Flight to Safety in Macroeconomics: Role of Cyclical Risk Aversion

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To teachers, family and friends.

Introduction

"No matter how evolved humans think they are, we still have the same flight-or-fight instincts of our caveman ancestors."

— Ernessa T. Carter, 32 Candles

Flight to Safety, Flight to Quality and Flight to Liquidity, have been searched in Google for over 14,000 times since 2004. The economic literature uses these terms loosely and interchangeably as the 'short-run and sudden' perverse movement of capital from risky to safe assets in search of safety, quality and liquidity. On uncertain, volatile and illiquid market trading days, patterns of multi-asset correlations known as 'risk off' and 'risk on' emerge. In risk off, investors favour safer assets like bonds, gold and safe-haven currencies, while simultaneously most risky assets as equities, commodities and emerging market currencies run out of favour. In risk on, the reverse phenomenon occurs as investors become more willing to indulge in risky ventures. These episodes of correlated flight of investment capital from risk to safety or in the reverse direction have been frequently occurring since the global financial crisis. They have been more frequent than could be defended by the central tenet of neoclassical macroeconomics, the inter temporal substitution of consumption which posits capital revaluation arises from a desire to consume more today vs tomorrow. The long-run covariance in daily returns of US 10-year treasury bonds and S&P 500 has been negative since 2005. Incidentally, in this period, the US has witnessed a severe crisis and prolonged recovery. The two seem related. It seems there is causality running from asset-prices to business cycles and not in the other way where macroeconomics generally searches for it.

The evidence is clear, risk on, risk off, capital flight and other asset prices phenomenon reflect our desire to change portfolio risk over time. Despite the extensive evidence linking risk aversion and precautionary behaviour to asset price adjustments and volatile fluctuations, the literature has mainly focused only on the passphrase 'short and sudden' of the Flight to Safety phenomenon. By exploring the capital flight phenomena only from an asset pricing and market imperfections perspective, it has mostly overlooked their implications for long term macroeconomic dynamics and policymaking. Evidence also suggests that these phenomena are rich in information about the way we form economic expectations, and therefore, it is the objective of this thesis to evaluate their long term impact on the macroeconomy.

The thesis contains three main chapters that investigate the Flight to Safety phenomenon and its impact on the macroeconomy from an empirical and theoretical perspective. In doing so, it advocates for a more significant role of investors' time-varying, state-varying and cyclical risk aversion in macroeconomic models and policy analysis. The first chapter establishes that Flight to Safety, a sudden and short term asset market phenomenon is worth the attention, as the identified Flight to Safety shocks have long-run adverse impact on US business cycles. The investigation then takes the direction to theoretically formalize the role of Flight to Safety in micro-founded DSGE models. The second chapter establishes that similar to the asymmetric impact of flight of capital (to safety or risk) on asset markets, the optimal policy for investors with cyclical risk-aversion should also be asymmetric in costly recessions. The third chapter brings the analysis back to Flight to Safety by demonstrating it as an optimal response in costly recessions when risk-averse investors have a low risk-bearing capacity. Abstract from each of the chapters follows.

Chapter 1 examines the dynamic effects and empirical significance of Flight to Safety (FTS) shocks in the context of US business cycles. FTS represents a sudden preference for safe over risky investments and contains important information on agents' time varying risk-aversion and their expectations for future economic activity. This analysis presents an identification for FTS shocks using vector autoregressions (VAR). Sign restrictions are applied, while controlling for monetary policy and productivity shocks, on the price differential series between stocks and bonds in the US. Identified positive disturbances to this differential series are characterised as FTS shocks. The business cycle impact of FTS is calculated by applying the structural VAR model to the US economic data from 1954 to 2019. A sudden increase in risk aversion, which is displayed through the FTS shocks in the identified VAR model, has played a significant role in keeping investments low in the US. FTS shocks explain more than sixty percent of the variation in US investments and they explain a higher proportion of macroeconomic fluctuations in periods around the Global financial crisis. This is a significant linkage when compared against results of DSGE models enriched with time-varying risk-premium and investment technology. FTS also comes up ahead of news shocks in providing early signals of shifts in total factor productivity. This analysis is consistent with other comparable high-frequency, kernel-based measures of identifying FTS. The results also reveal the asymmetric impact on the business cycle of Flight to Safety and its complement Flight to Risk phenomenon. This asymmetry lends support to pursuing a cyclical risk-aversion driven view of business cycles.

Chapter 2 explores the economic theoretical DSGE model based evidence that can explain the asymmetric result of the previous chapter. Standard analyses using the New-Keynesian (NK) model find that the policymaker cares far more about inflation stabilisation than any other policy objective. Empirical evidence, however, suggests that stabilisation of output, especially during recessions, is a more significant concern for policymakers than what the benchmark NK model implies. Evidence also suggests that the pre-Global financial crisis the Federal Reserve followed an asymmetric monetary policy. It involved slow reaction to increasing rates in expansionary phases but swift response to cutting rates during the recessionary periods. This chapter, therefore, solves for the globally optimal policy using Chebyshev's collocation method in an NK model which features costly recessions. The device used to generate costly recessions is taken from the literature on the equity risk premium which can generate such premia by assuming that household consumption habits make any loss of consumption in a bad state (or economic contraction) particularly harmful for households. The optimal discretionary policy result is non-linear and asymmetric during good and bad states of the economy. During bad states, it is geared towards addressing the output gap ahead of the inflationary gap. The asymmetry of the optimal policy is motivated to correct for the inherent asymmetries in the way households' precautionary behaviour is affected by the various states of the macroeconomy. Ignoring the households' precautionary motives could lead to costly policy mistakes.

Chapter 3 continues with the economic theoretical DSGE model based investigation into generating Flight to Safety mechanism as an optimal response to cyclical changes in individual risk aversion. The Flight to Safety phenomenon has been portrayed in literature as an aberrant reaction to some form of market externalities, valuation constraints or Knightian uncertainty. This consideration of Flight to Safety as a short-term and an asset market phenomenon has also resulted in it being overlooked for the purposes of macroeconomic theory and policymaking. Whereas, recent empirical evidence suggests that the Flight to Safety that exists even in low-frequency data, has significant information about future productivity and business cycle performance. Therefore this chapter produces a new micro-founded general equilibrium approach to model Flight to Safety phenomenon observed during recessions or bad states of the economy. It showcases Flight to Safety as an optimal policy response by individuals in an NK model which features costly recessions and an option to switch between risky and safer production technology. Due to the presence of consumption habits, there is a non-linear increase in marginal utility once consumption gets closer to the subsistence level of habits, this makes recessions costly. The households find it more hurtful losing a unit of consumption in bad states than losing a unit in boom times. However, making a precautionary switch towards a safer asset allocation safeguards the investors against any loss of future income. Global solution methods demonstrate that ex-ante Flight to Safety is the optimal response for individuals that are consuming closer to their subsistence, and that calls for a remedial policy response. These findings substantiate the macro-finance based explanations of recessions by demonstrating that Flight to Safety stems inherently from individual's risk aversion even in the absence of intermediate credit constraints or information asymmetry.

The thesis makes multiple contributions of which the three major ones are as follows. Firstly, its central conviction is to identify the role of Flight to Safety as the critical driver of business cycle fluctuations. By means of structural VAR model it shows that Flight to Safety shocks predate any regress in total factor productivity by several years, and account for more than 50% of fluctuations in macroeconomic variables at business cycle frequency. The finding that FTS can generate a long and sustained decline in investment-related macroeconomic variables asks several questions from the established tenets of business cycle dynamics. It proposes an alternative view that shocks to investors' risk aversion lead to booms and busts in business cycles. Secondly, the thesis shows that precautionary savings behaviour matters, and it changes non-linearly in the amount of information and control agents have over their future consumption. Therefore, optimal discretionary policy need to actively and asymmetrically manage this precautionary behaviour. A policymaker without an ability to observe output gap can also replicate the optimal strategy by restoring the equity premium to its steady-state level. Lastly, the thesis demonstrates that households near the upper or lower ends of their risk-bearing capacity, demonstrate sudden changes in their risk allocation that echo with the 'risk on' and 'risk off' phenomena. It also challenges the standing approach in literature to consider Flight to Safety phenomenon only as an aberrant reaction to some form of market externalities, constraints or uncertainty and thereby overlooking it for monetary policy.

Through identification and theoretical evidence posted in this thesis, we have learnt that the surge in FTS episodes over the last two decades is a crucial variable of significance in understanding business cycles. The explanatory power of FTS is promising and stands in contrast to the ineffectiveness of standard measures of productivity, sentiment, and expectations in explaining the slow recovery post-global financial crisis. There has been another surge in risk aversion during the Covid-19 phase, and therefore improved understanding of the Flight to Safety mechanism would be useful in making effective policies for recovery. In particular, what would be interesting to look out for in future research is an estimated micro-founded DSGE model with the causes and effects of the Flight to Safety phenomenon.

The thesis shows that the path forward to include asset pricing, time-varying risk aversion, non-linear solution methods, costly recessions and productive technology with heterogeneity in shock processes into economic modelling looks promising. It would go a long way in devising a risk aversion-based view of business cycles. It is a challenge worth surmounting to establish recessions and behaviours, as being driven from individual risk aversion, risk-bearing capacity and desire to shift risk allocation over time. There is conviction in that thought since in good times we show exuberance, and in bad times have an innate reaction to shed risk indiscriminately. Our risk-aversion and our risk-bearing capacity are situational; they determine whether we fight-or-flight and they can also reveal our future. Hopefully! Ms Carter would agree.

Jayant Yadav
Thursday 28th January, 2021

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List of Abbreviations

CHF	Swiss Franc
FEVD	Forecast Error Variance Decomposition
FOC	First order condition
FTR	Flight to Risk
FTS	Flight to Safety
GFC	Global financial crisis
JPY	Japanese Yen
MCMC	Markov chain Monte Carlo
MP	Monetary Policy
NK	New Keynesian
NNS	New Neoclassical Synthesis
SDF	Stochastic Discount Factor
SRatio	Surplus Ratio
S-VAR	Structural Vector Autoregressions
T-bonds	Treasury bonds
TFP/tfp	Total Factor Productivity
VAR	Vector Autoregressions
wrt	with respect to
ZLB	Zero Lower Bound

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All errors in the thesis are mine.

Samuel Taylor Coleridge said, *"The dwarf sees farther than the giant, when he has the giant's shoulder to mount on"*. There so much more that has gone into writing this thesis, if I have come this far, it's because of my teachers, family and friends who have shaped my life. With no disrespect, I am calling them by first names and in no particular order for all of them have been very dear to me and have guided me through various phases of life.

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Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution. All errors are mine.

Jayant Yadav

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1. Flight to Safety in Business Cycles

FTS in Business cycles examines the dynamic effects and empirical significance of Flight to Safety (FTS) shocks in the context of US business cycles. FTS represents a sudden preference for safe over risky investments and contains important information on agents' time varying risk-aversion and their expectations for future economic activity. This analysis presents an identification for FTS shocks using vector autoregressions (VAR). Sign restrictions are applied, while controlling for monetary policy and productivity shocks, on the price differential series between stocks and bonds in the US. Identified positive disturbances to this differential series are characterised as FTS shocks. The business cycle impact of FTS is calculated by applying the structural VAR model to the US economic data from 1954 to 2019. A sudden increase in risk aversion, which is displayed through the FTS shocks in the identified VAR model, has played a significant role in keeping investments low in the US. FTS shocks explain more than sixty percent of the variation in US investments and they explain a higher proportion of macroeconomic fluctuations in periods around the Global financial crisis. This is a significant linkage when compared against results of DSGE models enriched with time-varying risk-premium and investment technology. FTS also comes up ahead of news shocks in providing early signals of shifts in total factor productivity. This analysis is consistent with other comparable high-frequency, kernel-based measures of identifying FTS. The results also reveal the asymmetric impact on the business cycle of Flight to Safety and its complement Flight to Risk phenomenon. This asymmetry lends support to pursuing a cyclical risk-aversion driven view of business cycles.

1.1 Introduction

How can we identify Flight to Safety shocks? Do they have any significant effect on the economy and the business cycles? Does the Flight to Safety phenomenon warrant much attention if the shift of investments from risky to safe assets over one week reverses in the next week? These are some important questions of interest to both investors and policymakers alike. This chapter proposes answers to these and similar questions using sign-based restrictions in identifying the Flight to Safety shocks through vector autoregressions. Doing so also opens up new avenues to improve our understanding of macroeconomic shocks and their propagation mechanisms in business cycles.

The Flight to Safety shocks in this chapter represent unexpected positive innovations to households' risk aversion, or these are the shocks that lower investor's risk appetite. There are many instances in the financial markets, e.g. the Lehman crisis period and the Covid-19 period, when an increase in risk aversion and uncertainty has led to bursts of flight of investment capital from risky to safe assets. Such instances are commonly referred to as Flight to Safety (FTS). Similarly, any unexpected reduction in risk aversion, which then motivates a flight of investment capital in the reverse direction, i.e., from safe to risky assets, is known as Flight to Risk (FTR). This chapter investigates the impact and transmission of innovations to investors' risk aversion on financial markets, macroeconomic aggregates, and business cycles through the medium of Flight to Safety shocks.

The method for identifying Flight to Safety shocks in this chapter builds on Uhlig's (2005) agnostic identification strategy of imposing sign restrictions on the impulse response functions. This method identifies Flight to Safety shocks in a five variable structural vector autoregressions (S-VAR) model by imposing sign restrictions on impulse responses in four out of five macroeconomic series of the S-VAR, *viz.* TFP, price of risk, real rates, and surplus ratio. The TFP is adjusted for factor-utilization (Fernald, 2012). The price of risk series is obtained by differencing the S&P 500 index price from the price of long-term US treasury bonds. Any upside shift in this price of risk series will result from US Treasury bonds becoming expensive vis-à-vis US equities, which represents Flight to Safety. Real policy rates are obtained by adjusting effective federal funds rates for the rate of inflation. The surplus ratio is defined for this chapter[1] as one minus ratio

[1] The later chapters, which feature an NK model with internal habits, define surplus ratio differently as one minus ratio of subsistence habits to consumption. Table 2.1 on page 118

of the sum of non-durable goods and services consumption to total consumption. FTS shocks are identified by imposing orthogonality (or zero) restrictions between Flight to Safety and generic business cycle disturbances. The latter are shocks to the utilization-adjusted total factor productivity series and are ordered first in the five variable S-VAR. The fifth series that is ordered last in the S-VAR is a business cycle variable of interest (such as output, hours or investment, etc.). No restrictions are imposed on its impulse responses to any shocks in the model. In this manner, the identification method remains agnostic to the key variable of interest. Different sign restrictions, from the ones imposed to identify the FTS shocks, are included to single out the utilization-adjusted total factor productivity (TFP) shocks, and monetary policy (MP) shocks. These two disturbances are also restricted as orthogonal to each other.

The identification strategy is designed to overcome many challenges that are expected to arise in identifying the Flight to Safety shocks in vector autoregressions. At first, there is confusion over what one means by Flight to Safety shocks. There is no standard definition of the Flight to Safety phenomenon, and there is no actual price of risk series from which we can extract Flight to Safety shocks. The second concern is about the validity of the entire econometric procedure, which relates to whether the shocks in the price of risk series are purely led from a Flight to Safety (or Risk) motive or automatic movements resulting from other demand, business cycle, or monetary policy shocks. The next concern is avoiding the possibility that external shock impacts all variables in the model. Several shocks come to mind, which could have effects similar to those of a business cycle shock or a Flight to Safety shock. However, they result from sources exogenous to the model, e.g., a labour supply shock in another sector, an oil price shock, a fiscal policy shock. The fourth concern pertains to the choice of sign restrictions imposed and the credibility of identifying assumptions employed in this study. There may be agreement about some sign restrictions strategies, e.g., that positive monetary policy shocks raise interest rates in the short run. However, competing ideas come to mind when considering the impact of Flight to Safety on various macroeconomic time series. Finally, one also needs to consider that this study's results represent the impact of the intended Flight to Safety shocks and are not resulting from any other form of expectations, policy uncertainty, or sentiment shocks in disguise.

The first concern can be addressed by arguing that Flight to Safety is a phenomenon of forward looking expectations and that households prefer bonds

provides the definitions for use in later chapters.

or safer assets to equities or riskier assets, to safeguard their portfolio against expected loss of future income and wealth. By following this approach, we can obtain a price of risk series from the difference between the price of safe securities, which for this chapter is calculated by inverting the yield on 10-year US Treasury bonds and the price of risky securities (or the S&P 500 index). This price of risk series increases (or moves up) when investors favour safer investments (bonds) more than riskier investments (equities).

Developing a price of risk series in this manner has a clear advantage over other recent attempts in the literature to identify the incidence of Flight to Safety using likelihood and kernel-based methods. For e.g. Baele, Bekaert, Inghelbrecht, and Wei (2013 and 2019). The approach to developing the price of risk series followed in this chapter provides a smooth trend stationary series suitable for vector autoregressions. It can be replicated without relying on the researcher's inputs of the bandwidth and threshold criteria, as required in likelihood and kernel-based methods. The Sensitivity analysis section compares the Flight to Safety shocks obtained using methods of Baele, Bekaert, Inghelbrecht, and Wei (2019) with the Flight to Safety shocks obtained through S-VAR. These two procedures' key results are comparable, but working out a Flight to Safety series through VAR offers ease of replication and universal appeal.

For addressing the second concern, the orthogonality restriction imposed between total factor productivity shocks and Flight to Safety shocks filters out the possibility of automatic spillovers from business cycle surprises to Flight to Safety shocks. Orthogonality restriction between total factor productivity shocks and monetary policy shocks is also vital to keeping monetary shocks unrelated to generic movements in business cycles.

The third concern is addressed by repeating this exercise for other macro variables. The benchmark configuration's key variable of interest in subsequent experiments is replaced with other business cycle variables (such as hours, foreign portfolio flows, consumer prices, etc.). The purpose of such an examination is to seek common plausible explanations for changes in impulse responses between the replacement model and the benchmark S-VAR model and to avoid missing out on any explanatory contribution from common external factors.

The fourth concern is resolved by choosing policy-relevant and theoretically robust signs and zero restrictions. The identifying assumptions are based on the results of Smets and Wouters (2007). They consider a medium scale NK or

NNS (New Neoclassical synthesis) model that is consistent with the balanced steady-state growth path and is estimated using Bayesian methods. Their model has 7 structural shocks in: total factor productivity, risk premium, investment technology, wage, price markup, exogenous spending, and monetary policy. Several features of their model such as labour augmenting technological progress, investment adjustment costs, variable capacity utilization, and real rigidity in intermediate goods and labour market, make it a standard workhorse model of monetary policy analysis and also make it relevant for obtaining economic theory backed sign restrictions for total factor productivity, Flight to Safety and monetary policy shocks for the analysis made in this chapter.

Some of the critical results from Smets and Wouters (2007) that are useful in driving sign restrictions for this chapter are as follows. Technology shocks lead to an increase in output and consumption but a small decrease in nominal and real interest rates. On the initial impact, the fall in real rates is insufficient to prevent a decline in inflation and opening up of the output gap. Flight to Safety shocks restrictions in this chapter follow from risk premium shocks of Smets and Wouters (2007) where these innovations result in a fall in output, hours, and an increase in the real interest rate. One could argue that the risk premium shocks are not the same as a Flight to Safety shocks. Nevertheless, suppose the increase in risk premium is not uniform across investments of different risk profiles. In that case, for this chapter and to generate comparable results with standard DSGE models, it is within reason to characterize an increase in risk premium as a motivating factor for investors' preference for safety. The monetary policy shocks in that model (Smets and Wouters, 2007) on impact lead to an increase in nominal and real interest rates but a decrease in output, inflation, and hours. The monetary policy shocks in S-VAR model of this chapter include similar restrictions.

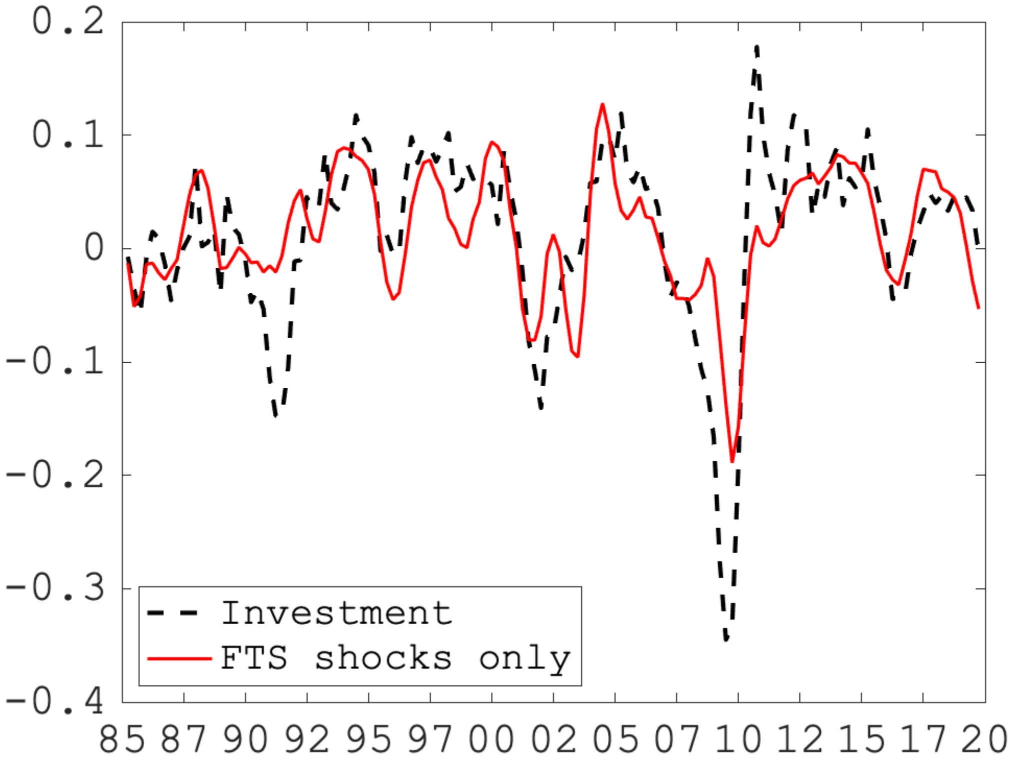
The fifth and final concern is addressed in the robustness section, where results from the economic policy uncertainty series [from Baker, Bloom, and Davis (2016) and Bloom (2014)], corporate bonds spread, liquidity spread, investment-specific technological productivity, and the relative price of the investment in terms of consumption are included in the analysis. Their results depict a reasonable likeness between the share of investment growth explained through Flight to Safety shocks and other series that capture the phenomenon of Flight to Safety. The Granger causality running from the price of risk series to these alternate series reaffirms the notion that Flight to Safety plays a significant role in business cycles.

The analysis contributes by motivating us to rethink about the role played by Flight to Safety in the most severe US recession since the Great depression. This chapter's empirical exercise extends research on Flight to Safety into new directions in the following manner. Firstly, it explores the long and short-run impact of Flight to Safety shocks on key macroeconomic variables through a structural vector autoregression study where Flight to Safety and monetary policy shocks are orthogonal to any business cycle shocks. The structural VAR makes minimal assumptions about the existence of any prior ordering of structural shocks. Secondly, it evaluates the strength of results by comparing them with the business cycle phenomenon in other sectors such as labour market and investments and in different periods such as the Great moderation period running up to the global financial crisis and the pre-Great moderation period. Thirdly, it evaluates the empirical exercise and identification strategy by obtaining comparable results from series that, in principle, have similar impact phenomena as the Flight to Safety. Some of the other asset market phenomena that coincide with episodes of FTS are: an increase in policy uncertainty, a widening of yield spreads between Baa and Aaa corporate bonds yields, an increase in liquidity spread between the yields on long-term and short-term treasury bonds, and an increase in the ratio of the price of investment goods to the price of consumption goods. The Robustness section makes a comparison between each series' business cycle impacts with the results obtained from the FTS series. Fourthly, the analysis brings to light the asymmetric economic impact of Flight to Safety and the capital flight in the reverse direction, Flight to Risk. Lastly, it develops a Flight to Safety shock series for the US economy that is intuitive and simple.

The contribution of results comes from providing a sound justification and closure to the main objective of undertaking this study to estimate the business cycle impact of FTS. Flight to Safety shocks significantly affect the long-term dynamics of the business cycle and economic activity. The impact of Flight to Safety shocks on the economy has increased in the post-Global financial crisis period. Hours, output, consumption, and investments all display negative responses to identified Flight to Safety shocks.

A one standard deviation Flight to Safety shock can account for a statistically significant 3% decline in private investments and a 4% decline in residential investments over a couple of years. Identified FTS shocks also lead to an imminent fall in total factor productivity by 8-10 quarters. Thus, they dispel the notion that FTS shocks are a neutral TFP or investment related TFP shock in disguise. FTS shocks also account for over sixty percent variation in relevant macroeco-

Figure 1.1: Contribution of FTS to Investment growth



Notes: Y-axis: Year on year growth rate (%) of US Investments and the benchmark model fitted with only Flight to Safety (FTS) shocks identified using Sign and Zero restrictions discussed in Strategy 1. X-axis label is Years (in YY format). Investment - is the US investment data series (PInv). FTS shocks only - is the Investment series in the benchmark model fitted with FTS shocks only.

nomic variables (hours, output, consumption, and investments) at business cycle frequencies, indicating that such shocks are an essential part of business cycles.

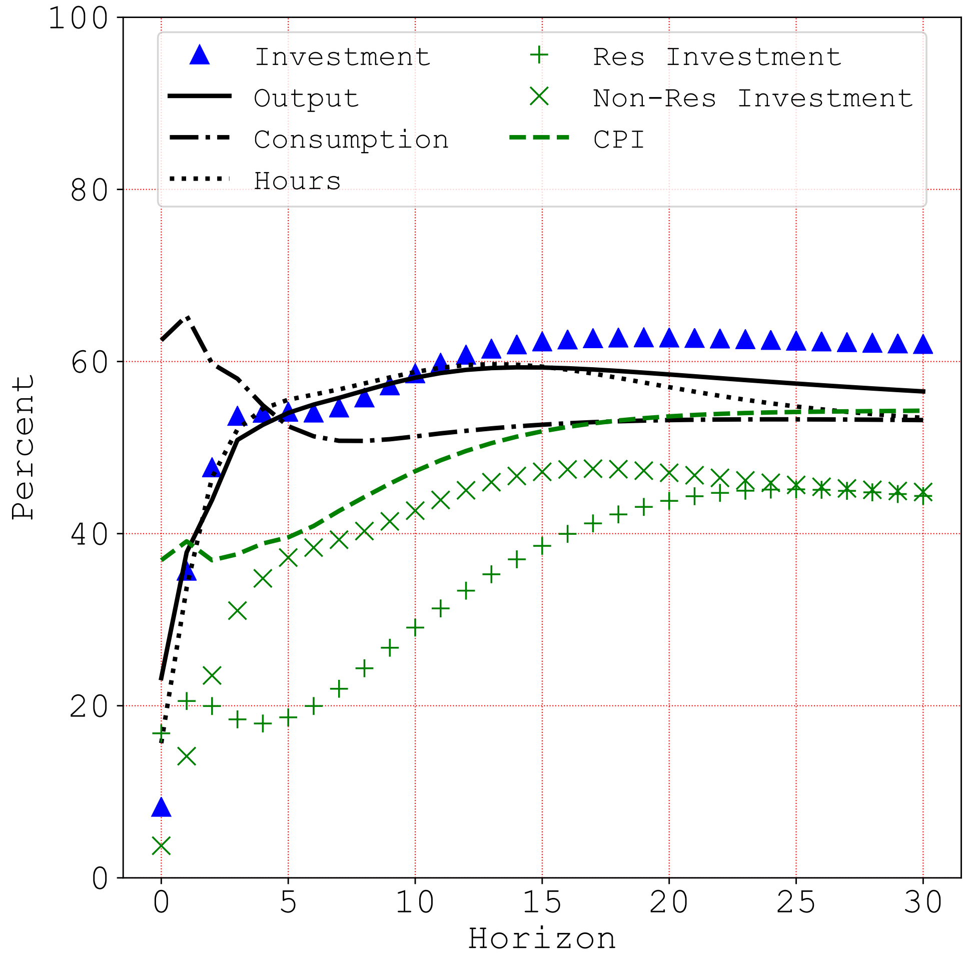
The identified Flight to Safety shocks in the benchmark VAR configuration can explain majority of the historical decomposition of Investments in the US. See Figure 1.1 on page 7, which plots the year on year growth in the investment data (dashed line) and the one that can be explained by only the Flight to Safety shocks in the benchmark VAR model (solid line). The co-movement between the two series is striking. In particular, during economic downturns such as previous recessions of the 80s, 90s, the Dotcom bust, and the Global financial crisis, the Flight to Safety shocks seem to be running the investment growth lower.

Besides FTS shocks, the benchmark model whose identification restrictions we will discuss in later sections, has shocks in total factor productivity, monetary policy, and consumer demand. Despite the presence of these other three keys shocks such a significant contribution of the Flight to Safety shocks, signifying that perceived risk aversion and precautionary motives manifested in Flight to Safety have a more prominent role in developing our understanding of business cycles. The significant contribution of FTS shocks to business cycles can be further corroborated from the k-period ahead forecast error variance decomposition (from Figure 1.2 on page 9) of crucial macro variables from identified Flight to Safety shocks. The figure shows that FTS explains more the sixty percent of forecast error variance in investment and a significant portion of it in other key macro variables such as output, hours, CPI, and consumption at business cycle frequency (8-32 quarters).

A breakdown of historical decomposition (Figure 1.3 on page 10) of the Investment series shows that FTS shocks were a significant driver of the increase in private investments in the years 1994-2007 running up to the global financial crisis. The significance of FTS shocks has been at its maximum during the period following the global financial crisis. Post-2009, there is a sluggish response of Investment series in the US due to negative FTS shocks hitting the economy. The recovery period had a reduced incidence of FTS shocks, but only after 2015 did we see any positive deviation from the Investment series trend.

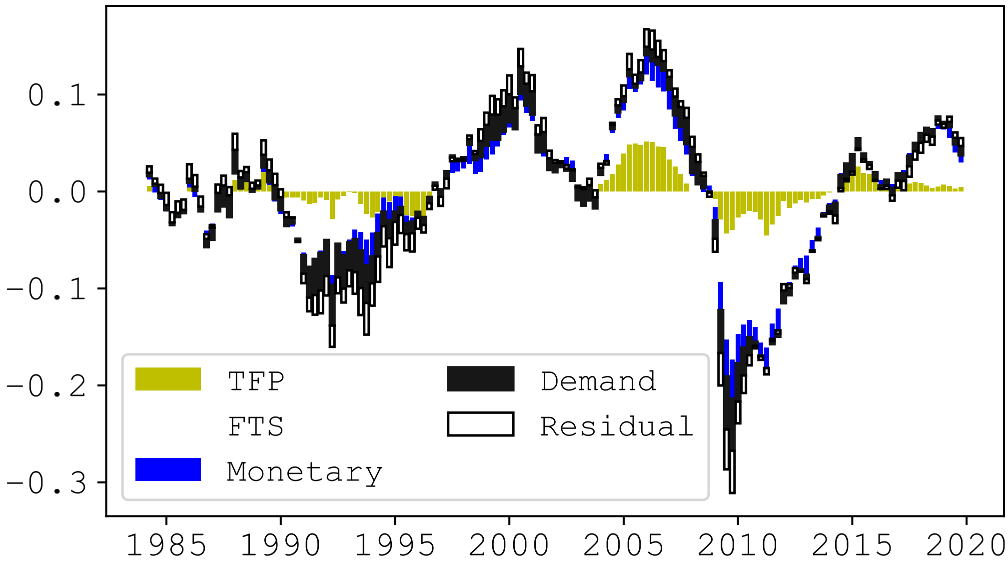
An underlying expectation hypothesis that this vector autoregression analysis could influence is that there are possibly two channels through which Flight to Safety may be relevant to study investments and business cycles. Firstly, *Expectations channel*, which explains Flight to Safety shocks as being caused

Figure 1.2: FEVD (%) explained by FTS shocks



Notes: The k-step ahead Forecast error variance decomposition (FEVD %) explained by FTS shocks, in the 5-variable VAR, which is identified using Sign and Zero restrictions Identification Strategy 1. The 5 variables in the benchmark model are: TFP, Price of risk (Bond minus Equity price), Real rates, Surplus Ratio and Investments. In other iterations of the 5-variable model, 'Investment' is replaced with other macro variables of interest. The result of 'Investment' and only those variables that replace 'Investment' in the benchmark VAR are reported.

Figure 1.3: Historical Decomposition of Investments



Notes : Historical Decomposition of Investment in VAR model Y_t = [tfp$_t$, Bond-Equity$_t$, real rate$_t$, Surplus ratio$_t$, Investment$_t$]$'$, which is identified using Sign and Zero restrictions discussed in Strategy 1. X-axis label: years.

by a build-up of expectations of an impending deterioration in the economy or an expected downturn in the business cycle. Flight to Safety is thus only an indication of a negative total factor productivity shock expected to hit the economy. Therefore, rational, forward-looking households respond in the short run by moving investments from risky equities to safer bonds to safeguard their portfolio against future expected loss of income and wealth.

A similar approach is followed in news shocks driven business cycle literature [see Nam and Wang (2019), Beaudry, Nam, and Wang (2011), Beaudry and Portier (2006) and (2014)] which postulates that cyclical fluctuations emerge when economic agents update their expectations from news about future productivity.

Secondly, the *Speculative channel*, according to which investors and businesses that have an understanding of the economy, pre-empt others by taking speculative positions. A Flight to Safety shock then is an over-correction by speculators to disappointing economic news. If enough speculators, investors, customers, and entrepreneurs over-correct, they can engineer a crisis of its own. Put, as per the expectations channel, Flight to Safety shock is a response to the expected future state of the economy, whereas in line with the speculative channel, a Flight to Safety shock results from an optimistic bet on the economy that turned-sour in the current state. The technology used in the analysis in this chapter is neither sufficient to identify the role of expectations, nor is it the central objective of this study to identify such channels. Still, by looking at the VAR study's evidence, there is some inclination to favour the Expectations channel. Aligning the results with estimation from a theoretical DSGE model could enhance our understanding of these channels.

This chapter contributes to growing macro-finance literature by developing facts about the Flight to Safety mechanism and linking it with time-varying risk aversion and precautionary savings motive of agents/investors and financial intermediaries. The linkages between FTS mechanism and behavioural features related to investments have not been included even in the following literature, which is considered seminal on Flight of Safety. Vayanos (2004) provides a theoretical introduction to the FTS phenomenon in asset pricing literature. Cochrane (2016, 2017) presents a macro-finance model that links changes in investor excess consumption to Flight to Safety. Baele, Bekaert, Inghelbrecht, and Wei (2019) and Baur and Lucey (2009) document the incidence of Flight to Safety across major countries. And Boudry, Connolly, and Steiner (2019) provides impact of Flight to Safety on liquidity returns, revenues and valuation of the commercial real estate industry. The previous literature has also explored the Flight to Safety mechanism in a limited way. Two common themes stand out. First of all, it is to consider FTS as being generated through market imperfections or externalities. These have been explored in the form of liquidity constraints [He and Krishnamurthy (2013), Brunnermeier and Sannikov (2014)], Knightian uncertainty (Caballero and Krishnamurthy, 2008), margin requirements of speculators (Brunnermeier and Pedersen, 2008), and intermediary balance sheets (Adrian, Boyarchenko, and Shachar, 2017). The second perspective is to consider FTS as a short term asset market phenomenon. The kernel-based approach discussed in Baele, Bekaert, Inghelbrecht, and Wei (2013) exemplies this perspective. It measures the likelihood of observing an FTS event. Either through a threshold-based method that calculates how each trading day compares to the distribution of return differential between bond minus equity returns. Alternatively, measuring it by an ordinal index-based method ranks each trading day on how it complies with the weak and strong symptoms attributed to an FTS day. This chapter relies on a quarterly time series of equity index and government bond yields to identify Flight to Safety shocks. It stays away from extreme but for short duration only movements in volatility, asset prices, and other high-frequency variables. Working with high-frequency data (like Baele, Bekaert, Inghelbrecht, and Wei (2013)) would not have answered one of the primary purposes of this study, which is to evaluate the long-term impact of FTS. If an extreme flight of investment from stocks to bonds over one week reverses in another week, that would leave the aggregate investment allocation for the quarter as unchanged and, therefore, less relevant for our macroeconomic analysis. The Flight to Safety shocks identified in this chapter are robust to similar results from kernel-based methods of identifying such shocks. However, compared to other approaches, the vector autoregressions approach followed here has higher universal applicability.

This chapter pursues a novel approach to develop stylised facts about Flight to Safety. I use agnostic identification which follows from Uhlig (2005) and Arias, Rubio-Ramírez, and Waggoner (2018). The sign restrictions based identification, building on the work of Canova and Nicoló (2002), Faust (1998) and Uhlig (2005) restricts the signs of specific impulse responses in the structural VAR, and without undercutting scientific inquiry, it keeps them in line with tenable priors accepted in theory. This strategy contributes to the literature by imposing minimal sign-based restrictions on the structural VAR and not using the orthogonal ordering approach. The latter assumes some macro variables as sluggish to react to shocks. The identification in this chapter imposes no restrictions on the key business cycle variable of interest.

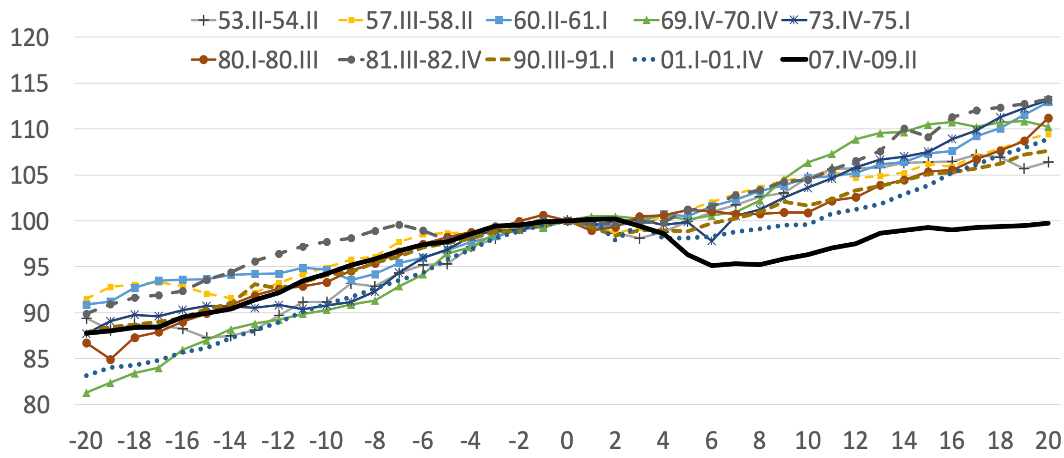
The success of an identification strategy depends on the plausibility of impulse responses it generates. This chapter's reasonable sign restrictions do not contradict with established results from DSGE models (Smets and Wouters, 2007). This impulse response-based identification strategy generates bounds on key variables of interest responses to structural shocks in the VAR. The identification strategy does not impose any sign restrictions on the key variables of interest, such as Output, Investment, Consumption, Hours, Unemployment, Labour productivity, Residential Investment, Non-Residential Investment, Capital Expenditure, R&D Expenditure, Consumer Prices, Government consumption and Expenditure, Prices of Investment goods in terms of Consumption goods, Consumer Sentiment, Capacity Utilisation, Vacancy rate, Participation rate, Foreign flows, and Wages. So in words of Uhlig (2005), the business cycle variable of interest remains agnostic to the identification. On the one hand, identifying FTS through S-VAR and sign restrictions differs from the existing approach to address this topic. On the other hand, this approach to study this phenomenon through vector autoregressions has a universal appeal.

The primary motivation for looking at the FTS phenomenon and obtaining its long term impact on the US macroeconomy comes from the unprecedented nature of the global financial crisis of 2007-08 and its long-lasting impact on consumption and investment data series. Compared to previous post-war recessions, the US's recovery following the global financial crisis has been muted and unprecedented for many quarters. Investment per capita in the US (as shown in Figure 1.4 on page 14), fell sharply after the Global financial crisis of 2008. Similarly, Consumption of Non-durable goods and services[2] was most distinctly impacted by the global financial crisis than any previous post-war recession. These series have been sluggish to return to its pre-crisis levels, as shown in the Figure 1.4 on page 14. During the global financial crisis, the performance was the worst performance of Investment, Consumption of non-durable and services, and many other macroeconomic aggregates in comparison to any post-war recession. The global financial crisis also dampened business sentiment (as shown in Figure 1.5 on page 15), but intriguingly, the impact was not so severely different from other previous recessions. Therefore sentiment alone cannot account for the significant decline in macroeconomic aggregate variables (esp. Investments) during the global financial crisis, and we have to look for alternate causes. This search for alternative explanations for the GFC was one of the chief motivations behind undertaking this study.

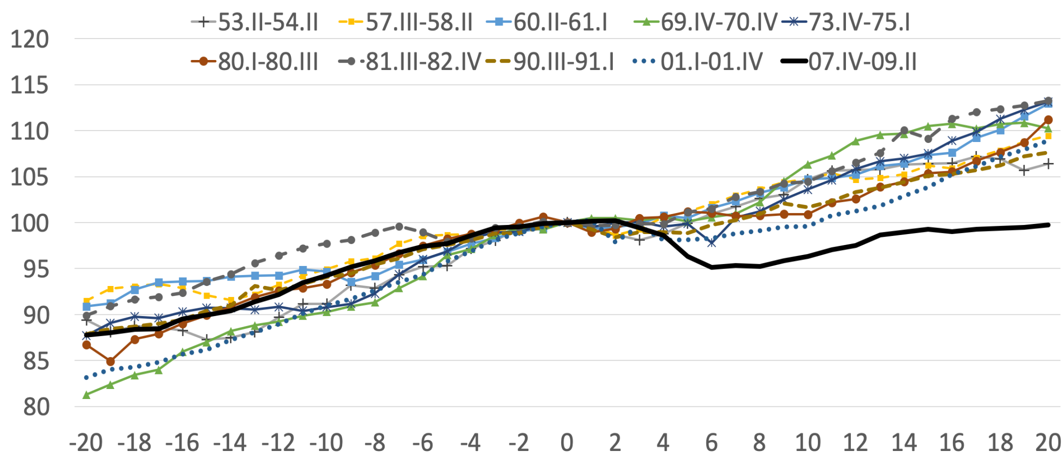
[2]For this chapter only, we can characterize this series as Consumption habits.

Figure 1.4: Global financial crisis and US recessions

(a) Private Investment per capita

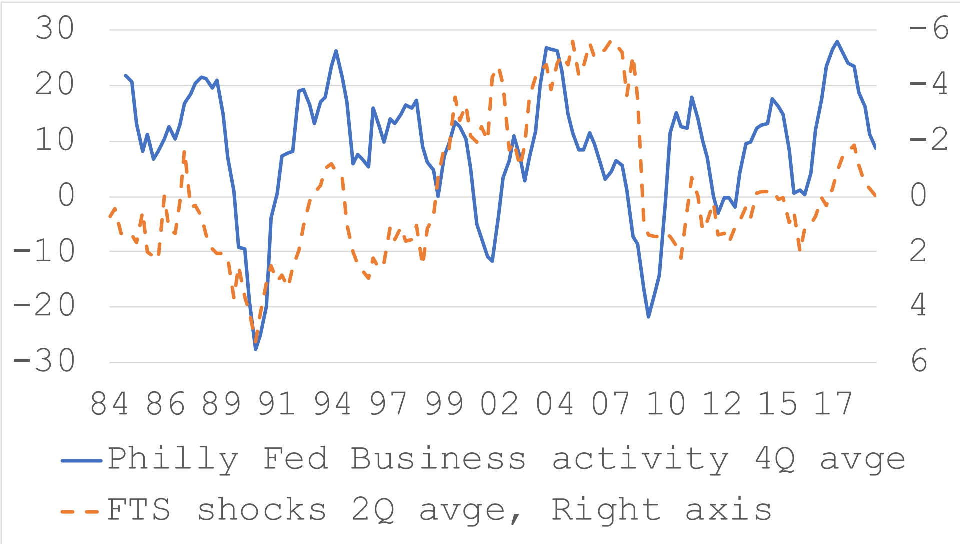


(b) Habits (Services and Non-Durables Consumption) per capita



Notes : Value at the peak of the each recession is indexed at 100, and corresponds to 0 on the X-axis, which denotes the time period in quarters pre and post the peak of a recession. The legend denotes peak to trough duration of a recession, for.eg. 07.IV-09.II is the peak to trough period 2007Q4-2009Q2 of the 2007-09 recession. Source: US FRED, NBER

Figure 1.5: Identified FTS shocks and Business outlook index



Notes: The figure shows Flight to Safety shocks (identified with restriction strategy 1) to a 5-variable benchmark VAR model, for Data: 1983:Q1 to 2019:Q3. The correlation between the two series is -52%. Two quarter average of identified FTS shocks (dashed, right inverted axis), and 4 quarter average of the Philadelphia Fed Manufacturing Business outlook survey (solid, left axis) are plotted.

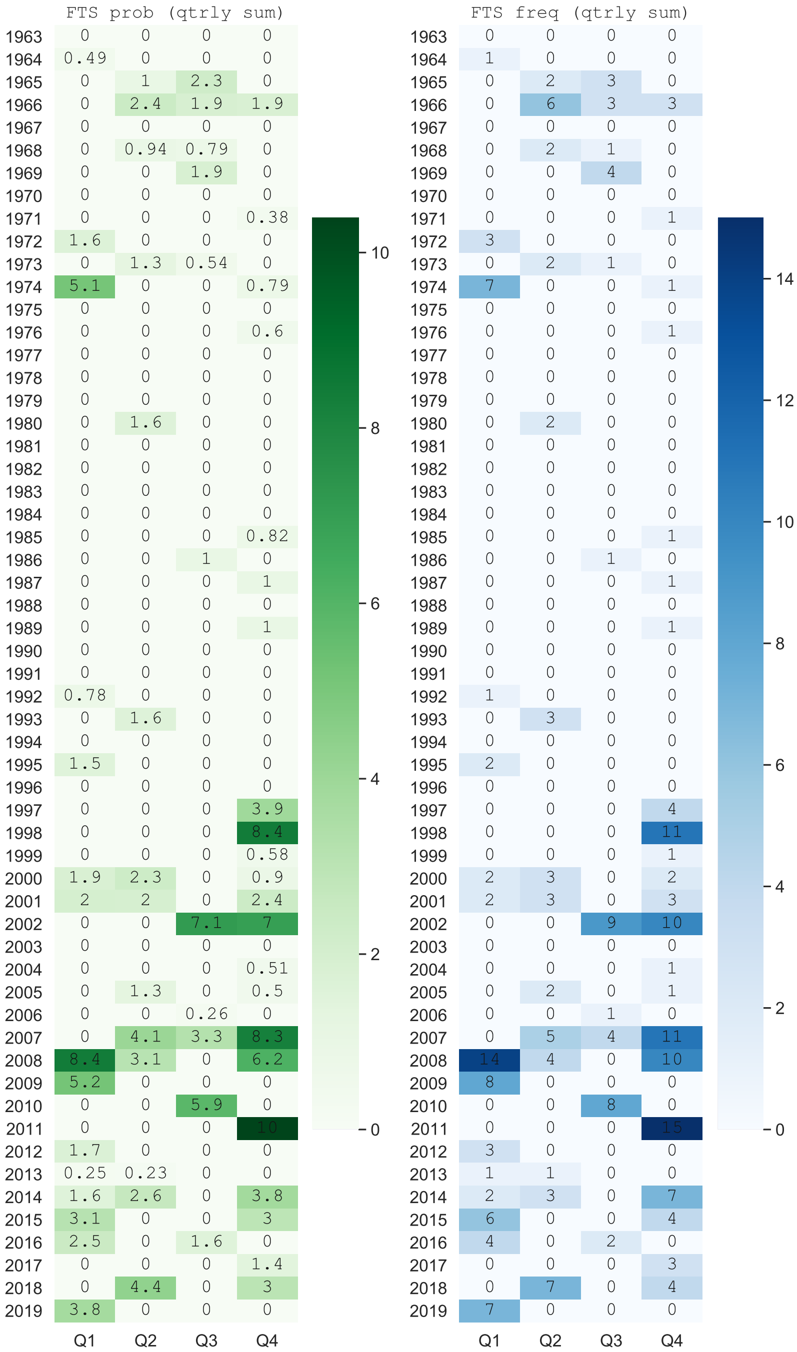
This unprecedented impact of the global financial crisis on the macroeconomic aggregates befits looking at explanations/mechanisms that could have been more relevant in the recent crisis than in any previous ones. Therefore, the search is for variables that had a somewhat more noticeable impact on the business cycle during the global financial crisis than the impact they had on the macroeconomic aggregate variables in any other post-war recession. Figure 1.6 on page 17 and results of Baele, Bekaert, Inghelbrecht, and Wei (2013) methods from Appendix A.2 show that the incidence of Flight to Safety has increased in the last two decades, which makes it a phenomenon of significance in recent events and a likely differential that could explain the impact of the global financial crisis.

Let us look at some anecdotal evidence that the identified FTS shocks from the empirical study performed in this chapter, a 5-variable VAR with the sign and zero restrictions, capture the changes in US Business conditions expectations. The identified FTS shock series, as seen in Figure 1.5 on page 15, has a negative 52% correlation to US Philadelphia Fed Manufacturing Business Outlook Survey (MBOS). The latter is an index tracking the direction of change in overall business activity. This figure plots a cumulative sum of two-quarters of FTS shock on an inverted scale, i.e., the dips on the chart refer to Flight to Safety, and the peaks are a shock in the reverse direction or a Flight to Risk. The two series closely track each other; they slowly increase through the Dotcom period of the 90s, ending with a bust in 2001. The housing sector bubble build-up in the 2000s, and the recession of 2007-08 are also visible. Most importantly, the inverted cumulative FTS shock has a peak to trough drop of near eight percentage points during 2007. It marks the worst peak to trough for identified series in the tested period (1983 to 2019). However, the worst peak to trough drop of Business confidence series during 2007 was similar to the one in the late 80s crisis. Monetary easing in the 1990s and 2000s was able to restore business confidence. However, we notice that despite the unprecedented monetary policy response following the Lehman crisis, there is a lack of recovery in business confidence, and it did not return to its peak for nearly a decade. Therefore, reasons/factors that were not prevalent in the 1980s were the main reason behind the dip in consumer sentiment during the GFC. This supports our faith in the original expectation that the identified FTS shocks from our VAR exercise, which have been unprecedented in times preceding the GFC, capture the GFC's driving mechanism, which was an unprecedented post-war economic downturn.

This chapter takes a small step forward in emphasizing the Flight to Safety and changes in investors' risk aversion as variables of significance in the business cycle. It would be interesting to see, in future research, if the findings of this chapter are consistent with results from an estimated DSGE model or whether there is a need to impose any additional restrictions based on general equilibrium modeling to bolster this chapter's findings further.

The rest of the paper is structured as follows. The next section 1.2 makes a brief discussion of related literature. Section 1.3 describes the data. It is followed by a description of the benchmark 5 variable model's identifying assumptions in section 1.4. This section also discusses the theoretical and numerical algorithm for sign and zero restriction identification employed in this chapter. Section 1.5 discusses the results from empirical analysis, their impact on business cycle variables, the forecast-error variance, and brings to light the asymmetry between Flight to Safety and Flight to Risk shocks. Section 1.6 performs sensitivity exercise on the results. It also discusses the alternative identifications of Flight to Safety shocks and their significance in various periods. Section 1.7 discusses the identified FTS shocks with methods propounded in literature in explaining business cycle fluctuations while section 1.8 concludes.

Figure 1.6: Frequency and Likelihood of Flight to Safety days.



Notes: Heatmap for frequency (freq) and aggregate likelihood (prob) of Flight to Safety (FTS) days in each quarter from 1963 to 2019. FTS days are calculated by Ordinal index approach (see Appendix A.2.3), for $k = 1.25$. Aggregated likelihood or strength of FTS, for each quarter in the studied period is calculated by adding the likelihood for each FTS day during that period.

1.2 Literature review

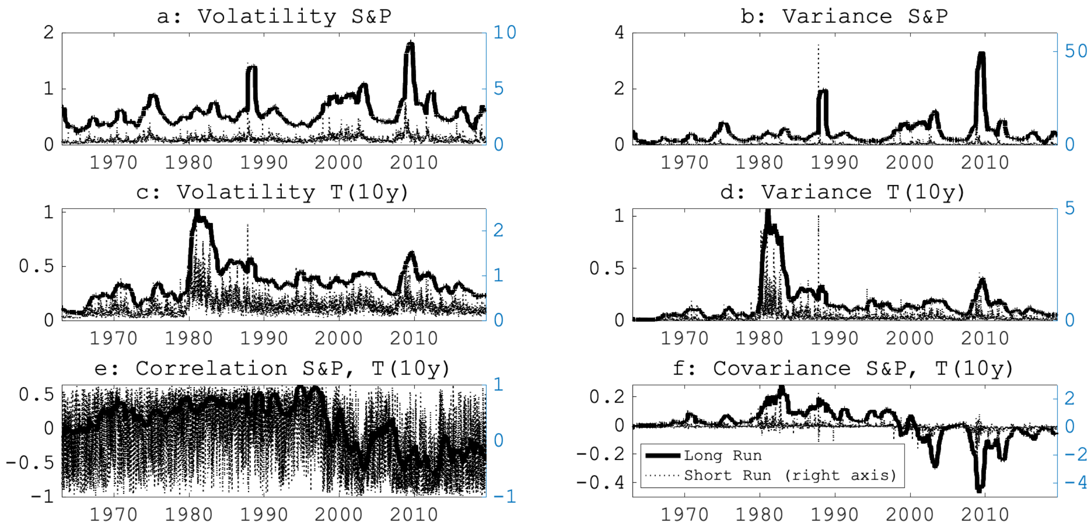
This section discusses both theoretical and empirical research related to Flight to Safety. In the wake of the global financial crisis, uncertainties about the growth and economic outlook have led to many pronounced episodes of negative correlation between returns on government bonds and stock returns. The returns on government bonds (safe assets) were positive, while the stock returns (risky asset) were negative. During such instances, there was also deterioration in market liquidity and an increase in volatility (See Figure 1.7 on page 19). It is such episodes that are commonly referred to as Flight to Safety (or FTS). However, the economic literature also uses FTS as an inclusive term for two other phenomena, *viz. Flight to Quality* (FTQ) and *Flight to Liquidity* (FTL), where the key difference between these two terms results from the underlying preference (whether for liquidity or quality) of investors that has motivated them to re-balance their portfolio from risky to safe assets.

Precautionary motives and risk aversion

A typical FTS episode is signified by a sudden increase in appetite for safe assets with respect to risky assets. The idea that during times of uncertainty, economic agents change their behaviour, by exhibiting caution towards excessive consumption and increase their savings, is quite old and one of the defining reasons for the study of macroeconomics. Agents that are uncertain about their future income and employment, exhibit precautionary behaviour to increase savings today to smooth out their consumption path and ameliorate the impact of realisation of a bad state in future. Modern understanding of the precautionary motives often refers to discussion in Keynes (1936), however there is an even earlier precedent in Marshall (1890), "The thriftlessness of early times was in great measure due to the want of security that those who made provision for the future would enjoy it".

Theoretical macro-finance literature [pioneered by Bernanke, Gertler, and Gilchrist (1996), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997)], demonstrates the impact of small shocks on the macro economy. Persistent effects from these shocks can permanently damage agents' net worth through a drop in prices of assets they hold. Moreover, it feeds into a feedback loop where the fall in prices amplifies the initial mechanism and reduces agents' net worth even further.

Figure 1.7: Conditional volatility in financial markets



Notes: Long run and Short run (right axis, in all charts) conditional volatility, correlation and covariance for daily returns (%) of S&P 500 and US 10-year Treasury bonds. The long run variances are calculated by using a backward looking kernel of 250 days, whereas short run variances are calculated by backward looking kernel of 5 days. See Appendix A.2.1 for a detailed methodology.

A rise in precautionary motives and increased risk aversion reduces investment demand and fuels deflationary pressure, which can be self-fulfilling. As per Brunnermeier and Sannikov (2014) when prices drop in a crisis due to higher expected returns, agents hold on to cash for buying assets later at fire-sale prices, which elongates periods of low growth and asset misallocation.

The theoretical asset pricing literature on investor related Flight to Safety is pioneered by Vayanos (2004). In his model, fund managers face a funding constraint in the form of the likelihood of withdrawal of managed funds by individuals that are investing in these funds. The fund managers' funding constraint depends on the fund (i.e., whether it outperforms a threshold set by investors). This funding constraint also evolves endogenously on the level of market liquidity and volatility. During volatile times, there is an increase in probability with which the fund's performance could fall short of investors' threshold set. In volatile periods, this feature increases fund managers' risk aversion and leads to a preference for more liquid (FTL) and safer (FTS) assets.

Financial constraints based models in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), show that the amplification from small shocks grows for these financially constrained agents when their wealth is at a distance from the steady-state. When the endogenous risk is driven by financial constraints instead of being driven by fundamentals, it increases agents' precautionary savings motive. Precautionary demand and savings are shown to play an essential role in other industries as well. Empirically Kilian (2009) decomposes oil prices and addresses that from 1975-2007 precautionary demand shocks, which is uncertainty about future supplies, had an immediate and persistent impact on oil prices. Whereas during that period, the contribution to oil prices from disruptions due to supply shocks had been often short lived, and contribution from global demand shocks had been persistent but somewhat delayed.

Wealth preservation and liquidity preference are other two commonly studied motives for FTS. Boucher and Tokpavi (2019) study whether (high or low) bond yield environment affects the strength of these motivations. In their quantile regression model, an environment of low bond yields, expansionary monetary policy, and low inflation jeopardize the well-known diversification benefits of holding US treasury bonds. There is a decrease in the traditional Flight to Safety channel's strength, i.e., switching investments from equity to bond. It gets substituted by a pick up in the strength of Flight to Safety from equity to other safe-haven assets viz. Gold, Japanese Yen (JPY) and Swiss Franc (CHF).

The CAPM literature establishes FTS as the joint occurrence of higher economic uncertainty, low cash flows, high risk premium, high precautionary savings motive and low real rates. FTS episode in asset pricing literature results from the precautionary response of investors to changes in economic conditions. When uncertainty (quantity of risk) or stock market volatility increases, then the agents, investment managers, speculators in asset pricing models become more risk-averse and prefer safe and high-quality assets.

Liquidity, volatility and market externalities

There is a macro, asset pricing, and CAPM literature pattern that the Flight to Safety mechanism is linked with precautionary behaviour and changes in risk aversion of household/investor. But other theoretical studies have explored FTS in relation to speculation, liquidity, and market volatility. Acharya and Pedersen (2005) study the relation between liquidity shocks and adverse economic and financial episodes. Illiquid securities in their results have a higher sensitivity to market liquidity and market returns. Stocks with higher liquidity risk on average command an additional 1.1% risk premium.

Previously observed episodes of Flight to Quality, including Penn central transportation company default of 1970, Crash of 1987, Russian default, the Asian crisis, Bailout of LTCM, and events following attacks of 9/11 were not just instances of capital/liquidity shortages. These episodes also witnessed disengagement by investors from risky activities. In the words of Caballero and Krishnamurthy (2008), these were instances of Knightian uncertainty or immeasurable risk. Their model demonstrates that agents and intermediaries faced with an increase in Knightian uncertainty or liquidity shortages give more consideration to the worst possible outcomes. Such agents become self-protective and conservative in the allocation of risk capital, thus rendering capital markets inflexible. A resolution to this situation exists in the form of a massive policy involvement from the government acting as lender of last resort and transferring some of its collateral, trust, and liquidity to the distressed capital market (Caballero and Kurlat, 2009).

Speculators during FTS suffer from the destabilizing effect of margin requirements (Brunnermeier and Pedersen, 2008). When speculators' funding liquidity is tight and has higher margin requirements, they provide more liquidity for assets with low margin requirements (and are safer). This generates FTS and co-movement of risk premiums in capital markets. In Brunnermeier and Pedersen (2008), high market volatility creates a differential between the liquidity of safe and risky stocks. In other related studies, there is a deterioration of aggregate liquidity due to either lower net worth of intermediaries (Adrian and Shin, 2010), or effect of regulation on intermediary balance sheets (Adrian, Boyarchenko, and Shachar, 2017), or dynamic adverse selection (Guerrieri and Shimer, 2014). When combined with uncertainty averse behaviour of agents, any of these could lead to portfolio reallocation from risky to safe assets, or FTS.

The empirical literature mainly studies the FTS as a high-frequency event and focuses on its impact on returns, liquidity and volatility of the asset markets (Acharya, Pedersen, Philippon, and Richardson, 2016). The distinction between liquidity profile of corporate bonds impacts the way different bonds react to FTS (Acharya, Amihud, and Bharath, 2013). There is a notable reduction in illiquid or stress regimes' asset prices, particularly for less liquid assets. The prices for investment-grade US corporate bonds are less affected than those of speculative (junk) grade bonds. A similar effect is visible in stocks with low book-to-market value ratio.

Baur and Lucey (2009) find that FTS episodes are a regular occurrence in crisis periods and show contagion for eight developed countries that they study. FTS demonstrates the resilience and diversification benefits of financial markets for investors, as it shows that there is an asset class that can absorb excess capital in times of crisis. Markets lacking an FTS absorbing asset class suffer more significant losses and are less resilient in a crisis period. Using daily data for 23 countries, Baele, Bekaert, Inghelbrecht, and Wei (2013) further provide some stylized impact of FTS. FTS phenomenon coincides with an increase in the Volatility Index (VIX), a decrease in consumer sentiment, a higher preference for holding safe-haven assets (Gold) and currencies (Japanese Yen JPY, and Swiss Franc CHF), and a rise in the TED spread, which is the difference between the 3-month Treasury bill and the 3-month US dollar LIBOR. Boudry, Connolly, and Steiner (2019) provide the impact of FTS on high-frequency responses such as short-term liquidity and returns, and the low-frequency response variables such as long term revenues and valuation, of the commercial real estate industry. Brunnermeier and Nagel (2008) demonstrate that individual investors do not depict any portfolio reallocation towards more risky assets as their wealth increases; instead, they re-balance slowly following capital gains/losses. The regression of daily returns of Gold prices on stock and bond returns of UK, US, and German markets, in Baur and Lucey (2010) demonstrates that Gold has short-lived safe haven (safer asset) properties during extreme stock market events. Whereas in normal times, Gold is a hedge (uncorrelated) to stock markets.

Sign restrictions and expectations dynamics

Cochrane (2017) finds that changes in economic fundamentals lead to FTS, and it has a long-run impact on the asset valuations and macroeconomic variables. Despite that, there is no empirical study explaining the impact of FTS on short and long-term fundamentals of the economy. One possible reason for this could be the difficulty in separating the endogeneity of business cycles to FTS. Boucher and Tokpavi (2019), is a notable exception, as they find that the strength of FTS from stocks to bonds weakens as the interest rate on government bond maturities fall towards ZLB. They highlight the endogenous mechanism, which translates the state of the business cycle and the level of interest rates to the strength and likelihood of the FTS. In their analysis, during a low yield environment, the FTS from stock to bond is substituted by the FTS from stock to other safe-haven assets such as Gold and Swiss Franc. In comparison, this chapter aims to understand both the long and short-term impact of FTS on the macroeconomy, especially the short and long-term impact of FTS episodes on depressing private investments, including residential and non-residential investments, in the US economy.

The sign restrictions based identification used in this paper also complements with significant empirical work done using sign restrictions in identifying the impact of macroeconomic shocks in explaining the business cycles. Previous research has used sign restrictions to identify monetary policy shocks (Uhlig, 2005), government spending shocks (Mountford and Uhlig, 2009), financial shocks [Hristov, Hülsewig, and Wollmershäuser (2012), Gambetti and Musso (2017)], oil price shocks [Baumeister and Peersman (2013), Kilian and Murphy (2012)], technology shocks [Dedola and Neri (2007) and Peersman and Straub (2009)], uncertainty shocks (Bloom, 2014), changes in the volatility from monetary policy shocks (Theodoridis and Mumtaz, 2015), financial and uncertainty shocks (Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek, 2016), news shocks [Beaudry and Portier (2006), Barsky and E. R. Sims (2011), Beaudry and Portier (2014)], and mood swings [Beaudry, Nam, and Wang (2011), Nam and Wang (2019)].

Sign based identifying restrictions command an active space in empirical research and has kept pace with new techniques, such as using agnostic priors (Arias, Rubio-Ramírez, and Waggoner, 2018) and in combination with Bayesian FAVARs (Ahmadi and Uhlig, 2015). Sign restrictions have been useful in enhancing our understanding of macroeconomic shocks and their propagation, and this paper contributes to its growing arsenal.

A related view of this chapter's risk-driven business cycle hypothesis is the news-driven business cycle hypothesis (Beaudry and Portier, 2006), which posits that booms and bust cycles can arise from better or worse expectations about future fundamentals. According to the news-driven business cycles hypothesis, favourable news about future factor productivity can ignite a boom today via creating an incentive to accumulate and install new capital for future demand. In contrast, a less than expected realization of total factor productivity today could lead to bust even without any actual fall in total factor productivity or changes in fundamentals.

The empirical evidence supported in VAR based identification schemes in [Beaudry and Portier (2006) and Beaudry, Dupaigne, and Portier (2011)] backs this claim, and noise shocks, i.e., misinformation about future total factor productivity, are related to positive co-movement in macroeconomic aggregates observed in data. Jaimovich and Rebelo (2009) provides an alternate viewpoint that positive news may reduce the incentive to accumulate capital and instead favour an increase in utilization of capital. Increased capacity utilization in itself leads to a boom. However, the news-driven hypothesis is questioned under a different identification scheme (Barsky and E. R. Sims, 2011). The impact of noise shocks is similar to that of a standard DSGE model. A better than expected total factor productivity leads to an increase in consumption but a decline in output, hours, and investment. The VAR-based evidence of news based literature is criticized (Arias, Rubio-Ramírez, and Waggoner, 2018) for its non-fundamental, non-invertibility, the sensitivity of cointegration assumptions, and choice of variables in the system. If stock prices are not included in the system, the results do not confirm news shock driven business cycles.

This chapter proposes another view to this debate that it is not noise shocks to news about future total factor productivity. Instead, investor risk aversion shocks lead to booms and busts in the business cycles. The Flight to Safety shocks identified in this analysis lead expectation formation and TFP changes by much longer periods than postulated by either news shocks or uncertainty shocks. The macroeconomic theory-based identification employed in the benchmark analysis is robust to alterations in sign restrictions. The fact that other time series can account for the Flight to Safety phenomenon can also produce results comparable to the benchmark analysis, reassuring our faith in this analysis's results. The significant results are not entirely dependent on the inclusion of the singular price of risk series. In this manner, we avoid some criticism levied on the replicability of news shocks results with other series. Moreover, it appeals to the universality of the cyclical risk aversion phenomenon captured in Flight to Safety.

As global financial markets have grown increasingly more interconnected, the last couple of decades have seen FTS episodes (See figures A.4 and A.5 in Appendix A.2.3) occurring more frequently and commonly around the world. Baele, Bekaert, Inghelbrecht, and Wei (2013) report 2.7% FTS days in the US, and similar instances of FTS in 23 other countries.

Economic literature witnessed an increase of interest in the phenomenon of Flight to Safety. However, it is still fledgling, and its stylized facts are being developed. More importantly, there is a lack of understanding about how Flight to Safety shocks would work in the context of standard business cycle models. This chapter embarks on a step in this direction by attempting to identify Flight to Safety shocks and their business cycle impact.

Table 1.1: Data series used in VAR models

Series	**Source Code**	**Definition**
Output	GDP	Gross domestic product
Disp. Income	DPIC	Disposable Personal Income
Consumption	PCEC	Personal Consumption Expenditures
Durable	PCDG	PCEC: Durable Goods
Non-Durable	PCND	PCEC: Non durable Goods
Services	PCESV	PCEC: Services
Investment	GPDI	Gross Private Domestic Investment
Res Investment	PRFI	Private Res. Fixed investment
Non-Res Investment	PNFI	Private Non-Res. Fixed investment
Gov CI	GCE	Govt. Consumption Exp. & Gross Investment
Policy rate	FEDFUNDS	Effective Federal Funds Rate
CPI	CUSR0000SA0	Consumer prices - Urban
Participation	CIVPART	Labour force participation ratio
Vacancy rate	composite HWI	Help-Wanted Index*
Unemployment rate	UNEMP	Unemployment rate
Wages per hour	$\frac{\text{A576RC1}}{\text{TOTLQ}}$	$\frac{\text{Compensation}}{\text{Hours}}$ from Non-farm payrolls
Hours	NFBUS	PNFS (excl. nonprofit): Hours
Employees	NFBUS	PNFS (excl. nonprofit): Employees
TFP	dtfp_util	Utilisation-adj TFP#
TFP(EqDur)	dtfp_I _util	Utilisation-adj TFP Equipment and Durables#
Rel. px Cons to Equ	relativePrice	Rel. price Consumption to Equipment
Disp Inc / capita	A229RC0Q0-52SBEA	Disposable Income per capita
Capex	BOGZ1FA38-5050005Q	Capex Domestic non-fin. sectors
Capacity Util	TCU	Capacity Utilisation: Total industry
R&D expenditure	Y694RC1Q0-27SBEA	GDP: Research and Development
Foreign portfol. flows	Equity & Debt	Cumulative flows (+ inflow, - outflow)
Uncertainty	EPU	Economic policy Uncertainty§
Inv share	invShare	Equipment and Cons Dur share of output
Cons.Sentiment	UMCSENT	Consumer sentiment index
Term spread	DGS10, DGS1	10 year and 1 year Treasury bond
Corp bond spread	DBAA_AAA	Baa minus Aaa yield
Equity	^GSPC	S&P 500 Index

Table 1.1 – continued from previous page

Series	Source Code	Definition
Deflator	DPCERD3Q0-86SBEA	PCEC (implicit price deflator)
Investment deflator	A006RD3Q0-86SBEA	GPDI deflator
Non Durables deflator	DNDGRD3Q0-86SBEA	Non-durables cons deflator
Durables deflator	DDURRD3Q0-86SBEA	Durables cons deflator
Services deflator	DSERRD3Q0-86SBEA	Services deflator

* Barnichon (2010) # Basu, Fernald, and Kimball (2006) § Baker, Bloom, and Davis (2016)

Sources: US FRED, BEA, BLS, IMF, Univ of Michigan, Datastream, FRBSF, Yahoo! Finance

1.3 Data

The source, purpose and definition of US data series that are considered in this empirical exercise are described in Table 1.1 on page 26, while Figure A.6 on page 262 hosts the logarithmic time series plots of those data series.

This empirical study investigates the role of Flight to Safety or Flight to Risk shocks on the business cycle in the US from 1983 Q1 to 2019 Q3. The selected period includes the Great moderation period 1983-2007, which in the macroeconomic history witnessed a reduction in business cycle fluctuations in the developed world, esp. the US. The selection of the beginning date (the year 1983) for the data is made to ignore the impact of Oil shocks in the 1970s and the subsequent change in the Fed's monetary policy stance and communication strategy. The data also accounts for the global financial crisis period and the subsequent recovery period from 2009 to 2019.

The macro time series are obtained from US FRED database of the Federal

Reserve bank of St. Louis. Long-run macroeconomic data studied in this chapter (as produced in Table 1.1 on page 26) are the fundamental series that represent the long term macro-economic health of the US economy at any point in time. These are relevant to our research question of decoding the short and long-run impact of Flight to Safety shocks and Flight to Risk shocks on business cycles.

The empirical analysis is conducted through five variable vector autoregressions, of which the first four key variables are: Total factor productivity, Price of risk (bond minus stock prices), Real rates and Surplus ratio, and the fifth variable is the variable of interest. In the benchmark configuration the variable of interest is Investments. The variable of interest in subsequent iterations of the VAR model is changed to one of the following variables of interest: Hours (total), Output, Disposable Income, Term spread (or difference between 1-year and 10-year T-bill rate), Consumer prices Consumption (Total) and Consumption (only of Non-Durable goods and Services) which for this chapter only is also defined as Consumption Habits.

The series on total factor productivity (tfp) is adjusted for factor-utilisation using Fernald (2012), which follows Basu, Fernald, and Kimball (2006) and its updated version in (Basu, Fernald, Fisher, and Kimball, 2013) with the purpose to create quarterly growth-accounting database for the US business sectors[3] Utilization adjusted tfp series is a better approximation of true technological progress and has been previously used to identify the impact of news and expectations shocks (Jaimovich and Rebelo, 2009). This source also provides for the relative share of investment to output (invshare), the relative price of Consumption to equipment (pxceq), and the utilization adjusted tfp series for equipment and durables (TFP(EqDur)). All of which are considered in the sensitivity and discussion sections of this chapter.

The difference between log of bond prices and the log of equity prices is considered for series on Price of risk. An increase over time in the price of bond minus the price of equity signifies consumers' preference for safer asset vis-a-vis risky asset. Therefore identified positive shocks to this series are characterized as Flight to Safety shocks while identified adverse shocks to this series are Flight to Risk shocks.

There are two ways in which we can get the price of equity, or the risky asset,

[3]See Fernald's website, US Fred pages for data and comparable series of total factor productivity (Fernald 2012).

first, by merely using S&P 500 Composite Index. The data on the adjusted closing price of S&P 500 Composite Index is sourced from Yahoo! finance (series ˆGSPC). An alternate way to construct relative price for risk is to consider the 10-year P/E (price to earnings) ratio of S&P 500, which is available on Robert Schiller's webpage[4], and use it to deduce a ten years earnings yield for S&P 500. Consider this earning yield of S&P 500 as a yield to maturity of a 10-year S&P 500 index and use it to reverse engineer the price for Risky asset (S&P). US 10-year Treasury bond is considered a safe asset, and the data for 10-year nominal bond yield to maturity is sourced from US FRED database (series DGS10). The price for safe asset (10-year T-Bond) is reverse calculated from its yield to maturity.

By taking the difference of log series of nominal bond prices and the log of S&P prices (either through the index value or through 10-year price to earnings yield), a series for Price of risk can be generated. The former approach is considered in the benchmark VAR, and the sensitivity of results to the latter approach is discussed in later sections.

The ex-post real rate of return is given by the Federal funds (nominal) rate minus the US all Urban consumers (indexed 2012 = 100) inflation rate. The term spread is calculated by taking the difference between US 1-year and US 10-year yield, while corporate bond spread (Baa-Aaa) is sourced from Moody's. The surplus ratio is a series developed by considering One minus the ratio of Non-durable and Services consumption to total Consumption.

$$Surplus\ Ratio = \frac{Consumption_{Total} - Consumption_{NonDurables+Services}}{Consumption_{Total}} \tag{1.1}$$

Non-Durable and Services consumption has been considered by many as a measure of consumption habits and as a measure of permanent consumption [See for e.g. Justiniano, Primiceri, and Tambalotti (2010), Campbell and Cochrane (1999) and Cochrane (2017)]. Consumption habits series is developed by adding up personal consumption expenditure on non-durables (PCND) and personal consumption expenditure on services (PCESV) series from the BEA. The ratio of consumption habits to total consumption is a proportion of total consumption that satiates the household's habits or permanent level of consumption. One minus this ratio describes the proportion of consumption in excess of the habits or permanent consumption level. The literature on cyclical risk aversion considers the Surplus ratio as the key component of the household's utility (De Paoli and Zabczyk, 2013). The household feels a tighter pinch, or marginal utility of consumption

[4] http://www.econ.yale.edu/~shiller/data.htm

increases when the excess consumption gets closer to its long-term sustainable part [Cochrane (2017), De Paoli and Zabczyk (2013)]. The results of benchmark 5 VAR model are not sensitive to replacing the Surplus Ratio with Total Consumption.

The other series which replace the key variable of interest in the benchmark VAR are also taken from the US FRED database. The nominal series on Consumption is the real personal consumption expenditure (PCEC) from the Bureau of Economic Analysis (BEA). Durables consumption represents the personal consumption expenditure on Durable goods (PCDG) series of the BEA. Nominal output is measured by the Gross Domestic Product (GDP) of the BEA.

To further investigate the behaviour of the investments sector, this chapter also considers the following variables of interest: Durable goods consumption, Residential investment, Non-residential investment, Research and Development expenditure, Capital Expenditure, Price of Investment in terms of Price of Consumption and the Price of Investment and Consumption of Durable goods in terms of Price of Consumption habits (i.e., non-Durables consumption and services).

The nominal series on Investments is given by Gross private domestic investment (GPDI), the non-residential investment is represented by Private Nonresidential Fixed Investment (PNFI), and the series on Residential investment is Private Residential Fixed Investment (PRFI). The GDP Research and Development expenditure by BEA (Y694RC1Q027SBEA) is used for the R&D series. The above described nominal data series are converted into real data series by deflating using the Personal Consumption Expenditure (PCE) based implicit price deflator (DPCERD, indexed 2012=100). All the above variables are divided by population, which is reverse calculated from the Disposable income (DPI) and Disposable income per capita series (A229RC0Q052SBEA) of the BEA, to get respective real per-capita series that are used in the empirical analysis of this chapter. The relative price ratio of investment to consumption is calculated by using respective implicit price deflators (indexed 2012=100). Similarly, the ratio of the sum of deflators for investment and durables consumption to the sum of deflators for non-durables consumption and services gives the relative price of investment and durables.

The business cycle analysis also considers the insights of labour market variables: Hours (per worker), Output per labour (labour productivity), Labour force participation rate, Unemployment rate, Vacancy rate, and Vacancy to Unemployment ratio. Some other variables considered for robustness are consumer sentiment, capacity utilization, and Foreign portfolio flows.

Labour market indicators of Hours (total) and Hours per employee are taken from the Bureau of Labour Statistics (BLS). Total hours are measured by the hours of all non-farm businesses in the BLS data. Hours per employee are calculated by dividing total hours by the total non-farm business Sector employees. Real wages per hour are generated by deflating the Total Compensation of Employees (Wage and Salary Disbursements) personal income and outlays data (A576RC1) of BEA with the PCE deflator and dividing it by the total number of hours of employees on from non-farm payrolls, from BLS. Labour productivity is calculated by dividing real GDP with total hours. Labour force participation rate (CIVPART) and Unemployment rate series are from the BLS. The Government expenditure is Government consumption expenditure and gross investment (GCE1) data from the BEA and is deflated using the PCE implicit price deflator and divided by population to derive the Government expenditure per capita series of the empirical analysis. Quarterly data on Vacancy rate is from monthly Help-Wanted Index[5] (Barnichon, 2010).

Some other important sub-index series are considered in the empirical analysis. This includes the Total Industry Percent of Capacity Utilisation (TCU) series from the BEA, which is used for the Capacity Utilisation series. Capital expenditures for Domestic non-financial sectors from BEA (BOGZ1FA385050005Q) are used to develop the Capital Expenditure series. The economic policy uncertainty series constructed in (Baker, Bloom, and Davis, 2016) is used as a measure of policy uncertainty. Consumer surveys about consumer sentiment available for a long history from the University of Michigan are included in the analysis.

1.3.1 Downward trend

The Price of risk series as shown in Figure A.6 on page 262 has a downward trend, which shows that the relative price of safe asset to risky asset has been falling. It was higher in the 70s and 80s than it was during the global financial crisis. For the Price of risk series we can say that there are many other factors, besides inflation that manifest in the price of bond and equities responsible for the downward trend. Investors adjust the bond yields[6] to account for various forms

[5]Constructed in 'Building a composite Help-Wanted Index' (Barnichon 2010).

[6] Price of a bond is an inversion of its yield. The analysis that is performed on yields can be translated into prices. In this chapter the nominal price of bond is measured by inverting

of embedded risks and in case of the long-dated bond, the term-premium[7]. Bond yields, therefore, adjust for the changes in safety premium, interest rate premium, reinvestment premium, duration premium, default premium, inflation premium, liquidity premium and event premium. The yields of long-dated (10 years) bond also adjust for the term-premium. All of which would impact the long-term bond yield[8] and the Price of risk series that we have included in this analysis.

Similarly the equity price is impacted by the underlying factors of dividend payments, mergers and acquisitions, reinvestment of capital, productivity increases, market power (domestic and global) and management changes. Besides factors that result in the time-varying nature of investor risk aversions such as increased globalisation and savings glut, they also contribute in uncertain ways to the Price of Risk series. A net effect of these over the years has been that the price of buying a real long term bond has been relatively falling to the real price of a unit

the 10 year bond yield, but I haven't come across any empirical work that works with bond prices by inverting bond yields. Shiller (2015) is an influential study on bond and equity return dynamics but it also uses yields (not prices) to develop a total return index for the long and short term bonds.

[7] Term premium is the amount by which the yield on a long bond is greater than the yield on a shorter dated bond. It is the premium that investors demand for holding a long term bond versus rolling over shorter term bonds for the same holding period. Ever since Fama and Bliss (1987) and Campbell and Shiller (1991), among others, found little evidence to support the Expectations Hypothesis which stated that the forward yield is market's expectation of short term rates in the future, it has become a widespread practise to decompose yields on long term bonds into two components: (i) expectations of future path of short-term interest rates and (ii) the term-premium. Term premium is deemed as a key ingredient in the optimal maturity structure of government debt (Greenwood et al. 2015) and an instrument through which monetary policy can influence long-term yields (Crump et al. 2016). Term-premia [See Kim and Wright (2005), Diebold et al. (2005) and Cohen et al. (2018)] are also the main driver of cross-sectional variation in yields, and contributed to co-movements between the US and the Euro area yields. The general equilibrium models of term-premium [Cox et al. (1985), or Lee (1995)], imply that the term-premium is a function of investors' attitudes toward risk. The term structure of interest rates varies according to market expectations and are very sensitive to the expected future path of growth, inflation, and monetary policy (Kliem at el. 2017), so a macroeconomic approach to understand it is warranted (Koop and Williams 2018).

[8] Common risk factors with respect to bond and bond fund investments, (as per FINRA https://www.finra.org/investors/learn-to-invest/types-investments/bonds/understanding-bond-risk) are as follows. Interest rate risk (or market risk, or holding -period risk) which is the possibility of an increase in interest rates during the holding period that would increase yields and lower the final selling price of the bond. Call risk (or reinvestment risk) which is the risk from an early repayment by the bond issuer. As interest rates fall, the bond issuer has an incentive to reissue at the prevailing low rates and repay the previously issued high interest bonds. Thus leaving the bond investor with no suitable reinvestment opportunities. Duration risk results from the sensitivity of bond prices to interest rate fluctuations. It is a non-linear function of bond's time to maturity. A shorter term bond has lower duration risk than longer dated bonds. Credit risk (or default risk) accounts for the possibility of default by the borrower. Inflation risk also impacts the price of the bond as rising inflation reduces the purchasing power of the returns on bond investments. Liquidity risk is the risk of not being able to find a buyer easily when required. Event risk from significant events such as war, trade-wars, mergers and acquisitions etc. is also a factor for valuing long term bonds.

of US equity index.

The benchmark 10 year treasury bond, whose price is used in the Price of risk series, is updated every year and so only the most liquid and relevant bonds are used. This avoids the changes in duration risk of the bond but it stays exposed to changes in interest rate risk, inflation risk and term premium. The equity index which is a market capitalisation based index is also regularly updated and that leads to a survivorship bias which means only the successful and large companies remain, while the failed ones are weeded out. The regular upkeep of the benchmark bond and equity series, keeps them relevant but exposes them to other biases. An ideal experiment would be to decompose each of the individual price effects of bond and equity prices and only compare them on the basis of safety premium. However in absence of such a natural data decomposition, the Price of risk series that we have considered in this chapter is a respectable alternative. The series could deviate from trend for the reasons that are peculiar to either bond or equity prices, but it would also deviate due to Flight to Safety and Flight to Risk. By way of the identification strategy employed in this chapter I present evidence that changes in individual risk aversion are a major source of such deviations.

1.3.2 Non-stationarity and Structural VAR

The variables considered in the VAR except unemployment rate, cpi and yield spreads, display a trend and are integrated of order one (see results in Table 1.2). The macro variables can be detrended by removing a deterministic trend or by first differencing before conducting OLS, otherwise one runs into the problems of spurious regressions, where the econometric procedure indicates relationship between two variables when none may exist. In that case single or joint hypothesis tests to examine the statistical significance of the coefficients and their standard errors are biased and the residuals violate the Gauss-Markov assumptions of no heteroscedastic and no auto-correlation. Therefore it is recommended especially while conducting unrestricted VAR and when working to devise point forecasts and causality that all of the components in the VAR have at least weak stationarity, that is they don't have time-varying first and second moments. But the spurious regression problem does not apply if variables are cointegrated[9] with one another.

[9]Two variables are said to be cointegrated if they are each unit root processes, but if a linear combination of them is stationary.

In such a case first differencing is not appropriate[10]. C. A. Sims (1980b), J. Stock, C. A. Sims, and Watson (1990) and Luetkepohl (2011) and others[11] have suggested that a way to avoid the problems of spurious regressions without first differencing is to obtain the structural VAR with higher number of lags, and that the forecast errors, historical decomposition, impulses responses and Granger causality based results are valid even for integrated representations. Therefore the structural analysis employed in this chapter avoids spurious regression by using 4-lags, imposing sign based restrictions and developing robust standard errors. Alternatively if the variables are co-integrated the Vector-error correction mechanism as presented in EC-VAR of Bansal et al. (2007, 2011) could be followed. They demonstrate that as cash-flow and consumption series are cointegrated, an error correction VAR describes a completely different optimal portfolio allocation, in particular for long term horizon, than a commonly used VAR approach. The return betas from their EC-VAR account for cross-sectional variation in equity returns, which is not the case if cointegration is ignored.

The next challenge is to devise an identification scheme to overcome the endogeneity in macroeconomic variables and other shocks that generate FTS behaviour. We answer these questions by isolating the business cycle and monetary policy shocks and only considering the FTS shocks that are orthogonal.

[10]Chris Brooks in his (2019) textbook Introductory Econometrics for Finance says, ".....many proponents of the VAR approach recommend that differencing to induce stationarity should not be done. They would argue that the purpose of VAR estimation is purely to examine the relationships between the variables, and that differencing will throw information on any long-run relationships between the series away" .

[11]Eric Sims proposes, "Differencing often feels like the right thing to do, but can result in serious mis-specifications if variables are cointegrated. Estimating in levels (provided there are lags of the dependent variable on the right hand side, which takes care of the spurious regression problem) is always safer" . Source: Eric sims' lecture notes `https://www3.nd.edu/~esims1/time_series_notes_sp13.pdf`.

Table 1.2: Unit root test results of various series

	Test Statistic	p-value	#Lags	#Obs	Critical values 1%	5%	10%
Util-Adj TFP	-0.801	0.819	0	146	-3.476	-2.882	-2.577
Price of risk	-1.204	0.672	3	143	-3.477	-2.882	-2.578
Real rate(%)	-2.086	0.25	11	135	-3.48	-2.883	-2.578
Consumption	-1.261	0.647	6	140	-3.478	-2.882	-2.578
Hours	-1.251	0.651	5	141	-3.478	-2.882	-2.578
Output	-1.22	0.665	2	144	-3.477	-2.882	-2.578
Investment	-1.967	0.301	2	144	-3.477	-2.882	-2.578
Cons (NDur+Svc)	-1.119	0.708	3	143	-3.477	-2.882	-2.578
Hrs per emp	-1.417	0.574	4	142	-3.477	-2.882	-2.578
Outp per Emp	-0.493	0.893	2	144	-3.477	-2.882	-2.578
CPI	-4.209	0.001*	0	146	-3.476	-2.882	-2.577
Surplus ratio	-0.877	0.795	3	143	-3.477	-2.882	-2.578
Participation rate(%)	-1.515	0.526	10	136	-3.479	-2.883	-2.578
Unemployment rate(%)	-2.762	0.064*	9	137	-3.479	-2.883	-2.578
Disp Income	-1.557	0.505	2	144	-3.477	-2.882	-2.578
Inv + Durables Cons	-1.472	0.547	3	143	-3.477	-2.882	-2.578
Rel.Px Inv	-3.711	0.004*	0	146	-3.476	-2.882	-2.577
Rel.Px Inv+DurC	-1.233	0.659	1	145	-3.476	-2.882	-2.578
Res Investment	-2.855	0.051*	5	141	-3.478	-2.882	-2.578
Non-Res Investment	-1.75	0.406	2	144	-3.477	-2.882	-2.578
Gov. Expenditure	-1.727	0.417	4	142	-3.477	-2.882	-2.578
R & D	-0.672	0.854	2	144	-3.477	-2.882	-2.578
Capex	-1.168	0.687	4	142	-3.477	-2.882	-2.578
1y-10y spread(%)	-3.58	0.006*	3	143	-3.477	-2.882	-2.578
Corp bond spread(%)	-4.744	0*	1	145	-3.476	-2.882	-2.578
Util-Adj TFP(Eq.Dur)	0.025	0.961	4	142	-3.477	-2.882	-2.578
Equity px	-1.632	0.467	1	145	-3.476	-2.882	-2.578
Bonds px	-3.006	0.034	10	136	-3.479	-2.883	-2.578
TR 10y-earn px	-3.352	0.013*	7	139	-3.478	-2.883	-2.578
10y-Cape px	-3.489	0.008*	3	143	-3.477	-2.882	-2.578
CAPE	-2.72	0.071*	1	145	-3.476	-2.882	-2.578
LT 10y-earn px$^{\#}$	-1.527	0.520	1	114	-3.489	-2.887	-2.580

Notes: ADF unit root test results for data series used in S-VAR. The null hypothesis of the test is, H_0 : The series has Unit root.* indicates results that are significant at 90%. $^{\#}$ series is available for period 1983-2011, all other results are for 1983-2019. Maximum 12 lags are considered for each series. #Lags describes the lag order which gives the best result to reject the Null hypothesis. #Obs is the number of observations left after accounting for lags.

1.4 Empirical analysis

This section describes the sign restrictions methodology that is adopted in this chapter to identify Flight to Safety shocks in the US. Sign and zero restrictions are derived from Uhlig's agnostic identification (Uhlig, 2005) and the methodology below is also supported by additional derivations presented in Appendix A.1. After describing the identification methodology, the section also presents the identifying assumptions and the theoretical rationale behind choosing them.

1.4.1 Methodology for identifying Flight to Safety shocks

The reduced form VAR and Uhlig's agnostic identification: Consider a VAR model in reduced form as in Uhlig (2005)

$$\boldsymbol{Y_t} = c_t + \sum_{k=1}^{p} \boldsymbol{B}_{(k)} \boldsymbol{Y_{t-k}} + \boldsymbol{u_t} \tag{1.2}$$

where $\boldsymbol{Y_t}$ is an m x 1 vector of endogenous variables, at date $t = 1, ...T$, $\boldsymbol{B}_{(k)}$ are reduced-form square coefficient matrices of size m x m and $\boldsymbol{u_t}$ is m x 1 one-step prediction error with m x m variance-covariance matrix $\boldsymbol{\Sigma}_u$. This reduced form VAR has a moving-average representation,

$$\boldsymbol{Y_t} = c_t + \sum_{h=0}^{\infty} \boldsymbol{\Phi}_{(h)} \boldsymbol{u_{t-h}} \tag{1.3}$$

where $\boldsymbol{\Phi_{(0)}} = I_m$. For simplification we can ignore the c_t term which comprises of intercept and/or time trend. This is mostly an agreeable representation of the reduced form VAR. The challenge in identification lies on how we decompose the forecast error $\boldsymbol{u_t}$. If we assume that there are m fundamental, mutually independent innovations, i.e. one shock for each endogenous variable, which are normalised to be of variance 1. They can be represented by a m x 1 vector v such that

$$E[vv'] = \Sigma_v = I_m. \tag{1.4}$$

The identification problem is then restricted to finding a linear mapping between reduced form innovations $\boldsymbol{u_t}$ and structural shocks v_t. The mapping is represented by an unknown matrix $\boldsymbol{A}$ such that

$$\boldsymbol{u_t} = \boldsymbol{A} v_t \tag{1.5}$$

The jth column of $\boldsymbol{A}$ represents the impact on all variables of VAR of the jth fundamental innovation. However identifying such an $\boldsymbol{A}$ matrix is not straightforward as the variance-covariance matrix of $\boldsymbol{u_t}$ which now can be written as

$$\boldsymbol{\Sigma}_u = E[\boldsymbol{uu'}] = \boldsymbol{A}E[vv']\boldsymbol{A'} = \boldsymbol{AA'} \tag{1.6}$$

also holds true for $\tilde{\boldsymbol{A}}$ which is any arbitrary orthogonalisation of Σ_v . For instance consider a Cholesky decomposition of $\Sigma_v = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{A}}'$ and some orthonormal matrix P such that

$$\boldsymbol{A} = \tilde{\boldsymbol{A}}P \tag{1.7}$$

$$\boldsymbol{AA'} = \tilde{\boldsymbol{A}}PP'\tilde{\boldsymbol{A}}' = \tilde{\boldsymbol{A}}\tilde{\boldsymbol{A}}' \tag{1.8}$$

Therefore the identification of the structural shocks v_t requires us to pin down the orthonormal matrix P, which means there are another $m(m-1)/2$ restrictions required to achieve this identification. Commonly used procedures in this identification are recursive ordering of variables (C. A. Sims, 1986), breakup of components into transitory or permanent (Blanchard and Quah, 1989), structural relations between fundamental innovations and prediction errors (Bernanke and Mihov, 1998), and sign restrictions [Uhlig (2005) and Mountford and Uhlig (2009)]. I use Uhlig (2005) sign-restriction based agnostic identification method to find the effects of FTS on the economy. A clear advantage of the approach is that sign-restriction limits identification exercise to only k shocks of interest, and the other $m-k$ fundamental innovations can be ignored. By not imposing any sign restrictions on the response of variables of interest, the procedure remains 'agnostic' (in the manner of Uhlig, 2005) with regards to these variables and can be used in finding the effect of shocks on such variables. Following from the equations (1.3) to (1.8) we can deduce the structural VAR in moving average form as

$$\boldsymbol{Y_t} = c_t + \sum_{h=0}^{\infty} \boldsymbol{\Psi_{(h)}} v_{t-h} \tag{1.9}$$

where $\boldsymbol{\Psi}_{(h)} = \boldsymbol{B}_{(h)}P$, $\boldsymbol{B}_{(h)} = \boldsymbol{\Phi}_{(h)}\tilde{A}$ and v_{t-h} are the structural shocks. The impulse response of jth shock at horizon $h \in h^-, ..., h^+$ is given by the jth column $\boldsymbol{\Psi}^j_{(h)}$ of matrix $\boldsymbol{\Psi}_{(h)}$:

$$\boldsymbol{\Psi}^j_{(h)} = \boldsymbol{B}_{(h)}p^j \tag{1.10}$$

where p^j is the jth column of P. The response of the ith element of the system is thus given by the ith element on the impulse response vector

$$\boldsymbol{\Psi}^{i,j}_{(h)} = \boldsymbol{B}^{i,}_{(h)}p^j \tag{1.11}$$

$\boldsymbol{B}^{i,}_{(h)}$ is the ith row of $\boldsymbol{B}_{(h)}$ and p^j is the jth column of P. Thus sign and zero restrictions can be applied to response $\boldsymbol{\Psi}^{i,j}_{(h)}$ for some horizons h to identify any jth structural shock. It follows from Uhlig (2005) that the problem comes down to identifying the unit vector p^j which comes closest to meeting the sign and zero restrictions (See Appendix A.1).

The identification can be achieved by either selecting from Markov chain Monte carlo (MCMC) simulation a certain number of impulse responses that meet our pre-determined sign restrictions or by choosing a Penalty (or criterion) function $\Theta(p)$ which increases in the size of violation by impulse response to the selected sign and zero restrictions. For ease of notation I am dropping the j superscript in the equations that follow from here on. As discussed in Mountford and Uhlig (2009), in the Penalty function approach, selecting between sign restrictions then becomes a minimisation problem that solves,

$$p^* = \underset{p}{argmin}\ \Theta(p) \text{ s.t. } p'p = 1, \tag{1.12}$$

for the Penalty function,

$$\Theta(p) = \sum_{i \in i^+} \sum_{h=h^-}^{h^+} f\left(-\frac{\boldsymbol{B}^{i,}_{(h)} p^j}{\sigma_i}\right) + \sum_{i \in i^-} \sum_{h=h^-}^{h^+} f\left(\frac{\boldsymbol{B}^{i,}_{(h)} p^j}{\sigma_i}\right) \tag{1.13}$$

where i^+ is the set of variables whose impulse responses $\boldsymbol{B}^{i,}_{(h)} p^j$ are set to be positive and i^- is the set of variables whose impulse responses are set to be negative for the horizon $h \in h^-, ..., h^+$. Any zero restrictions on impact of the variable ordered zth in the VAR system, can be included in the minimisation problem by adding an additional constraint on the unit vector p as:

$$p^* = \underset{p}{argmin}\ \Theta(p) \text{ s.t. } p'p = 1 \text{ and } R_{zero} p = 0 \tag{1.14}$$

where R_{zero} is zth row of $\boldsymbol{B}_{(0)}$, i.e. $R_{zero} = \boldsymbol{B}^{z,}_{(0)}$. The minimisation is solved in Matlab using Simplex and inbuilt methods, and the estimation is conducted using Bayesian procedures as in Uhlig (2005).
The algorithm for both MCMC and Penalty based methods is thus given by:
1. Draw from Normal-Wishart prior.
2.a. For a given draw either solve (1.14) to get a candidate solution for p^* , or
2.b. Apply the stereographic inversion on the candidate solution to solve other constraints, i.e. unit length and meeting Zero restrictions.
3. Obtain statistical inference on the basis of draws that solve step 2.

Table 1.3: Identification restrictions in S-VAR

variables / *shocks*	TFP	Price of Risk	Real rate	Surplus ratio	Variable of interest
Strategy 1					
Productivity	+	.	−	+	.
Flight to safety	0	+	+	−	.
Monetary policy	0	.	+	−	.
Demand	.	.	.	.	.
Residual	.	.	.	.	.
Strategy 2					
Productivity	+	.	−	+	.
Flight to safety	0	+	+	0	.
Monetary policy	0	.	+	0	.
Demand	.	.	.	.	.
Residual	.	.	.	.	.
Strategy 3					
Productivity	+	.	−	+	.
Flight to safety	0	+	.	0	.
Monetary policy	0	.	+	0	.
Demand	.	.	.	.	.
Residual	.	.	.	.	.

Notes: The benchmark VAR has 5 variables: TFP, Price of risk (which is Bond price minus Equity price), Real rate, Surplus ratio (which is One minus the ratio of Consumption of services and Non-durables to Total consumption), and a variable of interest which in the benchmark case is Investments but is replaced in different iterations by other business cycle variables for e.g. Output, Consumption, Hours, CPI etc. On impact of the respective shock, + means the variable is restricted to be positive and − means that the variable is restricted to be negative for the impact horizon. The impact horizon is shock period plus one more period. Symbol 0 signifies a zero restriction when the response variable is restricted to not respond to the shock contemporaneously, and Dots . signify that the impact response of that variable is left unrestricted.

1.4.2 Sign and Zero restrictions based identification

The benchmark 5-variable VAR model consists of: Utilisation-adjusted Total factor productivity, Price of risk, Real rate, Surplus ratio and the variable of interest which in the benchmark configuration is Investments. In other iterations of the model the variable of interest is replaced by: Hours, Output, Consumption, Disposable income, Term spread between 1-year and 10-year T-Bill rate, CPI and Habits. Residential investments, Non-residential Investments, Disposable income, Wages (per hour), consumption of non-durables and services, Foreign portfolio flows, Government expenditure, Labour productivity. The structural model is given by:

$$\boldsymbol{Y_t} = c_t + \boldsymbol{B}_{(0)} p v_t + \boldsymbol{B}_{(1)} p v_{t-1} + ... + \boldsymbol{B}_{(\infty)} p v_{t-\infty} \tag{1.15}$$

where

$$\boldsymbol{Y_t} = \begin{bmatrix} \text{TFP,} \\ \text{Price of risk,} \\ \text{Real rate,} \\ \text{Surplus ratio,} \\ \text{Investment} \end{bmatrix} \tag{1.16}$$

and

$$v_t = \begin{bmatrix} \text{TFP shock,} \\ \text{FTS shock,} \\ \text{Policy shock,} \\ \text{Demand shock,} \\ \text{Residual shock} \end{bmatrix} \tag{1.17}$$

Three different identification strategies are adopted (See Table 1.3 on page 39) for considering impact, impulse response restrictions from FTS shock on the first 3 or 4 variables of the benchmark 5 variable VAR. The impulse responses from the 5th variable or the variable of interest in all strategies are left unrestricted. In this manner, the impact analysis is agnostic to the effect on the variable of interest. The identification strategies (in Table 1.3) are a set of sign and zero restrictions that are supported by theoretical models of the business cycles and by observations from data.

A prominent model that has been used in deriving sign restrictions in this chapter is the NK (New Keynesian) or NNS (New Neoclassical synthesis) Smets and Wouters (2007) model that is consistent with the balanced steady state growth path and is estimated by them using Bayesian methods. Their model has 7 orthogonal structural shocks in: total factor productivity, risk premium shocks, investment technology shocks, wage shocks, price markup shocks, exogenous spending (policy) shocks, and monetary policy shocks. It also incorporates many relevant policy analysis features, such as labour augmented technological progress, investment adjustment costs, variable capacity utilization, and real rigidity in intermediate goods and labor markets.

Several other interesting features of the Smets and Wouters (2007) model are as follows. Household chooses over consumption and labour effort. Labour is differentiated by a union, which gives monopoly power over determination of wages, which are Calvo-style sticky. Consumption habits are exogenous. Households make decisions to rent and accumulate capital based on the rental rate and capital adjustment costs. Firms decide on producing differentiated products and set prices based on present and expected marginal costs (wages and rental rate of capital), and past and expected inflation. Wages depend on past wages and future inflation. The medium-scale of this model, its micro-theoretic foundations and relevance of the variables it considers with the business cycle phenomenon, all together make it a standard workhorse model of monetary policy analysis and a reliable resource to base the identifying restrictions of the empirical study of this chapter.

The results of Smets and Wouters (2007) are consistent with the great moderation period 1984 to 2004 and demonstrate a fall in the volatility of shocks (related to total factor productivity, monetary policy, and price markup), the volatility of output growth and inflation, and the sensitivity of response of output variables to monetary policy shocks. They also show that during the great moderation, the monetary policy response to output changes has slowed. The reaction of policy to inflation has slightly increased, but the output gap's reaction has reduced by half. The sensitivity of their results to investment adjustment costs and consumption habits is high. Investment shocks result in the hump-shaped responses in output, hours, inflation, and interest rates.

The sign restrictions in this paper are based on the key results of Smets and Wouters (2007). In particular that technology shocks lead to a fall in nominal and real interest rates. However, for the monetary policy reaction function that they estimate, this fall is not sufficient to prevent a drop in inflation and an opening of the output gap. Risk premium shocks result in a fall in output, hours, and an increase in the real interest rate. Furthermore, monetary policy shocks lead to an increase in interest rate, real interest rate but decrease output, inflation, and hours. The sign restrictions are applied for a horizon of shock period and the next period, while the zero restrictions are applied for the shock period itself. These restrictions are represented in the $\boldsymbol{B}_{(0)}$ and $\boldsymbol{B}_{(1)}$ matrices, which are discussed next for the three different identification strategies.

1.4.2.1 Identification strategy 1

Identification strategy 1 (See Table 1.3 on page 39), applies zero impact restriction on TFP from monetary policy shock and Flight to Safety shock. Recall, that the ordering of the 5 variable VAR in benchmark configuration is: TFP, Price of risk, Real rate, Surplus ratio and Investment. Therefore on applying the zero restrictions the impact matrix $\boldsymbol{B}_{(0)}$ becomes

$$\boldsymbol{B}_{(0)} = \begin{bmatrix} . & 0 & 0 & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \tag{1.18}$$

where dots (.) symbolise entries that remain unrestricted. On including sign restrictions given in Identification strategy 1 (see Table 1.3 on page 39) the $\boldsymbol{B}_{(0)}$ and $\boldsymbol{B}_{(1)}$ matrix are modified to

$$\boldsymbol{B}_{(0)} = \begin{bmatrix} + & 0 & 0 & . & . \\ . & + & . & . & . \\ - & + & + & . & . \\ + & - & - & . & . \\ . & . & . & . & . \end{bmatrix} \quad \text{and} \quad \boldsymbol{B}_{(1)} = \begin{bmatrix} + & . & . & . & . \\ . & + & . & . & . \\ - & + & + & . & . \\ + & - & - & . & . \\ . & . & . & . & . \end{bmatrix} \tag{1.19}$$

The sign restrictions in 5 variable VAR are implemented for a maximum horizon of one period after the shock, i.e. the restrictions are valid for the shock period and one period after. Any zero restrictions are imposed only on the impact period.

Identification strategy 1: restrictions to TFP shock In the 5 variable benchmark VAR model, Identification strategy 1 imposes negative sign restrictions from TFP shock (the first shock in VAR) on Real rates (the third variable in VAR) and imposes a positive restriction on Surplus ratio (the fourth variable in VAR). In models of business cycle, an increase in technological progress is expected to increase the household's desire for consumption and leisure. If we assume that consumption habits are slow to change, then it follows that an increase in household's discretionary consumption would raise the surplus consumption and the surplus ratio. However, the impact of real rates from a TFP shock is not that unambiguous. A positive TFP shock leads to decrease in real interest rate. This is consistent with the logic of standard Euler equation, where the current period real interest rates is inversely related to current consumption. For the policy maker with a dual mandate of price and output stability, in standard forward looking 3-equation New Keynesian (NK) model[12], it is optimal to consistently set interest rates equal to natural rate of interest. Therefore a TFP shock in models of price-rigidity warrants a cut in real rates to keep the output gap closed.

The literature based evidence points to a fall in natural rate of interest with increases in total factor productivity (Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez, 2017). But this causality is believed to run both ways, and low real rate through its effect on resource allocation affects TFP growth that drives natural rate lower (Cette, Fernald, and Mojon, 2016). Following from the work of Smets and Wouters (2007), in the identification strategy 1 I choose to keep the response of Real rates as negative on impact from the positive TFP shock. Results from Smets and Wouters (2007) demonstrate that both nominal interest rates and real rates fall on impact from a TFP shock, but by not enough to avoid a fall in prices.

Identification strategy 1: restrictions to FTS shock The impact from FTS shock is considered as orthogonal to the TFP shock and is included in the VAR through a zero restriction of the TFP (first variable in VAR) to FTS shock (second shock in VAR). If we cannot control for the state of business cycle (TFP shocks), then there could be confusion whether the increase in price of the safe asset is due to FTS or from a TFP linked economic downturn. The news shocks literature [Barsky and E. R. Sims (2009) and (2011), Beaudry and Portier (2005) and (2006)] also chooses some form of zero restriction on TFP from news/sentiment shocks.

[12]The forward looking 3-equation NK model (Galí, 2015) is given by:
the IS relation: $y_t = E_t[y_{t+1}] - \frac{1}{\theta}\left(i_t - E_t[\pi_{t+1}] + u_t^{IS}\right)$, $\theta > 0$
the Philips curve: $\pi_t = \beta E_t[\pi_{t+1}] + \kappa\left(y_t - y_t^n\right)$, $0 < \beta < 1$
and the policy rule: $i_t = r_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t$, $\kappa > 0$
where u_t^{IS} is an AR(1) shock process, y^n is the flexible price level output, r^n is the natural rate that prevails flexible prices, and $\tilde{y} = y - y^n$.

Flight to Safety by definition is a phenomenon when capital moves away from risky assets to safer assets. Therefore on impact from a positive FTS shock the Price of risk (the second variable of the benchmark VAR), which is difference between Bond price and Equity price, is restricted with a positive sign restriction. Favourable (or positive) news shocks, in some TFP related news shock literature (Gambetti and Musso, 2017) are also linked to a fall in nominal rates after the 1980s. If we assume that FTS episodes are linked with pessimistic news about future TFP, then similar restrictions that are advocated on Real rates from a negative TFP news shock can be placed for identifying FTS shocks. Real rates responses are therefore restricted to be positive on impact from FTS shock. On impact of a Flight to Safety shock agents look to save for precautionary reasons and therefore their surplus consumption would reduce. This provides us with the third restriction which is a negative response of surplus ratio (the fourth variable in benchmark VAR) on impact of a positive FTS shock. This is also consistent with the results from Smets and Wouters (2007). That model has shocks to risk premium such that an increase in the premia leads to fall in variables of business cycle activity, such as hours and output and leads to an increase in the real interest rates. An increase in risk-premia does not translate one-to-one into preference for safety that is a hallmark of FTS. However, a risk premium shock leads to increase in expected rate of return from risky assets and a fall in their prices. As long as the shift in risk premium after the shock, is not uniform across the whole spectrum of investments (of range least to highest risk), it can be safely argued that the price gap between most risky and least risky investment widens after the risk premium shock and use of the results from Smets and Wouters (2007) is justified.

Identification strategy 1: restrictions to Policy shock To distinguish monetary policy (MP) shock from TFP shock, a zero restriction is placed on the response of TFP series (first variable in VAR) to monetary policy shock (third shock in VAR). This is a commonly used restriction in identifying structural monetary shocks in TFP and news shock literature (Beaudry, Nam, and Wang, 2011). No restrictions from the policy shock are placed on the Price of risk (second series in VAR). A positive monetary policy surprise will lead to an increase in nominal rates and fall in inflation expectations, therefore the Real rate responses (third variable in VAR) are sign restricted to be positive. In Smets and Wouters (2007) monetary policy shock leads to increase in interest rate, real interest rate, and decrease in inflation. Structural model based evidence, including Smets and Wouters (2007) supports the view that positive monetary policy surprises negatively impact consumption and surplus consumption and other business cycle variables including hours and output, and so the response of surplus ratio (fourth series in VAR) to positive monetary policy shock is sign restricted to be negative.

The shock to fourth variable (surplus ratio) which is a Demand shock is not strictly identified in either of the identification strategies 1, 2, and 3. Therefore no restrictions are imposed on the response of any variables in the VAR from demand shocks that impact the Surplus ratio. There may be a case to include an additional restriction that positive demand shocks lead to increase in Surplus ratio. However I resist from doing so, for the main reason that the impact of demand shocks on Surplus ratio in not straightforward as any changes in demand need to be further decomposed into durables, non-durables or services to get their impact on the Surplus ratio. Besides keeping the number of identifying restrictions small has some philosophical backing based on the principle of Occam's razor. It's the notion of 'nominalism or reductionism' (attributed to William of Occam) that in explaining something no more than necessary assumptions should be made. The identification strategy remains agnostic to the response of the 5th variable in VAR or the variable of interest. No restrictions are imposed on this variable from shocks to the remaining 4 variables in the VAR. In the benchmark model the 5th variable, or the variable of interest, is Investments. The response of the variable of interest stays agnostic in terms of Uhlig (2005) representation. Table 1.3 describes the identification restrictions for various variables in benchmark 5 variable VAR.

1.4.2.2 Identification strategy 2

The Identification strategy 2 differs from Identification strategy 1 in the following manner. The negative sign restrictions on the surplus ratio (the fourth variable in the VAR) on impact of FTS and Policy shock are removed. They are replaced by zero restrictions for both FTS shock and Monetary policy shock. So the Surplus ratio which is a ratio of the difference between total consumption and non-durable and services consumption (habits) to total consumption, does not react contemporaneously to Flight to Safety shock and Monetary policy shocks. This is done to acknowledge an alternative empirical literature which argues for a lack of contemporaneous effect of monetary policy shocks on the macroeconomic variables such as investment, output and consumption, as their data is only available with a lag. On including sign restrictions given in Identification strategy

2 (see Table 1.3 on page 39) the $\boldsymbol{B}_{(0)}$ and $\boldsymbol{B}_{(1)}$ matrix are modified to

$$\boldsymbol{B}_{(0)} = \begin{bmatrix} + & 0 & 0 & . & . \\ . & + & . & . & . \\ - & + & + & . & . \\ + & 0 & 0 & . & . \\ . & . & . & . & . \end{bmatrix} \quad \text{and} \quad \boldsymbol{B}_{(1)} = \begin{bmatrix} + & . & . & . & . \\ . & + & . & . & . \\ - & + & + & . & . \\ + & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \tag{1.20}$$

1.4.2.3 Identification strategy 3

Identification strategy 3 has one less restriction from the previous two Identification strategies 1 & 2. In this strategy the positive impact restriction of Flight to Safety shocks on Real rates, the third variable of the VAR, is removed. So Real rates is only positively restricted to Monetary policy shocks and is left unrestricted for FTS shocks. In both Identification strategies 1 & 2, the response of Real rates had same restriction (to be positive on impact) to both FTS and Monetary policy shocks. That assumption was open to the possibility that some latent shock, which could meet the imposed restrictions of both FTS and Monetary policy shock to real rates, could be responsible for the results of the model. Identification strategy 3, breaks that link by restricting the impulse responses to Real rates from Policy and FTS shocks differently and therefore avoids the possibility of responses being driven form any common external shock. By analysing the differences between the impulse response from two different Identification strategies (1 and 3, or 2 and 3), we can get further convincing evidence of whether the shocks that this empirical exercise identifies are in fact FTS shocks.

On including sign restrictions given in Identification strategy 3 (see Table 1.3 on page 39) the $\boldsymbol{B}_{(0)}$ and $\boldsymbol{B}_{(1)}$ matrix are modified to

$$\boldsymbol{B}_{(0)} = \begin{bmatrix} + & 0 & 0 & . & . \\ . & + & . & . & . \\ - & . & + & . & . \\ + & 0 & 0 & . & . \\ . & . & . & . & . \end{bmatrix} \quad \text{and} \quad \boldsymbol{B}_{(1)} = \begin{bmatrix} + & . & . & . & . \\ . & + & . & . & . \\ - & . & + & . & . \\ + & . & . & . & . \\ . & . & . & . & . \end{bmatrix} \tag{1.21}$$

A reason for choosing different identification strategies 1, 2 and 3 in this study, is the inherent assumptions in those strategies about the key drivers and propagation channel behind the model. By assuming a direct and contemporaneous link between the shock variable and macroeconomic variables, we are assuming that forward looking rational households can anticipate the shocks and respond immediately, on the other hand by excluding such contemporaneous relationship we are testing if the shock variable is a primary driver of any change in the macroeconomic series.

Recall that the sign restrictions in 5 variable VAR are implemented for a maximum horizon of one period after the shock, i.e. the restrictions are valid for the shock period and one period after. Whereas, zero restrictions are imposed only on the impact period. The maximum lag length considered on the basis of information criteria is 4 quarters. The results of all models are tested for robustness have been tested over various horizons and other lags.

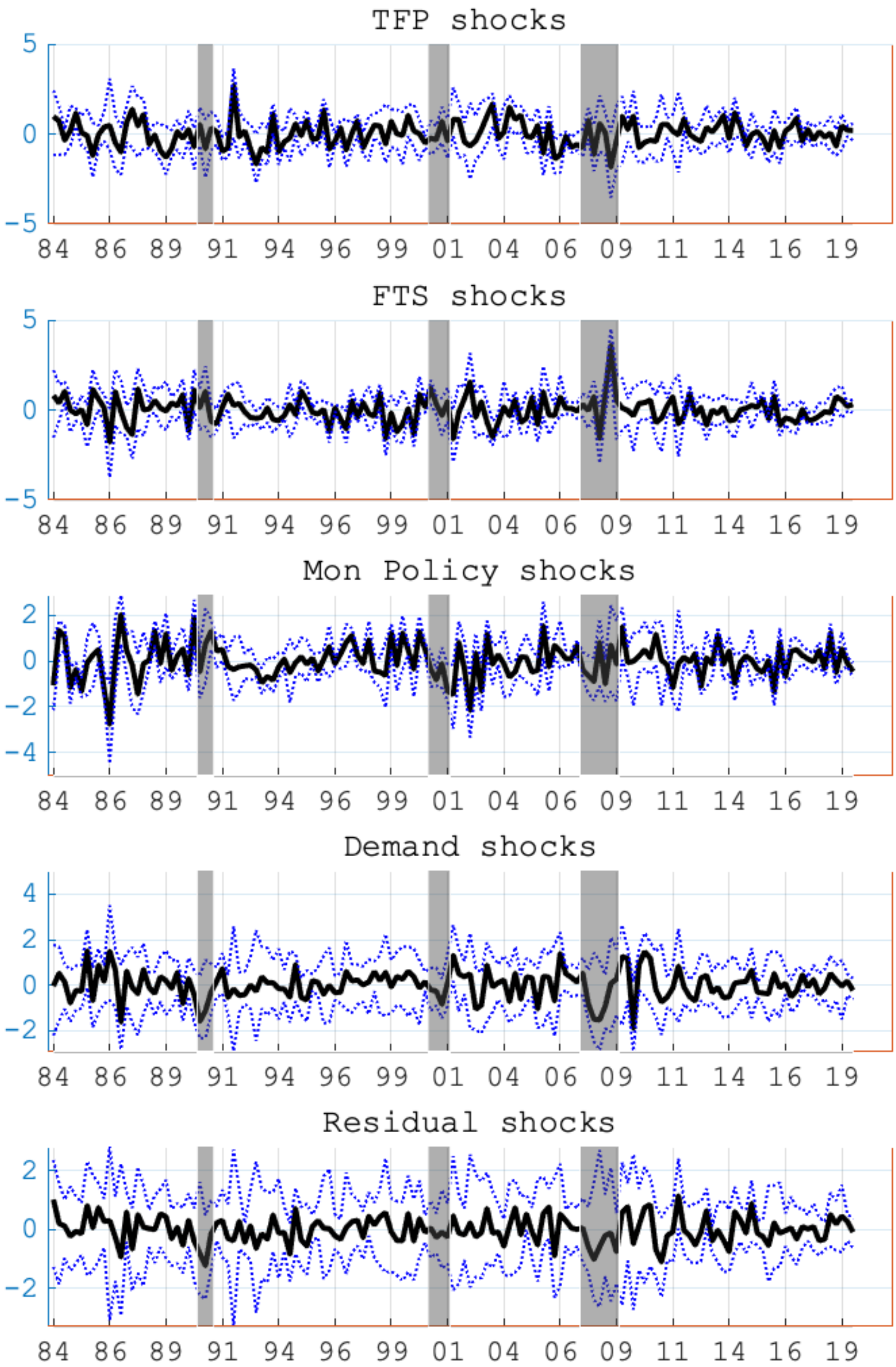
1.5 Structural VAR Results

This section describes the benchmark structural VAR results to show that Flight to Safety shocks inflict a long and lasting impact on the business cycle variables. The results from some investment-related variables compare well with the observed lack of investment growth in periods following the global financial crisis when economic pessimism or Flight to Safety was at higher levels. The section also shows how effective are various identification strategies in picking up innovations in Flight to Safety. It discusses the impact this identification has on beliefs about economic channels that guide investor behaviour during business cycles.

Let us first look at the identified structural shocks from benchmark VAR, which are plotted in Figure 1.8 on page 49, where the shaded areas represent the peak to trough dates of NBER recessions. The identified shocks correspond well with recessionary periods. The scale and size of shocks in the 1990-91 recession is smaller than in the other two recessions of 2000-01 and 2007-09. Most interestingly, only the Flight to Safety shock displays the most significant jump of its entire history during the 2007-09 recession. This brings us back to the original purpose of identifying data series and events related to the unprecedented global financial crisis more than they do so with any other post-war recession. The initial assessment portrays FTS shock as a likely candidate.

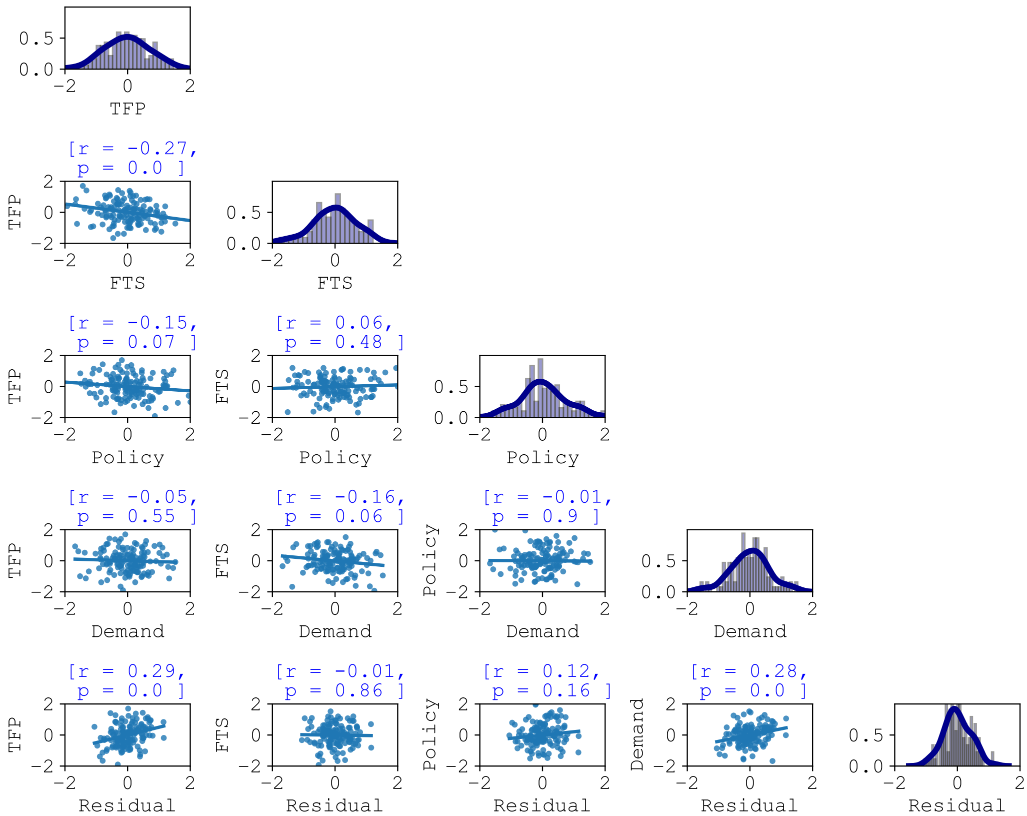
We can further assess the usefulness of the vector autoregressions in the benchmark configuration of the model by looking at identified structural shocks that are quite orthogonal (see Figure 1.9 on page 50) and uncorrelated. Figure 1.9 shows the probability distribution of these shocks, the regression line of one shock on another, and the correlation coefficients between the identified shocks. The regression line and scatter plots display a lack of meaningful relationships between the shocks. Correlation coefficients for most of them are not significant. There is some linkage in identified TFP and FTS shocks as their correlation coefficient has a small negative slope, which shows that positive FTS shocks appear to be leading to adverse TFP shocks. We further explore this feature in the following section through the results of impulse responses to FTS shocks.

Figure 1.8: Identified structural shocks



Notes: Standardised median structural shocks from 1983:Q1 to 2019:Q3 and their 68% confidence bands. Shocks are identified through the benchmark 5-variable VAR, using Sign and Zero restrictions discussed in Strategy 1. The 5 variables in that model are: TFP, Price of risk (Bond minus Equity price), Real rates, Surplus Ratio and Investments. The shaded areas represent peak to trough period of NBER recessions. Respective shocks are: TFP, FTS, Monetary Policy, Demand and Residual.

Figure 1.9: Orthogonality of structural shocks



Notes: Regression (line) of one structural shock on another, and Probability distribution of structural shocks identified in benchmark 5 variable VAR using Identification strategy 1. r is Pearson's correlation coefficient. p is Two-tailed p-value of the correlation coefficient. The 5 variables in the benchmark model are: TFP, Price of risk (Bond minus Equity price), Real rates, Surplus Ratio and Investments. Respective shocks are: TFP, FTS, Monetary Policy, Demand and Residual.

1.5.1 Economic contractions from Flight to Safety

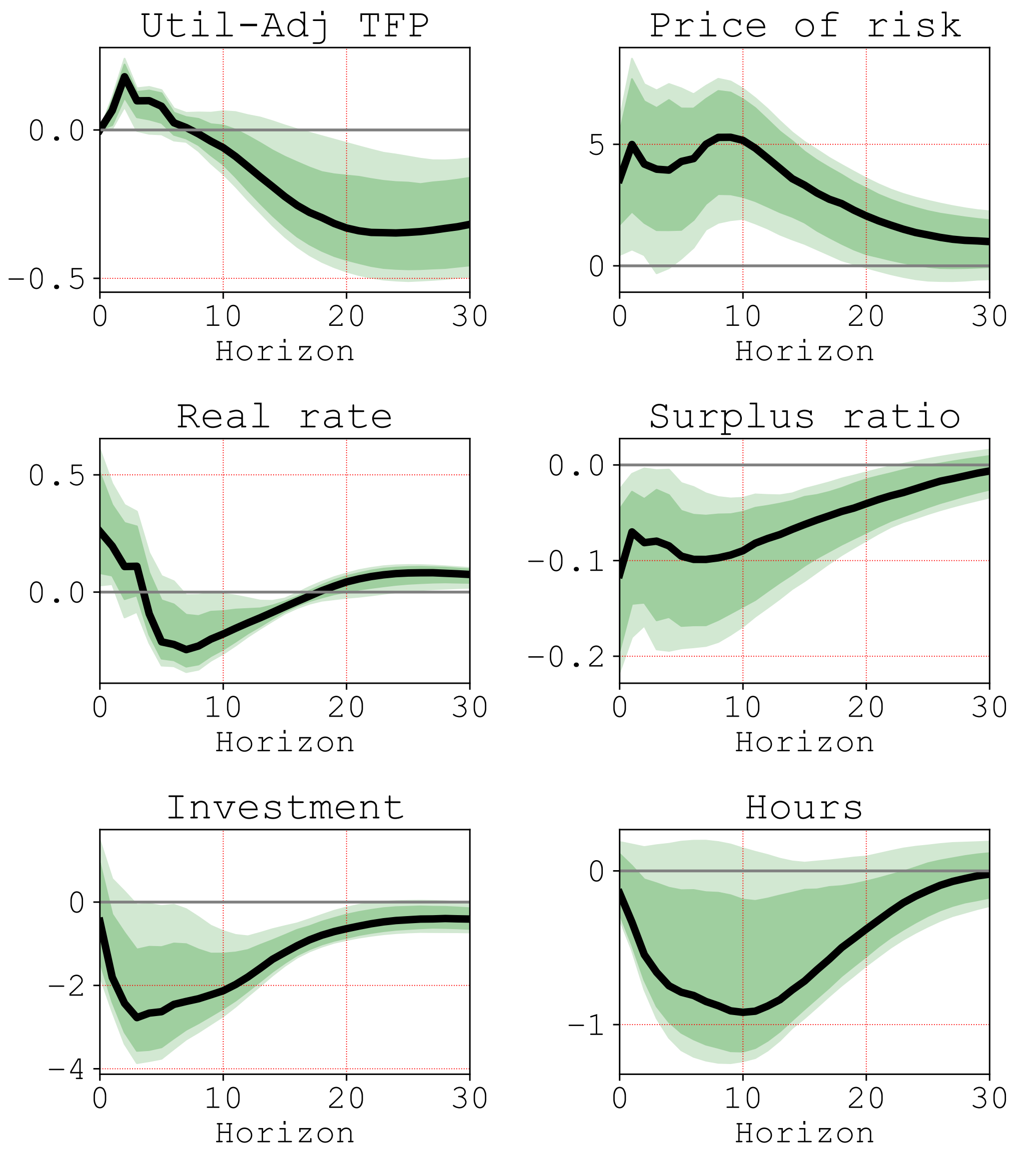
In this section, the results from identification strategy 1 (as discussed in Table 1.3 on page 39) for the 5 variable VAR with variables: Total factor productivity, Price of risk, Real rate, Surplus ratio, and Investments are discussed. Investments are a key variable of interest, and in later studies, it is replaced with other variables such as Output, Consumption, Hours, and CPI.

Figure 1.10 on page 52 plots the impulse responses from identified Flight to Safety shocks on all variables of the benchmark 5-variable VAR with Investments as the key variable that is unrestricted and considered agnostic in the identification strategy. Additionally, the figure also reports 'Hours' when replacing the variable 'Investment' in the benchmark model. Similarly, in Figure 1.11 on page 54 results of other variables of interests as they replace the fifth variable of the benchmark VAR are reported. The impulse response results for the first 4 variables in these alternate models are similar to those of the benchmark model where 'Investment' is the variable of interest. So, preserving space and time, they are only reported once, i.e., for the benchmark model.

The results (in Figure 1.10) show that the pessimism and risk aversion resulting from Flight to Safety shocks has a long term impact on the business cycle. The Bond minus Equity price (Price of risk) series jumps upon impact from FTS shocks. As described earlier, an unexpected increase in the price of a safe asset (10-year T-bond) vis-á-vis the price of a risky asset (S&P 500) symbolizes an increase in investors' preference for safety over risky investments, or Flight to Safety. So as expected from sign and zero restrictions set out in Identification strategy 1, we witness an immediate jump in this series of 4-5%. An impact of this size cannot be fully generated from an increase in the safe asset price alone. The yields on safe asset (bonds) are meager, and a 4-5% increase in prices (or a drop in yields of this magnitude) cannot be obtained without violating the Zero lower bound on bond yields. Therefore a steeper fall in S&P 500 index on impact from FTS shock is the primary driver of this sudden jump in the Price of risk series. Once the FTS shock has hit, it takes near 28 quarters for the series to return to its pre-shock levels. The identified FTS shock thus demonstrates a long term impact on the investors' preference for safer assets.

Identification strategy 1 (discussed in table Table 1.3 on page 39) restricts the response of Real rate to FTS shocks to be positive and Surplus ratio to be negative

Figure 1.10: Impulse responses of Benchmark VAR to FTS shocks



Notes: Median impulse response and 68% and 95% confidence bands of variables in benchmark VAR model to Flight to Safety (FTS) shocks of 1 s.d. which are identified with Sign and Zero restrictions strategy 1. Data: 1983:Q1 to 2019:Q3. In all charts, Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. Benchmark VAR has five variables: Total factor productivity, Price of risk (which generates the FTS shocks), Real rate, Surplus ratio is one minus ratio of consumption (non-durable goods and services) to total consumption and 'Investment'. In another iteration of the 5 variable VAR model 'Investment' is replaced with 'Hours'.

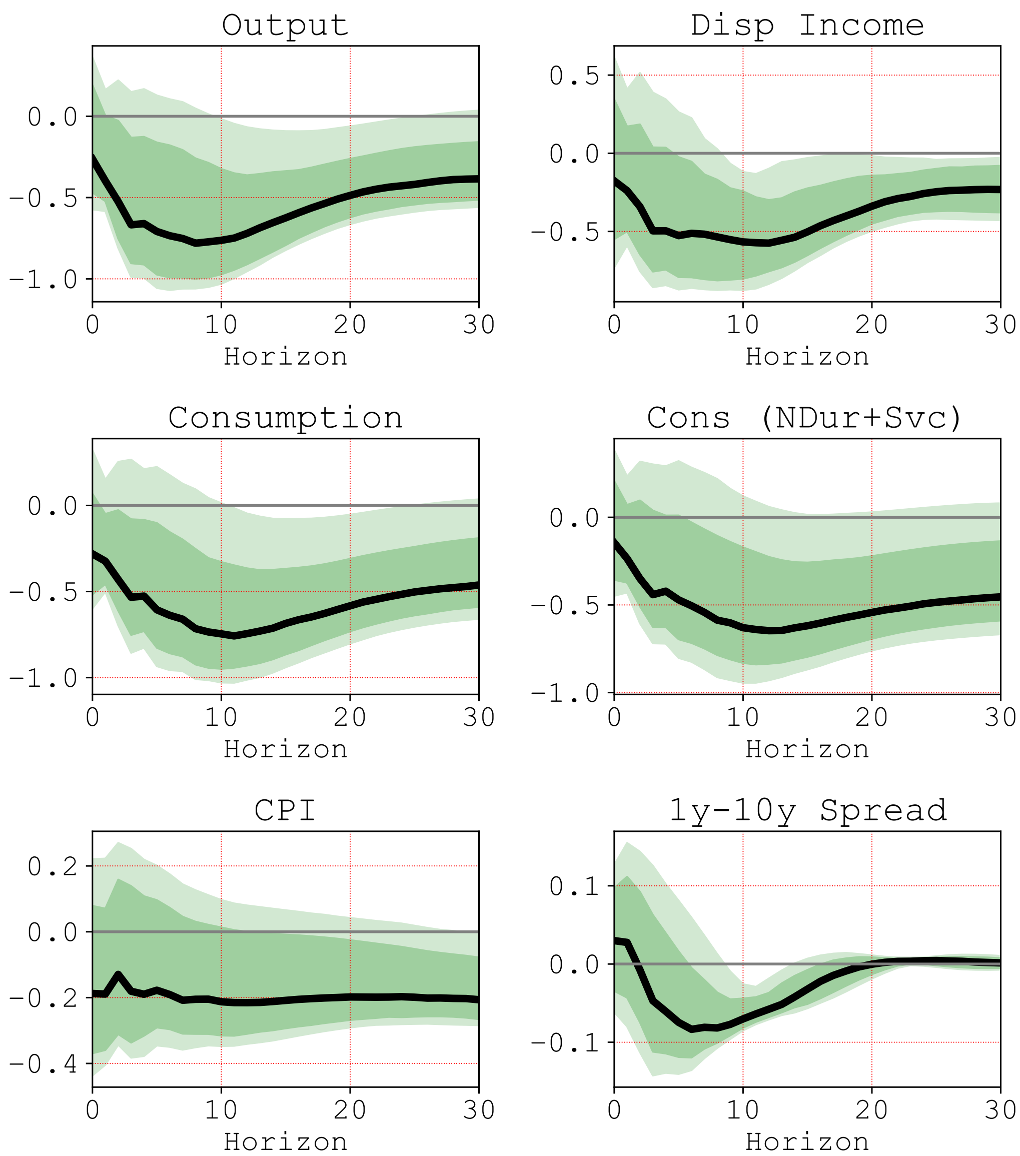
for the shock period and one period following the shock. Both these variables behave as expected. The Real rate rises, and the Surplus ratio falls on impact. The median impulse response of Real rates increases by 0.3 percent, whereas the fall in the median Surplus ratio response is about -0.1 percent. The latter is a significant fall for two reasons, the first of which is that there is not much scope for the Surplus ratio to fall. This ratio in Data in the long run averages around 12 percent, as habits (i.e., consumption of services and non-durable goods) form a significant (∼90%) portion of total consumption. Furthermore, the second reason is that the consumption-based models [Campbell and Cochrane (1999), Cochrane (2017), De Paoli and Zabczyk (2013)] which consider surplus ratio in the utility function, demonstrate that even for a small decline in this ratio there is relatively big spike in consumers' marginal utility.

The identification strategy 1 also puts a zero restriction on the response of total factor productivity so that the identified Flight to Safety shock is orthogonal to TFP. As a result, we see that the impulse response of TFP is muted on the impact of the FTS shock. However, it begins to turn negative around 8-10 periods after the shock and slides after that for another 12 quarters. For the length of business cycle frequency (8-32 quarters) after the FTS shock, there is a persistent decline in TFP.

Investment, which is the variable of interest in the benchmark 5-variable VAR, is kept unrestricted as no sign or zero restrictions are imposed on it from any shocks in the model. The impulse response in Investment is negative on the initial impact of FTS shock. Investment decisions are made many periods in advance, explaining the smaller immediate drop in Investments on the impact of the Flight to Safety shocks. However, the response reaches a median quarterly fall of around -3% in a short span of 2-3 periods after the shock. It sustains this L-shaped response and stays below -2% for about 10-12 quarters. The impulse responses also show that Investments are very slow to recover to their pre-shock levels. What is probably more significant is that after the impulse response in Investment reaches its lowest, i.e., 3-4 quarter after the impact of FTS shock, only then the Total factor productivity (TFP) begins to decline.

To better understand the mechanism through which FTS impacts the business cycle, 'Investment' in the benchmark configuration is replaced with other macroeconomic variables. They are 'Hours' whose impulse responses to FTS shocks are presented in Figure 1.10 on page 52 itself and six other variables. Impulse responses to FTS shock for these six variables are presented in Figure 1.11 on

Figure 1.11: Impulse responses of Macro variables to FTS shocks



Notes: Median impulse response (in %) and 68% and 95% confidence bands of Macro variables of interest to Flight to Safety (FTS) shocks of 1 s.d. in benchmark VAR model identified with Sign and Zero restrictions strategy 1. Data: 1983:Q1 to 2019:Q3. In all charts, Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. Only the responses of variable of interest in the VAR model are reported. 'Disp Income' stands for Disposable Income. '1y-10y spread' is the difference between yields of 1 year and 10 year US Treasury bonds. 'Cons (NDur + Svc)' is Consumption of Non-durable goods and Services.

page 54. These additional variables include four business cycle variables: 'Output', 'Disposable income', 'Consumption (Non-Durables and Services)' and total 'Consumption', a pricing variable 'CPI' and a liquidity variable '1y-10y spread' which is the difference in the yields of 1-year and 10-year US treasury bonds.

The impulse responses to these variables represent a negative hump-shaped response. 'Hours' exhibits a negative response that is slow to begin and decreases gradually, which resembles a reverse hump shape. However, except for hours neither of the other business cycle variables fully recover to their pre-shock levels. The median response in hours is small and reaches a minimum of approximately -0.9% around 10 periods after the shock. In contrast, the TFP for the first 8-10 quarters has been non-negative, which means the decline in hours is not driven by technical regress. Similarly, the responses in other variables of interest - Output, Disposable Income, Consumption (Non-Durables and Services), and Consumption - cannot be linked to a fall in TFP. It can be argued that their fall is more synchronous with Flight to Safety. Since after the Flight to Safety (FTS) shock has hit, the impulse response in these variables keeps on reducing for 10 periods, whereas for the same period after the shock, the TFP is non-negative.

It also seems that the impact of Flight to Safety shocks on macro variables must be playing out through a long-term decline in Investments. The impulse response in Investments has a faster and a more significant decline, almost 3-4 times of the decline witnessed in Hours, Consumption, Output, Consumption (of Non-durables and Services) and Disposable income. The negative impact on these macro variables of interest continues for 8-10 quarters before reaching its nadir and recovering after that. 8-10 quarters is also the time around when the steep fall in Investments begins to ameliorate. Therefore we can safely say that TFP shocks do not bring the economic gloom that is shown in 'Investments' and other variables of interest. Instead, it is driven by the pessimism in the economy brought out from a structural FTS shock.

This highlights the investment-oriented effects of FTS shocks, and provides evidence to one of the hypotheses set out at the inception of this study. Flight to Safety, which is a sudden change in preference for safer investments by investors through resource re-allocation, leads to an eventual decline in overall TFP. As when only safer (or less productive) economic investments are undertaken, the entire economy's production and productivity suffers.

One could also argue that the economic contraction is unrelated to TFP declines and is caused by FTS shocks. This contradicts other studies in the literature that follow the (Barro and King, 1984) conjecture, which has argued that investment re-allocation is not a major driving force of the business cycles. Several empirical studies on the news [Beaudry, Nam, and Wang (2011) Nam and Wang (2019)] and expectations shocks identify that those shocks only lead a TFP shock by 1-2 quarters. Therefore, the TFP shock appears to cause changes to macro variables and business cycles in those studies. The long lead time between FTS shock and TFP shock that we get in impulse responses of Identification strategy 1 breaks that link and leads us in seeking an alternate driving mechanism.

Habits or Consumption (Non-durables and Services) are also slow to react initially upon the impact of the FTS shock, but they end up mimicking the decline in Consumption and after 30 periods end up about -0.5% lower that their pre-shock levels. This is significant for two reasons. Firstly, it shows that the precautionary motives exhibited during FTS generate a correction in even the most hard-wired of consumption behaviour (habits). Secondly, for a consumer looking for her utility maximization, a stabilization of the surplus ratio (i.e., one minus the ratio of habits to total consumption) matters more than stabilization of total Consumption or Habits. This is further explored in later chapters.

The median response of term spread, in Figure 1.11, which is measured as the difference between the short-dated 1-year US Treasury bond and the long-dated 10-year US Treasury Bond yield, picks up slightly on the impact of the FTS shock. However, the move is not significant, especially when compared to the rise in the Price of risk. It demonstrates that the preference for safety that is a hallmark of an FTS shock does not significantly translate into a preference for liquidity, i.e. preference for the short term over long term safe bonds. This is not surprising since most long-term investors such as pension funds would match the duration of the risky assets that they are shedding from their portfolio with the duration of the safe investments they are undertaking in response to the FTS shock.

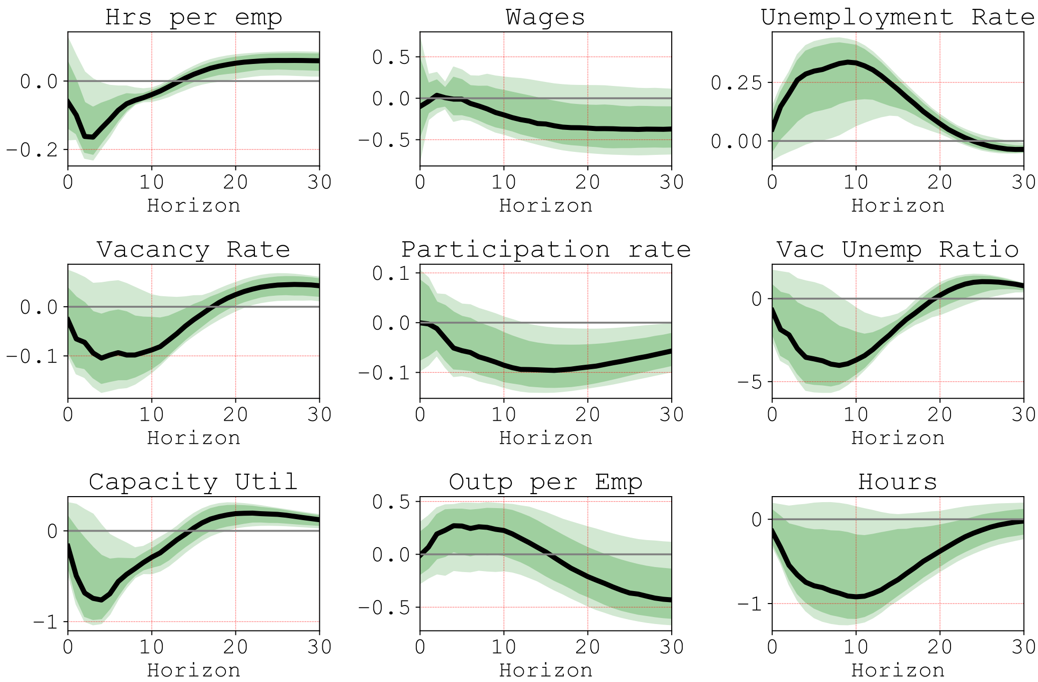
The impact of FTS shock on consumer prices (CPI) supports our conviction of imposing a positive restriction (in Strategy 1) on Real rates from the impact of FTS shock. The rationale for such restriction was that a Flight to Safety shock is dis-inflationary, and the evidence in the impulse response of CPI confirms it. The median CPI falls around 0.2% per quarter, or we can say there is a deflation of 0.2% quarterly deflation on the impact of FTS shock. The median CPI does not recover for the tested horizon of 30 periods, which for the quarterly inflation rate means that a few quarters after the Flight to Safety shock, the inflation rate consolidates around zero percent. This shows that FTS shock is deflationary on impact and non-inflationary in the long run, a sign of economic pessimism. Thus looking at the impulse responses of various macro variables, we can make an assessment that a Flight to Safety shock leads to an overall economic gloom in the economy.

The results highlight the fact that either one of the two possible channels of *speculation* or *expectations* through which FTS can impact the economy is at play. The *speculation channel* relies on FTS resulting from over-correction by speculators from a realization of negative news or information shock about the economy. If that were true, then the economic adjustment to FTS would have been swift. Nevertheless, given the slow but significant response of key Investment and activity-related variables to FTS shock and the long lead time of 8-10 quarters in the decline of total factor productivity after the FTS shock indicate that *speculation* may not be the primary mechanism driving this effect. The *expectations channel* relies on the notion that Flight to Safety emerges from expectations being formed about the deteriorating future state of the economy. It posits that FTS shocks are based on rational expectations and present a warning signal of an eventual decline in economic activity. If this were true, then FTS is an even earlier warning signal than news and sentiment shocks, where the lead time is usually 3-4 quarters. This seems plausible, as FTS represents an increase in risk-aversion, so providing a warning of impending deterioration in the economy motivates risk-averse rational agents to reallocate investment capital from risky to less risky (and also less productive) sectors. This re-allocation of risk over time feeds into a decline in economic activity and a decline in total factor productivity. This paper's empirical approach is not sufficient to distinguish between the two, but some clarity can emerge from looking at various impulse responses from different identification schemes. Through evidence posted by various macroeconomic variables, there is some inclination towards favouring the latter *expectations driven* explanation that FTS shock predates an economic downturn. We can further strengthen this conviction from the results of Investment and Labour related business cycle variables.

1.5.2 Cyclicality of Labour and Investment sector

To provide further evidence that identified FTS shocks consistently explain the properties of US business cycles, this section compares the impulses responses in

Figure 1.12: Impulse responses of Labour variables to FTS shocks



Notes: Median impulse response and 68% and 95% confidence bands of Labour related variables of interest to Flight to Safety (FTS) shocks of 1 s.d. in benchmark VAR model identified with Sign and Zero restrictions strategy 1. Data: 1983:Q1 to 2019:Q3. In all charts, Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. Only the responses of variable of interest in the VAR model are reported. 'Hrs per Emp' is Hours worked per employee. 'Vac Unemp' is ratio of Vacancy rate to Unemployment rate.

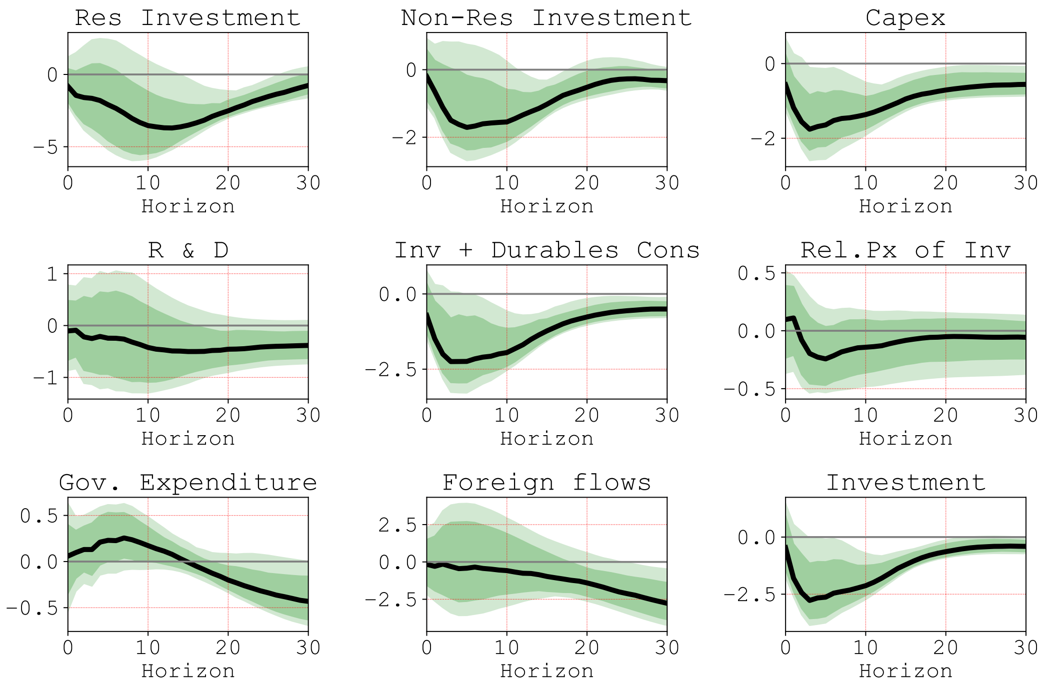
Identification strategy 1 (see Table 1.3), of a 5-variable VAR model where either a labour market variable (Figure 1.12) or an investment sector variable (Figure 1.13) is the variable of interest. Recall that in this identification the impact of TFP shock is restricted to be positive on the utility-adjusted TFP and Surplus ratio, and is restricted to negative on Real rates. The impact response of FTS shock is positive on the Price of risk (Bond-Equity price), positive on Real rates, and negative on Surplus ratio. The Monetary policy shock impact is restricted to positive on Real rate and negative on the Surplus ratio. Both FTS and Monetary policy shock are orthogonal to TFP shock.

The identified FTS shocks substantially impact unemployment rate and labour force participation rate (Figure 1.12 on page 58). Hours per worker decline slightly on the FTS shock impact and recover fast; also, they are not the major driver of total hours. These responses agree with Shimer (2005) empirical result that the major driver of the decline in total hours is not hours per worker but the unemployment rate. The intensive margin, given by hours per worker, is only partially responsible for fluctuations in aggregate hours and other labor market constituents.

The impulse responses to the Vacancy rate and the Unemployment rate are opposite to each other, signifying the negative correlation between the two at business cycle frequencies. Search based models of unemployment and business cycles (of which Mortensen and Pissarides (1994) is a key example) cannot generate the high negative correlation between the unemployment and vacancy rate. Due to the Nash bargaining mechanisms, the real wage determined in these models is too flexible (Shimer, 2005).

The results from Flight to Safety shocks generate a vacancy to unemployment ratio that is procyclical and a real wage rate that is highly sticky and in line with the *Beveridge curve* that portrays a downward sloping relation between the Vacancy rate and the Unemployment rate in the US data. The Labour force participation rate in our results is also procyclical. The impulse responses (in Figure 1.12) for labour productivity, which is the ratio of output to hours, exhibits an increase on the impact of FTS shocks. Whereas Hours, Output, Consumption reduce during the first 10 periods after the shock. This incongruity of response of labour-related business cycle variables with the response of TFP and labour productivity shows that the key driver behind this economic gloom is not the decline in TFP but rather an increase in economic pessimism breeding the Flight to Safety. This finding contradicts the economic models that ignore the role of

Figure 1.13: Impulse responses of Investment variables to FTS shocks



Notes: Median impulse response and 68% and 95% confidence bands of Investment related variables of interest to Flight to Safety (FTS) shocks of 1 s.d. in benchmark VAR model identified with Sign and Zero restrictions strategy 1. Data: 1983:Q1 to 2019:Q3. In all charts, Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. Only the responses of variable of interest in the VAR model are reported. 'Res' is Residential, 'Non-Res' is Non-Residential, 'Inv + Durables Cons' is sum of Investments and Durables Consumption, 'Rel.Px of Inv' is the Relative price of Investment in terms of Consumption, 'Gov.' stands for Government.

expectations and cyclical risk-aversion in business cycles.

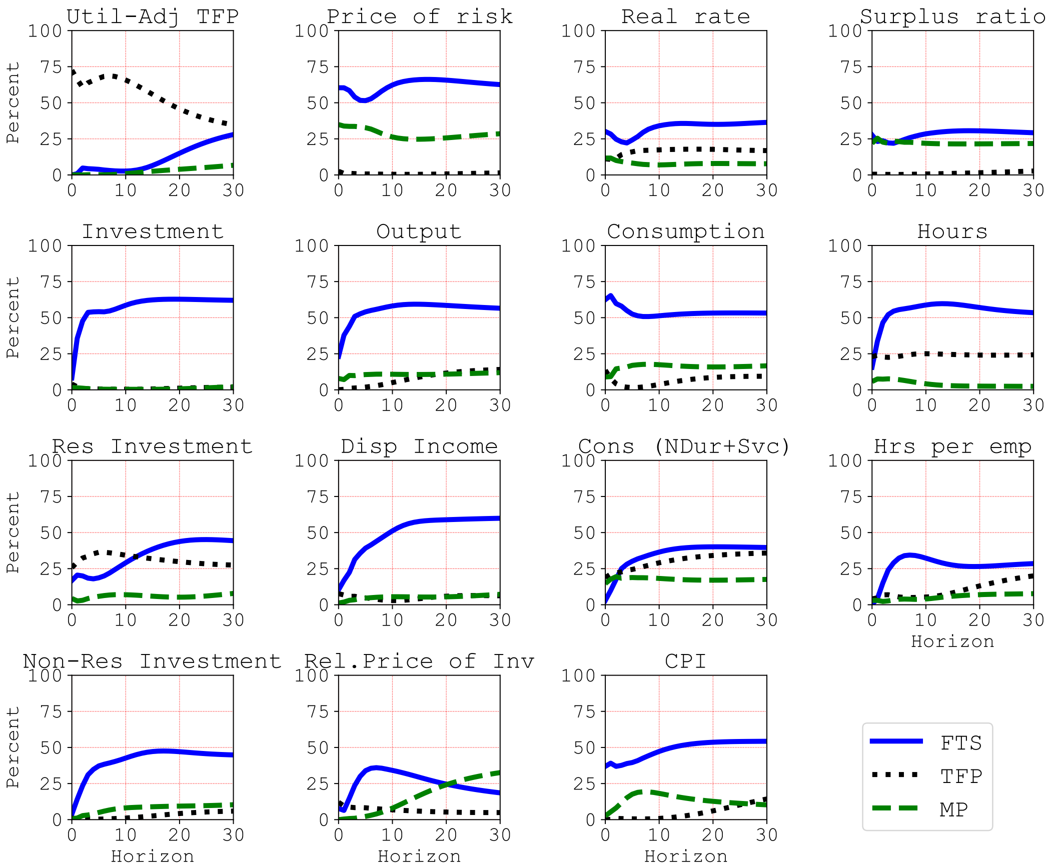
Further investigation into the constituent variables of investments, i.e., residential and non-residential investment (See Figure 1.13) shows that the residential investments react slowly to the FTS shock. However, after around 10 periods, it reaches a median reduction of -4% per quarter. Most housing-related investments are planned for many periods in advance. Once the residential investment projects are approved, they are slow to roll back, and once these plans are stalled or abandoned, they are even more challenging to get re-approved. On the other hand, non-residential investments that are more agile in comparison react early on the shock's impact. They consolidate after about 5 quarters at a median response of -2% per quarter and recover faster than residential investments. Capital expenditure is held back by -2% and is faster to react than residential and non-residential investments. The FTS shocks have a lesser impact on R&D expense which is a more stable form of investments. Similarly the consumption of the Durable goods of households, a part of Investment plus Durable series, is steadier than total investments. It declines by a little over -2% upon the impact of an FTS shock. The impact of FTS shocks on both Government expenditure, and Foreign flows are felt only in the long run. More importantly, FTS shock on impact lowers the relative price of investment good (in terms of consumption).

The variable 'Relative price of investment' in business cycle literature has been used as a series to develop investment-specific technology shocks and the marginal efficiency of investment shocks. The Flight to Safety shocks identified in this study do not run against the investment shocks literature-based evidence. A positive shock to investments lowers the relative price of investment in terms of the consumption good in models of Justiniano, Primiceri, and Tambalotti (2010) and (2011), and J. Greenwood, Hercowitz, and Krusell (2000) among others. The procyclical relative price of investments, which is a hallmark of investment-specific technology shocks-based explanations of business cycles, is also visible in this chapter's impulse responses. The ratio of investment price to price of consumer good falls on impact of the FTS shock, i.e. consumption becomes relatively expensive.

An immediate application of the findings is to get decomposition of forecast error-variance (See Table A.1 to A.5 in appendix pages 252 - 256) and to determine what proportion (Figure 1.14 on page 62) of the k-step ahead forecast variation, esp. at business cycle frequency (8-32 quarters) is explained by identified innovations to FTS.

The FTS shock explain a major share of Forecast error variance at business cycle frequency for each of the key business cycle variables: Output (58%), Consumption (50%), Investment (60%), Residential Investment (40%), Income (55%), Hours (55%), TFP (20%), Surplus ratio (30%), Real rate (35%). The FEV contribution for FTS shocks to key macro variable peaks before an increase in contribution from TFP shocks. Suggesting that FTS shocks rather than TFP shocks drive the highlighted business cycle features.

Figure 1.14: FEVD explained by TFP, FTS and MP shocks



Notes: The k-step ahead Forecast error variance decomposition (FEVD %) explained by FTS (Solid), TFP (Dots) and Monetary Policy (Dash) shocks, in the 5-variable VAR, which is identified using Sign and Zero restrictions discussed in Strategy 1. The 5 variables in the benchmark model are: TFP, Price of risk (Bond minus Equity price), Real rates, Surplus Ratio and Investment. In other iterations of the model, 'Investment' is replaced with other variables of interest. The result of benchmark VAR and other variables of interest are reported.

Table 1.4: Identification strategy for Flight to Risk shocks

variables / *shocks*	TFP	Value of Risk	Real rate	Surplus ratio	Variable of interest
FTR Strategy 1					
Productivity	+	.	−	+	.
Flight to Risk	0	+	−	+	.
Monetary policy	0	.	+	−	.
Demand	.	.	.	.	.
Residual	.	.	.	.	.

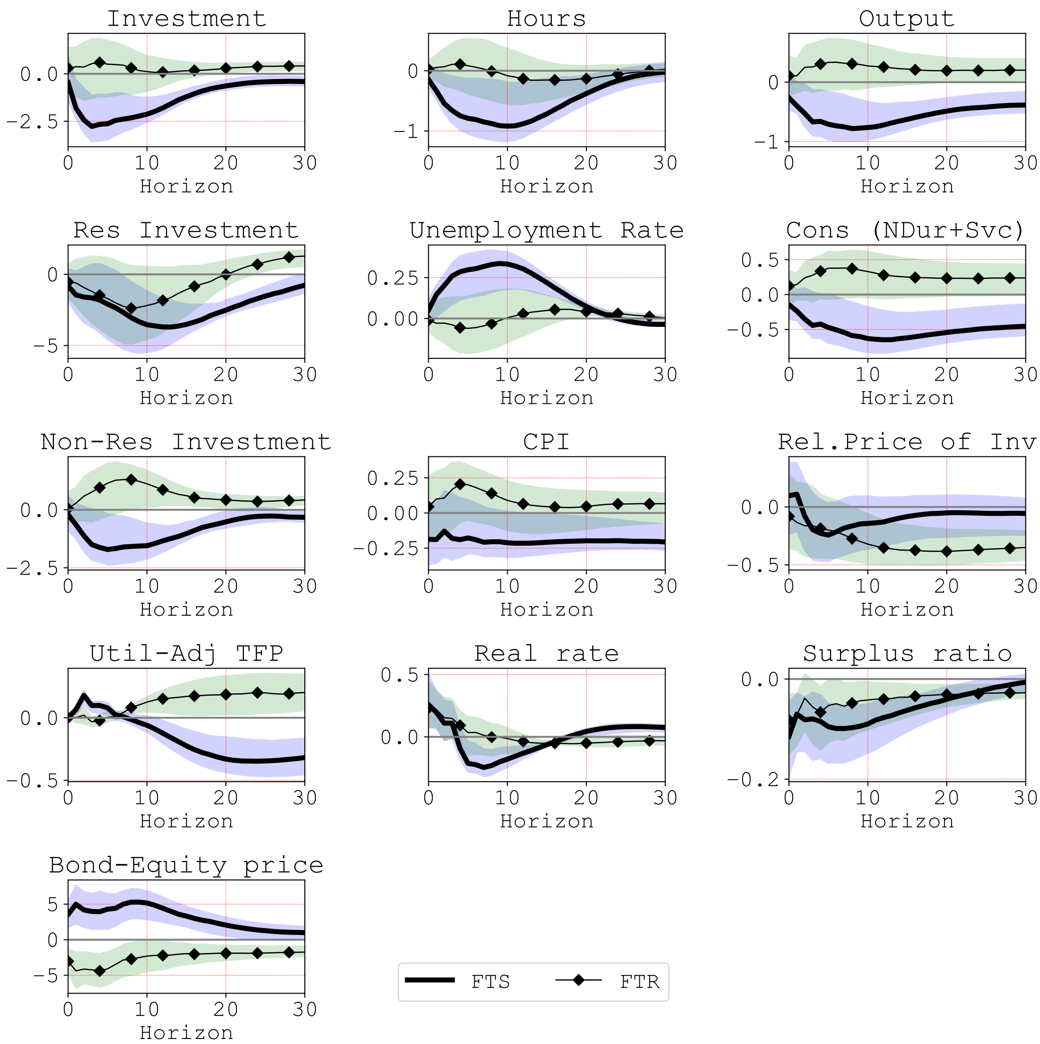
Notes: The SVAR for identifying Flight to Risk shocks has 5 variables: TFP, Value of risk (which is Equity price minus Bond price), Real rate, Surplus ratio (which is One minus the ratio of Consumption of services and Non-durable to Total consumption), and a business cycle variable of interest for e.g. Investments , Output, Consumption, Hours, CPI etc. On impact of the respective shock, + means the variable is restricted to be positive and − means that the variable is restricted to be negative for the impact horizon. The impact horizon is shock period plus one more period. Symbol 0 signifies a zero restriction when the response variable is restricted to not respond to the shock contemporaneously, and Dots . signify that the impact response of that variable is left unrestricted.

1.5.3 Asymmetry in FTS and FTR

One of the peculiar and significant features of Flight to Safety (FTS) or a sudden increase in preference for safer investments is that the market volatility and uncertainty experienced in such episodes are not reciprocated during its complementary market phenomenon of Flight to Risk (FTR). FTR is when investors prefer risky investments to safer bets. Such differential behaviour is based on the human psyche and behavioural biases that are out of this study's scope. However, for the purpose of our analysis, it is interesting to consider whether the impact of FTS is reversed for shocks of opposite magnitude. An FTR shock series is developed by choosing a complementary identification to the one imposed in the benchmark study. Impulse response results from this series help uncover FTR shocks' impact and keep the analysis relevant and comparable to the benchmark FTS model results.

The FTR shocks are identified by changing the sign of the Price of risk series, i.e. by calculating it as the difference between the Price of S&P 500 and the

Figure 1.15: Asymmetry between Flight to Safety and Flight to Risk



Notes: The figure shows impulse responses to Flight to Safety (solid) and Flight to Risk (diamonds) shocks and their 68% confidence bands (shaded). FTS shocks are identified with restriction strategy 1 in the benchmark VAR model. The FTR shocks are identified by imposing restriction strategy discussed in the Results section Table 1.4. Data: 1983:Q1 to 2019:Q3.

10-year Treasury bond. This could be called a Value of Risk series or negative Price of risk series. Therefore an increase in the Value of Risk or Equity minus Bond price series occurs when Equities get more expensive compared to bonds, and a positive shock to this series is studied as Flight to Risk. The identification strategy imposed in identifying FTR shocks is also adjusted to account for the complementary changes to benchmark strategy 1. A positive shock in the Value of Risk series for the impact horizon is restricted to a fall in Real rates and an increase in the Surplus ratio. Both these sign restrictions are opposite in signs to the sign restrictions imposed on these variables in the benchmark strategy 1 for identifying FTS shocks. Similar to FTS shocks the FTR shocks are also restricted to be orthogonal to TFP and monetary policy shocks. Table 1.4 on page 63 presents the identification restrictions imposed in identifying FTR shocks. The impulse responses to FTR shocks, as shown in Figure 1.15 on page 64, highlight the asymmetry in the response of Flight to Safety and Flight to Risk shocks. These impulse responses to FTR shocks are smaller in magnitude and short-lived compared to the impulse responses to FTS shocks. The increase in Equity minus Bond price (or a decrease in Bond minus Equity price) from an FTR shock leads to very slow adjustments in investments, hours, output, non-residential investments, inflation, and TFP and real rates when compared with their response to FTS shocks. The lack of negative response in Real rates is because monetary policy response is exogenous to the model. An increase in inflation after Flight to Risk does not warrant policy cuts from the inflation-targeting central bank. Real rates become negative when inflation stabilizes. The residential investment shows a lack of response, which is not that puzzling when considering that residential purchases are long-term decisions. These can be put away easily when faced with a Flight to Safety phenomenon, but they are not immediately put on board after a Flight to Risk. Housing is a long-term investment, and in the case of Flight to Risk shocks, the immediate response should be felt in more volatile (risky) but liquid options. The residential investment increases after 5 years of risk-taking. There is also a counter-intuitive fall in surplus ratio after an FTR shock. However, it can be explained by the slow response of output and consumption in comparison to the more significant increase in consumption (of Non-Durables and Services) habits. The increase in habits is also responsible for the relative price of investment goods in terms of consumption good getting into the negative territory. The surplus ratio slowly returns to its pre-shock levels. There is also feedback from FTR shocks to TFP, which reacts positively 5-8 quarters after the shock. Interestingly the response is delayed as it was in response to FTS shocks, highlighting the expectations channel through which the capital flight can impact productivity in an economy.

1.6 Sensitivity and Robustness analysis

The benchmark results are identified with Strategy 1 and using 1983:Q1 to 2019:Q3 period US data. This section analyses the sensitivity of those results to alternative identification strategies *viz.* Strategy 2 and Strategy 3 (given in Table 1.3). And to alternative data periods *viz.* pre-Great Moderation data (1954:Q3 to 1978:Q4) and pre-financial crisis or Great Moderation period of 1983:Q1 to 2007:Q2. This section also compares FTS results with estimation output from DSGE models and explores the relationship of FTS shocks with other News and Uncertainty shocks.

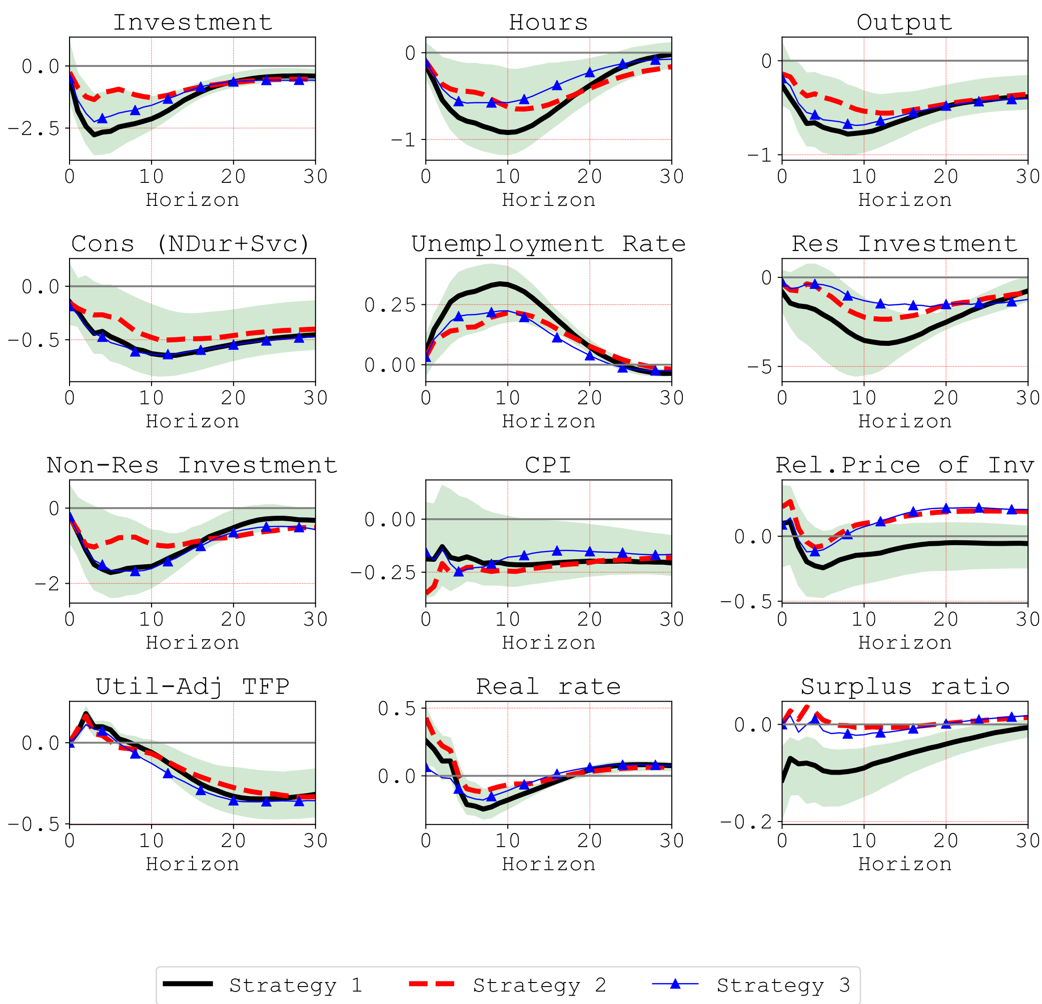
1.6.1 Alternative identification strategies

In Identification strategy 2 (refer Table 1.3 on page 39), the negative restriction on the impact of FTS shock to surplus ratio, which was included in Identification strategy 1, is removed. In place of that, a zero restriction is imposed on the Surplus ratio on impact from the FTS shock. The impact of FTS shock on Real rates is still kept, as it was in Identification strategy 1, at positive for the impact horizon, i.e., the shock period and the next period. In addition, the response of the Surplus ratio in the identification strategy 2 has a Zero restriction on impact from Monetary policy shock as well.

Changes in identification strategy 2 can be summarised in the following way. An increase in FTS is restricted to a positive impact on Price of risk and Real rates, Monetary policy shock is restricted to have a positive impact on Real rates, and both FTS and Monetary shock are orthogonal to the Surplus ratio. Whereas, in this strategy, the impact restrictions for TFP shocks are unchanged from Identification strategy 1 and are orthogonal to both FTS and Monetary policy shock. The responses of the variable of interest, which is Investment or any other macro variable, are kept agnostic.

The impulse responses to FTS shocks under Identification strategy 2, which are plotted as dashes in Figure 1.16 on page 67, are not significantly different from the responses to Identification strategy 1, which are represented by a solid line with its 68% confidence band is shaded in the same fig. 1.16. The decline in Investments is severe and keeps around -2% for around 10 quarters. The impulse responses in

Figure 1.16: Sensitivity of impulse responses to identification strategies



Notes: Sensitivity of impulse responses to FTS shocks under identification with Benchmark Strategy 1 (Solid) with responses from Strategy 2 (Dash) and Strategy 3 (Triangles). Data: 1983:Q1 to 2019:Q3. 68% confidence band of Benchmark strategy is shaded in light green. Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. 'Cons (NDur+Svc)' is Consumption of Non-durable goods and Services. 'Res' is residential, 'Non-Res' is non-residential, and 'Rel.Price of Inv' is Relative price of investment good in terms of consumption good.

key business cycle variables of interest such as hours, output, and consumption are immediate (See Figure 1.16) and are shaped in an inverse hump. The reduction in Consumption (non-durables and services) and the Surplus ratio is delayed by 2 periods, and the response is now less severe than it was in identification strategy 1. The response of the Surplus ratio stabilizes to pre-shock level as Habits fall in line with Consumption. The median response reaches a minimum of only -0.04%, which is due to the differences in identifying restrictions between Strategy 1 and Strategy 2. More importantly, the response of TFP is muted for around 10 periods and is nearly as severe as it was during the identification strategy 1. This exercise in identification strategy 2 shows that the Surplus ratio restriction was not driving the future decay in TFP. Instead, it further supports the argument that the FTS shocks predate, by a significant period, an imminent fall in TFP.

Identification strategy 3 is different from identification strategy 2 and strategy 1 concerning restrictions based on FTS shocks. The restriction imposed (in strategy 1 and 2) on the real rate response to be less than zero on the FTS shock impact is removed, and this variable is made unrestricted to the FTS shock. But the zero restrictions to TFP and Surplus ratio variables are kept, as they were in Identification strategy 2. Therefore Price of risk (Bond minus Equity prices) is the only variable that is restricted to increase on the impact of a positive FTS shock. The impact restrictions for TFP shocks are unchanged and are kept the same as they were in identification strategy 1 and identification strategy 2. The monetary shock has restrictions for only the Real rate to be positive for the impact period (shock period and the next period) horizon and has zero restrictions on the impact of the TFP series and the Surplus ratio. The impact response from the variable of interest is unrestricted and kept agnostic.

Thus Identification strategy 3 is a way to distinguish between the responses of FTS and Monetary Shocks. By removing the restriction on Real rate upon the impact of FTS shocks, we would like to ascertain if the shocks that we have identified are FTS shocks and are not driven by some exogenous shocks that impact both FTS and Monetary policy shocks similarly.

The results from identification strategy 3 are represented by triangles in Figure 1.16 on page 67. They differ from our findings from the results of the first 2 identification strategies. Once we take out the impact restriction on Real rate to be positive for an FTS shock, the real rates' median response is only slightly positive +0.1% on impact, whereas the fall in CPI upon impact from the FTS shock is -0.2%. This result should not be puzzling as it shows that the jump in Real rates from FTS shock is cut short by a simultaneous response of the Monetary policy, or a cut in nominal rates. It signifies that FTS shocks are co-incidental with cuts in policy rates, which further strengthens our initial hypothesis of the FTS shock being a pre-cursor of worsening economic climate.

By removing the restriction on response of Real rates in Identification Strategy 3, we can also remove the constraints on contemporaneous changes in monetary policy in response to FTS shocks. This leaves the interest rate policy some scope to be pre-emptive. Therefore the cuts in real rates in this strategy occur earlier compared to rate cuts in strategy 1 and 2. Through this, we witness that the delay in the response of TFP to FTS shocks is also shortened. Evidently, the TFP begins to decline after 5 periods, whereas it declines 8-10 periods after FTS shocks in identification strategy 1. Hours, output, consumption, and investment all exhibit a faster decline (or a shorter slump) and reach the minimum around the same time, i.e., 5 periods after the shock, when total factor productivity growth becomes less than zero. The main result from different identification strategies reinforces the conviction of results of identification strategy 1, which is that the innovations in Price of Risk (Bond minus Equity prices) predate a general economic slump, which in around 8-10 quarters also leads to an eventual decline in TFP.

Next, we look to address the possible criticism that the chosen data period in our benchmark configuration period of 1983:Q1 to 2019:Q3 is heavily influenced by the increase in the supply of safe assets around the world since the global financial crisis and by the impact of the crisis itself. This criticism is addressed by curtailing the analysis to the pre-crisis period of 1983:Q1 to 2007:Q2, which is also the Great Moderation period. We also consider the sensitivity of the results to the pre-Great Moderation period of 1954:Q3 to 1978:Q4, intending to understand the influence of great moderation on the strength of Flight to Safety shocks.

1.6.2 Impact of Great moderation period

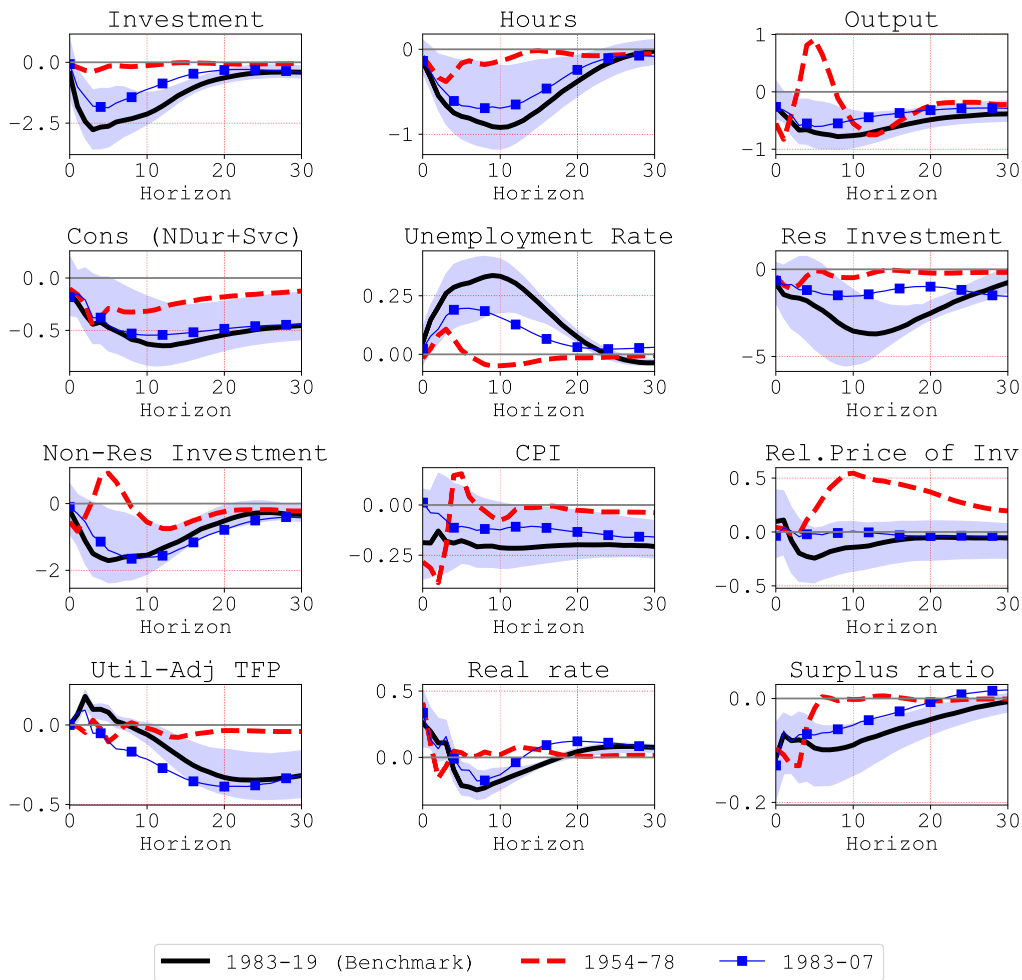
From the mid-80s to global financial crises, the Great moderation period represents a period in the macroeconomic history of developed economies when the incidence and volatility of business cycle fluctuations were significantly reduced from the decades that preceded it, 1954 to 1978. Structural developments in public policy and the Federal reserve's policy commitment and communication strategy were influential in bringing out this change in business cycle incidence and volatility during the Great Moderation period. It is argued that economic agents were better able to form expectations about future economic and policy uncertainty during great moderation than in the decades that preceded it.

The results of this empirical study show us that FTS shocks breed in pessimism about the general economic climate that predates an eventual decline in future TFP. One of the reasons this is possible is if FTS shocks signal households to expect future TFP and economic growth to be weaker. If these expectations are well-formed, then in line with the Great moderation literature, they must be significantly well-formed in the Great moderation era 1983-2007 than in the volatile era 1954-1978. Figure 1.17 on page 71 shows the median impulse responses to FTS shocks for business cycles variables of macroeconomic variables of interest for the pre-Great moderation period (1954-1978) in dashes, and the Great Moderation period (1985-2007) in squared line. This figure also compares these two periods with the benchmark period (1985-2019) median response given in a solid line. The shaded area depicts 68% confidence bands for the benchmark period responses.

It is clear from the results that the FTS shock has a limited impact during the pre-Great moderation period (dashes in Figure 1.17). This suggests that FTS shocks have been more critical in the Great moderation period. Due to the available technology, structural policy changes and low inflation in this period, it was easier to form expectations about the future TFP growth. Moreover, through improved trading opportunities and increased participation from retail investors, there has been an increase in availability and awareness about risky and safe assets in the Great moderation period as compared to the earlier period. The impact of the global financial crisis is also visible through the comparison of Great Moderation (squares) and the benchmark period (solid) results in fig. 1.17. The inclusion of post-Global financial crisis data accentuates the responses in unemployment and residential investments. What stands out most from the result is that during the Great Moderation, the response of investment, hours, real rates, utilization-adjusted total factor productivity is similar in scale and scope as predicted by the benchmark model. It reinstates our conviction in the results that FTS shocks generate business cycle fluctuations.

Thus from the various identification strategies, periods, and business cycle variables employed in this empirical study, we can confirm that the identified FTS shocks of the Price of risk (Bond minus Equity price) series are linked to

Figure 1.17: Sensitivity of impulse responses to different time periods



Notes: Sensitivity of impulse responses to FTS shocks during Benchmark period 1983:Q1 to 2019:Q3 (Solid) with responses from Pre-Great moderation period 1954:Q3 to 1978:Q4 (Dashes) and Great Moderation period 1983:Q1 to 2007:Q2 (Squares). 68% confidence band of Benchmark period are shaded. Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. 'Cons (NDur+Svc)' is Consumption of Non-durable goods and Services. 'Res' is residential, 'Non-Res' is non-residential, and 'Rel.Price of Inv' is Relative price of investment in terms of consumption.

an increase in investors' risk aversion. They enforce a chilling effect on economic activity and predate a decline in total factor productivity. The results reinforce our initial contention for undertaking this empirical study, which was that FTS shocks have an immediate and long-term negative impact on economic activity esp. Investments. The delayed response of TFP to FTS shocks brings to light the two channels through which FTS impacts the business cycles. Either FTS shock is an over-correction to a speculative bet gone wrong. Or FTS is grounded in rational expectations and represents a risk-aversion shock that induces a re-allocation of capital to safer assets, usually less productive. This increase in pessimism or fear of risk in a sufficient number of agents leads to an imminent long-term decline in economic activity. It is neither the objective nor the scope of this study to decide which of the two expectations channel is more crucial. However, the large and varied set of data included in this empirical exercise and the different assumptions imposed through identification strategies incline us towards the latter.

1.6.3 Comparison with Shiller data

Comparable series to the Price of Risk series that could account for a part of the term and inflation premium, also have the feature that relative price of safer asset was higher in the 80s and that risk taking has been relatively cheaper in the recent past. For this analysis I additionally consider Robert Shiller's (2015) series which includes:

(A) **LT 10y-earn px:** Long term real excess return (Bond minus Equity) series that adjusts long term nominal yields for growth in CPI,

(B) **TR 10y-earn px:** Total return index (bond minus equity) which compares the price difference between total return real bond index and total return real equity index,

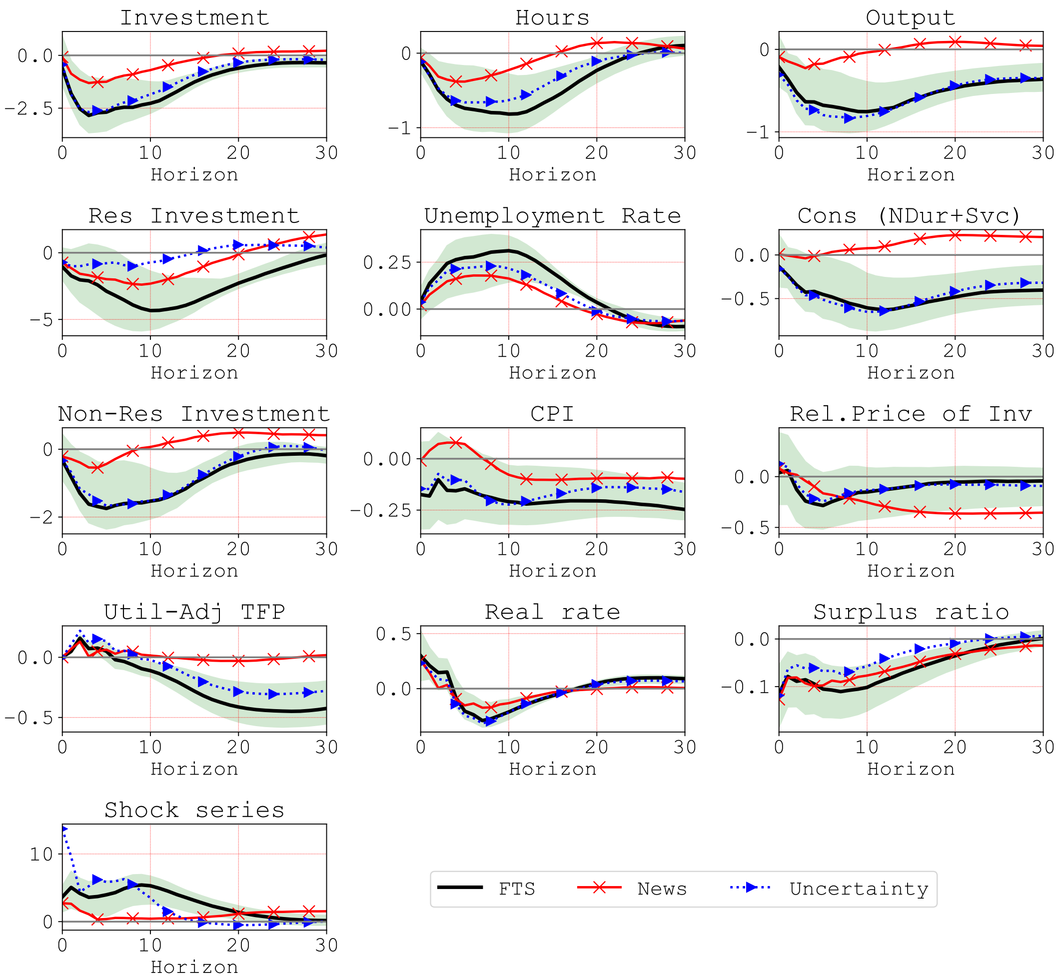
(C) **10y-Cape px:** Excess real bond returns which subtracts the cyclically adjusted price to earnings (CAPE) yield from real bond returns, and

(D) **CAPE:** a series on CAPE itself that has been a hallmark of long term price to earnings ratio.

Additional adjustments[13] are made into these four series to make them compatible

[13]The alternative series from Robert Shiller's database are adjusted as: A. LT 10y-earn px: Log Price difference between assets paying Long-term (LT) 10y yield and LT S&P Earnings

Figure 1.18: News, Uncertainty and FTS shocks



Notes: Comparison of impulse responses to FTS shocks in benchmark model (Solid) and their 68% Confidence band (shaded) with impulse responses to News shocks (Cross) and Uncertainty shocks (Right arrow). The fourth variable in the benchmark model is 'Price of Risk' and it is replaced with 'Price of Equity' and 'Uncertainty' series to obtain comparable results to the News and Uncertainty shocks. Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock. 'Cons(NDur+Svc)' is Consumption of Non-durable goods and Services. 'Res' is residential, 'Non-Res' is non-residential, and 'Rel.Px of Inv' is Relative price of investment in terms of consumption.

with this chapter' analysis. Of these, the first series (LT 10y-earn px) is the total return series showing a big spike around the global financial crisis (See Figure A.6 on page 262). These alternate series using the identification strategy 1 explain around 15-40% of forecast-error variance decomposition which is much lower (See Figure 1.19 on pages 75 - 76 for comparison) than the 50-60% FEVD decomposition that is explained by the Price of Risk series of this chapter. Of all the 4 Shiller's series that are considered the (LT 10y-earn px) Long term cpi-adjusted bond yield minus long term cpi-adjusted earnings yield series (see fig. 1.19a) explains the maximum, around 40% of forecast error-variance in investments and related variables. This series is similar in composition to the Price of risk that we have considered in this chapter. Shiller adjusts the long term bond and earnings yield for growth in cpi, like this chapter does it for the Price of risk series, but Shiller's series is only available till 2011, as it adjusts for the long-term future earnings yield of the equity index.

1.6.4 News, Uncertainty and Risk premium shocks

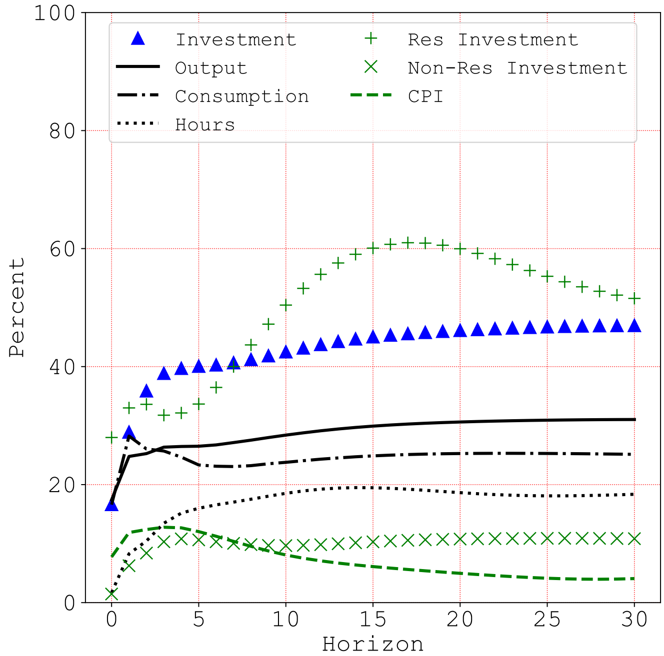
In this section, we check for the robustness of the identification strategy. The analysis starts by looking to answer the criticism that information present in Flight to Safety shocks identified in this chapter is comparable to other News and Uncertainty shocks. Therefore, pursuing the Flight to Safety line of inquiry may not extend our understanding of the business cycle by much. Beaudry and Portier (2006) and Beaudry, Nam, and Wang (2011) uncover the effect of news shocks on the business cycles by identifying news surprises as the shocks to an equity index (S&P 500) series. One of the key results from that analysis is news shocks predate fall in TFP by a couple of quarters. Following their classification of choosing equity index to identify news shocks, we make a comparative analysis in this chapter. The second variable in the benchmark model is replaced from the 'Price of risk' series to S&P 500, and sign and zero restrictions set out in Identification strategy 1 are used to identify 'News' shocks to the latter series.

Similarly, another comparison is made by replacing the 'Price of risk' series in the benchmark model with Bloom (2009) Economic policy Uncertainty (EPU) index, and sign and zero restrictions set out in Identification strategy 1 are used

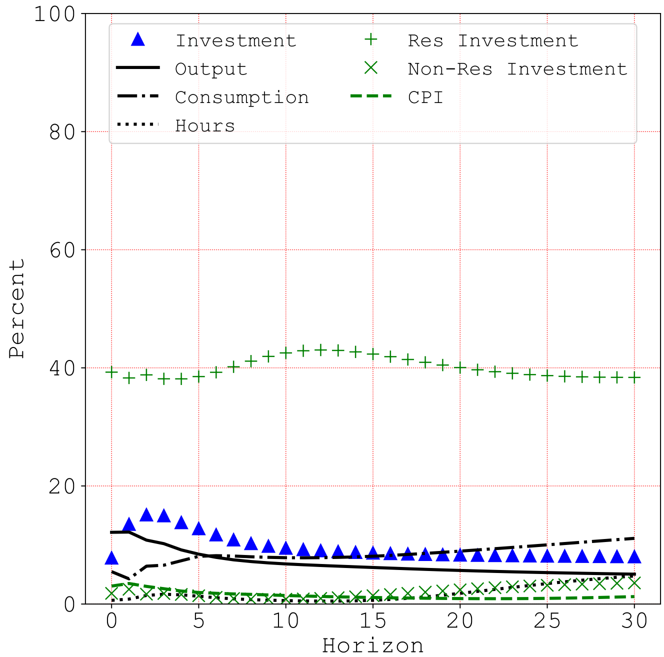
yield, B. TR 10y-earn px: Log Price difference between Total return (TR) 10y bond index and S&P index, C. 10y-Cape px: Log Price difference between assets paying 10y bond yield and Cyclically adjusted S&P Price to Earnings (CAPE) yield, D. CAPE: Log Price of asset paying Cyclically adjusted S&P Price to Earnings (CAPE) yield.

Figure 1.19: FEVD (%) explained by shocks to alternative Price of Risk series

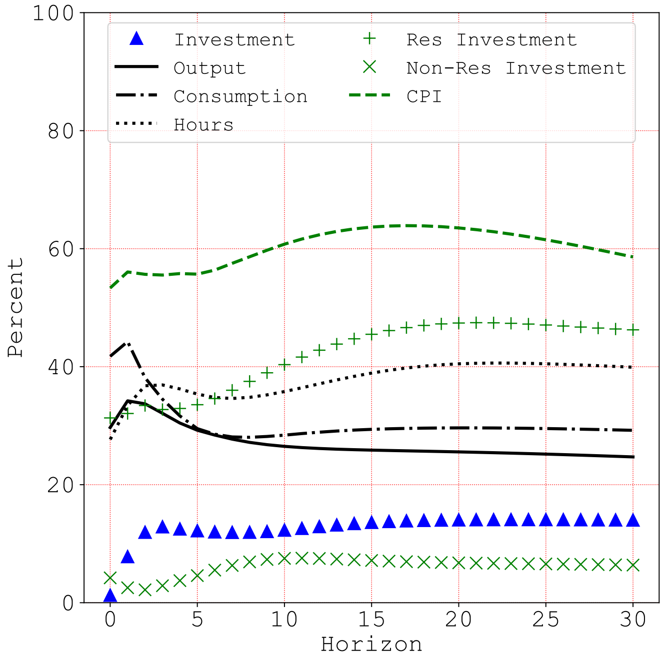
(a) LT 10y yield - Earnings px



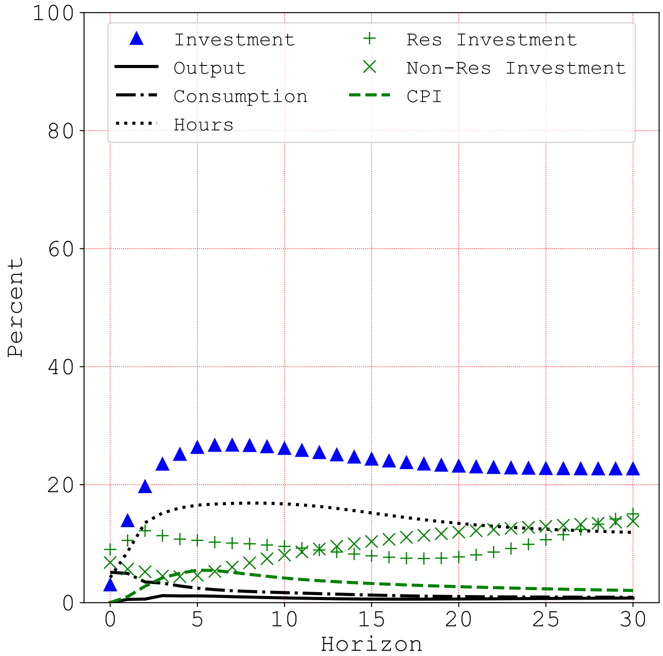
(b) TR 10y yield - Earnings px



Notes: The k-step ahead Forecast error variance decomposition (FEVD %) explained by FTS shocks in series that can act as alternative to Price of Risk series. The alternative series are developed form Robert Shiller's database as: A. Price difference between assets paying Long-term (LT) 10y yield and LT S&P Earnings yield, B. Price difference between Total return (TR) 10y bond index and S&P index, ...continued on next page...



(c) 10y yield - CAPE px



(d) CAPE

Notes (continued):........ C. Price difference between assets paying 10y bond yield and Cyclically adjusted S&P Price to Earnings (CAPE) yield, D. Price of asset paying Cyclically adjusted S&P Price to Earnings (CAPE) yield. The 5 variables in the benchmark model are: TFP, alternative Price of Risk series, Real rates, Surplus Ratio and Investments and it is identified using Sign and Zero restrictions Identification Strategy 1. In other iterations of the 5-variable model, 'Investment' is replaced with other macro variables of interest. The result of 'Investment' and only those variables that replace 'Investment' in the benchmark VAR are reported.

to identify 'Uncertainty' shocks to the latter series.

The impulse response comparison of thus obtained news shocks and uncertainty shocks with the Flight to Safety shocks is made in Figure 1.18 on page 73.

The careful reader may notice the slight difference in the impulse responses for FTS in Figure 1.18 with respect to the ones presented in the Results section earlier. This is because the impulse responses in Figure 1.18 are for the period 1985-2019, which is the entirety of the time period for which Bloom (2009) EPU data is available.

Using only the information present in Equity prices (S&P500) for identifying News shocks, we miss out on much information in the Price of risk series. An impact of that loss of information is visible in impulse responses comparison of News shocks with FTS shocks. The business cycle responses to News shocks are less pronounced and short lived in comparison to the Flight to Safety and Uncertainty shocks. One of the shortfalls of Uncertainty based explanations of business cycle Bloom (2009) is that they have a short-lived impact. The response of business cycle variables to Uncertainty shocks is similar in direction to their response to FTS shocks, but it is significantly short lived and smaller in magnitude for some of the key variables such as residential investments, hours, unemployment, and surplus ratio. The explanatory power of news in Beaudry and Portier (2006) and Beaudry, Nam, and Wang (2011) comes from news disturbances in leading the eventual change in TFP by 2-4 quarters. Through the lens of the identification strategy assumed in this chapter, we see that news shocks have minimal impact on the TFP. As discussed earlier, the FTS shocks lead to decline in the TPF growth rate by 8-10 quarters. From the impulse responses of TFP to uncertainty shocks (in fig. 1.18), we can say that the median response of TFP to uncertainty shocks is also delayed by 8-10 quarters. This could signify that the uncertainty shocks are an even earlier warning system for TFP decline than FTS shocks. Alternatively, since the magnitude of response of TFP to uncertainty shocks is smaller than its response to FTS shocks, one could counter-argue that it is the latter (FTS) shock that is leading TFP decline through an increase in uncertainty.

Before attributing a lot of the explanatory power to uncertainty shocks, we must also consider that the Economic policy uncertainty index (EPU), which measures uncertainty shocks, is back-calculated by counting policy uncertainty related words in news articles (Bloom, 2009). The EPU index already accounts for most of the speculation and expectation feeding into the TFP decline and

Flight to Safety. There is a significant overlap between these, and the impulse responses in Figure 1.18 may not be sufficient to resolve this confusion. So I use the Granger-causality test [See Table 1.5 on page 78] for identifying the lead-lag relation between FTS and Uncertainty shocks. The exercise shows that the Price of risk series, responsible for FTS shocks, Granger causes Uncertainty series at lags 2-3 while the Uncertainty series does not Granger cause the Price of risk at those lags. It demonstrates a lagged relationship between the two where the long-run impact of the Uncertainty shocks on TFP is a derivative of lagged FTS shocks.

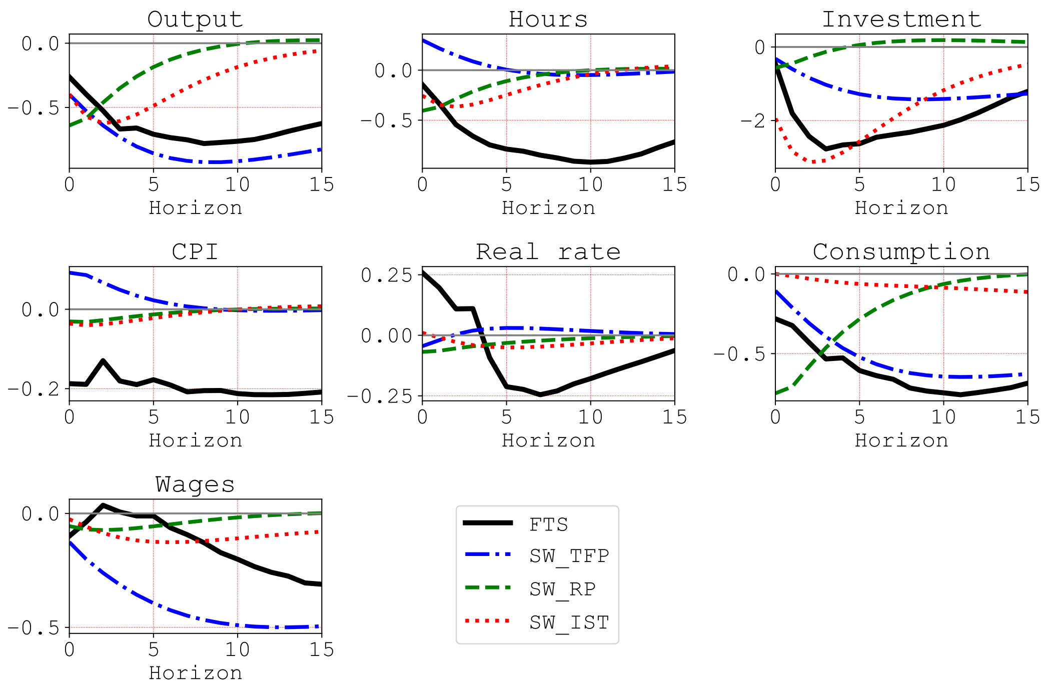
Table 1.5: Granger Causality results for FTS shocks

A. Does lagged 'Price of Risk' series Granger cause any of these variables							
lags	EPU	S&P 500	1y-10y	Baa-Aaa	TFP	TFP(EqDur)	Inv Share
1	0.658	0.709	0.134	0.09	0.017*	0.001**	0.865
2	0.016*	0.82	0.05*	0.0**	0.022*	0.002 **	0.96
3	0.048*	0.0**	0.101	0.002**	0.048*	0.008**	0.94
4	0.091	0.0**	0.067	0.003**	0.056	0.009**	0.57
5	0.146	0.0**	0.063	0.007**	0.087	0.006 **	0.657
6	0.063	0.0**	0.045*	0.002**	0.154	0.011*	0.586
7	0.121	0.0**	0.126	0.004**	0.136	0.022*	0.69
8	0.121	0.0**	0.135	0.005**	0.121	0.031*	0.67
12	0.207	0.0 **	0.18	0.009**	0.223	0.083	0.403

B. Does any of these lagged series Granger cause 'Price of Risk'							
lags	EPU	S&P 500	1y-10y	Baa-Aaa	TFP	TFP(EqDur)	Inv Share
1	0.334	0.713	0.28	0.236	0.579	0.478	0.695
2	0.081	0.432	0.159	0.323	0.277	0.131	0.718
3	0.062	0.494	0.281	0.263	0.063	0.007**	0.802
4	0.11	0.568	0.348	0.355	0.086	0.011*	0.831
5	0.202	0.571	0.034*	0.446	0.091	0.011*	0.864
6	0.166	0.631	0.07	0.546	0.014*	0.025*	0.605
7	0.151	0.795	0.046*	0.756	0.023*	0.033*	0.748
8	0.185	0.804	0.055	0.282	0.051	0.067	0.758
12	0.649	0.38	0.19	0.35	0.113	0.247	0.666

Notes: p-values are reported. * signifies 95% confidence, ** signifies 99% confidence. EPU: economic policy uncertainty index. 1y-10y: liquidity spread between 1 year and 10 year yields. Baa-Aaa: corporate bond spread. TFP(EqDur): TFP in Equipment and Durables consumption. Inv Share: equipment and consumer durables share of total output.

Figure 1.19: Comparison with Smets & Wouters (2007)



Notes: Comparison of median impulse responses to FTS shocks from Benchmark model (Solid) with updated (to 2019:Q3) results from Smets and Wouters (2007) model. SW_TFP is negative shock to TFP (Dash-dots), SW_RP is positive shock to Risk Premium, and SW_IST is negative shock to Investment-Specific Technology. Y-axis label is percentage points and X-axis label is Time (in quarters) horizon after the shock.

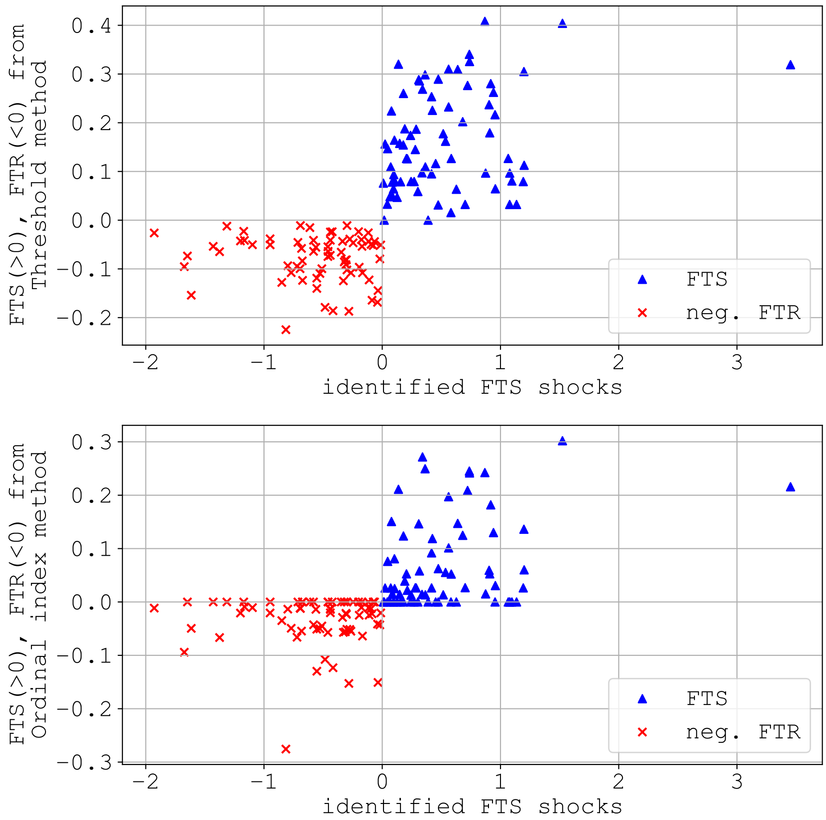
The Granger causality relationship of Price of Risk to News shocks, estimated from the S&P 500 series, is even stronger in comparison and runs to 12 lags. Table 1.5 on page 78 also reports Granger causality results from FTS to time-series, which are similar to the Price of risk developed in this chapter in containing information about Flight to Safety and which have been explored in literature for capturing sentiments and uncertainty about investments. These are liquidity spread between 1-year and 10-year yields (1y-10y), corporate bond spread (Baa-Aaa), TFP total, TFP in Equipment, and Durables consumption [TFP(EqDur)] and equipment and consumer durables share of total output (Inv Share). There is significant Granger causality running from the Price of Risk series to these alternate series. None of the series except for TFP in Equipment and Durables consumption [TFP(EqDur)] series Granger causes the Price of risk, and even for TFP(EqDur) series, the Granger causality is significant for lags greater than 3. However, the Granger causality running from the Price of risk is significant even for shorter lags.

In Figure 1.19, the findings of this chapter are compared with the established results from workhorse DSGE models equipped with investment technology and risk premium shocks (Smets and Wouters, 2007). The impulse response impact of adverse shocks to TFP, negative shocks to investment-specific technology, and positive shock to risk premium in an updated 2019 model version of Smets and Wouters (2007) are slow and less severe when compared with the impulse responses to FTS shocks studied in this chapter. The impulse responses to Smets and Wouters (2007) model also have the typical macroeconomic puzzles, such as a negative correlation of output, consumption, and investment with hours, which the results from the FTS shocks identified in this chapter seem to avoid.

The identified shocks from the benchmark configuration compare well against the FTS and FTR obtained by alternate methods such as the Ordinal index approach and the Threshold approach (Baele, Bekaert, Inghelbrecht, and Wei, 2013). Comparing the two methods with the identified shocks to the Price of risk series of the benchmark VAR is made through scatter plots in Figure 1.20 on page 81. It shows two interesting phenomena. Firstly, that during the (quarterly) periods when the VAR identifies a positive shock to the Price of risk series, there is also a higher likelihood of FTS days in those periods. And secondly, the likelihood of Flight to Safety days is higher than Flight to Risk days' likelihood. Obviously, the measure of FTS and FTR likelihood, which are calculated using Threshold (Appendix A.2.2) and Ordinal index (Appendix A.2.3) method are dependent on the selected parameter (κ or k), and a higher choice of that parameter would limit the likelihood of observing the extreme flight of capital in either direction (towards risk or safety). For the purpose of this analysis, it is important to note that in the Threshold method, the κ is chosen to be equal to 1, which results in 11% of total days being classified as FTS, and in the Ordinal index method, a choice of k is also equal to 1 gives 3.4% FTS days. Detailed methodology of finding the FTS and the FTR days using the two methods is made in this chapter's Appendix A.2.1. A comparison of FTS and FTR days using the ordinal index method with k of 1, as shown in Figure A.4 on page 260, also provide us more significant and more frequent spikes during FTS days than during FTR days.

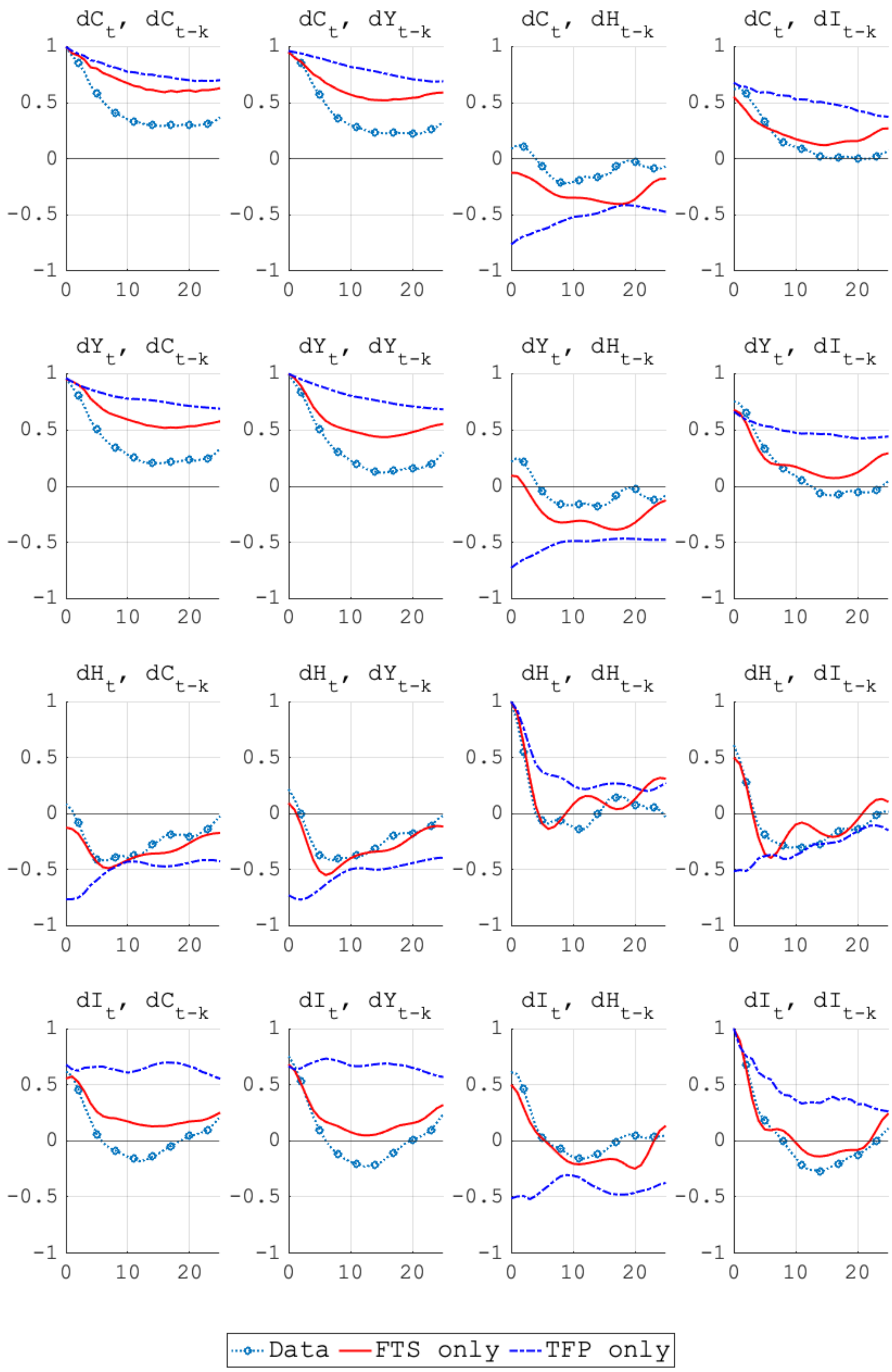
Lastly, we take a look at the correlation between the obtained shocks and the data, which is presented in the correlogram [See Figure 1.21 on page 82]. This investigation shows that the benchmark model fitted with only FTS shocks captures most of the correlation in data, which the model fitted with TFP shocks only fails to do.

Figure 1.20: FTS/FTR from Threshold and OI method



Notes: Scatter plot comparing the median structural shocks to price of risk series from benchmark VAR for period 1983:Q1 to 2019:Q3 (X-axis) with Quarterly average of FTS probability and negative (neg.) of FTR probability values (both on Y-axis). The FTS and FTR are calculated using Threshold method with $\kappa = 1$ in top figure and are calculated using Ordinal index method with $k = 1$ in bottom figure. The calculations are discussed in detail in Appendix A.2.1. FTS are only reported for quarters when identified shocks were positive, while negative (neg.) of FTR are only reported for quarters when identified shocks were negative.

Figure 1.21: Correlogram of Data and Model with FTS and TFP shocks



Notes: Y-axis: Cross-Correlation of the growth rate of Consumption (dC), Output (dY), Hours (dH) and Investment (dI) in US Data and the benchmark model fitted with only Flight to safety (FTS) shocks and TFP shocks identified using Sign and Zero restrictions discussed in Strategy 1. X-axis: k lag lengths.

1.7 Discussion

Our understanding of macroeconomic shocks and their propagation has come a long way since the early days of C. A. Sims (1980a) and Kydland and Prescott (1982). Most of the US business cycles' features have been well established in the Real business cycle literature. Ramey (2016) has a detailed discussion on this topic. The key results are that consumption is less volatile than output, output is less volatile than investment, but output is similarly volatile to hours. The US macroeconomic data series also shows co-movement and pro-cyclicality (J. H. Stock and Watson, 1999). However, not so well established in the RBC literature is the role of asset prices and the causes of business cycles (Rebelo, 2005).

The standard neoclassical paradigm of real business cycle models cannot rationalize the high expected risk premium observed in the US equity market returns or the low risk-free rate in US bond returns. It has resulted in puzzles [for e.g. Mehra (2003) and Donaldson and Mehra (2007) evaluate attempts to resolve the 'Equity premium puzzle' since it was first introduced in Mehra and Prescott (1985)].

With regards to our understanding of the nature and cause of business cycle fluctuations, much progress has been made through structural vector autoregressions and DSGE based research, but the literature is still inconclusive. This section discusses the developments of the last two decades in explaining business cycle fluctuations from the perspective of technology shocks, investment-adjusted technology shocks, the marginal efficiency of investment shocks, financial shocks, news shocks, information shocks, risk shocks, and uncertainty shocks, and compares them in light of the results from FTS shocks, as identified in this chapter.

1.7.1 Neutral Technology shocks

The usual explanations for the cause of business cycles for a long time were policy (monetary and fiscal) shocks and cost shocks. Kydland and Prescott (1982) added technology shocks to the list, and the literature that followed controversially attributed the cause of recessions to a fall in total factor productivity. These technology shocks raised productivity in all factors of production (labour and capital), which is why they are also known as neutral technology shocks.

Galí (1999) fuelled the opposition to this thought of understanding recessions as periods of technological regress. By identifying technology shocks as the only source for long-term labor productivity changes in his model, he could show that 'hours' at least in the short run fell in response to technology shocks. King and Rebelo (1999) and Baxter and King (1999) defend the RBC models by showing pro-cyclicality of hours to output. Francis and Ramey (2006) reconstruct the US historical data from WWII to find that technology shocks in later periods only raise productivity gradually, and this gradual rise provides an inventive to reduce hours in short-run for the anticipation of increasing hours in response to higher productivity in the long-run. Like Galí (1999), Francis and Ramey (2005), too, assume that hours have a unit root, and they show that their results are robust to over-identification tests.

Nevertheless, the assumption of considering a unit root in hours per capita is criticized by Christiano, Eichenbaum, and Vigfusson (2003), and they find that Gali's results fail if hours are used in levels instead of first-differences. Chari, Kehoe, and Mcgrattan (2000) dispel Gali's findings as being driven by measurement error. By bringing additional methods Basu, Fernald, and Kimball (2006) defended Gali's results. They use hours per worker as a proxy for making utilization adjustment to the Solow residual. This series is regularly updated and made available online [see Fernald (2012)]. Shocks to this adjusted series lead to a reduction in hours worked. In contrast, the vector autoregressions approach by Alexopoulos (2011), using a new series based on books published in the field of technology, finds that positive technology shocks lead to an increase in total factor productivity, investment, and to an extent, hours. Unsettled, the debate moves on to look for alternative explanations for business cycle fluctuations.

1.7.2 Investment-specific technology and MEI shocks

A prime alternative explanation to technology shocks is raised in investment-specific technology shocks (IST). As the name suggests, the investment-specific technological productivity shocks raise the productivity of only new capital (investment goods). The impact of a positive investment-specific technology shock (IST) is visible in the price of additional investment, becoming lower in terms of the price of consumption goods. It also increases the real rate of investment.

J. Greenwood, Hercowitz, and Krusell (1997) and (2000) were the first to examine the marginal efficiency of investment (MEI) shocks and the investment-specific technology (IST) shocks in calibrated DSGE models. They found them to account for around 30% of the variation in output. Over the last decade, several influential papers have argued that IST and MEI shocks are the key drivers of business cycles. See Fisher (2006) for long run restrictions VAR, Altig, Christiano, Eichenbaum, and Lindé (2011) for structural VAR, Justiniano et al (2010, 2011) for DSGE model and Araújo (2012) for emerging markets based evidence. Fisher (2006) extends Galí (1999) by assuming that only IST shocks impact relative price of investment and finds these shocks to explain more than 60% of variation in output growth and hours. Bernanke, Gertler, and Gilchrist (1999) establish the role of credit frictions in investment-specific technology shocks.

In models with an RBC core Barro and King (1984) conjecture (or curse) that only technology shocks can account for the observed co-movement among output, consumption, investment, and hours. They conjecture that after a positive investment (IST or MEI) shock, there would be a tendency to raise investment much higher than could be afforded by increasing labour effort. Therefore, any difference between labour income and investment would have to be made up by reducing consumption. This fall in consumption runs against the evidence of pro-cyclicality of consumption with output in data. Therefore investment shocks cannot be the primary driver of business cycles in models with inseparable preferences and an RBC core. Justiniano et al (2010, 2011) overcome this curse by including nominal and real rigidity in their model. Other researchers too, such as Khan and Tsoukalas (2011), J. Greenwood, Hercowitz, and Krusell (2000), Furlanetto and Seneca (2014) have extended their models with capital utilization cost, non-separable preferences, habits persistence, adjustment costs in intermediate inputs and investments adjustment costs. Such extensions introduce some form of real rigidity in their models to generate an explanatory power for investment shocks.

Medium scale DSGE model of Smets and Wouters (2007), on the one hand, confirms that productivity shocks have a negative short-run impact on hours worked. This is consistent even for a flexible price economy as argued by Galí (1999). However, on the other hand, they also raise doubts about the significance of investment-specific technology shocks. 'Demand' shocks, which in their model are captured via risk premium shock, exogenous spending, and investment-specific technology shocks, explain significant variation in macro data but only in the short-term. Whereas, only technology shocks explain most of the long-term variation in data.

In contrast, results from DSGE models of Justiniano, Primiceri, and Tambalotti (2010) and (2011) show that investment shocks are the primary driving force in output fluctuations. Their source of discrepancy with Smets and Wouters (2007) results from the different ways in which they measure investment and consumption in their models. They include durable goods consumption and inventories into Investments. Consumption of durables goods has also been commonly included in the definition of investments by other estimated DSGE models [See, Cooley and Prescott (1995), Christiano, Eichenbaum, and Vigfusson (2003) and Del Negro, Schorfheide, Smets, and Wouters (2007)]. Whereas, the Smets and Wouters (2007) model, unlike Justiniano, Primiceri, and Tambalotti (2010) and (2011), does not include inventories in investments but includes purchases of consumer durable goods in consumption. Both durable goods consumption and inventories are the more cyclical components of GDP (J. H. Stock and Watson, 2016) and including durable goods consumption and inventories in investment makes that investment series more volatile and procyclical.

Justiniano, Primiceri, and Tambalotti (2010) argue that it is the first of the two changes they make that is significant in driving their results. The first change they make to Smets and Wouters (2007) is by excluding durables goods from consumption and including them into investment. This change is majorly responsible for the increase in the variance decomposition at the business cycle frequency, which is explained by the investment-specific technology shock in their paper compared to Smets and Wouters (2007). Whereas in the case of the second change, when Justiniano, Primiceri, and Tambalotti (2010) exclude inventories from investment and move their investment series closer to Smets and Wouters (2007) definition of investment, it leads to two changes. First, the investment shock parameter becomes large, dampening investment shocks' impact on investment, output, and consumption. Second, the preference shock has less impact as there is a weaker response from both output and consumption.

The understanding from investment and the marginal efficiency of investment based literature is that investment-specific technology shocks explain most of the variance in output at business cycle frequency. However, the case for investment shocks as a significant contributor to business cycles, as shown in the significant result of Justiniano, Primiceri, and Tambalotti (2010) and (2011), seems less convincing if we include financial frictions in the model.

1.7.3 Financial frictions and Risk shocks

The rate at which investment goods are converted into consumption goods is identified as the main driver of business cycles, but this rate of change is controlled by the investment adjustment cost parameter in investment-specific technological shocks models, such as Justiniano, Primiceri, and Tambalotti (2011). Investment shocks are also linked to financial markets, and frictions in financial markets determine the pace of conversion of investment goods into consumption goods. Kamber, Smith, and Thoenissen (2015) exploit this feature by introducing a collateral constraint (Kiyotaki and Moore, 1997) into Smets and Wouters (2007) type of model and demonstrate that the ability of positive investment shocks in raising entrepreneurial consumption is attenuated in the presence of collateral constraints.

In the presence of binding collateral constraints, investment shocks' ability as a key driver of business cycles is diminished, and identification of robust investment specific structural shocks in DSGE models is difficult as per Kamber, Smith, and Thoenissen (2015). They also show that with investment shocks, there is no co-movement between consumption and output, going back to the original argument of Barro and King (1984). A positive investment shock lowers the relative price of investment goods, Tobin's Q. When the collateral constraint is binding, the entrepreneur, due to a positive investment shock, loses her collateral value. An increase in borrowing cannot finance a further increase in investment. Therefore, the binding collateral constraint stalls entrepreneurs' borrowing ability, and additional investment can only be made by reducing entrepreneurial consumption.

Risk shocks replace the role of investment shocks in explaining the variance in output data once borrowing constraints are included (as in Kamber, Smith, and Thoenissen (2015)). Their estimated risk premium shock rises sharply at the beginning of each post-war recession. The effective interest rate, as in Kamber, Smith, and Thoenissen (2015) is highly counter-cyclical. In the presence of a favourable risk shock, they find that there is an increase in investment demand. A negative risk premium shock lowers households and entrepreneurs' interest cost, even for those investors for whom the borrowing constraint is binding. This reduction in borrowing rates and lower debt-service cost allows them to undertake additional capital purchases simultaneously, raise their consumption, and avoid the co-movement puzzle of consumption not being pro-cyclical with output. An increase in Tobin's Q following a favourable risk shock loosens the borrowing constraint and further amplifies this mechanism. In this manner, we can see that as financial frictions are included, the significance of Investment specific technology shocks is reduced, and that of risk premium shocks is improved.

Similar mechanisms in Christensen and Dib (2008) and Merola (2015), which include an external finance premium that is impacted by firms' net worth, suggest that the variance decomposition contribution of Investment specific shocks is reduced in the presence of financial frictions. In a related paper, Nolan and Thoenissen (2009) describe that shocks to the financial sector in the form of entrepreneurial net worth play a significant role in business cycles, much more than TFP or monetary shocks, and these shocks are negatively correlated with external finance premium. Equity payouts are pro-cyclical, while debt payouts are countercyclical for US firms, as studied by Jermann and Quadrini (2012). They also show that events originating in the financial sector end up tightening firms' financing conditions and are quantitatively crucial in explaining the dynamics of real and financial variables and contributing to the great recession of 2007-08. Amano and Shukayev (2012) show that risk premium shocks are particularly important in driving the economy to ZLB.

DSGE model in Christiano, Motto, and Rostagno (2014) through agency problems of asymmetric information and costly monitoring (like Townsend (1979) and Bernanke, Gertler, and Gilchrist (1999)) introduces idiosyncratic uncertainty to the way entrepreneurs can convert raw capital into useful capital. Entrepreneurs pay a premium to borrow capital. This premium represents the riskiness of bet on each entrepreneur. By estimating their model using macroeconomic and financial variables, they conclude that shocks to the volatility of idiosyncratic uncertainty or risk shocks account for most fluctuations in GDP at business cycle frequency. The risk shocks-based view of business cycles is compelling; however, in DSGE models without some form of financial frictions, it gets challenging to disengage between risk shocks and the marginal efficiency of investment shocks.

1.7.4 News and Uncertainty shocks

The hypothesis that future technology expectations play an important role in driving business cycles was formalized in Beaudry and Portier (2004). By using stock prices as the basis for forming expectations about future economic conditions and by using two sequential identification schemes; first which makes innovations to stock prices orthogonal to TFP shocks, and second which drives the long term movement in TFP, Beaudry and Portier (2006) demonstrate that their news-driven shocks anticipate TFP growth by a couple of years. Beaudry and Portier (2005), Beaudry and Lucke (2010), Beaudry, Dupaigne, and Portier (2011) reach similar conclusions. Jaimovich and Rebelo (2009) in a DSGE model further posit that news shocks about economic fundamentals generate comovement in aggregate productivity and account for comovement in sectoral productivity as well. However, in a different VAR scheme by identifying news shocks as orthogonal to technology innovation and one which maximizes future variation in technology, Barsky and E. R. Sims (2009) show that the positive wealth effect generated from positive news about future productivity cannot lead to an expansion in RBC models. The increase in consumption and leisure from the wealth effect leads to a fall in output and hours. Suppose instead, because of the high elasticity of intertemporal substitution, the real rate of return effect dominates. In that case, investment and hours increase, but the increase in output does not compensate for the increase in investment, and so consumption falls.

The strength of the news-driven business cycle is also challenged when considering the significant relation between periods of economic downturn and high uncertainty. The volatility of the stock market or GDP is an often-used measure of uncertainty. This volatility surges during recessions. However, this surge cannot be explained by a measure of bad news or an increase in risk aversion (during recessions) alone. Only 1 of the 17 instances of volatility jumps from 1962 to 2008 that lowered the expected GDP growth and led to an increase in economic uncertainty was due to 'bad news' (Bloom, 2009). It is not surprising then that the macroeconomic research since the Global financial crisis has emphasized considering uncertainty, volatility, information, and sentiment-based shocks to understand the business cycle fluctuations.

On the one hand, uncertainty negatively impacts growth and spending. Romer (1990) says uncertainty near the Global Depression is responsible for a fall in durable consumer spending. In an influential paper Bloom (2009) depicts the cyclical variation in the standard deviation of firm-level stock returns, which he calls as uncertainty, to be an important determinant of business cycles. Uncertainty results in cautious decision-making on behalf of firms, as they deliberate on hiring and investment decisions since adjustment costs make those decisions expensive to reverse. It also results in cautious decision-making on behalf of consumers, as during high uncertainty, they delay consumption, especially of durable goods. Both these responses also reduce the efficacy of monetary and fiscal policy.

On the other hand, negative growth creates uncertainty. Period of negative growth, or recessions, also raise uncertainty by slowing down trading activity, difficult forecasting ability, policy miscommunication, and hyper-activism. Baker, Bloom, and Davis (2016) show that due to slackness in business activity, there is an increase in micro-level uncertainty since businesses try out new ideas for the reason that they are now cheaper to try. Nakamura, Sergeyev, and Steinsson (2017) using consumption and growth data from 16 OECD countries find that periods of lower growth have high fluctuations in long-run volatility. Income and wages, especially for low-wage earners, show volatility surge during recessions.

Contrary to traditional business cycle models (Kydland and Prescott, 1982), the uncertainty based results from Bloom (2014) and Baker, Bloom, and Davis (2016) further provide evidence that a fall in productivity is an effect of an increase in uncertainty, rather than a response to technological regress. They find that increase in uncertainty has a chilling effect on the productivity-enhancing reallocation of high productivity and low productivity firms. As uncertainty increases, high productivity firms do not want to be aggressive in their productivity allocation, and low productivity firms do not want to cut back on their aggressive propositions. We know that the reallocation of resources tends to drive most of the observed productivity growth. Therefore this hiatus in productivity reallocation during high uncertainty stalls productivity growth and such a stalling effect of uncertainty underlies the theory of uncertainty driven business cycle.

Uncertainty driven business cycle hypothesis finds support from micro-level evidence of Panousi and Papanikolaou (2011), which discusses the impact of CEO level decision making from an increase in uncertainty. CEOs do not make risky investments if their net worth is tied to or highly exposed to the firm's equity valuation and its risk valuation. The structural model in Bloom (2014) estimates that an average uncertainty shock has reduced the GDP by 1.3%. The uncertainty after the great recession was thrice as compared to previous uncertainty shocks. So around 3-4% of the fall in GDP during GFC could be attributed to uncertainty. A sudden increase in uncertainty due to natural disasters, terrorist events explain about 50% of the variation in output (Baker, Bloom, and Davis, 2016) following the event.

Arguably, the business cycle impact of uncertainty is limited only for the short term (Bloom, 2014). In the short run, investment and output reduce, but as uncertainty is reduced and once pent-up demand increases, an increase in hiring and investment leads to a rebound. Similarly, L. Stein and Stone (2010) show uncertainty accounts for a third of the fall in capital investment during 2008-10. However, uncertainty also seems to increase spending in R&D. Many new ventures are undertaken in uncertain times because there are more avenues for growth but less certainty about which avenues would be successful. The surge in R&D activity in 2020 to devise a vaccine that could eradicate the Covid-19 virus is one example.

1.7.5 Reconciling Business cycles with Flight to Safety

In a nutshell, various attempts undertaken over the past three decades to explain the co-movement in macroeconomic variables through business cycles have been successful in some parts. They have led to inconsistencies and puzzles in others. Neutral technology shocks explain most of the fluctuations for output and consumption but do not account for their co-movement with hours. Investment-related technology shocks using real frictions in the transformation of raw capital into meaningful capital can break the Barro-King (1984) curse, explain most of the output fluctuations, and reconcile hours to business cycles, but the results diminish in the presence of financial frictions. Agency cost, collateral constraints, asymmetric information, and risk-based models explain co-movements in business cycles. However, there is difficulty in specifying structural shocks robust to modest changes in these financial frictions. Expectations and uncertainty shocks are promising indicators of the future economic climate and driving economic fluctuations, but their impact on business cycles is mostly limited to the short run.

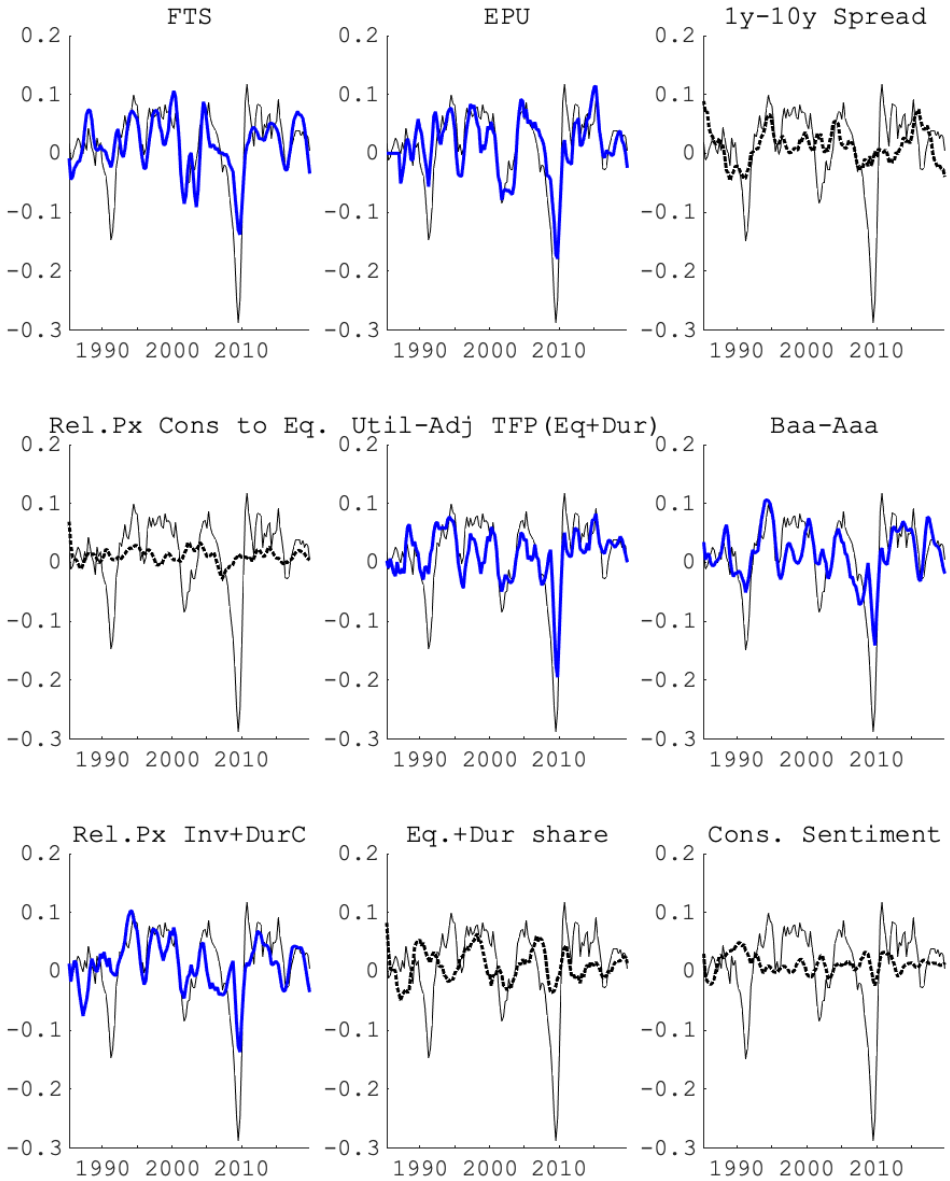
As presented in this chapter, the research on business cycles showcases - Flight to Safety - shocks as the major driver of economic fluctuations in the long run. There is a striking similarity between an increase in uncertainty and Flight to Safety. An increase in uncertainty leads to precautionary savings, which reduces consumption (Bansal and Yaron, 2004) and leads to Flight to Safety. Some of this increase in savings also flies abroad, as Fernández-Villaverde et al. (2009) show that an increase in uncertainty can lead to a flight of capital from small and open economies to larger and more closed ones, such as the United States. Greater uncertainty leads to higher default risk, an increase in risk premia, and makes ambiguity averse investors (Hansen, Sargent, and Tallarini, 1999) act as if the worst possible outcome is expected to occur. If uncertainty led expectations are the key drivers of investment rationale and business cycles fluctuations, then one mechanism where the impact of uncertainty is immediately reflected is FTS.

Measures of uncertainty developed in the last decade are at best proxies of the central phenomenon; one additional potent measure to that list could be FTS. Figure 1.22 on page 93 describes the growth of US investment series that can be explained by shocks to other comparable series. These alternate time-series are similar to the Price of risk developed in this chapter in containing information about Flight to Safety and which have been explored in literature for capturing sentiments, uncertainty, and risk aversion pertaining to investments.

The thick solid lines in that figure represent variables that closely resemble the contribution to Investment growth made by FTS shocks. Shocks to investment-related variables series that are used in investment-specific TFP shocks literature, such as the TFP in Equipment and Durables consumption [TFP(EqDur)], and the Relative price of Investment and Durables Consumption to price of Consumption of Non-Durables and Services (Rel.Px Inv+DurC). These shocks exhibit higher contribution to deviation in investment growth. In comparison shocks related to consumer sentiment and liquidity spread between 1-year and 10-year yields (1y-10y) show lower contribution. There is also a similarity in results from the corporate bond spread (Baa-Aaa) and economic policy uncertainty (EPU) as both these series reflect the impact of Flight to Safety. The significant Granger causality (which we have already noticed in table 1.5 on page 78) running from Price of Risk series to these alternate series reassures the faith in pursuing Flight to Safety as germane to understanding the nature of business cycles.

The main finding of this paper is to show that Flight to Safety has a long term impact on the economy. Flight to Safety shocks can also provide additional fillip of generating a long-term impact on the economy, which is missing in the uncertainty based literature. It would be interesting to establish the relevance of Flight to Safety through an estimated DSGE model, which has a safe and risky investment technology and a precautionary mechanism for investors to allocate between those two. That would be a fascinating avenue for future research.

Figure 1.22: Contribution from sentiment variables



Notes: Investment growth in data (thin solid line in all charts, %y/y) in comparison with the investment series fitted with shocks identified from Sign and Zero restrictions discussed in Strategy 1. Data: 1985:Q1 to 2019:Q3. The benchmark configuration of the model identifies 'FTS' shocks as disturbances to 'Price of Risk' series. In other iterations of the model this is replaced with other interesting proxies for consumer expectations/sentiment and results from shocks to that replacement series of interest are reported. Such as: 'EPU' is Baker, Bloom and Davis (2016) Economic Policy Uncertainty index. '1y-10y spread' is liquidity spread on US 1 year and10 year T-bills. 'Rel.Px Cons to Eq.' is Relative price of Consumption to price of Equipment. 'Util-Adj TFP (Eq+Dur)' is the Utilisation-adjusted TFP in producing Equipment and Consumer Durables. 'Baa-Aaa' is the spread on corporate bonds rated Aaa and Baa. 'Rel.Px Inv+DurCons' is the Relative price of Durables Consumption and Investment to Consumption of Non-Durables and Services. 'Eq.+Dur' share is the share of Equipment and Consumer Durables to Output. 'Cons. Sentiment' is US Consumer Sentiment indicator.

1.8 Summary and Conclusions

This chapter's central conviction is to identify the role of Flight to Safety as the critical driver of business cycle fluctuations. It achieves so by using a Price of risk series that measures the price differential of a safe and risky asset in a five variable structural vector autoregressions model identified using Uhlig (2005) sign based restrictions. The results show that Flight to Safety shocks predate any regress in total factor productivity by several years. Flight to Safety shocks can account for more than 50% of fluctuations in macroeconomic variables at business cycle frequency. This analysis is robust to small alterations in the identification strategy and excluding the post-Global financial crisis period data from this investigation. FTS shocks have gained a more prominent role in the past three decades as expectations formation, and policy communication has improved in that time.

Flight to Safety shocks lead to a long and sustained decline in investment-related macroeconomic variables, the mechanism with which they can achieve so asks several questions from the established tenets of business cycle dynamics. Various business cycle researchers at different stages have posited neutral technology, investment-specific technology, financial frictions, among others, as the main reason for economic fluctuations. Recently there is a focus on looking at expectations formation, in the form of news shocks and uncertainty shocks, as the main drivers of business cycles. This chapter proposes an alternative view that shocks to investor risk aversion lead to booms and busts in business cycles.

A typical FTS episode is signified by a sudden increase in appetite for safe assets with respect to risky investments. The notion that during times of uncertainty, economic agents change their behaviour, by exhibiting caution towards consumption and increasing their savings is quite old and one of the defining reasons for the study of macroeconomics. How FTS shocks impact the long term fluctuations in business cycle variables inclines us to support the view that Flight to Safety works through the *Expectations channel*. Rational, risk-averse investors, in response to an increase in uncertainty about their future income and employment, exhibit precautionary behaviour to increase savings today to smooth out their consumption path and alleviate the impact of realization of a bad state in the future. Investors smooth their consumption by shifting long term savings from risky to safer assets. Such a shift by a large number of people leads to a drop in macroeconomic activity that is visible through a fall in output, investment, productivity, hours, consumption, and in a broader economic gloom.

One of the primary motivations for pursuing this empirical exercise was searching for explanations for the unprecedented fall in output and the long-term decline in investment after the global financial crisis. Through identification and evidence posted, we can safely say that the surge in FTS episodes over the last two decades is a crucial variable of significance in understanding business cycles. The explanatory power of FTS is promising and stands in contrast to the ineffectiveness of standard measures of productivity, sentiment, and expectations in explaining the slow recovery post-global financial crisis. This chapter has merely scratched the surface of the possible channels in which the Flight to Safety mechanism impacts both closed and open economies. There has been another surge in risk aversion during the Covid-19 phase, and therefore improved understanding of the Flight to Safety mechanism would be useful in making effective policies for recovery. In particular, what would be interesting to look out for in future research is an estimated micro-founded DSGE model with the causes and effects of the Flight to Safety phenomenon.

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2. Asymmetry of Optimal Policy in Costly Recessions

Standard analyses using the New-Keynesian (NK) model find that the policymaker cares far more about inflation stabilisation than any other policy objective. Empirical evidence, however, suggests that stabilisation of output, especially during recessions, is a more significant concern for policymakers than what the benchmark NK model implies. Evidence also suggests that the pre-Global financial crisis the Federal Reserve followed an asymmetric monetary policy. It involved slow reaction to increasing rates in expansionary phases but swift response to cutting rates during the recessionary periods. This chapter, therefore, solves for the globally optimal policy using Chebyshev's collocation method in an NK model which features costly recessions. The device used to generate costly recessions is taken from the literature on the equity risk premium which can generate such premia by assuming that household consumption habits make any loss of consumption in a bad state (or economic contraction) particularly harmful for households. The optimal discretionary policy result is non-linear and asymmetric during good and bad states of the economy. During bad states, it is geared towards addressing the output gap ahead of the inflationary gap. The asymmetry of the optimal policy is motivated to correct for the inherent asymmetries in the way households' precautionary behaviour is affected by the various states of the macroeconomy. Ignoring the households' precautionary motives could lead to costly policy mistakes.

2.1 Introduction

Standard analyses using the NK model find that the policymaker cares far more about inflation stabilisation than any other policy objective. The micro-founded utility-based welfare functions of NK models, regardless of whether we use techniques which are linear, fully non-linear or higher-order approximations, lead to a policy reaction function (Taylor rule) where policy rates have far greater sensitivity to deviations[1] in inflation than to deviations in output or to any other policy variable[2]. Policymaker's objective in the linear-quadratic model can be reduced to minimising a loss function that weighs loss from inflation deviations far higher, by a multiple of hundred, than loss due to output deviations.

In contrast, most empirical and practical evidence suggests that policymakers care far more about stabilising output than the benchmark model implies, especially during contractions. Former Bank of England governor Mervyn King consistently overshot the Bank's inflation target. He was dismissive of 'Inflation nutters', who were so obsessed with hitting the inflation target that they were in danger of destabilising the economy[3]. During contractions, policymakers are more concerned about the impending costs to consumers and move their attention away from inflation stabilisation. The US Federal Reserve has a dual mandate of stable prices and maximum sustainable employment, which it modified[4] in August 2020 to allow for inflation to run moderately above 2 per cent for some time, to address the shortfalls in employment from the Covid-19 pandemic and to accommodate for previous periods when inflation had been running persistently below 2 per cent.

This paper seeks to resolve the discrepancy between empirical observations and theoretical policy responses by introducing costly recessions to the standard NK model and solving it numerically for the optimal discretionary policy. The device used to generate costly recessions is taken from the literature on the equity risk premium. A higher premium for investments in risky assets is generated in these models by assuming that household consumption habits make any loss

[1] from trend or target or any other measure

[2] Both Woodford (2003) and Gali (2015) are good resources on this topic.

[3] Source: Financial Times - Don't let the 'inflation nutters' get to you, https://www.ft.com/content/fbcb98dc-016b-11e6-99cb-83242733f755.

[4] Soure: US Fed - Guide to changes in the Statement on Longer-Run Goals and Monetary Policy Strategy, https://www.federalreserve.gov/monetarypolicy/....\guide-to-changes-in-statement-on-longer-run-goals-monetary-policy-strategy.htm.

of consumption in an economic contraction particularly harmful for households [Mehra and Prescott (1985) and Cochrane (2017)]. Therefore in a costly recession, the policymaker, armed with only one policy instrument: the interest rate, cannot simultaneously stabilise both output and inflation fluctuations and faces a trade-off.

The results provide a theoretical foundation for the asymmetric response of monetary policy that is observed in empirical analyses. The optimal policy recommends placing higher significance on output stabilisation when households are in adverse states of the economy. The discretionary policymaker bears a higher cost of inflation in order to obtain a faster normalisation of output deviations during contractions. She chooses this allocation to jointly respond to the inefficient wedge in the economy and the precautionary behaviour of its agents. Both of which are significant for stabilisation policy and are generated by the mechanism of slow-moving consumption habits which makes recessions costly, a feature that is lacking in standard NK models.

2.1.1 Related Literature

The two distortions of monopolistic competition and nominal rigidity in conventional NK models lead to inefficiently low output and to a lack of adjustments in output to changes in preferences and productivity. An optimal policy, therefore, seeks to vary competitive output one-for-one with changes in the economy's natural (or flexible prices) output. However price stability, even though there is no importance to it in the policy objective, emerges as a strong feature of optimal policy [See Clarida, Galí, and Gertler (1999) Galí (and 2015)]. A price stabilisation policy through 'Divine coincidence' (Blanchard and Galí, 2007) also coincides with an efficient level of output. The recommendation for monetary policy in micro-founded NK models is twofold, to minimise a loss function which penalises, through a higher weight any deviations from the objective of price stability and to communicate this objective well to enhance welfare (Woodford, 2003).

The empirical literature, in contrast, has shown that not only do policymakers care for output deviations but their response is asymmetric, as they care far more of output deviations, during recessions. Inflation parameter in Taylor rule shifts between periods of inflation targeting or recession targeting preferences (Cukierman and Muscatelli, 2008); Monetary policy reacts strongly during asset price busts (Ravn, 2014) in comparison to booms. The transmission of monetary policy is asymmetric (Santoro, Petrella, Pfajfar, and Gaffeo, 2014); depending on the sign of intervention (Barnichon and Matthes, 2016). Estimated policy transmission asymmetry in the US (Tenreyro and Thwaites, 2016) is unusually large during booms than in recessions. European central banks after the 1980s made asymmetric interest rate increases when inflation & output are above target in comparison to when they are below (Dolado, María-Dolores, and Naveira, 2005);

Several theoretic models can generate costly recessions. Barlevy (2004) notes that by including different consumer preferences (such as Epstein and Zin (1989)), higher weights of shock persistence, high shock volatility, incomplete markets and individual heterogeneity; the literature arrived a cost of business cycles at around 10% of consumers' lifetime consumption which is significantly higher than Lucas (1990) estimate. There are a few studies to optimal policy that include costly recessions [De Paoli and Zabczyk (2013), Amato and Laubach (2004), Santoro, Petrella, Pfajfar, and Gaffeo (2014)] but they only evaluate the Ramsey policy. The objective of this paper is to fill this gap in costly recessions literature by investigating the optimal discretionary policy in case of a distortionary steady state. The model also has distortions in the from nominal rigidities, habits externalities and revenue taxes.

Slow-moving consumption habits, as pioneered by Campbell and Cochrane (1999), are used to make marginal utility of a consumer from an excess unit of consumption in recessions higher in comparison to booms. It results in a state-dependent risk aversion. Costly recessions make the consumer more averse to risky assets and provide validation for equity risk premia. Most of the literature with consumption habits is based on external habits. Some linearized models suggest there is not much difference in results, whether habits are formed at an internal and external level [See Cochrane (2017) and Dennis (2009)]. However, external habits lead to externalities (Leith, Moldovan, and Rossi, 2012) as consumers are not able to recognize the impact of their individual consumption choices on the aggregate level of habits. It is thus creating a chance for policy intervention to neutralize the externalities by a tax or subsidy (De Paoli and Zabczyk, 2013). In contrast, the empirical (Chen and Ludvigson, 2009) and behavioural (Yogo, 2008) evidence recommends that internal habits are better representative of consumer preferences. The reason for parameterizing habits as mostly internal in this paper is to make the competitive economic output smaller than social planner output and to generate a small positive steady-state inflation that is relevant to observations in data. The model is calibrated to match observations in US macroeconomic data form 1985-2006.

In a setup similar to this paper De Paoli and Zabczyk (2013) use higher-order perturbations to account for investor's precautionary behaviour in explaining the observed equity premia. By including non-linearities inherent in NK models, they shed light on the role of uncertainty in devising optimal policy. They find that ignoring precautionary motive leads to welfare loss and that loss increases with the volatility of shocks in the economy.

This paper differs from De Paoli and Zabczyk (2013) in two aspects. The benchmark economy in their paper is made efficient by including a subsidy. Instead, the benchmark output here is smaller than efficient output, while the tax regime and inflationary bias in the model are relevant to data. De Paoli and Zabczyk (2013) demonstrate the significance of including higher-order approximations around a steady-state in capturing uncertainty and its effects on Ramsey policy. Whereas given the time-inconsistency offered by the policy problem, this paper uses numerical methods (Chebyshev's polynomials) to obtain a global solution for the discretionary policymaker in the absence of any commitment technology.

Structure of the rest of this chapter is as follows. The next section 2.2 describes the model and obtains the maximization conditions for Social planner and Discretionary policymaker. Section 2.3 describes an apparatus to solve the model numerically. Calibration, steady-state and impulse responses are discussed in section 2.4. Section 2.5 discusses the results in light of the inflationary bias, the inefficiency wedge, the output gap and the asymmetry of optimal policy. The last section 2.6 concludes.

2.2 A New-Keynesian model with Habits

We work with a simple variant of the standard new Keynesian model that can account for costly recessions. The model is set up in a way that for agents losing a unit of consumption during recessions has a higher cost than losing a unit of consumption during booms. This is achieved by including a slow-moving subsistence level of habits in household consumption. Contractions or bad states of the economy are represented by periods where agents' consumption is closer to their subsistence level of habits. And vice-versa, Expansions or good states of the economy are represented by periods when the agents' consumption is higher than their subsistence level of habits.

2.2.1 Description

This set-up is similar to De Paoli and Zabczyk (2013), but with the exception that this paper uses contemporaneous or *Keeping up with Joneses* habits. In contrast, their paper had lagged or *Catching up with Joneses* habits. The macroeconomic literature has more commonly relied on the latter *Catching up with Joneses* habits (X_t) of the kind $X_t = \theta C_{t-1}$ i.e. where habits X_t depend only on past consumption and the consumer's utility depends on habits. This habits representation has also been used to successfully generate the hump-shaped impulse response in consumption [for.e.g. in Leith, Moldovan, and Rossi (2012)]. However, such choice of modelling consumption habits also results in large variations in asset prices that are not observed in macroeconomic data. This paper on the other hand relies on *Keeping up with Joneses* habits or which evolve on the basis of both current and lagged consumption, for.e.g. $X_t = (1 - \theta)C_t + \theta C_{t-1}$. Representing habits in this manner has two significant advantages. Firstly, it allows us to avoid the additional state-variable of past period consumption C_{t-1}, which can be very helpful in avoiding the *curse of dimensionality* which has its bearing on enormously slowing down the numerical solution estimation in models that involve more than two states. Secondly, the contemporaneous habits in consumption are slow-moving which makes them useful in generating smooth cycles of risk aversion[5], and in generating a trend[6] that evolves with the growth in aggregate

[5] See Fig.2 in Cochrane (2016) which makes a demonstration of the slow-moving habits and the resulting cycles of risk aversion.

[6] In models with time-varying subsistence level of habits 'external habits' that adapt to current and past consumption, Ljungqvist and Uhlig (2015) show that government interventions

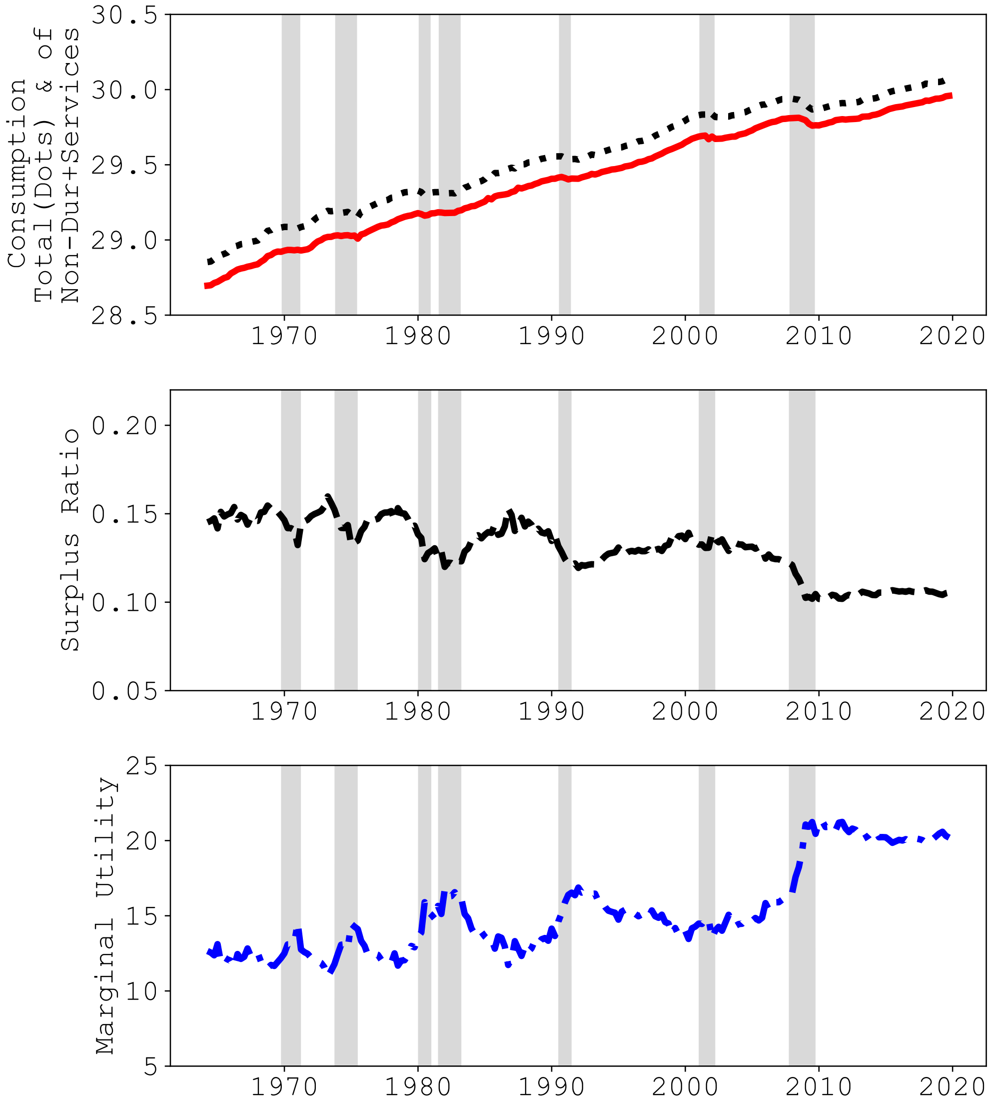
and individual consumption. Besides, the marginal utility from an additional unit of consumption depends on the agent's surplus consumption which is defined as the consumption in excess of subsistence level of habits. Slow evolution of habits leads to smooth transitions in marginal utility and therefore less volatile risk-aversion and asset prices.

For a simple illustration of slow-moving habits and changes in marginal utility, consider Figure 2.1 on page 116 which presents the log of total per-capita consumption and log of per-capita consumption of non-durable goods and services. The latter forms a steady composition of around eighty-five per cent of the total consumption. For the sake of argument consider that consumers find it challenging to give up on consumption habits that are formed in choosing non-durables and services. In contrast, they find it easier to give up on their habits of durables goods consumption. Although, this may not seem an innocuous and straightforward assumption to that every reader (including myself) who may have deep habits in consumption of durables goods, such as mobile phones and laptops, but who could easily change their mode of transport from taking Uber to riding a bicycle. Nevertheless, the argument that consumption habits formed in non-durables and services would be more challenging to give up has its merits. For an example, consider reducing the internet speed of a user of fibre broadband or as Cochrane (2016) puts it in a quote from the wife of a hedge-fund manager, "I cannot fly commercial again".

Figure 2.1 demonstrates that the gap between aggregate consumption and the consumption of non-durables and services in the US reduces during the NBER recession periods. The surplus ratio, i.e. one minus the ratio of consumption of non-durables and services to total consumption too goes down in US recessions and hovers around the 15% region. A marginal utility of consumption function that is inversely related to the surplus ratio rises through the peak to trough periods of US recessions.

that destroy aggregate endowment lead to large welfare gains. Campbell and Cochrane (2015) contradict that by showing that any welfare gains from endowment destruction result from a particular discrete approximation of the underlying continuous-time model which allows for jumps in consumption without allowing for jumps in habits. They show that other calculations of the discrete-time approximation easily overturn those results.

Figure 2.1: Consumption, Surplus Ratio and Marginal Utility for the US



Notes: Dots: Log of US real per-capita Total consumption ($ln(C_{Total})$). Solid: Log of US real per-capita Non-durables and Services consumption ($ln(C_{NDur+Svc})$). Dash: Surplus Ratio calculated as $1 - \frac{C_{NDur+Svc}}{C_{Total}}$. Grey Bars: peak to trough period of NBER Recessions. Dash Dots: Marginal Utility of consumption, calibrated using the parameters of the model as $[ln\,(C_{Total}) - ln\,(C_{NDur+Svc})]^{-\rho}$ for $\rho = 1.37$. Marginal utility is a decreasing function of the Surplus Ratio.

2.2.2 Household

The model economy inhabits a continuum j of agents/households that are uniformly distributed on the unit interval, $j \in [0,1]$. Under the assumption of complete markets, all households through risk sharing seek to maximize similar expected lifetime utility which is additively separable in consumption and labour, given by

$$U_j = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{(C_{j,t} - hX_{j,t})^{1-\rho} - 1}{1-\rho} - \frac{N_{j,t}^{1+\eta} - 1}{1+\eta} \right) \tag{2.1}$$

where $h \in [0,1]$ a parameter that is constant across households, and determines surplus consumption in excess of habits X_t. Let us call h as the 'habit size' parameter. Habits in the model are assumed to be 'superficial', partly 'internal' and partly 'external'. The extent of habits externality depends on the parameter ω for agents. As discussed earlier in the Introduction section, the parameter ω is chosen to keep the output of competitive economy smaller than efficient output and to generate small positive steady state inflation which matches with the observations in data. Agents' habits evolve on the basis of past individual habits $X_{j,t-1}$, their own current period consumption $C_{j,t}$ and aggregate per-capita consumption C_t

$$X_{j,t} = (1-\phi)\left(\omega C_{j,t} + (1-\omega)\, C_t\right) + \phi X_{j,t-1}. \tag{2.2}$$

In this set-up, all habits become external to agents when the internal habits parameter $\omega = 0$ and agents can internalise all habits when $\omega = 1$. The first term in agent's utility function shows that utility is derived from Surplus consumption i.e. Consuming in excess of the Subsistence level of habits, the latter being defined as hX. Second term in Utility function captures disutility from providing $N_{j,t}$ units of labour. β is the discount factor, ϕ is the parameter for habit persistence, ρ is inverse of elasticity of inter-temporal substitution (EIS) and η is inverse of the Frisch-elasticity of labour supply which measures the inter-temporal substitution of willingness to work. The ability of agent's current level of consumption to influence her habit formation is captured through the free parameter $(1-\phi)$. The coefficient of relative risk aversion (CRRA) for household j is:

$$v_j\left(C_{j,t}, X_{j,t}\right) = -C_{j,t}\frac{U_{j,cc}}{U_{j,c}} = \frac{\rho\left(1 - h(1-\phi)\omega\right)}{S_{j,t}} \tag{2.3}$$

where $S_{j,t}$ is the Surplus Ratio[7], a ratio of Surplus consumption to consumption. It is defined as

$$S_{j,t} = \frac{C_{j,t} - hX_{j,t}}{C_{j,t}} \tag{2.4}$$

and can be used to modify agent's utility function to

$$U_j = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{(S_{j,t} C_{j,t})^{1-\rho} - 1}{1-\rho} - \frac{N_{j,t}^{1+\eta} - 1}{1+\eta} \right). \tag{2.5}$$

At this time we can take a moment to ignore the j classification and consolidate all the habits related terminology that have been introduced in this paper in Table 2.1, so it can serve as a quick reference guide for future.

Table 2.1: Habits terminology

Notation	Definition
X_t	Habits
h	Habits size parameter
hX_t	Subsistence level of habits
$C_t - hX_t$	Surplus Consumption
$(C_t - hX_t)/C_t$	Surplus Consumption Ratio or Surplus Ratio
ω	Internal habits parameter
ϕ	Habits persistence parameter

Various states of the economy

In steady-state the level of Habits (from habit eq. (2.2)) should be equal to Consumption. Therefore in the steady state Surplus Ratio (from eq. (2.4)) simply turns out to be $\overline{S} = 1 - h$. Household's marginal utility during expansionary states should be lower than its level in the contractionary states. But the marginal utility of consumption is a decreasing function of Surplus Ratio. So in this utility specification, we could define *good states* (or expansions) as the states of the economy when the Surplus Ratio is higher than steady-state Surplus Ratio of $1 - h$. This is achieved when either consumption is way too much in comparison to habits or habits are way too low in comparison to consumption. In both those cases, the Surplus consumption S should be higher than its steady-state level. *Bad states* (or Recessions) by the same analogy are then defined as the states when agent's Surplus Ratio is below the steady-state level $(1 - h)$. This is the case when the agent's consumption gets closer to the level of subsistence level

[7] The careful reader may notice, that in the previous chapter Surplus ratio was defined differently as one minus the ratio of non-durables and services consumption to total consumption.

of habits (hX), or when subsistence level of habits are high in comparison to consumption. In both those cases, the marginal utility rises, which is another feature of contractions. Formally for j^{th} household, in steady state

$$\overline{S}_j = \frac{\overline{C}_j - h\overline{X}_j}{\overline{C}_j} = 1 - h \tag{2.6}$$

in Bad state

$$\frac{C_{j,t} - hX_{j,t}}{C_{j,t}} = S_{j,t} < \overline{S_j} = 1 - h \tag{2.7}$$

and in Good state

$$\frac{C_{j,t} - hX_{j,t}}{C_{j,t}} = S_{j,t} > \overline{S_j} = 1 - h \tag{2.8}$$

Household's Budget

Consumption allocation for any household j are limited by the period by period budget constraint

$$\begin{aligned}\int_0^1 P_t(i) C_{j,t}(i)\, di + V_t^n B_{j,t}^n + P_t V_t^r B_{j,t}^r + P_t V_t^{eq} F_{j,t} \leqslant W_t N_{j,t} + B_{j,t-1}^n \\ + P_t B_{j,t-1}^r + P_t \left(V_t^{eq} + d_t\right) F_{j,t-1} + \tau_{c,t}\end{aligned} \tag{2.9}$$

which shows that expenditure made in period t cannot exceed wealth and income brought into that period. $\int_0^1 P_t(i) C_t(i)\, di$ is the nominal consumption in period t of all goods of variety $i \in [0,1]$.

Household issue bonds to each other and save by buying one period zero coupon bonds, indexed as $B_{j,t}^r$ and $B_{j,t}^n$, paying one real and nominal unit in the next period, respectively. There is also an option to buy units (shares) of real Stock index F, which is a claim on aggregate profits of all firms in the economy, and pays a real dividend per share at rate d. Variable V^r describes real prices of real bonds, V^n is the nominal price of nominal bonds and V^{eq} is the real price of the real stock index. $\tau_{c,t}$ are per capita transfer from government, which are outside of the decision purview of the individual agent. A distinction in various asset holdings has been made in Table 2.2 to define necessary asset returns.

Consumption aggregation

Household j consumes a continuum of goods $i \in [0,1]$ where each variety in the Dixit and Stiglitz (1977) aggregation has same weight. Household's consumption

Table 2.2: Asset Prices and Returns

Variable	Definition
$B^r_{j,t}$	number of one-period real bonds, paying 1 real unit in period $t+1$
V^r_t	real price in period t of a one-period real bond
$B^n_{j,t}$	number of one-period nominal bonds, paying 1 nominal unit in period $t+1$
V^n_t	nominal price in period t of a one-period nominal bond
$F_{j,t-1}$	number of units (shares) of Real Stock index (S&P) held in period $t-1$
V^{eq}_t	real price per unit of Real Stock index in period t
d_t	real dividends per unit of Real Stock index, paid in period t

basket is

$$C_{j,t} = \left(\int_0^1 C_{j,t}\left(i\right)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{2.10}$$

and $\sigma > 1$ is the constant elasticity of substitution (CES) between any two varieties of i. The representative household is able to choose between different varieties by adjusting their shares in the consumption bundle to minimize its total cost. Optimization of household expenditure results in the following demand function for a good of variety i

$$C_t\left(i\right) = C_t \left(\frac{P_t(i)}{P_t} \right)^{-\sigma} \tag{2.11}$$

where the C_t is the aggregate consumption. While the economy-wide price index P_t is aggregated across all varieties i as

$$P_t = \left(\int_0^1 P_t\left(i\right)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{2.12}$$

The budget constraint for household j can therefore be modified as:

$$\begin{aligned} P_t C_{j,t} + V^n_t B^n_{j,t} + P_t V^r_t B^r_{j,t} + P_t V^{eq}_t F_{j,t} \leqslant W_t N_{j,t} + B^n_{j,t-1} + P_t B^r_{j,t-1} \\ + P_t\left(V^{eq}_t + d_t\right) F_{j,t-1} + \tau_{c,t} \end{aligned} \tag{2.13}$$

Solving for Households Utility optimisation

Considering the specification of the model by which the households cannot fully internalize the effects of their choice on aggregate consumption and their habit formation, we write the Lagrangian for household's problem by using $\lambda^m_{j,t} \geqslant 0$ and $\lambda^x_{j,t} \leqslant 0$ as multipliers of its budget constraint (2.13) and habit formation (2.2) as,

$$\mathcal{L}^j = \max_{\substack{\{C_{j,t},\\ N_{j,t},\\ X_{j,t}\\ B^n_{j,t},\\ B^r_{j,t},\\ F_{j,t}\}_{t=1}^{t=\infty}}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left[\begin{array}{c} \frac{(C_{j,t}-hX_{j,t})^{1-\rho}-1}{1-\rho} - \frac{N_{j,t}^{1+\eta}-1}{1+\eta} \\ +\lambda^m_{j,t} \left(\begin{array}{c} W_t N_{j,t} + B^n_{j,t-1} + P_t B^r_{j,t-1} \\ +P_t \left(V^{eq}_t + d_t\right) F_{j,t-1} \\ -P_t C_{j,t} - V^n_t B^n_{j,t} - P_t V^r_t B^r_{j,t} \\ -P_t V^{eq}_t F_{j,t} + \tau_{c,t} \end{array} \right) \\ +\lambda^x_{j,t} \left(\begin{array}{c} X_{j,t} - \phi X_{j,t-1} \\ -(1-\phi)\left(\omega C_{j,t} + (1-\omega) C_t\right) \end{array} \right) \end{array} \right]. \quad (2.14)$$

First order conditions for all $t \geqslant 1$

$$C_{j,t}: \quad \lambda^m_{j,t} P_t = (C_{j,t} - hX_{j,t})^{-\rho} - (1-\phi)\omega\lambda^x_{j,t} \quad (2.15)$$

$$X_{j,t}: \quad \lambda^x_{j,t} = h\left(C_{j,t} - hX_{j,t}\right)^{-\rho} + \beta\phi E_t\left[\lambda^x_{j,t+1}\right] \quad (2.16)$$

$$N_{j,t}: \quad \lambda^m_{j,t} = N^\eta_{j,t}/W_t \quad (2.17)$$

$$B^n_{j,t}: \quad V^n_t = E_t\left[\beta \frac{\lambda^m_{j,t+1}}{\lambda^m_{j,t}}\right] \quad (2.18)$$

$$B^r_{j,t}: \quad V^r_t = E_t\left[\beta \frac{\lambda^m_{j,t+1}}{\lambda^m_{j,t}} \frac{P_{t+1}}{P_t}\right] \quad (2.19)$$

$$F_{j,t}: \quad V^{eq}_t = E_t\left[\beta \frac{\lambda^m_{j,t+1}}{\lambda^m_{j,t}} \frac{P_{t+1}}{P_t} \left(V^{eq}_{t+1} + d_{t+1}\right)\right] \quad (2.20)$$

show that a unit increase in real wealth (2.15) generates marginal utility from an additional unit of consumption but it also tightens the habit constraint further. The agent loses utility by increasing her habits in the current period (2.16) and it also tightens the habit constraint for future. Marginal loss of utility from an additional unit of labour (2.17) gives W_t units of nominal income. (2.18) and (2.19) are standard bond pricing equations for nominal and real bonds respectively, and (2.20) relates the real price of equity to future price appreciation and expected profitability (dividends) of all firms listed on the stock index. We can define a real multiplier $\lambda^r_{j,t} \equiv \lambda^m_{j,t} P_t$ for consumer's budget constraint, which is the disutility from generating a real unit of wage (2.17). Working from (2.15),

$$\frac{1}{(1-\phi)\omega}\left((C_{j,t} - hX_{j,t})^{-\rho} - \lambda^r_{j,t}\right) = \lambda^x_{j,t} \tag{2.21}$$

$$\frac{1}{(1-\phi)\omega} E_t\left[(C_{j,t+1} - hX_{j,t+1})^{-\rho} - \lambda^r_{j,t+1}\right] = E_t\left[\lambda^x_{j,t+1}\right] \tag{2.22}$$

and (2.16) we can derive an inter-temporal substitution between consumption and real income:

$$(C_{j,t} - hX_{j,t})^{-\rho}(1 - h(1-\phi)\omega) - \lambda^r_{j,t} = \beta\phi E_t\left[\ (C_{j,t+1} - hX_{j,t+1})^{-\rho} - \lambda^r_{j,t+1}\ \right] \tag{2.23}$$

The Consumption Euler Equation can be extracted by dropping the j classification for homogeneous agents as

$$(1 - h(1-\phi)\omega)\ \ (C_t - hX_t)^{-\rho} \ = \ \lambda^r_t + \beta\phi E_t\left[\ (C_{t+1} - hX_{t+1})^{-\rho} - \lambda^r_{t+1}\ \right] \tag{2.24}$$

As the agent is able to (partly) internalise her habit formation, from eq. (2.24) she can see that the utility gains from an extra unit of consumption in current period has to be provided from increasing the lifetime real income or from expected loss of utility in future periods. Household's Labour supply (from Equation (2.17)) as a function of real wage $w_t \equiv W_t/P_t$, upon dropping the j classification for homogeneous households is simply

$$N^\eta_t = w_t \lambda^r_t \tag{2.25}$$

Stochastic Discount factor, hereafter SDF, is derived by removing j classification in homogeneous agents, for nominal pay-offs $M^n_{t,t+1}$ and for real pay-offs $M^r_{t,t+1}$ respectively,

$$M^n_{t,t+1} = E_t\left[\beta \frac{\lambda^r_{t+1}}{\lambda^r_t}\frac{P_t}{P_{t+1}}\right] \tag{2.26}$$

$$M^r_{t,t+1} = E_t\left[\beta \frac{\lambda^r_{t+1}}{\lambda^r_t}\right] \tag{2.27}$$

In absence of internal habits or when $\omega = 0$, habits are determined by a combination of previous period individual habits and aggregate consumption.

$$X_{j,t} = (1-\phi)C_t + \phi X_{j,t-1}. \tag{2.28}$$

The habit constraint (2.2) then is non-binding for the household. Habits are exogenous to household's maximisation problem and the first order conditions with respect to consumption and labour result in

$$\lambda_t^r = (C_t - hX_t)^{-\rho} \tag{2.29}$$

$$N_t^\eta = w_t\,(C_t - hX_t)^{-\rho}\,. \tag{2.30}$$

Equation (2.30) is familiar labour-supply relation which shows that real wage rate is equal to the ratio of the marginal loss in utility from labour to the marginal gain in utility from consumption. The SDFs too (when $\omega = 0$) achieve their standard representations

$$M_{t,t+1}^{n'} = E_t\left[\beta\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\frac{P_t}{P_{t+1}}\right] \tag{2.31}$$

$$M_{t,t+1}^{r'} = E_t\left[\beta\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right] \tag{2.32}$$

In absence of internal habits (2.32) the agent's real discount rate is a ratio of the expected marginal utility between consuming an extra unit tomorrow vis-á-vis consuming that unit today.

Whereas, if the agents are aware (or when $\omega \in (0,1]$) of the impact of an extra unit of consumption today on reducing marginal utility in future then they are less willing to over-consume today, have a lower discount rate, and require a higher expected return from investments. From the Consumption Euler equation (2.24)

$$\frac{1}{\phi}\left((1 - h(1-\phi)\omega) + \frac{\beta\phi E_t\left[\lambda_{t+1}^r\right] - \lambda_t^r}{(C_t - hX_t)^{-\rho}}\right) \;=\; \beta E_t\left[\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right] \tag{2.33}$$

The ratio of intertemporal discounted utitliy on the RHS is decreasing in the weight of internal habits (ω), the size of habits (h) and it demonstrates the precautionary nature of agents consumption choices. The presence of internal habits produces a precautionary wedge in the Consumption Euler equation.

Local-non satiation of preferences ensures that in equilibrium the budget constraint binds with equality, and the structure of constant EIS utility conforms to the usual Inada conditions. Additionally, we also need the no-Ponzi game condition (2.34) to hold with equality in the equilibrium, so that the present discounted value of household's nominal wealth at infinity is non-negative or there is no over-accumulation of debt.

$$\lim_{T\to\infty} E_t \left[\frac{\varpi_{j,T}}{R^n_{t,T}}\right] \geq 0 \tag{2.34}$$

The inada condition builds on eq. (2.18) which gives the one-period nominal rate of return $R^n_{t,t+1} = (V^n_t)^{-1}$, to arrive at the nominal rate of return (2.20) for $T \geq t+1$ as

$$R^n_{t,T} = \prod_{s=t}^{T-1} \left(\frac{P_{s+1}}{P_s} \frac{V^{eq}_{s+1} + d_{s+1}}{V^{eq}_s}\right). \tag{2.35}$$

while the nominal wealth brought by household into the period $t \geq 1$ is defined as $\varpi_{j,t} = B^n_{j,t-1} + P_t B^r_{j,t-1} + P_t \left(V^{eq}_t + d_t\right) F_{j,t-1}$.

2.2.3 Firms

There is a continuum $i \in [0,1]$ of monopolistic competitive firms with zero fixed costs. Productivity $\xi_{y,t}^{\frac{1}{1+\eta}}$ is non-idiosyncratic across firms and follows an autoregressive process (2.37) with $\varepsilon_{y,t} \equiv \log(\xi_{y,t})$ and uncorrelated $i.i.d$ random disturbances $\epsilon_{y,t}$ of mean zero and variance σ^2_y. Each firm produces a variety $Y_t(i)$ by employing labour $N_t(i)$ at the market determined wage W_t.

$$Y_t(i) = \xi_{y,t}^{\frac{1}{1+\eta}} N_t(i) \tag{2.36}$$

$$\varepsilon_{y,t} = (1-\gamma_y)\,\overline{\varepsilon_y} + \gamma_y \varepsilon_{y,t-1} + \epsilon_{y,t} \tag{2.37}$$

$$\epsilon_{y,t} \sim N\left(0, \sigma^2_y\right) \tag{2.38}$$

Firms face the following Rotemberg (1982) style price adjustment costs while adjusting prices in period $t+s$, with ψ being the degree of price stickiness.

$$\frac{\psi}{2}\left(\frac{P_{t+s}(i)}{P_{t+s-1}(i)} - 1\right)^2 Y_{t+s} \tag{2.39}$$

Being monopolistic, firms can choose to sell at a price above marginal cost φ_t which is non-idiosyncratic across i firms as both productivity and real wage rate are non-idiosyncratic. Firms' profit maximization problem in period t is to choose a sequence of prices $\{P_{t+s}(i)\}_{s=0}^{s=\infty}$ to maximize nominal profits $Q_t(i)$:

$$Q_t(i) = \max_{P_{t+s}(i)} E_t \sum_{s=0}^{\infty} M_{t,t+s}^n \left[\begin{array}{c} (1-\tau_{p,t+s})\, P_{t+s}(i) Y_{t+s}(i) - \varphi_{t+s} P_{t+s} Y_{t+s}(i) \\ -\frac{\psi}{2}\left(\frac{P_{t+s}(i)}{P_{t+s-1}(i)} - 1\right)^2 P_{t+s} Y_{t+s} \end{array} \right] \tag{2.40}$$

subject to

$$Y_{t+s}(i) = Y_{t+s}\left(\frac{P_{t+s}(i)}{P_{t+s}}\right)^{-\sigma} \tag{2.41}$$

$$\varepsilon_{\tau,t} = (1-\gamma_\tau)\,\overline{\varepsilon_\tau} + \gamma_\tau \varepsilon_{\tau,t-1} - \epsilon_{\tau,t} \tag{2.42}$$

$$\epsilon_{\tau,t} \sim N\left(0, \sigma_\tau^2\right) \tag{2.43}$$

The nominal stochastic discount factor $M_{t,t+s}^n$ from eq. (2.26) is also firm's nominal discount rate for s periods. Firms also pay $\tau_{p,t}$ an exogenous time-varying revenue tax rate which follows an exogenous process (2.42) with $\varepsilon_{\tau,t} \equiv \log(1-\tau_{p,t})$ and $\overline{\varepsilon_\tau} \equiv \log(1-\overline{\tau_p})$, where $\overline{\tau_p}$ is the steady rate of revenue tax. Gross inflation rate is given by $\pi_{t+s} \equiv P_{t+s}/P_{t+s-1}$ and in a symmetric equilibrium as $P_{t+s}(i) = P_{t+s}$ and all firms employ similar amount of labour so $Y_{t+s}(i) = Y_{t+s}(i')$ and the FOC of firm's optimisation problem which is also the NKPC is given by

$$0 = E_t \left\{ \begin{array}{c} M_{t,t+s}^n Y_{t+s}\left[(1-\sigma)(1-\tau_{p,t+s}) + \sigma\varphi_{t+s} - \psi(\pi_{t+s}-1)\pi_{t+s}\right] \\ +M_{t,t+s+1}^n Y_{t+s+1}\psi(\pi_{t+s+1}-1)\pi_{t+s+1}^2 \end{array} \right\} \tag{2.44}$$

substituting the value of nominal stochastic discount factor from eq. (2.26) we get,

$$\frac{M_{t,t+s+1}^n}{M_{t,t+s}^n} = \frac{\lambda_{t+s+1}^r}{\lambda_t^r}\frac{\lambda_t^r}{\lambda_{t+s}^r}\frac{P_{t+s}}{P_{t+s+1}} \tag{2.45}$$

which after substituting for $s = 0$ gets reduced to the following, familiar NKPC form,

$$0 = (1-\sigma)(1-\tau_{p,t}) + \sigma\varphi_t - \psi(\pi_t - 1)\pi_t + \psi\beta E_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{\lambda_{t+1}^r}{\lambda_t^r}\frac{Y_{t+1}}{Y_t}\right]. \tag{2.46}$$

A rearrangement of the NKPC leads us to the symmetric equilibrium Rotemberg pricing mark-up which is also the reciprocal of firm's marginal cost (φ_t).

$$\varphi_t = \left(\frac{\sigma-1}{\sigma}\right)(1-\tau_{p,t}) + \frac{\psi}{\sigma}\left((\pi_t-1)\pi_t - \beta E_t\left[(\pi_{t+1}-1)\pi_{t+1}\frac{\lambda_{t+1}^r}{\lambda_t^r}\frac{Y_{t+1}}{Y_t}\right]\right). \tag{2.47}$$

In the absence of price-stickiness and revenue taxes, the mark-up for the monopolistic firm reaches its desired level.

$$[\varphi_t]^{-1} = \frac{\sigma}{\sigma - 1} \text{ when } \psi = 0 \text{ and } \tau_{p,t} = 0. \tag{2.48}$$

2.2.4 Market Clearing

In symmetric equilibrium all firms produce the same output, and use the same amount of labour, hence the aggregate output is given as

$$Y_t = \xi_{y,t}^{\frac{1}{1+\eta}} N_t \tag{2.49}$$

and the real marginal cost is

$$\varphi_t = \frac{W_t}{P_t \xi_{y,t}^{\frac{1}{1+\eta}}}. \tag{2.50}$$

Homogeneous households provide an equal amount of labour to all (i) firms and own all (i) firms. After dropping the j subscript in all First-order conditions (from eq. (2.15) to eq. (2.20)), the following aggregated real profits across all firms,

$$\int_0^1 q_t(i)di = \int_0^1 (1 - \tau_{p,t}) \xi_{y,t}^{\frac{1}{1+\eta}} N_t(i)di - \int_0^1 \frac{W_t}{P_t} N_t(i)di - \int_0^1 \frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 Y_t di \tag{2.51}$$

could be accumulated for all homogeneous (i) firms as

$$q_t = (1 - \tau_{p,t}) \xi_{y,t}^{\frac{1}{1+\eta}} N_t - \frac{W_t}{P_t} N_t - \frac{\psi}{2} (\pi_t - 1)^2 Y_t \tag{2.52}$$

or simply

$$q_t = Y_t - \tau_{p,t} Y_t - w_t N_t - \frac{\psi}{2} (\pi_t - 1)^2 Y_t \tag{2.53}$$

The policy maker makes an equal lump-sum transfer $\tau_{c,t}$ to agents, inclusion of which does not affect agents' relative choices and FOCs. By making these modifications to the household budget eq. (2.13) we can derive the aggregate household budget equation as

$$C_t = w_t N_t + d_t F_{t-1} + \tau_{c,t} \tag{2.54}$$

In every period there is a net zero supply of one-period bonds. As there is no entry/exit of firms, so in every period there is a constant number of firms operating in the economy whose common shares $F_{t-1} = F_t$ are wholly owned by the household. Firms distribute all q_t profits as dividends to their shareholders. The aggregate dividend per share d_t received by households is equal to q_t/F_{t-1}. The total transfer from the government to households $\tau_{c,t}$ equates to total taxes collected $\tau_{p,t}Y_t$ by the government from the firms. After substituting for $d_tF_{t-1} = q_t$ in the aggregate household budget eq. (2.54) and for $\tau_{p,t}Y_t = \tau_{c,t}$ in the aggregate firm's profit eq. (2.53) we arrive at the following resource constraint which shows that Rotemberg pricing leads to an inefficiency wedge in utilization of full output.

$$C_t = Y_t \left[1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right]. \tag{2.55}$$

2.2.5 Social Planner equilibrium

In order to get an understanding of the output gap in decentralised economy, we have to determine the social planner equilibrium. By ignoring the nominal rigidities, taxes and habits externalities of the model, a benevolent social planner can derive the first-best outcome. She chooses real allocations to maximise household utility subject to aggregate constraints, and all habits are now internalised. Social planner's problem is to choose for all j households, $\{C_{j,t}, Y_{j,t}, X_{j,t}\}_{t=1}^{t=\infty}$ to maximize each household's utility (2.1) subject to internal habit formation (2.2) and the aggregate resource constraint (2.55); for an exogenous sequence $\{\xi_{y,t}\}_{t=1}^{t=\infty}$ of productivity (2.37). The Lagrangian for the social planner policy problem, using $\lambda_{j,t}^{x^*}$ as the multiplier for respective habit constraint for each household and $\lambda_{j,t}^{y^*}$ as the multiplier for aggregate resource constraint is given by

$$\mathcal{L}^{sp} = \max_{\left\{\begin{smallmatrix} C_{j,t}, \\ Y_{j,t}, \\ X_{j,t} \end{smallmatrix}\right\}_{t=1}^{t=\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \begin{array}{c} \sum_j \left(\frac{(C_{j,t} - hX_{j,t})^{1-\rho} - 1}{1-\rho} - \frac{\xi_{y,t}^{-1} Y_{j,t}^{1+\eta} - 1}{1+\eta} \right) \\ \\ +\lambda_t^{y^*} \left(\sum_j Y_{j,t} - \sum_j C_{j,t} \right) \\ \\ +\sum_j \left[\lambda_{j,t}^{x^*} \left(\begin{array}{c} X_{j,t} - (1-\phi)\omega C_{j,t} \\ -(1-\phi)(1-\omega)\sum_j (C_{j,t}) \\ -\phi X_{j,t-1} \end{array} \right) \right] \end{array} \right\}. \tag{2.56}$$

This representation of the social planner optimisation problem incorporates exclusion of pricing inefficiency wedge in the aggregate resource condition (2.55), and includes individual contribution to aggregate output $Y_{j,t}$ instead of its supply side counterpart $\xi_{y,t}^{\frac{1}{1+\eta}} N_{j,t}$ since this constraint always binds. The following FOCs with respect to social planner choice variables are:

$$\begin{aligned} C_{j,t}: &\quad 0 = (C_{j,t} - hX_{j,t})^{-\rho} - \lambda_{j,t}^{x^*}(1-\phi) - \lambda_t^{y^*} &(2.57)\\ Y_{j,t}: &\quad 0 = \lambda_t^{y^*} - \xi_{y,t}^{-1} Y_{j,t}^{\eta} &(2.58)\\ X_{j,t}: &\quad 0 = \lambda_{j,t}^{x^*} - h\left(C_{j,t} - hX_{j,t}\right)^{-\rho} - \beta\phi E_t\left\{\lambda_{j,t+1}^{x^*}\right\}. &(2.59)\end{aligned}$$

From (2.58), for a non-zero production in social planner equilibrium $\lambda_t^{y^*} > 0$ so the production constraint binds and aggregate output equates to aggregate consumption in the social planner equilibrium. By corollary per capita output should also be equal to per capita consumption, or $Y_t = C_t$.

Proposition 2.2.1. The Social planner equilibrium allocation which has equal units of Consumption, Habits and Output for all j households is efficient.

Proof. For each period t consider an allocation (C_t, X_t, Y_t) s.t. for all j households $C_{j,t} = C_t$, $Y_{j,t} = Y_t$ and $X_{j,t} = X_t$. The efficient social planner equilibrium allocation should satisfy FOCs (2.57 to 2.59). For non-negative output and non-zero productivity, from eq. (2.58) the constraint multiplier $\lambda_t^{y^*}$ is non-zero. So $\forall j$, $Y_{j,t} = \left(\xi_{y,t}\lambda_t^{y^*}\right)^{\frac{1}{\eta}}$. Let's call it Y_t. From the FOC with respect to $C_{j,t}$, i.e. eq. (2.57) $\forall j$ households we can derive $\lambda_{j,t}^{x^*} = \frac{1}{1-\phi}\left[(C_t - hX_t)^{-\rho} - \xi_{y,t}^{-1} Y_t^{\eta}\right]$ which is common across households, let's call it $\lambda_t^{x^*}$. Similarly from eq. (2.59) we can derive the conditional expectation of future habits multiplier to be common across j households as, $E_t\left\{\lambda_{j,t+1}^{x^*}\right\} = \frac{1}{\beta\phi}\left[\lambda_t^{x^*} - h\left(C_t - hX_t\right)^{-\rho}\right]$.

Therefore, the common equilibrium allocation (C_t, X_t, Y_t) satisfies the individual constraints for each j household and is efficient. □

By choosing the common allocation (as per Proposition 2.2.1) we can rewrite the first order conditions (2.57 to 2.59) as:

$$\begin{aligned} C_{j,t}: &\quad 0 = (C_t - hX_t)^{-\rho} - \lambda_t^{x^*}(1-\phi) - \lambda_t^{y^*} &(2.60)\\ Y_{j,t}: &\quad 0 = \lambda_t^{y^*} - \xi_{y,t}^{-1} Y_t^{\eta} &(2.61)\\ X_{j,t}: &\quad 0 = \lambda_t^{x^*} - h\left(C_t - hX_t\right)^{-\rho} - \beta\phi E_t\left\{\lambda_{t+1}^{x^*}\right\} &(2.62)\end{aligned}$$

If for the social planner the habits constraint was non-binding $\lambda_{j,t}^{x^*}$ would be equal to zero, and the optimal allocation from (2.57) for household j would equate marginal utility from an additional unit of consumption to marginal dis-utility from producing an extra unit of output. When the habits constraint binds (from 2.57), the marginal utility of a unit of consumption gets reduced by the shadow price of increasing the stock of habits. The shadow price of increasing habits is given from eq. (2.59) as the sum of the loss of utility in the current period and discounted sum of the loss of utility from future increases of habits.

When the habits constraint binds ($\lambda_{j,t}^{x^*} > 0$), from the Kuhn-tucker conditions we get

$$X_{j,t} = (1-\phi)\omega C_{j,t} + (1-\phi)(1-\omega)\sum_j C_{j,t} + \phi X_{j,t-1}$$

which is aggregated for j households as

$$\sum_j X_{j,t} = (1-\phi)\sum_j C_{j,t} + \phi \sum_j X_{j,t-1}$$

to give

$$X_t = (1-\phi)C_t + \phi X_{t-1}$$

where X_t is per-capita habits.

Proposition 2.2.2. Maximizing a common utility function for aggregate constraints of habits and production the social planner can obtain efficient allocation $(C_t, X_t, Y_t, \lambda_t^{y^*}, \lambda_t^{x^*})$ for all households.

Proof. A social planner maximizing the utility by ignoring the nominal rigidities and other inefficiencies, but obeying the habits and production constraint achieves the first best outcome. All habits are now internalised. The Lagrangian for the social planner policy problem, using $\lambda_t^{\hat{i}}$ as the multiplier for respective constraints is

$$\mathcal{L}^{sp} = \max_{\{C_t, Y_t, X_t\}_{t=1}^{t=\infty}} E_1 \left[\beta^{t-1} \left\{ \begin{array}{c} \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{\xi_{y,t}^{-1} Y_t^{1+\eta} - 1}{1+\eta} \\ + \lambda_t^{\hat{x}} \left(X_t - (1-\phi)C_t - \phi X_{t-1} \right) + \lambda_t^{\hat{y}} \left(Y_t - C_t \right) \end{array} \right\} \right] \tag{2.63}$$

which gives the following FOCs:

$$C_t: \qquad 0 = (C_t - hX_t)^{-\rho} - \lambda_t^{\hat{x}}(1-\phi) - \lambda_t^{\hat{y}} \tag{2.64}$$

$$Y_t: \qquad 0 = \lambda_t^{\hat{y}} - \xi_{y,t}^{-1} Y_t^{\eta} \tag{2.65}$$

$$X_t: \qquad 0 = -h\left(C_t - hX_t\right)^{-\rho} + \lambda_t^{\hat{x}} - \beta\phi E_t \left\{ \lambda_{t+1}^{\hat{x}} \right\} \tag{2.66}$$

that are equal to the efficient allocation in equations (2.60) to (2.62) in Proposition 2.2.1. □

We can eliminate $\lambda_t^{\hat{y}}$ from FOC with respect to C_t and Y_t in Proposition 2.2.2 to get

$$(C_t - hX_t)^{-\rho} - \lambda_t^{\hat{x}}(1-\phi) = \xi_{y,t}^{-1} Y_t^{\eta} \tag{2.67}$$

This is different from Household maximisation result (eq. (2.15)) as the social planner can internalise all habits in the economy. The tightening of habit constraint $\lambda_t^{x^*} \geq 0$ affects labour-leisure equilibrium condition. There is a wedge between marginal utility lost from generating an extra unit of consumption and net marginal benefit received from consuming that extra unit. This wedge is given by $\lambda_t^{x^*}(1-\phi)$ is increasing when the habit constraint binds tightly, or habits are fast-moving, i.e. current consumption has a more significant impact on future habits when ϕ is low. Thus social planner's decision to raise consumption by extra units today (t) has an externality and impacts surplus ratio S_{t+s} in future as well. The inter-temporal allocation (2.59) shows that the wedge (or loss of utility) from tightening the habit constraint today can be written as the discounted sum of future losses.

$$(1 - h(1-\phi))\,(C_t - hX_t)^{-\rho} - \xi_{y,t}^{-1} Y_t^{\eta} = \beta\phi E_t\left\{(C_{t+1} - hX_{t+1})^{-\rho} - \xi_{y,t+1}^{-1} Y_{t+1}^{\eta}\right\}. \tag{2.68}$$

In deterministic steady-state, habits $X^* = C^*$ and the corresponding social planner equilibrium, using $\xi_y^* = 1$ is

$$(1 - h(1-\phi) - \beta\phi)\,(C^* - hC^*)^{-\rho} = (1-\beta\phi)\,Y^{*\eta} \tag{2.69}$$

as $Y^* = C^*$, we get the deterministic steady state output level

$$Y^* = \left(1 - \frac{h(1-\phi)}{1-\beta\phi}\right)^{\frac{1}{\eta+\rho}} (1-h)^{\frac{-\rho}{\eta+\rho}} \tag{2.70}$$

Proposition 2.2.3. The Competitive equilibrium has an efficient steady state when $\overline{\tau_p} = 1 - \frac{\sigma}{\sigma-1}\left(\frac{1-k}{1-\omega k}\right)$, where $k = \frac{h(1-\phi)}{1-\beta\phi}$.

Proof. In Appendix B.2. □

Whether the level of output is greater in social planner equilibrium or in the decentralized equilibrium, this can be decided through monopoly power of firms, degree of habit formation and the prevailing output tax rate. The non-stochastic steady state of competitive equilibrium will be efficient when the prevailing tax-rate is equal to the efficient tax rate: $\overline{\tau_p} = 1 - \frac{\sigma}{\sigma-1}\left(\frac{1-k}{1-\omega k}\right)$, and it will produce more than efficient level of output if taxes are lower i.e. $\overline{\tau_p} < 1 - \frac{\sigma}{\sigma-1}\left(\frac{1-k}{1-\omega k}\right)$. In the benchmark calibration the tax-rate is higher $\tau_p > \overline{\tau_p}$ and that leads to a permanent wedge which cannot be overcome by removing the inefficiencies of nominal rigidities and habits.

Corollary 2.2.4. In absence of habits externalities $\overline{\tau_{p'}} = 1 - \frac{\sigma}{\sigma-1}$ is sufficient to make competitive steady state allocation efficient (conditional on $k \neq 1$).

Proof. Trivial from the above Proposition 2.2.3 after substituting for $\omega = 1$. □

If habits are fully internal ($\omega = 1$) for agents as they are for a social planner, then only distortions from monopolistic competition and taxes reduce the output from its efficient level. For a tax rate in benchmark calibration that is greater than $\overline{\tau_{p'}}$ there is permanent wedge between the competitive output with internal habits and the social planner outcome, that cannot be overcome by removing nominal rigidities. Notice that, as $k < 1$, the tax rate that ensures efficiency of the competitive economy (with $\omega = 1$) is lower than the tax rate were $\omega \neq 1$. From Proposition (2.2.3)

$$\frac{\partial \overline{\tau_p}}{\partial \omega} < 0 \tag{2.71}$$

The inefficiency wedge increases in the proportion of internal habits or ω approaching to 1.

2.2.6 Optimal policy under Discretion

In our model the evolution of habits makes the decision variables history dependent; future policy decisions depend on today's habits. Policy-maker cannot establish commitment over future policy decisions and she would have to re-optimize in each period. The optimal monetary policy under discretion can be described as a set of decision rules for 5 control variables $\{C_t, Y_t, \pi_t, X_t, \lambda_t^r\}$ as functions of 3 state variables $\{X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t}\}$ which maximise the following Value function

$$\begin{aligned} V^D\left(X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t}\right) = \max_{C_t, Y_t, \pi_t, X_t, \lambda_t^r} \Bigg(\frac{\left(C_t - hX_t\right)^{1-\rho} - 1}{1-\rho} - \frac{\xi_{y,t}^{-1} Y_t^{1+\eta} - 1}{1+\eta} \Bigg) \\ + \beta E_t \left\{ V^D\left(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1}\right)\right\} \end{aligned} \tag{2.72}$$

subject to the constraints of habit formation (2.2), resource constraint (2.55), NKPC (2.46), Consumption euler equation (2.24). By defining an auxiliary function Ω^P that depends on next period state variables

$$\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1}) \equiv (\pi_{t+1} - 1)\, \pi_{t+1}\lambda^r_{t+1}Y_{t+1} \tag{2.73}$$

we can re-write the policy maker's NKPC constraint as:

$$0 = (1-\sigma)(1-\tau_{p,t}) + \sigma\varphi_t - \psi(\pi_t - 1)\,\pi_t + \frac{\psi\beta}{\lambda^r_t Y_t} E_t\left[\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right] \tag{2.74}$$

Similarly using other auxiliary function

$$\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1}) \equiv ((1 - h(1-\phi))C_{t+1} - h\phi X_t)^{-\rho} - \lambda^r_{t+1} \tag{2.75}$$

we can modify the Consumption Euler Equation (2.24) to

$$(1 - h(1-\phi)\omega)\ (C_t - hX_t)^{-\rho} \ = \ \lambda^r_t + \beta\phi E_t\left[\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right] \tag{2.76}$$

This auxiliary function captures the impact that policy decisions have on future state but policy-makers are limited in their commitment beyond the current state, as a result we have a time-consistent policy.

Therefore by substituting φ_t in terms of wages and real multiplier from Marginal cost relation eq. (2.50) and Labour supply relation eq. (2.25), and by using respective lagrange multipliers $\{\Lambda_{1,t}, \Lambda_{2,t}, \Lambda_{3,t}, \Lambda_{4,t}\}$ for the following four constraints,

Habit:

$$X_t = (1-\phi)C_t + \phi X_{t-1} \tag{2.77}$$

Resource Constraint:

$$C_t = Y_t\left(1 - \frac{\psi}{2}(\pi_t - 1)^2\right) \tag{2.78}$$

NKPC modified:

$$0 = (1-\sigma)(1-\tau_{p,t}) + \sigma\varphi_t - \psi(\pi_t - 1)\,\pi_t + \frac{\psi\beta}{\lambda^r_t Y_t} E_t\left[\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right] \tag{2.79}$$

CEE modified:

$$(1-h(1-\phi)\omega)\ \ (C_t - hX_t)^{-\rho} \ \ = \ \ \lambda_t^r + \beta\phi E_t\left[\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right], \qquad (2.80)$$

the Lagrangian for the discretionary policy problem can be written as,

$$\begin{aligned}\mathcal{L}^D = \; & \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{\xi_{y,t}^{-1} Y_t^{1+\eta} - 1}{1+\eta} + \beta E_t\left\{\mathcal{V}^D\left(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1}\right)\right\} \\ & + \Lambda_{1,t}\left(X_t - (1-\phi)C_t - \phi X_{t-1}\right) \\ & + \Lambda_{2,t}\left(Y_t\left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right) - C_t\right) \\ & + \Lambda_{3,t}\begin{pmatrix} (1-\sigma)\left(1-\tau_{p,t}\right) + \frac{\sigma Y_t^{\eta}}{\lambda_t^r \xi_{y,t}} - \psi\left(\pi_t - 1\right)\pi_t \\ + \frac{\psi\beta}{\lambda_t^r Y_t} E_t\left[\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right] \end{pmatrix} \\ & + \Lambda_{4,t}\begin{pmatrix} \lambda_t^r - \left(1 - h(1-\phi)\omega\right)\left(C_t - hX_t\right)^{-\rho} \\ + \beta\phi E_t\left[\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right] \end{pmatrix}. \qquad (2.81)\end{aligned}$$

The first order conditions with respect to control variables are given as,

$$\begin{aligned} C_t : \qquad & 0 = (C_t - hX_t)^{-\rho} - \Lambda_{1,t}(1-\phi) - \Lambda_{2,t} \\ & \quad + \Lambda_{4,t}\left(1 - h(1-\phi)\omega\right)\rho\left(C_t - hX_t\right)^{-\rho-1} \qquad (2.82)\end{aligned}$$

shows that an increase in consumption (2.82) increases utility, tightens both the habit constraint ($\Lambda_{1,t} \geqslant 0$), and the resource constraint ($\Lambda_{2,t} \geqslant 0$), and lowers the stochastic discount factor ($\Lambda_{4,t} \geqslant 0$) but improves the inflation-output trade-off ($\Lambda_{3,t} \leqslant 0$).

$$\begin{aligned} Y_t : \qquad & 0 = -\xi_{y,t}^{-1} Y_t^{\eta} + \Lambda_{2,t}\left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right) \\ & \quad + \Lambda_{3,t}\left[\frac{\sigma\eta Y_t^{\eta-1}}{\lambda_t^r \xi_{y,t}} - \frac{\psi\beta}{\lambda_t^r Y_t^2} E_t\{\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\}\right] \qquad (2.83)\end{aligned}$$

Similarly producing an extra unit of output (2.83) generates (dis)utility, relaxes the resource constraint and improves the inflation-output trade-off ($\Lambda_{3,t} \leqslant 0$).

$$\pi_t \quad : \quad 0 \quad = \quad \Lambda_{2,t} Y_t \psi \left(\pi_t - 1\right) \quad + \quad \Lambda_{3,t} \left[\psi \left(2\pi_t - 1\right)\right] \tag{2.84}$$

Increase in inflation (2.84) tightens the resource constraint and its impact on the inflation-output trade-off also depends on the level of output.

$$X_t \quad : \quad 0 \quad = \quad \left[\begin{array}{c} -h\left(C_t - hX_t\right)^{-\rho} + \beta \frac{\partial E_t\left\{\mathcal{V}^D(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right\}}{\partial X_t} \\ +\Lambda_{1,t} + \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t} \frac{\partial E_t\left\{\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right\}}{\partial X_t}\right] \\ +\Lambda_{4,t}\left[\begin{array}{c} \beta\phi\left[\frac{\partial E_t\left\{\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right\}}{\partial X_t}\right] \\ -\left(1 - h(1-\phi)\omega\right)\rho h\left(C_t - hX_t\right)^{-\rho-1} \end{array} \right] \end{array} \right] \tag{2.85}$$

using Envelope condition,

$$\frac{\partial \mathcal{V}^D\left(X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t}\right)}{\partial X_{t-1}} = -\phi \Lambda_{1,t} \tag{2.86}$$

and one-step forward iteration

$$\frac{\partial E_t\left\{\mathcal{V}^D\left(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1}\right)\right\}}{\partial X_t} = -\phi E_t\left\{\Lambda_{1,t+1}\right\} \tag{2.87}$$

we can re-write the FOC for Habit as,

$$X_t \quad : \quad 0 \quad = \quad \left[\begin{array}{c} -h\left(C_t - hX_t\right)^{-\rho} - \beta\phi E_t\left\{\Lambda_{1,t+1}\right\} + \Lambda_{1,t} \\ +\Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t} \frac{\partial E_t\left\{\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right\}}{\partial X_t}\right] \\ +\Lambda_{4,t}\left[\begin{array}{c} \beta\phi\left[\frac{\partial E_t\left\{\Omega^C(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right\}}{\partial X_t}\right] \\ -\left(1 - h(1-\phi)\omega\right)\rho h\left(C_t - hX_t\right)^{-\rho-1} \end{array} \right] \end{array} \right] \tag{2.88}$$

and FOC for λ_t^r, shows that rise in real income leads to higher SDF and worse inflation-output trade-off

$$\lambda_t^r : \quad 0 = -\Lambda_{3,t}\left[\frac{\sigma Y_t^\eta}{(\lambda_t^r)^2 \xi_{y,t}} + \frac{\psi\beta}{(\lambda_t^r)^2 Y_t} E_t\left[\Omega^P(X_t, \varepsilon_{y,t+1}, \varepsilon_{\tau,t+1})\right]\right] + \Lambda_{4,t} \tag{2.89}$$

The discretionary equilibrium is given by the following system of equations:
a) FOCs (2.82), (2.83), (2.84), (2.88) and (2.89)

b) multipliers on constraints of: Habits (2.2), resources (2.55), NKPC (2.46) and Consumption Euler relation (2.24)
c) asset pricing conditions (B.3, B.4, B.5, B.11, B.12) for 5 variables $(R^f_{t,t+1}, R^n_{t,t+1}, V^{eq}_t, E[R^{eq}_{t,t+1}], d_t)$, respectively, without any additional multipliers as the respective constraints are binding,
d) two exogenous processes of productivity (2.37) and taxes (2.42),
e) and for a given value of parameters $\rho, \eta, \omega, \sigma, \psi, \phi, h, \beta$.

To solve this non-linear problem we need to find time-invariant Markov perfect policy rules for these 14 variables $\left\{C_t, Y_t, \pi_t, X_t, \lambda^r_t, \Lambda_{1,t}, \Lambda_{2,t}, \Lambda_{3,t}, \Lambda_{4,t}, R^n_{t,t+1}, R^r_{t,t+1}, E[R^{eq}_{t,t+1}], V^{eq}_t, d_t\right\}$ as functions of state variables $\{X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t}\}$. Since the policy functions cannot be computed in closed form, we solve it by numerically using Chebyshev's collocation method in the following section.

2.3 Numerical Solution

In the extended NK model of this chapter there are externalities in the form of consumption habits, nominal rigidity in the form of Rotemberg adjustment costs and distortions in the form of output taxes. There are three state variables *viz.* an internal state of last period habits (X_{t-1}), an external state of technological productivity ($\xi_{y,t}$) and an external state of output taxes ($\tau_{p,t}$). The previous section presents the various optimal policy functions as functions of the three states. This section describes the global solution methodology adopted to solve the discretionary optimal policy problem numerically.

2.3.1 Chebyshev Collocation

Chebyshev collocation method uses time iteration on a state subspace of Chebyshev's polynomials to approximate global optimisation solutions for non-linear partial differential equations. Due to the inherent non-linearities of the optimal policy and the uncertainty involved therein, solving for global optimisation using this numerical method is a preferred way to obtain the functional optimisation of our model and to investigate its economic dynamics in depth. This investigation is an improvement on DePaoli and Zabczyk's (2013) higher-order local approximations to the policy problem as this approach approximates for the solution at bigger deviations from the steady-state, such as the good and bad states of the model economy which are +/- 2 s.d. from the stochastic steady state. The Chebyshev collocation solution approach proceeds in the following manner. The two external states are transformed into their AR(1) representations of $\varepsilon_{y,t}$ $\varepsilon_{\tau,t}$, respectively. The state spaces for $(X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t})$ are discretized into set of (n_1, n_2, n_3) collocation nodes. Where, each collocation node in the state space is a mapping of the roots of Chebyshev's polynomial of the respective order (n_1, n_2, n_3) on the space $\in [-1, 1]$. Accordingly, the space of approximating functions is generated by the tensor products of these three sets of Chebyshev polynomials of order $(n_1 \otimes n_2 \otimes n_3)$. The residuals equations are obtained from the 14 Euler equations wherein Gauss-Hermite Quadrature is used to approximate the expectation terms and first differencing of the Chebyshev polynomials[8] is used to approximate the partial derivatives with respect to the habits state.

[8] Appendix of Leith and Liu (2016) is a good resource to learn about this procedure and provides algorithms to solving global optimal policy for endogenous variables of upto 2 states.

2.3.2 Solution procedure

Following Leith and Liu (2016) a 3-dimensional grid of internal state, productivity state and the tax state are formed. The order of polynomial selections is [8,8,8] for all the state variables of the model: $[X_{t-1}, \varepsilon_{y,t}, \varepsilon_{\tau,t}]$. Hence Chebyshev's collocation technique generates a square matrix of orthogonal polynomials of order $8^3 = 512$ which when mapped into the range $[1.40, 1.55]$ or similar-sized range for the internal state variable X_{t-1}, into the range $\left[-2\sigma_y/\sqrt{1-\gamma_y^2}, +2\sigma_y/\sqrt{1-\gamma_y^2}\right]$ for technology shock, and similarly into the range $\left[-2\sigma_\tau/\sqrt{1-\gamma_\tau^2}, +2\sigma_\tau/\sqrt{1-\gamma_\tau^2}\right]$ for cost shock form the state space of 512 grid points or collocation nodes which are later used for interpolating the optimal functions for our 14 policy variables.

Christopher A. Sims' non-linear solver[9] is employed to solve the resultant system of residual equations. It evaluates them at each collocation node to obtain the coefficients for each policy function. For a function evaluated on n points, we can obtain a polynomial of order $n-1$ that will fit its value at those n points with accuracy, that is why we are able to generate those coefficients. Gauss-Hermite quadrature nodes (of order 12) are used to form conditional expectations of the functions at time $t+1$, whereas any partial derivatives are calculated by first-differencing the Chebyshev's polynomials, and the polynomial matrix is simultaneously updated. By repeatedly iterating on the set of coefficients until they reach a convergence level of 10^{-8}, the procedure can narrow down on the coefficients. As per Judd (1998) any convergence smaller than 10^{-5} is acceptable. The solution has high convergence at off the grid points. Sims' non-linear solver is employed to check off-the-grid convergence at equally spaced 21 points on each grid.

The size of errors off-grid points is of the order 10^{-7}. The maximum errors at off-grid points is smaller than 1.4 x10^{-6}. The mean of errors is below 1.1 x10^{-7}. The standard deviation of Euler errors is smaller than 2.2 x10^{-7} and the median of errors is less than 1.1 x10^{-8}. Considering that this procedure was performed on a wide grid of internal state variable as the selection of grid for the internal state variable X_{t-1} captures more than 99.99% of the values that it can take after a million simulations, the convergence obtained through this procedure has a highly acceptable level of accuracy.

[9]Source page url: http://sims.princeton.edu/yftp/optimize/

2.4 Quantitative analysis

The model is calibrated to have reasonable asset pricing properties and to match observed level and volatility of key macroeconomic data in the US for 1986-2006. This section briefly discusses the parametrization strategy and the stochastic steady state of the model, which is followed by an assessment of the model in response to productivity and cost shocks. The model is then used to make a comparison of optimal policy responses during various states of the economy.

2.4.1 Calibration

The 'benchmark' model calibration strategy (See Table 2.3) follows from De Paoli and Zabczyk (2013). As a standard calibration, one-period discount rate is set as $\beta = 0.99$ to generate near $\sim 4\%$ p.a. steady-state risk-free real interest rate. The calibration of EIS in dynamic macro-economic models is close to 1, but micro evidence relates it closer to zero. An average consumer is different from an average stockholder (Guvenen, 2006) the varying estimates of EIS can be reconciled by heterogeneity in these two classes of representative agents. A middle ground can be chosen by using the fact that near 20 per cent of US consumers own 80 per cent of its assets, but constitute for only 30 per cent participation in consumption. A simple average between EIS of 0.1 for non-stock owners and 1.0 for stock owners results in an EIS of approx 0.40. Therefore, the benchmark choice of the inverse of EIS is $\rho = 2.37$ in this chapter. This calibration is slightly higher than Campbell and Cochrane (1999) but in line with De Paoli and Zabczyk (2013).

Habit persistence parameter ϕ, is set at high levels of 0.97 similar to De Paoli and Zabczyk (2013). Another comparable reference for habit persistence parameter can be found in sensitivity function $\lambda(.)$ from the seminal paper on habits (Campbell and Cochrane, 1999) and that sensitivity function too refers to a high estimate of habit persistence. The slow-moving habit specification $(1-\phi)$ and high limit of habit persistence ϕ agrees with the notion that consumption habits once formed are difficult to cut-back. Due to the specification of habit formation (in eq. (2.2)), the parameter ϕ and its free counterpart $(1 - \phi)$ determine which of the two, current consumption or past habits respectively, is more significant for agents' habits.

Table 2.3: Parameters for the Benchmark habits model

Parameters	Symbol	Value	References
Inverse of EIS	ρ	2.37	De Paoli and Zabczyk (2013) & Campbell and Cochrane (1999)
Discount factor	β	0.99	Leith and Liu (2016)
Inverse of Frisch Elasticity	η	6	De Paoli and Zabczyk (2013)
Habits persistence	ϕ	0.97	De Paoli and Zabczyk (2013) & Campbell and Cochrane (1999)
Rotemberg Cost	ψ	30	Leith and Liu (2016)
CES	σ	7.5	Leith and Liu (2016) & De Paoli and Zabczyk (2013)
Habits size	h	0.85	De Paoli and Zabczyk (2013)
Internal habits proportion	ω	0.75	Chen and Ludvigson (2009)
Revenue tax	τ_p	0.35	US corporate tax rate
Persistence of prod. shock	γ_y	0.95	Smets and Wouters (2007)
Stdev. of prod. shock	σ_y	0.095	De Paoli and Zabczyk (2013)
Persistence of cost shock	γ_τ	0.95	Smets and Wouters (2007)
Volatility of shocks	σ_y^2/σ_τ^2	2.5	De Paoli and Zabczyk (2013)

Parameter ω describes the proportion of habits that are internal to agents. Chen and Ludvigson (2009) empirically find that internal habits fit the observations in consumption data better than external ones. A choice of ω (=0.75) closer to 1, in the benchmark model, also reduces the size of the competitive economy and generates a low positive rate of inflation similar to what we observe in data.

The level of Surplus consumption or consumption in excess of subsistence habits $C - hX$ is determined by habit size parameter h. In deterministic steady state the Surplus ratio $S = \frac{C-hX}{C}$ is solely determined by the habit size parameter h, as $S = 1 - h$. By calibrating h at 0.85 the model ensures that subsistence habits are sufficiently close to the agent's total consumption and she would be extremely uncomfortable with a 10-15% drop in current consumption. To put it in perspective, consider that during the Great Recession, the biggest quarterly drop in US per capita GDP was near 11% p.a. De Paoli and Zabczyk (2013) recommends a high habit size parameter because risk aversion alone is insufficient to generate precautionary savings. Customers must feel a sustainable drop in their standard of living for them to save for precautionary reasons. Persistent shocks and a high h parameter suits their model in which they obtain the precautionary behaviour through higher-order approximation of the consumption Euler equation.

In the benchmark model Tax rate τ_p is kept at the prevailing (state + federal) corporate tax rate in the US of 35%. For the period in consideration 1985-2006, Tax receipts or tax revenue to GDP ratio in the US is near 26-28%, while the share of total Expenditure to GDP is near 35%. The latter is chosen for calibration as there are no borrowings from the government to fund expenditure in the model. Rotemberg adjustment cost (ψ) is set to 30 which accounts for 8-10 months of price rigidity, depending on other parameters of the model and is within an acceptable range of Leith and Liu (2016) whereas, the elasticity of substitution parameter σ at 7.5 resulting in $\sim$ 15% of markup is in line with its standard estimate used in literature. For exogenous shocks of technology and taxes the persistence parameters γ_y, γ_τ are both set to be 0.95. Auto-regressive parameter value of 0.95 for technology γ_y is towards the higher end of Smets and Wouters (2007) and the choice of 0.95 for γ_τ is in line with Leith and Liu (2016). The standard deviation of exogenous shock $\sigma_y = 0.095$ and its ratio with the σ_τ are calibrated to match the observed level of volatility ($\sim$ 0.34%) in quarterly log Non-Durables and Services Consumption growth in US data from 1985-2006. The beginning date of that period is chosen to avoid changes in US monetary policy regime during pre and post-Volcker era, and the Great moderation, while the end date avoids the excess volatility and ZLB period since the onset of the Great Recession. A parametrization which replicates observed behaviour in data and generates equity premium of $\sim$ 6% without resulting in a risk-free rate puzzle, or in other words, parameters which capture the observed risk-dynamics accurately should sufficiently help the model account for policy outcome during fluctuations in risk-premium and precautionary savings.

2.4.2 Stochastic steady state

The stochastic steady state, also known as the Risky Steady State [See Coeurdacier, Rey, and Winant (2011)], is the point where given that the current realization of shocks is zero, the agents choose to stay if they expect future shocks. For a decision rule for policy variable P such that $P_t = f(X_{t-1}, \xi_t; \xi_{t+1})$ where X is endogenous state (habits), ξ_t the exogenous shock states of productivity and taxes, and ξ_{t+1} is the conditional expectional of next period exogenous state variables. The risky steady state is then defined as a steady state value of P denoted as $\overline{P}$ such that, $\overline{P} = f(\overline{P}; 0)$. The stochastic steady state results in Table 2.4 on page 141 highlight the role of uncertainty in both consumption and production decisions. Parameterization for volatility of technology (σ_y) and tax (σ_τ) states (in Table 2.3 on page 139) was selected so that the log-consumption

Table 2.4: Stochastic Steady State Results

	Benchmark $\omega = 0.75$	Internal $\omega = 0.99$	External $\omega = 0.01$	*U.S. Data* **1985-2006**
Stochastic Steady State				
Consumption	1.47	1.40	1.60	-
Output	1.47	1.40	1.60	-
Inflation (%p.a.)	1.12	2.58	-3.86	2.50
Habits	1.47	1.40	1.60	-
Real Income Multiplier	18.0	13.7	29.5	-
Nominal Rate (%p.a.)	2.18	3.36	-2.54	4.71
Real Rate (%p.a.)	0.96	0.68	1.29	2.21
Exp. Equity Return (%p.a.)	7.23	7.61	6.14	7.62
Equity Index	12.7	12.0	13.73	-
Dividends	0.128	0.121	0.138	-
Simulated Volatility %p.a.				
$\sigma_{\Delta c}$	0.74	0.58	0.51	$0.67^{*}, 1.16^{**}$
$\sigma_{\Delta y}$	0.72	0.56	0.58	1.01
σ_{r^n}	3.7	4.0	2.0	0.96
σ_{r^r}	3.2	3.3	1.4	0.81
$\sigma_{log(E[R^{eq}])}$	4.7	4.9	2.1	14.4
σ_{π}	0.89	1.50	1.19	0.57
Sharpe Ratio ann.	1.32	1.40	1.12	0.41

Source: Data column reports Annualised data from qtrly series of US FRED database for period Q1:1985 to Q4:2006: Consumption c is *Non-Durables and Services per capita and **Non-Durables per capita; Output y is Real per Capita GDP, r^n is 3m T-bill rate. Robert Shiller's database is used for Real S&P Index and π which is Personal Consumption Expenditure (PCE) excl. Food and Energy.
Notes: The annualized standard deviation σ of key variables in subscript results from simulating the model for 100,000 periods and calculating the variance from its Risky Steady State. Δz represents the log difference in variable Z, whereas z represents the logarithm of variable Z.

growth from the benchmark model would be similar to the observations in US data. As expected, the stochastic steady state for Consumption, Output and Habits is similar for all models (Top section in Table 2.4). Due to the inefficiency wedge (Proposition 2.2.3) there is small positive inflation is steady-state for the benchmark calibration. For a low value of EIS (ρ^{-1}) the benchmark calibration can generate a high equity risk premium without any significant increase in the level of the risk-free rate, and thus avoids both the equity premium puzzle and the risk-free rate puzzle.

The stochastic volatility of various variables is reported in the middle section of Table 2.4. The simulated stochastic volatility of both consumption and output is in range with the observations of data. As stated previously, the parameterization of the ratio of the variance of exogenous shock variables, σ_y^2/σ_τ^2 is chosen so that the model can match the stochastic volatility of consumption found in US data. Although these parameters were not selected to model the volatility in inflation, yet the benchmark model is also able to approximate the observed inflation volatility of the US data closely. This non-zero inflation volatility from the model stands in contrast to inflation stabilization recommendations of standard optimal policy results. The model generates slightly bigger than observed inflation volatility and slightly smaller than observed volatility in output. This volatility trade-off between output and inflation in the stochastic results reflects that optimal policy in the model favours a higher stabilization in output during costly recessions. Although the expected equity return from the simulated results matches the data yet the ratio of excess returns (i..e above the risk-free rate) from the stock index to the volatility of stock returns, also known as the Sharpe ratio, is higher for the model in comparison with US data. It could be explained by the excess volatility of stock returns that is prevalent in data but missing from the model due to lack of heterogeneity in risky investments. It could also be the reason for a similar level of volatility in risk-free and risky investments in the model.

2.4.3 Impact of Productivity shocks

The composition of Habits in the benchmark model is mostly internal (as the parameter $\omega = 0.75$). Standard analyses show that nominal rigidities, if habits are entirely internal, do not pose a concern to the policymaker in the presence of productivity shock as the efficient allocation can always be replicated [See Amato and Laubach (2004) and Leith, Moldovan, and Rossi (2012)]. Therefore given that the size of habits externalities is relatively small in the benchmark calibration, we should expect 'Divine coincidence' to hold[10], i.e. in response to productivity disturbance, the change in policy rates should be able to stabilise inflation immediately and which coincides with output stabilisation.

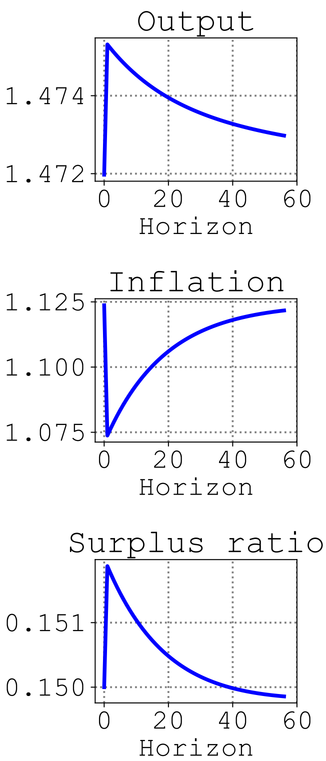
The impulse responses to productivity shocks for the benchmark model are presented in Figure 2.2 on page 144. Each policy variable is a function of three states, viz. past habits (X_{t-1}), technology ($\varepsilon_{y,t}$) and revenue taxes ($\tau_{p,t}$). The impulse responses in Figure 2.2 consider habits to be at its steady-state in the beginning of the period when shocks hit the economy and all the responses are conditioned on the tax state being at its steady-state level of 35%. In response to a +1% quarterly increase in firms' productivity, the central banker lowers real rates by more than 1% per annum. The cut in real rates is essential to stimulate demand otherwise firms would have to bear adjustment costs (due to nominal rigidity) to lower prices in order to sell the additional output generated from the increase in productivity. The resulting drop in inflation after the rate cut by the central banker is small (approx 0.1% p.a.) but it is not transitory. Inflation stays low for more than 20 quarters after the shock. The optimal discretionary policy does not immediately remove inflation deviations, and there appears to be a trade-off between output and inflation stabilisation.

The responses in Consumption and Output to the productivity shock are immediate and continue for long periods after the shock. This is unsurprising if we consider that a policymaker whose objective is to maximise household's utility would focus attention not on agent's consumption but instead on her Surplus consumption, i.e. consumption in excess of subsistence level of habits ($C_t - hX_t$). Habits are slow-moving which can be confirmed by the hump in habits chart in Figure 2.2. Therefore an immediate increase in Consumption raises the Surplus ratio (S_t). The optimal policy seeks to stabilise this ratio by reducing the increase in consumption which can be achieved by normalising real rates to their pre-shock levels as the impact of the positive productivity shock begins to fade. In selecting the real rate path, the policymaker also acknowledges the humped increase of habits due to the initial rise in consumption; therefore, she can opt for a faster normalisation. As a result Real rate, Inflation and Surplus ratio revert to the pre-shock state faster than Output or Consumption.

Additional impulses responses in Figure 2.3 on page 145 show that a positive productivity shock is beneficial for the firms as they employ less units of labour

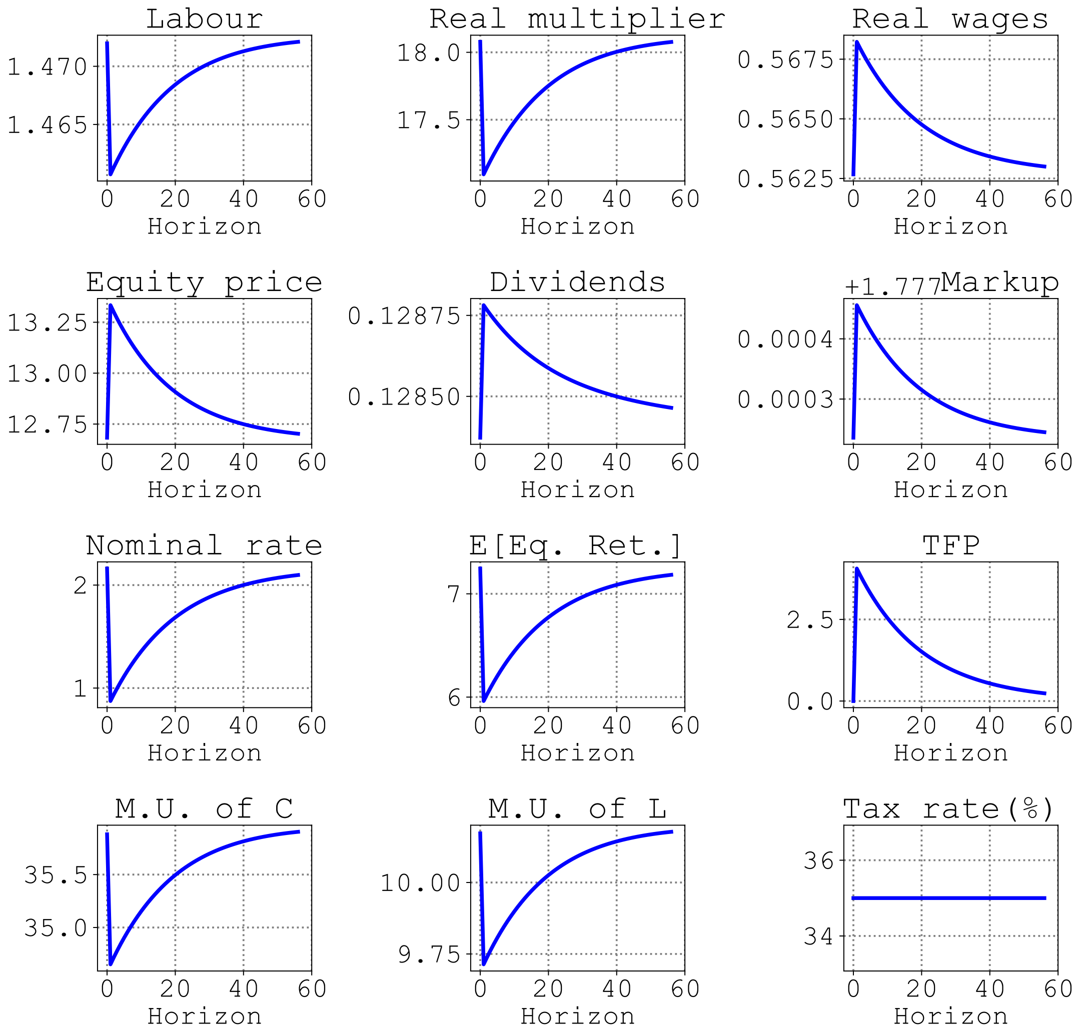
[10]See Blanchard and Gali (2007) for 'Divine coincidence'.

Figure 2.2: Impulse responses to Productivity Shocks I



Notes: Impulse responses to +1% quarterly increase in Productivity. Inflation, Real rate and Equity premium (Equity pr.) are captured in % p.a., while all others are in their standard representation. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. Habits are at their Stochastic Steady state in the shock period and all the responses are conditional on assuming no change in Tax rate for the entire horizon. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure 2.3.

Figure 2.3: Impulse responses to Productivity Shocks II



Notes: Impulse responses to +1% quarterly increase in Productivity. Nominal rate and Expected Return on Equity (E[Eq. Ret.]) and TFP are captured in % p.a., while all others are in their standard representation. M.U. is marginal utility of Consumption (C) and Labour (L). Policy functions depend on three states *viz.* Habits, Productivity and Taxes. Habits are at their Stochastic Steady state in the shock period and the responses are conditional on assuming no change in Tax rate for the entire horizon. X-axis: time horizon (in quarters).

(first row in fig. 2.3). Although there is an increase in real wages as households' productivity increase, which also relaxes the Real multiplier on households lifetime income (λ_t^r), yet the net effect of higher wages and lower workforce is to reduce the marginal cost for firms. Marginal cost is an inverse of the markup, which is plotted in the second row in fig. 2.3. Fall in marginal cost improves firms' profitability and the expected dividend stream (second row in fig. 2.3). This further leads to an increase in the Equity index, which is a function of the discounted value of firms' future dividends. Household investors become more risk-taking and accept a lower expected rate of returns from equity investments, which is denoted in fig. 2.3 as E[Eq. Ret.]. Despite that the equity premium slightly increases, because the real rate cut by the policymaker to clear markets from an increase in supply leads to drop in real rates which more than compensates for the drop in equity premium from high productivity. With interest rates being cut and staying low for a longer period, the optimal policy also results in maintaining a stable equity premium. The marginal utility is inversely related to Surplus consumption; it is decreasing when households are consuming in excess of the steady-state Surplus ratio (15%). Another way in which agents optimize their utility in response to a positive technology shock is by increasing their leisure, or by reducing their labour supply.

2.4.3.1 Productivity shocks in different habits externalities

Figure 2.4 on page 149 and Figure 2.5 on 150 compare the impulse responses of benchmark model in fig. 2.2 - 2.3 to varying degrees of habits externalities. The benchmark calibration of $\omega = 0.75$ is compared with the other extremes of habits calibration, namely Internal habits ($\omega = 1.0$) and External[11] habits ($\omega = 0.0$). The impulse responses in fig. 2.4 - 2.5 are reported in their percent deviation from their pre-shock steady state.

Similar to the previous exercise, all policy variables in fig. 2.4 - 2.5 consider habits to be at their respective steady-state at the beginning of the period when shocks hit the economy, and all the responses are conditional on the tax state being at its steady-state rate of 35%. The general feature of most of the impulse responses are similar, but the size of the impact of the productivity shock to output is bigger when habits are external. Habits continue with the slow-moving

[11] Calibrating $\omega = 0$ and $\omega = 1$ changes nature of households utility maximisation as the habits constraint (2.2) reaches its corners. To keep the results comparable External habits model is calibrated at $\omega = 0.01$. and internal habits at $\omega = 0.99$.

humped response and Surplus Ratio stabilises faster than output or interest rates. Markup is the inverse of marginal cost, and it is increasing in equilibrium output. The responses of inflation are different, and for internal habits model, the optimal policy can contain inflation deviations [See inflation chart in Figure 2.4].

Monetary policy has real effects. Figure 2.4 shows that in the presence of habits externalities and with only one policy instrument interest rates at hand, the policymaker cannot ensure that efficient allocation is replicated without inflationary repercussions. The inflation chart in Figure 2.4 shows that with an increase in habits externalities the trade-off between inflation and Surplus ratio stabilisation worsens. The real rate cut required in response to a 1% quarterly increase in productivity is increasing in size of habits externalities. The more households become unaware of the impact of their consumption on aggregate habits, the more they increase their consumption when productivity increases. This observation differs from the existing optimal policy literature [See Amato and Laubach (2004), Leith, Moldovan, and Rossi (2012), De Paoli and Zabczyk (2013)] which recommends a smaller rate cut for the Ramsey planner as the size of habits externalities increases. After a positive productivity shock, Ramsey Optimal policy in De Paoli and Zabczyk (2013) favours a smaller rate cut when habit externalities are large and persistent. The size of a rate cut during external habits for Optimal discretionary policy is larger than Optimal commitment policy in Leith, Moldovan, and Rossi (2012). However, in their model no comparison has been made between the discretionary policy results for internal and external habits. The central banker in these models curbs the temptation to lower interest rates following a rise in productivity since an increase in current consumption would lower the household's lifetime utility due to the externalities of habits. The model in this chapter which features costly recessions has similar properties to previously cited habits related literature and the deterioration of household's lifetime utility should worsen for an increase in consumption when habits externalities increase.

We must, therefore understand the anomaly that Optimal policy responses in Figure 2.4 poses. When agents are not aware of the impact of increasing current period consumption on future aggregate habits and therefore lower surplus ratio in future then why does the central banker persist with a high rate cut than she would if habits were internal? Or why doesn't she respond with a 'lean against the wind' smaller rate cut when habits are external and by doing so reduce the increase in output and habits, despite the high cost of not doing so to household's welfare? We will look to answer the next section, but before that, we also need to discuss the optimal policy responses to cost shocks.

2.4.4 Impact of Cost shocks

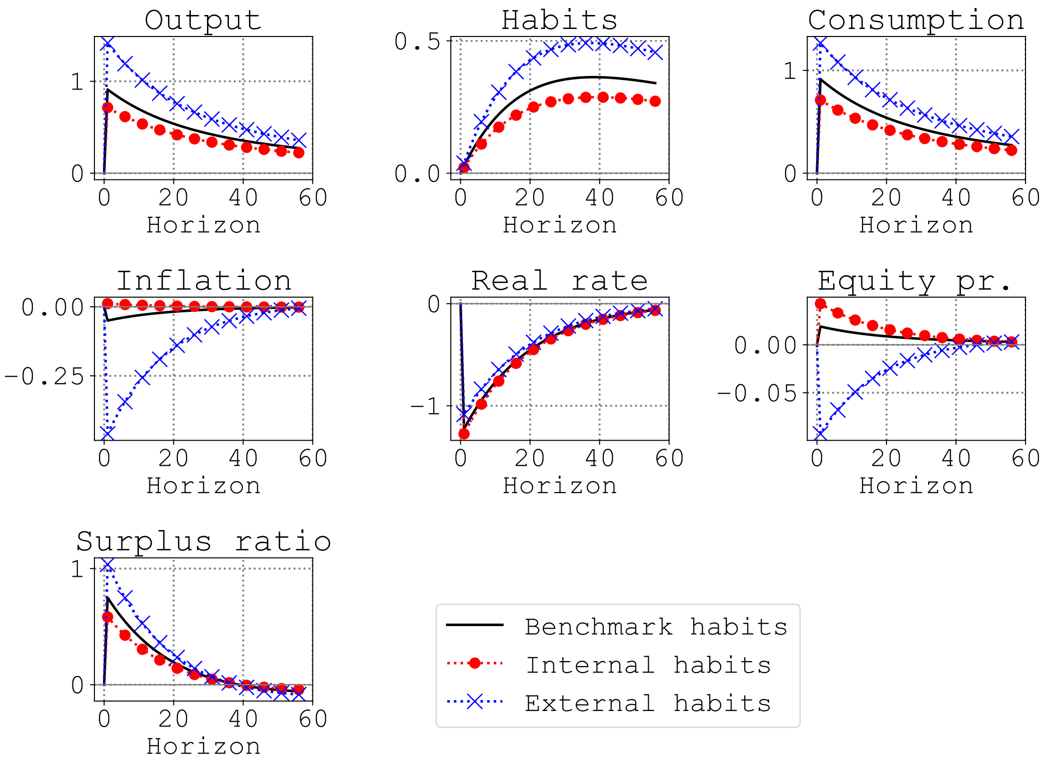
In standard NK models cost shocks offer a trade-off between output and inflation by generating a time-varying gap between the efficient and the flexible prices output, a gap that is not generated by demand/productivity shocks, and this gap creates a trade-off for monetary policy. The prevailing tax rate directly impacts the wedge between the competitive and efficient (Social Planner) output, as discussed in Proposition 2.2.3. Therefore the cost shock, which is administered by changes in the prevailing revenue taxes rate[12], also impacts the inefficiency wedge. In standard NK analyses, optimal policy in the presence of persistent cost shocks has higher volatility in inflation (Galí, 2015) as the policymaker with only one policy instrument cannot simultaneously attain zero inflation and efficient output allocation. Addressing inflationary cost shocks with an increase in real rates to lower inflation, also worsens the negative output gap when the steady-state competitive output is inefficient as it is so in our benchmark model. Therefore the policymaker has to mitigate the interest rate response in the face of this output-inflation trade-off.

Figure 2.6 on page 151 presents the impulse responses for key variables to cost shocks for the benchmark model. Additional impulse responses to cost shocks are presented in Figure 2.7 on page 152. As stated earlier, each policy variable is a function of three states of past habits (X_{t-1}), productivity ($\varepsilon_{y,t}$) and revenue taxes ($\tau_{p,t}$). These impulse responses in fig. 2.6 and fig. 2.7 consider habits to be at its steady-state at the beginning of the period the shock hits the economy and all responses are conditioned on productivity state being constant at its steady-state level of 0% growth.

In response to a +1% quarterly increase in the revenue tax rate, the central banker raises the real rates by 0.3% per annum approximately. The hike in interest rates is essential to reduce demand following the cost shock; otherwise, firms would have to bear adjustment costs (due to nominal rigidity) to raise their prices. Demand falls due to the increase in the interest rate, both consumption and output follow suit. Households' habits reduce slowly, and therefore we observe a steep fall in the Surplus Ratio. As stated earlier, the objective of the policy is

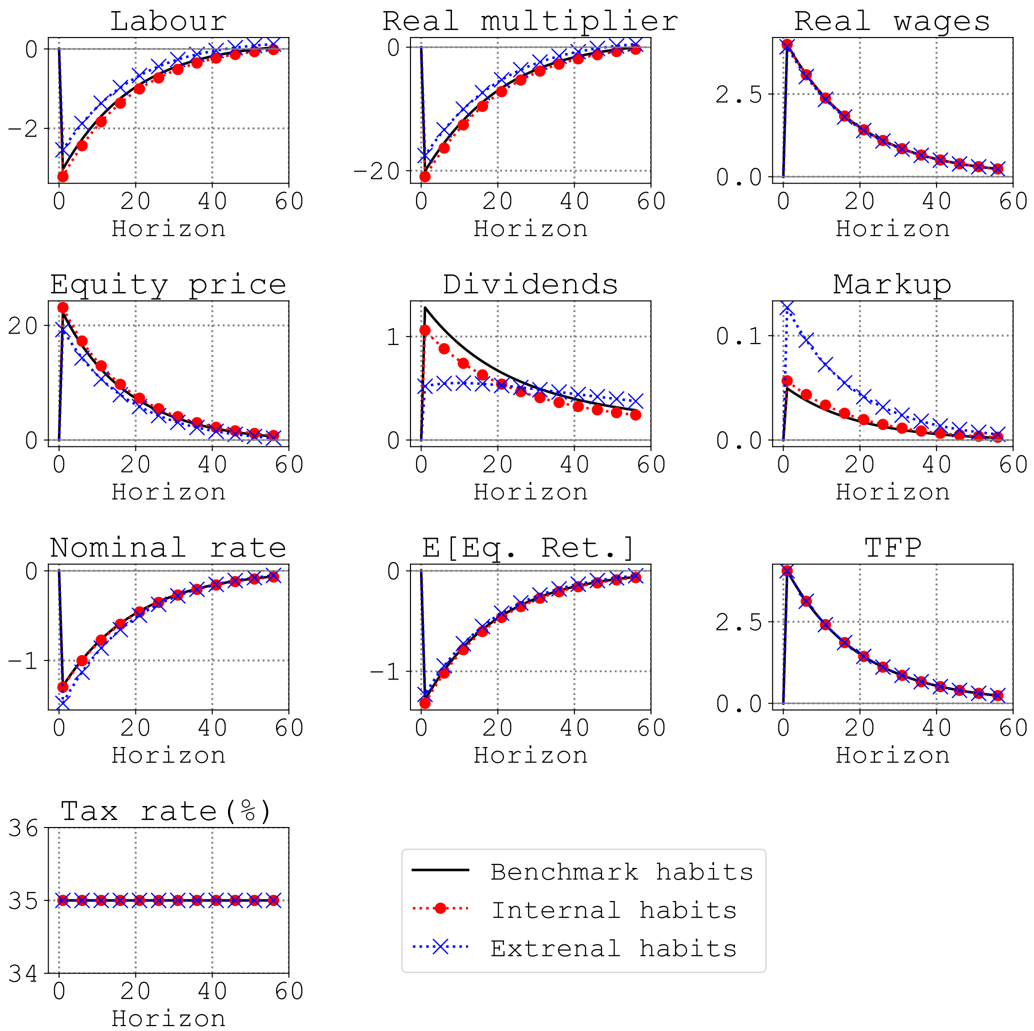
[12]Refer the AR(1) Equation (2.42) for revenue taxes

Figure 2.4: Productivity Shocks and Habits Externalities I



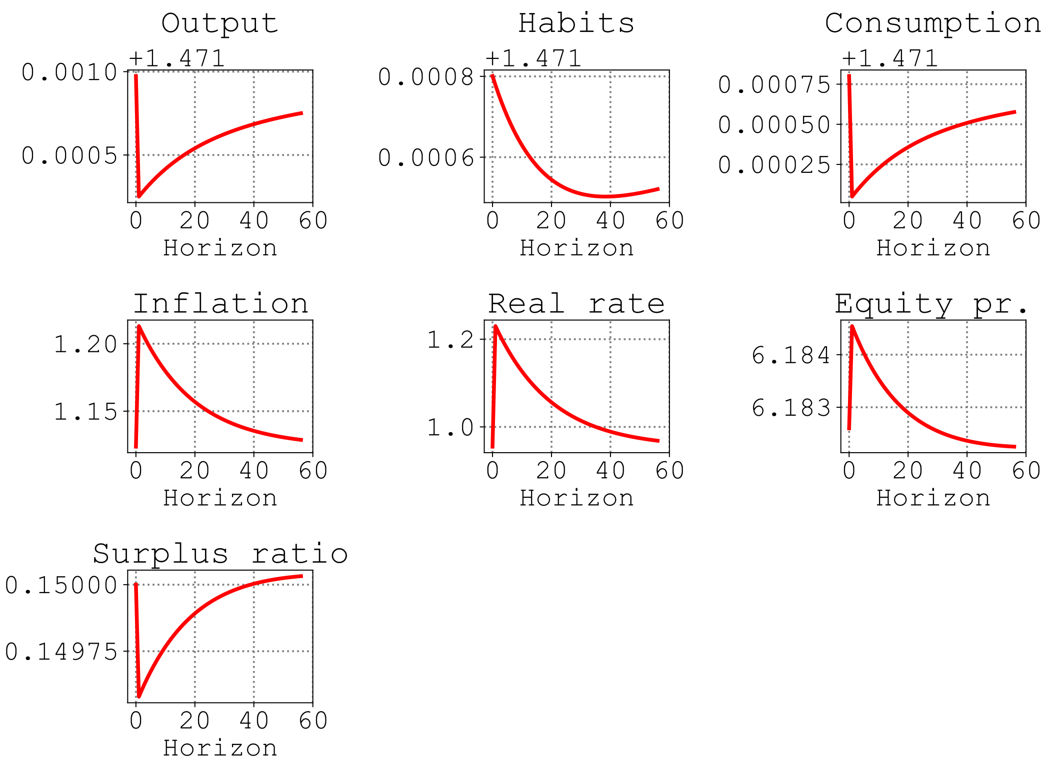
Notes: Impact of Productivity Shocks in models of various Habits Externalities. Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly productivity shock. For Surplus ratio the deviations are in basis points (0.01% = 1 bps), while for all others they are in %. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in tax rate for the entire horizon. The various models are defined as: Benchmark habits - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal habits - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External habits - when Habits are 1% internal ($\omega = 0.01$) and 99% external. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure 2.5.

Figure 2.5: Productivity Shocks and Habits Externalities II



Notes: Impact of Productivity Shocks in models of various Habits Externalities. Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly productivity shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in tax rate for the entire horizon. The various models are defined as: Benchmark habits - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal habits - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External habits - when Habits are 1% internal ($\omega = 0.01$) and 99% external. X-axis: time horizon (in quarters).

Figure 2.6: Impulse responses to Cost Shocks I



Notes: Impulse responses to +1% quarterly shock to the Tax rate. Inflation, Real rate and Equity premium (Equity pr.) are captured in % p.a., while all others are in their standard representation. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. Habits are at their Stochastic Steady state in the shock period and the responses are conditioned by assuming no change in Productivity (TFP) for the entire horizon. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure 2.7.

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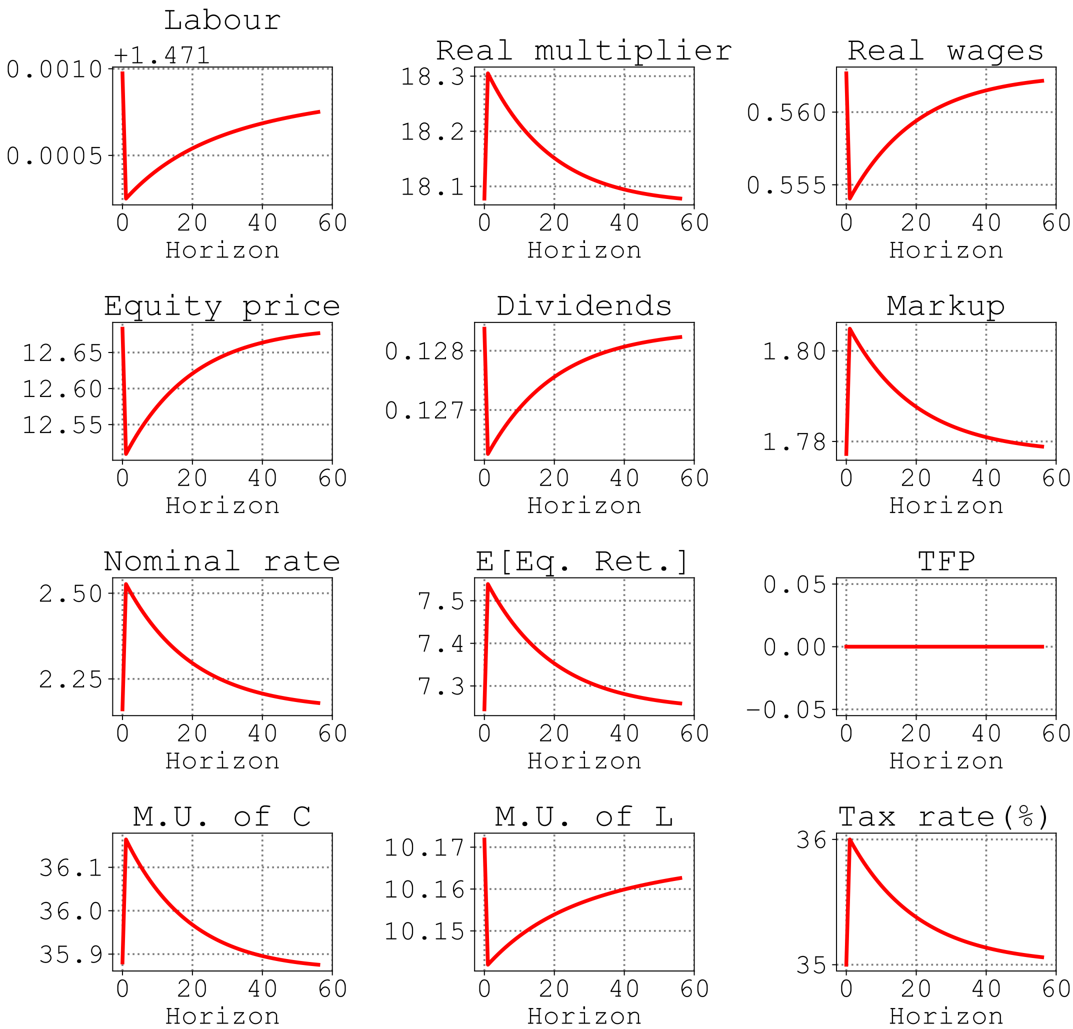
to stabilize the Surplus ratio, and policymaker could achieve that by reducing the interest rates once the impact of cost shock subsides.

From charts related to the firm's operating environment (in Figure 2.7 on page 152), we notice that the cost shock increase firm's operating expenses and lower its profits/dividends. Firms employ less labour as their supply curve shifts to the left. After being hit by a cost shock firms produce at less than the profit-maximizing level of output and want to raise their markup in order to raise profitability. The net impact of higher markup, high costs, and low demand is a fall in a firm's profits, and future expected dividend stream (profitability). Lower profitability of firms reduces the dividend income for households, and the equity index falls.

Lower production by firms widens the wedge between the efficient and compet-

Figure 2.7: Impulse responses to Cost Shocks II



Notes: Impulse responses to +1% quarterly shock to the Tax rate. Nominal rate and Expected Return on Equity (E[Eq. Ret.]) and TFP are captured in % p.a., while all others are in their standard representation. M.U. is marginal utility of Consumption (C) and Labour (L). Policy functions depend on three states *viz.* Habits, Productivity and Taxes. Habits are at their Stochastic Steady state in the shock period and the responses are conditioned by assuming no change in Productivity (TFP) for the entire horizon. X-axis: time horizon (in quarters).

itive output. Figure 2.6 on page 151 confirms this by showing that an increase in real rates (borrowing costs) by the central banker can reduce the spike in inflation. However it also leads to a drop in consumer demand and worsens the gap between efficient and competitive output.

Because if the steady-state output is inefficiently low, then interest rate hike by the central banker would lower output further and worsen the inefficiency. However, by responding with a lower rate hike (see rates and inflation charts in fig. 2.6) and letting the inflation to rise the optimal policy acknowledges the presence of a trade-off between output and inflation and favours output stabilization. The drop in consumption has a gradual impact on reducing habits. Therefore the Surplus ratio also recovers gradually and reverts to its pre-shock levels in 60 quarters. Due to a rise in their borrowing costs and a drop in Surplus ratio, the household's marginal utility from an additional unit of consumption is high. As a result, households become more risk-averse, they demand extra returns for investing in risky assets, and that increases the Equity premium. The increase in risk-free real rates mitigates the impact of higher expected returns from equity investments on the Equity risk premium.

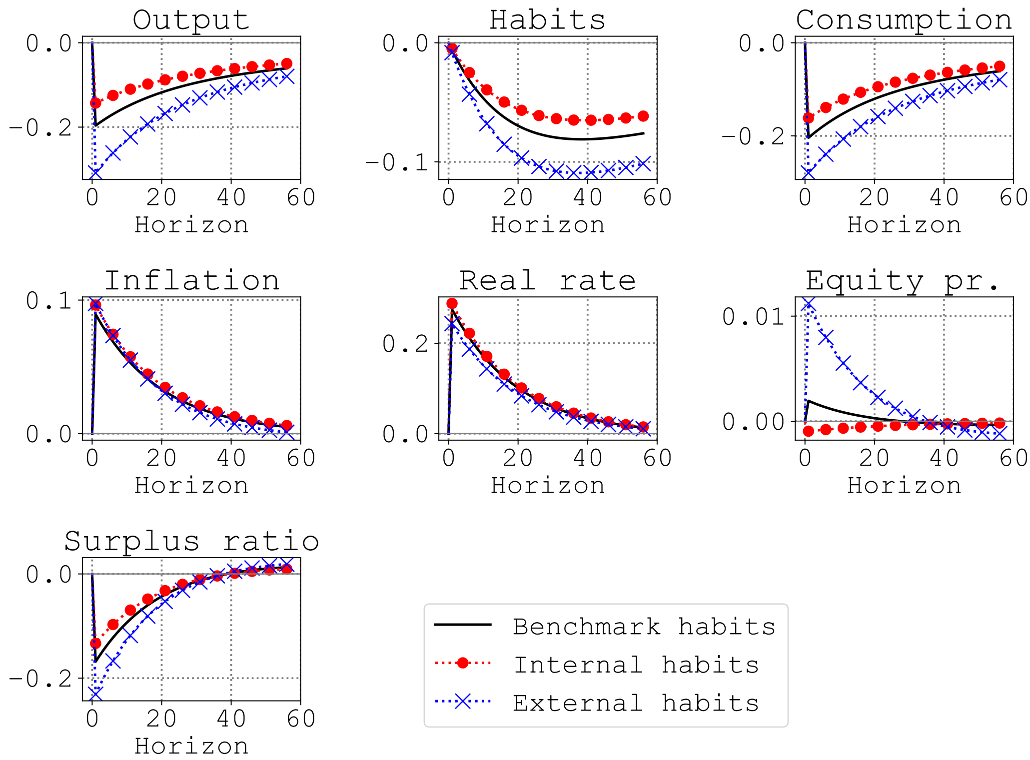
The policymaker faces a trade-off between output and inflation stabilization and seems to choose an interest rate path that seeks to normalize Surplus ratio. By raising the real rates, the policymaker is also able to curb any runway increase in Equity premium.

2.4.4.1 Cost shocks in different habits externalities

The trade-off between inflation and Surplus consumption exists for all levels of habits externalities (See Figure 2.8 - 2.9 on page 155 - 156). However, the trade-off expectedly worsens in the size of externalities. Cost shocks result in a smaller decrease in consumption and habits when the size of internal habits is larger; in comparison, the reduction in both consumption and habits is bigger when the habits are external. This is because as households become aware of the impact of their consumption on aggregate habits so they begin to complement the efforts of the policymaker in leading the Surplus ratio back to its steady-state level. Contrary to standard optimal policy recommendation of divine coincidence, we observe that as the size of habits externalities reduces there is an increased tolerance in optimal policy for inflation volatility. In Figure 2.8 the policymaker responds with similar size of a rate hike in response to a +1% change in the tax rate for various habits (externalities) models. However, it can achieve the best output-inflation trade-off when habits are fully internal.

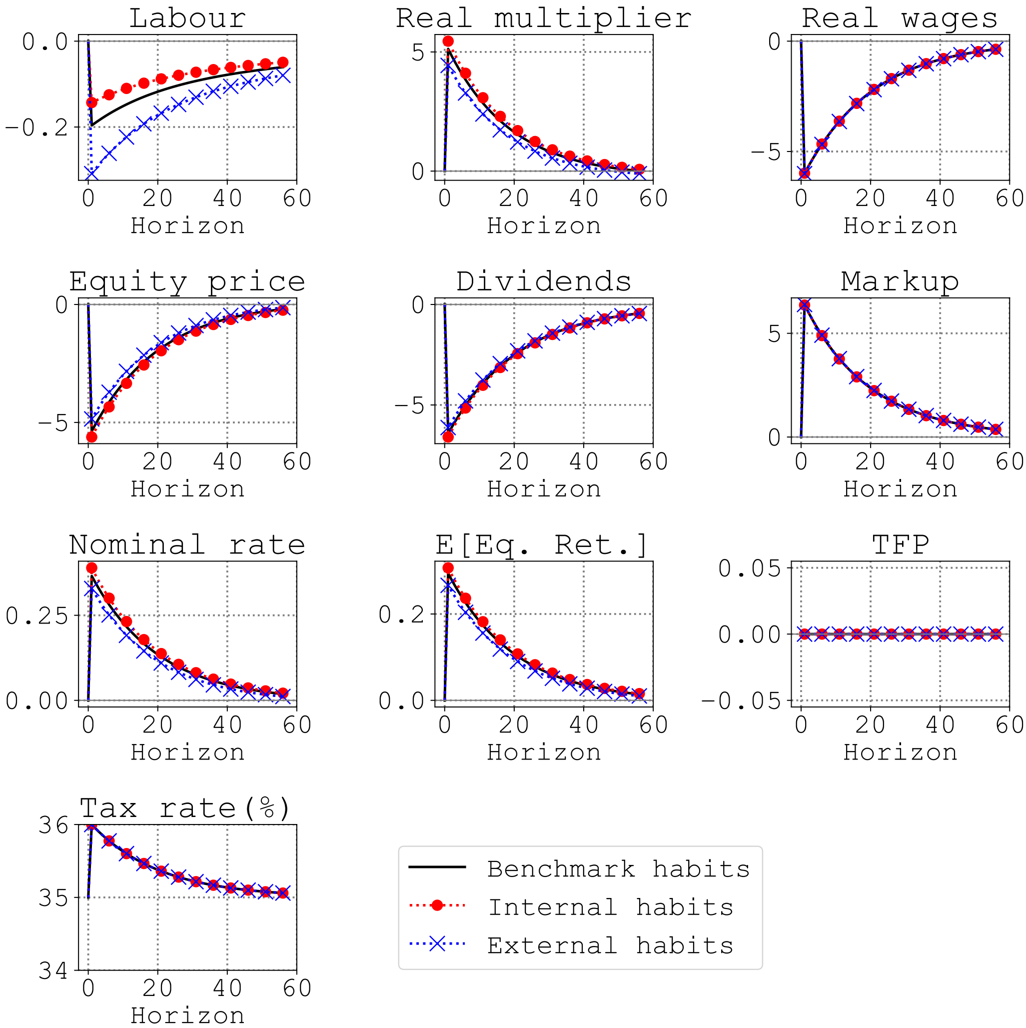
This abnormality begs the question as to why does the policymaker not respond with a smaller rate hike in case of external habits and control the drop in output and habits when the households are not aware of these externalities? Or its complementary question as to why does the policymaker not choose a lower interest rate hike when habits are internal to agents and thus accept a more considerable drop in output that she would if habits were external? Plausible explanations are sought in the following section.

Figure 2.8: Cost Shocks and Habits Externalities I



Notes: Impact of Cost Shocks in models of various Habits Externalities. Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly Tax rate shock. For Surplus ratio the deviations are in basis points (0.01% = 1 bps), while for all others they are in % p.a. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in Productivity for the entire horizon. The various models are defined as: Benchmark habits - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal habits - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External habits - when Habits are 1% internal ($\omega = 0.01$) and 99% external. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure 2.9.

Figure 2.9: Cost Shocks and Habits Externalities II



Notes: Impact of Cost Shocks in models of various Habits Externalities. Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly Tax rate shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in Productivity for the entire horizon. The various models are defined as: Benchmark habits - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal habits - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External habits - when Habits are 1% internal ($\omega = 0.01$) and 99% external. X-axis: time horizon (in quarters).

2.5 Discussion

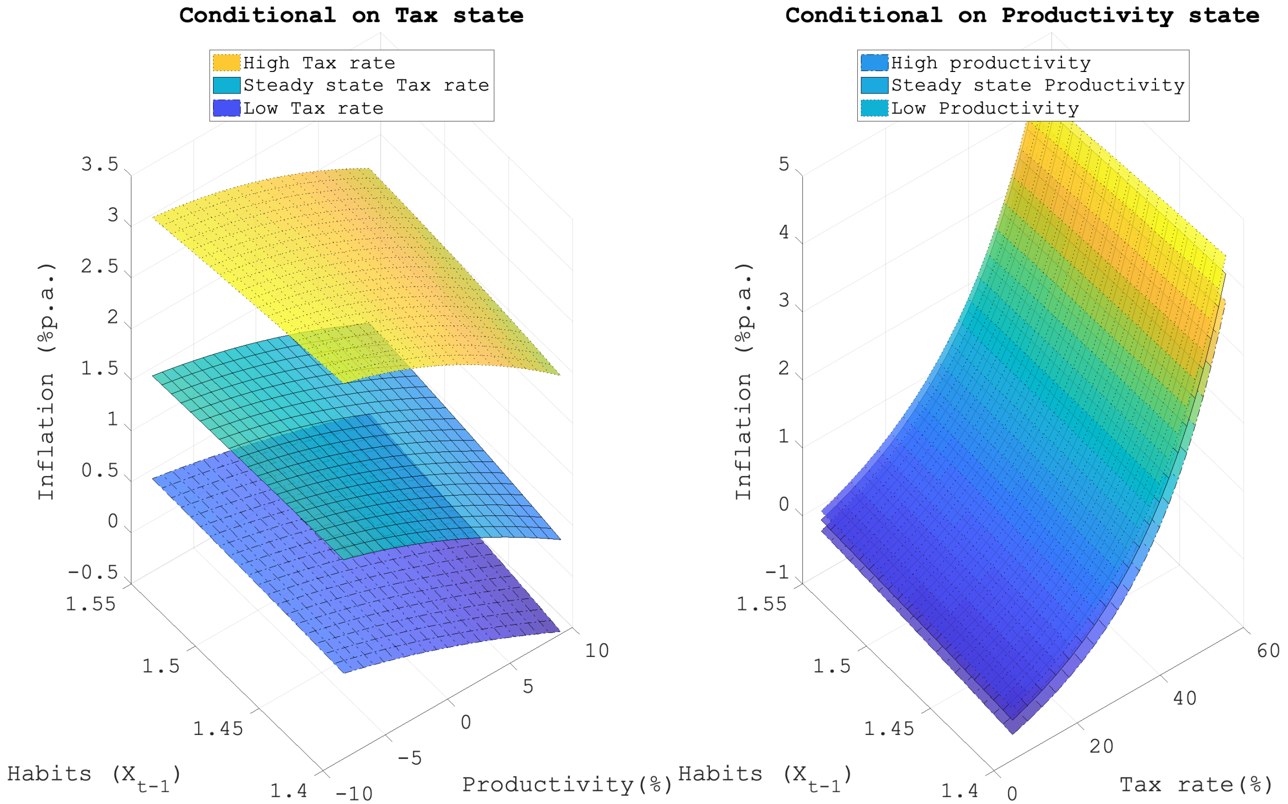
The answer to the atypical findings in analysis of productivity and cost shocks can be found in the steady-state inflationary bias of the economy and the precautionary behaviour of its household when contractions are costly.

2.5.1 The Inflationary bias

The presence of distortions via nominal rigidity, monopolistic competition and taxes make the steady-state output in our benchmark calibration (refer Table 2.4 on page 141, benchmark column) inefficient, as it is lower than Social planner's output. As stated in Proposition 2.2.3, even the flexible price output is inefficient due to persistent wedge in the economy generated by monopolistic competition and taxes. The benchmark calibration does not have any significant externalities from habits ($1 - \omega = 0.25$). So there is negligible counter-impact of habits' externalities in raising the competitive economy output (Leith, Moldovan, and Rossi, 2012) to negate the inefficiencies from monopolistic competition and taxes.

Since the competitive output is smaller than efficient output the discretionary policymaker has temptation to inflate the economy which results in the classical inflationary bias problem [Clarida, Galí, and Gertler (1999) Leith and Liu (2016) and Galí (2015)]. In the benchmark calibration, we see that the steady-state inflationary bias is 1.12%, and it is increasing in the size of internal habits ω (See Table 2.4). Figure 2.10 on page 158 shows the Optimal policy response of Inflation as a function of its state variables. As there are 3 states (Habits, Productivity, Tax rate) so we can only see the 3-dimensional policy surface charts in two states at a time, and conditional on a particular realization of the third state. The inflationary bias in the benchmark model is non-linear. Figure 2.10 also shows that positive productivity shocks reduce inflationary bias (left chart) and the volatility of inflation increases when tax rates (right chart) increase. The gap between various surfaces shows that inflation is more volatile over changes in tax rates than it is over changes in productivity growth. As shown in the right-hand chart of fig. 2.10, the increase in tax rates has an overwhelming effect on the inflationary bias irrespective of the level of habits or productivity state. The inflationary bias reduces when there is high productivity and low rate of taxes, as then the equilibrium output is bigger than the efficient output.

Figure 2.10: Inflationary bias in Benchmark habits model



Note: Optimal response of Inflation in the Benchmark habits model. Optimal policy of Inflation is a function three states, *viz.* Habits, Productivity and Taxes. The surface plots demonstrate the conditional optimal policy when either the Tax state (in left chart) or the Productivity state (in right chart) is constant at Steady state, High or Low level. Steady state Tax rate is 35%, High Tax rate is 48.4%, Low Tax rate is 18.1%. Steady state rate of productivity growth is 0%, High productivity is +5.3% and low productivity is -5.0%.

Leith and Liu (2016) note that the inherent non-linearities in the NKPC result in an inflationary bias. This bias increases non-linearly in the size of monopolistic distortion for Rotemberg price models. It is due to firms raising their markup when the firms' monopolistic power increases. Whereas, the inflationary bias problem in Leith and Liu (2016) reduces when nominal inertia increases as firms also discount their future adjustments costs and delay raising markup in the presence of existing inflation. But the inflationary bias in our model can adjust without any increase in the firm's monopolistic power or changes in nominal inertia. It is because the benchmark economy in our model has an inflationary bias. The presence of habits externalities, monopolistic competition, nominal rigidities and revenue taxes reduce equilibrium output. An increase in taxes reduces the competitive output further, thus increasing the wedge and strengthening the central banker's motive to increase output by inflating the economy.

Average markup from firm's marginal cost relation (2.47) is

$$\varphi^{-1} = \left[\left(\frac{\sigma - 1}{\sigma} \right) (1 - \tau_p) + \frac{\psi}{\sigma} (\pi - 1) \, \pi (1 - \beta) \right]^{-1} \tag{2.90}$$

a function of monopolistic competition, nominal rigidity, taxes and the prevailing average rate of inflation. It is increasing in taxes τ_p (or cost), but the interpretation is not straight forward. It can be easily seen that $\frac{\partial \varphi^{-1}}{\partial \tau_p} > 0$, it is also highly non-linear and relies on the average inflation rate in the economy. This representation of the firm's average markup can explain the impact of cost shocks on the price level and the resulting drop in demand and output. The action taken by firms in raising their markup to overcome the shock also pushes the central banker to be gentle with the interest rate hike and to avoid further reducing the equilibrium output.

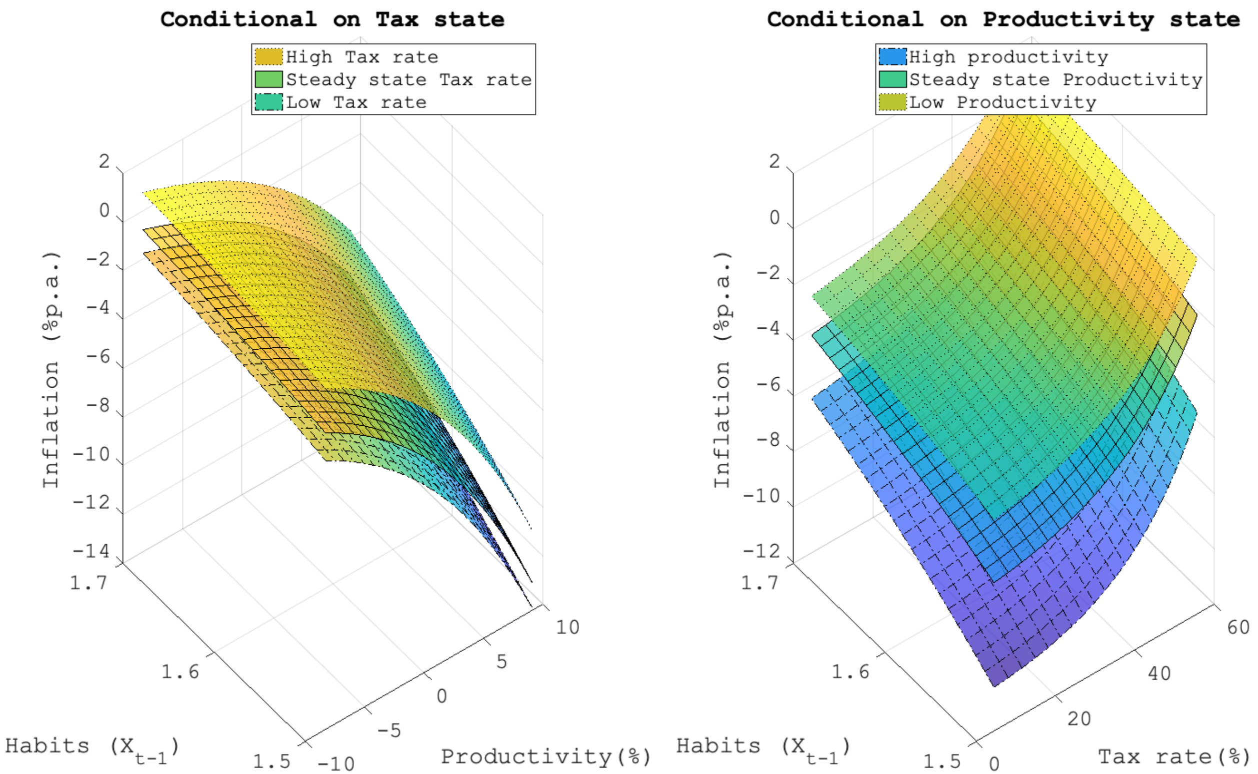
The steady-state inflationary bias reduces as habits externalities increase. The size of externalities in External habits model makes the equilibrium output bigger than the efficient. Therefore, there is a deflationary bias, as shown in Figure 2.11 on page 161. The deflationary bias worsens in the External habits model when there are high productivity or low taxes.

Whereas, the Inflationary bias, as shown by the internal habits model, plotted in Figure 2.12 on page 162, increases as taxes or productivity increase.

A comparison between the Inflationary bias charts of the Benchmark model (fig. 2.10) and the Internal habits model (fig. 2.12) shows a contrasting optimal policy for inflation when habits are high, and productivity is high. In the internal habits model, we notice that optimal policy response in a positive productivity state is to create more inflation (left chart fig. 2.12) rather than reducing the inflationary bias by raising output as it does in the benchmark economy (left-hand chart Figure 2.10). Therefore the optimal response to positive productivity shocks, when agents can fully internalize their habits, restricts any increase in output by not lowering rates as the policymaker would do if the habits were less than fully internalized by agents (for.e.g in the benchmark model).

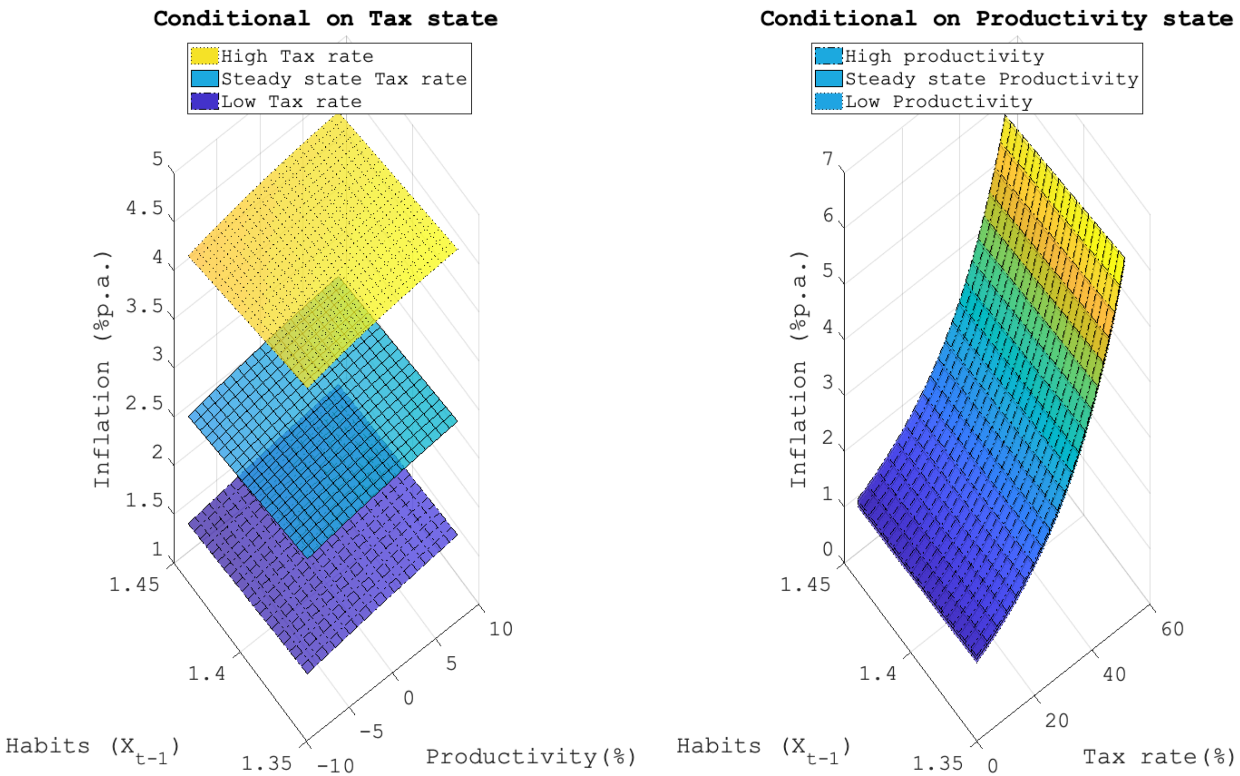
In order to understand these subtle changes in optimal policy and the inflationary bias, we need to look at another representation of the firm's average markup decision, which is in terms of the inefficiency wedge.

Figure 2.11: Inflationary bias in External habits model



Note: Optimal response of Inflation in the External habits model. Optimal policy of Inflation is a function three states, *viz.* Habits, Productivity and Taxes. The surface plots demonstrate the conditional optimal policy when either the Tax state (in left chart) or the Productivity state (in right chart) is constant at Steady state, High or Low level. Steady state Tax rate is 35%, High Tax rate is 48.4%, Low Tax rate is 18.1%. Steady state rate of productivity growth is 0%, High productivity is +5.3% and low productivity is -5.0%.

Figure 2.12: Inflationary bias in Internal habits model



Note: Optimal response of Inflation in the Internal habits model. Optimal policy of Inflation is a function three states, *viz.* Habits, Productivity and Taxes. The surface plots demonstrate the conditional optimal policy when either the Tax state (in left chart) or the Productivity state (in right chart) is constant at Steady state, High or Low level. Steady state Tax rate is 35%, High Tax rate is 48.4%, Low Tax rate is 18.1%. Steady state rate of productivity growth is 0%, High productivity is +5.3% and low productivity is -5.0%.

2.5.2 The Inefficiency wedge and Markup

The competitive economy equilibrium has a steady-state inflationary (or dis-inflationary) bias, if taxes (τ_p) are not efficient and create a distortion in the competitive output (Y) by making it either smaller (or bigger) than the Social planner's efficient output allocation (Y^*). Proposition 2.2.3 discusses the presence of a permanent wedge due to monopolistic competition which makes even the flexible price ($\psi = 0$) output ($\overline{Y}$) inefficient. We formalize that wedge as:

$$\nabla_\omega \equiv 1 - \frac{Y}{Y^*} = 1 - \nabla_R \left(\frac{\sigma - 1}{\sigma}\right)(1 - \tau_p)\left(\frac{1 - \omega k}{1 - k}\right) \tag{2.91}$$

where ∇_R is the Rotemberg pricing wedge $\left[1 - \frac{\psi}{2}(\pi - 1)^2\right]$. For fully internal ($\omega = 1$) habits, the wedge is reduced[13] to

$$\nabla_{\omega_1} = 1 - \nabla_R \left(\frac{\sigma - 1}{\sigma}\right)(1 - \tau_p), \tag{2.92}$$

if habits are fully external the wedge increases as

$$\nabla_{\omega_0} = 1 - \nabla_R \left(\frac{\sigma - 1}{\sigma}\right)(1 - \tau_p)\left(\frac{1}{1 - k}\right). \tag{2.93}$$

Whereas, if taxes are efficient $\tau_p = 1 - \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1-k}{1-\omega k}\right)$ then by definition in the flexible price equilibrium the Rotemberg pricing wedge should dissolve and we see

$$\nabla_{\overline{\omega}} = 0. \tag{2.94}$$

The size of the wedge ∇_ω changes when the competitive output changes vis-à-vis the efficient allocation output. A change in the size of the wedge also represents a shift in the output gap and inflationary bias. Firms account for this wedge in their decision making through the average markup relation which can now be modified using the tax rate,

$$1 - \tau_p = \frac{1 - \nabla_\omega}{\nabla_R}\left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1 - k}{1 - \omega k}\right), \tag{2.95}$$

to

$$\varphi^{-1} = \left[\frac{1 - \nabla_\omega}{\nabla_R}\left(\frac{1 - k}{1 - \omega k}\right) + \frac{\psi}{\sigma}(\pi - 1)\,\pi(1 - \beta)\right]^{-1}, \tag{2.96}$$

[13] Note that $k = h\frac{(1-\phi)}{1-\beta\phi}$ and for the chosen calibration of h, β, ϕ all < 1 we get $0 < k < 1$.

and if the taxes are efficient (Proposition 2.2.3) the average flexible price mark-up becomes

$$\varphi^{-1} = \left[\left(\frac{1-k}{1-\omega k}\right) + \frac{\psi}{\sigma}\,(\pi - 1)\,\pi(1-\beta)\right]^{-1}, \tag{2.97}$$

as by construction the average inflationary bias should cease to exist in the flexible price efficient economy ($1 - \nabla_\omega = \nabla_R = 1$). It shows the firms markup in a flexible price economy with efficient taxes is determined by the size of habits, habits persistence, and the proportion of internal habits (ω). The average markup for the efficient economy is increasing in the proportion of internal habits. It is not difficult to see that the firms' market power[14] increases if households become more aware of their consumption habits.

The transition dynamics of firms' average markup in an inefficient economy depend on the initial condition of the wedge, inflationary bias and internal habits. In response to these variables, the behaviour of average markup is not unambiguous. A negative output gap ($\nabla_\omega < 1$) in steady-state implies that competitive output is smaller than efficient output. And there is inflationary bias. So with an increase in the size of internal habits, the competitive economy output is getting smaller, and the inflation bias is getting larger. Ceteris paribus the optimal response by firms should be to reduce markup. Whereas if the initial condition was different and the competitive economy had a deflationary bias ($\nabla \omega > 1$) then firms' optimal response would be to increase the markup when households habits become more internal.

The response of firms and their markup can then be worked out through the following analogy. If the existing output gap between competitive and efficient output is negative, then it widens as firms charge a higher markup. On the other hand, if the gap is positive, then it narrows down due to higher markup. Since the inflationary bias [See fig. 2.10 to 2.12] is non-linear and state dependent, the output gap must be non-linear and state dependent. Therefore firms' optimal responses are non-linear and dependent on various states of the economy, on the prevailing inflationary bias and the output gap during those states. However, that is not all. The firms' optimal policy is also impacted by the demand side behaviour, determining that is not straightforward either. The presence of costly recessions also makes consumption behaviour non-linear and state-dependent. A complete picture cannot emerge without considering the behaviour of the output gap during various states of the economy.

[14] Leith, Moldovan and Rossi (2012) show a similar feature with deep habits.

2.5.3 Gap variables of the model

The welfare relevant output gap[15] of the model, can be obtained from the difference between the competitive economy output and the social planner's allocation (first best). We begin by considering the responses of gap variables for various models to +1% shock to productivity conditional on different initial states of habits and conditional on no changes in the tax state from its steady-state level (of 35%). Here we take a moment to recall that Surplus Consumption, is C above subsistence Habits (hX) and that Consumption equals Habits in steady-state. Based on this we defined Normal (economic) times as when the Surplus ratio is equal to $1 - h$, Expansion when the Surplus ratio is more than the $1 - h$ and Contraction is when households surplus is less than $1 - h$. In the Normal state, habits are at their steady-state when a shock hits the economy. In Contraction state, habits are +2 s.d. above their steady-state, so the surplus ratio is lower than its level in the normal state ($= 1 - h$). In Expansion state, habits are -2 s.d. below their steady-state, while the surplus ratio is $> 1 - h$.

Figure 2.13 on page 168 presents the response of gap variables for the benchmark model. It shows that the output gap is close to -2.5% and improves (i.e. becomes smaller) when positive productivity shock hits the economy. Presence of inherent inflationary bias in the benchmark model leads to a bigger increase in output after being hit by a productivity shock than the increase in output for the social planner. This results in the shrinking of the output gap. The Consumption gap and Labour gap also reduce due to this increase in the level of production. The habits gap follows its expected slow-moving trend. In the long term, there is an improvement in gap variables as over 100 quarterly periods later they return to their pre-shock Normal state values.

In the normal state by construction, there is no gap in the Surplus ratio. After being hit by a productivity shock, the Surplus ratio in normal state stabilises in less than 20 quarters. This is because the objective of the optimal policy after the positive productivity shock is to stabilise the Surplus ratio by lowering rates initially (Refer impulse responses in fig. 2.2 on page 144) and normalising them to curtail any runaway increases of habits.

On the consumer side of the economy, if there is an existing negative output

[15] Throughout the chapter, the term Gap refers to the difference between results of the respective model and its Social planner outcome. A negative gap shows that the allocation of a variable in the model is less than Social planner's allocation.

gap when there is a positive inflationary bias, the optimal policy response to positive productivity shocks is to reduce that gap or surplus ratio by using the policy instrument of interest rates.

In comparison for the External habits models after a +1% productivity shock, as shown in Figure 2.14 on page 169, there is an increase in the output gap, which is already very high at +5.5% in the normal state. As the underlying competitive economy has habits externalities and households are not aware of the impact of their consumption on external habits, after a productivity shock, there is a jump in consumption and output that further widens this gap. The increase in the gap is also bigger in comparison to the Benchmark model (fig. 2.13). In the Benchmark model, when households can control a high proportion of ($\omega = 0.75$) of their habit formation, they exhibit precaution in increasing consumption in response to positive productivity shocks. But consumption in the steady-state of the External habits model is very high. Therefore Optimal policy needs to respond to positive productivity shocks with a smaller rate cut as a result of which the firms bear price adjustment costs and cut prices to clear their excess produce, and there is a deflationary bias.

We have already seen from the impulse response comparisons in various habits externalities from fig. 2.4 that the rate cut implemented after a positive productivity shock is most effective in stabilising output and inflation when habits are internal. The deviation in inflation from its pre-shock level returns to zero, and there is very little change in Surplus ratio. The gap variables charts once again confirm all these features of the Internal habits model in Figure 2.15 on page 170. There is less than efficient output in the Internal habits model as the output gap with respect to the efficient output is −6.9% in the normal state. However, there is very little difference in the output gap between Expansion and Contraction states. As mentioned earlier, these are the states when the habits are $-/+2$ s.d. their Stochastic steady-state level.

There is a clear indication that if households are aware of their habits, then they run a very tight hold over their surplus consumption and their marginal utility. The Surplus ratio after the productivity shock is less than 0.01 bps[16] for internal habits, while the spread in the Surplus ratio between Expansion and Contraction states is below 0.02 bps, the least in comparison to Benchmark and External habits model.

[16] 1 basis point or 1 bps = 0.01%

This also indicates that households actions and precautionary motives complement the optimal interest rate policy after an expansionary shock. Agents realise the long term impact of increasing their consumption and therefore the central banker can obtain a better output-inflation trade-off. The discussion of gap variables of the various Habits externalities models in this sub-section has shown that the size and the nature of agents' precautionary motives play a crucial role in setting up the optimal policy and ignoring it could lead to policy mistakes.

Now we turn to explain, why is there better convergence of inflation to its pre-shock levels for internal habits model during productivity shocks and cost shocks. The efficient tax condition from Proposition 2.2.3 states that the flexible price competitive output is inefficiently smaller when

$$\tau_p > 1 - \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1-k}{1-\omega k}\right) \tag{2.98}$$

for given values of parameters $\sigma, \omega, h, \phi, k$ and β. We can rewrite it to say that competitive output is inefficiently smaller when

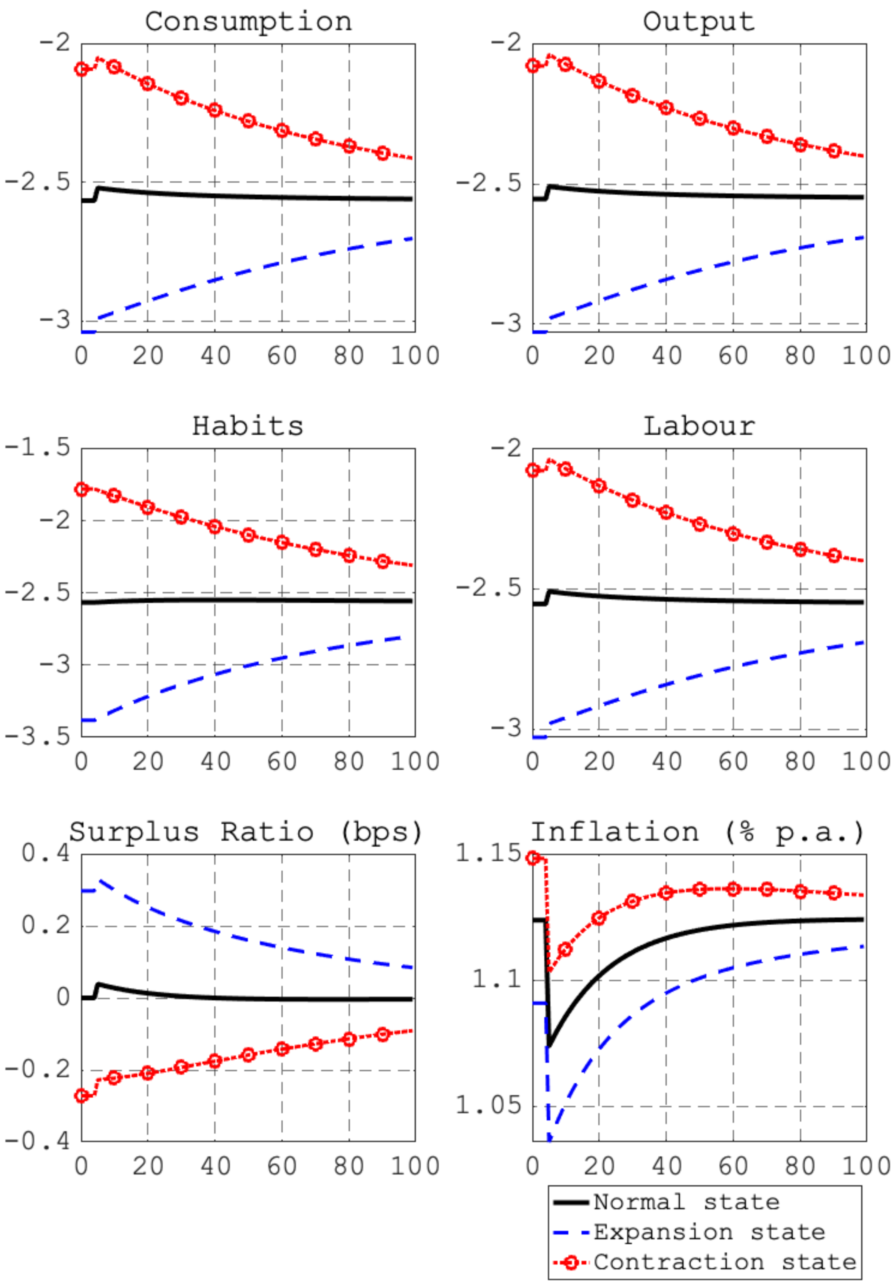
$$\omega > \frac{1}{k}\left[1 - \left(\frac{\sigma}{\sigma - 1}\right)\left(\frac{1-k}{1-\tau_p}\right)\right] \tag{2.99}$$

for given values of parameters σ, h, ϕ, β and τ_p. For the benchmark calibration given in Table 2.3 this value for ω comes out to be 0.5683 i.e. when the calibration of the proportion of internal habits becomes greater than 0.5683 the competitive output is then lower than efficient output and vice-versa.

2.5.4 Precautionary motives

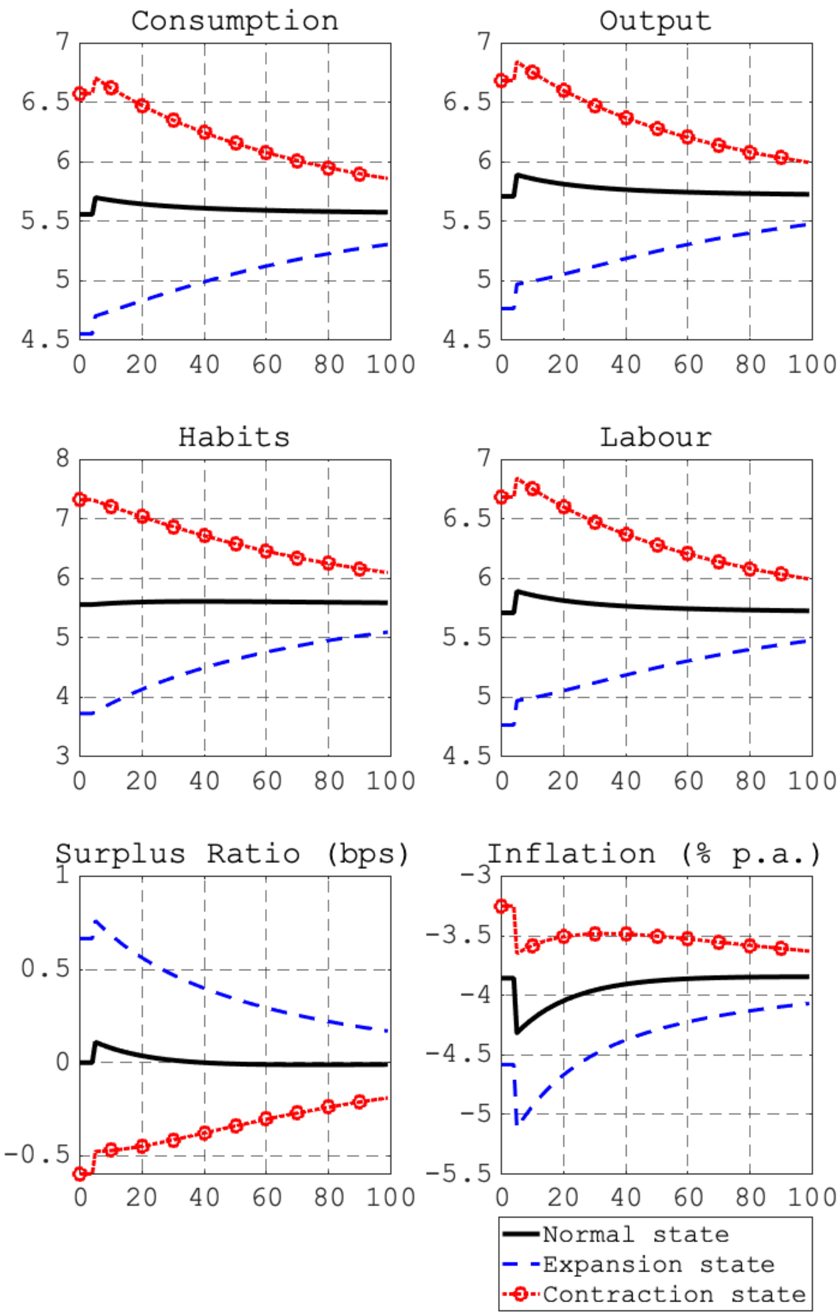
Households save for precautionary reasons to smooth out their consumption path given the uncertainty of future income. The benchmark model expects to generate precautionary behaviour in agents from costly recessions as loss of consumption during recessions is costlier than during normal times. If recessions are costly, then we should expect to see a consumer's precautionary savings as a function of varying degrees of economic inefficiency and to various states of the economy. De Paoli and Zabczyk (2013) show that in a non-linear NK model with cyclical risk aversion, an increase in perceived riskiness of the economic environment by investors after a negative supply shock raises their willingness to save. They recommend that optimal policy should account for precautionary savings by responding less than

Figure 2.13: Optimal policy gap in Benchmark habits model



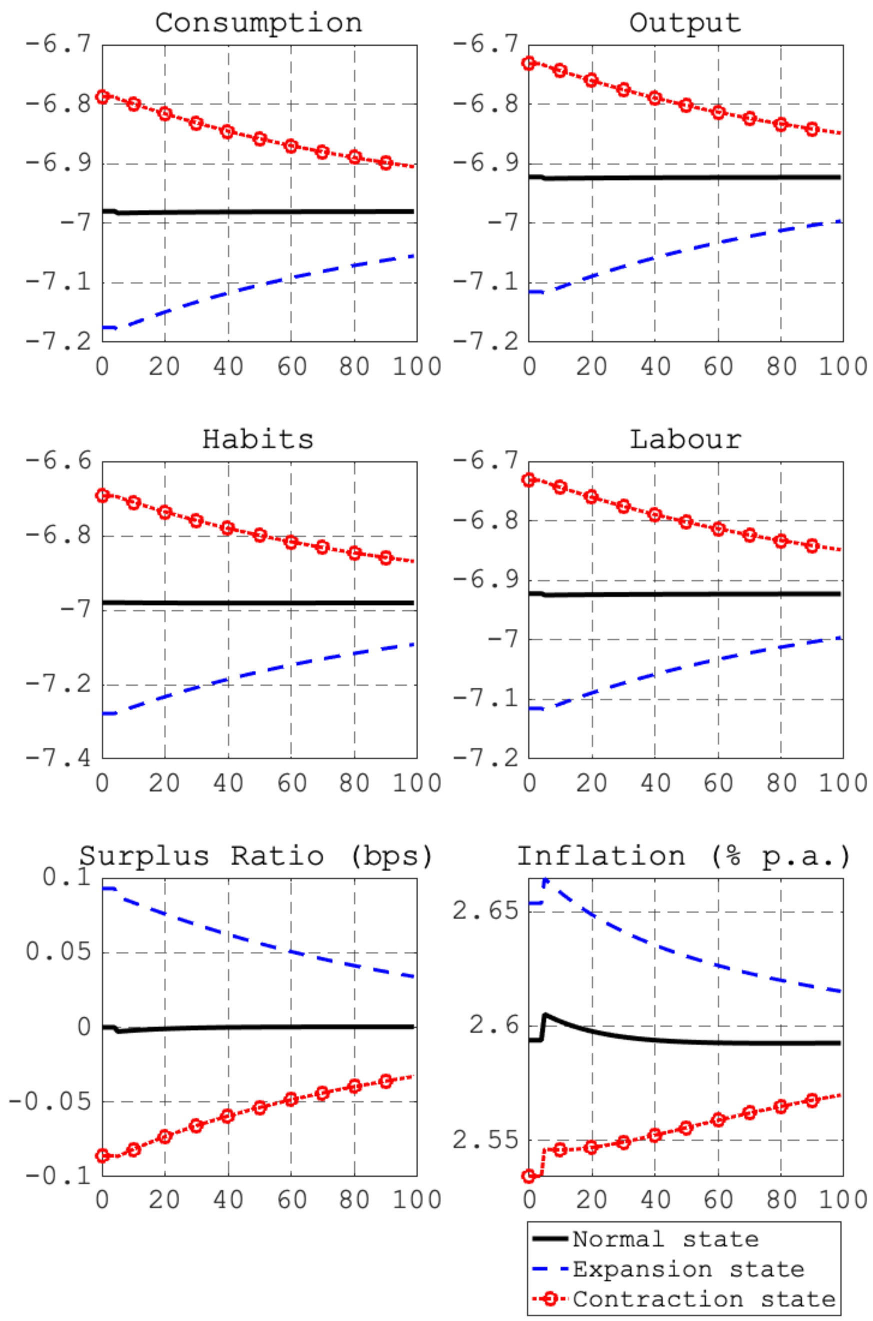
Notes: Gap refers to the difference between competitive output of the model and the Social planner's optimal outcome. Charts represent gaps variables in % deviation from the efficient outcome, and how they respond to a +1% quarterly Productivity Shock, which is administered in period 5. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) Contraction state - when Habits are above +2 s.d. of their SSS, and Expansion state - when Habits are below -2 s.d. below their SSS, in the shock period. X-axis: time horizon (in quarters).

Figure 2.14: Optimal policy gap in External habits model



Note: Gap refers to the difference between competitive output of the model and the Social planner's optimal outcome. Charts represent gaps variables in % deviation from the efficient outcome, and how they respond to a +1% quarterly Productivity Shock, which is administered in period 5. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) Contraction state - when Habits are above +2 s.d. of their SSS, and Expansion state - when Habits are below -2 s.d. below their SSS, in the shock period. X-axis: time horizon (in quarters).

Figure 2.15: Optimal policy gap in Internal habits model



Note: Gap refers to the difference between competitive output of the model and the Social planner's optimal outcome. Charts represent gaps variables in % deviation from the efficient outcome, and how they respond to a +1% quarterly Productivity Shock, which is administered in period 5. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) Contraction state - when Habits are above +2 s.d. of their SSS, and Expansion state - when Habits are below -2 s.d. below their SSS, in the shock period. X-axis: time horizon (in quarters).

it would in a linear model which has no precautionary behaviour.

It is not straightforward to determine whether the household's desire to save for precautionary motives is increasing or decreasing in size of internal habits in our model.

To obtain an analytical representation of the precautionary savings[17], let's consider the simplified case of no internal habits in the model, $\omega = 0$. The stochastic discount factor in absence of internal habits is,

$$M^{r'}_{t,t+1} = E_t\left[\beta\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right] \tag{2.100}$$

and the real rate relation from real bond pricing equation $[R^r_t]^{-1} = V^r_t$ is,

$$[R^r_t]^{-1} = E_t\left[\beta M^{r'}_{t,t+1}\right] \tag{2.101}$$

Under the assumptions of log-normality of $M^{r'}_{t,t+1}$, the euler eq. (2.101) becomes,

$$-r_t = E_t\left[m_{t+1}\right] + \frac{1}{2}var_t\left[m_{t+1}\right] \tag{2.102}$$

$$-r^*_t = E_t\left[m^*_{t+1}\right] + \frac{1}{2}var_t\left[m^*_{t+1}\right] \tag{2.103}$$

where $logR^r_t = r_t$, $logM^{r'}_{t,t+1} = m_{t+1}$ and the asterix(*) super-script describes the equilibrium interest rate consistent with price stability, or flexible price equilibrium rate. Therefore the interest rate that clears the bond market in price stability is affected by consumption smoothing $E_t\left[m^*_{t+1}\right]$, and precautionary savings effect $var_t\left[m^*_{t+1}\right]$.

Appendix B.3 on page 268 discusses the derivation of precautionary savings. As per Appendix B.3 we can write, the variance,

$$var_t(m^*_{t+1}) = \left[\sigma^2_\tau + \eta^2\sigma^2_y\right]\Phi_0\left[1 + 2\Psi^2_0 + 3\Psi^2_0\left(\kappa_x x^*_t - \kappa_\tau\varepsilon_{\tau,t} - \kappa_y\eta\varepsilon_{y,t}\right)\right]. \tag{2.104}$$

[17] Precautionary savings capture the effect of uncertainty on intertemporal wealth allocation. The concept was formalised by Leland (1968) which found that there is 'extra saving' when income is uncertain in comparison to the certainty equivalent framework if the third derivative of utility is positive. See Baiardi, Magnani and Menegatti (2020) for a literature review of the concept. In NK models based analyses of precautionary savings, such as in Baele, Bekaert, and Inghelbrecht (2010) which show these in a habits formulation that is similar to Campbell and Cochrane (1999), and in De Paoli and Zabczyk (2013) which demonstrate these in habits formulation that is similar to the one used in this chapter, a distinction between precautionary savings and consumption smoothing has been made through the log-normality of market clearing rate of interest from the consumption euler equation.

where we have used $\Psi_0 = \frac{h}{1-h}$, $\Phi_0 = \left[\frac{(1-h)\rho}{\eta(1-h)+\rho}\right]^2$ and $\Phi_1 = \eta(1-h)+\rho$, and

$$\kappa_x = \frac{(\eta+\rho)\phi(1-h)+\rho h(1-\phi(2-h))}{(1-h)\Phi_1} \tag{2.105}$$

$$\kappa_\tau = \frac{\rho h(2-h)(1-\phi)+\Phi_1\gamma_\tau(1-h)+\Phi_1(\gamma_\tau+\phi-1)}{\Phi_1^2} \tag{2.106}$$

$$\kappa_y = \frac{\eta^{-1}\rho h(2-h)(1-\phi)+\Phi_1\gamma_y(1-h)+\Phi_1(\gamma_y+\phi-1)}{\Phi_1^2} \tag{2.107}$$

The above condition for $var_t(m^*_{t+1})$ highlights three channels through which uncertainty affects the investors' behaviour: the macroeconomic volatility $\left[\sigma_\tau^2+\eta^2\sigma_y^2\right]$, the overall risk aversion which determines Φ_0, and the past (state x^*) and current economic conditions $(\varepsilon_{\tau,t}, \varepsilon_{y,t})$. On close inspection we can see that without habit formation $h = 0$, the precautionary saving motive is time invariant over the cycle $\Psi_0 = 0$. Moreover we can see that since all other terms are positive the signs of κ_y and κ_τ are determined by the conditions whether,

$$\gamma_y + \phi > 1 \tag{2.108}$$

$$\gamma_\tau + \phi > 1 \tag{2.109}$$

which specify that when shocks are persistent and habits are slow to adjust, adverse shocks to cost or technology raise uncertainty and precautionary motives.

2.5.5 Policy asymmetry

Due to inherent nonlinearities in the model and policy functions, it is difficult to explicitly break down the stochastic discount factor into inter-temporal and precautionary parts which are explicit functions of various states and parameters. Therefore any analytical explanation of the transmission mechanism of optimal policy needs to be extracted from differences in responses obtained during various states and shocks to the economy.

This section discusses the results of simulations run to look for changes in optimal policy responses for various:
a) Models of Habits externalities, i.e. External, Benchmark or Internal habits,
b) Initial conditions of the pre-shock economy, i.e. whether in Expansion, Normal or Contraction state,
c) The nature of shock hitting the economy, i.e. whether productivity or cost, and

d) The magnitude and direction of the shock hitting the economy, i.e. whether it is +/-1% or +/-3%.

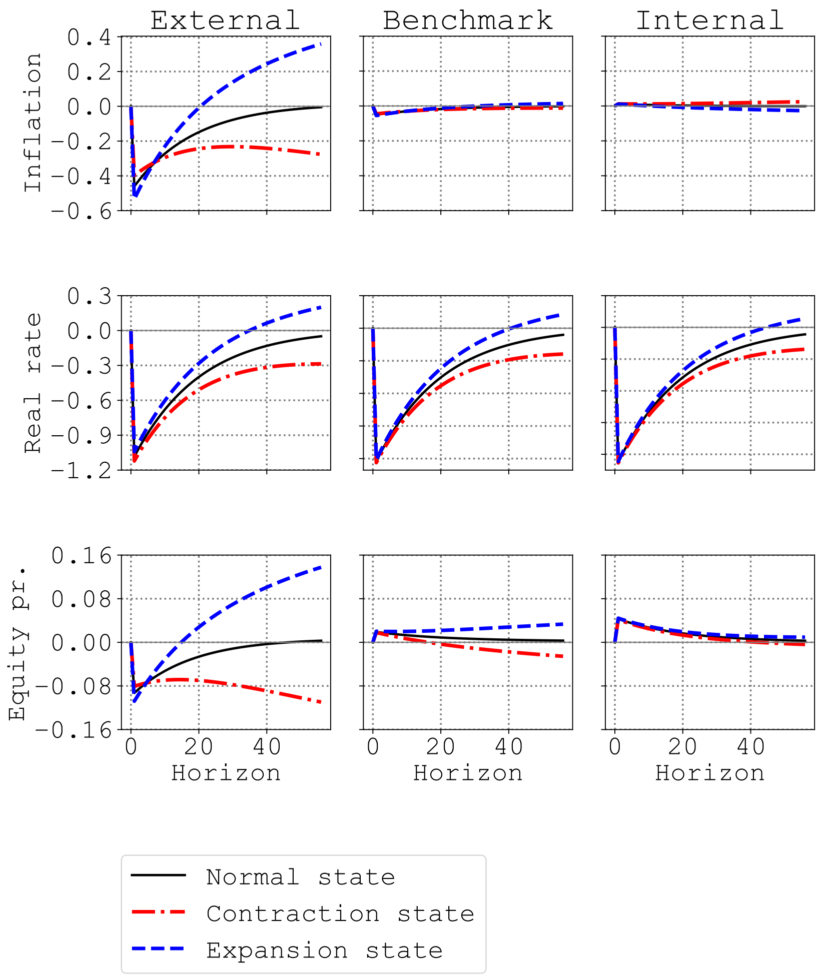
Figure 2.16 and 2.17 look at how the policy instrument (interest rates) accounts for the changes in agent's precautionary behaviour which is increasing in uncertainty about the future income and alters agent's expected consumption path. The central banker can limit interest rate response to both productivity and cost shocks when habits become more internal, and we notice that across all models (Figure 2.16 to 2.17) policy interest rates are kept lower during contractionary states.

The optimal policy response to +ve productivity shocks is also state-dependent, as seen in Figure 2.16. If habits are fully external (first column of fig. 2.16) then agents cannot acknowledge the impact of their present consumption on future habits. Therefore, we see a faster normalization in interest rate when the economy is in Expansion. In contrast, if the economy's initial state is one of Contraction and its hit by a positive shock, then interest rate remains low for longer. Similarly, after the initial rate cut, the optimal policy recommends normalizing rates back to their pre-shock levels slowly when the economy is in contraction. The simulations reveal that this normalization of interest rates is enhanced when the size of externalities reduces and agents precautionary motives are taken into account.

As agents can better internalize their habit formation, the trade-off between inflation and Surplus ratio stabilization improves. Recall that the Surplus ratio is above its steady-state level when the economy is in expansions and vice versa during contractions. So the optimal policy sets out an interest rate path to restore the Surplus ratio, which simultaneously also stabilizes the Equity premium.

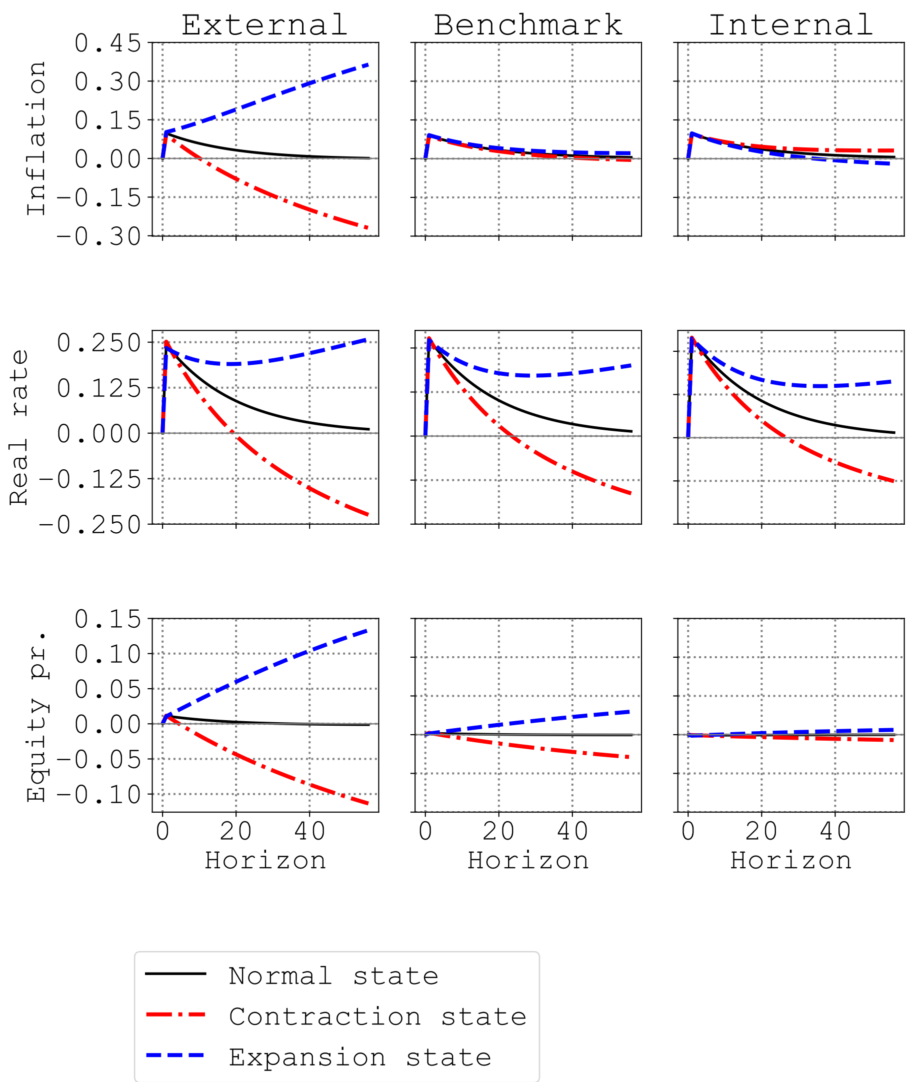
During inflationary cost shocks, as Figure 2.17 suggests, the central banker with only interest rates in hand as a policy tool should be less aggressive in both raising and cutting rates when habits are more internal, or when agent's precaution is high. The comparison between Benchmark (column 2 fig. 2.17) and Internal habits (column 3 fig. 2.17) is interesting as we observe that when habits are fully internal, then central bank responds with a bigger rate hike in the shock period. But as agents precautionary savings (due to internal habits) leads to drop in consumption and surplus ratio the central banker corrects its interest rates faster (during expansions) and responds with lower rates for longer when the economy is in contractions. This results is substantiated by comparing the optimal policy responses of positive and negative shocks to the economy.

Figure 2.16: Productivity shocks in varying Habits externalities and States I



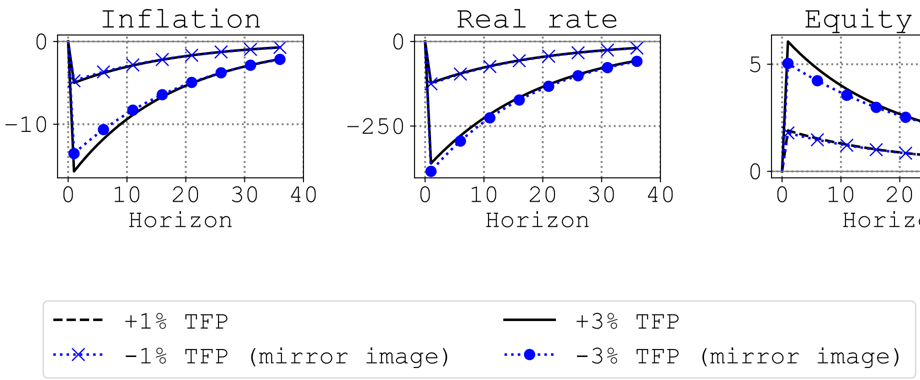
Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly Productivity shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in tax rate for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 1% internal ($\omega = 0.01$) and 99% external. All charts in a row share a common Y-axis which is labelled for charts in first column. All charts in a column share a common X-axis which is labelled for charts in the last row. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure B.3.

Figure 2.17: Cost shocks in varying Habits externalities and States I



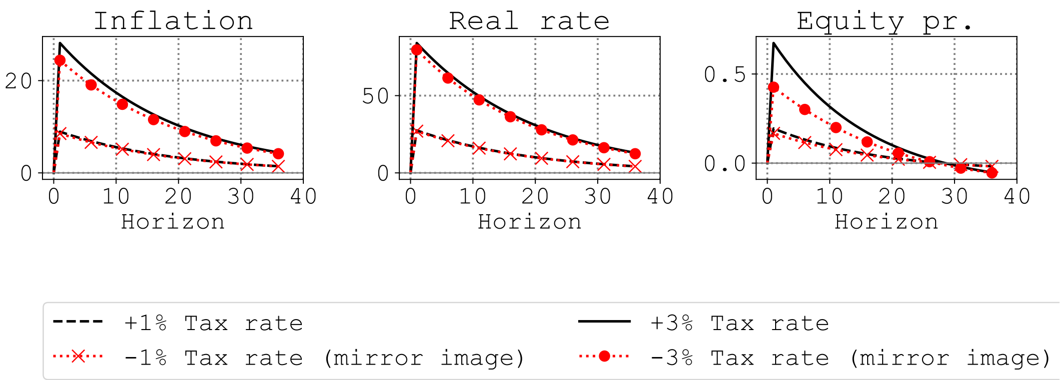
Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly Tax rate shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in Productivity for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 1% internal ($\omega = 0.01$) and 99% external. All charts in a row share a common Y-axis which is labelled for charts in first column. All charts in a column share a common X-axis which is labelled for charts in the last row. X-axis: time horizon (in quarters). Impulse responses for the remaining variables are posted in Figure B.4.

Figure 2.18: Asymmetric response to Productivity shocks



Notes: Impulse responses in annualised basis point (1bps = 0.01%) from their pre shock levels. The negative of impulse response to -1% and -3% productivity shocks and plotted. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in Tax rate for the entire horizon. Habits are assumed to be at their Stochastic steady state level in the shock period. X axis : Time horizon in quarters.

Figure 2.19: Asymmetric response to Cost shocks



Notes: Impulse responses in annualised basis point (1bps = 0.01%) from their pre shock levels. The negative of impulse response to -1% and -3% Tax rate shocks and plotted. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in Productivity for the entire horizon. Habits are assumed to be at their Stochastic steady state level in the shock period. X axis : Time horizon in quarters.

Not only is the optimal policy more accommodative of agents' precautionary motives during recessions, but it is also asymmetric between positive and negative shocks hitting the economy in any state. Figure 2.18 and 2.19 on page 176 show that policy response is asymmetric to the size of disturbance hitting the economy. Whereas, Figure 2.20 and 2.21 on pages 179 to 180 highlight the policy asymmetry to productivity and cost shocks during in the different Habits externalities models when the economy in pre-shock period is in Contraction state, i.e. its Habits are above +2 s.d. and its Surplus ratio is above $1 - h$. In these figures, mirror image (i.e. -1 times) of the impulse response to adverse shocks Productivity (TFP) and Taxes (Cost) are plotted.

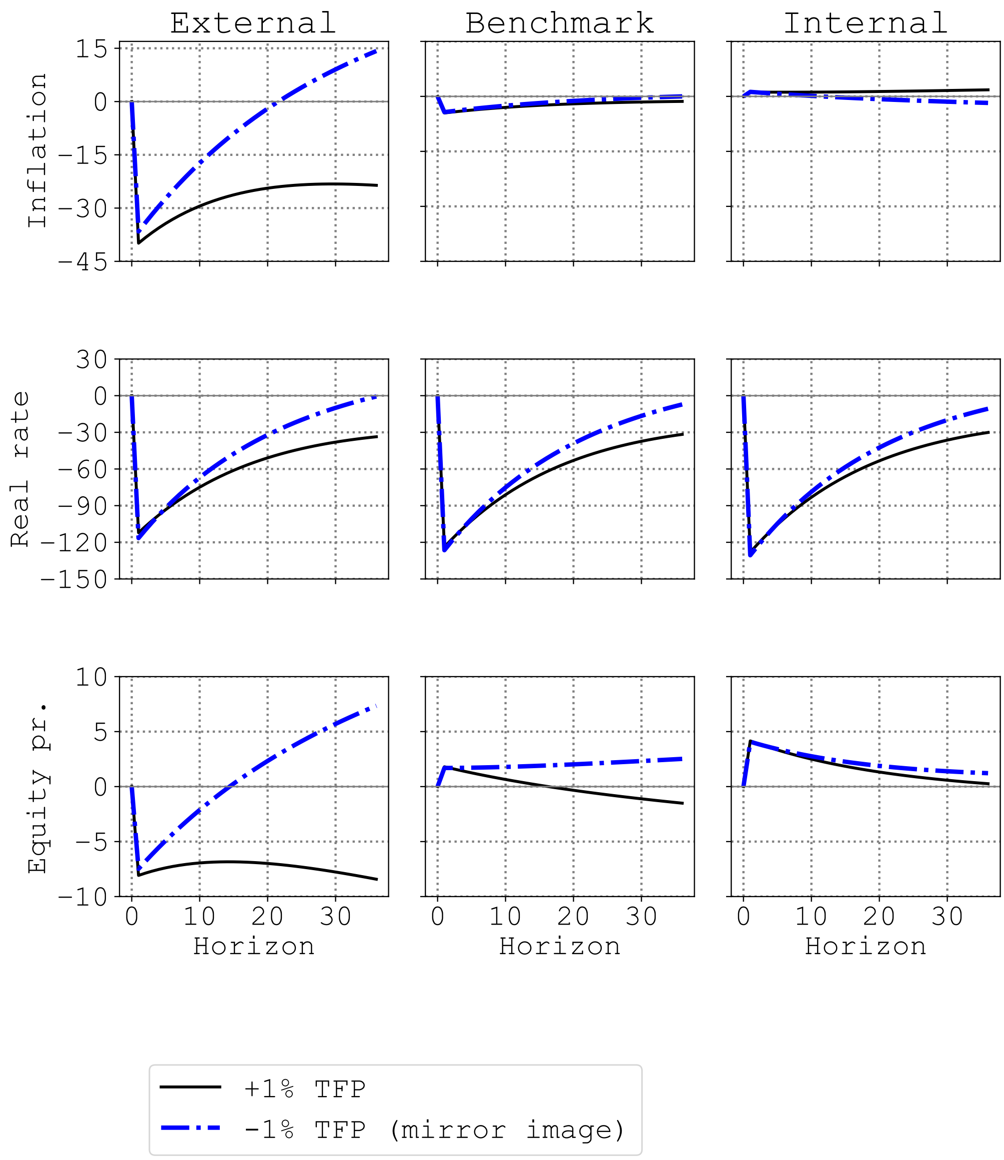
The following features in these charts that stand out. Firstly, a three times potent shock requires more than three times the optimal policy response, which highlight the inherent non-linearity of optimal policy. Secondly, the size of real rate adjustment decreases in habits externalities. Thirdly, the model with entirely internal habits achieves the best stabilisation of inflation and Equity premium. Lastly, the initial responses to positive and negative shocks are symmetric. However, there is an asymmetric pace of real rate normalisation, i.e. rollback to its pre-shock level, which depends on the pre-shock state and the nature of the shock.

In the presence of asset bubbles the 'leaning against the wind' policy utilizing a stronger interest rate response may raise the volatility of asset prices and their bubble component as per the backward-looking sunspot solution around a stable bubble state (Galí, 2014). So stabilisation of bubble component takes precedence in optimal monetary policy over stabilisation of aggregate demand. In contrast the forward-looking solution around an unstable bubbly state (Miao, Shen, and Wang, 2019), esp when bubble shocks are serially correlated, shows that asset bubbles respond similar to asset prices to increase in interest rates, and therefore the conventional 'lean against the wind' policy is optimal.

Recall, that our benchmark habits model has an inflationary bias. Its competitive output is inefficiently smaller to the first-best outcome. Therefore, the discretionary policymaker is more accommodative to expansionary (or deflationary) shocks, and pursues a faster normalisation of interest rates in response to the contractionary (or inflationary) shocks. This asymmetry is particularly visible in the various impulse responses during contractions (fig. 2.20 - 2.21). Optimal policy shows a greater tolerance for keeping rates 'low for longer' period during positive productivity shocks and cut in the Tax rates, and it normalises the real rates faster when hit by a negative Productivity and a rise in Taxes.

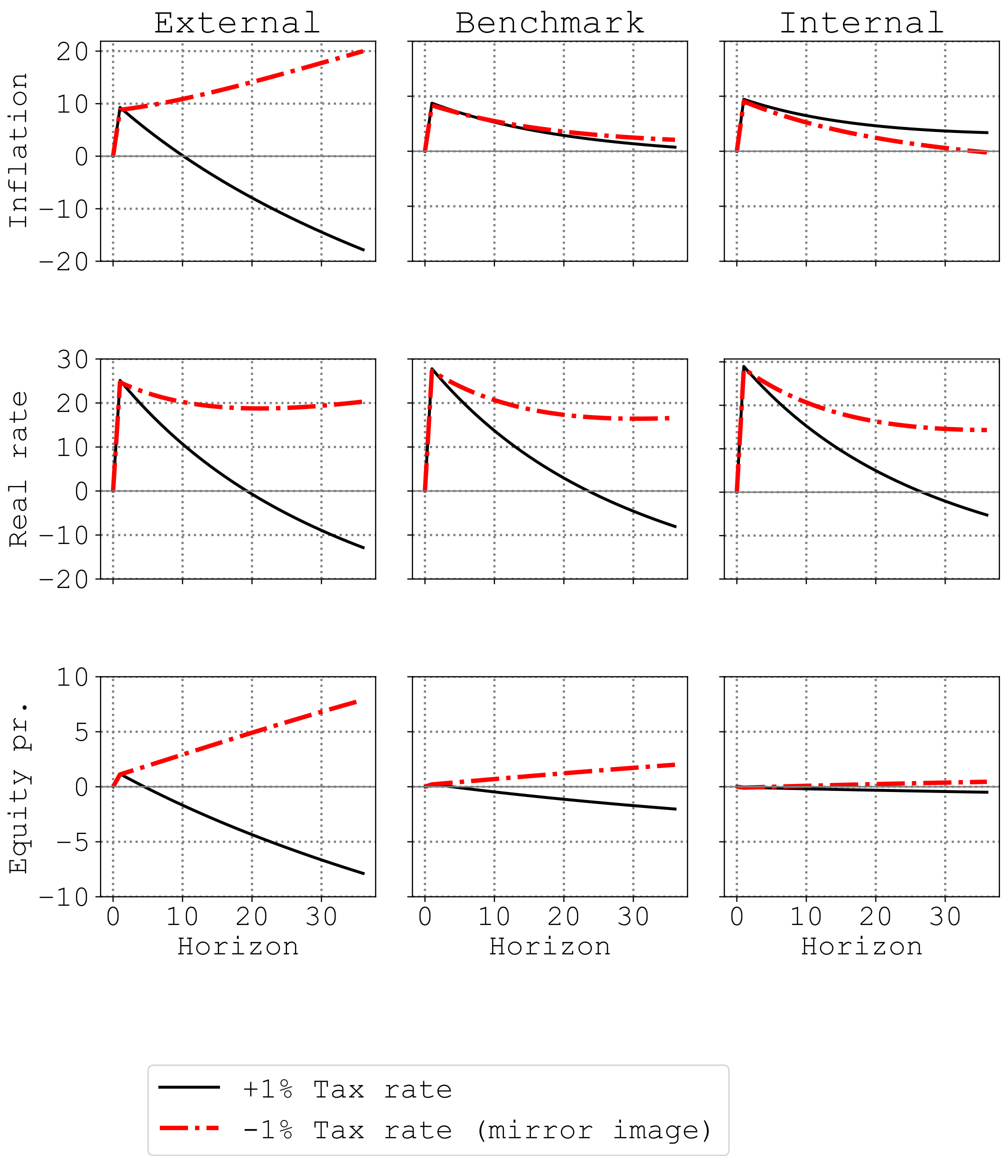
We learn from these simulations that in costly recessions household's display higher precautionary motives: when they have more control over the composition of their habits and their utility, and when they have a low surplus ratio. The optimal policy recommends asymmetric responses to alleviate the agent's precautionary behaviour. Ignoring the agent's behaviour in optimal policy decisions could reduce welfare and prove to be costly.

Figure 2.20: Asymmetric response to Productivity shocks during Contractions



Notes: Impulse responses in annualised basis point (1bps = 0.01%) from their pre shock levels. The negative of the impulse response to -1% productivity shocks is plotted. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in Tax rate for the entire horizon. Contraction state is defined as the state when Habits in the pre-shock period are above +2s.d. of their Stochastic steady state level. X axis : Time horizon in quarters. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 0% internal ($\omega = 0.01$) and 100% external.

Figure 2.21: Asymmetric response to Cost shocks during Contractions



Notes: Impulse responses in annualised basis point (1bps = 0.01%) from their pre shock levels. The negative of the impulse response to -1% productivity shocks is plotted. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in Productivity for the entire horizon. Contraction state is defined as the state when Habits in the pre-shock period are above +2s.d. of their Stochastic steady state level. X axis : Time horizon in quarters. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 0% internal ($\omega = 0.01$) and 100% external.

2.6 Summary and Conclusions

This chapter introduces costly recessions and cyclical risk aversion to standard New Keynesian analysis. Its global optimisation solution shows that optimal discretionary policy during bad states, is geared towards stabilising the output gap and the surplus ratio ahead of its usual care for inflation stabilisation that standard New Keynesian models attribute to it.

Precautionary savings matter, since agents are more protective of their consumption during recessions in comparison to booms. This precautionary behaviour is asymmetric, and it is increasing non-linearly in the amount of information and control agents have over their future consumption.

There is, therefore, need for active policy management which complements this precautionary behaviour. The optimal discretionary policy result is non-linear and asymmetric during good and bad states of the economy. During adverse productivity and cost shocks, it seeks to stabilise agent's surplus consumption (or consumption in excess of the subsistence level of habits), which subsequently neutralises the Equity premium.

In the presence of cyclical risk aversion, equity premium is a manifestation of the agent's precautionary motives. They have a similar impact to an agent's utility, which is that as the surplus consumption falls, risk-aversion increases, and both equity premium and marginal utility rises. Thus a policymaker without an ability to observe output gap can also sufficiently replicate the optimal strategy by restoring the equity premium to its steady-state.

The results hinge on the ability of the policymaker to reduce rates when required. Therefore including zero lower bound, capital with adjustment costs, availability of unconventional policy with the policymaker, heterogeneity of investment technology, and idiosyncratic income shocks could be other exciting extensions for future.

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3. Flight to Safety as Optimal Policy in Cyclical Risk Aversion

The Flight to Safety phenomenon has been portrayed in literature as an aberrant reaction to some form of market externalities, valuation constraints or Knightian uncertainty. This consideration of Flight to Safety as a short-term and an asset market phenomenon has also resulted in it being overlooked for the purposes of macroeconomic theory and policymaking. Whereas, recent empirical evidence suggests that the Flight to Safety that exists even in low-frequency data, has significant information about future productivity and business cycle performance. Therefore this chapter produces a new micro-founded general equilibrium approach to model Flight to Safety phenomenon observed during recessions or bad states of the economy. It showcases Flight to Safety as an optimal policy response by individuals in an NK model which features costly recessions and an option to switch between risky and safer production technology. Due to the presence of consumption habits, there is a non-linear increase in marginal utility once consumption gets closer to the subsistence level of habits, this makes recessions costly. The households find it more hurtful losing a unit of consumption in bad states than losing a unit in boom times. However, making a precautionary switch towards a safer asset allocation safeguards the investors against any loss of future income. Global solution methods demonstrate that ex-ante Flight to Safety is the optimal response for individuals that are consuming closer to their subsistence, and that calls for a remedial policy response. These findings substantiate the macro-finance based explanations of recessions by demonstrating that Flight to Safety stems inherently from individual's risk aversion even in the absence of intermediate credit constraints or information asymmetry.

3.1 Introduction

Macro-finance literature has shown that both risk premia and precautionary savings motives increase during recessions or economic *bad* times. A deterioration of the near term economic growth outlook and periods of poor employment prospects are accompanied by an enhanced preference for *safe* vis-à-vis less safe or *risky* investments. Such sudden movement of capital during *bad* economic times from *risky* to *safe* assets is commonly referred to as Flight to Quality or Flight to Safety, and a counter phenomenon of sudden movement of capital from *safe* to *risky* assets during *good* economic times is known as Flight to Risk. Conventional attempts to model the Flight to Safety (or Risk) have relied on some form of assumptions of credit or information constraints or Knighting uncertainty. This paper proposes a new micro-founded model of Flight to Safety during costly recessions and cyclical risk aversion. The model is a modification of the standard New Keynesian model used in policy analysis, with the following modifications. It accounts for an individual's cyclical (or time-varying) risk aversion. Also, it has the provision of investment options for that individual to safeguard them against any expected loss of future income. The parsimonious model is solved non-linearly using Chebyshev's numerical approximation, and it captures fundamental asset price dynamics observed in data.

The model economy incorporates costly recessions, the mechanism of which emerges from including internal habits in household consumption. The previous chapter has shown that in a standard new-Keynesian model with Rotemberg costs, a drop in consumption with respect to habits during *bad* states makes it particularly hurtful to consumers and they require a higher premium for undertaking risky investments even in *good* states. That same mechanism is carried forward in this chapter. A detailed description of the various *good* and *bad* states of the model is presented in Box 3.2.1.1 of section 3.2. Another distinguishing feature of the model is that the risk-averse representative agent has an option to alter the composition of risk in the production technology, by changing the proportion of two kinds of firms, *safe* firms and *risky* firms that operate in the economy.

In macroeconomic models of costly recessions, as we have seen from Chapter 2, the agent's risk aversion is countercyclical, and it leads to non-linear pricing of risk and returns. In these models, as the current consumption of the household becomes low, they reduce their habits and alter their stochastic discount factor. We have seen in the previous chapter that such actions have an asymmetric impact on the firm's marginal cost and optimal choice of production. The homogeneity in production opportunity set for the household in such models restricts them from making an influence on the riskiness of the production process itself. There is a lack of a feedback mechanism from household utility decision to risk allocation in such models. Whereas, empirical evidence and asset price observations from data suggest that investors align the risk composition of the portfolio often, as they expect an adverse or favourable future state of nature. Therefore, this chapter aims to address this gap and contributes to the literature by proposing a model of costly recessions which provides the representative agent with the option to alter the composition of risk operating in the economy. Moreover, in doing so, which can provide a theoretical grounding for the sudden changes in portfolio risk compositions that are observed in Flight to Safety and Flight to Risk episodes.

Establishing a model where Flight to Safety is an innate development as a reaction to the individual's utility optimisation and the producer's profit maximisation behaviour, requires introducing heterogeneity in production and giving control to the consumer to alter the output in future states by acting on it today. In the model economy, there are two kinds of intermediate firms that operate they are either *Safe* or *Risky*. *Safe* firms in the model offer less productive but also less risky technology or one which has a smaller unconditional variance of the technology process, whereas the *Risky* firms offer a more productive but also an ex-ante riskier or with a higher unconditional variance of the technology process. Based on her state-dependent risk aversion, the representative agent can decide upon the proportion of *safe* or *risky* firms that will operate in the economy. The ratio of *risky* firms that operate in the economy determines the aggregate risk in the economy. The representative agent's decision to ex-ante shutdown firms of *risky* category with respect to increasing state-varying risk aversion is called Flight to Safety. Furthermore, instead, if she chooses to shut down safe firms in favour of risky firms, then that phenomenon is captured as Flight to Risk.

The general equilibrium set up in this paper shows how a representative profit-seeking agent when acting as a producer maximises the firm's value in each state of nature by equating marginal rate of transformation with the stochastic discount factor. Therefore she chooses higher production (employing more risky technology) in states when consumption (output) is more valuable, or the stochastic discount factor is low. However, when the same producer dons the hat of a risk-averse consumer, she also equates the stochastic discount factor to her marginal rate of substitution. She, therefore, stays away from indulging in risky ventures in periods when consumption is dear.

The model also depicts how an increase in habits with respect to consumption or fall in consumption with respect to habits raises the marginal utility from consumption and the risk-aversion of the agent. Increase in habits lowers the stochastic discount factor and raises the expected return from a risky investment. As the agents become risk-averse, they shut down risky firms. This trade-off leads to a shift in the composition of risky and safe firms in the economy and impacts lifetime production, employment and welfare. This desire for investors to shift production from risky to safe technology as their cyclical risk aversion increases are captured in the model as Flight to Safety.

Similar to the solution approach in the previous chapter of this thesis, this model, too, is solved non-linearly using numerical methods. A key implication of pursuing global solution methods in macro-finance models is that they can avoid asset pricing puzzles such as equity premium puzzle and risk-free rate puzzle for plausible levels of risk aversion coefficient.

A necessary corollary that emerges from the results of this chapter is that Flight to Safety can be studied without using credit or information constraints or assumptions of Knightian uncertainty. This chapter also investigates the policy implications from the viewpoint of an efficient planner and find that competitive allocation has excessive risk-taking when the risk premium is low and not enough risk allocation when risk premium is high. Inefficient allocation of risk generates an inflationary bias in the steady-state.

The methodology used here to include intra-investment flows between high and safe investment opportunities to capture Flight to Safety has applications in empirical studies in estimating the welfare loss from recessions.

Finally, this model contributes to the policy debate in choosing the right micro-foundations to model Flight to Safety. As different micro assumptions would have different policy outcomes. If Flight to Safety is emerging from a lack of liquidity in intermediary capital, then bailouts are a justified policy response. Otherwise, if the observed Flight to Safety is a measure of balance sheet constraint faced by intermediaries, then regulatory involvement will include setting up alternate markets, removing inefficiencies and taking the role as lender of last resort. Instead, suppose Knightian uncertainty leads to Flight to Safety, as speculators take excessive risk in complex instruments that they do not understand and expect to being bailed out as they fail. In that case, the policymakers could intervene to prevent systemic fallout but also not intervene to stop moral hazard by letting a few rotten ones fail. However, suppose Flight to Safety is a common phenomenon of business cycles as modelled in this paper and it can occur due to widespread individual risk-aversion, even in the absence of financial intermediaries, financial innovation, credit constraints, or information asymmetry. In that case, policy interventions to modulate the process of risk allocation by investors are needed.

3.1.1 Related Literature

The Flight to Safety phenomenon does not have a single definition, but it is a particular term related to the process of 'sudden' restructuring of risk allocation during times of economic uncertainty as investors divest from risky assets in favour of safe assets. In recent memory, the Global financial crisis presented us with bouts of high volatility and uncertainty where capital allocation between safe and risky assets fluctuated leading into episodes[1] where there were excessive risk-taking or 'risk on' and where there were very little risk-taking or 'risk off'. This restructuring of investments from risky to safe assets is known as FTS, and its counterpart in the reverse direction, from safer to risky assets is known as FTR. Capital flights leading into re-allocation of risk can occur between two investment opportunities within the same sector, the same asset class. They can also occur between different issues (new and old) of the same asset, or between different assets, or between different classes of assets in the same country, or between domestic and international portfolios. There is also a wide range of assets that could classify as 'high' risk (risky) or 'low' risk (safe). In a panel study Apergis (2015) demonstrate that FTS and FTR are global phenomena and certain currencies such as the US Dollar (USD), the Swiss Franc (CHF) and the Japanese Yen (JPY) offer a safer haven than others to safeguard investments.

A feature of this 'sudden' nature of Flight to Safety episode is that it results in 'sharp' short term rise in prices (or fall in expected returns) of the sought after, i.e. less risky asset, and vice-versa for the more risky asset. This is the reason there is a vast literature that has focused on pricing and quantity behaviour of assets to understanding the Flight to Safety, and the Great Recession has once again brought this research to forefront [See Caballero (2010), Caballero

[1]The HSBC risk on risk off (RORO) index points to a significant shift in 'risk on' and 'risk off' during 2008-2010 period. Source: https://impact.ref.ac.uk/casestudies/CaseStudy.aspx?Id=20291. Whereas Fed Atlanta research article argues the periods of correlation in positive stock prices and negative treasury prices during the 'risk on' and vice versa during 'risk off' has lost its appeal in recent times. Source: https://www.frbatlanta.org/cenfis/publications/notesfromthevault/11-risk-on-risk-off-in-the-long-run-2017-11-28.

and Kurlat (2009), DeLong (2012), Cochrane (2011), DeLong (2010)]. But the FTS episodes have an impact beyond short term asset price fluctuations. In the shorter term, the supply of assets is fixed, and so the Flight to Safety behaviours have led to savings glut (Bernanke, Gertler, and Gilchrist, 1996) and hitting of Zero-lower bound (Caballero, Farhi, and Gourinchas, 2017) on eligible securities. Safe assets are subject to lower margin requirements as their prices deviate less from their fundamental values. Therefore during volatile times, large deviations of prices of risky assets from their fundamental price lead to hoarding of safe asset (Brunnermeier, 2009).

The theoretical literature on Flight to Safety argues that agency problems lead to Flight to Safety during times of risk aversion. Several attempts to model Flight to Safety mechanisms, especially in the theoretical literature following the great recession, revolve around a common theme of investigating FTS from the lens of market imperfections (Caballero and Kurlat, 2009). These commonly include introducing imperfections in the form of information asymmetries, incentive asymmetries, principal-agent problem, uncertainty-aversion, Knightian-uncertainty, the complexity of financial products, in generating the risk and illiquidity premia which motivates the Flight to Safety behaviour.

Fund managers incentives are modelled by Vayanos (2004) to benefit from a management fee which they receive in the proportion of assets under management. These managers depict liquidity preference and risk aversion when market volatility is high as then there is a higher chance of their fund value falling below its liquidation threshold. The drop in asset prices during Flight to Safety periods can be more severe if fund managers have margin requirements (as hedge funds do). A unique feature of periods of high illiquidity and uncertainty is that it raises margin requirements on collaterals. If then in order to meet margin requirements, overleveraged managers would look to resell their assets to the banks, then banks with limited capital would drag down their prices further (Krishnamurthy, 2010) and exacerbate the spiral. Besides, there exists a typical principal-agent problem where investors lack sufficient control over the fund managers to act in their best interests. He and Krishnamurthy (2013) model that investors cannot expect specialist (bank/fund managers) to exert full effort. Investors could forgo a portion of realised returns to provide incentives for fund managers. Even then, the intermediaries may choose their own desired level of market risk and ignore the investors' risk allocation recommendation. In order to then alter risk in their portfolio, the investors have to provide more incentives to fund managers, such as de-linking the managers' compensation with their performance. However, there is a limit to which this could be achieved, and that restricts the investor's ability to make meaningful changes to portfolio risk.

When faced with the worst-case possibility, the investment intermediaries look to minimise losses. Brock and Manski (2011) confirm Flight to Safety effect is more significant when specialists choose to maximise their payoff in the worst scenario (max-min) or minimise the maximum possible regret (min-max regret) from the worst case. Caballero and Krishnamurthy (2008) describe that Knightian uncertainty or a lack of knowledge about complex financial instruments or uncertainty of available information, forces the specialists to follow a max-min strategy where they focus on the worst possible outcome and hoard safe assets as a safeguard. It leaves them underinsured in other adverse scenarios. Therefore when a negative shock hits the market, liquidity dries too soon.

In a nutshell, theoretical studies focus on intermediaries and their liquidity preferences, as a critical mechanism that builds into Flight to Safety and freezing of credit markets. The story goes, that due to an external shock, there is information friction such as lack of knowledge of complex investment products or uncertainty about future, that hinders the intermediaries' assessment of the present situation. There is some real or perceived weakness in the wealth of intermediary, which builds into a sudden market reaction, preference for liquidity and Flight to Safety.

The intermediary based perspective of Flight to Safety places too high a value of information asymmetry and credit constraints. This approach has been useful in explaining the extraordinary turn of events such as the inter-bank credit freeze that occurred during the Great Recession. However, it ignores the more common Flight to Safety episodes when assets flow towards banks (intermediaries) instead of risky opportunities. Therefore, it is also problematic in determining the impact of Flight to Safety on the real economy from these models. Moreover, in this literature, the role of individual investors as being in control of their investment decisions is often ignored.

This paper borrows from the production side models of asset pricing, to provide a different perspective on Flight to Safety hypothesis. Key motivating literature for this paper is based on the partial equilibrium models such as Cochrane (2016), Belo (2010), Jermann (2010) and Cochrane (1991) that use producers' ability to transform production across various states of nature to establish the observed asset price behaviour. These papers have shown that a production-based measure of stochastic discount factor (SDF) is created by allowing consumers with a choice of productivity set in various states of nature. Such a measure of the SDF can also be useful in explaining the observed asset-pricing phenomenon. For e.g. Cochrane (1991) and Jermann (2010) consider the implications of including two production technologies in the model on asset prices. By giving producers access to two technologies, one for each state of nature, they extract a stochastic discount factor purely from the producer's first-order condition and marginal rate of transformation. They also match key moments of consumption-based asset-pricing literature. Further on this line of investigation Belo (2010) allows for an arbitrary number of states of nature in the economy and strengthens the results of production side models of asset pricing by matching for cross-sectional variance in stock returns.

However, asset pricing literature or models of the production side of the economy have not explained the risk on, risk off behaviour observed during bad states. There is a brief mention of it in a stylised example in Cochrane (2016) as it elucidates the expected changes in consumption and de-leveraging during Flight to Safety. However, his ad-hoc representation lacks the backing of a fully solved DSGE model. This chapter extends the research on Flight to Safety and Cochrane (2016) example of an endowment based economy into a general equilibrium model. The substitution of risk in an agent's production technology considered in this chapter results in ex-ante Flight to Safety that is endogenous and precedes changes in total factor productivity.

Rest of the chapter is structured as follows. Next section 3.2 describes the model economy, and it is solved for the discretionary planner in section 3.3. Section 3.4 describes the calibration and numerical solution. Section 3.5 discusses the results and policy implications, while the last section 3.6 concludes.

3.2 Model with habits and two technologies

A standard new Keynesian model[2] with Rotemberg cost is extended to account for costly recessions. Internal habits in consumption are included to make the economic *bad* times in the model costly for the consumer, as they make losing a unit of consumption in bad times costlier to losing a unit of consumption in *good* times. The consumption habits included in this model are contemporaneous or *Keeping up with the Joneses* instead of the more common approach of using *Catching up with Joneses* form of habits in macroeconomic literature. *Keeping up with the Joneses* form of habits are used to make current subsistence level of habits a function of past habits and present consumption. The dependence of present habits on the present period consumption is small enough to keep consumption habits as slow-moving, but the parameter is sufficient enough to make the consumer take into account the impact of an increase in present period consumption on present period habits and future utility.

The marginal utility from an additional unit of consumption derives from the difference between the level of consumption and the subsistence level of habits. Subsistence habits are assumed to be slow-moving. Therefore current changes in consumption have little impact on the subsistence level of habits. In this representation, the marginal utility and thus risk-aversion increases when consumption falls closer to a subsistence level of habits. Using the slow-moving contemporaneous representation of habits has a distinct advantage. It generates lower variation in asset prices with smooth cycles in risk aversion and presents a trend that confirms with the growth in aggregate and individual consumption observed in data. In contrast, the more common approach in macroeconomics is to model habits of the kind $U_t = C_t - \theta C_{t-1}$, i.e. where utility U_t depends on the difference between current and lagged consumption. It successfully generates the slow, hump-shaped response in consumption Leith, Moldovan, and Rossi (2012) but also leads to the large variation in asset prices that we do not observe in data. Slow-moving habits [Cochrane (2016), Fig.2] in comparison, produce smooth cycles in risk aversion.

Additionally, the model economy also involves a choice set in the TFP of intermediate good firms, and they operate with either *risky* or *safe* technology. The *risky* firms operate with higher productivity but also higher volatility of

[2] The central framework is similar to the New Keynesian DSGE model with internal habits that was developed in Chapter 2.

productivity than *safe* firms. The final goods firm in the economy then aggregates intermediate firms' output over a combination of the two technologies and sells it to households. Households own representative firms and receive firms' profits as dividends.

In every period the representative household exercises its operational or *risk* allocation and decides over the composition of *risky* or *safe* intermediate firms that would operate in the economy in the following period. Other intermediate firms that do not receive this operating permission from households cease to exist in that period. Another way to look at the risk composition of the intermediate firms is to consider that there is only a single intermediate firm which could produce using risky and safe technology and in each period. The household then in each period, decides the risk composition of its operational technology of the next period. The household bases this decision on its stochastic discount factor and the expected returns from both types of technology. It is this choice of operational allocation between the two types of technology that creates the possibility of a Flight to Safety when the household's precautionary motives increase.

In this chapter, the letter $\mathcal{R}$ represents variables in relation to *risky* technology and the letter $\mathcal{S}$ represents variables with *safe* technology. The next sub-section describes the model which has two agents: households and firms. Firms are further divided into categories: intermediate (*risky* and *safe*) and final (with adjustment costs). Risky technology is an explicit function of safe technology. So we have 3 states in the model: consumption habits, safe technology, and proportion of *risky* technology firms in the economy.

3.2.1 Household

There is a continuum of identical j households on unit measure, $j \in [0, 1]$. Each household then makes their consumption, labour and risk allocation decisions which maximise their expected lifetime utility

$$U_j = E_1 \sum_{t=1}^{\infty} \beta^{t-1} \left(\frac{(C_{j,t} - hX_{j,t})^{1-\rho} - 1}{1-\rho} - \frac{N_{j,t}^{1+\eta}}{1+\eta} \right), \tag{3.1}$$

subject to the their habits that accumulate on the basis of past habits $X_{j,t-1}$, their own consumption $C_{j,t}$ and aggregate per-capita consumption C_t as

$$X_{j,t} = (1 - \phi)\left(\omega C_{j,t} + (1 - \omega) C_t\right) + \phi X_{j,t-1}, \tag{3.2}$$

where 'habit size' parameter h determines the Surplus Consumption $C_{j,t} - hX_{j,t}$. We assume habits are 'superficial', partly 'internal' and partly 'external' depending on the parameter ω for households. As discussed earlier in the Introduction section of Chapter 2, the parameter ω is chosen to keep the output of competitive economy smaller than efficient output and to generate small positive steady state inflation that matches with observations in data. Let us include all habits related terms in one place.

Table 3.1: Habits terminology - repeat

Notation	Definition
X_t	Habits
h	Habits size parameter
hX_t	Subsistence level of habits
$C_t - hX_t$	Surplus Consumption
$(C_t - hX_t)/C_t$	Surplus Consumption Ratio or Surplus Ratio
ω	Internal habits parameter
ϕ	Habits persistence parameter

Notes: Same as Table 2.1 on page 118, which has the description of habit related terminology for the previous chapter

The budget constraint of all j households is given as

$$C_{j,t} + V_t^r B_{j,t}^r + \frac{\chi}{2}\left(q_{j,t} - \frac{1}{2}\right)^2 \leqslant w_t N_{j,t} + B_{j,t-1}^r + \tau_{c,t} + D_t^f + \left[D_t^{\mathcal{R}} q_{j,t-1} + D_t^{\mathcal{S}}\left(1 - q_{j,t-1}\right)\right]. \tag{3.3}$$

The Real bonds $(B_{j,t}^r)$ are the number of one-period zero-coupon bonds that are issued by households to each other. And $\tau_{c,t}$ are the time varying per-capita lump-sum transfers from the government. For a real bond, the j^{th} household pays today the discounted value of $V_t^r B_{j,t}^r$ which gives $B_{j,t}^r$ in the next period. $C_{j,t}$ is the consumption and $N_{j,t}$ is the labour supplied by the household which is paid at the prevailing real wage rate w_t.

The household receives three forms of dividends. D_t^f is the real dividends paid by final firms. $D_t^{\mathcal{R}}$ and $D_t^{\mathcal{S}}$ are total dividends received from a *risky* and a *safe* technology intermediate firms. In any period, as $0 \leqslant q_{j,t-1} \leqslant 1$, the household receives $q_{j,t-1}$ share of the dividends from the *risky* technology firm and $(1 - q_{j,t-1})$ share of dividends from *safe* technology firm. The household can change its risk allocation and choose to receive a different proportion of dividends from the *risky* firm in period $t+1$ by altering $q_{j,t}$. But choosing a $q_{j,t}$ different than $q_{j,t-1}$ would have an exogenous real (strictly non-zero) cost to the household, which is given by product of the non-zero parameter χ and the quadratic adjustment in proportion of risky technology as $\frac{\chi}{2}\left(q_{j,t} - 1/2\right)^2$. The inclusion of quadratic risk adjustment costs disallows for big reduction on risk allocation $q_{j,t}$.

Before solving for the households optimization I would like to present a case[3] for subsistence habits and various states of nature in the economy (See Box 1).

[3]The reader will notice similarity in this and the model in Chapter 1.

3.2.1.1 Good and Bad states of the economy

In steady state the level of Habits (from habit equation 3.2) should be equal to Consumption. Therefore the steady state Surplus Ratio is simply $S = 1 - h$. As during Expansions household's marginal utility should be low, so in this utility specification we define *good states* (or expansion) as the states of the economy when the Surplus Ratio is higher than steady state Surplus Ratio of $1 - h$. This can be achieved when either the consumption is way too much in comparison to the level of habits or habits are way too low in comparison to level of consumption in the economy. In both cases the Surplus consumption S should be higher. *Bad states* (or Recesssions) by corollary are then defined as the state when agent's Surplus Ratio is below the steady state level $(1 - h)$. It can happen when agent's consumption falls (or is low) so that it gets closer to sustainable habits (hX), or when habits are high in comparison to consumption. In both cases the marginal utility rises which is another feature of recessions. Formally for j^{th} household, in steady state

$$\overline{S}_j = \frac{\overline{C}_j - h\overline{X_j}}{\overline{C}_j} = 1 - h \tag{3.4}$$

in Bad state

$$\frac{C_{j,t} - hX_{j,t}}{C_{j,t}} = S_{j,t} < \overline{S_j} = 1 - h \tag{3.5}$$

and in Good state

$$\frac{C_{j,t} - hX_{j,t}}{C_{j,t}} = S_{j,t} > \overline{S_j} = 1 - h \tag{3.6}$$

An example of subsistence habits

As an illustration in Figure 2.1 on page 116 showcases the real consumption data from the US and create an aggregate per-capita habit process: $X_t = (1 - \phi)X_{t-1} + \phi C_t$. Habits are slow moving for $\phi = 0.97$. During NBER recessions the gap between Consumption (C_t) and Subsistence habits (hX_t) reduces. As expected the marginal utility rises during recessions or whenever per-capita consumption gets closer to per-capita subsistence habits.

3.2.1.2 Household's Utility optimisation

A household does not internalize fully the effects of its choice on aggregate consumption and its habit formation. Hence we write the Lagrangian for household's problem, using $\lambda^r_{j,t} \geqslant 0$ and $\lambda^x_{j,t} \leqslant 0$ as multipliers of its budget constraint (3.3) and habit formation (3.2), as

$$\mathbb{L}^j = \max_{\{C_{j,t},N_{j,t},X_{j,t},B^r_{j,t},q_{j,t}\}_{t=1}^{t=\infty}} E_1 \sum_{t=1}^{\infty} \beta^{t-1} \frac{(C_{j,t}-hX_{j,t})^{1-\rho}-1}{1-\rho} - \frac{N_{j,t}^{1+\eta}}{1+\eta}$$
$$+\lambda^r_{j,t} \left(\begin{array}{c} w_t N_{j,t} + B^r_{j,t-1} + \tau_{c,t} + D^f_t + \left[D^{\mathcal{R}}_t q_{j,t-1} + D^{\mathcal{S}}_t (1-q_{j,t-1}) \right] \\ -C_{j,t} - V^r_t B^r_{j,t} - \frac{\chi}{2}\left(q_{j,t}-\frac{1}{2}\right)^2 \end{array} \right)$$
$$+\lambda^x_{j,t} \left(X_{j,t} - \phi X_{j,t-1} - (1-\phi)\left(\omega C_{j,t} + (1-\omega) C_t\right) \right)$$
$$+\lambda^q q_{j,t} + \lambda^{\hat{q}}(1-q_{j,t}) \tag{3.7}$$

First order conditions for all $t \geqslant 1$

$$C_{j,t}: \quad \lambda^r_{j,t} = (C_{j,t}-hX_{j,t})^{-\rho} - (1-\phi)\omega\lambda^x_{j,t} \tag{3.8}$$
$$X_{j,t}: \quad \lambda^x_{j,t} = h\left(C_{j,t}-hX_{j,t}\right)^{-\rho} + \beta\phi E_t\left[\lambda^x_{j,t+1}\right] \tag{3.9}$$
$$N_{j,t}: \quad \lambda^r_{j,t} = N^\eta_{j,t}/w_t \tag{3.10}$$
$$B^r_{j,t}: \quad V^r_t = E_t\left[\beta\frac{\lambda^r_{j,t+1}}{\lambda^r_{j,t}}\right] \tag{3.11}$$
$$q_{j,t}: \quad q_{j,t} = \frac{1}{2} + \frac{1}{\chi}E_t\left[\beta\frac{\lambda^r_{j,t+1}}{\lambda^r_{j,t}}\left(D^{\mathcal{R}}_{t+1} - D^{\mathcal{S}}_{t+1}\right)\right] + \lambda^q - \lambda^{\hat{q}} \tag{3.12}$$

The Kuhn-tucker conditions on $q_{j,t} \in [0,1]$ are :

$$\lambda^q q_{j,t} \geq 0 \tag{3.13}$$
$$\lambda^{\hat{q}}(1-q_{j,t}) \geq 0 \tag{3.14}$$

The pricing equations with respect to Real bonds ($B^r_{j,t}$) is provided to define asset returns. All j households face the same constraints and have similar optimization results, so we can drop the j classification for homogeneous households. The Consumption Euler equation from FOC's wrt Consumption and Habits for homogeneous households is then given by

$$(1-h(1-\phi)\omega)\left(C_t - hX_t\right)^{-\rho} = \lambda^r_t + \beta\phi E_t\left[(C_{t+1}-hX_{t+1})^{-\rho} - \lambda^r_{t+1}\right], \tag{3.15}$$

which shows that marginal utility from an extra unit of consumption is provided from increase in the lifetime real income and from discounting the expected loss of utility in future periods for increasing habits by ϕ. Representative household's Labour supply (from equation 3.10) as a function of real wage w_t, once we drop the j classification for homogeneous households is simply

$$N_t^{\eta} = w_t \lambda_t^r, \tag{3.16}$$

Similarly, the representative household (from equation 3.12) changes the proportion of *risky* firms (from q_{t-1} to q_t) in the economy to accommodate the changes in expected discounted lifetime dividend returns after covering the cost χ of this current (q_t) and future ($q_{t+n}; n > 0$) changes in proportion of risky firms. The quadratic nature of the adjustments costs implies that in any period small changes in risk proportion cost less than large changes. From the Euler equation (3.12) when the constraints on q_t do not bind, i.e. $\lambda^q = \lambda^{\hat{q}} = 0$. $q \in (0,1)$ is given by:

$$q_t - \frac{1}{2} = \frac{\beta}{\chi} E_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} \left(D_{t+1}^{\mathcal{R}} - D_{t+1}^{\mathcal{S}} \right) \right] \tag{3.17}$$

When $q_t \leq 0$ or it hits the lower bound the multiplier on the upper bound of q_t constraint no longer binds $\lambda^{\hat{q}} = 0$, and there are zero expected dividends from risky firms $D_{t+1}^{\mathcal{R}} = 0$. In that case the multiplier λ^q on lower bound or non-negativity constraint that binds is given by:

$$\lambda^q = \frac{\beta}{\chi} E_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} \left(D_{t+1}^{\mathcal{S}} \right) \right] - \frac{1}{2} \tag{3.18}$$

Similarly if the choice of $q_t \geq 1$ hits the upper bound, then the constraint multiplier on the lower bound of q_t no longer binds and $\lambda^q = 0$ and there are zero expected dividends from *safe* firms $D_{t+1}^{\mathcal{S}} = 0$. In that case the value of the multiplier on the bonding upper bound constraint $\lambda^{\hat{q}}$ is:

$$\lambda^{\hat{q}} = \frac{\beta}{\chi} E_t \left[\frac{\lambda_{t+1}^r}{\lambda_t^r} \left(D_{t+1}^{\mathcal{R}} \right) \right] - \frac{1}{2} \tag{3.19}$$

Stochastic Discount factor, hereafter SDF, is derived by removing j classification in homogeneous agents, for nominal pay-offs $M_{t,t+1}^n$ and for real pay-offs $M_{t,t+1}^r$ respectively, as

$$M_{t,t+1}^n = E_t \left[\beta \frac{\lambda_{t+1}^r}{\lambda_t^r} \frac{P_t}{P_{t+1}} \right], \tag{3.20}$$

$$M_{t,t+1}^r = E_t \left[\beta \frac{\lambda_{t+1}^r}{\lambda_t^r} \right]. \tag{3.21}$$

Local-non satiation of preferences ensures that in equilibrium the budget constraint binds with equality, and the structure of constant EIS utility conforms to the usual Inada conditions. Additionally, we also need the no-Ponzi game condition (3.22) to hold with equality in the equilibrium, so that present discounted value of household's nominal wealth at infinity is non-negative or there is no over-accumulation of debt.

$$\lim_{T\to\infty} E_t \left[\frac{\varpi_{j,T}}{R^r_{t,T}}\right] \geq 0. \tag{3.22}$$

Household optimisation with respect to real bonds (3.11) gives one-period real rate of return $R^r_{t,t+1} = (V^r_t)^{-1}$ and $\varpi_{j,t} = B^r_{j,t-1} + D^f_t + D^{\mathcal{R}}_t q_{j,t-1} + D^{\mathcal{S}}_t (1 - q_{j,t-1})$ is the real wealth brought by household j into the period t. For the representative household as the net supply of bonds is zero the real wealth brought into the period is, $\varpi_t = D^f_t + D^{\mathcal{R}}_t q_{t-1} + D^{\mathcal{S}}_t (1 - q_{t-1})$, and the **no-Ponzi games** condition for the representative household is,

$$\lim_{T\to\infty} E_t \left[\frac{\varpi_T}{R^r_{t,T}}\right] \geq 0, \tag{3.23}$$

where the FOC wrt Bonds (3.11) is used to write the real rate as

$$R^r_{t,T} = \prod_{s=t}^{T-1} \left[\beta \frac{\lambda^r_{s+1}}{\lambda^r_s}\right]. \tag{3.24}$$

The value of lifetime earnings V^D_t from dividends for the household is

$$V^D_t = \sum_{i=1}^{\infty} E_t \left[\beta^i \frac{\lambda^r_{t+i}}{\lambda^r_t} \left(D^f_{t+i} + D^{\mathcal{R}}_{t+i} q_{t+i-1} + D^{\mathcal{S}}_{t+i} (1 - q_{t+i-1})\right)\right]. \tag{3.25}$$

Household's choice of q_t in all periods obeys the optimisation result (3.12) such that the any change in q_t is justified by the change in discounted earnings from *risky* and *safe* technology in the next period. We can write the **transversality condition** as

$$\lim_{T\to\infty} \beta^{T-1} \lambda^r_T \left(D^f_T + q_T D^{\mathcal{R}}_{T-1} + (1 - q_T) D^{\mathcal{S}}_{T-1}\right) V^D_T = 0 \tag{3.26}$$

which rules out any over-accumulation of wealth.

3.2.1.3 Household's expenditure minimisation

The household bundles the consumption good C_t by aggregating the final goods C_{it} produced from monopolistic final goods firms using the Dixit-Stiglitz aggregator:

$$C_t = \left[\int_0^1 C_{it}^{\frac{\sigma-1}{\sigma}} di\right]^{\frac{\sigma}{\sigma-1}}, \tag{3.27}$$

where σ is the CES between any two varieties of i. The household looks to benefit from any price differential between goods provided by different firms and minimizes total expenditure

$$min \int_0^1 P_{it} C_{it} di,$$

subject to the Dixit-Stiglitz aggregator (3.27). We can derive household's demand for good of variety 'i' as C_{it},

$$C_{it} = \left[\frac{P_{it}}{\lambda_c}\right]^{-\sigma} C_t,$$

where λ_c is the Lagrange multiplier or the marginal value from an extra unit of bundled good. By substituting the value of C_{it} in Dixit-Stiglitz aggregator (3.27), we get the cost P_t of obtaining an extra bundled good is equal to its marginal value to the household λ_c,

$$\lambda_c = \left[\int_0^1 P_t(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \equiv P_t,$$

which also gives the demand curve for final goods as

$$C_{it} = \left[\frac{P_{it}}{P_t}\right]^{-\sigma} C_t. \tag{3.28}$$

3.2.2 Firms

There are two types of firms in the model economy. Firstly, there is a continuum $[0, 1]$ of monopolistic intermediate firms producing imperfect substitutes, of which, in any period t, there are two types. The first type is $r \in [0, q_{t-1}]$ that operate with *risky* technology and the second type is $l \in [q_{t-1}, 1]$ that operate with *safe* technology. Secondly, there is a continuum i of monopolistic final good firms on the unit measure $i \in [0, 1]$, producing imperfect substitutes from goods produced by the risky and safe intermediate firms.

3.2.2.1 The monopolistic final goods firms

There is a continuum $i \in [0,1]$ of monopolistic firms that are owned by households. The ith firm does not hire any labour but produces the monopolistic good Y_{it} by putting together the intermediate goods from both *risky* and *safe* firms by using the following Dixit-Stiglitz aggregator :

$$Y_{it} = \left(\int_0^{q_{t-1}} \left(Y_{rt}^{\mathcal{R}} \right)^{\frac{\sigma-1}{\sigma}} dr + \int_{q_{t-1}}^1 \left(Y_{lt}^{\mathcal{S}} \right)^{\frac{\sigma-1}{\sigma}} dl \right)^{\frac{\sigma}{\sigma-1}} \tag{3.29}$$

where $\sigma > 1$ is the constant elasticity of substitution (CES) between any two varieties of intermediate goods produced within and between (*risky* or *safe*) intermediate firms.

Cost minimization of final firms:

$$\min_{Y_{rt}^{\mathcal{R}}, Y_{lt}^{\mathcal{S}}} \int_0^{q_{t-1}} P_{rt}^{\mathcal{R}} Y_{rt}^{\mathcal{R}} dr + \int_{q_{t-1}}^1 P_{lt}^{\mathcal{S}} Y_{lt}^{\mathcal{S}} dl,$$

subject to the production aggregator (3.29) at given prices of *risky* and *safe* intermediate goods. Using the Lagragian method,

$$\begin{aligned} L_{it} &= \int_0^{q_{t-1}} P_{rt}^{\mathcal{R}} Y_{rt}^{\mathcal{R}} dr + \int_{q_{t-1}}^1 P_{lt}^{\mathcal{S}} Y_{lt}^{\mathcal{S}} dl \\ &+ \mu_t^{'} \left(Y_{it} - \left(\int_0^{q_{t-1}} \left(Y_{rt}^{\mathcal{R}} \right)^{\frac{\sigma-1}{\sigma}} dr + \int_{q_{t-1}}^1 \left(Y_{lt}^{\mathcal{S}} \right)^{\frac{\sigma-1}{\sigma}} dl \right)^{\frac{\sigma}{\sigma-1}} \right) \end{aligned} \tag{3.30}$$

we write the multiplier $\mu_t^{'}$ as marginal cost for firm i of producing an extra unit of Y_{it}. The FOCs result in demand function for intermediate goods as,

$$Y_{rt}^{\mathcal{R}} = \left(\frac{P_{rt}^{\mathcal{R}}}{\mu_t^{'}} \right)^{-\sigma} Y_{it} \tag{3.31}$$

$$Y_{lt}^{\mathcal{S}} = \left(\frac{P_{lt}^{\mathcal{S}}}{\mu_t^{'}} \right)^{-\sigma} Y_{it}, \tag{3.32}$$

and the real marginal cost $\mu_t \equiv \mu_t^{'}/P_t$ as

$$\mu_t = \left[\int_0^{q_{t-1}} \left(\frac{P_{rt}^{\mathcal{R}}}{P_t} \right)^{1-\sigma} dr + \int_{q_{t-1}}^1 \left(\frac{P_{lt}^{\mathcal{S}}}{P_t} \right)^{1-\sigma} dl \right]^{\frac{1}{1-\sigma}}. \tag{3.33}$$

Profit maximization of final firms: Final goods monopolistic firm in order to set prices P_{it} faces nominal rigidity in terms of Rotemberg price adjustment costs

$$\left[\frac{\psi}{2}\left(\frac{P_{it}}{P_{it-1}}-1\right)^2 Y_t\right].$$

It chooses prices P_{it} and Y_{it} in order to maximize the discounted present value ($\mathbb{P}_{it}$) of nominal profits for its owner (jth) household

$$\mathbb{P}_{it} = \max_{P_{it+s}} E_t \sum_{s=0}^{\infty} M^n_{t,t+s} \left[\begin{array}{l} (1-\tau_p) P_{it+s} Y_{it+s} \\ -\mu_{it+s} P_{t+s} Y_{it+s} - \frac{\psi}{2}\left(\frac{P_{it+s}}{P_{it+s-1}}-1\right)^2 P_{t+s} Y_{t+s} \end{array} \right] \tag{3.34}$$

subject to Consumer's demand (3.28). The term τ_p is the rate of revenue tax that is set by the government. μ_{it+s} is the firm's marginal cost. Y_{t+s} and P_{t+s} are the level of final output and prices in the economy. Households own all firms and they use their s-period stochastic discount factor, $M^n_{t,t+s}$ from optimization result (3.20) to discount firm's profits. From firm's FOC wrt P_{it}, for $s = 0$ we can derive the final goods firms supply curve or NKPC as

$$0 = (1-\sigma)(1-\tau_p) + \sigma\mu_t - \psi(\pi_t - 1)\pi_t + \psi\beta E_t \left\{ \frac{\lambda^r_{t+1}}{\lambda^r_t} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\pi_{t+1} \right\} \tag{3.35}$$

where μ_t is real marginal cost of the firm, and in a symmetric equilibrium $\forall i$, $P_{it} = P_t$, $\mu_{it} = \mu_t$ and $Y_{it} = Y_t$.

3.2.2.2 Safe intermediate firms' problem

A *safe* retail firm $l \in [q_{t-1}, 1]$ hires labour to maximize its real profits $\mathbb{P}^{\mathcal{S}}_{lt}$ given by

$$\mathbb{P}^{\mathcal{S}}_{lt} = \frac{P^{\mathcal{S}}_{lt}}{P_t} Y^{\mathcal{S}}_{lt} - w_t N^{\mathcal{S}}_{lt} \tag{3.36}$$

where w_t is the real wage. Subject to following Production function (for *safe* firms)

$$Y^{\mathcal{S}}_{lt} = \xi^{\frac{1}{1+\eta}}_{\mathcal{S},t} N^{\mathcal{S}}_{lt} \tag{3.37}$$

$$log(\xi_{\mathcal{S},t}) = \varepsilon_{\mathcal{S},t} = (1-\gamma_{\mathcal{S}})\overline{\varepsilon_{\mathcal{S}}} + \gamma_{\mathcal{S}}\varepsilon_{\mathcal{S},t-1} + \epsilon_{\mathcal{S},t} \tag{3.38}$$

$$\epsilon_{\mathcal{S},t} \sim N\left(0, \sigma^2_{\mathcal{S}}\right). \tag{3.39}$$

where η is inverse of Frisch elasticity of labour, and s.t. the final demand constraint[4]

$$Y_{lt}^{\mathcal{S}} = \left(\frac{1}{\mu_t}\frac{P_{lt}^{\mathcal{S}}}{P_t}\right)^{-\sigma} Y_t. \tag{3.40}$$

We can derive firm's optimization condition by substituting for both $P_{lt}^{\mathcal{S}}$ and $N_{lt}^{\mathcal{S}}$ in firm's profit equation[5]. The FOC wrt $Y_{lt}^{\mathcal{S}}$ gives the inverse supply curve for the firm producing good of variety l with *safe* technology as

$$Y_{lt}^{\mathcal{S}} = Y_t \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{w_t}{\mu_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}}\right]^{-\sigma} \tag{3.41}$$

Since all *safe* firms have the same technology they all hire the same amount of labour ($N_t^{\mathcal{S}}$), produce the same output ($Y_t^{\mathcal{S}}$), face the same demand curve, get same price ($P_t^{\mathcal{S}}$) which is a markup over its marginal cost, and make the same amount of real profits ($\mathbb{P}_t^{\mathcal{S}}$):

$$P_t^{\mathcal{S}} = \left(\frac{\sigma}{\sigma-1}\right)\frac{w_t}{\xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}} P_t \tag{3.42}$$

$$\mathbb{P}_t^{\mathcal{S}} = \frac{1}{\sigma} Y_t \mu_t^{\sigma} \left[\left(\frac{\sigma}{\sigma-1}\right)\frac{w_t}{\xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}}\right]^{1-\sigma}. \tag{3.43}$$

3.2.2.3 Risky intermediate firms' problem

Similarly there is a continuum of perfectly competitive *risky* firms $r \in [0, q_{t-1}]$ which hire labour to maximize their real profits $\mathbb{P}_{rt}^{\mathcal{R}}$ which are given by

$$\mathbb{P}_{rt}^{\mathcal{R}} = \frac{P_{rt}^{\mathcal{R}}}{P_t} Y_{rt}^{\mathcal{R}} - w_t N_{rt}^{\mathcal{R}} \tag{3.44}$$

subject to following Production function for *risky* firms

$$Y_{rt}^{\mathcal{R}} = \vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}} N_{rt}^{\mathcal{R}} \tag{3.45}$$

[4]by using $\forall i, P_{it} = P_t$, the marginal cost $\mu_{it} = \mu_t$ and $Y_{it} = Y_t$.

[5]Therefore firm's problem is: $\underset{Y_{lt}^{\mathcal{S}}}{max} \left(\frac{Y_{lt}^{\mathcal{S}}}{Y_t}\right)^{\frac{-1}{\sigma}} \mu_t Y_{lt}^{\mathcal{S}} - \frac{w_t}{\xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}} Y_{lt}^{\mathcal{S}}$

and the respective demand constraint from retail firm

$$Y^{\mathcal{R}}_{rt} = \left(\frac{1}{\mu_t}\frac{P^{\mathcal{R}}_{rt}}{P_t}\right)^{-\sigma} Y_t. \tag{3.46}$$

The total factor productivity of *risky* firm $\vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}$ is dependent on the time-varying multiple ϑ_t which is proportional to the tfp of the *safe* firm. The risky firm tfp is,

$$\vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}} = k \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}} + 1 - k \tag{3.47}$$

$$\text{where} \quad k = \frac{\vartheta^h \xi_{\mathcal{S},h}^{\frac{1}{1+\eta}} - \vartheta^l \xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}}{\xi_{\mathcal{S},h}^{\frac{1}{1+\eta}} - \xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}}. \tag{3.48}$$

ϑ^h is the maximum value of ϑ_t and ϑ^l is the minimum value of ϑ_t. Similarly the $\xi_{\mathcal{S},h}^{\frac{1}{1+\eta}}$ and $\xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}$ are the maximum and minimum values of low risk firm's tfp. All of ϑ^h, ϑ^l , $\xi_{\mathcal{S},h}^{\frac{1}{1+\eta}}$ and $\xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}$ are constant parameters. Therefore k becomes the constant slope parameter of $risky$ firm TFP to $safe$ firm TFP. With value of $k > 1$ the multiple ϑ_t and *risky* firm's tfp are an increasing function of *safe* firm's tfp.

Similar to the *safe* firm, all *risky* firms too hire the same amount of labour ($N^{\mathcal{R}}_t$), produce the same output ($Y^{\mathcal{R}}_t$)

$$Y^{\mathcal{R}}_t = Y_t \left(\frac{\sigma}{\sigma - 1} \frac{w_t}{\mu_t \vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}}\right)^{-\sigma}. \tag{3.49}$$

Firms facing same demand curve, charge same price ($P^{\mathcal{R}}_t$) and make similar real profits $\mathbb{P}^{\mathcal{R}}_t$ as,

$$P^{\mathcal{R}}_t = \left(\frac{\sigma}{\sigma - 1}\right) \frac{w_t}{\vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}} P_t \tag{3.50}$$

$$\mathbb{P}^{\mathcal{R}}_t = \frac{1}{\sigma} Y_t \mu_t^{\sigma} \left[\left(\frac{\sigma}{\sigma - 1}\right) \frac{w_t}{\vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}}\right]^{1-\sigma}. \tag{3.51}$$

3.2.3 Aggregate relations

Pricing relations for both *risky* and *safe* firms can be substituted into final goods firms' cost minimization (3.33) to derive the real marginal cost in terms productivity differential as,

$$\mu_t = \left(\frac{\sigma}{\sigma-1}\right) w_t \left[\xi_{\mathcal{S},t}^{\frac{1}{1+\eta}} \left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{1}{\sigma-1}}\right]^{-1}. \tag{3.52}$$

The market power of intermediate firms increases marginal cost by the markup $\frac{\sigma}{\sigma-1}$ while the productivity differential between *risky* and *safe* firms lowers the marginal cost. In absence of this productivity differential (or when risk allocation is zero, $q_{t-1} = 0$) the marginal cost is given by the usual setup of markup times the ratio of wage to productivity.

Aggregate production function (see derivation Section C.2.4 in appendix) can be put in the form of productivity differences of intermediate firms as,

$$Y_t = N_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}} \left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{1}{\sigma-1}} \tag{3.53}$$

which is increasing in the share of more risky intermediate firms in the economy. Therefore the labour income in model economy is given by

$$w_t N_t = \mu_t \left(\frac{\sigma-1}{\sigma}\right) Y_t. \tag{3.54}$$

We can substitute wages from the above relations and use the labour market equilibrium (3.16) to define the marginal cost in terms of aggregate output, productivity differential and the markup as,

$$Y_t^{\eta} = \left(\frac{\sigma-1}{\sigma}\right) \mu_t \lambda_t^r \xi_{\mathcal{S},t} \left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{1+\eta}{\sigma-1}} \tag{3.55}$$

Similarly, by substituting out the real wages by (3.52), total profits (3.51) for as q_{t-1} *risky* firms, which are distributed to owner households in the form of dividends $D_t^{\mathcal{R}}$ are given by,

$$q_{t-1} D_t^{\mathcal{R}} = q_{t-1} \frac{\mu_t}{\sigma} \frac{\vartheta_t^{\sigma-1}}{\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]} Y_t. \tag{3.56}$$

These are increasing in the monopoly power and the productivity multiple ϑ_t of risky firms. Similarly total profits (3.43) from $1-q_{t-1}$ *safe* firms which are distributed to owner households as dividends $D_t^{\mathcal{S}}$ are,

$$(1-q_{t-1}) D_t^{\mathcal{S}} = (1-q_{t-1}) \frac{\mu_t}{\sigma} \frac{1}{\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]} Y_t. \tag{3.57}$$

Therefore sum of dividends from q_{t-1} *risky* and $1-q_{t-1}$ *safe* firms is reflected in the monopoly power of these firms and the marginal cost as,

$$q_{t-1}D_t^{\mathcal{R}} + (1-q_{t-1})D_t^{\mathcal{S}} = \frac{1}{\sigma}Y_t\mu_t \tag{3.58}$$

Recall, from (3.52) that the marginal cost is decreasing in the ratio of *risky* firms and the productivity differential between *risky* and *safe* firms. Therefore, household dividend earnings are increasing in the proportion of risky firms. Difference of dividends (when $q_{t-1} \notin \{0,1\}$) is

$$D_t^{\mathcal{R}} - D_t^{\mathcal{S}} = \frac{\mu_t}{\sigma}\frac{\left(\vartheta_t^{\sigma-1}-1\right)}{\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]}Y_t \tag{3.59}$$

.

Therefore q_t, by substituting for marginal cost[6] as:

$$q_t = \frac{1}{2} + \left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}E_t\left[\frac{Y_{t+1}^{1+\eta}}{\xi_{\mathcal{S},t+1}}\frac{\left(\vartheta_{t+1}^{\sigma-1}-1\right)}{\left[1+q_t\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]^{\frac{\sigma+\eta}{\sigma-1}}}\right] \tag{3.61}$$

In case $q_t = 0$, only safe firms operate and their dividends are

$$D_t^{\mathcal{S}} = \frac{Y_t^{1+\eta}}{\xi_{\mathcal{S},t}\lambda_t^r}\left(\frac{1}{\sigma-1}\right). \tag{3.62}$$

Similarly, in case $q_t = 1$, only risky firms operate and their dividends are

$$D_t^{\mathcal{R}} = \frac{Y_t^{1+\eta}}{\vartheta_t^{1+\eta}\xi_{\mathcal{S},t}\lambda_t^r}\left(\frac{1}{\sigma-1}\right). \tag{3.63}$$

Dividends (profits) from final firms are given by the difference between total output and total costs (viz. taxes, marginal and price adjustment),

$$D_t^f = \left[(1-\tau_p) - \mu_t - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right]Y_t. \tag{3.64}$$

[6] Here we substitute $\frac{\mu_t}{\sigma}$ from(3.55) is:

$$\frac{\mu_t}{\sigma} = \left(\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\lambda_t^r}\frac{1}{\sigma-1}\right)\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}} \tag{3.60}$$

3.2.4 Market clearing

In the economy there are at any point in time only $q_{t-1} \in [0,1]$ *risky* firms and $1-q_{t-1}$ *safe* firms that are operating. The aggregate budget constraint for the economy is found by integrating the following household budget constraint for all j households, $j \in [0,1]$:

$$C_{j,t} + V_t^r B_{j,t}^r + \tfrac{\chi}{2}\left(q_{j,t} - \tfrac{1}{2}\right)^2 = \begin{array}{c} w_t N_{j,t} + B_{j,t-1}^r + \tau_{c,t} + D_t^f \\ + \left[D_t^{\mathcal{R}} q_{j,t-1} + D_t^{\mathcal{S}}\left(1 - q_{j,t-1}\right)\right]. \end{array} \tag{3.65}$$

The net supply of real bonds is zero. All taxes collected are transferred to households so in aggregate $\tau_{c,t} = \tau_p Y_t$ and the aggregate household budget constraint becomes

$$C_t + \frac{\chi}{2}\left(q_t - \frac{1}{2}\right)^2 = w_t N_t + \tau_p Y_t + \left[D_t^f + D_t^{\mathcal{R}} q_{t-1} + D_t^{\mathcal{S}}\left(1 - q_{t-1}\right)\right], \tag{3.66}$$

The last term on RHS of the above equation captures total real profits from monopolistic final firms (D_t^f), from *risky* ($q_{t-1}D_t^{\mathcal{R}}$) intermediate firms and from *safe* ($\left(1-q_{t-1}\right)D_t^{\mathcal{S}}$) intermediate firms. We can use the equations (3.54 to 3.64) in aggregate household's budget constraint to arrive at

$$C_t + \frac{\chi}{2}\left(q_t - \frac{1}{2}\right)^2 = \mu_t\left(\frac{\sigma-1}{\sigma}\right)Y_t + \tau_p Y_t + \left[\left(1-\tau_p\right) - \mu_t - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right]Y_t + \frac{\mu_t}{\sigma}Y_t \tag{3.67}$$

which gives the **aggregate resource constraint**

$$C_t = Y_t\left[1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right] - \frac{\chi}{2}\left(q_t - \frac{1}{2}\right)^2. \tag{3.68}$$

3.2.5 Summary of the model

1. Habits accumulation:

$$X_t = (1-\phi)C_t + \phi X_{t-1} \tag{3.69}$$

2. Aggregate Resource constraint

$$C_t = \left[1 - \frac{\psi}{2}(\pi_t - 1)^2\right] Y_t - \frac{\chi}{2}\left(q_t - \frac{1}{2}\right)^2 \tag{3.70}$$

3. NKPC :

$$\begin{aligned} 0 = {} & (1-\sigma)(1-\tau_p) + \mu_t \sigma \\ & - \psi(\pi_t - 1)\pi_t + \psi\beta E_t\left[(\pi_{t+1} - 1)\pi_{t+1}\frac{\lambda^r_{t+1}}{\lambda^r_t}\frac{Y_{t+1}}{Y_t}\right] \end{aligned} \tag{3.71}$$

4. The Consumption Euler Equation:

$$[1 - h(1-\phi)\omega](C_t - hX_t)^{-\rho} = \lambda^r_t + \beta\phi E_t\left[(C_{t+1} - hX_{t+1})^{-\rho} - \lambda^r_{t+1}\right] \tag{3.72}$$

5. Marginal cost

$$\mu_t = \frac{Y_t^{\eta}}{\lambda^r_t \xi_{\mathcal{S},t}}\left(\frac{\sigma}{\sigma-1}\right)\left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}} \tag{3.73}$$

6. Proportion of risky technology

$$q_t = \frac{1}{2} + \left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda^r_t}E_t\left[\frac{Y_{t+1}^{1+\eta}}{\xi_{\mathcal{S},t+1}}\frac{\left(\vartheta_{t+1}^{\sigma-1} - 1\right)}{\left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right]^{\frac{\sigma+\eta}{\sigma-1}}}\right] \tag{3.74}$$

7. Nominal Rates or Fischer equation

$$\left[R^n_{t,t+1}\right]^{-1} = E_t\left[\beta\frac{\lambda^r_{t+1}}{\lambda^r_t}\frac{1}{\pi_{t+1}}\right] \tag{3.75}$$

3.3 Optimal policy

The presence of habits (X_{t-1}) and discretion between technologies (q_{t-1}) makes the optimum decision variables history dependent; future policy decisions depend on current period's habits (X_t) and proportion of *risky* technology firms (q_t). Policy-maker cannot establish commitment over future policy decisions and she would have to re-optimize in each period. The optimal monetary policy under discretion can be described as a set of decision rules for 6 control variables $\{C_t, Y_t, \pi_t, X_t, \lambda_t^r, q_t\}$ as functions of 3 state variables $\{\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}\}$ which maximise the following Value function,

$$\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}\right) = \max_{\substack{C_t, Y_t, \pi_t,\\ X_t, \lambda_t^r, q_t}} \left(\begin{array}{c} \frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} \\ -\frac{Y_t^{1+\eta}}{1+\eta}\left[\xi_{\mathcal{S},t}\left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]^{\frac{1+\eta}{\sigma-1}}\right]^{-1} \\ +\beta E_t\left\{\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t\right)\right\} \end{array} \right) \tag{3.76}$$

subject to the following 5 constraints.
Habits accumulation:

$$X_t = (1-\phi)C_t + \phi X_{t-1} \tag{3.77}$$

Aggregate Resource allocation:

$$C_t = \left[1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right] Y_t - \frac{\chi}{2}\left(q_t - \frac{1}{2}\right)^2 \tag{3.78}$$

Inflation NKPC constraint,

$$\psi\left(\pi_t - 1\right)\pi_t = (1-\sigma)\left(1-\tau_p\right) + \frac{Y_t^\eta}{\xi_{\mathcal{S},t}\lambda_t^r}\left(\frac{\sigma^2}{\sigma-1}\right)\left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]^{\frac{-1-\eta}{\sigma-1}} + \frac{\psi\beta}{\lambda_t^r Y_t} E_t\left[\left(\pi_{t+1} - 1\right)\pi_{t+1}\lambda_{t+1}^r Y_{t+1}\right] \tag{3.79}$$

Consumption Euler equation CEE :

$$\left[1 - h(1-\phi)\omega\right]\left(C_t - hX_t\right)^{-\rho} = \lambda_t^r + \beta\phi E_t\left[\left(C_{t+1} - hX_{t+1}\right)^{-\rho} - \lambda_{t+1}^r\right] \tag{3.80}$$

and Risk allocation constraint :

$$q_t = \frac{1}{2} + \left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r} E_t\left[\frac{Y_{t+1}^{1+\eta}}{\xi_{\mathcal{S},t+1}}\frac{\left(\vartheta_{t+1}^{\sigma-1} - 1\right)}{\left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right]^{\frac{\sigma+\eta}{\sigma-1}}}\right] \tag{3.81}$$

By defining the following auxiliary functions that take on future states as their argument,

$$\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv (\pi_{t+1}-1)\,\pi_{t+1}\lambda^r_{t+1}Y_{t+1} \tag{3.82}$$

$$\Omega^C(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv (C_{t+1}-hX_{t+1})^{-\rho}-\lambda^r_{t+1} \tag{3.83}$$

$$\Omega^Y(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv \frac{Y^{1+\eta}_{t+1}}{\xi_{\mathcal{S},t+1}}\left(\vartheta^{\sigma-1}_{t+1}-1\right) \tag{3.84}$$

we can rewrite the constraints (3.79, 3.80 and 3.81) as:

$$(1-\sigma)(1-\tau_p)+\frac{Y^\eta_t}{\xi_{\mathcal{S},t}\lambda^r_t}\left(\frac{\sigma^2}{\sigma-1}\right)\left[1+q_{t-1}\left(\vartheta^{\sigma-1}_t-1\right)\right]^{\frac{-1-\eta}{\sigma-1}}$$
$$=\psi(\pi_t-1)\,\pi_t-\frac{\psi\beta}{\lambda^r_tY_t}E_t\left[\Omega^P(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\right] \tag{3.85}$$

$$(1-h(1-\phi)\omega)\,(C_t-hX_t)^{-\rho}=\lambda^r_t+\beta\phi E_t\left[\Omega^C(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\right] \tag{3.86}$$

$$q_t-\frac{1}{2}=\left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda^r_t}E_t\left[\frac{\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\left[1+q_t\left(\vartheta^{\sigma-1}_{t+1}-1\right)\right]^{\frac{\sigma+\eta}{\sigma-1}}}\right] \tag{3.87}$$

These auxiliary functions show that the policy maker recognises the impact of their actions on the endogenous state, but they cannot commit to future policy i.e. we have a time-consistent policy. Notice that the NKPC has marginal cost[7] substituted into it, and the risk allocation constraint has the real dividends from safe and risky firms substituted into it from the the firm's maximisation problem [See 3.59 in section 3.2.3, reproduced here[8]]. The policymakers constraint on nominal rates is always binding and so it excluded from the discretionary policy problem.

[7] $\mu_t=\frac{Y^\eta_t}{\lambda^r_t\xi_{\mathcal{S},t}}\left(\frac{\sigma}{\sigma-1}\right)\left[1+q_{t-1}\left(\vartheta^{\sigma-1}_t-1\right)\right]^{\frac{-1-\eta}{\sigma-1}}$

[8] $D^{\mathcal{R}}_t-D^{\mathcal{S}}_t=\frac{\mu_t}{\sigma}\frac{\left(\vartheta^{\sigma-1}_t-1\right)}{\left[1+q_{t-1}\left(\vartheta^{\sigma-1}_t-1\right)\right]}Y_t.$

3.3.1 Solving for Optimal policy with Discretion

The Lagrangian for the discretionary problem is formed as

$$\begin{aligned}
\mathcal{L}^d = & \left(\frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta} \xi_{\mathcal{S},t}^{-1} \left[1 + q_{t-1} \left(\vartheta_t^{\sigma-1} - 1 \right) \right]^{\frac{-(1+\eta)}{\sigma-1}} \right) \\
& + \beta E_t \left\{ \mathcal{V}^d \left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t \right) \right\} \\
& + \Lambda_{1,t} \left(X_t - (1-\phi) C_t - \phi X_{t-1} \right) \\
& + \Lambda_{2,t} \left(\left[1 - \frac{\psi}{2} (\pi_t - 1)^2 \right] Y_t - \frac{\chi}{2} \left(q_t - \frac{1}{2} \right)^2 - C_t \right) \\
& + \Lambda_{3,t} \begin{pmatrix} (1-\sigma)(1-\tau_p) - \psi (\pi_t - 1) \pi_t \\ + \frac{Y_t^\eta}{\xi_{\mathcal{S},t} \lambda_t^r} \left(\frac{\sigma^2}{\sigma-1} \right) \left[1 + q_{t-1} \left(\vartheta_t^{\sigma-1} - 1 \right) \right]^{\frac{-1-\eta}{\sigma-1}} \\ + \frac{\psi\beta}{\lambda_t^r Y_t} E_t \left[\Omega^P (\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right] \end{pmatrix} \\
& + \Lambda_{4,t} \begin{pmatrix} \lambda_t^r - \left[1 - h(1-\phi)\omega \right] (C_t - hX_t)^{-\rho} \\ + \beta\phi E_t \left[\Omega^C (\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right] \end{pmatrix} \\
& + \Lambda_{5,t} \left(\frac{1}{2} - q_t + \left(\frac{1}{\sigma-1} \right) \frac{\beta}{\chi \lambda_t^r} E_t \left[\frac{\Omega^Y (\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\left[1 + q_t \left(\vartheta_{t+1}^{\sigma-1} - 1 \right) \right]^{\frac{\sigma+\eta}{\sigma-1}}} \right] \right)
\end{aligned} \tag{3.88}$$

where $\{\Lambda_{1,t}, \Lambda_{2,t}, \Lambda_{3,t}, \Lambda_{4,t}, \Lambda_{5,t}\}$ are the Lagrange multipliers. The policy rate in constraint (3.75) is chosen to optimize other policy functions. It is always binding and therefore not included in the Lagrangian specification.

First order conditions In order to compactly write the FOCs we make use of the notation $\mathcal{Q}_t \equiv \left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]$ and $\mathcal{Q}_{t+1} \equiv \left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right]$.

The FOCs with respect to the decision variables.

$$\mathbf{C_t}: \; 0 = (C_t - hX_t)^{-\rho} - \Lambda_{1,t}(1-\phi) - \Lambda_{2,t} + \Lambda_{4,t}\left[1 - h(1-\phi)\omega\right]\rho\left(C_t - hX_t\right)^{-\rho-1} \quad (3.89)$$

This shows that an increase in consumption (3.89) increases utility, tightens both the habit constraint ($\Lambda_{1,t} \geqslant 0$), and the resource constraint ($\Lambda_{2,t} \geqslant 0$), and lowers the stochastic discount factor ($\Lambda_{4,t} \geqslant 0$).

$$\mathbf{Y_t}: \quad 0 = -\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}}\left[\mathcal{Q}_t\right]^{\frac{-(1+\eta)}{\sigma-1}} + \Lambda_{2,t}\left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right) + \Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\eta\frac{Y_t^{\eta-1}}{\xi_{\mathcal{S},t}\lambda_t^r}\left[\mathcal{Q}_t\right]^{\frac{-(1+\eta)}{\sigma-1}} - \frac{\psi\beta}{\lambda_t^r(Y_t)^2}E_t\left[\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\right]\right] \quad (3.90)$$

Similarly producing an extra unit of output (3.90) generates (dis)utility, relaxes the resource constraint and improves the inflation-output trade-off ($\Lambda_{3,t} \leqslant 0$).

$$\pi_{\mathbf{t}}: \qquad 0 = -\Lambda_{2,t}Y_t\psi\left(\pi_t - 1\right) - \Lambda_{3,t}\left[\psi\left(2\pi_t - 1\right)\right] \quad (3.91)$$

Increase in inflation (3.91) tightens the resource constraint and its impact on the inflation-output trade-off also depends on the level of output. The FOC wrt Habits using Envelope theorem (See Appendix C.4 for derivation), is

$$\mathbf{X_t}: \; 0 = -\frac{h}{(C_t - hX_t)^{\rho}} - \beta\phi E_t\left\{\Lambda_{1,t+1}\right\} + \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t}E_t\left\{\frac{\partial\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\}\right] + \Lambda_{1,t} + \Lambda_{4,t}\left[\beta\phi E_t\left\{\frac{\partial\Omega^C(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\} - \frac{\left[1 - h(1-\phi)\omega\right]\rho h}{(C_t - hX_t)^{\rho+1}}\right] + \Lambda_{5,t}\left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}E_t\left[\left\{\frac{\partial\Omega^Y(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right] \quad (3.92)$$

The FOC for λ_t^r, shows that rise in real income leads to higher SDF and worse inflation-output trade-off.

$$\lambda_t^r:\ \ 0=-\Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\left(\lambda_t^r\right)^2}\left[\mathcal{Q}_t\right]^{\frac{-(1+\eta)}{\sigma-1}}+\frac{\psi\beta}{Y_t\left(\lambda_t^r\right)^2}E_t\left[\Omega^P(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\right]\right]$$

$$+\Lambda_{4,t}-\Lambda_{5,t}\frac{\beta}{\chi(\sigma-1)\left(\lambda_t^r\right)^2}E_t\left[\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right] \quad (3.93)$$

The FOC, using envelope theorem, for second endogenous state in the model (q_t) which is the proportion of risky firms (See Appendix C.4 for derivation) is

$$\mathbf{q_t}:\ \ 0=\beta E_t\left[\frac{Y_{t+1}^{\eta}}{\xi_{\mathcal{S},t+1}}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\eta+\sigma)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)\left(\frac{Y_{t+1}}{\sigma-1}-\Lambda_{3,t+1}\frac{\sigma^2(1+\eta)}{(\sigma-1)^2\lambda_{t+1}^r}\right)\right]$$

$$-\Lambda_{2,t}\chi\left(q_t-\frac{1}{2}\right)+\Lambda_{3,t}\frac{\psi\beta}{\lambda_t^rY_t}E_t\left\{\frac{\partial\Omega^P(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}$$

$$+\Lambda_{4,t}\beta\phi E_t\left\{\frac{\partial\Omega^C(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}$$

$$-\Lambda_{5,t}\left(1-\left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}\left[E_t\left[\left\{\frac{\partial\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right]\right]\right)$$

$$-\Lambda_{5,t}E_t\left[\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\frac{\partial\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}}{\partial q_t}\right] \quad (3.94)$$

As mentioned earlier, we make use of $\mathcal{Q}_t\equiv\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]$ and

$\mathcal{Q}_{t+1}\equiv\left[1+q_t\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]$ to simplify the notation.

This gives $\frac{\partial[\mathcal{Q}_{t+1}]^{\frac{-(\sigma+\eta)}{\sigma-1}}}{\partial q_t}=\frac{-(\sigma+\eta)}{\sigma-1}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(2\sigma+\eta-1)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)$.

The optimal policy results for the special cases when q_t hits the bounds of $[0,1]$ are mentioned in appendix sections Appendix C.5 and Appendix C.6.

To solve for the optimal policy equilibrium we need to find time-invariant Markov perfect optimal policy rules for :

a) Six policy variables $C_t, Y_t, \pi_t, X_t, \lambda_t^r, q_t$

b) Five multipliers $\Lambda_{1,t}, \Lambda_{2,t}, \Lambda_{3,t}, \Lambda_{4,t}, \Lambda_{5,t}$, on binding constraints

c) Two pricing conditions for real rates $R_{t,t+1}^r$, nominal rates $R_{t,t+1}^n$

d) Two pricing condition for the expected return on equity $E_t[R_{t,t+1}^{eq}]$ and the Value of equity holdings V_t^{eq}.

For given exogenous processes of productivity for *safe* (3.38) firms, a time-varying multiple (ϑ_t) for *risky* firms, and calibrated values of parameters (given in Table 3.2),

$\beta, \rho, \eta, h, \omega, \phi, \sigma, \psi, \chi, \tau_p, \gamma_{\mathcal{S}}, \sigma_{\mathcal{S}}, \vartheta^h, \vartheta^l, k$. The discretionary optimal policy is numerically solved in following section 3.4.

3.3.2 Solving for Efficient planner

The social planner looks to optimise aggregate household utility subject to feasibility constraints. The value function with respect to a representative household for the efficient planner is:

$$\mathcal{V}^{sp}\left(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}, \vartheta_t\right) = \max_{\left\{\begin{array}{c} C_t, \\ Y_t, \\ X_t, \\ q_t \end{array}\right\}} \left[\begin{array}{c} \frac{(C_t-hX_t)^{1-\rho}-1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\xi_{\mathcal{S},t}^{-1}\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}} \\ +\beta E_t\left\{\mathcal{V}^{sp}\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t, \vartheta_{t+1}\right)\right\} \\ +\Lambda_{1,t}^{sp}\left(X_t-(1-\phi)C_t-\phi X_{t-1}\right) \\ +\Lambda_{2,t}^{sp}\left(Y_t-\frac{\chi}{2}\left(q_t-\frac{1}{2}\right)^2-C_t\right) \end{array}\right] \tag{3.95}$$

the first order conditions are:

$$\mathbf{C_t}: \ \Lambda_{2,t}^{sp} = (C_t-hX_t)^{-\rho} - \Lambda_{1,t}^{sp}(1-\phi) \tag{3.96}$$

$$\mathbf{X_t}: \ \Lambda_{1,t}^{sp} = h\left(C_t-hX_t\right)^{-\rho} + \beta\phi E\left[\Lambda_{1,t+1}^{sp}\right] \tag{3.97}$$

$$\mathbf{Y_t}: \ \Lambda_{2,t}^{sp} = Y_t^{\eta}\xi_{\mathcal{S},t}^{-1}\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}} \tag{3.98}$$

$$\mathbf{q_t}: \ \Lambda_{2,t}^{sp}\chi\left(q_t-\frac{1}{2}\right) = \frac{\beta}{\sigma-1}E\left[Y_{t+1}^{1+\eta}\xi_{\mathcal{S},t+1}^{-1}\left[1+q_t\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right] \tag{3.99}$$

By eliminating $\Lambda_{2,t}^{sp}$ in the FOC w.r.t. Y_t and q_t we can get the optimal risk allocation Euler equation as,

$$q_t = \frac{1}{2} + \frac{\beta}{\chi\left(\sigma-1\right)}\frac{E_t\left[Y_{t+1}^{1+\eta}\xi_{\mathcal{S},t+1}^{-1}\left[1+q_t\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]}{Y_t^{\eta}\xi_{\mathcal{S},t}^{-1}\left[1+q_{t-1}\left(\vartheta_t^{\sigma-1}-1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}}} \tag{3.100}$$

and using

$$\Lambda^{sp}_{1,t} = \frac{1}{1-\phi}\left[(C_t - hX_t)^{-\rho} - Y_t^{\eta}\xi^{-1}_{\mathcal{S},t}\left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}}\right] \quad (3.101)$$

the Consumption Euler equation is

$$\left[(1 - h(1-\phi))\left(C_t - hX_t\right)^{-\rho} - Y_t^{\eta}\xi^{-1}_{\mathcal{S},t}\left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}}\right] =$$
$$\beta\phi E_t\left[(C_{t+1} - hX_{t+1})^{-\rho} - Y_{t+1}^{\eta}\xi^{-1}_{\mathcal{S},t+1}\left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right]^{\frac{-(1+\eta)}{\sigma-1}}\right] \quad (3.102)$$

The above two euler equations alongwith the aggregate habits and the resource constraints solve for the Social planner equilibrium.

Definition 3.3.1. The social planner's deterministic equilibrium is given by a path of allocations for $\{C_t, Y_t, X_t, q_t\}$ such that for all t, when $\overline{\xi}_{\mathcal{S},t} = 1$, $\vartheta_t = \overline{\vartheta}$ the allocation is stable at $\{\overline{C}, \overline{Y}, \overline{X}, \overline{q}\}$.

In deterministic steady-state, from the aggregate habits constraint we get that the steady state of habits is equal to the steady state of consumption, So: $\overline{X} = \overline{C}$ and from the aggregate resource constraint $\overline{Y} = \overline{C} + \frac{\chi}{2}\left(\overline{q} - \frac{1}{2}\right)^2$. Therefore the steady state Euler equations are:

$$\left(\overline{q} - \frac{1}{2}\right) = \frac{\beta}{\chi(\sigma-1)}\overline{Y}\left[1 + \overline{q}\left(\overline{\vartheta}^{\sigma-1} - 1\right)\right]^{-1}\left(\overline{\vartheta}^{\sigma-1} - 1\right) \quad (3.103)$$

and

$$\overline{Y}^{\eta}\left[1 + \overline{q}\left(\overline{\vartheta}^{\sigma-1} - 1\right)\right]^{-\frac{1+\eta}{\sigma-1}} = \left(1 - \frac{h(1-\phi)}{1-\beta\phi}\right)\overline{C}^{-\rho}(1-h)^{-\rho} \quad (3.104)$$

which can be simplified to give

$$\left[1 + \overline{q}\left(\overline{\vartheta}^{\sigma-1} - 1\right)\right]^{-1} = \left[\overline{Y}^{-\eta}\left(1 - \frac{h(1-\phi)}{1-\beta\phi}\right)\overline{C}^{-\rho}(1-h)^{-\rho}\right]^{\frac{\sigma-1}{1+\eta}} \quad (3.105)$$

using this expression we can derive

$$\left(\overline{q} - \frac{1}{2}\right)^2 = \left[\frac{\beta}{\chi(\sigma-1)}\overline{Y}\left[\overline{Y}^{-\eta}\left(1 - \frac{h(1-\phi)}{1-\beta\phi}\right)\overline{C}^{-\rho}(1-h)^{-\rho}\right]^{\left(\frac{\sigma-1}{1+\eta}\right)}\left(\overline{\vartheta}^{\sigma-1} - 1\right)\right]^2 \quad (3.106)$$

Therefore from the resource constraint

$$\overline{Y}-\overline{C}=\frac{1}{2\chi}\left[\frac{\beta}{(\sigma-1)}\overline{Y}\left[\overline{Y}^{-\eta}\left(1-\frac{h(1-\phi)}{1-\beta\phi}\right)\overline{C}^{-\rho}\left(1-h\right)^{-\rho}\right]^{\left(\frac{\sigma-1}{1+\eta}\right)}\left(\overline{\vartheta}^{\sigma-1}-1\right)\right]^{2} \tag{3.107}$$

which does not have a straightforward solution.

Proposition 3.3.2. The social planner's deterministic equilibrium exists for any $\overline{\vartheta}$.

Proof. See lemma 3.3.3. □

Lemma 3.3.3. (i) If $\overline{\vartheta}=1$, then A: $\overline{Y}=\overline{C}=\overline{X}$ and B: $\overline{q}=\frac{1}{2}$ are equivalent.
(ii) If $\overline{\vartheta}>1$, then $\overline{q}\in(\frac{1}{2},1]$.

(iii) If $\overline{\vartheta}<1$, then $\overline{q}\in[0,\frac{1}{2})$.

Proof. (i) A implies B, B implies A. Also if monopoly power of the firm increases, $\sigma\sim 1$ even then the previous solution holds as $\overline{\vartheta}^{\sigma-1}\simeq\overline{\vartheta}^{0}=1$.
(ii) If $\overline{\vartheta}>1$, then $\left(\overline{\vartheta}^{\sigma-1}-1\right)>1$ the LHS of the following equation

$$\left(\overline{q}-\frac{1}{2}\right)\left[1+\overline{q}\left(\overline{\vartheta}^{\sigma-1}-1\right)\right]=\frac{\beta}{\chi(\sigma-1)}\overline{Y}\left(\overline{\vartheta}^{\sigma-1}-1\right) \tag{3.108}$$

can only be negative for feasible values of $q\in(\frac{1}{2},1]$.
(iii) If $\overline{\vartheta}<1$, then $\left(\overline{\vartheta}^{\sigma-1}-1\right)<1$ and the LHS of the following equation

$$-\left(\frac{1}{2}-\overline{q}\right)\left[1+\overline{q}\left(\overline{\vartheta}^{\sigma-1}-1\right)\right]=\frac{\beta}{\chi(\sigma-1)}\overline{Y}\left(\overline{\vartheta}^{\sigma-1}-1\right) \tag{3.109}$$

can only be positive for feasible values of $q\in[0,\frac{1}{2})$. □

Based on the above results it is clear that the first best risk allocation is $>\frac{1}{2}$ when the expected value of risky firm's productivity multiple ϑ is >1, and it is $<\frac{1}{2}$ when the expected value of risky firm's productivity multiple is <1. Therefore in the first best equilibrium a risk allocation $q\neq\frac{1}{2}$ produces a wedge between $\overline{C}$ and $\overline{Y}$.

3.4 Calibration and Numerical solution

The calibrations of parameters are similar to the ones chosen in Chapter 2 and are mostly standard (as described in Table 3.2). The key difference in these two chapters is the choice of parameter k (from section 3.2.2.3) which controls for the slope of *risky* firm tfp vis-a-vis the *safe* firm tfp, and the transaction cost parameter χ.

The slope parameter k depends on the choice of maximum (ϑ^h) and minimum (ϑ^l) values of the *risky* firm tfp multiple ϑ_t, and the chosen extreme values of the low firm tfp, i.e., $\xi_{\mathcal{S},h}^{\frac{1}{1+\eta}}$ and $\xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}$. The calibrated value of quarterly parameters of ϑ^h is 1.06, and ϑ^l is 0.94. As a result, the risky firms provides an annual productivity gain (above safe firm productivity) of around 25% if it stays in the high multiple state ϑ^h and an annual loss of around 25% in productivity if it produces at low multiple ϑ^l state. Whereas, the value of $\xi_{\mathcal{S},h}^{\frac{1}{1+\eta}}$ and $\xi_{\mathcal{S},l}^{\frac{1}{1+\eta}}$ are determined from the extreme ends of the tfp state grid which is at +/- 3 standard deviations from its mean.

This calibration of productivity parameters is formed on the average productivity in the US. The trend multi-factor productivity growth in non-farm manufacturing sector in the US from 1948-2017 is around 1.02%. The average multi-factor tfp growth goes up to 1.36% for periods when it is above trend and falls to -1.15% in years with below-trend productivity growth. Whereas, the labour productivity in the US from 1988-2017 is more volatile and rises to 100% annual growth in some periods and falls to -10% in others. On average the labour factor productivity is +70% in years when it is above trend and -50% in years when it is below trend. Based on these facts, there is a wide range of calibration available for high state ϑ^h and low state ϑ^l multiples. The calibration chosen in this paper aims to replicate the stationary trend of US multi-factor productivity.

Quarterly S&P returns are non-negative only around 55% of the times, therefore an equally likely choice of $\vartheta > 1$ and $\vartheta < 1$ is not unusual. In this calibration, both high multiple and low multiple states are equally likely, and it offers a neutral bet to the risk-neutral agent. A risk-neutral consumer should opt for a 50% proportion of *risky* firms in the steady-state, which is not far from the international panel study Apergis, 2015 evidence which puts the risky investments form 35-45% of the total portfolio.

The transaction cost parameter χ is freely chosen at 0.05, which effectively generates a cost of $\chi/2$ or 0.025 that is equal to the ten per cent of steady-state dividend income. So agent's choice of $q \in [0,1]$ would generate a loss of $\in \left[0, \frac{\chi}{2}\frac{1}{4}\right] = [0, 0.625\%]$ unit of notional capital. This is a standard transaction cost faced by investors in financial markets. The calibration of other parameters is reasonably standard and in line with evidence from micro and macro data. The inverse of elasticity of intertemporal substitution parameter ρ is set at 2 as in Campbell and Cochrane (1999), and the inverse of Frisch-elasticity η is set at 4, most estimates place the inverse of Frisch-elasticity for intensive margin in the range 0 to 0.7 with median around 0.2 (Keane, 2011). The Rotemberg cost parameter ψ is chosen at 30 to map [as per (Leith and Liu, 2016)] an 8-9 month nominal rigidity under Calvo prices. The CES parameter σ is equal to 6 to give a stationary markup of 20%. The discount factor β is set at 0.99 to generate a 4.01% annual risk-free rate. Similar to the previous chapter, the habit parameters for persistence ϕ is set high at 0.97 to generate the costly recession effect in agents' marginal utility. The habit size parameter is set high at 0.85 to smooth out volatility in asset prices, and the internal habits parameter ω is set at 0.75 which makes the bulk of the habits as internal and generates an inflationary bias that is closer to observations in data. The corporate tax rate is set at pre-Trump effective US corporate tax rate of 35%. These parameters solve for global optimisation of the non-linear model.

3.4.1 Solution procedure

We cannot write the equilibrium policy functions for the discretionary planner as explicit functions of states and so we cannot solve them in closed form. Therefore, the policy problem is solved by using Chebyshev's collation technique[9]. The polynomial approximation method looks to identify the Markov-perfect equilibria where the policy functions evolve according to various states of the model. The majority of the solution procedure is similar to one described in Section 2.3.1. In order to avoid the risk of repetition, this section presents only a brief description of the solution approach and any significant changes to it.

The state space is three dimensional. One of the states is external ($\varepsilon_{\mathcal{S},t}$) and the remaining two (X_{t-1}, q_{t-1}) are internal. The state spaces for ($\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}$)

[9] Chebyshev's polynomial method used in this thesis derives from its version described in Leith and Liu (2016), the appendix of that paper also provides algorithms for applying this technique to optimal policy problems with 1 and 2 state variables.

Table 3.2: Calibration of Parameters

Parameters		Value	Target	Source
Inverse of EIS	ρ	2.0	avg data	Campbell et al. (1999)
Discount factor	β	0.99	4% p.a. avg return	Leith et al. (2016)
Inv. of Frisch Elasticity	η	4.0	avg data	Keane (2011)
Habits persistence	ϕ	0.97	low vol of asset px	Cochrane (2016)
Habits size	h	0.85	vol in MU	De Paoli et al. (2013)
Internal habits	ω	0.75	Chapter 2 result	Chen et al. (2009)
Rotemberg Cost	ψ	30	8-10m rigidity	Leith et al. (2016)
CES	σ	6	avg data	De Paoli et al. (2013)
Tech adj. Cost	χ	0.05	6 period decumu.	Rognlie et al. (2018)
Revenue tax	τ_p	0.35	corp tax rate	US statistics
safe firms Prod				
AR(1) coefficient	$\gamma_{\mathcal{S}}$	0.95		Smets et al. (2007)
Stdev. prod. shock	$\sigma_{\mathcal{S}}$	0.015	vol of tfp = 2.6%	De Paoli et al. (2013)
risky firms Prod				
High Prod Multiple	ϑ^h	1.06	+25% in high state	US multi tfp, std. cal.
Low Prod Multiple	ϑ^l	0.94	−25% in low state	US multi tfp, std. cal.
Safe tfp Grid	$\varepsilon_{\mathcal{S},t}$		[-0.144, 0.144]	
Habit Grid	X_{t-1}		[1.46, 1.52]	
Risk Grid	q_{t-1}		[0.01, 0.99]	

Notes : avg data = average micro macro data , MU = marginal utility, px = price, std. cal. = standard calibration. decumu = Decumulation refers to the de-accumulation period of assets. Prod = productivity

Table 3.3: Stochastic Steady state results

Variable	Symbol	SSS
Consumption	C	1.4771
Output	Y	1.4791
Inflation % p.a.	π	3.9
Habits	X	1.4771
Real multiplier	λ^r	10.60
Risk allocation %	q	49.76
Nominal rates % p.a.	R^n	7.95
Real rates % p.a.	R^r	3.9
Return on Equity % p.a.	R^{eq}	4.14
Value of Equity	V^{eq}	28.6
Simul. volatility	**US qtrly data**	**Model**
Log Cons growth	0.5 - 0.6 %	0.24%
Log Real rate	0.4%	2.22%
Log Output growth	0.6%	0.30%
Log Inflation	0.3%	1.04%
Sharpe ratio	0.2	0.0329

Notes : Parameters of the model are in Table 3.2.

are each divided into set of 9 collocation nodes. The state space is given by their tensor product or $(9 \otimes 9 \otimes 9)$. Each node in the state space is a mapping of the roots of chebyshev's polynomial of a selected order on the space $\in [-1, 1]$. The space of approximating functions is three-dimensional and given by the three sets of Chebyshev polynomials of order 9^3. The residual equations are defined from the 15 Euler equations. Gauss-Hermite Quadrature is used to approximate the expectation terms and first differencing of the Chebyshev polynomials is used to approximate the partial derivatives to internal states of habits and risk allocation. Sims' non-linear solver is employed to solve the resulting non-linear equations for the choice of parameters given in Table 3.2. The solver guesses the coefficients that would solve the residual equations in the state space. The guess is updated until the residual equations converge. According to Judd (1998) convergence criteria below 10^{-5} is acceptable. The policy problem is solved when the residual equations at grid points result in an error of size smaller than 10^{-8}. Size of errors at off grid points is also very small, the maximum error is below 7.6 x 10^{-4}, mean error is below 4.6 x 10^{-5}, standard deviation below 9.2x10^{-5}, and considering that we have included a wide grid of all states where the the optimal policy result runs into significant non-linearity, this is a very acceptable degree of accuracy.

The stochastic steady-state results of the procedure are given in Table 3.3. The stochastic steady-state for consumption and habits is similar and slightly lower than that of output. The inefficiency wedge from adjustment costs and transaction costs creates small positive inflation is steady-state for the benchmark calibration. The simulated stochastic volatility of inflation is higher while both volatilities of consumption growth and output growth are lower than the observations of data. This volatility trade-off between output and inflation in the stochastic results reflects that optimal policy in the model favours a higher stabilization in output during costly recessions. The expected equity return from the simulated results is smaller than the 5-6% equity premium observed in data. Including a higher CES parameter, or an independent stochastic process for tfp for the *risky* firm could raise this premium. The ratio of excess returns from the stock index to the volatility of stock returns, also known as the Sharpe ratio, is smaller for the model in comparison with US data. It is a result of the excess volatility of stock returns generated in the model due to a lack of heterogeneity in risky investments. Including more heterogeneity in risky investments can raise the Sharpe ratio but will also raise the complexity of the benchmark model. We leave that for future research.

3.5 Results and Discussion

The main contribution of this chapter is to demonstrate a Flight to Safety or reduction in risk allocation q_t when risk aversion increases. Risk aversion is state dependent and varies inversely with the Surplus ratio i.e. ratio of Consumption in excess of subsistence level of Habits to Consumption. $S_t = \frac{C_t - hX_t}{C_t}$. The optimal path for surplus ratio like its constituents consumption and habits is a function of all three states *viz.* safe firm technology ($\varepsilon_{\mathcal{S},t}$), habits ($X_{t-1}$) and risk allocation ($q_{t-1}$), and it responds to various shocks and changes in these states. An increase in surplus ratio symbolises a reduction in marginal utility and a decrease in risk aversion and vice versa. However, it is not straightforward to identify what determines an increase in surplus ratio. For a given combination of habits state (whether it be low, high or at steady state), the surplus ratio could also be low or high. It will also be motivated by the level of consumption in those states, which in itself is determined by the level of risk allocation state q_{t-1} and safe firms technology ($\varepsilon_{\mathcal{S},t}$). Therefore instead of looking at the three states separately a better picture of the state of the economy emerges if we consider the optimal policy choice in the space of the surplus ratio.

3.5.1 Optimal risk allocation policy

Figure 3.1 on page 228 demonstrates the optimal path for risk allocation q_t, which shifts non-linearly with respect to changes in the surplus ratio. The steady-state level of surplus ratio for the chosen calibration in this model is $1 - h = 0.15$, and marked with the red asterisk in fig. 3.1. The first impressions from the fig. 3.1 show that risk allocation is stable at level 1 or there are only risky intermediate firms in the economy when the surplus ratio is comfortably above the steady-state level. Whereas, the risk allocation is at 0, or there are 0% risky intermediate firms in the economy when the surplus ratio is uncomfortably below the steady-state level. Additional insights from the figure come by reading it in two parts, from going right to left for lines marked with 'left arrows', and from going left to right for lines marked with 'squares'.

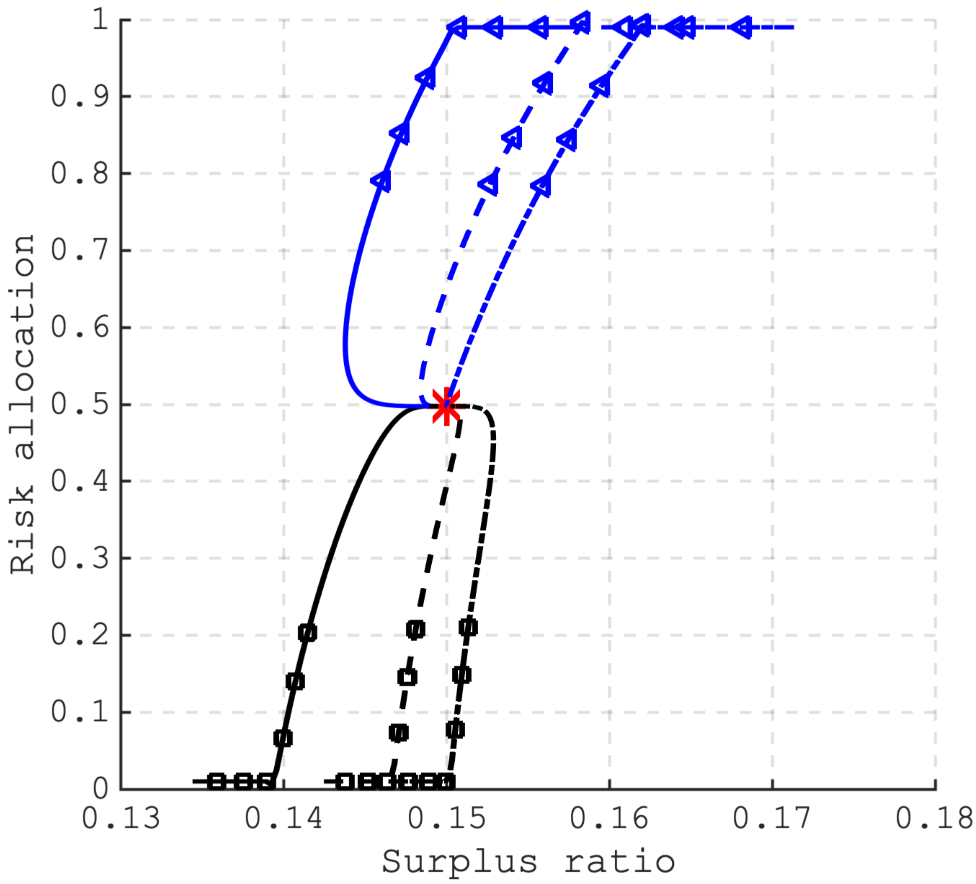
In the 'left arrows' marked lines, there is a change in optimal risk allocation when the surplus ratio reduces to its steady-state level. The drop in risk allocation is non-linear and suddenly there is a preference for *safe* assets (firms) vis-à-vis *risky* assets. The sudden change from 100% risk allocation is a Flight to Safety.

Similarly going from left to right on the solid lines marked with 'squares' in the figure, we can see that at low levels of surplus ratio there is no change in risk allocation. It is stable at level 0. The household is keeping all investments in *safe* technology when its surplus ratio is far lower than the steady-state value of 0.15. As the surplus ratio increases, there is an increase in risk allocation, and the investor now prefers some *risky* firms in the portfolio. This abrupt shift in risk allocation captures a Flight to Risk. As we follow from left to right, improvements in surplus ratio up to the point of the steady-state level lead to an increase in the optimal choice of risk allocation.

Figure 3.1 presents a stylised example of optimal policy convergence. Any point on the Risk-Surplus ratio space is an optimal path depending on the three different states of the model, *viz.* safe firm technology ($\varepsilon_{\mathcal{S},t}$), past habits ($X_{t-1}$) and past risk allocation (q_{t-1}). This figure shows convergence in optimal policy over time, for a given initial value of all three states. In the first period, the 'left arrows' marked lines represent states where the safe firm's tfp is +2 s.d. above its steady-state, and the habits state is either high (solid lines), low (dash-dots lines) or at steady state (dotted lines). That is why there are three lines marked with 'left arrows' where each one represents a different starting state of habits for a common (+2 s.d.) starting state of safe firm's tfp. Similarly, in 'squares' marked lines, the safe firm's tfp in the first period begins at -2 s.d. away from its steady-state level. The habits state is either high (solid lines), low (dash-dots lines) or at steady state (dotted lines) in the first period. Value of risk allocation state in the first period is at its steady-state level in all lines of this figure. The surplus ratio varies inversely with an increase of habits. Therefore the initial position of habit state can also be determined from the starting points of various lines. After the initial setup, all three states are allowed to evolve optimally, and the figure plots the resulting optimal policy paths of risk allocation and surplus ratio. Each line in this figure shows the quarterly time evolution of the optimal policy, where 'left arrows' or 'squares' markers mark the first 24 quarterly periods that are separated by a distance of 4 quarters.

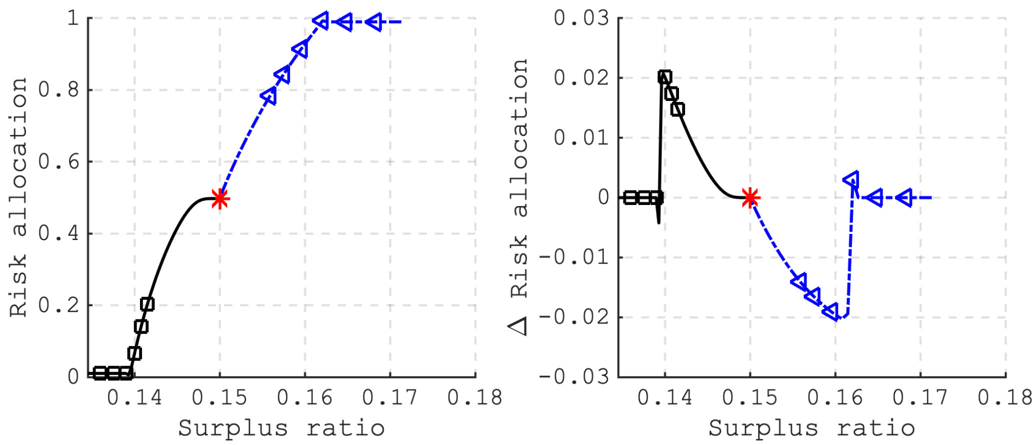
A better picture of the Flight to Safety mechanism emerges in Figure 3.2 on page 229 which zooms in on one section of Figure 3.1. Instead of plotting three separate lines for three (high, low and steady-state) different first period allocation of habits, this figure focuses on one initial state of habits. In both charts of this fig. 3.1 the 'left arrows' marked lines represent the transition when the initial

Figure 3.1: Optimal path for Risk allocation



Notes: Convergence of Risk allocation in the Surplus ratio space. Risk allocation and Surplus ratio are a function of 3 states, *viz.* current TFP, past Habits and past Risk allocation. This stylised example chooses the first period value of these three states and thereafter plots the optimal policy evolution for risk allocation and surplus ratio over time. In the 'Left arrows' marked lines the first period value of TFP state is at 2 s.d. above its steady state level, Risk allocation is at steady state and the Habits state is at a level that is either above, below or at its steady state. The 3 different 'left arrows' marked lines represent this evolution for various initial states of habits. Similarly, in the 'Squares' marked lines the first period value of TFP state is at 2 s.d. below its steady state level, Risk allocation is at steady state and the Habits state is at a level that is either above, below or at its steady state. The 3 different 'squares' marked lines represent this evolution for various initial states of habits. The Red asterix represents the Stochastic steady state allocation.

Figure 3.2: Optimal path for Flight to Safety



Notes: Convergence of Risk allocation in the Surplus ratio space. Δ represents one-period change in the variable. Risk allocation and Surplus ratio are a function of 3 states, *viz.* current TFP, past Habits and past Risk allocation. This stylised example chooses the first period value of these three states and thereafter plots the optimal policy evolution for risk allocation and surplus ratio over time. In the 'Left arrows' marked lines the first period value of TFP state is at 2 s.d. above its steady state level, Risk allocation is at steady state and the Habits state is below its steady state. Similarly, in the 'Squares' marked lines the first period value of TFP state is at 2 s.d. below its steady state level, Risk allocation is at steady state and the Habits state is above its steady state. The Red asterix represents the Stochastic steady state allocation.

(first period) state of habits is below its steady-state allocation, and safe firm's technology is 2 s.d. above its steady-state level. Whereas the 'squares' marked lines in both charts of this figure represent the transition where the first period allocation of habits is higher than the steady-state level of habits and the safe firm's technology is 2 s.d. below its steady-state level. The risk allocation state in all lines of this figure begins from its steady-state level. The surplus ratio is inversely related to habits and starts above 0.15 level for 'left arrows' marked lines and below 0.15 for 'squares' marked lines. The risk allocation exhibits Flight to Safety as there is a sudden preference for safety when the surplus ratio falls from the 0.16 level towards its steady-state level. Risk aversion decreases, or there is a Flight to Risk when surplus ratio increases above 0.14 in 'squares' marked line.

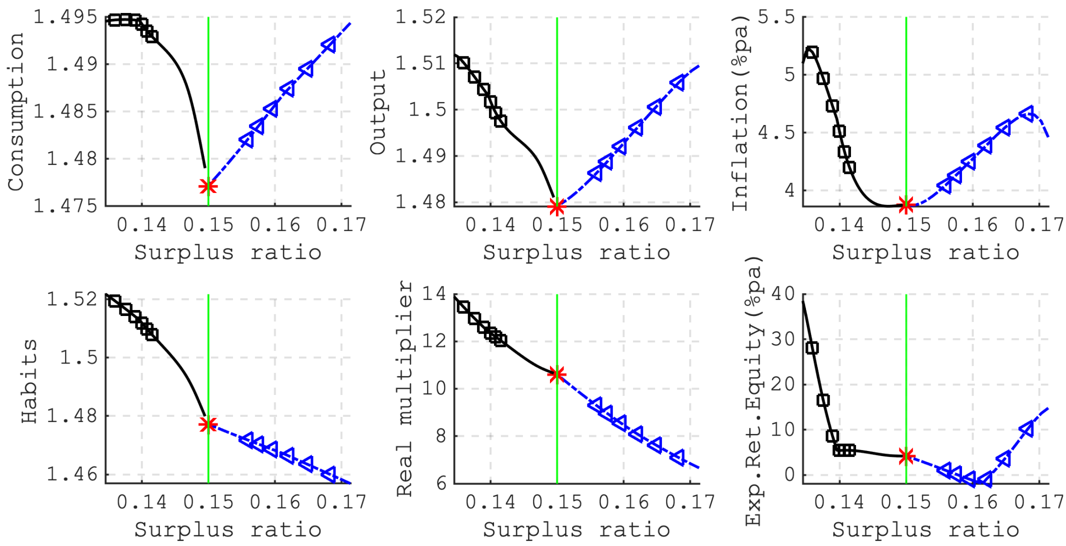
Interestingly the right-hand chart in Figure 3.2 captures this Flight to Safety and Flight to Risk phenomena better. It showcases the one-period changes in risk allocation for various surplus ratio. Around 0.16 and 0.14 levels, there is a sudden structural shift in risk allocation. Reading the chart from right to left for the 'left arrows' marked lines, we can see that the investor does not change her risk allocation until the time it nears 0.16 and then there is a sudden re-allocation of the portfolio to favour *safety*. This significant and sudden drop in risk allocation captures Flight to Safety.

Similarly, while reading the right-hand chart in Figure 3.2 form left to right, the 'squares' marked lines show a sudden increase in risk allocation when the surplus ratio increases above 0.14. It demonstrates a Flight to Risk for the investor whose marginal utility or stochastic discount factor improved for an increase in the surplus ratio. We can further corroborate these findings from the optimal policy choices for other variables of the model.

3.5.2 Optimal policy and Surplus ratio

Figure 3.3 on page 231 demonstrates further the non-linearity in optimal policy choices based on shifts in surplus ratio. In these charts, the vertical green line highlights the steady-state level (0.15) of Surplus ratio for the current choice of parameters of the model. Let us read the charts in this figure in two parts. Going from left to right on the 'squares' marked solid lines, and going from right to left on the 'left arrows' marked dash dots lines. Each policy variable is a function of three states. This figure showcases the convergence of optimal policy

Figure 3.3: Optimal path for Policy variables



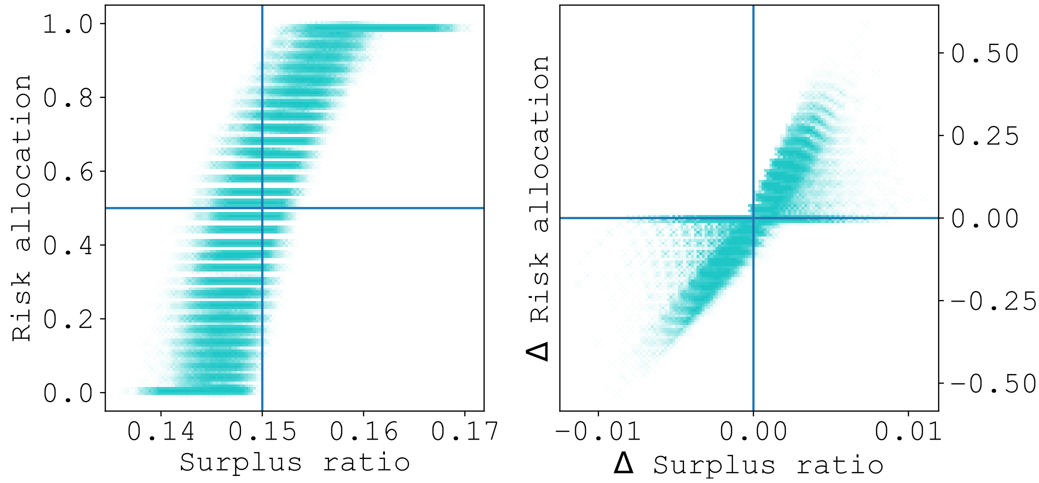
Notes: Convergence of optimal policy choices in the Surplus ratio space. Various policy functions and Surplus ratio are a function of 3 states, *viz.* current TFP, past Habits and past Risk allocation. This stylised example chooses the first period value of these three states and thereafter plots the optimal policy evolution for risk allocation and surplus ratio over time. In the 'Left arrows' marked lines the first period value of TFP state is at 2 s.d. above its steady state level, Risk allocation is at steady state and the Habits state is below its steady state. Similarly, in the 'Squares' marked lines the first period value of TFP state is at 2 s.d. below its steady state level, Risk allocation is at steady state and the Habits state is above its steady state. Exp.ret.Equity is the Expected return from equity investments. The vertical green line in all charts, marks the steady state level of surplus ratio (0.15). The Red asterix represents the Stochastic steady state allocation.

functions for decision variables in the surplus ratio space, for given initial/starting value of the three states. The 'left arrows' marked lines showcase the transition to steady-state when the first period safe firm's tfp state was 2 s.d. above its steady-state, habits state was below its steady-state level, and the risk allocation state was at its steady-state level. Whereas the 'squares' marked lines showcase the transition to steady-state when the first period safe firm's tfp state was 2 s.d. below its steady-state, habits state was below its steady-state level, and the risk allocation state was at its steady level. The 'left arrows' or 'squares' markers also demonstrate the first 24 quarters/periods of the transition, where the spacing period between two markers represents a lapse of 4 quarters.

Following from right to left in charts of fig. 3.3, any reduction in surplus ratio leads to a reduction in the optimal choice of consumption and output but an increase of habits. This increase of habits is not surprising since the high level of surplus ratio in 'left arrows' is partly due to habits being low in comparison to consumption. Habits keep up with the higher level of consumption and stabilise as surplus ratio reaches its steady-state mark of 0.15. Following from left to right in the various charts of fig. 3.3, we see that habits, consumption and output fall very dramatically to stabilise the surplus ratio. In the presence of costly recessions, and the way marginal utility is set up in this chapter, and the previous one, a surplus ratio smaller than 0.15 means the consumption C_t is getting closer to a subsistence level of habits hX_t, and the marginal utility is very high. In that situation, the objective of the optimal policy is to quickly stabilise the surplus ratio, which could be done from reducing habits and by avoiding risky ventures, as shown in previous fig. 3.2. The marginal value of an additional unit of income, or the real multiplier (λ_t^r) increases as surplus ratio reduces.

The competitive economy is inefficient, and there is an inflationary bias. The bias is increasing in the deviation from steady-state of surplus ratio on either side, and it is minimised when the surplus ratio stabilises. The Expected return on equity is very high towards both extreme ends of the surplus ratio, but for different reasons. The high value of the expected return on equity in 'left arrows' marked line is due to irrational exuberance, the marginal utility of the agents is low, the surplus ratio is high, and there is an expectation of the positive TFP growth to continue. The risk allocation in this phase of the economy is hitting the upper bound of 1 as equity investors expect to make higher returns on their investments. Whereas on the left portion of the chart, in 'squares' marked line, investors are exhibiting precautionary behaviour and extreme risk aversion; the surplus ratio is below steady-state, and the risk allocation is closer to the lower

Figure 3.4: Scatter plot of Optimal Risk allocation and Surplus ratio



Notes: Results from a 100,000 simulations. Δ is one-period change in the variable.

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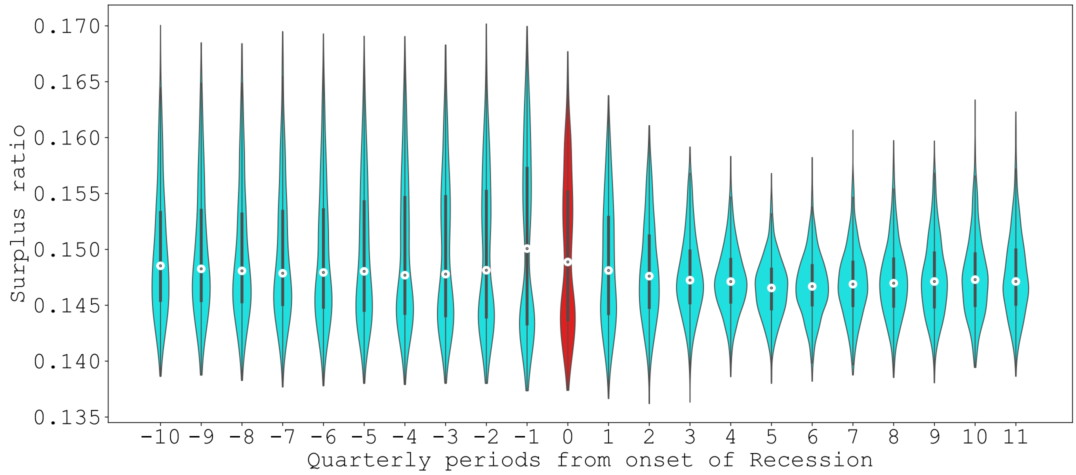
bound of 0. Investors in this situation require very high expected return on equity to part with their savings.

3.5.3 Simulations and Flight to Safety

The optimal policy results have shown us that risk aversion and risk allocation change with changes in the surplus ratio or agent's marginal utility. Nevertheless, the stylised nature of policy results and their dependence on the initial condition of safe firm's tfp could make one doubt the feasibility and believability of these paths or points on the optimal policy - surplus ratio space. Therefore we should look at optimal policy results from the simulated model, and identify if they corroborate the previous findings. Figure 3.4 on page 233 describes the optimal policy results from 100,000 simulations of the model. Markov Chain Monte Carlo methods are used to simulate the model for various possible initial states of safe tfp, habits state and risk allocation state. The chart on the left-hand side of the fig. 3.4 shows the scatter plots of optimal policy results for risk allocation and surplus ratio in the simulated model. A key theme which emerges from this chart is that at high levels of surplus ratio, there is 100% risk allocation and at low levels of surplus ratio, there is 0% risk allocation. The chart on the left hand has the presence of a flatter scatter pattern in the surplus ratio for each level of risk allocation, this chart by itself provides inconclusive information about the

Figure 3.5: Violin plots for Surplus ratio



Notes: Distribution of Surplus ratio in quarters leading up to and following the onset of Recession. The onset of recessions are mapped to period 0 on the X-axis. The recessionary periods are identified as those sequences in the simulated model which initiate periods of negative output growth that last for at least 4 quarters.

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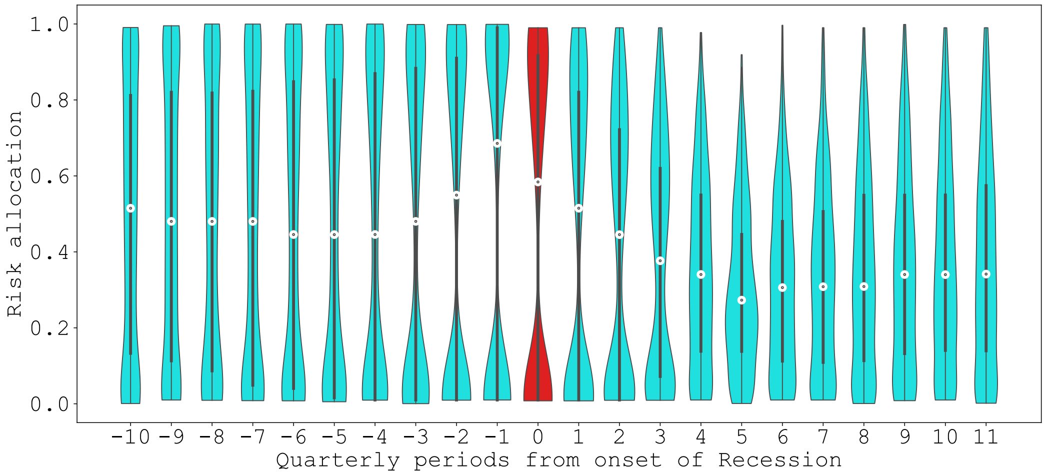
transition dynamics of optimal risk allocation. The left-hand chart is inconclusive because risk allocation could be high, low or steady depending on initial values of various states, for any level of surplus ratio.

A clear message emerges from the right-hand chart in this Figure 3.4 on page 233, which plots the one period change in risk allocation vs one period change in surplus ratio. It demonstrates an increasing relationship between the change in risk allocation and change in surplus ratio. When surplus ratio decreases, whether it be from habits being too high in comparison to consumption, or from a windfall drop in output/consumption due to an adverse tfp surge, or low dividends resulting from risky equity holdings, or a combination of such factors, such a decrease in the surplus ratio is coincident with a fall in risk allocation. There are also occasional values in the right-hand chart of fig. 3.4 when there is no change in risk allocation (i.e. scatter marks on the y-axis value of zero) for both positive and negative changes in surplus ratio. These periods represent the phenomenon similar to the flat lines in fig. 3.1. The risk allocation in these periods was near its lower or upper bounds. There is either a total aversion of risk or a total acceptance of risk as the surplus ratio (and marginal utility) is either at a highly comfortable of highly uncomfortable level for the investor.

Figure 3.6: Violin plots for Risk allocation



Notes: Distribution of Risk allocation in quarters leading up to and following the onset of Recession. The onset of recessions are mapped to period 0 on the X-axis. The recessionary periods are identified as those sequences in the simulated model which initiate periods of negative output growth that last for at least 4 quarters.

3.5.4 Ex-ante Flight to Safety

A key finding of the empirical results of the Chapter 1 is that Flight to Safety leads TFP changes by many quarters, esp. during recessions. The distribution of risk allocation in the simulated model reveals a similar pattern during periods before and after the onset recession. The onset of recessions is the beginning period of a sequence in the simulated model, which initiates periods of negative output growth that last for at least four quarters. Figure 3.6 on 235 demonstrates that the simulated model presents dichotomous risk allocation in periods running up to a recession. The violin (probability) distribution of risk allocation in the recession period in this figure is highlighted in red colour and mapped to zero on the x-axis. The violin plots for ten periods before the recession show normal distribution with fat tails. However, as we approach the period of onset of a recession, the distribution turns into bimodal. As the economy approaches the recessionary period there are two kinds of investors, they either have more than 50% or less than 50% levels of risk allocation. The median risk allocation, which the fig. 3.6 shows as white circles marks on each violin, is increasing as we approach the onset period of recessions. This dichotomy is corroborated from the median Surplus ratio responses, displayed as white circles marks on each violin in Figure 3.5 on 234. The surplus ratio distribution displays right-skewness, it has thin tails on the right side (i.e., above 0.15) several quarters before onset of recessions. It becomes somewhat bi-modal with a larger density below the steady-state level of 0.15, as we approach the onset of recessions. The median surplus ratio recovers quickly post-recessions, as expected since it is revealed to us as the main objective of optimal policy, and it has a near normal distribution thereafter.

The distribution pattern of risk-allocation is interesting. Considering that pre-recession there was a normal distribution of risk allocation, with a median of around 50%, the change of risk allocation distribution as we approach the onset of recessions from uniform to bimodal represents that some investors exhibited a Flight to Safety and other exhibited a Flight to Risk in anticipation of a recessions. The median allocation of risk gets below 0.5 or 50% following the onset of recessions. It continues falling for some quarters before a full recovery sets in and the distribution of risk allocation gets back to normal around eight quarters after the recession. In the case of a single representative agent, the bimodal distribution of risk allocation before a recession also represents that agent fluctuating between high and low levels of risk allocation. Thereby exhibiting the ‘risk on’, ‘risk off’ type of pattern that is a feature of episodes of high uncertainty. The existing models of macro-finance do not exhibit such ex-ante adjustment in risk allocation.

3.6 Summary and Conclusions

We have learned that Flight to Safety is an Optimal policy when risk aversion increases for investors who are facing costly recessions but which also have the option to control the future riskiness of their income-generating process. The inclusion of time-varying internal habits which is a device of producing costly recessions also generates a non-linear increase in the household's marginal utility and risk aversion. Households at either end of their risk-bearing capacity, those that consume too close to their subsistence levels or those very well to do, demonstrate sudden changes in their risk allocation. Such non-linear changes of risk allocation echo with the 'risk on' and 'risk off' phenomenon observed in data during periods of economic uncertainty surrounding the global financial crisis.

This chapter demonstrates the possibility of exploring Flight to Safety from the perspective of a micro-founded DSGE mechanism and by using non-linear solution methods to solve for optimal policy. It develops a New Keynesian model which features internal habits in consumption and an option to switch between investment projects of different riskiness. It also challenges the standing approach in literature to consider Flight to Safety phenomenon only as an aberrant reaction to some form of market externalities, valuation constraints or Knightian uncertainty and thereby overlooking it for monetary policy.

The simulation results from the model also lend support to the ex-ante changes in time-varying risk-bearing capacity, which pre-empt the onset of recessions. The results reinforce the findings from empirical studies, including the ones from this thesis, that the Flight to Safety that exists even in low-frequency data, has significant information about future productivity and business cycle performance. Habits provide an endogenous and elegant way to generate time-varying risk aversion in the model. A similar parsimonious solution that works with a continuous set of productive technology with heterogeneity in shock processes would go a long way in devising a risk-based view of business cycles. It is a challenge worth surmounting to establish recessions and behaviours, thereby being driven from individual risk aversion, risk-bearing capacity and desire to shift risk allocation over time.

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A. Appendix for Chapter 1

A.1 Numerical approach to sign restrictions

Uhlig (2005) shows that $a \in \mathbb{R}^m$ is an impulse vector iff a is a column of $\boldsymbol{A}$ such that $\boldsymbol{A}$ is a decomposition of the variance-covariance matrix $\Sigma = \boldsymbol{A}\boldsymbol{A}'$ (see Equ. 1.6). Let $\tilde{A}\tilde{A}'$ be a Cholesky decomposition of Σ, then a is an impulse vector iff

$$a = \tilde{A}\alpha \tag{A.1}$$

for some vector α of unit length. Let $r_i(k) \in \mathbb{R}^m$ be the vector response at k horizon to ith shock in Cholesky decomposition of Σ. Then the impulse response to a is given by

$$r_a(k) = \sum_{i=1}^{m} \alpha_i r_i(k) \tag{A.2}$$

Given some $\boldsymbol{B} = [\boldsymbol{B}'_{(1)}, ..., \boldsymbol{B}'_{(l)}]$, variance-covariance matrix Σ and some K period ahead forecast horizon after the shock, we need to obtain $\mathcal{A}(\boldsymbol{B}, \Sigma, K)$ the set of all such impulse vectors. One approach is to use the OLS estimate of VAR, $B = \hat{B}$ and $\Sigma = \hat{\Sigma}$, for a few choices of K. Monte-Carlo methods can be applied to pick impulse vectors that lie in set $\mathcal{A}$ and obey the sign restrictions. The maximum and minimum of those impulse responses can be plot to get an estimate of the bounds of the impulse responses. Analytically Uhlig (2005) discusses a way of making inference on these impulse responses, by making the following assumption.

Assumption A.1.1. Let $\tilde{A}(\Sigma)$ be the lower-triangular Cholesky factorisation of Σ. Let $\mathcal{P}_m$ be the space of positive-definite $m \times m$ matrices and $\mathcal{S}_m$ be the unit sphere in $\mathbb{R}^m$, $\mathcal{S}_m = \{\alpha \in \mathbb{R}^m : ||\alpha|| = 1\}$. The parameters $(\boldsymbol{B}, \Sigma, \alpha)$ are jointly drawn from a prior on $\mathbb{R}^{l \times m \times m} \times \mathcal{P}_m \times \mathcal{S}_m$. For both criteria function and sign-restrictions approach, this prior is proportional to a Normal-Wishart prior in $(\boldsymbol{B}, \Sigma)$ whenever the impulse vector $a = \tilde{A}(\Sigma)\alpha$ satisfies $a \in \mathcal{A}(\boldsymbol{B}, \Sigma, K)$ and zero elsewhere.

If we make this assumption, the impulse vector is parameterized in $\mathcal{A}(\boldsymbol{B}, \Sigma, \alpha)$ space rather than $\mathcal{A}(\boldsymbol{B}, \Sigma, a)$ space. So the prior is proportional to the Normal-Wishart prior times an indicator function on $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(\boldsymbol{B}, \Sigma, K)$. Therefore, the posterior is given by usual posterior on $(\boldsymbol{B}, \Sigma)$ times the indicator function $\tilde{A}(\Sigma)\alpha \in \mathcal{A}(\boldsymbol{B}, \Sigma, K)$.

To draw inference (in the sign-restrictions approach) from this posterior, jointly draw from unrestricted Normal-Wishart posterior for $(\boldsymbol{B}, \Sigma)$ as well as a uniform distribution over the unit sphere $\alpha \in \mathcal{S}_m$. Construct impulse vector using (A.1) and calculate impulse responses $r_a(k, j)$ for $k = 1, 2...K$ horizons, for the j variables (which are restricted in sign for the K horizon periods). If all the impulse responses obey sign restrictions, keep the draw, otherwise discard it. Repeat sufficiently (for 200 times) and do inference on the draws kept.

In the criteria function approach, a penalty function $f(j)$ is written which penalises the impulse response of a variable in the expected direction (positive or negative after the shock) significantly less than what it penalises the response in the opposite of expected direction. The impulse vector a is then defined as the vector that minimises the total penalty function $\psi(a)$ which is the sum of all impulse responses for all variables (of interest) to the shock. Calculating the impulse response in this way requires numerical minimisation. Save both the draw which is made from the Normal-Wishart prior on $(\boldsymbol{B}, \Sigma)$ and the resulting impulse vector for statistical analysis. In order to obtain inference in this approach, make Assumption A.1.1 and use Monte-Carlo method to pick $n = 100$ draws from it. Do numerical minimisation of the total penalty function $\psi(a)$ on each draw and save the results.

A.2 High-frequency approach to FTS

The structural VAR analysis of the FTS shocks on the macroeconomy has identified the FTS shocks for the benchmark model with respect to identification strategy 1. In this section, I discuss another method from the literature that has been considered in generating FTS series. This comparative analysis follows from Baele, Bekaert, Inghelbrecht, and Wei (2013) in defining FTS as days of high stress and high volatility in asset markets that coincide with days of negative stock-bond return correlation where equity returns are negative and bond returns are positive. Baele, Bekaert, Inghelbrecht, and Wei (2013) used a plethora of econometric techniques to develop specific measures of FTS. Based on daily returns data of benchmark equity index (risky asset) and benchmark 10-yr treasury bond (safe and liquid asset) they have found that on average FTS days are less than 3% of the sample (trading days between 1980 to 2013 of 23 developed countries), and that bond returns exceed equity returns by 2.5 to 4% on FTS days.

One of their techniques is to develop a continuous signal between interval $[0, 1]$ that signifies the probability or likelihood of FTS occurring on that day. This measure identifies FTS as a day of occurrence of extremely negative stock return and simultaneously an extremely positive bond return. Therefore, for country i FTS is given by

$$FTS_{i,t} = \mathbb{1}\{r_{i,t}^{b} > z_{i,b}\} \text{ x } \mathbb{1}\{r_{i,t}^{s} > z_{i,s}\} \tag{A.3}$$

where $\mathbb{1}$ is the indicator function, $r_{i,t}^{b}, r_{i,t}^{s}$ are country specific return on bond and stocks index, and $z_{i,b}, z_{i,s}$ are time-varying country specific thresholds that are directly related to time varying volatility of bond/equity returns. The key variable in derivation of the FTS days then is the time-varying threshold parameters, z_b, z_s that is k standard deviations from time-varying volatilites $\sigma_{b,t}$, $\sigma_{s,t}$.

$$z_b = k\sigma_{b,t} \quad \text{and} \quad z_s = k\sigma_{s,t} \tag{A.4}$$

There are a couple of methods that can be used to generate this time-varying volatility of stock and bond prices. A common approach is to fit a Garch (1,1) on the daily return series. This approach is based on using the Gaussian kernel, which treats both past and future observations as equally significant in generating the volatility of any given date. This paper instead relies on Baele, Bekaert, Inghelbrecht, and Wei (2019) approach that identifies the time-varying long and short-run volatility by employing backwards-looking Gaussian kernel density estimator[1]. For long-run volatility estimation, a normal density kernel is used over past 255 (i.e. near 1 year of trading days), and the weights on the previous 5 days are ignored. The short-run volatility is estimated by using the backwards-looking kernel over the last 5 days (i.e. 1 week of trading days). The volatility is modelled using a simple kernel method is discussed in the next subsection.

[1]Page 5 of Baele, Bekaert, Inghelbrecht and Wei (2013)

A.2.1 Kernel density based conditional volatility

Kernel methodology from Baele, Bekaert, Inghelbrecht, and Wei (2019) for finding the short and long run variances and covariances is reproduced for the avid reader. Given any date t_0 in a sample $t = 1, ..., T$. The kernel method calculates the variances of returns $r_{i,t}$ at a normalised date, $\tau = t_0/T \in [0, 1]$ as

$$\sigma^2_{i,\tau} = \sum_{t=1}^{T} K_h(t/T - \tau) r^2_{i,t} \qquad i = s, b \tag{A.5}$$

where the weights $K_h(z) = K(z/h)/h$ are dependent on how close the observations are to the point of interest τ, the size of chosen bandwidth $h > 0$, and the specific functional form of kernel density estimator. Baele, Bekaert, Inghelbrecht, and Wei (2019) suggest the following backward looking Gaussian Kernel.

$$K(z) = \frac{\psi}{2} exp\left(\frac{z^2}{2}\right) \quad \text{if} \quad z \leq \frac{t-l}{T} \tag{A.6}$$

$$K(z) = 0 \quad \text{if} \quad z > \frac{t-l}{T} \tag{A.7}$$

The scaling factor ψ is used to make the weights sum to 100% and l is the number of past days that are ignored in the variance calculation. For long-run variances, a bandwidth h of 255 days is chosen, and the last 5 observations ($l = 5$) are ignored. In short-run variances the size of h is 5 days and no observations are ignored, ($l = 0$). The time-varying short and long-run covariance between equity and bond return is calculated using the same Kernel density estimator to the cross-product of bond and equity returns. The time-varying correlation between these assets is then given by the ratio of the stock-bond covariance to the product of stock and bond volatilities. Once the time-varying volatilities and covariances are determined, we proceed in the next section to generate an FTS incidence indicator by using either a Threshold-based or an Ordinal rank-based approach. Figure 1.7 on page 19 highlights the results from short and long-run variance and covariance produced from the backwards-looking Kernel methods. A comparison between this nuanced backwards-looking Kernel-based method and a trivial alternative Rolling days method is made in Figure A.1 on page 257. The rolling days method chooses past 250 days of returns for calculating long run time-varying volatility and chooses past 5 days of returns for calculating the short run time-varying volatility. The rolling days method places equal weights to all past observations and therefore has higher peaks and lower troughs.

A.2.2 Threshold approach to FTS

The first approach to identify FTS days is based on the coexeedance approach in Baele, Bekaert, Inghelbrecht, and Wei (2013) that studies contagion across emerging markets by counting joint occurrences of extreme (or beyond a certain threshold) events. The significance of the events is compared to what would be expected under certain normal distributions.

Denote $r_t = (r_{e,t}, -r_{b,t})$ as the series of observed equity returns and negative of observed bond returns. The change of sign of the bond return observation is crucial. Recall that the FTS event is marked by a joint observation of bond returns being above a defined positive threshold and equity returns being below a defined negative threshold. If such a cumulative density function under the assumption of joint normality between stock and negative of bond returns were to be plotted on a two-dimensional grid with equity returns on the horizontal axis and the negative of bond returns on the vertical axis. It would showcase an FTS event to be lying in the bottom left, or south-west corner also known as the third quadrant of the two-axis coordinate system. Assuming that both equity and bond returns are normally distributed, with mean zero, so $r_{i,t} \sim N(0, \Omega_t)$. In this joint normal distribution the cumulative probability of a threshold that is set equal to negative (κ) times the standard deviation of the respective equity and bond returns is given as,

$$Prob_{ths,t} = N_{cdf}\left(\begin{bmatrix} -\kappa\sigma_{e,t} \\ -\kappa\sigma_{b,t} \end{bmatrix}, 0, \Omega_t\right) \tag{A.8}$$

where the variance-covariance matrix Ω_t consists of the variances of bond and equity return, $\sigma_{e,t}$ and $\sigma_{b,t}$ are respective time-varying volatilities, and the covariance between equity and negative of bond returns $\rho_{(e,-b),t}$ is,

$$\Omega_t = \begin{bmatrix} \sigma^2_{e,t} & \rho_{(e,-b),t} \\ \rho_{(e,-b),t} & \sigma^2_{b,t} \end{bmatrix} \tag{A.9}$$

The threshold κ can be calibrated to deliver on a targeted number of events that are considered extreme or demonstrate FTS. An observation is considered extreme (or demonstrates FTS) when the $Prob_{obs,t}$ is less than $Prob_{ths,t}$, i.e. the observed equity returns and negative of bond returns are below the threshold,

$$Prob_{obs,t} < Prob_{ths,t} = Prob\begin{bmatrix} r_{e,t} < -\kappa\sigma_{e,t} \\ -r_{b,t} < -\kappa\sigma_{b,t} \end{bmatrix} \tag{A.10}$$

Therefore the probability of FTS in the threshold method can be calculated as

$$FTS_t = \mathbb{1}\{r_{b,t} > 0\} \text{ x } \mathbb{1}\{r_{s,t} < 0\} \text{ x } \mathbb{1}\{Prob_{obs,t} < Prob_{ths,t}\} \text{ x } (1 - Prob_{obs,t}) \tag{A.11}$$

where the first term checks if the bond returns are positive, the second term checks if the stock return is negative and the third indicator function marks extreme events, i.e. when the cumulative probability of the event is lower than the cumulative probability of the threshold event. Once a day is deemed to have exhibit FTS, the last term puts a higher probability to more extreme events. So for a threshold probability of 0.04, only an observation that has strictly negative stock return, strictly positive bond return and has a cumulative probability density of $Prob_{obs} < 0.04$ can be counted as an FTS event. The FTS probability of that date is then set to be $1 - Prob_{obs}$, which will be greater than 0.96.

Whether an event would be extreme enough to be characterised as an FTS depends on the threshold κ, for instance, if we choose the threshold κ to be equal to 1.25 (time-varying) standard deviation below the mean equity or negative bond return, this would lead to a higher number of FTS days than we would get by setting a threshold κ of 2.0. Figure A.2 on page 258 plots the FTS incidence, calculated using the Threshold method, for different choices of the threshold (κ). In fig. A.2, the threshold probability is time-varying as it depends on the time-varying volatilities of stock and bond returns.

A.2.3 Ordinal index approach to FTS

Another way to identify FTS using kernel density based measure of time-varying volatility of returns and a threshold criteria, is the Ordinal index approach. This alternative method is based on Kremer, Lo Duca, and Holló (2012) composite measure of determining stress in financial system. Their method generates the composite index by aggregating the empirical (ranked) cumulative probability distribution over several indicators. So far, we have used the definition of FTS as the extreme chance of generating excess positive bond returns and excess negative equity returns. To this, using Baele, Bekaert, Inghelbrecht, and Wei (2019), we can add several other features of the market that accompany any FTS event, such as:

Strong features:

a) the size of difference between bond and equity return,

b) the dip in the difference between long and short run stock-bond correlation,

c) the spike in the difference between short term and long term equity return volatility.
and Weak features:
d) strictly negative bond return and strictly positive equity return,
e) short-term bond equity return correlation is negative and strictly below the long-term level,
f) ratio of short term to long term equity volatility is above a threshold k such that $k > 1$.

Once we have the time-varying long and short term volatility, the value of these six features for any day in the sample can be easily calculated using daily stock and bond market returns. The three strong features are all continuous data and are structured so that a high values occurs on days of FTS. The three weak features are binary, indicating whether the feature is achieved or not on any day in the sample, these are expected to take their 'true' value for FTS days. A combination of these results can be used to develop a continuous series of FTS likelihood for any day in the sample.

The procedure begins by generating ordinal ranks for each observation on each of the three strong features in the entire sample. The ordinal rankings for each feature are normalised to be $\in [0, 1]$. Thereafter a composite rank $\in [0, 1]$ for each day in the sample is obtained by averaging its ranking on the three strong features. Therefore an ordinal incidence rank of 0.9 or of 90% for a day in our sample signifies that only 10% of the total days in the sample have higher average ordinal rank than that particular day. This ordinal number can be read as a FTS likelihood series, where the closer the averaged rank gets to 1, the higher is the likelihood of FTS event occurring on that day. This estimate of the FTS likelihood can be futher improved by considering the weak features of FTS days.

Consider the Ordinal rank of all days that satisfy the weak symptoms of FTS, and consider the lowest of these ranks as a threshold. All days in the sample with ordinal rank below the rank of threshold are allocated an FTS probability of zero. The FTS probability of the threshold is assigned as 1 minus the probability of its 'false positives'. In order to obtain the 'false positives' for the threshold number, first determine the number of days in the sample that have an ordinal rank larger than the rank of the threshold, and then calculate the percentage of those days that fail to satisfy all of the three weak symptoms of FTS. Similarly we could rely on the weak features to obtain FTS probability measure for the observations that have an ordinal rank above the ordinal rank of threshold. FTS Probability of those observations is also equal to one minus the probability of observing their 'false positives'.

For instance, suppose that the ordinal rank on 3rd Aug 2018 is 0.7 which is greater than the ordinal rank of threshold 0.62 for that day. And of the 300 days in the sample that have an ordinal rank above 0.7, near 23% fail to satisfy all three weak features of FTS, then the 'false positive' probability of 3rd Aug 2018 is 23% and its FTS probability is given by $(1 - 23\%) = 67\%$.

The choice of threshold in this setup is determined by the lowest ordinal rank that satisfies the 3 weak features, however the third of those weak features involves subjectivity in the choice of $k > 1$, i.e. the ratio of short to long term equity volatility should be above k on FTS days, where k could be any number greater than 1 that could be chosen by the researcher. Based on different choices of k, Figure A.3 on page 259 plots the FTS incidences and their probabilities.

A.2.4 FTS and FTR days

The results of the threshold approach are presented for various levels of threshold κ in fig. A.2, and the results from the ordinal approach are presented in fig. A.3. In both these approaches as we increase the threshold criteria (κ or $Prob_{ths}$ or k), there is a decrease in FTS days. Therefore to obtain the desired level of FTS likelihood, a relevant threshold can be calibrated.

The daily FTS series is aggregated to generate a monthly and quarterly series. The charts in Figure A.5 describe this aggregation for the Ordinal index approach for $k = 1.25$. While it is straightforward to interpret the frequency charts in fig. A.5, as the total number of FTS events that occurred in a given time frame, interpreting the quarterly aggregate of FTS probability as given in fig. A.5 can be a bit confusing, but it is relevant to get the full incidence and impact of FTS events that may be missing from an aggregated series that is only based on frequency. In a frequency-based aggregation, a quarter with 4 FTS events is weighted higher than a quarter with 3 FTS events. However, it is possible that the 3 FTS events were all very extreme occurrences while the 4 FTS events were mild. Therefore an aggregate measure that sums up the extremity of FTS events is required. One approach to achieve this is by aggregating the sums of FTS probability in each quarter and dividing it by 62.5, which is the number of trading days in a quarter. This gives us a measure of the likelihood or probability of FTS days in that quarter.

Based on this aggregation we witness from both the heatmaps (in fig. 1.6 on page 17 and the bubbles chart in fig. A.5 on page 261) that there is a higher likelihood of FTS cases in periods surrounding the events of global market distress, such as the inflation of 70s, the Russia, Tequilla and Asian crisis, the Dot-com bubble and the Great Recession. There is also an upsurge in FTS events and probability in the last 5 years.

Similarly, we can run a complimentary exercise using the same methods, i.e. the ordinal index and the threshold approach to generate a Flight to Risk (FTR) variable. This can be achieved by considering periods when the return on equity index is positive and return on the bond index is negative, and this difference is above a threshold when using the threshold method, or if using the Ordinal method, the ordinal ranking of such days lies is in tails of a fitted cumulative probability distribution. A comparison of FTS and FTR days using the ordinal index method with threshold k of 1, as shown in Figure A.4 on page 260, demonstrates that incidence of both FTS and FTR events has increased since the late 1990s and that FTS events have bigger and more frequent spikes than FTR.

A.3 Tables

Table A.1: FEVD of Investment with Identification Strategy 1

Horizon	TFP	FTS	MP	SRatio	Residual
1	12.97	17.51	13.08	7.67	9.26
	(1.46, 44.78)	(2.47, 49.93)	(1.45, 45.92)	(0.89, 32.67)	(0.81, 40.68)
4	9.39	38.13	9.62	11.46	8.00
	(1.77, 34.73)	(8.38, 64.47)	(1.89, 26.36)	(1.73, 35.46)	(2.00, 30.86)
8	7.54	37.35	8.69	13.96	8.92
	(1.22, 30.02)	(6.29, 64.09)	(1.75, 22.89)	(2.35, 38.12)	(2.16, 30.59)
12	7.75	40.45	8.39	11.80	8.21
	(1.69, 31.57)	(8.90, 66.37)	(1.61, 20.80)	(2.15, 35.60)	(2.02, 28.34)
16	8.64	42.46	7.76	11.15	7.93
	(2.15, 32.39)	(10.77, 66.06)	(1.94, 20.29)	(2.17, 34.50)	(1.95, 26.81)
20	9.25	42.77	7.63	10.97	7.77
	(2.69, 33.52)	(11.45, 65.29)	(2.16, 20.13)	(2.44, 34.26)	(2.27, 26.13)
24	9.82	42.53	7.71	11.00	7.91
	(2.93, 33.87)	(11.86, 64.32)	(2.49, 20.16)	(2.60, 33.54)	(2.34, 25.81)
28	10.17	42.17	8.04	11.03	8.01
	(3.17, 33.64)	(12.16, 63.72)	(2.75, 20.16)	(2.80, 32.99)	(2.47, 25.58)
30	10.22	41.99	8.16	11.05	8.11
	(3.37, 33.64)	(12.27, 63.50)	(2.94, 20.21)	(2.82, 32.74)	(2.48, 25.50)

Notes: FEVD of Private Investment from all shocks in the benchmark model identified with Identification Strategy 1. Median level and 68% confidence bands are reported.

Table A.2: FEVD of Investment in Great Moderation period

Horizon	TFP	FTS	MP	SRatio	Residual
1	10.56	10.62	19.96	11.90	11.44
	(0.72, 42.80)	(1.24, 30.78)	(3.09, 66.15)	(0.65, 38.26)	(0.70, 41.78)
4	9.18	20.23	14.90	13.40	12.41
	(2.05, 34.70)	(5.44, 48.47)	(3.43, 41.11)	(2.56, 44.71)	(1.94, 34.98)
8	7.86	24.51	14.38	12.04	10.33
	(1.29, 35.73)	(5.46, 57.31)	(3.15, 37.26)	(1.92, 42.57)	(1.23, 32.53)
12	7.84	24.18	14.51	11.84	10.62
	(1.09, 35.76)	(5.14, 56.29)	(3.30, 38.31)	(1.82, 42.67)	(1.47, 32.39)
16	8.27	24.18	15.97	12.27	10.73
	(1.51, 35.47)	(4.78, 54.72)	(3.85, 38.15)	(2.13, 41.80)	(1.86, 33.13)
20	8.62	23.51	16.84	12.14	10.67
	(2.20, 36.61)	(4.57, 53.21)	(4.65, 39.58)	(2.94, 40.57)	(2.55, 31.64)
24	9.62	22.70	17.59	12.77	10.81
	(3.08, 35.45)	(4.60, 51.32)	(5.60, 39.82)	(3.57, 39.84)	(3.34, 31.11)
28	10.63	21.70	17.62	12.52	11.84
	(3.84, 34.26)	(4.53, 49.54)	(6.58, 39.68)	(3.97, 39.09)	(4.08, 31.35)
30	10.97	21.37	17.74	12.77	11.81
	(4.11, 33.72)	(4.53, 48.56)	(6.84, 39.41)	(4.18, 39.17)	(4.44, 31.55)

Notes: FEVD of Private Investment from all shocks in the benchmark model in Great Moderation period data (1983-2007) identified with Identification Strategy 1. Median level and 68% confidence bands are reported.

Table A.3: FEVD of Investment in Pre-Great Moderation period

Horizon	TFP	FTS	MP	SRatio	Residual
1	8.78	7.45	14.98	23.12	16.82
	(2.14, 23.52)	(0.61, 25.97)	(1.02, 48.08)	(2.73, 57.36)	(1.22, 47.26)
4	5.50	9.67	8.56	32.62	21.52
	(3.11, 15.10)	(1.82, 34.20)	(1.47, 29.15)	(5.28, 57.94)	(4.79, 56.39)
8	18.76	6.32	5.86	29.73	25.60
	(12.32, 27.66)	(1.32, 21.99)	(2.35, 23.02)	(9.04, 50.18)	(8.14, 49.35)
12	27.13	5.68	5.69	26.08	24.57
	(15.84, 36.87)	(1.21, 19.14)	(2.44, 20.88)	(8.40, 45.55)	(8.57, 43.53)
16	27.22	5.80	6.21	25.54	25.10
	(15.25, 37.94)	(1.53, 18.98)	(3.15, 20.05)	(8.36, 45.12)	(8.60, 43.74)
20	27.25	5.79	6.33	25.05	26.26
	(14.67, 38.51)	(1.67, 18.65)	(3.46, 19.55)	(8.39, 45.11)	(8.89, 44.67)
24	27.96	5.69	6.21	24.54	26.61
	(14.41, 39.69)	(1.64, 18.12)	(3.46, 18.76)	(8.32, 44.81)	(8.87, 46.03)
28	28.52	5.57	6.24	24.28	26.50
	(14.32, 40.98)	(1.59, 17.63)	(3.50, 17.95)	(8.35, 44.32)	(9.14, 45.79)
30	28.65	5.56	6.32	24.16	26.57
	(14.27, 41.36)	(1.57, 17.42)	(3.48, 17.63)	(8.38, 44.55)	(9.29, 46.45)

Notes: FEVD of Private Investment from all shocks in the benchmark model in pre Great Moderation period data (1954-1978) identified with Identification Strategy 1. Median level and 68% confidence bands are reported.

Table A.4: FEVD of Investment with Identification Strategy 2

Horizon	TFP	FTS	MP	SRatio	Residual
1	7.31	15.18	23.61	12.34	4.19
	(0.34, 32.83)	(2.36, 51.54)	(2.62, 72.04)	(0.93, 38.08)	(0.52, 17.76)
4	23.55	16.18	19.95	11.15	5.66
	(7.08, 47.50)	(2.99, 43.74)	(3.63, 50.98)	(2.71, 33.19)	(1.65, 21.85)
8	25.32	11.75	21.49	11.89	6.74
	(5.24, 48.81)	(1.70, 36.50)	(3.10, 47.29)	(1.99, 35.21)	(1.19, 26.75)
12	27.06	12.95	18.40	11.63	7.25
	(5.91, 51.14)	(2.90, 37.11)	(2.66, 43.81)	(1.70, 34.47)	(1.01, 26.05)
16	27.83	13.59	17.88	11.92	7.68
	(6.77, 50.80)	(3.99, 37.90)	(2.81, 40.85)	(1.75, 33.98)	(1.02, 25.92)
20	28.17	14.21	18.46	11.81	7.71
	(6.76, 49.62)	(4.90, 37.91)	(3.31, 39.49)	(2.00, 33.51)	(1.11, 25.81)
24	28.21	14.39	18.66	11.85	7.87
	(7.02, 48.75)	(5.64, 38.18)	(3.72, 38.71)	(2.33, 33.02)	(1.23, 25.52)
28	27.92	14.60	18.54	11.85	8.00
	(7.19, 48.00)	(6.23, 38.15)	(4.14, 38.14)	(2.65, 32.56)	(1.48, 25.38)
30	27.78	14.64	18.48	11.81	7.97
	(7.21, 47.64)	(6.48, 38.15)	(4.25, 38.05)	(2.68, 32.34)	(1.59, 25.43)

Notes: FEVD of Private Investment from all shocks in the benchmark model identified with Identification Strategy 2. Median level and 68% confidence bands are reported.

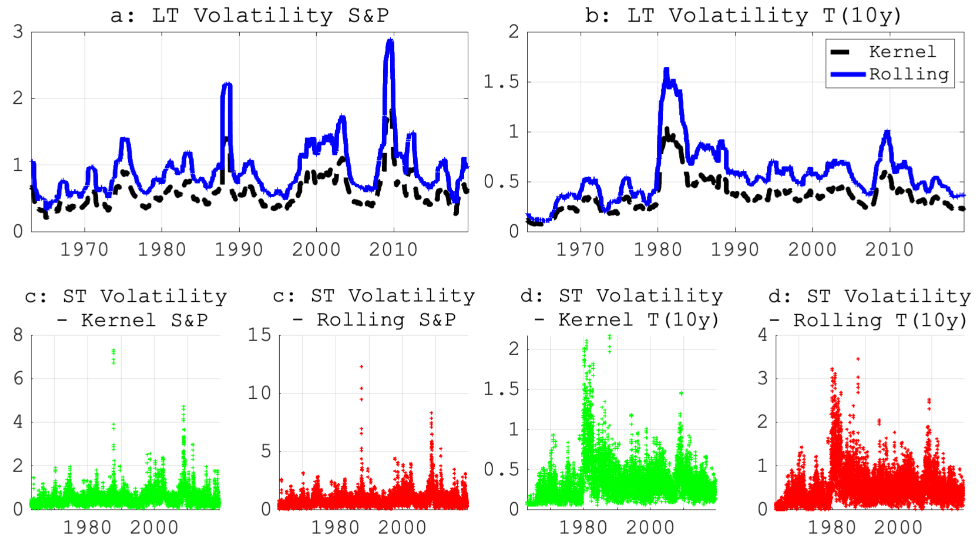
Table A.5: FEVD of Investment with Identification Strategy 3

Horizon	TFP	FTS	MP	SRatio	Residual
1	4.05	14.09	39.70	6.25	2.61
	(0.13, 19.80)	(1.71, 58.41)	(8.94, 78.99)	(0.66, 32.25)	(0.45, 11.70)
4	28.01	24.99	17.91	9.59	5.12
	(7.74, 48.65)	(4.50, 50.14)	(4.23, 37.20)	(2.67, 27.08)	(1.11, 19.03)
8	33.34	24.37	17.36	10.51	5.62
	(7.39, 51.82)	(4.35, 43.88)	(3.80, 31.07)	(1.97, 29.27)	(1.18, 23.71)
12	34.61	25.82	14.18	11.46	5.88
	(8.40, 55.76)	(5.25, 43.42)	(3.37, 26.43)	(2.10, 30.79)	(1.22, 24.93)
16	34.97	26.54	12.75	11.99	5.89
	(9.14, 56.37)	(6.06, 43.17)	(3.40, 23.90)	(2.26, 31.37)	(1.16, 25.32)
20	34.64	27.22	12.39	12.20	6.01
	(9.26, 56.75)	(6.59, 42.86)	(3.40, 23.06)	(2.23, 31.53)	(1.33, 25.39)
24	34.05	27.37	12.26	12.23	6.15
	(9.16, 56.36)	(7.13, 42.74)	(3.36, 22.74)	(2.35, 31.60)	(1.39, 24.91)
28	33.74	27.55	12.10	12.53	6.37
	(9.30, 56.15)	(7.62, 42.60)	(3.33, 22.45)	(2.67, 31.63)	(1.66, 24.61)
30	33.70	27.35	12.01	12.58	6.45
	(9.50, 55.90)	(7.90, 42.57)	(3.31, 22.28)	(2.83, 31.61)	(1.72, 24.47)

Notes: FEVD of Private Investment from all shocks in the benchmark model identified with Identification Strategy 3. Median level and 68% confidence bands are reported.

A.4 Figures

Figure A.1: Volatility from Kernel and Rolling days method



Notes: Long term (LT) and Short term (ST) conditional volatility, variance, correlation and covariance for daily returns on S&P 500 and US 10 year Treasury bonds. The long run variances are calculated by using a backward looking kernel of 250 days, whereas short run variances are based on backward looking kernel of 5 days. The rolling days volatility calculation puts equal weight on past 250 days for Long run and on past 5 days for Short run calculation. See Appendix A.2.1 for a detailed methodology.

Figure A.2: Threshold approach to FTS

Notes: Cumulative probability density of observation (in circles) and time-varying threshold (solid line) on days that are considered as FTS (i.e. when $Pr_{obs} < Pr_{ths}$). The threshold is set below 'kappa' times the conditional volatility of asset returns $r_t = (r_{e,t}, -r_{b,t})$. Parentheses in chart titles signify the percentage of total days from 1965 to 2019 that count as FTS for a particular choice of threshold criteria (or kappa). Y-axis: the probability of each FTS event, X-axis: date (in years).

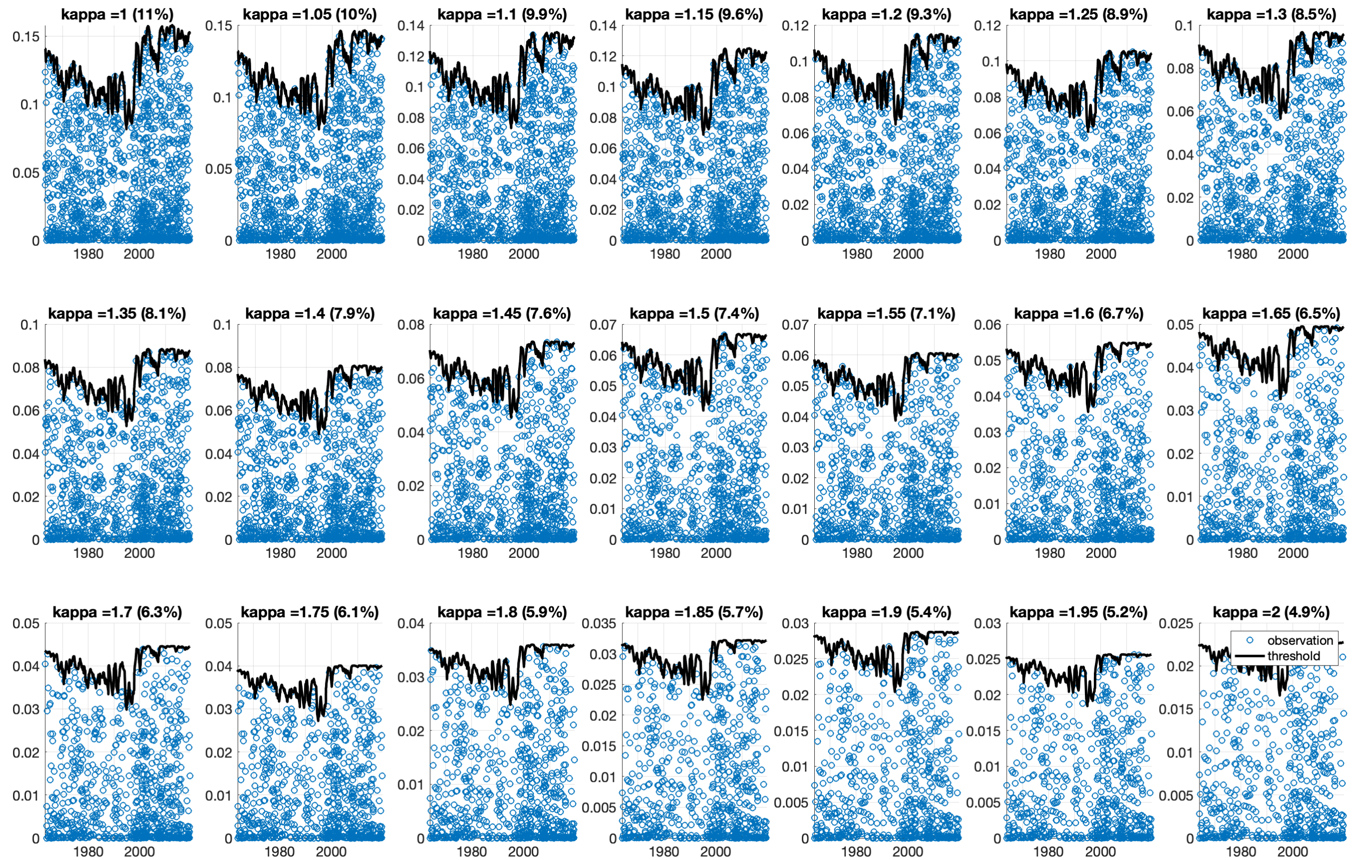


Figure A.3: Ordinal index approach to FTS

Notes: FTS probability of observations whose Ordinal rank is above the rank of chosen threshold k. Varying levels of threshold are set by choosing different k, or the 3rd weak feature of FTS, which requires the short to long run ratio of stock return volatility on FTS day to be greater than k, s.t $k > 1$. Y-axis: the probability of each FTS event, X-axis: date (in years).

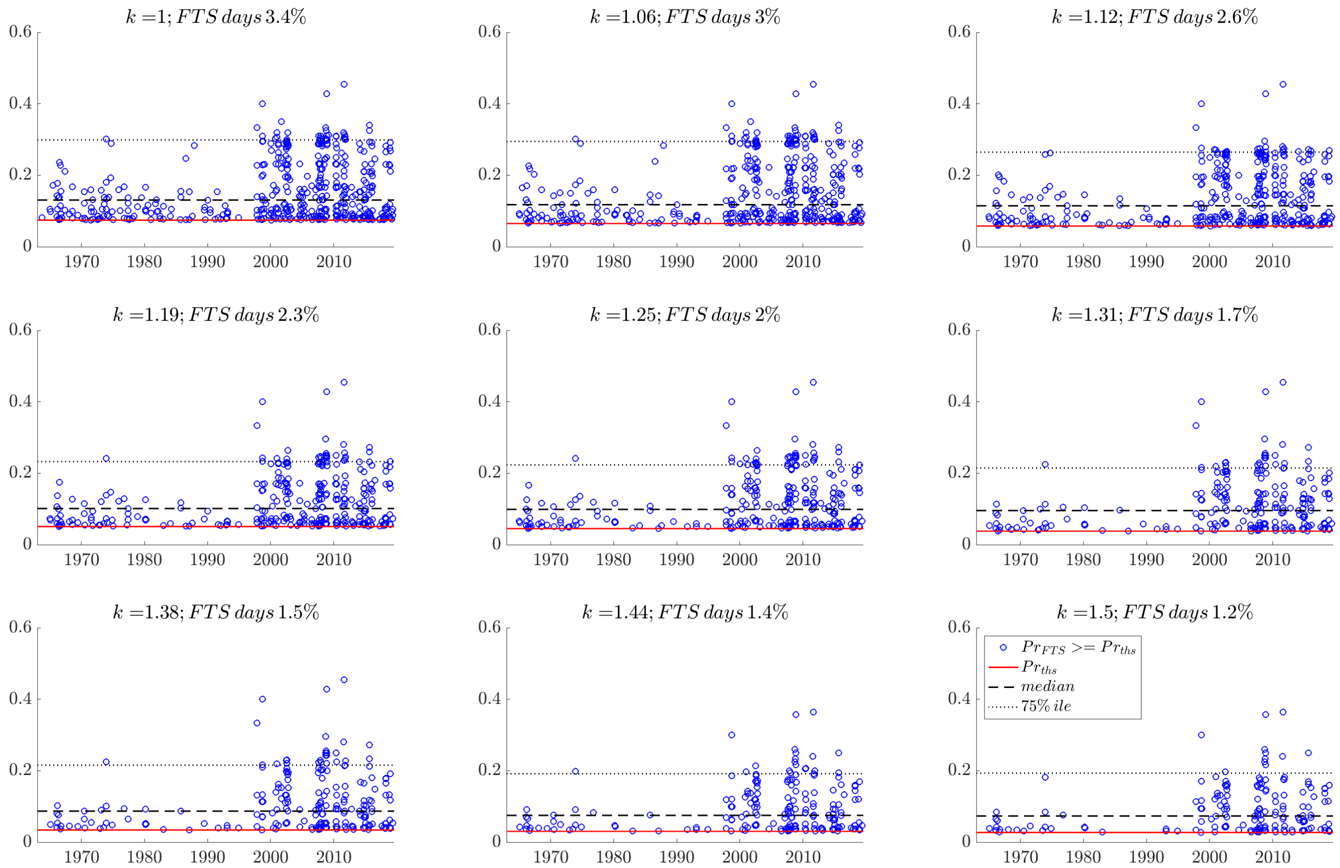
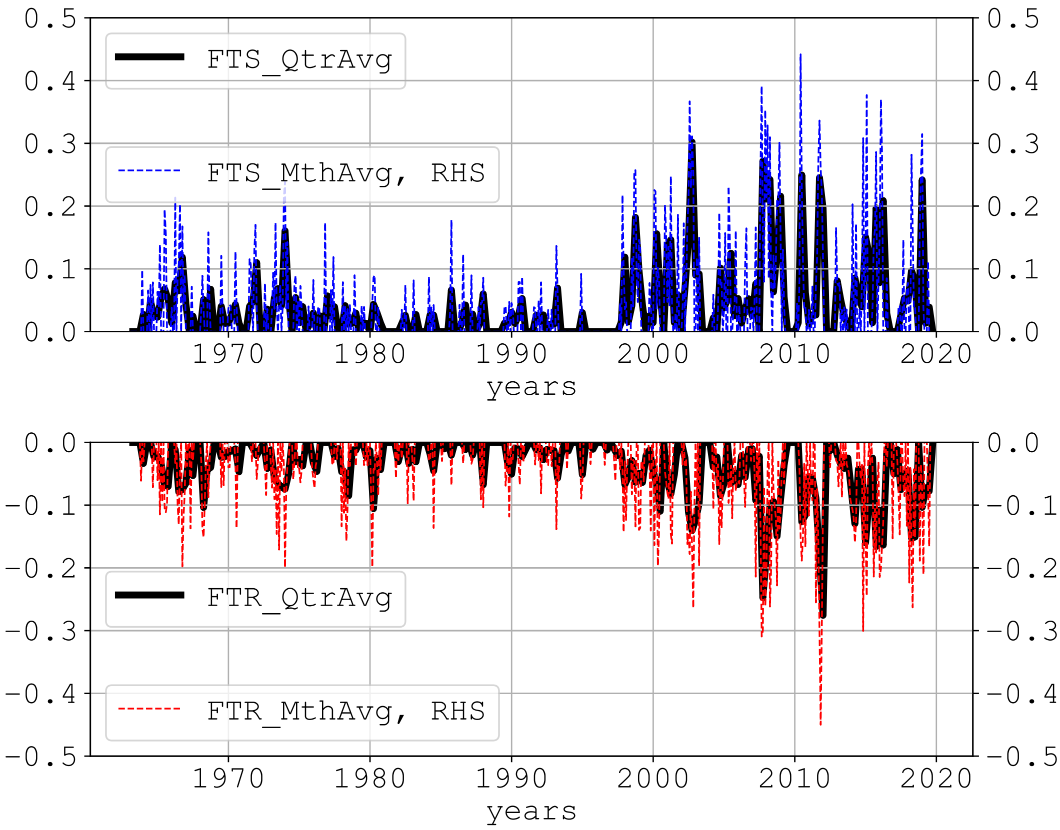
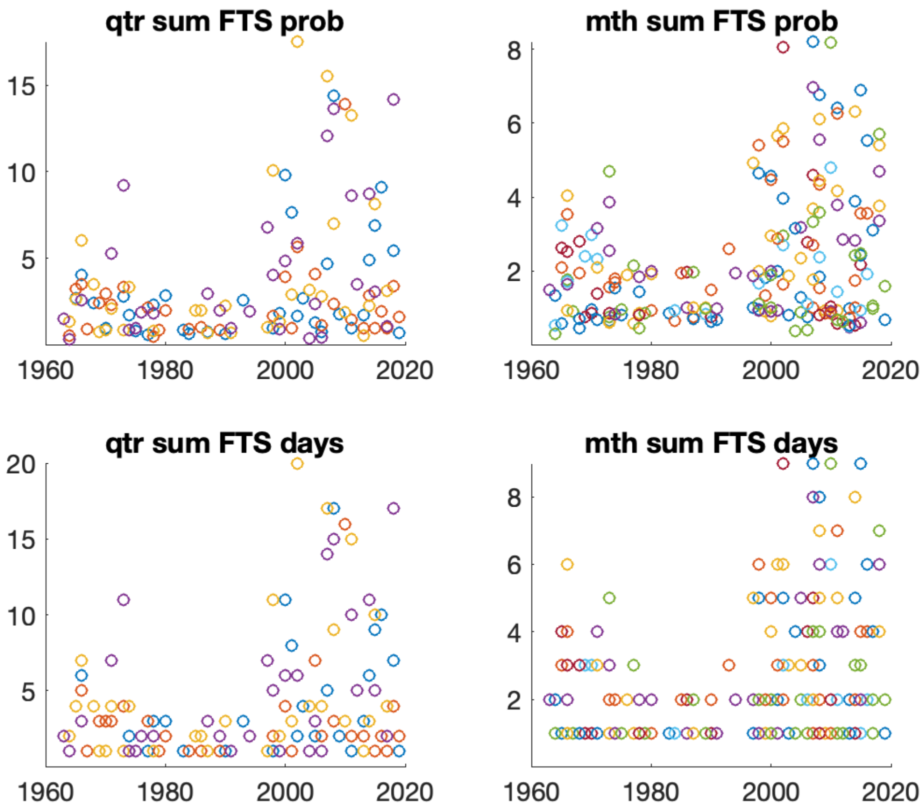


Figure A.4: FTS and FTR shocks from Ordinal index method



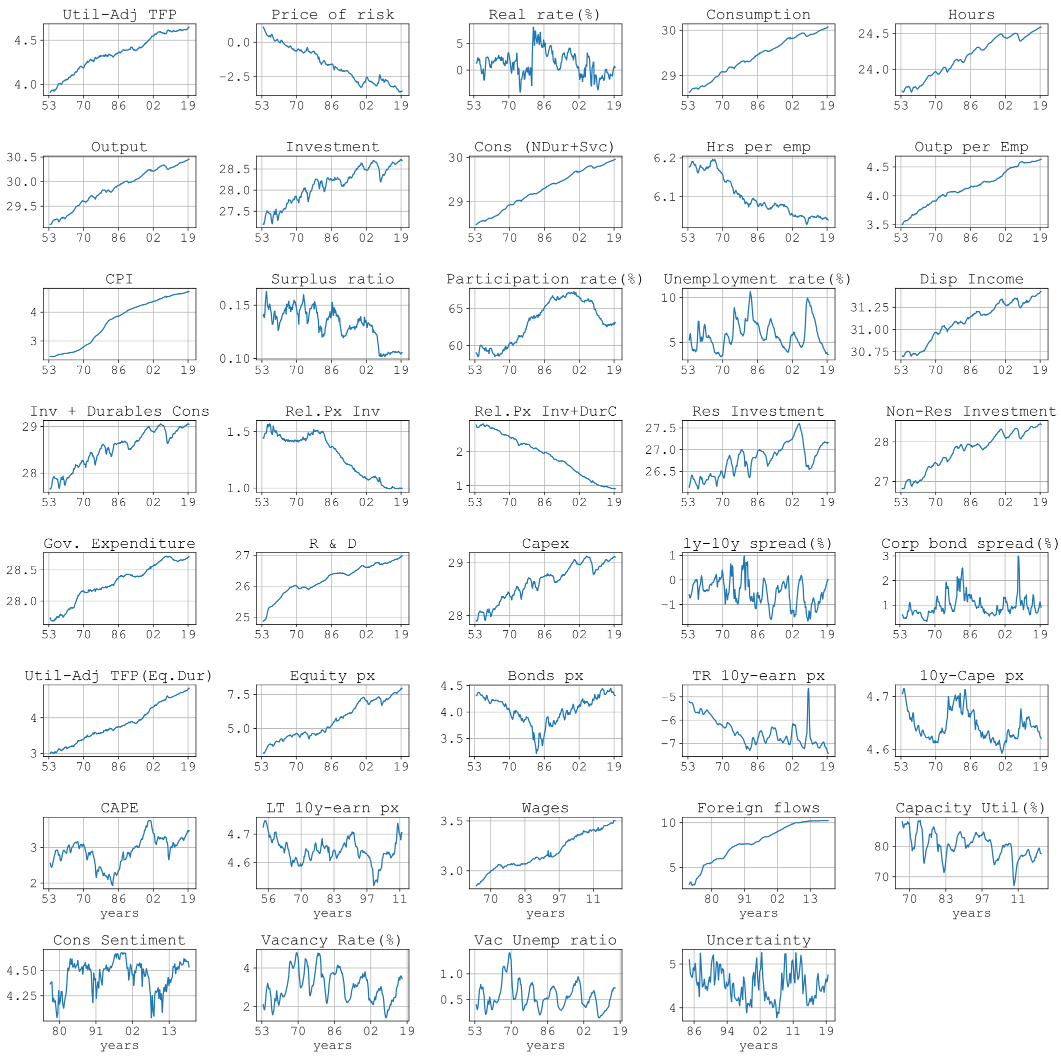
Notes: Quarterly and monthly average of FTS and FTR probability calculated from Ordinal index method discussed in Appendix A.2.3 with k equal to 1.

Figure A.5: Frequency and Likelihood of Flight to Safety days II



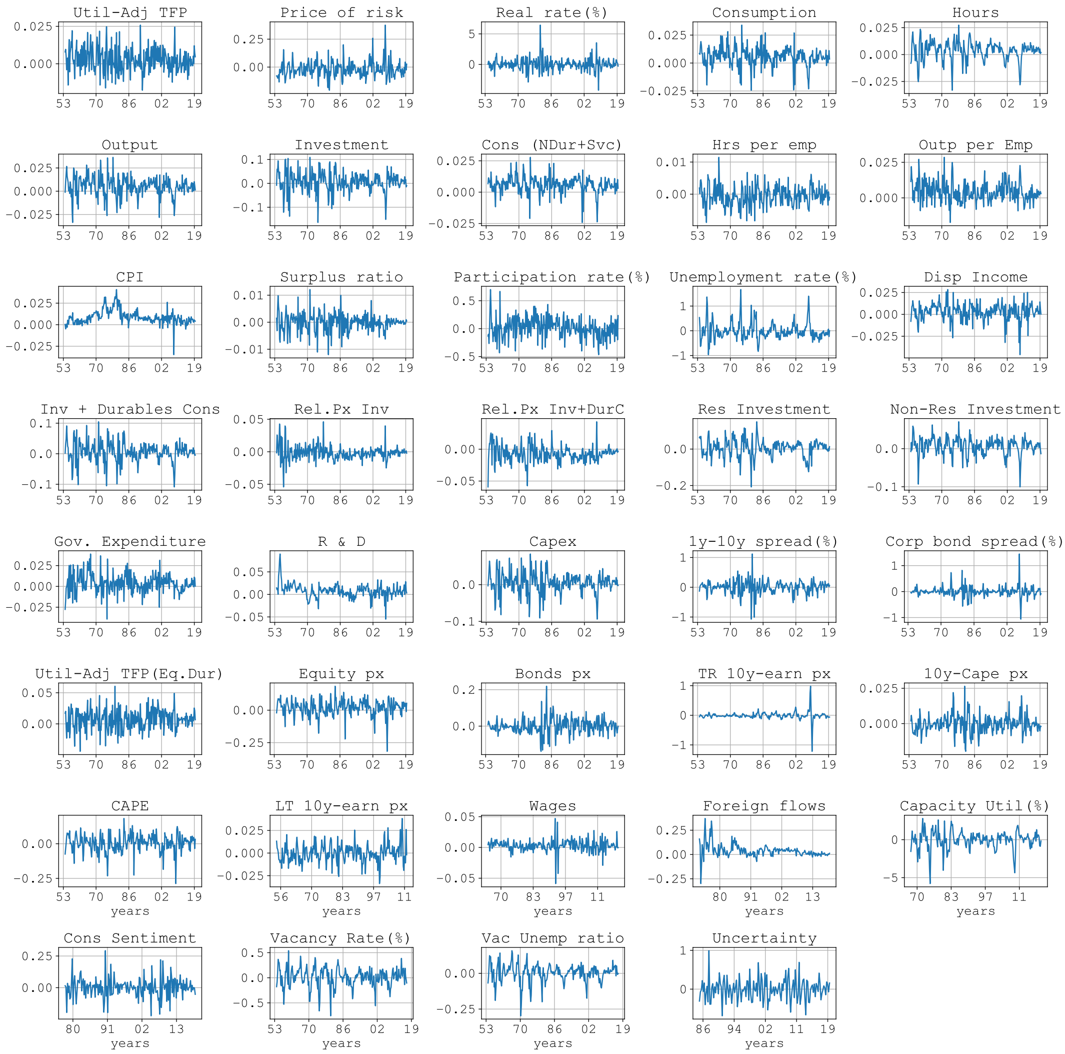
Notes: Aggregated Likelihood (prob) & frequency (days) of Flight to Safety (FTS). Aggregated by month (mth) & quarter (qtr) from 1963 to 2019. FTS days are calculated by Ordinal index approach (see Appendix A.2.3), for $k = 1.25$. Aggregated likelihood, or strength of FTS, for each month or quarter in the studied period is calculated by adding the cdf probability value for each FTS day during that period.

Figure A.6: Time-series plots of US macroeconomic variables



Notes : Cons(NDur+Svc) is the Non-surables and Services consumption, Rel.Px Inv is relative price of Investment in terms of the price of Consumption. Rel.Px Inv+Dur is relative price of Investment and Durables consumption in terms of price of Services+Non durables consumption. px stands for price, Cons for consumption, Inv for investment, Res for residential, emp for employee, vac for vacancy, TR for total return, earn for earnings, Cape for Cyclical adjusted price to earnings. Except for ratios and series labelled in %, all other time series are in logs. X-axis: years in last 2 digits (YY) format.

Figure A.7: First-differenced plots of US macroeconomic variables



Notes : Cons(NDur+Svc) is the Non-surables and Services consumption, Rel.Px Inv is relative price of Investment in terms of the price of Consumption. Rel.Px Inv+Dur is relative price of Investment and Durables consumption in terms of price of Services+Non durables consumption. px stands for price, Cons for consumption, Inv for investment, Res for residential, emp for employee, vac for vacancy, TR for total return, earn for earnings, Cape for Cyclical adjusted price to earnings. Except for ratios and series labelled in %, all other time series are in logs. X-axis: years in last 2 digits (YY) format.

B. Appendix for Chapter 2

B.1 Equity Premium

Defining auxiliary functions for next period variables

$$\Omega^{MN} = \frac{\lambda_{t+1}^r}{\pi_{t+1}} \tag{B.1}$$
$$\Omega^{MR} = \lambda_{t+1}^r \tag{B.2}$$

We can update the asset pricing Euler equations (2.18, 2.19 and 2.20) respectively as

$$\beta \frac{E_t\left[\Omega^{MN}\right]}{\lambda_t^r} R_{t,t+1}^n = 1 \tag{B.3}$$
$$\beta \frac{E_t\left[\Omega^{MR}\right]}{\lambda_t^r} R_{t,t+1}^r = 1 \tag{B.4}$$
$$\beta \frac{E_t\left[\Omega^{MR}(V_{t+1}^{eq} + d_{t+1})\right]}{\lambda_t^r V_t^{eq}} = 1 \tag{B.5}$$

with risk-free returns: nominal $R_{t,t+1}^n \equiv \frac{1}{V_t^n}$ and real $R_{t,t+1}^r \equiv \frac{1}{V_t^r}$. the Return on equity

$$E_t[R_{t,t+1}^{eq}] \equiv \frac{E_t[V_{t+1}^{eq} + d_{t+1}]}{V_t^{eq}} \tag{B.6}$$

Comparing the above equations for $R_{t,t+1}^r$ and $R_{t,t+1}^{eq}$, The Value of equity

$$V_t^{eq} = \frac{E_t\left[\Omega^{MR}(V_{t+1}^{eq} + d_{t+1})\right]}{R_{t,t+1}^r E_t[\Omega^{MR}]} \tag{B.7}$$

where Expectation of product on the RHS is

$$\begin{aligned} E_t\left[\Omega^{MR}(V^{eq}_{t+1}+d_{t+1})\right] &= E_t\Omega^{MR}E_tV^{eq}_{t+1}+E_t\Omega^{MR}E_td_{t+1} \\ &\quad + cov(\Omega^{MR},(V^{eq}_{t+1}+d_{t+1})) \qquad \text{(B.8)} \\ cov(\Omega^{MR},(V^{eq}_{t+1}+d_{t+1})) &= E_t\left[\left(\Omega^{MR}-E_t\Omega^{MR}\right)(V^{eq}_{t+1}-E_tV^{eq}_{t+1})\right] \\ &\quad +E_t\left[\left(\Omega^{MR}-E_t\Omega^{MR}\right)(d_{t+1}-E_td_{t+1})\right] \qquad \text{(B.9)} \end{aligned}$$

so we can say

$$V^{eq}_t = \frac{E_t[V^{eq}_{t+1}+d_{t+1}]}{R^r_{t,t+1}} + \frac{cov(\Omega^{MR},(V^{eq}_{t+1}+d_{t+1}))}{R^r_{t,t+1}E_t[\Omega^{MR}]} \qquad \text{(B.10)}$$

similarly we calculate the expected return on equity

$$E_t[R^{eq}_{t,t+1}] = R^r_{t,t+1} - \frac{cov(\Omega^{MR},(V^{eq}_{t+1}+d_{t+1}))}{V^{eq}_tE_t[\Omega^{MR}]} \qquad \text{(B.11)}$$

Equity Premium is the difference

$$ln(E_t[R^{eq}_{t,t+1}]) - ln(R^r_{t,t+1})$$

and dividends from (2.13) are

$$d_t = C_t - wN_t - \tau_pY_t \qquad \text{(B.12)}$$

B.2 Efficient competitive equilibrium

In steady state the marginal cost $\overline{\varphi}$ is given by

$$0 = (1-\sigma)(1-\overline{\tau_p}) + \sigma\overline{\varphi} - \psi(\overline{\pi}-1)\overline{\pi} + \psi\beta(\overline{\pi}-1)\overline{\pi} \tag{B.13}$$

$$\overline{\varphi} = \left(\frac{\sigma-1}{\sigma}\right)(1-\overline{\tau_p}) + \frac{\psi}{\sigma}(\overline{\pi}-1)\overline{\pi} - \frac{\psi}{\sigma}\beta(\overline{\pi}-1)\overline{\pi} \tag{B.14}$$

where $\overline{\pi}$ is the desired rate of inflation. Firm's marginal cost is equal to the wage rate which in the steady state using consumption-leisure MRS (2.25) and resource constraint is given as

$$\overline{\varphi} = \overline{w} = \overline{Y}^{\eta}\overline{C}^{\rho}\left(1-\frac{h(1-\phi)}{1-\phi}\right)^{\rho}\left(1-\frac{h(1-\phi)\omega}{1-\beta\phi}\right)^{-1} \tag{B.15}$$

$$\overline{\varphi} = \overline{Y}^{\eta+\rho}\left[1-\frac{\psi}{2}(\overline{\pi}-1)^2\right]^{\rho}\left(1-\frac{h(1-\phi)}{1-\phi}\right)^{\rho}\left(1-\frac{h(1-\phi)\omega}{1-\beta\phi}\right)^{-1} \tag{B.16}$$

$$\overline{Y}^{\eta+\rho} = \overline{\varphi}\left[1-\frac{\psi}{2}(\overline{\pi}-1)^2\right]^{-\rho}\left(1-\frac{h(1-\phi)}{1-\phi}\right)^{-\rho}\left(1-\frac{h(1-\phi)\omega}{1-\beta\phi}\right) \tag{B.17}$$

As $\overline{\pi} = 1$, i.e. zero percent inflation in the efficient result, the steady state marginal cost is

$$\overline{\varphi} = \left(\frac{\sigma-1}{\sigma}\right)(1-\overline{\tau_p}) \tag{B.18}$$

Using $k = \frac{h(1-\phi)}{1-\beta\phi}$, the first-best outcome in comparison to decentralized steady state allocation is

$$\frac{Y^*}{\overline{Y}} = \left(\frac{1-k}{1-\omega k}\right)^{\frac{1}{\eta+\rho}}\left(\frac{\sigma}{\sigma-1}\left(\frac{1}{1-\overline{\tau_p}}\right)\right)^{\frac{1}{\eta+\rho}} \tag{B.19}$$

whether the efficient output Y^* (2.70) is $\gtrless$ than steady state competitive equilibrium (with constant taxes) output $\overline{Y}$ depends upon[1]:

$$\left(\frac{\sigma}{\sigma-1}\right)\left(\frac{1}{1-\overline{\tau_p}}\right) \gtrless \left(\frac{1-\omega k}{1-k}\right) \tag{B.20}$$

[1]We are looking for $(\frac{Y^*}{\overline{Y}})^{\eta+\rho} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-\overline{\tau_p}}\right)\frac{1-k}{1-\omega k} \gtrless 1$

B.3 Variance of SDF

The stochastic discount factor in absence of internal habits is,

$$M^{r'}_{t,t+1} = E_t\left[\beta\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right] \tag{B.21}$$

By ignoring the constant β, and using $S_t = \left(\frac{C_t - hX_t}{C_t}\right)$,

$$\begin{aligned} log(M^{r'}_{t+1}) &= log\left(\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{(C_t - hX_t)^{-\rho}}\right) && \text{(B.22)}\\ &= log\left(\frac{(C_{t+1} - hX_{t+1})^{-\rho}}{C^{-\rho}_{t+1}}\frac{C_t^{-\rho}}{(C_t - hX_t)^{-\rho}}\frac{C^{-\rho}_{t+1}}{C_t^{-\rho}}\right) && \text{(B.23)}\\ &= log\left(S^{-\rho}_{t+1}S^{\rho}_t C^{-\rho}_{t+1}C^{\rho}_t\right) && \text{(B.24)}\\ &= -\rho\left(log(C_{t+1}) - log(C_t) + log(S_{t+1}) - log(S_t)\right) && \text{(B.25)} \end{aligned}$$

by using lowercase letters for log notation,

$$m_{t+1} = -\rho\left(c_{t+1} - c_t + s_{t+1} - s_t\right) \tag{B.26}$$

and the variance becomes,

$$\begin{aligned} var_t(m_{t+1}) &= \rho^2 E_t\left[c_{t+1} - E_t c_{t+1} + s_{t+1} - E_t s_{t+1}\right]^2 && \text{(B.27)}\\ &= \rho^2\left(var_t(c_{t+1}) + var_t(s_{t+1}) + 2cov_t(c_{t+1}, s_{t+1})\right) && \text{(B.28)} \end{aligned}$$

where the conditional expectations of all time t variables are eliminated. Approximating the surplus ratio to second order is sufficient to get a usable third order term for the variance and co-variance terms in $var(m_{t+1})$. The surplus ratio,

$$s_{t+1} = log(C_{t+1} - hX_{t+1}) - log(C_{t+1}). \tag{B.29}$$

Second order Taylor expansion around the steady state is,

$$s_{t+1} = log(1-h) + \frac{h}{1-h}\left[\begin{array}{c} \hat{c}_{t+1} - \hat{x}_{t+1} + (1-h)^{-1}\hat{c}_{t+1}\hat{x}_{t+1} \\ -\frac{1}{2}(2-h)(1-h)^{-1}\hat{c}^2_{t+1} - \frac{1}{2}h(1-h)^{-1}\hat{x}^2_{t+1} \end{array}\right] \tag{B.30}$$

where the superscript ˆ over variables describes their deviation from steady state, and in steady state $x = c$. We write, $\hat{x}_{t+1} = \tilde{x}_t$ as habits in period t+1 depend on variables known in period t. By defining the notation $\Psi_0 := \frac{h}{1-h}$, and by removing

$\hat{}$ superscript over variables for ease of notation, we can specify,

$$s_{t+1} = log(1-h) + \Psi_0 \left[c_{t+1} - x_{t+1} + \frac{1}{1-h} c_{t+1} \tilde{x}_t - \frac{2-h}{2(1-h)} c_{t+1}^2 - \frac{h}{2(1-h)} \tilde{x}_t^2 \right] \tag{B.31}$$

any time t variables can be eliminated to obtain the approximation for variance,

$$var_t(s_{t+1}) = \Psi_0^2 var_t \left(c_{t+1} - \frac{2-h}{2(1-h)} c_{t+1}^2 + \frac{1}{1-h} \tilde{x}_t c_{t+1} \right) \tag{B.32}$$

$$var_t(s_{t+1}) = \frac{\Psi_0^2}{(1-h)^2} \left(\begin{array}{c} (1-h)^2 var_t(c_{t+1}) \\ + \frac{(2-h)^2}{4} var_t(c_{t+1}^2) + \tilde{x}_t^2 var_t(c_{t+1}) \\ -(1-h)(2-h) cov_t(c_{t+1}, c_{t+1}^2) \\ +2(1-h)\tilde{x}_t var_t(c_{t+1}) \\ -(2-h)\tilde{x}_t cov_t(c_{t+1}^2, c_{t+1}) \end{array} \right) \tag{B.33}$$

and covariance as,

$$\begin{aligned} cov_t(c_{t+1}, s_{t+1}) &= \Psi_0^2 cov_t \left(c_{t+1}, c_{t+1} - \frac{2-h}{2(1-h)} c_{t+1}^2 + \frac{1}{1-h} \tilde{x}_t c_{t+1} \right) \\ &= \Psi_0^2 \left(var_t(c_{t+1}) - \frac{2-h}{2(1-h)} cov_t(c_{t+1}, c_{t+1}^2) + \frac{\tilde{x}_t}{(1-h)} var_t(c_{t+1}) \right). \end{aligned} \tag{B.34}$$

We need the expressions for $var_t(c_{t+1})$ and $cov_t(c_{t+1}, c_{t+1}^2)$ to get the conditional variance of m_{t+1},

$$var_t(m_{t+1}) = \rho^2 \left[\begin{array}{c} var_t(c_{t+1}) \\ + \frac{\Psi_0^2}{(1-h)^2} \left(\begin{array}{c} (1-h)^2 var_t(c_{t+1}) + \frac{(2-h)^2}{4} var_t(c_{t+1}^2) \\ + \tilde{x}_t^2 var_t(c_{t+1}) \\ -(1-h)(2-h) cov_t(c_{t+1}, c_{t+1}^2) \\ +2(1-h)\tilde{x}_t var_t(c_{t+1}) \\ -(2-h)\tilde{x}_t cov_t(c_{t+1}^2, c_{t+1}) \end{array} \right) \\ + \Psi_0^2 \left(var_t(c_{t+1}) - \frac{2-h}{2(1-h)} cov_t(c_{t+1}, c_{t+1}^2) + \frac{\tilde{x}_t}{(1-h)} var_t(c_{t+1}) \right) \end{array} \right] \tag{B.35}$$

it can be simplified as,

$$var_t(m_{t+1}) = \Psi_1 var_t(c_{t+1}) + \Psi_2 cov_t(c_{t+1}, c_{t+1}^2) + \Psi_3 var_t(c_{t+1}^2) \tag{B.36}$$

where $\Psi_0 = \frac{h}{1-h}$ and,

$$\Psi_1 = \rho^2 \left[1 + \Psi_0^2 \left(2 + \frac{\tilde{x}_t^2}{(1-h)^2} + 3\frac{\tilde{x}_t}{(1-h)}\right)\right] \tag{B.37}$$

$$\Psi_2 = -\rho^2 \left[\frac{\Psi_0^2(2-h)}{(1-h)} \left(\frac{3}{2} + \frac{\tilde{x}_t}{(1-h)}\right)\right] \tag{B.38}$$

$$\Psi_3 = \rho^2 \left[\frac{\Psi_0^2(2-h)^2}{4(1-h)^2}\right] \tag{B.39}$$

To compute the variance and covariances, let's consider the flexible price allocation of output, from the NKPC for internal habits,

$$0 = (1-\sigma)(1-\tau_{p,t}) + \sigma \xi_{y,t}^{-1} Y_t^{\eta} (C_t - hX_t)^{\rho} \tag{B.40}$$

it can be simplified to get an expression for flexible price allocation $c^* = y^*$,

$$c_t^* = y_t^* = \Upsilon^{-1} \left[\varepsilon_{\tau,t} + \eta \varepsilon_{y,t} + \rho(1-h)^{-1} h \tilde{x}_{t-1}\right]. \tag{B.41}$$

by defining $\Upsilon := \left(\eta + \frac{\rho}{1-h}\right)$, and avoiding the * for ease of notation the variance,

$$var_t(c_{t+1}) = \Upsilon^{-2} var_t \left[\gamma_\tau \varepsilon_{\tau,t} + \epsilon_{\tau,t+1} + \eta \gamma_y \varepsilon_{y,t} + \eta \epsilon_{y,t+1} + \rho \Psi_0 \tilde{x}_t\right] \tag{B.42}$$

$$= \Upsilon^{-2} \left[\sigma_\tau^2 + \eta^2 \sigma_y^2\right]. \tag{B.43}$$

and the covariance,

$$cov_t(c_{t+1}, c_{t+1}^2) = cov_t \left[\begin{array}{c} \Upsilon^{-1} \left(\gamma_\tau \varepsilon_{\tau,t} + \epsilon_{\tau,t+1} + \eta \gamma_y \varepsilon_{y,t} + \eta \epsilon_{y,t+1} + \rho \Psi_0 \tilde{x}_t\right), \\ \Upsilon^{-2} \left(\gamma_\tau \varepsilon_{\tau,t} + \epsilon_{\tau,t+1} + \eta \gamma_y \varepsilon_{y,t} + \eta \epsilon_{y,t+1} + \rho \Psi_0 \tilde{x}_t\right)^2 \end{array}\right] \tag{B.44}$$

As shocks are Gaussian and uncorrelated, we can write

$$cov_t(c_{t+1}, c_{t+1}^2) = 2\Upsilon^{-3} \left(\gamma_\tau \varepsilon_{\tau,t} + \eta \gamma_y \varepsilon_{y,t} + \rho \Psi_0 \tilde{x}_t\right) \left[\sigma_\tau^2 + \eta^2 \sigma_y^2\right]. \tag{B.45}$$

Substituting equation (B.43), (B.45) into (B.36), and ignoring the $var(c_{t+1}^2)$ terms,

$$var_t(m_{t+1}) = \Upsilon^{-2} \left[\Psi_1 + 2\Psi_2 \Upsilon^{-1} \left(\gamma_\tau \varepsilon_{\tau,t} + \eta \gamma_y \varepsilon_{y,t} + \rho \Psi_0 \tilde{x}_t\right)\right] \left[\sigma_\tau^2 + \eta^2 \sigma_y^2\right] \tag{B.46}$$

which is expanded by ignoring higher order deviation terms,

$$var_t(m_{t+1}) = \left[\sigma_\tau^2 + \eta^2 \sigma_y^2\right] \Phi_0 \left[\begin{array}{c} 1 + 2\Psi_0^2 + \frac{3\Psi_0^2}{(1-h)} \tilde{x}_t \\ -\Upsilon^{-1} \left(3\frac{\Psi_0^2(2-h)}{(1-h)} + 2\frac{\Psi_0^2(2-h)}{(1-h)^2} \tilde{x}_t\right) \\ \left(\gamma_\tau \varepsilon_{\tau,t} + \eta \gamma_y \varepsilon_{y,t} + \rho \Psi_0 \tilde{x}_t\right) \end{array}\right] \tag{B.47}$$

by using $\Phi_0 := \rho^2 \Upsilon^{-2} = \left(\frac{(1-h)\rho}{\eta(1-h)+\rho}\right)^2$ and as $\tilde{x}_t = x_{t+1} = (1-\phi)c_{t+1} + \phi x_t$

$$var_t(m_{t+1}) = \left[\sigma_\tau^2 + \eta^2\sigma_y^2\right]\Phi_0 \left[\begin{array}{c} 1 + 2\Psi_0^2 + \frac{3\Psi_0^2}{(1-h)}\left(1 - \frac{\rho h(2-h)}{(\eta(1-h)+\rho)}\right)\tilde{x}_t \\ -\left(\frac{3\Psi_0^2(2-h)}{(\eta(1-h)+\rho)}\right)(\gamma_\tau\varepsilon_{\tau,t} + \eta\gamma_y\varepsilon_{y,t}) \end{array}\right] \tag{B.48}$$

$$var_t(m_{t+1}) = \left[\sigma_\tau^2 + \eta^2\sigma_y^2\right]\Phi_0 \left[\begin{array}{c} 1 + 2\Psi_0^2 + \frac{3\Psi_0^2\phi}{(1-h)}\left(1 - \frac{\rho h(2-h)}{(\eta(1-h)+\rho)}\right)x_t \\ +\frac{3\Psi_0^2(1-\phi)}{(1-h)}\left(1 - \frac{\rho h(2-h)}{(\eta(1-h)+\rho)}\right)c_{t+1} \\ -\left(\frac{3\Psi_0^2(2-h)}{(\eta(1-h)+\rho)}\right)(\gamma_\tau\varepsilon_{\tau,t} + \eta\gamma_y\varepsilon_{y,t}) \end{array}\right]. \tag{B.49}$$

Substituting the value of c^*_{t+1} from Equation (B.41),

$$var_t(m_{t+1}) = \left[\sigma_\tau^2 + \eta^2\sigma_y^2\right]\Phi_0. \left[\begin{array}{c} 1 + 2\Psi_0^2 + \frac{3\Psi_0^2\phi}{(1-h)}\left(1 - \frac{\rho h(2-h)}{(\eta(1-h)+\rho)}\right)x_t \\ +\frac{3\Psi_0^2(1-\phi)}{(\eta(1-h)+\rho)}\left(1 - \frac{\rho h(2-h)}{(\eta(1-h)+\rho)}\right)\left[\varepsilon_{\tau,t} + \eta\varepsilon_{y,t} + \frac{\rho h}{(1-h)}\tilde{x}_{t-1}\right] \\ -\left(\frac{3\Psi_0^2(2-h)}{(\eta(1-h)+\rho)}\right)(\gamma_\tau\varepsilon_{\tau,t} + \eta\gamma_y\varepsilon_{y,t}) \end{array}\right] \tag{B.50}$$

reverting back to the * notation, the variance in expanded form is,

$$var_t(m^*_{t+1}) = \left[\sigma_\tau^2 + \eta^2\sigma_y^2\right]\Phi_0. \left[\begin{array}{c} 1 + 2\Psi_0^2 + 3\Psi_0^2 \\ \left[\begin{array}{c} \left(\frac{(\eta(1-h)+\rho)\phi - \phi\rho h(2-h) + (1-\phi)\rho h}{(1-h)(\eta(1-h)+\rho)}\right)x^*_t \\ -\left(\frac{\rho h(2-h)(1-\phi) - (1-\phi)(\eta(1-h)+\rho) + (2-h)(\eta(1-h)+\rho)\gamma_\tau}{(\eta(1-h)+\rho)^2}\right)\varepsilon_{\tau,t} \\ -\left(\frac{\rho h(2-h)(1-\phi) - (1-\phi)(\eta(1-h)+\rho) + (2-h)(\eta(1-h)+\rho)\eta\gamma_y}{(\eta(1-h)+\rho)^2}\right)\varepsilon_{y,t} \end{array}\right] \end{array}\right] \tag{B.51}$$

By the previously defined $\Psi_0 = \frac{h}{1-h}$, $\Phi_0 = \left[\frac{(1-h)\rho}{\eta(1-h)+\rho}\right]^2$, $\Phi_1 = \eta(1-h) + \rho$, and making use of the notations,

$$\kappa_x = \frac{(\eta+\rho)\phi(1-h) + \rho h(1 - \phi(2-h))}{(1-h)\Phi_1} \tag{B.52}$$

$$\kappa_\tau = \frac{\rho h(2-h)(1-\phi) + \Phi_1\gamma_\tau(1-h) + \Phi_1(\gamma_\tau + \phi - 1)}{\Phi_1^2} \tag{B.53}$$

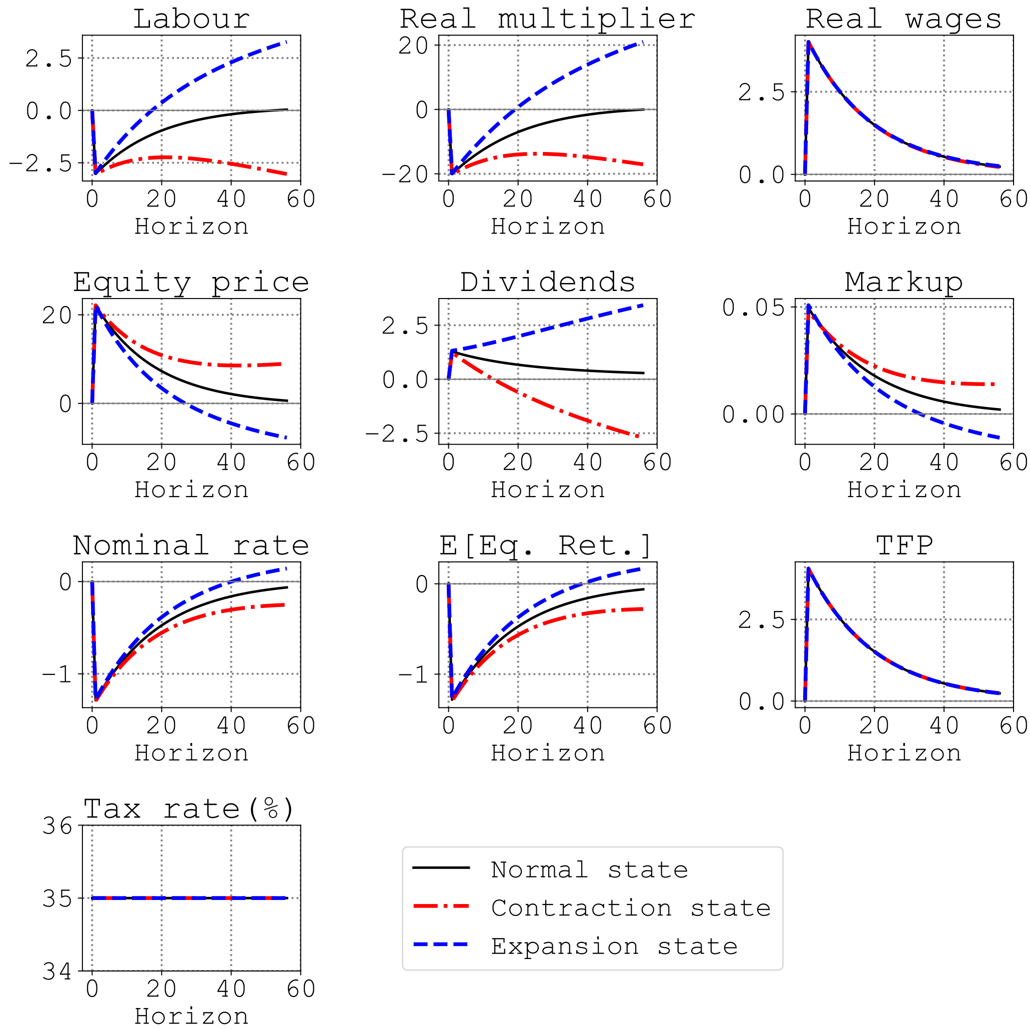
$$\kappa_y = \frac{\eta^{-1}\rho h(2-h)(1-\phi) + \Phi_1\gamma_y(1-h) + \Phi_1(\gamma_y + \phi - 1)}{\Phi_1^2} \tag{B.54}$$

the variance is simplified in following form, which is reported in the main text.

$$var_t(m^*_{t+1}) = \left[\sigma_\tau^2 + \eta^2\sigma_y^2\right]\Phi_0\left[1 + 2\Psi_0^2 + 3\Psi_0^2\left(\kappa_x x^*_t - \kappa_\tau\varepsilon_{\tau,t} - \kappa_y\eta\varepsilon_{y,t}\right)\right]. \tag{B.55}$$

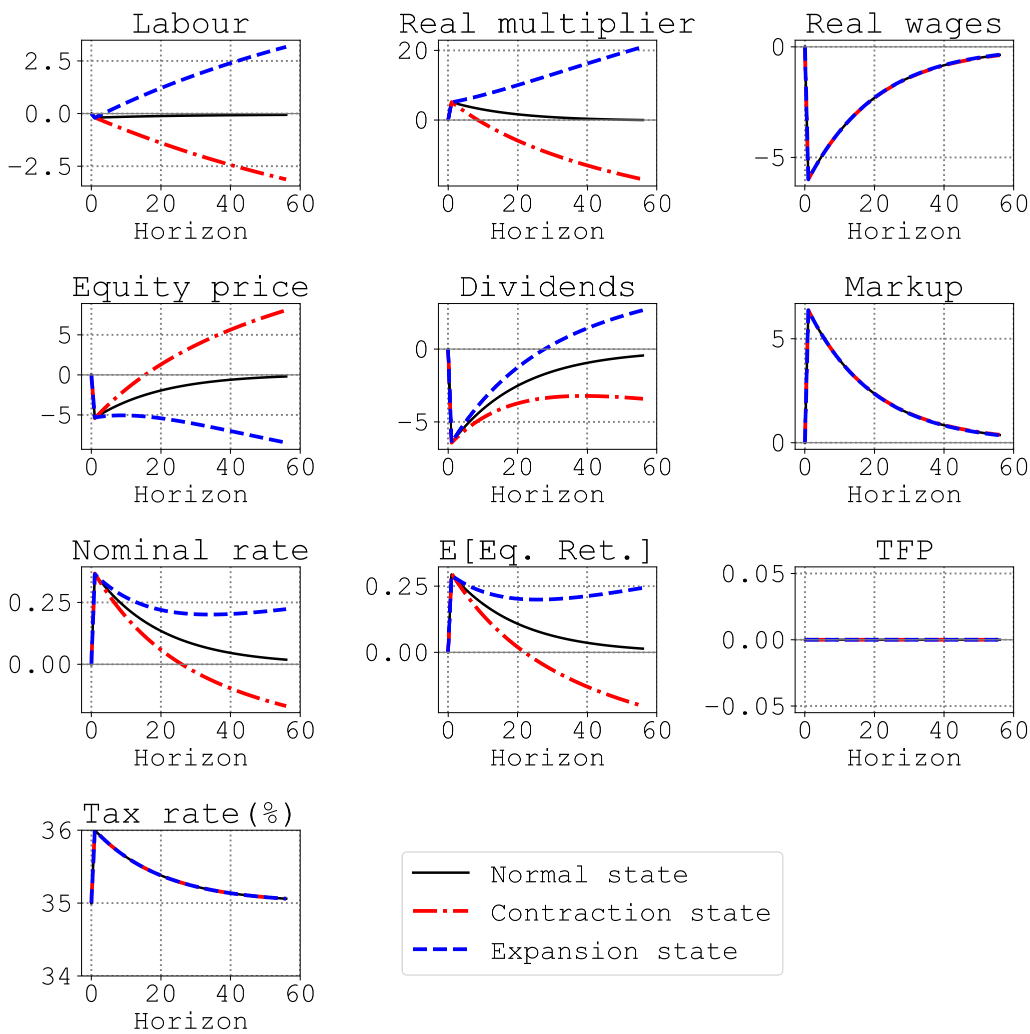
B.4 Figures

Figure B.1: Deviation caused by Productivity Shocks in various states



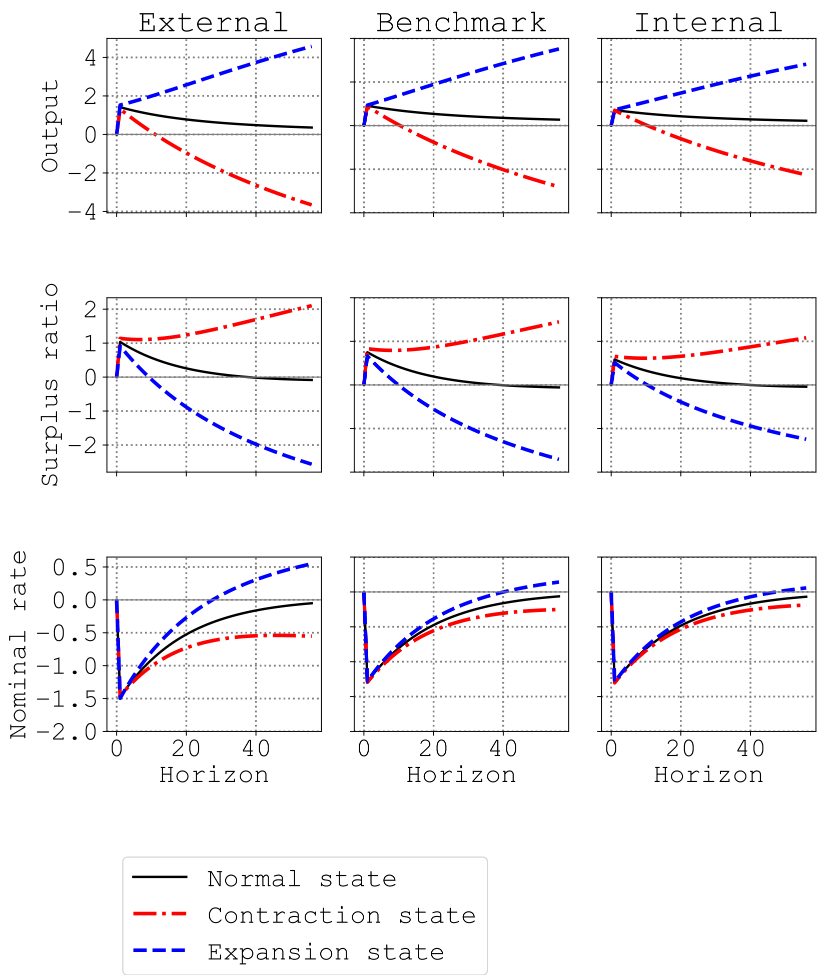
Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly productivity shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in tax rate for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. X-axis: time horizon (in quarters).

Figure B.2: Deviation caused by Cost Shocks in various states



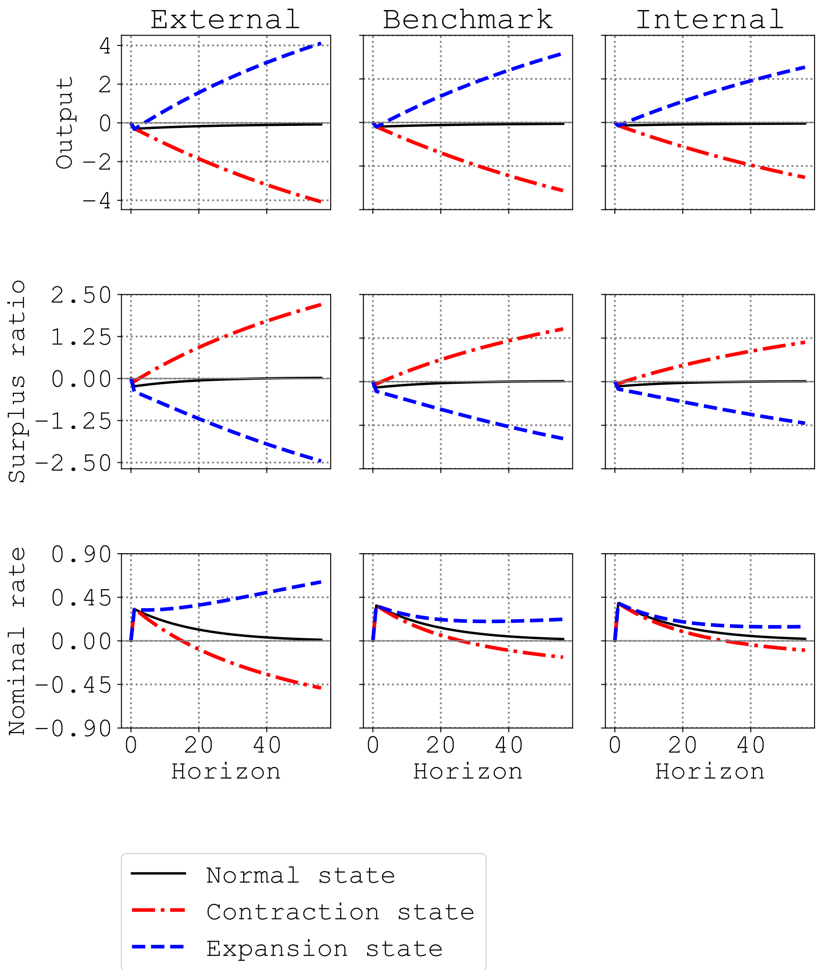
Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly shock to the Tax rate. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in productivity for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. X-axis: time horizon (in quarters).

Figure B.3: Productivity shocks in varying Habits externalities and States II



Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly productivity shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditioned by assuming no change in tax rate for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 1% internal ($\omega = 0.01$) and 99% external. All charts in a row share a common Y-axis which is labelled for charts in first column. All charts in a column share a common X-axis which is labelled for charts in the last row. X-axis: time horizon (in quarters).

Figure B.4: Cost shocks in varying Habits externalities and States II



Notes: Annualised deviation (%) from their pre shock levels, upon impact of a +1% quarterly Tax rate shock. Policy functions depend on three states *viz.* Habits, Productivity and Taxes. All the responses are conditional on assuming no change in Productivity for the entire horizon. The various states are defined as: Normal state - when Habits are at their Stochastic Steady state (SSS) in the shock period, Contraction state - when Habits are above +2s.d. of their SSS, and Expansion state - when Habits are below -2.s.d. below their SSS. The various models are defined as: Benchmark - when Habits are 75% internal ($\omega = 0.75$) and 25% external. Internal - when Habits are 99% internal ($\omega = 0.99$) and 1% external. External - when Habits are 1% internal ($\omega = 0.01$) and 99% external. All charts in a row share a common Y-axis which is labelled for charts in first column. All charts in a column share a common X-axis which is labelled for charts in the last row. X-axis: time horizon (in quarters).

C. Appendix for Chapter 3

C.1 Cost minimisation of final goods firms

We can substitute the relation for $Y_{rt}^{\mathcal{R}}$ and $Y_{lt}^{\mathcal{S}}$ from FOCs into the aggregator (3.29) to get

$$Y_{it} = \left[\int_0^{q_{t-1}} \left(Y_{it} \left(\frac{P_{rt}^{\mathcal{R}}}{\mu_{it}^{'}} \right)^{-\sigma} \right)^{\frac{\sigma-1}{\sigma}} dr + \int_{q_{t-1}}^{1} \left(Y_{it} \left(\frac{P_{lt}^{\mathcal{S}}}{\mu_{it}^{'}} \right)^{-\sigma} \right)^{\frac{\sigma-1}{\sigma}} dl \right]^{\frac{\sigma}{\sigma-1}} \tag{C.1}$$

which further gives the Lagrange multiplier or marginal cost as

$$\mu_{it}^{'} = \left[\int_0^{q_{t-1}} \left(P_{rt}^{\mathcal{R}} \right)^{1-\sigma} dr + \int_{q_{t-1}}^{1} \left(P_{lt}^{\mathcal{S}} \right)^{1-\sigma} dl \right]^{\frac{1}{1-\sigma}}. \tag{C.2}$$

C.2 Profit maximisation of final goods firms

By substituting for demand of final goods (3.28) a firm maximises nominal profits,

$$\mathbb{P}_{it} = \max_{P_{it+s}} E_t \sum_{s=0}^{\infty} M_{t,t+s}^{n} \left[\begin{array}{l} (1-\tau_p) P_{it+s} \left[\frac{P_{it+s}}{P_{t+s}} \right]^{-\sigma} Y_{t+s} - \mu_{it+s}^{'} \left[\frac{P_{it+s}}{P_{t+s}} \right]^{-\sigma} Y_{t+s} \\ -\frac{\psi}{2} \left(\frac{P_{it+s}}{P_{it+s-1}} - 1 \right)^2 P_{t+s} Y_{t+s} \end{array} \right]. \tag{C.3}$$

Firm's first order condition wrt P_{it} is

$$0 = E_t \left\{ \begin{array}{c} M_{t,t+s}^{n} Y_{t+s} \left[\begin{array}{c} (1-\sigma)(1-\tau_p) \left(\frac{P_{it+s}}{P_{t+s}} \right)^{-\sigma} + \sigma \frac{\mu_{it+s}^{'}}{P_{t+s}} \left(\frac{P_{it+s}}{P_{t+s}} \right)^{-\sigma-1} \\ -\psi \left(\frac{P_{it+s}}{P_{it+s-1}} - 1 \right) \frac{P_{t+s}}{P_{it+s-1}} \end{array} \right] \\ +\psi M_{t,t+s+1}^{n} Y_{t+s+1} \left(\frac{P_{it+s+1}}{P_{it+s}} - 1 \right) \frac{P_{it+s+1}}{P_{it+s}} \frac{P_{t+s+1}}{P_{it+s}} \end{array} \right\}.$$

All homogeneous final firms face a similar demand function, set the same prices demand same amount of intermediate goods, and therefore have the same marginal cost and the same markup. So we can derive the NKPC (3.35) in given in main text.

C.2.1 Wage index

Substituting the supply function for both *risky* (3.49) and *safe* (3.41) intermediate firms in the Dixit-Stiglitz aggregator for the final firm (3.29) and by using $\forall i \;\; P_t(i) = P_t$,

$$Y_{it} = \left[q_{t-1} Y_{it}^{\frac{\sigma-1}{\sigma}} \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\mu_t \vartheta_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}} \right)^{1-\sigma} + (1-q_{t-1}) Y_{it}^{\frac{\sigma-1}{\sigma}} \left(\frac{\sigma}{\sigma-1} \frac{w_t}{\mu_t \xi_{\mathcal{S},t}^{\frac{1}{1+\eta}}} \right)^{1-\sigma} \right]^{\frac{\sigma}{\sigma-1}}$$

which gives the real wage index w_t as

$$w_t = \left(\frac{\sigma-1}{\sigma} \right) \mu_t \left[q_{t-1} \vartheta_t^{\sigma-1} \xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1}) \xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} \right]^{\frac{1}{\sigma-1}} . \tag{C.4}$$

C.2.2 Labour index

Each household supplies equal units of labour ($N_t^{\mathcal{R}}$) to all (q_{t-1}) *risky* intermediate firms and equal units of labour ($N_t^{\mathcal{S}}$) to all ($1-q_{t-1}$) *safe* intermediate firms. The Labour market equilibrium ($N_t = q_{t-1} N_t^{\mathcal{R}} + (1-q_{t-1}) N_t^{\mathcal{S}}$) is written[1] in terms of intermediate firm's technology as

$$N_t = q_{t-1} Y_t^{\mathcal{R}} \vartheta_t^{-1} \xi_{\mathcal{S},t}^{\frac{-1}{1+\eta}} + (1-q_{t-1}) Y_t^{\mathcal{S}} \xi_{\mathcal{S},t}^{\frac{-1}{1+\eta}} \tag{C.5}$$

$$N_t = \left[q_{t-1} \vartheta_t^{\sigma-1} \xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1}) \xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} \right] Y_t \mu_t^{\sigma} w_t^{-\sigma} \left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} . \tag{C.6}$$

[1]Parameter η when equal to $\sigma-2$ can eliminate the exponential terms on $\mathcal{Q}_t$ and $\xi_{\mathcal{S},t}$.

C.2.3 Labour income

Total labour income is given by multiplying (C.6) on both sides by real wage (from 3.52) as

$$w_t N_t = \left(\frac{\sigma-1}{\sigma}\right)\mu_t Y_t. \tag{C.7}$$

C.2.4 Production index

Substituting real wage into the labour income (C.7) we can solve for the aggregate output as

$$Y_t = N_t\left[q_{t-1}\vartheta_t^{\sigma-1}\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1})\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}}\right]^{\frac{1}{\sigma-1}}, \tag{C.8}$$

or we can substitute the Labour in consumer utility function with

$$N_t^{1+\eta} = Y_t^{1+\eta}\left[q_{t-1}\vartheta_t^{\sigma-1}\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1})\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}}\right]^{\frac{-(1+\eta)}{\sigma-1}}. \tag{C.9}$$

From labour supply relation (3.16) and wages (3.52) we can transform

$$Y_t = (\lambda_t^r w_t)^{\frac{1}{\eta}}\left[q_{t-1}\vartheta_t^{\sigma-1}\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1})\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}}\right]^{\frac{1}{\sigma-1}}, \tag{C.10}$$

the Output into a function of Real income multiplier λ^r and marginal cost μ as,

$$Y_t = \left(\left(\frac{\sigma-1}{\sigma}\right)\mu_t\lambda_t^r\left[q_{t-1}\vartheta_t^{\sigma-1}\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}} + (1-q_{t-1})\xi_{\mathcal{S},t}^{\frac{\sigma-1}{1+\eta}}\right]^{\frac{1+\eta}{\sigma-1}}\right)^{\frac{1}{\eta}}. \tag{C.11}$$

C.3 Partial derivatives of auxiliary functions

$$\frac{\partial\Omega^{P}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial X_t} = \begin{array}{c}(\pi_{t+1}-1)\,\pi_{t+1}\left(\lambda^r_{t+1}\frac{\partial Y_{t+1}}{\partial X_t}+Y_{t+1}\frac{\partial\lambda^r_{t+1}}{\partial X_t}\right)\\ +\,(2\pi_{t+1}-1)\,\lambda^r_{t+1}Y_{t+1}\frac{\partial\pi_{t+1}}{\partial X_t}\end{array}$$

$$\frac{\partial\Omega^{C}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial X_t} = \left(\frac{\partial X_{t+1}}{\partial X_t}h-\frac{\partial C_{t+1}}{\partial X_t}\right)\rho\,(C_{t+1}-hX_{t+1})^{-\rho-1}-\frac{\partial\lambda^r_{t+1}}{\partial X_t}$$

$$\frac{\partial\Omega^{\lambda}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial X_t} = \frac{1}{\pi_{t+1}}\frac{\partial\lambda^r_{t+1}}{\partial X_t}-\frac{\lambda^r_{t+1}}{\pi^2_{t+1}}\frac{\partial\pi_{t+1}}{\partial X_t}$$

$$\frac{\partial\Omega^{Y}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial X_t} = \frac{\partial Y_{t+1}}{\partial X_t}(1+\eta)\frac{Y^{\eta}_{t+1}}{\xi_{\mathcal{S},t+1}}\left(\vartheta^{\sigma-1}_{t+1}-1\right)$$

$$\frac{\partial\Omega^{P}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t} = \begin{array}{c}(\pi_{t+1}-1)\,\pi_{t+1}\left(\lambda^r_{t+1}\frac{\partial Y_{t+1}}{\partial q_t}+Y_{t+1}\frac{\partial\lambda^r_{t+1}}{\partial q_t}\right)\\ +\,(2\pi_{t+1}-1)\,\lambda^r_{t+1}Y_{t+1}\frac{\partial\pi_{t+1}}{\partial q_t}\end{array}$$

$$\frac{\partial\Omega^{C}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t} = \left(\frac{\partial X_{t+1}}{\partial q_t}h-\frac{\partial C_{t+1}}{\partial q_t}\right)\rho\,(C_{t+1}-hX_{t+1})^{-\rho-1}-\frac{\partial\lambda^r_{t+1}}{\partial q_t}$$

$$\frac{\partial\Omega^{\lambda}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t} = \frac{1}{\pi_{t+1}}\frac{\partial\lambda^r_{t+1}}{\partial q_t}-\frac{\lambda^r_{t+1}}{\pi^2_{t+1}}\frac{\partial\pi_{t+1}}{\partial q_t}$$

$$\frac{\partial\Omega^{Y}(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t} = \frac{\partial Y_{t+1}}{\partial q_t}(1+\eta)\frac{Y^{\eta}_{t+1}}{\xi_{\mathcal{S},t+1}}\left(\vartheta^{\sigma-1}_{t+1}-1\right)$$

C.4 Derivation of Optimal policy

The FOC wrt to endogenous state variables.

$$\mathbf{X_t}: \qquad 0 = -h\left(C_t - hX_t\right)^{-\rho} + \beta \frac{\partial E_t \left\{ \mathcal{V}^d\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t\right)\right\}}{\partial X_t} + \Lambda_{1,t}$$

$$+ \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t}\frac{\partial E_t\left\{\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\right\}}{\partial X_t}\right]$$

$$+ \Lambda_{4,t}\left[\beta\phi\left[\frac{\partial E_t\left\{\Omega^C(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\right\}}{\partial X_t}\right] - \left[1 - h(1-\phi)\omega\right]\rho h\left(C_t - hX_t\right)^{-\rho-1}\right]$$

$$+ \Lambda_{5,t}\left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}\frac{\partial E_t\left[\Omega^Y(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right]^{\frac{-\sigma-\eta}{\sigma-1}}\right]}{\partial X_t} \tag{C.12}$$

using Envelope condition,

$$\frac{\partial \mathcal{V}^d\left(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}\right)}{\partial X_{t-1}} = -\phi\Lambda_{1,t} \tag{C.13}$$

and one-step forward iteration

$$\frac{\partial E_t\left\{\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t\right)\right\}}{\partial X_t} = -\phi E_t\left\{\Lambda_{1,t+1}\right\} \tag{C.14}$$

therefore by calling

$$\mathcal{Q}_t \equiv \left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right] \tag{C.15}$$

by calling,

$$\mathcal{Q}_{t+1} \equiv \left[1 + q_t\left(\vartheta_{t+1}^{\sigma-1} - 1\right)\right] \tag{C.16}$$

and by the taking the derivative inside the integral, we can re-write the FOC for Habit as,

$$\mathbf{X_t}: \qquad 0 = -\frac{h}{(C_t - hX_t)^{\rho}} - \beta\phi E_t\left\{\Lambda_{1,t+1}\right\} + \Lambda_{1,t}$$

$$+ \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t} E_t\left\{\frac{\partial\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\}\right]$$

$$+ \Lambda_{4,t}\left[\beta\phi E_t\left\{\frac{\partial\Omega^C(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\} - \frac{\left[1 - h(1-\phi)\omega\right]\rho h}{(C_t - hX_t)^{\rho+1}}\right]$$

$$+ \Lambda_{5,t}\left(\frac{1}{\sigma - 1}\right)\frac{\beta}{\chi\lambda_t^r} E_t\left[\left\{\frac{\partial\Omega^Y(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)}{\partial X_t}\right\}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right] \quad \text{(C.17)}$$

FOC wrt proportion of *risky* firms is

$$\mathbf{q_t}: \qquad 0 = \beta\frac{\partial E_t\left\{\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t\right)\right\}}{\partial q_t} - \Lambda_{2,t}\chi\left(q_t - \frac{1}{2}\right)$$

$$+ \Lambda_{3,t}\frac{\psi\beta}{\lambda_t^r Y_t}\frac{\partial E_t\left\{\Omega^P(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\right\}}{\partial q_t}$$

$$+ \Lambda_{4,t}\beta\phi\frac{\partial E_t\left\{\Omega^C(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\right\}}{\partial q_t}$$

$$- \Lambda_{5,t}\left(1 - \left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}\frac{\partial E_t\left\{\Omega^Y(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right\}}{\partial q_t}\right) \quad \text{(C.18)}$$

using Envelope condition,

$$\frac{\partial\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}\right)}{\partial q_{t-1}} = \frac{Y_t^{1+\eta}}{\sigma-1}\xi_{\mathcal{S},t}^{-1}\left[\mathcal{Q}_t\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left[\vartheta_t^{\sigma-1} - 1\right]$$

$$- \Lambda_{3,t}\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\lambda_t^r}\frac{\sigma^2(1+\eta)}{(\sigma-1)^2}\left[\mathcal{Q}_t\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left(\vartheta_t^{\sigma-1} - 1\right) \quad \text{(C.19)}$$

where we have made use of the notation, $\mathcal{Q}_t \equiv \left[1 + q_{t-1}\left(\vartheta_t^{\sigma-1} - 1\right)\right]$. Using one-step forward iteration,

$$\frac{\partial E_t\left\{\mathcal{V}^d\left(\varepsilon_{\mathcal{S},t+1},X_t,q_t\right)\right\}}{\partial q_t}=E_t\left[\begin{array}{c}\frac{Y_{t+1}^{1+\eta}}{\sigma-1}\xi_{\mathcal{S},t+1}^{-1}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left[\vartheta_{t+1}^{\sigma-1}-1\right]\\ \\ -\Lambda_{3,t+1}\frac{Y_{t+1}^{\eta}}{\xi_{\mathcal{S},t+1}\lambda_{t+1}^{r}}\frac{\sigma^2(1+\eta)}{(\sigma-1)^2}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)\end{array}\right] \tag{C.20}$$

which uses the one step forward notation, $\mathcal{Q}_{t+1}\equiv\left[1+q_t\left(\vartheta_{t+1}^{\sigma-1}-1\right)\right]$.

Thereafter taking the derivative inside the integral, FOC for *risk* allocation q_t becomes,

$$\mathbf{q_t}:\qquad 0=\beta E_t\left[\frac{Y_{t+1}^{\eta}}{\xi_{\mathcal{S},t+1}}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\eta+\sigma)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right)\left(\frac{Y_{t+1}}{\sigma-1}-\Lambda_{3,t+1}\frac{\sigma^2(1+\eta)}{(\sigma-1)^2\lambda_{t+1}^{r}}\right)\right]$$

$$-\Lambda_{2,t}\chi\left(q_t-\frac{1}{2}\right)+\Lambda_{3,t}\frac{\psi\beta}{\lambda_t^r Y_t}E_t\left\{\frac{\partial\Omega^P(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}+\Lambda_{4,t}\beta\phi E_t\left\{\frac{\partial\Omega^C(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}$$

$$-\Lambda_{5,t}\left(1-\left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi\lambda_t^r}\left[E_t\left[\left\{\frac{\partial\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)}{\partial q_t}\right\}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}\right]\right]\right)$$

$$-\Lambda_{5,t}E_t\left[\Omega^Y(\varepsilon_{\mathcal{S},t+1},X_t,q_t)\frac{\partial\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}}{\partial q_t}\right] \tag{C.21}$$

where

$$\frac{\partial\left[\mathcal{Q}_{t+1}\right]^{\frac{-(\sigma+\eta)}{\sigma-1}}}{\partial q_t}=\frac{-(\sigma+\eta)}{\sigma-1}\left[\mathcal{Q}_{t+1}\right]^{\frac{-(2\sigma+\eta-1)}{\sigma-1}}\left(\vartheta_{t+1}^{\sigma-1}-1\right). \tag{C.22}$$

C.5 Solving for Optimal policy when $q = 0$

Value function for the discretionary problem as $q \in [0,1]$ reaches lower bound,

$$\mathcal{V}^{d^-}(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}) = \max_{\{C_t, Y_t, \pi_t, X_t, \lambda_t^r\}} \left[\begin{array}{c} \left(\frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta} \xi_{\mathcal{S},t}^{-1} \right) \\ + \beta E_t \left\{ \mathcal{V}^{d^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right\} \end{array} \right] \tag{C.23}$$

It is subject to the 4 constraints on Habit accumulation (3.69), Aggregate Resource allocation(3.70), Inflation NKPC (3.71), and Consumption Euler equation CEE (3.72). The optimal policy problem can then be formulated as the Lagrangian,

$$\begin{aligned}
\mathcal{L}^{d^-} = & \left(\frac{(C_t - hX_t)^{1-\rho} - 1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta} \xi_{\mathcal{S},t}^{-1} \right) + \beta E_t \left\{ \mathcal{V}^{d^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right\} \\
& + \Lambda_{1,t} \left(X_t - (1-\phi) C_t - \phi X_{t-1} \right) \\
& + \Lambda_{2,t} \left(\left[1 - \frac{\psi}{2} (\pi_t - 1)^2 \right] Y_t - \frac{\chi}{8} - C_t \right) \\
& + \Lambda_{3,t} \left(\begin{array}{c} (1-\sigma)(1-\tau_p) - \psi (\pi_t - 1) \pi_t + \frac{Y_t^\eta}{\xi_{\mathcal{S},t} \lambda_t^r} \left(\frac{\sigma^2}{\sigma - 1} \right) \\ + \frac{\psi \beta}{\lambda_t^r Y_t} E_t \left[\Omega^{P^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right] \end{array} \right) \\
& + \Lambda_{4,t} \left(\begin{array}{c} \lambda_t^r - [1 - h(1-\phi)\omega] (C_t - hX_t)^{-\rho} \\ + \beta \phi E_t \left[\Omega^{C^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right] \end{array} \right)
\end{aligned} \tag{C.24}$$

where $\Omega^{P^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)$ and $\Omega^{C^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)$ are the auxiliary functions that depend on future policy decisions.

$$\Omega^{P^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv (\pi_{t+1} - 1) \pi_{t+1} \lambda_{t+1}^r Y_{t+1} \tag{C.25}$$

$$\Omega^{C^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv (C_{t+1} - hX_{t+1})^{-\rho} - \lambda_{t+1}^r. \tag{C.26}$$

Defining the auxiliary function

$$\Omega^{\lambda Y^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \equiv \frac{\lambda_{t+1}^r Y_{t+1}^{1+\eta}}{\varepsilon_{\mathcal{S},t+1}}, \tag{C.27}$$

the value of the multiplier $\lambda_t^q > 0$ is given by,

$$\lambda_t^q = \left(\frac{1}{\sigma - 1} \right) \frac{\beta}{\chi (\lambda_t^r)^2} E_t \left[\Omega^{\lambda Y^-}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t) \right] - \frac{1}{2}. \tag{C.28}$$

The policy rate in constraint (3.75) is chosen to optimize other policy functions. It is always binding and therefore not included in the Lagrangian specification.

First order conditions

The FOCs with respect to the decision variables.

$$\mathbf{C_t}: \quad 0 = (C_t - hX_t)^{-\rho} - \Lambda_{1,t}(1-\phi) - \Lambda_{2,t} + \Lambda_{4,t}\left[1 - h(1-\phi)\omega\right]\rho\left(C_t - hX_t\right)^{-\rho-1} \tag{C.29}$$

$$\mathbf{Y_t}: \quad 0 = -\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}} + \Lambda_{2,t}\left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right) + \Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\eta\frac{Y_t^{\eta-1}}{\xi_{\mathcal{S},t}\lambda_t^r} - \frac{\psi\beta}{\lambda_t^r(Y_t)^2}E_t\left[\Omega^{P^-}(\varepsilon_{\mathcal{S},t+1}, X_t)\right]\right] \tag{C.30}$$

$$\pi_{\mathbf{t}}: \quad 0 = -\Lambda_{2,t}Y_t\psi\left(\pi_t - 1\right) - \Lambda_{3,t}\left[\psi\left(2\pi_t - 1\right)\right] \tag{C.31}$$

$$\mathbf{X_t}: \quad 0 = -\frac{h}{(C_t - hX_t)^{\rho}} - \beta\phi E_t\left\{\Lambda_{1,t+1}\right\} + \Lambda_{1,t} + \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t}E_t\left\{\frac{\partial\Omega^{P^-}(\varepsilon_{\mathcal{S},t+1}, X_t)}{\partial X_t}\right\}\right] + \Lambda_{4,t}\left[\beta\phi E_t\left\{\frac{\partial\Omega^{C^-}(\varepsilon_{\mathcal{S},t+1}, X_t)}{\partial X_t}\right\} - \frac{[1-h(1-\phi)\omega]\rho h}{(C_t - hX_t)^{\rho+1}}\right] \tag{C.32}$$

$$\lambda_{\mathbf{t}}^{\mathbf{r}}: \quad 0 = \begin{bmatrix} -\Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\left(\lambda_t^r\right)^2} + \frac{\psi\beta}{Y_t\left(\lambda_t^r\right)^2}E_t\left[\Omega^{P^-}\left(\varepsilon_{\mathcal{S},t+1}, X_t\right)\right]\right] \\ +\Lambda_{4,t} \end{bmatrix} \tag{C.33}$$

C.6 Solving for Optimal policy when $q = 1$

Value function for the discretionary problem as $q \in [0,1]$ reaches upper bound

$$\mathcal{V}^{d^{+}}\left(\varepsilon_{\mathcal{S},t}, X_{t-1}, q_{t-1}\right) = \max_{\{C_t, Y_t, \pi_t, X_t, \lambda_t^r\}} \left[\begin{array}{c} \left(\frac{(C_t - hX_t)^{1-\rho}-1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\xi_{\mathcal{S},t}^{-1}\right) \\ +\beta E_t\left\{\mathcal{V}^{d^{+}}\left(\varepsilon_{\mathcal{S},t+1}, X_t, q_t\right)\right\} \end{array} \right] \quad \text{(C.34)}$$

It is subject to the 4 constraints on Habit accumulation (3.69), Aggregate Resource allocation(3.70), Inflation NKPC (3.71), and Consumption Euler equation CEE (3.72). The optimal policy problem can then be formulated as the Lagrangian,

$$\begin{aligned} \mathcal{L}^{d^{+}} = & \left(\frac{(C_t - hX_t)^{1-\rho}-1}{1-\rho} - \frac{Y_t^{1+\eta}}{1+\eta}\xi_{\mathcal{S},t}^{-1}\vartheta_t^{-(1+\eta)}\right) + \beta E_t\left\{\mathcal{V}^{d^{+}}\left(\varepsilon_{\mathcal{S},t+1}, X_t\right)\right\} \\ & +\Lambda_{1,t}\left(X_t - (1-\phi)C_t - \phi X_{t-1}\right) \\ & +\Lambda_{2,t}\left(\left[1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right]Y_t - \frac{\chi}{8} - C_t\right) \\ & +\Lambda_{3,t}\left(\begin{array}{c} (1-\sigma)\left(1-\tau_p\right) - \psi\left(\pi_t - 1\right)\pi_t + \frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\vartheta_t^{(1+\eta)}\lambda_t^r}\left(\frac{\sigma^2}{\sigma-1}\right) \\ +\frac{\psi\beta}{\lambda_t^r Y_t}E_t\left[\Omega^{P^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t)\right] \end{array} \right) \\ & +\Lambda_{4,t}\left(\begin{array}{c} \lambda_t^r - \left[1 - h(1-\phi)\omega\right]\left(C_t - hX_t\right)^{-\rho} \\ +\beta\phi E_t\left[\Omega^{C^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t)\right] \end{array} \right) \end{aligned}$$

where $\Omega^{P^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)$ and $\Omega^{C^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t, q_t)$ are the auxiliary functions that depend on future policy decisions.

$$\Omega^{P^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t, qt) \equiv \left(\pi_{t+1} - 1\right)\pi_{t+1}\lambda_{t+1}^r Y_{t+1}, \quad \text{(C.35)}$$

$$\Omega^{C^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t, qt) \equiv \left(C_{t+1} - hX_{t+1}\right)^{-\rho} - \lambda_{t+1}^r. \quad \text{(C.36)}$$

Defining the auxiliary function

$$\Omega^{\vartheta Y^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t, qt) \equiv \frac{\lambda_{t+1}^r Y_{t+1}^{1+\eta}}{\varepsilon_{\mathcal{S},t+1}\vartheta_t^{(1+\eta)}}, \quad \text{(C.37)}$$

the value of the multiplier $\lambda_t^q > 0$ is given by,

$$\lambda_t^{\hat{q}} = \left(\frac{1}{\sigma-1}\right)\frac{\beta}{\chi(\lambda_t^r)^2}E_t\left[\Omega^{\vartheta Y^{+}}(\varepsilon_{\mathcal{S},t+1}, X_t)\right] - \frac{1}{2}. \quad \text{(C.38)}$$

The policy rate in constraint (3.75) is chosen to optimize other policy functions. It is always binding and therefore not included in the Lagrangian specification.

First order conditions

The FOCs with respect to the decision variables.

$$\mathbf{C_t}: \ 0 = (C_t - hX_t)^{-\rho} - \Lambda_{1,t}(1-\phi) - \Lambda_{2,t} + \Lambda_{4,t}\left[1 - h(1-\phi)\omega\right]\rho\left(C_t - hX_t\right)^{-\rho-1} \quad \text{(C.39)}$$

$$\mathbf{Y_t}: \ 0 = -\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\vartheta_t^{(1+\eta)}} + \Lambda_{2,t}\left(1 - \frac{\psi}{2}\left(\pi_t - 1\right)^2\right) + \Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\eta\frac{Y_t^{\eta-1}}{\xi_{\mathcal{S},t}\vartheta_t^{(1+\eta)}\lambda_t^r} - \frac{\psi\beta}{\lambda_t^r(Y_t)^2}E_t\left[\Omega^{P^+}(\varepsilon_{\mathcal{S},t+1}, X_t)\right]\right] \quad \text{(C.40)}$$

$$\pi_{\mathbf{t}}: \ 0 = -\Lambda_{2,t}Y_t\psi\left(\pi_t - 1\right) - \Lambda_{3,t}\left[\psi\left(2\pi_t - 1\right)\right] \quad \text{(C.41)}$$

$$\mathbf{X_t}: \ 0 = -\frac{h}{(C_t - hX_t)^{\rho}} - \beta\phi E_t\left\{\Lambda_{1,t+1}\right\} + \Lambda_{1,t} + \Lambda_{3,t}\left[\frac{\psi\beta}{\lambda_t^r Y_t}E_t\left\{\frac{\partial\Omega^{P^+}(\varepsilon_{\mathcal{S},t+1}, X_t)}{\partial X_t}\right\}\right] + \Lambda_{4,t}\left[\beta\phi E_t\left\{\frac{\partial\Omega^{C^+}(\varepsilon_{\mathcal{S},t+1}, X_t)}{\partial X_t}\right\} - \frac{\left[1 - h(1-\phi)\omega\right]\rho h}{(C_t - hX_t)^{\rho+1}}\right] \quad \text{(C.42)}$$

$$\lambda_{\mathbf{t}}^{\mathbf{r}}: \ 0 = \begin{bmatrix} \Lambda_{4,t} \\ -\Lambda_{3,t}\left[\left(\frac{\sigma^2}{\sigma-1}\right)\frac{Y_t^{\eta}}{\xi_{\mathcal{S},t}\vartheta_t^{(1+\eta)}\left(\lambda_t^r\right)^2} + \frac{\psi\beta}{Y_t\left(\lambda_t^r\right)^2}E_t\left[\Omega^{P^+}(\varepsilon_{\mathcal{S},t+1}, X_t)\right]\right] \end{bmatrix} \quad \text{(C.43)}$$

Bibliography for Appendix

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