The Financing Innovation in Entrepreneurship and Hedge Funds

By

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A thesis submitted in fulfillment of the requirements

for the degree of Doctor of Philosophy

in the

Economics

Adam Smith Business School

University of Glasgow

October, 2016

Declaration of Authorship

I, Hai ZHANG, declare that this thesis titled, "The Financing Innovation in Entrepreneurship and Hedge Funds" and the work presented in it are my own. I confirm that:

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"It always seems
impossible until
it's done."

Nelson Mandela

Abstract

This thesis has developed appropriate dynamic models to shed more light on how pervasive innovation (e.g., equity-for-guarantee swaps) in entrepreneurship/hedge funds could alleviate the severe financing constraints for entrepreneurs/hedge fund managers who plan to launch new businesses.

Chapter 2 considers a risk-averse entrepreneur who invests in a project with idiosyncratic risk and takes debt financing via equity-for-guarantee swaps for diversification benefits. In contrast to the literature, we assume the entrepreneur is unable to get a loan from a bank directly because of the low creditability of the entrepreneur and lack of collateral and therefore, an innovative financial contract, equity-for-guarantee swaps, is signed among a bank, an insurer, and the entrepreneur. The new swap not only solves the serious problems of widespread financing constraints, but also significantly improves the welfare level of the entrepreneur.

Chapter 3 develops a new financial derivative product called fees-for-guarantee swap to alleviate financing constraints of ESFs managers as well as mitigate the manager's risk-shifting behaviour. Numerical results indicate that the incentive compensation, managerial ownership and the possibility of fund liquidation significantly mitigate the manager's risk-shifting incentive.

In Chapter 4, a dynamic valuation model of the hedge fund seeding business has been built to study the consumption and portfolio choice problem for a risk-averse manager who launches a hedge fund via a seeing vehicle. This vehicle, i.e. fees-for-seed swap, specifies that a strategic partner (seeder) provides a critical amount of capital in exchange for participation in the funds revenue. If structured properly this seeding vehicle could lead to Pareto improvement, as it alleviates the ESFs manager's financial constraint, helps seeder get high potential return for good performance and ordinary investors are more willing to invest in funds backed up by seeding investment.

Acknowledgements

At the end of this thesis, I would like to take this opportunity to thank all the people without whom the project would never have been possible.

First and foremost I wish to thank my principal supervisor, **Christian-Oliver Ewald**, head of Economics at Adam Smith Business School, for your always strong support since the days I began to apply the PhD programme here at ASBS. I would like to express my deep thanks to you, for accepting me as a PhD student, for the freedom you have given me to find my own path, and especially for the guidance you offered whenever needed.

I also want to take a moment to thank **Zhaojun Yang**, my MSc supervisor, who likes a life mentor for me and always encourages me to pursue my dream.

I extend my sincere gratitude to **Neng Wang** who supervised my research when I was visiting at Columbia Business School. I greatly acknowledge your thoughtful guidance and critical comments which lead to a large improvement of the last chapter of my thesis.

Many people have helped and taught me immensely at ASBS. Just to name a few: Mrs **Christine Athorne** welcomed me to the school in 2013 and has always been supportive. **Francesca Flamini** is very kind and helped me with my overseas fieldwork applications. In addition to his support in my research, **Mario Cerrato** provided me a strong recommendation letter when I applied GTA job at Glasgow International College.

I am indebted to all my friends and family who were always so helpful in numerous ways. Special thanks goes to **Youngmin Ha**, **Binghua Huang**, **Huamao Wang**, and **Jinqiang Yang**.

I wish to say a heartfelt thank you to my Mum, **Aifang**, for always believing in me and encouraging me to follow my dreams.

Lastly, I want to express my gratitude and deepest appreciation to my wife, **Chen**, and our son, **Hudson**. Without your supports and encouragements, I could not have finished this work, it was you who kept the fundamental of our family.

Hai Zhang

October 2016, Glasgow, UK

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List of Abbreviations

AUM	**A**sset **U**nder **M**anagement
CAPM	**C**apital **A**sset **P**ricing **M**odel
CARA	**C**onstant **A**bsolute **R**isk **A**version
ESFs	**E**arly **S**tage **F**unds
GBM	**G**eometric **B**rownian **M**otion
HFR	**H**edge **F**und **R**esearch
HWM	**H**igh **W**ater **M**ark
LLC	**L**imited **L**iability **C**ompany
ODE	**O**rdinary **D**ifferential **E**quation
PDE	**P**artial **D**ifferential **E**quation
SEC	**S**ecurities (and) **E**xchange **C**ommission
SMEs	**S**mall (and) **M**edium **E**nterprises

List of Symbols

AUM/HWM ratio	s
Bankruptcy loss rate	κ
Capital gains tax rate	τ_g
Cash-out threshold	y_u
Cash-out timing	T_u
Coefficient of absolute risk aversion	γ
Consumption	c
Corporate effective marginal tax rate	τ_m
Correlation coefficient	ρ
Debt coupon rate	b
Default threshold	y_d
Default timing	T_d
Effective liquidation barrier	$\tilde{b} \equiv l(1-\Phi)$
Effective marginal tax rate	τ_m
Entrepreneurial business profits tax rate	τ_e
Expected return of market portfolio	μ_M
Expected return of entrepreneurial business	μ_y
Exogenous liquidation barrier	l
Guarantee cost	φ
Guaranteed yield	y
Guarantee level	ϕ
Hedge fund expected return	μ
High water mark	H
Hurdle rate	h
Idiosyncratic volatility	ϵ

Incentive fee	k
Investment strategy	Π
Liquidation barrier	l
Lock-up period	T
Lump-sum cost	I
Management fee	m
Managerial skills	α
Market Sharpe ratio	η
Profit tax rate	τ_e
Recover rate	$\iota \equiv 1 - \kappa$
Reduction costs	m'
Risk-free rate	r
Systematic volatility	ζ
Subjective discount rate	δ
Seed capital ratio	Φ
Transaction cost	K
Volatility of market portfolio	σ_M
Volatility of the ESFs	σ
Wealth process	W
Withdrawal rate, the liquidation parameter	$\omega + \lambda$

Dedicated to my wife, Chen, and our son Hudson.

Chapter 1

Introduction

This thesis has developed appropriate dynamic models to shed more light on how pervasive innovation (e.g., equity-for-guarantee swaps and fees-for-seed swaps among others) in entrepreneurship/hedge funds could alleviate the severe financing constraints for entrepreneurs/hedge fund managers who plan to launch new businesses. A fundamental theme in this thesis is the recognition that markets are generally incomplete with frictions. This fact is often neglected in the context of real options, mostly for the reason that the appropriate models which take account of incompleteness are far more complex and mathematically more difficult to deal with. However, this thesis has presented a number of analytically tractable models and identified key effects of market incompleteness on the timing of investment and hedge fund management.

More specifically, this thesis pays particular attention to market incompleteness or frictions, e.g. financing constraints, and intends to give more insights on how financial innovation could relieve entrepreneurs'(hedge fund managers') borrowing constraints in terms of starting a new business. Although equity-for-guarantee swaps and fees-for-seed swap become increasingly popular in practice, few studies has provided a thorough research on this topic. There is too much unknown in this developing subject: "What role has the financial innovation played in relaxing financing constraints? How to price the guarantee costs and seed costs under the arrangements of innovative contracts? Why would all partners (the entrepreneur, bank and insurer involved in equity-for-guarantee swaps) like to participate in this innovative scheme? What is the potential and risks for partners who have signed the contract?", just to name a few. Therefore, to answer these important questions and shed more light on this topic is the main goal of this research.

In Section 1.1, I have provided essential background information about financial innovation both in entrepreneurship and hedge funds as well as the motivation of this research. Section **??** gives a short description about the primary approaches employed by this thesis. Finally, the outline of this thesis is provided in Section 1.2.

1.1 Research background and motivation

The 2008 financial crisis has devastating effects on finance accessibility in real investment. Campello, Graham, and Harvey (2010)'s survey [1] indicates that the inability to borrow externally caused many firms to bypass attractive investment opportunities, nearly 90 % of constrained companies say that financial constraints restrict their pursuit of attractive projects, and more than half of these firms are forced to cancel valuable investments during the crisis. Using a data set of over 10,000 UK small- and medium-sized enterprises (henceforth SME) employers, Lee, Sameen, and Cowling (2015) find that it is harder for innovative firms to access finance than other firms, which may lead to a long-term drag on the economy. Using detailed firm-level survey data on twenty-four hundred firms in China, Ayyagari, Demirg-Kunt, and Maksimovic (2010) find that only a relatively small percentage of firms (20%) utilize bank loans and most firms in the sample rely on a large informal sector and alternative financing channels (i.e. financing innovation).

In this section, I will focus on how the financial innovation in practice could relax these constraints so as to drive long-term growth in entrepreneurship and hedge funds respectively. Financial innovation like equity-for-guarantee swap(fees-for-seed swap) aiming to alleviate the financing constraints of the entrepreneur (the hedge fund manager) have been barely studied, thus, research on how these swaps affect firms' investment plans and financial policies is immensely important and undoubtedly worthwhile to investigate.

1.1.1 Entrepreneurship innovation

The increasing research interest in SMEs is driven by the recognition that SMEs play a vital role in promoting economic growth.[2] However, due to low credibility and lack of effective

[1] citeCampello2010 survey 1,050 Chief Financial Officers (CFOs) in the U.S., Europe, and Asia to directly assess whether their firms are credit constrained during the global financial crisis of 2008.

[2] More discussion about the importance of SMEs finance can be found in Wehinger (2013) and Lee, Sameen, and Cowling (2015) among others.

guarantee, many entrepreneurs, let alone fresh graduates, are unable to get a bank loan or get other debt financing cheaply. Under such situation, traditional financial theory on optimal capital structure is not reasonable since the entrepreneur has no other choice beyond starting her business with her own money only or simply giving up the business.

To alleviate theses borrowing constraints placed on SMEs, some insurers, entrepreneurs and bankers in China have developed innovative financial contracts like equity-for-guarantee swap which could benefit all if it is designed properly. equity-for-guarantee swaps is an agreement between a lender (bank), an insurer, and a borrower (entrepreneur), where the bank lends at a given interest rate to the entrepreneur and if the entrepreneur defaults on the debt, the insurer will make a compensatory payment to the creditor so that the creditor will always be paid up-to a certain guarantee level. In return for the guarantee, the firm needs to allocate a percentage of the firm's equity to the insurer. This contract was first signed in 2002 in the city of Shenzhen in China and it has become increasingly popular in the country.

The financial theory of public firms is quite different from that of SMEs, as SMEs' managers are more likely to subject to financing constraints and their investment lack of diversification.[3] These frictions result in incomplete-markets and cause non-separability between value maximization and consumption smoothing, which invalidates the classic complete-market valuation analysis widely applied to public firms.

Inspired by empirical evidences, Chapter 2 extends the dynamic incomplete-market framework of Chen, Miao, and Wang (2010) to endogenously model the impacts and interactions of the two frictions on entrepreneurs' consumption, portfolio, financing and exit, by introducing equity-for-guarantee swaps. First, our setting improves a generalized model of capital structure trade-off among borrowing constraints, tax, diversification benefits, and costs of financial distress. Second, the equity-for-guarantee swap fundamentally raises the entrepreneur's borrowing capacity and therefore the entrepreneur optimally issues more debt and takes higher leverage than that without the equity-for-guarantee swap. Higher leverage leads to larger tax shields and diversification benefits because the entrepreneur faces less equity exposure to the project and thus her portfolio (consisting of private equity and her liquid wealth) is less risky. Third, the entrepreneur with the swap receives more welfare increments and has more investment opportunities because of being more willing

[3] See Evans and Jovanovic (1989), Gentry and Hubbard (2004) and Buera (2009) among others.

to invest. Higher risk-averse entrepreneurs under higher non-diversifiable idiosyncratic risk gain more benefits resulting from the equity-for-guarantee swap.

1.1.2 Hedge fund seeding innovation

There has been a significant increase in both the number of hedge fund seeders and the amount of capital available for hedge fund seeding since the aftermath of 2008's market upheaval. According to HFM-Week research, in November 2011 seeders had approximately $4.6 billion in available capital, compared to approximately $1 billion just one year earlier. However, there still remains a tremendous shortage of capital for new and early-stage funds (ESFs). This is mainly because most capital providers or institutional investors increasingly focus on larger established hedge funds whose assets under management (AUM) are usually larger than 1 billion and who are considered highly credible. Additionally a larger talent pool of ESFs managers is now competing for the scarce available seed capital. Worse still, barriers to entry for ESFs are much higher today than in the period before the 2008 financial crisis.

Therefore, navigating the terrain to a successful launch of a hedge fund has become more difficult and the financing constraints faced by ESFs managers nowadays is much more serious than before. In order to reach the initial AUM target and cover organizational expenses, more and more ESFs managers are likely to turn to seed investors for early stage of capital through a seeding vehicle. This is an arrangement to which we refer as fees-for-seed swap that specifies that a seed investor (or seeder) commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced economics" which is usually the fees that the ESFs manager generates from the entire pool of assets in the fund.

A thumb rule for fees-for-seed swap ratio states that a seeder can expect about 1% of revenues for each $1 million of seed capital for seed transactions no larger than $50 million. However, seed arrangements can vary substantially based on factors such as the experience of the manager, the alpha record, the amount of seed capital provided, the withdrawal and lock-up period terms, and the relative negotiating power of each party.

Although the seeding vehicle in practice has, to some extent, solved ESFs managers' financing constraints, there is still a huge gap between the seeding capital demand and supply in the hedge fund seeding business. One main reason could be the severe information asymmetry between seeders and ESFs managers, which has always been the key factor holding back the development of the hedge fund seeding business, especially in the aftermath of the 2008 financial crisis. Inspired by small and medium-sized enterprises'experience of overcoming borrowing constraints in China as discussed in Yang and Zhang (2013), the financing constraints due to information asymmetry faced by ESFs managers will be largely alleviated by introducing the fees-for-guarantee swap supported by a commercial guarantee-company or an insurer. Therefore, an insurer was firstly introduced in Chapter 3, through fees-for-guarantee swaps, to solve the problem of information asymmetry by enhancing the ESFs manager's credibility. Unlike the traditional credit support scheme, however, ESFs managers in the new setting have to pay to the guarantee company a certain portion of her management fees, as guarantee costs instead of regular guarantee fees. Once the hedge fund is exogenously liquidated by a crisis shock or endogenously by investors, the seeder can still get compensatory payment from the insurer.

As there is no publicly available data on the historical performance of seeding strategies, there are only very few simple models in practice focusing on hedge fund seeding return, volatility and liquidity profile.[4] We are unaware, however, of any existing model that both captures the managerial skill (alpha) and the convex compensation in a hedge fund seeding business. Capturing these institutional features in a model that is sufficiently tractable to evaluate the costs of these innovative swaps in hedge fund seeding business is one of the main contributions of this study. Chapter 3 mainly focuses on the innovation of the hedge fund seeding business like fees-for-seed and fees-for-guarantee swaps. Moreover, Chapter 3 provides a dynamic framework for valuing the costs of these swaps, as well as modeling the impact of the so-called high-water mark, managerial ownership and fund liquidation on ESFs managers' risk shifting behavior, at the hedge fund seeding stage. Moreover, Chapter 4 provides the first dynamic framework on valuation of the hedge fund seeding business by solving the portfolio-choice problem for a risk-averse manager. One striking finding is that this seeding vehicle could lead to Pareto improvement, as it alleviates the ESFs manager's financial constraints, helps seeder get high potential return for good performance and ordinary investors are more willing to invest in funds backed up by seeding investment.

[4] Larch Lane Advisors LLC constructed a simple model to project returns and cash flows for a seeded fund featuring an innovative seeding strategy.

1.2 Outline of the thesis

This thesis is a collection of three independent research papers which represent the following three chapters respectively. Chapter 2 considers a risk-averse entrepreneur who invests in a project with idiosyncratic risk and takes debt financing via equity-for-guarantee swaps for diversification benefits and it has been published at **European Journal of Operational Research** (Wang, Yang, and Zhang 2015a). Chapter 3 develops a new financial derivative product called fees-for-guarantee swap to alleviate financing constraints of ESFs managers as well as to mitigate the manager's risk-shifting behaviour. Chapter 3 has been accepted by**the Fourth Chinese Capital Markets Conference-The European Journal of Finance**,2014, University of Nottingham Ningbo China. In Chapter 4, a dynamic valuation model of the hedge fund seeding business has been built to study the consumption and portfolio choice problem for a risk-averse manager who launches a hedge fund via the fees-for-seed swap, this chapter has been published at **Journal of Economic Dynamics and Control** (Ewald and Zhang 2016).

Chapter 2

Entrepreneurial Finance with Equity-for-Guarantee Swap and Idiosyncratic Risk

Coauthored with Huamao Wang and Zhaojun Yang[1]

Abstract

We consider a risk-averse entrepreneur who invests in a project with idiosyncratic risk and takes debt financing for diversification benefits. In contrast to the literature, we assume the entrepreneur is unable to get a loan from a bank directly because of the low creditability of the entrepreneur and lack of collateral and therefore, an innovative financial contract, named equity-for-guarantee swap, is signed among a bank, an insurer, and the entrepreneur. We build a dynamic incomplete model to investigate the effects of the swap on the entrepreneur's consumption, portfolio, financing and exit from the business. We find that the new swap leads to higher leverage, which brings more diversification and tax benefits. The new swap not only solves the serious problems of widespread financing constraints, but also significantly improves the welfare level of the entrepreneur. The growth of welfare level increases dramatically with risk-aversion index and the volatility of idiosyncratic risk.

[1] Dr Huamao Wang is currently Lecturer in Finance at Kent University (UK) and Dr Zhaojun Yang is Associate Professor in Finance at Southern University of Science and Technology (China)

2.1 Introduction

Borrowing constraints and lack of diversification are two typical frictions of entrepreneurship (e.g. (Evans and Jovanovic 1989; Gentry and Hubbard 2004; Buera 2009)). These frictions result in incomplete-markets and cause nonseparability between value maximization and consumption smoothing, which invalidates the classic complete-market valuation analysis widely applied to public firms. Inspired by empirical evidences, we extend the dynamic incomplete-market framework of Chen, Miao, and Wang (2010) (henceforth **CMW**) to endogenously model the impacts and interactions of the two frictions on entrepreneurs' consumption, portfolio, financing and exit, by introducing an innovative financial product, called equity-for-guarantee swap.

One of fundamental characteristics of entrepreneurship is lack of diversification. Specifically, the revenue of an entrepreneurial firm (private firm) suffers systematic and idiosyncratic risks. Entrepreneurs can trade risk-free bonds and the diversified market portfolio to diversify the systematic business risk but not the idiosyncratic risk. Therefore the diversification benefit of risky debt is important to entrepreneurs in addition to the standard trade-off between tax benefits and costs of financial distress, see Heaton and Lucas (2004) and Wang, Wang, and Yang (2012) among others.

In addition, there are many small and medium enterprises (SMEs) and fresh graduates, who are hungry for money to start a new business. Such investment is generally extremely high-risk, and to compensate for that risk, the entrepreneur comes with the potential for high returns. However, due to low credibility and lack of effective guarantee[2], many entrepreneurs, let alone fresh graduates, are unable to get a bank loan or get other debt financing cheaply. Under such situation, traditional financial theory on optimal capital structure is not reasonable since the entrepreneur has no other choice beyond starting her business with her own money only or simply giving up the business.

To overcome borrowing constraints, some insurers and entrepreneurs in China have developed equity-for-guarantee swap. This is an agreement between a lender (bank), an insurer, and a borrower (entrepreneur), where the bank lends at a given interest rate to the

[2]OECD ("OECD Economic Surveys: China 2005") finds that more than 50% of the firms surveyed reported their lack of collateral as a major barrier to bank borrowing.

entrepreneur and if the entrepreneur defaults on the debt, the insurer will make a compensatory payment to the creditor so that the creditor will always be paid up-to a certain guarantee level. In return for the guarantee, the firm needs to allocate a percentage of the firm's equity to the insurer. This contract was first signed in 2002 [3]in the city of Shenzhen in China and it has become increasingly popular in the country. Although, equity-for-guarantee swaps only exists in China, this chapter conducts theoretical research with focuses on its role in alleviating entrepreneurs' borrowing constraints and leading to pareto improvement. Therefore, the applications of this research is not limited in China and could apply to any markets which suffers severe asymmetric information problems.

In Chapter 2, we extend **CMW**'s model to take into account both idiosyncratic risk and the equity-for-guarantee swap. Chapter 2 is related to Yang and Zhang (2013), who provide the first formal study on equity-for-guarantee swap. However, Yang and Zhang (2013) merely discuss traditional capital structure issues in the classic framework of Leland (1994). Our model examines this contract in a more general context with idiosyncratic risk and cash-out option. Although we focus on entrepreneurial firms here, our model can be employed by public firms with concentrated managerial ownership who want to enhance the debt capacity and to increase liquid wealth.

The majority of results in **CMW** are based on the assumption that the entrepreneur has "deep pockets", i.e. she can issue debt with the coupon rate being higher than the project's revenue since she can inject cash into the firm to pay coupons. However, this assumption is not feasible for many entrepreneurs, not to mention fresh graduates. Actually, **CMW** point out that entrepreneurs may be liquidity-constrained, i.e. no external funds are available to cover the firm's debt service, and hence an earlier liquidation will be forced by the creditor. We argue that the assumption becomes practical thanks to the equity-for-guarantee swap. In fact, under the swap, the entrepreneur is equivalent to the one who has deep pockets and the default threshold can be lower than the coupon level because the claim owned by the creditor is guaranteed by the insurer. In exchange for the guarantee, the entrepreneur needs to pay the insurer a proportion of equity of the firm. In addition, since the insurer guarantees the debt, the creditor under the swap does not demand a protective covenant.

[3]For more information, please go to the Web(Chinese): http://news.sina.com.cn/s/2002-11-21/0931813745.html

Following **CMW**, we consider a risk-averse entrepreneur having access to standard financial investment opportunities, see Merton (1971), with a chance to invest in a project. The objective of the entrepreneur is to maximize her expected lifetime utility over intertemporal consumption. We choose the exponential utility primarily for analytical tractability. While constant absolute risk aversion (CARA) utility does not capture wealth effects, it reduces the dimension, especially for the double-barrier boundary problem, see Henderson (2002), Miao and Wang (2007), Henderson (2007), Ewald and Yang (2008), and Yang and Yang (2012) among others. Similar to **CMW**, our model adapts to any regular utility functions since the precautionary savings effect which is captured by utility functions with convex marginal utility is the driving force behind the results.

The entrepreneur pays a lump-sum investment cost (sunk cost) to start a project, which generates stochastic revenue cash flow with both systematic and idiosyncratic risks. The entrepreneur acts two roles of a consumer and a firm. As a consumer, she allocates her liquid wealth between a risk-free asset and a diversified market portfolio as in Merton (1971). As a firm, she makes investment, budgeting, financing, and exit decisions.

The entrepreneur has a default option and a cash-out option to exit the business. On one hand, the entrepreneur defaults on the debt and incurs inefficient liquidation when revenue is sufficiently low. Different to the literature, e.g. **CMW**, after default has taken place, the insurer makes a compensatory payment to the lender. On the other hand, the entrepreneur might repay the debt in full and sell the firm to cash out if the firm performs sufficiently well. The two options are nontradable American-style options on the illiquid project with endogenous double-threshold policies. These exit options and the equity-for-guarantee swap make the entrepreneur effectively bear much less risk and help the entrepreneur to achieve diversification benefits. After exiting, the entrepreneur behaves as a household and lives on her financial wealth only.

The main results of Chapter 2 are as follows. First, our setting improves a generalized model of capital structure trade-off among borrowing constraints, tax, diversification benefits, and costs of financial distress. Second, the equity-for-guarantee swap fundamentally raises the entrepreneur's borrowing capacity and therefore the entrepreneur optimally issues more debt and takes higher leverage than that without the equity-for-guarantee swap. Higher leverage leads to larger tax shields and diversification benefits because the entrepreneur faces less equity exposure to the project and thus her portfolio (consisting of private equity and her liquid wealth) is less risky. Third, the entrepreneur with the swap receives more welfare increments and has more investment opportunities because of being more willing to invest. Higher risk-averse entrepreneurs under higher nondiversifiable idiosyncratic risk gain more benefits resulting from the equity-for-guarantee swap.

Chapter 2 is organized as follows. Section 2.2 presents the model. Section 2.3 solves the model. Section 2.4 discusses the numerical results. Section 2.5 concludes. Appendix A provide equilibrium valuation of corporate securities.

2.2 Model setup

2.2.1 Investment Opportunities

We consider an infinitely-lived risk-averse entrepreneur who has an option to invest in a take-it-or-leave-it project at present time 0, which requires a one-time investment cost I. All sources of uncertainty arise from two independent standard Brownian motions B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$, where $\mathbb{F} \equiv \{\mathcal{F}_t : t \geq 0\}$ describes the flow of information available to the entrepreneur.

In addition to the project opportunity, the entrepreneur has access to standard financial investment opportunities, see Merton (1971). Let W denote the entrepreneur's liquid (financial) wealth process. The entrepreneur invests an amount of Π_t in a diversified market portfolio and the remaining amount $W_t - \Pi_t$ in the risk-free asset with a constant interest rate r at any time $t \geq 0$. The return of the diversified market portfolio is denoted by R which satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \tag{2.1}$$

where μ_M and $\sigma_M > 0$ are constants, and $\eta \equiv (\mu_M - r)/\sigma_M$ is the Sharpe ratio of the market portfolio.

We assume the project generates a stochastic revenue process $\{y_t : t \geq 0\}$ that follows a geometric Brownian motion (GBM):

$$\frac{dy_t}{y_t} = \mu_y dt + \rho\sigma dB_t + \epsilon dZ_t, \quad y_0 \text{ given,} \tag{2.2}$$

where μ_y is the expected growth rate, σ is the total volatility of revenue growth and $\rho \in [-1, 1]$ is the correlation coefficient between the project payoff and the return on market portfolio given by Equation (2.1).

The Brownian motions B and Z provide the sources of market risk (systematic) and idiosyncratic risk of private business, respectively. A higher absolute value $|\rho|$ of the correlation coefficient implies that the systematic volatility has a larger weight, *ceteris paribus*. The parameters $\zeta \equiv \rho\sigma$ and $\epsilon \equiv \sqrt{1-\rho^2}\sigma$ are respectively the systematic and idiosyncratic volatility of revenue growth.

2.2.2 Entrepreneurial Financing with Equity-for-Guarantee Swap

We assume the entrepreneur runs the project by setting up a limited liability entity, such as a limited liability company (LLC) or an S corporation, which allows her to face single-layer taxation for her business income and makes the debt nonrecourse. We follow the simple tax system in **CMW**. Entrepreneurial business profits incur taxes at a rate τ_e. A public firm is subject to a double taxation which is captured by an effective marginal tax rate τ_m. In addition, the capital gains upon cash-out are taxed at a rate τ_g.

The entrepreneur finances the initial one-time lump-sum cost I via her own funds and external financing. We assume that the main source of external financing is debt, e.g. bank loans. Due to the high default probability and relative lack of collateral, it is much more difficult for the entrepreneurial firm to take debt financing than for a large company. Unlike **CMW** who do not consider borrowing constraints, we study the entrepreneur who is constrained in borrowing due to protected covenants demanded by the lenders. This financing constraint is alleviated by introducing the equity-for-guarantee swap supported by a commercial guarantee company or insurer. Unlike the traditional credit hypothecation, though, the entrepreneurial firm in the new credit guarantee scheme must pay to the guarantee company a portion (φ) of equity as guarantee costs instead of regular guarantee fees.

Under the guarantee, the entrepreneur chooses to issue a interest-only consol with coupon b and par value $F_0 = F(y_0)$ at time 0 and remains unchanged until the entrepreneur exits from the project. The value $F(y)$ of the debt is equal to the equilibrium value of the cash flow generated by the debt, which is given by[4].

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right)\bar{q}(y) + (\phi - 1)\frac{b}{r}\tilde{q}(y), \tag{2.3}$$

After the debt is in place, at any time $t \geq 0$, the entrepreneur has three choices: (1) She runs the firm and receives a fraction $(1-\varphi)$ of cash payments from the firm; (2) She defaults once the default threshold y_d of the revenue process is reached and then the insurer must make a compensatory payment to the creditor so that the creditor is paid up-to a certain guarantee level; (3) She cashes out by selling the firm to a diversified buyer at the cash-out threshold y_u, which incurs a fixed transaction cost K. The default timing T_d and cash-out

[4]See computational details at Appendix A

timing T_u are endogenously determined by the entrepreneur in order to maximize her own expected total utility under the guarantee contract signed at time 0.

Once the entrepreneur defaults, the debt holders (lenders) take control and liquidate/sell the firm. Bankruptcy ex post is costly and the bankruptcy loss can be interpreted in different ways, such as loss from selling real assets, asset fire-sale losses, legal fees, etc. We assume that $\kappa \equiv 1 - \iota$ is the bankruptcy loss rate, i.e. ι is the recovery rate. Then the remaining liquidation/sale value of the firm is equal to $\iota A(y_d)$, where $A(y_d)$ is the equilibrium value of an unlevered (all-equity without debt) public firm given by (A.4). Moreover, the debt holders will gain the compensatory payment from the guarantee company so that under the arrangement of equity-for-guarantee contract the debt holders gain $\phi b/r$ once the entrepreneur defaults instead of the remaining value $\iota A(y_d)$ only, where ϕ is the guarantee level. Therefore, the value, denoted by P_{guar}, of the compensatory payment is given by

$$P_{guar} = (\frac{\phi b}{r} - \iota A(y_d))\tilde{q}(y), \tag{2.4}$$

where $\tilde{q}(y)$ is the value of a security that claims one unit of account at the default time. [5]

While selling the firm to cash out, the entrepreneur needs to retire the firm's debt obligation at par F_0 given by Equation(A.18) in Appendix A. Similar to **CMW**, we assume that the buyer is well diversified who will optimally relever the firm, see Leland (1994). The value of the firm after sale is the value of an optimally levered firm in the complete market, i.e. $V^*(y_u)$ given by

$$V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{\kappa(1-\tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}\right] \frac{y}{r-\nu}, \tag{2.5}$$

After the entrepreneur exits from her business through either default or cash-out, she lives on her own financial income and faces a standard complete-markets consumption and portfolio choice problem.

An investor is characterized by her initial wealth W_0, a time-discount rate δ and her preference $U(\cdot)$. She seeks to choose a bond coupon b and a consumption process c and make investment decisions so as to maximize her expected lifetime time-additive utility of

[56] for details. Equation(A.6) for the default option only and Equation(A.14) for the case with cash-out option.

consumption:

$$\mathbb{E}\left[\int_0^{\infty} \exp\left(-\delta s\right) U(c_s) ds\right]. \tag{2.6}$$

In the following, we consider the optimization problem Equation(2.6) under CARA utility to reduce the dimension of the double-barrier problem. That is

$$U(c) = -\exp(-\gamma c)/\gamma, \quad c \in \Re, \tag{2.7}$$

where $\gamma > 0$ is the absolute risk aversion parameter.

2.3 Model Solution

In this section, we analyze the entrepreneur's consumption, portfolio choice, default / cash-out, and financing decisions under borrowing constraints and the equity-for-guarantee swap. The idiosyncratic risk of the entrepreneurial firm invalidates the standard two-step complete-markets analysis due to the nonseparability between value maximization and consumption smoothing.

We note that the entrepreneur's problem is significantly different from that considered by **CMW**. In fact, the problem here is much more challenging than **CMW** since the guarantee cost depends on both the default threshold and the cash-out threshold, which conversely depend on the guarantee cost at the same time.

In order to derive the solution, in the first step, we solve the standard Merton consumption and portfolio choice problem faced by the entrepreneur, see Merton (1971), after she exits from her business via either cashing out or defaulting on the debt. In the second step, we solve a mixture of optimal control and optimal stopping problem. To achieve this goal, we note that the entrepreneur's optimization problem is a time-homogeneous Markov control problem. Therefore, we first derive the guarantee cost, which is a function of the default threshold y_d and the cash-out threshold y_u. Then, for any given y_d and y_u, we find the maximum for

$$\mathbb{E}\left[\int_0^{\tau_{\mathcal{D}}} \exp\left(-\delta s\right) U(c_s) ds + \exp(-\tau_{\mathcal{D}}) J^e(W_{\tau_{\mathcal{D}}})\right], \quad y_d \leq y_0 \leq y_u, \tag{2.8}$$

where W is the entrepreneur's financial wealth process, $\mathcal{D} = \{(w, y) \in \Re^2 | y_d \leq y \leq y_u\}$, $\tau_{\mathcal{D}} \equiv \inf\{t \geq 0 | (W_t, y_t) \notin \mathcal{D}\} = \min\{T_d, T_u\}$ is stoping time[7], and the function $J^e(\cdot)$ is derived by solving the standard Merton problem (Merton 1971) in the first step and given by

$$J^e(w) = -\frac{1}{\gamma r} \exp\left[-\gamma r \left(w + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{2.9}$$

. It is clear that this maximum is also a function of the default threshold y_d and the cash-out threshold y_u. Therefore, we need to solve a constrained nonlinear programming problem to obtain the optimal default threshold y_d^* and the cash-out threshold y_u^*. Finally, we determine the entrepreneur's initial investment and optimal capital structure.

Remark 1. *Throughout the text, from an application's point of view, we assume the initial state of the system belongs to the domain $\mathcal{D}$, i.e. $(W_0, y_0) \in \mathcal{D}$ for a practical reason. For example, if $y_0 \leq y_d$, there is no need to start a non-profit business.*

2.3.1 Guarantee Costs and Equity-for-Guarantee Swaps

Unlike the traditional credit hypothecation, though, the entrepreneurial firm in the new credit guarantee scheme must pay to the guarantee company a portion (φ) of equity instead of regular guarantee fees. Thus we call φ the guarantee cost of the equity-for-guarantee swap.

Generally speaking, a guarantee company is usually a diversified investor who signs such contracts with a large number of firms and therefore the idiosyncratic risk of a entrepreneurship firm is well-diversified. This means that the value (P_{guar}) of compensatory payment must be equal to the market value (equilibrium value) of the equity allocated to the insurer. That is

$$P_{guar} = \varphi E_0(y; y_d, y_u), \tag{2.10}$$

where $E_0(y; y_d, y_u)$ is the market value of equity of the entrepreneurial firm, which depends on the exit threshold pair (y_d, y_u) as well as revenue y. The value $E_0(y; y_d, y_u)$ is given by Equation(A.8) if the cash-out option is prohibited and given by Equation(A.17) if the cash-out option is admissible.

[7]Let $\mathcal{T}$ be the set of $\{\mathcal{F}_t : t \geq 0\}$-stopping times. Thus the default timing and cash-out timing are $T_d \in \mathcal{T}$ and $T_u \in \mathcal{T}$ respectively.

Therefore, the guarantee cost φ is given by

$$\begin{aligned}\varphi \equiv \varphi(y; y_d, y_u) &= \left(\frac{\phi b}{r} - \iota A(y_d)\right)\tilde{q}(y)/E_0(y; y_d, y_u) \\ &= \frac{\left(\frac{\phi b}{r} - \iota(1-\tau_m)\frac{y_d}{r-\mu}\right)\tilde{q}(y)}{(1-\tau_e)\left(\frac{y}{r-\mu} - \frac{b}{r}\right) - (1-\tau_e)\left(\frac{y_d}{r-\mu} - \frac{b}{r}\right)\tilde{q}(y) + \left[V^*(y_u) - (1-\tau_e)\left(\frac{y_d}{r-\mu} - \frac{b}{r}\right)\right]\bar{q}(y)},\end{aligned} \tag{2.11}$$

where $V^*(y)$, $\tilde{q}(y)$ and $\bar{q}(y)$ are defined in Equations (A.12), (A.14) and (A.15).

Remark 2. *The guarantee cost here is fundamentally different from that given by Yang and Zhang (2013), which does not take into account that the entrepreneur is a risk-averse individual and has the option to cash out. For this reason, thanks to game theory, the equilibrium value of equity must be related to the entrepreneur's decisions on the cash-out option and a default threshold, which is clearly different from that in Yang and Zhang (2013) based on a risk-neutral world.*

2.3.2 Consumption and Portfolio Choice after Exit

After exiting from her business, the entrepreneur lives on her own financial wealth and faces the standard consumption and portfolio choice problem in Merton (1971).

The entrepreneur's wealth process follows

$$dW_t = (rW_t + \Pi_t(\mu_M - r) - c_t)dt + \Pi_t \sigma_M dB_t. \tag{2.12}$$

The consumption and portfolio rules are given by

$$c^*(w) = r\left(w + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{2.13}$$

$$\Pi^* = \frac{\eta}{\gamma r \sigma_M}. \tag{2.14}$$

The maximum of the expected lifetime time-additive utility of consumption is given by Equation (2.9).

2.3.3 Entrepreneur's Decisions and Utility Indifference Prices

Before exit, the entrepreneur's financial wealth evolves as follows,

$$dW_t = (rW_t + \Pi_t(\mu_M - r) + (1-\tau_e)(1-\varphi)(y-b) - c_t)dt + \Pi_t \sigma_M dB_t, \quad 0 < t < \tau_{\mathcal{D}}. \tag{2.15}$$

Compared to the exogenously given fraction of equity retained by the entrepreneur in **CMW**, the fraction $1-\varphi$ in our model is endogenously determined by the equity-for-guarantee swap in Equation (2.11), which also depends on the entrepreneur's decisions on default and cash-out. For this reason, we first solve the optimization problem Equation(2.8) for any given exit threshold pair (y_d, y_u). Under this case, the entrepreneur's value function $J^s(w,y)$ satisfies the following Hamilton-Jacobi-Bellman equation according to Bellman's principle of optimality:

$$\begin{aligned}
&\sup_{c\geq 0,\Pi} \{U(c) + (rw + \Pi(\mu_M - r) + (1-\tau_e)(1-\varphi)(y-b) - c)J^s_w(w,y) \\
&+\tfrac{(\Pi\sigma_M)^2}{2} J^s_{ww}(w,y) + \Pi\sigma_M\sigma\rho y J^s_{wy}(w,y) + \mu_y y J^s_y(w,y) + \tfrac{\sigma^2 y^2}{2} J^s_{yy}(w,y) \\
&-\delta J^s(w,y)\} = 0,
\end{aligned} \tag{2.16}$$

with the value-matching conditions

$$J^s(w, y_d) = J^e(w), \tag{2.17}$$

$$J^s(w, y_u) = J^e(w + (1-\varphi)V^*(y_u) - F_0 - K - \tau_g((1-\varphi)V^*(y_u) - K - I)). \tag{2.18}$$

The first-order conditions for the optimal consumption and portfolio choice are:

$$U'(c) = J^s_w(w,y), \tag{2.19}$$

$$\Pi = \frac{-J^s_w(w,y)}{J^s_{ww}(w,y)}\left(\frac{\mu_M - r}{{\sigma_M}^2}\right) + \frac{-J^s_{wy}(w,y)}{J^s_{ww}(w,y)}\frac{\rho\sigma y}{\sigma_M}. \tag{2.20}$$

According to utility indifference pricing principle, for the current revenue $y_d < y < y_u$, the utility indifference price (also called subjective value)[8] of equity owned by the entrepreneur, denoted by $G(y)$, satisfies

$$J^s(w,y) = J^e(w+G(y)) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(w+G(y)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right)\right]. \tag{2.21}$$

Substituting Equations (2.9) and (2.21) into Equations (2.16) - (2.18), we obtain the following theorem immediately:

Theorem 2.3.1. *The entrepreneur exits from her business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever comes first. For any given exit threshold pair (y_d, y_u), liquid wealth level w and revenue $y_d < y < y_u$, the optimal consumption and portfolio rule is given by*

$$c^*(w,y) = r\left[w+G(y)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right], \tag{2.22}$$

$$\Pi^*(w,y) = \frac{\eta}{\gamma r\sigma_M} - \frac{\rho\sigma}{\sigma_M}yG'(y), \tag{2.23}$$

where $G(y)$ is the utility indifference price of equity owned by the entrepreneur and is a solution of the following ordinary differential equation:

$$\begin{aligned} rG(y) &= (1-\tau_e)(1-\varphi)(y-b)+(\mu_y-\rho\sigma\eta)yG'(y) \\ &\quad +\tfrac{\sigma^2 y^2}{2}G''(y) - \tfrac{\gamma r(1-\rho^2)\sigma^2 y^2}{2}G'(y)^2, \end{aligned} \tag{2.24}$$

subject to the boundary conditions:

$$G(y_d) = 0, \tag{2.25}$$

$$G(y_u) = (1-\varphi)V^*(y_u) - F_0 - K - \tau_g((1-\varphi)V^*(y_u)-K-I). \tag{2.26}$$

In order to complete the computation of the entrepreneur's optimization problem, we now need to derive the optimal default threshold y_d^* and optimal cash-out threshold y_u^*, such that the value function $J^s(w,y)$ is maximized. Equivalently, we need only to find the

[8] Thanks to the exponential utility assumption, the utility indifference price is independent of the wealth level of the entrepreneur.

maximum point (y_d^*, y_u^*) of the function $G(y; y_d, y_u)$, i.e. the utility indifference price of equity owned by the entrepreneur, with regard to independent variables y_d and y_u for any given revenue level[9]. The constrained nonlinear programming problem is solved by numerical methods.

Remark 3. *At first sight this theorem is similar to* ***CMW****, but the fraction $1-\varphi$ of equity owned by the entrepreneur here is endogenously determined under the newly invented equity-for-guarantee swap.*[10] *As a result, one of the distinctions from* ***CMW*** *is that the state transition Equation (2.15) itself depends on the optimal stopping times. Therefore, to derive the optimal default threshold and cash-out threshold more effectively, we solve a nonlinear programming problem instead of utilizing the smooth-pasting conditions as done by* ***CMW****.*

Equations (2.22) and (2.23) indicate that the entrepreneur will consume the implied value $G(y)$ and use the market portfolio to dynamically hedge the entrepreneurial business risk. Equation (2.24) implies that if the absolute risk-aversion index equals 0 (i.e. the entrepreneur is risk-neutral towards the idiosyncratic risk), or the idiosyncratic risk volatility ϵ is 0, Equation (2.24) becomes the standard equilibrium pricing equation.

Similar to **CMW**, Equation (2.24) implies that both systematic and idiosyncratic risks contribute to the risk premium for an entrepreneurial firm. This is different from a public firm, where the risk premium is determined by the firm's systematic risk through the classic CAPM model. Specifically, the systematic and idiosyncratic risk premium, denoted by $\xi^s(y)$ and $\xi^i(y)$ respectively, are given by

$$\xi^s(y) = \rho\sigma\eta y \frac{G'(y)}{G(y)}, \tag{2.27}$$

$$\xi^i(y) = \frac{\gamma r}{2} \frac{(\sqrt{1-\rho^2}\sigma y G'(y))^2}{G(y)}. \tag{2.28}$$

As we expected, the idiosyncratic risk premium is zero once the risk-averse index (γ) is zero or the absolute value ($|\rho|$) of the correlation coefficient between the project payoff and the

[9]Obviously, the maximum point will not depend on the current revenue level in our model. Thus we find the maximum point at $y_0 = 1$, and we set the constraint $(0, y_0)' \le (y_d, y_u)' \le (y_0, \bar{y})'$, where $\bar{y}$ is a sufficiently large boundary to include y_u^*.

[10]Here *endogenously* means that the guarantee cost φ is given explicitly in Equation (2.11).

return on market portfolio is one, i.e. idiosyncratic risk disappears. On the contrary, the systematic risk premium does not depend on the risk-averse index (γ) directly.

2.3.4 Capital Structure under Equity-for-Guarantee Swap

Now we turn to the entrepreneur's initial decision on the optimal debt borrowed from the lender for investing in the project in order to maximize the entrepreneur's net profit under the equity-for-guarantee swap at time $t = 0$.

The cash amount that the entrepreneur borrows from a lender is the value $F(y_0)$ of the cash flow received by the lender from the borrower, which depends on the coupon b of the debt. After the initial one-time lump-sum cost I, the entrepreneur acquires $F(y_0) - I$ with her own funds in the project. For this reason, the entrepreneur's initial optimization problem or optimal capital structure to start the project is to find optimal coupon level b^*, which maximizes the entrepreneur's net profit (target function)

$$P(y_0; b, I) \equiv G(y_0) + [F(y_0) - I]. \tag{2.29}$$

The entrepreneur should abandon the project if the maximum $P^*(y_0; b^*, I)$ is less than zero. Furthermore, we identify a breakeven investment cost I^* by solving a root-finding problem $P^*(y_0; b^*, I^*) = 0$.

Denote by $\hat{P}^*(y_0)$ the maximum net profit obtained by an entrepreneur without signing an equity-for-guarantee swap contract, and by $W^s(y_0)$ the welfare loss of this entrepreneur because the swap is not signed, i.e.

$$W^s(y_0) \equiv P^*(y_0) - \hat{P}^*(y_0). \tag{2.30}$$

In other words, W^s is the loss of utility without equity-for-guarantee swap. It is the additional endowment that an entrepreneur without equity-for-guarantee swap needs in order to achieve the same expected utility under the case where the equity-for-guarantee swap is signed. Furthermore, we will distinguish two cases without equity-for-guarantee swap. One case is that the firm can issue debt with the protective covenant $y_d = b$ rather than optimal default decision. While in another case, the firm is an unlevered firm since the entrepreneur is unable to issue debt because of the borrowing constraint of low credibility.

After the entrepreneur invests in the project, a natural measure of leverage for the entrepreneur is the private leverage ratio $L(y_0)$ defined by

$$L(y_0) = \frac{F(y_0)}{S(y_0)}, \tag{2.31}$$

where $S(y_0)$ is the total subjective value of the entrepreneurial firm. Under our equity-for-guarantee swap arrangement, there are three claims: inside equity held by the entrepreneur, diversified outside equity held by the insurer, and outside debt held by the lender. However, in contrast to **CMW**, the total subjective value in our model is given by only

$$S(y_0) = G(y_0) + F(y_0), \tag{2.32}$$

which excludes the value φE_0 of the outside equity held by the insurer because the value of the outside debt held by the lender has taken it into account already.

2.4 Discussion of Entrepreneurial Finance

In this Section, we use a numerical example to give strong evidence that EGS plays a vital role in alleviating entrepreneurs's financing constraints. In order to make a comparison, following **CMW**, our numerical results are based on the following annualized baseline parameter values: risk-free interest rate $r = 3\%$, expected growth rate of revenue $\mu_y = 4\%$, market price of risk, i.e. the Sharpe ratio of the market portfolio, $\eta = 0.4$, asset recovery rate $\iota = 0.6$. The entrepreneur's rate of time preference is $\delta = 3\%$ and the initial level of the project value is $y_0 = 1$. The initial investment cost for the project is $I = 10$ and the cash-out cost is $K = 27$. The effective marginal Miller tax rate τ_m of a public firm is set to 11.29% as in Graham (2000) and Hackbarth, Hennessy, and Leland (2007)[11]. The tax rate τ_e of entrepreneurial firm is set to equal τ_m for emphasizing the the entrepreneur's nondiversifiable idiosyncratic risk. The effective capital gains tax rate from selling the business is $\tau_g = 10\%$.

In addition, we set the guarantee level $\phi = 50\%$, the systematic volatility of growth rate $\zeta = 10\%$, and idiosyncratic volatility $\epsilon = 10\%$. Hence, the total volatility of the project $\sigma = 0.02^{0.5}$, correlation coefficient $\rho = 0.5^{0.5}$. We consider three values of the risk-aversion parameter $\gamma \in \{0, 1, 2\}$.

Similar to **CMW**, the baseline parameter values are carefully selected in order to make sure that the assumption $y_d < y_0 < y_u$ holds. That is, we excludes two special cases: One corresponds to a sufficiently large asset recovery rate ι, together with a sufficiently high guarantee level ϕ and a large risk aversion γ, which will lead to an immediate default (i.e. $y_d = y_0$); The other corresponds to a sufficiently small cash-out cost K, which will make the entrepreneur sell the firm immediately (i.e. $y_u = y_0$).

At the very beginning, a sensitivity analysis of $G(y)$ and$G'(y)$ is provided in Table 2.1.

2.4.1 Equity Value with Equity-for-Guarantee Swap

We apply Figure 2.1 to illustrate the properties of the utility indifference price G of equity, i.e. the value of the cash flow $(1 - \tau_e)(1 - \varphi)(y - b)$, received by the entrepreneur with the equity-for-guarantee swap. Figure 2.1 shows the results under the case of default option

[11]Similar to **CMW**, the effective Miller tax rate here integrates the corporate income tax, individuals equity, and interest income tax. Using Millers formula for the effective tax rate, and setting the interest income tax at 0.30, corporate income tax at 0.31, and the individuals long-term equity (distribution) tax at 0.10, we obtain an effective tax rate of 11.29%.

TABLE 2.1: The table presents the sensitivity analysis of the subjective value $G(y)$ and its derivative $G'(y)$ to the changes of guarantee level ϕ, idiosyncratic volatility ϵ, and cash-out cost K for risk aversion $\gamma = 1$ and the current project revenue $y_0 = 1$.

ϕ	0.40	0.45	0.50	0.55	0.60	0.65	0.70
$G(y)$	12.96	10.60	9.76	3.40	1.38	0.87	0.50
$G'(y)$	21.86	20.59	19.81	11.17	5.69	3.68	2.95
ϵ	0.00	0.05	0.10	0.15	0.20	0.25	0.30
$G(y)$	18.16	16.37	9.76	3.92	3.63	3.20	3.16
$G'(y)$	28.72	26.20	19.81	11.16	9.44	7.83	7.17
K	10.00	20.00	30.00	40.00	50.00	60.00	70.00
$G(y)$	10.02	9.80	9.75	9.72	9.71	9.70	9.70
$G'(y)$	20.27	19.90	19.78	19.73	19.70	19.69	19.68

only and under the case with both default option and cash-out option respectively. After that, we present the risk premiums demanded by the entrepreneur in Figure 2.2. Here we let $\tau_e = 0$ to exclude the effects of tax.

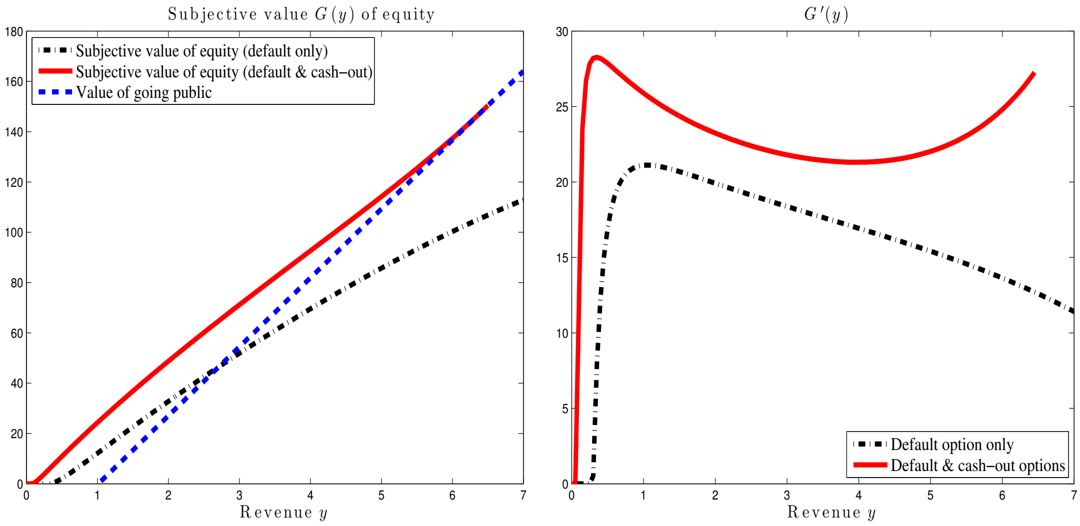

FIGURE 2.1: The figure depicts the subjective values $G(y)$ of equity held by the entrepreneur, its derivatives and the value of going public, i.e. the intrinsic value of the cash-out option, with the equity-for-guarantee swap under the case of taking default option only and the case of taking both default option and cash-out option respectively, under the assumption of $\gamma = 1$, $\tau_e = 0$ together with the baseline parameter values.

The results in Figure 2.1 under the equity-for-guarantee swap are similar to **CMW** if we think the amount of equity allocated to the insurer in our model as the outside equity in **CMW**, who do not take into account the swap. We find that the subjective values $G(y)$ of equity are convex functions of revenue y when it is sufficiently low, i.e. the default option is deep in the money, or it is sufficiently high, i.e. the cash-out option is deep in the money.

Under other situations, the subjective values are concave because the two options are not deep in the money and the risk aversion of the entrepreneur (precautionary saving demand) dominates the impact of a growth of revenue y.

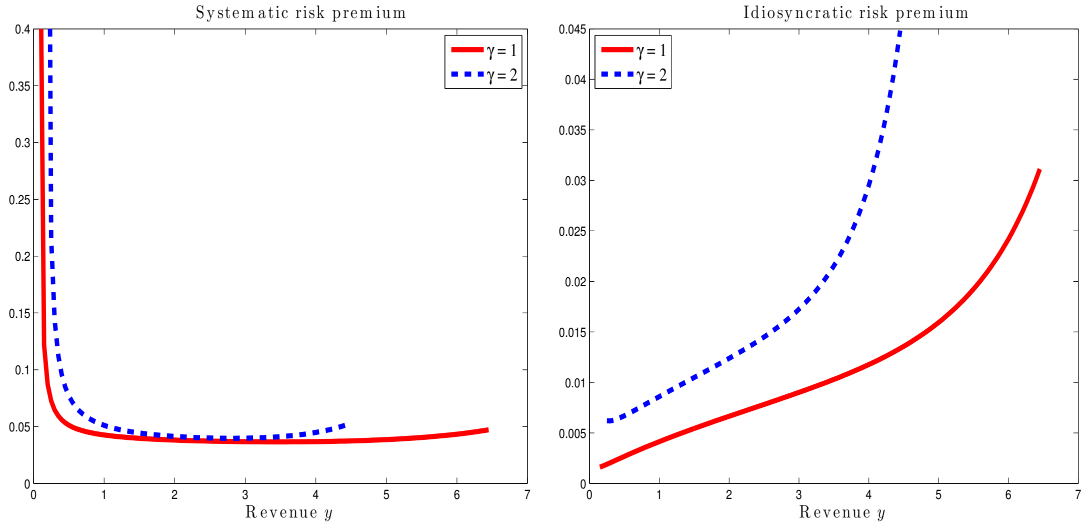

FIGURE 2.2: This figure shows the systematic and idiosyncratic risk premium under equity-for-guarantee swap with default option, cash-out option, $\tau_e = 0$ and baseline parameter values.

The properties of risk premium demanded by the entrepreneur shown in Figure 2.2 are similar to **CMW** as well. On the one hand, when the revenue y is small and approaches default threshold y_d, the systematic risk premium $\xi^s(y)$ is similar to the standard valuation model and it diverges to infinity as seen in (2.27) because of the significant leverage effect (the equity is a levered position). The systematic risk premium $\xi^s(y)$ rises when the revenue y approaches the cash-out threshold y_u because the cash-out option makes the value G more sensitive to cash flow shocks. On the other hand, the idiosyncratic risk premium $\xi^i(y)$ is small when y is close to y_d. When revenue y is large, its growth leads to a fast growth of $\xi^i(y)$ because the conditional idiosyncratic variance $(\sqrt{1-\rho^2}\sigma y G'(y))^2$ rises faster than $G(y)$, as seen in Equation (2.28).

2.4.2 Comparison of Capital Structures

We compare three capital structures under different financing arrangements in Table 2.2 without taxes $\tau_e = 0$ and in Table 2.3 with taxes $\tau_e = \tau_m = 11.29\%$ respectively. In Tables 2.2 and 2.3, Panel A represents the unlevered entrepreneurial firm; Panel B is the entrepreneur who is able to choose optimal risky debt but is unable to determine optimal default due to the protective covenant $y_d = b$; Panel C exhibits the entrepreneurial firm with optimal leverage and optimal default supported by the equity-for-guarantee swap. Table 2.2 shows the special case where entrepreneurs acquire only diversification benefits without tax benefits from risky debt.

Similar conclusions to **CMW** are found from the two tables. First, the model is equivalent to the complete-market benchmark when the risk aversion $\gamma \to 0$ and $\tau_e = 0$, and hence the firm is valued at the market value 33.33. Second, the subjective value for a risk-averse entrepreneur decreases because they discount the nontradable equity due to nondiversifiable idiosyncratic business risks. Third, more risk-averse entrepreneurs issue more debt in order to achieve greater diversification benefits, and as a result, it leads to a less subjective value of equity held by the entrepreneur.

TABLE 2.2: The table gives guarantee cost φ^*, optimal coupon b, debt value F_0, equilibrium value φE_0 of equity held by the insurer, subjective value G_0 of equity held by the entrepreneur, optimal leverage L_0, credit spread CS, 10 years default probability $p_d(10)$, 10 years cash-out probability $p_u(10)$, and welfare loss W^s under the tax rate $\tau_e = 0$ and optimal capital structure with cash-out option and equity-for-guarantee swap for risk aversion $\gamma = 0, 1, 2$ respectively.

γ	φ^* (%)	b	F_0	φE_0	G_0	L_0 (%)	CS (bp)	$p_d(10)$ (%)	$p_u(10)$ (%)	W^s
Panel A: without guarantee, no leverage										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	28.43	0.00	0.00	0.00	0.07	0.34
2.00	0.00	0.00	0.00	0.00	26.10	0.00	0.00	0.00	6.09	0.54
Panel B: without guarantee, optimal leverage and $y_d = b$										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.05	1.65	0.00	27.05	5.75	2.89	0.00	0.04	0.08
2.00	0.00	0.11	3.57	0.00	22.92	13.46	8.35	0.00	0.60	0.15
Panel C: with guarantee, optimal leverage and optimal default										
0.00	0.00	0.00	0.00	0.00	33.33	0.00	0.00	0.00	0.00	0.00
1.00	0.05	0.13	4.48	0.01	24.30	15.57	1.25	0.00	0.04	0.00
2.00	0.73	0.29	9.11	0.18	17.53	34.19	16.81	0.00	0.60	0.00

More importantly, our results reveal large increased benefits for the entrepreneur through introducing equity-for-guarantee swap. Under this swap, the entrepreneur is not forced to default by the protective covenant even when the revenue y is lower than the coupon b as demanded by the protective covenant. Accordingly, the entrepreneur is able to choose the endogenous optimal default threshold y_d that is generally lower than b. In other words, thanks to the swap, the entrepreneur has deep pockets now without liquidity constraints, which is an assumption widely applied by **CMW**. For this reason, we argue that the equity-for-guarantee swap in our model makes the important assumption in **CMW** feasible.

Admittedly, the entrepreneur with the swap faces higher credit spreads and default probabilities due to higher optimal leverage if the debt is guaranteed by only $\phi = 50\%$ level, i.e. the insurer only ensures that the debt holder receives 50% of the value b/r. Certainly, credit spreads and default probabilities can be reduced by raising the guarantee level ϕ if it is agreed by the three parties. However, a higher guarantee level ϕ demands that the entrepreneur should pay a higher portion of equity to the insurer in exchange for the guarantee, i.e. the guarantee cost gets higher. If the guarantee cost is greater than or equal to one, then it means that no insurer wants to sign the swap contract with the entrepreneur.

From Tables 2.2 and 2.3, thanks to the equity-for-guarantee swap, the entrepreneur with the guarantee achieves a considerable welfare increment, i.e. the welfare losses W^s of the entrepreneurs without the guarantee are significant. The welfare increment is enlarged by a higher tax rate and a higher risk-aversion index of the entrepreneur. In particular, the welfare increment gets even more relative to the case of no leverage, which is particularly common in China among others, since a large number of entrepreneurs of SMEs are fundamentally unable to issue debt directly.

TABLE 2.3: This table gives guarantee cost φ^*, optimal coupon b, debt value F_0, equilibrium value φE_0 of equity held by the insurer, subjective value G_0 of equity held by the entrepreneur, optimal leverage L_0, credit spread CS, 10 years default probability $p_d(10)$, 10 years cash-out probability $p_u(10)$, and welfare loss W^s under the tax rate $\tau_e = \tau_m = 11.29\%$, optimal capital structure with cash-out option and equity-for-guarantee swap for risk aversion $\gamma = 0, 1, 2$ respectively.

γ	φ^* (%)	b	F_0	φE_0	G_0	L_0 (%)	CS (bp)	$p_d(10)$ (%)	$p_u(10)$ (%)	W^s
Panel A: without guarantee, no leverage										
0.00	0.00	0.00	0.00	0.00	29.51	0.00	0.00	0.00	0.00	0.84
1.00	0.00	0.00	0.00	0.00	25.77	0.00	0.00	0.00	0.10	1.52
2.00	0.00	0.00	0.00	0.00	23.93	0.00	0.00	0.00	0.88	2.43
Panel B: without guarantee, optimal leverage and $y_d = b$										
0.00	0.00	0.18	5.85	0.00	24.07	19.54	16.43	0.00	0.00	0.44
1.00	0.00	0.20	6.42	0.00	19.79	24.49	19.32	0.00	0.21	1.08
2.00	0.00	0.24	7.53	0.00	16.87	30.88	25.21	0.02	1.28	1.97
Panel C: with guarantee, optimal leverage and optimal default										
0.00	3.14	0.46	14.02	0.53	16.33	46.21	30.79	0.05	0.00	0.00
1.00	7.17	0.61	17.53	0.99	9.76	64.24	49.46	0.55	0.10	0.00
2.00	21.10	0.84	21.72	1.99	4.64	82.39	84.53	4.93	0.21	0.00

2.4.3 Analysis of Welfare Loss

To further demonstrate the increased benefits resulting from the equity-for-guarantee swap for the entrepreneur, we present comparative statics on welfare loss W^s for different parameter values in Figures 2.3 and 2.4. We only show that the results of welfare loss W^s incurred by the entrepreneur without leverage due to the pervasive low borrowing capacity of an SME. The findings on the welfare loss W^s of the entrepreneur with the protective covenant are similar.

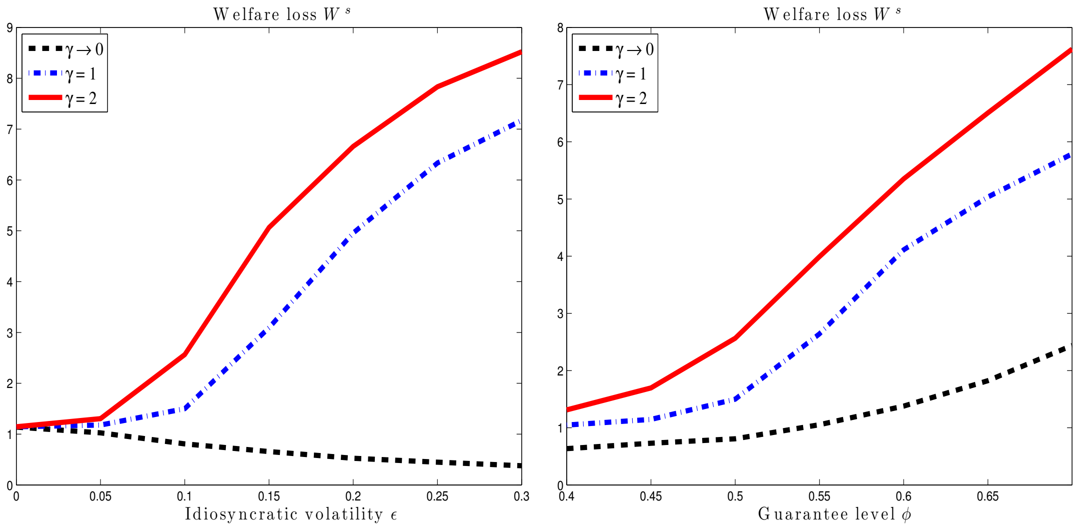

FIGURE 2.3: The figure gives comparative statics for welfare loss W^s with respect to idiosyncratic volatilities ϵ and guarantee levels ϕ.

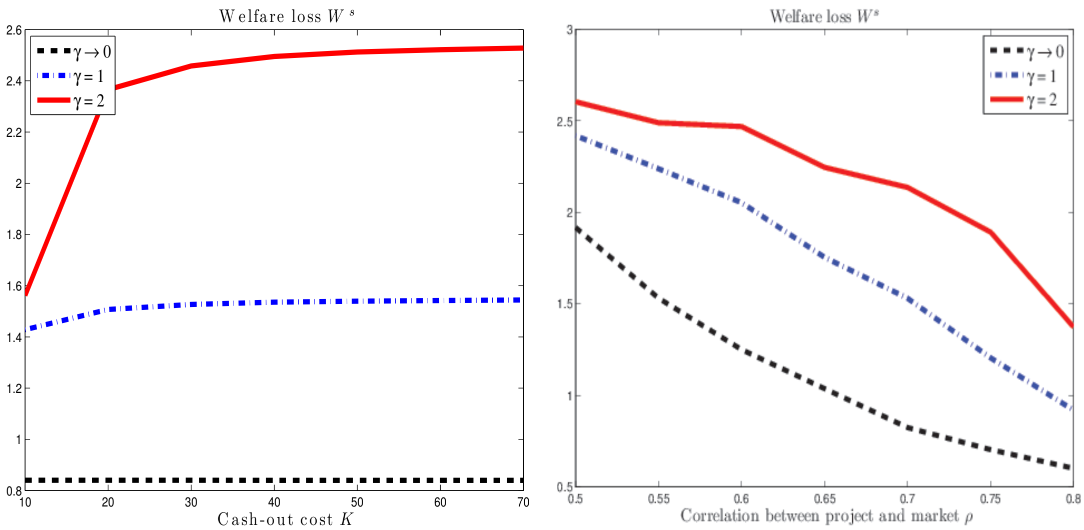

FIGURE 2.4: This figure presents comparative statics for welfare loss W^s with respect to cash-out costs K and correlation coefficients ρ.

Figures 2.3 and 2.4 highlight large benefits resulting from the equity-for-guarantee swap. Compared with the unlevered firm, the equity-for-guarantee swap raises net profit, and the welfare increment ascends for more risk-averse entrepreneurs who optimally take higher leverage. First, for a risk-averse entrepreneur, the welfare loss W^s increases substantially with the idiosyncratic volatility ϵ and the guarantee level ϕ. Second, if the cash-out cost K is small, the welfare loss W^s increases quickly with K for a sufficiently risk-averse entrepreneur, and then keep unchanged if K is sufficiently large. It also shows that the welfare increment increases quickly with the risk-aversion index and does not depend on the cash-out cost if the cost is sufficiently large. Finally, the welfare loss W^s decreases with the correlation coefficient ρ since a large absolute value of the correlation coefficient ρ means a less idiosyncratic risk, keeping parameter σ unchanged.

2.4.4 Breakeven Investment Cost

The previous subsection explores the benefits of the equity-for-guarantee swap in financing decisions. In this subsection, we focus on the effects of the equity-for-guarantee swap on an entrepreneur's investment decisions. Under the assumption of $\tau_e = 0$, **CMW** examine the effects of idiosyncratic volatility $\epsilon \in \{0.15, 0.20, 0.25\}$ on the breakeven investment cost I^* since the NPV analysis is not applicable in incomplete markets. Here we provide more detailed analysis on the cut-off rule I^* and emphasize the benefits of the equity-for-guarantee swap.

As before, we compare the breakeven cost I^* under three different capital structures, i.e. no leverage without the swap, optimal leverage with bankruptcy protection but without the equity-for-guarantee swap, and the optimal capital structure with the swap. As reported in Table 2.4, generally speaking, the breakeven investment cost I^* decreases for a more risk-averse entrepreneur and/or a higher idiosyncratic volatility of the revenue. For instance, if $\epsilon = 0.90$, an entrepreneur with risk aversion $\gamma = 0$ (risk-neutral) under the swap (Panel C) will invest in the project even the investment cost is as much as 27.26. By contrast, if the entrepreneur is risk-averse enough, say $\gamma = 2$, she will give up the project once the investment cost is greater than 21.55.

However, the trend of I^* does not hold true all the time. In fact, we notice that in Panels B and C of Table 2.4, as idiosyncratic volatility ϵ grows further, breakeven investment costs fluctuate slightly. This is because the subjective values of equity held by the entrepreneur include the values of the default option and the cash-out option, both of which increase with a growth of the volatility of idiosyncratic risk although for a risk-averse investor, the subjective value of larger risk asset will be less in general.

In addition, we find that in Panel B under risk aversion $\gamma = 2$, the breakeven investment cost is somewhat higher than that under $\gamma = 1$ if the volatility of idiosyncratic risk is large enough, say $\epsilon \geq 0.4$. This is because a more risk-averse entrepreneur would borrow more money from the bank and get more diversified benefits and more tax shields under bankruptcy protection $y_d = b$.

TABLE 2.4: This table gives breakeven investment cost I^* under tax rate $\tau_e = \tau_m = 11.29\%$ for different volatility ϵ of idiosyncratic risk and risk aversion $\gamma = 0, 1, 2$ respectively.

$\epsilon =$	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Panel A: without guarantee, no leverage									
0.00	29.51	29.17	28.59	28.00	27.52	27.15	26.87	26.66	26.50
1.00	25.78	20.99	17.83	15.92	14.73	13.98	13.48	13.12	12.86
2.00	23.94	18.44	15.18	13.30	12.14	11.37	10.92	10.60	10.37
Panel B: without guarantee, optimal leverage and $y_d = b$									
0.00	29.97	29.39	28.78	28.14	27.63	27.24	26.96	26.74	26.58
1.00	26.29	21.31	18.04	17.61	17.78	17.73	17.70	17.68	17.66
2.00	24.49	18.68	17.71	17.84	17.81	17.80	17.79	17.78	17.77
Panel C: with guarantee, optimal leverage and optimal default									
0.00	30.35	29.69	28.97	28.51	28.20	27.55	27.51	27.35	27.26
1.00	27.29	25.42	24.43	23.69	23.65	22.56	22.92	22.93	22.69
2.00	26.36	24.90	23.10	22.78	22.24	21.97	21.58	21.52	21.55

Actually, under the equity-for-guarantee swap, for a more risk-averse entrepreneur, the breakeven investment cost decreases gradually and it is considerably greater than the corresponding breakeven investment cost if the swap contract is not signed. These results imply that an entrepreneur armed with the equity-for-guarantee swap is more willing to invest, since the swap provides the entrepreneur with the eligibility not only to issue debt but also to freely choose the time to default.

More importantly, a high breakeven investment cost means a large net profit obtained by the entrepreneur after investing in the project. By comparing the three financing schemes in Table 2.4, we find that a risk-averse entrepreneur obtains considerable welfare increments resulting from the equity-for-guarantee swap, although the welfare increments are very limited if the entrepreneur is risk-neutral.

In addition, the stable values appear earlier for higher risk aversion. For instance in Panel C, the breakeven cost I^* falls to around 27 after $\epsilon = 0.7$ for the risk aversion level $\gamma \to 0$, and it is close to 23 after $\epsilon = 0.6$ for $\gamma = 1$, while the level value is about 22 after $\epsilon = 5$ for $\gamma = 2$. In fact, $I^* = G(y_0|I^*) + F(y_0|I^*)$, and equity is a call option on levered firm's operating assets with the value of the debt as the strike price. Growing volatilities make the option become out of the money from in the money, therefore I^* decreases firstly and then the loss is limited.

We notice the exception that the breakeven cost I^* grows slightly at $\gamma = 2$ and $\epsilon \in [0.3, 0.35]$ in Panel B, which is attributed to the protective covenant $y_d = b$. It falls slightly with increasing idiosyncratic risk ϵ difference in I^* between Panel B and A increases with idiosyncratic volatility ϵ, and the difference in I^* between Panel C and A increases more.

2.5 Conclusion

In China, there are a large number of small- and medium-sized enterprises (SMEs), who are almost impossible or difficult to get bank loan to start a new business because of too low credibility and lack of guarantee, even if the business is substantially profitable.

Usually, it is difficult or even impossible for SMEs to get bank loan to start a new business because of low credibility and lack of guarantee, even if the business is substantially profitable. There are a large number of SMEs encountering such obstacles and moreover, a great many students graduate from college every year (e.g. record-high 6.99 million in China in 2013), and with the increasingly serious employment situation, many governments encourage them to start their own businesses. Clearly all of them need funds, which in general cannot be borrowed directly from a bank. In order to overcome such financing constraints, a financial product, called equity-for-guarantee swap, which was invented in China since 2002, is becoming more and more popular. However, to the best of our knowledge, there is no quantitative study on such swaps apart from Yang and Zhang (2013), who consider only the equilibrium pricing problem for a firm with the equity-for-guarantee swap.

Chapter 2 provides a dynamic incomplete-market framework that models the impact and interactions of the two frictions: borrowing constraints and lack of diversification, on entrepreneurial investment, interdependent consumption, portfolio allocation, financing, and business exit decisions. We show that the equity-for-guarantee swap brings great benefits to the entrepreneur. The equity-for-guarantee swap improves the capital structure trade-off among tax, diversification benefit and financial distress costs. Hence, issuing covered risky debt generates substantial diversification benefits and tax benefits. The more risk-averse the entrepreneur, or the higher the idiosyncratic risk, the greater the benefit. Naturally, the most important advantage of the newly invented swap is to effectively overcome borrowing constraints on an entrepreneur, who are unable to invest in a project without the swap, even though the project is valuable.

Chapter 3

Hedge Fund Seeding Innovation with Fees-for-seed and Fees-for-guarantee Swaps

Coauthored with Binghua Huang [1]

Abstract

To alleviate the financing constraints resulting from severe asymmetric information in the hedge fund seeding stage, we have developed a new financial derivative product called fees-for-guarantee swap. This swap works together with the well-known fees-for-seed swap and actually plays a role in forcing managers to put a fraction of their earnings aside in advance in order that they refund the initial investment of the seeder once they default. We set up a dynamic continuous-time framework and provide closed-form prices for seed capital, guarantee costs and other claims. Our numerical results indicate that the incentive compensation, managerial ownership and the possibility of fund liquidation significantly mitigate the manager's risk-shifting incentive.

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3.1 Introduction

Over the past few years, there has been a significant increase in both the number of hedge fund seeders and the amount of capital available for hedge fund seeding. According to HFM-Week research, in November 2011, seeders had approximately $4.6 billion in available capital, compared to approximately $1 billion just one year earlier.

In China, institutional investors also show increasing interest in hedge funds seeding as the nation's wealthy look for more ways to invest their money and slow but steady financial reform offers budding managers more ways to play the markets. CITIC Securities Futures, China's biggest investment bank, has launched a fund that will seed new and established hedge fund firms in the country.[2] Separately, the asset management arm of another local brokerage, Hwabao Securities, is launching a similar fund with one billion yuan to invest in partnership with KKM Capital, a Shanghai company dedicated to helping Chinese hedge fund.

Despite the recent growth in available seed capital, there still remains a tremendous shortage of capital for the early-stage funds (ESFs, henceforth) because most capital allocators increasingly focus on larger established hedge funds considered highly credible. In addition, a larger talent pool of ESFs managers is now competing for the available seed capital. Moreover, barriers to entry for ESFs are much steeper today than that before the 2008's financial crisis[3]. Consequently, the financing constraints faced by the ESFs managers nowadays are much more serious than in the past.

Given this state of affairs, more and more emerging managers are likely to turn to seed investors for early stage capital. To be involved in the hedge fund seeding business, a seed investor or a seeder commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced

[2]CITIC Securities is no stranger to seeding hedge funds, as it invested $20 million in Hong Hong-based Vision Gain Capital in 2008. The newly-established fund has about 2 billion yuan ($330 million) in capital that will invest in Chinese hedge funds prompted by the country's growing interest in the sector as its growing number of billionaires are looking for alternative ways to invest their money, according to Financial Times's report.

[3]Title IV of the Dodd-Frank Wall Street Reform and Consumer Protection Act compels the U.S. Securities and Exchange Commission (SEC) to impose reporting requirements on all hedge funds as it deems necessary or appropriate in the public interest or for the assessment of systemic risk. The article, "Launch bad; Hedge funds", published in The Economist 20 Apr. 2013: 79 (US) states that it is much harder to break into the hedge-fund world than before, because of the rising expenses, more risk-averse investors and enhanced regulation.

economics" which is usually the fees[4] that the ESFs manager generates from the entire pool of assets in the fund. It is not uncommon that seeders receive a portion of the hedge fund's revenue stream to get greater return potential than that of the ordinary investor through the hedge fund seeding vehicle. If fees-for-seed swap is structured properly, the seeding vehicle can be highly beneficial to both ESFs managers and seeders[5].

Although the seeding approach in practice has, to some extent, solved ESFs managers' financing constraints, there is still a huge gap between the seeding capital demand and supply in the hedge fund seeding business. This is mainly because of the severe information asymmetry between seeders and ESFs managers, which has always been the key factor holding back the development of the hedge fund seeding business, especially in the aftermath of the 2008 financial crisis. Therefore, an insurer was firstly introduced in Chapter 3, through an innovative contract, to solve the problem of information asymmetry by improving the ESFs manager's credibility.

Inspired by small and medium-sized enterprises' experience of overcoming borrowing constraints in China as discussed in Yang and Zhang (2013), the financing constraints result of information asymmetry faced by ESFs managers will be largely alleviated by introducing the fees-for-guarantee swap supported by a commercial guarantee-company or an insurer. The rationale is that fees-for-guarantee swap can significantly mitigate the information asymmetry between a manager and a seeder, thus encourages the seeder to invest in the hedge fund seeding stage with confidence, as facing little credit risk. Unlike the traditional credit hypothecation, however, ESFs managers in the new credit guarantee scheme must pay to the guarantee company a certain portion of her management fees, as guarantee costs instead of regular guarantee fees. Once the hedge fund is exogenously liquidated by a crisis shock or endogenously by investors, the seeder can still get compensatory payment from the insurer. In order to guarantee the seeder's payoff and control her downside risk

[4]Hedge fund managers normally receive 20% of the increase in fund value in excess of the last recorded maximum, i.e. high-water-mark as incentive fees in addition to 2% of the asset under management as annual fees, which "two-twenty" viewed as the industry's norm. Several academic articles study the characteristics of hedge fund fees, such as Fung and Hsieh (1997), Fung and Hsieh (1999)), and Aragon and Nanda (2012), as well as several articles in the Wall Street Journal.

[5]A widely accepted rule of thumb is for a seeder to expect 1% of revenues for each $1 million of seed capital for seed transactions no larger than $50 million. However, it would not be surprising to see a 25% revenue share on a seed investment of US $100 million as seed arrangements can vary substantially based on factors such as the experience of the manager or alpha strategy, the amount of seed capital provided, and the relative negotiating power of each party.

well, it is not unreasonable to define that the present value of the compensatory payment plus the annual fees received from the manager to the seeder should be breakeven or equal to her initial investment under the arrangement of these innovative swaps.

Although there is a thumb rule in designing seed arrangements and a few simple models featuring hedge fund seeding return in practice[6], the theoretical research on the contract costs and optimal designing of these contracts is still unavailable. In this article, we firstly provide closed-form solutions to the seed capital costs and guarantee costs using standard continuous-time methodology under the arrangements of these innovative swaps. Moreover, we develop a numerical procedure to analyze the impact of managerial ownership, fund liquidation on the risk shifting behavior of the ESFs manager who has a convex payoff compensation structure.

Chapter 3 is closely related to Goetzmann, Ingersoll, and Ross (2003), which provides the first quantitative intertemporal valuation framework of investors' payoff and managers' fees in a setting where the fund's value follows a log-normal process and the fund managers have no discretion over the choice of portfolio. However Chapter 3 mainly focuses on the innovation of the hedge fund seeding business like fees-for-seed and fees-for-guarantee swaps. Moreover, we provide a dynamic framework for valuing the costs of these swaps, as well as modeling the impact of the so-called high-water mark, managerial ownership and fund liquidation on ESFs managers' risk shifting behavior, in hedge fund seeding stage.

Ewald and Zhang (2016) have done a similar research on the hedge fund seeding businesses, however, they focus on a risk-averse manager's decision and highlight the effect of frictions of incomplete markets. They provide a dynamic valuation model of the hedge fund seeding business, by solving the consumption and portfolio-choice problem for a risk-averse manager, who launches a hedge fund via a seeding vehicle. Chapter 3 introduces a new financial product called fees-for-guarantee swap, to solve the serious asymmetric information problem between ESFs managers and seeders. Yang and Zhang (2013) provide a formal study on equity-for-guarantee swap on SMEs' financing problems, which share some similarities of the financing problems of ESFs managers. Similarly, ESFs managers face serious financing constraints and these financial constraints can be largely solved by introducing fees-for-guarantee swap and fees-for-seed swap innovative swaps. Moreover,

[6]For example, Larch Lane Advisors (LLC) has constructed a simple model to project returns and cash flows for a seeded fund, featuring the innovative seeding strategy.

our model focuses on the impacts of these swaps on ESFs manager's risk shifting behavior and highlights hedge fund management at the hedge fund seeding stage.

Our article is also related to the substantial literature that examine the effects of convex payoff compensation on the risk choices of hedge fund managers.[7] Carpenter (2000) asserts that it is optimal for hedge fund managers, who faces no explicit downside risk, to choose infinite volatility as asset value goes to zero. On the contrary, managers should reduce the volatility to ensure that liquidation does not occur. Similarly, Basak, Pavlova, and Shapiro (2007) and Aragon and Nanda (2012) argue that the convex payoff structure of a manager does not necessarily induce risk shifting when fund undertakes poor performance, as long as she is exposed to downside risk, either through her ownership of fund share or through her annual fees. Aragon and Nanda (2012) empirically analyze the relationship between risk shifting by a hedge fund manager and the manager's incentive contract, personal capital stake, and the risk of fund closure. Other related empirical research include the work of Agarwal, Daniel, and Naik (2009) and Agarwal, Daniel, and Naik (2011). We are unaware, however, of any existing model that both captures the managerial skill (alpha) and the convex compensation in a hedge fund seeding business. Capturing these institutional features in a model that is sufficiently tractable to evaluate the costs of these innovative swaps in hedge fund seeding business is a main contribution of this study.

Chapter 3 is organized as follows: Section 3.2 presents two models of hedge fund seeding innovations and studies the impact of incentive contracts, managerial stake, and hedge fund liquidation on managers' risk shifting behavior; Section 3.3 discusses the quantitative results of our model; and Section 3.4 presents our conclusions. The equilibrium price of contingent claims, such as fees and investors' payoff can be found in the Appendix B.

[7]See, for example, Ross (2004), Panageas and Westerfield (2009) Hodder and Jackwerth (2007) and Lan, Wang, and Yang (2013) among others.

3.2 Modeling hedge fund seeding innovation

Similar to Yang and Zhang (2013)'s paper which focuses on the borrowing constraints of an entrepreneur, Chapter 3 assumes that the financing constraint faced by ESF managers will be largely alleviated by introducing the fees-for-guarantee swap supported by a commercial guarantee company or an insurer.

In this section we first specify the hedge fund dynamics and present a benchmark model with only fees-for-seed swap which is popular in the hedge fund seeding industry. Then we develop a dynamic framework to model the hedge fund seeding innovation with both the fees-for-seed and fees-for-guarantee swaps and explain why the fees-for-guarantee swap is beneficial to both an ESFs manager and a seeder. Last, we turn to the breakeven alpha strategy of the hedge fund seeding business.

3.2.1 Hedge fund dynamics and valuation

Dynamics of the asset under management As usual, we assume the cumulative-return process R of the diversified market portfolio satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \quad t \geq 0, \tag{3.1}$$

where μ_M and $\sigma_M > 0$ are constants. We denote by S the value process of the asset under management (AUM, henceforth) and by H the current high-water mark (HWM, henceforth) which is the highest level that the AUM has reached subject to certain adjustments. We assume for $t \geq 0$ that

$$\frac{dS_t}{S_t} = (\mu - \omega - m)dt + \rho\sigma dB_t + \sqrt{1-\rho^2}\sigma dZ_t, \quad S_0 \text{ given}, \quad S_t < H_t, \tag{3.2}$$

where ω is the regular withdrawal rate, m is the management fee rate, and $\rho \in [-1, 1]$ is the correlation coefficient between the hedge fund and the market. A higher absolute value of the correlation coefficient $|\rho|$ means a less idiosyncratic risk faced by ESFs managers: The values $\rho\sigma$, and $\epsilon \equiv \sqrt{1-\rho^2}\sigma$ are the systematic and idiosyncratic volatility of AUM respectively. All sources of uncertainty arise from two independent standard Brownian Motions

B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$, where $\{\mathcal{F}_t : t \geq 0\}$ describes the flow of information available to investors.

Management compensation contracts ESFs managers who run the fund are paid via both annual and incentive fees. The annual fees are specified as a constant fraction m of the net asset value while the incentive fees are commonly accompanied by the HWM provision. We denote by H the HWM process. If H is higher than the value of AUM, it evolves deterministically according to

$$dH_t = (g - \omega - m')H_t dt, \tag{3.3}$$

where g is the rate of interest or an other contractually stated rate and m' is the cost or fees allocated to its reduction. If the asset value reaches a new high, the HWM is reset to this higher level.

Hedge fund liquidation There are normally two possibilities for the hedge fund to be liquidated. First, following Goetzmann, Ingersoll, and Ross (2003), we assume the fund will be liquidated with an exogenously given probability λ per unit of time. Such liquidation time is donated by τ_1. Second, an endogenous liquidation required by ordinary investors may occur if the fund performance is sufficiently poor. In contrast to Grossman and Zhou (1993) and Lan, Wang, and Yang (2013) among others, who assume the liquidation threshold is bH_t, our liquidation threshold is a less value $l \equiv b(1 - \Phi)H_t$, where Φ is the fraction of AUM owned by the managers. The hitting time of the endogenous liquidation threshold is denoted by τ_2. Thus, unlike the existing literature, our model assumes a lower liquidation boundary, due to the managers' ownership of AUM under the arrangement of the innovative contract. In view of this, the new financing scheme of hedge fund generates more profits and leads to a significant welfare improvement[8].

[8] A number of independent studies have concluded that on average, ESFs outperform more established funds as being either smaller and nimbler, or hungrier for returns. Hedge Fund Research (HFR) found that over the 10-year period from 1994 to 2004, funds with less than a three-year track record outperformed older funds by over 5% annually, with nearly identical volatility. Similarly, a 2009 study by PerTrac Financial Solutions finds that younger and smaller funds have outperformed larger and older funds, over the long term. Other studies, Aiken, Clifford, and Ellis (2013) and Aggarwal and Jorion (2010) made a number of adjustments to raw performance data, to mitigate survivorship or backfill biases. Aggarwal and Jorion (2010) reached the conclusion that managers generate abnormal (excess) performance of 2.3% during their first two years relative to later years.

In short, the fund is liquidated either exogenously at stopping time τ_1 or endogenously at τ_2, depending on which comes first. Thus we define liquidation time $\tau \equiv \min\{\tau_1, \tau_2\}$. Naturally, after liquidation, the managers receive nothing but his share ΦS_τ of AUM.

The pricing of fees and the investor's claim To determine a linear pricing rule we must specify a *martingale pricing operator*, see Ingersoll (2006), or a *state-price deflator*, see e.g. Duffie (2001).

We denote by $\eta \equiv (\mu_M - r)/\sigma_M$ the Sharpe ratio of the market. Following Goetzmann, Ingersoll, and Ross (2003), we define $\alpha \equiv \mu - r - \beta(\mu_M - r) \equiv \mu - r - \rho\sigma\eta$ as the premium return of ESFs, which is considered as an excess return generated by the managers' skills. The risk-adjusted expected growth of the hedge fund under $\mathbb{Q}$ is then given by $\nu \equiv \mu - \rho\sigma\eta = \alpha + r$. Accordingly, the net asset value of the hedge fund follows

$$\frac{dS_t}{S_t} = (\alpha + r - \omega - m)dt + \rho\sigma dB_t^{\mathbb{Q}} + \epsilon dZ_t, S_t < H_t, \tag{3.4}$$

where Z and $B^{\mathbb{Q}}$ defined by $B_t^{\mathbb{Q}} \equiv B_t + \eta t$ are $\mathbb{Q}-$Brownian motion. Therefore, at any current time $t \geq 0$, according to (Equation (3.4), we get the present values of annual fee, the performance fee and the investors' claim respectively as follows:

$$A(S_t, H_t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^\tau e^{-r(s-t)} c S_s ds \right], \tag{3.5}$$

$$P(S_t, H_t) = \mathbb{E}_t^{\mathbb{Q}} \left\{ \int_t^\tau e^{-r(s-t)} k[dH_s - (g - \omega - m')H_s ds] \right\}, \tag{3.6}$$

$$I(S_t, H_t) = \mathbb{E}_t^{\mathbb{Q}} \left[\int_t^\tau e^{-r(s-t)} \omega S ds + e^{-r(\tau - t)} S_\tau \right]. \tag{3.7}$$

Thus, the value of the total management fees is given by

$$F(S_t, H_t) \equiv A(S_t, H_t) + P(S_t, H_t). \tag{3.8}$$

All of them can be explicitly derived and for the computation details, please refer to the Appendix B.

3.2.2 A benchmark model of hedge fund seeding innovation

In order to reach the initial AUM target and cover organizational expenses, ESFs managers usually seek help from seeders by swapping part of their fees with seed capital. Under the arrangement of the fees-for-seed swap, the ESFs manager obtains the amount ΦS_0 of seed capital at the cost of the fraction ψ of her total fees. Thus, the manager's total compensation denoted by $\widetilde{MV}(S_t, H_t)$ is given by

$$\widetilde{MV}(S_t, H_t) = (1-\psi)F(S_t, H_t) + \Phi I(S_t, H_t), \tag{3.9}$$

where the total fees $F(S_t, H_t)$ and the investor payoff $I(S_t, H_t)$ are defined in Equations (B.11) and (B.13) respectively.

In view of the scarcity of seed capital, a seeder often has more bargaining power when signing a swap contract. Thus in our model we assume that the fraction ψ is determined by

$$\psi A(S_0, S_0) = \Phi S_0, \tag{3.10}$$

where the annual fee $A(S_0, H_0)$ is defined in Equation (B.14). In other words, only harvesting the fraction ψ of the regular annual fees is enough for redeeming seed capital of the seeder, and the extra return $\psi P(S_0, S_0)$ allocated to the seeder is due to her strong bargaining power. Either the seeder may recognize this as a positive signal of gifted ESFs managers. Thus, the fraction ψ, referred to as seed cost in the following text, is given by

$$\psi = \frac{\Phi(\omega+m+\lambda-\alpha)\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}}{m\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}-(1+k)m(\theta_2-\theta_1)l^{1-\theta_1}+ckl^{\theta_2-\theta_1}-ck}, \tag{3.11}$$

where θ_1 and θ_2 are respectively the smaller and larger roots of a characteristic quadratic equation defined in the Appendix B.

The seed cost in practice is determined via the thumb rules; however, it is endogenously derived in our model. Interestingly, the relationship between fees-for-seed ratio and seed capital is linear when there is no exogenous liquidation risk ($l = 0$)[9], which is quite similar to the thumb rules. Clearly, the realistic value $\widetilde{SV}(S_t, H_t)$ of the seeder's claim is given by

$$\widetilde{SV}(S_t, H_t) = \psi F(S_t, H_t). \tag{3.12}$$

[9] One can obtain the results without effort by substituting $l = 0$ into Equation (3.11).

Finally, the total present value of hedge fund $\widetilde{HFV}(S_t, H_t)$ is the sum of the values of the managerial compensation and the seeder's and ordinary investors' claim, i.e.

$$\begin{aligned}\widetilde{HFV}(S_t, H_t) &\equiv \widetilde{MV}(S_t, H_t) + \widetilde{SV}(S_t, H_t) + (1-\Phi)I(S_t, H_t) \\ &= F(S_t, H_t) + I(S_t, H_t),\end{aligned} \tag{3.13}$$

which is similar to Goetzmann, Ingersoll, and Ross (2003) but has a major distinction in asset structure.

3.2.3 Hedge fund seeding innovation with both fees-for-seed swap and fees-for-guarantee swap

To attract sufficient capital a fees-for-guarantee swap is further introduced in our model. In this swap, a seeder provides the prescribed amount of seed capital (i.e. ΦS_0) to the manager in exchange for the fraction ψ of the manager's total fees. Once the hedge fund liquidates, an insurer, instead of the manager, must pay the amount ΦS_0 of cash to the seeder. As a return for the guarantee, the insurer gets a fraction, denoted by φ, of the manager's fees[10].

Evidently, thanks to the guarantee scheme, the seeder is exposed to less risk and they have therefore a stronger incentive to invest in the fund. As a matter of fact, many fund managers do so to attract investors. Under the guarantee swap, the value $MV(S_t, H_t)$ of the manager' compensation is changed as

$$MV(S_t, H_t) = (1 - \psi - \varphi)F(S_t, H_t) + \Phi I(S_t, H_t), \tag{3.14}$$

since the manager must pay $\varphi F(S_t, H_t)$ more to an insurer. Naturally, the value $GV(S_t, H_t)$ of the insurer's claim is

$$GV(S_t, H_t) = \varphi F(S_t, H_t). \tag{3.15}$$

Generally speaking, an insurer will sign a great many contracts of the swaps with many hedge funds and accordingly, to an insurer, the idiosyncratic risk of an hedge fund will be well-diversified. For this reason, we assume that the present value of insurer's compensatory payment V_{guar} to a seeder is equal to the present value of the fraction of fees allocated

[10] Unlike the traditional credit hypothecation, however, the hedge fund manager in the new credit guarantee scheme must pay to the guarantee company a portion (φ) of her fees(including both management fees and incentive fees) as guarantee costs instead of the regular guarantee fees.

to the insurer which is $GV(S_0, S_0)$, i.e.

$$V_{guar} \equiv GV(S_0, S_0). \tag{3.16}$$

We assume that the seeder will redeem her seed capital once the fund is liquidated. This assumption is straightforward as the seeder demands that her downside risk is well controlled at the level of her initial investment if the fund is liquidated. At liquidation the seeder will get nothing from the manager but redeem ΦS_0 compensatory payment from the insurer. Therefore, the value V_{guar} should satisfy

$$V_{guar} \equiv p_b(b, s_0)\Phi S_0, \tag{3.17}$$

where $p_b(b, s_0)$, the market value of a security that claims one unit of of account at the hitting time $\tau(b) = \inf\{t \geq 0 : s_t \leq b\}$, is given by[11]

$$p_b(b, s_0) = \left(\frac{s_0}{b}\right)^{\theta_1}. \tag{3.19}$$

Now we denote φ as the guarantee costs. We can easily compute guarantee costs by submitting Equations (3.16) and (3.17) into Equation (3.15):

$$\varphi = \frac{p_b(b, s_0)\Phi S_0}{F(S_0, S_0)}. \tag{3.20}$$

Then we define the value of seeder's return $SV(S_t, H_t)$ by:

$$SV(S_t, H_t) = \psi F(S_t, H_t) + p_b(b, s_0)\Phi S_0. \tag{3.21}$$

Equation (3.21) states that the seeder's payoff in the guarantee scheme is greater than the baseline model by the amount of $p_b(b, s)\Phi S_0$. The additional part is due to the bargaining power of the seeder especially for the case of start-up hedge funds. In the guarantee scheme,

[11] One can find the computational details in Duffie (2001). Similar to Wang, Wang, and Yang (2012) and Song, Yang, and Yang (2013) among others, we denote s as AUM/HWM ratio, i.e. $s = S/H$, an effective variable in our model that is given by

$$\frac{ds_t}{s_t} = (\alpha + r - \omega - m)dt + \rho\sigma dB_t + \epsilon dZ_t, s_t < 1. \tag{3.18}$$

the seeder will get her original investment back at fund liquidation instead of nothing in the baseline model. This means the seeder controls her downside risk well at fund liquidation in the guarantee scheme. Moreover, when the fund has good performance, she generates remarkable benefits by sharing the"enhanced economics", i.e. a portion of incentive fees. In our innovative model with guarantee arrangements, the seeder does not have to worry about the risk shifting behavior of the ESFs manager because the fees-for-seed swap contract aligns the manager's interest with hers completely.

Finally, the total present value of hedge fund $HFV(S_t, H_t)$ is given by the sum of managerial compensation, the seeder's payoff and ordinary investors' value:

$$\begin{aligned} HFV(S_t, H_t) &\equiv MV(S_t, H_t) + SV(S_t, H_t) + (1 - \Phi)I(S_t, H_t) \\ &= F(S_t, H_t) + I(S_t, H_t), \end{aligned} \tag{3.22}$$

which is the same as in Equation (3.13). This is true because the insurer does in fact nothing but forces the manager to put a part of his earnings aside for refunding the initial investment of the seeder.

3.2.4 Breakeven alpha strategy

A important question for both the ESFs manager and ordinary investors is what is the minimum α should be generated by the manager who receives a given portion (k) of incentive fees in order to make ordinary investors break even, i.e $I(S, S) = S$, when the contracts commences as in (Goetzmann, Ingersoll, and Ross 2003). This question is the same as how large a performance fee should be charged for a given level of performance. Using Equation (B.13) to solve $I(S, S) = S$ for k gives the maximum high water performance fee justified by a particular α:

$$k(\alpha) = \frac{1 - \theta_2 - (\theta_1 - \theta_2)l^{1-\theta_1} + (\theta_1 - 1)l^{\theta_2 - \theta_1}}{\theta_2 - \theta_1 l^{\theta_2 - \theta_1} - \theta_2 - \theta_1 l^{1-\theta_1} - (\omega + \lambda)(1 - l^{\theta_2 - \theta_1})/(\alpha - m)}. \tag{3.23}$$

Other than the breakeven alpha strategy case, it is not uncommon that investors either make losses or collect surplus depending on whether they hire a more skilled manager than the breakeven case or not. Therefore, it is quite essential for investors to access managers' skill correctly.

3.3 Quantitative Results

In this section we numerically analyze the impact of HWM, managerial stake, and the possibility of fund liquidation on ESFs managers' risk shifting behavior under the arrangements of the innovative contracts, i.e. the fees-for-guarantee swap and the fees-for-seed swap. In order to make a comparison, following Goetzmann, Ingersoll, and Ross (2003), Psillaki and Daskalakis (2009) and Lan, Wang, and Yang (2013) among others, our numerical examples are based on the following annualized baseline parameter values: typical parameter values $r + m' - g = 5$ percent, $k = 20$ percent, $m = 1.5$ percent, $\omega + \lambda = 10$ percent, fund volatility $\sigma = 20$ percent, alpha skills $\alpha = 5$ percent, and liquidation barrier $l = 0.5$, managerial stake $\Phi = 10$ percent thus the effective liquidation barrier $\tilde{b} \equiv l(1 - \Phi) = 0.45$.

3.3.1 The seeder's payoff and contract designing of fees-for-seed ratio

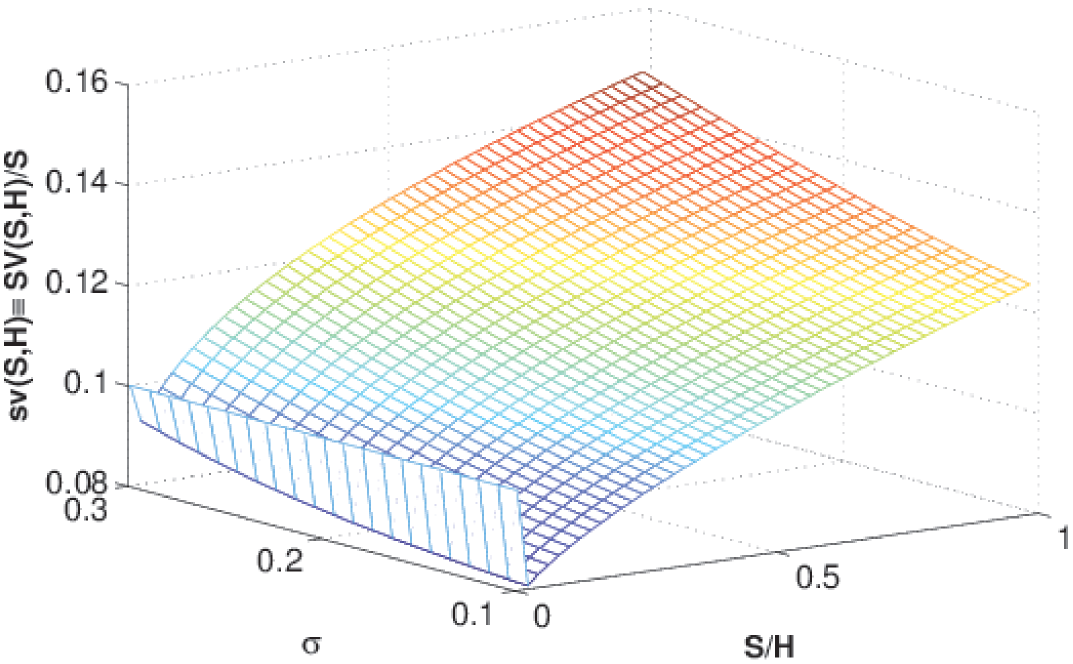

FIGURE 3.1: The impact of fund performance S/H, fund volatility σ on the value of the seeder's claim. In this case the liquidation barrier $l = 0$.

The seeder's return profile Under the arrangements of fees-for-guarantee swap and fees-for-seed swap contracts, a seeder's return potential is greater than that of ordinary investors by sharing the "enhanced economics" and her downside risk is transferred to an insurer.

As discussed in Section 3.2, the seeder's return profile is guaranteed at the level (equaling to her initial investment) once fund liquidation occurs. These innovative swaps effectively align the interests of the seeder with those of the ESFs manager. In our model the seeder's payoff is similar to a hedge fund manager except for the case of fund liquidation.

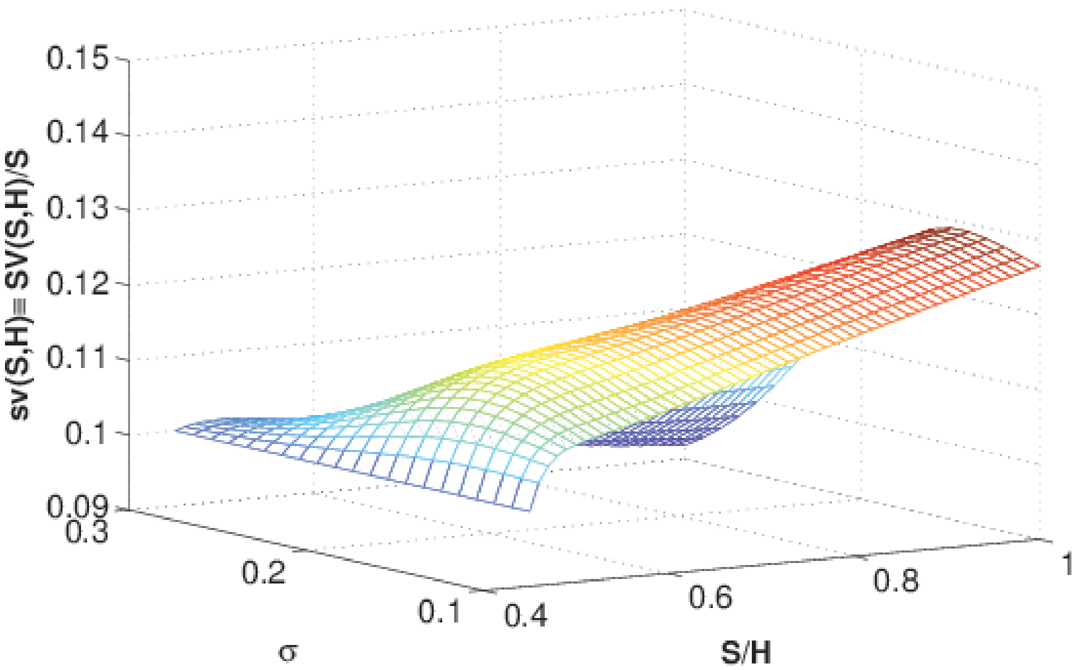

FIGURE 3.2: The impact of fund performance S/H, fund volatility σ on the value of the ESFs manager's total compensation. In this case the liquidation barrier $l = 0.5$.

As indicated in Figure 3.1 [12], the seeder's payoff is an increasing function of volatility and fund performance. By contrast, Figure 3.2 shows a completely different type of value function. Specifically, the seeder's payoff gets worse for even a better performance of the hedge fund with significantly high risk. At first glimpse it seems beyond belief, however, the reason is quite simple that extremely high level of risk outweighs the benefit from the fund's performance. Therefore, the seeder prefers a liquidation to redeem her investment than struggling in mud. Moreover, the seeder's value near the HWM has a slightly rise to reaches its maximum (top at almost 13%) and then follows a dramatic fall. The intuition is straightforward because the risk of endogenous liquidation moderates the seeder's risk attitude as if she acts like a risk averse agent, which is in accordance with the results of

[12] The 3-D figures here in this chapter and in Chapter 4 are ploted by Matlab 2016a software and as the color changes from light to dark (i.e. form green to yellow, then to red) the values increases.

Panageas and Westerfield (2009). Moreover, both Figure 3.1 and 3.2 show the seeder obtains great potential profits if the ESFs has good performances (15% and 13% at $S/H = 1$ for $l = 0$ and 0.5 respectively) under an appropriate level of risk and she still collects her initial investment at liquidation time, thus her downside risk is well controlled.

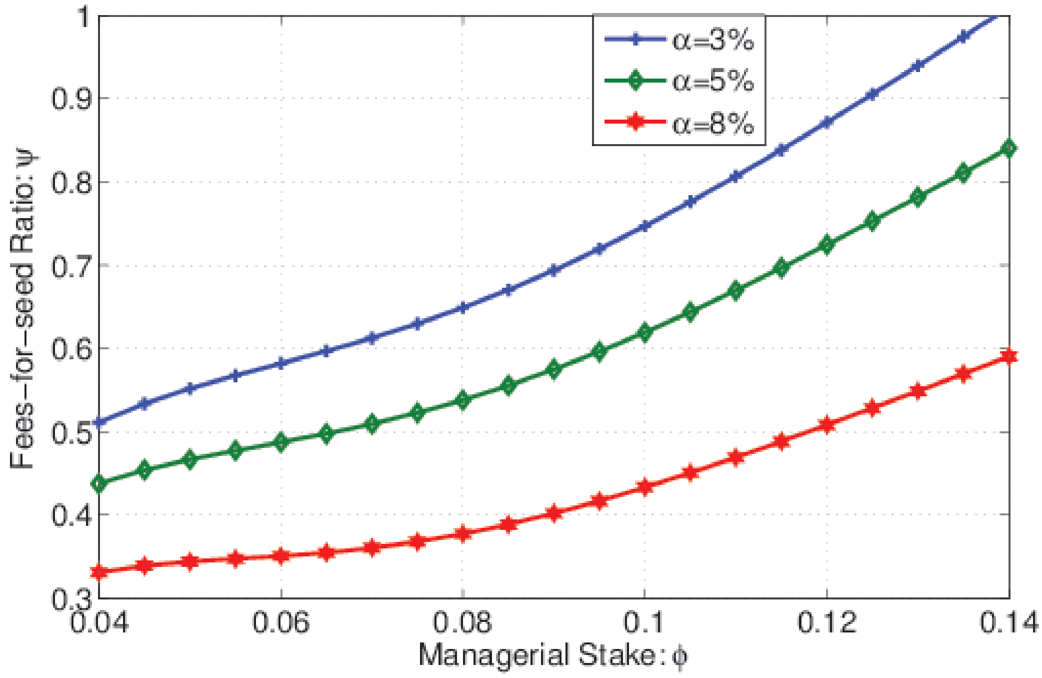

FIGURE 3.3: The impact of managerial stake Φ on the initial designing of fees-for-seed swap ratio ψ with two different liquidation barriers. In this case, we choose a higher volatility ($\sigma = 30\%$) in order to highlight the liquidation effect.

Contract designing: fees-for-seed ratio Unlike traditional swaps whose value should be zero at the commencement of the swap for both the seeder and the manager, the seeder in our innovative model obtains additional ψ fraction of performance fees because she has greater bargaining power than ESFs managers, who desperate for money. As Goetzmann, Ingersoll, and Ross (2003) states,"In particular, if investment capital is a scarce resource relative to potential hedge fund managers, virtually all benefits of the hedge funds may go to the investors". Our numerical results back up the above statements: a manager with lower alpha skills ($\alpha = 3\%$) have to give up all her performance fees to a seeder to swap the seed capital equaling to 14% ownership of the fund. However, to attract the same amount of seed capital, a much more skilled manager ($\alpha = 8\%$) gives up only 60% of her performance fees, as indicated in Figure 3.3. Both Figure 3.3 and 3.4 indicate that a manager has to give

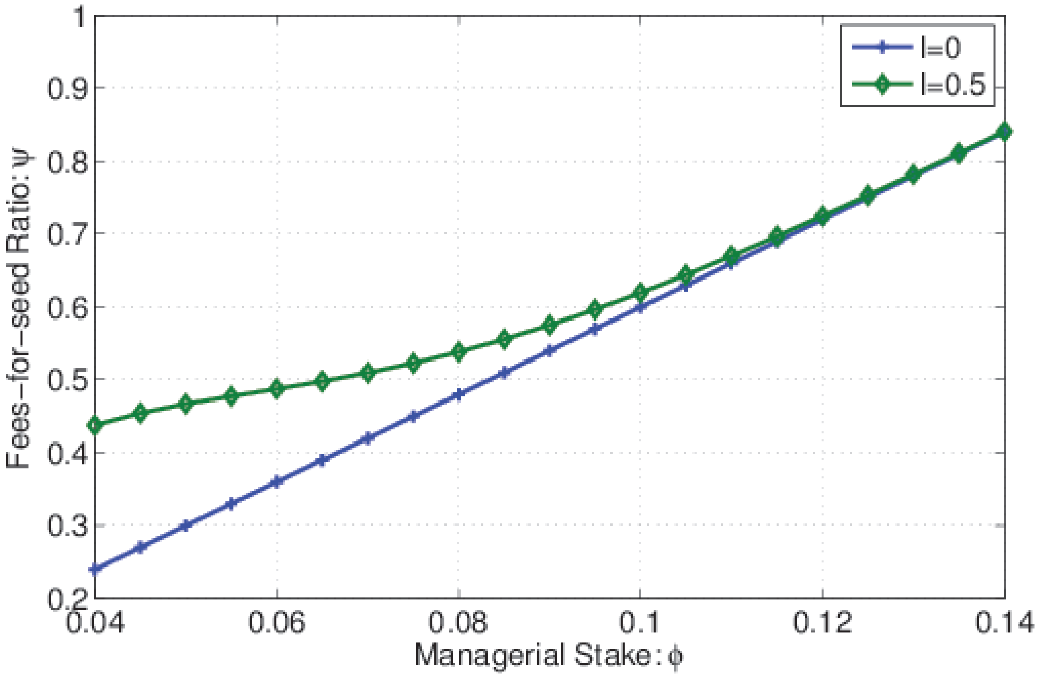

FIGURE 3.4: The impact of managerial stake Φ on the initial designing of fees-for-seed swap ratio ψ with three different levels of manager skills. Similar to Fig.3, fund volatilityσ is 30%.

up a larger fraction of her performance fees to exchange a greater amount of seed capital, as we expect. Interestingly, most of our numerical results from Figure 3.3 and 3.4 reveal the nonlinear relationship between the fees-for-seed ration and managerial stake except the case when ESFs managers face no endogenous liquidation risks. The rationale is that managerial stake has nonlinear effects of ψ in our model through affecting the liquidation barrier and these effects disappear once the liquidation b is zero.

3.3.2 The insurer's claim, i.e. guarantee costs φ

Generally speaking, an insurer will sign a great many contracts for the swaps with many ESFs managers and accordingly, to an insurer, the idiosyncratic risk of a hedge fund will be well-diversified. Figure 3.5 shows that as the increase of the fund risk, the guarantee costs increases, as we expect. These observations accord with economic intuition since a manager with better skills generates more fees and profits. For the case $\alpha = 5\%$, as the fund risk increases from 10% to 30% the guarantee costs rises up slowly form naught to 5%. Interestingly, the guarantee costs experiences a remarkable rise from 4% to 26% in the case

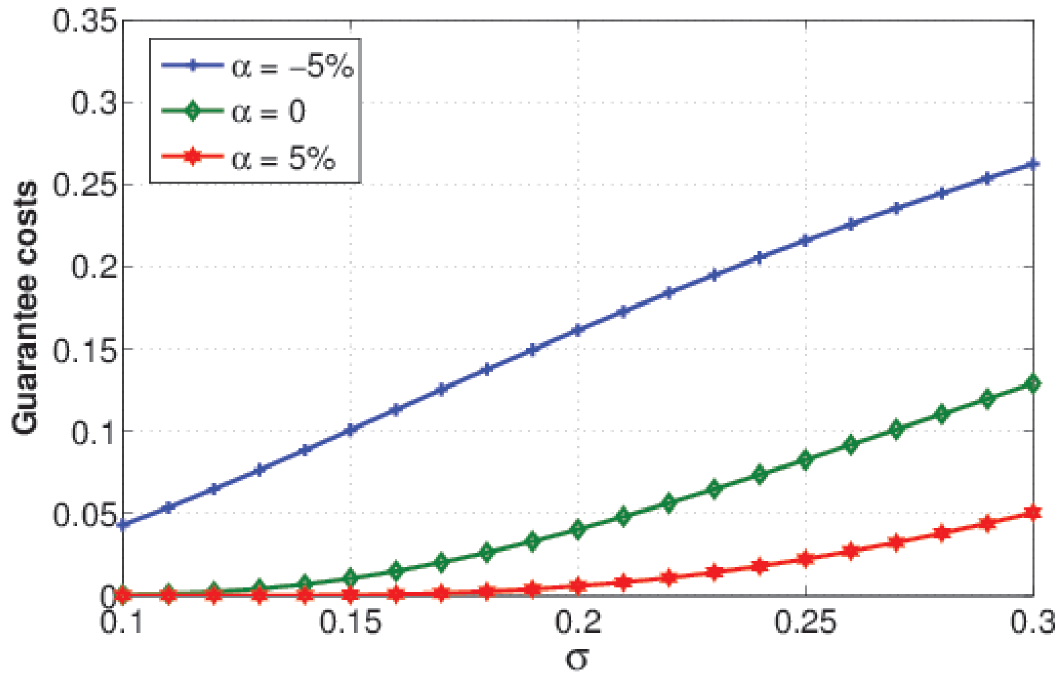

FIGURE 3.5: The impact of fund risk σ on the value of guarantee costs φ with three different values of alpha skills.

$\alpha = -5\%$, as the fund volatility rises from 10% to 30%.

In addition, we present illusive results on how fund liquidation barrier influences the guarantee costs. As shown in Figure 3.6, the guarantee costs of a higher liquidation barrier ($l = 0.8\%$) is almost ten times bigger than that of fund with a lower barrier ($l = 0.5\%$) for the case that the fund volatility is high at 30%. The rationale is that a higher liquidation barrier gives a higher probability of fund liquidation, thus the increase of the probability of an insurer's compensatory payment by who thus a greater guarantee costs is demanded. However, as fund risk decreases the cost gap is not noticeably because in the case of lower fund risk the probability of fund liquidation is very small even for a higher liquidation barrier. Interestingly, if the liquidation barrier is naught, then there is no need for an insurer in our scheme as the hedge fund will never be liquidated which is not the case in reality.

3.3.3 Risk shifting behavior of ESFs managers at the stage hedge fund seeding

The effect of liquidation barrier Similar to Carpenter (2000), it is optimal for ESFs managers who is compensated through an asymmetric bonus fee and faces no explicit downside risks to take extreme risks when she is further away from the money.

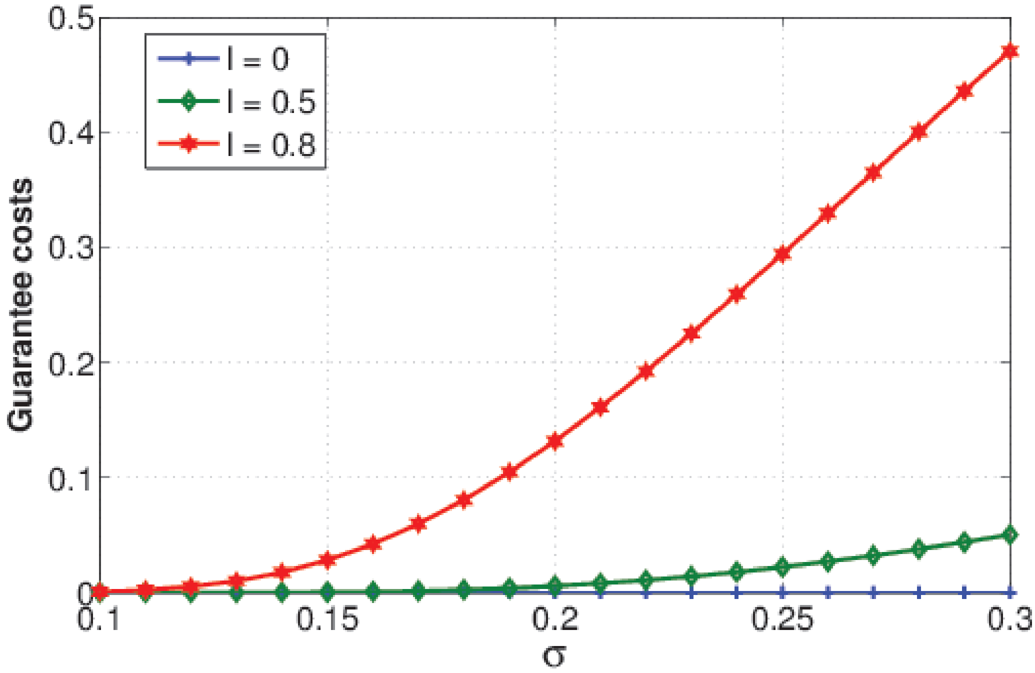

FIGURE 3.6: The impact of fund risk σ on the value of guarantee costs φ with three different values of liquidation barrier.

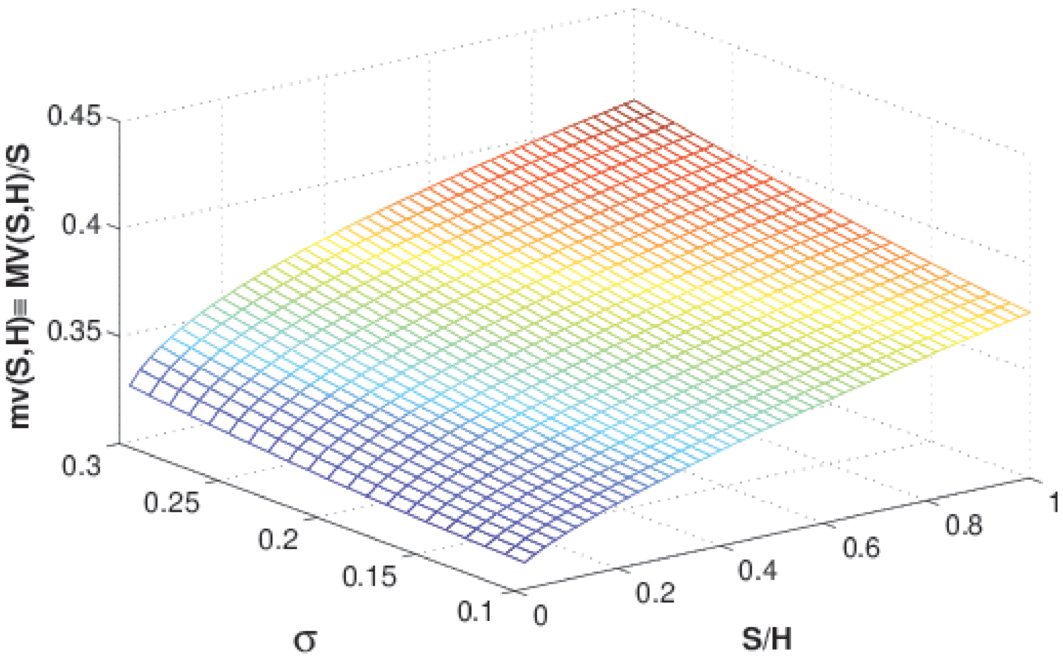

FIGURE 3.7: The impact of fund performance S/H, fund volatility σ on the value of the ESFs manager's total compensation when the manager faces no downside risk, i.e. liquidation barrier $l = 0$.

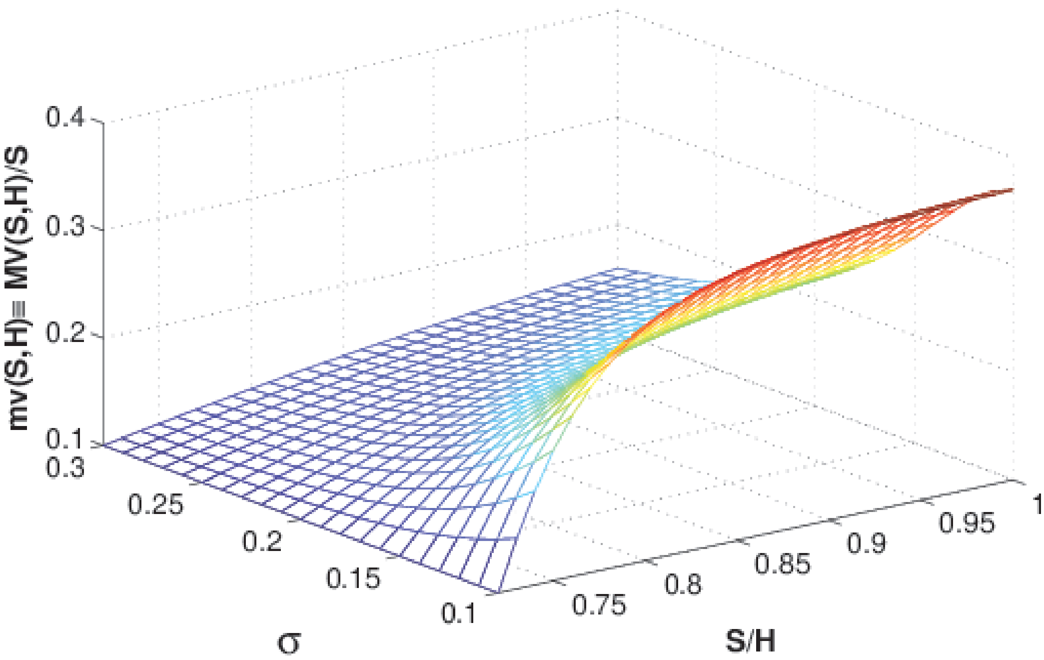

FIGURE 3.8: The impact of fund performance S/H, fund volatility σ on the value of the ESFs manager's total compensation. In this case the liquidation barrier $l = 0.8$.

The risk taking behavior in our model is illustrated in Figure 3.7: the total compensation value of the ESFs manager is always an increasing function of fund volatility, whether the fund is likely to liquidate or not, which means the ESFs manager has great incentive to shift risk. By contrast, risk shifting behavior is completely cubed when the ESFs manager faces explicit downside risk, as is shown in Figure 3.8. This result is generally consistent with Hodder and Jackwerth (2007), Basak, Pavlova, and Shapiro (2007) and Aragon and Nanda (2012). When the fund is likely to liquidate and as the fund volatility increases the performance fees value rises while the manager's annual fees and her managerial stake value decrease. As the loss effect of annual fees and ownership value outweigh the risk shifting benefits from the HWM, thus risk shifting behavior leading to a lower value of the total compensation.

Our model suggests that optimal contract designing not only moderate risk shifting following poor performance, but also give managers incentive to chasing profits. As is illustrated in Figure 3.9, the manager will choose low volatility in order to dramatically reduce the chance of hitting the liquidation boundary. However, the manager's optimal strategy is to increase risk slightly when the AUM is near its HWM. The intuition is quite simple: the

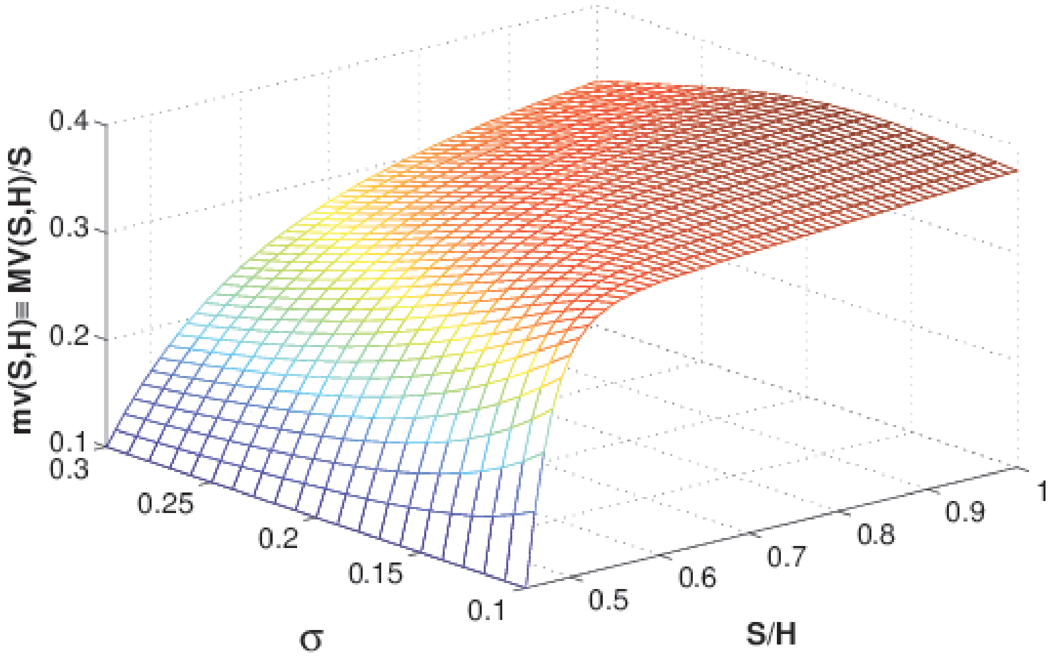

FIGURE 3.9: The impact of fund performance S/H, fund volatility σ on the value of the ESFs manager's total compensation. In this case the manager's ownership is $\Phi = 10\%$.

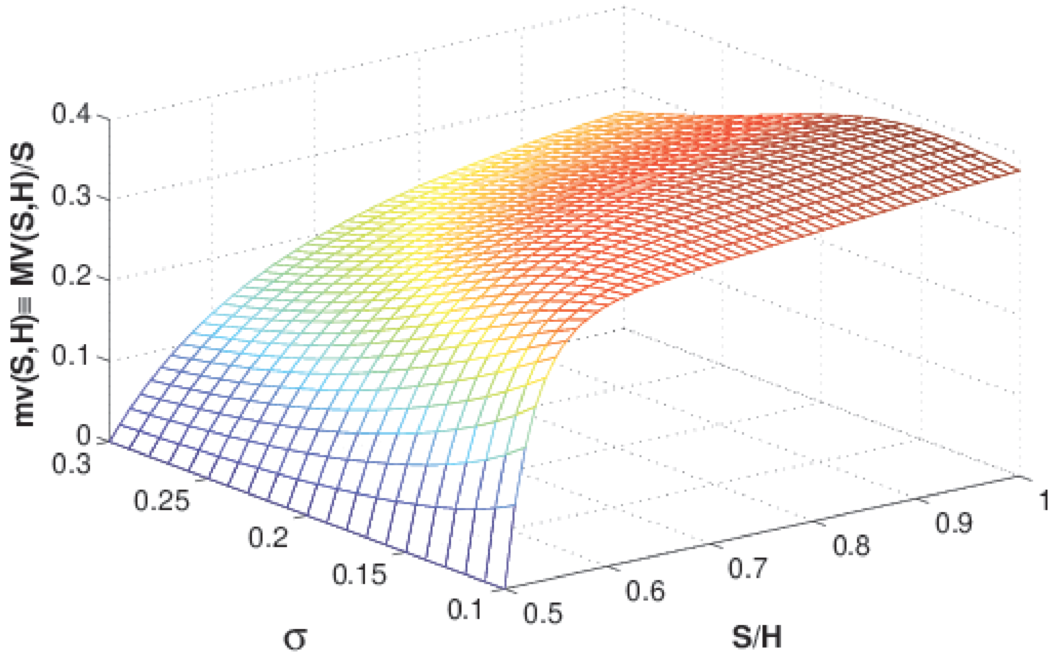

FIGURE 3.10: The impact of fund performance S/H, fund volatility σ on the value of the ESFs manager's total compensation. In this case the manager's ownership is $\Phi = 0\%$.

manager will take gambles to collect the huge profits once AUM reaches the HWM. However, the ESFs manager who faces liquidation constraints does not take extreme risks in our model when AUM is near its HWM because performance fees, as shown in Fig 3.2.

The effect of the managerial stake Now we turn to analyze the impact of managerial stake on a manager's incentive to shift risk. Generally speaking, an ESFs manager will not take great gambles following poor fund performance when having their own investment in the fund (See e.g. Carpenter (2000), Basak, Pavlova, and Shapiro (2007), Hodder and Jackwerth (2007) and Aragon and Nanda (2012) among others.). Our numerical results show that the managerial stake is effective in moderating the ESFs manager's incentive to shift risk when the fund is likely to liquidate endogenously.

As Figure 3.10 shows, the manager who has no ownership of fund share has a small increase of 8% in total compensation value, as she lowers the fund volatility from 30 percent to 10 percent when AUM is near the liquidation barrier ($x \to 0.5$). On the contrary, the total compensation value experience a bigger increase of nearly 14%, and in this case the liquidation boundary is 10% lower than that of the case without a managerial stake. The illusive results show that the manager who has her own investment in will largely improve her welfare when AUM near the initial liquidation barrier ($l = 0.5$) by decreasing the fund volatility dramatically, thus mitigating the manager's risk shifting behavior.

3.3.4 The breakeven α strategy

Finally, we show illustrative results on how α is important both for managers and investors. As shown in Figure 3.11 and 3.12, the ESFs manager should have much better managerial skills to justify a higher performance fee[13]. More specifically, Figure 11 indicates that the minimum α required for three different volatility ($\sigma = 10\%$, 20% and 30%) to justify a performance fee rate (e.g. the most usual case i.e. $k = 20\%$) is 270 and 330 and 450 basis points respectively. Now we turn to analyze the effect of the exogenous liquidation barrier on the justified performance fee rate. For a given performance fee rate, the required excess return is 390 and 450 basis points for two different liquidation barrier ($l = 0, 0.5$). The intuition is simple, a lower liquidation barrier contributes to higher values for investors, thus a smaller

[13]In this case, we choose a higher volatility ($\sigma = 30\%$) in order to highlight the liquidation effect.

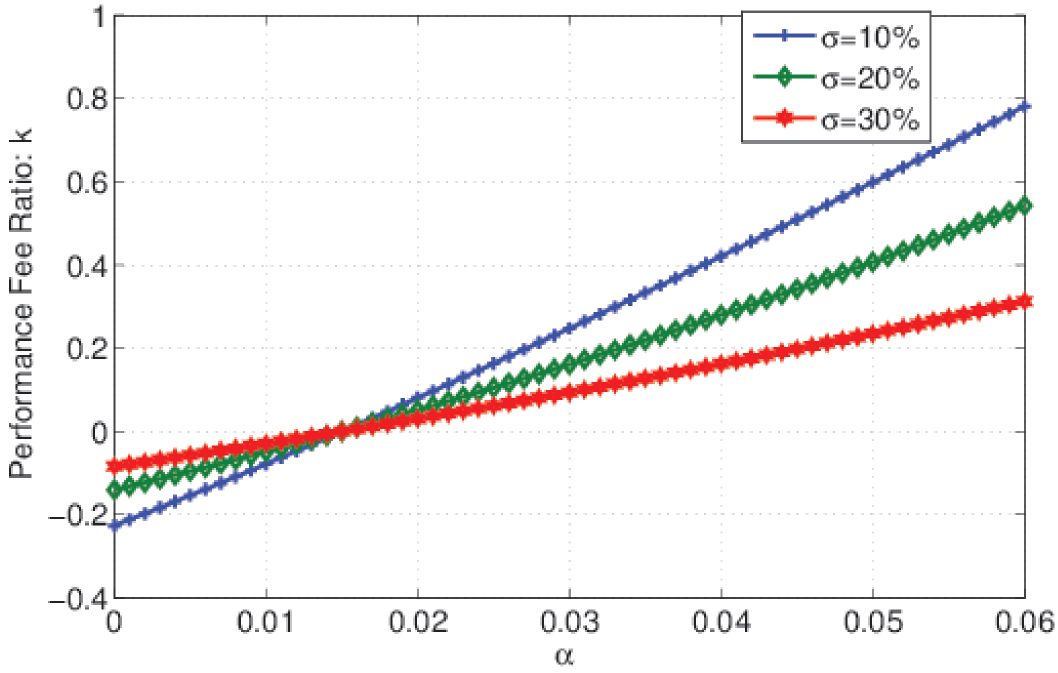

FIGURE 3.11: The maximum performance fee k rate for different managerial skills α with three levels of fund volatility.

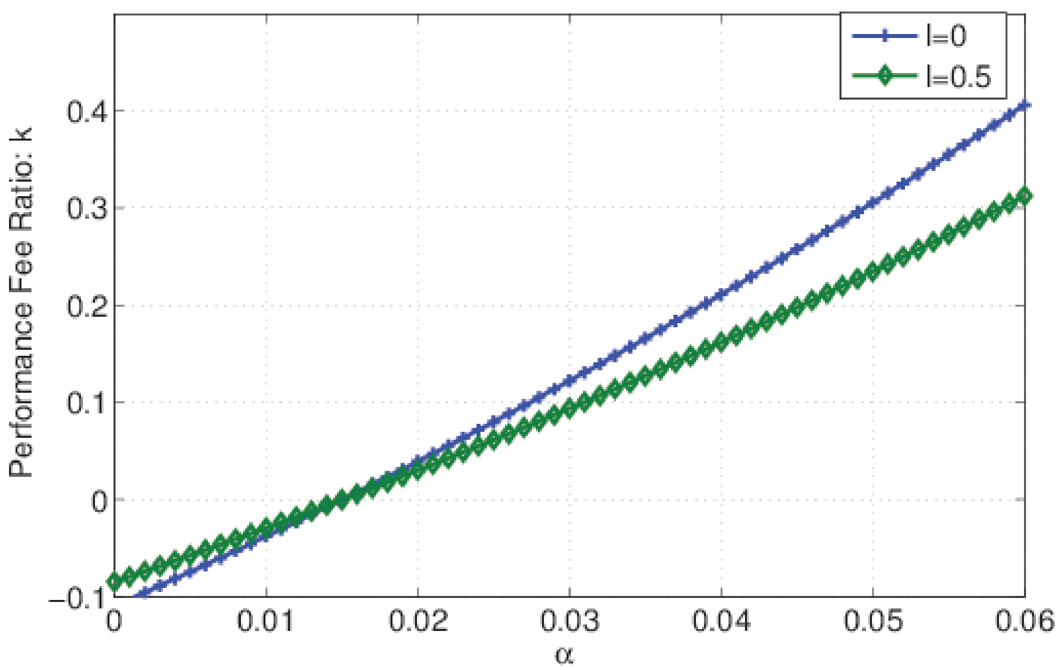

FIGURE 3.12: The sensitivity of the maximum performance fee k rate with respect to correlation coefficient ρ at two different levels of liquidation barrier.

alpha is required. Interestingly, our numerical results reveal that charging a positive performance fee is never justified for an ESFs manager who generates a α less than 150 basis points regardless of different values of fund volatility and liquidation barriers.

3.4 Conclusion

Chapter 3 deals with the pricing of seed costs and the risk shifting behavior of an ESFs manager who signs new three party agreements, here called the fees-for-guarantee swap and the fees-for-seed swap, with a seeder and an insurer. The main conclusions are detailed explicitly while numerical examples are provided.

We find that the seeder in the case with both the fees-for-guarantee swap and fees-for-seed swap faces less uncertainty, due to the guarantee than the case only with fees-for-seed swap in practice. This intuition is simple as the seeder still gets her initial investment from the insurer at the termination of the hedge fund. Thanks to fees-for-guarantee swap and fees-for-seed swap, the seeder not only controls her downside risk well, but also has remarkable potential profits by sharing the "enhanced economics" if ESFs has a good performance. Under the arrangements of fees-for-guarantee swap and fees-for-seed swap contracts, a seeder's return potential is greater than that of ordinary investors by sharing the "enhanced economics" and her downside risk is transferred to the insurer. In our model, the seeder's return profile is guaranteed at level (equaling to her initial investment) even when fund liquidation occurs. These innovative swaps effectively align the interests of the seeder with those of the ESFs manager. Interestingly, the seeder's payoff in our model is similar to a hedge fund manager who only charges the incentive fees to manage the fund. Unlike traditional swaps whose value should be zero at the commencement of the swap for both the seeder and the manager, the seeder in our innovative model obtains additional ψ fraction of performance fees because she has greater bargaining power than that of ESFs managers who desperate for money.

Closed-form solution of fees-for-seed ratio is derived in our model, which follows detailed numerical examples. The greater the seed capital obtained, the more fees the manager should give up, as we expect. Surprisingly, our quantitative results reveal the nonlinear relationship between the fees-for-seed ration and managerial stake, which is different from the "thumb rule" in practice except the case when ESFs managers face no endogenous liquidation risks. In addition, when a hedge fund whose AUM near its endogenous barrier, the guarantee costs rises as the fund volatility increases since a higher volatility increases the probability of compensatory payment of an insurer by who, thus a greater guarantee costs is demanded.

Interestingly, our numerical results reveal that charging a positive performance fee is never justified for an ESFs manager who generates an α less than 150 basis points regardless of different values of fund volatility and liquidation barriers. Other than the breakeven alpha strategy case, it is not uncommon that investors either make losses or collect surplus depending on whether they hire a more skilled manager than the breakeven case or not. Therefore, it is quite essential for investors to access managers' skill correctly.

At last, our model suggests that optimal contract designing, i.e. the determination of swap ratios e.g. ψ and φ not only moderate risk shifting following poor performance, but also give managers incentive to chasing profits. More specifically, managerial stake is effective in moderating the ESFs manager's incentive to shift risk when the fund is likely to liquidate endogenously.

Chapter 4

Hedge Fund Seeding via Fees-for-Seed Swaps under Idiosyncratic Risk

Coauthored with Christian Ewald [1]

Abstract

We develop a dynamic valuation model of the hedge fund seeding business by solving the consumption and portfolio-choice problem for a risk-averse manager who launches a hedge fund through a seeding vehicle. This vehicle, i.e. fees-for-seed swap, specifies that a strategic partner (seeder) provides a critical amount of capital in exchange for participation in the funds revenue. Our results indicate that the new swap not only solves the serious problem of widespread financing constraints for new and early-stage funds (ESFs) managers, but can be highly beneficial to both the manager and the seeder if structured properly.

[1]Professor Christian Ewald is head of Economics at Adam Smith Business School, Univeristy of Glasgow

4.1 Introduction

There has been a significant increase in both the number of hedge fund seeders and the amount of capital available for hedge fund seeding since the aftermath of 2008's market upheaval.[2] However, there still remains a tremendous shortage of capital for new and early-stage funds (ESFs). This is mainly because most capital providers or institutional investors increasingly focus on larger established hedge funds whose assets under management (AUM) are usually larger than 1 billion and who are considered highly credible. Additionally a larger talent pool of ESFs managers is now competing for the scarce available seed capital. Worse still, barriers to entry for ESFs are much higher today than in the period before the 2008 financial crisis.[3]

Therefore, navigating the terrain to a successful launch of a hedge fund has become more difficult and the financing constraint faced by ESFs managers nowadays is much more serious than before. In order to reach the initial AUM target and cover organizational expenses, more and more ESFs managers are likely to turn to seed investors for early stage of capital through a seeding vehicle. This is an arrangement to which we refer as fees-for-seed swap that specifies that a seed investor (or seeder) commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced economics" which is usually the fees that the ESFs manager generates from the entire pool of assets in the fund. If structured properly, the seeding approach can be highly beneficial to the ESFs manager and to investors who provide the seed capital. It is not uncommon that the hedge fund seeder receives a portion of the hedge fund's revenue stream to get greater return potential than an ordinary investor.

In general a seeder can expect about 1% of revenues for each \$1 million of seed capital for seed transactions no larger than \$50 million. However, seed arrangements can vary substantially based on factors such as the experience of the manager, the alpha record, the amount of seed capital provided, the withdrawal and lock-up period terms, and the relative

[2]HFM-Week research reported in November 2011 that seeders had approximately \$4.6 billion in available capital, compared to approximately \$1 billion just one year earlier.

[3]The Dodd-Frank Wall Street Reform and Consumer Protection Act (Title IV) compels the U.S. Securities and Exchange Commission (SEC) to impose reporting requirements on all hedge funds as it deems necessary or appropriate in the public interest or for the assessment of systemic risk. According to "Launch bad; Hedge funds" (The Economist 20 Apr. 2013: 79), it is much harder now to break into the hedge-fund business than it used to be because of the rising expenses, more risk-averse investors and enhanced regulation.

negotiating power of each party.[4]

While the seed investor will often demand the flexibility to redeem her[5] investment as soon as possible, the manager needs (and should require) the seed capital to remain invested for a period sufficient to set its strategy, create a track record, and procure other investors. Generally, during the lock-up period, the seed investor should be prohibited from redeeming the investment if, in the reasonable judgment of the manager, doing so could adversely affect the interests of the other investors in the fund.[6]

The ordinary investors may withdraw capital if the fund shows poor performance. For simplicity , we assume that the withdrawal rate is constant. This is a common assumption in the hedge fund literature and has been employed by Goetzmann, Ingersoll, and Ross (2003) and Lan, Wang, and Yang (2013). Also, depending on the terms of the deal, the seeder will generally commit to keep the investment in the fund for a defined lock-up period, typically two to four years. It makes sense to assume that during this initial phase, idiosyncratic risks take a more pronounced role as compared to later stages in the fund's life, possibly due to ordinary investors entering (or leaving) the fund and/or the fund manager experimenting with different asset classes in order to set up a successful strategy. As seed commitments expire, AUM will be divided among the ordinary investors, the seeder, and the ESFs manager according to a "waterfall" schedule.[7] After the initial seeding stage, the fund becomes more stable and in our idealized setup we assume that the ESFs manager no longer bears any idiosyncratic risk after the lock-up period has been completed. Therefore, we can apply Goetzmann-Ingersoll-Ross' (Goetzmann, Ingersoll, and Ross (2003)) model to calculate the market value of the fund at termination of the lock-up period. While the manager's performance incentives during the lock-up period are implemented through a waterfall schedule, performance incentives after the lock-up period are provided by a high-water-mark (HWM) incentive, compare Goetzmann, Ingersoll, and Ross (2003).

As there is no publicly available data on the historical performance of seeding strategies,

[4] Source form Larch Lane Advisors: Hedge Fund Seeding: A Compelling Alternative.

[5] Rather than using the gender neutral "she/he" and "her/his" we have chosen for simplicity to simply use the pronouns "she" and "her" throughout the manuscript.

[6] Infrequently, a seeder may identify certain conditions under which it will be permitted to redeem investment, prior to the lock-up period, such as in the event of poor fund performance, management attrition, sale or transfer of control of the management entity, criminal convictions or regulatory violations and material breaches of the investment parameters or provisions of the seed investment agreement.

[7] See details in the next Section.

there are only very few simple models in practice focusing on hedge fund seeding return, volatility and liquidity profile.[8] To our knowledge, Chapter 4 provides the first dynamic framework on valuation of the hedge fund seeding business by solving the portfolio-choice problem for a risk-averse manager.

Several other studies evaluate the performance of hedge funds focusing on different aspects. Goetzmann, Ingersoll, and Ross (2003) provide the first quantitative inter-temporal valuation framework of investors' payoff and managers' fees in a setting where the fund's value follows a log-normal process and the fund managers have no discretion over the choice of portfolio. Carpenter (2000) shows that it is optimal for hedge fund managers who face no explicit downside risk to choose infinite volatility as asset value goes to zero. This behavior is referred to as risk-shifting. Basak, Pavlova, and Shapiro (2007), Hodder and Jackwerth (2007) and Aragon and Nanda (2012) argue though that a manager's convex payoff structure does not necessarily induce risk shifting when the fund shows poor performance as long as the manager is exposed to downside risk, either through her ownership of fund share or through her annual fees. Panageas and Westerfield (2009), and Lan, Wang, and Yang (2013) analyze the impact of management fees and high-water mark based incentive fee on leverage and valuation. None of these studies, however, model the hedge fund seeding innovation in the context of the ESFs manager's portfolio choice problem, and hence they do not assess the costs of illiquidity and unspanned risk of hedge fund seeding investments.

Our article also relates to the literature about valuation and portfolio choice with illiquid assets, such as restricted stocks, executive compensation, illiquid entrepreneurial businesses, and private equity (PE) investments. For example, Kahl, Liu, and Longstaff (2003) analyze a continuous-time portfolio choice model with restricted stocks. Both Chen, Miao, and Wang (2010) and Wang, Wang, and Yang (2012) study entrepreneurial firms with unspanned idiosyncratic risks under incomplete markets. For PE investments, Sorensen, Wang, and Yang (2014) develop a dynamic valuation model of PE investments by solving the portfolio-choice problem for a risk-averse investor, who invests in a private equity fund, managed by a general partner. We are unaware, though, of any existing models that capture the illiquidity, managerial skill (alpha), risk attitude and compensation of the hedge fund seeding business. Capturing these important features in a model that is sufficiently

[8] Larch Lane Advisors LLC constructed a simple model to project returns and cash flows for a seeded fund featuring an innovative seeding strategy.

tractable to determine the subjective value of fees in the hedge fund seeding business is one of the main contributions of this study.

Chapter 4 is organized as follows. Section 4.2 presents a dynamic valuation framework for modeling hedge fund seeding innovation and the impact of incentive contracts, managerial stake and hedge fund liquidation on a risk-averse ESFs manager's consumption and portfolio-choice behavior. A solution for this model is derived in Section 4.3. Section 4.4 and Section 4.5 discuss numerical results for breakeven alphas, seed costs and subjective value of management compensation. The main conclusions are summarized in Section 4.6. The appendix provides detailed computations relating to the market value of the hedge fund after the initial seeding stage.

4.2 Model Setup

4.2.1 Hedge fund seeding investment opportunities

We consider an infinitely-lived risk-averse ESFs manager who has the opportunity to launch a take-it-or-leave-it hedge fund at present time 0, which requires to raise the target AUM S_0. All sources of uncertainty arise from two independent standard Brownian motions B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, \mathbb{P})$, where $\mathbb{F} \equiv \{\mathcal{F}_t : t \geq 0\}$ describes the flow of information available to the seeder.

In addition to the opportunity of launching a fund, the manager has access to standard financial investment opportunities, see Merton (1971). Let W_t denote the ESFs manager's liquid (financial) wealth process. At any time $t \geq 0$ the manager invests an amount of Π_t in a diversified market portfolio and the remaining amount $W_t - \Pi_t$ in the risk-free asset with a constant interest rate r. The return of the diversified market portfolio is denoted by R and satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \tag{4.1}$$

where μ_M and $\sigma_M > 0$ are constants, and $\eta \equiv (\mu_M - r)/\sigma_M$ is the Sharpe ratio of the market portfolio.

We assume that AUM of the hedge fund $\{S_t : t \geq 0\}$ follows a geometric Brownian motion (GBM):

$$\frac{dS_t}{S_t} = (\mu - \omega - m)dt + \rho\sigma dB_t + \sqrt{1-\rho^2}\sigma dZ_t, \quad S_0 \equiv S \text{ given}, \tag{4.2}$$

where μ, ω, m and σ are constants; μ is the expected growth rate, ωS is the regular withdrawals or distribution among investors, mS is the management fee continuously occurring at the rate mS[9], σ is the total volatility of hedge fund growth and $\rho \in [-1, 1]$ is the correlation coefficient between the hedge fund and the return on the market portfolio given by Equation (4.1).[10] The parameters $\zeta \equiv \rho\sigma$ and $\epsilon \equiv \sqrt{1-\rho^2}\sigma$ are respectively the systematic

[9] Hedge fund managers normally receive 20% of the increase in fund value in excess of the last recorded maximum, i.e. high-water-mark (henceforth, HWM) as incentive fee in addition to 2% of AUM as annual fees, a compensation structure often referred to as two-twenty and considered as the industry standard. Several academic articles study the characteristics of hedge fund fees, such as Fung and Hsieh (1997), Fung and Hsieh (1999)), andAragon and Nanda (2012).

[10] Unlike the standard literature which uses a HWM incentive structure throughout the whole lifetime of the fund, our model assumes a two-to-twenty rule under which the ESFs manager obtains incentive fees at the end

and idiosyncratic volatility of the hedge fund. Similar to Goetzmann, Ingersoll, and Ross (2003), we define $\alpha \equiv \mu - r - \beta(\mu_M - r) \equiv \mu - r - \rho\frac{\sigma}{\sigma_M}(\mu_M - r) \equiv \mu - r - \rho\sigma\eta$ as the premium return on ESFs, i.e. the managers' skills in CAPM context.[11]

The Brownian motions B and Z provide the sources of market risk (systematic) and idiosyncratic risk of the hedge fund, respectively. A higher absolute value $|\rho|$ of the correlation coefficient implies that the systematic volatility has a larger weight, *ceteris paribus*.

4.2.2 Seeding innovation with fees-for-seed swap

Since the aftermath of the 2008 financial crisis, there has been a tremendous shortage of capital available to ESFs. Investors have learnt their lessons from the financial crisis and become smarter and more cautious about their investments. This has tightened the financing constraints of hedge fund managers even further. According to the Seward & Kissel New Hedge Fund Study (2014), 65% of funds within the Study obtained some form of founders capital (significantly higher than the 43% in the 2013 study). Moreover, based on conversations with various industry participants, the study estimates, that within the entire hedge fund industry for the calendar year 2014, at least 40% of all launches greater than $75 million (and an estimated 15% of all fund launches) had some form of seed capital.

In order to attract sufficient capital to cover organizational expenses and be considered credible, hedge fund managers may seek a strategic partner or a seeder who provides a critical amount of seed capital ΦS_0 in exchange for economic participation in the funds revenue, i.e. a proportion ψ of the manager's fees including both management fee and performance fees in the seeding stage.

4.2.3 Waterfall schedule upon the expiration of seed investments

At the end of the lock-up period at time T, AUM S_T will be divided among the ordinary investors, the seeder, and the manager according to a so-called "waterfall" schedule, similar as in Sorensen, Wang, and Yang (2014). More specifically, let y denote the hurdle rate during

of the lock-up period through a waterfall schedule only, and then for the remaining lifetime of the fund obtains incentive fees through a HWM rule.

[11] According to Hedge Fund Research (HFR) the 10-year period between 1994 and 2004 saw funds with less than a three-year track record outperform older funds by over 5% annually, with nearly identical volatility. Other studies, Aiken, Clifford, and Ellis (2013) and Aggarwal and Jorion (2010) made a number of adjustments to raw performance data to mitigate survivorship or backfill biases.

the lock-up period for the ordinary investors, whose maximum payment at maturity T in consequence is:

$$Z_0 = (1 - \Phi) S_0 e^{yT}. \tag{4.3}$$

Any remaining proceeds after deducting the ordinary investors' share, Z_0, and returning the seed capital with preferred hurdle rate h, i.e. $\Phi S_0 e^{hT}$, constitute the profits of the ESFs, given by:

$$S_T - Z_1, \tag{4.4}$$

where $Z_1 \equiv Z_0 + \Phi S_0 e^{hT}$ is the upper boundary that equals to the sum of the maximum payment to the ordinary investors and the preferred return for the seeder. These profits are divided between the ordinary investors and the ESFs manager. The manager receives her carried interest, while the ordinary investor's share along with his maximum payments remain in AUM after the seeding stage. Therefore, there are three regions of the waterfall structure, depending on the amount of AUM at the end of the lock-up period.

Region 1: Hurdle rate for the ordinary investors $(S_T \leq Z_0)$ In our model, the ordinary investors' payoffs is senior to the seeder's investment, thus the seeder and the manager receive nothing if S_T falls below the boundary of Z_0. The guaranteed payment to the ordinary investors is given by:

$$OP_1(A_T, T) = min\{S_T, Z_0\}. \tag{4.5}$$

Region 2: Preferred return $(Z_0 \leq S_T \leq Z_1)$ At the upper boundary of this region, the seeder gets her seed capital back with a prescribed hurdle rate h, $\Phi S_0 e^{hT}$, and the seeder's payoff in this region, at maturity T, is:

$$SP(S_T, T) = max\{S_T - Z_0, 0\} - max\{S_T - Z_1, 0\}. \tag{4.6}$$

Region 3: The ESFs manager's carried interest $(S_T > Z_1)$ After deducting the guaranteed payment and preferred return of the seed capital, the ESFs manager claims her carried interest, the fraction k of the profits $S_T - Z_1$, given by:

$$GP(S_T, T) = k * max\{S_T - Z_1, 0\}. \tag{4.7}$$

Denote $OP_3(S_T, T)$ as the ordinary investors' share in this region which is given by:

$$OP_3(S_T, T) = (1 - k) * max\{S_T - Z_1, 0\}. \tag{4.8}$$

One can easily compute the sum of the payoffs of all agents, at maturity T, which satisfies:

$$OP_1(A_T, T) + OP_3(S_T, T) + SP(S_T, T) + GP(S_T, T) = S_T. \tag{4.9}$$

After returning the seed capital, only the ordinary investors' payoff remains in the fund, and the adjusted AUM S_T^* is:

$$S_T^* \equiv OP(A_T, T) = OP_1(A_T, T) + OP_3(S_T, T). \tag{4.10}$$

Therefore, the ordinary investors' claim at maturity T, denoted by $OP^*(A_T, T)$, satisfies:

$$OP^*(A_T, T) = I(S_T^*, S_T^*), \tag{4.11}$$

where $I(S_T^*, S_T^*)$, the investors' claim in the Goetzmann-Ingersoll-Ross model, is determined by Equation (C.11) in Appendix C. Appendix C presents more details about the market value of the ordinary investors' claim after the seeding stage.

4.2.4 The manager's problem

The ESFs manager's standard time separable preference is characterized by her initial wealth W_0 and a pure subjective discount rate δ, and her utility function $U(C)$, represented by:

$$\max_{c_s} \mathbb{E}\left[\int_0^{\infty} \exp\left(-\delta s\right) U(c_s) ds\right]. \tag{4.12}$$

For tractability, we assume the manager has constant absolute risk aversion (CARA) utility preference, given by

$$U(c) = -\exp(-\gamma c)/\gamma, \tag{4.13}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion. The overall time horizon $[0, \infty)$ entails the lock-up period $[0, T]$ during which the manager faces idiosyncratic risk (possibly due to assets in the fund being less liquid and proprietary) as well as the remaining period $[T, \infty)$, during which it is assumed that the underlying risks are fully spanned by public assets, as in Sorensen, Wang, and Yang (2014) section 3.[12]

4.2.5 Manager's liquid wealth dynamics

During the lock-up period, the manager's financial wealth evolves according to,

$$dW_t = (rW_t + (1-\psi)mS - c_t)dt + \Pi_t((\mu_M - r)dt + \sigma_M dB_t), \quad 0 < t < T. \tag{4.14}$$

The first term in Equation (4.14) is the wealth accumulation when the manager fully invests in the risk free asset, plus the revenue of managing the ESFs net of her consumption. The second term is the excess return from the manager's investment in the market portfolio.

At the end of the lock-up period T, the manager's wealth (including current portfolio wealth and futures management fees) jumps from W_{T^-} to W_T, with

$$\begin{aligned} W_T &= W_{T^-} + G(S_T, T) \\ &= W_{T^-} + (1-\psi)[k * GP(S_T, T) + F(S_T^*, S_T^*)]. \end{aligned} \tag{4.15}$$

The second term on the right hand side of Equation (4.15) represents the carried interest of the hedge fund seeding business. The term $F(S_T^*, S_T^*)$ consists of the market value of future fees paid to the manager after the lock-up period. Per assumption, the risk to which the fund is exposed after the lock-up period is fully spanned by public assets, as such the market value of the fees can be computed under the appropriate risk neutral measure as in Goetzmann, Ingersoll, and Ross (2003). An explicit expression for $F(S_T^*, S_T^*)$ is provided in Equation (C.12) in Appendix C. The dynamic programming principle and the fact that we have full spanning over the period $[T, \infty)$ implies that the solution of problem (12) can now been obtained from the solution of the corresponding problem starting at time T with wealth dynamics

$$dW_t = (rW_t + \Pi_t(\mu_M - r) - c_t)dt + \Pi_t \sigma_M dB_t, \tag{4.16}$$

[12]This is an idealization of the fact that idiosyncratic risks in the start up of the fund and in particular during the lock-up period are significantly higher than when the fund has established itself.

and initial wealth W_T according to (15), and then by backward induction over the interval $[0, T]$ as in the following section.

4.3 Model Solution

In this section, we first derive seed costs with fees-for-seed swaps for the ESFs manager and the breakeven alpha for the ordinary investors. Then we analyze the manager's consumption and portfolio choice in a dynamic valuation model taking account of illiquidity, ESFs managers' value-adding skills (alpha), incentive compensation, and the fees-for-seed swap. However, the idiosyncratic risk which is present in the hedge funds seeding business invalidates the standard two-step complete-markets (Arrow-Debreu) analysis (first value maximization and then optimal consumption allocation)[13] due to the non-separability between value maximization and consumption smoothing. In order to derive the solution, we first solve the standard Merton consumption and portfolio choice problem faced by the manager after the expiration of the lock-up period, similar as in Merton (1971) and Goetzmann, Ingersoll, and Ross (2003). Following this, we solve an optimal control problem maximizing the ESFs manager's utility during the seeding stage.

4.3.1 Market value of total fees and call options on the seeding investment

We assume that the market prices cash-flows attached to liquid assets by using a risk neutral measure $\mathbb{Q}$ equivalent to the measure $\mathbb{P}$ when restricted to all $\mathcal{F}_t$ for any $t \geq 0$.[14] The corresponding state-price deflator π satisfies $d\pi = -r\pi dt - \eta\pi dZ,\ \pi_0 = 1$ and restricted to $(\Omega, \mathcal{F}_t)$, we have $\lambda_t = \frac{d\mathbb{Q}}{d\mathbb{P}}\Big|_{\mathcal{F}_t}$ and $\lambda_t = \exp(rt)\frac{\pi_t}{\pi_0}$, see Duffie (2001).

Denote by $\nu \equiv \mu - \omega - m - \rho\sigma\eta \equiv \alpha + r - \omega - m$ the risk-adjusted drift rate of AUM, and $B_t^{\mathbb{Q}}$ a standard Brownian motion satisfying $dB_t^{\mathbb{Q}} = dB_t + \eta dt$. Then under $\mathbb{Q}$, the dynamics of AUM in Equation (4.2) can be rewritten as

$$\frac{dS_t}{S_t} = \nu dt + \rho\sigma dB_t^{\mathbb{Q}} + \epsilon dZ_t. \tag{4.17}$$

Let $G^*(S_t, t)$ be the market value of the claim underlying S_t with a payment flow mS_s for $s \in [t, T]$ and a terminal payoff $G^*(S_T, T) \equiv k * \max\{S_T - Z_1, 0\} + F(S_T^*, S_T^*)$. According to the dynamic asset pricing theory (Duffie (2001)), it can be written as a conditional

[13] See more details in C and Huang (1989)

[14] Such a measure may not be unique due to incompleteness, but we assume here that the market has chosen a risk neutral measure, which in consequence becomes the market measure.

expectation under the risk-adjusted measure $\mathbb{Q}$:

$$G^*(S_t, t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^T e^{-r(s-t)} m S_s ds + e^{-r(T-t)} G^*(S_T, T)\right]. \tag{4.18}$$

By using Ito's formula, $G^*(S_t, t)$ satisfies the follow PDE:

$$rG^*(S_t, t) = mS + G_t^* + \nu S_t G_S^*(S_t, t) + \frac{1}{2}\sigma^2 S_t^2 V G_{SS}^*(S_t, t), \tag{4.19}$$

with two boundary conditions

$$\begin{aligned} G^*(0, t) &= 0, \\ G^*(S_T, T) &= k * \max\{S_T - Z_1, 0\} + F(S_T^*, S_T^*), \end{aligned} \tag{4.20}$$

where $F(S_T^*, S_T^*)$ is the market value of total fees generated after the lock-up period expires at time T, given by Equation (C.12) in the Appendix C.

Similar to Sorensen, Wang, and Yang (2014), the value at time t of a plain-vanilla European call option with strike price K and terminal payoff $\max\{S_T - K, 0\}$ at T, denoted by $Call(S_t, t, \alpha, K)$, is given by:

$$\begin{aligned} Call(S_t, t, \alpha, K) &= \mathbb{E}_t^{\mathbb{Q}}\left[e^{-r(T-t)} \max\{S_T - K, 0\}\right], \\ &= e^{(\alpha-\omega-m)(T-t)} \mathbb{E}_t^{\mathbb{Q}}\left[e^{-\nu(T-t)} \max\{S_T - K, 0\}\right], \\ &= e^{(\alpha-\omega-m)(T-t)}\left[S_t N(d_1) - K e^{-\nu(T-t)} N(d_2)\right], \\ &= S_t e^{(\alpha-\omega-m)(T-t)} N(d_1) - K e^{-r(T-t)} N(d_2), \end{aligned} \tag{4.21}$$

where

$$\begin{aligned} d_1 &= d_2 + \sigma\sqrt{T-t}, \\ d_2 &= \frac{ln(S_t/K) + (\nu - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}. \end{aligned} \tag{4.22}$$

The pricing formula here is different from the classic Black-Scholes formula as we assume that the investment of the hedge fund seeding business could generate excess alpha.

4.3.2 Seed costs with fees-for-seed swaps

Under the arrangement of a fees-for-seed swap during the hedge fund seeding stage, the ESFs manager must give up a portion (ψ) of her fees in exchange for the seed capital (ΦS_0).

The cost ψ of the seed capital with fees-for-seed swap, is determined endogenously within the model, in such a way that the value of the contract is zero at initiation.

Generally speaking, a seeder is usually a diversified investor who signs such contracts with a large number of ESFs and therefore the idiosyncratic risk of the hedge fund seeding business is well-diversified.[15] Thus, the seed capital provided by the seeder must be equal to the market value (equilibrium value) of the fees allocated to the seeder when the contract commences. That is

$$\begin{aligned} \Phi S_0 &= \psi G^*(S_0, 0), \\ \psi &= \tfrac{\Phi S_0}{G^*(S_0,0)}. \end{aligned} \tag{4.23}$$

In addition to revenue sharing, seeding investments will be returned to seeders more or less by $SP(S_T, T)$ depending on the performance of the fund at the end of the lock-up period. Using the pricing formula defined in the last subsection, one can derive an explicit expression for $SP(S_t, t)$, the time t value of this claim, that is

$$SP(S_t, t) = Call(S_t, t, \alpha, Z_0) - Call(S_t, t, \alpha, Z_1). \tag{4.24}$$

Therefore, the time t value of the seeders' claim, denoted by $SP^*(S_t, t)$, is given by:

$$SP^*(S_t, t) = SP(S_t, t) + \psi * G^*(S_t, t). \tag{4.25}$$

4.3.3 Break-even alpha

In Section 4.2, we considered the terminal payoffs ($OP^*(S_T, T)$) for the ordinary investors at maturity T. Its present value, denoted by $I^*(S_t, t)$, satisfies:

$$I^*(S_t, t) = E_t^{\mathbb{Q}}\left[e^{-r(T-t)} I(S_T^*, S_T^*)\right]. \tag{4.26}$$

By using Ito's formula, we obtain the following PDE:

$$rI^*(S_t, t) = \omega S_t + I_t^* + \nu S_t I_S^*(S_t, t) + \frac{1}{2}\sigma^2 S_t^2 V I_{SS}^*(S_t, t), \tag{4.27}$$

[15]For example, Larch Lane Advisors LLC ("Larch Lane") , one of the first dedicated providers of hedge fund seed capital, has seeded 25 hedge funds and continues to be an active capital provider for the hedge fund industry.

with the following two boundary conditions defined below:

$$\begin{aligned} I^*(S_T, T) &= I(S_T^*, S_T^*), \\ I^*(0, t) &= 0. \end{aligned} \tag{4.28}$$

In order to break-even at the start of the fund, the ordinary investors' claim has to be equal to their initial investment, i.e. $I^*(S_0, 0) = (1 - \Phi)S_0$. Particularly, ordinary investors benefit (suffer the loss) from their investment in the ESFs if $I^*(S_0, 0) \geq (\leq)(1 - \Phi)S_0$ when the contract commences. Using Equation (4.27) to solve $I^*(S_0, 0) = (1 - \Phi)S_0$ for α provides the minimum alpha that should be generated by the ESFs manager for the ordinary investors to break-even when the contract commences.

4.3.4 Consumption and portfolio choice after the lock-up period

As indicated in section 2.5, the manager's investment problem after the lock-up period has expired at time T is equivalent to the classical Merton problem studied in Merton (1971), where the initial wealth W_T consists of realized portfolio value prior to time T, waterfall schedule payoff at time T and the market value of future management and performance fees after time T. The optimal consumption and portfolio rule is therefore given by

$$c^*(W) = r\left(W + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{4.29}$$

$$\Pi^* = \frac{\eta}{\gamma r \sigma_M}, \tag{4.30}$$

where W is the liquid wealth level.

The maximum of the expected utility of consumption after the seeding stage can be computed as

$$J^e(W) = -\frac{1}{\gamma r} \exp\left[-\gamma r\left(W + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{4.31}$$

4.3.5 Manager's decisions and certainty equivalent valuation

Define $J^s(W,S,t)$ as the manager's value function before the the end of the seeding stage, i.e.

$$J^s(W,S,t) = \max_{(c_s,\Pi_s)} \mathbb{E}\left[\int_t^T \exp\left(-\delta(s-t)\right) U(c_s)ds + e^{-\delta(T-t)} J^e(W_T)\middle| W_t = W, S_t = S\right], \tag{4.32}$$

where W is the manager's financial wealth process, and the function $J^e(\cdot)$ is given by Equation (4.31). In light of section 2.5. the value function $J^s(W_0,S_0,0)$ coincides with the value of the problem (12). During the lock-up period, the manager's financial wealth evolves according to,

$$dW_t = (rW_t + \Pi_t(\mu_M - r) + (1-\psi)mS - c_t)dt + \Pi_t\sigma_M dB_t, \quad 0 < t < T, \tag{4.33}$$

while as previously discussed, at the end of the lock-up period the wealth jumps to

$$\begin{aligned} W_T &= W_{T^-} + G(S_T,T) \\ &= W_{T^-} + (1-\psi)[k * GP(S_T,T) + F(S_T^*, S_T^*)]. \end{aligned} \tag{4.34}$$

Compared to the exogenously given fraction of management fees retained by the seeder in practice, the fraction ψ in our model is endogenously determined by the fees-for-seed swap. In this case, the manager's value function $J^s(W,S,t)$ satisfies the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned} \delta J^s(W,S,t) &= \sup_{c\geq 0,\Pi} \{U(c) + J_t^s + (rW + \Pi(\mu_M - r) + (1-\psi)mS - c)J_W^s \\ &+ \tfrac{1}{2}(\Pi\sigma_M)^2 J_{WW}^s + \Pi\sigma_M\sigma\rho S J_{WS}^s + \mu S J_S^s + \tfrac{1}{2}\sigma^2 S^2 J_{SS}^s\}. \end{aligned} \tag{4.35}$$

The first-order conditions for the optimal consumption and portfolio choice are:

$$U'(c) = J_W^s(W,S,t), \tag{4.36}$$

$$\Pi(S,t) = \frac{-J_W^s}{J_{WW}^s}\left(\frac{\mu_M - r}{{\sigma_M}^2}\right) + \frac{-J_{WS}^s}{J_{WW}^s}\frac{\rho\sigma S}{\sigma_M}. \tag{4.37}$$

According to the utility indifference pricing principle, the utility indifference price [16] of the management fee owned by the ESFs manager in the seeding stage, denoted by $G(S,t)$ [17], satisfies

$$J^s(W,S,t) = J^e(W+G(S,t)) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(W+G(S,t)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right)\right]. \quad (4.38)$$

Substituting Equations (4.36), (4.37), and (4.38) into Equation (4.35), we obtain the following theorem immediately[18]:

Theorem 4.3.1. *During the seeding stage for $t \in [0,T]$, the optimal consumption and portfolio rule is given by*

$$c^*(W,S,t) = r\left[W+G(S,t)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right], \quad (4.39)$$

$$\Pi^*(S,t) = \frac{\eta}{\gamma r\sigma_M} - \frac{\rho\sigma}{\sigma_M}SG_S(S,t), \quad (4.40)$$

where $G(S,t)$ is the utility indifference price of the fees owned by the manager, which is the solution of the following partial differential equation (PDE):

$$rG(S,t) = (1-\psi)mS + G_t + \nu SG_S(S,t) + \frac{1}{2}\sigma^2S^2G_{SS}(S,t) - \frac{\gamma r}{2}\epsilon^2S^2G_S(S,t)^2, \quad (4.41)$$

subject to the following two boundary conditions:

$$G(S_T,T) = (1-\Phi)[kGP(S_T,T)+F(S_T^*,S_T^*)] \quad (4.42)$$

$$G(0,t) = 0, t\in[0,T]. \quad (4.43)$$

The first boundary condition states that, at maturity T, the ESFs manager collects her carried interest plus the market fees paid by an outside manager. The second boundary condition reflects that the value of the manager's fees falls down to zero as the underlying AUM converges to zero during the lock-up period.

[16] It is sometimes called certainty equivalent wealth, which is the risk-adjusted subjective value of managing the hedge fund seeding business.

[17] Thanks to the exponential utility assumption, the utility indifference price is independent of the wealth level of the fund manager.

[18] The proof here is the same as Theorem 2.3.1, one can find proof hints in Appendix A.

Equations (4.39) and (4.40) indicate that the hedge fund manager will consume the implied value $G(S,t)$ and use the market portfolio to dynamically hedge the hedge fund seeding business risk. More specifically, Equation (4.39) indicates that the manager's consumption is equal to the annuity value of the sum of financial wealth W and the implicit value of the hedge fund seeding business $G(S,t)$ plus two constant terms which appear in the classical Merton rule, see Merton (1971). The portfolio-choice rule is given by Equation (4.40) in which the second term represents the manager's hedging demand in the context of her hedge fund seeding business. Equation (4.41) implies that if the absolute risk-aversion index equals zero (i.e. the ESFs manager is risk-neutral towards the idiosyncratic risk), Equation (4.41) becomes the standard equilibrium pricing equation. Therefore, the last term on the right side of Equation (4.41) captures the idiosyncratic risk effect on the managers valuation of the hedge fund business.

4.4 Breakeven Alphas and Seed Costs

We have derived the solution of the fees-for-seed swap portion ψ referred to as the seed cost and the subjective value of the ESFs manager's compensation in Section 4.3 In this section we provide some numerical results in order to develop more economic intuition. Baseline breakeven parameters are chosen according to Sorensen, Wang, and Yang (2014). Table 4.1 summarizes the parameter values used in our baseline breakeven case.

TABLE 4.1: Summary of key parameters in baseline breakeven case

Parameter	Symbol	Value
Risk-free rate	r	5%
Expected return of market portfolio	μ_M	11%
Expected return of the ESFs	μ	
Volatility of market portfolio	σ_M	20%
Volatility of the ESFs	σ	25%
Market Sharpe ratio	η	30%
Guaranteed yield	y	5%
Hurdle rate	h	8%
Management fee	m	2%
Incentive fee	k	20%
Lock-up period	T	2
Managerial skills	α	
Subjective discount rate	δ	5%
Idiosyncratic risk	ϵ	23%
Seed capital ratio	Φ	5%
Correlation coefficient	ρ	40%
Exogenous liquidation barrier	l	50%
Withdrawal rate, the liquidation parameter	$\omega + \lambda$	5%
Coefficient of absolute risk aversion	γ	2

4.4.1 Breakeven alphas, compensation contracts, and seed capital involvement

How important is the managerial ability of producing superior performance (alpha)? In order to develop more economic intuition, Table 4.2 presents breakeven alphas in different compensation contracts for various levels of seed capitals. We first consider the case without any fees, $m = k = 0\%$. No positive alpha is then required by the ordinary investors for the case of no seed capitals. Interestingly, the ordinary investors can even bear some loss if the fund gets some seed capitals. For example, the ordinary investors' investments breakeven for a negative alpha, $\alpha = -0.77\%$, in the case that the seeder provides 15% of AUM seed capital as the anchor investor, which means the ordinary investors are more willing to invest in the fund by following the seeders.

Moreover, some typical compensation contracts are compared in Table 4.2. Naturally, increasing either m or k increases the breakeven alpha, for a given level of seed investments. More specifically, holding the seed capital ratio fixed, increasing the management fee rate m by 0.5 percentage-points results in an increase of the breakeven alpha by the same amount. However, increasing k by 10 % increases the breakeven alpha by 0.55% to 0.75%, depending on seed capital ratio.

Table 4.2 provides evidence that an ESFs manager would easily reach the target AUM level to launch a fund via a seeding vehicle as the ordinary investors benefit from the scheme. However, the ESFs manager backed up with seed capital has to give certain fraction (ψ) of her fees revenues as compensation to the seeder, and we will analyze these seed costs in the following subsection.

TABLE 4.2: The table gives breakeven alphas for different levels of seed capital ratio, Φ, incentive fees k and management fees, m. Other parameters are $\beta = 0.5$, $h = 8\%$, $T = 2$ years, and $S_0 = 100$.

m	k	$\Phi = 0$	$\Phi = 5\%$	$\Phi = 10\%$	$\Phi = 15\%$
0.0%	0%	0.00%	-0.24%	-0.50%	-0.77%
1.5%	10%	2.14%	1.89%	1.62%	1.35%
1.5%	20%	2.79%	2.50%	2.22%	1.93%
1.5%	30%	3.54%	3.16%	2.83%	2.51%
2.0%	10%	2.65%	2.39%	2.12%	1.85%
2.0%[a]	20%	3.29%	3.00%	2.72%	2.43%
2.0%	30%	4.03%	3.66%	3.33%	3.01%
2.5%	10%	3.15%	2.89%	2.62%	2.35%
2.5%	20%	3.78%	3.50%	3.21%	2.93%
2.5%	30%	4.54%	4.16%	3.83%	3.51%

[a] Indicates the baseline breakeven case.

4.4.2 The seeder's value and seed costs

We refer to the last section for the seeder's value given by Equation (4.25). Not surprisingly, the seeder's value is significantly affected by the fund's performance as the left hand sub-figures in Figure 4.1 and Figure 4.2 indicate. For example, the seeder's value has more than

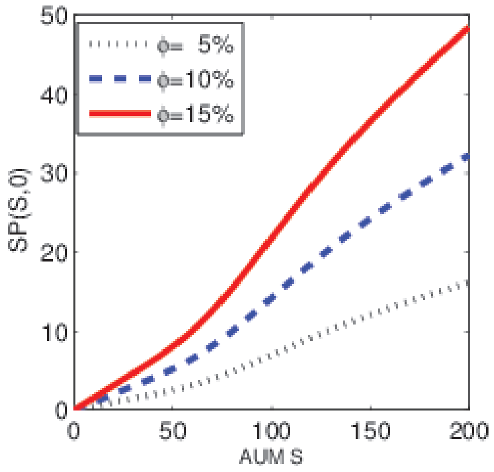

FIGURE 4.1: The figure provides comparative statics for the seeder's payoffs with respect to time t and AUM S for different levels of seed capital.

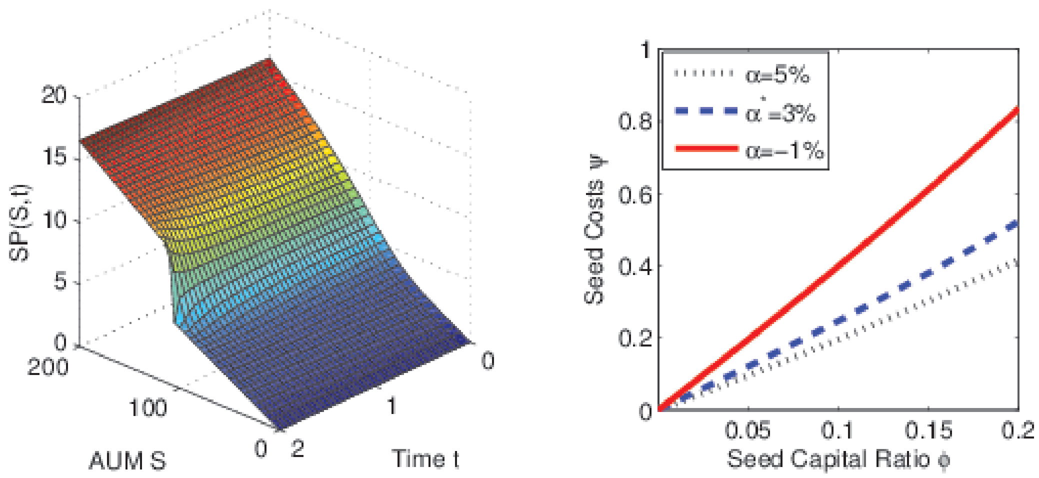

FIGURE 4.2: The figure presents comparative statics for the seeder's value with respect to t and S, and seed costs ψ with respect to seed capital ratio for three different levels of alpha.

tripled as the fund AUM is doubled (from $S_0 = 100$ to 200). On the other hand, the right subfigure in Figure 4.1 shows that the seeder's value is almost insensitive to time t, given AUM fixed at 100. The seeder's value changes only with time t near the maturity and AUM staying around Z_0 and Z_1. This is mainly because the seeder's positions in these options are in the money when AUM goes to the interval around Z_0 and Z_1.

Unlike an ad-hoc "rule of thumb" decision on the fees-for-seed ratio which is often common in practice, Chapter 4 provides a closed-form solution for the seed cost ψ which is informed by a number of factors such as the amount of the seed capital, the manager's alpha, the risk of the fund, etc. and takes account some key principles from Finance theory. Interestingly, the right hand subfigure in Figure 4.2 shows that the fees-for-seed ratio ψ is indeed a linearly increasing function of the seed capital which could be interpreted as a rule of thumb in practice. However note that the slope of the function varies with parameters such as the managerial skills. In details, the seeder demands more fund revenue share for a manager with negative alpha (-1%) than that for a more talented manager (5%) and the difference can be up to 40 % if providing 20 % of the initial AUM (S_0).

4.5 Seeding Investments with Lock-up Period, Subjective Value of Fees, and Idiosyncratic Risk

In this section, we further analyze the effects of the lock-up period, seeding investment, and idiosyncratic risk on management compensation, illiquidity discount, and the economic value of the fund.

4.5.1 Lock-up period effects, illiquidity discount and management compensation

Table 4.3 presents valuations for the ordinary investors, the seeder, and the ESFs manager for various levels of alpha. Panel A of Table 4.3, with a short lock-up period, shows that the ordinary investors breakeven with $\alpha = 3.13\%$, while the seeder's values for various alphas outperform her initial seeding investment $\Phi \cdot S_0 = 5$. This is straightforward because the seeder has an extra option value in addition to the breakeven swap when the seeding contract commences. As the alpha decreases from 5% to -1% the seed cost ψ increases from 9.98% to 19.94 %, which makes sure the seeder breakeven at the beginning of the contract. Due to the unspanned risk of the seeding business, the ESFs manager bears the illiquidity costs. Similar to Wang, Wang, and Yang (2012), the illiquidity discount for the manager is defined as the difference of the market value and the certainty-equivalent value of fees, which is given by $ID = G^*(S_0, 0) - G(S_0, 0)$. This discount is the amount the manager would be willing to pay for not bearing the idiosyncratic risk during the initial seeding stage. As we can see from Table 4.3, the amount of ID increases by 1.8 to 2.44 as increasing alpha from -1% to 5%.

In Table 4.3, Panel B, with a relative long lock-up period $T = 4$ years, shows the illiquidity discount is much greater than that in Panel A. For the case of $\alpha = 5\%$, the discount for $T = 4$ around is six times of that in Panel A. This means the manager bears more illiquidity discount for longer lock-up periods. Moreover, the seeder also suffers a loss for a long lock-up period. For example, fixing $\alpha = -1\%$, the seeder suffers losses of 1.03 (around 20% of her initial investment) by extending the lock-up period from 6 months to 4 years.

TABLE 4.3: The table presents valuations of different agents' claims with non-spanned risk for various levels of alpha. The columns refer to: the ordinary investors' interest (I^*), the seeders' payoffs (SP^*), the market value of fees (G^*), the economic value of the ESFs ($V = I^* + SP^* + G^*$), the ESFs manager's certain-equivalent valuation (G), the illiquidity discount ($ID = G^* - G$), and the seed costs (ψ). Parameter values are $\gamma = 2$, $S_0 = 100$, $m = 2\%$, $k = 20\%$, $\Phi = 5\%$, $\beta = 0.5$, and $h = 8\%$. Panel A and B report the results for the case with the lock-up periods $T = 0.5$ and $T = 4$, respectively.

$\alpha(\%)$	I^*	SP^*	G^*	V	G	ID	$\psi(\%)$
Panel A: $T = 0.5$							
-1.0	64.53	7.19	20.07	91.79	19.63	0.44	19.94
0.00	69.96	7.25	22.79	100.00	21.19	0.65	17.99
2.00	84.38	7.36	30.19	121.93	29.11	1.08	14.21
3.13 [a]	95.00	7.42	35.91	138.33	34.42	1.49	12.22
5.00	106.38	7.53	45.09	159.01	42.86	2.24	9.98
Panel B: $T = 4$							
-1.0	64.04	6.16	20.54	90.74	18.43	2.11	19.58
0.00	70.15	6.29	23.56	100.00	20.04	2.30	17.51
2.00	86.36	6.58	31.77	124.71	25.60	6.17	13.60
2.83 [a]	95.00	6.70	36.31	138.02	28.44	7.87	12.10
5.00	114.48	7.06	49.53	171.06	35.77	13.76	9.17

[a] Indicates baseline breakeven case.

4.5.2 Seed capitals effects, economic values of the fund

Table 4.4 presents results of economic values of the fund and subjective values of fees with or without seeding vehicle (shown in Panel A and Panel B respectively) for various levels of alpha. More specifically, the economic value of the fund with seed capital is less (more) than a fund without seed capital for a positive (negative) alpha, holding alpha fixed. This is because the seed capital will be returned at a hurdle rate and no longer earns the premium alpha. However, if we take the illiquidity discount into consideration, the adjusted economic value, denoted by $V^* = V - ID$, of the fund with seed capital is always greater than that without seed capital. Take the case $alpha = 5\%$ for example, the adjusted economic value of the ESFs with 5% of seed capital is 156.12, compared to 154.44 for a fund without seed capital. The main reason is that ψ percent of the ESFs manager revenue is transferred to the outside investors via fees-for-seed swap, thus generating diversification benefits for the fund.

One may note that the ESFs manager may be better off without any seeding investments. This is true only when the ESFs manager can reach the target AUM level for a successful launch. If it is hard for the manager to achieve the target, she may turn to the seeder and give up some fraction of her fees revenue due to lack of bargaining power. For example, Panel B of Table 4.4 illustrates that the manager should give up 12.08% of her fees in exchange of 5% of AUM from the seeder.

For an unskilled ESFs manager, $\alpha = 0$, both Table 4.3 and Table 4.4 show that the economic value of the fund equals to the initial investment $S_0 = 100$, which means the ordinary investor bears the loss to pay fees to the manager. As alpha increases, both the economic value of the fund and the ordinary investors' payoffs grow as we expected.

TABLE 4.4: The table presents valuations of different agents' claims with non-spanned risk for various levels of alpha. Panel A and B report the results for the case with $\Phi = 0$ and $\Phi = 5\%$, respectively.

$\alpha(\%)$	I^*	SP^*	G^*	V	G	ID	ψ (%)
Panel A: the seed capital ratio $\Phi = 0$							
-1.0	65.28	0.00	25.90	91.18	22.46	3.45	0.00
0.00	71.16	0.00	28.84	100.00	24.72	4.11	0.00
2.00	86.76	0.00	36.80	123.56	30.70	6.10	0.00
3.29 [a]	100.00	0.00	43.95	143.95	35.88	8.08	0.00
5.00	112.09	0.00	53.36	165.45	42.37	10.99	0.00
Panel B: the seed capital ratio $\Phi = 5\%$							
-1.0	64.27	6.53	20.51	91.31	19.08	1.43	19.60
0.00	70.00	6.63	23.37	100.00	20.45	1.60	17.62
2.00	85.17	6.85	31.14	123.15	27.38	3.76	13.84
3.00 [a]	95.00	6.96	36.39	138.35	31.34	5.05	12.08
5.00	109.73	7.19	47.25	164.16	39.21	8.04	9.57

[a] Indicates baseline breakeven case.

4.5.3 Idiosyncratic risk effect, risk aversion and fees

It is obvious that the (subjective) value for the ESFs manager is generally an increasing function of AUM and time t. Unlike the risk-neutral case, in the case of risk aversion the subjective values are concave functions of AUM. This is mainly because of the nonlinear terms in our pricing PDE (4.41). As shown in Figure 4.3, the more risk-averse the manager is, the greater the illiquidity discount the manager has to bear. More specifically, the illiquidity discount is very small when AUM stays at a very low level (e.g. below 40), while it increases quickly and reaches its peak at 33 (for the case $\gamma = 4$) as AUM increase to 200. Moreover, the right hand subfigure in Figure 4.3 indicates that the gap of the illiquidity discount between the case $\gamma = 4$ and the case $\gamma = 2$ converges to zero as t is approaching the end of the lock-up

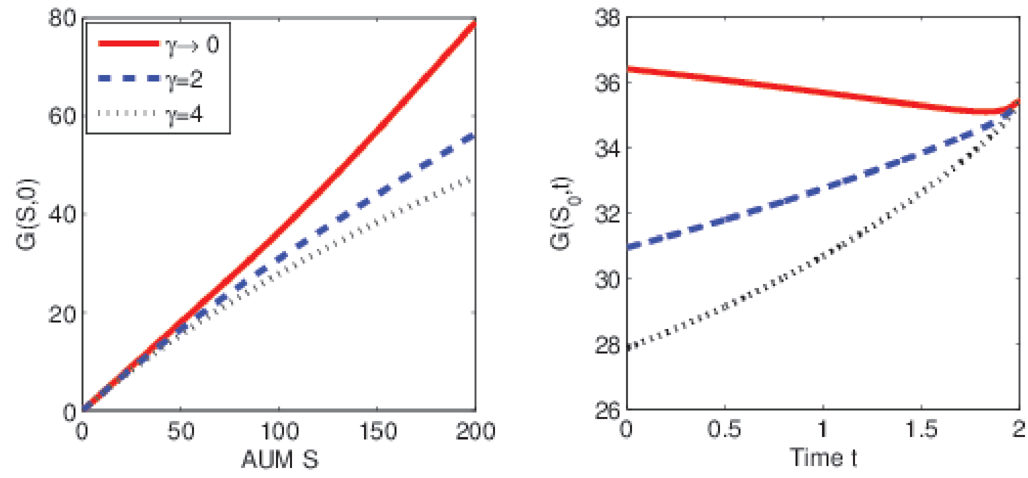

FIGURE 4.3: Comparative statics for management compensation with respect to AUM and time t for various levels of risk aversion.

period T. Once the lock-up period expires, the ESFs manager is out of the seeding business and bears no idiosyncratic risk any more.

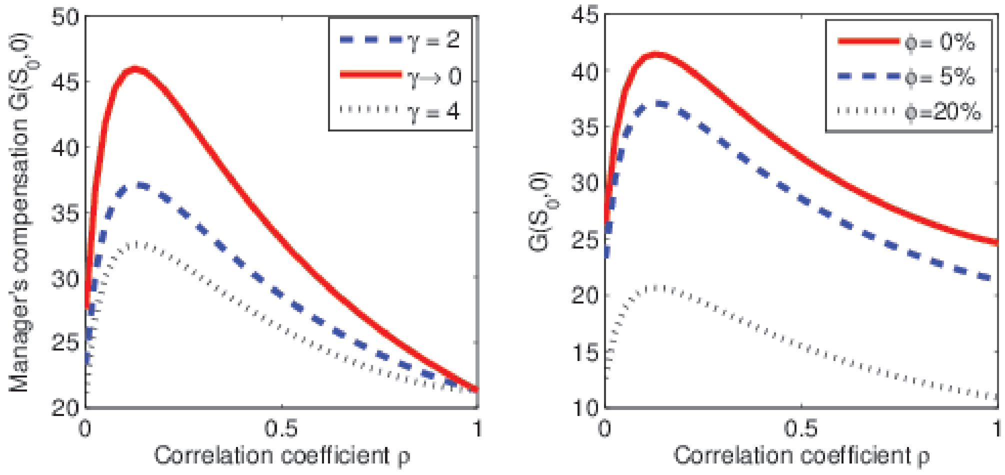

FIGURE 4.4: Comparative statics for management compensation with respect to correlation coefficients ρ.

Finally, Figure 4.4 shows the effect of the correlation coefficient ρ between the market and the fund on the subjective value of the management compensation. Interestingly, the management compensation is a not a monotonous function. It first increases with ρ to its maximum point and then decreases afterward. On the one hand, managerial skills alpha drops ($\alpha = \mu - r - \rho\sigma\eta$)) as ρ increases, which causes a negative effect on the subjective value G. On the other hand, the idiosyncratic risk decreases as ρ increases (keeping total fund volatility σ unchanged), which is a positive effect on management compensation. Therefore, the ESFs manager faces a trade off between the correlation coefficient and idiosyncratic risk. Particularly, management compensation is an increasing function as the positive effect of the idiosyncratic risk dominates the negative effect of diminishing alpha for ρ less than around 20 %. As ρ continues to increase, the negative effect dominates, and the function turns into a decreasing function.

4.6 Conclusion

In this article, we developed a dynamic valuation model for the hedge fund seeding business by solving the consumption and portfolio-choice problem of a risk-averse manager who launches a hedge fund through a seeding vehicle. As traditional approaches to attract the initial AUM and covering of organizational expenses becomes much harder for ESFs managers in a much tighter financial landscape, nowadays more and more ESFs managers are likely to turn to seed investors for early stages of capital by offering a certain proportion of their fees through a seeding vehicle. The new swap specifies that a seeder commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced economics" which is usually a proportion of the fees that the ESFs manager generates from the entire pool of assets in the fund. Our results indicate that the new swap not only solves the serious problems of widespread financing constraints for ESFs managers, but can also be highly beneficial to both the manager and the seeder if structured properly.

Moreover, we derived a closed-form solution for the fees-for-seed ratio, i.e. the seed cost, as well as the manager's value attached to the hedge fund seeding business. In addition we presented a detailed numerical analysis in which we discussed sensitivity effects of various model parameters such as the risk aversion coefficient as well as the skills factor α on our results. Our analysis showed that, as we would expect, the greater the seed capital obtained, the more fees the manager should give up. The fees-for-seed ratio is a linear increasing function of the amount of the seed capital. However, more interestingly, unlike an uninformed "rule of thumb", the slope term in this linear relationship depends on factors such as fund volatility and managerial skills. Therefore, the closed-form solution of the seed costs in our model is much more informed and can be regarded as a theoretical guide to the design of a seeding vehicle contract.

Our model assumes the ESFs manager is risk averse towards the hedge fund seeding business, thus she suffers the illiquidity discount for her valuation due to unspanned idiosyncratic risk. Once the ESFs is out of the seeding stage and enters into normal stage, the manager bears no idiosyncratic risk. We found that the more risk-averse the manager is, the greater the illiquidity discount the manager has to bear, thus the lower her subjective value is. In addition, the manager bears more idiosyncratic risk for longer duration of the lock-up period, i.e. the hedge fund seeding stage. Given fixed expected return and volatility of the fund, the ESFs manager faces a trade off between the correlation coefficient and idiosyncratic risk. As the correlation coefficient increases, the positive effect of decreasing idiosyncratic risk at first dominates the negative effect of diminishing alpha for low level of ρ, but then is dominated by the negative effect of diminishing alpha.

More importantly, the ordinary investors are more willing to invest in an ESFs backed up by seeders via a fees-for-seed swap. As our numerical results illustrate, the more seed capital the fund gets the smaller breakeven alphas the ordinary investors demand for their investment. Therefore, the seeding vehicle helps the ESFs attract more investors and get sufficient capital for a successful launch. Moreover, our results show that the adjusted economic value, denoted by $V^* = V - ID$, for a fund with seed capital is always greater than that without any seed capitals due to ψ percent of the ESFs manager revenue is transferred to the outside investors via fees-for-seed swaps.

Chapter 5

Conclusion & Further Research

The fact that markets are generally incomplete and with frictions (e.g. borrowing constraints and lack of diversification, ect.) is often neglected. Having built dynamic incomplete-market models, this thesis provides deep insights of what role does financing innovation play in alleviating severe financing constraints and driving economic growth. More specifically, I have developed a number of analytically tractable models (e.g. consumption-based CAPM, CCAPM henceforth) and identified key effects of market incompleteness on the timing of investment both in entrepreneurial finance and hedge funds management. In the following text, I will first outline the main contribution of this research and then I will provide details of further research which mainly based on this thesis.

5.1 Conclusion

Chapter 2 improves a generalized model of capital structure for SMEs with equity-for-guarantee swaps trade-off among borrowing constraints, tax, diversification benefits, and costs of financial distress. Our model predicts that these innovative swaps fundamentally raises the entrepreneur's borrowing capacity and therefore the entrepreneur optimally issues more debt and takes higher leverage than that without the equity-for-guarantee swap. Our numerical results show that: **(1)** the entrepreneur with the swap receives more welfare increments and has more investment opportunities because of being more willing to invest; **(2)** higher leverage leads to larger tax shields and diversification benefits because the entrepreneur faces less equity exposure to the project and thus her portfolio (consisting of private equity and her liquid wealth) is less risky; **(3)** higher risk-averse entrepreneurs under higher nondiversifiable idiosyncratic risk gain more benefits resulting from the equity-for-guarantee swap.

Chapter 3 deals with the pricing of seed costs and the risk shifting behavior of an ESFs manager who signs new three party agreements, here called the fees-for-guarantee swap and the fees-for-seed swap, with a seeder and an insurer. An insurer was firstly introduced in Chapter 3, through fees-for-guarantee swaps, to solve the problem of information asymmetry by enhancing the ESFs manager's credibility. These innovative swaps effectively align the interests of the seeder with those of the ESFs manager. Moreover, quantitative results reveal the nonlinear relationship between the fees-for-seed ration and managerial stake, which is different from the "thumb rule" in practice except the case when ESFs managers face no endogenous liquidation risks.

Chapter 4 has developed a dynamic tractable framework based on the ESFs manager's portfolio choice problem to capture important factors in the hedge fund seeding business such as illiquidity, risk attitude, managerial skill (alpha) and compensation. Numerical results illustrate that: **(1)** the risk averse ESFs manager suffers the illiquidity discount for her valuation due to unspanned idiosyncratic risk; **(2)** the ordinary investors are more willing to invest in an ESFs backed up by seeders via a fees-for-seed swap; **(3)** the more seed capital the fund gets the smaller breakeven alphas the ordinary investors demand for their investment, which means the seeding vehicle helps the ESFs attract more investors and get sufficient capital for a successful launch.

5.2 Further research

Having been built on the foundation of thesis, my plans for further research will continue using the above CCAPM approach to study the effects of financing innovation on managerial compensation, investment allocation and risk management under incomplete information framework. In this section, I will conduct further research in the following three aspects:

(1) Hedge funds seeding innovation under incomplete information

According to Grossman (September 29,2005), an investment in a hedge fund is really an investment in a manager and the specialized talent he possesses to capture profits from a unique strategy. Therefore, it is vital for both investors and managers to correctly assess the manager's skill, alpha, from a quantitative perspective, see in Lan, Wang, and Yang (2013). Unlike Lan, Wang, and Yang (2013) who assume the managerial skill, alpha, is observable constant variable, I develop a model under incomplete information setting featuring the uncertainty of alpha that is unobservable for both investors and the manager. Therefore, correctly updating beliefs via learning of the fund's alpha based on the history information of the fund is essential for agents to price contingent claims. In the spirit of Goetzmann, Ingersoll, and Ross (2003) and Lan, Wang, and Yang (2013), we extend their models into the hedge fund seeding business in an incomplete market, which facilitates quantitative analysis of dynamic learning, hedge fund seeding costs, management compensation, and dynamic investment strategies in a partially observable economy. Similar Dothan and Feldman (1986), Detemple (1986), and Gennotte (1986) among others, we have established a particular partial information setup by simply declaring the alpha unobservable, meanwhile offering a pivotal deviation from a complete information setting.

(2) Entrepreneurial finance with asymmetric information

Both Yang and Zhang (2013) and Wang, Yang, and Zhang (2015b) have shown the merits of the swap, one might ask why equity-for-guarantee swaps have not gained widespread acceptance. One possible answer is that, due to asymmetric information among three parties: i.e. the entrepreneur, the bank, and the insurer, the swap might involve significant risks in terms of both moral hazard and adverse selection. To shed more light on this, I would continue using the partial information framework to build a dynamic model that captures asymmetric information and its effect on entrepreneurs' consumption and investment decisions.

(3) Optimal long term financial contracting

Current research in this area feature learning as uncertainty arises if the project quality or agent ability is unobservable, see He et al. (forthcoming) and Prat and Jovanovic (2014) among others. DeMarzo and Fishman (2007) presents a theory of a firms investment dynamics in the presence of agency problems and optimal long-term financial contracts. As stated in DeMarzo and Fishman (2007) that"Firms and investors seek contractual ways to mitigate these problem", I plan to extend DeMarzo and Fishman (2007)'s model to analyze the effects of innovative financial contracts (e.g. equity-for-guarantee swaps and fees-for-seed swaps) on firms investment and its role of mitigating agency conflicts This future work will help to explain the popularity of high-water-mark used in hedge funds as benchmarks for profit sharing.

Appendix A

Equilibrium Valuation and Proof of Theorem 2.3.1

A.1 Equilibrium Valuation of Corporate Securities

In the previous text the entrepreneur's subjective value $G(y)$ of equity owned by the entrepreneur is obtained by solving numerically the nonlinear ODE (2.24) with two free boundaries y_d and y_u, which is derived from a utility maximization problem. Except for $G(y)$, other corporate securities summarized below are valued by equilibrium valuation approach which provides market values since these securities are held by diversified investors.

Generally speaking, to determine equilibrium valuation, one needs to fix a *martingale pricing operator* (Ingersoll (2006)), or a *state-price deflator* (Duffie (2001)). There are infinite state-price deflators in an incomplete market. To fix a deflator one can solve a single-agent optimization problem and take the marginal utility of the agent as the special state-price deflator (Duffie (2001)). This deflator is the equilibrium martingale pricing operator used in Ingersoll (2006), Goetzmann, Ingersoll, and Ross (2003) and essentially also in Merton (1976) if the agent selected is the representative agent.

With regard to the problem discussed here, the specially chosen state-price deflator π satisfies $d\pi = -r\pi dt - \eta\pi dZ,\ \pi_0 = 1$. The risk-neutral probability measure $\mathbb{Q}$ is equivalent to $\mathbb{P}$ with the restrictions to $\mathcal{F}_t$ for any $t \geq 0$. Restricted to $(\Omega, \mathcal{F}_t)$, we have $\lambda_t = \frac{d\mathbb{Q}}{d\mathbb{P}}$ and $\lambda_t = \exp(rt)\frac{\pi_t}{\pi_0}$, see Duffie (2001).

Denote by ρ the correlation coefficient between the project and the market portfolio, by $\nu \equiv \mu_y - \rho\sigma\eta$ the risk-adjusted drift rate of the project, and $B_t^{\mathbb{Q}}$ a standard Brownian motion satisfying $dB_t^{\mathbb{Q}} = dB_t + \eta dt$. Then under $\mathbb{Q}$, the dynamics of the revenue in (2.2) can be rewritten as

$$dy_t = \nu y_t dt + \rho\sigma y_t dB_t^{\mathbb{Q}} + \epsilon y_t dZ_t, \quad y_0 \text{ given.} \tag{A.1}$$

According to the dynamic asset pricing theory (Duffie (2001)), one can derive the following equilibrium price (value)

$$V^f(y) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} \exp\left(-r(s-t)\right) f(y_s) ds \,|y_t = y\right] \tag{A.2}$$

for any time-independent claim underlying the revenue y of the project with a payment flow $f(y_t)$ to the claimant. If $f(\cdot)$ is a linear function, $V^f(y)$ can be determined by solving the integral directly. Alternatively, the function $V^f(y)$ satisfies the following ODE from Ito's formula:

$$\nu y V_y^f(y) + \frac{1}{2}\sigma^2 y^2 V_{yy}^f(y) + f(x) - rV^f(y) = 0. \tag{A.3}$$

One can identify the solution of function $V^f(y)$ by specifying the general solution of the homogeneous ODE with boundary conditions.

Firstly, the unlevered public firm value $A(y)$ is immediately given by

$$A(y) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} \exp\left(-r(s-t)\right)(1-\tau_m) y_s ds \,|y_t = y\right] = (1-\tau_m)\frac{y}{r-\nu}. \tag{A.4}$$

Secondly, the two real roots, denoted by θ_1 and θ_2, of the quadratic equation $\frac{1}{2}\sigma^2\theta(\theta - 1) + \nu\theta - r = 0$ are given by

$$\theta_{1,2} = \frac{-(\nu - \sigma^2/2) \mp \sqrt{(\nu - \sigma^2/2)^2 + 2r\sigma^2}}{\sigma^2}, \tag{A.5}$$

where obviously $\theta_1 < 0$ and $\theta_2 > 1$.

Define stopping time $\tau_d = \inf\{t \geq 0 : y_t \leq y_d\}$ which is the first passage time when the stochastic cash flow hits the bankruptcy threshold y_d. Denote by $\tilde{q}_d(y)$ a security that claims one unit of account at the default time without cash-out option, then we have

$$\tilde{q}_d(y) = \mathbb{E}^{\mathbb{Q}}[e^{-r(\tau_d - t)}|y_t = y] = \left(\frac{y}{y_d}\right)^{\theta_1}. \tag{A.6}$$

Thus, the equilibrium value of equity of a levered public firm is given by

$$\begin{aligned} E(y) &= \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\tau_d} e^{-r(s-t)}(1-\tau_m)(y_s - b)ds \,|y_t = y\right] \\ &= (1-\tau_m)\left(\frac{y}{r-\nu} - \frac{b}{r}\right) - (1-\tau_m)\left(\frac{y_d}{r-\nu} - \frac{b}{r}\right)\tilde{q}_d(y). \end{aligned} \tag{A.7}$$

Naturally, the market value of the outside equity of an entrepreneurial firm without cash-out option is given by

$$E_0(y; y_d) = (1-\tau_e)\left(\frac{y}{r-\nu} - \frac{b}{r}\right) - (1-\tau_e)\left(\frac{y_d}{r-\nu} - \frac{b}{r}\right)\tilde{q}_d(y). \tag{A.8}$$

The endogenously determined default threshold of levered public firm, denoted by y_d^p, solves a maximum-equity-valuation problem, which is given by (see Chapter 11 of Duffie (2001))

$$y_d^p = \frac{\theta_1}{\theta_1 - 1}\frac{(r-\nu)b}{r}. \tag{A.9}$$

Prior to default, the creditors receive continuous coupon payment b, and after default has taken place, they are entitled to the unlevered value of the firm deducting bankruptcy cost. Thus the value, denoted by $D(y)$, of the debt of the public firm is given by

$$\begin{aligned} D(y) &= \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\tau_d} e^{-r(s-t)}bds + \int_{\tau_d}^{\infty} e^{-r(s-t)}\alpha(1-\tau_m)y_s ds \,|y_t = y\right] \\ &= \frac{b}{r}\left(1 - \left(\frac{y}{y_d}\right)^{\theta_1}\right) + \alpha A(y_d)\left(\frac{y}{y_d}\right)^{\theta_1}. \end{aligned} \tag{A.10}$$

Therefore, the sum of $E(y)$ in (A.7) and $D(y)$ in (A.10) gives the total firm value as:

$$V(y) = A(y) + \frac{\tau_m b}{r}\left(1 - \left(\frac{y}{y_d}\right)^{\theta_1}\right) - \kappa A(y_d)\left(\frac{y}{y_d}\right)^{\theta_1}, \tag{A.11}$$

where the first term $A(y)$ is the unlevered firm value in (A.4), the second term is the expected present value of tax shields and the last term is the expected present value of bankruptcy costs.

The optimal coupon b^* is the solution of maximizing the total firm value over all admissible parameter values $b^* = \arg\max_b V(y)$. Then, the public firm value with the optimal coupon is (Chen, Miao, and Wang (2010)):

$$V^*(y) = \left[1 - \tau_m + \tau_m \left(1 - \theta_1 - \frac{\kappa(1-\tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}\right] \frac{y}{r-\nu}, \tag{A.12}$$

which is just the value of the firm when the entrepreneur exercises cash-out option.

Thirdly, the market value $F(y)$ of outside debt and the market value $E_0(y; y_d, y_u)$ of equity of the entrepreneurial firm with cash-out option are derived by equilibrium valuation as well. They depend on $\tilde{q}(y)$ (resp. $\bar{q}(y)$) which is the market value of a security that claims one unit of account at the default time before cash-out (resp. when cash-out occurs before default).

For a given domain $\mathcal{D} \equiv (y_d, y_u)$, we define a stopping time $\tau_{\mathcal{D}} = \inf\{t \geq 0 : y_t \notin \mathcal{D}\}$, which is the time of the first departure of y_t from the domain $\mathcal{D}$. At the stopping time, when default occurs before cash-out (resp. cash-out occurs before default), the security $\tilde{q}(y)$ (resp. $\bar{q}(y)$) claims one unit, which define the boundary conditions of the pricing equation (A.3) as follows.

$$\tilde{q}(y_d) = 1; \tilde{q}(y_u) = 0; \bar{q}(y_d) = 0; \bar{q}(y_u) = 1 \tag{A.13}$$

By a standard approach, the market values of $\tilde{q}(y)$ and $\bar{q}(y)$ are given by

$$\tilde{q}(y) = \frac{y^{\theta_2} {y_u}^{\theta_1} - y^{\theta_1} {y_u}^{\theta_2}}{{y_u}^{\theta_1} {y_d}^{\theta_2} - {y_u}^{\theta_2} {y_d}^{\theta_1}}, \tag{A.14}$$

and

$$\bar{q}(y) = \frac{y^{\theta_1} {y_d}^{\theta_2} - y^{\theta_2} {y_d}^{\theta_1}}{{y_u}^{\theta_1} {y_d}^{\theta_2} - {y_u}^{\theta_2} {y_d}^{\theta_1}}. \tag{A.15}$$

At the default trigger y_d, debt was guaranteed with level ϕ, in that $F(y_d) = \phi \frac{b}{r}$, and the shareholder gets nothing, i.e. $E_0(y_d) = 0$. At the cash-out trigger y_u, debt is retired and recovers its initial value F_0, in that $F(y_u) = F_0$, and the equity value $E_0(y_u) = V^*(y_u)$. Thus solving the pricing equation (A.3) with boundary conditions defined above gives

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right) \bar{q}(y) + (\phi - 1)\frac{b}{r}\tilde{q}(y), \tag{A.16}$$

and

$$E_0(y;y_d,y_u) = (1-\tau_e)\left(\frac{y}{r-\nu}-\frac{b}{r}\right)-(1-\tau_e)\left(\frac{y_d}{r-\nu}-\frac{b}{r}\right)\tilde{q}(y) + \left[V^*(y_u)-(1-\tau_e)\left(\frac{y_d}{r-\nu}-\frac{b}{r}\right)\right]\bar{q}(y). \quad (A.17)$$

Therefore the initial outside debt is given by solving the equation $F_0 = F(y_0)$, which gives

$$F_0 = \frac{b}{r} + (\phi-1)\frac{b}{r}\frac{\tilde{q}(y_0)}{1-\bar{q}(y_0)}. \quad (A.18)$$

The prediction of default is characterized by the cumulative probability of the firm going bankrupt over the period $(0,T)$, which is equivalent to the probability that the cash flow process y_t hits the default boundary y_d from above given the constant level y_0 within T. This is calculated as (see, e.g. Leland and Toft (1996))

$$\mathcal{P}_{y_d,T} = Prob(T_d < T) = N(\frac{x_d-\hat{\mu}T}{\sigma\sqrt{T}}) + exp(\frac{2\hat{\mu}x_d}{\sigma^2})N(\frac{x_d+\hat{\mu}T}{\sigma\sqrt{T}}), \quad (A.19)$$

where $x_d = log(\frac{y_d}{y_0})$, $\hat{\mu} = \mu_y - \sigma^2/2$, and $N(\cdot)$ denotes the standard normal cumulative distribution function. Similarly, the cash-out probability is

$$\mathcal{P}_{y_u,T} = Prob(T_u < T) = N(\frac{-x_u+\hat{\mu}T}{\sigma\sqrt{T}}) + exp(\frac{2\hat{\mu}x_u}{\sigma^2})N(\frac{-x_u-\hat{\mu}T}{\sigma\sqrt{T}}), \quad (A.20)$$

where $x_u = log(\frac{y_u}{y_0})$.

A.2 Proof hint of Theorem 2.3.1

The proof here is quite similar to that of Chen, Miao, and Wang (2010). After exit (via cash-out or default), the entrepreneur solves standard complete markets market comsumption/portfolio choice problem and her value function $J^e(w)$ is explicitly given by Equation (2.9), i.e.

$$J^e(w) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(w+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right)\right]. \quad (A.21)$$

Before exit, the entrepreneur bears unspaned idiosyncratic risk and her value function $J^s(w,y)$ satisfies the HJB Equation (2.16) due to the principle of optimality.

Now we conjecture that $J^s(w,y)$ takes the following exponential form

$$J^s(w,y) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(w + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right], \tag{A.22}$$

where $G(y)$ is the entrepreneur's certainty equivalent wealth.Under this conjectured value function, it is easy to show that the optimal consumption rule and the portfolio rule are given by Equations (2.22) and (2.23), respectively. Substituting these expressions back into the HJB Equation (2.16) gives the differential Equation (2.24) for $G(y)$.

Appendix B

Pricing a hedge fund

In this appendix, we here provide the details of computing the equilibrium values of contingent claims associated with a hedge fund.

In order to compute the equilibrium value of fees and investor claim (Equations (3.5) to (3.7)) defined in Section (3.2.1), two boundary conditions are required. The first boundary condition indicates that once the asset value falls to the liquidation level, $\underline{S}(H_t)$, then investors will withdraw all his money from the hedge fund and there are no further costs or fees generated.

$$A(\underline{S}(H_t), H_t) = 0, P(\underline{S}(H_t), H_t) = 0, I(\underline{S}(H_t), H_t) = \underline{S}(H_t), \tag{B.1}$$

where$\underline{S}(H_t) = lH_t$.

The other condition applies along the boundary $S_t = H_t$. The HWM is reset to $H_t + \varepsilon$, while the net asset value excess the HWM to $H_t + \varepsilon$ and then the manager obtain a performance fee of $k\varepsilon$, reducing the asset value to $H_t + \varepsilon(1 - k)$. So, we have

$$P(H_t + \varepsilon, H_t) - P(H_t + \varepsilon - k\varepsilon, H_t) = k\varepsilon, \tag{B.2}$$

In the limit, as $\varepsilon \to 0$, and using Taylor's expansion rule, giving the second boundary condition

$$kP_S(H_t, H_t) = k + P_H(H_t, H_t), \tag{B.3}$$

and boundary conditions for the regular annual fees and the investor's claim are given by

$$kA_S(H_t, H_t) = A_H(H_t, H_t), kI_S(H_t, H_t) = I_H(H_t, H_t). \tag{B.4}$$

According to the dynamic asset pricing theory (Duffie (2001)), one can derive the following equilibrium price

$$V^f(S_t, H_t) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\tau} \exp\left(-r(s-t)\right) f(y_s) ds \,|S_t = S_0, H_t = H_0\right] \tag{B.5}$$

for any contingent claim underlying AUM (S_t) and HWM(H_t) with a payment flow $f(S_t, H_t)$ to the claimant. If $f(\cdot)$ is a linear function, $V^f(y)$ can be determined by solving the integral directly. Similar to Goetzmann, Ingersoll, and Ross (2003), the value function is time independent, i.e. $V_t^f = 0$. Thus, the function $V^f(y)$ satisfies the following ODE from Ito's formula

$$(\alpha + r - \omega - m)SV_S^f + \frac{1}{2}\sigma^2 S^2 V_{SS}^f + (g - \omega - m')HV_H^f + \lambda(f(M) - V^f) - rV^f = 0. \tag{B.6}$$

One can identify the solution of function $V^f(S_t, H_t)$ by specifying the general solution of the homogeneous ODE with boundary conditions defined in Equations (B.1), (B.3) and (B.4).

Further, it is clear by the economics of the problem that V^f is homogeneous of degree one in S and H, so the solution has the form $V^f(S_t, H_t) = H_t G(s)$. Substituting this and its derivatives into Equation (B.6) gives an ODE

$$\frac{1}{2}\sigma^2 x^2 G_{ss} + (\alpha + r + m' - g - m)sG_s - (r + m' - g + \omega + \lambda)G + \gamma s = 0, \tag{B.7}$$

where $\gamma = 0$ for $f = P$, $\gamma = m$ for $f = A$ or $f = F$, and $\gamma = \omega + \lambda$ for $f = I$.

The solution to Equation (B.7) is given by

$$G(s) = \frac{\gamma s}{m + \omega + \lambda - \alpha} + As^{\theta_1} + Bs^{\theta_2}, \tag{B.8}$$

where A and B are constants of integration and the two real roots, denoted by θ_1 and θ_2 solve the following quadratic equation:

$$\frac{1}{2}\sigma^2\theta(\theta - 1) + (\alpha + r + m' - m - g)\theta - (r + m' - g + \omega + \lambda) = 0. \tag{B.9}$$

Solving the above equation and with the no bubble conditions $m + \omega + \lambda \geq \alpha$ easily gives

$$\theta_{1,2} = \frac{-(\varpi - \sigma^2/2) \mp \sqrt{(\varpi - \sigma^2/2)^2 + 2\sigma^2(\varpi + m - \alpha + \omega + \lambda)}}{\sigma^2}, \tag{B.10}$$

where$\varpi \equiv \alpha + r + m' - m - g$ obviously $\theta_1 < 1 < \theta_2$.

Therefore the equilibrium value of the total fees, performance fee and the investor's claim are

$$\begin{aligned} F(S_t, H_t) &= \tfrac{m}{m+\omega+\lambda-\alpha} S_t + \tfrac{(\omega+\lambda-\alpha)k+[\theta_1(1+k)-1]ml^{1-\theta_1}}{(m+\omega+\lambda-\alpha)\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}} {H_t}^{1-\theta_2} {S_t}^{\theta_2} \\ &- \tfrac{l^{\theta_2-\theta_1}(\omega+\lambda-\alpha)k+[\theta_2(1+k)-1]ml^{1-\theta_1}}{(m+\omega+\lambda-\alpha)\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}} {H_t}^{1-\theta_1} {S_t}^{\theta_1}. \end{aligned} \tag{B.11}$$

$$P(S_t, H_t) = k \frac{{H_t}^{1-\theta_2} {S_t}^{\theta_2} - l^{\theta_2-\theta_1} {H_t}^{1-\theta_1} {S_t}^{\theta_1}}{\theta_2(1+k) - 1 - l^{\theta_2-\theta_1}[\theta_1(1+k) - 1]}. \tag{B.12}$$

$$\begin{aligned} I(S_t, H_t) &= \tfrac{\omega+\lambda}{m+\omega+\lambda-\alpha} S_t - \tfrac{(\omega+\lambda)k+[\theta_1(1+k)-1](m-\alpha)l^{1-\theta_1}}{(m+\omega+\lambda-\alpha)\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}} {H_t}^{1-\theta_2} {S_t}^{\theta_2} \\ &+ \tfrac{l^{\theta_2-\theta_1}(\omega+\lambda)k+[\theta_2(1+k)-1](m-\alpha)l^{1-\theta_1}}{(m+\omega+\lambda-\alpha)\{\theta_2(1+k)-1-l^{\theta_2-\theta_1}[\theta_1(1+k)-1]\}} {H_t}^{1-\theta_1} {S_t}^{\theta_1}. \end{aligned} \tag{B.13}$$

Thus the annual fees $A(S_t, H_t)$ is given by

$$A(S_t, H_t) = F(S_t, H_t) - P(S_t, H_t). \tag{B.14}$$

Appendix C

Market Value of the Hedge Fund

After the lock-up period, we assume that the fund enters into a normal stage and the manager no longer bears the idiosyncratic risk, which is quite similar to the case discussed in Goetzmann, Ingersoll, and Ross (2003). In the normal stage, the manager is paid via both management and performance fees. The management fee is specified as a constant fraction m of the net asset value while the incentive fee is commonly accompanied by a high water mark (HWM) provision. Intuitively, the HWM H_t is the running maximum of net asset value S when $g \equiv \omega + c^{'}$, i.e. $H_t = max\{S_u; u \in [0,t]\}$. In a more general setting, for $S_t < H_t$, the HWM H_t evolves deterministically as

$$dH_t = (g - \omega - c^{'})H_t dt, \tag{C.1}$$

where g is the contractual growth rate at which H changes (generally zero or r) and $c^{'}$ is the cost or fees allocated to reducing the HWM.

At any time $t \geq 0$, we can compute the value of the total fees $F(S,H,t)$ and the ordinary investors' value $I(S,H)$ respectively as follows:

$$F(S,H,t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} e^{-r(s-t)}[mS + k(dH_s - (g - \omega - c^{'})H_s ds)]ds\right], \tag{C.2}$$

$$I(S,H,t) = \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{\tau} e^{-r(s-t)}\omega S + e^{-r(s-t)}S_{\tau} ds\right], \tag{C.3}$$

where τ, the stochastic liquidation time, is defined as $\tau = \min\{\tau_1, \tau_2\}$ where τ_1 is the exogenous liquidation time and $\tau_2 \equiv inf\{t; S_t/H_t = l\}$ is the endogenous liquidation time.

Using the same valuation approach as in subsection 4.3.1, the market values of the contingent claims defined above satisfy the following ODE[1]

$$(\alpha + r - \omega - m)SV_S + \frac{1}{2}\sigma^2 S^2 V_{SS} + (g - \omega - c^{'})HV_H + \lambda V - rV + f(S,t) = 0, \tag{C.4}$$

where $f(S,t)$ represents any payment made to the claims to be valued for the two different cases, and two boundary conditions apply as stated below.

Further, it is clear that the underlying economics of the problem implies that $V(y)$ is homogeneous of degree one in S and H, hence the solution has the form $V(S,H) = HQ(x)$, where $x \equiv S/H$. Substituting this expression as well as its derivatives into Equation (C.4) gives an ODE

$$\frac{1}{2}\sigma^2 x^2 Q_{xx} + (\alpha + r + c^{'} - g - m)xQ_x - (r + c^{'} - g + \omega + \lambda)Q + \theta x = 0, \tag{C.5}$$

where $\theta = m$ for the case of management fee or total fees, and $\theta = \omega + \lambda$ for the case of payoffs allocated to the ordinary investors.

The solution to Equation (C.5) is given by

$$Q(x) = \frac{\gamma x}{m + \omega + \lambda - \alpha} + Ax^{\vartheta_1} + Bx^{\vartheta_2}, \tag{C.6}$$

where A and B are constants of integration and the two real roots, denoted by ϑ_1 and ϑ_2 solve the following quadratic equation:

$$\frac{1}{2}\sigma^2 \vartheta(\vartheta - 1) + (\alpha + r + c' - m - g)\vartheta - (r + c^{'} - g + \omega + \lambda) = 0. \tag{C.7}$$

Solving the above equation and imposing the no bubble conditions $m + \omega + \lambda \geq \alpha$ [2]leads to:

$$\vartheta_{1,2} = \frac{-(\varpi - \sigma^2/2) \mp \sqrt{(\varpi - \sigma^2/2)^2 + 2\sigma^2(\varpi + m - \alpha + \omega + \lambda)}}{\sigma^2}, \tag{C.8}$$

where$\varpi \equiv \alpha + r + c' - m - g$ obviously $\vartheta_1 < 1 < \vartheta_2$.

[1]Similar to Goetzmann, Ingersoll, and Ross (2003), the values of the contingent claims are independent of time in our setup, i.e. $V_t \equiv 0$

[2]Similar to Goetzmann, Ingersoll, and Ross (2003) who state that, "the total withdrawals from the assets, $m + \omega + \lambda$, must exceed the superior performance, α; otherwise, the fund will have a residual value at infinity whose present value is infinite."

In order to solve the ODE, two boundary conditions are required. One boundary condition is determined as the asset value falls to the liquidation barrier, $\underline{x} \equiv l$:

$$\begin{aligned} i(l) &\equiv I(lH,H)/H = l, \\ f(l) &\equiv F(lH,H)/H = 0. \end{aligned} \tag{C.9}$$

The other condition applies along the boundary $\bar{x} \equiv 1$ when the HWM is reset to $H+\varepsilon$, while the net asset value exceeds the HWM at the level of $H+\varepsilon$ and then the manager obtains a performance fee of $k\varepsilon$, reducing the asset value to $H+\varepsilon(1-k)$. For $\varepsilon \to 0$, we have

$$\begin{aligned} i(1) &= (k+1)i'(1), \\ f(1) &= (k+1)f'(1) - k. \end{aligned} \tag{C.10}$$

One can identify the solution $V(S,H)$ by specifying the general solution of the homogeneous ODE with the two boundary conditions defined in equations (C.9) and (C.10).

Solving the above equation and applying the no bubble conditions $m+\omega+\lambda \geq \alpha$ provides us with:

$$\begin{aligned} I(S,H) &= \tfrac{\omega+\lambda}{m+\omega+\lambda-\alpha}S - \tfrac{(\omega+\lambda)k+[\vartheta_1(1+k)-1](m-\alpha)l^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_2}S^{\vartheta_2} \\ &+ \tfrac{l^{\vartheta_2-\vartheta_1}(\omega+\lambda)k+[\vartheta_2(1+k)-1](m-\alpha)l^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_1}S^{\vartheta_1}, \end{aligned} \tag{C.11}$$

$$\begin{aligned} F(S,H) &= \tfrac{m}{m+\omega+\lambda-\alpha}S + \tfrac{(\omega+\lambda-\alpha)k+[\vartheta_1(1+k)-1]ml^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_2}S^{\vartheta_2} \\ &- \tfrac{l^{\vartheta_2-\vartheta_1}(\omega+\lambda-\alpha)k+[\vartheta_2(1+k)-1]ml^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_1}S^{\vartheta_1}. \end{aligned} \tag{C.12}$$

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