THE VELOCITY OF SOUND IN AIR

Thesis submitted by

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Above - general view of the experiment.

Below - appearance of the oscillograph screen.
Upper trace - loud speaker pulse.
Lower trace - microphone pulse.

Contents

In the descriptions of previous experiments on the velocity of sound, the date given is that of the first publication of the results; where this date is not known with certainty, the date given is that of the journal or book consulted by the present writer.

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The lengths, temperatures and velocities which occur in the descriptions of previous experiments are expressed in the units employed by the original authors. A table showing the results of previous experiments, reduced to metres per second and to standard atmospheric conditions, is given in Appendix D. Here, as throughout this work, attention is confined to measurements made in free air.

Chapter 1

EARLY MEASUREMENTS OF THE VELOCITY OF SOUND

In considering the history of Physics, it is not surprising to find that no determinations of the velocity of sound were made until the seventeenth century. There were no accurate timepieces until the middle of that century, and measurements of intervals less than a few minutes were subject to considerable error. In the pages which follow, we shall be concerned chiefly with experiments conducted during the last three centuries; references to the subject in earlier times are neither numerous nor important.

The Greeks had some knowledge of sound, mostly based on speculation rather than on experiment. Aristotle considered that the speed of a sound depended on its pitch, a high note travelling more quickly than a low note. In this, he seems to have confused the speed of the sound with the speed of the vibrating element producing it.

Pliny the Elder expressed the opinion that the speed of light was very much greater than that of sound, and the same view was held by the Arab, Al-Biruni (see Sarton, 1927). The origin of these speculations was probably in the observation that there is a delay between a lightning flash and the sound of the thunder corresponding to it; Leonardo da Vinci considered this phenomenon in 1490 or thereabouts, and is said to have put forward a rule for finding the distance of a thunderstorm by observing the interval between the flash and the sound (see Richter, 1939).

The Seventeenth Century: Mersenne.

The first numerical determinations of the velocity of sound appear to have been made by Mersenne (1588-1648). Mersenne, who was a monk of the Fransiscan Order, lived in Paris and carried out many investigations in sound, as well as in mathematics and philosophy. His acoustical writings are contained principally in the great Harmonie Universelle, published in Paris in 1636. This work, which is written in French, consists of a number of tracts bound together in one volume;

parts of it were later revised and abridged for publication in Latin. Mersenne's experiments are discussed in detail in Appendix A, but they may be summarised as follows.

He found the velocity of sound by observing, with a pendulum, the time between the flash when a gun was fired and the arrival of the sound at an observer some distance away. The value obtained was 230 toises per second. In another series of experiments he found that a wall at a distance of 81 toises sent back an echo with a delay of one second, giving a value of 162 toises per second for the velocity of sound; thus a reflected sound travelled more slowly than a direct sound.

It is often stated that the velocity of sound was measured by Gassendi (1592-1655), who was a friend of Mersenne. This statement, however, appears to have no foundation in fact; its origin is discussed in Appendix A.

Accademia del Cimento. The next experiments to be considered were carried out in 1660 or thereabouts by members of the Accademia del Cimento.

An account of their experiments was published in Florence in 1667 and was translated into English by Richard Waller (1684). Their measurement of the velocity of sound is described in Waller's translation as follows (p.138):

> We made this experiment in the night, with three several sorts of Pieces, with a Harquebuss, a Falconet and a Demicannon, planted at Three Miles distance from the Place of Observation, whence we could discern the flash of the powder in firing the pieces; from the flash then, we counted always an equal Number of vibrations of the Pendulum of a Clock, whether the shot was of the Harquebus, or the Falconet or the Demicannon, and that upon all Levels and Directions of the Barrels of those pieces. . . . . . One of our Academy took occasion from these experiments to think that the motion of all sounds might be equable, as well as equally swift; we arguing that thence, if true, many curious and profitable hints might be gained, but first, to be fully satisfy'd if there were really any such equability, we made the

> following Experiments. At the distance of one of our Miles exactly measured, which is about 3000 of our Braccia or 5925 foot, we fired several pieces, that is, six Harquebusses and as many chambers; at each whereof from the flash to the arrival of the report we counted teh whole Vibrations of the pendulum, each of which was half a second. Repeating the experiment at half a mile distance, that is, at the midway, we observed it to be in exactly half the time, always counting at each Report, about five vibrations, wherefore we rested satisfyed of the certainty of this equability.

Since the sound in these experiments travelled 5925 feet in five seconds, its velocity was 1185 feet per second.

Boyle. The velocity of sound was mentioned by Boyle (1690) in his Essay of the Great Effects of Even Languid and Unheeded Motion (p.24):

> I have more than once diligently observed, that the motion of Sound passes above four hundred yards in .... a second here in England; which I therefore add, because Mersennus relates, that in France he observed a Sound to move in that

> time many yards more; which may possibly proceed from the differing consistence of the English air and the French.

There is no evidence that Boyle conducted any serious experiments to obtain this value.

Walker. Walker (1698) carried out some experiments in Oxford. He used a half-second pendulum, and made his measurements as follows:

> I took this pendulum and, standing over against a high Wall, clapt Two small pieces of Boards together and observed how long it was ere the Echo returned and I removed my Station till I found the place whither the Echo returned in about half a second.

Walker found an average value of 1305 feet per second.

Derham. A series of careful experiments was conducted by Derham (1708), who was Vicar of Upminster in Essex. His paper described experiments in timing the sounds of guns fired at Blackheath and heard at Upminster, using a half-second pendulum. In other experiments he timed the passage of sound over distances between 2 miles and 12·5 miles, and found the average velocity to be one mile in 9¼ beats of the pendulum, or 4·625 seconds. This corresponds to 1142 feet per second. Flamsteed and Halley, whose work is mentioned by Derham, but does not appear to have been published separately, found the same value, using a base line of three miles.[×]

To Derham is commonly attributed the discovery of a rule for finding the distance of a thunderstorm by counting the number of seconds between the flash of lightning and the sound of the thunder. This rule is, however, very much older, and its history is discussed in Appendix B.

The Eighteenth Century: The Paris Academy.

Two experiments were carried out by members of the Academie Royale des Sciences in Paris. Cassini and others (1700), by observations on gunfire, found that sound travelled 1280 toises in 7 seconds - a rate of 182·9 toises per second.

More accurate experiments were made by Cassini, Maraldi and La Caille (Cassini, 1738), who used base

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[×] Derham says that in the experiments of Flamsteed and Halley, sound took 13½ seconds to travel a distance of three miles between Shooter's Hill and Greenwich Observatory; these figures give 1173 feet per second as the velocity of sound.

lines between 1756 and 5788 toises, and made observations during a period of several months, which led them to the following conclusions:

1. The velocity of sound does not depend on the atmospheric pressure.
2. The velocity is the same at all distances from the source (that is, the sound travels at a constant rate).
3. Sound travels more quickly with the wind than against it.

Their result for the velocity of sound in still air was 173 toises per second. Later experiments (Cassini, 1739), at Aiguemortes, near Nimes, gave the value 173½ toises per second. The results obtained in these two experiments were recalculated by Le Roux (1867), who gives the value 332 metres per second.

Bianconi. Bianconi conducted a series of experiments near Bologna in 1740, timing the sound of gunfire with the aid of a pendulum. His experiments were described in Della diversa velocita del suono, one of two tracts in Due lettere di fisica . . . scritte dal Signor Gian-Lodovico Bianconi . . . Venice, 1746 - a rare work which the writer has not had the opportunity to consult.

A brief description was published in 1744 (Bologna, 1744) and the experiments have been discussed by Govi (1883).

Bianconi had one station on a hill to the South-West of Bologna and another at Fort Urbana, on the road to Modena, North-West of the city. He did not measure the distance accurately, but knew it to be about 13 Bologna miles; Govi (1883) calculated the base line, from maps, to be 25813 metres. In his two principal experiments, Bianconi found that sound from a gun travelled between the two stations in 76 seconds when the temperature was 20° Reaumur, and in 78 seconds at a temperature of -1·2°R. His results, recalculated by Govi (1883), who applied a correction for temperature, lead to a mean result at 0°C of 330·8 metres per second.[x]

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1745b, 1751) made two measurements of the velocity of sound. At Cayenne, firing a small cannon over a base line of 20230 toises, he found the velocity to be 183½ toises per second, after correcting for the influence of the wind. At Quito, he found the velocity to be 175 toises per second.

Kästner and Mayer (1778), experimented near Göttingen, with a base line which varied between 1649·2 Paris feet and 3218·8 Paris feet. They found values of 1034 and 1037 Paris feet per second for the velocity of sound at 47°F.

Müller (1791), made similar experiments near Göttingen, with a base line of 9116 Calenberg feet (8222·3 Paris feet) and found the velocity of sound to be 1040·3 Paris feet per second.

Chapter 2

MEASUREMENTS OF THE VELOCITY OF SOUND SINCE 1800.

The Nineteenth Century: Espinosa and Bauza.

Espinosa and Bauza made observations on the velocity of sound while exploring in Chile in 1794. Their results were published in 1817. Using base lines between 13841 and 50316 Paris feet, they reported an average value for the velocity of sound of 190·6 toises per second at a temperature of 23·5°C.

Benzenberg (1810) observed the firing of guns from distances of 14241 and 27906 Paris feet, and found the velocity of sound to be 1028·3 Paris feet per second, after a correction for temperature. After the publication of these results, Gilbert, who was then Editor of the *Annalen der Physik*, made some suggestions for greater accuracy and Benzenberg, acting on this advice, repeated the experiments and found, for the velocity at 0°C, 1026·8 and 1027·1 Paris feet per second. Benzenberg made his temperature correction by using the formula:

$$V_t = V_0(1 + 0{\cdot}00375t)^{\frac{1}{2}}$$

Applying this correction to the results obtained by the members of the Paris Academy in 1738, and taking the temperature then to have been 6°C, he recalculated their result to give 1024·9 Paris feet per second.

Arago (1822). In earlier experiments, there was usually an error due to the influence of the wind, which might accelerate or retard the progress of the sound. To eliminate this inaccuracy, it is necessary to make observations simultaneously from both ends of a base line. (The Academy in 1738 had observed from the two ends of their base line, but not simultaneously). In experiments for the Bureau des Longitudes, directed by Arago, observations were made with this precaution between Monthlery and Villejuif, near Paris, over a distance of 9549·6 toises. The average value for the velocity of sound was 174·9 toises per second at a temperature of 16°C.

Goldingham (1823). These experiments were carried out in Madras, where guns were fired every morning and evening from Fort St. George and from St. Thomas's Mount. Two base lines, of 29547 and 13932 feet, were used and the average result, corrected by Goldingham for humidity and reduced to 0°C, was 1079·65 feet per second.

Moll and van Beek (1824) made experiments with observers on two hills, Zevenboompjes and Kooltjesberg, near Utrecht. Their routine was as follows. Shortly before 8 p.m. a rocket was fired from the first station to indicate that the necessary preparations had been made. At 8 p.m. a cannon was fired at this station, and at 8.5 p.m. another. Both explosions were seen and heard at the second station, where the chronometer was set and checked by the two flashes. Finally each station fired a gun when its chronometer recorded 8.10 p.m., and the time between the flash and the sound was measured at the other station. The distance between the two observation points was 17669 metres and the final result, corrected for humidity and reduced to 0°C, was 332·05 metres per second.

These results were recalculated by van der Kolk (1865) who found 332·77 metres per second.

Gregory (1824) carried out experiments at Woolwich with cannons and muskets, the sound being timed over distances between 2100 and 6550 feet. The results varied between 1094·2 feet per second at 27°F and 1117 feet per second at 68°F.

Parry, Nias and Fisher (1825). The Rev. George Fisher was Chaplain and Astronomer to the second expedition made by ships of the Royal Navy, under the command of Captain Parry, with the purpose of finding a North-West Passage, in 1821 - 1823. He directed experiments on the velocity of sound, which were made in 1821 and 1822, using base lines between 2880 and 8466 feet, marked out on the frozen sea near Melville Island (65°N, 85°W, approximately). Fisher's results are interesting as the first to be made at low temperatures. He found the following values for the velocity of sound:

T°F	V, ft/sec
-41·3	985·9
-33·3	1011·2
-27·2	1009·2
-21	1031·2
- 2	1039·8
33·3	1069·9

Parry and Foster (1826). Lieutenant H. Foster, F.R.S., was a member of Parry's expedition to North-West Canada in 1824-5 and made a number of determinations of the velocity of sound with a six-pounder gun at Port Bowen, using a base line of 12982·9 feet. He found the following results:

T°F	V, ft/sec
- 7	1040·5
- 9	1037·3
-37	1029·0
-24·5	1021·0
-18	1039·2
-37·5	1010·3
-38·5	1010·4
-21·5	1026·6
33·5	1098·3
35	1118·1

Kendall (1828). Kendall was a member of Franklin's Arctic expedition in 1825-7. In 1825 he made measurements of the velocity of sound on the frozen surface of a small lake near Fort Franklin (65°N, 123°W, approximately), and found the following results:

$T^\circ C$	V, ft/sec
- 2·5	1112
-14·17	1089
-19·44	1079
-37·64	1036·1
-40·56	1030·3

Bravais and Martens (1845) considered the possibility that the velocity of sound might be different in the horizontal and vertical directions. They timed the sound of gunfire between a station on the Faulhorn and another on the shore of Lake Brienz, in Switzerland; their distance apart was 9560 metres and the difference in elevation was 2079 metres. The final value for the velocity, corrected for humidity and reduced to 0°C, was 332·37 metres per second.

Bosscha (1854) described a new method. Two small hammers, controlled by electromagnets, strike a pair of metal plates at a constant rate. The plates are some distance apart, and the sounds are heard at a point on the line through the two tappers. It is easily seen that the sounds will coincide if the distance between the tappers is equal to the distance travelled by sound in the time between two strokes. Bosscha did not obtain any numerical results.

König (1862) discussed the method and suggested some improvements, again without quoting any results. Szathmari (1877) made a series of careful experiments in the open, using two electric bells controlled by a pendulum, instead of the two tappers in the original proposal. His mean value, reduced for dry air at 0°C was 331·57 metres per second.

Regnault (1868) conducted a long series of experiments both in free air and in tubes. His methods involved finding the time taken for an explosive sound to traverse a known distance, and in this respect was not essentially different from that used by previous workers. In an attempt to eliminate the personal error of the observer, Regnault used electrical methods of recording the time.

A fragile diaphragm was arranged so that it broke when the sound wave passed it, on its way from the explosion; the rupture of the diaphragm opened an electrical circuit and permitted a pen to touch the surface of a rotating cylindrical drum. Also in the path of the sound, but further from the source, was a second diaphragm, more rigid than the first, which bent as the sound passed and, closing another circuit, caused a break to be made in the tracing of the pen on the drum. A comparison wave was also impressed on the drum by a vibrating tuning fork of standard frequency. In this way it was possible to measure the time occupied by the sound in travelling between the two diaphragms, and Regnault's result, reduced for dry air at 0°C, was 330·6 metres per second. Although this method appears to remove the personal error of the observer, it really does no more than to replace it by a similar delay which arises from the inertia of the diaphragms, electromagnets and other components which have been introduced. The low value obtained by Regnault for the velocity of sound suggests that this delay may be even more than that of a human observer.

Stone (1872) measured the velocity of sound in experiments with gunfire as the source. His base line, near Cape Town, was 14,808 feet and the value found for the velocity was 1090·6 feet per second.

Greeley (1890) made observations at low temperatures at Franklin Bay (70°N, 125°W approximately) and found the following results:

T°C	V, metres/sec.
-10·9	326·1
-25·7	317·1
-37·8	309·7
-45·6	305·6

The length of the base line was 1279·2 metres.

Frot (1898) made experiments of the conventional kind and found the velocity of sound reduced to 0°C (but without correction for humidity) to be 330·7 metres per second. His base line was 5565 metres long.

The Twentieth Century: Hebb (1905, 1919)

Hebb measured the velocity of sound indoors, under conditions approximating to those of free air. Two paraboloids, A and B, of plaster of Paris, were placed with their axes coinciding and their concavities facing one another. A carbon microphone was placed at the focus of A and another near the focus of B; each microphone was connected to one of the two primary windings of a transformer, which had a telephone receiver connected to its secondary coil. The source of sound was a whistle, blown with constant pressure at the focus of the paraboloid B. A stationary wave pattern was formed in the space between the two paraboloids and, on moving A, the sound received in the telephone went through a succession of maxima and minima. By measurements on the stationary wave system, the wavelength and hence the velocity of the sound could be estimated. Hebb used frequencies between 1280 and 3072 c/s and found the average value for the velocity of sound in dry air at $0^{\circ}C$ to be 331·41 metres per second.

Esclangon (1919) made experiments in the open air. Sound from a gun was detected at various distances by hot wire microphones, fitted with tuned resonators to increase the sensitivity. The average velocity measured over distances up to 14 km, was 339·8 metres per second at a temperature of 15°C.

Angerer and Ladenburg (1922) conducted similar experiments on a more extensive scale. Small charges of explosive were detonated from each end of a base line (nearly, though not quite simultaneously) and the sound was detected by a number of hot wire microphones situated along the line. Measurements made close to the explosion showed that, in the first 3 metres, the velocity was as much as 1150 metres per second, falling quickly to 340 metres per second at 50 metres from the source. The measurements used in the final calculation were all made between points at a distance from the explosion, where the abnormal velocity did not introduce errors, and the average result, for dry air at 0°C, was 330·8 metres per second.

Miller (1937) made observations on the velocity of sound from three different types of guns, fired at Sandy Hook, in New Jersey. The sound was picked up by a microphone, at a distance of 20,312 feet, connected to a recording string galvanometer. The mean value for the velocity of sound, from eleven observations made on three days in 1919, was 331·36 metres per second (for a temperature of $0^{\circ}C$ and zero humidity).

Finally, we consider the experiment of Kukkamäki (1922), who fired small explosive charges from the ends of a base line 3 km ling, and detected the sound by means of two microphones, situated 1 km apart on the base line. The microphones were connected to recording galvanometers, so that the time taken by the sound to pass between them could be estimated. The average value found for the velocity, reduced to $0^{\circ}C$ in dry air, was 330·77 metres per second.

In studying this long series of experiments, covering three centuries, we find that the method employed has, with one or two exceptions, remained the same throughout. Unfortunately, it is not a very good method and, although the precision of timing devices has greatly increased since Mersenne, the results of gunfire experiments are still subject to serious uncertainty. The principal weakness of the method lies in the impossibility of controlling -

or even of knowing - the temperature at every point along the path of the sound. When a base line of several miles is used, the fluctuations of temperature along it are, in the most favourable conditions, as much as 1°C and the uncertainty in the velocity of sound is therefore about 0·6 m/sec. This difficulty could be reduced only by placing a great number of recording thermometers along the path of the sound, and taking simultaneous observations of the temperature every few yards during the experiment.

It must be stated also that many authors have expressed their results in a way which implies an accuracy not possible with the methods used. For example, Colwell (1938), obtained sound pulses from a device connected to the 60 cycle main supply, (this frequency was not measured or checked), and detected them with a microphone which could be moved to various distances from the source. The position of the microphone was presumably measured with a ruler, though no mention is made of any instrument for this purpose. The temperature was recorded to 0·2°C - presumably by mercury thermometers, though no mention is made of the method used. The humidity was not measured at all, though, at the temperatures prevailing during the experiment, the presence of water vapour in the air could affect the velocity of sound by an appreciable fraction of one per cent. These uncertainties do not, of course, detract from the fundamental advantages of Colwell's method; but he is not justified in giving his result - 331·54 metres per second - in a form which suggests control of temperature to 0·02°C, measurement of humidity to the nearest millibar, and constancy of the frequency of the sound to one part in 30,000. This is an extreme case, but similar errors of lesser magnitude have been made by many others.

ADDENDUM

Myrbach and Stampfer made experiments near Salzburg over a base line of 30601 Paris feet, and found the velocity of sound to be 1025.9 Paris feet per second. Their experiments, which are mentioned by Gehler (1836, p. 397), are described in two obscure journals, which it was not possible to consult (Myrbach and Stampfer, 1824).

Chapter 3

PRINCIPLE OF THE METHOD

The principal difficulty in any free air determination of the velocity of sound is that of keeping a constant temperature in the experimental region. If an accuracy of one part in ten thousand is sought, the temperature must be known to 0·05°C - a condition not satisfied by any of the experiments so far described. Any experiment to measure the velocity of sound must be conducted in a space having one of its dimensions equal to several wavelengths, and a considerable volume of air is therefore necessary if an audible frequency is used. If a frequency in the supersonic region is used, the dimensions of the apparatus may be reduced to such an extent that the control and measurement of temperature with the required precision are readily attained. The advantage thus gained is partly offset by the necessity of applying corrections for dispersion (particularly at very high frequencies), and for the error introduced by the proximity of the walls of the containing vessel.

The object of the present experiment was to measure the velocity of sound in free air, at an audible frequency, with an accuracy of one part in ten thousand - comparable with that of the best supersonic measurements.

Consideration of the technique used in radar for the measurement of short intervals of time suggests that the cathode ray oscillograph might be useful in making precise measurements; experiments using this device have, indeed, been carried out by a number of investigators, though with no attempt at great accuracy. Wold and Stephenson (1923) used a loud speaker connected to an audio-frequency oscillator, delivering sound to two tubes placed side by side, each having a microphone at the other end. The output from one microphone was amplified and applied to the X deflecting plate of the oscillograph, while the output from the other, after amplification, provided a Y deflecting voltage, thus producing a Lissajous figure on the screen. One tube was adjustable in length and it was possible by observation on the Lissajous figure to find the distance corresponding to a given phase difference between the sound arriving at the two microphones. The wavelength and velocity could then be calculated; results obtained by this method have not been published.

Patchett's method (1943) is essentially similar, and uses one microphone, which is moved relative to the loud speaker to produce phase changes. A potential difference applied to the loud speaker is also applied between the X plates of an oscillograph and the microphone output, after amplification, provides the Y deflection. The procedure adopted by Patchett is to vary the distance between the loud speaker and the microphone, observing the position at which the Lissajous figure is a straight line, corresponding to phase identity. The interval between two successive positions of the microphone at which this occurs is equal to the wavelength of sound at the frequency employed; Patchett quotes only meagre results. It is not to be expected that either of these methods will give an accurate value for the velocity of sound. The phenomenon on which they depend - the production of a particular Lissajous figure - is not very sharply defined, and is affected by distortion in the loud speaker, microphones and amplifiers. A considerable uncertainty in the observations is therefore unavoidable.

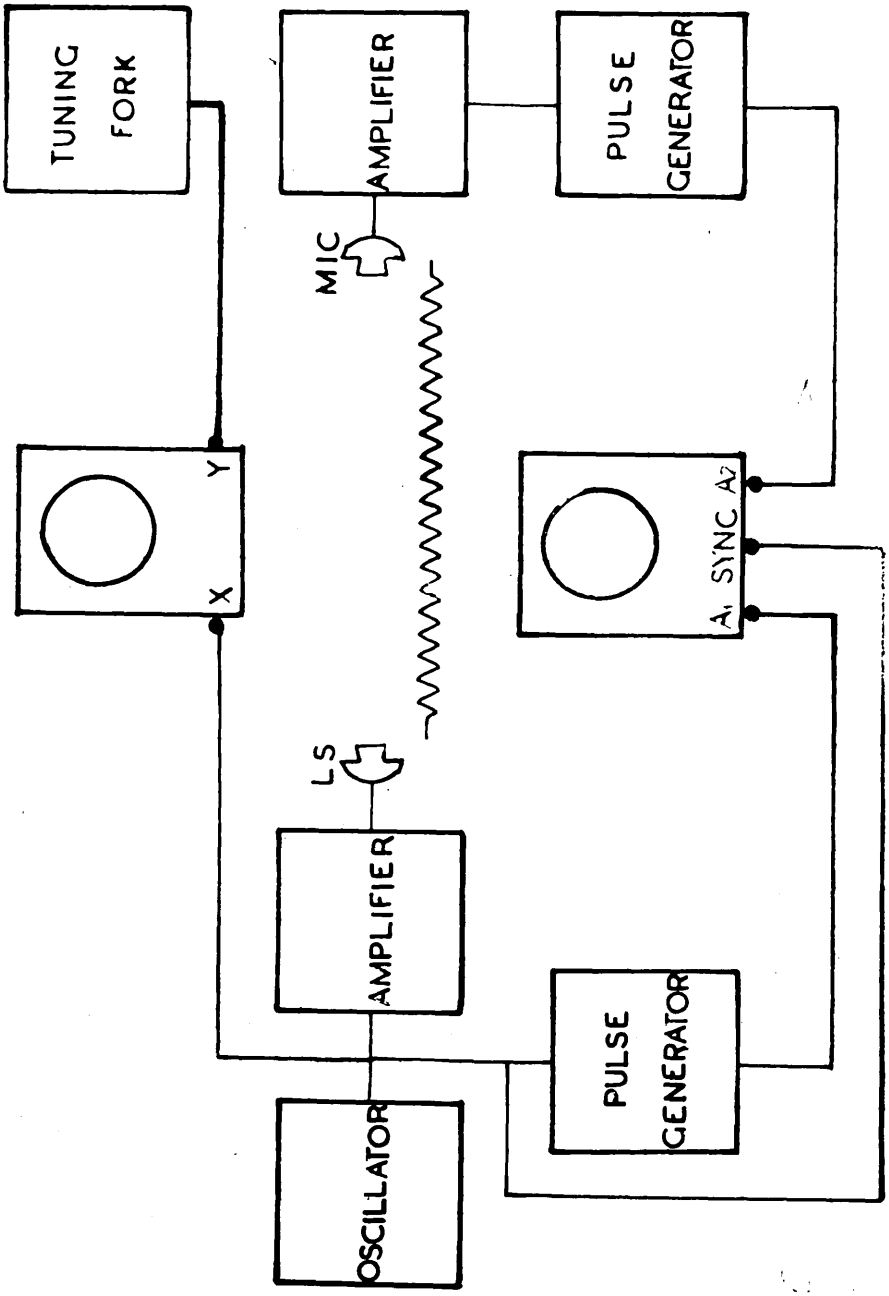
A more precise method has been described in several papers by Colwell and his collaborators (Colwell, 1938, 1939, 1940). Pulses derived from the alternating main supply are fed to a loud speaker. The sound produced is picked up by a microphone, amplified, and applied as a deflecting voltage to the Y plates of an oscillograph. The sweep voltage is also derived from the main supply and is therefore synchronised with the microphone pulses, so that a stationary trace appears on the screen. By observing the position of this trace, relative to a reference mark ruled on the screen, at various positions of the microphone, the wavelength can be found. The trace will cross the reference mark twice if the microphone is moved through one wavelength, since the fly-back takes place in the same time as the forward stroke in a sinusoidal sweep. The distance which the microphone moves between alternate transits of the trace across the reference mark is thus equal to the wavelength.

Using 60 c/s mains, the wavelength is about five and a half metres, and can be accurately measured. The position of the trace on the screen cannot be estimated with the same exactness because the pulse, if it is originally sharp, is broadened and distorted in its passage through the loud speaker, microphone and amplifier, and it becomes difficult to fix exactly the point on the trace corresponding to the beginning of the loud speaker pulse.

In a modification of this experiment, a sinusoidal tone, of various frequencies between 440 c/s and 1760 c/s was used as the source, measurements being made by observing coincidences of the crests of the received waves with the reference mark on the screen. In this way, several readings of the wavelength could be obtained on a bench a few metres long. It is, however, difficult to estimate precisely the position of the crest, since a sine wave has practically no slope near its peak.

The method now to be described is similar in principle to that of Colwell, but is capable of greater accuracy. An alternating voltage from an audio-frequency oscillator is amplified and fed to a loud speaker. The output from a movable microphone which picks up the sound is operated on by a pulse-shaping circuit and the pulses obtained are applied to the Y deflection plates of a cathode ray oscillograph, which may have an internal time base operating at a suitable frequency or may use the oscillator output for its sweep voltage. In either case, a stationary pattern is formed on the screen. The wavelength of the sound employed is found by measuring the distance which the microphone must be moved between successive positions at which the pulse coincides with an arbitrary reference mark on the screen. To avoid errors of observation due to parallax, it is advisable to use a double beam oscillograph in the following way. The microphone pulses are applied to one Y deflecting plate, as already described. The loud speaker voltage is connected to a second pulsing circuit and the pulses so formed are used to deflect the second beam of the oscillograph. One of the loud speaker pulses may then be used as a reference mark; a photograph of the trace obtained in this way is shown opposite. Measurements may also be made by observing coincidences of the fly-back trace with the reference mark.

Schematic arrangement of apparatus



Details of screw and traversing mechanism

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Chapter 4

APPARATUS AND EQUIPMENT

In order to carry out an experiment on these lines with a high degree of accuracy, the following requirements must be considered:

1. The location of the experiment.
2. The source of sound.
3. The control and measurement of its frequency.
4. The receiver.
5. The mechanism for moving the receiver, and measuring its position.
6. The control and measurement of temperature in the experimental region,
7. The measurement of humidity in the air.
8. The measurement of the carbon dioxide content of the air.
9. The power supplies and electrical circuits.

1. Location of experiment. It would, of course, be best to conduct the experiment in the open air, and an attempt to do so was made during the summer of 1946. It was found impossible to obtain a

sufficiently constant temperature and a sufficient calmness of the atmosphere for a reasonable length of time, and it was decided to set up the apparatus indoors. To simulate the conditions prevailing in free air, it is desirable to have a very large room, so that reflections of sound from the walls and ceiling will not be troublesome, and it is necessary also that there should be only small variations of temperature from hour to hour.

The site finally adopted for this experiment was the Bute Hall of the University of Glasgow, a room which is well suited to the purpose. It is a large chamber with walls of stone, bounded on the East and West by the open air, on the North by an empty landing leading to the Hunterian Museum, and on the South by the Randolph Hall, which was empty during the period of the experiment. It is isolated from the ground by a series of stone pillars, on which the floor rests.

The hall is 115 feet long, 69 feet wide and 67 feet high.

Before the experiments began, the Hall was cleared of all moveable furniture, and it was then possible to arrange the apparatus with a very large air space all around it.

2. Source of sound. The source should be fairly powerful, to avoid confusion at the receiver from extraneous sounds, and it should emit a narrow beam, to reduce the possibility of reflections from surfaces nearby. After a number of trials, a small moving coil headphone, two inches in diameter, was chosen. It was supplied with energy from an audio frequency oscillator, through a power amplifier.

The choice of frequency requires careful thought. In this experiment it was found that considerable reflections were produced by the walls and floor at frequencies below 10,000 c/s. At higher frequencies, the source emitted a narrower beam and the reflections diminished, but the efficiency of the source and the receiver also diminished; after a number of trials in the Bute Hall, a frequency of 13,500 c/s was adopted.

3. Control and measurement of frequency. The accuracy of this determination of the velocity of

sound depends to an important extent on the constancy of the frequency used, and on the accuracy with which it is known. The standard of frequency here was a valve-maintained tuning fork, supplied by Muirhead and Co., Ltd., and operating at 1000 c/s. This fork was run continuously during the experiment and was used to give the X deflecting voltage for a small oscillograph. The output from the audio frequency oscillator provided the Y deflection voltage, and a Lissajous figure was thus formed on the screen. This figure was observed at frequent intervals during the taking of observations, and was restored to a stationary form when necessary, by adjustment of the oscillator. It was found that after the fork and the oscillator had been running for about a week, the drift in the oscillator frequency was very slow, and its output could be kept to within one part in fifty thousand of the desired frequency without much difficulty.

The makers of the fork certified its frequency to be within three parts in a million of 1000 c/s on leaving the factory, and considered that the frequency stability, after the fork had been running for a few days, should be a few parts in a million.

4. The receiver. The receiver was a moving coil headphone identical with the transmitter, and was connected through a high-gain amplifier to a pulse shaping circuit, which will be described later.

5. Traversing mechanism and distance measurement. To obtain a reliable value for the wavelength of sound, and therefore for the velocity, it is necessary to measure very accurately the distance moved by the microphone between successive coincidences of the two pulses on the screen. The length measurement in this experiment was performed as follows. A steel screw, six feet long (actually the lead screw of a lathe, made for this experiment by Noble and Lund, Felling-on-Tyne) was rigidly mounted in a frame consisting of two lengths of steel girder with suitable end pieces welded on. A brass pillar, half an inch in diameter and a foot long, was fixed vertically at one end of the screw frame, and carried at its upper end the loud speaker. The microphone was attached by a similar pillar to a brass platform, six inches square, which rested on the screw, and could be moved back and forth along the screw by turning a handle at one end of the frame. This handle carried a circular scale divided into 500 parts, each representing a movement

of a thousandth of an inch. The screw had a small backlash, equal to about thirty divisions of the scale, but the error which might have arisen from this circumstance was obviated by taking one complete set of readings with the handle turning clockwise and the next with a counter-clockwise rotation; since the lengths required were differences between successive positions of the microphone, the backlash did not appear in the final readings.

With this arrangement, the microphone could be moved to within six inches of the loud speaker, or to a greater distance up to about 5' 6". Its position was recorded by counting the number of whole turns of the handle since the beginning of the traverse, and observing on the circular scale the number of thousandths remaining.

6. Temperature measurement. The control and measurement of temperature during the experiment is perhaps the most important single factor requiring attention if an accurate result is to be obtained. When using a large mass of air, it is difficult to keep the temperature constant by any system of

heaters and thermostats unless there is a vigorous circulation of air. A disturbance of this kind is undesirable in the present experiment, since it would produce movement of air in the space between the loud speaker and the microphone, and it is therefore necessary to rely on the structure of the building to keep the temperature constant. The Bute Hall is well insulated from the atmosphere outside, and from adjoining buildings, and it was found that the variations of temperature were not at all large. In many instances the temperature change during a period of ten minutes, which was sufficient to take one set of observations, was only 0·02°C.

Having thus obtained a room in which the temperature remained fairly constant, it was necessary next to arrange for its accurate measurement. For this purpose mercury thermometers were chosen, after some preliminary experiments with resistance thermometers and thermocouples. The thermometers used were made by C.F. Casella and Co., Ltd., to British Standard Specification 593 of 1935 (A40C/total), and had a range from -0·5°C to 40·5°C.

The thermometers, of which four were obtained, were divided to 0·1°C and were calibrated at the National Physical Laboratory to 0·02° C. Two of them, chosen from among the four for the smallness of their errors, were suspended by long threads from the ceiling of the building, so that their bulbs were at the same level as the loud speaker and microphone, and a few inches to the side of the direct path of the sound. It was found that the small amount of solar radiation which entered the Bute Hall was sufficient to produce a noticeable error in the readings of the thermometers, which were therefore fitted with radiation shields, consisting of cylinders of copper foil about an inch in diameter and two inches long, surrounding the bulb and attached to the stem by means of a cork. These shields also afforded protection against any draughts which might arise.

It was found that the reading of the thermometer changed by 0·02 or 0·04°C if an attempt was made to observe it directly, and it was therefore necessary to obtain the temperatures by observation from a telescope about six feet away. This method had the additional advantage that a magnified image was observed, with a corresponding increase in the accuracy of the readings. The temperature was estimated to 0·01°C, and was corrected in accordance with the N.P.L. certificate before the final results were calculated. There was also a correction for parallax, arising from the difference in level between the telescope and the thermometer bulbs. In order that the temperature of the air between the loud speaker and the microphone might not suffer variations due to currents near the floor, the apparatus was mounted on two wooden boxes about six feet high.

7. Measurement of humidity. The presence of water vapour in the air affects the velocity of sound in two ways, firstly by causing a change in the density of the atmosphere, and secondly by causing a change in the specific heat, C_v. The humidity during this experiment was measured by a small whirling hygrometer, which was read at intervals of about half an hour during the observations. The method of calculating the humidity, and of correcting the velocity of sound for the presence

of water vapour in the air, is described in a later chapter.

8. Influence of carbon dioxide in the air.
The air contains a small proportion of carbon dioxide, usually about three parts in ten thousand. The density of carbon dioxide is so large, compared with the density of air, that even this small amount produces a noticeable difference in the velocity of sound. Although there was no reason to believe that the carbon dioxide content of the air in the Bute Hall would be different from the accepted average value, it was considered necessary to make a determination, so that there should be no doubt on this point. A Haldane portable apparatus for carbon dioxide detection was used, and it was found that the proportion remained steady at three and a half parts in ten thousand, with slight variations from day to day.

9. Electrical circuits. The amplifiers and pulsing circuits were of conventional kinds, and are described in appendix C. A Cossor Double Beam Oscillograph was used for the observations on the pulses. The internal time base was employed, running at the same frequency as the audio frequency

oscillator, which provided a synchronising voltage. The pulses derived from the oscillator were applied to the A1 terminal of the oscillograph, and appeared on the upper beam, while the pulses derived from the microphone output were delivered to the A2 terminal and displayed on the lower beam.

Chapter 5

METHOD OF MAKING OBSERVATIONS

No readings were recorded until the oscillator and the tuning fork had been running continuously for a week, and could therefore be expected to have settled down at a stable frequency. The detailed procedure in making observations was as follows.

1. The fine frequency control of the oscillator was adjusted until the Lissajous figure on the screen became stationary at the shape corresponding to a ratio of 27:2.
2. The two thermometers were read in turn, observation being made through the telescope.
3. The hygrometer was whirled vigorously for about two minutes, and its two thermometers were read.
4. The telescope was moved to a position from which it could be used to observe the screen of the double beam oscillograph.
5. The platform carrying the microphone was moved to a position about five feet from the loud

speaker and was then moved forward, (towards the loud speaker), until the tips of the two pulses coincided on the screen of the oscillograph. The reading of the circular scale was noted.

6. The microphone was moved forward until the tips of the two pulses again coincided, and note was taken of the number of whole turns of the handle and of the fractional residue, indicated by the circular scale. The distance between this position of the microphone and the preceding one was equal to the wavelength of sound at 13,600 c/s.

7. This process was repeated until five coincidences of the two pulses had been recorded. The positions of the microphone at the coincidences were recorded, and numbered 1, 2, 3, 4, 5.

8. The microphone was then moved forward steadily and the coincidences of the pulses were counted; at number 41, the microphone was halted and its position was recorded, after allowing an interval of about a minute for any turbulence in the air to settle down.

9. The next four coincidences of the pulses were observed in the manner described, and the position of the microphone was noted at each. The number of whole turns between 5 and 41 having been counted,

it was now possible, by subtracting the first reading from the 41st, the 2nd from the 42nd, and so on, to obtain five estimates of a length equal to 40 wavelengths.

10. The frequency of the oscillator was checked by examining the Lissajous figure on the screen of the auxiliary oscillograph. The difference in frequency between the fork and the oscillator at this stage was usually less than one cycle in three seconds - that is, one part in 40,000. By adjustment of the oscillator fine frequency control, the ratio was restored to 27:2.

11. The microphone, now quite near the loud speaker, was moved forward two or three turns of the screw, and brought back to position 45 - approached in a direction opposite to that used in the forward traverse. The circular scale reading was noted, and differed from that found in the forward traverse by an amount representing the backlash of the screw.

12. The traverse was continued in the backward direction, until the microphone returned to its original position. Readings of its position were taken at coincidences 45 to 41 and again at

5 to 1. A further five values for 40λ were thus obtained.

The whole cycle of operations was then repeated to obtain further sets of values for 40λ. The wet and dry bulb thermometer readings varied only slightly during a period of two or three hours, and it was not necessary to record them after every set of readings; the hygrometer was operated about every half hour, and the humidity at intermediate times was found by interpolation.

Chapter 6

CALCULATION OF THE RESULTS.

In an experiment such as this, the final result must be referred to dry air under standard conditions of temperature and pressure; that is, a temperature of $0^{o}C$ and a pressure of one atmosphere. It is not possible in a free air measurement to maintain the atmospheric conditions at the standard values, and it is necessary to apply at the values of velocity observed a series of corrections, representing the changes brought about by the departure of the atmospheric conditions from the standard values. These corrections must be applied separately to each observation; in this experiment nearly a thousand observations were made, and the correction process was a considerable task.

To assess the various corrections, we must first know how the velocity of sound in air depends on the atmospheric conditions. For sound waves of small amplitude in a gas, the velocity of sound S is given by the equation

$$S = \left(\frac{C_p}{C_v} \cdot \frac{E}{d} \right)^{\frac{1}{2}}$$

where C_p and C_v are the thermal capacities of one mole of the gas at constant pressure and at constant volume respectively, E is the bulk modulus of the gas for isothermal changes and d is the density of the gas. Using the relations familiar in thermodynamics

$$E = -V \left(\frac{\partial p}{\partial V} \right)_T \qquad \text{and}$$

$$\frac{C_p}{C_v} = 1 - \frac{T}{C_v} \left(\frac{\partial p}{\partial T} \right)^2_V \left(\frac{\partial V}{\partial p} \right)_T$$

we find, following Hardy (1942):

$$S = \left[\frac{RT}{M} \left(f + \frac{gR}{hC_{V\infty}} \right) \right]^{\frac{1}{2}}$$

in which R = gas constant for one mole

T = temperature on Kelvin scale

M = molecular weight of the gas

C_{V_∞} = specific heat of the gas at constant volume for infinite dilution

V = volume of one mole

f, g, and h are constants equal to unity for a perfect gas and differing from unity (by an amount depending on the temperature) for any real gas. They are related to the other constants as follows:

$$f = \frac{EV}{RT}$$

$$g = \left(\frac{V}{R}\frac{\partial p}{\partial T}\right)^2_V$$

$$h = \frac{C_v}{C_{v\infty}}$$

The values of these constants for air at $0^\circ C$ and a pressure of one standard atmosphere are:

$f = 0{\cdot}998753$

$g = 1{\cdot}004503$

$h = 1{\cdot}000224$

The velocity of sound in air is influenced by the following factors.

1. The temperature, which produces three separate effects -
   (a) on the density, which is inversely proportional to the temperature on the Kelvin scale (for a perfect gas),

(b) on the imperfection of the gas,
that is, on the constants f, g,
and h discussed above,

(c) on the specific heat $C_{v\infty}$.

2. The presence of water vapour, which affects
   (a) the density of the air,
   (b) the specific heat $C_{v\infty}$.

In the present experiment it is necessary also to correct for

3. The known error of the screw.
4. The known error of the thermometers.

During this experiment, the temperature varies between 17·3°C and 18·7°C, and the humidity (expressed as the vapour pressure of the water vapour in the air) varies between 13·0 millibars and 14·2 millibars. Within these narrow limits, the corrections enumerated under (1) and (2) above all show a linear variation. This being so, the separate corrections may be calculated as though the others did not exist, and the total correction will be the sum of these components. The correction is carried out in the following stages:

A. The corrections to be applied under headings (1) and (2) above, for a temperature of 18°C and a humidity of 13·6 millibars are accurately found.

B. Calculations are then made of the differences in these corrections made by a change of 1°C in the temperature, above or below 18°C, and a change of 1 millibar in the humidity, above or below 13·6 millibars.

C. Knowing these differences, a first correction is applied to each of the values of 40λ, amending it for a temperature of 18°C and a humidity of 13·6 millibars. The values thus obtained all correspond to the same atmospheric conditions, and may be used in the statistical analysis which follows. When their mean has been found, this value is corrected for the change from 18°C to 0°C and from 13·6 millibars to 0 millibar, to give the final result of the experiment.

Before any of these corrections are made, it is necessary to define a standard atmospheric pressure, temperature and density, and to decide on the values to be adopted for other physical constants involved in the calculations. The following definitions were made, following the recommendations of Birge (1941):

1. Pressure. Standard atmosphere = 1013·2 millibars (1 millibar = 1000 dynes per sq. cm.)
2. Temperature. Ice point = 0°C = 273·16° Kelvin.

Composition of the atmosphere. The densities of the gases present in the atmosphere are as follows:

Nitrogen	1·251 grams per litre
Oxygen	1·429 grams per litre
Argon	1·783 grams per litre
Carbon dioxide	1·977 grams per litre.

It was found difficult to reconcile the percentage composition of the atmosphere given by the various authorities consulted, with the density of air given by the same, or other authorities; it was not practicable to analyse the air, and since a small uncertainty as to the composition is not significant in the present circumstances, the following compromise was adopted: the composition of the atmosphere by volume is

Nitrogen	78·04%
Oxygen	20·99%
Argon	0·94%
Carbon dioxide	0·03%

Using this composition, the density is calculated to be 1·29310 grams per litre.

Calculation of humidity. The vapour pressure of the water vapour in the air is obtained from the wet and dry bulb readings by the use of Regnault's formula (Regnault, 1845):

$$p = s - AP(t_1 - t_2)$$

in which p = vapour pressure

s = saturation vapour pressure at the temperature of the wet bulb

A = a constant, depending on the way in which the wet bulb is ventilated

P = atmospheric pressure

t_1 = dry bulb temperature

t_2 = wet bulb temperature

The constant A is taken to have the value 0·37 (Meteorological Office, 1940).

Density of air during the experiment. We next calculate the ratio of the density of dry air at 0°C to the density of air 18°C with a humidity of 13·6 millibars. Correcting first for the presence of the water vapour we have:

	volumes	density at 0°C.	contribution to density of air.
water vapour	13·6	0·8045	10·9
air	999·6	1·2931	1292·6
	1013·2		1303·5

The density at 0°C of air with humidity 13·6 millibars is thus

$$\frac{1303\cdot5}{1013\cdot2} = 1\cdot2865 \text{ grams per litre.}$$

When corrected for the change in density between 0°C and 18°C, this figure becomes

$$1\cdot2865(1 + 18/273\cdot16)^{-1} = 1\cdot2071 \text{ grams per litre.}$$

This calculation involves the assumption that the air and the water vapour behave as perfect gases.

Proceeding in the same way, we find the following values:

			Vapour pressure of water vapour in the air	
			13·6 mb	14·6 mb
T	17°C	density of air	1·2113	1·2109
	18°C		1·2071	1·2066
	19°C		1·2028	1·2024

This table refers to an atmospheric pressure of 1013·2 millibars; further calculation shows that the ratio between the density of dry air at 0°C and the density of moist air near 18°C and 13·6 millibars humidity is not altered significantly by a change of 10 millibars in the atmospheric pressure; the variations of this quantity are ignored in the calculations which follow.

Numerical considerations. The wavelength of sound at a frequency of 13500 c/s, under the atmospheric conditions prevailing in this experiment, is very nearly one inch, and the value of 40 λ, expressed in the units of 0·001 inch, given by the screw scale, is 40,000.

It will be convenient from this point to express the corrections in the form n parts in 40,000; the final value of 40 λ is so close to 40,000 (actually about 39998) that this procedure is quite accurate.

The correction to be applied for a change of temperature of 1°C near 18°C is found as follows:

density of dry air at 0°C = 1·2931

density of air at 18°C with

humidity 13·6 millibars = 1·2071

ratio of densities = $\frac{12071}{12931}$ = 0·93350 factor by which velocity at 18°C must be reduced

$$= (0{\cdot}93350)^{\frac{1}{2}} = 0{\cdot}96618$$

if the temperature is 19°C and the humidity is still 13·6 millibars, the factor is

0·96645

if the temperature is 19°C, the factor is

0·96785

Thus the factor by which the experimental value of 40 λ must be multiplied to give the value corresponding to dry air at 0°C is

0·96785 when the temperature is 17°C
and 0·96618 when the temperature is 18°C

The difference between these factors is 0·00167 and the difference between the correction terms for air at 18°C and air at 17°C is therefore

$$40{,}000 \times 0{\cdot}00167 = 66{\cdot}8 \text{ units.}$$

Similarly, the difference between the correction terms at 18°C and 19°C is

69·2 units.

The variations of temperature above and below 18°C are quite small in this experiment, and we therefore take a mean value of 68 units. The first correction for temperature is therefore: for every degree above or below 18°C at the time of observation, we subtract 68 units from, or add 68 units to the figure actually found for 40 λ.

Effect of humidity on density of the air. From the foregoing calculations (page 55), we see that, if the density of air at 18°C and a humidity of 13·6 millibars is taken as unity, the density at other hygrometric conditions is as follows:

18°C and 14·6 mb	1·0004
17°C and 13·6 mb	1·0035
17°C and 14·6 mb	1·0031
19°C and 13·6 mb	0·9965
19°C and 14·6 mb	0·9961

Thus the effect on density of a change of 1 millibar in the humidity near 18°C and 13·6 millibars is 4 parts in 10,000. The velocity of sound, depending on the square root of the density, will alter by 2 parts in 10,000 or 8 parts in 40,000.

Variation of C_{v_∞} with humidity and temperature.

The influence of temperature on the value of C_{v_∞} is very small, and is adequately dealt with by a single correction applied at the end of the calculations. The effect of humidity on the value of C_{v_∞} is more important, and is more difficult to evaluate, since it depends both on the humidity and on the temperature. We find first the value of C_{v_∞} for dry air of the standard composition, using the known values of C_{v_∞} for the component gases

	volumes	C_{v_∞}	contribution to C_{v_∞} of air
nitrogen	0·7804	4·9704	3·8789
oxygen	0·2099	5·0083	1·0512
argon	0·0094	2·98	0·0279
carbon dioxide	0·0003	6·6	0·0020
	1·0000		4·9600

For moist air, at 13·6 millibars humidity, the proportion of water by volume is 13·6/1013·2 = 1·342%, and the value of C_{v_∞} is found by a similar calculation:

	volumes	$C_{v\infty}$	contribution to $C_{v\infty}$ of air
air	0·98658	4·9600	4·8934
water vapour	0·01342	6·2	0·0833
	1·00000		4·9767

The term on which the velocity of sound depends is (page)

$$\left(f + \frac{gR}{nC_{v\infty}}\right)^{\frac{1}{2}} = \left(0{\cdot}998753 + \frac{1{\cdot}004503 \times 1{\cdot}9868}{1{\cdot}000224 \times C_{v\infty}}\right)^{\frac{1}{2}} = X$$

For dry air, $C_{v\infty}$ = 4·9600 and X = 1·18364. For air with humidity 13·6 millibars, X = 1·18308. The difference between these terms is 0·00056, and the corresponding correction to be applied to the observed values of 40 λ is therefore

$$\frac{0{\cdot}00056}{1{\cdot}18364} \times 40{,}000 = \underline{18{\cdot}9 \text{ units}}$$

This correction is to be applied to the average value of 40 λ after the other corrections have been made.

We now consider whether this correction term depends on the humidity. Proceeding as before, we find that the value of $C_{v\infty}$ for air with a humidity of 14·6 millibars is

4·9780

and the value of X at this humidity is

$$1{\cdot}18301.$$

For a humidity of 13·6 millibars, we have already found that

$$X = 1{\cdot}18308$$

The difference between these two terms is 0·00007, and the correction to be applied to the observed values of 40 λ is therefore

$$\frac{0{\cdot}00007}{1{\cdot}18364} \times 40{,}000 = 2{\cdot}4 \text{ units, for a change}$$

of 1 millibar in the humidity near 13·6 millibars.

Correction due to error of the screw. The screw was checked by means of gauge blocks and a dial micrometer. The blocks were correct at 20°C and the calibration was performed at 17°C, when it was found that the screw was 0·003" short in its length of 6 feet. Now, the blocks, of steel, have a coefficient of linear expansion of 10^{-5} and at 17°C they are shorter than the indicated value by 3 parts in 10^5. In 6 feet, the length measured by the blocks is therefore 0·00216" short and the total deficiency of the screw is 0·00516". At 18°C, the temperature to which the measurements have been referred in the experiment, the screw is longer than at 17°C, by $72.1.10^{-5}$ = 0·00072". Thus the screw error at 18°C is 0·00444" in 6 feet = 2.5 parts in 40,000, and measurements made with the screw must be reduced by this amount to find the true value of 40 λ.

Correction due to error of thermometers. The two thermometers used in the experiment both gave readings which were 0·03°C high at temperatures near 18°C. The parallax error, produced by the difference in level between the thermometers and the telescope used to read them, made the temperatures apparently 0·08°C too low. The procedure adopted to find the correct temperature was therefore to add 0·05°C to the mean reading of the two thermometers.

Summary of corrections. The corrections which must be made to the observed values of 40 λ may now be summarised as follows:

Corrections to be applied to every observation of 40λ

1. Effect of temperature on density:

   for a rise of 1°C above 18°C,

   correction -68 units

2. Effect of humidity on density:

   for a rise of 1 millibar above 13·6 millibars

   correction - 8 units

3. Effect of humidity on C_v : for a rise of 1 millibar above 13·6 millibars,

   correction +2·4 units

Total effect of humidity: for a rise of 1 millibar above 13·6 millibars,

correction -5·6 units

After all of these corrections have been applied, the mean value of 40λ may be calculated and the following corrections are then applied to the mean:

4. Effect of temperature and humidity on density: multiply mean value of 40λ by 0·96618
5. Effect of humidity on C_v : correction to mean value of 40λ = +18·9 units
6. Effect of screw error: + 2·5 units

Two further corrections remain to be considered.

7. Variation of f, g, and h with temperature. This correction is small and it is sufficiently accurate to take Hardy's estimate, which is -0·032 metres per second in the final value of S, for a change in temperature from 18°C to 0°C.

8. Variation of C_v of air with temperature. Following Hardy again, this correction, when applied to the final value of S, is +0·046 metres per second.

Chapter 7

THE RESULTS

Using the procedure described in chapter 5, 94 sets of ten readings were taken, giving 940 values of 40 λ . The preliminary calculations made with the first set will be given in full as an example.

Positions of microphone, indicated by screw scale, at coincidences of pulses on the oscillograph screen.

	microphone moving forward		microphone moving backward
	whole turns	thousandths	
No. 1	0	407	367
2	2	406	368
3	4	405	368
4	6	404	366
5	8	404	366
•••			
41	80	366	332
42	82	366	331
43	84	365	331
44	86	364	330
45	88	363	330

Values of 40 λ , in thousandths of an inch (1 turn of screw = 500):

41 - 1 = 39959 39965
42 - 2 = 39960 39963
... etc.

Thermometer readings:

at beginning of observations: 1 = 17·40°C
2 = 17·36°C

at end of observations: 1 = 17·46°C
2 = 17·42°C

Mean = 17·41°C
thermometer error - 0·03°C
parallax error + 0·08°C
corrected mean temperature 17·46°C

Humidity: readings of whirling hygrometer -

dry bulb 66°F
wet bulb 59·3°F
depression 6·7°F

(The thermometers in the whirling hygrometer were not very accurate individually - 66°F is 18·9°C, and the dry bulb thermometer thus had an error of -1·5°C.

It was established, by observations with both bulbs dry, that the two thermometers had almost identical errors, and the depression of the wet bulb obtained from their readings was therefore considered to be reasonably accurate).

True dry bulb temperature = 17·46°C = 63·43°F

True wet bulb temperature = 63·43 - 6·7 = 56·73°F

S.V.P. at wet bulb temperature = 15·73 mb

relative humidity = 15·73 - 0.37.6.7.

= 13·2 mb

Corrections: temperature - 0.46 x 68 = 36.8 (positive)

humidity - 0.4 x 5.6 = 2.2 (positive)

These corrections, together with the amended values for 40 λ at 18°C and 13.6 mb, to which they lead, are summarised in sheet 1 of the results, and similar calculations were carried out for each of the 94 sets. These observations now follow.

sheet 1 temperature 17.46° C humidity 13.2 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39959	+36.8	+3.0	39998.8
60			99.8
60			99.8
60			99.8
59			98.8
64			40003.8
64			03.8
63			02.8
63			02.8
65			04.8

sheet **2** temperature **17.49°** C humidity **13.3** mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
39966	+34.7	+1.7	39992.4
66			92.4
67			93.4
67			93.4
69			95.4
67			93.4
67			93.4
67			93.4
67			93.4
65			95.4

sheet 3 temperature 17.54° C humidity 13.3 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39964	+31.3	+1.7	39997
64			97
66			99
66			99
68			40001
66			39999
68			40001
66			39999
67			40000
68			40001

sheet **5** temperature **17.66°** C humidity **13.5** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39971	+23.1	+0.6	39994.7
71			94.7
71			94.7
71			94.7
72			95.7
75			98.7
73			96.7
72			95.7
72			95.7
73			96.7

sheet 6 temperature 17.73° C humidity 13.5 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39973	+18.4	+0.6	39992
75			94
75			94
76			95
77			96
77			96
75			94
75			94
76			96
75			95

sheet 7 temperature 17.77° C humidity 13.6 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39971	+15.6	0	39986.6
73			88.6
73			88.6
74			89.6
74			89.6
78			93.6
78			93.6
78			93.6
80			95.6
78			93.6

sheet 8 temperature 17.86° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39970	+9.5	-0.6	39978.9
74			82.9
75			83.9
75			83.9
75			83.9
89			97.9
88			96.9
85			93.9
82			90.9
82			90.9

sheet 9 temperature 17.92°C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39983	+5.4	−0.6	39987.8
84			88.8
85			89.8
84			88.8
84			88.8
84			88.8
83			87.8
83			87.8
83			87.8
86			90.8

sheet 10 temperature 17.93° C humidity 13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39987	+4.8	-1.1	39990.7
86			89.7
85			88.7
87			90.7
89			92.7
90			93.7
92			95.7
91			94.7
90			93.7
90			93.7

sheet 11 temperature 17.96° C humidity 13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39987	+2.7	-1.2	39988.5
87			88.5
90			91.5
88			89.5
89			90.5
92			93.5
92			93.5
91			92.5
92			93.5
90			91.5

sheet 12 temperature 17.97° C humidity 13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39991	+2.0	-1.1	39991.9
90			90.9
91			91.9
92			92.9
92			92.9
93			93.9
94			94.9
92			92.9
90			90.9
89			89.9

sheet **13** temperature **18.12** C humidity **13.7** mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40000	-8.2	-0.6	39991.2
40001			92.2
39998			89.2
39998			89.2
39998			89.2
40000			91.2
40003			94.2
40001			92.2
40004			95.2
40003			94.2

sheet 14 temperature 18.13 C humidity 13.7 mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
40006	-8.9	-0.6	39996.5
05			95.5
04			94.5
03			93.5
04			94.5
00			90.5
01			91.5
01			91.5
05			95.5
03			93.5

sheet 15 temperature 18.14° C humidity 13.7 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40001	-9.5	-0.6	39990.9
02			91.9
00			89.9
01			90.9
01			90.9
05			94.9
00			89.9
03			92.9
04			93.9
04			93.9

sheet **16** temperature **18.15°** C humidity **13.7** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40002	-10.2	-0.6	39991.2
05			94.2
03			92.2
05			94.2
07			96.2
05			94.2
06			95.2
06			95.2
07			96.2
04			93.2

sheet 17 temperature 18.15° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40003	-10.2	-0.6	39992.2
03			92.2
02			91.2
04			93.2
04			93.2
06			95.2
06			95.2
07			96.2
06			95.2
06			95.2

sheet 18 temperature 18.37 ° C humidity 13.8 mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
40015	-25.2	-1.1	39988.7
15			88.7
16			89.7
16			89.7
17			90.7
19			92.7
18			91.7
18			91.7
17			90.7
16			89.7

sheet 19 temperature 18.38° C humidity 13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40022	-26.2	-1.1	39994.7
20			92.7
21			93.7
22			94.7
23			95.7
14			86.7
14			86.7
15			87.7
14			86.7
13			85.7

sheet	temperature C	humidity mb
20	18.32°	13.8

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40010	-21.8	-1.1	39987.9
13			90.9
14			91.9
15			92.9
17			94.9
19			96.9
21			98.9
22			99.9
21			98.9
21			98.9

sheet 21 temperature 18.30°C humidity 13.8 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40012	-20.4	-1.1	39990.5
14			92.5
13			91.5
15			93.5
15			93.5
18			96.5
18			96.5
19			97.5
19			97.5
16			94.5

sheet 22 temperature 18.25° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40013	-17	-0.6	39995.4
12			94.4
14			96.4
16			98.4
17			99.4
13			95.4
12			94.4
12			94.4
11			93.4
10			92.4

sheet 23 temperature 18.22° C humidity 13.7 ^{mb}

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40012	-15	-0.6	39996.4
12			96.4
12			96.4
15			99.4
16			40000.4
14			39998.4
17			40001.4
17			01.4
16			00.4
16			00.4

sheet **24** temperature **18.21°** C humidity **13.7** mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40015	**-14.3**	**-0.6**	**40000.1**
14			**39999.1**
15			**40000.1**
15			**00.1**
15			**00.1**
15			**00.1**
15			**00.1**
15			**00.1**
17			**02.1**
16			**01.1**

sheet 25 temperature 18.21° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40006	-14.3	-0.6	39991.1
07			92.1
09			94.1
11			96.1
11			96.1
07			92.1
06			91.1
07			92.1
07			92.1
08			93.1

sheet 26 temperature 18.20° C humidity 13.7 mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
40004	-13.6	-0.6	39989.8
06			91.8
05			90.8
07			92.8
08			93.8
05			90.8
08			93.8
08			93.8
05			90.8
06			91.8

sheet 27 temperature 18.20 C humidity 13.7 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40010	-13.6	-0.6	39995.8
08			93.8
08			93.8
09			94.8
09			94.8
10			95.8
08			93.8
07			92.8
06			91.8
07			92.8

sheet 28 temperature 17.90° C humidity 13.3 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39992	+6.8	+1.7	40000.5
89			39997.5
87			95.5
89			97.5
88			96.5
89			97.5
88			96.5
86			94.5
85			93.5
85			93.5

sheet **29** temperature **17.92**° C humidity **13.3** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39994	+5.2	+1.7	40000.9
91			39997.9
89			95.9
89			95.9
90			96.9
91			97.9
92			98.9
92			98.9
92			98.9
93			99.9

sheet 30 temperature 17.97° C humidity 13.4 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39997	+2	+1.1	40000.1
93			39996.1
93			96.1
93			96.1
91			94.1
96			99.1
95			98.1
94			97.1
95			98.1
95			98.1

sheet 31 temperature 18.00° C humidity 13.4 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39995	0	+1.1	39996.1
93			94.1
92			93.1
93			94.1
93			94.1
40000			40001.1
39997			39998.1
98			99.1
96			97.1
96			97.1

sheet 32 temperature 18.00° C humidity 13.4 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39999	0	+1.1	40000.1
97			39998.1
96			97.1
96			97.1
97			98.1
40004			40005.1
40001			02.1
39998			39999.1
40000			40001.1
39999			00.1

sheet 33 temperature 18.00° C humidity 13.4 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39996	0	+1.1	39997.1
92			93.1
94			95.1
96			97.1
94			95.1
40001			40002.1
01			02.1
01			02.1
03			04.1
02			03.1

sheet 34 temperature 17.93° C humidity 13.4 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39987	+4.8	+1.1	39992.9
85			90.9
85			90.9
86			91.9
90			95.9
92			97.9
88			93.9
89			94.9
90			95.9
91			96.9

sheet 35 temperature 17.98° C humidity 13.4 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39989	+1.3	+1.1	39991.4
87			89.4
86			8.4
89			91.4
89			91.4
91			93.4
91			93.4
90			92.4
91			93.4
94			96.4

sheet 36   temperature 18.00° C   humidity 13.4 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39996	0	+1.1	39997.1
97			98.1
99			40000.1
95			39996.1
94			95.1
96			97.1
96			97.1
99			40000.1
40001			02.1
02			03.1

sheet 37 temperature 18.03 C humidity 13.5 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39991	-2	+0.6	39989.4
92			90.4
94			92.4
95			93.4
97			95.4
98			96.4
96			94.4
95			93.4
94			92.4
92			90.2

sheet 38 temperature 18.03° C humidity 13.5 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39980	-2	+0.6	39978.6
82			80.6
84			82.6
86			84.6
86			84.6
92			90.6
94			92.6
97			95.6
93			91.6
92			90.6

sheet 39 temperature 18.04° C humidity 13.5 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39987	-2.7	+0.6	39984.9
91			88.9
95			92.9
94			91.9
93			90.9
40000			97.9
02			99.9
01			98.9
39998			95.9
40002			99.9

sheet 40 temperature 18.02°C humidity 13.5 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39992	-1.1	+0.6	39991.5
95			94.5
97			96.5
95			94.5
95			94.5
96			95.5
89			88.5
88			87.5
88			87.5
89			88.5

sheet **41** temperature **18.02**° C humidity **13.6**$^{\text{mb}}$

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39989	-1.3	0	39987.7
89			87.7
90			88.7
88			86.7
87			85.7
40004			40002.7
01			39999.7
39999			97.7
97			95.7
94			92.7

sheet 42 temperature 18.09° C humidity 13.6 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39999	-6.1	0	39992.9
99			92.9
40002			95.9
04			97.9
08			40001.9
01			39994.9
00			93.9
01			94.9
01			94.9
01			94.9

sheet **43** temperature **18.12** C humidity **13.6**$^{\text{mb}}$

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40004	-8.2	0	39995.8
04			95.8
05			96.8
02			93.8
07			98.8
07			98.8
07			98.8
05			96.8
04			95.8
02			93.8

sheet **44** temperature **18.12**° C humidity **13.6** mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39995	-8.2	0	39986.8
97			88.8
97			88.8
99			90.8
40002			93.8
10			40001.8
07			39998.8
05			96.8
05			96.8
02			93.8

sheet 45 temperature 18.30° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40020	-20.4	-0.6	39999
18			97
18			97
20			99
24			40003
19			39998
18			97
18			97
20			99
18			97

sheet 46 temperature 18.30°C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40014	-20.4	.0.6	39993
15			94
18			97
18			97
17			96
19			98
20			99
20			99
22			40001
25			04

sheet	temperature	humidity
47	18.33°C	13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40019	-22.4	-1.1	39995.5
16			92.5
16			92.5
17			93.5
18			94.5
13			89.5
12			88.5
13			89.5
13			89.5
13			89.5

sheet 48 temperature 18.35° C humidity 13.8 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40016	-22.4	-1.1	39992.5
18			94.5
18			94.5
19			95.5
19			95.5
13			89.5
14			90.5
16			92.5
14			90.5
14			90.5

sheet 49 temperature 18.33°C humidity 13.8mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40016	-22.4	-1.1	39992.5
16			92.5
13			89.5
11			87.5
11			87.5
16			92.5
15			91.5
14			90.5
16			92.5
16			92.5

sheet 50 temperature 18.32° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40021	-21.7	-0.6	39998.7
22			99.7
19			96.7
19			96.7
22			99.7
18			95.7
19			96.7
19			96.7
18			95.7
16			93.7

sheet 51 temperature 18.31°C humidity 13.7mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40011	-21.1	-0.6	39989.3
12			90.3
15			93.3
15			93.3
13			91.3
15			93.3
14			92.3
11			89.3
11			89.3
10			88.3

sheet 52 temperature 18.28 C humidity 13.7 mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
40015	-19.1	-0.6	39995.3
15			95.3
15			95.3
16			96.3
17			97.3
18			98.3
16			96.3
16			96.3
15			95.3
14			94.3

53	18.21°		13.7	
sheet	temperature	C	humidity	mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40006	-14.3	-0.6	39991.1
10			95.1
11			96.1
13			98.1
13			98.1
14			99.1
12			97.1
11			96.1
10			95.1
09			94.1

sheet 54 temperature 18.20°C humidity 13.7 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40008	-13.6	-0.6	39993.8
08			93.8
08			93.8
08			93.8
10			95.8
12			97.8
13			98.8
13			98.8
12			97.8
11			96.8

sheet 55 temperature 18.20°C humidity 13.7 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40011	-13.6	-0.6	39996.8
10			92.8
08			93.8
09			94.8
10			95.8
06			91.8
04			89.8
03			88.8
02			87.8
04			89.8

sheet 56 temperature 18.18° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40006	-12.3	-0.6	39993.1
06			93.1
07			94.1
05			92.1
03			90.1
05			92.1
05			92.1
04			91.1
03			90.1
03			90.1

sheet 57 temperature 18.12° C humidity 13.7 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
40005	-8.2	-0.6	39996.2
03			94.2
04			95.2
03			94.2
02			93.2
03			94.2
02			93.2
01			92.2
01			92.2
01			92.2

sheet **58** temperature **18.10°** C humidity **13.7** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39999	**-6.8**	**-0.6**	**39991.6**
99			**91.6**
98			**90.6**
98			**90.6**
98			**90.6**
40001			**93.6**
39999			**91.6**
40000			**92.6**
00			**92.6**
00			**92.6**

sheet **59** temperature **18.10°C** humidity **13.7 mb**

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39997	-6.8	-0.6	39989.6
96			88.6
97			89.6
97			89.6
97			89.6
40001			93.6
00			92.6
39999			91.6
40000			92.6
39999			91.6

sheet 60 temperature 17.56° C humidity 13.0 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39964	+29.7	+3.4	39997.1
61			94.1
61			94.1
59			92.1
60			93.1
73			40006.1
69			02.1
68			01.1
69			02.1
70			03.1

sheet 61 temperature 17.62° C humidity 13.0mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39966	+25.8	+3.4	39995.2
64			93.2
62			91.2
61			90.2
61			90.2
70			99.2
68			97.2
68			97.2
69			98.2
67			96.2

sheet **62** temperature **17.67**°C humidity **13.0** mb

observed value of 40 λ	corrections temperature	humidity	corrected 40 λ
39964	+22.4	+3.4	39989.8
63			88.8
62			87.8
62			87.8
62			87.8
70			95.8
67			92.8
66			91.8
67			92.8
66			91.8

sheet 63 temperature 17.70°C humidity 13.0 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39965	+20.4	+3.4	39988.8
63			86.8
64			87.8
63			86.8
62			85.8
78			40001.8
76			39999.8
77			40000.8
76			39999.8
73			96.8

sheet 64   temperature 17.70° C   humidity 13.2 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39972	+20.4	+2.2	39994.6
70			92.6
67			89.6
68			90.6
66			88.6
68			90.6
71			93.6
72			94.6
71			93.6
71			93.6

sheet 65 temperature 17.70° C humidity 13.2 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39966	+20.4	+2.2	39988.6
66			88.6
64			86.6
64			86.6
65			87.6
69			91.6
71			93.6
72			94.6
70			92.6
69			91.6

sheet **66** temperature **17.80°** C humidity **13.3** mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39982	+13.6	+1.7	39997.3
82			97.3
81			96.3
80			95.3
80			95.3
87			40002.3
87			02.3
85			00.3
87			02.3
85			00.3

sheet 67 temperature 17.81° C humidity 13.3 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39983	+12.9	+1.7	39997.6
81			95.6
79			93.6
78			92.6
80			94.6
86			40000.6
84			39998.6
83			97.6
83			97.6
83			97.6

sheet 68 temperature 17.82° C humidity 13.3 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39984	+12.2	+1.7	39997.9
84			97.9
82			95.9
82			95.9
83			96.9
81			94.9
80			93.9
80			93.9
81			94.9
81			94.9

sheet **69** temperature **17.83°** C humidity **13.3 mb**

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39980	**+11.6**	**+1.7**	**39993.3**
80			**93.3**
79			**92.3**
78			**91.3**
78			**91.3**
88			**40001.3**
85			**39998.3**
87			**40000.3**
85			**39998.3**
85			**98.3**

sheet 70 temperature 17.97 °C humidity 13.5 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39988	+2.0	+0.6	39990.6
89			91.6
90			92.6
90			92.6
90			92.6
95			97.6
94			96.6
93			95.6
92			94.6
92			94.6

sheet **71** temperature **17.99°** C humidity **13.5** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39990	**+0.7**	**+0.6**	**39991.3**
92			**93.3**
92			**93.3**
91			**92.3**
92			**93.3**
98			**99.3**
97			**98.3**
96			**97.3**
94			**95.3**
94			**95.3**

sheet 72 temperature 18.00°C humidity 13.5 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39995	0	+0.6	39995.6
96			96.6
95			95.6
96			96.5
98			98.6
98			98.6
96			96.6
95			95.6
96			96.6
96			96.6

sheet 73 temperature 18.12° C humidity 13.6 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40004	-8.2	0	39995.8
04			95.8
06			97.8
05			96.8
07			98.8
12			40003.8
09			00.8
08			39999.8
07			98.8
06			97.8

sheet 74 temperature 18.15° C humidity 13.7 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40004	-10.2	-0.6	39993.2
06			95.2
08			97.2
08			97.2
07			96.2
10			99.2
11			40000.2
10			39999.2
07			96.2
08			97.2

sheet 75 temperature 18.16°C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40006	-10.9	-0.6	39994.5
06			94.5
06			94.5
06			94.5
05			93.5
10			98.5
07			95.5
08			96.5
08			96.5
08			96.5

sheet **76** temperature **18.17°** C humidity **13.7** mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40010	**-11.6**	**-0.6**	**39997.2**
11			**98.2**
11			**98.2**
12			**99.2**
13			**40000.2**
11			**39998.2**
13			**40000.2**
11			**39998.2**
10			**97.2**
08			**95.2**

sheet 77 temperature 18.19° C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40013	-12.9	-0.6	39999.5
15			40001.5
14			00.5
16			02.5
16			02.5
19			05.5
18			04.5
16			02.5
17			03.5
16			02.5

sheet 78 temperature 18.20°C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40009	-13.6	-0.6	39994.8
14			99.8
12			97.8
14			99.8
14			99.8
14			99.8
15			40000.8
15			00.8
15			00.8
14			39999.8

sheet 79 temperature 18.73° C humidity 14.2 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40048	-49.6	-3.4	39995
47			94
48			95
49			96
49			96
51			98
52			99
52			99
52			99
51			98

sheet 80 temperature 18.73° C humidity 14.2 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40049	-49.6	-3.4	39996
52			99
51			98
52			99
51			98
53			40000
54			01
54			01
53			00
53			00

sheet **81** temperature **18.00°** C humidity **13.7** mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39987	0	-0.6	39986.4
89			88.4
89			88.4
90			89.4
89			88.4
94			93.4
93			92.4
91			90.4
91			90.4
92			91.4

sheet 82 temperature 18.00°C humidity 13.7 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39992	0	-0.6	39991.4
94			93.4
94			93.4
93			92.4
96			95.4
93			93.4
94			92.4
96			94.4
96			94.4
94			92.4

sheet **83** temperature **18.01**°C humidity **13.7** mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39992	-0.7	-0.6	39990.7
95			93.7
96			94.7
95			93.7
95			93.7
97			95.7
99			97.7
98			96.7
96			94.7
97			95.7

sheet **84** temperature **18.00°**C humidity **13.7** mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39989	0	-0.6	39988.4
88			87.4
90			89.4
88			87.4
89			88.4
91			90.4
91			90.4
92			91.4
91			90.4
90			89.4

sheet **85** temperature **18.01** C humidity **13.7** mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39987	-0.7	-0.6	39985.7
87			85.7
86			84.7
89			87.7
88			86.7
88			86.7
87			85.7
80			78.7
88			86.7
88			86.7

sheet 86 temperature 18.07° C humidity 13.6 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
39995	-4.8	0	39990.2
94			89.2
93			88.2
95			90.2
95			90.2
97			92.2
98			93.2
94			89.2
96			91.2
97			92.2

sheet 87 temperature 18.08° °C humidity 13.6 mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
39997	-5.4	0	39991.6
97			91.6
98			92.6
98			92.6
40001			95.6
00			94.6
39998			92.6
97			91.6
97			91.6
40000			94.6

sheet 88 temperature 18.74°C humidity 14.2 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40047	-51	-3.4	39992.6
49			94.6
48			93.6
48			93.6
46			91.6
51			96.6
53			98.6
52			97.6
52			97.6
52			97.6

sheet 80 temperature 17.73° C humidity 13.2 mb

observed value of 40 λ	corrections temperature	corrections humidity	corrected 40 λ
39971	+20.6	+2.2	39993.8
70			92.8
69			91.8
69			91.8
66			88.8
69			91.8
69			91.8
69			91.8
68			90.8
70			92.8

sheet **90** temperature **18.74** C humidity **14.2** mb

observed value of 40 λ	corrections temperature	 humidity	corrected 40 λ
40049	-51	-3.4	39994.6
50			95.6
51			96.6
51			96.6
50			95.6
56			40001.6
53			39998.6
55			40000.6
55			00.6
56			01.6

sheet 91 temperature 18.73° C humidity 14.2 mb

observed value of 40 λ	corrections		corrected 40 λ
	temperature	humidity	
40050	-50.3	-3.4	39996.3
51			97.3
51			97.3
48			94.3
50			96.3
50			96.3
51			97.3
53			99.3
53			99.3
52			98.3

sheet 92 temperature 18.03°C humidity 13.6 mb

observed value of 40	corrections		corrected value of 40
	temperature	humidity	
39996	-2.0	0	39994
98			96
98			96
97			95
96			94
40005			40003
02			00
04			02
04			02
04			02

sheet 93 temperature 18.06°C humidity 13.6 mb

observed value of 40	corrections		corrected value of 40
	temperature	humidity	
40002	-4.1	0	39997.9
04			99.9
03			98.9
02			97.9
01			96.9
39994			89.9
93			88.9
93			88.9
94			89.9
96			91.9

sheet 94 temperature 18.00°C humidity 13.7 mb

observed value of 40	corrections		corrected value of 40
	temperature	humidity	
39988	0	-0.6	39987.4
88			87.4
89			88.4
90			89.4
89			88.4
91			90.4
91			90.4
92			91.4
91			90.4
89			88.4

Chapter 8

STATISTICAL EXAMINATION OF THE RESULTS.

We now make a statistical examination of the 940 results for 40 λ, obtained by the methods outlined in the foregoing chapters. The table on the following page shows the distribution of the results, set out in a convenient form for calculating their mean value and their standard deviation; the method used is as follows. The results all lie between 39978 and 40006 (in units of 0·001 inch, as usual). They are enumerated in groups, occupying the intervals 39978 - 39978·9, inclusive; 39979 - 39979·9: and so on, by steps of one unit up to 40006. For convenience in calculation, the mean value of the results contained in the interval between N and N + 0·9 is taken to be N + 0·45; this introduces slight errors in the smaller groups - but the algebraic sum of all such errors is not enough to affect the final calculation significantly. Inspection of the results and preliminary calculation shows that their mean value is about 39994.5; an approximate mean value of

39994.45 is assumed as a basis for the calculation, and is corrected to give the exact value, as will be shown later.

The first column in the table specifies the interval, x, within which there lie the number of values, y, given in the second column. The third column indicates the difference ξ between the assumed mean value and the centre of the interval x. The fourth column, used to find the true mean value, shows the values of $y\xi$; if the assumed mean is correct, the algebraic sum of all of the $y\xi$ values will be zero - otherwise, the difference between the true mean and the assumed mean will be

$$\frac{\Sigma y \xi}{R}$$

(R = total number of observations).

The fifth column, used in finding the standard deviation, contains the values of $y\xi^2$ for each interval. The standard deviation σ is given by the equation

$$\sigma^2 = \frac{\Sigma y \xi^2}{R}$$

- a result which is proved in the standard works on statistics (e.g. Whittaker, 1944).

x	y	ξ	$y\xi$	$y\xi^2$
39978	3	-16	- 48	768
39979	0	-15	0	0
39980	1	-14	- 14	14
39981	0	-13	0	0
39982	2	-12	- 24	288
39983	3	-11	- 33	363
39984	4	-10	- 40	400
39985	6	- 9	- 54	486
39986	15	- 8	-120	960
39987	23	- 7	-161	1127
39988	38	- 6	-228	1366
39989	42	- 5	-210	1050
39990	56	- 4	-224	896
39991	61	- 3	-183	549
39992	78	- 2	-156	312
39993	88	- 1	- 88	88
39994	86	0	0	0
39995	81	1	81	81
39996	78	2	156	312
39997	70	3	210	630
39998	53	4	212	848
39999	49	5	245	1225
40000	36	6	216	1296
40001	19	7	153	931
40002	23	8	184	1472
40003	15	9	135	1215
40004	4	10	40	400
40005	5	11	55	605
40006	1	12	12	144
	940		+ 116	17826

Thus the mean value of 40λ is $39994.45 + (116/940)$ $= 39994.57$ or, with an accuracy sufficient for the present purpose, 39994.6.

The standard deviation of these results is

$$\sqrt{\frac{17826}{940}} = \underline{4.35} \text{ units}$$

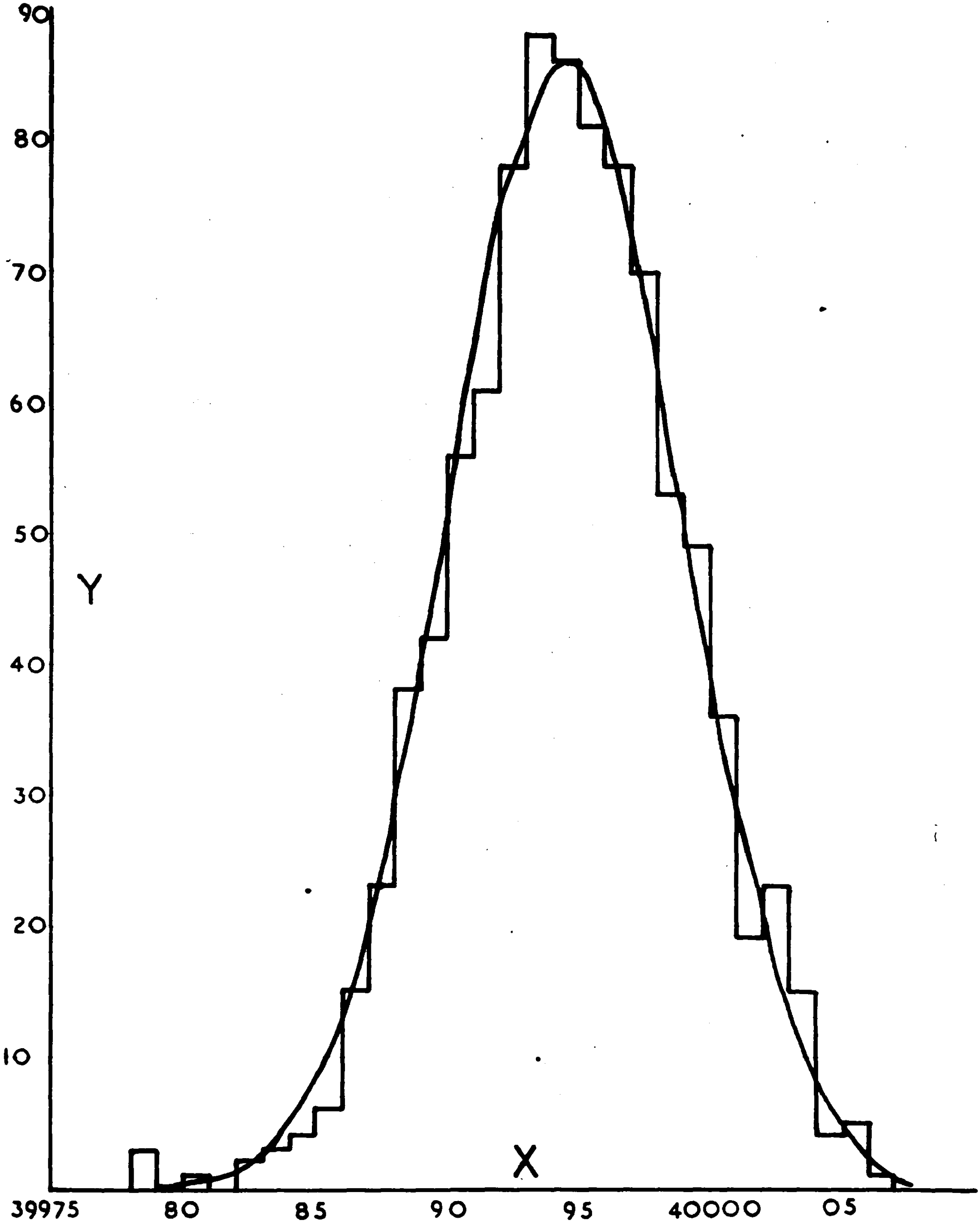
The normal frequency distribution for R results, with a mean value of a and a standard deviation of σ, is given by the equation

$$y = \frac{R}{\sigma\sqrt{2\pi}} e^{-\frac{(x-a)^2}{2\sigma^2}}$$

which in the present case becomes

$$y = 86.1\, e^{-0.0265(x-a)^2}$$

This curve is drawn on the next page, with the actual frequency distribution of the experimental results superimposed for comparison. The close correspondence between the two graphs indicates that the corrections have been fairly accurate, and that no important systematic errors have escaped attention.


90
80
70
60
50
40
30
20
10
Y
X
39975
80
85
90
95
40000
05

Chapter 9

CALCULATION OF THE VELOCITY OF SOUND.

The mean value found for 40 λ at 18°C and 13.6 mb is 39.9946 inches. We now correct this value for a temperature of 0°C and zero humidity.

40 λ at 18°C and 13.6 mb = 39994.6 units

correction for effect of humidity on $C_{v\infty}$ = + 18.9

correction for error of screw = + 2.5

40016.0

effect of temperature and humidity on density - multiply this value by 0.96618 = 38662.7

divide by 40 to obtain 966.567

= 0.966567 inch

multiply by 13500 to obtain velocity = 13048.35 inches per second

multiply by 0.0254001 to find velocity in metres per second = 331.437 metres per second

correction for variation of f, g and h with temperature = - 0.032 metres per second

correction for variation of Cv with temperature = + 0.046 metres per second

velocity of sound = 331.451 metres per second

Accuracy of this determination. The various factors influencing the accuracy of the experiment will now be briefly considered again, and an estimate of the uncertainty attaching to the result just found will be attempted.

1. Temperature. The calibration of the thermometers (p 40) left an uncertainty of 0.02°C and, in estimating the readings with the telescope, a further uncertainty of about 0·01° was introduced. These circumstances may partly account for the fact that the readings of the two thermometers differed by about 0·04°C in most of the observations - after the known errors had been corrected. Part of this difference may have been due to an undiscovered difference in temperature between different points in the hall; a number of factors including, for example, the ventilating system and, indeed, the structure of the hall, may have contributed. However this may be, it will be wise to allow for an uncertainty of 0.04°C in the temperature

measurements. Since a change in temperature of 1°C brings about a change of about 0.6 metres per second in the velocity of sound, the final determination has an uncertainty of about 0.025 metres per second on account of the deficiencies in temperature measurement.

2. Humidity. The readings of the wet bulb depression were reliable only to 0.1°F; inspection of the calculations previously described shows that this uncertainty represents about 0.01 metres per second in the final result.

3. Frequency. The accuracy of the valve maintained tuning fork was very much better than the one part in 10,000 which is aimed at in this experiment, and no correction need be considered. There was, however, a slight uncertainty in the oscillator frequency amounting, in the interval between successive checkings, to about one part in 50,000, or about 0.006 metres per second in the final result.

4. Carbon dioxide content. The carbon dioxide content as measured with the Haldane apparatus was subject to an uncertainty of about one part in 20,000; the proportion of carbon dioxide found in the air was 3½ parts in 10,000 and, within the limits of accuracy obtainable, this is the same as the usual figure of 3 parts in 10,000 which is generally accepted for the open air. No correction is therefore needed here.

5. Errors introduced by the electrical circuits. This is a difficult point. It was, for example, not certain that the delay introduced by the pulsing circuits remained the same for all distances of the microphone from the loud speaker. The following precautions were, however, taken:

(a) the circuit of the microphone amplifier was adjusted, after assembly, so that the amplitude and waveform of the output voltage delivered to the pulse generator was the same whether the microphone was close to the loud speaker or at the most remote position allowed by the screw;

(b) the pulse generators were adjusted so that the pulse shape remained the same whatever the position of the microphone;

(c) the amplifier driving the loud speaker adjusted so that its output had a sinusoidal waveform; when distortion was allowed in this amplifier, the pulse shape changed with the position of the microphone.

It is difficult to be sure that no error was produced by variations in the response of the various circuits employed. It is, however, safe to conclude that any errors of this kind were very small, partly because no variation in the oscillograph traces was detected by the eye as the microphone moved, and partly because the distribution curve (chapter 7) shows that no large systematic errors have been overlooked.

6. Errors in the computations. It is possible that uncertainty has been introduced in the course of the calculations. Some of the many physical and chemical constants used may, for example, be capable of measurement with

greater accuracy than has so far been applied to them, and it may therefore be necessary at a future time to modify the numerical results obtained at various stages in the calculation of corrections.

Considering all of these factors, and bearing in mind the evidence provided by the distribution curve, it seems reasonable to conclude that the velocity of sound, at a frequency of 13,500 cycles per second in dry air at a temperature of $0^{o}C$ and a pressure of 1013.2 millibars, does not differ from 331.45 metres per second by more than 0.04 metres per second. This value may be compared with the result of Hardy's experiments at supersonic frequencies (Hardy, 1942), which is 331.44 ± 0.05 metres per second, and with the result of Hardy's theoretical calculation of the velocity of sound, which is 331.45 ± 0.05 metres per second.

APPENDIX A

THE EARLY HISTORY OF THE VELOCITY OF SOUND

The experiments of Mersenne and of Gassendi have been a source of confusion to subsequent writers, as the following extracts show:-

1. Musschenbroek gives a table of values of the velocity of sound in his account of the experiments of the Accademia del Cimento (*Tentamina experimentorum naturalium captorum in Academia del Cimento*, Leyden, 1731, p. 113). This table includes the entries:-

Gassendus	1473
Mersennus	1474

The units are feet per second.

2. The same writer, in his *Essai de Physique*, Leyden, 1739, vol. 2, p. 716, says:

> Gassendi seems to have been one of the first to give attention to the space described by sound in a given time; and, according to his observations, sound travels 1473 feet in a second.

3. This statement is repeated, without alteration, in Musschenbroek's major work, the Introductio ad Philosophiam Naturalem, Leyden, 1742, p. 920, 2231.

4. Gehler, (Physikalisches Wörterbuch, Leipzig, 1836, vol. 8, p. 390), writes:

> Gassendi did not himself find the value 1473 feet per second, but quoted one of the observations of Mersenne, who found this value, and also 1380 feet per second.

5. Wolf, (History of Science, Technology and Philosophy in the 16th and 17th Centuries, London, 1935, p. 286), states:

> Of the various problems relating to acoustics, that of the velocity of sound attracted the greatest amount of attention during the seventeenth century. The first experiments in this connection appear to have been made by Gassendi (1592-1655) .... A cannon and a musket were fired towards suitably distant points, and measurements were made of the time which elapsed between

> the moment when suitably placed observers saw the flash and the moment when they heard the explosion. The velocity appeared to be the same in both cases, namely 1473 Paris feet per second. The result was much too high. Mersenne repeated the experiment and obtained a somewhat better result, namely, 1380 feet per second.

6. Alexander Wood, (Acoustics, London, 1940, p. 246,) writes:

> The simplest and most obvious method for determining the velocity of sound is to time the passage of a sound over as great a distance as possible. Mersenne (1588-1648) and Gassendi (1592-1655) both determined the velocity by noting the time which elapsed between the arrival of the flash and the report from a distant gun.

7. Poggendorff, (Geschichte der Physik, Leipzig, 1879, p. 301), says:

> (Aristotle) thought the velocity of

> propagation of a sound to be a function of its pitch, and believed that deep notes moved more slowly than high notes. The error of this theory was shown by Gassendi in a celebrated experiment. He caused a cannon and a musket to be fired at a considerable distance, and measured the time between the perception of the flash and of the sound. Since light travels over any distance possible on the Earth in an immeasurably small time, the interval between the flash and the sound, divided into the distance, gives the velocity of sound. In this way he made the first numerical determination, namely 1473 feet per second.

The values quoted in these extracts may be summarised as follows:-

Mersenne	1380 feet per second
	1473
	1474
Gassendi	1473

None of the authors mentioned above gives any reference to the place in the works of Mersenne or of Gassendi from which his conclusions are drawn and, in order to resolve the apparent confusion, it was thought necessary by the present writer to make some study of the original writings of these two men.

Gassendi's writings, in Latin, are contained in six large volumes, Opera Omnia, published in Lyons in 1647. His friend Bernier published an abridged translation in French (Bernier, 1684). This translation is accurate in places, but is marred by a number of irresponsible omissions and interpolations. Examination of Gassendi's works leads to the conclusion that he did not publish any numerical measurements on the velocity of sound. The belief that he had done so appears to rest on a passage in vol. 1, p. 418:-

> Quo loco tacenda non est Marsenni nostri observatio, qui velocitatem soni studiose emensus, deprehendit ipsum uno horae secundo pervadere ducentas trigintas parisinas orgyias, seu hexapodas, ac uno proinde minuto horae primo, seu sexagesima

> horae parte, supra orgyiarum quatordecim millia.

(We must not fail to mention the observations of our friend Mersenne, who studied the velocity of sound diligently, and knew that it covered two hundred and thirty Paris toises in a second, so in a minute, which is the sixtieth part of an hour, less than fourteen thousand toises).

On the same page, Gassendi describes his own experiment:

> I draw attention to an altogether surprising fact in regard to the movement of the air from a sounding body to the ear; it is that, with whatever violence it is set into motion by the sounding body, it travels always with the same speed. It is a matter of experience that sounds small or large, made at the same place, are carried in the same time to the place where they are heard; this can easily be observed from the sounds of artillery heard over a distance of two or three miles if, having observed the instant when the flash is produced, one counts the pulse beats or

> the oscillations of a pendulum until the sound arrives at the ear; one finds that the oscillations, which are of course of equal duration, are of equal number whether the sound is made by a large weapon, such as a cannon, or by a small weapon, such as a musket. (No numerical measurements are given here, and none have been found in a careful search through Gassendi's writings).

Musschenbroek appears to have been the first to give Gassendi credit for the experiment which he did not perform and it will be seen that a hasty reading of the two extracts given above might lead to such an error. It is apparent that subsequent writers on the history of physics have, when dealing with sound, relied on Musschenbroek or on one another, rather than on the original documents.

In support of the opinion here put forward, it may be mentioned that Mersenne makes no mention of any experiment by Gassendi in his writings, although the two men were friends, and both lived in Paris for many years. It is also significant that the lists of previous measurements which are given by Walker (1698) and by Derham (1708) include the value found by Mersenne, but make no mention of Gassendi. If he had actually measured the velocity of sound, it is unlikely that no knowledge of it should have been shown by his contemporaries, or by Walker and Derham. Musschenbroek does not indicate the place in Gassendi's works from which he draws his conclusions, and it seems quite clear that he was mistaken.

It remains to examine the three values for the velocity of sound which are attributed to Mersenne - 1380, 1473 and 1474 feet per second. These figures refer to his experiments with gunfire, which are mentioned several times in the course of his works. (His estimate of the velocity of sound from echo measurements has been curiously neglected by the historians). He gives everywhere the value 230 toises per second = 1380 feet per second (Mersenne, 1636a, 1644a, 1644b). Now, the toise was six <u>Paris</u> feet and 230 toises are therefore equal to 1380 Paris feet; here we have one of the three values. The other two result from the conversion of 1380 Paris feet into English feet. It is now generally considered that the toise was 1·949 [x] metres and, using this ratio, 230 toises = 1471 feet. The conversion ratio used in the seventeenth century was, however, slightly different. Walker (1698), quotes the value 1474 as having been given by Mersenne in Ballistica, Proposition 39. Turning to this place (Mersenne 1644b) we find that the figure actually given is 230 toises, and Walker evidently did not think it necessary to explain that he had converted into feet. During the seventeenth and eighteenth centuries, the value of the toise, and of other units of length, varied from time to time, and from place to place (see Marion, 1923; Preclin, 1943), and it is not surprising that the figure 1473 should have been obtained at some other time.

Summarising the conclusions reached, we find:-

1. that the evidence for Gassendi's determination of the velocity of sound is insufficient, though he referred to the measurements made by Mersenne;

[x] Larousse; Brittanica. (See bibliography).

2. that Mersenne found, from experiments with gunfire, that the velocity of sound was 230 toises per second;

3. that the various values attributed to him have been obtained by expressing this figure in different units, or in the same units with slightly different conversion factors.

APPENDIX B

THE THUNDERSTORM RULE.

Derham (1708) describes how the distance of a thunderstorm may be estimated, by multiplying the velocity of sound by the number of seconds elapsing between the lightning flash and the arrival of the sound corresponding to it. Subsequent writers have regarded Derham as the originator of this rule (see, for example, Poggendorff, 1879; Wolf, 1935).

The object of this note is to record two enunciations of the same rule, much before Derham.

1. The Accademia del Cimento (1667) describe the method as follows - we quote from Waller's translation (Waller, 1684):

> We may also by a single stroak made upon Wood, Stone or Metal, or any other sounding body; judge how far off he is that gives the blow; telling the vibrations between the stroak seen, and the hearing of the Noise, which if the Wind be favourable, may be heard for some miles, and it will

> be easie as well as curious to find the Distance of clouds from us, and at what height from the Earth, thunder is generated, counting the vibrations between the lightning and the blow.

2. <u>Mersenne</u> (1644b) describes the same process at some length:

> ... Secondly, from observations on the flash and the sound, it will be easy to find the distances of guns in a siege or a blockade, and the ingenious will not fail to profit by this. Thirdly, the thunder follows the lightning in the same way: for each second (whether measured by the pulse-beat, which lasts exactly a second, or by a pendulum, or by any other instrument) which elapses between the lightning and the thunder, 230 toises should be allowed, the distance being half a league if the time is five seconds, and a league, if ten seconds are counted; whether the distance is vertical, lateral, or oblique, is of no concern.

3. Leonardo da Vinci appears to have known the same rule. According to Richter, 1939, it is mentioned in Document A, a fragment of manuscript in the library of the Institut de France, in the Treatise on Light and Shade, 1490, also in the Institut de France, and in document K, a notebook in the same library. These notebooks have been transcribed by Ravaisson-Mollier, (1891), but it was not possible to obtain access to a copy of them during the present investigation. Govi, (1883), throws some light on the matter:

> Leonardo da Vinci measured the velocity of sound, but no one has been able to discover the units which he used.

There the matter must remain for the time being, though it is obviously one for further investigation when opportunity permits.

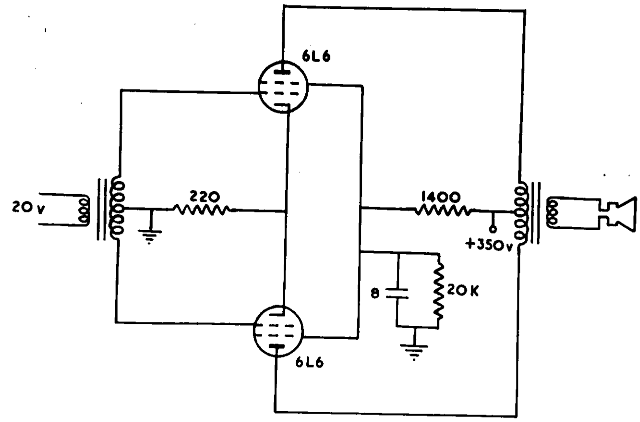
APPENDIX C

POWER SUPPLIES, AMPLIFIERS AND PULSE GENERATING CIRCUITS

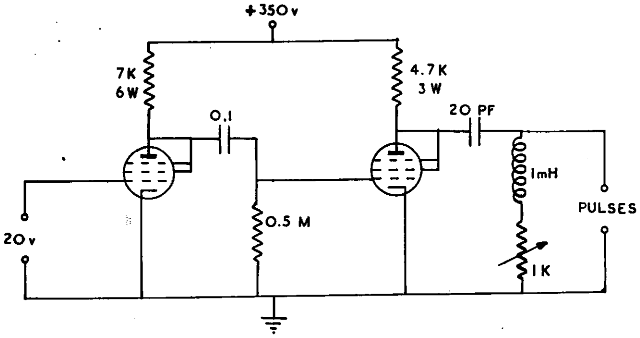
Power Supplies. The oscillator, tuning fork and oscillographs required alternating current, which was not available in the Bute Hall. A supply was brought in by a cable, connected to the nearest available outlet - about four hundred yards away. The current carried by this cable was about 2½ amperes, and the voltage delivered in the Bute Hall was about 235 volts.

The amplifiers and pulsing circuits required a D.C. supply at about 350 volts, and two power packs of conventional design were made - one serving the transmitting circuits and one the receiving circuits. The microphone amplifier, which had a low input and a high gain, was supplied by a separate power unit, with additional shielding and smoothing.

Amplifiers. The transmitter amplifier was of simple design, containing two beam tetrode valves (6L6) operated in push-pull. The oscillator output was kept at a pressure of 20 volts,



TRANSMITTER - POWER AMPLIFIER



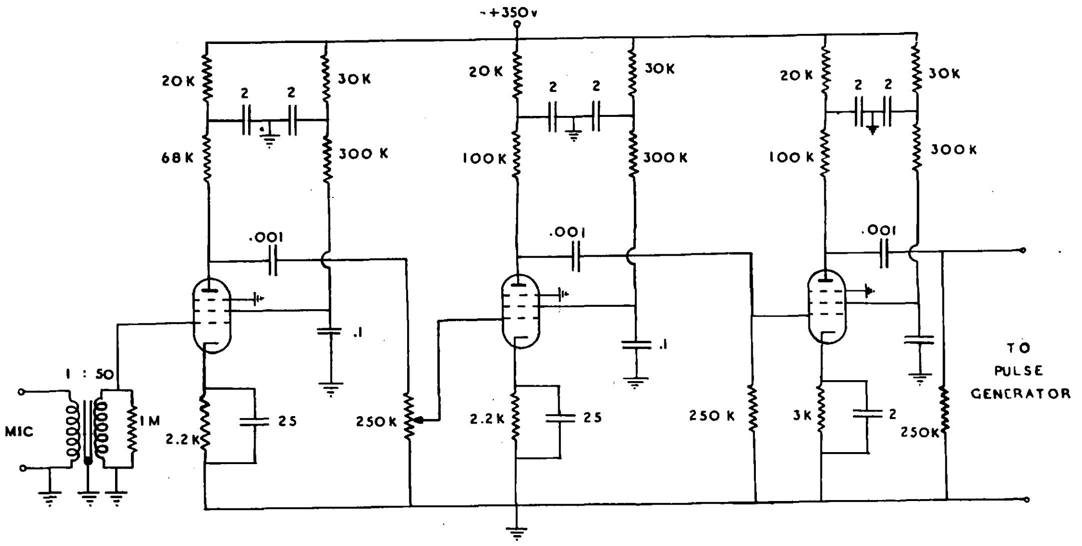
TRANSMITTER-PULSE GENERATOR

and was applied to the grids of the valves, through the input transformer, without any initial amplification.

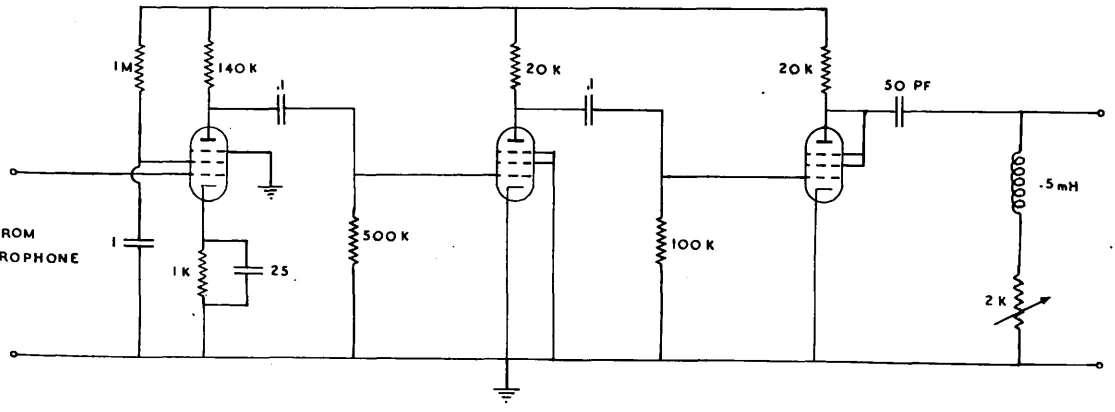
The microphone amplifier presented some interesting problems. The input voltage available from the microphone was about 100 microvolts, and a gain of about 50,000 was sought. The circuit contained three pentode valves, EF36, in cascade, with careful decoupling for the anodes and screens. Since only high audio frequencies were to be dealt with, the coupling condensers between the stages were made rather small (0.001 microfarad), thus reducing the amplification for hum and other unwanted signals. Gain control was obtained by means of a potentiometer in the grid circuit of the second valve.

Pulse Generators. Simple pulse generating circuits were used in the receiver and in the transmitter, and operated as follows. The sinusoidal input voltage was amplified by one or more valves (EF36), biased so as to produce a square wave output in the anode circuits.

The final square wave, of amplitude about 100 volts, was applied to a "ringing" circuit, consisting of a capacitance and an inductance, in series with a variable resistance of 1000 ohms. A momentary disturbance in this circuit, produced for example by the steeply rising front of the square wave, caused a short burst of oscillations at the natural frequency of the ringing circuit. These oscillations decayed at a rate depending on the value of the variable resistance, which could be adjusted to a critical value at which only one half cycle of the oscillations occurred. This half-cycle burst of current constituted a short pulse - about a microsecond long - which was applied to the input terminals of the oscillograph. The transmitter pulse generator, which had an input of 20 volts from the oscillator, used two stages of amplification, and the corresponding circuit in the receiver side had an extra valve, to compensate for the smaller input voltage available.



RECEIVER – MICROPHONE AMPLIFIER



RECEIVER – PULSE GENERATOR

APPENDIX D

TABLE OF VALUES OF THE VELOCITY OF SOUND

In this table, the results have been recalculated where necessary, to convert the velocity of sound to metres per second, in dry air at 0°C. In some cases, this information is obtainable from the original account of the experiment; where it has been deduced by the present writer, the temperature is indicated: 0$^+$. Where the information provided in the published account of an experiment has been insufficient to allow of this recalculation, the temperature and humidity are indicated: - .

MEASUREMENTS OF THE VELOCITY OF SOUND IN FREE AIR

Name	Place	Date	Baseline metres	Velocity m/sec	Temp °C	Humidity	
Mersenne	Paris	1636	158	316	-	-	**Echo**
Mersenne	nr Paris	1636	various	447	-	-	
Accademia del Cimento	Florence	1667	1806	361	-	-	
Walker	Oxford	1698	100	398	-	-	**Echo**
Cassini	nr Paris	1700	2495	356	-	-	
Derham	Essex	1708	3219-20012	348	-	-	
Flamsteed & Halley	Greenwich	1708	4828	358	-	-	
Cassini	nr Paris	1738	3421-11276	334	0^+	-	
Cassini	nr Nimes	1739	26492-43992	338	-	-	
Bianconi	Bologna	1744	25813	331	0^+	-	
Condamine	Cayenne	1745	39428	358	-	-	
Condamine	Quito	1745	?	337	-	-	
Kästner & Mayer	Göttingen	1778	536-1045	332	0^+	-	
Müller	Göttingen	1791	2670	338	-	-	
Espinosa & Bauza	Chile	1817	4496-14082	354.5	0^+	-	
Benzenberg	Dusseldorf	1810	4626-9072	334.0	0	-	
Benzenberg	Dusseldorf	1812	9069	333.5	0	-	
Arago	nr Paris	1822	18602	331.2	0^+	-	
Goldingham	Madras	1823	4247 & 9006	329.1	0	0	
Moll & van Beek	Utrecht	1824	17669	332.0	0	0	
Gregory	Woolwich	1824	640-1981	330.1	0^+	-	
Parry	Canada	1825	878-2636	327.3	0^+	-	
Parry	Canada	1826	3957	333.7	0^+	-	
Kendall	Canada	1828	805-1873	340.6	0^+	-	
Bravais & Martens	Switzerland	1845	9560	332.4	0	0	
Regnault	Paris	1868	1447-4891	330.6	0	0	
Stone	Cape Town	1872	4873	332.4	0	0	
Szathmari	Hungary	1877	-	331.6	0	0	Coincidence method
Greeley	Canada	1890	1279	333.3	0^+	-	
Frot	France	1898	5565	330.7	0	-	
Hebb	Chicago	1905	-	331.9	0	0	Stationary wave method
Hebb	Chicago	1919	-	331.4	0	0	- do -
Esclangon	France	1919	12600	330.8	0^+	0	
Angerer & Ladenburg	Belgium & Germany	1919	966-13075	330.8	0	0	
Miller	New Jersey	1937	6191	331.4	0	0	
Kukkamäki	Finland	1938	1000	330.8	0	0	

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