PROPERTIES OF BOSE-MESONS

by

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Chapter Three, on the production of mesons in nucleon-nucleon collisions, consists substantially of work done conjointly with Professor Gunn, and Dr. B.F.Touschek, and published under the title "The Production of Mesons in Proton-Proton Collisions" by J.C.Gunn, E.A.Power and B.F.Touschek. (Philosophical Magazine, 72 , 328 , 523 May 1951).

The calculations presented are claimed to be original. After the work contained in Chapter Five, on the decay of heavy mesons, had been completed, however, an abstract was received reporting similar computations made by Ozaki (Progress of Theoretical Physics, Japan, 5 , 1950).

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ABSTRACT.

The Thesis contains an analysis of the properties of the π- meson. Current meson theory is used in an attempt to distinguish between possible meson types.

After a survey of the experimental data involving mesons as real particles, there is a brief summary of the theory involved. An approximate method is proposed for the computation of cross-sections for meson processes involving matrix elements over exact nucleon eigenstates.* The validity of this approach, the distorted wave approximation, is considered in detail for spin zero mesons. Comparison is also made with other phenomenological approaches.

The distorted wave approximation is used to calculate cross-sections for the production of π - mesons in nucleon-nucleon collisions under certain simplifying assumptions concerning the nuclear forces. For a final continuous nuclear relative motion, it is found that low energy nucleon states are favoured. This, in the case of a final neutron-proton system, together with a large contribution from transitions to a final bound deuteron, leads to a meson spectrum well peaked at the highest allowed energies in agreement with recent experiments. The total cross-section depends more on the shape and size of the inter-nucleon potential than on the meson type, but the spectrum at a given angle, and, more

particularly, the angular distribution, are critically dependent on the parity of the meson produced. A comparison with experiment favours scalar mesons. There is also a consideration of the relatively small cross-section for the production of neutral π- mesons in the observations of simple collisions.

The absorption of π^--mesons by nuclei is then discussed on the assumption that capture takes place from the close shells of the meson-nuclear system. Detailed calculations are presented for the direct capture by heavy nuclei, and for various capture processes in Deuterium. When compared with recent observations, the selection rules for the absorption of mesons in light nuclei favour pseudoscalar mesons.

The final problem considered is of a different character, and consists of a field theoretical calculation using Pauli regulators with the Feynman technique. It concerns the decay of heavy neutral bosons to two π- mesons through an assumed nucleon coupling. Comparison is made with the various V-meson decays reported in cosmic ray photographs.

Chapter Six summarises the contemporary position of the relation of meson theory in the light of the calculations presented in this paper, with experiment.

(* A paper covering this part of the thesis has been submitted to the Royal Society of London, for publication.)

CHAPTER ONE

INTRODUCTION.

§ 1. The Meson

The existence of the meson was postulated on theoretical grounds by Yukawa (1935) in order to provide theoretical physics with an "explanation" of nuclear forces in terms of concepts acceptable by analogy with electromagnetic theory. The meson was to be a bose particle - the quantum of the nuclear force field - and it was to have a mass of about 150 electron masses in order to give the magnitude of the short range required for the internucleonic forces. In this case, as with other nuclear phenomena in which the meson was to play a part, such as magnetic moments of nucleons, and β-activity of certain nuclei, its rôle was that of a 'virtual' particle.

The concept of virtual particles (see Rosenfeld (1948)), grew, in quantum electrodynamics, when direct interaction between two states of a system was not allowed by the assumed interaction energy, but could take place through intermediate steps. Particles created and annihilated in these intermediate steps were labelled virtual. A very successful application of this idea was in the interchange of virtual longitudinal electromagnetic quanta giving rise to the static Coulomb potential between two charged particles. (Fermi (1932)).

The success went further when Möller showed that the interchange of transverse quanta added the retardation effects to the Coulomb interaction. The well known calculations of quantum electrodynamics, such as that obtaining the Klein-Nishina formula, rest on such concepts. Similar ideas have been applied by many authors to other possible fields, called 'meson fields', with the potential some other irreducible representation of the Lorentz group than the vector, uncharged, and zero rest mass field of quantum electrodynamics. The calculations have always appeared qualitatively reasonable but never quantitatively sound. This has been put down to the fact that no method has been found of obtaining solutions of the fundamental equations except in terms of a power series in the coupling constant (or its inverse). Also, until recently, even the higher order terms in these series have been ambiguous. With the development of the covariant formalisms of Tomonaga-Schwinger-Dyson it has been possible to calculate unambiguously corrections to the zero order approximations and these often turn out large. Dyson (1951) has lately put forward new ideas which may lead to the overcoming of these difficulties and to a real quantitative test of meson field theory.

§ 2. Experimental Evidence and Properties of Mesons.

The history of the discovery of real particles with mass intermediate between electron and proton and short lifetime is well known and will only be briefly given. In 1936 Anderson discovered that, among the particles of the 'hard' or penetrating component of cosmic radiation, there were present particles of mass about 200 electron masses which were able to penetrate many centimetres of lead. These are now known as μ- mesons and measurements by Brode (1949), give their mass as 215 $\pm$ 5 electron masses.§§ These mesons were originally identified with those of Yukawa, a belief strengthened by the observations of Williams and Roberts (1940) and Rossi (1942) who showed that mesons suffer ß-decay with a lifetime $\sim 10^{-6}$ sec., a value only slightly greater than that required by Yukawa in order to account for the ß-decay of certain nuclei. Many experiments showed, however, that the μ- meson seldom interacts with nuclei in passing through matter. Particular difficulty arose over the distinction between positively and negatively charged mesons. Tomonaga and Araki (1940) suggested that the Coulomb barrier between like charges would be sufficient to ensure that the μ^{+}- meson is stopped from reaching the nucleus of some atom it encounters and thus remaining free to suffer ß-decay. On the other hand, it would be expected that the μ^{-}- mesons would interact strongly with nuclei. Experiments showed that for heavy nuclei this was so, for only μ^{+}- mesons showed ß-activity; but for light

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§§ Latest measurements by Lederman, Tinlot, Booth (1951) give 210 $\pm$ 3 electron masses.

nuclei such as carbon, it seems that nearly all mesons, of either charge, decay. For reference to this see Rossi (1948). Marshak and Bethe (1947) therefore concluded that the interaction between mesons and nucleons was much less than that proposed in the Yukawa hypothesis. They suggested that the primary particles of nuclear explosions in cosmic ray bursts were heavy mesons, and that these subsequently decayed to μ- mesons.

This hypothesis of the existence of a heavier meson decaying very rapidly μ- wise was immediately verified by Powell and his co-workers at Bristol. (Lattes, Occhialini and Powell (1947)) with the development of more sensitive photographic plates. The mass of these heavy-charged mesons were estimated by grain counts at 300 electron masses. Latest measurements of their mass (Panofsky, Aamodt and Hadley (1951); Gardner, Barkas, Smith and Bradner (1950)) give $\sim$ 275 $\pm$ 3 electron masses. These mesons, referred to now as π- mesons, when stopped in the photographic emulsion sometimes lead to the emission of a μ- meson of unique energy. The constancy of the velocity of the μ- particles leads to the assumption of a direct coupling between the π- and μ- mesons with one further neutral particle (of very little mass) taking the momentum generated away. Besides the μ- decay mesons in the plates, stars are seen at the end of π- meson paths. These stars result from the strong interaction between the slowed down π- mesons and the nuclei which compose the plate. It was seen that the arguments of Tomonaga and Araki (1940) would substantially agree with observation if applied to these mesons. If the π- mesons are charged positively they suffer μ- decay; if negatively charged they are captured from their K shells about the nuclei of the plates and produce stars. The μ^- mesons observed would then be the result of $\pi^- \to \mu^-$ decay in air, where the π^-- meson, being free, does not interact with nucleons. The π- meson thus appears to play a part much nearer to Yukawa's meson than the μ- meson, and today it is customary to identify the π- meson with the quantum of the nuclear force field.

With the development of high energy particle accelerators it has been possible, during the past three years, to produce mesons under controlled conditions in the laboratory. The results of these experiments confirm the identification of the π- meson with the bose particle of Yukawa - first on account of the primary meson produced being a π- meson and secondly from the striking discrete π^+- meson spectrum following from its production in proton-proton collisions. (Figure 1). The first artificially produced mesons were obtained at Berkeley by the 384 inch synchrocyclotron by Gardner and Lattes (1948). In the past two years other high energy machines have been built for studying the properties of mesons. π- mesons have been produced by fast nucleon collisions (Richman and Wilcox (1949), Cartwright, Richman, Whitehead and Wilcox (1950), Peterson, Iloff and Sherman (1950)), and by highly energetic γ- rays (McMillan, Peterson and White (1949), Bishop, Steinberger and Cook (1950)) incident on various targets. The original experiments were carried out by 390 Mev. beams of α- particles. The proton beam used is obtained from the 184 inch cyclotron which produces 345 Mev. protons. The γ -rays which produce mesons, have come from the 335 Mev. electron synchrotron; others have recently begun operation. These artificially produced mesons have given detailed information about the production of mesons by particles of relatively, compared with cosmic ray primaries, low energies, these being just above the threshold (Barkas (1949)) for π- meson production. New machines in Britain and the United States of America, constructed in order to produce accelerated electron and proton beams of 300-3000 Mev. will enable comparison to be made between production cross-sections from incident particles over a range of energies. These machines may also lead to artificially produced mesons of mass greater than that of the π- meson for which evidence is forthcoming from cosmic ray photographs.

Particles, with mass between that of the π- meson and that of the proton, and with mass greater than that of the proton, have been recently demonstrated in experiments of differing kinds. The properties of these particles are only partially known, and no one can yet say how many types of particle are involved. Only the so-called V (or τ) mesons will be considered here. About 100 of these have been reported by various authors; first by Leprince-Ringuet and Lheritier (1944) and later by Seriff, Leighton, Hsiao, Cowan and Anderson (1950) Rochester and Butler (1947) and Leprince-Ringuet (1949). The mass of these mesons have been estimated roughly to be of the order of 700 - 1000 electron masses. With a very short lifetime, of about 3×10^{-10} sec., these V mesons decay to two particles. The evidence concerning the nature of the decay products is very meagre. Since several appear to produce stars in the photographic plates not unlike π^-- mesons produced stars, and it has been suggested that they are in fact π^- mesons. Thirty of the V mesons obtained in cloud-chamber photographs by Seriff, Leighton, Hsiao, Cowan, Anderson (1950) are neutral particles for which evidence is indirect. These V_o are apparently produced either in the lead block above, or in the lead block within, the chamber. Their mode of decay is to two charged particles of opposite sign whose mass is much greater than that of an electron.

Armenteros, Barker, Butler, Cachon and Chapman (1951) have reported 43 photographs of events, classified as V-events, in the cloud chamber in Pic-du-Midi. Over 90% demand explanation through the existence of heavy particles undergoing spontaneous decay. Conclusive evidence for the existence of a proton as one decay product of some of these neutral particles gives a mass of ~ 2,250 electron masses. The negative decay product is probably a π^-- meson. Some, at least three, of the neutral heavy V_0 decays give a positive decay product which is definitely not a proton and likely to be a meson. Unless the existence of a negative proton is assumed, the decay $V_0 \rightarrow \pi^+ + \pi^-$, as suggested by Anderson et al. for many of their photographs, is a possible explanation, with the mass of this V^0 meson at 1000 electron masses. Photographs of 4 charged heavy meson decays have been examined by these authors and the decay is possible to $\mu + \pi$ (with corresponding mass of heavy meson ~ 2,350 electron masses), or to $\pi^0 + \pi$ (with corresponding mass of heavy meson ~ 920 electron masses).

Returning to considerations of π- mesons, the evidence for the existence of neutral π^0- mesons is now very strong. The first sign of such particles arose during the study of the quanta produced when fast protons strike nuclei. These were reported by Bjorkland, Moyer, Crandell and York (1950). These photons, which appear at the threshold for production of charged π- mesons, give a very intense beam (much more, by a factor 100, than bremsstrahlung expected). The spectrum, in the centre of mass coordinates, is of symmetrical shape with peak at 70 Mev. Steinberger, Panofsky and Steller (1950) showed that these quanta are also produced when high energy gammas impinge on nuclei. They showed further that two photons are produced as the unique decay products of neutral particles produced in a manner similar to charged π- mesons. Carlson, Hooper and King (1950) have examined photographic emulsions exposed in the upper atmosphere, and have found a distribution of gammas (made visible by the electron pairs they produce), which shows that neutral mesons are produced under natural conditions with a frequency similar to that for charged π- mesons. The time of decay has been estimated from the experiments using artificially produced π^0- mesons at $< 10^{-11}$ sec., and by photographic plate measurements at $\sim$ 2.5×10^{-14} sec. The mass of the π^0- meson has been determined by Panofsky, Aamodt and Hadley (1951), at 264.6 $\pm$ 3.2 electron masses, from the absorption of π^- mesons in Hydrogen, where the reaction $\pi^- + p \rightarrow n + \pi^0 \rightarrow n + 2\gamma$ takes place.

Finally, in this summary of the experimental position concerning mesons, a brief résumé of the properties of the π- mesons will be given.

Charge:- The magnitude of the charge on charged π- mesons is that of the fundamental charge on the electron. Bradner (1949) has shown that the consistency of mass determinations obtained by different methods shows that the charge is, to within 3%, that of the electron.

Mass:- The mass quoted above at 275 $\pm$ 5 electron masses, is that given by Panofsky, Aamodt and Hadley (1951) and is well within the range given by Powell (1950) of between 270-290 electron masses from the results of various authors. The method of Panofsky, Aamodt and Hadley (1951) is to measure the position of the single γ- peak of the spectra of gammas produced by the capture in Hydrogen of π^-- mesons. The process involved is $\pi^- + p \rightarrow n + \gamma$ and hence, together with energy-momentum conservation, the position of this peak gives a precise measurement of the meson mass. Barkas, Bishop, Gardner and Lattes (1950) determined the mass of the charged mesons by measuring the momentum of individual particles by magnetic deflection and the residual range in a stopping medium. These experiments were possible for charged π- mesons of both signs and gave the mass of the negative mesons at 280.5 ± 6 electron masses and 278 ± 8 electron masses for that of the positive meson.

Lifetime:- The lifetime for decay μ -wise is quoted by Noyes (1951) as 1 - 3 x 10^{-8} sec. from experiments by Chamberlain, Mozley, Steinberger and Weigand (1950). The results of Kraushaar, Thomas and Henri (1950) give a lifetime of 1.6 x 10^{-8} sec., while those of Martinelli and Panofsky (1950) give 1.97 ± .25 x 10^{-8} sec. The accurate measurements have been made by capturing π- mesons emerging from cyclotron target in a magnetic field and causing them to spiral in a channel cut in a block of metal and then to be incident on photographic plates. If the particles decay in flight the number reaching the plates will be reduced and the lifetime can be deduced from the reduction ratio observed between these and non-decaying particles.

Nuclear Capture:- In heavy elements the capture of π^-- mesons leads characteristically to stars. High energy γ emission occurs in less than 10% of the absorptions in Helium and in less than .5% in Carbon. In Deuterium however γ emission is observed with a frequency comparable with direct nuclear capture. The quanta produced are peaked at 125 Mev. In Hydrogen, where no direct nuclear capture is possible, there are two γ peaks, at 70 Mev. and 130 Mev. The former peak is associated with scattering of charged π^-- meson into a neutral π^0- meson which subsequently decays to two quanta as discussed above in the evidence for the determination of the mass of the π^0- meson.

§ 3. Programme

Since Yukawa's postulate concerning the existence of a heavy quantum of the nuclear force field much evidence for the existence of real particles of intermediate mass has been acquired. It is thus desirable that his suggestion, and the theoretical consequences following from it, could be tested by the computation of lifetimes and cross-sections for certain processes occurring in nature in which mesons play a part as real particles. Only the decay process corresponds field theoretically to a single process in first order. This life-time has been calculated by Chang (1942) and agreement is possible with a small coupling $\pi-\mu \sim 10^{-8}$. The other processes in strict field-theoretic calculation, even in the lowest order, depend on virtual intermediate processes occurring. As has been stated above, no calculation involving heavy bose particles as intermediate virtual particles has been quantitatively successful. It is thus expedient to remove, as far as it is possible, such coneepts from calculations. Any theory which sets out to do this must, of necessity be of a phenomenological nature. Some of the first calculations of meson absorption, such as those by Yukawa and Sakata (1937), reduced the order of the meson field calculation to the first by taking transitions between eigenstates of a differing character - in a manner analogous to the calculations of the photoelectric effect in atomic theory. In the calculations of meson production, those of Foldy and Marshak (1949) give a phenomenological treatment in strict analogy with the bremsstrahlung calculation, in contrast to the third order field theoretical calculation of Morette (1949). In the bremsstrahlung analogy treatment only the actual meson emission is specified by meson field theory; the nucleon scattering process being described from nucleon-nucleon scattering data. In doing so the fact that nuclear forces are, at least in part, due to π - meson exchange is ignored. Similarly the calculations by Gunn, Power and Touschek (1951) of π^+- meson production near the threshold in proton-proton collisions makes use of the ordinary interaction between nucleonic and mesonic fields, but describes the interaction between the nucleons as due to a phenomenological potential. The method employed is similar to that used by various authors, such as Mott (1931), for the discussion of bremsstrahlung. On the face of it, it appears that in doing so, the fact that nuclear forces are due partially to π- meson exchange is ignored again. Here such approaches are considered in detail.

Chapter Two, after giving a brief introduction to meson field theory, gives a partial justification of the Gunn, Power and Touschek method of attack, and shows that, to first order perturbation theory, the method analogous to the distorted wave approximation in atomic physics is valid. In the next two chapters the application of this method to the meson production and the meson absorption problem is considered. The main part of the third chapter is based on the paper by Gunn, Power and Touschek (1951) with an extension to neutral meson production. In both of these chapters comparison is made with the work of other authors and with experiment. Especial consideration is given to the Foldy and Marshak (1949) approach to the production problem.

Chapter Five is concerned with a different problem and the calculation presented there, based on the Pauli regulator technique applied to the Feynman method of calculation of transition probabilities in quantum electrodynamics, is on the possible decay of a heavy bose particle to two like π- mesons. The purpose of this computation is to consider the possible mode of decay of the V_0 meson to two π- mesons.

The final chapter summarises the conclusions of the thesis and attempts an integration of the present day experimental and theoretical knowledge concerning the meson considered as a real particle of nature.

CHAPTER TWO

MESON FIELD THEORY AND THE DISTORTED WAVE APPROXIMATION.

Units.

Throughout the work natural units are used. All physical quantities are in dimensions dependent on length. This is equivalent to replacing $\hbar$ and c by unity in expressions in normal units. For the meson-nucleon couplings Lorentz-Heaviside units of charge are used during the calculations but conversion to normal units is made on final lifetimes and cross-sections.

Greek suffices take the values 1, 2, 3 and 4; English suffices take the values 1, 2 and 3 only. Repeated suffices are summed over their range of values.

(a) Meson Field Theory.

§ 1. Classical.

In Chapter One a meson field was defined as a general field over the quasi-Euclidean metric of special relativity with the potential of the field given by an irreducible representation of the Lorentz group of transformations. The meson field type is defined by the nature of these representations. The elementary types, associated with mesons of spin 0 or 1, are scalar, vector, antisymmetric tensor of order 3 (pseudovector) and antisymmetric tensor of order 4 (pseudoscalar). A transformation of the fundamental metric tensor to the four Krönecker delta can be carried out by the transformation $x_4 = it$. With this, the distinction between covariant and contravariant tensors vanishes and so will be disregarded. The physical properties of the field are deducible from the field Lagrangian L, the integral of a field Lagrangian density $\mathcal{L}_0$ which is a function of the potential $\psi(x)$ and its first space and time derivatives.

$$\mathcal{L}_0 = \mathcal{L}_0 (\psi(x), \partial_x^\nu \psi(x)) \qquad (2,1)$$

The physics of such a field is given by defining the energy-momentum tensor:-

$$T_{\mu\nu} = \int \left\{ \mathcal{L}_0 \delta_{\mu\nu} - \frac{\partial \mathcal{L}_0}{\partial \partial_x^\mu \psi(x)} \frac{\partial \psi(x)}{\partial x_\nu} \right\} dx \qquad (2,2)$$

The components of which give the three stress tensor, energy, momentum and energy current namely

$$T_{ik}, \quad H_0 = -T_{44}, \quad G_k = -iT_{4k}, \quad S_k = -iT_{k4}. \qquad (2,3)$$

The field equations follow from the condition that

$$\int \mathcal{L}_0 \, dx_1 \, dx_2 \, dx_3 \, dx_4$$

shall be stationary under arbitrary variations of the potential This is an Euler variation problem and gives the field equations:-

$$\frac{\partial \mathcal{L}_0}{\partial \psi(x)} = \frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}_0}{\partial \partial^\mu_x \psi(x)} \tag{2,4}$$

which are, by the restriction on $\mathcal{L}_0$, partial differential equations of the second order.

From the field equations it follows that the conservation laws given by

$$\frac{\partial T_{\mu\nu}}{\partial x_\mu} = 0 \tag{2,5}$$

are valid.

From the field equations, it follows also that if the field potential is complex it is possible for the field to be interpreted as a charge carrying field, in so far as invariance under gauge transformations of the first kind

$$\psi \rightarrow \psi e^{i\alpha}$$

being required of $\mathcal{L}_0$, a charge transport three vector and charge density defined by

$$S_\nu = -ie\left\{\frac{\partial \mathcal{L}_0}{\partial \partial^\nu_x \psi(x)} \psi(x) - \frac{\partial \mathcal{L}_0}{\partial \partial^\nu_x \psi^*(x)} \psi^*(x)\right\} \tag{2,6}$$

satisfy the continuity equation Div S = 0.

In developing particular fields it is necessary to choose an invariant Lagrangian to ensure the covariance of the field equations. Another condition, necessary in order that a canonical formalism can be developed, is imposed; namely that the Lagrangian is quadratic in the meson potential.

Differences between charged and uncharged field are for non-interacting fields trivial and only the charged field is considered in detail here.

For the charged meson field described by a scalar or scalar density potential the Lagrangian density is

$$-\left\{\mu^2\psi^\times(x)\psi(x) + \partial^\nu_x\psi^\times(x)\partial^\nu_x\psi(x)\right\} \tag{2,7}$$

and for the charged vector or vector density field it is

$$-\left\{\mu^2\psi^\times(x).\psi(x) + \tfrac{1}{2}\,\mathrm{Curl}\,\psi^\times(x).\mathrm{Curl}\,\psi(x)\right\} \tag{2,8}$$

where μ is a parameter of dimension of an inverse length. The classical equations of motion follow at once and are

$$[\Box^2-\mu^2]\psi_\sigma(x) = 0\ ; \quad [\Box^2-\mu^2]\psi^\times_\sigma(x) = 0 \tag{2,9}$$

That is to say each component of the field potential obeys the Klein-Gordon equation i.e. the Schrödinger equation for a free particle of rest mass μ .

It is usual to go a stage further by introducing a canonical formalism. This immediately distinguishes the time axis of the space-time coordinate system by defining a conjugate field $\pi(x)$ by the equations:-

$$\pi(x) = \frac{\partial\mathcal{L}}{\partial\dot\psi(x)}\ ; \quad \pi^\times(x) = \frac{\partial\mathcal{L}}{\partial\dot\psi^\times(x)} \tag{2,10}$$

With these definitions the energy density can be written

$$\mathcal{H}_0 = \pi(x)\,\dot{\psi}(x) + \pi^*(x)\,\dot{\psi}^*(x) - \mathcal{L}_0(x) \tag{2,11}$$

and the canonical equations describing the field, deducible from the Lagrangian equations (2,4) above, are

$$\dot{\psi}(x) = \frac{\partial \mathcal{H}_0}{\partial \pi(x)}, \quad \dot{\pi}(x) = -\frac{\partial \mathcal{H}_0}{\partial \psi(x)}; \quad \text{conjugate.} \tag{2,12}$$

§ 2. Quantum Mechanical.

The canonical field quantization of Heisenberg and Pauli (1929) on the classical meson fields follows from analogy with the quantization of a system of a finite number of degrees of freedom. The reduction from the continuum of meson field variables can be brought about by enclosing the field inside a finite box and subsequently making a Fourier analysis of the field variables. The new dynamical variables are the amplitudes of the components in the Hilbert space defined by the complete set of Fourier functions. Such canonical quantization is non-covariant and follows formally from the postulates

$$[\psi_\sigma(x), \psi_{\sigma'}(x')] = [\pi_\sigma(x), \pi_{\sigma'}(x')] = 0$$

$$[\pi_\sigma(x), \psi_{\sigma'}(x')] = -i\,\delta_{\sigma\sigma'}\,\delta(x - x') \tag{2,13}$$

holding between the meson field variables, now to be interpreted as quantum mechanical operators. The operators depend only on the spacial coordinates. Here such canonical quantization will be, very briefly, carried through in order to give a basis for the demonstration of the validity of the phenomenological treatment proposed and to prepare for the calculations of the meson-nucleon interaction problems of subsequent chapters.

Quantization Of The Scalar Field By Resolution Into Eigenwaves.

The complex scalar meson potential $\psi(x)$ in the absence of nucleons satisfies the equations (2,9)

$$[\Box^2 - \mu^2]\psi(x) = 0 \; ; \; [\Box^2 - \mu^2]\psi^*(x) = 0 .$$

Assuming the field is enclosed in a volume V, the field is expanded as

$$\psi(\underline{x}, t) = \sum_\lambda q_\lambda(t)\, \phi_\lambda(\underline{x})$$
$$\pi(x, t) = \sum_\lambda p_\lambda(t)\, \phi_\lambda(\underline{x}) \qquad (2,14)$$

where $\phi_\lambda(\underline{x})$ are an orthogonal normalised set of functions in V, vanishing§§ on S the boundary of V, and satisfy the equations

$$(\nabla^2 + k_\lambda^2)\,\phi_\lambda(\underline{x}) = 0$$
$$\int \phi_\lambda^*(\underline{x})\, \phi_\mu(\underline{x})\, d\underline{x} = \delta_{\lambda\mu} \qquad (2,15)$$

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§§ If expansion is in rectangular box, less restrictive conditions, namely periodicity on parallel sides, are sufficient.

It follows immediately that the amplitudes q_λ vary simple harmonically with frequency $\omega_\lambda/2\pi$ where $\omega_\lambda^2 = \mu^2 + k_\lambda^2$. i.e.

$$\ddot{q}_\lambda + \omega_\lambda^2 q_\lambda = 0$$

The boundary condition on the $\phi_\lambda(\underline{x})$ gives an allowed spectrum for k_λ and hence for ω_λ. From the definition of the conjugate field $p_\lambda = \dot{q}_\lambda^*$,

and the Hamiltonian density of the field is

$$\left\{ \mu^2 \sum_{\lambda,\mu} q_\lambda^* q_\mu \phi_\lambda^* \phi_\mu + \sum_{\lambda,\mu} q_\lambda^* q_\mu \underline{\nabla} \phi_\lambda^* . \underline{\nabla} \phi_\mu + \sum_{\lambda,\mu} p_\lambda^* p_\mu \phi_\lambda \phi_\mu^* \right\}$$

Whence the total energy

$$H_o = \sum_\lambda \left\{ \omega_\lambda^2 q_\lambda^* q_\lambda + p_\lambda^* p_\lambda \right\}$$

This is the energy of a set of oscillators the frequency of the λ'th. being $\omega_\lambda/2\pi$.

The field is now quantised in the normal manner. It is assumed that the canonical coordinates of the classical Hamiltonian are now quantum quantities obeying the commutation laws

$$p q - q p = -i$$
$$p^* q^* - q^* p^* = -i \qquad (2,16)$$

The allowed energy states are given by finding a diagonal representation for H_o. A suitable such representation is given by defining

$$q_\lambda = \sqrt{\frac{1}{2\omega_\lambda}}\,(a_\lambda + b_\lambda^\times) \quad ; \quad p_\lambda = \sqrt{\frac{\omega_\lambda}{2}}\, i\,(a_\lambda^\times - b_\lambda) \tag{2,17}$$

where

$$a = \begin{bmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \end{bmatrix}$$

and b similar in a skew space.

The energy operator is now

$$H_0 = \sum_\lambda \omega_\lambda (a_\lambda^\times a_\lambda + b_\lambda^\times b_\lambda + 1)$$

and is diagonal. The eigenvalues of $a_\lambda^\times a_\lambda$ are the positive integers and zero, $\overset{+}{N}_\lambda$ say; the eigenvalues of $b_\lambda^\times b_\lambda$ are the positive integers and zero, $\bar{N}_\lambda$ say. The allowed values of the energy, above that of the vacuum field, are thus

$$\sum_\lambda \omega_\lambda (\overset{+}{N}_\lambda + \bar{N}_\lambda) \tag{2,18}$$

and the stationary states of the field are characterised by the pairs of integers $\overset{+}{N}_\lambda$, $\bar{N}_\lambda$. The energy of the field is that of ($\overset{+}{N}_\lambda$ + $\bar{N}_\lambda$) corpuscles of energy ω_λ summed over λ .
With the same representation the operator for the total charge is diagonal and has eigenvalues

$$e \sum_\lambda (\overset{+}{N}_\lambda - \bar{N}_\lambda) \tag{2,19}$$

Thus the allowed values for the total charge are the values of the charge on $\overset{+}{N}_\lambda$ corpuscles of charge e and $\overset{-}{N}_\lambda$ corpuscles of charge -e, summed over the various λ .

By considerations of these results (2,18; 2,19) they may be interpreted as follows:- Corresponding to each wave , of frequency ω_λ , there are $\overset{+}{N}_\lambda$ mesons of charge e and energy ω_λ and $\overset{-}{N}_\lambda$ mesons of charge -e and energy ω_λ . Depending on the actual resolution carried out other physical quantities may be simultaneous eigenstates with the energy and charge; e.g. in a plane wave resolution the momentum is such a quantity; in spherical wave resolution the angular momentum commutes with the energy. However in any orthogonal resolution the energy and charge can be diagonalized to discrete values, and it is upon this that the particle nature of the meson is demonstrated.

§ 3. Interaction With Nucleons.

The fields so far considered have been vacuum fields. The particle meson interacts with nucleons as the quantum of the field. Nucleons are considered as sources and sinks of mesons as electrons are sources and sinks of photons. In field theory the nucleon is a singularity in the field which can be allowed for by adding terms to the Klein-Gordon equations making them inhomogeneous. This is analogous to adding density terms ($4\pi\rho$, $4\pi \underset{\sim}{j}$) to Maxwell's equations to allow for charge distributions. In this heuristic development the interaction is specified by adding terms involving the nucleon to the Lagrangian density. Bearing in mind the Lorentz invariance required and the assumption made above that the Lagrangian as a function of the meson field contains only the potential and its first space and time derivatives, only two simple terms are available for each field. These interaction terms are given in the appendix 1. for the four fields: they depend on tensors constructed from the nucleon spinor field.

The conjugate field and Hamiltonian density are defined as in the field free case and it follows, neglecting contact terms (Kemmer (1937)), that H the total energy of the field with its interaction with nucleons is $H = H_o + H_i$, where H_i is an interaction energy following from the assumed interaction Lagrangian. The possible energies for the four fields are given in appendix 1.

(b) A PHENOMENOLOGICAL APPROACH TO MESON-NUCLEON INTERACTION PROBLEMS.

§ 1. Introduction

It is well known that in the calculation of bremsstrahlung the phenomenon is accurately described by a second order field theoretic process, namely the scattering of the electron by a centre of force and the emission of the light quantum. That such a reduced order process gives the correct cross-section to the strictly third order process involved is a consequence of the possible, though non-covariant, division of the four vector potential describing the electro-magnetic field into longitudinal and transverse parts. The interchange of virtual longitudinal quanta being responsible for the Coulomb potential which acts as the scattering force; the transverse interaction, being $-e\underset{\sim}{\alpha}.\underset{\sim}{A}$ in the usual notation, having matrix elements corresponding to the emission and absorption of light quanta. Such a division is not possible in meson theory and it can be asked to what extent the calculation of meson production by a description analogous with bremsstrahlung, using the scattering interaction between the nuclear particles involved and a meson interaction, is valid.

The calculation of meson production using the ordinary meson-nucleon interaction together with a nucleon-nucleon potential was carried out for vector mesons by Massey and

and Corben (1939) and has been recently investigated by Foldy and Marshak (1949) for pseudoscalar mesons. These authors assumed a phenomenological rather than mesic force as acting between the nucleons.

The calculation of bremsstrahlung by Mott (1931), again depending on the possible division of the interaction energy between electron and the electromagnetic field into longitudinal and transverse parts, was to compute matrix elements between two electron states, which were not momentum states, but eigenstates of the energy of the electrons including their Coulomb energy in the scattering field. Similarly the calculation near the threshold for the production of π^+- mesons in proton-proton collisions by Gunn, Power and Touschek (1951), detailed calculations of which form chapter three of the thesis, makes use of the ordinary interaction between nucleonic and mesonic fields, but describes the interaction between the nucleons as due to a phenomenological potential.

It is the purpose of this chapter to consider the validity of such approaches in meson theory. Attention is concentrated mainly on the field theoretic problem raised by the use of the potential between nucleons together with the interaction term when the latter is itself responsible, if only in part, for the former. That is to say for a system whose total Hamiltonian is

$$H = H_f + H_n + H_i \qquad (2,20)$$

where H_f is the energy of the meson field, H_n is the energy of the nucleon field and H_i is the interaction energy between them, a system whose Hamiltonian is

$$H = H_f + H_n + H_i + U , \qquad (2,21)$$

where U is the interaction potential between nucleons, has been used. This is done despite U being included in part at least by H_i , which, by allowing virtual meson exchange between nucleons, has matrix elements between states differing only by the states of individual nucleons.

The Foldy and Marshak treatment considers the production process in strict analogy with bremsstrahlung, the stationary states being eigenstates of the Hamiltonian $H_n + H_f$ the H_i and U acting as perturbations. Such a calculation ignores, a priori, contributions which may arise from H_i alone in a third order process. The method used here for the production and absorption problems, after the style of the method of distorted waves, proceeds from stationary states of the system derived from the Hamiltonian $(H_n + U) + H_f$. H_i is here a perturbation acting between these stationary states. In either case the internucleon potential can be taken in the form given by meson theory, or by phenomenological considerations making use of the analysis of nucleon-nucleon scattering data by various authors.

The extension of the treatment involved may lead to some understanding of the ability to separate parts of processes and consider them as distinct - an ability obviously justified in phenomena occurring at macroscopic intervals. The Feynman technique is not used in the formalism as the transformations involved lead to non-point interactions between the two fields and hence there are no simple rules for a vertex as in Feynman diagrams. Attention is confined to the scalar and pseudoscalar fields it being probable, from recent evidence, that the π- meson has zero spin.

§ 2. General Formalism

Let $\mathcal{H}$ be the energy density of the total system

$$\mathcal{H} = \mathcal{H}_f + \mathcal{H}_n + \mathcal{H}_i \tag{2,22}$$

so that

$$H = \int \mathcal{H}(x)\, dx$$

where $\mathcal{H}_f$, $\mathcal{H}_n$ and $\mathcal{H}_i$ are the energy densities of the meson field, nucleon field and interaction respectively. If $\Psi(x)$, $\Pi(x)$ are the Fermi-Dirac spinor field and conjugate, M the mass of the nucleon and γ's are the 4 x 4 Dirac matrices then

$$\mathcal{H}_n = -i\,\Pi(x)\left\{\gamma^4\gamma^k\partial^k_x + M\gamma^4\right\}\Psi(x) . \tag{2,23}$$

With $\psi(x)$, $\pi(x)$ the meson field and conjugate and μ the mass of the meson

$$\mathcal{H}_f = \frac{1}{2}\left\{\mu^2\,\psi(x)\psi(x) + \partial^k_x\psi(x)\,\partial^k_x\psi(x) + \pi(x)\pi(x)\right\}$$

for neutral spin zero meson fields, and

$$\mathcal{H}_f = \mu^2 \psi^x(x)\psi(x) + \partial^k_x \psi^x(x)\partial^k_x \psi(x) + \pi^x(x)\pi(x)$$

for charged spin zero meson fields; these being given above (2,11). The interaction energy density depends on the reflexion properties of the meson field and, ignoring possible differences of type between Fermi-Dirac particles (Yang and Tiomno 1950), the usual association of scalar and pseudoscalar quantities is made. If q, q^x are isotopic operators in the Fermi field corresponding to proton-neutron exchange and the charge units are Lorentz-Heaviside, the possible energy interaction densities are as listed in appendix 1.

The field operators obey the commutation relations

$$[\pi(x), \psi(x')] = -i\delta(x-x'); \quad [\psi(x), \psi(x')] = [\pi(x), \pi(x')] = 0$$

$$[\Pi_\alpha(x), \Psi_\beta(x')]_+ = i\delta_{\alpha\beta}\delta(x-x'), \quad [\Psi_\alpha(x), \Psi_\beta(x')]_+ = [\Pi_\alpha(x), \Pi_\beta(x')]_+ = 0 \quad (2,24)$$

These being given for the meson field in equations (2,13), the nucleon spinor-field commutation laws following from Dirac electron theory. (See Wentzel (1943)).

§ 3. Contact Transformation

In order to show the explicit dependence, to order g^2, of the total Hamiltonian on U a contact transformation of variables after Möller and Rosenfeld (1940) is made to transform out the static interaction V , the first approximation to U . This is defined by the relation

$$X' = e^{iS} X e^{-iS} \tag{2,25}$$

where S is a scalar, between any variable X and its transform X′. The right hand side can be expanded as

$$X + i[S,X] + \frac{i}{2!}[Si,[S,X]] + \cdots \tag{2,26}$$

a proof being given in appendix two (a).
The transformed energy density is given by

$$\begin{aligned} \mathcal{H}' &= \mathcal{H}'_f + \mathcal{H}'_r + \mathcal{H}'_i \\ &= \mathcal{H}_f + i[S,\mathcal{H}_f] + \frac{i}{2!}[Si,[S,\mathcal{H}_f]] + \cdots \\ &\quad + \mathcal{H}_r + i[S,\mathcal{H}_r] + \frac{i}{2!}[Si,[S,\mathcal{H}_r]] + \cdots \\ &\quad + \mathcal{H}_i + i[S,\mathcal{H}_i] + \frac{i}{2!}[Si,[S,\mathcal{H}_i]] + \cdots \end{aligned}$$

S is chosen so that in the transformed Hamiltonian there is no term, except those due to recoil, linear in the coupling constant. For this to be, it is sufficient that S is linear in the coupling constant and obeys the relation

$$\mathcal{H}_i + i[S,\mathcal{H}_f] = 0. \tag{2,27}$$

Neglecting recoil, i.e. H′ for the moment, the term quadratic in the coupling constant is $\frac{i}{2}[S,\mathcal{H}_i]$ (2,28)
and as terms linear in the coupling constant are no longer present in the Hamiltonian, description of all second

order processes must be contained in this term. Such second order processes are scattering of nucleons by nucleons, corresponding to the interchange of virtual mesons, and scattering of mesons by nucleons together with similar processes such as nucleon pair creation by two mesons. Let V and C be the interaction energies, to order g^2, corresponding to nucleon-nucleon scattering and meson-nucleon scattering respectively. V , the first approximation to the accurate nucleon potential U , is independent, while C is quadratic in the meson field operators.

It is easily seen that terms cubic in the coupling constant are given by

$$\frac{2}{3}\, i\,[S, V+C] \tag{2,29}$$

and again all third order processes are contained therein. The term of prime interest in this energy is that giving rise to meson production with scattering of nucleons. Higher terms can be written down immediately, the n'th being

$$2\,\frac{n-1}{n!}\; i\,[S\,i,\,[S\,i, \ldots\; i\,[S, V+C]\ldots] \tag{2,30}$$

where there are n-2 commutators.

In order to determine S the equation which it satisfies is examined in the two cases considered.

a/ <u>Neutral Fields</u>

Here

$$H_f = \frac{1}{2}\left\{\mu^2\varphi^2(x) + |\partial_x^k \varphi(x)|^2 + \pi^2(x)\right\}$$

and letting S be of the form

$$\int A(x')\psi(x')\,dx' + \int B(x')\,\pi(x')\,dx'$$

it follows from the commutation laws for meson and conjugate fields that

$$i\,[S, \mathcal{H}_f] = -A(x)\,\pi(x) - \psi(x)\,(\nabla^2 - \mu^2)\,B(x)$$

where here, and throughout the work, the addition of divergence terms to the energy densities is justified by suitable periodicity conditions on the boundary enclosing the total system. Thus the equation for S is reduced to

$$A(x)\,\pi(x) + \psi(x)\,(\nabla^2 - \mu^2)\,B(x) = \mathcal{H}_i(x) \qquad (2,31)$$

A(x) and B(x) being defined above.

b/ Charged Fields

In this case

$$\mathcal{H}_f = \mu^2\,\psi^\times(x)\,\psi(x) + \partial_x^k\,\psi^\times(x)\,\partial_x^k\,\psi(x) + \pi^\times(x)\,\pi(x)$$

and assuming S to be of the form

$$\int A(x')\,\psi(x')\,dx' + \int B(x')\,\pi(x')\,dx' + \int A^\times(x')\,\psi^\times(x')\,dx' + \int B^\times(x')\,\pi^\times(x')\,dx'$$

the equation determining S is

$$A(x)\,\pi^\times(x) + A^\times(x)\,\pi(x) + \psi(x)\,(\nabla^2 - \mu^2)\,B^\times(x) + \psi^\times(x)\,(\nabla^2 - \mu^2)\,B(x) = \mathcal{H}_i(x) \qquad (2,32)$$

Consideration will be given later to the effect of recoil by calculating the effect of the transformation on the nucleon energy. It will appear that in certain cases this leads to the only interaction giving meson production.

§ 4. Scalar Meson with Scalar Coupling.

As an elementary example the calculation for the neutral scalar meson field with scalar coupling i.e. with

$$\mathcal{H}_i = -ig\,\Pi(x)\gamma^4\Psi(x)\psi(x)$$

is carried through. The equation determining S does not require A(x) for a solution, and A(x) is set to zero. The equation for B(x) is

$$(\nabla^2-\mu^2)B(x) = -ig\,\Pi(x)\gamma^4\Psi(x) \quad .$$

The required solution is obtained by means of the Green's source function for the operator $\nabla^2-\mu^2$, namely

$\dfrac{e^{-\mu|x-x'|}}{4\pi\,|x-x'|}$ which is written as $e(x-x')$, and is

$$B(x) = -ig\int\Pi(x')\gamma^4\Psi(x')\,e(x-x')\,dx'. \qquad (2,33)$$

It follows at once that

$$S = -ig\int\Pi(y)\gamma^4\Psi(y)\bar{\pi}(y')e(y-y')\,dy\,dy' \qquad (2,34)$$

from which V and C are calculated by means of the relation

$$\frac{i}{2}[S, H_i] = V + C$$

Expanding

$$\frac{i}{2}[S, H_i] = -i\frac{g^2}{2}\int e(y-x')\left[\Pi(y)\gamma^4\Psi(y)\,\pi(x'),\,\Pi(x)\gamma^4\Psi(x)\,\psi(x)\right]dx'dx\,dy$$

the commutator under the integral is

$$-i\,\delta(x-x')\,\Pi(y)\gamma^4\Psi(y)\,\Pi(x)\gamma^4\Psi(x)$$
$$+\,\psi(x)\pi(x')\left[\Pi(y)\gamma^4\Psi(y),\,\Pi(x)\gamma^4\Psi(x)\right] .$$

Now the last commutator vanishes on integrating over y; lemma 1a., (appendix 2b.) so that C = 0, as is well known, for, neglecting recoil there is no scattering of scalar mesons in neutral scalar theory. On the other hand

$$V = -\frac{g^2}{2}\int e(x-y)\,\Pi(x)\gamma^4\Psi(x)\,\Pi(y)\gamma^4\Psi(y)\,dy\,dx \qquad (2,35)$$

and is the well known Yukawa potential between nucleons as given by scalar theory.

To find the meson production term similar calculations are carried through for $\frac{2}{3}i[S,V]$ which vanishes identically since it equals

$$-\frac{1}{3}g^2\int e(y'-x')\,e(y-x)\,\pi(x')\left[\Pi(y')\gamma^4\Psi(y'),\,\Pi(x)\gamma^4\Psi(x)\,\Pi(y)\gamma^4\Psi(y)\right]dy\,dy'dx\,dx'$$

This result, in agfeement with that found by McPhee (1949), is that there is no meson production by neutral scalar theory when recoil is neglected. When the scalar field is charged, however, the non-commutativity of the isotopic factors fives a meson scattering term C and also a production term.

For the charged scalar field A(x) can be again set equal to zero and

$$B(x) = -ig\int \Pi(y)\gamma^4 q^\times \Psi(y) e(x-y)\,dy$$

$$B^\times(x) = -ig\int \Pi(y)\gamma^4 q\, \Psi(y) e(x-y)\,dy$$

First V and C are required from the relation $\frac{i}{2}[S,H_i] = V + C$. The commutator $[\Pi(y)\gamma^4 q^\times \Psi(y)\pi(x'), \Pi(x)\gamma^4 q\,\Psi(x)\psi(x)]$ is evaluated from lemma 1. as

$$i\,\Pi(x)(q^\times q - q q^\times)\Psi(x)\delta(x-y)\psi(x)\pi(x') - i\,\delta(x-x')\Pi(y)\gamma^4 q^\times \Psi(y)\Pi(x)\gamma^4 q\,\Psi(x)\,.$$

Using this and similar commutators it follows that

$$V = -\frac{g^2}{2}\int \Pi(y)\gamma^4 q^\times \Psi(y)\Pi(x)\gamma^4 q\,\Psi(x)\, e(x-y)\,dy\,dx \quad + \text{complex conjugate}$$

(2,36)

$$C = \frac{g^2}{2}\int \Pi(x)(q^\times q - q q^\times)\Psi(x)\,\psi(x)\pi(y)\, e(x-y)\,dy\,dx \quad + \text{complex conjugate.}$$

To obtain the meson production, or absorption, term from $\frac{2}{3}i[S, V+C]$, $i[S,V]$ is first considered. Now

$$i[S,V] = -\frac{g^3}{2}\int\big[\Pi(y)\gamma^4 q^\times \Psi(y)\pi(x') + \Pi(y)\gamma^4 q\,\Psi(y)\pi^\times(x'),$$
$$\Pi(y')\gamma^4 q^\times\Psi(y')\Pi(x)\gamma^4 q\,\Psi(x) + \Pi(y')\gamma^4 q\,\Psi(y')\Pi(x)\gamma^4 q^\times\Psi(y')\big]$$
$$e(x'-y)e(y'-x)\,dy\,dy'\,dx\,dx'.$$

To proceed to evaluate the commutators under the integrals use is made of the trivial relation:-

$$[A,\ BC] = A\,[B,C] + C\,[A,B] + [C,B]\,A + [A,C]\,B$$

Hence, neglecting contact terms,

$$i[S,V] = -i\frac{g^3}{2}\int\pi(x')e(x-y')\,e(x-x')\,\Pi(x)(q^\times q - q q^\times)\Psi(x)\Pi(y)\gamma^4 q^\times\Psi(y')\,dy'\,dx'\,dx$$

$$-i\frac{g^3}{2}\int\pi^\times(x')\,e(x-y')\,e(x-x')\,\Pi(x)\,(q^\times q - q q^\times)\,\Psi(x)\,\Pi(y')\gamma^4 q\,\Psi(y')\,dy'\,dx'\,dx$$

+ conjugate complex.

For $i[S,C]$ the term linear in π is contained in

$$\frac{g^3}{2}\int\big[\Pi(y)\gamma^4 q^\times\Psi(y)\pi(x'),\Pi(x)\,(q^\times q - q q^\times)\,\Psi(x)\,\psi(x)\,\pi(y')\big]e(x-y')e(x'-y)\,dy\,dy'\,dx\,dx'$$

and using the commutation laws above this term is

$$-i\frac{g^3}{2}\int\{\Pi(y)\gamma^4 q^\times\Psi(y)\,\Pi(x)\,(q^\times q - q q^\times)\,\Psi(x)\}\pi(y')e(x-y')e(x-y)\,dy'\,dy\,dx$$

and the $\pi^\times$ term similarly is

$$-i\frac{g^3}{2}\int\Pi(y)\gamma^4 q\,\Psi(y)\Pi(x)\,(q q^\times - q^\times q)\,\Psi(x)\,\pi^\times(y')\,e(x-y')\,e(x-y)\,dy'\,dy\,dx.$$

Now

$$\int [\Pi(y)\gamma^4 q^\kappa \Psi(y), \Pi(x)(q^\kappa q - q q^\kappa)\Psi(x)]\pi(y')\, e(x-y')\, e(x-y)\, dy'\, dy\, dx$$

is a contact term, hence the term linear in the meson field to third order in g is $i[S,V]$. It is to be noticed that no higher order terms in g are linear in the meson field operators, for the n'th term is $2\frac{n-1}{n!}i[S;,[S;,[\cdots ;S,[S,V+C]]\cdots]$ which gives rise to terms of order $n-2$ in the meson field operators from the commutator with V, and both terms of order $n-2$ and n from the commutator with C. Terms containing the meson field operators to odd order can give rise to single meson production, or absorption in first order perturbation theory. Two such terms, trilinear in the meson field occur in $\frac{2}{3}i[S,C]$ and are

$$-ig^3\frac{2}{3}\Pi(x)\gamma^4 q^\kappa \Psi(x)\left\{\varphi(x)\underline{I}^2(x) - \varphi^\kappa(x)\underline{I}^\kappa(x)\underline{I}(x)\right\} \quad + \text{ complex conjugate} \tag{2,37}$$

where

$$\underline{I}(x) = \int \pi(x')\, e(x-x')\, dx' \, .$$

Such terms will be discussed in paragraph 7.

§ 5. Pseudoscalar Meson with Pseudoscalar Coupling.

This case follows in exact parallel that given in paragraph 4. Here the interaction energy density is

$$\mathcal{H}_i = -g\,\Pi(x)\gamma^4\gamma^5\Psi(x)\,\psi(x) \qquad \text{(neutral)}$$

$$\mathcal{H}_i = -g\,\Pi(x)\gamma^4\gamma^5 q\,\Psi(x)\,\psi(x) - g\,\Pi(x)\gamma^4\gamma^5 q^*\Psi(x)\,\psi^*(x) \qquad \text{(charged)}$$

and again only B(x) is required being

$$B(x) = -g\int \Pi(x')\gamma^4\gamma^5\Psi(x')\,e(x-x')\,dx' \qquad \text{(neutral)}$$

$$B(x) = -g\int \Pi(x')\gamma^4\gamma^5 q^*\Psi(x')\,e(x-x')\,dx'. \qquad \text{(charged)}$$

Thus

$$S = -g\int \Pi(y)\gamma^4\gamma^5\Psi(y)\,e(y-x')\,\pi(x')\,dx'dy \qquad \text{(neutral)}$$

$$S = -g\int\left\{\Pi(y)\gamma^4\gamma^5 q^*\Psi(y)\pi(x') + \Pi(y)\gamma^4\gamma^5 q\,\Psi(y)\pi^*(x')\right\} \times e(y-x')\,dx'dy. \qquad \text{(charged)}$$

The calculation for V and C proceeds as before giving

$$V = \frac{g^2}{2}\int e(y-x)\,\Pi(y)\gamma^4\gamma^5\Psi(y)\,\Pi(x)\gamma^4\gamma^5\Psi(x)\,dy\,dx \qquad (2,38)$$

$$C = 0 \qquad \text{(neutral)}$$

while

$$V = \frac{g^2}{2}\int \epsilon(y-x)\,\Pi(y)\gamma^4\gamma^5 q^x \Psi(y)\,\Pi(x)\gamma^4\gamma^5 q\,\Psi(x)\,dy\,dx \text{ + complex conjugate}$$

(2,39)

$$C = \frac{g^2}{2}\int \epsilon(y-x)\,\Pi(x)\,(q^x q - q\,q^x)\,\Psi(x)\psi(x)\bar{\psi}(y)\,dy\,dx \text{ + complex conjugate}$$

for charged mesons.

As in the neutral scalar case there is no production term, neglecting recoil, for neutral pseudoscalar mesons with pseudoscalar coupling as V commutes with S. In the case of charged mesons the meson production, or absorption, term linear in the meson field operators, contained only in the third order term, is $i[S,V]$, as the relevant part of $i[S,C]$ is $\frac{1}{2}i[S,V]$. Further terms in $\frac{2}{3}i[S,C]$ are again present and are

$$\frac{2}{3}g^3\,\Pi(x)\gamma^4\gamma^5 q^x\,\Psi(x)\left\{\psi(x)\,I^2(x) - \psi^x(x)\,I^x(x)\,I(x)\right\} \text{ + complex conjugate}$$

(2,40)

§ 6. Recoil Terms

To consider the effect of recoil examination is made of H'_n. For neutral fields this can be calculated in closed form and this is done below. Before proceeding to this it is noted that in the expansion for $\mathcal{H}'_n$ namely

$$\mathcal{H}_n + i[S,\mathcal{H}_n] + \frac{i}{2!}[Si,[S,\mathcal{H}_n]] + \cdots$$

only the second term contains the meson field operators linearly when S contains either, but only one, of the meson or conjugate fields; this being the case for non-derivative couplings. The n'th term containing the meson operator in n-1 th. power.

For neutral fields, S is given by

$$S = \int B(x')\pi(x')\,dx'$$

where

$$B(x) = -ig\int \Pi(x')\gamma^4\Psi(x')\,e(x-x')\,dx' \qquad \text{(scalar)}$$

$$= -g\int \Pi(x')\gamma^4\gamma^5\Psi(x')\,e(x-x')\,dx'. \qquad \text{(pseudoscalar)}$$

Continuing the calculation

$$i[S, H_n] = -i\int [B(x')\pi(x'), \Pi(x)\{i\gamma^4\gamma^k\partial_x^k + iM\gamma^4\}\Psi(x)]\,dx'$$

$$= \int \pi(x')\,[B(x'), \Pi(x)\gamma^4\gamma^k\partial_x^k\Psi(x)]\,dx'$$

$$+ M\int \pi(x')\,[B(x'), \Pi(x)\gamma^4\Psi(x)]\,dx'$$

In the scalar case the commutators are evaluated by the use of lemmas 1 and 2, giving

$$i[S, H_n] = g\,I(x)\{\Pi(x)\gamma^k\partial_x^k\Psi(x) - \partial_x^k\Pi(x)\gamma^k\Psi(x)\}$$

where

$$I(x) = \int \pi(x')\,e(x-x')\,dx'.$$

In the same manner

$$\frac{i}{2!}\left[S;,\left[S,\mathcal{H}_n\right]\right] = i\frac{g}{2!}\int I(x)\,\pi(x')\left[B(x'),\Pi(x)\gamma^k\partial_x^k\Psi(x) - \partial_x^k\Pi(x)\gamma^k\Psi(x)\right]dx'$$

which is eventually reducible to

$$\lambda\,\frac{ig^2}{2!}\,I^2(x)\left\{\Pi(x)\gamma^4\gamma^k\partial_x^k\Psi(x) - \partial_x^k\Pi(x)\gamma^4\gamma^k\Psi(x)\right\}$$

It is fairly clear that

$$\mathcal{H}_n' = -iM\,\Pi(x)\gamma^4\Psi(x) - \frac{i}{2}\,\Pi(x)\gamma^4\exp\{2ig\gamma^4 I(x)\}\gamma^k\partial_x^k\Psi(x)$$
$$+\frac{i}{2}\,\partial_x^k\Pi(x)\gamma^4\exp\{2ig\gamma^4 I(x)\}\gamma^k\Psi(x). \qquad (2,41)$$

For pseudoscalar mesons with pseudoscalar coupling the transformation is similar, only here the M term gives a contribution. The final transformed $\mathcal{H}_n$ is

$$\mathcal{H}_n' = -iM\,\Pi(x)\gamma^4\exp\{-2g\gamma^4\gamma^5 I(x)\}\Psi(x)$$
$$-\frac{i}{2}\,\Pi(x)\gamma^4\exp\{-2g\gamma^4\gamma^5 I(x)\}\gamma^k\partial_x^k\Psi(x)$$
$$+\frac{i}{2}\,\partial_x^k\Pi(x)\gamma^4\exp\{2g\gamma^4\gamma^5 I(x)\}\gamma^k\Psi(x) \quad . \qquad (2,42)$$

§ 7. Validity of the Phenomenological Approaches.

The initial Hamiltonian H which describes the coupled nucleon-meson system is $H = H_f + H_n + H_i$ and the interaction energy H_i allows a potential U between the nucleons due to virtual emission and absorption of mesons. U is approximated to order g^2 by V. In order to obtain more accurate wave functions for the nucleon system U is taken from the observed nuclear forces, deducible from scattering data etc., rather than by the approximation V given by meson theory. In this way the distorted wave approach takes as eigenstates, between which the interaction energy allows transitions, the simultaneous eigenstates of H_f and $H_n + U$. Near the threshold for production, when the final nucleon state is of low energy, the cross-sections are quite susceptible to changes in the shape and size of the internucleon potential well. Experiments at 350 Mev. will thus still give vital information about nuclear forces since the final meson wave spans the potential well once or twice only. It appears then necessary to take accurate account of the nuclear wave functions in the meson production problem. This is possible if U is taken as the phenomenological potential given by experiment and thus only the meson emission and not the nucleon scattering is described by meson theory. Such a programme appears in contrast to a strict field theoretic calculation of the third order, such as given by Morette (1949).

As the principal directions of the Hilbert space of the system are the eigenstates of H_f and $H_n + U$ it is necessary to find the interaction energy causing transitions between these directions. It is shown that to first order it is possible to use H_i as this interaction energy. Careful examination of the order in g to which a consistent programme can be carried out is required. Since the approach is aimed at reducing the dependence of the result on the meson theory and laying more emphasis on the internucleon interaction, only transitions of first order in g are considered. Thus only terms in the Hamiltonian linear in g are retained, except, since by the assumption that U is comparable with H_n so that eigenstates of $H_n + U$ are significantly different from free states, it is necessary to consider terms describing the nuclear potential and products of g with such terms. It is clear that to this approximation the Hamiltonian after transforming out the lowest order interaction energy V in the manner of the previous paragraphs, is

$$H_n + V + H_f + i[S, H_n + V] \qquad (2,43)$$

Here the approximate potential V is replaced by the accurate energy U so that the Hamiltonian for the system is taken to be

$$H_n + U + H_f + i[S, H_n + U] \qquad (2,44)$$

and the validity, to the first order, of using H_i as a perturbation between simultaneous states of H_f and $H_n + U$ follows immediately. For the correct matrix element for the transition from state $|0\rangle$ to state $|f\rangle$ is

$$\langle 0|i[S, H_n+U]|f\rangle = i(E_f - E_0)\langle 0|S|f\rangle$$

where E_0, E_f are energies of the Fermi field in initial and final states. If ω is the energy of the meson involved

$$\langle 0|\psi(x)|f\rangle = \pm i\omega\langle 0|\mathcal{I}^{\kappa}(x)|f\rangle$$

and since energy is conserved in the transition, $E_f - E_0 = \pm\omega$

$$\langle 0|i[S, H_n+U]|f\rangle = \langle 0|H_i|f\rangle. \qquad (2,45)$$

So far it has been shown that the matrix elements for single meson processes using the normal interaction energy H_i, gives correct result to first order perturbation theory using only terms linear in the meson field. It is possible to obtain non zero matrix elements to first order through terms containing the meson field operators to any odd power, although mone appear in the approximation given above. It is clear that for processes involving single mesons these terms are divergent, but not all of the simple contact divergence. They are logarithmically divergent due to possible creation and annihilation of high energy virtual mesons. On the other hand it is inconsistent to consider terms of high power in g arising in this way without also considering higher order perturbation theory. It is of interest to note however that for neutral mesons such divergences, which appear only in the expansion for H'_n may be summed to renormalise the first term $i[S, H_n]$. Also for charged mesons besides the corrections due to higher order terms in H'_n there are others in the series $\sum \frac{i^{n-1}}{n!} i[S_i,[S_i, \ldots [S, V+C]]\ldots]$

Contact divergences appear together with logarithmic divergences due to products $I^\times(x) I(x)$ even to order g^3, in $\frac{2}{3} i[S, C]$. These were obtained explicitly in paragraphs 4 and 5. The logarithmic divergence in this term is $\frac{2}{3} g^2 I^\times(x) I(x) \mathcal{H}_i(x)$ which for processes involving one real meson is the interaction term with infinite factor.

Similarly the validity of the bremsstrahlung analogy approach can be examined. Here the matrix element giving rise to meson production is taken to be

$$\langle o|H|f\rangle = \sum_{I} \frac{\langle o|H_i|I\rangle\langle I|V|f\rangle}{E - E_I} + \sum_{II} \frac{\langle o|V|II\rangle\langle II|H_i|f\rangle}{E - E_{II}}.$$

The summations are over possible intermediate states. Neutral scalar theory gives no production if recoil is neglected. If $|a\rangle$, $|b\rangle$ are states differing by the emission of a meson of momentum $\underline{k}$ and energy ω,

$$\langle a|H_i|b\rangle = -ig\int \langle a|\Pi(x)\, \gamma^4 F(x)\, \psi(x)|b\rangle\, dx$$

and

$$\langle a|\ldots|b\rangle = \frac{1}{\sqrt{2\omega}} e^{ik.x}$$

giving

$$\langle a|H_i|b\rangle = -i\sqrt{\frac{1}{2\omega}}\, g\int \langle a|\,\Pi(x)\,\gamma^4\,\Psi(x)\,e^{i\underline{k}.\underline{r}}\,|b\rangle .$$

Now at threshold it is easily seen that $E - E_{I} = -\omega$ and $E - E_{II} = \omega$ if the intermediate states are of positive energy. It is clear therefore that the possible validity of this approach is bound up with the mixing of large and small components of the spinor field by H_i. For if there is no mixing $\langle o|H|f\rangle$ vanishes in agreement with paragraph 4. For charged mesons $i[S, U]$ does not vanish and for the approach to be valid

$$|\langle o|H|f\rangle| = |\langle o|\,i[S, H_n + U]\,|f\rangle| .$$

Again, the energy denominators being as before and using the sum rule, it follows that validity is ensured for non-mixing interactions.

Summarising the results: it is plausible to assume that the transformed Hamiltonian which give rise to first order processes is $H_n + U + H_f + i[S, H_n + U]$. From this the conclusions are that the bremsstrahlung calculation from the outset ignores the recoil term and is then valid for certain fields. The distorted wave approximation on the other hand, considers the recoil terms throughout and is generally applicable to mesons of differing parity. In the case of the neutral fields considered the recoil terms give the only contribution and consequently only the distorted wave approximation gives a finite probability for the production of neutral mesons. These conclusions are for non-derivative couplings between the fields.

Similar considerations are possible for derivative couplings and for example the neutral pseudoscalar field with pseudovector coupling has been investigated. It can be shown that in nonrelativistic approximation the term giving rise to meson production, neglecting recoil is $i[S,V]$ and arguments as above follow for this field.

CHAPTER THREE

THE PRODUCTION OF π-MESONS IN NUCLEON-NUCLEON COLLISIONS

§ 1. Introduction

A problem of considerable interest in meson theory, especially relevant since the high energy machines at Berkeley have been giving detailed experimental information, is the production of mesons in nucleon-nucleon collisions. The close connection between this process and nucleon-nucleon scattering provides an opportunity for testing the fundamental assumption of meson theory. As has been pointed out in chapter two, the meson theory of nuclear forces assumes that the coupling between nucleons takes place via a meson stream of virtual exchanges. If sufficient energy is available it is possible for a real meson to be produced in intimate interactions between nucleons.

This problem has previously been considered by Heitler (1943-5) and his collaborators, and received more recent treatment by Morette (1949) and by Foldy and Marshak (1949). The emphasis of the earlier work is on the effect of radiation damping in reducing the otherwise divergent cross-sections calculated by perturbation theory. The calculations of Morette (1949) followed a full third order field theoretical treatment using the Feynman-Dyson technique. This suffers from the failure of meson theory to describe (in more than a qualitative way) the interaction between nucleons, from the general failure of perturbation theory to give more than first order approximations to cross-sections which is very suspect for large couplings and also the inability of the S-matrix formalism to deal satisfactorily with closely interacting systems as initial or final states. This latter difficulty is of considerable importance in the region of the present day experiments where the energy available for the resultant nucleon system is small. The approach of Foldy and Marshak (1949) based on a phenomenological approach which separates the meson emission, which it describes by field theory, from the nucleon-nucleon scattering which is given in terms of the experimental potentials. The validity of this method was considered in Chapter Two. Even where the method has validity i.e. for non-mixing interactions, the difficulty still arises as in the field theoretic treatment, that the final nuclear state is treated in Born Approximation.

The method outlined in Chapter Two is used here to compute the cross-sections for the production of spin zero mesons in proton-proton collisions near the threshold and to compare with the observed cross-sections for 345 Mev. incident protons by Cartwright, Richman, Whitehead and Wilcox (1950) and Peterson, Iloff and Sherman (1950). Here the kinetic energy of the nucleons never becomes greater than their rest mass, so that some approximate non-relativistic treatment should be applicable to the nucleons motion. Thus this motion is described by the Schrödinger Equation with the potential chosen from nucleon scattering data; i.e. in the notation of Chapter Two, the nuclear eigenstates between which the interaction energy causes transitions are solution of the equation

$$(H_n + U)\Psi = E\Psi \tag{3,1}$$

where H_n is replaced by non-relativistic approximation

$$-\sum_i \frac{\nabla_i^2}{2M_i}$$

The advantage of the phenomenological approach is that the internucleon wave functions can then be described with an accuracy limited only by the inadequacy of existing data on scattering to provide nuclear potentials. These scattering experiments have been very much studied recently, and the analysis of the experimental results by various authors, see for example Jackson and Blatt (1950), has led to several at least qualitative conclusions regarding the internucleon potential.

In the present calculation it is taken as a working assumption that :-

1. The interaction in states of odd parity is so small that it may be neglected. This assumption has been introduced by Serber and is based on the approximate

symmetry about 90° of the angular distributions in neutron-proton scattering.

2. The interaction in states of even parity is charge independent and may be best represented by a singular form of potential with a long tail.

3. No account is taken of tensor forces, or of possible spin orbit forces.

The recent measurements on π^{+}-meson production in 345 Mev. proton-proton collisions have shown that the energy spectrum of the mesons has a strong maximum near the upper energy limit. In figure 1. the differential cross-section for the production of π^{+}-mesons by 345 Mev. protons a proton in the direction of the beam as observed by Cartwright, Richman, Whitehead and Wilcox (1950) is reproduced. Barkas (1949) has attributed this peak to the interaction between the resultant proton-neutron system and to the possible formation of a deuteron. The distorted wave approach outlined previously allows the calculation to proceed even with a final bound nuclear system and cross-sections are calculated below for a final bound deuteron state and for a continuous (but closely interacting) neutron-proton system.

§ 2. General Formalism

The Hamiltonian of the interaction between nucleon and the meson field will generally be represented by

$$H_i = \int \psi^*(x)\, Q\, \psi(x) \phi\, dx \quad + \quad \text{conjugate} \tag{3,2}$$

where $\psi(x)$ is the field operator of the nucleon, $\psi^*(x) = -i\,\Pi(x)$ and $\phi(x)$ the field operator of the mesons. The operator differs according to the charge and spin dependence of the meson theory. Explicit forma are given in appendix 1.

It has been shown in Chapter Two that it is possible to compute matrix elements from this interaction energy between interacting nucleon eigenstates. The matrix element (3,2) may be evaluated by expanding the nucleon operator $\psi^* Q \psi$ into a many particle operator following a method discussed by Becker and Leibfried (1946), the term appropriate to a transition from initial state o to a final state f each with A nucleons being

$$H_i = \int dx_1 \ldots \int dx_A\, \psi_o^*(x_1 \ldots x_A) \left\{ \sum_{i=1}^{A} \left(Q_i \phi(x_i) + Q_i^* \phi^*(x_i) \right) \right\} \psi_f(x_1 \ldots x_A)\,. \tag{3,3}$$

Here ψ_o, ψ_f are the properly antisymmetrised and normalised wave-functions of the initial and final states of the nucleon system, expressed as functions of the spin, space and charge variables. The term with ϕ^* corresponds to the annihilation, while the term with ϕ corresponds to the creation of a positive π-meson.

Calculations have been carried out in detail for scalar and pseudoscalar meson fields, using a scalar coupling in the first case and a pseudovector coupling in the second. The equivalence theorems show that, in the approximation used here, pseudoscalar and pseudovector coupling should give identical results in the pseudoscalar meson theory. With the usual Fourier expansion of the meson field it is then found for the matrix-element describing the emission of a positive π - meson with momentum $\underset{\sim}{k}$ and energy ω from a two nucleon system:

$$H_{of} = g\sqrt{\frac{1}{2\omega}}\iint d\underset{\sim}{r}_1\, d\underset{\sim}{r}_2\, \psi_o^+(r_1,r_2;\tau_i,s_i)\left\{\sum_{i=1}^{2}\pi^{(i)}\beta^{(i)}e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}_i}\right\}\psi_f(\underset{\sim}{r}_1,\underset{\sim}{r}_2;\tau_f\, s_f) \tag{3,4S}$$

for scalar mesons and

$$H_{of} = i\frac{f}{\mu}\sqrt{\frac{1}{2\omega}}\iint d\underset{\sim}{r}_1\, d\underset{\sim}{r}_2\, \psi_o^+\left\{\sum_{i=1}^{2}\pi^{(i)}(\underset{\sim}{\sigma}^{(i)}\underset{\sim}{k} - \rho_1^{(i)})e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}_i}\right\}\psi_f. \tag{3,4P}$$

Here and in the following S denotes scalar mesons, P pseudoscalar ones; $\beta^{(i)}$, $\underset{\sim}{\sigma}^{(i)}$, $\rho_1^{(i)}$, are the usual Dirac matrices applicable to the i'th nucleon, $\pi^{(i)}$ is the charge operator changing nucleon $\underline{i}$ from neutron into a proton, τ_i and s_i are the isotopic and spin variables of the i'th nucleon, μ is the rest energy of the meson.

The frame of reference will be chosen so that in the initial state the total momentum is zero, i.e. if $\underset{\sim}{p}_1$, $\underset{\sim}{p}_2$ denote the momenta of the two nucleons

$$(\underset{\sim}{p}_1)_o = -(\underset{\sim}{p}_2)_o = \underset{\sim}{p}_o .$$

In the final state it follows from the conservation of energy and momentum that

$$(\underset{\sim}{p}_1 + \underset{\sim}{p}_2)_f = \underset{\sim}{P} = -\underset{\sim}{k}.$$

The motion of the nucleon mass centre corresponding to $\underset{\sim}{P}$ is fairly slow (i.e. non relativistic), so that an approximate separation

$$\psi_f(\underset{\sim}{r}_1, \underset{\sim}{r}_2; s_i, \tau_i) = e^{i\underset{\sim}{P}.\underset{\sim}{R}}\, u_f(\underset{\sim}{r}; s_i, \tau_i)$$

is justified.

Here $\underset{\sim}{R}$ is the position of the mass-centre and $\underset{\sim}{r}$ the relative position vector of the two nucleons. The isotopic factor in u_0 and u_f may also be separated out. In the problem considered the initial state contains two protons so that $\psi_0 = u_0(r, s_i)\, {}^3(\tau)$, ; the final state may be either the charge singlet ${}^1(\tau)_0$ or charge triplet ${}^3(\tau)_0$ state. Using the properties

$$\pi^{(1)}\, {}^1(\tau)_0 = -\pi^{(2)}\, {}^1(\tau)_0 = -\frac{1}{\sqrt{2}}\, {}^3(\tau),$$

$$\pi^{(1)}\, {}^3(\tau)_0 = \pi^{(2)}\, {}^3(\tau)_0 = \frac{1}{\sqrt{2}}\, {}^3(\tau),$$

of the change of charge operator π one may write for the matrix elements (4)

$$H_{0f} = g\sqrt{\frac{1}{\omega}}\int d\underset{\sim}{r}\, u_0^+(\underset{\sim}{r}, s_i)\left\{\mp\beta^{(1)} e^{\frac{i}{2}\underset{\sim}{k}.\underset{\sim}{r}} + \beta^{(2)} e^{-\frac{i}{2}\underset{\sim}{k}.\underset{\sim}{r}}\right\} u_f(\underset{\sim}{r}, s_i) \qquad (3,5S)$$

$$H_{of} = \frac{if}{\mu}\sqrt{\frac{1}{4\omega}}\int d\underset{\sim}{r}\, u_o^+(r,s_i)\left\{ \mp(\underset{\sim}{\sigma}^{(1)}\cdot\underset{\sim}{k} - \omega\rho_1^{(1)})e^{\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}} + (\underset{\sim}{\sigma}^{(2)}\cdot\underset{\sim}{k} - \omega\rho_1^{(2)})e^{-\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}}\right\} u_f(\underset{\sim}{r};s_f) \quad (3,5P)$$

In these expressions the alternative - and + signs hold for the final charge singlet and charge triplet states respectively. In accordance with the approximate non-relativistic treatment of the nuclear motion the wave-function u_o and u_f may be reduced to their large components. To the first order in the velocity this corresponds to replacing β and $\underset{\sim}{\sigma}$ (which do not mix large and small components) by 1 and the 2x2 σ - matrices respectively. On the other hand

$$u_o^+\rho_1^{(1)}u_f = \frac{i}{2M}\, v_o^+\underset{\sim}{\sigma}^{(1)}\cdot(\overleftarrow{\underset{\sim}{\nabla}} - \overrightarrow{\underset{\sim}{\nabla}})\, v_f$$

where v_o and v_f denote the large components of u_o and u_f and $\underset{\sim}{\nabla}$ is the gradient with respect to the relative coordinate $\underset{\sim}{r}$. Introducing these expressions into (3,5S) and (3,5P) the final form for the matrix elements is

$$H_{of} = \frac{g}{2}\sqrt{\frac{1}{\omega}}\int d\underset{\sim}{r}\, v_o^+\left\{\mp e^{\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}} + e^{-\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}}\right\} v_f \quad (3,6S)$$

$$H_{of} = \frac{if}{2\mu}\sqrt{\frac{1}{\omega}}\int d\underset{\sim}{r}\, v_o^+\left\{\mp\underset{\sim}{\sigma}^{(1)}\cdot(\underset{\sim}{k} - \frac{i\omega}{M}\overleftarrow{\underset{\sim}{\nabla}})e^{\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}} + \underset{\sim}{\sigma}^{(2)}\cdot(\underset{\sim}{k} + \frac{i\omega}{M}\overleftarrow{\underset{\sim}{\nabla}})e^{-\frac{1}{2}i\underset{\sim}{k}\cdot\underset{\sim}{r}}\right\} v_f \quad (3,6P)$$

for scalar and pseudoscalar meson theories. The - and + signs hold for the final charge singlet and triplet states respectively.

The cross sections for the various processes by which a pair of protons may produce a positive meson must now be evaluated by using equations (3,6). The chief problem that remains is the selection of suitable wave-functions to describe the motion of the nucleons. In this choice, guidance is possible from the information acquired in the fitting of the deuteron and two body scattering data by inter-nuclear potentials. It is found that the cross sections are in fact rather sensitive to the choice of potential.

§ 3. Transitions to the discrete state.

At first consider the production of a meson when the transition of the nucleons leads to the ground state of the deuteron. Separate consideration must be given to the two possibilities that the protons are initially in the spin triplet or spin singlet states. In the case of an initial spin triplet the wave function v_o will be of the form

$$v_o = {}^3(\sigma)_{m_s} v_o$$

so that the matrix element in the pseudoscalar case becomes

$$H_{of} = \frac{if}{2\mu}\sqrt{\frac{1}{\omega}}\int d\underset{\sim}{r}\,{}^3(\sigma)_{m_s} v_o^+ \Big\{ -\underset{\sim}{k}.(\underset{\sim}{\sigma}^{(1)} e^{\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}} - \underset{\sim}{\sigma}^{(2)} e^{-\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}})$$

$$+ \frac{i\omega}{M}(\underset{\sim}{\sigma}^{(1)}.\overleftarrow{\nabla} e^{\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}} + \underset{\sim}{\sigma}^{(2)}.\overleftarrow{\nabla} e^{-\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}}) \Big\} (\sigma)_{m_s} v_f .$$

The evaluation of this matrix element will in general require the consideration of integrals such as

$$\int v_o^{\times} e^{\frac{1}{2} i \underset{\sim}{k} \cdot \underset{\sim}{r}} v_f \, d\underset{\sim}{r} \; ; \quad \int \underset{\sim}{\nabla} v_o^{\times} \cdot e^{\frac{1}{2} i \underset{\sim}{k} \cdot \underset{\sim}{r}} v_f \, d\underset{\sim}{r} \tag{3,8}$$

where v_o and v_f are solutions of Schrödinger-equations, say,

$$\begin{aligned} \nabla^2 v_o + (p_o^2 + M U_o) v_o &= 0 \\ \nabla^2 v_f + (p_f^2 + M U_f) v_f &= 0 \end{aligned} \qquad M = \text{nucleon-mass} \tag{3,9}$$

which follow from (3,1).

The potential functions U_o and U_f will in general be different on account of the spin and charge dependence of nuclear forces. Here, however, restriction is made to central forces of the same range and shape, so that

$$U_o = J_o w(r) \qquad \text{and} \qquad U_f = J_f w(r)$$

where the J's are constants of the dimension of an energy and w(r) is a dimensionless function of r depending only on one parameter - the range of the forces.

It can be immediately shown from equation (3,9) that

$$(p_o^2 - p_f^2) \int v_o^{\times} v_f \, d\underset{\sim}{r} = M \int (U_f - U_o) \, v_o^{\times} v_f \, d\underset{\sim}{r} \tag{3,10}$$

and this identity allows one to restrict the space integration to an interval of the order of the range of the nuclear forces. There is, however, no transformation directly available for the integrals (3,8). It is here that the assumption of "no interaction" in odd parity states leads to some simplification, for according to this assumption $U_0 = 0$ in the initial spin triplet states, and so v_0 may be written

$$v_0 = \frac{1}{\sqrt{2}}\left(e^{i\underset{\sim}{p}_0\cdot\underset{\sim}{r}} - e^{-i\underset{\sim}{p}_0\cdot\underset{\sim}{r}}\right).$$

The matrix element (3,7P) can now be written

$$H_{of} = -i\frac{f}{2\mu}\sqrt{\frac{1}{\omega}}\left\langle {}^3\nabla_{m_s}\right| \underset{\sim}{\sigma}^{(1)} + \underset{\sim}{\sigma}^{(2)} \left| {}^3\nabla_{m_s'}\right\rangle .$$

$$\left[\underset{\sim}{k}\left\{I(\underset{\sim}{p}_0 + \tfrac{1}{2}\underset{\sim}{k}) - I(\underset{\sim}{p}_0 - \tfrac{1}{2}\underset{\sim}{k})\right\} + \frac{\omega}{M}\underset{\sim}{p}_0\left\{I(\underset{\sim}{p}_0 + \tfrac{1}{2}\underset{\sim}{k}) + I(\underset{\sim}{p}_0 - \tfrac{1}{2}\underset{\sim}{k})\right\}\right], \tag{3,11P}$$

where as an abbreviation

$$I(\underset{\sim}{q}) = \int e^{i\underset{\sim}{q}\cdot\underset{\sim}{r}}\, v_f\, d\underset{\sim}{r}\,. \tag{3,12}$$

If the initial state is a singlet state its parity will be even so that $U_0 \neq 0$ and no simple approximation will be possible to the integrals (3,8). However, as an estimate of the order of magnitude of the even parity contribution use is made of the approximation

$$H_{of} = \frac{if}{2\mu}\sqrt{\frac{1}{\omega}}\;\frac{J_f - J_0}{J_f}\left\langle {}^1\nabla_0\right| \underset{\sim}{\sigma}^{(1)} - \underset{\sim}{\sigma}^{(2)} \left| {}^3\nabla_{m_s}\right\rangle . I(\underset{\sim}{p}_0)\,\underset{\sim}{k} \tag{3,13}$$

which is equivalent to assuming that of all states with even parity only the 2 proton S-state contributes and that the momentum of the meson in the integrals (3,8) can be neglected. The expression (3,13) can then be expected to give the right order of magnitude and to represent an upper limit in the sense that consideration of the meson momentum will tend to decrease the value of the integrals (3,8) .

For the calculation of the cross-section $\sum |H_{of}|^2$ is summed over the final spin states of the deuteron and averaged over the orientations of the original spin. This is now given by

$$\sum |H_{of}|^2 = \frac{f^2}{4\mu^2\omega}\left[\left\{ \underline{k}\left(I(\underline{p}_0+\tfrac{1}{2}\underline{k}) - I(\underline{p}_0-\tfrac{1}{2}\underline{k})\right) + \frac{\omega}{M}\underline{p}_0\left(I(\underline{p}_0+\tfrac{1}{2}\underline{k}) + I(\underline{p}_0-\tfrac{1}{2}\underline{k})\right)\right\}^2 + 2k^2 I^2(\underline{p}_0)\left(\frac{\Delta J}{J}\right)^2\right]$$

where $\Delta J = J_f - J_0$, and J is written for J_f .
The differential cross-section can be determined from

$$\frac{d\sigma}{d\Omega} = \frac{k\omega}{4\pi^2 V}\sum_f |H_{of}|^2 \tag{3,14}$$

where V is the initial relative velocity of the nucleons, and $d\Omega$ is the element of solid angle for the meson.

For the evaluation of $\underline{I}$ some assumption has to be made about the inter-nucleon potential. Present scattering evidence on the whole favours a fairly singular, long tailed potential, for which can be used as a simple analytical expression the potential suggested by Hulthén (1942) ;

$$U(r) = J\, e^{-kr}/(1-e^{-kr}). \qquad (3,15)$$

The normalised wave function of the ground state of the deuteron is then

$$v_f = \sqrt{\left\{\frac{kb(b^2-1)}{8\pi}\right\}}\, \frac{1}{r}\, e^{-\frac{1}{2}(b-1)kr}(1-e^{-kr})$$

with $b = \dfrac{MJ}{k^2}$. (3,16)

The depth and range of the potential well must be adjusted so as to fit the deuteron data. As parameters for the well the values

$$1/k = 1.17\times 10^{-13}\ \text{cm.},\quad {}^{(3)}J = 46.6\ \text{Mev.},\quad {}^{(1)}J = 27.2\ \text{Mev.},$$

are used, where ${}^{(3)}J$ and ${}^{(1)}J$ refer to the spin triplet and singlet states respectively. The Integral $\underline{I}$ from equation (3,12) may now be easily evaluated yielding:

$$I(\underline{q}) = \frac{\sqrt{\{2\pi\, b^3(b^2-1)k^5\}}}{(q^2+MW_D)(q^2+\frac{(b+1)^2}{4}k^2)}.$$

Here W_D is the binding energy of the deuteron.
In the energy region of experiment $p \simeq 400$ Mev. and $k \simeq 90$ Mev and, therefore, in good approximation

$$I(\underline{p}_0+\tfrac{1}{2}\underline{k}) + I(\underline{p}_0-\tfrac{1}{2}\underline{k}) \doteq 2\, I(\underline{p}_0)$$

$$I(\underline{p}_0+\tfrac{1}{2}\underline{k}) - I(\underline{p}_0-\tfrac{1}{2}\underline{k}) \doteq -4\,\frac{k}{p_0}\cos\theta\, I(\underline{p}_0)$$

where θ is the angle between $\underset{\sim}{k}$ and $\underset{\sim}{p}$. This gives finally an approximate expression for the cross section describing the formation of a deuteron under meson emission:

$$\frac{d\sigma}{d\Omega} = \frac{f^2}{V\mu^2} \frac{b^3(b^2-1)}{\left\{1+\frac{(b+1)^2k^2}{4p_0^2}\right\}^2} \quad \frac{k^3K^5}{p_0^8} \left\{\left(\frac{\Delta J}{J}\right)^2 + 2\left(\frac{\omega p_0}{Km}\right)^2 - 8\frac{\omega}{m}\cos^2\theta\right\} \tag{3,17P}$$

where f^2 denotes the square of the coupling constant in 'ordinary' units. A numerical discussion of this result will be given in section 6.

The corresponding result for scalar meson theory is:

$$\frac{d\sigma}{d\Omega} = 12\,\frac{g^2}{V} \frac{b^3(b^2-1)}{\left\{1+\frac{(b+1)^2k^2}{4p_0^2}\right\}^2} \quad \frac{k^3K^5}{p_0^{10}}\cos^2\theta. \tag{3,17S}$$

In this case no nuclear spin changes are allowed owing to the assumed central character of the forces. The only effective initial states, therefore, are triplet states. For the transition to be allowed the mesons must be emitted in states of odd parity. The approximate formula (3,17S) represents the emission of a p-wave meson, though some contributions from interference with waves of higher angular momentum are included.

The cross-section for scalar mesons increases more slowly just above the threshold than for pseudoscalar mesons which may be emitted in s-waves. However, with 345 Mev. protons (in the laboratory system) the two cross-sections have become comparable. If $g^2 \doteq f^2$ the pseudoscalar cross-section is about twice the scalar cross-section at this energy. Further discussion of this point is given in section 6 and illustrated in figure 2. In the forward direction, however, the differential scalar cross-section is larger than the pseudoscalar one. The angular variation will be discussed in detail later but here it will be noted that the variation $\cos^2\theta$ for scalar mesons is more in accord with present experimental evidence than that for pseudoscalar mesons. In the angular distribution of cross-section for pseudoscalar meson, no p-waves are emitted and there is an interference between s- and d-waves which lead to an approximate cancellation in the forward direction.

§ 4. Production of neutral mesons in proton-neutron collisions

The absence of π^0- mesons in simple nucleon-nucleon collisions has been reported from Berkeley by Bjorkland, Crandell, Moyer, York (1950). It was pointed out in chapter two that neutral meson production is forbidden for non-derivative couplings between nucleon and meson fields if recoil is ignored. The present method allows a comparison to be made between charged meson and neutral meson production cross-sections when recoil is considered. As an example the calculation of the cross-section for the production of neutral scalar mesons by collisions of fast protons with neutrons is briefly sketched. Restriction is made to the discrete transition, i.e. to final state of the nucleon system the deuteron ground state; and the cross-section is compared with that for the production of π^+- mesons in proton-proton collisions of comparable energy.

The coupling allowing the transition is given in appendix 1

$$-ig\Pi(x)\gamma^4\Psi(x)\phi(x) \tag{3,18}$$

Expanding by the Becker-Leibfried method (1946) as in paragraph 3 the matrix element can be written

$$\frac{g}{\sqrt{2\omega}}\int\chi_0^+(\underline{r}_1,\underline{r}_2)\langle s_0,\tau_0|s_f,\tau_f\rangle(e^{i\underline{k}.\underline{r}_1}+e^{i\underline{k}.\underline{r}_2})\chi_f(\underline{r}_1,\underline{r}_2)\,d\underline{r}_1\,d\underline{r}_2\,. \tag{3,19}$$

The emission of a neutral scalar meson does not change the isotopic or spin state of the nuclear system. Thus for transitions to final ^1S deuteron state the initial n-p system must be isotopic singlet and spin triplet. The non-vanishing matrix elements are each

$$\frac{g}{\sqrt{2\omega}}\int\chi_0^+(\underline{r}_1,\underline{r}_2)(e^{i\underline{k}.\underline{r}_1}+e^{+i\underline{k}.\underline{r}_2})\chi_f(\underline{r}_1,\underline{r}_2)\,d\underline{r}_1\,d\underline{r}_2\,.$$

Transforming to centre of mass and relative coordinates, $\underset{\sim}{R} = \frac{1}{2}(\underset{\sim}{r}_1 + \underset{\sim}{r}_2)$; $\underset{\sim}{r} = \underset{\sim}{r}_1 - \underset{\sim}{r}_2$, the integration over $\underset{\sim}{R}$ gives the conservation of total momentum and the resulting matrix element is

$$\frac{g}{\sqrt{2\omega}} \int g(\underset{\sim}{r})(e^{\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}} + e^{-\frac{1}{2}i\underset{\sim}{k}.\underset{\sim}{r}}) f(\underset{\sim}{r}) d\underset{\sim}{r}$$

where $g(\underset{\sim}{r})$ and $f(\underset{\sim}{r})$ are the initial and final wave functions of the relative n-p system. Only mesons of even parity are allowed. Since $g(\underset{\sim}{r})$ and $f(\underset{\sim}{r})$ are orthogonal the first nonzero term in ascending powers of $\underset{\sim}{k}$ is proportional to $\underset{\sim}{k}^2$. This term is

$$\frac{g}{\sqrt{\omega}} \left\{ I(\underset{\sim}{p} + \tfrac{1}{2}\underset{\sim}{k}) + I(\underset{\sim}{p} - \tfrac{1}{2}\underset{\sim}{k}) - 2I(\underset{\sim}{p}) \right\} \qquad (3,20)$$

where $\underset{\sim}{p}$ is the relative momentum of the initial system and from (3,12),

$$I(\underset{\sim}{p}) = \int e^{i\underset{\sim}{p}.\underset{\sim}{r}} f(\underset{\sim}{r}) d\underset{\sim}{r} .$$

As an approximation valid near the threshold

$$I(\underset{\sim}{p} + \tfrac{1}{2}\underset{\sim}{k}) + I(\underset{\sim}{p} - \tfrac{1}{2}\underset{\sim}{k}) - 2I(\underset{\sim}{p}) = 4 \frac{k^2}{p^2} I(p) \left\{ 6\cos^2\theta - 1 \right\}$$

is used, θ being the angle, in centre of mass system; the meson is emitted from the forward direction. This result is for a Hulthén potential well binding the nucleons of range $1/\kappa$ and depth $\kappa^2 b/M$, (3,16) and $I(q)$ is given explicitly as

$$\sqrt{\{2\pi\, k^5 b^3(b^2-1)\}} \Big/ \left(p^2 + \frac{(b+1)^2}{4}k^2\right)\left(p^2 + \frac{(b-1)^2}{4}k^2\right) .$$

If v is the relative velocity of the nucleons before the collision, the differential cross-section for the process is

$$\frac{d\sigma}{d\Omega} \doteq \frac{3}{2v} g^2 \frac{k^5 K^5}{p^{12}} \frac{b^3(b^2-1)}{\left\{1 + \frac{(b+1)^2}{4}\frac{k^2}{p^2}\right\}^2} \{6\cos^2\theta - 1\} . \qquad (3,21)$$

The ratio between the cross-sections for production of neutral and charged scalar mesons in such collisions is $\sim \frac{3}{2}\frac{k^2}{p^2} \simeq 0.06$ for 350 Mev. incident protons.

Experiments by Bjorkland, Crandall Moyer and York (1950) indicate that the cross-section for π^0- meson production in proton-proton collisions, if non-vanishing, is less by a factor 20 than the cross-section for π^+- meson production. This is in agreement with the above calculation as it is to be expected that the cross-section for π^0- meson production is greater for neutron-proton collisions than proton-proton collisions as in the former a bound final nuclear state is possible, and as is shown in paragraph 5 more probable.

§ 5. Transitions to Continuous States

The transitions to the continuous final states of the proton-neutron system in the case of π^+ -production in proton-proton collisions can be treated in a manner very similar to that applied in the discrete case. In the energy region considered the relative motion of proton and neutron in the final state is very slow, so that it will be sufficient to deal only with final S-states of the neutron-proton system. For pseudoscalar mesons the final S-states of the system can be reached either from the odd initial triplet states under emission of an even parity meson, or from the even initial singlet states with emission of an odd parity meson. The ^1S final states can only be reached from initial triplet states the transition $^1S \rightarrow {}^1S$ being totally forbidden. The contribution to the total cross-section from transitions to the final nucleon ^1S-states will be found to be negligible. For scalar mesons the only allowed transitions are triplet-triplet and singlet-singlet transitions, the latter giving a negligible contribution.

Turning to the detailed treatment of the case of pseudoscalar mesons and dealing first with transitions to the final triplet states, the matrix elements may be calculated as in the paragraph three. The integrals can be taken over, provided that v_f is normalised so that

$$v_f \sim \frac{1}{pr} \sin (pr + \delta) \qquad (3,22)$$

asymptotically, where $\underset{\sim}{p}$ denotes the momentum of relative motion of the nucleons in the final state. In the evaluation of $\underline{I}$ a transformation of the type (3,10) is always made so that only the behaviour of v_f near the origin is important. In this region v_f can be adequately represented by the form of the ground-state solution

$$v_f = \frac{N}{r} e^{-\frac{1}{2}(b-1)\kappa r}(1 - e^{-\kappa r})$$

The normalisation factor $\underline{N}$ has to be derived from the asymptotic behaviour of the true continuous s-waves in a Hulthén potential. It is then found that

$$N = \frac{1}{\kappa}\left[\frac{b\pi \sinh 2\pi\alpha}{\alpha\{\cosh 2\pi\alpha - \cos 2\pi\sqrt{(\beta^2-\alpha^2)}\}}\right]^{1/2}$$

where $\alpha = p/\kappa$ and b has been defined in equation (3,16). In the region of interest a valid approximation is

$$N = (b\pi/\kappa p)^{1/2}$$

and it is then found that

$$I(\underset{\sim}{q}) = \frac{4}{p_0^4}\left(\frac{\pi^3 b^3 \kappa^3}{p}\right)^{1/2} F(\underset{\sim}{q})$$

where

$$F(\underset{\sim}{q}) = \frac{p_0^4}{(p_0^2 - q^2)(p_0^2 + \frac{(b+1)^2}{4}\kappa^2)} .$$

The differential cross section for a transition to a final ^3S state with the production of a meson in the energy interval

ω to $\omega + d\omega$ is then given by

$$\frac{d\sigma}{d\omega\, d\Omega} = \frac{f^2}{4\pi V} \frac{b^3 k^3 M \kappa}{\mu^2 p_0^8} \Big[\Big\{ \underset{\sim}{k} \big(F(\underset{\sim}{p_0} + \tfrac{1}{2}\underset{\sim}{k}) - F(\underset{\sim}{p_0} - \tfrac{1}{2}\underset{\sim}{k}) \big)$$

$$+ \frac{\omega}{M} \underset{\sim}{p_0} \big(F(\underset{\sim}{p_0} + \tfrac{1}{2}\underset{\sim}{k}) + F(\underset{\sim}{p_0} - \tfrac{1}{2}\underset{\sim}{k}) \big) \Big\}^2 + 2k^2 \left(\frac{\Delta J}{J}\right)^2 F^2(p_0) \Big]. \qquad (3,23P)$$

Using the same approximation as in the treatment of the discrete state one obtains

$$\frac{d\sigma}{d\omega\, d\Omega} = \frac{4 f^2}{V} \frac{b^3}{\left\{ 1 + \frac{(b+1)^2 k^2}{4 p_0^2} \right\}^2} \frac{\kappa k^3 \omega}{\mu^2 p_0^6} \left\{ \frac{\omega}{M} - 4 \frac{k^2}{p_0^2} \cos^2\theta + \frac{2 k^2 M}{p_0^2 \omega} \left(\frac{\Delta J}{J}\right)^2 \right\} \qquad (3,24P)$$

where f^2 is in 'ordinary' units.

The result exhibits the same tendency towards cancellation in the forward direction as was found for the cross-section in the discrete case.

Transitions to the final ^1S states can be similarly treated. The resulting cross-section, however, as is evident from equation (3,24P) is proportional to the cube of the final state well depth. This reduces the contributions from singlet transitions by a factor of at least 8 and as a result they may be neglected in calculations of the present approximate character.

In the case of scalar mesons the same method leads to a cross-section

$$\frac{d\sigma}{d\omega\, d\Omega} = \frac{3}{2\pi} \frac{g^2}{V} \frac{b^3 \kappa k^3 M}{p_0} \left\{ F(p_0 + \tfrac{1}{2} k) - F(p_0 - \tfrac{1}{2} k) \right\}^2, \qquad (3,23S)$$

which can be approximated by

$$\frac{d\sigma}{d\omega\, d\Omega} = 24\, \frac{g^2}{V}\, \frac{b^3}{\left\{1 + \frac{(b+1)^2 k^2}{4 p_0^2}\right\}^2}\; \frac{k^3 k'^3 M}{p_0^{10}}\, \cos^2\theta, \tag{3,24S}$$

with g^2 in 'ordinary'units.

§ 6. Numerical Results

The variation of the total cross-section for the production of scalar and pseudoscalar mesons have been calculated. In transitions to the ground-state of the deuteron the cross-section has been plotted against the energy of the incident proton in the laboratory system and this is shown in figure 2. Corresponding to the possibility of production of mesons in an s-state the cross-section for pseudoscalar mesons is considerably higher near the threshold (E_p = 290 Mev) A maximum is reached at $E_p \sim 370$ Mev. It has already been noted that the results are rather dependent on the inter-nucleon well shape. For comparison these cross-sections have also been evaluated for a square well potential. In this case

$$V(r) = J \quad r < a$$
$$\phantom{V(r) = {}} 0 \quad r > a.$$

and

$$\underline{I}(q) = \frac{2\pi J}{i(p^2 + MW_1)}\; \frac{\sin(q-\lambda)a - \sin(q+\lambda)a}{\sqrt{2\pi}\,(a + \alpha')}$$

where $\lambda = \sqrt{\{M(J - W_D)\}}$

$\alpha = \sqrt{(M W_D)}$ and a is range of well.

Numerical calculation shows that for the parameters ([3] $J = 41$ Mev. $a = 1.85 \times 10^{-13}$ cm.) the resulting cross-sections are smaller by a factor $^{1}/_{25}$.

Figure three represents the energy dependence of the integrated total cross-section for transitions to the continuous part of the deuteron spectrum. The result in each case naturally increases more slowly above the threshold than in the discrete case. At $E_p = 350$ Mev. approximately the energy of the Berkeley experiments σ(continuous) is about $^{1}/_{2}$ of σ(discrete) for scalar mesons and $^{3}/_{4}$ for pseudoscalar mesons. The fall in the continuous cross-section indicated figure three should not be taken too seriously. At these energies contributions from waves with higher angular momentum as well as higher order corrections to the meson field must be considered. The ratio of the continuous to the discrete cross-section is consistent with experiment for both types of mesons.

The transformation to the laboratory system has the effect of throwing more mesons into the forward direction. Denoting by k', ω', θ', the momentum, energy and the angle at which the meson is emitted in the centre of mass system and by κ, ω, and θ the corresponding quantities in the laboratory coordinate system the Lorentz transformation is elementary. For example, taking the differential cross-section for the production of scalar discrete mesons to be

$$d\sigma/d\Omega' = A\cos^2\theta',$$

in the centre of mass system, the corresponding cross-section in the laboratory system is

$$\frac{d\sigma}{d\Omega} = A\gamma\,\frac{k^2}{k'^3}\,\frac{(k\cos\theta - v\omega)^2}{(k - v\omega\cos\theta)},$$

where v is the relative velocity of the two frames of reference and $\gamma = (1-v^2)^{1/2}$. The results have been evaluated for incident 350 Mev. protons for the production of both types of mesons. They are shown in figure four. The cross-section for scalar mesons (4a) is strongly peaked in the forward direction where it obtains its maximum value of 49 $g^2 \times 10^{-30} cm^2$/sterad. The cancellation in the forward direction is still apparent in the laboratory system for pseudoscalar mesons (4b). The maximum of the angular distribution of 39 $f^2 \times 10^{-30} cm^2$/sterad. is reached at an angle about 40^o . The magnitude of the cross-section at 30^o is of the observed order (if an experimental resolution of 6 Mev. is assumed). At 0^o the argument is not so good for pseudoscalar mesons.

The determination of the continuous contributions in the laboratory system is more laborious. In figure 5 $d\sigma/d\omega d\Omega$ is plotted against the kinetic energy for scalar mesons - assuming a proton energy of 350 Mev. - for observation at 0^o and at 30^o . The differential cross-sections are both peaked towards the upper end favouring high meson energies. Integrating over all energies it follows that at 0^o

$$\frac{d\sigma}{d\Omega} = 32\, g^2 \times 10^{-30} \text{ cm}^2/\text{sterad.}$$

and at 30^o

$$\frac{d\sigma}{d\Omega} = 3.4\, g^2 \times 10^{-30} \text{ cm}^2/\text{sterad.}$$

where g^2 is expressed in 'ordinary' units.

The cross-section falls away very rapidly from the forward direction. Pseudoscalar mesons show a very differently shaped differential cross-section, approximately proportional to $(\omega - \mu)^{1/2}$ so that there is not the same favouring of high meson energies. The approximate integrated contributions to the differential cross-section are for pseudoscalar mesons at $\theta = 0^o$

$$\frac{d\sigma}{d\Omega} = 30\, f^2 \times 10^{-30} \text{ cm}^2/\text{sterad.}$$

while at $\theta = 30^o$

$$\frac{d\sigma}{d\Omega} = 20\, f^2 \times 10^{-30} \text{ cm}^2/\text{sterad.}$$

where f^2 is expressed in 'ordinary' units.

Recent experiments have all been carried out with 345 Mev. protons and so the correct dependence of the cross-sections on energy cannot yet be discussed. The angular distributions at this energy can, however, be compared with experiment as cross-sections have been reported at angles 0^o, 18^o and 30^o to the beam. Agreement appears possible only with a $\cos^2\theta$ type of distribution at these angles and so favours scalar rather than pseudoscalar type mesons. The experimental points are shown on figure (4a) where g has been chosen to fit one reading. Also the marked peak in the continuous spectrum at high meson energies is predicted by scalar theory.

The forward cross-sections appear to require rather a large coupling constant ($g^2 \sim 3$; $f^2 \sim 5$) however these cross-sections are rather susceptible to fine changes in shape and size of the nuclear well. These considerations will be dealt with in chapter five but it is perhaps also of interest to note that Brueckner (1950) found $f^2 \sim .2$ to fit the observed meson production by photons (assuming the mesons to be pseudoscalar).

§ 7. Conclusions

It has not been the purpose of this paper to give the most complete description possible of meson production with the use of all the available data on the n-p and p-p interactions. Rather, by means of relatively simple analytical approximations for the potentials, and by using approximate methods, it has been shown how important it is, in the low energy region (say up to 500 Mev.), to take accurate account of the nucleon wave functions. In this region indeed allowance for the detailed behaviour of the nucleons is likely to be more important than the inclusion of any field theoretical refinements. Increasingly at higher energies the neglect of higher order reactive terms from the meson field will affect the results and simultaneously the concept of inter-nucleon potential will lose its validity. At the same time multiple meson production will begin affecting the results. It is likely, however, that in the low energy region the methods applied in this chapter should give at least qualitatively correct results.

If the use of Serber forces can be regarded as satisfactory then a considerable difference has been established between the behaviour of pseudoscalar and scalar mesons. The angular distributions are quite different: for scalar mesons the distribution goes with $\cos^2\theta$ in the centre of mass system; for pseudoscalar mesons the angular distribution though isotopic near the threshold has come nearer to a $\sin^2\theta$ law at the maximum of the total cross-sections. These results can only be fitted to present experimental evidence with the assumption of scalar mesons. On the other hand no exhaustive effort has been made to discover how, by alteration of the inter-nucleon potential - say by addition of spin orbit forces - the results of the pseudoscalar meson theory might be affected.

CHAPTER FOUR.

ABSORPTION OF π^--MESONS BY NUCLEI

§ 1. Introduction

The absorption of π^- mesons by nucleons is the mesonic process analogous to the electromagnetic photoeffect where photons, the quanta of the electromagnetic field, are absorbed by electrons. In this effect the electrons must be bound to some atom to allow for the conservation of energy and momentum and similar arguments apply to the nonradiative absorption of π^- mesons by nucleons. Thus such absorption is energetically possible in all nuclei save Hydrogen. It was in the photoelectric effect in atomic physics where the use of exact electron wave functions rather than the Born Approximation was shown to be necessary to give correct cross-sections in the neighbourhood of the absorption edge; the calculation being given by Stobbe (1930). The calculation was one of the earliest using matrix elements between states, which although eigenstates of the energy are not both eigenstates of the momentum or angular momentum.

The cosmic ray π^- mesons, observed by Powell and Occhialini (1948) in photographic plates, produced as a result of primary radiation interacting with nuclei in the neighbourhood of the emulsion, are observed to interact with the atoms of the emulsion to produce stars if negatively charged while suffering $\pi \rightarrow \mu$ decay if positive. Similarly the artificially produced mesons of the Berkeley 184 inch cyclotron have been absorbed in targets of Hydrogen, Deuterium and Carbon etc. under controlled conditions and the π^--mesons have been shown to interact strongly with the nuclei of the targets the π^+ meson again decaying to a μ^+-meson. This is due to the electrostatic repulsion between the π^+-meson and the nucleus not allowing sufficient overlap for capture on the one hand, while the π^- meson on the other, is attracted by the positively charged nucleus and can easily fall into Coulomb orbits about the nucleus and be captured therefrom. The first Bohr radius for the π^- meson-proton system is $\frac{m}{\mu}$ × first Bohr radius for the Hydrogen atom, and is thus $a_0 = 1.92 \times 10^{-11}$ cm. (exactly $\frac{1}{2}$ the Compton wave length of the electron), thus for meson in the K-shell its orbit is one practically inside the nucleus. The details of the slowing down process in the material have been considered by Fermi and Teller (1947) and Wightman (1950). At first the π^--meson is slowed down by elastic collisions estimated taking $10^{-13} - 10^{-9}$ sec., the subsequent stages are collisions with the atomic electrons which are emitted in an Auger effect, capture into an excited π^--proton system and finally cascade down to the inner shells. Wightman gives a time for the latter of about 10^{-9} sec., in Hydrogen, the principal contribution to this process being collisions with Hydrogen molecules. Thus the overall time is of the order of 10^{-9}sec., Since the lifetime of the $\pi \rightarrow \mu$ decay is

1.6×10^{-8} sec., (Kraushaar, Thomas and Henri (1950)), while the capture times from the tightly bound orbits turns out to be very small it is expected that the π^-- meson is captured before decay. The cascade process and Auger effect will not be considered here in detail. It should however be noticed that it is necessary to have a sufficiently dense target else the cascade down can take a time longer than the decay. Once the π^- meson is in fairly low orbits reduction of energy is by radiative transition and, as it turns out times for these processes are critical in subsequent discussion, these will be calculated in more detail.

The non-radiative capture of plane wave mesons by nuclei have been investigated by Yukawa and Sakata (1937) for scalar mesons, Massey and Corben (1939) and Sakata and Tanikawa (1939) for vector, and Tanikawa and Yukawa (1941) for pseudoscalar. Calculations using more exact wave functions for the nucleons were given by Bruno (1948). The effective of the Coulomb force in the plane wave case was estimated by Tomonaga and Araki (1940) by use of the well known factor multiplying the amplitude of the wave function of the incident meson at the nucleus. The capture in Deuterium from Coulomb orbits have been considered by Tamor and Marshak (1950) in a manner after Marshak and Wightman (1949), and by Brueckner, Serber and Watson (1951) using the principle of detailed balance and the observed cross-sections for the inverse processes. The capture from the K-shell by heavy nucleus was considered by Power (1949).

The radiative capture has been considered by Heitler (1938), Chang (1942) and Bruno (1948), and in Deuterium, again from the principle of detailed balance and the observed γ - production cross-sections by Brueckner, Serber and Watson (1951). The radiative capture in Hydrogen has special significance since direct absorption is not allowed.

The interaction of μ - meson is a two step process - possibly through the π-meson field. Such interaction is not considered here as the bose meson field is then virtual. For such interaction see Marty and Prentki (1948), Lopes (1948) and d'Espagnat (1948).

In this chapter two calculations are presented; the first considers the absorption in Deuterium in some detail and comparison with Brueckner, Serber and Watson (1951); and Tamor and Marshak (1950), is given; the second is an extension of the absorption in heavy nuclei with a simplified nuclear model. The results of these together with other recent experimental and theoretical results are subsequently discussed.

Possible processes in the interaction of a π^- meson with a proton are

$$\pi^- + p \rightarrow n \qquad a)$$

$$\pi^- + p \rightarrow n + \gamma \qquad b)$$

$$\pi^- + p \rightarrow n + 2\gamma \qquad c)$$

$$\pi^- + p \rightarrow n + \pi^0 \rightarrow n + 2\gamma \qquad d) \qquad (4,1)$$

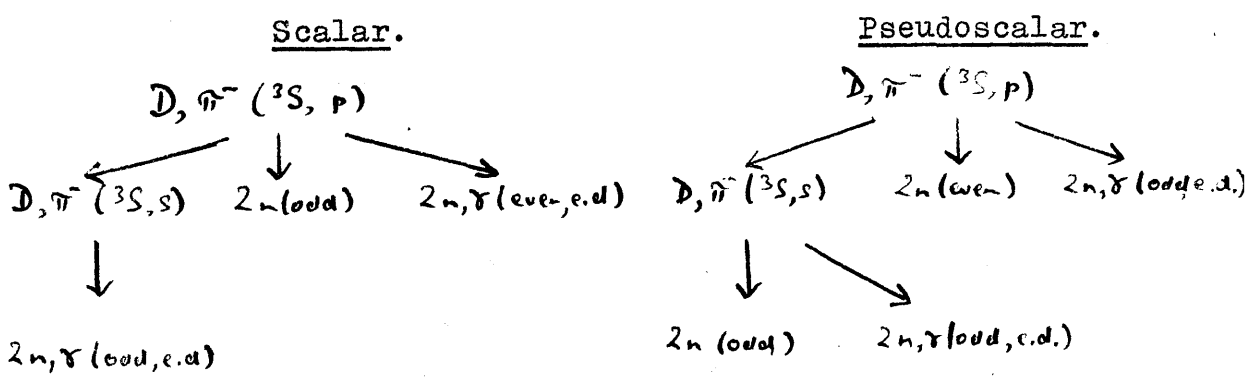
Other processes - such as those involving two more spin $\frac{1}{2}$ particles - are not considered being too slow for comparison. Process a/ is only possible for a bound proton. Absorption in Hydrogen must lead to the emission of one or more additional particles. Process c/ is expected to be small and not compete with single γ emission. Experiments by Panofsky, Aamodt and Hadley (1951) confirm this. These experiments show also that process d/ is practically certainly confined to absorption in Hydrogen where it competes with process b/, the ratio being of the order unity. In capture by nuclei with $A > 1$ observation leads to the conclusion that there are two competing processes namely a/ and b/ and for $A > 3$ upper limit for the fraction of absorptions giving rise to high energy γ emission is 10%. In Deuterium the ratio between a/ and b/ is ~ 2 with complete absence of d/. As pointed out by Brueckner, Serber and Watson the non-radiative capture in Deuterium depends on the probability of finding a high relative momentum of the nucleons in the deuteron. In more tightly bound systems this probability is larger.

§ 2. Capture in Deuterium

Ferretti (1946) pointed out that if the π^--meson is captured from its lowest Coulomb orbit about a deuteron nucleus the direct non-radiative capture

$$\pi^- + D \rightarrow 2n$$

is forbidden by parity and angular momentum conservation if the meson is scalar. This has led to the conclusion that the meson involved, if of spin zero, must be pseudoscalar since in fact only 30% of the absorption processes are radiative. (Tamor and Marshak (1950)). However absorption could take place from higher angular momentum orbits and it is thus vital to estimate the lifetime of negative π-mesons in p-states about the deuteron. Possible processes involved are the fall to the K-shell and capture by the nucleon with or without emitting radiation. These transitions, together with allowed s-orbit processes, are shown diagramatically below.



The predominating nucleon parity state and the meson (or radiation) state are given in the brackets.

§ 3. Fall to K-Shell

Consider first the mean lifetime of π^- meson in (n' , l' , m' ,) state falling to the state with quantum numbers (n, l, m,) with the emission of a γ ray. The energy of the γ for a π^- meson about a nucleus of charge Z is

$$\frac{Ze^2}{2a}\left(\frac{1}{n'^2}-\frac{1}{n^2}\right).$$

For a deuteron the energies of the K , L , M shells are

n = 1	-3.49 Kev.	
n = 2	- .87 Kev.	(4,2)
n = 3	- .39 Kev.	

The eigenfunctions of the state (n , l , m) are well known and are

$$R_{nl}(r)\,Y_l^m(\theta,\phi) \qquad (4,3)$$

where $Y_l^m(\theta,\phi)$ are the normalised tesserial harmonics and $R_{nl}(r)$ are the normalised radial Coulomb wave functions.

The interaction energy density causing transitions between the meson and electromagnetic field is

$$-ie\,\partial_k\psi^*(x)\,A_k(x)\,\psi(x)+ie\,\psi^*(x)\,A_k(x)\,\partial_k\psi(x)+e^2A_k^2(x)\,\psi^*(x)\psi(x) \qquad (4,4)$$

where $A_k(x)$ is the three vector potential of the transverse e.m. field. The longitudinal quanta are supposed gauged away to give the static potential. The process considered is a single quantum transition for which the interaction energy is

$$ie\,A_k(x)\left(\psi^*\partial_k\psi-\partial_k\psi^*\,\psi\right)$$

Expanding $A_k(x)$ into emission and absorption operators in the usual manner; the matrix element for the emission of a quanta of momentum $\underset{\sim}{k}$ and polarisation $\underset{\sim}{\varepsilon}$ is

$$\langle 1|H|2\rangle = \frac{ie}{\sqrt{2k}}\int \langle 1|\underset{\sim}{\varepsilon}.\,\psi^{*}\underset{\sim}{\nabla}\psi - \underset{\sim}{\nabla}\psi^{*}\psi|2\rangle e^{i\underset{\sim}{k}.\underset{\sim}{r}}\,d\underset{\sim}{r} \qquad (4,5)$$

where the units are Lorentz Heaviside.

Expanding the meson field into absorption and emission operators as in Chapter Two (2,14,17), the matrix element corresponding to the absorption of π^-- in state (n' , l' , m') and the emission of a π^-- in state (n , l , m) is

$$\frac{ie}{\sqrt{\{2k\omega_n\omega_{n'}\}}}\int\left\{\underset{\sim}{\varepsilon}.\,\underset{\sim}{\nabla}\left[R_{nl}(r)\,Y_l^{m*}(\theta,\phi)\right]\right\}R_{n'l'}(r)\,Y_{l'}^{m'}(\theta,\phi)\,e^{i\underset{\sim}{k}.\underset{\sim}{r}}\,d\underset{\sim}{r} \qquad (4,6)$$

where $-\int\underset{\sim}{\nabla}\psi^{*}\psi e^{i\underset{\sim}{k}.\underset{\sim}{r}}d\underset{\sim}{r}$ has been integrated by parts.

ω_n , $\omega_{n'}$ are the energies of the mesons in states with principal quantum numbers n and n' , both are in very good approximation equal to the mass of the meson.

Here consideration is given to the fall from p-states to the K-shell. The wave function of the latter is

$$\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{5/2} e^{-\frac{Z}{a_0}r}$$

from which the matrix element for the transition is

$$\frac{ie}{\sqrt{k}}\,\frac{1}{\mu}\,\frac{1}{\sqrt{\pi}}\left(\frac{Z}{a_0}\right)^{5/2}\int \underset{\sim}{\varepsilon}.\,\hat{\underset{\sim}{r}}\;e^{-\frac{Z}{a_0}r}\,R_{n1}(r)\,Y_1^{m'}(\theta,\phi)\;e^{i\underset{\sim}{k}.\underset{\sim}{r}}\,d\underset{\sim}{r}$$

k is small compared with Z/a_0 and only dipole radiation is considered. For $e^{i\underset{\sim}{k}\cdot\underset{\sim}{r}}$ the first term in the Raleigh expansion is used, namely $j_0(kr)$.
If $\underset{\sim}{\varepsilon}$ is in direction α , β with respect to the atomic azimuthal direction then

$$\underset{\sim}{\varepsilon}\cdot\underset{\sim}{\hat{r}} = \left\{\cos\theta\cos\alpha + \sin\theta\sin\alpha\cos(\phi-\beta)\right\}$$

If, further $S_n = \int e^{-\frac{Z}{a_0}r} R_{n1}(r)\, j_0(kr)\, r^2 dr$ (4,7)

the matrix elements which do not vanish are approximately

$$\frac{ie}{\sqrt{3\pi k}}\ \frac{S_n}{\mu}\left(\frac{Z}{a_0}\right)^{5/2}\begin{cases}\sqrt{2}\cos\alpha & \text{for } m = 0\\ -\sin\alpha\, e^{\mp i\beta} & m = \pm 1\end{cases} \qquad (4,8)$$

The transition probability per unit time is given by perturbation theory, and summing over final states, averaging over initial, and using natural units for the charge is

$$\omega = \frac{2^4}{3^2}\left(\frac{Z}{a_0}\right)^5 \frac{ke^2}{\mu^2} S_n^2 . \qquad (4,9)$$

The final S integrations are elementary and give

$$S_2 = \left(\frac{Z}{a_0}\right)^{-3/2}\frac{2^4}{3^3\sqrt{6}}$$

$$S_3 = \left(\frac{Z}{a_0}\right)^{-3/2}\frac{3^2}{2^5\sqrt{6}} . \qquad (4,10)$$

Thus transition probabilities in the two cases are

$$\omega_2 = 4\left(\frac{2}{3}\right)^9 e^6 k Z^2$$

$$\omega_3 = \frac{3}{2^7} e^6 k Z^2 \qquad (4,11)$$

remembering $a_0 = 1/e^2\mu$

For Deuterium it is found the lifetime for 2p meson to fall to K shell is 6.2×10^{-12} sec. and for 3p meson is 2.3×10^{-11} sec. These are, of course, independent of the meson parity.

§ 4. Non-Radiative Capture of π^- Meson from p and s Orbits about a Deuteron Nucleus.

As shown above the allowed transitions are for scalar mesons confined to direct absorption from p-states while for pseudoscalar mesons capture can take place from p- and s- orbits the resulting 2n system being even and odd in the two cases. The calculation of the mean life for such processes proceeds in the manner outlined in Chapter Two. Here the meson field operators are expanded in the orthogonal set of Coulomb orbit states about the nucleus, the quantization of the field being in terms of occupation numbers for mesons in such states.

§ 4.a Scalar

The coupling through which the transition takes place is given in Appendix 1 and, with the notation defined there, is

$$H_i = -g\bar{\Psi}^{\times}\rho_1\bar{\tau}\Psi\varphi \qquad + \quad \text{conjugate}$$

In non-relativistic approximation for the nucleon motion, is taken as 1 and Ψ's are then four component Dirac spinors.

$$H_i = -g \bar{\Psi}^{\times} q \bar{\Psi} \qquad + \quad \text{conjugate} \qquad (4,12)$$

Using the Becker-Liebfried method of expansion of this energy in configuration space of the many nucleon system, the matrix element for the absorption process can be written:-

$$-g \int \chi_i^{\times} \left(q^{(1)} \langle i | \psi(1) | f \rangle + q^{(2)} \langle i | \psi(2) | f \rangle \right) \chi_f \, dx + \quad \text{conjugate} \quad (4,13)$$

Here χ_i, χ_f are the properly symmetrised wave functions for the two nucleon system before and after the transition, respectively. Expanding the meson field operators in the usual manner

$$\langle i | \psi(x) | f \rangle = \frac{1}{\sqrt{2\omega}} \phi_\omega(x)$$

where $\phi_\omega(x)$ is the normalised eigenwave of the absorbed meson. The energy of this meson is ω and this is approximated to by the meson mass at the energies considered.

The matrix element can thus be written as

$$-\frac{g}{\sqrt{2\omega}} \int \chi_i^{\times} \left(q^{(1)} \phi_\omega(r_1) + q^{(2)} \phi_\omega(r_2) \right) \chi_f \, dx \; .$$

The matrix element over the isotopic spin states follows from

$$\langle {}^1\tau_0 | q^{(i)} | {}^3\tau_1 \rangle = (-)^i \frac{1}{\sqrt{2}}$$

and since there is no spin change the total matrix element is

$$\frac{g}{2\sqrt{\omega}} \int \chi_i^* \left(\phi_\omega(\underline{r}_1) - \phi_\omega(\underline{r}_2) \right) \chi_f \, dx.$$

Separating out the centre of mass coordinates

$$\chi_i = g(r)$$

g(r) is the deuteron ground state spacial wave function.

(4,14)

$$\chi_f = \frac{e^{i\underline{p}.\underline{r}} - e^{-i\underline{p}.\underline{r}}}{\sqrt{2}} \, e^{i\underline{P}.\underline{R}}$$

this representing the odd triplet state of the 2n system being assumed non-interacting (Serber)

$\underset{\sim}{p}$, $\underset{\sim}{P}$ are the relative and total momentum of the 2 neutron system. Integration over R gives $\underset{\sim}{P} = 0$.

$\phi(\rho)$ is the meson eigenwave in its Coulomb orbit about the centre of mass:-

$$\phi(\rho) = R_{nl}(\rho) \, Y_l^m(\theta, \phi)$$

S-state $\phi_\omega(\underline{r}_1) = + \phi_\omega(\underline{r}_2)$ and matrix element vanishes.

P-state $\phi_\omega(\underline{r}_1) = - \phi_\omega(\underline{r}_2) = R_{n1}\left(\frac{r}{2}\right) Y_1^m(\theta, \phi)$

and matrix element is

$$\frac{g}{\sqrt{2\omega}} \int g(r) \, R_{n1}\left(\tfrac{r}{2}\right) Y_1^m(\theta, \phi) \left(e^{i\underline{p}.\underline{r}} - e^{-i\underline{p}.\underline{r}} \right) d\underset{\sim}{r} \, .$$

Using the expansion theorem

$$e^{i\mathbf{p}.\mathbf{r}} = \sum_{\ell=0}^{\infty} (2\ell+1)\, i^{\ell}\, j_{\ell}(pr)\, P_{\ell}(\cos\theta')$$

where θ' is the angle between $\mathbf{p}$ and $\mathbf{r}$, and integrating over angular variables this becomes

$$2\sqrt{\frac{6\pi}{\omega}}\; g\; I_n \cos\gamma$$

$$I_n = \int g(r)\, R_{n1}\left(\tfrac{r}{2}\right) j_1(pr)\, r^2 dr \qquad (4,15)$$

and γ is the angle between $\mathbf{p}$ and the azimuthal direction of the π - meson-deuteron system.

Transition probability per unit time is given by perturbation theory; converting g to natural units and integrating over angles of emission of relative momentum this is

$$w = \frac{2^4}{3}\pi g^2 \frac{pM}{\omega} I_n^2 \qquad (4,16)$$

Now

$$R_{21}\left(\tfrac{r}{2}\right) = \left(\frac{1}{a}\right)^{5/2} \frac{1}{4\sqrt{6}}\; r\, e^{-r/4a}$$

and

$$R_{31}\left(\tfrac{r}{2}\right) = \left(\frac{1}{a}\right)^{5/2} \frac{1}{27\sqrt{6}} \left(4 - \frac{r}{3a}\right) r\, e^{-r/6a}$$

so that

$$I_2 = \sqrt{\left\{\frac{b(b^2-1)\kappa}{8\pi}\right\}}\,\frac{1}{4\sqrt{6}}\left(\frac{1}{a}\right)^{5/2}\int\left(e^{-\lambda r}-e^{-\lambda' r}\right)j_1(pr)\,r^2 dr$$

$$\lambda = \frac{1}{4a} + \frac{b-1}{2}\kappa \quad ; \quad \lambda' = \frac{1}{4a} + \frac{b+1}{2}\kappa \tag{4,17}$$

$$I_3 = \sqrt{\left\{\frac{b(b^2-1)\kappa}{8\pi}\right\}}\,\frac{4}{27\sqrt{6}}\left(\frac{1}{a}\right)^{5/2}\int\left(e^{-\lambda r}-e^{-\lambda' r}\right)\left(1-\frac{r}{12a}\right)j_1(pr)\,r^2 dr$$

$$\lambda = \frac{1}{6a} + \frac{b-1}{2}\kappa \quad ; \quad \lambda' = \frac{1}{6a} + \frac{b+1}{2}\kappa . \tag{4,18}$$

b and κ are defined in the two body discussion of chapter three. The difference between λ and λ' in the two cases is small since $1/a \sim 1.02$ Mev. and $\kappa(b-1)/2 \sim 21.5$ Mev. Also the large term in I_3 is that of I_2 with factor 16/27. Substituting approximate numerical values

$$\omega_2 = .27 \times 10^{10}\, g^2 \sec^{-1}.$$

$$\omega_3 = .95 \times 10^{9}\, g^2 \sec^{-1}.$$

Thus the lifetimes for the direct absorption of π^- from p orbits are

$$n = 2 \qquad \tau = 3.7 \times 10^{-10}/g^2 \text{ sec.}$$

$$n = 3 \qquad \tau = 1.05 \times 10^{-9}/g^2 \text{ sec.}$$

The ratio between them is 2.85 in full agreement with that found by Brueckner, Serber and Watson by their method. The absolute magnitudes agree with $g^2 \sim 3$, this value is that found necessary in the production problem (Chapter Three).

§ 4.b <u>Pseudoscalar.</u>

If the meson field is assumed to be pseudoscalar the axial vector coupling through which the transition can take place is given in Appendix 1 and, with the notation defined there, is

$$H' = -\frac{f}{\mu}\Psi^{\times}(\underset{\sim}{\sigma}.\nabla\psi q + \rho_1 \pi^{\times} q)\Psi \qquad + \text{ conjugate} \quad (4,19)$$

In non-relativistic approximation for the nuclear motion the Dirac matrices are written in the well known form of partioned matrices and the Ψ 's are divided into large and small components in the usual manner. The approximate energy density causing transitions can then be written in terms of the four component Dirac spinors as

$$-\frac{f}{\mu}\left\{\Psi^{\times}\underset{\sim}{\sigma}.\nabla\psi q\Psi - i\frac{\Psi^{\times}\pi^{\times}\underset{\sim}{\sigma}.\nabla q\Psi}{M} - i\frac{\Psi^{\times}\underset{\sim}{\sigma}.\nabla\pi^{\times}q\Psi}{2M}\right\}$$

$$+ \quad \text{conjugate.}$$

Expanding by the Becker-Liebfried method, as in the scalar case above, the matrix element for the absorption of a negative meson from a state characterised by the eigenwave $\phi_\omega(r)$ is approximately

$$-\frac{f}{\sqrt{2\omega}}\frac{1}{\mu}\int\chi_i^{\times}\langle i|\left\{\underset{\sim}{\sigma}^{(1)}.\underset{\sim}{\nabla}_1\phi_\omega(1)q^{(1)} + \underset{\sim}{\sigma}^{(2)}.\underset{\sim}{\nabla}_2\phi_\omega(2)q^{(2)}\right.$$

$$\left. + \frac{\omega}{M}\phi_\omega(1)\underset{\sim}{\sigma}^{(1)}.\underset{\sim}{\nabla}_1 q^{(1)} + \frac{\omega}{M}\phi_\omega(2)\underset{\sim}{\sigma}^{(2)}.\underset{\sim}{\nabla}_2 q^{(2)}\right\}|f\rangle\chi_f\, d\tau \qquad (4,20)$$

χ_i , χ_f are the properly symmetrised wave functions for the two nucleon system before and after the transition. The matrix element over the isotopic spin variables is evaluated as in the scalar case - the resulting matrix element for the transition is thus

$$\frac{f}{2\sqrt{\omega}\,\mu}\int \chi_i^* \langle i | \{ \underset{\sim}{\sigma}^{(1)}\cdot\underset{\sim}{\nabla}_1 \phi_\omega(1) - \underset{\sim}{\sigma}^{(2)}\cdot\underset{\sim}{\nabla}_2 \phi_\omega(2)$$

$$+ \frac{\omega}{M}\phi_\omega(1)\underset{\sim}{\sigma}^{(1)}\cdot\underset{\sim}{\nabla}_1 - \frac{\omega}{M}\phi_\omega(2)\underset{\sim}{\sigma}^{(2)}\cdot\underset{\sim}{\nabla}_2 \} | f \rangle \chi_f \, d\tau \ .$$

The spacial part of the meson eigenwave is unaltered and for an s-state meson

$$\phi_\omega(1) = \phi_\omega(2) = R_{10}\left(\frac{r}{2}\right).$$

Writing

$$\chi_i = g(r)$$

$$\chi_f = f(r)\, e^{i \underset{\sim}{P}.\underset{\sim}{R}}$$

and integrating over $\underset{\sim}{R}$ gives conservation of total momentum. The matrix element can now be written in terms of an integral over the relative coordinate $\underset{\sim}{r}$ as:-

$$\frac{f}{2\sqrt{\omega}}\,\frac{1}{\mu}\int g(r) \langle i | \underset{\sim}{\sigma}^{(1)}\cdot \underset{\sim}{\nabla} R_{10}\left(\frac{r}{2}\right) + \underset{\sim}{\sigma}^{(2)}\cdot\underset{\sim}{\nabla} R_{10}\left(\frac{r}{2}\right)$$

$$+ \frac{\omega}{M} R_{10}\left(\frac{r}{2}\right)\underset{\sim}{\sigma}^{(1)}\cdot\nabla + \frac{\omega}{M}R_{10}\left(\frac{r}{2}\right)\sigma^{(2)}\cdot\underset{\sim}{\nabla} | f \rangle f(r)\, d\underset{\sim}{r} \, .$$

The large term is

$$\frac{f}{\mu}\frac{\sqrt{\omega}}{2M}\int g(r)\,R(\tfrac{r}{2})\,\langle i|\,\underset{\sim}{\sigma}^{(1)}+\underset{\sim}{\sigma}^{(1)}|\,f\rangle\,.\,\underset{\sim}{\nabla}\,f(\underset{\sim}{r})\,d\underset{\sim}{r}\,.$$

The spin matrix element follows from the fact that the initial spin state is triplet $^3\sigma_m$. If the final spin state is singlet $^1\sigma_0$ the matrix element vanishes, while if triplet $^3\sigma_{m'}$ it is

$$\frac{f}{\mu}\frac{\sqrt{\omega}}{M}\int g(r)\,R_{10}(\tfrac{r}{2})\,\nabla f(\underset{\sim}{r})\,d\underset{\sim}{r}\quad\delta_{m,\,m'}.$$

Allowed transitions are thus to odd triplet states of the two neutron system. The integral over $\underset{\sim}{r}$ is calculated by assuming $g(\underset{\sim}{r})$ is the deuteron wave function defined in chapter three and $f(\underset{\sim}{r})$ is given by $(e^{i\underset{\sim}{p}.\underset{\sim}{r}}-e^{-i\underset{\sim}{p}.\underset{\sim}{r}})\,2^{-1/2}$ assuming no interaction in odd states.

The integration is rather tedious but finally leads to a transition probability per unit time

$$\omega=\frac{16}{3}f^2\,b^3(b^2-1)\,\frac{k^5p^3\omega}{\mu^2M}\left(\frac{1}{a}\right)^3\left(p^2+\frac{(b-1)^2}{4}k^2\right)^{-2}\left(p^2+\frac{(b+1)^2}{4}k^2\right)^{-2}. \tag{4,21}$$

Here the units for f are now 'natural' and b , κ are defined in terms of the depth and width of the binding well for the deuteron system as in the discussion of the two body problems in Chapter Three. A factor 2/3 arises from summing over the final spin states and averaging over the initial spin states of the two nucleon system. Numerical values for the constants in the probability are taken as in previous calculations, and lead to a transition probability per unit time of

$$\omega \doteqdot 1.1 \times 10^{15} f^2 \quad \sec.^{-1} \qquad (4,22)$$

and thus a very short lifetime for direct nuclear capture from the K shell is expected. Estimation of the lifetime for the capture from 2p orbit gives a multiplying factor $\sim (pa)^2 \sim 1.2 \times 10^5$ and thus a lifetime from this orbit of $\sim 10^{-10}\ 1/f^2$ sec. The ratio between differing principal quantum numbers for p-orbits is as in the absorption of scalar type mesons from similar orbits.

§ 5. Absorption of π^--mesons by heavy nuclei.

As a model to consider this problem it is assumed as before that the π^--meson is captured from Coulomb orbits about the nucleus but that the energy gained by the nucleus $\omega \sim$ 140 Mev. is transferred to one proton of the nucleus. The capturing proton is considered bound to the nucleus. The energy gained by the capture of the meson is large compared with the binding energy $I \sim 10$ Mev. of the proton in the nucleus; thus the nucleon is supposed to be emitted with energy

$$E = M - I + \omega .$$

It assumed also that the neutron emitted is in a free momentum state with momentum $\underset{\sim}{p}$, so that

$$E^2 = M^2 + p^2$$

and the density of allowed energy states for the final system is

$$\frac{pE\, d\Omega}{(2\pi)^3} \quad \text{/unit volume.}$$

For the initial binding of the proton to the nucleus it is assumed that the proton moves in a simple potential well of geometrical range $1/\alpha$.

The transition probability per unit time for the absorption of a pseudoscalar meson of energy ω is

$$w = \frac{pE\, d\Omega}{8\pi^2 \omega} \left| \int f^*(r) \langle 1 | g\rho_2 \phi_\omega + \frac{f}{\mu} \underset{\sim}{\sigma} \cdot \underset{\sim}{\nabla} \phi_\omega + i \frac{f}{\mu} \rho_1 \omega \phi_\omega | 2 \rangle e^{i \underset{\sim}{p} \cdot \underset{\sim}{r}} d\tau \right|^2 , \tag{4,23}$$

$f(\underset{\sim}{r})$ is the special wave function of the bound proton and $e^{i \underset{\sim}{p} \cdot \underset{\sim}{r}}$ is the spacial wave function of the final free neutron emitted. A long calculation gives the transition probability per unit time for capture from a Coulomb orbit with quantum numbers n , l , m to be

$$\omega = \frac{2pE}{\omega N^2} N_1^2 \sum_{k=0}^{n-\ell-1} \sum_{k'=0}^{n-\ell-1} M(k) M(k') \Big[p^2 \frac{\{f \frac{\omega}{\mu} - g\}^2}{(E+M)^2} J_{t_k,\ell} J_{t_{k'},\ell}$$

$$+ 2p \frac{\{f \frac{\omega}{\mu} - g\}}{E+M} \frac{f}{\mu} J_{t_k,\ell} \frac{\ell-m+1}{2\ell+1} \left(k' J_{t_{k'}-1,\ell-1} - \frac{Z}{na} J_{t_{k'},\ell+1} \right)$$

$$+ \frac{f^2}{\mu^2} \Big\{ \frac{\ell-m+1}{2\ell+1} \left(k J_{t_k-1,\ell+1} - \frac{Z}{na} J_{t_k,\ell+1} \right)$$

$$+ \frac{\ell+m}{2\ell+1} \left([2\ell+k+1] J_{t_k-1,\ell-1} - \frac{Z}{na} J_{t_k,\ell-1} \right) \times$$

$$\left([2\ell+k'+1] J_{t_{k'}-1,\ell-1} - \frac{Z}{na} J_{t_{k'},\ell-1} \right) \Big\} \Big] \qquad (4,24)$$

where

$$f(r) = N_1 e^{-\alpha r} r^s$$

$$M(k) = (-)^k \left\{ \frac{(n-\ell-1)!}{2n} \right\}^{1/2} \left(\frac{2Z}{na} \right)^{\ell+k+3/2} \frac{[(n+\ell)!]^{1/2}}{(n-\ell-1-k)!\,(2\ell+1+k)!\,k!}$$

$$J_\ell(t) = \sum_{s=0}^{\ell} \left\{ c(s,\ell) \frac{[n-(2s+1)]!}{(t-ip)^{n-2s}} + d(s,\ell) \frac{[n-(2s+2)]!}{(t-ip)^{n-(2s+1)}} + \text{conjugate} \right\}$$

where

$$c(s,l) = \frac{-1}{2p^{2s+1}} e^{-i\frac{\ell+1}{2}\pi} a_{2s}(\ell)$$

$$d(s,l) = \frac{i}{2p^{2s+2}} e^{-i\frac{\ell+1}{2}\pi} b_{2s+1}(\ell)$$

and J has argument $t = \beta_n = \alpha + Z/na$; $t_k = l + 2 + s$

For capture from s state with $l = 0$, $m = 0$, $n = 1$ and an exponential well binding so that $N_1 = \sqrt{\alpha^3/\pi}$ and $s = 0$ this reduces to (f coupling only)

$$\omega = \frac{128 p^3 E}{(\beta^2+p^2)^4} \left(\frac{Z}{a}\right)^3 \frac{\alpha^3}{\omega} \frac{f^2}{\mu^2} \left[\frac{\omega\beta}{E+M} - \frac{Z}{a}\right]^2 \frac{1}{N^2} , \tag{4,25}$$

while for the p-state $l = 1$, $n = 2$, $m = 0$ and a Yukawa well binding so that $N_1 = \sqrt{\alpha/2\pi}$ and $s = -1$, it reduces to

$$\omega = \frac{pE}{2N^2} \frac{\alpha}{\omega} \left(\frac{Z}{a}\right)^5 \left[\frac{f^2}{\mu^2} - \frac{4}{3}\frac{f}{\mu} \frac{(f\frac{\omega}{\mu} - g)}{E+M} \frac{p^2}{(\beta^2+p^2)} + \frac{4}{3} \frac{\{f\frac{\omega}{\mu} - g\}^2}{(E+M)^2} \frac{p^4}{(\beta^2+p^2)^2} \right] \frac{Z}{(\beta^2+p^2)^2} . \tag{4,26}$$

Approximate numerical values give, for Z not too large, transition probabilities:-

from s-orbit $\omega = Z^4\, 7 \times 10^{16}\, f^2 \text{ sec.}^{-1}$

from p-orbit $\omega = Z^4\, 3 \times 10^{11}\, f^2 \text{ sec.}^{-1}$

Comparing these times with accurate times calculated for Deuterium, $Z = 1$, it is seen that this model underestimates the time of capture for very light nuclei.

§ 6. Discussion

The method of calculating non-radiative absorption times for a π^--meson around heavy nuclei show that for nuclei, other than comparatively light nuclei, such absorption will be very rapid indeed. For pseudoscalar and vector mesons in S orbits about Chromium for example the lifetime is less than 10^{-21}sec./f^2. For light nuclei the probability of absorption varies as Z^4 while for heavy as Z^6. A factor $\sim (Z/pa)^2$ reduces the probability for capture from p-orbits as expected.

For Deuterium the interest lies in the branching ratio between absorption with and without radiation. If the meson is described by a scalar wave function the factor between the probability of absorption non-radiatively to radiatively is (assuming meson initially in L shell)

$$g^2 \; 6.2/370$$

which for $g^2 \sim 3$ (to give agreement with non-radiative capture times found phenomenologically by Brueckner, Serber and Watson and also to give correct absolute magnitudes for meson production by proton-proton collisions c.f. Chapter Three) is a factor 40 times too small to give the experimentally determined ratio of Panofsky, Aamodt and Hadley (1951) 7/3. If the meson is supposed described by a pseudoscalar wave function the corresponding ratio of importance is between radiative and non-radiative capture from the K shell.

Probability for absorption per second with no γ emission was $1.1 \times 10^{15} f^2$ sec.$^{-1}$ Probability for radiative capture per second has been estimated by Brueckner, Serber and Watson from the observed cross-section for π^+ meson production by high energy photons (Steinberger and Bishop (1950)) on protons as

$$2.7 \times 10^{14} \text{ sec.}^{-1}$$

A method of calculation similar to that given in above paragraphs does not apply straightforwardly here since it is necessary to consider transition involving intermediate states. The radiative capture, (as radiative production), can take place not only through a quadratic mixed term in the energy density, but also through a two stage process of similar order in the coupling constants. Bruno (1949) gives reasons for possible neglect of these second order terms but it is not at all clear (since partial integrations can cause derivatives to act on rapidly oscillating terms) that these are valid. An order of magnitude estimation taking direct coupling only, gives probabilities which are rather large. For example with the final two neutron system assumed in a triplet P state this probability is $4.2 \times 10^{15} f^2$ sec.$^{-1}$ Using the Brueckner, Serber and Watson value the ratio is f^2 11/2.7 which gives the observed branching ratio for $f^2 \sim .57$ - a reasonable value.

It thus appears that, assuming the π^- - meson has spin zero, the evidence concerning its character from absorption experiments in Deuterium points to a meson of odd parity. This conclusion is in agreement with Tamor and Marshak (1950), who obtain a ratio 2.1 for pseudoscalar mesons and quote a ratio 1/30 for scalar. These authors have extended their calculations to vector mesons and find again a predominance of non-radiative capture to radiative of 55.

Finally to complete the discussion of present day evidence on the nature of the π- meson obtainable from absorption experiments, reference is made to those of Panofsky, Aamodt and York (1950), who have measured the γ ray spectra resulting from the absorption of π^--mesons in Hydrogen. These experiments show that the two processes $\pi^- + P \rightarrow N + \gamma$ (b) and $\pi^- + P \rightarrow N + \pi^0$ (d), suggested by Marshak and Wightman (1949), actually do take place, with a frequency between the competing processes of the order of unity ($\sim$ 1.06). Interpretation of these results with the theoretical implications have been discussed at length by Marshak, Tamor and Wightman (1950). It is clear that for the mesic scattering to take place in lowest order from Coulomb s-states of the π^-- meson-proton system the parities of π^- and π^0 should be alike. Assuming that both have spin zero - an assumption highly probable for π^0 on account of its decay into two quanta and probable for π^- from its absorption in Deuterium. This is confirmed by the absorption probabilities estimated by Marshak, Tamor and Wightman who, combining the evidence from the absorption in Deuterium and the absorption in Hydrogen, state that: 'The only consistent weak coupling theory which is possible is that with a pseudoscalar π^0 '.

CHAPTER FIVE

THE DECAY OF A HEAVY NEUTRAL MESON TO PAIRS OF π - MESONS

§ 1. Introduction

In Chapter One the evidence at present known concerning the decay of a heavy neutral V-meson into two charged particles is summarised. It is the purpose of this chapter to consider one tentatively accepted view that a V_0 meson, of mass ~ 1000 electron masses, decays into two particles these being π - mesons. If no third neutral decay particle is present, partially verified by the coplanarity of the V tracks with the initial nuclear explosion, then the assumption is made that the heavy neutral particle, which must be a boson, has a direct trilinear coupling to the nuclear field. The calculations are analogous to those of Steinberger (1949) who considered the lifetime of a π°- meson decaying to two quanta.

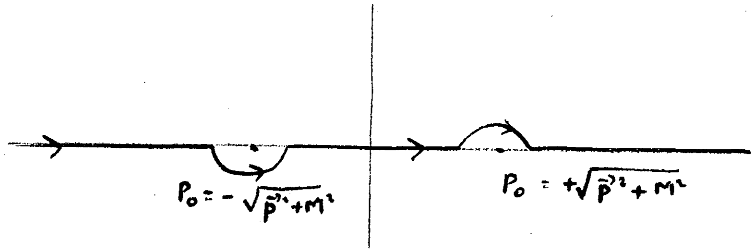
With a reasonable magnitude for the coupling constant between the V-meson and nucleon fields, consistent with the frequency of the production of these heavy particles, it is shown that the result of a first order calculation is to give lifetimes, where the decay is allowed, much shorter than the observed time of ~ 3×10^{-10} sec. Consideration of possible alternative descriptions of the observed process will be given in the subsequent discussion.

§ 2. Notation

A four-vector is denoted by $\underline{x}$ and the three space vector formed from its spacial components is denoted by $\vec{x}$. The infinitesimal element of four space $dx_1\ dx_2\ dx_3\ dx_0$ is written d^4x. μ_τ is the mass of the heavy decaying meson while μ is the mass of the either decay product, both in natural units. $\underline{k}_0$ is the four-vector momentum of the heavy meson; thus $k_4 = i\mu_\tau$ in rest frame. $\underline{k}_1$, $\underline{k}_2$ are the four-vector momenta of the decay products. g_τ, g are the coupling constants, in Lorentz-Heaviside units, between the heavy and π meson fields respectively with the nucleon field. $S_+(x)$ is the kernel of the Dirac wave equation consistent with hole theory and is defined by

$$S_+(\underline{x}) = \frac{1}{(2\pi)^4}\int_{I_2} \frac{i\gamma_\sigma p_\sigma - M}{p^2 + M^2}\, e^{ip.\underline{x}}\, d^4p \qquad (5,1)$$

where γ's are the Dirac matrices, M the mass of the quantum of the spinor-field i.e. the nucleon mass in the present problem. I_2 is the contour in the p_0-plane over which the integration of dp_0 is carried out, which, because of the poles on the real p_0 axis, needs special definition. It is shown in Feynman's paper (Feynman 1949) that the contour for this integration, to give matrix elements consistent with the hole theory of the spinor field, I_2, is given by:-



p_0-plane

(5,2)

F_τ and F are the coupling terms between the nucleon field and the various meson fields and are given from appendix 1. in the following table:-

Meson Type.	Coupling.	F.
Scalar	scalar	1
	vector	$-i\frac{\gamma_\mu k_\mu}{\mu}$
Pseudoscalar	pseudoscalar	γ_5
	pseudovector	$-i\frac{\gamma_5\gamma_\mu k_\mu}{\mu}$
Vector	vector	$e_\mu\gamma_\mu$
	tensor	$\frac{-i\gamma_\lambda\gamma_\mu + i\gamma_\mu\gamma_\lambda}{\mu} k_\mu e_\lambda$
Pseudovector	pseudovector	$e_\mu\gamma_5\gamma_\lambda$

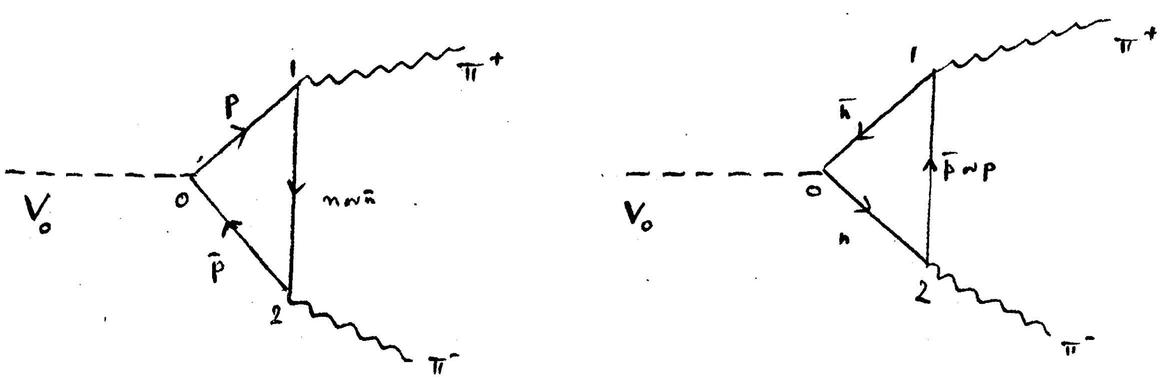
(5,3)

§ 3. General Formalisml

It is convenient in this problem to use the Feynman technique to obtain the matrix elements for the transition and the calculations arising from the older methods are equivalent to this procedure. With the kernels defined as in paragraph two the matrix element can be written down on inspection of the Feynman diagrams (see Dyson (1949)) for the process.

Consider the disintegration of a V-meson (dotted line) into two lighter π mesons (wavy line) through an intermediate nucleonic field (straight line). The lowest order Feynman diagrams are as follows:-

The initial meson with four-vector momentum $\underline{k}_0$ creates a virtual nucleon-antinucleon pair, the nucleon radiates a π - meson of four momentum $\underline{k}_1$, and finally the nucleon and antinucleon mutually annihilate emitting the second π- meson of four momentum $\underline{k}_2$. It is clear that for the production of charged π mesons the two resulting graphs are:-



Other graphs differ only by the position of the vertices 0 , 1, 2 relative to the time axis.

The ambiguity in the type of intermediate particle in the third side of the triangle is trivial being merely dependent on the orientation of this virtual nucleon line with respect to the time axis.

The transition element for the process is given by

$$g_\tau g^2 \Big[S_+(0-2)\, F^{(1)}\, S_+(2-1)\, F^{(1)}\, S_+(1-0) F_\tau + S_+(0-1)\, F^{(1)}\, S_+(1-2)\, F^{(2)}\, S_+(2-0) F_\tau \Big] \tag{5,4}$$

where isotopic factors have been neglected. It is assumed that the V-coupling is neutral while the π -coupling is charged thus the matrix element over the isotopic space is unity for both diagrams. It is noted that a symmetric theory (van Wyk (1950)) may give rise to additional selection rules; but as here the interest lies in the magnitude of the lifetimes where allowed this theory will not be considered.

With certain couplings between the nucleonic and the two types of mesonic field the matrix elements for such processes diverge and some convergence procedure is necessary to give finite lifetimes. In the formal computation of this chapter use is made of the Pauli regulator technique in eliminating these divergences. The subtraction method, for details see Pauli and Villass (1949), can be easily understood as consisting of the introduction of several fictitious intermediate fields as well as the nucleon field. The matrix element is regarded as a function of M , the nucleon mass, and to this matrix element are added or subtracted several others for the same process through the additional intermediate fields with masses M.. The infinities arising in each matrix element are of the same type and it is possible to choose the masses of the intermediate fictitious particles so as to give a final convergent matrix element. The conditions imposed on the fictitious fields will be discussed when the divergent integrals for the problem under consideration are obtained.

For decay into photons the result for decay into transverse vector mesons with vector coupling are immediately applicable to give Steinberger's (1949) results.

§ 4. The transition probability per unit time.

The matrix element for the decay process is between meson states:-

Initially $\sqrt{\frac{1}{2\nu_0}}\; e^{i\,\underline{k}_0 . \underline{x}_0}$

Finally $\sqrt{\frac{1}{2\nu_1}}\; e^{i\,\underline{k}_1 . \underline{x}_1} \cdot \sqrt{\frac{1}{2\nu_2}}\; e^{i\,\underline{k}_2 . \underline{x}_2}$

where ν_0 , ν_1 , ν_2 are the energies of the real mesons disintegrating and produced.

The transition element is thus from (5,1 and 5,4)

$$\frac{g_\tau g^2}{\sqrt{\{8\nu_0\nu_1\nu_2\}}} \frac{1}{(2\pi)^{12}} \int e^{-i\underline{k}_1.\underline{x}_1} e^{-i\underline{k}_2.\underline{x}_2} e^{i\underline{k}_0.\underline{x}_0} \times$$

$$\left\{ \frac{e^{i\underline{p}_{02}.(\underline{x}_0-\underline{x}_2)}}{i\gamma_\sigma p_\sigma^{02}+M} F^{(2)} \frac{e^{i\underline{p}_{21}.(\underline{x}_2-\underline{x}_1)}}{i\gamma_\tau p_\tau^{21}+M} F^{(1)} \frac{e^{i\underline{p}_{10}.(\underline{x}_1-\underline{x}_0)}}{i\gamma_\rho p_\rho^{10}+M} F \right.$$

$$\left. + \frac{e^{i\underline{p}_{01}.(\underline{x}_0-\underline{x}_1)}}{i\gamma_\rho p_\rho^{01}+M} F^{(1)} \frac{e^{i\underline{p}_{12}.(\underline{x}_1-\underline{x}_2)}}{i\gamma_\tau p_\tau^{12}+M} F^{(2)} \frac{e^{i\underline{p}_{20}.(\underline{x}_2-\underline{x}_0)}}{i\gamma_\sigma p_\sigma^{20}+M} F \right\} \times$$

$$d\underline{x}_0\, d\underline{x}_1\, d\underline{x}_2\, d\underline{p}_{02}\, d\underline{p}_{21}\, d\underline{p}_{10}\,.$$

$$= \frac{g_\tau g^2}{\sqrt{\{8\nu_0\nu_1\nu_2\}}} \int \delta(\underline{p}_{02}-\underline{p}_{10}+\underline{k}_0)\,\delta(\underline{p}_{10}-\underline{p}_{21}-\underline{k}_1)\,\delta(\underline{p}_{21}-\underline{p}_{02}-\underline{k}_2)$$

$$\times \Big[(i\gamma_\sigma p_\sigma^{02}-M) F^{(2)} (i\gamma_\tau p_\tau^{21}-M) F^{(1)} (i\gamma_\rho p_\rho^{10}-M) F$$

$$+ (-i\gamma_\rho p_\rho^{10}-M) F^{(1)} (-i\gamma_\tau p_\tau^{21}-M) F^{(2)} (-i\gamma_\sigma p_\sigma^{02}-M) F \Big] \frac{d\underline{p}_{02}\, d\underline{p}_{21}\, d\underline{p}_{10}}{D}$$

where

$$D = (p_{02}^2+M^2)(p_{21}^2+M^2)(p_{10}^2+M^2)$$

Now defining N

$$= (i\gamma_\sigma q_\sigma - M) F^{(2)} (i\gamma_\tau r_\tau - M) F^{(1)} (i\gamma_\rho p_\rho - M) F$$

$$+ (-i\gamma_\rho p_\rho - M) F^{(1)} (-i\gamma_\tau r_\tau - M) F^{(2)} (-i\gamma_\sigma q_\sigma - M) F$$

and $\underline{q} = \underline{p} - \underline{k}_2$; $\underline{r} = \underline{p} + \underline{k}_1$

so that

$$D = [q^2 + M^2][p^2 + M^2][r^2 + M^2],$$

the transition element becomes

$$\frac{g_\tau g^2}{\sqrt{\{8 \nu_0 \nu_1 \nu_2\}}} \; \delta(\underline{k}_0 - \underline{k}_1 - \underline{k}_2) \int \frac{N d^4 p}{D} .$$

All spin energy states of the intermediate nucleons contribute so summation is carried out over all such possible states. The δ function shows overall conservation of energy and momentum and the transition element is interpreted as giving a transition probability per unit time

$$\omega = 2\pi \rho(E) \left| \frac{g_\tau g^2}{4\sqrt{2\nu_0 \nu_1 \nu_2}} \frac{1}{(2\pi)^4} \int \frac{S_p N}{D} d^4 p \right|^2 , \qquad (5,5)$$

where $\rho(E)$ is the density of final states at an energy E .

§ 5. Evaluation of the Integral

Let

$$J = \frac{1}{8} \int \frac{S_p N}{D} d^4 p \qquad (5,6)$$

and using the result

$$a^{-1}b^{-1}c^{-1} = \int_0^1\int_0^1 \frac{2y\,dy\,dx}{[axy + b(1-x)y + c(1-y)]^3} \tag{5,7}$$

which is an extension of various of Feynman's results, it follows that

$$\frac{1}{D} = \int_0^1\int_0^1 \frac{2y\,dy\,dx}{[(q^2+M^2)xy + (p^2+M^2)(1-x)y + (r^2+M^2)(1-y)]^3}$$

$$= \int_0^1\int_0^1 \frac{2y\,dy\,dx}{[p^2 - 2\underline{p}.\underline{k} + \Delta]^3}$$

where $\underline{k} = \underline{k}^2 xy - \underline{k}^1(1-y)$

and $\Delta = M^2 + k_2^2 xy + k_1^2(1-y)$.

For the couplings which are considered in detail in this chapter SpN can be written

$$\mathrm{SpN} = 8\,M(A p^2 + B_{\lambda\mu} p_\lambda p_\mu + C_\lambda p_\lambda + R + S M^2 \quad) \tag{5,8}$$

where A , B , C , R and S are (dimensional) functions of $\underline{k}^1$, $\underline{k}^2$ but not of M , x , y or p . This is easily seen from the form of N

$$(i\gamma_\sigma q_\sigma - M)F^{(2)}(i\gamma_\tau p_\tau - M)F^{(1)}(i\gamma_\rho r_\rho - M)F$$

$$+ (-i\gamma_\rho r_\rho - M)F^{(1)}(-i\gamma_\tau p_\tau - M)F^{(2)}(-i\gamma_\sigma q_\sigma - M)F,$$

as any non-vanishing spur, for the F's chosen (not for example pseudoscalar with pseudovector coupling) will have factor M or M^3 as terms dependent on even functions of M cancel in the two parts. Thus final form for Spur N is as given in (5,8). A , B , C , R and S are determined on fixing the meson types and their couplings to the nuclear field - they are not dimensionless.

In order to evaluate J , I is defined by

$$I = \int_{I_2} \frac{M(Ap^2 + B_{\lambda\mu} p_\lambda p_\mu + C_\lambda p_\lambda + R + SM^2)}{[p^2 - 2\underline{p}.\underline{k} + \Delta]^3} d^4p$$

so that

$$J = \int_0^1\int_0^1 I\, 2y\, dy\, dx. \qquad (5,9)$$

In I $\underline{p}$ is replaced by $\underline{q} + \underline{k}$ and thus

$$I = \int_{I_2} \frac{M(Aq^2 + B_{\lambda\mu} q_\lambda q_\mu + SM^2 + T)}{[q^2 + M^2 - L]^3} d^4q$$

where $\Delta - k^2 = M^2 - L$

$$\text{so that} \quad L = k^2 - (k_2^2\, xy + k_1^2\,(1 - y))$$

$$\text{and where} \quad T = R + C_\lambda k_\lambda + B_{\lambda\mu} k_\lambda k_\mu + A k^2 \qquad (5,10)$$

In deducing the last form for I the factors $A\underline{q}.\underline{k}$; $B_{\lambda\mu}\ q_\lambda k_\mu$ etc. vanish on integration over $\underline{q}$, as

$$\int \frac{q_\lambda}{[q^2 + M^2 - L]^3} d^4q$$

is a vector with no preferred direction.

I is, in general, improper and must be regulated.

Case I. Suppose $A = 0 = B_{\lambda\mu}$.

Integral is proper and can be evaluated as

$$I = \int_{I_2} M \frac{SM^2+T}{[q^2+M^2-L]^3} d^4q$$

$$= M \frac{SM^2+T}{M^2-L} \frac{\pi^2 i}{2} ; \qquad (5,11)$$

for

$$\int_{I_2} \frac{d^4q}{[q^2+M^2-L]^3} = \int_{I_2} \frac{d\vec{q}\, dq_0}{[\vec{q}^2+M^2-L-q_0^2]^3}$$

and integrating first over I_2 the dq_0 variable, the integrand has poles at $q_0 = \pm\sqrt{\{\vec{q}^2+M^2-L\}}$.

Integration is carried out by completing I_2 into a closed contour by a large semi-circle below the real q_0 axis.

$$\int_{I_2} \frac{d\vec{q}\, dq_0}{[\vec{q}^2+M^2-L-q_0^2]^3} = -2\pi i \operatorname*{Res}_{q_0=+\sqrt{\ }} \int \frac{d\vec{q}}{[\vec{q}^2+M^2-L-q_0^2]^3}$$

$$= 2\pi i \frac{3}{16} \int \frac{d\vec{q}}{[\vec{q}^2+M^2-L]^{5/2}}$$

$$= \pi i \frac{3}{8} \int \frac{4\pi \vec{q}^2 dq}{[\vec{q}^2+M^2-L]^{5/2}}$$

$$= \frac{\pi^2 i}{2} \frac{1}{M^2-L} .$$

The latter result being trivial.

In the approximation which is used for all processes considered, namely that $M \gg |k|$

$$I \doteq \frac{\pi^2 i}{2M}(SM^2+T), \tag{5,12}$$

<u>Case II</u> Suppose one of A , B not zero - two subclasses to consider

i) $H = A + \frac{B_{\mu\mu}}{4} = 0$

ii) $H \neq 0$

Before proceeding to evaluate the integrals involved for these cases it is seen that

$$\int \frac{q_\lambda q_\mu}{[q^2+M^2-L]^3} d^4q = \Theta\, \delta_{\lambda\mu}$$ as it is a tensor

with no preferred directions. It follows at once that

$$\int \frac{q^2}{[q^2+M^2-L]^3} d^4q = 4\,\Theta$$

and

$$\int \frac{B_{\lambda\mu} q_\lambda q_\mu + A q^2}{[q^2+M^2-L]^3} d^4q = B_{\mu\mu}\Theta + 4A\Theta = 4H\Theta.$$

<u>Case II (i)</u>

The remaining term is $\int \frac{M(SM^2+T)}{(q^2+M^2-L)^3} d^4q$ which is finite and is the term considered in Case 1. (5,11) .

However for a consistent regularisation procedure this term requires regularisation as it is part of an infinite term.

Θ being of course infinite.

$$\text{Reg} \int \frac{M(SM^2+T)}{[q^2+M^2-L]^3} d^4q$$

$$= \sum C_i M_i \int \frac{SM_i^2+T}{[q^2+M_i^2-L]^3} d^4q \qquad \text{where } C_i = \pm 1$$

the sign depending on whether the auxiliary field with carrier mass M_i is to be added or subtracted.

$$\text{Reg. I} = \sum C_i M_i \frac{\pi^2 i}{2} \left\{ S + \frac{T+SL}{M_i^2-L} \right\}.$$

The conditions imposed on the fictitious fields have now to be considered. No real processes involving these additional fields are allowed and one requires the masses corresponding to these extra fields to be very large. The final term then gives

$$\frac{\pi^2 i}{2} M \frac{(T+SL)}{M^2-L}.$$

The first term will be zero - the condition $\sum M_i C_i = 0$ being applied. Then

$$\text{Reg. I} = \frac{i\pi^2}{2} \frac{T+SL}{M} \tag{5,13}$$

<u>Case II (ii)</u> $H \neq 0$

$$I = M\int \frac{Hq^2 + SM^2 + T}{[q^2 + M^2 - L]^3}\, d^4q$$

$$= MH\int \frac{(q^2+M^2-L) + S/_H\, M^2 + T/_H - M^2 + L}{[q^2 + M^2 - L]^3}\, d^4q$$

$$= MH\int \frac{d^4q}{[q^2+M^2-L]^2} + M\int \frac{(S-H)M^2 + T + LH}{[q^2 + M^2 - L]^3}\, d^4q.$$

Reg. I =

$$\sum C_i M_i H\int \frac{d^4q}{[q^2+M_i^2-L]^2} + \sum C_i M_i \int \frac{(S-H)M_i^2 + T + LH}{(q^2 + M_i^2 - L)^3}\, d^4q.$$

The final regulated integral is of the form considered in obtaining (5,13) and gives

$$i\frac{\pi^2}{2M}\left\{(T + LH) + (S-H)L\right\}$$

$$= \frac{i\pi^2}{2M}(T + SL).$$

The former integral is logarithmetrically divergent as it stands and a further condition on the regularisation procedure is required. Now $\sum_i C_i M_i \log M_i$ would become infinite as M_i are made large unless it is required that $\sum_i C_i M_i \log M_i$

is constant. This constant seems to be arbitrary, and as long as it is so, the subtraction is not unique. It has been taken here to be zero. This gives the intuitively correct result that the final matrix element will be small if the intermediate masses M_i are large.

Thus Reg. I = $\frac{i\pi^2}{2M}(T+SL)$. (5,14)

Returning to the evaluation of J : the definition (5,9) of J in respect to I is

$$J = \int_0^1\int_0^1 2y\,\bar{I}\,dy\,dx$$

The analysis is fairly long when the integration over x and y is carried out and is merely sketched below.

<u>Case I.</u>

$$\int_0^1\int_0^1 2y\,dy\,dx\ \frac{\pi^2 i}{2M}(SM^2+T)$$

$$= \pi^2 i\, SM\int_0^1\int_0^1 y\,dy\,dx + \frac{\pi^2 i}{M}\int_0^1\int_0^1 (R + C_\lambda k_\lambda + B_{\lambda\mu}k_\lambda k_\mu + Ak^2)\,y\,dy\,dx$$

$$= i\pi^2\frac{SM}{2} + \frac{i\pi^2 R}{2M} + \frac{i\pi^2}{M}\int_0^1\int_0^1 \Big\{ C_\lambda\left[k_\lambda^2 xy - k_\lambda^1(1-y)\right]$$

$$+ B_{\lambda\mu}\left[k_\lambda^2 xy - k_\lambda^1(1-y)\right]\left[k_\mu^2 xy - k_\mu^1(1-y)\right]$$

$$+ A\left[k_\lambda^2 xy - k_\lambda^1(1-y)\right]\left[k_\lambda^2 xy - k_\lambda^1(1-y)\right]\Big\}\,y\,dy\,dx$$

$$= i\pi^2\Big[\frac{SM}{2} + \frac{R}{2M} + \frac{C_\lambda}{M}\Big\{k_\lambda^2\,\frac{1}{2}\cdot\frac{1}{3} - k_\lambda^1\left(\frac{1}{2}-\frac{1}{3}\right)\Big\}$$

$$+ \frac{B_{\lambda\mu}}{M}\Big\{k_\lambda^2 k_\mu^2\,\frac{1}{3}\cdot\frac{1}{4} + k_\lambda^1 k_\mu^1\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right) - k_\lambda^2 k_\mu^1\,\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right) - k_\lambda^1 k_\mu^2\,\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)\Big\}$$

$$+ \frac{A}{M}\Big\{k_\lambda^2 k_\lambda^2\,\frac{1}{3}\cdot\frac{1}{4} + k_\lambda^1 k_\lambda^1\left(\frac{1}{2}-\frac{2}{3}+\frac{1}{4}\right) - 2\,k_\lambda^2 k_\lambda^1\,\frac{1}{2}\left(\frac{1}{3}-\frac{1}{4}\right)\Big\}\Big]$$

$$= \frac{i\pi^2}{12}\Big[6SM + 6R/M + 2\frac{C_\lambda}{M}(k_\lambda^2 - k_\lambda^1) + \frac{B_{\lambda\mu}}{M}(k_\lambda^2 k_\mu^2 + k_\lambda^1 k_\mu^1 - \tfrac{1}{2}k_\lambda^2 k_\mu^1 - \tfrac{1}{2}k_\lambda^1 k_\mu^2)$$

$$+ A/M\,(k_\lambda^2 k_\lambda^2 + k_\lambda^1 k_\lambda^1 - k_\lambda^2 k_\lambda^1)\Big]$$

which is written shortly as

$$J = i\frac{\pi^2}{2}SM + \frac{i\pi^2}{12M}T' \qquad (5,15)$$

for T' given by the expression

$$6R + 2C_\lambda(k_\lambda^2 - k_\lambda^1) + B_{\lambda\mu}(k_\lambda^2 k_\mu^2 + k_\lambda^1 k_\mu^1 - \tfrac{1}{2}k_\lambda^2 k_\mu^1 - \tfrac{1}{2}k_\lambda^1 k_\mu^2) \qquad (5,16)$$
$$+ A(k_\lambda^2 k_\lambda^2 + k_\lambda^1 k_\lambda^1 - k_\lambda^2 k_\lambda^1).$$

Case II. The evaluation of J proceeds similarly to case I and

$$J = -\frac{i\pi^2 S}{12M}(k_\lambda^2 k_\lambda^2 + k_\lambda^1 k_\lambda^1 + k_\lambda^2 k_\lambda^1) + \frac{i\pi^2}{12M}T'. \qquad (5,17)$$

§ 6. Special Reference Frame

A simplification is obtained by working in a coordinate system in which the initial particle is at rest. In the rest frame of the V-meson

$$\vec{k}_0 = 0 \,, \quad k_{00} = \nu_0 \quad = \mu_\tau$$

and

$$\vec{k}_1 = -\vec{k}_2 = \vec{k} \text{ (say)} \quad \text{and} \quad k_{10} = k_{20} = \nu \quad = \sqrt{\vec{k}^2 + \mu^2} \,.$$

There is a simplification of T' (5,16) and thus also of J. For example in case II

$$J = \frac{i\pi^2 S}{12M}(\mu^2 + 2\nu^2) + \frac{i\pi^2}{12M} T' \,.$$

In this frame the density of final states is

$$\rho(\epsilon) = \frac{k\nu\, d\Omega}{(2\pi)^3}$$

where for mesons with internal degrees of freedom further factors must be considered.

The transition per unit time is given from (5,5) and is

$$2\pi \frac{k\nu\, d\Omega}{(2\pi)^3} \left| \frac{g_\tau\, g^2}{4\sqrt{\{2\mu_\tau \nu^2\}}} \frac{1}{(2\pi)^4}\, 8J \right|^2 .$$

Integrating over all angles of emission $d\Omega$ and going over to ordinary units for the coupling constant $g \rightarrow \sqrt{4\pi}\, g$ this becomes:-

$$\omega = \frac{1}{2}\pi^{-6} g_\tau^2 g^4 \frac{k}{\mu_\tau \nu} J^2 \sec^{-1} \qquad (5,18)$$

The calculation is thus reduced to calculating J, for given types of meson in the V-meson rest frame.

§ 7. Calculation of Spurs

Detailed calculations are not given here for all possible cases of which there are forty nine. Typical examples are considered and results quoted for some others. Restriction is made to non-derivative couplings but selection rules for derivative couplings will be considered. Pseudovector mesons are not considered.

1/ Disintegration of Scalar Meson with Scalar Coupling

i/ Decay products scalar π -mesons with scalar coupling.

$$SpN = Sp\left[(i\gamma_\sigma q_\sigma - M)(i\gamma_\tau p_\tau - M)(i\gamma_\rho t_\rho - M) + (-i\gamma_\rho t_\rho - M)(-i\gamma_\tau p_\tau - M)(-i\gamma_\sigma q_\sigma - M)\right]$$

$$= Sp\left[2M\{\gamma_\sigma\gamma_\tau q_\sigma p_\tau + \gamma_\sigma\gamma_\rho q_\sigma t_\rho + \gamma_\tau\gamma_\rho p_\tau t_\rho\} - 2M^3\right]$$

$$= 8M\{ q_\sigma p_\sigma + q_\sigma t_\sigma + p_\sigma t_\sigma - M^2\}$$

$$= 8M\{3p^2 + 2p_\sigma(k_\sigma^2 - k_\sigma^1) - k_\sigma^1 k_\sigma^2 - M^2\}$$

(5,19)

ii/ Decay products pseudoscalar π -mesons with pseudoscalar coupling.

$$SpN = Sp\left[(i\gamma_\sigma q_\sigma - M)\gamma_5(i\gamma_\tau p_\tau - M)\gamma_5(i\gamma_\rho t_\rho - M) + (-i\gamma_\rho t_\rho - M)\gamma_5(-i\gamma_\tau p_\tau - M)\gamma_5(-i\gamma_\sigma q_\sigma - M)\right]$$

$$= Sp\left[(i\gamma_\sigma q_\sigma - M)(-i\gamma_\tau p_\tau - M)(i\gamma_\rho t_\rho - M) + (-i\gamma_\rho t_\rho - M)(i\gamma_\tau p_\tau - M)(-i\gamma_\sigma q_\sigma - M)\right]$$

$$= 8M\left\{-q_\sigma p_\sigma + q_\sigma t_\sigma - p_\sigma t_\sigma - M^2\right\}$$

$$= 8M\left\{-p_\sigma p_\sigma - k^2_\sigma p_\sigma - p_\sigma p_\sigma + k^1_\sigma p_\sigma + p_\sigma p_\sigma - k^1_\sigma k^2_\sigma - k^1_\sigma p_\sigma + k^2_\sigma p_\sigma - M^2\right\}$$

$$= 8M\left\{-p^2 - k^1_\sigma k^2_\sigma - M^2\right\}. \qquad (5,20)$$

iii/ Decay products vector π -mesons with vector coupling.

$$SpN = Sp\left[(i\gamma_\sigma q_\sigma - M)e^2_\lambda\gamma_\lambda(i\gamma_\tau p_\tau - M)e^1_\mu\gamma_\mu(i\gamma_\rho t_\rho - M) + (-i\gamma_\rho t_\rho - M)e^1_\mu\gamma_\mu(-i\gamma_\tau p_\tau - M)e^2_\lambda\gamma_\lambda(-i\gamma_\sigma q_\sigma - M)\right]$$

$$= Sp\,2\left[-M^3 e^2_\lambda e^1_\mu\gamma_\lambda\gamma_\mu + \left\{\gamma_\sigma\gamma_\lambda\gamma_\tau\gamma_\mu q_\sigma p_\tau + \gamma_\sigma\gamma_\lambda\gamma_\mu\gamma_\rho q_\sigma t_\rho + \gamma_\lambda\gamma_\tau\gamma_\mu\gamma_\rho p_\tau t_\rho\right\} M^2 e^2_\lambda e^1_\mu\right]$$

$$= -8M^2 e_\lambda^2 e_\lambda^1 + 8M e_\lambda^2 e_\mu^1 [q_\lambda p_\mu + q_\mu p_\lambda - \delta_{\lambda\mu} q_\sigma p_\sigma$$

$$+ p_\mu r_\lambda + q_\lambda r_\mu + q_\sigma r_\sigma \delta_{\lambda\mu} - q_\mu r_\lambda + p_\lambda r_\mu - \delta_{\lambda\mu} p_\sigma r_\sigma]$$

$$= 8M [e_\lambda^2 e_\lambda^1 \{-M^2 - p^2 - k_\sigma^2 p_\sigma - p^2 + k_\sigma^1 p_\sigma + p^2 - k_\sigma^1 p_\sigma + k_\sigma^2 p_\sigma - k_\sigma^1 k_\sigma^2\}$$

$$+ e_\lambda^2 e_\mu^1 \{4 p_\lambda p_\mu + 2k_\lambda^2 p_\mu + 2k_\lambda^1 p_\mu + k_\mu^2 k_\lambda^1 - k_\lambda^2 k_\mu^1\}]$$

$$= 8M [e_\lambda^2 e_\mu^1 \{4 p_\lambda p_\mu + 2k_\lambda^2 p_\mu + 2k_\lambda^1 p_\mu + k_\mu^2 k_\lambda' - k_\lambda^2 k_\mu^1\}$$

$$- e_\lambda^2 e_\lambda' \{M^2 + p^2 + k_\sigma' k_\sigma^2\}] . \qquad (5,21)$$

2/ Disintegration of Pseudoscalar Meson with Pseudoscalar Coupling.

1/ Decay products scalar π -mesons with scalar coupling.

$$SpN = Sp[(i\gamma_\sigma q_\sigma - M)(i\gamma_\tau p_\tau - M)(i\gamma_\rho r_\rho - M)\gamma_5$$
$$+ (-i\gamma_\rho r_\rho - M)(-i\gamma_\tau p_\tau - M)(-i\gamma_\sigma q_\sigma - M)\gamma_5]$$

$$= 0 \qquad (5,22)$$

ii/ Decay products pseudoscalar π -mesons with pseudoscalar coupling.

$$SpN = Sp[(i\gamma_\sigma q_\sigma - M)\gamma_5(i\gamma_\tau p_\tau - M)\gamma_5(i\gamma_\rho r_\rho - M)\gamma_5$$
$$+ (-i\gamma_\rho r_\rho - M)\gamma_5(-i\gamma_\tau p_\tau - M)\gamma_5(-i\gamma_\sigma q_\sigma - M)]$$

$$= 0 . \qquad (5,23)$$

iii/ Decay products vector π -mesons with vector coupling.

$$\mathrm{SpN} = \mathrm{Sp}\Big[(i\gamma_\sigma q_\sigma - M)\gamma_\lambda e^2_\lambda (i\gamma_\tau p_\tau - M) e^1_\mu \gamma_\mu (i\gamma_\rho t_\rho - M)\gamma_5$$

$$+ (-i\gamma_\rho t_\rho - M) e^1_\mu \gamma_\mu (-i\gamma_\tau p_\tau - M) e^2_\lambda \gamma_\lambda (-i\gamma_\sigma q_\sigma - M)\gamma_5 \Big]$$

$$= 2\mathrm{Sp}\Big[M e^2_\lambda e^1_\mu \{ \gamma_\sigma \gamma_\lambda \gamma_\tau \gamma_\mu \gamma_5 q_\sigma p_\tau$$

$$+ \gamma_\sigma \gamma_\lambda \gamma_\mu \gamma_\rho \gamma_5 q_\sigma t_\rho$$

$$+ \gamma_\lambda \gamma_\tau \gamma_\mu \gamma_\rho \gamma_5 p_\tau t_\rho \} \Big]$$

$$= 8M e^1_\lambda e^1_\mu \left[f_{\sigma\lambda\tau\mu} q_\sigma p_\tau + f_{\sigma\lambda\mu\rho} q_\sigma t_\rho + f_{\lambda\tau\mu\rho} p_\tau t_\rho \right]$$

$$\text{where } f_{\sigma\lambda\tau\mu} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ \sigma & \lambda & \tau & \mu \end{pmatrix}$$

$$= 8M e^2_\lambda e^1_\mu f_{\sigma\lambda\tau\mu} \left(q_\sigma t_\rho - q_\sigma p_\rho - p_\sigma t_\rho \right)$$

$$= -8M e^2_\lambda e^1_\mu k^2_\sigma k^1_\rho f_{\sigma\lambda\mu\rho} . \tag{5,24}$$

3/ Decay of Vector V_0-Meson with Vector Coupling.

i/ Decay to two scalar π -mesons with scalar coupling.

$$\mathrm{SpN} = \mathrm{Sp}\Big[(i\gamma_\sigma q_\sigma - M)(i\gamma_\tau p_\tau - M)(i\gamma_\rho t_\rho - M) e^0_\lambda \gamma_\lambda$$

$$+ (-i\gamma_\rho t_\rho - M)(-i\gamma_\tau p_\tau - M)(-i\gamma_\sigma q_\sigma - M) e^0_\lambda \gamma_\lambda \Big]$$

$$= 0$$

ii/ Decay to two pseudoscalar π- mesons with pseudoscalar coupling.

$SpN = 0$

iii/ Decay to two vector π-mesons with vector coupling

$SpN = 0$

Decay of vector meson is thus forbidden. (5,25)

If the decaying V-particle is scalar with a vector coupling to the intermediate nucleon field the transition is forbidden; although a scalar V_0 with scalar coupling can decay to two scalar π-mesons with vector coupling, or to two pseudoscalar mesons with pseudovector coupling. For the decay of a pseudoscalar V_0 meson with pseudovector coupling the analysis is different to that completed above since the Spur is not dependent on odd M. A pseudoscalar V_0 meson with pseudovector coupling cannot decay into scalar or pseudoscalar π-mesons nor can a vector V_0 meson with tensor coupling decay into vector π-mesons. These results follow not only from the vanishing of the spurs but from more general considerations of angular momentum and parity conservation. (Yang 1950). The results can be expressed in the form of a table.

Primary V_0		Scalar.		Pseudoscalar.		Vector.
		S	V	PS	PV	V
Decay Products						
	S	✓	X	X	X	X
Scalar	V	✓	X	X	X	X
	PS	✓	X	X	X	X
Pseudoscalar	PV	✓	X	X	X	X
Vector	V	✓	X	✓ (Convgt.)	✓	X

§ 8. Calculation of Transition Probabilities

From the definition of A , B , C , R and S (5,8) and the results of paragraph 7 their values in the cases considered above follow at once.

1. Scalar (s) → Scalar (s)

$A = 3$, $B = 0$, $C_\lambda = 2(k^1_\lambda - k^2_\lambda)$, $R = -k^1_\sigma k^2_\sigma$, $S = -1$.

To solve for J reference is made to Case II in paragraph 6 .

$$J = -\frac{i\pi^2}{12M}(\mu^2+2\nu^2) + \frac{i\pi^2}{12M}\left[-6\underline{k}^1.\underline{k}^2 + 4(\underline{k}^1-\underline{k}^2).(\underline{k}^2-\underline{k}^1) + 3(-2\mu^2+k^2+\nu^2)\right].$$

The algebra is elementary and the J vanishes.

Thus this case needs special attention, the approximation (5,12) for I breaking down.

$$I = \frac{i\pi^2}{2} \frac{T+SL}{M^2-L} M$$

and

$$J = \frac{i\pi^2}{2M} \int (T+SL)(1-L/M^2)^{-1} 2y\,dy\,dx$$

from (5,9).

$$J \doteq \frac{i\pi^2}{2M} \int (T+SL)(1+L/M^2)\, 2y\,dy\,dx$$

and in this case $\int (T+SL)\, 2y\,dy\,dx = 0$.

So that $J \doteq \frac{i\pi^2}{2M^3} \int (T+SL)L\, 2y\,dy\,dx$

and this is approximately evaluated in appendix 3. to

$$\sim \frac{1}{45} \frac{i\pi^2}{M^3} \nu^4 \qquad (5,26)$$

whence from(5,18)

$$\omega \doteq \frac{1}{\pi^2} g_\tau^2 g^4 \frac{10^{-3}\nu^8}{4\mu_\tau M^6} \qquad (5,27)$$

1,ii/ Scalar (s) $\longrightarrow$ Pseudoscalar (ps)

From (5,20) it follows that

$A = -1$, $B = 0$, $C = 0$, $R = -k^1_\sigma k^2_\sigma$, $S = -1$

and Case II of paragraph 6 is needed to evaluate J .

$$J = -\frac{i\pi^2}{12M}(\mu^2+2\nu^2) + \frac{i\pi^2}{12M}\left\{\left[-6k'_\sigma k^2_\sigma - (-2\mu^2 - k'_\sigma k^2_\sigma)\right]\right\}$$

$$= \frac{i\pi^2}{12M}(\mu^2+3\nu^2+5k^2)$$

$$= \frac{i\pi^2}{3M}(2\nu^2-\mu^2)$$

which gives a transition probability

$$\omega = \frac{1}{18}\frac{1}{\pi^2} g_\tau^2 g^4 \frac{k(2\nu^2-\mu^2)^2}{\mu_\tau \nu M^2} . \qquad (5,28)$$

1,iii/ Scalar (s) → Vector (v)

In this case the values of spur N (5,21) together with (5,8) give

$A = -e_\lambda^2 e_\lambda^1$ $R = e_\lambda^2 e_\mu^1 (k_\mu^2 k_\lambda^1 - k_\lambda^2 k_\mu^1) - k'_\sigma k^2_\sigma e_\lambda^2 e_\lambda^1$

$B_{\lambda\mu} = 4e_\lambda^2 e_\mu^1$ $S = -e_\lambda^2 e_\lambda^1 .$

$C_\mu = 2e_\lambda^2 e_\mu^1 (k_\lambda^1 + k_\lambda^2)$

Let vector meson be transverse so that

$$e_\lambda^2 k_\lambda^2 = 0 ; \quad e_\lambda^1 k_\lambda^1 = 0$$

$A = -\cos\theta = -e_\lambda^2 e_\lambda^1$; $B_{\lambda\mu} = 4e_\lambda^2 e_\mu^1$;

$C = 0$ $R = -\cos\theta \, k'_\sigma k^2_\sigma$; $S = -\cos\theta .$

J reduces after long analysis to

$$\frac{i\pi^2}{3M}\cos\theta\,(-\mu^2+2\nu^2)$$

whence

$$\omega = \frac{1}{18}\,\frac{1}{\pi^2}\,g_\tau^2 g^4\,\frac{k(2\nu^2-\mu^2)^2}{\mu_\tau\,\nu\,M^2}\cos^2\theta\,.$$

Summing over polarisations.

$$\omega = \frac{\pi^{-2}}{9}\,g_\tau^2 g^4\,\frac{k(2\nu^2-\mu^2)^2}{\mu_\tau\,\nu\,M^2}\,. \qquad (5,29)$$

For decay into photons $\nu = \mu_\tau/2$

$$\omega = 4\pi\,g_\tau^2\,\frac{e^4}{\pi^3}\,\frac{\mu_\tau^3}{12^2 M^2}\,.$$

in agreement with Steinberger (1949).

2,iii/ Pseudoscalar (ps) $\longrightarrow$ vector (v)

$R = -k_\sigma^2 k_\rho^1 e_\lambda^2 e_\mu^1 f_{\sigma\lambda\mu\rho}$, $\qquad A = B = C = S = 0$

and J converges

$$J = -i\,\frac{\pi^2}{2}\,k_\sigma^2 k_\rho^1 e_\lambda^2 e_\mu^1 f_{\sigma\lambda\mu\rho}/M$$

whence

$$\omega = \frac{1}{2}\,\pi^{-2} g_\tau^2\, g^4\,\frac{k^3\nu}{\mu_\tau M^2}\cos^2\theta,$$

and summing over directions of polarisation

$$\omega = \pi^{-2}\,g_\tau^2 g^4\,\frac{k^3\nu}{\mu_\tau M^2}, \qquad (5,30)$$

which for photons becomes

$$\omega = 4\pi\,g_\tau^2\,\frac{e^4}{\pi^3}\,\frac{\mu_\tau^3}{M^2}\,\frac{1}{2^6}\,.$$

Summarising these results (5, 25, 27, 28, 29, 30.)

<u>Primary Meson</u>

<u>Product</u>	<u>Scalar(sc)</u>	<u>Pseudoscalar (psc)</u>
Scalar	$\sim \frac{10^{-3}}{4} \frac{1}{\pi^2} g_\tau^2 g^4 \frac{\nu^8}{\mu_\tau M^6}$. $g^6\, 2.4 \times 10^{15} \text{ sec}^{-1}$.	Forbidden
Pseudoscalar	$\frac{1}{18} \frac{1}{\pi^2} g_\tau^2 g^4 \frac{k(2\nu^2-\mu^2)^2}{\mu_\tau \nu M^2}$. $g^6\, 1.6 \times 10^{21} \text{ sec}^{-1}$.	Forbidden
Vector (transverse)	$-\frac{1}{9} \frac{1}{\pi^2} g_\tau^2 g^4 \frac{k(2\nu^2-\mu^2)^2}{\mu_\tau \nu M^2}$. $g^6\, 3.2 \times 10^{21} \text{ sec}^{-1}$.	$\frac{1}{\pi^2} g_\tau^2 g^4 \frac{k^3 \nu}{\mu_\tau M^2}$. $g^6\, 7.3 \times 10^{20} \text{ sec}^{-1}$.
Photon	$4\pi g_\tau^2 \frac{e^4}{\pi^3} \frac{\mu_\tau^3}{M^2\, 12^2}$. $(4\pi g^2)\, 7.2 \times 10^{13} \text{ sec}^{-1}$.	$4\pi g_\tau^2 \frac{e^4}{\pi^3} \frac{\mu_\tau^3}{M^2} \frac{1}{2^6}$. $(4\pi g^2)\, 1.6 \times 10^{14} \text{ sec}^{-1}$.

where ordinary units have been used in all cases. Numerical computation is elementary - μ_τ is assumed $\sim$ 1000 electron masses, except for photon decay $\mu_\tau \sim 270$ electron masses.

§ 9. Discussion.

From the resultant lifetimes predicted by the above first order calculation it is seen that, unless the coupling constant g_τ^2 is assumed to be small ($\sim 10^{-12}$), the decay time of a heavy meson to two π-mesons as predicted is much shorter than the experimental estimate of 3×10^{-10} sec. An assumption of such a small coupling constant is in contradiction to the frequency of the production of V^0 mesons, which, if the coupling were small, would appear in nuclear collisions very seldom. Assuming a reasonable coupling a possibility for decreasing the theoretical transition probability would arise if processes forbidden in first order would be allowed in higher order. The pseudoscalar meson-nucleon coupling gives selection rules which are those arising from parity conservation, and decay to scalar or pseudoscalar mesons is forbidden to all orders. The vector coupling gives vanishing matrix elements from Furry's theorem and again this is independent of insertion of additional internal meson lines in the Feynman diagrams, as this insertion would always introduce two further fermion lines. It is seen also that even where fortuitous cancellation occurs, as in the scalar to scalar transition above, the result shows a discrepancy with experiment of the order of 10^6 .

A further possibility which could lengthen the predicted lifetime is that the higher order corrections to allowed first order processes interfere with the latter. Since no method is available for obtaining probabilities summed to all orders, such a cancellation would, rather than have the desired effect, tend to lessen any trust in such field theoretical calculations altogether. It is perhaps of interest to note here that the magnitude of the coupling constant cannot be the only significant factor determining whether a reasonable approximation or otherwise is given by low order calculations. For with an increase in the number of vertices by n, the matrix element gains a factor g^n but the number of graphs increases as $n!$

The most recent evidence on heavy mesons by Armenteros, Barker, Butler, Cachon and Chapman (1951) points to the existence of a neutral meson of mass ~ 2,250 electron masses which decay to a proton and negative π^--meson. First order calculations of the lifetime for such processes, where it is necessary to assume the heavy meson is a fermion, if no further neutral particles are involved, are elementary and give lifetimes again too small by factors ~ $g^2 10^{12}$. That g^2 should be of the order of the Fermi quadrilinear interaction constant, (made dimensionless with the π-meson mass) is less reasonable in this process than in the $\pi \to \mu$ decay where similar magnitude for the μ-meson π-meson coupling $\sim \frac{1}{2} \times 10^{-14}$ is required to give the experimental decay time. That this is so is seen by considering inverse processes. Such a small coupling between the very heavy meson field and the bose meson field would make production of such fermions unlikely.

The μ - meson coupling to the π-meson field being small in no way contradicts the observed π- meson production cross-sections as these never arise as the result of μ-meson collisions with nuclei.

The only other process possible in which one particle decays into two particles is a bose decay into two fermions. If this were indeed happening in either of the V-type decays the meson involved as a secondary particle would probably be a μ -meson. The decay process is exactly as in the $\pi \to \mu$ decay and only the mass values need be changed in the calculations to give either decay time. Here again the decay time is very rapid unless a small coupling constant ($g^2 \sim 10^{-12}$) is chosen. The production argument does not hold here and it is conceivable that the μ- meson is only weakly coupled to the heavy meson field. The difficulty still remains, in a different form however. For, unless the V-mesons are themselves the decay products of even heavier particles, the plausible theory of their creation would be through the π-meson field when high energy cosmic ray primaries are incident on nuclei in the upper atmosphere. Then the V-mesons would have a reasonable sized coupling with the π-meson field and a very rapid decay to two π-mesons would inevitably follow.

Finally it is possible, although most of the evidence at present is against this, that a fourth neutral particle is present as a third decay product. Three possibilities are suggested, for besides the straightforward four Fermi interaction in strict analogy with ß-decay , there are the processes of a fermion decay to two bose particles and a lighter fermion and of a boson decay into two fermions and a boson. These will not be considered here.

CHAPTER SIX

CONCLUSION.

1/ The Spin of the π Meson.

The nucleon- π -meson interaction in current theory is assumed trilinear and starting from the allowed Lorentz-invariant interactions and first order perturbation theory certain empirical results, in which mesons play a part as real particles, can be compared with the theory. A restriction is made, for the purposes of simplicity and economy of hypothesis, to a meson of spin 0 or 1 , both able to carry an explicit parity.

The order of experiments to which appeal is made in this Chapter, may at first sight appear rather arbitrary. Experiments will, however, be considered in an order in which they demonstrate explicitly the theoretical point under discussion.

The π°-meson decay to two gammas is a decisive experimental result eliminating the possibility of a spin one π°-meson. This can be seen by the considerations of Chapter Five applied to the decay of a neutral vector meson decay to two zero-massed, neutral, transverse vector mesons. It was shown first by Wigner (1949) and Yang (1950) under more general considerations. Both authors have shown that a spin one particle of either parity cannot decay into two photons with conservation of angular momentum. The lifetimes for the allowed decays calculated by Steinberger (1949), also particular cases of the bose to two bose decays considered in Chapter Five, are for reasonable ($\sim \frac{1}{2}$) nucleon- π -meson couplings those observed $2\text{-}5 \times 10^{-14}$ sec.

It would be hoped that the charged π - meson would have the same spin as the neutral π - meson, and this has been confirmed by the calculations of Tamor and Marshak (1950) combined with the observations of Panofsky, Aamodt and Hadley (1951) on the absorption of π^- -mesons in Deuterium. In Chapter Four it was pointed out that in the absorption by Deuterium of π^- -mesons two competing processes are observed - namely direct and radiative absorption in the ratio 7/3 : while the calculations predict an absolute predominance of direct absorption for vector mesons.

Most of the work in this thesis follows from these conclusions and assumes a spinless π - meson.

2/ The Parity of the π - Meson.

Assuming that the π - meson has spin zero the question of its parity is one of vital importance. This conclusion considers this question in the light of the evidence of Chapter Three - Four .

The calculations of the cross-sections for the production of π^+ -mesons in proton-proton collisions given in Chapter Three support the assumption that the π - meson is of even character. The evidence can be summarised very briefly as follows. At the energies of the primary proton considered, if a meson of either parity is produced it will probably (2,1) come off with maximum energy. For those mesons which come off with less than the maximum energy, i.e. where a continuous state of the proton-neutron system results, the cross-section is well peaked in favour of closely interacting nucleons. The peak is more marked for scalar mesons (figure 5) than for pseudo-scalar. The latter follows a $(\omega - \mu)^{1/2}$ law. The angular distribution for maximum energy mesons is far more critical; for scalar mesons the predominating state is p and the distribution follows a $\cos^2\theta$ law in the centre of mass system, while for pseudoscalar mesons they come off in even states and the distribution is more isotropic. An interference of s - and d - waves gives a $\sin^2\theta$ distribution, in centre of mass system, for small angles. The experimental evidence, consisting of cross-sections at 0° , 18° and 30° is in definite agreement with a $\cos^2\theta$ law. These angular distributions are shown in laboratory system of coordinates in figures 4a and 4b .

On the other hand consideration of the lifetimes for the possible modes of absorption of π^- - mesons in Deuterium favour pseudoscalar mesons. Since the direct absorption occurs the process cannot be the capture of a scalar meson from the

K-shell about the nucleus. The detailed calculations of Chapter Four show that, unless the coupling $g^2/\hbar c$ is very large $\sim$ 120 , a scalar meson will be absorbed with the emission of radiation . The observed branching ratio between non-radiative and radiative capture is in agreement with pseudoscalar meson theory for a coupling $f^2/\hbar c \sim .57$. Tamor and Marshak (1950) quote a ratio 2.1 : 1 for pseudoscalar mesons independent of f .

Other evidence on the parity of the π- meson arises from the production of mesons by high energy gammas incident on nucleons. This problem has been considered by Brueckner (1950) in detail, and his results are convincingly in favour of an odd meson when compared with the experiments of Steinberger and Bishop (1950) . The experiments for π- meson production by gammas on nucleons give a nearly isotropic distribution at angles between 40° and 135° . The predictions of scalar and pseudoscalar theory are given by Brueckner (1950). The scalar meson theory prediction is anisotropic, being of the form of an angular distribution cross-section for an electric dipole transition. The pseudoscalar theory prediction is isotropic in the observed region and gives reasonable agreement with experiment.

It appears that at present no definite conclusion can be drawn concerning the parity of the π- meson. In fact a rather definite paradox appears.§§

Finally it should be stated that the properties required of the π - meson as a virtual particle, as for example in a meson theory of nuclear forces, go beyond that of a scalar particle. In order to give even a qualitative explanation of the spin dependence of nuclear forces the π- meson must interact with the nucleon spin and allow a spin flip.

3/ Heavy Mesons.

Paragraph nine in Chapter Five contains a fairly complete account of the contemporary relation of theory to experiment concerning the V-mesons . There is again, an inconsistency in this latter case, however, the evidence being less reliable since it is based on examination of only a few V tracks. If the V-meson of $\sim$ 1000 electron masses decays to two mesons and is a boson it cannot have spin one. If

§§ (Footnote: Since this work was completed an unpublished paper by Brueckner and Watson has been received. The paper entitled 'The analysis of π - meson production in nucleon-nucleon collisions' is not based on meson theory. The authors, assuming the absorption and γ ray production evidence, state in reference to nucleon π - meson production "It appears, however, that a $\cos^2\theta$ angular distribution presents a real difficulty......... This would seem to be a rather fundamental discrepancy".)

it is of scalar or pseudoscalar type its decay time is far too long compared with that predicted for a reasonable coupling between the V-meson and nucleon fields. It can be stated with some certainty that the existence in any numbers of heavy bose particles with long lifetimes for spontaneous decay into nucleons, leptons or mesons cannot be explained on present theory.

APPENDICES

1. The Interaction Between Meson Fields ...... (i)
and the Nucleon Field.

2.a Proof of Expansion(2,26)..................... (v)

b Commutation Lemmas.

3. Special Case of the Decay of a Scalar ......(ix)
Meson to Two Scalar Mesons.

§§§§§§§§§§§

APPENDIX ONE

The Interaction Between Meson Fields and Nucleon Field.

The possible Lorentz-invariant Lagrangian densities determining a trilinear interaction between a spin one or spin zero meson field and the nucleon field are listed below. The interaction energy densities which follow from them are also given.

a. Spin Zero Field.

Neutral $\mathcal{L}(x) = \eta(x)\psi(x) + \xi_\nu(x)\frac{\partial\psi(x)}{\partial x_\nu}$

$$\mathcal{H}(x) = -\eta(x)\psi(x) - \vec{\xi}.\vec{\nabla}\psi(x) - \xi_0(x)\pi(x)$$

Charged $\mathcal{L}(x) = \eta(x)\psi(x) + \xi_\nu(x)\frac{\partial\psi(x)}{\partial x_\nu}$ + conjugate

$$\mathcal{H}(x) = -\eta(x)\psi(x) - \vec{\xi}.\vec{\nabla}\psi(x) - \xi_0(x)\pi^\times(x) \quad + \text{ conjugate.}$$

b. Spin One Field.

Neutral $\mathcal{L}(x) = \xi_\nu(x)\psi_\nu(x) + \frac{1}{2}\,\zeta_{\mu\nu}(x)\,f_{\mu\nu}(x)$

$$\mathcal{H}(x) = -\vec{\xi}.\vec{\psi} + \frac{\xi_0}{\mu^2}\,\mathrm{div}\,\vec{\pi} - \vec{\zeta}.\mathrm{curl}\,\vec{\psi} - \vec{\zeta}_0.\vec{\pi}.$$

Charged $\mathcal{L}(x) = \xi_\nu\psi_\nu(x) + \frac{1}{2}\,\zeta_{\mu\nu}(x)\,f_{\mu\nu}(x)$ + conjugate

$$\mathcal{H}(x) = -\vec{\xi}.\vec{\psi} + \frac{\xi_0}{\mu^2}\,\mathrm{div}\,\vec{\pi}^\times - \vec{\zeta}.\mathrm{curl}\,\vec{\psi} - \vec{\zeta}_0.\vec{\pi}^\times \quad + \text{ conjugate.}$$

where for vector fields

$$f_{\mu\nu} = \frac{\partial \varphi_\nu(x)}{\partial x_\mu} - \frac{\partial \varphi_\mu(x)}{\partial x_\nu}$$

$\mathfrak{f}_{\mu\nu}$ is antisymmetric - sixvector

The source functions $\eta(x)$, $f_\mu(x)$, and $\mathfrak{f}_{\mu\nu}(x)$ are functions, of the requisite tensor form, of the Dirac-Fermi field. The following are the irreducible tensors, bilinear in the nucleon field potential Ψ .

Scalar	$\Psi^* \beta \Psi$
Pseudoscalar	$\Psi^* \rho_2 \Psi$
Vector	$\Psi^* (\vec{\alpha}, i) \Psi$
Pseudovector	$\Psi^* (\vec{\sigma}, i\rho_1) \Psi$
Antisymmetric Tensor (Six Vector)	$\Psi^* (\beta\vec{\sigma}, i\rho_2 \vec{\sigma}) \Psi$.

In the above tensors the matrices β , $\vec{\alpha}$, $\vec{\sigma}$, and ρ_i are the well known Dirac matrices. The final form for the energy densities are given below in terms of the γ matrices, which are defined by the relations

$$\gamma^4 = \beta = \rho_3 \; ; \quad \gamma^5 = \rho_1 = i\alpha_1 \alpha_2 \alpha_3 \; ;$$

$$\gamma^k = -i\beta\alpha^k ; \quad \rho_1 \alpha_k = \sigma_k .$$

With the coupling constants in Lorentz-Heaviside units and q , q^* isotopic operators in the nucleon charge space these energy densities are:-

a (i). Scalar meson with scalar coupling.

$$-ig\,\Pi(x)\gamma^4\bar{\Psi}(x)\psi(x)$$ (neutral)

$$-ig\,\Pi(x)\gamma^4 q\,\bar{\Psi}(x)\psi(x) + \text{conjugate}.$$ (charged)

a (ii). Scalar meson with vector coupling.

$$-\frac{f}{\mu}\Pi(x)\gamma^4\gamma^k\bar{\Psi}(x)\partial^k_x\psi(x)+i\frac{f}{\mu}\Pi(x)\bar{\Psi}(x)\pi(x)$$ (neutral)

$$-\frac{f}{\mu}\Pi(x)\gamma^4\gamma^k q\,\bar{\Psi}(x)\partial^k_x\psi(x)+i\frac{f}{\mu}\Pi(x)q^*\bar{\Psi}(x)\pi(x)+\text{conjugate}.$$ (charged)

a (iii). Pseudoscalar meson with pseudoscalar coupling.

$$-g\,\Pi(x)\gamma^4\gamma^5\bar{\Psi}(x)\psi(x)$$ (neutral)

$$-g\,\Pi(x)\gamma^4\gamma^5 q\,\bar{\Psi}(x)\psi(x)$$ + conjugate. (charged)

a (iv). Pseudoscalar meson with pseudovector coupling.

$$-\frac{f}{\mu}\Pi(x)\gamma^4\gamma^5\gamma^k\bar{\Psi}(x)\partial^k_x\psi(x)-i\frac{f}{\mu}\Pi(x)\gamma^5\bar{\Psi}(x)\pi(x)$$ (neutral)

$$-\frac{f}{\mu}\Pi(x)\gamma^4\gamma^5\gamma^k q\,\bar{\Psi}(x)\partial^k_x\psi(x)-i\frac{f}{\mu}\Pi(x)\gamma^5 q^*\bar{\Psi}(x)\pi(x)+$$ conjugate. (charged)

b (i). Vector meson with vector coupling.

$$-g\,\Pi(x)\gamma^4\gamma^k\bar{\Psi}(x)\psi_k(x)-ig\,\Pi(x)\bar{\Psi}(x)\frac{\operatorname{div}\vec{\pi}}{\mu^2}$$ (neutral)

$$-g\,\Pi(x)\gamma^4\gamma^k q\,\bar{\Psi}(x)\psi_k(x)-ig\,\Pi(x)q^*\bar{\Psi}(x)\frac{\operatorname{div}\vec{\pi}}{\mu^2}$$ + conjugate. (charged)

b (ii). Vector meson with tensor coupling

$$\frac{f}{\mu}\Pi(x)\gamma^5\gamma^k\bar{\Psi}(x)\operatorname{curl}_k\vec{\psi}(x)-i\frac{f}{\mu}\Pi(x)\gamma^k\bar{\Psi}(x)\pi_k(x)$$ (neutral)

$$\frac{f}{\mu}\Pi(x)\gamma^5\gamma^k q\,\Psi(x)\operatorname{curl}_k\vec{\psi}(x)-i\frac{f}{\mu}\Pi(x)\gamma^k q^*\bar{\Psi}(x)\pi_k(x)+$$ conjugate. (charged)

b (iii) Pseudovector meson with pseudovector coupling.

$$g\,\Pi(x)\gamma^4\gamma^5\gamma^k\,\Psi(x)\,\psi_k(x) - i g\,\Pi(x)\gamma^5\,\Psi(x)\,\frac{d\vec{w}_\pi}{\mu^2} \qquad \text{(neutral)}$$

$$g\,\Pi(x)\gamma^4\gamma^5\gamma^k q\,\Psi(x)\,\psi_k(x) - i g\,\Pi(x)\gamma^5 q^\times\Psi(x)\,\frac{d\vec{w}_\pi}{\mu^2} + \text{conjugate} \qquad \text{(charged)}$$

APPENDIX TWO

a/ Proof of the Expansion (2,26)

By definition

$$e^{A} B e^{-A} = \sum_{s=0}^{\infty} \frac{A^s}{s!} B \sum_{t=0}^{\infty} (-)^t \frac{A^t}{t!}$$

Summing by diagonals

$$= \sum_{s=0}^{\infty} \sum_{r=0}^{s} \frac{A^r B A^{s-r} (-)^{s-r}}{r!\,(s-r)!}$$

Now

$$[A, e^A B e^{-A}] = \sum_{s=0}^{\infty} \left\{ \sum_{r=1}^{s+1} \frac{A^r B A^{s+1-r}}{(r-1)!\,(s+1-r)!} (-)^{s+1-r} - \sum_{r=0}^{s} \frac{A^r B A^{s+1-r}}{r!\,(s-r)!} (-)^{s-r} \right\}$$

which may be written

$$= \sum_{s=0}^{\infty} \sum_{r=0}^{s+1} A^r B A^{s+1-r} (-)^{s+1-r} \frac{s+1}{r!\,(s+1-r)!},$$

by extending the two sums at one end and interpreting $1/_{-1!} = 0$

putting $s = s + 1$ this becomes

$$= \sum_{s=0}^{\infty} s \sum_{r=0}^{s} \frac{A^r B A^{s-r}}{r!\,(s-r)!} (-)^{s-r}.$$

Let $$g_s = \sum_{r=0}^{s} \frac{A^r B A^{s-r}}{r!\ s-r!} (-)^{s-r}$$

so that $g_0 = B$

and let $g(x) = \sum g_s x^s$

then $f(0) = B$

$$f(1) = e^{A} B e^{-A}$$

and $f'(x) = \sum s g_s x^{s-1}.$

From the above commutator. $[A, f(x)] = f'(x).$

Now $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \ldots\ldots$

therefore

$$f(1) = B + [A,B] + \frac{[A[AB]]}{2!} + \ldots$$

$$= e^{A} B e^{-A}.$$

b/ <u>Commutation Lemmas</u>.

The following commutation relations are a consequence of the spinor commutation law given in (2,24) . A and B are products of γ matrices and P and Q are products of isotopic operators.

Lemma 1 $[\Pi(x) A P \Psi(y), \Pi(x') B Q \Psi(y')] =$

$$i\delta(y-x')\,\Pi(x) A B P Q \Psi(y') - i\delta(y'-x)\,\Pi(x') B A Q P \Psi(y).$$

1 a. $[\Pi(x) A \Psi(x), \Pi(x') B \Psi(x')] =$

$$i\delta(x-x')\,\Pi(x)[A,B]\Psi(x).$$

2 $[\Pi(x) A P \Psi(y), \Pi(x') B^{k} Q \partial^{k}_{y'} \Psi(y')] =$

$$i\delta(y-x')\,\Pi(x) A B^{k} P Q \partial^{k}_{y'} \Psi(y') - i\partial^{k}_{y'}\delta(x-y')\,\Pi(x') B^{k} A Q P \Psi(y).$$

2 a. $[\Pi(x) A \Psi(x), \Pi(y) B^k \partial^k_y \Psi(y)] =$

$$i\,\delta(y-x)\Pi(x) A B^k \partial^k_y \Psi(y) - i\,\partial^k_y \delta(y-x)\,\Pi(y) B^k A \Psi(x).$$

As an example lemma 2 is proved.

$$[\Pi(x) A P \Psi(y), \Pi(x') B^k Q \partial^k_{y'} \Psi(y')]$$

is written in full form with upper suffices referring to charge variables and lower suffices with spinor components.

$$[\Pi^i_\alpha(x)\, A_{\alpha\beta} P^{ij}\, \Psi^j_\beta(y),\ \Pi^\ell_\gamma(x')\, B^k_{\gamma\delta}\, Q^{\ell m}\, \partial^k_{y'} \Psi^m_\delta(y')]$$

$$= [\Pi^i_\alpha(x) \Psi^j_\beta(y), \Pi^\ell_\gamma(x')\, \partial^k_{y'} \Psi^m_\delta(y')]\, A_{\alpha\beta}\, B^k_{\gamma\delta}\, P^{ij}\, Q^{\ell m}$$

Now

$$[\Pi^i_\alpha(x) \Psi^j_\beta(y), \Pi^\ell_\gamma(x')\, \partial^k_{y'} \Psi^m_\delta(y')] =$$

$$\Pi^i_\alpha(x) \Psi^j_\beta(y)\, \Pi^\ell_\gamma(x')\, \partial^k_{y'} \Psi^m_\delta(y')$$

$$- \Pi^\ell_\gamma(x')\, \partial^k_{y'} \Psi^m_\delta(y')\, \Pi^i_\alpha(x)\, \Psi^j_\beta(y)$$

$$= \Pi^i_\alpha(x) \{ -\Pi^\ell_\gamma(x') \Psi^j_\beta(y) + i\,\delta_{j\ell}\, \delta_{\beta\gamma}\, \delta(y-x')\}\, \partial^k_{y'} \Psi^m_\delta(y')$$

$$- \Pi^\ell_\gamma(x')\, \partial^k_{y'} \Psi^m_\delta(y')\, \Pi^i_\alpha(x)\, \Psi^j_\beta(y)$$

$$= -\Pi^{\ell}_{\gamma}(x')\,\Pi^{i}_{\alpha}(x)\,\partial^{k}_{y'}\Psi^{m}_{\delta}(y')\,\Psi^{j}_{\beta}(y) + i\,\delta_{j\ell}\,\delta_{\beta\gamma}\,\delta(y-x')\,\Pi^{i}_{\alpha}(x)\,\partial^{k}_{y'}\Psi^{m}_{\delta}(y')$$

$$- \Pi^{\ell}_{\gamma}(x')\,\partial^{k}_{y'}\Psi^{m}_{\delta}(y')\,\Pi^{i}_{\alpha}(x)\,\Psi^{j}_{\beta}(y)$$

$$= -\Pi^{\ell}_{\gamma}(x')\left\{-\partial^{k}_{y'}\Psi^{m}_{\delta}(y')\,\Pi^{i}_{\alpha}(x) + i\,\delta_{ik}\,\delta_{\alpha\delta}\,\partial^{k}_{y'}\delta(x-y')\right\}\Psi^{j}_{\beta}(y)$$

$$+ i\,\delta_{j\ell}\,\delta_{\beta\gamma}\,\delta(y-x')\,\Pi^{i}_{\alpha}(x)\,\partial^{k}_{y'}\Psi^{m}_{\delta}(y') - \Pi^{\ell}_{\gamma}(x')\,\partial^{k}_{y'}\Psi^{m}_{\delta}(y')\,\Pi^{i}_{\alpha}(x)\,\Psi^{j}_{\beta}(y)$$

Thus

$$\left[\Pi(x)\,A\,P\,\Psi(y),\ \Pi(x')\,B^{k}\,Q\,\partial^{k}_{y'}\Psi(y')\right]$$

$$= i\,\delta(y-x')\,\Pi(x)\,A\,B^{k}\,P\,Q\,\partial^{k}_{y'}\Psi(y')$$

$$- i\,\partial^{k}_{y'}\delta(x-y')\,\Pi(x')\,B^{k}\,A\,Q\,P\,\Psi(y).$$

APPENDIX THREE.

Special Case of Scalar Meson Decay to Two Scalar Mesons.

In Chapter Five it was seen that the approximation used to evaluate J (5, 9)(5,12) breaks down in the case of scalar meson decay to two scalar mesons each coupling being scalar. Here the next approximation is considered and the result of (5,26) obtained.

$$I = \frac{i\pi^2}{2} \frac{M(T+SL)}{M^2-L}$$

and

$$J = \int I \, 2y \, dy \, dx \doteqdot i\pi^2 \frac{M^{-1}}{2} \int (T+SL)(1 + L/M^2) \, 2y \, dy \, dx$$

and in this case

$$\int (T+SL) \, 2y \, dy \, dx = 0$$

So that

$$J \doteqdot \frac{i\pi^2}{2M^3} \int (T+SL) \, L \, 2y \, dy \, dx$$

and this is evaluated in this special case.

Calculate first T + SL

$$= -\underline{k}^1.\underline{k}^2 + 2(k_\lambda^1 - k_\lambda^2)(k_\lambda^2 xy - k_\lambda^1(1-y)) + 3k^2 - k^2 + k_2^2 xy + k_1^2(1-y)$$

$$= -\underline{k}^1.\underline{k}^2 + 2\underline{k}^1.\underline{k}^2 xy - 2k_1^2((1-y) - 2k_2^2 xy + 2k_\lambda^2 k_\lambda^1 (1-y) + 2k^2$$
$$- \mu^2 xy - \mu^2(1-y)$$

$$= \underline{k}^1.\underline{k}^2(-1 + 2xy + 2(1-y) - 4xy(1-y)) + \mu^2[2(1-y) + xy - (1-y) - 2(1-y)^2 - 2x^2y^2]$$

and it is easily seen that the integral of (T + SL)y vanishes.

Now $L = k^2 - k_2^2 xy - k_1^2(1-y)$

$$= -2xy(1-y)\,\underline{k}'.\underline{k}^2 + \mu^2\left[(1-y) + xy - x^2y^2 - (1-y)^2\right].$$

Make an approximation $\mu < \mu_\tau$ so that in L only term $\underline{k}^2.\underline{k}'$ need be considered.

In this approximation

$$(T + SL)L \doteqdot -\underline{k}^2.\underline{k}'\,\underline{k}^2.\underline{k}'\left[-1 + 2xy + 2(1-y) - 4xy(1-y)\right]2xy(1-y)$$

$$= -2(k^2+\nu^2)^2\,(1 - 2y - 2xy + 4xy^2)\,xy\,(1-y)$$

and

$$J \doteqdot -i\frac{\pi^2}{M^3}(k^2+\nu^2)^2\int\left[2xy(1-y) - 4xy^2(1-y) - 4x^2y^2(1-y) + 8x^2y^3(1-y)\right]y\,dy\,dx$$

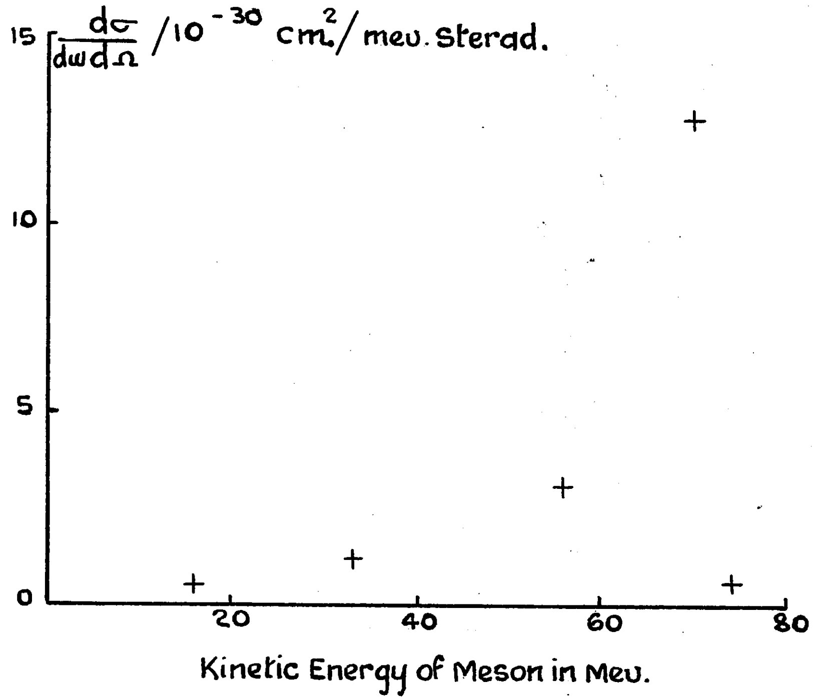
$$= -\frac{i\pi^2}{M^3}(k^2+\nu^2)^2\left(\frac{1}{12} - \frac{1}{10} - \frac{1}{15} + \frac{4}{45}\right)$$

$$= -\frac{(k^2+\nu^2)^2}{180}\,i\frac{\pi^2}{M^3},$$

which is the value used in the text.

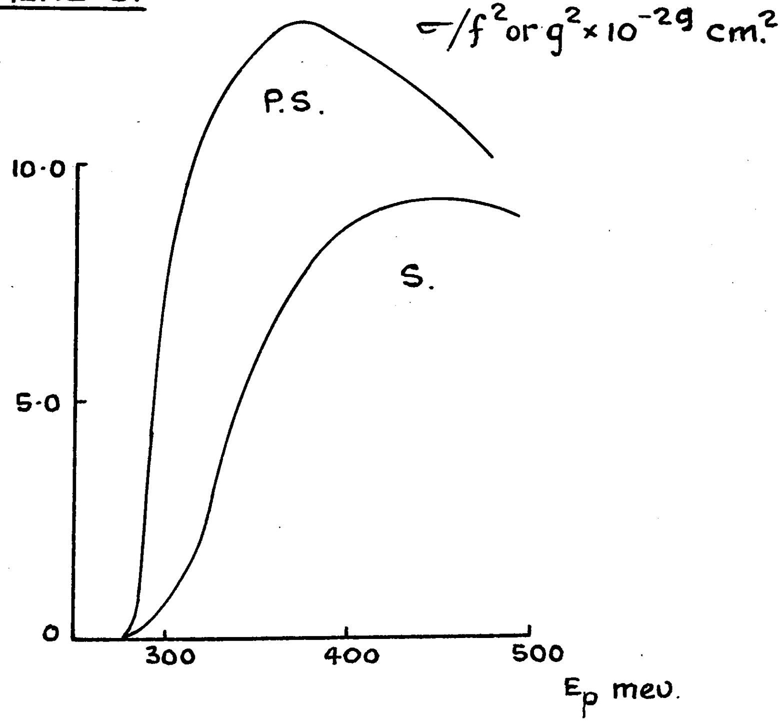
F I G U R E S

FIGURE I.



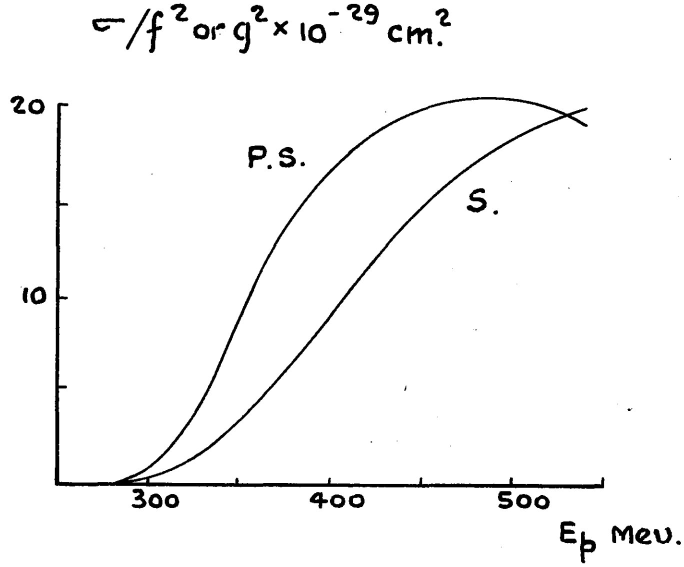
The differential cross-section for production of π^+ -meson by 345 MeV protons on protons in the direction of the beam. (Cartwright, Richman, Whitehead and Wilcox (1950)).

FIGURE 2.



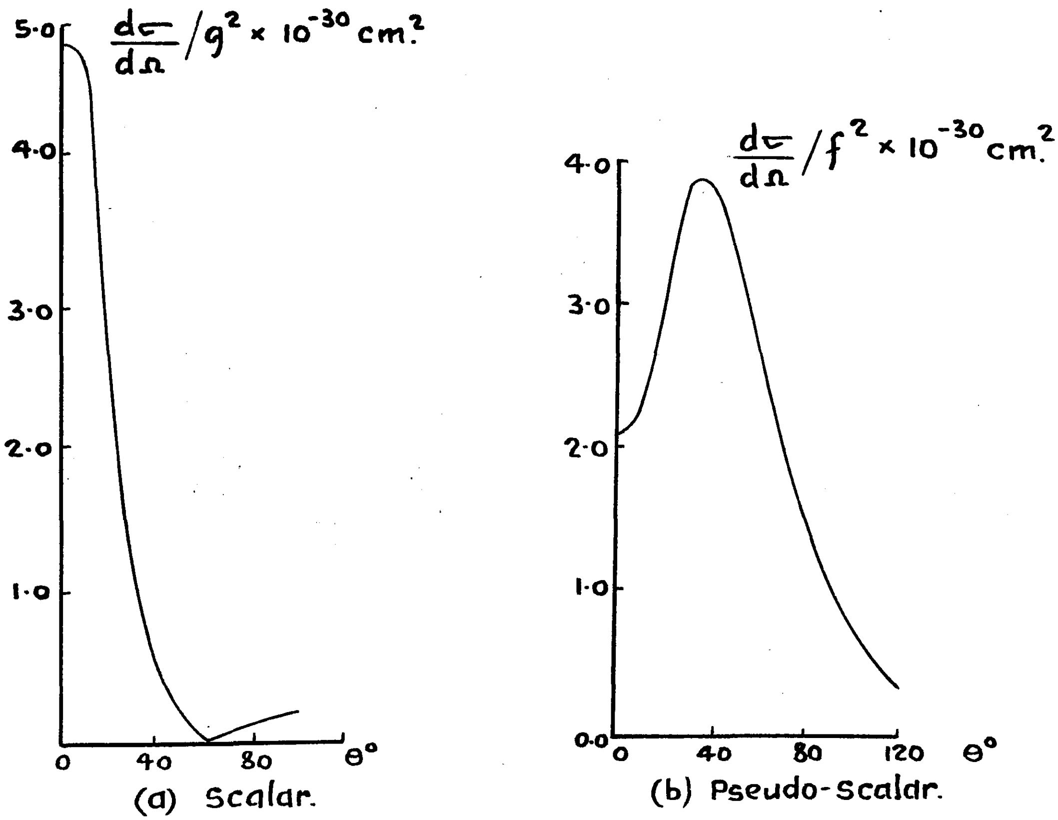
The cross-section for meson production, scalar and pseudoscalar, in proton-proton collisions against energies of incident proton, in cases where the final nucleons form a bound system.

FIGURE 3.



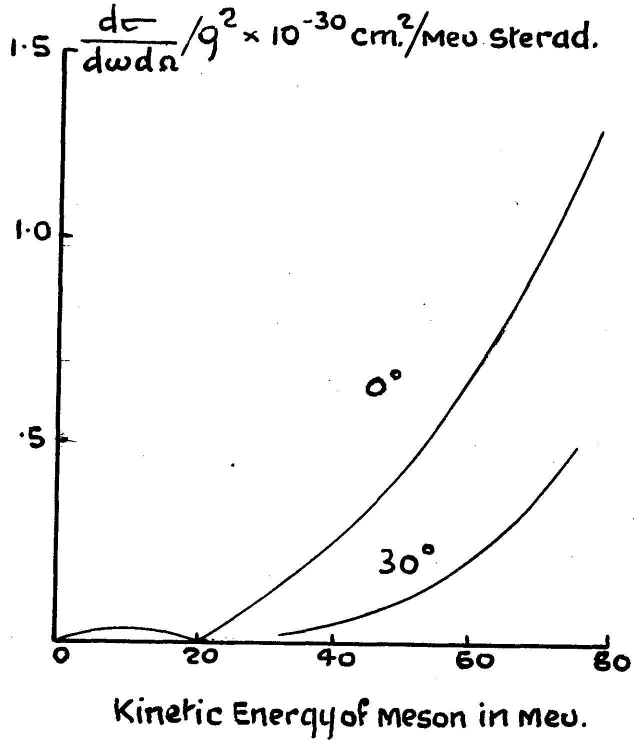
The integrated cross-sections for meson production in proton-proton collisions against energies of incident proton, in cases where final n - p system is in continuous state.

FIGURE 4.



Variation of the differential cross-section for meson production with the angle of emission of the meson (in laboratory system), is shown for incident 350 MeV protons and mesons of maximum energy.

FIGURE 5.



The cross-section/sterad per unit energy is shown against kinetic energy of the emitted scalar meson in the forward direction and at 30^0 to the beam of 350 MeV protons. The differential cross-section in angle falls off rapidly from the forward direction as the results show. E.g.

$$\left(\frac{d\sigma}{d\Omega}\right)_{30^\circ} \sim \frac{1}{10}\left(\frac{d\sigma}{d\Omega}\right)_{0^\circ}.$$

R E F E R E N C E S

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