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APPROXIMATE ANALYSIS OF LOAD  
DISTRIBUTION IN THREE-DIMENSIONAL  
TALL BUILDING STRUCTURES

by

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A thesis presented for the degree of  
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## SYNOPSIS

An approximate method is presented for the analysis of the distribution of lateral forces among the components of a three-dimensional tall building structure that consists of assemblies of shear walls, coupled walls, rigidly-jointed frames and cores, subjected to both bending and torsion.

The load distribution on each element is assumed to be represented sufficiently accurately by a concentrated interactive force at the top together with a polynomial in the height coordinate.

A set of flexibility influence coefficients, relating the deflection at any level to any particular load component, is established for each element, the continuum approach being used to analyse individual cores and coupled shear walls, and the shear cantilever analogy for the frame elements.

By making use of the equilibrium and compatibility equations at any desired set of reference levels, the load distribution on each assembly may be determined.

The accuracy of the technique, and the number of reference levels, were examined by comparing the results with those obtained from an 'exact' solution and those from two example structures which were analysed previously by various investigators. Finally, a parameter study has been carried out also to study the effect on the number of reference levels used.

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## NOTATIONS

The following symbols are used in this thesis.

$A_1, A_2$	cross-sectional areas of walls '1' and '2'
A	$A_1 + A_2$
b	Clear distance between walls in line
$b_1, b_2$	bay widths
C	Torsional stiffness of wall element
E	Young's modulus of elasticity
e	clear distance between spandrel beams
f	flexibility coefficient for deflection
f'	flexibility coefficient for rotation
G	Shear modulus
H	Total height of building
h	Storey height
i	any integer
$I_1, I_2$	second moment of areas of wall '1' and '2'
$I_c$	second moment of area of connecting beam
$I_h$	second moment of area of column
j	suffix denoting the $j^{\text{th}}$ element
l	distance between centroids of wall '1' and wall '2'
$l_1, l_2$	clear distances between columns
M	applied moment at any level
$M_1, M_2$	bending moments in wall '1' and '2'
M(1)	Base moment
m	largest number of terms in the series

$n$	particular term in the series
$n(\epsilon)$	axial force in the connecting medium
$P_0$	concentrated load at the top
$p_n$	polynomial load coefficients
$Q(1)$	base shear force
$q$	intensity of shear force in connecting beam
$s$	integral coefficient
$T$	axial force in wall
$T_0$	concentrated torque at the top
$t_n$	polynomial torque coefficient
$t_1$	width of column
$t_2$	depth of spandrel beam
$x$	height coordinate from the top of the building
$y, z$	horizontal coordinates
$l_j, r_j$	distance of the $j^{\text{th}}$ element from the datum
$\alpha, \beta, \gamma$	structural parameters
$\eta, \eta'$	supplementary variable of non-dimensional height coordinate
$\theta$	rotation of floor slab in horizontal plane
$\epsilon$	non-dimensional height coordinate
$[F]$	Square matrix of flexibility coefficients for the $j^{\text{th}}$ element
$[F']$	Square matrix of rotation flexibility coefficients for the $j^{\text{th}}$ element
$[M_T]$	matrix of applied twisting moments
$[S]$	matrix of coefficients $s$

Other subsidiary symbols are defined locally where they occur in the text

## INTRODUCTION

1.1 General

The existence of tall buildings usually reflects the socio-ideological and political trends of the society in which they occur. The reasons to build tall range from the natural urge to build up, to be at the centre of things, high land costs, and dense population up to a show of wealth and power. The economy, aesthetic effect, efficiency and above all, the prestige associated with tall buildings have in recent years, increased their rate of construction in many regions of the world.

While there is no general agreement as to what constitutes a tall building, from the structural engineer's point of view, a tall building is one in which lateral forces due to wind or earthquake play an important or dominant role in the structural design.

In general, low-rise buildings are designed to resist gravitational loads, and thereafter the influence of wind forces is checked using Building Design Codes, which generally allow some overstress due to the transient nature of the wind. However, the structure of high rise buildings must be designed from the beginning to resist vertical as well as horizontal forces, and an optimum system sought to minimise the influence of the former.

Therefore, it is essential to ensure adequate lateral stiffness to resist horizontal loads. The structural units which may be adopted for providing this stiffness to buildings are classified as frames, shear walls, cores or their combinations, in conjunction with the floor systems.

Any of the structural units, singly or in combination, form a structural system. Each of these systems tends to be more suitable for a particular range of height from an economic point of view. For example in the United States, for medium height buildings up to twenty storeys or so, the use of concrete frames could be adequate. Higher than that, up to about forty storeys, a system of shear walls, which may be solid or perforated, can be incorporated giving additional advantages as they can act also as functional partitions, fire resisting elements and acoustic insulations between specific dwelling areas. A combination of shear walls and frames may be used for buildings up to fifty storeys. Above these heights, tube-in-tube or multiple frame-tube systems appear to be more economical (1).

Fig. 1.1 and Fig. 1.2 show typical plan layouts of combinations of the various structural units.

## 1.2 PRINCIPAL METHODS FOR ANALYSING THREE-DIMENSIONAL TALL BUILDINGS

An important first step in analysing a three-dimensional tall building is to decide on an appropriate idealised model, to include all the significant load-resisting elements and their modes of behaviour. The distribution of load between the elements is usually determined by an elastic analysis, regardless of the eventual method of design. This is because of the magnitude of the analytical problem involved. The slabs are usually assumed to be rigid in their own plane, so that each floor is subjected to a rigid-body movement in plan. Consequently the vertical elements at any floor level undergo horizontal and rotational components of displacement in the horizontal plane.

The analytical problem can be simplified as far as possible into the following categories.<sup>(2)</sup>

- (a) Symmetric overall plan with parallel identical assemblies of walls, columns, frames, etc. (an example shown in Fig. 1.3(a)) subjected to a symmetrical load system. Because of the identical behaviour of the elements, the analysis of one only is sufficient subjected to a proportion of the load.
- (b) Symmetric overall plan consisting of non-identical plane assemblies (as in Fig. (1.3(b))), subjected to symmetrical

loading. In this case the structure can be analysed as a plane system by assembling the elements in series for example with connecting rigid pin-ended links at each floor level. The links simulate the behaviour of floor slabs in constraining the assemblies to deform identically, and allow the resulting distribution of load to be determined.

- (c) Symmetric plan as in (a) or (b), but subjected to eccentric loading. The load may then be replaced by a concentric lateral load and a twisting moment, whose effects can be considered separately and superimposed. The former can be treated as in (a) or (b), whilst the torsional moment can be treated as an equivalent system of pairs of concentrated forces at each floor level applied at convenient corresponding points on opposite sides of the axis of symmetric.<sup>(3)</sup> By transforming the stiffnesses and displacements of the other elements into the same locations, the entire structure can be assembled in the same plane, and a plane analysis performed as described in (b).
- (d) Non symmetrical plan as shown in Fig. 1.3c . In this case the structure will generally undergo simultaneous bending and torsional displacements, and a three-dimensional analysis is required to determine the load distribution among the elements.

After the form of action is established, the dominant modes of behaviour of the various components and assemblies are considered in order to choose the most appropriate method of analysis.

In the past two decades, much research work has been carried out on tall buildings, some of which have assisted the designer with the provisions of sufficient data to produce a safe and economic design. Some of the methods are applicable to elastic analysis, while some others are suitable for elasto-plastic analysis of tall buildings. Since this thesis is concerned with elastic analysis, the publications considered for literature review relate only to these analyses.

The two basic methods of three-dimensional analysis of tall buildings can be classified as the continuous method and the discrete method<sup>(4)</sup>.

In the continuous methods, the horizontal elements which connect the vertical elements are substituted by a continuous medium of equivalent stiffness continuously distributed along the height of the building. These methods lead to a system of differential equations which, after being integrated, give displacements and internal forces in the whole structure.

In the discrete methods the well-known matrix techniques are used. The majority of the authors

prefers the displacement method. The discrete methods lead to a system of many linear equations which, after being solved, gives displacements and internal forces in the whole structure.

The discrete methods are more general dealing with structures of variable form, either in plan or in elevation. However, they always use a large number of parameters and variables, making difficult to perceive the behaviour of the whole structure and the way in which the variation of parameters affects the results<sup>(4)</sup>.

The continuous methods on the other hand, assume an essential uniformity of the structure in plan and elevation, so that the behaviour of the structure can be represented as functions of a small number of elastic and geometric properties.

In the preliminary design stages, a number of initial plans may have to be considered. To enable the engineer to reach quick decisions regarding the dimensions and layout of the structural members, approximate methods of analysis are essential. When the final design is accepted, then it should be rigorously analysed for final checking of the design.

The continuous method of analysis is adopted throughout this work because it fulfils the basic requirements of the initial design stage, that is, a good

approximate solution to the problem is required with minimal computational effort. It has also the advantage that the accuracy of the solution increases with the number of storeys without additional computational effort.

### 1.3 Review of previous research

Many investigators have presented simplified theories for the analysis of three-dimensional symmetric structures which may be reduced to equivalent plane systems as described in 1.2(b). Based on the continuous connection technique, Heidebrecht and Stafford Smith<sup>(5)</sup> devised a method for the analysis of symmetric structures consisting of shear walls and rigidly jointed frame assemblies. The method is suitable for the static analysis of uniform and non-uniform structures, and for dynamic analysis of uniform structures. Coull<sup>(6)</sup> presented a method, also using the continuum approach for the analysis of regular symmetric structures consisting of coupled shear walls and cores. The shear flow intensity in the connecting beams of the coupled shear walls is considered as the unknown variable. The solution for the shear flow was then used to determine the deflections and the internal forces. Stafford Smith and Abergel<sup>(7)</sup> analysed coupled shear walls and cores by transforming them into a single coupled shear wall with modified parameters. Expressions

were given for the horizontal deflections and the internal forces. Base on the continuum approach and complementary energy theory, Arvidsson<sup>(8)</sup> devised a method for structures consisting of coupled shear walls and frames. The solution was obtained by Euler's formula.

Despite a large amount of research carried out on the behaviour of tall building structures, published studies which deal with the analysis of unsymmetric three-dimensional systems as described in 1.2(d) are few in number. In an asymmetric structure which consists of different load-bearing units, such as independent and coupled shear walls, rigidly jointed frames, and open box-type cores, lateral forces resulting from wind or earthquake action produce both lateral and torsional displacements. Relatively little work has been done in this particular area. Winokur and Gluck<sup>(9)</sup> presented a method which considered the structure subdivided into main structural units for which the separate in-plane stiffness matrices were determined. The translations of the units in two arbitrary orthogonal horizontal axes and their rotations about an arbitrary vertical axis form matrix equations with regard to the equilibrium of the floors. Their solution gave values for translations and rotations of each floor and hence the in-plane displacements of each

unit, from which the unit actions were determined. Stamato and Mancini<sup>(10)</sup> used the continuous approach and matrix analysis to derive solutions for deflections, rotations and internal forces. In the analysis frame assemblies were replaced by equivalent shear cantilevers. Wynhoven and Adams<sup>(11)</sup> used slope-deflection equations to formulate equations of equilibrium for three-dimensional wall-frame structures. The equations were then arranged in matrix form and solved for the unknown displacements by using a modified Gauss Elimination technique. Rutenburg and Heidebrecht<sup>(12)</sup> analysed asymmetric wall-frame structures by an approach which is based on the decoupling of the coupled torsion-bending differential equations using an orthogonal transformation. The deformations and stress resultants in the wall and frame assemblies were obtained by combining the respective coefficients, tabulated from the solved decoupled equations.

An approximate analysis of wall-frame assemblies was devised by Mortelmans, Roeck and Van Gemert<sup>(13)</sup>. The method was for the combined bending and twisting of long, tall buildings, subjected to wind loading. The method reduced to a solution of a linear system of four equations with four unknowns, enabling the determination of all bending and twisting moments in the elements of the structure, regardless of the number of floors.

Coull and Irwin<sup>(14)</sup> presented an approximate method for the torsional analysis of three-dimensional structures consisting of parallel assemblies of coupled shear walls and core elements. Using the continuum technique, the flexibility matrix of each assembly is determined and by inversion the stiffness matrix is obtained. After the component stiffness matrices are determined, the complete structure is solved by matrix analysis.

A simplified method of analysis of three-dimensional buildings whose structure consists of essentially parallel systems of shear wall assemblies and box-core elements was presented by Coull and Adams<sup>(15)</sup>. The bending and twisting distribution on each element was assumed to be represented with sufficient accuracy by a polynomial in the height coordinate. The method was later extended by Coull and Mohammed<sup>(16)</sup>. The solution was improved by including a top concentrated interactive force in addition to the polynomial load distribution. Rigidly jointed frames were included in the analysis as well by representing them by shear cantilevers.

Coull and Khachatoorian<sup>(17)</sup> presented closed form solutions for the elastic 'exact' analysis of three-dimensional structures based on the continuous connection technique. The analysis are for symmetric and assymmetric structures consisting of cores, coupled shear walls and rigidly jointed framework assemblies arranged in two

orthogonal directions, with only one form of coupled wall unit present.

#### 1.4 Reasons for present study

Most of the methods of analysis for three-dimensional tall buildings are related to either symmetrical structures or the torsion-bending analysis of structural systems with relatively simple structural layouts. Few of the methods presented are suitable for rapid hand calculations or use minimal computations for the preliminary proportioning of components at the initial stages of the design process.

Therefore a method of analysis which requires as little computation as possible and also which yields reasonably accurate results is highly desirable. The present research forms an extension to the work of Mohammed<sup>(16)</sup>, in attempting to produce such an analysis for non-symmetrical three-dimensional structures with elements arranged in two orthogonal directions or with skew orientations.

#### 1.5 Scheme of the Thesis

This thesis is concerned with the investigation of multi-storey three-dimensional structures which consist of shear walls and frames under the action of lateral loads.

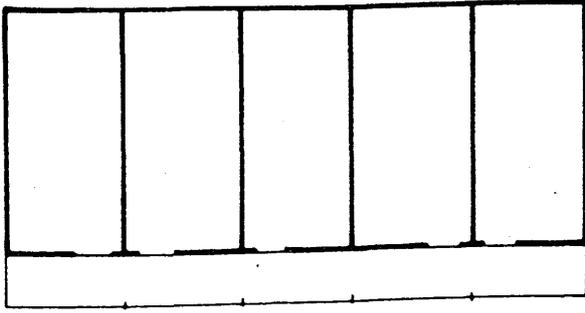
In Chapter 2 the approximate method of analysis is presented. The elements are assumed to be loaded in their own planes by a combination of a concentrated load at the

the top and a distributed load described by a polynomial series in the height coordinate.

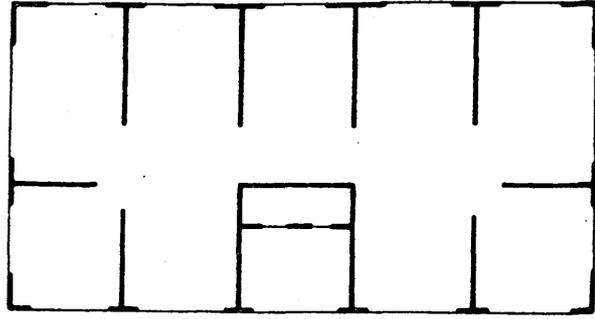
Chapter 3 deals with the analysis of two-dimensional elements or systems. Frames are represented by a shear cantilever of infinite flexural rigidity and of equivalent uniform shear rigidity. The continuous connection technique is used in the analysis of coupled shear walls and cores.

In Chapter 4, examples of numerical computations of the analytical method is presented to examine the convergence, accuracy and validity of the solutions, and to study the effects of the various parameters involved in the analysis. Comparisons of the results with other published works are also used to assess the accuracy of the technique.

Chapter 5 presents the relevant discussions on the results and the conclusions. Finally, suggestions are included for future work.



(a) No openings in walls: access from outside



(b) Coupled walls

Fig. 1.1 — Typical layouts of shear walls

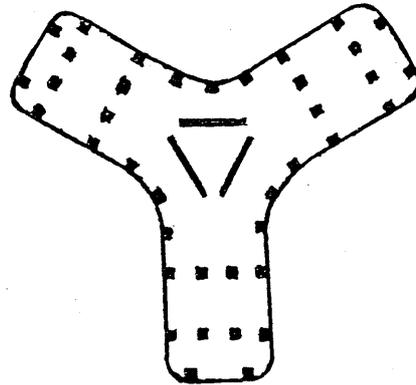
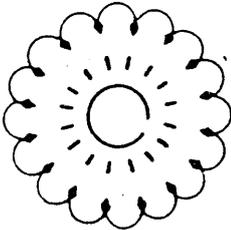
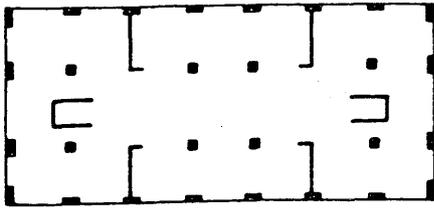
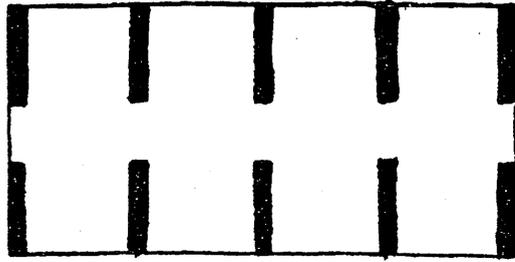
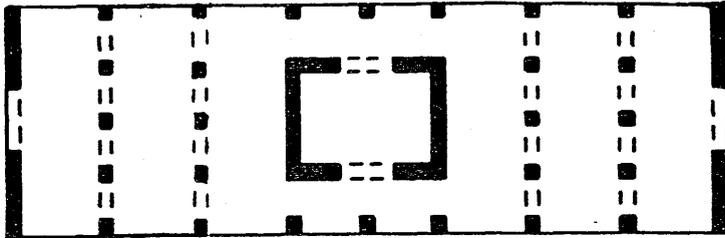


Fig. 1.2 Typical layouts of high-rise buildings with shear wall-frame interaction



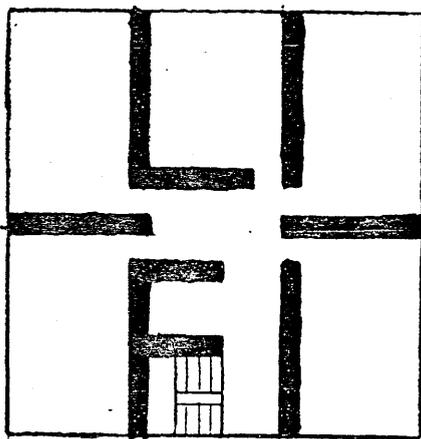
(a)



(b)

Fig. 1.3

Plan-forms of  
buildings



(c)

APPROXIMATE ANALYSIS OF  
THREE-DIMENSIONAL TALL BUILDINGS

2.1 Introduction

Most of the studies and analyses of shear wall structures have been concentrated on the problem of plane walls subjected to a known system of loads in their own plane. In the case of a three-dimensional complete building, the results are strictly accurate only if the structure consists of parallel systems of identical wall assemblies and is loaded symmetrically, so that any lateral loading is shared equally between them.

However, if the structure is made up of various forms of load-bearing elements such as independent and coupled shear walls, rigidly jointed frames, columns and box-type core structures surrounding lift shafts and stair wells, or if the elements are themselves arranged obliquely, considerable redistribution of lateral load may take place, particularly if torsional deformations of the building occur.

A commonly applied design rule is to assume that lateral loads are distributed among the elements in proportion to their stiffnesses, or their top deflection due to a unit laterally distributed loading. This is true provided that the lintel beams or floor slabs are effectively 'ball jointed' to the walls, so that no bending can be induced in any plane, but this is obviously not the case.

If coupling occurs between the walls and bending induced in the lintel beams and floor slabs, the design rule can give rise to significant errors. This can be seen by comparing the modes of deformation of a shear wall and frame structure subjected to a uniformly distributed load. The former bends in an essentially bending mode, while the latter deflects in a predominantly shear mode as illustrated in Fig. 2.1(a) and Fig. 2.1(b). If the two are constrained to deflect equally by a system of floor slabs, tensile linking forces are introduced in the upper levels and compressive forces in the lower regions, indicating a redistribution of load between the two throughout the height <sup>(18)</sup>. This is shown in Fig. 2.1(c).

The present analysis provides a relatively simple method of evaluating the distribution of lateral loads within a complete three-dimensional building where load-bearing elements consist of assemblies arranged in two orthogonal directions or with skew orientations.

The method depends basically on the form of load distribution which is assumed to be carried by each element in the structure.

In the first instance a method is presented for multi-storey buildings whose load-bearing elements consist of parallel systems subjected to bending and torsion. After that, the method is extended to include elements in two orthogonal directions and with inclined applied loading.

## 2.2 Assumptions

The following assumptions are made in the analysis.

(1) The floors are assumed to be so stiff in their own plane that each floor undergoes a rigid body displacement in its own plane. Out of plane they are completely flexible and there are no coupling of assemblies.

(2) Any form of wind pressure distribution may be considered. The effect of the wind load above the level  $\xi_i = \frac{x_i}{H}$  may be represented by a resultant horizontal load  $W_i$  and a twisting moment  $MT_i$  acting at the datum position, 0 as shown in Fig. 2.2.

(3) The resultant force and moment at any level are resisted by a combination of differential shearing forces and torsional moments on the assemblies.

### 2.2.1 Representation of load distribution on each element

The load distribution on each element  $j$ , may be described by a combination of

- i) a horizontal point load  $P_{0j}$  at the top (2.1 a)  
and ii) a distributed load  $p_j$  whose intensity is defined by a polynomial series of the form

$$\begin{aligned} p_j &= p_{0j} + p_{1j}\xi + p_{2j}\xi^2 + \dots + p_{mj}\xi^m \\ &= \sum_{i=0}^m p_{ij}\xi^i \end{aligned} \quad (2.1 b)$$

where  $m$  is some arbitrary integer, for a simplified

analysis  $m$  is assumed to be small (say 10 or less)

$p_{ij}$  is a constant coefficient

$\xi$  is a non-dimensional height coordinate  $x/H$

Similarly, the twisting moment on any element  $j$  may be assumed to consist of a combination of

i) a point torque  $T_{Oj}$  at the top (2.2 a)

ii) a distributed twisting moment distribution

defined by a power series of the form

$$t_j = \sum_{i=0}^m t_{ij} \xi^i \quad (2.2 \text{ b})$$

The height coordinate is chosen to be measured downwards from the top of the structure. This is due to the anticipated pattern of load distribution whereby the load on the elements increases downwards.

The total shear force  $Q_j$  and the twisting moment,  $T_j$  carried by any element at level  $\xi_i$  are given by

$$\begin{aligned} Q_j &= P_{Oj} + H \int_0^{\xi} p_{ij} d\xi = P_{Oj} + H \sum_{i=0}^m \frac{p_{ij} \xi^{i+1}}{i+1} \\ &= P_{Oj} + \sum_{i=0}^m s_i p_{ij} \end{aligned} \quad (2.3)$$

and

$$\begin{aligned} T_j &= T_{Oj} + H \int_0^{\xi} t_{ij} d\xi = T_{Oj} + H \sum_{i=0}^m \frac{t_{ij} \xi^{i+1}}{i+1} \\ &= T_{Oj} + \sum_{i=0}^m s_i t_{ij} \end{aligned} \quad (2.4)$$

where the coefficient  $s_i = \frac{H \xi^{i+1}}{i+1}$  (2.5)

### 2.3 Complete three-dimensional analysis of structures subjected to bending and torsion

Suppose that a structure consists of a number of wall and frame assemblies as shown in Fig. 2.2.

For horizontal equilibrium the total applied shear  $W_i$  at any level must equal the sum of the shear forces on the individual elements at that level, or

$$W = Q_1 + Q_2 + \dots + Q_J = \sum_{j=1}^J Q_j \quad (2.6)$$

For rotational equilibrium, the total applied moment  $M_T$  must equal the sum of the moments of the shear forces and the twisting moments on the individual elements at that level, so that,

$$\begin{aligned} M_T &= (Q_1 l_1 + Q_2 l_2 + \dots + Q_J l_J) + (T_1 + T_2 + \dots + T_J) \\ &= \sum_{j=1}^J Q_j l_j + \sum_{j=1}^J T_j \end{aligned} \quad (2.7)$$

For each individual assembly, a unique linear load-displacement relationship exists, in which the horizontal deflections at any chosen set of reference levels may be determined for each load component of equations (2.1). The set of deflections corresponding to a unit value of the component will yield a set of flexibility influence coefficients  $f_{ij}$  defined as the horizontal deflection at level  $\xi_i$  due to a unit value of component  $P_{0j}$  or  $P_{ij}$ .

Similarly, a corresponding set of flexibility influence coefficients  $f'_{ij}$ , defined as the rotation at level  $x_i$  due to a unit value of twisting moment component  $T_{oj}$  or  $t_j$  may be derived for each assembly. The load-deflection and torque-rotation relationships for the individual elements will be considered in Chapter 3.

The load-deflection relationship for the  $j^{\text{th}}$  element may be expressed in matrix form as

$$[V]_j = [F]_j [P]_j \quad (2.8)$$

where  $[V]_j$  is a column vector of deflections at any arbitrary set of reference levels.

$[P]_j$  is a column vector of load coefficients  $P_{oj}$  and  $p_{ij}$

$[F]_j$  is a square matrix of influence coefficients  $f_{ij}$

The total shear force  $Q_{ij}$  at each reference level may be expressed in terms of the load coefficient in matrix form as

$$[Q]_j = [S][P]_j \quad (2.9)$$

where  $[Q]_j$  is a column vector of shear forces  $Q_{ij}$  at each reference level on element  $j$ , and  $S$  is a square matrix of integration coefficient  $s_i$  given by equation (2.5).

The deflection at any reference level of the  $j^{\text{th}}$  element may then be related to the applied shear forces through the flexibility matrix of equation (2.8), giving

$$[V]_j = [V_0] + 1_j [\theta] = [F]_j [P]_j \quad (2.10)$$

where  $[V_0]$  and  $[\theta]$  are column vectors of the deflection at the datum position 0 and the rotation of the structure at each reference level respectively. (Fig 2.2)

The first equilibrium equation (2.6) then becomes

$$[W] = \sum_{j=1}^J [Q]_j = [S] \sum_{j=1}^J [F]_j^{-1} ([V_0] + 1_j [\theta]) \quad (2.11)$$

in which  $[W]$  is the column vector of the total applied shear forces  $W_i$  at the reference levels  $x_i$  and the summation is carried out over the  $J$  separate elements in the structure.

The torque-rotation relationship for the  $j^{\text{th}}$  element may be expressed in the form

$$[\theta]_j = [F']_j [t]_j \quad (2.12)$$

where  $[\theta]_j$  and  $[t]_j$  are column vectors of rotations  $\theta_{ij}$  and torque coefficients  $T_{0j}$  and  $t_{ij}$  at any set of reference levels, and  $[F']_j$  is a square matrix of influence coefficients  $f'_{ij}$ .

The total torque  $M_T$  at each reference level may be related to the torque coefficients  $T_{0j}$  and  $t_{ij}$  for all chosen levels by the expression

$$[M_T]_j = [S][T]_j \quad (2.13)$$

where  $[M_T]_j$  is the column vector of twisting moments at

each reference level on element  $j$ , and  $[T]_j$  is the column vector of torque coefficients on the element.

The condition of rotational equilibrium (2.7) then becomes

$$[M_T] = [S] \sum_{j=1}^J \left\{ [F]_j^{-1} ([V_0] + 1_j [\theta]) 1_j + [F']_j^{-1} [\theta] \right\} \dots\dots (2.14)$$

By solving equations (2.11) and (2.14) simultaneously, the expressions for the deflections at 0 and the rotation of the structure may be shown to be

$$[V_0] = \left\{ [G_2] - [G_3][G_2]^{-1}[G_1] \right\}^{-1} \left\{ [M_T] - [G_3][G_2]^{-1}[W] \right\} \dots\dots (2.15)$$

$$[\theta] = \left\{ [G_2] - [G_1][G_2]^{-1}[G_3] \right\}^{-1} \left\{ [W] - [G_1][G_2]^{-1}[M_T] \right\} \dots\dots (2.16)$$

where

$$[G_1] = [S] \sum_{j=1}^J [F]_j^{-1}$$

$$[G_2] = [S] \sum_{j=1}^J [F]_j^{-1} 1_j$$

$$[G_3] = [S] \sum_{j=1}^J \left\{ [F]_j^{-1} 1_j^2 + [F']_j^{-1} \right\}$$

Having determined the deflections and rotations of each element at all reference levels, the loads on different elements follow from equations (2.8) and (2.12) as

$$[p]_j = [F]_j^{-1} [V]_j \quad (2.17)$$

The distribution of the twisting moments is obtained from

$$[t]_j = [F^t]_j^{-1} [\theta] \quad (2.18)$$

Having determined the applied loading on each assembly, the internal stress-resultants and deformations can be obtained from the analysis of the individual structural assembly, using techniques such as those presented in Chapter 3.

#### 2.4 Complete three-dimensional analysis of structures with assemblies in two orthogonal directions.

Consider an assymmetrical building structure which consists of a number (J) of elements-arranged as shown in Fig. 2.3(a).

Under the action of wind forces, W, which act at a distance L from the left hand corner O, the structure will undergo translational displacements  $U_j$  and  $V_j$  in the y and z directions, and the floors will undergo a rotational displacement  $\theta$  in the Oyz plane. Any datum position can be chosen, for this analysis O is chosen as the datum point, and all displacements are referred to O. If wind forces W act at an angle, it can be resolved into the two orthogonal directions as  $W_y$  and  $W_z$ . (Fig. 2.3(b))

The displacements of any element at level  $\epsilon$  in the two directions may be expressed as

$$\begin{aligned} U_j &= U_o - r_j \theta \\ V_j &= V_o + l_j \theta \end{aligned} \quad (2.19)$$

where  $U_o$  and  $V_o$  are the deflections of point 0 in the Ozy plane and  $r_j$  and  $l_j$  are the perpendicular distances between the centroids or the shear centres of each element and the datum axis Oyz respectively. (Fig. 2.3(c))

The total shear force  $Q_{ij}$  at each reference level may be expressed in terms of the load coefficient in matrix form as

$$\begin{aligned} [Q]_{jy} &= [S][P]_{jy} \\ [Q]_{jz} &= [S][P]_{jz} \end{aligned} \quad (2.20)$$

where  $[Q]_{jy}$  and  $[Q]_{jz}$  are column vectors of shear forces in the y and z directions respectively at each reference level on element j, and S is a square matrix of integration coefficient  $s_i$  given by equation (2.5).

The deflection at any reference level of the  $j^{\text{th}}$  element may then be related to the applied shear forces through the flexibility matrix of equation (2.8), giving

$$[U]_j = [U_o] - r_j [\theta] = [F]_{jz} [P]_{jz} \quad (2.21)$$

$$[V]_j = [V_o] + l_j [\theta] = [F]_{jy} [P]_{jy} \quad (2.22)$$

where  $[U_0]$ ,  $[V_0]$  and  $[\theta]$  are column vectors of the deflection in the z and y directions at the datum position 0 and the rotation of the structure at each reference level respectively.

Proceeding as before, the equilibrium equations become

$$[W]_z = \sum_{j=1}^J [Q]_{jz} = [S] \sum_{j=1}^J [F]_{jz}^{-1} ([U_0] - r_j [\theta]) \quad (2.23)$$

$$[W]_y = \sum_{j=1}^J [Q]_{jy} = [S] \sum_{j=1}^J [F]_{jy}^{-1} ([V_0] + l_j [\theta]) \quad (2.24)$$

in which  $[W]_z$  and  $[W]_y$  are column vectors of the total applied shear forces in the two directions z and y, and the summation is carried out over the J separate elements.

For rotational equilibrium, the total applied moment  $M_T$  is given by

$$[M_T] = \sum_{j=1}^J [Q]_{jy} l_j + \sum_{j=1}^J [Q]_{jz} r_j + \sum_{j=1}^J [T]_j \quad (2.25)$$

Using a similar derivation for equation (2.14) the condition of rotational equilibrium then becomes,

$$[M_T] = [S] \sum_{j=1}^J \left\{ [F]_{jy}^{-1} ([U_0] - r_j [\theta]) r_j + [F]_{jz}^{-1} ([V_0] + l_j [\theta]) l_j + [F]_j^{-1} [\theta] \right\} \quad (2.26)$$

By solving equations (2.15), (2.16) and (2.19) simultaneously the expressions for the deflections at 0 and the rotation of the structure may be shown to be,

$$\begin{aligned}
 [U_0] &= ([G_2]^{-1}[G_4][G_3]^{-1}[G_1] - [G_4]^{-1}[G_5][G_3]^{-1}[G_1] - \\
 & \quad [G_4]^{-1}[G_3])^{-1} \times \{ ([G_2]^{-1}[G_4][G_3]^{-1} - \\
 & \quad [G_4]^{-1}[G_5][G_3]^{-1}) [W]_y + [G_2]^{-1}[W]_z - [G_4]^{-1}[M_T] \} \\
 & \quad \dots\dots\dots (2.27)
 \end{aligned}$$

$$\begin{aligned}
 [V_0] &= ([G_1]^{-1}[G_3][G_4]^{-1}[G_5] + [G_3]^{-1}[G_5][G_4]^{-1}[G_2] - \\
 & \quad [G_3]^{-1}[G_4])^{-1} \times \{ [G_1]^{-1}[W]_y - [G_3]^{-1}[M_T] + \\
 & \quad ([G_1]^{-1}[G_3][G_4]^{-1} + [G_3]^{-1}[G_5][G_4]^{-1}) [W]_z \} \\
 & \quad \dots\dots\dots (2.28)
 \end{aligned}$$

$$\begin{aligned}
 [\theta] &= ([G_3]^{-1}[G_4][G_2]^{-1}[G_4] - [G_1]^{-1}[G_3] - [G_3]^{-1}[G_5])^{-1} \\
 & \quad \times \{ [G_1]^{-1}[W]_y - [G_3]^{-1}[M_T] + [G_3]^{-1}[G_4][G_2]^{-1}[W]_z \} \\
 & \quad \dots\dots\dots (2.29)
 \end{aligned}$$

where

$$[G_1] = [S] \sum_{j=1}^J [F]_{jy}^{-1} \quad (2.30)$$

$$[G_2] = [S] \sum_{j=1}^J [F]_{jz}^{-1} \quad (2.31)$$

$$[G_3] = [S] \sum_{j=1}^J [F]_{jy}^{-1} r_j \quad (2.32)$$

$$[G_4] = [S] \sum_{j=1}^J [F]_{jz}^{-1} l_j \quad (2.33)$$

$$[G_5] = [S] \sum_{j=1}^J \left\{ -[F]_{jy}^{-1} r_j^2 + [F]_{jz}^{-1} l_j^2 + [F']_j^{-1} \right\} \quad (2.34)$$

Having determined the deflections and rotations of each element at all reference levels, the loads on different elements follow from equations (2.11) as

$$[P]_{jy} = [F]_{jy}^{-1} [U]_j \quad (2.35)$$

$$[P]_{jz} = [F]_{jz}^{-1} [V]_j \quad (2.36)$$

in the y and z directions respectively. The distribution of the twisting moments is obtained from

$$[T]_j = [F']_j^{-1} [\theta] \quad (2.37)$$

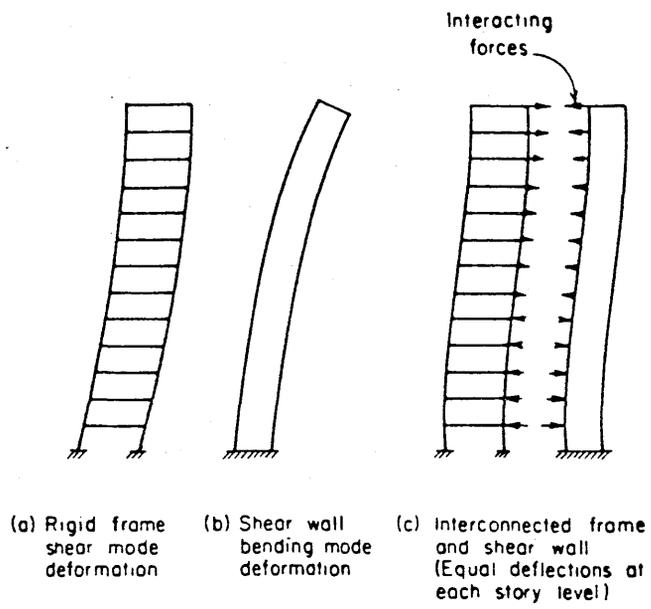


Fig. 2.1 Wall-Frame interaction

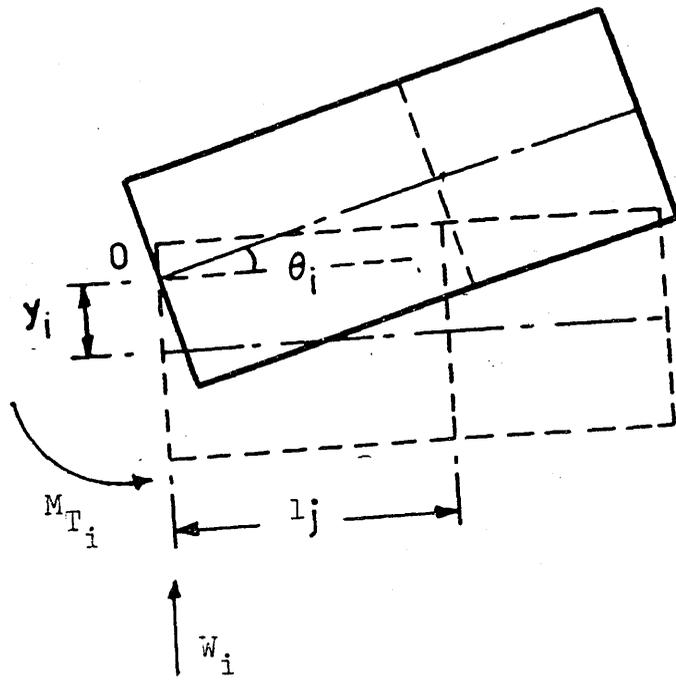
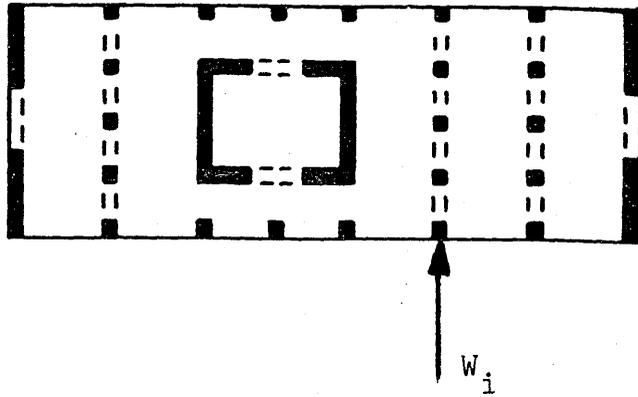


Fig. 2.2

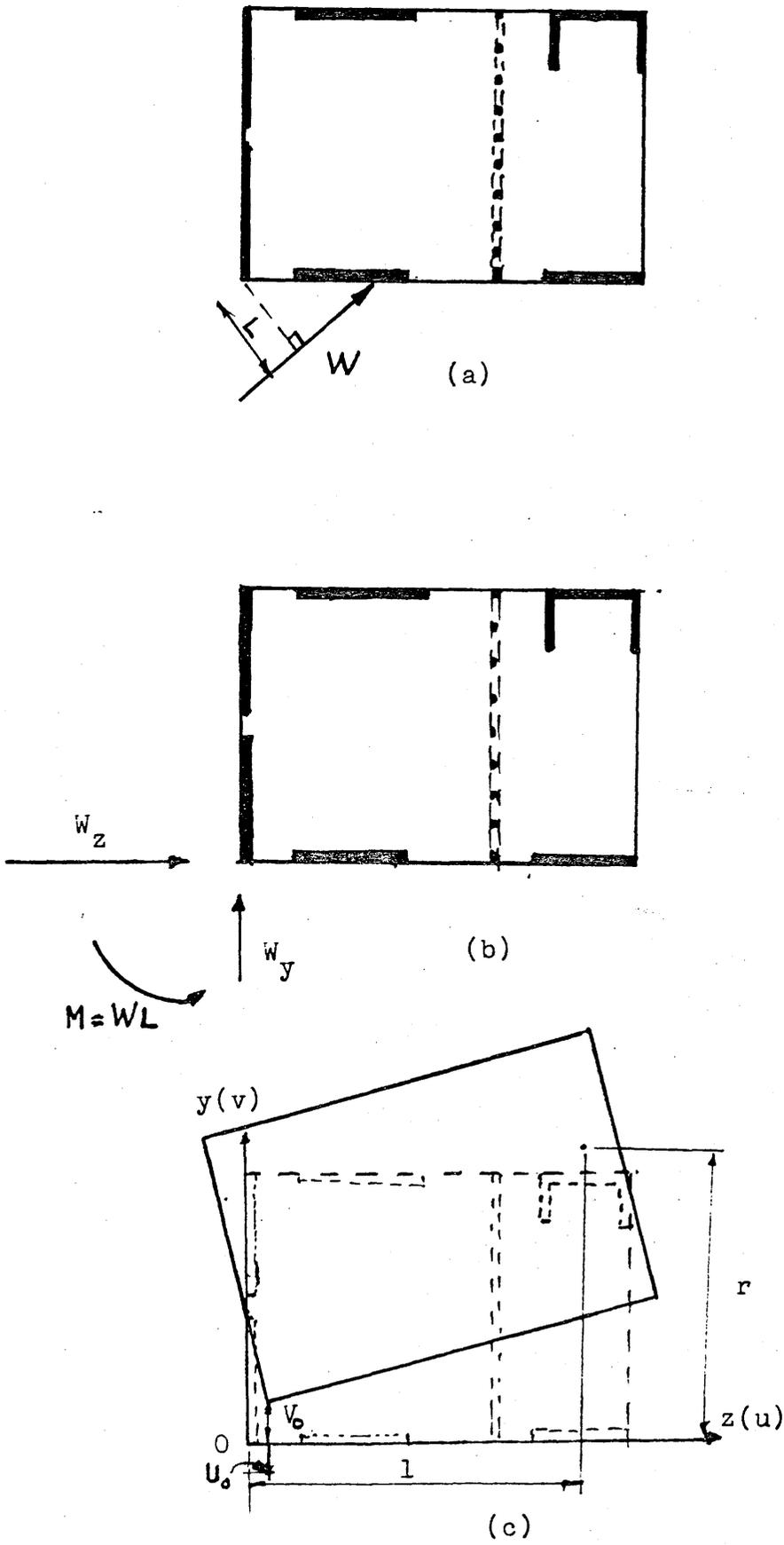


Fig. 2.3

ANALYSIS OF INDIVIDUAL ELEMENTS  
OF TALL BUILDING STRUCTURES3.1 Introduction

The resistance to lateral forces in many tall buildings is provided by coupled shear walls, cores, frames and single shear walls. When these assemblies are subjected to distributed lateral loads, their modes of behaviour are different. A cantilever core or an independent shear wall deform in a bending mode, a frame in a shear mode, and a coupled shear wall bends with a reversal of curvature in the higher levels. When constrained to act together by floor slabs, a considerable redistribution of load may occur between these elements throughout the height of the building under the influence of lateral forces.

In general, these elements are loaded with any lateral force distribution in the plane being considered. However, the lateral load distribution considered in this analysis are expressed by a combination of a top concentrated load at the top and a polynomial series form of distribution.

In this Chapter, expressions are derived for the load-deflection and torque-rotation relationships for the different individual structural elements considered. These enable sets of influence coefficients, relating the

deflection and rotation at any level to a unit value of applied load or torque component.

To simplify the analysis it is assumed that a frame assembly may be replaced by an equivalent shear cantilever which has the same effective lateral stiffness. Single shear walls, which are not coupled to any other shear bearing elements act as simple cantilevers. In the case of coupled walls, the continuous connection technique is used to derive the differential equations governing its behaviour.

### 3.2 Structural Frames

The most fundamental component of a tall building is the rigid frame, which achieves its lateral stiffness from the rigidity of the joints between columns and beams or slabs.

#### 3.2.1 Replacement of tall frames by an equivalent shear cantilever

When subjected to lateral forces, tall frames deform in a predominantly shearing mode, due to the racking action over each storey height. The frames may therefore be replaced by an equivalent 'shear cantilever' of effective shearing rigidity  $GA$ , and infinite flexural rigidity. This shearing rigidity must be chosen such that the horizontal deflection

of both frame panel and beam are the same under the action of the same shear force.

Consider the single storey segment of a frame shown in Fig. 3.1(a). Since the columns may be closely spaced, and the spandrel beams relatively deep, the finite size of the joint relative to the free column height and beam span must be taken into account. This can be done by assuming that short rigid arms exist at each node, of width equal to the width of the column, and of height equal to the depth of the beams.

Due to the high in-plane rigidity of the floor slabs, the columns are assumed to be constrained to deflect equally at each floor level and the beams deflect with a point of contraflexure at their mid-span position. In addition to these, the columns are assumed to bend with points of contraflexure at their mid-height positions. The forces on the frame segment, and effective boundary conditions are shown in Fig. 3.1(c).

If a horizontal force  $Q$  is applied at the node  $D$ , the resulting horizontal deformation  $\Delta$  can be calculated from the moment-deformation characteristics of the frame segment. The load-displacement (19) relationship is

$$\frac{Qh}{2} = \frac{6EI_h}{e^2} \left(1 + \frac{t_2}{e}\right) \frac{\Delta}{1 + \frac{\frac{2I_h}{e} \left(1 + \frac{t_2}{e}\right)^2}{\frac{I_{d1}}{l_1} \left(1 + \frac{t_1}{l_1}\right)^2 + \frac{I_{d2}}{l_2} \left(1 + \frac{t_1}{l_2}\right)^2}}$$

in which

$I_h$  = second moment of area of column

$h$  = storey height,  $I_{d_1}$  and  $I_{d_2}$  are the second moments of area of the adjacent beams of total lengths  $d_1$  and  $d_2$  respectively,  $t_1$  and  $t_2$  are the length and height of the rigid arms, and

$$e = h - t_2$$

$$l_1 = d_1 - t_1$$

$$l_2 = d_2 - t_1$$

For an equivalent shear cantilever element of the same bay width, subjected to the same shearing force  $Q$ , Fig. 3.1(b) the load-displacement relationship is,

$$\Delta = \frac{Q}{GA} h$$

On equating the two relationships, the shearing rigidity  $GA$  becomes

$$GA = \frac{1}{Z} \frac{12EI_h}{e^2} \left( 1 + \frac{t_2}{e} \right)$$

where

$$Z = 1 + \frac{2I_h(1 + t_2/e)^2}{e \left[ \frac{I_{d_1}}{l_1} \left( 1 + \frac{t_1}{l_1} \right)^2 + \frac{I_{d_2}}{l_2} \left( 1 + \frac{t_1}{l_2} \right)^2 \right]}$$

This relationship is applicable also to an exterior column if the second moment of area of one of the beams is taken to be zero. The total stiffness of the equivalent cantilever is then equal to the sum of the individual GA values of the bay.

If, as is frequently the case with tall buildings,  $I_{d_1} = I_{d_2} = I_d$  and  $d_1 = d_2 = d$ , or  $l_1 = l_2 = l = d - t_1$ , the shear rigidity becomes

$$GA = \frac{1}{Z} \frac{12EI_h}{e^2} (1 + t_2/e)$$

Where

$$Z = 1 + \frac{2I_h (1 + t_2/e)^2}{e \left[ \frac{I_{d_1}}{l_1} \left(1 + \frac{t_1}{l_1}\right)^2 + \frac{I_{d_2}}{l_2} \left(1 + \frac{t_1}{l_2}\right)^2 \right]}$$

If the spans of the beams and heights are relatively large in comparison with the joint dimensions,  $t_1$  and  $t_2$  may be taken to be zero in the above expression.

### 3.2.2 Assumed loads on the equivalent shear cantilever on rigid foundation

The structural behaviour of the shear cantilever is defined by the relationship

$$\frac{dy}{dx} = -\frac{Q}{GA} \quad (3.1)$$

where  $Q$  = the shearing force at level  $x$ .

Using the non-dimensional height coordinate  $\xi = \frac{x}{H}$

equation (3.1) becomes

$$\frac{dy}{d\xi} = - H \frac{Q}{GA} \quad (3.2)$$

### 1) Concentrated Load at the Top

For a shear cantilever which has a point load, P, applied at the free end, the shear force at level  $\xi$  is given by

$$Q = P \quad (3.3)$$

Therefore equation (3.2) becomes

$$\frac{dy}{d\xi} = - H \frac{P}{GA} \quad (3.4)$$

Integrating equation (3.4) gives

$$y = - H \frac{P}{GA} \xi + C_2 \quad (3.5)$$

At the base, there is no horizontal displacement, that is

$$\text{at } \xi = 1 \quad y = 0 \quad (3.6)$$

Using the boundary conditions (3.6) in equation (3.5) the deflection becomes,

$$y = \frac{PH}{GA} (1 - \xi) \quad (3.7)$$

### 2) Polynomial load distribution

Let the applied load be expressed by a polynomial series of the general form,

$$p = p_0 + p_1 + p_2^2 + \dots + p_m \xi^m = \sum_{m=0}^{\infty} p_m \xi^m \quad \dots \dots \dots (3.8)$$

For simplicity, consider the action of any particular term  $p_n \xi^n$  of the series. The shearing force at level  $\xi$  of the equivalent shear cantilever is given by

$$Q = H \int_0^{\xi} p_n \lambda^n d\lambda = \frac{H p_n \xi^{n+1}}{n+1} \quad (3.9)$$

where  $\lambda$  is an arbitrary non-dimensional height coordinate. Substituting equation (3.9) into equation (3.2) and using the boundary condition (3.6) the deflection at level  $\xi$  becomes,

$$y = \frac{p_n H^2}{GA} \frac{n!}{(n+2)!} (1 - \xi^{n+2}) \quad (3.10)$$

The combination of both the top concentrated force and the polynomial load distribution gives

$$y = P_0 \frac{H}{GA} (1 - \xi) + p_n \frac{H^2}{GA} \frac{n!}{(n+2)!} (1 - \xi^{n+2}) \quad \dots \dots \dots (3.11)$$

This expression enables the deflection  $y$  at any level  $\xi_i$  due to a unit value of load component ( $P_0$  or  $p_n$ ) to be determined, thus furnishing a set of flexibility influence coefficients for the frame.

### 3.3 Shear Walls

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The other basic component of a tall building is the structural shear wall, which occurs in a variety of cross-

sectional shapes. Although the term shear wall is used, its deflection under lateral forces will be a flexural one and shearing effects will generally be insignificant. This is because the wall panel, when viewed in the context of the whole building, will appear as a slender cantilever beam.

### 3.3.2 Assumed loads on the shear walls on rigid foundation.

#### 1) Concentrated load $P_0$ at the top

The bending moment-curvature relationship is given by

$$M = \frac{EI}{H^2} \frac{d^2y}{d\xi^2} = P_0 H \xi \quad (3.12)$$

Integrating twice with the boundary conditions,

$$\text{at } \xi = 1, \quad y = 0 \text{ and } \frac{dy}{d\xi} = 0 \quad (3.13)$$

the deflections can be shown to be

$$y = \frac{P_0 H^3}{EI} \left( \frac{\xi^3}{6} - \frac{1}{2} \xi + \frac{1}{3} \right) \quad (3.14)$$

#### 2) Polynomial load

Again, considering any term,  $p_n \xi^n$  of the polynomial series (3.8), the bending moment is given by

$$M = H^2 \int_0^\xi p_n \lambda^n (\xi - \lambda) d\lambda = p_n H^2 \frac{n!}{(n+2)!} \xi^{n+2} \dots \dots \dots (3.15)$$

Therefore

$$M = \frac{EI}{H^2} \frac{d^2y}{d\xi^2} = p_n H^2 \frac{n!}{(n+2)!} \xi^{n+2} \quad (3.16)$$

Integrating twice and using the boundary conditions (3.13)

the deflection at level  $\xi$  becomes

$$y = \frac{p_n H^4}{EI} \left\{ \frac{n!}{(n+4)!} \xi^{n+4} - \frac{n! \xi}{(n+3)!} + \frac{n!(n+3)}{(n+4)!} \right\} \dots (3.17)$$

The combined deflections due to the top concentrated load  $P_0$  and the polynomial load becomes

$$y = \frac{H^3}{EI} \left[ P_0 \left\{ \frac{\xi^3}{6} - \frac{1}{2} \xi + \frac{1}{3} \right\} + p_n H \left\{ \frac{n!}{(n+4)!} \xi^{n+4} - \frac{n! \xi}{(n+3)!} + \frac{n!(n+3)}{(n+4)!} \right\} \right] (3.18)$$

### 3.4 Coupled Shear Walls

Coupled shear walls are walls containing openings. These openings normally occur in vertical rows to accommodate doorways, windows and corridors in the essentially regular layout of a tall residential building. The connection between the wall sections is provided by either connecting beams which form part of the wall, or floor slabs, or a combination of both.

#### 3.4.1 General Theory

As the basic continuous medium theory of coupled shear walls is fully documented (20,21), only the fundamental assumptions and equations are given here.

### 3.4.2 Assumptions

- 1) Plane sections of both walls and beams which are plane before bending remain plane after bending.
- 2) The individual connecting beams of flexural rigidity  $EI_p$  are replaced by an equivalent continuous medium or set of laminae of flexural rigidity  $EI_p/h$  per unit height.
- 3) The axial deformation in the coupling beams and hence of the continuous medium are negligible.
- 4) The stiffnesses of the walls are so much greater than those of the coupling beams that their slopes are not affected locally by the action from the discrete beams. Consequently, the slopes and deflections of the two walls are equal at all levels. Therefore each of the coupling beams and hence each lamina will have a point of contraflexure at the mid-span position.
- 5) The coupled walls have uniform sectional properties throughout the height and are rigidly built in at the base.

### 3.4.3 Action of the connecting laminae

The discrete set of connecting beams may be replaced by an equivalent continuous medium as in Fig. 3.2. The discrete set of shear forces and axial forces in the beams may then be replaced in the substitute system by a distributed shear and an axial distribution of intensity 'q' and 'n' per unit height respectively.

Suppose that the connecting system has been 'cut' along the vertical axis through the points of contraflexure at the mid-span positions of the connecting laminas, where only shear and axial forces occur. Under the action of the applied lateral loads and the internal forces, the condition of vertical displacement compatibility at the 'cut' is

$$1 \frac{dy}{dx} - q \frac{b^3 h}{12EI_c} - \frac{1}{E} \left( \frac{1}{A_1} + \frac{1}{A_2} \right) \int_x^H \int_0^{\eta'} q(\lambda) d\lambda d\eta = 0 \quad (3.19)$$

The three terms of equation (3.19) are respectively the relative displacement due to the bending of the walls, the relative displacement of the connecting beams and the vertical displacement due to the axial deformations of the walls.

Equation (3.19) is in terms of two variables  $y$  and  $q$  and may be written in terms of the non-dimensional height coordinate  $\xi$  as

$$\frac{dy}{d\xi} - \frac{b^3 h}{12EI_c} Hq - \frac{H^2}{EI} \left( \frac{A}{A_1 A_2} \right) \int_{\xi}^1 \int_0^{\eta} q(\lambda) d\lambda d\eta = 0 \quad (3.20)$$

where  $\xi = \frac{x}{H}$ ,  $\eta = \frac{\eta'}{H}$  and  $A = A_1 + A_2$

#### 3.4.4 Governing Differential Equation

From the condition of vertical equilibrium for wall 1, the integral of the laminar shear force distribution above the section  $\xi$  is equal to the axial force on the wall at

that level, that is

$$T = H \int_0^{\xi} q(\eta) d\eta \quad (3.21)$$

The bending moments  $M_1$  and  $M_2$  in the two walls are related to the curvature by their respective flexural rigidities,  $E_1 I_1$  and  $E_2 I_2$  as follows,

$$M_1 = \frac{E_1 I_1}{H^2} \frac{d^2 y}{d\xi^2} = M - \frac{T_1}{2} - M_{a1} \quad (3.22)$$

$$M_2 = \frac{E_2 I_2}{H^2} \frac{d^2 y}{d\xi^2} = -\frac{T_1}{2} + M_{a2} \quad (3.23)$$

where  $M_{a1}$  and  $M_{a2}$  are the moments due to the axial forces in the connecting medium. They are equal in magnitude and are given as

$$M_{a1} = M_{a2} = H^2 \int_0^{\xi} n(\xi - \lambda) d\lambda \quad (3.24)$$

where  $\lambda$  is an arbitrary non-dimensional height coordinate. The summation of equations (3.22) and (3.23) gives the moment-curvature relationship for the coupled walls as,

$$\frac{d^2 y}{d\xi^2} = \frac{H^2}{(E_1 I_1 + E_2 I_2)} (M - T_1) \quad (3.25)$$

Equation (3.20), (3.21) and (3.25) give the relationship between the three variables  $T, q$  and  $y$ . By selecting any of the functions as a redundant and eliminating the other two, a governing differential equation may be set up in terms of the redundant function.

Consider T as the redundant. Substituting equation (3.25) into the first derivative of equation (3.20) and rearranging in terms of the second derivative of T gives the general governing differential equation as

$$\frac{d^2T}{d\xi^2} - \gamma^2 T = -\beta^2 H^2 M \quad (3.26)$$

where

$$\gamma^2 = \beta^2 H^2 \left( \frac{AI}{A_1 A_2 l} + 1 \right) \quad (3.27)$$

$$\beta^2 = \frac{12I_c l}{hb^3 I} \quad (3.28)$$

and

$$I = I_1 + I_2$$

Similarly, expressions for the variables q and y can be derived.

#### 3.4.5 Boundary Conditions

At the top of the building there is no axial force acting on the wall, so that,

$$\text{at } \xi = 0 \quad T = 0 \quad (3.29)$$

at the base where  $\xi = 1 \quad \frac{dy}{d\xi} = 0$

and from (3.20) and (3.21)

$$\frac{dT}{d\xi} = 0 \quad \text{at the base.} \quad (3.30)$$

### 3.4.6 Assumed interactive load on coupled walls

#### 1) Concentrated load at the top

If a pair of coupled shear walls of height  $H$  (Fig. 3.2) is subjected to a point load  $P_0$  applied at the top; i.e at  $\xi = 0$ , the applied external moment at any level is then given by

$$M = P_0 H \xi \quad (3.31)$$

Substituting equation (3.31) into equation (3.26) gives the governing differential equation as

$$\frac{d^2 T}{d\xi^2} - \gamma^2 T = -\beta^2 H^3 P_0 \xi \quad (3.32)$$

Equation (3.32) is a second order linear equation with constant coefficients. The solution of equation (3.31) is given by

$$T = P_0 H^3 \beta^2 (C_1 \sinh \gamma \xi + C_2 \cosh \gamma \xi + \frac{\xi}{\gamma^2}) \quad (3.33)$$

On using the boundary conditions (3.29) and (3.30) the constants  $C_1$  and  $C_2$  can be found as follows

$$C_1 = -\frac{1}{\gamma^3 \cosh \gamma} \quad (3.34)$$

$$C_2 = 0 \quad (3.35)$$

The deflection at any level can be found by integrating equation (3.25) and by substituting the boundary conditions at  $\xi = 1$ , that is

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$$y = 0 \quad \text{and} \quad \frac{dy}{d\xi} = 0 \quad (3.36)$$

to give

$$y = \frac{P_o H^3}{EI} \left\{ \frac{1}{6} \left( 1 - \frac{1\beta^2 H^2}{\gamma^2} \right) (\xi^3 - 3\xi + 2) + \frac{1\beta^2 H^2}{\gamma^4} \left( 1 - \xi + \frac{\sinh \delta \xi - \sinh \delta}{\gamma \cosh \delta} \right) \right\} \quad (3.37)$$

## 2) Polynomial load distribution

Again, considering the action of any term  $p_n \xi^n$  in (3.8), the applied moment at any level  $\xi$  on the coupled walls is given by,

$$M = p_n H^2 \frac{n!}{(n+2)!} \xi^{n+2} \quad (3.38)$$

On substituting equation (3.38) into equation (3.26), the differential equation becomes

$$\frac{d^2 T}{d\xi^2} - \gamma^2 T = -\beta^2 H^4 p_n \frac{n!}{(n+2)!} \xi^{n+2} \quad (3.39)$$

The solution of equation (3.39) may be shown to be

$$T = p_n H^4 \beta^2 \left\{ C_1 \sinh \delta \xi + C_2 \cosh \delta \xi + \frac{n! \sin^2(n+3) \frac{\pi}{2}}{\gamma^{n+4}} + \sum_{i=0}^{n+1} \frac{n! \sin^2(i+1) \frac{\pi}{2}}{\gamma^{i+2} (n-i+2)!} \xi^{n-i+2} \right\} \dots (3.40)$$

By applying the boundary conditions (3.29) and (3.30) the constants are found to be

$$C_1 = \frac{n!}{n+4} \tanh \delta \sin^2(n+3) \frac{\pi}{2} -$$

$$\frac{1}{\delta \cosh \delta} \sum_{i=0}^{n+1} \frac{n!}{\delta^{i+2}} \frac{\sin^2(i+1) \frac{\pi}{2}}{(n-i+1)!} \quad (3.41)$$

$$C_2 = \frac{-n!}{\delta^{n+4}} \sin^2(n+3) \frac{\pi}{2} \quad (3.42)$$

By integrating equation (3.25) and substituting in the boundary conditions (3.36), the deflection at level  $\xi$  is given by

$$y = \frac{p_n H^4}{EI} \left\{ \left( \frac{n!}{(n+4)!} \xi^{n+4} - \frac{n! \xi}{(n+3)!} + \frac{n!(n+3)}{(n+4)!} \right) - \right.$$

$$\frac{1}{\delta^4} \frac{p^2 H^2}{\delta^4} \left[ \left( \frac{n!}{\delta^n} \left( \frac{\xi^2}{2} - \xi + \frac{1}{2} \right) + \frac{n!(1 - \cosh \delta (1 - \xi))}{\delta^2 \cosh \delta} \right) \sin^2(n+3) \frac{\pi}{2} + \right.$$

$$\sum_{i=0}^{n+1} \left( \left( \frac{n!}{(n-i+4)!} \xi^{n-i+4} - \frac{n! \xi}{(n-i+3)!} + \frac{n!(n-i+3)}{(n-i+4)!} \right) \frac{1}{\delta^{i-2}} - \right.$$

$$\left. \left. \frac{n!}{\delta^i (n-i+1)!} \left( 1 - \xi + \frac{\sinh \delta \xi - \sinh \delta}{\delta \cosh \delta} \right) \sin^2(i+1) \frac{\pi}{2} \right] \right\}$$

..... (3.43)

The combined deflection due to the top concentrated load and the polynomial load gives

$$y = \frac{P_0 H^3}{EI} \left\{ \frac{1}{6} \left( 1 - \frac{1\beta^2 H^2}{\gamma^2} \right) (\xi^3 - 3\xi + 2) + \right.$$

$$\left. \frac{1\beta^2 H^2}{\gamma^4} \left( 1 - \xi + \frac{\sinh \delta \xi - \sinh \delta}{\delta \cosh \delta} \right) \right\} +$$

$$\frac{P_n H^4}{EI} \left\{ \left( \frac{n!}{(n+4)!} \xi^{n+4} - \frac{n! \xi}{(n+3)!} + \frac{n!(n+3)}{(n+4)!} \right) - \right.$$

$$\left. \frac{1\beta^2 H^2}{\gamma^4} \left[ \left( \frac{n!}{\gamma^n} \left( \frac{\xi^2}{2} - \xi + \frac{1}{2} \right) + \frac{n!(1 - \cosh \delta (1 - \xi))}{\gamma^2 \cosh \delta} \right) \sin^2(n+3) \frac{\pi}{2} + \right. \right.$$

$$\left. \sum_{i=0}^{n+1} \left( \left( \frac{n!}{(n-i+4)!} \xi^{n-i+4} - \frac{n! \xi}{(n-i+3)!} + \frac{n!(n-i+3)}{(n-i+4)!} \right) \frac{1}{\gamma^{i-2}} - \right.$$

$$\left. \left. \frac{n!}{\gamma^i (n-i+1)!} \left( 1 - \xi + \frac{\sinh \delta \xi - \sinh \delta}{\delta \cosh \delta} \right) \sin^2(i+1) \frac{\pi}{2} \right] \right\}$$

..... (3.44)

### 3.5 Torsion of individual elements

#### 3.5.1 Elements of constant torsional stiffness

The distribution of twisting moment on each element is assumed to be of an analogous form to the direct load distribution, that is, the twisting moment,  $t$ , carried by any particular elements for the different assumed distributions are expressed as follows

- 1) Concentrated torque at the top

$$t = T_0 \quad (3.45)$$

- 2) Polynomial torque distribution

$$t = \sum_{m=0}^n t_m \xi^m \quad (3.46)$$

The torsional moment on any assembly at any level is related to the resultant twist by,

$$T = H \int t \, d\xi = - \frac{C}{H} \frac{d\theta}{d\xi} \quad (3.47)$$

where  $C$  is the torsional rigidity of the cross-section.

The twisting moment-rotation relationship for any assumed torque distribution can be obtained by substituting the corresponding torque distribution into equation (3.47). Equation (3.47) may then be integrated, and, on putting in the boundary condition of zero twist at the base, the rotation at any level  $\xi$  due to the assumed applied torque distribution may be expressed as follows

1) Concentrated torque at the top

$$\theta = \frac{H}{C} T_0 (1 - \xi) \quad (3.48)$$

2) Polynomial torque distribution

$$\theta = \frac{H^2}{C} t_n \frac{n!}{(n+2)!} (1 - \xi^{n+2}) \quad (3.49)$$

3.5.2 Elements of variable torsional stiffness

For elements of variable torsional stiffness throughout the height, such as that of a shear core with openings, where the coupling effects of the linter beams and floor slabs restrain the rotation of the element to a variable degree (Fig. 3.3), the standard 'strength of materials' procedure fails to represent the situation, and resort to more rigorous methods has to be made. The method adopted in this analysis is similar to that for coupled shear walls where the continuous connection technique is employed, and is set out in Reference 22. On following the same procedure, the governing differential equation for the rotation at any level of a perforated core is found to be

$$\frac{d^3 \theta}{dx^3} - \alpha^2 \frac{d\theta}{dx} = -\beta_T^2 T \quad (3.50)$$

where

$$\alpha^2 = \beta_T^2 GJ_0$$

$$\beta_T^2 = \frac{1}{EI_w}$$

In the case of a doubly symmetrical perforated cores of the form shown in Fig.3.3, the values of  $I_w$ ,  $J_o$  and  $J$  become:

$$I_w = \text{warping constant} = \left( \frac{D^2 I_1}{2} + B^2 I_2 \right) + \frac{B^2}{4} (D+d)^2 dt_2$$

$$J_o = \text{torsional constant} = J + \frac{24B^2 D^2 I_c}{b^3 h} \cdot \frac{E}{G}$$

$$J = \text{St. Venant torsional constant} = \frac{2}{3} Bt_1^3 + \frac{4}{3} dt_2^3$$

and  $T$  = is the total applied torque.

Similar expressions for other cross-sectional forms can also be obtained (23).

Equation (3.50) may be rewritten in non-dimensional coordinates

as,

$$\frac{d^3 \theta}{d\xi^3} - \gamma^2 \frac{d\theta}{d\xi} = - \frac{B^2}{T} H^3 T \quad (3.51)$$

where  $\gamma = \alpha H$ .

The general solution of equation (3.51) may be shown as,

$$\theta = C_1 + C_2 \cosh \gamma \xi + C_3 \sinh \gamma \xi + \theta_p \quad (3.52)$$

where  $\theta_p$  is a particular integral solution which depends on the loading function.

### 3.5.2.1 Load cases

(I) Concentrated Torque at the top

The total applied torque at any level is,

$$T = T_0$$

In this case the particular integral solution of equation becomes:

$$\theta_p = \frac{\beta_T^2 H^3}{\gamma^2} T_0 \xi$$

and the complete solution may be written as,

$$\theta = C_1 + C_2 \cosh \gamma \xi + C_3 \sinh \gamma \xi + \frac{\beta_T^2 H^3}{\gamma^2} T_0 \xi \quad (3.53)$$

Assuming that the core is rigidly fixed at the base, which is a realistic assumption for most practical cases, the integration constants may be obtained from the following boundary conditions.

$$\text{At the base, } \xi=1, \theta=0 \text{ and } \frac{d\theta}{d\xi} = 0 \quad (3.54)$$

At the top,  $\xi = 0$ , the bending moment in each wall is zero and hence,

$$\frac{d^2\theta}{d\xi^2} = 0 \quad (3.55)$$

The constants are then found to be,

$$C_1 = \frac{\beta_T^2 H^3 T_0}{\gamma^3} \tanh \gamma - \frac{\beta_T^2 H^3 T_0}{\gamma^2}$$

$$C_2 = 0$$

$$C_3 = -\frac{\beta_T^2 H^3 T_0}{\gamma^3 \cosh \gamma}$$

The general solution for a coupled core structure fixed at the base, free at the top and subjected to a point torque at the top is thus given by,

$$\theta = \frac{\beta_T^2 H^3 T_0}{\gamma^2} \left\{ \xi - \frac{\sinh \gamma \xi}{\gamma \cosh \gamma} + \frac{\tanh \gamma}{\gamma} - 1 \right\} \quad (3.56)$$

## 2) Polynomial Torque distribution

In the case of a polynomial torque distribution of intensity  $t_n \lambda^n$ , the twisting moment at level  $\xi$  may be expressed as,

$$T = H \int_0^\xi t_n \lambda^n d\lambda = \frac{H}{n+1} t_n \xi^{n+1}$$

The particular solution of equation (3.52) may be shown to be

$$\theta_p = \frac{\beta_T^2 H^4 t_n}{\gamma^2} \sum_{i=0}^{n+2} \frac{n!}{\gamma^{2(n-i+2)!}} \xi^{n-i+2} \sin^2(i+1) \frac{\pi}{2}$$

The complete solution of the differential equation (3.52) is given by,

$$\theta = C_1 + C_2 \cosh \gamma \xi + C_3 \sinh \gamma \xi + \frac{\beta_T^2 H^4 t_n}{\gamma^2} \sum_{i=0}^{n+2} \frac{n!}{\gamma^i (n-i+2)!} \xi^{n-i+2} \sin^2(i+1) \frac{\pi}{2} \quad (3.57)$$

Assuming again that the core is fixed at the base and free at the top, the integration constants,  $C_1$ ,  $C_2$  and  $C_3$  are found to be,

$$C_1 = \frac{\beta_T^2 H^4 t_n}{\gamma^2} \left\{ \left( \sum_{i=0}^n \frac{n!}{\gamma^i (n-i)!} \cosh \gamma - \frac{1}{2} \tanh \gamma \sinh \gamma \right. \right.$$

$$\left. \sum_{i=0}^n \frac{n!}{\gamma^i (n-i)!} + \frac{\tanh \gamma}{\gamma} \sum_{i=0}^{n+1} \frac{n!}{\gamma^i (n-i+1)!} \right.$$

$$\left. - \sum_{i=0}^{n+2} \frac{n!}{(n-i+2)!} \right) \sin^2(i+1) \frac{\pi}{2} \left. \right\}$$

$$C_2 = - \frac{\beta_T^2 H^4 t_n}{\gamma^4} \sum_{i=0}^n \frac{n!}{\gamma^i (n-i)!} \sin^2(i+1) \frac{\pi}{2}$$

$$C_3 = \frac{\beta_T^2 H^4 t_n}{\gamma^2} \left\{ \left( \frac{1}{2} \tanh \gamma \sum_{i=0}^n \frac{n!}{\gamma^i (n-i)!} - \frac{1}{\gamma \cosh \gamma} \right. \right.$$

$$\left. \sum_{i=0}^{n+1} \frac{n!}{\gamma^i (n-i+1)!} \right) \sin^2(i+1) \frac{\pi}{2} \left. \right\}$$

Hence the general solution for a coupled core structure fixed at the base, free at the top and subjected to a general polynomial torque may be written as,

$$\theta = \frac{\beta_T^2 H^4 t_n}{\gamma^4} \left\{ \sum_{i=0}^n \left[ \frac{n!}{\gamma^{i-2}(n-i+2)!} (\xi^{n-i+2} - 1) + \frac{n!}{\gamma^{i-1}(n-i+1)!} \cosh \gamma (\sinh \gamma - \sinh \gamma \xi) + \frac{n!}{\gamma^i (n-i)!} (\tanh \gamma \sinh \gamma \xi - \cosh \gamma \xi - \tanh \gamma \sinh \gamma + \cosh \gamma) \right] \sin^2(i+1)\frac{\pi}{2} + \frac{n!}{\gamma^{n-1}} \left( \xi - \frac{\sinh \gamma \xi}{\gamma \cosh \gamma} + \frac{\tanh \gamma}{\gamma} - 1 \right) \sin^2(n+2)\frac{\pi}{2} \right\} \quad (3.58)$$

For a single channel-section without coupling, equations (3.56), (3.57) and (3.58) can again be obtained provided that the constants are redefined as follows:

$$\gamma^2 = \beta_T^2 H^2 GJ_0$$

$$\beta_T^2 = \frac{1}{EI_w}$$

$$J_0 = J = \text{st. Venenat torsional constant} = \frac{\beta t_1^3}{3} + \frac{dt_2^3}{3}$$

$$I_w = \text{warping constant} = I_1 e^2 + I_2 \beta^2 + \frac{\beta^2}{4} (e+d)(D+2e)dt$$

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and  $e = \text{the shear centre of the section from the web} = \frac{3d^2}{B(1 + \frac{6d}{B})}$

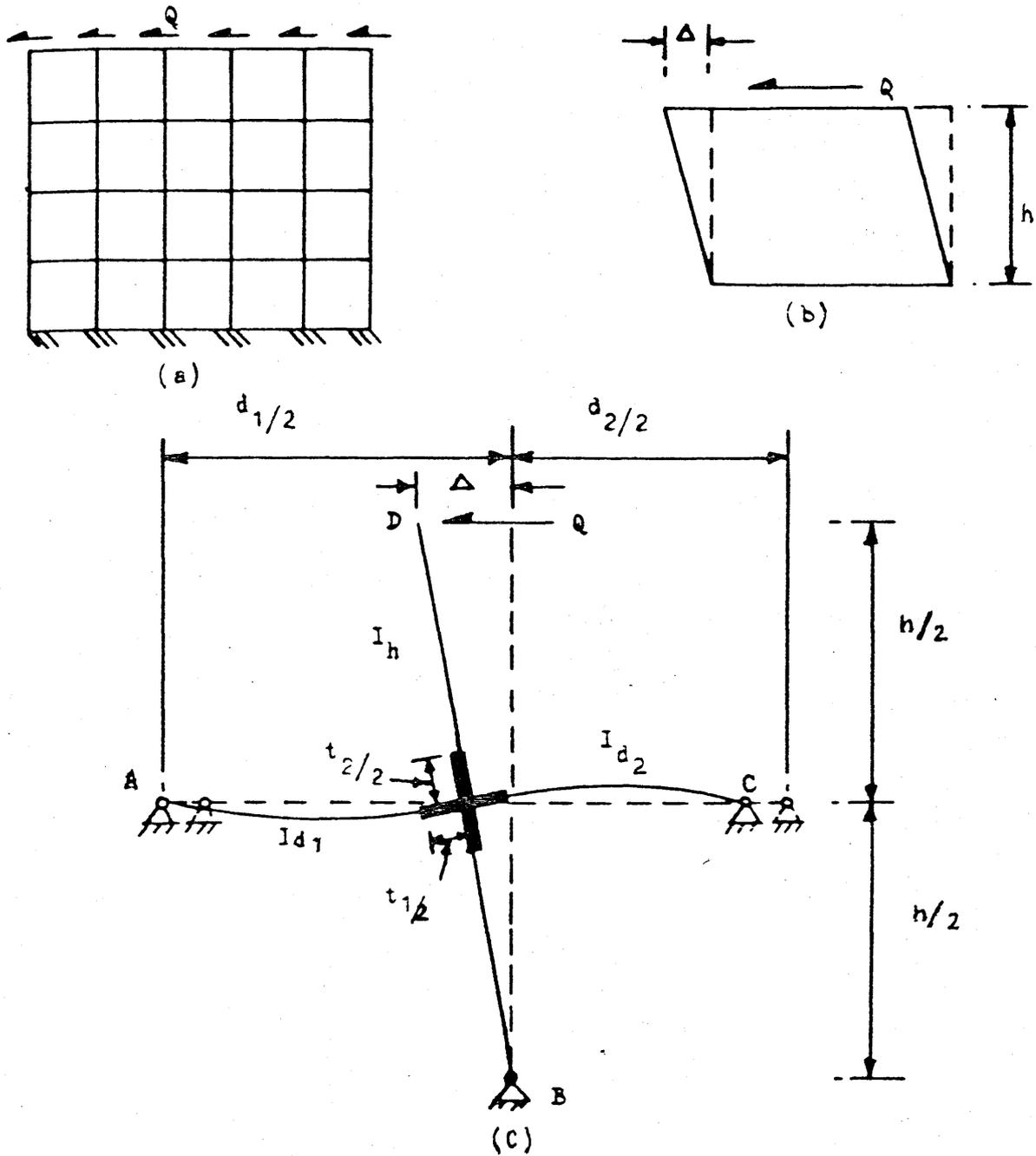
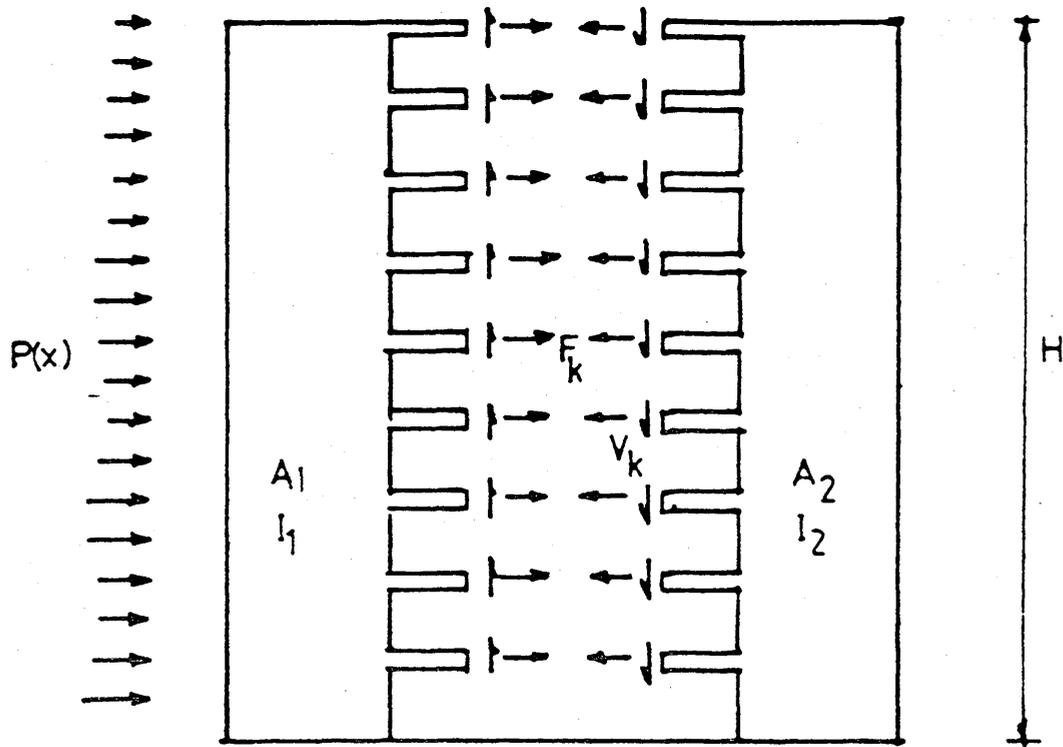
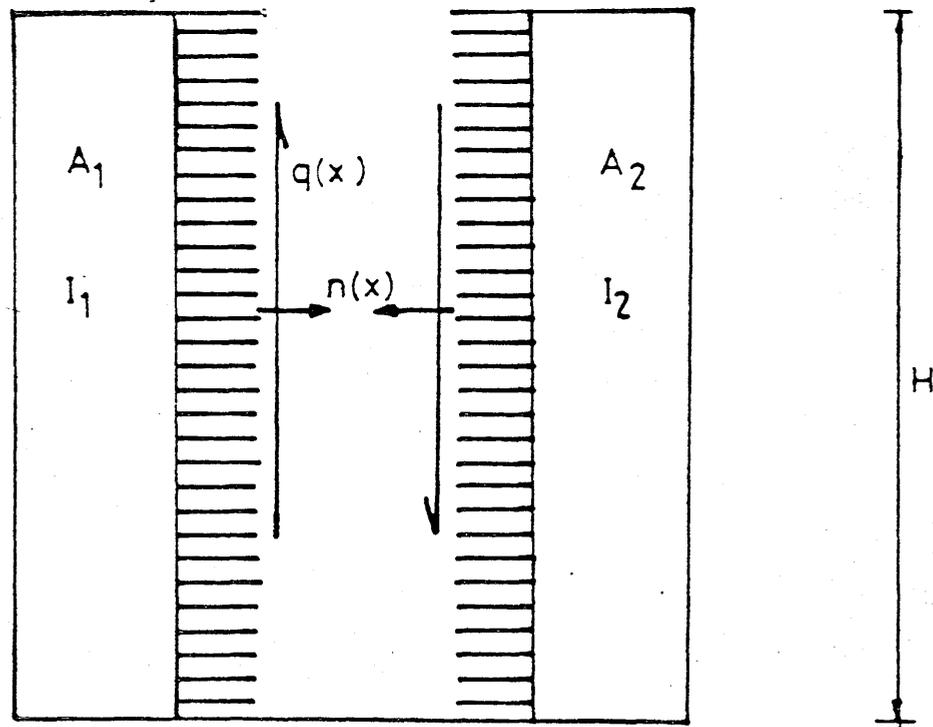


Fig. 3.1 Representation of frame panel by equivalent shear cantilever element



(a) Discrete forces on Cut ends of connecting beams



(b) Continuous force distribution on cut ends

Fig. 3.2 Replacement of connecting beams by an equivalent continuous medium.

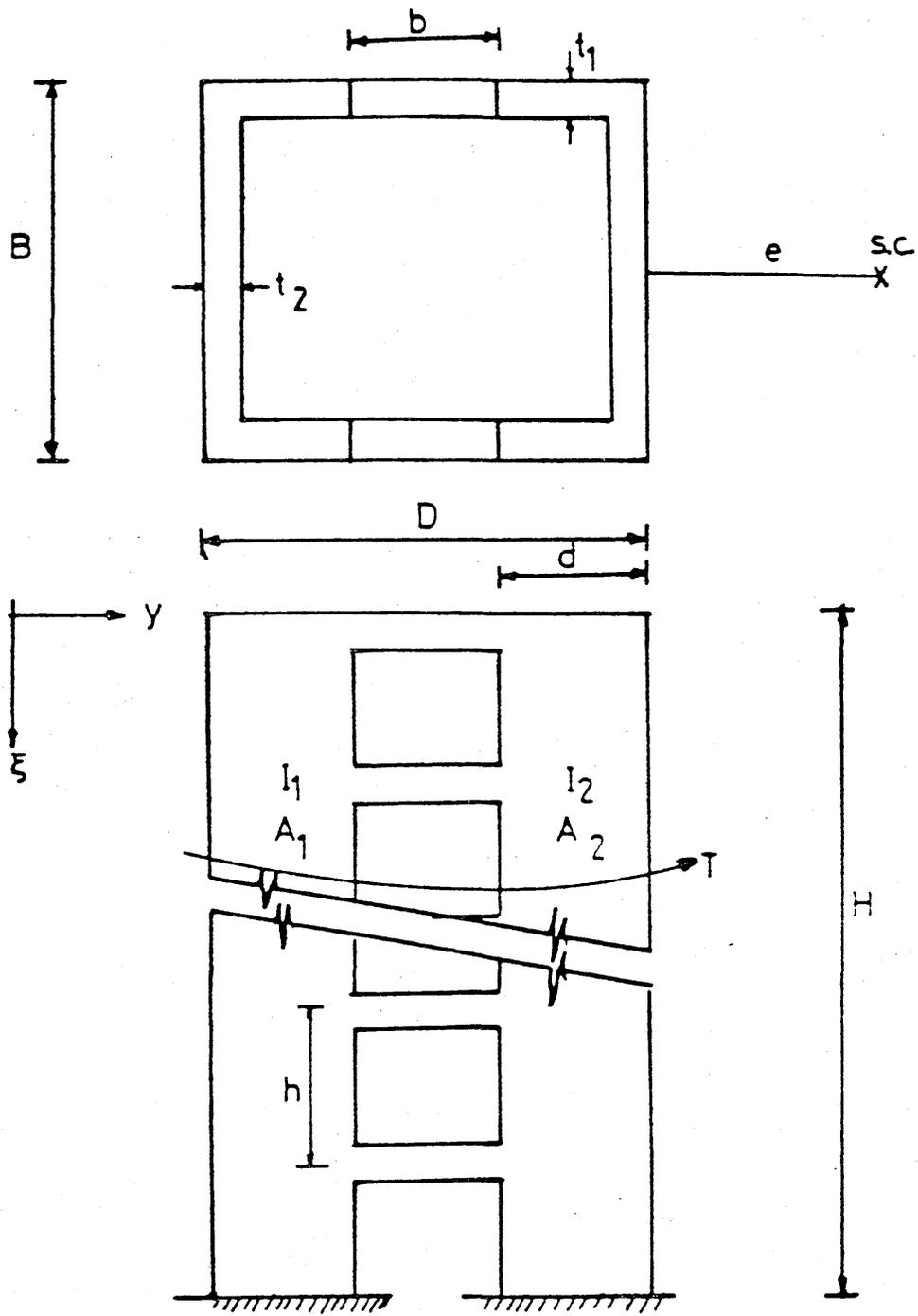


Fig. 3.3: Core Structure

## NUMERICAL PARAMETER STUDIES

4.1 Introduction

In order to examine the validity and accuracy of the approximate method described in Chapter 2 for the analysis of symmetric and asymmetric structures consisting of different load bearing elements, several numerical investigations of the structural behaviour have been carried out.

These investigations consist of two parts and concern wall-frame structures. The first part is an investigation of a general wall-frame structure with **symmetrical** plan form as shown in Fig. 4.1. A representative stiffness parameter and the number of reference levels are varied and the results of the analysis are presented in graphical form.

The second part is a comparison between the results of the present analysis and published data on tall building structures taken from different publications<sup>(10,13)</sup>. Because of the limited number of published data on structures of the forms considered it has proved possible to compare with only these publications. The structures in this part are asymmetrical in plan-form to produce the effects of torsion. Again, the number of reference levels

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is varied and the results compared.

#### 4.2 Interaction between walls and frames

As stated in section 2.1, walls and frames independently exhibit different behaviour when they deform under the action of similar horizontal loads. If they are constrained to deflect together by floor slabs, they will have to be 'pushed apart' at the top, and 'pulled together' near the base.

The equations governing the behaviour of the structure, subjected to a wind load of intensity  $w$  are, for a wall,

$$\frac{EI}{H^4} \frac{d^4 y}{d\xi^4} = w - n_1 \quad (4.1)$$

and for a frame,

$$- \frac{GA}{H^2} \frac{d^2 y}{d\xi^2} = n_1 \quad (4.2)$$

where  $n_1$  is the horizontal interactive force distributed over the height of the structure.

Adding equations (4.1) and (4.2) and dividing through by  $\frac{EI}{H^4}$  yields

$$\frac{d^4 y}{d\xi^4} - \gamma^2 \frac{d^2 y}{d\xi^2} = \frac{wH^4}{EI} \quad (4.3)$$

where  $\gamma^2$  is the relative stiffness ratio, given by

$$\gamma^2 = \frac{H^2 GA}{EI} \quad \text{and} \quad \xi = x/H$$

Equation (4.3) may readily be integrated to give a closed-form solution for any standard applied loading. The four constants of integration which arise in the solution may be determined from the boundary conditions at the base and the top of the structure.

In the particular case of a structure which is free at the top and rigidly built in at the base and subjected to a uniformly distributed load of intensity  $w$ , the complete solution becomes, in non-dimensional form

$$y = \frac{wH^4}{\gamma^2 EI} \left\{ \frac{1}{2}(1 - \xi^2) - \frac{1 - \cosh\delta(1-\xi) + \delta(\sinh\delta - \sinh\delta\xi)}{\gamma^2 \cosh\delta} \right\} \dots\dots\dots(4.4)$$

The shear forces in the wall and frame are

$$S_b = -wH \left\{ \frac{\sinh\delta(1-\xi) - \delta \cosh\delta\xi}{\gamma \cosh\delta} \right\} \quad (4.5)$$

$$S_s = wH \left\{ \xi + \frac{\delta \sinh\delta(1-\xi) - \delta^2 \cosh\delta\xi}{\gamma^2 \cosh\delta} \right\} \quad (4.6)$$

where the subscripts 'b' and 's' refer to the flexural and shear cantilevers respectively. The bending moment in the wall and frame are

$$M_b = \frac{wH^2}{\gamma^2} \left\{ -1 + \frac{\cosh\delta(1-\xi) + \delta \sinh\delta\xi}{\cosh\delta} \right\} \quad (4.7)$$

$$M_s = wH^2 \left\{ \frac{\xi^2}{2} + \frac{-\cosh\delta(1-\xi) - \delta \sinh\delta\xi}{\gamma^2 \cosh\delta} + \frac{1}{\gamma^2} \right\} \quad (4.8)$$

### 4.3 Symmetrical wall-frame structure

In order to investigate the degree to which, and the conditions under which, the present method of analysis are applicable, a wall-frame structure as shown in plan in Fig 4.1 is considered and analysed by the 'exact' method in the preceding section, which is independent of the number of reference levels. The results from the approximate method given in Chapter 2 using different number of reference levels are compared with those from the 'exact' analysis.

Equations (4.4) to (4.8) show that the load distributions are dependent on the relative stiffness  $\delta$  where  $\delta = H\sqrt{\frac{GA}{EI}}$ . It is then necessary to examine the effect of this parameter on the results.  $\delta$  is the significant parameter governing the behaviour of a wall-frame structure. Buildings that are predominantly shear walls with little or no frame action (i.e when  $\delta=0$ ) will have a low value of  $\delta$  and will behave as a flexural member. As the proportion of frame action increases,  $\delta$  increases, and the structure behaves more as a shear member. Similarly as the height of a structure of given cross-sectional proportions is increased, it behaves more as a shear member<sup>(5)</sup>.

In this example,  $\delta$  is varied from 0 to 6 in steps of 1.0.

#### 4.3.1 Deflections, bending moments and shear forces

Results were obtained by varying the number of reference levels from 2 to 8 and also the value of  $\delta$  for the deflections of the structure, bending moments in the walls and shear forces in the frames. These are plotted in Fig. 4.2 to 4.21. They are shown in non-dimensional form as  $Y/Y(0)$ ,  $M/M(1)$  and  $Q/Q(1)$  respectively.  $Y(0)$  is given by  $Y(0) = \frac{wH^4}{8EI}$  that is the deflection at the end of the cantilever wall due to load intensity  $w$ .  $M(1)$  and  $Q(1)$  are the applied moment and shear at the base respectively.

Points of deflections using corresponding number of reference levels and their chosen positions are plotted in the graphs of Fig. 4.2 to 4.8. The positions of the reference levels are chosen to be at equal intervals from the top of the assembly. Some points overlap each other and where this occurs they are labelled for identification accordingly with figures representing the number of reference levels.

Generally the deflections (Fig 4.2 to 4.8) correspond closely to the 'exact' solution for the complete range of  $\delta$ . The bending moment values agree with the 'exact' solution for about 6 reference levels. At the base, for  $\delta = 0$  (Fig. 4.9) and using 6, 7 and 8 levels, the difference between the results obtained and the 'exact' solution are 3%, 8% and 12% respectively. As  $\delta$  increases these

differences decreases.

For the shear forces distribution, Fig. 4.16 to 4.21 for the range of values of  $\delta$ , the results agree closely for about 4 to 6 reference levels. Hence it is accurate for this central range only. At other number of reference levels the solutions fluctuate especially at the top and bottom of the structure, producing unreliable results.

In general it can be seen from these Figures that the solution varies with the number of reference levels used in the analysis. Using 2 and 3 reference levels seems to be insufficient to portray the shear force distributions in the frames accurately. The instability of the solution using more than 6 reference levels may be due to the shape of the load distribution curves being indistinct for the higher exponent terms and the successive lines in the matrix of flexibility coefficients become increasingly similar. The matrix then tends towards singularity and the solution fluctuates.

#### 4.4 Comparison with published data

In order to examine the accuracy and validity of the approximate method, two example structures which were analysed by previous investigators are considered. They are essentially wall-frame structures, a continuation to section 4.3, but with asymmetrical plan-forms.

Therefore an investigation was carried out for structures under lateral loads and torsional moments. These examples have been taken as originally presented in the publications thus the unit of applied load in example 1 is in imperial units and in example 2 all units are metric.

#### Example 1

This example is a ten-storey model structure of the plan-form shown in Fig. 4.22(a) which was first considered by Stamato and Mancini<sup>(10)</sup>. The model of this structure was first tested by Stamato and analysed by his proposed method. It was based on the continuous approach whereby the floor slabs are assumed a 'continuous medium' consisting of an infinite number of horizontal diaphragms with no transverse stiffness but infinite in-plane rigidity. A matrix analysis was used to derive solutions for the deflection, rotation and the internal forces. Frame assemblies were replaced by equivalent shear cantilevers. Because of the basic assumptions of this example are similar to the present analysis this example is taken to compare with present results.

All the columns were  $\frac{3}{4}$  in. square, the beams were  $\frac{1}{4}$  in. thick and  $\frac{5}{4}$  in. deep and the wall dimensions were  $\frac{1}{4}$  in. and 4 in. in the z and y directions respectively, and have a constant cross-section throughout the height of the building. The horizontal uniformly distributed load was  $w=0.2$  lb/in. applied in the plane of frame 2.

All frames are identical and they are considered to act independently<sup>(10)</sup>, though they appear to form a closed tube. The relevant structural data are

Storey height  $h = 5$  in.

Total model height,  $H = 50$  in.

For shear wall  $I_z = 1.3333$  in<sup>4</sup>.

$E = 4.2 \times 10^5$  lb/in<sup>2</sup>.

For each frame  $GA = 2960$  lb.

For this analysis the datum axis is chosen at point O, the centroidal axis of the shear wall (fig. 4.22(a)). The choice of datum is the same as Stamato's.

The model is analysed using the approximate method in Chapter 2 as follows

- 1) Using the two-dimensional analysis with displacements of the elements in one direction only (the y direction) and without taking into account the stiffnesses of frames 4 and 5.
- 2) Using the above analysis but incorporating the stiffnesses of frames 4 and 5.
- 3) Using the complete three-dimensional analysis with displacements in two orthogonal directions (the y and z directions).

Hereafter the above analyses are referred to 2D, 2DS and 3D respectively.

The distributions of the deflections of frame 2, rotations, bending moments in the wall, and shear forces in frames 2,3,4 and 5 for each of the three analyses above are shown in Fig. 4.23 to 4.39. The results are compared to those given by Stamato.

From the graphs, comparing the 2D and 2DS analyses it is shown that incorporating the orthogonal stiffnesses of frames 4 and 5 in the two-dimensional analysis gives a closer agreement to the datum results. The 3D analysis is very much similar to the 2DS one.

The graphs in the 3D analysis (Fig. 4.34 to 4.39) show that by using 4 to 6 reference levels close agreement to the results of Stamato is achieved. Tabulated below are the differences to Stamato's analysis for the base bending moment in the wall and the base shears in frames 2,3 and 4 using the various number of reference levels.

Number of ref. levels	Percentage Difference			
	Base moment in wall 1	Base shear in frame 2	Base shear in frame 3	Base shear in frame 4
2	1%	50%	6%	63%
3	1%	16%	2%	20%
4	0%	1%	0%	1%
5	0%	0%	0%	0%
6	2%	2%	3%	3%
7	15%	4%	6%	12%
8	9%	30%	28%	16%

Table 1. Percentage difference between the results of present analysis and that of Stamato's results.

Therefore again, it can be concluded that close agreement, and consistent results are achieved using 4 to 6 levels. Although Stamato's analysis is itself an approximate analysis, nevertheless, this central range of number of reference level gives consistent results with it. Using 2,3,7 and 8 levels give fluctuating results, especially at the top and bottom of the structure. These are probably due to the reasons given earlier on in section 4.3.1.

### Example 2

This second example is another wall-frame structure. The plan form of the building considered is shown in Fig. 4.22(b). It has been analysed by Mortelmans et al<sup>(13)</sup> initially and was later used by Khachatoorian<sup>(17)</sup> in his 'exact' analysis. It consists of nine frames and a core. The dimensions of the columns were 0.28m and 0.7m in the z and y directions, horizontal beams were 0.2m thick and 0.4m deep and the dimensions of the core were 6.5m and 4m in z and y directions. The wall thickness of the core was 0.18m and the lateral uniformly distributed wind load was  $w = 1 \text{ KN/m}^2$ . The relevant structural data are

Storey height  $h = 3\text{m}$

Total building height  $H = 30\text{m}$

For core  $I_2 = 9.9897\text{m}^4$

$J_0 = 31.3172\text{m}^4$

$E = 2.5 \times 10^7 \text{ KN/m}^2$

$$G_0 = 1.0417 \times 10^7 \text{ KN/m}^2$$

For each frame  $GA = 60194 \text{ KN}$

As the structure is asymmetric in plan, the effects of bending and torsion upon the results of the present approximate analysis can be investigated.

In many tall buildings box cores provides an important contribution both to the bending and the torsional stiffness of the building. Mortelmans et al have assumed that, due to crack formation, the bending and torsional rigidities of the core are reduced to one third and one tenth of the original value respectively.

In this example variations of the bending and torsional stiffnesses of the core is included. They are classified as Case I ( $I_z, J_0$ ), Case II ( $I_z/3, J_0$ ), Case III ( $I_z, J_0/10$ ) and Case IV ( $I_z/3, J_0/10$ ), where ( $I_z/3, J_0/10$ ) for example denotes a case where one third of the bending stiffness is used in combination with one-tenth of the torsional stiffness.

The results using the present analysis are compared with both results from Khachatoorian and Mortelmans et al. Khachatoorian presented an 'exact' analysis with the frames modelled as a shear cantilever.

The analysis was for three-dimensional uniform asymmetric structure consisting of cores, coupled walls and rigidly jointed frameworks in two orthogonal directions.

By considering the general equilibrium conditions of structure three governing differential equations (two translational and one rotational) were generated. By integrating them and using known boundary conditions, they are solved simultaneously. Closed-form solutions were obtained for the deflections, rotation and the internal forces.

Mortelmans et al analysed the structure using an approximate frame method of analysis which is based on the following assumptions:

- (i) the rotations at all the junction nodes of one beam have the same size - i.e. the slopes are equal.
- (ii) the beams are of constant stiffness.
- (iii) the bending moments concentrated in the nodes and exerted by the beams may be spread over the height of the floor.

From the bending and twisting equilibrium equations of the wall and frames, a system of six equations were set up and later reduced to four simultaneous linear equations by expressing two of them as a function of the other four. The equations then be solved simultaneously using a pocket calculator.

Fig. 4.40 to 4.47 show the distributions of the deflections in Frame 1 and the rotations for the various

cases. From the graphs, by using a range of **number** of reference levels from 2 to 8, it is seen that most of the points, especially those using 2 to 6 levels appear to lie on a smooth curve (not shown in graphs), for each of the graphs. These curves correspond more closely to the 'exact' analysis of Khachatoorian.

Fig. 4.44 to 4.47 for cases III and IV (for the torsional stiffness reduce by a tenth), show that there is a large error in using 8 levels. This may be due to the instability of the matrices as a larger number of levels is used.

Fig. 4.49 to 4.50 show the distribution of bending moments in the core (Case 1), and the distribution of the torsional moments in the core (Case I and III) respectively. They all show a closer agreement with the results of Khachatoorian. As a larger number of levels are used, for example 7 and 8, fluctuations occur especially at the top and bottom of the structure.

On the whole, the results show that using about 4 to 6 levels produces sufficiently accurate results and they conform more closely to the 'exact' analysis. This is most probably due to the similar assumptions in the 'exact' analysis whereby the frame actions are modelled as a 'shear cantilever'.

Results from Mortelmans et al are those from another approximate method, using a different set of assumptions as mentioned earlier for the frame action.

For cases I and II there is close agreement between the present method and that of Mortelmans et al, while there are significant differences for cases III and IV. These differences were most probably due to the assumption from Mortelmans et al that at the base of the structure

$$\frac{dy_0}{dx} = \frac{d\theta}{dx} = 0 \text{ where } y_0 \text{ is the displacement of frame 1}$$

and  $\theta$  is the rotation of the structure.

By considering the twisting equilibrium at the base of the structure with mid point of frame 1 as the datum, (Fig.2.2) the equilibrium equation becomes

$$T = \sum_{i=2}^8 (GA)_{f_i} l_{f_i} \frac{d}{dx} (y_0 + l_{f_i} \theta) +$$

$$GJ_0 \frac{d\theta}{dx} = w \cdot 20 \cdot H$$

where  $GA_{f_i}$  is the shear rigidity of one frame and  $l_{f_i}$  is the distance of the frames from from Frame 1.

at  $x = 0$  ,  $\frac{dy_0}{dx} = 0$

therefore

$$\frac{d\theta}{dx} = \frac{w \times 20 \times H}{\sum_{i=2}^8 (GA)_{f_i} l_{f_i}^2 + G_0 J_0}$$

If the torsional effects of the columns are considered negligible,

$$\sum_{i=2}^8 GA_{f_i} l_{f_i}^2 = 0.1709 \times 10^8 \text{ KNm}^2$$

and

$$G_o J_o = 3.2623 \times 10^8 \text{ KNm}^2$$

From the above it is obvious that for this case  $G_o J_o$  is a more important part of the denominator, for any reduction of its value will in turn result in a higher value of  $\frac{d\theta}{dx}$  and hence the overall analysis.

For case I where  $G_o J_o$  is unaltered  $\frac{d\theta}{dx} = 7.0 \times 10^{-5} \text{ m}^{-1}$  while when  $G_o J_o$  is reduced to one tenth of its original value (for cases III and IV) the value of  $\frac{d\theta}{dx} = 4.8 \times 10^{-4} \text{ m}^{-1}$  which is almost seven times larger.

Therefore it may be deduced that the large differences between the results obtained by Mortelmans et al and the present method together with the 'exact' analysis of Khachatoorian for cases III and IV are mainly due to the boundary conditions chosen.

Fig. 4.49 and 4.50 illustrate the distribution of the torsional moments in the core for case I and III respectively. There are significant differences at the base of the core due to the boundary conditions chosen as explained above.

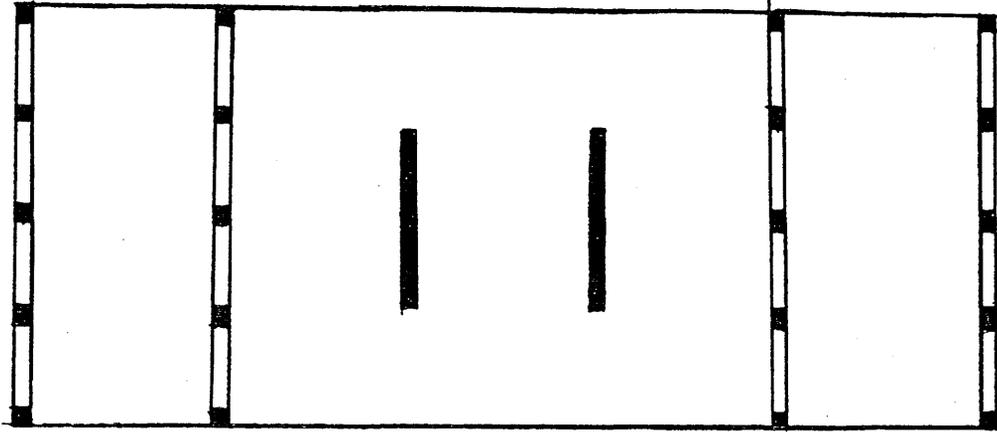


Fig. 4.1 : Example - Wall-Frame Structure

2 3 4 5 6 8

7

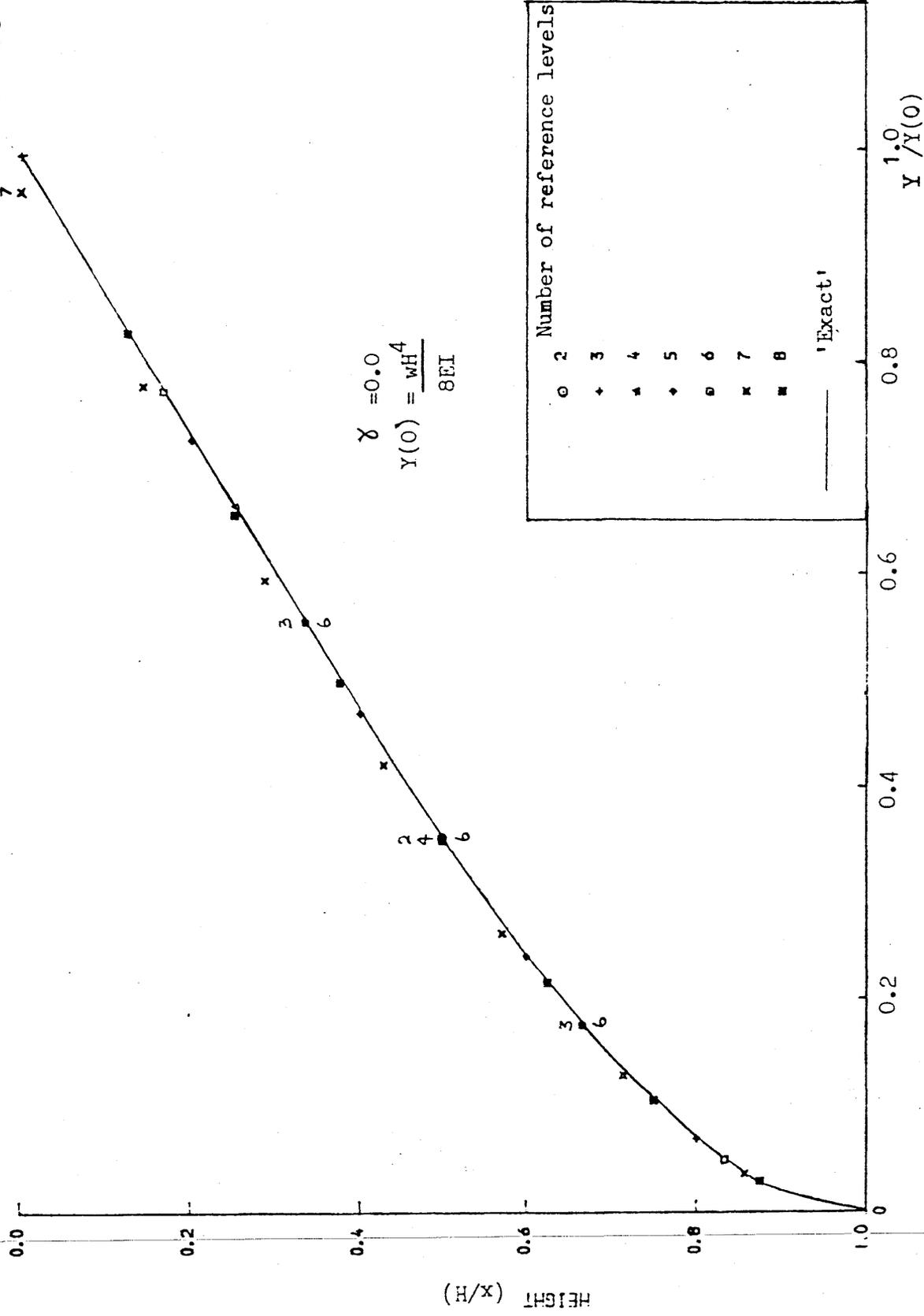


FIG 4.2 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\gamma=0.0$ )

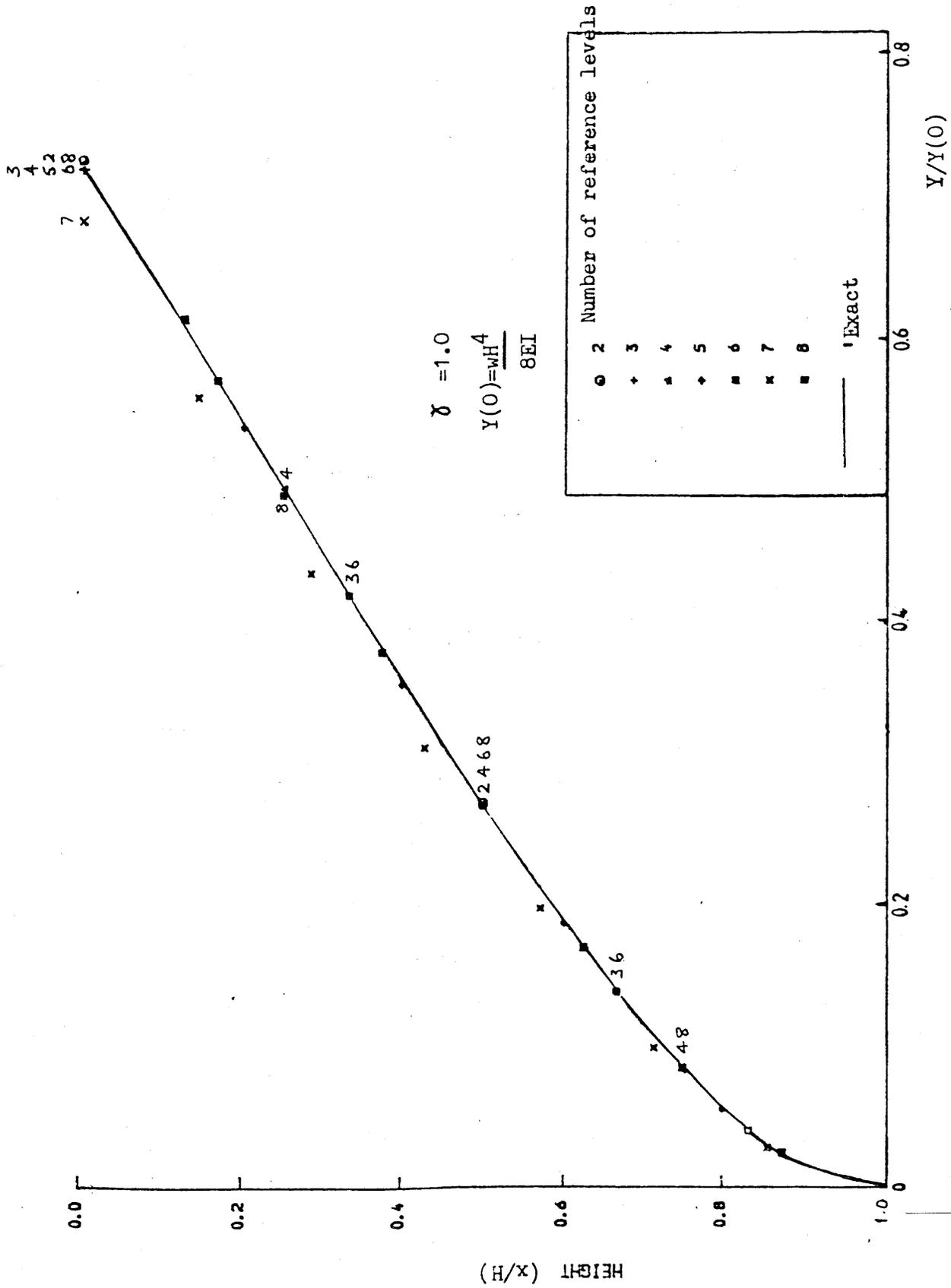


FIG 4.3 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\gamma=1.0$ )

5  
4  
52  
768  
x 10

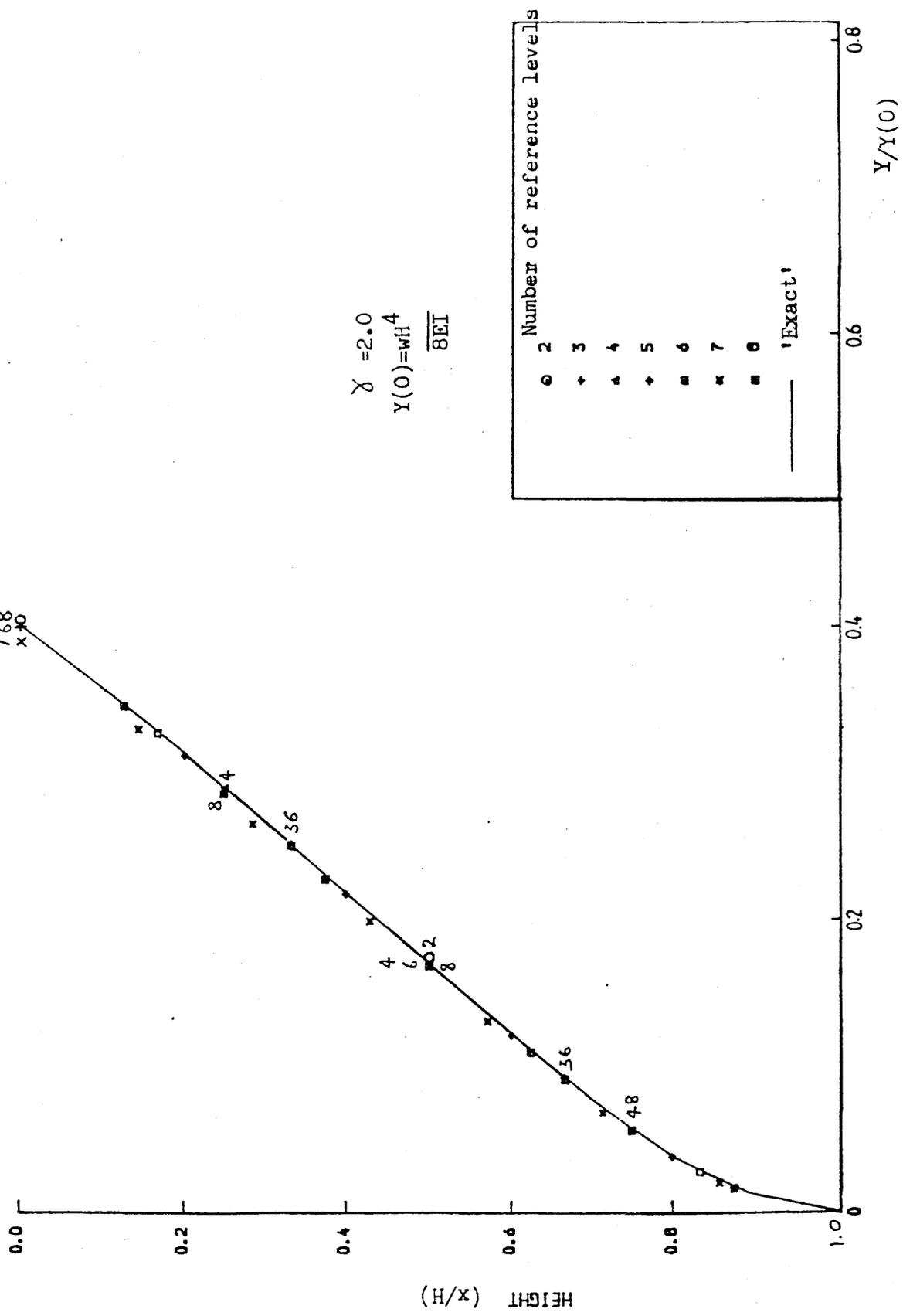


FIG 4.4 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\gamma=2.0$ )

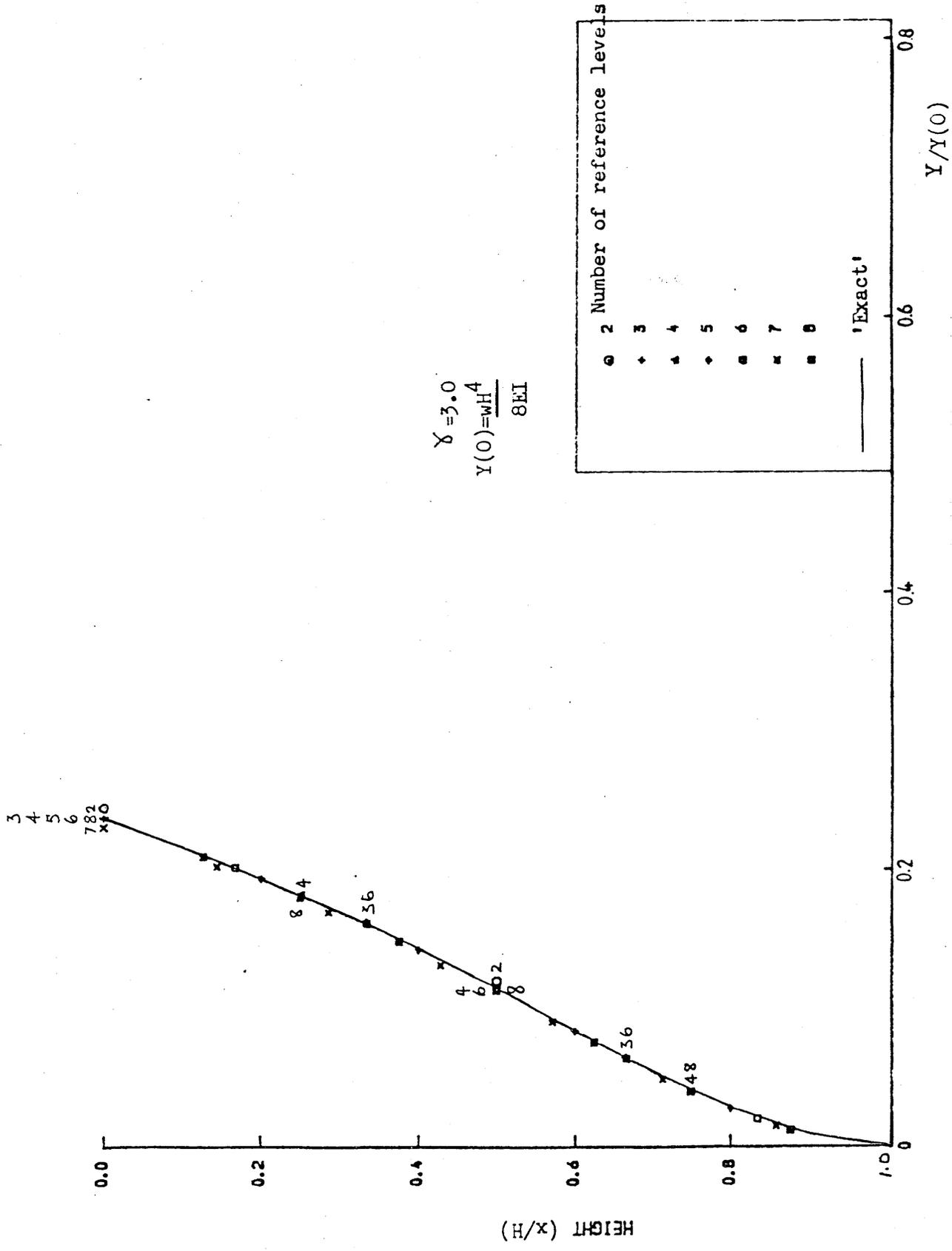


FIG 4.5 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\gamma = 3.0$ )

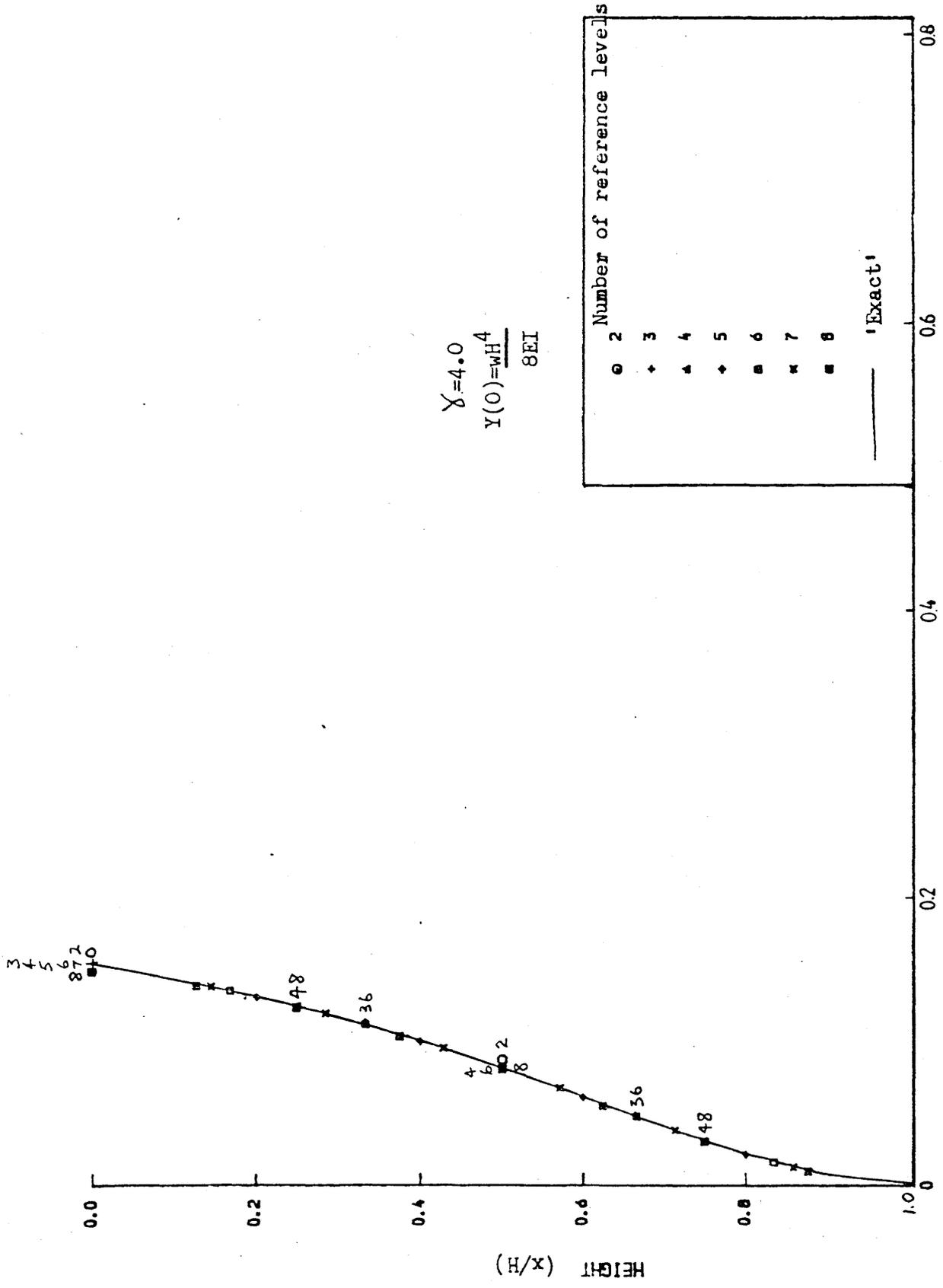


FIG 4.6 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\chi=4.0$ )

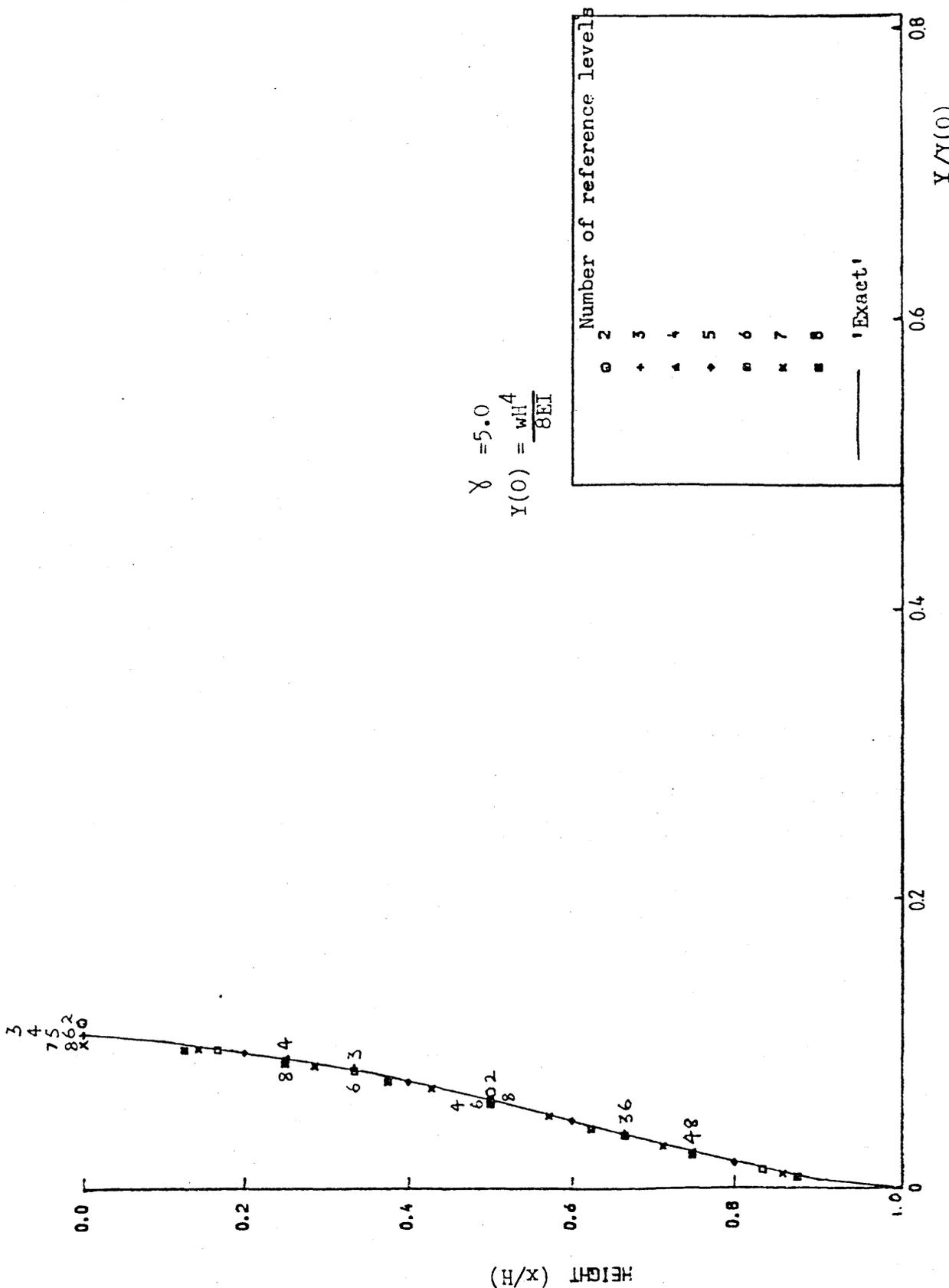


FIG 4.7 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\chi=5.0$ )

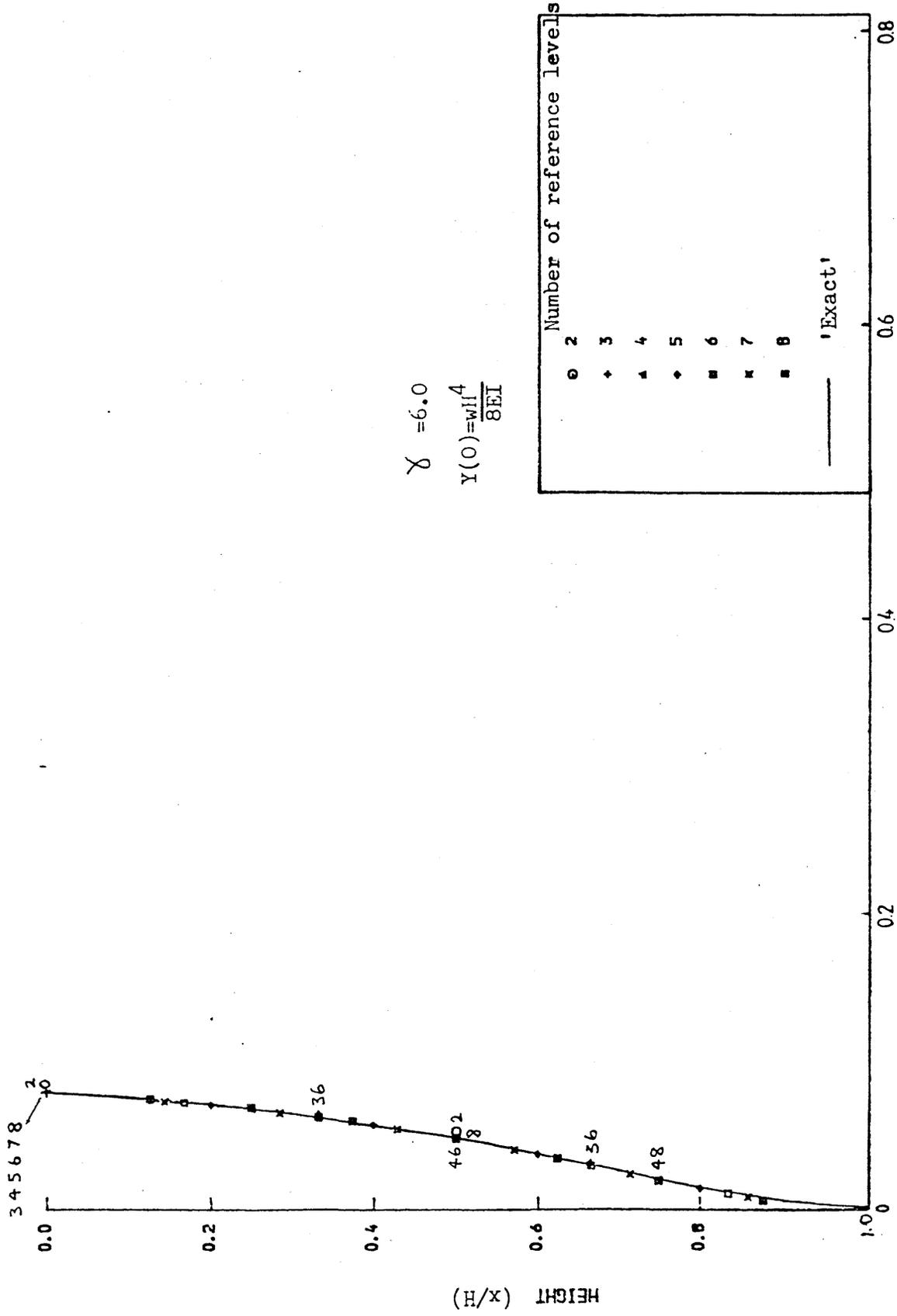


FIG 4.8 DISTRIBUTION OF LATERAL DEFLECTION OF STRUCTURE ( $\chi = 6.0$ )

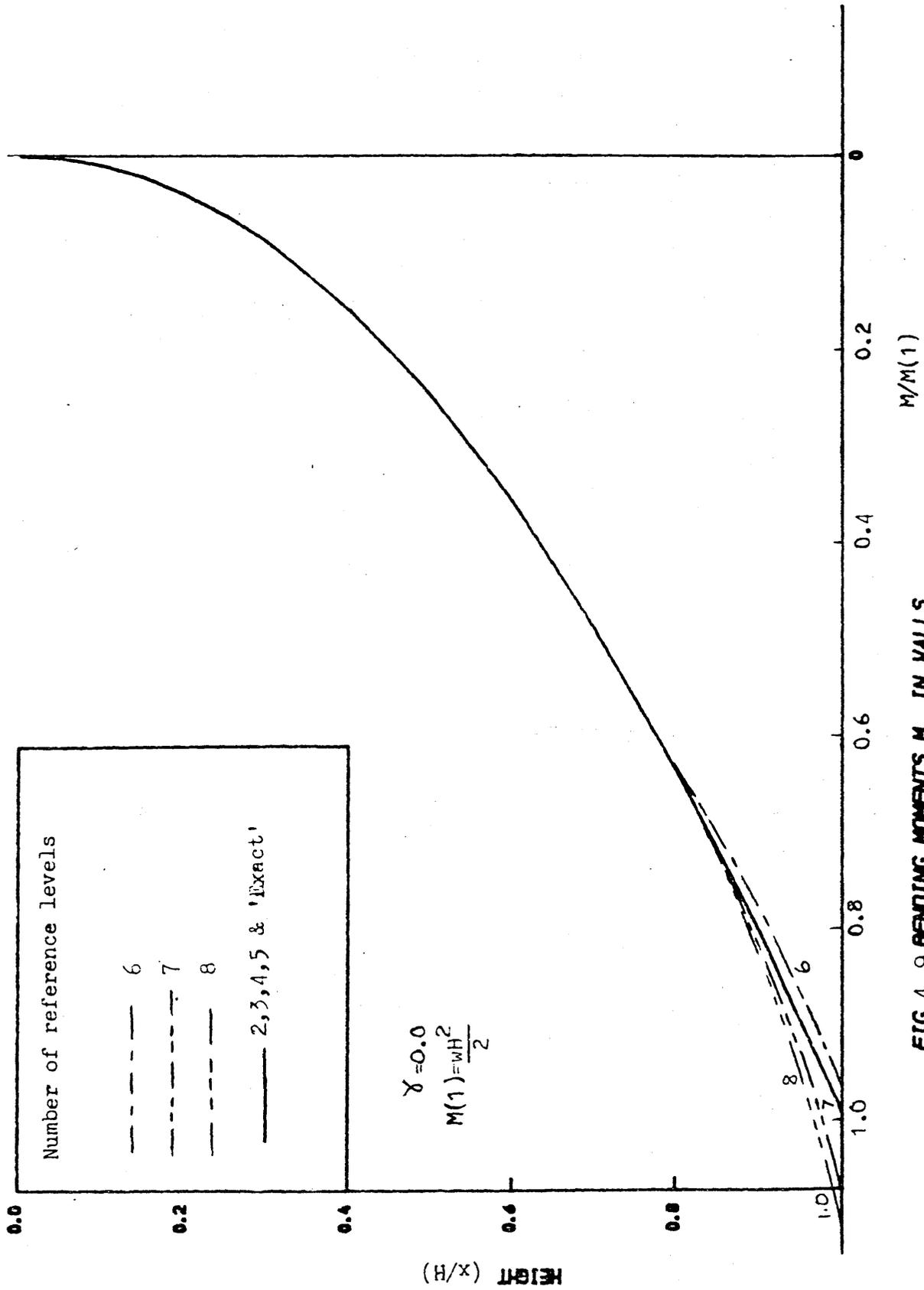


FIG 4.9 BENDING MOMENTS  $M$  IN WALLS

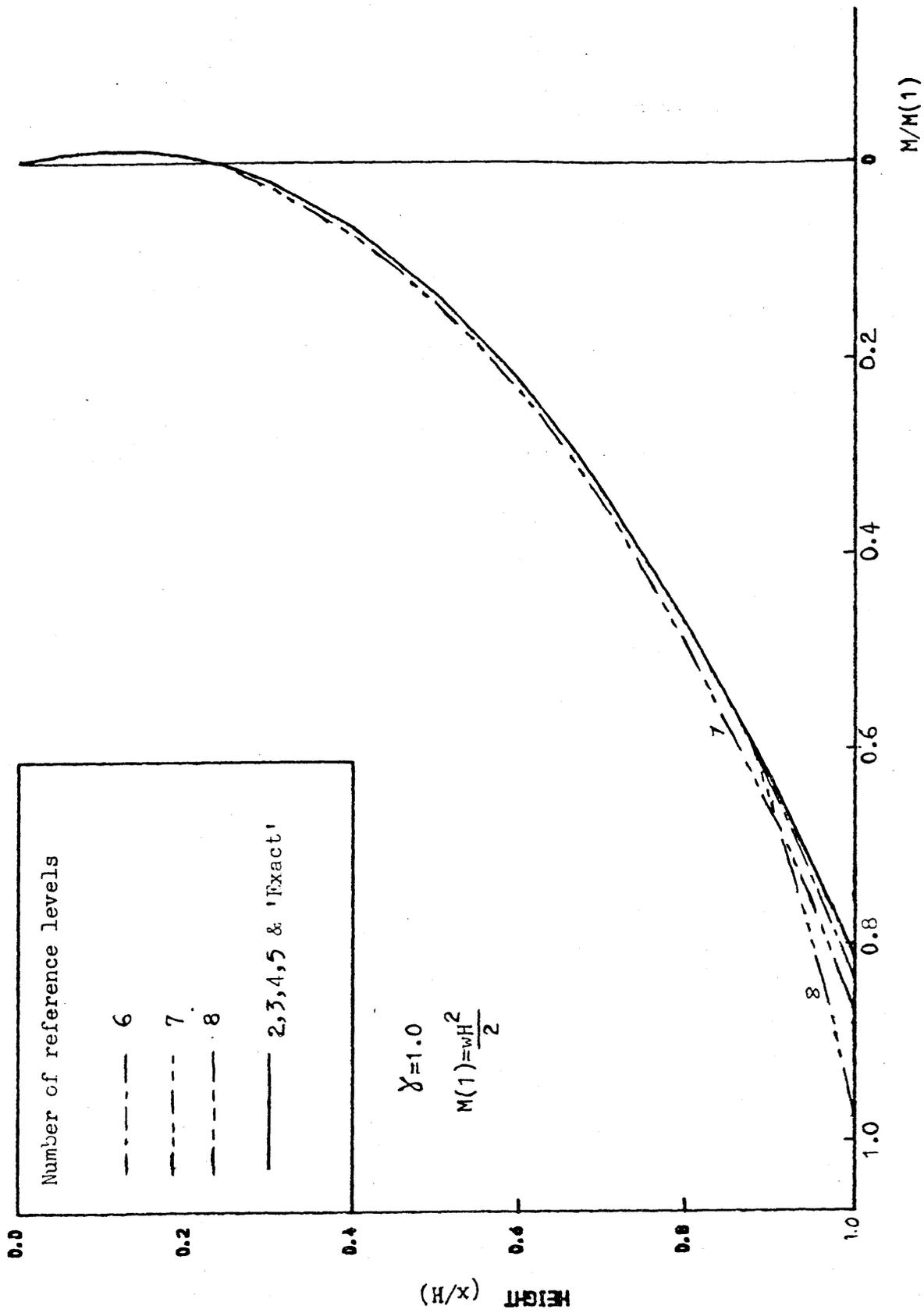


FIG 4. 10 BENDING MOMENTS M IN VALLS

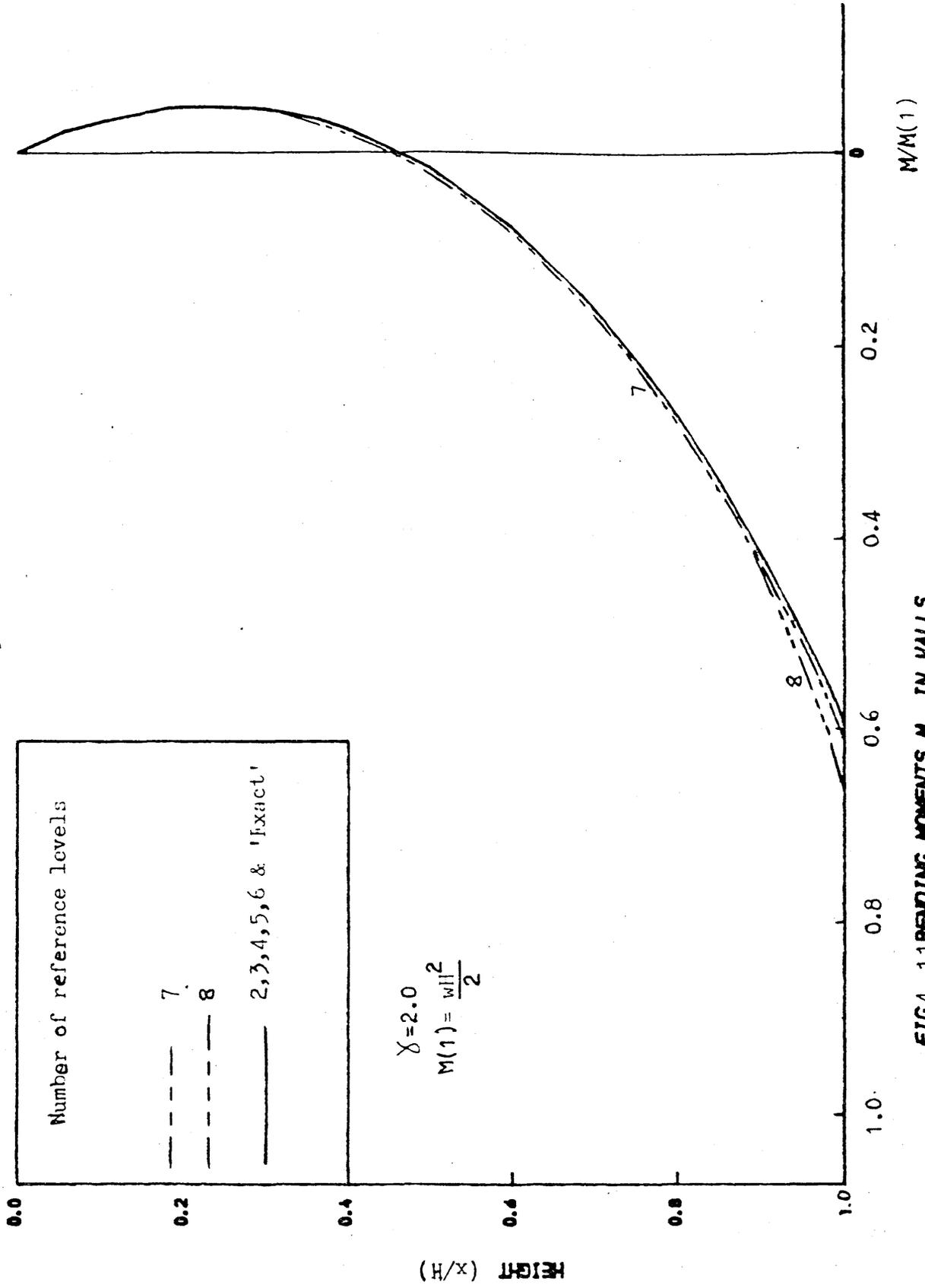


FIG. 4.11 BENDING MOMENTS M IN WALLS

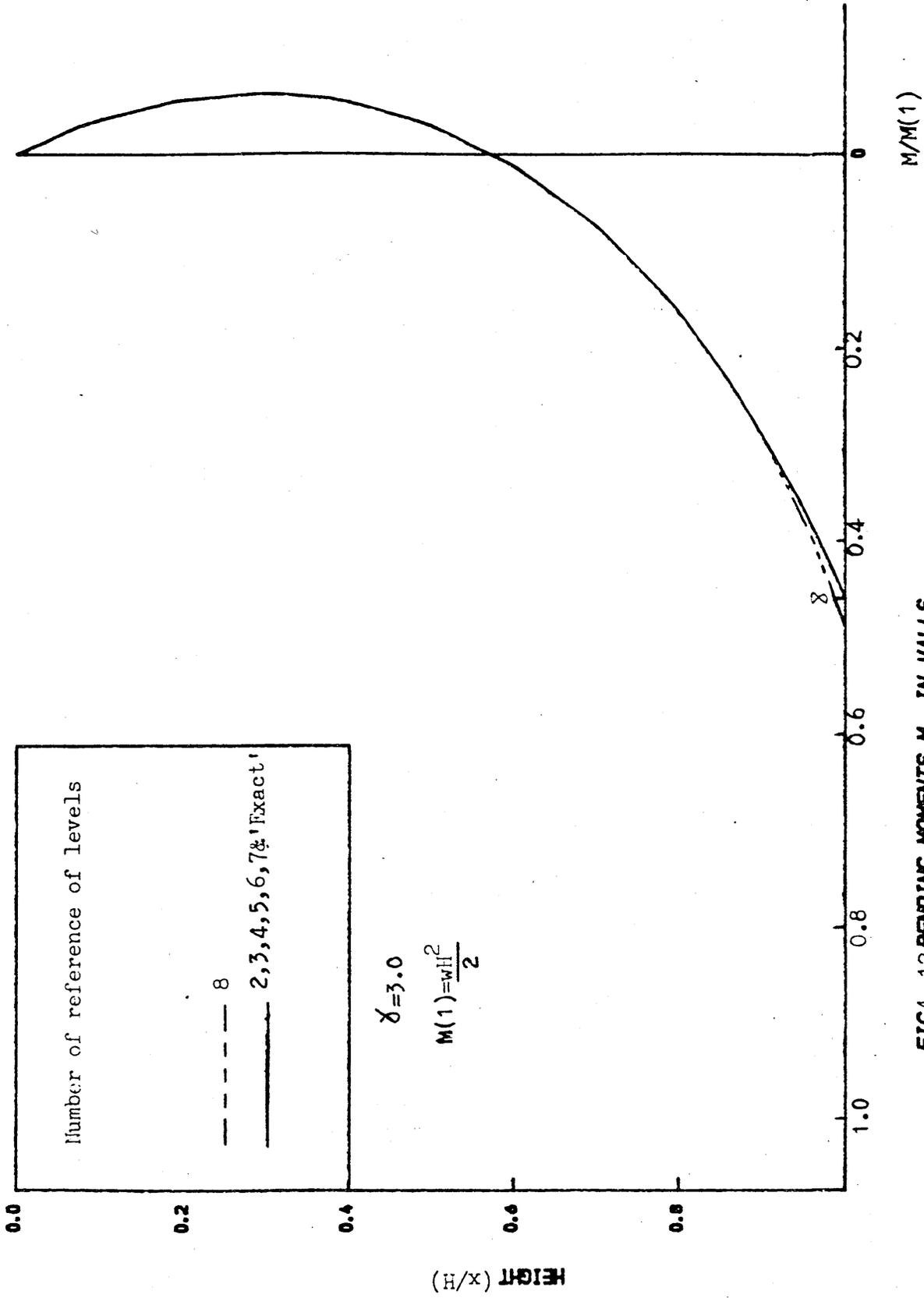


FIG4. 12 BENDING MOMENTS M IN WALLS

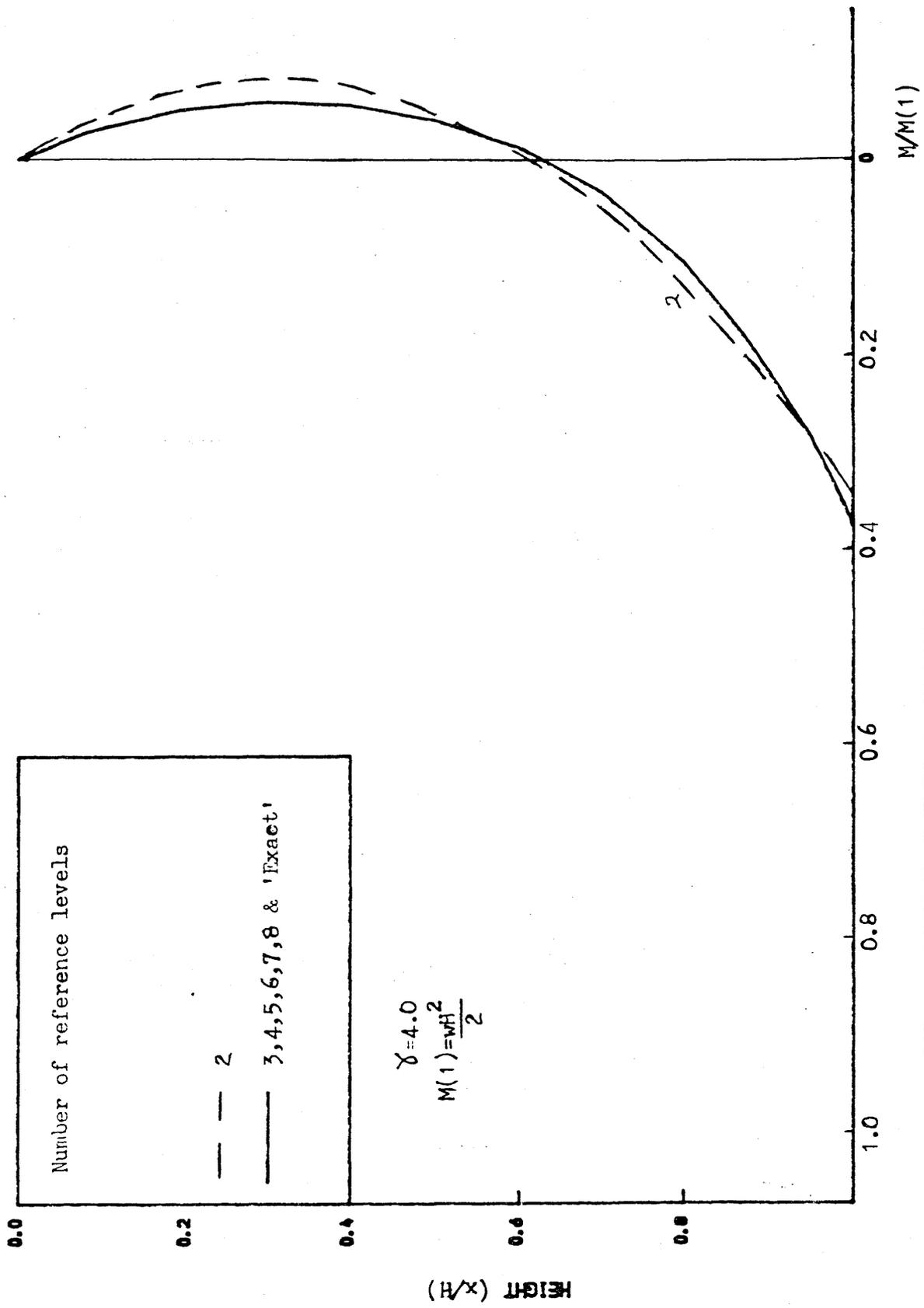


FIG. 4. 13 BENDING MOMENTS  $M$  IN WALLS

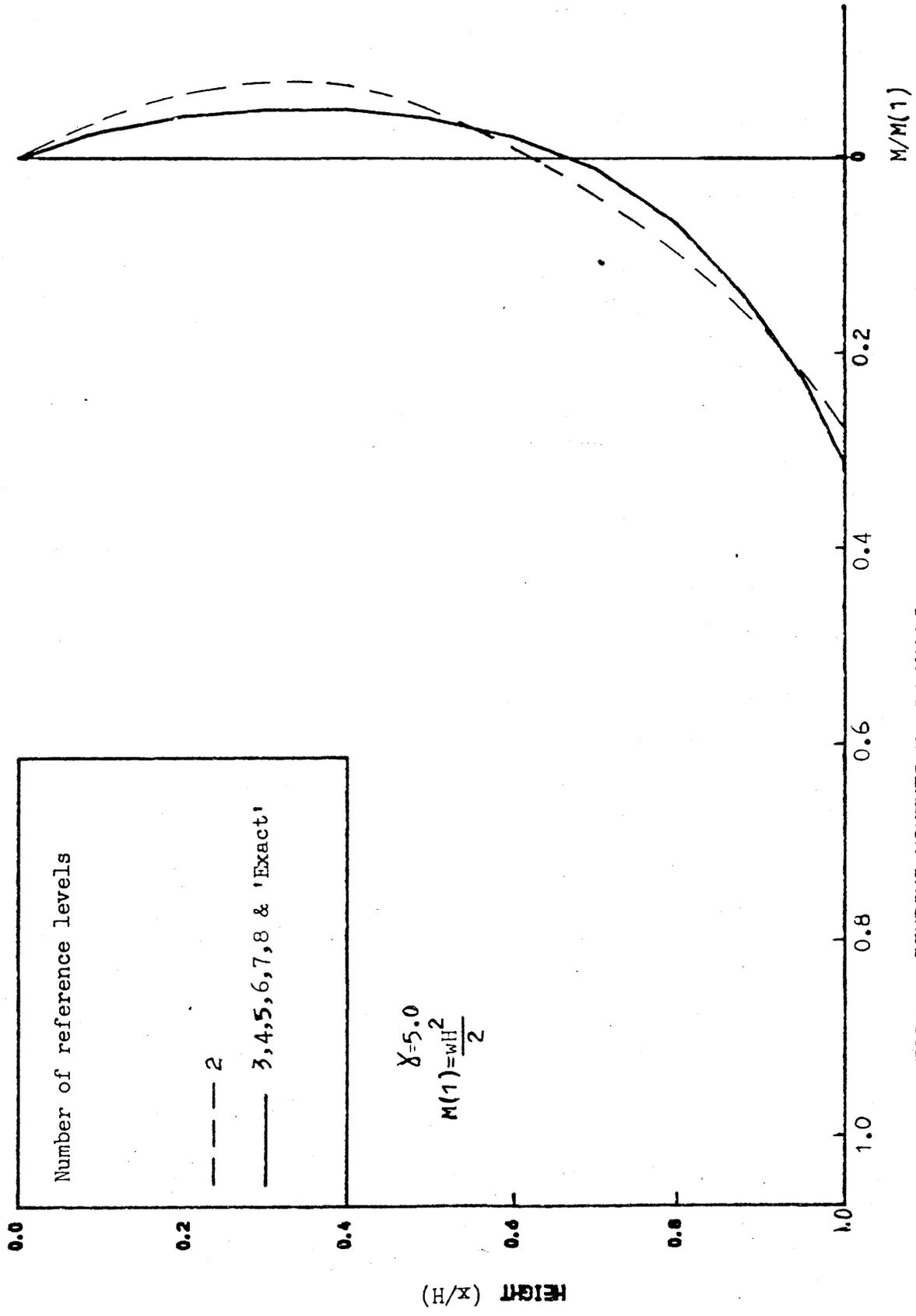


FIG. 14 BENDING MOMENTS M IN WALLS

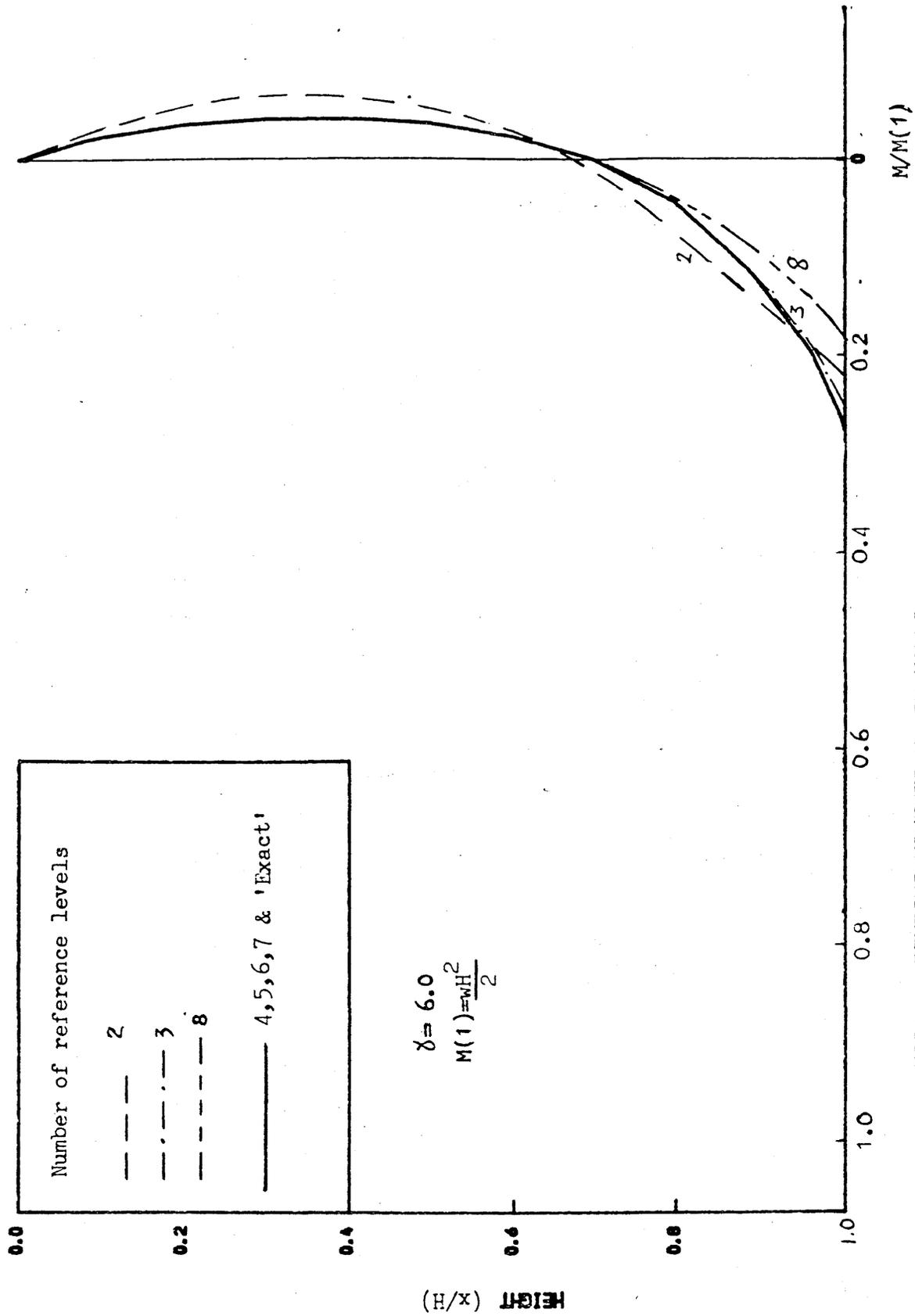


FIG. 4. 15 BENDING MOMENTS  $M$  IN WALLS

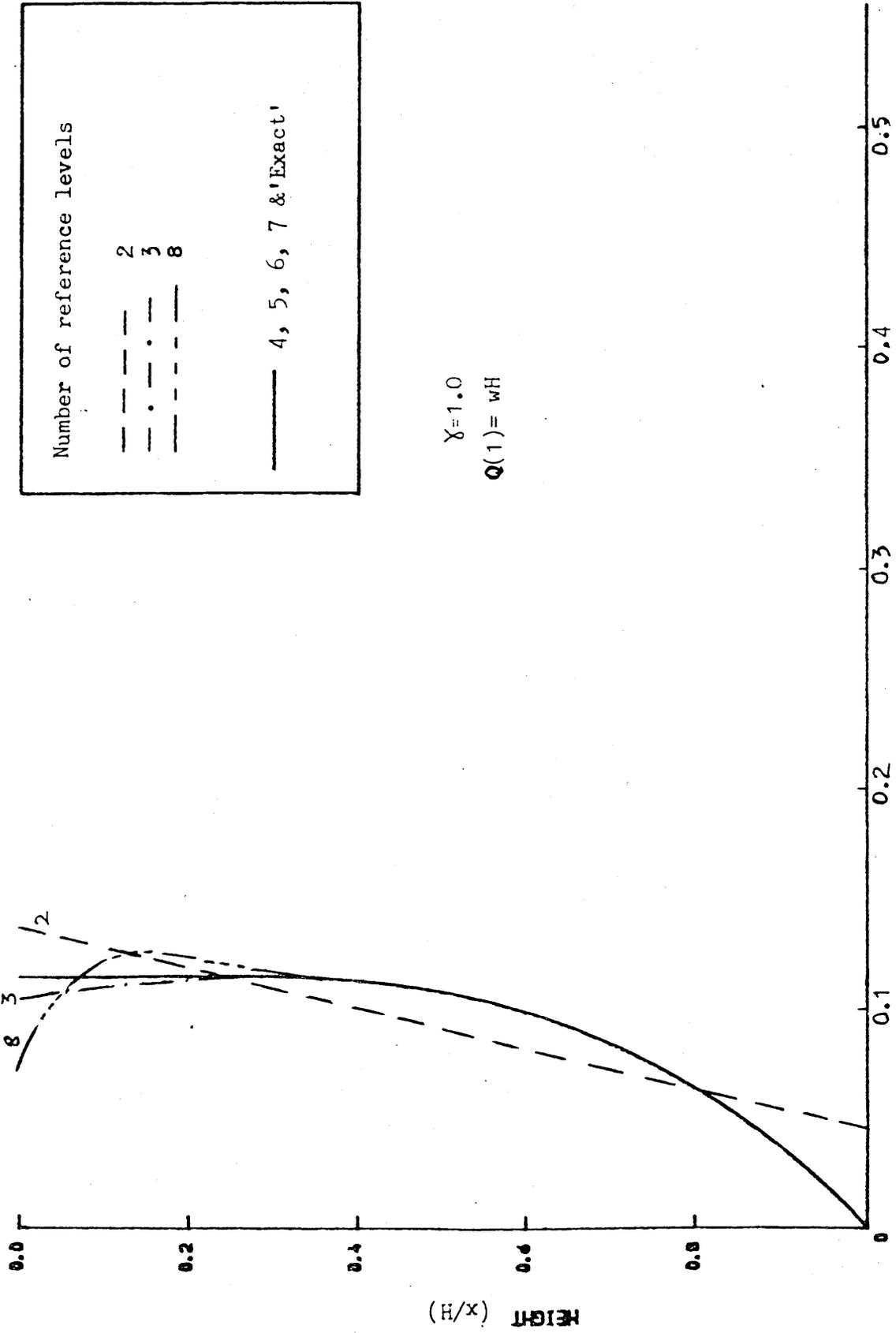


FIG 4.16 SHEAR FORCE IN FRAMES

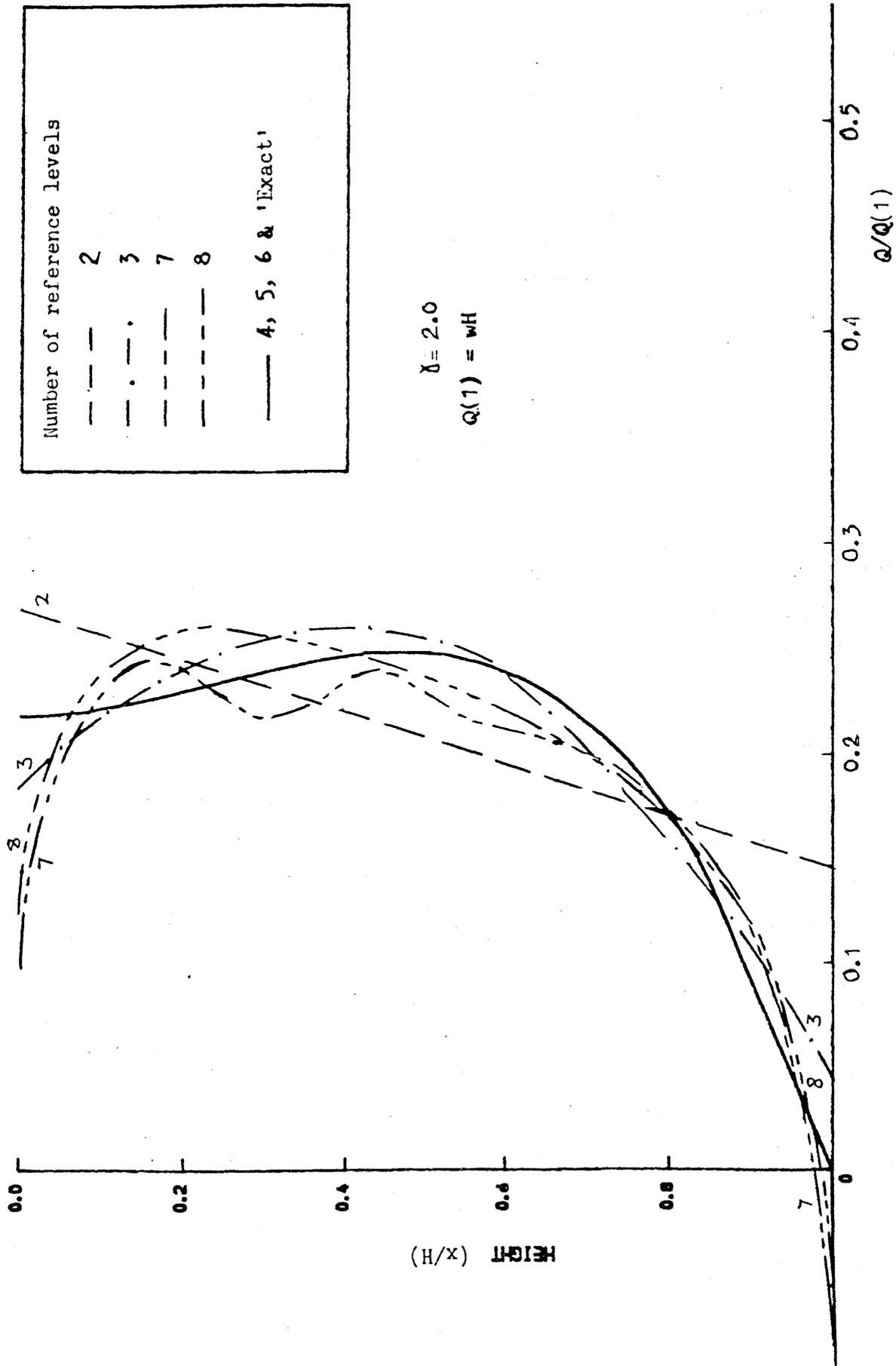


FIG 4.17 SHEAR FORCE IN FRAMES

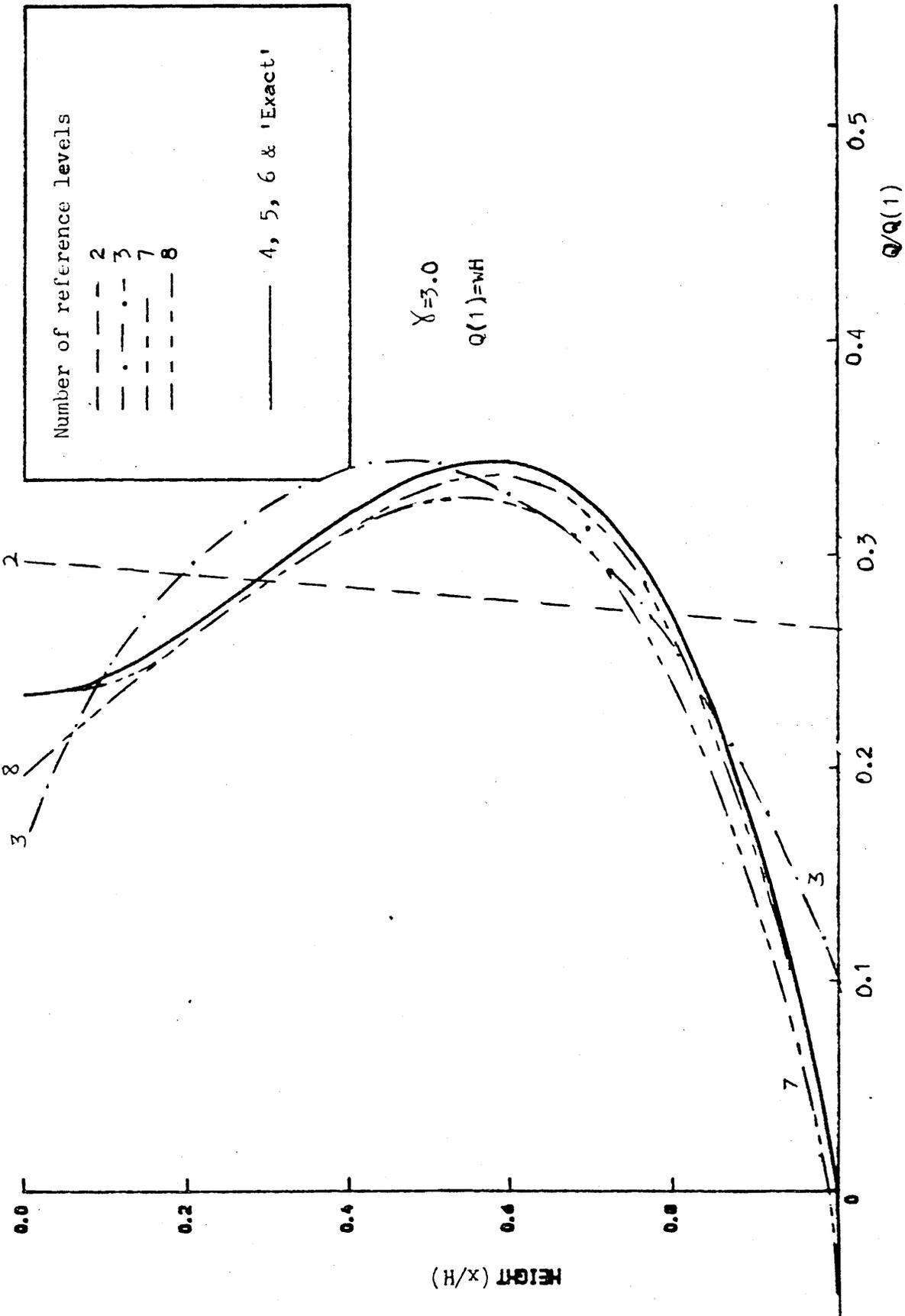


FIG 4.18 SHEAR FORCE IN FRAMES

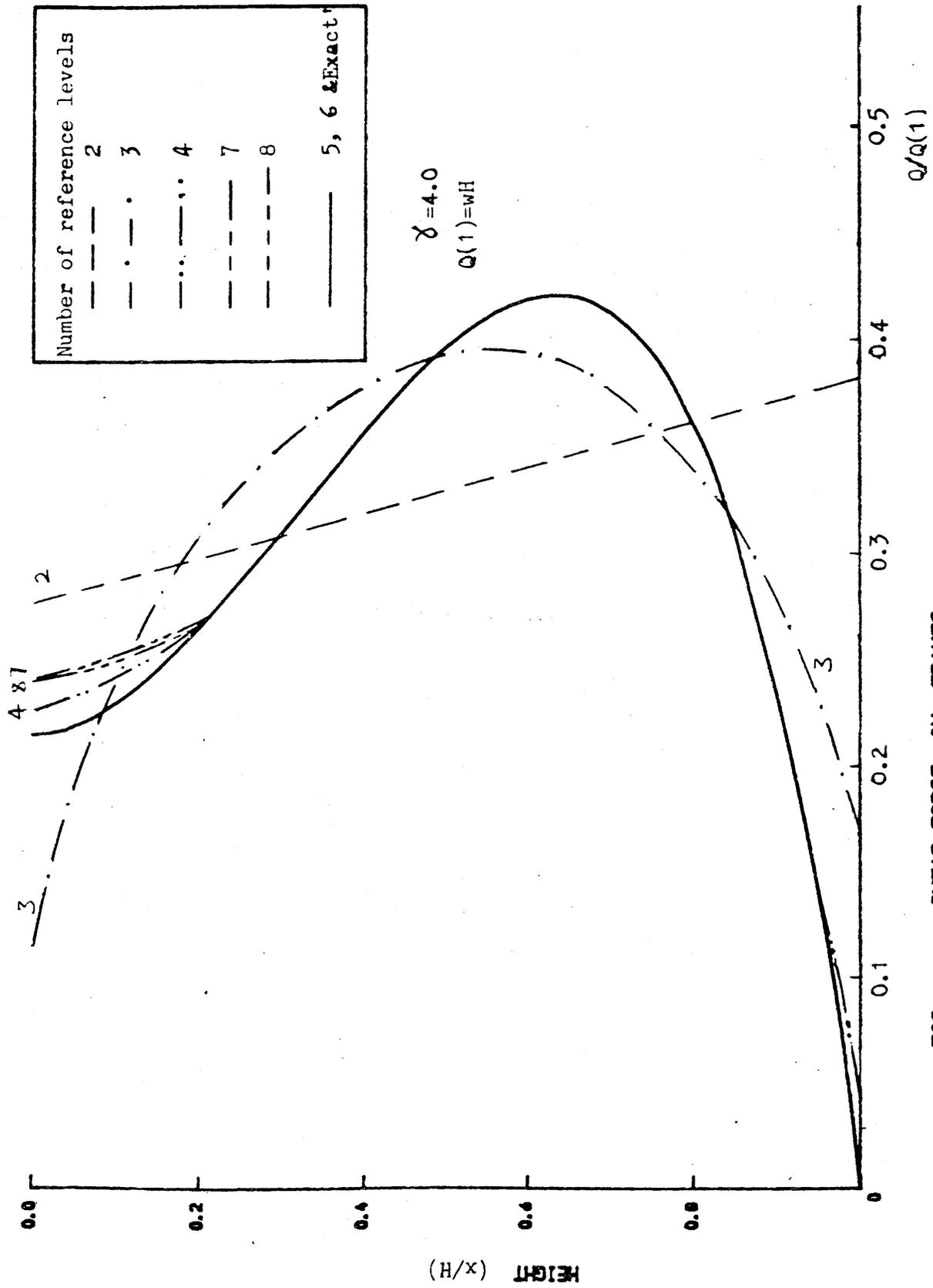


FIG 4.19 SHEAR FORCE IN FRAMES

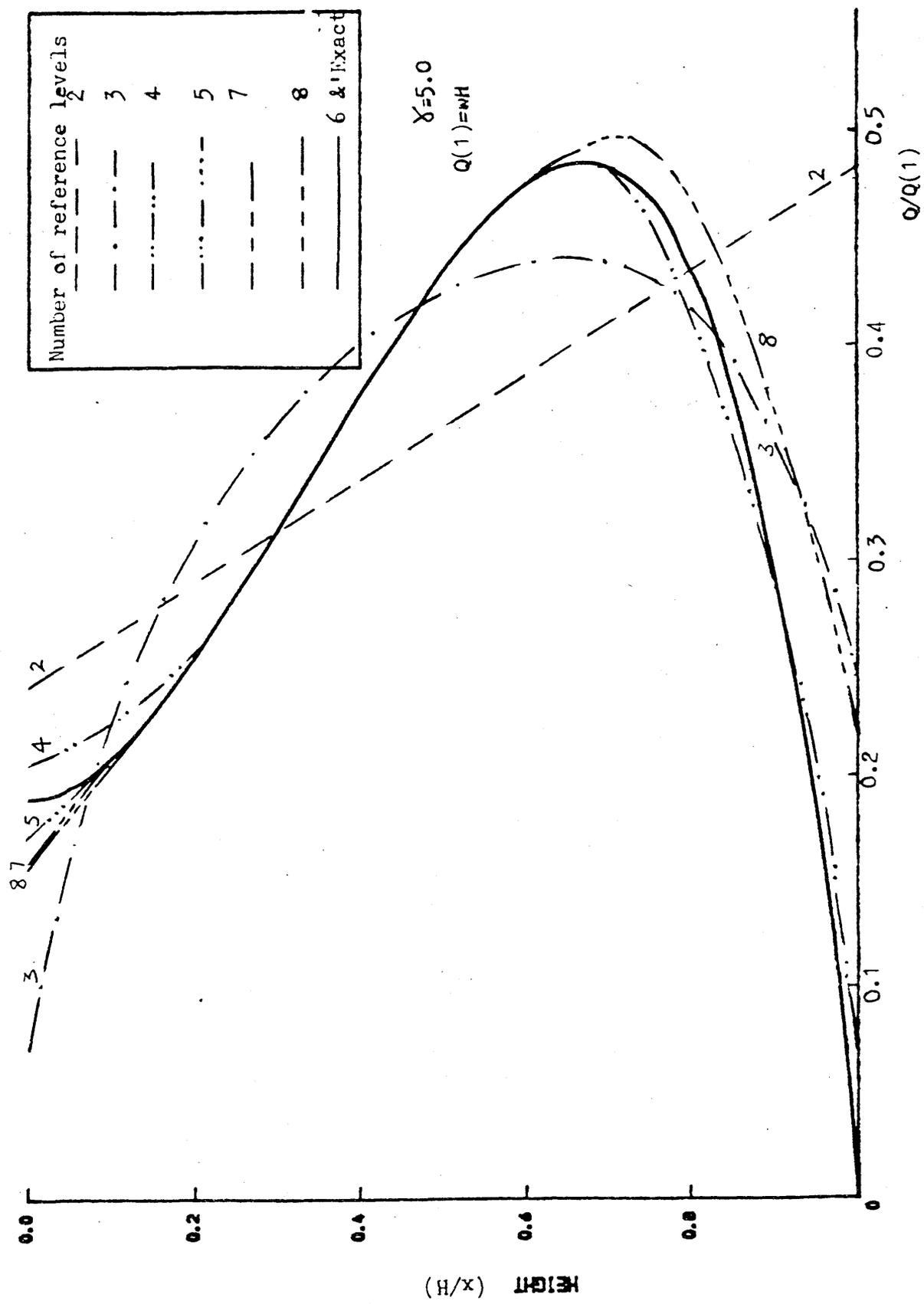


FIG 4.20 SHEAR FORCE IN FRAMES

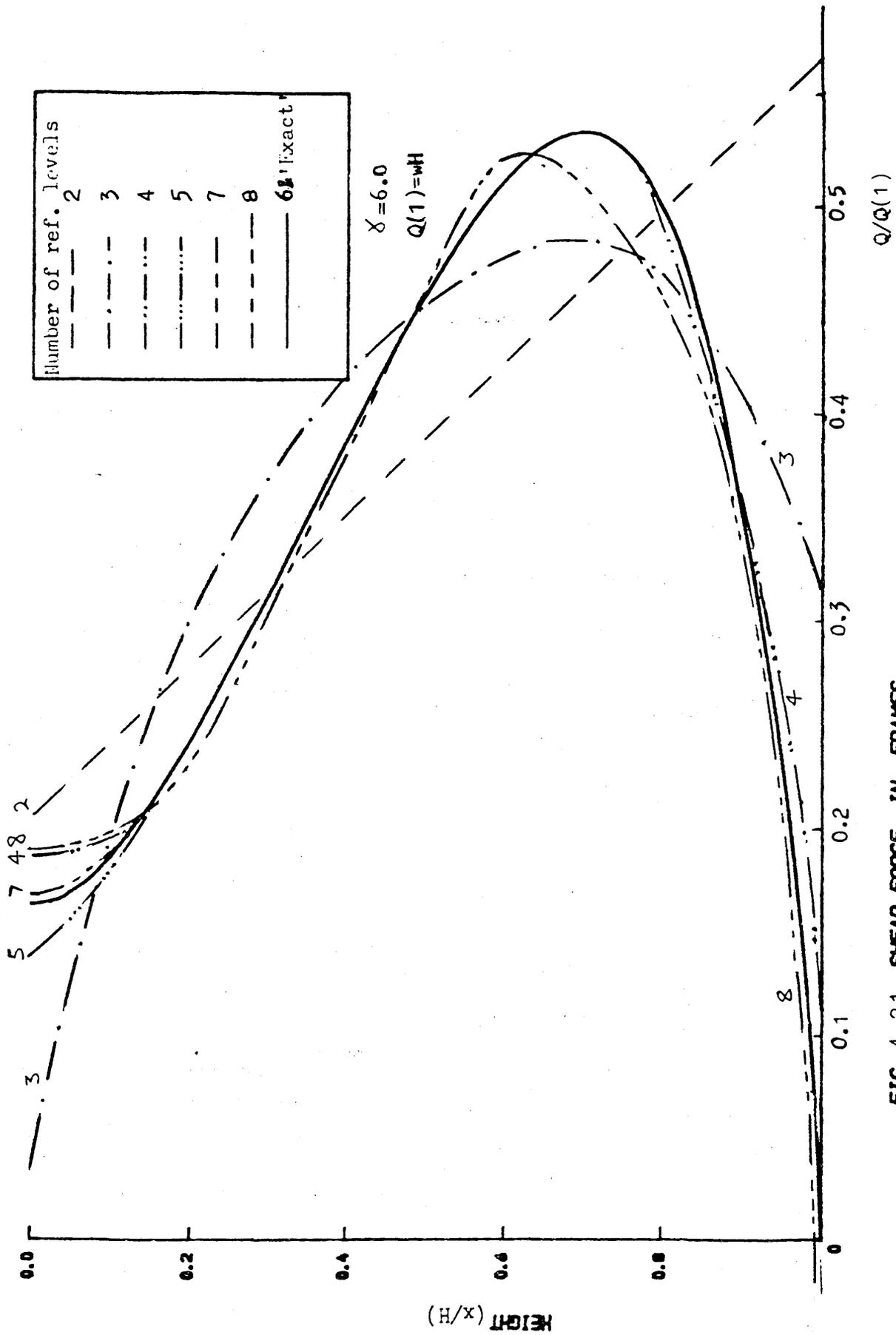
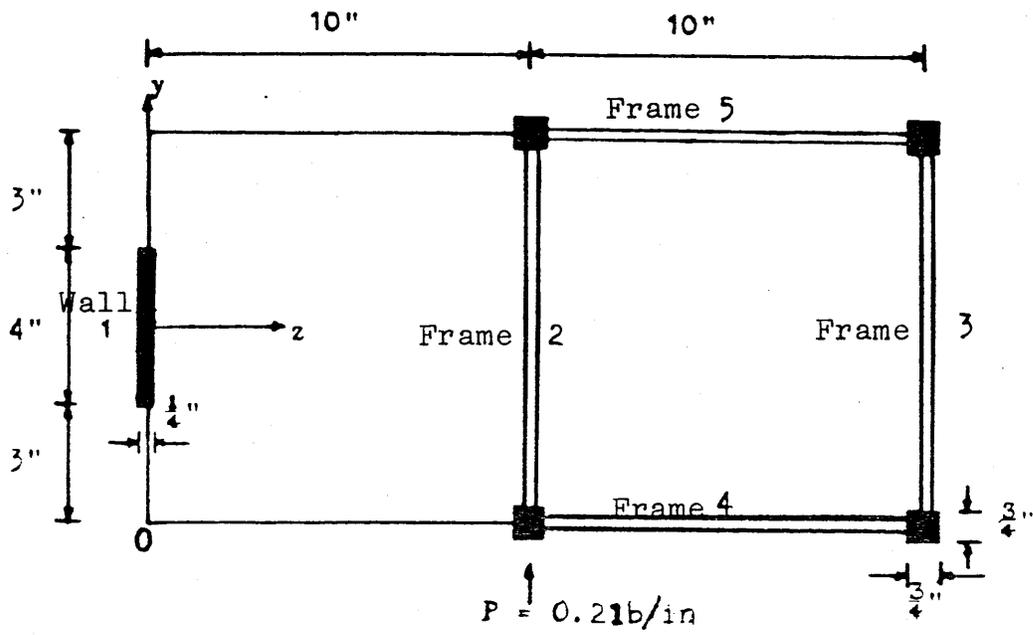
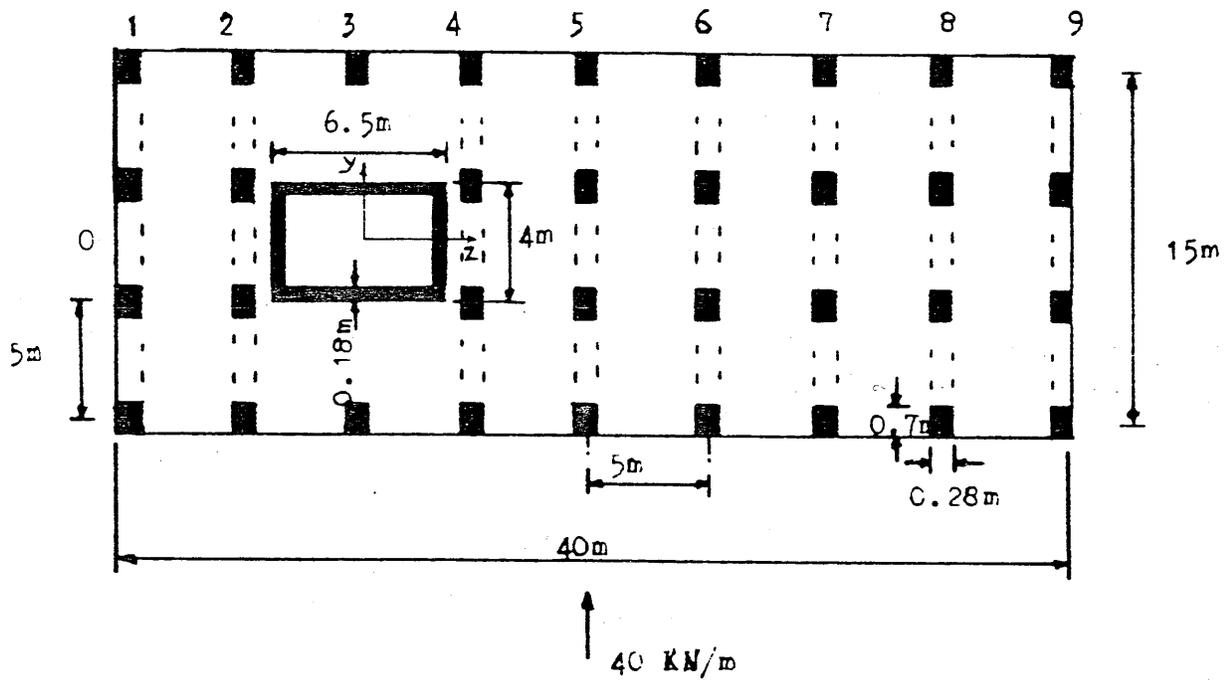


FIG 4.21 SHEAR FORCE IN FRAMES



(a)



(b)

Fig. 4.22 Plan forms of example structures I and II

3  
4  
5  
762 8  
MO - ■

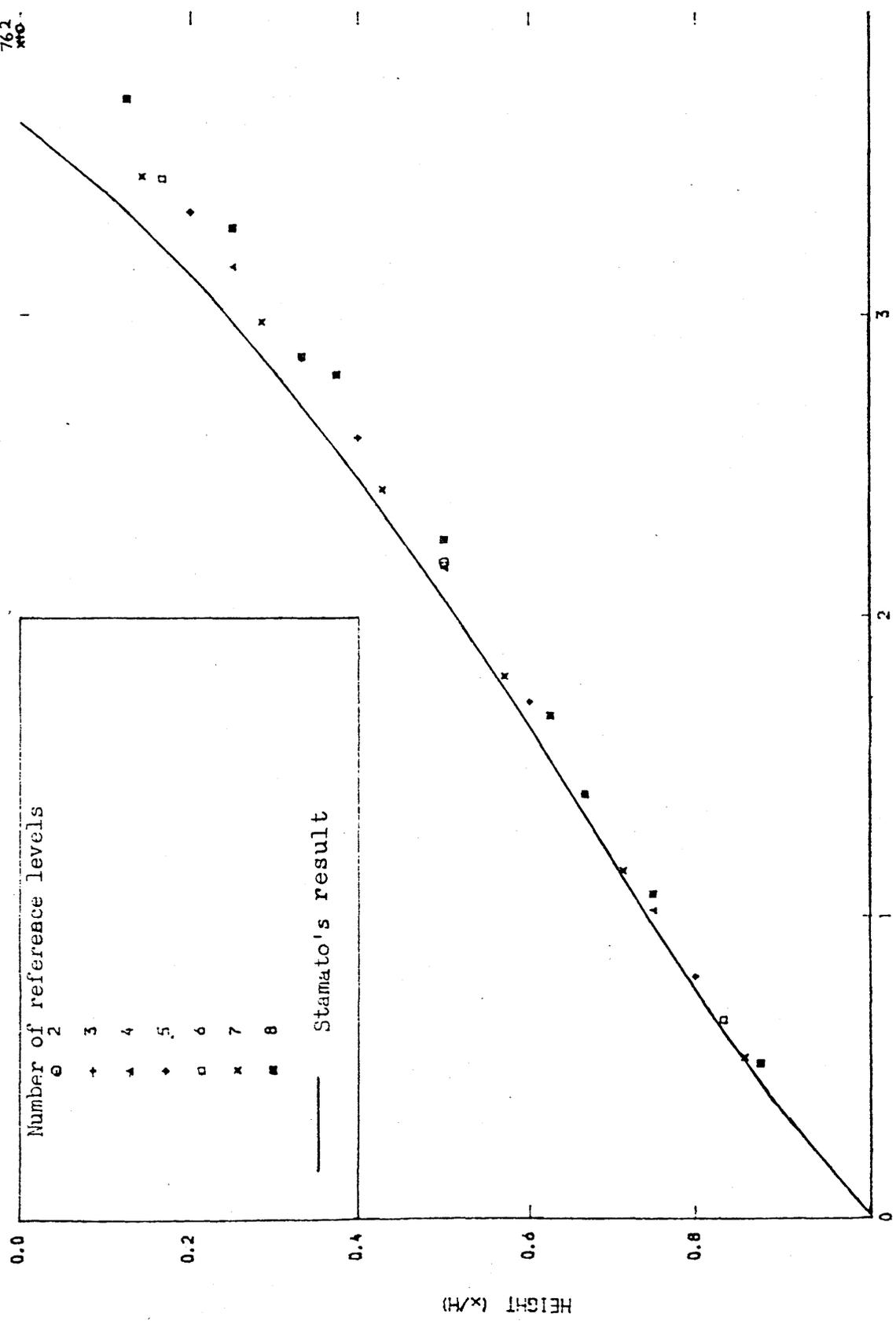


FIG 4.23 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 2 ("2-D")

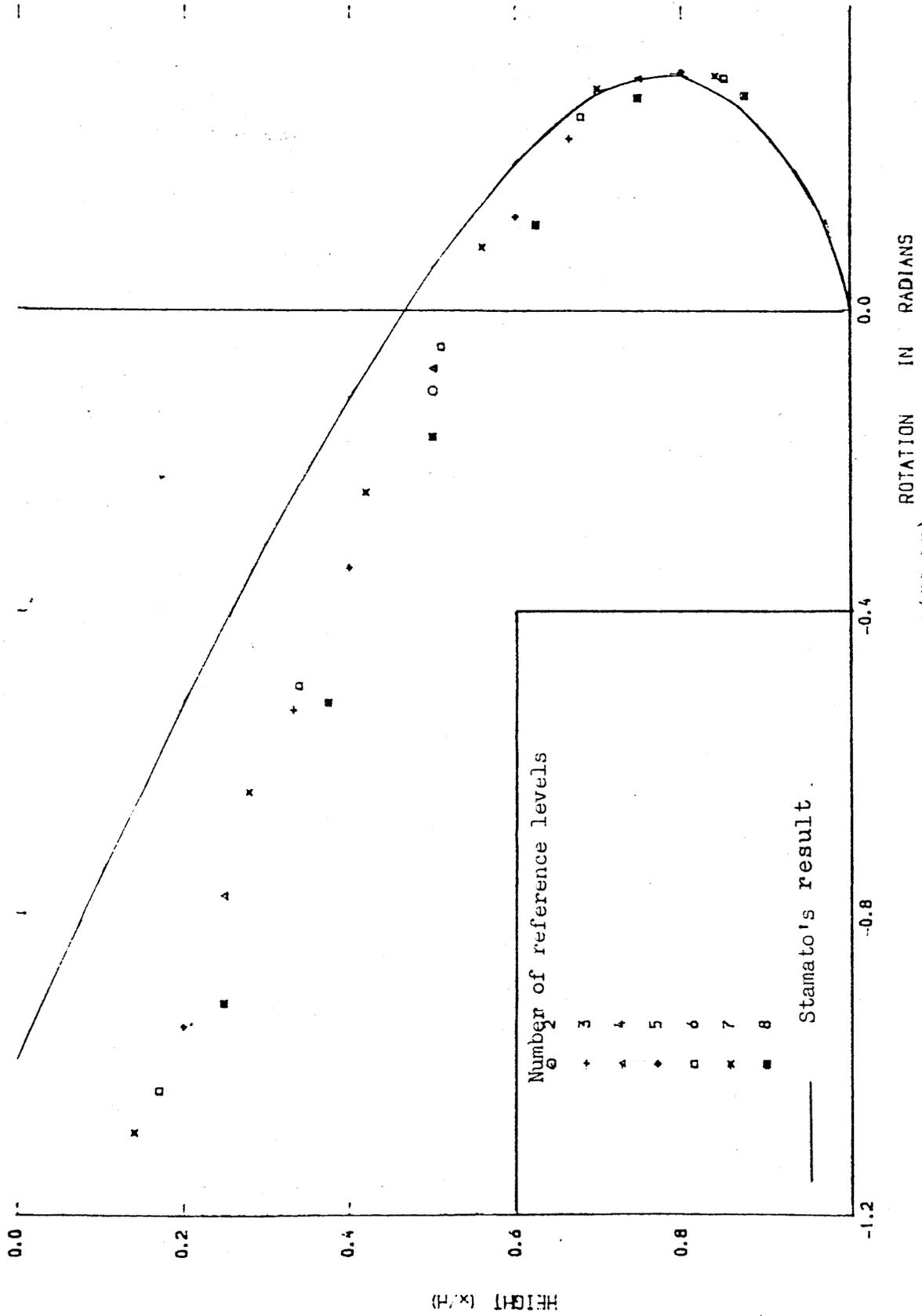


FIG 4.24 ROTATIONS OF DIAPHRAGMS ("2-D")

Number of reference levels

- 2
- .-.- 3
- .-.- 4,5,6
- .-.- 7
- 8
- \_\_\_\_\_ Stamato's result

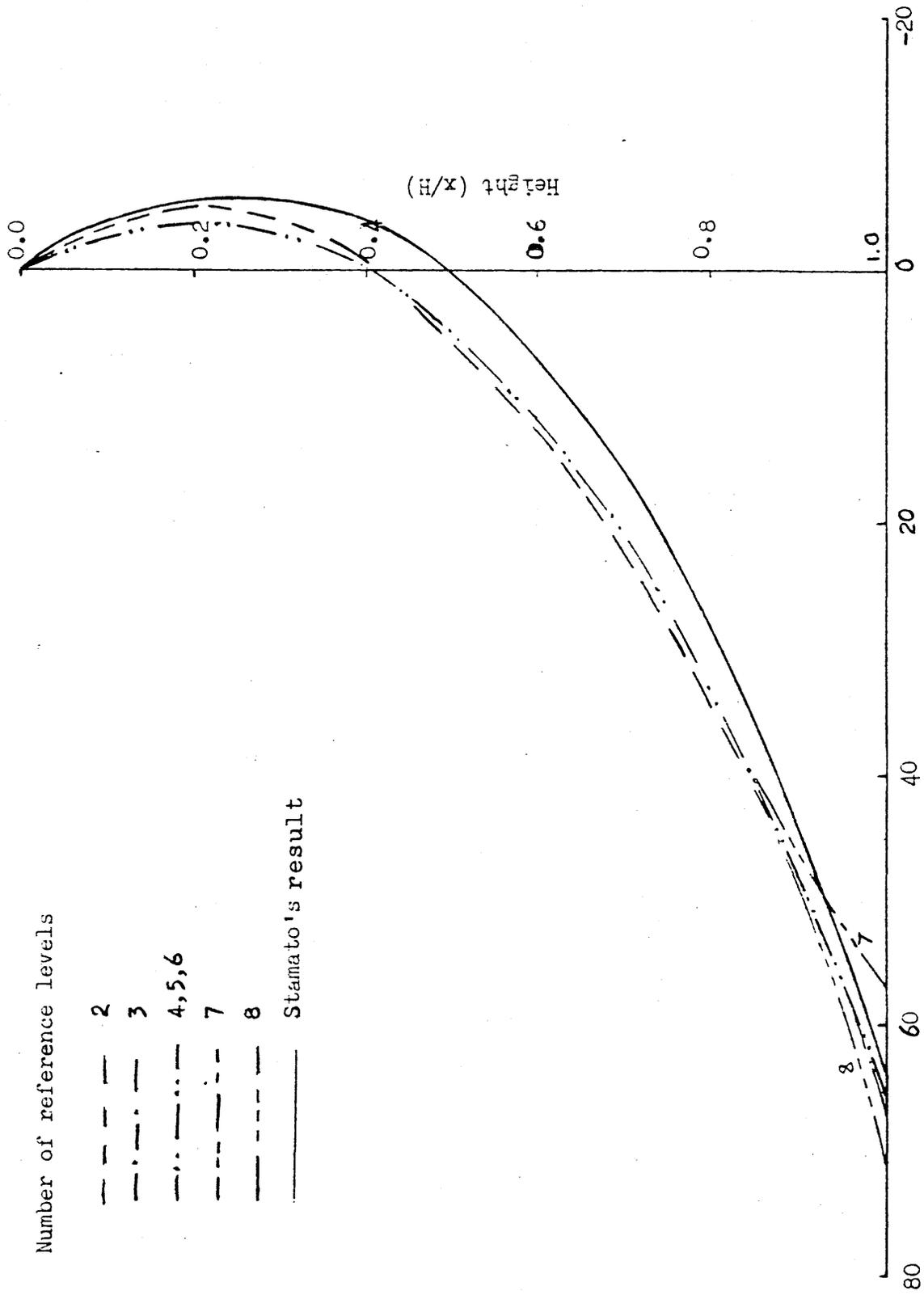


Fig 4.25 Bending Moments in Wall ("2-D")

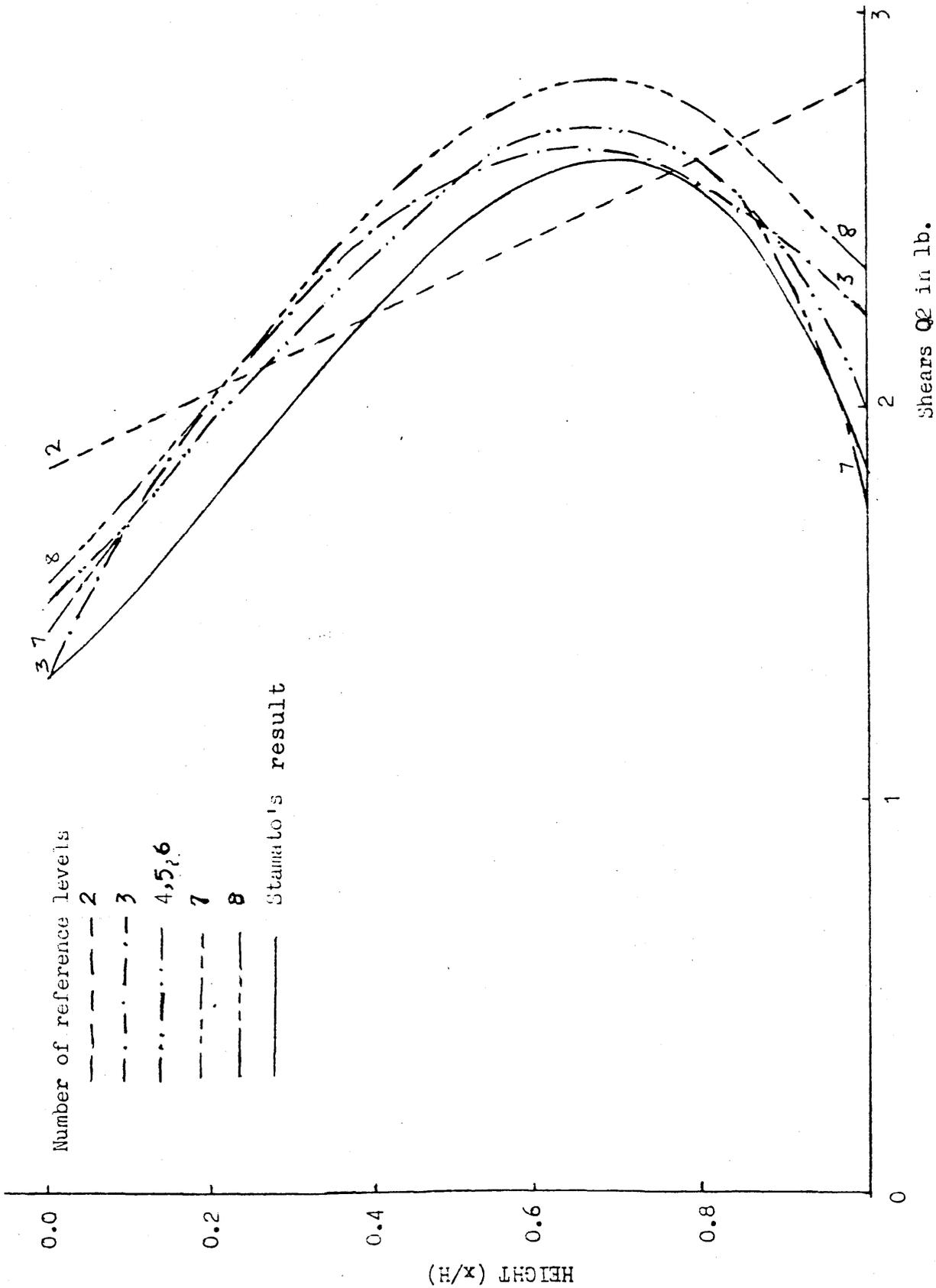


FIG 4.26 Shear Forces in Frame 2 ("2-D")

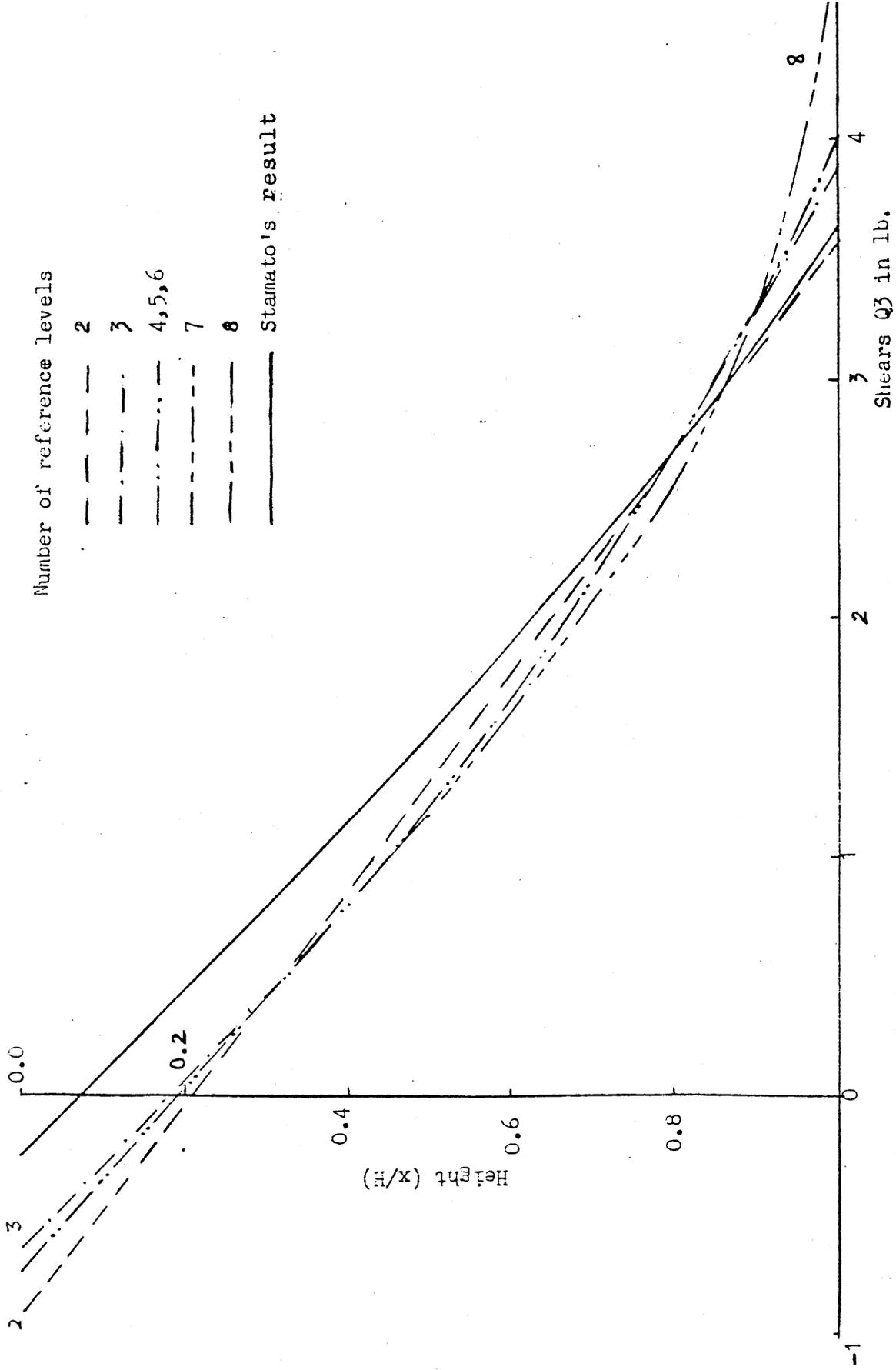


Fig. 4.27 Shear Forces in Frame 3 ("2-D")

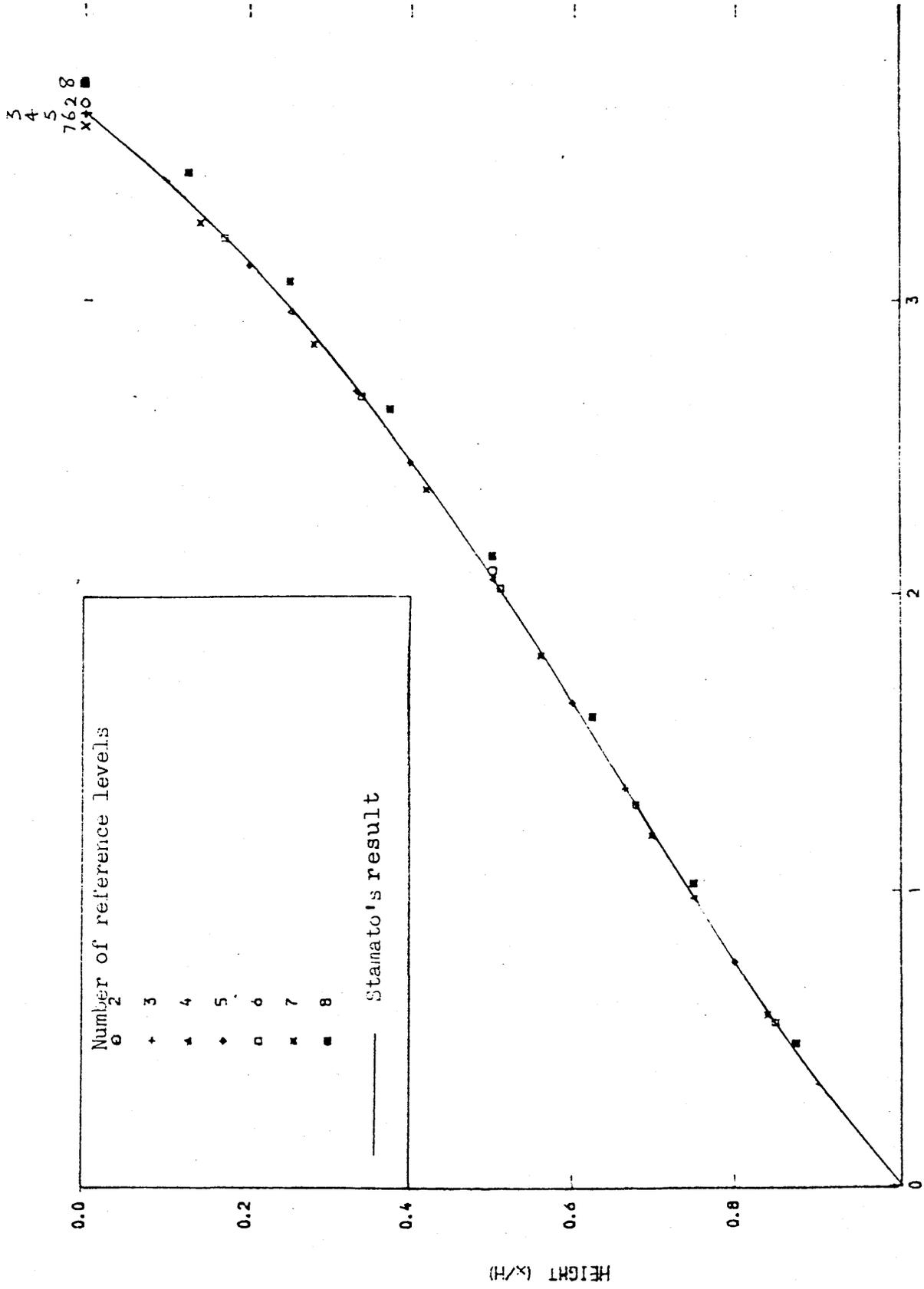


FIG 4.28 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 2 ( "2- DS" )

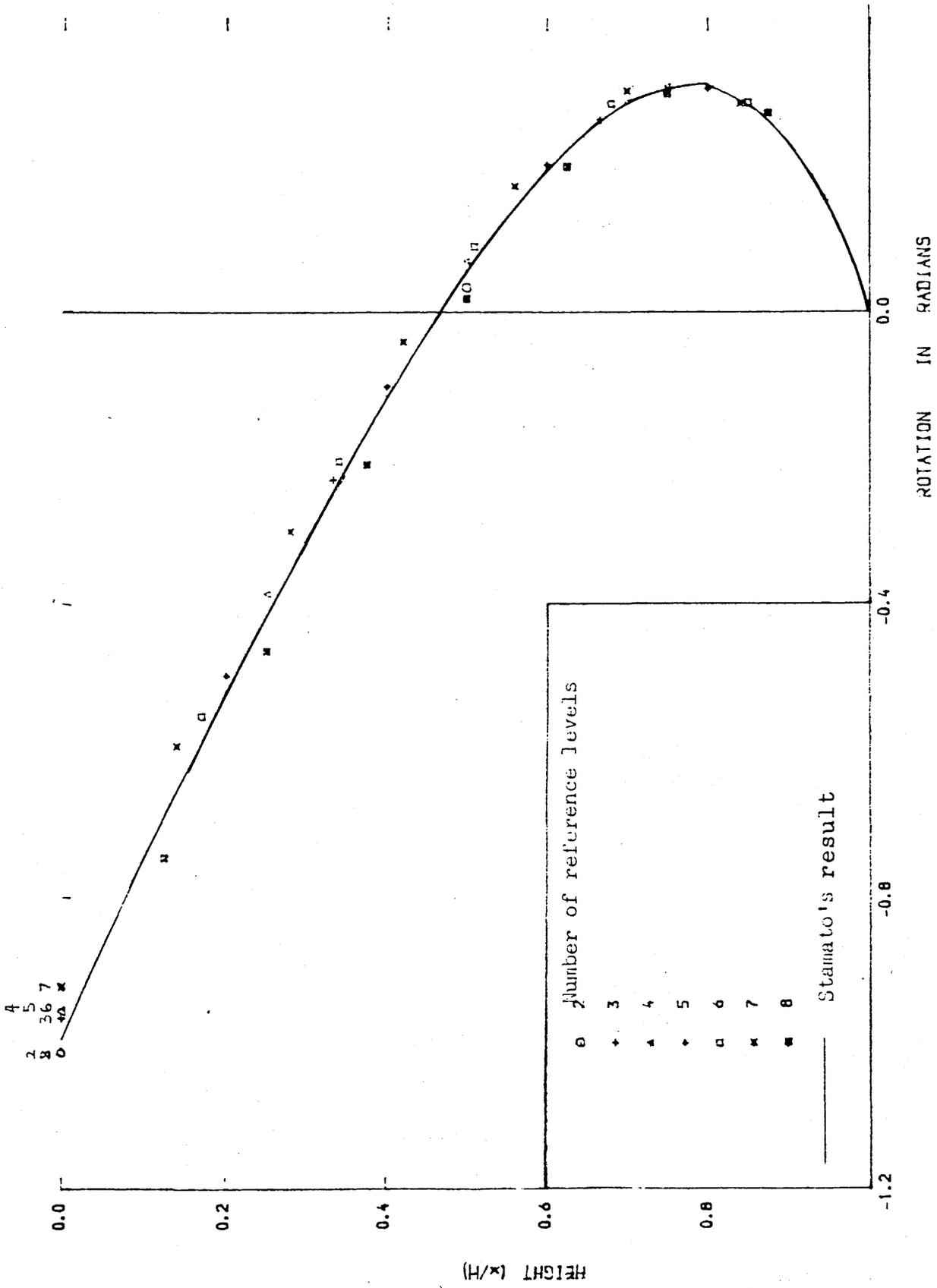


FIG 4.29 ROTATIONS OF DIAPHRAGMS ("2-DS")

Number of reference levels

--- 2

— 3,4,5,6 & Stamatatos's result

--- 7

--- 8

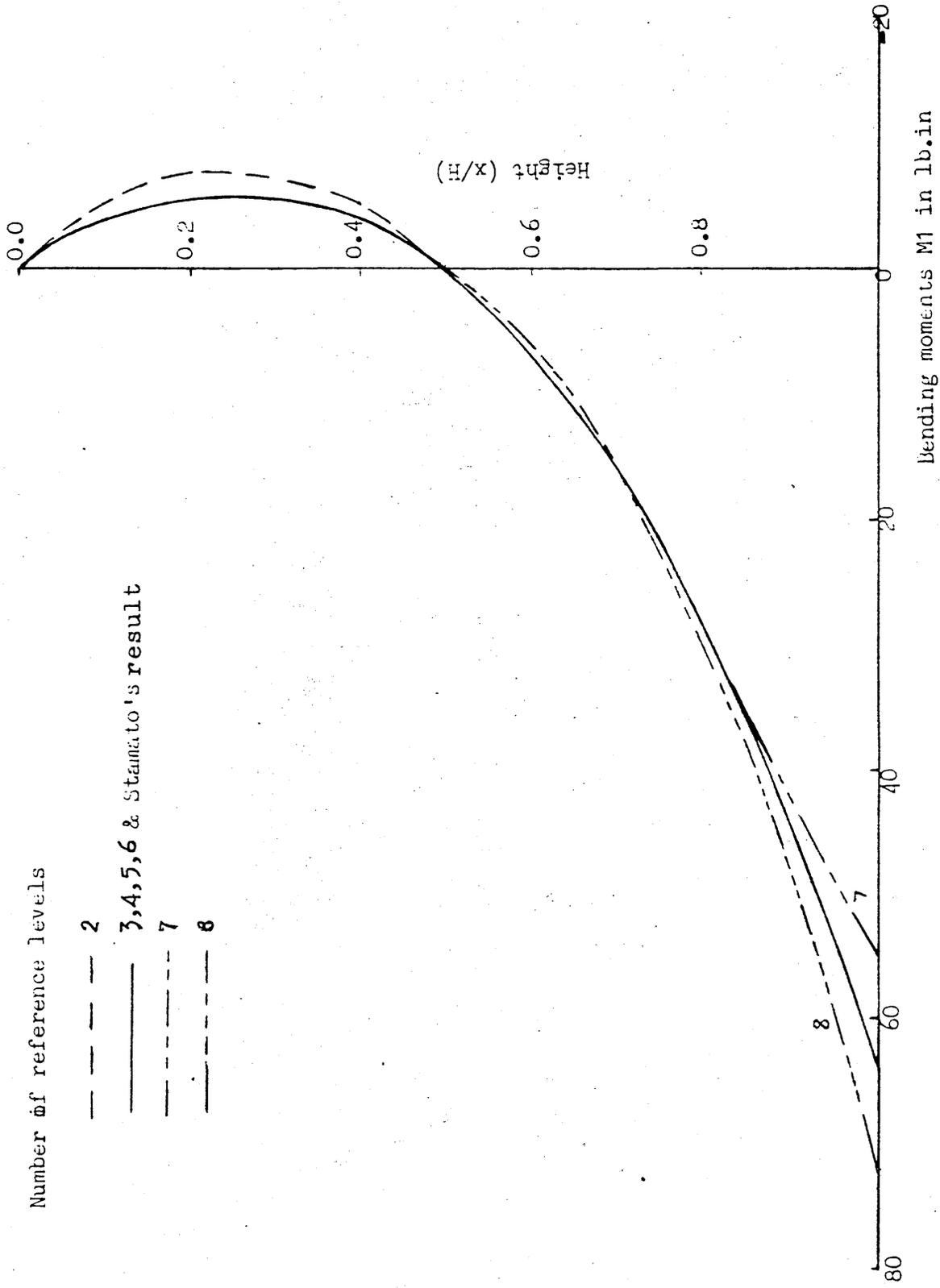


Fig 4.30 Bending Moments in Wall ("2-DS")

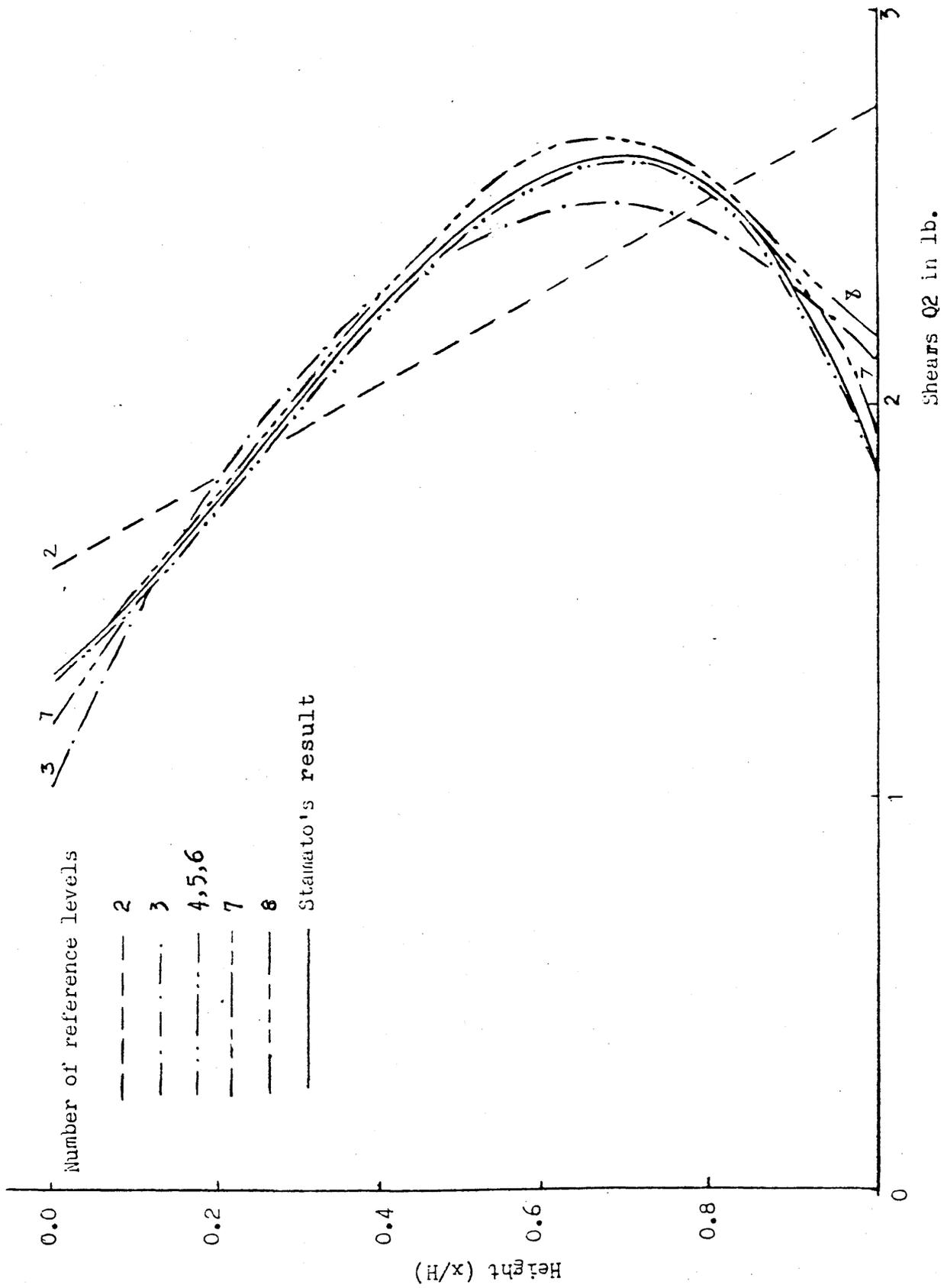


Fig. 4.31 Shear Forces in Frame 2 ("2-DS")

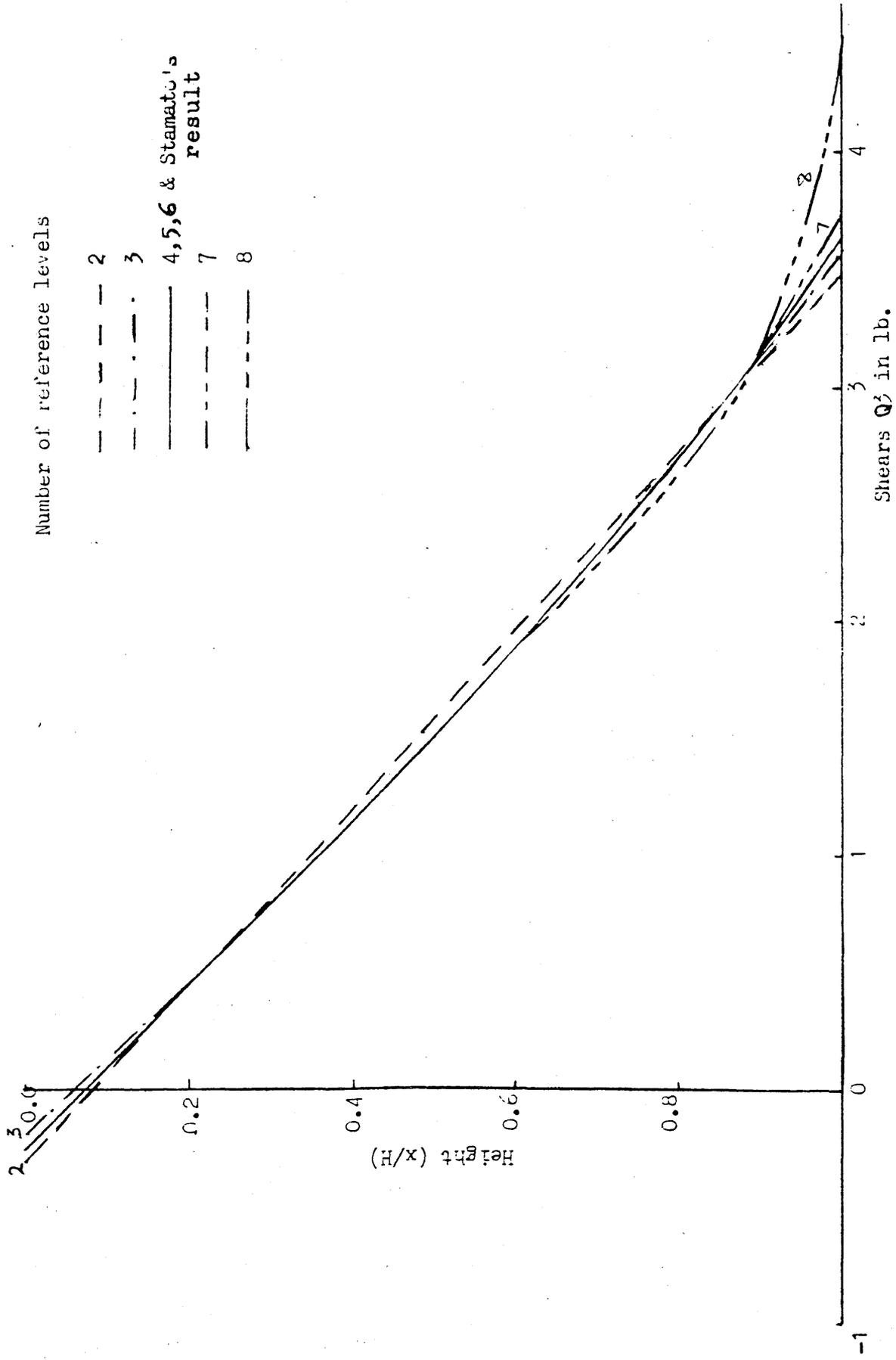


Fig 4.32 Shear Forces in Frame 3 ("2-DS")

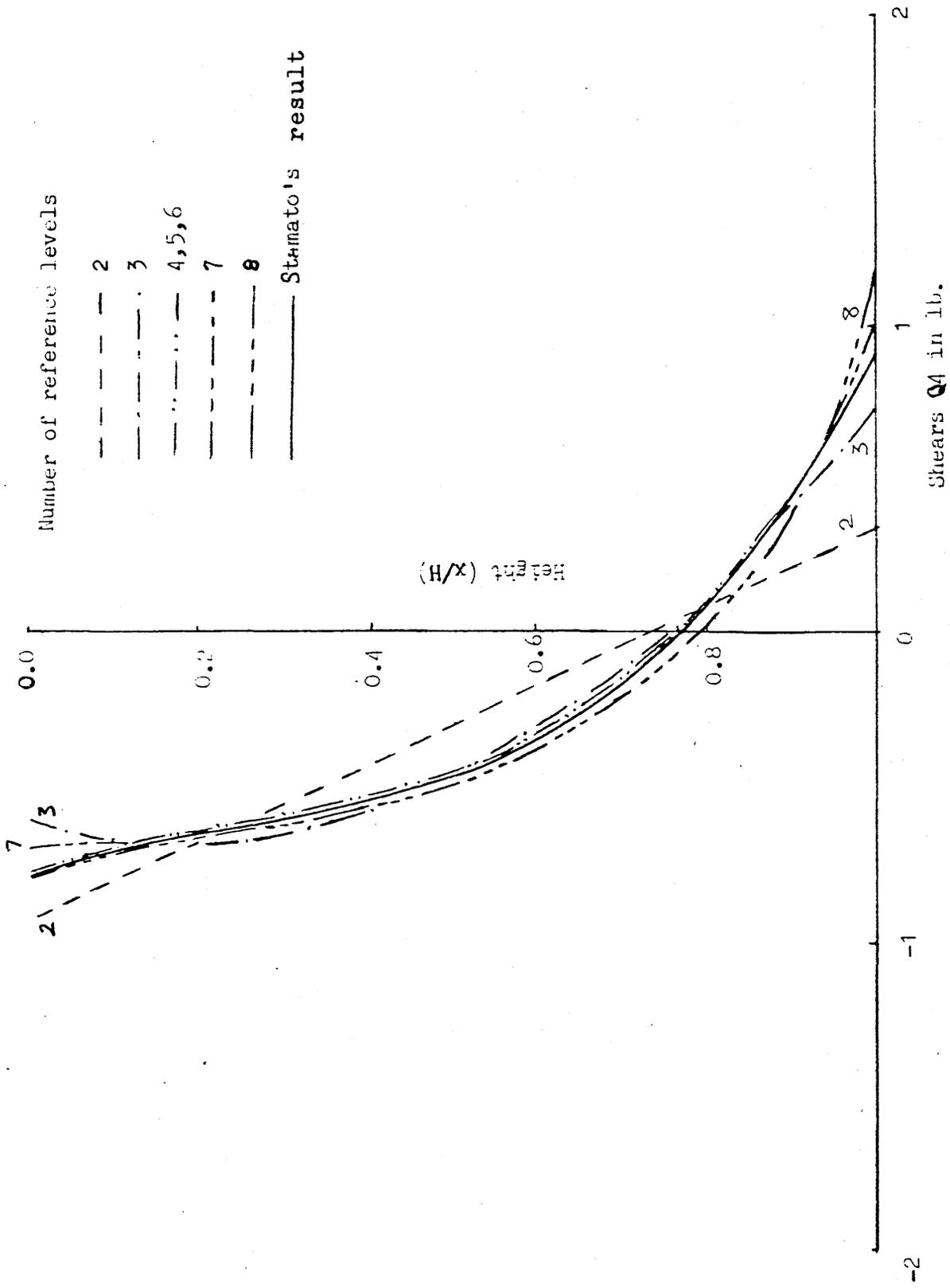


FIG. 4.33 Shear Forces in Frame 4 ("2-DS")

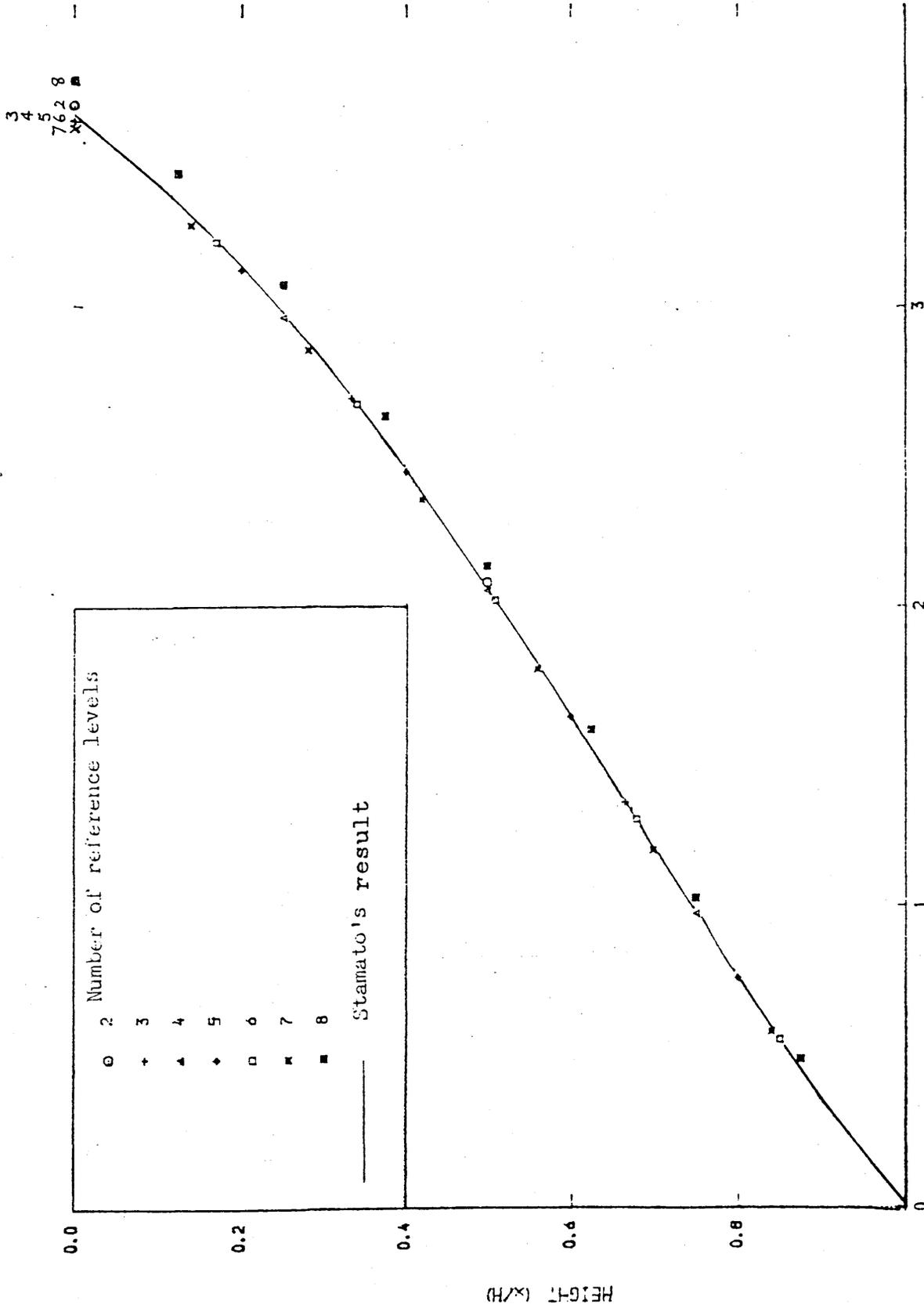


FIG 4.34 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 2 ("3-D")

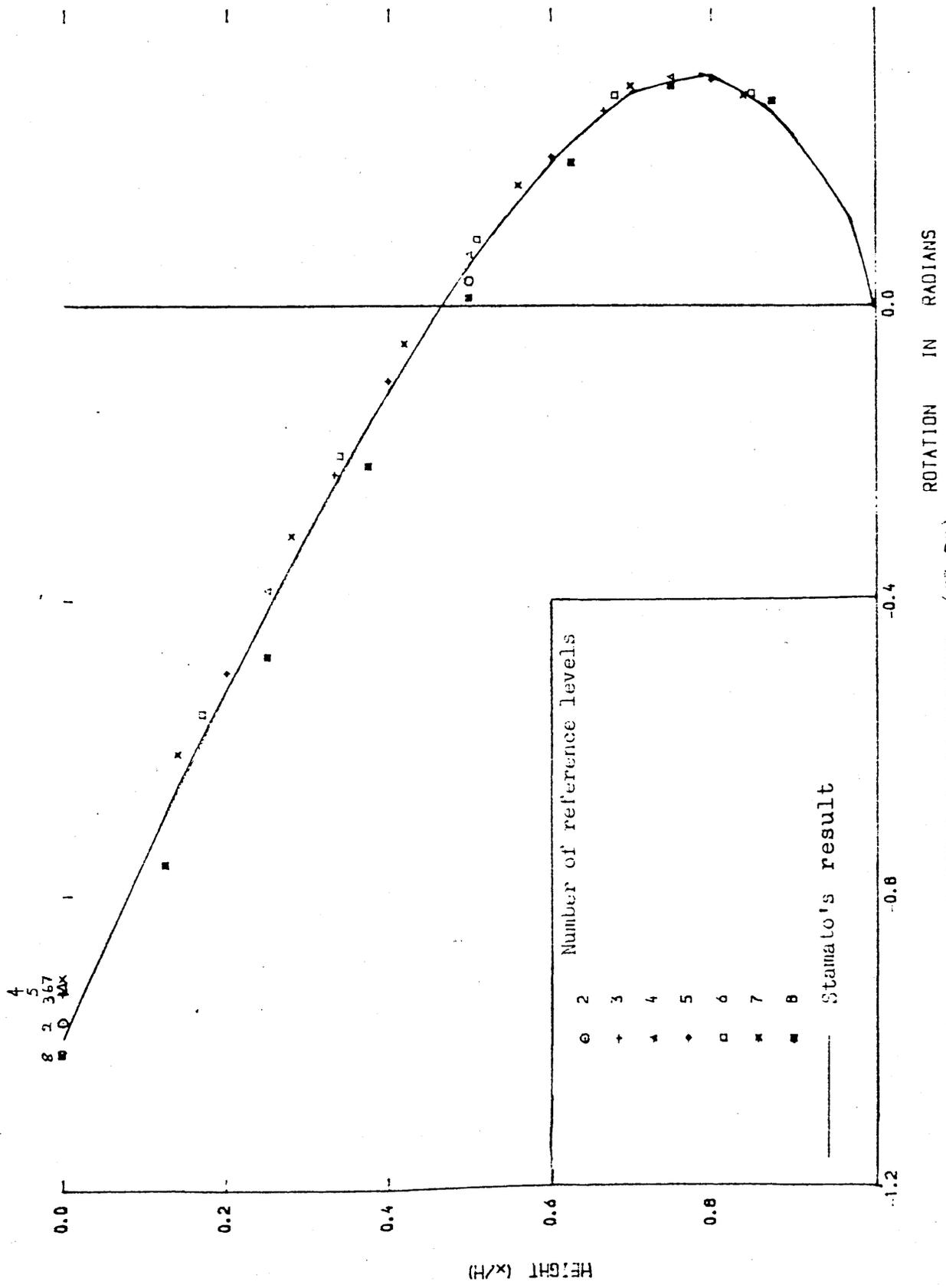


FIG 4.35 ROTATIONS OF DIAPHRAGMS ("3-D")

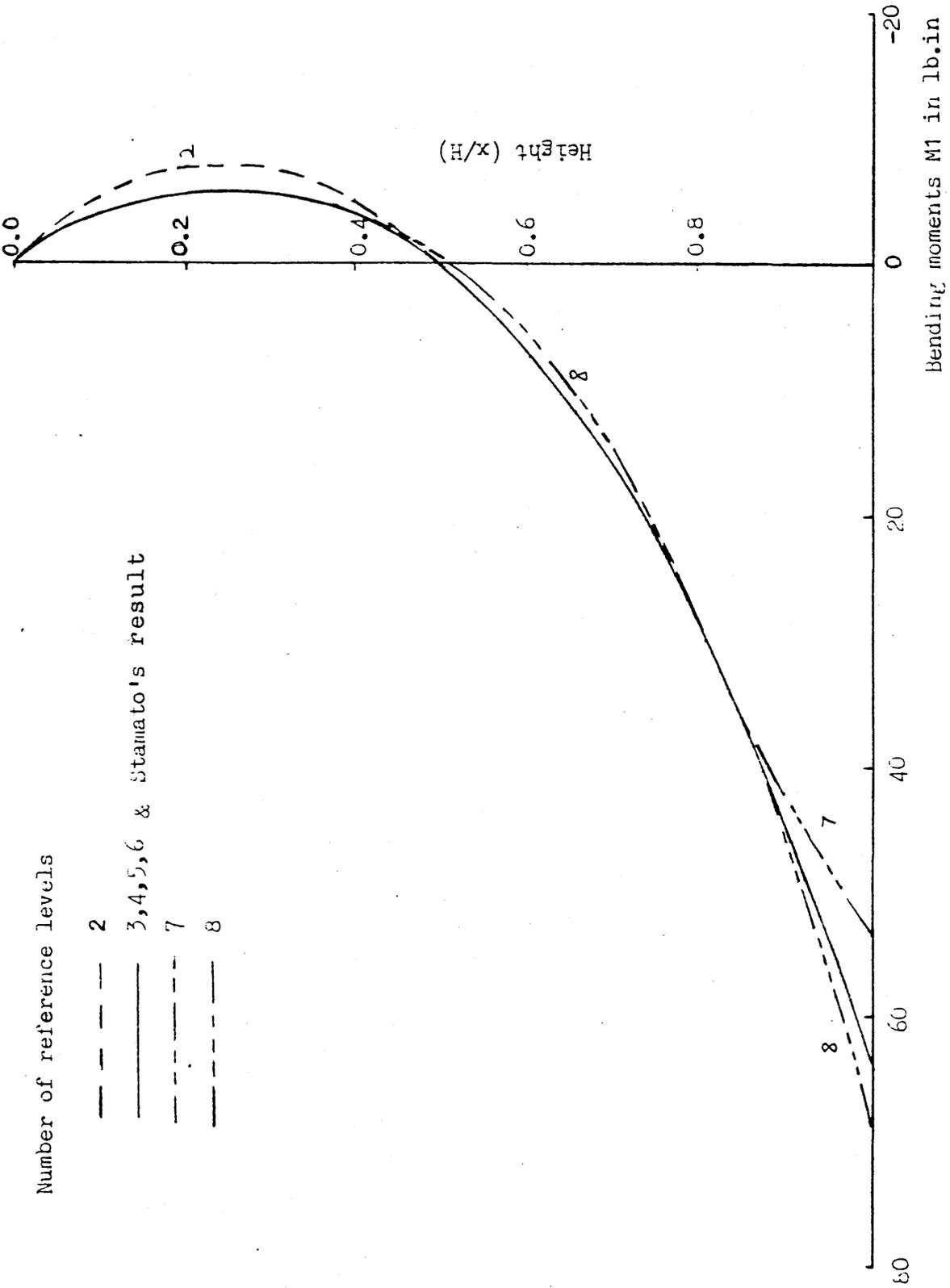


Fig. 4.36 Bending Moments in Wall ("3-D")

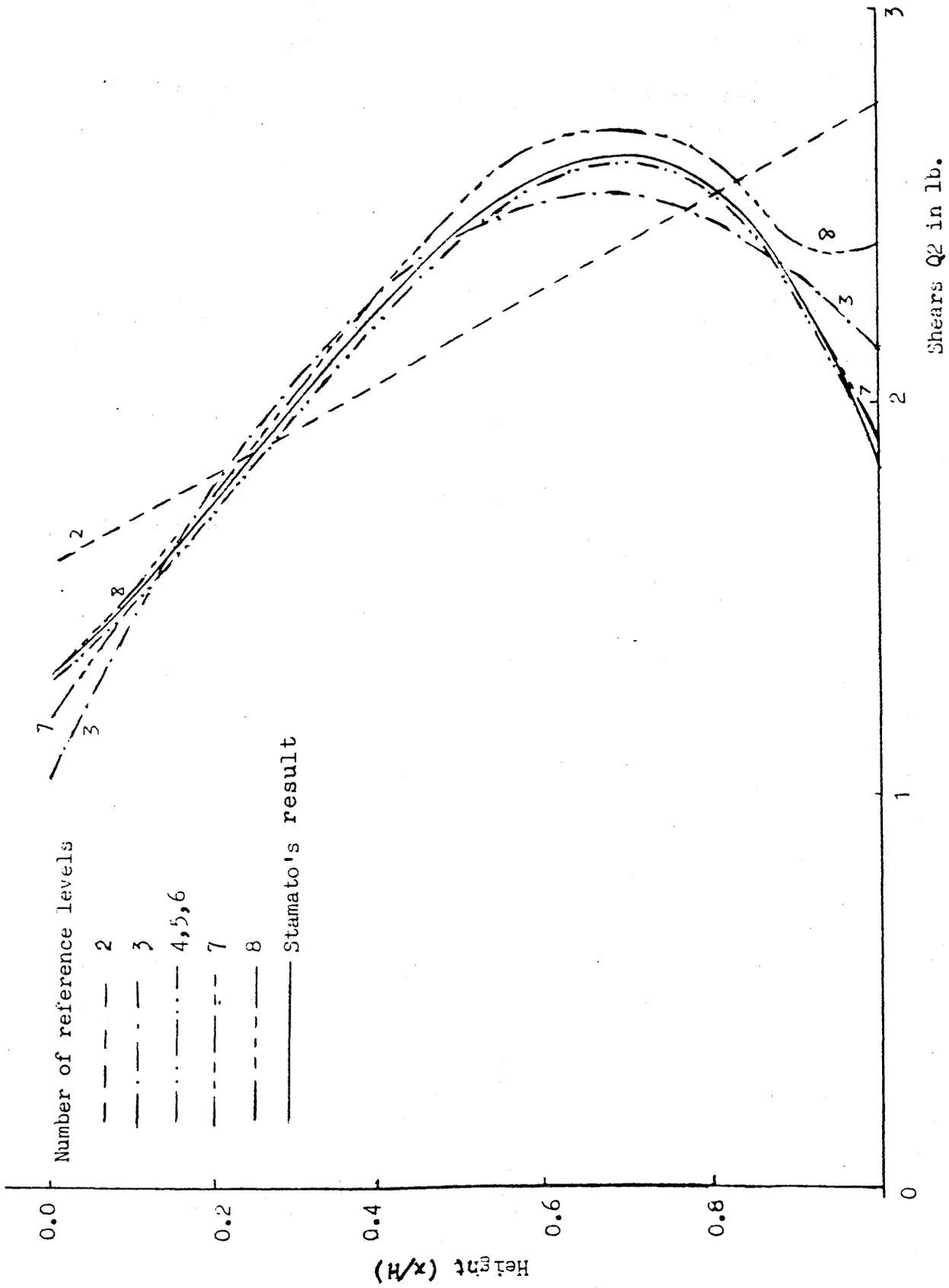


Fig. 4.37 Shear Forces in Frame 2 ("3-D")

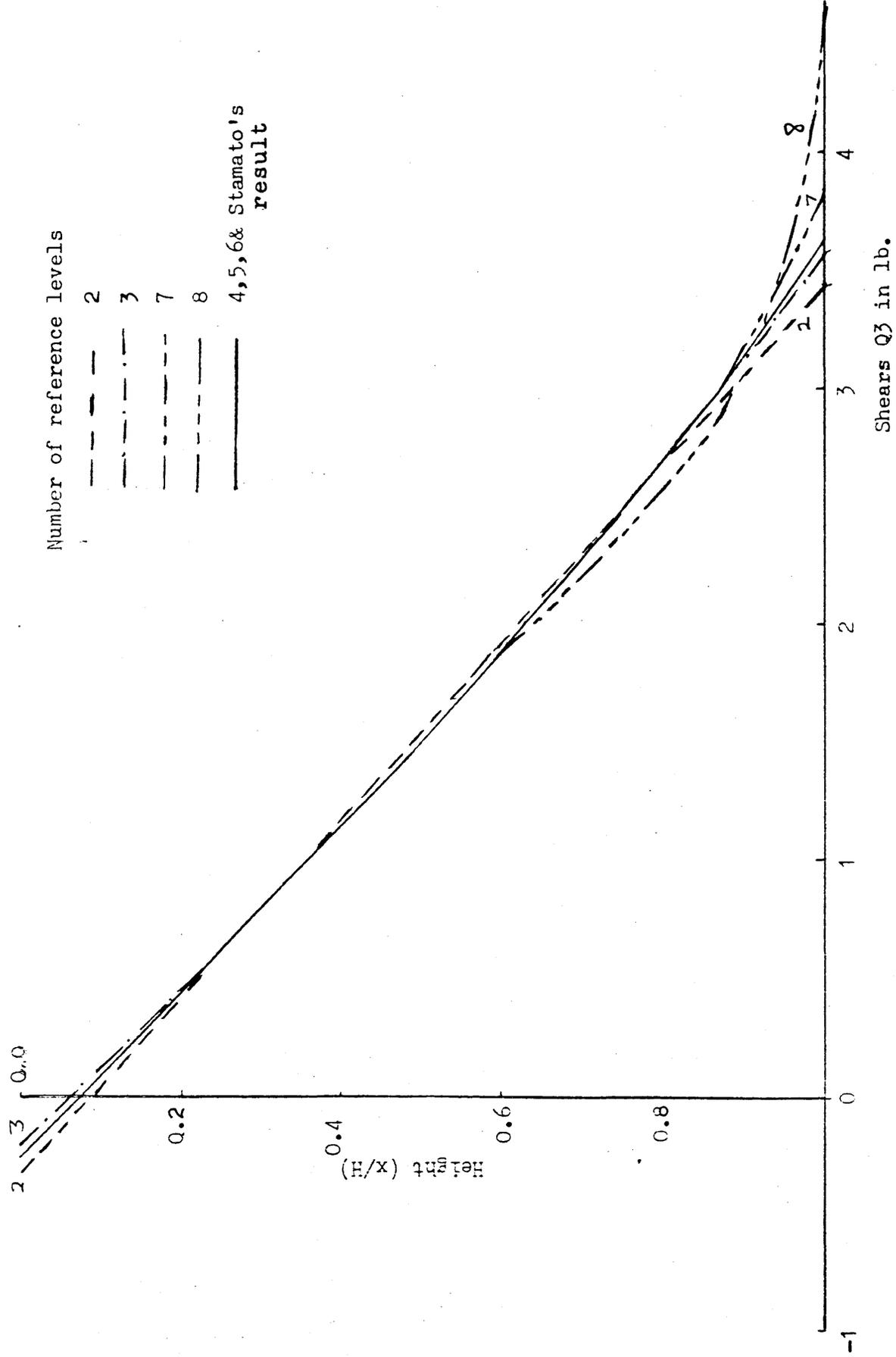


Fig. 4.38 Shear Forces in Frame 3 ("3-1")

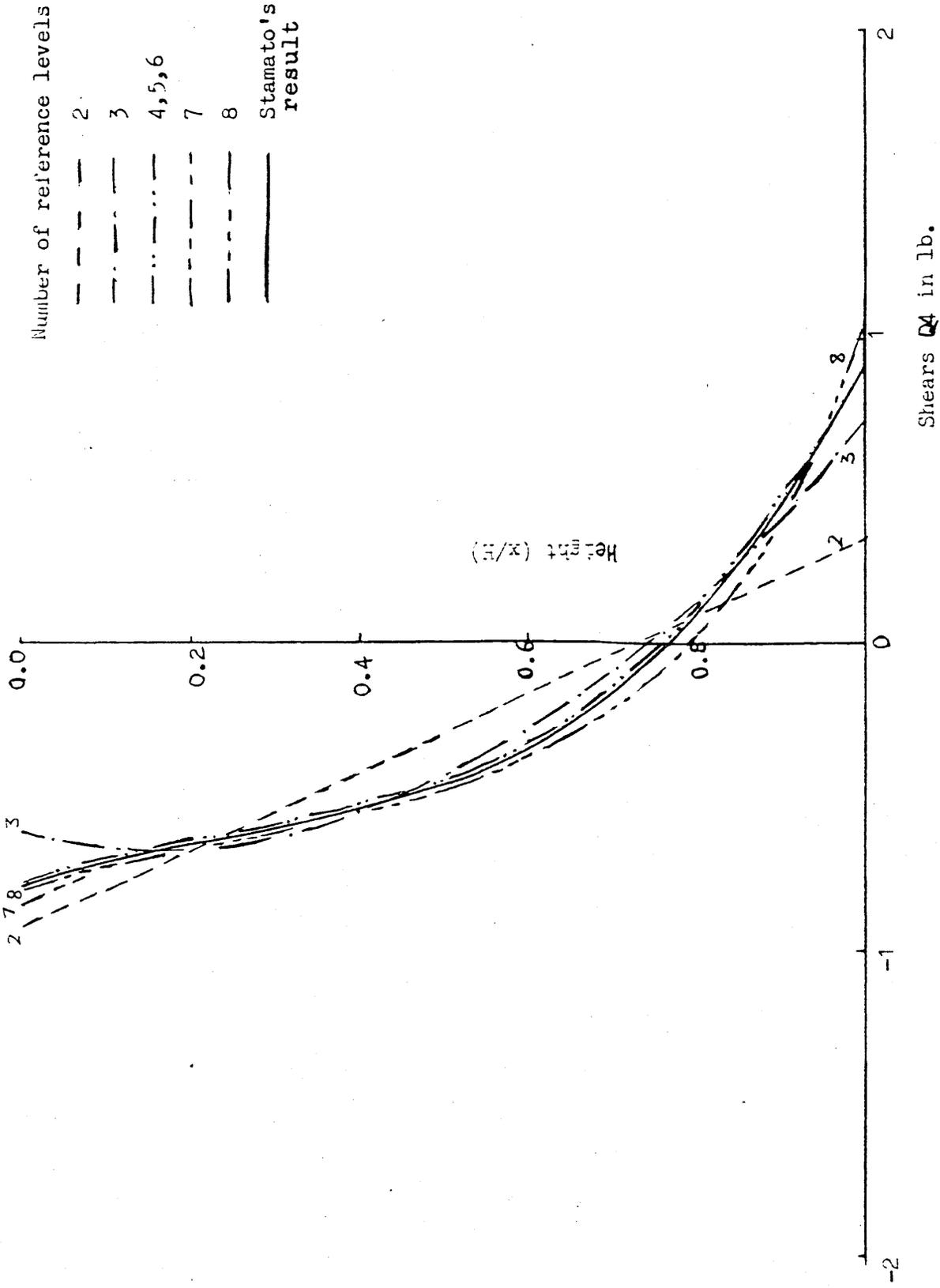


Fig. 4.39 Shear forces in frame 4 ("3-D")

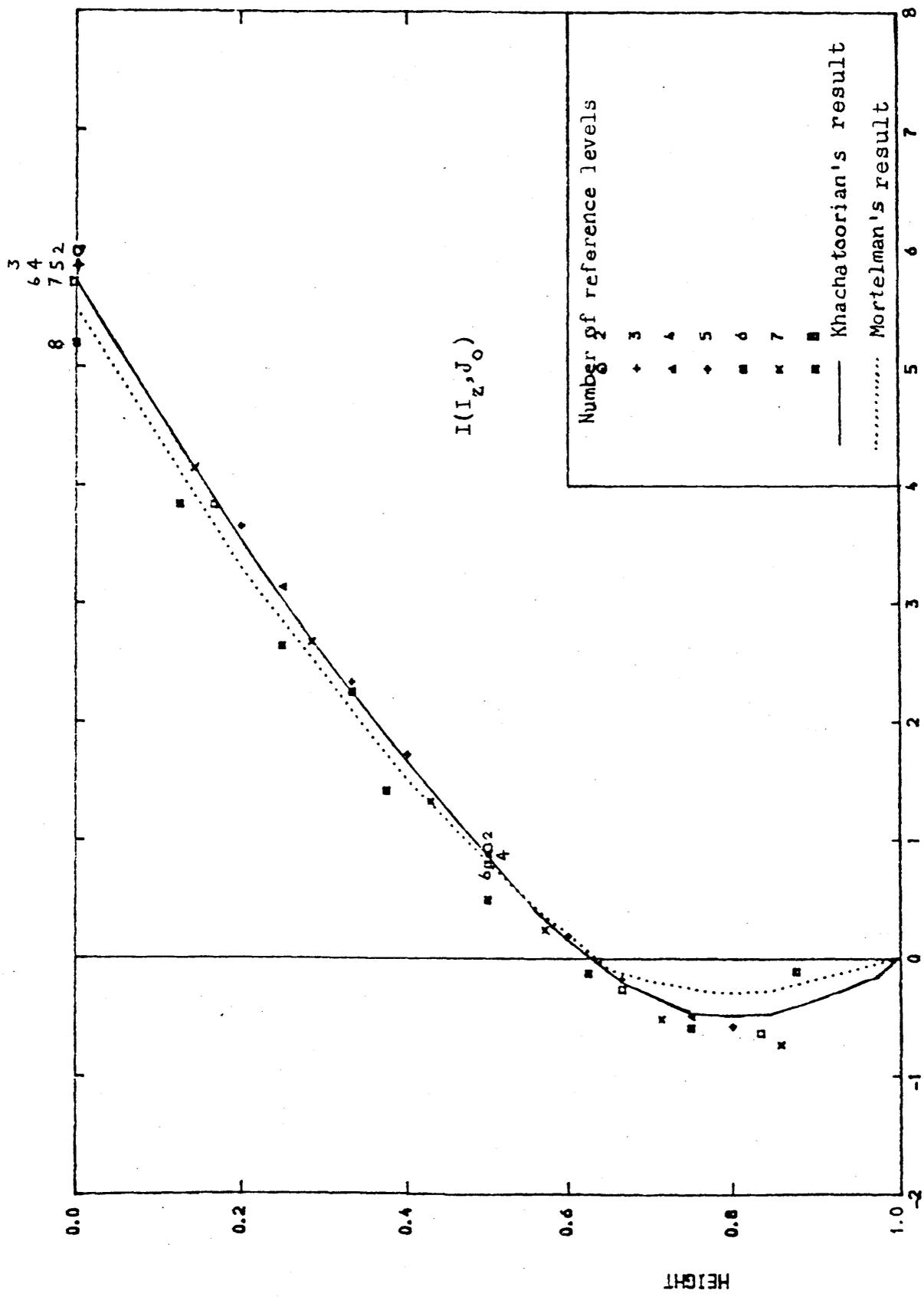
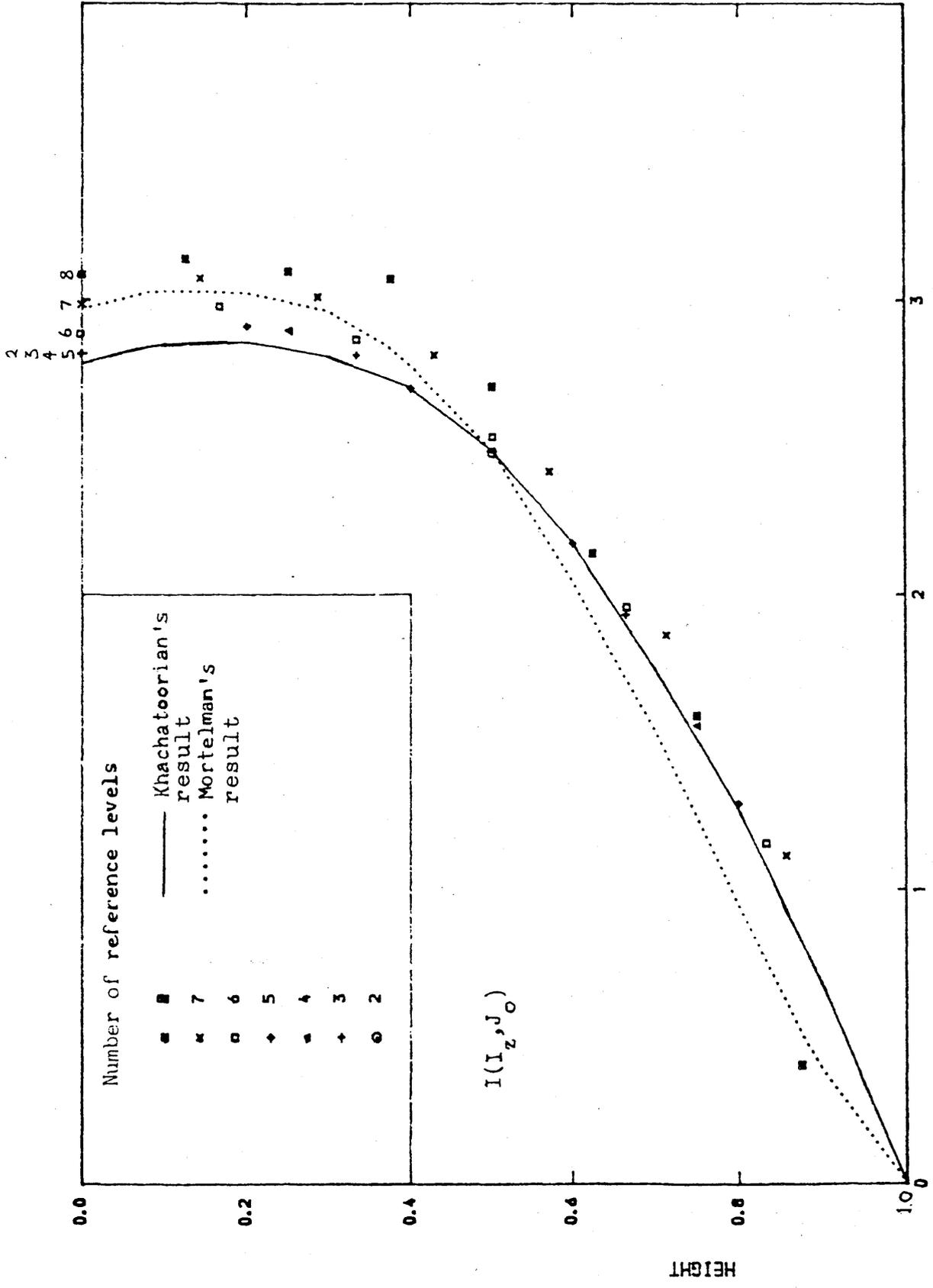
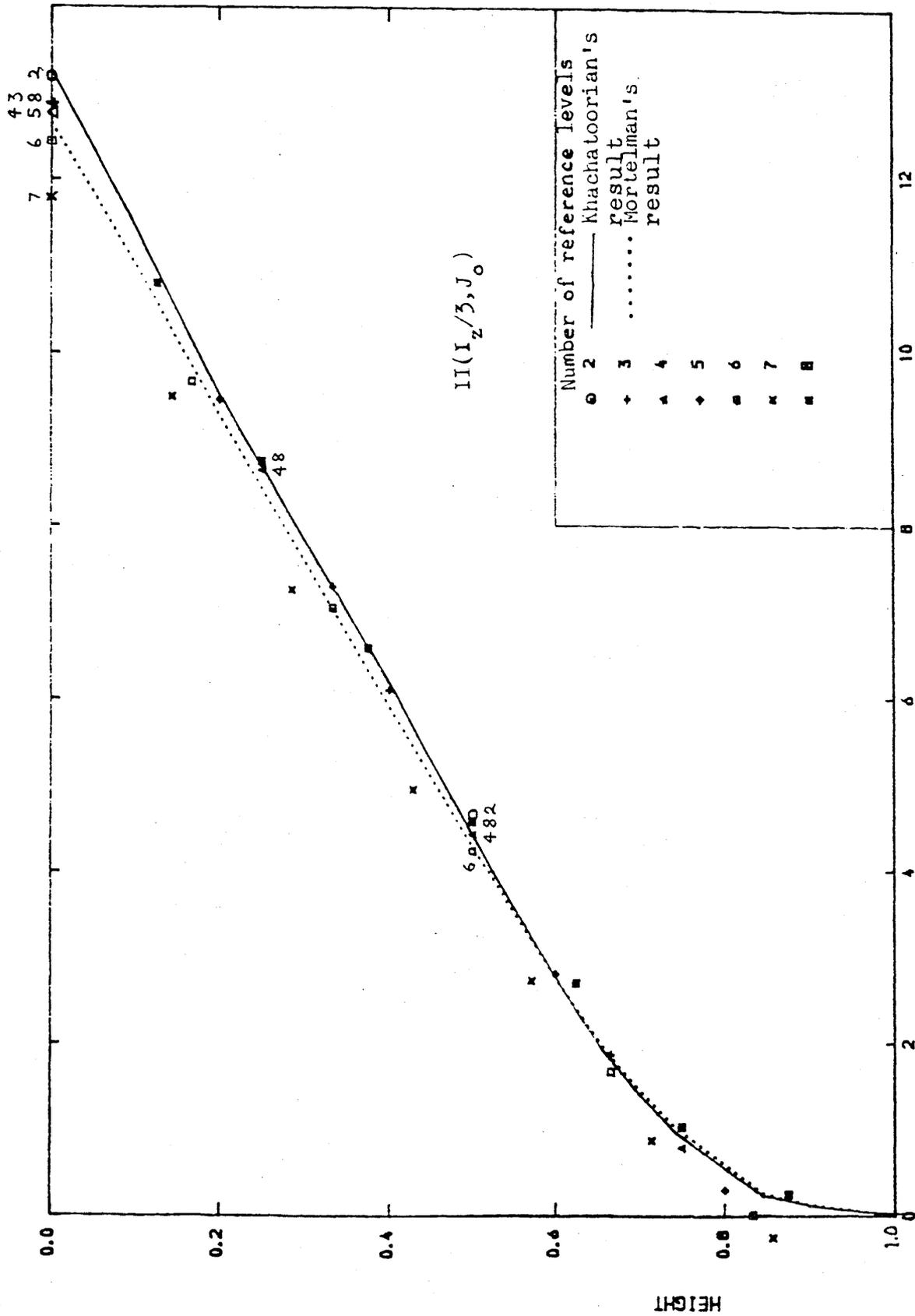


FIG 4.40 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 1 (Case I)

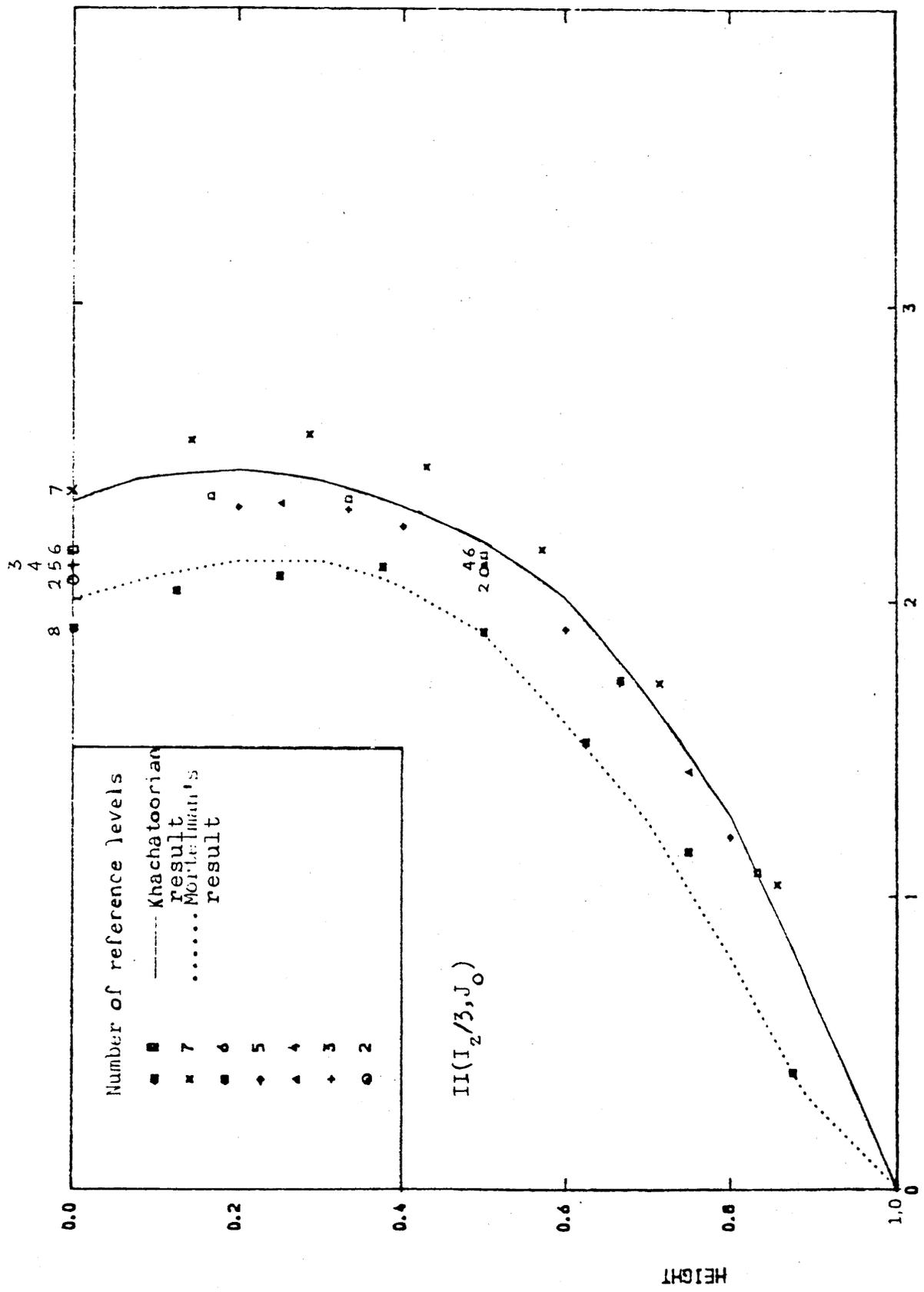


ROTATIONS x 1.0E-4 rad

FIG 4.41 ROTATIONS OF DIAPHRAGMS (Case I)



DEFLECTION Y (mm)  
 FIG 4.42 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 1 (Case II)



ROTATIONS x 1.0E-4 rad

FIG 4.43 ROTATIONS OF DIAPHRAGMS (Case II)

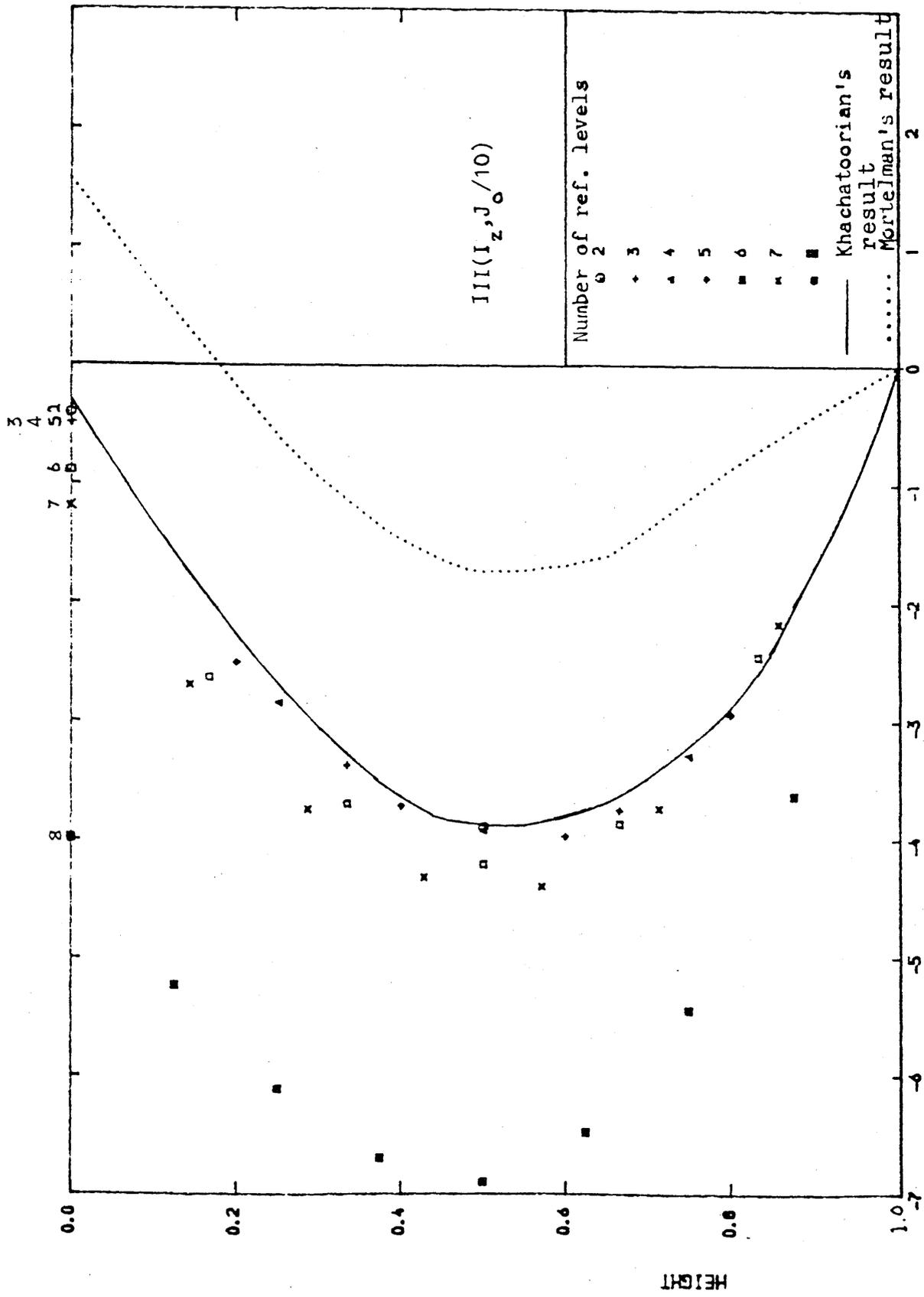


FIG4.44 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME I (Case III)

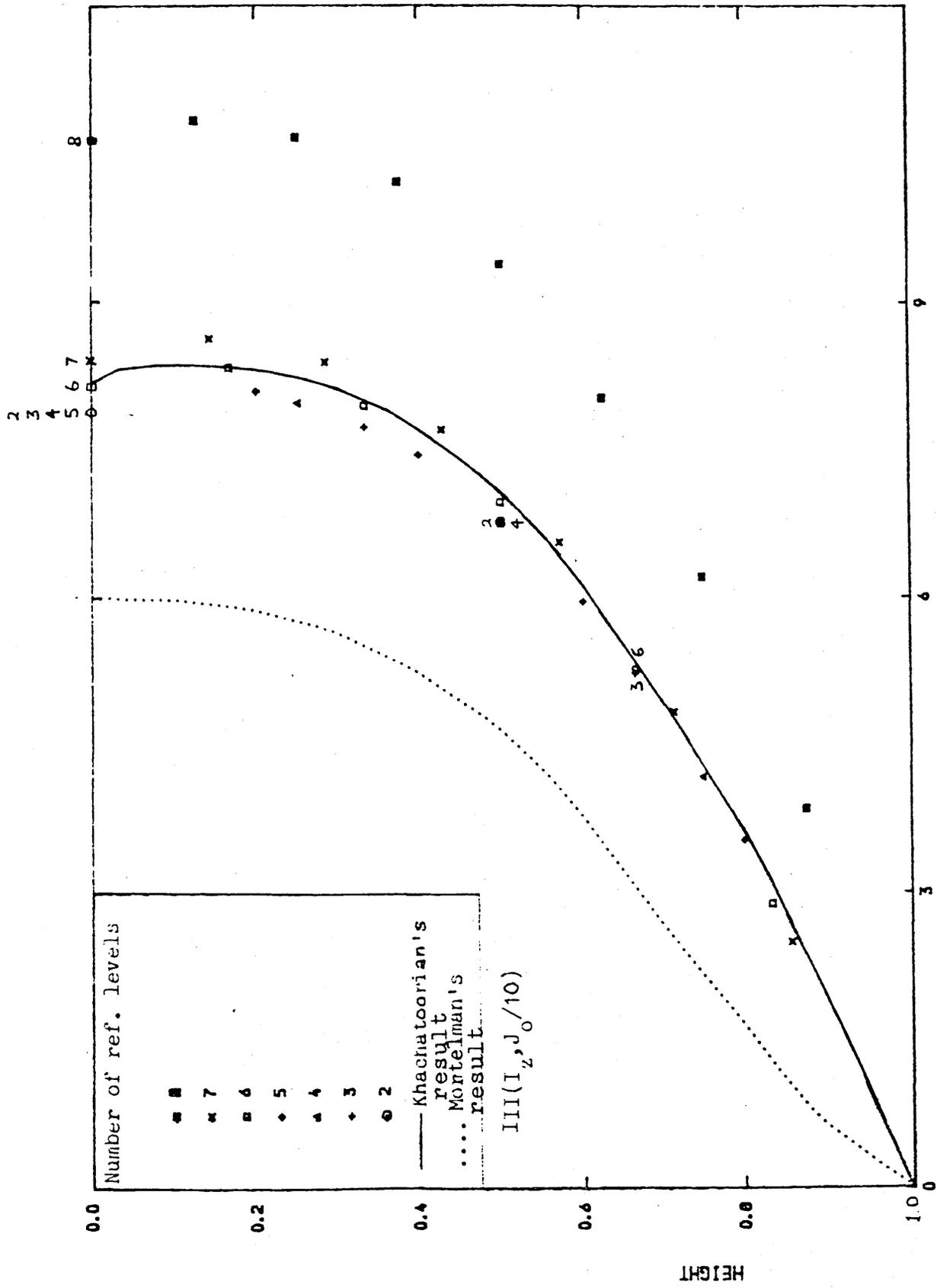


FIG 4.45 ROTATIONS OF DIAPHRAGMS (Case III)

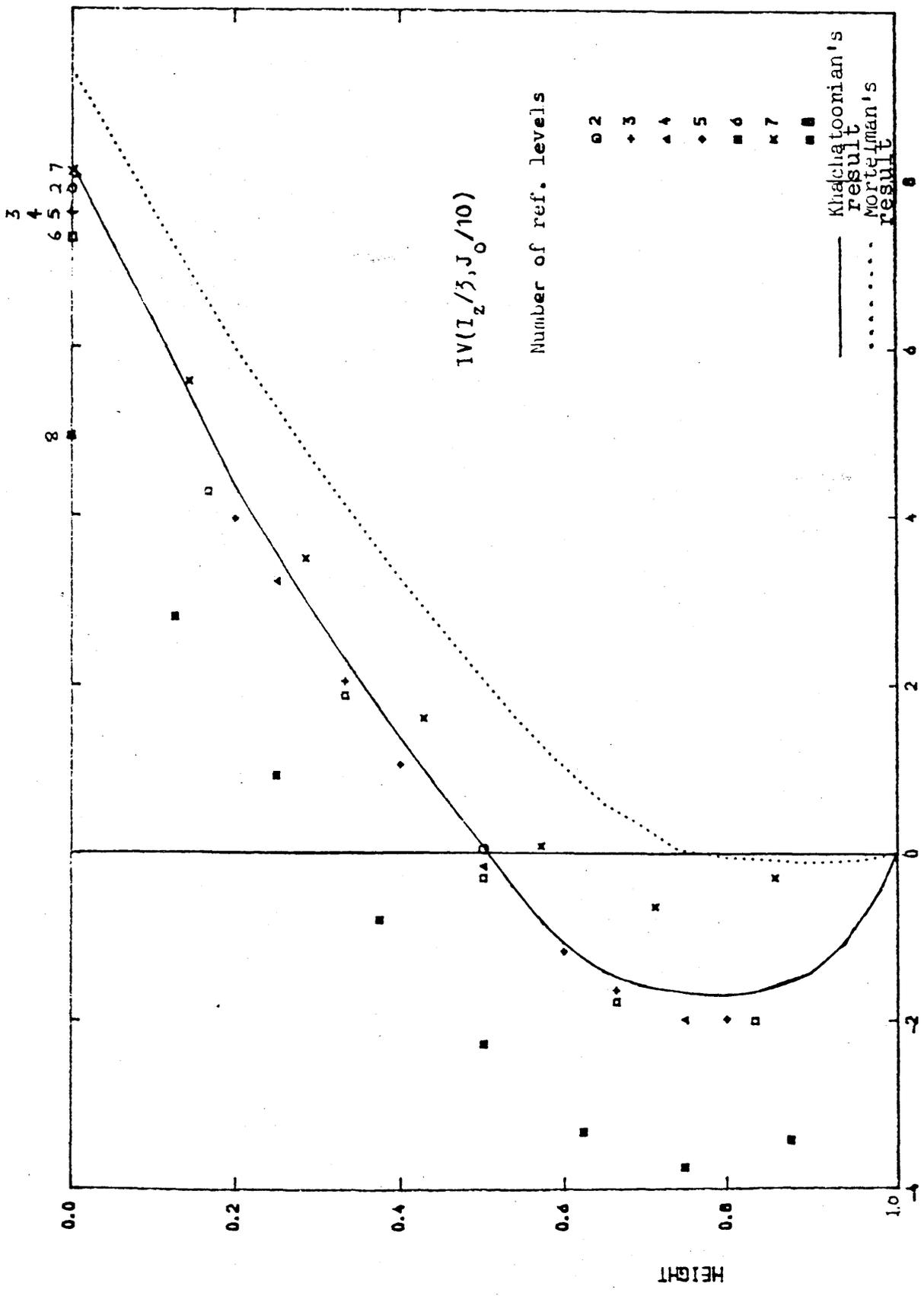
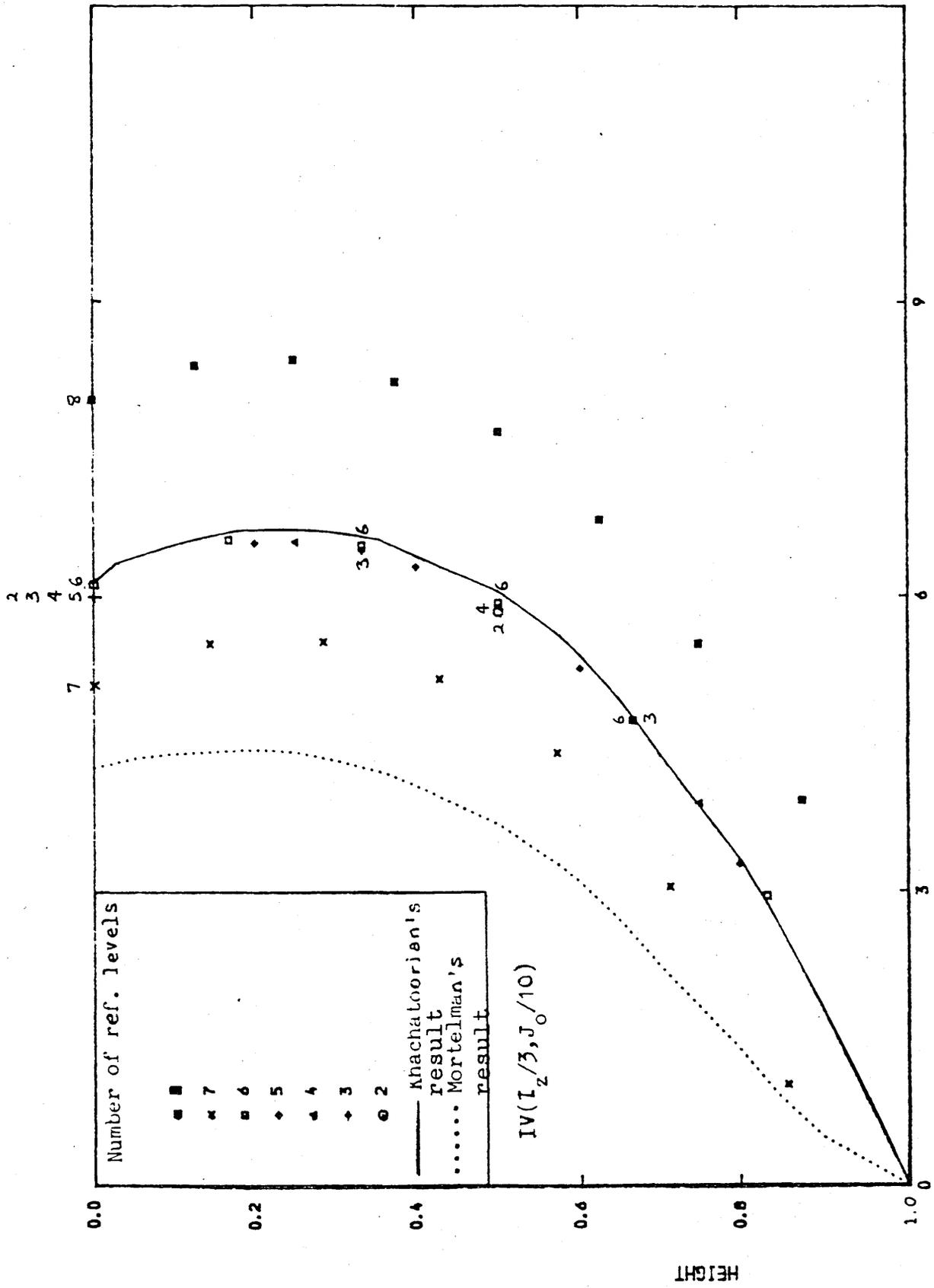


Fig. 4.46 DISTRIBUTION OF LATERAL DEFLECTION OF FRAME 1 (Case IV)



ROTATIONS  $\times 1.0E-4$  rad

Fig. 4.4 ROTATIONS OF DIAPHRAGMS (Case IV)

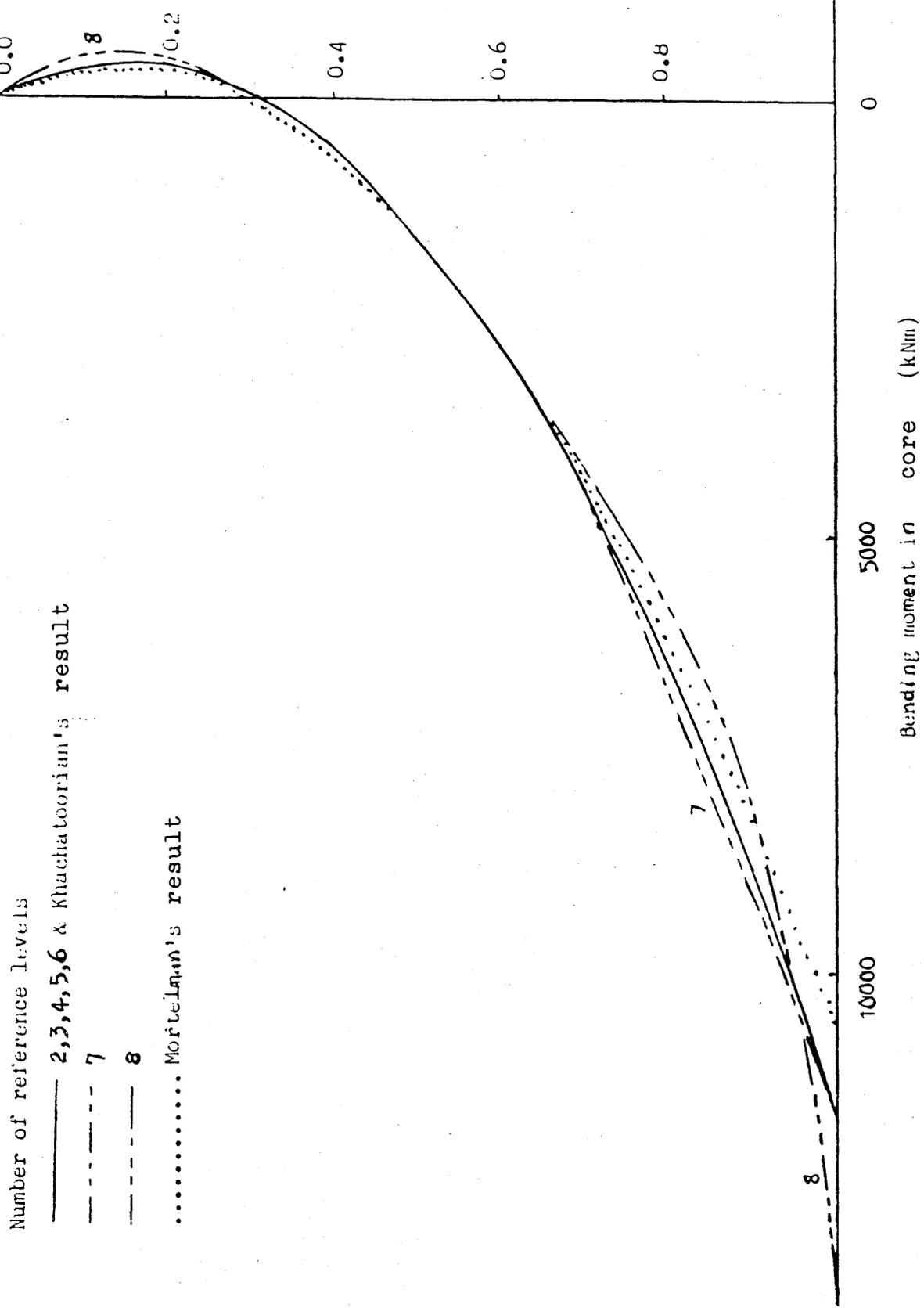


Fig.4. 48 Distribution of bending moment in core (Case I)

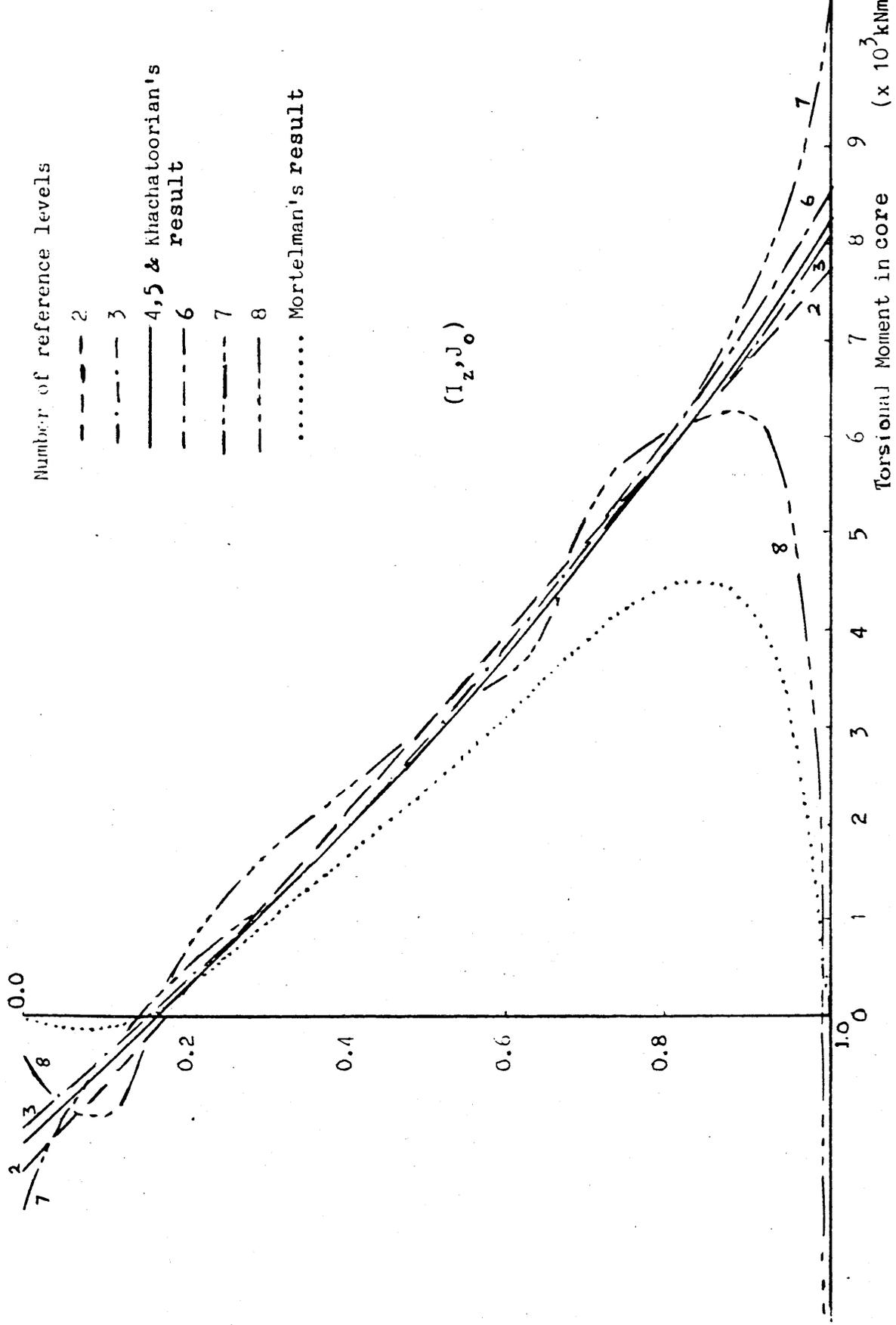
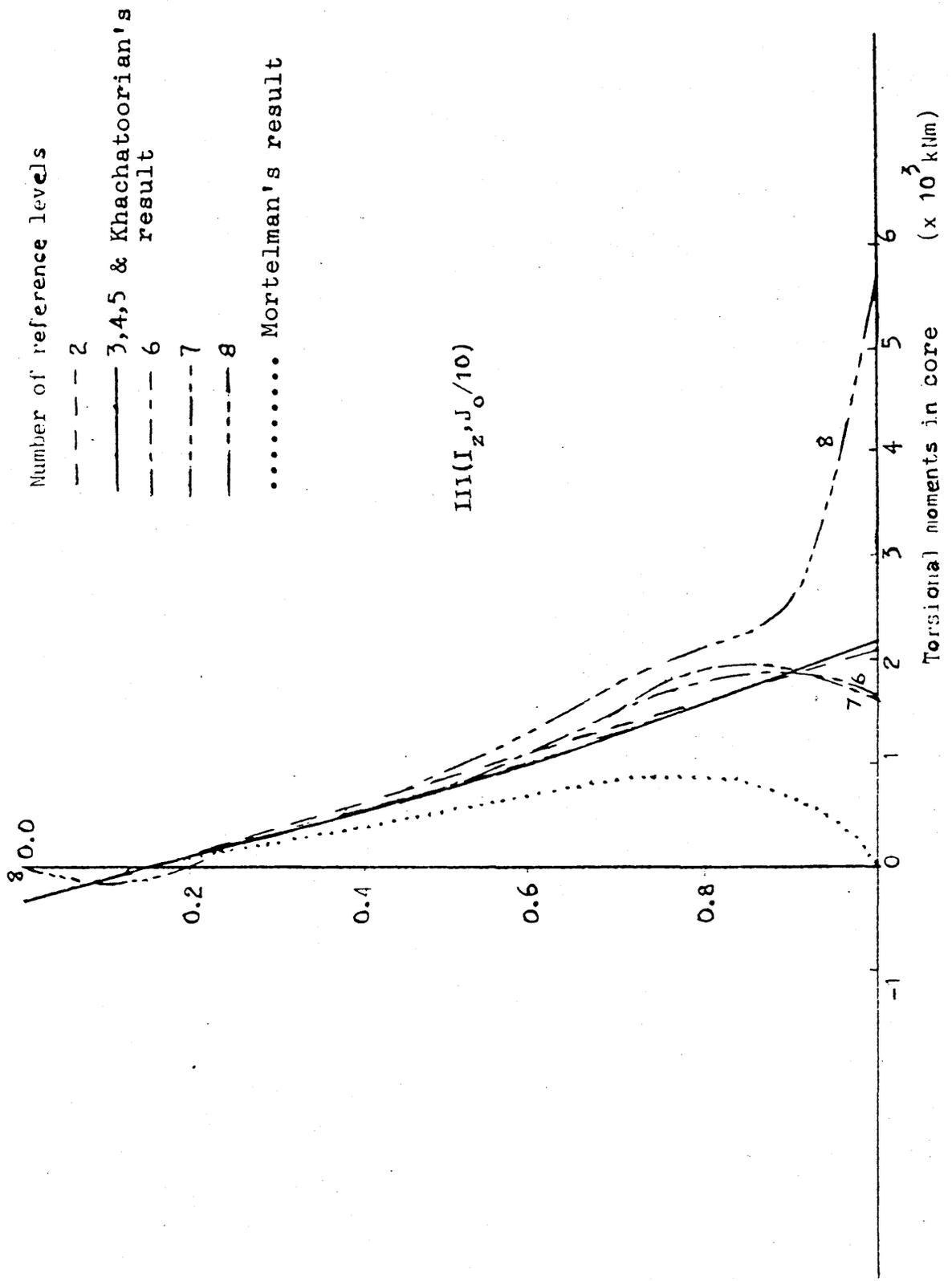


Fig. 4.49 Distribution of the torsional moments in core (Case I)



Number of reference levels

- - - - 2
- 3, 4, 5 & Khachatourian's result
- - - - 6
- - - - 7
- - - - 8
- ..... Mortelma's result

Fig 4.50 Distribution of torsional moment in core (Case III)

## CONCLUSIONS AND SUGGESTIONS

## FOR FUTURE WORK

5.1 Conclusions

A relatively simple approximate method has been presented for the solution of structural problems involving laterally loaded three-dimensional tall shear wall-frame systems.

The frame has been represented by an equivalent shear cantilever of infinite flexural rigidity which has the same shear stiffness as the frame. Single shear walls or isolated box cores have been analysed by means of elementary bending theory. Coupled walls have been analysed by using the widely accepted continuous connection technique to provide explicit solutions suitable for inclusion in a three-dimensional analysis. Solutions were given for the assumed concentrated load at the top together with a polynomial load distribution acting on the elements.

The complete structure was analysed by satisfying the compatibility and equilibrium conditions at a chosen number of reference levels throughout the height of the structure whereby the load-displacement characteristics of the individual assemblies may be evaluated in the loading forms assumed above. These loads may then be

used to evaluate the forces and displacements at any point on the individual elements in the structure.

The method may be extended to include any load form for which an explicit mathematical solution for the horizontal deflections of an element may be found at any level. The method may also be extended to incorporate any structural form for which the above explicit solution for the deflection may be described.

It was found that for a shear wall-frame system the recommended number of reference levels to be employed in the solution is 4 to 6, as shown by the consistency of all the results to produce a sufficiently accurate solution.

Although the positions of the reference levels were chosen to be at equal intervals from the top of the assembly, the forces and displacements of the elements may be evaluated at any desired level irrespective of the positions of the reference levels used.

Lastly, it can be concluded that the approximate method described in Chapter 2 may be used for the rapid analysis of the type of structures considered, particularly in the earlier design stages. Hence the method enables more plans and more economical schemes to be decided upon before adopting more complex and costly analysis.

## 5.2 Suggestions for future work

In this thesis, a relatively simple approximate method have been presented for the evaluation of lateral load distribution among the various elements of a three-dimensional structure, and the corresponding forces and displacements. However, there are still some related problems which need to be investigated. Some to these problems are detailed below.

- 1) In the present study the three-dimensional structures were only analysed under the action of uniform lateral loads. Both theoretical and experimental work should be carried out on such structures when irregular lateral loadings are applied.
- 2) The method presented are applicable to uniform regular structures. An extension of this method to structures with variable configuration along the height needs further research.
- 3) Consideration should also be given to the possibility of including the action of wind gusts and earthquake to the method.
- 4) The methods presented deal with structures consisting of various structural elements in two orthogonal directions. An investigation into the behaviour of structures consisting of obliquely placed load bearing elements is desirable.
- 5) Other structural systems for example coupled wall-frame systems need also to be investigated.

## REFERENCES

1. Monographs on the planning and design of tall buildings, 5 vols.  
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## APPENDIX A

This appendix consists of computer programmes for the numerical examples in Chapter 4. The subroutines for the inversion (MINV), multiplication(MPRD), addition (MADD) and subtraction (MSUB) of the matrices are also included.

In the programmes the dimension arrays are changed according to the number of reference levels used. For example, the programmes listed consist of the figure 7 in the dimension brackets. This means that the programme is set for seven reference levels. This should be changed if other number of reference levels are used.

Below are the programme names and their functions

- PROGRAM WF7 - to find the deflections and the internal forces for the wall-frame example of fig. 4.1
- PROGRAM DWT72 - to find the deflections, rotations and the internal forces for the structure of example 1. This programme is also for two-dimensional analysis.
- PROGRAM THREEED - similar to DWT72, but for three-dimensional analysis for example 1 structure.
- PROGRAM FC7 - to find deflections and rotations of the example structure 2.

The following are the abbreviations used in the programmes.

- N - no. of reference levels used.
- FW, FF and FT - the flexibility matrices, for the wall, frame and the rotational flexibility matrix for the wall/core.
- S - matrix of coefficients s
- BMS - matrix of coefficients to calculate base moment in wall
- WX , WY - matrix of applied load in the two orthogonal directions.
- Program SWALL1 - to compute FW, FF and FT respectively.  
FRAME1  
SWTOR1

```

1      PROGRAM WF7
2      DIMENSION FW(7,7), FF(7,7), G1(7,7), FWI(7,7), FFI(7,7), G1I(7,7)
3      , S(7,7), PW(7), PF(7), FWS(7,7), FWSI(7,7), FBF(7,7), W(7), L(7),
4      , B(7,7), BFI(7,7), SPF(7), BM(7), BS(7), BMS(7,7), BMW1(7),
5      , M(7), Y(7), FWFC(7,7)
6      READ (5,*) N
7      READ (5,*) ((FW(I,J),J=1,N),I=1,N)
8      READ (5,*) ((FF(I,J),J=1,N),I=1,N)
9      READ (5,*) ((S(I,J),J=1,N),I=1,N)
10     READ (5,*) ((BMS(I,J),J=1,N),I=1,N)
11     READ (5,*) (W(I),I=1,N)
12     DO 2 I=1,N
13     DO 2 J=1,N
14     2 FWI(I,J)=FW(I,J)
15     CALL MINV(FWI,N,D,L,M)
16     DO 3 I=1,N
17     DO 3 J=1,N
18     3 FFI(I,J)=FF(I,J)
19     CALL MINV(FFI,N,D,L,M)
20     CALL MADD(FWI,FFI,FBF,N,N,O,O)
21     CALL MPRD(S,FBF,G1,N,N,O,O,N)
22     DO 10 I=1,N
23     DO 10 J=1,N
24     10 G1I(I,J)=G1(I,J)
25     CALL MINV(G1I,N,D,L,M)
26     CALL MPRD(G1I,W,Y,N,N,O,O,1)
27     CALL MPRD(FWI,Y,PW,N,N,O,O,1)
28     CALL MPRD(FFI,Y,PF,N,N,O,O,1)
29     DO 55 I=2,N
30     IS=I-1
31     BS(I)=36.0/FLOAT(IS)
32     BS(1)=1.0
33     55 CONTINUE
34     DO 58 I=2,N
35     IM=I-1
36     BE=FACT(IM-1)/FACT(IM+1)
37     BM(I)=BE*1296.0
38     BM(1)=36.0
39     58 CONTINUE
40     CALL MPRD(BM,PW,BM1,1,N,O,O,1)
41     CALL MPRD(BMS,PW,BMW1,N,N,O,O,1)
42     CALL MPRD(BS,PF,BS1,1,N,O,O,1)
43     CALL MPRD(S,PF,SPF,N,N,O,O,1)
44     DO 100 I=1,N
45     WRITE (6,101) BMW1(I), Y(I)
46     101 FORMAT (2E13.5)
47     100 CONTINUE
48     WRITE (6,121) BM1
49     121 FORMAT (E13.5)
50     DO 106 I=1,N
51     WRITE (6,108) PW(I)
52     108 FORMAT (E13.5)
53     106 CONTINUE
54     STOP
55     END
56     FUNCTION FACT(I)
57     IF (I.LT.0) GOTO 110
58     FACT=1.0
59     IF (I.EQ.0) GOTO 109
60     DO 99 NN=0,I-1
61     TERM=FLOAT(I-NN)

```



```

1 PROGRAM DWT72
2 DIMENSION FW(7,7), FF(7,7), FT(7,7), FWI(7,7), FFI(7,7), FTI(7,7)
3 , B(7,7), C(7,7), C1(7,7), E(7,7), F(7,7), S(7,7), G2TSBI(7,7),
4 , BXFFI(7,7), FBF(7,7), CXFWI(7,7), C1XFFI(7,7), CWC1F(7,7),
5 , FXFFI(7,7), EWFF(7,7), EWFT(7,7), G1G2I(7,7), XG3(7,7), WSUB(7),
6 , EXFWI(7,7), G1(7,7), G2(7,7), G3(7,7), T(7), W(7), THETA(7),
7 , G2I(7,7), L(7), M(7), G3G2I(7,7), XG1(7,7), G2SUB(7,7),
8 , G2SUBI(7,7), G3G2IW(7), TSUB(7), Y(7), G2TSB(7,7), G1G2IT(7),
9 , HTHETA(7), H(7,7), Y1(7), F(7), SV(7), FFTI(7,7),
10 , H1(7,7), H1T(7), P2(7), SP2(7), Y2(7), P3(7), SP3(7),
11 , BS(7), BMS(7,7), BMW1(7), BM(7),
12 , S12(7), S123(7), T2(7), T3(7), T4(7), T23(7), T234(7),
13 , ST(7), TW1(7), STW1(7),
14 , Q(7,7), QTHETA(7,7), P1(7), SV1(7)
15 READ (5,*) N
16 READ (5,*) ((FW(I,J),J=1,N),I=1,N)
17 READ (5,*) ((FF(I,J),J=1,N),I=1,N)
18 READ (5,*) ((FT(I,J),J=1,N),I=1,N)
19 READ (5,*) ((S(I,J),J=1,N),I=1,N)
20 READ (5,*) ((BMS(I,J),J=1,N),I=1,N)
21 READ (5,*) (T(I),I=1,N)
22 READ (5,*) (W(I),I=1,N)
23 READ (5,*) ((B(I,J),J=1,N),I=1,N)
24 C READ (5,*) ((C(I,J),J=1,N),I=1,N)
25 READ (5,*) ((H(I,J),J=1,N),I=1,N)
26 C READ (5,*) ((E(I,J),J=1,N),I=1,N)
27 READ (5,*) ((C1(I,J),J=1,N),I=1,N)
28 READ (5,*) ((F(I,J),J=1,N),I=1,N)
29 READ (5,*) ((Q(I,J),J=1,N),I=1,N)
30 READ (5,*) ((H1(I,J),J=1,N),I=1,N)
31 DO 2 I=1,N
32 DO 2 J=1,N
33 2 FWI(I,J)=FW(I,J)
34 CALL MINV(FWI,N,D,L,M)
35 C WRITE (6,*) ((FWI(I,J),J=1,N),I=1,N)
36 DO 3 I=1,N
37 DO 3 J=1,N
38 3 FFI(I,J)=FF(I,J)
39 CALL MINV(FFI,N,D,L,M)
40 C WRITE (6,*) ((FFI(I,J),J=1,N),I=1,N)
41 DO 4 I=1,N
42 DO 4 J=1,N
43 4 FTI(I,J)=FT(I,J)
44 CALL MINV(FTI,N,D,L,M)
45 CALL MPRD(B,FFI,BXFFI,N,N,O,O,N)
46 CALL MADD(FWI,BXFFI,FBF,N,N,O,O)
47 CALL MPRD(S,FBF,G1,N,N,O,O,N)
48 C CALL MPRD(C,FWI,CXFWI,N,N,O,O,N)
49 CALL MPRD(C1,FFI,C1XFFI,N,N,O,O,N)
50 C CALL MADD(CXFWI,C1XFFI,CWC1F,N,N,O,O)
51 C CALL MPRD(S,CWC1F,G2,N,N,O,O,N)
52 CALL MPRD(S,C1XFFI,G2,N,N,O,O,N)
53 C CALL MPRD(E,FWI,EXFWI,N,N,O,O,N)
54 CALL MPRD(F,FFI,FXFFI,N,N,O,O,N)
55 C CALL MADD(EXFWI,FXFFI,EWFF,N,N,O,O)
56 C CALL MADD(EWFF,FTI,EWFT,N,N,O,O)
57 C CALL MPRD(S,EWFT,G3,N,N,O,O,N)
58 CALL MADD(FXFFI,FTI,FFTI,N,N,O,O)
59 C CALL MPRD(S,FXFFI,G3,N,N,O,O,N)
60 CALL MPRD(S,FFTI,G3,N,N,O,O,N)
61 C WRITE (6,*) ((G3(I,J),J=1,N),I=1,N)

```

```

62      DO 10 I=1,N
63      DO 10 J=1,N
64      10 G2I(I,J)=G2(I,J)
65      CALL MINV(G2I,N,D,L,M)
66      CALL MPRD(G3,G2I,G3G2I,N,N,O,O,N)
67      CALL MPRD(G3G2I,G1,XG1,N,N,O,O,N)
68      CALL MSUB(G2,XG1,G2SUB,N,N,O,O)
69      DO 20 I=1,N
70      DO 20 J=1,N
71      20 G2SUBI(I,J)=G2SUB(I,J)
72      CALL MINV(G2SUBI,N,D,L,M)
73      CALL MPRD(G3G2I,W,G3G2IW,N,N,O,O,1)
74      CALL MSUB(T,G3G2IW,TSUB,N,1,O,O)
75      CALL MPRD(G2SUBI,TSUB,Y,N,N,O,O,1)
76      1 FORMAT (7E15.5)
77      CALL MPRD(G1,G2I,G1G2I,N,N,O,O,N)
78      CALL MPRD(G1G2I,G3,XG3,N,N,O,O,N)
79      CALL MSUB(G2,XG3,G2TSB,N,N,O,O)
80      DO 30 I=1,N
81      DO 30 J=1,N
82      30 G2TSBI(I,J)=G2TSB(I,J)
83      CALL MINV(G2TSBI,N,D,L,M)
84      CALL MPRD(G1G2I,T,G1G2IT,N,N,O,O,1)
85      CALL MSUB(W,G1G2IT,WSUB,N,1,O,O)
86      CALL MPRD(G2TSBI,WSUB,THETA,N,N,O,O,1)
87      5 FORMAT (7E15.5)
88      CALL MPRD(H,THETA,HTHETA,N,N,O,O,1)
89      CALL MADD(Y,HTHETA,Y1,N,1,O,O)
90      11 FORMAT (7E15.5)
91      CALL MPRD(FWI,Y,P,N,N,O,O,1)
92      DO 55 I=2,N
93      IS=I-1
94      BS(I)=50.0/FLOAT(IS)
95      BS(1)=1.0
96      55 CONTINUE
97      DO 58 I=2,N
98      IM=I-1
99      BE=FACT(IM-1)/FACT(IM+1)
100     BM(I)=BE*2500.0
101     BM(1)=50.0
102     58 CONTINUE
103     CALL MPRD(S,P,SV,N,N,O,O,1)
104     25 FORMAT (7E15.5)
105     CALL MPRD(Q,THETA,QTHETA,N,N,O,O,1)
106     CALL MPRD(FFI,QTHETA,F1,N,N,O,O,1)
107     CALL MPRD(S,P1,SV1,N,N,O,O,1)
108     26 FORMAT (7E15.5)
109     CALL MPRD(FFI,Y1,P2,N,N,O,O,1)
110     CALL MPRD(S,P2,SP2,N,N,O,O,1)
111     CALL MPRD(H1,THETA,H1T,N,N,O,O,1)
112     CALL MADD(Y,H1T,Y2,N,1,O,O)
113     CALL MPRD(FFI,Y2,P3,N,N,O,O,1)
114     CALL MPRD(S,P3,SP3,N,N,O,O,1)
115     CALL MPRD(BM,P,BM1,1,N,O,O,1)
116     CALL MPRD(BMS,P,BMW1,N,N,O,O,1)
117     CALL MPRD(BS,P2,BS2,1,N,O,O,1)
118     CALL MPRD(BS,P3,BS3,1,N,O,O,1)
119     CALL MPRD(BS,P1,BS4,1,N,O,O,1)
120     CALL MPRD(FTI,THETA,TW1,N,N,O,O,1)
121     CALL MPRD(S,TW1,STW1,N,N,O,O,1)
122     CALL MADD(SV,SP2,S12,N,1,O,O)
123     CALL MADD(S12,SP3,S123,N,1,O,O)

```



```

1 PROGRAM THREEED
2 DIMENSION L(7), M(7), G1I(7,7), G1(7,7), G2I(7,7), G2(7,7),
3 , G3I(7,7), G3(7,7), G4I(7,7), G4(7,7), G4G3I(7,7), G2I43I(7,7),
4 , G5(7,7), G5G3I(7,7), G4I53I(7,7), G4IG3(7,7), AG1(7,7), B(7,7),
5 , BI(7,7), WX(7), AWX(7), WY(7), G2IWY(7), T(7), G4IT(7), C2(7),
6 , CG4IT(7), V1(7), U(7), G1IG3(7,7), G1I34I(7,7), G3IG5(7,7),
7 , G3IG4(7,7), Z(7,7), ZG2(7,7), E(7,7), EI(7,7), G1IWX(7), ZWY(7),
8 , G3IT(7), F(7), FZWY(7), V(7), G3I42I(7,7), G(7,7), H(7,7),
9 , QI(7,7), Q(7,7), R(7), X(7), XR(7), ROT(7), A(7,7), G3I54I(7,7),
10 , FW(7,7), FF(7,7), FT(7,7), FWI(7,7), FFI(7,7), FTI(7,7),
11 , B1(7,7), C(7), C1(7,7), E1(7,7), F1(7,7), S(7,7), EXFWI(7,7),
12 , BXFFI(7,7), FBF(7,7), CXFWI(7,7), C1XFFI(7,7), CWC1F(7,7),
13 , FXFFI(7,7), B2ROT(7), PF2(7), U1(7), EWFF(7,7), EWFT(7,7),
14 , SPW1(7), W5(7,7), W5ROT(7), B3(7,7), B3ROT(7), V2(7), PF3(7),
15 , SPF3(7), ST(7), TW1(7), STW1(7), PF4(7), SPF4(7),
16 , S12(7), S123(7), T2(7), T3(7), T4(7), T23(7), T234(7),
17 , BS(7), BMS(7,7), BMW1(7), BM(7), PW2(7),
18 , FW1(7,7), FW1I(7,7), BFW1I(7,7), W5FW(7,7), B25FW(7,7),
19 , S25(7,7), S25FW(7,7), E25FW(7,7),
20 , B2(7,7), PW1(7), UM(7), SPF2(7), PF5(7), SPF5(7), B2FFI(7,7)
21 READ (5,*) N
22 READ (5,*) ((FW(I, J), J=1, N), I=1, N)
23 READ (5,*) ((FF(I, J), J=1, N), I=1, N)
24 READ (5,*) ((FT(I, J), J=1, N), I=1, N)
25 READ (5,*) ((S(I, J), J=1, N), I=1, N)
26 READ (5,*) ((BMS(I, J), J=1, N), I=1, N)
27 READ (5,*) (T(I), I=1, N)
28 READ (5,*) (WX(I), I=1, N)
29 READ (5,*) (WY(I), I=1, N)
30 READ (5,*) ((B1(I, J), J=1, N), I=1, N)
31 READ (5,*) ((B2(I, J), J=1, N), I=1, N)
32 C READ (5,*) ((C2(I, J), J=1, N), I=1, N)
33 C READ (5,*) ((C1(I, J), J=1, N), I=1, N)
34 C READ (5,*) ((E1(I, J), J=1, N), I=1, N)
35 READ (5,*) ((F1(I, J), J=1, N), I=1, N)
36 READ (5,*) ((W5(I, J), J=1, N), I=1, N)
37 READ (5,*) ((B3(I, J), J=1, N), I=1, N)
38 READ (5,*) ((FW1(I, J), J=1, N), I=1, N)
39 READ (5,*) ((S25(I, J), J=1, N), I=1, N)
40 DO 2 I=1, N
41 DO 2 J=1, N
42 2 FWI(I, J)=FW(I, J)
43 CALL MINV(FWI, N, D, L, M)
44 DO 3 I=1, N
45 DO 3 J=1, N
46 3 FFI(I, J)=FF(I, J)
47 CALL MINV(FFI, N, D, L, M)
48 DO 4 I=1, N
49 DO 4 J=1, N
50 4 FTI(I, J)=FT(I, J)
51 CALL MINV(FTI, N, D, L, M)
52 DO 5 I=1, N
53 DO 5 J=1, N
54 5 FW1I(I, J)=FW1(I, J)
55 CALL MINV(FW1I, N, D, L, M)
56 CALL MPRD(B1, FFI, BXFFI, N, N, O, O, N)
57 CALL MADD(BXFFI, FW1I, BFW1I, N, N, O, O)
58 CALL MPRD(S, BFW1I, G1, N, N, O, O, N)
59 CALL MPRD(B2, FFI, B2FFI, N, N, O, O, N)
60 CALL MPRD(W5, FW1I, W5FW, N, N, O, O, N)
61 CALL MADD(B2FFI, W5FW, B25FW, N, N, O, O)

```

```

62      CALL MPRD(S,B25FW,G3,N,N,O,O,N)
63      CALL MADD(FWI,BXFFI,FBF,N,N,O,O)
64      CALL MPRD(S,FBF,G2,N,N,O,O,N)
65      CALL MPRD(C1,FFI,C1XFFI,N,N,O,O,N)
66      CALL MPRD(S,C1XFFI,G4,N,N,O,O,N)
67      CALL MPRD(F1,FFI,FXFFI,N,N,O,O,N)
68      CALL MADD(FXFFI,FTI,EWFT,N,N,O,O)
69      CALL MPRD(S25,FW1I,S25FW,N,N,O,O,N)
70      CALL MADD(EWFT,S25FW,E25FW,N,N,O,O)
71      CALL MPRD(S,E25FW,G5,N,N,O,O,N)
72      DO 10 I=1,N
73      DO 10 J=1,N
74      10 G1I(I,J)=G1(I,J)
75      CALL MINV(G1I,N,D,L,M)
76      DO 20 I=1,N
77      DO 20 J=1,N
78      20 G2I(I,J)=G2(I,J)
79      CALL MINV(G2I,N,D,L,M)
80      DO 30 I=1,N
81      DO 30 J=1,N
82      30 G3I(I,J)=G3(I,J)
83      CALL MINV(G3I,N,D,L,M)
84      DO 40 I=1,N
85      DO 40 J=1,N
86      40 G4I(I,J)=G4(I,J)
87      CALL MINV(G4I,N,D,L,M)
88      CALL MPRD(G4,G3I,G4G3I,N,N,O,O,N)
89      CALL MPRD(G2I,G4G3I,G2I43I,N,N,O,O,N)
90      CALL MPRD(G5,G3I,G5G3I,N,N,O,O,N)
91      CALL MPRD(G4I,G5G3I,G4I53I,N,N,O,O,N)
92      CALL MPRD(G4I,G3,G4IG3,N,N,O,O,N)
93      CALL MSUB(G2I43I,G4I53I,A,N,N,O,O)
94      CALL MPRD(A,G1,AG1,N,N,O,O,N)
95      CALL MADD(AG1,G4IG3,B,N,N,O,O)
96      DO 50 I=1,N
97      DO 50 J=1,N
98      50 BI(I,J)=B(I,J)
99      CALL MINV(BI,N,D,L,M)
100     CALL MPRD(A,WX,AWX,N,N,O,O,1)
101     CALL MPRD(G2I,WY,G2IWY,N,N,O,O,1)
102     CALL MPRD(G4I,T,G4IT,N,N,O,O,1)
103     CALL MADD(AWX,G2IWY,C,N,1,O,O)
104     CALL MSUB(C,G4IT,CG4IT,N,1,O,O)
105     CALL MPRD(BI,CG4IT,U,N,N,O,O,1)
106     CALL MPRD(G1I,G3,G1IG3,N,N,O,O,N)
107     CALL MPRD(G1IG3,G4I,G1I34I,N,N,O,O,N)
108     CALL MPRD(G3I,G5,G3IG5,N,N,O,O,N)
109     CALL MPRD(G3IG5,G4I,G3I54I,N,N,O,O,N)
110     CALL MPRD(G3I,G4,G3IG4,N,N,O,O,N)
111     CALL MSUB(G1I34I,G3I54I,Z,N,N,O,O)
112     CALL MPRD(Z,G2,ZG2,N,N,O,O,N)
113     CALL MADD(ZG2,G3IG4,E,N,N,O,O)
114     DO 60 I=1,N
115     DO 60 J=1,N
116     60 EI(I,J)=E(I,J)
117     CALL MINV(EI,N,D,L,M)
118     CALL MPRD(G1I,WX,G1IWX,N,N,O,O,1)
119     CALL MPRD(G3I,T,G3IT,N,N,O,O,1)
120     CALL MPRD(Z,WY,ZWY,N,N,O,O,1)
121     CALL MADD(G1IWX,G3IT,F,N,1,O,O)
122     CALL MADD(F,ZWY,FZWY,N,1,O,O)
123     CALL MPRD(EI,FZWY,V,N,N,O,O,1)

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124 CALL MPRD(G3IG4,G2I,G3I42I,N,N,O,O,N)
125 CALL MPRD(G3I42I,G4,G,N,N,O,O,N)
126 CALL MSUB(G3IG5,G,H,N,N,O,O)
127 CALL MSUB(H,G1IG3,Q,N,N,O,O)
128 DO 80 I=1,N
129 DO 80 J=1,N
130 80 QI(I,J)=Q(I,J)
131 CALL MINV(QI,N,D,L,M)
132 CALL MPRD(G3I42I,WY,R,N,N,O,O,1)
133 CALL MADD(G1IWX,G3IT,X,N,1,O,O)
134 CALL MSUB(X,R,XR,N,1,O,O)
135 CALL MPRD(QI,XR,ROT,N,N,O,O,1)
136 CALL MPRD(B2,ROT,B2ROT,N,N,O,O,1)
137 CALL MSUB(U,B2ROT,U1,N,1,O,O)
138 CALL MADD(V,B2ROT,V1,N,1,O,O)
139 CALL MPRD(W5,ROT,W5ROT,N,N,O,O,1)
140 CALL MSUB(U,W5ROT,UM,N,1,O,O)
141 CALL MPRD(FWI,V,PW1,N,N,O,O,1)
142 CALL MPRD(FW1I,UM,PW2,N,N,O,O,1)
143 DO 55 I=2,N
144 IS=I-1
145 BS(I)=50.0/FLOAT(IS)
146 BS(1)=1.0
147 55 CONTINUE
148 DO 58 I=2,N
149 IM=I-1
150 BE=FACT(IM-1)/FACT(IM+1)
151 BM(I)=BE*2500.0
152 BM(1)=50.0
153 58 CONTINUE
154 CALL MPRD(BM,PW1,BM1,1,N,O,O,1)
155 CALL MPRD(BM,PW2,BM2,1,N,O,O,1)
156 CALL MPRD(S,PW1,SPW1,N,N,O,O,1)
157 CALL MPRD(BMS,PW1,BMW1,N,N,O,O,1)
158 CALL MPRD(FFI,V1,PF2,N,N,O,O,1)
159 CALL MPRD(BS,PF2,BS2,1,N,O,O,1)
160 CALL MPRD(S,PF2,SPF2,N,N,O,O,1)
161 CALL MPRD(B3,ROT,B3ROT,N,N,O,O,1)
162 CALL MADD(V,B3ROT,V2,N,1,O,O)
163 CALL MPRD(FFI,V2,PF3,N,N,O,O,1)
164 CALL MPRD(BS,PF3,BS3,1,N,O,O,1)
165 CALL MPRD(S,PF3,SPF3,N,N,O,O,1)
166 CALL MPRD(FFI,U,PF4,N,N,O,O,1)
167 CALL MPRD(BS,PF4,BS4,1,N,O,O,1)
168 CALL MPRD(S,PF4,SPF4,N,N,O,O,1)
169 CALL MSUB(U,B2ROT,U1,N,1,O,O)
170 CALL MPRD(FFI,U1,PF5,N,N,O,O,1)
171 CALL MPRD(BS,PF5,BS5,1,N,O,O,1)
172 CALL MPRD(FTI,ROT,TW1,N,N,O,O,1)
173 CALL MPRD(S,TW1,STW1,N,N,O,O,1)
174 CALL MPRD(S,PF5,SPF5,N,N,O,O,1)
175 CALL MADD(SPW1,SPF2,S12,N,1,O,O)
176 CALL MADD(S12,SPF3,S123,N,1,O,O)
177 CALL MPRD(B2,SPF2,T2,N,N,O,O,1)
178 CALL MPRD(B3,SPF3,T3,N,N,O,O,1)
179 CALL MPRD(B2,SPF4,T4,N,N,O,O,1)
180 CALL MADD(T2,T3,T23,N,1,O,O)
181 CALL MADD(T23,T4,T234,N,1,O,O)
182 CALL MADD(T234,STW1,ST,N,1,O,O)
183 DO 100 I=1,N
184 WRITE (6,101) U(I), V(I), ROT(I), V1(I), PW2(I)
185 101 FORMAT (5E13.5)

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1 PROGRAM FC7
2 DIMENSION FW(7,7), FF(7,7), FT(7,7), FWI(7,7), FFI(7,7), FTI(7,7)
3 , B(7,7), C(7,7), C1(7,7), E(7,7), F(7,7), S(7,7), G2TSBI(7,7),
4 , BXFFI(7,7), FBF(7,7), CXFWI(7,7), C1XFFI(7,7), CWC1F(7,7),
5 , FXFFI(7,7), EWFF(7,7), EWFT(7,7), G1G2I(7,7), XG3(7,7), WSUB(7),
6 , EXFWI(7,7), G1(7,7), G2(7,7), G3(7,7), T(7), W(7), THETA(7),
7 , G2I(7,7), L(7), M(7), G3G2I(7,7), XG1(7,7), G2SUB(7,7),
8 , G2SUBI(7,7), G3G2IW(7), TSUB(7), Y(7), G2TSB(7,7), G1G2IT(7),
9 , PW1(7), BM(7), BS(7), BMS(7,7), BMW1(7),
10 , PF1(7), SPF1(7), TW1(7), STW1(7),
11 , HTHETA(7), H(7,7), Y1(7)
12 READ (5,*) N
13 READ (5,*) ((FW(I,J),J=1,N),I=1,N)
14 READ (5,*) ((FF(I,J),J=1,N),I=1,N)
15 READ (5,*) ((FT(I,J),J=1,N),I=1,N)
16 READ (5,*) ((S(I,J),J=1,N),I=1,N)
17 READ (5,*) ((BMS(I,J),J=1,N),I=1,N)
18 READ (5,*) ((B(I,J),J=1,N),I=1,N)
19 READ (5,*) ((C(I,J),J=1,N),I=1,N)
20 READ (5,*) ((C1(I,J),J=1,N),I=1,N)
21 READ (5,*) ((E(I,J),J=1,N),I=1,N)
22 READ (5,*) ((F(I,J),J=1,N),I=1,N)
23 READ (5,*) (T(I),I=1,N)
24 READ (5,*) (W(I),I=1,N)
25 READ (5,*) ((H(I,J),J=1,N),I=1,N)
26 DO 2 I=1,N
27 DO 2 J=1,N
28 2 FWI(I,J)=FW(I,J)
29 CALL MINV(FWI,N,D,L,M)
30 DO 3 I=1,N
31 DO 3 J=1,N
32 3 FFI(I,J)=FF(I,J)
33 CALL MINV(FFI,N,D,L,M)
34 DO 4 I=1,N
35 DO 4 J=1,N
36 4 FTI(I,J)=FT(I,J)
37 CALL MINV(FTI,N,D,L,M)
38 CALL MPRD(B,FFI,BXFFI,N,N,O,O,N)
39 CALL MADD(FWI,BXFFI,FBF,N,N,O,O)
40 CALL MPRD(S,FBF,G1,N,N,O,O,N)
41 CALL MPRD(C,FWI,CXFWI,N,N,O,O,N)
42 CALL MPRD(C1,FFI,C1XFFI,N,N,O,O,N)
43 CALL MADD(CXFWI,C1XFFI,CWC1F,N,N,O,O)
44 CALL MPRD(S,CWC1F,G2,N,N,O,O,N)
45 CALL MPRD(E,FWI,EXFWI,N,N,O,O,N)
46 CALL MPRD(F,FFI,FXFFI,N,N,O,O,N)
47 CALL MADD(EXFWI,FXFFI,EWFF,N,N,O,O)
48 CALL MADD(EWFF,FTI,EWFT,N,N,O,O)
49 CALL MPRD(S,EWFT,G3,N,N,O,O,N)
50 DO 10 I=1,N
51 DO 10 J=1,N
52 10 G2I(I,J)=G2(I,J)
53 CALL MINV(G2I,N,D,L,M)
54 CALL MPRD(G3,G2I,G3G2I,N,N,O,O,N)
55 CALL MPRD(G3G2I,G1,XG1,N,N,O,O,N)
56 CALL MSUB(G2,XG1,G2SUB,N,N,O,O)
57 DO 20 I=1,N
58 DO 20 J=1,N
59 20 G2SUBI(I,J)=G2SUB(I,J)
60 CALL MINV(G2SUBI,N,D,L,M)
61 CALL MPRD(G3G2I,W,G3G2IW,N,N,O,O,1)

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62     CALL MSUB (T,G3G2IW,TSUB,N,1,0,0)
63     CALL MPRD (G2SUBI,TSUB,Y,N,N,0,0,1)
64     CALL MPRD (G1,G2I,G1G2I,N,N,0,0,N)
65     CALL MPRD (G1G2I,G3,XG3,N,N,0,0,N)
66     CALL MSUB (G2,XG3,G2TSB,N,N,0,0)
67     DO 30 I=1,N
68     DO 30 J=1,N
69     30 G2TSBI (I,J)=G2TSB (I,J)
70     CALL MINV (G2TSBI,N,D,L,M)
71     CALL MPRD (G1G2I,T,G1G2IT,N,N,0,0,1)
72     CALL MSUB (W,G1G2IT,WSUB,N,1,0,0)
73     CALL MPRD (G2TSBI,WSUB,THETA,N,N,0,0,1)
74     CALL MPRD (H,THETA,HTHETA,N,N,0,0,1)
75     CALL MADD (Y,HTHETA,Y1,N,1,0,0)
76     CALL MPRD (FWI,Y1,PW1,N,N,0,0,1)
77     DO 55 I=2,N
78     IS=I-1
79     BS (I)=30.0/FLOAT (IS)
80     BS (1)=1.0
81     55 CONTINUE
82     DO 58 I=2,N
83     IM=I-1
84     BE=FACT (IM-1)/FACT (IM+1)
85     BM (I)=BE*900.0
86     BM (1)=30.0
87     58 CONTINUE
88     CALL MPRD (BM,PW1,BM1,1,N,0,0,1)
89     CALL MPRD (BMS,PW1,BMW1,N,N,0,0,1)
90     CALL MPRD (FFI,Y,PF1,N,N,0,0,1)
91     CALL MPRD (BS,PF1,BS1,1,N,0,0,1)
92     CALL MPRD (S,PF1,SPF1,N,N,0,0,1)
93     CALL MPRD (FTI,THETA,TW1,N,N,0,0,1)
94     CALL MPRD (S,TW1,STW1,N,N,0,0,1)
95     CALL MPRD (BS,TW1,BTW1,1,N,0,0,1)
96     DO 100 I=1,N
97     WRITE (6,101) BMW1 (I), SPF1 (I), STW1 (I), Y (I), THETA (I)
98     101 FORMAT (5E13.5)
99     100 CONTINUE
00     WRITE (6,121) BM1, BS1, BTW1
01     121 FORMAT (3E13.5)
02     DO 106 I=1,N
03     WRITE (6,108) PW1 (I), PF1 (I), TW1 (I)
04     108 FORMAT (5E13.5)
05     106 CONTINUE
06     STOP
07     END
08     FUNCTION FACT (I)
09     IF (I.LT.0) GOTO 110
10     FACT=1.0
11     IF (I.EQ.0) GOTO 109
12     DO 99 NN=0,I-1
13     TERM=FLOAT (I-NN)
14     99 FACT=FACT*TERM
15     109 RETURN
16     110 STOP 'FACTORIAL OF NUMBER LESS THAN 0'
17     END

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LISTING OF FILE :GNCV45.FRAMEC(1,\*,1).P7(8) FOR USER :GNCV45 AT 1987/07/13.

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BATCH :GNCV45 KHAIRUNAINI AT 1987/07/13\_\_12:28:32







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1 C
2 C .....
3 C
4 C SUBROUTINE MINV
5 C
6 C PURPOSE
7 C INVERT A MATRIX
8 C
9 C USAGE
10 C CALL MINV(A,N,D,L,M)
11 C
12 C DESCRIPTION OF PARAMETERS
13 C A - INPUT MATRIX, DESTROYED IN COMPUTATION AND REPLACED BY
14 C RESULTANT INVERSE.
15 C N - ORDER OF MATRIX A
16 C D - RESULTANT DETERMINANT
17 C L - WORK VECTOR OF LENGTH N
18 C M - WORK VECTOR OF LENGTH N
19 C
20 C REMARKS
21 C MATRIX MUST BE A GENERAL MATRIX
22 C
23 C SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
24 C NONE
25 C
26 C METHOD
27 C THE STANDARD GAUSS-JORDAN METHOD IS USED. THE DETERMINANT
28 C IS ALSO CALCULATED. A DETERMINANT OF ZERO INDICATES THAT
29 C THE MATRIX IS SINGULAR.
30 C
31 C .....
32 C
33 C SUBROUTINE MINV(A,N,D,L,M)
34 C DIMENSION A(*),L(*),M(*)
35 C
36 C .....
37 C
38 C IF A DOUBLE PRECISION VERSION OF THIS ROUTINE IS DESIRED, THE
39 C C IN COLUMN 1 SHOULD BE REMOVED FROM THE DOUBLE PRECISION
40 C STATEMENT WHICH FOLLOWS.
41 C
42 C DOUBLE PRECISION A,D,BIGA,HOLD,DABS
43 C
44 C THE C MUST ALSO BE REMOVED FROM DOUBLE PRECISION STATEMENTS
45 C APPEARING IN OTHER ROUTINES USED IN CONJUNCTION WITH THIS
46 C ROUTINE.
47 C
48 C THE DOUBLE PRECISION VERSION OF THIS SUBROUTINE MUST ALSO
49 C CONTAIN DOUBLE PRECISION FORTRAN FUNCTIONS. ABS IN STATEMENT
50 C 10 MUST BE CHANGED TO DABS.
51 C
52 C .....
53 C
54 C SEARCH FOR LARGEST ELEMENT
55 C
56 C D=1.0
57 C NK=-N
58 C DO 80 K=1,N
59 C NK=NK+N
60 C L(K)=K
61 C M(K)=K

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62      KK=NK+K
63      BIGA=A(KK)
64      DO 20 J=K,N
65      IZ=N*(J-1)
66      DO 20 I=K,N
67      IJ=IZ+I
68      10 IF( ABS(BIGA)- ABS(A(IJ))) 15,20,20
69      15 BIGA=A(IJ)
70      L(K)=I
71      M(K)=J
72      20 CONTINUE
73      C
74      C          INTERCHANGE ROWS
75      C
76      J=L(K)
77      IF(J-K) 35,35,25
78      25 KI=K-N
79      DO 30 I=1,N
80      KI=KI+N
81      HOLD=-A(KI)
82      JI=KI-K+J
83      A(KI)=A(JI)
84      30 A(JI)=HOLD
85      C
86      C          INTERCHANGE COLUMNS
87      C
88      35 I=M(K)
89      IF(I-K) 45,45,38
90      38 JF=N*(I-1)
91      DO 40 J=1,N
92      JK=NK+J
93      JI=JP+J
94      HOLD=-A(JK)
95      A(JK)=A(JI)
96      40 A(JI)=HOLD
97      C
98      C          DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
99      C          CONTAINED IN BIGA)
100     C
101     45 IF(BIGA) 48,46,48
102     46 D=0.0
103     RETURN
104     48 DO 55 I=1,N
105     IF(I-K) 50,55,50
106     50 IK=NK+I
107     A(IK)=A(IK)/(-BIGA)
108     55 CONTINUE
109     C
110     C          REDUCE MATRIX
111     C
112     DO 65 I=1,N
113     IK=NK+I
114     HOLD=A(IK)
115     IJ=I-N
116     DO 65 J=1,N
117     IJ=IJ+N
118     IF(I-K) 60,65,60
119     60 IF(J-K) 62,65,62
120     62 KJ=IJ-I+K
121     A(IJ)=HOLD*A(KJ)+A(IJ)
122     65 CONTINUE
123     C

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1      SUBROUTINE LOC(I, J, IR, N, M, MS)
2      IX=I
3      JX=J
4      IF(MS-1) 10,20,30
5      10 IRX=N*(JX-1)+IX
6      GO TO 36
7      20 IF(IX-JX) 22,24,24
8      22 IRX=IX+(JX*JX-JX)/2
9      GO TO 36
10     24 IRX=JX+(IX*IX-IX)/2
11     GO TO 36
12     30 IRX=0
13     IF(IX-JX) 36,32,36
14     32 IRX=IX
15     36 IR=IRX
16     RETURN
17     END

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F LISTING OF FILE :GNCV45.SOURCE(1,\*,1).LOC(1) FOR USER :GNCV45 AT 1987/07,

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F BATCH :GNCV45 KHAIRUNAINI AT 1987/07/13\_\_12:35:52

AA/
AA/
AA/

5 5K LISTED GPAD14 LPG14

AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME
AS 2980 EMAS*** FTPMAN SPOOLING	GLASGOW CENTRE VME

duration: 00:00:12      Packets out: 0      Packets in: 59  
leared

