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AUTOMATED SHIP HULL FORM DESIGN OPTIMISATION THROUGH MORPHING AND EVOLUTIONARY COMPUTATION

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ABSTRACT

Eco-friendliness and energy efficiency are now key criteria in the design and construction of all new ships due to stringent environmental regulations aimed at reducing their carbon footprint. An efficient hull form helps in reducing the overall drag acting on a ship, as well as reducing fuel consumption and harmful emissions to the environment. More recently, Simulation-Based Design (SBD) methods have been proposed to improve the efficiency of the hull form design process. However, the entire process still requires considerable human inputs, and success depends heavily on the designer's experience and knowledge.

To address these limitations, an industry-first design automation tool based on Morphing and Evolutionary Computation (MEC) was developed in this thesis. This was achieved by combining a novel two-dimensional (2D) curve morphing method and a Non-dominated Sorting Genetic Algorithm II (NSGA-II) using proven hull form designs as initial starting points. The 2D curve morphing method provides an efficient way to modify the hull form whilst providing more flexible geometric variations. NSGA-II is a multi-objective optimisation algorithm that uses nature-inspired operations to optimise a population of candidate solutions in a generative manner. This tool supplements experienced designers in modifying the shape of the hull form, in a more automated manner, helping them to achieve more diverse and optimal design solutions.

Central to the working of MEC are novel crossover and mutation mechanisms, developed using cross-morph and boundary-morph. Together, these two morphing techniques enable geometric manipulation to be performed directly on the hull, instead of modifying the design parameters as is the case in traditional approaches. By applying the two morphing techniques within NSGA-II, more diverse shapes can be discovered, and the entire design process can further be automated.

MEC is particularly well-suited for the initial design stage, where shape diversity and efficient performance evaluation are key to finding more optimal designs. The final output of this method is a set of Pareto optimal designs, which can further be refined by the designer. Through digitisation and computational intelligence, it is envisioned that automated design will help to improve the overall design efficiency and capability to produce more efficient and smarter ships in the future.

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AUTHOR'S DECLARATION

I declare this thesis is a record of original work carried out by myself, under the supervision of Professor Cindy Goh at University of Glasgow. The copyright of this thesis belongs to the author under the terms of the United Kingdom copyright acts. As such, due acknowledgement must be made of the use of any material contained or derived from this thesis. This thesis has not been submitted elsewhere in consideration for a higher degree.

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NOMENCLATURE

$M(t)$	Morphed shape
t	Morphing parameter
R_0	Source shape
R_1	Target shape
f_i	Objective function
χ	Vector of design variables
K	Equality constraints
J	Inequality constraints
R_F	Frictional resistance according to ITTC-1957 friction formula
$1+k_1$	Form factor describing the viscous resistance of hull form in relation to R
R_{APP}	Resistance of appendages
R_W	Wave-making and wave-breaking resistance
R_B	Additional pressure resistance of bulbous bow near the water surface
R_{TR}	Additional pressure resistance of immersed transom stern
R_A	Model-ship correlation resistance
C_F	Frictional resistance coefficient
Rn	Reynolds number
ν	Kinematic viscosity of water
C_P	Prismatic coefficient
LR	Form-factor
Y_S	Y value of source ship
Y_T	Y value of target ship
Y_R	Y values for reference (source) curve
Y_B	Y values for boundary (target) curve
$Y(t)_C$	New Y value of child ship
Y_{TS}	Y values of transition section between two boundaries
ρ	Water density
V	Speed of ship
S_i	Cubic spline
S_{APP}	Wetted area of appendages
$k = 1,3,5$	Coupled surge, heave and pitch

$k = 2,4,6$	Coupled sway, roll and yaw
$\ddot{\eta}_j$	Acceleration of harmonic oscillation in j direction
$\dot{\eta}_j$	Velocity of harmonic oscillation in j direction
η_j	Displacement of harmonic oscillation in j direction
M_{kj}	Solid mass or inertia coefficient
A_{kj}	Hydrodynamic mass or inertia matrix
B_{kj}	Hydrodynamic damping or radiation damping coefficients
C_{kj}	Hydrostatic stiffness or restoring coefficients
$\sum F_k$	Harmonic exciting wave force or moment in k direction
z	Plane of ship's cross-section
ζ	Plane of unit circle
M_s	Scale factor
a_{2n-1}	Mapping coefficients
N	Maximum parameter index number
θ	Corresponding angle in mapped unit circle plane
B_s	Sectional breadth on waterline
D_s	Sectional draught
H_0	Half breadth to draught ratio
σ_s	Sectional area coefficient
A_s	Area of cross-section
b_{max}	Maximum half breadth of the cross-section
d_{max}	Maximum draught of the cross-section
a_k	Two-dimensional added mass coefficients
b_k	Two-dimensional damping coefficients
A_0	Area of each section
B_0	Breadth of each section
ϕ_m	Velocity potential
ψ_m	Conjugate stream function
φ_c	2D radiation potential for harmonic motions of unit amplitude
ψ_c	Stream function cos component
φ_s	2D radiation potential for harmonic motions of unit amplitude
ψ_s	Stream function sin component
α	Radius of circular cylinder
β	Heading angle

γ	JONSWAP factor
μ	Linearised damping coefficient
σ	Area coefficient at a cross-section along the hull
ξ	Wave elevation
ω	Angular frequency (Strip-theory)
W	Weightage (Morphing)
ε	Phase angle
λ	Wave length
A_k	Three-dimensional added mass coefficients
B_k	Three-dimensional damping coefficients
b_0	Breadth of cross-section
x_{ATS}	x frame at aft transition section
x_{FTS}	x frame at forward transition section
x_{TS}	x frame between x_{ATS} and x_{FTS}
x_{APS}	x frame at aft-peak section
x_{FPS}	x frame at fore-peak section
x_B	Total number of transverse frame between reference and boundary
x_A	x frame adjacent to reference frame
$x_{F/AB}$	x frame at forward or aft boundary section
x_R	x frame at reference frame section
x_{MB}	x frame at mid-boundary section

LIST OF ABBREVIATIONS

2D	Two-Dimensional
3D	Three-Dimensional
AP	Aft perpendicular
B-Spline	Basis Spline
BEM	Boundary Element Method
CAD	Computer-Aided Design
CAE	Computer-Aided Engineering
CFD	Computational Fluid Dynamic
CPUs	Central Processing Units
DOE	Design of Experiment
EA	Evolutionary Algorithm
EC	Evolutionary Computation
EEDI	Energy Efficiency Design Index
EP	Evolutionary Programming (EP)
FP	Forward perpendicular
FFD	Free-Form Deformation
GA	Genetic algorithm
GPUs	Graphic Processing Units
HFDO	Hull Form Design and Optimisation
HPC	High-Performance Computing
IMO	International Maritime Organisation
ITTC	International Towing Tank Conference
LPP	Length between Perpendiculars
LOA	Length Overall
MEC	Morphing and Evolutionary Computation
NSGA-II	Non-dominated Sorting Genetic Algorithm II
NURBS	Non-Uniform Rational Basis Spline
PSO	Particle Swarm Optimisation
RANS	Reynolds-Averaged Navier-Stokes
SBD	Simulation-Based Design

SQP	Sequential Quadratic Programming
UQ	Uncertainty quantification
URANS	Unsteady Reynolds-averaged Navier–Stokes
VLCC	Very-Large Crude Carrier

1. INTRODUCTION

1.1. Importance of Hull Form Design and Optimisation

Hull Form Design and Optimisation (HFDO) is an integral part of the ship design process where the hull shape is modelled, modified and evaluated in an iterative manner to improve the overall design [1]. HFDO continues to be an important topic in the marine industry due to more stringent environmental regulations and commercial needs to reduce fuel consumption during ship operation. An effective solution towards reducing fuel and carbon emission is the design of more efficient hull forms, where factors that contribute to the ship resistance such as viscous drag, frictional and wave-making resistance, are effectively reduced [2]. As part of the Energy Efficiency Design Index (EEDI) regulation implemented in 2013 by the International Maritime Organisation (IMO), ships constructed after 2025 need to be at least 30% more fuel efficient [3]. In 2018, IMO further mandated the reduction of greenhouse gas emission by the shipping sector by at least 50% by 2050 [4]. Consequently, eco-friendly shipping and fuel efficiency are now key design criteria in the construction of all new ships.

Besides fuel and energy efficiency, motion-keeping or seakeeping is another important performance criterion. A well-designed hull form helps in constraining the motion of the ship against external environmental conditions such as wind, wave and currents, thereby ensuring maximum habitability, operability and survivability of human and other cargo onboard the vessel [5]. In many specialised ships such as naval, passenger and offshore vessels, seakeeping performance is a key consideration in the design of hull form.

1.2. Digitisation and Existing Limitations in Ship Hull Form Design

Digitalisation of ship design, construction and operation is gaining increasing attention in the marine industry. Ship production is currently shifting towards more automated and intelligent systems that are highly connected. Ships are also becoming smarter with more automated systems and progressively shifting towards fully-autonomous vessels. In contrast, the development and adoption of design automation has been slow.

Traditionally, ship design was viewed more as an ‘art’, where optimal hull form could only be created by ‘experienced’ designers, than a ‘science’, where optimal designs could be produced automatically without human intervention. However, this view is changing due to the increased availability of Computer-Aided Design (CAD) that simplify the

design of complex structures such as ships [6]. Fundamentally, the role of ‘computer-aided’ design is to assist the designer to achieve better or more optimal designs. It is, therefore, still essential to retain the ‘art’ in the designer, who can concentrate on producing more creative designs. ‘Science’ in this case can play a bigger role by automating the ‘laborious’ or repetitive aspects of the design process to improve the overall efficiency and increase the possibility of creating designs with better performance. Therefore, to achieve rapid development and adoption of digitisation in the entire marine industry, it is important for the ship design process to become more automated and efficient.

Practically, there is no single solution that can address all the aforementioned issues because there is always a trade-off regarding the extent of the ship design process that can be automated. For example, manual user input provides more flexibility in the ship design process but requires more design man-hours. Conversely, automation offers higher design efficiency but restricts the freedom of the designer. Localised geometry modification using conventional Non-Uniform Rational Basis Spline (NURBS) or Free-Form Deformation (FFD) allows highly flexible shape manipulation of the hull form but requires manual selection and modification of geometry parameters or variables by the designer. This is highly experience-based and cannot be automated. Similarly, CFD provides high accuracy to the hydrodynamic evaluation process, but requires longer computational time compared to empirical or analytical methods that are much faster but less accurate. These factors represent an important ship design problem currently facing the industry that this thesis aims to address.

The work presented in this thesis seeks to improve and address existing limitations in the design automation and optimisation of hull form on three main fronts. First, the dependence on the experience and expertise of a designer to set up, monitor and analyse results. Second, the lack of an efficient geometry modification method that can intuitively transform the hull shape with minimal designer input. Third, the high computational cost of CFD, especially when coupled with a highly iterative optimisation process such as HFDO.

1.3. Aim of This Thesis

The objectives of this thesis are:

- To propose an automated HFDO approach that supplement experienced designers’ knowledge to improve the shape of hull towards more optimal designs.

- To formulate an automated geometry modification technique based on 2D curve morphing that enables efficient geometry modifications suitable for HFDO.
- To combine the geometry modification technique with intelligent search for more efficient design and optimisation of the hull form.

1.4. Key Contributions

The key contributions of this thesis are:

- Introduction of an industry-first, fully-automated geometry modification and optimisation approach that optimises the shape of the hull with minimal user input and enables a more efficient creation of optimal and innovative hull form. Current simulation-based design (SBD) processes for HFDO are either semi- or highly automated and require an expert designer to input, adjust or post-process the design parameters or output throughout the geometry modification and optimisation process to achieve optimal designs.
- Derivation of a 2D shape modification and optimisation method that only requires information from the existing offset table, thereby allowing more efficiency and shape variations of the hull form. Current SBD schemes used in HFDO require the additional step of converting the 2D data into a 3D surface for geometry modification, optimisation and performance evaluation using CFD.
- Creation of a unique problem mapping scheme where the 2D offset data are directly mapped through phenotype-genotype mapping comprising the morphing parameter at respective transverse frames of the vessel. This enables the shape of the hull to be represented and modified by simply changing the morphing parameters at respective frames. Current mapping schemes primarily use binary code or design variables to represent and modify the hull geometry.
- Introduction of a cross-morph method that allows the combination of two or more vastly different hull forms to create new designs. Existing morphing methods morph the entire vessel while the proposed method can partially morph and retain the shape of both original vessels using interpolation techniques.
- Creation of an automated geometry modification method through a boundary-morph approach that allows efficient transformation using only one ship's geometry without any user input. Current morphing methods require two existing vessels to interpolate the hull forms or require certain user inputs to select the position to morph.

1.5. Publications

To promote and increase awareness of this research within the research community and marine industry, several articles were submitted and published in international peer-reviewed conferences and journals relating to the field of ship design and Evolutionary Computation. The list of conference presentations and journal publications is provided below.

- **Conference Publications**

1. **J.H. Ang**, C. Goh and Y. Li, “Key Challenges and Opportunities in Hull Form Design Optimisation for Marine and Offshore Applications,” *International Conference on Automation & Computing*, Glasgow, 11-12 September, 2015.
2. **J.H. Ang**, C. Goh and Y. Li, “Hull Form Design Optimisation for Improved Efficiency and Hydrodynamic Performance of Ship-Shaped Offshore Vessels,” *International Conference on Computer Applications in Shipbuilding*, Bremen, 29 September-1st October, 2015.
3. **J.H. Ang**, C. Goh and Y. Li, “Hybrid Evolutionary Shape Manipulation for Efficient Hull Form Design Optimisation,” *Conference on Computer Applications and Information Technology in the Maritime Industries*, Lecce, 9-11 May, 2016.
4. **J.H. Ang**, C. Goh and Y. Li, “Smart Design for Ships in a Smart Product Through-Life and Industry 4.0 Environment,” *IEEE World Congress in Computation Intelligence*, Vancouver, 25-29 July, 2016.
5. **J.H. Ang**, C. Goh and Y. Li, “On the Role of Smart Design in Future Manufacturing for the Marine Industry,” *Industrial Systems in the Digital Age Conference*, Glasgow, 20-21 June, 2017.
6. **J.H. Ang**, C. Goh, V.P. Jirafe and Y. Li, “Efficient Hull Form Design Optimisation Using Hybrid Evolutionary Algorithm and Morphing Approach,” *International Conference on Computer Applications in Shipbuilding*, Singapore, 26-29th September, 2017.
7. **J.H. Ang**, C. Goh, V.P. Jirafe and Y. Li, “Smart Design of Hull Forms Through Hybrid Evolutionary Algorithm and Morphing Approach,” *International Marine Design Conference*, Helsinki, 11-14 June, 2018.

8. **J.H. Ang**, C. Goh, C. Choo and Y. Li, “Smart Design for Digital Manufacturing in a Connected Marine Industry Value Chain,” *Connected Everything Conference*, Newcastle, 26-27 June, 2018.
9. **J.H. Ang**, C. Goh, C.T. Choo, Juveno, Z.M. Lee, V.P. Jirafe, and Y. Li, “Evolutionary Computation Automated Design of Ship Hull Forms for the Industry 4.0 Era,” *IEEE Congress on Evolutionary Computation*, Wellington, 10-13 June, 2019.

- **Journal Publications**

1. **J.H. Ang**, C. Goh, A.A. Saldivar and Y. Li, “Energy-Efficient Through-Life Smart Design, Manufacturing and Operation of Ships in an Industry 4.0 Environment,” *Energies*, 2017.
2. O. Hafiiz and **J.H. Ang**, “Gearing for New Ocean of Opportunities,” *38th Annual Journal of Society of Naval Architect and Marine Engineers Singapore*, 2019.

- **Patent Application**

Title: Automated Geometric Modification and Optimisation Method for Ship Hull Form Design

Country of filing: Singapore

Application number: Not available at the time of thesis submission (patent filing in progress)

1.6. Outline of Thesis

This thesis is structured as follows:

Chapter 1 highlights the importance of hull form design and presents the motivation and scope of this thesis. It summarises the key contributions and publications achieved through this thesis.

Chapter 2 provides the background to ship and hull form design, followed by simulation-based design. It details the state-of-the-art in simulation-based hull form design by examining the key methodologies from a design automation perspective. Methods that are already in development and those that need further development are identified.

Chapter 3 introduces the concept of 2D curve morphing and presents two novel 2D curve morphing approaches developed in this thesis. Some examples of how these approaches can be applied to modify hull geometry are also provided in this chapter.

The concept of combining 2D curve morphing and NSGA-II to automatically search and create optimal hull form designs is explored in Chapter 4. This technique is integrated with simplified hydrodynamic tools using the Holtrop method and strip theory to evaluate the resistance and seakeeping performance of vessels.

A case study of a single-objective optimisation problem to reduce the drag of a container vessel is presented in Chapter 5. The main objective is to test the effectiveness of MEC for different initial population sizes and GA settings. The results demonstrate that it is possible to simultaneously achieve a fully-automated hull form design process and produce optimal solutions.

Chapter 6 presents a case study involving a multi-objective optimisation problem. The objective is to reduce drag while improving the seakeeping function for an offshore cable-laying vessel. In this chapter, different objective functions comprising non-conflicting and conflicting functions are evaluated. The result validates the effectiveness of the proposed approach to automatically modify the hull form with conflicting objectives and simultaneously achieve noticeable improvements.

Chapter 7 summarises the thesis with a conclusion and key benefits, and provides recommendations for future work.

2. CRITICAL REVIEW OF EXISTING SIMULATION-BASED DESIGN TECHNIQUES

In this chapter, a critical review of existing SBD methodologies for HFDO is presented. The key processes involved in HFDO include design exploration, geometry modification and performance evaluation. As part of this review, methods that are already in development and areas that require further attention are identified. Finally, the topics of research are selected and further addressed as part of this thesis.

2.1. Background to Ship Design

Ship design is a complex and highly-iterative process comprising several stages and involving many design man-days. Typically, the entire design process for a new ship building project can span between 6 to 12 months depending on the scale of the project. Market competition and more efficient computer systems and software are constantly reducing this design cycle time.

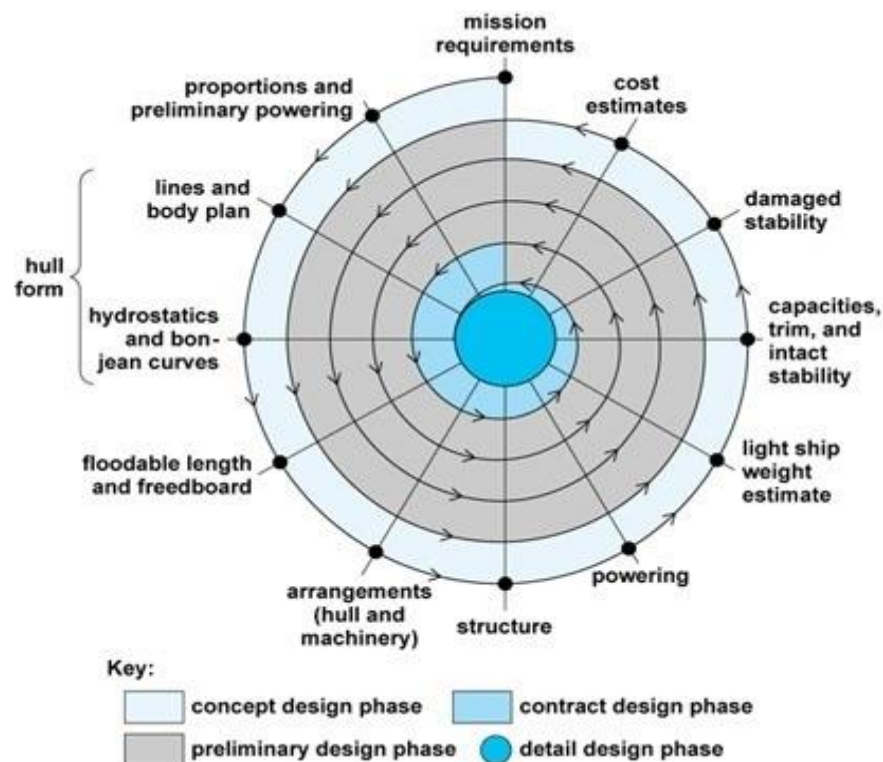


Figure 2.1 Ship design spiral [7].

There are four key phases in ship design, as shown in Figure 2.1. The process starts with the concept design phase. In the mission requirements stage, the ship owner must first

define the type of cargo to be transported and the intended trade route for the vessel. The vessel's desired design parameters and features such as main dimensions and capacity are then specified. Once the project is proven feasible, the process will move on to the preliminary design phase, which further defines the key ship characteristics that affect cost and performance. In this phase, the shape of the hull is one of the most important design parameters that needs to be determined as early as possible. This is due to its impact on the overall performance efficiency; any major changes to the hull form in later design or construction stages will have detrimental effects on the cost and delivery schedule for the vessel [8]. The next phase is the contract design stage, which presents a set of plans and specifications as part of a shipbuilding contract document. This is followed by the detail design phase, where construction details such as accommodation, machinery, piping and electrical arrangement are produced prior to actual construction of the vessel [7].

Traditionally, ships are designed using a 'trial-and-error' approach, where designers first create the new hull form design by hand or using CAD tools based on their experience and assess the performance using first principles or computer programs. The hull form is then adjusted or modified as necessary. This process is repeated multiple times to achieve better performance. This method is exhaustive, time-consuming and does not guarantee optimal design. In the current competitive landscape and with the pressure for shorter design cycles, traditional methods are progressively being replaced with faster and more effective SBD methods [9].

2.2. Simulation-Based Design

SBD is a process where simulation is used as the primary means of design evaluation and verification [10]. It has been applied extensively in geometry design and optimisation for different fields in transport engineering. These fields include automobile [11] - [13], aerospace [14], [15] and marine [16] - [18], where geometry affects overall performance due to aerodynamic, hydrodynamic and aero-elastic factors. The development of computer simulation and optimisation has led to the rise of SBD, which combines the process of modelling and CFD simulation with advanced optimisation techniques. In ship design, SBD is a useful approach for naval architects to conduct initial investigations and improve the vessel's performances such as resistance, seakeeping, manoeuvring and structural strength during the early stages of the design process. While it is recognised that SBD can be very efficient in assisting the designer in the decision-making process toward the development of more optimal hull forms, success in achieving optimal design

currently still depends heavily on the experience of the designer in setting up the optimisation problem.

Optimisation in ship design started as early as the 1970s [19]. Rapid developments in Computer-Aided Engineering (CAE) and CFD soon followed in the 1980s-1990s, as computers became more powerful and readily available. Fuelled by advancements in new optimisation and evaluation techniques, SBD found application in HFDO processes in the 2000s. Since the 2010s, with more efficient optimisation and CFD, SBD for both ship design and HFDO has increasingly been accepted and applied in the marine industry. Parallel computing and high-performance computers have also been applied with SBD to reduce the computation time. Towards the end of the 2010s, next-generation SBDs started to focus on increasing the design efficiency and accuracy of CFD methods applied in HFDO processes. A brief timeline and key milestones related to the development of CAD and SBD in ship design and HFDO are provided in Figure 2.2.

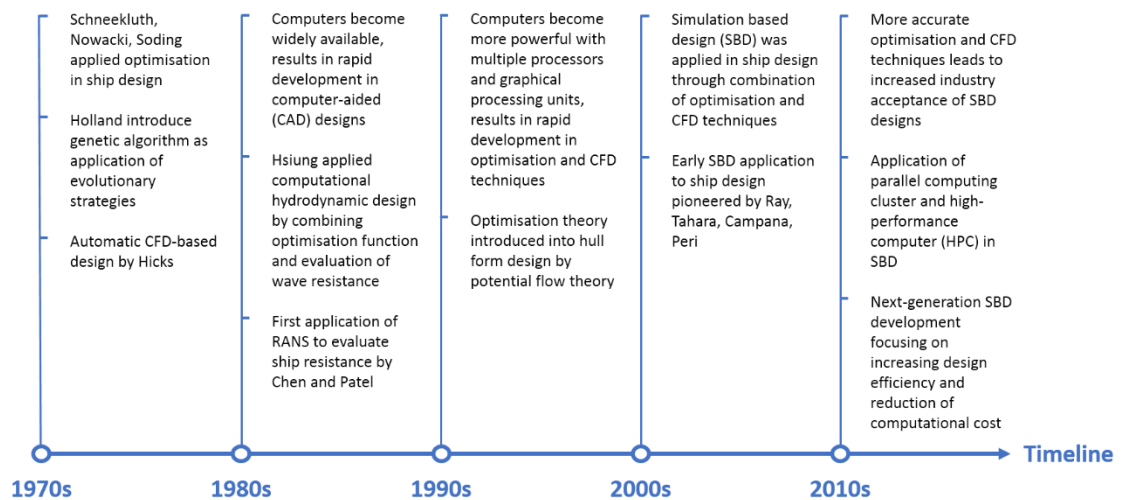


Figure 2.2 Timeline of CAD and SBD development in ship design and hull form optimisation (abstracted from [20]).

2.3. Hull Form Design and Optimisation

HFDO is part of the ship design process to derive the optimal hull form design, which can be achieved using various CAD or computer simulation tools, independently or in combination with SBD. The key aspects of HFDO are:

- **Objective function** - This is a process whereby the designer defines the design objectives to suit customer's requirements defined in the design specification. An example of design objective is improving vessel performance such as reducing drag, improving seakeeping performance, or both.

- **Design variables** - In HFDO, design variables are control parameters such as length, width, draft, block coefficient, entrance angle, width, that alter the shape of the hull and in turn affect the ship's performance such as drag, seakeeping, manoeuvrability, stability, strength, displacement and cost.
- **Design constraints** - Constraints in design determines the limit to which the design can be changed and still be considered feasible. It can be in the form of prescribed requirements given by ship owners such as maximum length, breadth or draft, displacement or deadweight. In general, the higher the number of constraints set, the fewer feasible designs the HFDO process can generate.
- **Performance evaluation** - Performances such as resistance and seakeeping need to be evaluated using numerical approximation. When linked to an optimiser, the candidates can be evaluated automatically to determine the most optimal hull form. Typically, simple methods are faster but possess less accuracy, while complex methods are more accurate but slower.

For a more automated HFDO process, SBD can be applied by combining three processes, namely, design exploration, geometry modification and performance evaluation, into a single platform. The process first starts with the design exploration phase where the hull shape is linked to an optimisation function to systematically search for optimal designs. Subsequently, the geometry modification phase changes the shape of the hull to create new designs. Finally, the performance of the new hull will be evaluated in the performance evaluation phase. This process is iterated until the termination criteria or the optimal hull design is achieved. An illustration of the SBD process for an automated HFDO is provided in Figure 2.3.

While SBD has been proven to reduce the initial design time and the need for physical prototypes or model testing, its adoption by the marine industry is still slow; this can be attributed to the following reasons. SBD in its current form, while highly automated, still relies heavily on the experience of the designer in developing new hull designs, and often requires expert knowledge in setting up the optimisation model and evaluating the performance using CFD. Furthermore, geometry modification is a key process in the transformation and generation of new hull variants. Currently, there are limited geometry modification methods that can intuitively modify the hull shapes without the input of an experienced designer. This restricts development towards fully-automated design.

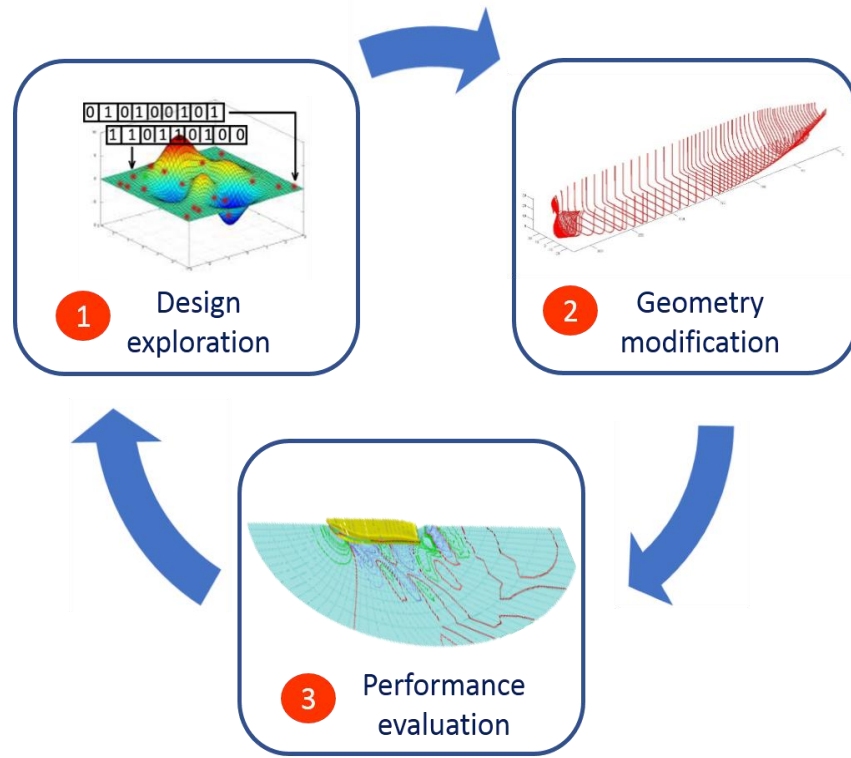


Figure 2.3 Simulation-based design process for HFDO [21].

Another major issue is the high computational cost and evaluation time involved in SBD using high-fidelity CFD methods such as the Reynolds-Averaged Navier-Stokes (RANS) equation. In particular, for the preliminary design stage, the hull form model needs to be modified and re-adjusted for many iterations so as to search for different combinations and the most optimal design. While the use of parallel computing clusters or High-Performance Computing (HPC) helps to reduce the evaluation time, it is still prohibitively expensive. This makes high-fidelity evaluation such as CFD unattractive for automated HFDO applications.

To better understand the limiting issues and provide direction towards more automated design, SBD approaches for each of the key HFDO processes outlined in Figure 2.3 are examined and potential solutions as well as areas requiring further attention identified in the following sections.

2.4. Design Exploration

Design exploration is a search process where key design parameters such as geometry of the hull are explored in a systematic manner to produce a set of possible candidate solutions. It can be considered the most important function in the automated HFDO process as it drives the optimisation framework towards finding optimal solutions.

2.4.1. Existing Design Exploration Approaches in HFDO

Optimisation algorithms can be classified in several categories. Within the context of search space exploration, they can either be gradient or non-gradient, deterministic or stochastic, local or global. Gradient-based methods depend on derivative information to guide the search process while gradient-free methods do not use any derivative information, rather, they rely on comparing objective function values at specific points of the search space. An algorithm is deterministic if it works in a non-random manner and will reach the same final solution if it starts from the same initial point. Conversely, an algorithm is stochastic if it involves randomness and the algorithm will reach a different point every time it runs [22]. Local optimisation methods aim at providing only one solution. They are often deterministic and typically converge towards a local optimum. Unfortunately, most real-world problems possess more than a single optimum solution, hence, local search techniques cannot guarantee the global optima. In contrast, global optimisation attempts to avoid getting stuck in local optima. Unlike local optimisation methods where a local optimum is ensured by zero gradients, these criteria do not exist in global optimisation methods to ensure the attainment of a near global optimum solution [16]. An illustration of local and global optima is provided in Figure 2.4. It can be seen that the search in the solution space, which starts from an initial parameter may lead to local optima, which are sub-optimal, thereby missing better solutions ‘hidden’ somewhere in the global optimum.

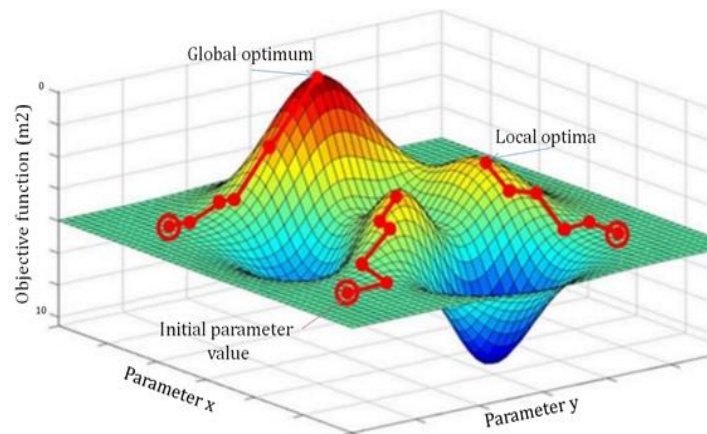


Figure 2.4 Illustration of global and local optima for two objective functions- parameter X and Y [23].

As the requirements of optimisation techniques evolve, application of optimisation has also become multi-form and multi-dimensional, incorporating the use of various cross-disciplinary techniques. Depending on the objective function and optimisation approach,

the most suitable algorithm can be selected based on its characteristics and applicability. Areas of consideration in the selection of optimisation algorithms include type of problem, nature of algorithm, quality of solution, computing resources, time limit, algorithm implementation, availability and even expertise of the decision maker [22]. In design exploration, most optimisation methodologies are able to run automatically, but setting up the process and the linkage to geometry modification and performance evaluation processes are still highly dependent on a user's input and experience. The search algorithms are classified here into two main types: gradient-based (or derivative-based) and gradient-free (or derivative-free) algorithms.

- **Gradient-Based Approach**

This is a class of local search algorithms where the search directions are defined by the gradient of the function at the current point. Gradient-based algorithms are highly efficient as local search algorithms and suited for solving problems with large numbers of design variables. However, they risk being trapped in a local optimum for problems which are not convex [24]. Gradient-based algorithms can be applied for global search but require estimations of first order derivatives, which increases their dependency on the number of variables and consequently their computational cost [25]. Classic gradient-based methods include Steepest Descent and Gauss-Newton methods [24]. Examples of gradient-based methods applied in HFDO include Conjugate Gradient [26], [27] and Sequential Quadratic Programming (SQP) method [17], [28] - [31]. The Conjugate Gradient method belongs to a class of Krylov subspace iteration methods, which are used mainly for finding numerical solutions to sparse symmetric positive definite systems of linear equations [24]. The main advantage of Conjugate Gradient is its simplicity. However, it requires high cycle time and can only reach a local minimum. SQP techniques possess a robust and stable scheme for any nonlinear programming problem to solve local quadratic programming problem through approximation by Taylor expansion around a set of design variables [28]. It produces a sequence of second order local models of the objective function in order to minimise it [25]. SQP is a highly efficient search method due to its fast convergence rate but suffers the limitation of getting stuck in local optima [32]. Another type of gradient-based approach applied to HFDO is the Tangent Search method [33], [34].

- **Gradient-Free Approach**

The gradient-free approach, also known as direct search, can be classified under local and

global search methods. The main advantages of this class of techniques are that they do not require gradient data (derivatives) and are able to handle noisy or discontinuous objective functions. Gradient-free global optimisation approaches are usually preferred to derivative-based local approaches when objectives are noisy, derivatives are unknown or the presence of multiple local optima cannot be excluded [35]. Key examples of gradient-free techniques applied in HFDO include Hooke and Jeeve [36], [37] and the Nelder and Mead method [36], [38] - [41]. The Hooke and Jeeves method belongs to a family of pattern search methods that do not require optimisation of the gradient of the problem. It incorporates the history of iterations to produce new search directions based on exploratory and pattern move [24]. The search strategy is simple and requires less storage for variables, but it may prematurely converge during exploration. It also requires a large number of evaluations to accurately find a solution [42]. The method developed by Nelder and Mead is a heuristic and derivative-free local search method, which uses the concept of a simplex to guide the search towards the local optimum [24]. It provides fast and accurate convergence but is not suitable for highly nonlinear and multimodal objective functions.

A popular class of gradient-free approaches is Evolutionary Algorithm (EA). EAs are a group of generic population-based heuristic optimisation techniques that are stochastic in nature (rather than deterministic). They are based on a collective learning process within a population of individuals that evolve towards a better region of the search space [43]. EAs are widely applied as they can solve very complex problems without getting trapped in local optima [44]. Compared to gradient-based methods which are deterministic and efficient for local searches, EAs are highly stochastic and their randomness increases the variety of search for the near-global optimum. However, they consume high computational time when coupled with CPU-time intensive solvers such as CFD for analysis. In addition, they are difficult to formulate due to their randomness and their results cannot be predicted in advance. Examples of key EA techniques applied in HFDO include Genetic Algorithm (GA) [18], [45] - [51] and Particle Swarm Optimisation (PSO) [28], [52] - [55].

GA, first developed by Holland [56], is a heuristic search algorithm that mimics the process of natural selection based on Darwin's evolution of biological systems. The three main components of GA are selection, crossover and mutation. Through these operators, GA performs multiple directional searches by maintaining a population of potential solutions in each generation [57]. Compared to traditional optimisation techniques, GA

is capable of performing near-global optimal search and do not require evaluation of gradients. It is also suitable for parallel implementation, which helps to enhance the computational efficiency [58]. Several extensions of GA have been introduced to improve the efficiency of the algorithm, including Non-dominated Sorting Genetic Algorithm (NSGA), Vector-Evaluated Genetic Algorithm, Strength Pareto Evolutionary Algorithm [59], and Pareto Archived Evolution Strategy [60]. Among these extensions, NSGA-II, an extension of NSGA appears to be a very promising method and has been applied in HFDO [61] - [63].

PSO was first developed by Kennedy and Eberhat [64] based on the swarm behaviour observed in fish and bird schooling in nature. PSO simulates the social behaviour of a population of individuals (particles) that share information while exploring the search space, according to the particles' position and velocity [24]. PSO has low memory usage and iterations converge based on attraction for near global optima without the need for first and second order local information [65]. Compared to GA, PSO can converge quickly to local optima, but does not guarantee global convergence [66]. Due to its random nature, PSO is also computationally very expensive when coupled with high-fidelity simulation tools such as CFD in HFDO [22]. Similar to GA, several extensions such as deterministic PSO and synchronous or asynchronous PSO were made to improve the performance of PSO [35], [67], [68].

Other derivative-free algorithms applied in HFDO include Simulated Annealing [16], Ant Colony Optimisation [69] and Evolution Strategies [43]. More derivative-free algorithms that have recently been applied in HFDO include Mesh Adaptive Direct Search [70], Firefly Algorithm [71], Cuckoo Search [72], Bat Algorithm [73] and Fish Shoal Algorithm [35].

2.4.2. Latest Developments

Recent developments in more powerful computing resources, digitisation, advance optimisation techniques and improved accuracies in CFD have opened up unprecedented opportunities for SBD that can be applied to HFDO. To overcome and cope with existing and new challenges, conventional manual hull design optimisation techniques are no longer sufficient. To keep up with new challenges, next-generation SBDs are already evolving towards more accurate and efficient hull form designs. Some of the latest and most advanced design exploration solutions applied in HFDO are listed below.

- **Approximation or surrogate models** - Approximation techniques or surrogate

models are increasingly used to reduce the overall computational cost without over-compromising on the accuracy of solutions. By using approximation techniques, optimal solutions of the original problem can be obtained through successive approximation by reducing the original complex problem into relatively smaller approximations of subproblems [74]. Some successful approximation techniques that are applied in HFDO include Kriging [75], [76], Response Surface Methodology [77] – [80], Artificial Neural Networks [81], [82] and Elman Neural Networks [74]. While approximation methods help to improve the efficiency of HFDO, the algorithm can be too complex to be effectively applied in the SBD process.

- **Uncertainty quantification and robust design optimisation** - Efficient optimisation techniques are important in ensuring that optimal solutions are achieved. They should also be easy to integrate with other modelling and evaluation components. The search for an optimal solution is made more challenging by the fact that uncertainties are always present in real-world systems. Therefore, it is useful to find a workable method to search for optimal solutions. Potential solutions to this problem include Uncertainty Quantification [83] and Robust Design Optimisation [84], both of which search for an optimum design using probabilistic scenarios.
- **Holistic optimisation** - Similar to other optimisation problems, it is not sufficient to consider only the shape of the hull in ship design optimisation. Hence, it is essential to find holistic design optimisations that consider other design objectives. Holistic design optimisation considers all aspects of a product over several stages of its lifecycle and across several disciplines, where multi-objective and multi-disciplinary optimisations are important contributors [85]. These objectives may include cost, structure strength, cargo capacity, safety, powering, etc. Some recent works on holistic optimisation include multi-objective optimisation of naval ship and container ship design [62], [85]. The challenge here is that as the number of objectives increased, it becomes harder to achieve good improvement due to trade-offs.
- **Design parameter sensitivity analysis** - To reduce the dependency on experienced designers, design parameter sensitivity analysis can be performed to provide clarity to the designer as to which parameters have the most impact on the overall design optimisation process. Design of Experiment (DOE) is often used [40], [54], [86], [87] as a preliminary exploration of the design space to investigate the influence of parameters on the objectives and to provide a first idea of the expected performance [40].

2.4.3. Areas Which Require Attention

It is well-known in the industry that appropriate optimisation techniques should be selected for different problems to avoid error in results, premature conclusion or failure of the optimisation procedure. However, this requires a good understanding of the problem and application of suitable optimisation techniques by the user. While most optimisation techniques are highly automated, they still require several design hours and good experience to set up the optimisation problem. Due to these limitations, there is no fully automated SBD methodology that does not require user input to set up and run the HFDO process with minimal user intervention. Hence, this is an important area to address in order to enable more automated and efficient design processes.

2.5. Geometry Modification

Geometry modification plays a pivotal role in the success of HFDO by ensuring that the hull shape can be modified to create a wide variety of new shapes. This is, however, a non-trivial task as every new shape generated must be smooth and of feasible design. Considering that the shape of the hull needs to be highly streamlined in order to reduce the resistance, this makes any modification to the hull surface extremely challenging, especially if it is to be carried out automatically without any designer input. While it is desirable to reduce the number of input parameters or ‘control points’, this may come at the expense of maintaining a consistently smooth surface or ‘feasible design’. There are several shape modification techniques introduced in the field of computer graphics and geometry modelling. The choice of technique will depend on properties such as design parameters, variables and diversity of admissible shapes [88]. In principle, the more the variety of potential designs the method can generate, the greater the improvements of the design the method can produce. There are broadly two main ways to generate a hull form. First, the hull geometry from a parent hull is varied, where main dimensions from proven designs are already determined and incremental changes can be applied by making small modifications to the parameters to achieve a more reliable hull form variation. Second, a totally new hull form is created using basic form or parameters. The latter is good for novel designs but can be extremely challenging to model and requires a long process to test and improve the design. Most HFDO processes use the former approach where the geometries are modified based on proven designs [20].

2.5.1. Existing Geometry Modification Approaches

There are two main approaches used to modify the geometry of the hull - direct or

systematic modification. Direct modification changes the hull geometry by directly adjusting the hull coordinates using control points through curve or surface representations such as Bézier curve and NURBS [20]. While this method is highly flexible, it requires a large number of control points to represent the shape and hence is not very efficient for modifying the entire hull shape. In addition, direct methods are usually applied when the designer requires more flexibility and design freedom to change localised hull geometry based on their preference or experience, hence, they are not automated. Conversely, systematic modification varies the hull shape using a function that considers global hull parameters such as block coefficient or series of local hull representations. This method is particularly useful for global modifications, which enable the entire hull form to be transformed more efficiently. However, changes in shape are more restrictive in some ways and not as flexible as direct modifications. Systematic variations are mostly automated and can be run automatically using computer software. However, it usually requires manual pre-assignment of modification parameters and the end result varies widely depending on the input and according to the experience of the designer. Key geometric modification techniques based on direct and systematic modification for HFDO are reviewed below.

- **Direct modification**

A hull form is mainly generated using a series of curves or surfaces that are defined and represented by various techniques. The predominant curve definition methodologies applied in hull form modelling are mostly in parametric polynomial forms, in particular Basis spline (B-spline) [29], [89] – [93] and Bézier surface [17], [40], [94], [95]. B-spline defines the shape of curves using a given set of order, knot vector and a set of control points.

NURBS is an extension of B-spline and has a set of control points with weightage, which define the shape of the curve. NURBS is extremely popular and has become the ‘de-facto’ standard for hull surface representation. This is because it can represent most shapes fairly accurately using a very small amount of definition data. The control points can also be manipulated easily using coordinate transformation. In addition, it allows a single definition to be used for hydrodynamic analysis [20]. However, this approach suffers from a number of drawbacks. First, hull surface definition using NURBS requires a large number of control points and therefore, increases the number of design variables for surface modification in the hull form optimisation [49]. Each NURBS control point acts locally on the geometry and may cause complications during surface fairing and result in

high computation time [96]. Second, the resulting geometry from NURBS does not allow designers to freely adjust the control points to achieve the desired shapes [97]. Third, NURBS and splines are difficult to express in admissibility conditions and this can cause CAD failures and overestimation of dimensionality for complex shapes [98]. While manual manipulation of mathematical surface definitions is possible, this method is usually not favoured. This is because individual vertices must be moved several times to achieve a fair surface after any changes are made.

Besides B-spline and NURBS, there are several other variations in spline definition such as cubic spline [43], [81], fairness-optimised B-Spline [99], T-spline [41] and spline-intension [100]. These variations help to overcome the limitations and to improve the main spline algorithm. Bézier surface is controlled by a fixed number of control points that are used as design variables or input parameters in the hull form optimisation process and is similar to interpolation in several aspects. The key advantages of Bézier surface are reduction in the number of design iterations and ease of achieving surface smoothness. However, this method is not suitable if a large part of the hull section needs to be modified because the computational requirement increases with the number of constraints and with larger grid nodes [101].

- **Systematic modification**

Before the introduction of computers, hull surfaces were created and modified by hand in 2D on a drawing board. This process, which ensures that 2D views are properly matched with 3D surfaces, is extremely time-consuming. Since the introduction of computers in ship design, systematic modification can now be performed easily within the ship's 3D model. Some common geometry modification methodologies used in HFDO include shifting [32], [36], [77], [102], parametric modification [38], [47], [61], [103] - [105], Free-Form Deformation (FFD) [53], [55], [75], [106], [107], and blending approach [34], [40], [47], [53], [58], [87], [90], [108] - [110].

One of the earliest hull form modification techniques developed was shifting, or '1-Cp' technique, introduced by Lackenby [111]. This technique changes a hull form by longitudinally translating the hull sections using manual calculation and simple scaling functions to deform forward and aft regions of the hull surfaces. Lackenby's shifting method is still used in modern hull designs as it is extremely easy to apply and provides effective hull form distortion. However, the technique needs to be extended to handle modern surface representations and distortion beyond the section area curve limits [112].

An example of extensions based on Lackenby's approach is sectional area curve transformation with F-Spline [99]. Parametric modification is another useful method that captures the essence of intended shapes and their possible variations. It reduces the number of unknown parameters within the modelling process and the shape is determined from fewer data that represent high level information [113]. Parametric modification can be applied rapidly and improves control over the hull shape to be optimised [114]. However, the shape modification can be quite restrictive due to parametric constraints. Both the Lackenby shift and parametric modification methods require parameters pre-defined by the designer and hence cannot be automated.

FFD is a technique developed by Seberg and Parry [115] for animation of objects. The working principle of FFD is to enclose a complex geometric shape within a simpler shape which goes under distortion [112]. Several rules can be applied in the movement of the control points so that the distortion can be controlled along prescribed directions or linked together with the movement of different control points [75]. While FFD is very flexible, hull form modifications are limited to the original shapes bounded by the box. As FFD variables have little physical significance for designers, it is difficult to establish an effective set of design variables [22]. Successful application of FFD also depends greatly on the experience and skill of the designer to control the 'shifting' directions.

Blending, also known as morphing, is widely used in computer graphics and industrial design to compute a continuous transformation from a source shape to a target shape. In ship design applications, it is defined by blending two proven hull form designs using a blending parameter with equal number of control points [32]. This method can be used to automatically transform an entire hull form with a few design variables and the surface smoothness is always assured. However, it lacks the ability to create a wide variety of hull forms, especially if there are limited existing designs that are available for blending.

In addition to the aforementioned methods, there are several other geometric modification techniques that are applied in HFDO. These include hull form parameters [105], radial basis function interpolation [46], [102] and orthogonal basis function [116].

2.5.2. Latest Developments

Some advanced geometry modification techniques are discussed below.

- **Design space variability and dimensionality reduction** - The use of an infinite number of variables in existing modification techniques inevitably increases the computational cost especially when coupled with large shape variations and high-

fidelity solvers. One solution to reduce the computational cost is the use of design-space dimensionality reduction techniques while retaining desired level of geometric variance [22]. Design space variability and dimensionality reduction such as Karhunen-Loeve expansion enables more efficient analysis and optimisation procedure in high-fidelity HFDO processes which are high-dimensional, large design space and computationally expensive CFD solvers [55]. This can effectively be deployed as a pre-optimisation tool to investigate and obtain a design space with the highest geometric variance, which in turn provides the highest chance of improvement.

- **Integration between CAD and CAE systems** - In any product design, seamless integration between CAD and CAE is crucial, where the geometry of digital models from CAD can be accurately mapped and transferred into CAE tools for detailed structural or hydrodynamic analysis. This is, however, a challenging task considering issues such as information losses due to over-simplification, incompatibility between CAD and CAE software and long computation times [117]. In recent years, Iso-Geometric Analysis (IGA) [118] has become a most efficient methods in bridging the gap between CAD and CAE. IGA directly applies basis functions of geometric modelling for structural or hydrodynamic analysis, and has the ability to preserve geometry at all levels, as well as detailed features without excessive mesh refinements [119]. An example of HFDO application using IGA include Ginnis et al. [120] where they developed a CAD-geometry-mesh mapping tool that is automated and tightly integrated with a performance analysis and optimisation solver.
- **Automatic grid generation** - The adaptation of existing surface or grid according to newly generated CAD geometry is crucial in enabling fully-automated HFDO. This is especially important when dealing with complex geometries that prohibit automated surface or mesh generation. Coupled with CFD solvers, the computational grid must regenerate or deform according to the prior performance evaluation of the new modified shape [22]. This is a particularly challenging operation to automate. So far, the new mesh quality still needs to be checked manually. This automation becomes even more important when using stochastic optimisation techniques such as EA, which require the creation and evaluation of a large number of design variations within a reasonable amount of time to find a feasible solution. To reduce the need to re-adapt the mesh each time the surface is regenerated, one possible solution is to adapt the mesh along with the initial surface so that the mesh can be deformed with

the modified surface at the same time. The advantage of this method includes eliminating the need to regenerate the entire volume grid each time the surface is modified. In addition, preservation of the initial shape and deformation can be carried out with a prescribed degree of continuity [22]. However, limitations still exist for mesh generation of complex shapes especially when performed automatically.

There are two main types of grid for different solvers - surface grid for low-fidelity hydrodynamic simulations such as potential flow and volume grid for high-fidelity hydrodynamic simulations such as the RANS equation [88]. Surface grid solvers involve the deformation of grids that are transposed onto the surface of the hull while volume grid solvers such as boundary layer grid require the surface grid to be transposed into a volume grid [22]. Another challenge for automated grid generation is the ability to robustly deal with defects during the automated conversion process. A possible solution to this problem is using a space deformation method which deforms the embedding space around the object and in so doing, deforms the object explicitly [88]. One example of automated grid manipulation method implemented in HFDO include simple adaptive algorithms such as the volume grid through elliptic-algebraic methods using exponential schemes [32], [58].

2.5.3. Areas Which Require Attention

For HFDO, a good geometry definition and modification technique should consist of the following capabilities: First, the curve or surface definition should be easy to code and decode by the selected optimiser and solvers. Second, for optimal hydrodynamic performance, the modified surface must be reasonably smooth. Third, it must be intuitive and sufficiently intelligent to select and modify shapes that are suitable for the selected objectives. There are both pros and cons to applying direct or systematic variations to hull geometry modifications. The direct method provides more flexibility but requires more control points or design parameters. Conversely, systematic variation requires less design parameters but is more restricted. Hence, an area which requires more attention is the development of more flexible geometric modification techniques. These techniques should enable the efficient creation of a large variety of hull forms while ensuring that the designs are feasible.

2.6. Performance Evaluation

As shown in Figure 2.3, the final stage in the HFDO process is performance evaluation. Here, each solution produced from the optimiser are evaluated based on the objective

functions, and results of the simulation are then fed back into the optimisation loop. This process continues until the search has converged or the termination criteria are reached. In HFDO, the performance parameters that are influenced by the shape of the hull include resistance, seakeeping and manoeuvring. Among these parameters, resistance and seakeeping are recognised as the most important and often selected as key objective functions in most HFDO applications. There are several hydrodynamic analytical techniques used for performance evaluation, depending on the flow problem and location to be investigated. Each method has its characteristics and is suited for different applications, as elaborated below.

2.6.1. Existing Performance Evaluation Approaches

Here, hydrodynamic performance prediction techniques are classified into three main categories - empirical, analytical and numerical-based approaches. Empirical-based approaches provide simple and reasonably accurate estimates of the ship performance such as resistance and power requirement using statistical data. Analytical-based approaches use simple analysis of basic hull form to estimate flow around the ship. More recently, numerical-based approaches such as CFD have been used extensively in hull form optimisation for the evaluation of resistance. These approaches have proven effective in simulating fluid flow around the vessel. While CFD is still not as accurate as model test, it provides early insights into the flow characteristics to enable the hull form design to be further improved numerically. Some key performance evaluation methodologies used in HFDO are reviewed below.

- **Empirical-Based Approaches**

Some of the most commonly used empirical methods for estimating the total resistance of a ship include the International Towing Tank Committee (ITTC) 1957 extrapolation line that was applied in [26], [43], [121] – [124] and the Holtrop and Mennen method [125] applied in [126], [127]. ITTC-57 is widely used to calculate the frictional drag acting on a ship while the Holtrop and Mennen method is used to estimate the viscous resistance [128]. Both empirical methods combine simple physical models and regression analysis to determine the required coefficients from a set of ships through accumulation of experimental results from model testing [129]. Their key advantage is that they are fast and simple to implement. However, their accuracy is often limited to only basic hull forms. For preliminary hull form optimisation and design, it is common to combine empirical and analytical or numerical approaches for the prediction of ship resistance. For

example, the frictional resistance can be estimated using ITTC friction line and wave making resistance computed by Michell integral, slender ship theory or Rankine source panel method. Empirical methods are best suited for automated performance evaluation as they are highly efficient for evaluating large iterations of design and do not require pre or post processing of the design for evaluation, unlike CFD. These evaluation methods are particularly useful for initial design stages where the priority is to explore and evaluate as many design variations as possible and does not require high accuracy which are more important at later stages of design where the hull forms are more finalised. Most empirical-based approaches are easily coded using a computer program and hence can be fully automated.

- **Analytical-Based Approaches**

There are several analytical-based approaches for hydrodynamic predictions, such as Michell integral thin-ship approximation [130] – [136] and slender-ship approximation [26], [137] – [140]. Linear free-surface potential flows are of fundamental importance in ship hydrodynamics as the flow due to a ship advancing in wave is effectively potential and linear beyond a small distance from the ship hull [141]. The Michell integral method evaluates the wave making resistance based on the assumption of a thin ship, potential flow and linearized free surface boundary. It is robust, simple to implement and highly efficient in predicting pressure distribution at a ship hull and related wave profile and wave drag and hence particularly suited for preliminary design and hull form optimisation. However, it is not sufficiently accurate for many applications, especially for late-stage designs and detail performance evaluation as it neglects viscous effects and over-simplifies wave motion [142].

Slender ship theory is based on zeroth-order approximation used to estimate the velocity potential of flow caused by a ship [130]. It approximates the far field wave spectrum function corresponding to nearfield disturbance velocity [26]. This method is ideally suited for hull form optimisation due to its extreme simplicity and high efficiency [140]. However, a key limitation of both the Michell integral and slender ship methods is the simplification of simple slender hull form, which may not be suitable for analysing modern hull designs. In addition, there are several other resistance prediction techniques that are derived from classical methods such as Fourier-Kochin function [139], [141], [143] and Neumann-Michell theory [52], [142].

In seakeeping analysis, the strip theory method [144] is widely used [50], [145] in

analysing the ship motion by solving a succession of 2D problems and integrating them into 3D results [129]. It predicts ship motions such as heave, pitch, roll and yaw for a ship advancing at constant speed in regular wave [144]. Strip theory determines the wave-induced motion of a 3D floating body by dividing the hull into a finite number of 2D strips or cross sections along its length. It can provide relatively good results compared to model tests and full-scale trials. Strip theory also allows faster computation time and simplified body geometry definition [146]. Most importantly, it can be applied automatically using a computer program. As strip theory is based on potential flow, it neglects viscous effects, which can be an issue when predicting roll motion at resonance frequencies [147]. More recently, CFD is used in seakeeping evaluation due to its ability to capture some viscous and non-linear effects such as green water and bottom slamming [148]. However, strip theory is still preferred due to its speed and fairly good accuracy compared to CFD, which requires high computation time.

- **Numerical-Based Approaches**

CFD is increasingly used by the industry to gain preliminary insights into local flow details of ship hull and to obtain recommendations on how to improve the design, prior to final validation through model testing in a towing tank. Key CFD methods applied in HFDO include Rankine source potential flow [17], [28], [35], [39], [75], [81], [86], [93], [100], [149] and RANS equation [32], [53], [89], [94], [113], [150].

The potential flow theory, also known as panel method, is governed by Laplace equation and discretised using body surface and free surface panels [89]. It uses Boundary Element Method (BEM) to solve potential flows with free surface around the hull and predict the wave pattern and estimate the wave resistance [61]. Potential flow transforms integrals over the whole fluid domain into integrals over the boundaries of fluid domain by dividing the surface of the ship into discrete elements or panels [129]. As the effects of viscosity are limited to small boundary layers, potential flow models are very useful, in particular for free surface flows. However, in many hydrodynamic cases, they can only be used as preliminary evaluation due to limitations such as transom immersion hypothesis, no computation of viscous drag and linear free surface [151]. Despite these limitations, potential flow is widely used by the industry in ship and propeller design due to its high evaluation efficiency.

RANS equations are time-averaged equations of motion for fluid flow and are primarily used to describe turbulent flows. Unsteady Reynolds-Averaged Navier–Stokes (URANS)

equations model the entire turbulence scale and resolve large scale motions for transient flows while solving for the mean flow velocity [152]. The RANS method is used in several ship design applications to model a wide variety of ship flows with relative accuracy compared to any other CFD methodology. The key advantage of the RANS method is that it can capture global and local wave patterns including complex breaking waves, and is capable of capturing viscous effects at full scale. However, it requires long computation time and the quality of the result may differ significantly depending on the user or commercial software used [151].

Despite the considerable efforts invested in the development of high-fidelity RANS solvers, they are generally not sufficiently accurate for solving ship hydrodynamics problems such as wave breaking, turbulent contact line, bubbly wake, and cavitation [152]. Moreover, numerical solutions obtained from CFD simulations can be polluted by errors arising from discretisation and iterative methods, thereby leading to noisy performance evaluations [41]. Conversely, currently available computers are still not able to handle turbulent eddies using direct numerical simulations where the computational requirements are very expensive and can take several years to solve [153]. For more accurate analysis, different flow codes can be used for analysing different locations of the vessel. For example, inviscid free-surface methods based on boundary element approach can be used to analyse the wave making effects at the forebody while viscous flow codes can be used to analyse viscous effects around the aft body or appendages [22]. Compared to empirical methods, CFD methods are harder to automate but can still be achieved with more sophisticated software or algorithms that enable automated meshing and fixing of errors.

2.6.2. Latest Developments

Despite the limitations of computational methods for performance evaluation, CFD is a valuable tool for assessing the flow characteristics of a proposed hull form design, particularly in the early ship design phase. CFD methodologies are getting more accurate and efficient due to recent advances in computing. Consequently, they offer tremendous opportunities and possibilities in the improvement and evolution of HFDO. Some advances made to improve the accuracy of CFD applied in HFDO include:

- **Uncertainty Quantification (UQ) assessment** – UQ is a process used to determine the accuracy of a mathematical model compared to the actual physics and the impact of uncertainty based on the output of the model [154]. UQ is used in CFD methods

for solutions and computational grid including documentation, verification and validation [155]. Examples of UQ include Monte Carlo simulation and importance sampling. These are connected with analysis tools such as high fidelity URANS solver or low fidelity potential solver using a metamodel [22].

- **High performance computing** - Computer performance has grown exponentially over time. Hardware has also become increasingly affordable [156]. This has enabled the realisation of several computationally intensive techniques such as GA and CFD that previously required extensive computer power. They are increasingly applied in ship design and HFDO. According to Jamshed [153], all systems with a computational power of more than 10¹² floating points per second (FLOPS/s) can be classified as HPC. As computer hardware continues to evolve, emerging Graphic Processing Units (GPUs) can be used to achieve up to ten times faster performance than Central Processing Units (CPUs) and can therefore, be useful in further reducing the computational requirement of CFD applications [152]. However, despite its increased computational speed and affordability, it is important to note that advancements in HPC alone is insufficient for HFDO applications unless optimisation and CFD evaluation techniques also evolve by becoming more accurate and efficient to capitalise on this growth [19].

2.6.3. Areas Which Require Attention

It should be noted here that the most important aspect in the selection of performance evaluation techniques for different objective functions is the accuracy of results and evaluation time. One key challenge in applying high-fidelity models such as RANS directly in the optimisation loop is the computational cost. This is especially true for HFDO where most of the search processes are highly iterative and thereby require large numbers of simulations. Another key limiting factor to applying CFD to HFDO is the need to convert 2D data into 3D surfaces, followed by meshing and automatically correcting the error in the modified geometry without human intervention. It is, therefore, useful to apply a performance evaluation tool that allows fast evaluation of a large number of solutions and at the same time provides reasonably accurate results. Such tools can be used for initial hull form designs.

2.7. Selection of Topics for Thesis

Based on the aforementioned identified gaps under each HFDO process, the most important topics for this thesis are listed in next page.

- High dependency on experienced user for automation of the SBD process
- Lack of a flexible geometry modification approach that can generate a wide variety of hull forms using minimal design parameters while simultaneously achieving a feasible design
- High computational cost when EA is coupled with time-intensive CFD evaluation

2.8. Chapter Summary

This chapter provided a background and detailed the state-of-the-art in SBD for HFDO. First, it highlighted the key phases in ship design including concept design, contract design, preliminary design and detail design. Among these, the design of the hull form is an important process that is performed during the initial design stage. Simulation-based design was also introduced in this chapter, followed by hull form design and optimisation. In addition, SBD approaches for each of the key HFDO processes, namely, design exploration, geometry modification and performance evaluation, were reviewed and their advance aspects and areas requiring attention identified. Finally, the areas identified for further investigation were selected for the thesis. These include high dependence on the experience of a user, lack of a flexible geometry modification approach and the high computational cost of CFD analyses.

3. GEOMETRY MODIFICATION THROUGH TWO-DIMENSIONAL CURVE MORPHING

This chapter introduces a novel concept of 2D curve morphing for the modification of hull form geometries. First, the general concept of morphing applied to 2D or 3D space is introduced. Next, some morphing examples applied to ship design are reviewed, followed by an illustration of current 2D representations of ship hull. Finally, the concept of 2D curve morphing is introduced and elaborated on. It includes constant-morph, cross-morph and boundary-morph. By using only 2D curves along transverse sections of the ship, it is demonstrated that the 2D curve morphing method can be applied in HFDO as a useful tool for geometry modification, thus enabling the creation of a wide variety of hull forms more efficiently.

3.1. Morphing Concept

Morphing, also known as metamorphosis, is the sequential construction of intermediate objects that represent the gradual transition from one object to another. It is widely used in the film industry to create animations. It is also applied in computer graphics and industrial design to compute a continuous transformation from a source to a target shape [157]. Morphing is a very useful tool that enables a designer to modify, manipulate and transform the shape or geometry of a design in pursuit of better design attributes such as performance, quality and aesthetics. A good morphing algorithm creates a smooth transition between source and target shapes by generating a series of intermediate shapes that preserve the geometric properties of both input shapes [158]. In this thesis, morphing is carried out in 2D. 2D morphing is particularly suited to ship design and HFDO applications due to its simple representation and modification of hull shape and wide availability of 2D data in the form of offset table. Compared to geometric modification using 3D surfaces, 2D morphing eliminates the need to translate the hull form into 3D surfaces prior to optimisation and performance evaluation.

3.1.1. Two-Dimensional Morphing

In 2D morphing, the source and target shapes can be described by 2D images or planar curves [159]. Image morphing captures the 2D image of an object and transforms the appearance into that of another object by producing intermediate images that are natural-looking [160]. The entire process comprises two main steps. First, the two images are

warped so that they have the same features. Second, the source and target images are cross-dissolved [161]. Warping is an image distortion technique that defines a function for mapping the corresponding pixels in the source to and from the target image [162]. There are several types of warp-based morphing. These include field-based morphing [163], [164], radial basis function [165] and free-form deformation [166]. In contrast, cross-dissolve is a process whereby one image is faded out while another is simultaneously faded in during the morphing of two images [167]. An example of image morphing is shown in Figure 3.1, where a series of intermediate images are created between the source and target images using the warping and cross-dissolving methods.



Figure 3.1 Image morphing using warping and cross-dissolving method (photo adapted from [168]).

There are two types of planar curve morphing, namely, elevation map and curve morphing [169]. The elevation map approach views a 2D image as a single colour map taken as an elevation map over a pixel grid and has important applications in terrain modelling and image morphing [170]. There are two types of curve morphing algorithms. One approach is based on an explicit point correspondence process between the source and target points while the other transforms the problem of curve interpolation into a functional interpolation using partial differential equations [171].

The 2D curve morphing method applied in this thesis is based on the explicit point correspondence process. This is due to the simplicity of its application in ship designs, where the hull from geometries are mainly described in 2D and can be mapped directly using the hull coordinate points from the offset table. This helps in the reduction of steps required for geometry modification, thereby increasing the overall design efficiency. The ability to work in 2D space also enables more hull form varieties to be created, thereby increasing the diversity of the design and generation of solutions with better performance. In view of its versatility and usefulness, 2D curve morphing is therefore selected for

geometry modification in this thesis. 2D curve morphing is described in further details in Section 3.2.

3.1.2. Three-Dimensional Morphing

In 3D morphing, a 3D source model is continuously transformed into a 3D target model. Similar to 2D morphing, the key steps involved in 3D morphing include warping followed by blending using cross-dissolve [172]. This method is used extensively in computer graphics to create special effects, especially in the entertainment industry such as filmmaking [160]. 3D objects can be characterised by two main features, namely, geometry and topology. Geometry determines the shape of an object and features such as size and curvatures while topology determines the structures of the object such as number of holes and disconnected components [173]. Existing 3D morphing techniques can be classified as surface-based or volume-based. In surface-based methods, the correspondence between the source and target surface is first established. This is done by assigning each point on the source surface to a corresponding point on the target surface. In most cases, linear interpolation is then carried out between the pair of corresponding points. This method provides accurate results within a short time and is less computationally intensive, as only the faces, and not the internal points of the object, are transformed. Other non-linear interpolation methods include Bezier splines, cubic and Hermite splines [158]. Conversely, volume-based methods interpolate two 3D volumetric models by transforming both the faces and internal points of the source and target objects [174]. Although volume-based morphing techniques are more versatile than surface-based techniques, the morphing sequence are often computationally expensive and it is harder to control the topology [158].

3.1.3. Morphing in Ship Design

In ship design applications, several researchers have applied morphing to modify the shape of the hull. Tahara applied morphing using a 3D patch (surface) model to transform a ship hull model into a target model [32], [53], [58], [155]. This is done by blending two basic hull forms using the same number of control points. The main advantage of this method is its ability to rapidly transform the shape of an entire hull form. However, the original source and target hull form designs constrain the design variations. Feng applied the ship blending method by merging two sets of NURBS curves into one [47], [90]. The advantage of this method is that it reduces the design variables. However, it requires manual pre-selection of the relevant NURBS curves to be transformed and hence cannot

be automated. In another application, Jacquin used morphing to interpolate between the meshes of old and new hull forms after geometry modifications [41]. In this application, morphing is carried out on the mesh rather than the shape of the hull prior to performance evaluation using CFD.

In another mesh-based morphing, Kang and Lee applied 3D mesh-based surface morphing to generate intermediate hull surface models between two source and target ships [108], [109]. They demonstrated an ability to generate a wide variety of hull surfaces using both interpolation and extrapolation. This method is very useful because it can automatically transform the entire mesh surfaces of two original ships. However, it suffers the same limitation as Tahara's method where the design variations are constrained by the two source and target ship designs. An example of a new hull form generated using this method is shown in Figure 3.2, abstracted from [109]. In this paper, M_1 and M_2 are the original meshes of the 3D hull forms. Firstly, both models are parameterised and mapped into 2D (P_1 and P_2). This is followed by merging into a single mesh (P_M) for easy handling. P_M is then remapped back to 3D (M_1' and M_2') so that they correspond to each other, and interpolation (morphing) can then be performed [109].

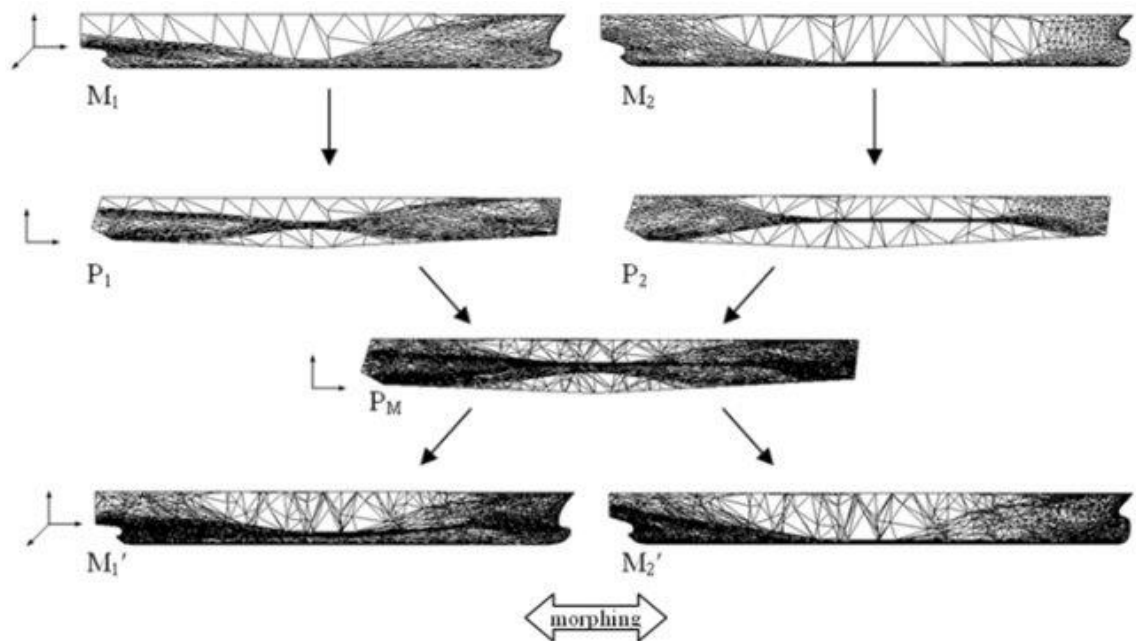


Figure 3.2 3D surface morphing of two hull form using the merging method. [109]

More recently, Hong and Zong [87], [110] applied a self-blending method to modify the cross-sections of the bulbous bow of a fishing vessel and trimaran. This was done by selecting several key coordinate points from the source and target cross-sections and generating a series of new cross-sections by merging in-between the two original cross-

sections. The advantage of this method is that it requires only one principle ship for blending compared to using two ships in previous methods. However, it requires manual input by the designer to pre-select the curves for transformation and is only limited to localised areas with large shape disparity.

3.2. Two-Dimensional Representation of a Ship Hull

Since the beginning of shipbuilding and the introduction of CAD, 2D offset table remains the most fundamental representation of a ship's hull form. Today, 2D offset tables are still widely used in shipyards and ship design firms as an integral part of the ship design process. An offset table comprises key information of the hull geometries that represents the ship's length (frames), breadth (half-breadth) and height (waterline). Figure 3.3 shows an example of a ship and its corresponding coordinate hull geometry points, which are clearly depicted. By linking up the coordinate points from the offset table, the entire shape of the hull can be represented and easily modified by adjusting the lines [8]. It should be noted that 2D representation cannot represent the entire hull form as accurately as a 3D surface, but this inaccuracy is considered negligible if it is only used as a reference during the initial design stage.

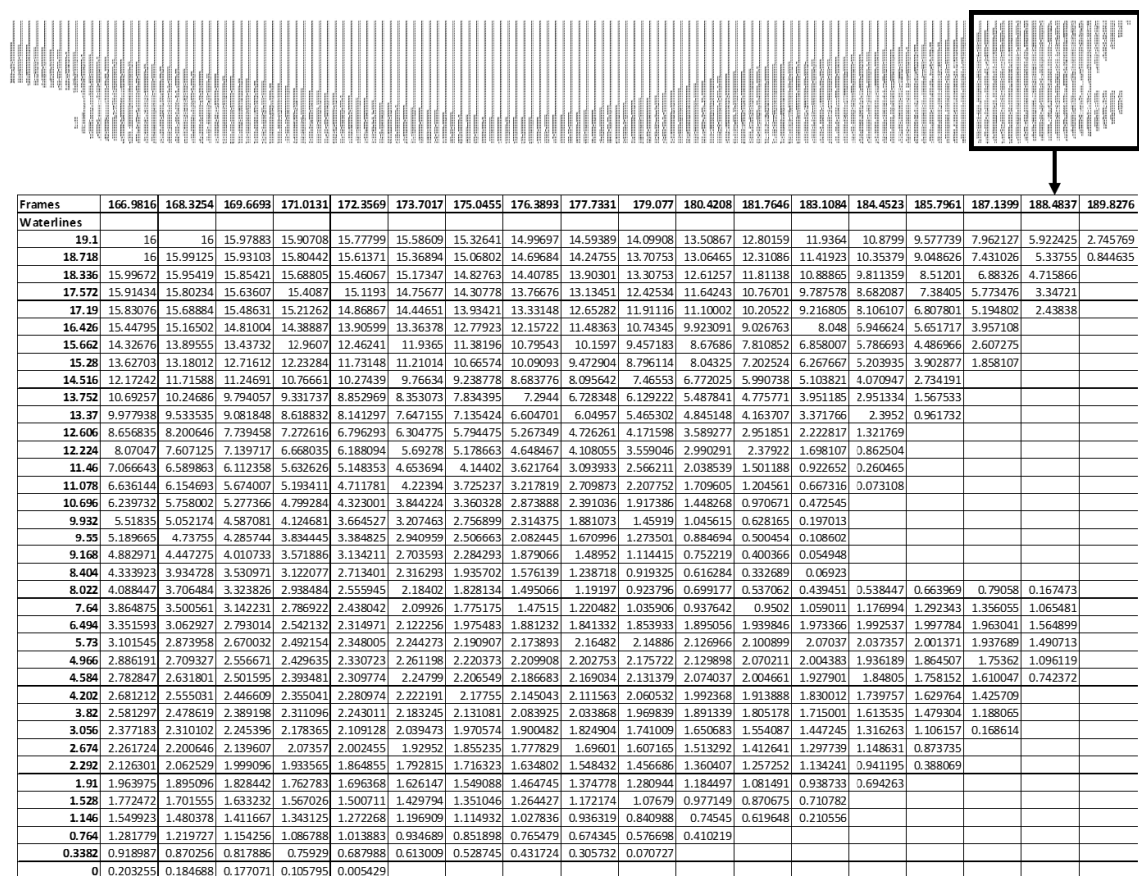


Figure 3.3 Sample of offset table of a ship comprising coordinate points representing the ship's width (half-breadth), length (frames) and height (waterlines) values in metres.

These 2D offset data can be mapped into a 2D lines plan comprising body plan, half-breath plan and sheer plan. To perform hull geometry modification, optimisation and hydrodynamic evaluation in HFDO, the 2D lines plan can be used to construct a 3D model of the hull form using existing CAD tools. An illustration of the mapping between offset table and its corresponding 2D lines plan and 3D hull form is provided in Figure 3.4.

Figure 3.4 shows how the 2D data abstracted from the two sets of offset tables are mapped into 2D lines plan, comprising half-breath plan, sheer plan and body plan. As illustrated, these three lines plans can effectively be used to represent the entire 3D model of the hull form. Following the same principle, the 2D lines plan may also be obtained from the 3D model and translated back into the offset table. Therefore, it can be seen that working in 2D using offset tables provides a simple and efficient way for representing and modifying the hull form.

3.3. Two-Dimensional Curve Morphing

In this thesis, curve morphing based on 2D offset was applied to transform the shape of the hull. The coordinate points from the offset table are connected to form the transverse hull lines, herein referred to as curves. Using Equation 3.1, one or more existing hull forms can be interpolated (or extrapolated) between the source and target curves to generate a series of new curves.

$$M(t) = (1 - t) \times R_0 + t \times R_1 \quad \text{for } -0.1 \leq t \leq 1.1 \quad (3.1)$$

where M is the morphed curve, t is the morphing parameter, R_0 denotes the source curve and R_1 denotes the target curve.

Equation 3.1 shows how a source shape (R_0) can be transformed into a target shape (R_1). Three observations are made. First, when t is set to 0, the morphed shaped $M(t)$ is the same as the source shape, R_0 . This means that no morphing has taken place. Second, when t is set to 1.0, the morphed shape $M(t)$ takes on the form of the target shape, R_1 . Third, by adjusting the value of t , many different shapes may be obtained between the source and target shapes. For values of t in the range of 0 and 1.0, hull lines are interpolated. In this case, any shapes created will be a merger of the source and the target shapes. For t values less than 0 or greater than 1, hull lines are extrapolated and any new shapes produced will be resemble closer to either the source or the target shape depending

2D lines plan and 3D model of hull form

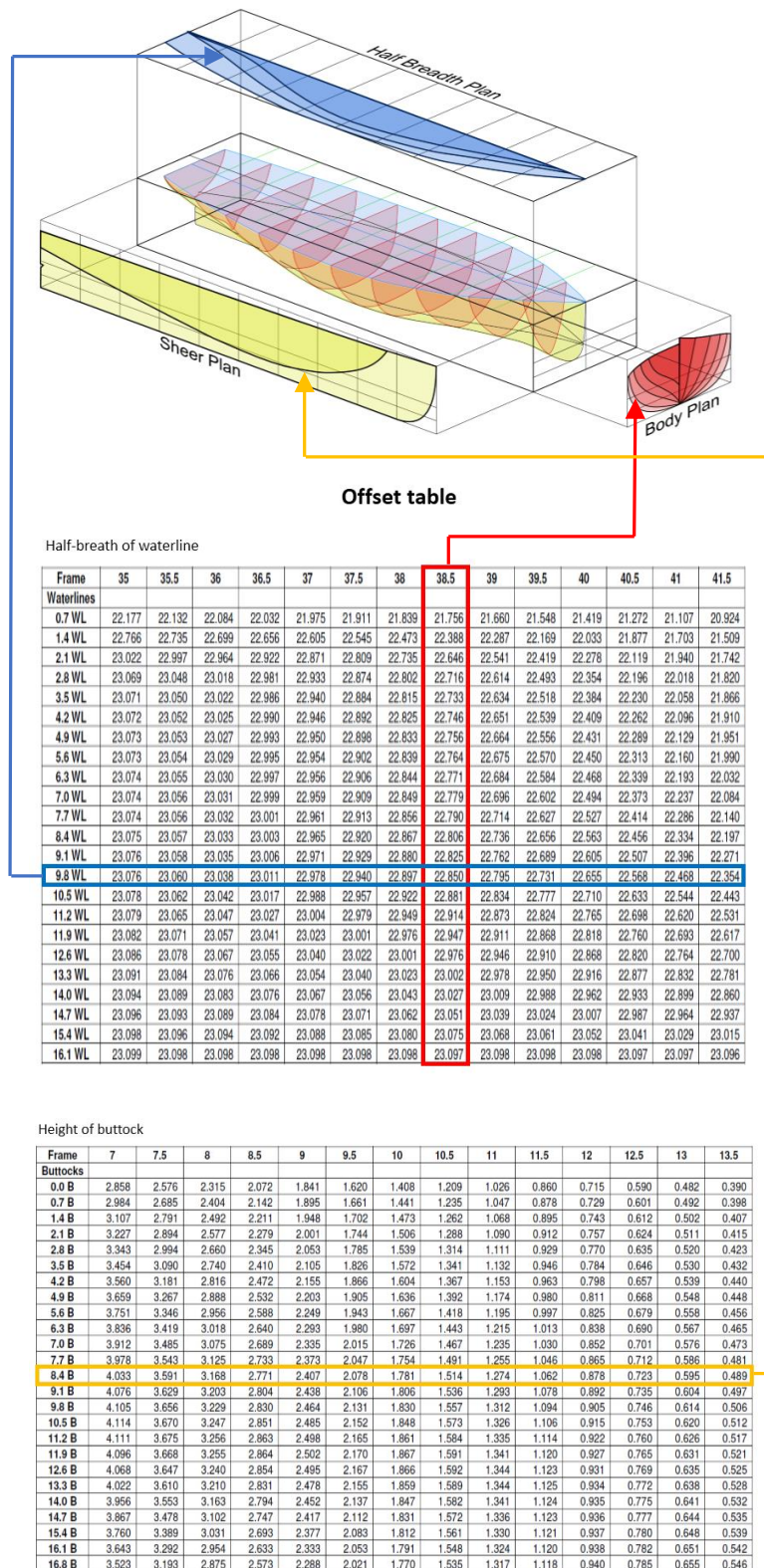


Figure 3.4 Hull form mapping between 2D lines plan and 3D hull form model and the corresponding offset tables (ship lines figure adapted from [175]).

on the value of t . These processes are illustrated in Figure 3.5, where 2D curve morphing is applied to morph (interpolate) two vessels, ship A (source) and ship B (target).

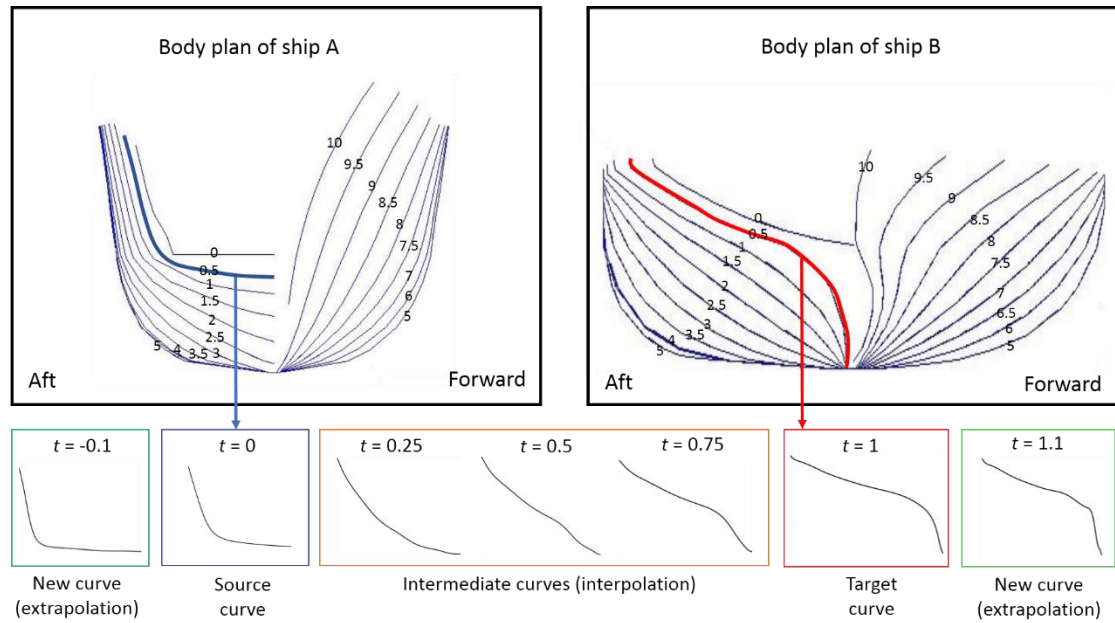


Figure 3.5 Intermediate (interpolation) and new hull curves (extrapolation) generated via 2D curve morphing between source curve from ship A and target curve from ship B at a station number of 0.5 [176].

As shown in Equation 3.1, the morphing process is linear where regular and ‘expected’ intermediate shapes are produced. Non-linear interpolation methods such as Gaussian distribution or additional weightage, can also be applied if more random outputs (shapes) are required. This is further elaborated in Section 3.6.

In Figure 3.5, the morphing parameter is applied to a hull line extracted from station 0.5 (see bold lines) in each vessel. It can be seen that by adjusting the morphing parameter t , a range of different shapes can be obtained. In this example, the blue line represents the curve from source vessel (ship A) and the red line represents the original transverse curve from target vessel (ship B). By applying different morphing parameter (t) between $t = 0$ and $t = 1$ (in this case $t = 0.25$, 0.5 and 0.75), three intermediate curves are created by interpolating between the source (Ship A) and target curves (ship B), as illustrated in the orange box. By setting the morphing parameter $t < 0$ or $t > 1$ (in this case $t = -0.1$ and 1.1), new curves are created through extrapolating beyond the source (Ship A) and target curve (ship B), as illustrated in the green box. This example demonstrates the usefulness of the 2D curve morphing, where the entire hull form or selected transverse section between the source and target ships can be transformed easily by simply changing the morphing parameter (t) along the transverse frames.

Nonetheless, it is recognised that the amount of modifications which can be made by interpolating and extrapolating hull lines may be limited. For example, in existing applications, morphing (interpolating and extrapolating) are applied on the entire hull form and hence the new hull forms are very ‘similar’ to the source and target ships. In order to improve its versatility, three variations of 2D curve morphing are developed in this thesis. These are presented in detail in Sections 3.4 to 3.6.

3.3.1. Regularisation of Hull Forms

As explained in Section 3.2, an offset table contains all the key data about a vessel’s geometry and different hull forms will have different offset tables. The main differences may be in the number of coordinate points or where the coordinate points are located (section and waterline levels). Therefore, it is important that prior to morphing, all vessels are regularised so that the corresponding points between source and target vessels are mapped to one another. In this thesis, cubic spline [114] interpolation was used to create the same number of points across all section curves of vessels. This method was selected due to its simplicity and fast discretisation. However, it is acknowledged that 2D-based cubic spline interpolation may result in lost accuracy and misrepresentation of hull shape when transformed into 3D. Other non-linear curve fitting techniques such as B-spline surface fairing and feature-preserving smoothing [177] can also be applied as alternatives for regularisation. These are recommended for future work.

The general formula for cubic spline interpolation is given in Equation 3.2.

$$S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i ; \text{ for } x \in [x_i, x_{i+1}] \quad (3.2)$$

where $S_i(x)$ denotes the spline and $[x_i, y_i]$ represents a table of points for $i = 0, 1, \dots, n$ for a function, $y = f(x)$.

As an example, the source curve from ship A contains 19 coordinate points while the corresponding target curve from ship B contains 14 coordinate points in their respective offset tables. To morph the sectional curve for both ship A and B, the coordinate points must correspond to each other by having the same number of coordinate points for both the source and target curves.

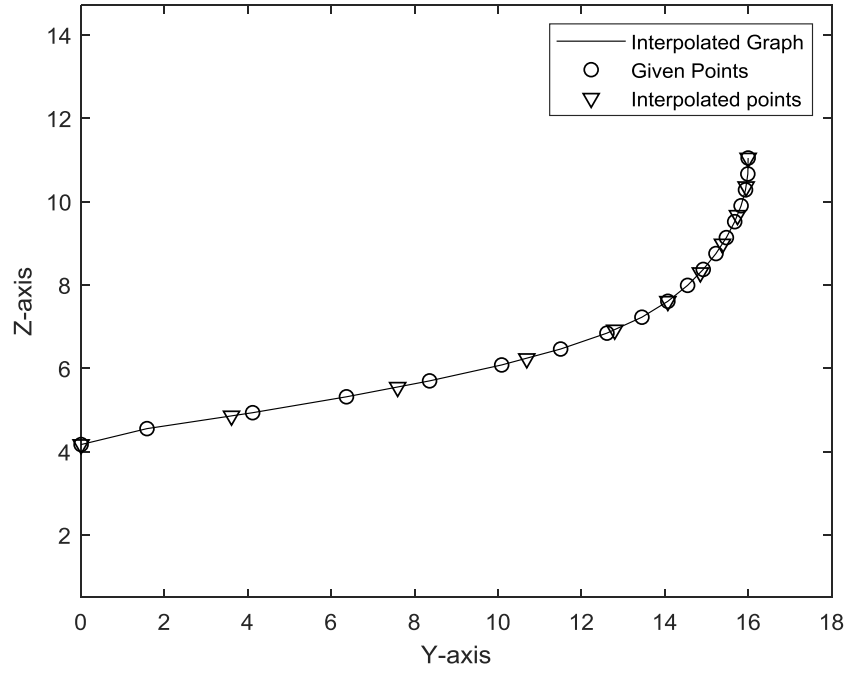


Figure 3.6 Curve correspondence using cubic spline interpolation.

As illustrated in Figure 3.6, 11 equidistant (based on waterline or Z-axis values) coordinate points are created using cubic spline interpolation for ship A, reducing the initial 19 coordinate points. The process is repeated for ship B to reduce the number of coordinate points from 14 to 11. In this manner, both ship A and B will have the same number of coordinate points, making them correspond to each other to enable the application of 2D curve morphing.

3.3.2. Application of 2D Curve Morphing to Hull Form Geometry Modification

To apply 2D curve morphing to hull form geometry modification, the entire 2D hull data can be simplified and mapped into a unique representation which allows the hull form to be modified using the morphing parameter t at each frame or station of the vessel. This is an important feature to ensure that 2D curve morphing can be applied to HFDO to create more diverse hull forms efficiently. As an example, the hull form is first divided into 21 frames. In each frame (or station) the morphing parameter, t , is a real-number which determines the extent to which the 2D hull lines within the frame are morphed. Figure 3.7 provides an illustration of the correspondence between two ships, its station number and morphing parameters.

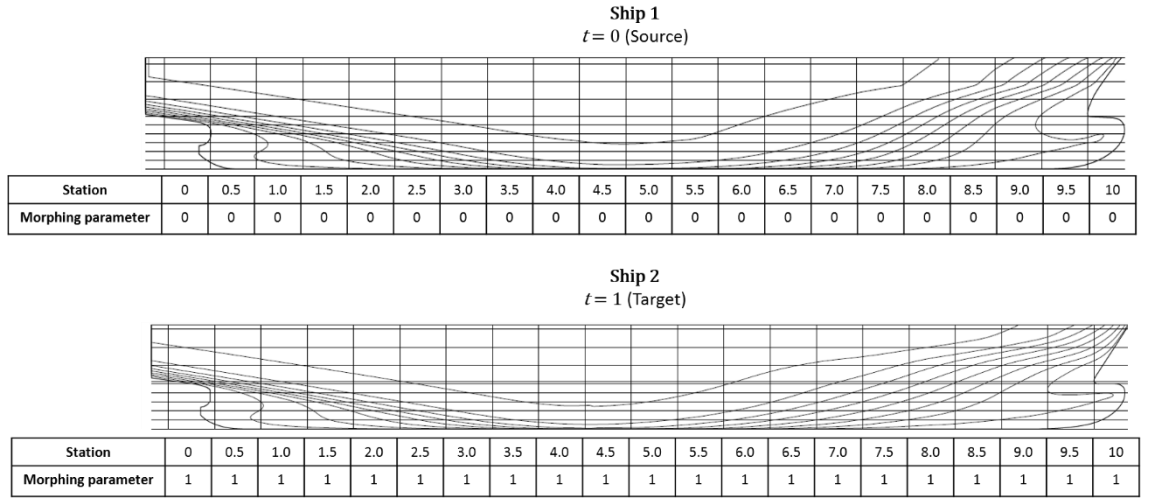


Figure 3.7 Illustration of two existing vessels (ship 1 and ship 2) and its corresponding station number and morphing parameters t .

In Figure 3.7, the geometries of two vessels are mapped onto their corresponding morphing parameters t , by setting $t = 0$ across all stations for ship 1 (source) and $t = 1$ across all stations for ship 2 (target). Using this representation, the geometry of the hull can easily be modified by changing the morphing parameters for each frame. In this thesis, three novel 2D curve morphing methods are introduced, namely, constant-morph, cross-morph and boundary-morph.

3.4. Constant-Morph

Constant-morph is applied in this thesis to transform the entire hull form using the same morphing parameter t . By applying constant (non-varying) morphing parameter t across the entire length of the ship, a considerable number of intermediate designs can be generated by interpolating the offset points from 2D curves between the source and target hull forms. This approach is formulated using a modified morphing equation, as shown in Equation 3.3.

$$Y(t)_{C_{x,z}} = (1 - t) \times Y_{S_{x,z}} + t \times Y_{T_{x,z}} \quad \text{for } -0.1 \leq t \leq 1.1 \quad (3.3)$$

where t is the morphing parameter, $Y(t)_{C_{x,z}}$ is the new Y value of the child ship at corresponding x and z coordinates, $Y_{S_{x,z}}$ is the Y value of the source ship at corresponding x and z coordinates and $Y_{T_{x,z}}$ is the Y value of the target ship at corresponding x and z coordinates. x is given as the distance from the aft perpendicular (AP) section in X - directions and z is the distance from baseline in Z - directions.

Here, the new Y value is obtained through interpolation between $Y_{S_{x,z}}$ and $Y_{T_{x,z}}$ when $0 \leq t \leq 1$. Conversely, extrapolation between $Y_{S_{x,z}}$ and $Y_{T_{x,z}}$ is achieved for $0 > t > 1$. To ensure feasibility of the extrapolation, a constraint is introduced to ensure that the value of t falls within the range between or equal to -0.1 and 1.1 .

To demonstrate the application of constant-morph, Equation 3.3 was applied to generate a series of ‘new’ hull form designs by morphing between two existing hull forms, as illustrated in Figure 3.8. Here, the source ship is shown (Figure 3.8(a)) in blue ($t = 0$) and the target ship is shown (Figure 3.8(e)) in red ($t = 1$). Intermediate hull forms between source and target ships are created by varying the morphing parameter, t , between -0.1 to 1.1 with typical values between 0 and 1 as shown (Figures 3.8(b) to (d)).

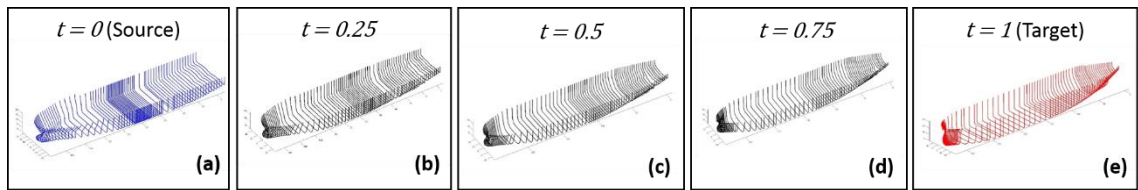


Figure 3.8 Wire mesh diagrams of (a) source ship ($t = 0$), (e) target ship ($t = 1$) and (b) to (d) intermediate ships ($t = 0.25, 0.5$ and 0.75) created using constant-morph [176].

The above example demonstrated that the entire hull form can be modified uniformly by changing the morphing parameter t across the entire length of the ship. In addition to interpolating the offset points between source and target ships, one can also extrapolate beyond the existing ships using constant-morph to create more ‘new’ designs. This morphing concept is similar to earlier works by other authors [47], [53], [58], [90], [108], [109], [155], however in this thesis, morphing is carried out directly on 2D curves rather than on 3D surfaces or meshes.

3.5. Cross-Morph

To create more variations of hull forms to increase the variety of solutions, a second 2D curve morphing method developed in this thesis is the cross-morph method. Cross-morph works by splitting hull forms from two different ships at a given transverse section and combining them by morphing their hull lines together. This can be achieved by setting a linear-varying morphing parameter t at different transverse curves along the stations or frame lines (x -planes) along the ‘split’ or interface sections. The working principle of cross-morph is illustrated in Figure 3.9.

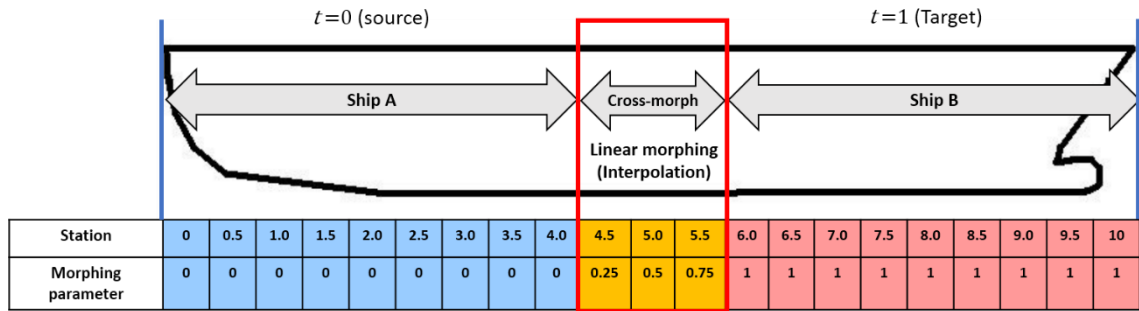


Figure 3.9 Working principle of cross-morph concept.

Cross-morph works using the ‘cut-and-join’ concept [178]. Essentially, the hull body of two vessels are ‘split’ along the transverse frame (or cross-section) of the vessel and joined together seamlessly resulting in a smooth transition at the joining site as shown in Figure 3.9. This is achieved through the following steps. Firstly, two principal vessels are assigned with two morphing parameters: $t = 0$ for source vessel (vessel A) and $t = 1$ for target vessel (ship B). This is similar to the constant-morph concept introduced in Section 3.4. Both vessels are then split in their mid-body based on a randomly selected site (in this case station 5) and joined together. In this example, ship B’s forward body are joined with ship A’s aft body.

Next, a range of linear-varying morphing parameter t is applied to provide a smooth transition from ship A ($t = 0$) to ship B ($t = 1$). As shown in Figure 3.9, the crossover section (in red box) falls into Stations 4.5 to 5.5. Here, varying parameters are set to each station by interpolating between the aft and forward ‘split’ cross-section of ship A and B respectively. The result is a smooth change in morphing parameters t as shown in orange in the morphing scheme. It should be noted the width of the transition frames (or stations) will vary depending on the difference in sizes between the two reference vessels. This width can be shorter for vessels that are very close in sizes and longer for vessels that are very different. As a general guide, the transition frames can be set at 10% of a vessel’s length (L_{pp}) for similar sized vessels to 90% of a vessel’s length (L_{pp}) for vessels of very different sizes. This is to ensure sufficient smoothing at the transition frames (or stations), which may otherwise make the end product infeasible.

The mathematical formula for cross-morph is obtained by extending the modified morphing equation, as presented in Equations 3.4 to 3.6.

For the aft body section (from source vessel) between Aft Perpendicular (AP) and Aft Transition (AT) frame:

$$Y(t = 0)_{x,z} = Y_{S_{x,z}} \quad \text{for } x_{AP} \leq x \leq x_{AT} \quad (3.4)$$

For forward body section (from target vessel) after Forward Transition (FT) frame and Forward Perpendicular (FP):

$$Y(t = 1)_{x,z} = Y_{T_{x,z}} \quad \text{for } x_{FT} \leq x \leq x_{FP} \quad (3.5)$$

For mid-body section between Aft Transition (AT) and Forward Transition (FT) frames:

$$Y(t = x_{TS} \times W)_{TS_{x,z}} = (1 - t) \times Y_{S_{x,z_{AT}}} + t \times Y_{T_{x,z_{FT}}} \quad (3.6)$$

for $x_{AT} \leq x_{TS} \leq x_{FT}$, $W = 0.1 \times (x - x_{AT})$

Where t is the morphing parameter, $Y_{S_{x,z}}$ is the Y value of the source ship at corresponding x and z coordinates, $Y_{T_{x,z}}$ is the Y value of the target ship at corresponding x and z coordinates, and $Y(t = x_{TS} \times W)_{TS_{x,z}}$ is the new Y value of the transition section at corresponding x and z coordinates. x is the distance from the AP section in x - directions, z is distance from baseline in z - directions, x_{AT} is x frame at the aft transition section, x_{FT} is x frame at the forward transition section, x_{TS} is x frame between x_{AT} and x_{FT} , x_{AP} is x frame at the aft perpendicular (AP) section, x_{FP} is x frame at the forward perpendicular (FP) section and W is the weightage.

Using ships A and B as an example, the body of both ships are ‘cut’ at mid-ship (mid body) before combining them, as shown in Figure 3.10. In this case, the aft of ship A and forward of ship B are merged, as shown in Figure 3.10(c).

From Figure 3.10, one can clearly see that the model cannot be realised by simply ‘cutting and joining’ two unrelated hull forms together because the forward and aft body of the vessels are asymmetrical. However, by applying a morphing parameter t that is gradually varied along different stations (X -axis), a smooth transition can be created. In Figure 3.11, cross-morph is applied to the mid body of the ship by linearly changing the morphing parameter t from 0.1 to 0.9 from Stations 3 to 7 (highlighted in orange) of the vessel. Note that there are no changes to the curve for ship A (source) when $t = 0$ and the curve for ship B (target) when $t = 1$.

The hull shape can be modified to take on more of ship A or more of ship B by varying the morphing parameter t at different parts of the ships, as illustrated in Figure 3.12. Note that there are no changes to the lines (or shape) of the aft body of ship A when cross-morph is applied to the forward body of ship B, as seen in Figure 3.12(a). Similarly, there

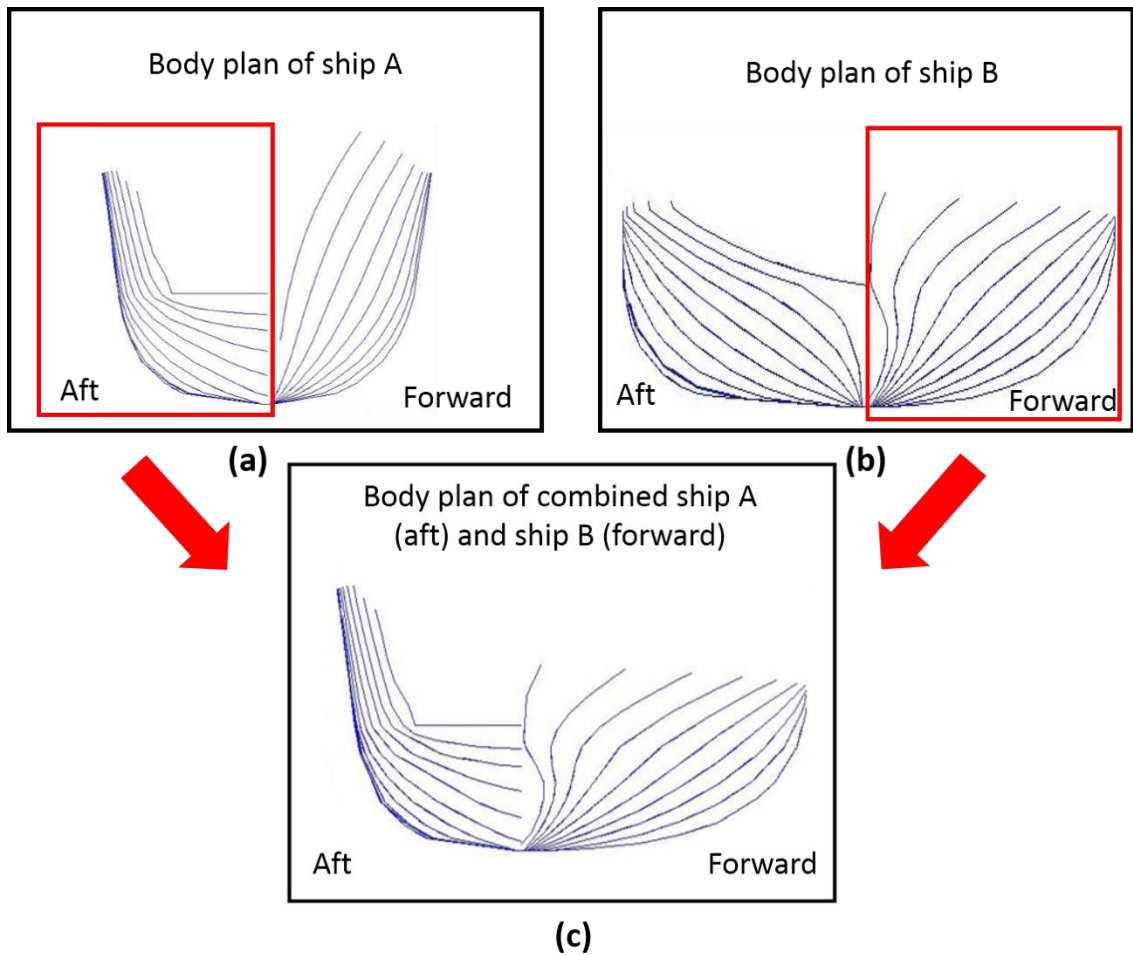


Figure 3.10 Combination of the aft body of ship A and the forward body of ship B.

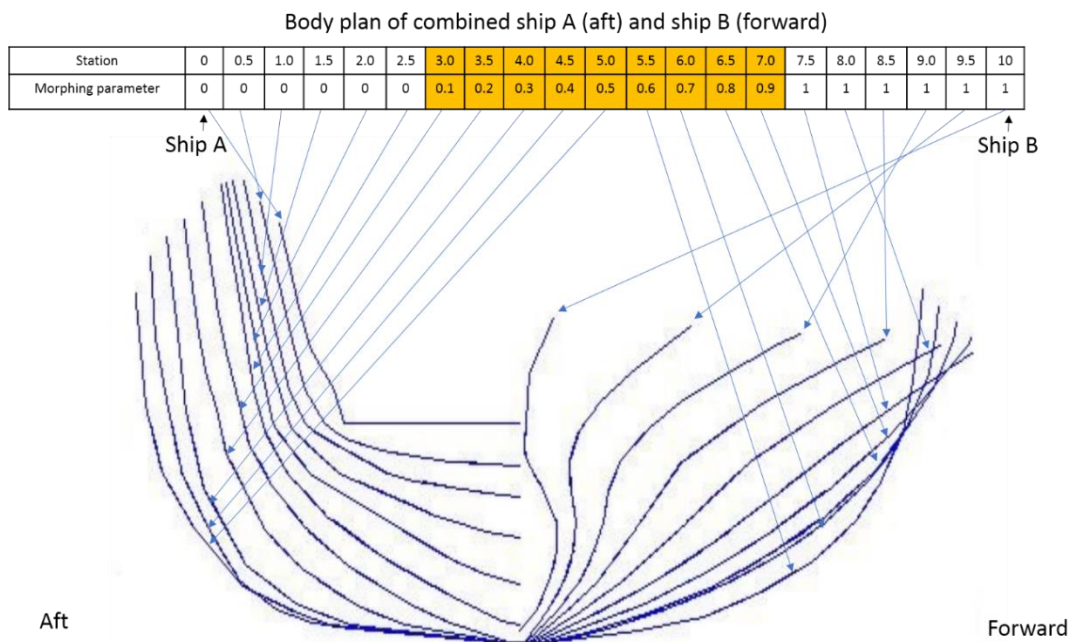


Figure 3.11 Cross-morph applied to mid body of combined ship A (aft) and ship B (forward).

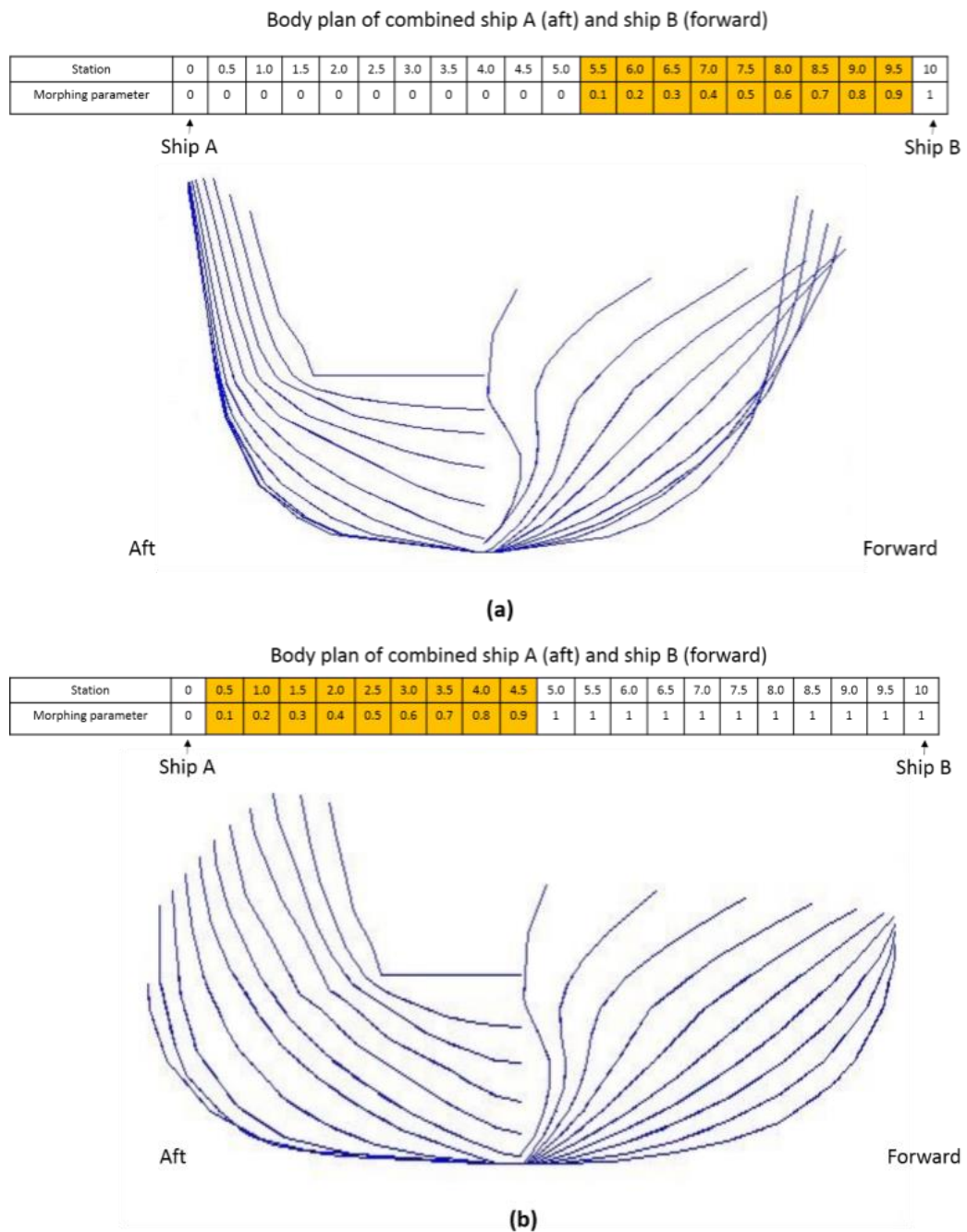


Figure 3.12 Cross-morph applied to (a) forward body of combined ship A (aft) and ship B (forward) and (b) aft body of combined ship A (aft) and ship B (forward).

are no changes to the forward body of ship B when cross-morph is applied to the aft body of ship A, as observed in Figure 3.12(b).

This approach provides great flexibility for changing the hull shape at any location of the ship while maintaining a smooth transition. Using the same principle, one can also ‘cut and join’ two or more ships at different sections, such as morphing the aft body of ship A to ship B mid body and the forward body of ship C. This is illustrated in Figure 3.13.

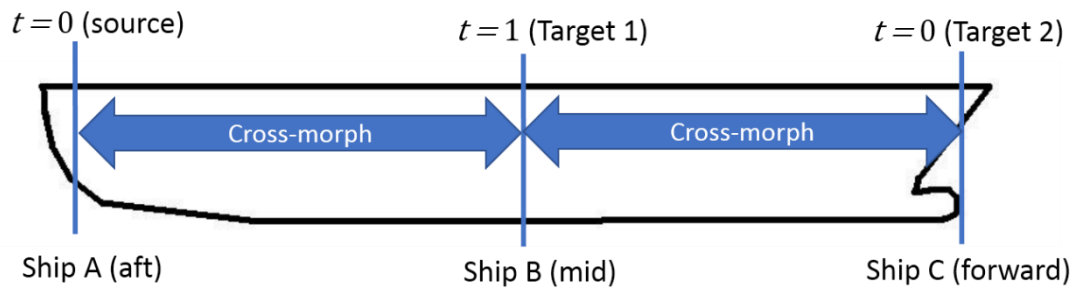


Figure 3.13 Combination of the aft body of ship A, mid body of ship B and forward body of ship C.

So far, cross-morph has been applied to merging vessels of similar sizes using either single-point or multi-point approach. The technique can also be extended to vessels of varying sizes. This is discussed in Sections 3.5.1 and 3.5.2.

3.5.1. Application of Single-Point Cross-Morph to Vessels of Varying Sizes

While cross-morph may be directly applied to geometrically similar ships, it can also be applied to ship sizes that are vastly different. To illustrate this point, two existing ships: a volume carrier (Very Large Crude Carrier or VLCC) and an offshore vessel (pipe-laying) are combined using cross-morph. The principal dimensions for both ships are provided in Table 3.1.

Table 3.1 Principal dimensions of a VLCC and a pipe-laying vessel

	VLCC (m)	Pipe-laying (m)
Length overall (LOA)	327	182
Length between perpendiculars (LPP)	314	168
Breadth (B)	58	46
Height (H)	31	23
Design draft (T)	20	11

The isometric views of both hull lines are illustrated in Figure 3.14.

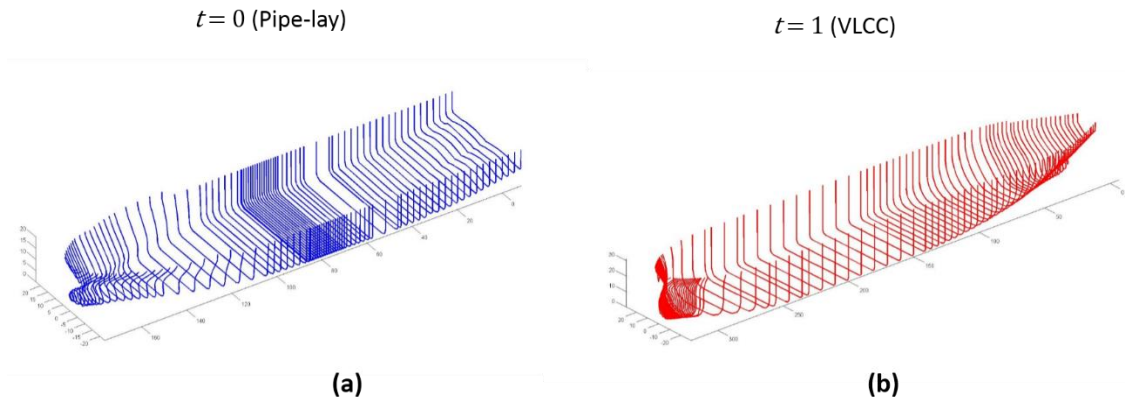


Figure 3.14 Wire mesh diagram of two existing ships (a) Pipe-laying (blue) and (b) VLCC (red).

One possible way to generate new hull combinations or variations is by ‘splitting’ both ships in the mid-ship section and combining them as illustrated in Figure 3.15.

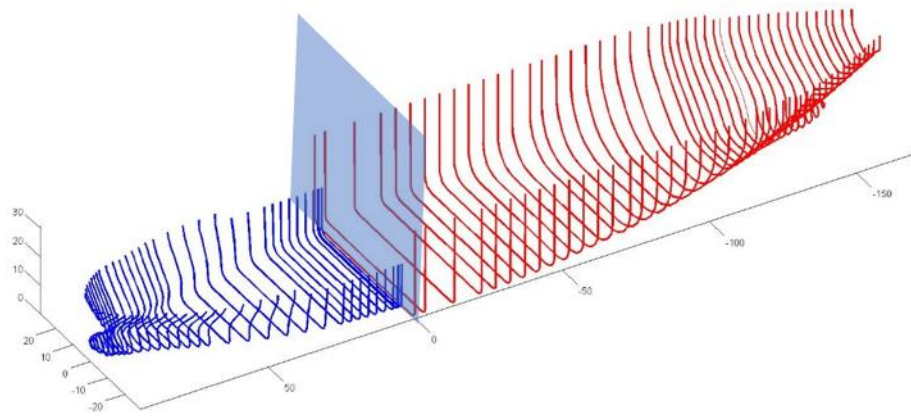


Figure 3.15 Combination of pipe-laying forward body and VLCC aft body. The shaded plane shows the intersection point.

From Figure 3.15, it is clear that a ‘cut and join’ model is not feasible as it results in sudden changes in the mid body. As a consequence, the forward and aft bodies of the end product would be asymmetrical rendering the design infeasible. This problem is resolved using cross-morph, where the morphing parameter t is applied on multiple curves along different stations (x -axis). This provided a gradual modification in joining the two vessels, as shown in Figure 3.16. Here, the values of t vary from 0.1 to 0.9 for Stations 1 to 9 of the vessels, respectively. These are highlighted in black, as shown in Figure 3.16. The original sections of the source, ship A (pipe-laying) and the target, ship B (VLCC) are highlighted in blue and red respectively. The shaded plane shows the two intersection points.

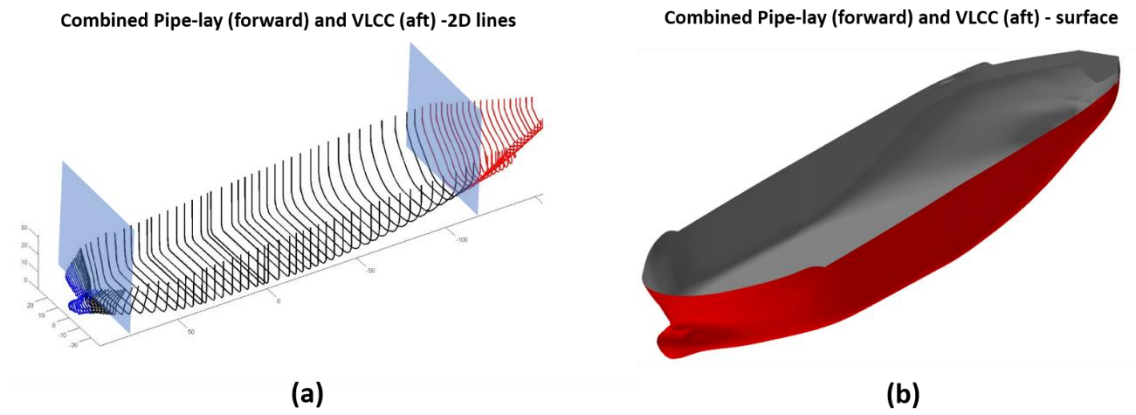


Figure 3.16 (a) Wire mesh diagram and (b) 3D surfaces of combined ship using cross-morph [171].

Figure 3.16(a) shows how two different hull bodies can be smoothly ‘joined’ together using cross-morph, even if their sizes are vastly different. To demonstrate the feasibility of the ship created using cross-morph, the surface model of the same vessel is created and shown in Figure 3.16(b). To improve the shape changes in transition area between the two intersections points, an additional smoothing function can be incorporated which is elaborated in Section 3.6.

3.5.2. Application of Two-Point Cross-Morph to Vessels of Varying-Sizes

Besides combining two ships, three different types of ships can equally be combined using two-point cross-morph. Two-point cross-morph was introduced in this thesis to increase the number of vessel combinations, thereby leading to more varied designs. To prevent drastic changes in the transition frame which may lead to infeasible designs, the main dimensions (length, breath and height) of selected vessels were rescaled to similar sizes before a two-point crossover was applied. As an example, two-point crossover was applied to create a new hull form by merging the hull forms of three different vessels, namely, a container vessel, a bulk carrier and an oil tanker. Prior to cross-morphing, each vessel is regularised using linear proportional approach to rescale the ships to the same principal dimensions. Subsequently, the vessels are ‘split’ into three separate sections and ‘joined’ using cross-morph. This process is illustrated in Figure 3.17 where the new hull form took the aft body from the container, the mid body of the bulk carrier and the forward body of the oil tanker. The isometric views of the new hull are shown in Figure 3.18.

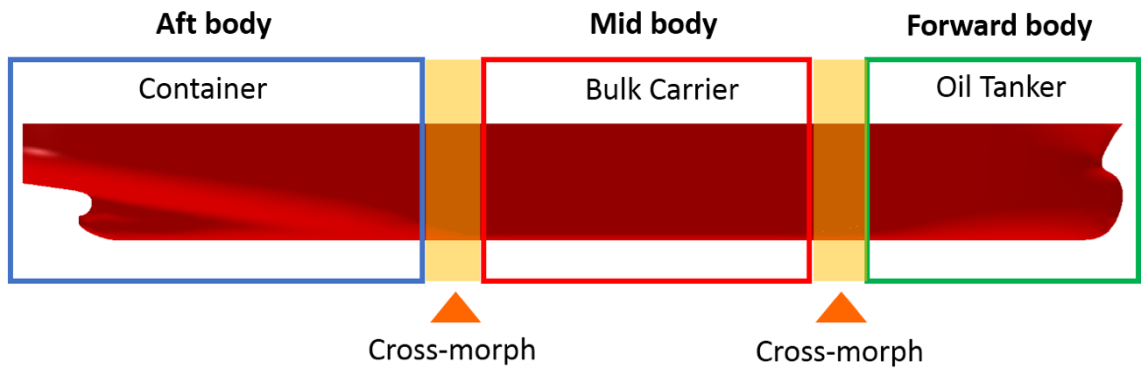


Figure 3.17 Combination of three different ship types using cross-morph [179].

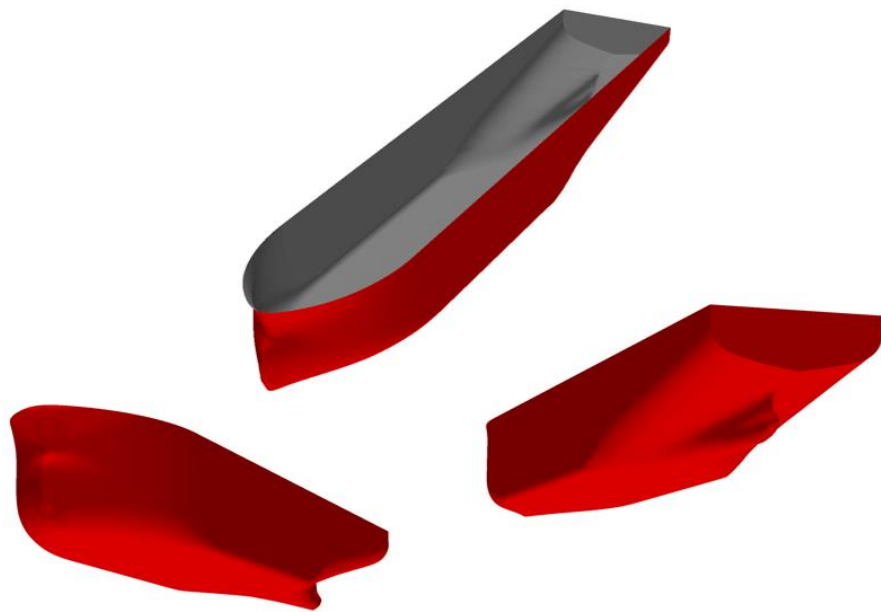


Figure 3.18 Isometric view of a ship incorporated from different sections of a container vessel, a bulk carrier and an oil tanker.

In the same manner, other hull forms can be created by mixing and matching sections of different vessels, as shown in Figure 3.19. Although the approach can only produce new designs based on existing ones, it has demonstrated its ability to seamlessly mix-and-match different vessel types, which hitherto could only be achieved by a human designer. A potential application could be incorporating this into a design software where a designer can rapidly explore different combinations of vessels.

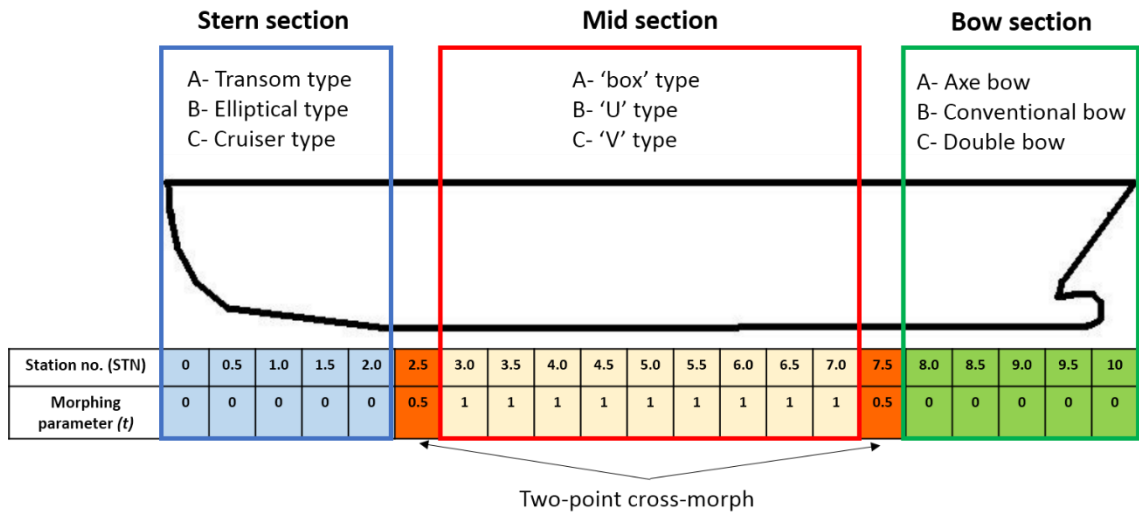


Figure 3.19 Mix-and-match of different designs using multi-point cross-morph.

3.6. Boundary-Morph

From the two 2D curve morphing methods introduced earlier, at least two or more reference vessels are required in order to produce new designs. This may not be practical where initial designs are limited. A novel boundary-morph method, an extension of 2D curve morphing, is developed in this thesis to modify the shape of hull forms using only one reference vessel. This is achieved by interpolating between the transverse sections of the maximum boundary of the forward, mid and stern sections of the principal vessel. Boundary-morph works by splitting the hull into two sections - forward and aft as shown in Figure 3.20.

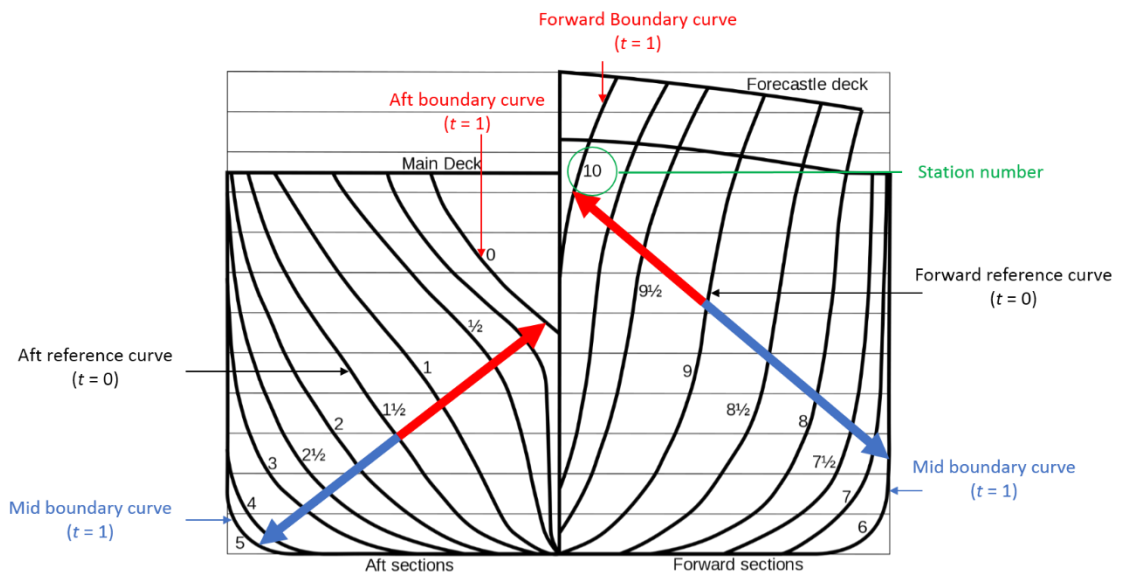


Figure 3.20 Boundary-morph for aft and forward sections with outward shifting indicated by the blue arrows and inward shifting indicated by the red arrows.

The workflow of boundary-morph is illustrated in Figure 3.21 and elaborated below.

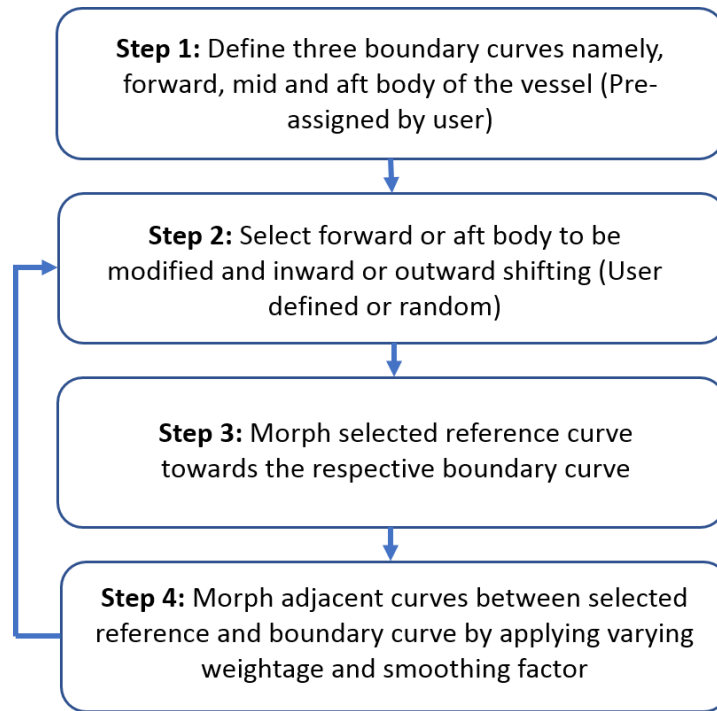


Figure 3.21 A step-by-step explanation of boundary-morph.

Firstly in step 1, three boundary curves were pre-defined: forward, mid and aft body of the vessel. This can be a one-time specification for all scenarios depending on user requirements.

In step 2, the i) forward body or aft body to be modified and ii) outward or inward shifting are selected. For forward body modification, the reference curve is set as the source curve ($t = 0$) and the boundary curve in mid body is set as the target curve ($t = 1$) for inward shifting. In the case of outward shifting, the reference curve is set as the source ($t = 0$) and the boundary curve in the forward body is set as the target ($t = 1$).

Similarly, for aft body modification, the reference curve is set as the source curve ($t = 0$) and the boundary curve in the aft body is set as the target curve ($t = 1$) for inward shifting. When outward shifting is applied, the reference curve is set as the source curve ($t = 0$) and the boundary curve in mid-body is set as the target curve ($t = 1$). The selection of the reference curve in the forward or aft body and inward or outward shifting can be defined by the user or selected randomly by the computer.

In step 3, the selected reference curve is morphed towards the respective boundary curve depending if it is inward or outward shifting. Finally, in step 4, the adjacent curves between the reference curve and boundary curve will also be morphed by applying varying

weightages and smoothing factors. This process is repeated to automate the shape modification process for HFDO application.

To illustrate its working principles, boundary-morph was applied to modify the hull form of a container vessel. Firstly, three boundary curves are selected as forward curve (Station 10), mid curve (Station 5) and aft curve (Station 0) as shown in Figure 3.22. The settings of the morphing parameter t for each curve at the respective stations are also provided.

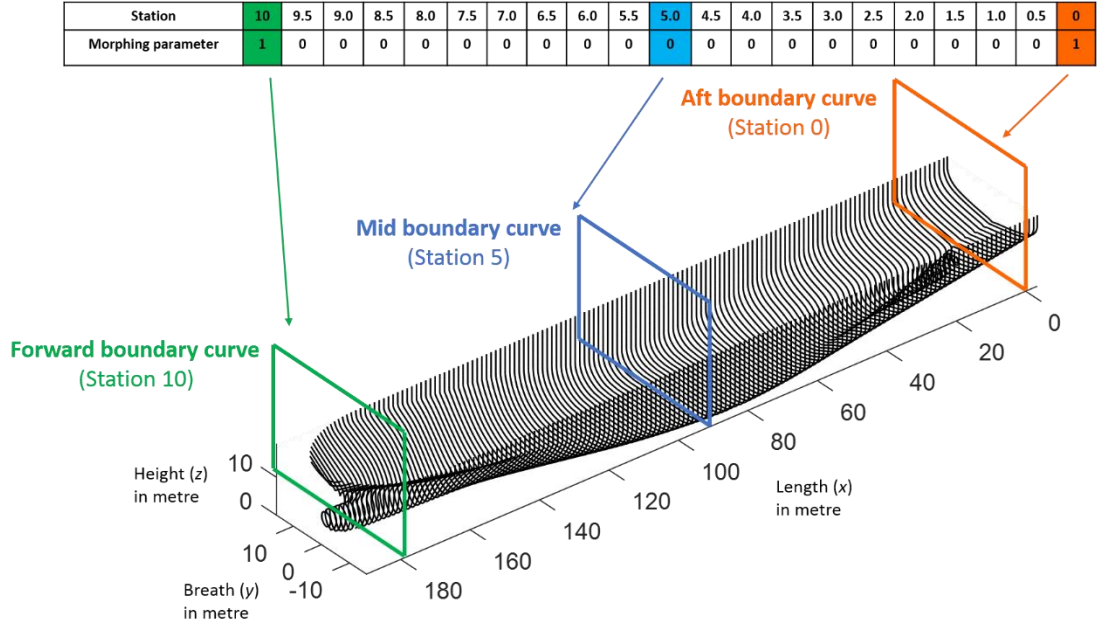


Figure 3.22 Wire-mesh diagram of original hull form of container ship with boundary curve set at station 10 (forward), station 5 (midship) and station 0 (aft).

The mathematical formula for boundary-morph uses the extended morphing equation, as presented in Equations 3.7 to 3.9.

For morphing the reference frame between the Mid-Boundary (MB) curve (source) to the Forward or Aft Boundary (F/AB) curve (target):

$$Y(t = x_R \times W)_{R_{x,z}} = (1 - (t \times S_{TS})) \times Y_{S_{x,z}} + (t \times S_{TS}) \times Y_{T_{x,z}}$$

where $W = 0.02 \times (x - x_{AB})$ when $x_{AB} \leq x \leq x_{MB}$ and $W = 0.02 \times (x - x_{MB})$ when $x_{MB} \leq x \leq x_{FB}$ (3.7)

For morphing adjacent frames between the Reference (R) curve (source) to the Mid-boundary (MB) or the forward and aft boundary (F/AB) curve (target):

$$Y(t = x_A \times W)_{A_{x,z}} = (1 - (t \times S_{TS})) \times Y_{S_{x,z}} + (t \times S_{TS}) \times Y_{T_{x,z}}$$

where $W = 0.04 \times (x - x_R)$ when $x_R \leq x \leq x_{MB}$ and $x_R \leq x \leq x_{FB}$

$$, \text{ and } W = 0.04 \times (x_R - x) \text{ when } x_{AB} \leq x \leq x_R$$

$$, \text{ and } W = 0.04 \times (x - x_{MB}) \text{ when } x_{MB} \leq x \leq x_R \quad (3.8)$$

$$\text{subject to } 0.2 \leq t \leq 0.8 \text{ and smoothing factor, } S_{TS} = 1 - \frac{Y_{TS}}{B_{max}} \quad (3.9)$$

where $Y(t)_{R_{x,z}}$ is the new Y value for the reference curve at corresponding x and z coordinates, $Y(t)_{A_{x,z}}$ is the new Y value for adjacent curves at corresponding x and z coordinates, $Y_{S_{x,z}}$ is the Y value for the reference (source) curve at corresponding x and z coordinates and $Y_{T_{x,z}}$ is the Y value for the boundary (target) curve at corresponding x and z coordinates. x_A is the x coordinate of the adjacent frame between the reference frame and boundary frames, $x_{F/AB}$ is x frame at the forward or aft boundary section, x_R is x frame at the reference frame section and x_{MB} is x frame at the mid boundary section. W is the weightage, Y_{TS} is the Y value of the transition section between the two boundaries and B_{max} is the maximum $\frac{1}{2}$ breadth.

From Equations 3.7 and 3.8, the weightage factor allows a gradual increase of morphing parameter t until it reaches the target curve. In the case of the smoothing factor, shown in Equation 3.9, it is applied to inject the non-linearity function into the morphing function so as to ensure smooth transitioning between the X-directions. This is achieved by benchmarking the Y -value of the selected frame with respect to the largest Y -value of the vessel (maximum half-breadth) at any given X -location.

Four scenarios are considered as shown in Figure 3.23. For the first scenario shown in Figure 3.23(a), a forward reference curve is selected at Station 7.5 and morphed towards forward boundary curve at Station 10 by setting the morphing parameter to $t = 0.5$. At the same time, it can be observed that the adjacent curves of the forward reference curve were also changed concurrently through varying weightage and smoothing factor (Equation 3.10) that was applied to change the morphing parameter t gradually for each curve corresponding to the morphing scheme. This resulted in a more slender hull form at the forward section of the vessel.

Using the same concept, the aft section was modified by selecting the aft reference curve (Station 2.5) and morphed towards the aft boundary curve (Station 10) with the morphing parameter set to $t = 0.5$, in the second scenario shown in Figure 3.23(b). This resulted in a more slender hull form at the aft section. In third scenario shown in Figure 3.23(c), the same forward reference curve (Station 7.5) is morphed towards the mid boundary curve

(Station 5) with the morphing parameter set to $t = 0.5$, resulting in a much fuller hull form at forward section.

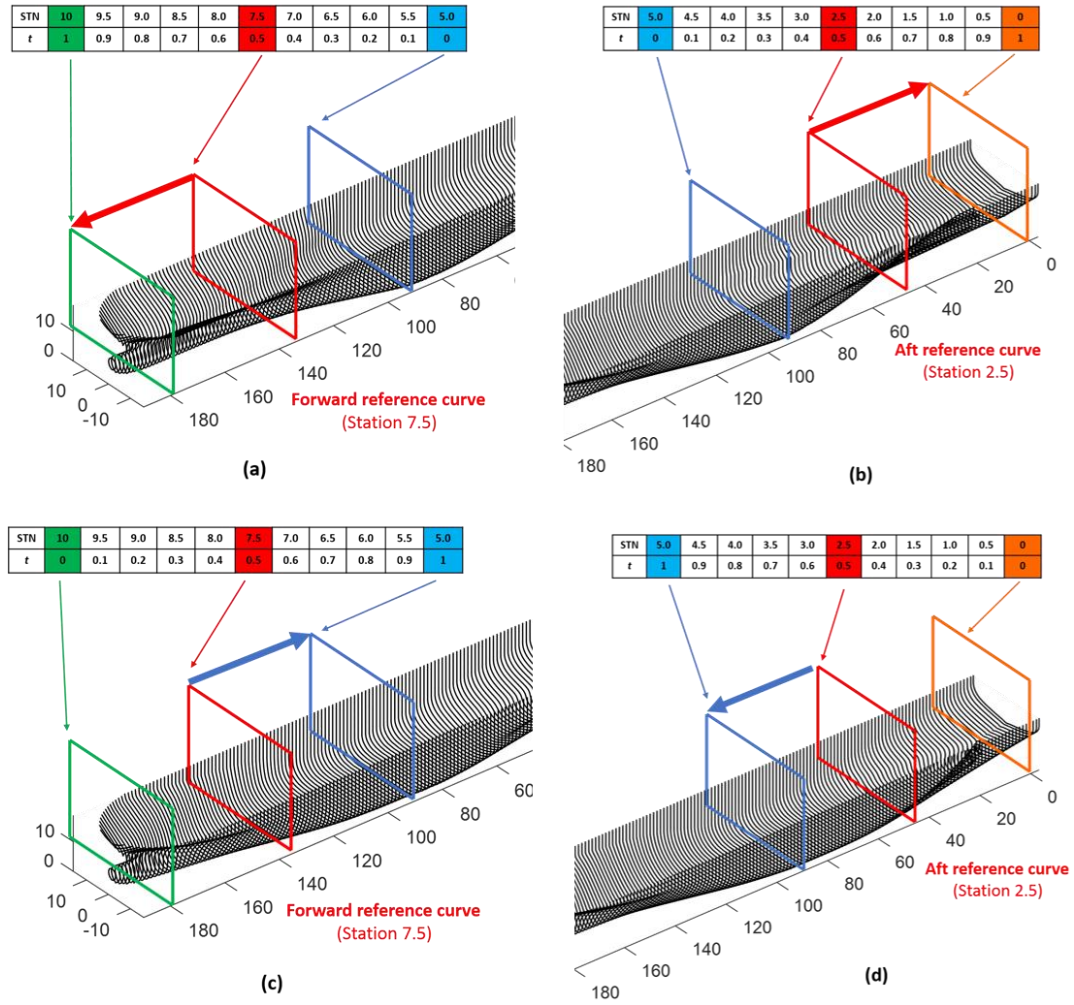


Figure 3.23 Wire-mesh diagrams of modified hull form using boundary-morph. (a) Inward shifting from forward reference curve (Station 7.5) to forward boundary curve (Station 10). (b) Inward shifting from aft reference curve (Station 2.5) to aft boundary curve (Station 0). (c) Outward shifting from forward reference curve (Station 7.5) to mid boundary curve (Station 5). (d) Outward shifting from aft reference curve (Station 2.5) to mid boundary curve (Station 5).

Likewise in the fourth scenario shown in Figure 3.23(d), the aft section (Station 2.5) was morphed towards the mid boundary curve (Station 5) using a morphing parameter of $t = 0.5$. This resulted in a fuller hull form in the aft section.

The four scenarios demonstrated that boundary-morph can be applied effectively to modify the shape of the hull using only one principal vessel. This is achieved by introducing different weightages to neighbouring curves of the reference curve and morphing them towards the boundary curve.

3.7. Chapter Summary

In this chapter, the concept of 2D curve morphing and its application to hull form design was presented. When applied to HFDO, geometry modification using 2D curve morphing can significantly reduce the computation time compared to 3D surface modifications. Three 2D curve morphing methods for intuitive and flexible modification of the shape of hull (using only one design variable – the morphing parameter t) were introduced. Constant-morph enables the entire hull form to be modified. Where two or more hull forms exist, cross-morph can be used to join them in a seamless manner. Unlike current applications that require at least two ships for morphing, boundary-morph enables modification of the hull form using only one reference ship. Together, these geometry modification methods can produce a wide variety of hull forms and are useful enablers of HFDO.

4. AUTOMATED HFDO INTEGRATING MORPHING AND EVOLUTIONARY COMPUTATION

By integrating the geometry modification techniques developed in Chapter 3 into an intelligent search method, an industry-first design automation tool based on Morphing and Evolutionary Computing (MEC) approach is developed in this chapter. This tool reduces the dependency on experienced ship designers to modify the shape of the hull form to efficiently achieve more diverse and potentially more optimal design solutions. The final output of this method is a set of optimal hull form designs, which can be used as a starting point for further design and improvement by a designer.

4.1. Genetic Algorithm

Genetic Algorithms (GA), first developed by Holland [56] is a heuristic search method based on the Darwinian theory of natural selection and the “survival-of-the-fittest” principle. GA is a subset of Evolutionary Computation (EC), a family of nature-inspired algorithms for global optimisation [24]. GA works based on the principle of genes and chromosomes. Using genetic operators such as selection, crossover and mutation, information represented by genes are exchanged between chromosomes over several iterations, typically with fitter solutions replacing weaker ones. This will eventually lead to a set of near-optimal solutions, which are represented in the form of a Pareto frontier. A Pareto frontier is a set of solutions where no solution is better off (or more superior) than the others at any given point within the solution space. Compared to other EC approaches such as Evolutionary Programming (EP) and PSO, GA can search within a large solution space within a relatively short time. It has been successfully applied to structural optimisation problems such as circuit synthesis [180], Neural Networks [181] and hull form designs [18], [45] - [49].

To fully appreciate the working principles of GA, it is important to first understand how it represents (maps) an optimisation problem. A solution in the actual problem space is termed a phenotype and its corresponding representation in the GA space is termed a genotype. For example in design optimisation, a phenotype can be a possible solution to the problem such as an object which can be represented in the form of either integer or float or bit string in the GA or genotype space. In GA, this representation is termed a

chromosome and is made up of building blocks known as genes. Figure 4.1 provides an illustration of this concept and the encoding and decoding of a solution.

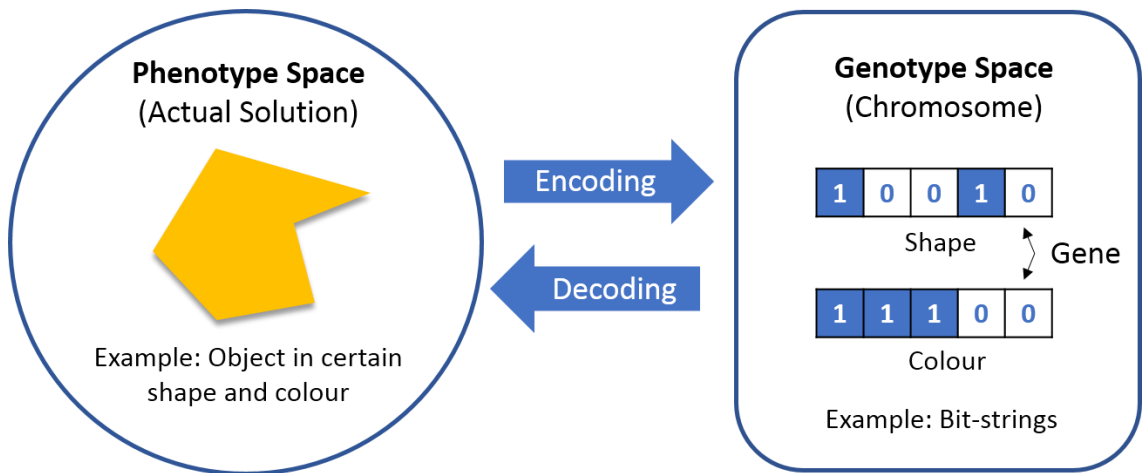


Figure 4.1 illustrates the concepts of (i) encoding, where a solution (phenotype) is mapped into a chromosome (genotype) and (ii) decoding, where a chromosome is converted back into its corresponding actual solution.

Once the phenotype – genotype mapping is established, information exchange takes place between chromosomes through genetic operators, namely, selection and replacement, crossover and mutation. This takes place over several iterations, typically with better solutions replacing weaker ones, eventually leading to a set of optimal solutions at the end of the optimisation process. The phenotype-genotype mapping of a problem is thus a crucial step in GA as an incorrect or inappropriate representation of the problem will result in failure of the optimisation. A GA consists of six key steps, as shown in Figure 4.2.

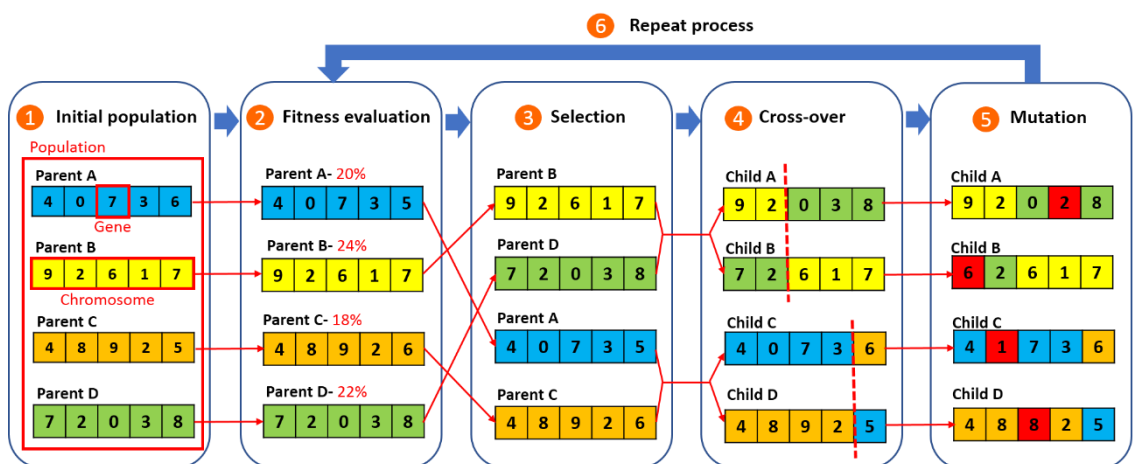


Figure 4.2 Working principles of a genetic algorithm [179].

With reference to Figure 4.2, the detailed steps of GA are:

Step 1 - Initial population: The first step in GA is to initialise a group of the population as initial parents. Similar to biological systems, each parent is made up of several different genes that form a chromosome. The genes can be represented in either binary or integers.

Step 2 - Fitness evaluation: The initial population will go through an evaluation process to determine the fitness value of each individual. The evaluation method will be based on the objective function defined in the optimisation problem.

Step 3 - Selection: A selection process takes place to pair individuals together based on their fitness values to produce the next generation or child (solutions).

Step 4 - Cross-over: The regeneration process takes place in cross-over processes in which part of each parent genes will be passed onto the new child (solution). This cross-over process can either be single or multiple point cross-over and adjusted by different ratings, which will affect the diversity of the new solutions generated.

Step 5 - Mutation: A mutation process takes place to infuse a unique trait into the child through minor modification of genes that are apart from the parents' genes. This process helps to introduce new elements into the child (solution) while ensuring the retention of good genes created during the cross-over process.

Step 6 - Repeat process: In the next generation, the newly created child (solutions) will form a new parent population. The entire process is repeated until the termination criteria are met.

GAs can be used on both single and multi-objective optimisation problems. In the former, only one optimal solution will be found at the end of the optimisation, while the latter can produce several solutions with equivalent performances, commonly known as a Pareto optimal set. Multi-objective GA is used in real-world design problems, as is the case in this thesis. Here, NSGA-II, first developed by Deb [182], [183], is used to optimise the design of hull forms.

NSGA-II is based largely on the working principles of the original GA but possesses additional features, namely, non-dominated sorting and a crowding distance measure to enhance the search process. When compared to standard GA, Niched-Pareto Genetic algorithm (NPGA) [184] or NSGA-I [182], NSGA-II is particular useful for multi-objective problems as it helps the population reach the true pareto-optimal front and sustains the spread of solutions even with large numbers of generations [183]. Therefore, it is popular for its ability to search a design space more effectively, leading to more

diverse and creative solutions. While the NSGA-II concept in this thesis is similar to earlier works, the main difference lies in the crossover and mutation operator, where real-value chromosomes (morphing parameters) are used to change the geometry of the vessel and create new solutions.

A basic illustration of non-dominated selection and crowding distance within NSGA-II is provided below in Figure 4.3.

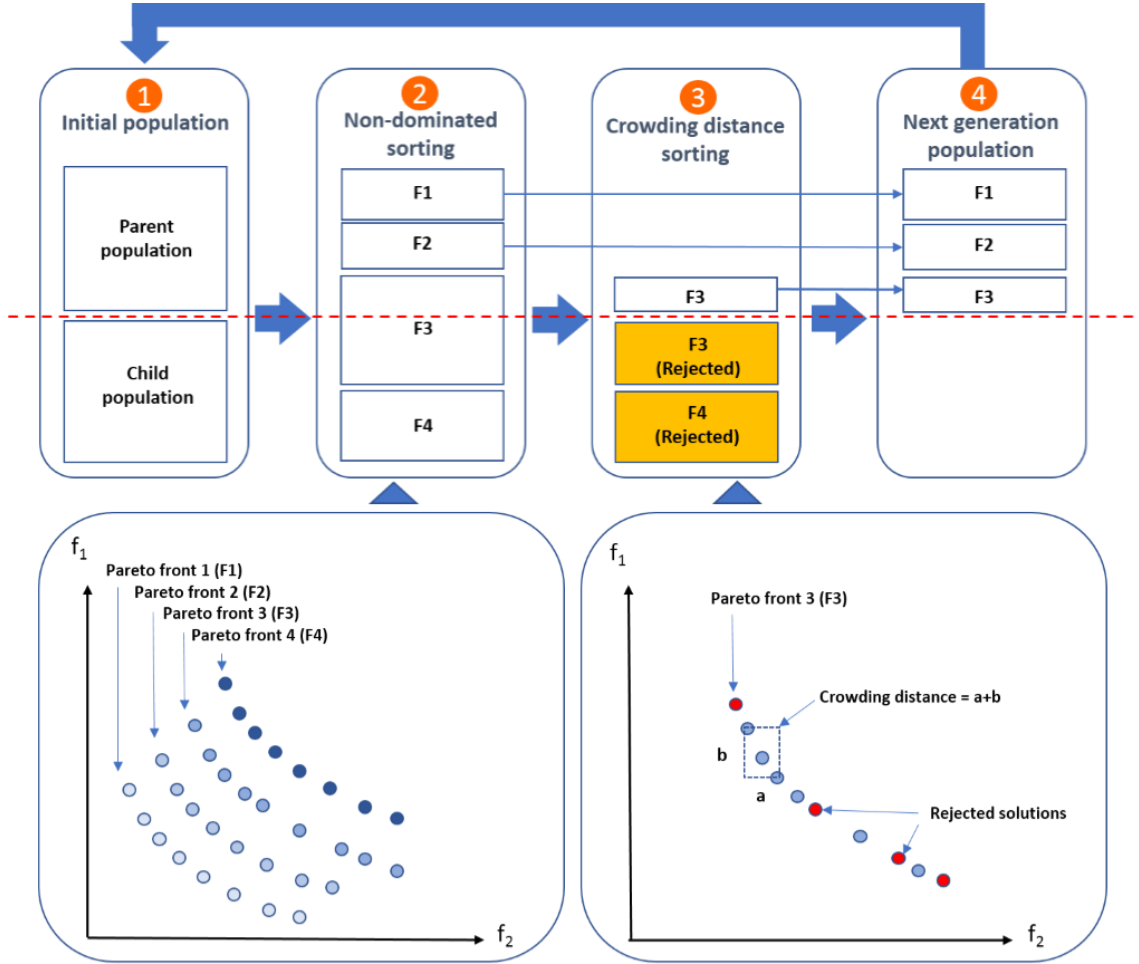


Figure 4.3 Working principle of NSGA-II featuring non-dominated and crowding distance sorting for multi-objective optimisation [adapted from 185].

As depicted in Figure 4.3, the key steps in NSGA-II are described as follows.

Step 1: Gather the same number of parent population (from previous generation) and the new offspring population to form the initial population.

Step 2: Sort the population into different Pareto fronts according to non-domination level (for example, F1 is the best level followed by F2 and so on). Apply binary tournament selection to randomly select two solutions from the current population and select the better solution with respect to non-domination ranking.

Step 3: Compare the solution within the same non-domination front using crowding distance to measure the density of neighbouring solutions.

Step 4: Create the next generation by copying the best solutions (elitism) from the mixed population of parents and off-springs. Solutions that are not copied to are rejected.

4.2. Morphing Evolutionary Computation (MEC)

In this section, the working principles of MEC developed in this thesis for hull form design optimisation is described in detail. An overview of MEC is provided in Figure 4.4.

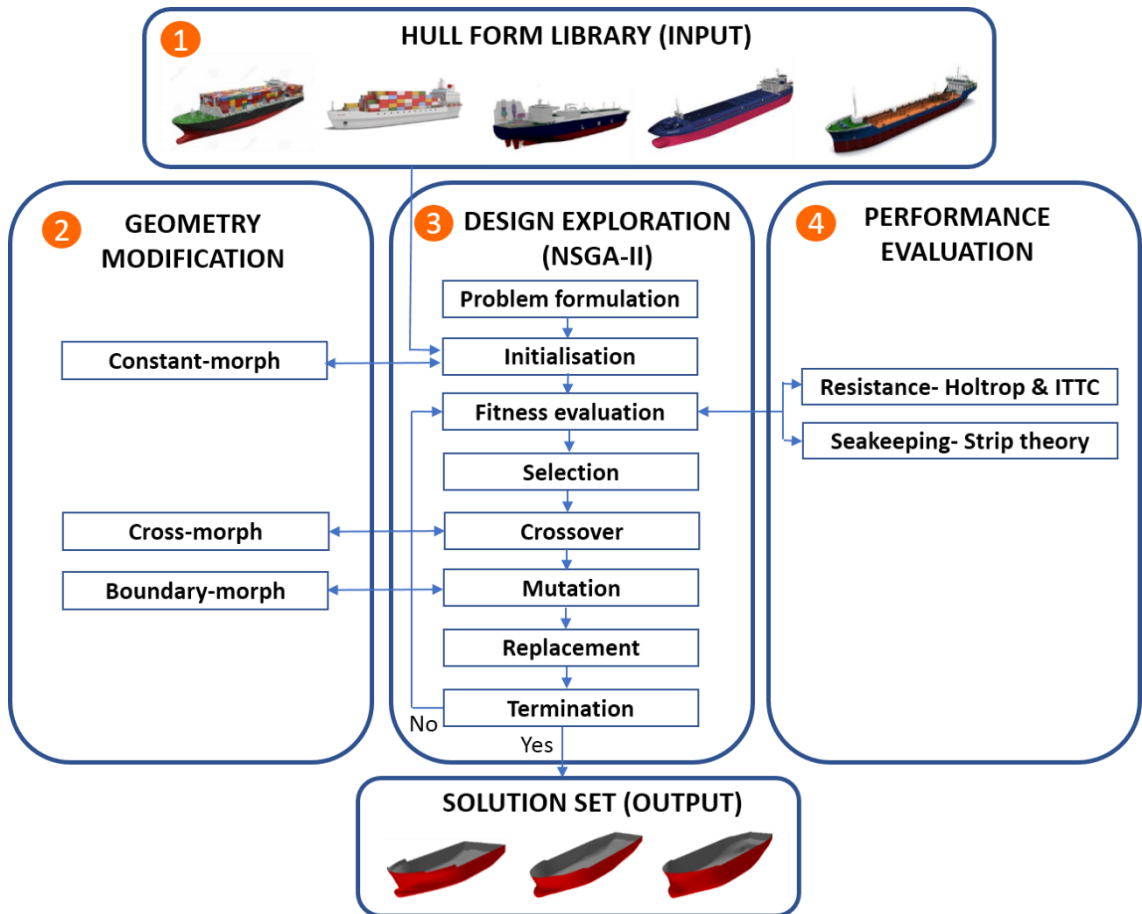


Figure 4.4 Overview of MEC.

As seen from Figure 4.4, MEC consists of 4 main parts, namely, hull form library, geometry modification, design exploration (NSGA-II) and performance evaluation. The hull form library consists of a set of proven designs (from experienced designers). These are used as inputs for the optimisation. In geometry modification, the three 2D morphing techniques – constant-morph, cross-morph and boundary-morph – described in Chapter 3 are used to modify the shape of hull forms. The central processing unit for the entire optimisation process is design exploration using NSGA-II [183]. Taking proven designs from the hull form library as “starting points”, MEC integrates morphing techniques in

geometry modification to iteratively change the shapes of candidate design solutions. The selection and replacement processes are implemented through non-dominated selection and crowding distance sorting, which determine the best individuals to be carried over to the next generation. The performance of each solution is evaluated in performance evaluation according to the design objectives using empirical methods such as the Holtrop and Mennen method [125] for resistance and strip theory [144] for seakeeping. The optimisation ends when a set of optimal design solutions is found or when a termination criterion is reached. Detailed workings of MEC is described in Figure 4.5.

In this research, MEC was implemented using MATLAB [186] as the proposed algorithms can be easily extended from the optimisation toolbox. Furthermore, MATLAB provided a seamless integration to commercial evaluation software and other resistance evaluation algorithms such as Holtrop and Mennod [125] and seakeeping (strip theory) [144]. This was a challenge with other programming languages.

4.2.1. Problem Formulation and Initialisation

In most optimisation problems, formulating the right objectives and constraints are paramount to the success of the optimisation. This is particularly true for complex problems such as hull form optimisation [187]. Therefore, it is important to identify input design specifications as well as objective functions before the optimisation commences. These include the ship type, principal dimensions such as Length between Perpendiculars (LPP), beam (B) and draft (T). Depending on the design specifications, objective functions may include minimising resistance and/or seakeeping. In addition, proven designs stored in the hull form library that are used as starting points for the optimisation are prepared during initialisation.

- **Objective Function**

In real world applications, there are two main types of optimisation problems- single and multi-objective. The main goal of single objective problems is to find the ‘best’ solution to a design criterion. This could correspond to the minimum or maximum value of the objective function and can be formulated as shown in Equation 4.1.

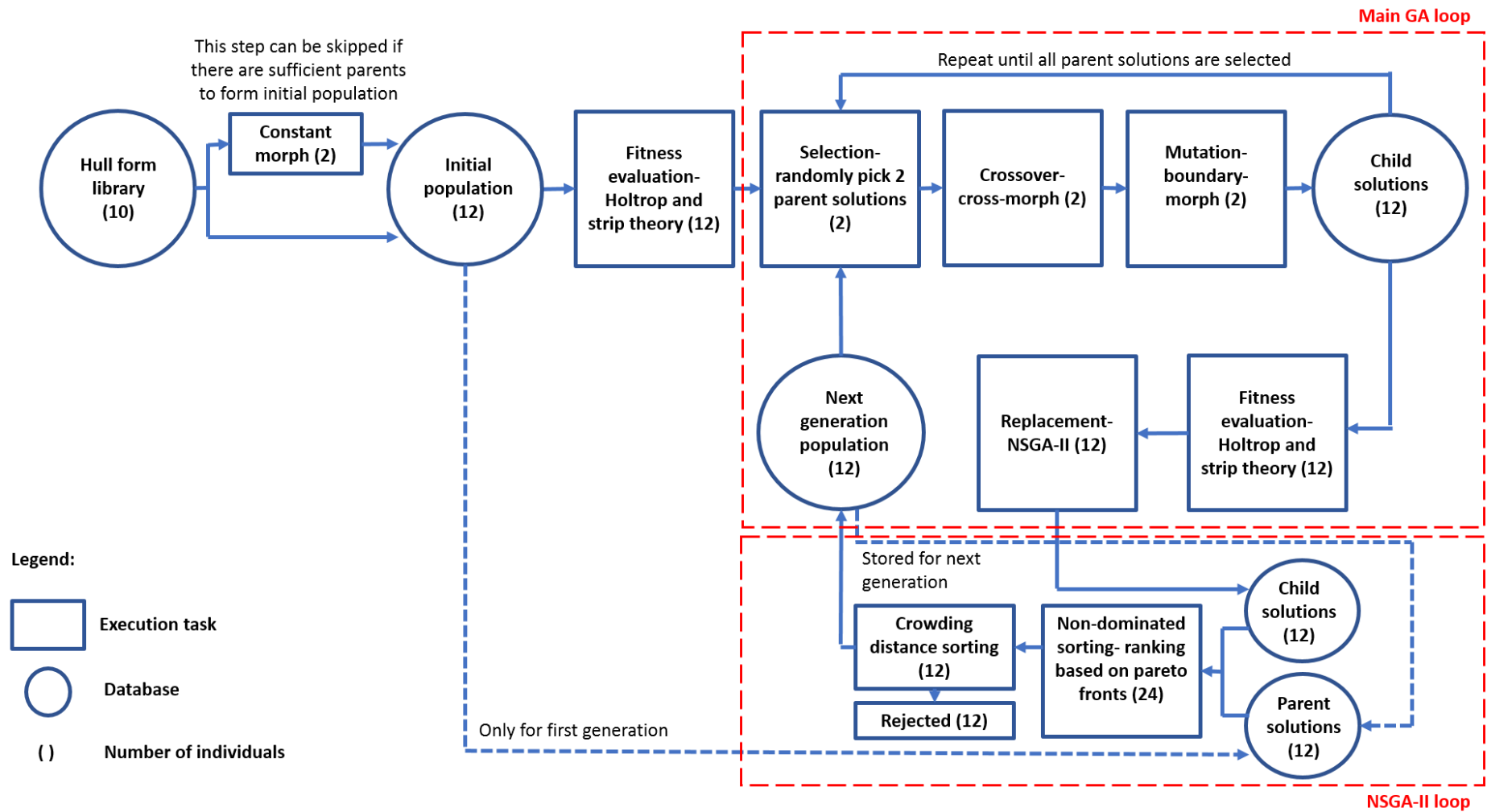


Figure 4.5 Schematic diagram detailing the step-by-step operation of MEC.

$$\begin{aligned}
&\text{Minimise or maximise} && f(x), \\
&\text{subject to} && h_k(x) = 0, && k = 1, \dots, K \\
&&& g_j(x) = 0, && j = 1, \dots, J \\
&&& x_{nl} \leq x_n \leq x_{nu}, && n = 1, \dots, N
\end{aligned} \tag{4.1}$$

where f is the objective function, K is the equality constraint, J is the inequality constraint, x represents the vector of design variables and x_{nl} and x_{nu} are the lower and upper bounds of the x th design variable, x_n , respectively.

Conversely, multi-objective optimisation problems usually deal with conflicting objectives where there is no unique global solution but a set of solutions [24]. In general, a mathematical problem to multi-objective optimisation can be represented as shown in Equation 4.2.

$$\begin{aligned}
&\text{Minimise or maximise} && f_i(x), && i = 1, \dots, I \\
&\text{subject to} && h_k(x) = 0, && k = 1, \dots, K \\
&&& g_j(x) = 0, && j = 1, \dots, J \\
&&& x_{nl} \leq x_n \leq x_{nu}, && n = 1, \dots, N
\end{aligned} \tag{4.2}$$

where f_i represents the objective functions, K is the equality constraint, J is the inequality constraint, x represents the vector of design variables and x_{nl} and x_{nu} are the lower and upper bounds of the x th design variable, x_n , respectively.

- **Constraints**

Design constraints determine the minimum and maximum allowable limits and thus have a huge influence on the design solution space [24]. Any design that does not meet the constraints will be deemed infeasible, as illustrated in Figure 4.6. Some examples of design constraints in hull form design optimisation include principal dimensions such as length, breadth and draft, displacement, deadweight, speed and cargo capacity. Only principal dimensions and displacements are considered in MEC. In MEC, only feasible solutions are carried over to the next generation while infeasible solutions are discarded.

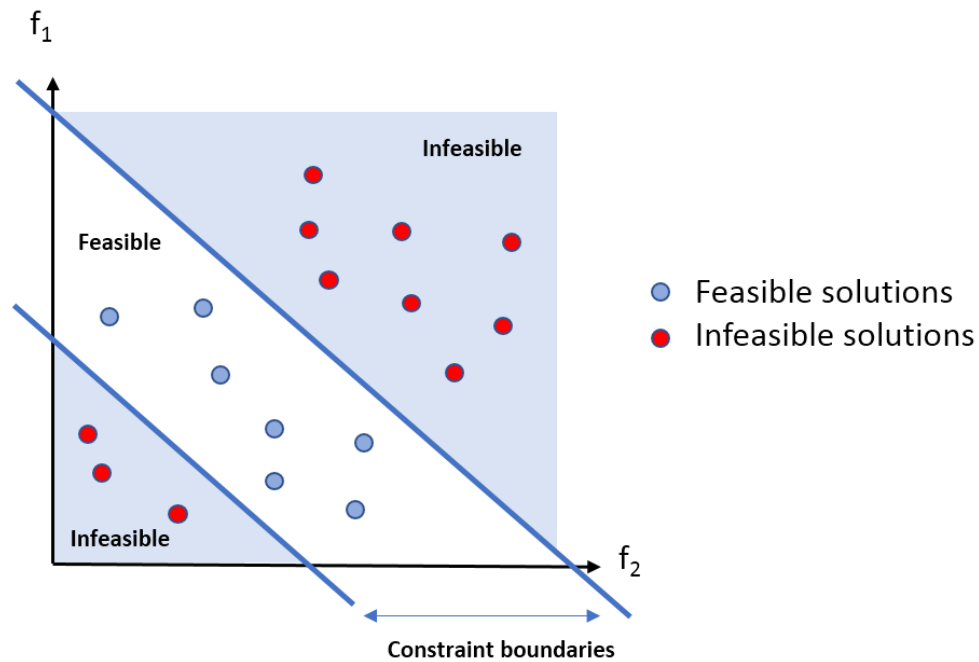


Figure 4.6 Feasible and infeasible solutions with constraint boundaries.

- **Initialising the Hull Form Library**

The first step in MEC is to create the first pool of individuals and map them into a unique encoding scheme. Similar to the current ship design process, this can be obtained from existing hull forms from the hull form library, which are designed by experienced designers. It should be noted that MEC does not replace experienced designers, but rather supplements them by creating more diverse and optimal designs. To further increase variation of the initial population, random design can be added by creating new models from initial concepts. The advantage of using existing designs is the assurance of performance since these designs have been validated to meet design objectives, hence shortening the design cycle. However, it should be noted that improvements made are often incremental. An alternative is to model a new hull form using proven designs as starting points while allowing/introducing more degrees of freedom in the design before adding it to the hull form library.

- **Linear Proportion Approach**

In MEC, existing ships in the hull form library, also known as parent solutions, are used to form the initial population. Based on the design requirements, the existing ships can be scaled up or down by applying the linear proportion approach to modify the hull form according to the desired length, beam and draft. This is to ensure that all the parent solutions are rescaled to the same principal dimensions before the geometry modification

and optimisation process. Linear proportion is one of the simplest methods for hull shape transformation where the breadth and draught are proportionally changed to maintain the form, similar to the basic form [188]. The linear proportion method is illustrated in Figure 4.7.

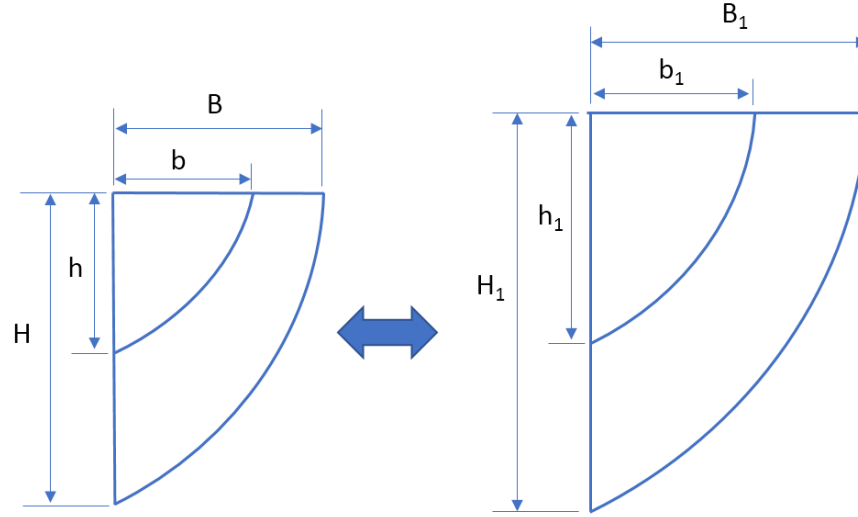


Figure 4.7 Changing the ship's principal dimensions using linear proportions approach.

The general formula for the linear proportion method is provided below:

$$h_1 = \frac{h \times H_1}{H}; b_1 = \frac{b \times B_1}{B} \quad (4.3)$$

where H , h , B and b represent the overall height, sectional height, overall breadth and half-breadth of the scaled-down ship, respectively while H_1 , h_1 , B_1 and b_1 are the overall height, sectional height, overall breadth and half-breadth of the scaled-up ship, respectively.

- **Phenotype to Genotype mapping**

In this thesis, candidate solutions (hull forms) are represented in the genotype space as real-valued chromosomes. As described in Section 3.3.2, the hull of the vessel is divided into x frames, each corresponding to a gene in the chromosome. Each gene contains two variables, namely, the station number and a morphing parameter, t . The station number represents the transverse frame in the X -plane and corresponding t value, which changes the hull geometries according to the 2D curve morphing methods described in Chapter 3. This phenotype – genotype mapping scheme is shown in Figure 4.8.

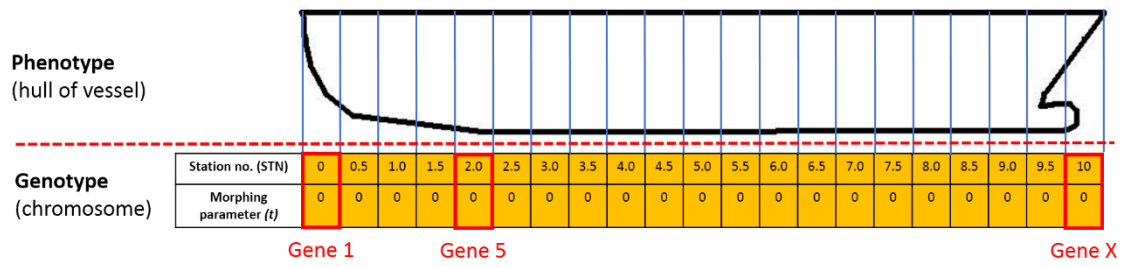


Figure 4.8 Phenotype - genotype mapping using real-valued chromosome (initial value of $t = 0$).

Incorporating the morphing scheme into the phenotype - genotype mapping for MEC enables the hull shape to be transformed globally by simply changing the morphing parameters t at various stations along the entire vessel when coupled with a GA. This unique process provides a simple yet direct representation of the ship geometry. This is a key novelty of this thesis. In other applications of GA in hull form optimisation, the mapping is typically represented by the design variables in the form of real or binary strings used to modify the hull geometry as defined by the designer.

- **GA parameters**

During initialisation, several GA parameters are specified. These parameters include the crossover and mutation rates, elitist size, population size and number of generations. Crossover and mutation rates determine the probability of information exchange between the selected chromosomes. The population size is the number of chromosomes generated in every generation. The number of generations determines how many times the process repeats before terminating. Elitist size determines the number of child solutions (as a factor of population size) to be carried over to the next generation. These settings have a strong impact on the outcome of the entire optimisation process and need to be adjusted accordingly to the optimisation problem. An example of GA settings is provided here.

- Population size: 10
- No of generations: 100
- Crossover rate: 0.9
- Mutation rate: 0.9
- Elitist size: $0.1 \times \text{population size}$

In this example, the population size was set to 10, considering the limited number of initial solutions available, which were derived from existing vessel designs. With this low population size in mind, 100 generations were set to allow sufficient time for the new solutions to grow and expand without pre-matured convergence. High crossover and

mutation rate of 0.9 were set to ensure a higher chance of new solutions being generated so as to increase the variety of solutions. In addition, a small percentage of the best individuals in the population were carried forward to the next generation via elitism to ensure that good individuals are preserved. However, it should be noted that too high crossover and mutation rates are very disruptive and the search may not converge [189]. Hence, depending on the optimisation problem and objectives, the parameters need to be adjusted accordingly.

4.2.2. Selection and Replacement

Selection is a process of picking individuals that will be used in the reproduction process for generating new solutions. The principle is to always select good solutions to increase the chances of obtaining better individuals. Current selection methods include roulette wheel, linear ranking and tournament selection. Roulette wheel randomly selects individuals according to their fitness values. The advantage of this method is that it preserves the diversity of the population by giving every individual a chance to be selected [190]. However, it tends to have premature convergence [189]. In tournament selection, individuals will compete in the entire population based on their fitness values to be selected as a parent. Its simplicity and efficiency make it the most popular selection method [191]. However, this method results in great reduction in diversity as individuals with the highest fitness values tend to be selected constantly while those with low fitness values will lose out either in the beginning or at immediate stages [192]. Linear ranking selects parents by ranking the fitness of the individuals. This selection method addresses the issue of premature convergence but requires high computation time and hence not favourable for the complex optimisation problem [189]. There are other selection methods that do not depend on the fitness of individuals. One such method is the random selection method which selects a parent on a purely random basis and provides equal probability for every individual to be selected in every generation. Considering that all parent or existing vessels in the hull form library are of proven designs and useful, random selection was adopted here for MEC.

Replacement methods are used to determine if the newly created solution will be used to replace some, all or none of the current solutions. There are different replacement methods that are applied to GA. These include tournament selection, deterministic crowding, first-in-first-out, conservative strategy and elitism [193]. In MEC, replacement is applied using the elitism approach. Elitism is based on the principle of the survival-of-the-fittest where best solutions in each generation are preserved and carried forward to

the next generation in hope of producing better solutions. In MEC, the replacement method for building the next generation of candidate solutions is compared here between GA without elitism and NSGA-II with elitism. Figure 4.9 shows a comparison when the two methods were applied to a minimisation problem.

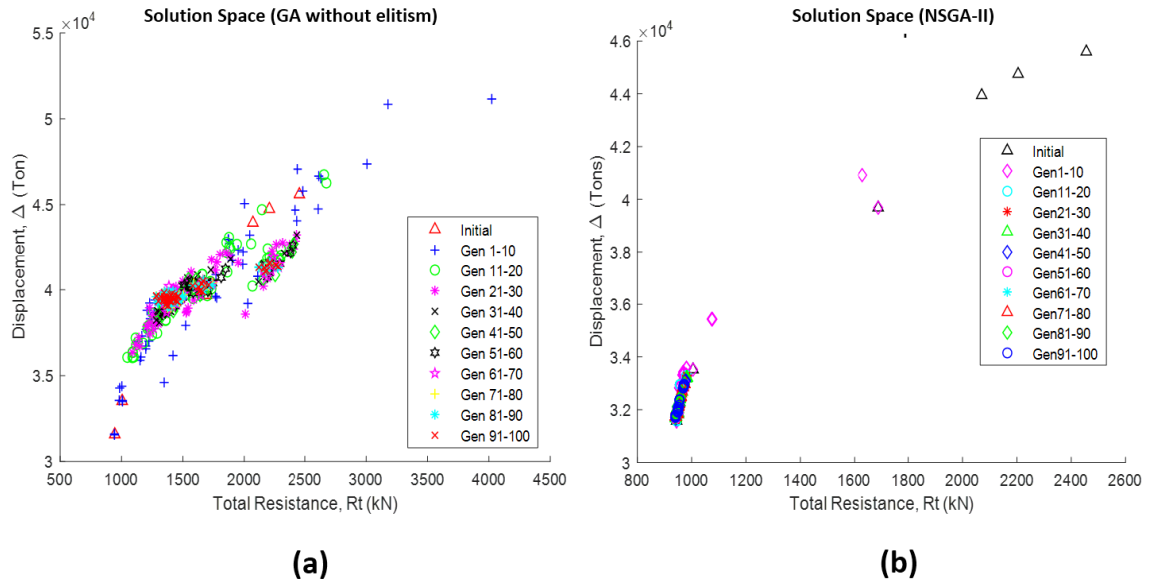


Figure 4.9 Solution space for (a) GA without elitism and (b) NSGA-II with elitism.

It can be observed from Figure 4.9(a) that the solutions found throughout the optimisation process are concentrated around the centre of the initial population. This means that there is little diversity and the solutions found may not be better than the initial designs. When elitism was implemented as shown in Figure 4.9(b), the solutions obtained converged towards the lower left corner of the solution space. It can thus be concluded that preserving the best individuals throughout the optimisation process could lead to better solutions at the end of the search.

4.2.3. Crossover

Crossover is an important operator in GA where the chromosomes of two parents are combined to form a new chromosome of the child. The principle of this operator is to create new individuals by mixing the good genes of their parents which will subsequently lead to fitter individuals. An important parameter for crossover in GA is the crossover rate. Crossover rate determines the percentage (or probability) of the solution going through crossover, with a range from 0 to 1. A crossover rate of 0 means that none of the solutions will go through crossover while 1 means all solutions will go through crossover.

In MEC, cross-morph (described in Section 3.5) is applied in crossover through the encoding scheme to create new (child) solutions. To demonstrate the effects of different crossover rates in MEC, a comparative study was carried out using crossover rates of 0.1 to 1.0. Six initial (existing) designs were used to form the initial population. The results obtained from this study are shown in Figure 4.10.

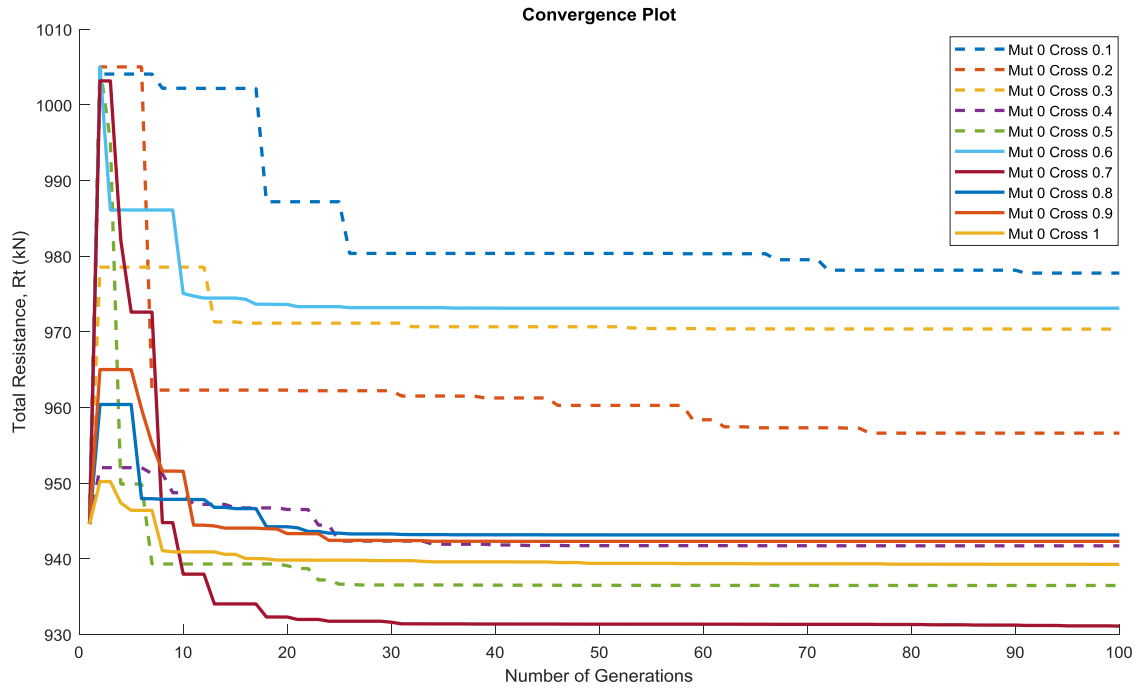


Figure 4.10 Convergence plot using crossover rate of 0.1 - 1.0. Mutation rate is set as zero to evaluate the effects of crossover only.

Figure 4.10 shows that the resistance of the solutions reduced as the crossover rate increased. For example, crossover rates of 0.1 to 0.3 resulted in higher resistance values than crossover rate of 0.7 to 1. This demonstrates that the higher the crossover rate, the higher probability the solutions will go through crossover and create new solutions that perform better. Next, it can be observed that the solutions in almost all crossover settings converge before the 20th generation and the search stagnates thereafter. This demonstrates that there are limited new solutions created. This can be due to lack of diversity in the solutions created in the crossover. This is a reasonable result considering that in this case, there are only six vessels in the initial population which limits the number of new shapes that can be created using cross-morph. Nevertheless, this example demonstrated the ability of crossover in MEC to generate new designs (solutions) with better overall performance.

4.2.4. Mutation

Mutation is the next reproduction process in the GA. Here, new genes created from crossover are randomly perturbed to produce new genetic structures. In the phenotype space, this can be seen as changes made to the object such as the hull. Similar to crossover, mutation in GA is controlled by the mutation rate and it determines the probability of the solutions going through mutation. By mutating a chromosome sparingly, new variations are introduced into existing solutions to form new ones which will still retain some traits of the solution. In MEC, boundary-morph (described in Section 3.6) is applied to randomly modify the shape of the hull and create chromosomes of new solutions. To demonstrate the effects of different mutation rates, a comparison was carried out using mutation rates from 0.1 to 1.0. The results obtained are shown in Figure 4.11.

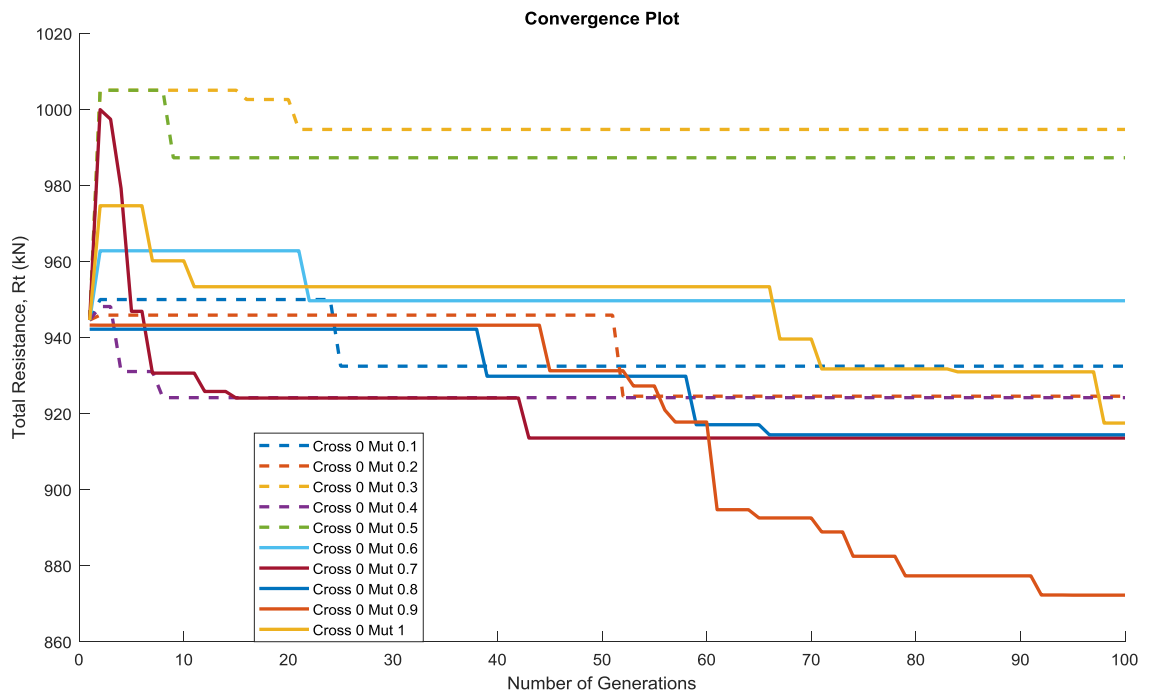


Figure 4.11 Convergence plot using mutation rate of 0.1 - 1.0. Crossover rate is set as zero to evaluate the effects of mutation only.

From Figure 4.11, it is observed that the solutions generally performed better as the mutation rate increases. For example, mutation rates of 0.3 and 0.5 have higher resistance values as compared to mutation rates of 0.7 to 0.9, where solutions with the lowest resistance values were found. This is expected as the higher the mutation rate, the higher the chances of the solutions being perturbed, leading to more new solutions discovered. However, unlike crossover, most solutions in mutation converged much later, around the 50th generation onwards. This demonstrated that mutation is a useful mechanism to inject

more diversity to the solution space. This is very beneficial after the optimisation has been running for some time and the search starts to stagnate.

4.3. Fitness Evaluation

To measure the ‘fitness’ of the parent ships, the performance of each hull form design is evaluated based on the objective function. In hull form optimisation, there are many objective functions relating to the shape of hull that can be improved, including resistance, seakeeping, stability, structural strength and cargo capacity. The selection of an objective function varies widely according to the type of ship and design specification prescribed by the ship owner. As this thesis focuses mainly on hull form optimisation, it will consider only objective functions that are affected by the hull shape (resistance and seakeeping).

In MEC, it was suggested that candidate design solutions be assessed using semi-empirical methods such as the Holtrop and Mennen method for resistance analysis and strip theory for seakeeping analysis. This is in view of the large number of candidate solutions to be evaluated and these methods can provide fast and good estimations, which are particularly suited for the initial ship design process. Higher fidelity CFD methods such as potential flow or RANS can be applied at later design stages to validate the optimal design.

4.3.1. Resistance

There are several methods for calculating the resistance pressure acting on the hull, including the empirical-based ITTC and Holtrop method, analytical-based Michell integral thin-ship approximation, slender ship approximation and numerical-based CFD methods such as potential flow and RANS. Each method has its pros and cons and the selection of the right method will depend on degree of accuracy, computation time available and the ability to interpret the results (in case of the CFD methods). During the initial design stage, it is important to obtain ‘sufficiently-optimal’ hull forms as early and fast as possible. Therefore, it is more practical to apply simpler and faster approaches such as empirical or analytical-based methods instead of numerical-based CFD methods, which are usually more computationally expensive. This thesis applies the Holtrop and Mennen method for resistance evaluation as it is fast, thereby ensuring that more design variations can be evaluated and the results can easily be calculated using programming tools such as MATLAB.

- **Holtrop and Mennen method**

According to [125], the total resistance can be estimated using:

$$R_T = R_F (1 + K_1) + R_{APP} + R_W + R_B + R_{TR} + R_A \quad (4.4)$$

where the terms in Equation 4.4 are defined as follows:

R_T	total resistance
R_F	frictional resistance according to ITTC-1957 friction formula
$(1 + K_1)$	form factor describing the viscous resistance of hull form in relation to R_F
R_{APP}	resistance of appendages
R_W	wave-making and wave-breaking resistance
R_B	additional pressure resistance of bulbous bow near the water surface
R_{TR}	additional pressure resistance of immersed transom stern
R_A	model-ship correlation resistance

- **Frictional resistance**

The frictional resistance based on ITTC-1957 is given by:

$$C_F = \frac{0.075}{(\log_{10}(Rn) - 2)^2} = \frac{Rf}{0.5 \times p \times S \times V^2} \quad (4.5)$$

where,

Reynolds number can be obtained by:

$$Rn = \frac{V \times Lwl}{\nu} \quad (4.6)$$

kinematic viscosity of water,

$$\nu = (43.4233 - 31.38 \times p) \times (t + 20)1.72p - 2.202 + 4.7478 - 5.779p) \times 10^{-6} \quad (4.7)$$

where t is the temperature of water in °C.

- **Form factor**

The prediction formula for the form factor of the hull is given by Equation 4.8.

$$1 + K_1 = c_{13} \times (0.93 + c_{12} \left(\frac{B}{L_R}\right)^{0.92497} (0.95 - C_P)^{-0.521448} (1 - C_P + 0.0225lcb)^{0.6906}) \quad (4.8)$$

where C_P is the prismatic coefficient based on waterline length L and lcb is the longitudinal position of centre of buoyancy forward of $0.5L$ as percentage of L . In the form-factor formula, L_R is a parameter that reflects the length of the run according to:

$$\frac{L_R}{L} = 1 - C_P + \frac{0.06 \times C_P \times lcb}{(4 \times C_P - 1)} \quad (4.9)$$

- **Appendage resistance**

The appendage resistance can be determined using:

$$R_{APP} = 0.5 \rho V^2 S_{APP} (1 + k_2)_{eq} C^F \quad (4.10)$$

where ρ is the density of water, V is the speed of the ship, S_{APP} is the wetted area of appendages, $1 + k_2$ is the appendage resistance factor and C^F is the coefficient of frictional resistance based on the ITTC-1957 formula.

- Wave resistance

The wave resistance can be determined using:

$$R_W = c_1 c_2 c_5 \nabla \rho g e^{\{m_1 F_n^d + m_2 \cos(\lambda F_n^{-2})\}} \quad (4.11)$$

- Additional resistance due to bulbous bow near surface

$$R_B = 0.11 e^{(-3 \times P_B^{-2})} \times \frac{F_{ni}^3 A_{BT}^{1.5} \rho g}{(1 + F_{ni}^2)} \quad (4.12)$$

- Additional pressure resistance due to immersed transom

$$R_{TR} = 0.5 \rho V^2 A_T c_6 \quad (4.13)$$

- Model-ship correlation resistance

$$R_A = 0.5 \rho V^2 S C_A \quad (4.14)$$

- **Validation of Holtrop and Mennen method with OpenFOAM**

To evaluate the accuracy of the Holtrop and Mennen method for resistance calculation in MEC, an existing container ship was evaluated using the RANS method in openFOAM [194], which is an open-source CFD program. The principal dimensions of the selected container ship are as follows:

- Length (LPP) = 185m
- Breadth = 32m
- Draft = 9m

First, the 2D offset data of the container ship was manually converted into a 3D surface and then evaluated using Kelvin waves with 4.9m mesh cells in OpenFOAM snappyHexMesh as shown in Figure 4.12.

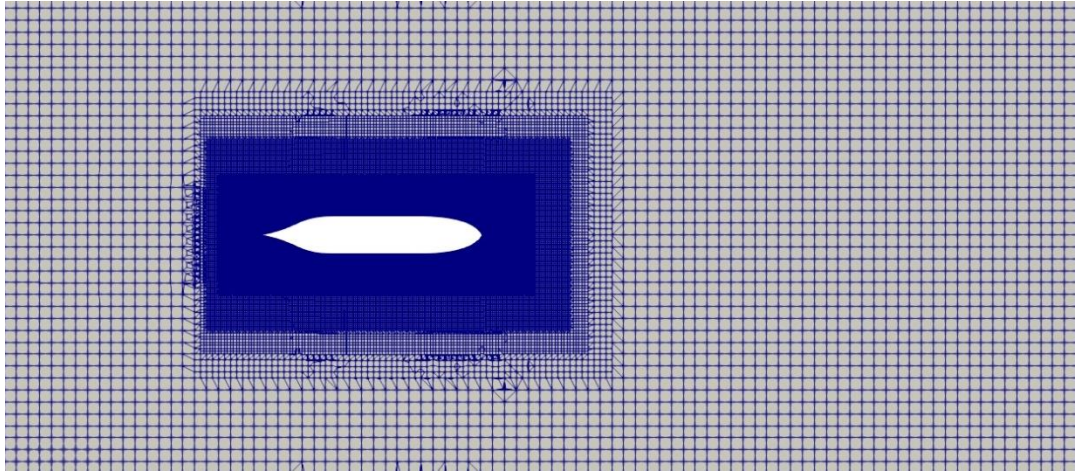


Figure 4.12 4.9m mesh cells generated using SnappyHexMesh.

Next, the RANS code was executed in OpenFOAM using ship speed of 20 knots. It took about 56 hours to run the entire simulation using 10 CPU-core. The results of the CFD evaluation comprising Kelvin wave and convergence chart are provided in Figures 4.13 and 4.14.

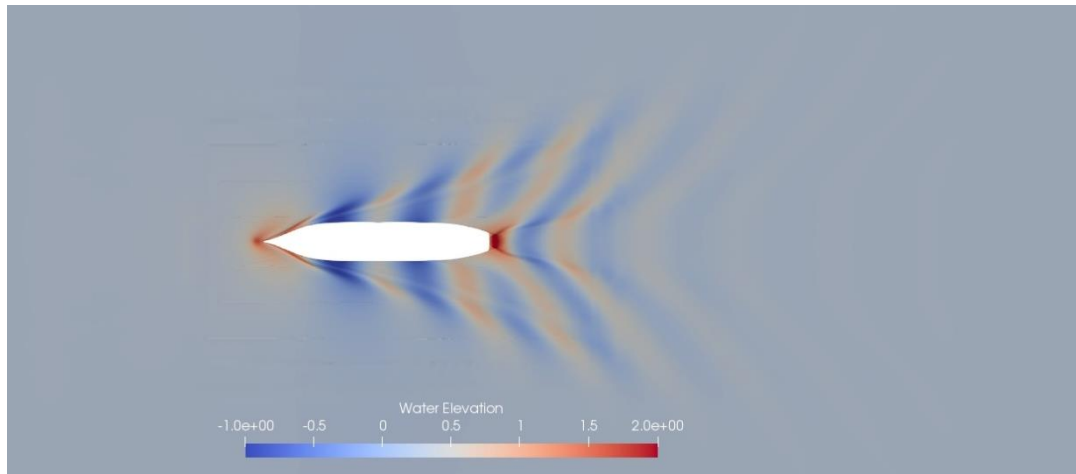


Figure 4.13 Kelvin waves generated from OpenFOAM.

From Figure 4.13, it is observed the Kelvin waves including the transversal waves and the diverging waves are well captured by the CFD simulation.

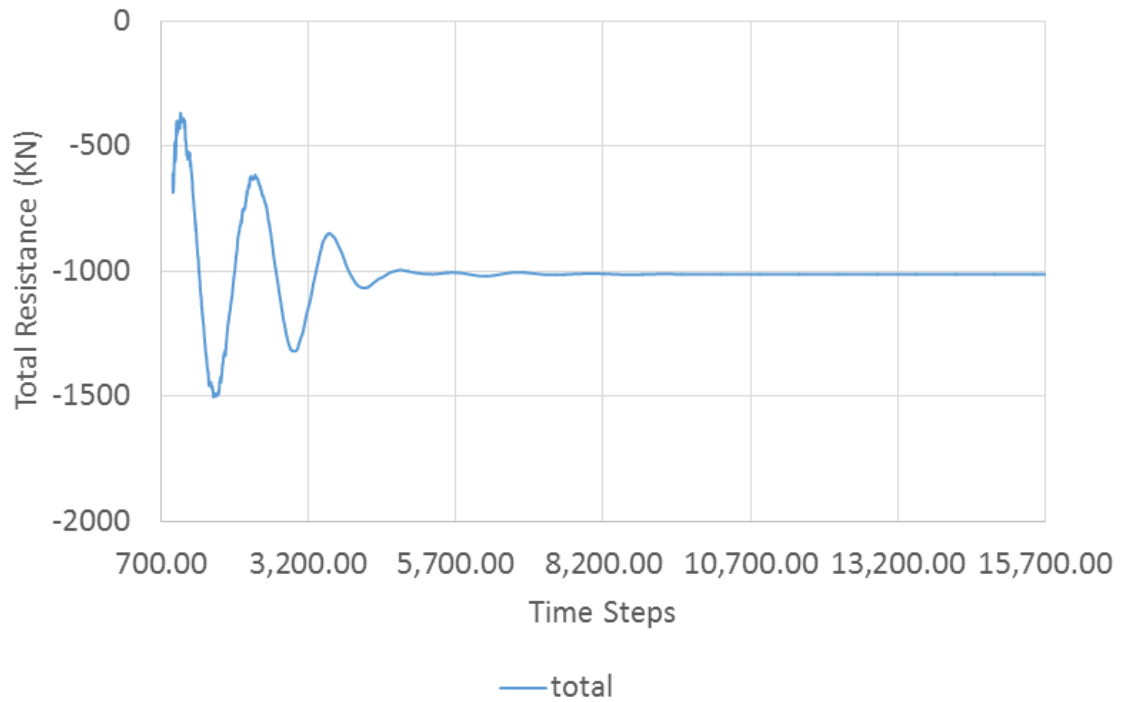


Figure 4.14 Convergence chart based on fine mesh (4.9m) and 3 prism layers.

Subsequently, the same ship was evaluated using the Holtrop and Mennen method, which is programmed in MATLAB using Equations 4.4 to 4.14. The results were compared and shown in Table 4.1.

Table 4.1 Resistance validation results between Holtrop & Mennen and openFOAM

Parameters	Holtrop and Mennen method (MATLAB)	RANS method (openFOAM)	Percentage difference %)
Evaluation time	30 Seconds (4 CPU-core)	56 hours (10 CPU-core)	671,900
Total resistance (kN)	948	1000	5.49

The results in Table 4.1 show that the empirical Holtrop and Mennen method can produce fairly accurate results (difference of 5.49%) and uses much less time than computationally expensive CFD methods such as RANS. This demonstrates that the Holtrop and Mennen method is suitable for evaluating the total resistance of a hull form especially during the early design stage, where the design would need to go through hundreds to thousands of iterations. The Holtrop and Mennen method is consequently selected for MEC.

4.3.2. Seakeeping

Seakeeping is an important evaluation function that predicts the motions of a ship at sea and this will affect the overall comfort and safety of personnel and cargo onboard the ship. There are several methods for calculating seakeeping functions, ranging from simple 2D strip theory, unified theory, green function method, panel method and more recently, to more sophisticated 3D methods such as Rankine panel method and CFD. Among the aforementioned methods, strip theory is very popular and has several commercial software designed based on it. In this thesis, a strip theory code based on pioneering works by [144] is integrated into MEC to evaluate seakeeping performance.

Similar to curve morphing that transforms the shape of the hull through individual cross-sectional curves, strip theory analyses the hydrodynamic forces acting on each strip of the hull cross-section and provides an approximation of the total hydrodynamic forces acting on the entire ship. Combining strip theory with MEC for evaluation allows the evaluation to be performed more efficiently without the need for meshing (which is required for CFD) or converting the surface back to lines after each geometry modification to evaluate the seakeeping performance. Another consideration for not using the commercial strip theory code is due to the ‘black-box’ function and challenges faced in linking up with the MEC program developed as part of this thesis.

- **Strip Theory**

There are several versions of the strip theory developed based on [195] to calculate the wave-induced motions of a ship, which include works by [144], [196], [197]. The strip theory by Salvesen, Tuck and Faltinsen (S-T-F) is the most recognised and widely used in the industry due to its ability to predict the heave, pitch, roll and yaw motions as well as wave-induced vertical and horizontal shear forces, bending moments and torsional moments for ships advancing at constant speed in regular waves. The strip theory by S-T-F is based on potential flow theory which solves the three-dimensional problem of hydrodynamic and exciting wave forces and moments acting on the ship by integrating the two-dimensional potential solution over the entire length of the ship [198]. It considers a ship to be made up of a finite number of transverse two-dimensional slices where each slice is treated hydro-dynamically as a segment of infinitely long floating cylinder, as shown in Figure 4.15. The key advantage of this strip theory method is its high speed and ability to produce accurate results for a wide variety of seakeeping

problems. However, it is limited to ships that are sufficiently slender and with relative low Froude number.

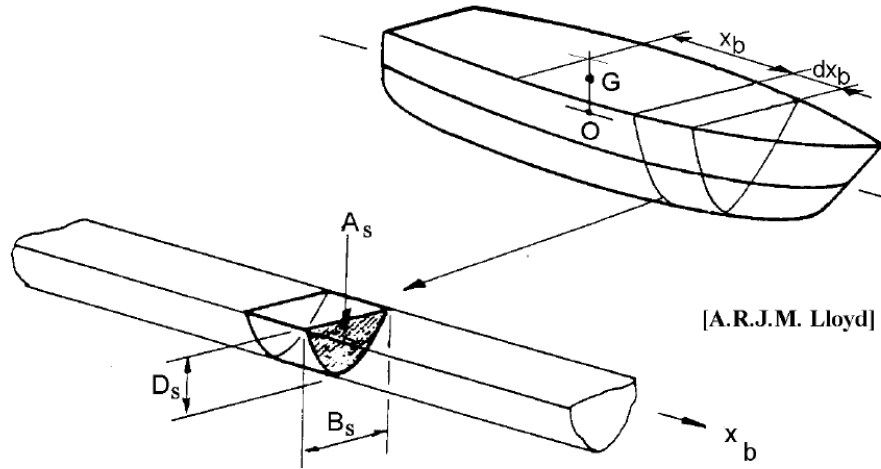


Figure 4.15 Strip theory represented by cross-sections [199].

• Ship Motions

A ship operates under six rigid body degrees of freedom, namely, surge η_1 , sway η_2 , heave η_3 , roll η_4 , pitch η_5 and yaw η_6 about the reference frame (x,y,z), as shown in Figure 4.16.

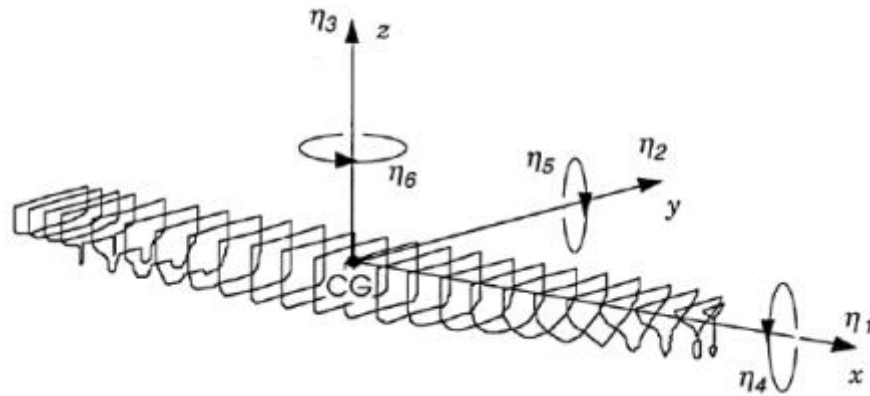


Figure 4.16 Definition of coordinate system and rigid body motions [200].

The response of the ship due to waves can be computed by solving the equation of motion in Equation 4.15.

$$\sum_{j=1}^6 [(M_{kj} + A_{ij})\ddot{\eta}_j + B_{kj}\dot{\eta}_j + C_{kj}\eta_j] = F_k e^{i\omega t} = F_k^I + F_k^D$$

for $k = 1, 2, 3, 4, 5, 6$

(4.15)

where the following definitions are used:

$k = 1,3,5$	Coupled surge, heave and pitch
$k = 2,4,6$	Coupled sway, roll and yaw
$\ddot{\eta}_j$	Acceleration of harmonic oscillation in j direction
$\dot{\eta}_j$	Velocity of harmonic oscillation in j direction
η_j	Displacement of harmonic oscillation in j direction
M_{kj}	Solid mass or inertia coefficient
A_{kj}	Hydrodynamic mass or inertia matrix
B_{kj}	Hydrodynamic damping or radiation damping coefficients
C_{kj}	Hydrostatic stiffness or restoring coefficients
ΣF_k	Harmonic exciting wave force or moment in k direction

- **Conformal Mapping**

Ursell [201] derived the potential flow around a circular cylinder involving heaving in free-surface, which satisfies the free-surface and boundary body conditions [202]. Hence, for determination of two-dimensional added mass and damping in strip theory, a cross-section of the arbitrary shape would first need to be conformally mapped to a unit semi-circle. The advantage of conformal mapping is that the velocity potential of the fluid around a complex arbitrary shape can be derived from a simpler cross-section such as a semi-circular section [203]. There are several types of conformal mapping used for the analysis of flow around marine structures. They include two-parameter Lewis conformal mapping [204], three-parameter conformal mapping [205] and other works by Journee and Adegeest [203] and Jong [206]. MEC applies the two-parameter Lewis conformal mapping due to its fast and considerable transformation of conventional mono-hull forms. The detailed calculations of Lewis conformal mapping are provided in Appendix I.

- **Two-Dimensional and Three-Dimensional Potential Coefficients**

There are several approaches to computing 2D potential coefficients, including approaches proposed by Tasai, Keil and Frank. Here, the determination of hydrodynamic coefficients for heaving, swaying and rolling is largely based on works by Lloyd [199], Ursell [201], Jong [206] and Taisai [207, 208]. The detailed calculations of two-dimensional and three-dimensional added mass and damping are provided in Appendix II and III respectively.

- **Equations of Motion in the Frequency Domain**

Using the added mass and damping coefficient obtained from Appendix II and III, the equations of motion in six degrees of freedom can be obtained from Equation 4.15 and expanded as follows:

$$(M + A_{11}) \cdot \ddot{\eta}_1 + A_{15}\ddot{\eta}_5 + B_{11}\ddot{\eta}_1 + B_{15}\ddot{\eta}_5 - M(\ddot{\eta}_2\ddot{\eta}_6 + \ddot{\eta}_3\ddot{\eta}_5) = F_1^{ext} \quad (4.16)$$

$$(M + A_{22}) \cdot \ddot{\eta}_2 + A_{24}\ddot{\eta}_4 + A_{26}\ddot{\eta}_6 + B_{22}\ddot{\eta}_2 + B_{24}\ddot{\eta}_4 + B_{26}\ddot{\eta}_6 - M(\ddot{\eta}_1\ddot{\eta}_6 + \ddot{\eta}_3\ddot{\eta}_4) = F_2^{ext} \quad (4.17)$$

$$(M + A_{33}) \cdot \ddot{\eta}_3 + A_{35}\ddot{\eta}_5 + B_{33}\ddot{\eta}_3 + C_{33}\ddot{\eta}_3 + B_{35}\ddot{\eta}_5 + C_{35}\ddot{\eta}_5 - M(\ddot{\eta}_1\ddot{\eta}_5 + \ddot{\eta}_2\ddot{\eta}_4) = F_3^{ext} \quad (4.18)$$

$$(I_{xx} + A_{44}) \cdot \ddot{\eta}_4 + A_{42}\ddot{\eta}_2 + A_{46}\ddot{\eta}_6 + B_{42}\ddot{\eta}_2 + B_{44}\ddot{\eta}_4 + B_{46}\ddot{\eta}_6 + C_{44}\ddot{\eta}_4 - ((I_{yy} + I_{zz})\ddot{\eta}_5\ddot{\eta}_6) = F_4^{ext} \quad (4.19)$$

$$(I_{yy} + A_{55}) \cdot \ddot{\eta}_5 + A_{51}\ddot{\eta}_1 + A_{53}\ddot{\eta}_3 + B_{51}\ddot{\eta}_1 + B_{53}\ddot{\eta}_3 + C_{53}\ddot{\eta}_3 + B_{55}\ddot{\eta}_5 \mp \ddot{\eta}_5 - ((I_{zz} + I_{xx})\ddot{\eta}_4\ddot{\eta}_6) = F_5^{ext} \quad (4.20)$$

$$(I_{zz} + A_{66}) \cdot \ddot{\eta}_6 + A_{62}\ddot{\eta}_2 + A_{64}\ddot{\eta}_4 + B_{62}\ddot{\eta}_2 + B_{64}\ddot{\eta}_4 + B_{66}\ddot{\eta}_6 - (I_{zxx} + I_{yy})\ddot{\eta}_4\ddot{\eta}_5 = F_6^{ext} \quad (4.21)$$

where F_{1-6}^{ext} are diffraction forces and Froude-Krylov forces.

- **Validation of the Strip Theory Method using Commercial Software**

To validate the results obtained using the selected strip theory method for seakeeping calculation, the same container ship was evaluated using the MATLAB codes that are developed in this thesis and the strip theory function in commercial software. NAPA [209] is a commercial marine design software that is widely used in the industry. The ship's details and settings are provided below:

- Length between perpendiculars, LPP = 185 m
- Beam, B = 32 m
- Mean draught, d = 9 m
- Ship velocity, U = 20 knots

As part of this validation, three degrees of freedom, namely, heave, roll and pitch motions, were evaluated using different heading angles. The results of the validation are provided in Figures 4.17 to 4.19.

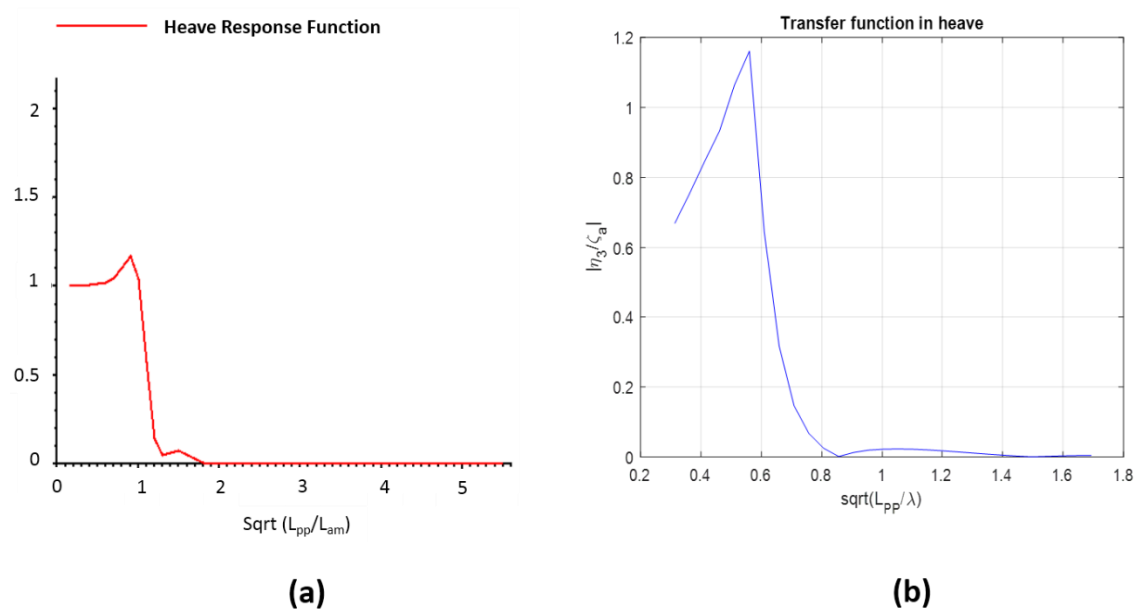


Figure 4.17 Transfer function for heave obtained using (a) NAPA and (b) MATLAB based on a heading angle of 135 degrees.

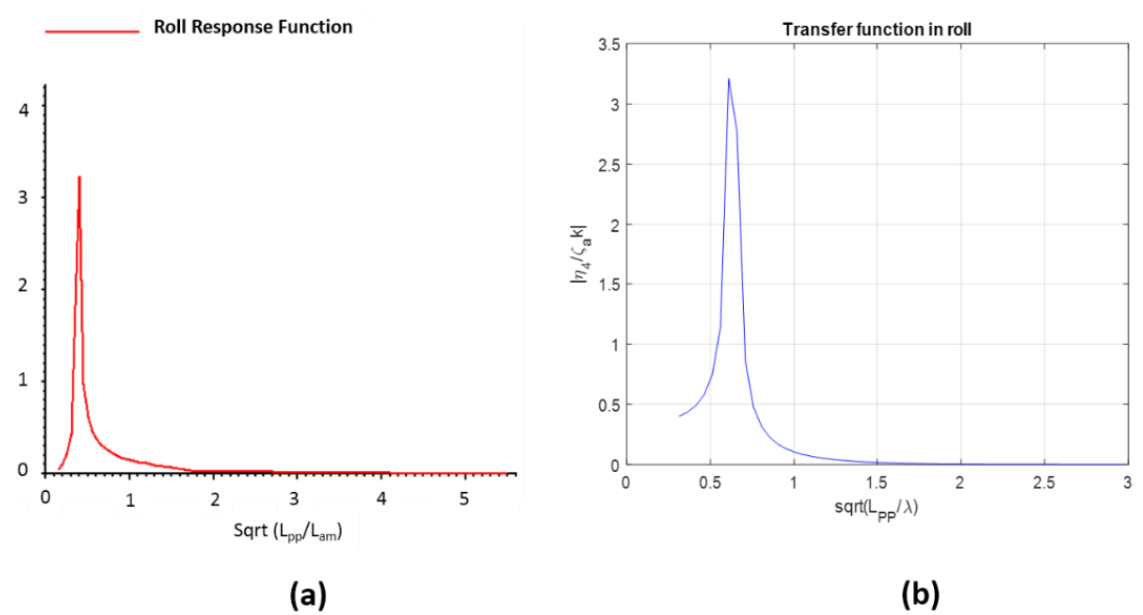


Figure 4.18 Transfer function for roll obtained using (a) NAPA and (b) MATLAB based on a heading angle of 90 degrees.

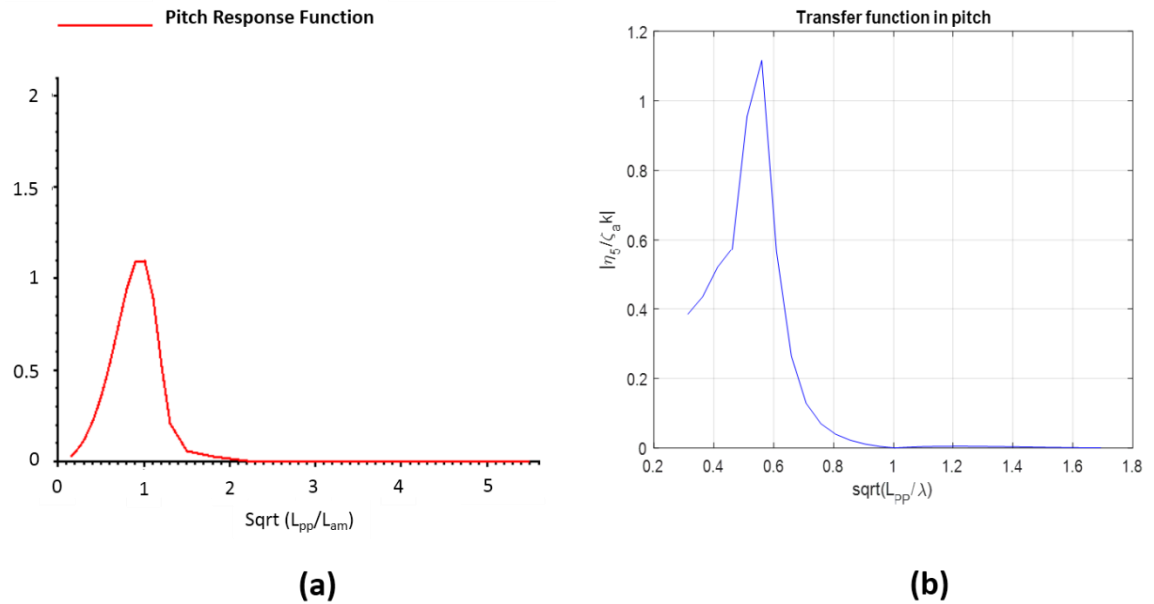


Figure 4.19 Transfer function for pitch obtained using (a) NAPA and (b) MATLAB based on a heading angle of 135 degrees.

Figures 4.17 to 4.19 show that the evaluated results are comparable for both the MATLAB codes and the commercial software, NAPA. The difference in maximum response function for the respective motions are compared in Table 4.2.

Table 4.2 Seakeeping (strip theory) results validation between MATLAB code and NAPA

Maximum response	NAPA	MATLAB	Difference
Heave motion	1.173	1.181	0.6%
Roll motion	3.228	3.210	-0.5%
Pitch motion	1.094	1.111	1.5%

The results presented in Table 4.2 show that the strip theory code generated using MATLAB for seakeeping evaluation is comparable to that obtained using the commercial software, NAPA, with a slight difference between -0.5% and 1.5%. From an industry perspective, this is within the acceptable range of less than 5%. To improve the accuracy, more refinements and validation of the MATLAB code can be performed in the future. This set of MATLAB codes will be used for evaluating the seakeeping functions, including heave, roll and pitch motions in MEC.

4.4. Chapter Summary

This chapter introduced the MEC approach, which combines morphing and evolutionary computation to automate the hull form design process. NSGA-II is a multi-objective optimisation algorithm that uses nature-inspired operations such as selection, crossover and mutation to optimise a population of candidate solutions in an iterative manner. Central to the working of MEC is a unique problem mapping scheme that maps the hull shape (phenotype) into GA chromosomes (genotype) and vice-versa. As a result, coupled with GA, it enables the hull shape to be transformed globally by simply changing the morphing parameters (t). The key advantages of MEC are that it can work with a small population size and that it uses cross-over and mutation process to inject the variation needed for optimisation. MEC applies simplified hydrodynamic evaluation methods such as Holtrop and Mennen and strip theory to estimate the overall drag and seakeeping performance of the hull. These approximation methods are directly applied to the geometries generated from the 2D curve morphing method, enabling the efficient turnaround of a significant number of evaluations. By combining 2D curve morphing and NSGA-II into MEC, it was demonstrated that the entire geometry modification, optimisation and performance evaluation process can be done automatically with minimal user intervention.

5. APPLICATION OF MEC TO THE DESIGN OF A CONTAINER SHIP

To demonstrate the effectiveness of MEC, automated single-objective optimisation was performed on the hull form of a container ship to minimise the total resistance of the vessel. Three case studies were used to assess the effects of different initial population sizes (and variety of vessels) and different crossover and mutation rates on the optimisation performance. In the first case study, a large initial population consisting of 12 vessels of different types was used. In the second case study, the initial population was reduced by 60% to four vessels of the same type. In the third case study, the initial population in case study 2 was expanded from four to 12 vessels using constant-morph. The results obtained from MEC were also validated with an existing SBD commercial software to compare the efficiency gain and performance improvement. The results demonstrate that MEC can be employed successfully to automate the hull form design process for different initial population sizes, with an ability to produce optimal solutions for single-objective optimisation.

5.1. Objective and Design Specification

In GA, the quality and size of initial (or parent) populations is important because they strongly influence the creation of new solutions and ultimately, their performances. It is established in the field that the smaller the initial population size, the more challenging it is to create more diverse and better solutions. In this chapter, three case studies will be carried out to evaluate the effects of different initial population sizes (and ship types) and different crossover and mutation rates using MEC.

Most established shipyards or ship design firms possess a database of hull forms, built up from their own proprietary designs or past projects delivered, which may consist of ten to a hundred different designs. For the first case study, 12 existing hull forms were selected as initial population in the hull form library of MEC. The reason for the small number is because limited proprietary designs are created and owned by the shipyard or design firms which can be used in the initial population. This comprises different ship types, including container ships, oil tankers and offshore vessels.

Smaller shipyards or ship design firms may have a limited number of vessels and ship types in their database. To address this scenario, the initial population is reduced to four

different container vessels. This configuration is explored in the second case study to demonstrate the effects of MEC on small population sizes using same vessel types.

To circumvent issues associated with small initial populations, a third case study was conducted by expanding the 4 container vessels from the second case study to 12 vessels using the constant-morph method. For all three cases, 36 different crossover and mutation rates were applied to compare and examine the impact of different crossover and mutation ratings on large and small initial populations. The MEC results obtained for these three case studies are presented in Section 5.2, 5.3 and 5.4, respectively.

The design specifications for the container ship are listed below:

- **Principal Dimensions**

Length between Perpendicular (LPP)	= 185m
Breadth (B)	= 32m
Draft (T)	= 9m
Design Speed (V)	= 20 knots
Displacement Volume (Δ)	$\leq 34,358.7$ tonnes

- **Objective Function and Constraints**

The single-objective function is used to minimise overall resistance with a given design specification. Displacement volume (unit: metric ton) is set as design constraint so that the ship's cargo capacity is not compromised (reduced) while improving the objective function.

The single-objective optimisation is formulated as follows:

$$\begin{aligned} \text{Minimise} \quad & f = R_t \\ \text{Subject to:} \quad & \text{LPP} = 185\text{m}, B = 32\text{m}, T = 9\text{m} \\ & \Delta \leq 34,358.7 \text{ tonnes} \end{aligned}$$

where, R_t is the total resistance, LPP is Length between Perpendicular, B is Breadth (moulded), T is Draft and Δ is the Displacement of the ship in tonnes.

- **Hull Form Library**

The principal dimensions of the 12 existing (parent) vessels are provided in Table 5.1.

Table 5.1 Principal dimensions for 12 existing (parent) ships

Ship	Ship type	Length between perpendicular, LPP (m)	Breadth (m)	Depth (m)
1	Container Ship A	202	32.2	19.1
2	Container Ship B	185	32	19.1
3	Container Ship C	120	23	11.2
4	Container Ship D	145	13.6	13.6
5	Roll-on/ roll-off ship (ROPAX)	210	34	17.1
6	Bulk Carrier	220	36	23.75
7	Product Tanker	171	31	15.6
8	Chemical Tanker	126.5	21.5	10.5
9	Very Large Crude Carrier (VLCC)	314	58	31
10	Pipe-laying Ship	242	45	21.1
11	Cable-laying Ship	121	23	12.5
12	Offshore Accommodation Ship	168	24	13

These 12 vessels are proven designs that are ‘hand-crafted’ by experienced designers and used as parents in the initial population. Wire-mesh and body plan diagrams for the 12 existing (parent) vessels are shown in Figure 5.1 and Figure 5.2, respectively.

5.2. Case Study 1: Large Initial Population

In the first case study, 12 existing vessels in the Hull Form Library were set as the initial population. It has been demonstrated in Chapter 4 that low crossover and mutation rates do not yield good designs. Hence, different crossover and mutation rates ranging between 0.5 and 1 were investigated to select the optimal settings. A total of 36 different combinations were assessed.

A summary of the initial settings used is provided as follows.

- Initial populations = 12 vessels
- Crossover rates = 0.5 to 1.0
- Mutation rates = 0.5 to 1.0
- Total generations = 100

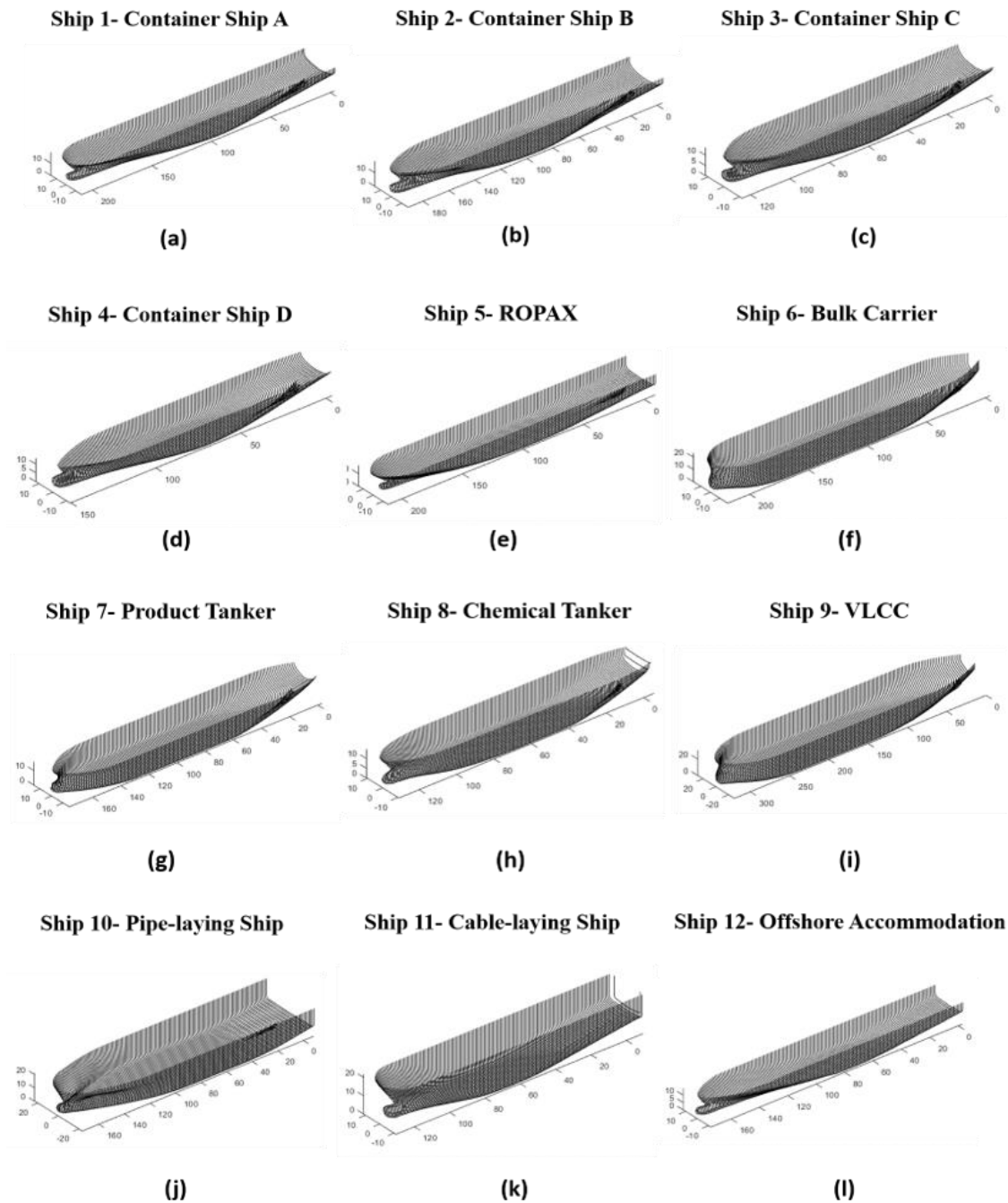


Figure 5.1 Wire-mesh diagrams of existing (parent) ships.

Based on the design criteria, the initial hull forms were first evaluated using the Holtrop method to estimate the total resistance, R_t . Subsequently, the hull shapes are modified using single-point cross-morph and boundary-morph. The new design solutions are then carried over to the next generation, where the entire process is repeated for 100 generations. Hull forms from the first generation are shown in Figure 5.3 to illustrate the geometric changes after cross-morph and boundary-morph.

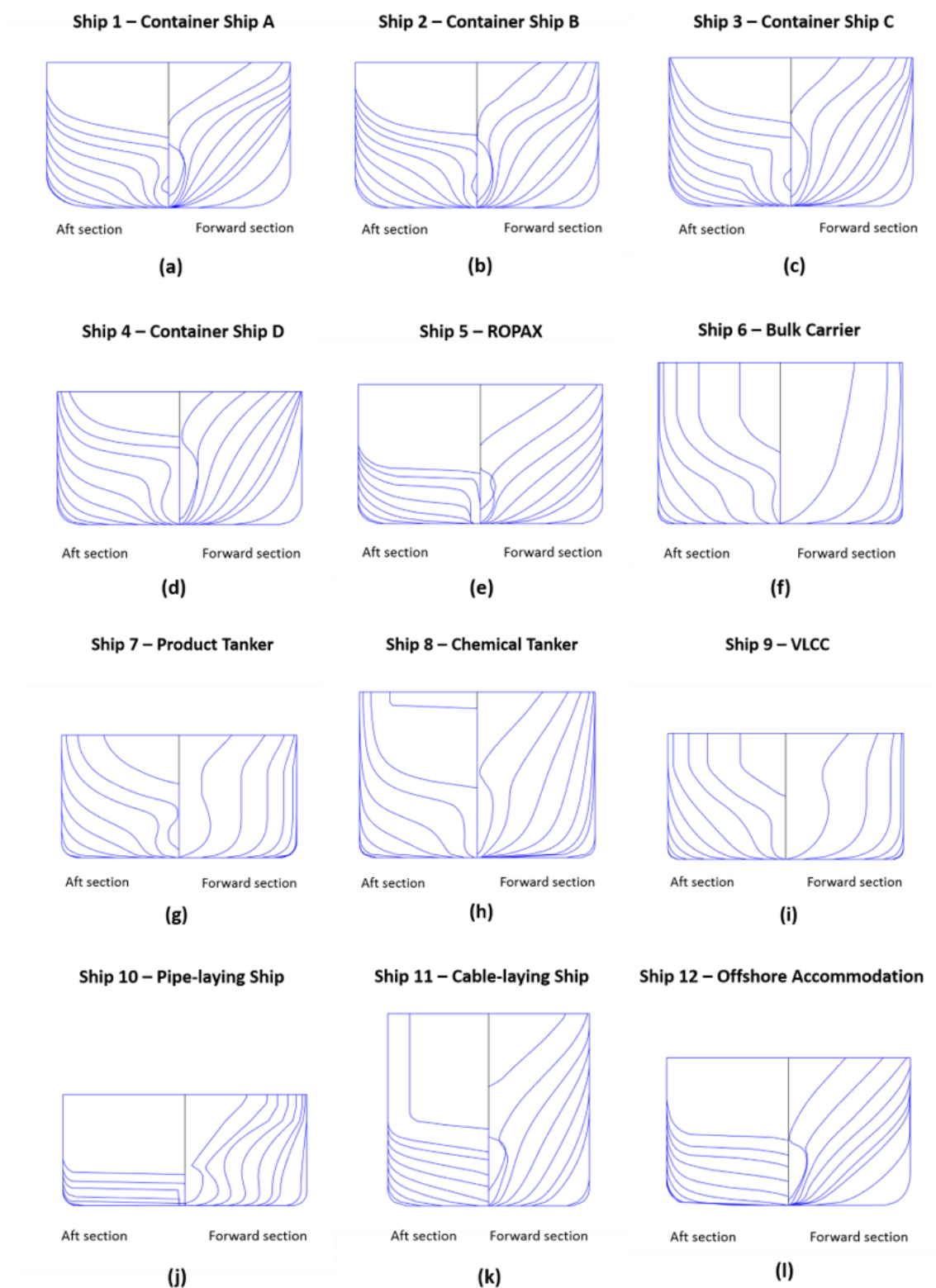


Figure 5.2 Body plan of existing (parent) ships.

Figure 5.3 shows the hull forms of two initial parent vessels and the geometric changes in the child vessel produced following cross-morph and boundary morph. Here, frames of the source vessel are represented in red while frames of the target vessel are shown in blue and frames modified through 2D curve morphing are represented in black.

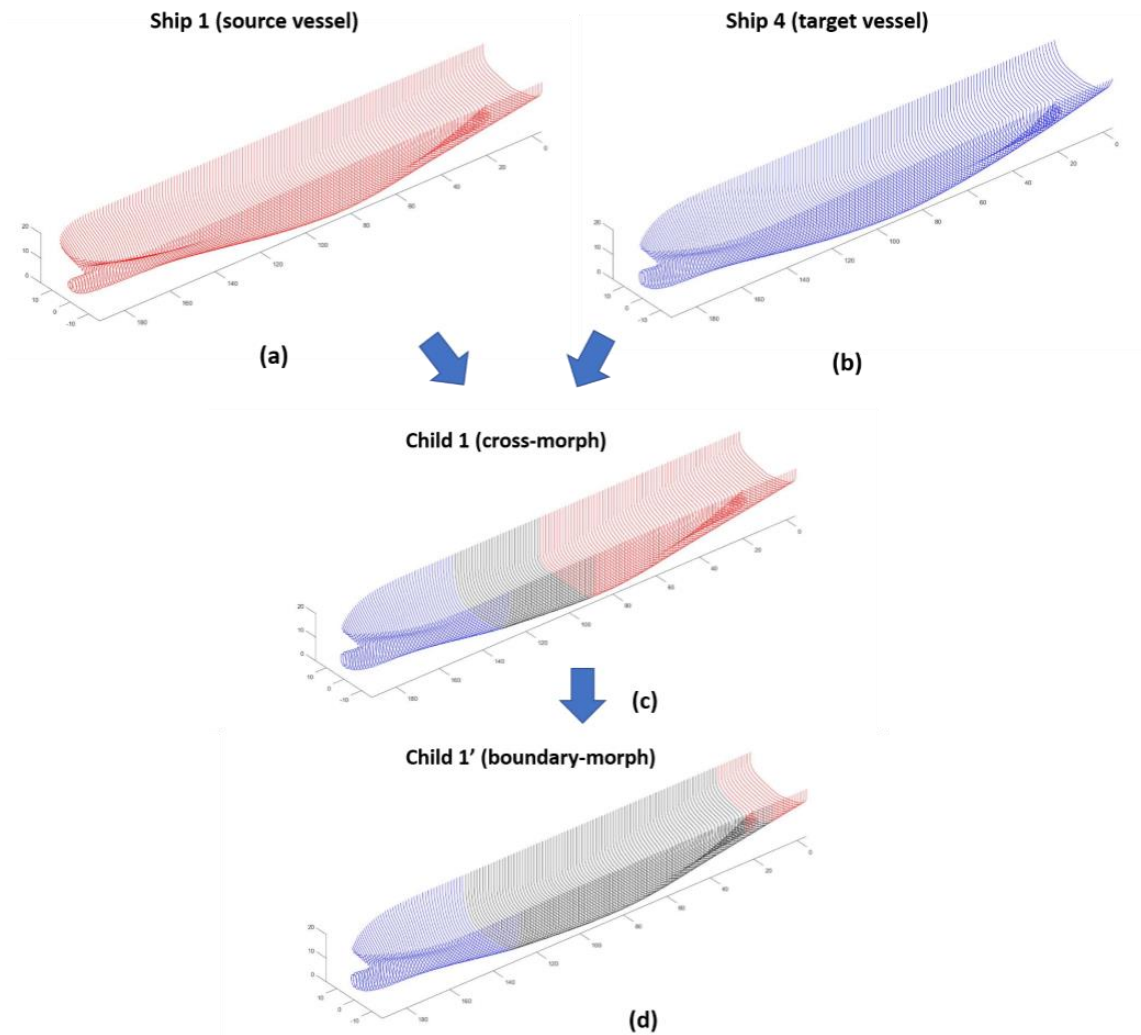


Figure 5.3 Wire-mesh diagrams of initial parents and child vessels from generation 1 of the optimisation.

Figures 5.3(a) and 5.3(b) show the parent vessels that were randomly selected from the initial population. These vessels are merged together using cross-morph resulting in child 1. As shown in Figure 5.3(c), it can be seen from the mid body section that cross-morph provided a smooth transition between the two parent vessels and retained a longer aft body of ship 1 (in red) and a shorter forward body of ship 4 (in blue).

Using boundary-morph, child 1 was mutated into child 1' as shown in Figure 5.3(d). It can be seen that child 1' now has a longer modified mid body that extended into the forward and aft body of the two parent vessels. The result is a new hull form, which had been modified extensively to retain only a small portion of forward and aft body from the two parent vessels. It can be seen that the shape of child 1' has a slightly slender forward body and fuller mid-body as the displacement volume was 'shifted' towards the aft body of the vessel. By doing so, it enables the shape of the forward body to be more

streamlined, thereby reducing the resistance while maintaining the minimum displacement.

A total of 36 simulations were performed using different combinations of crossover and mutation rates. The convergence charts shown in Figure 5.4 present the results obtained when the mutation rate was varied from 0.5 to 1 for each crossover rate starting from 0.5, with each increment of 0.1 until crossover rate reaches 1.0.

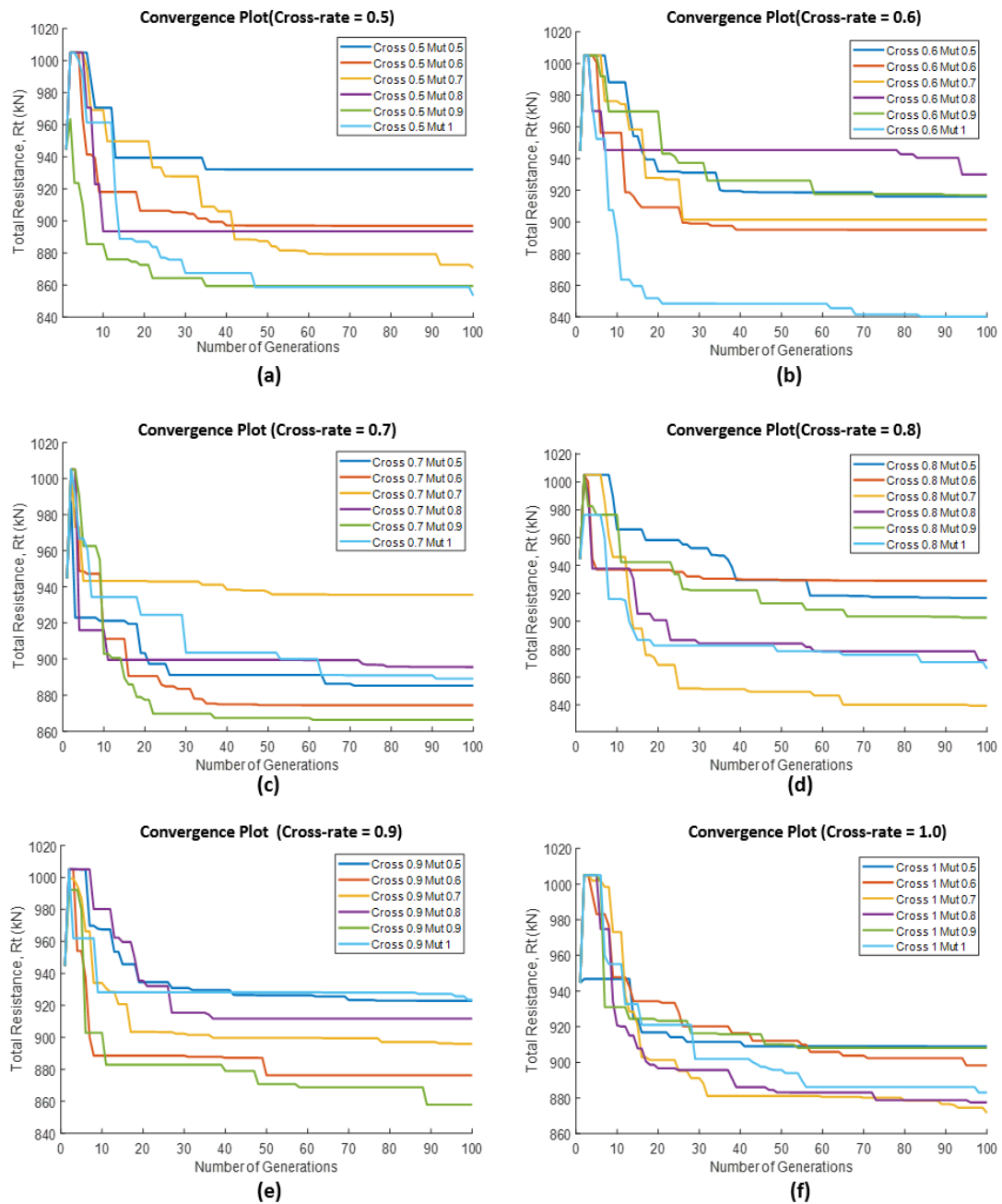


Figure 5.4 Convergence chart showing crossover rate of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.0.

Several key observations can be made from Figure 5.4. First, it is observed for crossover rates lower than 0.6, very high mutation rates were required to prevent premature convergence and poor performance. For instance, from Figures 5.4(a) (Cross 0.5, Mut 1) and 5.4(b) (Cross 0.6, Mut 1), it is evident that a mutation rate of 1 is needed to compensate for the low crossover probabilities (0.5 and 0.6). This is likely due to the initial population having a limited number of parent vessels available for crossover. Thus, the two parent vessels selected for crossover were very similar or identical, leading to child solutions having very similar or even cloned characteristics and performances. This coupled with a low crossover rate would result in solutions being “stuck” in a local optimum, resulting in premature convergence as the optimisation progresses.

Mutation, on the other hand, provided the more “bespoke” modifications where only small sections of the hull form are modified at a time. More importantly, it has a greater number of degrees of freedom. During mutation, sections of the hull can be modified in any directions. This is a key difference as crossover only morphs the hull forms in the x-direction. From Figures 5.4(a) and 5.4(b), it can be seen that with the exception of mutation rates of 0.9 and 1, the search started to converge very quickly around the 20th generation for all other settings. Furthermore, the solutions generated with higher mutation rates were also more superior.

For traditional optimisation, where the population size is larger (hundred of individuals), a high crossover rate coupled with a low mutation rate can provide good convergence [73]. In this case, the initial population is comparatively smaller and hence requires a higher mutation rate to increase shape variation. This can be seen from Figures 5.4(d) to (f), where crossover rates are 0.8 to 1.0, respectively and the mutation rate is 0.5 so that the search converges prematurely. Nonetheless, it is important to maintain a balance between crossover and mutation rates.

When a high crossover rate was used in conjunction with high mutation rate, the solutions found tended to be sub-optimal. This phenomenon is best observed in Figures 5.4(c) to 5.4(f) where the optimisation performance began to degrade when the mutation rates were increased from 0.7 to 1 for crossover rates of 0.7 and higher. For example, in Figure 5.4(e) for crossover rate of 0.9 and mutation rate of 1 (Cross 0.9, Mut 1), the optimisation stagnated at around the 10th generation and although the best solution found had better resistance performance (930 kN) as compared to the best initial parent (944.6 kN), it was sub-optimal. It is likely that this might have resulted from reasonably good chromosomes which were obtained after crossover was over-perturbed by mutation and as a

consequence, became too fragmented and lost its structure. This means that a good chromosome might not survive the current generation and be carried forward to the next as the optimisation progresses resulting in only sub-optimal solutions found at the end of the optimisation. In hull form design, this can be thought of as over-refinement of the geometry of the hull resulting in irregular and infeasible forms. Typically, both crossover and mutation should encourage construction of new solutions. However, this could prove counter-productive when overused. Balancing this “construction-destruction” paradox is important to the success of MEC.

Based on these findings, the crossover and mutation rate were found to be 0.8 and 0.7 respectively. The solution space for this setting (Cross 0.8, Mut 0.7) is shown in Figure 5.5.

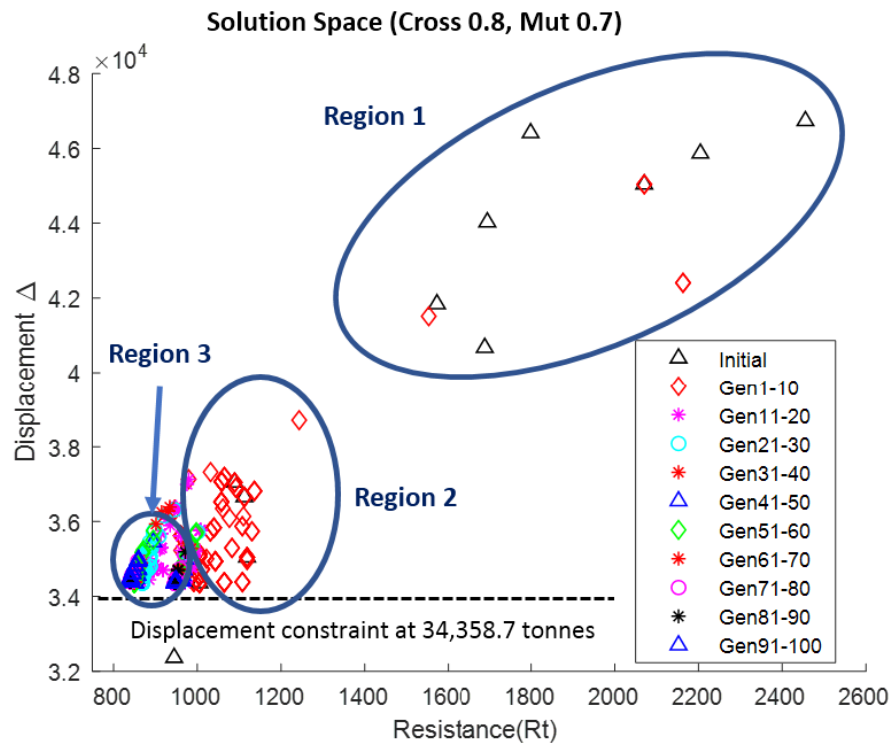


Figure 5.5 Solution space for crossover rate = 0.8 and mutation rate = 0.7.

From Figure 5.5, it can be seen that the solution space is divided broadly into 3 regions. The first (top right corner) covered the solutions from the initial population. The second region mainly comprised of solutions found in the first 10 generations and the third region located at the bottom left corner of the plot is made up of solutions starting from 11th to 100th generations. From the leftward and downward shift of the solution distribution and the distance between the three regions, it can be deduced that better solutions were found very quickly (within the first 10 generations of the optimisation) and the search began

slowing down beyond that with solutions having only incremental performance improvements. This characteristic is not uncommon in optimisation problems with limited individuals in the initial population which may sometimes be thought of as having converged prematurely. Nonetheless, as explained earlier, mutation might have resulted in a slightly more diverse pool of individuals which are carried over to the next generation for reproduction. As a consequence, incremental improvements were observed. The performance of the best initial parent and the six unique solutions found at the end of the optimisation are shown in Table 5.2.

Table 5.2 Best initial parent and child solutions optimising resistance function (high initial population)

Solution	Resistance (kN)	% improvement over best initial parent	Displacement (tonnes)
Best initial parent-Ship 1 (Container Ship A)	944.60	-	32,365.2
Solution 1189	839.32	11.14	34,460.16
Solution 1191	840.08	11.06	34,401.87
Solution 1199	852.05	9.79	34,821.34
Solution 1190	946.91	-0.24	34,416.65
Solution 1198	947.01	-0.25	34,360.05
Solution 1200	949.16	-0.48	34,435.97

It can be seen that all solutions were within the displacement constraint, of which three solutions had better resistance performances than the best parent design (Container Ship A, 944.6 kN) with the optimal solution (solution 1189, 839.32 kN) having an improvement of 11.14%. The wire-mesh diagrams of the best child solutions from the (a) 1st, (b) 5th, (c) 10th, (d) 20th, (e) 50th and (f) 100th generations are presented in Figure 5.6.

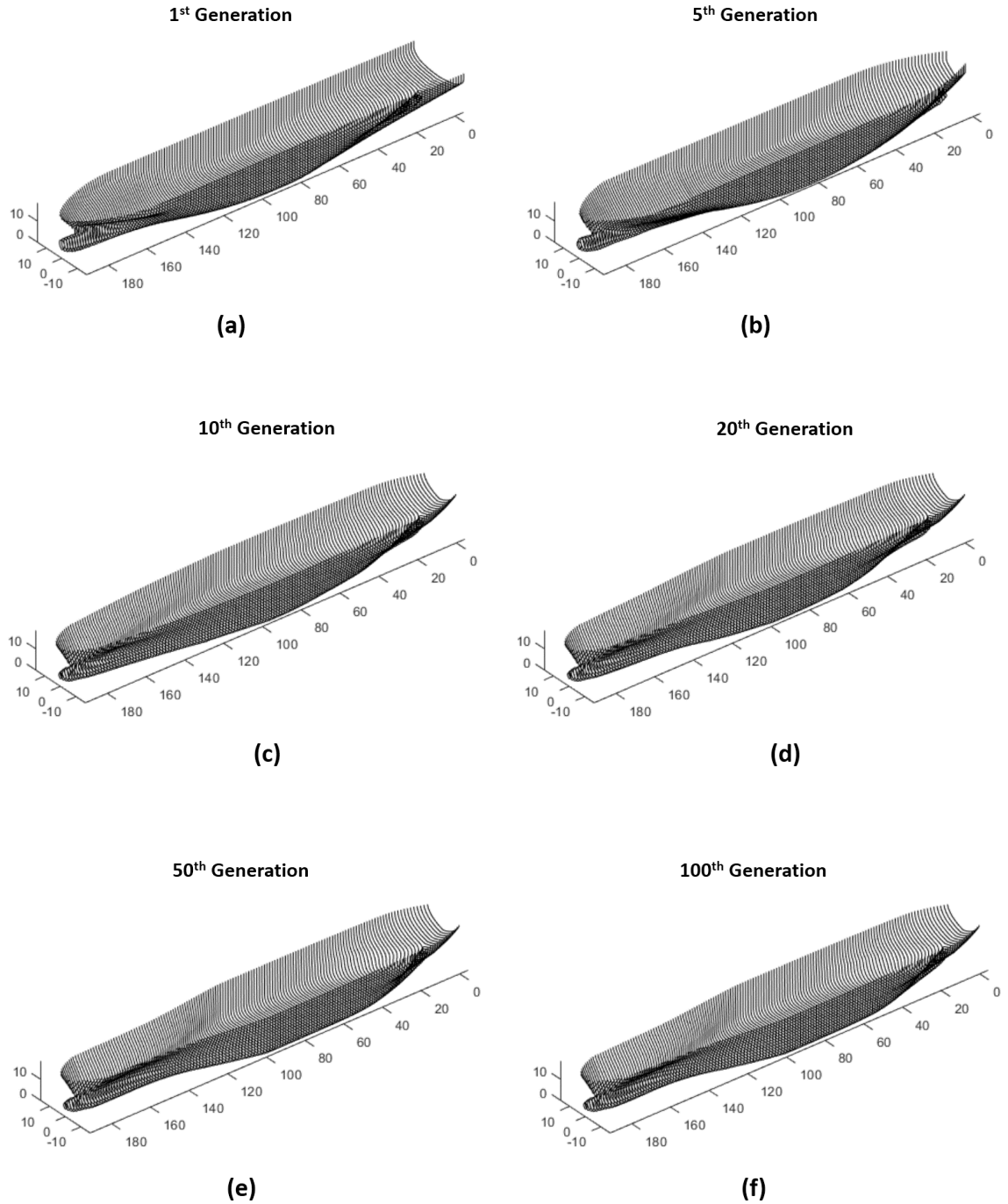


Figure 5.6 Wire-mesh diagrams of child solutions for (a) 1st, (b) 5th, (c) 10th, (d) 20th, (e) 50th and (f) 100th generation.

Figure 5.6 shows the evolution of the best solution as the optimisation progressed. At the start of the optimisation, parent ship 2 (Container Ship B) in the initial population was carried forward into the first generation. The wire mesh diagram of the hull is shown in Figure 5.6(a). Figure 5.6(b) shows the hull form obtained in the 5th generation after parent ship 2 was cross-morphed with the aft body section of initial parent ship 9 (VLCC). The effect of this new combination is that the volume displacement at the aft body was

significantly reduced. This is not ideal where cargo capacity is concerned. As the optimisation progresses, a new aft section which resembles initial parent ship 4 (Container Ship D) was introduced in the 10th generation during crossover and the bow area was modified in boundary-morph. This new combination provided more displacement volume at the aft section and created more cargo spaces. The resultant hull form is shown in Figure 5.6(c). It can be seen from Figures 5.6(d) to 5.6(f) that between the 20th to 100th generations, the bow body began to narrow and the mid body of the vessel started to expand. This is a direct effect of boundary-morph which bodes well with the distribution shown in Figure 5.5 where performance improvements from the 20th generation onwards were mainly driven by mutation.

The body plans of the best initial parent (ship 1- Container Ship A) and best solution (solution 1189) are shown in Figure 5.7.

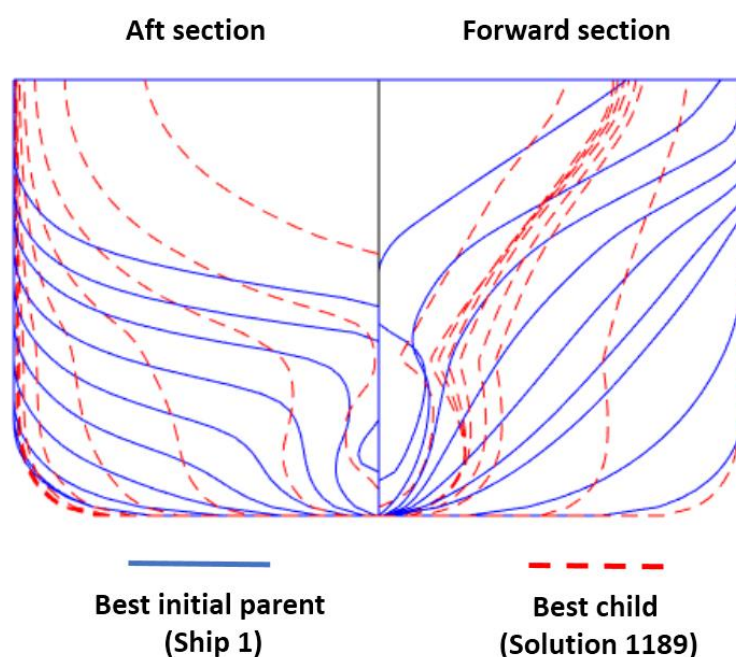


Figure 5.7 Body plan comparing best child (solution 1189) and best initial parent (ship 1 - Container Ship A).

It can be observed from Figure 5.7 that the hull lines from the best child are relatively smooth (without broken lines or bumps) and mostly feasible. This is a considerable achievement given that the hull form underwent approximately 1200 (100 generations with 12 solutions generated in each population) rounds of modifications. Comparing the two hull forms of the best solution and the best initial parent, it is clear that they are completely different. For example, it can be seen that the aft body of the best solution has

taken on the shape of the initial parent (ship 7 – Product Tanker) while the forward body resembles that of the initial parent (ship 9 - VLCC). This is a good illustration of how crossover and mutation has worked together to produce a highly feasible hull form that meets the design specifications.

The results showed that MEC has ‘intelligently’ sized the shape of the vessel to meet the design specifications. The optimisation was performed automatically without any input from the designer beyond the initial settings prior to the start of the process.

5.3. Case Study 2: Small Initial Populations (Four Vessels)

In the second case study, only four container ships from the Hull Form Library were used as parent vessels in the initial population. As with case study 1, 36 combinations of crossover and mutation rates (ranging from 0.5 to 1.0) were applied and the results are presented in Figure 5.8.

The settings used for the optimisation are shown below:

- Initial populations = 4 vessels
- Crossover rates = 0.5 to 1.0
- Mutation rates = 0.5 to 1.0
- Number of generations = 100

From Figure 5.8, a number of interesting observations can be made about the optimisation. Some exhibited similar trends to those observed in Figure 5.4 for case study 1 while others do not.

First, for crossover rates below 0.7, high mutation rates are required to improve the performance of the search. For example, in Figure 5.8(a) where the crossover rate was 0.5, better solutions were found as the mutation rate increased. This is most clearly observed for mutation rates from 0.5 to 0.7 and bodes well with the findings from the first case study. However, unlike case study 1 where sub-optimal results were obtained when both the crossover and high mutation rates were increased above 0.7 (Figures 5.4(c) to 5.4(f)), improvements in performance were observed in this case study as the crossover and mutation rates continue to increase. This is evident from Figures 5.8(c) to 5.8(f) that better solutions were found for crossover and mutation rates of 0.7 to 1. In almost all cases, the best solution was obtained when the mutation rate was 1.

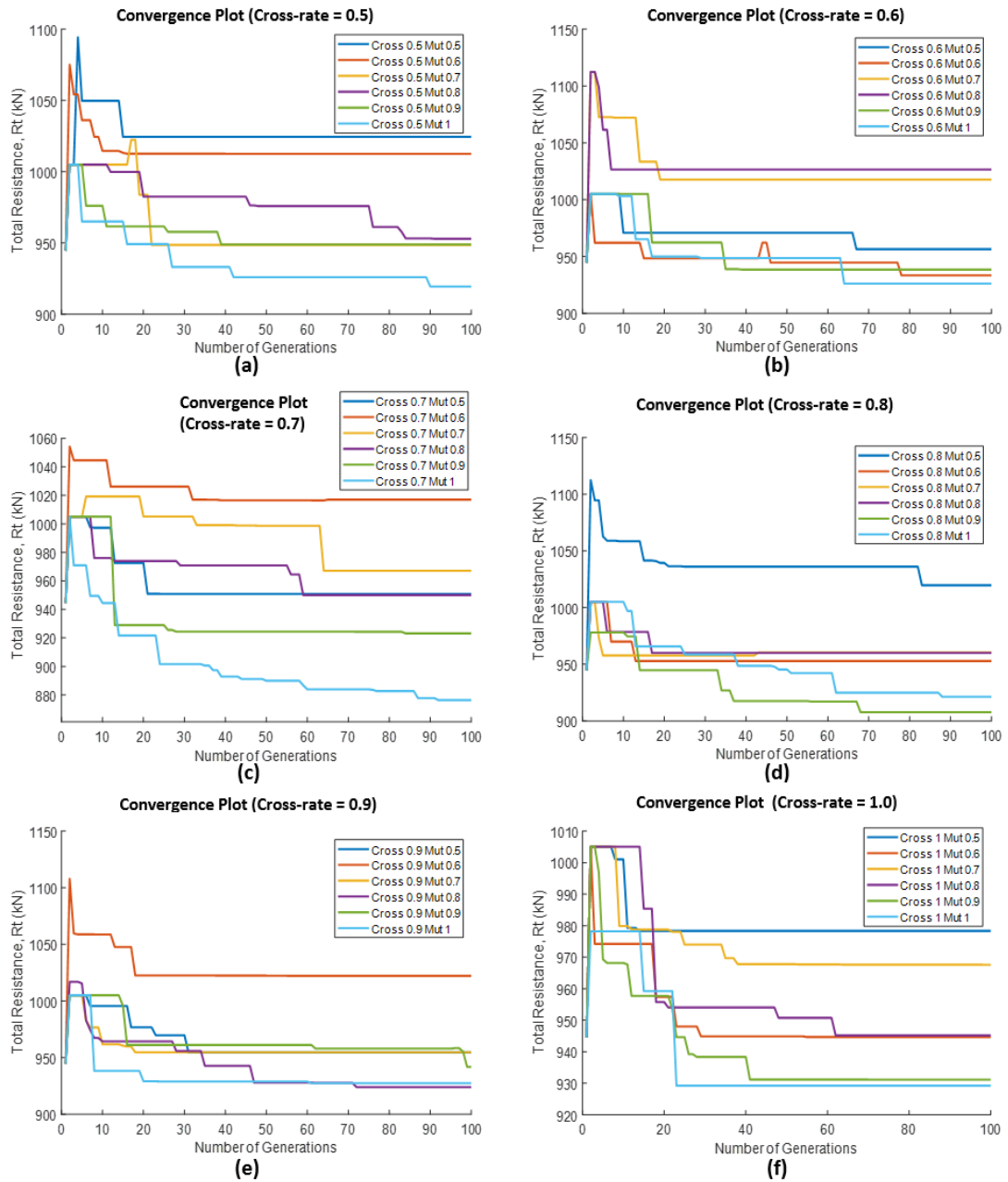


Figure 5.8 Convergence chart showing crossover rate of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.0.

Second, as with case study 1, a high mutation rate was needed to prevent premature convergence. For example, from Figures 5.8(c) and 5.8(d), it can be seen that for crossover rates of 0.7 and 0.8, performance improvements were found very early on in the optimisation. However, this tended to plateau around the 20th generation and little or no improvement was found beyond this point.

It is clear from both case studies that crossover on its own could only provide limited improvement to the solutions, especially as the optimisation progressed over a number of generations. This observation particularly resonates with the second case study when the

initial population consisted of only a handful of parent solutions with very similar structures. To validate this observation, another simulation was carried out by setting the mutation rate to 0 and crossover rates ranging from 0.5 to 1.0. The result of this simulation is provided in Figure 5.9.

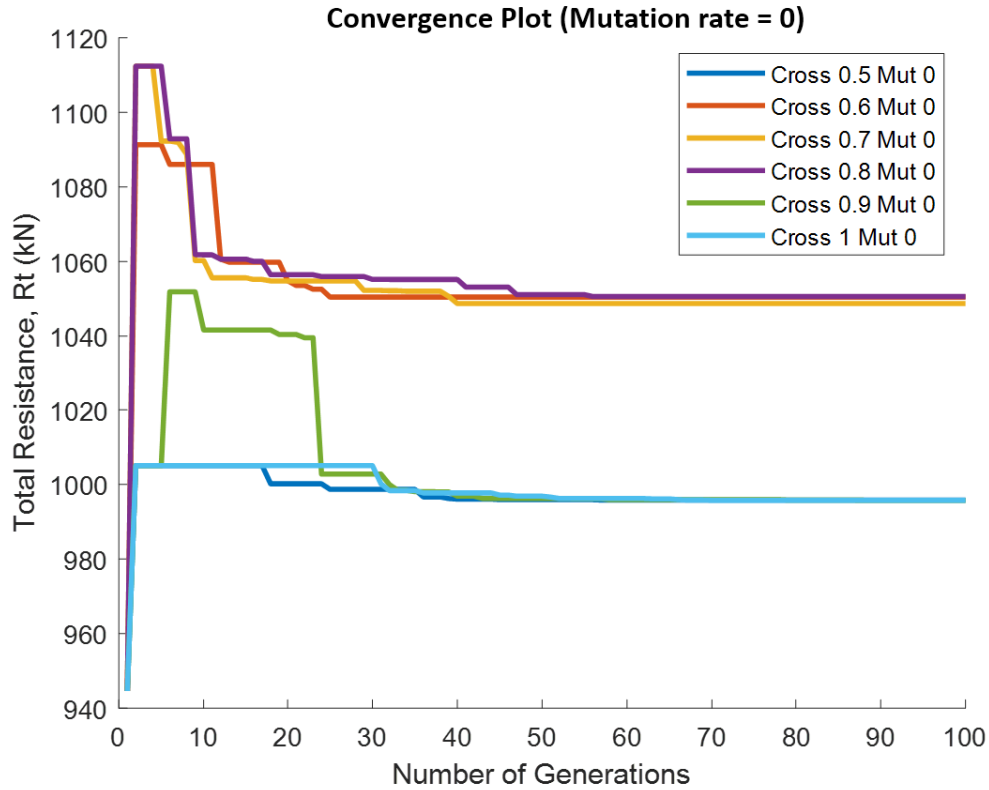


Figure 5.9 Convergence chart showing mutation rate = 0 and crossover rates from 0.5 to 1.0.

Figure 5.9 shows the effect of different crossover rates when no mutation was applied (mutation rate = 0). There were two distinct outcomes. The best resistance (1000 kN) was observed when the crossover rates were at the two extreme settings, i.e. 0.5, and 0.9 and 1.0. For crossover rates between 0.6 and 0.8, the solutions obtained exhibited worse (higher) resistance. The best vessel in this range had a resistance of 1050 kN. In comparison to Figure 5.8 where mutation rate was varied from 0.5 to 1.0, the resistance performance had worsened by approximately 10%. These results further confirmed that mutation is important to “refresh” the population of solutions for better improvements.

The solution space for a crossover rate of 0.7 and a mutation rate of 1.0 (Cross 0.7, Mut 1.0) is shown in Figure 5.10. Similar to case study 1, three distinct regions are identified. The first region, located to the right of the plot, consisted of solutions from the initial population and the first 10 generations of the optimisation. The second comprised of solutions found between 11th to 30th generations and the third region contained solutions

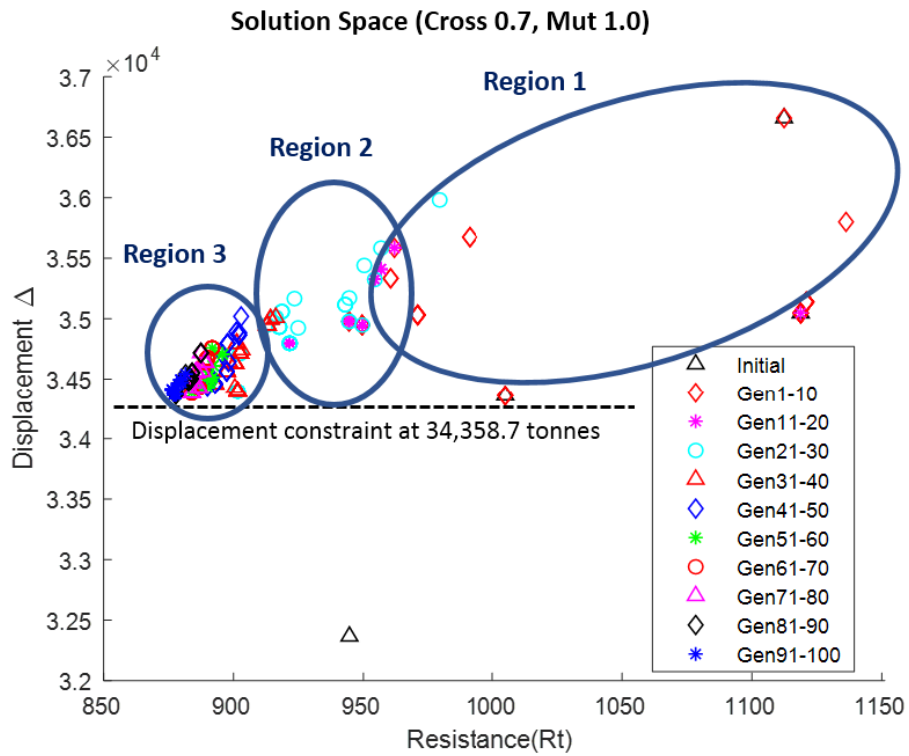


Figure 5.10 Solution space for crossover rate = 0.7 and mutation rate = 1.0.

from 31st to 100th generations. As with case study 1, the “movement” of solutions in the search space tended towards a leftward and downward shift, an indication of convergence as the optimisation progressed. Nonetheless, when one considers the rate of this shift, it becomes apparent that this was slower as compared to case study 1. In case study 1, the optimisation progressed very quickly toward the optimal solution space (Region 3) where solutions found were crowded around the optimal solution within the first 10 generations. However, in the current case study, it took another 10 more generations before the search started to converge towards Region 3. This is because there is a smaller number (and type) of parent vessels in the initial population as compared to case study 1. This limited the options available for crossover and mutation and further affirmed the need for higher mutation rate in this case.

The performance of the best initial parent and the four solutions found at the end of the optimisation are shown in Table 5.3. Here, the optimal solution (solution 397) has a resistance of 876.31 kN, an improvement of 7.22% over the best parent (ship 1). The hull of the best child solutions from the 1st, 5th, 10th, 20th, 50th and 100th generation are presented in Figure 5.11.

Table 5.3 Best initial parent and child solutions for resistance optimisation using a small initial population

Solution	Resistance (kN)	% improvement over best parent	Displacement (tonnes)
Best initial parent-ship 1 (Container Ship A)	944.60	-	32,365.2
Solution 397	876.31	7.22	34,412.99
Solution 398	876.99	7.15	34,365.52
Solution 399	877.74	7.07	34,368.46
Solution 400	878.83	6.96	34,458.39

Figure 5.11 shows the shape changes to the best child solutions during the optimisation process. It can be observed that the child solutions following crossover were very similar during the 1st to 5th generation as seen in Figures 5.11(a) and 5.11(b). This is because there were only four vessels of the same type in the initial population. Despite the similarities, the resemblance of the initial child solution to the forward end of initial parent ship 2 (Container Ship B) and aft end of initial parent ship 3 (Container ship C) is observed. The new hull form did not vary greatly from the original parent designs and hence there was no significant improvement in the performance. However, from the 10th generation onwards as shown in Figure 5.11(c) to Figure 5.11(f), the effect of mutation was more apparent where the bow section was reduced and the mid-section body was increased. This created significant shape changes especially to the forward and aft body of the new child solution as compared to initial parents.

It can be observed from Figure 5.11(f) that the bow of the final solution was very ‘sharp’ and streamlined, which helps to reduce the overall resistance of the vessel. This improved hull form is consistent with the previous case study, with a narrow bow and fuller mid-body, demonstrating the ability of MEC to modify the hull, resulting in lower resistance. The body plan of the best child (solution 397) and best initial parent (ship 1 – Container Ship A) are plotted in Figure 5.12.

From Figure 5.12, it can be seen that although there are changes in both the forward and aft sections of the ship, some attributes of the initial parent (ship 1 – Container Ship A) are retained. For example, the aft section of the best child (solution 397) is similar to that of the initial parent (ship 4 – Container Ship D) and the forward section of the initial parent (ship 2 – Container Ship B). Similar to case study 1, the bow section has been

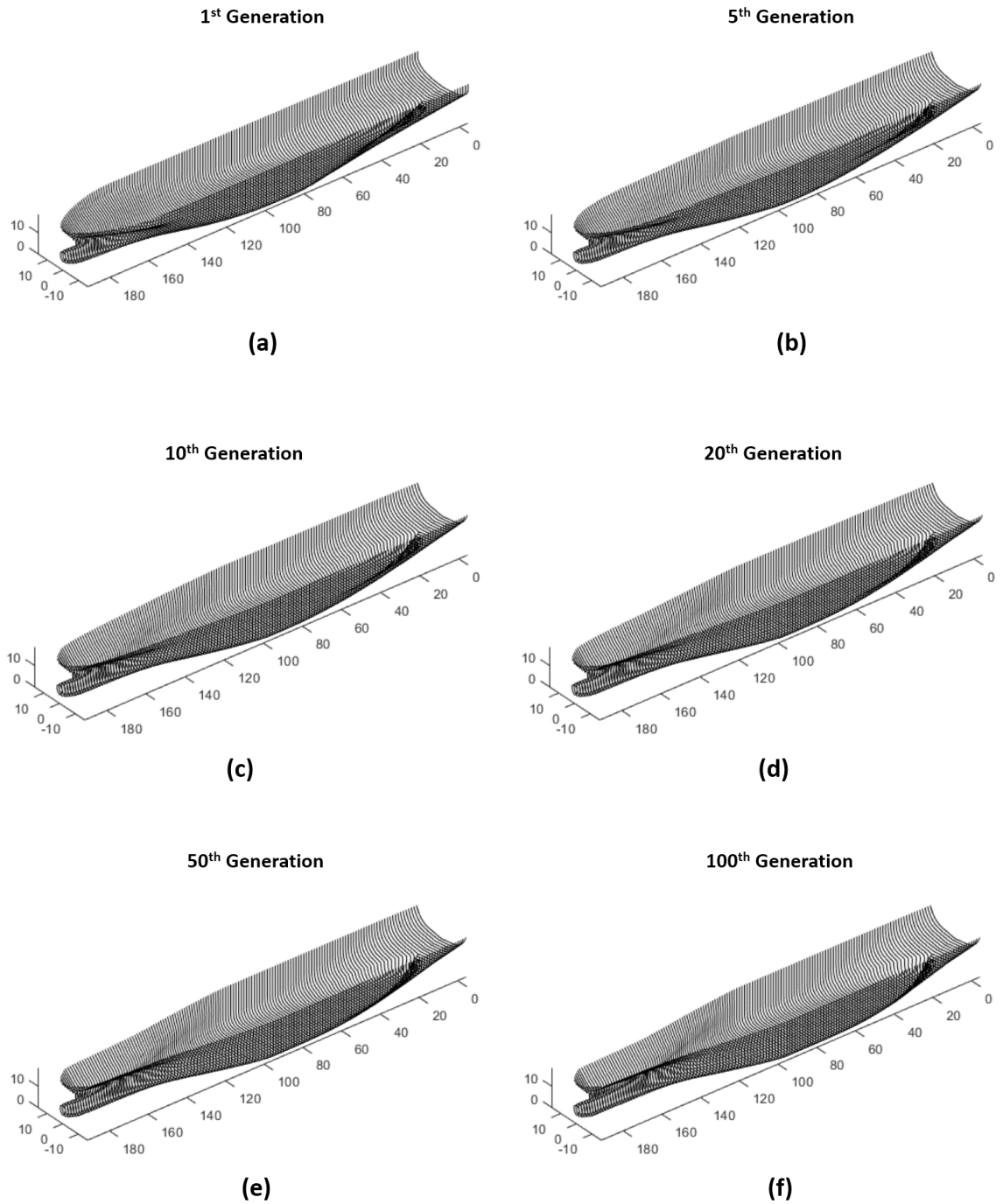


Figure 5.11 Wire-mesh diagrams of hull form for child solutions for (a) 1st, (b) 5th, (c) 10th, (d) 20th, (e) 50th and (f) 100th generation.

extensively modified during boundary-morph, resulting in a very slender bow which helps to reduce the resistance. All the modified lines are smooth which means that the generated solution is feasible. This result demonstrates that MEC is well-suited for optimisation problems where there are only a small number of individuals available in the initial population.

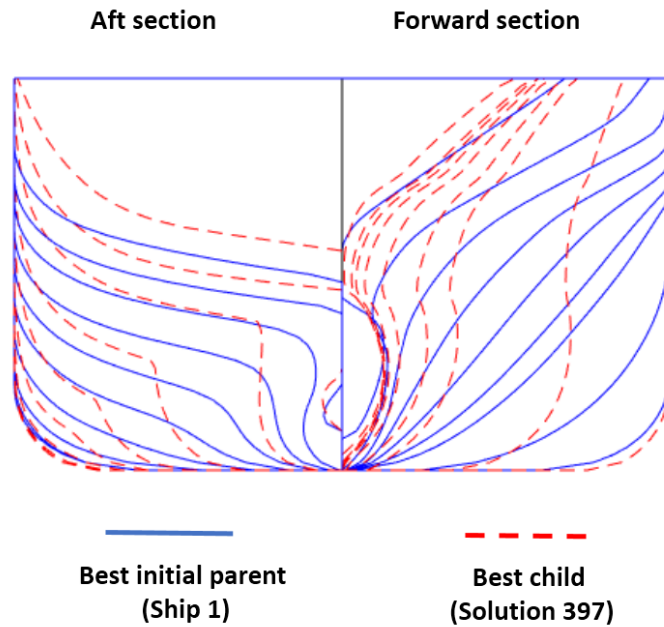


Figure 5.12 Body plan comparing best child (solution 397) and best initial parent (Ship 1 – Container Ship A).

5.4. Case Study 3: Expansion from Small to Large Initial Population

In this case study, the same initial population from case study 2 was used. However, eight new vessels were created using constant-morph and added to form an initial population of 12 vessels. This process is illustrated in Figure 5.13.

In Figure 5.13, the frames of the source and target vessels are represented in blue and red respectively while the frames of the child solutions created through constant-morph are represented in black. It can be seen that the hull forms of all child vessels are highly streamlined and have large deck spaces. This is an important requirement for a container vessel where lower resistance and higher payload are both highly desirable properties. This is a very efficient way to rapidly increase the size of the initial population. Similarly, 36 combinations of cross-over and mutation rates ranging from 0.5 to 1.0 were applied to select the optimum settings for this case study. The MEC settings are summarised below.

- Initial populations = 12 vessels
- Crossover rates = 0.5 to 1.0
- Mutation rates = 0.5 to 1.0
- Number of generations = 100

The convergence charts are shown in Figure 5.14. By comparing the convergence plots between case studies 2 and 3, two key observations can be made. First, as seen in Figure 5.14, most of the convergence took place after 50-60th generation as opposed to 20 - 30th



Figure 5.13 Increasing the number of parent vessels in the Initial population from four to 12 (8 new vessels represented in black) using constant-morph.

generation in Figure 5.8 in the second case study. The overall resistance was also improved from around 880 kN to around 850 kN for the best child solution. This demonstrated that by extending the initial population from 4 to 12 vessels using constant-morph, more shape variations and diversity of solutions are achieved, leading to delayed convergence and better performances.

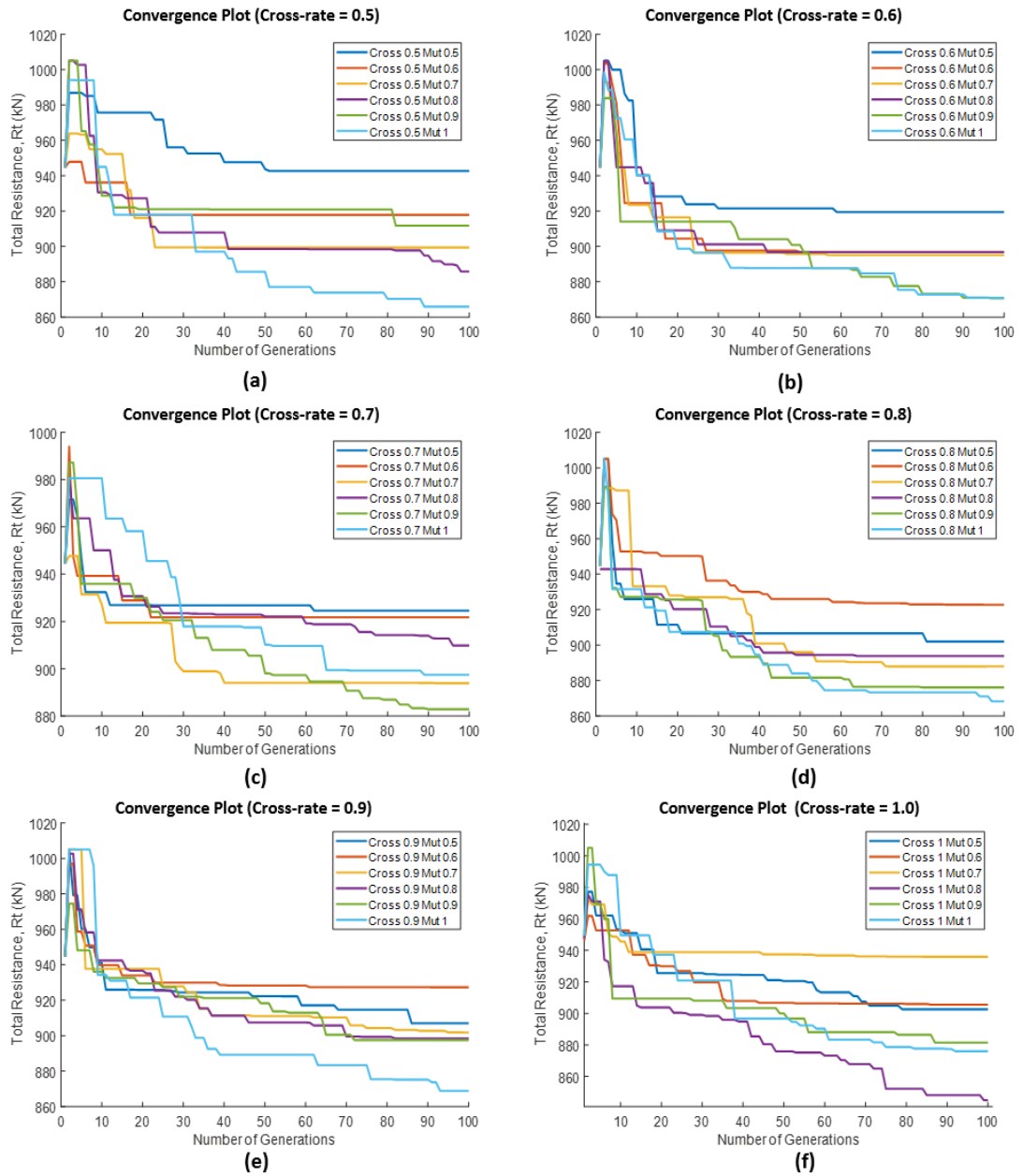


Figure 5.14 Convergence chart showing crossover rate of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.0.

Second, higher crossover rates are observed to result in better performances for the expanded initial population in case study 3 as compared to the smaller initial population in case study 2. For example, when crossover rate was increased from 0.5 to 1.0 with the mutation rate set at 0.8, there was no improvement observed. In this case study, the resistance reduced by -4.7% as crossover rate increased from 0.5 to 1.0 for the same mutation rate. This difference might be attributed to better crossover effect where more diverse solutions were created by combining a larger variety of parents from the increased initial population. The solution space for the most optimal setting (Cross 1.0, Mut 0.8) is shown in Figure 5.15.

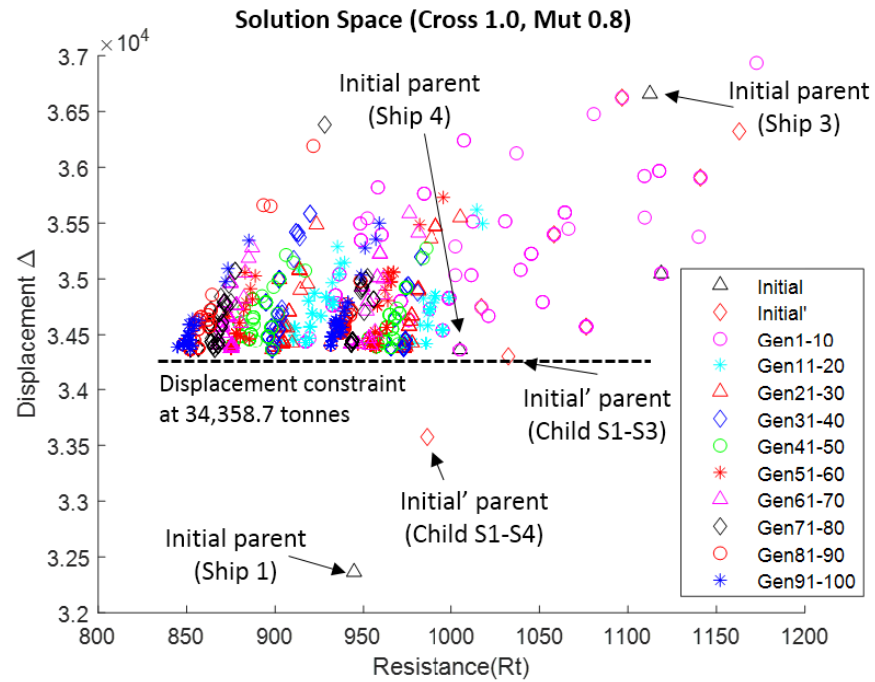


Figure 5.15 Solution space for crossover rate = 1.0 and mutation rate = 0.8. Initial' represents the initial population's solutions generated through constant-morph.

Figure 5.15 showed that the solutions found in this case study tended to be more spread out across the entire solution space as compared to those found in the previous two case studies. There are a few reasons for this. First, the absence of any distinct regions as observed in case study 3 suggests that there are no local optima or “traps” in the solution space. Second, in comparison to case study 2, the expanded initial population had more diverse individual. This demonstrated the efficiency of constant-morph to create more new initial parents for a larger initial population. As a consequence, there is wider coverage of the search space and increased likelihood of better solutions being discovered during the optimisation.

From Figure 5.15, it can also be observed that the increase in diversity might be likely attributed to the new solutions added to the initial population (Initial'). It can be seen that these solutions covered areas of the solution space previously not “discovered” by the initial population. For example, initial' parent (Child S1-S3) was located in between initial parents ship 1 and ship 3. Similarly, initial' parent (Child S1-S4) was located in the design space between ship 1 and ship 4. The performance of the best parent and 12 best solutions in the final generation are presented in Table 5.4.

Table 5.4 Best parent and the solutions found at the end of the optimisation ranked in order of performance.

Solution	Resistance (kN)	% improvement over best parent	Displacement (tonnes)
Best Parent- Ship 1	944.60	-	32,365.2
Solution 1189	844.86	10.55	34,384.72
Solution 1193	848.11	10.21	34,406.61
Solution 1195	848.50	10.17	34,426.19
Solution 1199	848.52	10.17	34,426.33
Solution 1190	931.38	1.39	34,401.73
Solution 1191	931.70	1.36	34,479.34
Solution 1192	931.93	1.34	34,486.79
Solution 1194	932.48	1.28	34,477.61
Solution 1196	932.61	1.26	34,487.38
Solution 1197	933.14	1.21	34,607.14
Solution 1198	934.04	1.11	34,601.83
Solution 1200	934.50	1.06	34,627.16

The best solution (solution 1189) achieved a resistance of 844.86 kN, an improvement of 10.55% over the best parent design. In comparison to case study 2, there was an improvement of 3.33%. This improvement further affirmed the importance of having diverse solutions in the initial population and the effectiveness of constant-morph to expand the initial population very rapidly. The hull form of the best solution taken from the (a) 1st, (b) 5th, (c) 10th, (d) 20th, (e) 50th and (f) 100th generations are presented in Figure 5.16.

It can be seen in Figure 5.16(a) that the 1st generation started off with a solution that is similar to the initial parent (ship 1). Following which, a new shape as shown in Figure 5.16(b) was obtained in the 5th generation where a bow body of the initial parent (ship 2) and an aft body of the expanded initial' parent (child S2-S4, $t = 0.8$) was merged during crossover. This combination continued over subsequent generations with the entire section of the hull form continuously 'fine-tuned' through mutation. Similar to the other two case studies, it can be observed from Figure 5.16(f) that the final shape of the best child at the 100th generation has a very fine and narrow forward body for low resistance.

In addition, the mid to aft body of the hull appeared to be much fuller as compared to the solution found in the 50th generation in Figure 5.16(e). These modifications compensated

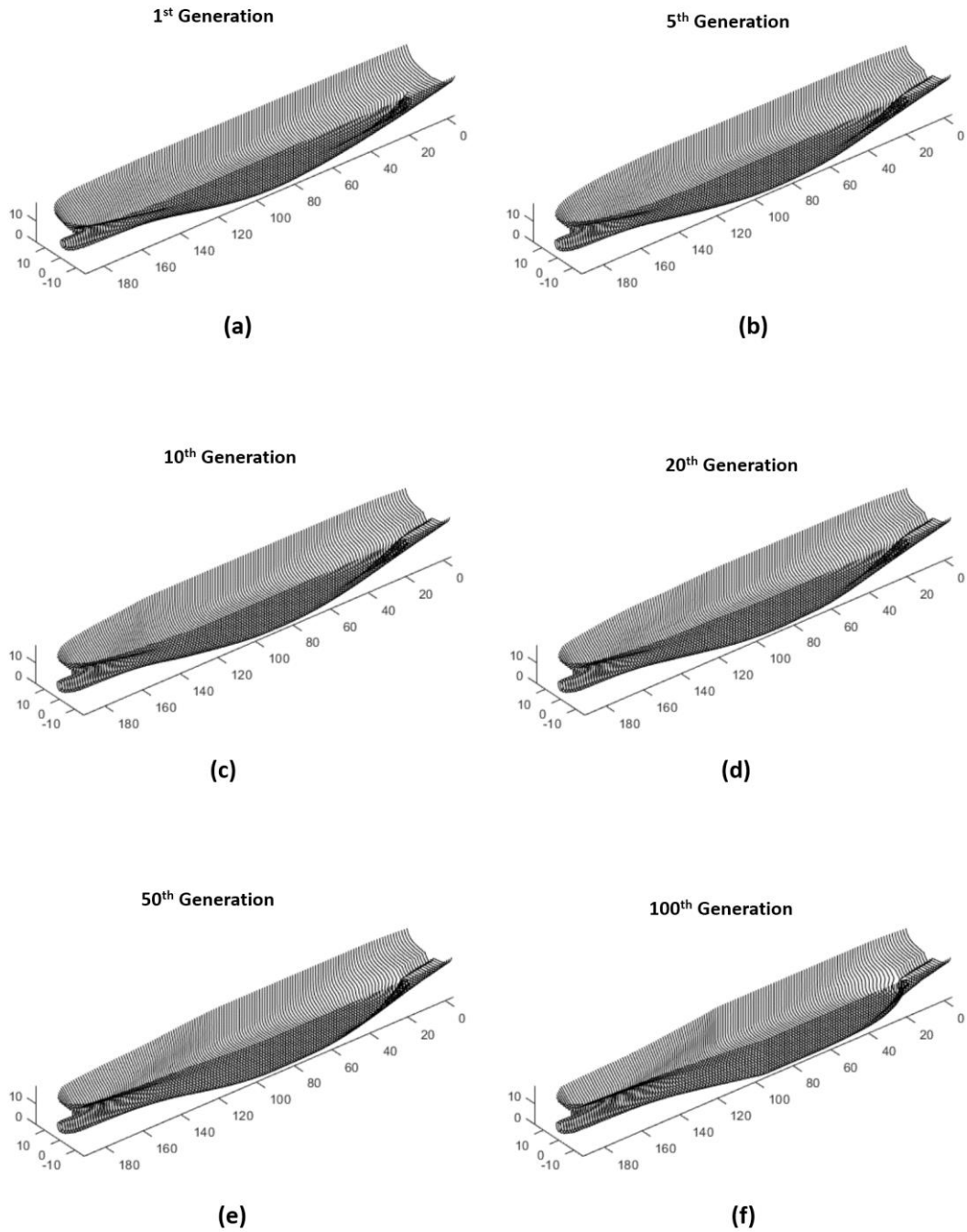


Figure 5.16 Wire-mesh diagram of child solutions for (a) 1st, (b) 5th, (c) 10th, (d) 20th, (e) 50th and (f) 100th generation.

for the loss of volume at the bow by shifting it to the aft body in order to maintain the minimum displacement. The body plans of the best child (solution 1189) and best initial parent (ship 1) are plotted in Figure 5.17.

Similar to previous cases, the best child solution and the best parent have rather distinct hull forms. Figure 5.17 shows that the aft section was similar to child S2-S4 ($t = 0.8$), which was created using constant-morph while the forward section took the shape of the

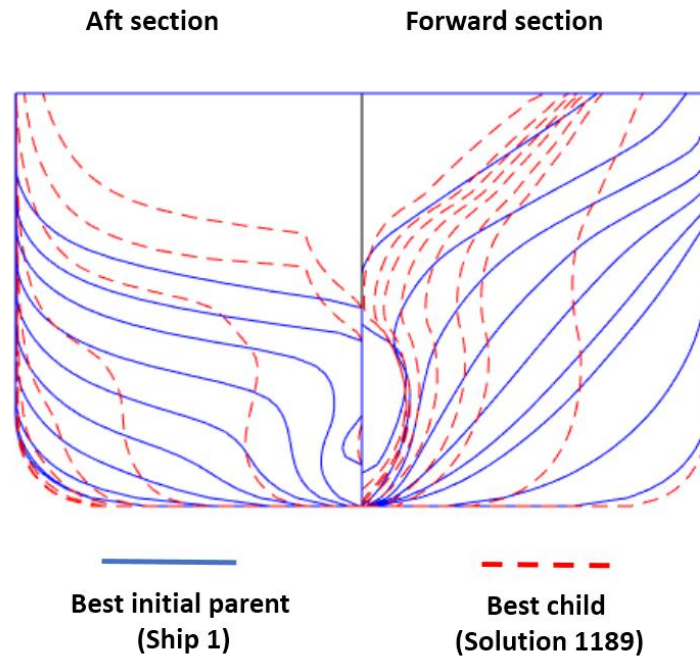


Figure 5.17 Body plan comparing best child (Solution 1189) and best initial parent (Ship no. 1 – Container Ship A).

initial parent (ship 2). As compared to the best parent (ship 1), the cross-sectional area at the aft of best child solution has expanded considerably. This is due to the shifting of displacement volume from the forward section to the aft of the vessel. At the same time, it can be observed that the bow section has shrunk significantly to create a slimmer cross-section thereby reducing the overall resistance. More importantly, the hull lines of the best child solution are still highly smooth and feasible.

5.5. Validation of Solutions from MEC with NAPA

In this section, the solutions created by MEC are validated against NAPA. NAPA provides highly customised manager functions where user can apply for hull form optimisation. These manager functions include geometric transformation using FFD, optimisation using Multi-Objective Genetic Algorithm (MOGA) [210] and resistance evaluation using the Rankine-source panel method [211]. An experienced designer was tasked with optimising the hull form of a container ship based on the same design specification. The designs were assessed based on the overall time taken and improvement achieved with MEC. The steps and results of the validation are provided in the following pages.

- **Step 1 - 3D Surface Modelling**

Firstly, the hull form was created manually in NAPA. As a starting point, an existing vessel within the initial population was used. This is one of the most time-consuming processes where the 2D offset data are imported using offset manager function into NAPA and then modelled and faired to ensure that the surfaces are fully intact. The final surface of the initial hull is shown in Figure 5.18.

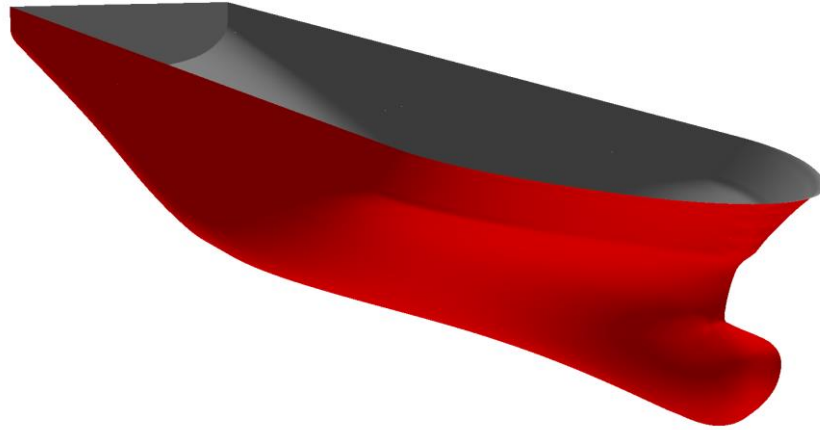


Figure 5.18 3D surface of a container vessel created in NAPA.

- **Step 2 - Geometry Modification**

In the optimisation function, the hull form was modified using FFD by extending the bulbous bow further forward. Firstly, a bounding box around bulbous bow was created by the designer. Then, three movement points were set: 5m towards +X direction, 3m towards +Z direction and 2m towards -Z direction. The outcome of the geometry transformation is shown in Figure 5.19.

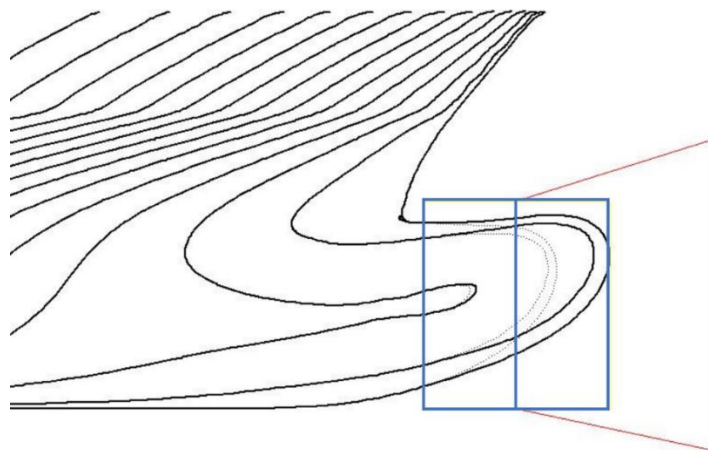


Figure 5.19 FFD transformation comparing initial (dotted) and transformed hull (bolded). The bounding box (blue) and direction of shifting (red).

- **Step 3 - Panelisation Of Hull Surface**

To evaluate the resistance of the hull, the model was first panelised using 1960 hull panels and 1000 free surface panels were selected for evaluating the fluid region. Potential flow was then used to estimate the flow field around the ship hull.

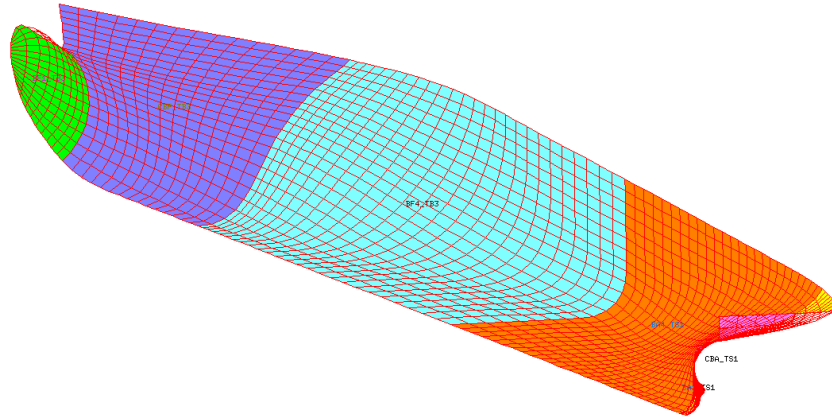


Figure 5.20 Panelised surface using 1960 hull panels and 1000 free surface panels.

- **Step 4 - Optimisation and CFD Evaluation**

Optimisation was carried out using the GA features provided in NAPA with the following settings. For comparison, these settings were set close to those in MEC. A higher crossover setting was applied to increase shape variations as mutation in NAPA is more localised using Free-form Deformation (FFD), when compared to global transformation of the entire hull using MEC. In this case, only the bulbous bow was mutated in NAPA, as modifying the entire hull form using FFD would be too disruptive to the hull shape, due to the limited number of control points available. The results of the optimisation are provided in Table 5.5.

- Number of individuals in a population = 10
- Number of generations = 100
- Crossover probability = 1.0
- Mutation probability = 0.5

From the results shown in Table 5.5, an improvement of 1.6% can be observed when comparing the best solution (ID 96) to the initial design. The wave profile of solution ID 96 is shown in Figure 5.21.

Table 5.5 Optimisation results comparing initial vessel and best five child designs.

Solution ID	Blend 1	Pressure resistance coefficient (CPRS)	Improvement from initial design (%)
Initial	0	0.254	-
103	8.2795	0.252	0.79
96	9.5307	0.25	1.6
160	8.8022	0.253	0.39
199	9.6480	0.251	1.19
82	9.8045	0.252	0.79

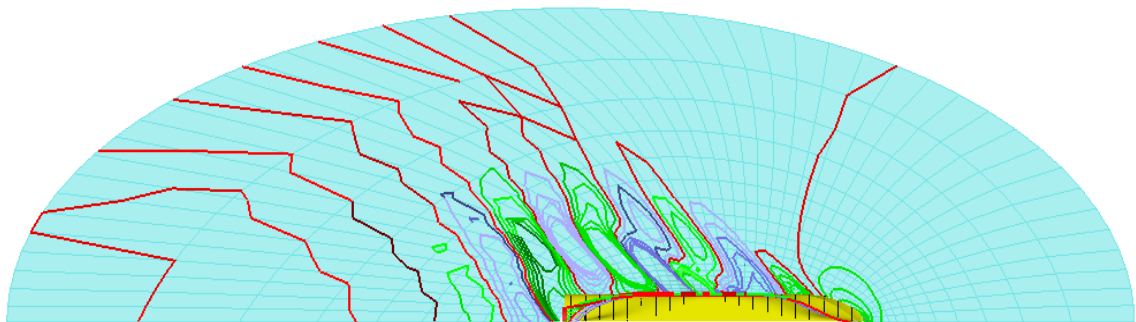


Figure 5.21 Wave profile of optimised design (solution ID 96).

The breakdown of tasks and the time taken from start to finish for one simulation using NAPA and MEC are compared in Table 5.6.

Table 5.6 Task breakdown and time taken for each simulation in NAPA and MEC

Step	Task	NAPA Time (hours)	MEC Time (hours)
1	3D hull form creation from offset table	17.6	0
2	Setting up free-form deformation manager function	0.5	0
3	Setting up Panelisation manager function	10	0
4	Setting up optimisation manger function	1.5	0
5	Setting up CFD analysis	1	0
6	MOGA optimisation	1.5	0.1
7	Extraction of results	0.5	0
	Total time taken	33.1	0.002 (10 sec)

- **Comparison of Results between MEC and NAPA**

Table 5.7 presents the results using MEC (from case study 1 described in Section 5.2) as compared to that from NAPA in terms of both the performance and time taken to complete the optimisation. It compares the results obtained from MEC and NAPA in terms of both performance (resistance) and the time taken to complete the optimisation in case study 1. In both aspects, it is clear that MEC had better performances over NAPA. Two key benefits are noteworthy. First, MEC was able to efficiently navigate the design space within a short period of time. It took six minutes to run through 43,200 design solutions as compared to NAPA where 1000 solutions took approximately 33 hours.

Table 5.7 Comparison of time taken and optimisation results between MEC and NAPA

Criteria	MEC (Case study 1)	Existing SBD (NAPA)	Differences (MEC – NAPA)
Total time taken	10 sec x 36 combinations = 6 min	33.1 hours	33 hours
Improvement in resistance compared to initial design	11.14%	1.6%	9.44%
Number of design solutions evaluated	1200 x 36 combinations = 43,200	1000	42,700

As shown from Table 5.7, the additional time required by NAPA was primarily due to the additional step for 3D surface modelling used in NAPA. This time saving highlights a key benefit of MEC. As it uses only 2D curves, the need to convert the 2D offset data into 3D surface for the performance evaluation is removed and hence the overall time taken is significantly reduced. Second, through its unique morphing schemes, it was able to mix-and-match and modify existing solutions to create newer ones automatically. These features are extremely useful as designers can now use MEC to carry out the initial optimisation for a design problem. This frees up time for them to focus on more important tasks which would otherwise not be afforded.

5.6. Interoperability of MEC between 3D CAD surfaces and 2D offset points

To demonstrate the complete design loop from the 3D CAD surface to discretised 2D offset data and vice-versa, the input data of the initial parent designs can also start from 3D CAD surfaces and convert to 2D offset points using hull-to-offset manager function in NAPA. The process is illustrated in Figure 5.22.

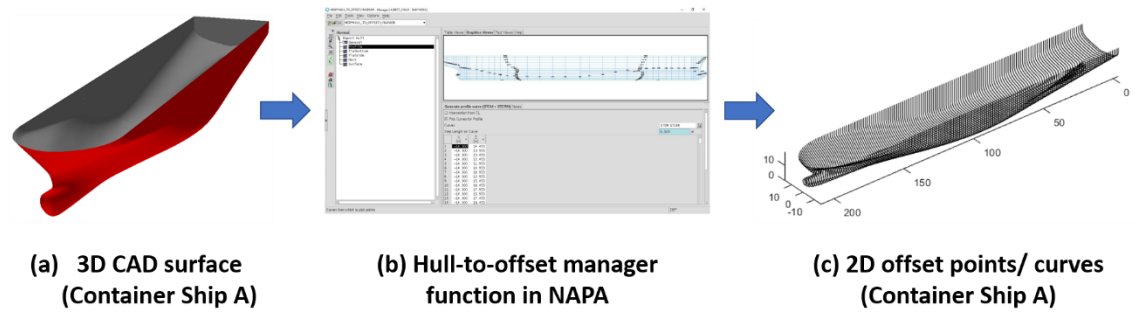


Figure 5.22 Conversion from 3D CAD surface to 2D offset curve format using hull-to-offset manager function in NAPA.

From Figure 5.22(a), the initial vessel model (container ship A) is in a 3D CAD format based on NURBS patch surfaces in NAPA. This surface is then discretised from 3D into 2D offset points using the hull-to-offset manager function in NAPA, as shown in Figure 5.22(b). The final output in Figure 5.22(c), is a 2D representation of the same vessel in offset data format that is ready to be included in the hull form library for automatic HFDO using MEC. In a similar fashion, after completion of the HFDO process, the optimised hull form (from case study 1) as shown in Figure 5.23(a), which is in 2D offset format (Figure 5.23(b)) can also be converted back to 3D CAD format using the offset manager function in NAPA, as shown in Figure 5.23(c). By converting the optimised hull form back into 3D CAD format, it allows the 3D model to be validated using CFD.

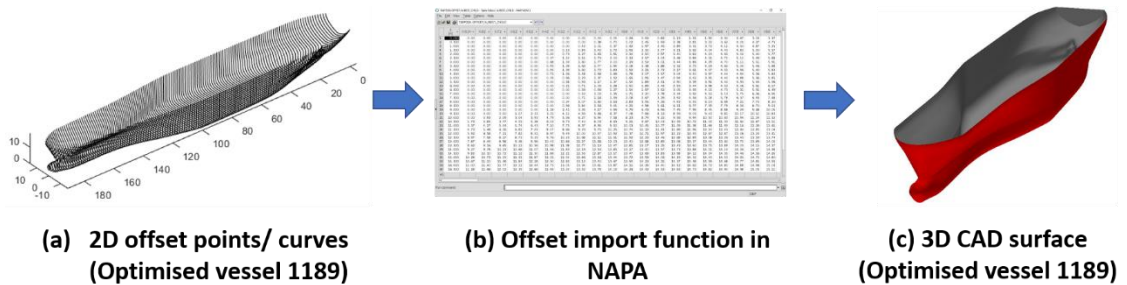
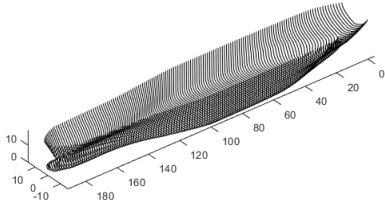
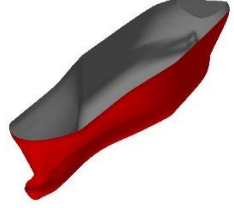


Figure 5.23 Conversion from 2D offset curve to 3D CAD surface format using the offset import function in NAPA.

- **Comparisons of vessel geometries and hydrostatics**

To further demonstrate the usability of the 2D offset curve format, vessel geometries were compared with the 3D CAD surface (from the optimised child vessel in case study 1). The results are shown in Table 5.8.

Table 5.8 Comparison of vessel geometries, hydrostatics and performance of the optimised child vessel (solution 1189) for the 2D offset curve and 3D CAD surface format

Parameters	2D offset curve	3D CAD surface
Vessel model		
Principle dimensions (m)	185(Lpp) x 32(Breath) x 9 (Draught)	185(Lpp) x 32(Breath) x 9 (Draught)
Displacement (tonnes)	34460.16	33903.3
Block coefficient, Cb	0.631	0.619
Prismatic coefficient, Cp	0.644	0.638
Waterplane area coefficient, Cw	0.673	0.683
Total resistance (kW)	839.328	1028.745

Here, differences between the two vessels in terms of displacement (556.69 tonnes) and total resistance (189.417kN) were observed. The discrepancy in displacement can be attributed to the additional volume that arises from the spacing between the transverse frames for the 2D offset curve calculations. This can be resolved by reducing the frame spacing of the offset table to the smallest value possible by creation of new transverse frames. The disadvantage of this measure would be more 2D offset points created and an overall increase in computation times. The difference in the total resistance could be due to two reasons. Firstly, the empirical method (Holtrop & Mennen) applied for calculating the total resistance of the 2D offset model was deployed using historical data from conventional vessels. This being the case, the resistance is very close between the Holtrop and RANS methods (for the original vessel container ship A) as validated in Section 4.3.1 (Table 4.1). However, the optimised child vessel may be different to the parent, following crossover and mutation. This could result in these discrepancies. Therefore, the potential

flow method used to calculate the total resistance of the 3D CAD surface model appears to be more accurate and of higher value, when compared to the 2D offset curve model. Secondly, some geometric details may not be accurately captured in the 2D offset model, when compared to the 3D CAD surface model. To further improve the accuracy, more detailed CFD analyses such as potential flow or RANS, can be used in place of the empirical method. This is recommended as future work for this research.

- **Time-cost analysis of the complete optimisation loop**

For completeness, a detailed time-cost analysis of the complete-loop comparing the start of the optimisation using the 2D offset curve and the 3D CAD surface model are provided in Table 5.9. The conversion from 3D to 2D adds approximately one hour to the optimisation.

Table 5.9 Comparison of vessel geometries, hydrostatics and performance of optimised vessel (solution 1189) for the 2D offset curve and 3D CAD surface format

Task	2D offset curve	3D CAD surface	Level of automation
Convert from 3D CAD surface to 2D offset curve format	-	1 hour	Semi-automatic
Optimisation process using MEC	0.1 hours	0.1 hours	Fully automatic
Convert 2D offset curve to 3D CAD surface	17.6 hours	17.6 hours	Semi-automatic
Total time (hours)	17.7 hours	18.7 hours	

5.7. Results and Discussion

To understand the effects of different initial population sizes, crossover and mutation settings, the combined results for the three case studies are summarised in Table 5.10. Four key conclusions are drawn from the three case studies. First, comparing high and low initial populations (case studies 1 and 2), it can be concluded that higher initial populations resulted in more diverse child designs and hence, lead to better improvements. This is demonstrated where the overall improvement decreased from 11.14% to 7.22% when the initial population was reduced from 12 vessels in case study 1 to 4 vessels in case study 2. Furthermore, the type of vessels in case study 1 were more varied as compared to case study 2 where all vessels were of the same type. Second, it

Table 5.10 Summary of Optimisation Results

Parameters	Case study 1- large initial population	Case study 2- small initial population	Case study 3- expanded initial population
MEC settings and constraints			
Initial population	12	4	12
Optimal crossover rate	0.8	0.7	1.0
Optimal mutation rate	0.7	1.0	0.8
Generations	100	100	100
Principal dimensions (m)	185 (LPP) x 32 (Breadth) x 9 (T)		
Minimum displacement volume (tonnes)	34,358.7		
Best initial parent- Ship 1 (Container Ship A)			
Displacement (tonnes)	32,365.2		
Resistance (kN)	944.6		
Best child solutions			
Displacement (tonnes)	34,460.16	34,412.99	34,384.72
Resistance (kN)	839.32	876.31	844.86
% improvement over best initial parent	11.14	7.22	10.55
Time taken for 36 runs (minutes)	6	6	6

was proven that MEC yielded better designs even for very low initial populations. Expanding the initial population to 12 vessels using constant-morph enabled more shape variations and as a result improved the performance from 7.22% to 10.55% from case study 2 to case study 3. This same concept may be applied to hull form libraries with limited sizes to rapidly increase their initial population and lead to better diversity and convergence.

Third, comparing case studies 1 and 3 suggested that more diverse ship types could potentially produce more shape variations and result in better (performance) solutions, though the improvement was marginal (11.14% for a mixture of vessels and 10.55% for all container ships).

Finally, it can be concluded that crossover and mutation operations as well as the balance in their settings are important influencers to the solutions generated by MEC. Comparing the convergence charts obtained from the three case studies, the optimal crossover and mutation settings varied in the range between 0.7 and 1.0. As demonstrated in the results,

the optimal setting depended greatly on the number of initial population and also the ship types available in the Hull Form Library. In the scenario where the initial population had a high mix (vessel types) as in case study 1, high crossover and mutation settings (0.7 - 0.8) are needed to prevent pre-mature convergence. Conversely, it is shown that very high crossover and mutation rates (e.g. 1.0) resulted in sub-optimal solutions due to over-perturbation. For an initial population with a small number and type of parent vessels, case study 2 showed that both high crossover (0.7) and very high mutation (1.0) rates were necessary to produce the most optimal solutions. This is because there were only limited samples within the initial population to select for crossover. Hence, using crossover on its own in such cases was not able to generate sufficient new shape variations and a higher mutation rate is paramount to produce the much-needed shape variations. In case study 3 which consists of a large initial population with limited vessel types, the best results were achieved when the crossover rate and mutation rate were increased to 1.0 and 0.8 respectively.

When the solutions obtained from MEC was benchmarked against NAPA in Section 5.5, there was an improvement of 9.44% in the resistance and the total design took 33 hours less to complete. The results demonstrated the potential in applying fully automated optimisation tools to hull form design.

Key Achievements

A number of noteworthy achievements were attained in this chapter. First, considering there were no changes to the main dimensions, significant improvements in the resistance performance from the three case studies (7.22% reduction in resistance in case study 2 to 11.14% reduction in case study 1) were obtained. This was achieved despite the fact that the original hull form used in this case study was already optimised and its performance proven in real applications. Furthermore, in most HFDO, the main dimensions and minimum displacements were adjusted in order to allow some 'room' for designers to explore. In all the case studies presented here, the minimum displacements were maintained.

Second, the results showed that lower resistances are obtained (with higher displacements) compared to the best parent designs (which have much lower displacements). This is a significant achievement because MEC has made it possible to automatically adjust the hull form to reduce the resistance while simultaneously maintaining minimum displacement. In actual practice, it would be very demanding for

a designer to achieve similar results using the manual ‘trial and error’ approach. Similarly, this would be very challenging for conventional SBD approaches where reducing the resistance would almost always result in the reduction of the overall displacement.

Third, the geometry of the final modified hull form (or child design) remains relatively smooth with no bumps or steps observed despite undergoing hundreds of rounds of iterations and modifications. This is a unique feature of the 2D curve morphing applied in this thesis which ensures the feasibility of the hull form. Furthermore, using 2D curves also helped to significantly reduce the overall design time.

Together these three case studies demonstrated the versatility and ability of MEC to automatically transform the shape of the hull while simultaneously yielding better and feasible designs for single-objective optimisation of container ships.

5.8. Chapter Summary

In this chapter, the hull form of a container ship was optimised using MEC in three different case studies. In the first case study, a large initial population was applied using a variety of ships. A total of 36 combinations of different crossover and mutation rate settings were used to evaluate the influence of different crossover and mutation settings. In the second case study, only four container ships were selected to demonstrate the effects of small initial populations. The results showed an improvement of 11.14% over the best initial parent for large initial population in case study 1 and 7.22% improvement over the best initial parent for small initial population in case study 2. The initial population of 4 vessels was expanded to 12 vessels using constant-morph in case study 3 and the performance improved to 10.55%. The results obtained from the first case study were then validated with an existing SBD solution and it was demonstrated that MEC is able to reduce the overall time significantly with better performance. From the three case studies conducted, it can be concluded that MEC can reduce the overall resistance of a container ship in a fully automated manner.

6. APPLICATION OF MEC TO MULTI-OBJECTIVE OPTIMISATION OF AN OFFSHORE CABLE-LAYING SHIP

In this chapter, MEC is applied to the design of an offshore cable-laying ship. In offshore cable-laying operations, it is critically important to have a hull with good seakeeping characteristics as they determine the ‘up-time’ the vessel can operate safely. Seakeeping motions, namely, pitch, roll and heave, are therefore important design considerations for offshore vessels. Another important design objective is the resistance of the vessel. It is expected that the hull should be streamlined to reduce the overall drag acting on the vessel, leading to lower fuel costs. These design criteria are used in the automated multi-objective optimisation of an offshore cable-laying ship presented in this chapter.

6.1. Objective and Design Specifications

There is no common standard in measuring seakeeping performance but it can be affected by aspects such as ship motions, accelerations, course-keeping and local sea load. Depending on the operation model and travel route, the design criteria varies largely for different types of ships [4]. Three degree of motions, namely pitch, heave and roll are considered in this case study. In multi-objective optimisation, objective functions can be classified into two main types: non-conflicting and conflicting. For non-conflicting objective functions, objectives work in tandem with one another such that improving one objective will result in the improvement of the other(s). In contrast, for conflicting objective functions, improving one objective would result in worsening the performance of the other(s). Two case studies are presented in this chapter to evaluate the performance of MEC in handling both non-conflicting and conflicting objective optimisation. In the first case study, the goal is to minimise the heave and resistance of the vessel. This case study demonstrated the performance of MEC for non-conflicting multi-objective optimisation. In the second case study, roll and pitch motions are added to further ascertain that the proposed methodology is capable of carrying out both non-conflicting and conflicting objective optimisation.

The design specifications for the offshore cable-laying ship are given below.

- **Principal dimensions**

Length between Perpendicular (LPP) = 121m

Beam (B) = 23m

Draft (T)	= 7.5m
Design Speed (V)	= 14.5 knots
Displacement (Δ)	$\leq 15,000$ tonnes

- **Objective functions and constraints**

The multi-objective optimisation is formulated as:

$$\begin{aligned} \text{Minimise} \quad & f = R_t \text{ and heave (Case study 1)} \\ & f = R_t, \text{ pitch, roll and heave (Case study 2)} \\ \text{Subject to:} \quad & \text{LPP} = 121\text{m}, B = 23\text{m}, T = 7.5\text{m} \\ & \Delta \leq 15,000 \text{ tonnes} \end{aligned}$$

where LPP represents Length between the Perpendicular, B is the Breadth (moulded), T is the Draft, R_t is the total resistance and Δ represents the Displacement of the ship in metric ton (tonnes).

- **Initial population**

The initial population consists of 12 vessels. These are proven designs preloaded into the Hull Form Library and are used as parent designs in the initial population. These vessels were also used in Chapter 5. For ease of comparison and reference, the body plans and wire-mesh diagrams of these vessels are also shown in Figures 6.1 and 6.2. The principal dimensions are provided in Table 6.1.

Table 6.1 Principal dimensions for 12 parent vessels

Ship	Type	Length between perpendicular, LPP (m)	Breadth (m)	Depth (m)
1	Container Ship A	202	32.2	19.1
2	Container Ship B	185	32	19.1
3	Container Ship C	120	23	11.2
4	Container Ship D	145	13.6	13.6
5	Roll-on/ roll-off ship (ROPAX)	210	34	17.1
6	Bulk Carrier	220	36	23.75
7	Product Carrier	171	31	15.6
8	Chemical Tanker	126.5	21.5	10.5
9	Very Large Crude Carrier (VLCC)	314	58	31
10	Pipe-laying Ship	242	45	21.1
11	Cable-laying Ship	121	23	12.5
12	Offshore Accommodation Ship	168	24	13

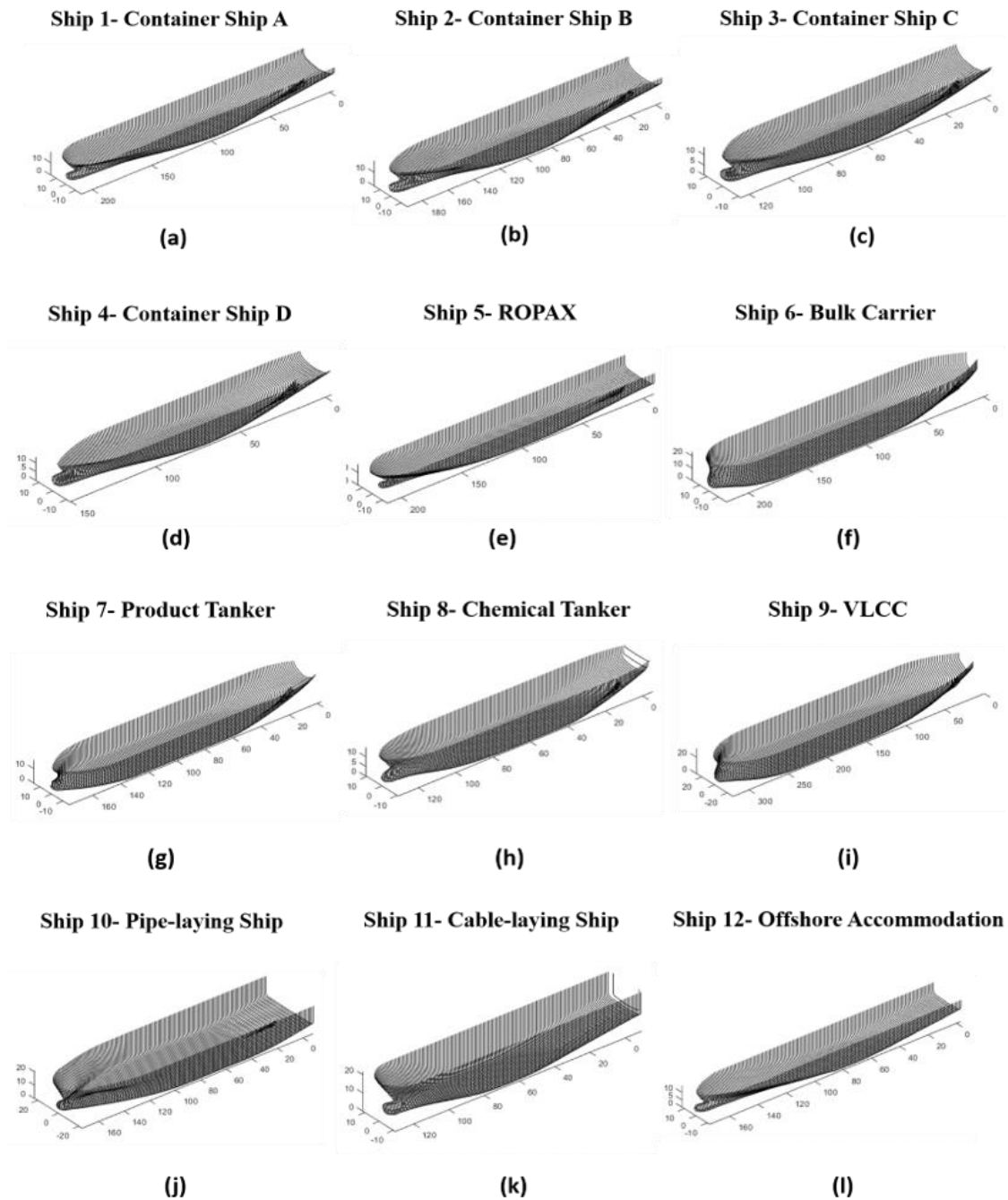


Figure 6.1 Wire-mesh diagrams of existing (parent) ships.

In both case studies presented in this chapter, the heading angle is set at 45 degrees (where head sea = 0 degree) for seakeeping calculation. Multi-point cross-morph, namely two-point, were applied to increase the variety of solutions in both case studies presented here as multi-objective optimisation problems tended to stagnate quickly. In this method, blending is conducted using two crossover points between three vessels, instead of one crossover point between two vessels, as in a single point crossover. Figure 6.3 shows an illustration of two-point crossover. This approach allows a larger variety of vessels to be combined and therefore ensures that more varied shapes are created.

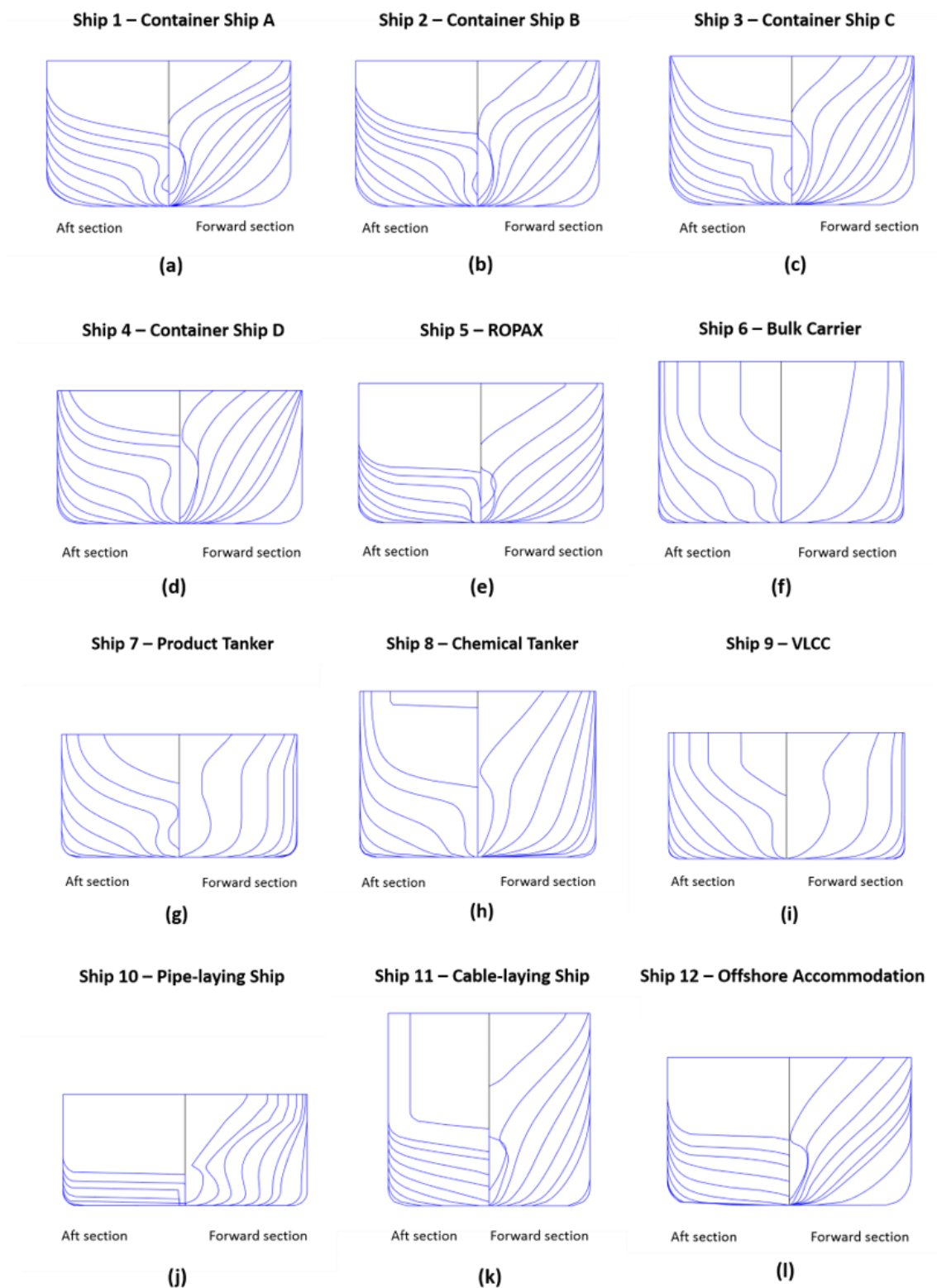


Figure 6.2 Body plans of existing (parent) ships.

This is achieved by altering the chromosome of the three different parent vessels and their connections (transition frames). The forward section of ship 9 (VLCC) is indicated in pink, the mid-body of ship 1 (Container Ship A) is indicated in blue and aft-body of ship 7 (Product Tanker) is indicated in red. The intersection (crossover) points are shown in

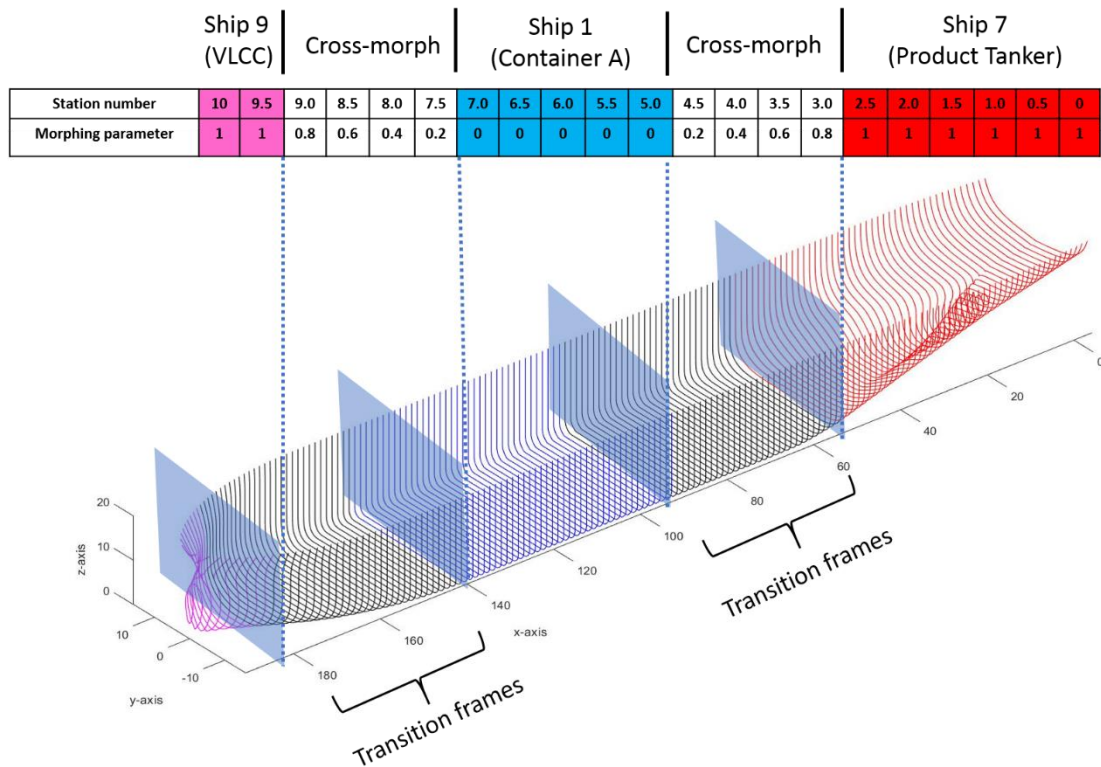


Figure 6.3 Two-point cross-morph combining ship 9 (VLCC), ship 1 (Container Ship A) and ship 7 (Product Tanker).

blue shaded planes and the transition frames that connects the existing vessel body is produced using two-point cross-morph indicated in black. From the encoding (mapping) scheme shown, it can be observed that cross-morph enables a smooth transition by varying the morphing parameters between the three different parent vessels. This example demonstrates the ability of two-point cross-morph to ‘cut-and-joint’ multiple hull sections into one seamless hull form which helps to create more shape variations.

6.2. Case Study 1: Two-Objective Functions (Resistance and Heave)

This section presents optimisation results from case study 1 for an offshore cable-laying vessel for two non-conflicting objectives. The goal is to minimise the resistance and heave of the vessel. The settings used are given as follows.

- Initial population = 12 vessels
- Crossover rate = 0.5 to 1.0
- Mutation rate = 0.5 to 1.0
- Number of generations = 100

Similar to the case studies performed in Chapter 5, 36 simulations were performed using different combinations of crossover and mutation rates that were varied from 0.5 to 1.0.

The convergence charts of total resistance and heave are shown in Figure 6.4 and Figure 6.5, respectively.

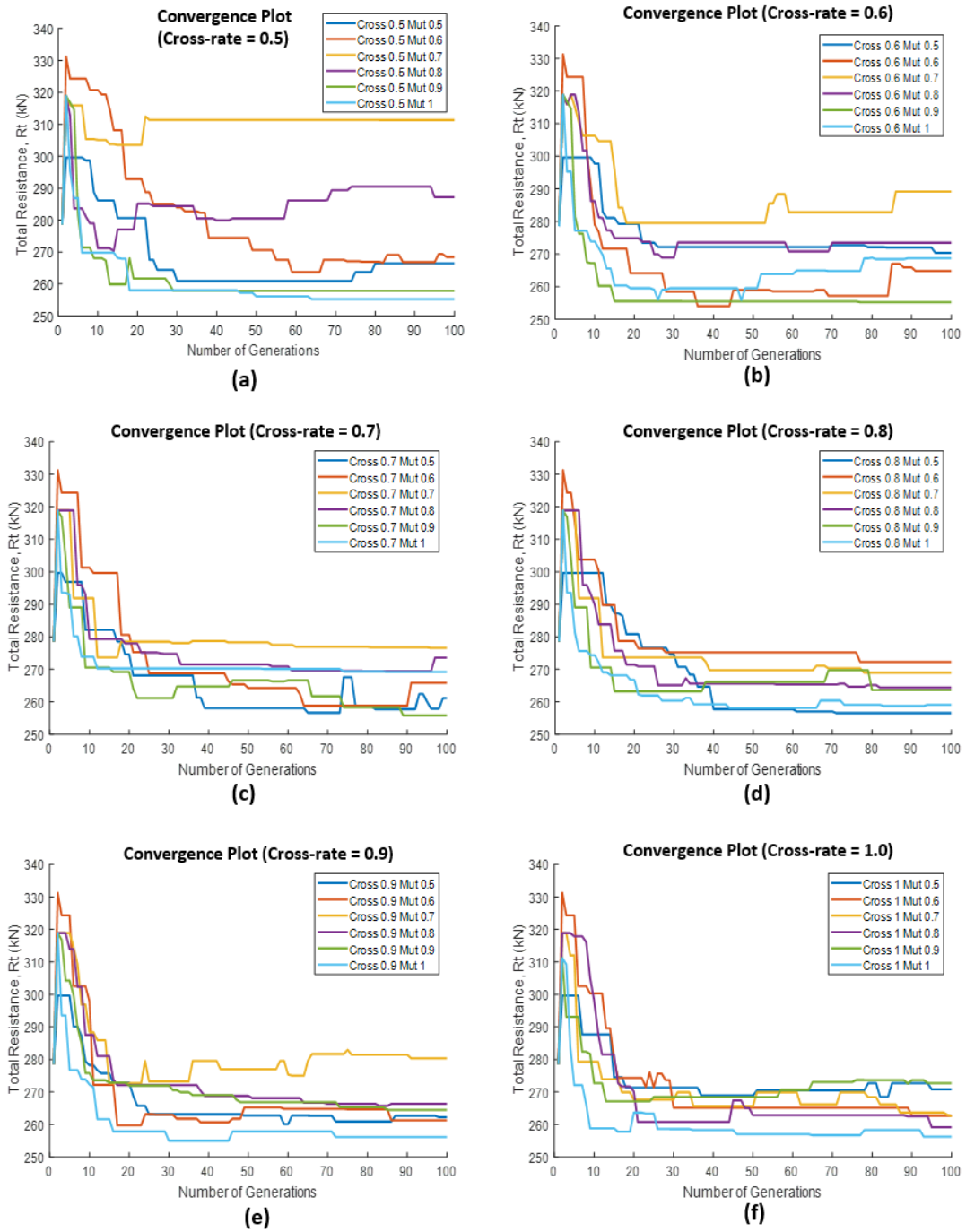


Figure 6.4 Convergence charts for total resistance showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1

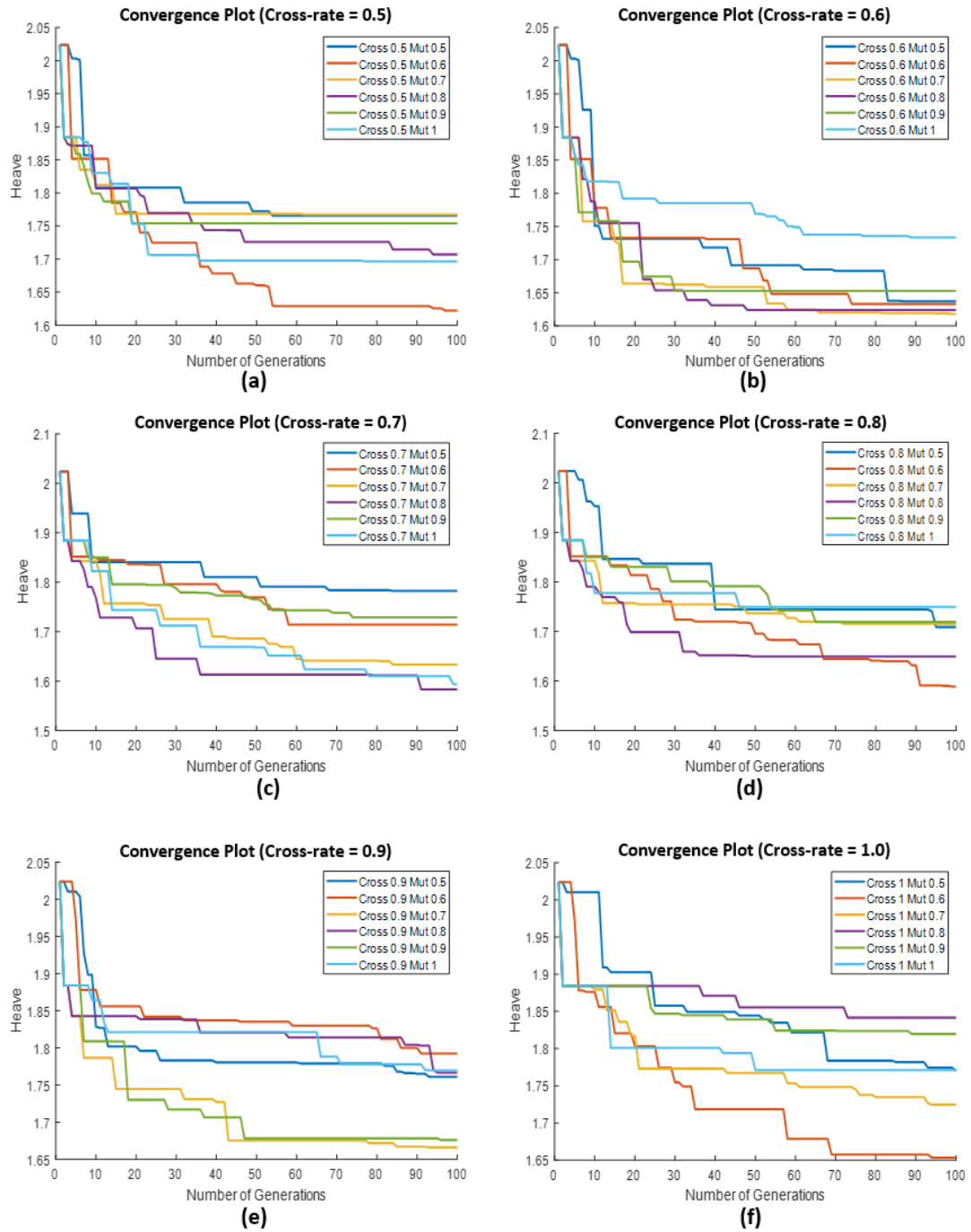


Figure 6.5 Convergence charts for heave showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.0.

From Figures 6.4 and 6.5, the resistance tended to stagnate for lower crossover and mutation rates (< 0.6). This could have limited shape variations and hence the number of good solutions that can be carried over to the next generation is reduced. Heave performance, however, continued to improve as the crossover and mutation rate increases. It was also observed that when the mutation rate increased to 0.9 and 1.0, the search converged prematurely. This was likely because a high mutation rate could have ‘over-manipulated’ the structures, and destroyed good features from reproduction and

crossover. Therefore, a balance between crossover and mutation rates is crucial to the success of optimisation. For ease of comparison, the best individual for each crossover-mutation rate combination is shown in Table 6.2. The best result for each crossover rate is highlighted in green and the overall best resistance and heave for all crossover and mutation rates are highlighted in yellow. It should be noted that no individual is best for both objectives.

The best resistance was obtained when the crossover rate was 0.6 and the mutation rate was 0.9. It was observed that a lower crossover rate appeared to have a better effect on resistance performance. Considering that a two-point crossover is used in this case, a high crossover rate may be too disruptive, resulting in the vessel shape becoming overly irregular. This has a direct impact on the hydrodynamic (flow) performance. Mutation, on the other hand, only introduces subtle changes to the hull. The best heave was obtained when the crossover rate was 0.7 and the mutation rate was 0.8. In this case, a slightly higher crossover rate may have helped to create more varied form factors, which had positive impacts on seakeeping performances. Conversely, it was also found that too high a crossover might have resulted in over-varied form factors or irregular shapes, hence worsened seakeeping performance.

For further investigation, the best solution for crossover at 0.6 and mutation at 0.9 was selected due to better performances for both objective functions. The combined convergence chart and solution space for resistance and heave is shown in Figures 6.6 and 6.7, respectively. From Figure 6.6, it can be observed that the resistance value increased at the beginning of the generation and came down gradually after 10 generations. This could be due to the trade-off of one objective (resistance) for the improvement of the other objective (heave). Subsequently, both objectives stabilised at lowest value after the 30th generation. This demonstrated the ability of MEC to find the best solution even in multi-objective case.

As seen in the zoomed-in section of the solution space (blue bounded box) shown in Figure 6.7, the solutions can be grouped into three regions. Together, these regions illustrate how the design space was explored. Region 1 shows the solutions found in the first 40 generations. It can be seen that the solutions in this region are well spread across the design space for both objectives. This suggests that the selection and reproduction mechanisms were able to find preliminary designs that could be further refined as the optimisation progressed.

Table 6.2 Best results at final generation for total resistance and heave for each crossover and mutation rates.

Crossover rate	Mutation rate	Total resistance (kN)	Heave
0.5	0.5	266.4	1.76
	0.6	268.4	1.62
	0.7	311.3	1.76
	0.8	290.4	1.70
	0.9	257.8	1.75
	1.0	255.3	1.69
0.6	0.5	270.3	1.63
	0.6	264.8	1.63
	0.7	289.1	1.61
	0.8	273.4	1.62
	0.9	255.2	1.65
	1.0	268.7	1.73
0.7	0.5	259.3	1.78
	0.6	265.8	1.71
	0.7	276.7	1.63
	0.8	273.5	1.58
	0.9	259.4	1.72
	1.0	269.1	1.59
0.8	0.5	259.9	1.70
	0.6	272.2	1.59
	0.7	268.8	1.71
	0.8	264.3	1.64
	0.9	263.5	1.71
	1.0	259.1	1.74
0.9	0.5	262.2	1.76
	0.6	261.3	1.79
	0.7	280.3	1.66
	0.8	266.3	1.76
	0.9	264.4	1.67
	1.0	256.1	1.76
1.0	0.5	270.7	1.77
	0.6	262.6	1.65
	0.7	262.5	1.72
	0.8	267.7	1.84
	0.9	272.7	1.81
	1.0	258.2	1.77

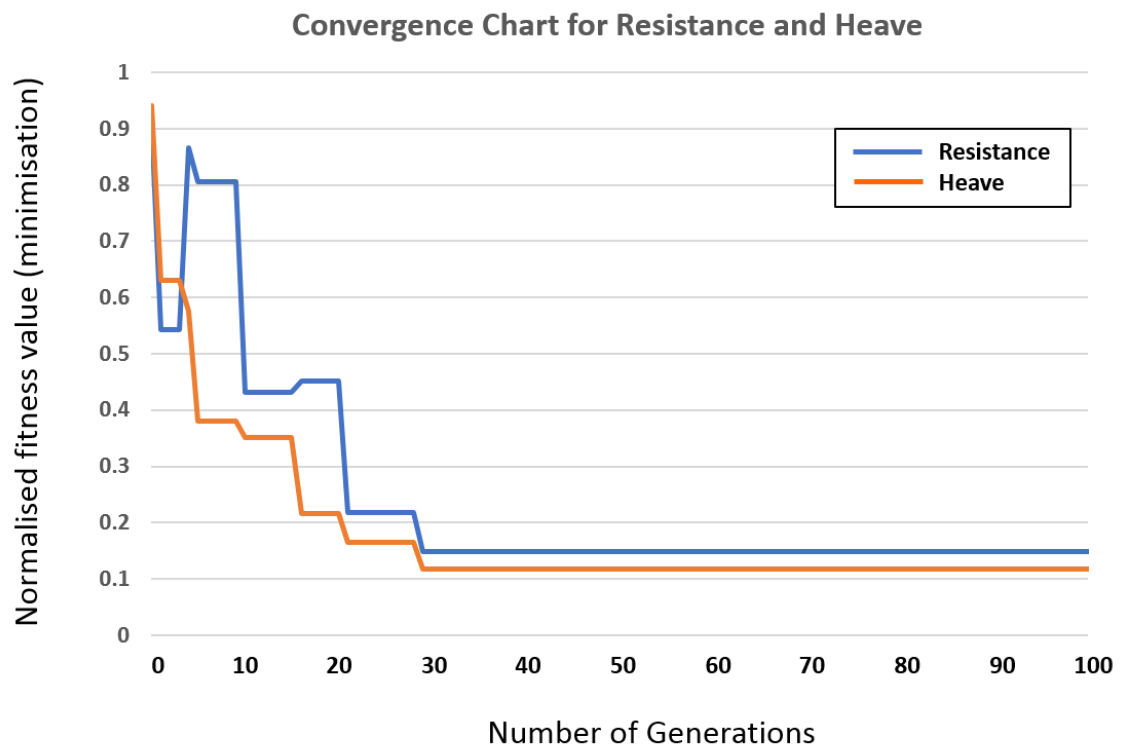


Figure 6.6 Convergence chart of resistance and heave.

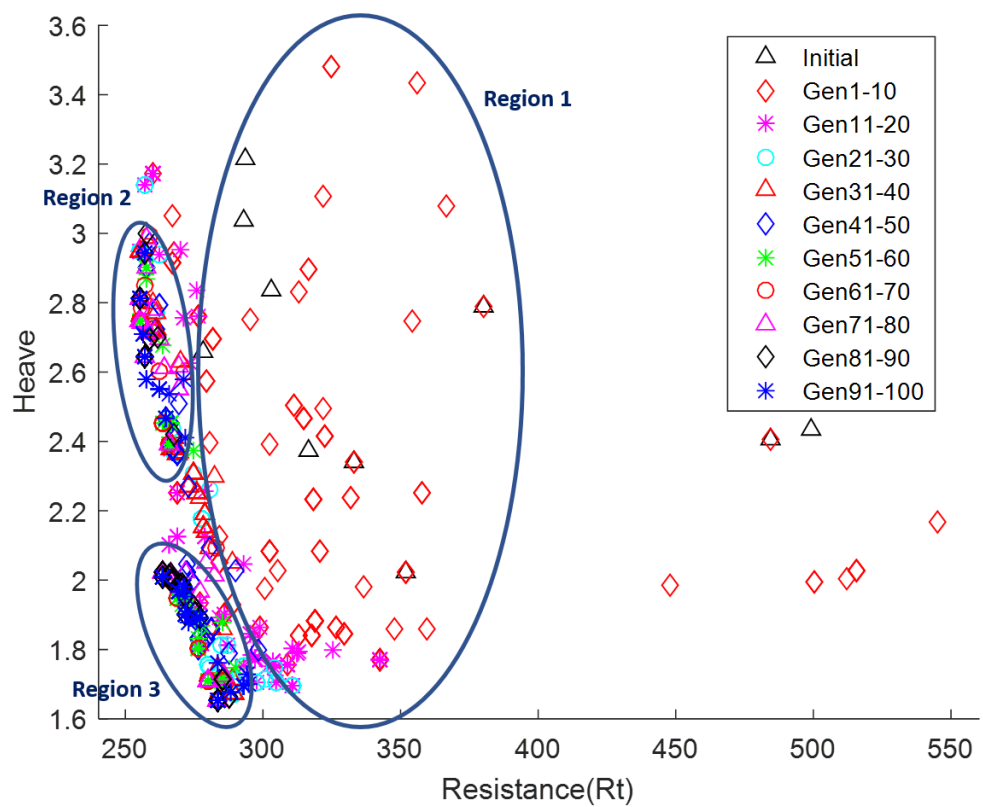


Figure 6.7 Solution space showing the performance of the initial parents and child solutions for optimising resistance and heave motion.

This is evident by observing the solutions in region 2 which have better resistance than those in region 1. Finally, in region 3, the optimisation converged to a set of Pareto optimal solutions satisfying both objectives for resistance and heave motions.

To ascertain the efficacy of MEC, three comparisons were conducted. First, the best solutions obtained from the optimisation were compared to the cable-laying vessel (ship 11) in the initial population in terms of their cable-laying performance. The latter was manually optimised by a designer. Second, for each objective, each of the best solutions was benchmarked against the best performing parent solution for that objective. For example, the resistance of all the best solutions found at the end of the optimisation was compared against that of parent vessel (ship 1 – Container Ship A) which has the best resistance amongst all parent solutions in the initial population. Third, the quality of the solutions obtained from MEC was compared against one another. This provided insights into their feasibility as well as the choice of the solutions available to the designer at the end of the optimisation.

The performance of the best initial parents and best solutions are presented in Table 6.3. Child solutions that outperformed the best parent solutions are highlighted in green, while those that are inferior are highlighted in orange. There are several key observations from Table 6.3. First, all the 12 near-optimal solutions outperformed the best initial parent (ship 11) in terms of resistance (mean improvement, $\mu = 23.23\%$). Of these solutions, seven child solutions showed an average improvement of 9.95% in heave. In comparison to ship 1 (Container Ship A) which was the best parent vessel in terms of resistance, nine of the child solutions showed an average improvement of 4.74% in resistance and 10 child solutions showed an average improvement of 23.93% in heave. By comparing all child solutions, solutions 1189, 1196, 1197, 1199 are considered optimal solutions, which demonstrated improvement over both best initial parent ship 1 and ship 11. To better understand the results from a naval architect's perspective, the hull forms of the best child solutions for heave (solution 1190) and resistance (solution 1194) are shown in Figure 6.8.

From Figure 6.8, the two child solutions appeared different from each other. It can be seen from Figure 6.8(a) that the cross-sectional areas were substantially equal across the entire length of the vessel, which helped in reducing the heave motion. In Figure 6.8(b), the forward and aft body had much lower cross-sectional areas when compared to the mid-body. As a result, the vessel had a narrower forward and aft section and therefore a lower resistance.

Table 6.3 Best initial parents and solutions for objectives - resistance and heave

Best Child	Best parent	Best heave – Ship 11 (Cable-laying Ship) Resistance = 352.04 kN Heave = 2.02		Best resistance - Ship 1 (Container Ship A) Resistance = 278.54 kN Heave = 2.65		Displacement, tonnes
		Resistance	Heave	Resistance	Heave	
Solution	(% improvement over best parent)					
1189	263.42 (25.17)	2.01 (0.44)	263.42 (5.42)	2.01 (21.11)	15062.23	
1190	283.38 (19.50)	1.65 (18.17)	283.38 (-1.73)	1.65 (37.62)	15018.99	
1191	283.38 (19.50)	1.65 (18.17)	283.38 (-1.73)	1.65 (37.62)	15018.99	
1192	256.7 (27.07)	2.70 (-34.09)	256.7 (7.83)	2.70 (-2.21)	15101.53	
1193	262.06 (25.55)	2.55 (-26.27)	262.06 (5.91)	2.55 (3.74)	15000.01	
1194	255.25 (27.49)	2.81 (-39.22)	255.25 (8.35)	2.81 (-6.12)	15019.29	
1195	288.20 (18.13)	1.67 (17.03)	288.20 (-3.47)	1.67 (36.75)	15257.98	
1196	271.72 (22.81)	1.89 (5.96)	271.72 (2.44)	1.89 (28.32)	15042.73	
1197	269.21 (23.52)	1.96 (2.74)	269.21 (3.34)	1.96 (25.86)	15123.11	
1198	264.55 (24.85)	2.46 (-22.00)	264.55 (5.02)	2.46 (6.99)	15067.05	
1199	273.09 (22.42)	1.87 (7.13)	273.09 (1.95)	1.87 (29.21)	15260.39	
1200	271.87 (22.77)	2.40 (-19.27)	271.87 (2.39)	2.40 (9.08)	15066.43	

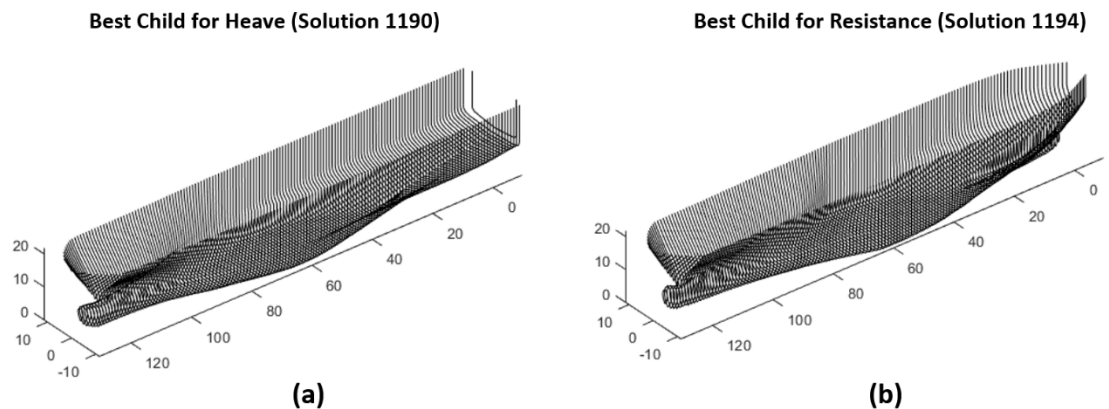


Figure 6.8 Wire-mesh diagrams of the best child (a) solution 1190 and (b) solution 1194.

Figure 6.9 shows the body plans for the best initial parents and best child solutions comprising of (a) best child (solution 1190) and best parent (Ship 11) with lowest heave and (b) best child (solution 1194) and best parent (Ship 1) with lowest resistance.

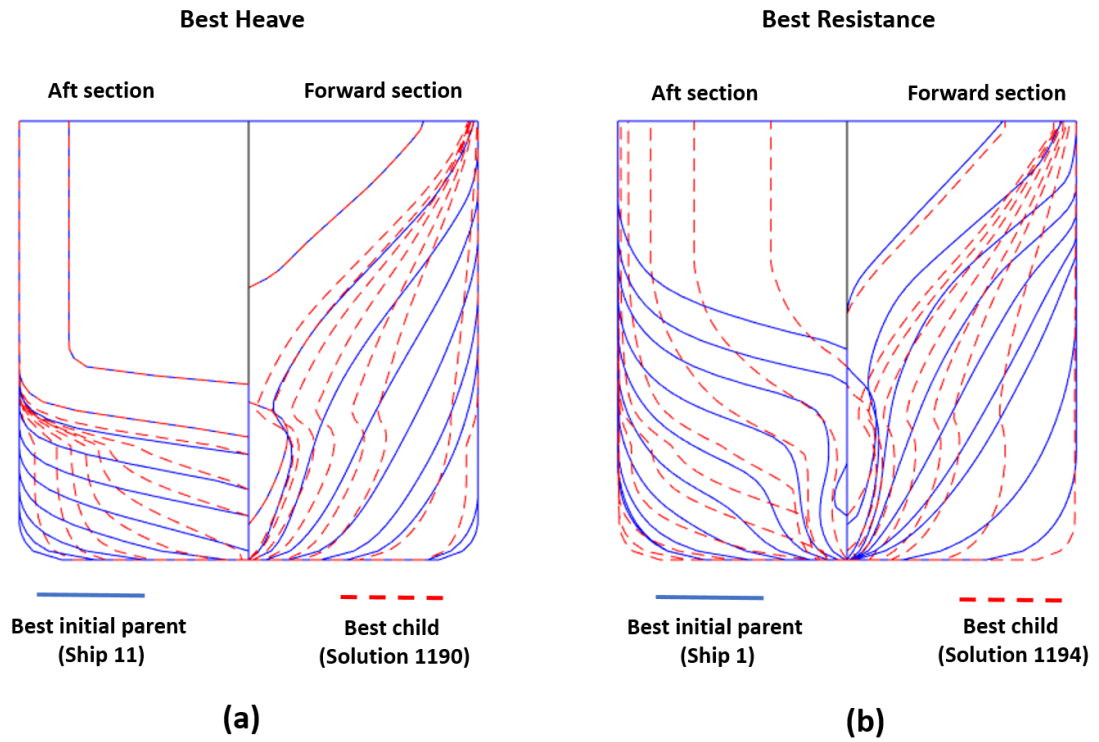


Figure 6.9 Body plans comparing (a) best child - solution 1190 with best initial parent- ship 11 for heave and (b) best child - solution 1194 with best initial parent- ship 1 for resistance.

From Figure 6.9, it can be seen that there is some resemblance in the shapes of both best child solutions to the initial parent vessels. In Figure 6.9(a), the extreme bow and aft section of solution 1190 are very similar to ship 11 (Cable-laying Ship). However, the mid-section looks rather different. This is because during the two-point cross-morph the child solution was created by merging ship 3 (Container Ship C) mid-body and ship 11 forward and aft body from the initial population. In Figure 6.9(b), the forward section of solution 1194 is similar to ship 1 (container ship A). It was observed that mid and stern sections were created by merging ship 8 (Chemical Tanker) and ship 6 (Bulk Carrier) using cross-morph. In both cases, the vessels were further modified with boundary-morph during mutation.

The total time taken for each optimisation was approximately 16 hours, using a 4-core CPU. 36 simulations were completed to compare results from varying crossover and mutation settings. As compared to the single-objective optimisation presented in Chapter 5, the duration has increased significantly due to the additional seakeeping performance evaluation using strip theory.

6.3. Case Study 2: Multi-Objectives Function (Resistance, Heave, Roll and Pitch)

In the second case study, two more objectives, roll and pitch are added to resistance and heave to demonstrate the performance of MEC for multi-objective optimisation involving both non-conflicting and conflicting objective functions. The same parent vessels in case study 1 were used as starting points in this case study. The following settings are used.

- Initial population = 12 vessels
- Crossover rate = 0.5 to 1.0
- Mutation rate = 0.5 to 1.0
- Number of generations = 100

As with the earlier case study, 36 simulations were performed using different combinations of crossover and mutation rates, which varied between 0.5 and 1.0. The convergence charts of total resistance, heave, pitch and roll are shown in Figure 6.10 to Figure 6.13, respectively.

For comparison of the four objective functions (total resistance, heave, pitch and roll), the best results at the final generation for each crossover-mutation rate combination are presented in Table 6.4. As no individual had the best performance in all four objectives, the best result for each crossover rate is highlighted in green and the overall best result for each objective is highlighted in yellow.

Several observations were made from Table 6.4. The best performance for each objective generally worsened (increased) as the crossover rate increased, except for pitch which remained constant. For example, the best performance for resistance increased from 245.5 kN at a crossover rate of 0.5 to 260.5 kN at a crossover rate of 1.0. The best performance for heave increased from 1.62 to 1.64, at crossover rates of 0.5 and 1.0, respectively. This was the same for roll, which increased from 0.08 at a crossover rate of 0.5, to 0.11 at a crossover rate of 1.0. This trend was consistent with the first multi-objective case studies in Section 6.2, where higher crossover rate tended to cause too many shape changes during cross-morph, resulting in poorer performances.

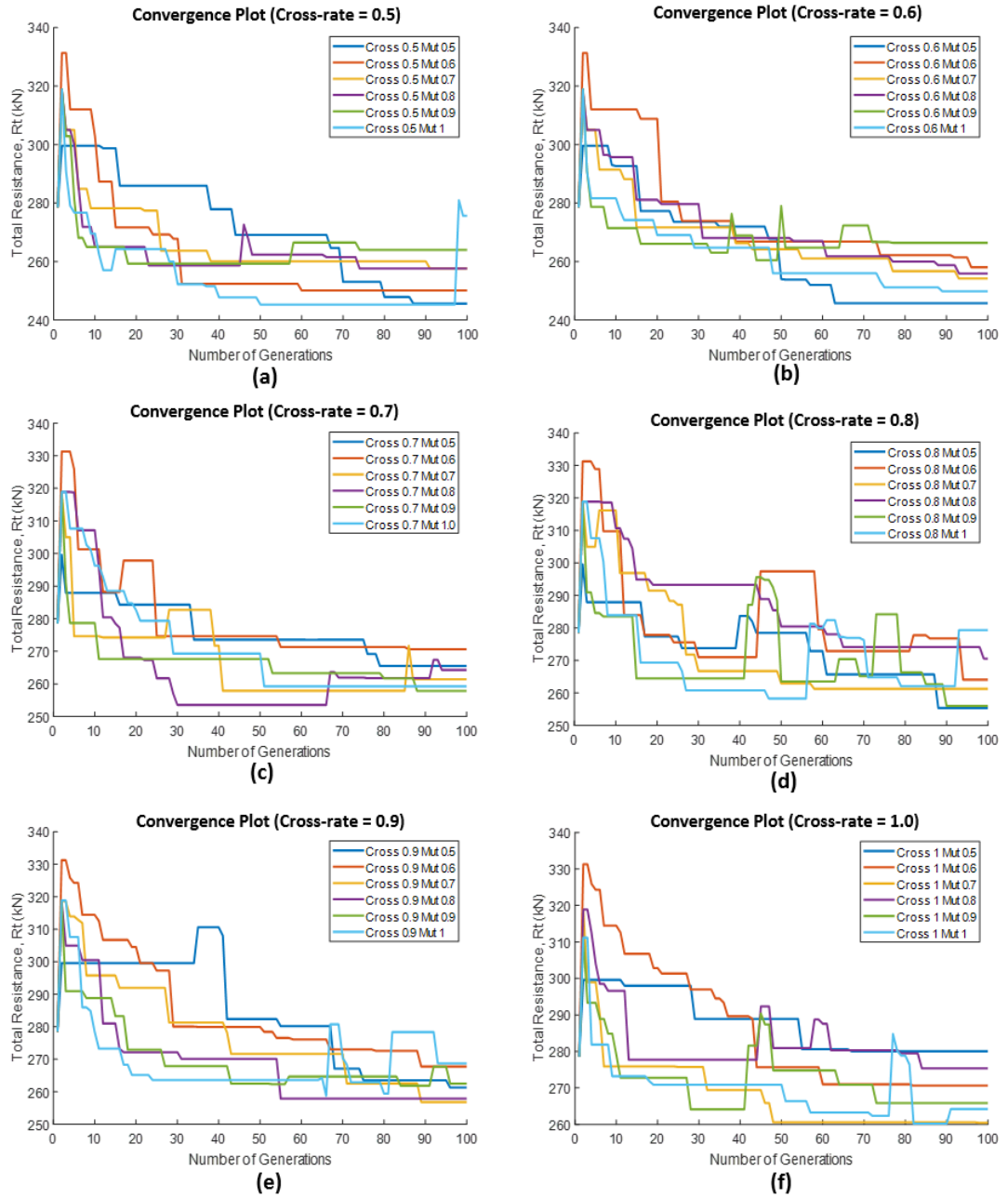


Figure 6.10 Convergence charts of total resistance showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.

Furthermore, by comparing the different mutation rates for each objective, better performances were observed when a higher mutation rate was used to balance a lower crossover rate. For example, better performances for pitch and roll were achieved at a crossover rate of 0.6 and a mutation rate of 0.9, and better performance for heave and roll were achieved at a crossover rate of 0.7 and a mutation rate of 1.0. Overall, this trend was consistent with the two-objective case in Section 6.2, where higher mutation rates were required to further refine the shape through subtle changes using boundary-morph.

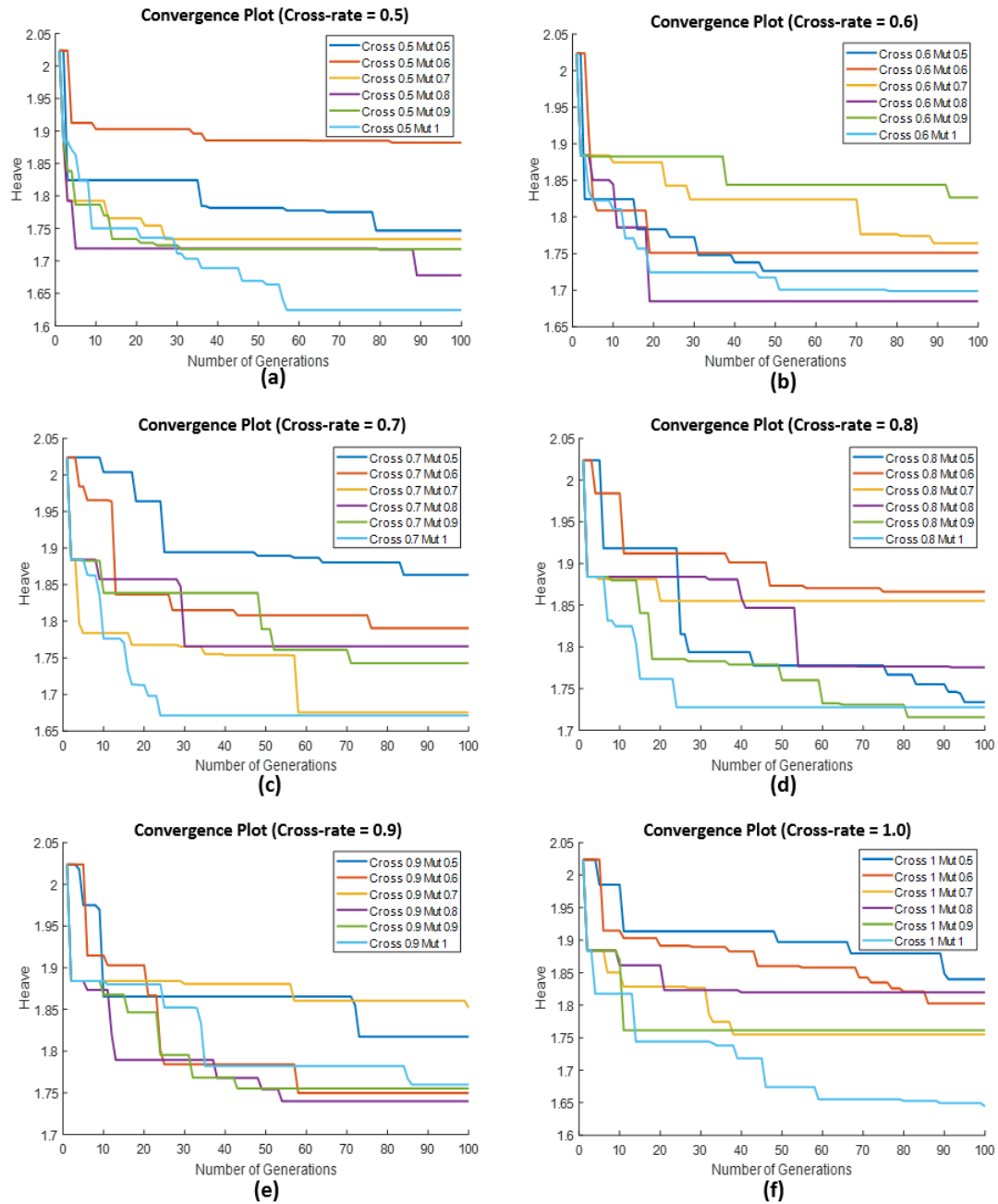


Figure 6.11 Convergence charts of heave showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.

However, where a high crossover rate was used, such as 0.8 or 0.9, a too high mutation will lead to too many shape changes and as a consequence, a poorer performance. Based on these findings, the crossover and mutation rate with the best (lowest) resistance were found to be 0.5 and 0.5, the best (lowest) heave was 0.5 and 1.0, the best pitch was 0.8 and 0.6 and the best roll was 0.7 and 1.0, respectively. For further investigations, the best solution for crossover at 0.7 and mutation at 1.0 was selected as they generated better results for the most objective functions.

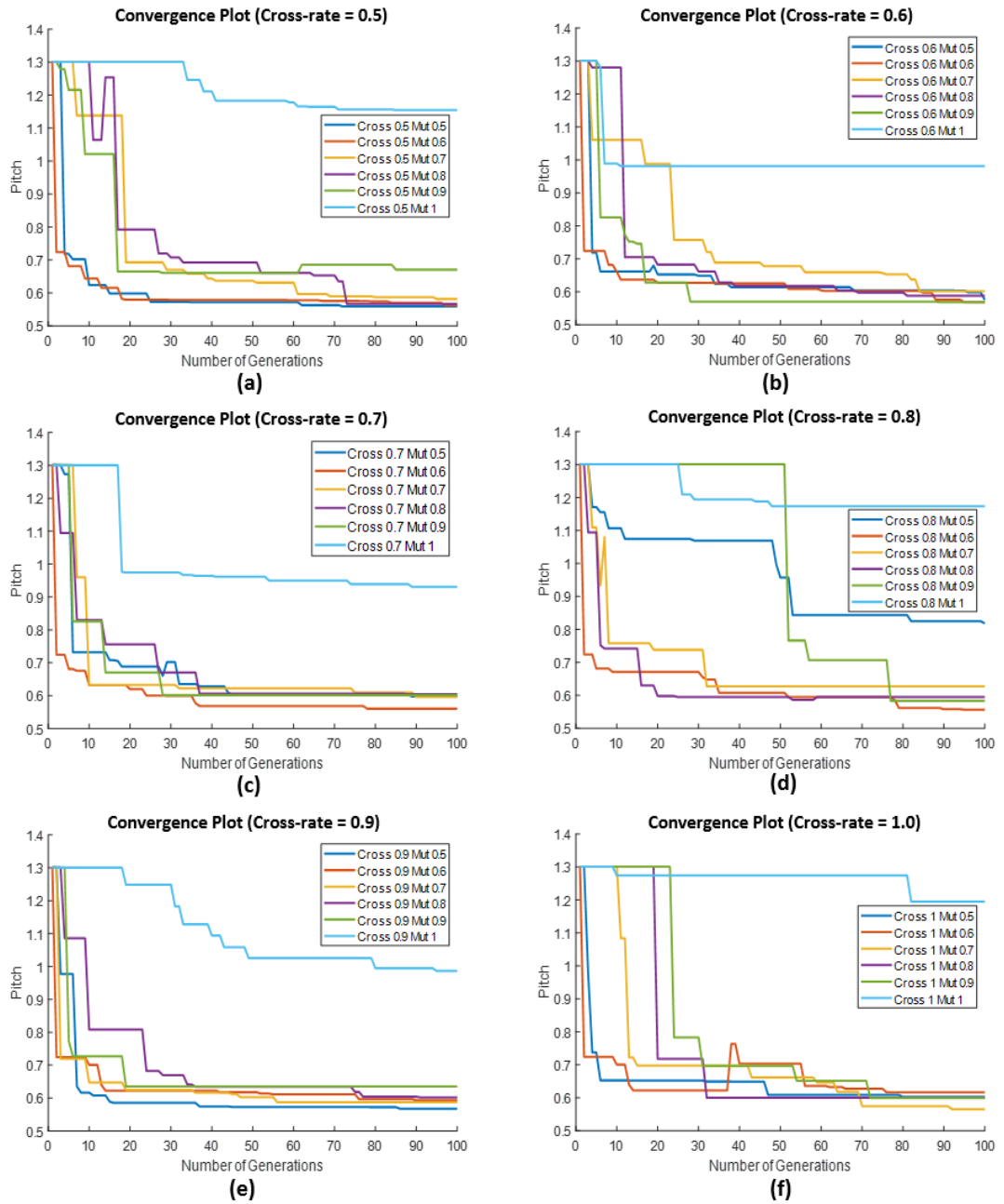


Figure 6.12 Convergence charts of pitch showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.

By comparing the results obtained for each objective function, it can be seen that the best results are obtained for both resistance and pitch in several settings (e.g. crossover 0.5 and mutation 0.5, crossover 0.9 and mutation 0.7), which demonstrated they are non-conflicting. However, it can also be seen that both resistance and pitch worsened (increased) when heave or roll objective function is at its peak (lowest). This shows that both resistance and pitch are conflicting to heave and roll function respectively.

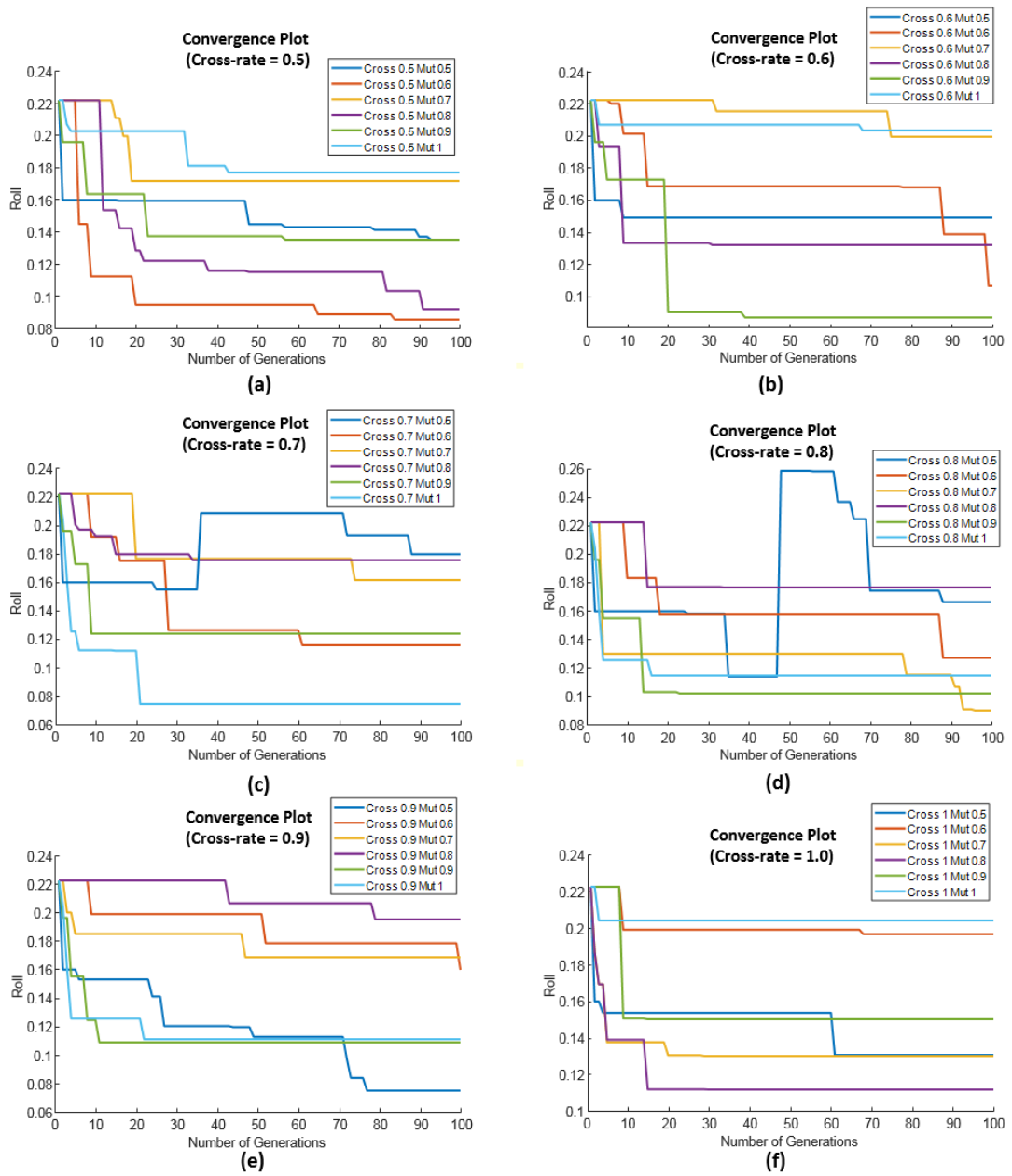


Figure 6.13 Convergence charts of roll showing crossover rates of (a) 0.5, (b) 0.6, (c) 0.7, (d) 0.8, (e) 0.9, (f) 1.0 and respective mutation rates ranging from 0.5 to 1.

The combined convergence chart for resistance, heave, pitch and roll is presented in Figure 6.14. It can be seen from the combined convergence chart that all the objectives improved (reduced) concurrently and reached the minimum after 20th generation. Similar to the two-objective case in Section 6.2, the fitness for pitch and roll were increased at the initial point due to performance trade-offs, but subsequently came down when it began to balance and reach its near-optimal point.

Table 6.4 Best results for total resistance, heave, pitch and roll for different combination crossover and mutation rates from 0.5 to 1.0.

Crossover rate	Mutation rate	Total resistance	Heave	Pitch	Roll
0.5	0.5	245.5	1.74	0.56	0.13
	0.6	250.0	1.88	0.56	0.08
	0.7	257.5	1.73	0.57	0.17
	0.8	257.5	1.67	0.56	0.09
	0.9	263.9	1.71	0.67	0.13
	1.0	261.7	1.62	1.15	0.17
0.6	0.5	245.7	1.72	0.58	0.14
	0.6	258.0	1.75	0.56	0.10
	0.7	254.1	1.76	0.59	0.19
	0.8	255.9	1.68	0.58	0.13
	0.9	266.4	1.82	0.56	0.08
	1.0	249.8	1.69	0.98	0.20
0.7	0.5	265.4	1.86	0.59	0.17
	0.6	270.5	1.79	0.56	0.11
	0.7	261.4	1.67	0.59	0.16
	0.8	264.2	1.76	0.60	0.17
	0.9	257.8	1.74	0.60	0.12
	1.0	259.2	1.67	0.93	0.07
0.8	0.5	255.3	1.73	0.81	0.16
	0.6	264.0	1.86	0.55	0.12
	0.7	261.2	1.85	0.62	0.08
	0.8	270.5	1.77	0.59	0.33
	0.9	256.0	1.71	0.58	0.10
	1.0	286.0	1.72	1.17	0.11
0.9	0.5	261.2	1.81	0.56	0.08
	0.6	267.7	1.74	0.59	0.15
	0.7	256.8	1.85	0.58	0.16
	0.8	257.8	1.73	0.60	0.19
	0.9	262.5	1.75	0.63	0.10
	1.0	268.7	1.74	0.98	0.11
1.0	0.5	280.0	1.83	0.60	0.13
	0.6	270.6	1.80	0.61	0.19
	0.7	260.4	1.75	0.56	0.13
	0.8	275.3	1.81	0.59	0.11
	0.9	265.8	1.76	0.59	0.15
	1.0	278.3	1.64	1.19	0.20

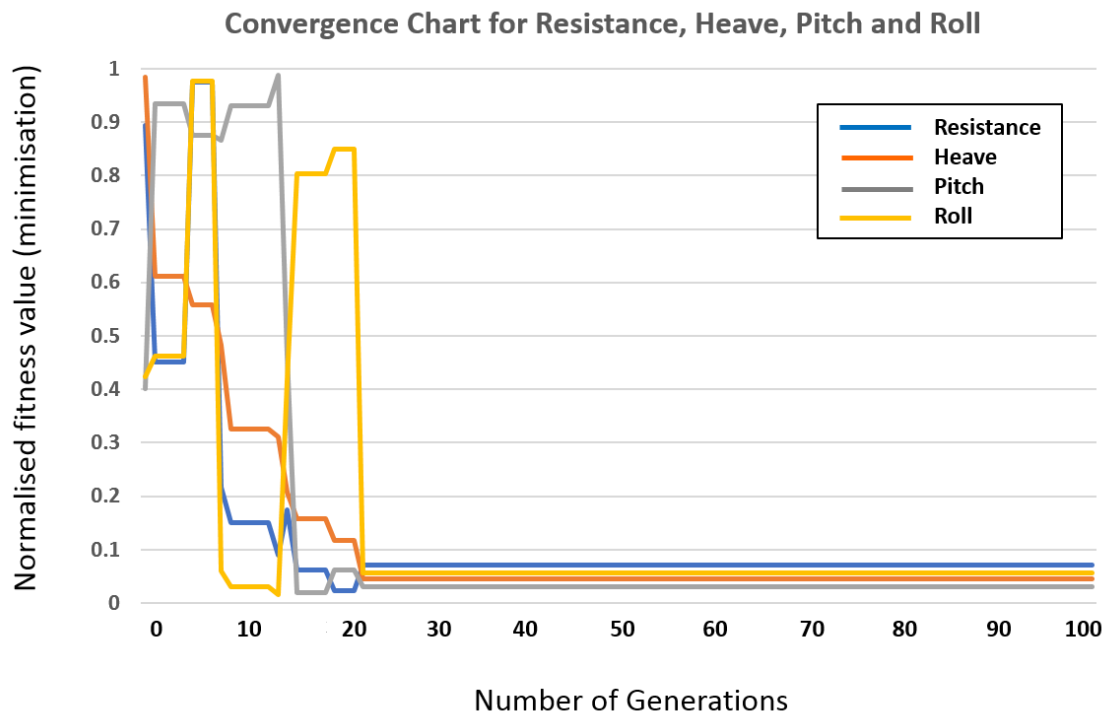


Figure 6.14 Combined convergence chart for resistance, heave, pitch and roll.

As there are altogether four objectives considered in this optimisation, a total of six pairs of objective comparisons is presented to cover all the possible cases. Table 6.5 shows the combination for comparing the four objective functions.

Table 6.5 Combinations for comparison of four objectives

	Resistance	Heave	Pitch	Roll
Heave	Figure 6.15			
Pitch	Figure 6.16	Figure 6.17		
Roll	Figure 6.18	Figure 6.19	Figure 6.20	

From Figures 6.15 to 6.20, initial observations on the overall distribution of the solutions for each objective pair showed that the solutions were dispersed across the design space. By examining the distributed solutions in generations 91–100, two main trends were observed. Firstly, from Figures 6.15 to 6.16, most of the solutions converged towards the bottom-left hand corner as optimisation progressed. In particular, from Figure 6.17 and 6.18, a clear Pareto front emerged at the final generations. This demonstrated an ability of MEC to search for near-optimal solutions.

Solution Space For Heave and Resistance

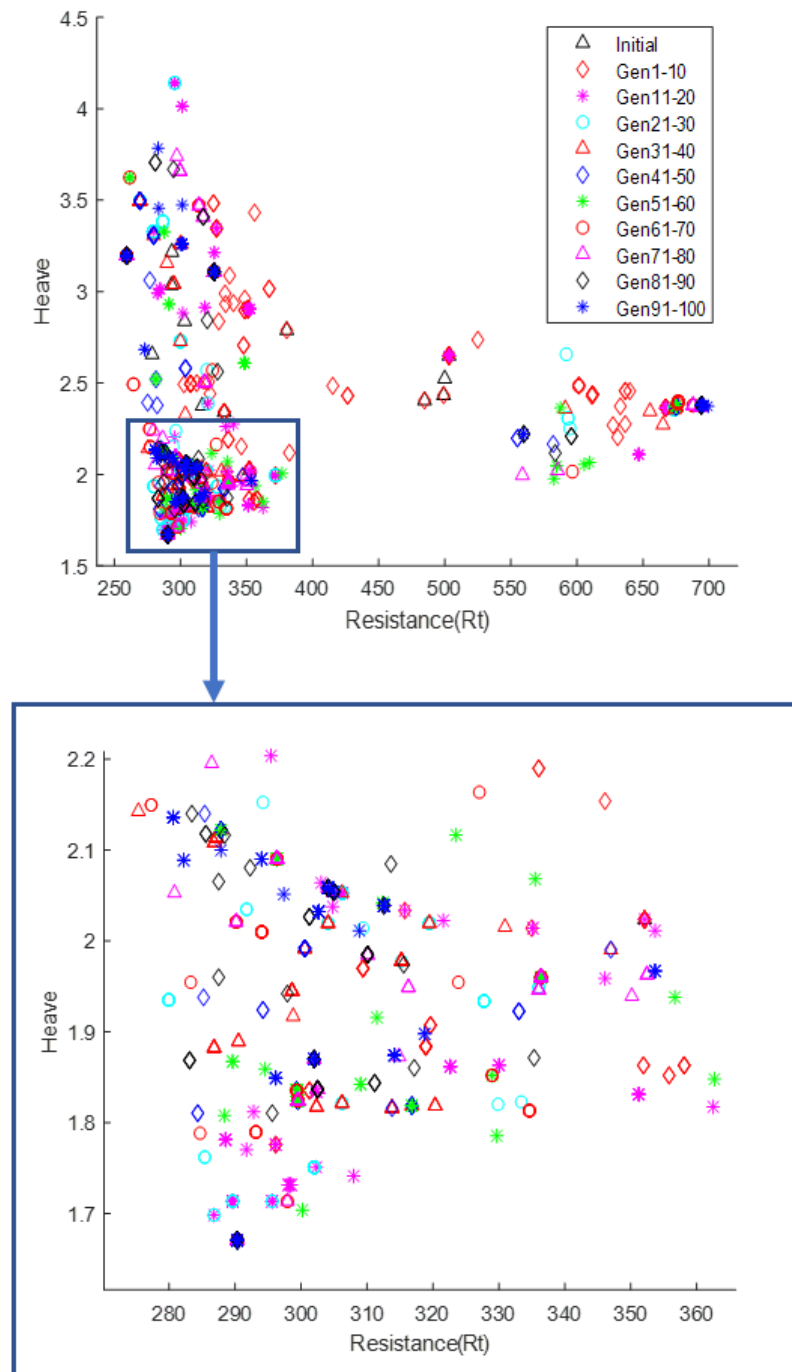


Figure 6.15 Solution space for heave and resistance.

Secondly, Figure 6.19 and Figure 6.20 showed that the solutions generally converged towards better performances of only one objective. This showed that most of the best solutions can only satisfy one design objective but they did not meet the requirements of the other. A possible interpretation of this is that while a larger Pareto optimal solution set is not uncommon for a higher number of objectives, it becomes more challenging to find solutions that meet all design criteria as the number of design criteria increases.

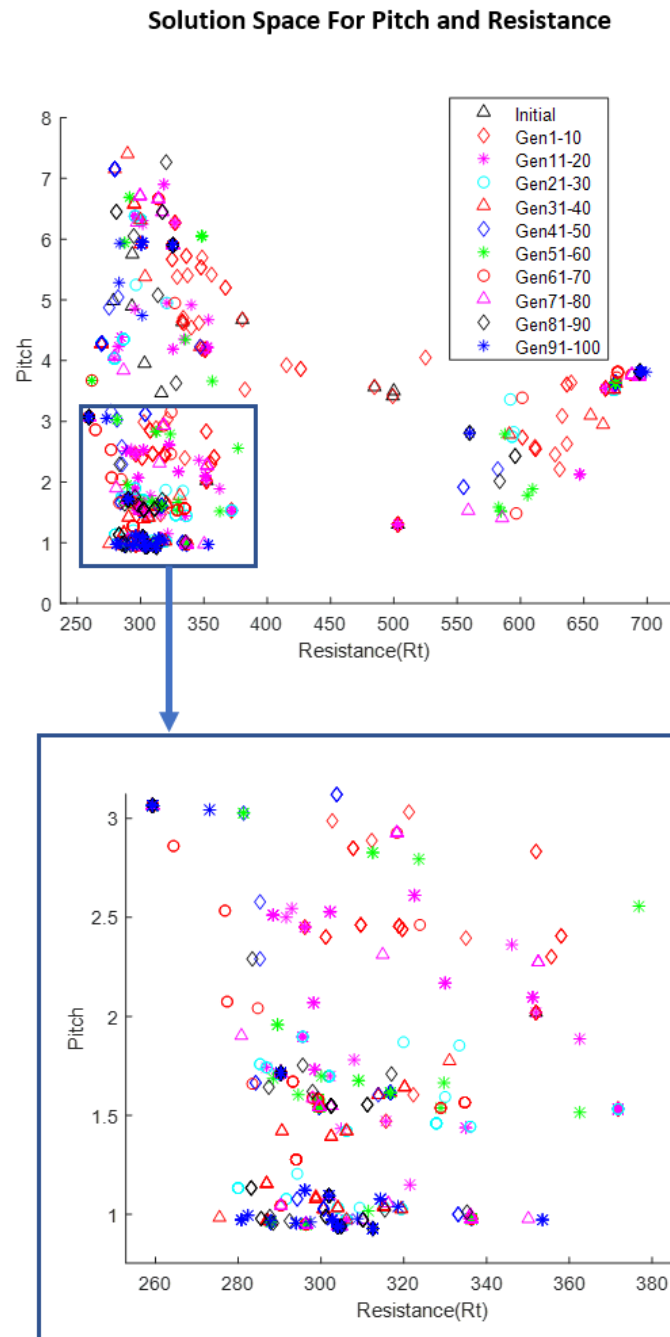


Figure 6.16 Solution space for pitch and resistance.

This phenomenon is best observed by comparing the convergence plots in Figure 6.15 and Figure 6.7 (case study 1) for resistance vs heave where the latter had only two objectives to optimise. It can be seen that the performance in the current case study is more levelled such that it may perform slightly worse in one objective and better in another to achieve a more balanced solution.

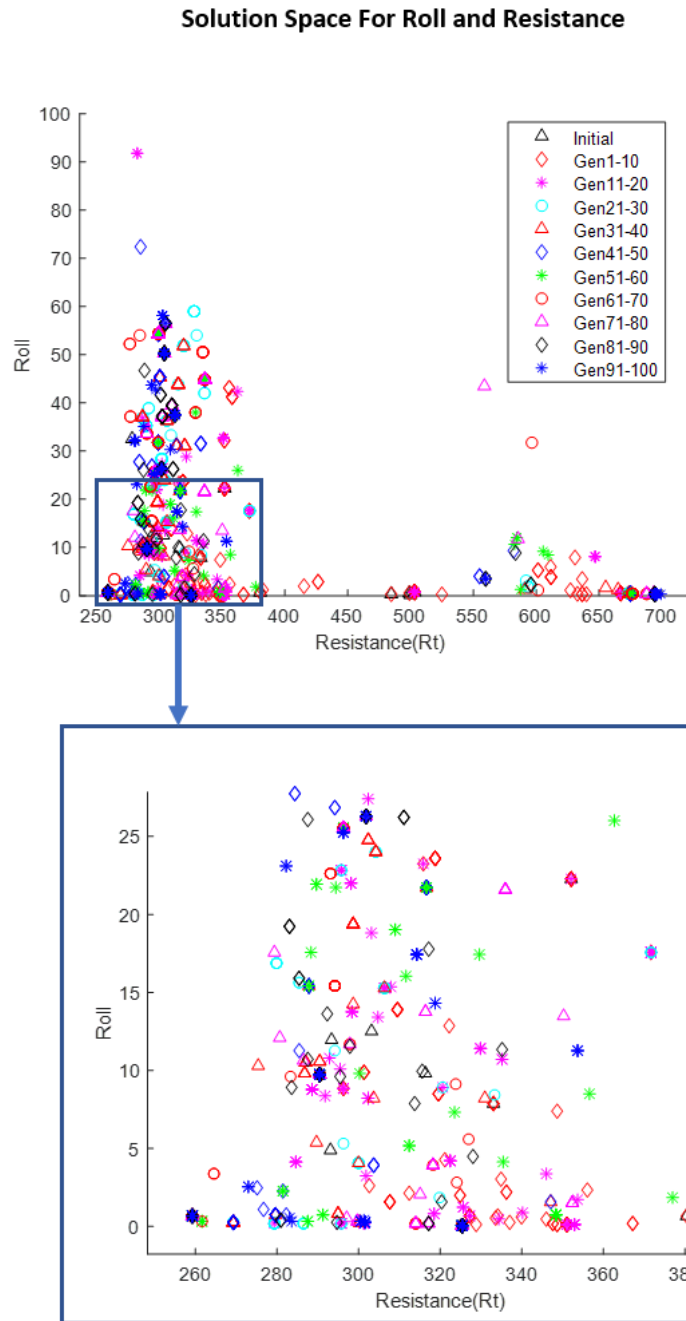


Figure 6.17 Solution space for roll and resistance.

The performance of the best parent and child solutions obtained at the end of the optimisation are presented in Table 6.6. Child solutions that outperform the best parent solutions are highlighted in green, while those that are inferior are highlighted in orange. The best solution for each objective is highlighted in yellow.

As with case study 1, three performance comparisons were done for this case study where the group of Pareto optimal solutions were first compared with the cable-laying vessel in the initial population. Comparisons were then conducted at the objective level where the pool of optimal solutions were benchmarked against the best performing parent vessel for

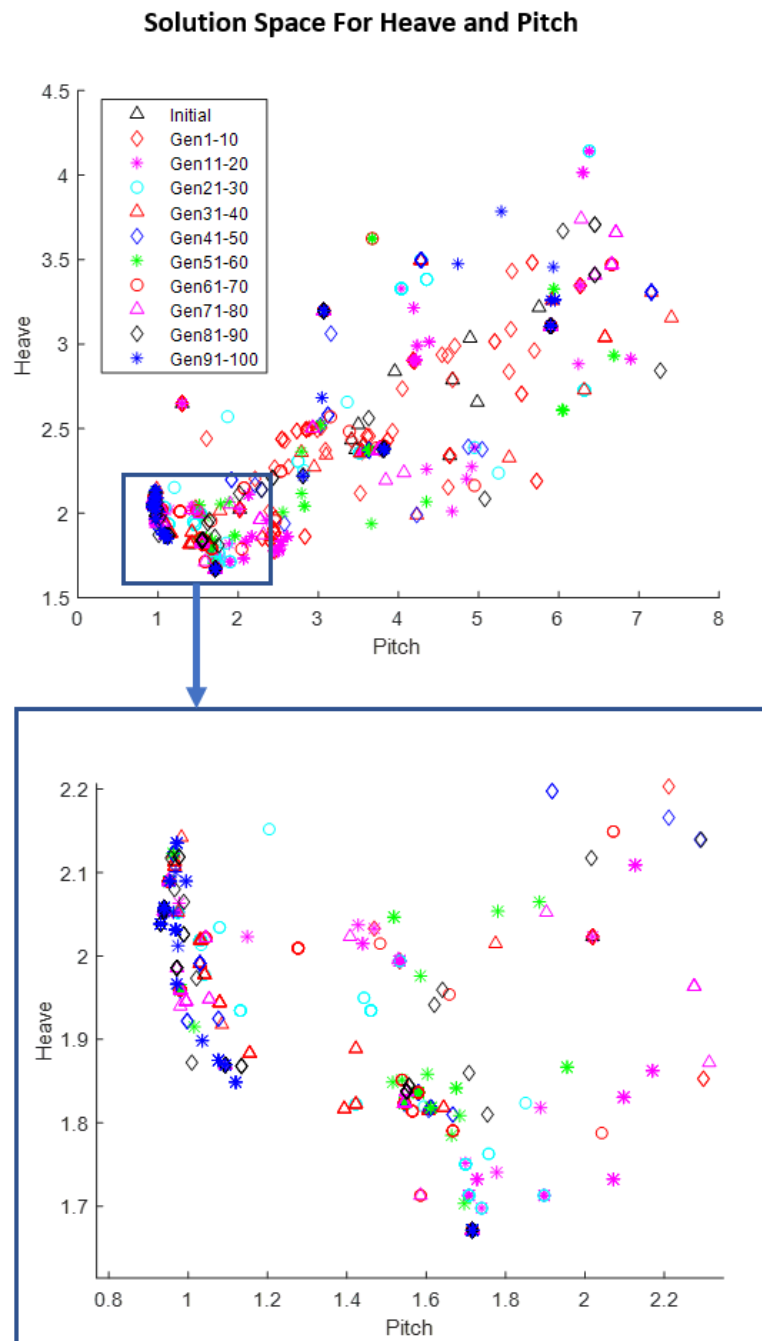


Figure 6.18 Solution space for heave and pitch.

each objective and finally the performance of each Pareto optimal solution in the Pareto was compared with one another.

From Table 6.6, three Pareto optimal solutions (1190, 1198, 1120) had at least three objective functions that were superior to ship 11. Among these, solution 1190 exceeded the performance of an existing cable-laying vessel, which was manually optimised by the designer, for all objective functions. This demonstrated that fully automated HFDO is possible and can meet all design criteria, including outperforming existing proven

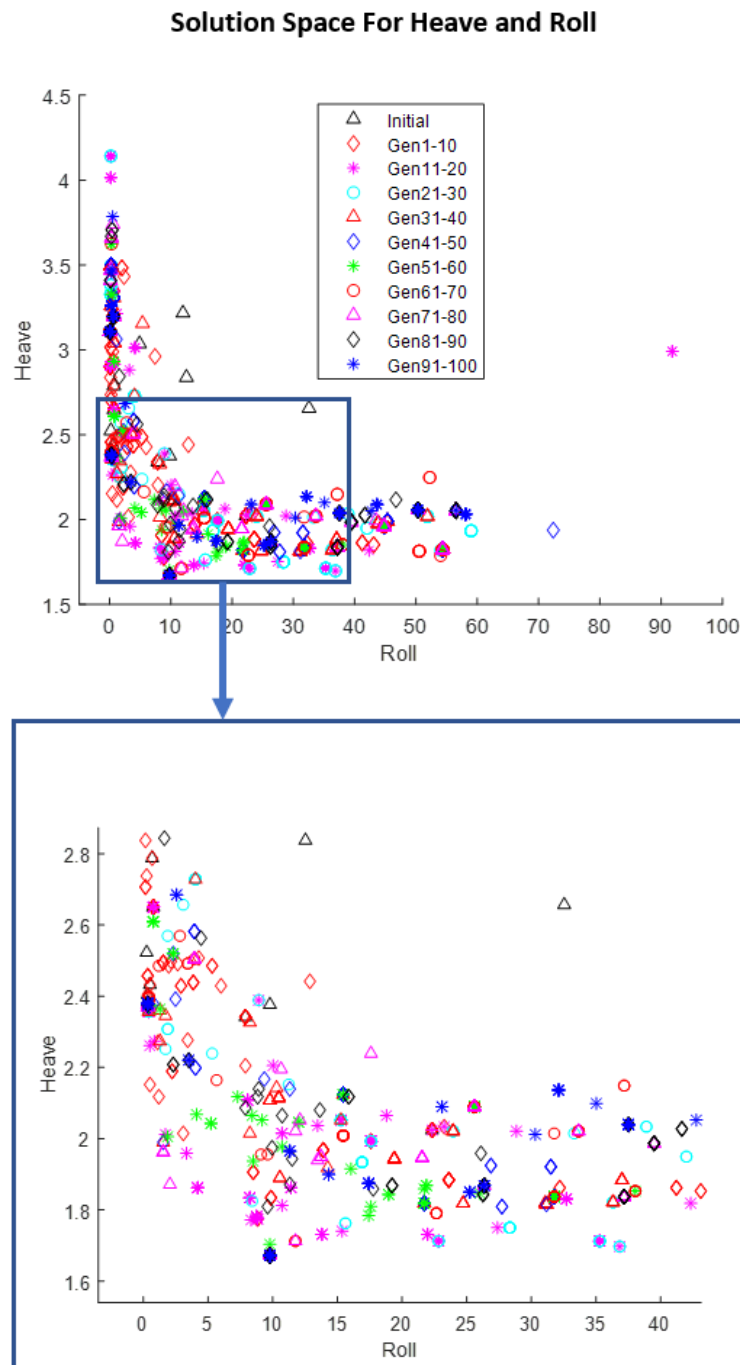


Figure 6.19 Solution space for heave and roll.

designs. On the whole, each child solution had at least two objectives that were superior to the parent solution.

Taking each objective in turn, performance between a child solution and the best parent solution were also compared. From Table 6.6, large numbers of solutions outperformed the best parent for each objective. For example, solutions 1190, 1198, 1120 for heave, solution 1192 for resistance, solutions 1191, 1195, 1196, 1197, 1198, 1199, 1120 for pitch and solution 1193 for roll. More notably, most child solutions had better performances

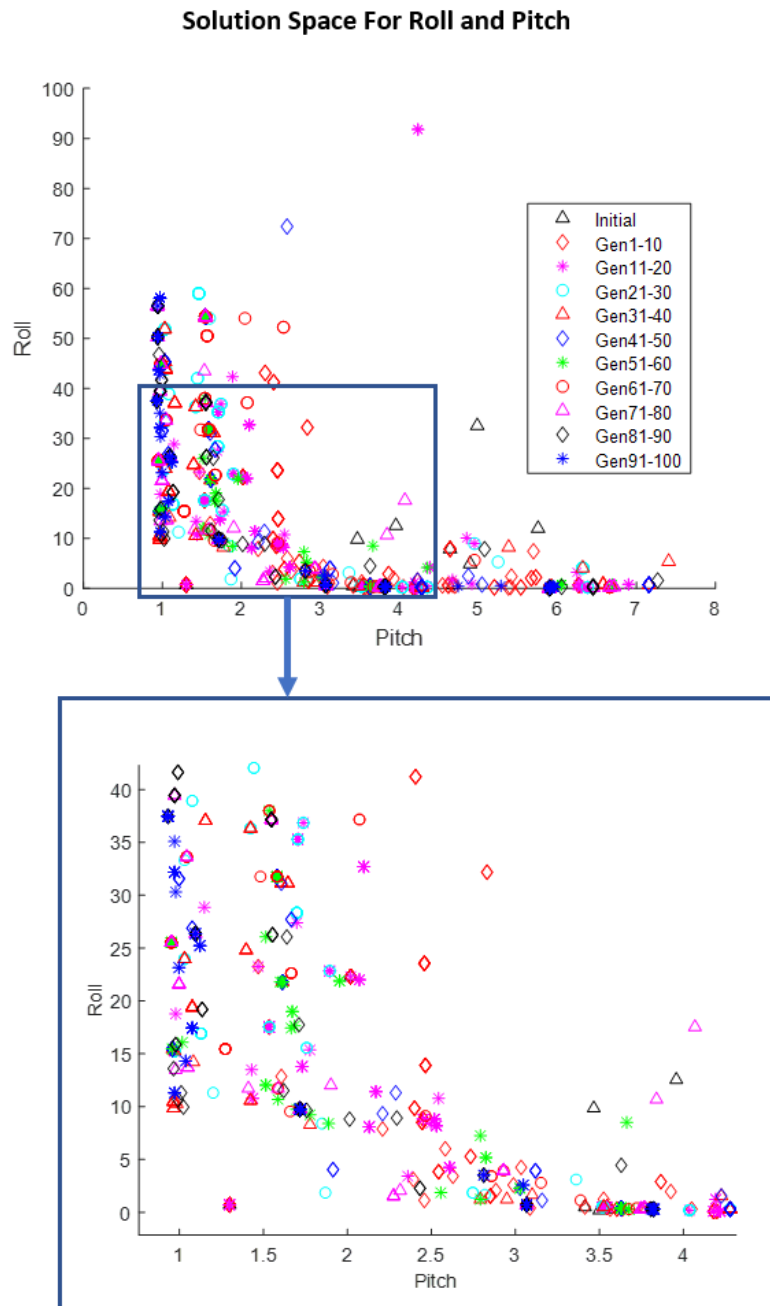


Figure 6.20 Solution space for roll and pitch.

where three of the four objective functions were satisfied, when compared to the best parent solutions.

It is acknowledged that the parent vessels were not optimised for all four objectives and may have otherwise performed better. Likewise, if the objective for this case study was to optimise for resistance only, then the solutions obtained may have performed better as seen in the results of the case studies presented in Chapter 5. Nonetheless, what these results showed is that through MEC, it is possible to automate the hull form design

Table 6.6 Best initial parent and child solutions for optimising four-objective functions

Parent Child	Ship 11 - Cable-laying Ship (Best heave) $R_t = 352$, Heave = 2.02, Pitch = 2.01, Roll = 22.28				Ship 1 - Container Ship A (Best resistance) $R_t = 278.5$, Heave = 2.65, Pitch = 4.98, Roll = 35.53				Ship 10 - Pipe-laying Ship (Best pitch) $R_t = 503.1$, Heave = 2.64, Pitch = 1.3, Roll = 0.72				Ship 7 (Product Carrier) Best roll $R_t = 499.9$, Heave = 2.52, Pitch = 3.49, Roll = 0.22					
	Soln.	R_t	Heave	Pitch	Roll	R_t	Heave	Pitch	Roll	R_t	Heave	Pitch	Roll	R_t	Heave	Pitch	Roll	Δ (tonnes)
1189		301.58 (14.33)	3.26 (-61.22)	5.95 (-194.8)	0.26 (98.83)	301.58 (-8.27)	3.26 (-22.84)	5.95 (-19.46)	0.26 (99.2)	301.58 (40.07)	3.26 (-23.15)	5.95 (-357.8)	0.26 (64.17)	301.58 (39.67)	3.26 (-29.27)	5.95 (-70.19)	0.26 (-17.75)	16097.62
1190		290.34 (17.53)	1.67 (17.43)	1.72 (15.07)	9.74 (56.27)	290.34 (-4.23)	1.67 (37.09)	1.72 (65.58)	9.74 (70.05)	290.34 (42.30)	1.67 (36.93)	1.72 (-31.91)	9.74 (-1235)	290.34 (41.92)	1.67 (33.80)	1.72 (50.97)	9.74 (-4288)	15156.84
1191		312.58 (11.21)	2.04 (-0.73)	0.93 (53.93)	37.51 (-68.36)	312.58 (-12.22)	2.04 (23.25)	0.93 (81.33)	37.51 (-15.29)	312.58 (37.88)	2.04 (23.06)	0.93 (28.45)	37.51 (-5040)	312.58 (37.47)	2.04 (19.24)	0.93 (73.41)	37.51 (-16792)	17299.19
1192		259.26 (26.36)	3.20 (-57.95)	3.07 (-51.86)	0.67 (97.01)	259.26 (6.93)	3.20 (-20.35)	3.07 (38.46)	0.67 (97.96)	259.26 (48.48)	3.20 (-20.65)	3.07 (-135.8)	0.67 (8.86)	259.26 (48.14)	3.20 (-26.65)	3.07 (12.33)	0.67 (-199.53)	15078.35
1193		325.49 (7.55)	3.11 (-53.57)	5.90 (-192.1)	0.07 (99.67)	325.49 (-16.85)	3.11 (-17.01)	5.90 (-18.39)	0.07 (99.77)	325.49 (35.32)	3.11 (-17.30)	5.90 (-353.7)	0.07 (89.88)	325.49 (34.89)	3.11 (-23.13)	5.90 (-68.65)	0.07 (66.75)	16781.44
1194		699.21 (-98.61)	2.37 (-17.24)	3.81 (-88.60)	0.30 (98.66)	699.21 (-151.0)	2.37 (10.67)	3.81 (23.58)	0.30 (99.08)	699.21 (-38.95)	2.37 (10.44)	3.81 (-192.9)	0.30 (58.94)	699.21 (-39.87)	2.37 (5.99)	3.81 (-8.87)	0.30 (-34.95)	21950.36
1195		302.69 (14.02)	2.03 (-0.37)	0.97 (51.98)	58.10 (-160.7)	302.69 (-8.67)	2.03 (23.52)	0.97 (80.54)	58.10 (-78.56)	302.69 (39.85)	2.03 (23.33)	0.97 (25.42)	58.10 (-7861)	302.69 (39.45)	2.03 (19.52)	0.97 (72.28)	58.10 (-26062)	16836.64
1196		282.30 (19.81)	2.09 (-3.22)	1.00 (50.73)	23.11 (-3.73)	282.30 (-1.35)	2.09 (21.35)	1.00 (80.03)	23.11 (28.97)	282.30 (43.90)	2.09 (21.15)	1.00 (23.48)	23.11 (-3067)	282.30 (43.53)	2.09 (17.24)	1.00 (71.56)	23.11 (-10307)	15918.61
1197		304.11 (13.62)	2.06 (-1.67)	0.94 (53.42)	50.32 (-125.8)	304.11 (-9.18)	2.06 (22.54)	0.94 (81.13)	50.32 (-54.65)	304.11 (39.56)	2.06 (22.34)	0.94 (27.66)	50.32 (-6795)	304.11 (39.17)	2.06 (18.48)	0.94 (73.11)	50.32 (-22559)	16931.43
1198		353.62 (-0.44)	1.97 (2.83)	0.97 (51.95)	11.29 (49.31)	353.62 (-26.95)	1.97 (25.97)	0.97 (80.53)	11.29 (65.29)	353.62 (29.73)	1.97 (25.78)	0.97 (25.37)	11.29 (-1447)	353.62 (29.26)	1.97 (22.09)	0.97 (72.26)	11.29 (-4985.8)	17542.06
1199		297.35 (15.54)	2.05 (-1.38)	0.96 (52.31)	42.74 (-91.84)	297.35 (-6.75)	2.05 (22.75)	0.96 (80.67)	42.74 (-31.37)	297.35 (40.91)	2.05 (22.56)	0.96 (25.93)	42.74 (-5757)	297.35 (40.52)	2.05 (18.71)	0.96 (72.47)	42.74 (-19148)	16650.06
1120		308.92 (12.25)	2.01 (0.60)	0.98 (51.69)	30.32 (-36.06)	308.92 (-10.91)	2.01 (24.26)	0.98 (80.42)	30.32 (6.83)	308.92 (38.61)	2.01 (24.07)	0.98 (24.96)	30.32 (-4053)	308.92 (38.20)	2.01 (20.30)	0.98 (72.11)	30.32 (-13551)	16764.19

Legend:

Child solutions that outperform the best parent solutions

Child solutions that are inferior than best parent solutions

Overall best child solutions for individual objective

optimisation process and produce solutions that matched what a designer can come up with.

When the Pareto optimal solutions were compared to one another, the best solution for pitch was 1190 (1.67), the best solution for resistance was 1192 (259.26 kN), the best solution for pitch was 1191 (0.93) and the best solution for roll was 1193 (0.07). When compared to the best solution (1190), solution 1192 had two objectives that were better (resistance and roll) and solutions 1191 and 1193 had one objective (pitch and roll respectively) that were better than solution 1190. The wire-mesh diagrams and body plans of the best child solutions for the objective functions are presented in Figures 6.21 and 6.22, respectively.

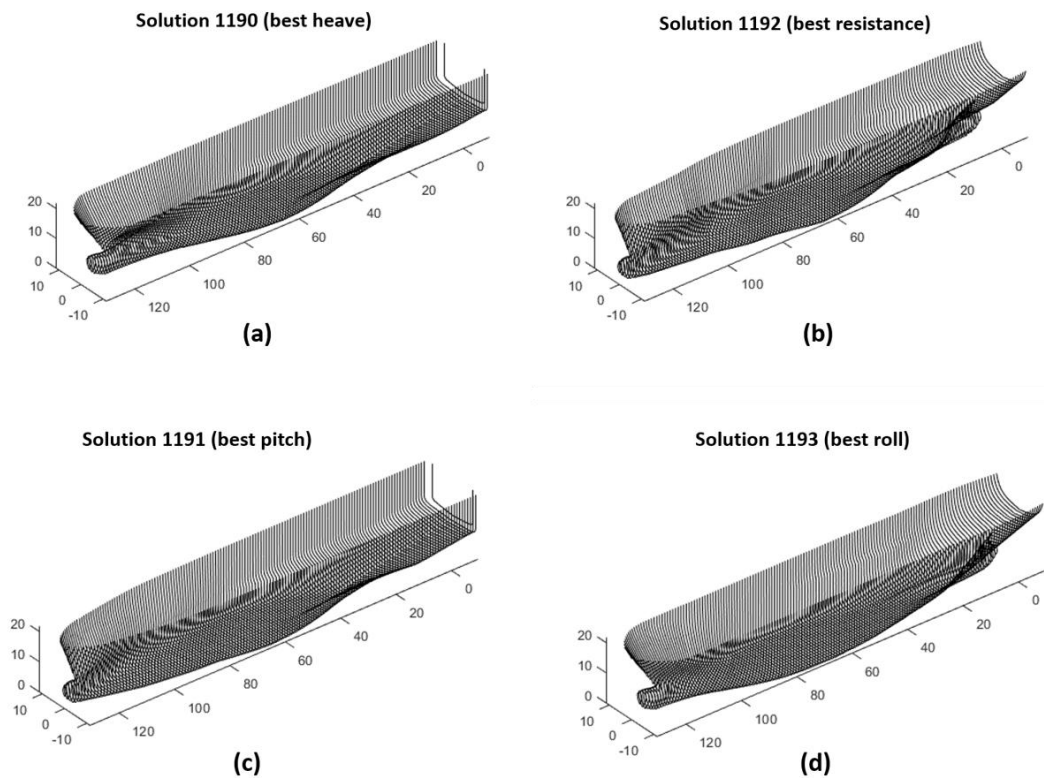


Figure 6.21 Wire-mesh diagrams of the best child for (a) heave (solution 1190), (b) resistance (solution 1192), (c) pitch (solution 1191) and (d) roll (solution 1193).

From Figure 6.21, it can be observed that each best child solutions had a different hull form for each objective. For example, Figure 6.21(a) shows a fuller hull form that is relatively uniform across the length of vessel, which resulted in lower heave. The hull form in Figure 6.21(b) is sharp at the bow and has a slender body which is good for resistance. Figure 6.21(c) shows a hull form with full bow and wide aft section which provided lower pitch. Figure 6.21(d) shows a hull form that is relatively full at the mid-

body and slender at the aft, which helped to reduce the roll motion. By comparing all figures, the hull forms are highly feasible with no abrupt shape changes or unevenness. It was observed that solutions 1192 and 1193 may not be suitable for an offshore cable-laying vessel due to the small transom that limits cable-laying operations.

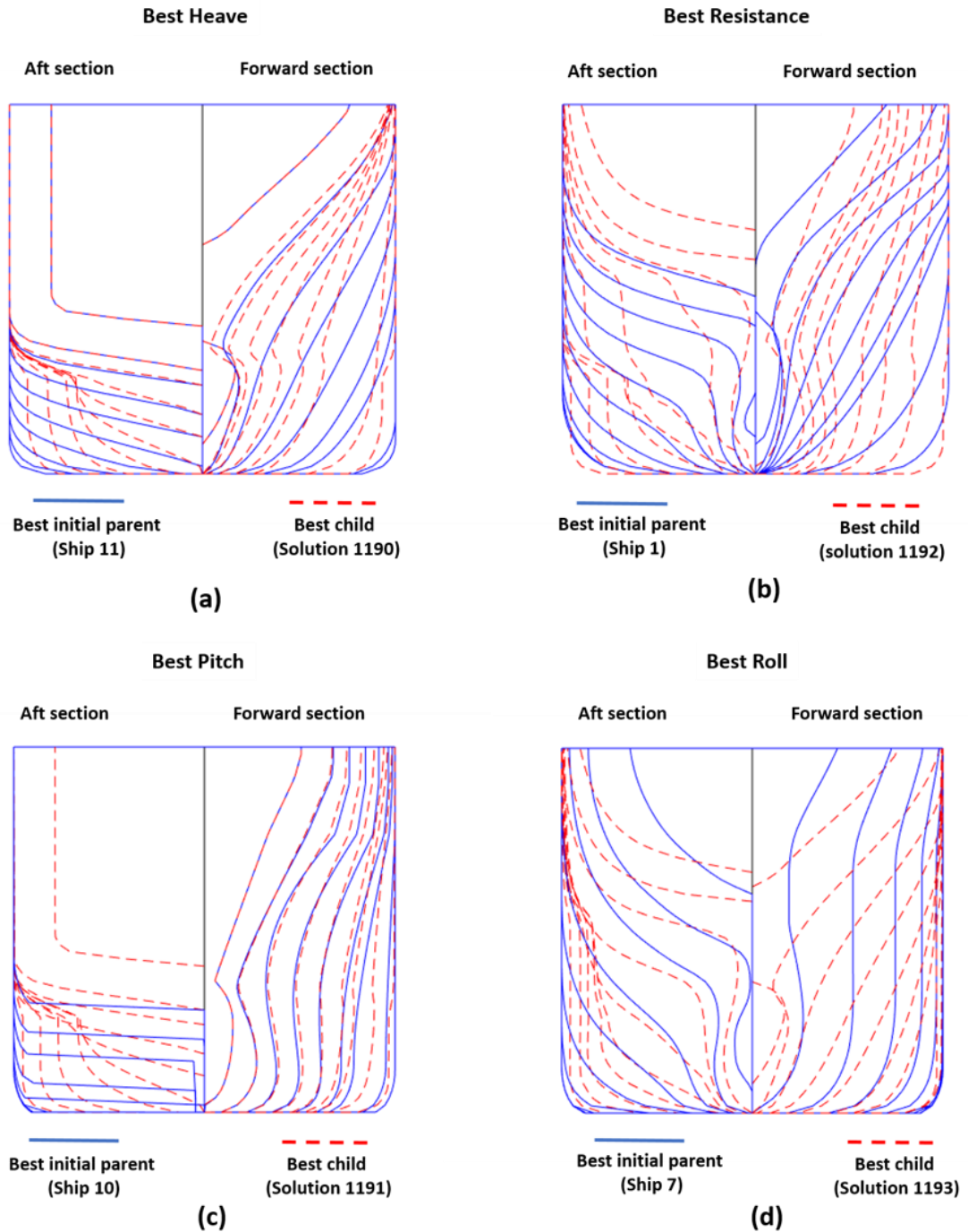


Figure 6.22 Body plans comparing (a) the best child (solution 1190) and the best initial parent (ship 11) for heave, (b) the best child (solution 1192) and the best initial parent (ship 1) for resistance, (c) the best child (solution 1191) and the best initial parent (ship 10) for pitch and (d) the best child (solution 1193) and the best initial parent (ship 7) for roll.

Figure 6.22 shows that the best child vessels have a very different hull form from their best parent vessels, although there was some resemblance at different parts of the body section. For example, it can also be seen from Figure 6.22(a) that both the forward and aft section of best child solution 1190 (heave) was inherited from parent ship 11, a cable-laying vessel. This highlights the fact that MEC was able to search the design space to find existing solutions which was close to the design specification and at the same time create new vessels through crossover and mutation. Solution 1192 (resistance) is a combination of ship 4, 6 and 10 as seen in Figure 6.20(b). The aft section took the shape of ship 4 (Container Ship D), while the mid-section resembled that of ship 6 (Bulk Carrier) and the forward section took its shape from ship 10 (Pipe-laying Ship).

For best child solution 1191 (pitch) in Figure 6.22(c), the aft section takes the shape of the best parent: ship 11 (Cable-laying Ship), mid-section resembles ship 6 (Bulk Carrier) and the forward section from ship 10 (Pipe-laying Ship). Finally, for best child solution 1193 shown in Figure 6.22(d) inherited properties from parent ships 4, 9 and 11. For example, the aft section resembles ship 4 (Container Ship D), the mid-section resembles ship 8 (Chemical Tanker) and the forward section takes the shape of ship 11 (Cable-laying Ship). The time taken for each optimisation case was approximately 16 hours on a 4-core CPU. In total, 36 simulations were completed using parallel computing, which reduced the processing time to under five days.

6.4. Optimisation using Goal-setting Approach

From the four-objective case study presented in Section 6.3, some of the solutions found in the final generation (in particular Figure 6.19 and 6.20) have extremities where individual objective tends to dominate. Although these individuals are near-optimal, in practice they may not be useful to the designer as they may not be considered all-rounded solutions that satisfy both the objective functions. To circumvent this problem, further investigation was conducted to narrow down the solution space by applying the goal-setting approach. This was performed by constraining the search space such that any individuals created that do not meet the constraints set are eliminated and are not carried over to the next generation.

To set the constraints and thereby the boundaries of the search space, the best performing parent in the initial population for each objective were used as benchmarks. The constraints were then set at $\pm 5\%$ of the best objective values. The additional constraints for each objective function are provided in Table 6.7.

Table 6.7 Upper and lower limit constraints set for each objective function.

Objective functions	Lower limit (minimum) constraint	Upper limit (maximum) constraint
Resistance	278.54 kN (ship 1) $\times$ 95% = 264.61 kN	503.19 kN (ship 10) $\times$ 105% = 528.34 kN
Heave	2.02 (ship 11) $\times$ 95% = 1.91	3.21 (ship 4) $\times$ 105% = 3.37
Pitch	1.3 (ship 10) $\times$ 95% = 1.23	5.75 (ship 4) $\times$ 105% = 6.03
Roll	0.22 (ship 7) $\times$ 95% = 0.2	32.53 (ship 1) $\times$ 105% = 34.15

The optimisation was then performed using the best crossover and mutation rates of 0.7 and 1.0 obtained from Section 6.3, and the results are presented in Figure 6.23.

It can be observed from Figure 6.23 that the individuals are generally scattered within the constraint limits, except in Figure 6.213(e), where extremities are still found on either side of the objective functions. This could be attributed to the conflicting nature of the heave and roll motion, where the improvement of one function will result in worsening the performance of the other. By comparing the goal-setting and non-goal-setting case in Section 6.3, it can be seen from Figures 6.23(a) to 6.23(f) that the individuals are generally more spread out as compared to Figures 6.15 to 6.20. In particular, the solutions found in the final population are also more scattered around the solution space as compared to those solutions obtained in Section 6.3, which converge towards the bottom-left hand corner. The best individuals for each objective are compared in Table 6.8.

Table 6.8 Comparison of best results obtained for each objective with and without goal-setting

	Best parent from initial population	Best child (without goal-setting)	Improvement over best parent	Best child (with goal-setting)	Improvement over best parent
Resistance (kN)	278.54	259.2	6.94%	264.61	5.13%
Heave	2.02	1.67	17.32%	1.91	5.94%
Pitch	1.3	0.93	28.46%	1.4	-7.69%
Roll	0.22	0.07	68.18%	0.22	0%

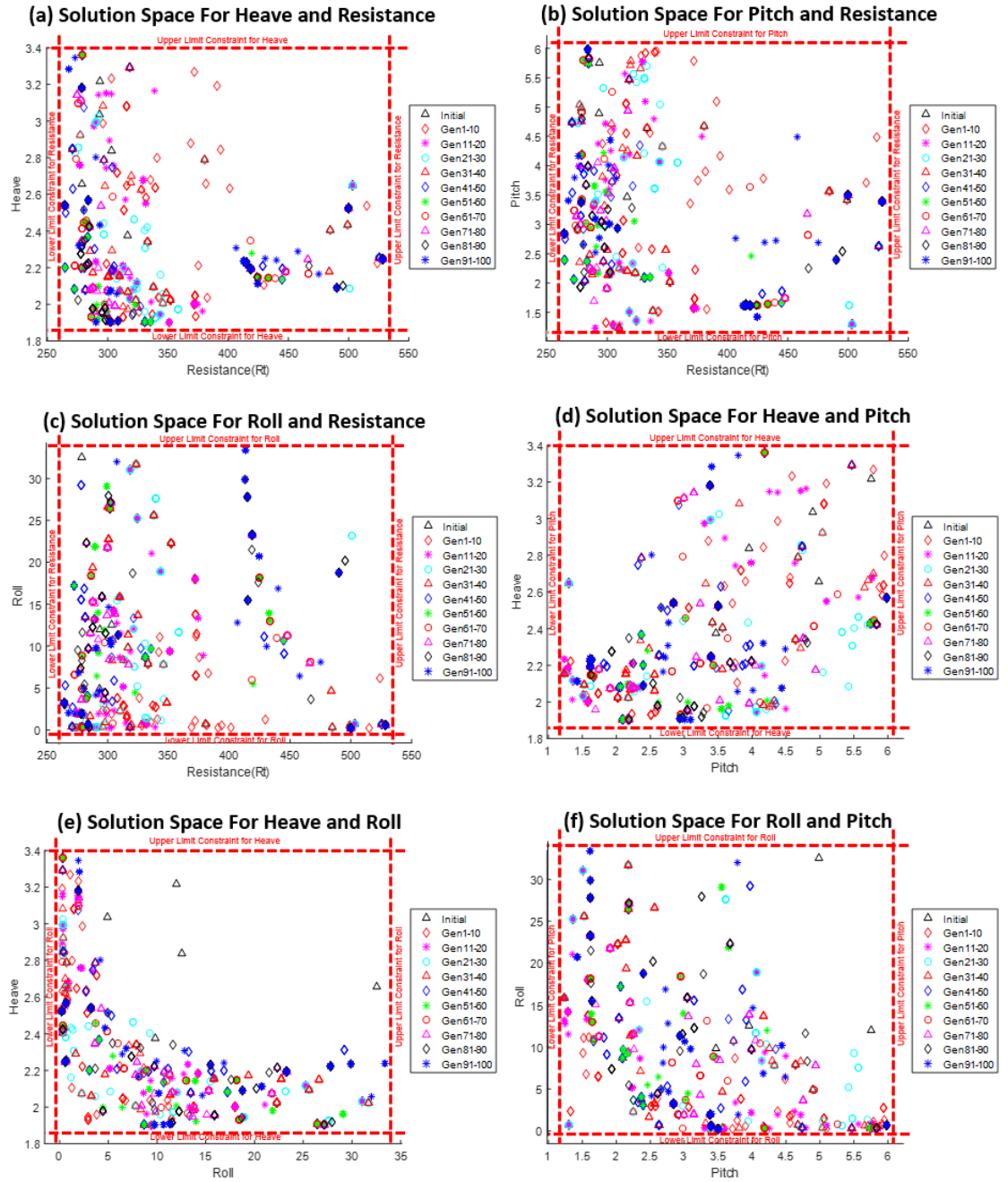


Figure 6.23 Solution space with goal setting approach for a) resistance and heave, b) resistance and pitch, c) resistance and roll, d) pitch and heave, e) pitch and roll and f) roll and heave. The constraint limits for each objective are shown in red line.

From the results shown in Figure 6.23 and Table 6.8, goal-setting does provide diversity in the solutions found, as they are more spread out. However, the overall performance is not improved. When the solutions obtained using goal-setting were compared to the best parent in the initial population, i.e. designs produced by experienced designers, only 50% of the objectives showed better performance. On the other hand, when no goal-setting was applied, the solutions found outperformed the best parent vessel for all four objectives.

Comparing the best child with and without goal-setting confirmed that the latter was inferior. The reason for this result could be due to the small constraints ($\pm 5\%$), which lead to a relatively small impact on the performance. Another possible reason could be due to the small population size where it cannot afford to eliminate any potentially good individual which could be carried forward to the future generation.

6.5. Results and Discussion

Several observations can be made from the two case studies. First, most of the solutions achieved better performance over the parent solutions in all objective functions in case study 1 while solutions in case study 2 typically could achieve good improvements for up to three out of four objectives. By comparing the performance improvement over best initial parents in case study 1, solution 1190 achieved 18.17% improvement in heave over best parent (ship 11) while solution 1194 achieved 8.35% improvement in resistance over best parent (ship 1). In case study 2, solution 1190 achieved 17.43% improvement in heave over best parent (ship 11), solution 1192 achieved 6.93% improvement in resistance over best parent (ship 1), solution 1191 achieved 28.45% improvement in pitch over best parent (ship 10) and solution 1193 achieved 66.75% improvement in roll over best parent (ship 7).

Second, it is clear from both case studies that the results produced by MEC for multi-objective optimisations were better with crossovers between 0.6 and 0.7 and mutation rates between 0.9 and 1.0. In the case of two-objective optimisation, it was found that crossover at 0.6 and mutation at 0.9 provided the best results. In the four-objective optimisation problem, crossover at 0.7 and mutation at 1.0 were found to provide the best results. It is observed that high crossover settings will cause “over-variation” of the hull shape, which may be too disruptive to the shape of the hull and lead to negative results. High mutation, on the other hand, provided subtle variations or shape refinements through boundary-morph, while keeping the main body of the vessel following crossover intact, leading to better performances.

Third, from Table 6.6 most solutions did not exhibit good roll and resistance. In fact, only one of the 12 child solutions have better resistance performance and one child solution has better roll motion when compared to the best parent solution for these objectives. When ranked according to the number of solutions which performed better than the best parent vessel, it was shown that roll and resistance was the hardest objective function to optimise, followed by heave and pitch.

Lastly, it was observed that where two optimisation problems have overlapping objective functions, the one with a larger set of objectives tended to perform worse than the problem with a smaller set of objectives when the same objective functions are compared. This is because a Pareto optimal solution in the larger set has more objectives to “balance” in order to find a more all-rounded solution.

Key Achievements

As seen in case study 2, it is very tedious to adjust the design parameters or hull geometry using conventional ‘trial-and-error’ method as improving one objective function would very likely result in worsening the performance of others. Automated optimisation as presented in this thesis offers the potential to replace this manual search process and thus freeing up designer time to focus on other more important task.

It was also observed from case study 2 that MEC could “mimic” the human designer by automatically deciding which objectives to “sacrifice” while improving others to achieve more balanced solutions. This is beneficial as it offers the designer a pool of feasible solutions which are already optimised toward some design criteria and could thus cut down the overall design time required. More importantly, a near-Pareto optimal solution (1190) exceeded the performance of an existing cable-laying vessel, for all objective functions.

Furthermore, by maintaining a small population of individuals throughout the optimisation, MEC provides a methodical way to derive the solutions. This allows the designer to trace the key steps of how a child solution was generated in a constructive manner and gained better insights into the design and could help towards alleviating the ‘designer’s dilemma’ in selecting the best design to a multi-objective optimisation problem.

6.6. Chapter Summary

Two case studies were conducted in this chapter to optimise the hull form of a cable-laying ship. In the first case study, two-objective optimisations were performed using resistance and heave. Improvements between 0.44% and 18.17% were achieved as compared to the best parent (ship 11) for heave and 1.95% to 8.35% improvements to resistance were made as compared to the best parent (ship 1). In the second case study, four-objective optimisation was performed comprising of resistance, roll, heave and pitch objective functions. The overall result achieved improvement in all the objective functions up to 17.43% for heave, 6.93% for resistance, 28.45% for pitch and 66.75% for

roll, when compared to their respective best initial parents. These results showed that MEC can be effectively applied to hull form design optimisation problem consisting of both non-conflicting and conflicting objective functions and produced solution that are either better than or on par with what a human designer can produce.

7. CONCLUSIONS AND FURTHER WORK

7.1. Overall Conclusions

Eco-friendly shipping and fuel efficiency are important concerns faced by the marine industry today. The shape of the hull affects the overall performance efficiency of a vessel and therefore ranks as a top priority in the design and optimisation of a ship. Digitalisation is gaining increased attention across the marine industry as ships become smarter and more eco-friendly. With the proliferation of the Internet, machines are becoming increasingly connected and automated, and ships are becoming more intelligent, unmanned and autonomous, gradually shifting towards the emerging “Fourth Industrial Revolution”. Relative to these advances, however, progress in design automation of ships has been slow. The reasons for this slow adoption can be attributed to limitations in current HFDO approaches such as high dependency on designers and the lack of efficient yet flexible geometry modification approaches.

To overcome these limitations, this thesis made three key advancements. Firstly, an automated HFDO approach is proposed to supplement experienced designer knowledge in improving hull form designs. Secondly, an efficient geometry modification technique to create more diverse and varied hull forms was developed. Thirdly, geometric modification was combined with intelligent search technique for more efficient and automated design of hull form.

By using existing proven hull forms as starting points, fully automated HFDO was enabled, where new designs were created automatically by modifying these designs from the hull form library. This significantly reduces the designer time and increases the chances of finding optimal designs when compared to modifying from only one design, as for traditional approaches. By working directly on offset data in the 2D domain, it eliminates a key step in existing design methods which requires 2D offset information to be re-modelled into three-dimensional (3D) surfaces manually by the designer before geometry modifications and performance evaluation can take place, thus streamlining the entire design process. This frees up designer time for more important or creative tasks, which cannot be performed by a computer.

Two novel methods of 2D curve morphing were applied to hull form modifications. Cross-morph enables the combination of different ship sections through ‘cutting and

joining' of two or more ships. This allows partial features of different ships to be retained, which is not possible using conventional methods. Boundary-morph is another innovative morphing method that enables the hull geometry to be modified by morphing a particular transverse frame into another frame using only one reference vessel. This is different from conventional morphing methods which require at least two reference ships to morph in-between one another.

The morphing scheme was merged with evolutionary computation to enable a unique and novel reproduction process (crossover and mutation), where the hull geometry can be modified directly instead of changing the design variables or parameters related to the hull geometry, as with other SBD methods. This is a very useful approach as it significantly reduces the computational effort required to modify the geometry and helps to increase the shape diversity. The final output of MEC is a set of near-Pareto optimal designs which the designers can further modify and carry over to the next design stage.

Two sets of applications were presented to illustrate how MEC was used to optimise the hull form of two ships with different functions and design objectives. In the first application, single objective optimisation focusing on reducing the drag of a container ship was carried out. The results demonstrated that fully-automated HFDO can be achieved with performance improvement between 7.22% and 11.14%. In the second application, three design objectives (pitch, roll and heave) relating to seakeeping were added on top of resistance to optimise the hull of an offshore cable-laying ship. The results proved that MEC can be applied effectively to modify the hull form automatically for multiple-objective scenarios and at the same time achieve improvements of up to 8.35% for resistance, 18.17% for heave, 28.45% for pitch and 66.75% for roll objective functions.

It was noted that there is no design automation solution that can totally outsmart or replace a human designer. While more research and developmental work are needed for ship design automation, it is hoped that this research will open up new avenues and possibilities towards more automated ship and hull form designs. A promising outcome from this thesis includes future work towards more fully automated and intelligent hull form designs, where design, construction and navigation processes for vessels become more integrated.

7.2. Key Benefits

Compared to the conventional hull form design process, there are several benefits of the MEC approach developed in this thesis. These are listed;

- **Industry's First Fully Automated Hull Form Design Tool**

MEC is the first fully automated hull form design and optimisation tool in the industry. Conventional SBD tools require expert input to set the direction for automated simulation, but this can be limited by the experience of the designer. Compared to traditional morphing that interpolates in between existing hull forms, MEC enables the creation of new hull form combinations that are superior to existing designs. This tool allows the designer to start optimising or improving the design from a more optimal hull form after automated optimisation process by MEC. This will help to reduce the total design duration, produce more optimal hull form designs and free up the designer's time for more creative task.

- **Starts from Proven Existing Designs**

The MEC approach starts from a pool of proven existing hull form designs instead of using improved designs from only one 'sister' ship hull design. This enables the exploration of wider solution spaces and creation of more optimal hull forms. For any designer, one of the most difficult tasks is to find the right location and to modify the geometry in a way that can provide the highest efficiency gains. This requires a lot of experience and trial and error, considering that any 'wrong move' (modification) may cause the performance to worsen and the final solution to be only sub-optimal. Unlike conventional geometry modification and optimisation approaches that require the designer to input the directions of shape variations and degree of changes for the optimiser to explore, MEC allows automatic geometry variation based on proven designs, which enables the generation of feasible designs with minimal human intervention. Some existing methods require additional pre-check steps such as DOE to ensure that variations defined by the designer helps to reasonably improve the design with no unfeasible designs. MEC eliminates the need for this pre-qualification check since all base-line designs used from the hull form library are feasible. By performing morphing via interpolation and extrapolation between these proven designs, it enables automated geometry modification with minimal designer input.

- **A Unique Problem Mapping Scheme**

The phenotype-genotype mapping developed in this thesis provides an elegant and effective way to transform the hull form into an optimisation problem. Hull lines are simple to represent in phenotypic spaces within GA and provide more flexibility for shape modifications compared to surfaces. Through unique crossover and mutation functions, an entire hull shape can be ‘mix-and-match’ and transformed using the morphing parameter t , which enables the creation of more innovative hull forms. This approach has the flexibility of combining two or more different hull forms together, thereby increasing the solution space. It is noted that the MEC approach is not limited to only ship hull forms. It can also be extended to other geometry-related design applications such as airplanes and automobile main body design.

- **More Innovative and Diverse Hull Forms**

Three variations of 2D curve morphing are applied to enable more unique hull form modifications and create more diverse hull forms. Constant-morph is applied in MEC to create a pool of initial population through interpolation and extrapolation of proven existing designs. By using only one constant design variable (t) applied across the entire hull length, intermediate designs can be generated very quickly. The benefit of starting from proven designs is that it reduces the design space exploration while simultaneously assuring the performance. Cross-morph is applied in the crossover process within MEC, which enables the combination of different ships at various parts of the hull sections. This unique geometry modification process allows two or more ships with very different sizes to be combined seamlessly. Cross-morph is particularly useful in generating a large variety of hull form designs by taking parts of the hull characteristics of proven ship designs. Boundary-morph is applied in the mutation process within MEC for small incremental changes to the hull geometry. Compared to other existing geometry modification processes, boundary-morph enables global modification of the entire hull form without any user input and therefore, can be fully automated.

- **Direct 2D mapping**

Compared to 3D models, morphing using 2D models offers the following advantages. First, 2D models for all constructed ships are always readily available. Currently, most ship design starts from either 2D offset data or from CAD models of hull lines that are represented in 2D. Very rarely are 3D models available due to proprietary designs and other considerations. By directly working in 2D, MEC eliminates the need to translate

the 2D data into 3D models for geometry modification and hull form optimisation. Second, analysis of 2D data is more efficient and less computationally demanding for geometry modification, optimisation and performance evaluation compared to 3D models. It is recognised that 2D models may not be as accurate as 3D models, nonetheless, the level of accuracy is sufficient for preliminary design stage, where process efficiency is more important.

- **Fast and Efficient Shape Modification and Performance Evaluation**

The main issue with using conventional methods is that manual modification of hull form design using conventional CAD tools is still a ‘trial-and-error’ process. This process is highly time consuming, inefficient and may not guarantee an improved hull form. For an experienced designer, the entire process from designing to evaluation of one hull form takes at least 2-3 days. MEC can address the limitations of conventional methods since the entire geometry modification and evaluation process can now be fully automated. MEC not only enables fast generation of intermediate hull forms as well as new combined designs of different vessels but also intelligently optimises the search for optimal solutions. This not only improves its efficiency but also increases the certainty of an optimal hull form design being generated.

7.3. Recommendations for Future Works

It is recognised that there are some limitations as well as areas for improvement to the work presented in this thesis. These are discussed below.

- **Existing Designs in the Hull Form Library**

The results of optimal designs produced from MEC depends on the number and quality of existing hull forms available in the Hull Form Library. The higher the quantity and quality of existing designs, the better the results produced. In some cases, there may be insufficient existing designs for special ship types, for example, cruise or navy ships, potentially limiting the results obtained. Hence, the design solution space here is closely linked to the size of the hull form library as against the experience or input of the designer. One solution is to create more parent designs by either manually adding unique hull forms designed from scratch by a designer or incorporating non-conventional ship-shaped ships such as multi-hull, submarine or even air-craft designs. This will help promote more diversity and design innovations, which would potentially complement those developed by a human designer.

- **Limited Control by the Designer and Unpredictability of Results**

As MEC is a fully automated process, there is limited control by a designer to influence or direct how and what changes are due to the settings of the algorithm. Control of the geometry and optimisation process can only be done in the initial settings prior to the start of optimisation and in the formulation of design objectives. This limits the creativity of the designer, which is especially crucial for novel designs or large and complex systems such as offshore structures [212]. Furthermore, due to the black box nature of EA, it is often challenging to interpret the results obtained with respect to why a certain outcome was reached. One solution is the use of the grey box models, which helps in providing more insights into the optimisation process.

- **Regularisation of Hull Data using Curve Fitting Functions**

One limitation was in the use of cubic spline interpolation for regularising two different offset data, where some accuracy was lost during data discretisation. There are many other curve fitting techniques such as B-spline surface fairing and feature-preserving smoothing functions, which can be considered and applied during the regularising process to improve shape accuracy [177]. These techniques are useful for complex shapes such as hull forms as they accurately capture shape changes at localised areas, when compared to applying cubic spline for the entire hull form. Some examples of curve fitting function with feature-preserving smoothing features include the Hermite interpolation, the Newton-Raphson method and conic splines [213]. Further investigation in this aspect can be conducted.

- **Geometric Modification using 2D Morphing**

Currently, 2D morphing is only performed along the X -plane, which is the transverse cross-section (body line) of the hull form. To generate more diverse hull shapes, morphing can also be performed along the Z -plane (waterline) or Y -plane (buttock line) or a combination of X , Y and Z -planes. This will generate more shape variations and increase the solution space, potentially leading to the discovery of more optimal hull designs. In some instances, some odd lines (curves) will appear during the 2D morphing process, resulting in ‘double hull’ issues. This issue occurs mainly at the extreme bow and stern areas where there is a large geometric disparity between the parent hull forms. To avoid this issue in MEC, the extreme bow and stern are not subjected to the morphing procedure during constant, cross and boundary-morph processes. Further investigations can be carried out to resolve this issue.

- **Performance Evaluation Accuracy**

The accuracy of performance evaluation using empirical methods such as the Holtrop method and strip theory is limited to standard type hull forms and may not be accurate for non-standard ship type vessels. While high resolution CFD methods remain too computationally expensive for evaluating large numbers of candidate solutions, empirical methods could be replaced with other numerical models or low fidelity CFD methods such as potential flow for resistance calculations. To incorporate these methods into the HFDO process, future research could investigate overcoming the conversion of 2D offset points to 3D surfaces before meshing. This is usually a show-stopper in the adoption of CFD in HFDO.

- **Rescaling Effects**

In MEC, the principal dimensions of existing (parent) ships are adjusted due to rescaling to enable all the ships to be joined together. The implication of this rescaling is that the modified hull form may not fully ‘inherit’ or carry over the actual hull form characteristics of the existing (parent) ship based on its original dimensions. In the case of cross-morph, it is still possible to skip the rescaling process and perform cross-morph using the original scales, as demonstrated in Section 3.5.1. However, in the case of MEC, there is a high possibility of generating infeasible designs when cross-morph is applied multiple times without first rescaling all the parents, especially if the principal dimensions are significantly different. Further study is needed to examine the effects of no rescaling in crossover and to develop solutions to reduce the number of infeasible designs generated.

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APPENDIX I - LEWIS CONFORMAL MAPPING

• Basic Formulation and Assumptions

The general transformation formula of Lewis Conformal Mapping is given in Equation A1.1.

$$z = M_s \times \sum \{a_{2n-1} \times \zeta^{-(2n-1)}\} \quad (\text{A1.1})$$

with,

$z = x + iy$ plane of ship's cross-section

$\zeta = ie^a \times e^{-i\theta}$ plane of unit circle

M_s scale factor

a_{-1} = +1

a_{2n-1} conformal mapping coefficients ($n = 1, \dots, N$)

N Maximum parameter index number

θ Corresponding angle in mapped unit circle plane

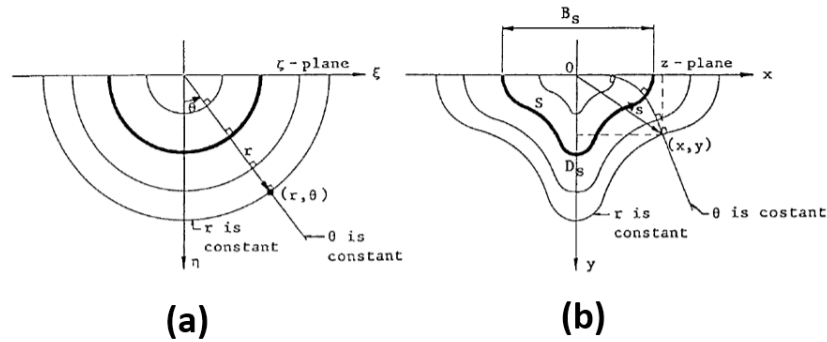


Figure A1- 1 Mapping relations between the (a) circle ζ -plane and (b) arbitrary z -plane [204].

The relation between the coordinates in the z -plane and variables in the ζ -plane are shown in Equation A1.2 to A1.3:

$$x = -M_s \times \sum_{n=0}^N \{(-1)^n \times a_{2n-1} \times e^{-(2n-1)a} \times \sin((2n-1) \times \theta)\} \quad (\text{A1.2})$$

$$y = +M_s \times \sum_{n=0}^N \{(-1)^n \times a_{2n-1} \times e^{-(2n-1)a} \times \cos((2n-1) \times \theta)\} \quad (\text{A1.3})$$

By putting $a = 0$, the contour approximated by conformal mapping of the ship's cross-section becomes:

$$x_0 = -M_s \times \sum_{n=0}^N \{(-1)^n \times a_{2n-1} \times \sin((2n-1) \times \theta)\} \quad (\text{A1.4})$$

$$y_0 = +M_s \times \sum_{n=0}^N \{(-1)^n \times a_{2n-1} \times \cos((2n-1) \times \theta)\} \quad (\text{A1.5})$$

The scale factor M_s is defined as:

$$M_s = \frac{B_s/2}{\sum_{n=0}^N \{a_{2n-1}\}} = \frac{D_s}{\sum_{n=0}^N \{(-1)^n \times a_{2n-1}\}} \quad (\text{A1.6})$$

where,

B_s sectional breadth on waterline

D_s sectional draught

In Lewis conformal mapping [205], the half breadth to draught ratio H_0 and the sectional area coefficient σ_s are defined as:

$$H_0 = \frac{B_s/2}{D_s} = \frac{1+a_1+a_3}{1-a_1+a_3} \quad (\text{A1.7})$$

$$\sigma_s = \frac{A_s}{B_s \times D_s} = \frac{\pi}{4} \times \frac{1-a_1^2-3 \times a_3^2}{(1+a_3)^2-a_1^2} \quad (\text{A1.8})$$

Where:

A_s area of cross-section

By using Equations A1.7 and A1.8, the following quadratic equation is derived:

$$c_1 \times a_3^2 + c_2 \times a_3 + c_3 = 0 \quad (\text{A1.9})$$

Where:

$$c_1 = 3 + \frac{4 \times \sigma_s}{\pi} + \left(1 - \frac{4 \times \sigma_s}{\pi}\right) \times \left(\frac{H_0 - 1}{H_0 + 1}\right)^2$$

$$c_2 = 2 \times c_1 - 6$$

$$c_3 = c_1 - 4$$

- **Boundaries of Lewis forms**

Some frequently used Lewis forms are shown in Figure A1-2.

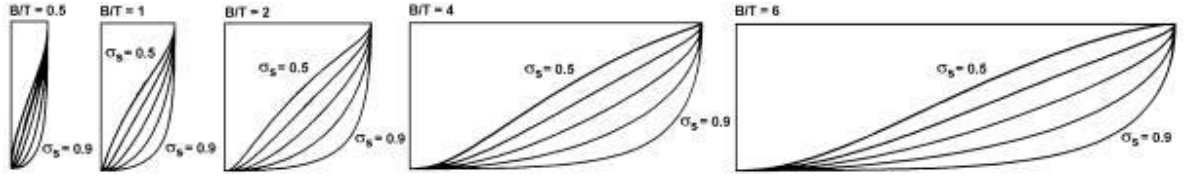


Figure A1- 2 Typical Lewis forms [204].

Lewis method, in sometimes, can give unreasonable results depending on the form of cross-sections. The following typical Lewis hull forms with the regions of the half breadth to draught ratio H_0 and the area coefficient σ_s to match can be classified.

- **re-entrant forms, bounded by:**

$$\text{for } H_0 \leq 1.0: \sigma_s < \frac{3\pi}{32} \cdot (2 - H_0) \quad (\text{A1.10})$$

$$\text{for } H_0 \geq 1.0: \sigma_s < \frac{3\pi}{32} \cdot \left(2 - \frac{1}{H_0} \right)$$

- **conventional forms, bounded by:**

$$\text{for } H_0 \leq 1.0: \frac{3\pi}{32} \cdot (2 - H_0) < \sigma_s < \frac{3\pi}{32} \cdot \left(3 + \frac{H_0}{4} \right) \quad (\text{A1.11})$$

$$\text{for } H_0 \geq 1.0: \frac{3\pi}{32} \cdot \left(2 - \frac{1}{H_0} \right) < \sigma_s < \frac{3\pi}{32} \cdot \left(3 + \frac{1}{4H_0} \right)$$

- **bulbous and not-tunneled forms, bounded by:**

$$H_0 \leq 1.0 \quad \text{and} \quad \frac{3\pi}{32} \cdot \left(3 + \frac{H_0}{4} \right) < \sigma_s < \frac{3\pi}{32} \cdot \left(3 + \frac{1}{4H_0} \right) \quad (\text{A1.12})$$

- **tunnelled and not-bulbous forms, bounded by:**

$$H_0 \geq 1.0 \quad \text{and} \quad \frac{3\pi}{32} \cdot \left(3 + \frac{1}{4H_0} \right) < \sigma_s < \frac{3\pi}{32} \cdot \left(3 + \frac{H_0}{4} \right) \quad (\text{A1.13})$$

- **combined bulbous and tunnelled forms, bounded by:**

$$\text{for } H_0 \leq 1.0: \frac{3\pi}{32} \cdot \left(3 + \frac{1}{4H_0} \right) < \sigma_s < \frac{\pi}{32} \cdot \left(10 + H_0 + \frac{1}{H_0} \right) \quad (\text{A1.14})$$

$$\text{for } H_0 \geq 1.0: \frac{3\pi}{32} \cdot \left(3 + \frac{H_0}{4}\right) < \sigma_s < \frac{\pi}{32} \cdot \left(10 + H_0 + \frac{1}{H_0}\right)$$

- **non symmetric forms, bounded by:**

$$0 < H_0 < \infty \quad \text{and} \quad \sigma_s > \frac{\pi}{32} \cdot \left(10 + H_0 + \frac{1}{H_0}\right) \quad (\text{A1.15})$$

Figure A1-3. illustrates these given ranges of H_0 and σ_s for the different typical Lewis forms.

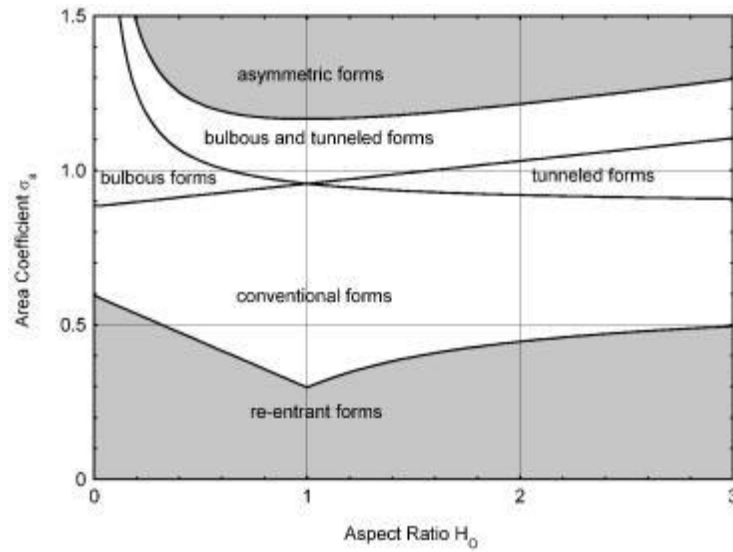


Figure A1- 3 Ranges of H_0 and σ_s of Lewis forms [204].

APPENDIX II - TWO-DIMENSIONAL POTENTIAL COEFFICIENTS

Using the Ursell and Tasai method, the two-dimensional added mass (a_i) and damping coefficients (b_i) for sway, heave, roll and coupling coefficient of sway into roll are described using the following equations:

$$a_{22} = \frac{\rho b_0^2}{2} \times \frac{M_0 Q_0 + N_0 P_0}{P_0^2 + Q_0^2} \quad (\text{A2.1})$$

$$a_{33} = \frac{\rho b_0^2}{2} \times \frac{M_0 B_0 + N_0 A_0}{A_0^2 + B_0^2} \quad (\text{A2.2})$$

$$a_{44} = \frac{\rho b_0^2}{8} \times \frac{Y_R Q_0 + X_R P_0}{P_0^2 + Q_0^2} \quad (\text{A2.3})$$

$$a_{42} = \frac{-\rho b_0^3}{2} \times \frac{Y_R Q_0 + X_R P_0}{P_0^2 + Q_0^2} \quad (\text{A2.4})$$

$$b_{22} = \frac{\rho b_0^2}{2} \times \frac{M_0 Q_0 - N_0 \theta_0}{P_0^2 + Q_0^2} \times \omega \quad (\text{A2.5})$$

$$b_{33} = \frac{\rho b_0^2}{2} \times \frac{M_0 A_0 - N_0 B_0}{A_0^2 + B_0^2} \times \omega \quad (\text{A2.6})$$

$$b_{44} = \frac{\rho b_0^2}{8} \times \frac{Y_R P_0 + X_R Q_0}{P_0^2 + Q_0^2} \times \omega \quad (\text{A2.7})$$

$$b_{42} = \frac{-\rho b_0^2}{2} \times \frac{Y_R P_0 - X_R Q_0}{P_0^2 + Q_0^2} \times \omega \quad (\text{A2.8})$$

where b_0 is the breadth of the cross-section.

• Heave motion and its coefficients

In heaving motion, the coefficients A_0 , B_0 , M_0 and N_0 are required to compute sectional potential coefficients a_{33} and b_{33} , which is described as below:

$$A_0 = \psi_c \left(\frac{\pi}{2} \right) + \sum_{m=1}^M p_m \psi_m \left(\frac{\pi}{2} \right) \quad (\text{A2.9})$$

$$B_0 = \psi_s \left(\frac{\pi}{2} \right) + \sum_{m=1}^M q_m \psi_m \left(\frac{\pi}{2} \right) \quad (\text{A2.10})$$

$$\begin{aligned} M_0 = & - \int_0^{\frac{\pi}{2}} \frac{\varphi_s}{1 + \sum_{n=1}^N a_{2n-1}} \sum_{n=0}^N (-1)^n (2n-1) a_{2n-1} \cos(2n-1) \theta - \\ & \sum_{m=1}^M \frac{(-1)^m q_m}{1 + \sum_{n=1}^N a_{2n-1}} \sum_{n=0}^N \frac{(2n-1)^2}{4m^2 - (2n-1)^2} a_{2n-1} + \frac{4\pi}{(1 + \sum_{n=1}^N a_{2n-1})^2} q_1 + \\ & \sum_{m=1}^M (-1)^m q_m \sum_{n=0}^{N-m} (2n-1) a_{2n-1} a_{2m+2n-1} \end{aligned} \quad (\text{A2.11})$$

$$\begin{aligned}
N_0 = & - \int_0^{\frac{\pi}{2}} \frac{\varphi_c}{1 + \sum_{n=1}^N a_{2n-1}} \sum_{n=0}^N (-1)^n (2n-1) a_{2n-1} \cos(2n-1)\theta - \\
& \sum_{m=1}^M \frac{(-1)^m p_m}{1 + \sum_{n=1}^N a_{2n-1}} \sum_{n=0}^N \frac{(2n-1)^2}{4m^2 - (2n-1)^2} a_{2n-1} + \frac{\xi_b \pi}{4(1 + \sum_{n=1}^N a_{2n-1})^2} p_1 + \\
& \sum_{m=1}^M (-1)^m p_m \sum_{n=0}^{N-m} (2n-1) a_{2n-1} a_{2m+2n-1}
\end{aligned} \tag{A2.12}$$

where,

$$\sum_{m=1}^M p_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_c(\theta) - h(\theta) \psi_c(\frac{\pi}{2})) f_{2n}(\theta) d\theta \tag{A2.13}$$

$$\sum_{m=1}^M q_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_c(\theta) - h(\theta) \psi_s(\frac{\pi}{2})) f_{2n}(\theta) d\theta \tag{A2.14}$$

in which,

$$f_{2m}(\theta) = -\psi_m(\theta) + h(\theta) \psi_m(\frac{\pi}{2}) \tag{A2.15}$$

$$h(\theta) = \frac{1}{1 + \sum_{n=1}^N a_{2n-1}} \sum_{n=0}^N (-1)^n a_{2n-1} \sin(2n-1)\theta \tag{A2.16}$$

and,

$$\begin{aligned}
\phi_m = & \cos(2m\theta) + \frac{\xi_0}{1 + \sum_{n=1}^N a_{2n-1}} \left(\frac{\cos(2m-1)\theta}{2m-1} + \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m+2n-1)} a_{2n-1} \cos(2m + \right. \\
& \left. 2n-1)\theta \right)
\end{aligned} \tag{A2.17}$$

$$\begin{aligned}
\psi_m = & \sin(2m\theta) + \frac{\xi_0}{1 + \sum_{n=1}^N a_{2n-1}} \left(\frac{\sin(2m-1)\theta}{2m-1} + \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m+2n-1)} a_{2n-1} \sin(2m + \right. \\
& \left. 2n-1)\theta \right)
\end{aligned} \tag{A2.18}$$

$$\varphi_c = \pi e^{-kz} \cos \frac{\omega^2}{g} y \tag{A2.19}$$

$$\psi_c = \pi e^{-kz} \sin \frac{\omega^2}{g} |y| \tag{A2.20}$$

$$\varphi_s = \pi e^{-kz} \sin \frac{\omega^2}{g} |y| + (Q \cos \frac{\omega^2}{g} y + (S - \pi) \sin \frac{\omega^2}{g} y) e^{-kz} \tag{A2.21}$$

$$\psi_s = -\pi e^{-kz} \cos \frac{\omega^2}{g} |y| + (Q \sin \frac{\omega^2}{g} y - (S - \pi) \cos \frac{\omega^2}{g} y) e^{-kz} \tag{A2.22}$$

Then, the porter series is:

$$Q = Y + \ln \left(\frac{\omega^2}{g} \sqrt{y^2 + z^2} \right) + \sum_{n=1}^{\infty} p_n \cos(n\beta) \tag{A2.23}$$

$$S = \beta + \sum_{n=1}^{\infty} p_n \sin(n\beta) \tag{A2.24}$$

where,

$$\beta = \arctan\left(\frac{y}{z}\right) \quad (\text{A2.25})$$

$$p_n = \frac{\left(\frac{\omega^2}{g}\sqrt{y^2+z^2}\right)^n}{n!} \quad (\text{A2.26})$$

$$Y = 0.57722$$

• Roll motion and its coefficients

In roll motion, the coefficients P_0 , Q_0 , Y_R and X_R are required to compute sectional potential coefficients a_{44} and b_{44} , which is described as below:

$$P_0 = \frac{\pi\chi_a}{2\eta_a} \xi_b \sin(\gamma) \quad (\text{A2.27})$$

$$Q_0 = \frac{\pi\chi_a}{2\eta_a} \xi_b \cos(\gamma) \quad (\text{A2.28})$$

$$\begin{aligned} Y_R = & \frac{1}{2(1+\sum_{n=1}^N a_{2n-1})^2} \times \int_0^{\frac{\pi}{2}} \varphi_s \sum_{n=0}^N \sum_{i=0}^N (-1)^{n+i} (2i-1) a_{2n-1} a_{2i-1} \sin((2n- \\ & 1)\theta) d\theta + \frac{1}{2(1+\sum_{n=1}^N a_{2n-1})^2} \sum_{m=1}^M (-1)^m q_m \sum_{n=0}^N \sum_{i=0}^N \frac{(2i-1)(2n-2i)}{(2m+1)^2-(2n-2i)^2} a_{2n-1} a_{2i-1} - \\ & \frac{\pi\xi_0}{8(1+\sum_{n=1}^N a_{2n-1})^3} \sum_{m=1}^M (-1)^m q_m \times \left(\sum_{n=m}^N \sum_{i=0}^{n-m} \frac{(-2m+2n-2i-1)(2i-1)}{(2n-2i)} \times \right. \\ & \left. a_{2n-1} a_{2i-1} a_{-2m+2n-2i-1} + \sum_{n=0}^{N-m} \sum_{i=m+n}^N \frac{(-2m-2n+2i-1)(2i-1)}{(2n-2i)} \times \right. \\ & \left. a_{2n-1} a_{2i-1} a_{-2m-2n+2i-1} \right) \end{aligned} \quad (\text{A2.29})$$

$$\begin{aligned} X_R = & \frac{1}{2(1+\sum_{n=1}^N a_{2n-1})^2} \times \int_0^{\frac{\pi}{2}} \varphi_c \sum_{n=0}^N \sum_{i=0}^N (-1)^{n+i} (2i-1) a_{2n-1} a_{2i-1} \sin((2n- \\ & 1)\theta) d\theta + \frac{1}{2(1+\sum_{n=1}^N a_{2n-1})^2} \sum_{m=1}^M (-1)^m p_m \sum_{n=0}^N \sum_{i=0}^N \frac{(2i-1)(2n-2i)}{(2m+1)^2-(2n-2i)^2} a_{2n-1} a_{2i-1} - \\ & \frac{\pi\xi_0}{8(1+\sum_{n=1}^N a_{2n-1})^3} \sum_{m=1}^M (-1)^m p_m \times \left(\sum_{n=m}^N \sum_{i=0}^{n-m} \frac{(-2m+2n-2i-1)(2i-1)}{(2n-2i)} \times \right. \\ & \left. a_{2n-1} a_{2i-1} a_{-2m+2n-2i-1} + \sum_{n=0}^{N-m} \sum_{i=m+n}^N \frac{(-2m-2n+2i-1)(2i-1)}{(2n-2i)} \times \right. \\ & \left. a_{2n-1} a_{2i-1} a_{-2m-2n+2i-1} \right) \end{aligned} \quad (\text{A2.30})$$

where,

$$\sum_{m=1}^M p_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_c(\theta) - \psi_c(\frac{\pi}{2})) f_{2n}(\theta) d\theta \quad (\text{A2.31})$$

$$\sum_{m=1}^M q_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_s(\theta) - \psi_s(\frac{\pi}{2})) f_{2n}(\theta) d\theta \quad (\text{A2.32})$$

in which,

$$f_{2m}(\theta) = -\psi_m(\theta) + \psi_m\left(\frac{\pi}{2}\right) \quad (\text{A2.33})$$

$$-\chi \frac{\pi \omega b_0}{4g\eta_a} (\mu(\theta) - 1)(P_0 \cos(\omega t) + Q_0 \sin(\omega t)) \quad (\text{A2.34})$$

and,

$$\begin{aligned} \phi_m &= \sin((2m+1)\theta) - \frac{\xi_0}{1+\sum_{n=1}^N a_{2n-1}} \left(\frac{\sin(2m-1)\theta}{2m} + \right. \\ &\left. \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m-2n)} a_{2n-1} \sin(2m+2n)\theta \right) \end{aligned} \quad (\text{A2.35})$$

$$\begin{aligned} \psi_m &= -\cos((2m+1)\theta) + \frac{\xi_0}{1+\sum_{n=1}^N a_{2n-1}} \left(\frac{\cos(2m)\theta}{2m} + \right. \\ &\left. \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m-2n)} a_{2n-1} \cos(2m+2n)\theta \right) \end{aligned} \quad (\text{A2.36})$$

$$\varphi_c = \mp \pi e^{-kz} \sin \frac{\omega^2}{g} |y| \quad (\text{A2.37})$$

$$\psi_c = \pi e^{-kz} \cos \frac{\omega^2}{g} y \quad (\text{A2.38})$$

$$\varphi_s = \pm \pi e^{-kz} \cos \frac{\omega^2}{g} y \mp (Q \sin \frac{\omega^2}{g} y + (S - \pi) \cos \frac{\omega^2}{g} y) e^{-kz} \pm \frac{|y|}{\frac{\omega^2}{g}(z^2+y^2)} \quad (\text{A2.39})$$

$$\psi_s = \pi e^{-kz} \cos \frac{\omega^2}{g} |y| + \left(Q \cos \frac{\omega^2}{g} y + (S - \pi) \sin \frac{\omega^2}{g} y \right) e^{-kz} - \frac{z}{\frac{\omega^2}{g}(z^2+y^2)} \quad (\text{A2.40})$$

- **Sway motion and its coefficients**

In sway motion, the coefficients M_0 , N_0 , P_0 and Q_0 are required to compute sectional potential coefficients a_{22} and b_{22} , which is described as below:

Coefficient A_0 and B_0 can be described as:

$$-\chi \frac{\pi \omega b_0}{4g\eta_a} (\mu(\theta) - 1)(P_0 \cos(\omega t) + Q_0 \sin(\omega t)) \quad (\text{A2.41})$$

Thus,

$$P_0 = \frac{-\chi a \pi}{\eta_a} \xi_b \sin(\varepsilon) \quad (\text{A2.42})$$

$$Q_0 = \frac{-\chi a \pi}{\eta_a} \xi_b \cos(\varepsilon) \quad (\text{A2.43})$$

$$\begin{aligned}
M_0 &= -\frac{1}{\sigma_a} \int_0^{\frac{\pi}{2}} \phi B O_s(\theta) \sum_{n=0}^N \{(-1)^n (2n-1) a_{2n-1} \sin(2n-1)\theta\} d\theta + \\
&\frac{\pi}{4\sigma_a} \sum_{m=1}^{N-1} \{(-1)^m Q_{2m}(2m+1) a_{2m+1}\} + \\
&\frac{\xi_b}{\sigma_a^2} \sum_{m=1}^M (-1)^m Q_{2m} \sum_{m=1}^M \sum_{n=0}^{N-m} \frac{(2n-1)(2i-1)}{(2m+2i)^2 - (2n-1)^2} a_{2n-1} a_{2i-1}
\end{aligned} \tag{A2.44}$$

$$\begin{aligned}
N_0 &= -\frac{1}{\sigma_a} \int_0^{\frac{\pi}{2}} \phi B O_c(\theta) \sum_{n=0}^N \{(-1)^n (2n-1) a_{2n-1} \sin(2n-1)\theta\} d\theta + \\
&\frac{\pi}{4\sigma_a} \sum_{m=1}^{N-1} \{(-1)^m P_{2m}(2m+1) a_{2m+1}\} + \\
&\frac{\xi_b}{\sigma_a^2} \sum_{m=1}^M (-1)^m P_{2m} \sum_{m=1}^M \sum_{n=0}^{N-m} \frac{(2n-1)(2i-1)}{(2m+2i)^2 - (2n-1)^2} a_{2n-1} a_{2i-1}
\end{aligned} \tag{A2.45}$$

where,

$$\sum_{m=1}^M p_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_c(\theta) - \psi_c(\frac{\pi}{2})) f_{2n}(\theta) d\theta \tag{A2.46}$$

$$\sum_{m=1}^M q_m \int_0^{\frac{\pi}{2}} f_{2m}(\theta) f_{2n}(\theta) d\theta = \int_0^{\frac{\pi}{2}} (\psi_s(\theta) - \psi_s(\frac{\pi}{2})) f_{2n}(\theta) d\theta \tag{A2.47}$$

in which,

$$f_{2m}(\theta) = -\psi_m(\theta) + \psi_m(\frac{\pi}{2}) \tag{A2.48}$$

and,

$$\begin{aligned}
\phi_m &= e^{-(2m+1)\alpha} \sin((2m+1)\theta) - \frac{\xi_0}{\sigma_a} \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m+2n)} a_{2n-1} \times \\
&e^{-(2m+2n)\alpha} \sin(2m+2n)\theta
\end{aligned} \tag{A2.49}$$

$$\begin{aligned}
\psi_m &= -e^{-(2m+1)\alpha} \cos((2m+1)\theta) + \frac{\xi_0}{\sigma_a} \sum_{n=1}^N (-1)^n \frac{(2n-1)}{(2m+2n)} a_{2n-1} \times \\
&e^{-(2m+2n)\alpha} \cos(2m+2n)\theta
\end{aligned} \tag{A2.50}$$

$$\varphi_{c \times j} = -\pi e^{-vz} \sin(vx) \tag{A2.51}$$

$$\psi_c = \pi e^{-vz} \cos(vx) \tag{A2.52}$$

$$\varphi_s \times j = \pi e^{-vz} \cos(v|y|) - \int_0^\alpha \frac{v \cdot \cos(kz) + k \cdot \sin(kz)}{k^2 + v^2} e^{-ky} dk - \frac{|y|}{v(z^2 + y^2)} \tag{A2.53}$$

$$\psi_s = -\pi e^{-vz} \sin(v|y|) - \int_0^\alpha \frac{v \cdot \sin(kz) + k \cdot \cos(kz)}{k^2 + v^2} e^{-ky} dk - \frac{|y|}{v(z^2 + y^2)} \tag{A2.54}$$

Then, the porter series is:

$$Q = Y + \ln(v\sqrt{y^2 + z^2}) + \sum_{n=1}^\infty p_n \cos(n\beta) \tag{A2.55}$$

$$S = \beta + \sum_{n=1}^\infty p_n \sin(n\beta) \tag{A2.56}$$

where,

$$\beta = \arctan\left(\frac{y}{z}\right) \quad (\text{A2.57})$$

$$p_n = \frac{\left(v\sqrt{y^2+z^2}\right)^n}{n \, n!} \quad (\text{A2.58})$$

$$Y = 0.57722$$

- **Coupling of roll into sway motion and its coefficients**

In coupling of roll into sway motion, the coefficients M_0 , N_0 , P_0 and Q_0 are required to compute sectional potential coefficients a_{24} and b_{24} , which are already computed above.

APPENDIX III - THREE-DIMENSIONAL ADDED MASS AND DAMPING COEFFICIENT

According to [201], the 3-dimensional added mass ' A_i ' and damping coefficient ' B_i ' can be calculated as follow:

$$A_{22} = \int_0^L a_{22} \cdot dx - \frac{U}{\omega_e^2} b_{22}^A \quad (A3.1)$$

$$A_{24} = \int_0^L a_{24} \cdot dx - \frac{U}{\omega_e^2} b_{24}^A = A_{42} \quad (A3.2)$$

$$A_{26} = \int_0^L x \cdot a_{22} \cdot dx + \frac{U}{\omega_e^2} \int_0^L b_{22} \cdot dx - \frac{U}{\omega_e^2} x_a b_{22}^A + \frac{U}{\omega_e^2} a_{22}^A = A_{62} \quad (A3.3)$$

$$A_{33} = \int_0^L a_{33} \cdot dx - \frac{U}{\omega_e^2} b_{33}^A \quad (A3.4)$$

$$A_{35} = \int_0^L x \cdot a_{33} \cdot dx + \frac{U}{\omega_e^2} \int_0^L b_{33} \cdot dx + \frac{U}{\omega_e^2} x_a b_{33}^A - \frac{U}{\omega_e^2} a_{33}^A \quad (A3.5)$$

$$A_{44} = \int_0^L a_{44} \cdot dx - \frac{U}{\omega_e^2} b_{44}^A \quad (A3.6)$$

$$A_{46} = \int_0^L x \cdot a_{24} \cdot dx + \frac{U}{\omega_e^2} \int_0^L b_{24} \cdot dx - \frac{U}{\omega_e^2} x_a b_{24}^A + \frac{U^2}{\omega_e^2} a_{24}^A \quad (A3.7)$$

$$A_{53} = \int_0^L x \cdot a_{33} \cdot dx + \frac{U}{\omega_e^2} \int_0^L b_{33} \cdot dx + \frac{U}{\omega_e^2} x_a b_{33}^A \quad (A3.8)$$

$$A_{55} = \int_0^L x \cdot a_{33} \cdot dx - \frac{U}{\omega_e^2} \int_0^L b_{33} \cdot dx - \frac{U}{\omega_e^2} x_a b_{33}^A + \frac{U^2}{\omega_e^2} a_{33}^A \quad (A3.9)$$

$$A_{64} = \int_0^L x \cdot a_{24} \cdot dx - \frac{U}{\omega_e^2} \int_0^L b_{24} \cdot dx - \frac{U}{\omega_e^2} x_a b_{24}^A \quad (A3.10)$$

$$A_{66} = \int_0^L x \cdot a_{22} \cdot dx - \frac{U}{\omega_e^2} \int_0^L a_{22} \cdot dx - \frac{U}{\omega_e^2} x_a b_{22}^A + \frac{U^2}{\omega_e^2} x_a a_{22}^A \quad (A3.11)$$

Where, super subscript 'A' and subscript 'a' refer to mid-ship cross-section.

$$B_{22} = \int_0^L b_{22} \cdot dx - U a_{22}^A \quad (A3.12)$$

$$B_{24} = \int_0^L b_{24} \cdot dx - U a_{24}^A = B_{42} \quad (A3.13)$$

$$B_{26} = \int_0^L x \cdot b_{22} \cdot dx + U \int_0^L a_{22} \cdot dx + U x_a a_{22}^A + \frac{U^2}{\omega_e^2} b_{22}^A \quad (A3.14)$$

$$B_{33} = \int_0^L b_{33} \cdot dx - U a_{33}^A \quad (\text{A3.15})$$

$$B_{35} = - \int_0^L x \cdot b_{33} \cdot dx + U \int_0^L a_{33} \cdot dx - U x_a a_{33}^A - \frac{U^2}{\omega_e^2} b_{33}^A \quad (\text{A3.16})$$

$$B_{44} = \int_0^L b_{44} \cdot dx + U a_{44}^A \quad (\text{A3.17})$$

$$B_{46} = \int_0^L x \cdot b_{24} \cdot dx - U \int_0^L a_{24} \cdot dx + U x_a a_{24}^A + \frac{U^2}{\omega_e^2} b_{24}^A \quad (\text{A3.18})$$

$$B_{53} = - \int_0^L x \cdot b_{33} \cdot dx - U \int_0^L a_{33} \cdot dx + U x_a a_{33}^A \quad (\text{A3.19})$$

$$B_{55} = \int_0^L x^2 \cdot b_{33} \cdot dx - \frac{U^2}{\omega_e^2} \int_0^L b_{33} \cdot dx + \frac{U}{\omega_e^2} x_a a_{33}^A + \frac{U^2}{\omega_e^2} x_a b_{33}^A \quad (\text{A3.20})$$

$$B_{64} = \int_0^L x \cdot b_{24} \cdot dx - U \int_0^L a_{24} \cdot dx + U x_a a_{24}^A \quad (\text{A3.21})$$

$$B_{66} = \int_0^L x^2 \cdot b_{22} \cdot dx + \frac{U^2}{\omega_e^2} \int_0^L b_{22} \cdot dx - U x_a a_{22}^A + \frac{U^2}{\omega_e^2} x_a b_{22}^A \quad (\text{A3.22})$$