Economic Dynamics of Defaults in a Heterogeneous World

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Abstract

This thesis investigates the inter-relationship between incomplete market arrangements, limited commitment in financial contracts and endogenous unsecured consumer bankruptcies in a heterogeneous agent general equilibrium framework. There are three chapters that complete this thesis. The following describes a brief abstract of the three chapters.

Chapter 1: We construct a general equilibrium heterogeneous agent model where a continuum of ex-ante identical agents face uninsurable idiosyncratic shocks. Consumption insurance is restricted to a financial contract in the form of a one period non-contingent bond that is not enforceable. The existence of sequential incomplete markets along with incomplete contracts results in endogenous defaults in equilibrium. A novelty in our framework is the presence of partial defaults and the specification of the stochastic labor endowment shock. We discretize a normal mixture autoregressive process that can capture higher order moments of the earnings distribution. The calibrated model delivers a significant mass of households amounting to 18.9% who hold zero or negative wealth. Furthermore, about 28.7% of the households have a history of bad credit implying that they have limited access to the credit markets. Importantly, these statistics match their data counterparts almost perfectly. Finally, we highlight the fragile assumptions of asymmetric default costs prevalent in the sovereign default literature.

Chapter 2: In the second chapter, we extend our model in two directions. Firstly, we incorporate a risk neutral financial intermediary that takes deposits and offers loans. In this way, we allow a price schedule that is contingent on the loan size, idiosyncratic earnings and employment state. Secondly, we model defaults that resemble the U.S. Chapter 13 bankruptcy law. In particular, defaulted agents face a temporary exclusion from the credit markets along with an earnings loss structured in the form of a wage garnishment. Our results highlight the role of financial intermediaries and risk-neutral pricing on the bankruptcy rates.

We also deliver wealth and income distribution moments including the inequality indexes that match household survey data estimates.

Chapter 3: In the third and final chapter, we capture the effects of a “credit crunch” and a “credit easing” shock on the real activity in our heterogeneous agent economic environment. Shocks take the form of a gradual tightening or loosening of the borrowing constraint. Periods of adjustment are set to mimic the Global Financial Crisis episode (2008-09) and the credit boom periods (2000-07). We confirm recent results in the post-crisis literature and find that tight credit generates a recession in the short run. However, we observe that in the long run a credit crunch increases productive capital in turn leading to an output growth. An interesting finding in this chapter is that prolonged periods of credit expansion drives an initial consumption driven output boom accompanied with excessive debt followed by increasing default rates leading to a destruction of productive capital and ending up in a sever recession.

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This thesis is dedicated to my parents,
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‘ Ἓν οἶδα ὅτι οὐδὲν οἶδα ’

— Socrates, 470 – 399 BC

“Any fool can know. The point is to understand.”

— Albert Einstein, 1879-1955

“We might have reason to be driven! We live for a short stretch of time in a world we share with others. Virtually everything we do is dependent on others, from the arts and culture to farmers who grow the food we eat.”

— Amartya Sen, 1933 -

“The major driver of economics is the equilibrium approach, which has taken various forms over the years. General equilibrium is the statement that all the different parts of the economy influence each other, even if it’s remote, like mortgage-backed securities and their demands on automobiles.”

— Kenneth Joseph Arrow, 1921 – 2017

Declaration

I declare that, except where explicit reference is made to the contribution of others, this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Printed Name: Ioannis D. Tsafos

Signature

Preface

The United States experienced the most dramatic boom and bust in household debt, during the periods 2000 to 2007, ever since the Great Depression. Household debt increased at a steady pace throughout the 1990s, and then jumped by $7 trillion between the years 2000 and 2007. This boom in debt ended badly. By the year 2009, delinquency rates on debt had reached above 10%, a level significantly higher than seen since the Great Depression. The subsequent recession has been termed as the U.S. Great Recession of 2007-09 and its international spillovers resulted in a Global Financial Crisis.

The Great Recession started with an initial decline in aggregate economic activity such as consumption and employment and the recovery was extremely slow. The debt fuelled boom in house prices and its eventual collapse triggered by widescale mortgage defaults has been identified as the primary reason behind this crisis. It is now understood that the steep growth in delinquencies on household debt in 2007 was driven mainly by lower credit score individuals. This event underscored the importance of the cross-sectional distribution of agents and their wealth, borrowing and income on aggregate downturns. In particular, the widespread growth of wealth and income inequality have raised concerns about their potential impact on societies and economies.

Although detailed analysis both in the academia and the popular press have come out detailing the causes and consequences of wealth inequality, relatively, little is known about the considerable proportion of people holding zero or negative wealth. Negative wealth arises when a household's total debt exceeds the total assets. The presence of negative wealth, even when such a status is temporary, affects the household's ability to save for durable goods, restricts access to further credit, and may require living in a state of limited consumption. We know little about the characteristics of such households or about what drives negative wealth. A better understanding of these factors could also prove valuable in explaining and forecasting the persistence of wealth inequality and aggregate economy.

The Survey of Consumer Expectations (SCE), introduced in 2013, has a special section on household finances fielded once a year. This module collects detailed data on the composition of respondents' financial assets and debt. The survey estimates that about 18.1 percent of the households in the U.S. population have net wealth less than or equal to zero, while 14.0 percent have strictly negative wealth. Interestingly, they find that the negative net worth households are characterized by huge amounts of unsecured debt in the form of credit cards or student loans rather than mortgage debt.

Motivated by these observations, we build a theoretical model in our first chapter that is designed to capture these salient features of the data. Our model incorporates a continuum of agents facing uninsurable idiosyncratic shocks to their labour earnings and unemployment risk in the spirit of Bewley (1983), Huggett (1993) and Aiyagari (1994) literature. Agents in this economy can only partially insure against these risks through borrowing and saving in a one period non-contingent bond. We extend this standard framework by allowing for limited commitment. The presence of sequentially incomplete markets accompanied with stochastic risks endogenously generates defaults in equilibrium. A novelty in our model is that following default only the amount of debt exceeding the household's assets is erased from the budget constraint, that is, defaults could be seen as partial. Essentially, the net debt exceeding the level of assets can be defaulted upon. In this way, our default parallels the structure of Chapter 7 U.S. bankruptcy law. Defaulted households are punished with a temporary exclusion from credit market. The equilibrium in our model is a stationary recursive competitive equilibrium.

A second highlight of our model is in the calibration of the stochastic process underlying the labor earnings risk. In influential work by Guvenen et al. (2015) analysing the labor earnings of over a million agents using adminstrative records stresses the importance of capturing higher order moments of the earnings distribution. Since standard $AR(1)$ processes cannot capture high skewness and kurtosis, we employ a novel methodology developed by Civale et al. (2016), and calibrate the parameters of the log labour earnings growth using a normal mixture autoregressive process (NMAR). This process is then discretized and used in our computation.

The incorporation of this process goes a long way in explaining the left tails of the distribution. Importantly, our computed results reveal that over 18.9% of households hold negative net worth consistent with the empirical evidence. Furthermore, the model produces unsecured default rates of about 7.4% similar to the levels observed in credit card charge-offs. Finally, our results bring into question the form of default punishment followed in the sovereign default literature. We observe that agents are more likely to default in a high income state when the default punishment is characterized by symmetric costs.

In the second chapter, we enhance the model by incorporating a risk-neutral financial intermediary that provides loans to borrower households and takes deposits from saver households. The pricing of these loans is contingent on three states: the size of the loan, the idiosyncratic level of labour earnings and the status of employment/unemployment. The optimal pricing thus depends on expected default probabilities. The presence of a price schedule contingent on idiosyncratic earnings and employment state is the novel feature in our model in comparison to others in the literature (see Athreya (2002), Li and Sarte (2006) and Chatterjee et al. (2007)).

Since the pricing decision of the intermediary is based on an expectation of the borrowers' default probability, we find, as expected that the equilibrium level of defaults is much lower. The level of defaults we obtain is about 0.65% consistent with the Chapter 13 bankruptcy filings observed in the U.S. The wealth and income distributions are much more realistic exhibiting a right skewness and a very long right tail. The inequality indexes we obtain for these are respectively, 0.76 and 0.50. The data counterparts for these statistics are 0.80 for wealth and 0.43 for income. Furthermore, we find that about 5.35% of total households are constrained, or in other words have a bad credit history (score). These results highlight the importance of the presence of a financial intermediary and moreover risk-neutral pricing in affecting the economy. The results further stresses on the importance of accurate pricing of loans in influencing default incentives. An intermediary facing informational constraints or some form of inadequate screening methods would fail to see key borrower characteristics that could impact repayment of the loan in the next period.

We conclude this thesis by coming back to the Global Financial Crisis. In the last chapter, we deviate from the steady state analysis and implement out of steady state experiments by focusing on the transition dynamics of our heterogeneous agent general equilibrium model with endogenous defaults. We examine the relevance of the credit supply view of the crisis, see Mian and Sufi (2016) using two computational experiments. The first is an unanticipated gradual tightening of the household's borrowing limit. The period of adjustment is calibrated to mimic the observed credit crunch in U.S. that lasted for a period of six quarters. The specific form of this gradual shock is inspired by related work in Guerrieri and Lorenzoni (2017). Our results reveal that a credit crunch causes a consumption and output drop (recession) in the short run consistent with most findings in the literature. However, of particular interest is the dynamics we obtain in the long run. We observe that this short run recession is followed by an output growth in the long run wherein the economy finally settles down to a better steady state characterized by improved aggregates. Aggregate consumption and capital is higher and debt and default rates lower.

Our second experiment involves relaxing the borrowing limit, also known as credit easing. As in the previous case, the unanticipated shock is adjusted gradually such that easier and easier credit lasts for a period of seven years. This experiment is something novel to the literature. The aim here is to match the loose borrowing environment prevalent in the U.S. during the years 2000-07 and analyse the consequences. There is a plethora of literature that analyse the effects of financial shocks and constraints on the aggregate economy. Although a good variety of these explain certain features of the crisis such as the expansionary phase or the contractionary phase, a complete characterization of the entire cycle is lacking. Our credit easing shock in the third chapter aims to fill this gap. We find that in the initial credit expansion phase, loose credit hikes both consumption and output generating a temporary boom. However, this is also accompanied with an accumulation of debt. Eventually, defaults kick in and deleveraging of heavily indebted households results in increasing defaults and destruction of capital. The aggregate output drops due to two reasons. First, households cut back on their spending habits or consumption because of excess debt and secondly due to the loss of productive capital from defaulted assets.

Chapter 1

Default motives and negative net worth households

1.1 Introduction

According to the Federal Reserve Bank of New York, 14% of households in United States have *negative net worth*, that is their total debt exceeds the total assets (see figure 1.1). The main research question of this chapter is the following - **How can we explain this significant mass of agents with negative net worth using a standard heterogeneous agent general equilibrium model?**

Different factors might contribute to this undesirable outcome. Mortgage debt is the natural answer to such a question. Surprisingly, New York Fed found that among households that feature high negative net worth the vast majority of debt comes from student and credit card loans and only 20% of them own their own house (see figure 1.2). In other words, it is non-collateralized debt and not collateral debt such as mortgage that is behind the low negative net worth. Understanding the factors that drive negative wealth is of particular importance, since it could prove to be a valuable tool in explaining the persistence of wealth inequality across major countries, particularly so in the United States.

In this chapter we present a general equilibrium model with unsecured consumer credit, where households are subject to idiosyncratic labour earnings shocks. In this manner, households are ex-ante identical but ex-post heterogeneous. Due to lack of commitment and the one asset available for trade in our economy, markets are sequentially incomplete.

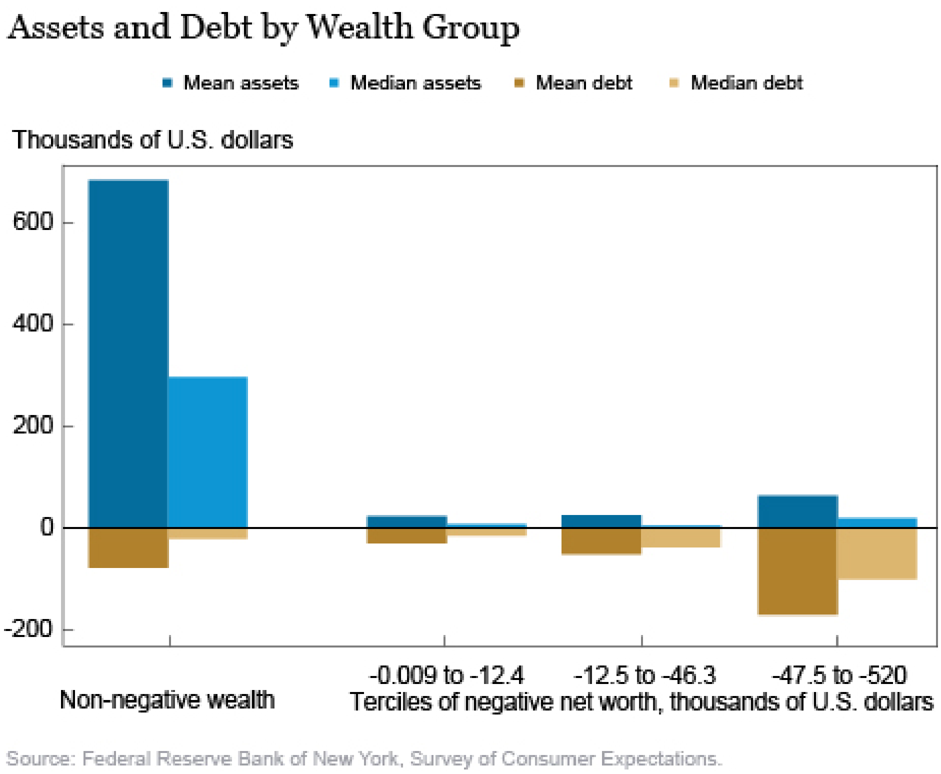

Figure 1.1: **Asset and Debt by Wealth Group:** This figure displays the average and median level of assets and debt across four different groups of households. Firstly those featuring non-negative wealth and secondly households in every tercile of negative net worth.

Our model could simply be characterized as an extension of Aiyagari (1994), where agents have a default option. All households in such economies seek to smooth intertemporal consumption, subject to borrowing constraints. Furthermore, in our model framework, unlike the complete markets case, default can arise as an equilibrium outcome endogenously. Default in our benchmark model is strategic and induces a temporary exclusion from credit markets. In particular, borrowers can default on their debt obligations whenever their expected value from defaulting is higher than their expected value to repay.

We make default option available only amongst households with negative net worth. This assumption is strong, yet reasonable, since in reality households with positive net worth, would not risk to loose their assets when defaulting.

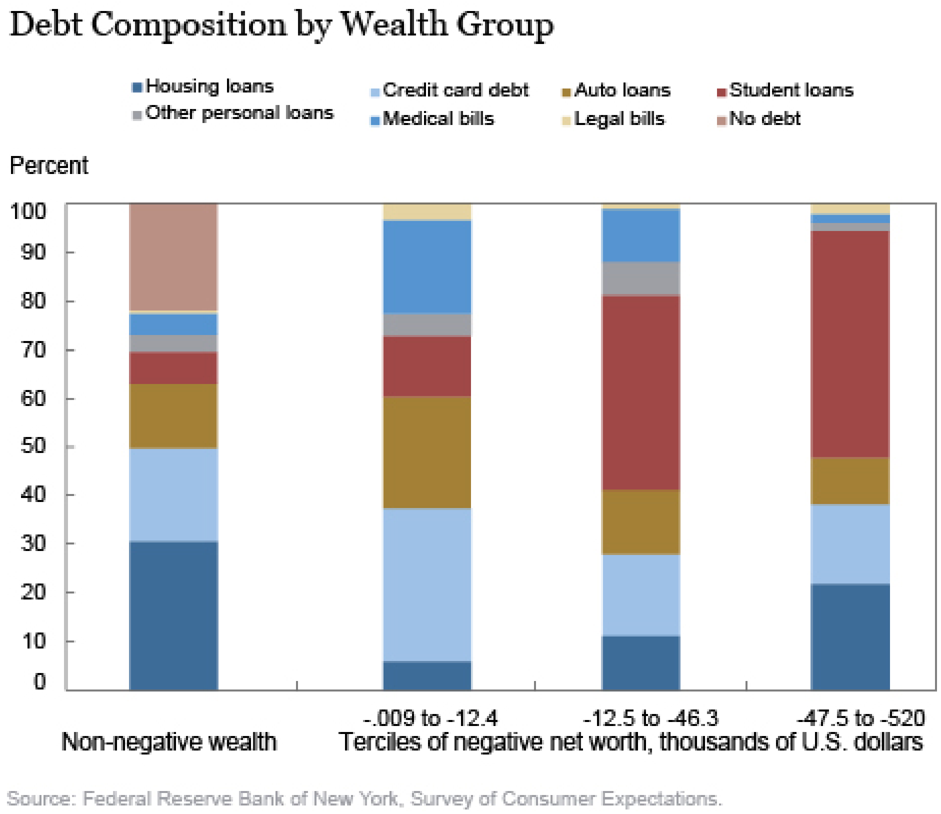

Figure 1.2: **Debt Composition by Wealth Group:** This figure displays the debt composition for four different groups of households. Firstly those featuring non-negative net wealth and secondly households in every tercile of negative net worth. Seven categories of household debt are presented, housing mortgages, credit cards, auto loans, student loans, medical bills, legal bills and other personal loans.

In support of this argument stands Chapter 7 of the U.S. Bankruptcy code, under which filing for bankruptcy results in seizure of all assets and a resulting discharge of household debt. Since in our model we only consider the net level of assets, namely the positive assets net off debt that a household holds, it is sensible to assume that households with positive net worth are not allowed to default. Following the default period a household faces difficulties in getting new credit, for a period of around 6.6 years.

A first key element of our model is the nature of default punishment. We depart from the literature on strategic defaults by assuming that a defaulted household moves to zero net worth, in other words his/her net (and not the entire debt) liability is discharged. In this sense, default could be characterised as "partial"[1] as only the fraction of debt exceeding his assets is discharged and thus erased from his budget constraint. From this point of view, we claim that even in cases of household bankruptcies, all defaults are partial[2].

This assumption of partial defaults is consistent in the context of sovereign default literature where novel evidence suggests that defaults are only partial (see Arellano and Mateos-planas (2013). Furthermore, this implies a shrinkage of the default incentives in contrast with the case under which a "unilateral debt write off" would be allowed (default in full).

A second key assumption of our model is that households trade amongst each other directly. Most of the research in the household bankruptcy literature, such as Chatterjee et al. (2007) and Li and Sarte (2006) amongst others, assume the presence of a financial intermediary. One advantage of this approach is that the interest rate for borrowing and savings is not the same, since the financial intermediary clears the financial market by charging a premium for loans. In this manner, financial intermediary internalises defaults and can guarantee non-risky savings/deposits.

However, the approach of risky savings, where households trade amongst each other depending on their financial position, allows for spill-overs of defaults in the production side of the economy, via the asset market clearing condition. In this manner, defaults can affect the equilibrium outcome and the whole economy, in a direct way and not only via the price channel.

1.2 Contribution and Main Results

Our model economy follows closely Li and Sarte (2006), but we drop the presence of financial intermediaries and exogenous labour supply choice. Instead, we enrich the earnings process by including non-Gaussian earnings shocks on top of an unemployment risk. The rich stochastic process is assumed to capture the

[1]The term partial in our framework can be mainly seen as entering a debt restructuring program following default.

[2]Under Chapter 7 bankruptcy Law in the U.S. some of your liquid assets transferred to the courts can be distributed among your creditors as partial repayment of the debt you owe. These are called non-exempt assets. Provided all your liquid assets have been distributed the **remaining** debt is discharged. **This is equivalent to assuming that following default your net asset position is zero.**

high skewness and kurtosis in actual earnings data. The absence of financial intermediaries is assumed so as to understand the implications of a uniform price for borrowing and lending. Lenders are not compensated for holding risky loans that might be defaulted on. In equilibrium, this would result in a higher risk free rate and less capital in equilibrium relative to the standard Aiyagari-type framework. Due to this high interest rate, households who faced a history of bad shocks would face high repayment costs and would more likely end up in the lower tail of the distribution with negative net worth. Li and Sarte (2006) separate risky borrowing from risk free deposits in the presence of a financial intermediary who charges a premium to carry the risky loans. They attempt to capture Chapter 7 and Chapter 11 filing rates and not negative net worth households.

We now discuss the main results of this chapter. Firstly, we match key statistics of the U.S. economy. Our theoretical framework generates a mass of non-positive net worth households of 18.9% with its counterpart in data being 18.1%. Furthermore, the default rate in our model economy of 7.4% replicates the default rate of credit card (unsecured) loans in data which is around 7.6%. Li and Sarte (2006) predict the filing rates for Chapter 7 and 13 to be 0.86% and 0.39% respectively. This substantial difference in our predictions can be mainly attributed to the mechanism prevailing in our model. Lenders have greater incentives to lend in our model as compared to Li and Sarte (2006), due to the higher lending rate (Li and Sarte (2006) finds a deposit rate of 2.516%). On the other hand, borrowing incentives are also stronger due to the lower predicted lending rate as compared Li and Sarte (2006) (around 13%). Therefore, trade is larger and so do defaults.

We also claim that a wide class of models failing to capture wealth inequality underestimate the mass of negative or non positive net worth households in the economy. Our model generates a wealth gini index of 0.795 even if it does not properly resembles the fourth and fifth quintile of the wealth distribution in data. This issue arises because of the assumption of a single interest rate.

Additionally, we capture the households with a bad credit record in the economy. We find that 28.7% of households are constrained or equivalently they face difficulties in having full access to credit. This value coincides to the value of 29% in data which measures the mass of borrowers that have a very bad credit score, specifically below 620.

Another key result of our model is that households in an unemployed state or a bad labour income state default less compared to households in a good state. On first inspection this result appears to be controversial. However, recent empirical studies (see Gerardi et al. (2015)) witness that the vast majority of financially distressed, or equivalently heavily indebted households do not default. The authors find that 80% of unemployed households, even when having less than one month of mortgage payments in a savings account are current, in other words they do not choose to default.

Our study serves as a theoretical pillar to the findings of Gerardi et al. (2015). Low income households in our model do not choose to default, except for the case that they are heavily indebted or are very close to the borrowing limit. An explanation of this behaviour is that low income households value more the access to credit, since accumulation of debt serves as a means to increase their current consumption above the subsistence level. These households presume that if the bad state persists in the future, no access to credit market will be pernicious.

The presence of partial and not full default is also a decisive factor for generating this result. The outside option is not so appealing for low income households, since only a specific fraction of debt will be discharged. Thus, default is more costly for them compared to high income households. Households with high disposable income are able to enjoy a high level of consumption at any case, even following the default event.

Finally, a theoretical contribution of this chapter is that we highlight the strength of a mechanism commonly used directly or indirectly in both the sovereign default and consumer bankruptcy literature. We show that asymmetric default costs can revert the above discussed results and generate more defaults in the low income state.

Firstly, we claim that the use of asymmetric costs as in Arellano (2008) is not empirically relevant and can only be based on implausible assumptions. We prove this by modifying our benchmark model with the condition that post-default, there exists an extra cost in the form of wage garnishment. This is in line with Chapter 13 bankruptcy law in the US.

The notion of asymmetry arises from our additional condition that wage garnishment will only occur, when a household's income will exceed a certain threshold. In this manner, we alter the default penalty to be conditional on the level of the borrower's labour earnings. Thus, high earnings households default less, since their penalty is much higher, or to express it differently they have a lot more to loose.

The rest of the chapter is organised as follows. In Section 1.2 we review the literature related to heterogeneous agents models, financial contracts and defaults/bankruptcies. We then describe our model economy in Section 1.3, presenting in detail the household's problem. In Section 1.4 we provide a brief description of the stationary recursive competitive equilibrium and state the market clearing conditions. We discuss the calibration strategy in Section 1.5. In Section 1.6 we present our results for our benchmark economy along with the case of asymmetric default costs. Properties of the model along with distributional effects are discussed. We conclude this chapter in Section 1.7. Further details of the equilibrium properties and the numerical procedures are summarized in an Appendix at the end of the second chapter.

1.3 Literature Review

In this section we will review the literature on each critical part of the model employed in this chapter. This will include, firstly literature related to heterogeneous agents models in a general equilibrium framework and secondly related to financial contracts and defaults/bankruptcies.

1.3.1 Heterogeneous Agents in General Equilibrium

The adoption of heterogeneity[3] as a concept by macro-economists finds its roots in the "rational expectations revolution". Until the 1970s macro-economists were using ad hoc aggregate relationships. Lucas and Sargent amongst others introduced the new agenda of dynamic stochastic general equilibrium models. The essence of this framework is the optimal decision making of the individual agents consistent with each others beliefs and nature.

[3]An extensive review of heterogeneous agents models in general equilibrium framework is given by Heathcote et al. (2009).

Despite that and mainly due to lack of appropriate mathematical tools, the first generation of macroeconomic models followed the famous work of Kydland and Prescott (1982), was along the lines of the representative agent framework. Since the realization that numerical methods can be used to study heterogeneous agents, and the data evidence which confirmed this necessity, macro-economists started modelling heterogeneity.

Heckman (2001) in his Nobel lecture states: "The most important discovery was the evidence of the pervasiveness of heterogeneity and diversity in economic life."

For instance, by introducing to a general equilibrium model, households that are ex-ante identical and ex-post heterogeneous due to facing idiosyncratic uninsurable income risk (see Huggett (1993),Aiyagari (1994)) , a precautionary savings motive is present which increases aggregate wealth and reduces the equilibrium interest rate.

An early work in this literature was by Huggett (1993) who constructed an economy where agents experience uninsurable idiosyncratic endowment shocks and aim to smooth consumption by holding a risk-free asset. He used a borrowing constraint of one year income and found that the resulting risk-free rate is 1% below the corresponding representative agent economy. In later work, Aiyagari (1994) addressed the impact of uninsured idiosyncratic risk on aggregate savings rate and highlighted the importance of asset trading to individuals and the implications of inequality in wealth and income on aggregate assets.

Additionally, as economists we find interesting to study welfare effects. Introducing heterogeneity provides a suitable workhorse for addressing welfare questions. A representative paradigm in this literature is Storesletten et al. (2001). This work finds that countercyclical variation in idiosyncratic risk, firstly amplifies the welfare cost of aggregate productivity shocks and secondly imposes a cost on its own. The magnitude of this effect is an order larger compared to the one documented in Lucas (1987). Additionally, the magnitude of these effects increases non-linearly in risk aversion.

In general a lot of macroeconomic questions cannot be addressed without the use of some kind of heterogeneity. For instance, the evaluation of large scale government programs, such as social security policies, minimum wage policies, tuition subsidies and any other sort of large scale reform, requires heterogeneous agents general equilibrium models.

In that manner, one can account not only for general equilibrium effects but also for the heterogeneous impact of policies across the population. Conesa and Krueger (1999) analyse the role of idiosyncratic uncertainty in an economy where rational agents vote on hypothetical social security reforms. In this model a transition to a fully funded social security system might have less political support than in the benchmark model without heterogeneity within generations.[4]

Heterogeneity with complete markets

This heterogeneity can be seen as the smallest deviation from the representative agent model. Naturally, the assumption of complete markets is unrealistic. From an empirical perspective, it is rejected at multiple levels. For instance, Attanasio and Davis (1996) find that large changes in the distribution of household consumption is due to changes in earnings.

Moreover, complete markets imply that when agents have identical preferences there should not exist consumption volatility. Furthermore, Jappelli and Pistaferri (2006) studying the empirical transition matrix of consumption using a panel drawn from the Bank of Italy Survey of Household Income and Wealth strongly reject the consumption insurance model, providing evidence of significant volatility.

In heterogeneous agents models with complete markets, there are cases that the source of heterogeneity will become irrelevant. In particular, even though agents might be subject *to idiosyncratic insurable shocks* the macroeconomic aggregates might do not depend on the wealth distribution.

For instance, Maliar and Maliar (2003) allow for idiosyncratic but fully insurable productivity shocks and show that aggregate dynamics coincide with the corresponding representative agent economy subject to "aggregate shocks". The heterogeneous agents economy behaves exactly as if there was a representative consumer, facing three different shocks (i.e. labour,preferences and technology

[4] Agents are ex-ante identical and the source of heterogeneity within generations is due to the realizations of the labour productivity shock.

shocks).

Heterogeneity with incomplete markets

The *"standard incomplete markets"* model (SIM) is the backbone of studying heterogeneous agents models. There exist a lot of variants within the SIM framework. However they all share two common features. Firstly, they all introduce imperfect insurance and secondly realistic risk-sharing mechanisms.

In these models, a large number of agents facing idiosyncratic shocks make independent choices (i.e consumption, savings etc.), while these choices determine the aggregates, thus equilibrium prices. This model resembles the stochastic growth dominating the business cycle literature. Agents maximize expected lifetime utility in response to shocks, by adjusting consumption, hours, savings and generally any choice variable.

The question which arises naturally is how this market incompleteness should be modelled. One way that this could be achieved is the use of information frictions. Attanasio and Pavoni (2011) present endogenous incomplete markets by introducing moral hazard and hidden savings. They show that these features generate what in consumption literature has been called *excess smoothness of consumption*.

An alternative way proposed by the literature is to work with enforcement frictions. For instance Krueger and Perri (2006) allow the agent to step away from a contract (i.e. limited enforcement). In particular, agents enter risk-sharing contracts, but at any point in time have the option to renege at the cost of losing their assets and being excluded from future risk-sharing.

These two approaches could be integrated. For instance, Chatterjee et al. (2007) present a model where the set of assets is exogenously determined, but borrowing costs are specified endogenously as a function of default incentives. Particularly, the default option results in debt discharge, yet bankruptcy remains on a household's credit record for some length of time. In this setting two things are clear. An indebted household will weigh the benefit of maintaining access to credit market and credit suppliers will have to price their loans accordingly, accounting for the likelihood of default (also taking into account the credit history).

Efficiency and constrained efficiency

A very interesting question in this class of models (SIM) is how close agents are to achieving perfect risk sharing. Levine and Zame (2002) examine whether market incompleteness matter for welfare, for prices and consumption. Intuitively, we could think that if the horizon is long enough it might not matter if the agents are able to self-insure, i.e., borrow in bad times and save in good times. Levine and Zame (2002)'s argument is based on a framework of an infinite horizon exchange economy with infinitely-lived agents. Trade takes place via short-lived real assets.

They find that as the discount factor β tends to 1, hence agents become increasingly patient, the welfare losses tend to 0. This holds only under the assumption that no aggregate risk is present. In particular, the source of risk stems from transitory and purely idiosyncratic shocks and only one consumption good is traded. This result is fragile:

(a) *when aggregate risk is introduced.*

(b) *more than one consumption goods are available.*

(c) *shocks are permanent.*

Along the lines of the previous discussion we would also be interested in understanding whether information constraints or limited enforcement structures exist such that *constrained efficient allocations* can be decentralized, with a risk free asset, but no state-contingent claims. Allen (1985) studies *Pareto efficient allocations* in a two-period model including information frictions.

In Allen (1985)'s environment, risk averse agents are able to hide their random labour endowment from the planner and borrow/save without being monitored. Hence, planner finds efficient to transfer to all individual at time zero, and leave intertemporal smoothing on them. Consumption allocations in this model can be decentralized via a competitive asset market, where a risk-free bond is traded by agents with the same initial level of wealth.

1.3.2 Financial Contracts, Limited Enforcement and Defaults

In the classic heterogeneous agents literature, founded on income fluctuation problem, agents' consumption smoothing is ultimately determined by their distance from the borrowing constraint.

In the typical *Bewley (1983) economy* agents are not allowed to borrow, but in Aiyagari (1994) borrowing serves as a means to smooth intertemporal consumption, while agents are constrained by the so-called natural borrowing limit. However, default option is not present in the *Aiyagari economy*.

Borrowing constraints are employed by ruling out defaults and also avoid Ponzi schemes. The latter is a prerequisite for the existence of equilibrium in an economy with incomplete markets. Zhang (1997) endogenises borrowing constraints seeking insights on their relevant economic drivers. For this purpose he introduces the so called "no default borrowing constraint". This constraint-type does not allow an individual to borrow more than the amount he would have the ability to repay.

In other words, the constraint itself incentivise individuals not to default. This is feasible by setting the borrowing limit such that the expected utility when the individual is not excluded from the asset market is never lower than the utility of autarky.[5] However, default in Zhang (1997) is not an equilibrium outcome, due to the very tight borrowing limits. This tightness guarantees that agents prefer to repay in every state of the nature.

There are several papers that modify the standard incomplete markets model (SIM) to allow for defaults. A significant part of the literature employs this modified model to study the welfare implications of different bankruptcy laws or changes in bankruptcy laws.[6] For instance, Athreya (2002) models an exchange

[5]The default penalty in Zhang (1997) is permanent exclusion from financial markets. This type of exclusion is called "autarky".

[6]There is a large body of literature along the lines of U.S. bankruptcy code. In U.S. there are different types of bankruptcies laws. The three main bankruptcy chapters are 7,11,13. Chapters 7 and 13 are primarily used for individuals while Chapter 11 for companies. Chapter 7 erases most types of unsecured debt. This includes credit card debts, medical bills, gasoline cards debt and few others. Under Chapter 13, depending on your disposable income, a debt restructuring payment plan is proposed, and the bankruptcy court appoints a trustee to supervise your case. First all the monthly payments, agreed under the debt restructuring plan, should be successfully completed and ultimately the rest of your outstanding debt is wiped off.

economy augmented with a default option which resembles the Chapter 7 "total liquidation" bankruptcy. Bankruptcy in his model induces a temporary exclusion from financial markets. Probability to re-enter is exogenously specified by the use of a lottery which determines the average length of time that a household is borrowing constrained (i.e. no access to credit market).

Li and Sarte (2006) explore the two types of bankruptcies, Chapters 7 and 13, that are allowed under U.S. law, in a general equilibrium framework. In their model bankruptcy induces a temporary exclusion from credit markets, while each of the U.S. bankruptcy chapters is modelled explicitly via the use of different value functions. In disagreement with previous work done in this literature complete elimination of bankruptcy provisions causes output drop and welfare decline, by decreasing capital and labour input in the production function.

Default Punishments

Default necessarily induces a penalty. We described above two different types of penalties, Zhang (1997) used a permanent exclusion from financial markets, while Athreya (2002) and Li and Sarte (2006) a temporary exclusion. Both punishment ideas, are influenced from the contributions of sovereign default literature, consisting of two different strands.

Firstly, Eaton and Gersovitz (1981) based on the stylized fact that default makes re-entry to international capital markets more difficult, they suggest that once a country defaults will be denied any access to capital markets and will be permanently reverted to autarky. In this model the default is an optimal choice, namely it doesn't happen accidentally and the benefits of default grow with the size of outstanding debt.

On the other hand, Bulow and Rogoff (1989) suggested that due to recent developments of financial markets it is more realistic to assume that following default, a country could still smooth its expenditures, by trading contracts which involve no credit.

This type of default punishment plays a major role in any limited enforcement environment, since it directly affects the incentives of agents (countries, firms or individuals). A reasonable question is how incentives are differently influenced in each punishment case. Literature provides a menu of different results. Hellwig and Lorenzoni (2009) allowing for trading no-credit involved contracts post default, show that consumers default much in the high endowment state. This allows them to attain a consumption bundle which gives them higher utility compared to the repayment option. On the contrary, Arellano (2008) applies an "asymmetric" punishment imposing fixed output costs in the defaulted country only when output is above a certain threshold. Furthermore, a temporary exclusion from "international intertemporal trading" is assumed. With this type of punishment default incentives are stronger the lower the endowment is.

Therefore, the importance of the default option along with the punishment nature is outstanding and hence modelling such mechanisms appropriately needs careful consideration.

1.4 The Model Economy

We build a modified version of deterministic growth model allowing to include a large number of households facing uninsured idiosyncratic shocks. Our model is closely related to the models of Li and Sarte (2006) and Chatterjee et al. (2007). Time in our model is discrete and indexed by $t \in (0, 1, \ldots, \infty)$. Economy is populated by a continuum of ex-ante identical infinitely-lived households distributed uniformly over $[0, 1]$.

1.4.1 Preferences

Individual households care about streams of consumption in the standard way, that is, they have time-separable preferences over streams of consumption.

$$U(c_0, c_1, c_2, \ldots) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1.1}$$

where for each period t, c_t is restricted to belong in $\mathcal{C}$, the per period consumption feasibility set of any individual household, which specifies non-negative consumptions. The discount factor $\beta \in (0, 1)$ is common across all households. The period utility function $u(c_t)$ is continuous, concave, strictly increasing, continuously differentiable and satisfies the Inada conditions[7]. For simplicity we assume that households supply labour inelastically (normalized to 1) in the sense that any increase or decrease in the price of labour (wage) does not result in a corresponding increase or decrease in its supply. Hence, the period utility function is assumed to be a $CRRA$, characterised by constant relative risk aversion.

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{1.2}$$

where the parameter σ measures the degree of relative risk aversion that is implicit in the utility function. Our model is stationary and in what follows, we do not use the time t index.

1.4.2 Endowment

Households do not value leisure, since it is not an input in the utility function. Thus, the time endowment (normalized to 1) is supplied to the market. Households face idiosyncratic labour income risk, or earnings risk. This risk arises from two different sources in this model.

Firstly, households face unemployment risk, in the sense that there exists an exogenous probability that a household will transit from employment to unemployment and vice-versa. We denote by $s \in \mathcal{S} = \{u, e\}$ the employment status of a household today. Hence, for each household there exist two distinct cases, either $s = e$, implying that the current *status* of a household is *employment*, or $s = u$ denoting that the current *status* of the household is *unemployment*. Employment follows a first order Markov chain with transition probabilities denoted as $\pi(s' \mid s)$.

Secondly, households conditional on being employed receive a stochastic idiosyncratic labour endowment $y \in \mathcal{Y}$ which follows a first order Markov process

[7] $\lim_{c \to 0} u'(c) = \infty$. Inada conditions are named after the Japanese economist Ken-Ichi Inada Inada (1963). The conditions as such have also been introduced by Hirofumi Uzawa Uzawa (1963).

with *finite support* $\mathcal{Y}$,[8] a set with *cardinality* N.[9] We denote by $\pi(y' \mid y)$ the conditional probability of transiting from state y today to state y' tomorrow. These conditional transition probabilities of the Markov chain are identical across households. We denote by $\Pi(y)$ the associated invariant distribution.

For both the idiosyncratic shocks s and y we assume that a law of large numbers holds[10], so that for instance $\pi(y' \mid y)$ conditional on being employed also represents the fraction of agents subject to this particular transition.

1.4.3 Technology and Production

There exists one representative firm in the economy producing output by the means of a neoclassical production function.

$$Y = F(K, N), \tag{1.3}$$

where Y is total output, K denotes the capital input, and N denotes labour. The representative firm takes factor prices as given, hence w and r choosing optimally capital and labour respectively. The production function is assumed to be a typical Cobb-Douglas with constant returns to scale.

$$Y = K^{\alpha} N^{1-\alpha}, \tag{1.4}$$

where α captures the share of capital income in output. We further assume that capital depreciates at a constant rate δ when used in production, where $\delta \in (0, 1)$.

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[8] A discrete random variable, say y, takes at most countably many values. In particular there exists a finite or countable set $\mathcal{Y} \subset \mathbb{R}$ such that $\mathbb{P}[y \in \mathcal{Y}] = 1$. Therefore, we know with certainty that the only values that y takes belong in $\mathcal{Y}$. The **smallest set $\mathcal{Y}$ with that property** is called the **support** of y. Since the number of elements living in $\mathcal{Y}$ is finite, we call $\mathcal{Y}$ the **finite support** of the random variable y.

[9] *Cardinality* of a set $\mathcal{Y}$, denoted as $|\mathcal{Y}|$ or $\mathbf{card}(\mathcal{Y})$ is a measure of the numbers of elements in $\mathcal{Y}$. It can be interpreted as the set size. However, sets can also be infinite. Thus, cardinality of set could also be infinite.

[10] A detailed discussion of how law of large numbers could be applied can be found in Judd (1985) and Uhlig (1996). Uhlig (1996) shows that if we let $X(i)$, $i \in [0, 1]$ be a collection of identically distributed and pairwise uncorrelated random variables with common finite mean μ and variance σ^2, then $\int_0^1 X(i)di = \mu$. To achieve that, he interprets the integral as a Pettis-integral. Gelfand-Pettis integral just extends the definition of Lebesgue integral to vector-valued functions on a measure space. It can be used in the case where the measure space is an interval with Lebesgue measure.

A single good is being produced every period and it can either be consumed or invested via the only available asset in the economy, which is a one-period non-contingent bond.

1.4.4 Aggregate Resource Constraint

The aggregate feasibility condition in the model economy can be expressed as

$$F(K, N) = C + I = C + K' - (1 - \delta)K, \tag{1.5}$$

where C denotes aggregate consumption, K represents the aggregate capital stock, N is the labour input and δ denotes the depreciation rate of capital when used in production. All aggregate variables are denoted by capital letters.

1.4.5 Financial Markets and Contracts

Trade takes place every period amongst households. The only available asset for trade in the market is a *one period non-contingent bond*.[11] Since a market for contingent claims on future labour earnings is not available, pay-offs to the existing asset cannot span the consumption space. Thus, markets are *sequentially incomplete*[12].

Unlike the complete markets case, where agents choose optimal portfolios holdings at period $t = 1$ (initial period) and no subsequent trade is necessary, incomplete markets favour trade amongst agents every period. It essentially serves as a means of smoothing intertemporal consumption, achieving the maximum lifetime utility.

[11] As Zhang (1997) states this assumption is consistent with the existing moral hazard or adverse selection problems in reality which leads to the missing market.

[12] Quinzii and Magill (1996) propose that the commodity space $\mathbb{R}^n$ can be written as a direct sum of the market subspace and its orthogonal complement, namely $\mathbb{R}^n = \langle W \rangle \oplus \langle W \rangle^{\perp}$. Since the market subspace must not offer arbitrage opportunities, there exists at least one vector of state prices $\pi \in \langle W \rangle^{\perp}$. Thus, $dim\langle W \rangle^{\perp} \geq 1$. Since $dim\langle W \rangle + dim\langle W \rangle^{\perp} = n = \#\mathbb{D}$ it follows that whenever the market subspace offers no arbitrage opportunities $dim\langle W \rangle \leq \#\mathbb{D} - 1$. Thus $\#\mathbb{D} - 1$ is the maximal dimension of the subspace. This leads to the following definition of the complete and incomplete markets
Definition: Let the market subspace $\langle W(q, V) \rangle$ be arbitrage free. If $dim\langle W(q, V) \rangle = \#\mathbb{D} - 1$ then the financial markets are said to be *complete* ; if $dim\langle W(q, V) \rangle < \#\mathbb{D} - 1$ then the markets are *incomplete*.

Purchasing a bond with positive face value a' implies that an individual household entered into a contract[13] where saves qa' units of period t good, getting a promise to receive a' units of good next period. On the other hand purchasing a bond with negative face value a' implies that a household entered into a contract in which by receiving $-qa'$ units of good at period t commits to deliver, conditional on not defaulting, a' units of good at period $t+1$.

Therefore financial markets are incomplete (SIM), while financial contracts are unenforceable, namely a limited commitment problem is present and any borrowing household can default at any time on the outstanding debt that holds.

1.4.6 Default Option and Penalty

Henceforth we will refer to a household that borrows as the "borrower" and a household that lends as the "lender". Furthermore, only households with negative net worth ($a' < 0$) are permitted to default. On defaulting, the amount of debt exceeding assets, in other words the net liability, is discharged and erased from the budget set. We thus, implicitly, assume that defaults are partial [14].

This default option closely resembles Chapter 7 of the U.S. bankruptcy code for unsecured debt, such as credit card debts, medical bills gasoline card debts and few others. In this model framework, a defaulted household faces temporary exclusion[15] from the credit market.

As in Athreya (2002) and Li and Sarte (2006) we model exclusion from credit markets using a lottery. Each period following default a household remains in the borrowing-constrained state with probability $\theta \in (0, 1)$. The mean waiting time for the household to regain full access to financial markets is given by $\frac{1}{1-\theta}$.

[13] Quinzii and Magill (1996) suggest that a contract can be defined as a reciprocal agreement between two or more parties to perform and/or to receive the transfer of some specified goods, services or income under specified contingencies from some initial date (its date of issue) until some terminal date (its date of maturity).

[14] An alternative approach to study partial defaults is to allow for two assets; a short-lived one period bond and a long-lived perpetuity bond with coupon payments that decay geometrically over time. Arellano and Ramanarayanan (2012) studies the maturity structure tracing important effects in emerging markets, attempting to give a role for different maturities in the dynamics of debt and default. However, the whole analysis occurs in a completely different framework than ours, as lenders are risk neutral and production is absent.

[15] Representative papers using temporary exclusion from the credit market as a default penalty is Athreya (2002) and Li and Sarte (2006).

The parameter θ is exogenously calibrated, employing the underlying Bernoulli process [16] with parameter $1-\theta$ capturing the mean waiting time.

Furthermore, in line with Bulow and Rogoff (1989), any defaulted borrower cannot borrow following default. However, is allowed to trade contracts involving no-credit. Therefore, in this model borrowers are allowed to smooth consumption to some extent even following default, by saving if possible via the one period non-contingent bond available in the market. Essentially they are allowed to become lenders, provided that their labour income is sufficient.

At any given state, an unconstrained borrower (i.e. he has access to credit markets) should choose either to repay or to default. Hence, *value of the option to default* can be described by the following equation.

$$V^o(a, y, s) = \max\left\{V^R(a, y, s), V^D(0, y, s)\right\}, \tag{1.6}$$

where $V^R(a, y, s)$ is the value associated with repayment and staying in the contract, $V^D(0, y, s)$ is the value associated with defaulting on your outstanding debt, while $V^o(a, y, s)$ as clearly shown from the above equation can take either of the two values, depicting the value of the option accordingly.

It is important to emphasize that in this environment default cannot be random (luck). Instead, it is defined as an optimal choice as described by equation 1.6. In particular if the value of the borrower when remaining to the contract is greater than his value when defaulting, his optimal decision will be to satisfy his repayment obligations.

In the model environment, borrowers can find themselves in two different groups. We follow the terminology of Athreya (2002) and Li and Sarte (2006) and we define the borrower who currently has full access to credit market as *unconstrained* and the borrower who defaulted and is borrowing constrained (with no access to credit market) the current period as *constrained*. By construction of this model lenders are always *unconstrained*, since they have full access to financial markets.

[16] Consider a Bernoulli process, the random variables are i.i.d Bernoulli with common parameter $1-\theta \in (0,1)$. The natural state space is $\Omega = \{0,1\}^\infty$. Let $S_n = X_1 + \ldots + X_n$ (number of successes or arrivals in n steps). The random variable S_n is binomial with parameters n and $1-\theta$. $\mathbb{E}[S_n] = np$. Let now T_1 be the time of first success. Formally $T_1 = \min\{n|X_n = 1\}$. We know that T_1 is geometric, hence $\mathbb{E}[T_1] = \frac{1}{1-\theta}$.

1.4.7 Household's Problem

Households maximize expected discounted lifetime consumption utility subject to their individual budget and borrowing constraints. Households take prices, namely the bond price and wage rate as given.

We only study *stationary recursive competitive equilibria*, namely we require the distribution of agents across states to be invariant. In this manner, probability measure will remain unchanged, even if individual households can move across the earnings or wealth distribution, thus social mobility is allowed.

Naturally, all aggregate variables are constant over time in equilibrium. The recursive formulation of household's problem dictates the use of *Contraction Mapping Theorem*[17]. Due to this formulation policy correspondences will be time invariant.

Households are endogenously divided into two groups. Firstly, households that have complete access to financial markets, that is to say they can both borrow and lend. This households belong to the *unconstrained group*. Secondly, households that have limited access to financial markets, namely they are borrowing constrained, being able only to lend if their labor earnings are sufficient. This households belong to the *constrained group*.

Borrowing and lending in this model is considered as net. In particular, a household is a borrower iff is a net borrower. Similarly a household is a lender iff is net lender.

[17]**Definition**: Let (S, ρ) be a metric space and $T : S \to S$ be an operator mapping S into itself. V is a **contraction mapping** if for some $\kappa \in (0, 1)$

$$\rho(Tx, Ty) \leq \kappa\rho(x, y), \quad \text{for all} \quad x, y \in S.$$

A contraction mapping simply brings the elements of the vector space S as close as possible.

Theorem 1.4.1. *Contraction Mapping Theorem: Let (S, ρ) be a complete metric space and $T : S \to S$ be a contraction. Then there exists a unique measure $\hat{\nu} \in S$ such that*

$$T\hat{\nu} = \hat{\nu}$$

1.4.7.1 The Unconstrained Households

The states for an unconstrained household can be described by its level of asset holdings, $a \in \mathcal{A} \subset \mathbb{R}$, its labour endowment draw, $y \in \mathcal{Y} \subset \mathbb{R}^+$ and its employment draw $s \in \mathcal{S} \in [0,1]$.

A household that is an unconstrained borrower faces two options. It can either repay its debt obligations in full, or default on the outstanding amount of its debt holdings. We denote by $V^R(a,y,s)$[18] the value function of a household that repays its debt in the current period. We denote by $V^D(0,y,s)$ the current value associated with defaulting. Finally, we denote $V^o(a,y,s)$ the value of the option to be unconstrained.

The recursive formulation of this problem could be presented by the following equations.

$$V^R(a,y,s) = \max_{\{c\geq 0,a'\}} \{u(c) + \beta \sum_{(y',s')\in(\mathcal{Y},\mathcal{S})} \pi(s'|s)\pi(y'|y)V^o(a',y',s')\} \tag{1.7}$$

subject to

$$\begin{aligned} c + qa' &= wy\left[1-(1-\rho)\mathbb{1}_{s=u}\right] + a \\ a' &\geq -\overline{B} \end{aligned} \tag{1.8}$$

where c denotes consumption in the current period, a' denotes the next period asset holdings, w is the household wage, q is the current price of the asset and $\mathbb{1}_{s=u}$ is an indicator function which takes the value 1 when household is in the unemployed state and 0 otherwise. In this case labour earnings are equal to the unemployment benefits, which are defined as a fraction of the average labour earnings, thus $b(y) = \rho w y$.

Equation 1.8 stands for the budget constraint and borrowing limit. Note that there is an upper bound $\bar{B}$ on borrowing. Constraints in asset holdings are generally necessary to rule out Ponzi schemes and ensure the existence of equilibrium[19]. In the literature a priori bounds are often chosen to be a certain

[18] For notational convenience we will not introduce a different value function for a household which is an unconstrained lender. However, it should be clear that whenever we compare the value of repayment and the value of default we do it only for values of $a < 0$.

[19] Aiyagari (1994) explains that if for instance the interest rate is negative a borrowing is necessary for a maximum to exist. Without a borrowing limit the present value of earnings is almost surely infinite. In Aiyagari (1994) the limit is chosen to be the present value of budget balance, also ensuring the non-negativity of consumption.

fraction of total income. In this model we specify the bound accordingly ensuring that aggregate borrowing can never exceed the total capital stock in the economy.

We turn now to the right hand-side of equation 1.7. It ensures that households that repay their debt in current period will be unconstrained the following period. In this fashion, they will enjoy full access to financial markets.

Let now $V^C(a, y, s)$ denote the value function of a borrower that defaulted and is currently borrowing constrained. In this case $a \in \mathbb{R}^+$, namely the household is allowed to hold only net positive assets in the constrained state. In other words, borrowing constrained households are only allowed to save and/or lend.

The recursive formulation of the problem associated with value of default could be presented by the following equations.

$$V^D(0, y, s) = \max_{\{c \geq 0, a' \geq 0\}} \{u(c) + \beta \sum_{(y',s') \in (\mathcal{Y},\mathcal{S})} \pi(s'|s)\pi(y'|y)V^C(a', y', s')\} \tag{1.9}$$

subject to

$$\begin{aligned} c + qa' &= wy\,[1 - (1 - \rho)\mathbb{1}_{s=u}] \\ a' &\geq 0 \end{aligned} \tag{1.10}$$

where similarly to the previous problem, c denotes consumption in the current period, a' denotes the next period asset holdings, w stands for household wage, q is the current price of the asset and $\mathbb{1}_{s=u}$ is an indicator function which takes the value 1 when household is in the unemployed state and 0 otherwise.

The budget constraint in equation 1.10 indicates that once a borrower (household that is a net borrower) defaults, his/her net asset position is zero ($a = 0$). In right hand-side of equation 1.9 we observe that households choosing to default are necessarily borrowing constrained in the following period.

From the second constraint in equation 1.10 it is obvious that following the default event the household can only save and/or lend. Consequently, a' should be restricted to belong only in the positive subset of a household's trading possibility set.

1.4.7.2 The Constrained Households

In this section we specify the problem of borrowing constrained households, namely households that have defaulted in previous periods. The default punishment is modelled as a temporary exclusion from credit market. Similarly to Athreya (2002) and Li and Sarte (2006) we model exclusion using a lottery. In particular, we assume that every period after the default event households face a probability θ to be borrowing constrained and $1-\theta$ to regain full access to financial markets.

The recursive formulation of the problem of constrained households could be presented by the following equations.

$$\begin{aligned} V^C(a,y,s) = \max_{\{c\geq 0, a'\geq 0\}} \{u(c) + \beta \sum_{(y',s')\in(\mathcal{Y},\mathcal{S})} \pi(s'|s)\pi(y'|y)\theta V^C(a',y',s') \\ +(1-\theta)V^o(a',y',s')\} \end{aligned} \tag{1.11}$$

subject to

$$\begin{aligned} c + qa' &= wy\left[1-(1-\rho)\mathbb{1}_{s=u}\right] + a \\ a' &\geq 0 \end{aligned} \tag{1.12}$$

where c denotes consumption in the current period, a' denotes the next period asset holdings, w stands for household wage, q is the current price of the asset and $\mathbb{1}_{s=u}$ is an indicator function which takes the value 1 when household is in the unemployed state and 0 otherwise.

We now move to the right-hand side of equation 1.11. We observe that constrained households will re-enter the credit market with probability $1-\theta$ and will remain constrained with probability θ. In the case of re-entry their value function will be the value of the option. Therefore, transiting to the unconstrained state households have the option to borrow while being able to choose between defaulting and repaying.

The budget constraint in equation 1.12 suggests that constrained households during the exclusion period can accumulate positive assets, by either saving or lending to other households, conditional on the sufficiency of their labour earnings, so that consumption is non-negative and the budget constraint is satisfied. The constraint in equation 1.12 guarantees that in the constrained state no borrowing is allowed, since households do not have access to credit market.

1.5 Stationary Recursive Competitive Equilibrium

Prior to the definition of equilibrium in our economy, it is useful to explain the notion of equilibrium in an infinite horizon model with a support of an underlying Markov process. Equilibrium in infinite horizon models is a matter of continual readjustment to the resolution of uncertainties, generated by a time homogeneous[20] Markov process (see Duffie et al. (1994)).

The concept of *Recursive Competitive Equilibrium* requires that all agents behave optimally and all markets clear. Additionally, the notion of *stationarity* requires in our economy that agents' distribution across states is invariant.

We denote the household's repayment decision as z, where $z = 1$ when a household decides to default and $z = 0$ when decides to repay its debt. We are now ready to describe the equilibrium in our economy.

Description of Equilibrium

A stationary recursive equilibrium in our economy can be described by a set of prices $\{w, r, p\}$, namely the wage, the interest rate and the asset price respectively, a set of policy functions for the *unconstrained group* $\{c, a, z\}$, thus consumption, borrowing/saving and default option respectively, and a set of policy functions $\{c, a\}$ for the *constrained group*, a set of value functions $\{V^R, V^D, V^C, V^o\}$ for repayment, default, constrained and the option to be unconstrained, a set of aggregate variables $\{C, A, K, L\}$ and finally a set of probability measures $\{\mu^{uc}, \mu^c\}$ where μ^{uc} denotes the measure of unconstrained households and μ^c denotes the measure of constrained households, such that:

(i) Taking prices as given, households policy functions solve their maximization problems.

[20] A Markov Chain $Y(t)$is said to be time homogeneous if

$$\mathbb{P}(Y(s+t) = j)|Y(s) = i)$$

is independent of s. When this holds, setting $s = 0$ yields $\mathbb{P}(Y(t) = j)|Y(0) = i)$.

(ii) Firm's maximize profits yielding

$$w = (1 - \alpha)\left(\frac{K}{N}\right)^{\alpha}$$
$$r = \alpha\left(\frac{K}{N}\right)^{\alpha-1} - \delta$$

(iii) The *labour market* clears and N is given by

$$N = (1 - \Pi(u))\sum_{y\in\mathcal{Y}} y\Pi(y)$$

(iv) The *goods market* clears

$$Y = C + \delta K$$

where C is given by

$$C = \int_{\mathcal{A}\times\mathcal{Y}\times\mathcal{S}} c(a, y, s)d\mu^{uc} + \int_{\mathcal{A}^{+}\times\mathcal{Y}\times\mathcal{S}} c(a, y, s)d\mu^{c}$$

(v) The *asset markets* clears and K is given by

$$K = \int_{\mathcal{A}\times\mathcal{Y}\times\mathcal{S}} a'd\mu^{uc} + \int_{\mathcal{A}^{+}\times\mathcal{Y}\times\mathcal{S}} a'd\mu^{c}$$

1.6 Calibration of the Model

Due to the inherent non-linearity of our model we are unable to derive analytical solutions. Hence, we are forced to use a numerical method to compute the equilibrium of our economy. To solve the model numerically one needs first to calibrate the parameters. We decide to set the model's period to *one year* and we calibrate all the parameters accordingly.

In this section, we describe our calibration strategy in setting the structural parameters. We set the risk aversion parameter σ to be 3. As is conventional in the literature (Krusell and Smith (1998)) we set the capital share to output ratio in the Cobb-Douglas production function as α=0.33. We further assume that the deprecation rate of capital is δ= 3.5% annually. The rate of time preference, β is set to 0.94. The probability of exclusion from the credit markets, θ is calibrated to 0.85 implying a mean exclusion period of 6.6 years consistent with Chapter 7 Bankruptcy law.

Since agents in this model are facing two different types of idiosyncratic risks, namely countercyclical unemployment risk and also acyclical earnings risk, we will analyse both types in the following sections.

1.6.1 Unemployment Risk

Idiosyncratic unemployment risk is determined by a 2×2 transition matrix. The probabilities composing this matrix represent the transition in and out of unemployment for each state that an agent belongs.

In this manner π_{uu} depicts the probability than an agent finding himself in the *unemployment state* today, will remain in this state the following period. Similarly, π_{ue} shows the probability that an agent will transit from *unemployment* today to *employment* the following period.

To calibrate the transition probabilities of this matrix we follow Krueger et al. (2016). Since, Krueger et al. (2016) examines the case of both idiosyncratic and aggregate risk, we simplify their approach by allowing only for idiosyncratic unemployment risk. The idiosyncratic transition matrix can be uniquely pinned down by calculating the *job separation rate* π_{eu}. Alternatively, we could use the *job finding rate* π_{ue}.

Krueger et al. (2016) uses the monthly Current Population Survey (CPS) dataset to compute quarterly values. Since the time period of our model is yearly, we use the quarterly job finding and separation rates to compute the yearly transition probabilities. We assume that an individual starts the first quarter as employed and calculate the probability that this individual will remain employed at the end of the fourth quarter, π_{ee}.

This can take place in multiple ways. We denote as s_1, s_2, s_3, s_4 the job separation rates in the first,second, third and fourth quarters of the year, respectively. There are in total 8 cases that we have to account for. These are listed below.

(I) The individual does not lose his job in any of the four quarters of the year $(1, 2, 3, 4)$. The associated probability is $(1-s_1)\times(1-s_2)\times(1-s_3)\times(1-s_4)$.

(II) He loses his job in the first quarter, finds a job in the second quarter and remains on job in the following two quarters with probability: $s_1 \times f_2 \times (1 - s_3) \times (1 - s_4)$.

(III) He loses his job in the first quarter, does not find a job in the second quarter, finds a job in the third quarter and does not lose his job in the last quarter with probability: $s_1 \times (1 - f_2) \times f_3 \times (1 - s_4)$.

(IV) He does not lose his job in the first quarter, loses his job in the second quarter, finds a job in the third quarter and doesn't lose the job in the last quarter with probability: $(1 - s_1) \times s_2 \times f_3 \times (1 - s_4)$.

(V) He does not lose job in the first quarter, loses his job in the second quarter, doesn't find a job in the third quarter and finds a job in the fourth quarter. The associated probability is : $(1 - s_1) \times s_2 \times (1 - f_3) \times f_4$.

(VI) He loses his job in the first quarter, doesn't find a job in the second and third quarters and finds a job in the last quarter with probability: $s_1 \times (1 - f_2) \times (1 - f_3) \times f_4$.

(VII) He does not lose his job in the first and second quarters, loses job in third quarter and finds a job in the last quarter with probability: $(1 - s_1) \times (1 - s_2) \times s_3 \times f_4$.

(VIII) Loses job in first quarter, finds a job in second quarter, loses job in the third and finds a job in the fourth quarter with probability: $s_1 \times f_2 \times s_3 \times f_4$.

Therefore, knowing the above probabilities we can calculate the probability of an individual *being employed* at the beginning of the first quarter and who becomes *unemployed* at the end of the quarter, π_{eu}, as follows:

$$\begin{aligned}
\pi_{eu} &= 1 - \pi_{ee} \\
&= 1 - \Big((1-s_1)\times(1-s_2)\times(1-s_3)\times(1-s_4) \\
&+ s_1 \times f_2 \times (1-s_3)\times(1-s_4) \\
&+ s_1 \times (1-f_2)\times f_3 \times (1-s_4) \\
&+ (1-s_1)\times s_2 \times f_3 \times (1-s_4) \\
&+ (1-s_1)\times s_2 \times (1-f_3)\times f_4 \\
&+ s_1 \times (1-f_2)\times(1-f_3)\times f_4 \\
&+ (1-s_1)\times(1-s_2)\times s_3 \times f_4 \\
&+ s_1 \times f_2 \times s_3 \times f_4\Big)
\end{aligned} \tag{1.13}$$

In a similar approach, we calculate the transition probability from unemployment to employment, π_{ue}. We use CPS data and we follow Shimer (2005) for the measurement of the *job finding* and *separations rates*.[21] By following this strategy and using the CPS data we find the transition matrix presented below, by calculating the average probabilities from $2006Q_1$ to $2014Q_4$, since we are interested in including the period of the global financial crisis of 2008. Off the principal diagonal is the *job finding rate* and *job separation rate* respectively.

$$\begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix} = \begin{pmatrix} 0.1700 & 0.8300 \\ 0.0420 & 0.9580 \end{pmatrix}$$

[21] We download the following series from CPS: The **unemployment level** (UNEMPLOY - Thousands of Persons, Monthly, Seasonally Adjusted), the **short term unemployment level** (UEMPLT5 - Number of Civilians Unemployed for Less Than 5 Weeks, Thousands of Persons, Monthly, Seasonally Adjusted) and the **employment level** (CE16OV - Civilian Employment Level, Thousands of Persons, Monthly, Seasonally Adjusted). Denoting by u_t the unemployment rate and by u_t^s the short term unemployment rate, we define the *job finding rate* $1 - \left(\frac{u_{t+1} - u_{t+1}^s}{u_t}\right)$ and the *separation rate* $\frac{u_{t+1}^s}{1-u_t}$.

1.6.2 Earnings Risk Conditional on Employment

A large body of literature calibrate the earnings using Gaussian $AR(1)$ processes. This Gaussian innovations inherently produce an income distribution with zero skewness and a kurtosis of three. However, recent studies, in particular by Guvenen et al. (2015) show that earnings shocks display a sizeable deviation from log-normality, which is the standard assumption in the incomplete markets literature.

In particular, Guvenen et al. (2015) using a large panel data set of earnings history drawn from U.S. administrative records have found that earnings shocks display strong negative skewness and extremely high kurtosis. The data produces a kurtosis of 30 and a skewness of -1.35.

These higher order moments cannot really be produced by a Gaussian $AR(1)$ process. Guvenen et al. (2015) recommends the use of a first order autoregressive process with normal mixture innovations (NMAR).

Following this, Civale et al. (2016) develop a discretization method, building on Tauchen (1986), which they refer to as Extended Tauchen (ET). This method improves on Tauchen (1986) in two directions. Firstly, the AR(1) innovations are distributed as a mixture of normals. This ensures that the stochastic process can capture the appropriate higher-order moments, namely non-zero skewness and high kurtosis. Secondly, the state space is selected optimally, so as to deliver a set of targetted moments. This gives remarkable precision levels while approximating.

We employ a first order autoregressive process with normal mixture innovations (NMAR) given by the following equation:

$$y_t = \rho y_{t-1} + \eta_t \tag{1.14}$$

where

$$\eta_t \sim \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2), & \text{with probability } p_1 \\ \mathcal{N}(\mu_2, \sigma_2^2), & \text{with probability } 1 - p_1 \end{cases} \tag{1.15}$$

In the above process y denotes log-earnings, ρ the persistence of the log-earnings process, and η_t stands for the Gaussian innovations.

We denote $\Delta y_t = y_t - y_{t-1}$ as the log-earnings growth. Following Civale et al. (2016) we calibrate an NMAR process targeting the data moments of Guvenen et al. (2015) reported in Table 1.1.

There are six parameters $(\rho, p_1, \mu_1, \mu_2, \sigma_1, \sigma_2)$ that govern the NMAR process. We follow the strategy of Civale et al. (2016) and fix $\rho = 0.99$, $p_1 = 0.9$ while imposing that $\mu_2 = \frac{-p_1 \mu_1}{1-p_1}$ to ensure that $\mathbb{E}(\eta) = 0$.[22]

Hence the remaining three parameters $(\mu_1, \sigma_1, \sigma_2)$ are left free to match the targeted data moments of Guvenen et al. (2015). The NMAR calibration that we find is reported in 1.2.

Table 1.1: Targeted Data Moments of Guvenen et al. (2015)

$\mathbb{E}(\Delta y)$	$\text{Var}(\Delta y)$	$\text{S}(\Delta y)$	$\text{K}(\Delta y)$
0	0.23	-1.35	17.8

Table 1.2: NMAR calibration following Civale et al. (2016)

ρ	p_1	μ_1	μ_2	σ_1^2	σ_2^2	N
0.99	0.9	0.0336	-0.3021	0.0574	1.6749	12

[1] N is the number of states.

[22]This is because the unconditional expectation is $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$.

Table 1.3: Calibrated Parameters

Description	Parameter	Value
Preferences		
Households Discount Factor	β	0.94
Coefficient of Relative Risk Aversion	σ	3
Technology		
Capital Share	α	0.33
Depreciation Rate of Capital	δ	0.035
Defaults		
Probability of Remaining Constrained	θ	0.85
Unemployment Rate*		7.25%

Notes: This table reports the calibrated parameters for our benchmark model economy. The unemployment rate that we obtain from the calibrated employment risk and calibrated earnings risk processes is 7.25%.

1.7 Results

In this section we present and discuss all the results obtained from the computational solution of our model. This includes the discussion of value, savings and consumption functions. We further discuss the default regions, that is, below which net worth to income ratio households decide to default. We compare households in different states, i.e., employed, unemployed, high labour earnings state and low labour earnings state.

We then turn to the case of asymmetric default costs, describing the nature of the mechanism that we have added to our benchmark model. We next present the new results obtained when we account for asymmetric default costs. This includes a discussion of value, policy functions and more importantly default regions. We then comment on the default regions and we compare with the corresponding regions in our benchmark model.

Finally, we examine the properties of our benchmark model economy. We present the statistics generated from our model and we compare with their counterparts in the U.S. economy. We discuss in more detail the distributional properties of our model, presenting the wealth distribution quintiles, a wealth histogram for households with good and bad credit history and finally a wealth histogram is reported for the employed and unemployed households in our economy.

1.7.1 Value and Policy Functions

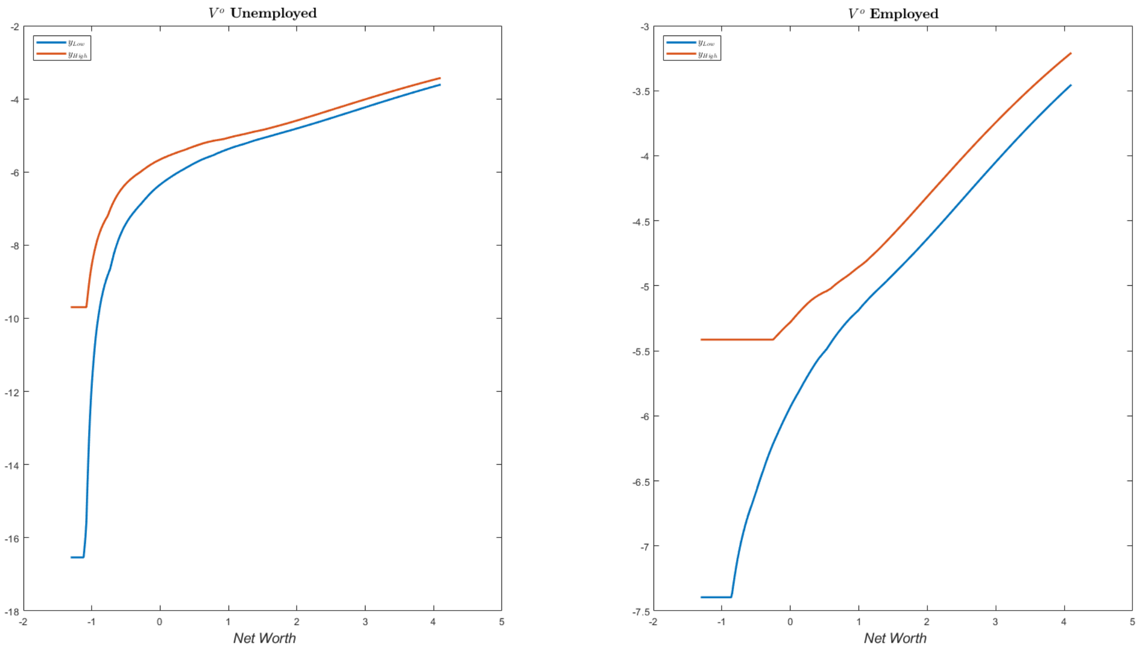

Figure 1.3: V^o **for Employed and Unemployed:** The left panel of this figure presents V^o for *unemployed* under two different realizations of labour earnings shocks. The right panel of the figure presents V^o for *employed* under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

In figure 1.3 we observe the behaviour of value of the option for both employed and unemployed households. We present the value of the option for two different realizations of labour earnings shock. Note that unemployed households (see Krueger et al. (2016)) carry the idiosyncratic state even if does not affect their current labour earnings directly, since they do not participate in the labour market. This occurs due to our definition of unemployment benefits, which are defined as a fraction of potential labour earnings of a household.

The value of the option, V^o , embodies the optimal choice between value of repayment and value of default for households with negative net worth[23], along with the regular value function for households with positive net worth. Recall that households with positive net worth are not allowed to default in this model.

A striking result is that households with higher labour earnings realization both in the employed and unemployed state default more compared to low labour earnings households. In figure 1.3 the straight horizontal line of value of the option indicates the choice of the default option.

At a first inspection, this result appears to be counter-intuitive. We pose the following question: Why would households with lower labour earnings choose to default less? Firstly, we should recall that in this model default is strategic. In this sense, we disregard cases where households *can't pay* and we only consider cases that households *won't pay*, as a consequence of their optimal choice.

Analysing the right-hand-side part of figure 1.3, we can indeed see that households with low earnings, find default optimal only when their net-worth is very negative. The cost of default bears an exclusion from credit market, hence an exclusion from borrowing for a period of around seven years[24]. This is in line with Chapter 7 bankruptcy law in the United States. A Chapter 7 bankruptcy remains on a household's credit report for at least seven years from the date of filing the Chapter 7 petition. This implies that during this period households will have very restricted or no access to credit.

It is clear that in this model borrowing serves as a means to achieve consumption smoothing. In particular, households with very low earnings find crucial for their survival to have some access to credit that enables them to achieve even a low consumption level. In support of this argument is the left panel of figure 1.3. The graph indicates that unemployed households default less compared to employed households, or more precisely default only when featuring lower levels of net worth compared to employed households.

To further comprehend this result, we should recall that the only source of income for a household in an unemployed state is unemployment benefits. However,

[23]Note that net worth is expressed in every graph as a ratio of net worth to mean income.
[24]We calibrate parameter θ in our model to 0.85 which captures an average exclusion period of 6.6 years.

this income might not be sufficient for a household to support even the essential lowest level of consumption while also repaying his debt.

One could argue that in such an event household should find it optimal to default and enjoy the partial debt relief arising from the exercise of this option. In this manner these households could possibly achieve higher consumption. However, as we observe in the left panel of figure 1.4 households in this state choose to borrow more and move to a higher level of negative net worth *without defaulting*. This behaviour confirms that unemployed households are willing to live in a state of limited consumption while maintaining their access to credit.

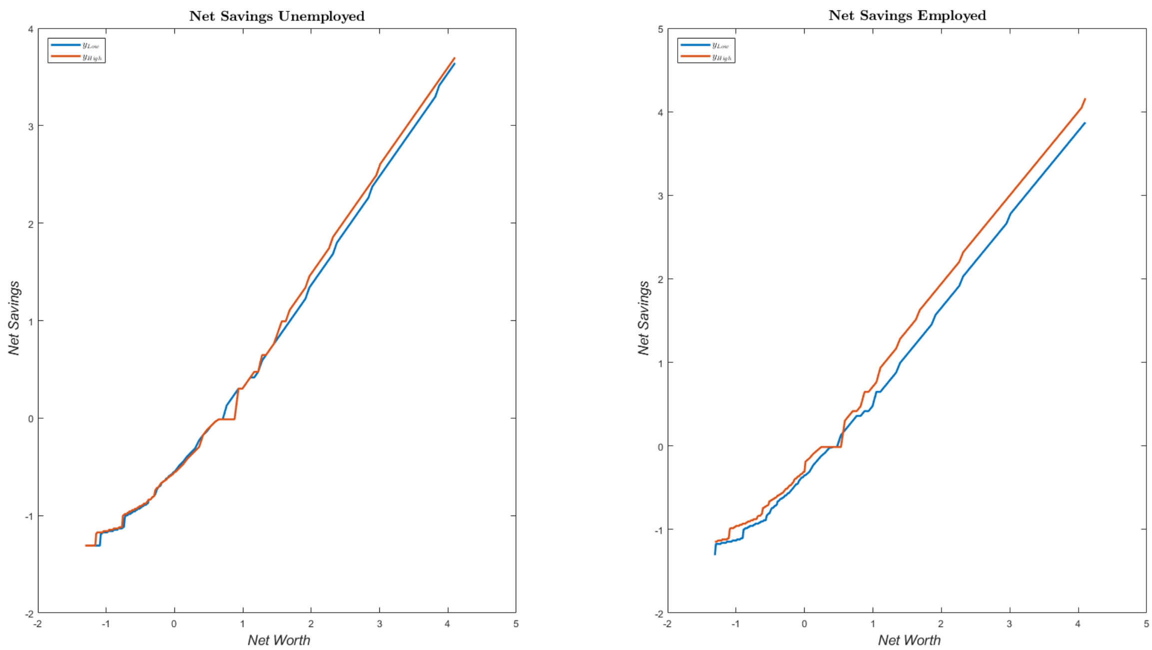

Figure 1.4: **Net Savings for Employed and Unemployed:** The left panel of this figure displays net savings for *unemployed* households under two different realizations of labour earnings shocks. The right panel of the figure displays net savings for *employed* households under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both figures.

In that sense, access to credit appears to be more valuable for households in a bad state. Since the probability to remain unemployed is slightly above five times the probability to remain employed, unemployed households perceive their current status as more “permanent”. Thus, they are not willing to take the risk of defaulting and loose their access to credit market. They are more concerned about the continuation of a bad state realization, under which they will not achieve even the lowest level of consumption if being excluded from credit. Hence, they adopt the strategy of accumulating more debt bearing its extra cost until a good state is realized in the future. These households only default when reaching the borrowing limit or more precisely when they lie very close to it. Essentially they are willing to borrow as long as someone is willing to lend them, mainly aiming to subsidise their consumption.

The finding of low default rates amongst the more financially distressed households, can shed light on why lenders rarely renegotiate loan modifications with very high risk borrowers. This is because most of them continue to pay. (see Adelino et al. (2013)). Furthermore, these results provide theoretical support to new empirical evidence introduced by Gerardi et al. (2015). They find that the vast majority of borrowers with very low ability to pay avoid default and prefer to decrease their consumption to subsistence levels.

At the opposite end, households that are employed and face a higher labour income shock as observed in the right panel of figure 1.3 default for almost any non-positive value of net worth . Interestingly, even if it is not apparent in figure 1.3 since we do not plot the highest income shock, they would be willing to default, if they were given the option, even for slightly positive values of net-worth.

This indicates that indirect cost of defaults for higher labour income employed households is not as large compared to the low income or unemployed households. This is because these households have enough earnings to support their consumption, even when being excluded from credit market.

We further argue that high labour income households will continue to borrow even for positive values of net worth to increase their consumption. This can be seen in the right panel of figure 1.4. Households with positive net worth and good realization of earnings shock, choose to borrow up to a net worth to income ratio of 1. However, and since in our framework positive net-worth households are not allowed to default, thus are forced to repay, they do not achieve perfect consumption smoothing. Clearly default option could serve as a means to smooth your consumption.

Consumption policy functions for employed and unemployed states are graphically illustrated in figure 1.5. Consumption exhibits spikes and higher volatility for positive values of net worth. Recall that, default is not an option for agents that hold positive net worth. Therefore, these borrowing households that would optimally decide to default and are not allowed, would be forced to reduce their consumption in order to repay. This would hinder their consumption smoothing, since the default option and not only borrowing/saving functions as a consumption insurance mechanism.

In the right panel of figure 1.5 it is evident that consumption is constantly larger for higher income employed households compared to lower income employed households up to a certain point of net worth. Above a level of net worth to income ratio around 1.5, consumption for both low and high labour income employed households is almost equivalent.

This can be explained by the fact that beyond a certain level of wealth the marginal propensity to consume from labour earnings is less than the marginal propensity to save. In support of this argument is figure 1.4. It is apparent that net savings is an increasing function of net worth independently of any labour earnings realization. Furthermore, households being in a higher income state tend to save more.

Finally, when a household is in an unemployed state while having positive net worth saves less compared to an employed household with equal net worth, since it finds necessary to subsidise his consumption.

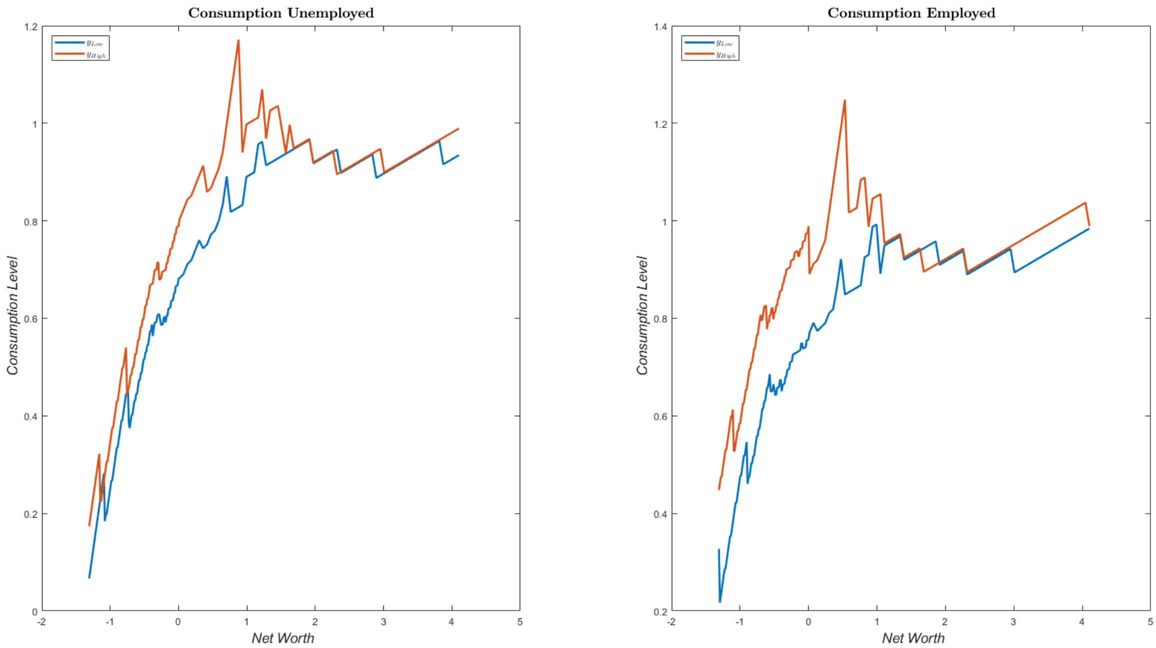

Figure 1.5: **Consumption Levels for Employed and Unemployed:** The left panel of this figure presents optimal consumption for *unemployed* households under two different realizations of labour earnings shocks. The right panel of the figure presents optimal consumption for *employed* households under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

1.7.2 Default Regions

To give a clear illustration of the our previous discussion regarding defaults, we present in figure 1.6 the default regions for a combination of employed/unemployed state and high/low earnings realization. Default regions are defined as the area in which value of default is higher than value of repayment. This is the area covering the left side of the intersection of the two value functions in all graphs presented below.

In the right bottom panel of figure 1.6 it is clear that employed households with high realized earnings, default in all levels of negative net worth up to -0.25. Employed households with low realized earnings default only above -0.85 negative net worth. This illustrates the finding that higher income households feature larger incentives to default compared to the lower income ones.

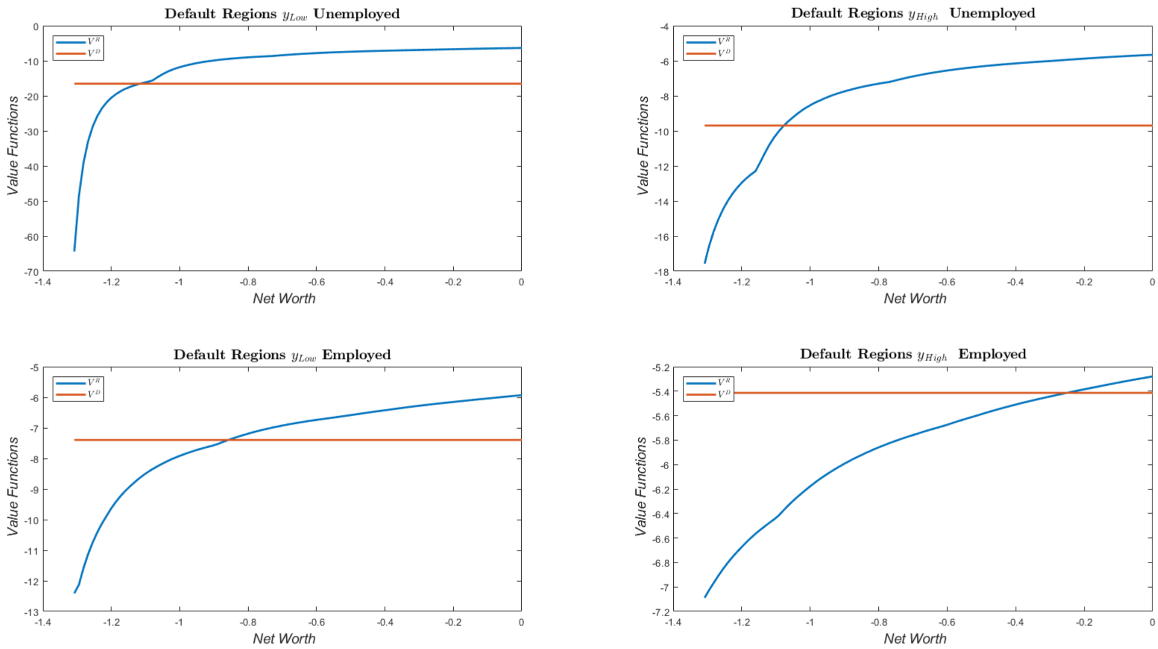

Figure 1.6: **Default Regions for Employed/Unemployed and y_{Low} and y_{High} earnings:** The left panel of this figure presents default regions for an unemployed/employed state when a low labour earnings shock is realised y_{Low}. The right panel of the figure presents default regions for an unemployed/employed state when a high labour earnings shock is realized y_{High}. To specify the default regions we compare value of repayment V^R to value of default V^D as depicted in the north-west part of all graphs.

We claim that this result occurs due to *symmetric default penalties* that are present in our model. In this sense, both high and low income households face the same default penalty, namely a temporary exclusion from credit market. Our results are in conflict with findings of some other studies.

For instance, Arellano (2008) in a sovereign debt and default framework, finds that default sets are stronger the lower the endowment is. In other words, defaults are higher in a low realized income state. We claim that this result is present due to an *asymmetric default cost* in her model. More precisely Arellano (2008) assumes that default entails some direct output costs only when output is above a certain threshold. Definitely this mechanism, unlike in the consumer bankruptcy framework, is fairly valid when studying defaults at a country level. In particular, it would be bizarre to assume that high income households face an extra cost compared to those with low income, especially when studying defaults in unsecured debt.

It is also evident in figure 1.6 that unemployed households default more than employed households. This follows naturally from our previous discussion, since these households have even less earnings, namely they only receive an unemployment benefit. Their behaviour is similar to employed low income households. In particular, unemployed households default only above a negative net worth to mean income ratio of -1.1. More generally, we could argue that low income or unemployed households tend to default only when they are heavily indebted, in the sense of featuring very negative worth. It is not clear however whether this very low income unemployed households would even default at this level of negative net worth, if they were a facing a less tight borrowing constraint.

To conclude this section, default is more costly in our model in the lowest states (i.e unemployment state or low earnings realization), even if post defaults savings remain an option. However, in a very low state, this option cannot even be exercised, unlike when being in a high state. This is reasonable since low earnings are hardly sufficient to support consumption. Indeed in our model very low earnings or unemployed households choose to have almost zero savings in the post-default era. Essentially, being in a constrained state does not enable them to benefit from the outside savings option.

Contrariwise, high earnings households find optimal to default and trade no-credit involved contracts across the post-default path. This allows them to attain a higher consumption bundle, since the outside option is both attractive and feasible for them. In a simpler way, their earnings are adequate to both maintain a satisfactory level of consumption, although choosing high positive savings.

1.7.3 The Case of Asymmetric Default Costs

In our benchmark model economy we assume that default costs are symmetric, in the sense that every household, independently of the idiosyncratic earnings shock realization and the level of indebtedness, will be equally penalised. Both high and low income households will suffer the consequences of default, and will be temporarily excluded from credit market. To prove the strength of asymmetric default costs assumption, we modify our benchmark model in the following way.

Our model employs twelve idiosyncratic earnings states. We assume that households facing a high income state, while choosing to default, will face a higher punishment relative to those households in low states. This punishment takes the form of an earnings loss or alternatively a wage garnishment. This loss intends to capture the pecuniary costs of default.

In our framework, we further assume that this pecuniary cost is dependent on the level of income. The direct costs of default in our modified framework, similarly to Arellano (2008), takes the following form:

$$\chi(y) = \begin{cases} wy & if \quad wy \leq w\gamma \\ w\gamma & if \quad wy > w\gamma \end{cases} \tag{1.16}$$

where γ is an exogenous parameter capturing the rate of labour income garnishment as in Chatterjee et al. (2007). The rationale behind the theory of pecuniary costs is the notion of a bad credit rating, which will bear a loss of reputation in the credit market. We assume, as in Chatterjee et al. (2008) that this pecuniary cost stems from the loss of one's reputation in the credit market.

Nevertheless, this cost will not be applicable if your labour income realization is below a certain threshold.

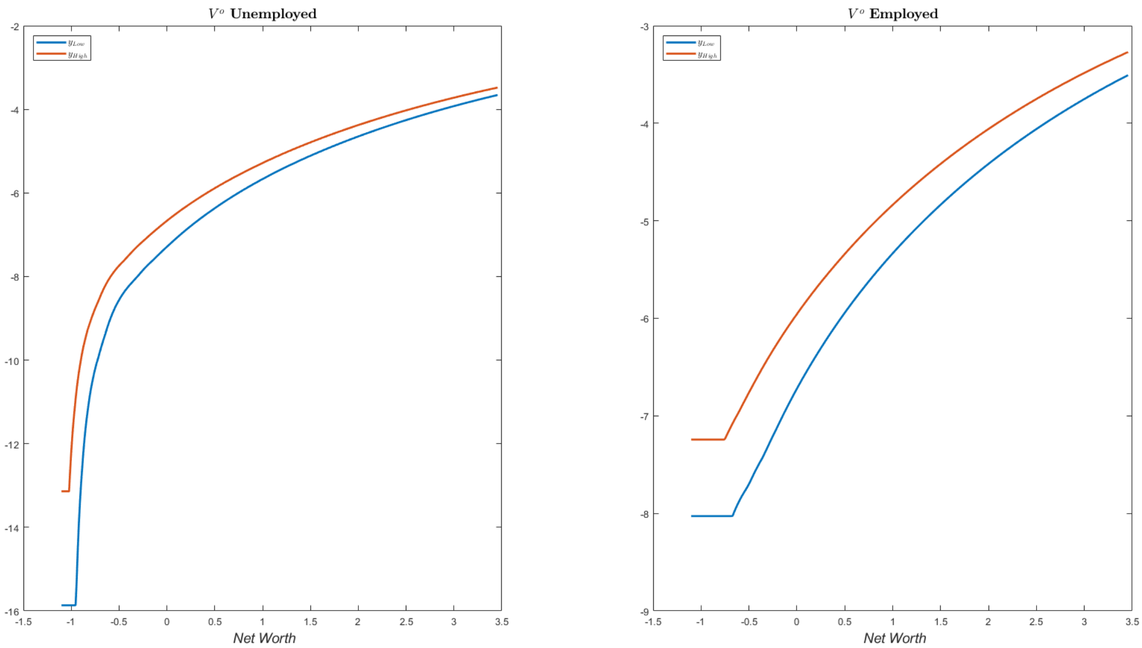

Figure 1.7: V^o **for Employed and Unemployed with** $\gamma = 0.75$ **:** The left panel of this figure presents V^o for *unemployed* households under two different realizations of labour earnings shocks. The right panel of the figure presents V^o for *employed* households under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

This assumption is in line with the *Federal Wage Garnishment Law* and the *Consumer Credit Protection Act* III. Title III of *CCPA* limits the amounts of an individual earnings that may be garnished and further protects an employee from being fired.

It is administrated by the U.S Department of Labour's Wage and Hour Division and applies in all fifty states. More importantly, if the disposable earnings of the individual are less than 217.5$ per week, there can be *no garnishment*. Based on this law we do not allow wage garnishment to take place, as presented in the previous equation when your realized labour earnings are below $w\gamma$. In this manner, the default penalty becomes larger for households facing high earnings shock.

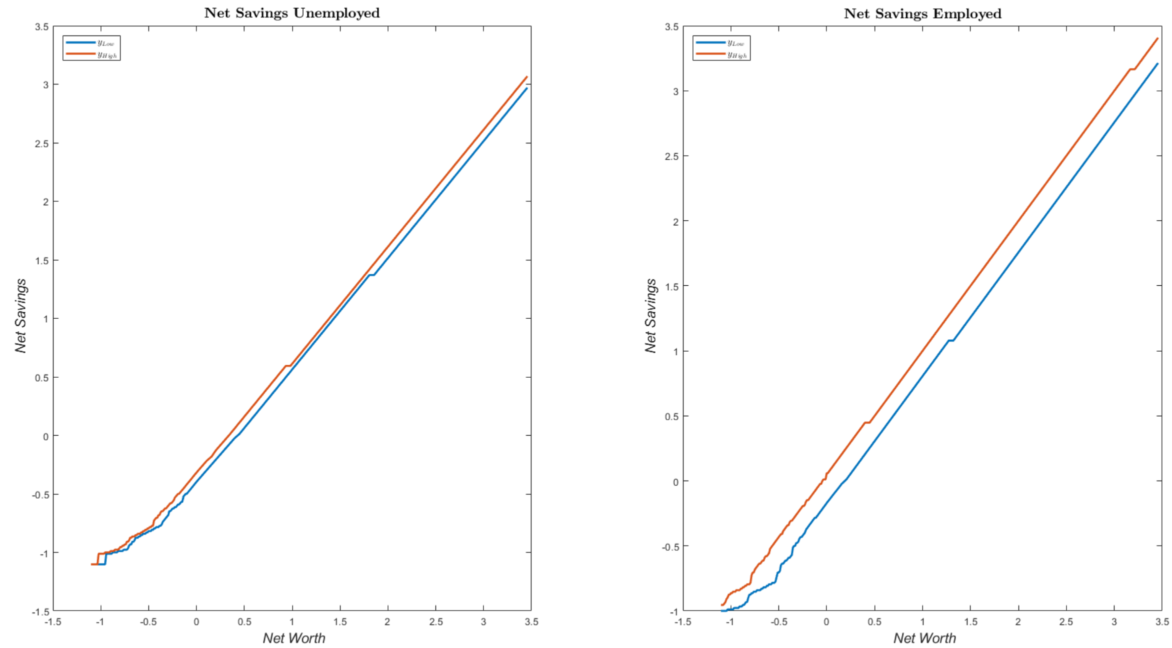

Figure 1.8: **Net Savings for Employed and Unemployed with** $\gamma = 0.75$**:** The left panel of this figure presents net savings for *unemployed* households under two different realizations of labour earnings shocks. The right panel of the figure presents net savings for *employed* households under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

Claiming that this mechanism has significant impact in the results of our model, we are expecting to see major changes in value, policy functions and equilibrium outcomes compared to our benchmark economy. We should state that all results presented in this section are based on a calibration of parameter $\gamma = 0.75$. This implies that 25%[25] of labour earnings will be garnished if you are at a high income state.

Indeed, as we observe in figure 1.7 and in contradiction to the results of our benchmark model, households with higher labour income default less compared

[25]Title III of the CCPA sets the maximum amount that could be garnished in any working week or pay period. This is independent to the number of garnishments orders that might be received by an employer. For ordinary garnishments orders including bankruptcy, any state or federal tax normally this rate would not exceed 25% of the employee's disposable earnings. There exist a few limitations or exceptions for cases of child support, alimony, non-tax debt owed to federal agencies, defaulted federal student loans, but 25% is in most cases the upper bound.

to those with lower. In the left panel of figure 1.7 is evident that employed households facing a high income shock default when their net worth to income ratio is below -0.7 while those facing a low income shock default when their net worth is below -0.55.

The results are similar for unemployed households, in the sense that households with higher earnings realization default more. Nevertheless, unemployed households as observed in the left panel of figure 1.7 default only when are highly indebted, or more precisely when their net-worth to income ratio is below -1.0 and very close to the model's borrowing limit.

This result remains unchanged compared to our benchmark model, since these households as previously discussed are willing to sacrifice a part of their consumption to repay their debts and consequently maintain their access to credit market.

This is due to their fear that a bad state will continue to be realized in the future and without access to borrowing, consumption might turn below subsidence level and very close to zero. In other words, "survival" is a key element in the decision making of these households.

For a better illustration of our results we present figure 1.8 where is clear that employed households under a high income realization and positive net worth do not choose to borrow. This is different compared to the results of our benchmark model in figure 1.4. This is because default option is not any more so attractive, particularly for high income households.

With symmetric default costs, these households were enjoying the benefits of defaults, without facing an extra cost. In this manner, they would prefer to borrow more and default, when they were given the option, while their high labour earnings would remain unaffected. With asymmetric default costs and when incorporating wage garnishment, labour earnings would be affected after default, since a part of them will be garnished and essentially lost from the household's budget set.

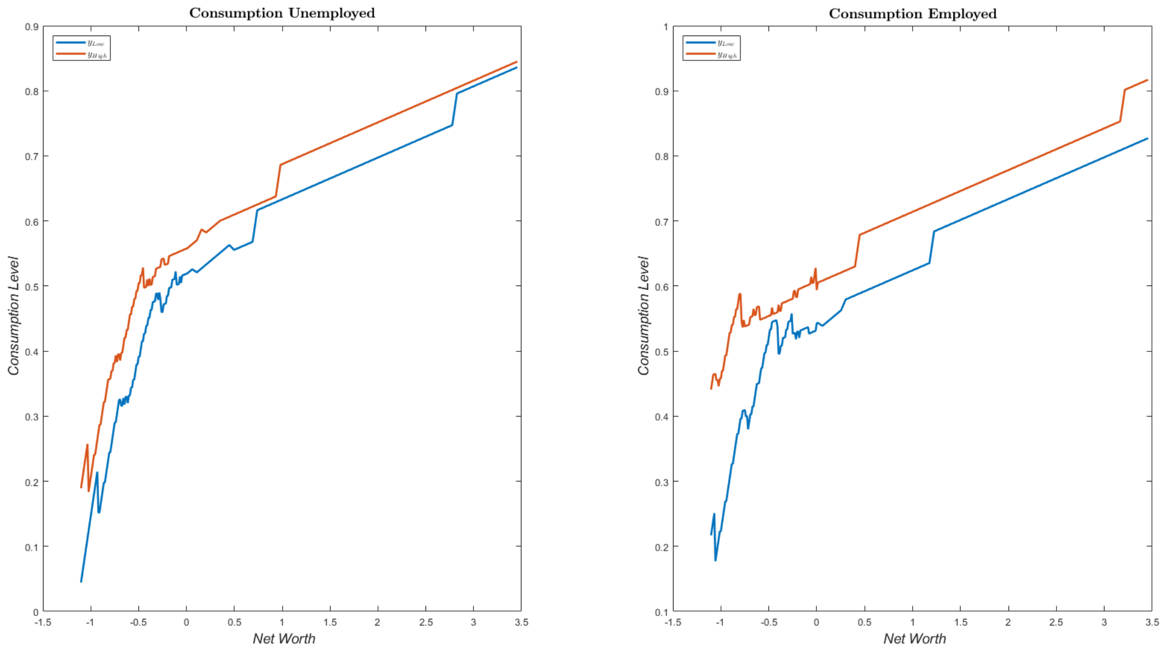

Figure 1.9: **Consumption Levels for Employed and Unemployed with** $\gamma = 0.75$**:** The left panel of this figure presents optimal consumption for *unemployed* households under two different realizations of labour earnings shocks. The right panel of the figure presents optimal consumption for *employed* households under two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

Precautionary savings motives for high earnings households strengthen, since borrowing and defaulting do not appear to be an optimal strategy to maximize consumption utility. For consumption smoothing purposes precautionary savings are less risky and guarantee that no earnings loss will be realized.

In a way, the introduction of this asymmetry dampens the strategic default motives and make default option appealing more in a bad state realization, when a household will have very little to lose and no option to borrow more, due to the model's constraint.

Turning to optimal consumption behaviour we realize that in presence of asymmetric default costs, consumption of high income employed households is on average less compared to the benchmark case of symmetric default costs. The right panel of figure 1.9 shows that consumption is smoother and on average lower compared to the right panel of figure 1.5. The same holds for unemployed households.

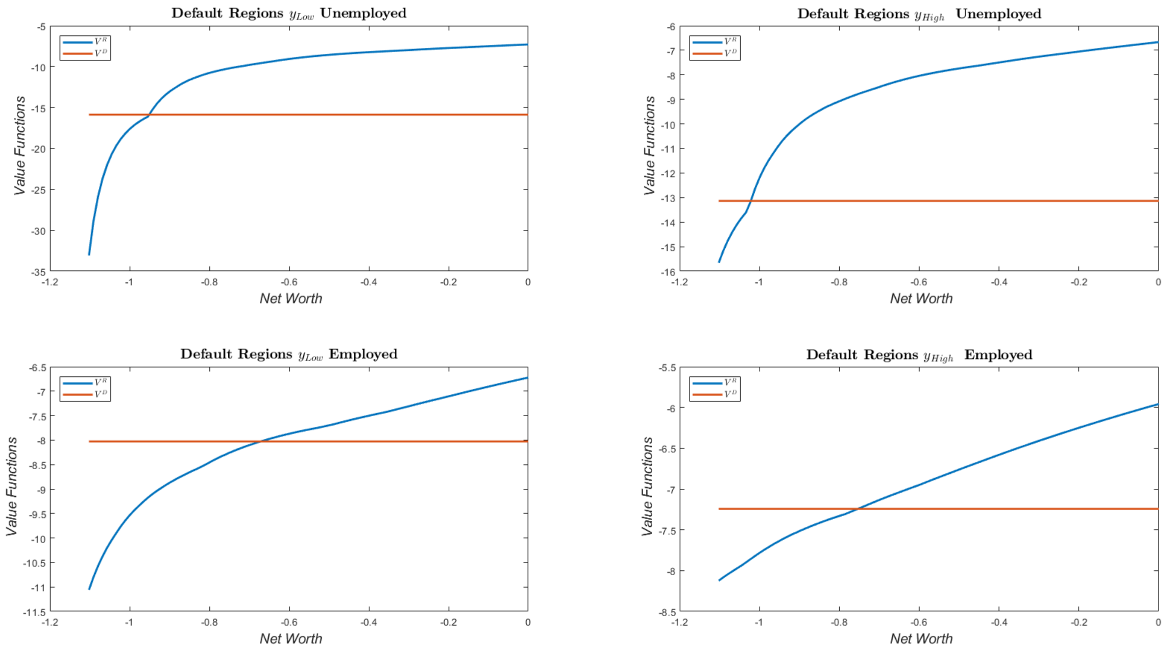

Figure 1.10: **Default Regions for Employed/Unemployed and y_{Low} and y_{High} earnings with $\gamma = 0.75$:** The left panel of this figure presents default regions for an unemployed/employed state when a low labour earnings shock is realised y_{Low}. The right panel of the figure presents default regions for an unemployed/employed state when a high labour earnings shock is realizes y_{High}. To specify the default regions we compare value of repayment V^R to value of default V^D as depicted in the north-west part of all graphs.

However, we should mention that optimal consumption behaviour of low-income or unemployed households resembles closely the results of our benchmark model. This however is not the case for high-income employed households, exactly due to the extra costs being apparent when choosing to default.

Essentially high income households prefer to live in a state of lower consumption, while expanding their savings buffer. The asymmetric default cost is internalized and the post default state features such high costs that becomes undesirable. In this respect, high income households strictly prefer to achieve permanently a lower consumption bundle, rather than enjoy a higher borrowing-boosted consumption temporarily and finally face a permanent earnings loss.

To conclude this section, we present the default regions in figure 1.10. Firstly it is apparent that our benchmark model's defaults result reverts as is depicted from the V^o in figure 1.7. Defaults occur more in the low idiosyncratic state, and as in Arellano (2008) default incentives are stronger the lower the income realization. This serves as a proof that the assumption of asymmetric default cost is very strong and can alter the results.

However, this assumption does not appear to hold in data, especially when studying consumer bankruptcies with unsecured debt. We should further comment that this result in our model is only feasible for very high values of wage garnishment. For this reason we have chosen to calibrate this parameter to the highest possible value that could have an empirical support.

Nonetheless, Chapter 7 bankruptcy which is closely resembled in this chapter does not induce a wage garnishment. This can only be met in Chapter 13 bankruptcy code. Chapter 13 Bankruptcy code provides debtors an opportunity to save their assets (i.e. houses) from foreclosure, by allowing to enter a new payment plan. During this payment plan a portion of your income is garnished as a contribution towards your repayment instalments.

1.7.4 Properties of the Model Economy

In this section we discuss the equilibrium outcome of our benchmark model. It matches fairly well key statistics of the United States economy. Table 1.4 reports statistics derived from our model economy and compares with their counterparts in the data. As we can observe our model successfully replicates some macro and distributional statistics[26].

To motivate the importance of (1.) defaults (2.) NMAR earnings process and the (3.) the employment-unemployment risk, we also report equilibrium values

[26] Most of the U.S statistics used for comparison with our model results are averages. In particular, capital-to-output ratio denotes the average from 1929-2014, consumption-to-output-ratio denotes the average value from 2006-2014, Negative and Non-Positive Net Worth Households are findings provided by the Federal Reserve Bank of New York, while these findings defer slightly across different surveys (i.e. SCF,PSID and SCE), Wealth, Income GINI and other measures of inequality are reported from quadrini rios rull .... Default Rate stands for the value calculated by the Research and Statistics Group-Microeconomic Studies of Federal Reserve Bank of New York. This value denotes the 90+ day delinquency rates for credit cards. Finally, Households with Bad Credit Record are households with very bad credit scores, specifically < 620, as calculated by the Federal Reserve Bank of New York using the Equifax Risk Score. The value reported is the average between 2006 and 2014.

for three other models for comparison. The last three columns of the table report these results. The first model that we use for comparison is our benchmark model with simple Gaussian AR(1) shocks instead of the NMAR process but with employment risk. The column titled Model-AR(1) reports the obtained equilibrium values. It is clear that compared to the benchmark model, this model performs poorly in matching U.S. data. The capital to output ratio is much lower while the consumption to output ratio is a lot higher. Importantly, this model over-predicts the mass of agents who have negative networth compared to the data. This is primarily because in equilibrium the default rate is an implausibly high value of 30%. Examining the results of two other models without defaults, reported in the last two columns, it is clear that the presence of defaults is a necessary ingredient to matching U.S. data statistics. For both the no-default models, the capital to output ratios are significantly higher. The absence of default implies that there is no loss of assets or in other words capital in these economies. Agents accumulate capital for precautionary reasons and this is reflected in higher levels of aggregate savings relative to the economies with defaults. Furthermore, these economies are also characterized by a zero mass of agents with negative networth contradicting data evidence.

Our base model that includes the NMAR earnings process, has employment-unemployment risk and the option of defaults replicates almost perfectly the share of non-positive net worth households in the US. We find that 18.9% of households in our model feature negative or zero net worth.
Across different surveys, this value varies from $18-19\%$. The Survey of Consumer Expectations, being held by the Federal Reserve Bank of New York estimates this value around 18.1%, while Panel Study of Income Dynamics (PSID) reports a value of 19.4%. We further calculate the probability to default in our economy and we compare with the ratio reported by New York Fed regarding credit cards being in above 90 days delinquency. Our model finding is 7.4%, while the corresponding value in data is 7.6%.

We should clarify that credit cards in the above described status serves as a good proxy for defaults in our model, since we are only studying unsecured loans. In data these loans are mainly comprised by credit card loans, student

Table 1.4: The Benchmark Model Economy

Economy	U.S (Data)	Model	Model-AR(1)	ND-NMAR	ND - NMAR+ Emp.
K/Y ratio	**2.9**	**2.67**	1.78	3.17	3.23
C/Y ratio	0.69	0.80	1.01	0.88	0.88
Negative NW (%)	14	11.6	19	--	--
Non-Positive NW (%)	**18.1**	**18.9**	0.49	--	--
Default Rate(%)	**7.6**	**7.4**	30	--	--
Bad Credit Record (%)	**29**	**28.7**		--	--
Net Interest Rate (%)	--	8.97	14	6.17	6.15
Wealth Gini	**0.85**	**0.795**			
Income Gini	0.55	0.64			
Location of Mean Wealth(%)	83	70			
Location of Mean Income (%)	73	77			
Coefficient of Variation in Wealth	**6.35**	**6.25**			
Coefficient of Variation in Income	3.45	4.9			
Variance of the log Income	0.92	3.2			
Variance of the log Wealth	**4.65**	**4.39**			
Mean/Median Income	1.70	2.55			
Mean/Median Wealth	6.42	10			

Notes: This table reports the equilibrium results of our benchmark economy and compares with values of the relevant variables in the U.S. economy. The second column contains the values from empirical data on the U.S. economy, the third column reports values obtained for the main model, the other columns report values for other versions of the model. ND stands for no defaults. The three models used for comparison are (1.) the base model with defaults but with AR(1) Gaussian earnings process instead of NMAR; (2.)no-default version of the model with only the NMAR earnings shocks; and (3.) no-default model with both NMAR and emp. risk shocks.

and auto loans. The corresponding value for student loans with an above 90 days delinquency status is slightly higher.

From a distributional aspect our model underpredicts the wealth GINI index and over-predicts the income Gini index. Our model generates a wealth gini of 0.795 while its counterpart in data is 0.85 as reported in Quadrini and Ríos-Rull (2015). Our model over-predicts the income GINI by around 16%. This is primarily due to the assumption that interest rate is the same for both borrowing and lending activities. Since the total income for any agent in our economy is the sum of exogenous labour income and income from asset returns, agents who accumulate savings will end up getting higher returns on their capital and agents who are indebted will face high repayment costs. The net effect is that this constant return assumption adversely impacts the income distribution with bigger tails thereby over-predicting the income GINI Index. In the second chapter, we will introduce a financial intermediary to price loans differently from deposits and we observe that the Income GINI Index drops down to 0.50. It is clear that the highly persistent ($\rho = 0.99$) exogenous NMAR process might accelerate this mechanism but is not the main driving process.

We also report other types of wealth and inequality measures following the literature, see Quadrini and Ríos-Rull (2015). These measures include the percentage location of mean wealth (income), the coefficient of variation in wealth (income), variance of log wealth (income) and mean/median wealth (income). The observed pattern is similar to the GINI indexes. Our model in general captures better the inequality measure of wealth, namely the coefficient of variation and variance of logs, than the measures of income.

Importantly, the model delivers that 28.7% of households have a bad credit record. Bad credit record in our model suggests that household have at least defaulted once. In other words we measure the mass of households being constrained in equilibrium, namely do not have full access to financial markets. We find that these households account for 28.7% of the population. The empirical equivalent of this finding is households that feature bad credit score. Specifically we compare our result with households that have a credit score below 620. It is fair to assume that these households do not have full access to credit market or at least have very limited access to it. In data this households with a bad credit score represent on average 29% of the population. Bad credit households are the mass of agents in the constrained state (the sum of constrained probability distribution) which is an endogenous equilibrium outcome and do not arise from any calibrated targets. Similarly, negative net worth households are calculated as the total measure (sum) of the distribution of households who have a net asset position that is lower than zero, which is again an equilibrium outcome and not a calibrated target.

The underlying assumption that assists us capturing these statistics is that we have only one interest rate. In this manner we assume that there are no transactions costs and no financial intermediary is present. In this case, households borrow from one to each other, depending on their net asset position and market price. Our model delivers an interest rate of 8.97%.

For equal interest rates on debt and assets, households will always accumulate as much debt as possible. Thereby, households are characterized by strong precautionary borrowing motive, while precautionary savings channel is not shut-down especially for households that face high labour income shocks. In a simpler way, we could argue that interest rates are not high enough to discourage borrowing and not low enough to deter households from savings.

A feature that is absent in our model is financial intermediaries. We assume that interest rate is the same for both borrowing and lending unlike Chatterjee et al. (2007) or Li and Sarte (2006) who explicitly model a risk neutral zero profit financial intermediary that charges a premium to risky loans ensuring that savings are safe.

A common interest rate for all households would imply that there is a cross-subsidization effect, wherein the highly indebted agents are subsidized by agents with low debt. This consequently creates a higher proportion of agents with negative net worth in equilibrium. The market clearing interest rate that we obtain is thus higher than an equivalent Aiyagari economy absent any default choice.

To acquire a better illustration of the distributional properties of our model we show in figure 1.11 the share of aggregate mean wealth that each quintile of households holds. Our model successfully captures the percentage of mean net wealth in the first and second quintile, and underestimates the share of mean net wealth in the fourth and fifth quintile.

According to U.S. data the first quintile of households holds negative net wealth representing -0.01 of the total mean net wealth, while the second quintile holds slightly positive net wealth representing -0.008 of total mean net wealth. The values generated from our model, as can be observed in 1.11 are -0.02 and 0.01 respectively.

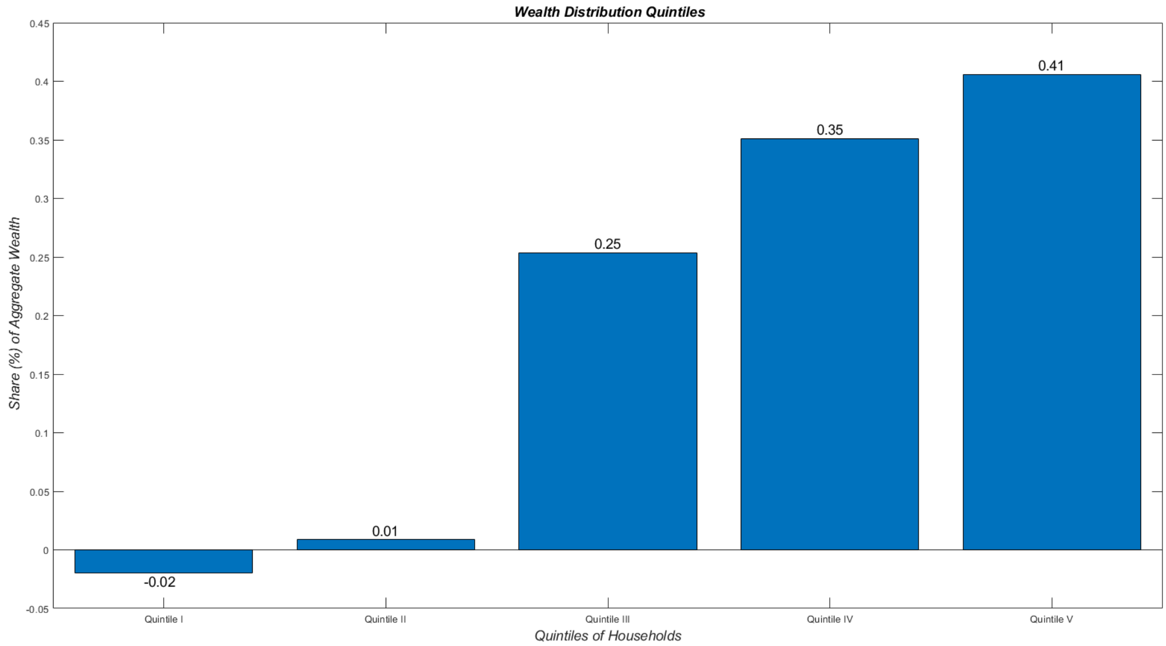

Figure 1.11: **Wealth Distribution Quintiles:** This figure presents the quintiles of the wealth distribution generated by our model. Values depict the share of aggregate wealth held by each quintile of households in our economy.

Nevertheless, we fail to capture the share of net wealth in the third, fourth and fifth quintiles. In data, these quintiles are respectively, 0.033, 0.10 and 0.86. The fact that our model captures well the lowest 40% of agents and not the top 60% is not surprising. In data, the portfolio holdings of positive net worth household include not just savings but also other financial assets. This would mean that the actual wealth of our model economy agents would be comparatively lower. However, for the negative net worth households, who have a higher debt relative to assets, this difference between our model and data does not apply. These agents do not possess any assets neither in the data nor in our model.

In figure 1.12 we present the net wealth histogram as a fraction of average households labour income. We exclude the right tail which comprises about 20% of the population. For households with bad credit record the model delivers a distribution which is right skewed and features a peak at zero, while a significant mass of household is located around zero.

This is reasonable since these households have defaulted and are consequently excluded from borrowing. Additionally, those are mainly households hit by bad labour income shocks or are unemployed. In this sense their labour income is not sufficient to increase their savings. The distribution of households with bad credit record is very similar to the findings of Chatterjee et al. (2007) that construct a much richer model, characterized by a schedule of loan prices rather than a single price.

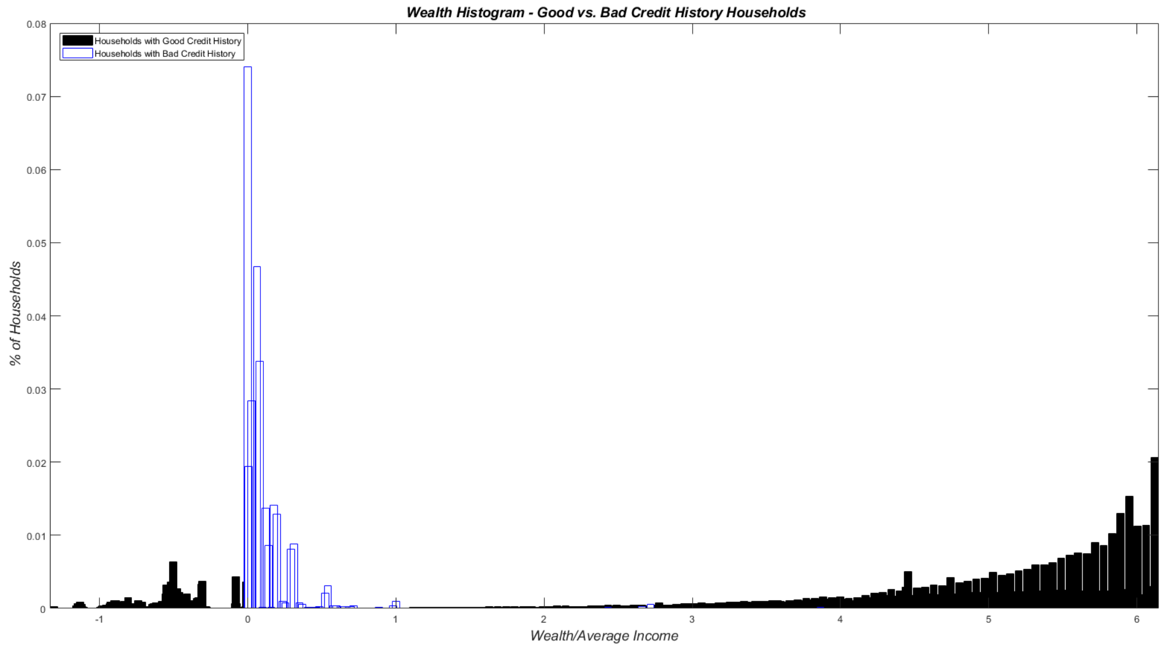

Figure 1.12: **Wealth Histogram - Good vs Bad Credit History Households:** This figure depicts the net wealth to average income distribution of households with Good Credit History (i.e. No Default) and the distribution of households with Bad Credit History (i.e Default). On $x-axis$ we express net wealth as a ratio of average income, while on $y-axis$ the mass of households at each level of wealth is depicted. Note that households in our economy have measure 1.

On the contrary our model implied distribution for households with good credit record, or equivalently households that are unconstrained is different from Chatterjee et al. (2007). In our model these households are mainly those facing a high labour income shock and are employed. Firstly, due to the fact that in our model there exists a common interest rate for borrowing and lending, that in equilibrium is relatively high, specifically around 9%, households with high disposable income are willing to save heavily and their optimal savings frequently meets the bound. This provides a good explanation regarding the fact that a significant mass of unconstrained households are characterized by relatively high wealth as a fraction of average income. We further observe in figure 1.12 that a significant mass of unconstrained households feature negative net worth. In particular these households represent 11.6% of the total population in our economy.

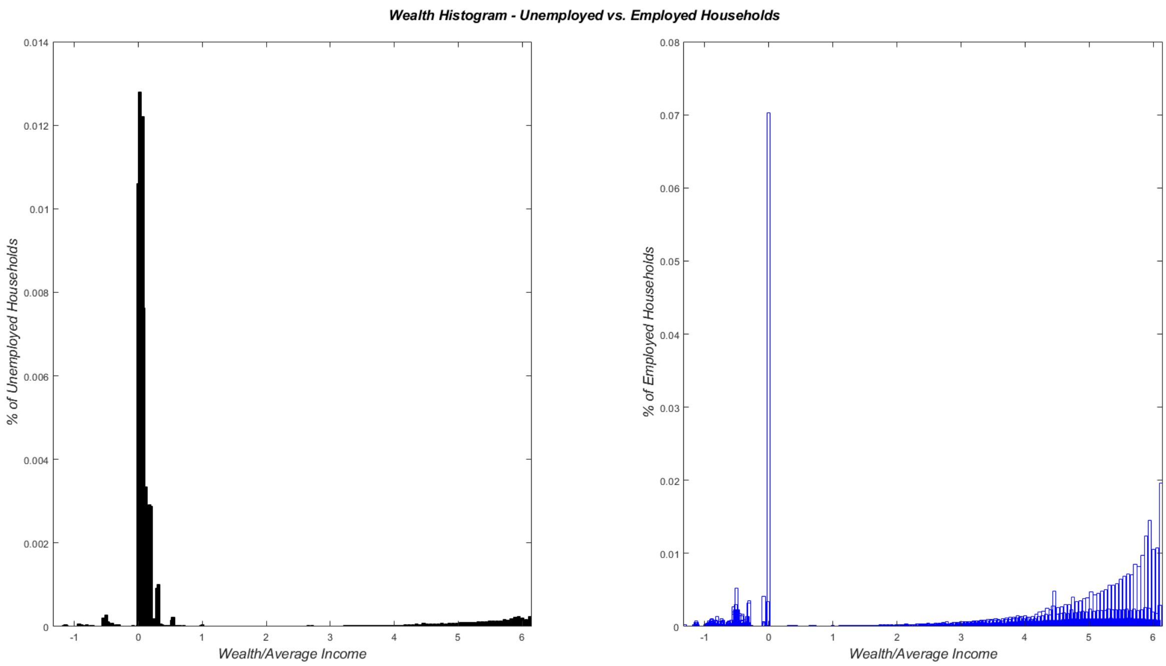

Figure 1.13: **Wealth Histogram - Unemployed vs Employed Households:** This figure presents the net wealth to average income distribution of employed (right panel) and unemployed (left panel) households. On $x - axis$ we express net wealth as a ratio of average income, while on $y - axis$ the mass of households at each level of wealth is depicted.

This value is very close to findings across different surveys in United States. The range of these values in data is between 12% and 14%. Overall we could argue that our model successfully replicates the left part of the distribution of households, along with the mass of households with bad credit record while fails to generate accurately the right part of the distribution.

As previously explained this result arrives due to our assumption of one common interest rate. Lastly we present figure 1.13 which describes the wealth distribution as a ratio of average income among unemployed and employed households. In the left panel of figure 1.13 is evident that unemployed households, as expected do not feature high level of wealth and they are mostly concentrated around zero and slightly positive or slightly negative net worth. Note that the unemployment rate that our model generates is 7.25%. This explains why a smaller mass of households is represented in the left panel of figure 1.13.

Employed households feature much higher disposable income. In that respect, and given the appealing equilibrium interest rate, they are able to accumulate more wealth. Indeed, we observe in figure 1.13 that a significant mass of households lies in the right side of employed household's distribution. Furthermore, a considerable mass of employed households lies at zero. This can be partly explained by the presence of default option. A defaulting household is forced to move to zero net worth, while also facing an exclusion from credit market. In the degree that default arises as an equilibrium outcome, it is anticipated that a considerable mass of households will lie at zero level of net worth.

1.8 Conclusion

In this chapter, we accomplished the following results. Firstly, we developed a theoretical framework that captures salient features of wealth distribution, consumer bankruptcy and the prevalence of a significant number of negative net worth households. The existence of a substantial mass of negative net worth households explains the rise in wealth inequality observed in the aftermath of the crisis in US.

Secondly, we introduced a novel feature in our model wherein indebted households are allowed to default on a part of the debt, specifically the outstanding debt that exceed their liabilities. Consequently, endogenous equilibrium defaults are partial and not full.

Thirdly, our benchmark model in contrast with the literature but consistent with recent empirical evidence produced lower default probabilities for low income states. Fourthly, our results bring into question the importance and relevance of the asymmetric default costs in this class of models. Our analysis indicates this use is mostly a convenient approach to generate low defaults for high income agents.

Finally, our model includes an additional idiosyncratic state (employment vs. unemployment) and applies a mixture of autoregressive process that matches higher order moments of the earnings distribution, two technical features that are absent in most of the heterogeneous agents literature.

Appendix

1.A NMAR Process

This appendix discusses the discretization method of calibrating a markov process with non zero skewnesss and high kurtosis. Civale et al. (2016) extends Tauchen (1986) AR(1) discretization to normal mixture autoregressive processes and thereby capture the skewness and high kurtosis observed in earnings data.

Bloom et al. (2018), Guvenen et al. (2015) and Bachmann et al. (2017) find empirical patterns that cannot be captured by a simple AR(1) process. On the other hand, standard economic models that attempt to mimic the data patterns stick mostly to Gaussian shocks and Tauchen (1986) discretization method. Our model is an attempt to fill this gap in the literature by using a rich NMAR earnings process in a modified Aiyagari type model where borrowing is defaultable under different pricing kernels.

A first order autoregressive process with normal mixture innovations (NMAR) is given by the following equation:

$$y_t = \rho y_{t-1} + \eta_t \tag{1.17}$$

where

$$\eta_t \sim \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2), & \text{with probability } p_1 \\ \mathcal{N}(\mu_2, \sigma_2^2), & \text{with probability } 1 - p_1 \end{cases} \tag{1.18}$$

In the above process y denotes log-earnings, ρ the persistence of the log-earnings process, and η_t stands for the Gaussian innovations. The moments of y are determined by ρ and η. In the first step of the exercise, the value for η is calibrated using the General Method of Moments (GMM). Civale et al. (2016) finds that the variance of η is independent of the skewness and kurtosis of η. This allows Civale et al. (2016) to produce the pair of values for skewness and kurtosis of η. They report different combinations of skewness and kurtosis of η and compare them with their empirical counterparts reported by Bloom et al. (2018), Guvenen et al. (2015) and Bachmann et al. (2017).

In the second step, they select different values of ρ so as to be able to map the pair of skewness and kurtosis of η to the observed skewness and kurtosis of the log earnings data by Guvenen et al. (2015) and Bloom et al. (2018). They report results for different values of $\rho = \{0.8, 0.9, 0.99\}$.

There are six parameters $(\rho, p_1, \mu_1, \mu_2, \sigma_1, \sigma_2)$ that govern the NMAR process. Civale et al. (2016) fix $\rho = 0.99$, $p_1 = 0.9$ while imposing that $\mu_2 = \frac{-p_1\mu_1}{1-p_1}$ to ensure that $\mathbb{E}(\eta) = 0$.[27]

Hence the remaining three parameters $(\mu_1, \sigma_1, \sigma_2)$ are left free to match the targeted data moments of Guvenen et al. (2015). In our model, the NMAR calibration that they find is used. This is reported in 1.2.

We now describe in brief the method that they use to discretize this NMAR process. They call it the Extended Tauchen (ET) method. A set of moments are first chosen from the data, Civale et al. (2016) uses Guvenen et al. (2015) earnings moments. The moments of the NMAR process conditional on an assumed set of parameters are then mapped to these data moment targets. A distance measure is minimized with an appropriate weighting matrix to find optimal parameters. The weighting matrix is chosen such that the distance between the model moments and the data moments is equal to the sum of squared percentage deviations of each moment from its target. The deviations are thus percentages and scale independent. Each moment condition is weighted equally. The moments of the markov chain can be analytically calculated and this allows easier mapping from the continuous process to the data targets.

Detailed description of this approach including the analytical moment equations and the process of discretization can be found in Civale et al. (2016) appendix.

[27] This is because the unconditional expectation is $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$.

Chapter 2

Distributional and Default Implications from Risk-Neutral Pricing

2.1 Introduction

In this chapter we resolve some of the limitations put forward by the model introduced in our first chapter. Similar to our previous chapter, we analyse a general equilibrium model that allows for unsecured consumer credit, in the presence of defaults. Defaults in this model, unlike the previous one, closely resemble the Chapter 13 bankruptcy law in the United States.

Furthermore, we explicitly model a financial intermediary that gives loans to households that are willing to borrow and takes deposits from households that are willing to save. The financial intermediary makes zero profits and prices loans based on *three key factors*. Firstly, prices depend on the size of the loan as in the paper of Chatterjee et al. (2007). Secondly, we extend this approach by allowing prices to depend on two more *important factors*, which are the idiosyncratic earnings realization and the employment-unemployment status.

Similar to our previous chapter, where we successfully tried to match the higher moments of the earnings distribution in United States, we employ the discretization technique of NMAR processes, proposed by Civale et al. (2016) to match the moments of the earnings distribution found by Guvenen et al. (2015).

Our economy is characterized by limited commitment and thus markets are *endogenously incomplete*. The main question that we pose in this chapter is the following:

Do the presence of a financial intermediary alter household decisions and macroeconomic aggregates?

In our previous chapter trade was taking place amongst households via a one-period not contingent bond and price does not depend on the size of borrowing, even though a large loan would reasonably be linked with a higher probability to default compared to a smaller loan. Under this assumption, a cross subsidisation effect comes forward, that is, small (loan size) borrowers subsidize large borrowers.

Furthermore, the underlying assumption of our previous chapter was that borrowing and savings induces the same price. Apart from the unrealistic nature of this assumption, from a theoretical perspective we found that this directly drives defaults incentives through an over-attractiveness of the outside option. To overcome these shortcomings, we use a risk neutral pricing approach, modelled via the existence of a financial intermediary.

We find that this *market arrangement* is important in *multiple ways*. Credit suppliers, represented in our economy by financial intermediaries price their loans taking into account the probability of default. An extra refinement in our model is that probability of default and the loan price schedule is dependent on the current states of employment and earnings and importantly on the amount of borrowing. In this manner, financial intermediaries are compensated for the objective default frequency of households with these certain state realizations.

2.2 Contribution and Results

The main contribution of this chapter is in the way the risky loans are priced. We have an endogenous loan price schedule extending the single price assumed in Li and Sarte (2006). Thus, we account for the employment/unemployment state of the agents and his/her earnings realizations. The presence of this loan schedule means that our endogenous bankruptcy filings are almost the same as in the data. Our approach of pricing is thus closer to Chatterjee et al. (2007). The difference in our model relative to Chatterjee et al. (2007) is in the assumption that this loan price is conditional on the earnings process and the employment status of the agent.

We turn now to the findings of this chapter. We answer the above posed question with an emphatic ***Yes, the presence of a financial intermediary results in lower defaults and a more realistic income distribution***. Although the wealth GINI is under-predicted, the top quintiles of the aggregate wealth share is now closer to empirical data than the previous chapter.

Our model generated distribution resembles much better the quintiles of the U.S. empirical wealth distribution. In particular, the fifth quintile of our model generated wealth distribution holds 67% of total wealth as opposed to around 85% of its data counterpart.

Furthermore, the model generated income inequality is much closer to the observed income inequality in US. More specifically, our model generates an income Gini index of 0.50 with its data counterpart being 0.55.

We now turn to our results related to defaults. Firstly, our model closely resembles the bankruptcy filings in United States in the period of $2009-2010$. Our model generated filing rate is 0.64% while its empirical counterpart is 0.65%. This finding is also of a theoretical interest in the following way.

The presence of a financial intermediary that takes into account the probability of default on every credit given and optimally prices these contracts based on the idiosyncratic income and employment level of the borrower results in a lower default rate in equilibrium. This is despite the fact that borrowing and hence total debt is higher compared to a no intermediary economy. This further highlights the importance of appropriate screening, from the side of financial intermediaries in reality. Mispricing of loans or miscalculation of default probabilities might bear serious consequences for the economy. Price is a crucial determinant of default incentives.

In this chapter we also highlight the link between price schedules and the attractiveness of the outside option, claiming that these are the major drivers of default incentives. We find that for some levels of debt and some idiosyncratic states, given the risk-free interest rate in our economy, that is relatively low, households prefer to borrow and repay, rather than borrow, default and then exercise the outside option. This is because in an economy characterized by low deposit rates, borrowing incentives exceed savings incentives, thus default become a less appealing option, and economy continuously accumulates debt.

This mechanism also sheds light on the current increase in household debt observed in different countries, and mainly in the United States. This has raised major concerns amongst the policy makers, since along with the fact that around 15% of households in the U.S. have negative net worth, might become a triggering factor for a next crisis. Our model successfully replicates the fraction of households with negative and non positive net worth and also delivers a high for the standards of general equilibrium heterogeneous agents models, household debt to GDP ratio, around 9.8%.

The rest of the chapter is organised as follows. In Section 2.2 we review the literature related to asset pricing, in various theoretical models. In particular we discuss asset pricing in a representative agent economy, asset pricing in heterogeneous agents economies, as well as the link between asset pricing and defaults. We then describe our model economy in Section 2.3, presenting the market structure and the households' problem, for both the *constrained* and the *unconstrained* group. In Section 2.4 we describe the problem and the role of the financial intermediary in this economy and we present the results of its first order conditions. In Section 2.5 we provide a brief description of the stationary recursive competitive equilibrium and state the market clearing conditions for our economy. We discuss the calibration strategy in Section 2.6. In Section 2.7 we present the key results obtained from the computational solution of the model, and we discuss the characteristics of the economy in equilibrium and the related model statistics. We conclude this chapter in Section 2.8.

2.3 Literature Review

In this section we will review the literature related to this chapter. Firstly we discuss studies which are relevant to the notion of asset pricing in general and in particular in the case of a representative agent economy.

We then turn to asset pricing in models where agents are heterogeneous. We provide useful explanations introduced by the literature related to the pricing kernel and its difficulties in such model environments.

Finally, we present the part of the literature which is closely related to this chapter, that is, literature in asset pricing in the presence of default options. We discuss both models related to consumers' bankruptcy and also to sovereign defaults.

2.3.1 Representative Agent Asset Pricing

The seminal paper of Lucas (1978) provides useful insights in the notion of asset pricing in an *exchange economy* with a representative agent and examines equilibrium prices. Lucas (1978) derives a functional equation, describing the dependence of prices on the marginal utility of consumption, in the so called Consumption based Asset Pricing model.

The underlying assumption in this economy is that prices reflect all available information in the economy. In other words, Lucas (1978)'s study is within the framework of *rational expectations* as defined by Muth (1961). The advantage of a simple representative agent model is that equilibrium quantities can be easily calculated. Thus, the only remaining challenge is to determine equilibrium prices. He suggests that price should be expressed as a function of the state of the economy.

On the one side, individual consumers take prices as given to solve their maximization problem, deciding optimally their policy functions. On the other side, this optimal individual behaviour results to the market clearing conditions of the economy, which consequently determine the price. In a simpler way, there exists a bidirectional relation, where the market clearing price is consistent with the individual decision rules, and decisions rules are consistent with the market clearing price.

This result can become more comprehensible by the means of an example. We pose the following question: How one can price a one period risk free bond, within the representative agent framework? The pricing equation indicates how price changes in response to expected growth of consumption. In general agents would be willing to borrow more to increase their consumption level and prices would depend on the inverse of the elasticity of intertemporal substitution. With high elasticity even small changes in expected growth would trigger large changes in equilibrium price. The pricing kernel in this case will be expressed as follows:

$$m_{t+1} = \beta \frac{u'(c_{t+1})}{u(c_t)} \tag{2.1}$$

where m_{t+1} denotes the pricing kernel.

We have analysed the case of a risk free asset. However, in real life most assets are risky. Thus, it is interesting to know how price would be adjusted, when the asset in the economy is for instance a one period defaultable bond. In this case price can be expressed as follows:

$$q_t = \beta \mathbb{E} \frac{u'(c_{t+1})}{u(c_t)} d_{t+1} \tag{2.2}$$

where d_{t+1} denote the payouts.

This equation can be generally transformed as follows:

$$q_t = \frac{\mathbb{E} d_{t+1}}{1 + r_f} + \mathrm{cov}(m_{t+1}, d_{t+1}) \tag{2.3}$$

where the second term captures the covariance of marginal utility and payouts. Note that in the limited case of *risk neutral agents* this last term disappears and consequently price is given only by the first component.

However, following the seminal work of Mehra and Prescott (1985), who suggested that the pricing implications of the representative agent general equilibrium model are not consistent with observed U.S. data, economists realized that an alternative modelling approach would be necessary. This puzzle that Mehra and Prescott (1985)'s study revealed is the so called *"equity premium puzzle"*. Essentially, this puzzle can be summarized in the following question. Why stocks feature such a higher return if they are not so risky?

Our inability to address this question in a representative agent economy, led us to the conclusion that a departure from the frictionless Arrow-Debreu setup is indispensable. In particular, heterogeneous agents models equipped with liquidity or borrowing constraints, transaction costs etc. appear to be the appropriate workhorse models. But how does the pricing kernel look like when heterogeneous consumers populate the economy?

2.3.2 Heterogeneous Agents Asset Pricing

Asset pricing difficulties frequently arise in heterogeneous agents models, due to the non-tractability of the pricing kernel. Only in some cases and under specific assumptions the pricing kernel can be analytically pinned down and prices can consequently be easily calculated.

The fundamental building block of finance is that financial markets do not allow for any arbitrage opportunities that are not risky. Following this assumption and the contributions of Ross (1976) and Harrison and Kreps (1979) one can argue that there exists a positive random variable m_t which satisfies the following moment condition:

$$p_t = \mathbb{E}(m_{t+1}X_{t+1}) \tag{2.4}$$

where p_t denotes a vector of asset prices and X_t represents a vector o payoffs, on a set of assets. As explained in our previous discussion m_t is known as the pricing kernel.

The common characteristic of the pricing kernel amongst all the heterogeneous agents models is that it is agent specific. In particular, it is given by the agent specific intertemporal marginal rate of substitution. Telmer (1993) explains that if individuals are at interior solutions to their allocation problems then this kernel should satisfy equation 2.4.

For instance, Telmer (1993) studies asset pricing in a complete market heterogeneous agents economy and shows that individuals are able to diversify their risk by riskless borrowing and lending. He further suggests that when restrictions are applied in the pricing kernel, incomplete markets cannot explain the properties of abnormal asset returns.

Duffie et al. (1994) state that the joint hypothesis of incomplete consumption insurance along with agents' heterogeneity can enrich the implications of the representative-agent model. A few studies supported the argument that economies with uninsurable income risk and borrowing, can possibly replicate or come very close to the case of complete-risk sharing.

Aiyagari and Gertler (1991) found that consumers in economies with uninsurable risk could replicate the complete markets perfect risk sharing, even when one accounts for transaction or other borrowing costs. However, Duffie et al. (1994) suggested that when one relaxes the stationarity assumption of labour to aggregate income, and without the use of borrowing constraints, transaction costs, or unrealistic assumptions on bond supply, could change the implications of a representative agent economy.

Lucas (1994) finds that when asset markets are closed entirely, the model can replicate both the risk free and the equity premium. More findings of her study suggests that asset prices will be similar to the representative agent economy. We should note that the paper examines the above discussed ideas in an infinite horizon model.

All the discussion of asset pricing in different contexts and alternative model frameworks is of great interest, since it has major applications in different research fields in economics. However, and as noted before, finding the general equilibrium price, or pinning down the pricing kernel is not a trivial task in heterogeneous agents and always requires a lot of assumptions. Frequently to face these difficulties, economists are forced to use computational techniques. For instance, in the previous chapter we have used an iterative bisection method to solve for the general equilibrium interest rate, since no closed form expression could be available.

More importantly, in the particular class of models which are related to defaults or consumer bankruptcies the structure of the markets along with the pricing equation, can significantly alter the generated model results. In the following section we present a few studies with alternative pricing approaches along the lines of this literature.

2.3.3 Asset Pricing and Defaults

Krueger and Perri (2006) addresses the relation of consumption and income inequality. Their analysis takes place in a framework in which agents can trade a full set of contingent consumption claims, while facing endogenously arisen constraints due to limited enforcement. Since households in this model trade a full set of Arrow securities and markets are complete, prices are state and time dependent, that is, they depend on the labour endowment history and time, reflecting changes in the income process. Since in their framework assets are Arrow securities, that is, state contingent zero coupon bonds, the non arbitrage condition implies that households should be indifferent between saving in risk-free capital or constructing a risk-free asset by using the full set of Arrow securities. In their framework price is described by the following relation:

$$q_t(y^t, y_{t+1}) = \frac{\pi(y_{t+1} \mid y_t)}{1 + r_{t+1}} \tag{2.5}$$

where $\pi(y_{t+1} \mid y_t)$ denotes the transition probabilities of the Markov chain related to the idiosyncratic part of the labour endowment process.

However, when financial markets are incomplete, agents cannot expunge completely the idiosyncratic risk (see Alvarez and Jermann (2000)). Due to this reason, *pricing kernels feature higher volatility* compared to complete markets. Thus, these models strongly depend on the available assets and they have limited tractability, since fixed point theorems become very complicated when an infinite number of agents and a lot of assets are present.

Furthermore, Alvarez and Jermann (2000) explain that in model economies that agents have the option to default, risk sharing might be reduced since agents with low income, will only borrow at most the amount that they will be able to repay. Due to the introduced solvency constraints, default does not arise as an equilibrium outcome. More importantly under their assumptions and the resulting equilibrium definition, prices of securities are characterized by a simpler and more intuitive representation.

An important property of their pricing kernel is that one period contingent claims are *priced by the agent with the highest marginal rate of substitution*. Prices can be described by the following relation:

$$q_{a,t}(z^t, z') \equiv \max_{i \in I} \beta(z) \frac{u'(e_{i,t+1}(z^t, z'))}{u'(e_{i,t}(z^t))} \pi(z' \mid z_t) \tag{2.6}$$

This agent is essentially the non-constrained agent. In this manner, the price of an Arrow security equals to the highest marginal valuation amongst all the agents. Alvarez and Jermann (2000) finally show that interest rates are necessarily lower when solvency constraints are present.

Arellano (2008) extends the approach of Eaton and Gersovitz (1981) and develops a stochastic general equilibrium model with endogenous default risk. Her model studies default events in a representative agent framework and loans are supplied by international risk neutral lenders. Risk neutral pricing implies that lenders break even in expected value in every bond contract. In particular, lenders choose the amount of loans they will supply taking prices as given. The price of a discount bond in this framework is equal to the risk adjusted opportunity cost. This reflects the risk neutral compensation for a lower expected payoff. Price in this environment depends on the size of new borrowing and the aggregate state. In this manner price can be expressed in the following form:

$$q(B', y) = \frac{(1 - \delta(B', y))}{1 + r} \tag{2.7}$$

where δ stands for the default probability that similarly to price depends on the size of the loan and the aggregate state.

Different pricing approaches are also adopted in models that study consumer bankruptcies. For instance Li and Sarte (2006) evaluates the effects of a reform on bankruptcy legislation in United States. They use a general equilibrium model with production where default option is available to households and markets are incomplete. They suggest that the complete elimination of bankruptcy would induce welfare losses and output decline.

In Li and Sarte (2006)'s environment households can hold positive and negative assets, or more specifically they can have deposits and take loans. Default on borrowing is strategic and the interest rate charged on debt it is assumed that features a risk premium which prices the default probability. Contrariwise, deposits are safe and risk free. Furthermore, the presence of financial intermediary that breaks even guarantees that an equilibrium with a positive endogenous risk premium exists.

One drawback of their approach is that financial intermediaries price every loan with the same price. Loan prices or alternatively their endogenous risk premium does not depend neither on the size of the loan supplied nor on the idiosyncratic income state.

Along the lines of the consumer bankruptcy literature, Chatterjee et al. (2007) study a general equilibrium economy where households smooth consumption using unsecured loans and a riskless asset. Financial intermediary is also present in their economy, although the problem that it solves is different compared to the economy of Li and Sarte (2006).

In Chatterjee et al. (2007)'s model, price is a function of the size of the loan and some ex-ante individual household characteristics. Financial intermediaries choose the number of loan contracts that will supply to households and they also process capital that they rent to firms. Interestingly, in their model, price does not depend on the idiosyncratic state of labour earnings.

However, in reality interest rates on loans depend on the size of the loan and more importantly on a household's labour income. Furthermore, deposit rate also depends of the size of the deposit. Thus, it is interesting to model these features in a general equilibrium production economy where default option is present.

In what follows, we try to bridge this gap in the existing literature by implementing an Arellano (2008) type pricing, where in an analogous way even if the framework is different bond prices also depend on the aggregate state. Her model equivalent, adjusted to the heterogeneous households framework, would require price to depend on the idiosyncratic income state.

2.4 The Model Economy

Our model *builds on the model of our previous chapter*. In particular, we employ an alternative way of asset pricing. We assume the presence of a financial intermediary in the economy, that prices loans based on the size of a loan similarly to Chatterjee et al. (2007). We extend the approach of Chatterjee et al. (2007) by further allowing the price to depend on the idiosyncratic earnings state. This implies that each loan is priced individually, based on its size and the current realization of the idiosyncratic earnings shock.

Time in our model is discrete indexed by $t \in (0, 1, \ldots, \infty)$. Economy is populated by a continuum of ex-ante identical infinitely lived households that are distributed uniformly over $[0, 1]$. In the beginning of the period all households are identical, since no ex-ante heterogeneity is present in our model. All households face two types of risks, that is, unemployment risk and also conditional on being employed they face acyclical earnings risk[1]. When households are unemployed we assume they receive unemployment benefits which are defined as a fraction of their potential labour earnings[2]. Households have time-separable preferences over streams of consumption.

$$U(c_0, c_1, c_2, \ldots) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2.8}$$

where for each period t, c_t is restricted to belong in $\mathcal{C}$, the per period consumption feasibility set of any individual household, which specifies non-negative consumptions. The discount factor $\beta \in (0, 1)$ is common across all households. The period utility function $u(c_t)$ is continuous, concave, strictly increasing, continuously differentiable and satisfies the Inada conditions.

Households do not value leisure, and the time endowment is normalized to 1. Since households face unemployment risk, we denote their employment status by

[1]More details regarding unemployment risk can be found on Krueger et al. (2016) and also our previous chapter.

[2]This stems from the assumption that if a government was present in our model it would implement a balanced budget unemployment insurance system.

$s \in \mathcal{S} = \{u, e\}$, where u denotes unemployment and e denotes employment. Let $\pi(s' \mid s)$ denote the transition of probabilities of the associated Markov chain. We should note that these probabilities are iid across households.

Furthermore households face idiosyncratic labour earnings risk $y \in \mathcal{Y}$ that follows a 12-state first order Markov process, with *finite support* $\mathcal{Y}$. Let $\pi(y' \mid y)$ denote the transition probabilities from state y today to state y' tomorrow. Note that these probabilities are firstly iid across households and secondly independent from the probabilities associated with unemployment risk. We further assume that a law of large numbers holds, so that our problem is well defined (see Uhlig (1996)).

There is one single good produced in the economy according to an aggregate *Cobb-Douglas production function* that can either be consumed or invested.

$$Y = K^{\alpha} N^{1-\alpha}, \tag{2.9}$$

where α captures the share of capital income in output and $\delta \in (0, 1)$ captures the depreciation rate of capital, when it is used in production.

Furthermore the *aggregate resource constraint* in this economy could be expressed as follows:

$$F(K, N) = C + I = C + K' - (1 - \delta)K, \tag{2.10}$$

where C denotes aggregate consumption, K represents the aggregate capital stock, N is the labour input and $\delta \in (0, 1)$ denotes the depreciation rate of capital when used in production.

2.4.1 The Market Structure

Firstly, markets are perfectly competitive. The real wage is given by w and the rental rate on capital is given by r. Furthermore, households feature borrowing and lending opportunities. They can take unsecured loans of different sizes. Similarly to Chatterjee et al. (2007) we treat these loans as distinct financial assets.

Households can *default* on their loans at any point of time, while default as in our previous chapter is strategic. In this sense, it arises as a result of an optimal household choice.

The default risk *varies with the size of the loan*. Furthermore, it *varies with the labour earnings idiosyncratic state* and the *individual employment* or *unemployment state*. In this manner, we extend previous work in the consumer bankruptcy literature and we differentiate from models as Li and Sarte (2006) and Chatterjee et al. (2007). The model of Li and Sarte (2006) does not allow price to depend on the size of the loan, while the model of Chatterjee et al. (2007) does not allow price to depend on the idiosyncratic earnings state or unemployment state.

Households can take borrowing or saving decisions every period. However, *unlike in our previous chapter* trade does not take place amongst households directly, due to the presence of a financial intermediary. Households can purchase a *one-period non-contingent discount bond* with a face value in the finite set $\mathcal{L} \subset \mathbb{R}$.

We turn now to the description of the *contract types*. A purchase of a discount bond with positive face value ℓ' implies that the individual household has entered into a contract under which it will receive *with certainty* ℓ' units of consumption good next period. In this sense deposits are assumed to be risk-free and guaranteed.

We next discuss the risky borrowing contract. A purchase of a discount bond with negative face value ℓ', individual labour earnings y and employment-unemployment state s indicates that the individual household has entered into contract under which, it will receive $q(\ell', y, s)(-\ell')$ units of consumption good today, promising to deliver, conditional on not defaulting $-\ell'$ units of consumption good tomorrow.

It is important to note that the total financial assets available in the market are $N_{\mathcal{L}} \cdot N_{\mathcal{Y}} \cdot N_{\mathcal{S}}$ where N denotes the cardinality of the associated sets. As we mentioned before trade does not take place amongst households, thus households can only purchase these bonds from financial intermediaries. We further assume that households are indifferent between investing in physical capital, or investing in bonds with positive face value. The underlying assumption of our statement is that unlike in Chatterjee et al. (2007)'s economy, financial intermediaries in our model do not own capital and consequently do not rent capital to firms.

This structure implies that loan contracts are unenforceable, that is, any borrower can default on his loan at any time period. Furthermore, default induces a punishment that similarly to our previous chapter is described as temporary exclusion from the credit market. A defaulted household will regain full access to the credit market, with probability $1-\theta \in (0,1)$ and remain in a constrained state with probability θ.

Furthermore, any household that has agreed on a loan contract with a financial intermediary decides whether to default or not based on the following value functions:

$$V^o(\ell, y, s) = \max\left\{V^R(\ell, y, s), V^D(0, y, s)\right\}, \tag{2.11}$$

where $V^R(\ell, y, s)$ is the value associated with repayment, $V^D(0, y, s)$ is the value associated with defaulting, while $V^o(\ell, y, s)$ depicts the value of the option which is always the maximum.

It is important to mention that households are still allowed to hold deposits after the default event, or in other words they are allowed to save in a risk-free asset. However, following the default event a proportion of a household's labour income will be seized. This could be simply modelled as a *wage garnishment*. This default type resembles Chapter 13 bankruptcy in United States.

Under Chapter 13 bankruptcy law, a repayment plan is developed for around five years. Monthly instalments are scheduled and being supported by a wage garnishment. This garnishment simply acts as wage tax. In reality courts make an estimation of five-year income based on current earnings. For simplicity we assume that the amount of garnishment will be captured by a parameter $\gamma \in (0,1)$. In line with the *Federal Wage Garnishment Law* and the *Consumer Credit Protection Act* III, we assume that the amount of earnings that could be garnished cannot exceed 25%.

Since our model differs only in the pricing approach and the default type, compared to the model used in our previous chapter, we will only briefly describe the household's problem. The reader for further details can refer to section 1.4.7 of our previous chapter.

2.4.2 Household's Problem

The individual household's problem is essentially to maximize expected discounted lifetime utility derived from consumption subject to the associated budget and borrowing constraints.

When households solve their optimisation problem, they take prices as given, that is, the loan price, the wage and the deposit rate. Note that we only study *stationary recursive competitive equilibria*. In this sense the distribution across different states should be invariant, thus policy function will be time invariant.

We will first describe the problem of the *unconstrained households* and the problem of *constrained households*[3] will follow.

2.4.2.1 The Unconstrained Households

The states for an unconstrained household can be described by the level of loans or deposits denoted by $\ell \in \mathcal{L} \subset \mathbb{R}$, the labour earnings draw denoted by $y \in \mathcal{Y} \subset \mathbb{R}^+$ and employment-unemployment draw denoted by $s \in \mathcal{S} \in [0, 1]$.

We denote by $V^R(\ell, y, s)$ the value function of a household that repays his loan in the current period. We denote by $V^D(0, y, s)$ the current value associated with defaulting. Finally,we denote $V^o(\ell, y, s)$ the value of the option to be unconstrained.

Let c be consumption in the current period and ℓ' denote the amount of loans or deposits that a households optimally chooses. Furthermore, w is the household wage, $q(\ell', y, s)$ is the price for a loan or a deposit, while $\mathbb{1}_{s=u}$ is an indicator function which takes the value 1 when household is in the unemployed state[4] and 0 otherwise. Following these definitions, the unconstrained household's problem could be presented as follows:

[3]The terms unconstrained and constrained households are described in detail in our previous chapter. More details can be found in the studies of Li and Sarte (2006) and Athreya (2002)

[4]In this case labour earnings are equal to the unemployment benefits, which are defined as a fraction of the average labour earnings, thus $b(y) = \rho w y$.

$$V^R(\ell, y, s) = \max_{\{c \geq 0, \ell'\}} \{u(c) + \beta \sum_{(y',s') \in (\mathcal{Y}, \mathcal{S})} \pi(s'|s)\pi(y'|y)V^o(\ell', y', s')\} \tag{2.12}$$

subject to

$$\begin{aligned} c + q(\ell', y, s)\ell' &= wy\left[1 - (1-\rho)\mathbb{1}_{s=u}\right] + \ell \\ \ell' &\geq -\overline{B} \end{aligned} \tag{2.13}$$

Let now $V^C(\ell, y, s)$ denote the value function of a household that defaulted on its loan and is currently borrowing constrained. In this case household is only allowed to hold deposits while also facing the wage garnishment. In other words a γ proportion of its labour earnings is garnished, thus the household only receives $1-\gamma$. Following these definitions, the problem associated with the value of default could be presented as follows:

$$V^D(0, y, s) = \max_{\{c \geq 0, \ell' \geq 0\}} \{u(c) + \beta \sum_{(y',s') \in (\mathcal{Y}, \mathcal{S})} \pi(s'|s)\pi(y'|y)V^C(\ell', y', s')\} \tag{2.14}$$

subject to

$$\begin{aligned} c + q(\ell', y, s)\ell' &= [1-\gamma]wy\left[1 - (1-\rho)\mathbb{1}_{s=u}\right] \\ \ell' &\geq 0 \end{aligned} \tag{2.15}$$

2.4.2.2 The Constrained Households

We now turn to the problem of borrowing constrained households, that is, households that have defaulted in some previous period. Recall that we denote by θ the exogenous probability related to the household remaining in the borrowing constrained state[5]. Following these definitions the problem of constrained households could be presented as follows:

$$\begin{aligned} V^C(\ell, y, s) = \max_{\{c \geq 0, \ell' \geq 0\}} \{u(c) + \beta \sum_{(y', s') \in (\mathcal{Y}, \mathcal{S})} & \pi(s'|s)\pi(y'|y)\theta V^C(\ell', y', s') \\ & +(1-\theta)V^o(\ell', y', s')\} \end{aligned} \tag{2.16}$$

subject to

$$\begin{aligned} c + q(\ell', y, s)\ell' &= [1-\gamma]wy\left[1-(1-\rho)\mathbb{1}_{s=u}\right] + \ell \\ \ell' &\geq 0 \end{aligned} \tag{2.17}$$

where c denotes consumption in the current period and ℓ' denotes the amount of deposits that a households optimally chooses. Furthermore, w is the household wage, θ is the probability that a household will remain in the constrained state and $\mathbb{1}_{s=u}$ is an indicator function which takes the value 1 when household is in the unemployed state and 0 otherwise.

[5]The term borrowing constrained in this framework is related to no access to credit markets, or in a simpler manner no ability to get loans.

2.5 The Financial Intermediary

The financial intermediary in this economy is closely related to Chatterjee et al. (2007)'s financial intermediaries with the difference that price also depends on the idiosyncratic earnings and unemployment shocks (y, s). The intermediary chooses the number of contracts $n_{\ell_{t+1},y_t,s_t} \geq 0$ of type (ℓ_{t+1}, y_t, s_t) to sell in each period, so to maximize the present discounted value of current and future cash flows. The maximisation problem can be presented as follows:

$$\max \sum_{t=0}^{\infty} \frac{1}{1+r-\delta} \pi_t \tag{2.18}$$

where π_t denotes profits at time t. Every period cash flow is given by the following equation:

$$\begin{aligned} \pi_t = \ell^+_{t+1} - (1+r-\delta)\ell^+_t + &\sum_{\ell_t,y_{t-1},s_{t-1}} n_{\ell^-_{t,y_{t-1},s_{t-1}}} \ell^-_t (1 - p_{\ell_t,y_{t-1},s_{t-1}}) \\ &+ \sum_{\ell_t,y_t,s_t} q_{\ell_{t+1},y_t,s_t} n_{\ell^-_{t+1,y_t,s_t}} \ell_{t+1} \end{aligned} \tag{2.19}$$

Note that we have departed from the notation used in the previous pages of this chapter, since it is more intuitive to think of the problem of the financial intermediary in terms of time subscript t.

The probability that a loan of type (ℓ_{t+1}, y_t, s_t) where obviously $\ell_{t+1} < 0$ will default is $p_{l_t,y_{t-1},s_{t-1}}$. Deriving the first order conditions with respect to n_{ℓ_{t+1},y_t,s_t} we get the following result:

$$q_{\ell_{t+1},y_t,s_t} = \begin{cases} \dfrac{1}{1+r-\delta} & \text{if} \quad \ell_{t+1} \geq 0 \\[2em] \dfrac{(1 - p_{\ell_t,y_{t-1},s_{t-1}})}{1+r-\delta} & \text{if} \quad \ell_{t+1} < 0 \end{cases} \tag{2.20}$$

2.6 Stationary Recursive Competitive Equilibrium

In this section we describe the equilibrium in our economy. As discussed before we only consider stationary equilibria in the sense that the distribution of agents across states is time invariant. However in the context of *incomplete markets*, unlike in the complete markets case, households can still move across the earnings or wealth distribution.

Description of Equilibrium

A stationary recursive equilibrium in our economy is a set of strictly positive prices w^*,r^*, a non-negative loan price vector q^*, a non-negative default vector p^* and strictly positive quantities of labour N^* and capital K^*. Furthermore equilibrium is characterized by a set of decisions rules ℓ'^*, d^*, c^* for the *unconstrained group* and a set of decision rules ℓ'^*, c^* for the *constrained group*, a set of value functions V^R, V^D, V^C, V^o and a set of probability measures μ^{uc}, μ^c such that:

(i) Taking prices as given, households policy functions solve their maximization problems.

(ii) The *labour market* clears and N^* is given by

$$N^* = (1 - \Pi^*(u)) \sum_{y \in \mathcal{Y}} y \Pi^*(y)$$

(iii) The *goods market* clears

$$Y^* = C^* + \delta K^* - \gamma w^* \int_{\mathcal{L}^+ \times \mathcal{Y} \times \mathcal{S}} ys \quad d\mu^c$$

where C^* is given by

$$C^* = \int_{\mathcal{L} \times \mathcal{Y} \times \mathcal{S}} c^*(\ell, y, s) d\mu^{uc} + \int_{\mathcal{L}^+ \times \mathcal{Y} \times \mathcal{S}} c^*(\ell, y, s) d\mu^c$$

(iv) The *asset markets* clears and K^* is given by

$$K^* = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} \ell' d\mu^{uc} + \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} \ell' d\mu^{c}$$

Given the above stated conditions and since N^* and K^* are strictly positive, the first order conditions for the firm imply:

$$w^* = (1-\alpha)\left(\frac{K^*}{N^*}\right)^{\alpha}$$
$$r^* = \alpha\left(\frac{K^*}{N^*}\right)^{\alpha-1} - \delta$$

and the first order conditions for the financial intermediary imply:

$$q^*_{\ell',y,s} = \frac{(1-p^*_{\ell',y,s})}{1+r^*-\delta}$$

where the default probability is zero when $\ell' > 0$. When $\ell' < 0$ and the optimal option is to choose default it is given by

$$p^*_{\ell',y,s} = \int \pi(s' \mid s)\pi(y' \mid y)\mathbb{1}(\ell' < 0, V^o = V^d)$$

2.7 Calibration

The calibration in this model follows the strategy in the first chapter. Certain parameters are left free while the others are calibrated to match key data statistics. The risk aversion parameter, the capital share and the depreciation rate of capital are all left free and have the same values as in the first chapter. These values are given by $\sigma = 3$, $\alpha = 0.33$ and $\delta = 0.035$. All the calibrated parameter values are reported in Table 2.7.1.

The time preference rate, β is calibrated to achieve a capital to output ratio of 3. Our model delivers 2.97 with a β of 0.953. The probability of remaining constrained, θ, and the labour garnishment rate, γ, is calibrated to match both the mean exclusion period outside the market and the yearly Chapter 13 bankruptcy filings.

According to the Fair Credit Reporting Act, bankruptcy filing remains on a defaulted household's record for 10 years. Furthermore, the US Courts administrative data for the personal cases during the 12 month Period ending December 31, 2009 gives a total bankruptcy filing amount of 1,420,000.[6] The total adult population, over the age of 20 years, measured from the US Census Bureau Current Population Survey for the year 2009 is 215 million. Consequently, the bankruptcy filing rate in the economy is 0.65%.

[6] http://www.uscourts.gov/statistics-reports

Table 2.7.1: Calibrated Parameters

Description	Parameter	Value
<u>Preferences</u>		
Households Discount Factor	β	0.953
Coefficient of Relative Risk Aversion	σ	3
<u>Technology</u>		
Capital Share	α	0.33
Depreciation Rate of Capital	δ	0.035
<u>Defaults</u>		
Probability of Remaining Constrained	θ	0.90
Labour Earnings Garnishment	γ	0.20

Notes: This table reports the calibrated parameters for our benchmark model economy.

The calibrated value of θ and the labour garnishment rate γ that delivers these statistics is 0.90 and 0.20. These values deliver a mean exclusion period of 10 years, equilibrium default rate of 0.64% and imply that 20% of your potential earnings are taken away post-default.

The garnishment rate is consistent with the *Federal Wage Garnishment Law* and the *Consumer Credit Protection Act* III which limit the income garnished at 25%.

The bounds for the debt/asset levels are motivated by the precautionary savings literature. Equilibrium in these models require the property that $\beta(1 + r^* - \delta) < 1$.

As long as this constraint on the effective discount rate is satisfied, the mean level of assets converges implying that an upper bound on the assets exists, l_{max}. The debt limit l_{min} can be set to any value less than or equal to the largest debt level that could be repaid by the employed household in the highest earnings state facing the lowest possible interest rate, i.e., $l_{min} \leq [y_{max} w_{max}]$.

Similarly to the first chapter, the unemployment/employment transition probabilities are taken from the job finding and job losing rates in Current Population Survey. The transition matrix is presented below and the calculation is based on average probabilities spanning the 2006 to 2014 time period.

$$\begin{pmatrix} \pi_{uu} & \pi_{ue} \\ \pi_{eu} & \pi_{ee} \end{pmatrix} = \begin{pmatrix} 0.1700 & 0.8300 \\ 0.0420 & 0.9580 \end{pmatrix}$$

The earnings risk conditional on employment, as before, is discretized using a 12 state Markov chain following Civale et al. (2016) and is calibrated to match higher order moments of log earnings estimated by Guvenen et al. (2015).

Finally the period utility function used for the computational solution of our model is assumed to be a $CRRA$:

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{2.21}$$

where the parameter σ measures the degree of relative risk aversion.

2.8 Results

In this section we discuss key results obtained from the computational solution of our model. Firstly, this involves a detailed discussion of the loan price schedule, default probabilities and spreads. We further discuss the results related to savings policy functions and we establish the link between borrowing, defaults and consumption behaviour.

In the second part of this section we discuss the behaviour of our economy in equilibrium and the extent to which our generated model statistics could match empirical facts in the U.S. economy. We provide arguments that support the idea that our results are explainable in the light of our new pricing approach, that is, the *introduction of a financial intermediary in our economy.* Finally, we highlight all the distributional aspects of our model, by presenting the wealth distributions, amongst different groups in our economy, namely employed households versus unemployed households, as well as households with good credit record versus households with bad credit record.

2.8.1 Loan Prices, Defaults and Policy Functions

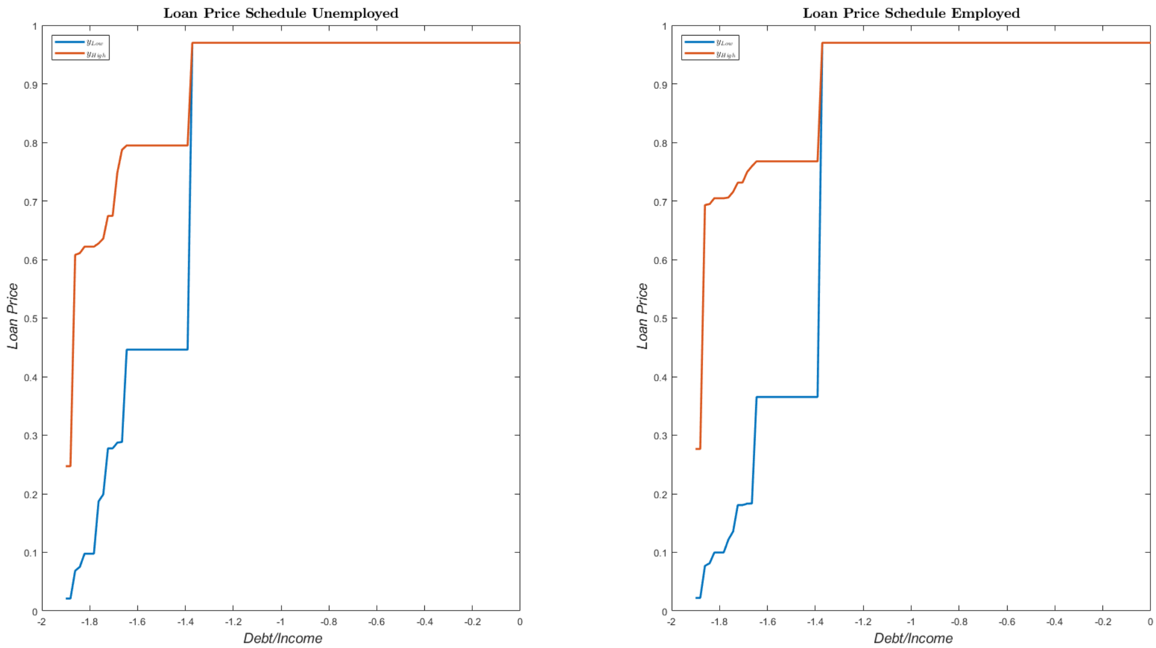

Figure 2.8.1: **Loan Price Schedule:** This figure displays the loan price schedule for two different realizations of labour earnings shocks for *unemployed* and *employed* households. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

Figure 2.8.1 plots the price schedule which actually determines the set of loans that each borrowing household can choose every period. The plot displays two different realizations of labour earnings shocks, both for the employed and unemployed state.

Firstly, we should observe that loan prices are an increasing function of assets. In other words, the more indebted a household the lower the loan price, and the higher the implied loan interest rate. This indicates that the cost of borrowing is much higher for a household that already holds large amounts of debt. Importantly, the low income households face lower loan prices, thus higher borrowing interest rates, compared to those households with higher income. This is because, households with low income are considered as more risky from financial intermediaries, since they face higher probability to default. Since in our model loan prices depend on three factors, that is, the size of loan, the idiosyncratic income state and the employment/unemployment state, the result presented in figure 2.8.1 is anticipated.

Financial intermediaries are risk neutral and loan prices are based on the probability that a household with certain income characteristics and employment status will default. In the right panel of figure 2.8.2, it is evident that default probability is much higher for households being in a low income state. Amongst heavily indebted households, that is households that hold above 160% of their income in debt, those with low income face 2.5 times higher probability to default.

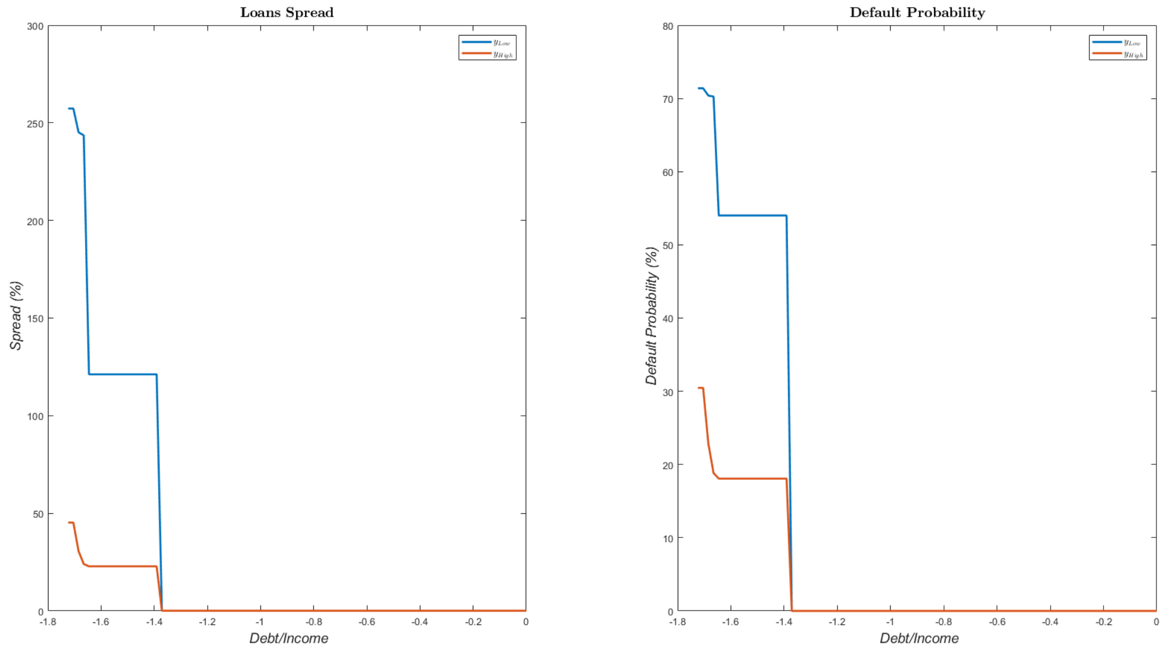

Figure 2.8.2: **Loan Spread and Default Probability:** The right panel of this figure displays *default probability* for two different realizations of labour earnings shocks. The left panel of this figure presents the *loans spread* for two different realizations of labour earnings shocks. The realization is denoted as y_{Low}, y_{High} and is located on the north-east part of both graphs.

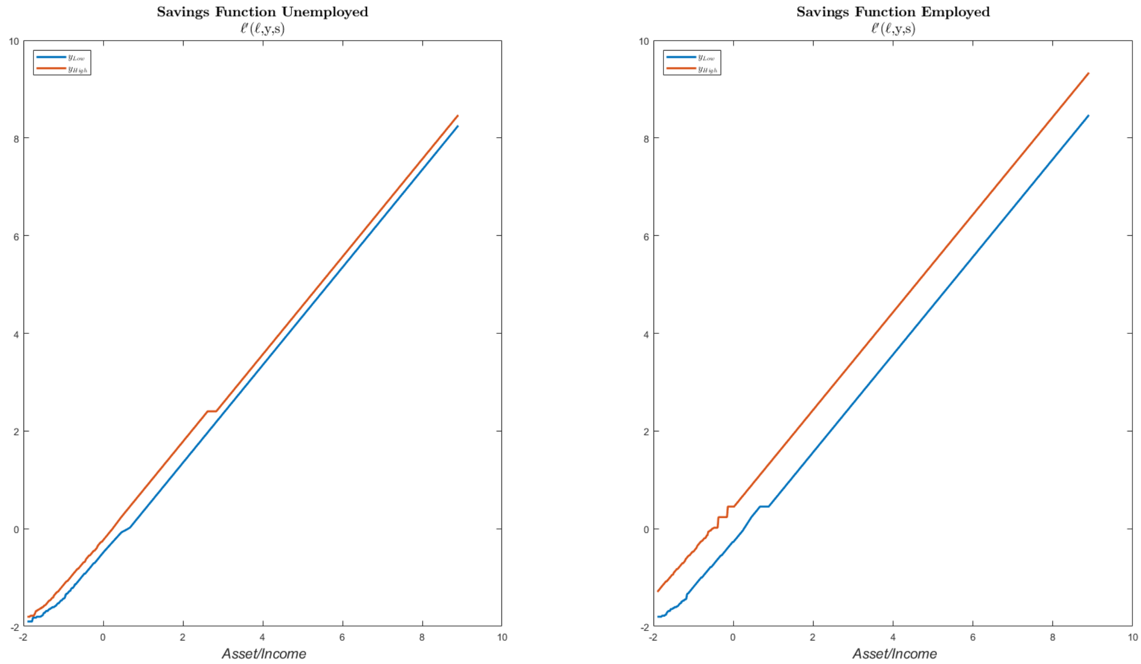

Figure 2.8.3: **Savings Function Employed Vs. Unemployed:** The right panel of this figure displays the optimal savings,conditional on not defaulting, for *employed households* as a function of assets to income ratio for two different realizations of labour earnings shocks. The left panel presents optimal savings for *unemployed households*. The realization is denoted as y_{Low}, y_{High} and is located on the north-west part of both graphs.

We further observe that households having a debt to income ratio less than 140% face high prices, that is, the loan interest rates are very low. Financial intermediaries perceive that these households is highly unlikely to default, hence they are offering loan contracts with a very low spread, as can be observed in the left panel of figure 2.8.2.

Indeed households with debt to income ratio less than 140% do not default in our model. This is because the service cost of debt is very low. Thus, the value of repayment under any income realization or any employment status, exceeds the value of default. In contrast to our previous chapter model, savings (deposits) are not *risky*, thus the generated return is much lower, around 3%.

The outside option is not appealing when the interest rate charged for a loan is very close to savings rate. Note that in our model framework, defaulted borrowers are allowed to save following the default event. However, savings with low interest rates do not appear to be an appealing alternative, when the service cost of debt is very low. This directly lessens the default incentives of households and thus results in less default decisions.

Note that our model delivers countercyclical debt limits, in the sense that higher income households borrow less than their lower income counterparts, as we can see in figure 2.8.3. In support of this argument is the observed fact that households with almost zero assets and high earnings choose to save, while low income households with the same asset level still choose to borrow. This is because *precautionary borrowing* serves as a means of consumption smoothing for households in the low income state. Contrariwise, high-income households develop a *precautionary savings* behaviour.

High spreads for heavily indebted households in the left panel of figure 2.8.2 reflect their high default risk captured by the high default probability. Indeed these households in our model optimally decide to default. This is because the service cost of debt becomes extremely high and consequently triggers defaults.

Essentially a borrowing household in our model has two instruments to control his consumption path. Firstly, a household can *borrow* in bad times, that is, periods in which unemployment or a low income state is realized so to maintain its consumption in a satisfactory level. Secondly when interest rates become very high and indebtedness is deep, a household should default, to achieve even the minimum level of consumption.

For a better illustration of our argument, we can think of a heavily indebted household that faces either bad income realization or transits to unemployment. In this case, household's wealth would be very small. In particular, in our model environment, wealth would be negative. In other words, no savings that could be utilised to boost consumption are available. The optimal behaviour of the borrowing household would be to borrow heavily in order to restrain his consumption. Indeed, as confirmed by figure 2.8.3, low income households choose to borrow more even when they are heavily indebted.

However the willingness of the household to borrow might not be accompanied by the availability of such financial contracts. This is the case in our model environment when a household is close to the borrowing limit. Household's only alternative under such circumstances is to default. This further explains why in these class of models default incentives are lower in low states and higher in high states. Low income households would be willing to borrow as long as supply of credit is available to them and postpone defaulting to the future, when better income shocks might be realized. Thus, we could argue that the marginal disutility of default is higher for low income households compared to those with high income. In a simpler way, in consumption terms default is less costly for households with high disposable income, under the model assumption that a good state is more persistent, i.e. when a household is currently, the probability that will remain employed next period, than the probability to become unemployed.

2.8.2 Equilibrium Economy and Model Statistics

An interesting feature of our model is that it matches very well key statistics of the U.S. economy. The model successfully reproduces macro, distributional and households bankruptcy statistics. It is important to mention that our alternative pricing approach employed in this chapter, had significantly enhanced the distributional results of our model.

In particular, by adopting the assumption that borrowing/savings does not take place via the trade of a bond amongst households, yet with the presence of a financial intermediary, we achieved to generate different borrowing and savings rate in our economy and make savings risk free. This had a significant effect, for instance in the accuracy of our generated statistics related to the last quintile of our net worth distribution.

Table 2.8.1: The Benchmark Model Economy

Economy	U.S Data	Model II	Model I
Capital-to-output-ratio	**2.9**	**2.97**	2.67
Consumption-to-output-ratio	**0.69**	**0.74**	0.80
Negative Net Worth Households (%)	14	17.9	11.6
Non-Positive Net Worth Households (%)	**18.1**	**19**	18.9
Wealth Gini Index	**0.85**	**0.76**	0.795
Income Gini Index	**0.55**	**0.50**	0.64
Bankruptcy Filings(%)	**0.65**	**0.64**	7.4*
Household Debt-to-GDP-Ratio(%)	17.7	9.8	--

Notes[1]: This table reports the equilibrium results of our benchmark economy, called Model II, the chapter I model results called Model I, and compares with values of the relevant variables in the U.S. economy. For the variable Bankruptcy filings, we have reported the default rates for Model I. We do not distinguish between debt and assets in Model I so there is no equivalent value for the household debt/GDP ratio.
Notes[2]: Household Debt-to-GDP-Ratio is calculated in U.S. Data, by using consumer credit to GDP ratio in the U.S. This ratio in the last 10 years ranges between 17% and 18%. The ratio reported in the above table refers to 2012.

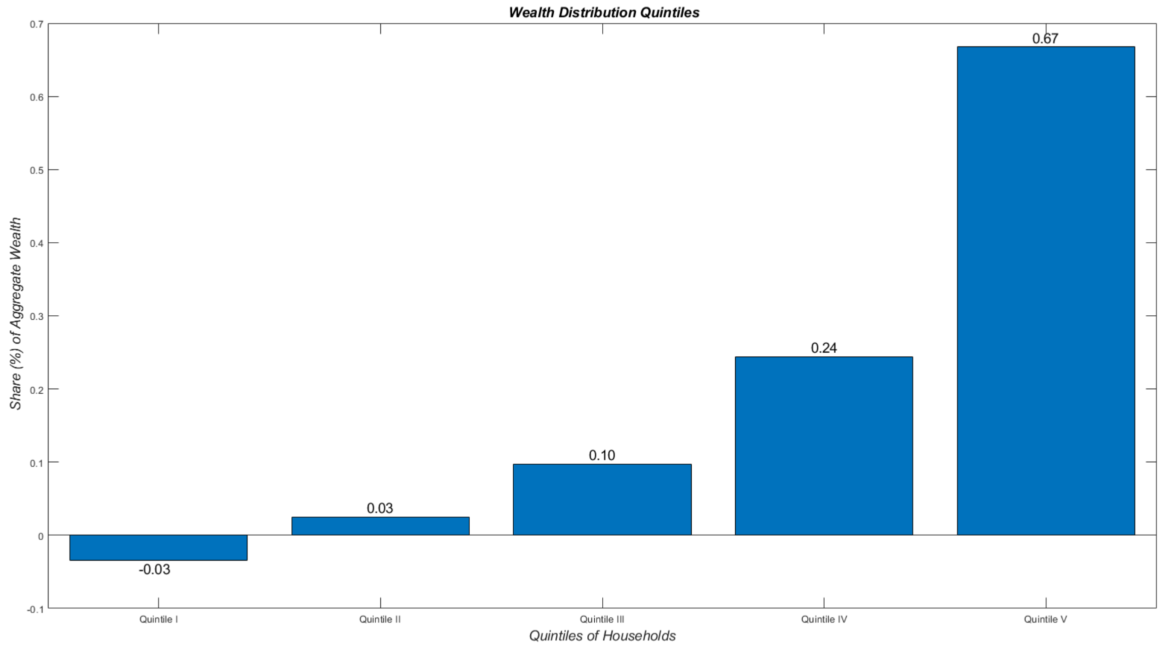

Figure 2.8.4: **Wealth Distribution Quintiles:** This figure presents the quintiles of the wealth distribution generated by our model. Values depict the share of aggregate wealth held by each quintile of households in our economy.

Figure 2.8.4 plots the obtained wealth distribution quintiles. In comparison with the first chapter, the third, fourth and fifth distribution quintiles are now much closer the data. In data, these quintiles are respectively, 0.033, 0.10 and 0.86. This better fit is because, in this model, deposit rate is much lower, compared to our previous chapter, since deposits are safe and guaranteed by the financial intermediary. In figure 2.8.4 we observe that the last quintile of the model generated wealth distribution holds around 67% of total wealth, while its empirical counterpart is around 86%. A better fit is possible if we enrich the household's portfolio with other financial and durable assets. This would ensure that agents belonging to the top part of the distribution would have enough savings alternatives and thereby higher wealth relative to the negative net worth households.

It is apparent from Table 2.2 that the inclusion of financial intermediaries and risk-neutral pricing had an impact more on the default or bankruptcy filings and not the wealth/income distribution quintiles. Despite the improvement in income GINI, the wealth GINI now under-predicts actual inequality in the data. [7]

With low deposit rate, households firstly do not have strong incentives to default and secondly the *precautionary savings channel* for households with a low income realization weakens. The outside option, namely the option to save following the default event is not so appealing, given the low deposit rate environment. This explains why less households are located in the last quintile of the distribution, and thus long right tails are present. This results in the high share of aggregate wealth being owned by households in the last quintile of the wealth distribution.

The model generates a household debt/gdp ratio of 9.8% while its empirical counterpart is 17.7%. Note that in our economy there exist earnings and debt states, where the interest rate charged from the financial intermediary equals the risk free interest rate. When households find themselves in these states they find optimal to borrow and not to save. Indeed, real world observations show that a very low interest rate environment fosters borrowing and thus accumulation of debt. In other words, when borrowing is very cheap everyone has stronger

[7] This means that a better calibrated NMAR process controlling for the persistence parameter ρ could shed some light on getting better fit of the wealth distribution.Calibration of an adequate NMAR process relevant to mimic the U.S. wealth distribution is left for future work.

incentives to borrow, while when borrowing is very expensive default incentives increase for heavily indebted households, and the outside option becomes more appealing.

Furthermore our model successfully replicates defaults in U.S., and in particular, bankruptcy filings. Our model generates filings of 0.64% while its counterpart in data is 0.65%.[8] From a theoretical perspective, it is interesting to note that this model delivers relatively low defaults in equilibrium. This can primarily be explained due to the presence of a financial intermediary, that prices optimally every loan, handling it as a distinct financial asset.

As we have previously explained, the vector of prices depends on the size of the loan, and the two idiosyncratic states. This implies that the intermediary prices in all the available information, and offers a loan contract under a price that essentially reflects the default probability of this particular loan contract, or in other words the default probability of the specific household. As a consequence defaults are minimized in equilibrium.

The second mechanism that underlies our model, affecting default incentives and contributing to the low defaults in equilibrium is the outside option. The risk-free interest rate/ deposit rate generated in equilibrium is 3%. This implies that the returns on savings are not high, while borrowing in some idiosyncratic states and levels of indebtedness is very cheap. This urges households to increase their borrowing and also strengthens their repayment incentives, making default option less attractive.

[8] The bankruptcy filing rates are genuine model predictions obtained endogenously in equilibrium and not a results of calibration.

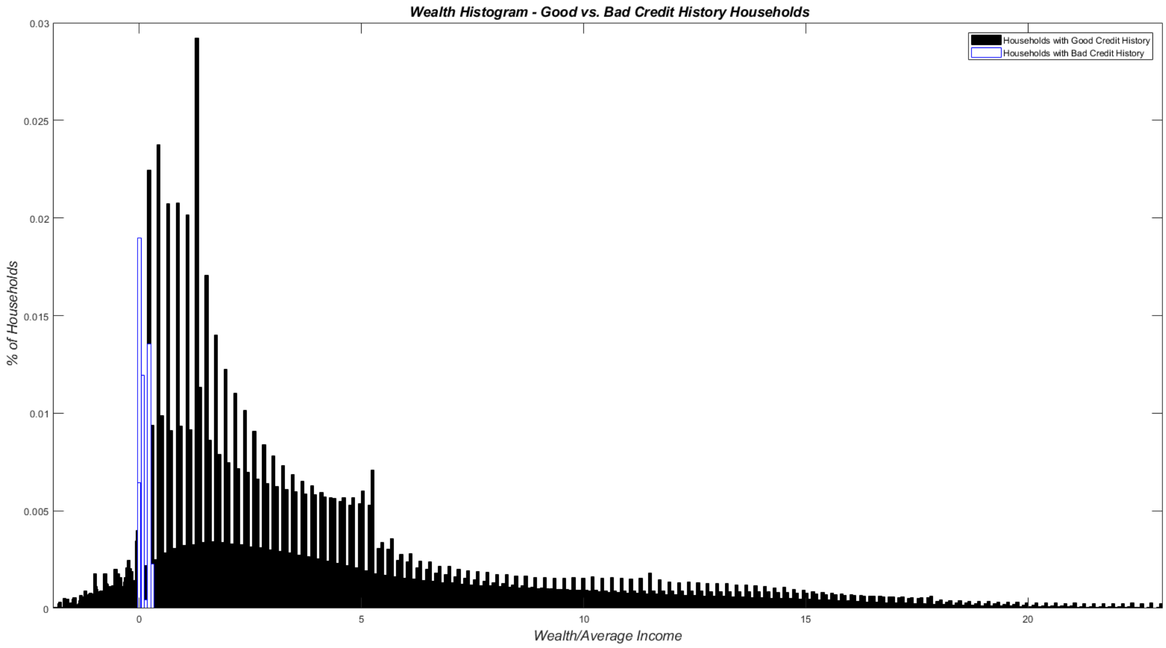

Figure 2.8.5: **Wealth Histogram - Good vs Bad Credit History Households:** This figure depicts the net wealth to average income distribution of households with Good Credit History (i.e. No Default) and the distribution of households with Bad Credit History (i.e Default). On $x-axis$ we express net wealth as a ratio of average income, while on $y-axis$ the mass of households at each level of wealth is depicted.

In addition our model has a good performance with respect to income inequality, measured by the associated gini index. In particular, the model delivers a Gini index of 0.50 while the associated value in data is 0.43. This result is mainly driven by the fact that our earnings process is exogenously specified to match the moments of Guvenen et al. (2015). However, in our income definition there exists an endogenous part, namely the asset returns. In this sense, we could argue that our model does not distort the earnings inherent to our exogenously specified earnings process, while also correctly identifies the appropriate asset returns so that the Gini index will be close to the observed data values.

We finally turn to the discussion related to the wealth distribution amongst households with *good credit history*, that is, households that have never defaulted, and *bad credit history*, namely households that are constrained in equilibrium. The shape of our model generated wealth distribution closely resembles the distribution of Chatterjee et al. (2007). The distribution for households with good credit history is right skewed and features a long right tail. Unconstrained households experience higher values of wealth to income ratio compared to the constrained households that are concentrated around zero. Our model suggests that a significant amount of the population has very low wealth. Note that a considerable amount of the population, 17.9% has negative wealth. In this manner, our model successfully replicates novel empirical findings which suggest that around 14% of the population in United States has negative net worth. This issue raises major concerns to the policy makers, since these households are very vulnerable to any unanticipated shock. Furthermore, it is evident that a small fraction of households holds most of wealth as also witnessed by the wealth quintiles presented in figure 2.8.4. We further find that around 5.35% of households is constrained in equilibrium, which indicates that a significant fraction of households in the economy do not have or have very limited access to credit, due to their bad credit history.

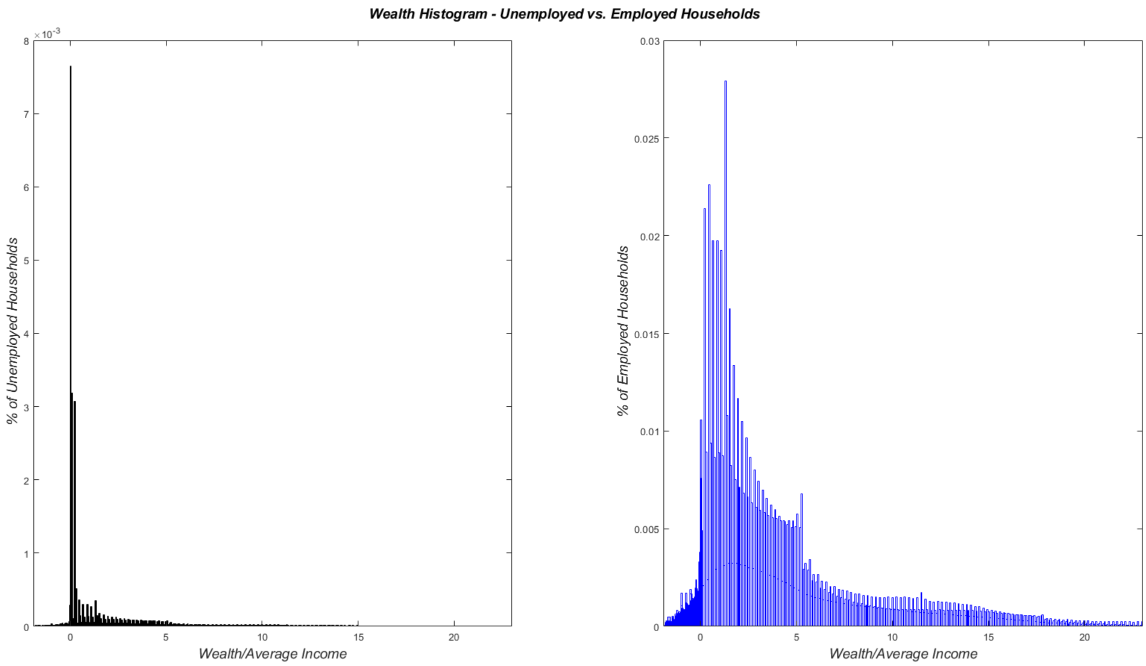

Figure 2.8.6: **Wealth Histogram - Unemployed vs Employed Households:** This figure presents the net wealth to average income distribution of employed (right panel) and unemployed (left panel) households. On $x - axis$ we express net wealth as a ratio of average income, while on $y - axis$ the mass of households at each level of wealth is depicted.

Since we are also interested in the composition of this wealth distribution by employed and unemployed households, we present Figure 2.8.6. Clearly, unemployed households have on average less wealth than employed households. This can be mainly seen as a low income effect. Unemployed households experience much lower income levels compared to employed. Thus accumulation of wealth is arduous, since unemployment benefits might even not be sufficient to attain a low level of consumption. This indicates that idiosyncratic shocks are important determinants of wealth inequality, indebtedness or very low wealth levels. Households that experience "life events" captured in our model from employment-unemployment status, are usually financially distressed, have negative levels of wealth and are willing to live in a state of limited consumption such that they maintain their access to credit. These households that represent in our model economy a significant fraction of the population, also borrow under very high interest rates, since their default probability is very high. This combination of very high interest rates, heavy indebtedness and bad “life events”, explains the failure of policy makers to tackle inequality successfully.

2.9 Conclusion

In this chapter we accomplished the following goals. Firstly we overcame some of the limitations inherent to the model of our previous chapter, by introducing a *risk-neutral* financial intermediary. By allowing the price vector to depend on the size of the loan, the idiosyncratic income state, and the employment status of the households, we generated different interest rates for savings and borrowing. We introduced a novel feature, that is, financial intermediaries take into account the households characteristics, captured by the two idiosyncratic shocks.

Secondly, we have showed that this model serves as a successful variant of the model developed in our previous chapter, since it successfully replicates key U.S. statistics related to wealth inequality, income inequality, bankruptcy filing rates, as well as main macroeconomic aggregates in the U.S. economy. It particularly delivers good results related to the share of wealth that each quintile of the wealth distribution holds, as well as the income Gini Index.

Thirdly, we construct an equilibrium in which the risk-free interest rate is low, thus default incentives are lower, due to the less attractive outside option. This mechanism delivers much lower default rates in equilibrium and is of particular interest, since it highlights the importance of meticulous pricing from the side of financial intermediaries.

Our results in the first two chapters indicate that non-collateralized debt and a risk-neutral financial intermediary can capture well the left-tail of the distribution. However, for an adequate description of the right tails, the number and type of assets available for households needs to be enriched. A richer model of portfolio holdings possibly with durable assets like housing and an appropriately calibrated NMAR process will be better suited for this task. This is left as a future exercise.

Finally, our results can serve as a theoretical pillar to the increasing wealth inequality and the large fraction of households in the United States that experience negative wealth, while economy accumulates debt and *precautionary borrowing* behaviour is present.

Appendix

"The unified character of mathematics lies in its very nature; indeed, mathematics is the foundation of all exact natural sciences".

David Hilbert (1862-1943)

This Appendix is meant to formalize the mathematical properties of our model economy in the first and second chapters. We restate the definition of the equilibrium in a more formalized way and describe its existence. We then proceed to the details of the computational algorithm used in the first two chapters along with the conditions for robustness.

2.A Existence of the Equilibrium

This section characterizies the conditions under which an equilibrium exists in our model economy. There are three markets in our model: the labor market, the asset market and the aggregate goods market. Equilibrium in the labor market is trivial since the aggregate labor supply is exogenous and labor demand is strictly decreasing in wages. Equilibrium in the resource constraint follows directly from the equilibrium in the other two markets by Walras' law (also proved in Chatterjee et al. (2007)). Before we discuss the asset market equilibrium conditions, we first specify the sets, functional spaces and the underlying measure used in our model. These would help understand the properties of our equilibrium. Where necessary, we also briefly describe the mathematical theorems used. The material detailed here would be self-sufficient to understand all the mathematical properties of our model.

The individual in our economy is characterized by the pair (ℓ, y, s) - the individual states. Let μ be the distribution of agents across states, this includes both the unconstrained and constrained types, $\mu = \mu^{uc} + \mu^{c}$. For the purposes of our model, we need this measure to be a probability measure. We now describe a mathematical structure which would produce this desired measure.

Define $\ell \equiv [-\overline{B}, \ell_{max}]$ as the compact set of possible asset holdings. The minimum bound, or the debt limit, is defined as any value less than or equal to the maximum earnings of the luckiest household ($y_{max}.s_{max}.w$) so this is the largest debt level that could be repaid by the luckiest household facing the lowest possible interest rate. The maximum bound exists as long as the effective discount rate is less than unity. This upper bound arises due to the fact that increasing wealth implies that the coefficient of variation of income goes to zero and thereby the role of consumption smoothing disappears. Define $\mathcal{Y} \equiv \{y^1, y^2, y^3 \dots y^{N_1}\}$ and $\mathcal{S} \equiv \{s^1, s^2, s^3 \dots s^{N_2}\}$ as countable sets containing the stochastic endowments of labour earnings and unemployment risks, respectively.[9] A finite cardinality of these countable sets implies that we are dealing with finite state Markov processes.

The state space of the model is defined as the Cartesian product of these three sets, $\mathscr{S} \equiv L \times Y \times S$ with Borel σ algebra $\mathcal{B}$ and a typical subset $(\mathcal{L} \times \mathcal{Y} \times \mathcal{E})$. The space $(\mathscr{S}, \mathcal{B})$ is a measurable space, and for any set $\mathcal{S} \in \mathcal{B}$, $\mu(\mathcal{S})$ is the measure of agents in the set $\mathcal{S}$. The set of all probability measures over $(\mathscr{S}, \mathcal{B})$ is denoted as χ.

To understand how individuals transit across the asset, earnings and unemployment states over times, we need to define a transition function. We define $Q((\ell, y, s), \mathcal{L} \times \mathcal{Y} \times \mathcal{S})$ as the probability that an individual agent with current state (ℓ, y, s) transits to the set $\mathcal{L} \times \mathcal{Y} \times \mathcal{S}$ next period. In other words, following Stokey et al. (1989), the transition function can be explicitly defined as follows:

Theorem 2.A.1. ***Transition Function:*** *Let $(\mathscr{S}, \mathcal{B})$ be a measurable space. A transition function is a function $Q : \mathscr{S} \times \mathcal{B} \to [0, 1]$ such that:*
(i.) for each $\ell \in \mathscr{S}$, $Q(\ell, .)$ is a probability measure on $(\mathscr{S}, \mathcal{B})$; and
(ii,) for each $L \in \mathcal{B}$, $Q(., L)$ is a $\mathcal{B}$ measurable function.

In our model, the transition function is expressed as follows:

$$Q((\ell, y, s), \mathcal{L} \times \mathcal{Y} \times \mathcal{S}) = \sum_{y' \in \mathcal{Y}, s' \in \mathcal{S}} \mathbb{1}\{\ell'(\ell, y, s) \in \mathcal{L}\}\pi(y'|y)\pi(s'|s) \tag{2.22}$$

where $\mathbb{1}$ is the indicator function which takes the value 1 when the optimal savings policy $\ell'(\ell, y, s)$ lies inside the asset set, $\mathcal{L}$, and 0 otherwise. The transition

[9]A set is called countable (or countably infinite) if it has the same cardinality as $\mathbb{N}$. Equivalently, a set A is countable if it can be enumerated in a sequence, i.e., if all of its elements can be listed as a sequence $a_1, a2, \dots$. A set is called uncountable if it is infinite and not countable.

probabilities for the two idiosyncratic states are denoted by $\pi(y'|y)$ and $\pi(s'|s)$. Thus Q is the transition function defined on the measurable space $(\mathscr{S}, \mathcal{B})$.

Any transition function in this space is associated with an operator defined on the probability measures over this space. Formally, for any probability measure μ on $(\mathscr{S}, \mathcal{B})$, the operator T^* associated with it gives:

$$\mu'(\mathcal{L} \times \mathcal{Y} \times \mathcal{S}) = T^*(\mu) = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} Q((\ell, y, s), \mathcal{L} \times \mathcal{Y} \times \mathcal{S}) d\mu(\ell, y, s) \tag{2.23}$$

Our definition of a **stationary recursive competitive equilibrium** then requires that this probability measure of agents in the economy be invariant, i.e., for all $(\ell, y, s) \in \mathcal{B}$, the invariant probability measure satisfies

$$\mu^*(\ell, y, s) = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} Q((\ell, y, s), \mathcal{L} \times \mathcal{Y} \times \mathcal{S}) \tag{2.24}$$

We have now established the mathematical structure of our model, the existence of an equilibrium here depends on three key properties:(1.) Compactness of the state space (2.) Continuity of the measure with respect to the interest rate (implying continuity of supply) and (3.) Monotonicity of the asset supply. We start our analysis by specifying the demand and supply functional that clear our asset market.

Compactness- Demand for Capital. The necessary condition for optimal choice of the firm gives us the aggregate capital,

$$K(r) = F_k^{-1}(r + \delta)$$

The Cobb-Douglas production function ensures that the demand for capital is a continuous, strictly decreasing function of the interest rate:

$$K(r) = \left(\frac{\alpha L}{\delta + r}\right)^{\frac{1}{1-\alpha}}$$

Supply of Capital.The aggregate supply of capital is the sum of assets supplied by the measure of unconstrained and constrained agents:

$$K = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} \ell'(\ell, y, s; r) d\mu^{uc} + \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} \ell'(\ell, y, s; r) d\mu^{uc} \tag{2.25}$$

The existence of the equilibrium then depends on proving the continuity of the two supply functions in the right hand side of the above equation. Convergence properties for the policy functions is best understood by examining the first order optimality conditions for the general model:

$$u'(c_t) = \beta\Big(\frac{1}{q}\Big)E_t[u'(c_{t+1})] + \lambda_t \tag{2.26}$$

where λ_t denotes the Lagrangian multiplier on the borrowing constraint. This condition implies the Euler equation

$$u'(c_t) \geq \beta(1+r)E_t[u'(c_{t+1})], \qquad 1+r = 1/q \tag{2.27}$$

Inspecting the Euler equation it is clear that the asset accumulation is going to be shaped by two forces: the effective rate of time preference $\beta(1+r)$ and labor income risk. The higher the rate of time preference, $\beta(1+r)$, the more agents prefer to accumulate wealth since they prefer future consumption. The idiosyncratic labor income risk fuels further their accumulation motives to insure their consumption against bad income realisations in the future.

The time horizon in our model is infinite, if T is finite, the boundedness of both c_t and a'_t is trivial since the effective rate of time preference for any $t > T$ is zero. In the case of infinite horizon, the convergence properties as described above depends on both the effective rate of time preference and the nature of risk. In the absence of risk, when income fluctuations are deterministic, the Euler equation clearly implies convergence of both consumption and savings as long as $\beta(1+r) < 1$. In the presence of stochastic income, we need to rely on a useful supermartingale theorem.

Multiplying both sides of eq. (2.27) by $\beta^t(1+r)^t$ and defining $M_t \equiv \beta^t(1+r)^t u'(c_t) > 0$, the Euler eq. (2.27) can be written as

$$M_t \geq E_t M_{t+1} \tag{2.28}$$

which asserts that M_t follows a supermartingale (a $\leq$ bounded martingale).We can then apply Doob's supermartingale convergence theorem for non-negative stochastic processes.

Theorem 2.A.2. *Let $(\mathscr{S}, \mathcal{B}, \mu)$ be a probability space and $X = (X_n)_{n\geq 0}$ be a supermartingale which is bounded in L^1, i.e., $\sup_n E[|X_n|] < \infty$. Then X_n converges a.s. towards an a.s. finite limit.*

By this theorem, the nonnegative stochastic process converges almost surely to a non-negative random variable M, i.e.,

$$\lim_{t\to\infty} M_t = M < \infty$$

Existence of a Stationary Distribution μ^*- To understand the continuity of the asset supply function, we need to first clarify under what conditions does the invariant measure expressed in eq. 2.24 exists or in other words the associate operator T^* defined by

$$(T^*(\mu))(\mathcal{L} \times \mathcal{Y} \times \mathcal{S}) = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} Q((\ell, y, s), \mathcal{L} \times \mathcal{Y} \times \mathcal{S}) d\mu(\ell, y, s) \tag{2.29}$$

has a unique fixed point. This requires firstly that the transition function Q to satisfy the Feller property.

Theorem 2.A.3. ***Feller Property-**: A transition function Q on $(\mathscr{S}, \mathcal{B})$ has the **Feller property** if the associated operator T maps the space of bounded continuous functions on $\mathscr{S}$ into itself; that is $T : C(\mathscr{S}) \to C(\mathscr{S})$, where $C(.)$ denotes continuous functions.*

This property basically means that the operator maps a continuous and bounded function into itself. Since the domain of the asset space is compact, the savings policy function $\ell'(\ell, y, s)$ is continuous and bounded implying that the Feller property is easily satisfied. However, the Feller property is not sufficient in ensuring the uniqueness of the invariant distribution. We have to resort to the Monotone Mixing Condition to guarantee this.

Continuity and the Monotone Mixing Condition- This condition essentially means that there should exist a positive probability to transit from ℓ_{max}, the highest asset level to some intermediate asset level in a finite N periods and an equally high probability to transit from the minimum asset level $(-\overline{B})$ to an intermediate asset level in N periods.

The theorem, stated by Hopenhayn and Prescott (1992), in a simplified version is as follows:

Theorem 2.A.4. ***Monotone Mixing Condition-*** *If Q is a transition function and it is increasing on the (ordered) measurable space $(\mathscr{S}, \mathcal{B})$, there exists $\mathscr{T}^* \in \mathscr{S}$, $\delta > 0$ and N such that*

$$P^N(d, \{\mathscr{T} : \mathscr{T} \leq \mathscr{T}^*\}) > \delta \quad \textit{and} \quad P^N(c, \{\mathscr{T} : \mathscr{T} \geq \mathscr{T}^*\}) > \delta \tag{2.30}$$

then the operator T^* has a unique fixed point μ and for all $\mu_0 \in \chi$ the sequence of measures defined by

$$\mu_n = (T^*)^n \mu_0 \tag{2.31}$$

converges weakly to μ.

An underlying assumotion behind this theorem is that $(\mathscr{S}, \geq)$ is an ordered space, where the order is defined as

$$\begin{aligned} \mathscr{T} \geq \mathscr{T}' \quad \text{iff} & \\ & \{(\mathscr{T}_1 \geq \mathscr{T}_1') \quad \text{and} \quad (\mathscr{T}_2 = \mathscr{T}_2') \\ & \text{or} \quad (\mathscr{T}' = c = (-\ell', y_1, s_1)) \\ & \text{or} \quad (\mathscr{T} = d = (\ell_{max}, y_N, s_N))\} \end{aligned} \tag{2.32}$$

This closed order along with the Euclidean metric is a compact metric space, where we have denoted $\geq$ as a closed order, $c \in \mathscr{S}$ and $d \in \mathscr{S}$ are the smallest and the largest elements in $\mathscr{S}$, under order $\geq$, and $(\mathscr{S}, \mathcal{B})$ is a measurable space.

Furthermore, we have defined $P^N\{\mathscr{T}, \mathscr{S}\}$ as the probability of transiting from state $\mathscr{T}$ to $\mathscr{S}$ in N steps. From our previous discussions, the first statement, that Q is a transition function, of this theorem is direct. The assumption that this function is also increasing means that in Stokey et al. (1989)'s words

Theorem 2.A.5. ***Monotonicity of Transition Function:*** *A transition function Q on $(\mathscr{S}, \mathcal{B})$ is* ***monotone*** *if the associated operator T has the property that for every nondecreasing function $f : Z \to \mathbb{R}$, the function Tf is also nondecreasing.*

it is monotonic and coupled with the fact that the savings policy function is increasing in the state space leads us to conclude that the transition function is also increasing. Convergence is then given by

Theorem 2.A.6. ***Monotone Convergence Theorem****: If $\{f_n\}$ is a monotone increasing sequence of functions in a space of nonnegative measurable real valued functions on $(\mathscr{S}, \mathcal{B}, \mu)$ converging pointwise to f then*

$$\int f d\mu = \lim_{n\to\infty} \int f_n d\mu$$

In an informal sense, the monotone mixing condition is satisfied in general dynamic models which require invariant distributions whenever the agent starting from a low level of assets gets hit by a sequence of long stream of bad shocks, the agent will deleverage debt until some neighbourhood of the lower bound is reached. The motive for the agent to decumulate assets comes from the agent's information that this low sequence of income realisations is well below average, permanent income is higher and consumption follows the permanent income.

The conclusion of the Monotone Mixing Condition Theorem is that existence of an invariant measure can be guaranteed by iterating on the operator T^*.

The Default Decision and Existence of Equilibrium- Our discussion until now was in a more general sense, to standard Bewley-Huggett-Aiyagari (BHA) models, and we did not consider the influence of default decisions. However, our model as in Chatterjee et al. (2007) extends these BHA models to include the possibility of defaults. Agents can choose optimally whether to repay their debt obligations or default on them. These decisions are made period by period. The presence of discrete decision making means that our mappings are not functions but correspondences. Informally, discrete choices imply that agents can be indifferent between different states such as being in the unconstrained or the constrained state. In other words, the value functions and policy functions map the state space system not to a point but a set. Chatterjee et al. (2007) proves that the optimal policy in these models is compact valued and upper hemi-continuous. This means that the measurable selection theorem of Stokey et al. (1989) can be applied to guarantee the existence of measurable policy functions for consumption, asset holdings and default decisions. Transition functions and the existence of an invariant measure of agents can be implemented by the appropriate construction of measurable spaces and continuity arguments.

Now that we have completely described the mathematical structure our model is based on, we move on to the details of our computational algorithm.

2.B Computation of the Equilibrium

In this section, we describe the numerical procedures used to compute the general equilibrium in our model. Our model involves a continuum of agents facing idiosyncratic uninsurable shocks who have to decide their level of consumption, savings and default decisions bsaed on an infinite time horizon. The inherent non-linearities and the theoretical complications involved in the existence of a unique equilibrium means that we have to resort to numerical procedures. We use a variety of standard tools to achieve this task. The high dimensionality of our model implies that it is computationally cumbersome. We modify existing procedures to accommodate the complexities and to lower the processing time while not sacrificing on accuracy and robustness.

Our algorithm involves a number of steps. We start by discretizing the state space $\mathscr{S}$ and then proceed to value function iteration to get the optimal policy functions. These optimal policy functions along with the exogenous markov processes are used to obtain the invariant distribution of agents in the economy. Finally, we implement a modified bisection method to capture the market clearing prices.

2.B.1 Discretization of the state space

The state space $\mathscr{S}$ of our model is discretized into finite intervals in a grid. The bounds of the grids are motivated by the theory discussed in the previous section. Minimum and maximum bounds for assets are chosen to ensure the convergence of asset supply. The exogenous shocks are discretized using a finite state Markov process. The labor endowment shocks have a cardinality of $N = 12$ while the unemployment/employment state naturally have 2 states. The associated probabilities for the employment/unemployment transition matrix is obtained from the empirical data as described in the main text while that of the endowment shock is discretized by approximating the NMAR process as described in Civale et al. (2016). The number of points in the grid is chosen optimally ensuring that the aggregates do not change when it is any more finer. Furthermore, we include more points in the borrowing part, especially in the neighbourhood of zero since the defaulted agents are reallocated to this point in the state space. Consequently, we expect and deliver a substantial mass of agents with zero assets validating our approach.

2.B.2 Numerical Algorithm

Our model as in other Bewley-Huggett-Aiyagari models, solution involves a number of steps, which we present below.

(i) Value Function must be solved in order to obtain the optimal policy rules

(ii) Optimal policy functions and the exogenous Markov process, are used for the approximation of the steady-state agents' distribution.

(iii) Using some moments (usually only the mean) of the distribution we find the aggregate variables.

(iv) Market clearing conditions are assessed under a given price.

(v) If market clearing does not hold, the process from (i)-(iv) is repeated for different prices, until general equilibrium is found.

The challenge of this algorithm is that every step involves numerical errors. For instance if numerical errors occur in the value function solution, this in turn will affect the convergence of the agents' distribution, where if we achieve convergence more errors will accumulate, causing convergence difficulties for price iteration and hence general equilibrium.

Computation of the Stationary Equilibrium

For computing the stationary equilibrium of our economy, we modify the algorithm presented in Heer and Maußner (2005) for the computation of stationary equilibrium of an Heterogeneous-Agents economy. Our modified algorithm is as follows

(A) Compute the invariant labor supply.

(B) Make an initial guess for the interest rate r.

(C) Place a grid on asset space such that more points exist in the negative interval of the grid.

(D) Compute wage rate w and the capital stock K.

(E) Compute household's decision functions including the default probabilities and the loan pricing schedule.

(F) Compute the invariant distribution of agents for all the idiosyncratic states.

(G) Compute aggregate asset holdings and check the aggregate consistency conditions.

(H) Update the interest rate if the market does not clear.

Step1: Dicretized Value Function Iteration and Optimal Policy Functions

The algorithm for discretized value function iteration is trivial and it is does not involve any difficulties in achieving convergence of the value function and obtain the policy rules.

Step2: Computation of the Invariant Density Function

The approximation of the agents distribution function relies upon the discretization of the state space. The asset grid should be finer, than the one used for the value function iteration for finding the policy rules. The algorithm can be described as follows

(A) Place a grid on the asset space A such that the grid is finer than the one used to to compute the optimal decision rules.

(B) Set $i = 0$. Choose initial discrete density functions $f_0(\epsilon, a)$ where ϵ is the idiosyncratic shock and a the asset holdings evaluated on the constructed grid.

(C) Set $f_{i+1}(\epsilon, a) = 0$ for all ϵ and a. For every $a \in A$ and every idiosyncratic shock ϵ, compute the optimal next period wealth $a_{j-1} \leq a' \leq a_j$. In turn, for all $a' \in A$ and ϵ' compute the following sums

$$f_{i+1}(\epsilon', a_{j-1}) = \sum_{\epsilon} \sum_{\substack{a \in A \\ a_{j-1} \leq a' \leq a_j}} \pi(\epsilon'|\epsilon) \frac{a_j - a'}{a_j - a_{j-1}} f_i(\epsilon, a)$$

$$f_{i+1}(\epsilon', a_j) = \sum_{\epsilon} \sum_{\substack{a \in A \\ a_{j-1} \leq a' \leq a_j}} \pi(\epsilon'|\epsilon) \frac{a' - a_{j-1}}{a_j - a_{j-1}} f_i(\epsilon, a)$$

(D) Iterate until f converges.

This algorithm requires almost half computational time compared to the classical algorithm, where we aim to compute the invariant distribution via the computation of the inverse of the decision rule. Furthermore, computing the inverse of decision rule does not guarantee convergence in complex models like ours (where the optimal decision rules are computed via VFI and not Euler equation, due to the presence of the discrete default choice).

Step3: Calculate Aggregate Variables and Evaluate Market Clearance

Having calculated the stationary agents' distribution of assets we are able to find the aggregate variables in economy. We achieve that by taking the weighted sum using the above found distribution as weights. The evaluation of market clearing is just the evaluation of asset market clearing condition, just requiring the related error to be less than a very low value.

Evaluating the market clearance can be implemented using two methods, a bisection method and a grid search method. These algorithms are described in the following section.

Algorithm I - Bisection for Asset Market Clearing Bisection methods are used to solve function equations of the form $f(x) = 0$ for $x \in [a, b]$, where the function is assumed to be continuous and as long as $f(a)$ and $f(b)$ are of opposite sign, the intermediate value theorem says that there is a x^* in the interval of a to b such that $f(x^*) = 0$. Essentially, the bisection constructs a series of shrinking intervals $I_j, j = 1, 2, \ldots$ that bracket the solution to a desirable degree. The problem of finding an interest rate that would clear the asset market in our model can be naturally mapped to the bisection approach.

Continuous and monotonic asset demand and supply functionals imply that intervals of switching signs are easily obtained. Prices can be naturally adjusted according to excess demand or excess supply criterion. A higher interest rate would increase savings and thus asset supply while lowering capital demand and vice-versa.

The Bisection method unlike other approaches such as Newton-Raphson or Gauss-Seidel is derivative free and hence widely applicable and easier to implement. The purpose of this algorithm is to approximate a solution to the market clearing condition $K(r) - \ell'(r) = 0$ for interest rate r in the interval $[-\delta, 1/\beta - 1 + \delta]$. The following steps describe the algorithm used.

1. Number of price iterations is set to N, a convergence tolerance is set to a very low value ε and an error in convergence is defined as ϵ.

2. We start with an initial guess for the interest rate r_0 corresponding to the complete markets case.

3. Aggregate assets for both unconstrained and constrained type agents are obtained from the invariant distribution μ induced from the optimal savings policy function ℓ'.

4. We then compute the error in market clearance as $\varepsilon = |K(r) - \ell'(r)|$.

5. Check for convergence: If $\varepsilon < \epsilon$ then stop the bisection procedure and save results else go to the next step.

6. Update the interest rate according to the bisection rule:

$$r_1 = g(r_0) + (1 - g)((F_k(A(r_0)), N) - \delta)$$

7. Update initial interest rate $r_0 = r_1$ and then go back to step C and keep iterating until the convergence criterion is met.

It should be noted that in step 4, we have checked for quantities, instead we could check for convergence in interest rates directly. Both ways produces robust results as long as the tolerance criterion is low enough. The parameter g indicates the dmapening coefficient and for fairly general models a value of 0.5 would suffice. However, high dimensional models such as ours with discrete default choices makes the supply of assets a highly non-linear function extremely sensitive to price movements and thus require the g value to be endogenous. It should be noted that as the error, ε gets smaller and smaller, the dampening coefficient g needs to go higher and higher meaning that more of the old interest rate should be weighed. The solution to this is to let g be free for sufficiently big error values and once it gets close, the dampening should increase on the ϵ, i.e., such that $g'(1/\epsilon) > 0$. Instead of explicitly specifying a functional form, it is useful to experiment with several and choose the most optimal one.

An alternative although time consuming approach is to specify a grid of interest rates in the interval and find the market clearing through a costly grid search. An advantage of this approach is that the possibility of multiple equilibrium can be investigated. Our application of this computational procedure, however, did not produce any other feasible interest rates that could be a candidate for an equilibrium.

Algorithm II - Grid Search for Asset Market Clearing

1. Place a grid for interest rates[10] $r = 1, \ldots, nr$.

[10]There is no need the grid to be very fine in the beginning of the procedure, as we just need to find an approximate region where the price lies, even if this results to a larger error in the market clearance, than the one we aim to achieve.

2. Given r we solve the VFI to get optimal policy functions.

3. Given r we iterate on the distribution to find the agents' distribution.

4. Given r we integrate to get the aggregate variables and evaluate market clearance.

5. If we find an r which makes the error of market clearance relatively small(but even a little higher than 10^-4) we understand that an equilibrium could possibly exist in this price region.

6. We go back to step (A) and make the r grid smaller and more dense. In particular, having obtained a rough idea from step (E) where the interest rate lies, we construct a grid in this region with more grid points. In turn, we repeat the process form (B)-(E).

7. If market clearance conditions for two consecutive points on the r grid switch sign, the price which has the lowest absolute error is an ϵ-equilibrium price.

Chapter 3

The Global Financial Crisis: Causes and Effects

3.1 Introduction

The recent *global financial crisis* of $2008 - 2009$ made clear that economists and macroeconomic models should give a more prominent role both to financial shocks and financial frictions. Any attempt to understand the causes and effects of the financial crisis would require an analysis of the interaction between economic or financial shocks and the enforcement issues regarding financial contracts. Importantly, agent heterogeneity is essential to understand the adverse impact of these shocks on the distributional aspects of the economy.

It is widely believed that the balance sheet adjustment in the household sector was the prominent feature of the Great Recession. A considerable group of economists believe that balance sheet re-adjustment by households was the main cause that held back the cyclical recovery of the U.S. economy. Furthermore, this recession missed the usual business cycle recovery path and was characterized by a prolonged period of persistent output below its trend, see Kozlowski et al. (2019).

In this chapter, we construct a general equilibrium model, where households are ex-post heterogeneous by facing idiosyncratic earnings and unemployment risk, asset markets are endogenously incomplete due to limited enforcement of financial contracts and economy features production via a representative firm. *We use the same model as in the second chapter but we do not focus on the case of a stationary equilibrium where the distribution is invariant. Instead, we compute* *the non-stationary state of an economy and examine the transitional dynamics when it is subject to financial shocks.*

In this context, we attempt to answer the following two research questions:

1. How does a credit boom/credit crunch affect real activity of the economy?

2. What is the differential impact of a credit boom/credit crunch on the short-run versus the long-run? or Can a credit boom (crunch) result in an output drop (growth) in the long run?

Due to the presence of idiosyncratic shocks, households seek to smooth intertemporal consumption, subject to borrowing constraints. A financial intermediary operates in the economy by taking deposits and supplying loans. Deposits in this economic environment are safe and their return is the risk free rate. Contrariwise, loans are risky, since households every period can exercise their default option. Default is strategic and induces a temporary exclusion from the credit market. During the exclusion period households are credit constrained, but can trade along the post default path contracts that involve no credit. Essentially, they can supply deposits to the financial intermediary. Firms own capital, and households are indifferent between having deposits or investing in physical capital implying that the return to capital and the safe deposit rate are the same by arbitrage arguments.

Financial intermediaries treat every loan as a distinct financial asset and thus they price it differently. Price depends on the size of the loan as in our Chapter 2 and in Chatterjee et al. (2007). Furthermore, price also depends on the individual household characteristics, that is, in the employment-unemployment status of the individual household, as well as in its current earnings realization. When households default except for the exclusion of the credit market, they also face a wage garnishment similar to our previous chapter. To conclude, our model remains the same as in the second chapter with the only difference being that we are interested in out of steady state dynamics.

We solve our model computationally to find **a recursive competitive equilibrium with transition**, namely, an equilibrium where a sequence of aggregate prices, quantities and probability measures is defined. We study two types of shocks that trigger this transition and that both affect the households' borrowing capacity, and consequently the individual household's balance sheet.

Firstly, similar to Guerrieri and Lorenzoni (2017) we study an **unanticipated permanent drop in the households' borrowing limit**, that we call *credit crunch*. We model this unanticipated drop in the borrowing limit as a gradual process that takes place over a period of six quarters mimicking the duration of economic downturn in U.S.

Secondly, we model *credit easing* as an **unanticipated permanent expansion in the households' borrowing limit**. Similarly to the credit crunch, the growth in borrowing limit is a gradual process taking place over a period of twenty eight quarters or seven years. This reflects the long expansionary path followed by the U.S. economy during the years of $2000Q1 - 2006Q4$. We now turn to the contribution and main results of this chapter.

3.2 Contribution and Results

The way we model the shocks in borrowing limits is similar to the approach of Guerrieri and Lorenzoni (2017). Our model is a production economy with endogenous credit defaults while Guerrieri and Lorenzoni (2017) assumes no production and no default option. However, households in their framework can supply labour endogenously and secure debt using durable assets. Despite these differences, our results are consistent with their findings in the following way. We both find that a *credit crunch* will cause a recession in the short run. However, the presence of a positive supply of assets or capital in equilibrium means that for the agents in our model, the recession is only a short/medium run phenomenon while in the long run we observe an output growth. Precautionary motives arising from uninsured idiosyncratic risk are stronger in the long run implying higher savings/capital and higher output at the new equilibrium.

We rationalize this finding by disentangling the two effects on output, the "consumption effect" and the "capital and defaults effect". In short run the *consumption effect* is predominant, since consumption declines due to repayment incentives that are developed by the low wealth and income households in our model, output experiences a drop. In the same time capital experiences a boost, due to the increase in savings driven by precautionary motives, since the deposits in our economy represented by a one period discount bond with positive face value, through the asset market clearing condition become productive capital. This endogenous mechanism along with low defaults accompanied with less debt results in an output growth in the long run. This is the first result of this chapter.[1]

The *credit easing* shock triggers an economic boom, during which the "consumption effect" generated by easy credit and more borrowing, will cause an increase in output. In the short run, movements in consumption dominate the "capital and defaults mechanism". Default rates gradually increase, due to the rapid accumulation of debt, predominantly utilised by low-income and negative wealth households. The credit driven consumption will gradually collapse, since it is not be accompanied by an increase in economic fundamentals, namely increase in the productive capital in the economy. Households debt to GDP will almost triple during this period, while default rates will double, and households with negative net-worth will rise up to almost 18%. The "capital and defaults effect" will finally dominate since economies in the long run revert to fundamentals, and economy will experience a large scale recession of almost -1.6% while aggregate consumption will decline by almost -1.4%. This is the second result that we obtain in this chapter.

Our model thus generates both a short run boom (recession) and a long run recession (boom) from credit easing (crunch) shocks. We claim that the two mechanisms proposed, consumption and capital-defaults, can account for the causes and effects of both the pre-crisis and after-crisis period. The interaction of defaults and financial shocks, in the light of households' balance sheet readjustment, provides valuable insights useful for optimal fiscal/macro-prudential policy-making.[2]

[1]While examining the transition dynamics, we restrict our analysis to just equilibrium aggregates of the economy. It goes without saying that we can also report distributional changes like GINI indexes for wealth and income along the transition path. However, we focus here mostly on the aggregate level of economic activity rather than the distributional properties of the economy.

[2]Kozlowski et al. (2019) is a recent work that studies the persistence of GDP below pre-crisis trends after the Great Recession. The usual business cycle recovery was not observed in the after recession U.S. output data. These authors argue that agents are ignorant of the true distribution of the shocks and the implied learning dynamics results in output persisting below the trend levels for an extended period. Our stationary equilibrium model assumes no aggregate shock and thus examining growth trends is beyond the scope of this chapter. Our

The rest of the chapter is organised as follows. In Section 3.2 we review the literature related to credit crises, liquidity traps and financial shocks. In Section 3.3 we describe the model environment and we present the individual household's problems. We then turn to the problem of financial intermediary in Section 3.5. In Section 3.6 we define the recursive equilibrium with transition of our economy. Section 3.7 provides a brief description of the calibrated strategies and values that we used to computationally solve our model. We then describe in Section 3.8 the concept of the shocks to the borrowing limit and explain the adjustment path that these shocks are following. In Section 3.9 we provide and explain the results of this chapter both in the case of *credit crunch* and the case of *credit easing*. We conclude this chapter in section 3.10.

3.3 Literature Review

In the aftermath of the *global financial crisis* of 2008-2009, economists show increasing interest in understanding the dynamics of credit crises along with their resulting effects in aggregate and individual activity. In particular, economists are interested in the micro-level of the economy, that is, how households adjust their consumption, borrowing or saving behaviour in periods of credit crunch or periods of recessions and also credit shocks might affect the firms balance sheet and investment spending. There exists a large body of literature, which initiated a few decades ago and has been revisited following the event of *global financial crisis* trying to shed light on these questions.

In this section we review and present the literature related to credit crises, liquidity traps, recessions and generally the interaction between credit constraints, asset prices and thus the macro economy. Different studies follow various approaches to address this set of questions. In the rest of this section we discuss the various mechanisms and interactions proposed by the literature.

Kiyotaki and Moore (1997) study the role of credit constraints and their interaction with the aggregate economy. They propose a mechanism that links the

model enriched with an aggregate productivity shock in the lines of Krusell and Smith (1998) might be better able to deal with this.

credit limits and the returns on assets. They find that via this mechanism amplified spill overs to different sectors of the economy might be triggered. Their model exhibits limited commitment, since borrowers cannot force lenders to repay their debts, unless these debts are collateralized.

Their economy features *credit constrained* and *non credit constrained* firms. Land is in fixed supply and serves a twofold role. Firstly, can be used as production factor and secondly can be used as a collateral to secure a loan. More importantly when they introduce lumpy investment to firms, economy adjusts in a slower pace while exhibiting damped oscillations. Kiyotaki and Moore (1997)'s approach gives a good flavour of the effects that a credit crunch could have in the aggregate economy.

Bernanke et al. (1999) introduced the concept of financial accelerator in a business cycle framework, aiming to highlight the role of credit market frictions in aggregate fluctuations. Their framework departs from the underlying assumptions of the famous Modigliani and Miller (1958) theorem, which in short implies that the structure of financial markets is irrelevant to the real economic activity. An alternative view has been proposed by Gertler and Hubbard (1988). They suggested that credit market conditions might have significant effects in economic activity. For instance, an increase in bankruptcies or increasing debt in some economic sector might cause recessions and affect the real economy in multiple ways.

These ideas have been revisited by Gertler and Hubbard (1988), Bernanke (1983) and Bernanke et al. (1999). Nevertheless, they date well back in the past, as early as the Great Depression. For instance, Fisher (1933) approached the causes of the Great Depression, proposing that the malfunction of financial markets often results in over-indebtedness and can cause great output declines, that are followed by a period of deflation.

Bernanke et al. (1999) suggest that the mechanism propagating and amplifying shocks to macroeconomic activity is the link between the *external finance premium* and the *borrowers' net worth*. They explain that external finance premium will be *countercyclical*, thus generating investment, consumption and production swings. Finally, by introducing price stickiness into their model further analyse the effects of monetary policy, in an economy that exhibits credit market frictions.

Earlier studies tried to comprehend the effects of *global financial crisis*, by introducing different mechanisms and choosing alternative stances towards the sectors that contributed to it. Brunnermeier (2009) analyses financial crisis from various perspectives. He suggests that the series of events unfolded during the crisis period, emerged due to a series of financial shocks that amplified leading to a liquidity crunch and thus a full-fledged financial crisis. Indeed, raising money by selling assets was difficult during the crisis period.

Brunnermeier (2009) defines this concept as *market liquidity* while also highlights the role of *funding liquidity*. He claims that even a small shock can cause a liquidity freeze, while he also discusses the effects on the borrowers' balance sheet. However, it was not only borrowers but also lenders that were affected during the financial crisis. As a result of the lower capital supply, lending was restricted. Banks had fears that they will also face interim shocks, thus adopted the strategy of precautionary hoarding, which finally led to a sharp increase in the interbank market rate and a *credit crunch*.

Eggertsson and Krugman (2012) precisely show that in periods of a credit crunch or equivalently *forced rapid deleveraging*, aggregate demand is enfeebled. In particular, they construct a flexible endowment economy, in which *patient agents* lend *impatient agents*, while also a debt limit is introduced. They explore the effects of a sudden decrease in the debt limit. They find that net borrowers are forced to decrease their spending which will eventually drive the economy towards the zero lower bound.

However in their paper, borrowing is caused by patience and impatience and not by idiosyncratic shocks, as in the seminal work of Bewley (1983), Huggett (1993) and Aiyagari (1994). Furthermore, Eggertsson and Krugman (2012) explain how deleveraging can affect both prices and output. Interestingly in their paper, deleveraging implies larger drop in investment and durable consumption as compared to regular consumption.

Along the same lines but using a different theoretical model, for instance Goldberg (2011) studies the effects that a limited firm's borrowing ability would have on the economy. In particular, the author studies the effects of credit crunch on an economy that features uninsured idiosyncratic investment risk. Under these circumstances, consumption becomes riskier and the risk free rate declines. Furthermore, with a relatively high elasticity of intertemporal substitution, the credit shock causes a drop in aggregate capital. The model further features limited enforcement and the set of financial contracts is constrained. Due to this, collateral constraints arise wherein entrepreneurs require a *high equity premium*.

From the perspective of the households' balance sheet, Guerrieri and Lorenzoni (2017) study the consequences of credit tightening on consumer spending in the framework of a heterogeneous agents and incomplete markets economy. They assume that credit crunch occurs by the means of an unanticipated cut in the borrowing capacity of consumers. As a result, those that are heavily indebted and close to the borrowing limit, are forced to repay their debts, while households that are not constrained are characterized by a precautionary savings behaviour. The mechanism works through the indebted consumers in their model, that are forced to readjust their positions to lower levels of debt.

In this environment, they firstly analyse the dynamics of interest rates and find that an initial fall takes place, while later a lower steady state is attained. This is because the agents located to the lower part of the distribution set a higher wealth target and increase their savings. Secondly they examine the responses of aggregate economic activity, particularly the output response. Guerrieri and Lorenzoni (2017) find that in the setting of flexible prices output decreases by 1.1% as a response to the credit crunch. However, throughout their approach they do not take into consideration **defaults and bankruptcy** that are clearly very important features during the adjustment to an environment with a tighter credit limit.

Hall (2011) also highlights the importance of tightening the households borrowing constraint as a means to explain and understand the financial crisis of 2008 − 2009 in the United States. He argues that in a market-clearing economy, when demand from one specific sector declines, output does not decline since other sectors might expand. However, in cases where the economy experiences low inflation, interest rates cannot lower, due to the presence of zero lower bound.

In the Great Depression, the presence of deflation caused a hike in interest rates while in the Great Slump, commenced the last months of 2007, low inflation resulted in a slightly negative real interest rate, even if the presence of unemployment and the anaemic demand, were asking for a much lower interest rate in the economy. In the above discussed framework Hall (2011) concentrates on three different, yet interrelated factors that could possibly cause declines in demand. These are the excess accumulation of consumer and housing durables, the rise in consumer debt that was financing this accumulation, and finally the various financial frictions that emerged from the decline in prices.

Mian and Sufi (2011) also attempt to explain the household leverage crisis in the United States by examining the home-equity based borrowing channel. They use a dataset which comprises from individual credit files and follow a random sample of around $74,000$ home owners. The data used span the period of 1997 to 2008. They estimate how borrowing of households that owned a house responded to the rise in house prices, aiming to identify the subset of homeowners that experienced the sharpest response.

Mian et al. (2013) investigate the consumption channel of housing market collapse in the United States. They pose the following question: What is the response of consumption when households face high negative shocks to their net worth? Additionally they aim to explain how households located in different positions across the wealth distribution experience different marginal propensities to consume, when they loose one dollar. They suggest that a representative agent model is not able to capture the salient features of data. For instance, marginal propensity to consume declines in wealth, and heterogeneity matters. Mian et al. (2013) answer these questions to shed light on times of severe recessions. They show that more leveraged households experience higher marginal propensities to consume.

Furthermore, Mian and Sufi (2014) explain via the housing net worth channel that the huge shrinkage of households balanced sheets resulted in an employment decline in the United States during the period of 2007 to 2009. In particular the employment to population ratio dropped from 63% in 2007 to 58% in 2009 which essentially means that around nine millions jobs were lost.

Their empirical approach is the first that takes the advantage of a detailed cross-sectional variation to capture the employment effects due to deterioration in the household balance sheet. Their work serves as a pillar for other theoretical studies, that explore and explain the decline in real activity via the weakening of household balance sheet. A few related studies are those previously discussed, that is, the paper of Guerrieri and Lorenzoni (2017), the study of Eggertsson and Krugman (2012) and the study of Hall (2011).

Other related studies is the paper of Farhi and Werning (2016) and the study of Midrigan and Philippon (2011). Farhi and Werning (2016) put forward a theory of macro-prudential and monetary policy in financial markets. Their study focuses on economies, where monetary policy is constrained from the zero-lower bound and feature nominal rigidities in goods and labour markets.

Essentially their paper provides an appropriate framework that combines monetary and macro-prudential policies. The motivation of the paper stems from the fact that when market incompleteness is introduced into an Arrow and Debreu (1954) framework, creates a pecuniary externality, that cannot be internalized by agents, and thus the **equilibrium allocations** in general **are constrained inefficient**. In this sense, there is room for improvement. Hence, interventions are meaningful and macro-prudential policies have a role.

Midrigan and Philippon (2011) suggest that regions having experienced the largest changes in household leverage, are those that have also experienced the highest declines in output. They model an economy with a cash in advance constraint, in which collateralized borrowing and public money are used for transactions.

In this framework house prices constrict the cash in advance constraint and this causes recessions.Interestingly, their findings suggest that real activity is very vulnerable in liquidity shocks. Monetary policy in their model can be exercised to mitigate the severity of recessions caused by credit.

In particular their model is built along the lines of *Lucas Islands Model* (see Lucas (1972) and Lucas (1973)), that is, there exists a continuum of islands that trade between each other. Every island can produce both tradable and non-tradable goods, while tradable goods are imperfectly substitutable.

Their model departs from other standard cash-in-advance models in the literature, in the sense that except for the supply of public money, households enjoy an extra line of credit. Furthermore, a collateral constraint is incorporated into the model, in the sense that households are allowed to borrow up to a certain fraction of their house value.

In this manner, houses play a twofold role. They firstly provide liquidity and secondly housing services. In this model house prices have an effect on the level of nominal balances that are utilised to finance consumption. This is exactly the channel via which nominal credit shocks can generate business cycles.

Their model predicts a decline in employment of 5.5%, a decline in non-durable consumption of 3.8% and durable consumption of 14%, that successfully matches the relevant values in the U.S. economy. Their last contribution is the extension of their analysis towards capturing the role of credit constraints and not only liquidity constraints.

Gertler and Kiyotaki (2010) built a canonical framework to study the effects of credit market frictions in real activity. Furthermore, they explore the role of credit market interventions to access their effectiveness in the mitigation of a crisis. They suggest that there are two aspects of financial crises that have not been captured in the literature.

Firstly, the literature developed prior to financial crisis, put more emphasis on credit market constraints of borrowers, excluding the financial sector borrowing. Secondly, it has been observed that during the crisis period, various unconventional policy measures have been employed, as a way to tackle the effects of an economic downturn and the financial crisis itself. These policies do not resemble the traditional approaches followed before the crisis period. In particular, central banks supplied imperfectly secured loans to financial intermediaries, while fiscal authorities in cooperation with the central banks were giving liquidity injections into "big" banks aiming to improve the availability of credit in the market.

A lot of economists around the world argue that these novel policies halted the rapid decline of economic activity. Taking this newly developed economic and policy environment into account, Gertler and Kiyotaki (2010) developed a model that incorporates financial intermediation in business cycles absent of frictions, addressing the following questions. Firstly, how disruptions in financial intermediation can trigger a financial crisis that will cause effects in real activity. Secondly, how policy interventions conducted by the central bank, could work in toning down the direct and indirect effects of a crisis.

Del Negro et al. (2011) introduce liquidity frictions into a DSGE model, that features nominal and real rigidities. They question whether a shock in liquidity can explain the decline in short-term interest rates and the resulting recession. In this framework they try to shed light into 2008 financial crisis and the recession that followed soon after that. Their paper is a departure from irrelevance result introduced by Wallace (1981), by incorporating frictions introduced by Kiyotaki and Moore (2012). They suggest that both the shock in liquidity and also liquidity policy can have large effects. In particular, the shock to the *"resaleability constraint"* accounts for more than half of the output drop observed in data, while fully explains the drop in inflation. In absence of any intervention they find that recession would be even larger.

On the contrary, when non- standard open market operations are also considered economy "escapes" from recessions. Interestingly, when they introduce flexible prices, financial frictions can only generate a drop in investment, while output does not change, since consumption counterbalances the drop in investment. This consumption boom forces the natural rate of interest to drop substantially while the loss of liquidity raises the premium that agents are willing to pay. Thus unconventional policies take a role, since they can directly target the loss of liquidity paper.

Justiniano et al. (2015) suggest that the leveraging and deleveraging cycle observed in $2000 - 2007$ and 2008 onwards respectively, cannot be explained by the relaxation and tightening of the loan to value ratio in mortgage markets. They arrive at this conclusion by a DSGE model, calibrated using micro data from the Survey of Consumer Finances.

Their model suggests that direct effects in house prices can explain the credit cycle. Interestingly, they find that aggregate effects of households' leveraging and deleveraging are minor, since the responses of borrowers and lenders fade out in the aggregate level.

Justiniano et al. (2015) adds on the debate on the causes and effects of leveraging cycles. In particular, their model features heterogeneous households where borrowing and lending gives a role to debt. Motivated by the U.S. economy characteristics where debt was primarily held via mortgages, they add collateral constraints that force debt to be a fraction of the house value. They focus on two main drivers of the leveraging cycle, that is, firstly a change in credit limits, given the value of houses, and secondly a change in the house value for a given credit limit. They deduce that a large increase in house prices will lead to a considerable rise in debt. When house prices fall, collateral value features a dive, while debt does not decrease. Therefore, the debt to collateral ratio increases as observed in the data in United States. Finally, they conclude that the macroeconomic effects of a leveraging cycle are relatively small, independent of the source of the shock.

Christiano et al. (2014) augment a monetary DSGE model by introducing a BGG (see Bernanke et al. (1999)) mechanism. They fit the model to the U.S. data allowing for a time-varying cross sectional idiosyncratic uncertainty. They draw the important conclusion, that is, the most important factor that drives the business cycle is the **variation of risk**.

More specifically they introduce the concept of effective capital, by assuming that when an entrepreneur purchases K units of capital, then Kw units of the initial capital become effective. They name the random variable w an experience shock. When w is realized the financial intermediary cannot observe the realization, except if it decides to undertake monitoring, that is obviously costly. They define risk to be the cross-sectional standard deviation of the logarithm of w. Fluctuations in this risk trigger responses of their economy that mimic the business cycle. They find that the degree of cyclical measures of uncertainty are very similar in both data and the model.

To conclude our literature review section, we discuss a recent influential study by Mian and Sufi (2016) that supports and provides new evidence towards the **credit supply view** of financial crisis and the consequent recession in the United States. They suggest that the large increase in credit supply, not accompanied by an analogous evolution of the economic fundamentals or productivity caused the household boom and bust. Supporting the supply view,try to explain the default crisis as mainly driven by households that had very bad credit score.

The boom that U.S. experienced from 2000 to 2007 ended up in an unprecedented **delinquency rate** on debt that has **surpassed** 10%. This is partly explained due to distributional issues of the financial sector, when transforming savings from individuals located in the higher end of the wealth distribution into loans to individuals belonging to the lower part of the wealth distribution. Their approach is clearly in contrast to the *passive view* towards the financial sector. This essentially states that finance and capital structure plays no role. The *credit supply view* suggests that equity-based contracts might work towards the reduction of the magnitude of the real estate booms and thus make busts less dramatic.

Naturally, this gives a further support in views discussed throughout this section that assign role to macro-prudential policies that for instance target household-debt-to-GDP ratios. Theoretical work that supports this argument had been done by Farhi and Werning (2016) or Korinek and Simsek (2016).

Mian and Sufi (2016) use a number of data sets to perform their analysis. The main data set is the individual level *Equifax Credit Bureau Data*. This dataset is based on a 0.45% random sample of individuals in 1997 that were residing in zip codes, in which the *Fiserv Case Shiller Weiss* data are also available. The aggregate debt patterns that for this specific sample match fairly well the aggregate debt from the Federal Reserve Flow of Funds.

They conclude that asset price bubbles depend on the evolution of credit growth and support the causal relationship of this credit expansion in explaining the $2008 - 2009$ financial crisis, preceded by a steep increase in house prices. This credit expansion played a very important role in the default crisis that was observed during the recent economic downturn.

However, they admit that there are still open questions regarding the fundamental driver that boosted credit supply. Different stands exist in the literature that explain the increase in credit supply adopting alternative approaches. Some researchers support the idea of the increase in global savings, alternatively called the *"Global Savings Glut"*. Some others, such as, Shin (2012) give alternative explanations. Shin (2012) states that the culprit for easy credit before the burst of financial crisis in United States is driven by the *"Global Banking Glut"*. Research by Rey (2015) or Bruno and Shin (2015) propose that monetary policy might also serve as an important driver of rapid changes in credit supply. In this chapter we try to shed light on some of the issues discussed throughout this literature review.

3.4 The Model Environment

Consider an economy with infinite horizon that is populated by a continuum of households that face two different types of risks, that is, countercyclical unemployment risk and conditional on being employed acyclical earnings risk. Households are infinitely lived and uniformly distributed over $[0, 1]$. Time in our model is discrete and thus indexed by $t \in (0, 1, \ldots, \infty)$.[3]

At the beginning of time all households are identical, that is, ex-ante heterogeneity is beyond the structure of our model in this chapter. It is important to note that households under the unemployment status receive unemployment benefits. These benefits, following Krueger et al. (2016) are defined as a fraction of each household's potential earnings growth. In this sense, unemployed households carry with them the idiosyncratic income state even though it does not really affect their current labour earnings, since they are under the unemployment status.

Households have time separable preferences over streams of consumption, while deriving utility solely from consumption, since they do not value leisure and the time endowment is normalized to unity. Thus, households' preferences are represented by the following utility function:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_{it}) \right], \tag{3.1}$$

where for each period t, c_{it} denotes consumption of household i and is restricted to belong in $\mathcal{C}$, that is, the per period consumption feasibility set of any individual household specifying non-negative consumptions.

The discount factor $\beta \in (0, 1)$ is common across all households and the period utility function $u(c_{it})$ is continuous, concave, strictly increasing, continuously differentiable and satisfies the Inada conditions. The functional form assumed for

[3]Unlike the previous two chapters, all variables here are time-varying and thus indexed with their time indexes.

the period utility in our model is a typical $CRRA$, that is characterised by constant relative risk aversion and can be expressed by:

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma}, \tag{3.2}$$

where the parameter σ measures the degree of relative risk aversion, embedded in our utility function.

We denote the employment status of each household i in the economy as $s \in \mathcal{S} = \{u, e\}$ where u stands for unemployment and e represents employment. Therefore, the associated transition probabilities can be defined as $\pi(s_{t+1} \mid s_t)$ consistently to the underlying Markov chain. We further assume that transition probabilities are *iid* across all households in our economy.

We now turn to idiosyncratic labour earnings risk, that every employed household in our economy faces. We denote it, by $y \in \mathcal{Y}$ and it follows a 12 state first order Markov process, with *finite support* $\mathcal{Y}$. Let now $\pi(y_{t+1} \mid y_t)$ denote the associated transition probabilities that describe the transition from the current to the future earnings state. These probabilities are *iid* across all households in the economy and also independent from the corresponding transition probabilities related to *unemployment risk*. This assumption is important for the computational tractability of our model. We should further assume that a *low of large numbers holds*[4] so that our formulation is *well-defined*.

In the economy operates one representative firm that produces output every period, utilising a neoclassical production function

$$Y_t = F(K_t, N_t), \tag{3.3}$$

where in every period t, Y_t denotes the total output, K_t represents the capital input, and N_t denotes labour input.

The representative firm takes factor prices w_t and r_t as given and chooses optimally K_t and N_t. The functional form that we used for our production function

[4]see Uhlig (1996).

is a typical *Cobb-Douglas* that features constant returns to scale and can be represented by the following expression:

$$Y_t = K_t^{\alpha} N_t^{1-\alpha}, \tag{3.4}$$

where α captures the share of capital income in output. We further assume that capital depreciates every period, when used for production at a constant rate $\delta \in (0, 1)$. Every period production takes place, and a single good is being produced that can either be consumed or invested.

Our economy also features a *risk neutral financial intermediary.* The intermediaries role is trivial, namely it collects savings, in the form of deposits from households that are able and willing to save, while giving loans to households that are able and willing to borrow. The intermediary in our economy makes zero profits.

In this manner, households are facing *borrowing* and *lending* opportunities in an economy that is characterized by *perfectly competitive markets.* Different size loans are supplied by the intermediary to households. These loans are assumed to be *unsecured*, in that no collateral is pledged, since our model does not feature, for instance, houses or land.

Each *unsecured* loan supplied by the intermediary is considered as a distinct financial asset. This assumption is similar to Chapter 2, and closely related to Chatterjee et al. (2007).

Households every period solve their constraint maximization problem and decide the following policy functions. In the beginning of the period, idiosyncratic shocks are realized and then households decide how much they consume, how much they will borrow or save, and conditional on carrying a loan from the previous period, households decide if they will *default* on their loan or not. In the framework of this chapter, default is only *strategic* and cannot occur by luck.

Due to the presence of limited commitment in our model, intermediaries face default risk. More importantly default risk varies with the size of the loan and the *set of idiosyncratic states* of each household, defined by their earnings and unemployment status.

Trade amongst households and the financial intermediary takes place via an *one-period non-contingent discount bond* with a face value defined in the finite set $\mathcal{L} \subset \mathbb{R}$. A purchase of a discount bond with positive value $\ell_{t+1} > 0$ implies that household i entered into a contract under which, household will receive with probability one, ℓ_{t+1} units of period $t+1$ consumption good. This holds because an implicit assumption of our model's structure is that depositing in the financial intermediary does not involve any risk, that is, deposits are secure.

A purchase of o discount bond with negative face value $\ell_{t+1} < 0$, individual labour earnings y_t and employment-unemployment state s_t implies that household i has entered into a contract under which, household will receive $q(\ell_{t+1}, y_t, s_t)(-\ell_{t+1})$ units of period t consumption good, promising to deliver, conditional on not defaulting $-\ell_{t+1} > 0$ units of period $t+1$ consumption good.

The financial assets available in the market are $N_{\mathcal{L}} \cdot N_{\mathcal{Y}} \cdot N_{\mathcal{S}}$ where N denotes the cardinality of the associated sets. We further assume that every household i is indifferent between investing in physical capital or depositing in the the financial intermediary, namely investing in a bonds with positive face value.

The choice of default induces a punishment that similarly to our previous chapters is modelled as a temporary exclusion from the credit market. Households that choose to default at current period t, will remain in the *credit-constrained state* with exogenous probability θ and thus will restore full access to financial markets with probability $1 - \theta$.

Following the default event, households are allowed to trade contracts that involve no-credit. In other words they are allowed to save, in the form of deposits with the intermediary but they are not allowed to borrow. When the individual household defaults the outstanding amount of his/her loan is erased from his/her budget constraint. However, from this period onwards the individual household faces a loss of income, that takes the form of a wage garnishment. In particular every period a specific fraction of household's labour income denoted by $\gamma \in (0, 1)$ will be confiscated and the remaining fraction of the household's labour income will remain disposable for consumption and investment.

Financial markets are endogenously *sequentially incomplete* (SIM) due to the presence of limited commitment and the aggregate recourse constraint reads every period

$$F(K_t, N_t) = C_t + I_t = C_t + K_{t+1} - (1-\delta)K_t, \tag{3.5}$$

where C_t denotes aggregate consumption at period t, K_t represents the aggregate capital stock at period t, K_{t+1} denotes the aggregate capital stock at period $t+1$, N_t is the labour input at period t, and δ denotes the depreciation rate of capital when used in production. All aggregate variables are denoted by capital letters.

This setting is a generalization of the model of Guerrieri and Lorenzoni (2017) in two ways. Firstly their model does not account for capital and they just set bond supply exogenously, calibrating it as the sum of all liquid assets hold by the household sector.

Secondly, their model lacks a crucial feature that could help us in understanding the effects of a credit crunch in the aggregate level, and this is the presence of limited commitment, namely the option of a household to default.

Finally, our model does not aim only to shed light in periods of credit crunch, but also in periods of credit expansion, which could give us useful insights in understanding how we ended up in the financial crisis of $2008 - 2009$.

3.5 The Household's Problem

Households solve a typical maximization problem, that is, they maximize their expected discounted lifetime utility, subject to the associated individual budget and borrowing constraints. Households take prices as given and they can find themselves being into two distinct endogenous states.

Firstly, a household can be *unconstrained*, namely it has full access to financial markets and secondly a household can be *credit constrained*, that is the household has defaulted in some past period and currently does not have access to the credit market, in other words households cannot borrow.

In what follows we present two distinct optimization problems, each of them related to the *unconstrained* and *credit constrained* households respectively.

3.5.1 The Unconstrained Household

The states for an unconstrained household can be described by his level of loans or deposits denoted by $\ell_t \in \mathcal{L} \subset \mathbb{R}$, his labour earnings draw denoted by $y_t \in \mathcal{Y} \subset \mathbb{R}^+$ and his employment-unemployment draw denoted by $s_t \in \mathcal{S} \in [0,1]$.

We denote by $V_t^R(\ell_t, y_t, s_t)$ the value function of a household that repays his loan at period t. We denote by $V^D(0, y_t, s_t)$ the value of default at period t and finally, we denote as $V^o(\ell_t, y_t, s_t)$ the value of the option to be *unconstrained* in the current period.

Let also c_t denote consumption in the current period and ℓ_{t+1} represent the amount of loans or deposits that a households optimally chooses. Furthermore, w_t is the household wage at time t, $q(\ell_{t+1}, y_t, s_t)$ is the price for a loan or a deposit, while $\mathbb{1}_{s_t=u}$ is an indicator function which takes the value 1 when household is in the unemployed state[5] and 0 otherwise. Following these definitions the unconstrained household's problem could be presented as follows:

$$V_t^R(\ell_t, y_t, s_t) = \max_{\{c_t \geq 0, \ell_{t+1}\}} \Big\{ u(c_t) + \beta \sum_{(y_{t+1}, s_{t+1}) \in (\mathcal{Y}, \mathcal{S})} \pi(s_{t+1}|s_t)\pi(y_{t+1}|y_t) V_{t+1}^o(\ell_{t+1}, y_{t+1}, s_{t+1}) \Big\} \tag{3.6}$$

subject to

$$\begin{gathered} c_t + q_t(\ell_{t+1}, y_t, s_t)\ell_{t+1} = w_t y_t \left[1 - (1-\rho)\mathbb{1}_{s_t=u}\right] + \ell_t \\ \ell_{t+1} \geq -\overline{B} \end{gathered} \tag{3.7}$$

It is useful to note that, unlike in the notation followed in our previous chapters, *the value and policy functions are also a function of time since aggregate prices are time-varying.*

[5] In this case labour earnings are equal to the unemployment benefits, which are defined as a fraction of the average labour earnings, thus $b(y) = \rho w y$.

3.5.2 The Value of Default

Prior to presenting the problem associated to the value of default, we should denote the value function of *credit constrained* households as $V_t^{cc}(\ell_t, y_t, s_t)$. This is the value function associated with a household that has defaulted some previous period and is currently credit constrained. Following these definitions the problem associated with the value of default could be presented as follows:

$$V_t^D(0, y_t, s_t) = \max_{\{c_t \geq 0, \ell_{t+1} \geq 0\}} \left\{ u(c_t) + \beta \sum_{(y_{t+1}, s_{t+1}) \in (\mathcal{Y}, \mathcal{S})} \pi(s_{t+1}|s_t)\pi(y_{t+1}|y_t) V_{t+1}^{cc}(\ell_{t+1}, y_t, s_t) \right\} \tag{3.8}$$

subject to

$$\begin{aligned} c_t + q_t(\ell_{t+1}, y_t, s_t)\ell_{t+1} &= [1-\gamma] w_t y_t \left[1 - (1-\rho)\mathbb{1}_{s_t = u}\right] \\ \ell_{t+1} &\geq 0 \end{aligned} \tag{3.9}$$

At any given time, an *unconstrained* household can choose either to repay or to default. Therefore, the *value of the option to default* can be described by the following equation.

$$V_t^o(\ell_t, y_t, s_t) = \max \left\{ V_t^R(\ell_t, y_t, s_t), V_t^D(0, y_t, s_t) \right\}, \tag{3.10}$$

where $V_t^R(\ell_t, y_t, s_t)$ is the value associated with repayment and staying in the contract, $V_t^D(0, y_t, s_t)$ is the value associated with defaulting, while $V_t^o(\ell, y_t, s_t)$ as clearly shown can take either of the two values, representing the value of the option accordingly.

3.5.3 The Credit Constrained Household

Finally we describe the problem of a credit constrained household, that is, a household that have defaulted in some previous period and is currently excluded from the credit market. We denote by θ the exogenous probability related to the household remaining in the *credit constrained state*. Therefore, the problem of *credit constrained* households could be presented as follows:

$$V_t^{cc}(\ell_t, y_t, s_t) = \max_{\{c^t \geq 0, \ell_{t+1} \geq 0\}} \Big\{ u(c_t) + \beta \sum_{(y_{t+1}, s^{t+1}) \in (\mathcal{Y}, \mathcal{S})} \pi(s_{t+1}|s_t)\pi(y_{t+1}|y_t) \\ \theta V_{t+1}^{cc}(\ell_{t+1}, y_{t+1}, s_{t+1}) + \\ (1-\theta) V_{t+1}^{o}(\ell_{t+1}, y_{t+1}, s_{t+1}) \Big\} \tag{3.11}$$

subject to

$$\begin{aligned} c_t + q_t(\ell_{t+1}, y_t, s_t)\ell_{t+1} &= [1-\gamma] w_t y_t \left[1 - (1-\rho)\mathbb{1}_{s_t = u}\right] + \ell_t \\ \ell_{t+1} &\geq 0 \end{aligned} \tag{3.12}$$

3.6 The Financial Intermediary

The role of the financial intermediary is the same in Chapter 2, and resembles the role of intermediary in the study of Chatterjee et al. (2007). The main difference is that in our economy the intermediary does not sell capital to firms.

The intermediary chooses the number of contracts $n_{\ell_{t+1}, y_t, s_t} \geq 0$ of type (ℓ_{t+1}, y_t, s_t) to sell in each period, so to maximize the present discounted value of current and future cash flows. The maximisation problem can be presented as follows:

$$\max \sum_{t=0}^{\infty} \frac{1}{1+r-\delta} \pi_t \tag{3.13}$$

where π_t denotes profits at time t. Every period cash flow is given by the following equation:

$$
\begin{aligned}
\pi_t = \ell_{t+1}^{+} - (1+r-\delta)\ell_t^{+} + \sum_{\ell_t, y^{t-1}, s^{t-1}} n_{\ell^-_{t,y^{t-1},s^{t-1}}} \ell_t^{-} (1 - p_{\ell_t, y^{t-1}, s^{t-1}}) \\
+ \sum_{\ell_t, y^t, s^t} q_{\ell_{t+1}, y^t, s^t} n_{\ell^-_{t+1,y^t,s^t}} \ell_{t+1}
\end{aligned}
\tag{3.14}
$$

The probability that a loan of type (ℓ_{t+1}, y_t, s_t) where obviously $\ell_{t+1} < 0$ will default is $p_{l_t, y_{t-1}, s_{t-1}}$. Deriving the first order conditions with respect to n_{ℓ_{t+1}, y_t, s_t} we get the following result:

$$
q_{\ell_{t+1}, y_t, s_t} = \begin{cases} \dfrac{1}{1+r-\delta} & \text{if} \quad \ell_{t+1} \geq 0 \\ \\ \dfrac{(1 - p_{\ell_t, y_{t-1}, s_{t-1}})}{1+r-\delta} & \text{if} \quad \ell_{t+1} < 0 \end{cases}
\tag{3.15}
$$

This completes the problem of the financial intermediary in this economy.

3.7 Recursive Competitive Equilibrium with Transition

In this section we provide a description of the equilibrium for this economy, considering also the time dimension of the transition. Since the transition is characterized by a sequence of aggregate prices and quantities, the definition of a *recursive competitive equilibrium* as described in the previous chapters should be modified.

Description of Equilibrium

Given a sequence of interest rates and wages $\{r_t, w_t\}_{t=0}^{\infty}$, a non-negative sequence of loan prices $\{q_t\}_{t=0}^{\infty}$ and a non-negative sequence of defaults $\{p_t\}_{t=0}^{\infty}$, a *recursive competitive equilibrium* is a sequence of value functions $\{V_t^R, V_t^D, V_t^{cc}, V_t^o\}_{t=0}^{\infty}$ and optimal policy functions $\{\ell_{t+1}, d_t, c_t\}_{t=0}^{\infty}$ for *unconstrained households*, a sequence of optimal policy functions $\{\ell_{t+1}, c_t\}_{t=0}^{\infty}$ for *credit constrained households*, optimal firm choises $\{L_t, K_t\}_{t=0}^{\infty}$ and a set of distributions $\{\mu_t^{uc}, \mu_t^{cc}\}_{t=0}^{\infty}$ such that for all t:

(i) Given prices $\{r_t, w_t, q_t\}_{t=0}^{\infty}$, the policy functions $\{\ell_{t+1}, d_t, c_t\}_{t=0}^{\infty}$ solve the *unconstrained households* problem and the policy functions $\{\ell_{t+1}, c_t\}_{t=0}^{\infty}$ solve the *credit constrained households* problem.

(ii) Given prices $\{r_t, w_t, q_t\}_{t=0}^{\infty}$, the firm chooses optimally its capital K_t and its labour L_t and the first order conditions imply:

$$w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^{\alpha}$$
$$r_t = \alpha\left(\frac{K_t}{N_t}\right)^{\alpha-1} - \delta$$

(iii) The *labour market* clears and N_t is given by

$$N_t = (1-\Pi(u))\sum_{y\in\mathcal{Y}} y\Pi(y)$$

(iv) The *goods market* clears

$$F(K_t, L_t) = C_t + K_{t+1} - (1-\delta)K_t - \gamma w_t \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} ys \quad d\mu_t^{cc}$$

where C_t is given by

$$C_t = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} c_t(\ell_t, y_t, s_t)d\mu_t^{uc} + \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} c_t(\ell_t, y_t, s_t)d\mu_t^{cc}$$

(v) The *asset markets* clears and K_{t+1} is given by

$$K_{t+1} = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} \ell_{t+1} d\mu_t^{uc} + \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} \ell_{t+1} d\mu_t^{cc}$$

(vi) For all $\mathcal{L} \times \mathcal{Y} \times \mathcal{S}$ the probability measure μ_{t+1}^{uc} satisfies

$$\mu_{t+1}^{uc} = \int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} Q_t((\ell_t, y_t, s_t), \mathcal{L} \times \mathcal{Y} \times \mathcal{S}) d\mu_t^{uc}$$

where Q_t is the transition function defined as

$$Q_t((\ell_t, y_t, s_t), \mathcal{L}\times\mathcal{Y}\times\mathcal{S}) = \sum_{y_{t+1}\in\mathcal{Y}} \sum_{s_{t+1}\in\mathcal{S}} \mathbb{1}\{\ell_{t+1}(\ell_t, y_t, s_t) \in \mathcal{L}\}\pi(s_{t+1}|s_t)\pi(y_{t+1}|y_t)$$

(vii) For all $\mathcal{L}^+ \times \mathcal{Y} \times \mathcal{S}$ the probability measure μ_{t+1}^{cc} satisfies

$$\mu_{t+1}^{cc} = \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} Q_t((\ell_t, y_t, s_t), \mathcal{L}^+ \times \mathcal{Y} \times \mathcal{S}) d\mu_t^{cc}$$

where Q_t is the transition function defined as

$$Q_t((\ell_t, y_t, s_t), \mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}) = \sum_{y_{t+1}\in\mathcal{Y}} \sum_{s_{t+1}\in\mathcal{S}} \mathbb{1}\{\ell_{t+1}(\ell_t, y_t, s_t) \in \mathcal{L}^+\}\pi(s_{t+1}|s_t)\pi(y_{t+1}|y_t)$$

(viii) The first order conditions for the *financial intermediary* imply that:

$$q_t = \frac{(1 - p_t)}{1 + r_t - \delta}$$

3.8 Calibration

This model is characterized by inherent non-linearity which makes its analytical solution impossible. Therefore we construct a numerical algorithm to find the initial and final equilibrium of the economy and then solving backwards we can compute the transitional dynamics of our economy, when a shock to the borrowing limit is realized. We depart from other papers in the literature, for instance Guerrieri and Lorenzoni (2017), by examining two distinct cases. Firstly we examine the case of *credit crunch* , i.e., a tightening in the borrowing limit that households face, and secondly the case of *credit easing*, i.e, an increase in the borrowing limit that households face.

To analyse the model we are forced to migrate to numerical simulations. Therefore, we need to specify preferences and also choose the values for a set of parameters. Preferences are described in section 3.4 and the period utility is assumed to be a $CRRA$

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{3.16}$$

where σ is the parameter capturing the relative risk aversion.

The time period of the model is set to be one quarter. The discount factor β is chosen to be 0.9428, targeting a yearly interest 4.15% in the initial steady state. The coefficient of relative risk aversion is set to be $\sigma = 3$. The coefficient of relative risk aversion plays a crucial role in driving the precautionary motives. However, different experiments that we have done with values between 2 and 4 do not alter qualitatively our results. Therefore, we have chosen the value that generates quantitatively better results. The earnings process is approximated, by the use of a 12-state Markov chain and similarly to the previous chapters, we have utilized the discretization method proposed by Civale et al. (2016), using an NMAR process, namely a first order autoregressive process with normal mixture innovations, in order to match the moments of Guvenen et al. (2015).

Transitions between employment and unemployment, similarly to our previous chapters were calculated in *quarterly frequency* following the approach of Shimer

Table 3.8.1: Calibrated Parameters

Description	Parameter	Value
Preferences		
Households Discount Factor	β	0.9428
Coefficient of Relative Risk Aversion	σ	3
Employment		
Transition to Unemployment	$\pi_{e,u}$	0.057
Transition to Employment	$\pi_{u,e}$	0.882
Unemployment Benefit	ρ	0.4
Technology		
Capital Share	α	0.33
Depreciation Rate of Capital	δ	0.025
Defaults		
Probability of Remaining Constrained	θ	0.95
Labour Earnings Garnishment	γ	0.20

Notes: This table reports the calibrated parameters for our model economy.

(2005) and Krueger et al. (2016). We used CPS data for the measurement of *job finding* and *separations rates*.[6] By following this strategy we calculate the average probabilities from $2000Q_1$ to $2014Q_4$, **since we are interested in including the period of the global financial crisis** of 2008. The probability of a household remaining on the constrained state following default, has been chosen to match an average exclusion period of 5 years. Labour earnings garnishment is set to $\gamma = 0.20$ so to guarantee that its value is below the upper permissible bound of 25%, as introduced by *Federal Wage Garnishment Law* and the *Consumer Credit Protection Act* III.

[6]We download the following series from CPS: The **unemployment level** (UNEMPLOY - Thousands of Persons, Monthly, Seasonally Adjusted), the **short term unemployment level** (UEMPLT5 - Number of Civilians Unemployed for Less Than 5 Weeks, Thousands of Persons, Monthly, Seasonally Adjusted) and the **employment level** (CE16OV - Civilian Employment Level, Thousands of Persons, Monthly, Seasonally Adjusted).

Denoting by u_t the unemployment rate and by u_t^s the short term unemployment rate, we define the *job finding rate* $1 - \left(\frac{u_{t+1} - u_{t+1}^s}{u_t}\right)$ and the *separation rate* $\frac{u_{t+1}^s}{1 - u_t}$.

3.9 Shocks to the Borrowing Limit

In this chapter we address two important questions. We firstly explore what is the response of the economy when experiencing a *credit crunch* aiming to explain the period of the recession following the financial crisis. This exercise is similar to Guerrieri and Lorenzoni (2017), though their model does not feature a production economy and also does not incorporate limited commitment. The economy starts at $t = 0$ in a steady state with a borrowing limit of $-B_{t=0} = 3.0$. At $t = 1$ a permanent unanticipated shock in the borrowing limit will be realized and will tighten the borrowing limit to $-B_{t=T} = 1.5$.

Following Guerrieri and Lorenzoni (2017) we assume that economy does not adjust to the the new borrowing limit in one period, namely in our model in one quarter. In particular, we assume that the borrowing limit $-B_t$ follows a linear adjustment path as follows:[7]

$$-B_t = \max\Big\{-B_{t=T}, -B_{t=0} - \Delta B \cdot t\Big\} \tag{3.17}$$

Similarly to Guerrieri and Lorenzoni (2017) we choose ΔB such that the adjustment of the economy last six quarters. Our motivation to follow the approach is firstly that in reality debt has much longer maturity than a quarter. Furthermore, the recession in the United States lasted almost 6 quarters following the blow up of the financial crisis.

We further explore in a similar manner the response of the economy when experiencing a *credit easing*. A big bulk of literature aims to explain the financial crisis by looking at the consequences that followed the actual event. However, a more interesting question is what led to the financial crisis, and if we could predict the recession. To gain useful insights regarding this question we should explore the period during which economy was booming, namely $2000 - 2007$. One of the

[7]This is a pure unanticipated shock in the sense that the agents inside the economy do not incorporate eq. 3.17 in their decision making process and thus cannot predict how or if the borrowing limit will change in the next period.

facts is that during this period, access to credit was easy and credit supply was huge. We claim that our model could provide useful insights in addressing this question.

Therefore we model *credit easing* as follows. The economy starts at $t = 0$ in a steady state with a borrowing limit of $-B_{t=0} = 3.0$. At $t = 1$ an unanticipated shock in the borrowing limit will be realized and will increase the borrowing limit permanently to $-B_{t=T} = 4.5$.

Again we assume that the borrowing limit $-B_t$ follows a linear adjustment path as follows:

$$-B_t = \max\Big\{ -B_{t=T}, -B_{t=0} + \Delta B \cdot t\Big\} \tag{3.18}$$

In this case since we are interested in the period before crisis, and particularly in the period from $2000-2007$ we choose ΔB such that the adjustment of the economy last 28 quarters, or in other words 7 years which the duration from $2000-2007$. During this period supply of credit increases progressively towards the new borrowing limit which as explained before is set to $-B_{t=T} = 4.5$ so to represent an increase of 50% in credit supply. In this framework we study the effects of these two shocks, namely the *credit crunch* and *credit easing* and we study their effects into real activity.

3.10 Results

In this section we discuss the key results obtained from the computational solution of the transitional dynamics of our economy, when facing a *credit easing* and a *credit crunch*. Our presentation does not involve any discussion of the properties of the *initial* and *final stationary* equilibrium since detailed description of the equilibrium can be found for similar models in our first and second chapter.

We discuss in detail the effects of both shocks in the aggregate quantities and prices of our model. In particular we present results related to the response of output, consumption, Household-Debt-to-GDP, ratio and aggregate capital.

Furthermore, we look at the responses of default rates, interest rates, and the proportion of households that feature negative net worth.

3.10.1 Credit Crunch

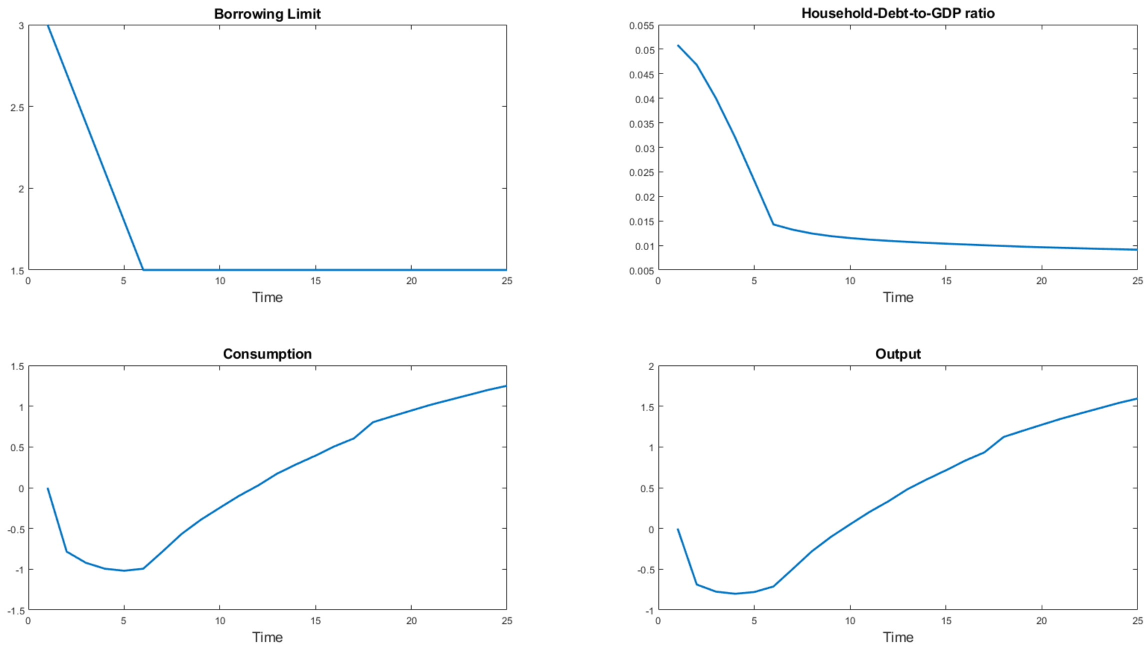

Figure 3.10.1: **Output, Consumption and Debt responses:** This figure displays the responses of output, consumption and household debt to GDP ratio, following a *tightening of the credit limit* of 50%. The adjustment of the borrowing limit is displayed in the upper left panel of the figure. Furthermore, output and consumption responses are expressed in terms of percentage deviation from the initial steady state, while Household-Debt-to-GDP ratio is expressed in actual values that by definition are ratios.

Figure 3.10.1 illustrates the response of our economy to an unanticipated decline to the households' borrowing limit, or as referred to in the literature as a credit crunch. In the upper left panel of our figure we present the exogenous adjustment of the borrowing limit that after six periods has a drop of 50% compared to the initial stationary equilibrium. In the upper right panel of our figure we show the response of the household debt to GDP ratio following the aforementioned shock. In the lower left panel of our figure we present the response of aggregate consumption to the decline of borrowing limit. It is presented in terms of percentage deviations from the initial steady state. Finally, the lower right panel of our figure illustrates the response of aggregate output to the debt limit contraction. This figure is also presented in terms of percentage deviation from the initial steady state, as this gives us a better illustration of the results.

Firstly we should stress that the qualitative properties of the results presented are not sensitive to any reasonable change of the calibration. This finding is in strong agreement with the study of Guerrieri and Lorenzoni (2017), that utilise a much simpler model, where firstly the option of default is absent, and secondly the supply of bonds in the economy is fixed, since their economy does not feature production, thus there is no link between the assets of the households' sector and the production side of the economy.

When one observes figure 3.10.1, it is evident that the consumption effect on output is very large. Following the decline in the borrowing limit, households that are heavily indebted are forced to adjust towards the new borrowing limit. One could expect that this could be achieved in our model framework via the mechanism of defaults. However, as one can observe in the right panel of figure 3.10.2 the default rate in the economy drops following the realization of the shock. In view of this result, we should also observe the behaviour of the risk free interest rate in the economy. Interest rate drops for almost 30 basis points. Guerrieri and Lorenzoni (2017) also identifies an interest rate overshooting after a debt contraction.

The mechanism that works in our model can be explained as follows. Firstly, when the borrowing limit tightens, households in the lower left part of the wealth distribution, are forced to adjust. Households that are far away from the borrowing limit, are not so affected from the shock. Note that households perfectly anticipate the path that the borrowing limit will follow. Therefore, with the decline in the risk-free interest rate the outside option does not give them higher utility and thus they do not decide to default. Recall that in our model default is strategic and households are allowed to save during the post default path. Highly indebted households prefer to maintain their access to credit market, since the outside option is not attractive, due to the interest rate decline. Furthermore, for consumption smoothing purposes access to borrowing is more valuable for households that face lower income. Households with low income in our model mainly represent the borrowers in our model. In this sense, these households are adjusting to the new borrowing limit by repaying their debt and maintaining their access to the credit market. However, for repayment to be feasible a sacrifice in consumption terms is necessary. This is exactly the reason that drives the drop in consumption that one can observe in figure 3.10.1. Consequently the drop in consumption leads to a temporary recession, that we can view in this model via the output drop. The magnitude of this recession is almost 1% drop in GDP, which is consistent with the empirical evidence in recession following the financial crisis in the United States, and also in agreement with the results of Guerrieri and Lorenzoni (2017).

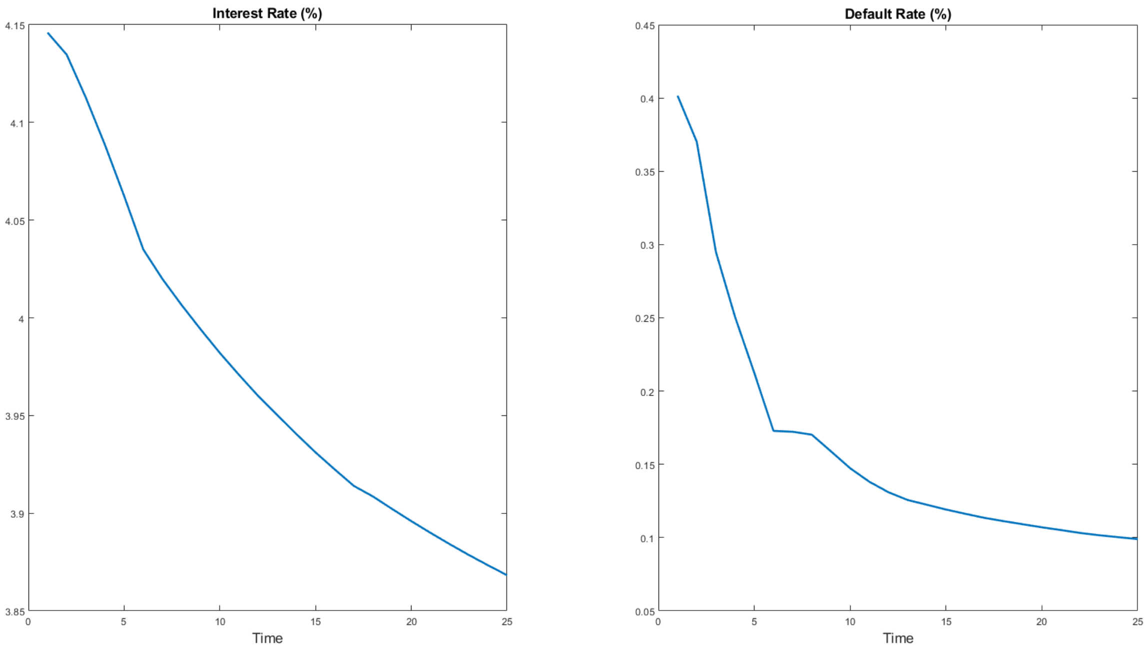

Figure 3.10.2: **Interest Rate and Default Rate responses:** This figure displays the responses of interest rate and default rate following a *tightening of the credit limit* of 50%. In left panel of the figure we display the response of the risk-free-interest rate following the shock. In the right panel of the figure we show the response of the default rate in the economy following the shock.

A second very interesting finding is that the economy after almost 14 quarters features a slow recovery and finally ends up to a better equilibrium in which output is slightly above 1.5% higher compared to the initial steady state.

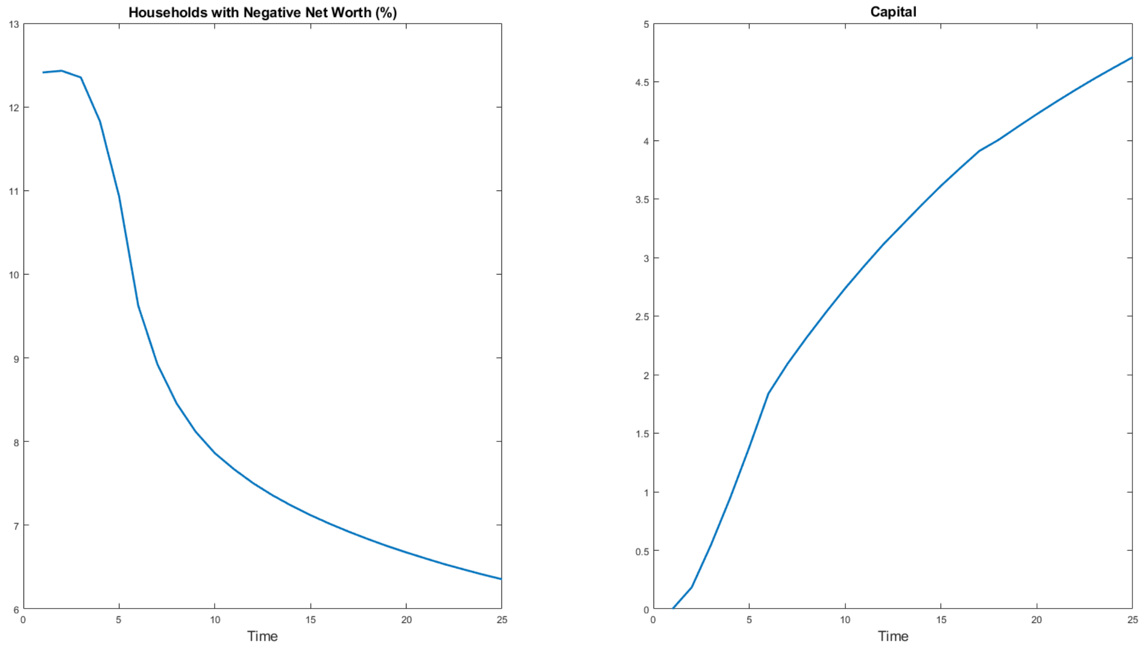

Figure 3.10.3: **Negative Net Worth and Capital responses:** This figure displays the responses of households with negative net worth in the economy and also aggregate capital following a *tightening of the credit limit* of 50%. In the left panel of the figure we show the change of the percentage of households with negative net worth following the shock. In the right panel of the figure we present the response of aggregate capital to the shock. It is expressed in terms of percentage deviations from the initial steady state.

We disentangle the output behaviour into two effects. In the "short run", output features the "consumption effect" which causes the drop via the mechanism that we have previously discussed. However in the *long-run* output growth is driven by the "capital and defaults effect". For a better illustration of this argument we can observe in figure 3.10.3 the capital response. Capital features a significant increase following the credit crunch, that amounts to almost 5% compared to the initial steady state. This because in this class of models the precautionary savings motive is very strong. Following the decline in the borrowing limit, households that experience good earnings shocks save more. Aggregate borrowing drops due to the tighter constraint, as observed in the top-right panel of figure 3.10.1. This generates a positive effect in the economy, since less households are now defaulting, more households are saving, and slowly the economic fundamentals enhance, as witnessed from the increase in the aggregate productive capital in the economy. This in turn boosts up consumption and economy experiences a boom in the long run, in which experiences higher capital, less defaults, more consumption and thus higher output.

An interesting side result of our model as one can see in the left panel of the figure 3.10.3 is that a credit tightening decreases the percentage of households in the economy that face negative net worth. This finding has policy implications related to the allocation of loans from the banks. For instance, households being in the lower part of the wealth distribution are those that ask for credit in order to boost their consumption. In other words, these loans are allocated for non-productive purposes, thus this does not improve the economic fundamentals and just leads to a consumption boom that as we will later see can trigger in the long run a recession. Contrariwise, when less credit is allocated to households in this part of the distribution, both capital increases and also the households that were willing to borrow up to the borrowing, simply they cannot, which consequently leads to an better steady state in long run, since these households are forced to not to live beyond their economic abilities, via an easy and cheap access to credit.

3.10.2 Credit Easing

Most of the existing literature related to the financial crisis of $2008-2009$ aims to understand how the severe financial shock led to a full-fledged recession. However, less studies examine how we ended up in such a severe financial crisis, which consequently had all this adverse effects in real activity. In this section we examine the case of a credit easing, or in a simpler a gradual relaxation of the borrowing limit, that results in an increasing households' borrowing capacity.

Similarly to the previous part we study the effects of this relaxation of the borrowing constraint on macroeconomic aggregates, that is, aggregate quantities and aggregate prices.

Figure 3.10.4 summarises the response of our economy to an increase to the households' borrowing capacity, representing the notion of "credit easing". In the upper left panel of figure we present the exogenous adjustment of the borrowing limit after 28 quarters or equivalently 7 years with an increase of 50% compared to the initial steady state. In the upper right panel of our figure we show the response of the households debt to GDP ratio on the same shock. In the lower left panel of the figure we display the response of aggregate consumption, that is expressed in terms of percentage deviations from the initial steady state. The lower right panel of the figure illustrates the response of aggregate output to the debt limit expansion, that is also presented in terms of percentage deviations from the initial steady state.

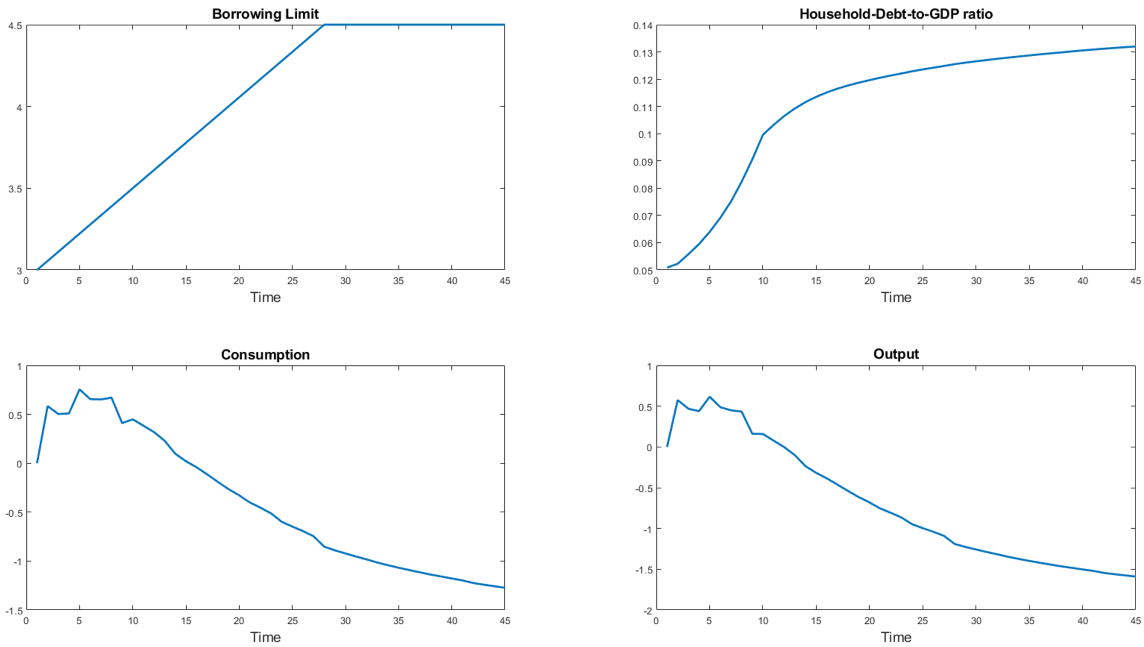

Figure 3.10.4: **Output, Consumption and Debt responses:** This figure displays the responses of output, consumption and household debt to GDP ratio, following an *easing of the credit limit* of 50%. The adjustment of the borrowing limit is displayed in the upper left panel of the figure. Furthermore, output and consumption responses are expressed in terms of percentage deviation from the initial steady state, while Household-Debt-to-GDP ratio is expressed in actual values that by definition are ratios.

Following the increase in the households' borrowing capacity, there is a jump in aggregate consumption. This is the effect that one can observe in bottom left panel of figure 3.10.4. Via the channel that previously defined as the "consumption effect" output experiences an analogous increase in the short run. This is because easier borrowing incentivise households with low and negative wealth to borrow more so to subsidise their consumption and derive a higher level of utility. This causes a boom in the economy in the short-run. However, any boom that is not based on the economic fundamentals unavoidably leads to a bust. Indeed we observe in figure 3.10.6 that capital increases in the short run, but as defaults increase (see figure 3.10.5) and interest rate hikes, a negative effect appears.

Defaults in our model have social costs, since the defaulted amount of assets is deducted from the amount of capital that will be available in the economy the following period. Therefore, when the cumulative effect of defaults exceeds a certain threshold, economy experiences capital losses which in turn will lower aggregate output and aggregate consumption.

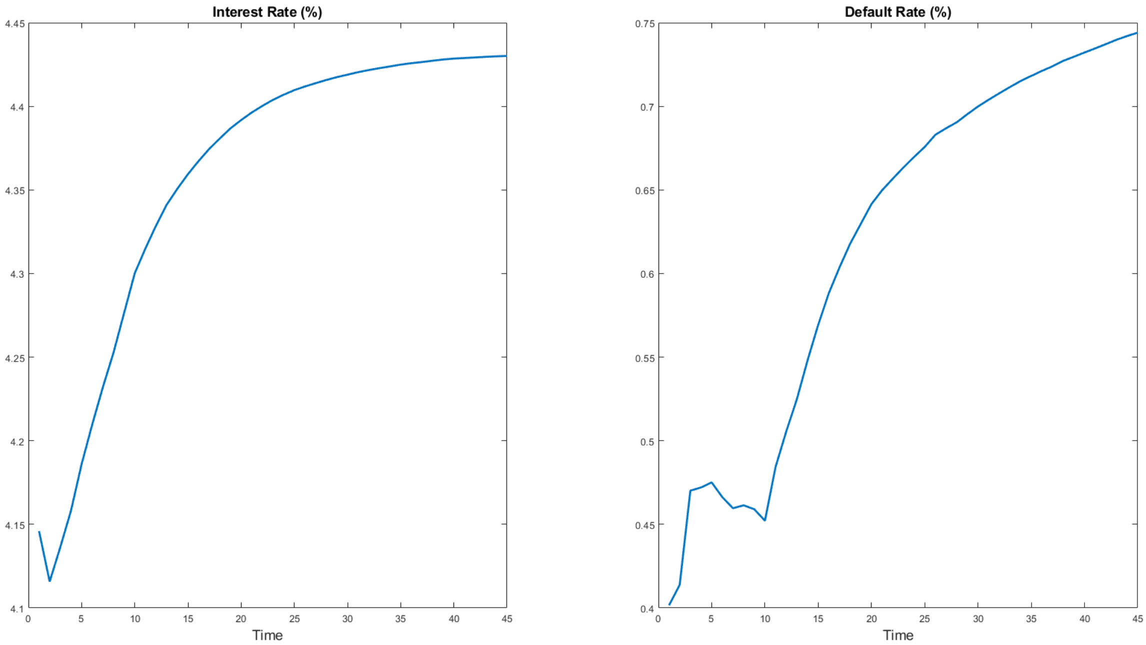

Figure 3.10.5: **Interest Rate and Default Rate responses:** This figure displays the responses of interest rate and default rate following an *easing of the credit limit* of 50%. In left panel of the figure we display the response of the risk-free-interest rate following the shock. In the right panel of the figure we show the response of the default rate in the economy following the shock.

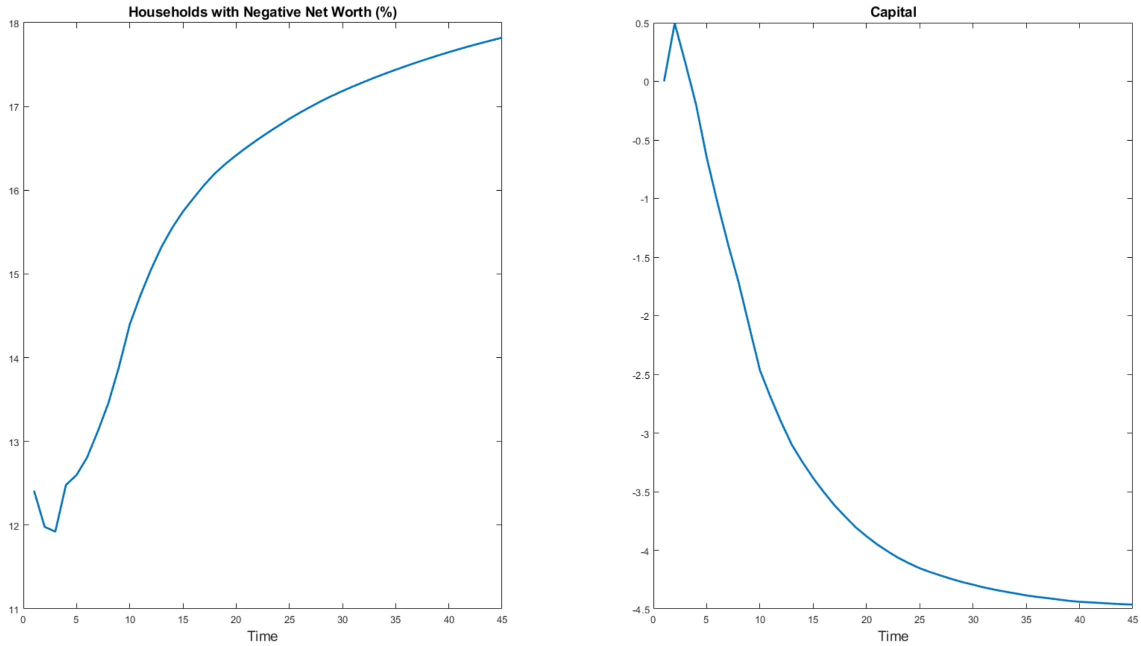

Figure 3.10.6: **Negative Net Worth and Capital responses:** This figure displays the responses of households with negative net worth in the economy and also aggregate capital following an *easing of the credit limit* of 50%. In the left panel of the figure we show the change of the percentage of households with negative net worth following the shock. In the right panel of the figure we present the response of aggregate capital to the shock. It is expressed in terms of percentage deviations from the initial steady state.

In particular, when credit is abundant in the economy, lower wealth and income households are willing to borrow to the limit. This fact, along with the fact that this type of credit is just used to increase the level of consumption of low income households gives an illusionary sense of prosperity that triggers a credit-driven boom that is orthogonal to the economic fundamentals. The huge level of borrowing will rise gradually the default rate in the economy which in turn will cause a drop in aggregate capital. One can see the major importance of defaults and the causal relation between defaults capital and output, when observing the shape of the line in figure 3.10.5. In the short run we can observe that the non smoothness of default rate is also reflected in the bottom panel of figure 3.10.4. The non-monotonicity of output and consumption response in the first periods, following the credit easing is exactly due to the non- monotonic adjustment of the default rate, that is caused by the household heterogeneity and the various default incentives in this framework, depending on the position of the household across the wealth distribution.

Households with negative net worth as can be observed in the left panel of figure 3.10.6 decline in the short run and steadily increase soon after. This spike can be explained by the excess borrowing that economy features as also can be seen in the upper right panel of figure 3.10.4. The household debt to GDP ratio increases almost three times in the long run, while the risk free interest rate also increases in the long run by around 30 basis points compared to the initial steady state.

As defaults accumulate and the default rate almost doubles in the long run, economy experiences a severe capital loss that amounts to about 4.5% compared to the initial steady state. This passes via the capital-output channel to aggregate output and consequently causes a severe recession, where the output drop is almost -1.6% lower compared to the initial steady state.

To summarize, the economy faces a consumption boom during the initial periods following the increase in the borrowing limit. This boom is credit-driven and leads to a temporary increase in output.

However, as interest rate hikes and the default rate of the economy increases, capital loses are taking place, that in turn affect directly output. Economy after a prolonged booming period ends up in a worse steady state characterized by higher default rates, lower consumption and capital stocks and households experiencing negative net worth rise to almost 18% of the population.

This is because credit in this model does not have a productive role, namely it is used only for subsidizing consumption. On the contrary, savings plays a productive role, since they are transformed via financial intermediaries to capital which is consequently used for production.

We claim that the mechanism described in this section can shed light on the roots of the financial crisis of $2008-2009$ that in our opinion has its origin in the credit-driven consumption and output boom that was observed during the period of $2000-2007$. To understand and explain this mechanism **the role of defaults and productive capital** is indispensable, which are exactly the elements that we highlight with our model structure.

3.11 Conclusion

In this chapter we accomplished the following goals. *Firstly* we extended the model of Guerrieri and Lorenzoni (2017) by also accounting for defaults and a production economy. Both these features provide useful insights in understanding and revealing the two transmission mechanisms working silently in the background of the model economy. The first mechanism that drives the short run effects in real activity is the "consumption channel" and the second that drives the long run effects in the real activity is the "capital-defaults channel".

Secondly instead of focusing only on periods of *credit tightening* we also studied periods of *credit expansion*. We have showed that prolonged periods of *credit expansion*, where credit is mainly fuelling consumption will unavoidably lead to a severe recession of a similar magnitude as in the Great Recession. This is because defaults during periods of easy credit increase and capital is getting destructed. In this manner the experienced consumption and output boom in the short-run is false, since it is not supported by an increase in productive capital, and is mainly driven by misallocated credit utilised for consumption purposes.

Finally, we confirmed the results found by Guerrieri and Lorenzoni (2017) when economy faces a "credit crunch". We have additionally showed that their results are correct in the short run, but in the long run economy would converge to a better equilibrium in which output would be higher by 1.6% and consumption almost 1.2%, provided that credit would keep being tight. This result stands in support of macro-prudential policies and indicates that the real cause of a financial crisis and a consequent recession is the misallocation of credit in the non-productive parts of the economy.

Our results are in support of the recent concerns related to the expansion of credit, households indebtedness and the high fraction of households with negative net worth in the United States. The possible directions for extending this work is to allow for aggregate shocks in the line of Krusell and Smith (1998) and introduce space for fiscal/macro-prudential policies so as to arrive at a complete narrative of a financial crisis. It would also be interesting to analyse how the wealth/income distribution quantiles evolve after the credit shock and how preventive measures can be constructed.

Appendix

3.A Computation of the Transition Path

The economy at time $t = 0$ is in steady state with stationary distribution $\mu_0 = \mu*$ over assets ℓ and idiosyncratic earnings y and s.Now consider the following thought experiment, the financial intermediaries decide to lower the borrowing limit (credit crunch) from $-\overline{B}$ to $-\overline{B}'$. Here we describe a on-off shock and not a sequence, the computation for a gradual sequence of shocks is similar. Hence, we need to compute the relevant interest rates for $t = 1, \ldots T$.

We will assume that after T periods with T arbitrarily large but finite, the economy will converge to the new steady state. This assumption is useful because it allows us to get a finite sequence of aggregate capital stocks, savings, consumption and prices. The principle of backward induction can be conveniently used to get the relevant time indexed value and the policy functions.

The computational algorithm is described below:

Algorithm

1. Fix the time period T big enough to allow for a smooth convergence to the new steady state.

2. Compute the initial steady state objects $\{v^*, c^*, \ell^*, K*\}$ corresponding to the initial borrowing limit $-\overline{B}$ and the final steady state aggregates $\{v^{**}, c^{**}, \ell^{**}, K^{**}\}$

3. Given a sequence of aggregate capital stocks $\{\hat{K}_t\}_{t=1}^{T}$ of length T such that $K_1 = K^*$ since capital t time 1 is predetermined at time 0 which is a steady state) and $K_T = K^{**}$. Since labour is exogenous in our model and there is no aggregate uncertainty, $N_t = N$ for all t.It is easy to determine for each t, the corresponding wage and :

$$\hat{w}_t = F_N(\hat{K}_t, N)$$
$$\hat{r}_t = F_K(\hat{K}_t, N)$$

   which are the necessary elements needed tin the budget constraint of the household to solve at time t.

4. We know that the $v_T(\ell, y, s) = v^{**}(\ell, y, s)$, we can use backward induction to solve for all value functions from time $T-1$ to 1 and the associated policy functions including the $\{\hat{\ell}_{t+1}\}_{t=1}^{T-1}$ and the default probabilities.

5. Loan prices are computed for each time $T-1$ to 1 using the default probabilities corresponding these periods.

6. Given the savings policy functions, we can reconstruct the sequence of transition functions $\{\hat{Q}_t\}_{t=1}^{T-1}$ and since we know that $\mu_0 = \mu^*$, we can recover the entire sequence of measures $\{\mu(\ell, \hat{y}, s)_t^{uc}\}_{t=1}^{T-1}$ and $\{\mu(\ell, \hat{y}, s)_t^{c}\}_{t=1}^{T-1}$ and compute the aggregate level of assets supplied:

$$\int_{\mathcal{L}\times\mathcal{Y}\times\mathcal{S}} \hat{\ell}_{t+1}(\ell, y, s) d\hat{\mu}_t^{uc} + \int_{\mathcal{L}^+\times\mathcal{Y}\times\mathcal{S}} \hat{\ell}_{t+1}(\ell, y, s; r) d\hat{\mu}_t^{c}$$

7. Market Clearing is checked in every period t, i.e., whether the guess of the equilibrium capital stocks is $\{\hat{K}_t\}_{t=1}^{T}$ is consistent with the level of aggregate assets supplied that households would accumulate when facing the sequence of borrowing limit and the prices induced by the assumed sequence of aggregate capital. Market Clearing is checked based on computing the absolute values of excess demand or supply.

8. If the error in market clearance do not satisfy the convergence criterion, we implement the bisection method modified with endogenous dampening g to update a new guess for the time series of capital $\{\hat{K}_t\}_{t=1}^{T}$.

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