Standard Stars for Polarimetric Measurement

and

Their Statistics

By

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Summary

The most important aspect of stellar polarimetry involves the studying of polarimetric standard stars. The techniques of polarimetry have advanced progressively with the associated improvement of accuracy. This has not been accompanied with establishment of better standards of reference. Two fundamental reductions must be made to the polarization data before any inferences can be drawn from observations: (A) Instrumental polarization corrections should be performed on the data by observing "zero" polarization standards, (B) Absolute position angle calibration, so that the data may be presented in a particular coordinate frame so allowing comparison of data collected on different instruments. The latter can be obtained from knowledge of the orientation of the polarimeter relative to North-South direction. Laboratory techniques should be used, where available, for absolute calibration of position angle. It is however, far more convienient to observe highly polarized stars with a well defined position angle of polarization; the equatorial coordinate frame is normally chosen for reference. These standards also provide means of testing the stability of the instrument.

The aim of this study is to present a scheme that may be used to quantify some criteria for establishment of future standard polarimetric stars. There have been only few authors that have paid any attention to establishing polarimetric standards. Discussions presented in Chapter 1 deal with the present standing of polarimetric standards and the areas where further improvements are required.

Since the discovery of interstellar polarization by Hall and Hiltner, there has been a significant increase in our precision of polarization measurements. It is therefore essential to investigate the statistical behaviour of errors involved with the

measurements of polarization and position angle; the former has been quantitatively well studied by previous workers, but the latter has lacked the detail it deserves. We have constructed accurate confidence intervals of position angle of polarization and have provided statistical formulation of the distribution. It is demonstrated in Chapter 2 that confidence values of position angle at low levels of signal-to-noise ratio are significantly different from the Gaussian distribution assumed in the literature. Also a data simulation method was performed in order to determine the distribution of differences in two values of position angle at low signal-to-noise, since analytical solutions would be too complex to handle for this situation.

The literature shows that some of the well established standards are challenged either because of their imprecise tabulated values or because of suspected polarimetric variability. Proper statistical techniques should be used however to assess correctly and accurately these findings. It is shown in Chapter 3 that because of inadequate statistical procedures applied to the data of polarimetric standards, many stars supposedly showing variations in their polarization and position angle, may still be considered as standards. We have performed more rigid statistical tests on some previously published broad-band data. It is therefore concluded that, prior to this work presented here, all data involving standard polarimetric stars have not been statistically interpreted correctly.

We have undertaken new observations on some well known polarization standards which are thought to have a variable polarization. Double channel polarimetry of 55Cyg, ϕCas, ρCas, oCas, ρLeo and 14Cep was performed and attempt were made to establish further standards in Cassiopeia and Perseus.

1. Introduction

1.1 Polarimetric Standard Stars

During the past few decades the measurement of optical polarization has become increasingly important in stellar astronomy. Since the discovery by Hall (1949) and Hiltner (1949) of interstellar polarization, it has been found that some stars show temporal variations or a peculiar wavelength dependence of the observed degree and angle of polarization. It was realized that these stars were intrinsically polarized. Study of these effects are an important part of astronomical polarimetry. Knowledge of the polarization associated with a star can give insight to the geometry, rotation, binary nature, magnetic fields, etc. or to the dust in the interstellar medium.

When performing a polarimetric observational run, it is important that the measurements are well calibrated. This is achieved by making observations of stars thought to be unpolarized and of stars with a well defined position angle associated with their polarization. The main purpose of polarimetric calibrations are :

(a) To remove systematic polarization effects introduced by the instrumentation (by observing "zero" polarization stars).

(b) To relate the polarization direction to an absolute reference frame (the equatorial reference system) by observing a standard star with large polarization and constant position angle at all wavelengths and both parameters free from time variability.

In polarimetry one way of investigating precision (observational consistency) is also to study polarimetric standards.

In the past standard stars have been proposed (e.g. see Serkowski, 1974a) for such work, but experience has revealed shortcomings in the lists. Techniques of measurements have also improved over the years and the accompanying accuracy has raised several problems with the establishment of standard stars. There are conflicting reports in literature as to which stars are polarimetric standards (see Tinbergen, 1979, 1982, Hsu and Breger, 1982 and Bastien et al., 1988). However most of the problems in establishing standards for use in polarimetry are due to lack of understanding of the statistics associated with the measurement of polarization parameters and not paying proper attention to careful data analyses and standardization.

It is, of course essential in any evaluations based on observational data that careful attention be paid to the reliability of the results. This is particularly the case for results related to the establishment of standards. We must pay particular attention to those aspects of the observations and their analyses that can, if sufficient care is not taken, lead to untrustworthy results (i.e. correct interpretation of the data).

There is also a lack of faint time-tested polarization standards. In general, polarimetric standards are not continuously monitored and therefore temporal variability can not be ruled out. To improve the situations in both respects, we need to acquire repeated measurements on a number of potentially acceptable targets over several years. The large telescopes now available will probably contribute very little to the establishment of faint standards, merely because of the short observing runs that are normally granted. The important work of establishing standard stars has to be carried out elsewhere, thus making it difficult for faint standards to be measured for use with new instruments. The new detectors now becoming available are sensitive enough to be used on stars brighter than 10^{th} or 12^{th} magnitudes. Consequently, there is a danger that the use of poor faint time-tested standards will increase in the future.

The most important criterion for a standard star to be included in any accepted list is that it should be constant with respect to the characteristic for which it is proposed as a standard (i.e. constant in time or in wavelength). This criterion can never be fully met. There is always a possibility that an apparently constant star is a long-period variable or, it might be revealed as being variable when observations of higher internal precision become possible. In the present century, particularly the last few decades has been a time of significant increases in the internal precision of virtually all forms of measurements. In polarimetry, this trend seems likely to continue for some time due to its comparatively recent discovery and it will be accelerated by the increasing use of space instruments (e.g. WUPPE). In general, as the internal precision of our measurements increases, fewer stars will be acceptable as standards and the harder it will become to establish them.

The work involved in this thesis concentrates on the possible improvements that can be made in order to establish more accurate polarimetric standards for future observations. A critical review of some previously published broad-band polarization data on polarimetric standards are presented. It is shown that because of inadequate statistical procedures applied to the polarization data, many supposedly standard stars claimed to be intrinsically polarized may still be considered as standards. This thesis also deals with the understanding of statistical error estimation on position angle of polarization, since most polarimetric literature does not cater for accurate statistical tests and estimation of position angle errors. Finally, we present some observational data obtained on polarization standards which are thought to exhibit variations in their polarization and position angle. It is shown that 55 Cyg and ϕ Cas show a definite polarization variability, the latter star exhibits significant rotation of its position angle with wavelength.

1.2 Zero Polarization Standard Stars

1.2.1 Introduction

In order to eliminate the effects of instrumental polarization when undertaking an observational run, it is regular practice to make some measurements of "zero" polarization stars. Instrumental polarization affects the measurement of both the degree of polarization and the position angle of any observed star; generally its effects are more important for stars with low polarization. Instrumental polarization is also likely to depend on wavelength. Data obtained by Gehrels (1960) for telescopes of different aperture are displayed in Fig. 1.1. It will normally be adequate to assume that instrumental polarization is produced by the variation over the telescope mirror of the reflection coefficient in, and perpendicular to, the planes of incidence of the rays. The instrumental polarization can also be produced by the polarimeter itself (i.e. from the optical system). It is important therefore to have a set of "zero" polarization stars known extremely precisely, so that the instrumental polarization can be subtracted from the observed polarization of any star of interest. It is also necessary to have these stars evenly spread over the sky (see Section 1.2.2) and to cover a range of apparent magnitudes. It is essential for high precision polarimetry to avoid instrumental effects.

One way to avoid the effect as introduced by the fore-optics is to use rotatable telescope (see Serkowski, 1974a), designed so that the whole telescope is rotated about its optical axis. This will reduce one of the main contributors (i.e. telescope mirror), but will not entirely overcome the problem, since polarization may arise from other components of the system. Another method which has been suggested is to use an alt-azimuth mounting, with such a system the field rotates and one could observe the same object with say 90° field rotation, thereby calibrating for the instrumental effects (see Serkowski, 1974b). This method is often used in radio astronomy since many radio telescopes have this type of mounting.

There have been several catalogues of "zero" polarization stars proposed by various workers such as Behr (1959), Appenzellar (1966), Walborn (1967), Serkowski (1974a), Schröder (1976), Piirola (1977) and Tinbergen (1979). Stars in these catalogues are presumed to show no polarization within the attainable errors. All these stars are in the neighbourhood of the Sun, their distances being less than 35 parsecs.

The catalogue which contains the largest sample of stars was undertaken by Tinbergen (1979). Observations of over 100 stars were carried out in broad-band at three different observing stations. Stars of 5^{th} magnitude and brighter were covered. Most of the stars chosen were confined to mainly in the A, F, G and K-type because least work had been done on these type of stars. Also, it is well known that many early type and late type stars exhibit intrinsic broad-band polarization variability as a result of circumstellar scattering shells. For example, Be star polarimetry (measurement and models) has been reviewed by Coyne and Mclean (1982) and M-type giants and supergiants have been reviewed by Schwarz (1986).

Tinbergen also compares his data with surveys of Behr (1959), Piirola (1977), Schröder (1976) and Serkowski et al. (1975). The results are summarized in Fig. 1.2, but the comparison is

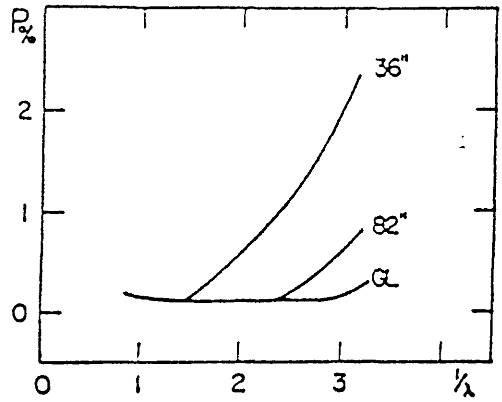


Fig. 1.1 Percentage polarization introduced by aluminized mirrors for different telescopes, versus the inverse of the effective wavelength in microns. For the McDonald 36-inch and 82-inch and for the Goethe Link telescope.
(Taken from Gehrels, 1960)

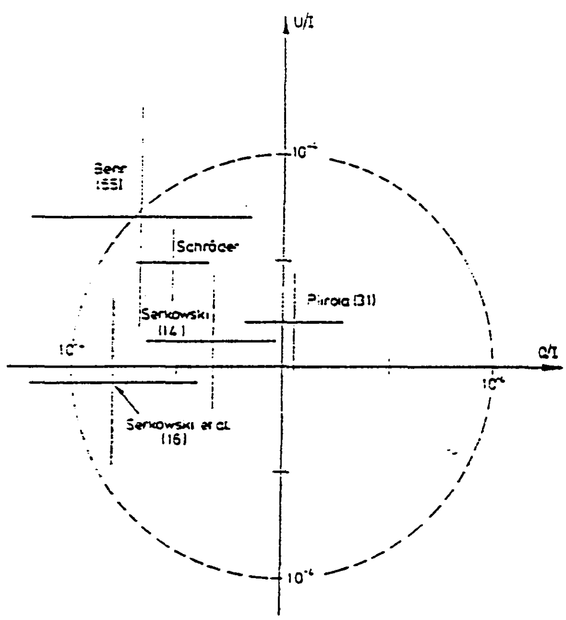


Fig. 1.2 Stokes parameter plot of the systematic difference between various surveys; the number of stars involved to provide the mean value and variance are shown in brackets. Where Tinbergen's survey is taken as reference. (Taken from Tinbergen, 1979)

inconclusive as all the data were collected on different telescopes. Each data set will have its own offset and therefore the mean value of polarization will be different in each individual survey. Tinbergen (1982) re-evaluated the 181 stars which had been presented in his previous work, with some further observations by Piirola (1977), in order to assess the possible existence of any level of polarization and determine any correlation between polarization and spectral class. He displays a binned distribution of the observed degree of polarization within 35 pc of the Sun (Fig. 1.3). Six stars apparently do not belong to the main statistical population and he concludes that they posses anomalous polarization. He attempted to remove instrumental polarization by using measurements at each observing station of a preferred set of "unpolarized" stars from within his data.

There are some underlying problems associated with the interpretation of Tinbergen's (1979, 1982) data. The main concern with his data analysis is that there is no apparent correction for the biasing that measurement noise introduces. He has assumed that the uncertainties on polarization values follow Gaussian distributions. Stewart (1984) has shown that biasing effects are large at small values of polarization (see Chapter 2). The errors (σ) on the Normalized Stokes Parameters (hereafter NSPs) obtained by Tinbergen depend on the observational site used. The value of σ for each measurement is prescribed through a weight which is inversely proportional to the square of the error with the value of unity corresponding to an error of 0.01% on q and u; weights of 1 to 6 are quoted. As Tinbergen (1982) points out, there may be systematic errors of the order of 0.003% in the NSPs, and also some scintillation noise may be present in the measurements of the brightest stars, this may cause an underestimation of the weights.

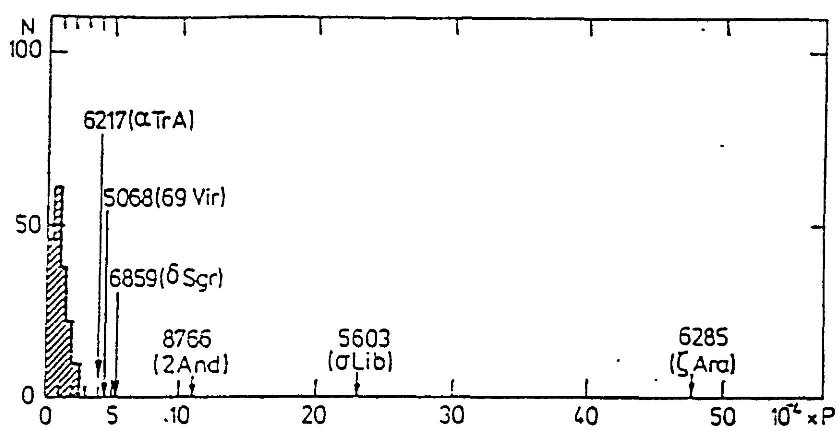


Fig 1.3 Distribution of observed degree of polarization in a sample of supposedly nearby stars. The concentration is towards low levels of polarization.
(Taken from Tinbergen, 1982)

1.2.2 Effect of Interstellar Polarization

In establishing a catalogue of unpolarized stars it is necessary to consider whether interstellar polarization effects might cause problems even within the solar neighbourhood. All the stars closer than 35 parsecs are usually regarded as sources of unpolarized light. Since the brighter stars are generally closer, it is reasonable to assume that the effect of interstellar polarization will be small compared to the stars situated further away. It has long been known that light passing through the interstellar medium is scattered by aligned interstellar dust grains, thereby producing interstellar polarization (e.g. Greenberg, 1974).

The statistical information on the distribution of interstellar dust is obtained by studying the interstellar extinction (A_υ) of the individual stars in the solar neighbourhood. There have been several studies of extinction values; it can be seen from Table 1.1 that A_υ changes from 0.0002 in Behr (1959) to 0.00008 in Tinbergen (1982). Differences are mainly due to the distribution of stars in a particular patch of sky in each survey. Due to selection effects of this kind, which may result in under or overestimating the real values of A_υ, it is essential in any survey to cover all parts of the sky as the distribution of the interstellar dust is certainly not random (Knude, 1979). Consequently, the polarization due to interstellar dust will vary from one region of the sky to another. Piirola (1977) and Tinbergen (1982) have suggested a value for the interstellar polarization within 35 pc of the Sun to be of the order of 0.005% .

Table 1.1

Summary of polarization and extinction data for the first 35 pc in the neighbourhood of the Sun. The stellar distance group is r; p/r is the r.m.s degree of polarization for the group, divided by an estimated mean distance. A_{υ}/r is p/r multiplied by (2.17/0.065)mag. (Taken From Tinbergen, 1982)

Author	r(pc)	$p/r(pc^{-1})$	A_{υ}/r(mag/pc)	Notes
Behr (1959)	12-25	$5\ 10^{-6}$	0.0002	Northern Sky
Walborn (1968)	<25	$5\ 10^{-6}$	0.00016	One-third of Northern sky
Piirola (1977)	<25	$4\ 10^{-6}$	0.00012	Northern sky
Tinbergen (1982)	10-35	$2.5\ 10^{-6}$	0.00008	Combination of Piirola+Tinbergen Northern+Southern sky

1.2.3 Correlation of Spectral Type with Polarization

Tinbergen (1982) has also raised the possibility that all spectral types later than F0 display polarization at about the level of 0.01% and that the values are time dependent. Having plotted polarization against spectral type, he puts a 95% confidence level of $p \simeq 2\varepsilon_p$ through the data set where ε_p is the average r.m.s error of the measured polarization (see Fig. 1.4). He concludes that 32 out of 181 stars have an observed degree of polarization greater than 95% confidence level.

The conclusion arrived at by Tinbergen (1982) may result from lack of stringent application of statistical concepts. As already mentioned he calculates ε_p assuming that the data have a Gaussian distribution and takes no account of the biasing that noise introduces. With the development of a better understanding of polarization statistics (e.g. see Clarke et al., 1983 and Simmons and Stewart, 1985) it is possible to reassess more realistically the polarization correlation with spectral type. In order to perform and draw any conclusions from any statistical tests, we need to have large sample of stars. Since we are interested in stars of particular spectral types and within a given distance from the Sun, we are limited in the number of stars that we can consider.

For a large sample of stars of particular spectral type, we could use the χ^2 test (see Simmons and Stewart, 1985). In this method the observed measurement (signal-to-noise $\frac{p}{\sigma}$) of the polarization can be binned to form a histogram and this can be compared with the theoretical distribution (Rician distribution, see Chapter 2). The goodness of fit can be assessed by using the

χ^2 test, even when the σ's (the standard deviation) have different values. Leroy and Le Borgne (1989) have performed this procedure with 54 late-type dwarfs covering G and early K spectral types. It can be seen from Fig 1.5 that it appears to follow a similar curve to the theoretical one. They concluded that on average the stars have no intrinsic polarization larger than the uncertainty (± 0.014%) of their measurement.

It is also possible to use a non-parametric method such as Kolmogorov-Smirnov test (see Conover, 1980) to assess if the data sample comes from a given continuous distribution by comparing the observed with the theoretical cumulative distribution (see Chapter 3). The advantage of using this method is that it can be applied to small sample of data (greater than 3 data points).

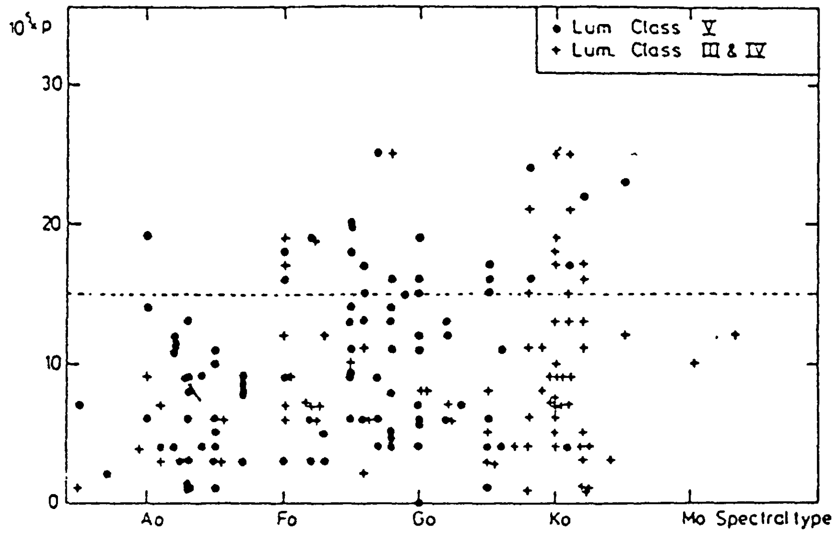


Fig 1.4 Observed degree of polarization plotted against spectral types. The broken line is at 15 . 10^{-5} ($p \approx 2\varepsilon_p$). (Taken from Tinbergen, 1982)

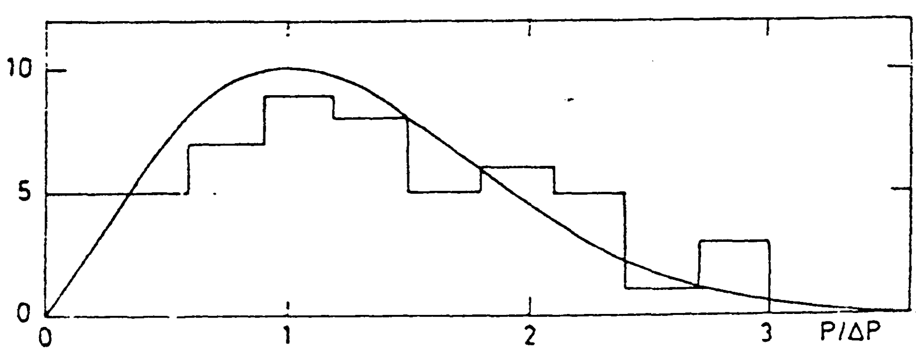


Fig 1.5 A histogram of p/σ for 54 observations; the continuous curve is the theoretical distribution (Rician distribution) expected if stars have no polarization. (Taken from Leroy and Le Borgne, 1989)

1.2.4 Systematic Errors and Their Effects

There are several sources of systematic error which could affect the results of high precision polarimetry of unpolarized standards. The main source of biasing is the parasitic effect of sky background, if not subtracted from the source could lead to unsatisfactory data. However, there are instrumental problems associated with observing such low levels of polarization. There has been some discussions whether the scattered moonlight could effect polarimetric data. Leroy and Le Borgne (1989) claim that moonlight can strongly bias broad-band polarization measurements, observations at U-band will be most strongly affected due to Rayleigh scattering and hence it can be anticipated that scattered moonlight will contribute strongly to the polarimetric noise. Huovelin and Piirola (1989) have claimed that there is no bias towards higher polarization during periods of full moon. They put an upper limit on the polarization contribution due to moonlight of 0.005%.

1.3 Standard Polarization Stars

1.3.1 Introduction

Most of the work in establishing polarization standards has been carried out by Serkowski (1960) and the revised version of a catalogue was presented in 1974. Since then, a few observers have measured these stars with relatively higher precision. A list of bright stars that are considered as polarization standards is given in Table 1.2.

There are conflicting problems in choosing the polarized standard stars. In recent years Hsu and Breger (1982) have stated that 3 (55Cyg, 9Gem and HD183143) out of 15 stars in Serkowski's list of standards show a definite variability with time. Dolan and Tapia (1986) and Bastien et al. (1988) also have claimed that most of these bright standard stars in Serkowski's catalogue show temporal variability in polarization and in position angle. Bastien et al. (1988) studied 15 stars including 10 contained in Hsu and Breger's (1982) catalogue and have found all to be variable accept one (LCar).

However, there are shortcomings in all these works, for example Hsu and Breger (1982) presented their results without proper statistical analysis of their data. Bastien et al. (1988) also fail to perform proper statistical tests; e.g. failing to put confidence level on their data after normality testing (see Chapter 3). It is also useful to study the particular form of polarization variability in these type of stars. The particular form of these variations (temporal variations or a peculiar wavelength dependence of the observed degree and angle of polarization) can than be used to investigate the mechanism

Table 1.2
Polarised Standard Stars

HD(Name)	m_v	P(%)±σ	θ(°)±σ	Spectral type	Variable in P	Variable in θ	Remarks
7927(φ Cas)	4.98	3.4±0.1[a]	94±1	F0 Ia	Y[c]	Y[cg]	Complex Δθ/Δλ
		3.41±0.02[b]	92.23±0.1				
		3.320±0.019[c]	92.6±0.2				
		3.35±0.05[f]	92±1				
11831(+59°364)	8.04	5.201±0.013[c]	95.8±0.1	A2 Ib	Y[c]	N[c]	
		5.37±0.11[d]	96				
14433(+56°568)	4.24	3.9±0.1[a]	112±1	A1 Ia	Y[c]	Y[c]	
		3.91±0.03[b]	112.5±0.1				
		3.831±0.011[c]	112.6±0.1				
21291(2HCam)	4.86	3.5±0.1[a]	115±1	B9 Ia	?[d]	Y[d]	Complex Δθ/Δλ
		3.53±0.02[b]	116.6±0.2				
		3.32±0.06[f]	115±1				
23512(+23°524)	3.55	2.3±0.1[a]	30±1	A0 V	Y[c]	Y[c]	
		2.29±0.01[b]	29.9±0.3				
		2.094±0.013[c]	30.5±0.2				
25443(+61°669)	6.74	5.223±0.010[c]	135.1±0.1	B0.5 III	Y[c]	?[c]	
		5.25±0.07[d]	132				
43384(9 Gem)	3.27	3.0±0.1[a]	170±1	B3 Ia	Y[b]	Y[b]	
		3.01±0.04[b]	169.8±0.7				
80558(HR 3708)	5.91	3.3±0.1[a]	162±1	B7 Iab	Y[c]	Y[c]	
		3.32±0.02[b]	162				
		3.111±0.008[c]	162±0.1				
84810(L Car)	3.3	1.6±0.1[a]	100±1	F8-K0 Ib	N[c]	N[c]	Cepheid
	-4.0	1.62±0.01[b]	100				
		1.578±0.010[c]	99.8±0.2				
111613(HR4876)	5.75	3.2±0.1[a]	81±1	A1 Ia	Y[c]	Y[c]	Complex Δθ/Δλ
		3.14±0.06[b]	81				
		3.057±0.008[c]	80.2±0.1				

147084(o Sco)	4.54	4.3±0.1[a] 4.46±0.03[b] 3.857±0.008[c] 3.42±0.06[f]	32±1 32.0±0.1 31.8±0.1 32±1	A5 II	Y[c]	?[c]	Complex $\Delta\theta/\Delta\lambda$
154445(HR6353)	5.63	3.7±0.1[a] 3.73±0.01[b] 3.556±0.015[c] 3.44±0.15[f]	90±1 90.1±0.1 90.3±0.1 88±1	B1 V	Y[c]	?[c]	
160529(-33°12361)	6.67	7.3±0.1[a] 7.31±0.02[b]	20±1 20.4±0.1	A2 Ia	?	N[c]	Complex $\Delta\theta/\Delta\lambda$
161056	6.32	3.886±0.009[c] 4.08±0.07[e]	68.2±0.1 69	B1.5 V	Y[c]	?[c]	
183143(+18°4085)	6.87	6.4±0.1[a] 6.08±0.05[b] 5.75±0.05[f]	0±1 178.3±0.2 180±1	B7 Ia	Y[b]	Y[g]	Complex $\Delta\theta/\Delta\lambda$
187929(η Aql)	3.5 -4.3	1.8±0.1[a] 1.67±0.03[b] 1.704±0.012[c] 1.60±0.08[f]	93±1 93.8±0.3 93.9±0.2 91±1	F6-G2 Ib	?[cg]	?[cg]	(Cepheid)Complex $\Delta\theta/\Delta\lambda$
198475(55 Cyg)	4.83	2.8±0.1[a] 2.72±0.02[b] 2.60±0.12[f]	3±1 3.2±0.2 2±1	B3 Ia	Y[bcg]	Y[cg]	Complex $\Delta\theta/\Delta\lambda$
204827(+58°2272)	7.93	5.7±0.1[a] 5.70±0.03[b] 5.614±0.017[c] 5.68±0.09[f]	60±1 59.3±0.1 59±0.1 58±1	B0 V	Y[c]	Y[cg]	Complex $\Delta\theta/\Delta\lambda$

Notes on Table 1.1

a) Serkowski (1974a) Given in p_{max} at λ_{max}

b) Hsu and Breger (1982) Given in p_{max} at λ_{max}

c) Bastien et al. (1988) Given λ at 4700 Å

d) Coyne (1974) Given in p_{max} at λ_{max}

e) Serkowski et al. (1975) Given in p_{max} at λ_{max}

f) Bailey and Hough (1982) Given in p at λ = 4300 Å

g) Dolan and Tapia (1986)

responsible for the intrinsic polarization and hence to study the stellar atmospheres and the circumstellar environment.

In order to find out whether or not an intrinsic component is present in the observed polarization of these standard stars, the following criteria should be investigated;

a) Does $p_{obs}(\lambda)$ vary with time?

b) Does $p_{obs}(\lambda)$ show a wavelength dependence which significantly differs from Serkowski's (Serkowski et al., 1975) law for interstellar polarization?

c) Does the position angle (θ) vary significantly with wavelength and is it time dependent?

d) Does the star differ from the average polarization of neighbouring objects which presumably show interstellar polarization only?

The important areas where improvements are needed for individual stars in any catalogue are as following;

We should systematically exclude those stars that are thought to show variability in their polarization. The process of finding standards with large interstellar polarization is biased towards luminous stars which are visible through great distances; however giant and supergiant, and stars with extended atmospheres often exhibit intrinsic linear polarization, which may be variable (Coyne 1971, Coyne and Mclean 1982, Serkowski 1968, 1970 and Coyne and Kruszewski 1969). Dyck and Jennings (1971) have investigated the polarimetric behaviour of giant and supergiant stars in the red domain of the H-R diagram. Of the stars undertaken in their survey all the supergiants observed displayed intrinsic polarization, but no giant earlier than M2 showed any effects (at their level of accuracy). There is, however, a need to have low luminosity stars with high mean polarization, but unfortunately these stars would be faint due to interstellar absorption at large distances and physically difficult to find.

Any candidate star will have to be monitored regularly over a long period of time (at least a few months). Observations should be carried out at different wavelengths in order to establish the variability of degree of polarization and position angle and their dependency on wavelength over a long period of time. Even, if the star is found to be variable, the information in itself is valuable, as it can be interpreted and provide insight as to the nature of the star.

It is also essential to have a range of apparent magnitudes, since there is a lack of faint polarization standard stars suitably established for use with large telescopes. Clemens and Tapia (1990) have observed some intermediate magnitude stars ranging from 6.9 to 9.7, these have been observed with high polarimetric precision. Unfortunately their observations span only a few days. However, fainter stars are needed for CCD-based polarimetry, since the quantum efficiency of most CCDs can make observations of polarization standards nearly impossible without the introduction of neutral density filters which might disturb the measurement say by increasing the scattered light in the optical system. The stars that have been chosen for use in calibrating the Hubble Space Telescope (hereafter HST) (see Bohlin et al., 1989) are not fainter than magnitude 10.34 and these have not been observed sufficiently to attain high precision required (e.g. in Table 1.3 only one star was observed 20 times, the rest fewer than 10 times). Some of the included standards have only been observed once or twice and there is a danger that they might be variable if observed over longer periods of time. We need to have fainter standards than those quoted in Table 1.3, since the dynamic range of the HST is 0 to 24 magnitudes.

It is also important to consider the possibility of variations in interstellar polarization over periods of time. Since standards are chosen because of their large interstellar polarization, any change in particle characteristics along the line of sight will have an effect on the observed polarization. Bastien et al. (1988) have claimed that variations could be due to interstellar scintillations similar to extragalactic radio sources (see Heeschen and Rickett, 1987). This obviously depends on the degree of correlation of the neighbouring interstellar particles. Any variation of interstellar polarization would be difficult to observe, as there is always the possibility of low levels of the intrinsic polarization being present in the star.

It has been suggested by Hsu and Breger (1982) that when making polarimetric observations, three or more polarized standards, having different position angles, should be used in order to prevent the systematic errors caused by unexpected variability of any of the standards. It is clear that future catalogues of standard polarization stars will require particular attention of the points described above.

Table 1.3
Polarized Standard Stars for use in HST

Name	α (2000)	δ (2000)	V	$B-V$	Spectral Type	$\%P_V(Err)$[a]	θ_V[b]	N[c]
BD+64° 106	$0^h\ 57^m\ 36^s.71$	+64° 51′ 35″.1	10.34	+0.69	B1V	5.65(.053)	96°.8	2
BD+59° 389	2 02 42.06	+60 15 26.5	9.07	+1.01	F0Ib	6.69(.027)	98.2	4
HD19820	3 14 05.35	+59 33 47.7	7.11	+0.51	O9IV	4.81(.047)	114.9	4
HD25443	4 06 08.07	+62 06 07.0	6.78	+0.29	B0III	5.13(.061)	134.2	1
BD+25° 727	4 44 24.90	+25 31 42.7	9.50	+0.72	A2III	4.27(.012)	33.8	1
HD251204	6 05 05.67	+23 23 38.9	10.28	+0.28	B0IV	4.04(.066)	147	1
HD298383	9 22 29.76	−52 28 57.4	9.68	+0.88	A0Ib	5.23(.009)	148.6	12
HD110984	12 46 44.91	−61 11 11.7	8.95	+0.44	B0IV	5.70(.007)	91.6	20
HD111579	12 51 03.61	−61 14 37.8	9.50	+0.78	B2Ib/II	6.46(.014)	103.1	6
HD126593	14 28 51.06	−60 32 24.8	8.50	+0.49	B0.5IV	5.02(.012)	75.2	6
o Sco	16 20 38.20	−24 10 10.3	4.57	+0.84	A5II	4.17(.008)	32.9	6
HD154445	17 05 32.24	−0 53 31.7	5.61	+0.12	B1V	3.80(.075)	88.03	4[d]
HD155197	17 10 15.62	−4 50 03.1	9.20		A0	4.38(.030)	103.2	4
HD161056	17 43 47.03	−7 04 46.2	6.32	+0.36	B1.5V	4.035(.038)	67.01	4[d]
HD204827	21 28 57.70	+58 44 24.0	7.93	+0.82	B0V	5.36(.025)	58.6	5

[a] $\%P_V(Err)$ is the percent polarization in the V filter with the uncertainty in parentheses.
[b] θ_V is the equatorial position angle in the V filter.
[c] Number of observations by Tapia or Schmidt
[d] Possibly variable

1.3.2 Wavelength Dependency of Polarization and Position Angle

The wavelength dependence of interstellar linear polarization over the optical region was first noted by Gehrels (1960); variations of position angle with wavelength for some stars were noted by Treanor (1963). The wavelength dependence of polarization has been used to investigate geometric and physical properties of interstellar medium such as size distribution of the grains and their chemical composition [e.g. Coyne et al. (1974), Serkowski et al. (1975), Wilking et al. (1980) and Nagata (1990)]. The linear polarization over optical wavelengths is well represented by the empirical relation

$$\frac{P}{P_{max}} = Exp[-K \ln^2 (\frac{\lambda_{max}}{\lambda})] \qquad 1.1$$

where λ_{max} is the wavelength at which the polarization is maximum (P_{max}) and K is a constant describing the peakiness of the curve (see Fig. 1.6). The value of K chosen by Serkowski (1974a) is 1.15 but more recently it has been proposed that it depends on the λ_{max} for the given star (see Wilking et al., 1980).

The standard stars in Serkowski's (1974a) and in Hsu and Breger (1982) catalogue are listed according to their P_{max} at λ_{max}. No analogous empirical law for the wavelength dependence of position angle exists due to the fact that it depends primarily on the alignment of the dust grains between the observer and the star (which is clearly different for each star), whereas the polarization depends simply on the number of dust grains along the line of sight, on their size, composition and on the alignment efficiency.

Dolan and Tapia (1986) have investigated the wavelength

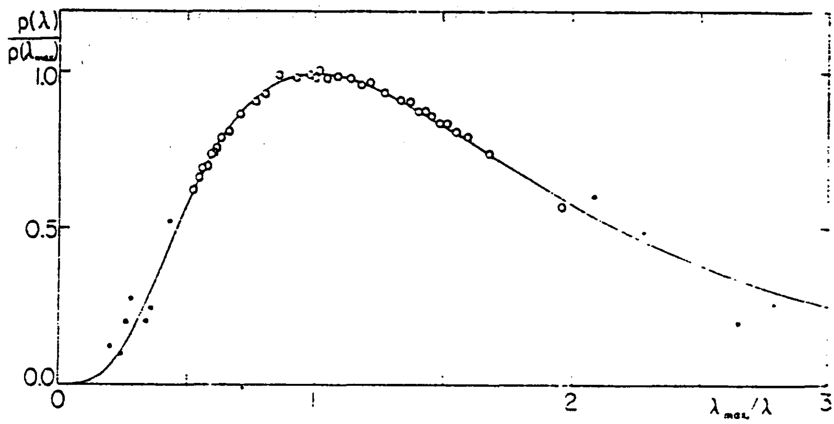


Fig 1.6 Wavelength dependence of interstellar polarization, where solid line is calculated from Eq. 1.1, with K = 1.15. (Taken from Serkowski, 1974a)

dependency of position angle in standard stars and have suggested that 9 out of 11 stars show a dispersion of position angle (see Table 1.2). Some of these show a complex $\Delta\theta/\Delta\lambda$ variation which may be associated with the effect of the interstellar medium. The shape of θ vs λ can be caused by multiple clouds of different particle size and different alignment in the magnetic field along the line of sight. This could be less important if the filter passband in which it were a standard were carefully defined and easily reproducible. However, if there is a temporal variability of θ vs λ over a time scale of few months or shorter then this is only attributed to the intrinsic variability of the source rather than variability in the interstellar medium.

1.3.3 Absolute Position Angle Calibration

The orientation of the plane of vibration of the linear polarization has to be specified in a standard celestial coordinate system in order that observations may be compared. Generally, the NSPs, q and u are determined in a frame described by the instrument as fixed to the telescope. Therefore it is necessary to determine the rotation angle, ϑ, between the internal coordinate system of the instrument and celestial coordinate system. The correct value of ϑ is necessary without systematic error to compare the results of different observers, since the instrumental frame orientation is arbitrary and controlled by one of the elements of the polarimeter. The required values of q and u can be determined from:

$$q = q' \operatorname{Cos}2\vartheta + u' \operatorname{Sin}2\vartheta$$
$$u = u' \operatorname{Cos}2\vartheta - q' \operatorname{Sin}2\vartheta \qquad 1.2$$

where q' and u' are the measured NSPs in instrumental frame, set at a particular angle, ϑ, with respect to the reference frame.

Several laboratory and astrometric methods have been applied to determining the rotation angle ϑ ; Lyot (1929, 1964) measured the position angle offset by observing the entire disc of the planets Mars, Venus and minor planets, since the plane of vibrations for these planets is always either perpendicular to or parallel to the scattering plane. For any epoch, the scattering plane can be calculated from coordinates tabulated in the Astronomical Almanacs (see Gehrels and Teska, 1960 and Gehrels et al., 1964). The value of position angle can be measured quite accurately, since the degree of polarization of the light from these objects is usually several percent.

Behr (1956) and Hiltner (1962) used a plane parallel stress free glass plate (where the polarization can be calculated according to Fresnel laws taking into account of multiple refractions from the refractive index of the glass plate). This plate is placed in the optical path, ahead of the analyzer. It can be tilted (this produces polarization) and, at the same time rotated around the optical axis (to any position angle). With the proper tilt and rotation of the glass plate, the polarization and the position angle can be determined respectively. One technique is to adjust the plate in order to cancel any polarization in the stellar radiation. A similar method has been used by Serkowski (1974a) where the viewing mirror of the eyepiece is replaced by a glass plate; the drive is stopped and the telescope pointed in a such a direction that a spirit level put on this glass plate indicates the exact horizontal orientation. The position angle of the plane of incidence of the telescope axis on a glass plate can be calculated from reading off the declination and hour angle circles. This is compared with the position angle of polarization measured for any unpolarized standard star through the glass plate remaining tilted to the telescope's optical axis (the tilted glass will introduce polarization of few percent). The position angle can be easily determined with an accuracy of a few minutes of arc.

The method employed by Gehrels and Teska (1960) involved pointing the telescope at the meridian towards a bright star or day light sky. A sheet of polaroid was hung from a plumb line in front of the telescope. After measuring the linear polarization the polaroid was rotated around the axis of the plumb line by 180° and the measurement repeated. By this method, the bisection of the two position angle corresponds to a vibration parallel to the N-S direction (see Rowell et al., 1969 and Aspnes, 1970). This position angle is subsequently used as a reference for the equatorial frame. Serkowski (1974b) has suggested that the polaroid may exhibit the equivalence of circular birefringence and rotate the plane of the polarization by as much as $0\overset{\circ}{.}2$. One way to avoid using a polaroid is to use a tilted stress free glass plate which can serve as a polarizer (see Behr, 1956, Hiltner, 1962, Serkowski, 1974a).

Dolan and Tapia (1986) use a method whereby the optical axis of the polarimeter is aligned with the optical axis of the telescope. A Glan-Thompson prism is aligned mechanically in a slide which can be inserted into the beam such that the plane of polarization of the transmitted light is parallel (to within $\pm 0\overset{\circ}{.}1$) to one of the cross hairs in the field eyepiece of the polarimeter. The polarimeter is rotated as a unit around its optical axis until the vertical thread of the cross hair is aligned N-S. This alignment is determined by driving the telescope north and south and keeping a star bisected by the cross hair. If the star remains bisected at both ends of the cross hair, the N-S alignment is accurate to $\leq 0\overset{\circ}{.}1$. When a source is observed through the Glan-Thompson prism the transmitted radiation is polarized in the E-W equatorial plane. The resulting position angle of subsequent observations are then measured directly in the equatorial system.

The calibration of the polarimeter position angle in any coordinate system must be done with high accuracy. All the above methods described can easily be adapted to any system of polarimetry. Since the accuracy achieved in all these methods are in order of $0\overset{\circ}{.}1$, there is little to choose between them. Unless a carefully selected set of standards stars can be found for this purpose, the calibration of the rotation angle between the internal coordinate system of the polarimeter and a celestial coordinate system must initially be done by one of these methods.

Since we are considering high accuracy in our studies of position angle, it is therefore important to understand the statistical behaviour associated in its measurements. In the next Chapter we will consider the ways in which the standard error (σ_θ) of position angle is calculated. The statistical distribution of position angle is derived and a method for constructing confidence intervals is undertaken.

Chapter 2: On the Statistical Behaviour of Polarimetric Position Angle

2.1 Introduction

2.2 The Statistical Distribution of Position Angle

2.3 Confidence Interval Estimation of Position Angle

2.4 Confidence Interval of Position Angle Differences

2.5 Conclusion

2. On the Statistical Behaviour of Polarimetric Position Angle

2.1 Introduction

As the techniques of optical polarimetry have progressed, it is now possible to determine the value of the position angle of the direction of vibration, θ, to a fraction of a degree. Dolan and Tapia (1986) suggest that the standard error describing the uncertainty of a measure, σ_θ, that can be achieved is typically $\pm$ $0^\circ\!.2$ or even $\pm$ $0^\circ\!.1$ for bright stars with large polarization. With such precision, it is claimed that many of the so-called standard stars, mainly established by Serkowski (1960), exhibit a wavelength dispersion of position angle or display temporal variations or suffer from both effects (see Hsu and Breger, 1982 and Dolan and Tapia, 1986). Although these works give adequate pictorial evidence of such effects, with error bars attached to the data points, they all generally fail to give proper confidence values on the disparate measures of position angle. The displayed error bars are usually based on a 1σ value which in turn is usually estimated on the assumption that the data can be considered as coming from a Gaussian distribution with the variance obtained from the associated Gaussian distribution of p. Eyeball estimates as described above can be misleading and, in addition, there well may be misinterpretations as a consequence of the underlying statistical behaviour of θ not following that of a normal distribution.

It is suggested here that, it would be useful to be able to ascribe proper confidence levels to possible differences in position angle, particularly in situations for which the values of polarization are small. Since optical polarization values for stars are usually very small, their repeated measures are frequently obtained with low values of signal-to-noise ratio. It is therefore essential to know the nature of the noise and any influence on the data so that its biasing effects can be removed.

Serkowski (1958, 1962) demonstrated that the usual experimental noise with a normal distribution introduces a bias in the determination of the degree of polarization, p. It has also been demonstrated by Clarke et al. (1983) that NSPs, being the components of p, are themselves not normally distributed when the photon count rate is low and when scintillation noise is affecting the signal. However, in most circumstances this further complication can be ignored. Stewart (1984) and Simmons and Stewart (1985) provided an in depth analysis of the statistical properties and the behaviour of p, at small levels of polarization measured with low signal-to-noise. They also investigated several techniques for compensating the well known (but frequently ignored) bias on the observed degree of polarization. They constructed confidence intervals for the degree of polarization of the light of a star when the value of the uncertainty (σ) on the NSPs, q and u, is known. However, this biasing effect can be ignored for large signal-to-noise ratios.

The biasing effect on p and the non-normal distribution of position angle in low signal-to-noise situations has also been appreciated in Radio Astronomy by Wardle and Kronberg (1974). In most cases σ_θ is estimated not by investigating the dispersion of θ, but by combining the dispersion of the p values (σ_p) with the best estimate for p, according to the formula (Equation 2.2) below.

The quoted formal errors on p and θ are (see Serkowski, 1958, 1962);

$$\sigma_p = \begin{cases} \sigma\left(2 - \frac{\pi}{2}\right)^{1/2} & p_o \approx 0 \\ \sigma & p_o \gg \sigma \end{cases} \qquad 2.1$$

where σ is the error on either of the measured NSPs, the errors assumed to be the same for both and,

$$\sigma_\theta = \begin{cases} \frac{\pi}{\sqrt{12}} \text{ rad} = 51^\circ\!.96 & p_o \approx 0 \\ \frac{\sigma_p}{2p} \text{ rad} = 28^\circ\!.65 \frac{\sigma_p}{p} & p_o \gg \sigma \end{cases} \qquad 2.2.$$

These error estimates are only valid for the cases of $p_o \approx 0$ and $p_o \gg \sigma$ where p_o is the true polarization. As we shall see, the error estimation on θ is more complicated than suggested above. The statement of formal uncertainties are couched in terms of a 1σ value and it is an easy matter, if the errors are normally distributed, to calculate the spread of θ to any appropriate chosen confidence limit. However, polarimetric measurements generally provide data which should not be considered in terms of these formal treatments. It is important to investigate the underlying behaviour of the errors so that accurate confidence values can be applied to any data set.

The statistical behaviour of p has been well studied with the production of confidence tables for estimating the associated uncertainties (see Simmons and Stewart, 1985), the general treatment of confidence interval estimation of position angle, however, seems to have been totally ignored. In most papers describing polarimetric results the assessment of position angle uncertainties (errors) are assumed to follow a normal distribution with σ_θ being calculated from the second part of Equation 2.2. In Section 2.2 we provide an analytical expression for the distribution of θ following its derivation from two normally distributed probability densities of q and u. Section 2.3 deals with the derivation of two extreme cases represented by Equation 2.2 and the production of confidence tables for estimating the uncertainties of the position angle for measurement made with different signal-to-noise ratios. In Section 2.4 a data simulation technique is used to investigate the statistical distribution of the differences of two position angle values.

2.2 The Statistical Distribution of Position Angle

It has been shown by Simmons and Stewart (1985) and Vinokur (1965) that if the NSPs, q and u are independent variables, and normally distributed around their true values of q_o, u_o, their probability densities may be represented in the usual way by P(q) and P(u) where;

$$P(q) = \left(\frac{1}{\sigma_q \sqrt{2\pi}} \right) e^{-\left(\frac{1}{2\sigma_q^2}\right)(q-q_o)}$$

$$P(u) = \left(\frac{1}{\sigma_u \sqrt{2\pi}} \right) e^{-\left(\frac{1}{2\sigma_u^2}\right)(u-u_o)} \qquad 2.3.$$

For convenience, a coordinate frame may be chosen such that $u_o = 0$, and assuming that $\sigma_u = \sigma_q = \sigma$, then their joint distribution will be

$$P(q, u) = \left(\frac{1}{2\pi\sigma^2}\right) e^{-\left(\frac{1}{2\sigma^2}\right)[(q-q_o)^2 + (u)^2]} \qquad 2.4.$$

The probability distribution of polarization is therefore found by transforming the above equation into polar coordinates, where $p = (q^2 + u^2)^{1/2}$, $p_o = (q_o^2 + u_o^2)^{1/2}$ and with q, u and q_o replaced by $p\text{Cos}\phi$, $p\text{Sin}\phi$ and p_o respectively

$$\therefore \quad F(p, \phi) = \left(\frac{1}{2\pi\sigma^2}\right) e^{-\left(\frac{1}{2\sigma^2}\right)[p^2 - 2pp_o\text{Cos}\phi + p_o^2]} \qquad 2.5.$$

The marginal distribution for p and ϕ, can be obtained by integrating over the appropriate parameter. Integrating over ϕ yields:

$$F(p) = \frac{p}{2\pi\sigma^2} e^{-(\frac{1}{2\sigma^2})[p^2+p_o^2]} \int_0^{2\pi} e^{(pp_o \cos\phi)/\sigma^2} d\phi \qquad 2.6$$

which can be rewritten as:

$$F(p, p_o) = \frac{p}{2\pi\sigma^2} e^{-(\frac{1}{2\sigma^2})[p^2+p_o^2]} I_o(ip_o p/\sigma^2) \qquad 2.7$$

where I_o is the zero order Bessel function. Equation 2.7 is known as the Rice distribution (Serkowski, 1958, Vinokur, 1965, Simmons and Stewart, 1985), and its properties are well understood. The Rice distribution is shown in Fig. 2.1 for different values of p_o.

The probability distribution of ϕ can also be determined by integrating Equation 2.5 with respect to p

$$F(\phi) = \int_0^{\infty} F(p, \phi) p \, dp \qquad 2.8.$$

Thus the distribution of position angle(ϕ) will be

$$F(\phi, p) = \frac{e^{-(\frac{p^2}{2})}}{2\pi} \{ 1 + \Psi e^{\Psi^2} (1 + ERF(\Psi))\} \qquad 2.9$$

where $\Psi = \frac{p\cos\phi}{\sqrt{2}}$, $p = \frac{p_o}{\sigma}$, $\phi = 2\theta$ and ERF is the Gaussian error function. The probability distribution of the position angle(ϕ) is shown in Fig. 2.2.

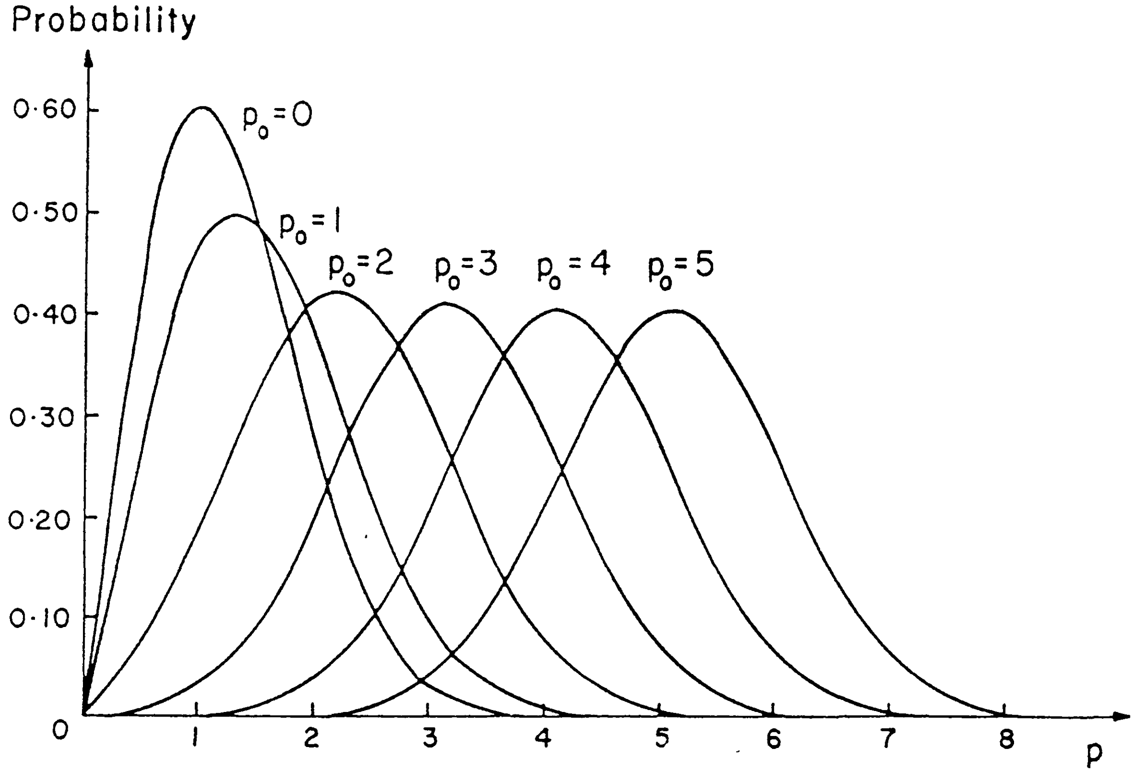


Fig. 2.1 The Rice distribution, $F(p, p_o)$ as a function of p is displayed for values of p_o = 1, 2, 3, 4 and 5. (Taken from Stewart, 1984)

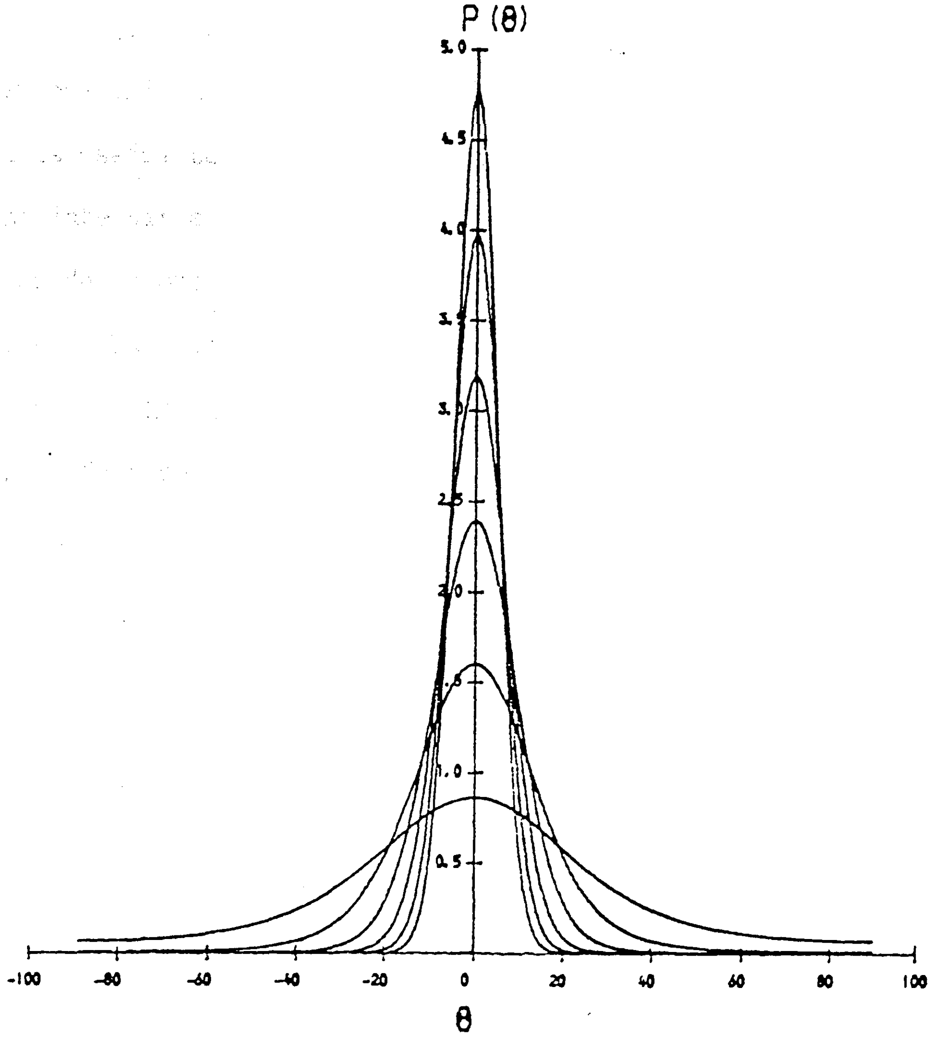


Fig. 2.2 The position angle distribution F(θ, p) as function of θ is displayed for values of p = 0.5, 1, 2, 3, 4 and 5.

2.3 Confidence Interval Estimation of Position Angle

The usual quoted formal errors on the estimated values of the position angle are given in Serkowski (1958, 1962), (see Equation 2.2). However, these formulae refer to two extreme circumstances and express only a 1σ value. Their derivation is somewhat obscure, and it is useful to understand how these are obtained (see below). Intermediate situations, more likely to be met in practice, are not considered properly in the literature and neither is the fact that the probability distribution for θ is non-normal, thus yeilding 1σ, 2σ and 3σ confidence intervals which will be different from the normal distribution case.

Any measurement that is made may be referred to as point estimate of the parameter, but it must be remembered that a point estimator is a random variable distributed in some way around the true value of the parameter. The true parameter value may be higher or lower than our estimate. It is useful therefore to obtain an interval within which we are reasonably confident that the true value will lie (i.e. to construct what are known as confidence limits). In this section we discuss how the error estimation for θ is calculated in the literature. We have also devised confidence interval tables for use in estimating the errors on the position angle at low levels of signal-to-noise ratio.

There are two conditions for which errors can be calculated readily on values of position angle viz: when $p_o \approx 0$ and when $p_o \gg \sigma$. We shall now consider both these cases.

(i) $p_o \approx 0$

If we assume that the underlying polarization is "zero"($p_o \approx 0$)

then σ_θ may be derived by considering that the values of θ come from a uniform distribution, i.e. $P(\theta)$ = constant. By definition we have

$$\int_0^\pi P(\theta)d\theta = 1 \qquad 2.10$$

thus, $P(\theta)= \frac{1}{\pi}$. Because the integral limits run from 0 to π and therefore the mean of our distribution will be at $\frac{\pi}{2}$ (see Fig. 2.3).

From the definition of the variance

$$\sigma_\theta = \int_0^\pi P(\theta)(\theta - \bar{\theta})^2 \, d\theta \qquad 2.11$$

where $\bar{\theta} = \frac{\pi}{2}$ and $P(\theta) = \frac{1}{\pi}$,

$$\sigma_\theta = \frac{\pi}{\sqrt{12}} \text{ rads} = 51.96^\circ \qquad 2.12.$$

(ii) $p_o \gg \sigma$

The error on position angle, frequently used by the observationalists is calculated from $\sigma_\theta = \frac{\sigma_p}{2p}$. This value however is only valid for high signal-to-noise ratio.

According to the usual definitions for the degree of polarization and position angle may be written as:

$$p = (u^2 + q^2)^{1/2} \quad \text{and} \quad \theta = \frac{1}{2} \text{Tan}^{-1}(\frac{u}{q}) \qquad 2.13.$$

The standard errors σ_p, σ_θ for p and θ may be written respectively as:

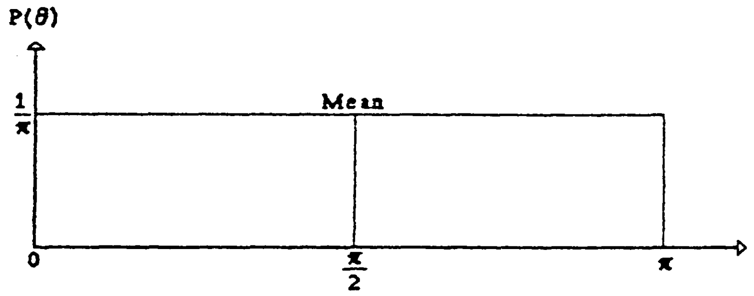


Fig. 2.3 Uniform distribution of θ as defined by Eq. 2.12.

$$\sigma_p = \frac{[q^2\sigma_q^2 + u^2\sigma_u^2]^{1/2}}{p} \qquad 2.14$$

and

$$\sigma_\theta = \frac{[q^2\sigma_u^2 + u^2\sigma_q^2]^{1/2}}{2p} \qquad 2.15.$$

If we assume that $\sigma_u = \sigma_q = \sigma_p$, then substituting Eq. 2.14 into Eq. 2.15 yields,

$$\sigma_\theta = \frac{\sigma_p}{2p} \text{ rads or } = 28.65^\circ \frac{\sigma_p}{p} \qquad 2.16.$$

At high signal-to-noise ratio (as will be demonstrated below) the distribution for θ essentially follows a normal distribution and the scaling of the error to $2\sigma_\theta$ and $3\sigma_\theta$ leads to the well known confidence intervals of 95.45% and 99.75% respectively.

The analytical methods above provide relatively simple values of the uncertainty for two extreme cases. In real situations, however the behaviour of the uncertainties involves more general expressions, as we have seen from Section 2.2. Few investigators have studied them and even then their explanation is incomplete. Wardle and Kronberg (1974) investigated the $1\sigma_\theta$ confidence limits associated with the analytical expression of θ (see Equation 2.9) and presented their results in graphical form. To extend the investigation and to make it more useful, we have evaluated $2\sigma_\theta$ and $3\sigma_\theta$ values of confidence limits and provide tabulated material giving more information on general confidence in more readily accessible form.

The investigation was performed using two different computer techniques viz; numerical integration and data simulation. Both studies were carried out with the Glasgow University's IBM 3090 mainframe computer. The outline of each method is discussed:

(i) Numerical Integration

This method gives exact values of confidence limits based on the numerical integration of Equation 2.9. Using two NAG (Natural Algorithm Group) routines by which S15AEF evaluated the term ERF(Ψ) and Pattersons approximation (D01AHF) evaluated the integral, the values of confidence were explored by adjustment of the prescribed limits. The limits of integration were changed in small steps to enable us to get the precise value of the integration at the 1σ, 2σ and 3σ confidence levels. The procedure was undertaken for a range of values of $\frac{p}{\sigma}$ between 1 and 10.

If θ_i were a sample measurement from a Gaussian distribution then the probability that it would lie within the interval of $\theta - \sigma \leq \theta_i \leq \theta + \sigma$ is

$$\int_{\theta-\sigma}^{\theta+\sigma} P(\theta, \frac{p}{\sigma})d\theta = 68.26\% \qquad 2.17.$$

Similar integrations involving 2σ and 3σ terms in the limits will produce 95.45% and 99.75% respectively. It is necessary to input a range of signal-to-noise ratios to appreciate the behaviour of θ. Since we know that θ_i does not originate from a Gaussian distribution, comparison between the Gaussian and non-Gaussian is required to see how significant the departures are. Using the definition of variance of the distribution which can be represented as

$$\sigma_\theta^2 = \int_{-\pi/2}^{+\pi/2} P(\theta, \tfrac{p}{\sigma})(\theta - \bar{\theta})^2 \, d\theta \qquad 2.18.$$

We can compare this with Equation 2.17. Since our limits run from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$, we would expect from this distribution, $\bar{\theta} = 0$. The results of the investigation of confidence intervals are shown in Figs. 2.4, 2.5 and 2.6, using Equations 2.17 and 2.18 as a non-normal and normal distributions respectively. They are also represented in tabulated form in Appendix A.

We can also plot the ratio of $\frac{\sigma_{\theta(\text{non-Gaussian})}}{\sigma_{\theta(\text{Gaussian})}}$ for the confidence interval of 68.26% against the signal-to-noise ratio, thus giving some indication as to where the largest discrepancies occur (see Fig. 2.7). One particular interesting feature of the comparison is that for $\frac{p}{\sigma} > 5$ the curves tend to come together in an asymptotic way. Thus if $\frac{p}{\sigma} > 5$ then the more simple Equation 2.2 can be used to describe the situation. The reason for choosing 5 as critical value is that in the range of $\frac{p}{\sigma}$ from 0 to 5, there are large swings either side of the true value. It is also interesting to note that Simmons and Stewart (1985) reported that the confidence interval values in p (polarization) for all values of $\frac{p}{\sigma} > 6$ can readily be obtained by assuming that the distribution of p is normal. The discrepancy between the Gaussian and true distribution of position angle at low signal-to-noise level becomes large when the 3σ confidence limit is explored (see Figs. 2.4, 2.5 and 2.6). It can be seen from Fig 2.4 that if a Gaussian distribution is assumed, then we are underestimating our errors when $\frac{p}{\sigma} < 1$, but when $\frac{p}{\sigma} > 1$ we are over estimating the errors. In the cases of the 2σ and 3σ

confidence limits the opposite occurs; when $\frac{p}{\sigma} < 1$ there is a over estimation of the true value, but when $\frac{p}{\sigma} > 1$ we under estimate the uncertainties. It is therefore recommended that these tables (see Appendix A) be used whenever errors are being quoted on position angle especially at low signal-to-noise ratio.

For further study of the problem of confidence on θ values, a method involving data simulation seemed appropriate and also serves as a check on the basic results above.

(ii) Data Simulation

A program was developed to generate 2000 values of q_i and u_i, the values being taken from normal distribution. Generated values of q_i, u_i were obtained by considering a distribution with true values of $\bar{q}$ and $\bar{u}$, each with an associated error ($\sigma = \sigma_q = \sigma_u$). For simplicity a value of $\bar{u} = 0$ was chosen so that the underlying value of $\theta(= \bar{\theta}) = 0^\circ$. The values of $\bar{q}$ and σ were supplied as input for each run of the program.

It may be noted that in the limit

$$\sigma_q^2 = \frac{\sum_{i=1}^{n} (q_i - \bar{q})^2}{n - 1} \qquad 2.19$$

and

$$\sigma_u^2 = \frac{\sum_{i=1}^{n} (u_i)^2}{n - 1} \qquad 2.20.$$

A set of position angle values θ_i were calculated according to Equation 2.13. In investigating the various confidence limits, the position angle data were first sorted in ascending order. The value of $\bar{q}$ was increased in small steps and the signal-to-noise ratio was calculated from $\frac{\bar{q}}{\sigma}$.

The program was written for the exercise employing NAG routines G05CCF, G05DDF and M01ANF. G05CCF sets the random number generator routine to non-repeatable starting positions. G05DDF returns a pseudo-random number taken from a normal distribution defined by user inputs of the mean and variance. M01ANF allows the generated values of θ_i to be sorted into ascending order.

The confidence limits were then obtained from the sorted generated data. For example, to find the values corresponding to the 99% confidence limit, we need to consider the 10^{th} and 1991^{st} value of θ ; the 95% and 68.2% limits require the 50^{th} and 1950^{th}, 318^{th} and 1683^{rd} respectively. The above procedure was repeated five times and the mean value of each confidence interval taken.

As expected the comparison between the confidence values obtained by numerical integration and data simulation gave similar results. Since numerical integration gives exact values of confidence level, there is really no need to undertake the simulation calculation. However, as the latter technique will be used in a later study, it was undertaken to confirm that the routines had been established correctly.

Frequently met situations are those for which a decision has be made as to whether two measured values of θ are different. i.e. the observations might have been made at different times, or at different wavelengths and with different signal-to-noise ratio. Since the two values of θ both come from non-normal distributions, the analytical expression describing the distribution of $\Delta\theta = \theta_1 - \theta_2$ would be complex and its investigation is best done by data simulation.

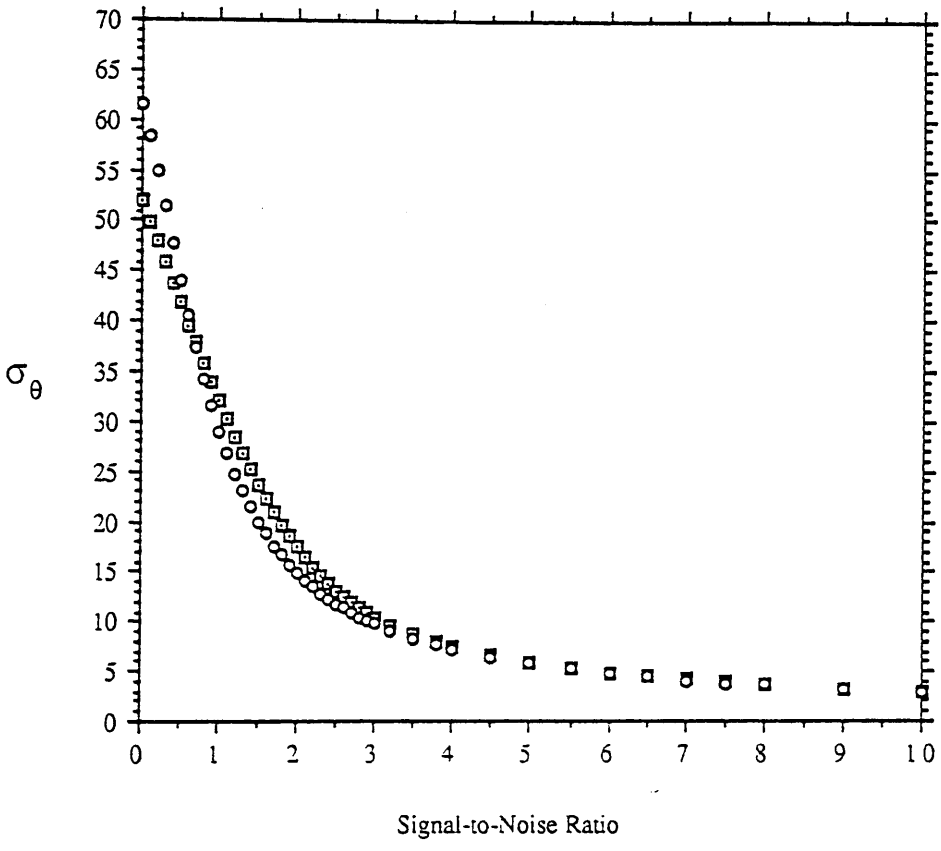


Fig. 2.4 Confidence interval for the position angle(θ) at 68.26%, where ◻ is the Gaussian value and ○ is the calculated value of θ from Eq. 2-9.

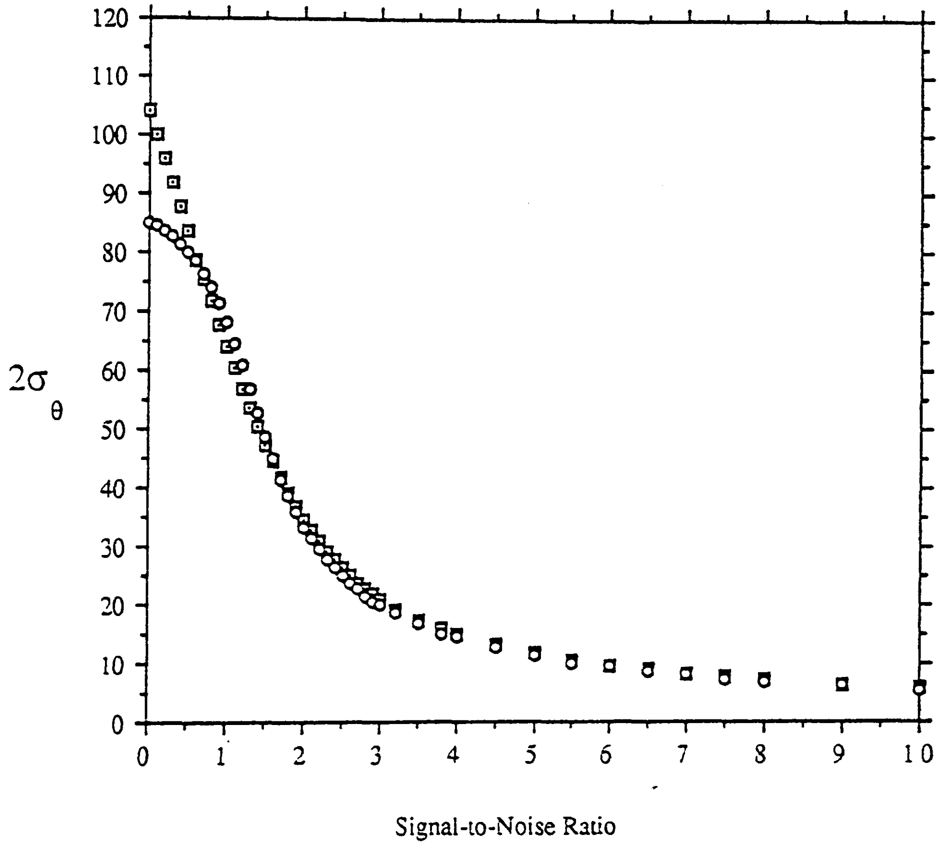


Fig. 2.5 Confidence interval for the position angle(θ) at 95.45%, where ▫ is the Gaussian value and ○ is the calculated value of θ from Eq. 2.9.

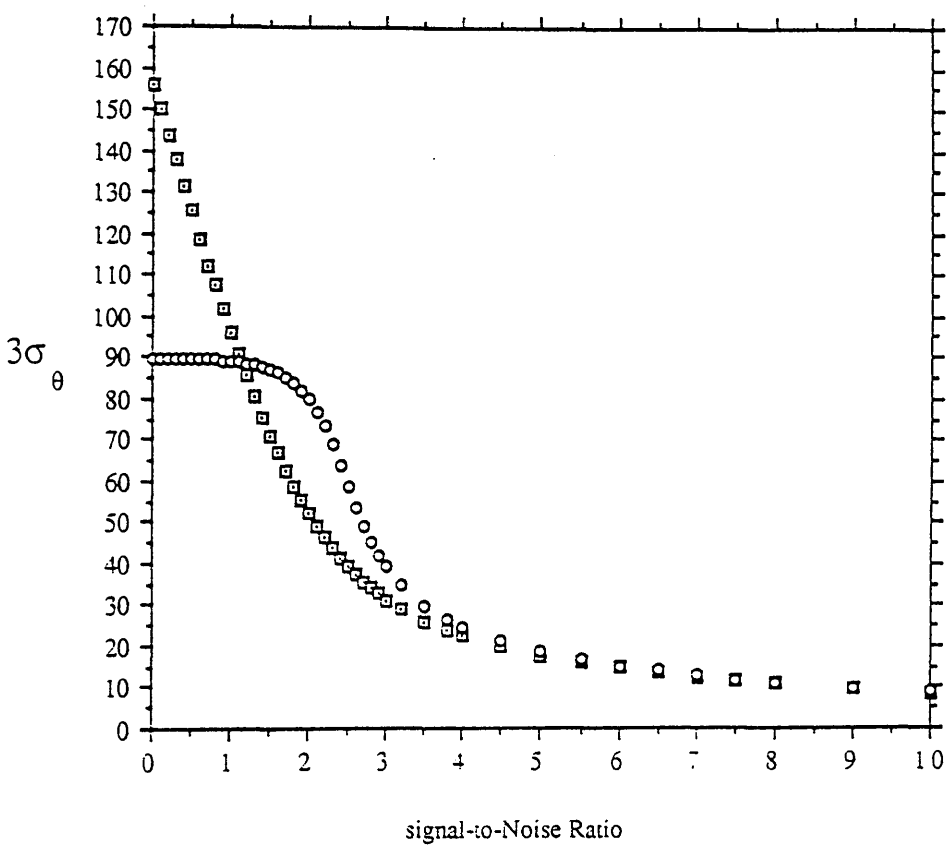


Fig. 2.6 Confidence interval for the position angle(θ) at 99.75%, where ▫ is the Gaussian value and o is the calculated value of θ from Eq. 2.9.

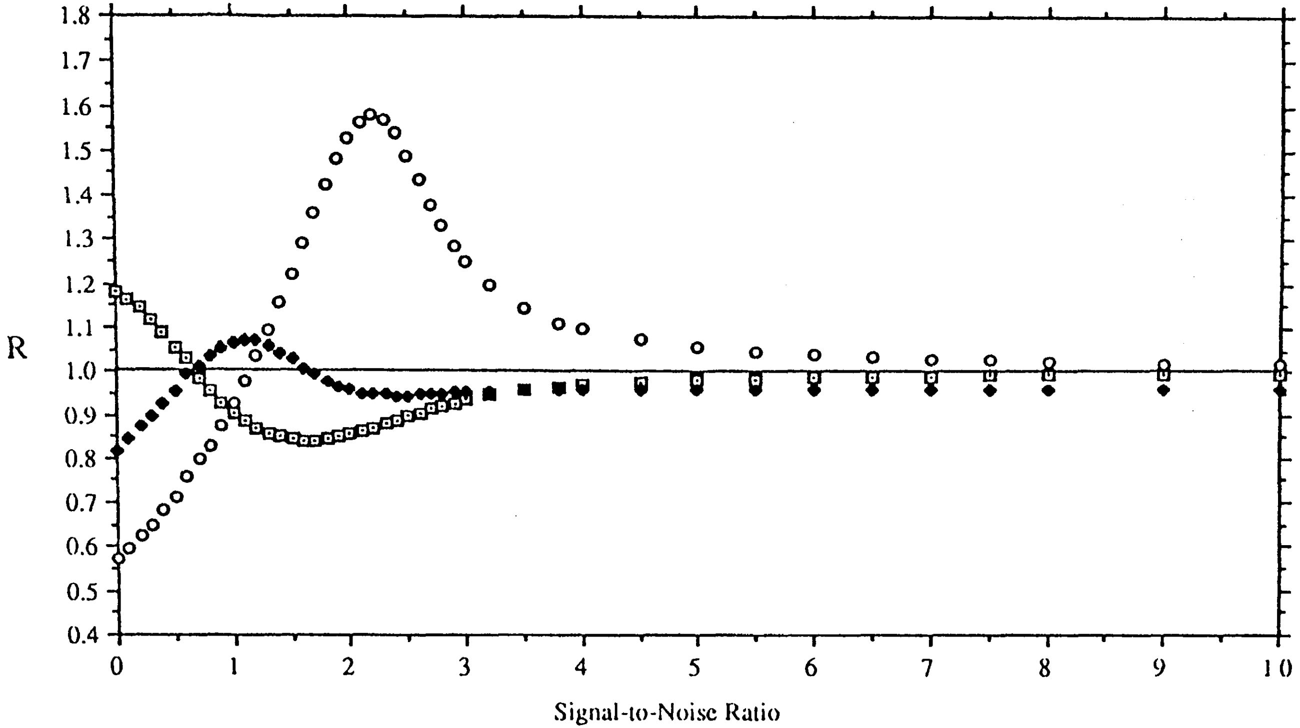


Fig. 2.7 The ratio of the true distribution and the Gaussian (R). This is plotted against the signal-to-noise ratio, at 1σ (▫), 2σ (◆) and 3σ (o).

2.4 Confidence Interval of Position Angle Differences

The confidence interval associated with measurements of the differences between two values of position angle was investigated by a data simulation method, where the same procedures as in Section 2.3 were adopted. The scheme described below is in terms of measurements made at two different wavelength values, B and R but could equally well relate to pairs of measurements made at two different times.

Sets of NSPs were generated given by q_{R_i}, u_{R_i} and q_{B_i}, u_{B_i}, where the subscripts R and B correspond to two distributions (say for measurement at the Red and Blue part of the spectrum). These distributions are described by $\bar{q}_R$, $\bar{q}_B$, σ_R and σ_B with $\bar{u}_R = \bar{u}_B = 0$. The values of θ_{R_i} and θ_{B_i} were determined from Eq. 2.13, and the value $\Delta\theta_{(R-B)_i}$ $(= \theta_{R_i} - \theta_{B_i})$ was obtained. The distribution of $\Delta\theta_{(R-B)_i}$ was then be used to obtain the confidence interval.

The problem was tackled by considering three different cases which one might encounter. The confidence intervals for these distributions were obtained and are tabulated in Appendix B.

(i) $\bar{q}_R = \bar{q}_B$ and $\sigma_R \neq \sigma_B$

This situation corresponds to measurements for which the underlying value is the same but the errors associated with the two observations are different. The value of the errors of the observations will depend on weather conditions and instrumental stability. The scheme of simulation is depicted in Fig. 2.8a, where signal-to-noise at S_B $(\frac{\bar{q}_B}{\sigma_B})$ is kept high and constant and the signal-to-noise of S_R $(\frac{\bar{q}_R = \bar{q}_B}{\sigma_R})$ is varied. The exercise was repeated for a range of values of S_B and the results are shown in Figs. 2.9a to 2.9j. These Figures show the distribution of differences in two position angle values at 99%, 95% and 68.2% confidence level. The simulated distributions are compared with the Gaussian distribution, where the latter was obtained from previous section;

$$\sigma^2_{\theta Gaussian} = (\sigma^2_{\theta_1} + \sigma^2_{\theta_2})$$

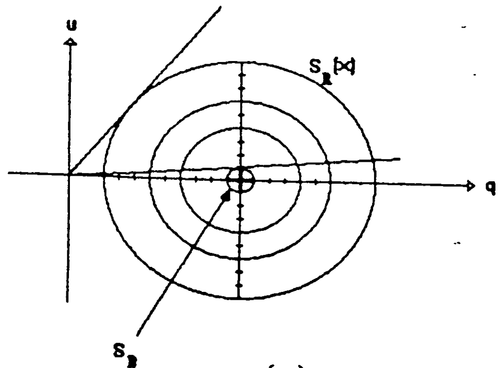
the non-Gaussian form of the position angle is plotted against the Gaussian form.

There are two signal-to-noise ratios involved in our simulation, one associated with S_B and another with S_R. Since the signal-to-noise at S_B is kept constant while S_R is varied, thus signal-to-noise ratio of 1 in Fig 2.9a refers to the signal-to-noise ratio of S_B, and S_R is shown along abscissa in each Figures. This is repeated for signal-to-noise ratio of 1 to 10.

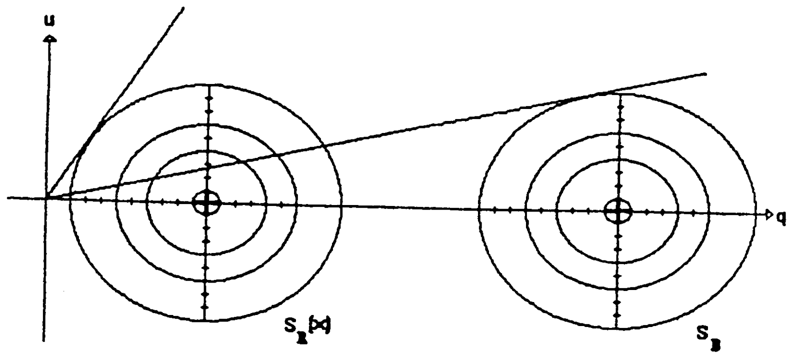
These Figures show that there are large differences between the Gaussian distribution and simulated distribution, even at high signal-to-noise ratio. In Fig. 2.9j the value of simulated distribution at 99% confidence level is underestimated according to Gaussian distribution.

(ii) $\bar{q}_R \neq \bar{q}_B$ and $\sigma_R = \sigma_B = \sigma$

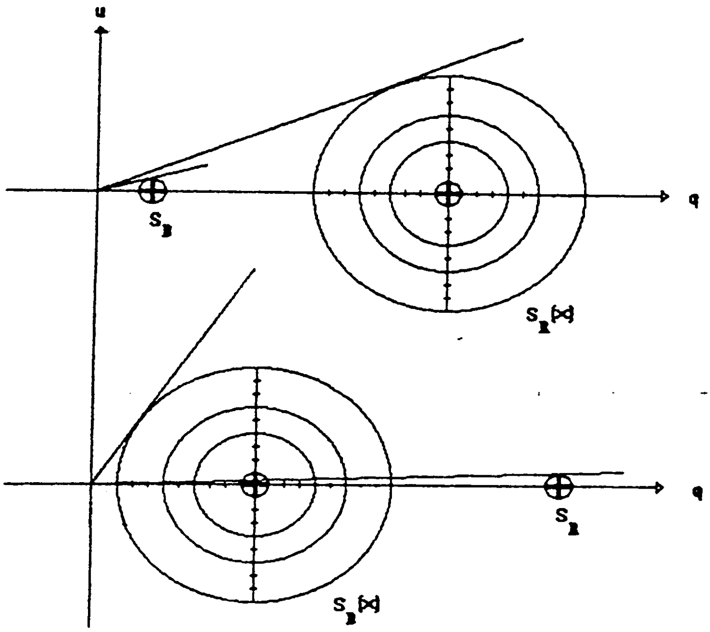
This corresponds to the case, for which the compared measurements have been taken with the same accuracy but that there is an underlying difference between the two values. The procedure was performed by fixing the value of S_B $(\frac{\bar{q}_B}{\sigma})$ with increasing



(a)



(b)



(c)

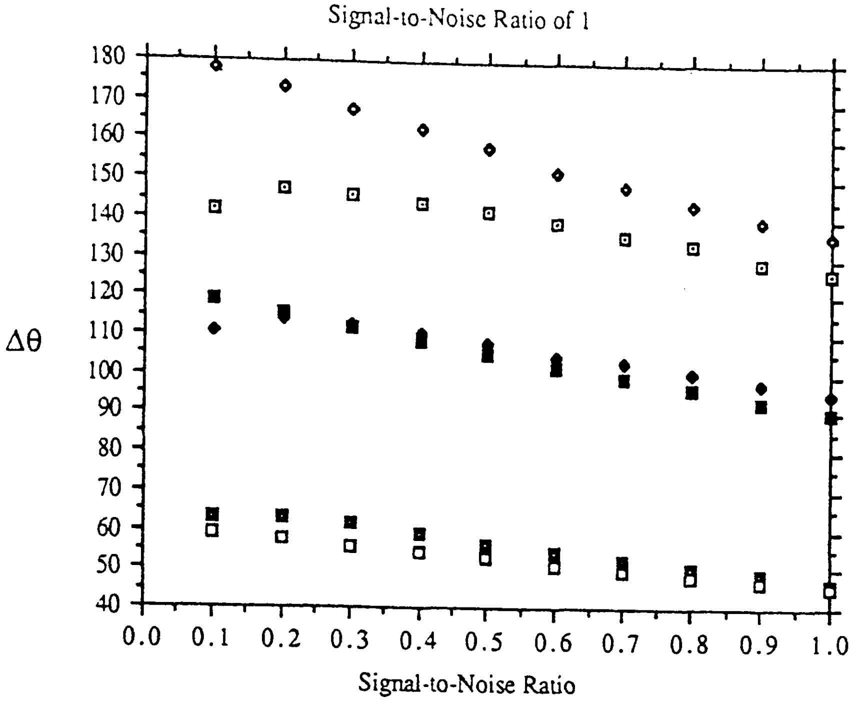
Fig. 2.8 Three cases where position angle differences can be obtained; where (a) $\bar{q}_R = \bar{q}_B$ with $\sigma_R \neq \sigma_B$, (b) $\bar{q}_R \neq \bar{q}_B$ with $\sigma_R = \sigma_B$ and (c) $\bar{q}_R \neq \bar{q}_B$ with $\sigma_R \neq \sigma_B$.

value of S_R ($\frac{\bar{q}_R}{\sigma}$) being taken with small increments (see Fig. 2.8b).

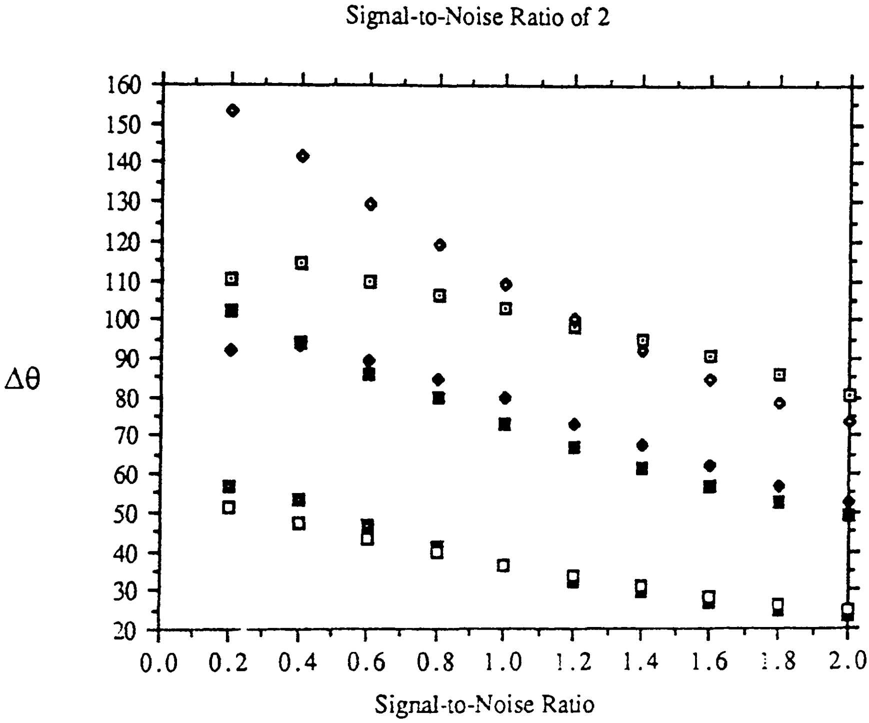
(iii) $\bar{q}_R \neq \bar{q}_B$ and $\sigma_R \neq \sigma_B$

The situation corresponds to the most general of cases, there being an infinite varieties of values that might be chosen and hence we chose to consider two extreme cases (see Fig. 2.8c). We consider $\bar{q}_B < \bar{q}_R$, with fixed S_B ($\frac{\bar{q}_B}{\sigma_B}$) which is assigned a large signal-to-noise and the signal-to-ratio at S_R ($\frac{\bar{q}_R}{\sigma_R}$) is then varied. We can do similar thing with varying the signal-to-noise ratio of S_B and fixing S_R. The results are shown in Figs. 2.10a and 2.10b.

It can be seen from Figs. 2.10a and 2.10b that the Gaussian values at 99% confidence level overestimates the true value obtained from simulation technique.

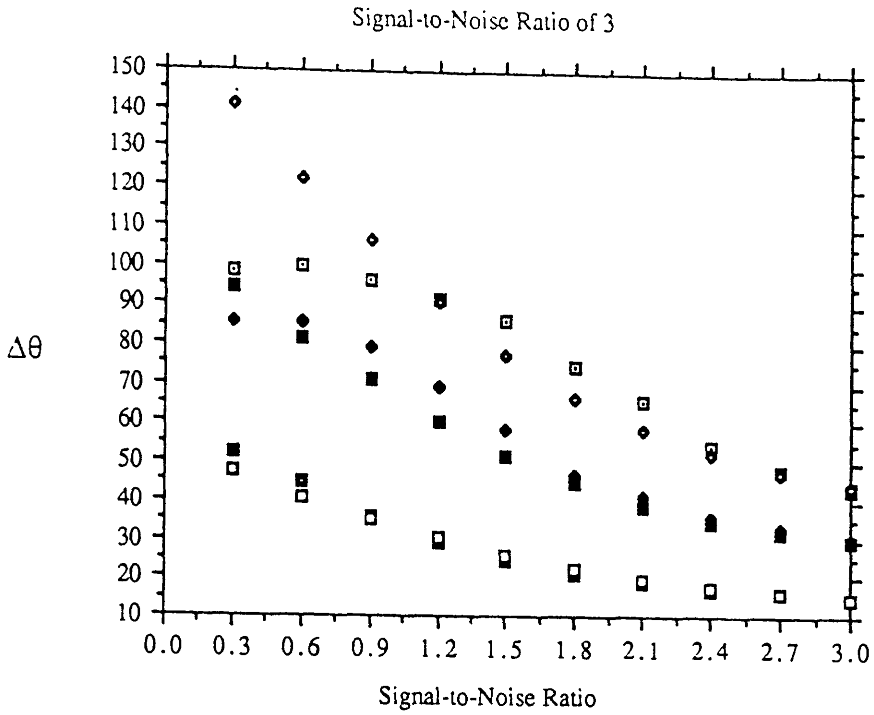


(a)



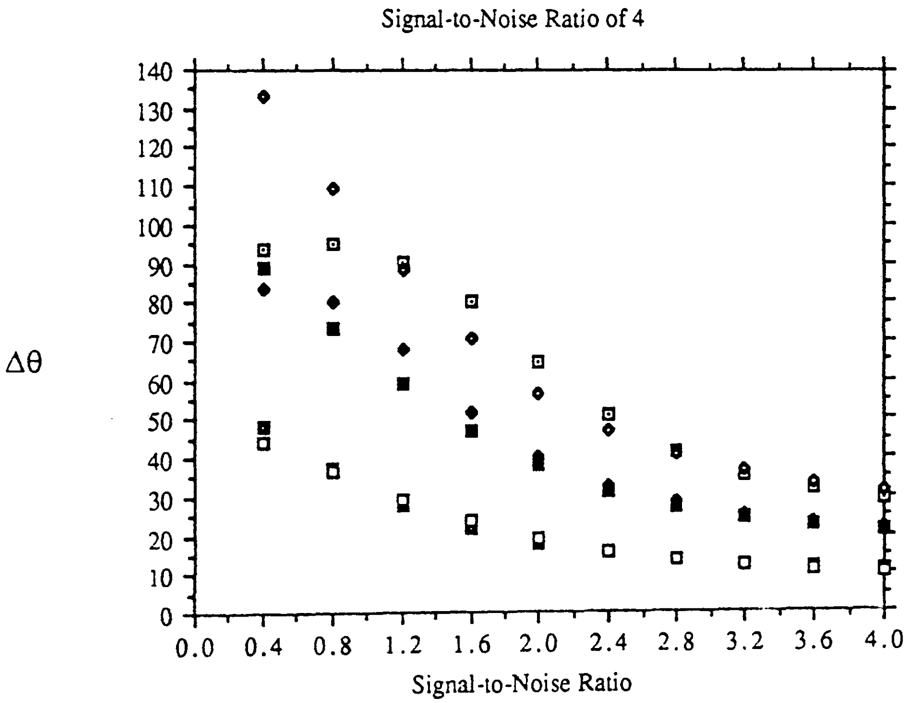
(b)

Fig. 2.9
Confidence interval of the differences in two position angle values, where signal-to-noise ratio of 1 to 10 above each graph refers to the signal-to-noise of S_B (see Fig. 2.8a); where ■, ◆ and ◘ are the simulated value at 68.2%, 95% and 99% with the associated 1σ(□), 2σ(■) and 3σ(◇) Gaussian value respectively.



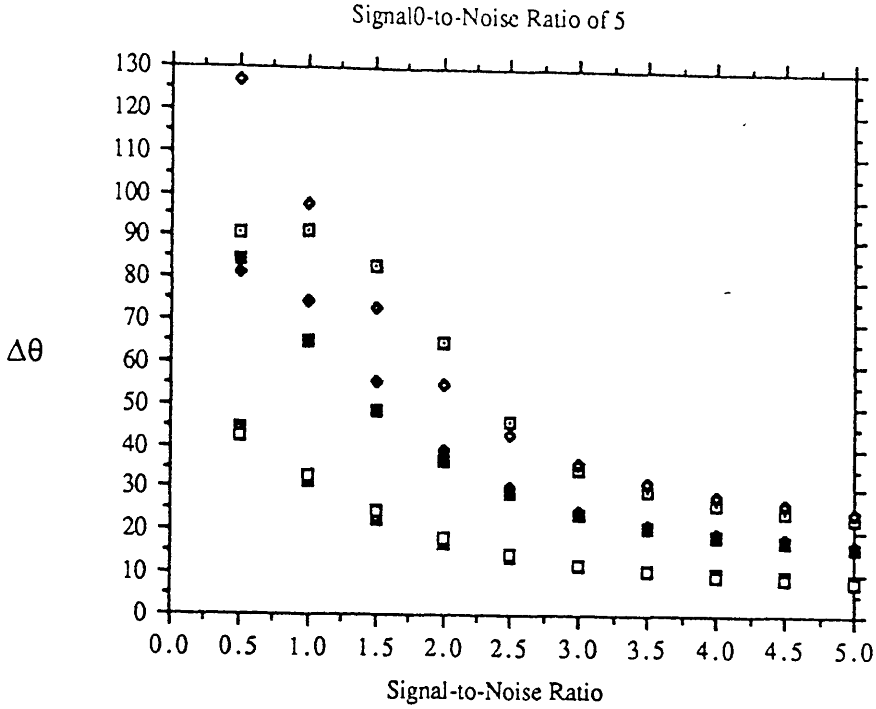
Signal-to-Noise Ratio of 3
Δθ
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0.0
0.3
0.6
0.9
1.2
1.5
1.8
2.1
2.4
2.7
3.0
Signal-to-Noise Ratio

(c)

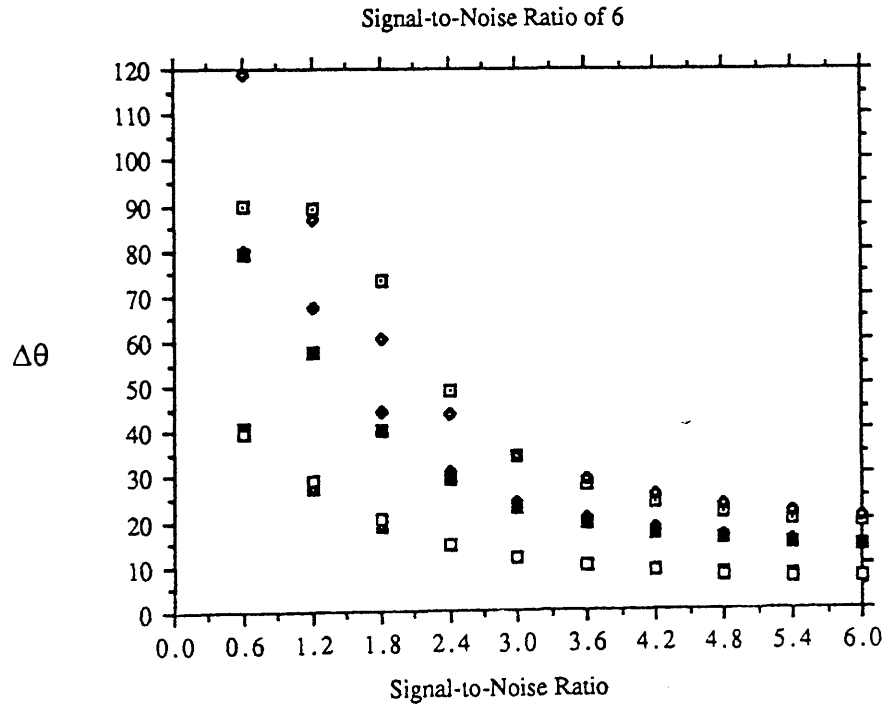


Signal-to-Noise Ratio of 4
Δθ
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0.0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6
4.0
Signal-to-Noise Ratio

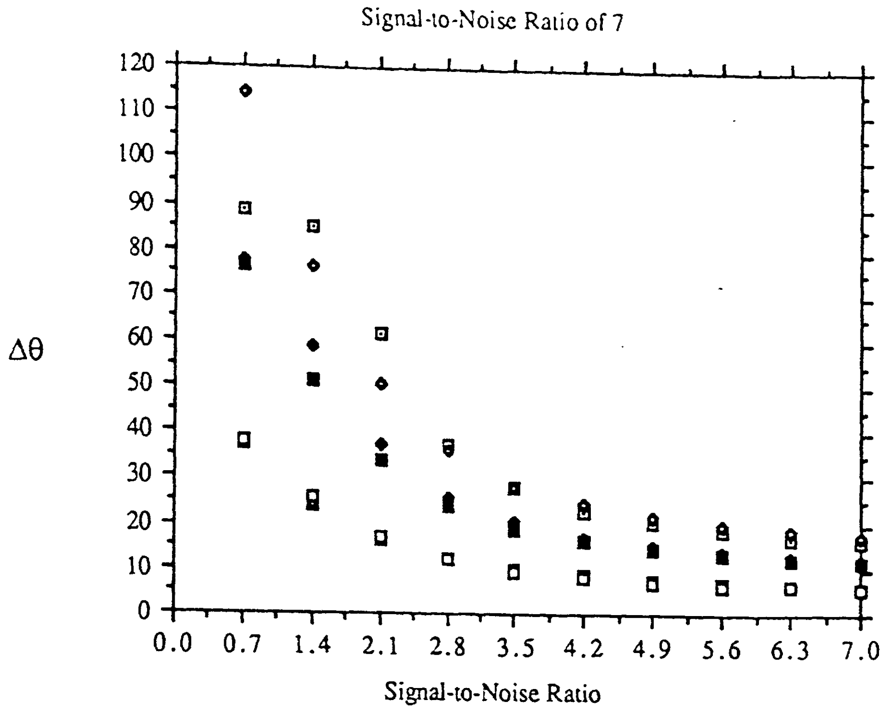
(d)


Signal0-to-Noise Ratio of 5
Δθ
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
Signal-to-Noise Ratio

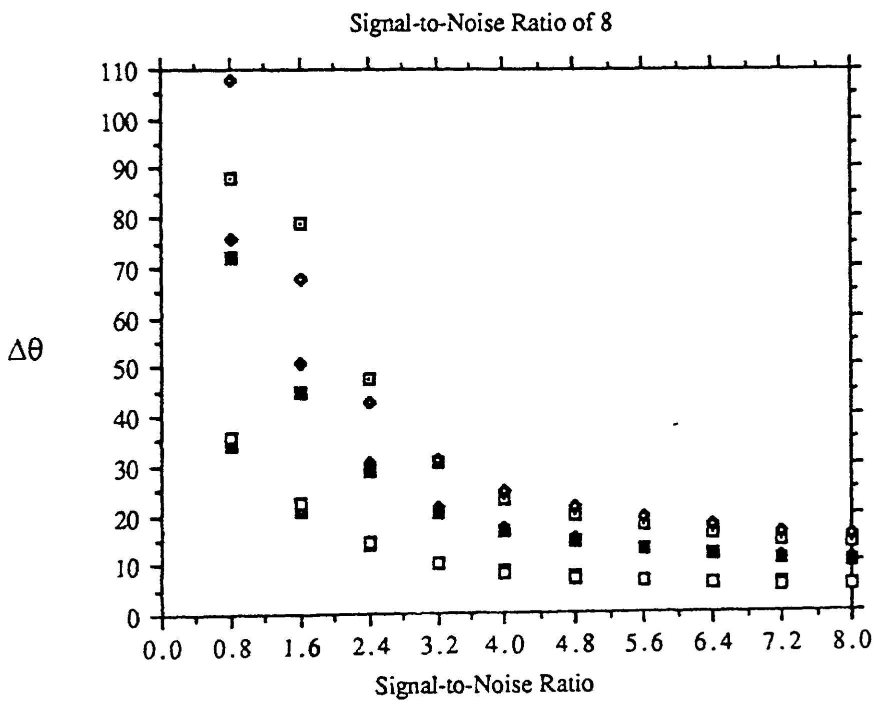
(e)


Signal-to-Noise Ratio of 6
Δθ
120
110
100
90
80
70
60
50
40
30
20
10
0
0.0
0.6
1.2
1.8
2.4
3.0
3.6
4.2
4.8
5.4
6.0
Signal-to-Noise Ratio

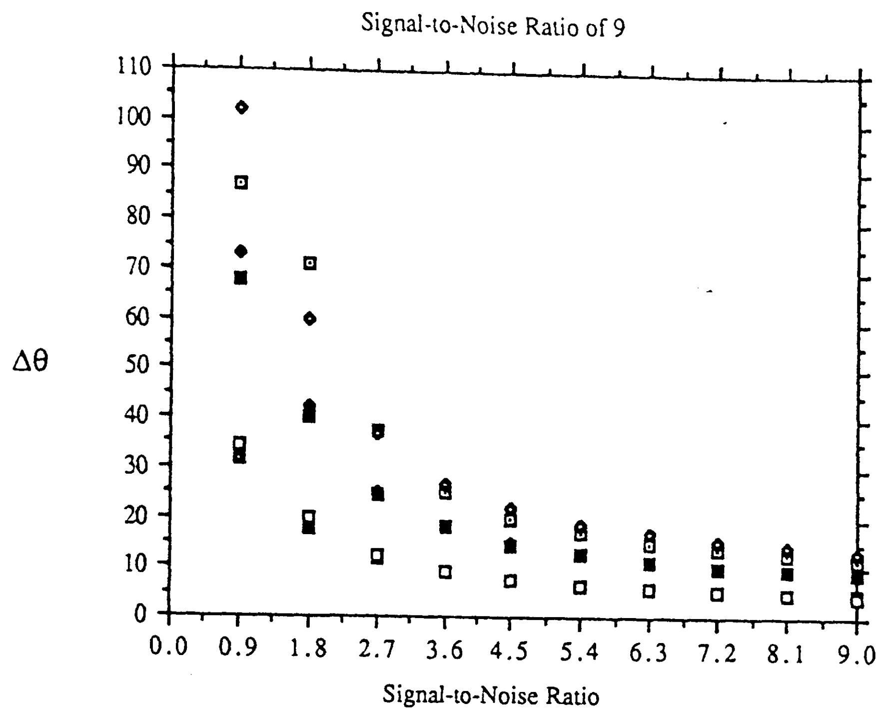
(f)


Signal-to-Noise Ratio of 7
120
110
100
90
80
70
60
50
40
30
20
10
0
Δθ
0.0 0.7 1.4 2.1 2.8 3.5 4.2 4.9 5.6 6.3 7.0
Signal-to-Noise Ratio

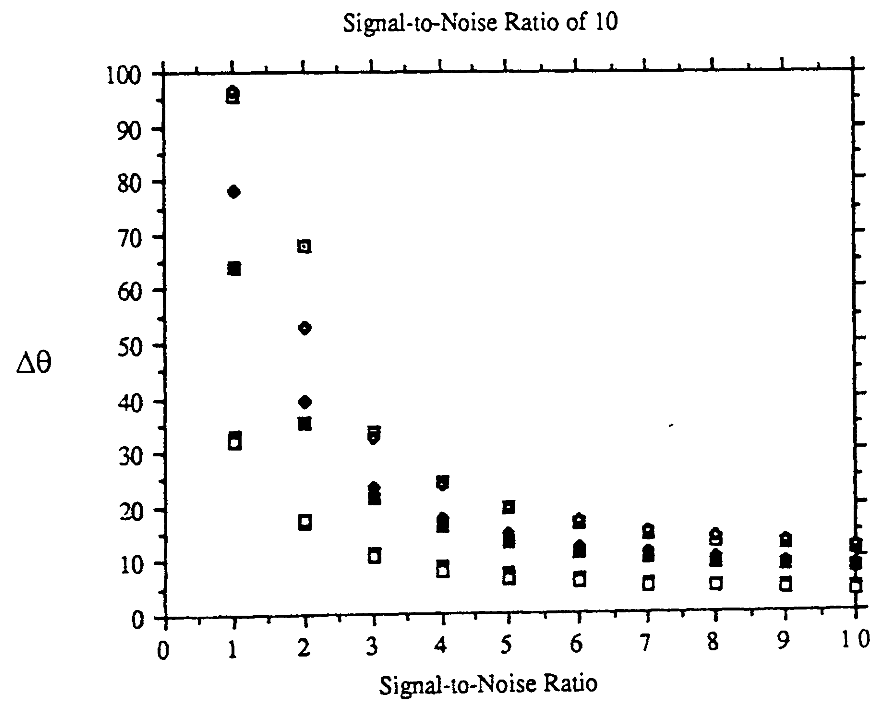
(g)


Signal-to-Noise Ratio of 8
110
100
90
80
70
60
50
40
30
20
10
0
Δθ
0.0 0.8 1.6 2.4 3.2 4.0 4.8 5.6 6.4 7.2 8.0
Signal-to-Noise Ratio

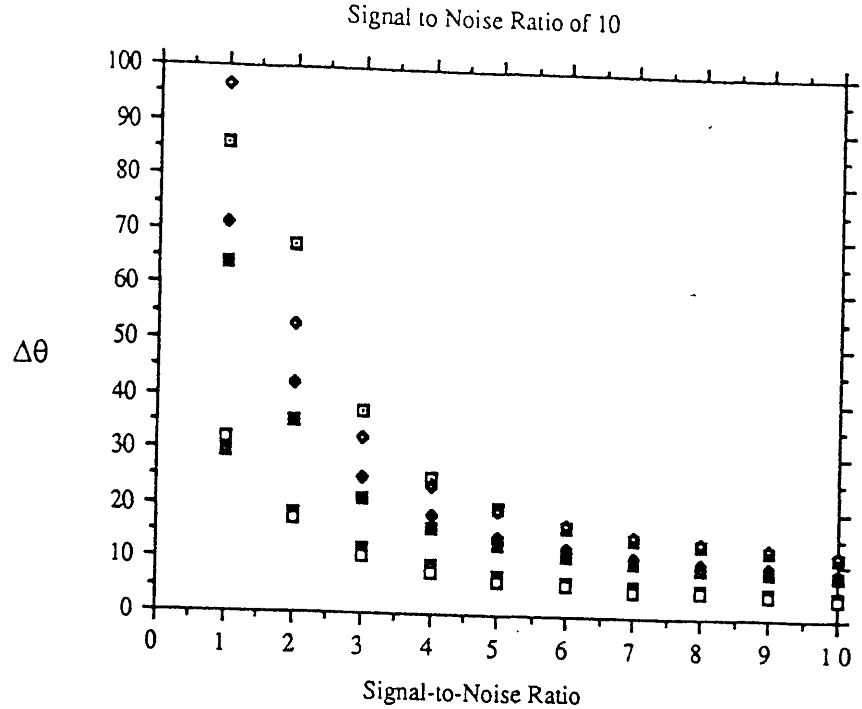
(h)


Signal-to-Noise Ratio of 9
Δθ
110
100
90
80
70
60
50
40
30
20
10
0
0.0 0.9 1.8 2.7 3.6 4.5 5.4 6.3 7.2 8.1 9.0
Signal-to-Noise Ratio

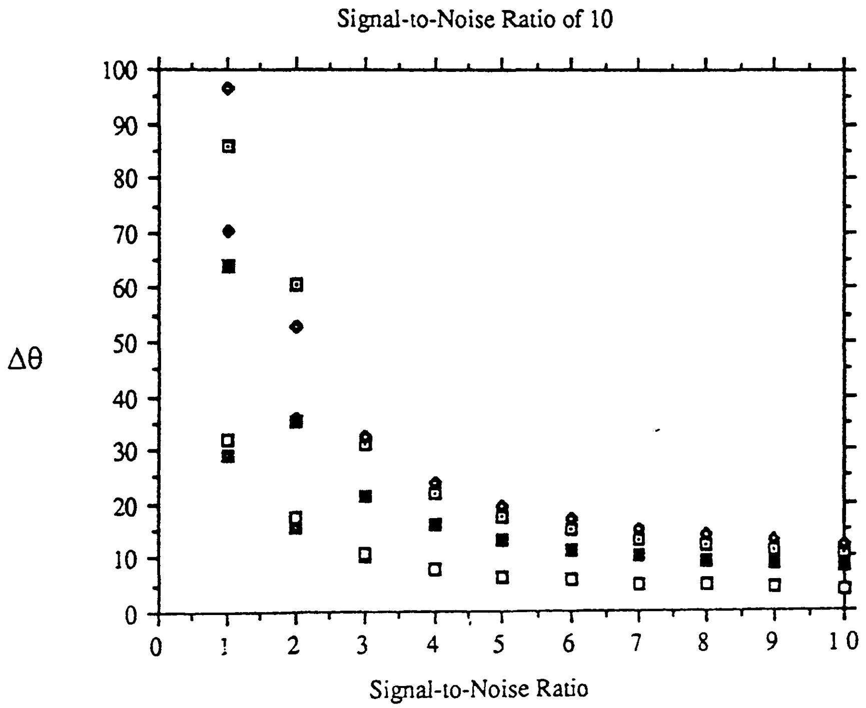
(i)


Signal-to-Noise Ratio of 10
Δθ
100
90
80
70
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10
Signal-to-Noise Ratio

(j)



(a)



(b)

Fig 2.10
Confidence interval of the differences in two position angle values, where signal-to-noise ratio of 10 above each graph refers to the signal-to-noise of S_B (see Fig. 2.8c); where ■, ◆ and ▫ are the simulated value at 68.2%, 95% and 99% with the associated 1σ(□), 2σ(■) and 3σ(◇) Gaussian value respectively.

2.5 Conclusion

Following the work of Serkowski (1958, 1962) and Wardle and Kronberg (1974), it was shown that the error estimation on the position angle is not a Gaussian distribution and that this is an important consideration for signal-to-noise ratios less than 5. The confidence interval of θ for the values of $\frac{p}{\sigma} < 10$ have been computed at 68.26 % (1σ), 95.45% (2σ) and 99.75% (3σ) level (see Appendix A). It is seen that for values of $\frac{p}{\sigma} > 10$ the error estimation of θ can be constructed by assuming a normal distribution of θ. The differences were most noticeable when considering the $3\sigma_\theta$ (99.75%) level.

Many workers base their estimates of position angle errors on Gaussian distribution and not the real distribution. As we have shown this could lead to an under (or over) estimation of the uncertainties on the values of position angle at low levels of signal-to-noise ratios. The treatment of position angle errors are usually done on $1\sigma_\theta$ which is in any case inadequate, particularly when $3\sigma_\theta$ value considerations are the most disparate. This has not been appreciated in all previous works, most of them provide only formal error calculations.

In addition, we have performed a data simulation for calculating the confidence interval on the differences of two values of position angle. This will allow us to see how the difference in two values of θ are distributed. The distribution significantly departed from Gaussian at 99% confidence level even at high signal-to-noise ratios (see Appendix B).

The value of p is generally biased for low signal-to-noise ratios and small levels of polarization (see Serkowski, 1958, 1962, Wardle and Kronberg, 1974 and Simmons and Stewart, 1985). If no correction has been made for this then the value of σ_θ will be biased as well, since σ_θ is usually calculated from $\frac{\sigma_p}{2p}$.

It has been suggested by Clarke and Stewart (1986) that when statistical analysis of data sets are to be performed (especially when combining or comparing) the NSPs should be used in preference to p and θ as the statistical properties of NSPs are more readily understood.

Chapter 3: A Statistical Reassessment of Polarimetric Standard Stars

3.1 Introduction

3.2 Statistical Tests on the Normalized Stokes Parameters

3.2.1 Normality Testing From the Values of Skewness and Kurtosis

3.2.2 Modified Kolmogorov-Smirnov Test (Non-parametric Test)

3.3 Application of Statistical Tests to the "Zero" Polarization Stars

3.4 Conclusion

3. A Statistical Reassessment of Polarimetric Standard Stars

3.1 Introduction

In this chapter we briefly review the structure of previous analyses of observations of polarimetric standard stars. We discuss new statistical techniques for dealing with polarimetric standard star data. As we will see, there are shortcomings in all previous works. In the investigation undertaken here, it is demonstrated that due to a lack of understanding of proper statistical procedures, the conclusions reached in the previous works are highly suspect. There is therefore a need to investigate these results with more correct statistical tests and reassess their current standing.

As we have mentioned in previous sections (see Chapter 1) the work involved in establishing standard stars has been mainly due to Serkowski (1960). Since then only a few authors (Tinbergen, 1979, Hsu and Breger, 1982, Dolan and Tapia, 1986 and Bastien et al., 1988) have tackled the problem in any substantial measure. Tinbergen (1979) concentrated on establishing a catalogue of zero polarization standard stars. Tinbergen (1982) concluded that stars later than spectral type F0 display polarization ~ 0.01%. Tinbergen based his conclusion on the number of stars which were outside the $2\sigma_p$ value of the distribution, this however being based on it being Gaussian. The other investigators consider the possible variability of polarization standards in Serkowski's list (see Section 1.3 Table 1.2), without undertaking an independent survey on additional stars.

Hsu and Breger (1982) stated that 3 stars in Serkowski's list showed time variability and recommended that these stars should no longer be used as standards. Hsu and Breger considered the systematic errors which may also affect the precise determination of polarization and position angle such as, accurate determination of the Dead-Time associated with photon counting, Faraday rotation due to the Earth's magnetic field, atmospheric refraction effects, change of position angle due to the astrometric motion and absolute zero point of position angle (for the latter see Section 1.3.3). Unfortunately the data presented by them lacked the necessary statistical underpinning.

Dolan and Tapia (1986) investigated the wavelength dependence of position angle of stars in Serkowski's list. Their conclusions were based on statistical evidence that 9 out of 11 stars exhibited rotation of the position angle with wavelength. Bastien et al. (1988) have reported that most of the stars they considered (in Serkowski's list) show temporal variability in polarization.

In Section 3.2 we examine Bastien et al.'s data (provided by private communication) and employ a more developed test to assess the validity of their findings. In Section 3.3 we shall use the data in various surveys such as, Tinbergen (1979), Leroy and Le Borgne (1989), and Huovelin et al. (1985) to determine if there are correlations between spectral type and polarization. The data in these surveys are taken at face value (no debiasing corrections were made on the value of p), mainly in order to illustrate our method of analysis.

3.2 Statistical Tests on the Normalized Stokes Parameters

In the work performed by Bastien et al. (1988) on supposedly polarized standard stars, it was concluded that 11 out of 13 stars in the survey showed signs of time-dependent variations. Their study is based on statistical techniques involving assessment of the skewness and kurtosis of the distribution of the repeated measurements. However the conclusions are arrived at by no more than hand waving assessment of these determined parameters. In this section, the assessments are made thoroughly. Before this is done, we should point out that we have converted their polarization and position angle values into NSPs, q and u. Therefore all the statistical tests are performed on NSPs rather than p and θ, so avoiding the difficulties these latter parameters hold in terms of their statistical behaviour. We also present further data obtained by McDavid (1990) on 2HCam (private communication).

In investigating polarimetric standard stars, the repeated q and u measurements of such stars would be expected to have a normal distribution if the observations are made with a constant signal-to-noise ratio. Application of normality assumptions to non-normally distributed data will affect the statistical interpretation and therefore the conclusions which may be drawn from any given set of observations.

In our analysis of the polarized standards data we shall use two independent statistical tests (one of which had been employed by Bastien i.e. taking moments about the mean), in order to decide whether the data can be considered as coming from a Gaussian distribution with a variance dictated by observational noise. Both of these methods can be used to indicate departure from normality. We will show that the NSPs for most of the stars in the catalogue, despite Bastien et al.'s conclusions pass normality testing. In addition to completing skewness/kurtosis tests we have employed the Kolmogorov-Smirnov test which is of Non-Parametric type. We discuss both methods in detail and present the outcome of the test for each star.

3.2.1 Normality Testing From the Values of Skewness and Kurtosis

Bastien et al. (1988) have claimed that 11 out of 13 stars show large deviations in their skewness and kurtosis values in respect of values that should ensue from Gaussian distribution. They also presented histograms of the data for five stars (see Fig. 3.1) with the largest number of observations and they interpret them as being significantly deviated from normal distributions, thus, making the stars unsuitable for calibration purposes. However, they failed to put confidence intervals on their conclusions, as the expected departure of skewness and kurtosis from 0 and 3 respectively for their small samples.

One way of testing whether data can be considered as being normal is by taking moments about the mean of the distribution. A normal distribution is completely characterized once the mean ($\bar{\mu} = \frac{1}{N}\sum_{i=1}^{N} x_i$) and σ^2(variance) are known. However for any distribution, it is possible to take moments defined by;

$$\mu_n = \frac{1}{N}\sum_{i=1}^{N} (x_i - \bar{\mu})^n \qquad 3.1$$

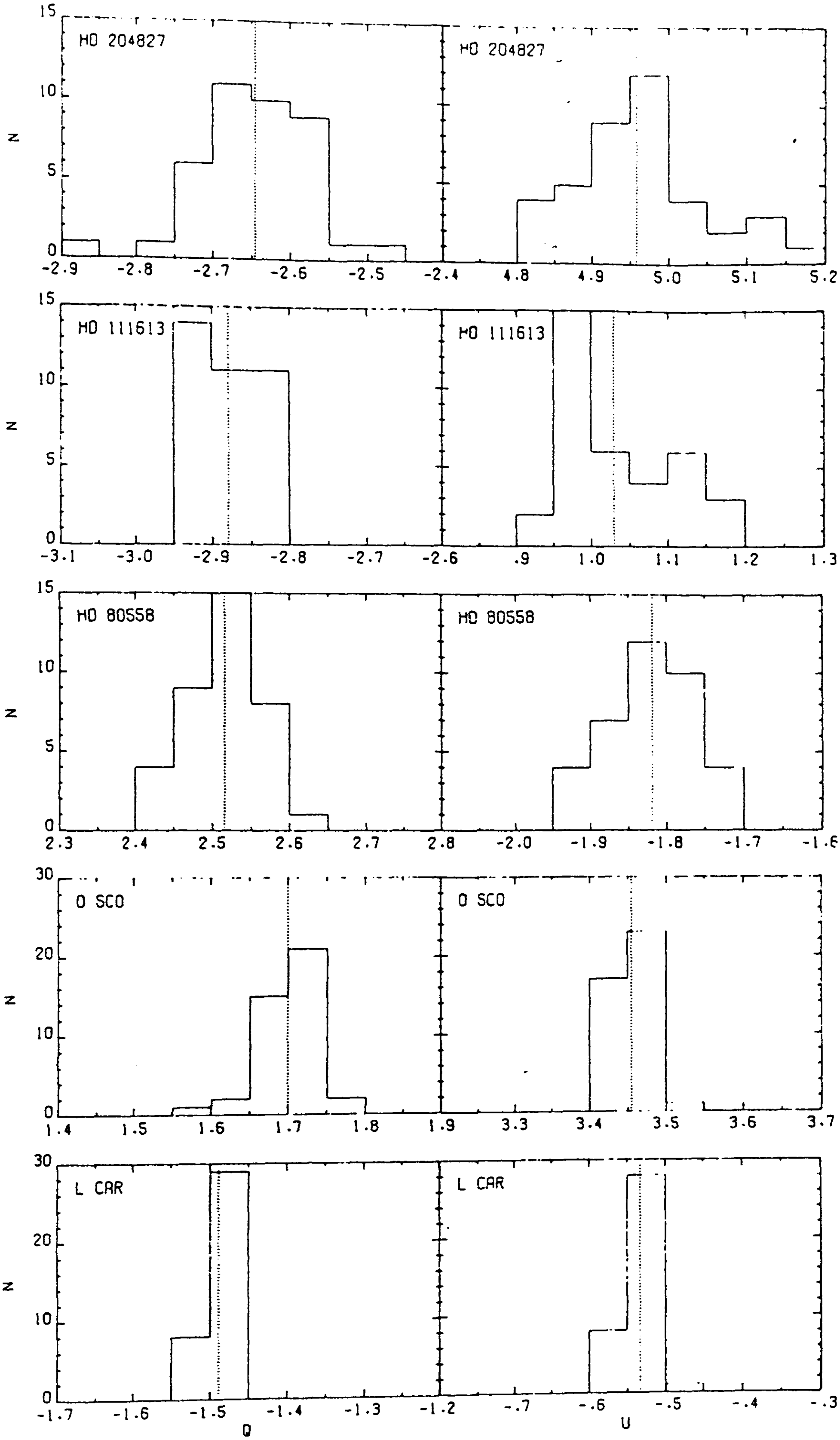


Fig. 3.1 Histogram of the Normalized Stokes Parameters for five stars; the dotted line is the unweighted mean values. (Taken from Bastien et al., 1988)

where μ_n is the n^{th} moment of the distribution. Note that $\mu_2 = \sigma^2$ provides a measure of the variance, and values of n = 3 and n = 4 allow investigation of skewness and kurtosis respectively (see Wall, 1979).

Skewness describes the symmetry of any distribution, and its assessment provides useful tests as, for example, a skewed distribution may result from a time drift in the measured values of a quantity. For the normal distribution, large sample determinations of x gives β (skewness), where:

$$\beta = \frac{\mu_3^2}{\mu_2^3} = 0 \qquad 3.2$$

If the distribution has $\beta > 0$ then the population density has a long tail to larger values and when $\beta < 0$, it has a long tail to smaller values.

Kurtosis describes the degree of peakiness, and for any normal distribution γ(kurtosis) is defined as:

$$\gamma = \frac{\mu_4}{\mu_2^2} = 3 \qquad 3.3$$

When $\gamma > 3$ the distribution is sharply peaked, and when $\gamma < 3$ the distribution curve is more flat-topped than a normal distribution.

For any series of repeated measurements, the determined values of skewness and kurtosis are unlikely to have the exact values of 0 and 3 respectively, because the data comprise small samples. For a given number of measurements making up a distribution, it is possible to assign bounds to the expected variations in the determined values of β and γ. Thus it is possible to put statistical confidence levels on the determined values of skewness and kurtosis.

Jones (1969) and Brooks (1984) have performed computer simulation studies on the normal distributions to obtain sample dependent confidence levels for these spreads, the sample size ranging from 3 up to 125 points. Given a sample size N, we can put a proper confidence level on the determined values obtained from Eqs. 3.2 and 3.3 . The hypothesis of normality is usually rejected at the 95% and greater confidence levels.

We have used Bastien et al.'s 1988 derived values of skewness and kurtosis (since we have used an alternative definition of γ, 3 is added to Bastien's kurtosis values) and tabulated them (see Table 3.1) together with the determined confidence levels on skewness and kurtosis, this latter exercise being neglected by Bastien et al.. It can be seen that only 4 (HD11831, HD14433, HD161056, HD204827) stars are rejected at the 99% level with the kurtosis test, and 3 (ϕ Cas, HD14433, HD161056) stars with the skewness test. The immediate conclusion of this is that Bastien has grossly exaggerated the apparent noisiness of the standard stars and only 5 (ϕ Cas, HD11831, HD14433, HD161056 and HD204827) can be rejected at the 99% confidence level. It is seen that 2 (107Psc, κ Cet) out of 3 unpolarized stars are also rejected at the 99% confidence level.

The analysis of data for 2HCam resulted in a failure at the 99% level in the B-band, but that for V and U the data passed the normality tests. The reason for this behaviour would be explained by the way data were collected. The observation of 2HCam at different wavelengths were not simultaneous. Most of the B-band observations were undertaken during 1986, but observations at V and U bands were obtained in 1987. This would indicate that 2HCam has a long term and erratic polarization variability.

Table 3.1

Statistical Confidence level on the values of skewness and kurtosis for Bastien et al.'s 1988 data

Name(N)	Skewness q	Skewness u	Confidence level 95% Lower	Confidence level 95% Upper	Confidence level 99% Lower	Confidence level 99% Upper	Kurtosis† q	Kurtosis† u	Confidence level 95% Lower	Confidence level 95% Upper	Confidence level 99% Lower	Confidence level 99% Upper
ϕ Cas(20)	-1.370	+0.849	-0.835	+0.929	-1.294	+1.281	5.334	4.753	1.747	4.628	1.587	5.808
HD11831(11)	+0.556	+0.178	-1.132	+1.136	-1.526	+1.538	1.338	2.701	1.513	4.516	1.380	5.464
HD14433(11)	+1.953	-1.856	-1.132	+1.136	-1.526	+1.538	7.742	8.715	1.513	4.516	1.380	5.464
HD23512(16)	+0.490	-0.766	-1.024	+1.018	-1.424	+1.375	3.807	3.184	1.661	4.649	1.514	5.859
HD25443(12)	-0.794	+0.626	-1.099	+1.114	-1.510	+1.528	4.767	4.439	1.546	4.546	1.406	5.633
HD80558(37)	-0.173	+0.068	-0.736	+0.735	-1.011	+1.012	3.010	2.075	1.965	4.473	1.817	5.509
L Car(37)	-0.540	-0.144	-0.736	+0.735	-1.011	+1.012	2.925	2.643	1.965	4.473	1.817	5.509
HD111613(36)	-0.016	+0.528	-0.738	+0.742	-1.027	+1.030	1.865	1.871	1.952	4.482	1.808	5.539
o Sco(41)	-0.538	-0.075	-0.699	+0.708	-0.875	+0.950	5.254	2.498	1.997	4.432	1.857	5.435
HD154445(19)	+0.690	-1.013	-0.959	+0.949	-1.336	+1.314	2.348	3.143	1.725	4.645	1.574	5.889
HD161056(8)	-1.319	-2.383	-1.203	+1.208	-1.601	+1.604	4.168	9.038	1.393	4.098	1.274	4.855
η Aql(13)	+0.551	-0.097	-1.096	+1.080	-1.480	+1.467	3.142	2.983	1.581	4.617	1.431	5.753
HD204827(40)	-0.748	+0.454	-0.707	+0.708	-0.990	+0.973	5.930	2.920	1.986	4.432	1.835	5.488
2HCam(43)^{B}	-2.797	-0.030	-0.692	+0.692	-0.931	+0.949	9.986	2.358	2.014	4.436	1.863	5.366
2HCam(31)^{V}	+0.407	-0.269	-0.798	+0.804	-1.093	+1.084	2.488	2.460	1.902	4.581	1.748	5.598
2HCam(6)^{U}	+0.178	+0.580	-1.239	+1.236	-1.521	+1.510	1.926	1.763	1.291	3.517	1.143	3.864
					Un-Polarized							
107Psc(6)	+2.009	+0.172	-1.239	+1.236	-1.521	+1.510	7.385	1.682	1.291	3.517	1.143	3.864
$\varkappa_1$ Cet(7)	-1.266	+1.878	-1.232	+1.229	-1.567	+1.556	3.487	6.970	1.328	3.867	1.215	4.385
χ^1 Ori(10)	+1.152	+0.540	-1.165	+1.155	-1.555	+1.568	4.112	1.168	1.479	4.412	1.343	5.365

N Corresponds to the number of measured values.

† We have used an alternative definition of kurtosis, where we have Added 3 to the Bastien's kurtosis values.

Data on 2HCam^{BVU} obtained by McDavid (1990) at B filter (4400A passband of 1000A), V filter(5500A passband of 900A) and U filter (3650 passband of 700A).

3.2.2 Modified Kolmogorov-Smirnov Test (Non-parametric Test)

This method is simpler to use than the skewness and kurtosis test (as described above). The principle of the Kolmogorov approach involves comparing the cumulative frequency curve of the data to be tested with the cumulative frequency curve of the hypothesized distribution. When the hypothetical and experimental curves have been drawn, the test statistic is obtained by finding the maximum vertical distance between them, and comparing this with a set of tabulated values (see Conover, 1980, Table A15). If the observational data depart substantially from the expected distribution, the two curves will be expected to be widely separated over parts of the cumulative frequency diagram. If, however, the data are closely in accord with the expected distribution, the two curves will never be very far apart.

Kolmogorov-Smirnov (hereafter K-S) type of statistics can be modified to test the hypothesis of normality. That is, the null hypothesis states that the population is one of the family of normal distributions without specifying the the true mean and variance. This test was first presented by Lilliefors (1967).

In this method, we consider that the Normalized Stokes Parameters consist of random sample of size N which is associated with a Gaussian distribution. The sample mean and variance is calculated from Equation 3.1. The original data are transformed into the standard normal variable Z, given by;

$$Z_i = \frac{x_i - \mu}{\sigma} \qquad 3.4.$$

The test statistic is computed from the Z_i instead of from the original random sample. These Z_i values are arranged in ascending order and plotted as a cumulative distribution function with step height of $\frac{1}{N}$.

A graph of standard normal distribution function $F^*(x)$ and the empirical distribution function of normalized sample S(x), (Z_i is defined by Equation 3.4) is drawn. The maximum vertical distance between two curves is evaluated.

$$T_1 = \sup_x |F^*(x) - S(x)| \qquad 3.5$$

The greatest (denoted by "sup" for supremum) vertical distance T_1 is compared with the tables provided by Lilliefors (1967) and the corrected version in Conover (1980)- see Table A15. The Lilliefors test calls for rejection of hypothesis at 99% confidence level. The graphs of the tests of all of the Bastien et al.'s and McDavid's data are depicted in Figs. 3.2a to 3.2s.

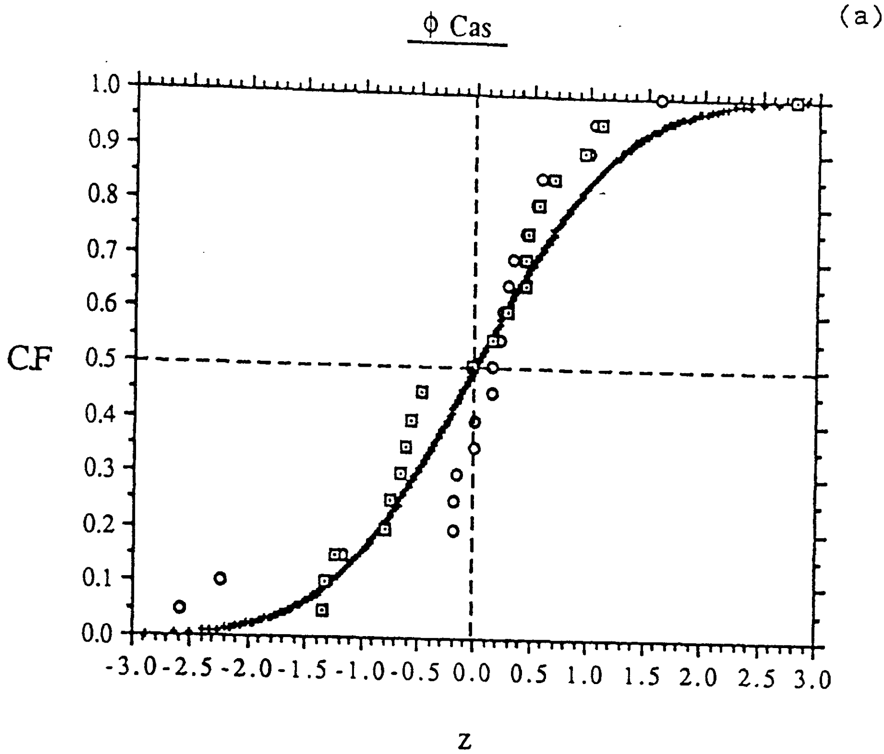
The numerical values for T_1 are shown in Table 3.2, for the NSPs, q and u. It is evident from Table 3.2 that 6 stars (ϕ Cas, HD14433, HD111613, HD154445, HD161056 and η Aql) are rejected at the 95% level. But only 3 stars (ϕ Cas, HD111613 and HD161056) are rejected at the 99% level.

It is seen from Table 3.2 that data of 2HCam for the B-band were again rejected at the 99% level. From Table 3.1 the value of Kurtosis obtained for q in B-band was 9.986 which is rejected at the 99% level. In K-S test the value of q(T_1) obtained was 0.310 which is also rejected at 99% level. For this star, there seems to be good agreement between two methods for the rejection of the data.

Table 3.2
K-S Test of Normality on Bastien's data

Name(N)	T_1 q	T_1 u	Confidence level† 95%	Confidence level† 99%
ϕCas(20)	0.273	0.133	0.190	0.231
HD11831(11)	0.128	0.228	0.249	0.284
HD14433(11)	0.187	0.271	0.249	0.284
HD23512(16)	0.138	0.098	0.213	0.250
HD25443(12)	0.164	0.177	0.242	0.275
HD80558(37)	0.106	0.107	0.145	0.169
LCar(37)	0.119	0.118	0.145	0.169
HD111613(36)	0.118	0.189	0.148	0.172
oSco(41)	0.114	0.089	0.138	0.161
HD154445(19)	0.199	0.279	0.195	0.235
HD161056(8)	0.224	0.369	0.285	0.331
ηAql(13)	0.171	0.239	0.234	0.268
HD204827(40)	0.101	0.117	0.140	0.163
2HCam(43)^B	0.310	0.054	0.135	0.157
2HCam(31)^V	0.040	0.088	0.159	0.185
2HCam(6)^U	0.240	0.238	0.319	0.364
107Psc(6)	0.311	0.195	0.319	0.364
κCet(7)	0.267	0.316	0.300	0.348
χ^1Ori(10)	0.239	0.178	0.258	0.294

† Rejection of null hypothesis if T_1 exceeds 99% level for particular sample size (N).



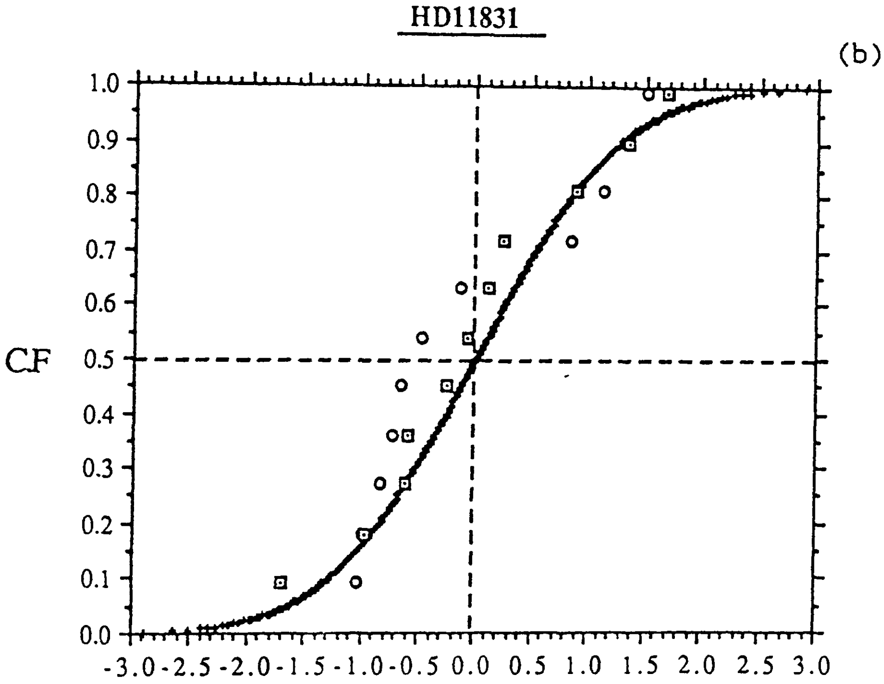
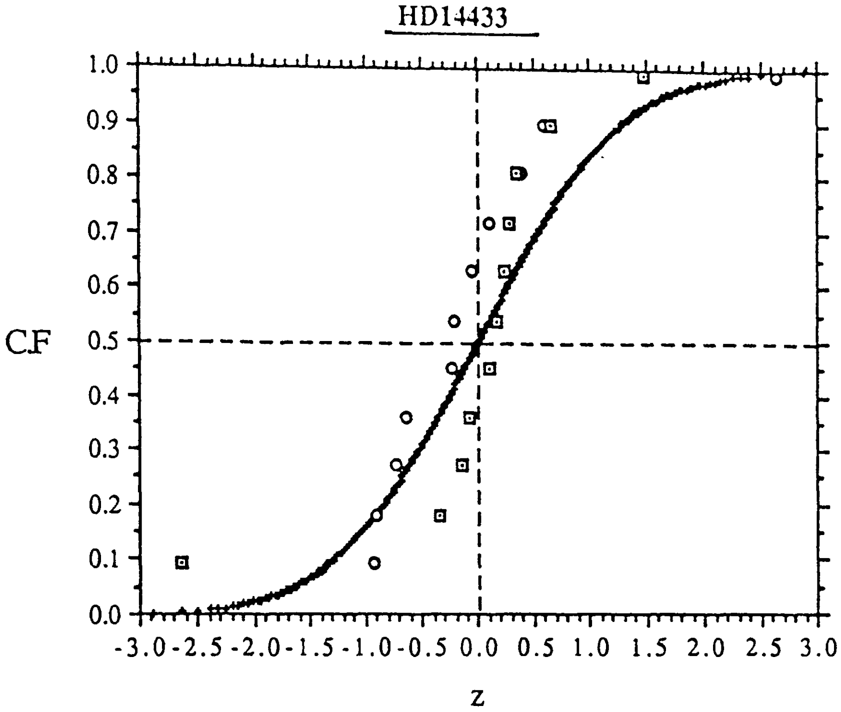


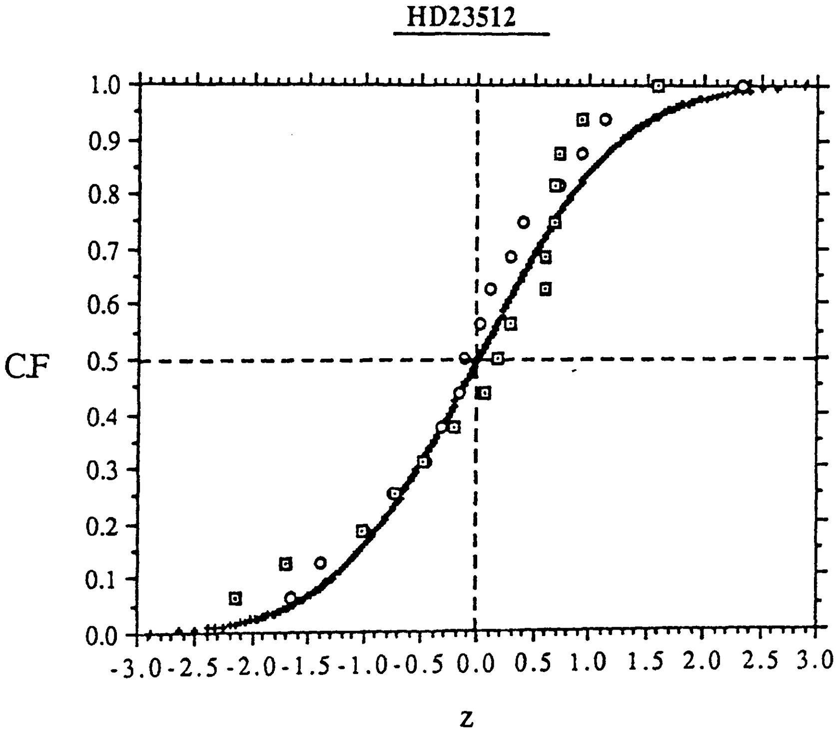
Fig. 3.2

K-S test is used to test for normality. Maximum differences between the theoretical cumulative frequency (•) and the two tested distributions; where (o) is q and (⊡) is u. All the data are Bastien et al.'s (1988) except 2HCam which was by McDavid (1990).

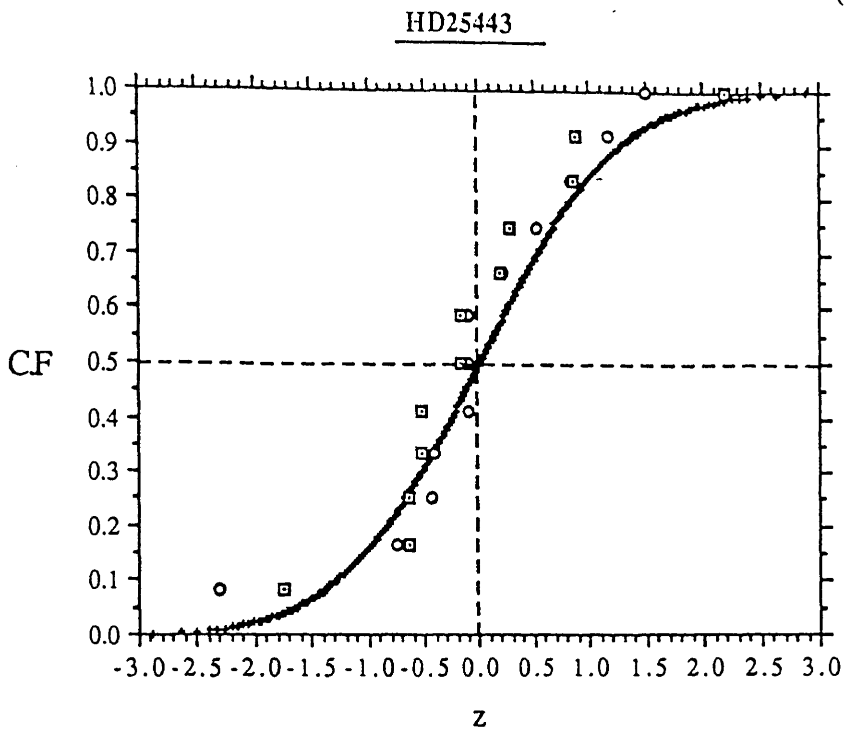
(c)


HD14433
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

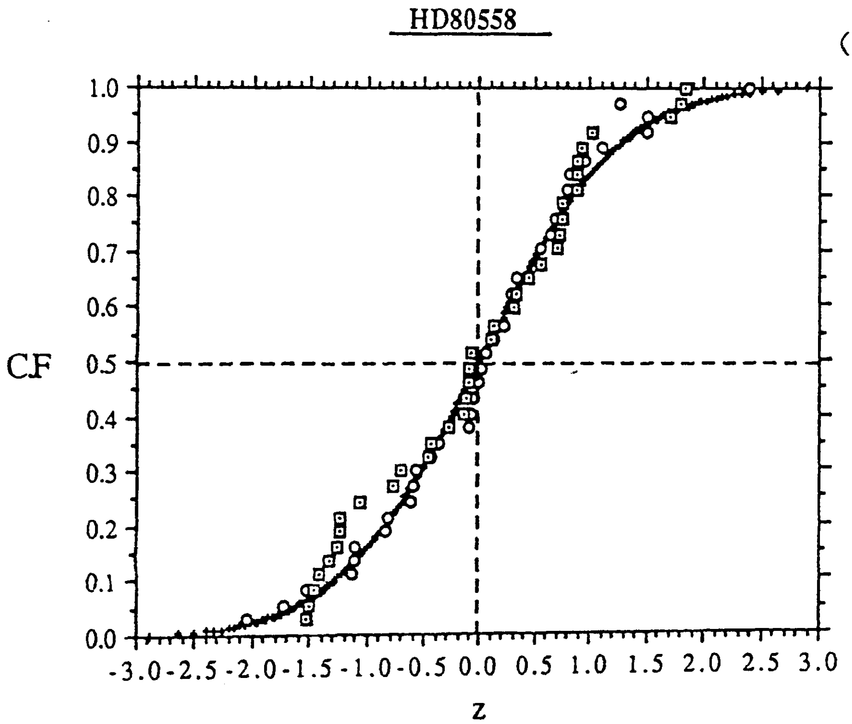
(d)


HD23512
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

(e)

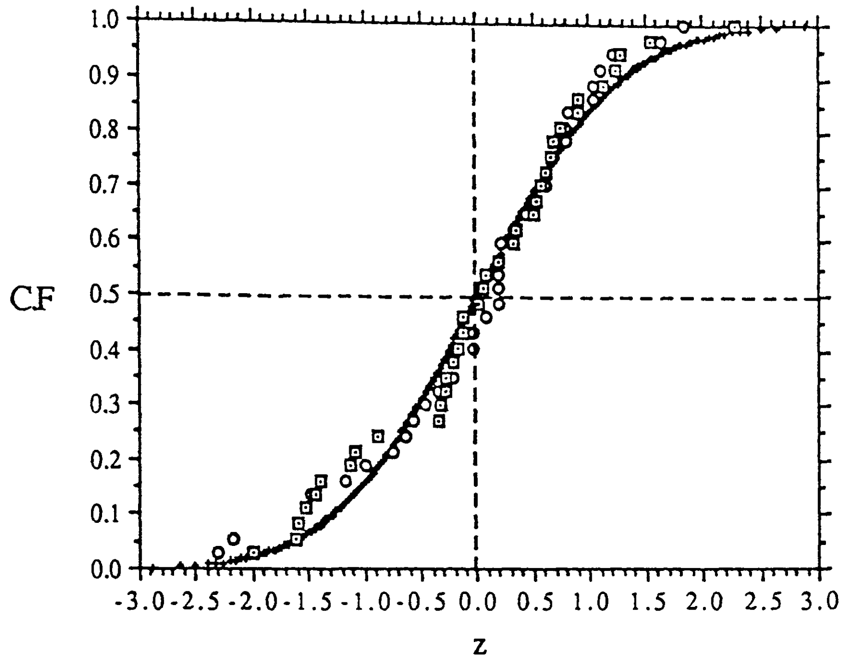

HD25443
CF
z
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0

(f)


HD80558
CF
z
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0

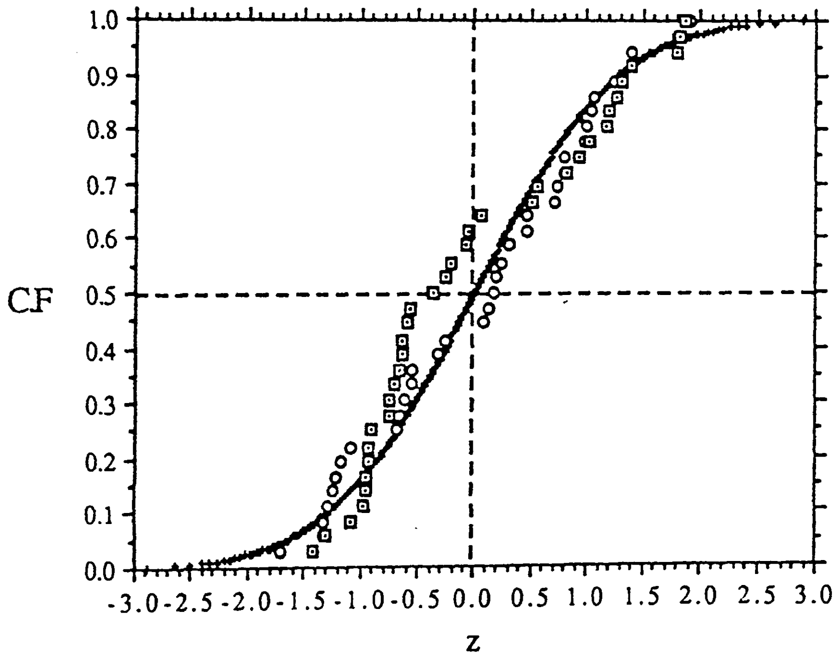
(g)

ℓ Car

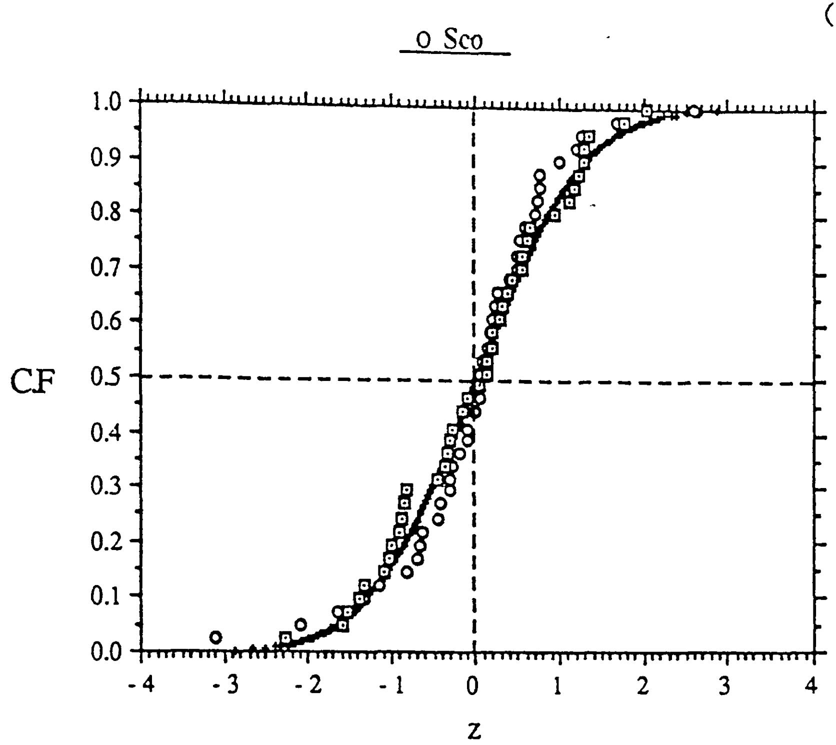


HD111613

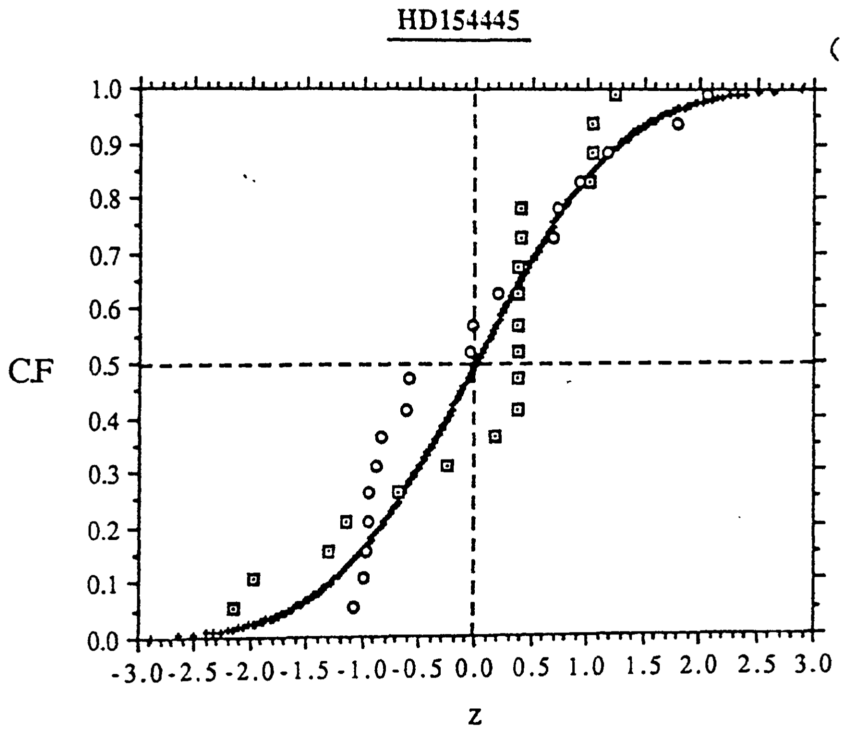
(h)



(i)

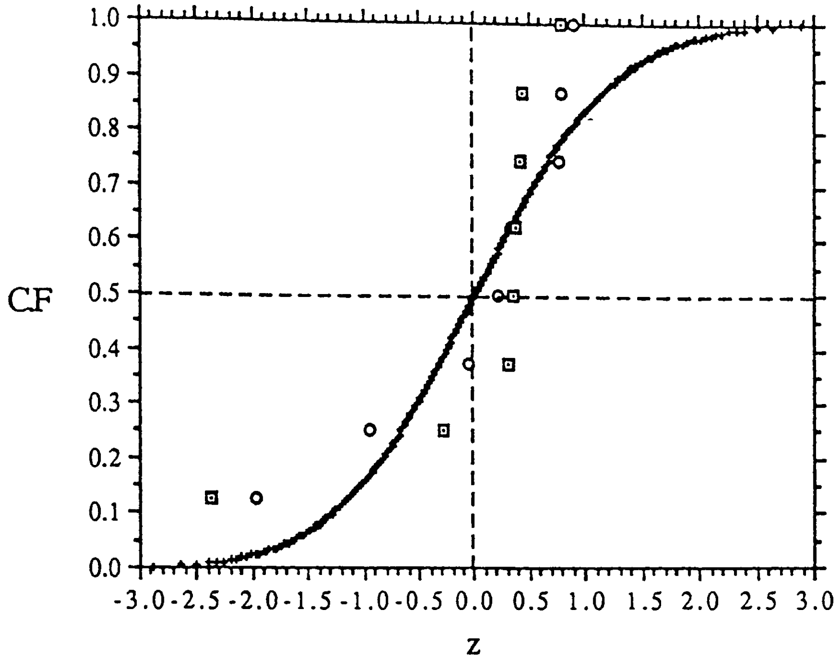

o Sco
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
CF
-4
-3
-2
-1
0
1
2
3
4
z

(j)


HD154445
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
CF
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

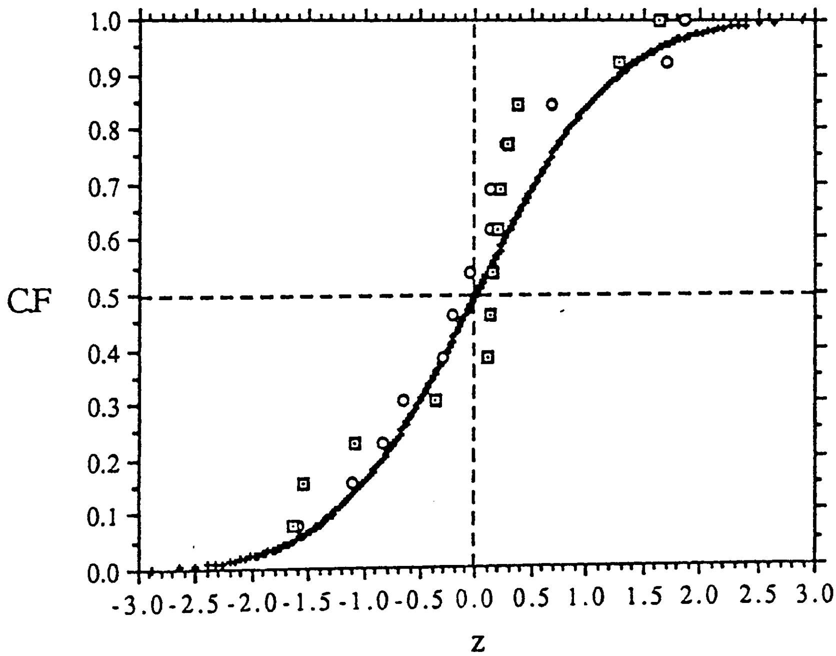
(k)

HD161056

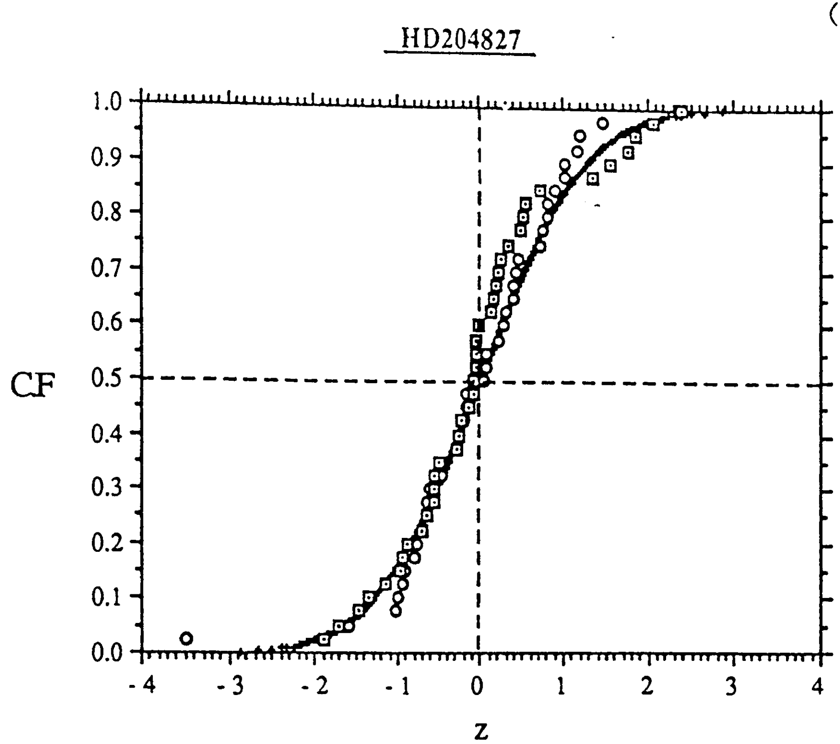


η Aql

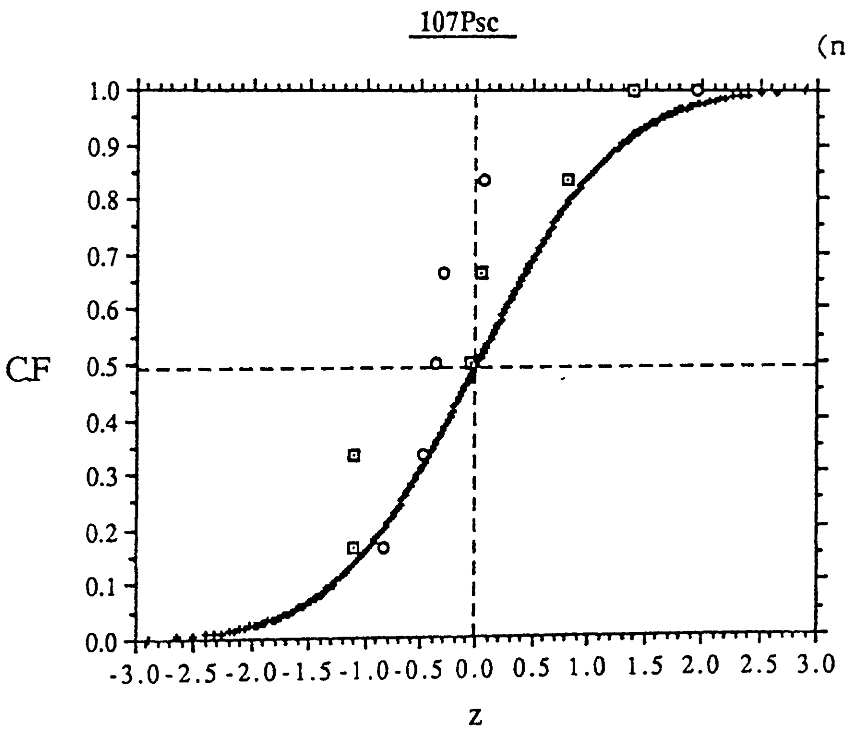
(l)



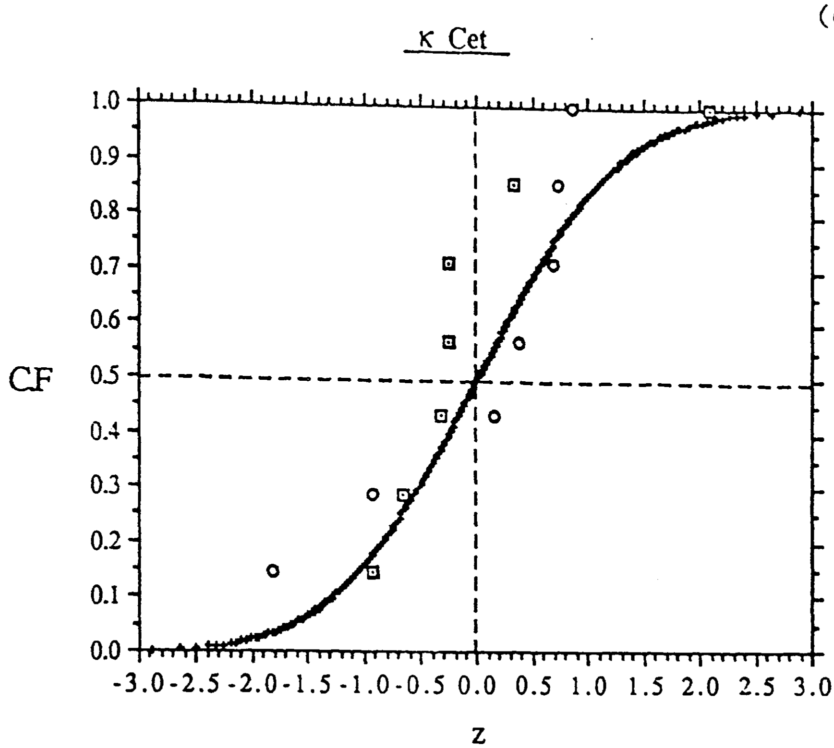
(m)


HD204827
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-4
-3
-2
-1
0
1
2
3
4
z

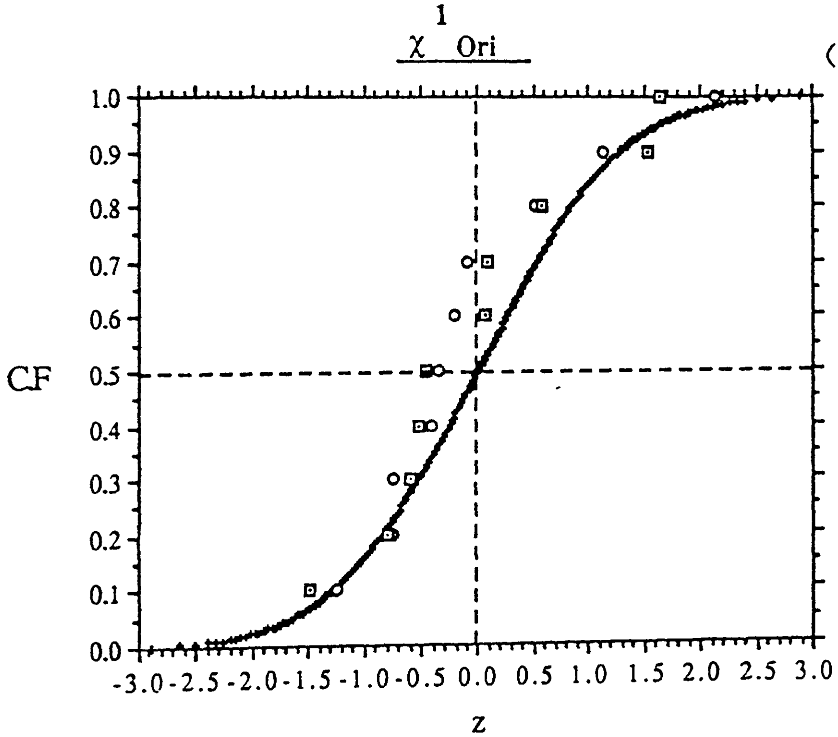
(n)


107Psc
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

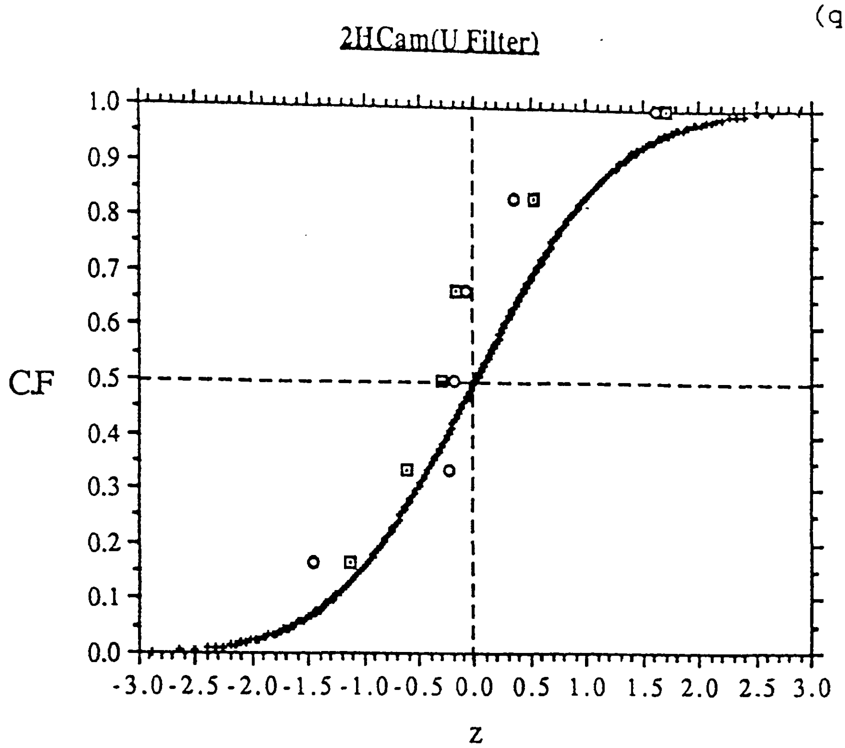
(o)


κ Cet
1.0
0.9
0.8
0.7
0.6
CF
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

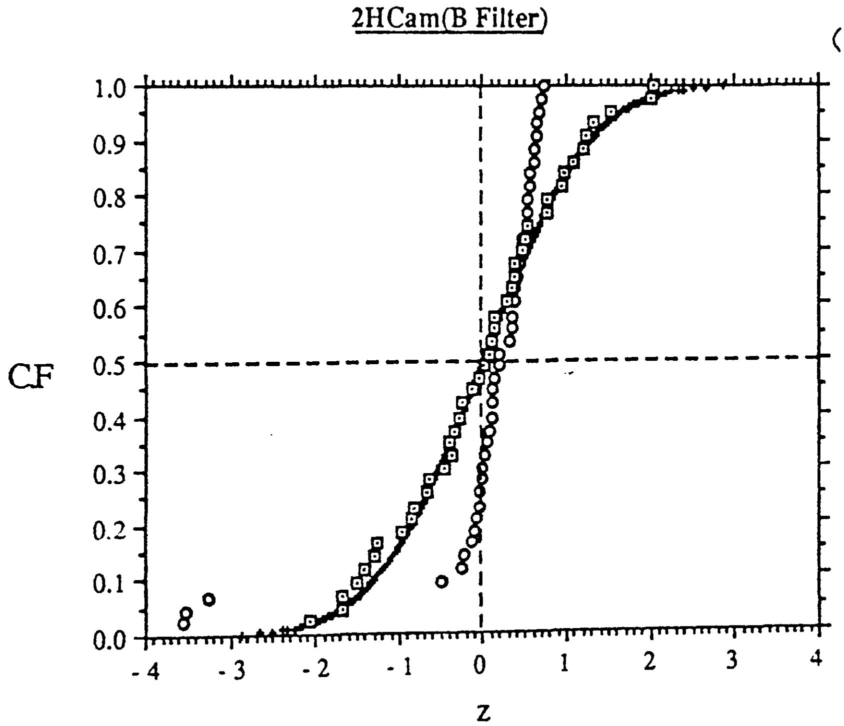
(p)


χ[1] Ori
1.0
0.9
0.8
0.7
0.6
CF
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
z

(q)

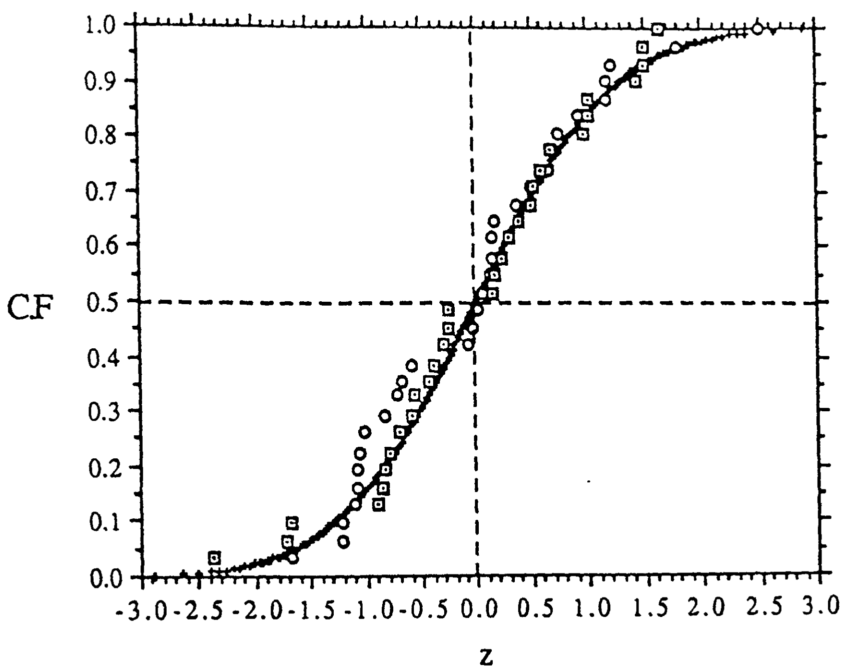

2HCam(U Filter)
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-3.0 -2.5 -2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5 2.0 2.5 3.0
Z

(r)


2HCam(B Filter)
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-4 -3 -2 -1 0 1 2 3 4
Z

(s)

2HCam(V Filter)



3.3 Application of Statistical Tests to the "Zero" Polarization Stars

In this section we reassess the data presented in various surveys (Tinbergen, 1979, Leroy and Le Borgne, 1989 and Huovelin et al., 1985) and offer some suggestions as to what effects might influence the behaviour of the distribution of p. As in Section 1.2.3 we discuss ways in which we might correlate polarization with spectral type. Tinbergen and Zwaan (1981) and Tinbergen (1982) have suggested that some stars with spectral type F0 and later show intrinsic polarizations. In these studies a $2\sigma_p$(95%) criterion for rejecting stars as being unpolarized was applied (see Fig. 1.3). In Chapter 2 we discussed that the distribution of p is not Gaussian at low levels of signal-to-noise ratio. Stewart (1984) has shown that a 95% confidence level on polarization being present is given by 2.45σ rather than 2σ. As a consequence Tinbergen's study underestimates the confidence level at 95% value. It is therefore possible that a greater percentage of the stars in Tinbergen's observations may actually arise from an unpolarized distribution than he suggests.

The probability distribution of p can be re-written as;

$$F(p, p_o) = p\, e^{\frac{-(p^2+p_o^2)}{2}}\; I_o(ipp_o) \qquad 3.6$$

where I_o is zero order Bessel function, p is the ratio of observed degree of polarization to the noise ($\frac{p}{\sigma}$) and p_o is the ratio of true polarization to the noise ($\frac{p_o}{\sigma}$). If we assume that the source is unpolarized, then $p_o = 0$, and the distribution function for p becomes;

$$F(p,\ 0) = p\ e^{\frac{-p^2}{2}} \qquad 3.7.$$

The above equation is a Rayleigh distribution (see Vinokur, 1965, Wardle and Kronberg, 1974), where the distribution has a mean of $\left(\frac{\pi}{2}\right)^{1/2}$ and variance of $\left(2 - \frac{\pi}{2}\right)$.

If the observations are performed on different unpolarized stars, the distribution function of p will follow the Rayleigh distribution. There are various statistical ways of testing the hypotheses that any sample does indeed come from a given population. We shall consider here two such statistics, namely the χ^2 test, used for large samples of data, and the K-S test which can be used on small data sets.

a) χ^2 Testing

When the sample of stars is larger than 50, χ^2 testing can be applied, whereby the observed polarization is binned to form a histogram and compared with the histogram of the theoretical distribution (Rayleigh Distribution). Leroy and Le Borgne (1989) have applied this procedure to their data (see Section 1.2.3 Fig. 1.5).

b) Kolmogorov-Smirnov Test

When the sample of stars is small, which is usually the case, a more appropriate test would be a Kolmogorov-Smirnov type as described in Section 3.2.2, with the theoretical cumulative distribution function being C.F. = $1 - e^{\frac{-p^2}{2}}$. This test has the great advantage of being exact even for small samples.

The outlined techniques have particular importance for the determination of an unpolarized star. This can also be extended to a group of supposedly zero polarized standard stars to decide whether their overall behaviour is consistent with zero polarization. We will solely use the K-S test, for investigating the influence (if any) of spectral type within the distribution of presupposed unpolarized stars.

We have taken Tinbergen's 1979 data and binned each star to its particular spectral type (taken from Hoffliet, 1964). Tinbergen (1979) included some data taken from another survey by Piirola (1977) and include these data in our reassessment. We also investigate the data on solar type stars obtained by Leroy and Le Borgne (1989) and Huovelin et al. (1985). We reassess their data in order to see if these stars could have come from a distribution of zero polarization stars.

The value of error σ was calculated for each star from the relation provided by Tinbergen, $\sigma = \frac{10^{-4}}{\sqrt{w}}$ where w is the weight given for each observation. The value of w is the same for q and u, and therefore $\sigma_p = \sigma_u = \sigma_q$. The value of $\frac{p}{\sigma_p}$ was then calculated for each star of particular spectral type. Leroy and Le Borgne (1989) and Huovelin et al. (1985) provide σ_p for each observation. However, we do not know how the noise, σ_p, was calculated nor if any debiasing has been applied to the data and are therefore forced to take their values of σ_p at face value.

The calculated $\frac{p}{\sigma_p}$ values, for the stars of Tinbergen's data were ranked in ascending order and plotted against the theoretical distribution for each spectral type and are depicted in Figs. 3.3a to 3.3d. From test statistics, A type stars pass at the 99%

Table 3.3
K-S Test performed on different Spectral Types

Spectral Type(N)	T_1	Confidence Level† 95%	99%
A (38)*	0.159	0.215	0.258
F (55)*	0.351	0.179	0.215
G (40)*	0.287	0.210	0.252
K (41)*	0.321	0.207	0.248
F & G Type‡ class V (13) At U-Filter	0.563	0.361	0.432
B-Filter	0.489	0.361	0.432
V-Filter	0.629	0.361	0.432
R-Filter	0.319	0.361	0.432
I-Filter	0.451	0.361	0.432
K & G Type§ class V (54) At U-Filter G (44)	0.136	0.200	0.240
K (10)	0.125	0.409	0.489

† Taken from Conover (1980), Table A14.

N Number of Stars in each Spectral Type.

* Calculated from Tinbergen's 1979 data.

‡ Calculated from Huovelin et al.'s 1985 Data.

§ Calculated from Leroy and Le Borgne 1989 Data.

confidence level indicating that the population comes from zero polarization stars (see Table 3.3) whereas F, G and K type stars fail the test at the 99% level.

There is one interesting feature of the shape of the curves in Figs. 3.3a to 3.3d. It is noticeable that there is a dip in the observed distribution. This dip occurs in all four spectral type stars. The dip seems to be an artifact of the data rather than any astrophysical phenomena. The above statistical test could easily be reapplied if necessary corrections are made to the data, or probability distribution of p is modified.

The K-S test was also applied to the Huovelin et al. (1985) data at different wavelengths (UBVRI). All resulted in failure at the 99% confidence level except at R-Band (see Figs. 3.4a to 3.4e). The data of Leroy and Le Borgne (1989) passed at the 99% confidence level (see Fig. 3.5a and b). However there is inconsistency in these findings, since both of the authors have observed solar type stars. This has been debated in the literature (see Huovelin and Piirola, 1990) by both camps and it seems appropriate that more observations are required in order to resolve the above inconsistency.

It is however impossible to argue from these statistics that the failure is due to intrinsic polarizations, until one has excluded possible biasing. We also have to consider what effects can influence the shape of the theoretical and observational cumulative distribution of polarization. These effects are summarized below:

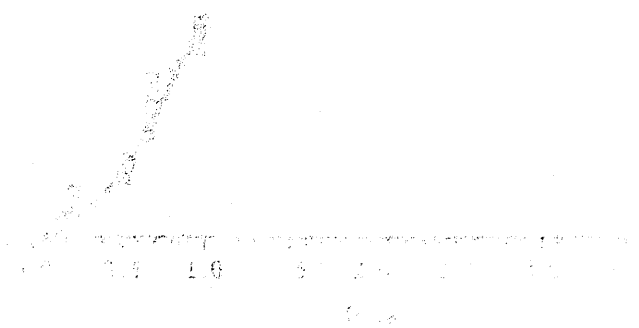
i) If the values of σ_p are underestimated, the resulting conclusions must be modified. The estimated value of σ_p on

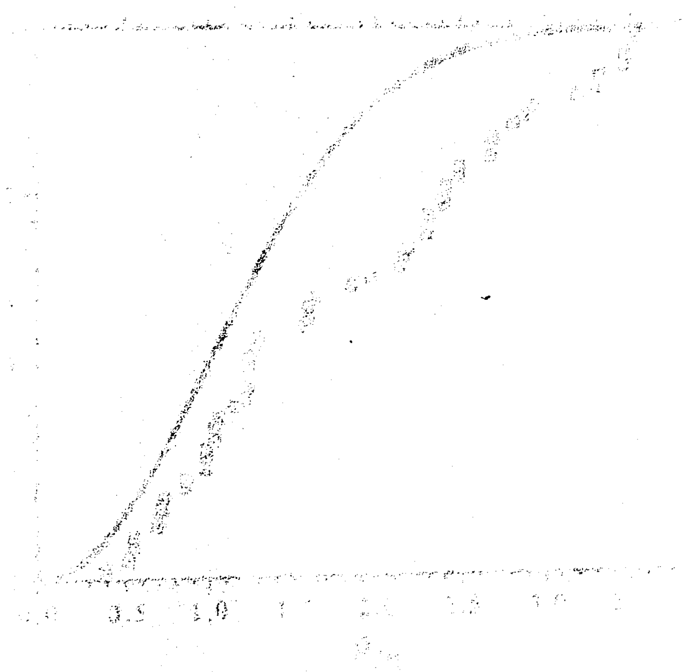
Tinbergen's data was obtained by a weight system as described above. Therefore we do not know exact values of σ on the NSPs. One could test this by increasing the value of σ_p, it is apparent that the observed distribution curve will shift towards smaller values. Tinbergen (1979) also suggested that there might be a systematic errors of the order of 0.003% on the NSPs.

ii) Since we are observing such low levels of polarization in all these catalogues, we do not know what systematic effects can affect the observed polarization (i.e. instrumental polarization). It would be appropriate to allow p_o (true polarization) to have a small value in Equation 3.7 (Rician distribution) and see how this effects the theoretical curve. Thus comparing the observed distribution to the new theoretical Rician distribution.

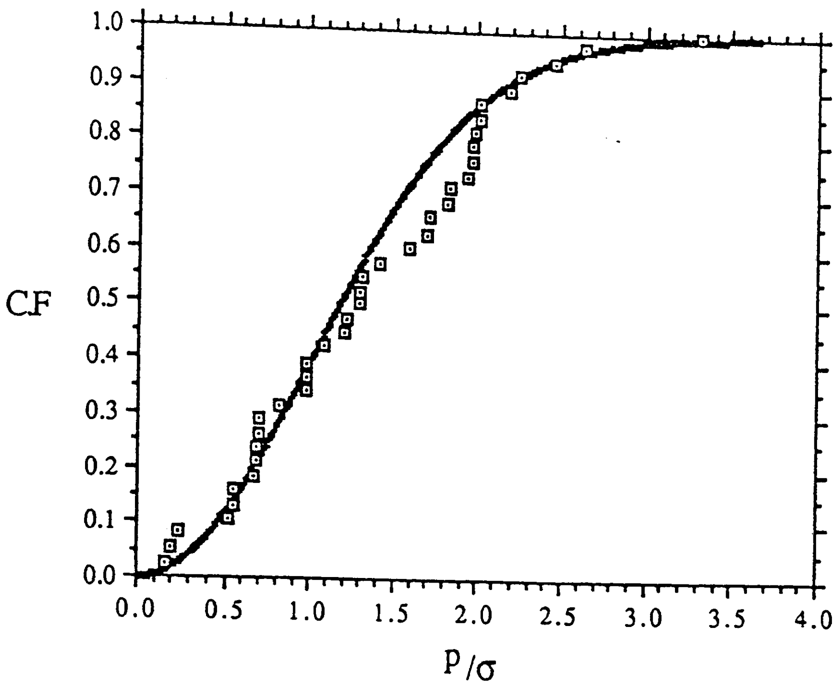
iii) If we assume that any observed polarization is intrinsic to these stars then, the value for each star will depend on the inclination of the axis of symmetry with respect to the line of sight. A net polarization may be caused by the scattering of starlight through a non-spherically symmetric circumstellar envelopes, thus we would expect that the degree of polarization will dependent on the inclination. It has been shown by Brown and Mclean (1977) and Simmons (1982) that most scattering mechanisms will produce a $\sin^2(i)$ dependency for the intrinsic polarization. If we assume that the angle of inclination is randomly distributed for these stars, then we would expect to have a $\sin^2(i)$ function superimposed on our observed distribution. We can investigate this further by obtaining a new theoretical probability distribution given by $p_o\sin^2(i)$, similar to Equation 3.6. However, it is very easy to show that the addition of this kind to the C.F. does not explain the kink (dip) displayed in Tinbergen's (1979) data.

It is therefore essential that in any future observations of this kind, the way in which the uncertainty of $\bar{p}$ is calculated should be stated; i.e. if any biasing corrections have been made to the data and which debiasing technique is performed on the data (see Simmons and Stewart, 1985).





A Type Stars (a)



F Type Stars (b)

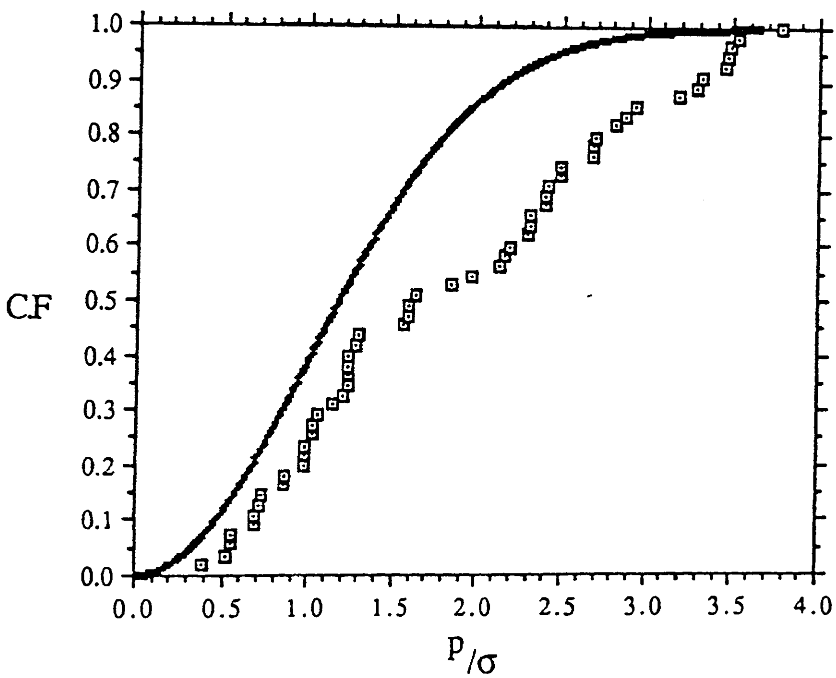
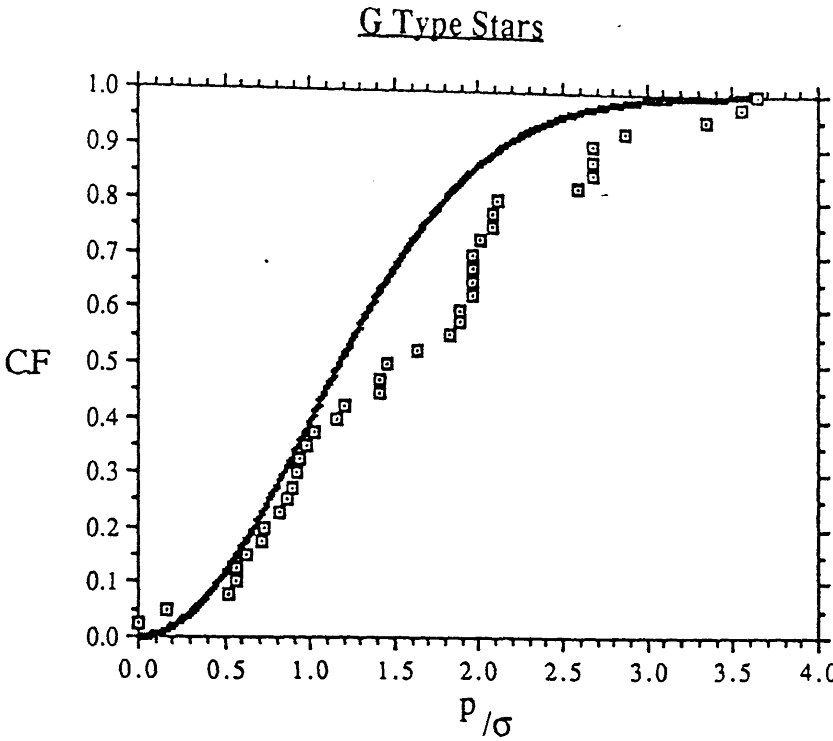


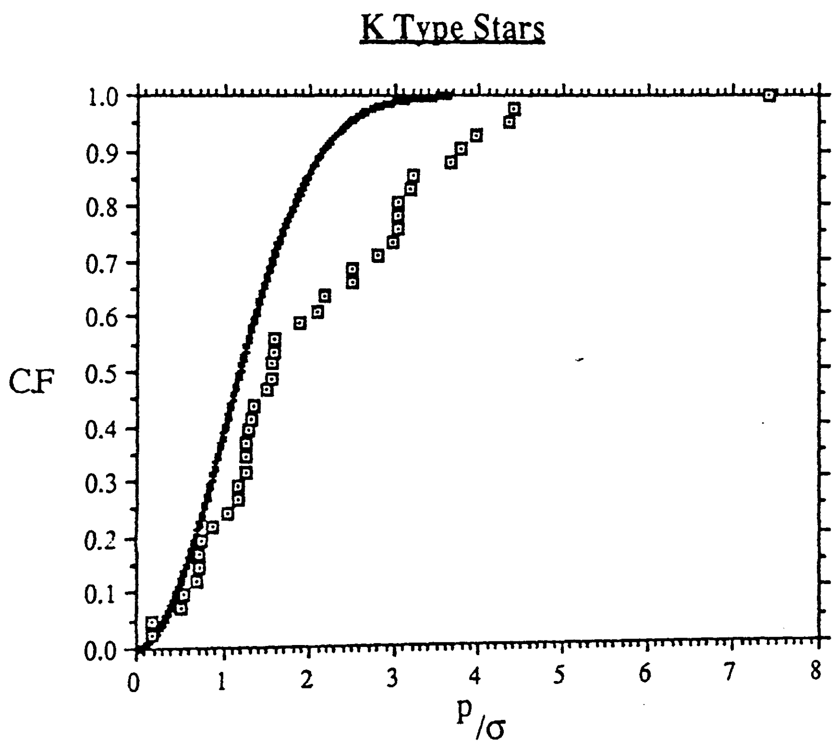
Fig. 3.3

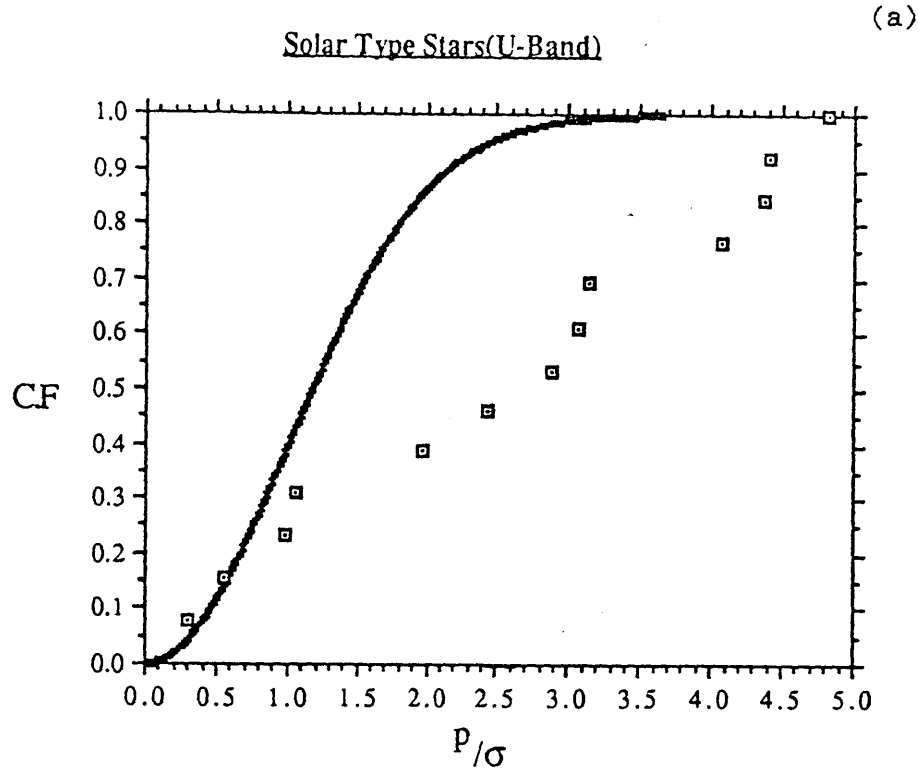
K-S test is employed to test for the deviations between theoretical (Rayleigh distribution) and empirical distribution obtained from observations, where (▫) is p/σ ; (a) A type, (b) F type, (c) G type and (d) K type stars.

(c)


G Type Stars
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
p/σ

(d)


K Type Stars
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0 1 2 3 4 5 6 7 8
p/σ



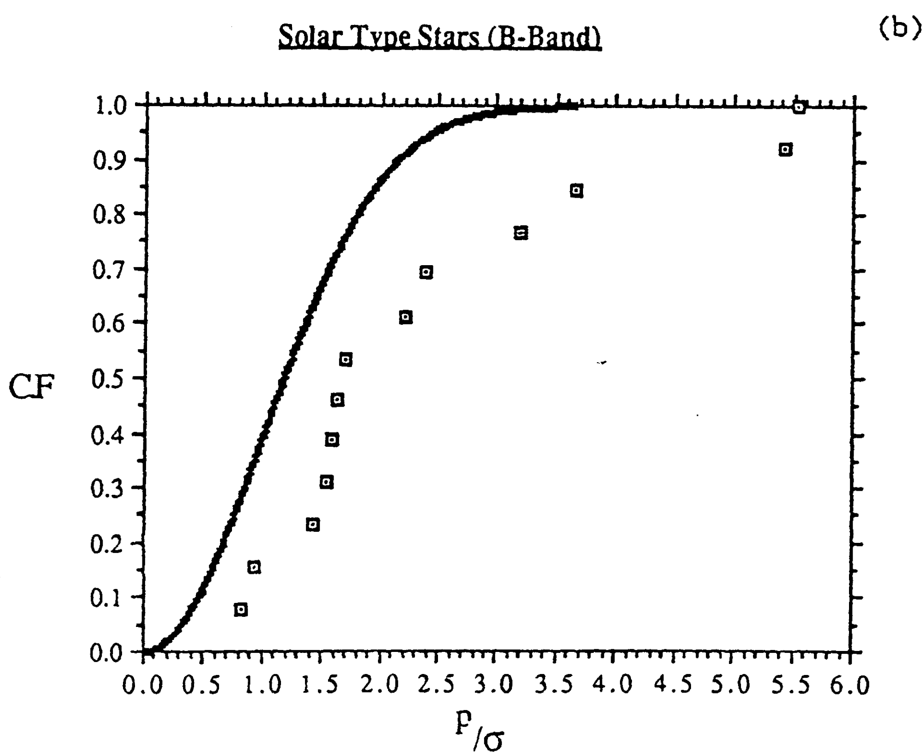
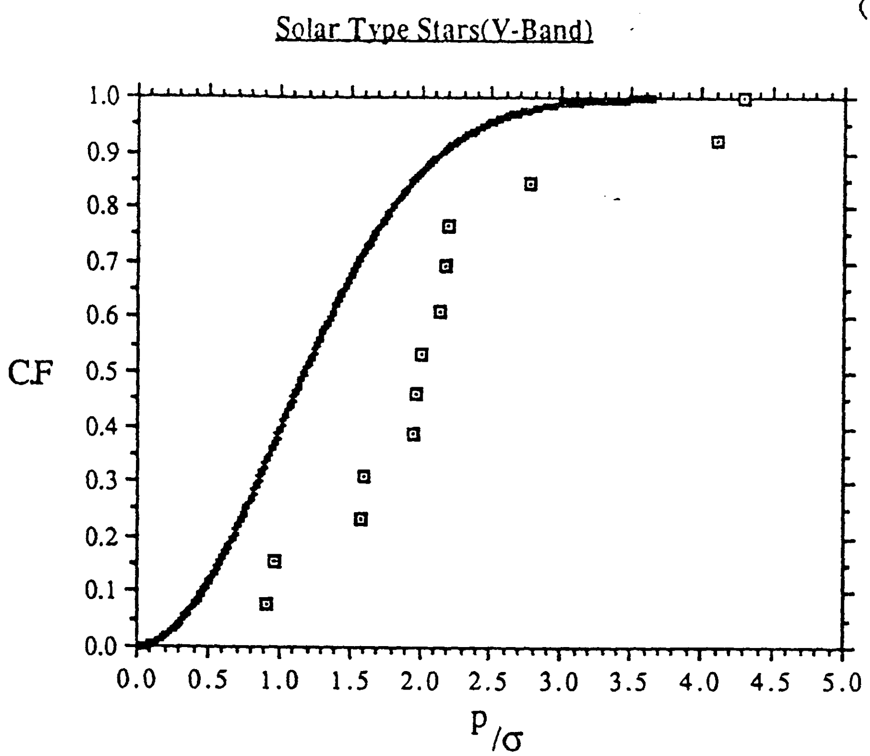
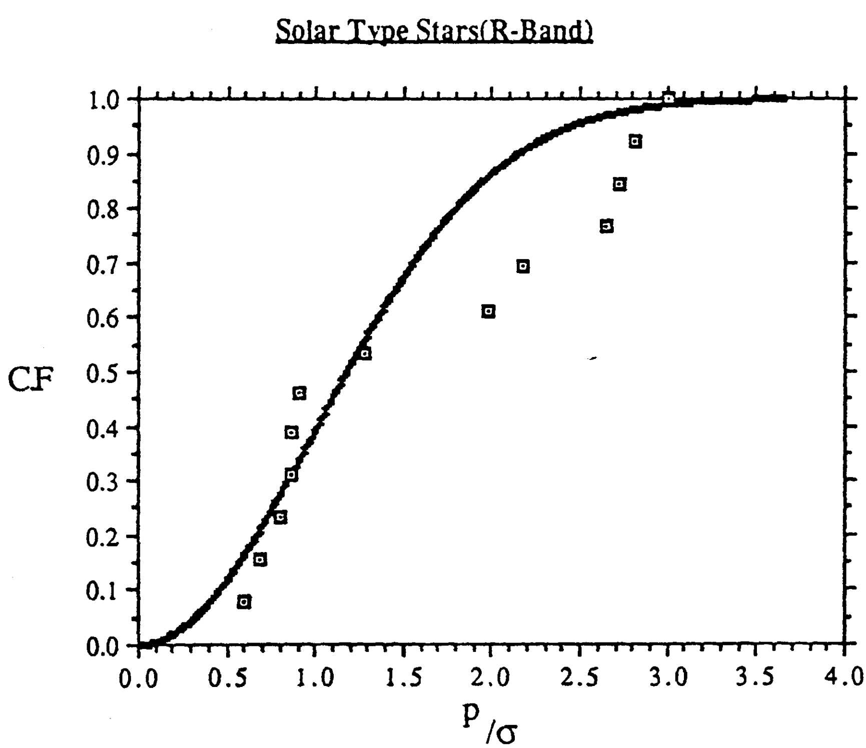


Fig. 3.4
K-S method is employed to test for zero polarization among solar type stars. Data taken from Huovelin et al. (1985); (a) U-filter, (b) B-filter, (c) V-filter, (d) R-filter and (e) I-filter.

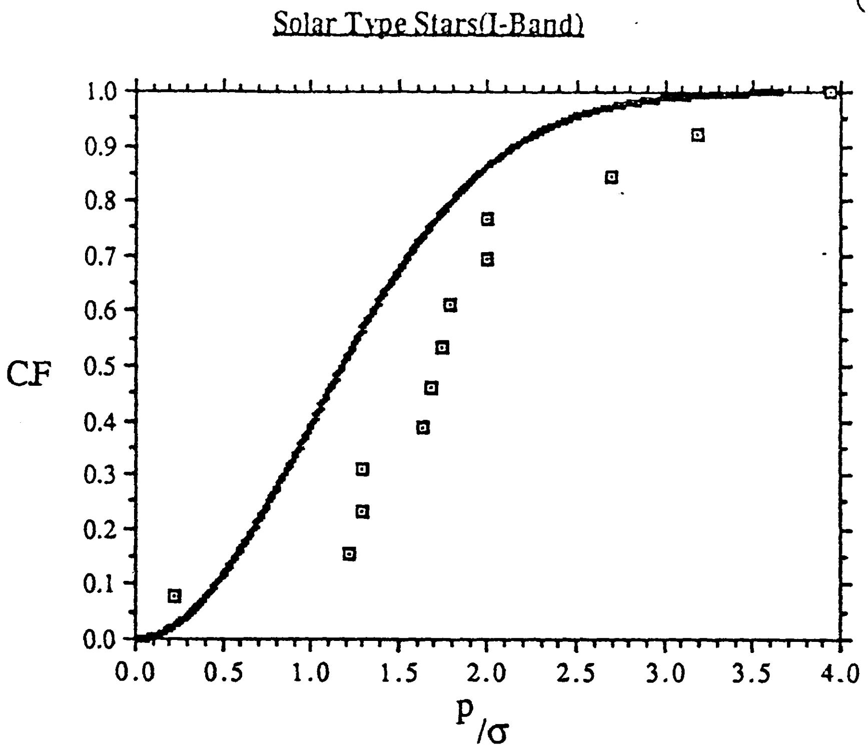
(c)


Solar Type Stars(V-Band)
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0
P/σ

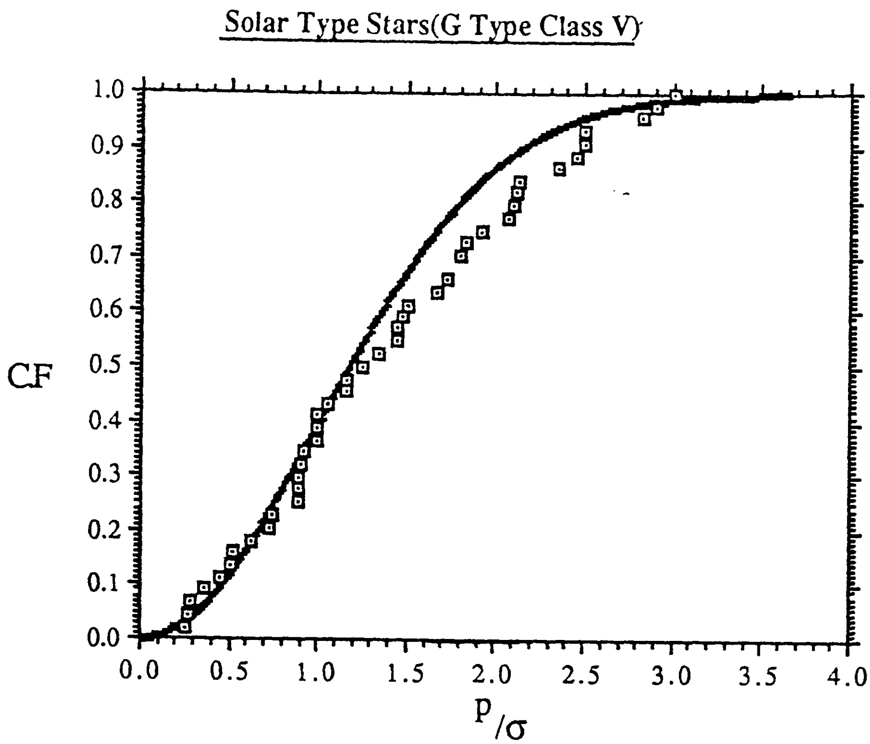
(d)


Solar Type Stars(R-Band)
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
P/σ

(e)


Solar Type Stars(I-Band)
CF
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
p/σ

(a)



(b)

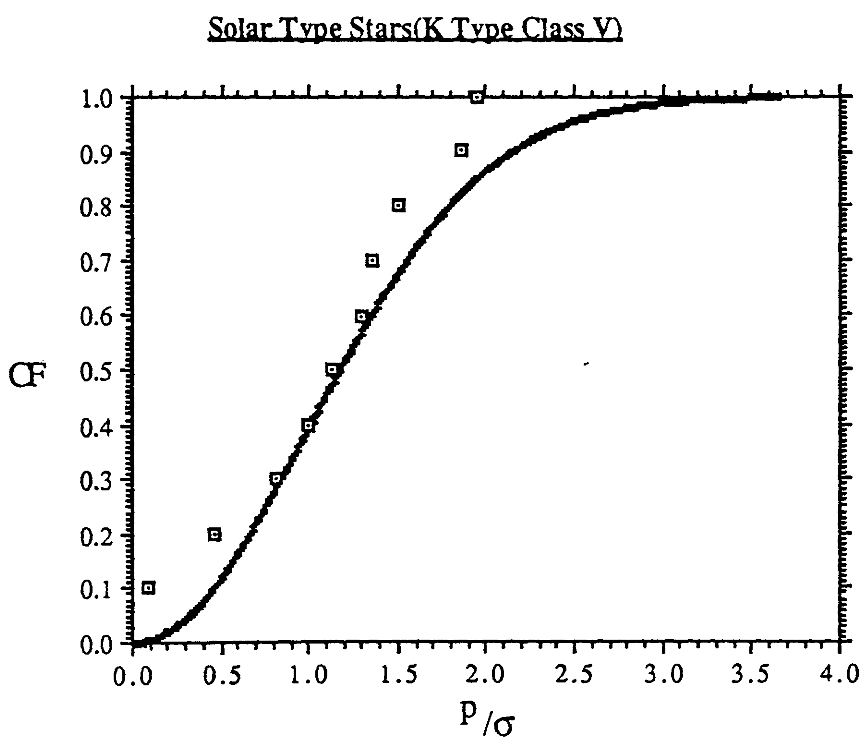


Fig. 3.5
K-S test applied to the solar type data obtained by Leroy and Le Borgne (1989); (a) G type class V stars and (b) K type class V.

3.4 Conclusion

The data on the 13 stars in the polarization standard catalogue presented by Bastien et al. (1988) and 1 by McDavid (1990) have been reassessed using proper rigid statistical tests. Using Bastien et al.'s own statistical analysis, we have shown that there is no strong evidence for most of the stars in the catalogue as having a variable polarization. By putting confidence levels on Bastien's skewness and kurtosis values only 4 (ϕ Cas, HD14433, HD161056, HD204827) stars were rejected at 99% confidence level. In our K-S test statistics, only 3 (ϕ Cas, HD161056 and HD111613) stars are rejected at 99% confidence level. It is suggested that long term observations of all these stars in the catalogue should be undertaken. It is also of fundamental importance that any evaluation of the observational data should have the correct statistical interpretation. It was seen that without confidence levels on the values of skewness and kurtosis, subjective assessment gives a false detection of variability in polarization.

We also looked at the data in Tinbergen's (1979) catalogue containing 181 "zero" polarization stars. These observations were reassessed with a more accurate statistical test than presented in Tinbergen's (1982) analysis. The only test presented by Tinbergen (1982) was that he put a $2\sigma_p$ (95% Gaussian) level on his data, however, this was incorrect as we pointed out. The K-S test was applied to the data, classifying the stars according to spectral type. It is concluded that a broad band polarization appears to exist within spectral types F, G and K. The data on A type stars however seem to come from a zero polarization population. This apparent trend gives rise to some interesting questions about the distribution of intrinsic polarization of stars of different spectral types. The above conclusion is in general agreement with Tinbergen (1982).

Our analysis of solar type data obtained from Leroy and Le Borgne (1989) and Huovelin et al. (1985) revealed that there are inconsistencies of their data; for example Leroy and Le Borgne's data passed the K-S test indicating that data come from a unpolarized population, but Huovelin et al.'s data failed the test. Further observations of solar type stars are required before any conclusions can be drawn from our statistical analysis.

However it is extremely important to emphasize that these conclusions depend on the exclusion of any systematic effects as described in Section 3.3. Our conclusions are not rigid as there are still uncertainties as to what role the imprecise values of σ_p are playing.

Chapter 4: Observational Studies of Standard Stars

4.1 Introduction

4.2 Instrumental Polarization

4.3 Observations, Data Reduction and Analysis of Polarized Standards

4.4 Conclusion

4. Observational Studies of Standard Stars

4.1 Introduction

There were several objectives in this program. The first was to search for new bright standards in Perseus and Cassiopeia and the second was to study some of the well known polarized standard stars. Due to adverse weather conditions in Glasgow few stars were in fact monitored and only small data sets, of the stars, were obtained. Nevertheless, although the quantity of material is considerably less than desirable for a survey, the polarimetric precision typically achieved was about ± 0.05% in the B-band and ± 0.1% in the red band. Table 4.1 lists the polarized and unpolarized stars under study.

All the observations presented here were obtained during November 1989 to August 1990 with the GUPP (Glasgow University photometer/polarimeter) instrument attached to the 20", f/8 telescope at the Cochno station outside Glasgow (Lat = 56° N, Long = 4 W). The observational work was carried out by myself with some support from Dr. Clarke.

The instrumentation of the GUPP has been thoroughly described by Clarke and Brooks (1984) and also by Schwarz and Clarke (1984), and is a improved version of the polarimeter described by Clarke and Mclean (1975). Polarimetric modulation was performed by a superachromatic quartz/magnesium fluoride half-wave plate, which was rotated continuously at 20 Hz producing a polarimetric modulation of 80 Hz. The light is collimated from the telescope prior to the half-wave plate. A double beam polarizing prism (a modified Foster prism) provides a geometric separation of 90° between the orthogonally polarized beams, whereby a simultaneous

Table 4.1

Program Stars

Name	$\delta_{(2000)}$	$\alpha_{(2000)}$	V	Spectral Type
		Polarized		
ρ Cas	+57°30′	$23^h54^m23^s$	4.54	G0 Iap
o Cas	+48 17	00 44 44	4.54	B5 III
ϕ Cas	+58 14	01 20 05	4.98	F0 Ia
55 Cyg	+46 07	20 48 56	4.84	B3 Ia
ξ Cas	+53 54	00 36 58	3.61	B2 V
σ Cas	+55 45	23 59 00	4.95	B1 V
ρ Leo	+09 18	10 32 49	3.85	B1 Iab
14 Cep	+58 00	22 02 05	5.56	09 V
9 Gem	+23 44	06 16 59	6.28	B3 Iab
η Per	+55 54	02 50 42	3.76	K3 Ib-IIa
α Per	+49 51	03 24 20	1.80	F5 Ib
		Unpolarized		
β Cas	+59 09	00 09 10	2.25	F2 III-IV
θ Boo	+51 51	14 25 12	4.05	F7 V
θ Cyg	+50 13	19 36 27	4.48	F4 V
α Cas	+56 32	00 40 31	2.23	K0 II-III
η Cep	+61 50	20 45 17	3.43	K0 IV
ι Per	+49 37	03 09 04	4.05	G0 V
γ Gem	+16 24	06 37 43	1.93	A0 IV

polarimetry can be performed in two colours. Interference filters were placed in the collimated beam after the polarizing prism. Fabry lenses are used to direct light onto photomultipliers, an EMI 9558Q for red channel, cooled to -20°C by means of a thermoelectric cooler and an uncooled EMI 9789B tube for the blue channel which has its peak quantum efficiency at about 4000Å.

Photon counting techniques were used to measure the polarizations. The generated photo-electron pluses were collected by six counters, three for each channel. These indicate the photon counts over different portions of the modulated signal in a similar way to Klare et al. (1972) and as described in Stewart (1984).

The control of the instrument and data collection were performed by a QL Sinclair microcomputer (see Clarke, 1989). Each determined value of q and u were obtained at 10 second integration times (the errors on the q and u were determined by taking the mean of several integrations of 10 second over the modulated signal)and a typical total integration time for a bright star such as β Cas was $\simeq$ 30 minutes. The integration time was increased further for fainter stars to achieve the same general levels of polarimetric accuracy.

Since photon counting was used in our polarimeter, it was therefore necessary to determine the value of dead-time for the amplifiers (see Appendix C) so that corrections for counting losses can be applied. The effects due to the dead-time could bias our polarization values especially for bright stars. The value of dead-time for the blue and red channels were calculated to be 265 and 83 nanoseconds respectively.

Double beam polarimetry was performed by using two filters where λ_R was centred on 6790 Å with passband of 130 Å (made by Grubb parsons), which is close to the Standard R-band and λ_B was standard B-band using a Corning 5030 Å filter. All the observations were carried out with a 30 arcsec circular non-conducting diaphragm which was placed in the focal plane of the telescope. Regular tracking checks were made after every 20 recorded data lines (200s).

The background light from the sky was recorded at the beginning, in the middle and end of each data file, each lasting about 4 minutes and subsequently subtracted form the source data.

The data presented here are in the form of the Normalized Stokes Parameters rather than the statistically biased quantities of p and θ (see Clarke and Stewart 1986).

4.2 Instrumental Polarization

The instrumental polarization was determined by observing solar neighbourhood stars which were also bright enough for the observation to be reasonably accurate with the limited telescope aperture. The NSP values of unpolarized standards are shown in Table 4.2 for the blue and red channels. These stars were chosen from different catalogues (see section 1.2). Their distances from the Sun range between 12 to 34 parsecs, thus any interstellar polarization contribution will be at minimum with our level of accuracy. These stars were monitored throughout the observations to check the stability of instrumental polarization (see Fig. 4.1). During each observing night, up to two unpolarized stars were observed one at the beginning of the observation and one at the end. The weighted means of the measurements were calculated by looking at the estimate of $\bar{q}_i$, which will have an associated error σ_i. Therefore the best estimate of $\bar{q}$ is

$$\bar{q}_W = \frac{\sum_{i=1}^{n} W_i \bar{q}_i}{\sum_{i=1}^{n} W_i} \qquad 4.1$$

where the weights are given by

$$W_i = \frac{1}{\sigma_i^2}$$

and the best estimate of the variance of $\bar{q}_W$ is the reciprocal of the sample variance

$$\sigma_W^2 = \frac{1}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}} \qquad 4.2.$$

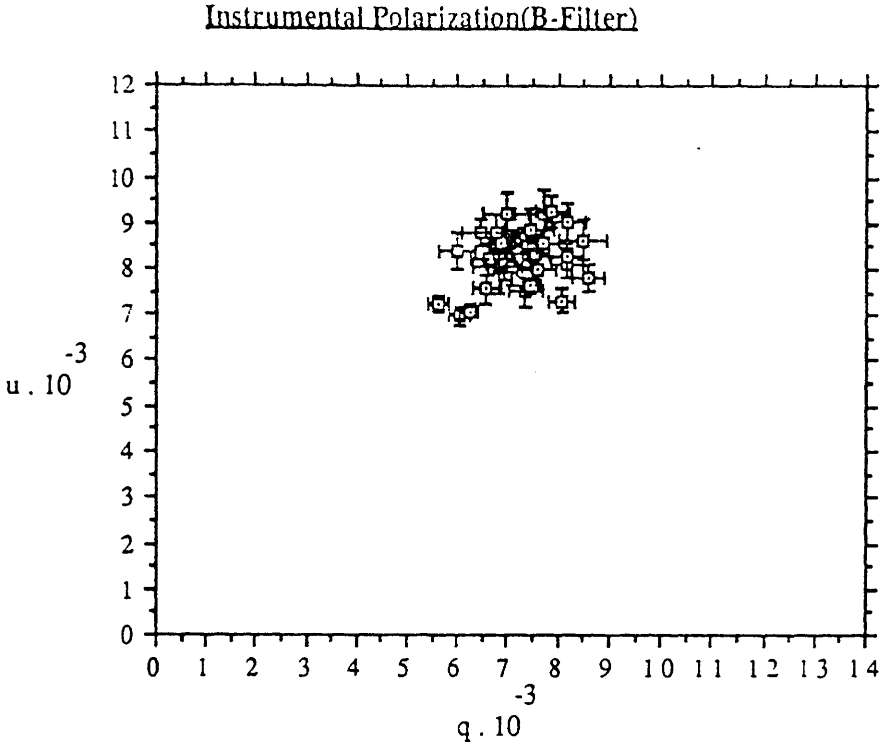
Similar formula apply to the u parameter. The mean instrumental polarization values taken from all the unpolarized stars over the whole observational run were 1.07% ± 0.03% and 1.08% ± 0.04% for blue and red channel respectively. The instrumental polarization is strong and this was attributed to the mirror coatings which were several years old and the mirrors were covered with dust at the time of my observations. It should also be noted that the telescope is designed with an f/8 ratio which exacerbates the problem.

Normality testing was performed on q and u for the blue and red channels. By computing the skewness and kurtosis values for β Cas, α Cas, θ Cyg and θ Boo, and comparing these with the sample dependent tables derived in Brooks (1984) these stars passed the test at the 95% confidence level.

Table 4.2
Instrumental Polarization

Name	N	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
β Cas	9	+0.006813	±0.000229	+0.007396	±0.000230	+0.006289	±0.000394	+0.008887	±0.000374
θ Cyg	5	+0.007222	±0.000457	+0.008538	±0.000443	+0.006630	±0.000940	+0.009012	±0.000947
θ Boo	6	+0.006992	±0.000321	+0.008492	±0.000316	+0.006036	±0.000679	+0.008325	±0.000666
α Cas	7	+0.007188	±0.000262	+0.008600	±0.000256	+0.006065	±0.000328	+0.008493	±0.000344
η Cep	2	+0.007474	±0.000398	+0.008732	±0.000399	+0.006164	±0.000616	+0.008419	±0.000661
ι Per	2	+0.008416	±0.000357	+0.008162	±0.000329	+0.006799	±0.000692	+0.009145	±0.000642
γ Gem	1	+0.006918	±0.000376	+0.008112	±0.000332	+0.006062	±0.000563	+0.008751	±0.000589
Weighted Mean		+0.007044	±0.000291	+0.008023	±0.000287	+0.006191	±0.000455	+0.008848	±0.000459

N Number of observations



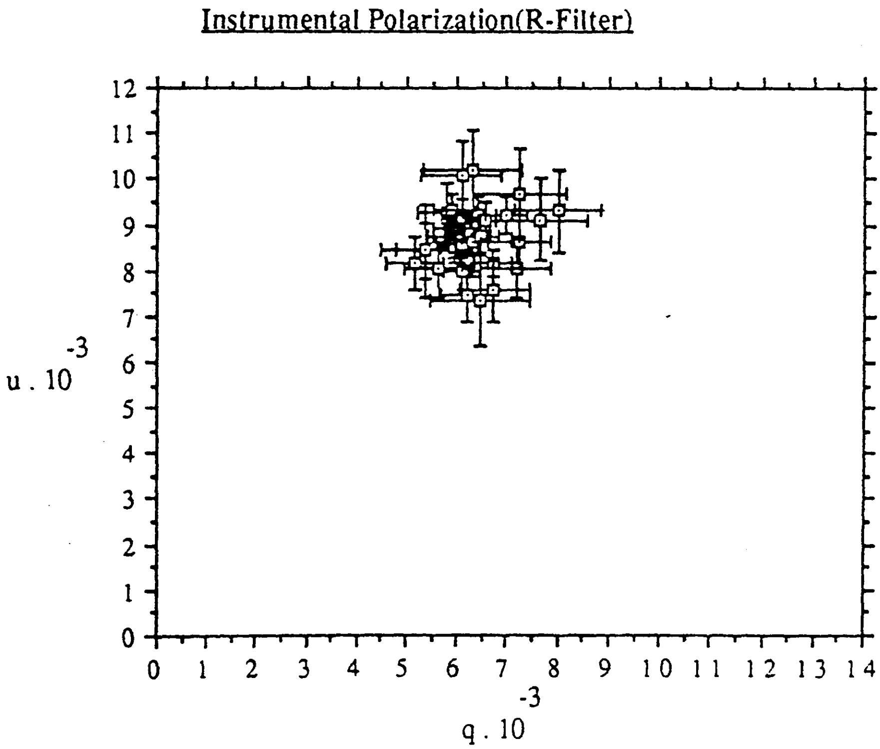


Fig. 4.1 plot of the instrumental polarization in the q, u plane.

4.3 Observations, Data Reduction and Analysis of Polarized Standards

The stars under consideration were taken from different catalogues. The primary reason for observing these stars was to determine their polarization properties and to identify polarization variables, thus making them unsuitable for being standards. The procedures adopted for reduction and assessment of the data were:

A) Calculate the mean value of the NSPs for the data run under study.

B) Remove effects of instrumental polarization.

C) Test for normality in the repeated measurements for stars with many observations.

The data reduction of the program stars was carried out in the instrumental frame, since any conversion to a standard frame might introduce a systematic error to our data (see Section 1.3.3). Due to the fact that we are primarily interested in the variations of polarization and position angle there is no need, at this stage, to compare our results with other investigators. We have not performed any correction to uncertainties of position angle, since our observations are presented in NSPs rather than p and θ, also measurements were done with high signal-to-noise ratio (see Chapter 2).

For each star, a q, u diagram was plotted to see what kind of changes occur. In addition the behaviour of q and u with time were also plotted. The results for the stars under study will be discussed individually.

55 Cyg (HD 198478)

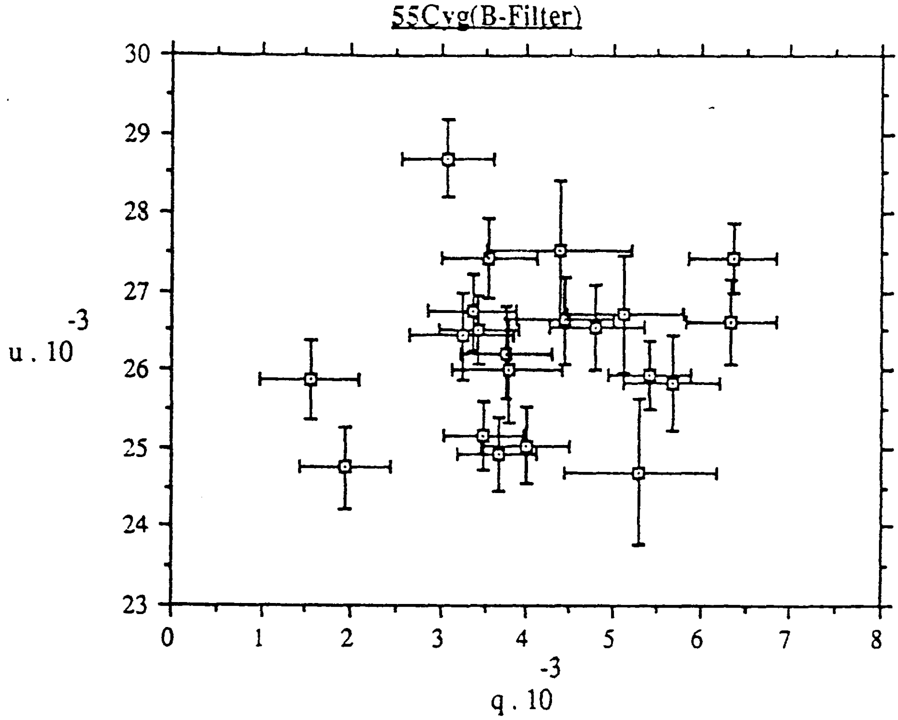
This star is one of the standard polarized stars which has been listed by Serkowski (1974a) with λ_{max} = 5300Å, $p_{\lambda_{max}}$ = 2.8% ± 0.1% and $\theta_{\lambda_{max}}$ = 3°± 1°. However, Hsu and Breger (1982) have claimed to have found temporal variability in both polarization and position angle. They have reported a change in p of ~ 0.06% and about 1° in position angle. Wilking et al. (1980) stated that 55 Cyg does not show any rotation of the position angle over the visible and infrared wavelength regions. The criterion used was $\Delta\theta \geq 5°$ as having a θ rotation. Avery et al. (1975) detected circular polarization which changed handedness from positive in the B-band to negative in the I-band. The origin of the circular polarization is explained, by there being a continuous change in the direction of grain alignment along the line of sight to the star. Dolan and Tapia (1986) reported that 55 Cyg exhibits a statistically significant rotation of its position angle with wavelength, the value of position angle which they have measured differs by 3° from those of Hsu and Breger (1982). Treanor (1963) has suggested that the polarization of 55 Cyg is unusually high for a star at its galactic longitude; thus a local anomaly in the dust distribution or in the magnetic field, might be present.

It should be noted that 55 Cyg is a supergiant of type B3Ia and we might compare this with another similar supergiant such as χ^2 Ori (B2Ia). χ^2 Ori has a similar polarization value (p_{max} = 2.79% ± 0.05% at λ_{max} = 5000Å) as 55 Cyg. Polarization variability for this star has been reported by many workers (see Lupie and Nordsieck, 1987) and indeed many of the early-type supergiant stars exhibit variable intrinsic polarization. Thus we would expect a similar effects in 55 Cyg. Underhill (1960) and Granes et al. (1971) have observed the Hα profile of 55 Cyg changes over a few days are readily apparent.

Polarimetric observations were made on 20 nights from Nov 1989 to Aug 1990, with 4 nights of consecutive observations. The maximum polarization changes for the B-band were $\Delta p_B \sim 0.4\%$ with position angle(θ) of $\Delta\theta_B \sim 4.^\circ8$, and for the red band $\Delta p_R \sim 1.1\%$ with $\Delta\theta_R \sim 4.^\circ2$ (see Table 4.3 and Figs. 4.2 and 4.3a, b). The overall position angle varies with time, but the individual position angles are relatively constant with wavelength to within observational errors (1°). Normality testing was performed on the q and u values in Table 4.3 for both channels. Although the data would not be considered as other than normal (with large variance dictated by experimental noise) at the 95% confidence level, it is very obvious that 55 Cyg is a polarimetric variable. It would have been useful to carry out a period search on the data, but this was not possible due to the small data sample.

Table 4.3
Normalized Stokes Parameters (Instrumental Frame) of 55 Cyg (HD 198478)

J.D (2440000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
7802.348	+0.001932	±0.000502	+0.024752	±0.000519	+0.007085	±0.001205	+0.025384	±0.001234
7805.324	+0.003240	±0.000587	+0.026438	±0.000547	+0.007198	±0.001255	+0.025559	±0.001326
7814.300	+0.003975	±0.000524	+0.025034	±0.000495	+0.008612	±0.001416	+0.030683	±0.001552
7814.322	+0.003492	±0.000457	+0.025164	±0.000434	+0.006028	±0.001809	+0.030277	±0.001787
7836.270	+0.003644	±0.000466	+0.024928	±0.000483	+0.003926	±0.001115	+0.026892	±0.001139
7839.435	+0.003550	±0.000545	+0.027435	±0.000514	+0.005662	±0.001077	+0.024831	±0.001076
7845.290	+0.005425	±0.000475	+0.025942	±0.000456	+0.005269	±0.000955	+0.024671	±0.000933
7847.278	+0.006346	±0.000504	+0.026615	±0.000529	+0.004653	±0.001100	+0.025917	±0.001016
7865.307	+0.003360	±0.000512	+0.026751	±0.000491	+0.006255	±0.001018	+0.026974	±0.001021
7867.256	+0.003736	±0.000537	+0.026224	±0.000595	+0.006160	±0.001134	+0.024395	±0.001205
7880.287	+0.005303	±0.000880	+0.024699	±0.000925	+0.008088	±0.001677	+0.026884	±0.002339
7976.624	+0.005126	±0.000678	+0.026720	±0.000741	+0.004034	±0.001528	+0.022366	±0.001293
7978.603	+0.004439	±0.000566	+0.026646	±0.000561	+0.005551	±0.001106	+0.025191	±0.001055
7985.577	+0.001532	±0.000555	+0.025878	±0.000507	+0.005770	±0.001030	+0.024649	±0.000946
8012.561	+0.003427	±0.000457	+0.026513	±0.000431	+0.004329	±0.000872	+0.025761	±0.000956
8013.545	+0.003764	±0.000635	+0.026004	±0.000659	+0.005366	±0.001159	+0.025021	±0.001956
8014.577	+0.004791	±0.000552	+0.026544	±0.000542	+0.004842	±0.001137	+0.024862	±0.001086
8015.523	+0.005670	±0.000559	+0.025847	±0.000599	+0.005453	±0.001039	+0.024010	±0.001093
8035.509	+0.003069	±0.000522	+0.028695	±0.000504	+0.006083	±0.001053	+0.026435	±0.001014
8105.484	+0.006354	±0.000497	+0.027434	±0.000442	+0.005126	±0.000963	+0.026657	±0.000978
8124.440	+0.004364	±0.000843	+0.027536	±0.000877	+0.004026	±0.001786	+0.020338	±0.001378



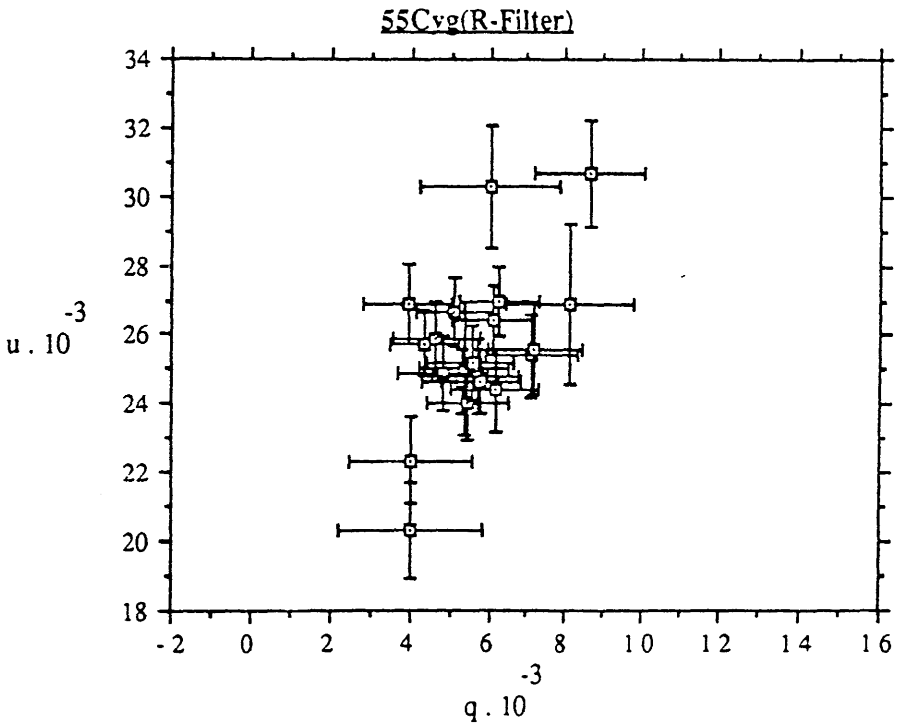


Fig. 4.2 Plot of the polarization of 55 Cyg in the q, u plane.

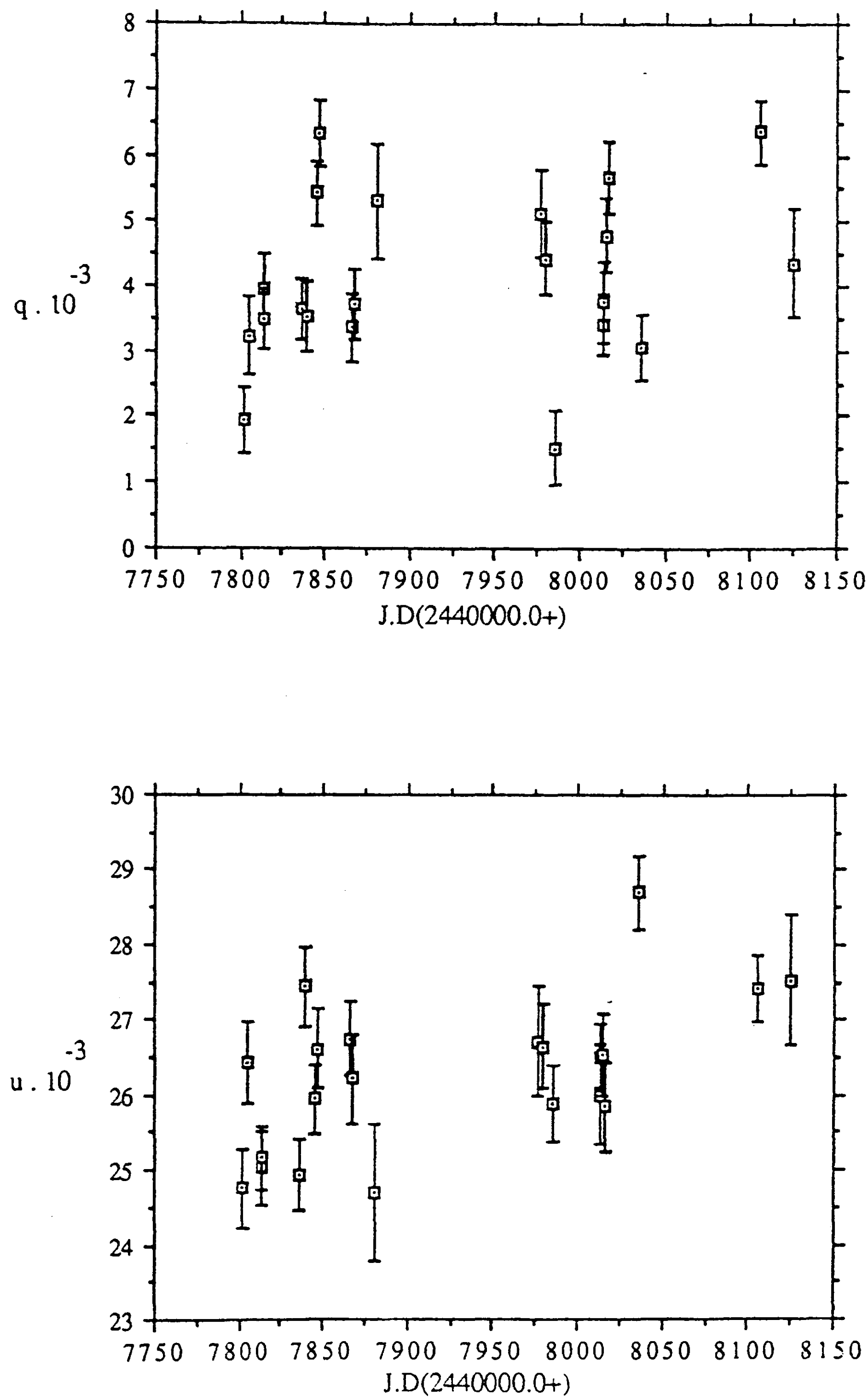
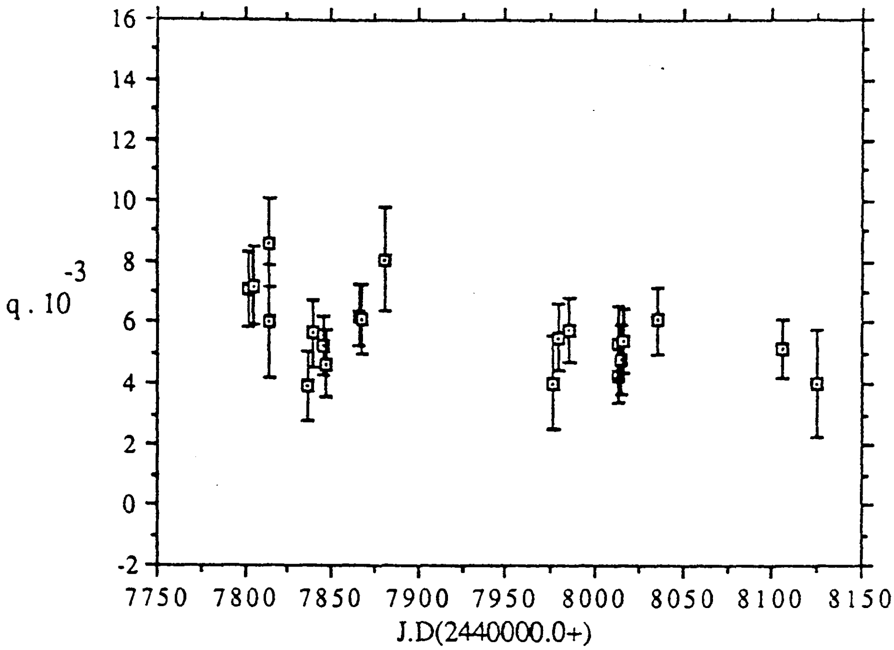


Fig. 4.3a Time Dependence of the Normalized Stokes Parameters q and u for 55 Cyg at B-band.



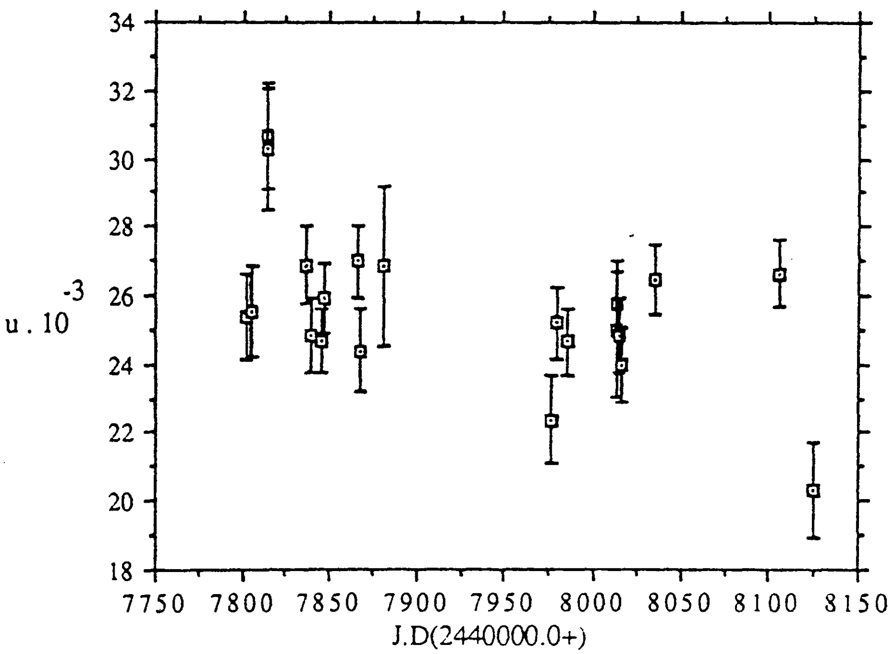


Fig. 4.3b Time Dependence of the Normalized Stokes Parameters q and u for 55 Cyg at red band.

ϕ Cas (HD 7927)

This star is a polarized standard according to Serkowski (1974a), with λ_{max} = 5100Å, p_{max} = 3.4% ± 0.1% and $\theta_{\lambda max}$ = 94° ± 1°. Observations by Hsu and Breger (1982) have refined the parameters further where λ_{max} = 5150Å ± 60Å, p_{max} = 3.41% ± 0.02% and θ_V = 92°.3 ± 0°.1. It has been known for some time that the position angle exhibits a wavelength dependency (see Gehrels and Silvester, 1965, Coyne and Gehrels, 1966). Dolan and Tapia (1986) have found that the $\theta(\lambda)$ dependence varies from night to night.

Bastien et al. (1988) have claimed that ϕ Cas exhibits temporal variations in both p and θ and this is supported by our statistical analysis of their data (see Chapter 3), which showed that the data did not come from a normal distribution.

During the present observational run, the star was observed on 7 nights at two different wavelengths (see Table 4.4 and Figs. 4.4 and 4.5a, b). From the 7 points recorded, the analysis in the instrumental frame revealed that q and u behaved as Gaussian at two wavelengths. The maximum differences between observations are $\Delta p_B \sim 0.1\%$ with $\Delta\theta_B \sim 3°$ for the B-band, and $\Delta p_R \sim 1.5\%$ with $\Delta\theta_R \sim 4°$ for the red band. The polarization wavelength dependence varies in the following way: the overall position angle varies with time, and there is a significant rotation of position angle with wavelength. Polarization values at the B-band are relatively constant but there is a large change of polarization in the red band (see Table 4.4).

Table 4.4
Normalized Stokes Parameters (Instrumental Frame) of ϕ Cas (HD 7927)

J.D (2447000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
801.517	-0.004859	±0.000527	-0.033051	±0.000504	+0.001655	±0.000872	-0.021141	±0.001016
802.442	-0.005286	±0.000708	-0.032716	±0.000713	+0.004079	±0.001960	-0.018281	±0.002078
836.466	-0.003019	±0.000625	-0.032380	±0.000665	+0.004692	±0.001551	-0.033644	±0.001373
839.563	-0.002146	±0.000594	-0.032402	±0.000606	+0.003550	±0.001100	-0.029792	±0.001078
847.376	-0.003205	±0.000837	-0.033533	±0.000785	+0.005567	±0.001443	-0.030294	±0.001591
853.515	-0.002041	±0.000471	-0.032660	±0.000491	+0.005833	±0.001000	-0.030440	±0.000894
857.388	-0.003212	±0.000522	-0.032683	±0.000510	+0.006480	±0.000978	-0.031815	±0.000943

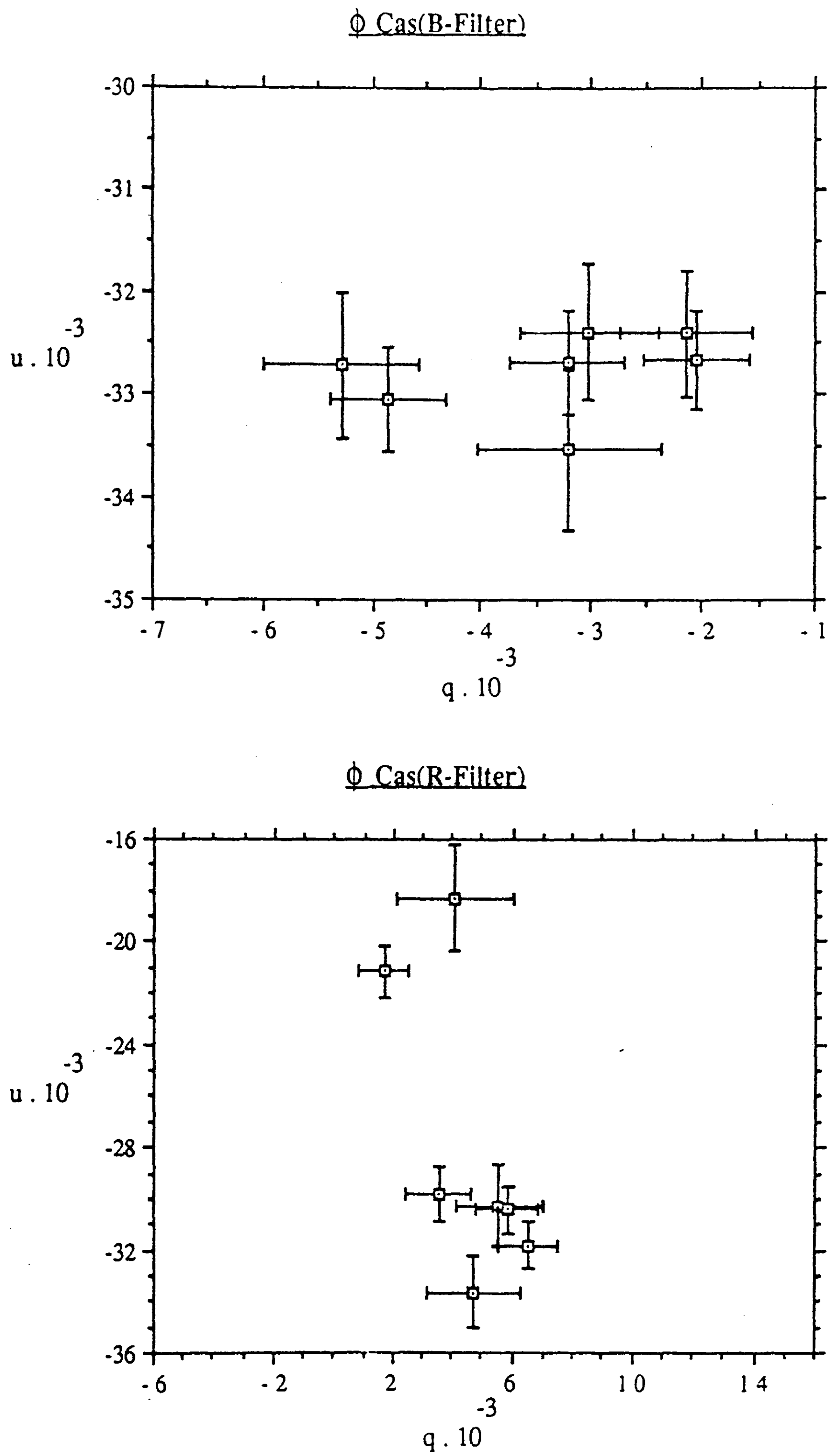
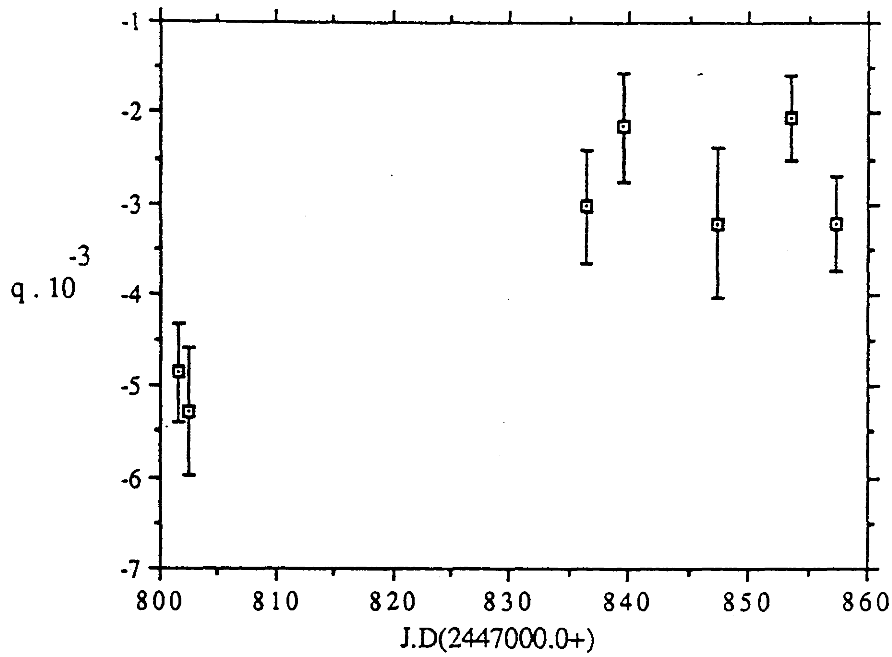


Fig. 4.4 Plot of the polarization of ϕ Cas in the q, u plane.



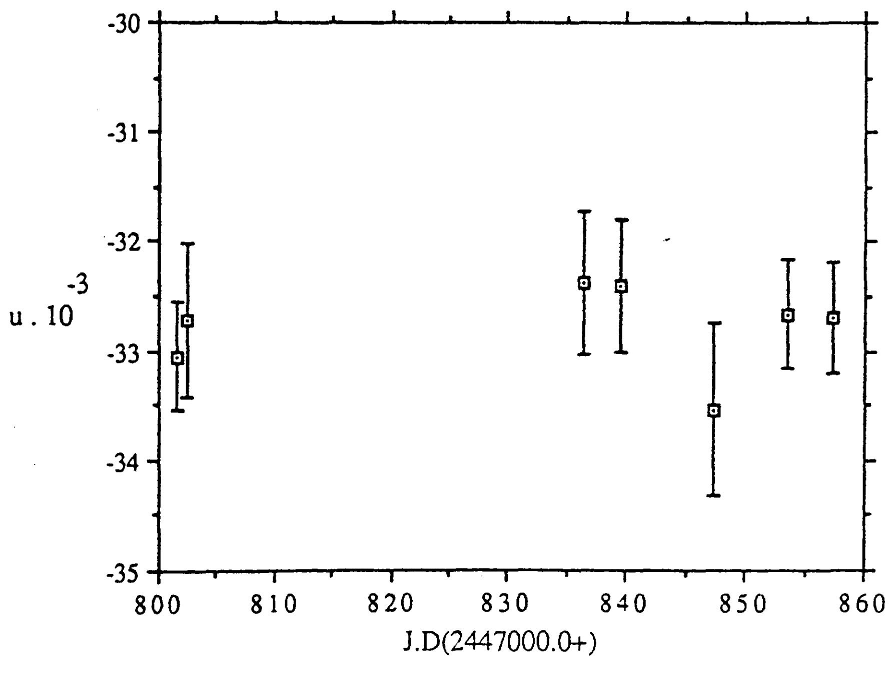


Fig. 4.5a Time Dependence of the Normalized Stokes Parameters q and u for ϕ Cas at B-band.

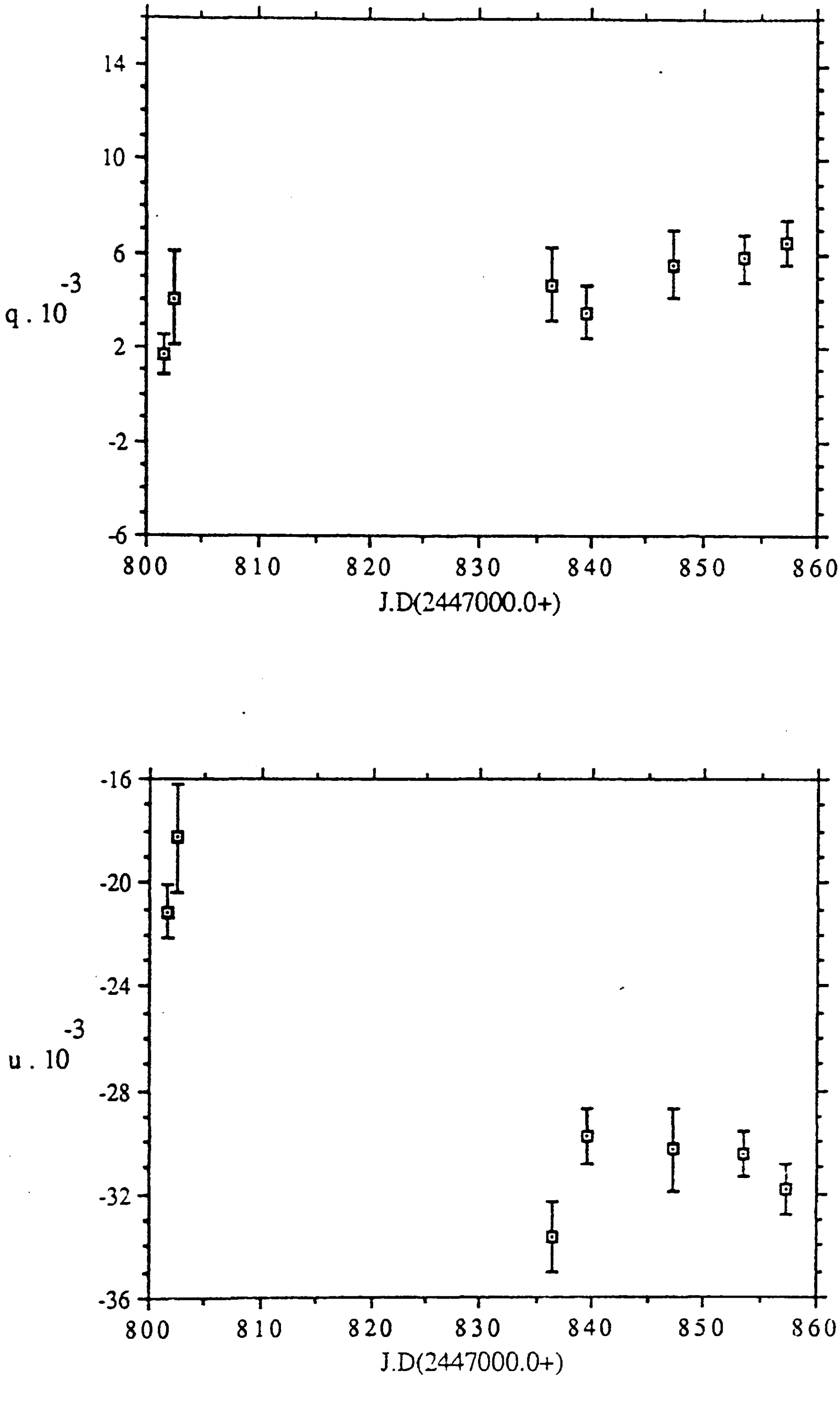


Fig. 4.5b Time Dependence of the Normalized Stokes Parameters q and u for ϕ Cas at red band.

ρ Cas (HD 224014)

ρ Cas is one of the original stars proposed by Serkowski (1960) as being a polarization standard. Hiltner (1951) gives p = 1.4% and θ = 51° at λ_{eff} = 5400A. Coyne and Gehrels (1966) have also observed this star and they conclude that there is a good agreement between their observation and previous workers.

This star was observed on 4 nights (Table 4.5), the polarization changes are of order of ~ 0.17% in the B-band and ~ 0.32% for the red measurements (see Figs. 4.6 and 4.7a, b). There is a rotation of position angle with wavelength. Statistical analyses were inappropriate because of the few data points collected.

o Cas (HD 4180)

This star is quoted by Behr (1959) as having a p = 0.76% and θ = 83°. Our polarization values taken over 3 nights (see Figs. 4.8 and 4.9a, b) differs significantly from Behrs of order of 0.7% (see Table 4.6). Polarization and position angle values are constant within our observational errors (± 0.05% with ± 1° in the B-band and ± 0.1% with ± 3° in the red band). However, there is a change of position angle in the red band on the second night (see Table 4.6). Long term observations of this star is required to establish whether this star is a standard polarization or not.

14 Cep (HD 209481)

Hall (1958) gives a polarization value of 1.98% with θ = 72° at λ = 4500A whereas Behr (1959) quotes a polarization value of 1.61% θ = 67° at λ = 4600A. Serkowski (1960) states that this star is a standard. This star was observed on 5 nights (see Table 4.7). We have found the maximum change of polarization of order of 0.28% in the B-band and 0.12% in the red band. There is a significant rotation of position angle in our measurements (see Figs. 4.10 and 4.11a, b). It is concluded that 14 Cep shows a definite variability in its polarization and position angle.

ρ Leo (HD 91316)

This star was used by Appenzeller (1966) as a standard, $p = 0.16\% \pm 0.03\%$ and $\theta = 120^\circ\!.8 \pm 5^\circ\!.1$. ρ Leo was observed on 6 nights (see Table 4.8). In our observations there are significant variations in polarization and position angle (see Figs. 4.12 and 4.13a, b). Polarization values vary significantly within few days and there is no correlation between polarization variability in the B-band and the red band. The maximum change in polarization in the B-band is 0.57% and in the red band 0.25%.

It has been reported by Underhill and Doazan (1982) that there is a slight variability in the Hα emission profile. It is therefore concluded that ρ Leo exhibits a variable polarization and position angle.

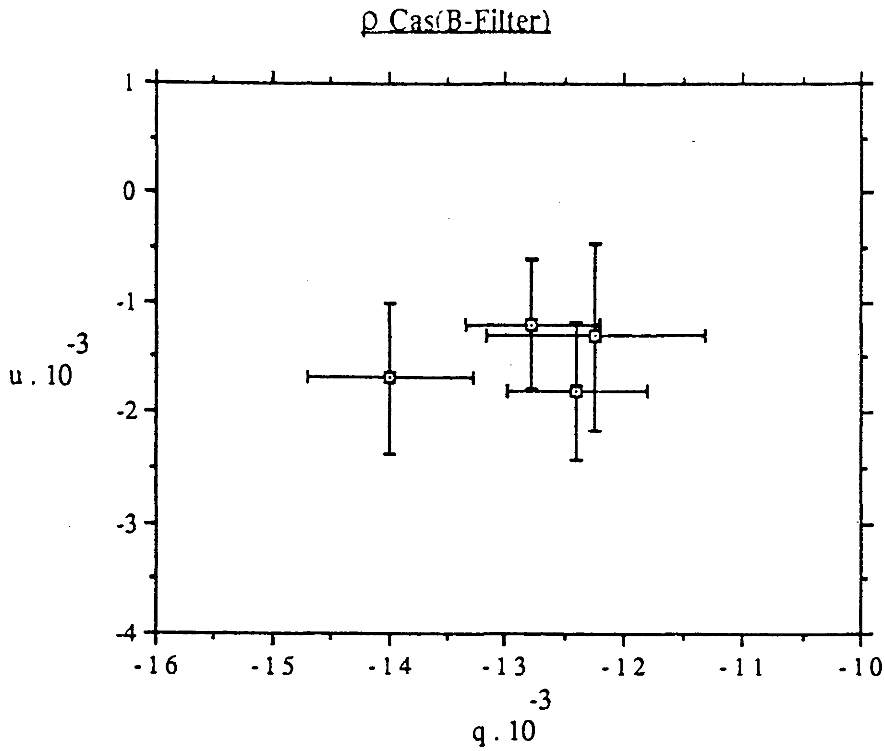
Other measured Stars

We included in our observations (see Table 4.9) some well known polarization variables such as Be stars (ζ Tau, χ^2 Ori, and 48 Per). Unfortunately few data points were obtained for these stars and statistical tests were not applied.

We have compared our polarization values in Table 4.9 with those of Hiltner (1951) and Behr (1959). Only two stars (9 Gem and ψ Per) showed significant differences. There was good agreement in the polarization values of ζ Cas, η Per, δ Per, and 29 Per.

Table 4.5
Normalized Stokes Parameters (Instrumental Frame) of ρ Cas (HD 224014)

J.D (2447000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
802.413	-0.013993	±0.000705	-0.001693	±0.000696	-0.011247	±0.001088	-0.001425	±0.001076
814.388	-0.012782	±0.000568	-0.001190	±0.000595	-0.014500	±0.001166	-0.001404	±0.001035
839.596	-0.012249	±0.000924	-0.001303	±0.000855	-0.012908	±0.001180	-0.006459	±0.001224
857.422	-0.012406	±0.000593	-0.001804	±0.000635	-0.012476	±0.000825	-0.006108	±0.000854



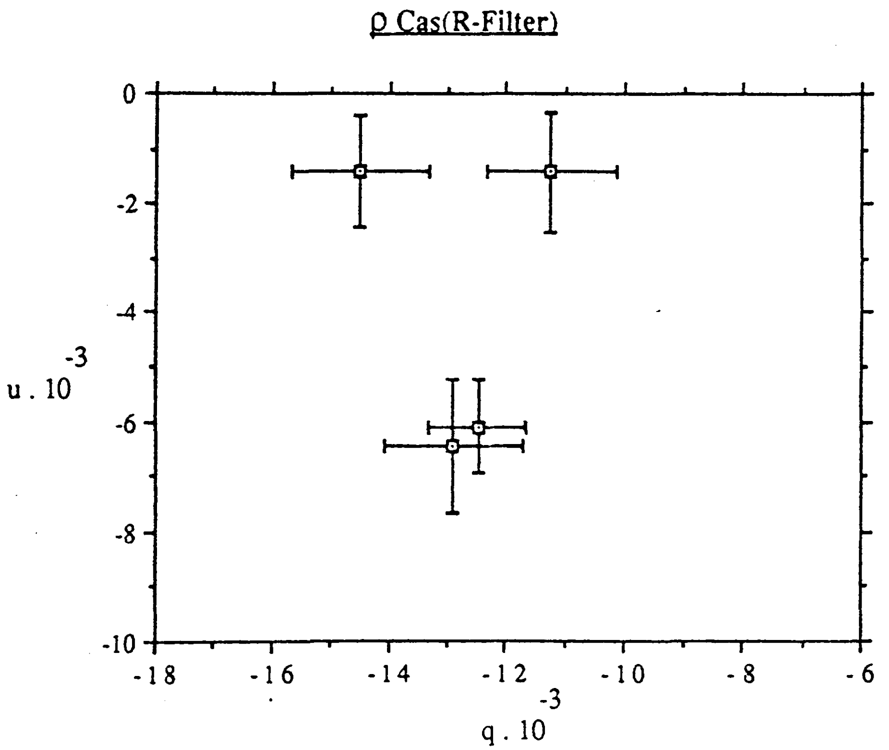
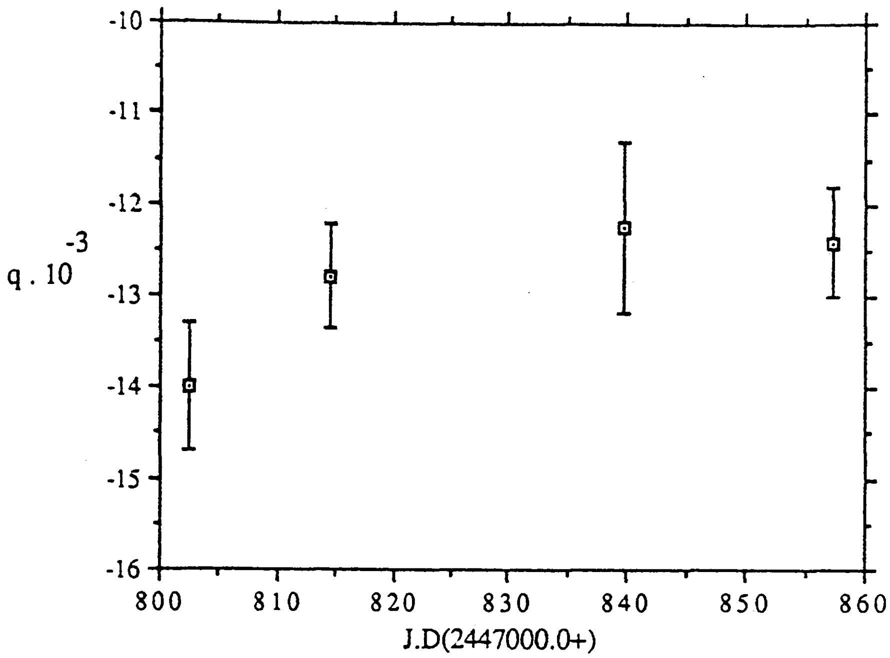


Fig. 4.6 Plot of the polarization of ρ Cas in the q, u plane.



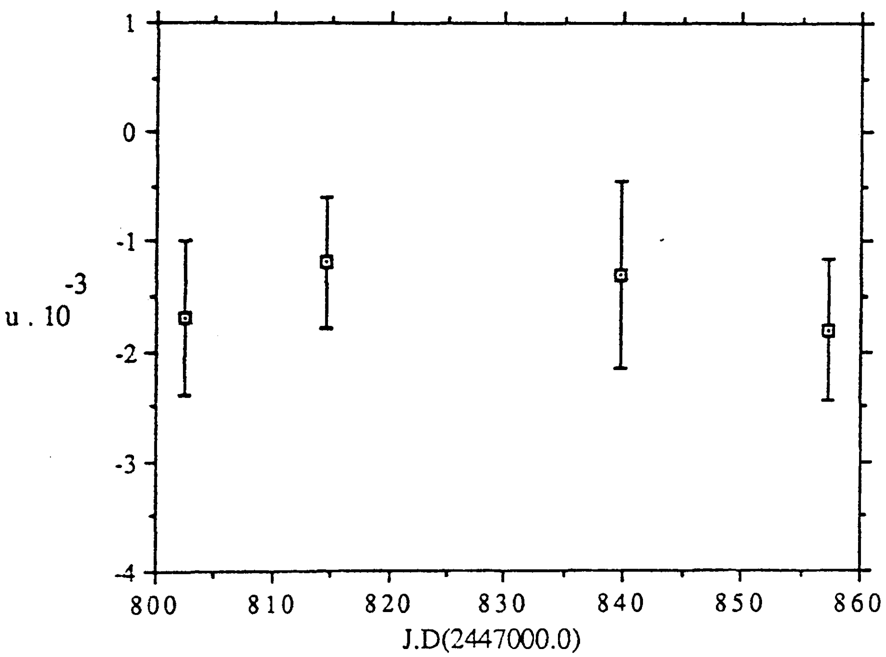
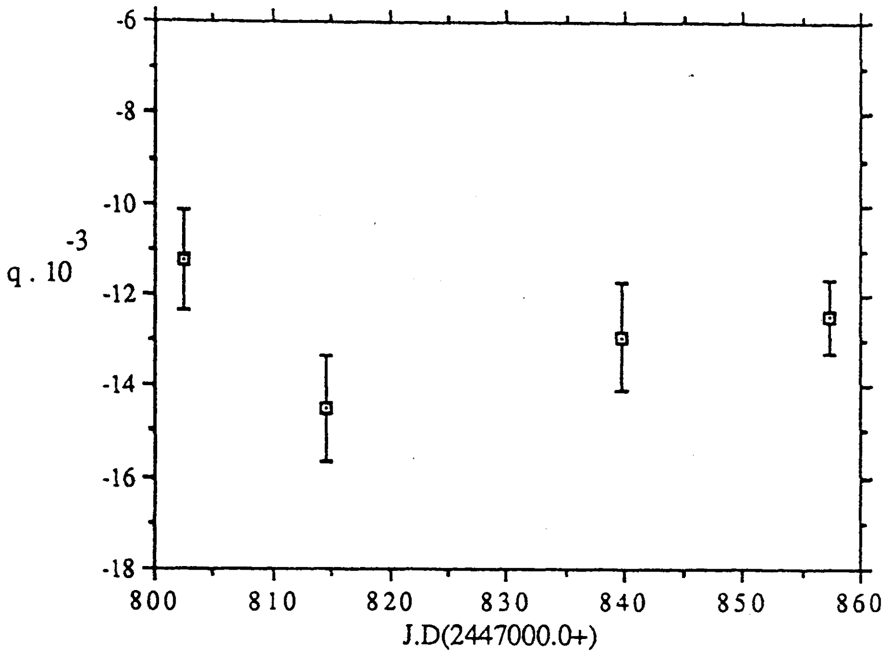


Fig. 4.7a Time Dependence of the Normalized Stokes Parameters q and u for ρ Cas at B-band.



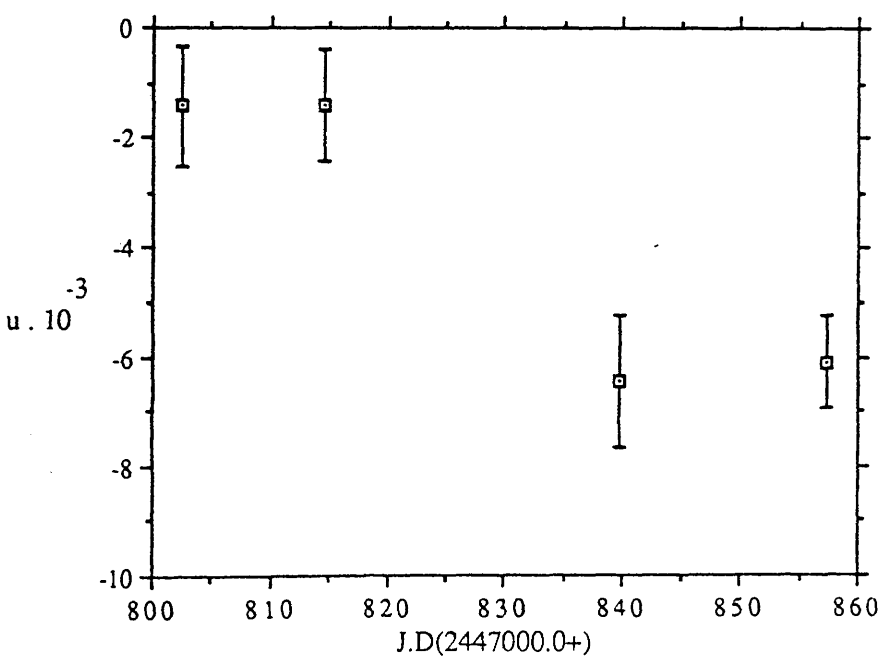


Fig. 4.7b Time Dependence of the Normalized Stokes Parameters q and u for ρ Cas at red band.

Table 4.6
Normalized Stokes Parameters (Instrumental Frame) of o Cas (HD 4180)

J.D (2447000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
814.503	-0.006465	±0.000467	-0.013049	±0.000417	-0.004104	±0.001759	-0.011727	±0.001661
835.542	-0.005378	±0.000562	-0.012610	±0.000545	-0.001018	±0.001275	-0.012915	±0.001473
857.515	-0.005337	±0.000444	-0.012544	±0.000425	-0.003770	±0.000902	-0.012415	±0.000942

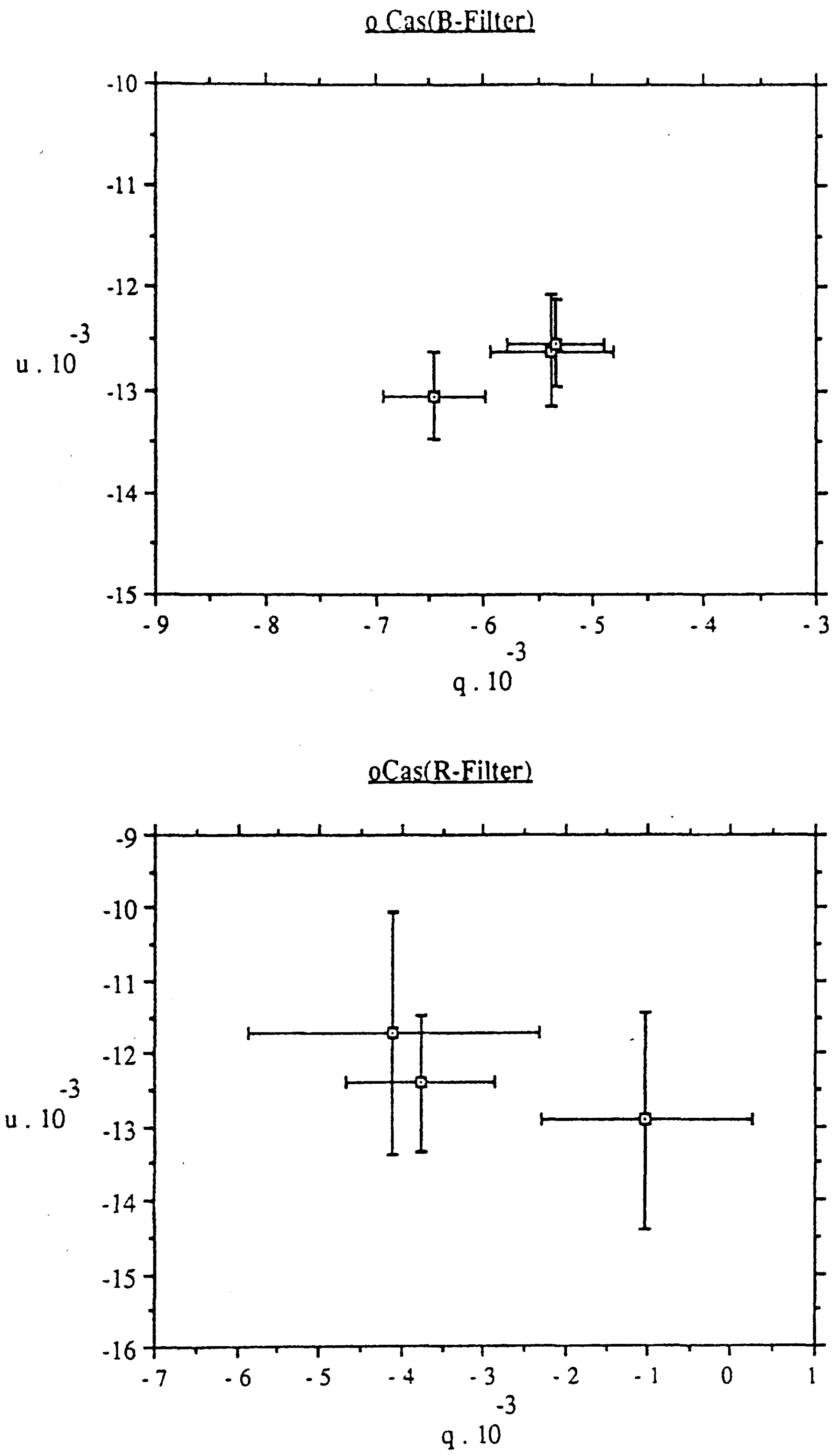
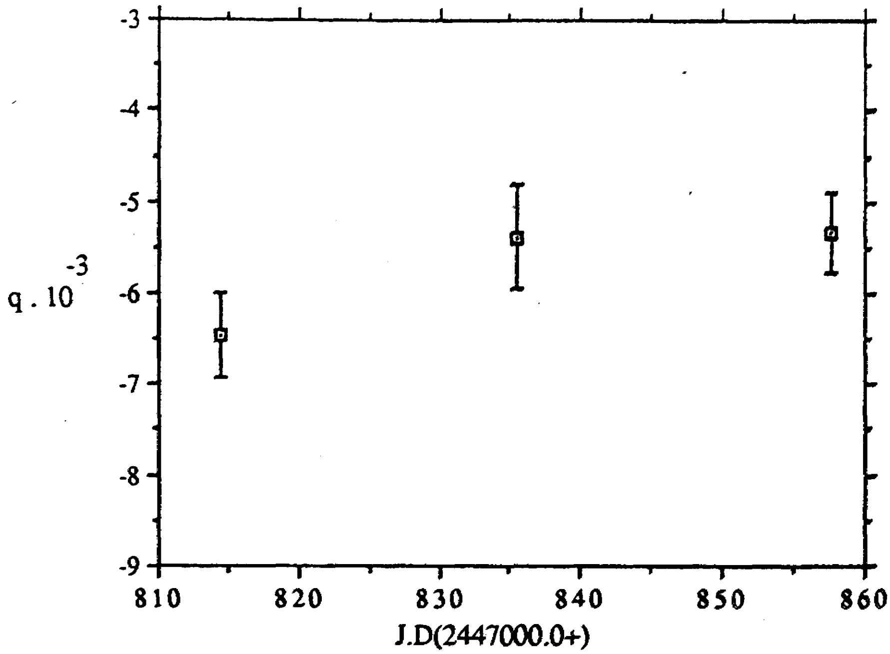


Fig. 4.8 Plot of the polarization of o Cas in the q, u plane.



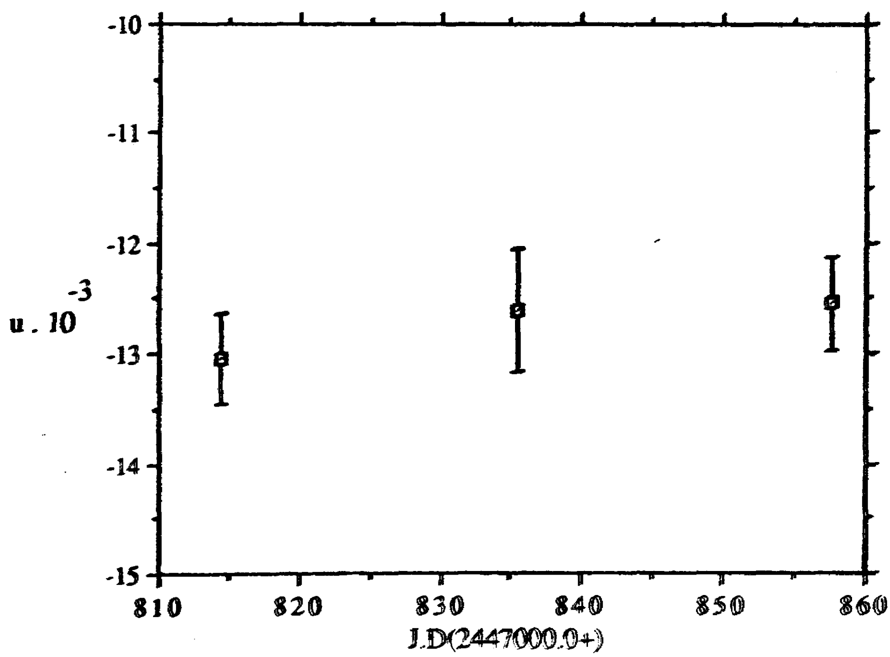
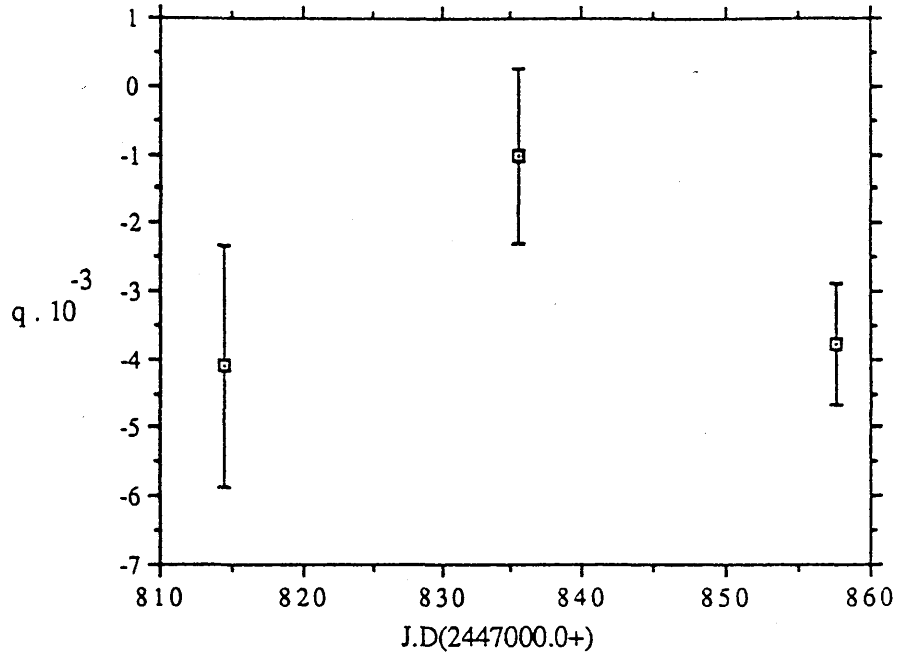


Fig. 4.9a Time Dependence of the Normalized Stokes Parameters q and u for o Cas at B-band.



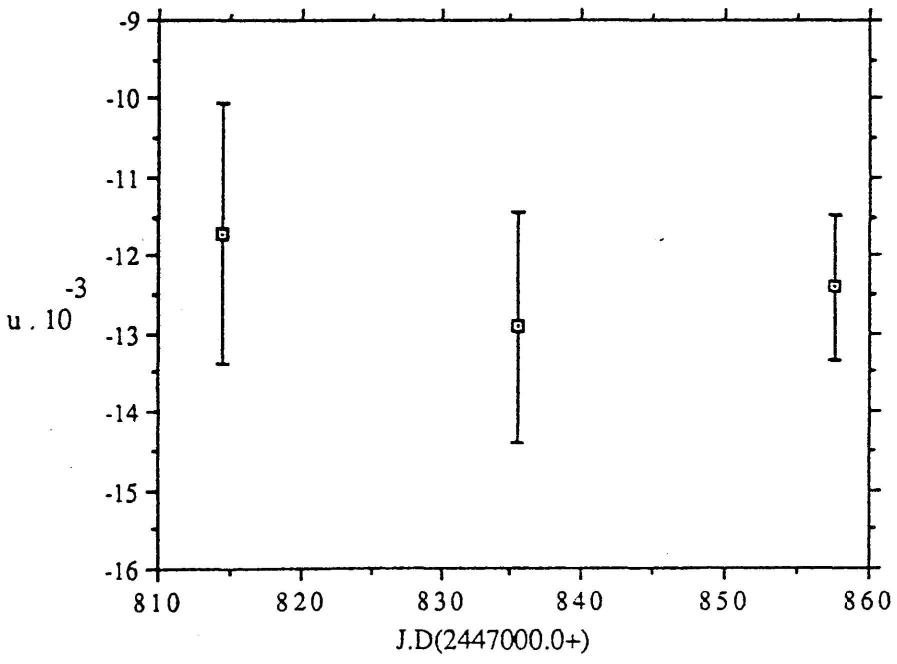
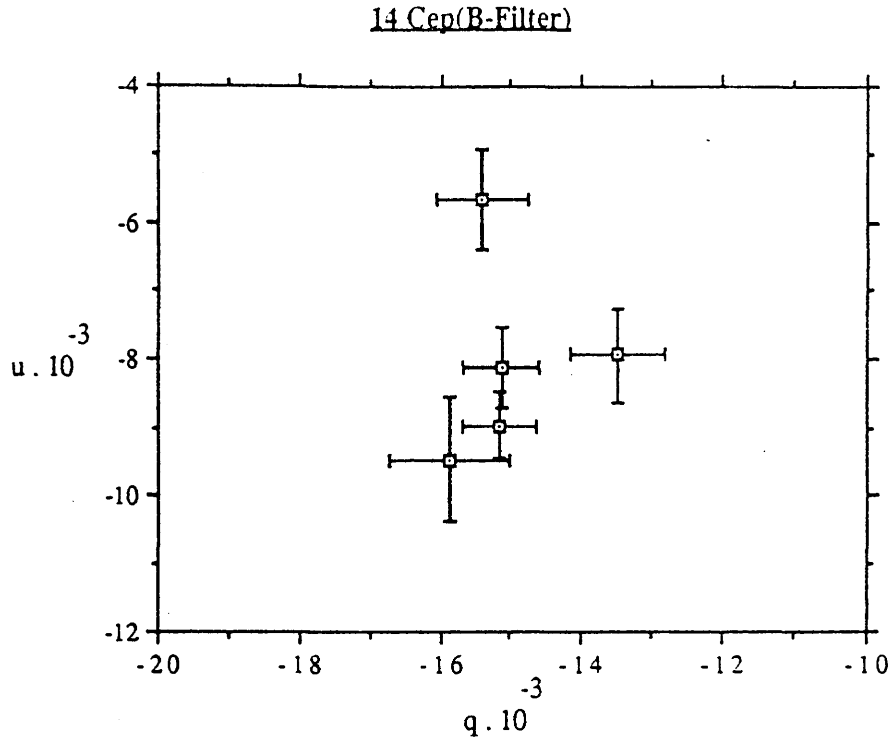


Fig. 4.9b Time Dependence of the Normalized Stokes Parameters q and u for o Cas at red band.

Table 4.7
Normalized Stokes Parameters (Instrumental Frame) of 14 Cep (HD 209481)

J.D (2440000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
7978.644	-0.015145	±0.000548	-0.008129	±0.000583	– – –	– – –	– – –	– – –
7985.624	-0.015417	±0.000655	-0.005668	±0.000727	-0.010363	±0.001619	-0.012664	±0.001916
8012.610	-0.015876	±0.000871	-0.009462	±0.000914	-0.011571	±0.003095	-0.011478	±0.002838
8013.581	-0.013495	±0.000659	-0.007941	±0.000681	-0.010731	±0.001659	-0.013473	±0.001679
8015.564	-0.015155	±0.000516	-0.008955	±0.000501	-0.012041	±0.001189	-0.012788	±0.001234



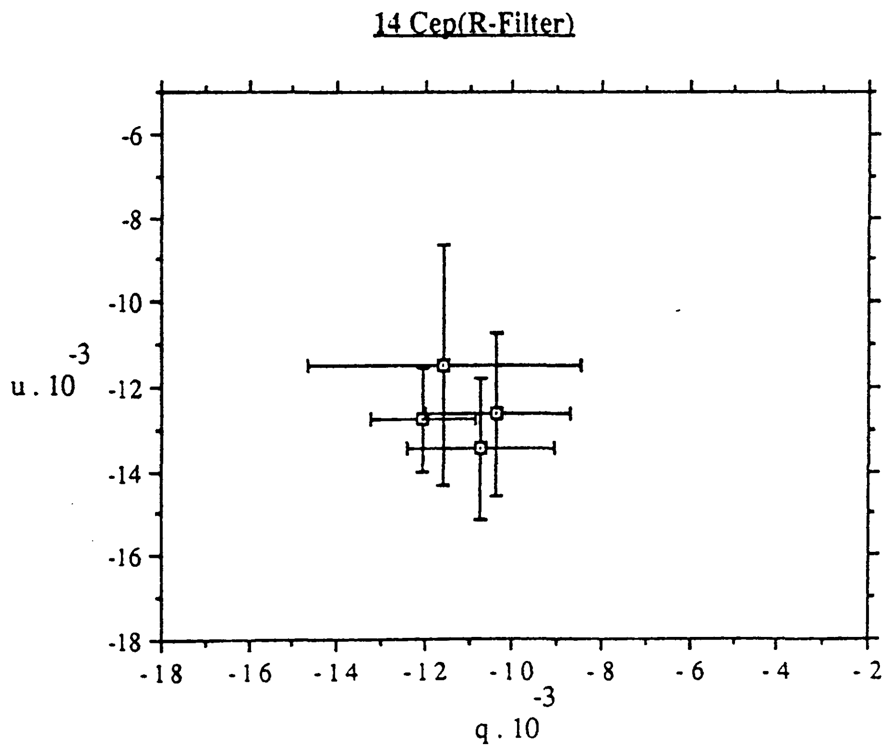


Fig. 4.10 Plot of the polarization of the 14 Cep in the q, u plane.

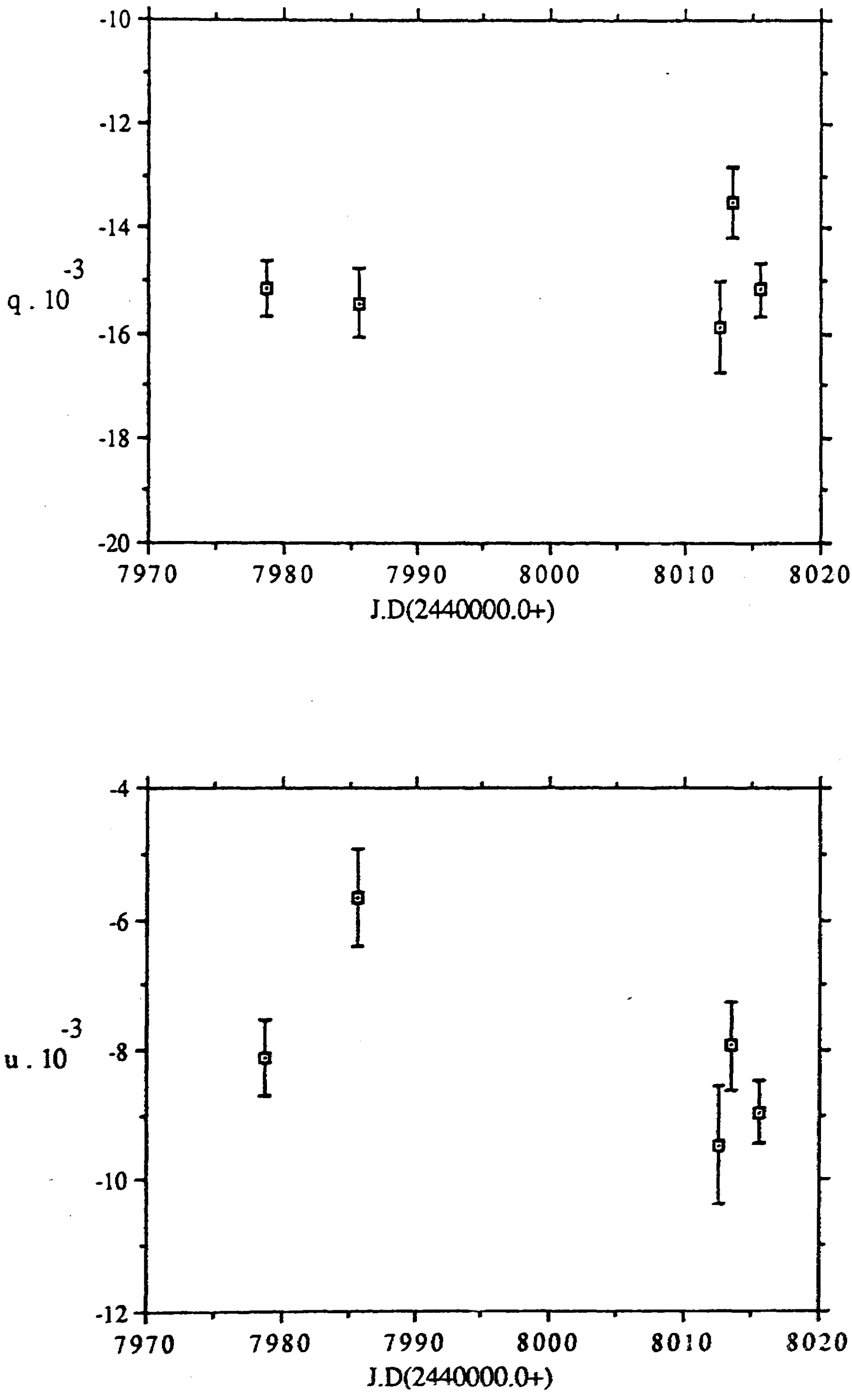
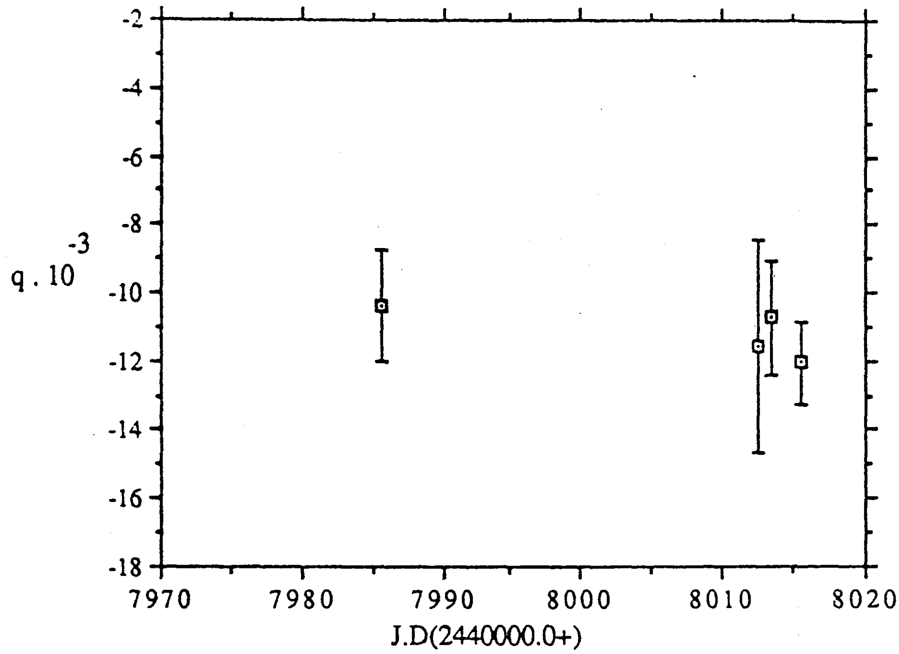


Fig. 4.11a Time Dependence of the Normalized Stokes Parameters q and u for 14 Cep at B-band.



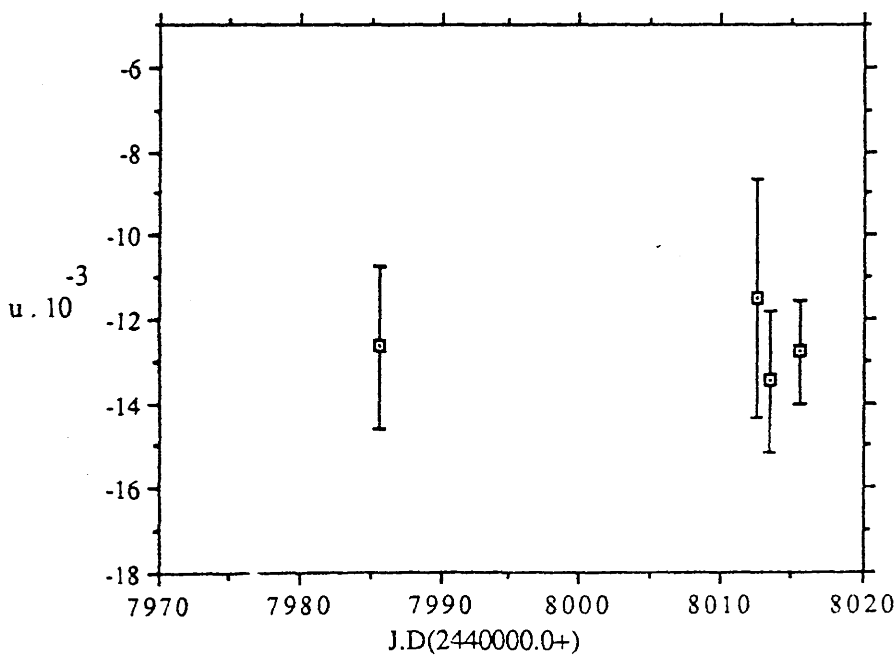
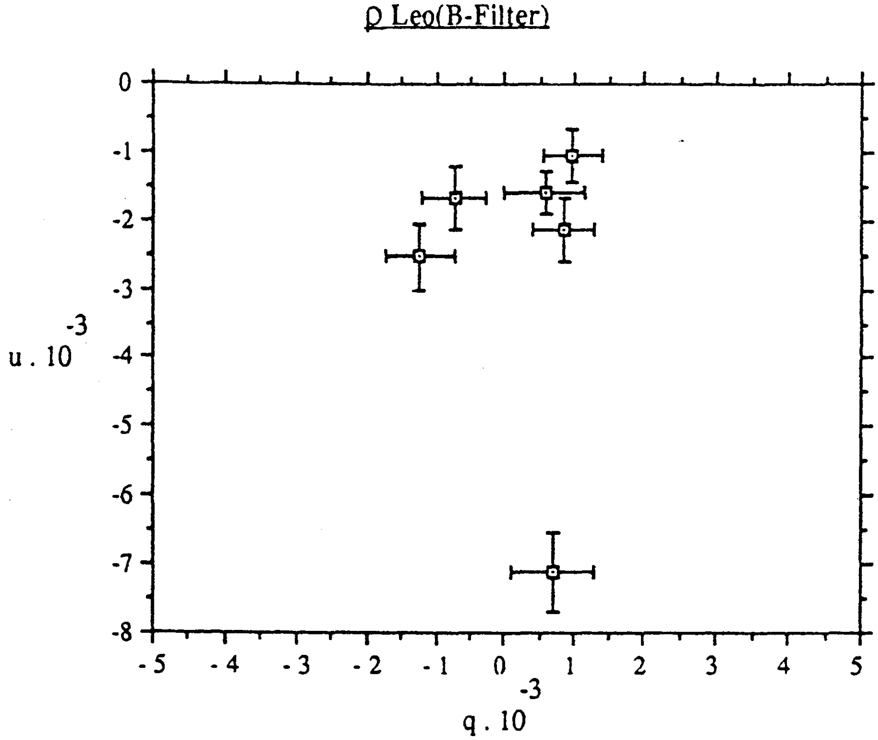


Fig. 4.11b Time Dependence of the Normalized Stokes Parameters q and u for 14 Cep at red band.

Table 4.8
Normalized Stokes Parameters (Instrumental Frame) of ρ Leo (HD 91316)

J.D (2440000+)	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u
7952.532	+0.000847	±0.000446	-0.002117	±0.000469	+0.003409	±0.000877	-0.001248	±0.000945
7976.580	+0.000685	±0.000589	-0.007100	±0.000577	+0.000352	±0.001001	-0.001043	±0.001134
7978.497	+0.000564	±0.000564	-0.001591	±0.000297	+0.000586	±0.000979	-0.001556	±0.000978
7985.473	-0.000732	±0.000471	-0.001659	±0.000471	+0.002004	±0.000887	-0.002343	±0.000899
8002.424	+0.000958	±0.000423	-0.001044	±0.000397	+0.000425	±0.000905	-0.001443	±0.000916
8042.473	-0.001229	±0.000491	-0.002520	±0.000479	+0.001497	±0.001186	-0.003090	±0.001078



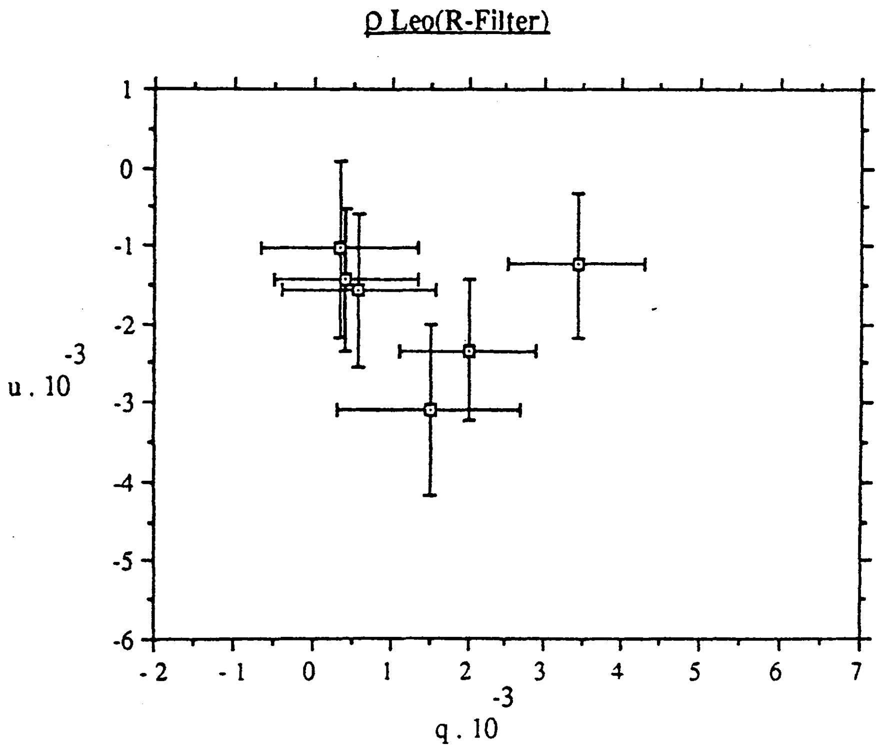


Fig. 4.12 Plot of the polarization of ρ Leo in the q, u plane.

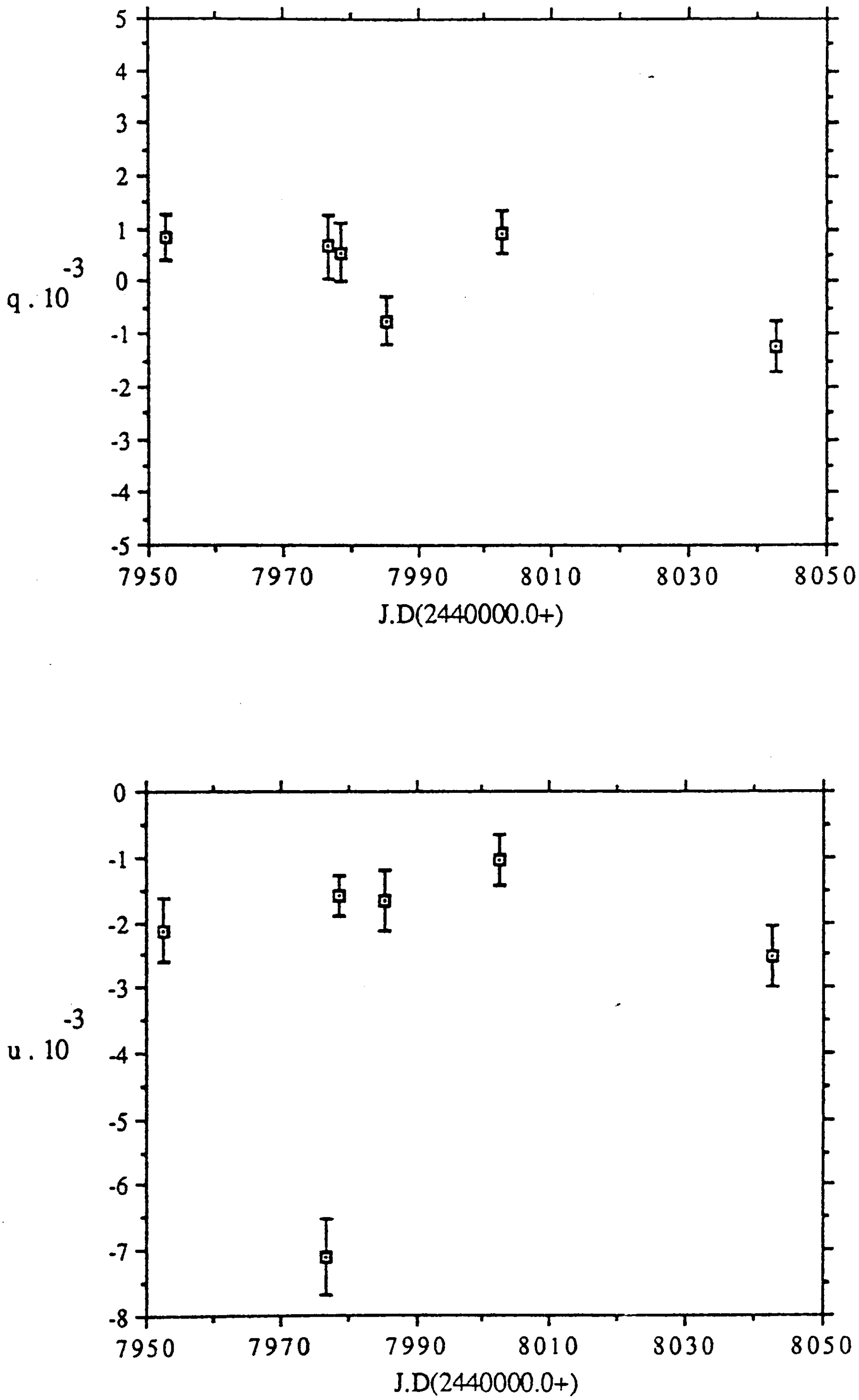


Fig. 4.13a Time Dependence of the Normalized Stokes Parameters q and u for ρ Leo at B-band.

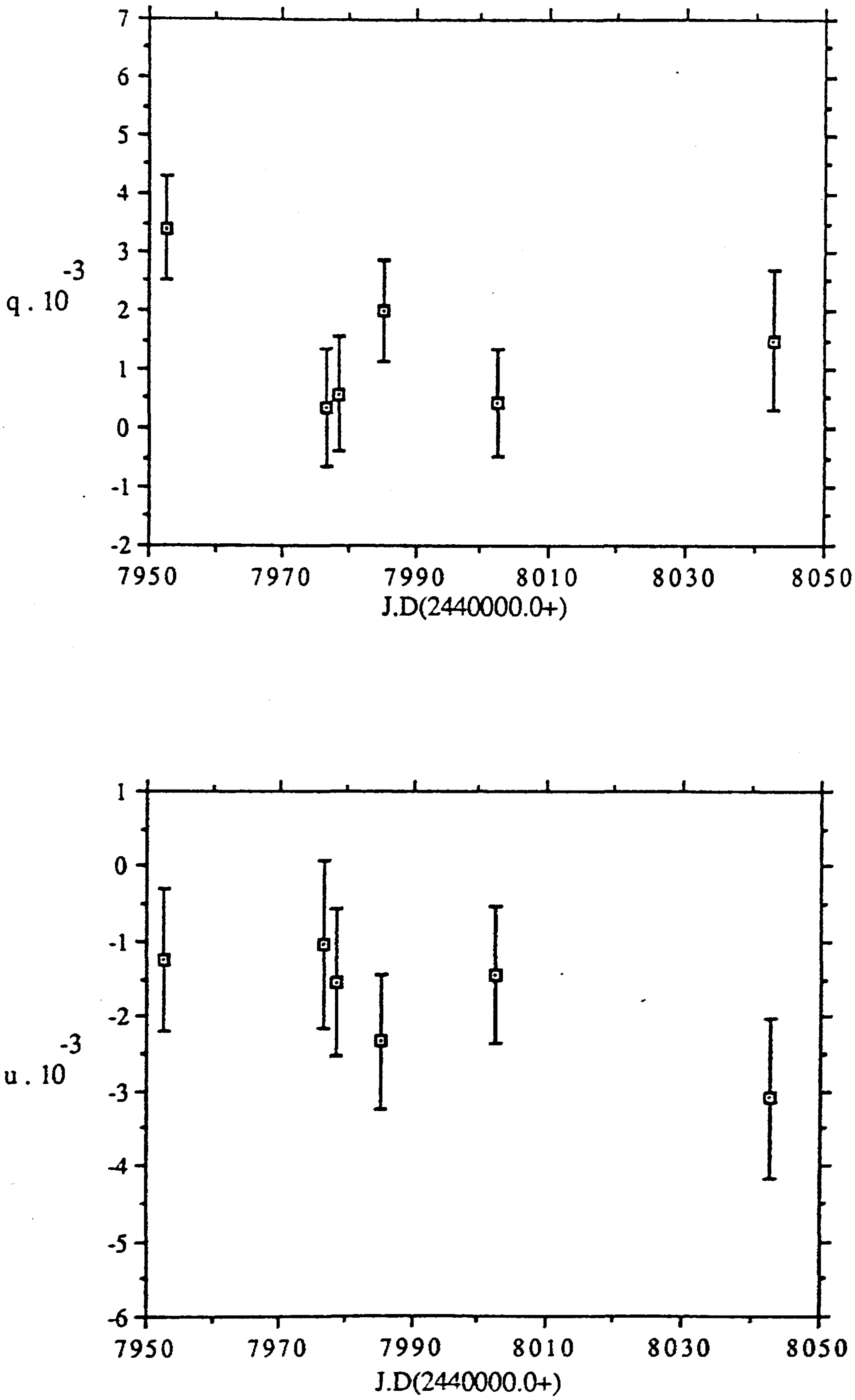


Fig. 4.13b Time Dependence of the Normalized Stokes Parameters q and u for ρ Leo at red band.

Table 4.9
Polarization of some Stars (Instrumental Frame)

Name	q_B	σ_q	u_B	σ_u	q_R	σ_q	u_R	σ_u	J.D(2447000+)
ξ Cas	-0.002727	±0.000549	-0.006877	±0.000555	-0.006174	±0.001516	-0.002586	±0.001763	835.513
	-0.002858	±0.000441	-0.006136	±0.000403	-0.001381	±0.001399	-0.007284	±0.001207	857.484
α Per	+0.001898	±0.000567	-0.003057	±0.000560	+0.001942	±0.000719	-0.002512	±0.000735	835.665
	+0.002454	±0.000347	-0.003813	±0.000343	+0.001749	±0.000549	-0.003397	±0.000544	857.548
η Per	+0.008374	±0.000663	-0.003807	±0.000598	+0.007167	±0.000762	-0.005200	±0.000752	835.638
	+0.006560	±0.000572	-0.004409	±0.000545	+0.007958	±0.000643	-0.004101	±0.000680	857.635
δ Per	+0.002520	±0.000641	+0.001593	±0.000526	+0.001252	±0.000939	+0.001792	±0.000998	835.758
	+0.002161	±0.000419	+0.001143	±0.000386	+0.000799	±0.000708	+0.001188	±0.000694	857.666
θ Gem	+0.009411	±0.001121	+0.015240	±0.000948	-0.017941	±0.007336	+0.031839	±0.007963	835.729
	+0.007940	±0.001025	+0.018942	±0.001077	-0.000796	±0.002782	+0.013037	±0.002998	952.441
σ Cas	-0.011332	±0.000459	-0.004530	±0.000452	-0.010907	±0.001178	-0.005473	±0.001329	857.452
ε Cas	+0.002598	±0.000345	-0.003952	±0.000343	+0.004091	±0.000709	-0.002000	±0.000718	858.271
29 Per	-0.000290	±0.000517	-0.006378	±0.000484	-0.000987	±0.001242	-0.007071	±0.001375	857.577
σ Per	+0.001210	±0.000657	+0.000885	±0.000591	+0.000813	±0.000839	+0.001100	±0.000800	857.715
ψ' Per	-0.000448	±0.000443	-0.000381	±0.000411	-0.001259	±0.001044	-0.001296	±0.000967	857.690
χ^2 Ori	+0.006664	±0.000512	+0.024356	±0.000490	+0.000890	±0.001094	+0.026945	±0.001095	952.339
48 Per	+0.005137	±0.000441	+0.007121	±0.000459	+0.003908	±0.000932	+0.009606	±0.000934	857.741
ζ Tau	-0.009503	±0.000423	+0.006283	±0.000464	-0.009748	±0.000813	+0.002472	±0.000768	835.694
	-0.013122	±0.000419	+0.008747	±0.000410	-0.010412	±0.000731	+0.003450	±0.000748	952.339
	-0.012476	±0.000409	+0.009022	±0.000383	-0.010807	±0.000743	+0.003884	±0.000710	952.404
	-0.010847	±0.000580	+0.007917	±0.000538	-0.009571	±0.000916	+0.003155	±0.000895	952.470
	-0.013205	±0.000426	+0.008617	±0.000429	-0.010733	±0.000774	+0.002855	±0.000777	976.365
	-0.013339	±0.000428	+0.008396	±0.000463	-0.011441	±0.000735	+0.003406	±0.000708	976.397

4.4 Conclusion

Two channel polarimetry has been performed on previously established standard polarization stars. The instrumental polarization was corrected by observing "zero" polarization stars and this was subtracted from the data.

The results of this investigation suggests that there are polarization variations in all the observed stars. Even though most of the samples are too small for rigorous statistical analysis (i.e. search for periodicity in the data), it appears by inspection that all these stars are variables, although in light of criticisms laid before other workers some caution must be applied to the statement. Many more observations are required in a better location than Glasgow.

There exists a definite polarization variability in 55 Cyg and ϕ Cas. The temporal polarimetric variability of these two stars is presumably due to the changes in their intrinsic polarization. Since, the process of finding stars with large interstellar polarization is biased towards supergiants (luminous) stars which would be visible over greater distances, we would expect these early type supergiants stars to have some small variations in their polarizations.

The polarization of 55 Cyg appears to vary over a period of few weeks rather than a few days as suggested by spectroscopy (Underhill, 1960 and Granes et al., 1971). Observations of 55 Cyg on 4 consecutive nights indicate that there were no changes in the magnitude of the polarization but there might be a rotation of position angle by 1°. Further investigations of 55 Cyg and ϕ Cas are definitely warranted. Not only would constraints be developed

as to their usefulness as standards but the fundamental cause of the variability might be established and modelled. To check this we have to observe these stars over a long period of time.
Most of the stars measured showed a significant change in polarization values from those of other investigators. Thus it is concluded that most of the stars in our survey exhibited a variable polarization. Therefore whole area of establishing accurate polarization standards for use in observations are currently wide open.

Chapter 5: Overall Conclusion and Future Work

5.1 Position Angle Statistics

5.2 Polarimetric Standard Stars

5. Overall Conclusion and Future Work

5.1 Position Angle Statistics

Serkowski (1958, 1962) demonstrated that observational noise introduces a bias in the determination of the degree of polarization, p. The statistical behaviour of polarization has been studied in great detail by Simmons and Stewart (1985). But in general, most polarimetric papers treat the assessment of polarimetric uncertainties (errors) in a half-hearted manner (see Chapter 2, Section 2.3). Most investigators apply standard formulae in calculating the uncertainties of the position angle assuming that repeated measurements follow a normal distribution.

Our investigation of the statistical properties of the polarization position angle (θ) was based on reviewing some previously published works in this area (Serkowski, 1958, 1962, Vinokur, 1965 and Wardle and Kronberg, 1974) and have shown that the quoted normal procedure resulting in confidence intervals in position angle values were inadequate for low signal-to-noise ratios. We have evaluated the precise confidence interval of position angle and showed that at low levels of signal-to-noise ratios the position angle distribution can not be represented by a Gaussian distribution.

Accurate confidence intervals for θ at the 68.26% (1σ), 95.45% (2σ) and 99.75% (3σ) levels were constructed using derived probability distribution of position angle (see Appendix A). The constructed confidence interval of θ was compared with a Gaussian distribution, and to all intents and purposes the distribution of θ for signal-to-noise ratio greater than 5 can be assumed to be Gaussian. However for low signal-to-noise levels the confidence intervals of θ are widely different than those associated with a Gaussian distribution. It is therefore, recommended that when quoting the uncertainties on position angle at low levels of signal-to-noise the true distribution of θ should be used rather than the Gaussian distribution.

A data simulation method was undertaken in order to see the behaviour of the differences of two position angle values, since the analytical solution would be too complex to handle (see Section 2.2). It was found that even at high signal-to-noise ratios the Gaussian distribution overestimated the real values according to our simulated distribution.

From the investigation undertaken, we were able to show that the distribution of p and θ are complicated than the Gaussian distribution assumed in the literature. It should also be noted that if the values of p are uncorrected for statistical bias, due to observational noise, the uncertainty of the position angle will also be biased. Due to these biasing effects it is preferable, therefore, to use the NSPs, q and u, instead of p and θ.

5.2 Polarimetric Standard Stars

The advance of the instrumentation in stellar polarimetry in the past few decades has made the accuracy of polarimetric measurements on the levels of 0.01% possible. There are, however, numerous considerations to be made when such an accuracy is pursued, for example, are the instrumental polarization, the accurate determination of the position angle offset, etc. (see Hsu and Breger, 1982).

The first systematic survey of polarimetric standard stars was performed by Serkowski (1974a). His results on these stars have remained a standard reference since that time. Since their establishment few investigators have scrutinized these stars. There are shortcomings in the work done by few authors (Tinbergen, 1979, 1982, Hsu and Breger, 1982, Dolan and Tapia, 1986 and Bastien et al. 1988) on Serkowski's list of standard stars. In many cases subjective approaches rather than a thorough statistical analysis is adopted by these authors to decide whether certain stellar sources show temporal polarimetric variability.

There are also several improvements needed for the establishment of polarimetric standard stars. We discussed areas where observational and statistical techniques, should be improved upon to establish polarimetric standard stars in future, so that these could be used for calibration purposes.

Polarization Standard Stars

The problem of finding accurate polarized standard stars is a difficult one indeed. We suggested that several improvements were needed to the existing set of polarized standards (e.g. excluding giant and supergiant stars from any lists). For any given list, polarized standard stars must possess large interstellar polarization, cover a wide range of apparent magnitudes, have a well defined position angles in different wavelength regions and these stars must be continuously monitored. It is also important to have correct statistical interpretation of the data.

Our initial statistical study of the data presented by Bastien et al. (1988) resulted in modifying some of their conclusions. It was seen (see Chapter 3, Section 3.2.1) that Bastien et al. (1988)

subjectively concluded that 11 out of 13 stars were unsuitable as standards due to intrinsic polarization variability. Their statistical analysis, however, was not performed with proper confidence levels, thus exaggerating their results. The independent statistical analysis of their data undertaken by us revealed that the data of only 3 (ϕ Cas, HD161056 and HD111613) stars can be rejected (not originating from a normal distribution) at 99% confidence level.

Our observations of two well known polarization standard stars 55 Cyg and ϕ Cas, revealed that both stars show a definite polarimetric variability, thus making them unsuitable for any calibration purposes. It should be noted that 55 Cyg (B3 Ia) and ϕ Cas (F0 Ia) are supergiants and as previously stated such stars may exhibit temporal polarimetric variability (due to the presence of a stellar wind).

Unpolarized Standard Stars

We discussed the possibility of the existence of small levels of polarization in unpolarized standard stars. Our statistical analysis of "zero" polarization data revealed that polarization is present at various levels with most stellar classifications (see Chapter 3, Section 3.3). In order to form a more definite conclusion a larger data set comprising measurements of different spectral types is required.

The data on different catalogues of "zero" polarization stars presented by Tinbergen (1979), Huovelin et al. (1985) and Leroy and Le Borgne (1989) was reassessed. In our investigation of the hypothesis that the recorded values represented the expected distribution of "zero" polarization measured with noise, we considered the normalized $\frac{p}{\sigma}$ values from each observation and compared the overall distributions with the expected theoretical distribution (see Section 3.3). To decide if the polarization is limited to any specific stellar types, we performed a Kolmogorov-Smirnov test to see if a correlation between spectral type and polarization existed. Therefore if a particular spectral type fails the K-S test (i.e. the existence of a net polarization) we could subsequently remove these spectral types from any catalogues of unpolarized standard stars.

Using Tinbergen's (1979) polarimetric data we concluded that broad band polarization appears to exist in spectral types F, G and K. Only A-type stars passed the K-S test as coming from "zero" polarization stars.

Inconsistencies in the data obtained by Huovelin et al. (1985) and Leroy and Le Borgne (1989) were found to exist. An analysis of the data of Huovelin et al. (1985) indicated that solar type stars appear to exhibit broad band polarization (failure in K-S test), whereas Leroy and Le Borgne (1989) data shows that the solar type stars come from "zero" polarization stars. New observations of solar type stars must be undertaken in order to resolve the above inconsistency.

We also recommended that data on "zero" polarization standard stars should be presented in the form of q and u and their associated errors rather than biased value of degree of polarization although several debiasing techniques do exist to correct for the bias (see Simmons and Stewart, 1985 and Stewart, 1991).

Future Work

There is scope for improvements in establishing a better set of polarimetric standards. It seems appropriate that with the statistical techniques now available a new polarimetric standard star survey should be undertaken in the near future. We suggest that a small sample of selected stars taken from various catalogues should initially be measured to an extremely high polarimetric precision (very few standard stars are characterized to a precision of say ± 0.005% at present) at different wavelength regions to establish if any time variability exists. These stars should be monitored over a long period of time, these could then form the basis whereby all standard measurements can be referred.

There are several systematic ways in which one can establish further polarized standard stars. It would be essential to avoid stars which show spectroscopic and photometric variability (e.g. binaries and photometric variables, (non) radial pulsators), also stars which show mass loss (i.e. Wolf Rayet, T Tauri, Herbig-Haro and young stars). Stars that have emission lines in their spectrum (i.e. Be stars) should systematically excluded from any survey, since they possess intrinsic polarization. The chosen stars should ideally be located on the galactic plane, to ensure a large degree of polarization.

Observations of unpolarized standard stars should be limited to those stars in the neighbourhood of the Sun (distances typically < 50 pc) and located away from galactic plane. The chosen stars must have normal spectral types (no red variables or supergiants).

The investigatory measurements and statistical analysis of standard stars described in this thesis are sufficiently encouraging to warrant further studies being made with improved polarimetric accuracy. The suggestions of future work hold a wealth of fascinating observational and theoretical studies of polarizations associated with polarimetric standard stars.

Appendices

Appendix A: Confidence Interval of Position Angle

The statistical confidence intervals of position angle were calculated by numerically integrating:

$$F(p,\ \theta) = \frac{e^{-(\frac{p}{2})}}{2\pi} \{ 1 + \Psi\ e^{\Psi^2} (1 + ERF(\Psi))\} \qquad A.1$$

where $p = \frac{P_o}{\sigma}$, $\Psi = \frac{p\mathrm{Cos}\theta}{\sqrt{2}}$, $\theta = 2\phi$ and ERF is the Gaussian Error function.

The probability that θ_i lies within the interval of $\theta - \sigma \le \theta_i \le \theta + \sigma$ is;

$$\int_{\theta-\sigma}^{\theta+\sigma} F(p,\ \theta)\ d\theta = 68.26\% \qquad A.2$$

similar expressions are obtained for 95.45% and 99.75% corresponding to 2σ and 3σ confidence interval respectively, Where p is the signal-to-noise ratio from 0 to 10. This is shown in Table A.1.

From the definition of variance, the confidence interval can also be obtained for a Gaussian distribution. The results in Table A.2 are calculated by;

$$\theta^2_{Gaussian} = \int_{\theta-\sigma}^{\theta+\sigma} F(p,\ \theta)(\theta - \bar{\theta})^2\ d\theta \qquad A.3$$

Therefore $1\sigma_\theta$, $2\sigma_\theta$ and $3\sigma_\theta$ are according to a Gaussian distribution.

Table A.1

Obtained from Eq. A.2

Signal-to-Noise($\frac{P}{\sigma}$)	68.26%($1\sigma_\theta$)	95.45%($2\sigma_\theta$)	99.75%($3\sigma_\theta$)
0.000	61.434	85.050	89.775
0.100	58.284	84.377	89.744
0.200	54.898	83.578	89.707
0.300	51.344	82.625	89.663
0.400	47.713	81.485	89.610
0.500	44.105	80.121	89.546
0.600	40.616	78.485	89.468
0.700	37.321	76.531	89.372
0.800	34.271	74.207	89.253
0.900	31.491	71.476	89.106
1.000	28.985	68.318	88.923
1.100	26.742	64.756	88.692
1.200	24.744	60.868	88.401
1.300	22.966	56.783	88.030
1.400	21.385	52.665	87.555
1.500	19.978	48.674	86.944
1.600	18.722	44.935	86.152
1.700	17.599	41.521	85.121
1.800	16.593	38.456	83.776
1.900	15.687	35.733	82.019
2.000	14.870	33.326	79.736
2.100	14.129	31.199	76.806
2.200	13.457	29.318	73.141
2.300	12.844	27.649	68.751
2.400	12.283	26.161	63.811
2.500	11.769	24.830	58.662
2.600	11.296	23.633	53.690
2.700	10.859	22.552	49.181
2.800	10.455	21.571	45.254
2.900	10.080	20.676	41.902
3.000	9.731	19.858	39.054
3.200	9.102	18.413	34.540
3.500	8.299	16.621	29.738
3.800	7.627	15.164	26.332
4.000	7.237	14.333	24.541
4.500	6.419	12.620	21.105
5.000	5.767	11.284	18.602
5.500	5.237	10.210	16.673
6.000	4.796	9.326	15.129
6.500	4.424	8.585	13.859
7.000	4.106	7.955	12.794
7.500	3.830	7.412	11.886
8.000	3.590	6.939	11.102
9.000	3.189	6.155	9.814
10.000	2.869	5.531	8.799

Table A.2

Obtained from Eq A.3

Signal-to-Noise($\frac{p}{\sigma}$)	$1\sigma_{\theta Gaussian}$	$2\sigma_{\theta Gaussian}$	$3\sigma_{\theta Gaussian}$
0.000	51.961	103.923	155.884
0.100	49.966	99.931	149.897
0.200	47.943	95.886	143.829
0.300	45.904	91.808	137.712
0.400	43.858	87.717	131.575
0.500	41.816	83.633	125.449
0.600	39.389	78.777	118.166
0.700	37.785	75.570	112.156
0.800	35.816	71.631	107.447
0.900	33.890	67.779	101.669
1.000	32.015	64.031	96.046
1.100	30.201	60.402	90.603
1.200	28.453	56.907	85.360
1.300	26.779	53.557	80.336
1.400	25.182	50.364	75.546
1.500	23.667	47.334	71.001
1.600	22.237	44.473	66.710
1.700	20.892	41.785	62.677
1.800	19.635	39.270	58.906
1.900	18.464	36.929	55.393
2.000	17.379	34.757	52.135
2.100	16.375	32.751	49.126
2.200	15.452	30.903	46.355
2.300	14.604	29.208	43.812
2.400	13.828	27.655	41.483
2.500	13.118	26.236	39.355
2.600	12.471	24.941	37.412
2.700	11.880	23.760	35.640
2.800	11.341	22.682	34.024
2.900	10.849	21.699	32.548
3.000	10.400	20.800	31.200
3.200	9.611	19.221	28.832
3.500	8.646	17.292	25.938
3.800	7.874	15.749	23.623
4.000	7.439	14.878	22.317
4.500	6.549	13.098	19.647
5.000	5.858	11.716	17.574
5.500	5.303	10.606	15.909
6.000	4.846	9.692	14.538
6.500	4.463	8.926	13.389
7.000	4.137	8.273	12.410
7.500	3.855	7.710	11.566
8.000	3.610	7.220	10.830
9.000	3.203	6.407	9.610
10.000	2.879	5.759	8.638

Appendix B: Confidence Interval of Differences in Position Angle

The confidence limits were obtained by data simulation method, where 2000 randomly generated numbers were used. This is compared with the Gaussian value obtained in Appendix A.
Column 1 is the signal-to-noise ratio, Column 2, 3 and 4 are the confidence intervals of position angle at 99%, 95% and 68.2% respectively, Column 5, 6 and 7 are the Gaussian values at 3σ, 2σ and 1σ respectively ($\sigma^2_{\theta Gaussian} = (\sigma^2_{\theta_R} + \sigma^2_{\theta_B})$).

Table B.1

The hypothesized case

$$\bar{q}_B = \bar{q}_R \quad \text{and} \quad \sigma_q \neq \sigma_u$$

Table B.2

The hypothesized case

$$\bar{q}_B \neq \bar{q}_R \quad \text{and} \quad \sigma_q \neq \sigma_u$$

for two extreme cases.

Table B.1

Signal-to-Noise	99%	95%	68.2%	3σ	2σ	1σ
0.100	142.115	110.553	63.390	178.028	118.685	59.343
0.200	147.565	114.218	63.420	172.950	115.300	57.642
0.300	146.315	112.744	61.642	167.897	111.931	55.966
0.400	143.874	110.399	59.013	162.901	108.601	54.300
0.500	142.051	108.094	56.502	157.995	105.330	52.665
0.600	138.897	104.683	54.216	152.276	101.517	50.759
0.700	135.696	102.865	52.080	148.574	99.049	49.525
0.800	133.609	100.580	50.326	144.117	96.078	48.039
0.900	129.229	97.771	48.661	139.862	93.241	46.621
1.000	126.618	95.118	45.989	135.830	90.553	45.276
0.200	110.545	91.833	56.398	152.987	101.991	50.996
0.400	114.622	93.717	53.030	141.527	94.352	47.176
0.600	109.837	89.370	46.530	129.156	86.104	43.052
0.800	106.143	84.459	41.048	119.428	79.618	39.809
1.000	102.766	79.741	36.226	109.284	72.856	36.428
1.200	98.416	72.971	32.299	100.022	66.682	33.341
1.400	94.433	67.279	29.446	91.790	61.193	30.597
1.600	90.653	61.987	26.956	84.666	56.444	28.222
1.800	85.768	56.990	24.806	78.664	52.442	26.221
2.000	80.668	52.666	23.329	73.731	49.154	24.577
0.300	98.207	85.684	52.240	141.202	94.134	47.067
0.600	99.977	85.654	44.568	122.216	81.477	40.739
0.900	96.578	79.332	35.903	106.348	70.899	35.449
1.200	91.753	68.830	28.991	90.883	60.589	30.294
1.500	86.081	58.173	24.328	77.554	51.702	25.851
1.800	74.381	47.023	20.978	66.658	44.439	22.219
2.100	65.947	41.000	18.887	58.196	38.797	19.399
2.400	54.440	36.032	17.059	51.906	34.604	17.302
2.700	47.742	32.788	15.840	47.367	31.578	15.789
3.000	43.014	30.417	14.897	44.123	29.415	14.708
0.400	93.760	83.509	48.194	133.454	88.969	44.485
0.800	94.957	79.982	37.544	109.740	73.160	36.580
1.200	90.292	68.291	28.147	88.229	58.820	29.410
1.600	80.322	51.665	21.719	70.344	46.896	23.448
2.000	64.568	39.908	17.849	56.711	37.807	18.904
2.400	50.892	32.756	15.449	47.105	31.403	15.702
2.800	41.267	28.508	13.801	40.690	27.127	13.563
3.200	35.344	25.432	12.519	36.460	24.307	12.153
3.600	31.965	23.269	11.513	33.595	22.397	11.198
4.000	29.420	21.939	10.887	31.561	21.041	10.520
0.500	91.185	81.094	44.237	126.674	84.449	42.225
1.000	91.477	74.219	31.557	97.640	65.094	32.547
1.500	83.561	55.514	22.664	73.144	48.762	24.381
2.000	65.044	38.998	17.142	55.018	36.679	18.339
2.500	45.967	30.161	14.115	43.100	28.734	14.367
3.000	34.415	24.729	12.151	35.809	23.873	11.936
3.500	29.956	21.701	10.815	31.331	20.887	10.444
4.000	26.754	19.764	9.871	28.406	18.937	9.469
4.500	24.599	18.294	9.183	26.360	17.573	8.787
5.000	23.100	17.291	8.620	24.854	16.569	8.285

0.600	89.712	79.886	41.010	119.057	79.371	39.686
1.200	89.088	67.466	27.400	86.589	57.726	28.863
1.800	73.295	44.516	18.696	60.673	40.449	20.224
2.400	48.918	30.900	14.309	43.957	29.304	14.652
3.000	34.091	24.140	11.680	34.421	22.947	11.473
3.600	27.867	20.259	10.049	29.016	19.344	9.672
4.200	23.945	17.793	8.971	25.673	17.115	8.558
4.800	21.657	16.223	8.229	23.408	15.605	7.803
5.400	19.937	15.074	7.629	21.778	14.519	7.260
6.000	19.037	14.178	7.186	20.560	13.707	6.853
0.700	88.416	77.551	37.424	114.033	76.022	38.011
1.400	84.966	58.948	23.693	76.559	51.039	25.520
2.100	61.654	37.046	16.048	50.669	33.779	16.890
2.800	37.467	25.455	12.079	36.216	24.144	12.072
3.500	27.999	20.252	9.931	28.249	18.832	9.416
4.200	22.887	17.054	8.501	24.531	16.354	8.177
4.900	20.416	15.324	7.706	21.824	14.549	7.274
5.600	18.389	13.800	6.990	19.945	13.297	6.648
6.300	17.047	12.812	6.486	18.578	12.385	6.193
7.000	16.068	12.184	6.095	17.550	11.700	5.850
0.800	87.988	75.861	34.358	107.992	71.994	35.997
1.600	79.039	50.548	20.566	67.583	45.055	22.528
2.400	47.747	30.221	13.734	42.873	28.582	14.291
3.200	30.608	21.389	10.332	30.799	20.533	10.266
4.000	23.155	17.067	8.579	24.806	16.538	8.269
4.800	19.654	14.745	7.414	21.305	14.203	7.102
5.600	17.533	13.024	6.573	19.003	12.668	6.334
6.400	15.896	11.982	6.060	17.388	11.592	5.796
7.200	14.647	11.100	5.642	16.207	10.805	5.402
8.000	13.844	10.436	5.318	15.316	10.211	5.105
0.900	86.980	73.125	31.696	102.122	68.081	34.040
1.800	71.007	41.988	17.571	59.684	39.789	19.895
2.700	37.369	25.032	11.688	36.913	24.609	12.304
3.600	24.949	18.227	8.924	26.887	17.925	8.962
4.500	19.886	14.748	7.330	21.871	14.581	7.290
5.400	16.902	12.685	6.339	18.849	12.566	6.283
6.300	15.104	11.311	5.693	16.837	11.225	5.612
7.200	13.743	10.352	5.220	15.418	10.279	5.140
8.100	12.727	9.620	4.867	14.378	9.585	4.793
9.000	11.926	9.078	4.603	13.580	9.060	4.530
1.000	85.865	70.359	29.125	96.434	64.289	32.144
2.000	60.662	35.795	15.444	52.846	35.231	17.615
3.000	30.961	21.520	10.350	32.373	21.582	10.791
4.000	21.704	15.951	7.911	23.931	15.954	7.977
5.000	17.310	12.973	6.545	19.582	13.055	6.527
6.000	14.981	11.239	5.677	16.911	11.274	5.637
7.000	13.252	10.012	5.065	15.120	10.080	5.040
8.000	12.150	9.158	4.664	13.853	9.235	4.618
9.000	11.348	8.589	4.359	12.922	8.614	4.307
10.000	10.695	8.097	4.104	12.216	8.144	4.072

Table B.2

Signal-to-Noise	99%	95%	68.2%	3σ	2σ	1σ
1.000	95.625	78.170	33.192	96.434	64.289	32.144
2.000	67.825	39.252	16.976	52.846	35.231	17.615
3.000	33.669	23.359	11.394	32.373	21.582	10.791
4.000	24.220	17.623	8.753	23.931	15.954	7.977
5.000	19.595	14.611	7.230	19.582	13.055	6.527
6.000	16.270	12.310	6.185	16.911	11.274	5.637
7.000	14.798	11.047	5.554	15.120	10.080	5.040
8.000	13.164	10.028	5.058	13.853	9.235	4.618
9.000	12.445	9.460	4.821	12.922	8.614	4.307
10.000	11.814	8.964	4.509	12.216	8.144	4.072
1.000	85.699	71.179	29.617	96.434	64.289	32.144
2.000	67.356	42.009	18.297	52.846	35.231	17.615
3.000	37.503	25.440	11.916	32.373	21.582	10.791
4.000	25.296	18.507	9.001	23.931	15.954	7.977
5.000	19.686	14.562	7.339	19.582	13.055	6.527
6.000	16.729	12.417	6.288	16.911	11.274	5.637
7.000	14.783	11.205	5.619	15.120	10.080	5.040
8.000	13.828	10.126	5.163	13.853	9.235	4.618
9.000	12.584	9.507	4.846	12.922	8.614	4.307
10.000	11.848	8.930	4.537	12.216	8.144	4.072

Appendix C: Dead-Time Correction for Photon Counting Losses

Polarization measurements are recorded by using photon-counting technique. The technique's greatest drawback is its inability to handle high counting rates in measuring the brighter stars. Since the electronic circuits of the amplifier units have a finite ability to resolve closely spaced pulses, thus counting more than one close pulse as a single pulse. Therefore, the data should be corrected in order to remove its effects.

A widely used formula for transforming the observed counting rate (n) to the true counting rate (N) is;

$$n = N\ e^{-(tN)} \qquad \text{C.1}$$

expanding the exponential term

$$n = N(\ 1 - tN + \frac{(tN)^2}{2!} \ldots\ldots\ldots) \qquad \text{C.2.}$$

This is approximated to

$$n = N(\ 1 - tN) \qquad \text{C.3}$$

where t has the dimensions of time and is usually referred to as the "dead time" of the system (see Fernie, 1976, Henden and Kaitchuck, 1982).

The "dead time" correction for our system was performed by measurements of bright and faint sources. The technique for finding t takes advantage of the fact that for low count rates the dead time correction is negligible. Our technique consisted of placing a white sheet of paper in front of the telescope aperture (to obtain uniform brightness). A tungsten lamp was used for illumination and various light intensity levels were achieved by changing the position of the lamp. The diaphragms in the photometer head were used, where 17 thou was used to measure n_L and 30 thou to measure n_H. Depending on the ratio of diaphragm areas we can obtain a relation whereby;

$$\frac{n_H}{n_L} = \frac{N_H}{N_L} = A \qquad \text{C.4}$$

$$\therefore \qquad \frac{n_H}{n_L} = \frac{N_H\ (\ 1 - tN_H\ )}{N_L\ (\ 1 - tN_L\ )} = \frac{N_H - tN_H^2}{N_L} \qquad \text{C.5}$$

where it is expected that $n_L = N_L$ for low counts.

Thus

$$\frac{n_H}{n_L} = A - AtN_H \qquad \text{C.6.}$$

At this stage we make an assumption, where we let $n_H = N_H$ and get an estimate for t. Applying an iterative technique to Eq. C.3 we can obtain a better estimate of t (dead time).

The dead time (t) used for our observations were 265 nanoseconds for Blue channel and 83 nanoseconds for Red channel.

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