# Investigations On

# SUPERSATURATION IN NOZZLE FLOW

by

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**CONTENTS**

## SYNOPSIS

This thesis is concerned with attempts to examine the problem of the supersaturated condition in the expansion of steam through simple nozzles. In particular two of the recognised difficulties of the subject are explored. One is the peculiar form of the flow curve originally established by Mellanby and Kerr. The other is the failure of direct temperature measurements in the jet.

Part I of the thesis endeavours to present the somewhat conflicting facts and opinions of the various authorities on the subject and reviews briefly the experimental evidence and the ideas that have been expounded. It emphasises the importance given, in all hypotheses, to the supersaturation ratio and the wide and uncertain variations that would appear to have been shown in this value.

Part II is wholly devoted to the experimental work which is in two sections, dealing respectively with flow and temperature measurements. In both two types of nozzle were used - convergent and divergent. In the flow and pressure tests the main object was to check the Mellanby and Kerr type of results over a wider range and with variable surface ratio as an additional factor. The temperature tests were based on a new method which gave very striking results.

In Part III, the main features of the tests are discussed. The flow curves are analysed by the different methods employed by Mellanby and Kerr and by Powell. It is shown that neither method is satisfactory and that a better agreement is obtained on the assumption of sudden reversion. The results of the temperature investigations are then advanced as proof of sudden reversion and an explanation is given of why, before reversion, the temperature readings are high. The reason for failure in the measurement of undercooled temperature is thus made clear. The supersaturation ratios and velocities at the point of reversion are examined but it is finally concluded that the sudden change to the equilibrium state is really caused by the shock wave in high speed flow.

## PART I - REVIEW

### A. THE EXPERIMENTAL EVIDENCE

An accurate knowledge of the expansion of steam through a nozzle is of outstanding practical importance and of great theoretical interest. It has been the subject of various analytical efforts and a multitude of experimental researches, and yet there still remain discrepancies between theory and experiment.

One of the most interesting is the well known experimental result, that the rate of flow of initially dry or slightly superheated steam through a nozzle is in excess of that calculated on the assumption of expansion in thermodynamic equilibrium.

This excessive discharge has so often been confirmed that the fact is well established. A result of this kind casts doubt on the soundness or adequacy of the underlying theory. The usual explanation rests on the assumption that rapidly expanding steam may lie in a state of supersaturation i.e. condensation is, for the time being, suspended and the steam is expanding as a dry gas.

Supersaturation is based upon the fact that the vapour pressure of a liquid at a given temperature is greater above a curved surface than over a plane one. Thus sufficiently small droplets of water will evaporate if they are placed in a space of saturated steam. The space, therefore, becomes supersaturated and the fluid density will be higher.

Equilibrium can exist between the steam and the droplets only if the vapour pressure of the droplets is equal to the pressure of the steam. This condition has been considered by Lord Kelvin[21] and by Von-Helmholtz[15] and leads to the well known equation:

$$\log_e \frac{P}{P_s} = \frac{2\sigma}{RTDr} \quad \text{———} \quad (1)$$

where $P_s$ is the saturation vapour pressure corresponding to the temperature T while $\sigma$ is the surface tension of water of density D and R is the usual gas constant. This relation gives the ratio of the actual pressure to the saturation pressure corresponding to the existing temperature if a droplet of certain radius (r) is to be in equilibrium with the vapour about it. If this ratio is lowered, the droplet will evaporate and if it is raised, the droplet will grow. This ratio between the two pressures is generally referred to as the supersaturation (S.S.) ratio.

Since surface tension decreases with increasing temperatures until it reaches zero value at the critical condition, it follows that the S.S. ratio at low temperatures is higher and decreases gradually to unity at the critical condition. On the other hand, if the droplets are very minute, the S.S. ratio will accordingly be very large, and finally in the limiting case where (r = zero) no droplets are present at all, it will be theoretically possible to obtain infinite supersaturation. Thus for a pure steam to condense, there must be some nuclei of finite dimensions.

## The Excessive Discharge and The Flow Curve Form

The flow of supersaturated steam, or steam which would normally become wet during expansion, in a nozzle is considered to be flow in a thermally unstable condition. Due to the high speed of flow it is unable to condense within the time limits available. Condensation requires nuclei of suitable size and a certain margin of time; in the absence of the first or the insufficiency of the second, the steam fails to develop its normal thermal changes and acts very nearly as a gas. Thus the relations for expansion of superheated steam can be applied to this condition of supersaturation.

Under these circumstances, the steam attains lower temperature. Thus its density is greater and it follows that the flow so determined is in excess of that given by the ordinary theory of wet expansion.

The theoretical maximum discharge can be expressed for this condition of supersaturation by the equation:

$$\frac{W}{A} = \left(\frac{P_1}{V_1}\right)^{\frac{1}{2}} \frac{r}{0.718} \left(\frac{1 - r^{\frac{n-1}{n}}}{r^{\frac{n-1}{n}}}\right)^{\frac{1}{2}} \qquad \text{———————} (2)$$

where $\frac{W}{A}$ = theoretical flow in lb./sec./sq.inch, $P_1$ and $V_1$ are the initial conditions, r = ratio of expansion $\frac{P}{P_1}$ where P is the pressure at the position where A is measured. n is the index of expansion in $P.V^n = C$ and in this case = 1.3.

Using equation (2) for $P_1$ = 65.9 lbs/sq.inch Abs. and r = 0.527, the curve AB fig.(1) is obtained showing the variation of flow rate with initial temperature and indicates a continuous

FIG. I.

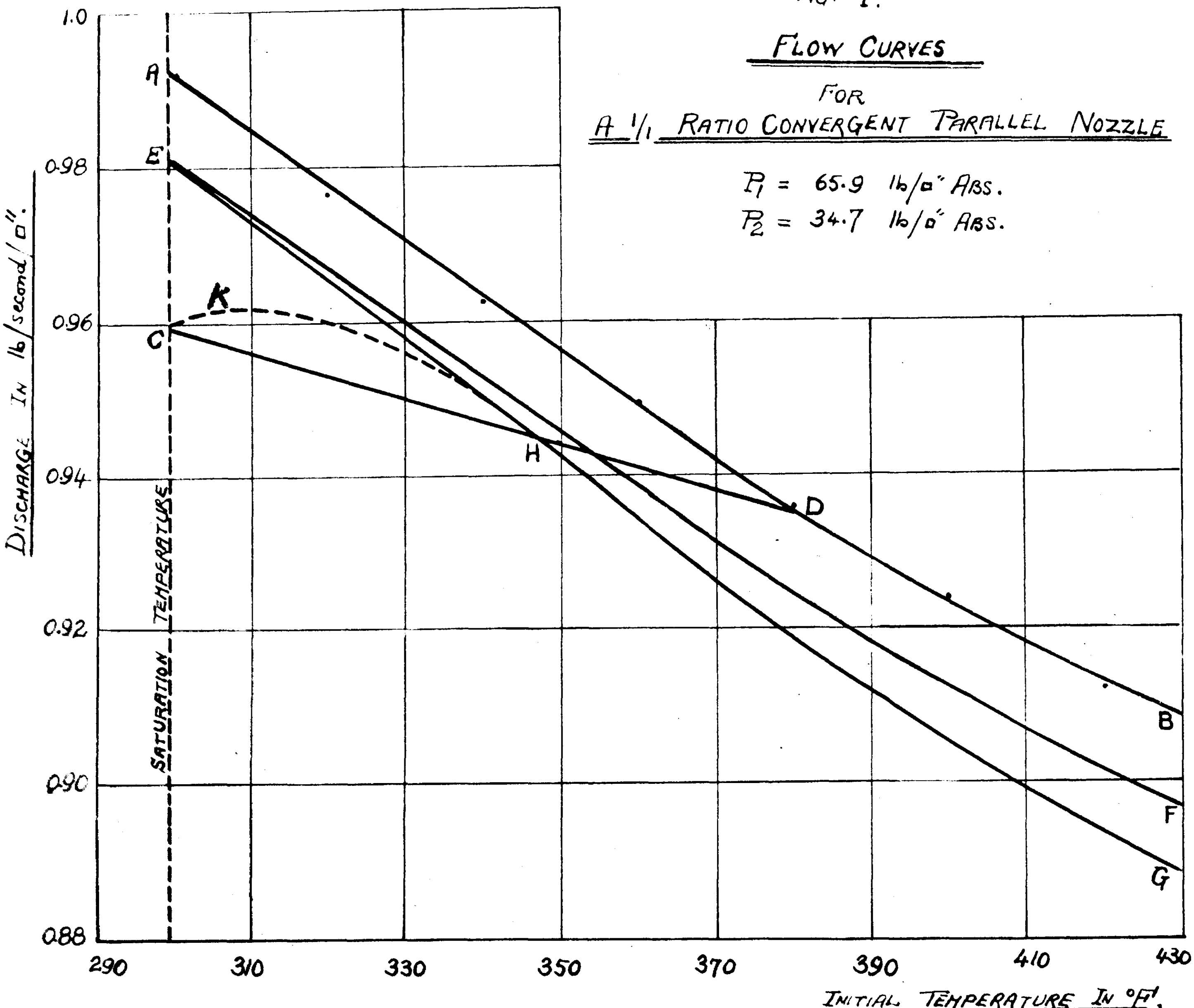
FLOW CURVES

FOR

A 1/1 RATIO CONVERGENT PARALLEL NOZZLE

$P_1 = 65.9$ lb/□" ABS.

$P_2 = 34.7$ lb/□" ABS.



drop with higher superheats.

In order to establish the corresponding curve for stable expansion, the law of adiabatic expansion is taken as approximating to $P.V^{m}$ = Const. where m = 1.035 + 0.1q where q is the dryness fraction and equal to unity in this case. Thus the curve CD is derived, the shape of which depends on the variation of the critical pressure ratio and it can be assumed a st. line. The point D marking the limiting amount of superheat that gives an expansion entirely within the superheat region is obtained by the Adiabatic relation

$$\frac{T_1}{T_2} = \frac{1}{r^{\frac{n-1}{n}}} \qquad \text{———} \qquad (3)$$

where $T_2$ is the absolute temperature of saturated steam at the throat pressure.

A curve representing the flow in which losses are taken into consideration is obtained by employing the equation

$$\frac{W}{A} = \left(\frac{P_1}{V_1}\right)^{\frac{1}{2}} \frac{r}{0.718} \frac{\left(1 - K - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}}}{K + r^{\frac{n-1}{n}}} \qquad \text{———} \qquad (4)$$

where K is the loss factor. With a constant K value, equation (4) gives such a curve as EF. It is nearly parallel to AB but indicates lower values of flow. It is found by experiment that nozzle efficiency decreases gradually with higher superheats and this could result in a curve such as EHG.

Any departure of the actual flow curve from this theoretical curve would appear to indicate a certain failure to achieve the full degree of undercooling. Such actual curves have been obtained by Mellanby and Kerr(26,d). These are of the form indicated by KHG and show at one and the same time high discharge rates - presumably due to supersaturated flow - and a flat top characteristic that seems to be in agreement with flow in thermal equilibrium.

This double feature of these flow curves has never been fully explained. It has to be emphasized that these flow experiments, establishing the fact of excessive discharge, give a curve form not apparently in keeping with the idea of complete supersaturation.

## The Cloud Chamber Phenomena

The cloud chamber method, well known to physists was first used by Wilson(46a) in 1897 to investigate the nature of this phenomenon by expanding air saturated with water vapour adiabatically in a glass cylinder provided with a light glass piston.

He observed, on expanding from an initial temperature of 20°C, that a persistent cloud was formed on sudden expansion when the vapour pressure was about 8 times the saturation value corresponding to the pressure. Thus steam can exist in a supersaturated condition until this limit is reached. After this condensation takes the form of a thick fog of very fine particles as if the vapour itself contained innumerable nuclei. If this value of 8 is substituted in the Kelvin-Helmholtz equation (1), and using an appropriate value for the surface tension, the effective radius of the nuclei is found to be of the order of $5.0 \times 10^{-8}$ cm. which is a value approaching molecular dimensions.

At Wilson's suggestion, the cloud chamber method has also been used by Powell[31,a] in 1928 to make an accurate test of the phenomenon at different initial temperatures. With an improved apparatus Powell investigated the limiting S.S. ratio at four temperatures and obtained the values of 7.8, 5.07, 3.74 and 2.87 at -16.4, 3.2, 19.1 and 47°C respectively.

To extend his results to higher temperatures, he made use of the two facts that the cloud limit S.S. ratio is unity at the critical temperature and that the S.S. ratio approaches this limit steadily. It follows then that the S.S. ratio is a function of the initial temperature.

Powell's calculations resulted in a droplet radius of $6.4 \times 10^{-8}$ cm. associated with condensation at these limiting conditions.

## The Optical Demonstrations

Stodola[41] seems to have been the first to utilize the optical method for the study of flow in transparent nozzles. He found that condensation took place immediately beyond the throat and it retained its position no matter how high the initial pressure, so long as the steam was initially dry and saturated. With initial superheat this position was found to move into the conical part of the nozzle to a small extent, noticeable only in very slender nozzles.

He recorded some tests with a high velocity jet, showing that there was not any visible sign of condensation until the steam had cleared the nozzle by a considerable distance. These tests, however, seem to be difficult to reconcile with others.

He concluded that in a nozzle, condensation takes place even before the throat though to a very slight extent, but that expansion to a pressure somewhat below the critical is the immediate cause of an almost sudden increase in condensation. He also estimated the time taken by the droplets to form to be of the order of $1 \times 10^{-4}$ of a second.

The flow of low pressure steam through nozzles similar to those used in turbines has also been studied optically by Yellot[47] assisted by pressure measurement along the nozzle. He found that the fall in pressure along the axis was not continuous but was momentarily arrested at a certain point in the expansion with which the beginning of condensation, as determined visually, coincided. He found that the limiting S.S. ratio varies from 5.3 at 82.4°F to about 3.04 at 198.4°F. The droplets radius at these limiting conditions varies from $(5.9 \text{ to } 7.0) \times 10^{-8}$ cm. respectively. These figures would have to be altered if allowance were made for flow losses.

Yellot assisted by Holland[illegible] carried out further tests with improved apparatus and under different conditions of expansion. These tests show higher S.S. ratios than those mentioned above, which led them to conclude that the velocity of the steam in the condensation region is the controlling factor.

Using the same apparatus as Yellot, Rettaliata[34] showed that wall roughness caused the condensation point to occur at higher pressure and farther downstream (upstream ?) than in a smooth nozzle. He suggested that the S.S. ratio depends on the rate of change of velocity along the nozzle and that it cannot be fixed for all conditions.

The optical method appears therefore to lead to various conclusions depending on apparatus and conditions. There is nothing very conclusive except the fact that the start of condensation can be observed. But there are differences as to where and how it starts and as regards the main deciding influences.

## Attempts At Temperature Measurement

Callendar and Nicholson[6A] seem to have been first to attempt the measurement of the temperature of steam expanding in an engine cylinder. They used a delicate platimum thermometer constructed of 0.001 inch wire. The temperatures recorded showed a departure from the saturation values. Under favourable conditions this was as much as 10°F.

Stodola endeavoured to measure the temperature along a nozzle by a copper constantan element stretched along the axis. This failed to record any degree of undercooling. On the contrary the results were rather in excess of the saturation values.

Martin[23,b] has recorded temperature measurements with an ordinary mercury thermometer shielded with hygroscopic substances fitted at the exhaust flange of a steam turbine. These temperatures were lower than the saturation values, in some cases by as much as $10^{\circ}F$.

Batho[2], by arranging a thermojunction in a wire stretched along the axis of a nozzle in such a way that it could be moved freely along the nozzle has obtained records of temperature along the length. These showed about 4 to $6^{\circ}F$ departure from the saturation values.

Muller[29] has used a measuring device made of magnesia tube containing the thermocouple wires in exploring the temperature along the nozzle. He obtained a temperature curve which was lower than the saturation curve at some points by as much as $10^{\circ}F$.

It is generally supposed that the failure to record the actual temperature is due to the formation of a film of moisture on the surface of the measuring device. The temperature of this film depends upon the rate of interchange of its molecules with the undercooled vapour. If more molecules condense than evaporate, the film will thicken and rise in temperature owing to the liberation of latent heat. In a quiet space, the film will soon surround itself with a jacket of molecules not differing much in temperature from itself, while if exposed to the full rush of undercooled steam, this jacket will be swept away and a larger number of low temperature molecules will reach the surface of the film.

The film will continue to increase in thickness and rise in temperature until a stage is reached at which the number of molecules evaporated in unit time is equal to the number condensed. The number condensed depends entirely upon the pressure of the surrounding vapour, so that the temperature indicated by a thermometer immersed in a supersaturated steam corresponds rather to the pressure of the surrounding steam than to its temperature.

The presence of a thermojunction, however delicate it may be, in a stream of undercooled steam, is thus supposed to upset the condition of supersaturation, at least in its vicinity, and all temperatures recorded by it are those corresponding to the pressure. This is the reason advanced to explain why Stodola was unable to detect any undercooling in his attempts. The same applies to the observations of Batho who has suggested that the impinging of the stream against the junction would cause too high a temperature to be registered.

It is, however, clear that no experimental attempt at temperature measurement has succeeded in obtaining direct evidence either of undercooling to the degree considered possible or of reversion to the saturated condition.

## B. THE VARIOUS HYPOTHESES

### Martin's Interpretation

Martin was the first to bring the phenomenon of supersaturation to the attention of engineers by his well known papers[23,b]. He assumed that condensation always commences with droplets of certain size, namely $5 \times 10^{-8}$ cm. as found from the classical experiments of Wilson. He allowed for the variation of surface tension with temperature and applying the Kelvin-Helmholtz equation he found that the S.S. ratio varies from 11 at 0°C to 4.3 at 100°C.

With the aid of Callendar's equations, he was able to calculate the properties of steam at the limiting condition of supersaturation, and his results where plotted on the $I-\phi$ Chart give a st. line which he called the "Wilson line". This Wilson line replaces the saturation line in the chart if Martin's theory is accepted, and divides it into two fields of dry and wet steam. This Wilson line lies between the 3 and 4 per cent moisture lines on the usual chart.

He demonstrated that steam as finally discharged from a modern steam turbine was still undercooled. He assumed the vapour itself remains at the temperature corresponding to the S.S. limit so that fresh nuclei can be formed continually throughout the whole expansion, but the temperature of the droplets rises very rapidly to that corresponding to the normal equilibrium condition and is maintained in the neighbourhood of this temperature by evaporation with further expansion.

He also pointed out that the occurence of condensation or even its completion does not necessarily imply the simultaneous establishment of thermal equilibrium, and he advanced strong reasons in favour of the view that there must be a sensible lag between the two.

Callendar's Assumptions

To apply the conception of supersaturation to the thermodynamic theory of the steam turbine Callendar(6,b) assumed that:-

a. The time interval of the flow through a nozzle is too small to allow condensation to occur.

b. The S.S. ratio at the cloud limit is equal to 8 at all times.

c. The co-aggregated molecules - required for explaining the deviation of the vapour from the laws of gases - act as suitable centres of condensation when this limit is reached.

d. At this limit, the droplets size is $5 \times 10^{-8}$ cm. as already suggested by Martin.

Callendar was also of the idea that some degree of supersaturation must persist throughout the whole range of expansion, since - as he believed - the transformation cannot be instantaneous.

Using the records of pressure distribution in the steam turbines of the S.S. Mauritania, Callendar showed that the limiting condition might be presented by the 3 per cent moisture line on the chart, which agrees very closely with the Wilson line at low pressures.

He realized that the application of Wilson's result at high pressures gives undercooling much in excess of what is really possible, and also gives results obviously impossible in the neighbourhood of the critical pressure. Thus while he adopted the new Wilson line, he suggested the revision of the S.S. ratio than commonly accepted.

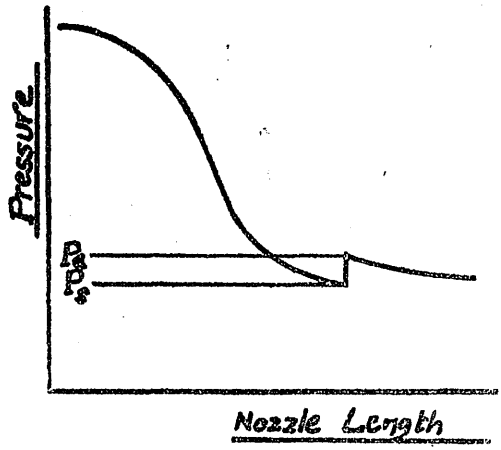
Keenan's Explanation

Keenan(20,b) defined the supersaturated condition of steam in two classes:-

a. Supersaturated steam of the first class is any steam entirely free from water droplets, but at a temperature lower than the steam table saturation temperature corresponding to its pressure.

b. Supersaturated steam of the second class is steam in thermal equilibrium with small droplets of water. In this case, despite the presence of water droplets, the steam is undercooled relative to the conventional saturation temperature which corresponds to a mixture of steam and relatively large droplets of water in equilibrium.

In his discussion on Yellot's paper, Kennan demonstrated that, if certain assumptions are made, the commencement of condensation must be accompanied by a sudden rise in pressure. If subscripts $S$ and $e$ refer to sections above and below the condensation point in a nozzle, then the equations of continuity, energy and momentum can be written successively as:-




$$\frac{v_s}{V_s} = \frac{v_e}{V_e} \quad \text{———} \quad (5) \text{ since } A_s = A_e$$

$$H_s + \frac{v_s^2}{2g} = H_e + \frac{v_e^2}{2g} \quad \text{———} \quad (6)$$

$$\frac{wv_s}{g} + P_s a_s = \frac{wv_e}{g} + P_e a \quad \text{———} \quad (7)$$

Where v = stream velocity, V = specific volume, a = cross sectional area of the stream, w = rate of flow and H = enthalpy.

Keenan assumed that at e the water droplets are relatively large, larger than $1 \times 10^{-5}$ inch diameter, and thus the ordinary steam tables provide a fourth relation and offer the possibility of solution by a "cut and try" process. For more exact calculations the size of the water droplets at e has to be assumed; then applying the Kelvin-Helmholtz equation for mixtures of vapour and droplets in equilibrium, with the Stodola's equation for capillary energy of the droplets, and using the adiabatic expansion equation $P.V^{1.316}$ = constant; the relation between $P_e$, $H_e$ and $V_e$ can be found, and the sudden pressure rise can be calculated.

He concluded that steam expanding in a nozzle involves complete supersaturation until a vapour state is reached which would exist in stable equilibrium with droplets of about $6.4 \times 10^{-8}$ cm. radius - based on Yellot's deductions, - but which is still undercooled compared with steam in equilibrium with larger droplets.

## Stodola's Conception

Stodola assumed that supersaturated steam always contains droplets of water of certain size which could exist in thermodynamic equilibrium according to the Kelvin-Helmholtz equation. If in the accidental play of the molecular impacts, a smaller droplet is formed it will vaporize very rapidly. On the other hand, if as a result of the collisions, a larger droplet is formed, though it has the same temperature as the steam the droplet is below its boiling point and thus forms a nucleus of condensation. This process continues until the droplet has become sufficiently large to render the capillary pressure negligible and thus upsets the supersaturated condition of steam with the complete transformation into the normal saturated condition.

This case is entirely different from the case when a large number of droplets of equal size are formed simultaneously and all of which grow uniformly. As long as the size does not exceed the limiting size necessary for thermodynamic equilibrium with the vapour, the steam remains undercooled.

Stodola found mathematically that the time necessary for equalizing the temperature in both the droplets and the surrounding steam is a small fraction of the duration of the flow in the nozzle and the temperature may be regarded as uniformly distributed. The temperature of the droplets, if they exceed the limiting size, will rise rapidly towards the saturation value and the corresponding duration of time is practically infinitesimal. On the other hand, with condensation setting in at an increasing rate as the expansion proceeds. They also doubted the possibility of any sudden condensation such as is usually assumed to take place at the limiting condition.

Powell's Method

Powell[31,b] has attempted, applying his new data on the S.S. ratio and its variation with the initial temperature, to explain the form of the flow curve disclosed by Mellanby and Kerr's experiments. He assumed that:- (a) The steam remains dry and expands according to Callendar's equation for dry and supersaturated steam up to the point at which the cloud condensation begins. (b) After the cloud limit is passed, the steam reverts gradually to its wet condition. The vapour is at the cloud limit temperature, while the droplets formed as the expansion proceeds rise suddenly to the normal saturation temperature.

His analysis is in fair agreement with the flow curve for the nozzle in which the frictional losses were least, but it has not been applied to the others. It must be admitted here that Powell's analysis gives a tendency in the right direction and leads to a better estimate of the flow.

But the results of such an analysis depend on the assumptions made and unless there is complete agreement with experimental facts there is no real proof that the assumptions are correct. It does not follow from the agreement established that the idea of gradual reversion after the cloud limit is passed, which is introduced in Powell's method, is conclusive. It tends to meet a peculiarity in curve form and that is its main justification.

Variation of the S.S. Ratio

Wilson afforded no evidence of the constancy of his limiting S.S. ratio of 8, although it is mathematically possible that it might hold throughout the practical range of pressures, but it appears more likely that there is a continuous diminution in this ratio with higher pressures until it reaches unity at the critical point.

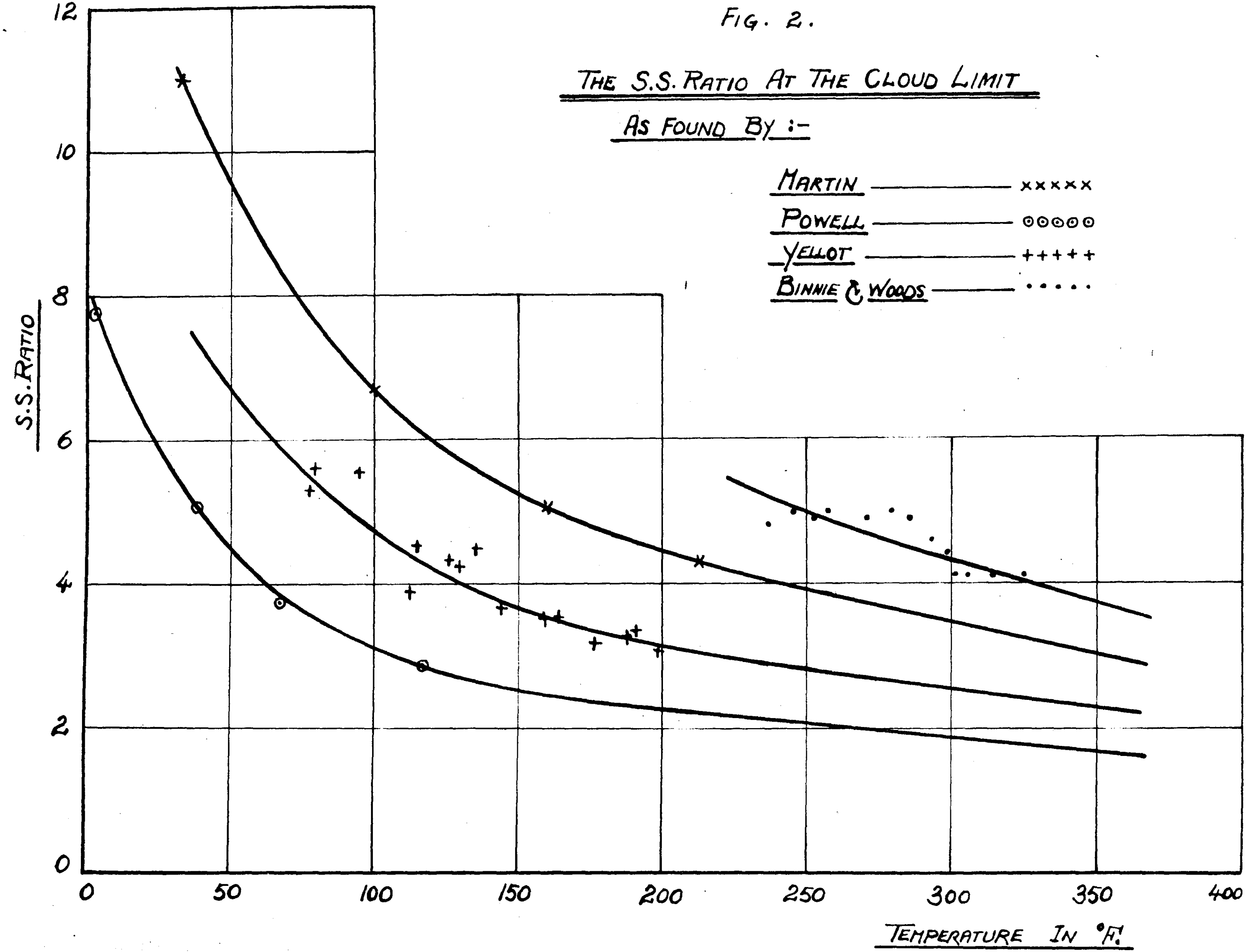
Martin, assuming the effective radius of the condensation nuclei to be constant and equal to $5 \times 10^{-8}$ cm., has found the limiting S.S. ratio to vary from 11 at $0^{\circ}$C to about 4.3 at $100^{\circ}$C.

Callendar has assumed that the S.S. ratio at the cloud limit is equal to 8 at all times, but he has admitted that further experiments were desirable especially at higher pressures.

Stodola, in his account of the subject, has shown that an expansion to a pressure somewhat below the critical is the immediate cause of an almost instantaneous reversion to the wet condition. In his tests the highest S.S. ratio attained was about (3.1 - 3.3) so long as the steam was initially dry and saturated.

Powell has found that the S.S. ratio is a function of the initial temperature of steam, reaching a value of 7.8 at $2.5^{\circ}$F,

FIG. 2.


THE S.S. RATIO AT THE CLOUD LIMIT
AS FOUND BY :-
MARTIN ×××××
POWELL ⊙⊙⊙⊙⊙
YELLOT +++++
BINNIE & WOODS .....
S.S. RATIO
0
2
4
6
8
10
12
0
50
100
150
200
250
300
350
400
TEMPERATURE IN °F.

decreasing gradually to 2.87 at 116°F and finally reaching unity at the critical temperature where naturally no condensation can exist.

Yellot by direct measurement in a nozzle has found that the S.S. ratio at the commencement of condensation varies from 5.3 at 82.4°F to 3.05 at 198.4°F. His results are slightly higher than Powell's ratios but lower than Martin's values.

Binnie and Woods[5] in the course of their pressure measurements along a nozzle, have detected a sudden pressure rise associated with the reversion of steam at its limiting condition; an observation first demonstrated mathematically by Keenan. They analysed these pressure "jumps" and deduced S.S. ratios varying from 5.0 at 240°F to about 4.1 at 320°F.

These various and rather widely different values for the S.S. ratio are brought together in fig.(2) from which it will be clear that the authorities are by no means in agreement about the all important limiting values supposed to govern the start of condensation in nozzle flow.

## Temperature Distribution Along the Nozzle

The temperature of supersaturated steam at the different stages of expansion along the nozzle can be calculated from Callendar's relation for dry expansion thus: $T_2 = T_1 \times (r)^{\frac{n-1}{n}}$ where $T_1$ = Absol. temp. of steam at inlet condition, $T_2$ that at a section where the pressure ratio is r and n is the index of expansion = 1.3.

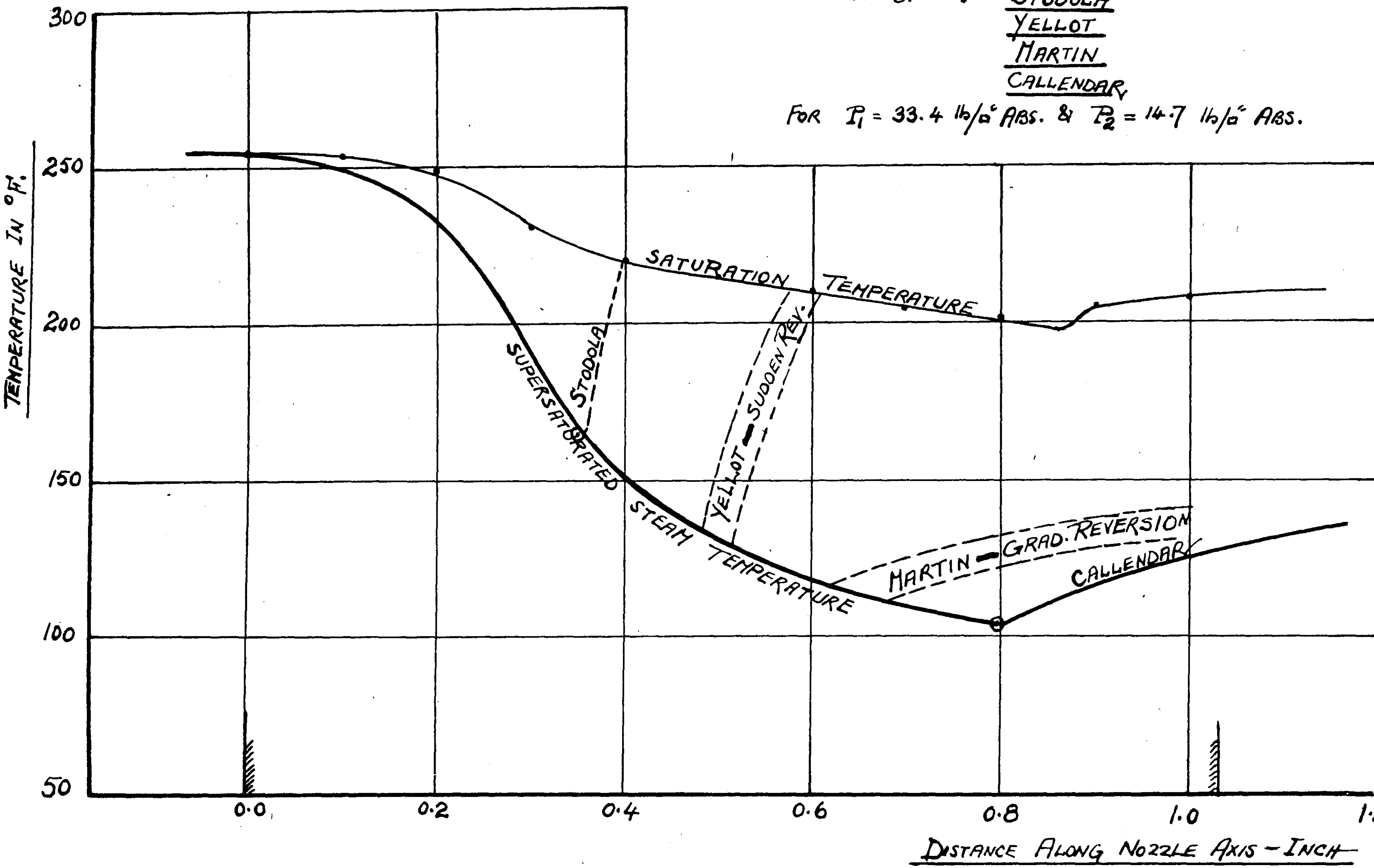
FIG. 3.

TEMPERATURE DISTRIBUTION

ALONG A CONVERGENT DIVERGENT NOZZLE

ON IDEAS OF : STODOLA, YELLOT, MARTIN, CALLENDAR

FOR $P_1 = 33.4$ lb/□" ABS. & $P_2 = 14.7$ lb/□" ABS.



Accordingly, the steam temperature along the nozzle can be calculated on the assumption of complete supersaturation. The saturation values can be obtained from the pressure distribution curve and thus the degree of undercooling at the various sections of the nozzle can be easily found as seen in fig.(3).

Using the temperature curve as a means to interpret the various ideas on the subject, the point of reversion to the wet condition according to Stodola appears to be at a pressure ratio of 0.54 where the steam temperature rises suddenly to the saturation value. Callendar's idea means a partial reversion only at about 0.35 pressure ratio where the S.S. ratio is about 8. Thereafter the temperature rises gradually. Martin's theory would indicate different degrees of undercooling depending on the initial temperature of steam and thus the point of reversion would appear between Stodola's and Callendar's. Again the temperature would rise gradually. According to Yellot and his colleagues, Binnie and Woods, the reversion point takes place before Callendar's limit and the steam temperature rises very rapidly to the saturation value.

The ideas and experimental investigations briefly summarized and illustrated have failed to show consistent and definite results regarding the degree of supersaturation to be expected in nozzle flow. The evidence in favour of the theory is convincing as regards the existence of supersaturation, but it does not provide definite views about the positive limitations and consequences. The important experimental evidence on flow quantities is not fully met, while the evidence on the temperature measurements along the nozzle is entirely inconclusive.

The investigations which are described in the next part of this thesis were undertaken to confirm and extend the evidence on flows and pressure distributions, and to attempt a further study of the temperature changes along the jet in the hope that these and their correlation would provide positive information of value in this subject.

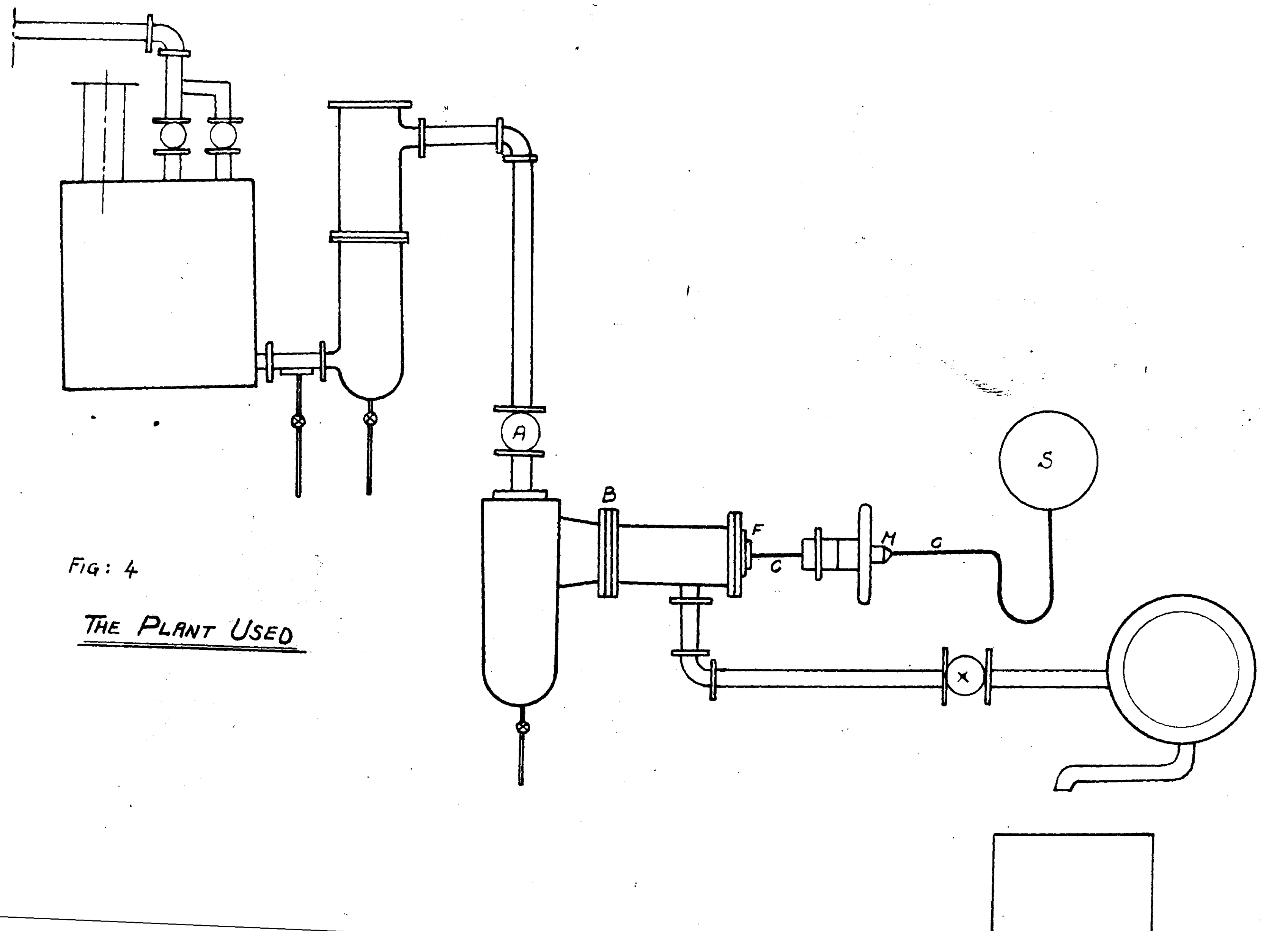



FIG: 4

THE PLANT USED

# PART II.

## EXPERIMENTAL INVESTIGATIONS

### A. PRESSURE AND FLOW EXPERIMENTS

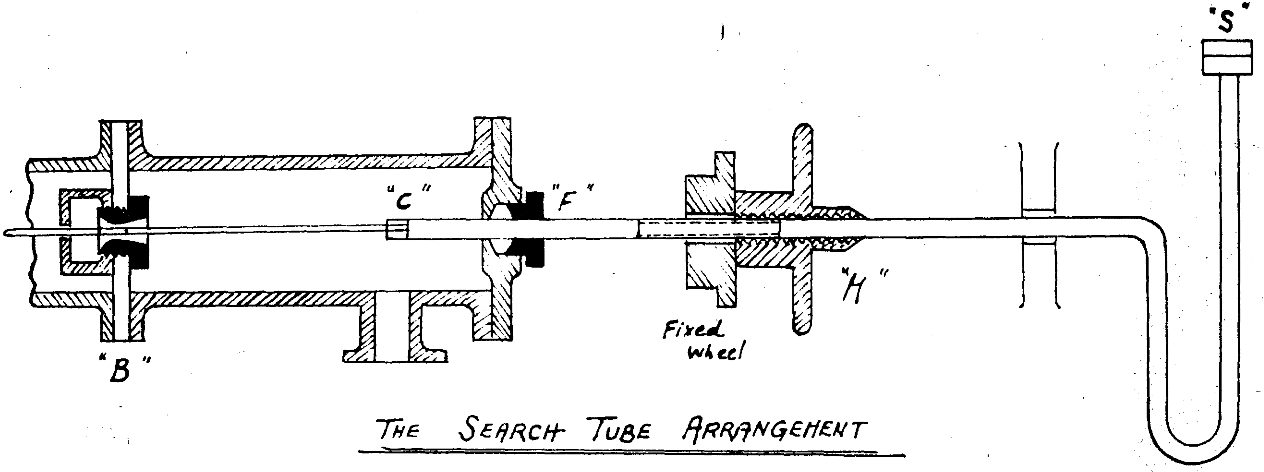
#### Apparatus and Procedure

The main features of the plant used are shown in fig.(4). The steam from the boiler passes through the two superheaters. The first is a double helical coil directly heated by gas jets from a two ring burner; each can be controlled separately. There is a cylindrical steel sheet between the two coils to control the draught. The second superheater is electrically heated. With the gas jets fully opened, a superheat of about 300°F at 75 lb/sq.in. absolute can be attained.

The receiver, directly supplying the nozzle, is fitted with a pressure gauge and a thermometer. These allow of the determination of the supply values of pressure and volume. The control of the pressure is by the stop valve (A) while the temperature is controlled by the adjustment of the gas jets. The superheaters and the receiver are provided with drains so that the establishment of superheat is not hindered by any collection of water.

The nozzle is screwed into the nozzle plate (B) and carries also, on the inlet side, a stirrup for centring the search

FIG. No. 5.




THE SEARCH TUBE ARRANGEMENT

IN DETAILS

tube. This tube is of copper, closed at the free end and fixed to the pipe (C) which passes out through the stuffing box (F) to the fixed head which is screwed on the pipe. By means of the handwheel, the relative motion between the tube and this fixed head results in moving the tube out. At one point near the free end of the search tube, a small hole 1/32" diameter is drilled diametrically through the walls, consequently the pressure at any position can be determined, the tube being set in the desired position by means of the handwheel. The pressure is indicated on the gauge (S).

The position of the hole is altered by the handwheel operating the tube and its exact location is obtained by the reading of the micrometer (M) which is centrally fixed to this handwheel. The zero reading is determined by setting the apparatus with the hole in the search tube in line with the inlet edge of the nozzle.

The pressure in the exhaust chamber is controlled by the valve (X). The steam from this chamber passes through a small atmospheric condenser and the issuing condensate is led to the tank on the platform balance. An alternative arrangement allows the exhaust chamber to be connected to a relatively large condenser, so that vacuum can be established. In this case the condensate is led to a measuring tank.

In carrying out a flow test, the required degree of superheat was approached slowly and steam allowed to pass freely through the system for some time. This continued until it was certain that the inlet steam temperature could not vary more than $\pm \frac{1}{2}^{\circ}$F with dry and slightly superheat tests and $\pm 1^{\circ}$F with higher superheats. In very few cases did the average differ from the extremes by as much as $1.5^{\circ}$F. The search tube was set so that the exploring hole was in a position about $\frac{1}{4}$ inch behind the inlet edge. Once a steady condition was established, the flow test commenced, the period of which was determined by the time necessary to pass about 70 lbs. of condensate which varied of course with the different conditions.

Jet pressures were read at axial intervals varying with the rapidity of the expansion. These were as low as 0.005 of an inch in the convergent part where the pressure gradient to the throat is steep and 0.025 of an inch in the parallel or divergent part where the expansion is slower. When the search tube was moved to a new position, the gauge needle fell slowly then rose with a jerk, then fell again and rose with a jerk. It was the custom when exploring the convergent parallel nozzle to consider the readings meanway between the extremities, but at certain positions along the divergent nozzle the needle vibrations were so violent that an ample time had to be given until the needle became more steady.

## Nozzles Used

Three nozzles have been used in these investigations, as follows:-

a. Convergent Parallel nozzle - I -, made of brass, has a throat diameter of 11/32 inch, an entry curve of 5/16 inch and the length of the parallel tail was equal to the throat diameter.

After establishing the flow and pressure curves for this nozzle, it was altered to provide a different tail length, this being reduced to 11/64 inch, thus giving a nozzle of parallel length to throat ratio ½ to 1. After experimenting with this second form it was again altered to provide a purely convergent type.

All three forms were tested without search tubes for flow results and with an ⅛ inch diam. search tube for combined flow and pressure data. This gives 6 flow curves and 3 pressure curves for the series.

b. Convergent Parallel nozzle - II -, made of Monel metal, of the convergent parallel type having the same dimensions as the first nozzle. This nozzle was operated first with the full bore open to flow then with a brass search tube of ⅛ inch diameter under the same conditions. The nozzle was also explored with a 3/16 inch diameter search tube and finally with a ¼ inch diameter search tube. In this way different hydraulic mean depths were obtained so that the effect of surface ratio on flow and pressure could be determined.

c. Convergent Divergent Nozzle - III -, on which only pressure and temperature measurements were made, is of brass having the same throat diameter of 11/32 inch and entry curve of 5/16 inch. Beyond the throat the nozzle profile has a divergent angle of $6^{\circ}$. The overall length is 1-1/32 inch.

Convergent Nozzle Results

The inlet pressure was kept constant at 45.5 lb/sq.in. Abs.,

Fig. No. 8.

PRESSURE DISTRIBUTION ALONG THE CON. PARALLEL 1/1 RATIO NOZZLE

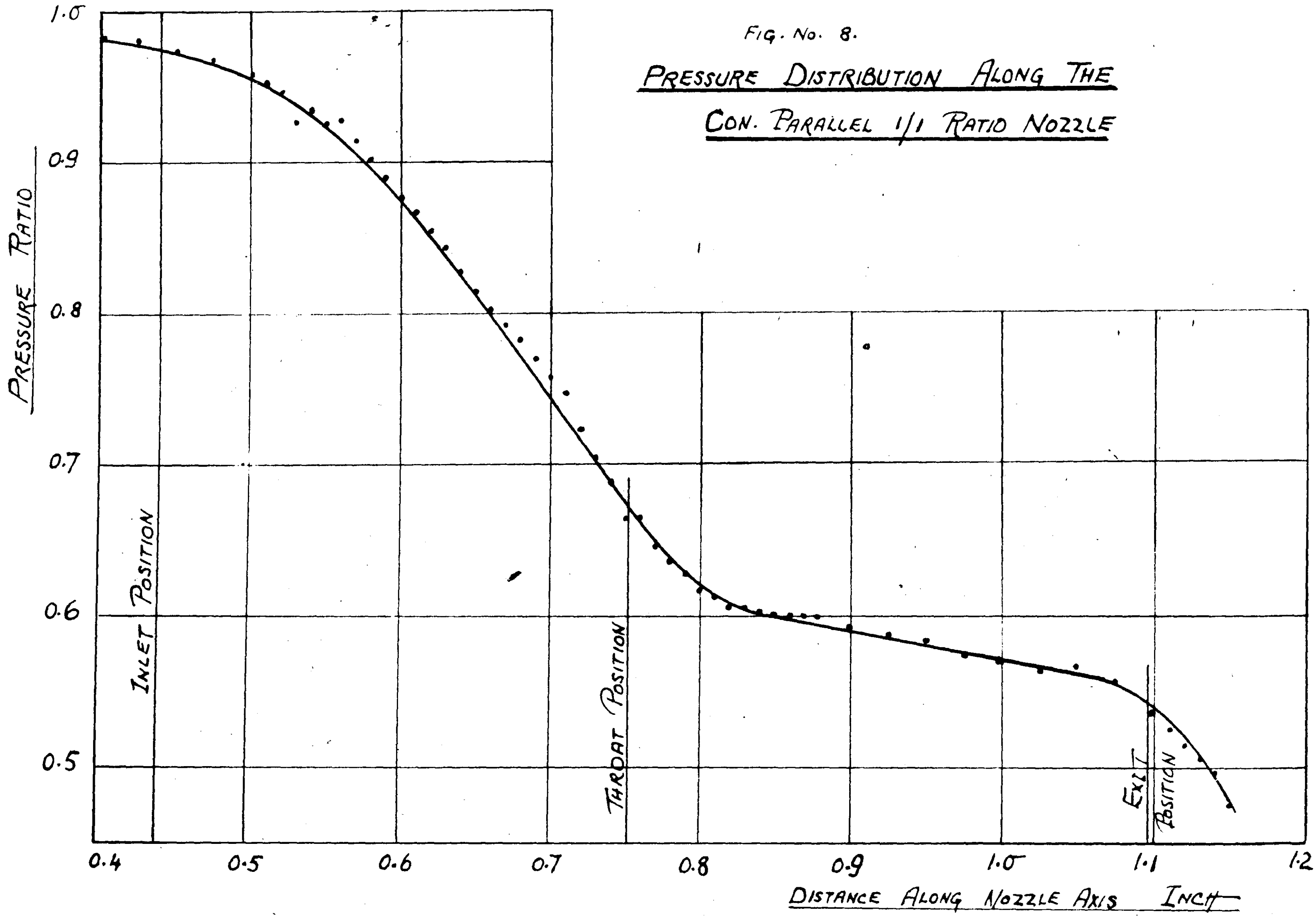
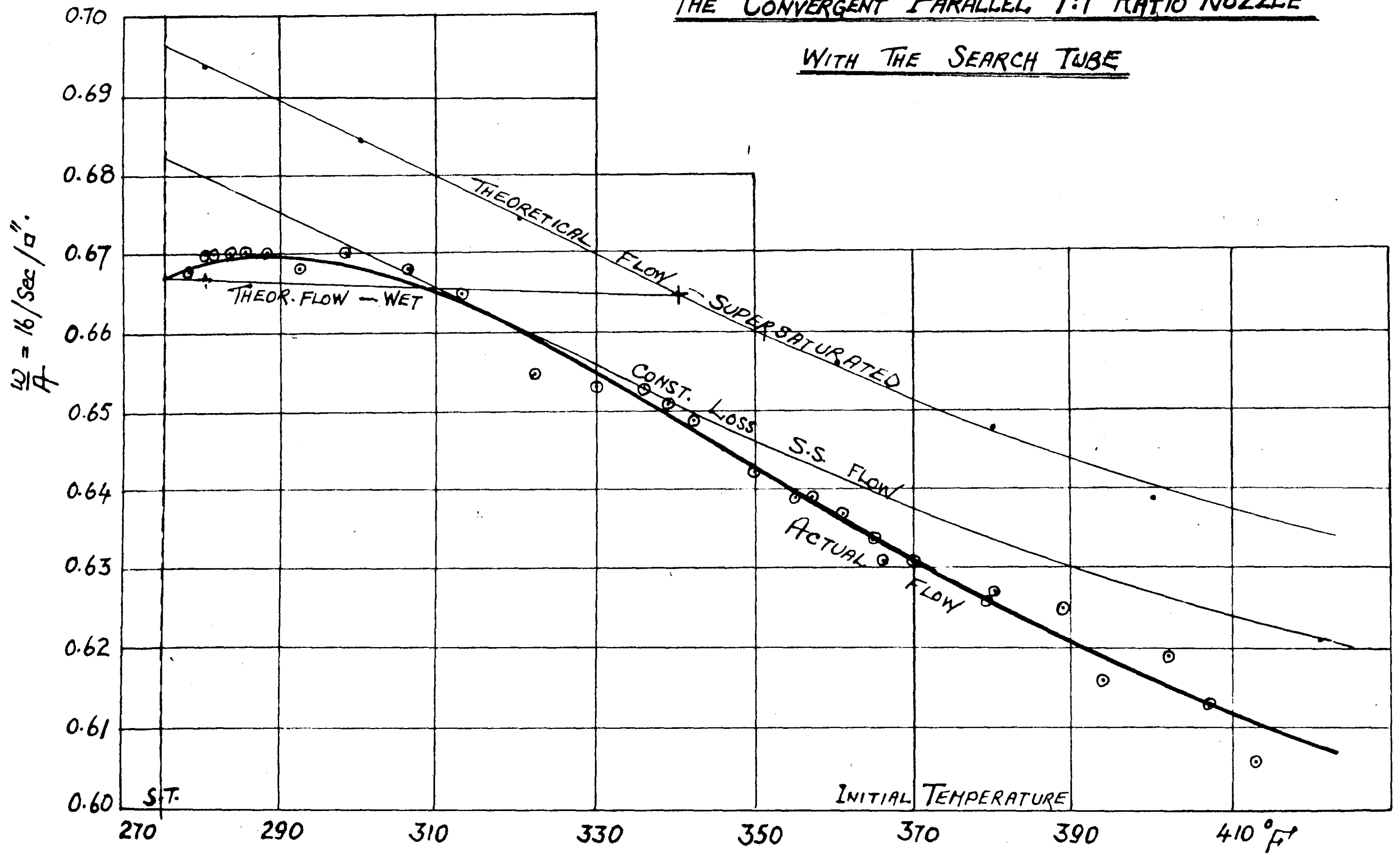


FIG. No. 7

FLOW CURVE FOR

THE CONVERGENT PARALLEL 1:1 RATIO NOZZLE

WITH THE SEARCH TUBE



and the back pressure was kept always at 22.7 lb/sq.in Abs. The inlet temperature was raised to give the required degree and kept steady at that throughout the period of the test. The condenser supply was also controlled to give a steady condensate temperature which was maintained constant for all tests. Changes in supply pressure during a test were very slight and easily controlled by the valve.

<u>The Convergent Parallel 1/1 Nozzle with Search tube:</u>

The flow curve for this nozzle with varying initial temperature and with the flow quantities reduced to the standard pressure of 45.5 lb/sq.in. Abs. are shown in fig.(7). The pressure distribution along the nozzle is shown in fig.(8) where the value indicated at any point represents the ratio of the pressure at that point to the standard pressure. This pressure curve represents the average of three explorations at 440, 360 and 350$^{\circ}$F initial temperature; although there is a tendency to rise with fall of superheat but too slight to affect the curve. The pressure ratio recorded at the outlet section viz. 0.54 is used later with discussion of theoretical flow curves.

The curve of theoretical flow rate on the assumption of stable expansion is indicated on fig.(7) and the fact that the actual flow is in excess of this is clearly in evidence. The theoretical supersaturated flow rate at the different initial temperatures is also indicated and a curve parallel to it and tangent to the actual flow curve will indicate a constant less S.S. flow rate. The point of tangency is about 320$^{\circ}$F initial temperature

FIG. No. 12.

PRESSURE DISTRIBUTION ALONG THE PURELY CONVERGENT NOZZLE

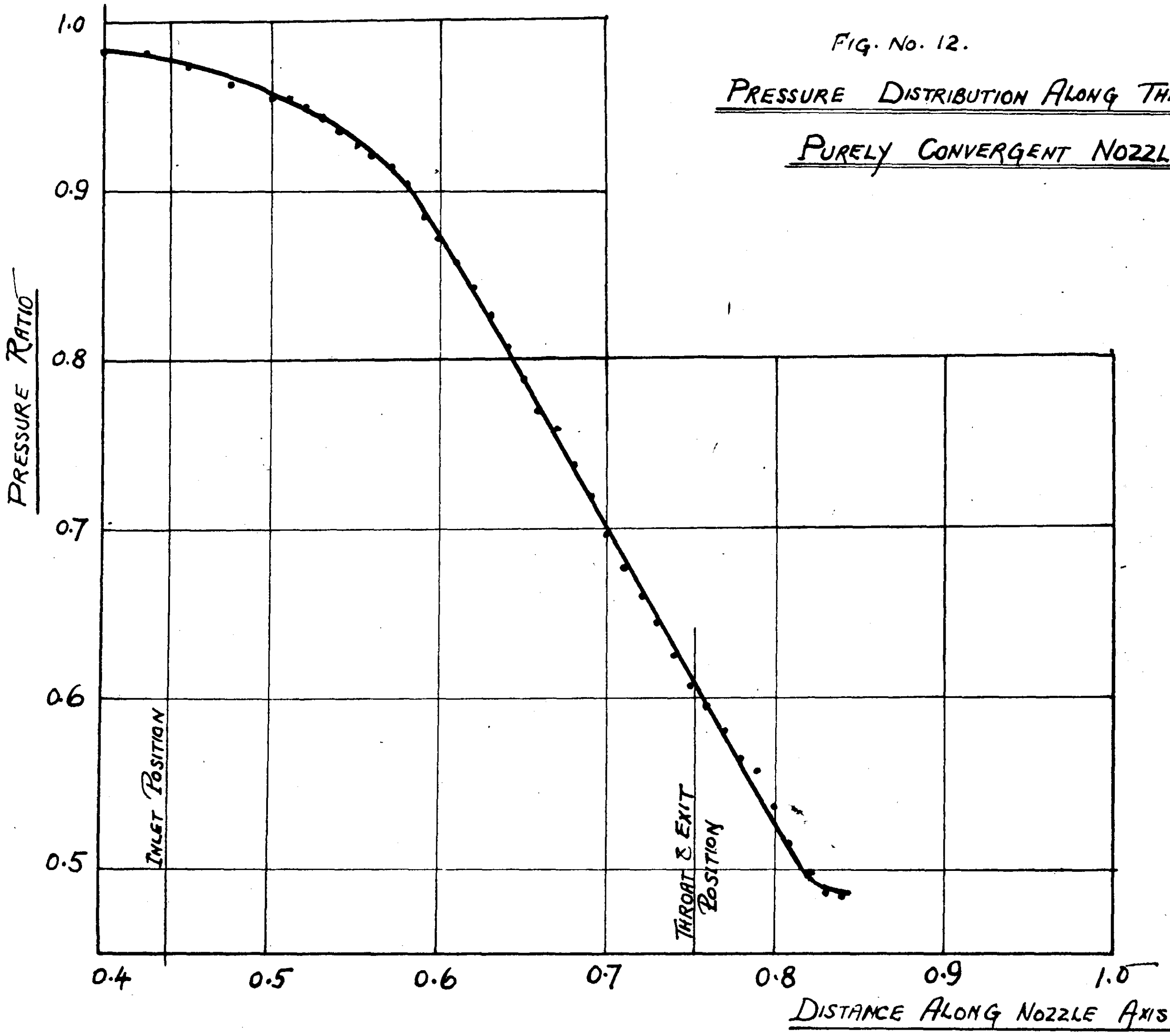


FIG. No. 11.

FLOW CURVE FOR

THE PURELY CONVERGENT NOZZLE

WITH THE SEARCH TUBE

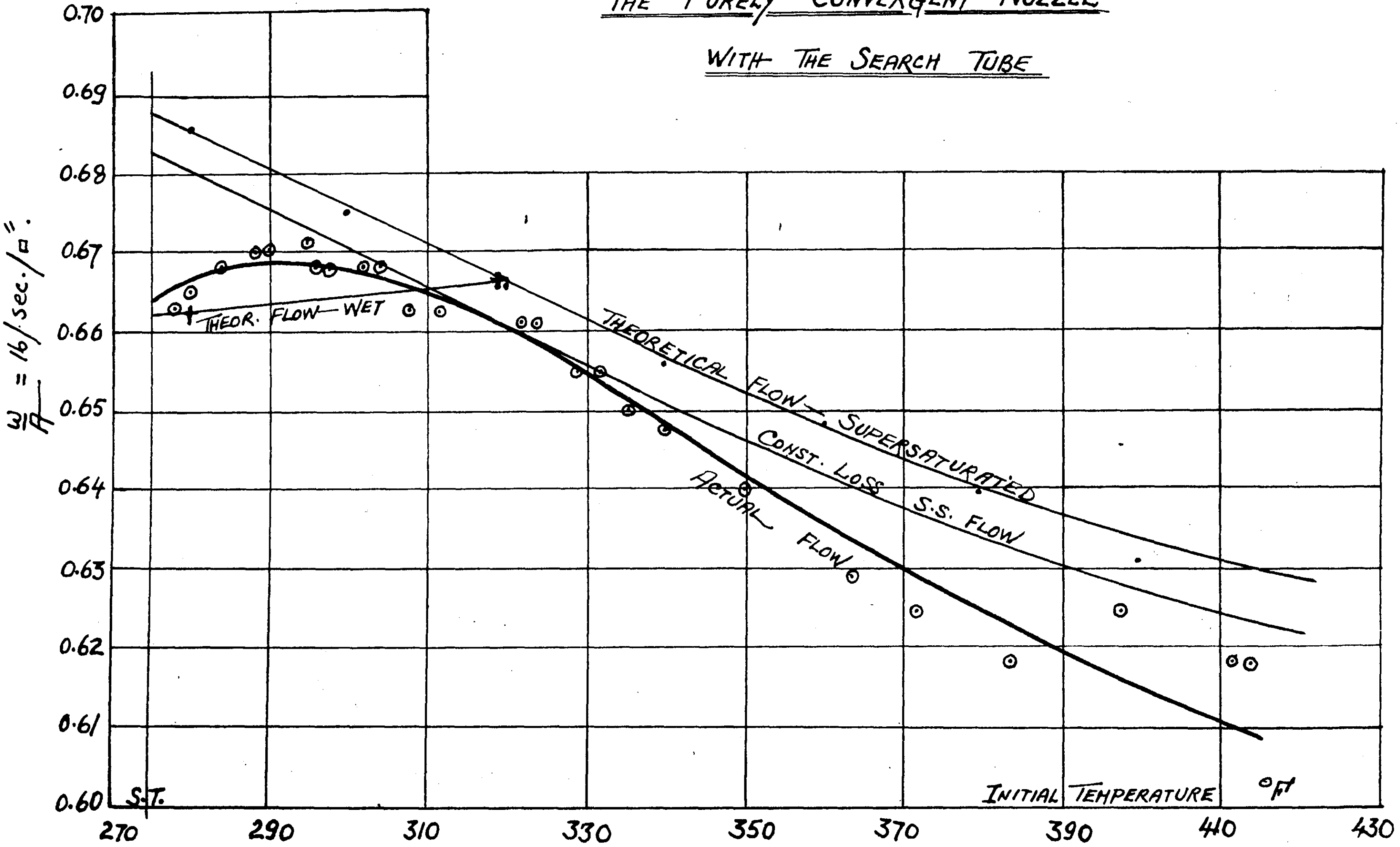


Fig. No. 10.

Pressure Distribution Along the Con. Parallel $\frac{1}{2}/1$ Ratio Nozzle

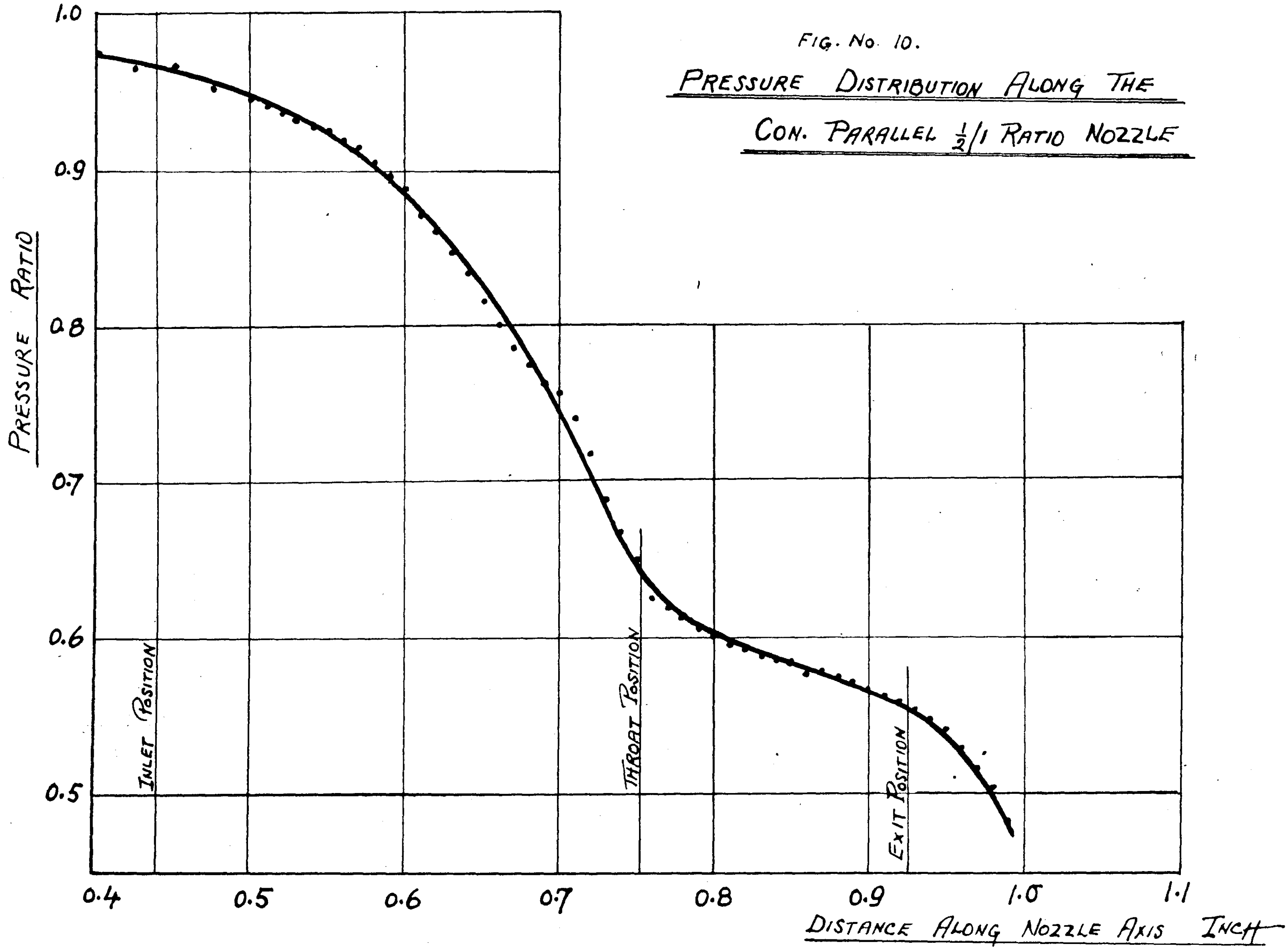
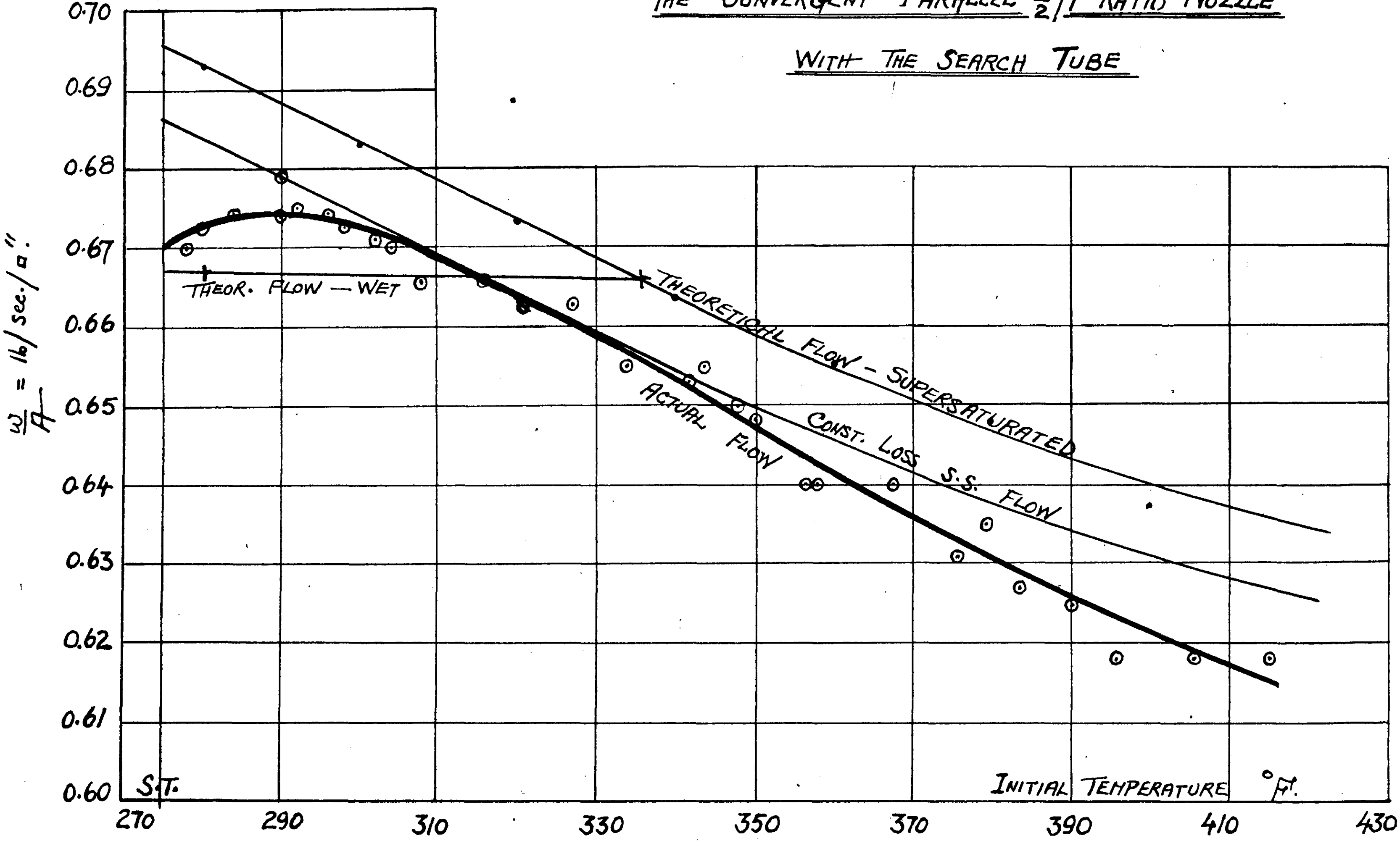


Fig. No. 9.

FLOW CURVE FOR

THE CONVERGENT PARALLEL $\frac{1}{2}/1$ RATIO NOZZLE

WITH THE SEARCH TUBE



and the actual flow curve departs from this constant loss curve both above and below this temperature. This departure will be dealt with fully in the discussion.

<u>The Convergent Parallel ½/1 Ratio nozzle with Search Tube</u>: The flow curve for this nozzle is shown in fig.(9) where the flow quantities are reduced to the standard pressure. This nozzle gives flow rates rather higher than those in the previous case and shows more clearly the excessive discharge condition.

The pressure curve fig.(10) indicates a higher pressure ratio at the outlet section of the nozzle. This ratio 0.555 is used later in determining the theoretical flow curves for this case. The constant loss curve based on the assumption of complete supersaturation is tangent to the actual curve at an initial temperature of about 320°F, and the actual curve departs in the same manner as in the previous case but with larger discrepancies especially at the low temperature side.

<u>The Convergent Nozzle with Search Tube</u>: The flow curves for this form are shown in fig.(11) and the pressure curve in fig.(12).

In this nozzle, the flow losses will be low, but it does not follow that the flow quantities measured with it are maximum. Since the pressure curve indicates a ratio of 0.606 at the exit section, slightly higher than the critical ratio with which the flow is maximum, the theoretical flow curve based on complete supersaturation shows lower values than the corresponding curves in the two previous cases.

FIG. No. 14.

FLOW CURVE FOR

THE CONVERGENT PARALLEL 1/2/1 RATIO NOZZLE

WITHOUT THE SEARCH TUBE

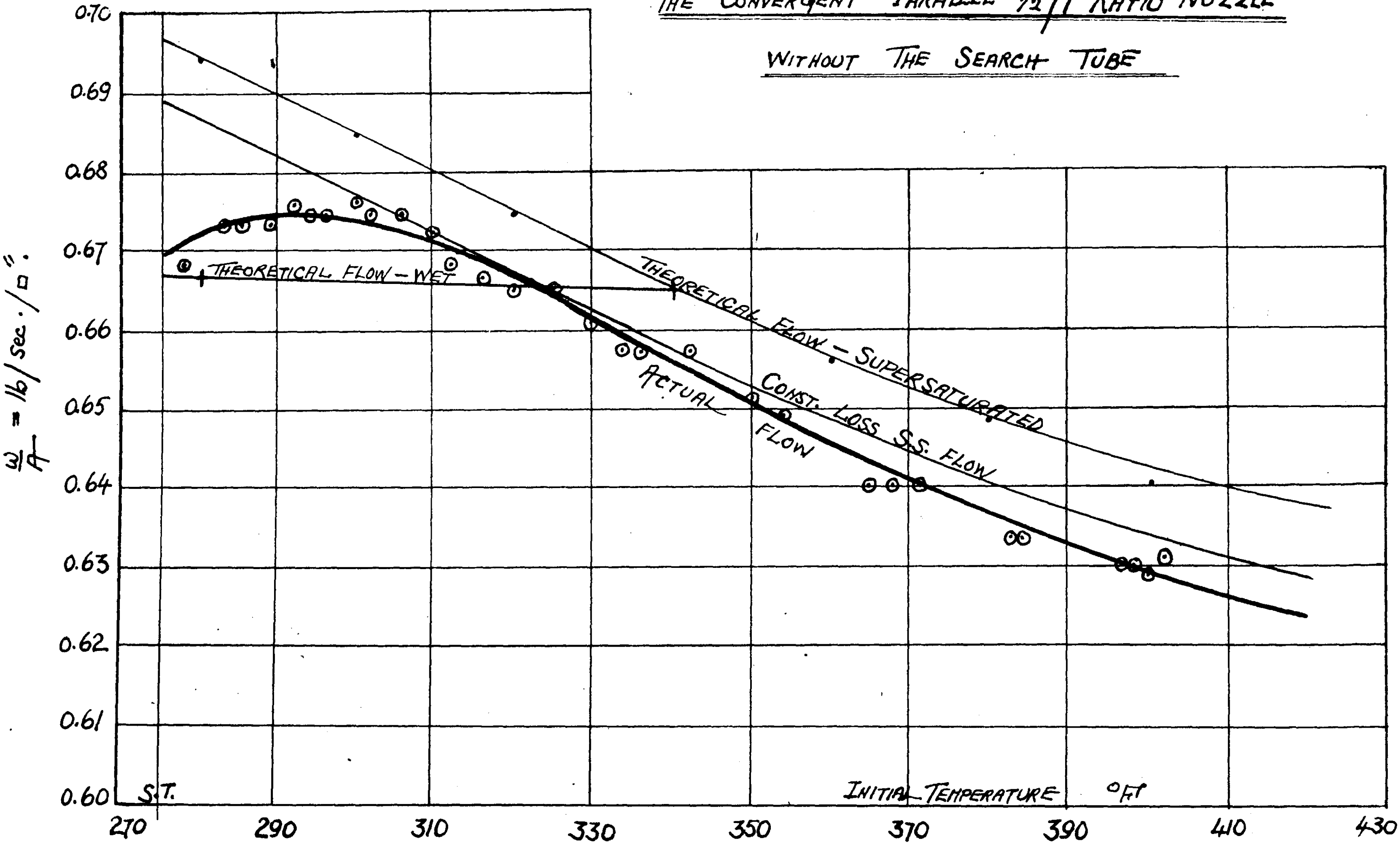
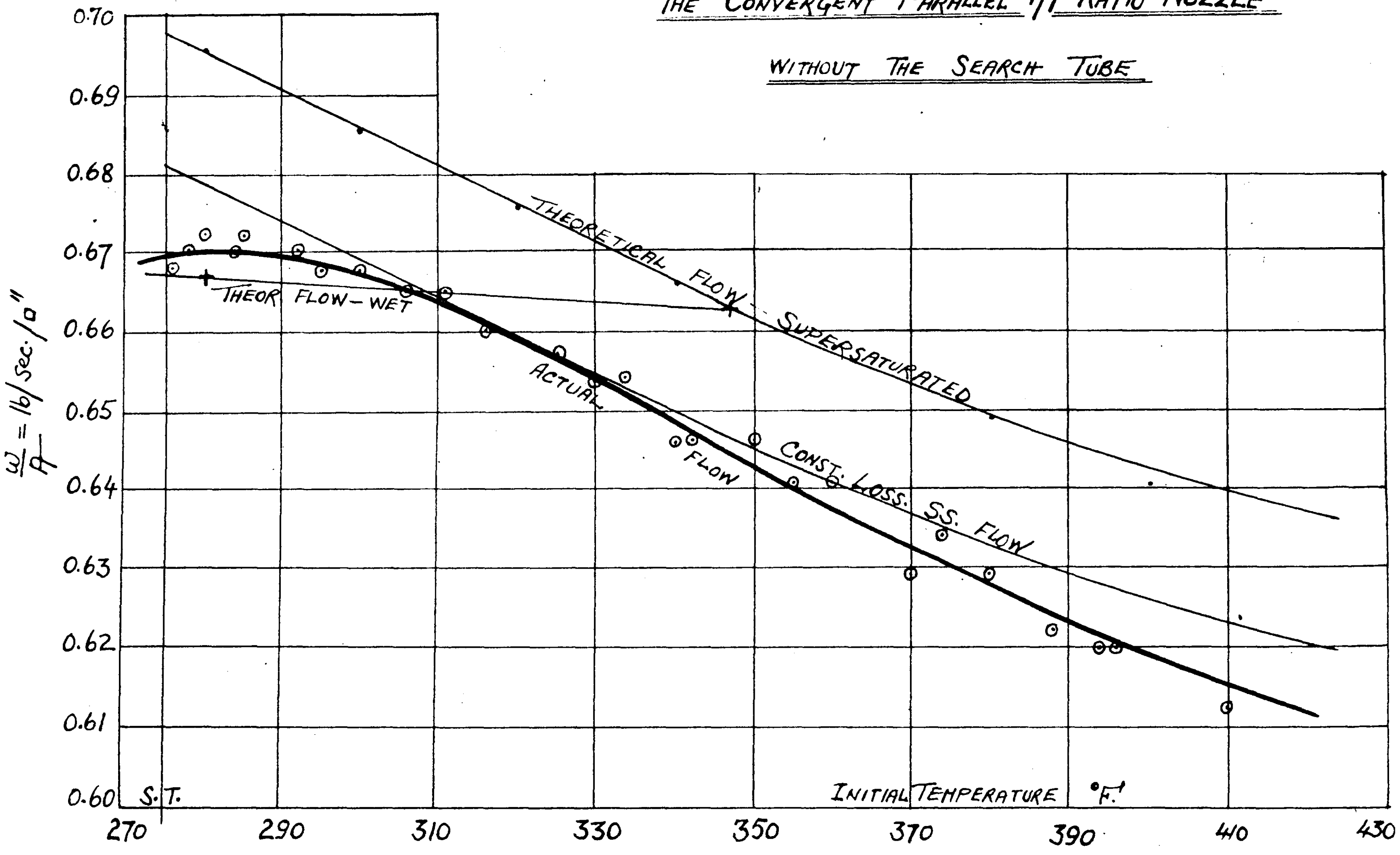


FIG. No. 13

FLOW CURVE FOR

THE CONVERGENT PARALLEL 1/1 RATIO NOZZLE

WITHOUT THE SEARCH TUBE



The constant loss curve is tangent to the actual curve at an initial temperature, also of about $320^{o}F$, but it is nearer to the theoretical curve than in the previous cases. The actual curve departs in the same way from the constant loss curve and its shape on the low temperature side is definitely more convex in form.

<u>The Convergent Parallel 1/1 Ratio Nozzle Without Search Tube</u>: When this nozzle is operated with the full bore open to the flow, no pressure readings are available to provide an outlet pressure ratio, but the theoretical flow curves are determined by assuming a ratio slightly lower than that obtained with the search tube. This ratio is taken as 0.52, and the flow curves are as shown in fig.(13).

The most noticeable difference with the previous results is the more definite flattening of the curve near the initially dry condition. The experimental points are as regular as in the previous cases and the curve form is well in line with the general type showing closely similar features.

<u>The Convergent Parallel ½/1 Ratio Nozzle Without Search Tube</u>: The actual flow curve is given on fig.(14) along with the theoretical curves based on the critical ratio of 0.54. The excessive discharge condition in this nozzle is more noticeable than in the previous cases. At the time when this nozzle was tested, the boiler conditions were such as to make it difficult to establish the flow curve for initial conditions close to the dry state.

The actual flow curve is tangent to the constant loss

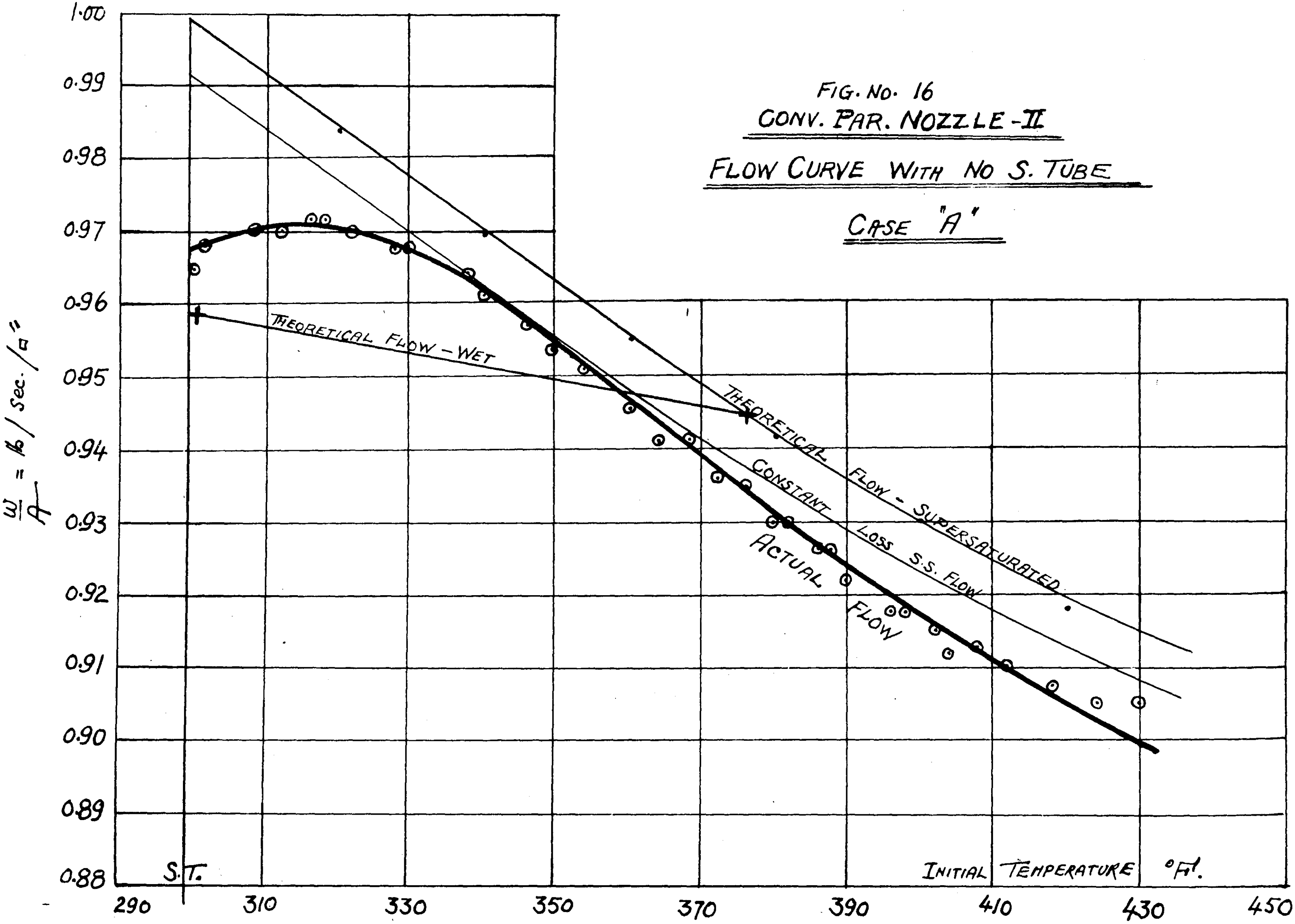
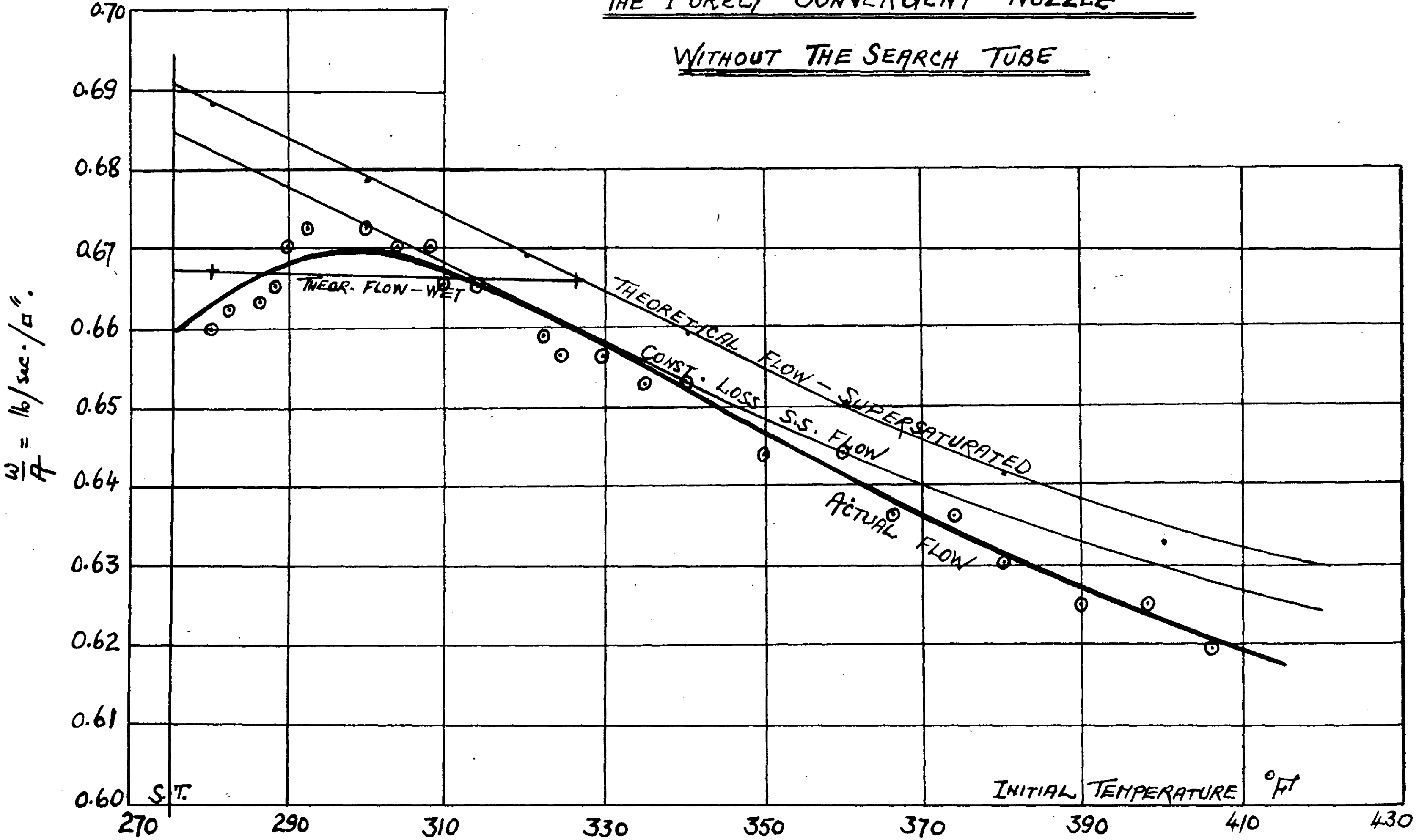


FIG. No. 16
CONV. PAR. NOZZLE - II
FLOW CURVE WITH NO S. TUBE
CASE "A"
THEORETICAL FLOW - SUPERSATURATED
CONSTANT LOSS S.S. FLOW
ACTUAL FLOW
THEORETICAL FLOW - WET
W/A = lb/sec./□"
1.00
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
S.T.
INITIAL TEMPERATURE °F.
290
310
330
350
370
390
410
430
450

FIG. No. 15.
FLOW CURVE FOR
THE PURELY CONVERGENT NOZZLE
WITHOUT THE SEARCH TUBE



curve at about 320°F and it departs from this in the usual way, and it is closely in line with the general type.

The Convergent Nozzle Without Search Tube: The flow curve for this nozzle is shown in fig.(15) and the value of 0.58 for the pressure ratio at exit section is assumed to determine the theoretical flow curves. The actual flow curve is tangent to the constant loss curve at the temperature of 320°F and it still maintains the general type, but the obvious difference with the previous results lies in the much pronounced curvature of the top part.

Tests With Varying Surface Ratio

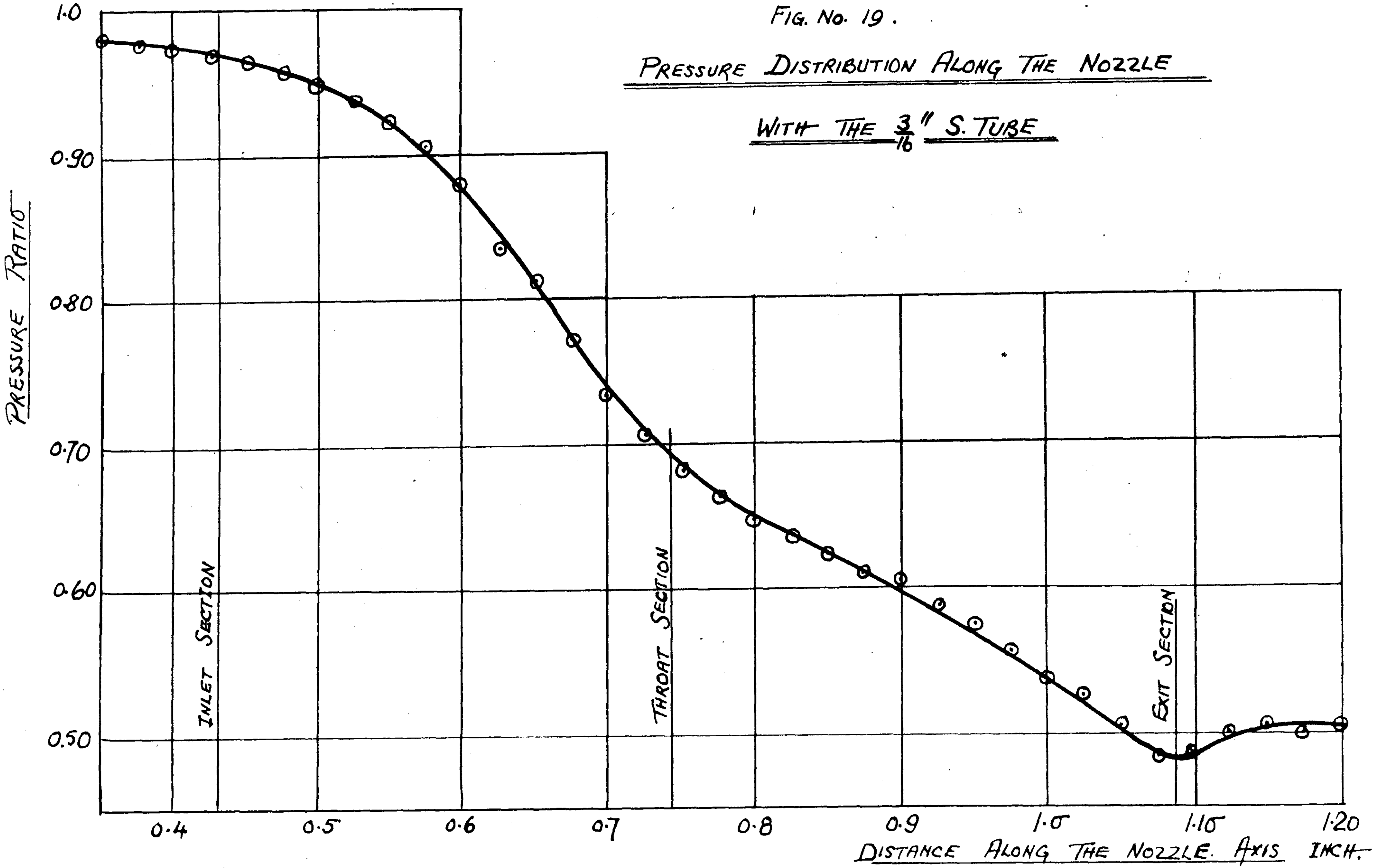
To study the variation of flow quantities with different surface ratio, the convergent parallel nozzle - II - is used with search tubes of various diameters. Care was taken to ensure that the conditions were the same in all tests. The inlet pressure was constant at 65.9 lbs/sq.in Abs. and the back pressure at 34.7 lbs/sq.in. Abs. As in other tests the condensate outlet was maintained at 140°F. The nozzle was first used without a search tube and thereafter with tubes of ⅛, 3/16 and ¼ inch diameter and thus providing four cases really correspond to hydraulic mean depths of 0.0861, 0.0548, 0.03915 and 0.0236 inch respectively. The results for the different cases are presented below.

Without Search Tube: The flow curve for this case is shown in fig.(16) for the standard pressure of 65.9 lbs/sq.in. Abs. The value of 0.52 for the pressure ratio at the outlet section has

Fig. No. 19.

PRESSURE DISTRIBUTION ALONG THE NOZZLE

WITH THE $\frac{3}{16}$" S. TUBE



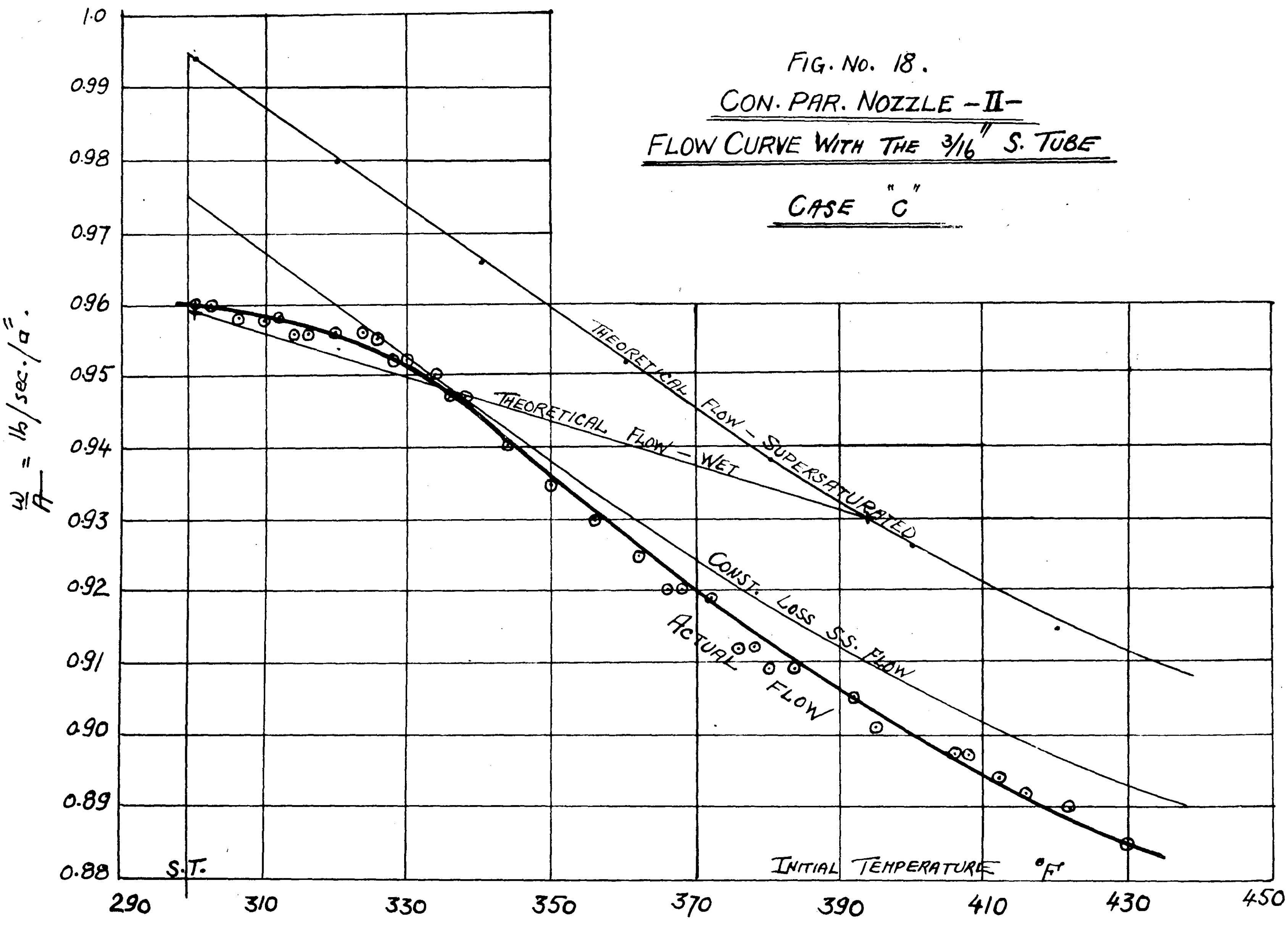


FIG. No. 18.
CON. PAR. NOZZLE -II-
FLOW CURVE WITH THE 3/16" S. TUBE
CASE "C"
THEORETICAL FLOW - SUPERSATURATED
THEORETICAL FLOW - WET
CONST. LOSS S.S. FLOW
ACTUAL FLOW
W/A = lb/sec./□"
1.0
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
S.T.
INITIAL TEMPERATURE °F
290
310
330
350
370
390
410
430
450

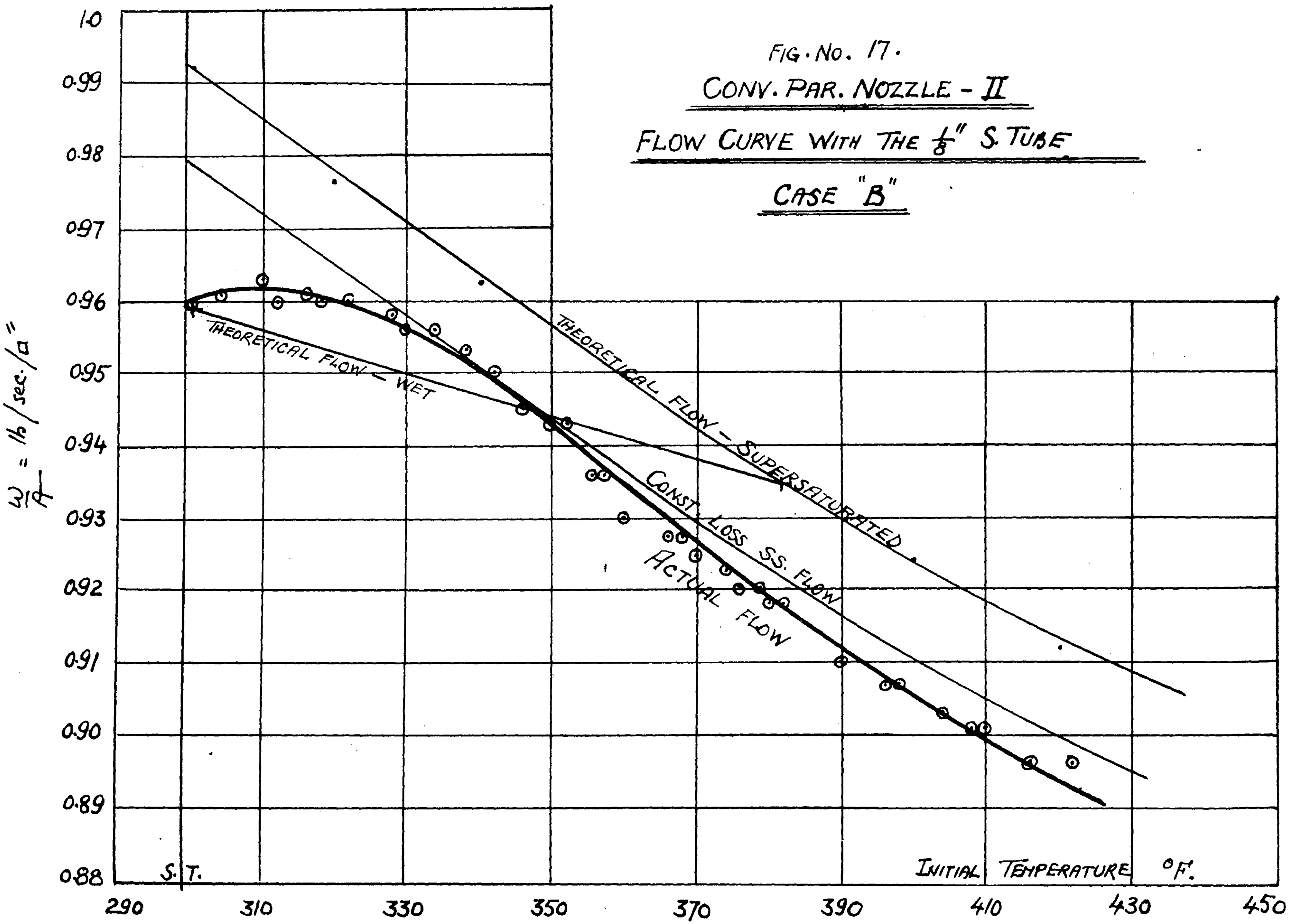


FIG. No. 17.
CONV. PAR. NOZZLE - II
FLOW CURVE WITH THE 1/8" S. TUBE
CASE "B"
THEORETICAL FLOW - SUPERSATURATED
THEORETICAL FLOW - WET
CONST. LOSS SS. FLOW
ACTUAL FLOW
W/A = lb/sec./□"
1.0
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
0.90
0.89
0.88
290
310
330
350
370
390
410
430
450
S.T.
INITIAL TEMPERATURE °F.

been taken for the establishment of the theoretical flow curves as shown in the same figure. The condition of excessive discharge is very clearly shown.

The actual curve is tangent to the constant loss curve at about 340°F. In general, the features are similar to those already presented for other cases but the much more strongly defined curvature in the lower temperature field will be noticed.

<u>With ¼ Inch Diameter Search Tube</u>: The flow curve for this case is shown in fig.(17) and is similar to the last except that the top part is more nearly horizontal and the flow rate is of lower value due to the increased losses. The value of 0.50 is used in establishing the theoretical flow curves as before, and the actual curve is tangent to the constant loss curve at about 340°F initial temperature.

Both above and below this temperature, the actual curve departs in the same way and is well in line with the general type.

<u>With 3/16 Inch Diameter Search Tube</u>: The flow quantities for this case are drawn for the same standard pressure as above in fig.(18). The pressure ratio curve for this nozzle has been obtained and is shown in fig.(19). It indicates a value of 0.48 at exit section which is used to establish the theoretical flow curves. The actual flow curve is tangent to the constant loss curve at about 340°F.

This curve, although well in line with the general type, slopes upwards in the low superheat region instead of being
convex
horizontal or curvex as in the previous two cases.

FIG. No. 22.

FIRST SERIES OF PRESSURE MEASUREMENT.

ALONG THE DIVERGENT NOZZLE

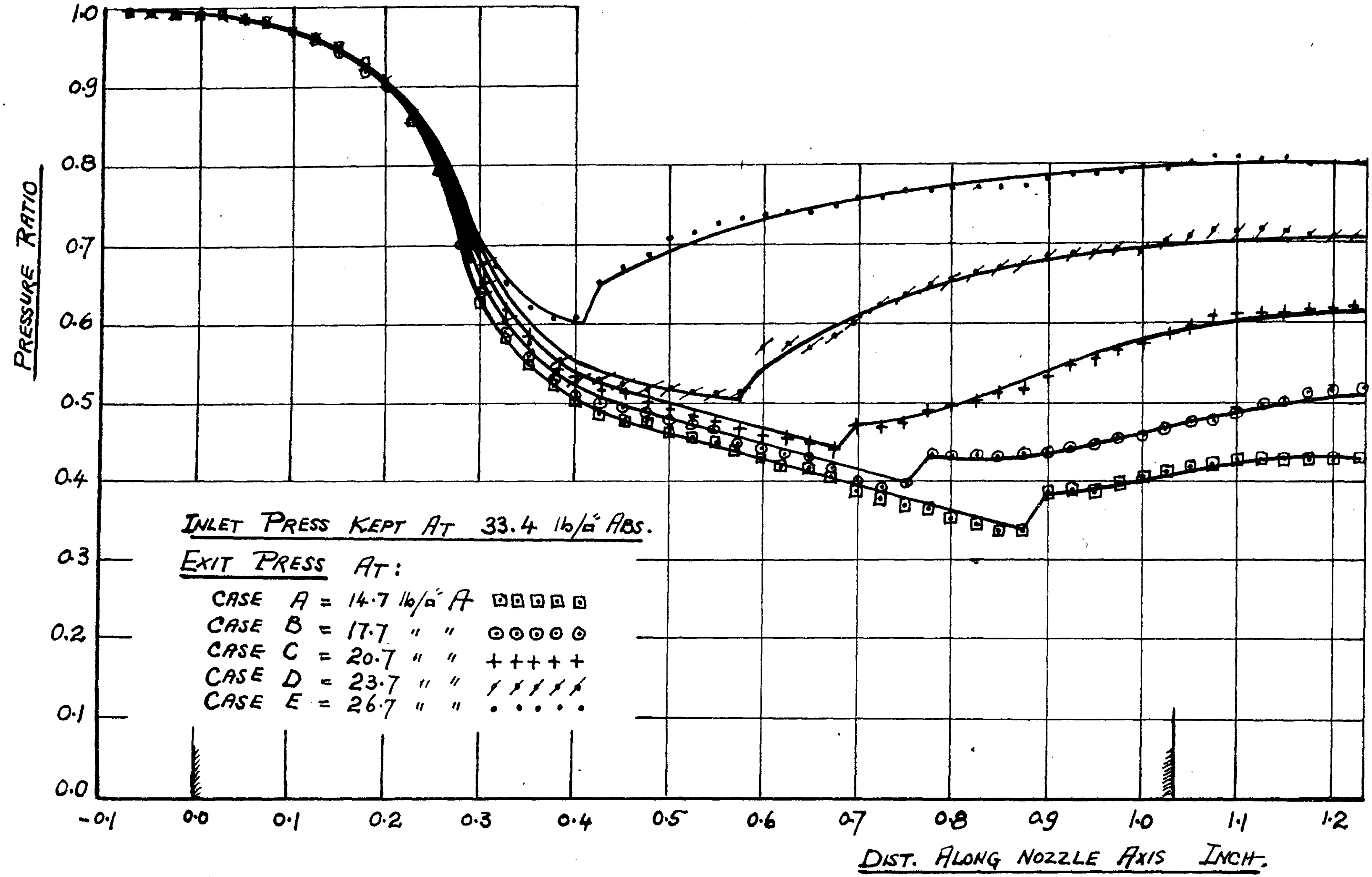
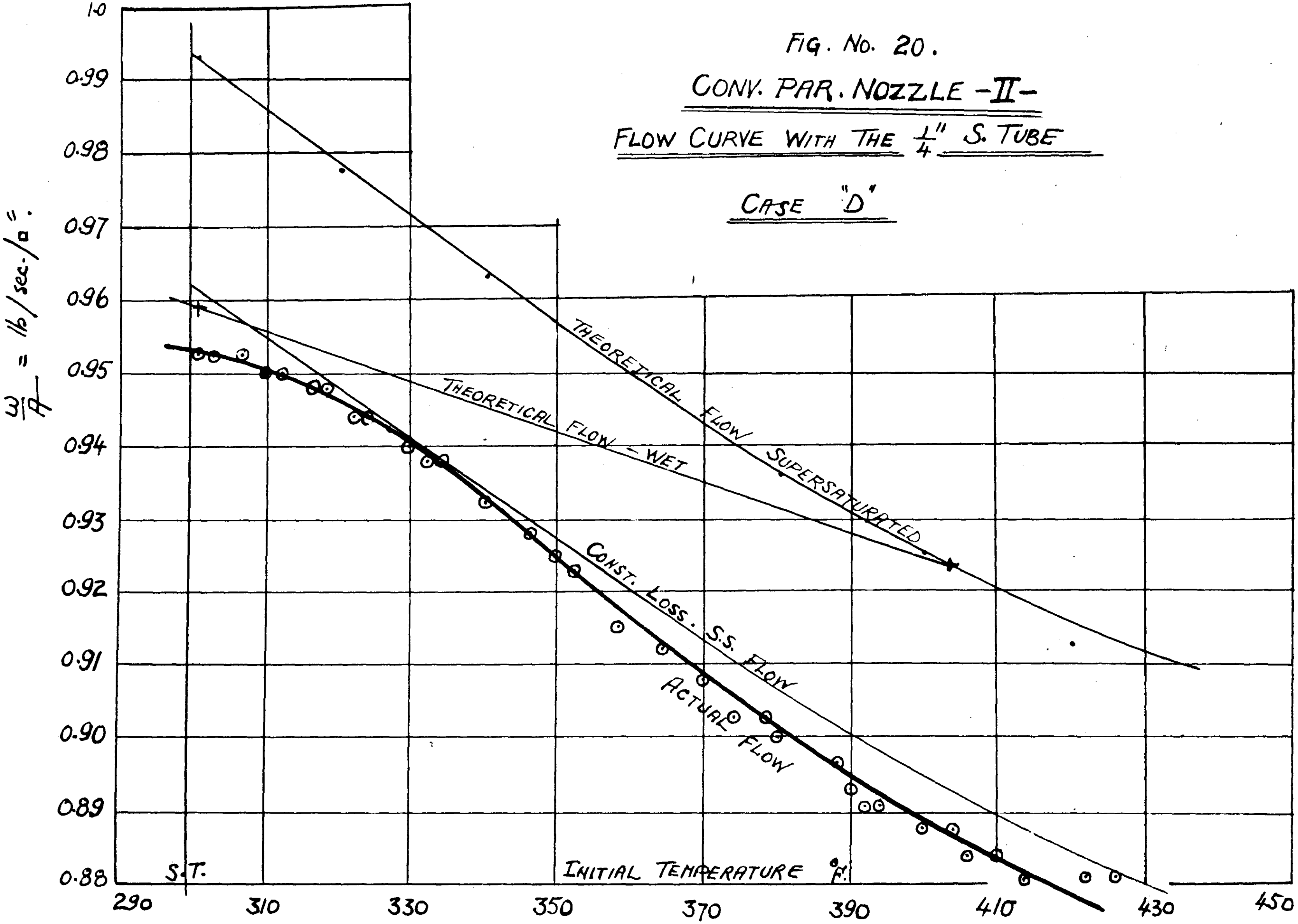


FIG. No. 20.

CONV. PAR. NOZZLE -II-

FLOW CURVE WITH THE $\frac{1}{4}''$ S. TUBE

CASE "D"



With $\frac{1}{4}$ Inch Diameter Search Tube: In this case the friction losses are highest and the flow quantities are naturally the lowest of the series. The higher loss tends to obscure the effect of excessive discharge and in fig.(20), in which the theoretical flow curves are drawn on a pressure ratio of 0.46 at exit section, the top points of the actual flow curve are well below the theoretical curve on the assumption of wet expansion.

The actual curve is tangent to the constant loss curve at a temperature of about 340°F and lies quite close to it, showing that the assumption of constant loss is perhaps more nearly correct the higher the surface ratio.

## Divergent Nozzle Results - Series I

The inlet pressure was kept constant at 33.4 lbs/sq.in Abs. and the steam was just dry or only very slightly superheated. The back pressure was varied to give 14.7, 17.7, 20.7, 23.7 and 26.7 lbs/sq.in. Abs. thus providing five different ratios of expansion under which the nozzle was tested. These different cases are distinguished by symbols A, B, C, D and E respectively.

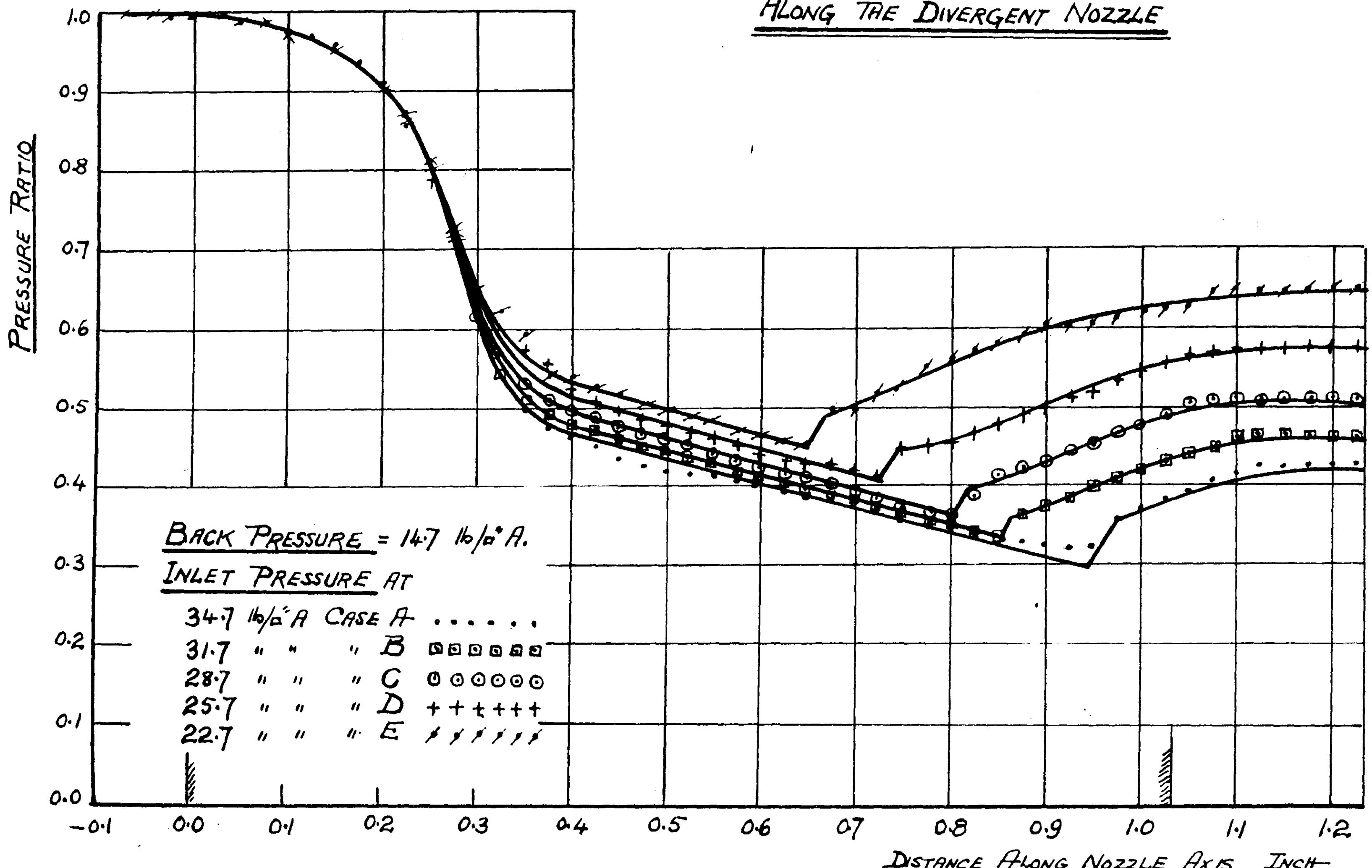
This series of tests consists of pressure measurements along the nozzle under these different cases of expansion. Ample time was given to the search tube pressure gauge to read the accurate final values especially in the divergent part of the nozzle. In these cases the temperature was also explored as is explained later.

The pressure curves are presented in fig.(22) in which the value indicated at any section in any case is the ratio of the

FIG. No. 23.

SECOND SERIES OF PRESSURE MEASUREMENT

ALONG THE DIVERGENT NOZZLE



pressure at that point to the standard pressure of 33.4 lbs/sq.in. Abs. The curves are of the usual type and show the recompression points very clearly. This sudden rise in pressure is nearest the outlet in case (A) where the expansion ratio is the lowest, and approaches the nozzle throat with higher back pressures until finally in case (E) it is within the throat section.

Divergent Nozzle Results - Series II

Tests similar to those already described were carried out but with constant back pressure and varying initial pressures. With the former kept at atmosphere, the supply conditions were controlled to give variations between 34.7 and 22.7 lbs/sq.in. Abs. In the five different tests made (cases A to E, series II) the inlet pressure was reduced by steps of 3 lbs/sq.in. The same procedure was followed and the same precautions taken. Temperature measurements were also made and these are considered later.

The pressure ratio curves are shown in fig.(23) from which it can be seen that they have the same characteristic as in series I. In fact they do little more than provide a verification of the practice of basing pressure studies in nozzle flow on pressure ratios instead of absolute values.

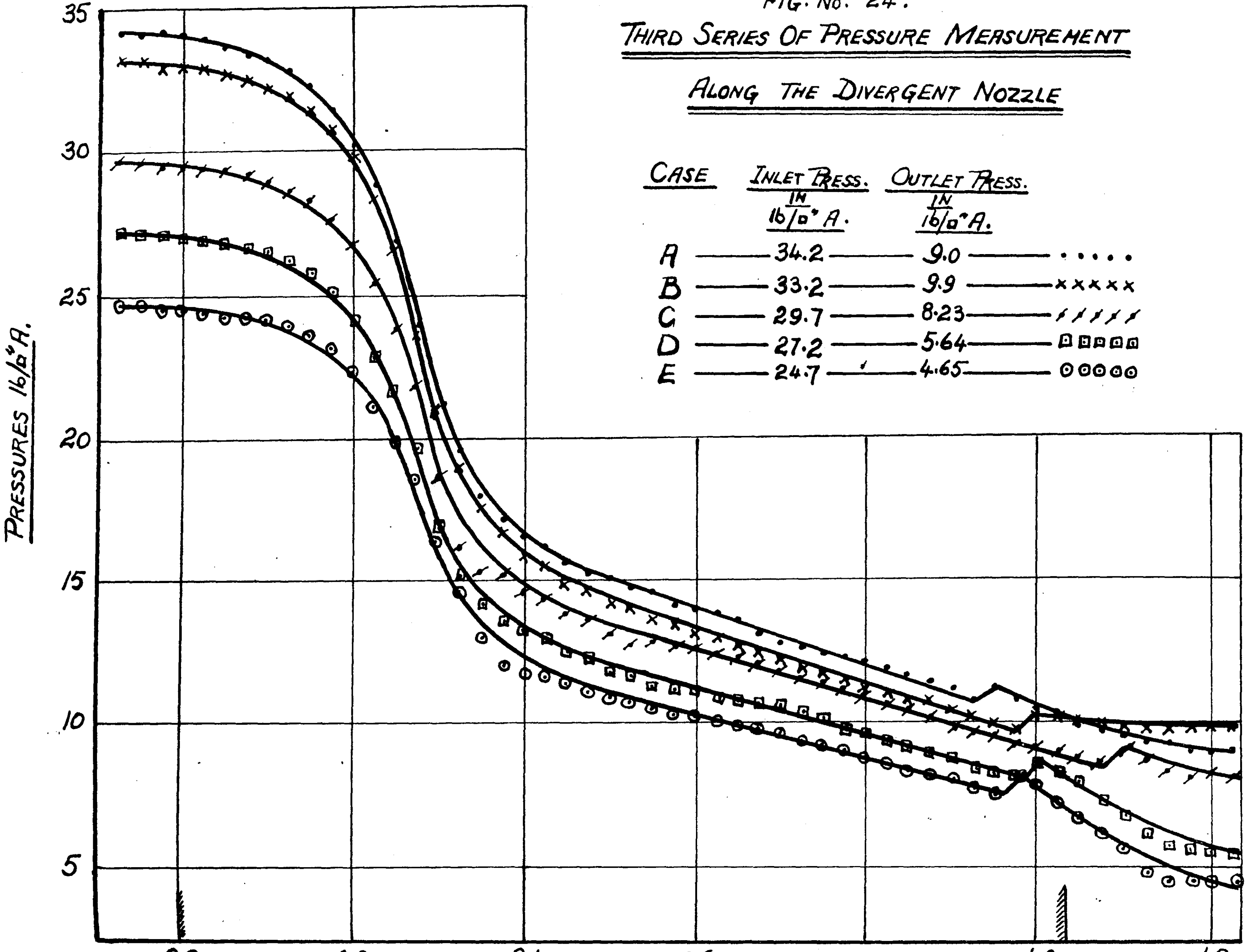
Divergent Nozzle Results - Series III

In this series, the apparatus was connected directly to a relatively large surface condenser where vacuum could be maintained to any degree. Chosing a pressure ratio allowing full expansion

FIG. No. 24.

THIRD SERIES OF PRESSURE MEASUREMENT

ALONG THE DIVERGENT NOZZLE



along the nozzle, the inlet and back pressures were determined for five different cases.

The resulting pressure curves are shown in fig.(24). The pressure drops steeply in the convergent part in the usual way and then gradually along the divergence, but near the outlet a pressure "jump" is recorded in all cases. Although in actual pressure terms this disturbance is slight - about ½ lb/sq.in. - it is clearly disclosed. These curves have been plotted in actual pressures so as to show up the features clearly. If pressure ratios had been used here the curves would nearly coincide.

Temperature exploration was also carried out in these cases and the records of temperature diagrams have to be examined in order to show the significance of the "jump".

## B. TEMPERATURE INVESTIGATIONS

### The "Hidden" Thermojunction

The fact that all experimental temperature determinations in high speed steam jets had failed to give results of value or certainty seemed to make any further attempts on same lines rather hopeless. The common argument that condensation came down on exposed surfaces was convincing in view of the general fact that all thermocouples inserted in the stream read either the saturation temperature or something close to that.

At the beginning of these investigations attention was concentrated on the flow curve type and its variations. It was early established that the Mellanby and Kerr form of curve could be fully substantiated. It was in fact found that the peculiarity of the original form was even more pronounced than had been supposed. The assumption made by the original authors that the curve shape implied early and partial condensation was at least plausible in view of this shape. It was in fact supposed that the "fractional reversion" apparently shown by the curves represented the condensation on the surface bounding the flow. In the small elementary type of nozzle, the surface ratio is, of course, high.

When, however, surface ratio was made a main variable - Convergent parallel nozzle series II - the results obtained, when analysed, upset this line of reasoning. The results of the analysis are given later and meantime it is sufficient to say that they showed a very marked reduction of the so called "fractional reversion" with increase of surface ratio. This did more than make the idea doubtful. It meant that it was in conflict with the surface condensation argument and that the failure to read undercooled steam temperatures correctly might not be due to any such cause.

Then remained the possibility that direct impingement on the thermojunction was the difficulty. Hence the conclusion was reached that if the surface were not maintained at saturation temperature by condensation, a thermojunction "hidden" from the stream and insulated from the wall of the search tube that carried it might read correctly.

FIG. 21 a.

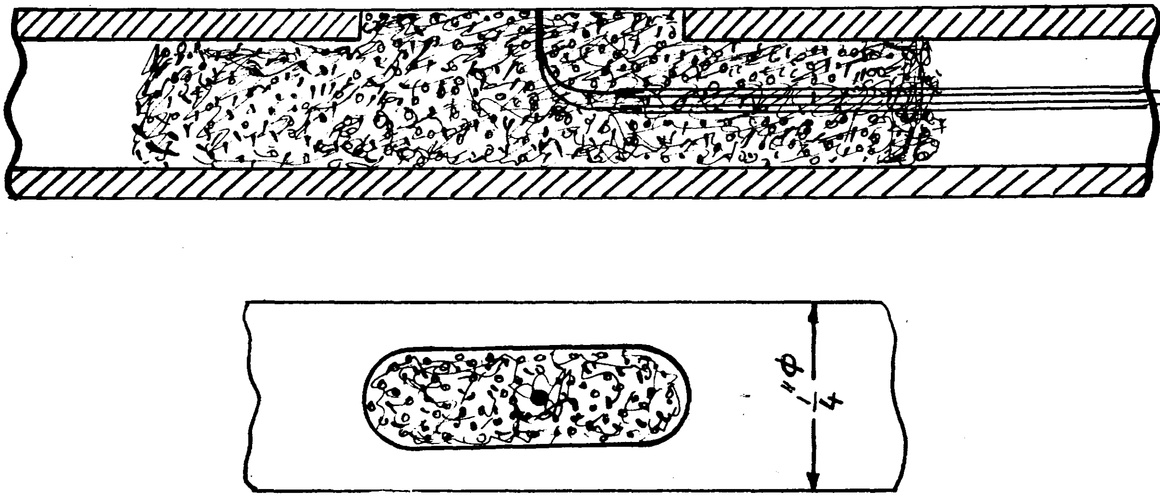
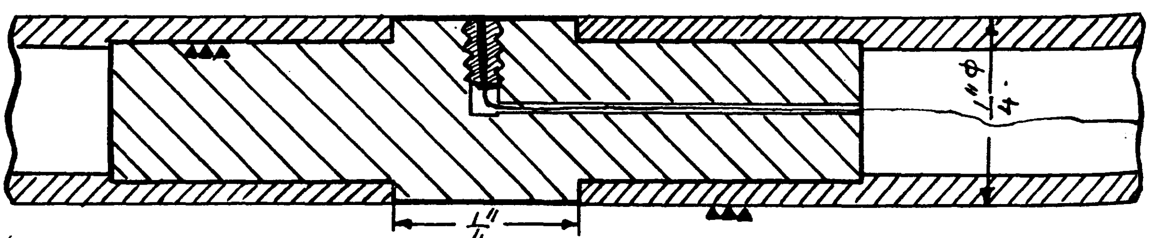



FIG. 21 b.



This led to the attempts and results now presented. It may be admitted that the line of reasoning, in view of the later interpretation of data, does not now appear wholly logical but it indicated a mere approach to the temperature problem and immediately on trial give results of interest and significance.

## First Thermojunction Type

The principle followed was that of insulating the thermo-junction and preventing exposure to the fluid. This was first attempted by "bedding" a silver soldered junction of fine constantan and copper wires in a small block of cement made of alundum powder and water and set in a slot cut in the wall of the search tube being thereafter hardened by indirect flame. The cement surface was smoothed off to the contour of the wall. Special care had to be exercised to ensure that the junction was not exposed. The scheme will be clear from the sketch in fig.(21a). Calibration was carried out in the usual way using a milli-voltmeter.

In the trial run, the tube was fitted in the nozzle and steam at 50 lbs/sq.in. gauge was supplied. After a few minutes drops of water were seen to issue from the free end of the search tube indicating leakage at the cement. This was taken to mean that the small cement inset could not withstand the pressure used and a low pressure of 13 lbs/sq.in. gauge was adopted for further tests. It was also decided to remake the cement using sodium nitrate in place of water to give more adhesion.

The convergent parallel nozzle II was used in these tests.

FIG. No. 25.

PRESSURE DISTR. WITH THE 3/16" S. TUBE

AVERAGE CURVE AT 300 °F INLET TEMP.

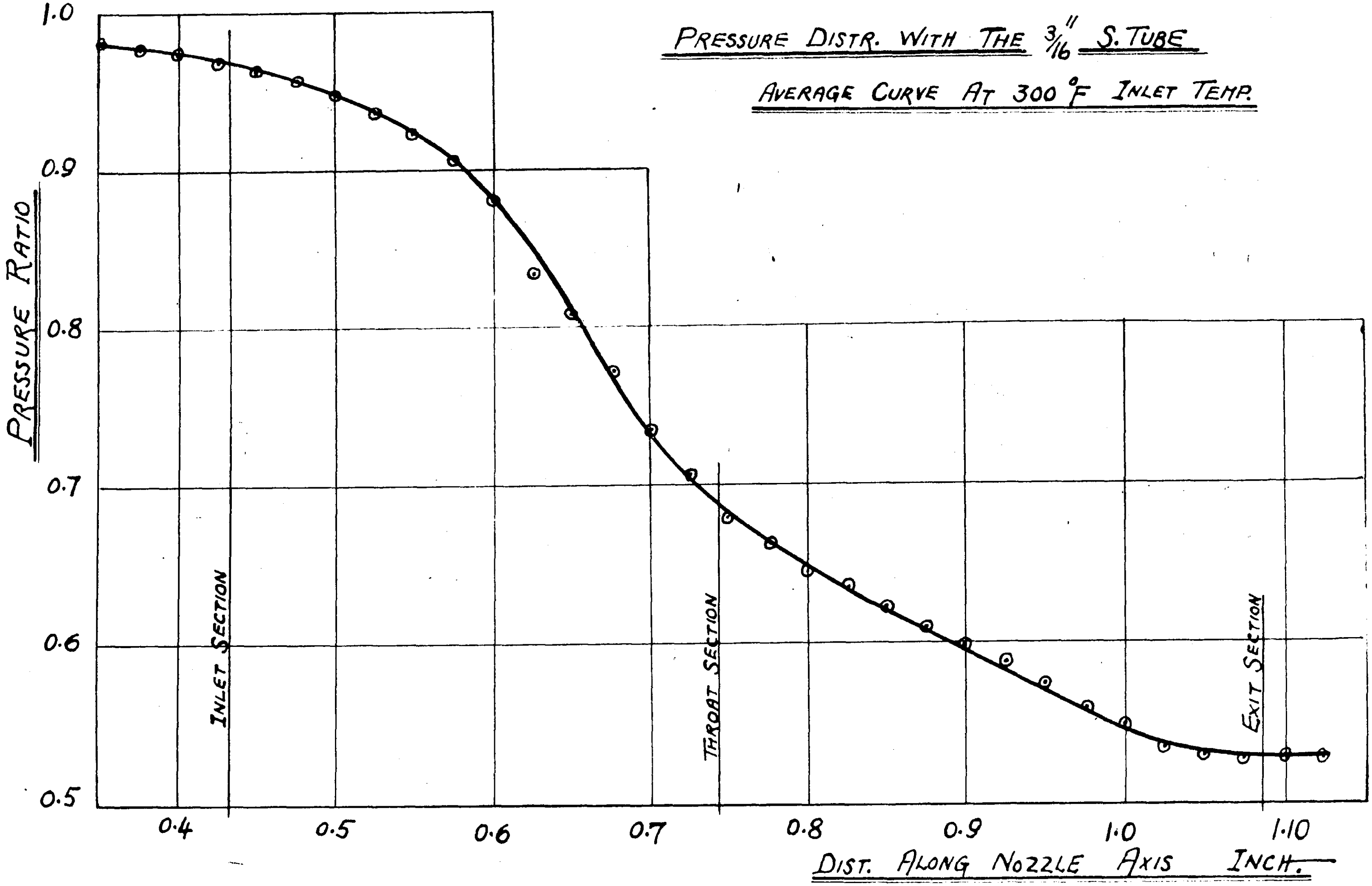
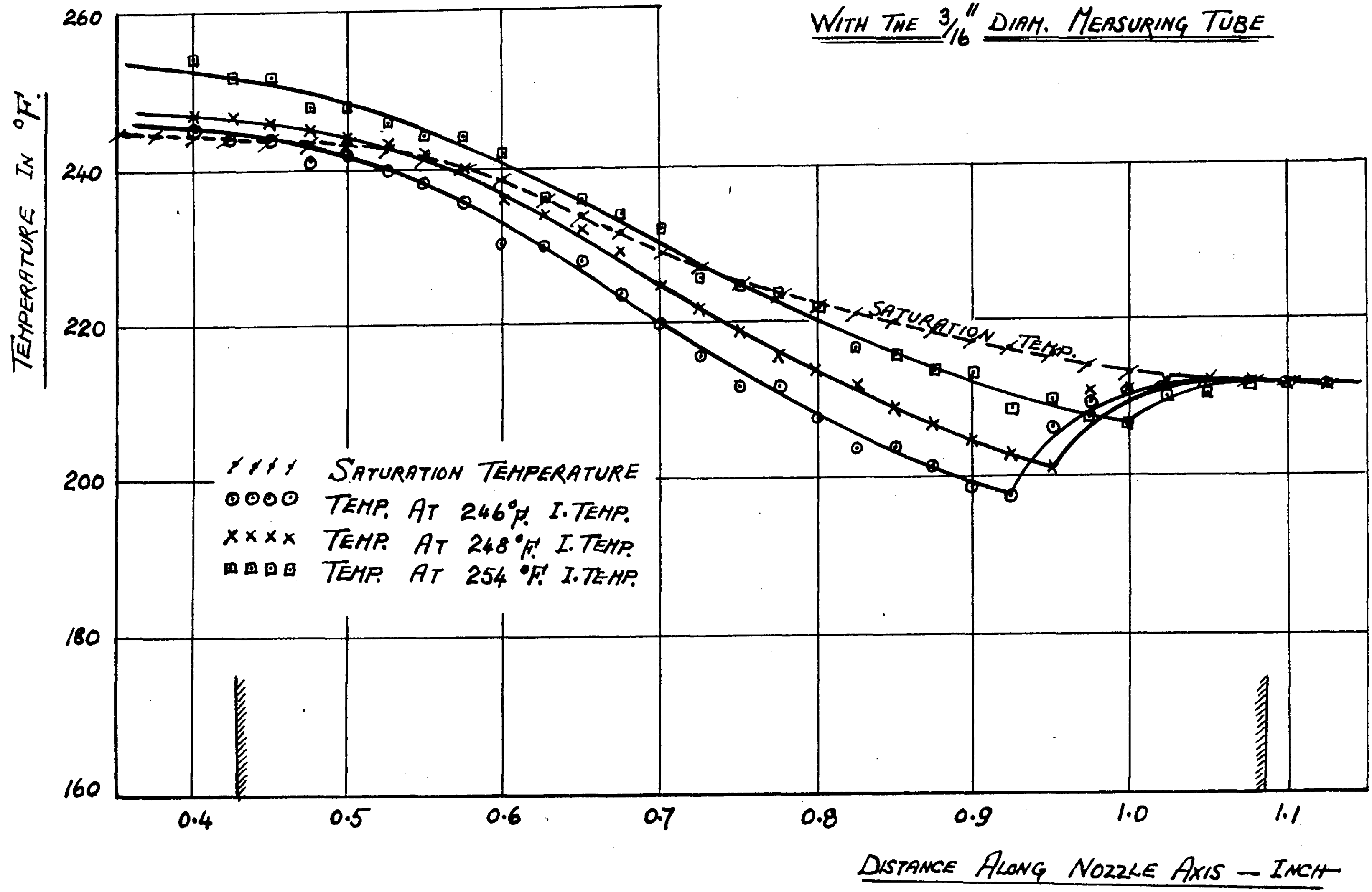


Fig. No. 26.

Temperature Along the Convergent Nozzle

With the 3/16" Diam. Measuring Tube



Before the temperature observations were made, the pressure distribution was explored so that the saturation temperatures at all points in the length could be compared with the actual readings. The pressure curve is shown in fig.(25).

<u>Test With Initial Temperature of 246°F.</u> The temperature was measured at intervals of 0.025 inch along the full nozzle length. The curve showing the readings is given in fig.(26) which also shows the saturation temperatures corresponding to the pressures.

It will be seen that some undercooling is recorded just beyond the inlet and that this increases thereafter reaching a value of about 11°F at the throat and attaining a maximum of 19°F at a position 0.15 inch before the outlet. At this point it takes a very sudden rise meeting the saturation temperature curve within a short distance.

This was the first indication that the method had merits as the degree of undercooling shown was distinctly higher than had been obtained in any other thermojunction application while the sudden rise was a new manifestation indicating that the method had the power to give positive evidence of reversion.

On a repeat of this test, however, it was discovered that the readings were different. This sudden rise was still shown but the recorded undercooling was reduced. Another repeat test showed a still further reduction. The fitting was then removed for examination and it was found that the high speed of fluid had swept away a layer of the cement and exposed the junction. A new junction

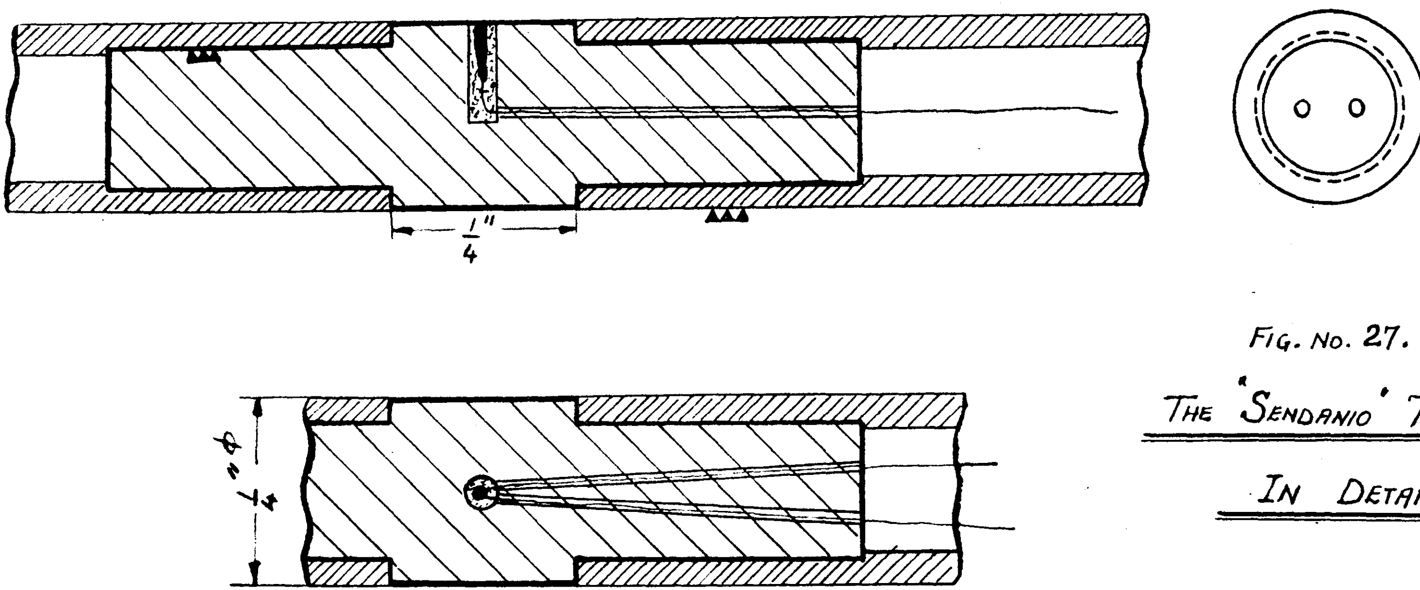



FIG. NO. 27.

THE "SENDANIO" THERMO-TUBE

IN DETAILS

had to be arranged and cemented but it was already clear that the scheme used, while giving promise of success, was going to be troublesome.

Test with Initial Temperature of 248. After renewal of the cemented junction the fitting was used with an initial temperature of 248°F. The curve of temperature for this case is also shown on fig.(26) and can be compared with the previous result. The undercooling appears less and the point of reversion occurs lower downstream.

Test with Initial Temperature of 254°F. The tube was again fitted with a new junction and cement filling and tested at the higher temperature of 254°F. The curve for this is included in with the others in fig.(26). This initial temperature is now high enough to limit considerably the range of possible undercooling and this is reflected in the curve which shows superheat to a point beyond the throat and a relatively small maximum undercooling of 6°F near the outlet where, however, reversion is again shown quite clearly.

### Second Thermojunction Type

Other synthetic resins were tried in place of the alundum powder, but they failed to withstand the high temperature and velocity of the flowing steam for even as short a time as the alundum powder. It was then decided to try a new arrangement using an asbestos bonded material commercially known as "Sendanio".

The design of this second type is shown in fig.(27). A tube $\frac{1}{4}$ inch diameter is used. It is cut transversally for the

FIG. No. 28.

PRESSURE DISTRIBUTION ALONG THE NOZZLE

WITH THE $\frac{1}{4}$ INCH S TUBE

AVERAGE CURVE AT (246–48) °F.

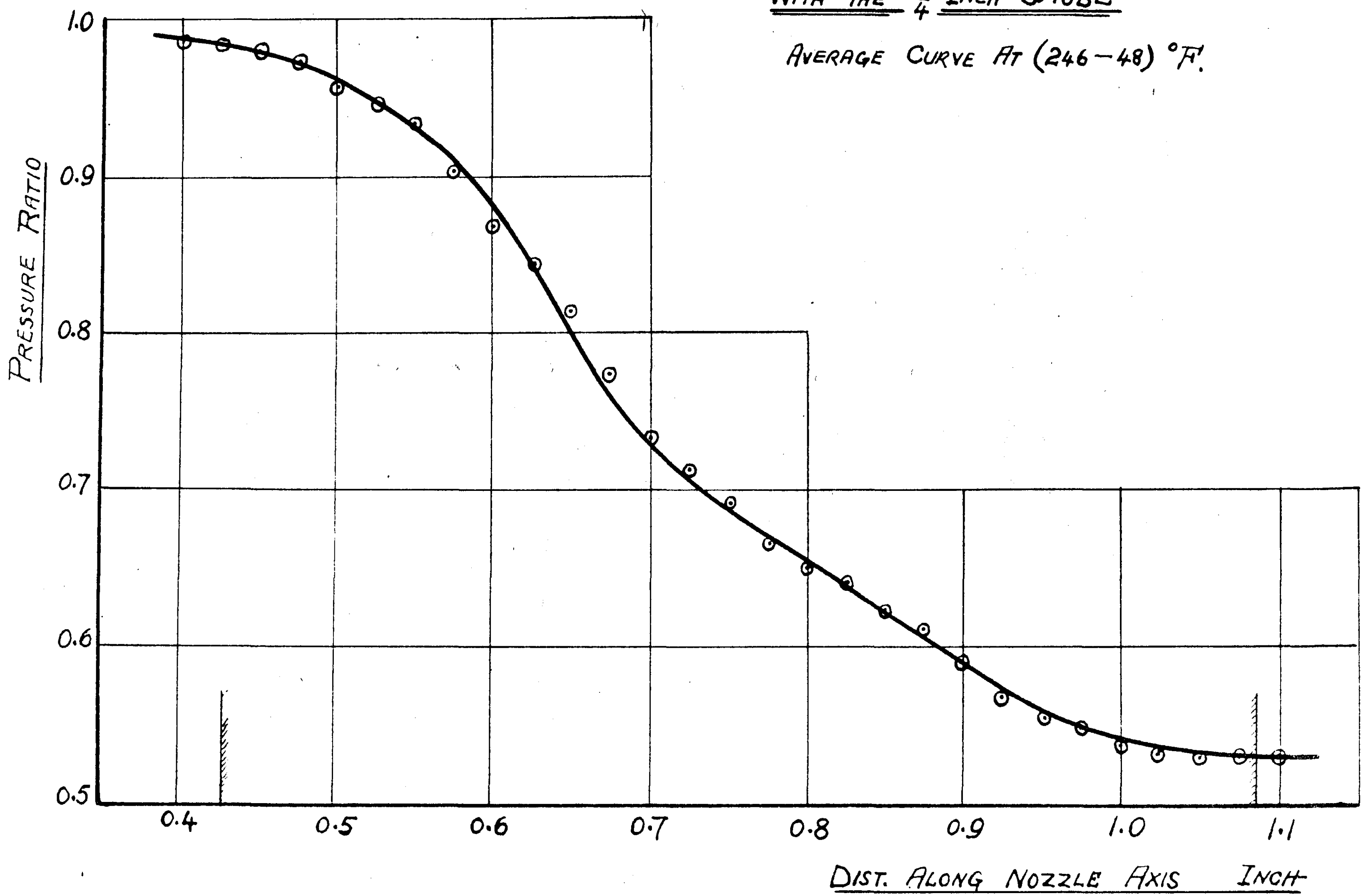
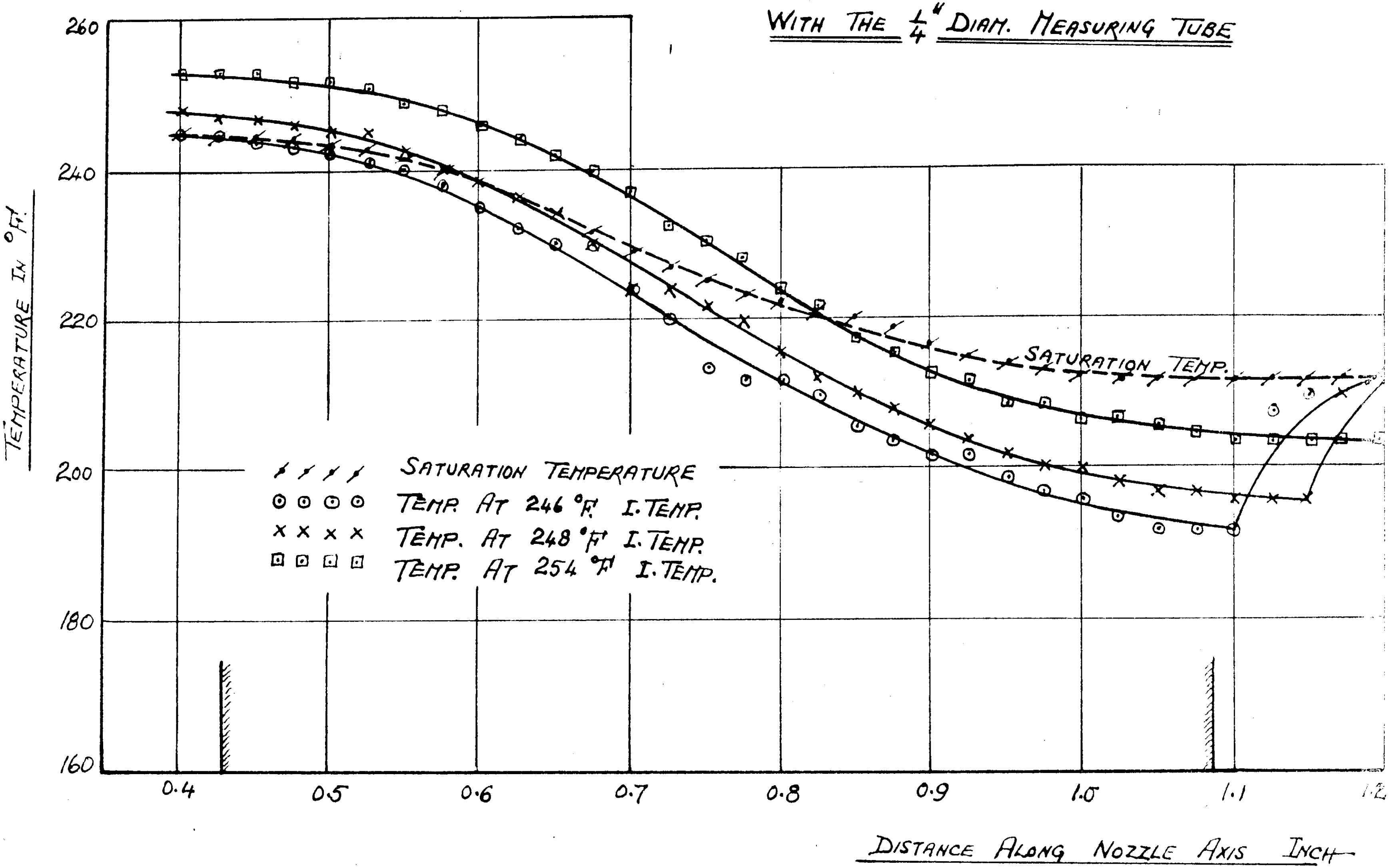


Fig. No. 29

Temperature Along The Convergent Nozzle

With The 1/4" Diam. Measuring Tube

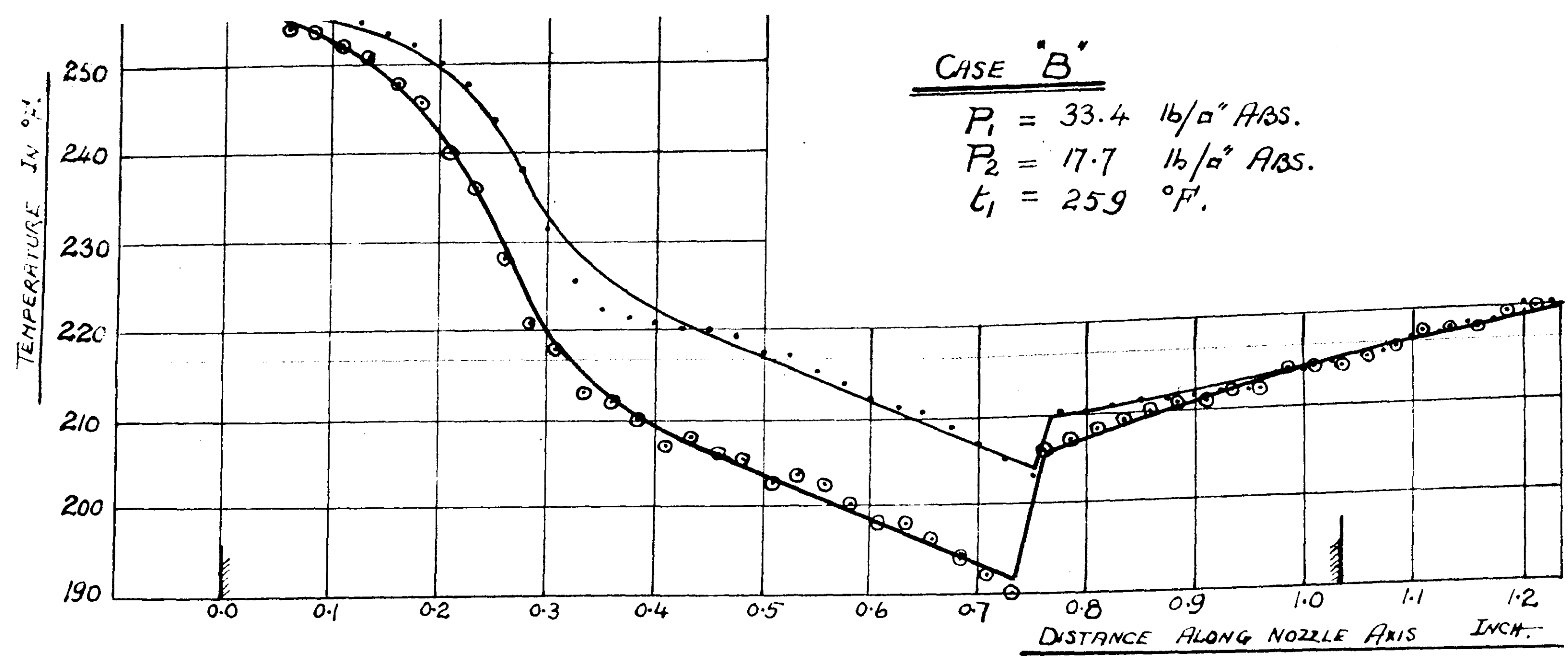


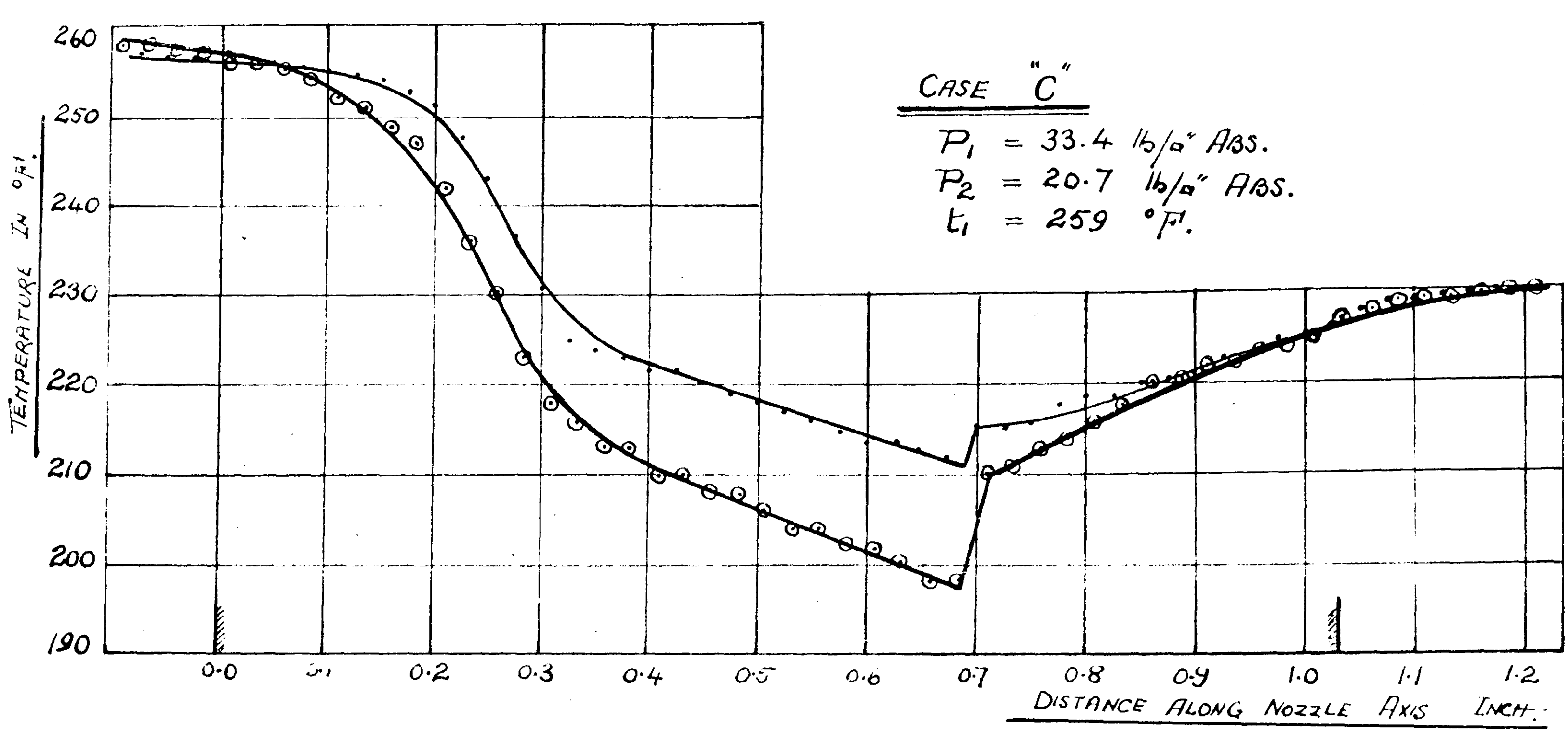
introduction of the "Sendanio" part which is specially fitted to the bore of the tube to ensure a steam tight joint. The thermo-junction wires are passed in as indicated through separate holes to meet in the small radial passage where the junction is formed and set close to the surface. Attempts to fill the radial hole with silver solder failed as the adhesion with the material was unsatisfactory and alundum powder was used for the purpose of shrouding the thermojunction.

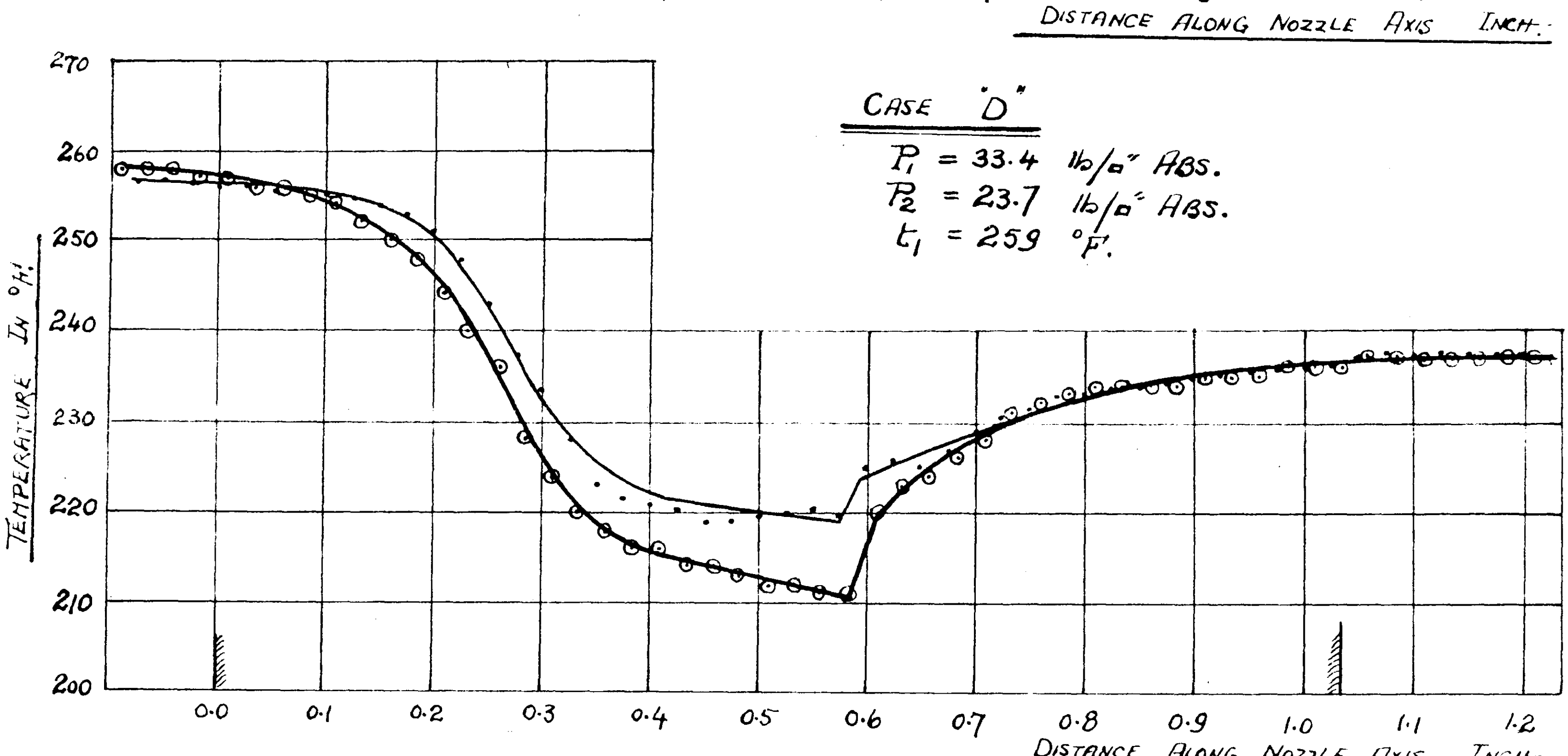
The nozzle used with the first type of junction was again investigated with the new arrangement and under the same conditions. The pressure distribution with a $\frac{1}{4}$" diameter search tube was first determined under the same conditions of expansion as with the first junction type. This is given in fig.(28) and the corresponding saturation curve in fig.(29)

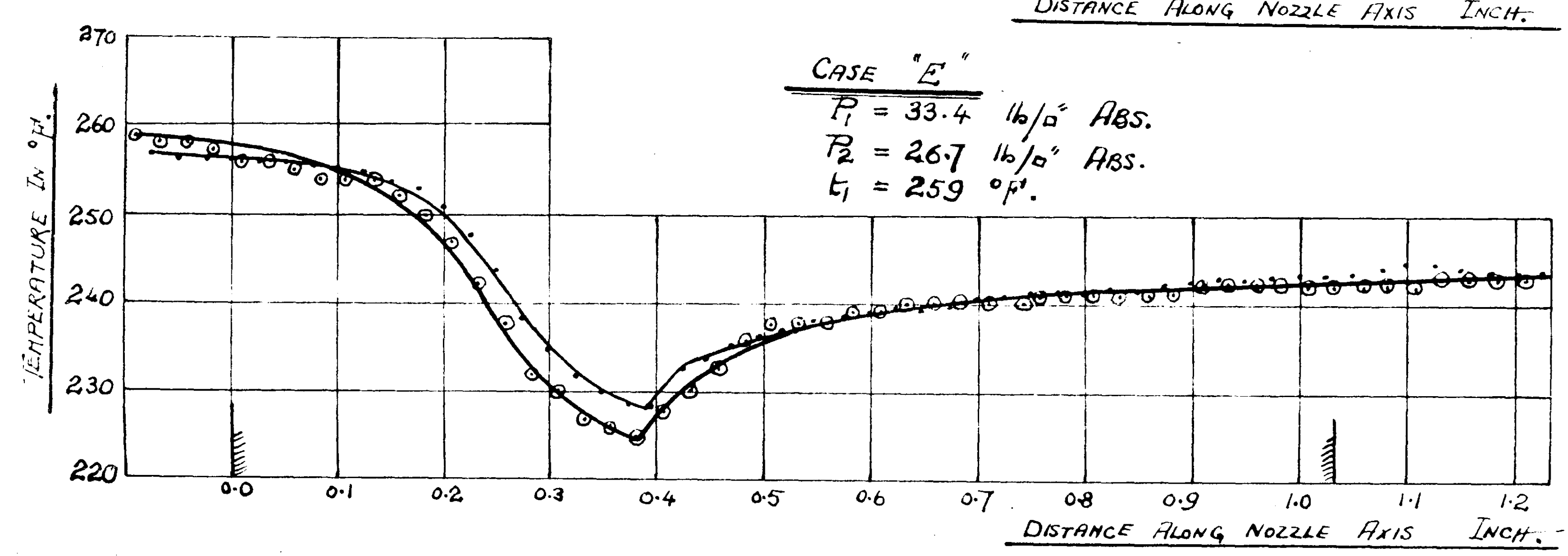
<u>Test with an Initial Temperature of 246°F.</u> The temperature was measured at intervals of 0.025 inch and the curve obtained is shown in fig.(29). The curve form is closely similar to that obtained with the first junction type and shows about 10°F undercooling at the throat. The degree, however, increases gradually along the parallel part until the steam leaves the nozzle where it suddenly reverts to the normal saturated temperature.

<u>Test with an Initial Temperature of 248°F.</u> The same test was repeated with an initial temperature of 248°F and the curve obtained is similar to the last but about 3-4°F higher and the point of reversion appears a little farther downstream.


CASE "B"
$P_1$ = 33.4 lb/□" ABS.
$P_2$ = 17.7 lb/□" ABS.
$t_1$ = 259 °F.
TEMPERATURE IN °F.
250
240
230
220
210
200
190
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2
DISTANCE ALONG NOZZLE AXIS INCH.


CASE "C"
$P_1$ = 33.4 lb/□" ABS.
$P_2$ = 20.7 lb/□" ABS.
$t_1$ = 259 °F.
TEMPERATURE IN °F.
260
250
240
230
220
210
200
190
0.0 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2
DISTANCE ALONG NOZZLE AXIS INCH.


CASE "D"
$P_1$ = 33.4 lb/□" ABS.
$P_2$ = 23.7 lb/□" ABS.
$t_1$ = 259 °F.
TEMPERATURE IN °F.
270
260
250
240
230
220
210
200
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2
DISTANCE ALONG NOZZLE AXIS INCH.


CASE "E"
$P_1$ = 33.4 lb/□" ABS.
$P_2$ = 26.7 lb/□" ABS.
$t_1$ = 259 °F.
TEMPERATURE IN °F.
270
260
250
240
230
220
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2
DISTANCE ALONG NOZZLE AXIS INCH.

Test with an Initial Temperature of 254°F. With this degree of superheat (about 10°F) there appears to be no reversion of the steam within the nozzle length or beyond the exit section so far as it has been explored. The undercooling attained at the throat is about 3°F and increases to 8°F at the extreme end of the nozzle.

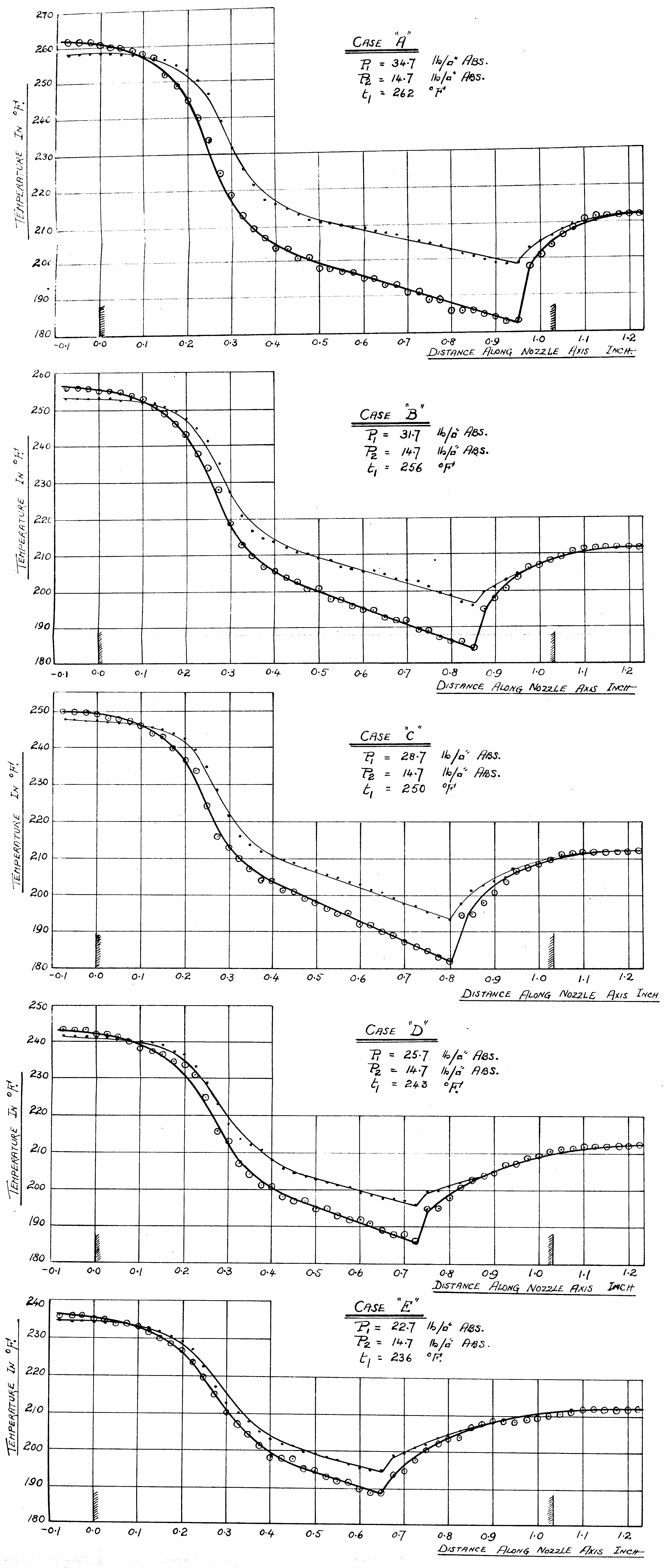
The results obtained with this second thermojunction have the same characteristic as those given by the first but whereas the earlier type had to be renewed for every test, the improved form displayed a somewhat higher degree of reliability and endurance. Nevertheless it proved faulty and for the same reason as before, viz. erosion of the alundum plug.

A further improvement was then effected. A small copper pin was screwed into the radial hole in the "Sendanic" part and the thermojunction wires were silver soldered to this pin. Thus, when smoothed and rounded off, the outer surface is wholly metallic and not affected by the flow. This scheme is shown in fig.(21b). The modification proved very successful and represents the final form used. It was employed in obtaining all the results with the divergent nozzle which are now presented.

Divergent Nozzle Results - Series I

The temperatures along the divergent nozzle when operating under the same conditions as for the first series of pressure measurements were explored. The temperature curve together with the saturation temperature curve corresponding to the pressures are shown together for each case in the series of diagrams marked cases

## Second Series Of Temperature Measurement On The Divergent Nozzle



A, B etc. Reference to fig.(22) already given will show the *how* pressure curves correspond.

Thus it would appear as though the degree of undercooling varies inversely with the pressure ratio at the point of reversion which would seem to indicate a dependence on velocity but this is not necessarily a legitimate deduction unless it is certain that undercooling is being properly recorded.

## Divergent Nozzle Results - Series II

The temperature curves for this series are given in the series of diagrams marked cases A, B etc. and correspond to the second series of pressure tests with variable inlet and constant back pressure conditions. They show the same features as in the previous case with increasing undercooling to the point of sudden reversion and again comparison with the corresponding pressure curves shows that the positions of the discontinuities are practically identical.

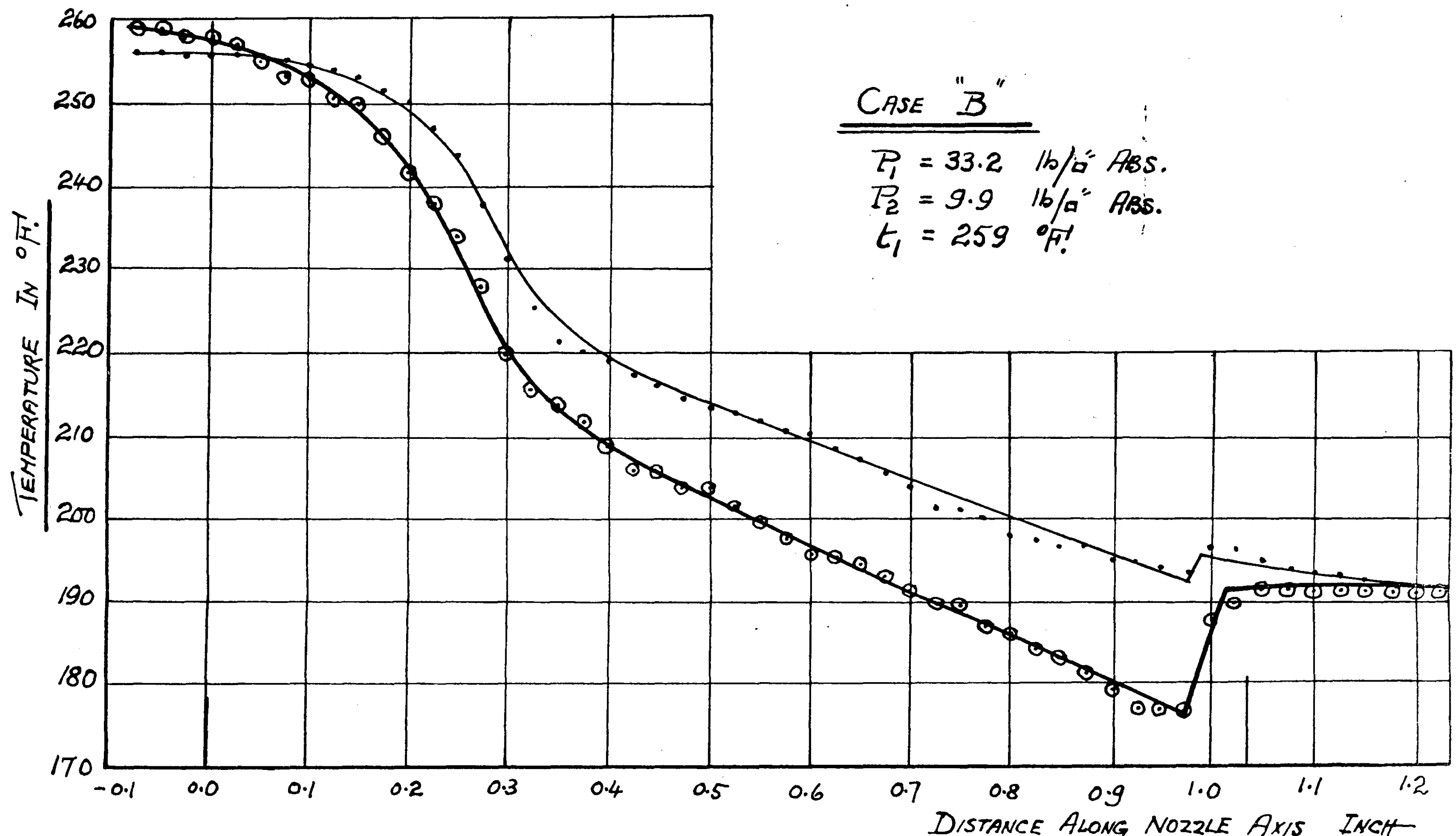
Although in this series as in the previous, the maximum measured undercooling is only about one third of the theoretically possible it still represents the highest value so far observed attaining 16,14,11,9 and 6$^{o}$F for cases A, B, C, D and E respectively.

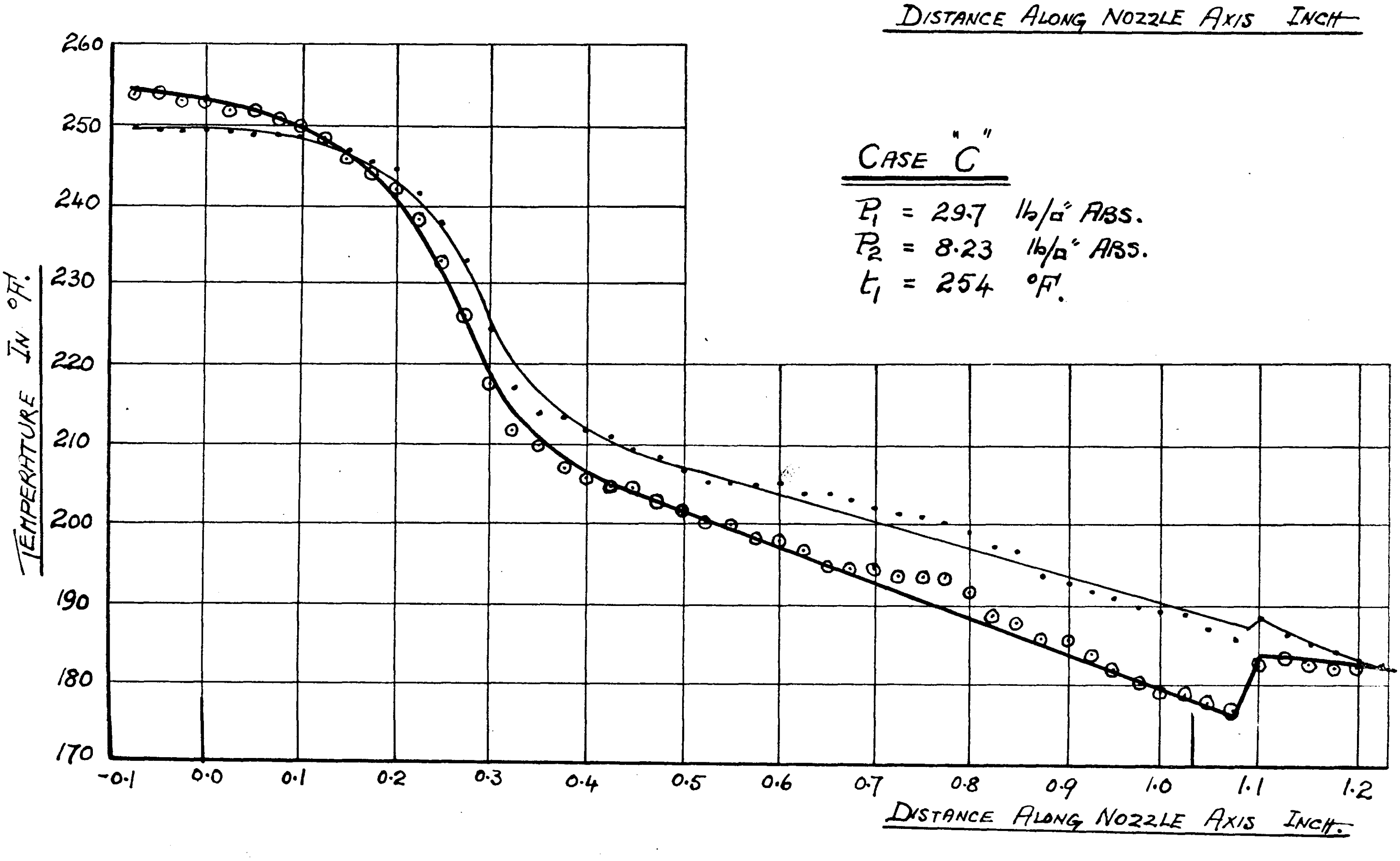
## Divergent Nozzle Results - Series III

The instability of the back pressure in this series made it difficult to obtain the temperature curves with certainty. But in two cases the conditions were favourable and the results of these

# Third Series Of Temperature Measurement

## Along The Divergent Nozzle

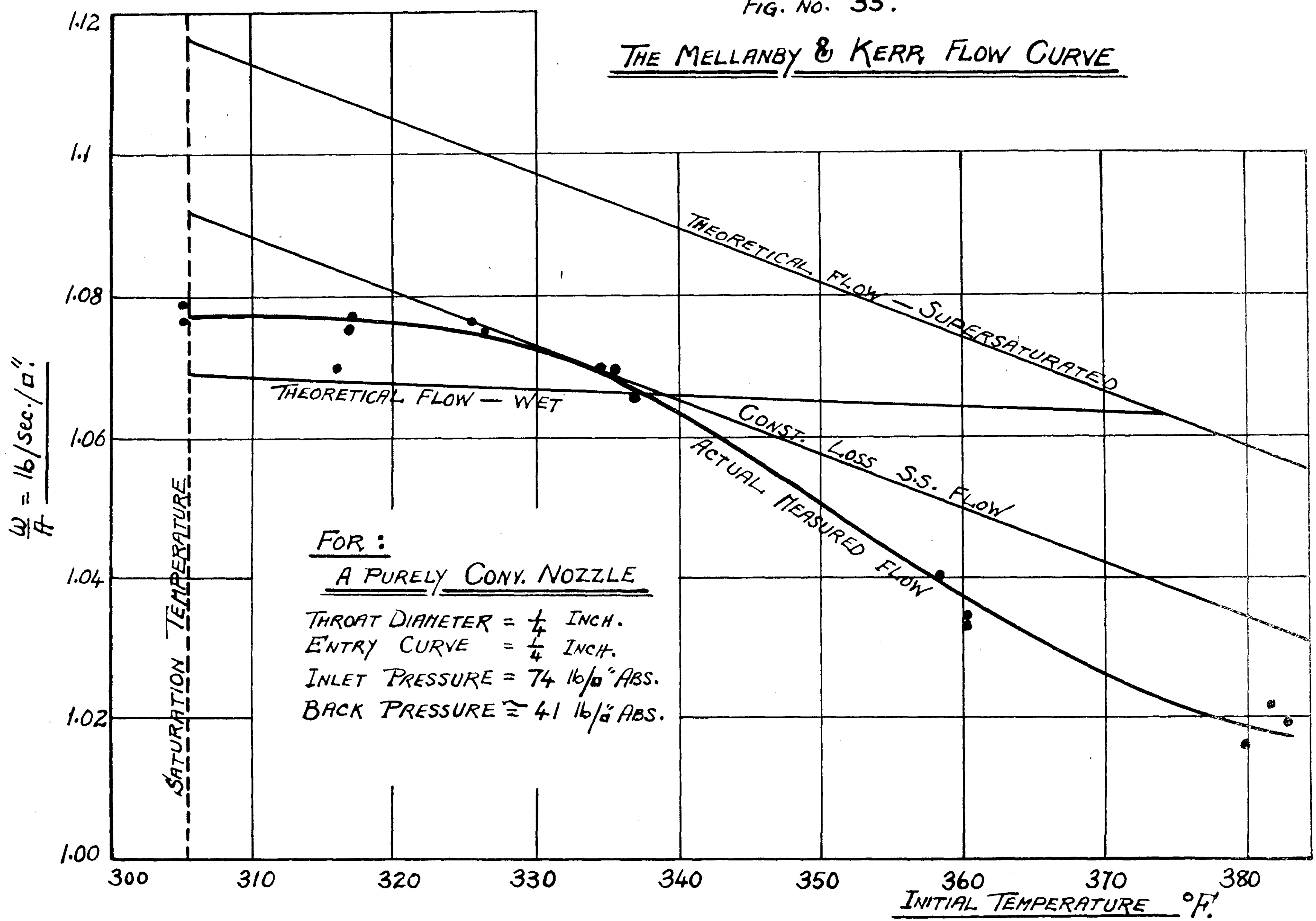


are shown in fig.(32). In this series, the pressure drop was continuous along the nozzle length and only very slight pressure irregularities were observed around the outlet. It will be seen that the temperature reversion points are in corresponding positions for the two cases explored.

FIG. NO. 33.

THE MELLANBY & KERR FLOW CURVE



# PART III

# DISCUSSION OF RESULTS

## A. THE FLOW CURVES

All the flow curves that have been presented here confirm the peculiar curve type originally established by Mellanby and Kerr in 1922. They measured the flow through two nozzles of the simple convergent and convergent parallel types, similar to those used in these investigations, and both with and without a search tube. In all cases, flow determinations were made with varying initial superheats at a supply pressure of about 75 lbs/sq.in. Absolute. The back pressure was always below the critical value.

The results they obtained with the purely convergent nozzle having the full bore open to flow are reproduced in fig.(33). This diagram is similar to figs.7,9,13 and 1 of this paper and is repeated here for convenience. This case is that of minimum loss effects and the excessive discharge feature is clearly shown. The flat top of the flow curve is particularly well defined in these tests.

In the more extensive tests of the present investigations, the curve top is found to be convex in form. This is well shown in figs.(11 and 15) for the purely convergent nozzle with and without the search tube respectively. As the losses increase, the curve top flattens somewhat but does not wholly lose the characteristic of convexity. This is particularly well shown in the results of the tests with varying surface ratio - ________ ).

The two facts established by all these flow curves would appear to be in conflict with each other. They may be stated concisely as follows:-

(a) The flow at and near the initially dry condition is excessive when compared with the theoretical as obtained on the assumption of stable expansion. The curves show the nature and magnitude of the excessive discharge which is supposed to be the consequence of expansion in the supersaturated state.

(b) The form of the curves at and near the initially dry condition, whether flat or convex, is not easily reconciled with the idea of complete supersaturation, being in fact rather similar to the theoretical form for stable expansion.

## Partial Reversion Before the S.S. Limit

Mellanby and Kerr, by comparing the measured flow curve with the theoretical based on normal equilibrium expansion and that based on complete supersaturation to a ratio of 8 fold, have suggested that steam reverts partially at conditions well within the limit. They have expressed doubts also of the possibility of any sudden reversion at the limit.

In view of the curves obtained in which excess flow is associated with a form like that to be expected from expansion in thermal equilibrium these assumptions would appear to be quite reasonable.

Their analytical method consists of measuring the discrepancy between the actual flow and the constant loss supersaturated flow - shown before - referred to as (y) and then introducing it into Callendar's steam formulae to obtain the actual volume $V_2$ at $P_2$. If x represents the actual wetness fraction at $P_2$ and v and $h_2$ are respectively the liquid volume and heat per pound at $P_2$ then:-

$$V_2(1 + y) = xv + (1 - x) V \quad \text{——————} \quad (1)$$

$$H_2 = xh_2 + (1 - x) H \quad \text{——————} \quad (2)$$

where V and H are the volume and total heat of steam at $P_2$.

Applying Callendar's relation between H and V, equation (2) becomes

$$H_2 = xh_2 + (1 - x) \left\{\frac{P_2}{a} (V - C) + B\right\} \quad \text{——————} \quad (3)$$

where a, C and B are constants.

Substituting for V and neglecting (v - c) there results

$$x = \frac{P_2 V_2 y}{a(B - h_2)} \quad \text{——————} \quad (4)$$

In this $V_2$ is the volume after dry expansion with loss. It could be written as:-

$$V_2 = \frac{V_1}{r} (K + r^{\alpha}) \quad \text{——————} \quad (5)$$

where K is the usual loss factor, $\alpha = \frac{n - 1}{n}$ and $r = \frac{P_2}{P_1}$

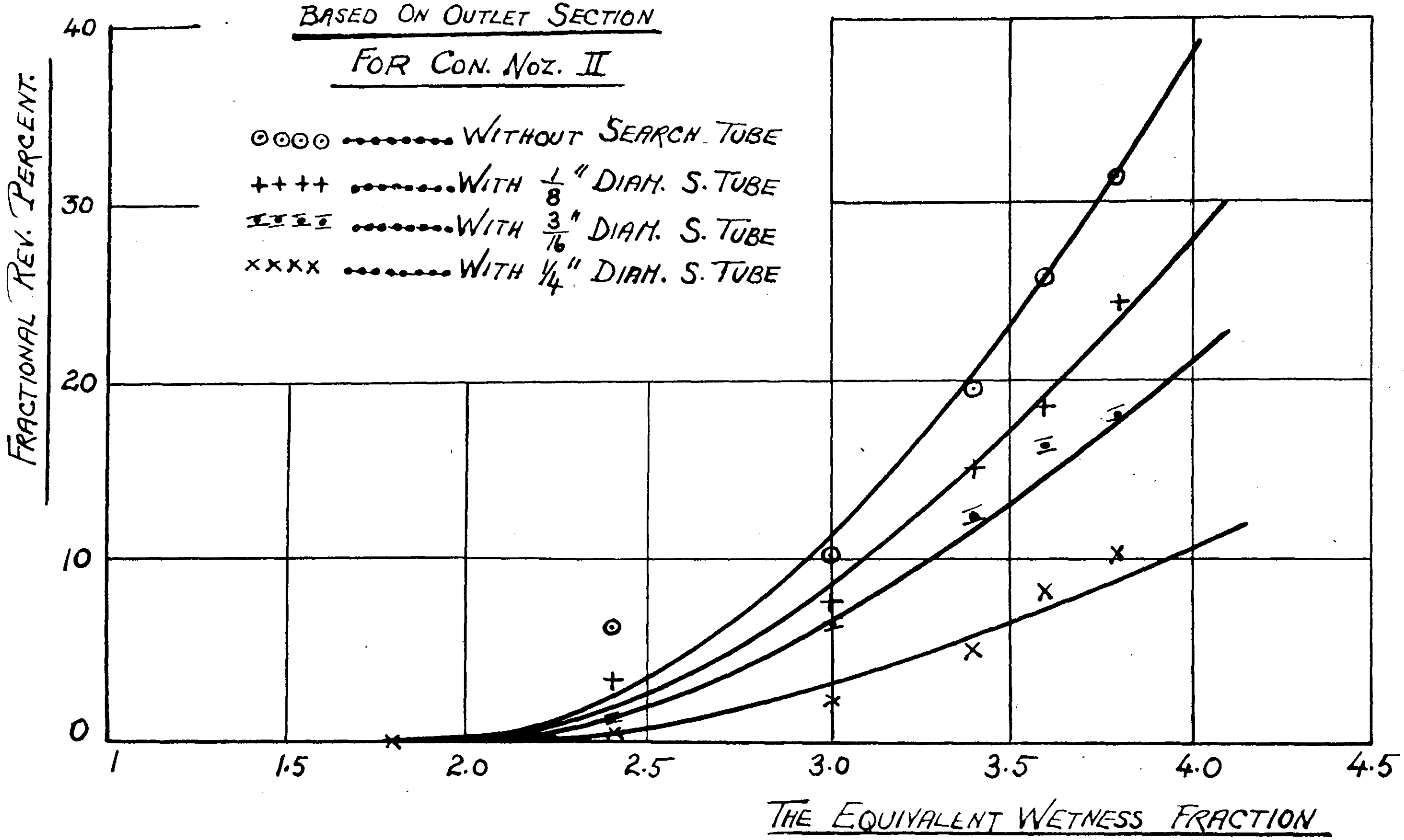
$$\text{Hence} \quad x = \frac{P_1(K + r^{\alpha})}{a(B - h_2)} V_1 y \quad \text{——————} \quad \text{I}$$

FIG. No. 34.

THE FRACTIONAL REV. PERCENT.

BASED ON OUTLET SECTION

FOR CON. NOZ. II



If complete reversion is assumed to take place, the water present in each lb. of steam would be:-

$$M = \frac{H_s - H_2}{L_2} \quad \text{———} \quad (6)$$

where $H_s$ and $L_2$ are the total and latent heats of dry steam at $P_2$, but

$$H_2 = \frac{P_2}{a}(V_2 - c) + B \quad \text{———} \quad (7)$$

then

$$H_2 = \frac{P_1V_1}{a}(K + r^{\alpha}) + B \quad \text{———} \quad (8)$$

And by substituting in (6) there results:-

$$M = \frac{1}{L_2}\left\{(H_s - B) - \frac{P_1}{a}(K + r^{\alpha})\right\} \quad \text{———} \quad II$$

By dividing I by II there results what Mellanby and Kerr have called the "fractional reversion" and used to indicate the extent of the effect which they thought most probable as an explanation of the flow curve shape.

When this method is applied to the present results, curves of fractional reversion percentages similar to those shown by Mellanby and Kerr are obtained. They are presented here by fig.(34) for the tests of the second series in which the surface ratio was varied. These have been chosen for illustration as they bring in the effect of surface more definitely than in the original study and thereby strongly emphasise the features that weaken this line of reasoning.

It will be noticed from fig.(34) that as the surface ratio increases the calculated degree of reversion diminishes. This reduction is very marked.

In all possibility the partial reversion argument was prompted by the established idea that condensation comes down on all surfaces in contact with the jet. If this is so, then greater surface should mean higher reversion. This is certainly not the case on the present basis of analysis. It must therefore be concluded that while the idea of partial reversion before the limit does serve to meet the peculiarity of the curve forms it fails to provide a rational explanation of the variations.

Gradual Reversion After the S.S. Limit

Powell, at Wilson's suggestion, used the cloud chamber method to make accurate tests at different temperatures since Wilson's experiments were all done at room temperature. He found that the S.S. ratio is not equal to 8 at all times, but a function of the initial temperature - see fig.(2). He employed his new data to establish the theoretical flow curve and compared this with the Mellanby and Kerr flow curves. He is the only other investigator who has endeavoured to explain the peculiar curve form.

Powell made the assumptions that:-

(a) Up to the pressure at which condensation starts to set in, as determined by his tests, the steam remains dry and expands in accordance with Callendar's equation for dry steam.

(b) After that pressure, the vapour is still undercooled, but any condensed water droplets will be at the saturation temperature corresponding to the existing pressure.

In calculating the theoretical curve, he ignored the losses that take place in nozzle flow. His curve has a flat top sloping upwards towards the saturated condition and thus giving a tendency in the right direction. The method may be briefly explained as follows:-

If the initial conditions of steam are $P_1$, $V_1$ and $H_1$ and the final known condition is $P_2$, then the expansion can be divided into two portions (a) from $P_1$ to $P_{ss}$ in dry condition (b) from $P_{ss}$ to $P_2$ in partially dry and wet condition, where $P_{ss}$ is the limiting pressure of the supersaturated condition. To determine $P_{ss}$, Callendar's equation for dry steam is used to calculate the total heat at successive pressures P lower than $P_1$, and the intersection of the curve of H against P with that established by Powell of $H_{ss}$ against $P_{ss}$ gives the required $P_{ss}$. It follows then that the heat drop in the first portion is simply $(H_1 - H_{ss})$.

The gradual reversion after this pressure necessitates the adoption of a step by step method of approximation in calculating the heat drop from $P_{ss}$ to $P_2$ by dividing it into successive steps as $(P_{ss} - p_1)$, $(p_1 - p_2)$, $(p_2 - p_3)$ etc., such that within each step the index n of the expansion equation $P.V^n$ = Constant can be assumed to be constant. Since n is not really known, the heat drop in each step can be calculated approximately at first and then used to calculate the mixture volume at the end of each step. This is introduced into the general equation for the heat drop to get a more accurate value. This method of calculation has been fully explained by Powell(31,b).

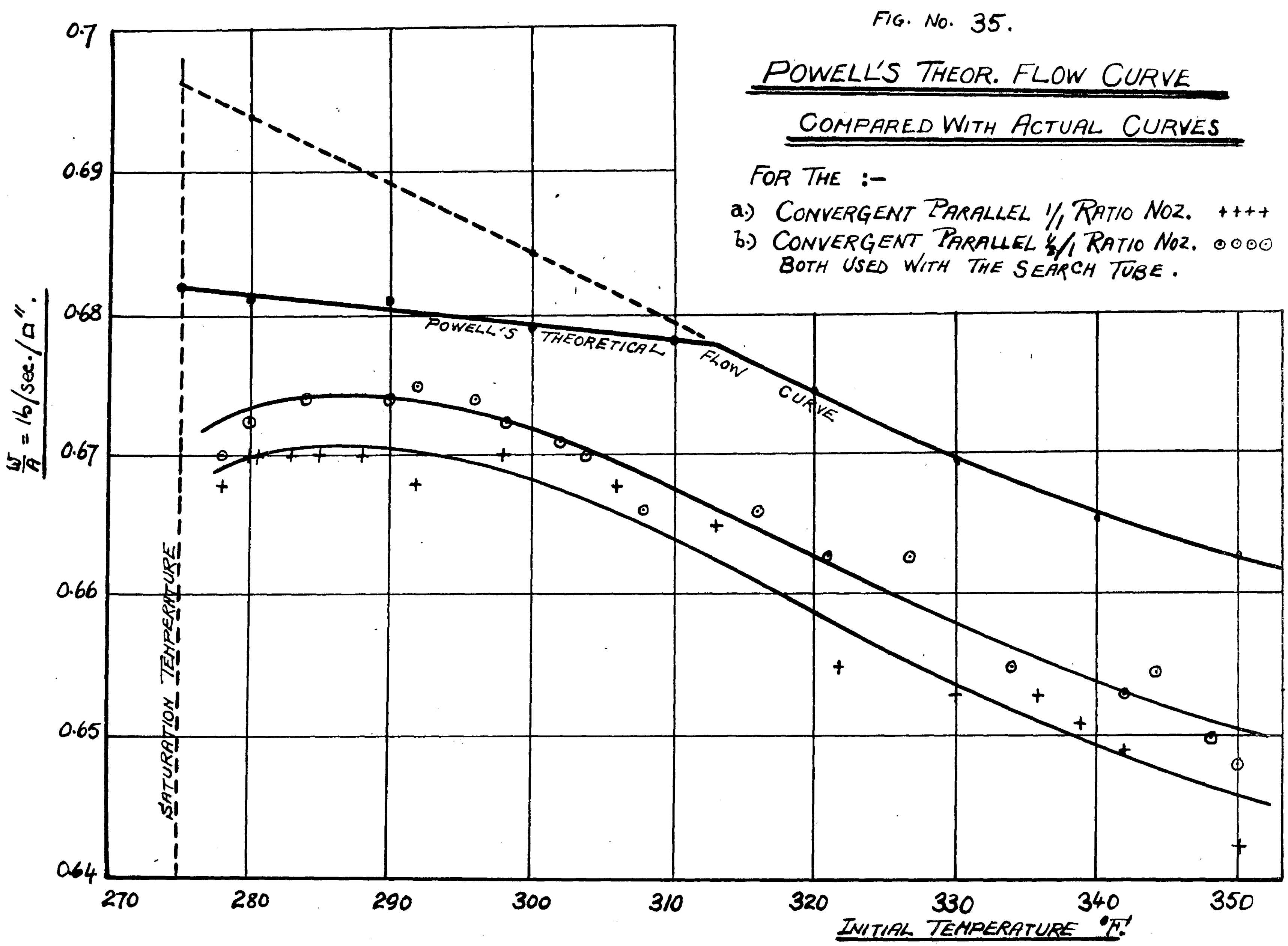


Fig. No. 35.
Powell's Theor. Flow Curve
Compared With Actual Curves
For The :-
a.) Convergent Parallel 1/1 Ratio Noz. ++++
b.) Convergent Parallel ½/1 Ratio Noz. ⊙⊙⊙⊙
Both Used With The Search Tube.
Powell's Theoretical Flow Curve
Saturation Temperature
$\frac{W}{A}$ = lb/sec./□".
0.7
0.69
0.68
0.67
0.66
0.65
0.64
270
280
290
300
310
320
330
340
350
Initial Temperature °F.

Summing up the heat drop in these successive steps gives the heat drop from $P_{ss}$ to $P_2$ and by adding that from $P_1$ to $P_{ss}$ the total heat drop through the nozzle is estimated. The equation of continuity is used then to determine the flow rate thus

$$\frac{W}{A} = \frac{\left[2gJ(H_1 - H_2)\right]^{\frac{1}{2}}}{V_2}$$

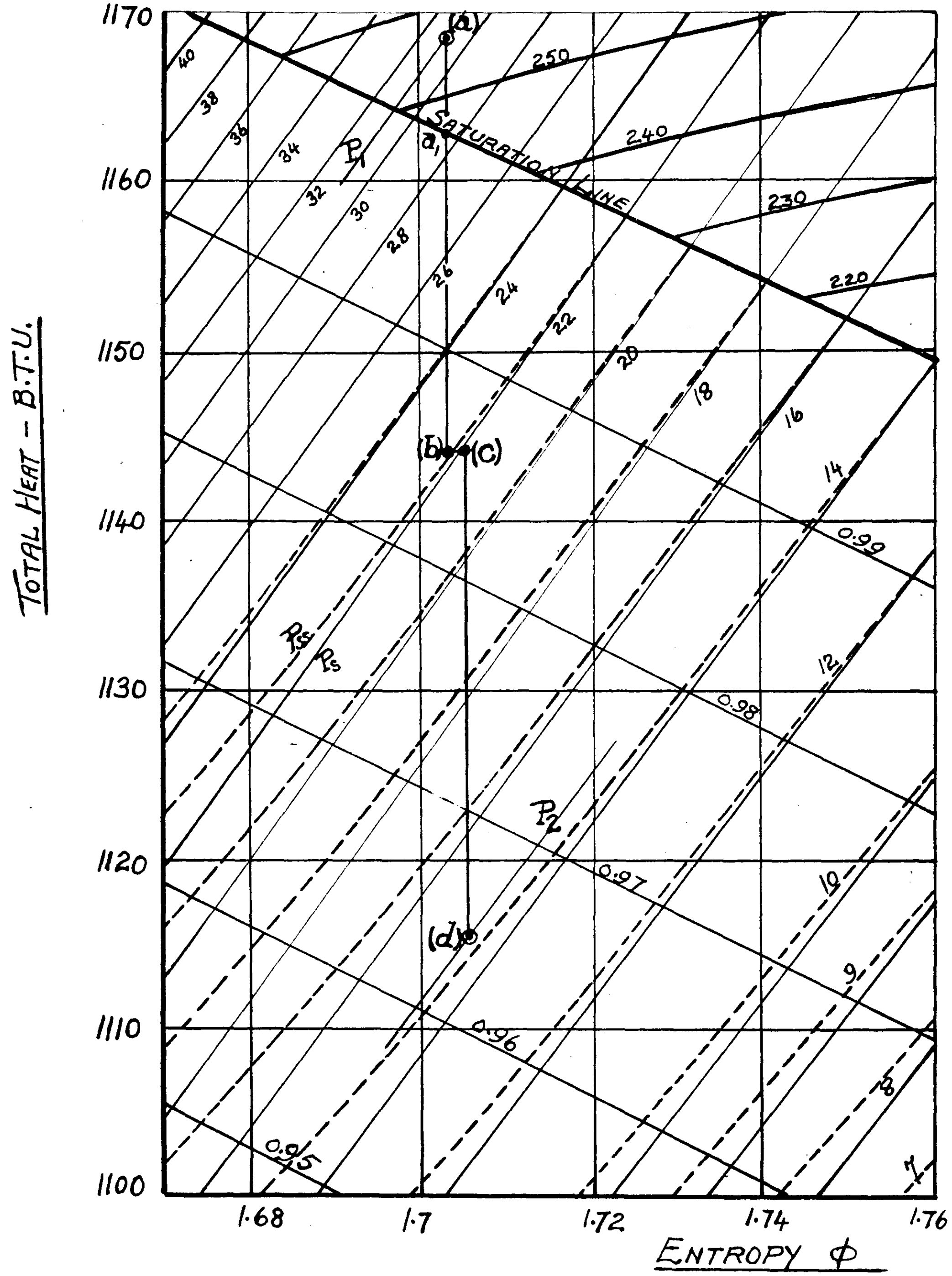
where $V_2$ is the final volume of the mixture at $P_2$ while g, J have their usual meanings and A is the exit area of the nozzle.

The theoretical maximum discharge based on this method for the conditions of expansion of the first series of flow experiments is shown in fig.(35). This curve has a straight top sloping upwards towards the saturation temperature and while the actual curves have a point of inflexion where this curve departs from theoretical curve for superheated flow, there is otherwise a marked difference in form.

Flow losses of course prevent the actual curves from approaching the theoretical more closely but no rational law of loss could serve to explain the disagreement in form. Thus an increase of loss with temperature such as is indicated by the forms in the high temperature region would, if systematic, lead to a closer resemblance in the supersaturated range of flow. It would seem, therefore, that Powell's assumption of partial reversion after the limiting condition is reached, while providing a better estimate of the theoretical flow than other methods, fails to give a complete explanation of flow curve type.

## Sudden Reversion At the S.S. Limit

There would not appear to be any particular merit in



TOTAL HEAT - B.T.U.
1170
1160
1150
1140
1130
1120
1110
1100
1.68
1.7
1.72
1.74
1.76
ENTROPY Φ
SATURATION LINE
250
240
230
220
40
38
36
34
32
30
28
26
24
22
20
18
16
14
12
10
9
8
7
0.99
0.98
0.97
0.96
0.95
(a)
$a_1$
(b)
(c)
(d)
$P_1$
$P_2$
$P_{ss}$
$P_s$

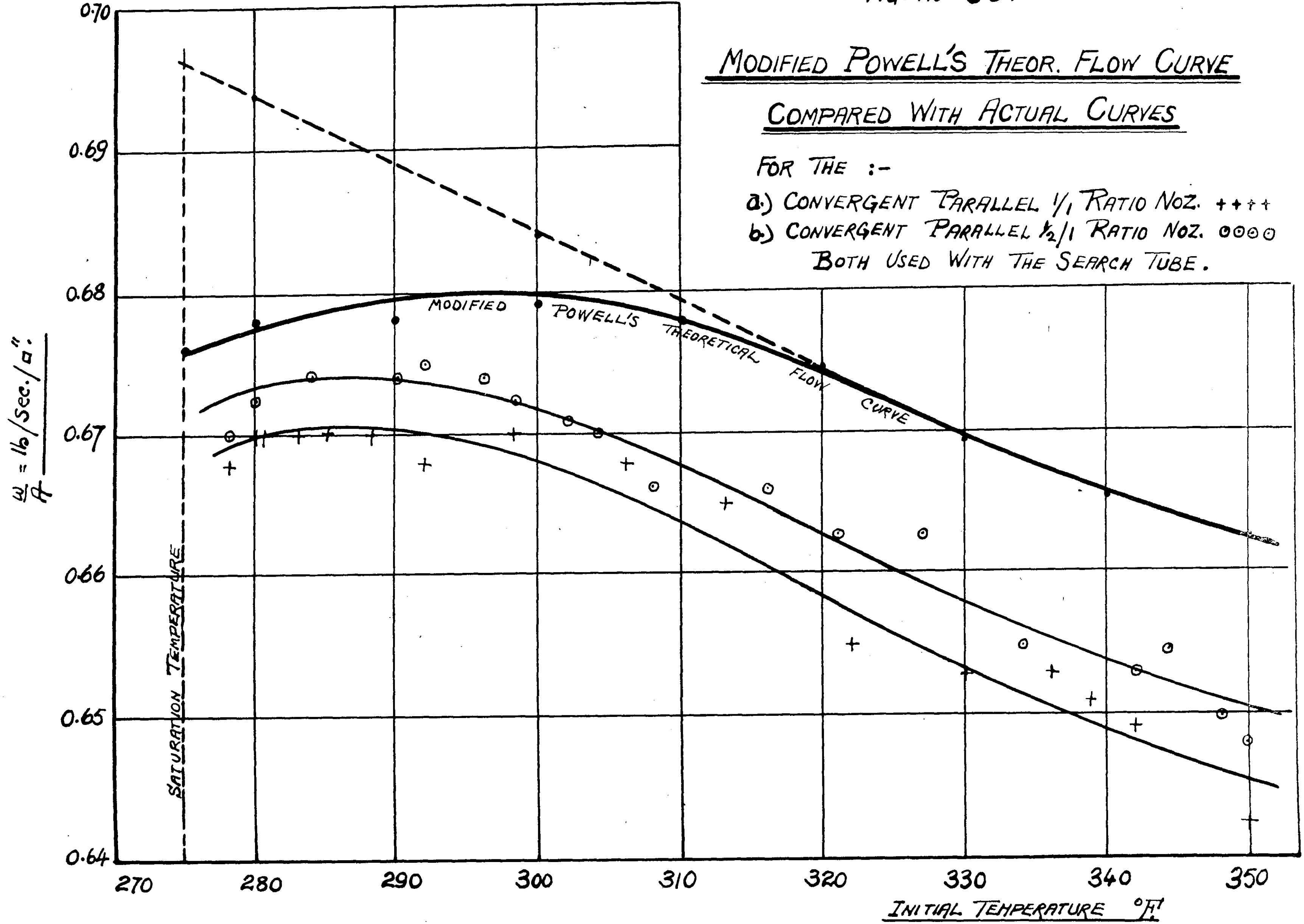
... of these ideas of premature or gradual reversion. The Mellanby and Kerr method considers the loss effects and meets the flow curve form, but is not rational in its consequences. The Powell method on the other hand neglects the losses and gives a theoretical curve only approximating to the actual form and therefore not wholly convincing.

The question naturally arises as to whether a study on the basis of sudden reversion at the limiting condition as established by Powell and including a consideration of loss effects would not give a closer agreement. This necessitates a modification of the Powell's method as follows:-

Steam expanding isentropically from point (a) crosses the saturation line in the chart at point ($a_1$) but remains dry and supersaturated until it reaches point (b) on the supersaturated pressure line $P_{ss}$. At (b) condensation takes place and thermal equilibrium is restored instantaneously. This is represented by the constant enthalpy line (bc) where (c) lies on the saturation pressure curve corresponding to $P_{ss}$ and represents the condition of wet steam. The assumption is made here that the transformation takes place at constant pressure, whereas probably some degree of recompression occurs. After (c) the expansion proceeds under thermal equilibrium conditions till point (d) is reached on the wet pressure line $P_2$.

In any expansion, the pressure $P_{ss}$ is determined as explained before and the heat drop from (a) to (b) is calculated from Callendar's equation for dry steam. The expansion from (c)

FIG. NO. 36.


MODIFIED POWELL'S THEOR. FLOW CURVE
COMPARED WITH ACTUAL CURVES
FOR THE :-
a.) CONVERGENT PARALLEL 1/1 RATIO NOZ. ++++
b.) CONVERGENT PARALLEL 1/2/1 RATIO NOZ. ⊙⊙⊙⊙
BOTH USED WITH THE SEARCH TUBE.
MODIFIED POWELL'S THEORETICAL FLOW CURVE
SATURATION TEMPERATURE
$\frac{W}{A}$ = lb/Sec./□"
0.70
0.69
0.68
0.67
0.66
0.65
0.64
270
280
290
300
310
320
330
340
350
INITIAL TEMPERATURE °F.

to (d) takes place under wet conditions and the heat drop is obtained from a Mollier Chart or can be calculated from the steam tables. Besides this, an estimate of the volume of wet steam at (d) is made and introduced together with the total heat drop into the equation of continuity to obtain the rate of flow through the nozzle.

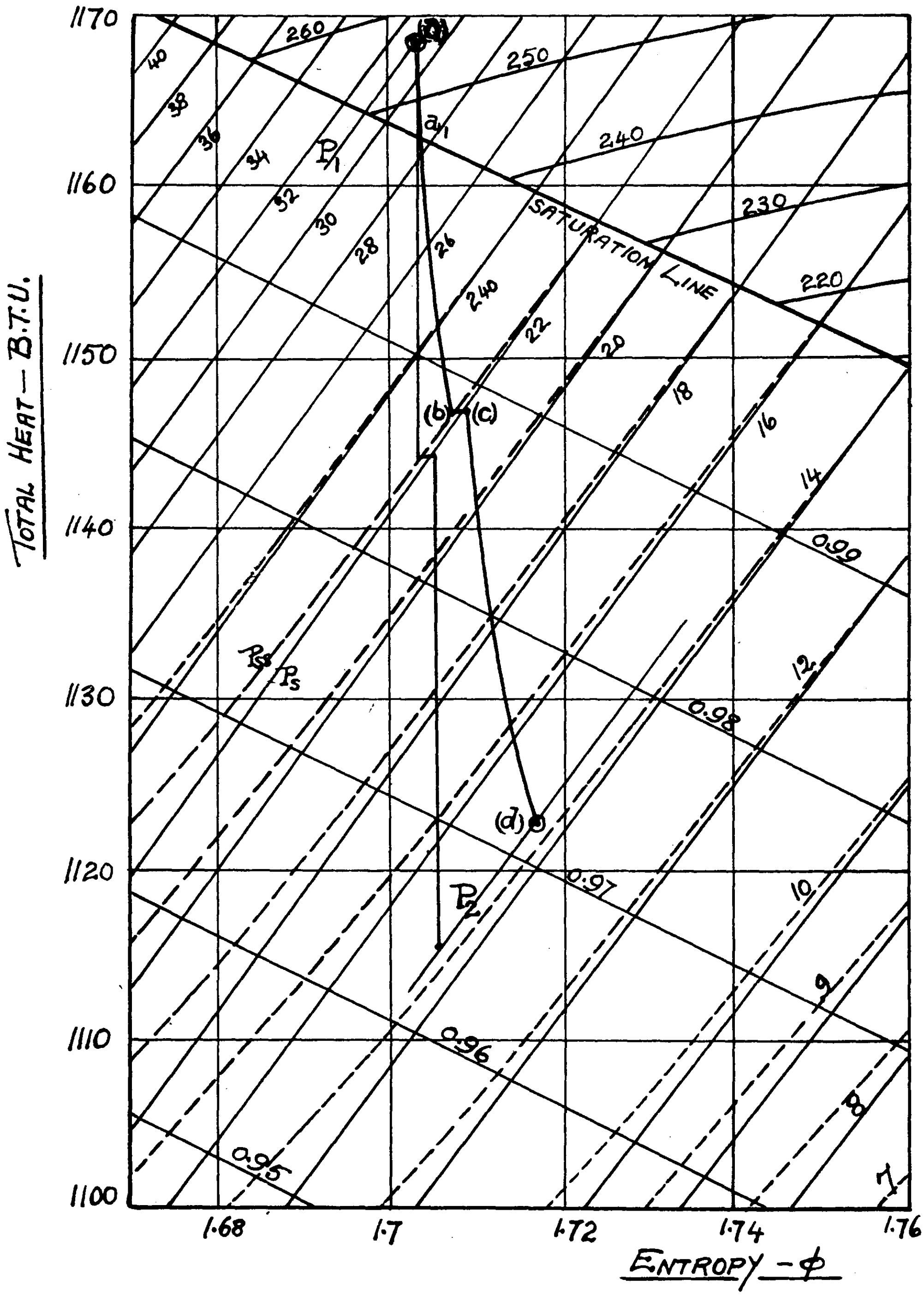
This method has been applied to the case of the convergent parallel nozzle previously considered. The resulting form for no loss effects is shown by the upper curve in fig.(36). The other curves indicate the influence of assumed loss factors.

It will be seen that the convex top now appears quite clearly in the theoretical form and that the decrease in the degree of convexity with loss, already noted as an experimental fact, is also demonstrated. These rather surprising results of a comparatively simple modification of Powell's method have probably hitherto been missed because flow tests had not established curves with the convex characteristic and because discussion has been limited to the condition of flow without loss.

It would appear to be a sound conclusion that as far as flow curves can give guidance, the sudden reversion idea at the limiting conditions is more satisfactory than others.

## Influence of Surface Ratio

In view of the agreement just established, it seems desirable to carry the comparison into this second series of flow results and it is necessary now to include the actual loss effects



TOTAL HEAT – B.T.U.
ENTROPY – φ
SATURATION LINE
1170
1160
1150
1140
1130
1120
1110
1100
1·68
1·7
1·72
1·74
1·76
260
250
240
230
220
40
38
36
34
32
30
28
26
22
20
18
16
14
12
10
9
8
7
0.99
0.98
0.97
0.96
0.95
$P_1$
$P_2$
$P_s$
(b)
(c)
(d)
$a_1$

in the analysis.

The relation for the rate of mass flow through a nozzle, previously given can be modified to:-

$$0.718 \left(\frac{V_1}{P_1}\right)^{\frac{1}{2}} \times \frac{W}{A} = \frac{\left(1 - K - r^{\frac{n-1}{n}}\right)^{\frac{1}{2}}}{K + r^{\frac{n-1}{n}}}$$

The right hand side of the equation is denoted by the jet function "F" in which when the experimental value of r is known it becomes possible to determine the velocity and volume conditions at the point considered, and the total expansion loss "K" that has occurred to that point.

In determining this "K" factor, expansions falling entirely in the superheat region were chosen and the curves of "K" against initial temperature as determined from the nozzles readings were constructed. These were slightly curved in form but for the present purpose an approximate linear relation is sufficiently accurate. Having thus obtained an estimate of the probable loss factor, it can be included in the calculation of heat drop and used with the assumption of sudden reversion at Powell's limiting condition.

As before, the expansion is divided into two portions; (ab) and (cd) in dry and wet conditions respectively. Accordingly, the heat drop in the first portion is associated with a loss of $\Delta H$ in heat units and the path of the state point (a - b) depends entirely upon the nature of these losses. This $\Delta H = \frac{144n}{n-1} P_1 V_1 \frac{K}{J}$ in heat unit per lb., where each term bears the same meaning as explained before. This loss of heat increases the specific volume by $\frac{V_1 K}{r}$

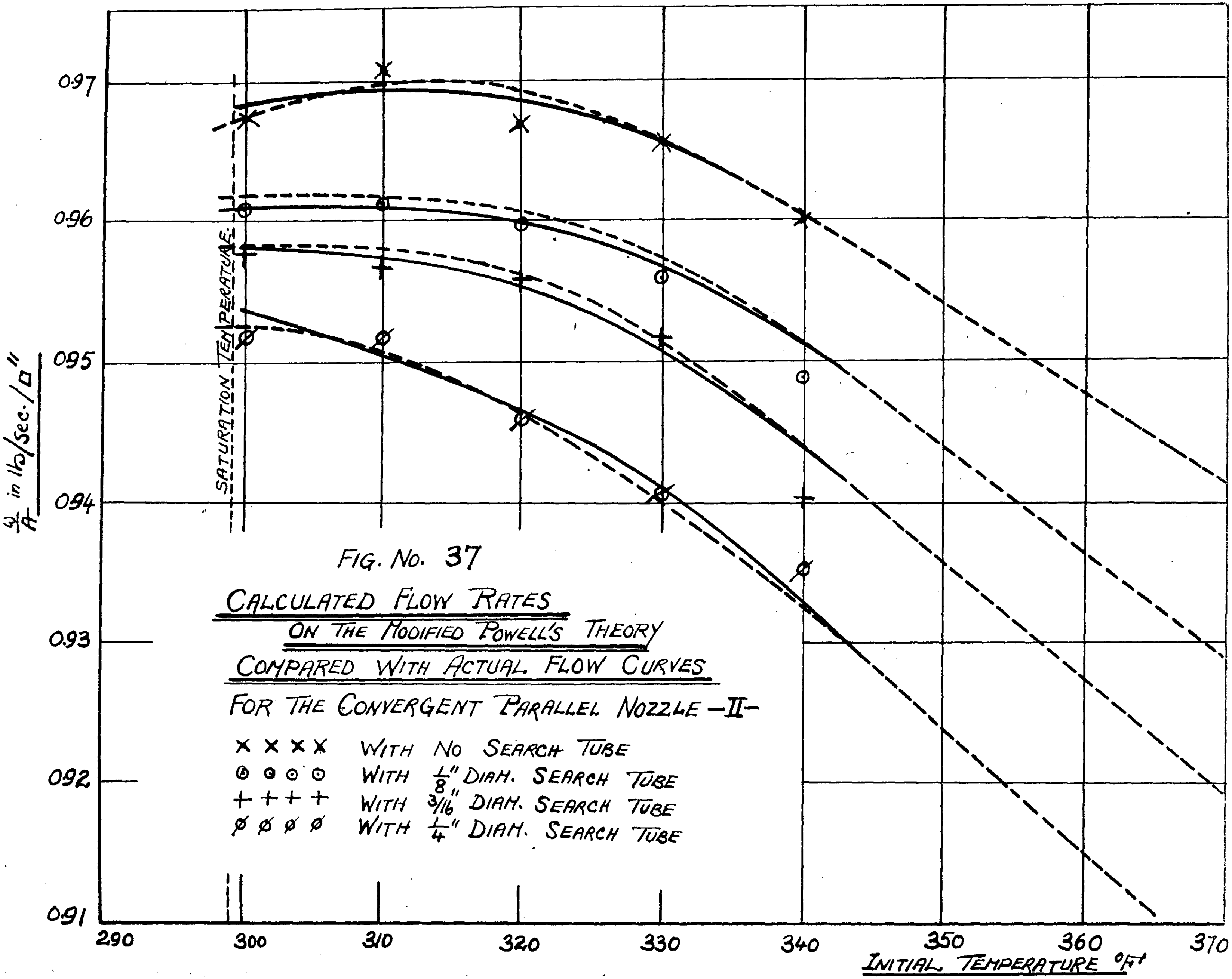



FIG. No. 37

CALCULATED FLOW RATES
ON THE MODIFIED POWELL'S THEORY
COMPARED WITH ACTUAL FLOW CURVES
FOR THE CONVERGENT PARALLEL NOZZLE -II-

beyond the value of $\frac{V_1 r^{\frac{n-1}{n}}}{r}$. The reversion takes place at constant total heat as shown before, and the expansion then proceeds under conditions of thermal equilibrium (cd) and the reheat is equal to $\frac{144n}{n-1} P_s V_s \frac{K}{J}$ where n = 1.135 in this case and $P_s$, $V_s$ are the conditions at (c). The final condition of steam can now be obtained.

This analysis is applied to the tests of the second series and the results are shown in fig.(37) where the full lines show flow curves calculated as explained above and the dotted lines show the corresponding experimental flow curves. The agreement in form in the slight superheat range beyond the just dry condition seems fairly satisfactory and indicates that the idea of sudden reversion at the limiting condition, with loss effect emboded in the analysis, gives flow curves that are more closely in agreement with the experimental types than can be produced on any basis of argument involving gradual reversion.

The maximum flow of steam expanding to the critical condition in a nozzle is attained only if there is uniformity of pressure across the throat. Any other pressure value above or below the critical, over even a minute portion of this area, would entail smaller flow quantities since such values require larger areas than when at the critical pressure condition.

In practically all nozzle calculations and theoretical considerations, the assumption is tacitly made that the flow is in parallel stream lines and hence the pressure and velocity across a flow section are uniform. But since the steam is compressible, its inertia is bound to produce a condensation at the axis near the throat and a corresponding rarefaction on the walls at the same time, with an inverse velocity range in keeping therewith. This effect would result in a wave flow beyond this point and thus the stream lines will not be parallel but sincous. The magnitude of this wave depends on the shape of the entry curve to the nozzle, being greater the shorter the curve, and it is a fact that with a sharp entry type nozzle the maximum flow is considerably altered and in the limit, with a hole in a thin plate, there is no maximum flow but the flow increases with the reduction of back pressure towards a limit.

The velocity curve, as shown by Stodola, is perfectly constant over 4/5th of the diameter and drops rapidly to zero towards the periphery. The velocity (v) at a radius (r) is given by:-

$$v = v_o \left(1 - \left(\frac{r}{r_o}\right)^n\right) \quad \text{———}$$

where ($v_o$) is the velocity at the axis and (n) is a constant varies between 20 and 30 according to the shape of the entry curve. Mellanby and Kerr also give a similar relation which gave fairly consistent results with their experiments.

The flat form of the velocity curve in the centre of the jet does not imply absence of frictional losses, but the type of this friction is different from the friction of the walls and irregular eddy currents different from the classical eddies of Helmholtz are superimposed upon the nozzle flow and usually referred to as turbulence. This turbulence has been studied by Lorentz(22) who concluded that the eddies projected from the periphery equalise the velocities across the different sections of the nozzle. Thus for a certain percentage of loss, the flow rate is higher the smaller the hydraulic mean depth of the jet.

Thus it becomes clear that the surface ratio of a jet affects the flow, since turbulence together with wall friction produce a partial superheat at the periphery while in the undisturbed inner region of the jet expansion takes place with considerable condensation. The approaching of the actual flow curve to the constant loss curve with increased surface ratio (figs.16-20) confirms this conclusion.

## Limitations Of Flow Curve Data

In his treatise on the "Properties of Steam" Callendar has used some results quoted by Mellanby and Kerr and represented them in such a way as to appear in complete agreement with the usual theory regarding supersaturation in nozzle flow. This theory based on Wilson's results assumes that steam expands under conditions of complete supersaturation with a limiting S.S. ratio of 8 at all times. In discussing the Mellanby and Kerr curve Callendar ignored the flat top.

Mellanby and Kerr in analysing their own results have accepted the Wilson limit and to account for the peculiarity of the flow curve form they found it necessary to assume early reversion.

Powell has demonstrated that the S.S. ratio is not fixed for all conditions, but is a function of the initial temperature. He explained the Mellanby and Kerr flow curve form by applying his revised data and the conception of gradual reversion beyond the limiting condition of supersaturation.

It has now been shown that none of these methods really gives a full explanation of the curve form. The idea of sudden and complete reversion at the correct S.S. limit applied in conjunction with a consideration of loss effects in the flow gives a fuller explanation of the more extensive data now obtained.

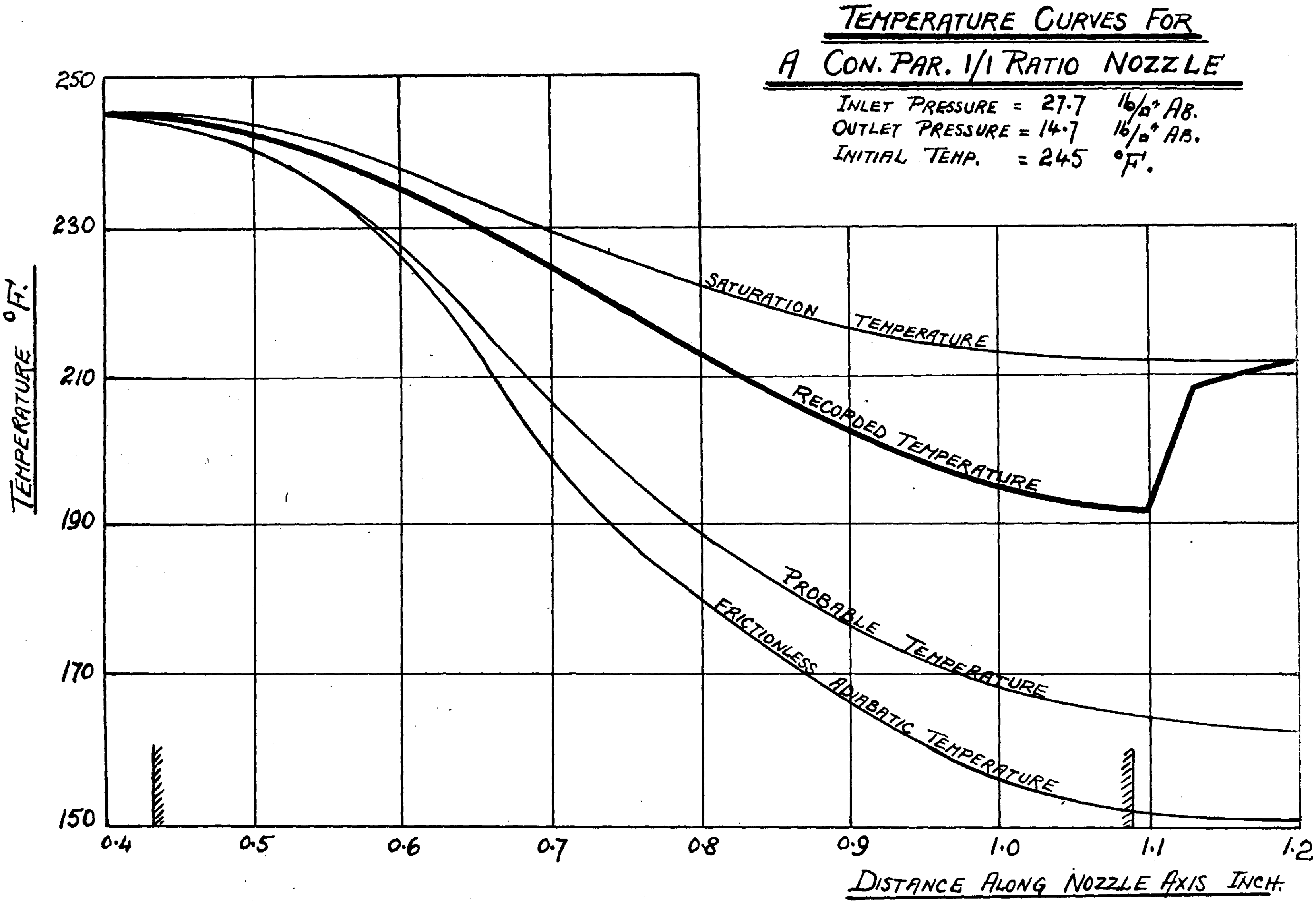
The truth is, however, that the flow curves are not in themselves adequate to explain the phenomenon. They demonstrate that some such phenomena occur, and give by their peculiarity and variation of form certain effects that must be met by any explanation. But it is contended that they do not deny the possibility of sudden reversion, and, if further evidence in favour of that type of change can be advanced, the flow curve evidence is in support. That additional evidence comes from the temperature experiments carried out in the course of these investigations and the various features of these will now be discussed.

## B. THE TEMPERATURE CURVES

The temperature curves obtained in these investigations and already presented show certain features that may be summarized concisely as follows:-

The thermojunction is "hidden" from the stream and so reads

FIG. No. 38.


TEMPERATURE CURVES FOR
A CON. PAR. 1/1 RATIO NOZZLE
INLET PRESSURE = 27.7 lb/□" AB.
OUTLET PRESSURE = 14.7 lb/□" AB.
INITIAL TEMP. = 245 °F.
250
230
210
190
170
150
TEMPERATURE °F.
SATURATION TEMPERATURE
RECORDED TEMPERATURE
PROBABLE TEMPERATURE
FRICTIONLESS ADIABATIC TEMPERATURE
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
DISTANCE ALONG NOZZLE AXIS INCH.

what is a search tube surface temperature and probably very close to the boundary layer temperature.

The combined pressure and temperature observations disclose a small amount of undercooling just beyond the nozzle inlet. This, however, increases along the nozzle reaching values that have not hitherto been obtained experimentally.

At some positions along the nozzle length or just around the nozzle outlet in the case of the convergent parallel nozzle, the temperature reading "jumps" to the saturation value. This occurs in such a way that the only interpretation is that condensation has suddenly occurred.

Thus the temperature curves give what would appear to be a positive demonstration of sudden reversion.

Although these effects are quite definite, the actual thermojunction readings only give a small proportion of the amount of undercooling theoretically possible. The maximum registered undercooling in one case is about $18^{o}F$, as against $60^{o}F$ theoretical undercooling. It has to be realized that losses taking place in expansion will raise the temperature above that of isentropic expansion, but even then there is a considerable discrepancy between the recorded temperatures and those that must hold in the flow. In the case mentioned it might be about $30^{o}F$.

The relationship between the various temperatures may be seen in fig.(38) which approximately represents the case of a convergent parallel nozzle. The recorded temperature curve for the case of initially dry and saturated steam is shown by the heavy line with the sudden step-up at the outlet, while the others allow comparison of this with the adiabatic temperature, the same with loss effects added and the saturation temperature corresponding to the pressure.

Readings before Reversion

Throughout the range within which undercooling is recorded, the measured degree of it is only a small amount of what is physically possible; in the case quoted above 18°F have been detected as against 60°F on theoretical considerations. In other words, if the steam is supersaturated, the thermojunction is reading surprisingly high. Nevertheless definite undercooling is shown, attaining much higher values than have hitherto been recorded by direct attempts many of which have failed even to record more than a few degrees.

While, therefore, this method positively detects undercooling it may also be accepted as proving that an exposed thermojunction in the supersaturated steam will read falsely because it is open to direct impingement. It is also clear, however, that the recorded temperatures even under the improved conditions of measurement are higher than the main stream temperatures can possibly be. This in itself makes the margin for detection small and this also tells against the direct method. These high readings in the supersaturated range, therefore, require explanation.

If the readings had been on or close to the saturation temperature all along that range, it would have been possible to claim that condensation came down on the metal surface, and so to substantiate further the idea generally accepted as to the reason for high readings. But the diagram of undercooling disclosed prevents this explanation. The sudden jump later is definitely the result of condensation and thus to that point, the temperatures do not mean condensation. Equally they do not mean actual steam temperatures. The question of what they do really mean becomes important.

It is natural to consider heat conduction along the tube wall as an influence, but the smallness of the tube and the very low temperature gradients involved indicate that this cannot be a full explanation. The tube is not at a high temperature at one end only, but there are high temperatures at both ends and conduction would be both ways with a balance of effect somewhere between.

Conduction, therefore, will not serve for more than a minute fraction of the effects considered and the explanation rests on a much more fundamental feature in the flow.

## Explanation of High Readings

The explanation is most simply given by recalling the conception of Osborne Reynolds regarding the mechanism of heat transmission and its inter-relation with friction. In this it is assumed that thermal conductivity in the thin layer at the boundary may be neglected and that all transmission takes place by eddy motion.

It then follows that the momentum lost by friction is to the total momentum of the fluid as the heat actually supplied is to that which would have been supplied if all the fluid had reached the surface. In symbols this may be expressed as:-

$$\frac{\Delta P.A}{M.v} = \frac{CW.\Delta\Theta}{CW.(T-\Theta)} \quad \text{———————} \quad (1)$$

In this equation $\Delta P$ is the pressure drop in a short length in which $\Delta\Theta$ is the temperature rise, M and W are mass and weight flow per second, v is the velocity, $\Theta$ fluid temperature and c is the specific heat.

As it stands, this is a statement for the transmission of heat from a wall at T to a fluid at $\Theta$. The wall is maintained at T by other agencies and the principle explains how the fluid picks up heat. In the present case, the situation is different but the mechanism is the same. The heat comes only from the fluid friction and a mutual interchange occurs. In such a case, the equation reduces to a very simple result. The aim is to obtain a clue to the value of $(T - \Theta)$ which is a representation of the difference being considered.

On the assumption that the resistance is proportional to the square of the speed and that the friction work is the heat taken up, we have that

$$\frac{\Delta P.A}{CW.\Delta\Theta} = \frac{J}{v} \quad \text{———————} \quad (2)$$

and then

$$(T - \Theta) = \frac{v^2}{cJg} \quad \text{———————} \quad (3)$$

This simple result is capable of considerable refinement by special consideration of the conditions within the boundary layer and more exact relationships for the dependence of friction on velocity. It is easily possible to bring into equation (3) factors dependent on the ratio of the speed in the boundary layer to that of the stream and on the viscosity and thermometric conductivity of the fluid.

The elaborations so possible are in effect the main features of such treatments as those by Stanton[41] and by Goldstein[5] The latter's treatment of the problem of the kinetic temperature is particularly appropriate here. By the kinetic temperature at a point of a body immersed in a stream of velocity $U_o$ is meant the temperature which is taken up by the body when it is non-conducting. Using the symbol $\Delta T_1$ for the kinetic temperature rise and from the fundamental equations of energy and heat, Goldstein established the formula

$$\Delta T_1 = \frac{U_o{}^2}{2gJCp}\left[1 - \frac{U^2}{U_o{}^2}\left(\sigma^{\frac{1}{3}} - 1\right)\right] \qquad \text{———} \quad (4)$$

where U is the velocity at the edge of the boundary layer and $U_o$ is that of the stream while $\sigma$ is the ratio of the kinematic viscosity to the thermometric conductivity and generally referred to as the Prandtl number, g and J have their usual significance and Cp is the specific heat at constant pressure. This equation could, however, be simplified by taking $U_o$ as nearly equal to U, and then it becomes:

$$\Delta T_1 = \frac{U_o{}^2}{2gJCp} \cdot \sigma^{\frac{1}{3}} \qquad \text{———} \quad (5)$$

The more complete treatment would be out of place here, since the data on frictional effects can only be approximately treated and the other conditions are not certain. In any case, the problem is one of providing a reasonable explanation and not of deducing precise facts on losses and temperatures.

The conception of the kinetic temperature as that established on a non-conducting body applies well to the present case as the measuring tube used in these experiments practically satisfies the condition.

To emphasize this, the ratio of the heat conducted to that interchanged may be considered briefly. This may be expressed as follows, for a short length $\delta x$

$$\frac{\text{Heat conducted}}{\text{Heat Interchanged}} = \frac{K.a.\frac{\delta^2 T}{\delta x^2}\,\delta x}{\frac{Cp}{J}\,s\,v^3\,\delta x} = \frac{KJ}{Cp}\cdot\frac{a}{s}\cdot\frac{1}{v^3}\,\frac{\delta^2 T}{\delta x^2} \qquad (6)$$

in which K is conductivity, C is a friction coefficient, a and s are respectively area and perimeter of tube, v is velocity and T is wall temperature.

The factors $KJ/Cp$ and $1/v^3$ would apply to any bodies immersed in the fluid and the latter in the case of the high speeds of flow considered here, would keep the ratio low in general. The dimensional factor $\frac{a}{s} = \frac{r}{2}$ where $r$ is the radius, is especially small for the search tube used, while examination of the curves of recorded temperature will show that $\delta^2 T/\delta x^2$ has a low value in the important length. Actually it may reach a high figure where reversion occurs but the question does not arise there.

An approximate consideration of the data available shows that the ratio, at these points on the temperature curves where the speeds are relatively low and the values of $\delta^2 T/\delta x^2$ relatively high, may amount to a few per cent, but is generally below 1 per cent. This justifies the neglect of conduction effects in the examination

FIG. No. 39.

THE $K_T$ FACTOR ALONG

THE CON. PAR. NOZZLE II WITH $\frac{1}{4}$" S.T.

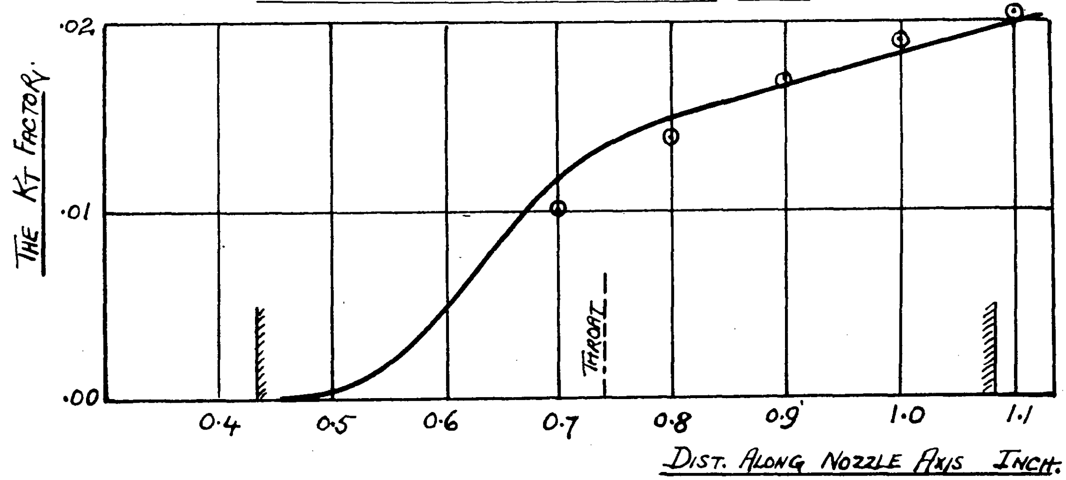
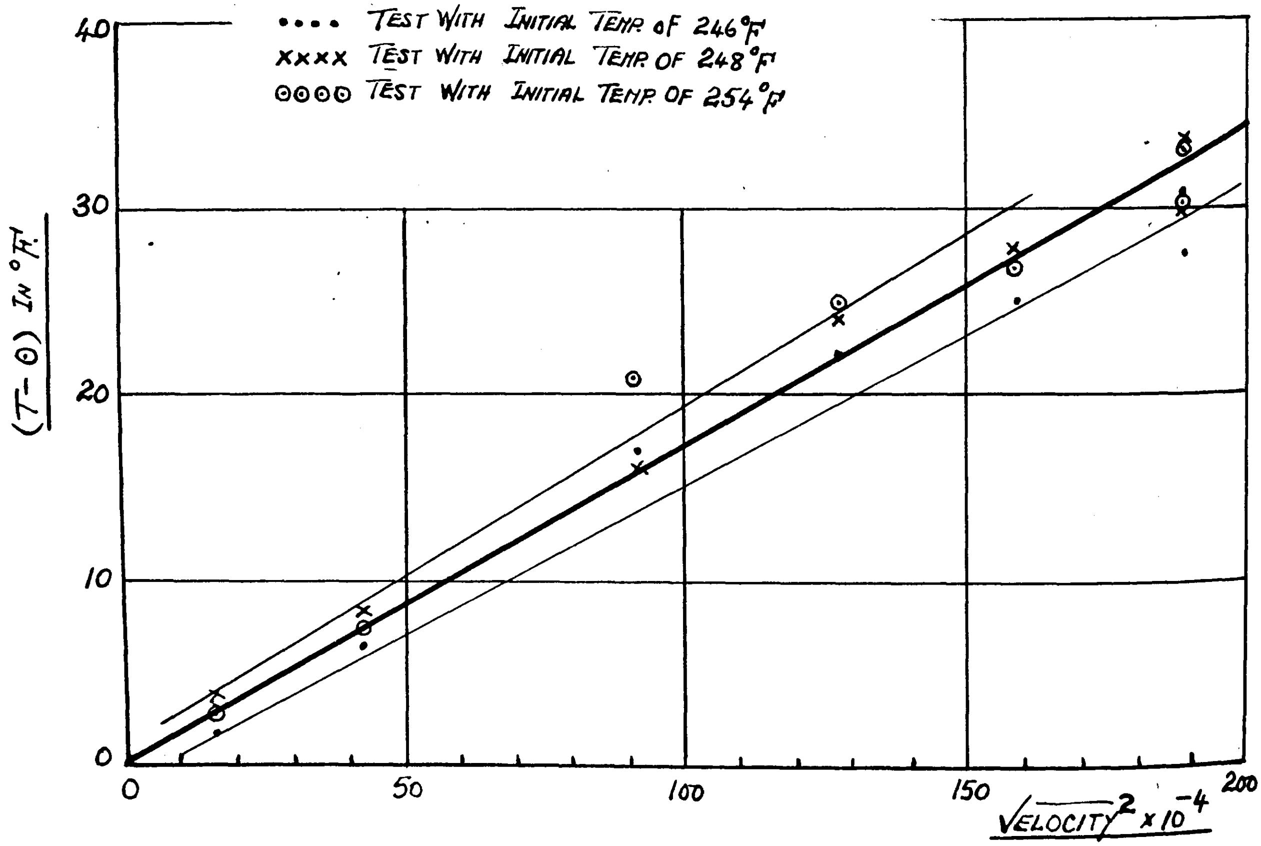



FIG. No. 40.

THE TEMPERATURE DISCREPANCY

FOR THE CON. PAR. NOZZLE II WITH $\frac{1}{4}$" S. TUBE



of the temperatures and in the general explanation of the high readings.

## The Temperature Discrepancy

The explanation advanced in the previous section means that the thermojunction as applied in these experiments will fail to read the true range of undercooling by an amount which depends on the square of the speed - equation (3). It is admitted that a more exact relationship may be suitable as is expressed by equation (4) but the data do not permit of the greater precision and in any case only qualitative verification is required.

To carry this out, it is necessary to make an estimate of the actual speeds of flow. This involves a determination of flow losses which can, in the cases available, only be deduced from the experimental pressure curves and mass flows.

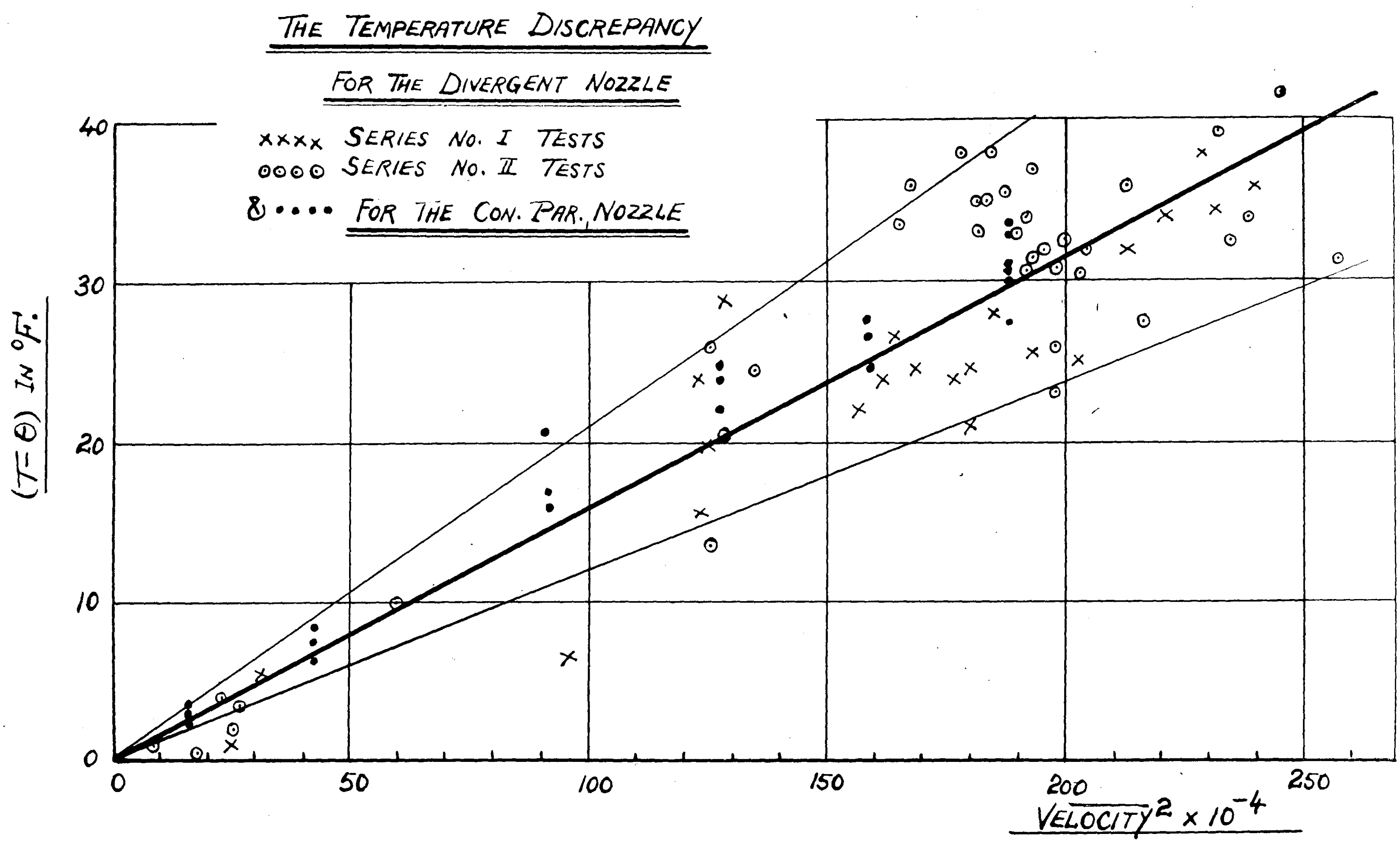
The convergent parallel nozzle with the $\frac{1}{4}$" diameter search tube is taken as an example. Values of the loss factor "K" at the various stages of expansion are determined for the test results. They are as shown by the curve in fig.(39) in which the shape of the part to the throat is uncertain but the remainder is reasonable both in form and magnitude.

With the "K" values known, the heat dissipated to any point in the expansion can be determined from the relation

$$\Delta H = \frac{144n}{n-1} P_1 V_1 \times \frac{1}{J} (K) \quad \text{———} \quad (7)$$

in heat units per lb. of flow and each item bears the same meaning

FIG. NO. 41.


THE TEMPERATURE DISCREPANCY
FOR THE DIVERGENT NOZZLE
×××× SERIES NO. I TESTS
⊙⊙⊙⊙ SERIES NO. II TESTS
& •••• FOR THE CON. PAR. NOZZLE
40
30
20
10
0
0
50
100
150
200
250
$(T-\Theta)$ IN °F.
$\text{VELOCITY}^2 \times 10^{-4}$

as before. Thus the rise in temperature above the theoretical can be estimated. This has been done using simply an average value for the specific heat. The relationship between the probable temperatures and the theoretical has already been illustrated in fig.(38).

The loss factors being known, the velocity at any point can be calculated in the usual way from the equation

$$v = \left[\frac{2gn}{n-1} P_1V_1\left(1 - K - r^{\frac{n-1}{n}}\right)\right]^{\frac{1}{2}} \qquad \text{—————— (8)}$$

and since the actual velocities and temperatures are now approximately known, the discrepancy in thermojunction readings can be related to the velocity. This is shown by the plot of temperature difference $(T - \Theta)$ against $(\text{velocity})^2$ as shown in fig.(40). It is clear from this that the difference is roughly as the square of the speed, and the agreement is probably as close as could be expected in view of the data available and the approximate method of treatment.

The divergent nozzle with its greater expansion and speed ranges provides more extensive data but somewhat more uncertain as regards loss effects. The results of the first two series of experiments on the divergent nozzle have been analysed in a similar way and altogether have provided 86 results giving probable corresponding values of velocity of flow and excess temperature readings. These, together with the convergent parallel nozzle results already presented, are shown in fig.(41) which indicates a massing of all results around the line denoting dependence on the square of the velocity.

It may therefore be allowed that the high readings of the thermojunction before the sudden "jump" at reversion are reasonably explained. If the explanation is valid, it follows that all arguments claiming that high temperature readings are due to condensation on exposed surfaces must be discarded. There is no proof of any condensation until the sharp rise in the temperature occurs. The curve shape, the values, and the explanation of these all combine to deny any prior condensation in the stream or on the boundary.

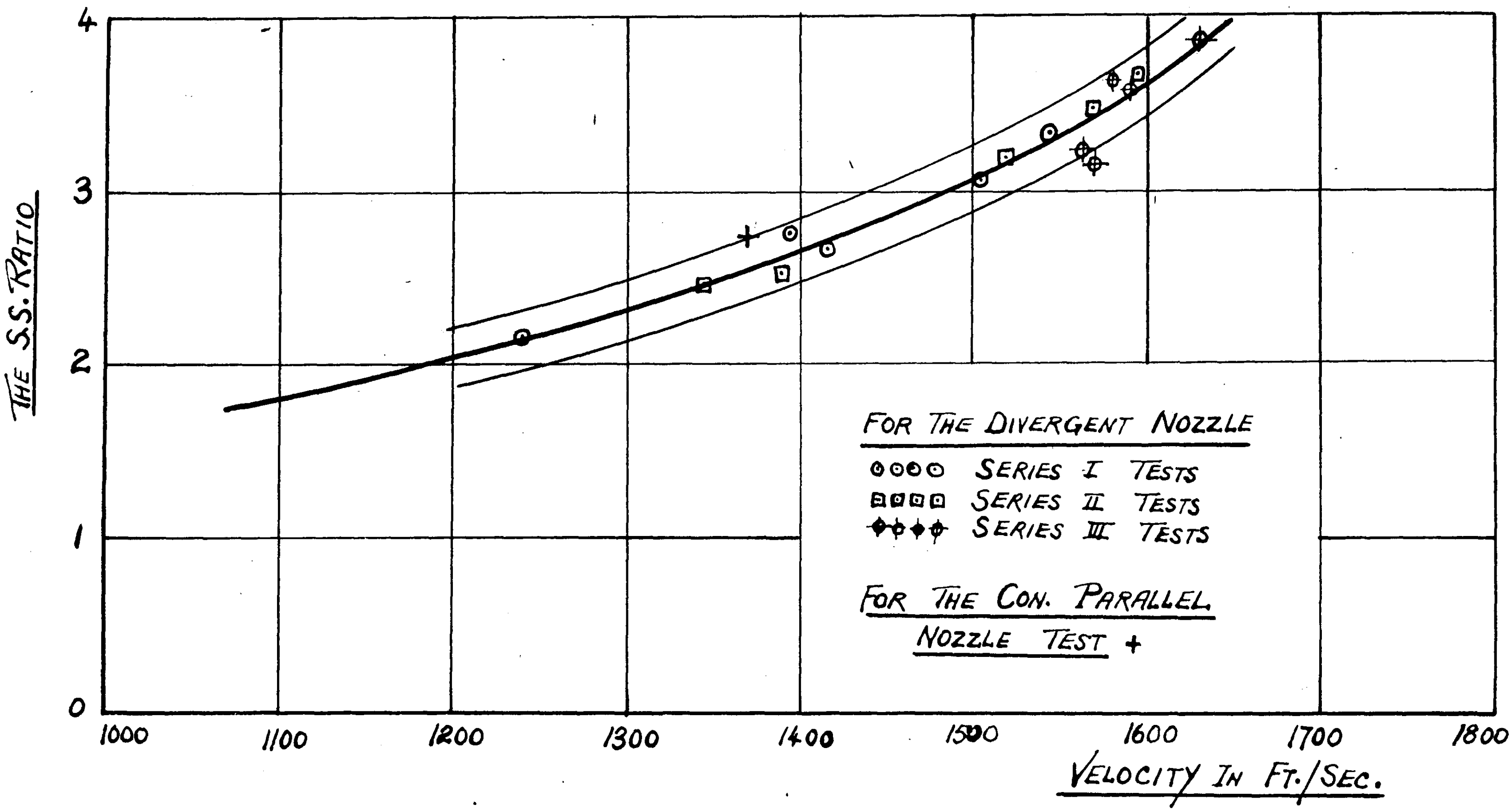
## Conditions At Reversion

Having established the fact that the thermojunction readings, in the period of supersaturated flow, are mainly influenced by heat interchange phenomena and cannot show the full extent of the undercooling, Attention may be directed to that aspect of the temperature curves which is so clearly disclosed by the experimental records. The sudden rise of temperature to the saturation value at some position in the nozzle length, shown in all cases, is unmistakeable in its meaning. It establishes the fact and defines the position of reversion with certainty. As the readings before that point are high, the full extent of the "jump" is not demonstrated but it is nevertheless clear and decisive.

Some
Sum previous calculations have established the approximate conditions at all points on the flow. The definition of the point of reversion by the temperature records allow the fact of reversion

FIG. No. 42.

THE S.S. RATIO & THE VELOCITY



to be related to the fluid conditions obtained at its position. It is possible thus to consider reversion in the light of the pressure, velocity or S.S. ratio existing at the point of occurrence.

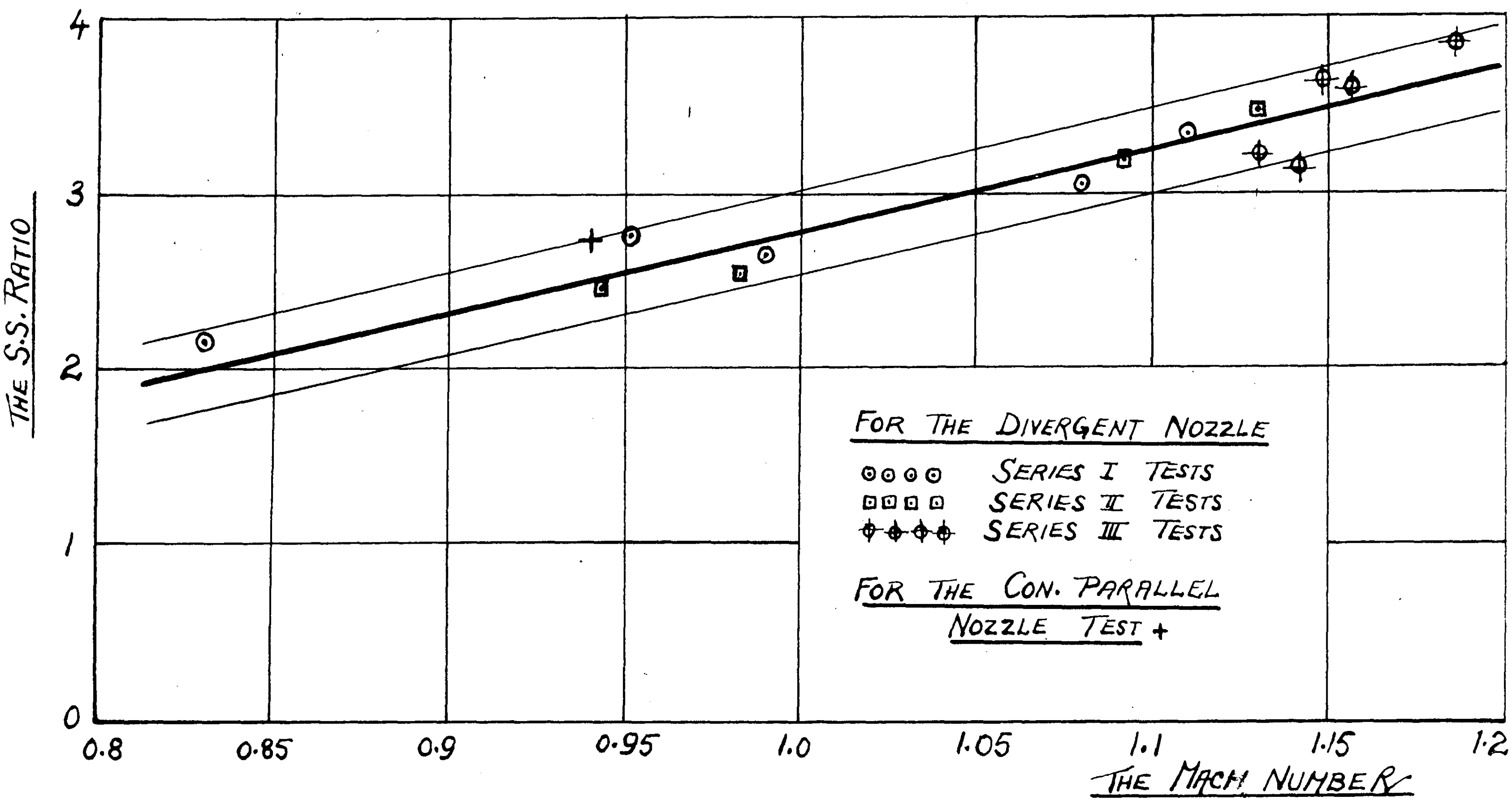
The method of estimating probable actual fluid temperatures already described can be applied to the fluid at the position of the temperature "jump" since the pressure there is known. If the pressure and temperature are P and T, then the ratio of P to the saturation pressure corresponding to T gives the S.S. ratio. The velocity at the position can also be approximately determined and so the simultaneous values of pressure, velocity and S.S. ratio are known reasonably well for each recorded case.

It will be appreciated that while the introduction of somewhat uncertain loss factors into these calculations renders all results approximate only, the neglect of loss effects would make them completely misleading. This is in fact a weakness of nearly all studies in supersaturated flow as S.S. ratios based on theoretical conditions are incorrect.

The S.S. ratios as determined in the manner explained are plotted against the fluid velocity at the point of reversion in fig.(42). The results for both the convergent parallel and divergent nozzles are included so that the range of velocity is wide. There would appear to be a systematic variation with velocity and the curve might reasonably be held as indicating a tendency towards unit S.S. ratio at zero velocity. It may also be remarked that at the higher velocities the recorded undercooling is also high, the variation with velocity being quite marked from the 18°F in case A

FIG. NO. 43.

THE S.S. RATIO & THE MACH NO.



of the 1st series on the divergent nozzle to only 4°F in case E of the same series.

These various features would appear to lend strong support to the arguments of those authorities who have considered that velocity exercises a dominant influence. In particular Rettaliata, who employed two rates of flow accounted for the difference between the S.S. ratios obtained as due to the different velocities in the condensation region. But actually, and in spite of the apparently well ordered relationship, there is no justification for such a conclusion.

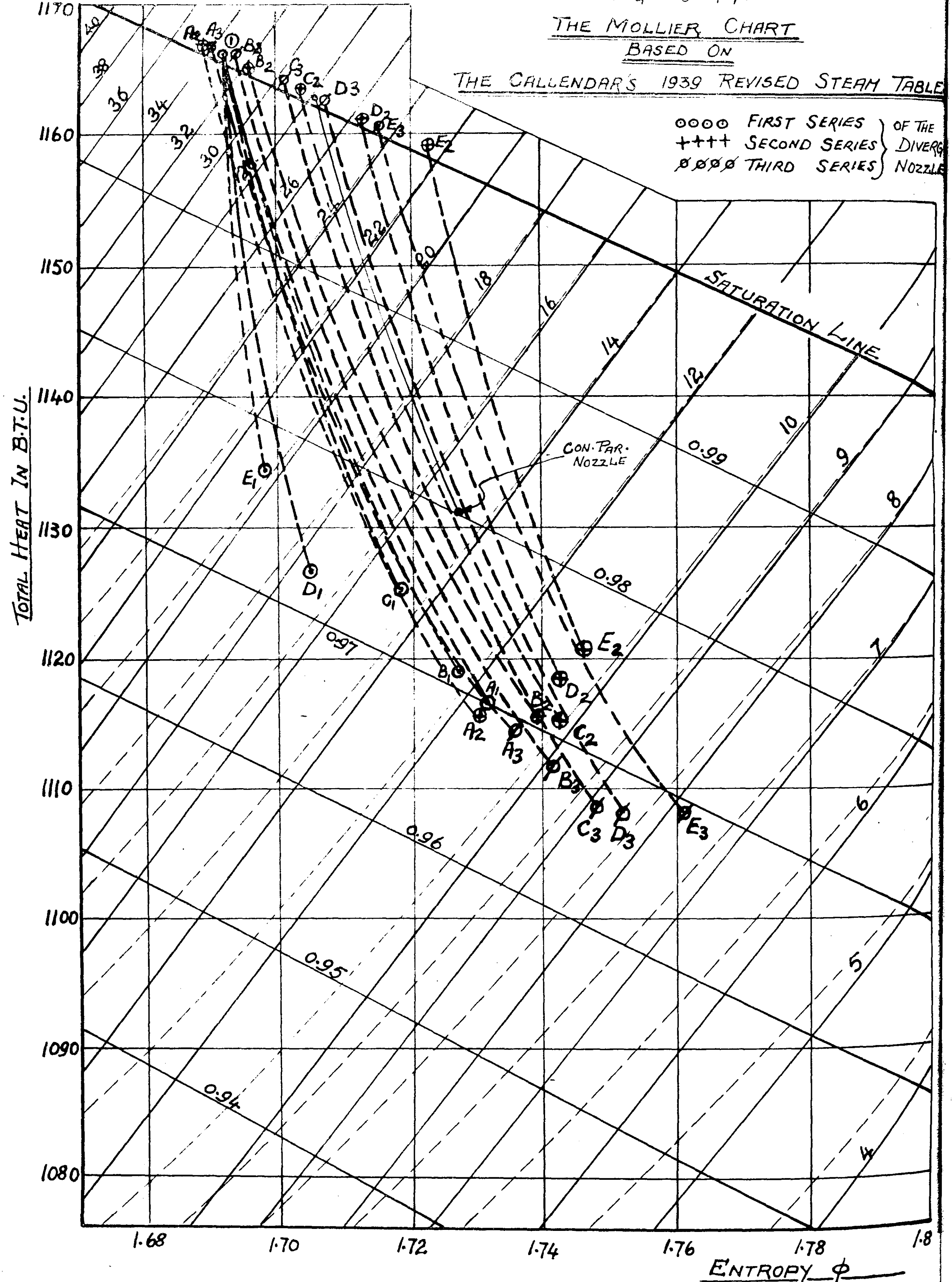
As very high speeds of flow, around and above the acoustic velocities are involved in these considerations it is natural to use the Mach number as a basis for plotting since, thereby, the velocity is represented by its ratio to the speed of sound which, of course, varies with the fluid conditions. The Mach No. in the present application is obtained from the equation:-

$$\text{Mach No.} = \frac{\left[\frac{2gn}{n-1} P_1 V_1 \left(1 - K - r^{\frac{n-1}{n}}\right)\right]^{\frac{1}{2}}}{\left[gnP_2V_2\right]^{\frac{1}{2}}} \qquad (9)$$

Where $P_2$ and $V_2$ are the conditions at reversion point and could be obtained by the adiabatic equation, and when evaluated for all the cases available gives the relationship between S.S. ratio and Mach No. shown in fig.(43). This of course merely gives the results of fig.(42) on an alternative base, but the simpler form obtained would seem to indicate that the Mach No. was a superior variable to employ in presenting the relationship.

The points of reversion can also be shown on the $I-\phi$

FIG. No. 44.



THE MOLLIER CHART
BASED ON
THE CALLENDAR'S 1939 REVISED STEAM TABLES
oooo FIRST SERIES
++++ SECOND SERIES
ØØØØ THIRD SERIES
OF THE DIVERGENT NOZZLE
SATURATION LINE
CON. PAR. NOZZLE
TOTAL HEAT IN B.T.U.
ENTROPY φ
1170
1160
1150
1140
1130
1120
1110
1100
1090
1080
1.68
1.70
1.72
1.74
1.76
1.78
1.8
0.99
0.98
0.97
0.96
0.95
0.94
40
38
36
34
32
30
26
24
22
20
18
16
14
12
10
9
8
7
6
5
4
A1
A2
A3
B1
B2
B3
C1
C2
C3
D1
D2
D3
E1
E2
E3

done for the sake of simplicity in treatment and because it has been necessary to emphasize their meanings and relative significance. Two have so far been considered, the high readings in the undercooled zone and the sudden rise at reversion. But there is a third, viz. the coincidence of reversion with the point of recompression in the divergent nozzle.

Comparison of the temperature and pressure ratio curves for the divergent nozzle in all cases in which the back pressure was such as to compel a recompression within the nozzle length, shows conclusively that the temperature "jump" and the rapid pressure rise practically coincide in position. This is highly significant. Without such positive measurements as these experiments have allowed the fact could not be assumed. With these it is unescapable.

The recompression that occurs in a divergent nozzle operating against too high a back pressure is a point of abrupt discontinuity. It is, in fact, a point of shock. There are extremely sudden changes in state and speed. It is the position where the shock wave is established.

The fact of coincident reversion leads at once to the conclusion that this occurrence is not primarily determined in high speed steam flow by the physical limit to the supersaturated state expressed by the S.S. ratio, but by the sudden obstruction of the shock wave. This at once transforms the problem into one in hydrodynamics and the significance of the S.S. ratio is greatly reduced. If it were possible to imagine a perfectly smooth undisturbed flow in the development of these high speeds, then the limiting S.S. ratio would eventually operate to end the supersaturated condition. With anything of the nature of a shock wave in the flow a compulsory change is imposed at that position.

There are two aspects of these phenomena which deserve notice as providing further arguments in favour of the conclusion. One is that the supersaturated condition is highly unstable and shock is probably the surest way of causing reversion. The other is that both the shock effect in fluid flow and reversion from the supersaturated condition are natural phenomena characterised alike by the feature of almost instantaneous entropy increase.

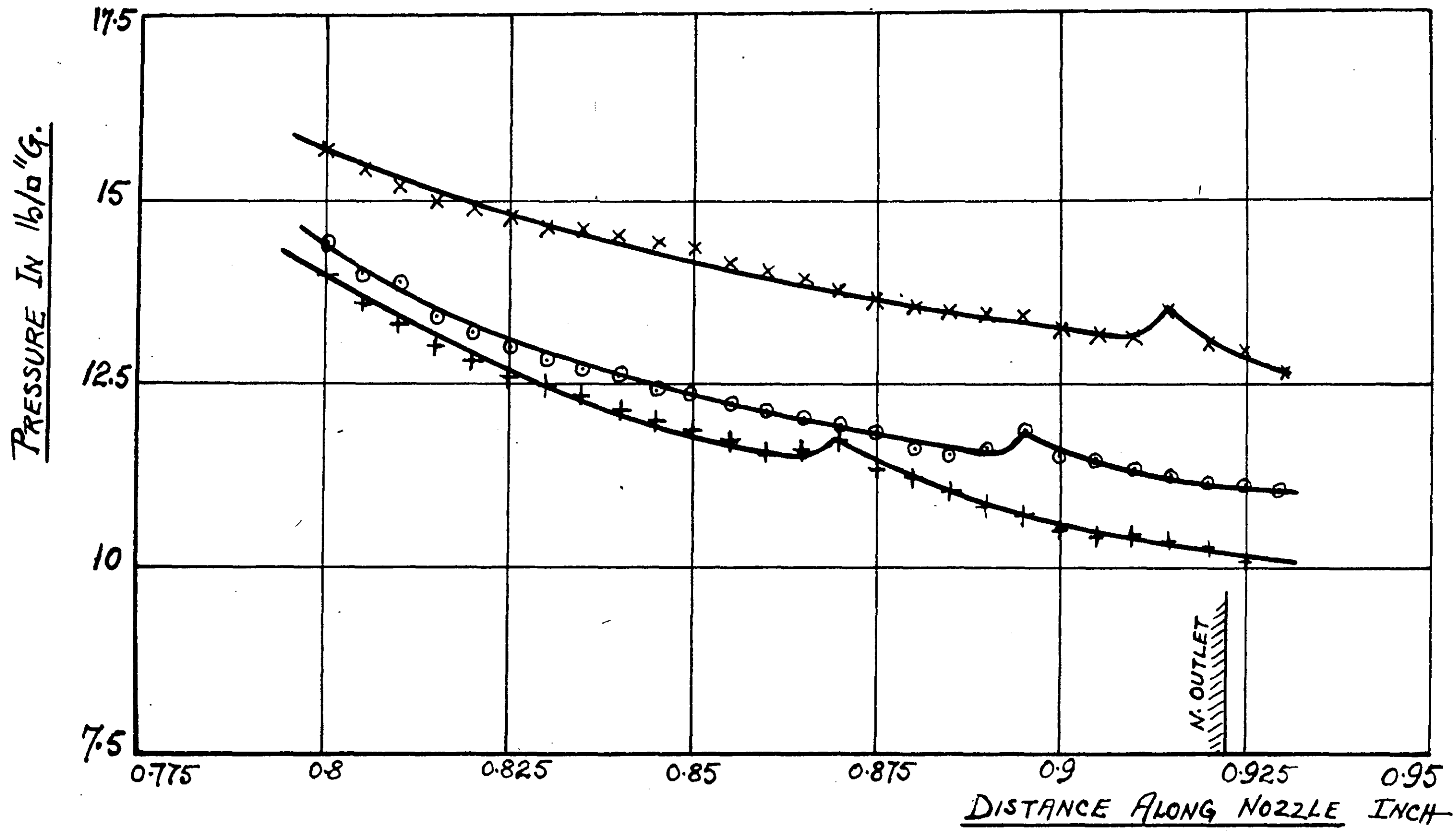
While the conclusion reached can hardly be questioned as a reasonable interpretation of the divergent nozzle results, examination of the convergent nozzle curves does not disclose the same definite facts. Well defined recompressions do not exist in the expansion with the convergent parallel type of nozzle and so the association made so clear by the divergent type cannot be detected in the same way in the other.

But the reasoning is based on the existence of shock waves and not on the establishment of major recompressions. But the convergent nozzle can expand to the critical condition and reversion has been detected by the temperature measurements in all cases in which it does. Since the acoustic velocity is reached shock waves are possible. These do not always disclose themselves by marked pressure rises. In fact the method of measuring pressure in these experiments is not able to detect slight pressure irregularities except occasionally or by special precautions.

FIG. No. 45.

PRESSURE DISTRIBUTION AROUND THE OUTLET OF THE CON. PARALLEL ½/1 RATIO NOZZLE

×××× WITH INITIAL TEMPERATURE = 300 °F.
⊙⊙⊙⊙ WITH INITIAL TEMPERATURE = 292 °F.
++++ WITH INITIAL TEMPERATURE = 286 °F.



That pressure "jump" does occur in such cases has been noticed by several investigations and has been under certain conditions observed in the present experiments. At an early stage in the first series of pressure and flow tests, pressure discontinuities were obtained in the Convergent Parallel nozzle. The nature of these is shown in fig.(45) in which with different initial conditions, the position of the pressure "jump" varied slightly. These cannot be related to the later reversion determinations as the same initial conditions were not repeated. But the fact of a transitory pressure rise would appear certain; and it is always fairly close to the nozzle outlet in which region reversion invariably occurs. It also shows that with different initial temperatures the reversion position varied slightly.

But this mild experimental demonstration is hardly necessary if it is allowed that the shock wave, when it exists, can compel reversion. There is always a shock wave associated with the outlet conditions in a convergent nozzle expanding to the critical conditions or delivering into a space well below the critical pressure. The wave may be anywhere around the nozzle outlet and that is precisely what has been found for the reversion points in that particular nozzle type, as can be seen by reference to figs. (29 and 32).

Hence, although the simple nozzle form does not provide the same positive evidence as the divergent type, the facts which it gives do not conflict with the conclusions drawn from the evidence of the latter. It may therefore be claimed that the temperature curves that have been obtained in these investigations do, in effect, establish the shock effect in high speed flow as the controlling factor in reversion.

Some reasons have already been advanced to show that the characteristic flow curve type is not in conflict with the idea of sudden reversion, it may be claimed that all of the experimental evidence is in accord with the simple conclusion reached. It may be added that this conclusion serves also to explain the variations and uncertainties of the S.S. ratio hitherto considered as the determining influence.

## C. CONCLUSIONS

This study of flow, pressure and temperature effects in small elementary nozzles under conditions of supersaturation leads to the following general conclusions:

1. The flow curve type originally established by Mellanby and Kerr is fully substantiated. The peculiarity they found is even more pronounced. It persists even in cases where high losses prevent flows above the theoretical on stable expansion.

2. The variation of flow curve form with surface ratio indicates that the Mellanby and Kerr idea of partial reversion before the S.S. limit is not rational, and the support which that theory gives to the argument that condensation comes down on exposed surfaces

is thereby lost.

3. There is no special justification for the idea, used by Powell, of gradual reversion beyond the S.S. limit. It would seem that the consideration of losses in the flow combined with the idea of sudden reversion is in better agreement with the facts in so far as flow curves can disclose them.

4. By the use of a thermojunction so arranged as to be hidden and insulated it is possible to obtain records of undercooling and, particularly, of the fact and position of sudden reversion. Temperature curves obtained in this way provide the evidence that flow determinations cannot give.

5. The undercooling in the range of supersaturated flow, as recorded by the special thermojunction, is much less than it will be in the fluid. The discrepancy lies in the fact that the junction is reading the boundary layer temperature and this is above the stream temperature by virtue of the transformation of losses into heat. The theoretical reasoning that establishes the difference as approximately dependent on the square of the velocity is fairly supported by the values deduced from these experiments.

6. Since the temperature measurements show some undercooling and the failure to show all of it has been rationally explained, there is no justification for the belief that condensation will occur on all surfaces exposed to the flow.

7. As the occurrence of reversion is clearly shown and its position specifically indicated by the temperature records, the values of pressure, velocity and S.S. ratio are all deducable for the

experiments. While systematic variation of the S.S. ratio with velocity would appear to be shown this is not fundamental. This velocity is merely reflecting the flow condition that actually enforces reversion.

8. Reversion is a consequence of the shock wave that exists in high speed flow around and above the acoustic velocity. The fact is fully demonstrated by the divergent nozzle tests where reversion is always coincident with the recompression. It is also supported by the convergent nozzle results where reversion takes place around the nozzle outlet which is the position of the shock wave in such a case.

9. It follows that the S.S. ratio hitherto always employed as determining the limit of the supersaturated flow cannot be so used in the case of high speed expansion where the limit is really set by these hydrodynamic conditions that create a shock wave.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