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MULTIVARIABLE SYSTEM CONTROLLER TUNING TECHNIQUES BASED ON SENSITIVITY MEASURES

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To my father's memory
and to my mother

ABSTRACT

A controller tuning technique using sensitivity functions of the controller parameters is proposed which does not require a detailed model of the plant either based on physical principles or through system identification. Only simple signal processing is required in the tuning process. The sensitivity information is used by an adjustment algorithm involving a least squares type of criterion function.

The generation of the sensitivity functions which are of central importance in this approach is described in this thesis, involving use of a signal convolution approach and a two-stage method both in the time-domain and frequency-domain. Three different forms of input signal, which involve the step input, the impulse input and the extended pseudorandom binary sequence (PRBS) input signal are selected in the calculation of the sensitivity functions.

The two-stage approach to generating the sensitivity functions of the controller parameters in systems with unknown plants has been investigated for the first time in this research work. The advantage of this novel approach is that there is no limitation on the form of the test input signal. The sensitivity functions can be obtained from measurements directly without any calculations. No problems of implementation arise with the sensitivity filters required.

In the controller tuning process, the least squared approach is used to provide the figure of merit for each projected system response. The changes of the controller parameters are altered to minimise the difference between the response of the actual system and the desired response.

The details of an application of the tuning procedure using the signal convolution approach for generating the sensitivity functions for a two-tank system with two inputs two outputs both in the time-domain and the frequency-domain are given. Special consideration is given to the accurate modelling of the two-tank system upon which

this work is based. Questions of plant nonlinearity and measurement noise and their effects on the tuning process are given careful consideration but no significant problems were encountered. In order to prove that the technique is suitable for more complex problems, the technique has also been applied successfully to helicopter flight control system design optimisation. This is of potential interest as a means of reducing the period of time for test flying and design modification for practical helicopter flight control systems. The tuning process is a very "visible" one and likely to be attractive for applications of this kind.

From the results of these two applications of the tuning technique it can be seen that the tuning process is very effective although the initial responses of the system may be far from the desired responses. In fact, the adjustment procedure provides fast is convergence in the cases considered. Significant progress is made at each adjustment without any oscillations in parameter values. The number of experiments needed to generate the sensitivity information needed for controller tuning is, in general, significantly smaller than that required for a traditional parameter perturbation method for sensitivity function generation.

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CHAPTER 1

Introduction on Controller Tuning in General

1.1 Introduction

Difficulties arise in designing multivariable control systems due to cross-coupling between control loops and in many cases because of the lack of detailed dynamic plant models which include accurate information about inherent cross-coupling effects within the plant. In the design of a control system a multivariable controller can provide superior performance in many applications because it co-ordinates the use of information between the various inputs and outputs and can provide decoupled closed-loop control. Unmodelled dynamics and inaccurate modelling of the plant frequently lead to control system performance deficiencies, particularly where the dynamical interactions are most significant. In order to achieve system performance and robustness it is frequently necessary to require adjustment of the controller during commissioning tests. Thus, before a multivariable controller can be implemented, techniques for field tuning of such a control system must be developed.

Most methods for solving the problems of control systems design require accurate knowledge of the structure and parameters of the system being controlled especially in the case of multivariable systems. In many practical situations such information is not available. System identification is a procedure by which a mathematical description of a plants dynamic behaviour is extracted from test data. Identification problems are known to occur due to cross-coupling, and in multivariable applications such as helicopter flight control accurate identification from open-loop tests may present major problems. In such cases the experienced control systems engineer may still arrive at a successful feedback design by way of adroit tuning of the control loops after preliminary design calculations based on time-domain or frequency domain methods.

Most controller tuning adaptive and self-tuning control methods depend upon the availability of a model or on the use of system identification techniques to derive a plant model from measurements. Fig. 1.1 shows a typical scheme of this kind based upon system identification.

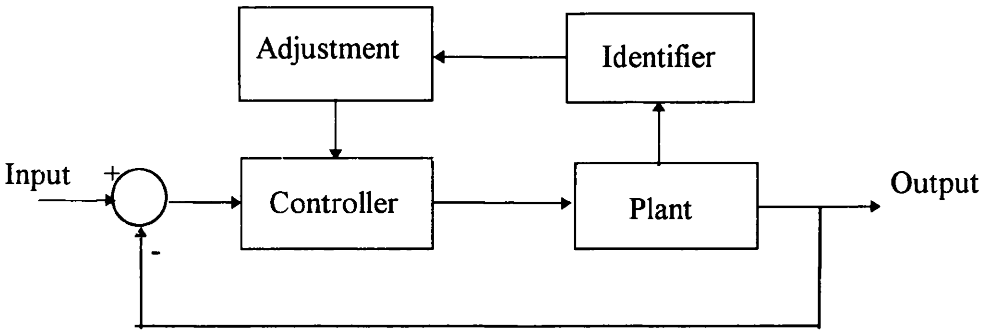


Fig. 1.1 Scheme of system self-tuning techniques

The self-tuning (Flynn,D., *et al.,* 1994) is an automatic process in which the controller parameters are adjusted using one of a number of possible algorithms based on experimental responses. Although it is an approach widely used in the process engineering field, self-tuning control, like other adaptive systems, is not seen as an attractive approach in safety-critical application areas such as aircraft flight control. In this thesis a controller tuning technique is proposed which does not require a detailed model of the plant either based on physical principles or through system identification. Only simple signal processing is required in the tuning process and it is assumed that the tuning process is based on manual intervention. The parameter changes calculated in this tuning process do not lead automatically to changes in the controller and thus the approach may be inherently more attractive for flight control system applications where safety is of parameter importance.

Previous work on this approach has been based upon use of a step function input and has involved time-domain signal processing (El-Shirbeeny, E.H.T. *et al.*, 1974, Winning, D.J., *et al.*, 1977, Murray-Smith, D.J., 1985, Manness, M.A., 1988). This research involves investigation of the use of other forms of test input and both time-domain and frequency-domain methods of analysis. The advantages of using a frequency-domain approach to controller tuning lie in the link which this then provides to frequency domain methods of controller design. This is particularly important for multivariable systems where techniques such as Individual Channel Design use a frequency domain approach and allow physical insight to be gained which may be lacking in a purely time-domain method.

In this thesis Chapter 1 provides a brief review of tuning techniques and in particular the problems of applying such techniques to multivariable systems and especially the helicopter flight control system. In Chapter 2 the author presents the theoretical basis of the tuning procedure used throughout this work. The calculation of sensitivity functions which are of central importance in this approach is described in Chapter 3, involving use of the signal convolution approach both in the time domain and the frequency domain. The selection of the form of input signal, which may involve the step input, the impulse input and the Pseudorandom Binary Sequence input signal is presented in Chapter 4. In Chapter 5 the author gives details of an application of the tuning procedure using a signal convolution method for generating the sensitivity functions for a two-input two-output system both in the time-domain and the frequency-domain. In order to prove this technique is suitable for more complex problems, the technique has been successful applied to helicopter flight control system design optimisation. Chapter 6 gives results of showing the application of the approach to a helicopter flight control system developed using Individual Channel Design methods. Finally, some discussion and conclusions and suggestion for further work are presented in Chapter 7.

1.2 Tuning Problems for Multivariable Control Systems

Much previous research on controller tuning has been based on the classical single-input single-output (SISO) control system. However, in a wide variety of industrial environments ranging from process control to the helicopter flight control many modern dynamical control systems are inherently multi-input multi-output (MIMO) cases.

Controller tuning presents particular difficulties in the case of multivariable systems because of the number of variables involved and the associated interactions. For example, much effort is currently being expended in attempts to design helicopter flight control systems using active control technology (ACT) and controller adjustment following preliminary flight testing is an important problem. The motivation behind this work on multivariable flight control is largely a desire to produce battlefield helicopters with better handling qualities and more extensive flight envelopes. Indeed, the lethality of the modern battlefield demands improved helicopter performance and ease of operations. Some of the manoeuvres required for a successful mission will only be possible through the use of ACT in control system design. The successfully design of control configured vehicles is essential in order to meet these requirements. However, at the centre of every design will be a mathematical model of the raw plant dynamics and the outcome of the design process will largely be determined by the accuracy with which the plant dynamics are known.

In the particular case of helicopter flight control system design, the problem is inherently multivariable in nature and modelling uncertainties arise in a number of different areas. The cross-coupling is largely associated with the dynamic characteristics of the main rotor. The dynamics of the main rotor are complex, particularly during transient manoeuvres and present considerable difficulties in terms of mathematical modelling. Problems associated with uncertainties in rotor models limit the performance which can be achieved in applying multivariable control design methods and controller adjustments may be needed at the prototype flight testing stage in the development of a new helicopter. The methods of controller tuning currently available are not particularly efficient and extensive re-adjustment of controller parameters from flight test results can be a lengthy and therefore expensive process.

1.3 Adjustment of Controller Using Parameter Sensitivity Measures

In order to optimise the system response there are a number of widely used methods to adjust the controller parameters. One of them is by attempting to improve the model from measured data and redesign. This can be a lengthy and time consuming process and may be impossible for control system design in cases where the controlled system is not yet available for testing. Trial and error techniques may also be used in attempts to optimise a system's response. However this type of an approach suffers from a lack of quantitative information concerning how the parameters should be adjusted. In a multivariable system with many parameters, it may be difficult to know which parameters significantly affect the performance and the manner in which the response will change with a shift in parameter values. Also the trial and error approach to on-line controller optimisation is highly iterative and becomes very difficult in complex controllers with many parameters. Using sensitivity function methods (Kokotovic', P.V., 1964, Kokotovic', P.V. and Rutman, R.S., 1965, Kokotovic', P.V. and Rutman, R.S., 1967, Winning, D.J.,*et al.*, 1977, Murray-Smith, D.J., 1985) it is possible to identity the changes which will lead the greatest improvement in response. The sensitivity functions give the rate of change of system responses to parameter variations. This is the quantitative information which is missing in the trial and error adjustment method. The sensitivity functions can be generated with a cosystem approach (El-Shirbeeny, E.H.T. *et al.*, 1974), by parameter perturbation methods or efficiently calculated by the signal convolution approach (Winning, D.J., *et al.*, 1977, Murray-Smith, D.J., 1985). Methods such as these have the advantage that the sensitivity functions of the controller parameters can be obtained one by one and all the controller parameters can be adjusted. Because the signal convolution method does not rely on knowledge of the plant and because it does not require a large number of tests on the system, sensitivity functions generated in this way may show important advantages when considered as a practical tool in the optimisation complex control system such as those which arise in aircraft flight control.

1.4 Application of Individual Channel Analysis and Design for MIMO Systems

A new approach - Individual Channel Analysis and Design (ICAD) - to an enduring problem - multivariable feedback control has recently been proposed by O'Reilly and Leithead (O'Reilly, J., and Leitnead, W.E., 1991, Leithead, W.E. and O'Reilly, J.,

1991) in 1991. This approach provides a basis for analysing multivariable feedback systems in the frequency domain using individual feedback channels. The initial publications on the method have all involved plant models which are assumed known and part of the current research is concerned with the development of techniques of the experimental determination of the multivariable structure functions which are the basis of the individual channel approach.

A second aspect of the project involves the investigation of frequency domain methods for controller tuning in multivariable systems which again avoids the need for any precise knowledge of the plant dynamics. The ICAD approach has been used by on a helicopter flight control system with a 19th order model (Dudgeon, G.J.W., *et al.*, 1995) and this control system provided a basis for some of the work on controller parameter tuning in this research. It is believed that this approach could allow to experimental on-site tuning using spectral analysis methods for frequency domain estimation of controller sensitivity functions.

The combination of experimental determination of structure functions and the plant-model-independent process for controller tuning could be of considerable practical interest in the development and commissioning of practical multivariable control systems.

1.5 Background to the Sensitivity Approach

1.5.1 Review of the Tuning Techniques

The tuning technique is based on the sensitivity functions of the controller parameters for a control system. The sensitivity tuning technique was initially suggested by Dr. R. Tomovic (Tomovic, R., 1964) in the early 1960s. Of the traditional approaches to generating the sensitivity functions of the controller parameters, there are drawbacks to both a sensitivity cosystem approach and parameter perturbation techniques. To use a sensitivity cosystem one must have accurate knowledge of both the structure and parameters of the plant. Such knowledge is rarely available for most practical systems. In parameter perturbation methods, the sensitivity functions are approximated through calculations of differences between system responses before and after a small change in a parameter. Although a detailed plant model is not necessary in this case, the calculations can be critically affected by the amplitude of the perturbations and for a system with many parameters, the generation of all of the sensitivity functions can require a large number of tests on the system. An alternative approach which generates the sensitivity functions by signal convolution has been developed for the special case of controller parameters in closed-loop systems. (Tomovic, R., 1964, El-shirbeeny, E.H.T. *et al.*, 1974, Winning, D.J., *et al.*, 1977) Most of the previous research involving the signal convolution approach (direct assessment) has involved single-input single-output system and multivariable applications have remained largely unexplored. The signal convolution method for the multivariable case was presented by Manness and Murray-Smith in 1988.

1.5.2 Previous Work at Glasgow University

Over a period of nearly 20 years research work on controller tuning has been carried out intermittently in the Dept. of Electronics and Electrical Engineering at Glasgow University. In early 1974 a method was presented for the determination of parameter sensitivity functions of a simple feedback system from measured response data obtained during a step test by EL-Shirbeeny, Murray-Smith and Winning. Results of simulation studies and a practical implementation involving a synchronous-generator-excitation system were presented. After three years in 1977 the tuning technique was developed further for on-line optimisation of a synchronous-generator-excitation control system using the sensitivity method by Winning etc. A further phase of development took place during the period 1984-1988 when a study was made by Manness and Murray-Smith which involved systematic procedures for adjusting the control in systems involving state variable feedback structures. The tuning technique which had been developed relied upon the signal convolution method to generate sensitivity functions of the state variables with respect to control system gains. In model reference tuning, computer simulation was used to validate both the application of the signal convolution method to multivariable control systems and the ability of the Least Integral Error Square Performance Index to tune a controller of the helicopter flight control system. The Least Integral Error Square Performance Index method linked the relation between the cost function and the sensitivity functions and was used to optimise controller parameters. But due to difficulties encountered in finding the sensitivity function this study involved only a step input and was concerned with a specific time-domain approach to controller design and tuning.

1.6 Statement of Originality

New developments which have taken place in the course of the work described in this thesis include the following:

a) Further development of the tuning technique based on controller parameter sensitivity functions through the investigation of test signals other than step inputs.

b) Further development and practical application of frequency-domain methods for determining controller sensitivities.

c) The derivation and practical application of a two-stage method for determining controller sensitivity functions.

d) Application of the sensitivity approach to controller tuning in a real system involving significant non-linearity and measurement noise.

e) External validation of a mathematical model of a two-tank system used the practical implementation.

f) Development of the potential value of the tuning approach for helicopter flight control system tuning.

CHAPTER 2

Controller Parameter Adjustment Methods

2.1 Introduction

In multivariable control system design the main difficulties are :

1. Cross - coupling between control loops of the dynamic systems.
2. Unmodelled dynamics and inaccurate modelling of the plant which frequently lead to control system performance deficiencies, particularly where the dynamical interactions are most significant.

Helicopter flight control system design provides an interesting example where the problem is inherently multivariable in nature and involves model structure and parametric uncertainties. The cross - coupling and uncertainties are both largely associated with the dynamic characteristics of the main rotor and interaction effects between the main rotor, tail rotor and fuselage. During large transient manoeuvres the helicopter aerodynamics is especially complicated.

Even well designed helicopter flight control systems may require some form of tuning as a result of preliminary flight tests. To reduce cost and minimise development time, it is important that this tuning process be carried out in an efficient fashion.

As has been mentioned in the previous chapter, controllers must often be adjusted on-site to optimise the controlled system. Most controller tuning techniques, for example those based on system identification techniques (Tischler, M.B., 1991) or trial and error methods, either require a mathematical model of the plant to be controlled or a series of tests which inevitably involve an iterative process which may converge slowly. If simple trial and error adjustments are made in the complex controllers which are needed in the helicopter application the problem is very difficult to manage (Tischler, M.B.,1987). What is needed is information about the influence of each parameter of the controller on each of the output variables. In this research sensitivity functions are used to predict the parameter changes necessary to move the various output responses of a control system towards some desirable responses and thus improve the overall performance of the system.

There are several advantages for tuning methods based on sensitivity measures. Firstly, the controller parameter sensitivity functions offer quantitative information, including magnitude and direction, for predicting the controller parameter changes necessary to improve the various output responses which enable us to tune controller parameters with least effort. Secondly, it can be shown that for some specific methods of sensitivity analysis there is no requirement for detailed knowledge about the dynamics of the plant being considered

2.2 Tuning Procedure of the Controller Parameters in the Time Domain

The procedure for the tuning of controller parameters of a multivariable control system usually can be split into three steps:

a) Selection of the optimisation technique

b) Method for minimisation of the chosen cost function and

c) Method for the adjustment of the controller parameters.

Each of these aspects of the problem will now be considered in turn.

2.2.1 Choice of the Optimisation Technique

In order to systematically tune a real control system, there must be a criterion by which improvements in system response can be measured. That is, for each set of possible control system parameter values, a figure of merit must be assigned to the system response. By comparing the figure of merit associated with various sets of parameter perturbations, it is possible to identity the changes which will lead to the greatest improvement in response. One possible cost function is called the Least Integral Error Square Performance Index which may be used to make the system responses as similar as possible to a desired or 'ideal' system response. Suppose the response of a system is $y(t,m_i)$ and after a small change of parameter m_i, Δm_i, the system response becomes $y(t,m_i + \Delta m_i)$, the relation between $y(t,m_i + \Delta m_i)$ and $y(t,m_i)$ can be found from a Taylor's series, that is

$$y(t,m_i + \Delta m_i) = y(t,m_i) + \sum_{i=1}^{n} \frac{\partial y}{\partial m_i} \Delta m_i + \varepsilon(t) \quad (2.1)$$

where $\frac{\partial y}{\partial m_i}$ is the sensitivity function

$\varepsilon(t)$ is the residue involving additional terms based on higher order sensitivities

n is the number of parameters under consideration

If the change of a parameter is small compared with the parameter itself, it is possible to use a first order approximation.. So the change of system response is given by

$$\Delta y_s = y(t,m_i + \Delta m_i) - y(t,m_i) \cong \sum_{i=1}^{n} \frac{\partial y}{\partial m_i} \Delta m_i \quad (2.2)$$

Suppose the desired change of the system response is Δy_d and from equ.(2.2) the actual change of the system response which corresponds to the alteration of parameters Δm_i is Δy_s. Then the error or residual between Δy_d and Δy_s is

$$E(t) = \Delta y_d - \Delta y_s \cong \Delta y_d - \sum_{i=1}^{n} \frac{\partial y}{\partial m_i} \Delta m_i \quad (2.3)$$

and the Integral Error Square Performance Index J is defined in the time domain as

$$J = \int_0^T (E(t))^2 dt \quad (2.4)$$

where T is a time greater, in general, than the settling time of the system.

We generally assume that the system to be tuned is modelled as a sampled data system. This implies that the input and output data are recorded in discrete time and between samples quantities are regarded as constant, through the action of a sample and hold circuit. In sampled-data form equ.(2.4) can be write as:

$$J = \sum_{l=0}^{q} [E(l\Delta t)]^2 \Delta t \quad (2.5)$$

where Δt = sampling interval and q = number of samples in time considered i.e., q = T/Δt.

Substituting from equ. (2.3) gives

$$J = \sum_{l=0}^{q} [\Delta y_d(l\Delta t) - \sum_{j=1}^{n} S_{mj}(l\Delta t)\Delta m_j]^2 \Delta t \tag{2.6}$$

where Δm_j is change of the controller parameter

and S_{mj} is the sensitivity function $\frac{\partial y}{\partial m_j}$ for the controller parameter m_j.

2.2.2 Method for the Minimisation of the Cost Function J

In order to minimise the index J and thus get values of all the parameter changes Δm_j, (Winning, D.J., *et al.*, 1977) the derivatives of J are taken with respect to each of the n parameter changes. Equating these derivatives $\frac{\partial J}{\partial \Delta m_k}$ to zero gives a set of n equations.

$$\sum_{l=0}^{q} \{2[\Delta y_d(l\Delta t) - \sum_{j=1}^{n} \{S_{mj}(l\Delta t)\Delta m_j\}]S_{mk}(l\Delta t)\Delta t\} = 0 \qquad \text{for all k} \tag{2.7}$$

This gives a set of algebraic equations which give the relations between the unknown variables, the magnitude of parameters adjustment, the known value of the desired change of system response, Δy_d, and the sensitivity functions S_{mj}. Since the desired change Δy_d is known, and since the sensitivity functions S_{mj} can be found, these n equations (2.7) are linear equations in the n variables Δm_1 to Δm_n.. (See Appendix 1). Solution of these equations will thus give values of the desired parameter changes Δm_1 to Δm_n which will minimise the difference between the synthesised and desired responses of the control system.

2.2.3 Method for the Adjustment of the Controller Parameters

In practice, stabilisation of the system may be achieved partly by cascade compensation and partly by feedback compensation using one of the output variables.

So it is desired to adjust simultaneously the response of a number n of the output variables of the system by suitable choice of the changes in all the adjustable parameters Δm. From equ.(2.7) a set of n linear equations can be formed and solved to give the necessary changes in all parameters Δm_1 to Δm_n. The process of parameter change computation, followed by implementation of these changes in the system, may need to be repeated several times before the performance index is minimised. The basis of equ.(2.1) and hence of the method described above, is that the parameter changes Δm_i are small. The method involves estimation of all the necessary parameter changes, but, as these may be relatively large, the accuracy of the predicted changes in output responses could be expected to be low. In some cases this can lead to a worsening of the responses and a consequent increase in the performance index. So in this, all parameter changes are normalised in terms of the maximum permitted changes, and the algebraic sum of these normalised changes is compared with another maximum permitted value. If the sum so calculated exceeds this maximum value, all parameter changes are reduced in the ratio of the actual sum to the maximum permitted value.

2.3 Adjustment of the Controller Parameters in the Frequency Domain

In the frequency domain the idea of the adjustment of the controller parameters is to fit a measured frequency response to a desired frequency response. If we have the magnitude $|W_c(\omega)|$ of the closed loop frequency response and m is a parameter dependent upon the controller C(ω) the frequency-domain equivalent of equ.(2.3) can be written

$$E_{wc}(\omega) = |W_d(\omega)| - |W_c(\omega, m_i)| - \sum_{i=1}^{n} \frac{\partial |W_c(\omega, m_i)|}{\partial m_i} \Delta m_i \qquad (2.8)$$

where $|W_d(\omega)|$ is the desired or ideal closed loop frequency response.

A performance index, I_p, similar to the time-domain index of equ.(2.4), gives a prediction of how well the transfer function can be expected to approximate the desired response once the predicted parameter changes have been implemented.

$$I_p = \int_{\omega_{min}}^{\omega_{max}} E_{wc}^2(\omega)d\omega \tag{2.9}$$

For the case of a finite number of frequency values Δω apart equ.(2.9) should be written as

$$I_p = \sum_{l=0}^{p}\left\{|W_d(\omega_{min}+l\Delta\omega)| - |W_c(\omega_{min}+l\Delta\omega,m_i)| - \sum_{i=1}^{n}\frac{\partial|W_c(\omega_{min}+l\Delta\omega,m_i)|}{\partial m_i}\Delta m\right\}^2 \tag{2.10}$$

where p is the number of frequency values considered, given by

$$p = \frac{\omega_{max}-\omega_{min}}{\Delta\omega}$$

The condition for minimisation of I_p involves the following equations :

$$\sum_{l=\omega_{min}}^{\omega_{max}}\left\{2\left\{\Delta W_d(\omega_{min}+l\Delta\omega) - \sum_{j=1}^{n}[S_{mj}(\omega_{min}+l\Delta\omega)\Delta m_j]\right\}S_{mk}(\omega_{min}+l\Delta\omega)\Delta\omega\right\} = 0$$

for all k (2.11)

where $\Delta W_d(\omega_{min}+l\Delta\omega) = |W_d(\omega_{min}+l\Delta\omega)| - |W_c(\omega_{min}+l\Delta\omega,m_i)|$

and S_m is the sensitivity function of the frequency response of the system.

As with the time domain case, this set of algebraic equations gives the relations between the unknown variables, the magnitude of parameter changes, the known value of the desired change of the magnitude $|\Delta W_d(\omega_{min}+l\Delta\omega)|$ of the frequency response of the system, and the sensitivity function S_m. Since the desired change $|\Delta W_d(\omega_{min}+l\Delta\omega)|$ is known, and the sensitivity functions S_m can be obtained, these n equations (2.11) are linear equations in the n variables Δm_1 to Δm_n . Solution of these equations will thus give values of the desired parameters changes Δm_1 to Δm_n which will minimise the difference between the synthesised and desired frequency response of the control system.

2.4 Desirable Properties of the Sensitivity Functions

From Sections 2.1 and 2.2 we know that the sensitivity functions of the output response both in the time domain and the frequency domain are very important. Several methods (Kokotovic', P.V. and Rutman, R.S., 1967, Ngo, Y.H., and Evans, F.J., 1972, Daniels, A.R., Lee, M. and Pal, M.K., 1977, Soudant, B., 1985, Murray-Smith, D.J., 1985, Manness, M.A., 1988, Oppen, I., *et al.,* 1995) can be used to determine parameter sensitivity functions of a dynamic system. Before discussing the calculation of sensitivity functions some useful properties of the sensitivity function should be presented here. In the particular case of controller parameters the properties are as follows :

Property 1 - To predict the effect of controller parameter changes in terms of the output response it is necessary for the calculated sensitivity function to provide information in terms of both magnitude and direction.

Property 2 - The sensitivity function should be calculated from measurable signals and ideally should not require an accurate plant model as this is often not available in practical engineering control system applications.

Property 3 - The sensitivity of a system in terms of all the parameter sensitivity functions should ideally be obtained simultaneously from the results of a single test on the system.

CHAPTER 3

Calculation of Sensitivity Function

3.1 Introduction

As described in Chapter 2 the adjustment algorithm is based on the idea that from controller parameter sensitivity functions it is possible to predict the changes which will occur in the system responses when changes are made in parameters of the controller. Therefore generating controller parameter sensitivity functions is central to the focus of the adjustment algorithm. In this chapter the theory of generating sensitivity functions will be presented and a number of methods to calculate sensitivity functions will be discussed. Both the time domain and the frequency domain implementations of one method based on signal convolution are explained. An associated two-stage process approach based on cosystem concepts in the time-domain is also described and the calculation of the sensitivity functions for the case of digital control are presented in this chapter as well. Possible advantages and disadvantages of these methods are discussed.

3.2 Theoretical Foundation of the Sensitivity Approach

Sensitivity functions provide a measure of the rate of change of system output response in the time and frequency domains with respect to changes in system parameters. Sensitivity functions not only provide a measure of the change of the system response corresponding to the change of controller parameters but also the direction of the change of the system response. These offer us quantitative information concerning how parameters should be adjusted to meet given closed-loop performance requirements. Suppose the response of a system is $y(t,m_i)$ and after a small change of parameter m_i, Δm_i, the system response becomes $y(t,m_i + \Delta m_i)$. Then if the change of a parameter is small compared with the parameter itself, it is possible to use a first order approximation.. So, as shown in Chapter 2, the change of system response is given by

$$\Delta y_s = y(t, m_i + \Delta m_i) - y(t, m_i) \approx \sum_{i=1}^{n} \frac{\partial y}{\partial m_i} \Delta m_i \tag{3.1}$$

For the simplest case, with only one parameter (n = 1), the change of parameter Δm to achieve a desired response $y_d(t)$ can be approximated by equating the desired change of system response Δy_d to the product of the sensitivity function $\frac{\partial y}{\partial m_i}$ and the parameter change Δm. Similarly, rewriting the equ.(3.1) as the change of transfer function of the system:

$$\Delta W(s) = W(s, m_i + \Delta m_i) - W(s, m_i)$$

it follows that

$$\Delta W(s) \approx \frac{\partial W}{\partial m_i} \Delta m_i \tag{3.2}$$

In general, there are several methods for calculation of the sensitivity functions of the system.

3.2.1 Parameter Perturbation Approach

This approach can be applied either in the time domain or in the frequency domain. It is presented here in terms of the Laplace variable s. If the output Y(s) in a system is dependent upon a set of parameters, $\underline{m}$, the sensitivity function for parameter m_i can be expressed as

$$\frac{\partial Y(s)}{\partial m_i} = \lim_{\Delta m_i \to 0} \frac{Y(s, m + \Delta m_i) - Y(s, m)}{\Delta m_i} \tag{3.3}$$

The disadvantages of this approach are as follows :

1. This method requires n+1 tests, where n is the number of sensitivity functions required. The sensitivity function is dependent upon small differences between the time responses or frequency responses for small parameter perturbations and the calculation is therefore prone to numerical inaccuracies.

2. If the measured signal of the system response Y(s) is corrupted by even small amounts of noise, the approach will fail to give the correct sensitivity function. However, it is very important to note that with an infinitely small perturbation Δm_i and

under the assumption of zero measurement noise this approach can provide the exact sensitivity functions. Even with finite parameter perturbations this method can provide a valuable point of reference for comparisons with results obtained from other approaches to the calculation of sensitivity functions.

3.2.2 A Cosystem Approach for Generating the Sensitivity Functions

The sensitivity analysis of systems which can be described by linear lumped-parameter equations with constant coefficients may be carried out using a technique, developed by Kokotovic' (Kokotovic', P.V. and Rutman, R.S., 1965, Kokotovic', P.V., 1964, Murray-Smith, D.J., 1970), which is known as the "sensitivity points" method.

The sensitivity of the output Y(s) to changes of any parameter m_i of a single input single output linear model with input U(s) is given by

$$\frac{\partial Y(s)}{\partial m_i} = \frac{\partial W(s)}{\partial m_i} U(s) = \frac{1}{W(s)} \frac{\partial W(s)}{\partial m_i} Y(s)$$

where W(s) is the system transfer function.

The sensitivity $\frac{\partial y(t)}{\partial m_i}$ can therefore be found if the system output y(t) is applied as input to a "cosystem model"(Wilkie, D.F., and Perkins, W.R., 1968) which has the transfer function

$$\frac{1}{W(s)} \frac{\partial W(s)}{\partial m_i}$$

It has been proved by Kokotovic' (Kokotovic', P.V., 1964) that for linear feedback structures the cosystem model has the same structural form as the system model itself and that the sensitivity coefficients are signals which appear at certain predetermined points in the block diagram of the cosystem model. In practical terms the cosystem approach to the assessment of controller parameter sensitivity in a closed loop system requires that an accurate plant model be available. The output of the real feedback system would then be applied to the cosystem model as shown in Fig. 3.1 below:

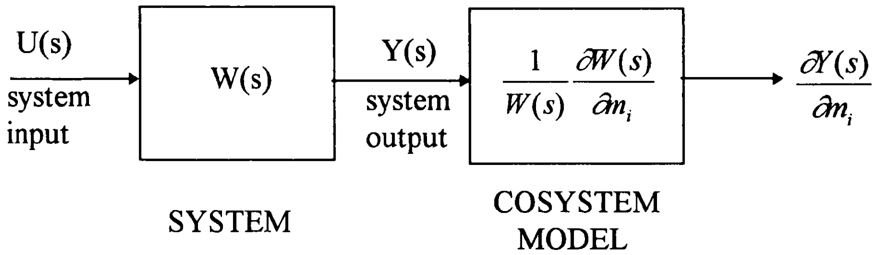


Fig. 3.1 Cosystem approach to parameter sensitivity analysis

3.2.3 The Signal Convolution Method

The use of sensitivity techniques for parameter adjustment in an on-line situation has been inhibited by the difficulties in generating the sensitivity functions, caused by the need for accurate knowledge of the structure and parameters of the system being considered. In order to avoid these difficulties, some efforts have made to develop an approach which is based entirely on signal processing and does not require detailed knowledge of the dynamics of the plant. Winning and co-authors (Winning, D.J., *et al.*, 1977) have derived a suitable relation for a particular case in which system is of the single-input single-output type and the test input signal is a step signal applied to the reference input. In the current work, the method has been further extended to multivariable systems and to more general forms of input.

3.2.3.1 Convolution in the Time domain

Here a sensitivity assessment method is presented for determination of the parameter sensitivity functions in the time domain for the response of a closed loop system to changes of controller parameters. Changes of plant parameters are not considered since the emphasis is on controller parameter adjustments.

To simplify the problem, we first consider a single-input single-output case.

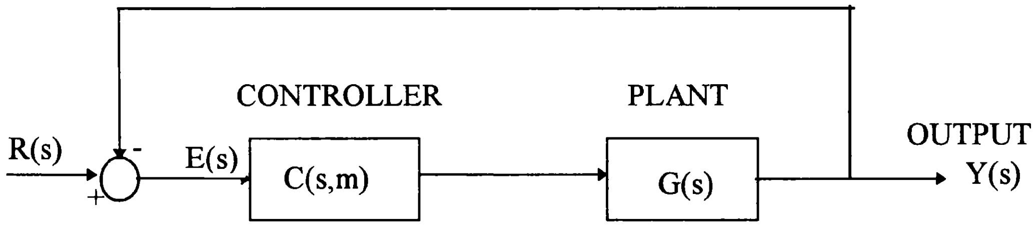


Fig. 3.2 A feedback loop system with controller for SISO case

In the case of a system which involves a linear plant G(s) and linear controller C(s,m) as shown in Fig. 3.2 the system response Y(s) can be described as follows

$$Y(s) = \frac{G(s)C(s,m)}{1+G(s)C(s,m)}R(s) \tag{3.4}$$

The transfer function of the system can be written as

$$W_c(s) = \frac{Y(s)}{R(s)} = \frac{G(s)C(s,m)}{1+G(s)C(s,m)} \tag{3.5}$$

where G(s) is transfer function of the plant

C(s,m) is transfer function of the controller

R(s) is reference input of the system

The sensitivity functions of the system response to the controller parameters $\frac{\partial Y(s)}{\partial m}$ can be expressed as

$$\frac{\partial Y(s)}{\partial m} = \frac{G(s)\frac{\partial C(s,m)}{\partial m}R(s)}{(1+G(s)C(s,m))^2} \tag{3.6}$$

$$\begin{aligned}\frac{\partial Y(s)}{\partial m} &= \frac{E(s)}{R(s)}\frac{Y(s)}{R(s)}\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}R(s)\\ &= E(s)\frac{Y(s)}{R(s)}\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}\\ &= E(s)W_c(s)\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}\\ &= Z(s)Y(s)/R(s) \end{aligned} \tag{3.7}$$

where E(s) is error between output and reference input of the system and $W_c(s)$ is the transfer function of the closed-loop system. The quantity Z(s) is defined by

$Z(s) = E(s)\dfrac{1}{C(s,m)}\dfrac{\partial C(s,m)}{\partial m}$ and is thus the output of a filter of transfer function $\dfrac{1}{C(s,m)}\dfrac{\partial C(s,m)}{\partial m}$ when subjected to an input which is the error signal E(s) in the closed-loop system; as shown in Fig. 3.3

The sensitivity functions can be calculated from equ.(3.7) in the time-domain using convolution if the input signal R(s) has a specific known form, e.g. a step input signal, an impulse input signal etc. For these specific known forms the calculation of the sensitivity functions will be described in Section 3.3.2 later.

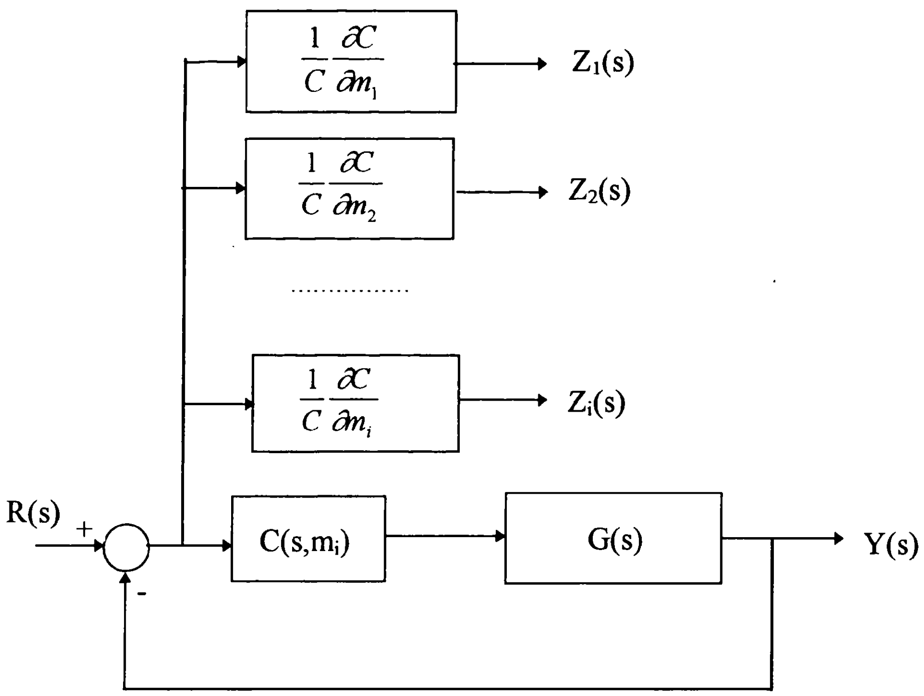


Fig. 3.3 Block diagram for the closed loop system with controller tuning

In the case of a transfer function description the expression for the sensitivity function of the closed-loop transfer function for the controller parameter m can be written as follows:

$$\frac{\partial W_c(s)}{\partial m} = \frac{E(s)}{R(s)} \frac{1}{C(s,m)} \frac{\partial C(s,m)}{\partial m} W_c(s) \qquad (3.8)$$

The block involving $\frac{1}{C(s,m)} \frac{\partial C(s,m)}{\partial m}$ is again a filter having a transfer function which involves only the controller. As in the case of the calculation of output sensitivities it is independent of the plant transfer function G(s). For most practical forms of controller there is no difficulty in implementation of such a filter.

3.2.3.2 Convolution Approach in the Frequency Domain

Suppose the time response y(t) of the closed loop system and the filter signal z(t) have been obtained by measurement or simulation, then the frequency responses of the system could be estimated using these discrete signals and the radix-2 fast Fourier transform (FFT) or more directly though frequency response testing based on testing using sinusoidal signals. The sensitivity function of the system can then be obtained by the convolution method in the frequency domain. Differentiating equ (3.5) with respect to controller parameter m_i

$$\frac{\partial W_c(j\omega)}{\partial m_i} = \frac{\frac{\partial C(j\omega)}{\partial m_i} G(j\omega)[1+G(j\omega)C(j\omega)] - G(j\omega)C(j\omega)G(j\omega)\frac{\partial C(j\omega)}{\partial m_i}}{[1+G(j\omega)C(j\omega)]^2}$$

$$= \frac{\frac{\partial C(j\omega)}{\partial m_i} G(j\omega)}{[1+G(j\omega)C(j\omega)]^2} \qquad (3.9)$$

This can be rewritten as

$$\frac{\partial W_c(j\omega)}{\partial m_i} = \frac{G(j\omega)C(j\omega)}{1+G(j\omega)C(j\omega)} \frac{1}{C(j\omega)} \frac{\partial C(j\omega)}{\partial m_i} \frac{1}{1+G(j\omega)C(j\omega)}$$

where the first term can be recognised as the closed loop transfer function from equ.(3.5). From the definition of Z(s) in equ.(3.7) it follows that

$$\frac{1}{C(j\omega)}\frac{\partial C(j\omega)}{\partial m_i}\frac{1}{1+G(j\omega)C(j\omega)}=\frac{Z_i(j\omega)}{R(j\omega)}$$

so that $\frac{\partial W_c(j\omega)}{\partial m_i}=W_c(j\omega)\frac{Z_i(J\omega)}{R(j\omega)}=P_i(j\omega)W_c(j\omega)$ (3.10)

where $P_i(j\omega)=\frac{Z_i(j\omega)}{R(j\omega)}$

In fact, the transfer function of the closed loop system can be expressed in polar form :

$$W_c(j\omega) = M(\omega)\, e^{j\alpha(\omega)}$$

and similarly $P_i(j\omega)=U_i(\omega)\, e^{j\beta(\omega)}$

Equ.(3.10) can then be rewritten as

$$\frac{\partial W_c(j\omega)}{\partial m_i}=M(\omega)e^{j\alpha}U_i(\omega)e^{j\beta}=M(\omega)U_i(\omega)e^{j(\alpha+\beta)} \tag{3.11}$$

Using the Euler formula

$$e^{j\phi} = \cos\phi + j\sin\phi$$

and a Taylor series, we can obtain from equ.(3.11) (See Appendix 2)

$$\frac{\partial|W_c(j\omega)|}{\partial m_i}=\frac{\partial M(\omega)}{\partial m_i}=M(\omega)U_i(\omega)\cos(\beta) \tag{3.12}$$

The equ.(3.12) is an exact expression for the sensitivity function in the frequency domain (Murray-Smith, D.J., 1985)

3.2.3.3 Two-Stage Convolution Process

Equ.(3.6) is the basic equation relating the sensitivity of the output of the closed-loop system to a controller parameter m. This equation can be rearranged to take the form

$$\frac{\partial Y(s)}{\partial m}=[\frac{G(s)C(s,m)}{1+G(s)C(s,m)}R(s)]\frac{\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}}{1+G(s)C(s,m)} \tag{3.13}$$

The factor in the square brackets represents the output Y(s) of the closed-loop system to the reference signal R(s)

Hence $$\frac{\partial Y(s)}{\partial m} = Y(s)\frac{\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}}{1+G(s)C(s,m)} \tag{3.14}$$

This equation shows that the sensitivity $\frac{\partial Y(s)}{\partial m}$ can be found by applying the output Y(s) to a cosystem having the transfer function

$$F(s,m) = \frac{\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}}{1+G(s)C(s,m)} \tag{3.15}$$

In practice the generation of such a cosystem would require knowledge of the plant transfer function G(s). It may be noted, however, that the factor

$$\frac{1}{1+G(s)C(s,m)}$$

is the transfer function of the closed-loop system itself from the reference input to the error signal and this immediately suggests that the closed-loop system could act its own cosystem if a two stage process were adopted involving storage of the closed-loop response Y(s) in the first stage test and then the application of the stored signal as the reference input in the second stage. This two stage approach is illustrated in Fig. 3.4(a) and 3.4(b).

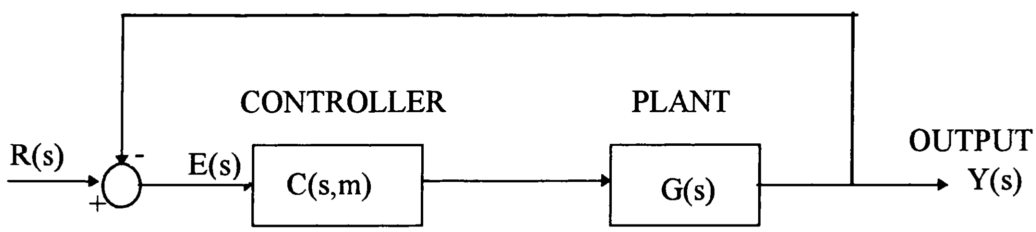


Fig. 3.4(a) First stage to obtain signal Y(s)

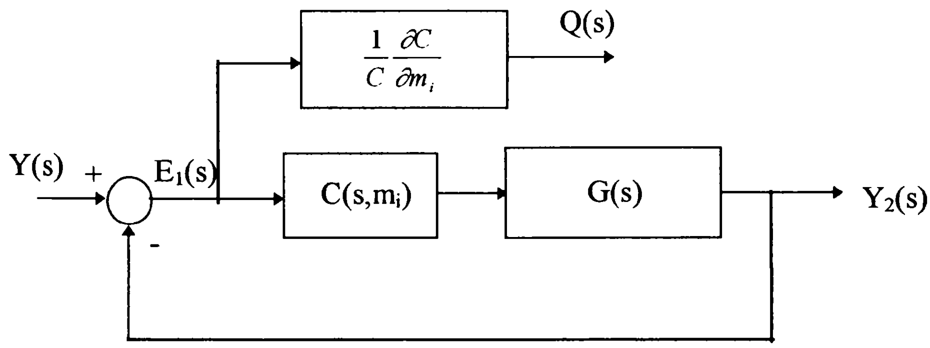


Fig. 3.4(b) Second stage to obtain sensitivity function

Assuming that the output of the closed-loop system in the second test is $Y_2(s)$ it is clear that the error signal $E_1(s)$ in the second test is

$$E_1(s) = Y(s) - Y_2(s) = \frac{1}{1+G(s)C(s,m)}Y(s) \tag{3.16}$$

and that the sensitivity $\frac{\partial Y(s)}{\partial m}$ is given by a signal

$$Q(s) = E_1(s)\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m} \tag{3.17}$$

formed by applying the signal $E_1(s)$ to a filter having a transfer function

$$\frac{1}{C(s,m)}\frac{\partial C(s,m)}{\partial m}$$

Since this filter is the only element which changes when different controller parameters are considered it is clear that n separate filters can be used to generate n controller parameter sensitivity functions.

It should be noted that this two-stage process is equivalent to the convolution process in the time domain discussed in Section 3.2.3.1 but in this case the properties of the closed-loop system transfer function are used to carry out the convolution process. It also avoids any need for the reference signal R(s) to have a specific ideal form such as a step or an impulse

3.3 Application of Controller Parameter Sensitivity Function Evaluation Methods in the Time Domain to Single-Input Single-Output Systems

Three methods of calculation of the sensitivity functions of a closed-loop system for controller parameters will be applied here. These methods are illustrated using two simple examples which involve linear single-input single-output applications.

3.3.1 The Parameter Perturbation Method

In general, the output of a closed-loop system is dependent upon a set of controller parameters, m. As shown in Section 3.2.1, if the structure of the system is given, the sensitivity function for controller parameter m_i, of this system can be assessed as

$$\frac{\partial Y(s)}{\partial m_i} = \lim_{\Delta m_i \to 0} \frac{Y(s, m_i + \Delta m_i) - Y(s, m_i)}{\Delta m_i} \tag{3.18}$$

This assessment requires n+1 tests, where n is number of sensitivity functions to be assessed. The method is dependent upon the difference between the time responses with only a small parameter perturbation. A simple closed-loop system with a three term (PID) controller provides a convenient basis for illustrating the parameter perturbation approach. The plant is expressed by the transfer function 1/(s+1). The transfer function of the PID controller was taken as

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + s\tau} \tag{3.19}$$

Thus the transfer function of this closed-loop system is expressed below as

$$W(s) = \frac{(K_p\tau + K_d)s^2 + (K_p + K_i\tau)s + K_i}{\tau s^3 + (K_p\tau + K_d + \tau + 1)s^2 + (K_p + K_i\tau + 1)s + K_i} \tag{3.20}$$

Figs. 3.5(a), 3.5(b) and 3.5(c) show the sensitivity functions, S_p, S_i and S_d, corresponding to the controller parameters $K_p = 5.24$, $K_i = 3.57$ and $K_d = 3.0$. The variables y_p, y_i and y_d are the step responses of the system for 10% change of the parameters K_p, K_i and K_d, respectively.

- --- time response y_1, • --- response for m 10% changed

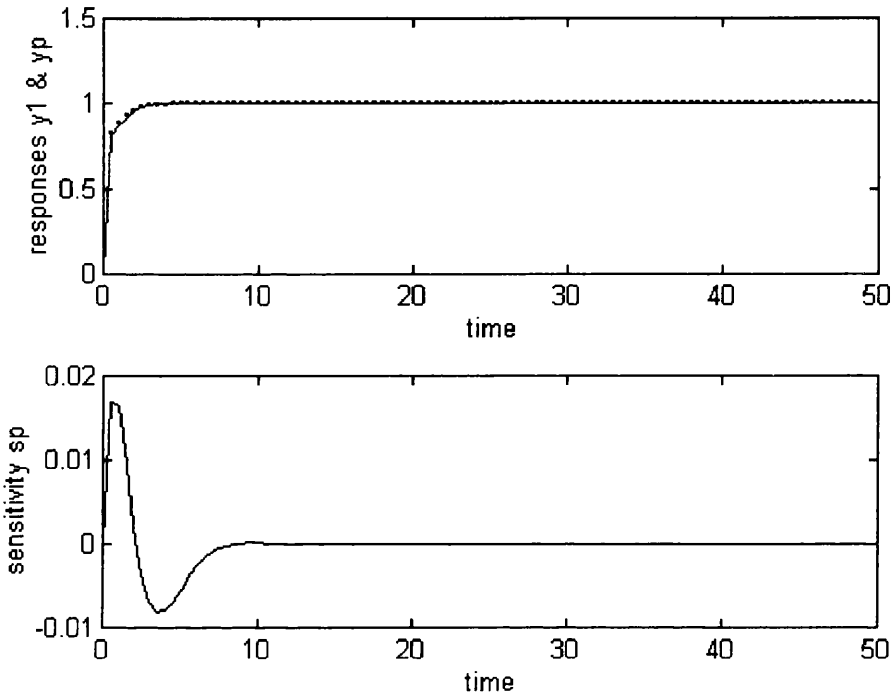


Fig.3.5(a) Original and perturbed responses for first order system with PID controller, together with sensitivity functions determined by the parameter perturbation method for 10% parameter changes in parameter K_p

- --- time response y_1,· --- response for m 10% changed

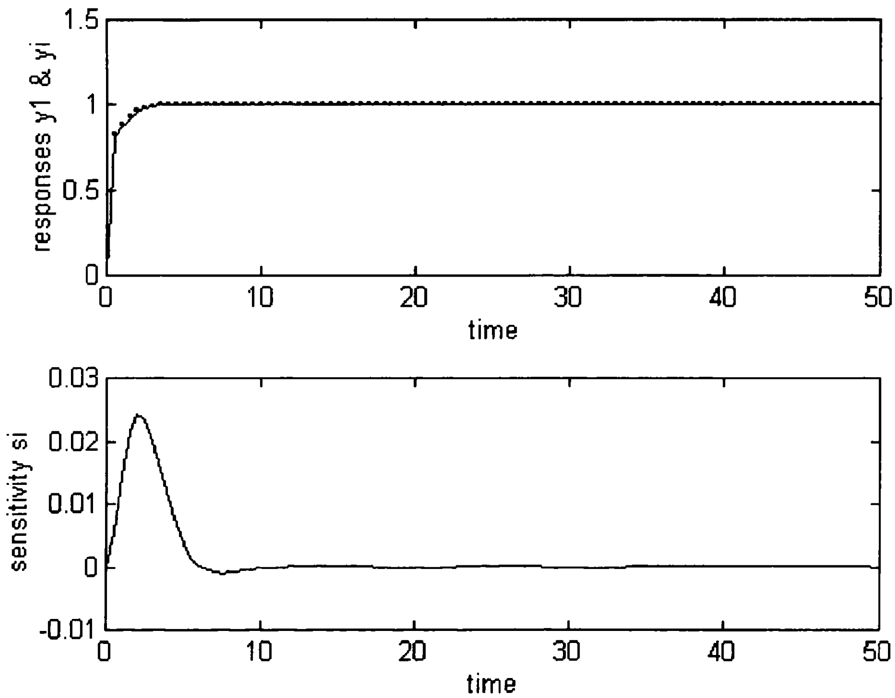


Fig.3.5(b) Original and perturbed responses for first order system with PID controller, together with sensitivity functions determined by the parameter perturbation method for 10% parameter changes in parameter K_i

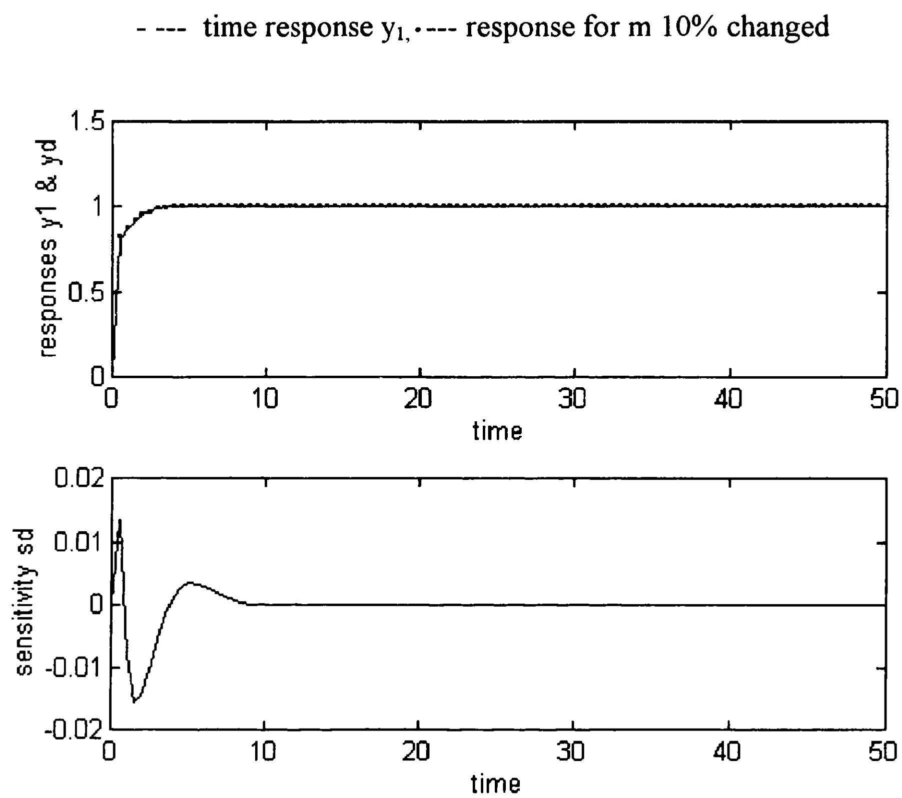


Fig.3.5(c) Original and perturbed responses for first order system with PID controller, together with sensitivity functions determined by the parameter perturbation method for 10% parameter changes in parameter K_d

The results show that the method is simple but it needs n+1 measurements to get n sensitivity functions for n parameters. This method is satisfactory in the case where measurement noise is negligible. It is should be noted that if the measured signal y(t) is corrupted by even small amounts of noise, the method will fail to give the correct sensitivity function. So, there is an assumption of zero noise and an infinitely small parameter perturbation in this approach to the determination of sensitivity functions.

3.3.2 The Convolution Method

In the special case of an input R(s) which is a unit impulse equ.(3.7) becomes

$$\frac{\partial Y(s)}{\partial m} = Z(s)\,Y(s) \tag{3.21}$$

This gives a sensitivity function in the time domain which can be expressed by the convolution integral as

$$\frac{\partial y(t)}{\partial m} = \int_0^T y(\tau)z(t-\tau)d\tau \tag{3.22}$$

Similarly, if the input R(s) is a unit step signal equ.(3.7) becomes

$$\frac{\partial Y(s)}{\partial m} = Z(s)[sY(s)] = Y(s)[sZ(s)] \tag{3.23}$$

The first of the relationships of equ.(3.23) gives a sensitivity function in the time domain in the form

$$\frac{\partial y(t)}{\partial m} = \int_0^T \frac{dy(\tau)}{d\tau} z(t-\tau)d\tau \tag{3.24}$$

and introduces the need to differentiate the system output with respect to time prior to carrying out the convolution operation.

The second relationship

$$\frac{\partial Y(s)}{\partial m} = Y(s)[sZ(s)]$$

may allow the need for numerical differentiation to be avoided by incorporating the differentiation process into the sensitivity filter. The sensitivity is then given by

$$\frac{\partial Y(s)}{\partial m} = Y(s)Z'(s) \tag{3.25}$$

where $Z'(s) = sZ(s) = \dfrac{s}{C(s,m)}\dfrac{\partial C(s,m)}{\partial m}E(s)$

The corresponding time-domain expression is

$$\frac{\partial y(t)}{\partial m} = \int_0^T y(\tau)z'(t-\tau)d\tau \tag{3.26}$$

Although previous work on this convolution approach to sensitivity function calculation has all been based on the use of step function inputs (El-Shirbeeny, E.H.T., *et al.*, 1974, Winning, D.J. *et al.*, 1977, Murray-Smith, D.J., 1985) it should be noted that numerical differentiation is a noise enhancing process and is generally regarded as undesirable.

In order to make comparisons easily with other methods we again use the system of section 3.3.1 as an example. The filter outputs $Z_p(s)$, $Z_i(s)$ and $Z_d(s)$ corresponding to parameters K_p, K_i and K_d are shown below:

$$Z_p(s) = \frac{\tau s^2 + s}{D_2 s^2 + D_1 s + D_0} \mathrm{E(s)} \tag{3.27}$$

$$Z_i(s) = \frac{\tau s + 1}{D_2 s^2 + D_1 s + D_0} \mathrm{E(s)} \tag{3.28}$$

$$Z_d(s) = \frac{\tau s^3 + s^2}{(D_2 s^2 + D_1 s + D_0)(1 + \tau s)} \mathrm{E(s)} \tag{3.29}$$

where $D_2 = K_p \tau + K_d$

$D_1 = K_p + K_i \tau$

$D_0 = K_i$

and $\mathrm{E(s)} = \dfrac{\tau s^3 + (\tau + 1)s^2 + s}{\tau s^3 + (K_p \tau + K_d + \tau + 1)s^2 + (K_p + K_i \tau + 1)s + K_i}$

Fig. 3.6 shows the sensitivity functions S_p, S_i and S_d corresponding to the controller parameters K_p, K_i and K_d using the convolution approach.

Fig. 3.6 and Fig. 3.5 show that the results using the convolution approach agree with the results using the parameter pertubation method perfectly.

In order to demonstrate the potential of this approach with a more realistic problem another slightly more complex example involving a synchronous generator excitation system with cascade compensation (Winning, D.J., *et al.*, 1977), as shown in Fig. 3.7, was used to compare the method of sensitivity generation using the convolution approach with results obtained by the parameter perturbation method. For the system of Fig. 3.7 , the terminal voltage response V_T is related to the voltage reference V_R by the equation

$$V_T(s) = \frac{C(s,a)W(s)}{1+C(s,a)W(s)}V_R(s)$$

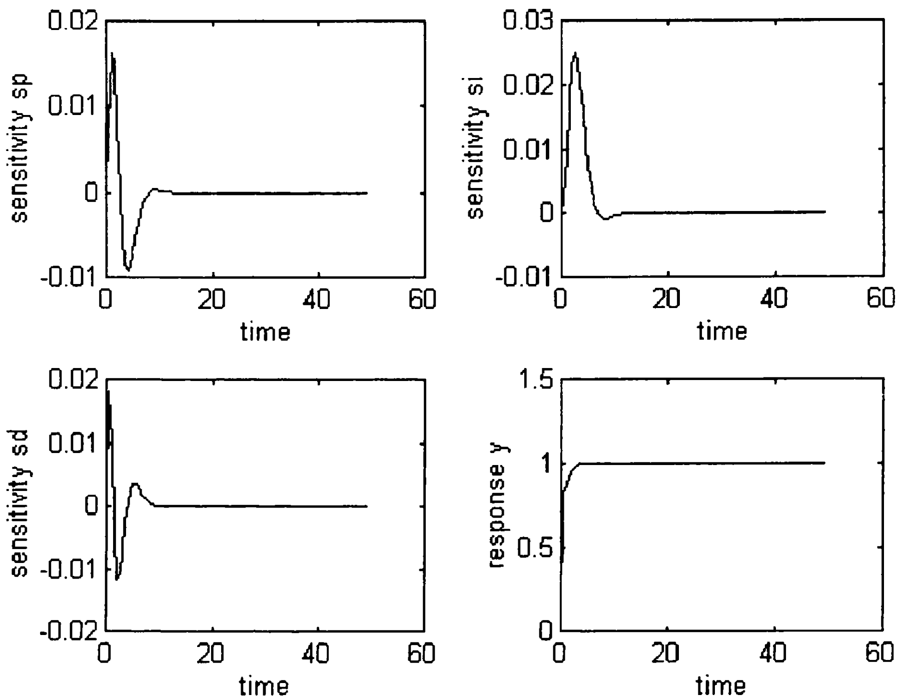


Fig. 3.6 Sensitivity functions for closed-loop system with PID controller calculated using the convolution approach (for comparison with Figs.3.5(a), 3.5(b) and 3.5(c))

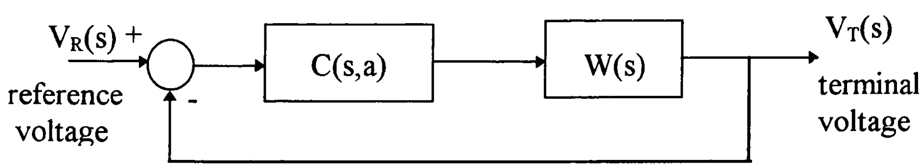


Fig. 3.7 Block diagram of a synchronous generator and excitation system

The transfer function of the controller used in this example was of the form shown in equ.(3.30) with parameters β and γ adjustable in the range 0-1.

$$C(s,a) = \frac{1+(0.68\beta+0.047)s}{1+(30\gamma+3)s} \tag{3.30}$$

The plant transfer function W(s) is expressed as :

$$W(s) = \frac{200(1+30s)}{(1+0.047s)(1+1.8s)(1+5.3s)}$$

The corresponding sensitivity filter transfer functions $F_\beta(s)$ and $F_\gamma(s)$ are :

$$F_\beta(s) = \frac{0.68s}{1+(0.68\beta+0.047)s}$$

$$F_\gamma(s) = \frac{-30s}{1+(30\gamma+3)s}$$

Fig. 3.8 shows two sets of the sensitivity functions for a step input signal. It may be seen that the result obtained by the convolution approach agrees with the result obtained by the parameter pertubation method. The parameter values β = 0.2 and γ = 0.2 were used. The sensitivity functions by the pertubation method are for 10% parameter changes.

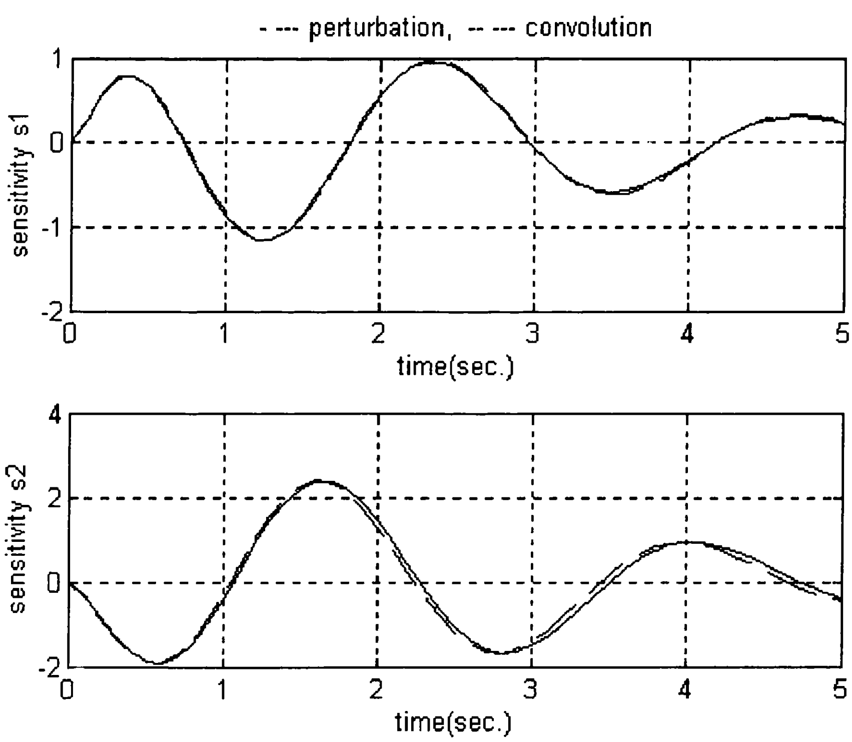


Fig. 3.8 Sensitivities using perturbation and convolution approach for a synchronous generator and excitation system (α = 0.2, β = 0.2, step input case)

3.3.3 Sensitivity Functions Using the Two-Stage Approach

Considering the same simulated example used in Section 3.3.1, the sensitivity functions can directly be obtained using the two stage approach. The output Y(s) of the closed loop system is obtained in the first stage. The filter outputs $Q_i(s)$ corresponding to the controller parameters K_p, K_i and K_d should give the sensitivity functions at the second stage in response to the measured signal Y(s) as reference input.

The system output Y(s) in the first stage is:

$$Y(s) = E(s)C(s)G(s)$$

$$= \frac{(K_p\tau + K_d)s^2 + (K_p + K_i\tau)s + K_i}{\tau s^3 + (K_p\tau + K_d + \tau + 1)s^2 + (K_p + K_i\tau + 1)s + K_i}$$

The filter outputs $Q_i(s)$ can be expressed as following :

$$Q_p(s) = \frac{\tau s^2 + s}{D_2 s^2 + D_1 s + D_0} E_1(s)$$

$$Q_i(s) = \frac{\tau s + 1}{D_2 s^2 + D_1 s + D_0} E_1(s)$$

$$Q_d(s) = \frac{\tau s^3 + s^2}{(D_2 s^2 + D_1 s + D_0)(1 + \tau s)} E_1(s)$$

where $E_1(s) = Y(s) - Y_2(s) = \frac{\tau s^3 + (\tau + 1)s^2 + s}{\tau s^3 + D_4 s^2 + D_3 s + D_0} Y(s)$

and where $Y_2(s)$ is the plant output in the second stage test

and $D_4 = K_p\tau + K_d + \tau + 1$

$D_3 = K_p + K_i\tau + 1$

$D_2 = K_p\tau + K_d$

$D_1 = K_p + K_i\tau$

$D_0 = K_i$

If the input used is the step signal, the sensitivity functions can be obtained immediately in MATLAB. The results shown in Fig. 3.9 agree perfectly with Figs. 3.5(a), 3.5(b) 3.5(c) and Fig. 3.6 using the parameter perturbation method and the convolution approach respectively.

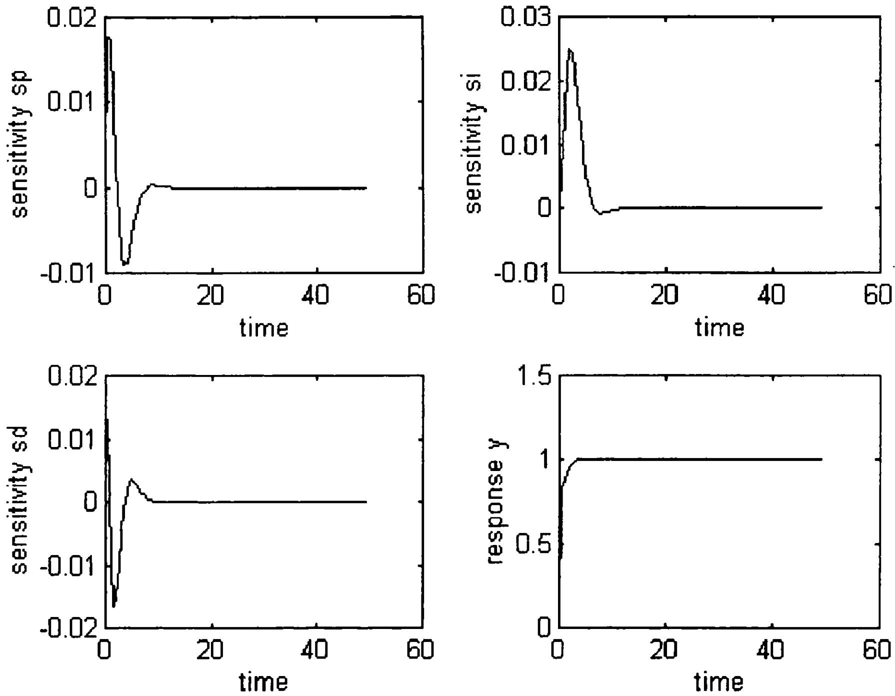


Fig. 3.9 Sensitivity functions obtained using two-stage approach for the closed-loop system with PID control

The sensitivity functions of the controller parameters β and γ for the synchronous generator excitation system described Fig. 3.7 in section 3.3.2 can also be obtained by the two-stage approach. We obtain the output response $V_T(s)$ from first stage :

$$V_T(s) = \frac{C(s,a)W(s)}{1+C(s,a)W(s)} V_R(s)$$

$$= \frac{109.8s^2 + 636.6s + 200}{4.0354s^4 + 89.3117s^3 + 183.9967s^2 + 652.747s + 201} V_R(s)$$

The filter outputs $Q_i(s)$ in the second stage can be expressed as following

$$Q_\beta(s) = \frac{0.68s}{(0.68\beta + 0.047)s + 1} E_1(s)$$

$$Q_\gamma(s) = \frac{-30s}{(30\gamma + 3)s + 1} E_1(s)$$

where the error of the system $E_1(s)$ is

$$E_1(s) = \frac{4.0354s^4 + 89.3117s^3 + 74.1967s^2 + 16.147s + 1}{4.0354s^4 + 89.3117s^3 + 183.9967s^2 + 652.747s + 201} V_T(s)$$

The sensitivity functions of the controller parameters β and γ are shown in Fig. 3.10. Fig. 3.10 shows the sensitivity functions using the two-stage approach agree very well with the sensitivity functions using the parameter perturbation method and the convolution approach (Fig. 3.8).

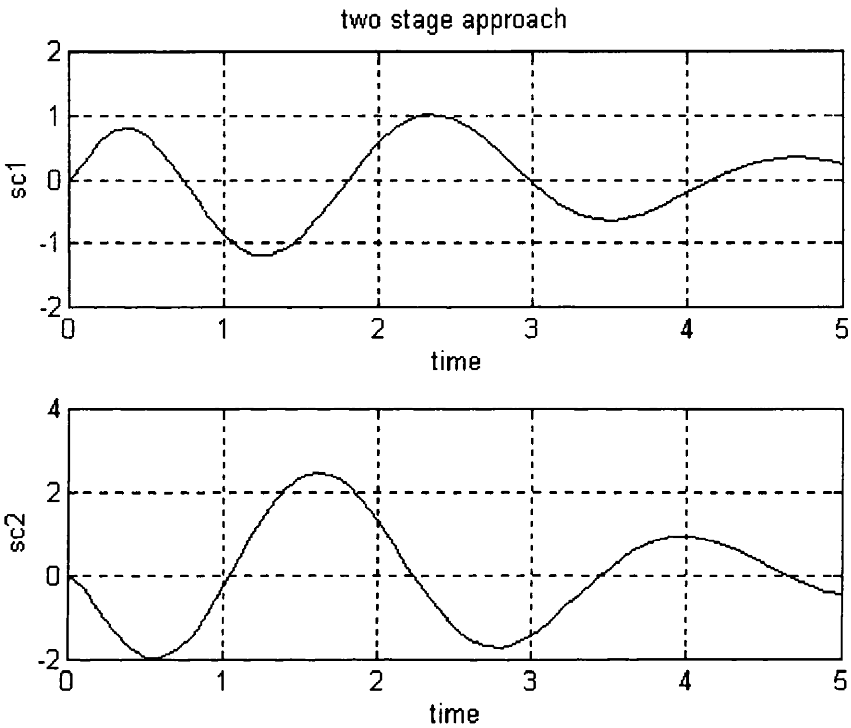


Fig. 3.10 Sensitivity functions obtained using two-stage approach for a synchronous generator and excitation system

It should also be noted that although these examples use a step function test signal the two-stage approach can be used for any type of input signal. The results suggest that it can provide an efficient means of assessing sensitivity in site tests if facilities for analogue to digital and digital to analogue conversion are available. Operating conditions in the tests for the two stages must be identical.

3.4 Investigation of Sensitivity in the Frequency Domain

In the frequency domain, the controller parameter sensitivity function for a closed loop system can be defined as the change of the system response in terms of magnitude and phase with respect to the change of a system parameter. The function thus provides a quantitative measure of the degree of influence a parameter change has on the system response. Controller parameter sensitivity functions can be generated by two methods. One of them involves the small parameter perturbation method. Another one is a frequency-domain equivalent of the convolution approach described in Section 3.2 and this overcomes the problems inherent in the parameter perturbation method. The two-stage approach could, in principle, also be applied.

3.4.1 Parameter Perturbation Method in the Frequency Domain

In the frequency domain the parameter perturbation method also can be used to generate the parameter sensitivity functions. If we consider the cases of sinusoidal inputs, s = jω, the equ.(3.3) becomes

$$\frac{\partial |W_c(j\omega)|}{\partial m_i} = \lim_{\Delta m_i \to 0} \frac{\|W_c(j\omega, m_i + \Delta m_i)\| - \|W_c(j\omega, m_i)\|}{\Delta m_i} \qquad (3.31)$$

where $W_c(j\omega)$ is the closed loop transfer function and m_i is the controller parameter of interest. It is important to notice that in the limit for an infinitely small perturbation equation (3.31) is an exact expression for the sensitivity function of the closed loop system. Fig 3.11 shows the closed-loop frequency response of the system described in Section 3.3.1 with three-term control for one parameter combination of the controller parameter.

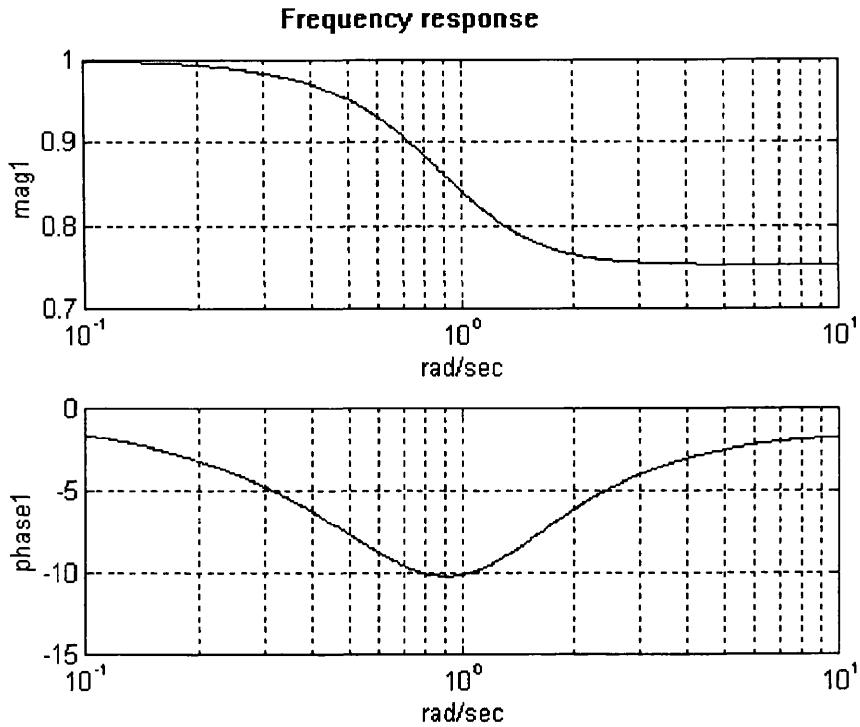


Fig. 3.11 Frequency response for the example in Section 3. 3.1

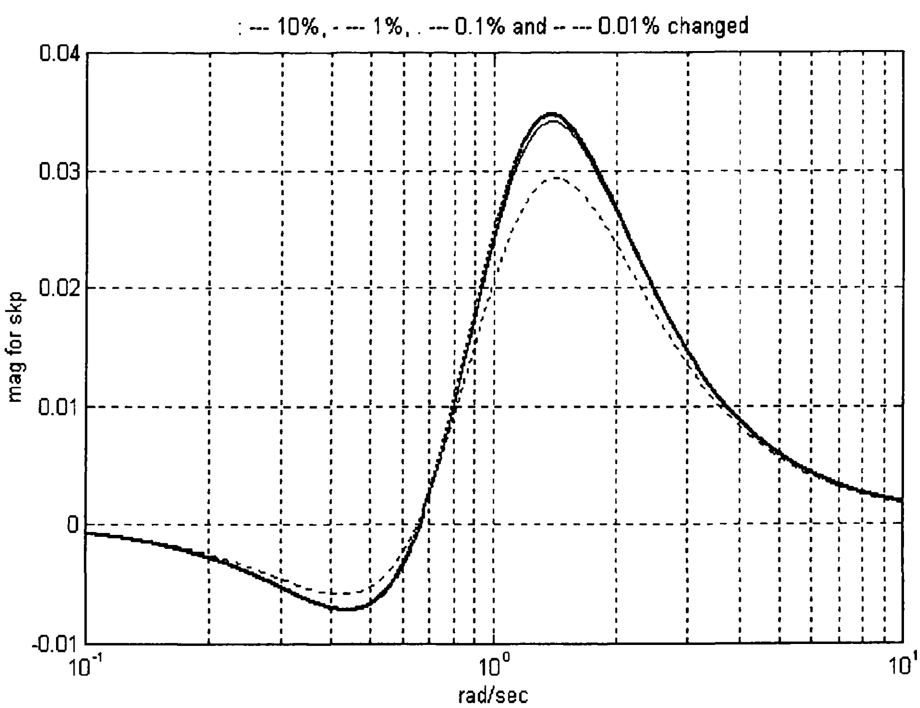


Fig. 3.12 Parameter perturbation method : sensitivity function for parameter K_p

Fig.3.12 shows the sensitivity function of the controller parameter K_p for this example. In this case graphics have been plotted for four different parameter perturbations (10, 1, 0.1, and 0.01 percent parameter perturbation). The difference is small for the 1% and 0.1% cases and the parameter perturbation method is thus seen to converge for small perturbations. A major constraint of the method is also shown. Fig. 3.12 suggests the use of a 1% parameter perturbation or less to generate the correct sensitivity function. It is easy to realise that problems would arise if equ.(3.19) was employed to assess the sensitivity functions in cases where noise and drift problems could corrupt measurements. Fig. 3.13 shows frequency domain parameter sensitivity functions for all three parameters of the PID controller.

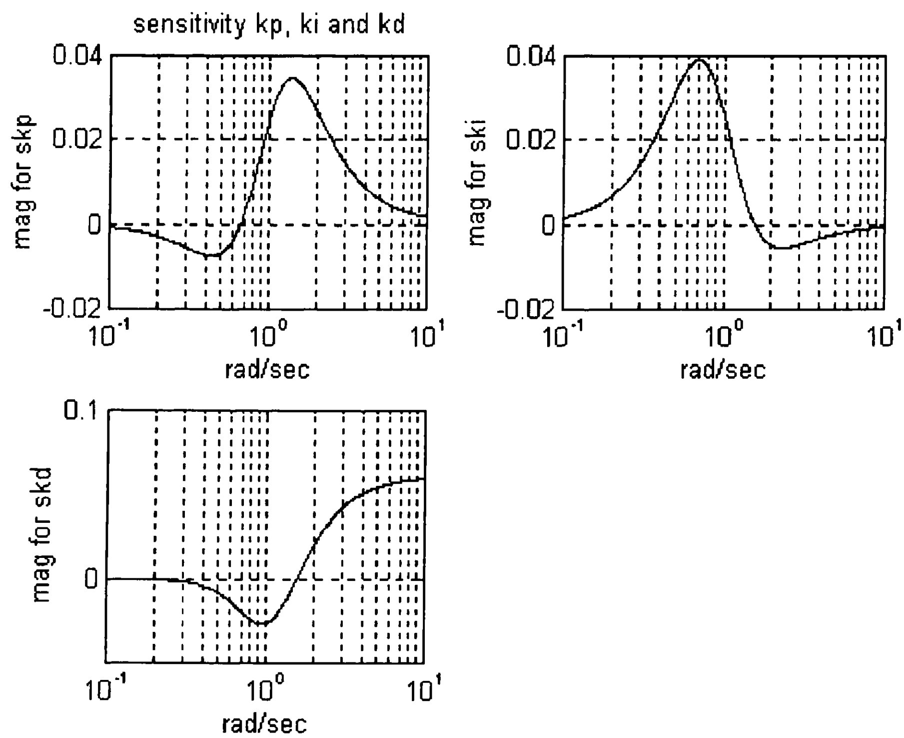


Fig. 3.13 Frequency domain sensitivity function using parameter perturbation method for changes of 1% in each parameter K_p, K_i and K_d of the three term controller

3.4.2 Convolution Approach in the Frequency Domain

To allow comparisons with the parameter perturbation method described in Section 3.4.1, the method for the simultaneous direct assessment of sensitivity functions in the frequency domain presented in Section 3.2.3.2 is applied to the system with three term control. The frequency response of the closed loop system with a parameter change Δm in the controller is given by

$$Y(j\omega, m + \Delta m) = Y(j\omega, m) + \frac{\partial Y(j\omega, m)}{\partial m}\Delta m \qquad (3.32)$$

From equ.(3.10) it follows that the sensitivity function can be expressed as

$$\frac{\partial Y(j\omega, m)}{\partial m} = Z_i(j\omega, m)W_c(j\omega, m) \qquad (3.33)$$

Hence to obtain the sensitivity of the closed loop system for the controller parameter m we may need to calculate (or measure) the responses $Y(j\omega)/R(j\omega)$ and $Z_i(j\omega)$. At every frequency we multiply these complex quantities together to give the sensitivity function $\frac{\partial Y(j\omega, m)}{\partial m}$. The important point here is that $Y(j\omega, m+\Delta m)$ can be found from the measured $Z_i(j\omega)$ and the measured $W_c(j\omega, m)$ without any need to know the transfer function of the closed loop system G(s). Fig. 3.14 shows the sensitivity functions for the PID control system example of the Section 3.3.1 using the convolution approach. The results shown in Fig. 3.14 agree well with the results shown in Fig. 3.13 using the parameter perturbation method. The technique can thus be used for tuning of the controller parameters.

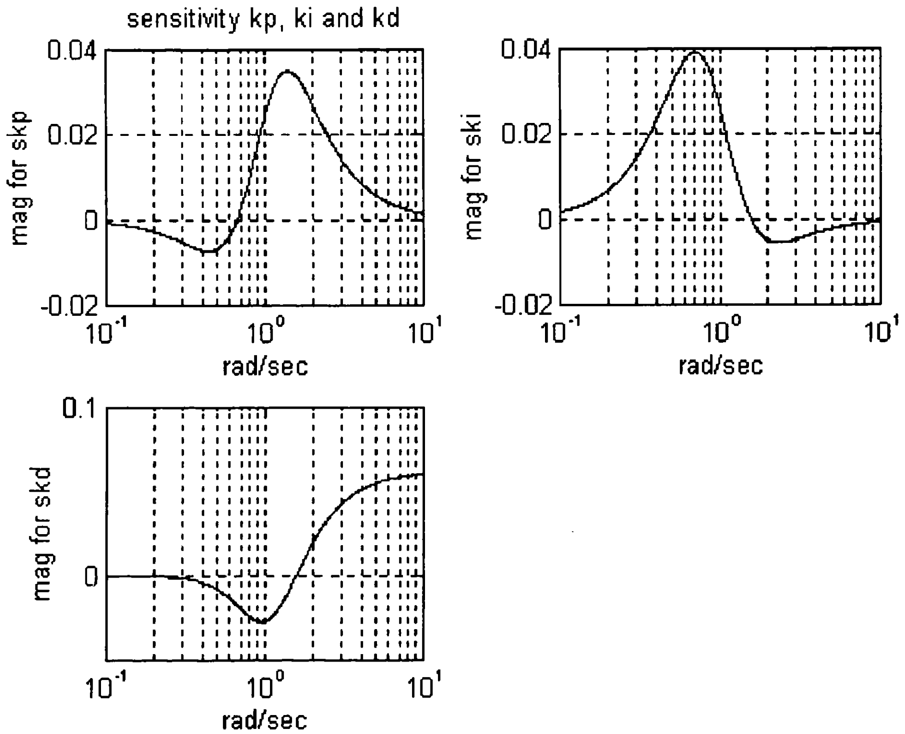


Fig.3.14 Sensitivity function using convolution approach for the three-term control system example

3.5 Sensitivity Functions for Digital Control Systems

A method for the assessment of sensitivity functions for a digital control system is presented below. The block diagram for a typical single input single output closed loop system is shown Fig.3.15.

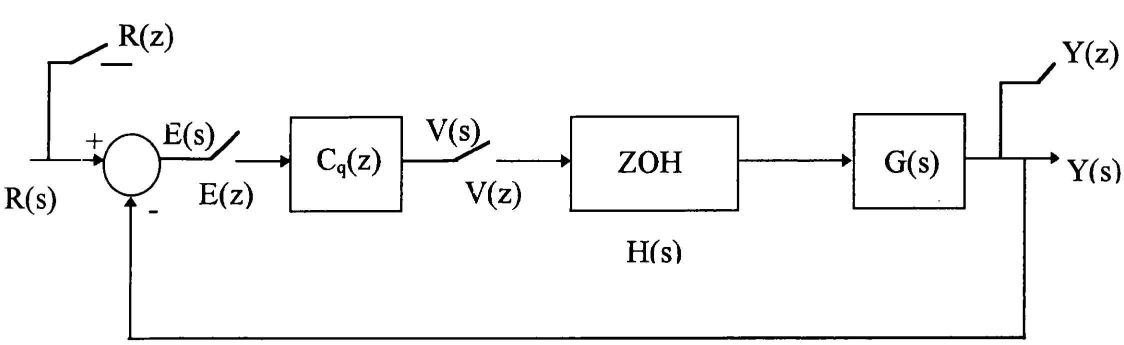


Fig. 3.15 Block diagram of a closed-loop digital control system

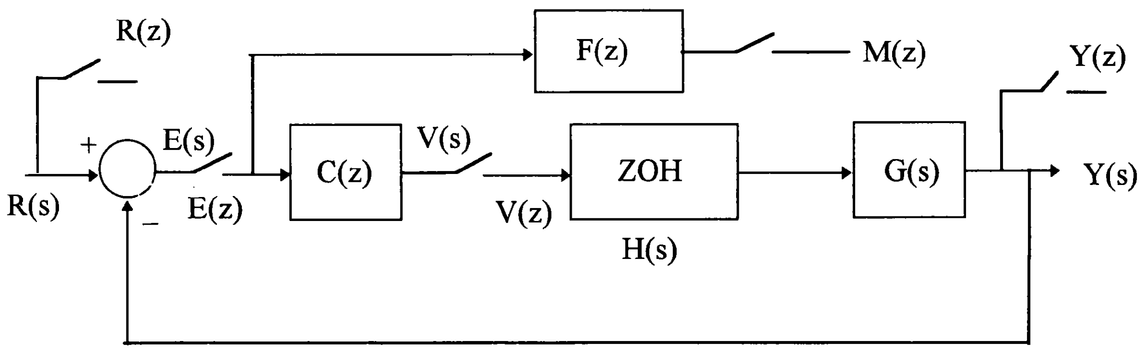


Fig. 3.16 A closed loop digital control system with sensitivity filter

In Fig. 3.15 the error between the response Y(z) and the reference R(z) of the digital control system E(z) is

$$E(z) = R(z) - Y(z) = \frac{1}{1 + C(z)GH(z)} R(z)$$

where $Y(z) = Z\{GH(s)\}V(z) = GH(z)V(z)$

$V(z) = C(z)E(z)$ is the output of the controller.

Therefore the output of the system Y(z) is

$$Y(z) = \frac{C(z)GH(z)}{1 + C(z)GH(z)} R(z)$$

$$\frac{\partial Y(z)}{\partial q} = \frac{[1 + C(z)GH(z)]GH(z)\frac{\partial C(z)}{\partial q} - C(z)GH^2(z)\frac{\partial C(z)}{\partial q}}{[1 + C(z)GH(z)]^2} R(z)$$

$$\begin{aligned}
&= \frac{GH(z)\frac{\partial C(z)}{\partial q}}{[1 + C(z)GH(z)]^2} R(z) \\
&= \frac{1}{C(z)} \frac{\partial C(z)}{\partial q} \frac{C(z)GH(z)}{1 + C(z)GH(z)} \frac{R(z)}{1 + C(z)GH(z)} \\
&= \frac{1}{C(z)} \frac{\partial C(z)}{\partial q} E(z) \frac{Y(z)}{R(z)} \qquad (3.34) \\
&= F(z)E(z)\frac{Y(z)}{R(z)} \\
&= M(z)\frac{Y(z)}{R(z)}
\end{aligned}$$

.

where $GH(z) = Z\{\frac{1-e^{-st}}{s}G(s)\} = (1-z^{-1})Z\{\frac{G(s)}{s}\}$

$$F(z) = \frac{1}{C(z)}\frac{\partial C(z)}{\partial q}$$

$$M(z) = E(z)F(z)$$

For a unit step input R(s) = 1/s or R(z) = z/(z-1) the sensitivity function of the digital control system could be expressed as

$$\frac{\partial Y(z)}{\partial q} = M(z)\frac{Y(z)}{R(z)}$$

$$= M(z)Y(z)\frac{z-1}{z}$$

$$= M(z)Y(z) - z^{-1}M(z)Y(z) \qquad (3.35)$$

where M(z) is the output of the filter F(z).

For a digital control system the output M(z) and Y(z) could be obtained by using a series method:

$$M(z) = m_0 + m_1 z^{-1} + m_2 z^{-2} + \cdots + m_n z^{-n}$$

$$Y(z) = y_0 + y_1 z^{-1} + y_2 z^{-2} + \cdots + y_n z^{-n}$$

The product of M(z) and Y(z) is given

$$M(z) \cdot Y(z) = m_0y_0 + (m_0y_1 + m_1y_0)\, z^{-1} + \cdots$$

$$= \sum_{j=0}^{n}(\sum_{i=0}^{j} m_i y_{i-1})z^{-j} \qquad (3.36)$$

The sensitivity functions of the controller parameters for the digital control system can thus be obtained by equ.(3.35). Fig.3.18 shows the results of sensitivity function calculation for a example which is shown in Fig. 3.17. The transfer functions of the filter $F_i(z)$ corresponding to the controller parameters k, a and b are expressed as :

$$F_k(z) = \frac{1}{C(z)}\frac{\partial C}{\partial k} = \frac{1}{k}$$

$$F_a(z) = \frac{1}{C(z)}\frac{\partial C}{\partial a} = \frac{-z^{-1}}{1-az^{-1}}$$

$$F_b(z) = \frac{1}{C(z)}\frac{\partial C}{\partial b} = \frac{z^{-1}}{1-bz^{-1}}$$

The special case considered involves k = 1, a = 0.5 and b = 1 with T = 0.347 sec. The calculation in this case is quite simple. It is only necessary to obtain the outputs M(z) and Y(z), to obtain the product of M(z) and Y(z) . The sensitivity function is then directly obtained. Fig. 3.18 shows the sensitivities to controller parameters k, a and b using the series method and the perturbation method. The sensitivity functions by the perturbation method are for 1% changes in the parameter k, a and b.

The measure of controller sensitivity provides information about the sensitivity of the system output at sampling instants only. However, this is appropriate in tuning of digital controller. The calculation of the sensitivity function for the digital controller is very simple involving only the multiplication of time series. This approach should allow optimisation of the digital controllers (such as dead - beat controllers) in which there is significant parametric interaction. One reason for dominance of three - term controller is the fact that effects of changing each gain factor in the controller can be easily predicted and thus the controller is easy to tune. With other digital controllers this is seldom true and tuning after installation is difficult and time consuming. The tuning technique based on easily calculated sensitivity functions should provide important advantages in this case.

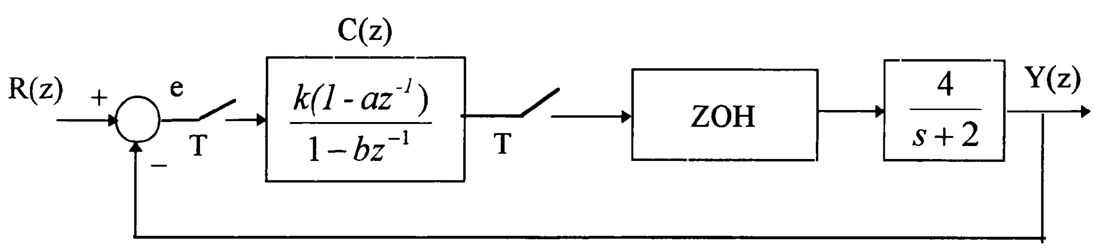


Fig. 3.17 A example for digital closed-loop control system

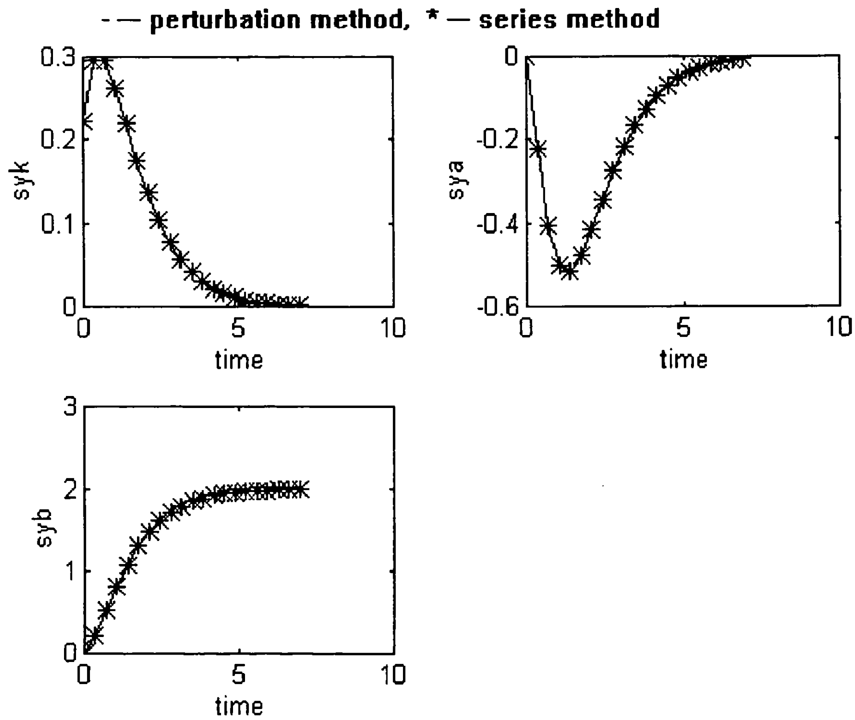


Fig. 3.18 Sensitivity functions for the digital control system of Fig. 3.17 for case where k = 1, a = 0.5, b = 1 and T = 0.347

CHAPTER 4

Selection of Input Test Signals for Controller Tuning

4.1 Introduction

As with any kind of system testing it is clear that the form of input signal used can have a significant influence on the determination of sensitivity functions and thus on the performance of the sensitivity tuning technique. Concerning the choice of input signal there are several aspects to consider, as in the case of system identification. Certain tuning methods require a special type of input and, in general, the selection of input test signals is dependent on the method of parameter tuning, the method of calculation of sensitivity functions, and some practical issues associated with experimental realization. For instance, a step or an impulse as input test signal may be appropriate in time-domain analysis. For frequency domain analysis the input test signal may be a series of sinusoids or some appropriate broad-band test input.

We generally assume that the system to being tuned is modelled as a sampled data system. This implies that the input and output data are recorded in discrete time and between samples quantities are regarded as constant through the action of a sample and hold circuit.

In order to obtained good information content in the response, the input signal should have the following properties:

1) a wide bandwidth which covers the complete frequency range of interest. The signal should have sufficient energy density to ensure an acceptable signal to noise ratio over the relevant frequency range.
2) a low peak to average power ratio. This is to avoid any nonlinearities when operating the system.
3) a zero mean value. This is of particular importance for a system with a free integrator. Here a small bias in the test input could cause gross errors in the sensitivity function estimate. Test inputs with non zero mean values also cause a

change in system operating point which may be undesirable if the system displays nonlinear behaviour.

There exist a number of signals with such properties. For example, a sine wave signal satisfies properties 2) and 3); an impulse signal satisfies property 1) while various forms of random or multi-frequency signals can satisfy properties 1), 2) and 3). For systems with a large signal-to-noise ratio, an step input signal can give valuable information about the dynamics.(Godfrey, K., 1993) Although the step input signal is easy to implement, it does not have a zero mean value and thus introduces a change of operating point over the period of the test.

4.2 Impulse Input Signal

4.2.1 Impulse Input Realised in Practice

For a linear closed loop dynamic system with reference, x(t), the output, y(t) depends upon the weighting function, w(t), and the input x(t). It is defined through the convolution integral as

$$y(t) = \int_{-\infty}^{t} w(t-\tau)x(\tau)d\tau \qquad (4.1)$$

An ideal impulse reference signal has the form

$$x(t) = R\,\delta(t)$$

where δ(t) is the Dirac function. Equation (4.1) can thus be rewritten as

$$y(t) = R\,w(t) \qquad (4.2)$$

It thus appears to be a straight-forward process to get the weighting function of the system if the amplitude R is known. In practice there are significant difficulties. Theoretically, for an impulse response an ideal Dirac function δ(t) is needed as input. Only then will the output be equal to the weighting function w(t) of the system. An impulse function is not really a true test signal in a practical sense since an ideal impulse cannot be realized. The impulse function has been justified mathematically using a theory of "generalized functions" but for practical system testing proposes an approximate impulse must be used. In this approach, we consider an example:

$$x(t) = \begin{cases} 1/\alpha & 0 \le t \langle \alpha \\ 0 & \alpha \le t \end{cases} \tag{4.3}$$

This input satisfies the condition $\int x(t)dt = 1$ as does the idealised impulse and should resemble it for sufficiently small values of the impulse length α. Use of the approximate impulse equ.(4.3) will give a distortion of the output compared with the ideal impulse response. However, if the duration α of the impulse (4.3) is short compared to the time constants of interest, then the distortion introduced may be negligible. This signal is actually a rectangular pulse of width α and height $1/\alpha$. Use of a large impulsive type of input may however be impractical for some applications where the system may be driven into a nonlinear region.

4.2.2 Calculation of the Sensitivity for an Impulse Input Signal

If the input signal is a unit impulse input, i.e. R(s) = 1, the equ.(3.6) may be rewritten in the simple form

$$\frac{\partial Y(s)}{\partial m} = Z(s)Y(s) \tag{4.4}$$

The sensitivity function in the time domain is then given simply by the convolution of the output y(t) and sensitivity signal z(t).

$$\frac{\partial y(t)}{\partial m} = \int_0^T y(\tau)z(t-\tau)d\tau \tag{4.5}$$

Problems can, however, arise in connection with impulse response calculations. The example introduced in Section 3.3.1 involving a first order plant with PID controller illustrates the difficulty. In this example :

G(s) = 1/(s+1)

$$C(s) = K_p + \frac{K_i}{s} + \frac{K_d s}{1 + \tau s}$$

where $K_p = 8.24$, $K_i = 3.57$, $K_d = 7.0$ and $\tau = 0.005$.

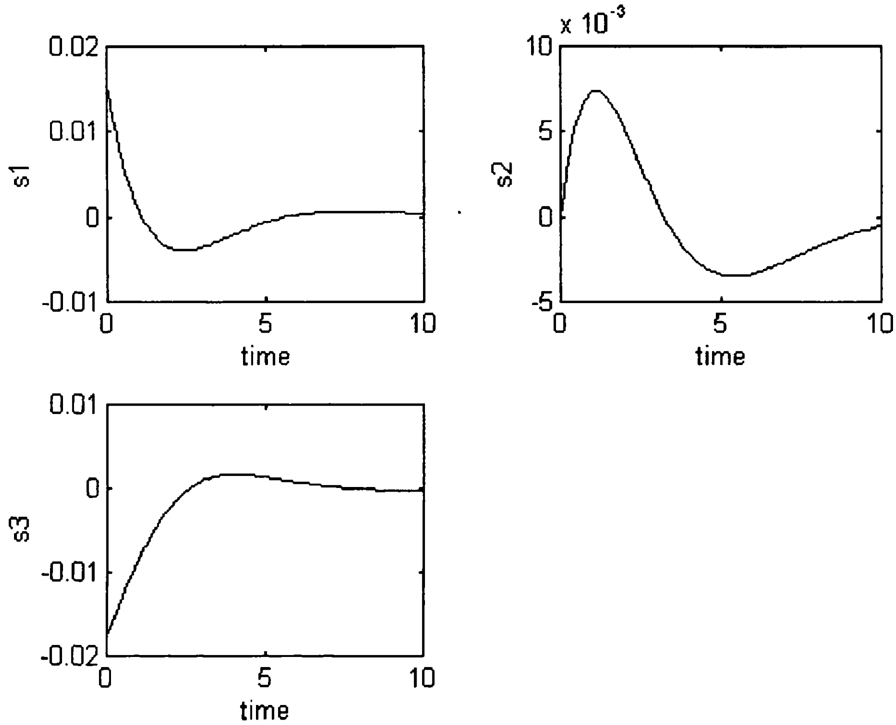


Fig.4.1 Sensitivity function for a three-term controller system using parameter perturbation approach (for unit impulse input)

Fig.4.1 shows a result using the perturbation approach.

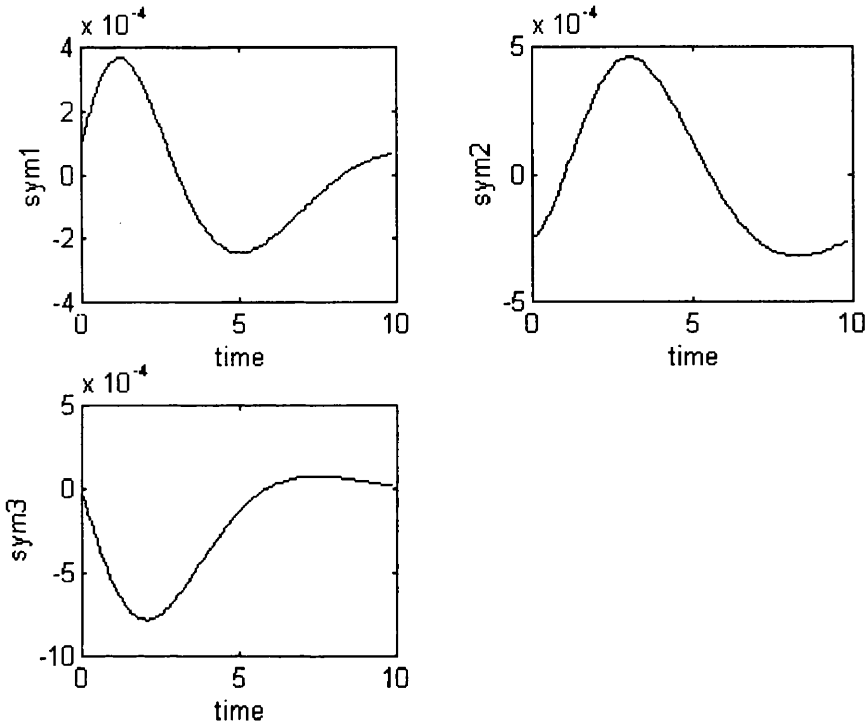


Fig.4.2 Sensitivity function for a three-term controller system using convolution approach (for unit impulse input case)

Fig.4.2 shows the corresponding result using the convolution approach above obtained using MATLAB software. These two results using two different methods should be the same, but Fig.4.3 shows that the convolution calculation is not correct. The inverse Laplace transform of the input R(s) = 1 is not equal to δ(t), so the results of the sensitivity calculation are not correct. In order to eliminate this problem relating to the impulse response calculation modified block diagram, equivalent to that shown in Fig.3.3, has been considered. This modified diagram is shown in Fig.4.4

From equ.(3.7) the sensitivity function of the system may be expressed as

$$S_m^Y(s) = \frac{\partial Y(s)}{\partial m} = \frac{1}{C}\frac{\partial C}{\partial m}E(s)\frac{Y(s)}{R(s)} \qquad (4.6)$$

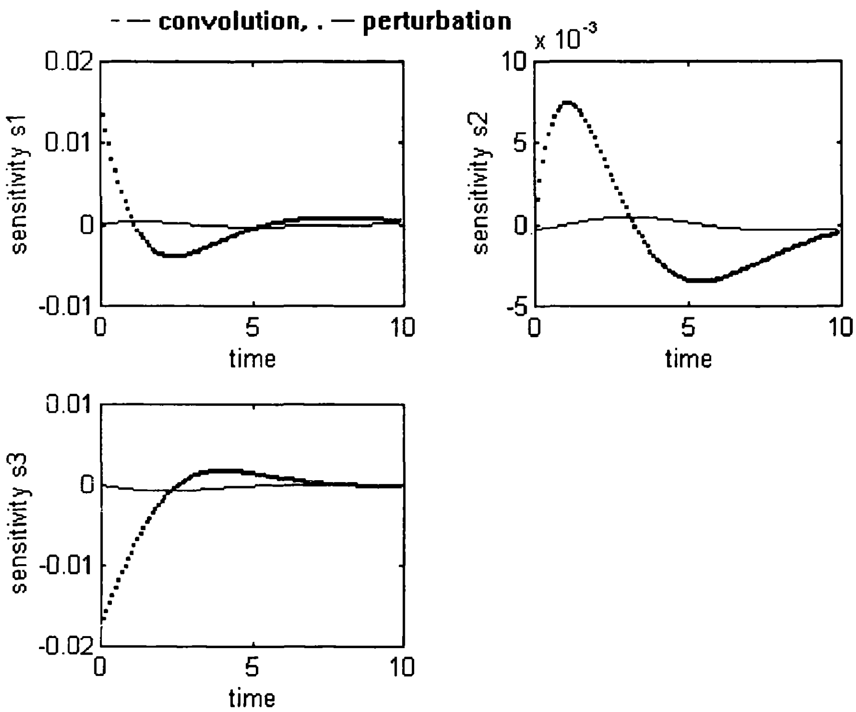


Fig.4.3 Comparison of the convolution method and perturbation method for unit impulse input case

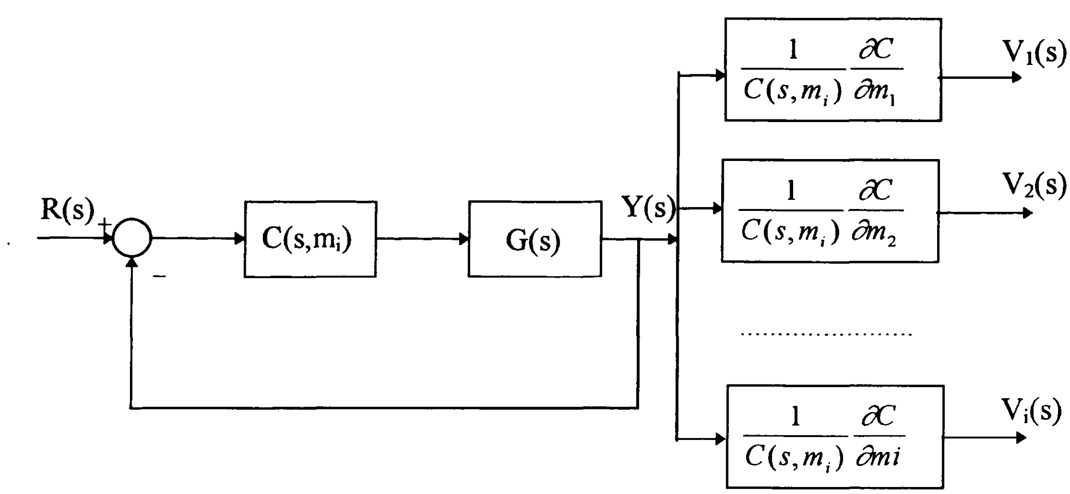


Fig. 4.4 An equivalent diagram to Fig. 3.3

From Fig.4.4 the output of the system V(s) is

$$V(s) = Y(s)\frac{1}{C}\frac{\partial C}{\partial m}$$

Hence in terms of the structure of Fig.4.4 it is possible to write

$$S_m^y(s) = E(s)\frac{V(s)}{R(s)} = E(s)V(s) \quad (\text{if } R(s) = 1)$$

where E(s) = R(s) - Y(s) (Laplace domain)

e(t) = δ(t) - y(t) (time domain)

To taking the inverse Laplace transform of $S_m^y(s)$, we have

$$S_m^y(t) = \int_0^t e(\tau)v(t-\tau)d\tau$$

$$= \int_0^t [\delta(\tau) - y(\tau)]v(t-\tau)d\tau$$

$$= \int_0^t \delta(\tau)v(t-\tau)d\tau - \int_0^t y(\tau)v(t-\tau)d\tau$$

Because $\int_0^t \delta(\tau)v(t-\tau)d\tau$ = v(t) when τ = 0, v(t - τ) = v(t), it follows that

$$S_m^y(t) = v(t) - \int_0^t y(\tau)v(t-\tau)d\tau \qquad (4.7)$$

So if the outputs of the system y(t) and v(t) can be obtained by measurement or simulation, the sensitivity function for the controller parameters of the closed loop system could be calculated by the convolution method.

Fig.4.5 shows some results using four different approaches for a very simple closed-loop system. G(s) = 1/s, proportional controller C(s) = k = 5 for a impulse input case. The analytical solution of the sensitivity function for the controller parameter k is shown in Fig.4.5 c) which Fig.4.5 b) shows the result from the convolution of the system output y(t) and the filter output z(t). There appear to be big differences between the analytical result b) and the convolution result c). The result b) is however similar to the response in Fig.4.5 a) which is a result for the function s1= - kt $e^{-k\,t}$. This just equals the second part of the analytical expression of the sensitivity function

$s3 = e^{-kt}(1-kt)$. As shown in Appendix 3 the MATLAB impulse function neglects the δ function in the convolution of the system output y(t) and the filter output z(t). Neglecting the δ function in the convolution is equivalent to making the error E(s) = -Y(s). Hence passing the error signal through the F(s) filter to give z(t) and then carrying out the convolution with y(t) is equivalent to the calculation of

$$-\int_0^t y(\tau)V(t-\tau)d\tau$$

Hence if we add v(t) to existing convolution results we should get the correct sensitivity function shown in Fig.4.5 d).

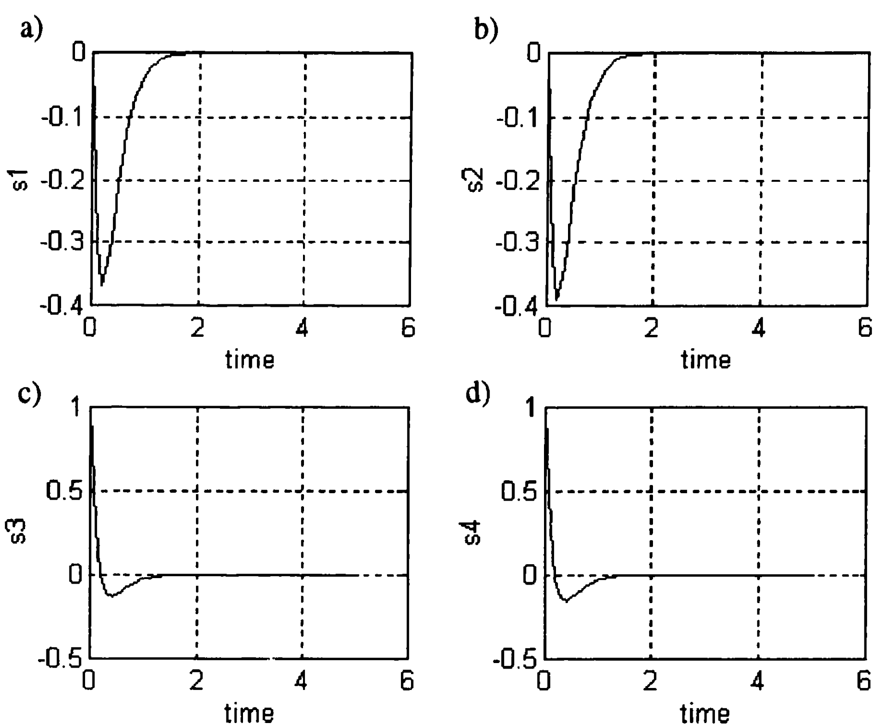


Fig. 4.5 Sensitivity functions for a simple example using different methods

a) s1 for result of $(-k\,t\,e^{-kt})$

b) s2 for convolution of y(t) & z(t)

c) s3 for analytical solution $s3 = e^{-kt}(1-k\,t)$

d) s4 for corrected method

4.3 PRBS Input Signal

It is well known that pseudo-random binary signals (PRBS) provide an alternative to impulsive inputs and have been used successfully in a wide range of practical applications. Godfrey (Godfrey, K., 1993) provides a useful review of such signals and their application in system identification and other forms of system testing.

4.3.1 Generation of PRBS

4.3.1.1 n-Sequences

Among others, Barker (Barker, H.A., and Davy, R.W., 1975), Davies (Davies, W.D.I., 1970), Eykhoff (Eykhoff, P., 1974), Golomb (Golomb, S.W., 1967), Lamb (Lamb, J.D., and Rees, D., 1973), Nichols (Nichols, S.T. and Dennis, L.P., 1971), Poussart (Poussart, D. and Ganguly, U.S., 1977) and Wellstead (Wellstead, P.E., 1975) have investigated the properties of n-sequences and the related Pseudo-Random Binary Sequences. This is a very widely discussed topic in the literature. An n-sequence {u} is defined by the recurrence relationship

$$c_0u_i + c_1u_{i-1} + \dots + c_nu_{i-n} = 0 \quad \text{for } c_0c_n \neq 0 \bmod p$$

where u_i and c_j are members of a Galois field, GF(p). The sequence has a period of

$$N = p^n - 1$$

where the characteristic polynomial

$$f(D) = c_0 + c_1D + \dots + c_nD^n$$

is primitive. Mapping the elements of the GF(p) into real numbers then defines the pseudo random sequence {x}.

4.3.1.2 Pseudo-Random Binary Sequences - (PRBS)

Pseudorandom binary sequences (PRBS) are defined as an n-sequence {x} with p = 2, values ± 1. The sequence is periodic with sequence length of $N = 2^n - 1$. The PRBS are two-state signals which may generated, for example, by using a shift register of order n as depicted in Fig.4.6 (Soderstrom, T. and Stoica, P., 1989).

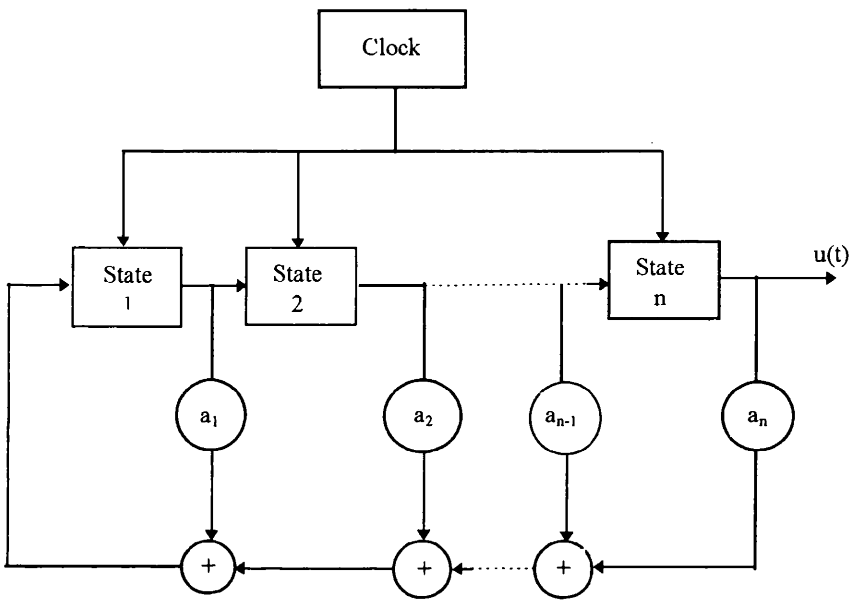


Fig.4.6 Shift register with module-two feedback path

The maximum possible sequence period length of a PRBS is $N = 2^n - 1$. For such a sequence is it shown by Golomb (Golomb, S.W., 1967) that

$$|X(\omega)| = \frac{a}{N}$$

when ω represents the discrete frequencies

$$\omega = \frac{2\pi k}{NT}, \quad k = 0, 1, 2, \ldots, N-1$$

Thus, the spectrum is constant with spectral lines occurring at ω. In order to use the direct spectral method is it necessary to perform a Discrete Fourier Transform (DFT) of N points (The PRBS is of period NT and the spectral lines must coincide with the DFT bins). However the discrete Fourier transform can be computed with much greater efficiency if the number of data points is a power of 2 (using the Fast Fourier Transform). In the next section a very easy method of extending the PRBS by one element is described, thus making the PRBS compatible with the Fast Fourier Transform (FFT).

4.3.1.3 Extended PRBS

In section 4.3.1.2 some properties of a PRBS were briefly described. However the sequence created was of length $2^n - 1$ which is not compatible with the FFT. Lamb (Lamb, J.D. and Rees, D., 1973) tried to make the PRBS compatible with the Fast Fourier Transform by taking an extra sample of the input- and output-signals. The period of the signals was $T(2^n - 1)$ but a 2^n FFT was calculated. Since this period of the test signal is not matched to the DFT data length only poor results were produced. But in many applications (Lamb, J.D. and Rees, D., 1973, Nichols, S.T. and Dennis, L.P., 1971) it would be advantageous to employ sequences of period 2^n instead of period 2^n -1 in order to fit better the system clock requirements. One very simple way to make a sequences of period is to extend the Pseudo-Random Binary Sequences of period 2^n -1 by one element per period, yielding an extended PRBS of period 2^n which is compatible with the FFT. The period of the signal is then going to be 2^nT and the FFT algorithm can be used.

Consider a binary sequence {x} of period N (PRBS). The elements of {x} take on the values -1 and +1 and are denoted by x_i, i being an integer, i.e.

$$\{x\} = \ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots, x_{N-1}, x_N, \ldots$$

An extended PRBS of period $N^{(ex)} = 2^n$ is derived from the PRBS by simply inserting one element per period (Fiebig, U.C.G. and Schnell, M., 1993). This element is a +1 or a -1 in order to obtain a balanced sequence. A sequence is said to be balanced if within a period, the extended sequence has zero mean. It is convenient to inset the additional element +1 such that the longest run of +1s is increased by one, Alternatively it is possible to insert the additional element -1 such that the longest run of -1s is increased by one. The favourable properties of the original PRBS are retained in this process.

4.3.2 Basic Properties of Pseudo-Random Binary Sequences (PRBS)

Pseudo-Random Binary Sequences have properties which approximate those of white noise in terms of the autocorrelation function and spectral properties of the output relative to the input. A PRBS is also easier to apply than white noise as a test signal since it involves a binary signal and there are some other advantages over both step input and impulse input. Firstly, the PRBS has lower amplitude of test input for a given noise level in the results after processing . The PRBS also avoids moving the operation point significantly away from the initial state. Compared with the impulse input, the PRBS has the advantage of being a more practical form of test signal since it is amplitude limited. Also an impulse signal may cause the system to run into non-linearity by its high peak to average power ratio, and its non-zero mean value may cause errors in a system which has one or more free integrators. In contrast the PRBS and extended PRBS avoid possible nonlinear effects and the symmetrical form of the extended signal reduce the error caused by free integrators in a system.

Pseudo-radom binary signals (PRBS) provide an approximation to white noise as shown by the autocorrelation function. A PRBS test signal has the following properties :

a) If a PRBS is generated from an n - stage shift register with feedback, the length of the PRBS N is 2^n- 1 shift clock pulses. The PRBS is periodic with period T = N Δt, where N is an odd integer.
b) The two possible clock pulse levels -c and +c are equally likely and the presence of -c or +c in any one clock pulse interval is statistically independent of that in all other intervals. In any period, there are $\frac{N+1}{2}$ intervals when the signal is at one level and $\frac{N+1}{2}$ intervals when it is at the other.
c) Whether the signal changes level at any particular event point is predetermined, so that the PRBS is deterministic and experiments are repeatable.

d) The autocorrelation function (Unbehauen, H. and Rao, G.p., 1987) approximates an impulse function and makes PRBS potentially attractive as a test signal.

e) The spectrum of the PRBS of period 15 is shown Fig. 4.7.

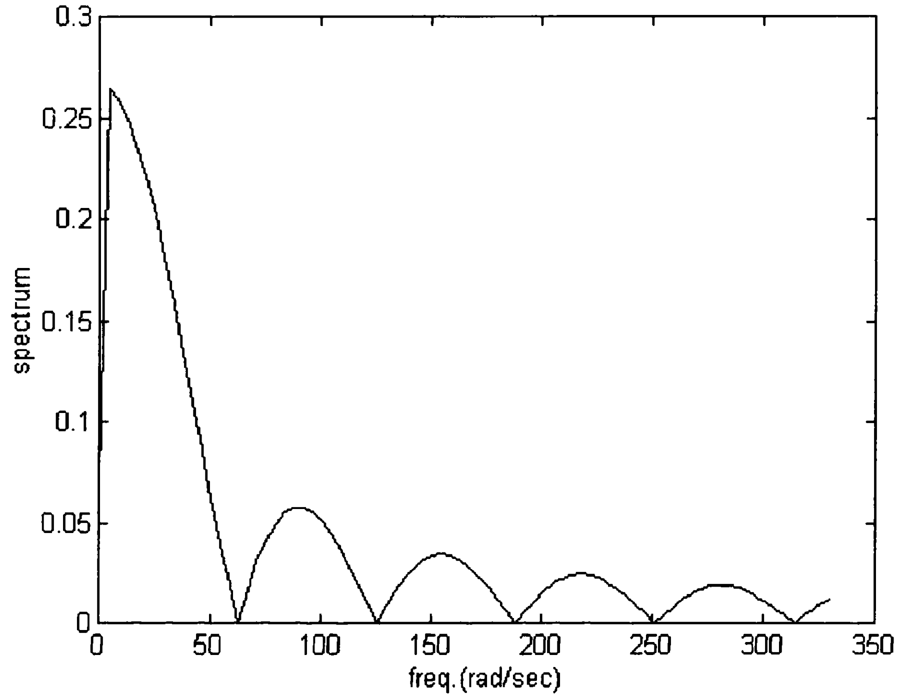


Fig.4.7 Spectrum for a PRBS of period 15

f) The crosscorrelation function directly determines the impulse response of a system. In general the cross-correlation function for a system with impulse response g(t) is given by

$$\Phi_{xy}(t) = \int_0^T g(\tau)\Phi_{xx}(t-\tau)d\tau \qquad (4.8)$$

where $\Phi_{xx}(t)$ is the autocorrelation function of the input signal x(t). When x(t) is white noise $\Phi_{xx}(t) = \delta(t)$ and thus

$$\Phi_{xy}(t) = \int_0^T g(\tau)\delta(t-\tau)d\tau$$

$$= g(t)$$

An extended PRBS has properties similar to those of PRBS. In particular the extended PRBS provides good autocorrelation properties similar to those of the original PRBS.

Fig.4.8 shows the calculation results of the autocorrelation properties for original PRBS and the extended PRBS (n = 4).

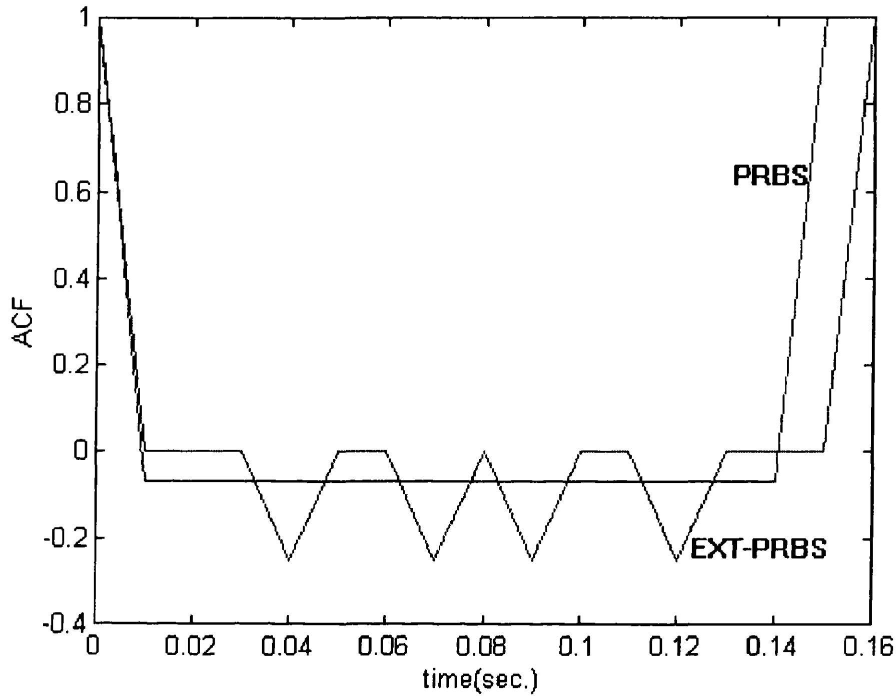


Fig 4.8 Auto-correlation for PRBS & EXT-PRBS

Fig 4.10 shows this property using an extended PRBS of period 128 and it is similar with the PRBS of period of 127 shown Fig.4.9. The autocorrelation is periodic with period $n = 2^{N-1}$ or $n = 2^N$ and symmetric such that $\Phi_{xx}(\tau) = \Phi_{xx}(n-\tau)$.

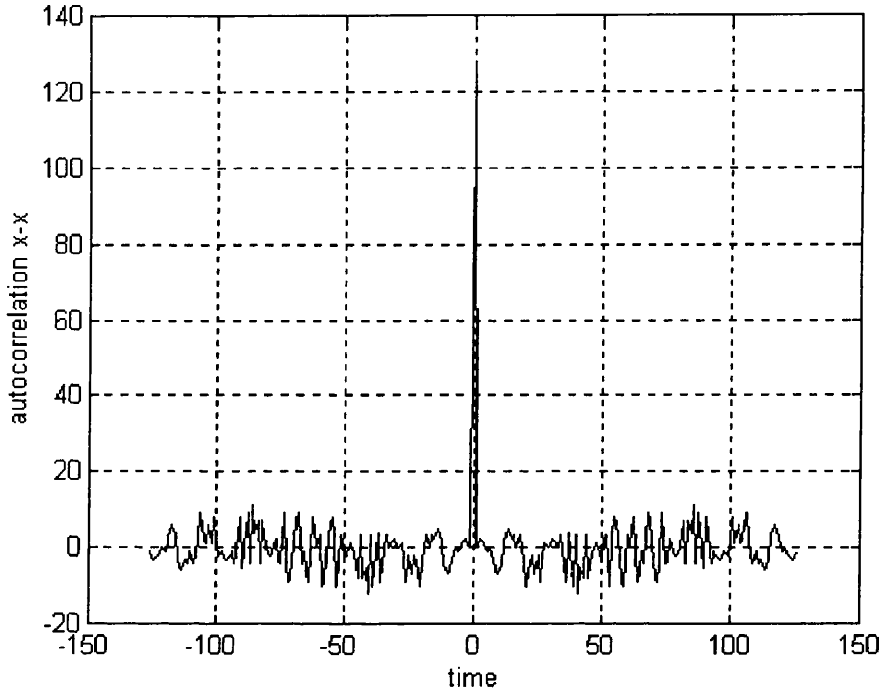


Fig.4.9 Autocorrelation of PRBS of period 127

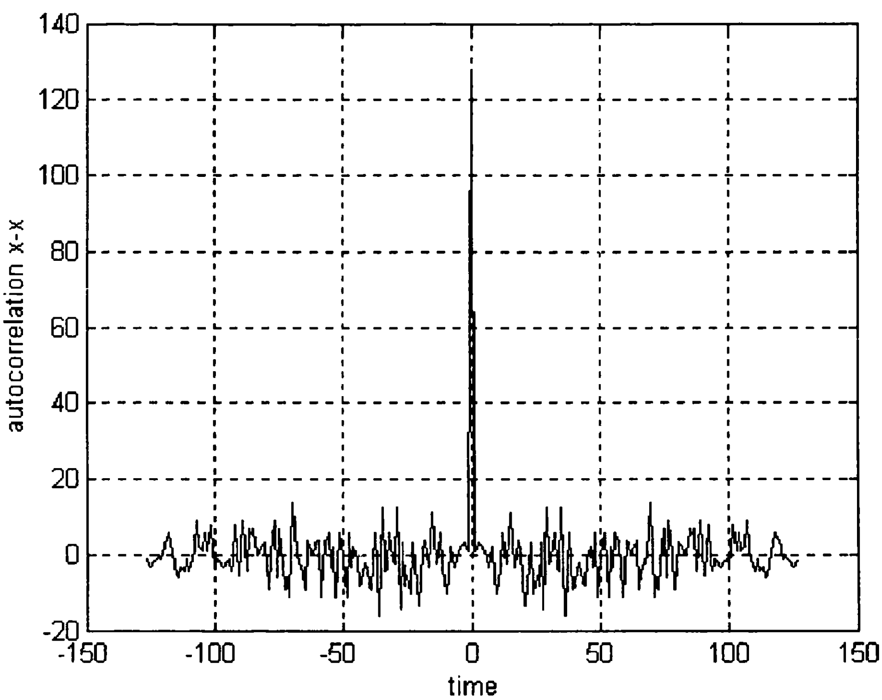


Fig.4.10 Autocorrelation of an extended PRBS of period 128

4.3.3 Calculation of the Sensitivity Function for the PRBS Input Signal

In general, from theory, it is not too difficult to get the sensitivity functions for an impulse input. We only need to form the product Z(s) and Y(s) for an impulse input and inverting the product, the sensitivity functions in time-domain are obtained directly. For the Pseudorandom Binary Sequence (PRBS) input case the sensitivity functions will be calculated by following steps :

First, the impulse responses y_{imp} and z_{imp} may be calculated if the time response of the closed loop system, y(t) and the filter signal z(t) have been obtained. Assuming the PRBS input is represented by a sequence x(t), the impulse responses y_{imp} and z_{imp} can be generated by the following formula (Unbehauen, H. and Rao, G.P., 1987).

$$y_{imp} = \frac{\Phi_{xy}}{c^2 \frac{n+1}{n} \Delta t} \tag{4.8}$$

$$z_{imp} = \frac{\Phi_{xz}}{c^2 \frac{n+1}{n} \Delta t} \tag{4.9}$$

where Φ_{xy} is the crosscorrelation function between the output and the input.

i.e. $$\Phi_{xy} = \frac{1}{T}\int_0^T x(\tau)y(t+\tau)d\tau$$

and Φ_{xz} is the crosscorrelation function between the input and the filter signal.

i.e. $$\Phi_{xz} = \frac{1}{T}\int_0^T x(\tau)z(t+\tau)d\tau$$

where c is the signal magnitude

n is the number of bits of the signal over complete period

and Δt is the time interval

Then, the sensitivity function is calculated by convolution of the impulse responses y_{imp} and z_{imp}. It has to be noted that different controller parameters give correspondingly different z_{imp}. variables and thus different sensitivity functions.

4.3.4 Spectral Analysis Method for Frequency Domain Identification.

Spectral analysis can in some cases give more physical insight than time - domain approaches. In the current work the Fast Fourier Transform (FFT) in MATLAB is used for spectral analysis. The complex coefficients of the Fourier series of the system X_k and Y_k can be obtained by spectral analysis methods from measurements, assuming the input signal x is an extended m-sequence (PRBS) with broad band properties and the output signal y is directly obtained from measurements. It may be shown that the following relation exists between Y_k and $S_{xx}(\omega)$ and $S_{xy}(\omega)$ for a random input of this kind, namely

$$S_{xy}(\omega) = X_k^* \, Y_k$$

$$S_{xx}(\omega) = X_k^* \, X_k$$

Here $S_{xx}(\omega)$ is the spectral density of the autocorrelation of the input signal and $S_{xy}(\omega)$ is the spectral density of the crosscorrelation between x and y. X_k and Y_k can be easily found from the FFT of x and y separately. The quantity X_k^* is the conjugate of X_k.

4.3.5 The Spectral Analysis Method for Controller Sensitivity Function Evaluation

Section 3.2.3.2 has provided an indication of the signal measurements needed in order to estimate the sensitivity of the frequency response of a closed loop system. In this section the direct spectral method will be developed. This method has an advantage over other methods like the perturbation method or single sine wave method (Newland, D.E., 1984) since the frequency response estimates at a number of frequencies are obtained simultaneously.

Assuming the output of the system y(t) and filter signal z(t) with PRBS as the reference x(t) have been measured, the FFT, using samples obtained over exactly one period of the test signal, of y(t), $z_i(t)$ and the reference x(t), gives

$$Y(\omega) = \mathrm{fft}(y(t)) = |Y(\omega)|\, e^{j\omega\alpha}$$

$$Z_i(\omega) = \mathrm{fft}(z_i(t)) = |Z_i(\omega)|\, e^{j\omega\beta}$$

$$X(\omega) = \mathrm{fft}(x(t)) = |X(\omega)|\, e^{j\omega\gamma}$$

The frequency response Mfω and the sensitivity function of the system for the controller parameters m_i, Mfs in the frequency domain, could be calculated by

$$|Mf\omega| = \frac{|Y(\omega)|}{|X(\omega)|} \tag{4.10}$$

$$|Mfs| = \frac{|Z_i(\omega)|}{|X(\omega)|}|Mf\omega|\cos(\beta - \gamma) \tag{4.11}$$

where β and γ are defined as above.

4.4 Discussion

For the identification of the system by crosscorrelation techniques such a signal provides a convenient white noise approximant. When such a signal is the input to a system, its crosscorrelation with the system output provides an estimate of the system weighting function. The estimation error due to linear drift signals could be made small. In earlier work (Winning, J.D., *et al*, 1977, Manness, M.A., 1988) step input test signals were used in the sensitivity tuning technique. In the current work, pseudorandom binary signals (PRBS) are also considered. The main theoretical advantage of this test signal is that the PRBS may be applied as a small amplitude test signal which has impulse-like properties in terms of its autocorrelation function. It is also a broad-hand test signal which can provide a useful basis for spectral analysis methods. Some disadvantages of the PRBS test signal will be discussed in Chapter 5 when such signals are used in a practical application.

CHAPTER 5

Application of the Techniques to Multivariable Control of Two - Tank System

5.1 Description & Theoretical Analysis of 2 - Tank System

As mentioned in Chapter 1, many engineering control system design problems are inherently multivariable in nature. The methods of controller tuning and sensitivity analysis outlined in Chapters 2 and 3 apply equally in the multivariable case (Murray-Smith, D. J., 1985) although they have been presented here in the context of single-input single-output systems. In order to investigate the applicability of the approach to real systems in which measurement noise and non-linearities are present a real 2 - input 2 - output system has been used as a test case. The chosen experimental system involved two coupled water tanks, a pump and liquid level sensors. The original system design was developed at the Control System Centre of the University of Manchester Institute of Science and Technology (UMIST) (Wellstead, P.E., 1981).

The biggest source of error in characterising the coupled-tank system lay in the original level measurement system. The problem was in two parts: the nonlinearity of the output signal and more importantly, the poor repeatability of this signal from the variable resistance depth sensors. The sensors worked on the principle that a varying depth of electrolyte caused a resistance change across the two track sensor. However, control of electrolyte conductivity was crude and the characteristics of the sensor changed with time. Repeated use would result in a build up of deposits on the sensor tracks, further degrading reliability. It was decided therefore that an alternative method of depth sensing be found, with the threefold aim of accuracy, linearity and repeatability (Details of the modified sensor system may be found in Appendix 5). Although the coupled tank system was designed as a single input system it is readily modified (Kane, P.A., 1992) by the addition of a second pump to provide two inputs.

This allows control of the liquid level in both tanks and that configuration has been used in this work. The two input two output system is shown in Fig. 5.1.

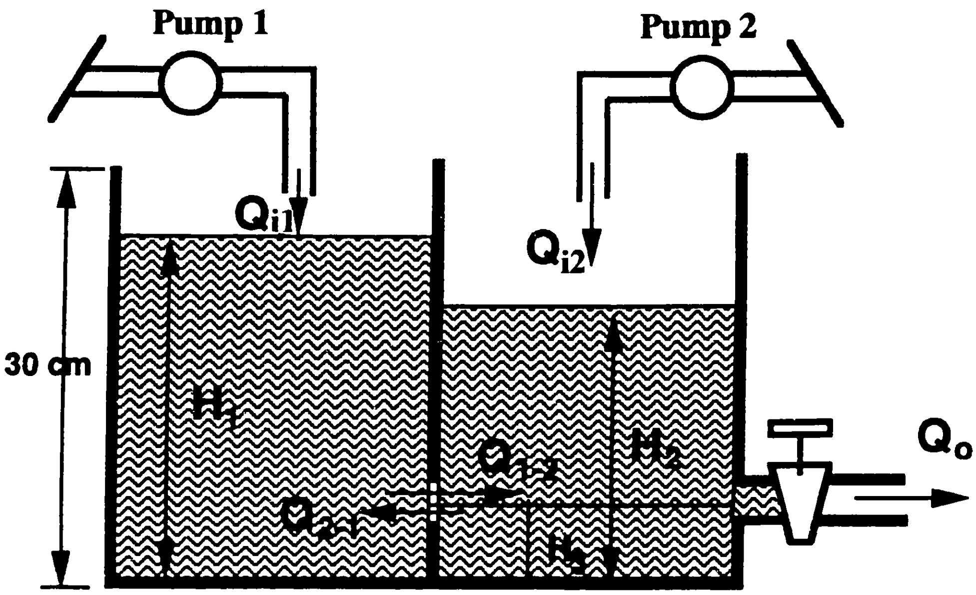


Fig 5.1 Two input & two output tank system

5.1.1 Theoretical Model

5.1.1.1 Nonlinear Model

Based on the Bernoulli Equation the water flow rate Q_1 from the tank 1 and the water flow rate Q_0 from the tank 2 could be obtained without difficulty. (See Appendix 4) :

$$Q_1 = C_{d1} a_1 \sqrt{2g(|H_1 - H_2|)} \quad (5.1)$$

$$Q_0 = C_{d2} a_2 \sqrt{2g(H_2 - H_3)} \quad (5.2)$$

The dynamic equations of the system are derived by taking flow balances for each tank. For tank 1, the rate of change of fluid volume can be determined from the difference between input and output flows, i.e.:

$$\frac{dV_1}{dt} = a\frac{dH_1}{dt} = Q_{i1} - Q_1 \tag{5.3}$$

where V_1 is the volume of water in tank 1

H_1 is height of water in tank 1

a is cross-sectional area of tank 1 and 2

Q_1 is flow rate of water from tank 1

Q_{i1} is pump flow rate to tank 1

Similarly for tank 2

$$\frac{dV_2}{dt} = a\frac{dH_2}{dt} = Q_{i2} + Q_1 - Q_0 \tag{5.4}$$

where V_2 is the volume of water in tank 2

H_2 is height of water in tank 2

Q_0 is flow rate of water out of tank 2

Q_{i2} is pump flow rate to tank 2

From equ(5.1), (5.2), (5.3) and (5.4) the nonlinear model can be obtained as follows:

$$\left.\begin{aligned} a\frac{dH_1}{dt} &= Q_{i1} - C_{d1}a_1\sqrt{2g(H_1 - H_2)} \\ a\frac{dH_2}{dt} &= Q_{i2} + C_{d1}a_1\sqrt{2g(H_1 - H_2)} - C_{d2}a_2\sqrt{2g(H_2 - H_3)} \end{aligned}\right\} \tag{5.5}$$

5.1.1.2 Linearlised Model

For control system studies, the system equations are linearised by considering only small variations q_{i1} in Q_{i1}, q_{i2} in Q_{i2}, q_0 in Q_0, q_1 in Q_1, h_1 in H_1 and h_2 in H_2.

In the steady state

$Q_{i1} = Q_1$, $Q_{i2} + Q_1 = Q_0$, and

$$a\frac{dh_1}{dt} = q_{i1} - q_1 \tag{5.6}$$

$$a\frac{dh_2}{dt} = q_{i2} + q_1 - q_0 \tag{5.7}$$

giving

$$q_1 = \frac{1}{2}\sqrt{2g}C_{d1}a_1(\frac{h_1 - h_2}{\sqrt{H_1 - H_2}}) \tag{5.8}$$

$$q_0 = \frac{1}{2}\sqrt{2g}C_{d2}a_2(\frac{h_2}{\sqrt{H_2 - H_3}}) \tag{5.9}$$

where a_1 & a_2 are cross sectional areas of orifices 1 & 2 respectively

C_{d1} & C_{d2} are discharge coefficients of orifices 1 & 2

H_3 is height of the drain tap (fixed)

g is the gravitational constant 9.81 m/s^2

substituting equ.(5.6), (5.7), (5.8) and (5.9) gives

$$a\frac{dh_1}{dt} = q_{i1} - \frac{1}{2}\sqrt{2g}C_{d1}a_1(\frac{h_1 - h_2}{\sqrt{H_1 - H_2}}) \tag{5.10}$$

$$a\frac{dh_2}{dt} = q_{i2} + \frac{1}{2}\sqrt{2g}C_{d1}a_1(\frac{h_1 - h_2}{\sqrt{H_1 - H_2}}) - \frac{1}{2}\sqrt{2g}C_{d2}a_2(\frac{h_2}{\sqrt{H_2 - H_3}}) \tag{5.11}$$

By rearranging equations (5.10) and (5.11), the following state-space model of the coupled tanks system is produced

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} -\frac{k_1}{a} & \frac{k_1}{a} \\ \frac{k_1}{a} & -\frac{k_1 + k_2}{a} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \end{bmatrix} \tag{5.12}$$

where $k_1 = \dfrac{\sqrt{2g}C_{d1}a_1}{2\sqrt{H_1 - H_2}}$

$$k_2 = \frac{\sqrt{2g}C_{d2}a_2}{2\sqrt{H_2 - H_3}}$$

and by taking Laplace transforms of this model, regarding Q_{i1} as input 1, H_1 as output 1, Q_{i2} as input 2 and H_2 as output 2, the transfer functions of the model are obtained as follows :

$$g_{11} = \frac{H_1(s)}{Q_{i1}(s)} = \frac{\frac{1}{a}(s + \frac{k_1 + k_2}{a})}{s^2 + \frac{2k_1 + k_2}{a}s + \frac{k_1 + k_2}{a^2}} \quad (5.13)$$

$$g_{12} = \frac{H_1(s)}{Q_{i2}(s)} = \frac{\frac{k_1}{a^2}}{s^2 + \frac{2k_1 + k_2}{a}s + \frac{k_1 + k_2}{a^2}} \quad (5.14)$$

$$g_{21} = \frac{H_2(s)}{Q_{i2}(s)} = \frac{\frac{k_1}{a^2}}{s^2 + \frac{2k_1 + k_2}{a}s + \frac{k_1 + k_2}{a^2}} \quad (5.15)$$

$$g_{22} = \frac{H_2(s)}{Q_{i2}(s)} = \frac{\frac{1}{a}(s + \frac{k_1}{a})}{s^2 + \frac{2k_1 + k_2}{a}s + \frac{k_1 + k_2}{a^2}} \quad (5.16)$$

where all parameters values are listed in Table 5.1

Table 5.1 Coefficients for 2 tank system

Coefficients	Value	Definition
a (m^2)	$9.7e^{-3}$	cross-sectional area of tank 1 and 2
g (m/sec^2)	9.81	gravitational constant
a_1 (m^2)	$3.956e^{-5}$	cross sectional area of orifices 1
a_2 (m^2)	$3.850e^{-5}$	cross sectional area of orifices 2
C_{d1}	0.63	discharge coefficient of orifices 1
C_{d2}	0.58	discharge coefficient of orifices 2
H_3 (m)	0.03	height of drain tap (fixed)

5.2 Validation of the Mathematical Model of the 2 - Tank System

5.2.1 Introduction to Model Validation Methods

In order to apply the individual channel analysis and design methods, it is necessary to have an externally validated mathematical model from which we can obtain data as a basis for comparison. There are several approaches to model validation which are relevant. Three of these are:

a) Checking the steady state behaviour of the system. This can be done by comparing theoretical results obtained by solving the simultaneous algebraic equations for $dH_1/dt = 0$ and $dH_2/dt = 0$ with the measured results on the real system under steady state conditions. If there exist large differences between the two results, it suggests that something is wrong with the model.

b) Checking dynamic response for large perturbations. This is a process for model validation which involves direct comparison of (open loop) responses for transients following a step change of flow in or flow out. Again large differences between theoretical and measured values suggest problems with the model but we must consider this in relation to the accuracy with which we can measure and set the input flow rates. Difficulties associated with the practical application of this approach are concerned with the fact that the model is inherently non-linear and it is not always clear what changes are needed to obtain an improved match.

c) Checking dynamic response for small perturbations. This involves measurement of time constants for small perturbations about a steady state condition. Tests are made on the real system and estimated time constants are compared with values from linear theory or from computer simulations for the same operating condition. Because one is dealing with a linearised description analytical insight can be helpful in establishing the source of any errors in the underlying mathematical description.

5.2.2 Model Validation Results

5.2.2.1 Steady State Points

Some steady state points have been obtained for the 2 tank system. Table 5.2 shows the steady state points from the experiments on the 2 - input 2 - output real water tank system. Table 5.2 also shows the calculated results from the non-linear model of the two- tank system.

Table 5.2 Steady state points of the 2 tank system

Case	Experimental Results				Calculated Results	
	Q_{i1} cm^3/sec	Q_{i2} cm^3/sec	H_1 mm	H_2 mm	H_1 mm	H_2 mm
1	15.8	27.56	220	206	224	209
2	15.0	20.33	163	150	162	149
3	19.0	19.00	188	165	189	167
4	19.0	16.00	163	142	167	147
5	12.0	28.30	185	177	193	185

From Table 5.2 it is can be seen that the results obtained from the real system are very close to the results from the calculation from non-linear model when values of H_1 and H_2 are relatively large (over 150 mm for H_1 and H_2). Differences have been found to be greater for cases involving smaller depth values. It is possible that the differences come from discharge coefficients C_{d1} and C_{d2} in the model (Murray-Smith, D.J. and Gong, M., 1994).

5.2.2.2 Dynamic Response for Small Perturbations

The time constant T_1 and T_2 obtained from the linearised model for an operating point involving H_1 = 70 mm and H_2 = 62 mm are:

T_1 = 75.75 sec. T_2 = 5.889 sec.

Frequency response analysis using sinusoidal testing of an analogue computer simulation based on the nonlinear two-tank model has provided estimates which are very close to these.

Corresponding time constant estimates from the step response of the real system are:

T_1 = 76sec. T_2 = negligible

The time constant T_1 estimated from a step applied to the real system is therefore close to the results from the analogue computer simulation and the theoretical analysis. From the steady state tests and the small perturbation tests it is clear that the model is of acceptable accuracy for operating conditions involving values of H_1 and H_2 greater than 150 mm. Agreement between steady-state measurements and steady-state model predictions is generally quite good for most parts of the operating range, but typical results with the nominal parameter set suggest immediately that the model is not perfect. Dynamic tests can also show significant differences between the simulation model predictions and the behaviour of the real system. Differences between the steady-state liquid levels in the simulation model and in the real system, for a given value of input flow rate, are found to vary slightly with H_1 and H_2 due to the limitations of equ.(5.1) and equ.(5.2) in describing the relationships between output flow and the liquid level in each tank.

5.3 Application of Individual Channel Analysis and Design

A new approach known as Individual Channel Analysis and Design (ICAD) has been developed for multivariable feedback control by O'Reilly and Leithead (O'Reilly, J. and Leithead, W.E., 1991, Leithead, W.E. and O'Reilly, J., 1991). Each individual channel is enclosed within a feedback loop with a compensator which must be designed to meet the channel specification. Let us, in particular, focus our attention on the 2 - input 2 - output multivariable problem with diagonal controller gain matrix C(s) as shown in Fig.5.2. This 2 - input 2 - output multivariable design problem can be decomposed into the two equivalent SISO individual channel design problems depicted in Fig.5.3(a) and Fig.5.3(b).

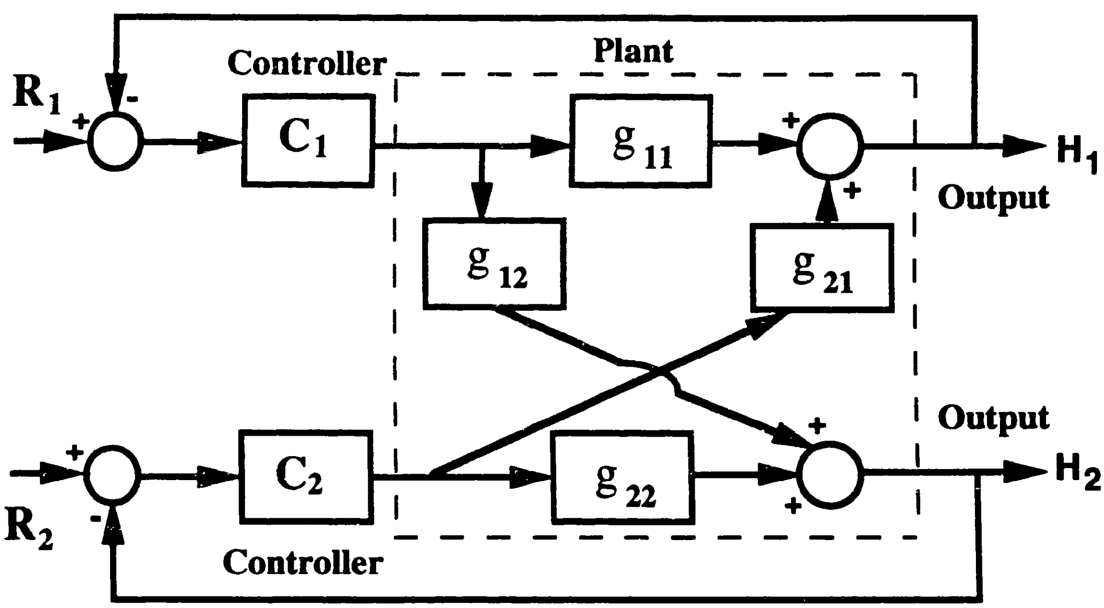


Fig.5.2 The 2-input 2-output multivariable control problem with diagonal feedback

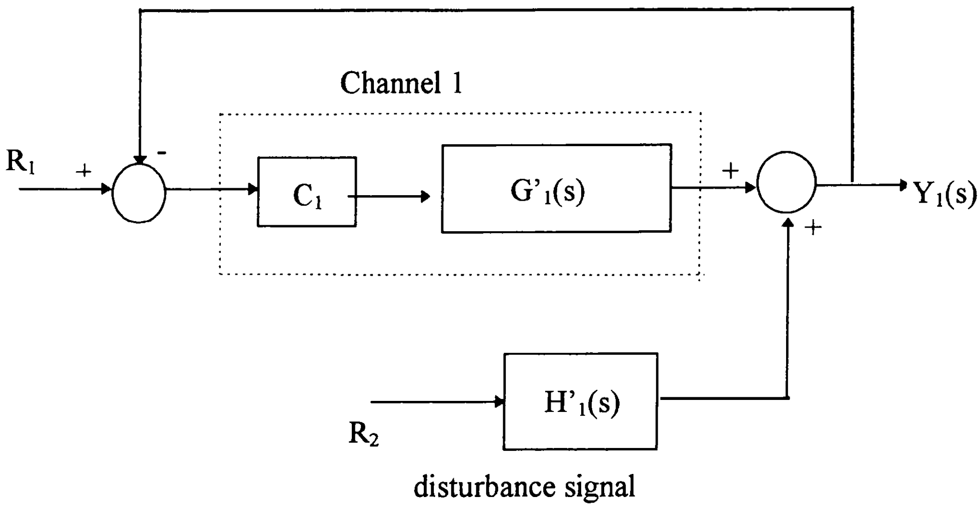


Fig.5.3(a) Channel 1 with cross-reference disturbance signal and unity negative feedback

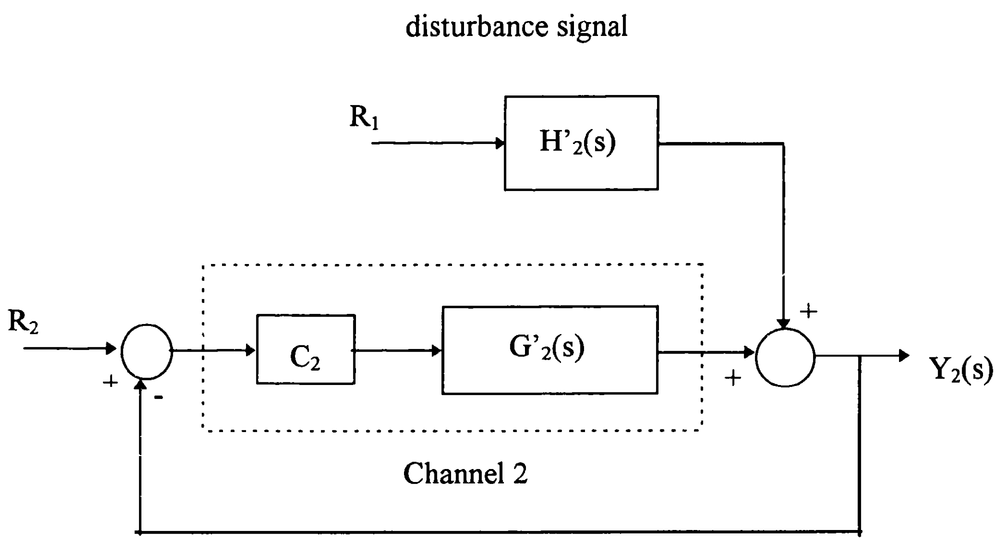


Fig.5.3(b) Channel 2 with cross-reference disturbance signal and unity negative feedback

where $G_1'(s) = g_{11} - g_{12}\, g_{22}^{-1}\, g_{21}\, h_2$

$H_1'(s) = g_{12}\, g_{22}^{-1}\, h_2$

$$h_1 = \frac{C_1 g_{11}}{1 + C_1 g_{11}}$$

$G_2'(s) = g_{22} - g_{21}\, g_{11}^{-1}\, g_{12}\, h_1$

$H_2'(s) = g_{21}\, g_{11}^{-1}\, h_1$

$$h_2 = \frac{C_2 g_{22}}{1 + C_2 g_{22}}$$

Assuming that the transfer functions g_{11}, g_{12}, g_{21} and g_{22} of the SISO systems have been given, channel 1 has the open - loop SISO transmittance

$$Ch_1(s) = C_1 g_{11}(1 - \gamma h_2) \qquad (5.17)$$

where $\gamma(s) = \dfrac{g_{12} g_{21}}{g_{11} g_{22}}$

and channel 2 has the open - loop SISO transmittance

$$Ch_2(s) = C_2\, g_{22}(1 - \gamma h_1)$$

Further, the multivariable nature of the channels is characterised by the complex-frequency multivariable structure function $\gamma(s)$ (O'Reilly, J. and Leithead, W.E., 1991): When the magnitude of γh_i ($i = 1, 2$) is much less than one the loop signal interaction is low; otherwise, the loop signal interaction is high.

Assuming the steady state operating point has been determined, for example,

$H_1 = 220$ mm, $\quad H_2 = 206$ mm,

from the equ.(5.12) the coefficients k_1, k_2, and eigenvalue of the matrix A in the state-space form and the transfer functions can be obtained:

$k_1 = 5.5536\text{e-}04\ m^2/\text{sec.}$ $\qquad k_2 = 1.2195\text{e-}04\ m^2/\text{sec.}$

eig(A) = - 0.0059

- 0.1211

$$g_{11} = \frac{103.0928s + 7.1985}{s^2 + 0.1271s + 0.0007}$$

$$g_{12} = g_{21} = \frac{5.9024}{s^2 + 0.1271s + 0.0007}$$

$$g_{22} = \frac{103.092s + 5.9024}{s^2 + 0.1271s + 0.0007}$$

In this case above the structure function $\gamma(s)$ in the frequency range $0.0001 < \omega < 10$ rad/sec. is shown by Fig.5.4.

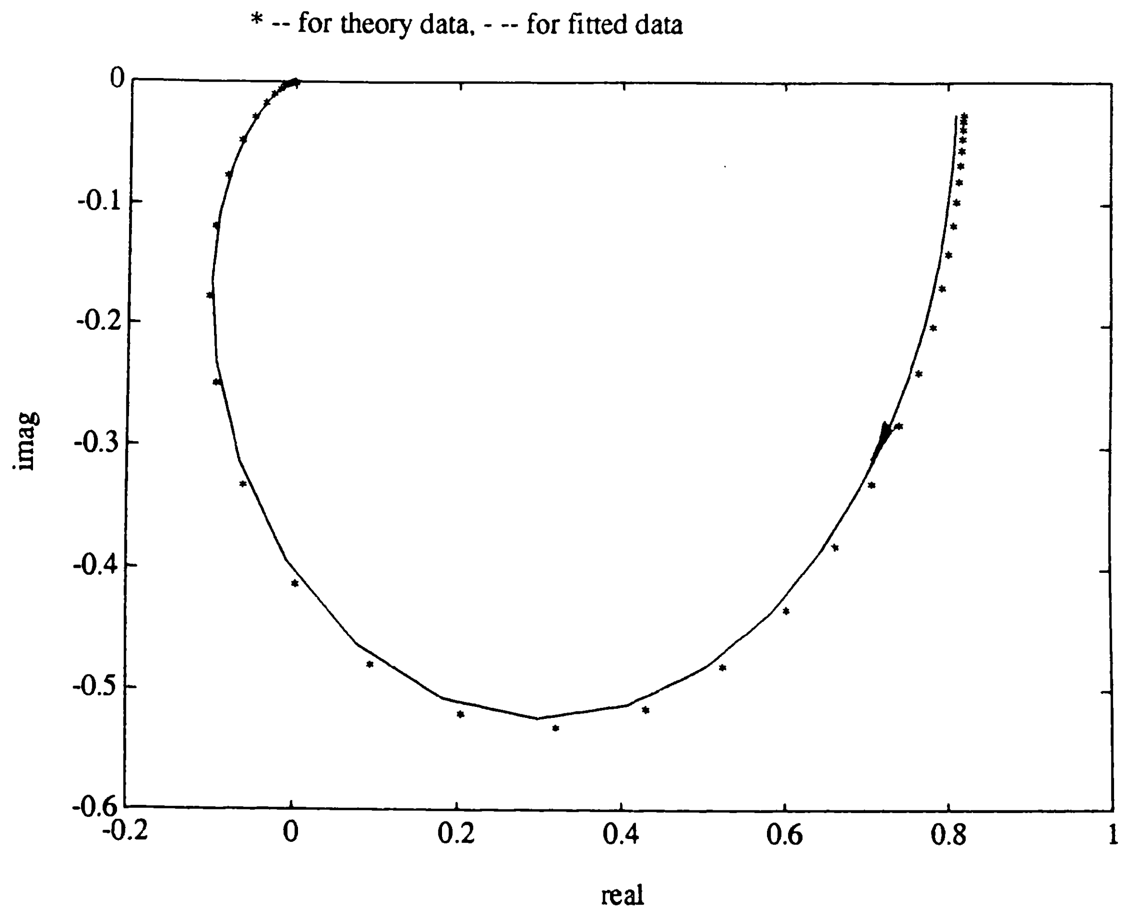


Fig 5.4 Structure function γ(s) from mathematical model

(0.0001<ω<10 rad/sec.)

Fig. 5.4 shows that the complex frequency multivariable structure function γ(s) describes the multivariable nature of the SISO Channel 1 and Channel 2. It can be seen that at low frequency the γ(s) function is close to point (1,0) both for the theoretical data for the linear model and the experimental data from analogue simulation with the non-linear mathematical model. The dynamic performance of the 2-tank control system shows γ(s) function values which are small in the upper part of the frequency range suggesting satisfactory properties in terms of robustness in the closed loop system. In fact we are only interested in γh_2. Fig. 5.5 shows the results of γh_2 for the simple case with crossed feedback which is only for a controller with gain constants C_1 and C_2. From this figure we can see the γh_2 are close to a value of one for the real part. This typical set of plots shows low values of magnitude in the frequency range of importance for this application, Proportional control is not, however, adequate as it does not eliminate steady state errors. The final selected controller structures were of the P+I type.

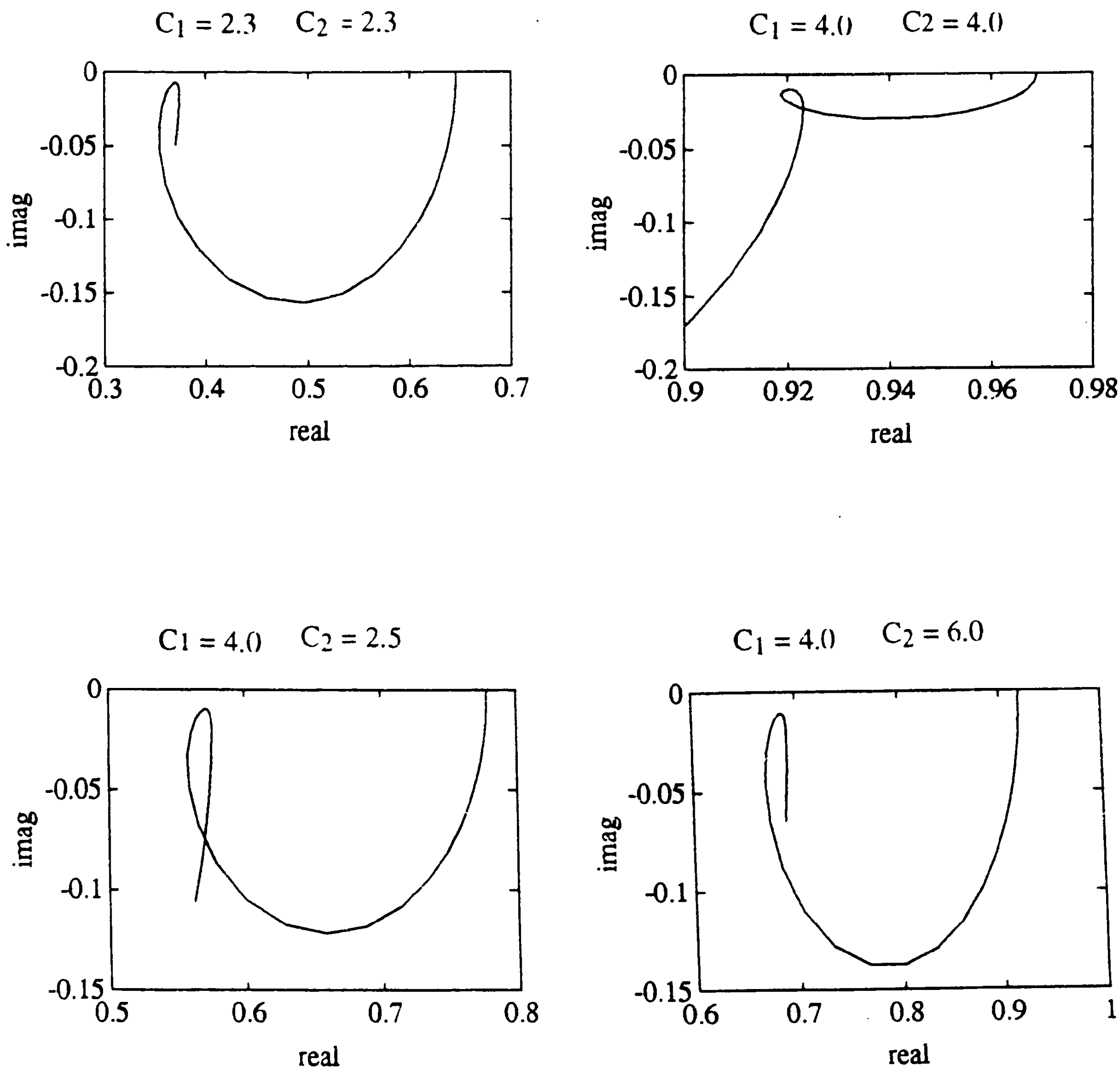


Fig. 5.5 Product γh2 for controller C_1 & C_2 = constant only with crossed feedback (ω = 10^{-3} - 10^{0} rad/sec.) for 4 sets of gain values

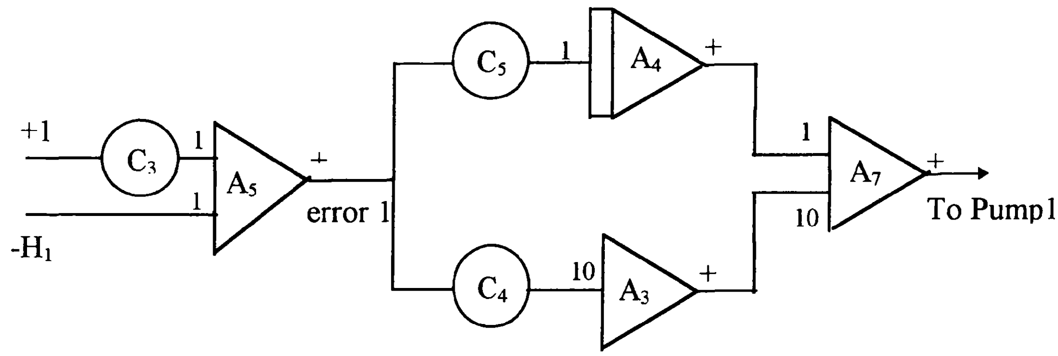
5.4 Analogue Controller Implementation of the ICAD for 2 - Tank System

To verify that the ICAD approach works well for 2 - tank system, the 2 - input 2 - output tank system with a PI controller has been implemented on an analogue tutor as a continuous controller, shown in Fig.5.6 The analogue tutor is a form of analogue computer which employs a number of basic units namely; summing amplifiers, integrators and coefficient units that have an adjustable range from 0 - 1.0. These units can be connected up to form a continuous controller.

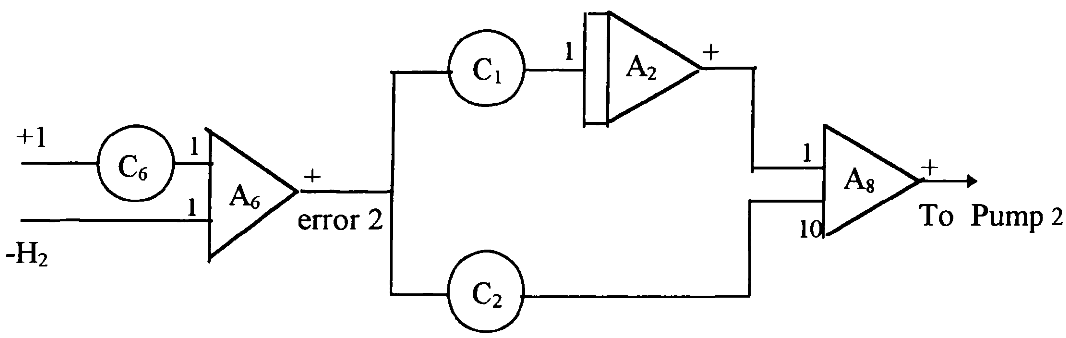
The coefficients of the analogue tutor taken in Fig 5.6 are listed in Table 5.3.

Table 5.3 Coefficients in the analogue tutor

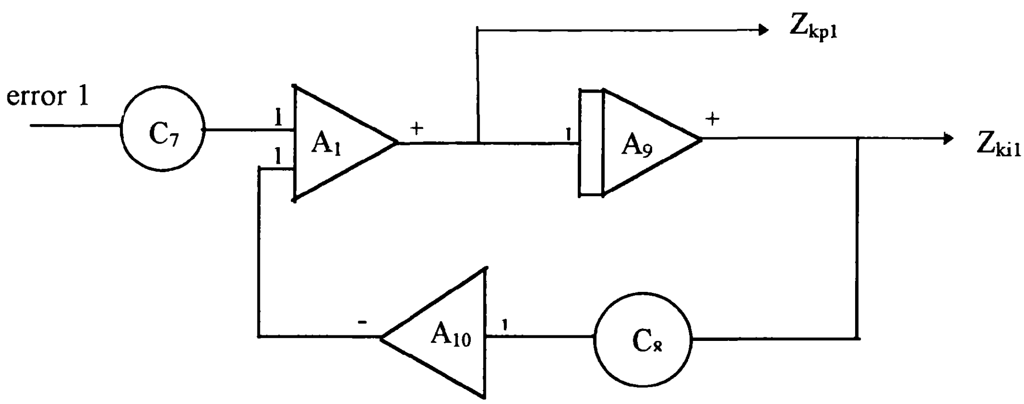
Coefficient		value
C_1	K_{i2}	0.4
C_2	$K_{p2}/10$	0.7
C_3	size of the input 1/100	
C_4	$K_{p1}/10$	0.824
C_5	K_{p2}	0.5
C_6	size of the input 2/100	
C_7	$1/K_{p1}$	0.12
C_8	K_{i1}/K_{p1}	0.061



a) Channel 1



b) Channel 2



c) Filter 1

Fig.5.6 Analogue PI control for 2-input 2-output case

5.5 Theory of Compensator Sensitivity Analysis for the Multi - Input Multi - Output Case

From the analysis of the SISO case, we have obtained the frequency responses and sensitivity functions in the frequency domain. Following the ICAD approach by O'Reilly and Leithead, the complete analysis of the Multi-input Multi-output system will be developed. Here we only use the two - input two - output case as an example for illustration.

From Fig.5.3(a) the output $Y_1(s)$ of the block diagram can be obtained easily:

The output can be expressed by

$$Y_1(s) = R_2(s)\,H_1'(s) + G_1'(s)\,C_1\,(R_1(s) - Y_1(s))$$

$$= \frac{G_1'(s)C_1}{1+C_1G_1'(s)}R_1(s) + \frac{H_1'(s)}{1+C_1G_1'(s)}R_2(s) \qquad (5.18)$$

The corresponding sensitivity function is

$$\frac{\partial y_1(s)}{\partial m} = \frac{G_1'(s)\frac{\partial C_1}{\partial m}}{(1+C_1G_1'(s))^2}[R_1(s) - H_1'(s)R_2(s)] \qquad (5.19)$$

However

$$E_1(s) = R_1(s) - Y_1(s)$$

$$= \frac{1}{1+C_1G_1'(s)}\;[R_1(s) - H_1'(s)R_2(s)] \qquad (5.20)$$

Consider the case where $R_2(s) = 0$, equ.(5.20) and equ.(5.19) become:

$$E_1(s) = \frac{R_1(s)}{1+C_1G_1'(s)}$$

$$\frac{\partial y_1(s)}{\partial m} = \frac{G_1'(s)\frac{\partial C_1}{\partial m}}{1+C_1G_1'(s)}E_1(s)$$

$$= \frac{1}{C_1}\frac{\partial C_1}{\partial m}\frac{Y_1(s)}{R_1(s)}E_1(s) \qquad \text{from equ.(5.18)}$$

$$\frac{\frac{\partial y_1(s)}{\partial m}}{R_1(s)} = \frac{Z_1(s)}{R_1(s)}\frac{Y_1(s)}{R_1(s)} \tag{5.21}$$

where $Z_1(s) = \frac{1}{C_1}\frac{\partial C_1(s)}{\partial m}E_1(s)$

Equ.(5.21) is the same as in the single-input single -output case.

Similarly, when $R_1(s) = 0$, equ.(5.19) and equ.(5.20) become:

$$E_1(s) = -\frac{H'(s)R_2(s)}{1+C_1G_1'(s)}$$

$$\frac{\partial y_1(s)}{\partial m} = -\frac{H'(s)G_1'(s)R_2(s)\frac{\partial C_1}{\partial m}}{(1+C_1G_1'(s))^2}$$

$$= \frac{E_1(s)}{1+C_1G_1'(s)}\frac{1}{C_1}\frac{\partial C_1}{\partial m}C_1G_1'(s)$$

$$\frac{\frac{\partial y_1(s)}{\partial m}}{R_2(s)} = \frac{Z_1(s)}{R_2(s)}\frac{Y'_1(s)}{R'_1(s)} \tag{5.22}$$

where $\frac{Y'_1(s)}{R'_1(s)} = \frac{C_1G_1'(s)}{1+C_1G_1'(s)}$

Hence the sensitivity function of the sensitivity filter structure is same for the two cases considered.

Now equ.(5.18) is equivalent to an expression

$$Y_1(s) = W_{11}(s)\,R_1(s) + W_{12}(s)\,R_2(s)$$

$$Y_1(s, m+\Delta m) = W_{11}(s)\,R_1(s) + \frac{\partial W_{11}(s)}{\partial m}R_1(s)\,\Delta m + W_{12}(s)\,R_2(s) + \frac{\partial W_{12}(s)}{\partial m}R_2(s)\,\Delta m$$

For the above analysis

$$\frac{\partial W_{11}(s)}{\partial m} = \frac{Z_1(s)}{R_1(s)}\frac{Y_1(s)}{R_1(s)} \quad \text{and} \quad \frac{\partial W_{12}(s)}{\partial m} = \frac{Z_1(s)}{R_2(s)}\frac{Y_1'(s)}{R'_1(s)}$$

From the ICAD concept the block diagram of the feedback loop sensitivity filter system with controller tuning for each channel is given in Fig.5.7. The completed block diagram of the closed loop system with controller tuning is shown Fig 5.8.

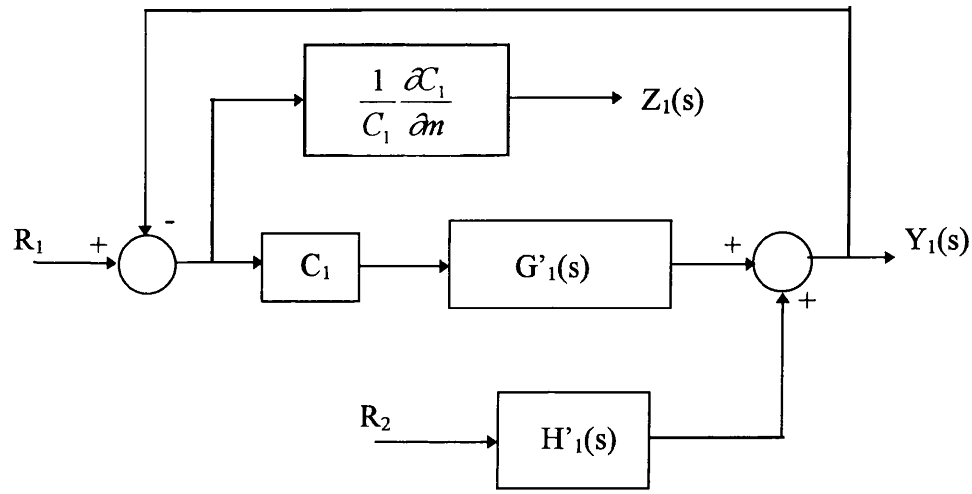


Fig 5.7 A feedback loop cosystem with controller tuning

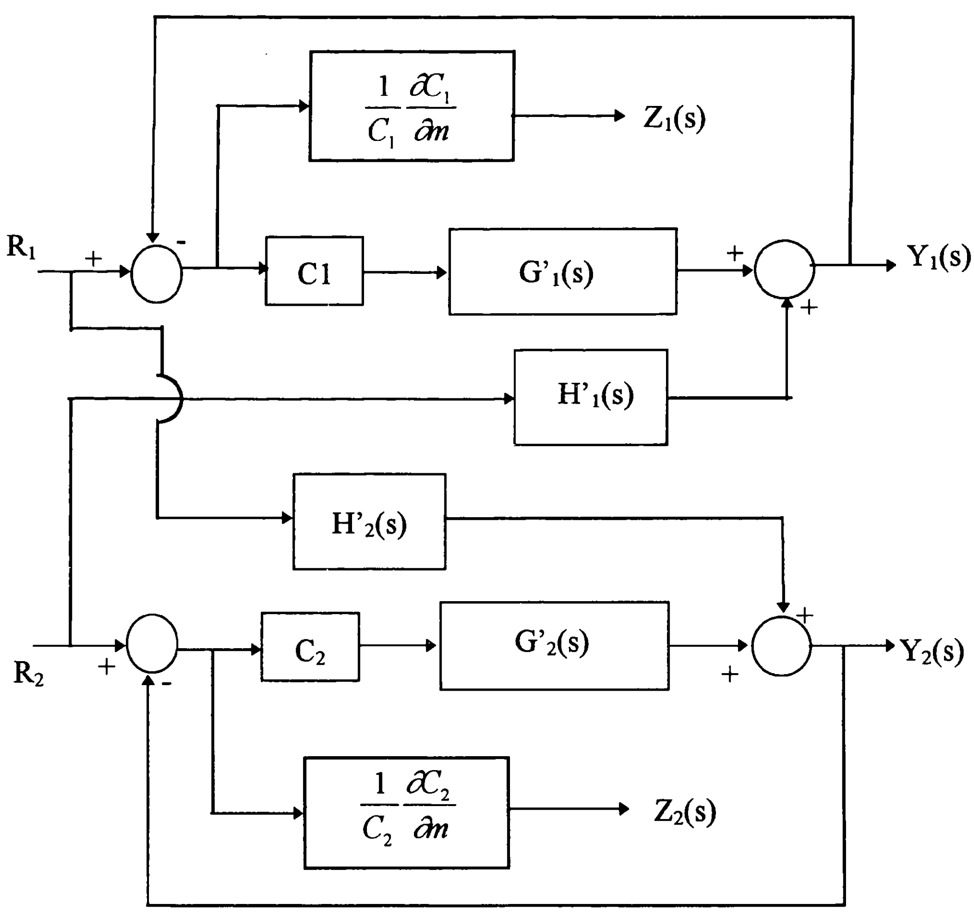


Fig 5.8 Completed diagram of the closed loop with controller tuning

two - input two - output case

Hence for complete analysis of the two-input two-output system, two tests could be undertaken. The first involves an input applied at input one with measurements at y_1, z_1 and z_2. The second involves an input applied at input two with measurements at y_2, z_1 and z_2.

5.6 Calculation of the Sensitivity Function in the Time - Domain

The sensitivity function can be calculated using equ.(3.7.) in Chapter 3 if the sampling data have been measured from the real system. Namely if the response y(t) and the filter output z(t) of the 2 - tank system have been obtained. This involves sampling and convolution operations. The analogue controller implementation of the complete 2 input 2 output water tank system is based on the SISO case. This involves the use of an ELONEX PC, with the PC-812 card, outputting analogue signals to control the pumps, sampling the analogue signals from the depth sensors and a modified program originally written by Dr. M. Macauley of Glasgow University. This modified program is written in Pascal. The SISO system is depicted in block diagram shown in Fig 5.9.

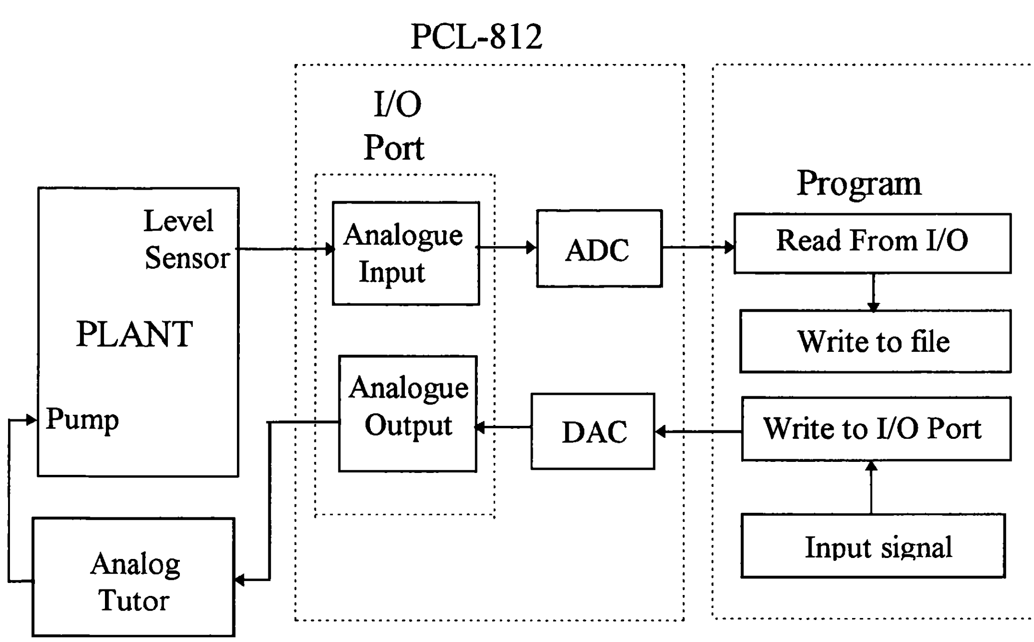


Fig 5.9 Block diagram showing process of test signal generation and sampling for data collection system

Three different types of input signals have been considered as reference input. The calculation results for these three cases have been obtained using the convolution approach and MATLAB software on the PC.

5.6.1 Step Input Case

In the step input case, based upon the equ.(3.7) the output response y(t) of the system and filter outputs z_{kp} and z_{ki} have been measured from the real system. There is a need to differentiate one of the signals with respect to time prior to the convolution operation. In this experiment the size of the step has been chosen as 90 mm for input R_1 and the PI controller parameters were $K_{p1} = 8.24$, $K_{i1} = 0.5$ with sampling time $\Delta t = 0.5$ sec. The output response y_1 and the sensitivity functions S_1 and S_2 corresponding to controller parameters K_{p1} and K_{i1} respectively are shown in Fig 5.10 which is only for tank 1.

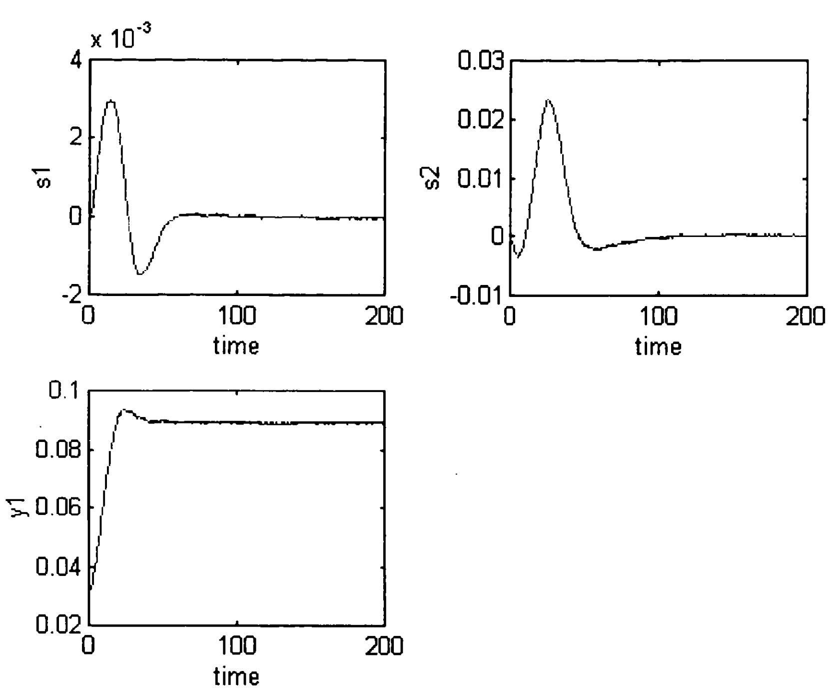


Fig 5.10 Sensitivity functions for PI controller parameters K_{p1} and K_{i1} (step input case)

5.6.2 Impulse Input Case

In fact, we cannot realise a perfect impulse input as considered in Chapter 3 and Chapter 4. But if we use a small enough α in the equ (4.3) the results still are accurate. Here we give results for a pulse input with $\alpha = 0.2$ which was taken as a approximation of an impulse input. After the calculation of the sensitivity functions the results for tank 1 are given in Fig.5.11.

The response of the system depends on the value of α taken. If the α taken is small enough, the pulse approximates the impulse but comparing the results with the step input case shown in Fig.5.10 the response of the pulse input case is seen to be significantly noisier in the steady state. However the sensitivity functions of the parameters are consistent with these found for the step in terms of dynamic characteristics.

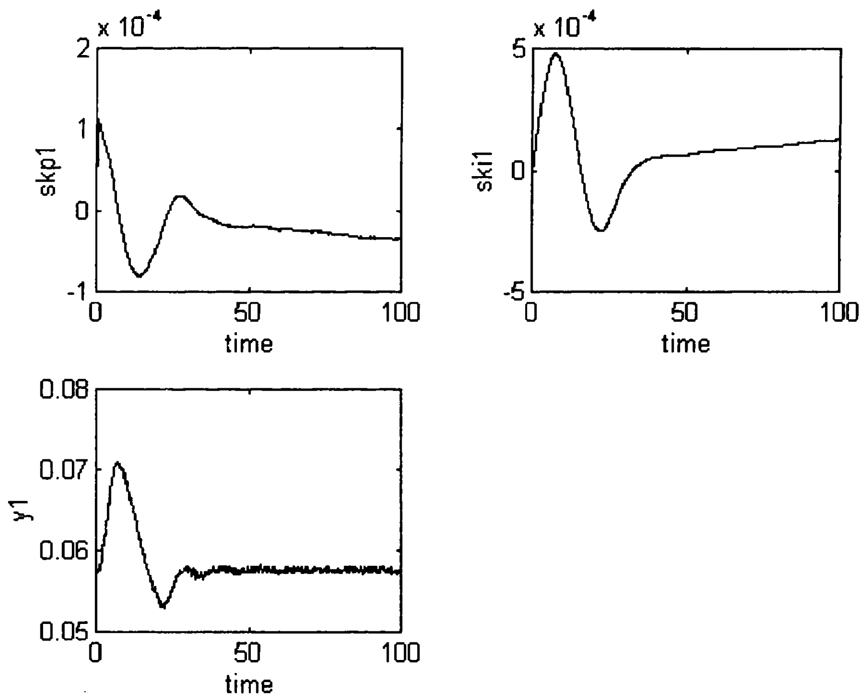


Fig 5.11 Sensitivity functions for pulse input

for tank1 only ($\alpha = 0.2$)

5.6.3 PRBS Input Case

An extended PRBS signal based on a seven stage shift register has been tested as input signal on this water tank system.
Results in terms of time-domain analysis were not encouraging since the calculated sensitivity functions were corrupted by noise. It is believed that this problem is due, in part at least, to the non-linear characteristics of the pump. Most attention has therefore been given to frequency-domain methods.

5.7 Calculation of the Sensitivity Function in the Frequency - Domain

In order to make the PRBS compatible with the FFT an extended PRBS must be taken. We still use a feedback shift register with 7 stages to generate the PRBS signal. The sampling rate was Ts = 0.5 sec. and with N = 64, giving a clock generating period for the PRBS of T = N*Ts = 32 sec. The real system data were collected using the technique described in Sections 5.4 and 5.5. The output response y(t) and filter outputs Z_{kp} and Z_{ki} corresponding to controller parameters K_p and K_i have to be obtained. Thus all data, system output y(t), reference r(t), PRBS, and filter outputs z(t) will be transferred to the frequency domain using the radix - 2 fast Fourier Transform (FFT) in MATLAB.

Based upon equ.(4.8) and equ.(4.9) preliminary the frequency responses of the system and the sensitivity functions for controller parameters were obtained by calculation in MATLAB. This was done both for a linear model and for the real two-tank system. Fig.5.12 and Fig.5.13 show the frequency responses and the sensitivity functions in the frequency domain. Fig.5.12 shows the calculated results from the theoretical linear model which Fig.5.13 shows calculation results from the real system data. From these figures we can see that the results from the real system measurements agree with the theoretical calculation results. There are only slight differences over the frequency range of interest ($0-10^{-2}$ rad/sec).

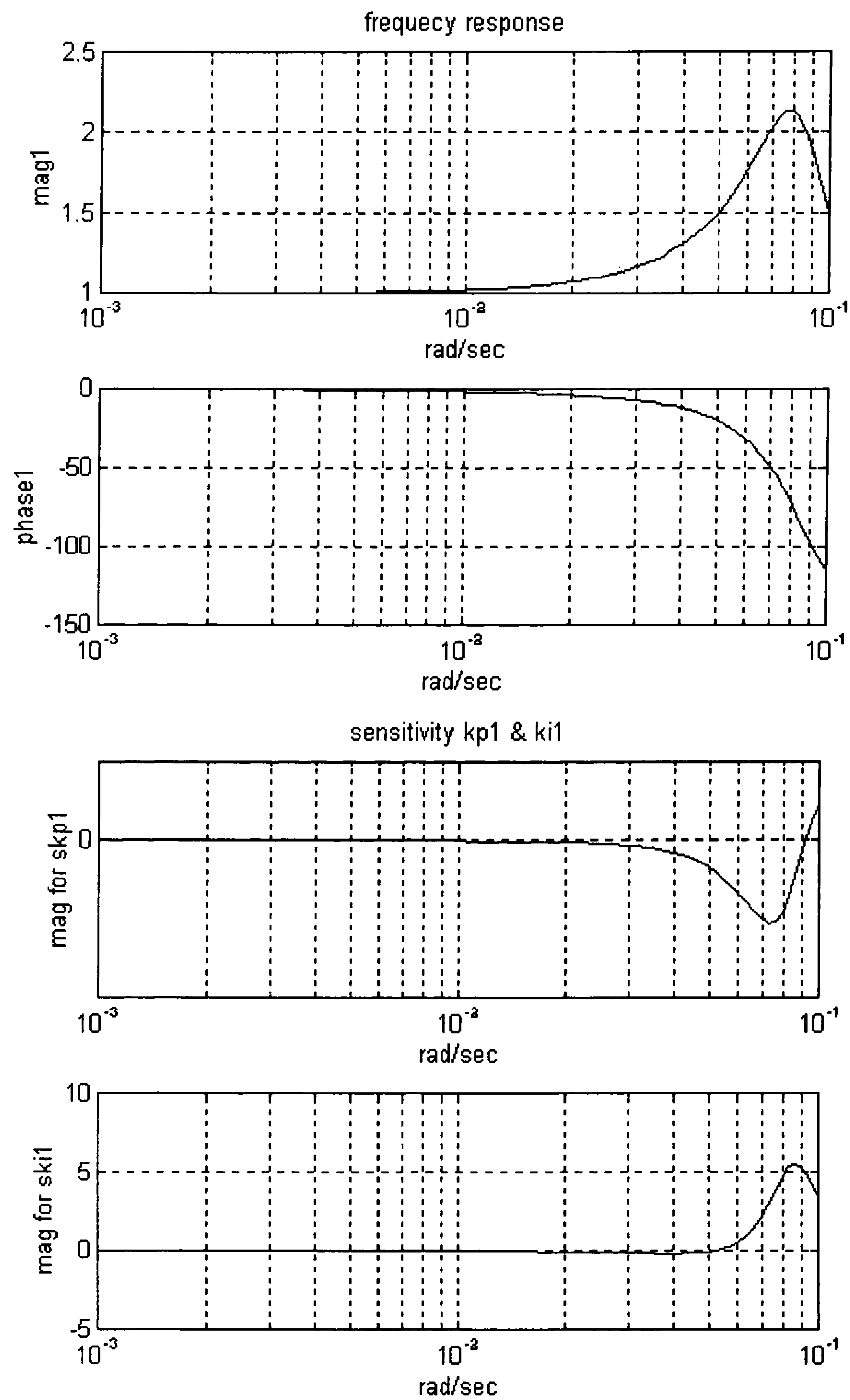


Fig 5.12 Theoretical results of the frequency response and sensitivity functions for SISO case

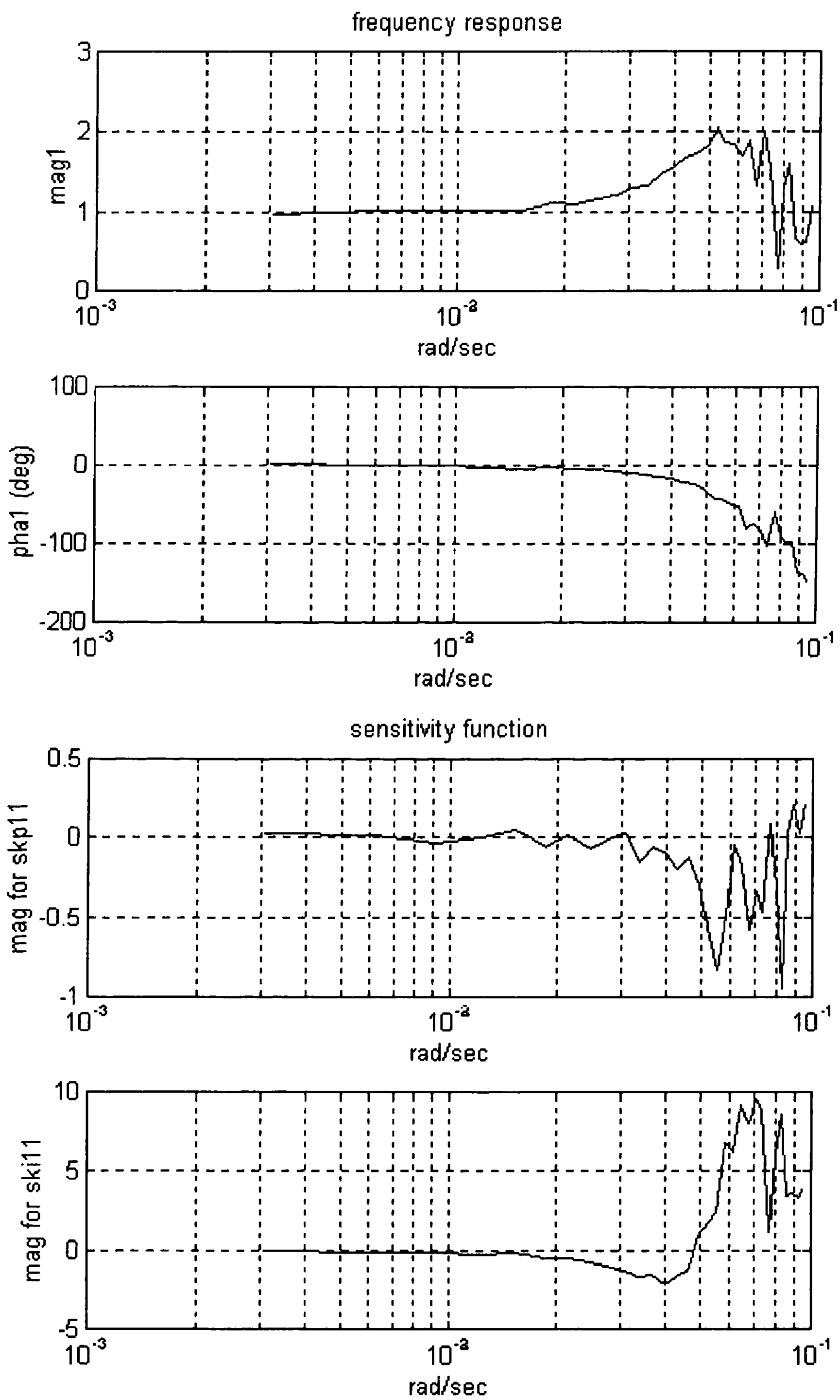


Fig 5.13 Frequency response and sensitivity functions

for the 2 - tank system for SISO case

5.8 Simulation Results of the Sensitivity Functions

Interpretation of the experimental results can be made easier through simulation studies in order to avoid the very slow process involved in experiments on real hardware. The calculation of the sensitivity functions by means of convolution depends on the model being linear. We can use simulation to assess the significance of this assumption. In the step input case the sensitivity calculation involves differentiation of the response signal. This will increase the effect of measurement noise. On the other hand the convolution operation involves smoothing, so there are possible advantages in this approach compared with parameter perturbation methods. Using simulation such advantages may be investigated in more detail. The non-linear model has been used for the simulation of the water tank system (equ.(5.5)) and the simulation was carried out using the Slim and MATLAB software packages on the PC computer.

5.8.1 Simulation Results for the Time Domain

In order to compare the differences between the results from the convolution approach and the results from the parameter perturbation method results from these two approaches are shown in this section as below:

5.8.1.1 Sensitivity Functions for the Step Input Case

The approached adopted is the same as that described in Section 5.6, only the output responses have been obtained from simulation instead of from measurement. The calculation results for the simulation are shown in Fig.5.14(a). The size of the step input was 90 mm. Simulation results from the non linear model agree well with the results from real system measurements. Fig.5.14(b) shows the response of the simulated system the only significant difference being that the measured results from real system are delayed by a few seconds compared with the simulation results. This difference is inevitably a result of model inadequacies.

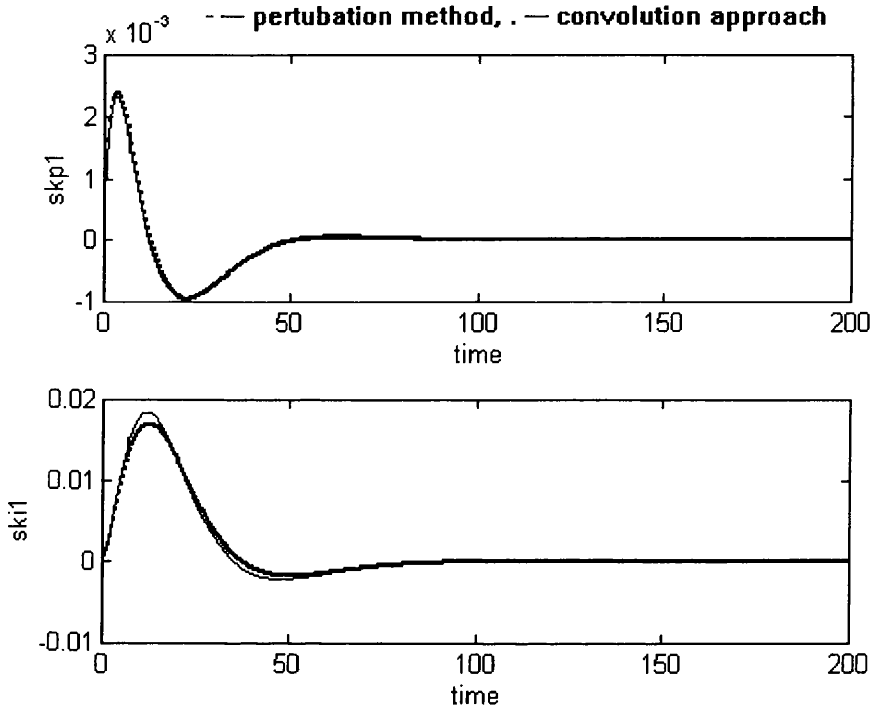


Fig. 5.14(a) Simulation results for step input case

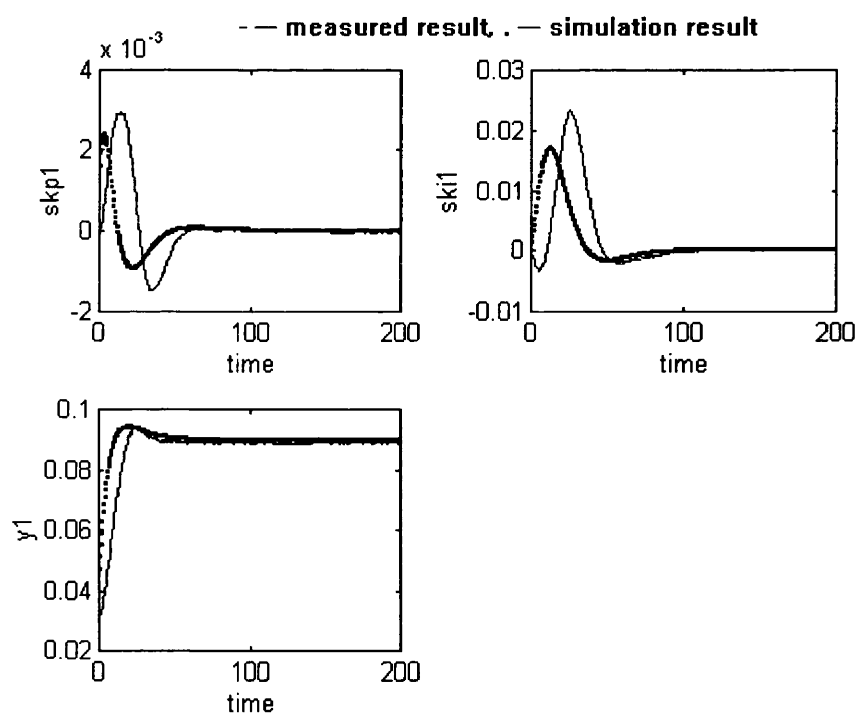


Fig 5.14(b) Comparison of measured and simulation cases for the step input case

5.8.1.2 Sensitivity Functions for the Pulse Input Case

Fig.5.15 shows the sensitivity functions for the controller parameters obtained for the pulse input signal using simulation. Comparing with the results of Fig. 5.11 there is a slight difference between the two methods but the sensitivities are similar. The magnitude of the response depends on the value α. differences between the perturbation results and the convolution results are more significant in the case of the parameter K_{i1} and are likely to be associated with nonlinear effects and the chosen pulse amplitude.

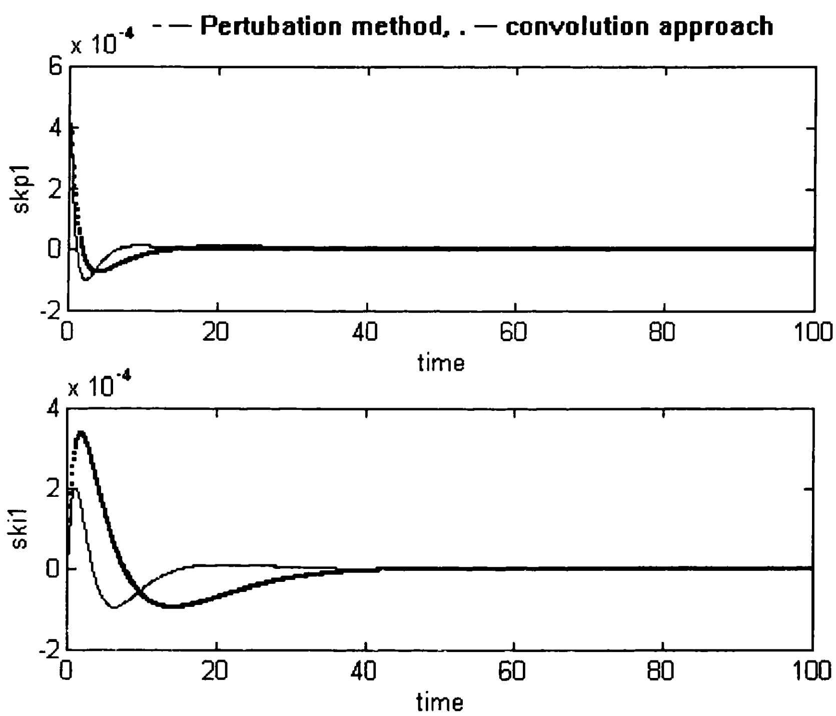


Fig 5.15 Simulation results for pulse input case

5.8.1.3 Effect of the size of the Input Signal

Different sizes of test signal will give different sensitivity functions. Fig.5.16 shows the sensitivity functions corresponding to the size 20 mm and 70 mm of the step test signals. This shows clearly that the form of the sensitivity functions depends on the magnitude of the test signal. Clearly the smaller the test signal magnitude the more accurate is the sensitivity function computed by the convolution method, provided noise is insignificant.

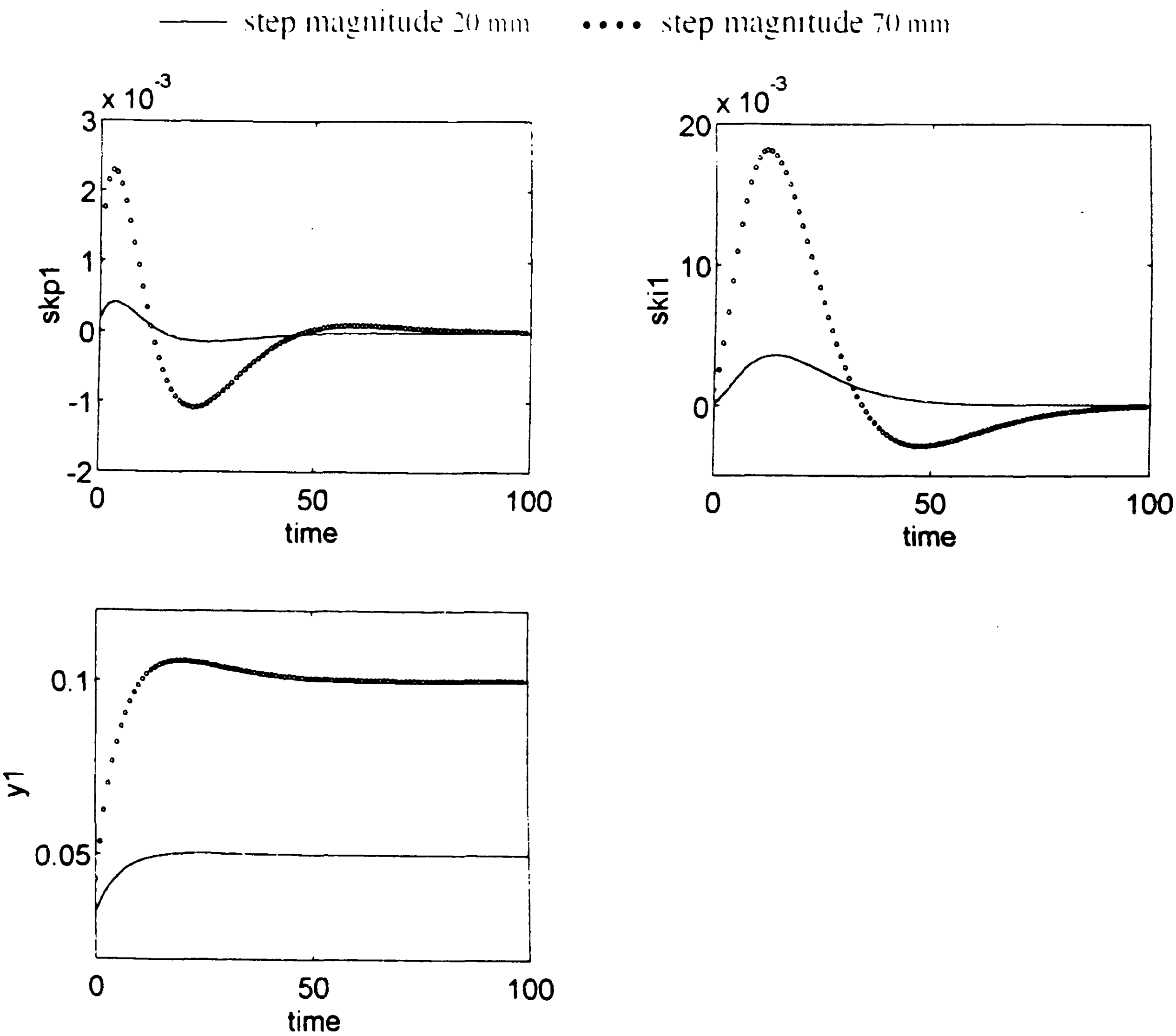


Fig.5.16 Effect of the size of the step input signal

5.8.2 Simulation Results for the Frequency Domain

Fig.5.17 shows simulation results for the frequency response and the sensitivity functions for the same condition as in Fig.5.12 and Fig.5.13 in Section 5.7. Although this is a SISO case only, comparison with the Fig.5.12 and Fig.5.13 shows that the theoretical results and the results from the real system measurements agree.

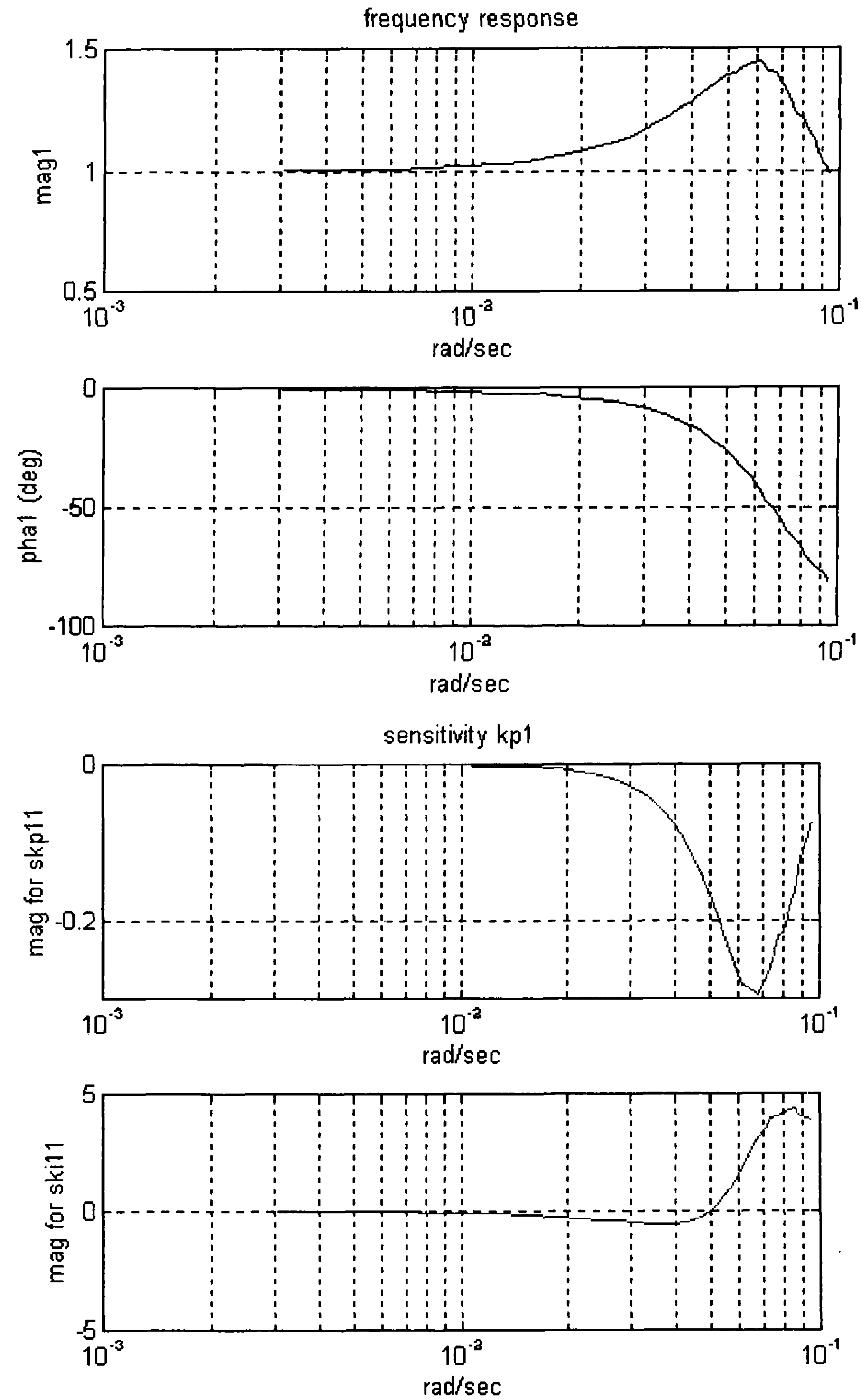


Fig 5.17 Simulation results of the frequency response and sensitivity function for SISO case

5.9 Generating Sensitivity Functions Using the Two-Stage Convolution Approach

In order to demonstrate the application of this method, the two input two output closed loop water tank system will be taken as an example. Simulation results for a step input for the sensitivity functions $\frac{\partial y_1 y(t)}{\partial k_{p1}}$ and $\frac{\partial y_1 y(t)}{\partial k_{i1}}$ calculated by the differential method, the convolution method and the two-stage process method are shown in Fig.5.18 and Fig.5.19. Results by all there methods are almost identical.

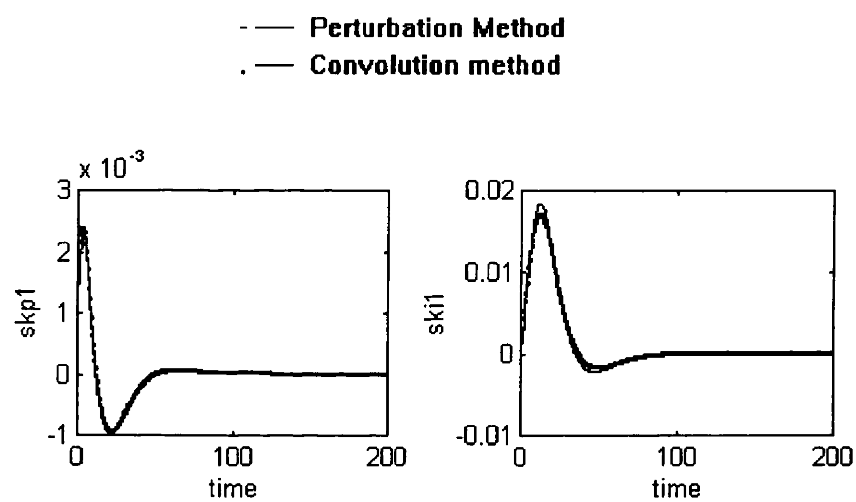


Fig 5.18 Simulation result of differential & convolution method

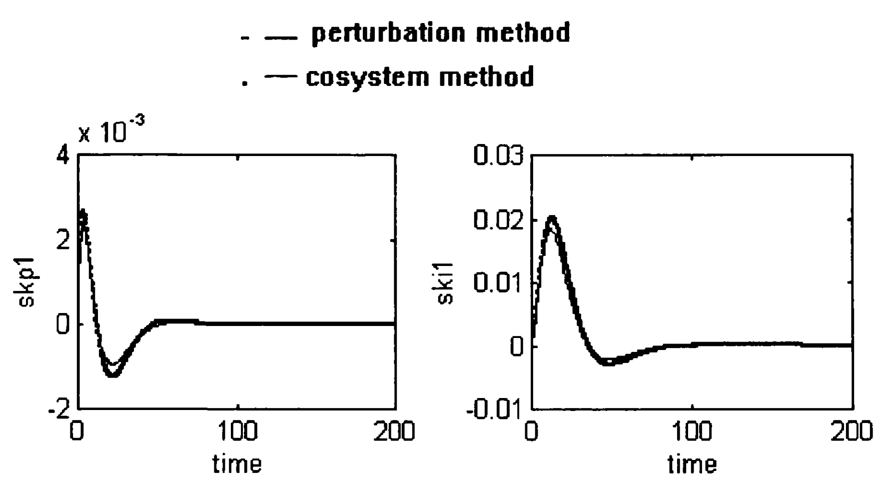


Fig.5.19 Simulation results of differential & two-stage process methods

5.10 Controller Parameters Tuning Procedure

Once the sensitivity functions of the controller parameters of the control system are obtained the controller parameters tuning can be carried out. From Chapter 3 we know that using the integral least-squared method a set of linear equations are obtained. By solving these linear equations the required change of each of the controller parameters is calculated and the first tuning adjustment is completed. If the responses of the system for these changed controller parameters are satisfactory the tuning procedure will be completed. Otherwise the procedure will be repeated by using new sensitivity functions corresponding to the changed controller parameters. Namely, after one application of the tuning procedure we got a new set of controller parameters. This adjustment process is repeated again until responses of the system reach the desired response.

5.10.1 Controller Tuning in the Time-Domain

A complete PI controller parameter tuning procedure for the 2 - tank system has been carried out using the sensitivity function tuning technique for a step input. The initial PI controller parameters were taken as $K_{p1} = 2.06$, $K_{i1} = 0.5$ for tank1 and $K_{p2} = 1.75$, $K_{i2} = 0.4$ for tank 2. Fig.5.20 - Fig.5.21 show the adjustment procedure. We can see that after three adjustments, the parameters $K_{p1} = 7.935$, $K_{i1} = 0.75$ and $K_{p2} = 6.95$, $K_{i2} = 0.45$. Comparing the desired responses of the system obtained corresponding to the controller parameters $K_{p1} = 8.24$, $K_{i1} = 0.5$ for tank1 and $K_{p2} = 7.0$, $K_{i2} = 0.4$ for tank 2 with the actual responses of the system obtained corresponding to the controller parameters $K_{p1} = 7.935$, $K_{i1} = 0.75$ for tank 1 and $K_{p2} = 6.95$, $K_{i2} = 0.45$ for tank2, the results are satisfactory.

5.10.2 Controller Tuning in the Frequency-Domain

A MATLAB simulation for a completed PI controller parameters tuning procedure in the frequency domain for the 2 - tank system has been carried out using sensitivity function tuning technique for a PRBS input signal. The PI controller parameters were taken as $K_{p1} = 4.0$, $K_{i1} = 1.0$ for tank 1 and $K_{p2} = 7.0$, $K_{i2} = 0.5$ for tank 2. After six iterations the controller parameters were $K_{p1} = 2.34$, $K_{i1} = 0.2973$ for tank1 and $K_{p2} = 7.0$, $K_{i2} = 0.5$ and agreement with the desired frequency responses was very good. The adjustment procedure of the controller parameters K_{p1} and K_{i1} is listed on Table 5.3. Fig.5.22 - Fig.5.27 show the sensitivity functions corresponding to each set of the controller parameters used. Fig.5.28 - Fig.5.29 show the frequency responses corresponding to each set of the controller parameters used. From the results it can be seen that the tuning process is very effective although the initial frequency responses of the system are far away from the desired frequency responses. In fact, the adjustment procedure converges fast. For the two-tank system six iterations are needed only. From Fig.5.28 and Fig.5.29 we can see that significant progress is made at each adjustment without any oscillations. The desired frequency responses for the multivariable system can be obtained using the ICAD approach.

Table 5.3 Adjustment of the controller parameters in the frequency domain

Iteration No.	**Parameter K_{p1}**	**Parameter K_{i1}**
Desired	2.0	0.25
Initial	4.0	1.0
1	4.0	0.125
2	2.6482	0.4464
3	2.466	0.0698
4	3.6310	0.3584
5	0.4927	0.0929
Final	2.34	0.2973

o --- Tuning result, - --- Desired response

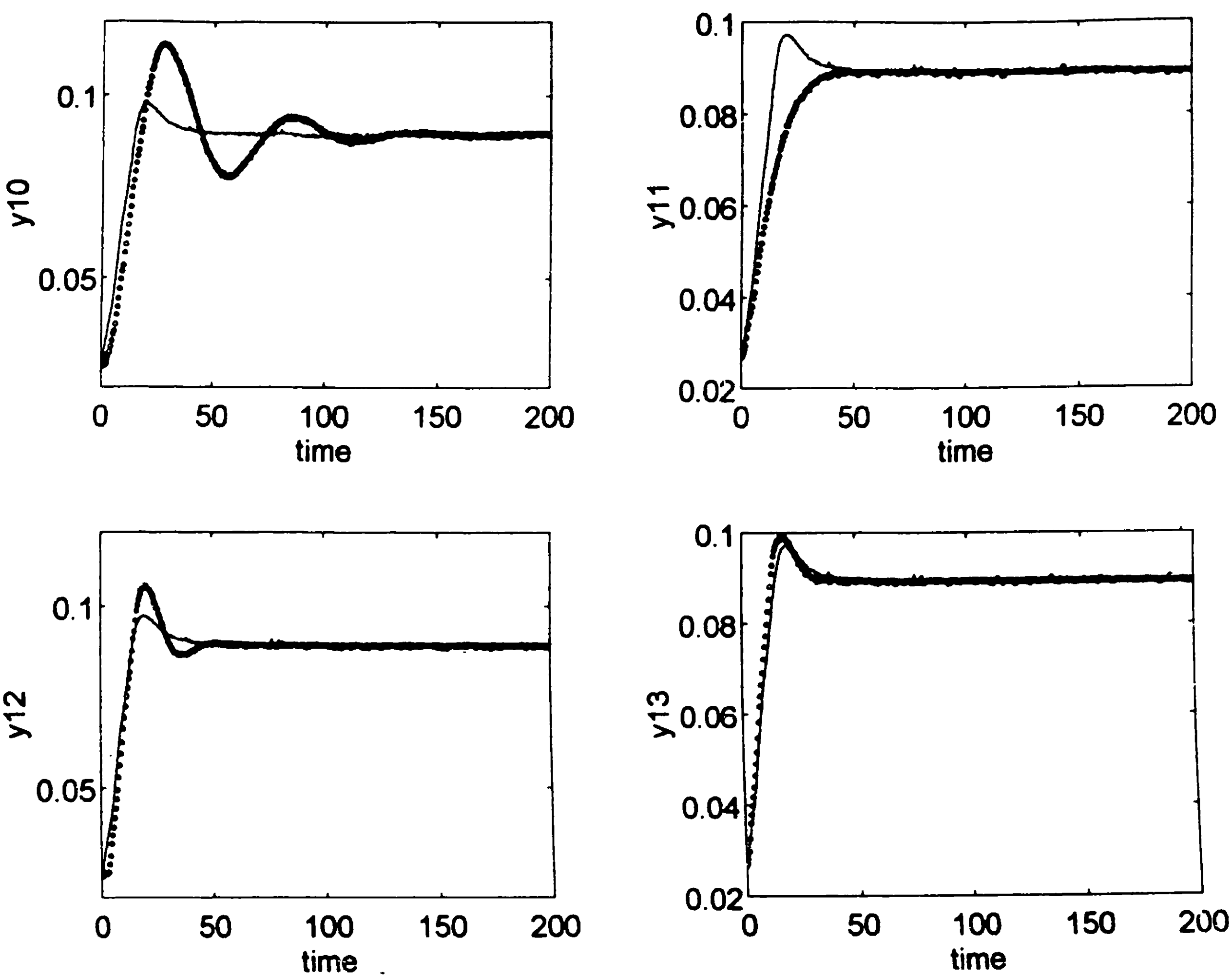
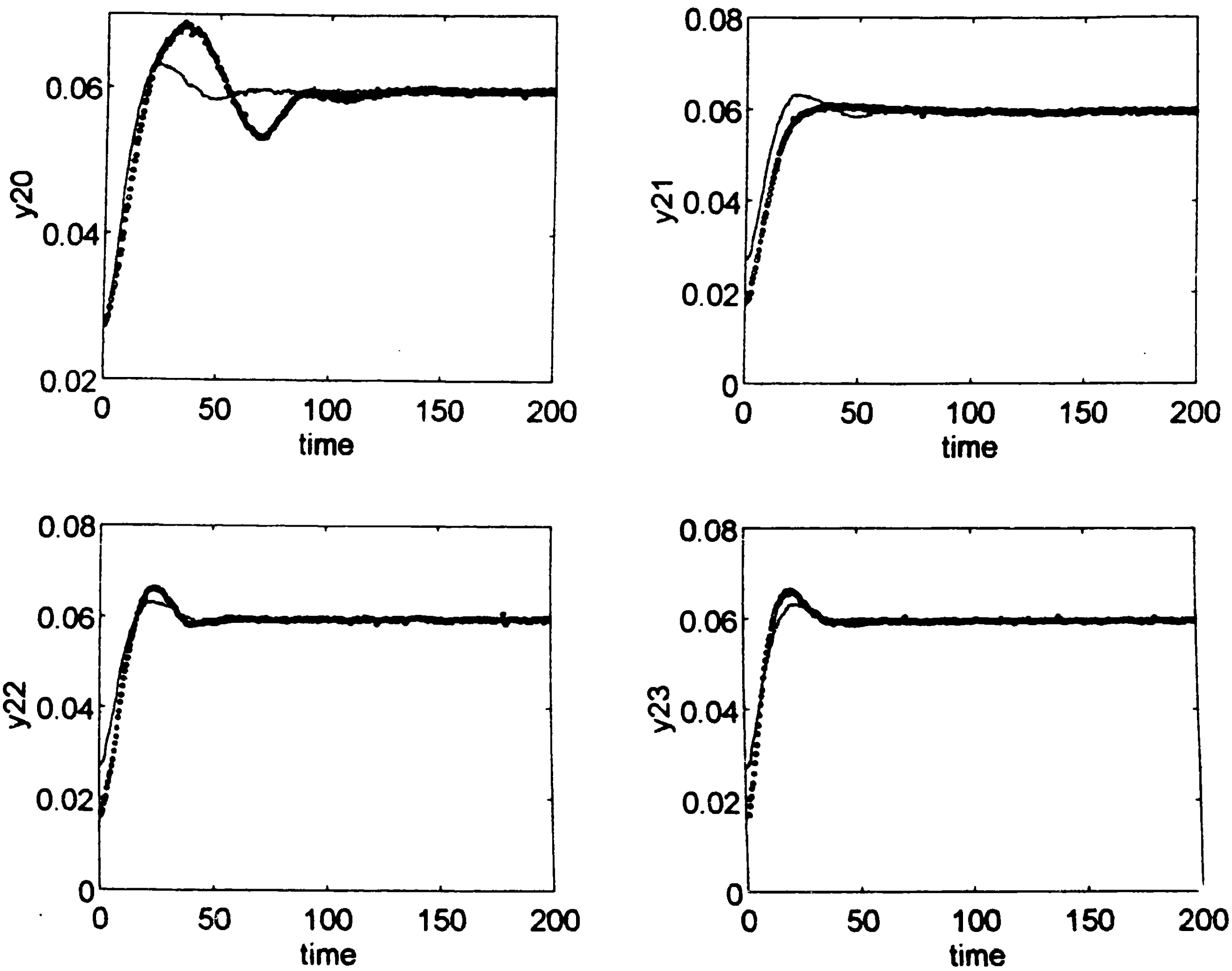


Fig. 5.20 The two tank control system tuning results for three iterations for tank 1 (time-domain)

Note that the initial response is highly oscillatory and that often three iterations the tuning process gives a response which is very close to the desired response.

o --- Tuning result, - --- Desired response



<u>Fig. 5.21 The two tank control system tuning results for</u>

<u>three iterations for tank 2 (time-domain)</u>

<u>Note that the close agreement between the actual</u>

<u>and desired responses after three iterations</u>

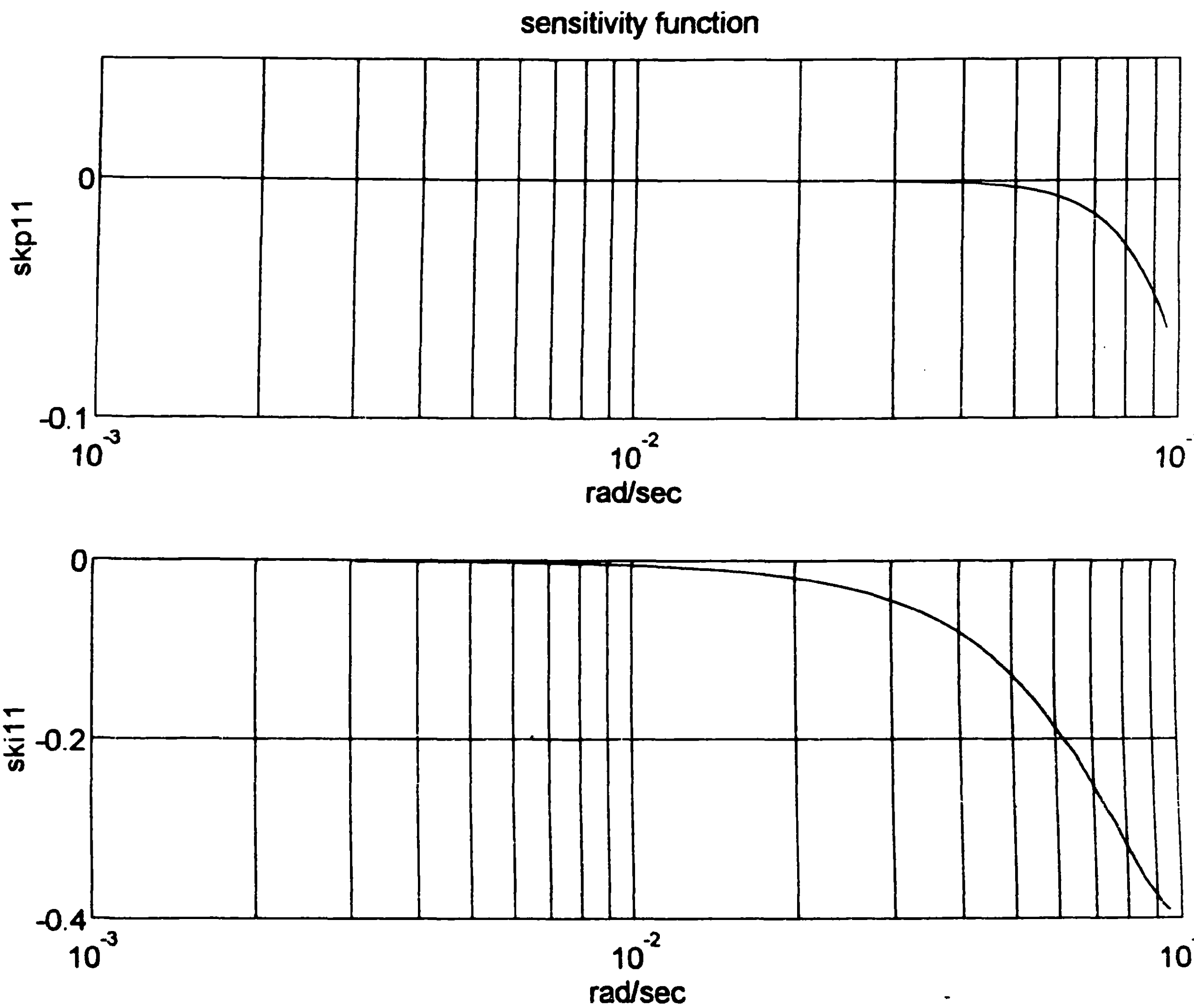


Fig. 5.22 Sensitivity functions of the initial parameters K_{p1} & K_{i1}

(frequency domain)

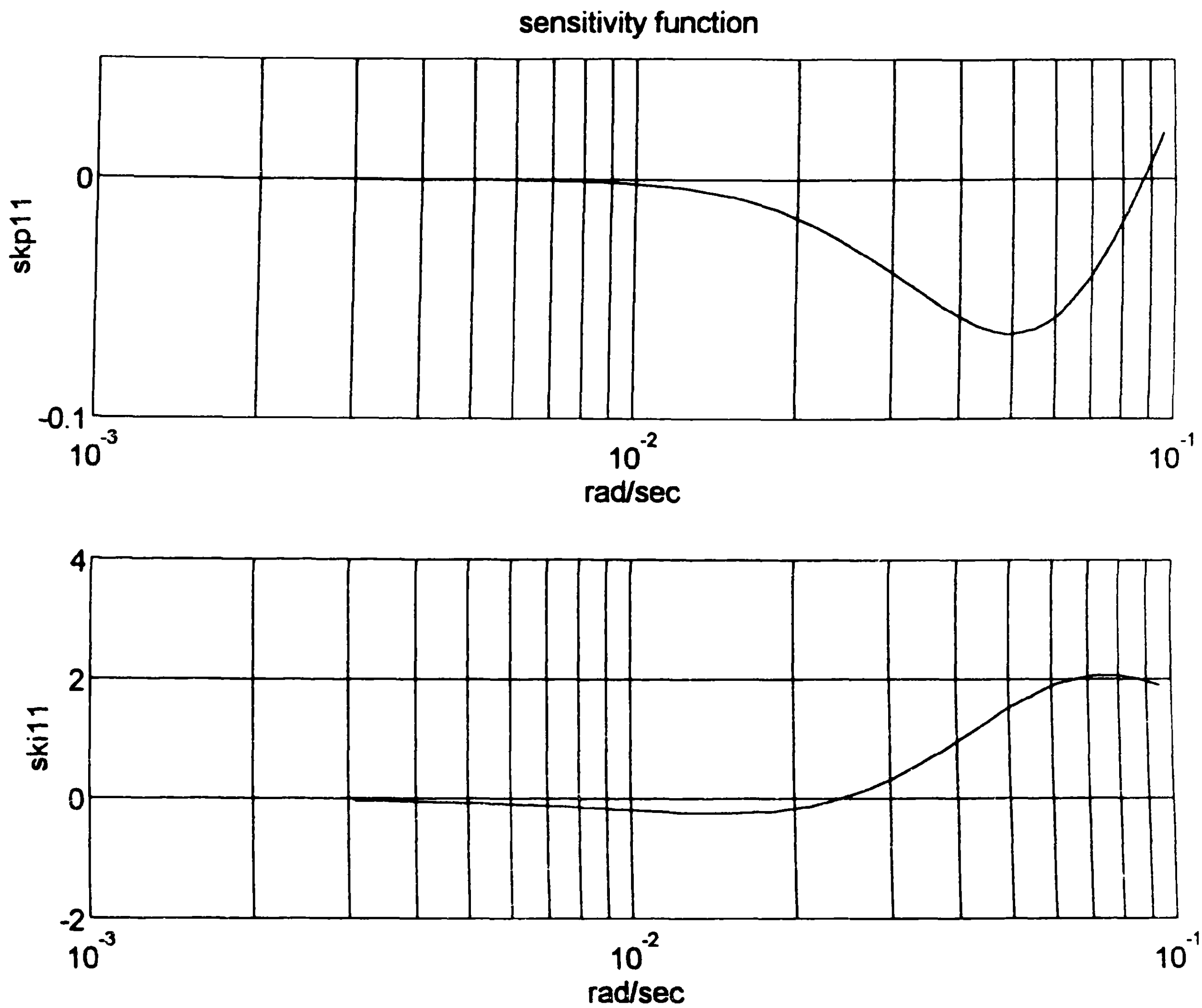


Fig. 5.23 Sensitivity functions of the parameters K_{p1} & K_{i1} after one iteration (frequency domain)

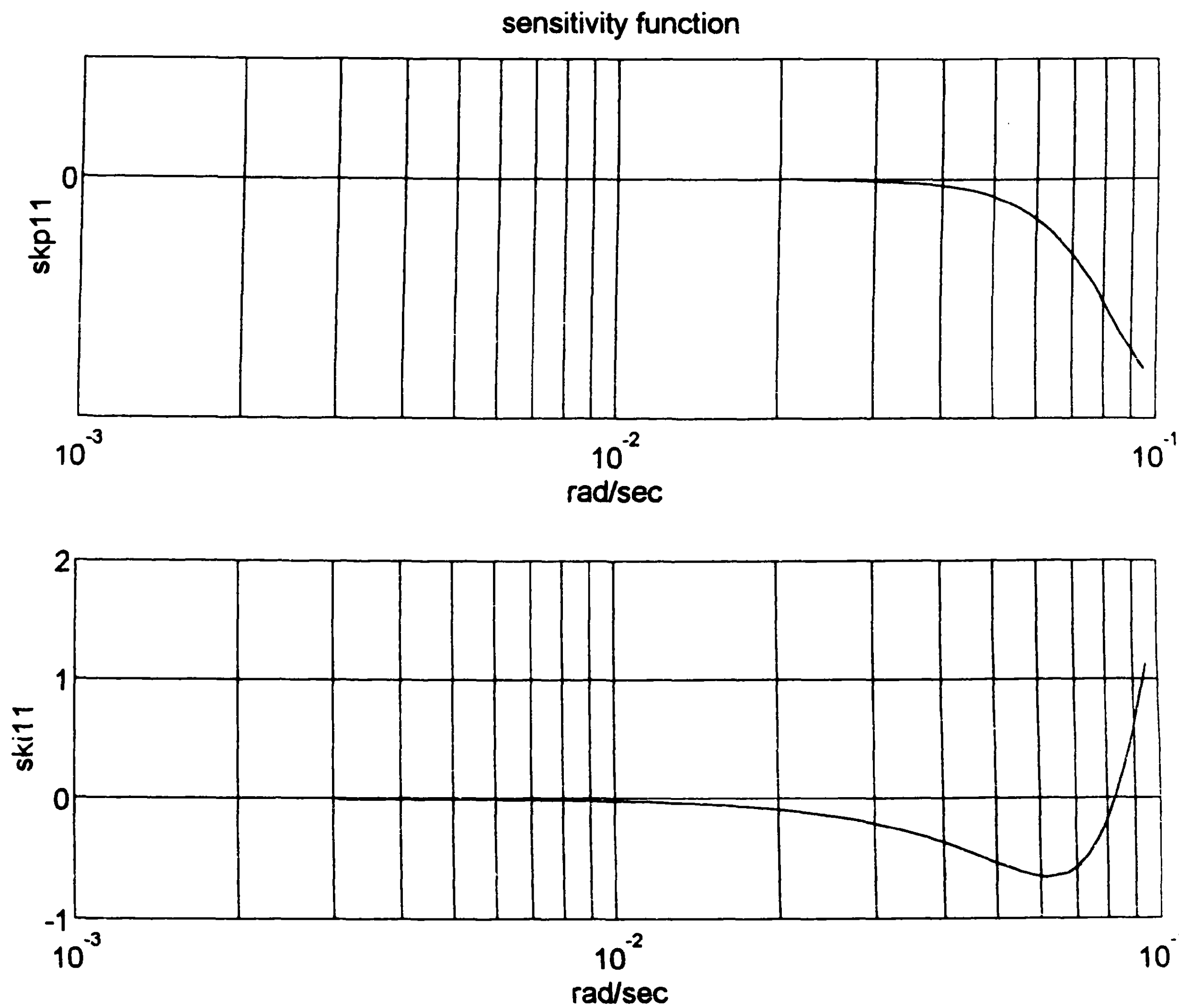


Fig. 5.24 Sensitivity functions of the parameters K_{p1} & K_{i1} after two iterations (frequency domain)

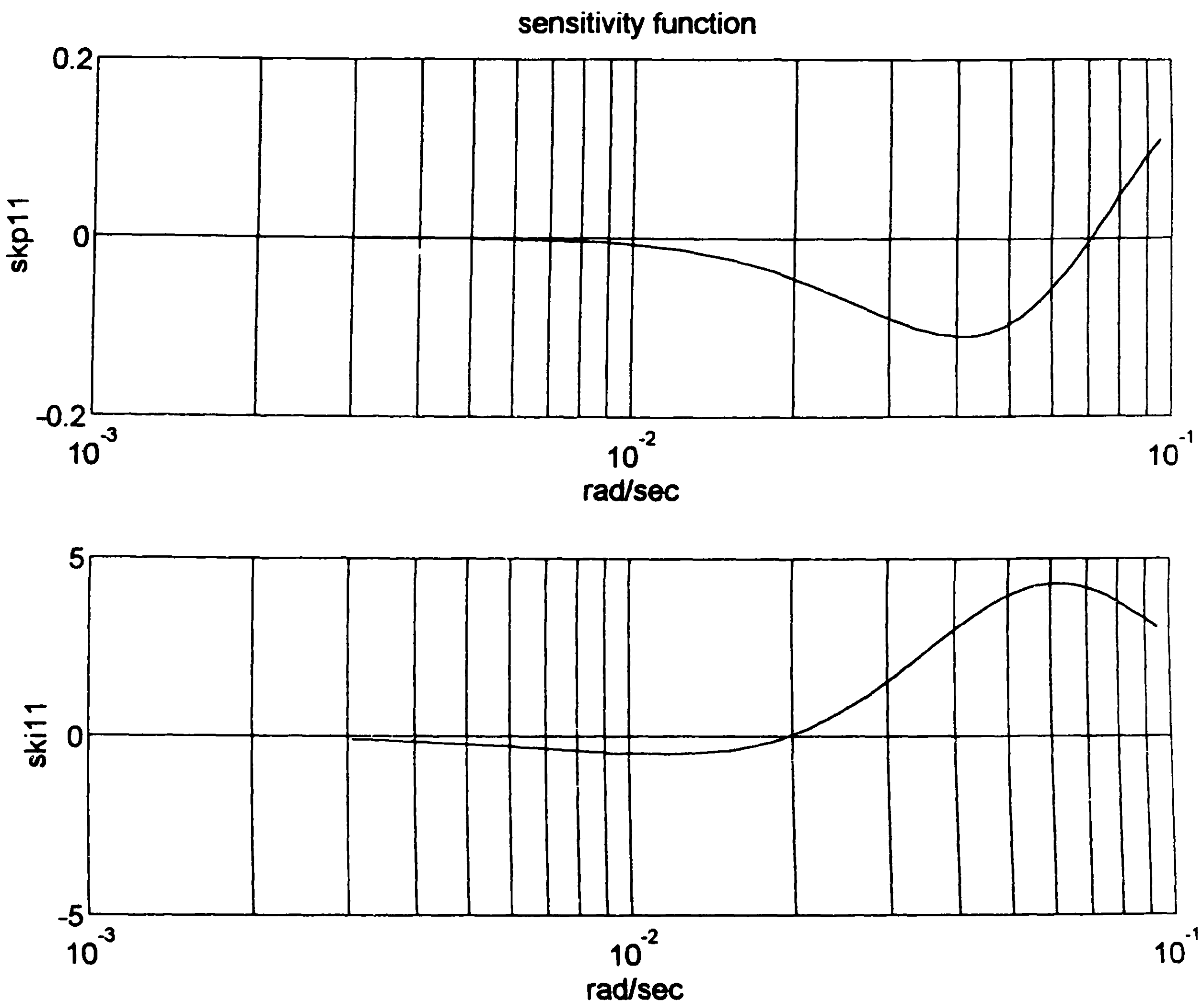


Fig. 5.25 Sensitivity functions of the parameters K_{p1} & K_{i1} after three iterations (frequency domain)

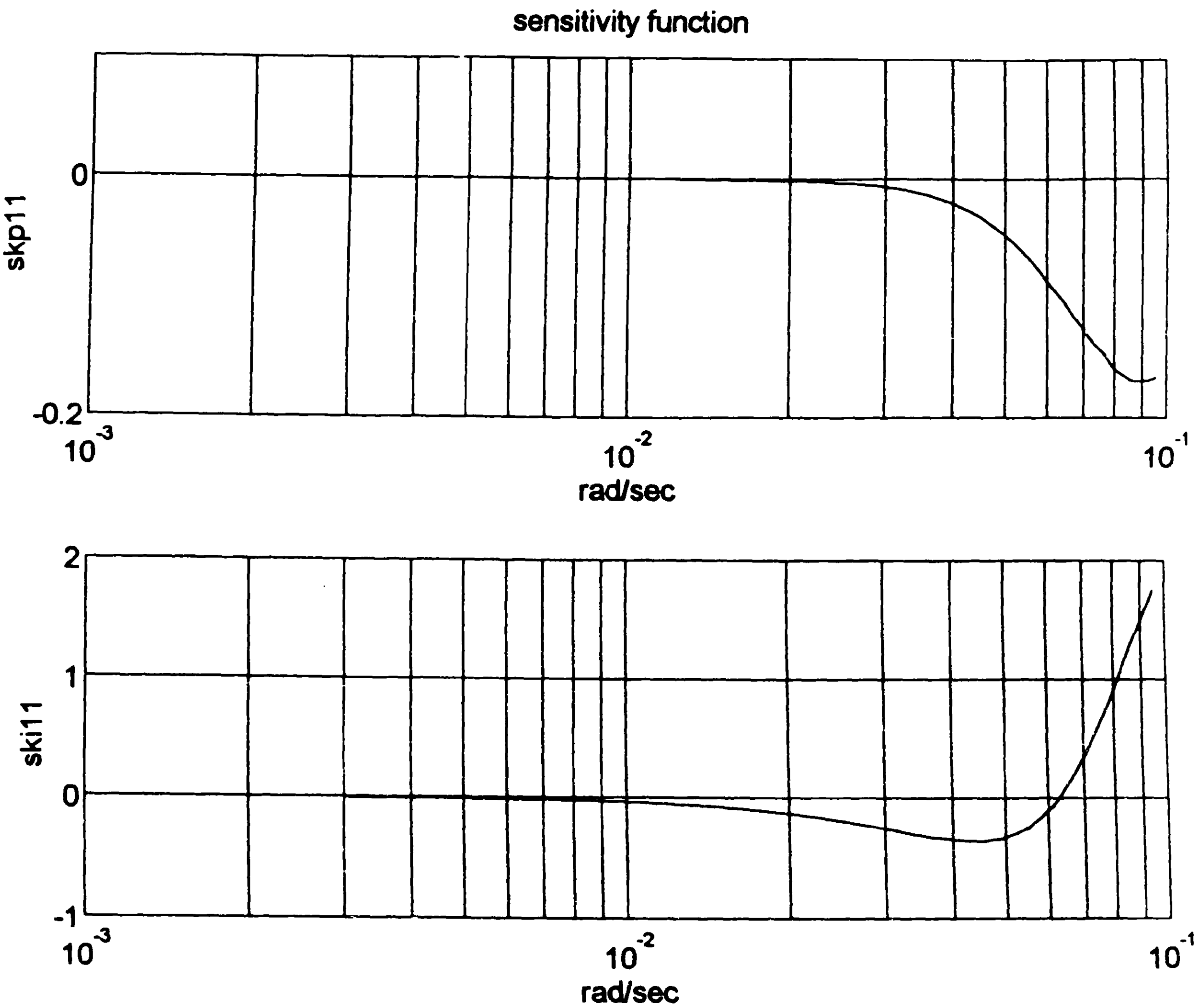


Fig. 5.26 Sensitivity functions of the parameters K_{p1} & K_{i1} after four iterations (frequency domain)

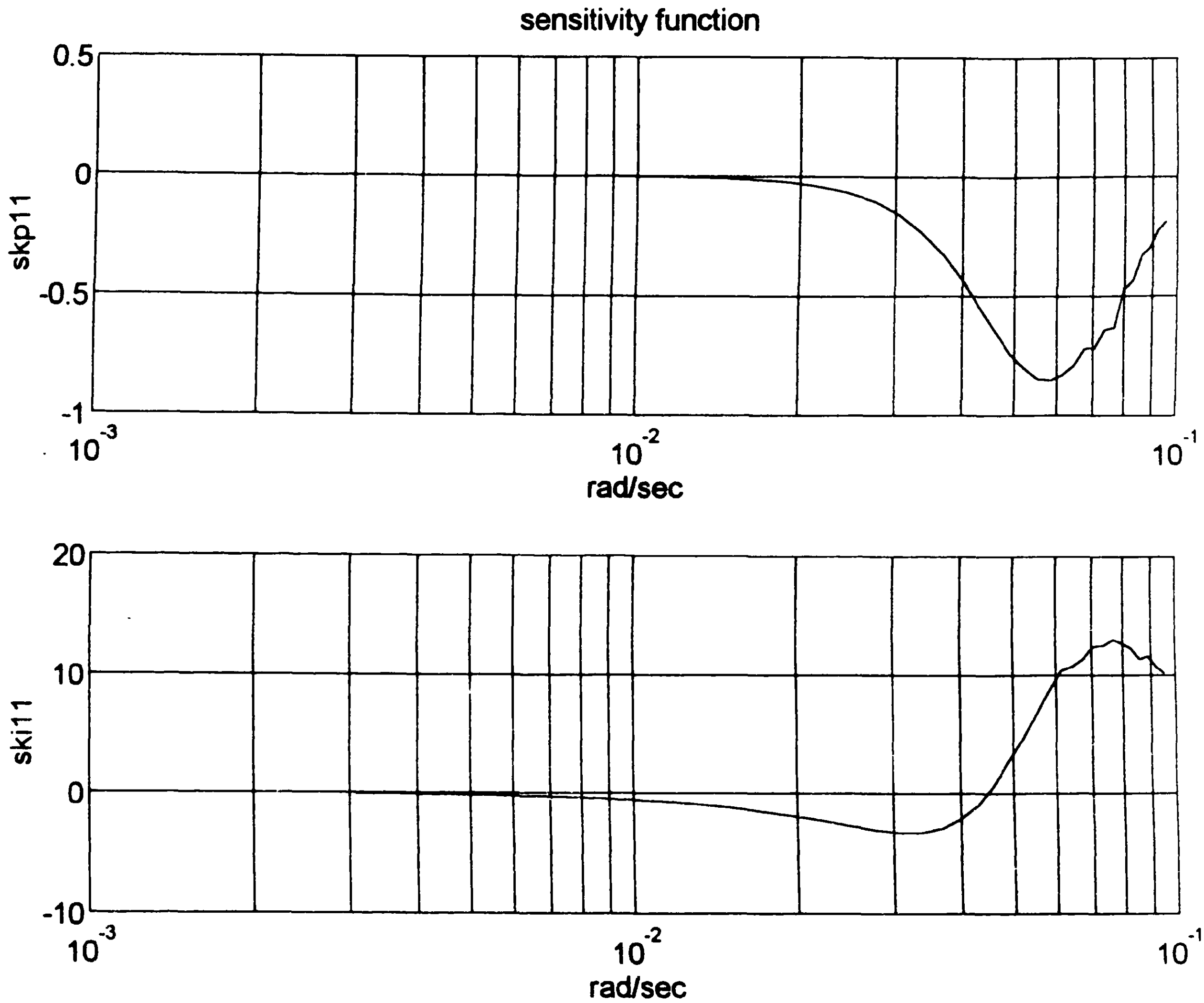


Fig. 5.27 Sensitivity functions of the parameters K_{p1} & K_{i1} after five iterations (frequency domain)

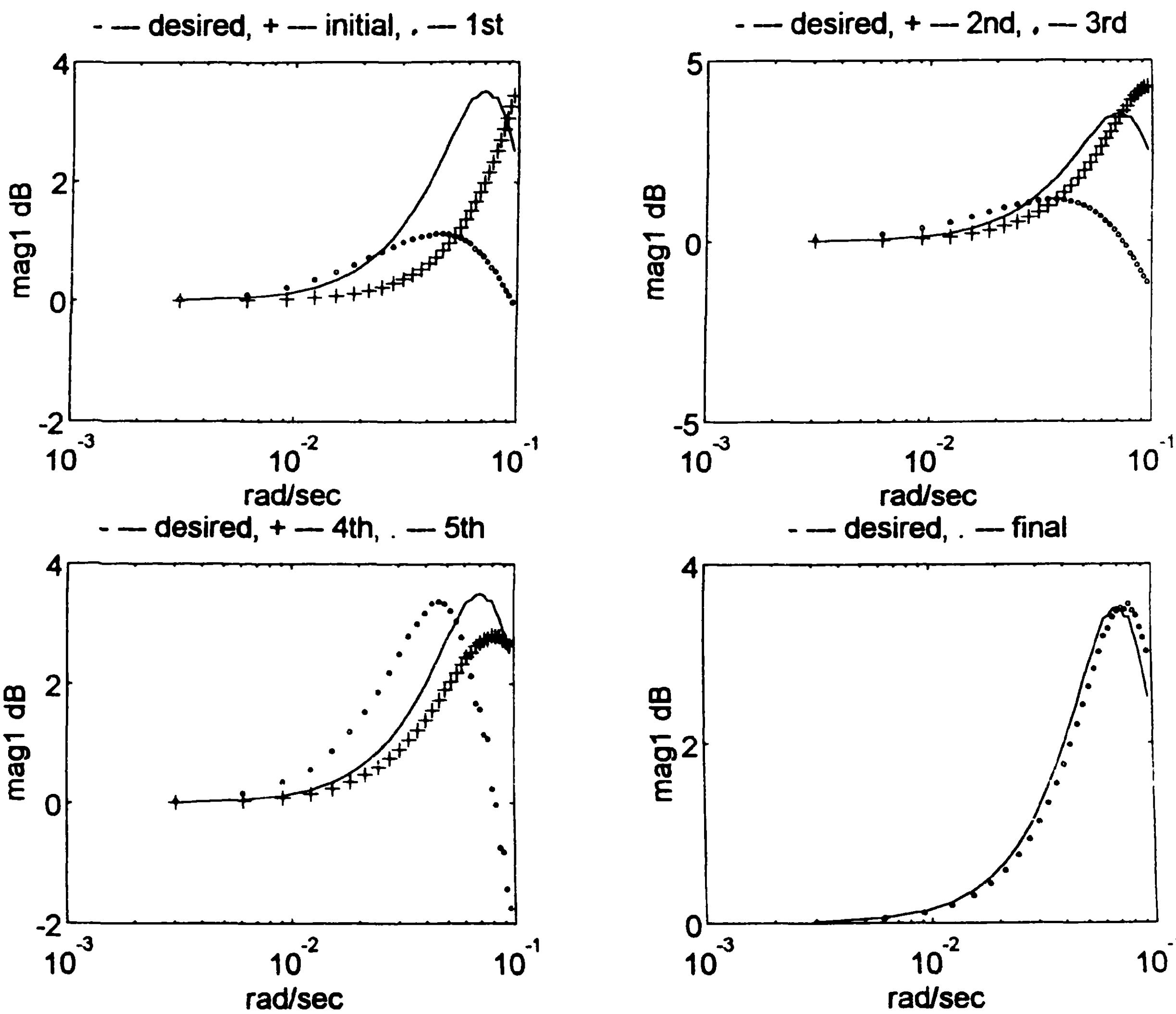


Fig. 5.28 The tuning results for tank 1 in the frequency domain

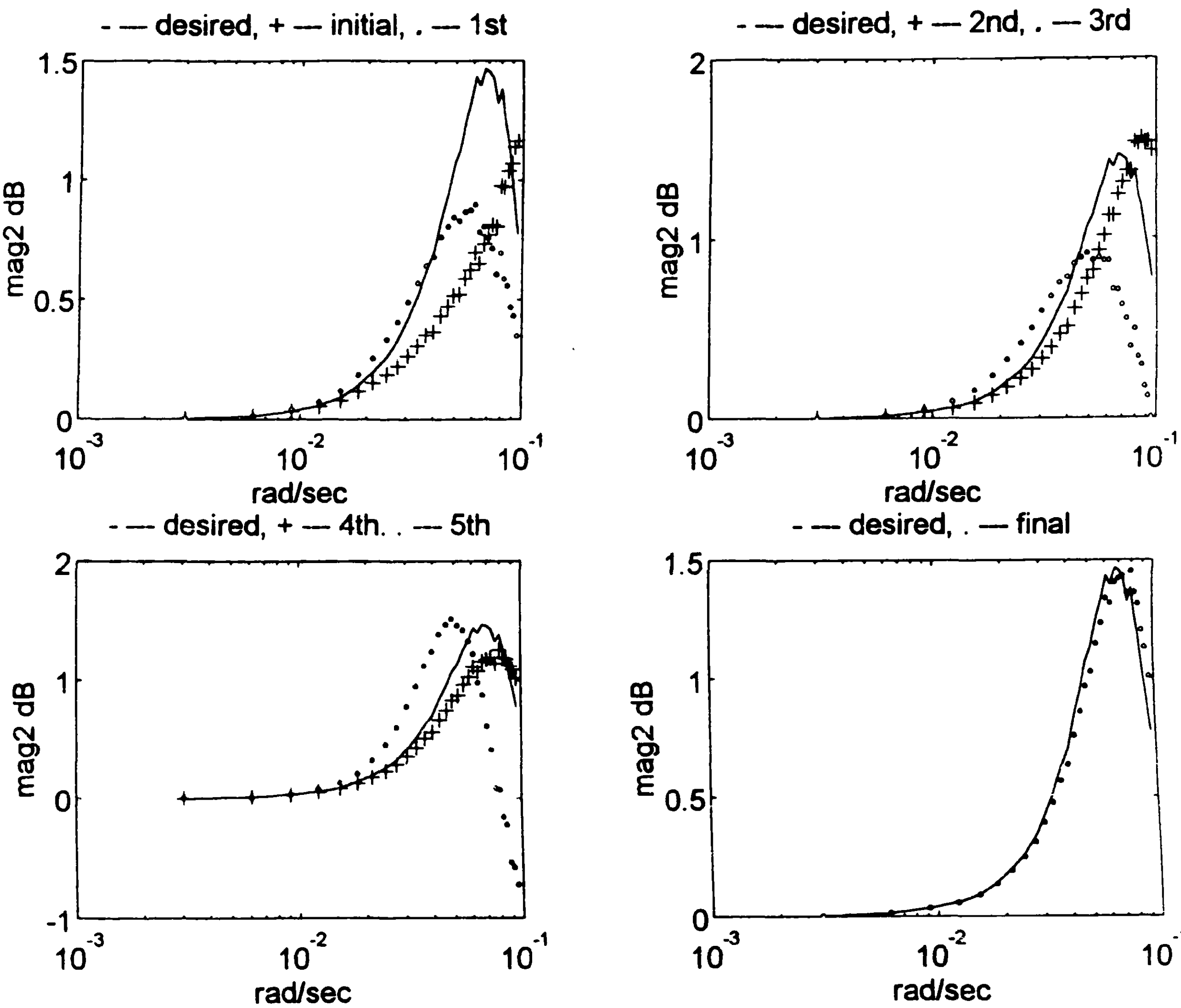


Fig. 5.29 The tuning results for tank 2 in the frequency domain

CHAPTER 6
Application of the Technique to Helicopter Flight Control System Tuning

6.1 Application of the Tuning Techniques to Helicopter Flight Control Systems

In Chapter 2 it was mentioned that the problem of helicopter flight control system design is inherently multivariable in nature and involves model structure and parametric uncertainties. The cross - coupling and uncertainties are both largely associated with the dynamic characteristics of the main rotor and interaction effects between the main rotor, tail rotor and fuselage. During large transient manoeuvres the helicopter aerodynamics is especially complicated. Even well designed helicopter flight control systems may require some form of tuning as a result of preliminary flight tests. To reduce cost and minimise development time, it is important that this tuning process be carried out in an efficient fashion.

Helicopter flight control systems which are developed to meet the demands of flight will undoubtedly require tuning to eliminate, as far as possible, adverse response characteristics on the controlled helicopter. The tuning requirement arises because of problems inherent in the use of present-day methods for flight controller design. The most important of these is that the use of a mathematical model of the helicopter plant inevitably introduces error into the design since descriptions of high order dynamics, particularly those of the rotor, are inaccurate. The effect of these dynamics will usually be noticeable to the pilot in terms of a decrease in system stability and undesirable cross-couplings.

6.2 Application of the Tuning Technique for a Flight Control System

In order to apply the tuning technique to helicopter flight control systems, a linearised model of a typical combat rotorcraft trimmed to 30 knots forward flight has been taken as an example. Control synthesis studies had earlier been performed for this vehicle using the method of Individual Channel Analysis and Design (ICAD) for this multivariable system (Dudgeon, G.J.W., *et al.*, 1995) The highly coupled nature of rotorcraft dynamics is reflected in the mathematical model of this multivariable system. Uncertainties in some aspects of the vehicle model mean that controller parameter tuning is potentially very important for optimisation of the control system design on the basis of preliminary flight test results.

6.2.1 Linearised Model of the Helicopter Control Flight System

The application of the tuning technique is based on a 19th order representation of a typical combat rotorcraft in straight and level flight at 30 knots. This linearised model of the helicopter control system has rigid body states, 6 rotor states and 4 actuator states. This 19th order model of a typical combat rotorcraft flight at 30 knots straight and level flight using state-space form is shown in Appendix 6. 30 knots was chosen as it is the midpoint of the low speed range.

6.2.2 Individual Channel Analysis and Design of the Helicopter Control Model

As mentioned in the previous chapter, the Individual Channel Analysis and Design method may be applied to the this control system for the multivariable case. Its most distinctive feature is the use of the so-called multivariable structure functions which make explicit the role of cross-coupling and quantify its effects. Any such problems can be solved within the ICAD framework straightforwardly. The aircraft is modelled as an 4-by-4 transfer function matrix $\underline{G}(s)$. A diagonal control matrix $\underline{C}(s)$ is in the forward path, immediately before $\underline{G}(s)$, and a feedback loop is closed around $\underline{G}(s)\ \underline{C}(s)$. Fig 6.1 shows the block diagram of this control system.

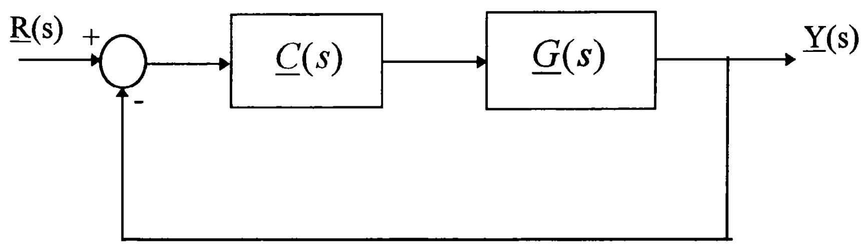


Fig. 6.1 Control Structure

6.2.3 Optimal Controller Design and Transfer Functions of the Filter for the Channels

The channel controllers so designed, using classical loop shaping (Dudgeon, G.J.W., *et al.*, 1995), are given by:

$$C_1(s) = \frac{0.13(s+1)}{s(s+10)} \tag{6.1}$$

$$C_2(s) = \frac{0.25(s+2)(s+2.1)}{s(s+3.4)(s+25)} \tag{6.2}$$

$$C_3(s) = \frac{-0.129(s+3.2)(s^2-0.16s-0.15)}{s(s^2+1.32s+0.69)(s+13)} \tag{6.3}$$

$$C_4(s) = \frac{-0.72(s+2)}{s(s+25)} \tag{6.4}$$

We rewrite the controller transfer functions in a more general form, relating the structure above Examples are

$$C_1(s) = \frac{K_{11}s+K_{12}}{K_{13}s^2+K_{14}s} \tag{6.5}$$

$$C_4(s) = \frac{K_{41}s+K_{42}}{K_{43}s^2+K_{44}s} \tag{6.6}$$

Other channels can be treated in the same way. For example, the transfer functions of the filters corresponding to channel controller $C_1(s)$ are expressed as

$$F_{11}(s) = \frac{1}{C_1(s)}\frac{\partial C_1(s)}{\partial K_{11}} = \frac{s}{K_{11}s+K_{12}} \tag{6.7}$$

$$F_{12}(s) = \frac{1}{C_1(s)}\frac{\partial C_1(s)}{\partial K_{12}} = \frac{1}{K_{11}s + K_{12}} \qquad (6.8)$$

The transfer functions of the filter corresponding to channel controller $C_4(s)$ are expressed as

$$F_{41} = \frac{1}{C_4}\frac{\partial C_4}{\partial K_{41}} = \frac{s}{K_{41}s + K_{42}} \qquad (6.9)$$

$$F_{42} = \frac{1}{C_4}\frac{\partial C_4}{\partial K_{42}} = \frac{1}{K_{41}s + K_{42}} \qquad (6.10)$$

$$F_{43} = \frac{1}{C_4}\frac{\partial C_4}{\partial K_{43}} = \frac{-s^2}{K_{43}s^2 + K_{44}s} \qquad (6.11)$$

$$F_{44} = \frac{1}{C_4}\frac{\partial C_4}{\partial K_{44}} = \frac{-s}{K_{43}s^2 + K_{44}s} \qquad (6.12)$$

6.2.4 Tuning Result

Because real flight data for this system are not available at the moment, the only approach possible is to use computer simulation methods based on the linear model and controllers to get the typical responses which emulate flight test data. Assuming a set of arbitrary controller parameters is given, the actual step responses y(t) and the sensitivity filter output z(t) from the linear model for this set of controller parameters can be obtained. Thus the sensitivity functions of the parameters can be generated using the convolution approach or the two stage approach discussed in Section 6.2.3.

Several examples have been considered for the tuning process. Case 1 involves tuning the two parameters K_{11} and K_{12} of the numerator for the controller $C_1(s)$. Table 6.1 shows the tuning process of the controller parameters K_{11} and K_{12} for Channel 1. Individual Channel Analysis & Design suggests that the control $C_1(s)$ for this case (Case 1) should be as given in equ.(6.5). The optimal values of the controller parameters for this controller are taken as

$K_{11} = 0.13$, $K_{12} = 0.13$, $K_{13} = 1.0$, $K_{14} = 10$.

Starting the tuning process for K_{11} = 1.3 and K_{12} = 0.26 with K_{13} and K_{14} at the given values the actual response approaches the desired response after two iterations only. The desired step responses of this system for the controller parameter of channel 1 are shown in Fig.6.2 for the augmented system at 30 knots.

Fig.6.2 and Fig.6.3 show the desired response and initial response for a) the height rate step response, b) the pitch attitude step response, c) the roll attitude step response and d) the yaw rate step response.
Fig.6.4 - Fig.6.9 show the sensitivity functions and the height rate step responses for the tuning process of this example.

Table 6.1 Tuning process for Case 1

Iteration No.	**Parameter K_{11}**	**Parameter K_{12}**
Desired value	0.13	0.13
Initial value	1.3	0.26
First iteration	0.123	0.2558
Second iteration	0.1301	0.124
Final value	0.1301	0.124

Several cases of the 4 controller parameters for the yaw rate step responses and two cases of the 2 parameters for the height rate step responses at 30 knots have also been chosen for adjustment. The controller $C_4(s)$ for this case (Case 2) is expressed as equ.(6.6). The optimal values of the controller parameter for this controller are taken as

$K_{41} = -0.72$, $K_{42} = -1.44$, $K_{43} = 1$ and $K_{44} = 25$.

Fig.6.10 shows the initial step response for the system using the controller parameters above (Case 2). Fig.6.11, Fig.6.13 and Fig.6.15 show the sensitivity functions against different parameter values K_{41}, K_{42}, K_{43} and K_{44} of the controller $C_4(s)$ of Case 2 (see Table 6.2). A corresponding set of linear equations of the form of equ.(2.7) can be developed using the sensitivity functions. Once the equations have been solved one iteration of the tuning process can be take place. The results of the parameter tuning process are shown in Table 6.2, Fig.6.12, Fig.6.14 and Fig.6.16.

Another example has been used for tuning (Case 3). The controller parameters K_{41}, K_{42}, K_{43} and K_{44} of the channel C_4 is chosen as

$K_{41} = -0.5$

$K_{42} = -5$

$K_{43} = 2.5$

$K_{44} = 10$

The sensitivity functions and initial responses are shown in Fig.6.18 and Fig.6.19.
We also consider a second flight condition at 50 knots (Case 4), keep the same controller structure as in Case 2 and attempt to re-tune the controller (Appendix 6). The results of the parameter tuning process are shown Table 6.3, Fig.6.20, Fig.6.21 and Fig.6.22.

Table 6.2 Tuning process for Case 2

Iteration No.	K_{41}	K_{42}	K_{43}	K_{44}
Initial	-1.0	-0.25	1.25	28.0
1	-0.7619	-0.375	0.977	24.8248
2	-0.8155	-0.5625	1.1965	27.2350
3	-0.8791	-0.8438	1.2878	29.8459
4	-0.9630	-1.2656	1.3809	33.2519
5	-1.0670	-2.1007	1.4869	37.8588
6	-1.0673	-2.1140	1.4735	37.1047
7	-1.0572	-2.1013	1.4572	36.6648
8	1.0492	-2.0900	1.4484	36.3905
9	-1.0438	-2.0821	1.4436	36.2127
10	-1.0401	-2.0767	1.4406	36.0965

Table 6.3 Tuning process for Case 4

Iteration No.	K_{41}	K_{42}	K_{43}	K_{44}
Initial	-1.0	-0.25	1.25	28.0
1	-0.7393	-0.3750	0.9116	24.8712
2	-0.7809	-0.5625	1.1033	26.5654
3	-0.8404	-0.8438	1.2108	28.8182
4	-0.9209	-1.2656	1.3211	31.8351
5	-0.9794	-1.9359	1.3484	34.6217
6	-0.9861	-1.9693	1.3604	34.3253
7	-0.9868	-1.9723	1.3674	34.2656
8	-0.9871	-1.9744	1.3699	34.2772
9	-0.9872	-1.9742	1.3709	34.2754

6.3 Discussion

Comparing Fig.6.2 with Fig.6.9 and Fig.6.16 show that the step responses correspond closely to the desired step responses after a few iteration. After only two iterations the actual height step response agrees with the desired step response very well for Case 1. The tuning technique using the parameter sensitivity function is efficient. From Fig.6.17 even tuning four parameters of the controller for Case 2 only ten iterations are needed. But Fig.6.23 shows that if the flight condition changed at 50 knots and keeping the same controller structure of Case 2 as the stating point the tuning process is needed for five iterations only. From Fig.6.17 and Fig.6.23, there are not any oscillations and the process is very fast converging after only one iteration. In fact, the responses of the system after tuning not only agree with the desired response of the system perfectly but the values of the controller parameter of final iteration are also equal to the desired values for Case 2. For example, the tuning result for controller 4 is expressed as :

$$C_4 = \frac{-1.0335s - 2.067}{1.4345s^2 + 35.8851s}$$

Rearranging by dividing throughout by 1.4345 gives which is consistent with equation (6.4)

$$C_4 = \frac{-0.72s - 1.44s}{s^2 + 25s} = \frac{-0.72(s+2)}{s(s+25)}$$

For Case 4 with the second flight condition at 50 knots (Appendix 6) the results are much better than for Case 2 with flight condition at 30 knots.

We have to be very careful in the choice of the initial parameter values. From the tuning procedure we found that although the sensitivity functions can be obtained, the tuning process is not viable for some cases when the controller parameters used are not suitable for the system responses. Also that will cause the tuning process to diverge. A particular case (Case 3) is shown in Fig.6.18 and Fig 6.19 which shows that, although the sensitivity functions still can be obtained, the output responses do not make sense. The starting point for the tuning operation is too far away, thus the tuning process will be unsuccessful.

- --- height rate, -- --- pitch attitude, -· --- roll attitude, · --- yaw rate

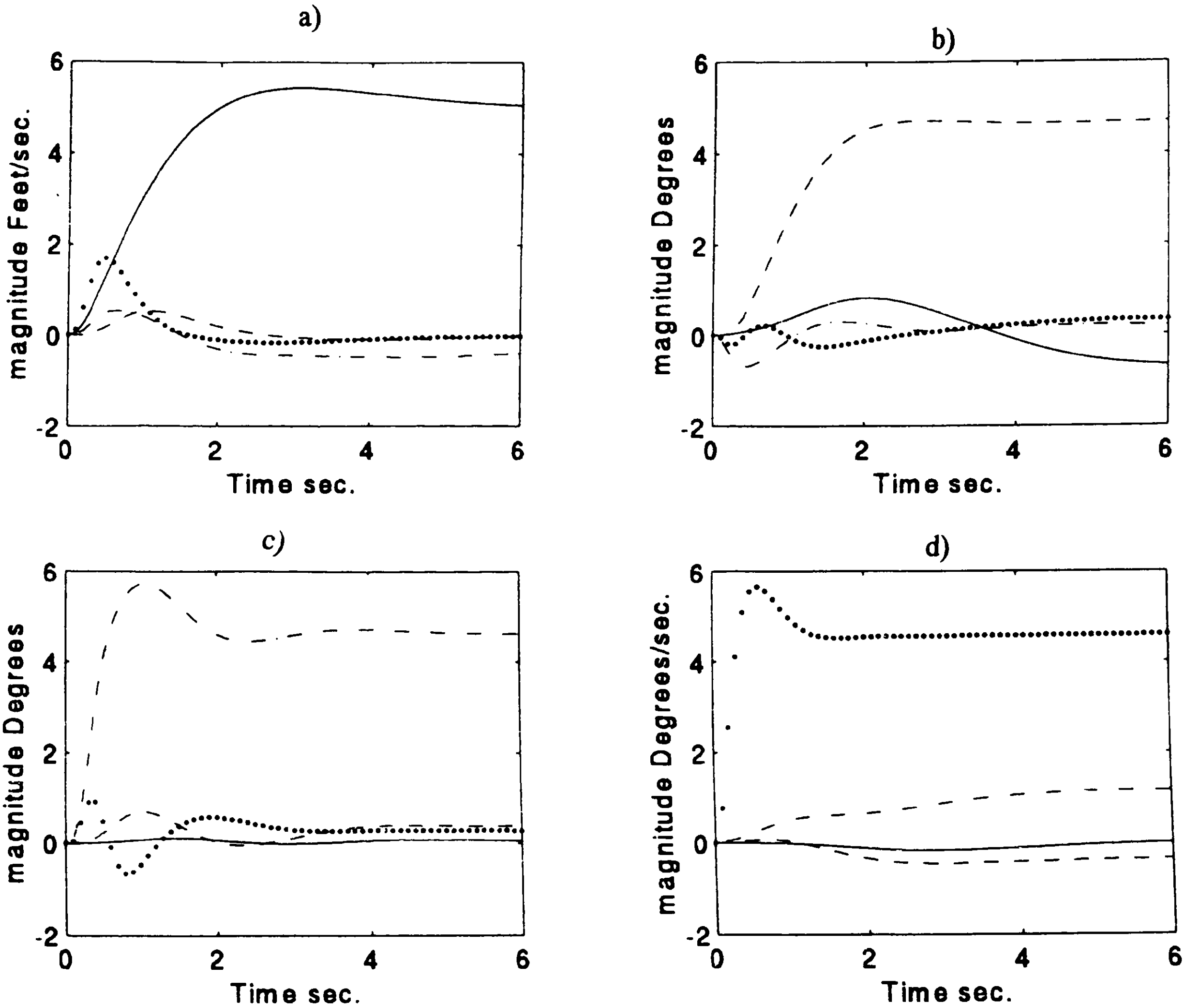


Fig. 6.2 Desired step responses for Case 1

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

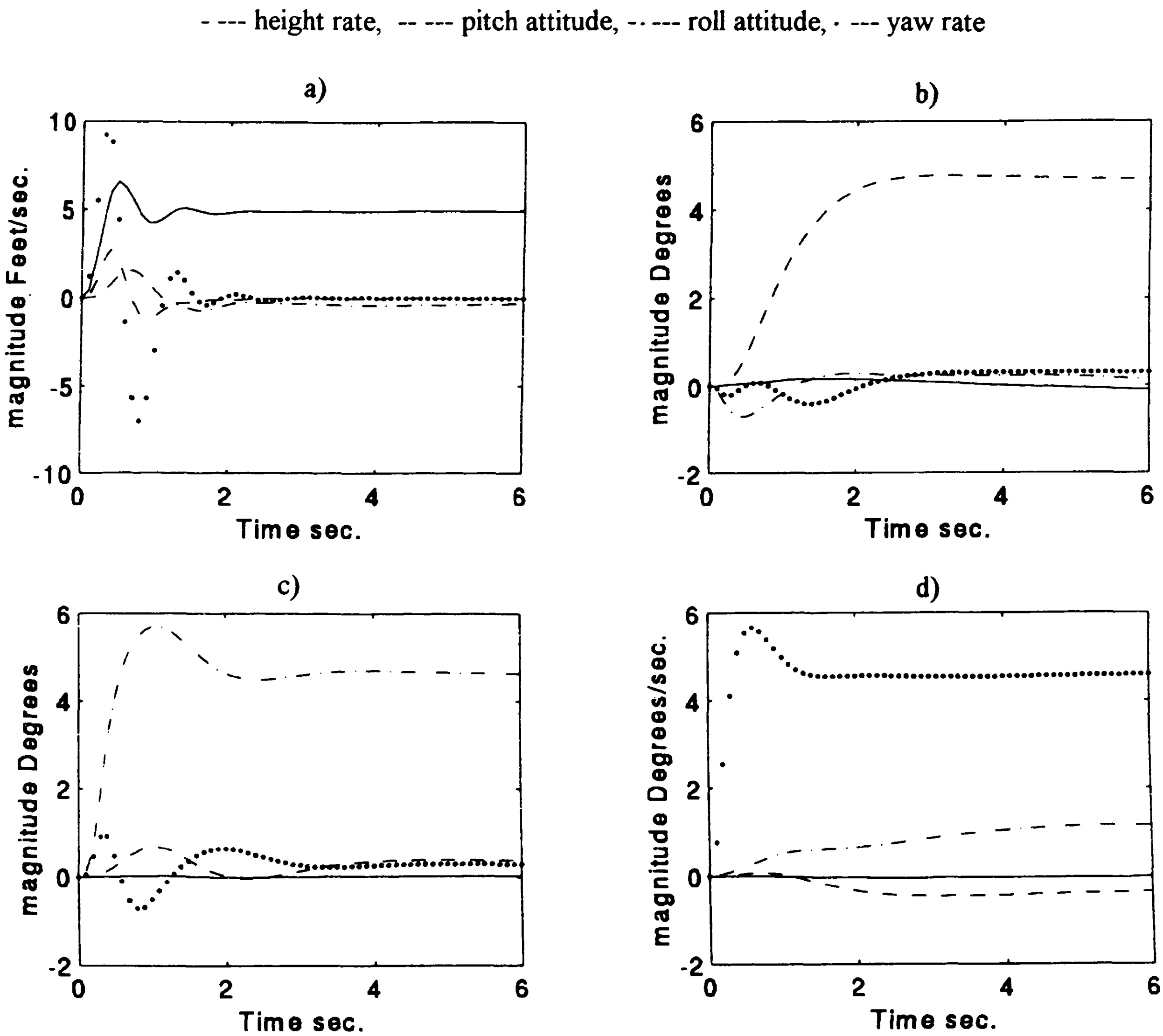


Fig. 6.3 Initial step responses for Case 1

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -· --- roll attitude, · --- yaw rate

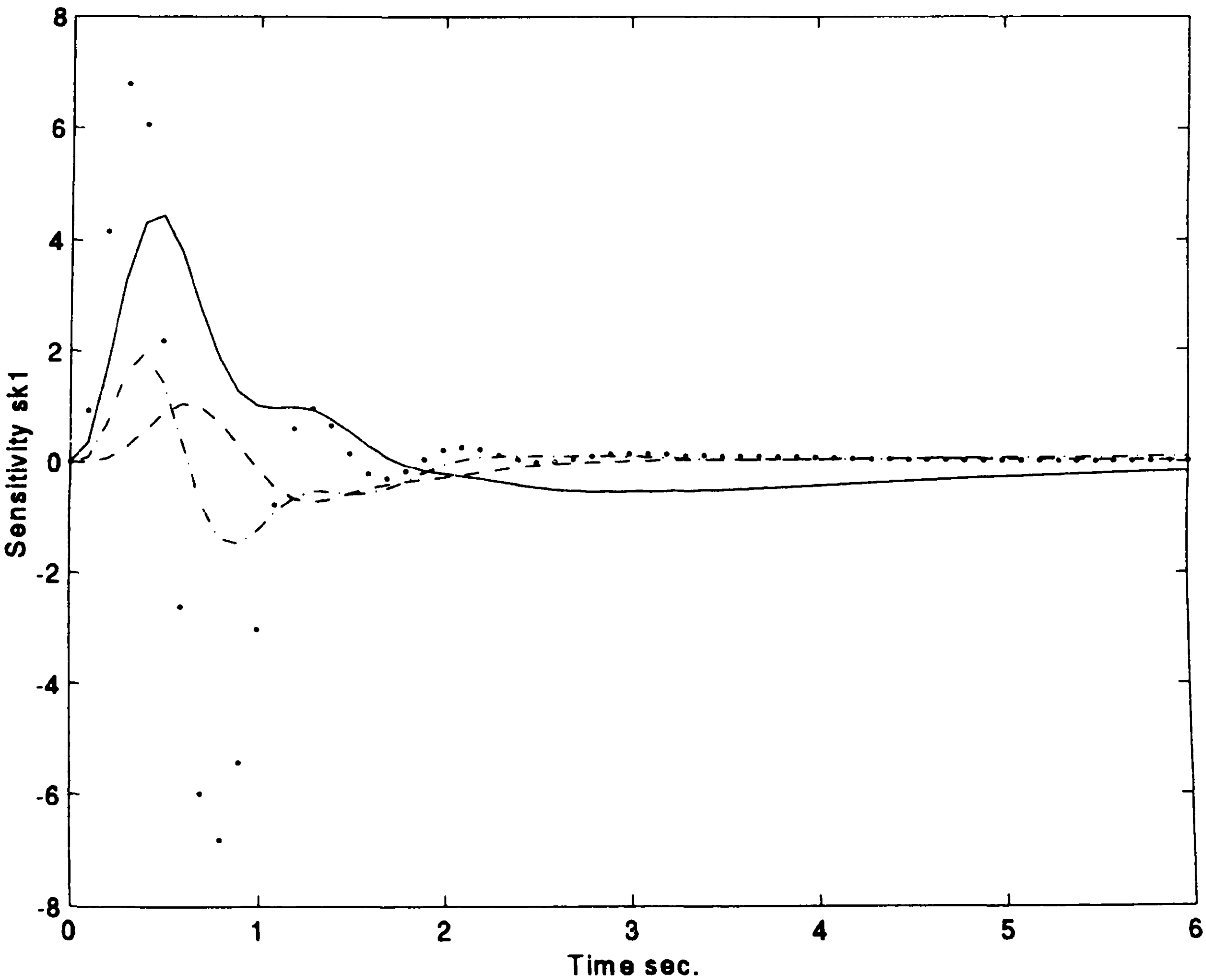


Fig. 6.4 Initial sensitivity for controller parameter K_{11} for Case 1

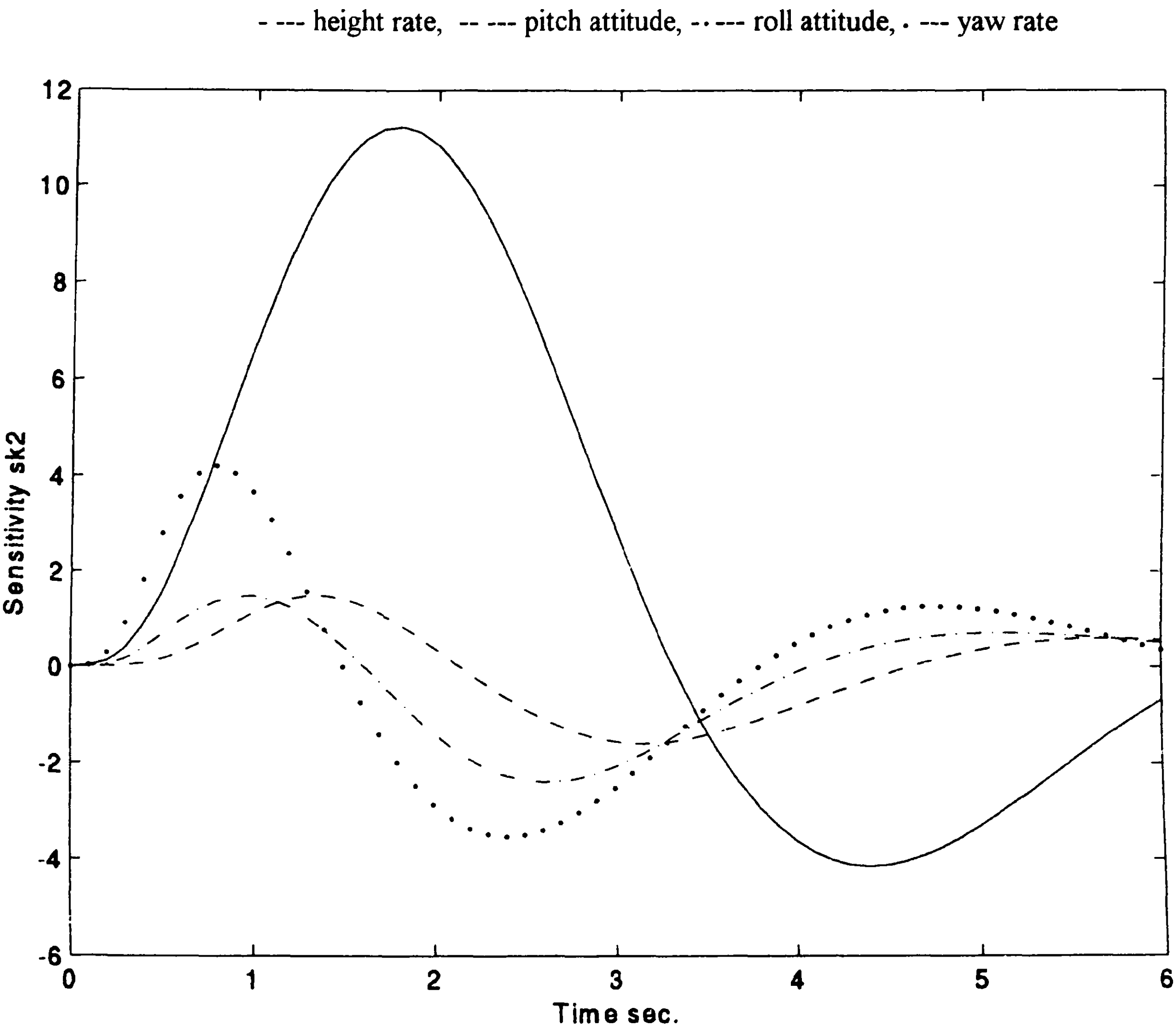


Fig. 6.5 Initial sensitivity for controller parameter K_{12} for Case 1

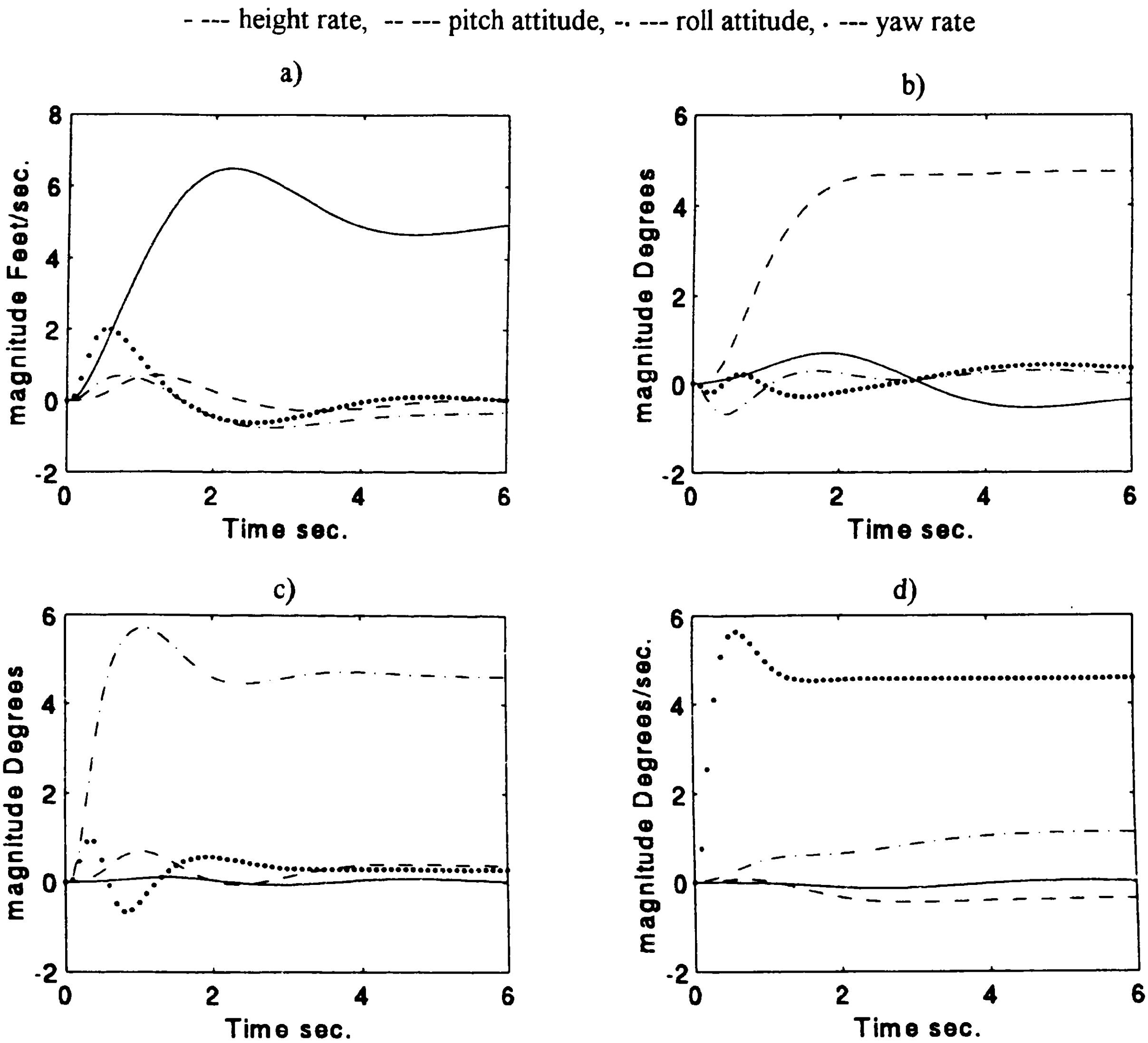


Fig. 6.6 Tuning results of one iteration for Case 1 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -. --- roll attitude, . --- yaw rate

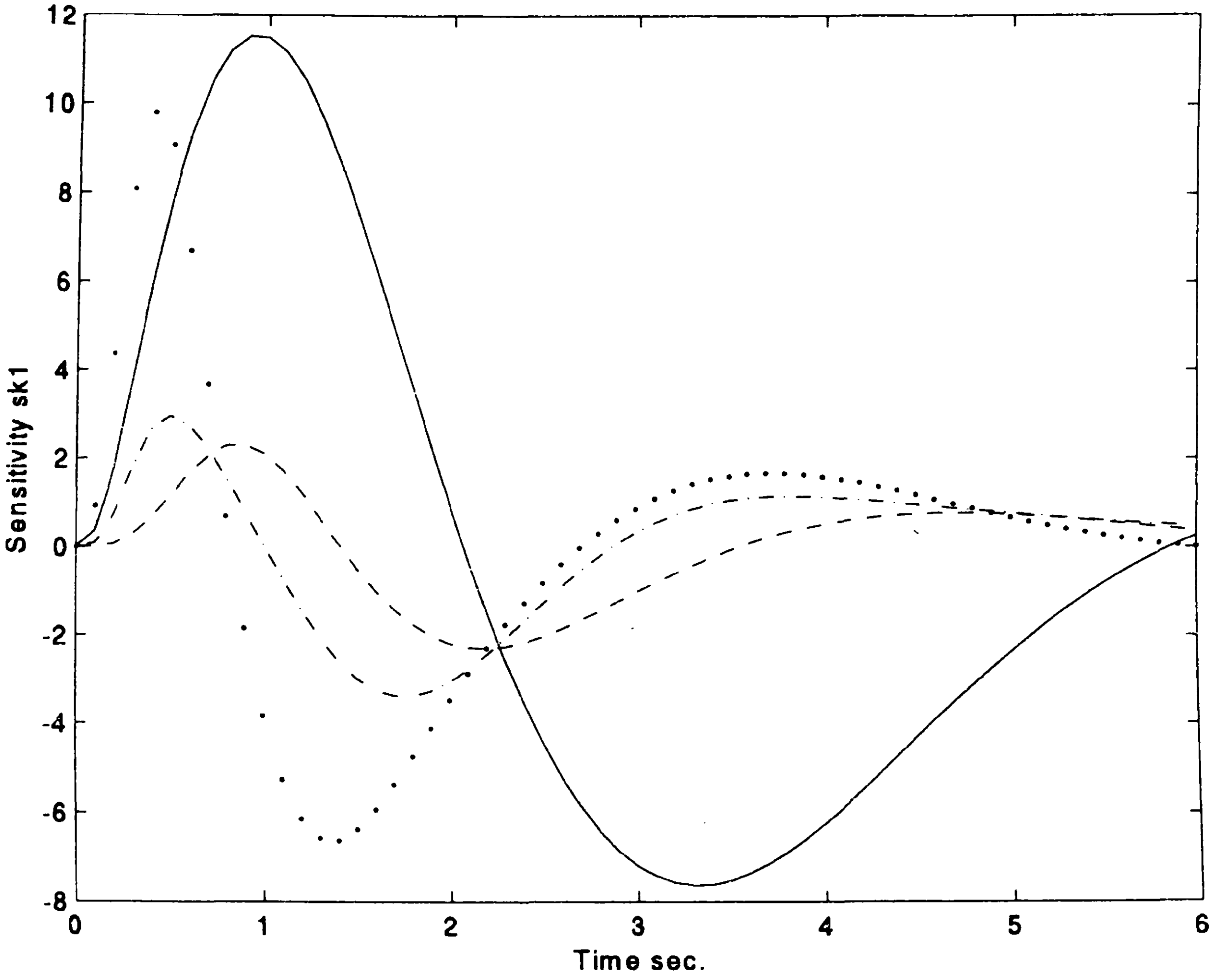


Fig. 6.7 Sensitivity functions for controller parameter K_{11} for Case 1 (one iteration)

- --- height rate, -- --- pitch attitude, -. --- roll attitude, . --- yaw rate

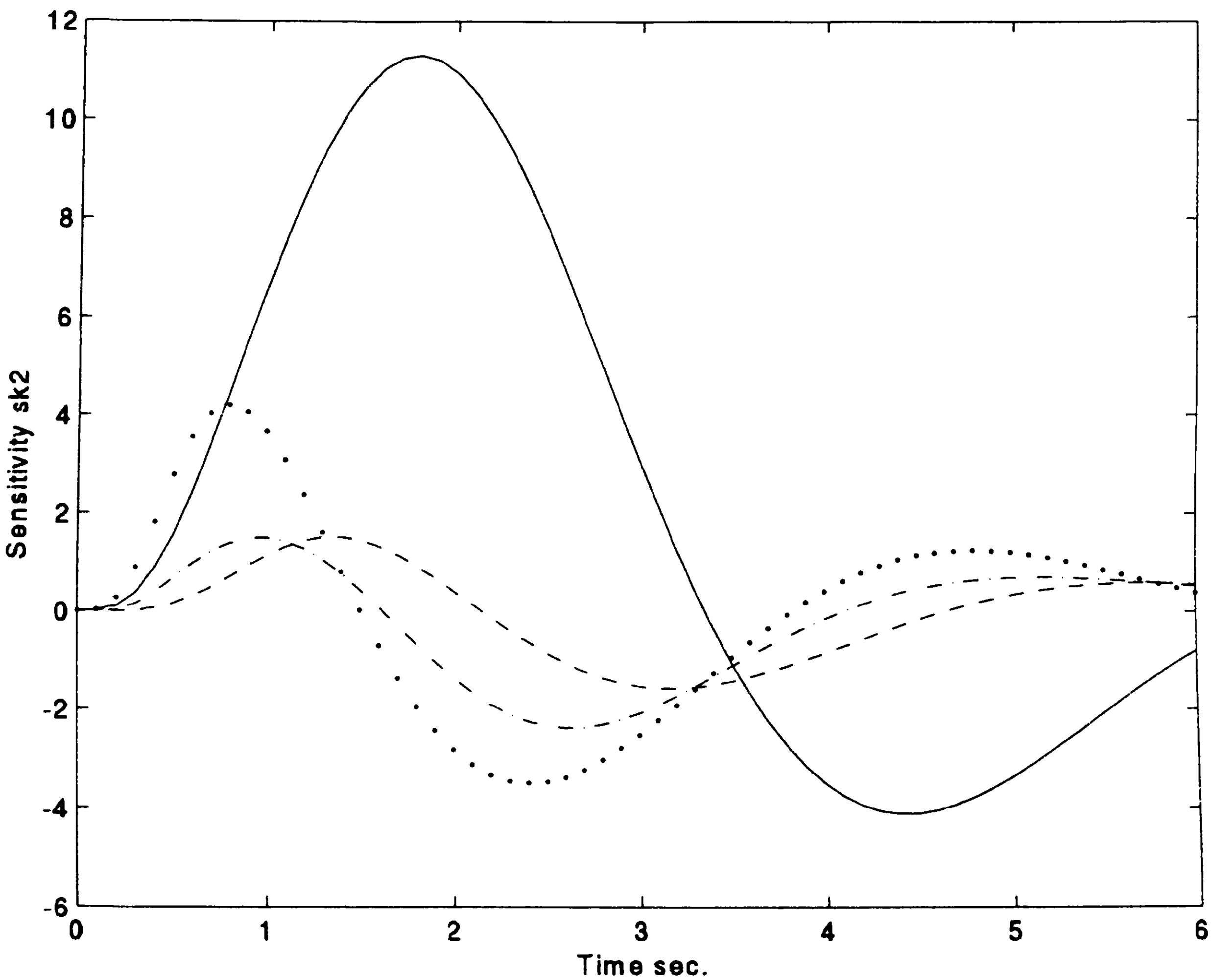


Fig. 6.8 Sensitivity functions for controller parameter K_{12}

for Case 1 (one iteration)

- --- height rate, -- --- pitch attitude, -. --- roll attitude, . --- yaw rate

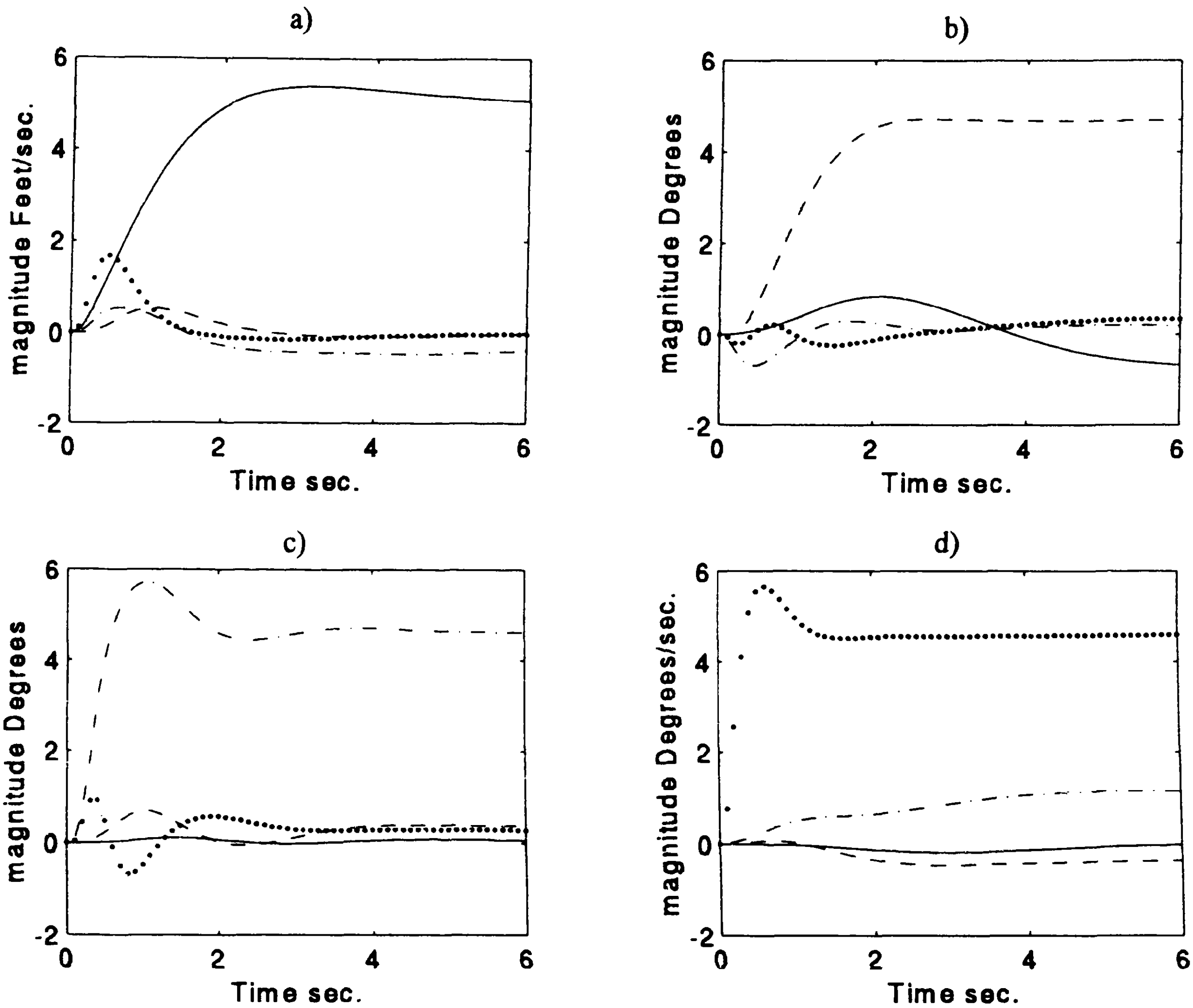


Fig. 6.9 Tuning results of two iterations for Case 1 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -· --- roll attitude, · --- yaw rate

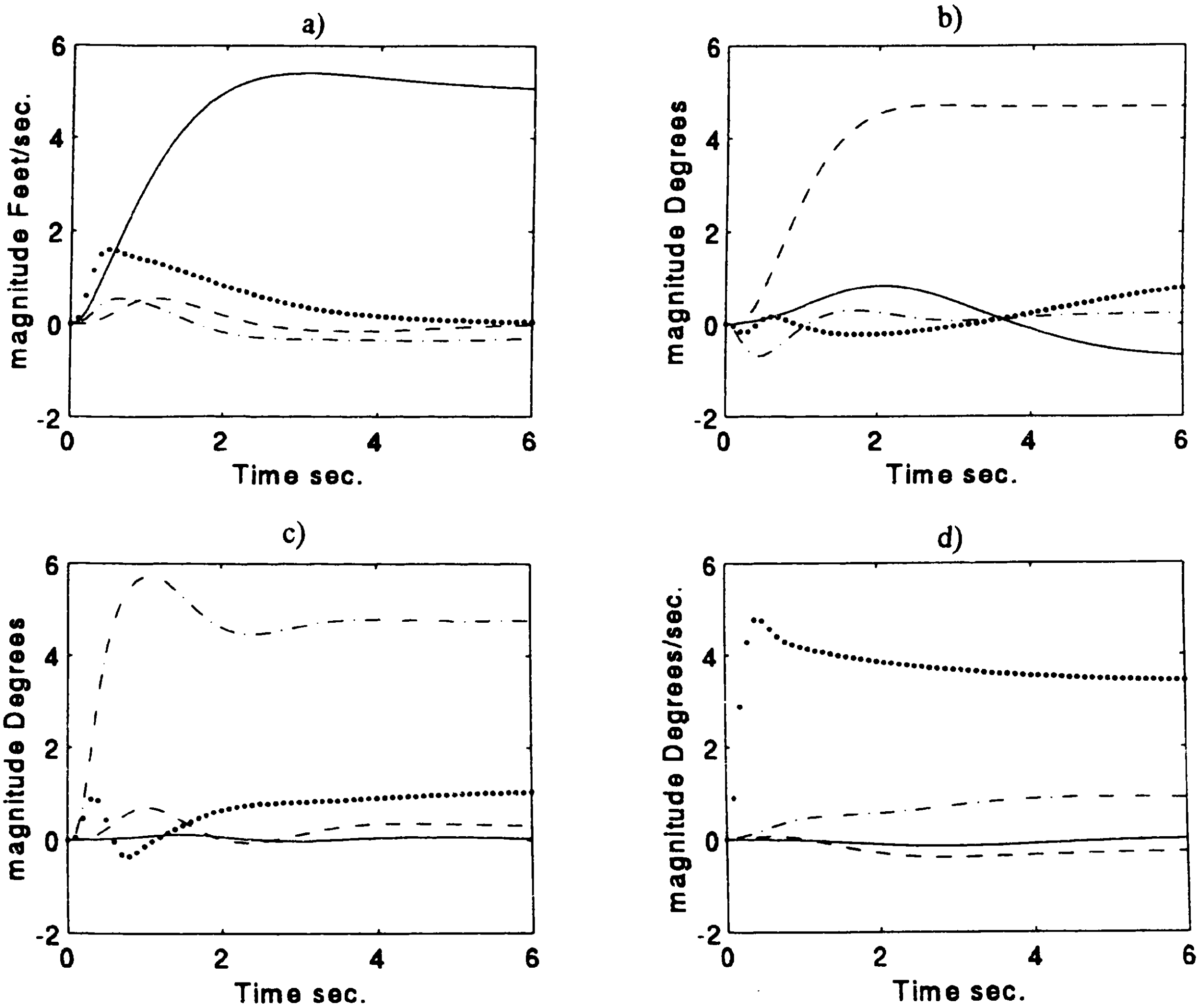


Fig. 6.10 Initial step responses for Case 2

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -· --- roll attitude, . --- yaw rate

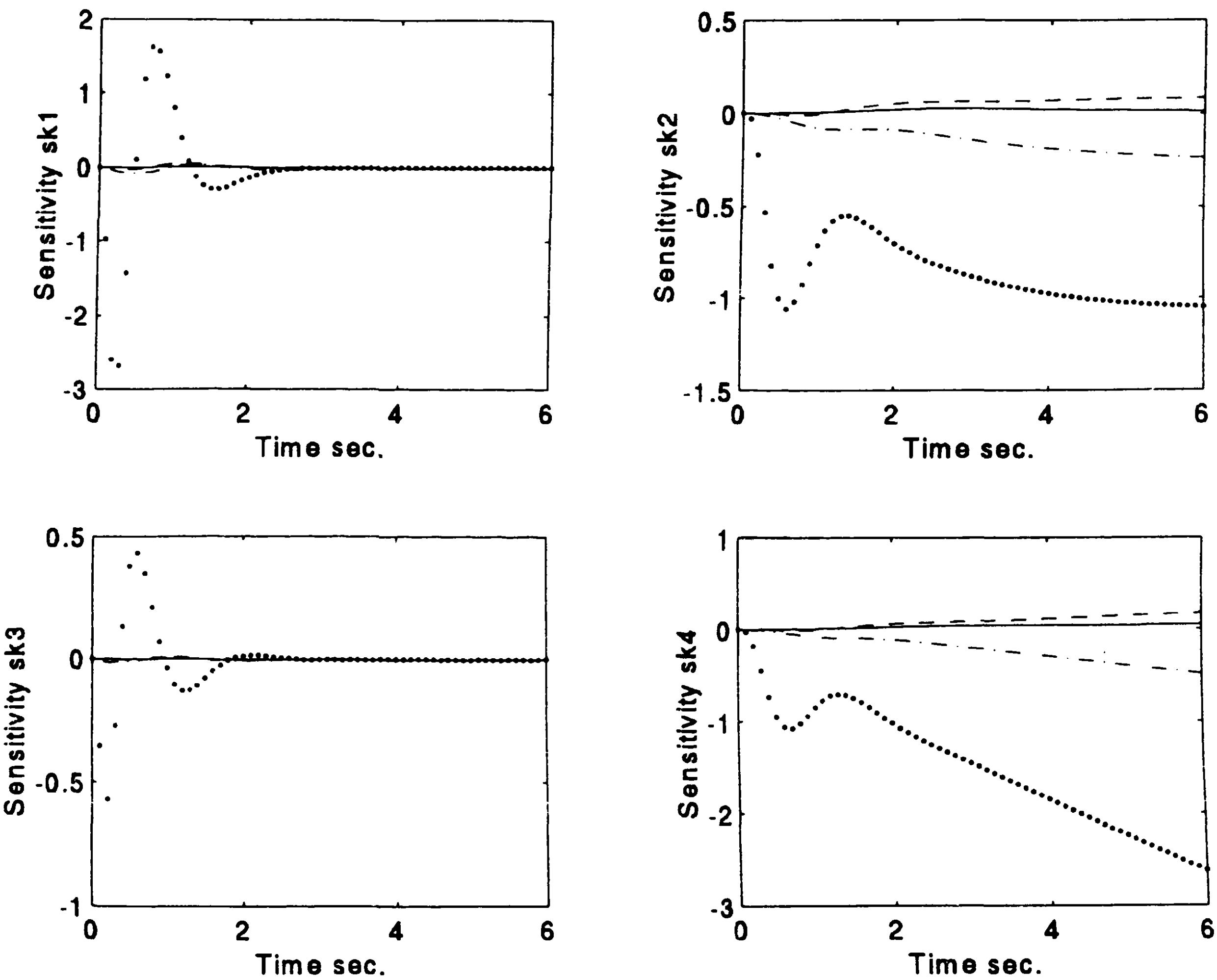


Fig. 6.11 Initial sensitivity for controller parameters

K_{41}, K_{42}, K_{43} & K_{44} for Case 2

- --- height rate, -- --- pitch attitude, -. --- roll attitude, . --- yaw rate

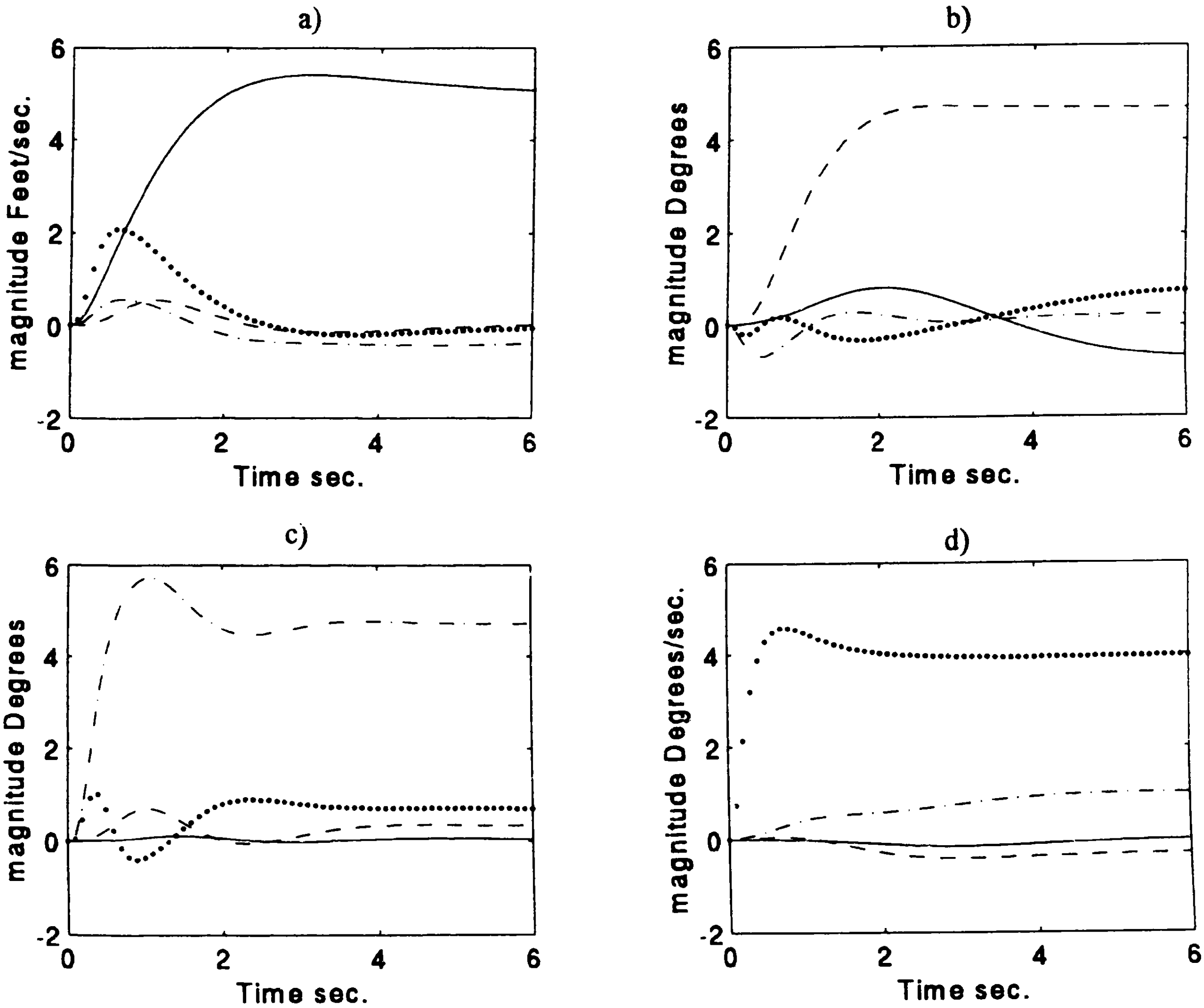


Fig. 6.12 Tuning results of one iteration for Case 2 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -. --- roll attitude, . --- yaw rate

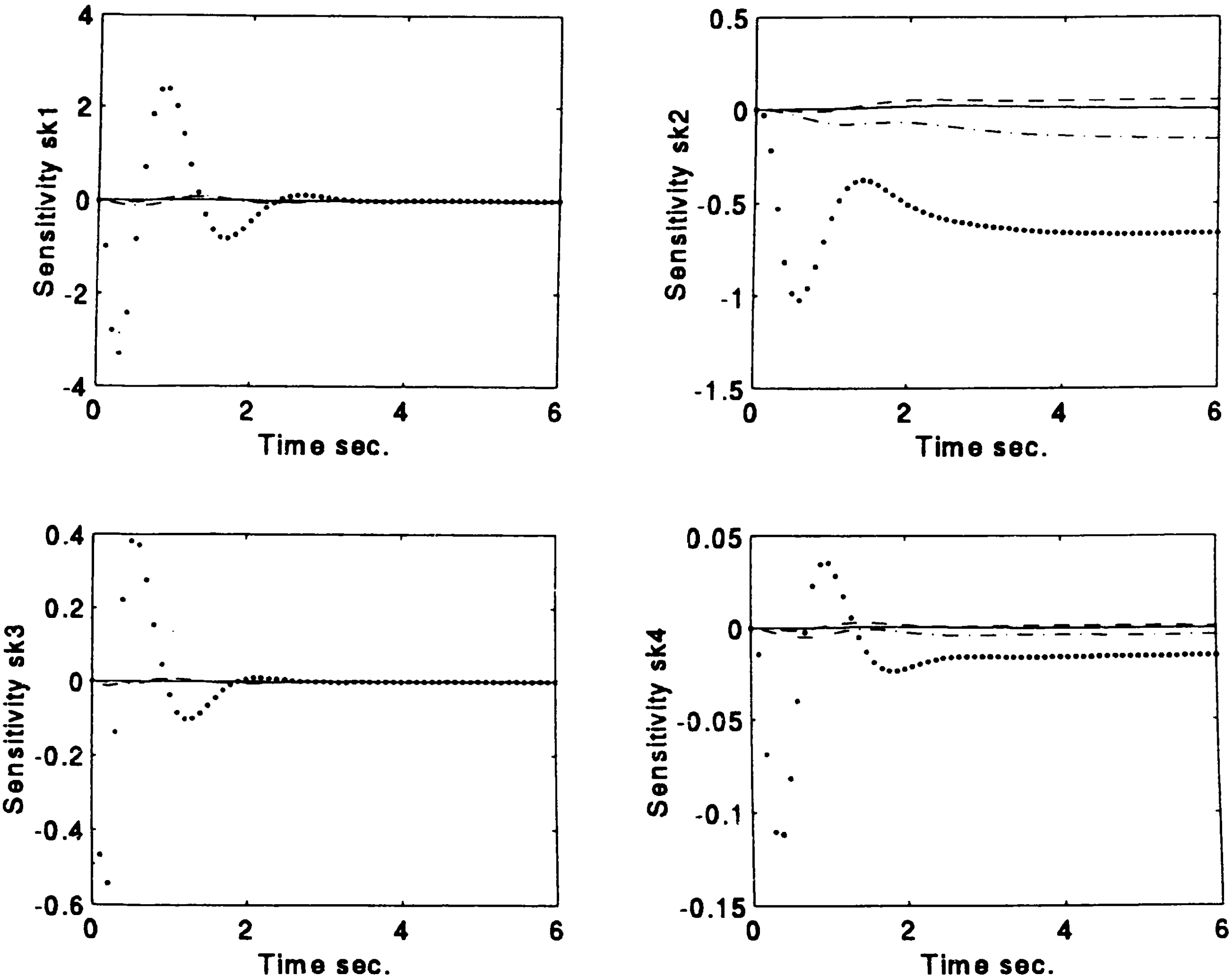


Fig. 6.13 Sensitivity functions for controller parameters

K_{41}, K_{42}, K_{43} & K_{44} for Case 2 (one iteration)

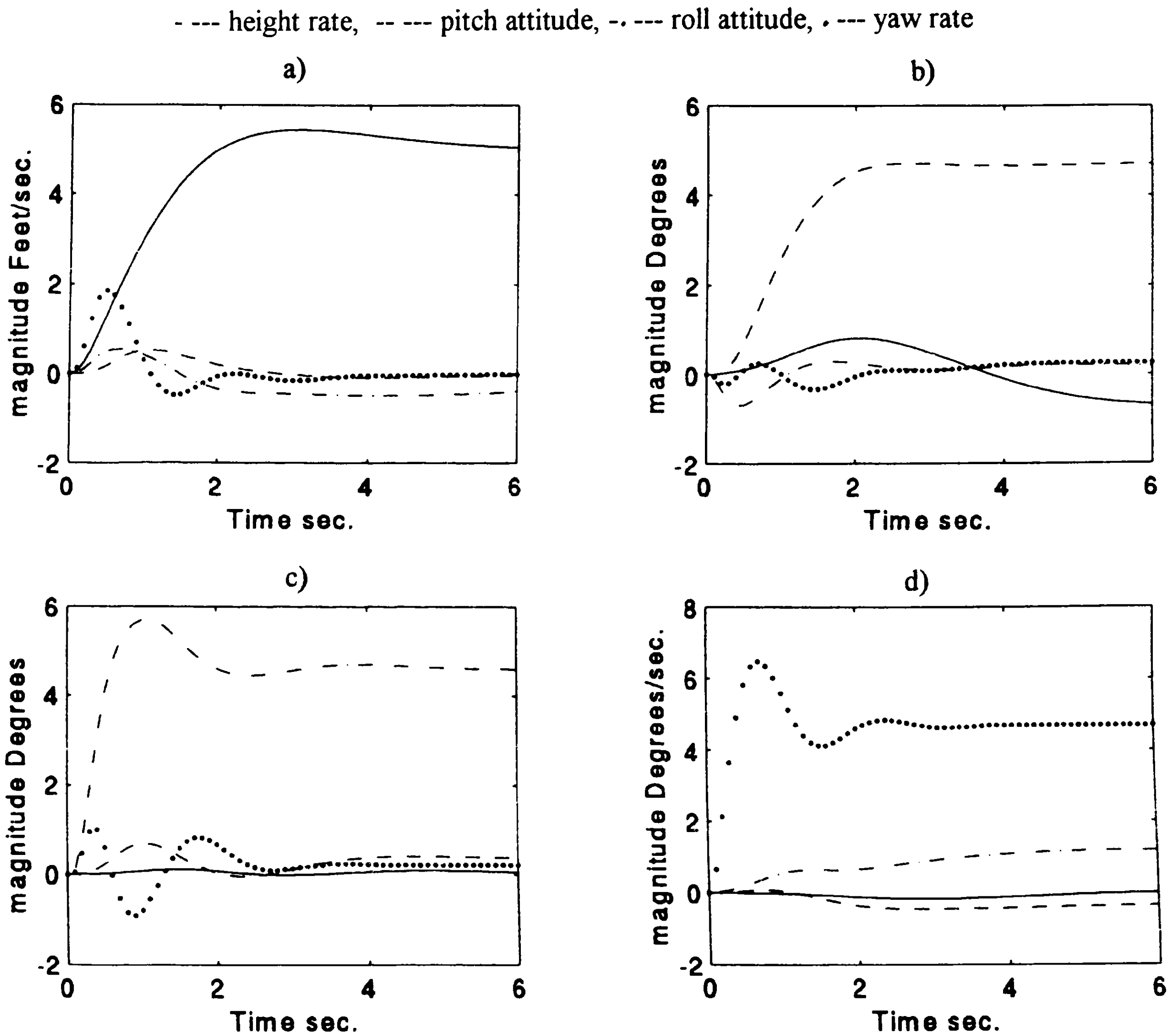


Fig. 6.14 Tuning results of two iterations for Case 2 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

- --- height rate, -- --- pitch attitude, -· --- roll attitude, · --- yaw rate

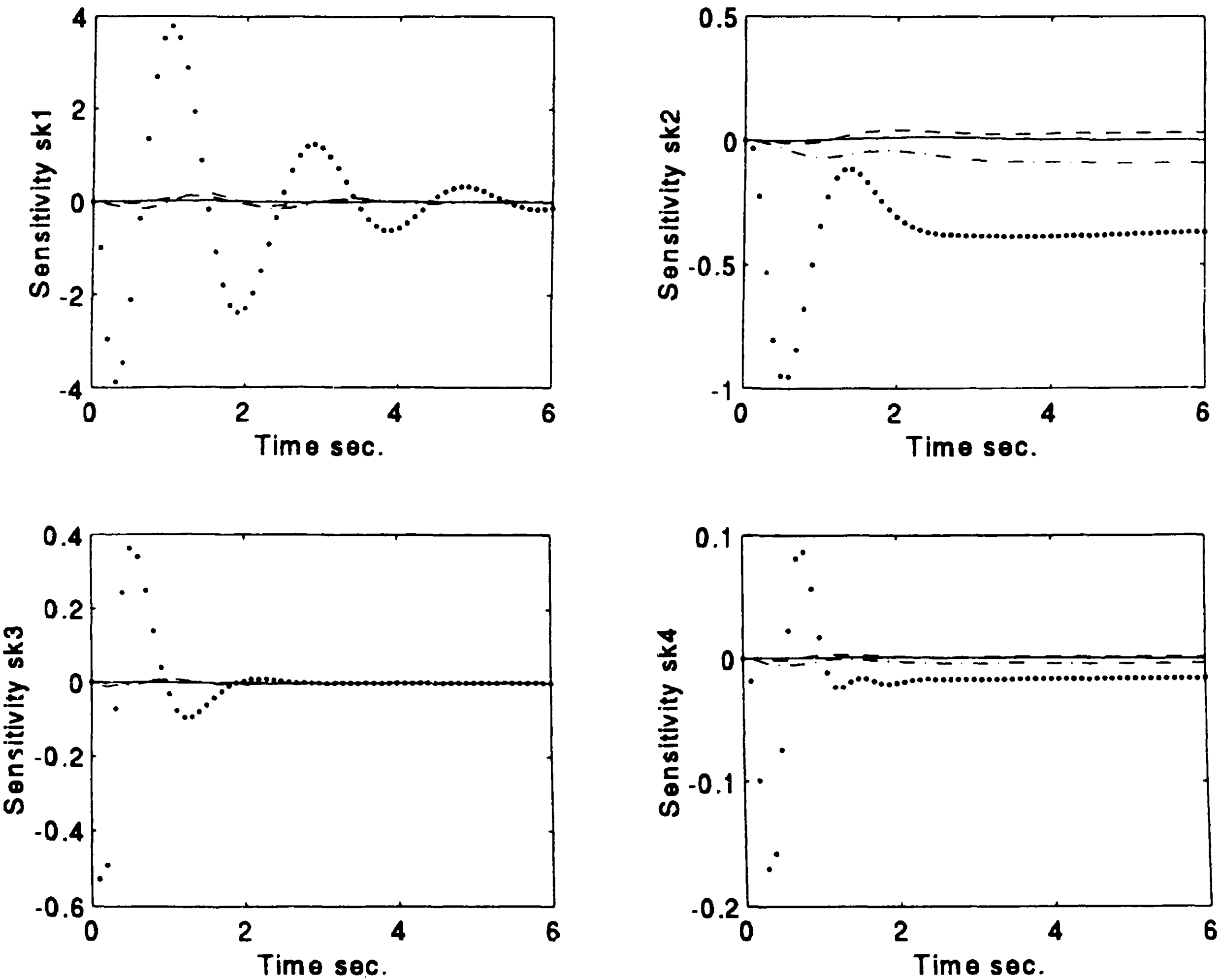


Fig. 6.15 Sensitivity functions for controller parameters

K_{41}, K_{42}, K_{43} & K_{44} for Case 2 (two iterations)

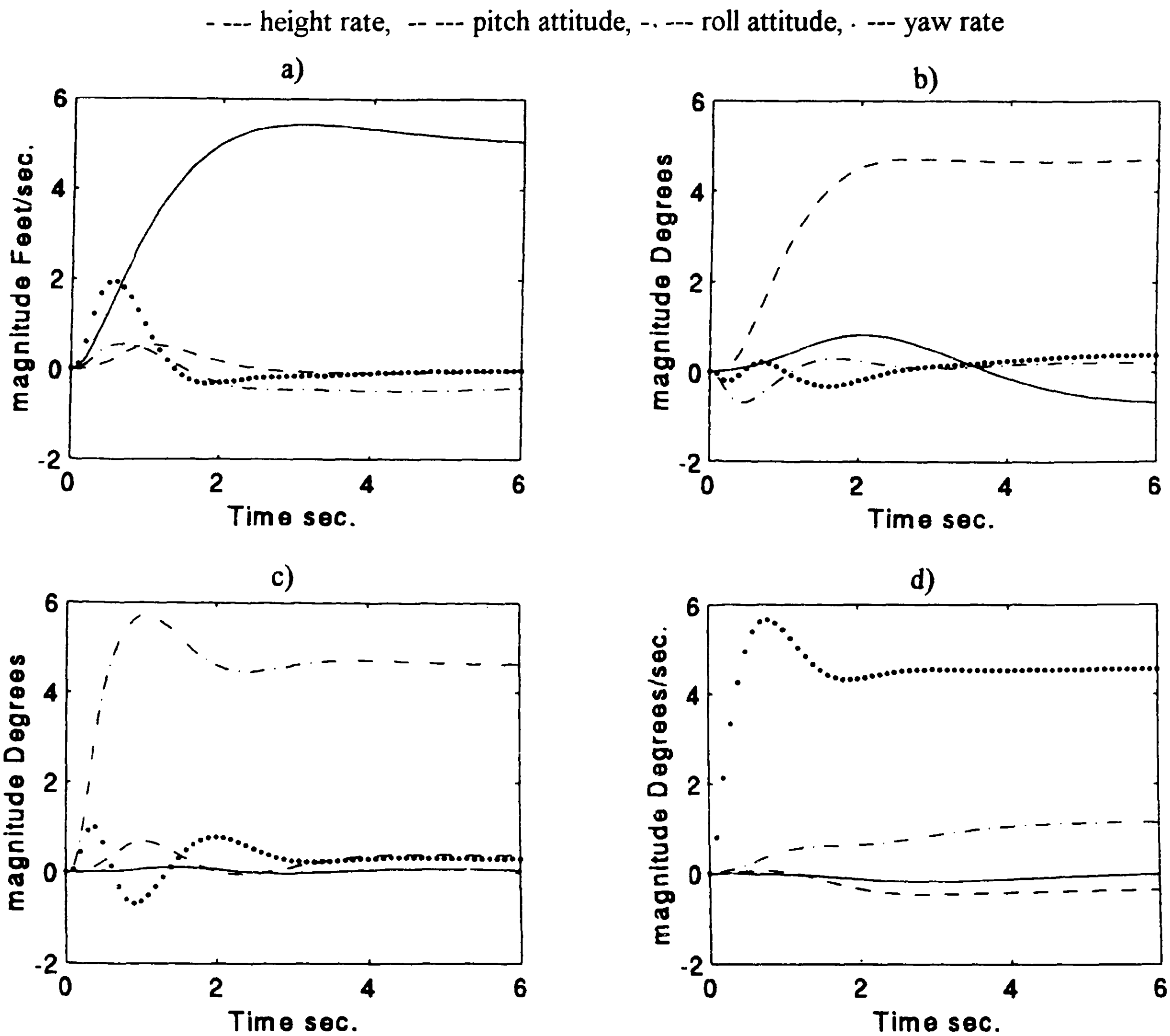


Fig. 6.16 Tuning results of three iterations for Case 2 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

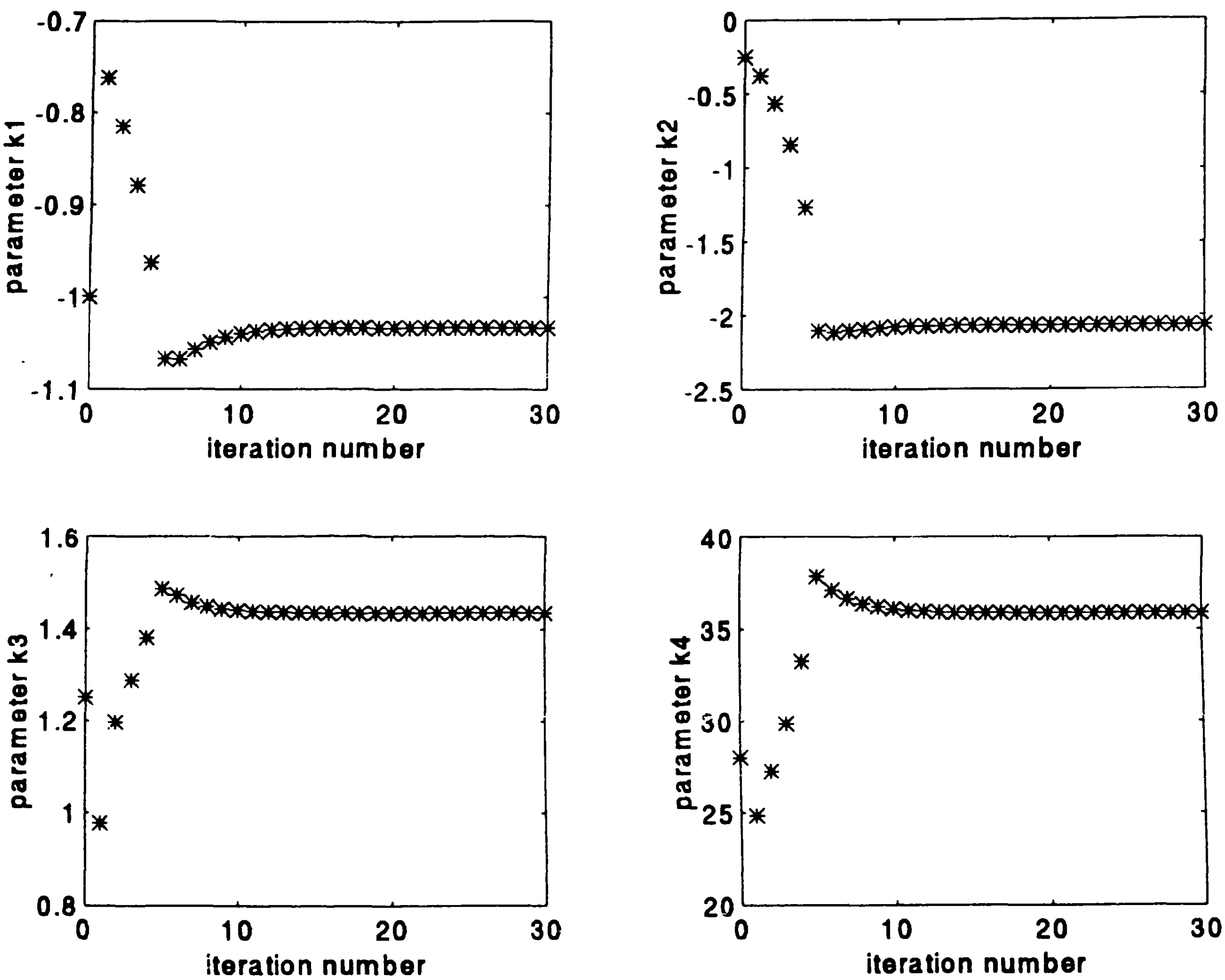


Fig. 6.17 Iteration of the controller parameters K_{41}, K_{42}, K_{43} & K_{44} using convolution approach for Case 2

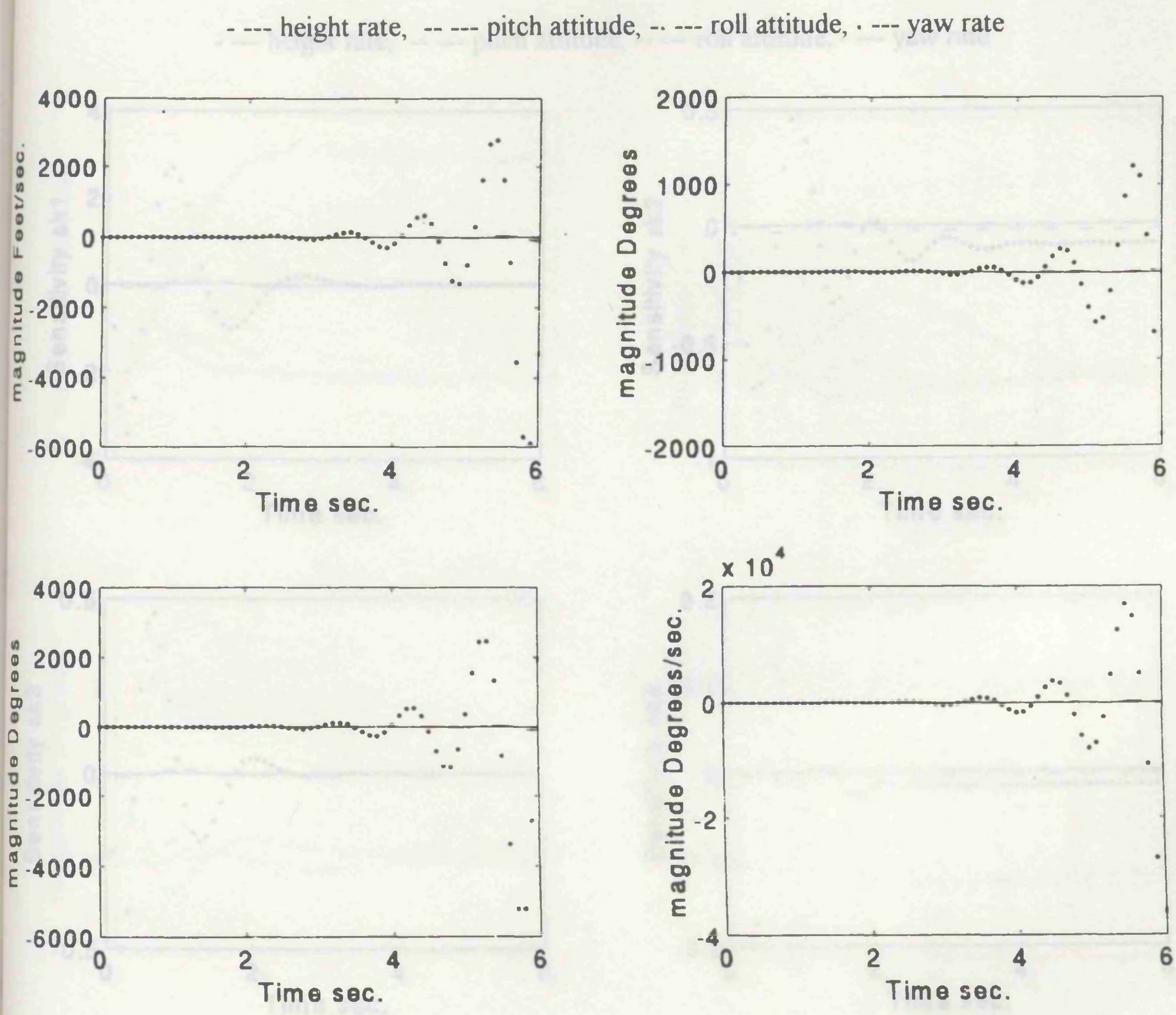


Fig. 6.18 Initial step responses for Case 3

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

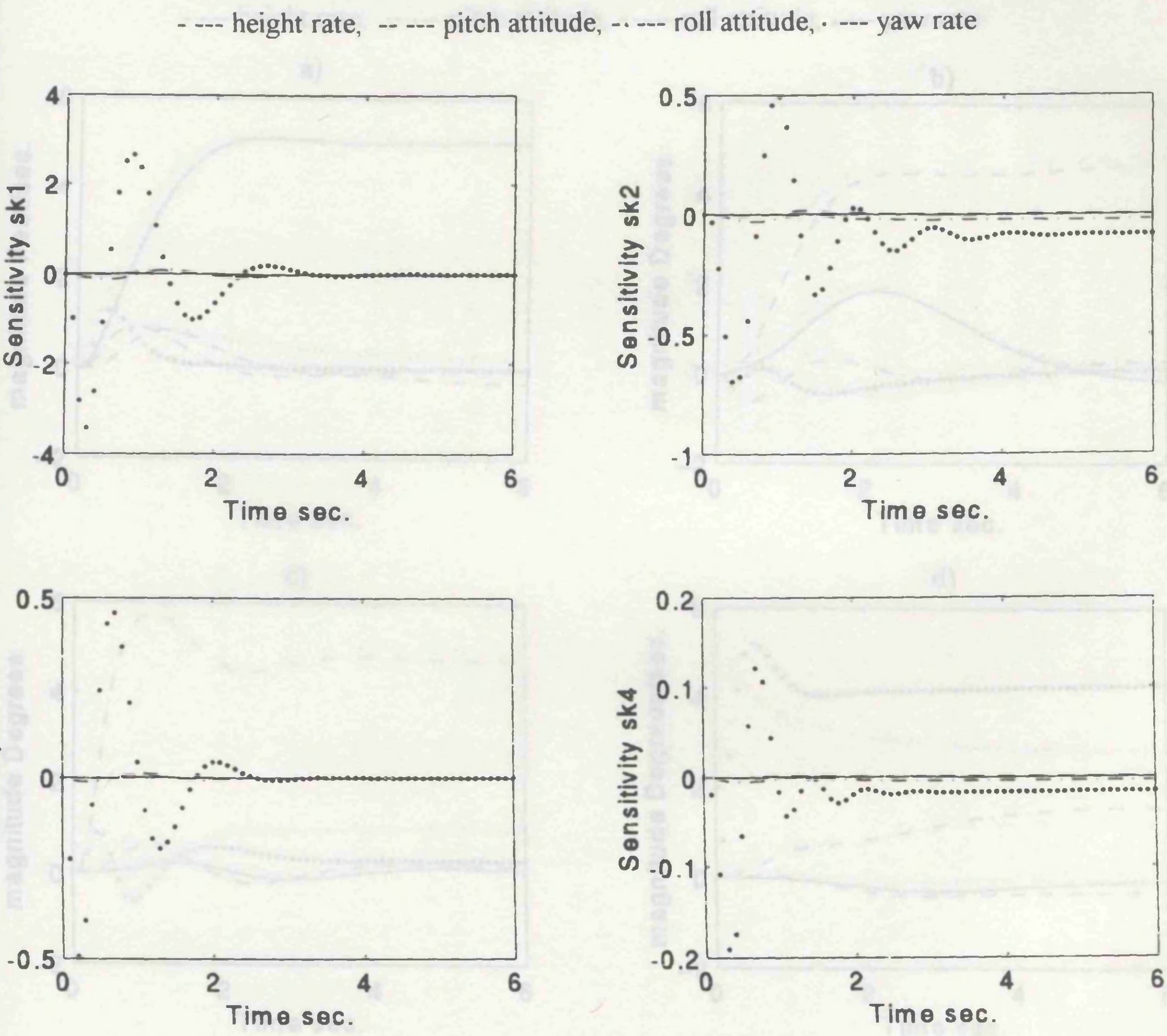


Fig. 6.19 Sensitivity functions for controller parameters

K_{41}, K_{42}, K_{43} & K_{44} for Case 3

- --- height rate, -- --- pitch attitude, -·--- roll attitude, · --- yaw rate

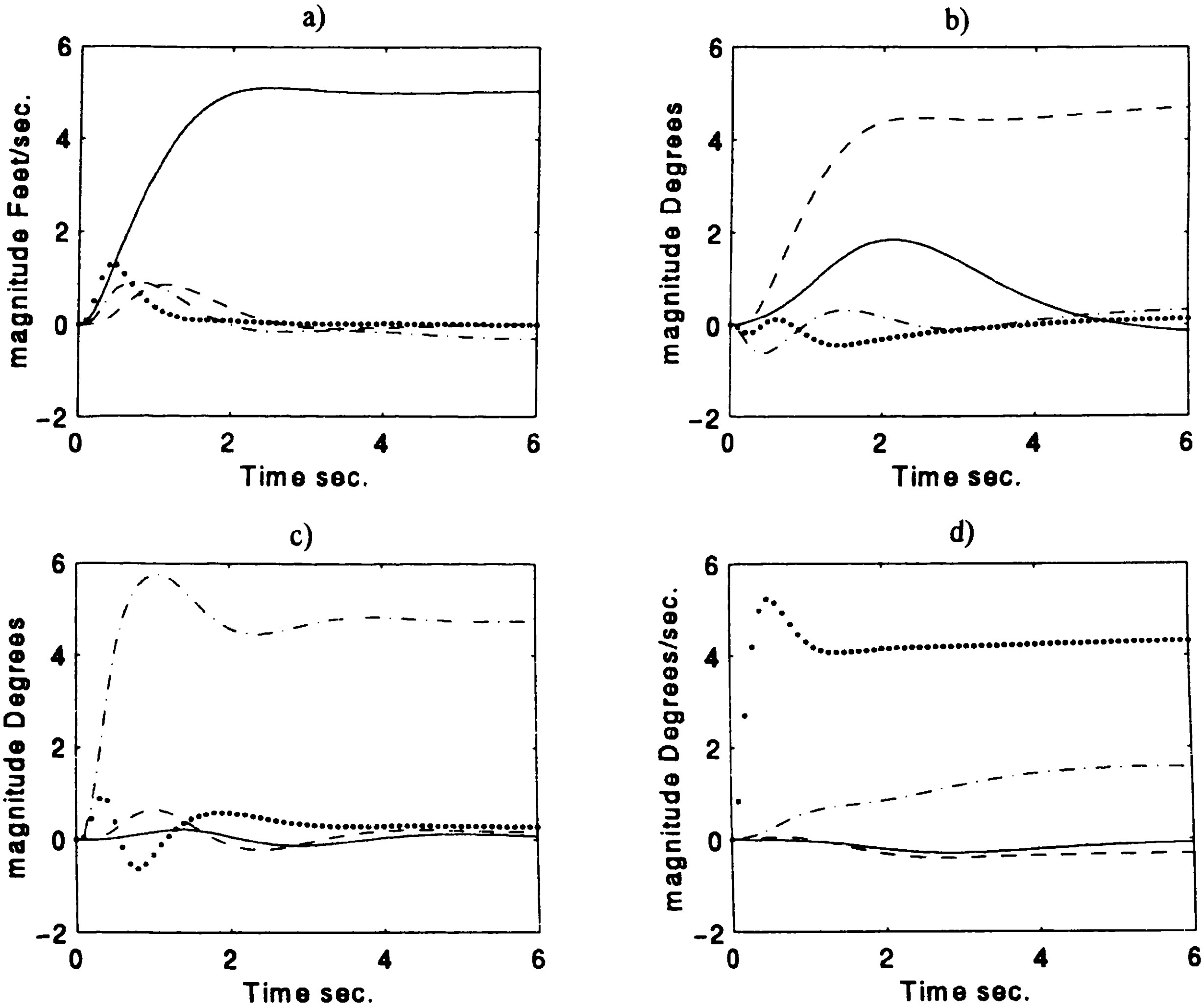


Fig. 6.20 Desired step responses for Case 4

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

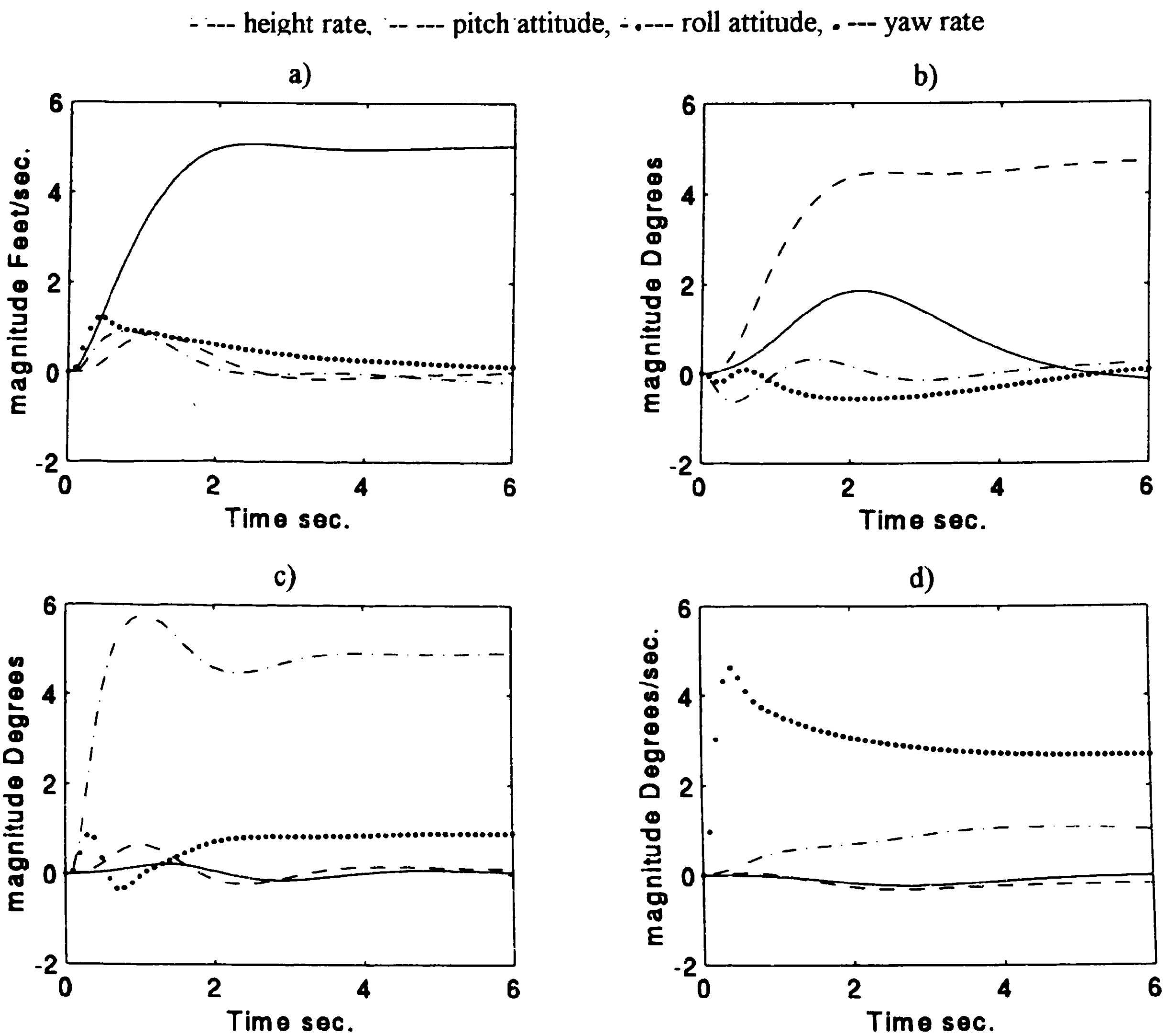


Fig. 6.21 Desired step responses for Case 4

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

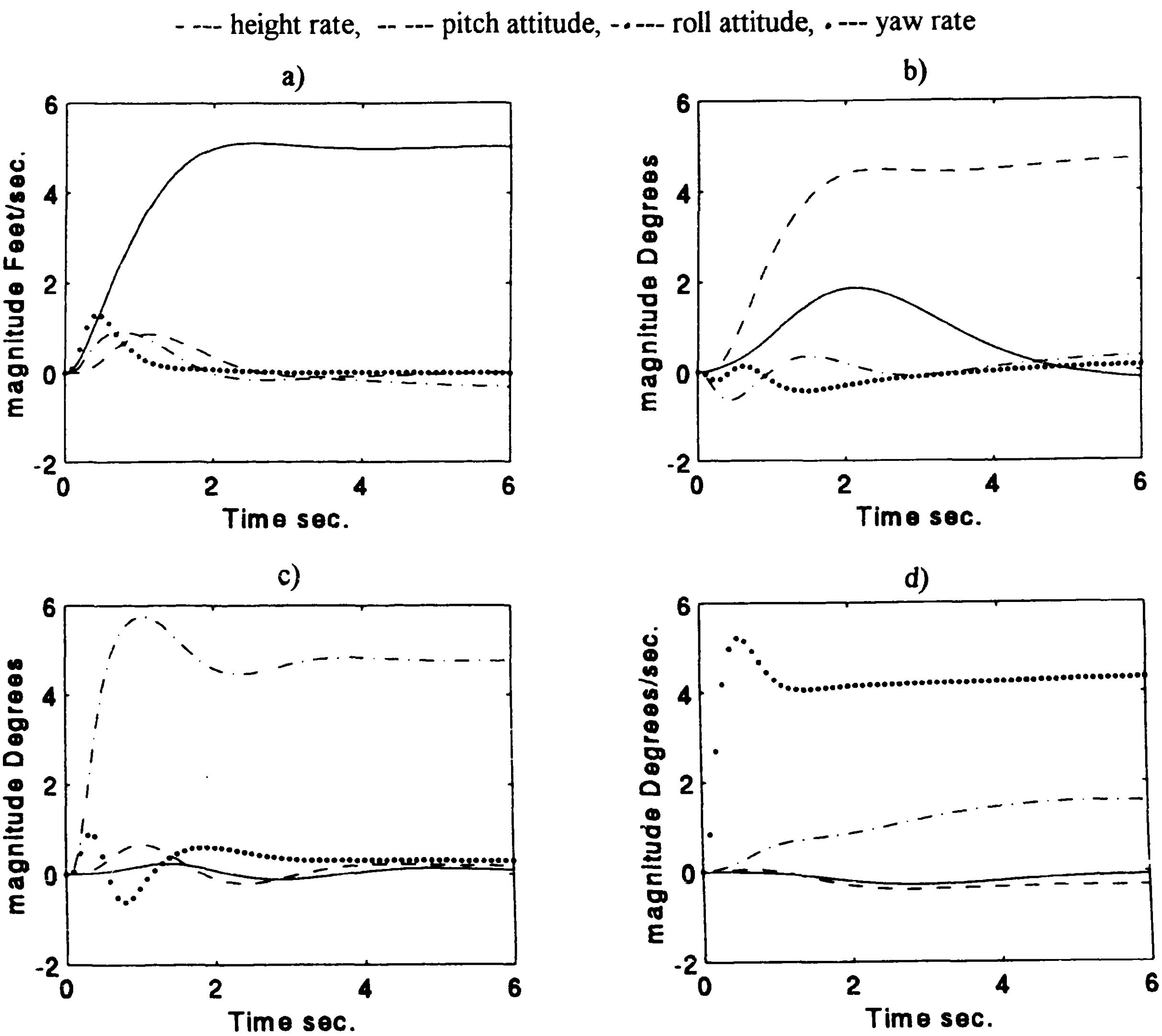


Fig. 6.22 Tuning results of two iterations for Case 4 (step responses)

a) Height rate step responses

b) Pitch attitude step responses

c) Roll attitude step responses

d) Yaw rate step responses

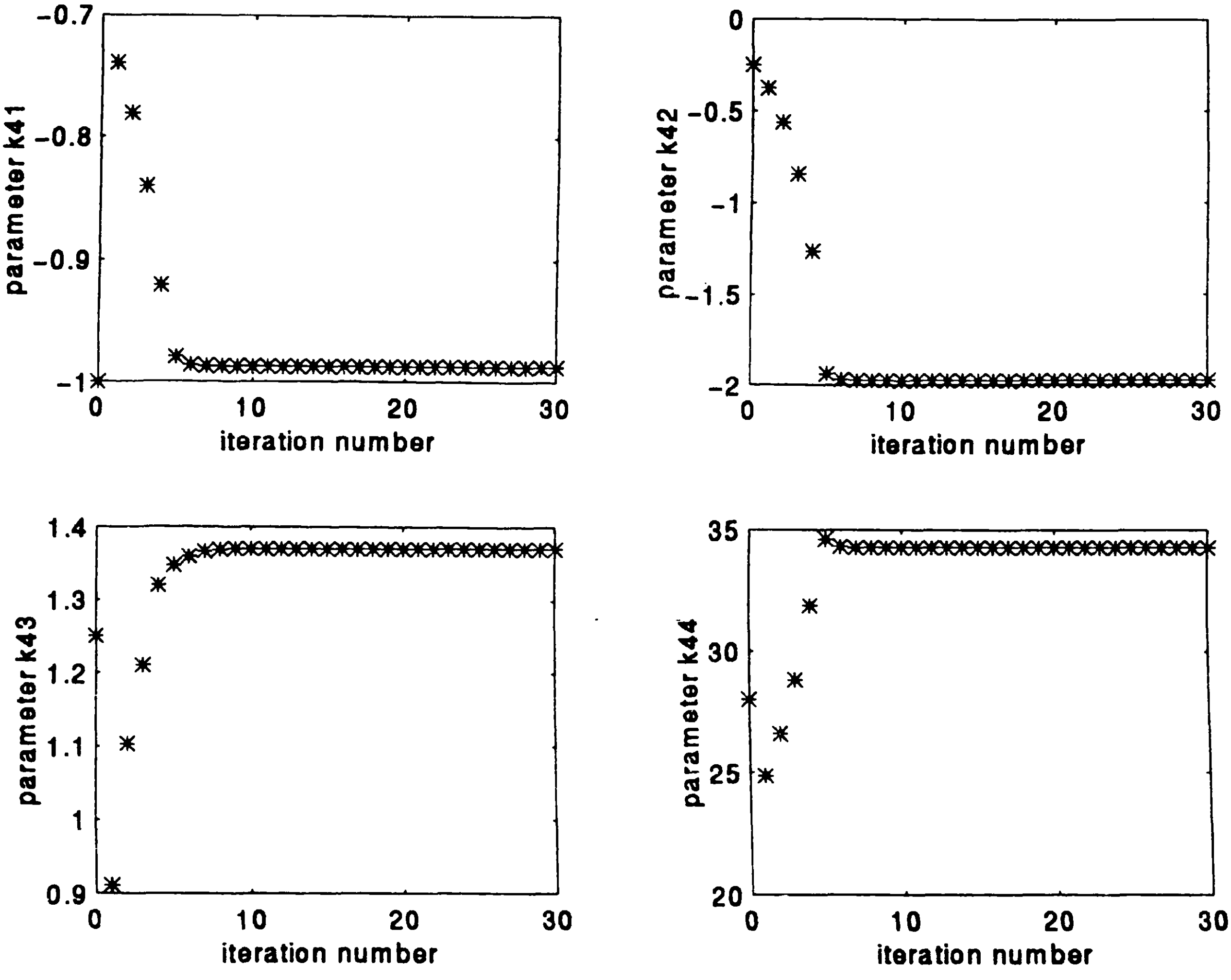


Fig. 6.23 Iteration of the controller parameters K_{41}, K_{42}, K_{43} & K_{44} using convolution approach for Case 4

CHAPTER 7

Discussion and Conclusion

7.1 Conclusions and Recommendations Concerning the Tuning Process

The sensitivity approach to controller tuning is not new. Applied initially to specialised applications in the electrical generation field the technique has now been shown to have much wider applicability. The tuning of the controller parameters for more general forms of control system may be carried out easily and efficiently through sensitivity analysis of the system response. The approach described in this thesis is significantly different from self-tuning control since it is not carried out automatically in an on-line fashion. The controller parameter sensitivity functions may be generated conveniently using a convolution calculation or a two-stage approach in which convolution is implicit. These approaches require, at most, only simple filtering of the error and output signals and are independent of the plant model. This approach has been shown to give the correct estimates of the parameter sensitivity functions for a closed loop system in both the time and frequency domains. The signal convolution method has also been shown to be an efficient and accurate technique for the generation of sensitivity functions for multivariable closed loop control systems. A parameter perturbation method with a very small parameter perturbation is used as a reference when assessment of these novel methods for sensitivity function calculation is carried out. The work which has been presented in this thesis attempts to extend the ideas to provide a coherent and generally applicable set of tools to :

1) allow more general forms of test input

In the current work, three test input signals, a step signal, an impulse signal and a pseudo random binary sequence, have been considered. The size of the step signal influences the sensitivity function of the controller parameters. On the other hand the step test input is easy to implement and is a widely used test signal. The unit impulse test input still can be used for some cases since it can be approximated by a pulse of unit area having finite amplitude and duration. For PRBS test signals, the length, amplitude C, time interval Δt and period of the PRBS taken will affect the sensitivity functions in the time domain.

In the frequency domain extended PRBS test inputs have been investigated. These signals have some special properties which allow them to be used as an approximation to white noise : a) same autocorrelation function and spectral properties, b) easier than white noise to apply as a test signal since it involves a binary signal. c) because of the fact that correlation and spectral analysis calculations can be carried out in situations where signal to noise ratios are relativity poor compared with those acceptable for direct analysis of transient response data, these PRBS signals may provide benefits in that satisfactory results may be obtained for smaller test signal amplitudes.

The novel two-stage approach described in Chapter 3 has particular attractions in that any form of test input may be used in the two-stage approach. Particular emphasis has been placed on step inputs in the work described in this thesis but this new approach can be used for many other forms of input signal.

2) allow application to multi-input multi-output systems

The tuning technique has been applied to practical Multi-Input Multi-Output control system problems where initial design was carried out using the ICAD approach. The tuning technique has been applied to the two particular multivariable examples, one of which is a real two-tank liquid level system implemented in the laboratory and the second involves a more theoretical study based on a linearised control model of a typical combat rotorcraft trimmed to 30 knots and 50 knots forward flight.

3) provide a basis for the practical application of frequency domain methods

The direct convolution approach using the FFT has been employed through the measurements of the system output y(t) and the filter output z(t) for generating sensitivity functions in the frequency domain and the tuning process has been realised satisfactorily using a least squared approach in the frequency domain.

4) allow digital controller tuning

It has been shown that the sensitivity functions can be generated satisfactorily by using the direct convolution approach for a digital control system. If the input signal is a unit step the sensitivity functions of the controller parameters for a digital control system may be obtained by the series method. Based on the sensitivity functions, the controller parameter tuning process can be applied to the digital control system in exactly the same way as in the case of continuous systems.

7.2 Evaluation of the Approach in a Practical Application

An important part of the current work is a practical application of the approach involving a real process on the two-tank system with measurement noise, actuator and sensor nonlinearities and other process uncertainties. The evidence from this application suggests that satisfactory tuning can be achieved in practical systems although the accuracy of the sensitivity functions are influenced by noise and nonlinearities.

7.2.1 Noise Influence on the Sensitivity Function from the Measurements

A important point is the fact that measured data from real systems, which are the basis of the theory of this approach, can have very noisy characteristics. Noise on the filter response z(t) can have an important influence on the sensitivity function because the signals z(t) are so small. The noise is often much bigger than the signal z(t), especially as the closed loop system error tends towards a zero value and thus the sensitivity functions of the controller parameters can show errors, both in the time domain and frequency domain approaches. Noise issues cause particular concern in the case of the step form of test input with the direct form of convolution calculation. In this case numerical differentiation of one measured signal is necessary and this is clearly a noise-enhancing process. The use of an impulse test signal avoids this problem but has been found to introduce other complicating factors and is a form of test input which is less widely used than step functions. The two-stage approach has potential benefits as it can be used with any form of input and does not involve explicit differentiation of measured signals. There is still some room of further work, although experience with the two tank system suggests that measurement noise is not a major problem, even with the use of step inputs and numerical convolution calculations.

7.2.2 Influences of the System Nonlinearities

Previous work involving tuning techniques based on parameter sensitivity functions has been concerned mainly with linear systems. The two-tank system involves a number of sources of nonlinearity which have an important influence on the behaviour of liquid level control systems applied to it. The plant is inherently non-linear because of the non-linear dependence of flow on liquid depth. Validation studies show that the traditional model based on orifice flow assumptions is of limited value. There are therefore significant uncertainties associated with the mathematical description of this system and these uncertainties relate mainly to the terms involving non-linear functions. Additional nonlinearity in the two-tank system arises from the pump characteristics which involve saturation effects.

Although no theoretical studies of the effects of non-linear elements on the calculation of controller parameter sensitivities has been attempted, simulation studies and the practical experience with the two-tank system suggest that the tuning process based on sensitivity calculations can be successfully applied to systems which can be approximated by continuously differentiable non-linear functions.

Experience with the pump saturation phenomenon suggested that successful tuning could only be achieved if test inputs were chosen to have an amplitude which ensured that saturation of the pump did not occur. This suggests that hard non-linear elements within the closed loop system may cause problems with this approach to sensitivity calculation and tuning.

7.3 Development and Testing of the Two-Stage Approach

Two methods of generating the sensitivity functions of the controller parameters in systems with unknown plants involving the direct convolution method and the two-stage approach have been investigated in this research work. The two-stage approach is a novel and potentially important aspect of the work. The advantage of this approach is that there is no limitation on the form of the test input signal. The sensitivity functions can be obtained from second stage measurements directly without any calculations. However, it needs twice the number of measurements for generating the sensitivity functions compared with the direct numerical convolution approach. It should be noted however that the number of tests required by the signal convolution method is much smaller then that involved in the use of parameter perturbation techniques and the two-stage approach is therefore potentially attractive for cases where the convolution method cannot be applied. The effects of parameter discretisation of the sensitivity filters and methods of minimising or correcting the introduced error has been investigated mathematically and through simulation studies.

7.4 Application of the Techniques to Helicopter Flight Control System

The controller parameter tuning technique using sensitivity functions has been successfully applied to a relatively complex MIMO problem (i.e. helicopter flight control system) in this research work. The development of good generic mathematical models of helicopters and other forms of rotorcraft suitable for use as a basis for the design of high-bandwidth full-authority active flight control systems presents interesting problems and some major difficulties. Problems arise because of the inherent complexities of the system dynamics and impose significant limitations on the design of high performance systems. One of the fundamental problems in the design of high-bandwidth highly-augmented flight control systems for helicopters is model uncertainty. This includes uncertainties in model structure and uncertainties in model parameters. (Murray-Smith, 1995) Although the tuning process is given in the time domain only, this work is novel and potentially important for the flight testing and certification of flight control systems. It also provides a link between time domain sensitivity and frequency domain sensitivity using the FFT basis of the ICAD approach. The γ function of the MIMO system can also be found experimentally using spectral analysis based on the extended PRBS test signal.

7.5 Additional Remarks

Some of the main points of the various methods of calculating sensitivity functions are summarised here:

1) Number of experiments needed to calculate sensitivity functions for n controller parameters :

Parameter perturbation method	n + 1
Convolution approach	1
Two-stage approach	2

2) The forms of sensitivity filter that arise show that there are no problems of implementation for practical controller transfer functions. In general, if the controller transfer function is C(s), the transfer function of the sensitivity filter corresponding on the controller parameters K is :

$$\frac{1}{C(s)}\frac{\partial C(s)}{\partial K}$$

For the forms of controller which arise in practice implementation of this sensitivity filter seldom presents any difficulties.

3) Susceptibility of the techniques to measurement noise very often affects the results

of the sensitivity function calculation. However the sensitivity functions still give parameter changes in the right direction and, although the tuning process may take more time, the controller parameter adjustments still converge.

4) Effect of the size of the step signal

   The different sizes of test signal will give different sensitivity functions, as shown in Chapter 5, and care must be taken to select a size of test signal which is appropriate for the system being considered, taking into account any measurement noise present and knowledge of the forms of nonlinearity likely to arise within the system.

7.6 Further Work

Although the principles of the methods of sensitivity analysis of the closed-loop systems described in this thesis have been established for some time, along with the method of using sensitivity functions in the tuning of controllers, there are several aspects of this approach which have been extended by the current investigation and provide a basis which could now allow the techniques to be applied in a general way to real control systems. This is especially relevant in the case of multivariable systems, for which only theoretical analysis had previously been carried out.

An assessment of how precise the calculation of the sensitivity functions should be has, however, not been performed. Clearly the sensitivity functions are only used for frequencies up to a little bit over the bandwidth of the closed loop system. Errors in the sensitivity functions for frequencies higher than this is thus not critical. For the frequencies actually used in the controller tuning technique is it intuitively clear that errors in the estimated sensitivity functions are going to slow down the convergence of the controller tuning method.

It appears that there are links between the γ function and the sensitivity functions. This requires further investigation.

The two-stage approach has been tried in the time domain in the current work. This approach has a potential benefit due to the freedom regarding the input signal and there is scope for the application of the approach in the frequency domain.

REFERENCES

Astrom, K.J. and Hagglund, T., "Automatic Tuning of PID Controllers", ISA, Research Triangle Park, N.C., 1988

Barker, H.A. and Davy, R.W., "System Identification Using Pseudorandom Signals and the Discrete Fourier Transform", Proc. IEE, Vol 122, No 3, pp 305-311, 1975

Daniels, A.R., Lee, M., and Pal, M.K., "Combined Suboptimal Excitation Control and Governing of A.C. Dynamic Turbogenerators Using Dynamic Sensitivity Analysis", Proc. IEE, Vol 124, No 5, 1977

Davies, W.D.T., "System Identification for Self Adaptive Control", Wiley, London, pp 25-83, 1970

Dudgeon, G.J.W., Gribble, J.J., O'Reilly, J. and Murray-Smith, D.J., "The Use of Individual Channel Analysis and Design to Meet Helicopter Handling Qualities Requirements", Proc. Twenty First European Rotorcraft Forum, Vol 3 , 1995

El-Shirbeeny, E.H.T., Murray-Smith, D.J., Winning, D.J., "Technique for Direct Evaluation of Parameter Sensitivity Functions", Electronics Letters, Vol 10, No 25/26, pp 530-531, 1974

Eykhoff, P., "System Identification", Wiley, New York, 1974

Fiebig, U.C.G., and Schnell, M., "Correlation Properties of Extended m-Sequences", Electronics letters, Vol 29, No 20, pp 1753-1755, 1993

Flynn D., Hogg, B.W. and Swidenbank, E., "Self-tuning Expert Control for Turbogenerator Systems", Trans. Inst M.C., Vol 16, No. 1, pp 40-47, 1994

Godfrey, K., "Perturbation Signals for System Identification", Prentice Hall, Hemel Hempstead, 1993

Golomb, S.W., "Shift Register Sequences", Holden-Day, San Francisco, 1967

Kane, P.A., "Computer Control of Liquid Level in an Inter Connected Tank System", Project Report, Dept. of the Electronics & Electrical Engineering of the University of Glasgow, 1992

Kokotovic', P.V., "Method of Sensitivity Points in the Investigation and Optimisation of Linear Control Systems", Automation and Remote Control, Vol 25, No 12, pp 1512-1518, 1964

Kokotovic', P.V. and Rutman, R.S., "Sensitivity of Automatic Control Systems", Automation and Remote Control, Vol 26, No 4, pp 727-749, 1965

Kokotovic, P.V. and Rutman, R.S., "A General-Purpose Optimiser Based on Sensitivity Points", Proc. Third All-Union Conference on Automatic Control, Nauka, 1967

Kreyzig, E., "Advanced Engineering Mathematics 6th ed." ,Wiley, New York, 1988

Lamb, J.D., "System Frequency Response Using P-n Binary Waveforms", IEEE Trans. on Automatic Control, pp 478-480, Aug. 1970

Lamb, J.D. and Rees, D., "Digital Processing of System Responses to Pseudorandom Binary Sequences to Obtain Frequency Characteristics using the Fast Fourier Transform" in "The Digital Use of Computers in Measurement", IEE Conf. Publ, 103, pp 141-146, 1973

Leithead, W.E., and O'Reilly, J., "Performance Issues in the Individual Channel Design of 2 - Input 2 - Output System", Int. J.Control, Vol 54, No 1, pp 46-82, 1991

Manness, M.A., "Techniques for Tuning of Helicopter Multivariable Flight Control Systems and Handling Qualities", University of Glasgow, Ph.D Thesis, 1988

Murray-Smith, D.J., "Parameter Sensitivity Analysis in System Modelling", Control Systems Group Report, No CSG/70/001, Department of Electronics and Electrical Engineering, University of Glasgow, 1970

Murray-Smith, D.J., "Investigations of Methods for the Direct Assessment of Parameter Sensitivity in Linear Closed Loop Control Systems", Preprints IMACS 11th World Congress, Norwegian Soc. Auto. Control, Vol 4, pp 41-44, 1985

Murray-Smith, D.J. and Gong, M., "A Practical Exercise in Simulation Model Validation", Proc. IMACS Symp. MATHMOD 1 Conference, Vol 2, pp 231-234, Vienna, 1994

Murray-Smith, D.J., "Modelling Limitations for Helicopter Flight Control System Design", in Breiteneckev, F and Husinsky, I. (editors), EUROSIM ' 95 Simulation Congress Proceedings, pp 397-402, Elsevier, Amsterdam, 1995

Newland, D.E., "Random Vibrations and Spectral Analysis", Second edition, Longman Scientific and Technical, New York, 1984

Nichols, S.T. and Dennis, L.P., "Estimating Frequency-Response Function Using Periodic Signals and the F.F.T.", Electronics Letters, Vol 7, No 22, pp 662-663, 1971

Ngo, Y.H. and Evans, F.J., "Optimal Control of Generator Excitation in Electric Power Systems", Proc. IEEE, No 33, pp 187-192, 1972

Oppen, I., Gong, M. and Murray-Smith, D. J., "The Development of Tuning Techniques for Single-Input Single-Output and Multivariable Control Systems Using Controller Sensitivity Measures", in Cheng, R.C.H. and Pooley, R.(editors), Proc. 2nd Conference of the UK simulation Souety, April 19-21 1995, North Berwick, pp 73-79, UKSS, Edinburgh, 1995

O'Reilly, J. and Leithead, W.E., "Multivariable Control by 'Individual Channel Design'", Int.J.Control, Vol 54, No 1, pp 1-46, 1991

Poussart, D. and Ganguly, U.S., "Rapid Measurement of System Kinetics - An Instrument for Real - Time Transfer Function Analysis", Proc. IEEE, Vol 65, No 5, pp 741-747, 1977

Porter, B., Jones, A.H. and McKeown, C. B., "Real - Time Expert Tuners for PI Controllers", IEE proc., Vol 134, part D, pp 260-263, 1987

Smith, J., "An analysis of helicopter flight mechanics part I - Users guide to the software package HELISTAB", Royal Aircraft Establishment, TM FS(B) 569, October 1984

Soderstrom, T. and Stoica, P., "System Identification", Prentice Hall International, New York, 1989

Soudant, B., "Investigations of Methods for Direct Assessment of Parameter Sensitivity in Closed Loop Control Systems", University of Glasgow, Project Report, 1985

Tischler, M.B., "Digital Control of Highly Augmented Combat Rotorcraft", NASA, TM-88346, May 1987

Tischler, M.B., "System Identification Requirements for High-bandwidth Rotorcraft Flight Control System Design", AGARD-LS-178, 1991

Tomovic', R., "Sensitivity Analysis of Dynamic Systems", McGraw-Hill, New York, 1964

Unbehauen, H. and Rao, G.P., "Identification of Continuous Systems", Elsevier, Amsterdam, 1987

Wang, H. and Daley, S., "Controller Tuning via Rule Based Identification", Trans., Inst., M. C., Vol 15, No. 1, pp 38-45, 1993

Wellstead, P.E., Coupled Tanks Apparatus: Manual, TecQuipment Ltd, 1981

Wellstead, P.E., "Pseudonoise Test Signals and the Fast Fourier Transform", Electronics Letters, Vol 11, No 10, pp 202-203, 1975

Wellstead, P.E., "Reference Signals for Closed-Loop Identification", Int.J.Control, Vol 26, No 6, pp 945-962, 1977

Wilkie, D.F., and Perkins, W.R., "Essential Parameters in Sensitivity Analysis", Second IFAC Symposium, Belgrade (Dubrovnik, Yugoslavia) August 1968

Winning, D.J., El-Shirbeeny, E.H.T., Thompson, E.C. and Murray-Smith, D.J., "Sensitivity Method for Online Optimisation of a Synchronous-Generator Excitation Controller", Proc. IEE Vol 124, No 7, pp 631-638, 1977

Zhuang, M. and Atherton, D.P., "Automatic Tuning of Optimum PID Controllers", Proc. IEE, Vol 140, No 3, pp 216-224, 1993

Appendix 1

Theory of the Parameter Adjustment

It is assumed here that the sensitivity functions of the controller parameters have been generated.

For simplicity we use a single input single output feedback control system with n controller parameters as a example, as in Sections 2.2.1 and 2.2.2.

From equ.(2.6) (for a diffirence Δy_d between the desired response y_d and the actual response y_a and controller parameters m_1, m_2, ... m_n) the cost function is

$$J = \sum_{l=0}^{q} [\Delta y_d(l\Delta t) - \sum_{j=1}^{n} S_{m_j}(l\Delta t)\Delta m_j]^2 \Delta t$$

where Δt is sampling interval

n is number of controller parameters

q is $\frac{T}{\Delta t}$

S_{m_j} is sensitivity function of parameter m_j

To minimise the cost function J

$$\frac{\partial J}{\partial \Delta m_j} = 0 \text{ for all m}$$

$$\text{i.e. } \sum_{l=0}^{q} \left\{ 2\left[\Delta y_d(l\Delta t) - \sum_{j=1}^{n}\left[S_{m_j}(l\Delta t)\Delta m_j \right]\right] S_{m_j}(l\Delta t)\Delta t \right\} = 0 \quad \text{for all m} \qquad \text{(A1-1)}$$

Assuming $n = 2$ equ.(A1-1) becomes as

$$\begin{cases} \sum_{l=0}^{q} \Delta y_{da}(l\Delta t) S_{m_1}(l\Delta t) - \sum_{l=0}^{q}\left[S_{m_1}(l\Delta t)\Delta m_1 + S_{m_2}(l\Delta t)\Delta m_2 \right] S_{m_1}(l\Delta t) = 0 \\ \sum_{l=0}^{q} \Delta y_d(l\Delta t) S_{m_2}(l\Delta t) - \sum_{l=0}^{q}\left[S_{m_1}(l\Delta t)\Delta m_1 + S_{m_2}(l\Delta t)\Delta m_2 \right] S_{m_2}(l\Delta t) = 0 \end{cases} \qquad \text{(A1-2)}$$

Equ..(A1-2) can be rewritten as

$$\begin{cases} A_{11}\Delta m_1 + A_{12}\Delta m_2 = 0 \\ A_{21}\Delta m_1 + A_{22}\Delta m_2 = 0 \end{cases} \qquad \text{(A1-3)}$$

where $A_{11} = \sum_{l=0}^{q} S_{m_1}{}^2(l\Delta t)$

$$A_{12} = \sum_{l=0}^{q} S_{m_2}(l\Delta t) S_{m_1}(l\Delta t)$$

$$A_{21} = A_{12}$$

$$A_{22} = \sum_{l=0}^{q} S_{m_2}{}^2(l\Delta t)$$

$$B_1 = \sum_{l=0}^{q} \Delta y_d(l\Delta t) S_{m_1}(l\Delta t)$$

$$B_2 = \sum_{l=0}^{q} \Delta y_d(l\Delta t) S_{m_2}(l\Delta t)$$

Suppose the desired change Δy_d is known and the sensitivity functions S_{mi} can be found the equ.(A1-3) are linear equations in the 2 variables Δm_1 and Δm_2.

$$AX = B \tag{A1-4}$$

where $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

$$X = \begin{bmatrix} \Delta m_1 \\ \Delta m_2 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Thus the solution of the equations is

$$X = A^{-1} B \tag{A1-5}$$

Appendix 2

Convolution Approach in the Frequency Domain

The sensitivity function of the closed-loop system can be obtained by the convolution method in the frequency domain. Differentiating equ.(3.7) with respect to controller parameter m_i the sensitivity function can be expressed as:

$$\frac{\partial W_c(j\omega)}{\partial m_i} = W_c(j\omega)\frac{Z_i(J\omega)}{R(j\omega)} = P_i(j\omega)W_c(j\omega) \qquad \text{(A2-1)}$$

where $P_i(j\omega) = \dfrac{Z_i(j\omega)}{R(j\omega)}$

In general, in terms of harmonic response, the transfer function of the closed-loop system can be expressed in polar form as

$$W_c(j\omega) = M(\omega)\, e^{j\alpha(\omega)} \qquad \text{(A2–2)}$$

and similarly the transfer function of the filter also can be expressed as

$$P_i(j\omega) = U_i(\omega)\, e^{j\beta(\omega)} \qquad \text{(A2–3)}$$

Equ.(A2-1) can be re-written as

$$\frac{\partial W_c(j\omega)}{\partial m_i} = M(\omega)e^{j\alpha}U_i(\omega)e^{j\beta} = M(\omega)U_i(\omega)e^{j(\alpha+\beta)} \qquad \text{(A2-4)}$$

Using a Taylor series we have

$$W_c(j\omega, m_i + \Delta m_i) = W_c(j\omega, m_i) + \frac{\partial W_c(j\omega,m_i)}{\partial m_i}\Delta m_i + \ldots\ldots$$

$$= M(\omega)e^{j\alpha} + M(\omega)U_i(\omega)e^{j(\alpha+\beta)}\Delta m_i + \ldots\ldots \qquad \text{(A2-5)}$$

From Kreyzig (Kreyzig, E., 1988), we have the Euler formula,

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Equ.(A2–5) becomes

$$W_c(j\omega,m_i+\Delta m_i) = M(\omega)\{\cos\alpha+j\sin\alpha+U_i(\omega)\Delta m_i[\cos(\alpha+\beta)+j\sin(\alpha+\beta)]+\ldots\}$$

$$= M(\omega)\{[\cos\alpha+U_i(\omega)\Delta m_i\cos(\alpha+\beta)+\ldots]+j[\sin\alpha+U_i(\omega)\Delta m_i\sin(\alpha+\beta)+\ldots]\} \qquad \text{(A2-6)}$$

Thus

$$|W_c(j\omega,m_i+\Delta m_i)| = M(\omega)\{[\cos\alpha+U_i(\omega)\Delta m_i\cos(\alpha+\beta)+\ldots]^2+[\sin\alpha+U_i(\omega)\Delta m_i\sin(\alpha+\beta)+\ldots]^2\}^{1/2} \qquad \text{(A2-7)}$$

$$= M(\omega)\{\cos^2\alpha+2U_i(\omega)\Delta m_i \cos\alpha\cos(\alpha+\beta)+U_i^2(\omega)\Delta m_i^2\cos^2(\alpha+\beta)$$
$$+\sin^2\alpha+2U_i(\omega)\Delta m_i \sin\alpha\sin(\alpha+\beta)+U_i^2(\omega)\Delta m_i^2\sin^2(\alpha+\beta)+\ldots\}^{1/2}$$
$$= M(\omega)\{1+2U_i(\omega)\Delta m_i[\cos\alpha\cos(\alpha+\beta)+\sin\alpha\sin(\alpha+\beta)]$$
$$+U_i^2(\omega)\Delta m_i^2+\ldots\}^{1/2} \quad \text{(A2-8)}$$

Because

$$\cos(z_1 - z_2) = \cos z_1 \cos z_2 + \sin z_1 \sin z_2$$

with

$$z_1 = \alpha + \beta$$

$$z_2 = \alpha$$

equ.(A2-8) can be re-written

$$|W_c(j\omega,m_i+\Delta m_i)| = M(\omega)\{1 + 2U_i(\omega)\Delta m_i\cos\beta + U_i^2(\omega)\Delta m_i^2 + \cdots\}^{1/2} \quad \text{(A2-9)}$$

Assuming a function is

$$f(x) = (1+x)^{1/2} \quad \text{(A2-10)}$$

A Tailor series is defined as (Kreyzig, E., 1988)

$$f(x) = \sum_{m=0}^{\infty}\frac{f^{(m)}(x_0)}{m!}(x-x_0)^m \quad \text{(A2-11)}$$

for equ.(A2-10) we have for $x_0 = 0$,

$$f^{(0)}(x) = (1+x)^{\frac{1}{2}}, \qquad f^{(0)}(0) = 1$$

$$f^{(1)}(x) = \frac{1}{2}(1+x)^{-\frac{1}{2}}, \qquad f^{(1)}(0) = \frac{1}{2}$$

Thus, the power series for equ.(A2-10) at $x_0 = 0$

$$f(x) = 1 + \frac{1}{2}x + \cdots \quad \text{(A2-12)}$$

Defining

$$x = 2U_i(\omega)\Delta m_i\cos\beta + U_i^2(\omega)\Delta m_i^2 + \ldots \quad \text{(A2-13)}$$

equ.(A2-9) can then be re-written as

$$|W_c(j\omega,m_i+\Delta m_i)| = M(\omega)\{1 + U_i(\omega)\Delta m_i\cos\beta + \frac{1}{2}U_i^2(\omega)\Delta m_i^2 + \ldots\} \quad \text{(A2-14)}$$

From the parameter perturbation method the sensitivity function can be expressed as

$$\frac{\partial |W_c(s)|}{\partial m_i} = \lim_{\Delta m \to 0_i} \frac{|W_c(j\omega, m_i + \Delta m_i)| - |W_c(j\omega, m_i)|}{\Delta m_i} \tag{A2-15}$$

And finally with equ.(A2-14) and (A2-2) inserted into equ.(A 2-15)

$$\frac{\partial |W_c(s)|}{\partial m_i} = \lim_{\Delta m_i \to 0} \frac{M(\omega)\left\{1 + U_i(\omega)\Delta m_i \cos\beta + \frac{1}{2}U_i^{\,2}(\omega)\Delta m_i^2 + \cdots\right\} - M(\omega)}{\Delta m_i}$$

$$\frac{\partial |W_c(\omega)|}{\partial m_i} = \frac{\partial M(\omega)}{\partial m_i} = M(\omega)U_i(\omega)\cos\beta \tag{A2-16}$$

It is clear that equ.(A2-16) is exact expression for the sensitivity function in the frequency domain.

Appendix 3

Investigation of the Problem for an Impulse Input

In order to investigate the problem discussed in Section 4.2.2 in detail a very simple example has been considered, in which the plant is chosen to be G(s) = 1/s, the transfer function of the controller C(s) = k and input R(s) = 1. In this case, using the notation established in Chapter 3 we have.

$$\frac{Y(s)}{R(s)} = \frac{k}{s+k};$$

$$\frac{E(s)}{R(s)} = \frac{s}{s+k};$$

$$\frac{\partial C}{\partial k} = 1; \quad \frac{1}{C}\frac{\partial C}{\partial k} = \frac{1}{k};$$

$$Z(s) = \frac{E(s)}{k} = \frac{s}{k(s+k)}R(s)$$

and $$S_k^y(s) = Z(s)\frac{Y(s)}{R(s)} = \frac{s}{(s+k)^2}R(s) = \frac{s}{(s+k)^2}$$

From the inverse Laplace transform, it can be shown that

$$S_k^y(t) = e^{-kt}(1-kt) \tag{A3-1}$$

We can however also use the convolution method to get the sensitivity function. Because $Y(s) = \frac{k}{s+k}$ and $Z(s) = \frac{s}{k(s+k)} = \frac{1}{k}(1-\frac{k}{s+k})$, the corresponding time domain quantities are $y(t) = ke^{-kt}$ and $z(t) = \frac{1}{k}\delta(t) - e^{-kt}$, respectively. Hence, the sensitivity function can be calculated by

$$\begin{aligned} S_k^y(t) &= \int_0^t y(\tau)z(t-\tau)d\tau \\ &= \int_0^t z(\tau)y(t-\tau)d\tau \\ &= \int_0^t [\frac{1}{k}\delta(\tau) - e^{-k\tau}]ke^{-k(t-\tau)}d\tau \end{aligned}$$

$$= e^{-kt}\int_0^t [\delta(\tau)e^{k\tau} - k]d\tau$$

$$= e^{-kt}[\int_0^t \delta(\tau)e^{k\tau}d\tau - kt] \qquad \text{(A3-2)}$$

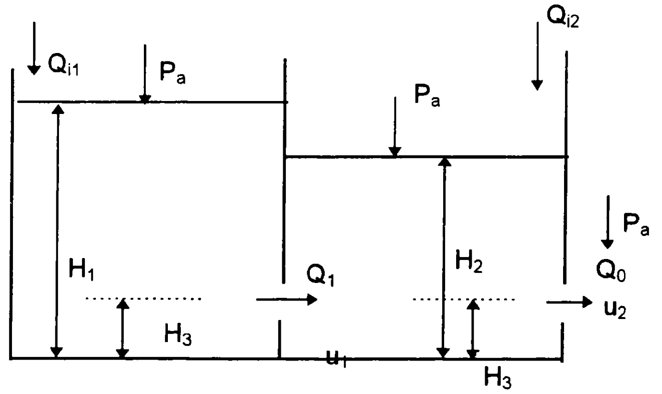
Comparison of the equ.(A3-1) and (A3-2) shows that for the two equations to be equivalent it is necessary that

$$\int_0^t \delta(\tau)e^{k\tau}d\tau = 1 \qquad \text{(A3-3)}$$

This is clearly true from the properties of the unit impulse function but results show that the MATLAB impulse response function cannot handle this. It appears that the MATLAB impulse function neglects the integral in equ.(A3-3) in calculating the expression in equ.(A3-2). Hence care has to be taken in applying MATLAB using the convolution method in this case.

Appendix 4

Two Tanks Modelling Deduction



A two tank system is shown in the diagram above.

Bernoulli's Theorem: In the steady motion of a liquid the quantity

$$P + \frac{1}{2}\rho v^2 + \rho gh$$

has the same value at every point of the same streamline where P, ρ and v are the pressure density and speed, g is the accretion due to gravity. Supposing that the volumes of the two tanks are large enough and the flow rates at their exits, Q_1 and Q_0 are small so that the flow inside the tank is negligibly small . Applying the Bernoulli theory to tank1, we can have

$$P_a + \rho g(H_1 - H_3) = \frac{1}{2}\rho u_1^2 + \rho g(H_2 - H_3) + P_a \qquad \text{(A4-1)}$$

where P_a is the ambient air pressure acting on the liquid surface

ρ is the density of liquid.

$\rho g(H_1 - H_3)$ is the pressure exerted by water at the height of H_3 in tank 1.

u_1 is the flow velocity at the exit of tank 1 and $1/2(\rho u_1^2)$ is its dynamic pressure.

$\rho g(H_2 - H_3)$ is the pressure exerted by water at the position adjacent to the exit of tank 1.

The velocity at the exit of tank 1, u_1, can be further deducted from equ.(A4-1), i.e.

if $H_1 > H_2$ $\quad u_1 = \sqrt{2g(H_1 - H_2)}$

if $H_1 < H_2$ $\quad u_1 = \sqrt{2g(H_2 - H_1)}$

The flow rate at the exit of tank 1 can be expressed as

$$Q1 = C_{d1}\, a_1\, u_1 = C_{d1}\, a_1 \sqrt{2g(|H_1 - H_2|)} \tag{A4-2}$$

where a_1 is the cross sectional area of exit 1

C_{d1} is the discharge coefficient for tank one

Similarly, application of the Bernoulli theory to tank 2 gives

$$P_a + \rho g(H_2 - H_3) = \frac{1}{2}\rho u_2^2 + P_a \tag{A4-3}$$

The velocity at the exit 2 can be obtained from equ.(A4-3) ,i.e.

$$u_2 = \sqrt{2g(H_2 - H_3)}$$

The flow rate at exit 2 is

$$Q_0 = C_{d2}\, a_2 u_2 = C_{d2}\, a_2 \sqrt{2g(H_2 - H_3)} \tag{A4-4}$$

where a_2 is the cross sectional area at the exit 2

C_{d2} is the discharge coefficient for tank 2

Appendix 5

Development of Tank Level Measuring System

As we know, the depth measuring has a very important role in the two tank system. Accuracy, linearity and repeatability are required for its measurement. The biggest source of error in characterizing the coupled-tank system lay in the original level measurement system.

The sensors worked on the principle that a varying depth of electrolyte caused a resistance change across a two track sensor. However, control of electrolyte conductivity was crude and its characteristics changed with time. Repeated use would also result in a build up of deposits on the sensor tracks, further degrading reliability.

An alternative method of depth sensing was found, providing of accuracy, linearity and repeatability.

A5.1 Pressure Sensing

A single pressure transducer was used for evaluation as a possible means of satisfying the above specification. Some essential parameters of the transducer are given as follows:

a) Supply Voltage $\cong$ +12V dc

b) Differential pressure range: 0 - 1 psi

c) Full scale span of o/p signal $\cong$ 48mV

d) Linearity & Hysteresis typically 0.2% FS

e) Repeatability typically 0.5% FS

f) Sensitivity @ 12V $\cong$ (36 - 60)mV/psi

A conversion showed that, a water height of 30cm in the tank would produce a pressure of 0.43 psi. This is just under half of the sensor's full range. According to the sensitivity of the transducer which is listed in point f), the (36 - 60) mV/psi $\times$ 0.43 psi range would yield a signal between (15 - 26)mV for a full tank. This dictated that the amplifier used to boost this signal to a working level would require a variable gain that would accommodate all signal levels in this range.

An output range of 0 - 10V was decided on as a reasonable signal to represent tank depth from empty to full. This required a gain range of :

$$\frac{V_o}{V_1} = \frac{10}{(15 \sim 26)10^3}$$

i.e. a variable gain in the range 400 to 600.

One important limitation on the transducer was that it must remain dry at all times. This was achieved by mounting the transducers on a paxolin board, hung from the tank side. From here, a push-fit nylon tube is led from each high pressure port to the bottom of each of the tank, so that the transducer acts by air compression. The mounting board arrangement is shown in Fig. A5.1.

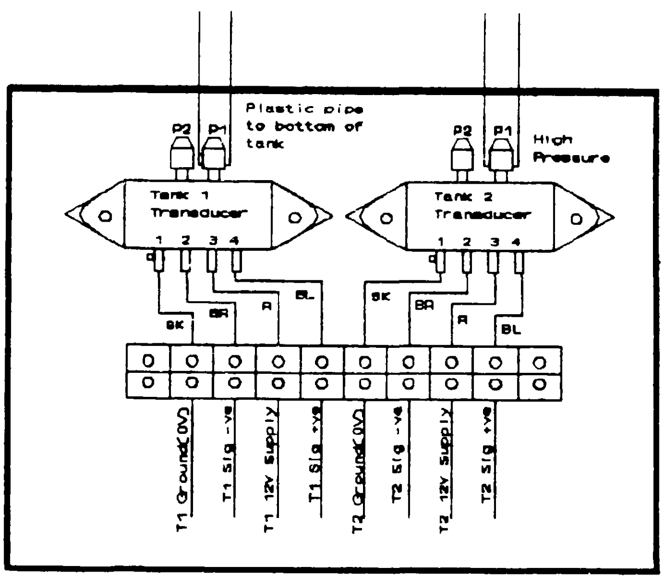


Fig. A5.1 Transducer wiring & tubing connections

Early tests of transducer response proved promising. However two things were noted: the calibration shifted with changes in supply voltage and the output signal worked in reverse. That is, a high tank level gave minimum mV (say 15), with a linearly increasing signal for a falling level, eventually ending with ≅ 42mV for 3cm w.g. These observations suggested two requirements of the measuring system. One was a fixed power supply for the transducer which would give a consistent and stable voltage on switch-on. The other being that, since the amplified signal would yield 10 - 0V for 0 - 30 cm w.g, an inverter would be needed to give the required 0 - 10V signal. This allowed work to progress on the design of the amplifier.

A5.2 Amplifier design

A full size circuit diagram of the final design is given in Fig. A5.2. The designed circuit processes the incoming transducer signal in two stages: an amplifying stage and an inverting stage. Both stages utilize the LF 351N operational amplifier.

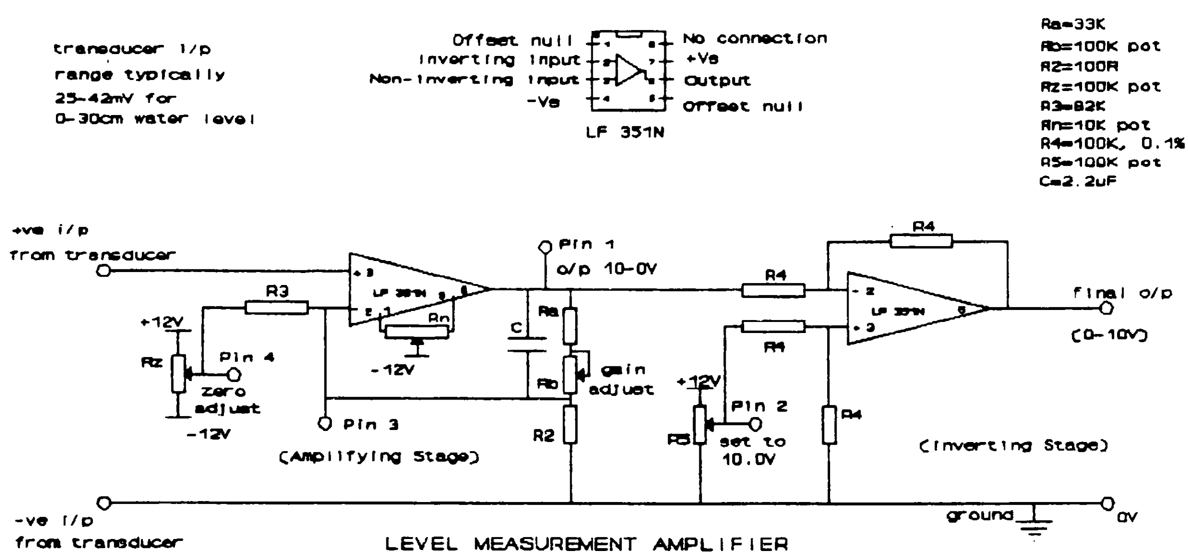


Fig. A5.2 Amplifier schematic Diagram

A5.2.1 Amplifying Stage

The amplifying stage is configured in the non-inverting mode, giving a gain of : $Av = 1 + R_1 / R_2^2$, here $R_1 = R_a + R_b$. Resistance values were chosen to maintain a high input impedance and minimising the current load from the power supply modules. R_2 was arbitrarily chosen to be 100 Ω, which set a value of :

$R_1 = 39.9$ KΩ for Av = 400

and $R_1 = 59.9$ KΩ for Av = 600

Hence, a fixed resistance of 33 kΩ for R_a in series with a 100 kΩ variable resistance for R_b, gives the required range of gain. The offset null provided by the 10 kΩ potentiometer (R_n) connected across opamp pins 1 & 5, did not give enough zero bias to compensate for the 15mV minimum signal. Therefore, a 100 kΩ potentiometer (R_2) connected between the positive and negative supply rails, was used to bias the inverting input and thus provide the required zero adjustment. R_3 acts to limit the current drain to the 0V rail via R_2.

The original design suffered from high frequency output oscillations. A 2.2μF capacitor connected across the feedback path eliminated this problem. Care was taken to ensure that the time constant of the RC combination, ($\approx 3s$), was enough to eliminate the stay AC, but did not interfere with the process response.

The amplifier PCB contains two measurement circuits for each tank. The component layout is given in Fig.A5.3.

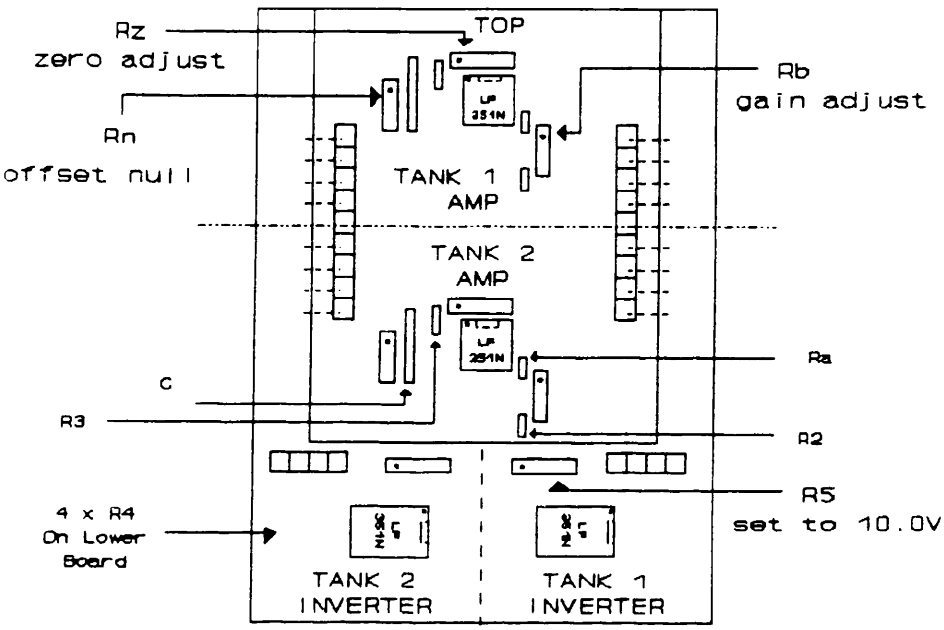


Fig. A5.3 Component layout diagram

A5.2.2 Inverting Stage

The inverting stage is connected as a simple 1:1 difference amplifier. This is achieved via the configuration of the four high precision resistors ($4 \times R_4$). Again, a 100kΩ pot (R_5) is used to set a reference level, which is adjusted here to give 10.0 V at pin 2. This ensures that the final output signal is the correct sense, i.e. 0 - 10 V for 0 - 30 cm of tank level.

Appendix 6

The following state space matrices form the 19th order linear model of a typical combat rotorcraft at 30 knots straight and level flight. The model was produced from HELISTAB (Smith, J., 1984)

a =

1.0e+003 *

Columns 1 through 7

0.0000	0.0000	-0.0033	-0.0321	0.0000	0.0000	0
-0.0002	-0.0005	0.0512	-0.0021	0.0000	-0.0005	0.0013
0.0000	0.0000	-0.0002	0	0.0000	0.0000	0
0	0	0.0010	0	0	0	0
0.0000	0.0000	0.0000	0.0001	-0.0001	0.0032	0.0321
0.0000	0.0000	0.0000	0	0.0000	0.0000	0
0	0	0.0000	0	0	0.0010	0
0.0000	0.0000	0.0000	0	0.0000	0.0001	0
0	0	0.0000	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0003	0.0010	-0.0018	0	0.0000	0.0010	0
-0.0002	-0.0007	0.0326	0	0.0004	0.0726	0
0.0004	0.0001	-0.0730	0	0.0006	0.0340	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0.0320	0	0	0
0	0	0	0	0	0	0
0	0	0	-0.0278	0	0	0
0.0000	0	0	0	0	0	0
-0.0498	0	0	0	-0.0320	0	0
-0.0001	0	0	0.0007	-0.1609	0	0
0.0001	0	0	0	0	0	0
-0.0007	0	0	0.0016	-0.0290	0	0
0.0010	0	0	0	0	0	0
0	0	0	0	0	0.0010	0
0	0	0	0	0	0	0.0010
0	0	0	0	0	0	0
0	0	-1.5148	0	0	-0.0317	0
0	0	-0.1021	-0.2453	-1.1331	0	-0.0317
0	0	0	1.1279	-0.2453	-0.0029	0.0713
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 19

0	0.0221	0.0022	0.0000	0
0	-0.2983	-0.0301	0.0000	0
0	0.0009	0.0001	0.0000	0
0	0	0	0	0
0	-0.0009	-0.0001	0.0000	0.0159
0	0.0057	0.0006	0.0000	-0.0010
0	0	0	0	0

0	0.0138	0.0014	0.0000	-0.0131
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0.0010	0	0	0	0
-0.0014	0.7405	0.0622	0	0
-0.0713	-0.1013	-0.0102	1.1331	0
-0.0317	0.1640	1.1342	0	0
0	-0.0126	0	0	0
0	0	-0.0126	0	0
0	0	0	-0.0126	0
0	0	0	0	-0.0250

b =

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
12.5786	0	0	0

0	12.5786	0	0
0	0	12.5786	0
0	0	0	25.0000

c =

Columns 1 through 7

0.0065	-0.0997	0	5.0666	0.0040	0	-0.0133
0	0	0	10.0000	0	0	0
0	0	0	0	0	0	5.0000
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
5.0000	0	0	0	0	0	0

Columns 15 through 19

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

d =

0	0	0	0
0	0	0	0

0	0	0	0
0	0	0	0

The following state space matrices form the 19th order linear model of a typical combat rotorcraft at 50 knots straight and level flight. The model was produced from HELISTAB (Smith, J., 1984)

a =

1.0e+003 *

Columns 1 through 7

0.0000	0.0000	-0.0044	-0.0321	0.0000	0.0001	0
-0.0001	-0.0007	0.0845	-0.0017	0.0000	-0.0009	0.0011
0.0000	0.0000	-0.0003	0	0.0000	0.0000	0
0	0	0.0010	0	0	0	0
0.0000	0.0000	0.0000	0.0001	-0.0001	0.0042	0.0321
0.0000	0.0000	0.0000	0	0.0000	0.0000	0
0	0	0.0000	0	0	0.0010	0
0.0000	0.0000	0.0000	0	0.0000	0.0001	0
0	0	0.0000	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0.0001	0.0013	-0.0011	0	0.0000	0.0018	0
0.0000	-0.0006	0.0319	0	0.0003	0.0723	0
0.0004	0.0002	-0.0728	0	0.0003	0.0330	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0.0320	0	0	0
0	0	0	0	0	0	0
0	0	0	-0.0278	0	0	0
0.0000	0	0	0	0	0	0
-0.0832	0	0	0	-0.0320	0	0
-0.0002	0	0	0.0012	-0.1609	0	0
0.0001	0	0	0	0	0	0
-0.0010	0	0	0.0027	-0.0290	0	0
0.0010	0	0	0	0	0	0
0	0	0	0	0	0.0010	0
0	0	0	0	0	0	0.0010
0	0	0	0	0	0	0
0	0	-1.5148	0	0	-0.0317	0
0	0	-0.1701	-0.2453	-1.1377	0	-0.0317
0	0	0	1.1233	-0.2453	-0.0048	0.0713
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Columns 15 through 19

0	0.0248	0.0041	0.0000	0
0	-0.3334	-0.0554	0.0000	0
0	0.0010	0.0002	0.0000	0
0	0	0	0	0
0	-0.0007	-0.0001	0	0.0174
0	0.0046	0.0008	0.0000	-0.0011

0	0	0	0	0
0	0.0113	0.0019	0.0000	-0.0142
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0.0010	0	0	0	0
-0.0024	0.8101	0.1145	0	0
-0.0713	-0.1640	-0.0272	1.1377	0
-0.0317	0.2835	1.1426	0.0000	0
0	-0.0126	0	0	0
0	0	-0.0126	0	0
0	0	0	-0.0126	0
0	0	0	0	-0.0250

b =

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
12.5786	0	0	0

0	12.5786	0	0
0	0	12.5786	0
0	0	0	25.0000

c =

Columns 1 through 7

0.0052	-0.0998	0	8.4444	0.0033	0	-0.0144
0	0	0	10.0000	0	0	0
0	0	0	0	0	0	5.0000
0	0	0	0	0	0	0

Columns 8 through 14

0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
5.0000	0	0	0	0	0	0

Columns 15 through 19

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

d =

0	0	0	0
0	0	0	0

0	0	0	0
0	0	0	0

The state vector is

$$x = [x_{rigidbody}\ x_{rotor}\ \ x_{actuator}]$$

where

$$x_{rigidbody} = [\ u\ w\ q\ \theta\ v\ p\ \phi\ r\ \psi]$$

$$x_{rotor} = [\ \beta_{\tau 0}\ \beta_{1c}\ \beta_{1s}\ \beta_{to}\ \beta_{1c}\ \beta_{1s}]$$

$$x_{actuator} = [\ \theta_{0act}\quad \theta_{1sact}\quad \theta_{1cact}\quad \theta_{0Tact}\ ]$$