H = 1915 metres  17 models	8	9
C.2	S = 1:12,500 H = 1915 metres  17 models	16	6
D.2	S = 1:28,000 H = 4290 metres  8 models	8	10

Table 5.13.2

Project No.	Number of Terrestrial Height Check Points	Expected Maximum Error, (‰ H)	Maximum height residuals detected at check points. (‰H)	
			Before Height Accuracy Improvement	After Height Accuracy Improvement
A.2	62	±0.725	-1.420	0.535
B.5	26	±0.70	-1.385	0.739
B.6	28	±1.28	-7.378	2.663
B.7	44	±1.28	-17.946	4.015
C.1	13	±0.53	-1.950	0.522
C.2	16	±0.94	-14.920	2.661
D.2	65	±0.525	-0.342	0.224

Table 5.13.3

Project No.	σ_o = the Standard Error or unit Weight (%oH)		
	σ_o used in computing the expected maximum Error	σ_o obtained	
		Before Height Accuracy Improvement	After Height Accuracy Improvement
A.2	0.20	0.39	0.15
B.5	0.20	0.40	0.21
B.6	0.20	1.15	0.42
B.7	0.20	2.80	0.63
C.1	0.20	0.74	0.20
C.2	0.20	3.17	0.57
D.2	0.20	0.13	0.09

Table 5.13.4

Project No.	The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Points (%oH)		Height Accuracy Improvement Factor
	Before Height Accuracy Improvement	After Height Accuracy Improvement	
A.2	0.832	0.185	4.5
B.5	0.929	0.432	2.2
B.6	5.398	1.814	3.0
B.7	11.874	1.842	6.4
C.1	1.149	0.540	2.1
C.2	11.077	1.252	8.8
D.2	0.204	0.090	2.3

Table 5.14

(Tables 5.14.1 & 5.14.2) - Comparison of Procedures A & B, All Blocks

Table 5.14.1

Project No.	Control Pattern	Procedure	The Standard Deviation (σ) of the Height Residuals Detected at the Terrestrial Height Check Points (%oH).		Height Accuracy Improvement Factor	Number of Adjustments Carried Out
			Before Height Accuracy Improvement	After Height Accuracy Improvement		
A.2	2	A	0.832	0.156	5.3	4
		B		0.185	4.5	2
B.5	1	A	0.929	0.405	2.3	3
		B		0.432	2.2	3
B.6	2	A	5.398	0.418	12.9	4
		B		1.814	3.0	2
B.7	2	A	11.874	1.844	6.4	4
		B		1.842	6.4	2

Table 5.14.2

Project No.	Control Pattern	Procedure	The Standard Deviation (σ) of the Height Residuals Detected at the Terrestrial Height Check Points (‰H).		Height Accuracy Improvement Factor	Number of Adjustments carried Out
			Before Height Accuracy Improvement	After Height Accuracy Improvement		
C.1	1	A	1.149	0.546	2.1	3
		B		0.540	2.1	3
C.2	2	A	11.077	0.747	14.8	4
		B		1.252	8.8	2
D.2	2	A	0.204	0.086	2.4	4
		B		0.090	2.3	2

CONCLUSIONS

5.3 Conclusions

From the analysis of results of all the tests we can conclude the following:

(1) The results obtained in the tests have verified well-known conclusions regarding the presence of systematic error in block triangulation. In particular the following points may be made:-

(i) The existance of systematic height errors spoils the overall accuracy.

(ii) Systematic height error is independent of block size, but depends more on the bridging distance between bands of control.

(iii) Maximum systematic height error occurs midway between bands of control.

(iv) If the pattern of control is irregular, then the accuracy of an adjusted block depends on the density of control points.

(2) The TP technique can detect and eliminate systematic height errors.

(3) If the height accuracy is poor, then improvement of height accuracy results in an improvement of planimetric accuracy.

(4) In these tests, the systematic height residuals show that the values which occur at the adjusted photogrammetric height points are larger than the corresponding values for the terrestrial heights of the same points. The explanation for this probably lies in the fact that no account was taken of Earth curvature and refraction before entering the adjustment phase.

(5) The results of theoretical accuracy studies can be considered sufficiently realistic since an accuracy better than the expected one could be obtained after elimination of existing systematic height errors. That is to say, the errors of real photographs behave

according to the theoretical assumptions and that the mathematical model sufficiently predicts reality. (The results obtained by Ebner (1976) also support this conclusion).

(6) The TP technique saves the cost of providing the additional ground control that would have been required to improve the accuracy by an equivalent amount if the technique had not been used.

(7) The arrangement of height control affects the results. The ideal arrangement will result in obtaining optimum improved accuracy.

(8) Only two bands of terrestrial height control together with an additional terrestrial height check point lying midway between them are enough to obtain the optimum height accuracy. That is to say, if n is the number of parallel strips in a rectangular block, then the terrestrial heights of only $(2n + 3)$ points will be required to obtain the optimum height accuracy after the application of the TP technique.

(9) The determination of corrections (for the photogrammetric tie points that are to be used as height control) is more accurate for Procedure A than Procedure B in Control Pattern 2. The results of projects involving Control Pattern 1 do not show this difference in accuracy between Procedure A and Procedure B. An explanation for this could be either that the bridging distances are too short, or that the Pattern of Control is too irregular. Further tests with other blocks would be required to substantiate this explanation.

CHAPTER VI

A Comparison of the TP technique with other methods

6. A comparison of the TP technique with other methods.

Any comparison between methods of compensating for systematic error in block adjustment must take into account the mathematical models used in the block adjustment, the extent to which systematic error is removed before adjustment and the success achieved by the technique. Further considerations include the additional cost and complexity involved in applying the technique, though these become less important if the technique is really effective in terms of the accuracy of the final results.

Most modern block adjustment programs are based on sound least squares principles, and although some regard the parameters of three-dimensional models as unknowns, others regard the parameters of each bundle as unknowns. Theoretically, the difference in approach concerns what is actually regarded as the observations which are to be minimized - i.e. whether they are the observed model co-ordinates or the observed plate co-ordinates. In practice, the difference in the results obtained by the two approaches is not excessive. Of greater significance is the magnitude of the systematic error that arises in any block adjustment with sparse control, whether it is based on bundles or models. Hence the comparison in this chapter includes the results from tests which are based on both bundles and models. It also includes results from tests using data which have had preliminary corrections for systematic error (such as Earth's curvature and refraction), as well as those which have not been corrected in this way.

The comparison between the TP technique and the other methods is made from the following three points of view:-

(a) the ability to produce the required results;

(b) the cost involved in producing such results, and conversely the saving in the cost of ground control - i.e. the economic aspects.

(c) the ease with which the TP technique may be applied.

6.1 The ability to produce the required results

There are few references in existing literature which discuss the different approaches to the compensation of systematic error in block adjustment. Strictly speaking, such a comparison would only be valid if exactly the same input data was used for each approach. Unfortunately, circumstances did not make this possible; however some tests have been carried out by different authors using data from the same test area - namely the Oberschwaben Test Block. Details of these tests are summarized in Table 6.1, and the results of the tests are given in Table 6.2. Technique No. 1 in these tables is the TP technique.

Table No. 6.1 (Details)

Oberschwaben Block; 1:28,000; Sidelap = 20%.

Author	Technique No.	Project No.	Block (Executing Organisation)	Photography	Block Adjustment Method
Abdel Rahim (1980)	1	1.1	Frankfurt (F)	WA	Independent Models (I.M)
Ebner (1976)	2	2.1	Frankfurt (F)	WA	Bundle
		2.2	Frankfurt (F)	WA	Ind. Models
		2.3	Frankfurt (F)	WA	Ind. Models
Bauer, Muller (1972)	3	3.1	Frankfurt (F)	WA	Bundle
Bauer (1973)	4	4.1	Frankfurt (F)	WA	Bundle
		4.2	Frankfurt (F)	WA	Bundle
		4.3	Frankfurt (F)	WA	Bundle
		4.4	Frankfurt (F)	WA	Bundle
		4.5	Frankfurt (F)	WA	Bundle
Haug (1976)	5	5.1	The Hague (T.H)	SWA	Ind. Models
		5.2	Delft (Dt)	SWA	Ind. Models
Schneider (1978)	6	6.1	Frankfurt (F)	WA	Ind. Models

Table No. 6.2 (Accuracy Comparison)

Technique No.	Project No.	Bridging Distance	*μz/μm/ Before the height accuracy improvement	*μz/μm/ After the height accuracy improvement	Number of times of the height accuracy improvement	Accuracy Comparison Ratio (between Technique No. 1 and the other techniques)
1	1.1	8	31.2	13.2	2.4	
2	2.1	8	22.2	14.6	1.5	1.1
2	2.2	8	19.0	17.1	1.1	1.30
2	2.3	4	14.7	14.1	1.0	1.1
3	3.1	5	20.0	14.0	1.4	1.1
4	4.1	5	19.7	16.1	1.2	1.2
4	4.2	5	19.6	14.7	1.3	1.1
4	4.3	2	15.7	15.5	1.0	1.2
4	4.4	2	15.7	14.4	1.1	1.1
4	4.5	2	15.7	15.8	1.0	1.2
5	5.1	2	14.9	14.9	1.0	1.1
5	5.2	2	14.6	14.5	1.0	1.1
6	6.1	4	15.1	13.8	1.1	1.05

*μz/μm/ = The standard deviation of the height residuals detected at the terrestrial height check points (in micrometers (μm) in negative scale).

Inspection of the results in Table 6.2 may be made, bearing in mind at the same time the circumstances associated with each project given in Table 6.1. For example, the poor results (31.2μm) before accuracy improvement in project 1.1 (which is Project D.2 in Table 5.10 of Chapter 5) may be explained by the fact that no preliminary corrections for Earth's curvature and refraction were applied to the input data. What is very clear from Table 6.2 however, is that the greatest improvement is obtained using the TP technique (namely from 31.2μm to 13.2μm), though the differences in the final accuracies for all the methods is marginal. Although the accuracy before improvement with the TP technique is far worse than for any other method, the accuracy after applying the technique is marginally better.

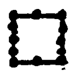
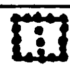
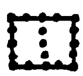
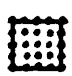
6.2 Economic aspects

One of the most important factors determining the cost of an aerial triangulation project is the expense involved in obtaining ground control. Thus any method which reduces the required number of control points is likely to reduce the overall cost.

As described earlier in Chapter 4, the TP technique improves the height accuracy by reducing the original bridging distance. It achieves this by using additional bands of corrected photogrammetric tie points as height control, together with the original bands of terrestrial height control. It was shown in Chapter 5 that, in principle, only two bands of height control together with one additional terrestrial height check point, lying midway between the bands, was enough to obtain the optimum height accuracy. Thus if n is the number of parallel strips in a block, then only (2n + 3) terrestrial height control points would be required to obtain the optimum height accuracy. Furthermore, the pattern of control required by the technique is more consistent with that required for planimetric adjustment. Thus the organisation and completion of the field survey work to establish the control becomes an easier and less costly process.

Column 7 (the last column) in Table 6.3 compares the number of control points required as a ratio of the TP technique (Technique No.1).

Table No.6.3 (Control point Comparison)

Technique No.	Project No.	Bridging Distance	Terrestrial Control Point patterns for blocks which have been reduced in size to approximately that of Project 1.1	*μz/ μm/ After the height accuracy improvement	Number of the terrestrial height points used for the same block size	Control Requirements Ratio between Technique No.1 and the other techniques
1	1.1	8		13.2	11	
2	2.1	8		14.6	12	1.1
2	2.2	8		17.1	12	1.1
2	2.3	4		14.1	19	1.7
3	3.1	5		14.0	19	1.7
4	4.1	5		16.1	19	1.7
4	4.2	5		14.7	19	1.7
4	4.3	2		15.5	25	2.3
4	4.4	2		14.4	25	2.3
4	4.5	2		15.8	25	2.3
5	5.1	2		14.9	25	2.3
5	5.2	2		14.5	25	2.3
6	6.1	4		13.8	19	1.7

• - Terrestrial height control

 - Terrestrial height check point for detection of maximum systematic error after first adjustment.

The comparison is based on the number of height control points which are required to be established by field survey methods, and although strictly speaking this is not an exact measure of expense, it does give some indication of relative costs in the establishment of the necessary control points.

Another important point which must be considered when discussing the economic aspects of the TP method is that of computational costs. Certainly, more computation is required with the TP technique, since the block adjustment procedure has to be repeated several times. This adds to the overall costs of the block adjustment process. However, with large efficient computers (such as the machine used in this research) and a modern simultaneous direct least-squares solution such as the S.B.A.I.M method, the cost per model is a relatively small amount both in absolute terms and as a proportion of the whole aerial triangulation process. So the extra computational costs are almost certainly relatively small as compared with the savings in the provision of additional ground control points produced by the TP method.

The use of the TP technique would show a larger computational overhead if a method of block adjustment was employed which was not efficient from a computational point of view. This might arise for example with iterative solutions on small computers where much segmentation of the block takes place and there is a great deal of input/output and transfer of data. Such solutions are not efficient in the first place and therefore their computational costs per model are much higher than those which result when an efficient modern solution is adopted. Obviously, it would make less sense to then multiply this higher unit cost several times by repitition of the process. The conclusion is that from an economic point of view, the TP technique is really best employed with an efficient block adjustment such as those provided by the PAT-M or the S.B.A.I.M programs.

In those organisations where an older, less efficient and less economic type of block adjustment is still in use, it is of course still possible to use the TP technique for removal of the systematic errors. To keep the number of repititions to a minimum, and therefore computational costs to the smallest level possible, Procedure B would be recommended for use instead of Procedure A.

6.3 The ease of application of the TP technique.

The overall ease of application of the technique may be considered by making a comparison with each of the other methods in turn. Details of the other methods are given in Chapter 3; this section considers only the relative merits between these methods and the TP technique.

(a) The method of additional parameters

Certain principles must be followed (Bauer and Müller (1972)) when introducing additional parameters with a block adjustment. These include

(i) The assumption that all photos in the block obtain the same correction;

(ii) The requirement that the number of additional parameters must be as small as possible, and that their mutual correlation, as well as their correlation with orientation unknowns, must be as small as possible;

(iii) The desirability of having parameters which are insensitive to a rotation of the photo through 180°, so that either of the two positions in the measuring instrument may be assumed without affecting the results.

Bauer and Müller criticise these principles in the following manner:-

The assumption (i) above that conditions are the same for all photos in the block is probably invalid, and attempting to assess the differences for different parts of the block and assigning parameters accordingly, makes the method unduly complicated. More control points are required to solve for the additional unknowns and it is not easy to define the number and best arrangement of these control points. What is more, it is not an easy matter to choose parameters which have minimum correlation with one other and with the orientation unknowns. Failure in this respect leads to worse results than if they had not been introduced at all.

It is clear therefore, that the method of additional parameters is complicated and indeed dangerous if it is implemented incorrectly. The TP technique on the other hand assumes the use of normal photography, and ordinary block adjustment programs based on conventional mathematical models.

(b) The method involving common image deformation parameters. Bauer (1973) has also reported on this method and concludes that it is desirable

(i) to carry out the flight on a single day;

(ii) to prescribe a regular flight performance in terms of direction and sidelap; and

(iii) to use only one camera.

Requirement (i) above is clearly not very practical and many factors (such as the size of the area to be covered, the prevailing weather conditions, technical difficulties which may arise, etc.) normally prevent the completion of the photography in one day. Similar arguments apply as far as requirement (ii) is concerned, though perhaps to a lesser extent than in (i). The use of only one camera is not such a demanding requirement, though there are occasions, such as when technical problems arise, when more than one camera would be required to complete a photographic mission. Thus for this method also it is clear that the use of common image deformation parameters is invalid.

6.4 Conclusions

The main conclusions that can be drawn from the comparison with other methods is firstly, that the additional parameters or self-calibration methods require either more control, or assumptions that are often unacceptable. Secondly they involve complications as far as the computer programs are concerned - mainly over the matter of which additional parameters should be included in the adjustment. This clearly demands the attention of personnel with a level of expertise above that normally required for block adjustment operations. The third point and probably the most significant, is the fact that with a reduction in the number of control points that have to be provided, the saving in field and office work could reduce the expense of the triangulation considerably.

Against these advantages there are one or two slight disadvantages. The first is that, for the TP method to be effective, the control and check points must be distributed in a regular pattern. Although at first sight this might appear to be a retrograde step, it is in fact not a serious handicap, because the pattern is similar to that which is normally required anyway for rectangular blocks. The second point concerns the extra computational effort required by the TP method. However, as discussed previously, this should not add significantly to the overall costs of the triangulation since the computational costs are a relatively small proportion of the whole aerial triangulation process, especially if an efficient modern block adjustment is employed.

The overall conclusion may be drawn that the TP technique is simple and effective in its application, requiring no change to conventional block adjustment programs, and no additional information (such as from auxiliary instruments - statoscope, horizon camera, etc.,). It could also be regarded as the least expensive of all the procedures for compensation of systematic error.

CHAPTER VII

Conclusion

7. CONCLUSION

7.1 Experience gained from the tests on real data

The comments listed below are based on the results obtained after applying the TP technique to a number of test blocks.

(1) The results have verified well-known conclusions regarding the presence of systematic error in block triangulation. In particular the following aspects have been verified:-

(i) Systematic error is independent of block size, but depends on the bridging distance between bands of control.

(ii) Maximum systematic error occurs midway between bands of control.

(iii) If the pattern of control is irregular, then the accuracy of an adjusted block depends on the density of control points.

(2) The TP technique can detect and eliminate systematic height error. It does so by using a regular pattern of height control, arranged in bands across the strips. Only two bands, at the extreme edges of the block, plus a check point lying midway between them, are enough to obtain the optimum height accuracy. Thus if n is the number of parallel strips in a rectangular block, then the number of terrestrial height control points would be $(2n + 3)$. The arrangement of height control affects the results and anything less than an ideal arrangement reduces the effectiveness of the technique.

(3) If the height accuracy is poor in the first instance, then application of the technique to improve the height accuracy will also produce an improvement in the planimetric accuracy.

(4) Throughout the tests conducted in this research it was found that the adjusted photogrammetric heights were larger than the known terrestrial values for the same points. The explanation probably lies in the fact that no account was taken

of Earth's curvature and refraction before entering the adjustment phase. Clearly though, the sign of the systematic error will depend on the nature of the source.

(5) After the elimination of systematic errors, it was found that the results were as good as, if not better than, those predicted by theory. Thus the theoretical predictions, based on certain assumptions and mathematical models, may be considered sufficiently realistic at least for these test blocks.

7.2. Recommendations

The experience gained in applying the technique to the specific test blocks mentioned earlier has highlighted certain aspects which should, or could, be considered when further tests are contemplated, or when the method is applied in practice.

(1) The technique relies heavily on control and check point values being correct, and it allows very little room for error in these values, particularly when the method involving only one check point between bands of control is used. Thus field and office methods should be devised which ensure that the values are to all intents and purposes error-free. As an additional safeguard, it is recommended therefore that more than one check point is used.

(2) The successful application of the technique depends on the blocks being rectangular in shape, and also on the control (both terrestrial and photogrammetric) being distributed in band in a regular fashion. Any major deviation from these requirements will cause a deterioration in the accuracy of the results. Clearly, as with other analogous control situations, the determination of values outside the area defined by control should be avoided.

(3) In order to test the effectiveness of the technique further, it is recommended that suitable test blocks with long bridging distances are processed several times. On each occasion, a

particular source (or several sources) of systematic error (such as Earth curvature, or lens distortion, or refraction) could be deliberately left in the raw data (the measured co-ordinates), and the results after application of the technique on each occasion could then be assessed. Certain conclusions could then be drawn as to whether or not image refinement is really necessary before block adjustment. It is recommended further that the results of these tests are compared with those obtained by the use of self-calibration methods on the same data.

(4) The successful elimination of systematic error from adjusted blocks of aerial triangulation open up new possibilities regarding the reliable identification of sources of such systematic error. Confident predictions may then be made of the likely results of a triangulation carried out under certain conditions (of photography, flying height, measuring instruments, etc.,)

(5) In the tests carried out so far it has been found that a reduction of the bridging distance to less than 4 models does not improve the accuracy any further. It would appear therefore that the optimum results are obtained for this bridging distance. However it is recommended that further tests are conducted, with blocks having other characteristics, in order to establish whether or not this is a general rule.

(6) It is recommended that further tests should be carried out to assess the results of combining the TP technique with a self-calibration method. Self-calibration could be used in the initial block adjustment, and then the TP technique could be applied to the results. It is felt that the advantages of both methods - self-calibration for planimetry, and the TP technique for height - might then be seen in the final results.

7.3 Final Conclusion

The limited amount of testing carried out by the author has produced results which enable the following general predictions, regarding the application of the technique, to be made.

(1) The effectiveness of the TP technique should lead to a considerably improved accuracy in and better reliability of the results of aerial triangulation. Such improved results may be applied in the following fields:

(a) Numerical determination of co-ordinates in the applied fields of cadastral surveys, the planning of road construction and large-scale precision mapping for topographic and engineering purposes.

(b)A network of additional terrain points determined photogrammetrically;

(c) A combination of photogrammetric network densification and photogrammetric cadastral surveys in a single (joint) system.

(2) The technique saves the cost of additional Ground Control, that would have been required to improve the height accuracy by an equivalent amount, if the technique had not been used. It may be used in one or more of the following ways:

(a) to <u>raise the accuracy of the photogrammetric solution</u> without changing the scale or the amount of the ground control;

(b) to <u>reduce the ground control requirement</u> for a specified accuracy and scale;

(c) to <u>reduce the scale of aerial photography</u> without affecting the specified accuracy. This will lead to a reduction in the required number of ground control points, since there will be fewer photographs required at the smaller scale than at the larger scale. Thus use of a smaller photo scale while maintaining the specified accuracy will result in a significant saving in the cost of ground control, cost of aerial photography, cost of aerial triangulation, and cost of stereo-plotting;

(d) to dispense with the additional information from auxiliary instruments such as the statoscope, horizon camera, etc., which are often advocated to improve the accuracy of the aerial triangulation process. This will result in a saving in the additional costs due to equipment, operation, modification of aerial triangulation procedure, and complications in the adjustment procedure. In fact, the reduction in cost could be very considerable for mapping vast areas, as for instance in the Sudan (the author's country) where aerial triangulation has been carried out to provide the necessary control for small scale topographic mapping (1/100,000 map series project (see Simmons (1976)), aimed at covering the whole country.

(3) Since existing systematic height errors affect the accuracy of block aerial triangulation, it will be essential to eliminate them when testing, for instance:

(a) the existing theories (see Kubik, Kure (1972), Abdel Rahim (1971)),

(b) the quality of ground control, and adjusted photogrammetric co-ordinates (see Hvidegaard (1976), Tegeler (1976)),

(c) the criteria for rejection of observations (see Förstner (1976), Molenaar (1976)), and

(d) new theories concerning, for example, the solution of the problem of additional unknown parameters (see Ebner (1976)).

The elimination of the systematic errors will make the conclusions in such investigations more reliable, and thus help different organisations establish a strategy for aerial triangulation which suit their own circumstances. They might also help towards establishing a general policy on aerial triangulation based on computation methods, amount of control, size of computer, etc. (see Ackermann (1976)).

(4) The various complications involved in the other methods (for the detection and compensation of systematic errors after triangulation) discourage their practical application, and a method which can be more readily applied in practice would be welcome. It is hoped that the TP technique will fulfil this need.

A P P E N D I C E S

APPENDIX A

Accuracy of Aerial Triangulation

Appendix A

Accuracy of Aerial Triangulation

A.1 Remarks

A.1.1 Remarks on the theoretical accuracy of planimetric strip and block triangulation

The results of theoretical accuracy investigations of planimetric block triangulation are given by Ackermann (1966). They are based on the mathematical and stochastic model used in the Anblock method of block adjustment (see Ekhart (1967) and Van den Hout (1966)). The well-known rules of least squares adjustment and propagation of random error have been applied in the investigations.

The results are given in the units $\frac{\sigma}{\sigma_o}$, where σ_o is the standard error of unit weight for a single observation. The standard error of unit weight can be interpreted as describing the accuracy of the co-ordinate measurements of the corner points of the stereograms together with the effects of model deformations, etc., Fig.A.1 shows some of these results, and the following conditions are relevant to the investigations:-

(i) The presented results are valid for all planimetric block adjustment procedures which work with, or are equivalent to, simultaneous linear orthogonal transformations of independent models.

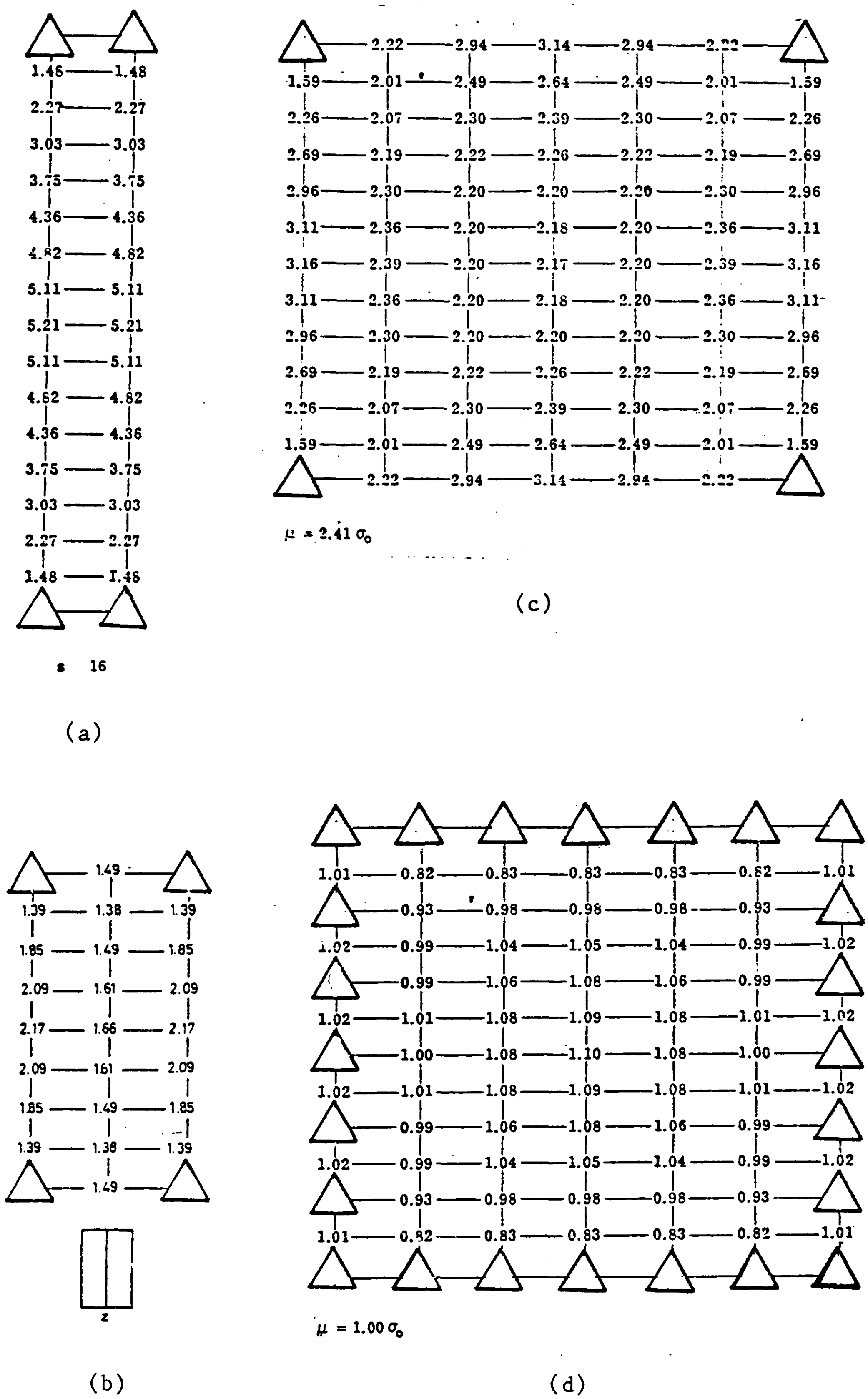
(ii) The model co-ordinates of both the control and tie points are treated as observations which are assumed to be mutually independent and of equal weight. The terrestrial control co-ordinates are treated as error-free.

(iii) Ideal conditions are assumed regarding the geometry of the blocks - i.e. the size of the models, the overlap between them, and the overlap between strips is considered ideal. Control and tie points are also assumed to be located ideally in the corners of the models.

(iv) The results are basically obtained as weight coefficients of adjusted co-ordinates and are presented in units of the "standard error of unit weight" (σ_o) which amounts, according to experience, to 16μm at photo scale for film photography and to 10μm at photo scale for plate photography.

FIG.A.1

Planimetric accuracy $\frac{\sigma_x}{\sigma_o} = \frac{\sigma_y}{\sigma_o}$



A.1.2 Remarks on the theoretical accuracy of height strip and block triangulation

Fig.A.2 represents the results of a theoretical accuracy investigation of height strip and block triangulation carried out by Jerie (1968). This theoretical investigation is based on the Vermeir - Jerie mathematical model. The assumptions that have been made are as follows:-

(1) The height errors in an unadjusted aerotriangulation strip depend on the following four groups of pseudo-observation errors:-

(a) Longitudinal tilt transfer errors $d\phi_i$ between consecutive models. These errors cause the well-known double-summation effect.

(b) Scale transfer errors dS_i, assumed to occur between each stereo-model.

(c) Lateral tilt-transfer errors $d\omega_i$, assumed to occur between each stereo-model.

(d) Photogrammetric height measuring errors dz_k in each individual point, caused by local model deformations and observation errors.

(2) The following free parameters are assumed:-

(a) ΔZ_o constant height error (index error).

(b) $d\phi_o$ error in absolute longitudinal tilt.

(c) $d\omega_o$ error in absolute lateral tilt.

These parameters take care of the error in absolute orientation of the first stero-model.

(3) All pseudo-observation errors are assumed to be uncorrelated and have equal variances within each group of errors.

(4) Translation errors between consecutive models as well as actual model deformations are neglected since their influence is not as significant as that of the pseudo-observation errors.

(5) The influence of most systematic errors is considered to be negligable.

The following expression for the height error of a point r in model k of an unadjusted aerial triangulation strip is obtained when the mathematical model is that described above:-

$$\Delta Z_r = \Delta Z_o + x_r \, d\phi_o + y_r d\omega_o$$

$$+ \sum_1^{k-1} (x_r - \bar{x}_i)(d\phi_i + \Delta\phi)$$

$$+ z_r \left(\sum_1^{k-1} dS_i + \frac{x_r - \bar{x}_{k-1}}{\bar{\bar{x}}_k - \bar{x}_{k-1}} dS_k\right)$$

$$+ y_r \left(\sum_1^{k-1} d\omega_i + \frac{x_r - \bar{x}_{k-1}}{\bar{\bar{x}}_k - \bar{x}_{k-1}} d\omega_k\right) + dz_r$$

where,

x_r, y_r are the strip co-ordinates of point r (reduced to the first nadir point).

$\bar{x}_i$ represents the strip co-ordinates of the first nadir point of the particular model ...

$d\phi_i$, $d\omega_i$, dS_i, dz_r are the Pseudo-observation errors.

ΔZ_o, $d\phi_o$, $d\omega_o$ are the Free parameters, representing the error in absolute orientation of the first model.

$\Delta\varphi$ is the Free parameter for systematic Longitudinal tilt correction.

The above equation serves as condition-equation for the adjustment if ΔZ_r is the contradiction between the terrestrial height and the machine height obtained during triangulation.

FIG.A.2

Height Accuracy
(%oH)

(Standard deviation of minor control points)

Strip

.80	.80
1.30	1.30
1.64	1.64
1.82	1.82
1.86	1.86
1.75	1.75
1.51	1.51
1.15	1.15
.70	.70
.70	.70
1.15	1.15
1.51	1.51
1.75	1.75
1.86	1.86
1.82	1.82
1.64	1.64
1.30	1.30
.80	.80

Block

.59	.47	.45	.45	.45	.45	.45	.47	.59
.87	.72	.68	.68	.68	.68	.68	.72	.87
1.05	.89	.84	.84	.84	.84	.84	.89	1.05
1.14	.98	.92	.92	.92	.92	.92	.98	1.14
1.16	1.00	.94	.94	.94	.94	.94	1.00	1.16
1.10	.94	.89	.89	.89	.89	.89	.94	1.10
.98	.83	.78	.78	.78	.78	.78	.83	.98
.79	.65	.61	.61	.61	.61	.61	.65	.79
.54	.42	.40	.40	.40	.40	.40	.42	.54
.54	.42	.40	.40	.40	.40	.40	.42	.54
.79	.65	.61	.61	.61	.61	.61	.65	.79
.98	.83	.78	.78	.78	.78	.78	.83	.98
1.10	.94	.89	.89	.89	.89	.89	.94	1.10
1.16	1.00	.94	.94	.94	.94	.94	1.00	1.16
1.14	.98	.92	.92	.92	.92	.92	.98	1.14
1.05	.89	.84	.84	.84	.84	.84	.89	1.05
.87	.72	.68	.68	.68	.68	.68	.72	.87
.59	.47	.45	.45	.45	.45	.45	.47	.59

	STRIP	BLOCK	BLOCK EXCL. BORDER
Mean Std. Dev.	1.35 °/ooH	0.75 °/ooH	0.70 °/ooH
Max. Std. Dev.	1.86 °/ooH	1.16 °/ooH	1.00 °/ooH

A.1.3 Remarks on the theoretical accuracy of 3-dimensional spatially adjusted blocks

Figs. (A.3 - A.8) represent the results $\frac{\sigma}{\sigma_o}$ of a theoretical accuracy investigation of a three-dimensional block adjustment carried out by Kunji (1968). The results for planimetry are the same as those obtained by Ackermann and given earlier; therefore they are not repeated here. The following conditions apply to the investigation:-

(i) The mathematical model considered uses the spatial intersection of conjugate rays as the condition.

(ii) The photogrammetric coverage is assumed to yield a schematically ideal block with a regular layout of exposure stations, tie points, etc., as is common in theoretical studies.

(iii) All systematic errors of image co-ordinates such as lens distortion, refraction, film shrinkage, etc., are supposed to have been corrected before the adjustment phase.

It would appear that the investigation and the presentation of the results by Kunji is restricted to square-shaped blocks only.

Height Accuracy.

Fig. A. 3 Distribution of errors ($\frac{\sigma_z}{\sigma_0}$) of Z coordinates (20% Lat. overlap; SWA Photography).

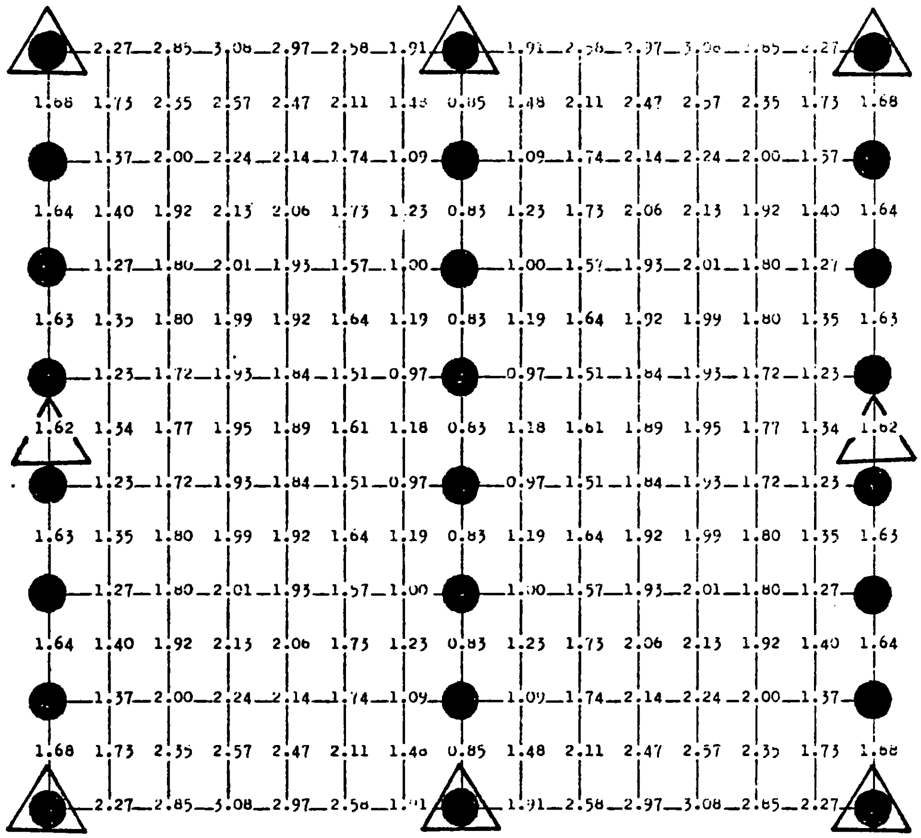
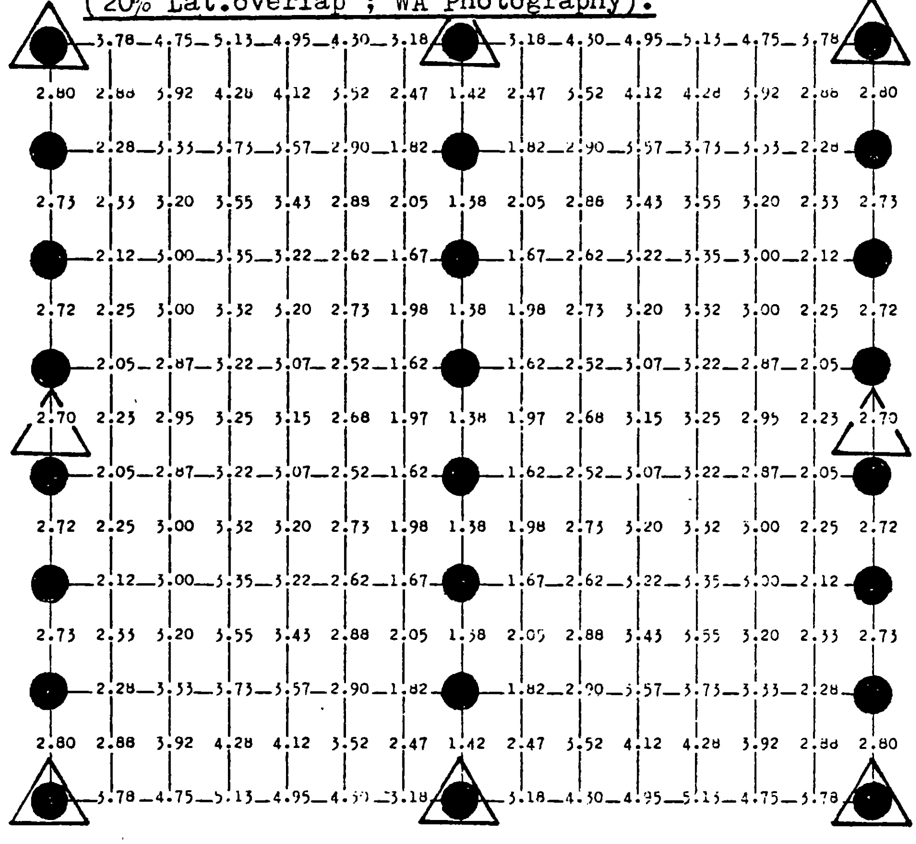


Fig. A.4 Distribution of errors ($\frac{\sigma_z}{\sigma_0}$) of Z coordinates (20% Lat.overlap ; WA Photography).



Height accuracy

Fig. A. 5 Distribution of errors ($\frac{\sigma_z}{\sigma_o}$) of Z coordinates
(20% Lat. overlap; SWA Photography)

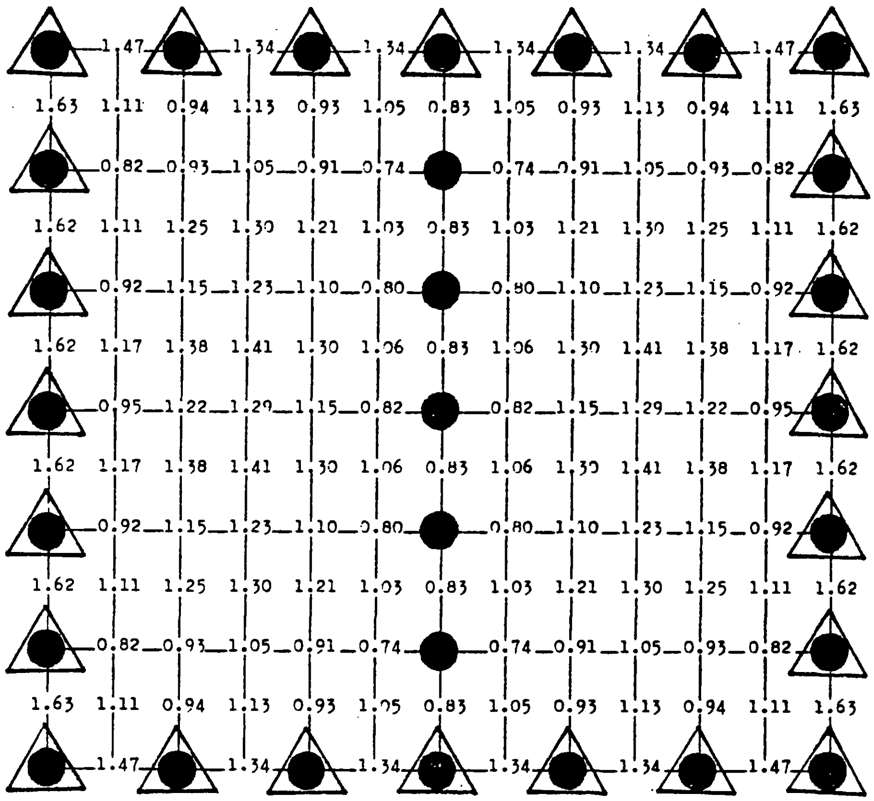
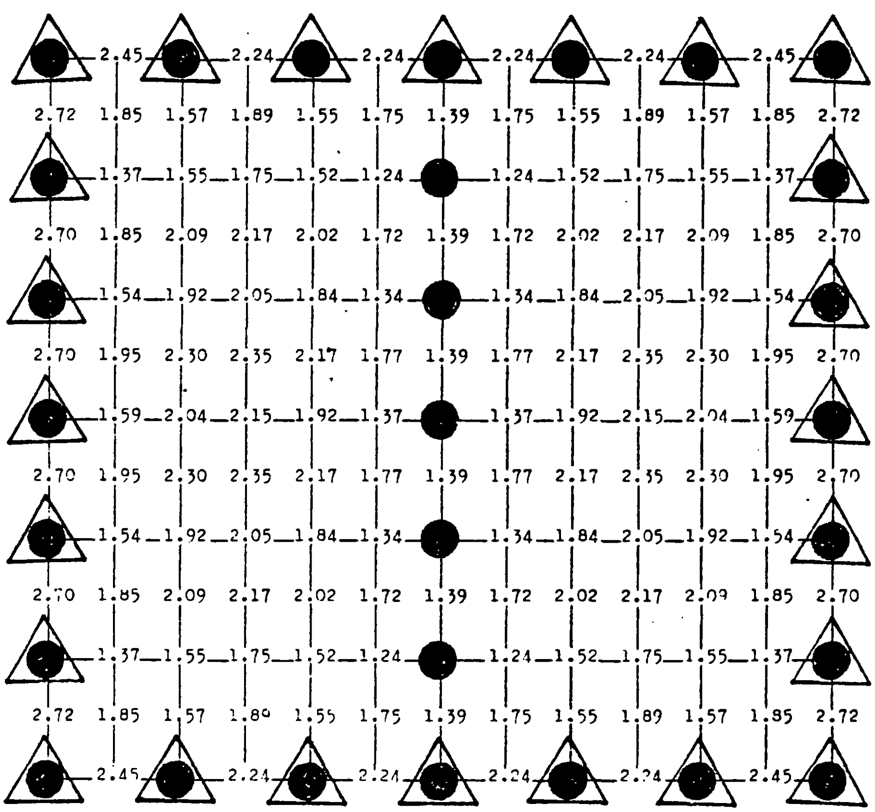


Fig. A. 6 Distribution of errors ($\frac{\sigma_z}{\sigma_o}$) of Z coordinates
(20% Lat. overlap; WA Photography).



Height accuracy

Example for bordering of blocks beyond the control frames.

Fig. A. 7 Distribution of errors ($\frac{\sigma_z}{\sigma_o}$) of Z coordinates.

(60% Lat. Overlap; SWA Photography).

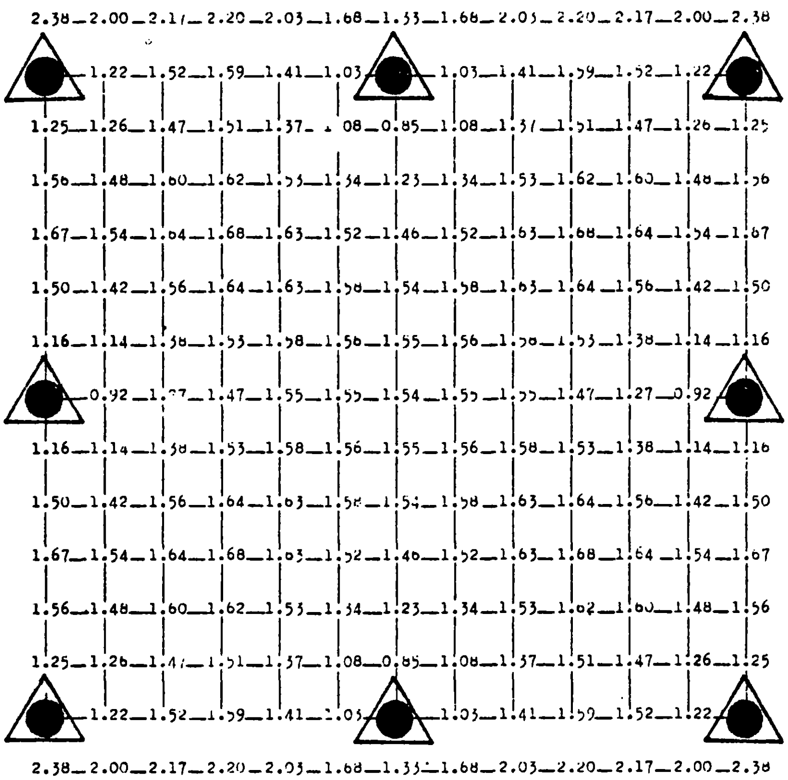
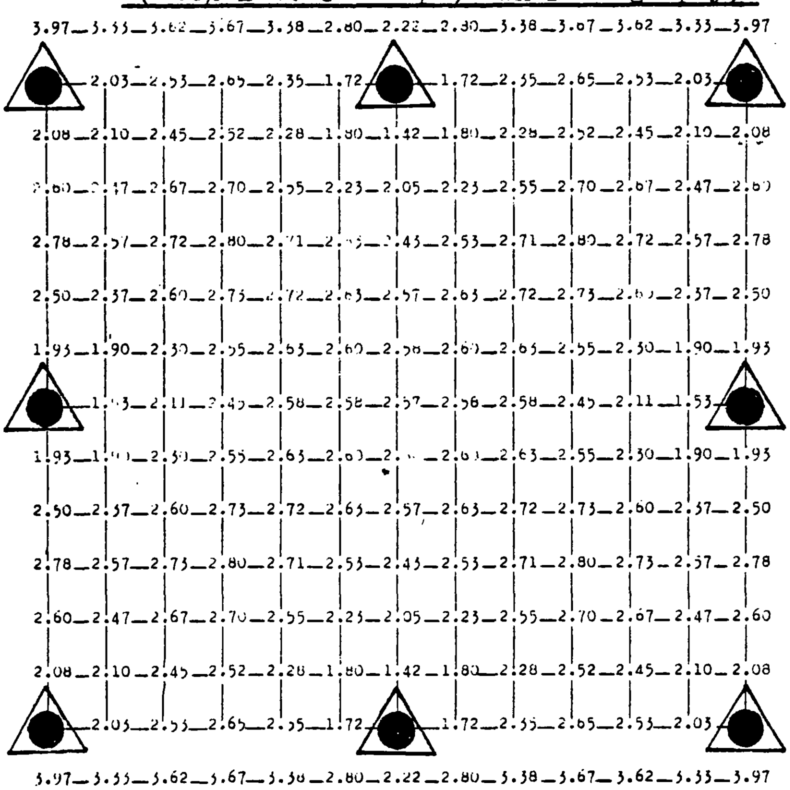


Fig. A. 8 Distribution of errors ($\frac{\sigma_z}{\sigma_o}$) of Z coordinates

(60% Lat. Overlap ; WA Photography).



A.2 Conclusions

A.2.1 Conclusions from the theoretical accuracy investigation of planimetric strip and block triangulation

(1) Accuracy is low in a block that is controlled at its corners only and it decreases as the size of the block increases.

(2) The accuracy is high, homogenous, and independent of block size and shape, in a block with complete perimeter control.

(3) The errors increase at the edges of a block with relaxed perimeter control and the largest errors occur between control points, while the central part is not seriously affected.

(4) The accuracy can be improved towards the edges of a block with incomplete perimeter control by using border photographs in the adjustment instead of additional perimeter control.

(5) An additional control point in the centre of a block improves the accuracy only slightly and locally, since the accuracy is always quite stable in the central part of the block.

(6) Compared with methods which require control for each model or each strip, an appropriate block adjustment will mean a considerable reduction in the number of required ground control points.

A.2.2 Conclusions from the theoretical height accuracy investigation of strip and block triangulation

(1) The maximum standard errors, in a block with bands of control across the strips, will occur at the edges, if there is no perimeter control.

(2) If there is perimeter control in a block, the maximum standard errors will occur in the centre between the bands.

(3) Maximum and mean standard errors in a block are almost independent of the size of block, but depend mainly on the bridging distance between the bands of control.

(4) The accuracy along the edges of a block without perimeter control, can be improved by adding control points in the first and last

strips, but the accuracy towards the centre of the block does not improve.

(5) The accuracy can be improved towards the edges of a block with no perimeter control by using border photographs in the adjustment instead of using additional perimeter control.

If several blocks adjacent to each other are to be joined, then it is recommended to adjust each block with an overlap of one strip with the adjoining block. This procedure avoids the necessity of having control points along the edges of the blocks. The accuracy of a block is considerably higher when the marginal strips are not actually used.

(6) The bridging distance between bands of control must be reduced if the overall accuracy of the block is to be improved.

The standard arrangement of height control in a block is in bands across the strips. A control point should be located in each lateral overlap, or close to it, in order to control the lateral tilts of the strips in cases where no auxiliary data are used.

APPENDIX B

General remarks concerning the S.B.A.I.M. Computer Program used in testing the TP technique

Appendix B

General remarks concerning the S.B.A.I.M Computer Program used in Testing the TP technique (See EL maleeh (1976)).

1. The S.B.A.I.M Program is a procedure for the simultaneous block adjustment of independent models.

2. The adjustment procedure is based on:-

   (a) the concept of spatial similarity transformation of models, in which seven transformation parameters per model are determined separately and successively in groups of 4 planimetric parameters and 3 vertical parameters per model;

   (b) the least squares approach and a direct solution of reduced normal equations pertaining to the transformation parameters.

3. The mathematical formulation of the block adjustment problem is given by the following observation equations which represent the relationship between the co-ordinates of a terrain point i and the associated photogrammetric co-ordinates i j of the same point determined in model j.

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = -\lambda j . Rj . \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} - \begin{bmatrix} X_o \\ Y_o \\ Z_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i \qquad \text{..........} \quad (1)$$

where

i = Point number

j = The model number

$[x\ y\ z]^T_{ij}$ = The vector of model co-ordinates of point i measured in model j.

$[X\ Y\ Z]^T_i$ = The vector of terrain co-ordinates of point i (unknown in the case of control points).

$[Vx\ Vy\ Vz]^T_{ij}$ = The vector of residuals (corrections) to the transformed co-ordinates of point i in model j.

λ_j = The scale factor

Rj = 3 x 3 orthogonal matrix (three independent unknowns)

$\begin{bmatrix} X_o & Y_o & Z_o \end{bmatrix}_j^T$ = Shift vector (shifts of origin of model co-ordinate system j)

The last three terms λj, Rj, and $\begin{bmatrix} X_o & Y_o & Z_o \end{bmatrix}_j^T$ are the orientation parameters of model j consisting of seven unknowns.

The 3-dimensional matrix is expressed in terms of the following Rodrigues-Cayley matrix:-

$$Rj = \frac{1}{k} \begin{bmatrix} 1+\frac{1}{2}(a^2 - b^2 - c^2) & -c + \frac{1}{2}ab & b + \frac{1}{2}ac \\ c+\frac{1}{2}ab & 1+\frac{1}{2}(-a^2+b^2-c^2) & -a+\frac{1}{2}bc \\ -b+\frac{1}{2}ac & a+\frac{1}{2}bc & 1+\frac{1}{2}(-a^2-b^2+c^2) \end{bmatrix} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where $k = 1 + \frac{1}{2}(a^2 + b^2 + c^2)$

(a, b, c) = the tilt parameters.

The observation equations in (1) are non-linear in the unknown parameters and the linearised form of equation (1) is as follows:-

$$\begin{bmatrix} V_x \\ V_y \\ Vz \end{bmatrix}_{ij} = \begin{bmatrix} 0 & -z & y & -x \\ z & o & -x & -y \\ -y & x & o & -z \end{bmatrix}_{ij} \cdot \begin{bmatrix} da \\ db \\ dc \\ d\lambda \end{bmatrix}_j - \begin{bmatrix} dX_o \\ dY_o \\ dZ_o \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

The symbols in equation (3) have the same meaning as in equation (1); da, db, dc, $d\lambda$, dXo, dYo, dZo represent increments to the parameters a, b, c, λ , Xo, Yo, Zo respectively.

x, y, z are the model co-ordinates (transformed approximately to the terrain system) with which the iteration process starts. They change their meaning to become model co-ordinates from the previous iteration during the successive iterations of the adjustment.

Equation (3) is applicable to control points and all tie points including the projection centres, but instead of the general approach of this equation, a simpler approach has been chosen for the S.B.A.I.M Program. The approach iterates between planimetric and height adjustment.

Groups of 4-parameters and 3-parameters are determined in sequence instead of a single group of 7-parameters.

The mathematical formulation of the observational equations for planimetry are as follows:-

$$\begin{bmatrix} V_x \\ \\ V_y \end{bmatrix}_{ij} = - \begin{bmatrix} x & -y \\ \\ y & x \end{bmatrix}_{ij} \bullet \begin{bmatrix} r \\ \\ S \end{bmatrix}_j - \begin{bmatrix} X_o \\ \\ Yo \end{bmatrix}_j + \begin{bmatrix} X \\ \\ Y \end{bmatrix}_i \quad \ldots\ldots\ldots\ldots(4)$$

The symbols of equation (4) have the same meaning as in equation (1).

The projection centres are excluded from the list of points i and are not used for the determination of the planimetric transformation parameters, as the convergence of the plan-height iterations would be adversely affected.

The mathematical formulation of the observational equations for heights of model points are as follows:-

$$\begin{bmatrix} vz \end{bmatrix}_{ij} = \begin{bmatrix} -y & x \end{bmatrix}_{ij} \bullet \begin{bmatrix} da \\ db \end{bmatrix}_j - \begin{bmatrix} dZ_o \end{bmatrix}_j + \begin{bmatrix} Z \end{bmatrix}_i - \begin{bmatrix} z \end{bmatrix}_{ij} \ldots\ldots(5)$$

The mathematical formulation of the observational equation for projection centres are as follows:-

$$\begin{bmatrix} Vx \\ Vy \\ Vz \end{bmatrix}_{ij}^{PC} = \begin{bmatrix} o & -z \\ z & o \\ -y & x \end{bmatrix}_{ij} \bullet \begin{bmatrix} da \\ db \\ \end{bmatrix}_j - \begin{bmatrix} o \\ o \\ dZo \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i - \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij} \ldots\ldots\ldots(6)$$

Equation (5) is applicable to model tie points, to height control points and full control points (planimetric control points do not participate in the height adjustment).

Equation (6) is applicable to common projection centres (PC).

The increments da, db refer to the tilt parameters of the rotation matrix of equation (2) and so they correspond directly with the symbols of equation (3).

dZ_o is the increment to the vertical shift parameter.

x, y, z are the model co-ordinates resulting from the previous plan adjustment.

As seen from above, the mathematical formulation of the S.B.A.I.M procedure replaces the basic approach of equations (1) and (3) by equations (4), (5) and (6), but the simultaneous determination of the seven transformation parameters of all models is nevertheless maintained. The models are transformed rigorously after each determination using, in principle, the full spatial similarity transformation formula. The increments of the parameters just determined are substituted and so the model co-ordinates $[x \quad y \quad z]^T_{ij}$ would refer always to the latest stage of transformation. This will allow the restarting of the computations of adjustment after any iteration.

4. Although the numerical solution is direct, it requires 2-3 iterations to converge (counting a plan-height sequence as one iteration).

One reason for the need of an iteration process is that the models are not levelled prior to the adjustment, and so the plan adjustment does not represent the final adjustment.

A second reason is that the height adjustment is based on linearised equations and it starts from zero approximations for the tilt parameters and so an iteration process is essential to compensate for the actual tilts.

A third reason is that the mutual influence between horizontal position and elevation makes it necessary to update the model co-ordinates during the successive iterations.

5. The following assumptions are made in the plan and height adjustment phases of the S.B.A.I.M Program:-

(a) The models are independent.

(b) The model co-ordinates are of equal weight and so they are given weight 1.

(c) The ground control co-ordinates are error free. That is to say, the errors are attributed to the photogrammetric measurements.

(d) The model and terrain co-ordinates are measured or computed in orthogonal co-ordinate systems.

6. The steps of the height adjustment are as follows:-

(a) The 3-dimensional co-ordinates resulting from the previous plan adjustment are reduced to the respective centre of gravity of each model.

(b) The reduced normal equations pertaining to the vertical parameters (3 per model) are solved using Cholesky Decomposition.

(c) Spatial similarity transformation of each model (j) with the three determined transformation parameters $(da, db, dZ_o)j$.

7. The concepts of eliminating the unknown co-ordinates of tie points and of working with reduced normal equations are applicable to both the plan and height adjustments.

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