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**Generation and Transmission Maintenance
Scheduling considering the impact of Renewable
Energy**

by

Luis Adolfo Salinas San Martin

B. S. Electromechanical Engineering, Universidad Privada Boliviana
(UPB), Cochabamba, Bolivia, 2002

MBA International Business, Florida International University (FIU),
Florida, United States of America, 2004

M. S. Electrical Engineering with Business, University of Strathclyde,
Glasgow, United Kingdom, 2006

Dipl. Economics of the Electricity Sector - Integration of Latin
America, Univ. Federal do Rio de Janeiro (UFRJ), Rio de Janeiro,
Brazil, 2011

A dissertation submitted in fulfilment of the requirements for the degree of
Doctor
of Philosophy in the School of Engineering at the University of Glasgow
Glasgow, United Kingdom.

Abstract

The generation and transmission maintenance scheduling (GTMS) problem, in a competitive electricity market environment, presents electricity utilities scheduling their facilities for maintenance to improve productivity and maximize profits, and an independent system operator (ISO) pushing for maintenance schedules (MS) of generators and transmission facilities that keep the system reliability and minimizes operation cost. Thus, the GTMS is inherently a high-dimensional, non-linear, non-convex, and multi-objective optimization problem that contains mixed integer-real variables and conflicting objectives related to the goals of the different parties in the market.

The GTMS problem is crucial in power systems operation and planning due to the increasing complexity of today's power grid, the aging of current operating electricity facilities, and the increasing share of renewable energy in the network and the market. In that sense, this thesis proposes to solve the GTMS problem using hybrid models that combine in a novel way multi-objective evolutionary algorithms (MOEAs) and classical optimization techniques to obtain a set of feasible non-dominated MS solutions.

These hybrid models solve the GTMS problem in systems with thermal, hydro, and wind generation, handling maintenance and operation variables separately and sequentially, considering transmission congestion and losses, the opportunity cost in the future of water stored in reservoirs, the stochastic nature of wind generation and the impact of MS in electricity prices in the market. The models used match accepted industry maintenance practices with cutting-edge optimization techniques developed in the academia. The models are evaluated in the IEEE-RTS 24 test system, complemented with hydro units and wind farms belonging to two Bolivian electricity utilities. GENCO's profits, system adequacy, and operation costs are used as objective functions, and their conflicting relationships are evaluated in the obtained set of MS solutions. Finally, the models allow the ISO to use this set to identify the best MS solution using the technique for ordering preferences according to similarity to an ideal solution (TOPSIS) decision-making tool.

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Preface

This PhD thesis has been written following the requirements established by the University of Glasgow and the corresponding program inside the School of Engineering and has been sponsored by the Bolivian State and the author of the present document.

This research work stemmed from the collaboration and supervision of Dr. Jin Yang and his research group, who has been working over the last year in re-organizing a power systems group inside the University of Glasgow, so that it can make research in the areas of optimization methods applied to power systems, renewable energy impact in today's electricity markets and future power networks and smart grids. My main contributions to the group aims have been to develop a multi-objective generation and transmission maintenance scheduling tool, combining metaheuristic and classical optimization techniques in a novel way, to provide power systems planners and decision-makers with a set of non-dominated maintenance schedules of electricity facilities that ensure a reliable supply of electricity to final consumers at a minimum cost, and from which the best schedule can be chosen.

As electricity facilities get older around the world, they need to be replaced or to undergo maintenance and refurbishment works to extend their life cycle to adapt them to cope with the stochastic nature of renewable energy. Thus, there is a great need to combine different optimization techniques in novel ways to tackle the maintenance scheduling problem, incorporating the effect of renewable energy, the impact on electricity prices, and considering common practices of the power industry today. This work is a further step in this direction.

Acknowledgements

And now, nearly at the end of my research, I find the necessity to make a stop on the way to look back for a short while and see how far I have travelled, how many things I have lost, how many difficulties I have endured and overcome to arrive finally to this point. In that sense, I believe it is important to mention the people and institutions that have assisted me during this lonely and difficult journey.

First, I really would like to express my gratitude to Dr. Jin Yang, my present supervisor. Even though he has supervised my research since last year, he has managed to assist and support me in every way possible to tackle my research and personal difficulties. What is more, it has been an honour to assist him in giving tutorials, lectures, and demonstrations at the University of Glasgow as part of his research group. I really hope in the future we can keep on working together. I would like to thank sincerely Dr. Ying Liu, for reviewing and supervising my work at critical moments and providing suggestions to improve my research quality. I also would like to thank Dr. Keliang Zhou and Prof. Yun Li, for supervising my research during the second and the first year of my stay at the University, respectively.

Second, I would like to thank the University International Student Support team, for making my stay enjoyable with the organization of lunch gatherings and family trips around beautiful Scotland. I am also in debt with the Financial Aid team at the University, who have assisted me with a Hardship Fund when my sponsor could not comply with its obligations. Without that help, I would not have been able to finish my research in 3 years. My sincere thanks go also to all the staff at the School of Engineering, for assisting me all the way through until completion of my research, especially during the difficult times of the COVID-19 pandemic. Please, be sure that I tried my best to make myself worthy of your assistance.

Most of all, I would like to dedicate this research project to my family. To my parents, for always being there for me, supporting my decisions, following my steps with enthusiasm, love, and care. To my brothers, sisters, and their families, for lending me their hands when I really needed them and tearing smiles from my

face at difficult moments. To all of them, sorry for all the inconvenience and problems that happened. Things can go wrong, expectations may not be realized, but our family values always remain: Character, strength, hard work, and honesty.

This work is also dedicated with huge gratitude to my country Bolivia, for giving me the chance to make this research project. Today, more than ever, my motherland needs its best people working hard and united, to lead her down the right path again, far away from ignorance, corruption, and tyranny. Only in this way, its traditional republican and democratic values may endure, and freedom and justice may prevail again.

Finally, thanks to God almighty, who has been my partner and co-pilot during this travel and adventure. Faith, charity, and hope are the fuel needed to overcome all difficulties. Now, at last, it is time to move ahead, look forward and hope that all the effort made will bear fruits someday in the future. In the meantime, I am only left to say: Mission Accomplished!

Author's Declaration

I declare that this thesis work named “Generation and Transmission Maintenance Scheduling considering the impact of Renewable Energy” has been composed exclusively by myself and that it has not been submitted, as a whole or in part, in for any other degree at the University of Glasgow or any other institution. The work contained in this document is of my own, except where explicitly stated otherwise in the text.

List of Symbols

The symbols listed below are reserved for a specific use, unless specified otherwise in any section where their meaning is different. Other symbols may be used throughout the dissertation in an unreserved fashion.

Symbol	Meaning
Indices	
i	Hydroelectrical generation unit
j	Thermal generation unit
b	Demand block or sub-period of demand
l	Transmission line or transformer
e	Water reservoir
t	Period of analysis (week)
n	Electrical node or bus
Sets	
Φ_G	Set of generation units owned by GENCO G
Θ_e	Set of hydro units upstream to reservoir e (Hm3)
Ψ_e	Set of reservoirs upstream to reservoir e (Hm3)
Φ_e	Set of hydro units connected to reservoir e (Hm3)
Variables	
$V_{j,t}, W_{j,t}$	Availability binary variables of thermal unit j at time t
x_j^g	Maintenance starting time of thermal unit j
$X_{j,t}^g$	Thermal unit j 's binary intermediate availability variable at time t
$P_{j,t}^g$	Power output of thermal unit j at time t (MW)
$P_{j,b,t}^g$	Power output of thermal unit j in subperiod b at time t (MW)
$[P^g]_t$	Vector of power output of thermal generation units at time t (MW)
x_i^h	Maintenance starting time of hydroelectrical unit i
$X_{i,t}^h$	Hydroelectrical unit i binary intermediate availability variable at time t

$P_{i,t,b}^h$	Power output of hydroelectrical unit i in subperiod b at time t (MW)
$[P^h]_t$	Vector of power output of hydro generation units (MW)
$u_{i,b,t}$	Hydroelectrical unit i water discharge in subperiod b at time t (Hm3)
$v_{e,t}$	Final volume of reservoir e at time t (Hm3)
$v_{e,t}^o$	Initial volume of reservoir e at time t (Hm3)
$s_{e,t}$	Water spilled of reservoir e at time t (Hm3)
y_l	Maintenance starting time of transmission line l
$Y_{l,t}$	Transmission line l 's binary intermediate availability variable at time t
$f_{l,t,b}$	Line l real power flow in subperiod b at time t (MW)
$[f]_t$	Vector of transmission lines real power flows at time t (MW)
$f_{l,t}^{loss}(P_{j,t}^g)$	Network power losses at time t as a function of generators power output (MW)
$f_{n,t}^{loss}(P_{j,t}^g, P_{i,t}^h)$	Transmission losses at time t as a function of generators power output assigned as additional loads to each node (MW)
$f_{l,b,t}^{loss}$	Line l losses in subperiod b at time t (MW)
$[f^{loss}]_t$	Vector of transmission losses assigned as additional loads to each node at time t (MW)
$[\emptyset]$	Power Losses Auxiliary Matrix
$[\varphi]$	Nodal power losses sensitivity matrix
$\tau_{b,t}$	Electricity price in subperiod b at time t (US\$/MWh)
$\tau_{n,b,t}$	Electricity price in node n in subperiod b at time t (US\$/MWh)
$\tau_{n,t}$	Electricity spot price in node n at time t (US\$/MWh)
$NF_{n,t}$	Penalty factor at node n at time t
$NF_{n,b,t}$	Penalty factor at node n in subperiod b at time t
$\alpha(v_{t+1})$	Cost to go Function in the future for using water in the present (US\$)
λ	Lagrange multiplier Variable (US\$/MWh)
$[\tau]_t$	Nodal prices vector in subperiod b at time t (US\$/MWh)
$[\tau]_{b,t}$	Vector of Nodal Energy Prices during subperiod b at time t (US\$/MWh)

$\pi_{b,t}$	System marginal cost during subperiod b at time t (US\$/MWh)
$[\pi]_{b,t}$	Vector of System Marginal Cost during subperiod b at time t (US\$/MWh)
$NF_{n,t}$	Nodal Penalty Factor in node n at time t
$[NF]_{b,t}$	Vector of Nodal Penalty Factors during subperiod b at time t
$\mu_{l,t}$	Congestion cost of line l at time t (US\$/MWh)
$[\mu]_{b,t}$	Vector of transmission lines congestion cost during subperiod b at time t (US\$/MWh)
$\vartheta_{e,t+1}$	Water cost-to-go dual multiplier at time $t + 1$ (US\$/Hm3)
MS_t	Market merchandising surplus (US\$)
U_t	Uplift or extra cost of operating a constrained electricity system (US\$)
E_t^u	Payments made in an unconstrained electricity market by DISCOs (US\$)
E_t	Adjusted payments made in a constrained electricity market by DISCOs (US\$)
R_t^u	Revenues made in an unconstrained electricity market by GENCOs (US\$)
R_t	Adjusted revenues made in a constrained electricity market by GENCOs (US\$)
ΔR_t	Extra revenues made by GENCOs in a constrained electricity market (US\$)
F_t^u	Unconstrained system operation cost (US\$)
F_t^c	Constrained system operation cost (US\$)
F_1	Adequacy Index (%)
F_2	System total operation cost (US\$)
F_G	Profits of GENCO G (US\$)
d_m	Diversity Measure
D_M	Diversity Performance Metric

Parameters

M	Total Number of Objective Functions
H	Number of reference points of the NSGA III method
Np	Number of individuals in the population
p	Number of partitions considered along each objective function

p_c	Crossover probability
p_m	Mutation probability
$h(i)$	Diversity Array
η_c	Crossover probability distribution index
η_m	Mutation probability distribution index
u	Random Number between [0, 1]
u'	Integer Random Number
d_j^g	Duration of maintenance of thermal unit j
s_j^g	Earliest start of maintenance of thermal unit j
k_j^g	Latest start of maintenance of thermal unit j
d_i^h	Duration of maintenance of hydroelectrical unit i
s_i^h	Earliest start of maintenance of hydroelectrical unit i
k_i^h	Latest start of maintenance of hydroelectrical unit i
d_l	Duration of maintenance of line l
s_l	Earliest start of maintenance of line l
k_l	Latest start of maintenance of line l
$P_{n,b,t}^r$	Renewable generation in node n during subperiod b at time t (MW)
$P_{n,t}^d$	Power Demand at node n at time t (MW)
$P_{n,b,t}^d$	Electricity demand at node n in subperiod b at time t (MW)
$P_{b,t}^d$	Electricity demand in subperiod b at time t (MW)
$P_{n,t}^{d,max}$	Maximum electricity demand in node n at time t (MW)
$P_t^{d,max}$	Maximum electricity demand at time t (MW)
$P_j^{g,max}$	Maximum generation output of thermal unit j (MW)
$P_j^{g,min}$	Minimum generation output of thermal unit j (MW)
$P_i^{h,max}$	Maximum generation output of hydroelectrical unit i (MW)
$P_i^{h,min}$	Minimum generation output of hydroelectrical unit i (MW)
β_{nl}	Sensibility Matrix row corresponding to node n
$[\beta]_t$	Sensibility matrix at time t
r_l	Resistance of transmission line l (pu)
$[r]$	Vector of resistances of transmission lines and transformer (pu)

$[Sg]$	Incidence matrix that relates nodes and generators in the system
$[St]$	Incidence matrix relating connections with their respective nodes
f_l^{max}	Maximum capacity of transmission line l (MW)
f_l^{min}	Minimum capacity of transmission line l (MW)
v_e^{max}	Maximum volume at reservoir e (Hm3)
v_e^{min}	Minimum volume at reservoir e (Hm3)
u_i^{max}	Maximum water turbinated of hydroelectrical unit e (Hm3)
u_i^{min}	Minimum water turbinated of hydroelectrical unit e (Hm3)
$a_{e,t}$	Water inflow in reservoir e at time t (Hm3)
G	Number of Generation Companies
N	Number of nodes in the system
Ng	Number of thermal generation units
Nh	Number of hydroelectrical generation units
Nl	Number of transmission lines
Ne	Number of reservoirs considered
Nb	Number of demand sub-periods considered
T	Duration of the period under analysis
C_{0j}, C_{1j}, C_{2j}	Generation cost curve coefficients of unit j given in (MBtu/h), (MBtu/MWh) and (MBtu/MW ² h), respectively
$B_{j,k}, B_{0j}, B_{00}$	Transmission network loss coefficients
c_j^f	Thermal unit j fuel price (US\$/MBtu)
c_j^g	Unit j marginal generation cost (US\$/MWh)
$c_{j,b}^g$	Unit j marginal generation cost during subperiod b (US\$/MWh)
$c_{e,t}^s$	Water Spilled cost in reservoir e at time t (US\$/Hm3)
$C_{j,t}^g$	Maintenance cost of thermal unit j at time t (US\$)
$FC_{j,t}$	Fixed Cost of thermal unit j at time t
$T_{b,t}$	Duration of sub-period b at time t (hrs)
R^{min}	Minimum reserve margin (MW)
N^{min}	Minimum number of units available
ρ_i	Productivity factor of hydro unit i (MW/Hm3)
$SR_{b,t}$	Percentage of spinning reserve during subperiod b and time t

List of Abbreviations

AEP	Annual Energy Production
ANN	Artificial Neural Network
ASCE	The American Society of Civil Engineers
ASF	Argument Scalarization Function
ATM	Adaptive Trade-off Model
B&B	Branch-and-bound
BLX-a	Blend Crossover
CC	Correlation Coefficient
CD	Crowding Distance
CVR	Close Value Range
FCF	Future or Cost-to-go Function
DC	Direct Current
DD	Dual Dynamic Method
DISCO	Distribution Company
DP	Dynamic Programming
DS	Dual-Simplex Method
EA	Evolutionary Algorithms family
ED	Economic Dispatch
ENS	Energy not Supplied
ERA	Electricity Regulation Agency
GA	Genetic Algorithms
GP	Goal Programming
GENCO	Generation Company
GENOCOP	Genetic algorithm for numerical optimization problems with linear constraints
GMS	Generation Maintenance Schedule
GSO	Group Search Optimizer
GSOMP	Group Search Optimizer with Multiple Procedures
GTMS	Generation and Transmission Maintenance Schedule
HTWGS	Hydro-Thermal-Wind Power
ICF	Immediate cost Function
IEC	International Electrotechnical Commission
ISO	Independent System Operator

LIM	Lambda Iteration Method
LMBP	Levenberg-Marquardt Back Propagation
LOLP	Loss of Load Probability
MOEA	Multi-objective Evolutionary Algorithm
MOGA	Multi-objective genetic
MOOP	Multi-objective optimization problems
MPSO	Modified particle swarm optimization
MOPSO	Multi-objective particle swarm optimization
MS	Maintenance Schedule
MSE	Mean Squared Error
MX	Masked Crossover
NGC	National Grid Company
NGET	National Grid Electricity Transmission
NSGA	Non-dominated sorting genetic algorithm
OECD	Organization for Economic Co-operation and Development
PAES	Pareto Archived Evolution Strategy
PESA	Pareto Envelope-based Selection Algorithm
PMX	Partially-matched crossover
PSO	Particle Swarm Optimization
RDGA	Rank-Density Based Genetic Algorithm
RWGA	Random Weighted Genetic Algorithm
SACH	Self-adaptive Constraint Handling technique
SAM	Self-Adaptive Mechanism
SANUX	Statistics-based adaptive non-uniform crossover operator
SDP	Stochastic Dynamic Programming
SDDP	Stochastic Dual Dynamic Programming
SPEA	Strength Pareto Evolutionary Algorithm
TOPSIS	Technique for Order Preference by Similarity to the Ideal Solution method
TRANSCO	Transmission Company
UC	Unit Commitment
VEGA	Vector Evaluated Genetic Algorithm
WBGA	Weight-based Genetic Algorithm

CHAPTER 1

INTRODUCTION

1.1 Background

In today's electricity markets, cutting operations and maintenance (O&M) costs and preserving service security and adequacy are among the top priorities for network operators and planners, energy authorities, and electricity companies. On the other side, electricity network assets' health and proper operation require constant monitoring and routine preventive maintenance to increase their life span. Maintenance timing and duration are critical parameters that define maintenance and operation costs. In that sense, how much maintenance do assets require involves a trade-off: too much of it and the facility becomes uneconomical to own, too little of it and the facility runs at the risk of breaking down involving expensive overhaul works. It is perceived that careful planning and good coordination among electricity utilities and system operators in a restructured power system, are essential to determine a maintenance schedule of electricity facilities that guarantee an optimal trade-off between utilities' cost reductions and electrical service reliability.

Electricity utilities have evolved from a reactive "break and fix" towards a preventive maintenance approach [1]. The business-as-usual practice of preventive maintenance is equipment being replaced or overhauled as they age or on a schedule recommended by their manufacturers. This approach has been facilitated by many maintenance systems developed for the purpose, which consider important network planning aspects as well. In the quest to continually reduce the cost of asset maintenance, new asset management technologies offer realistic and tangible near-term benefits, but not without navigating and overcoming some hurdles related to assets monitoring. This progress in assets management techniques leads to predictive maintenance practices based on condition monitoring technologies. Predictive maintenance is a big step forward in the evolution of asset management and involves moving from planned

preventive maintenance to a state where required maintenance is predicted, and course of actions are prescribed by asset-tracking systems. There are clear gains for utilities to move in this direction, such as (i) getting ahead of equipment failures to avoid major breakdowns, outages, or replacement of perfectly good parts when not required, (ii) having a greater understanding of what kind of maintenance is required and the personnel and tools needed to perform it. Asset focus maintenance systems of this calibre provide knowledge that drastically improve repair time and effectiveness. However, electricity assets preventive maintenance deals with power system planning and operation, while predictive maintenance is more asset-focused.

Generation and transmission maintenance schedule is an important part of a power system operation planning. In centralized electric power systems, traditional maintenance is determined by the system operator and imposed on a single utility that handles the generation, transmission, and distribution of electricity in a system. Maintenance programs in this structure rely heavily on duration, timing, costs, and manufacturer recommendations. After the restructuring of the power industry in many countries, vertically integrated utilities were divided into several generation (GENCO), transmission (TRANSCO), and distribution companies (DISCO), which compete in an electricity market to supply the demand. In this competitive electricity market environment, each utility determines its maintenance schedule considering its particular interests, related to assets overhaul and replacement, to achieve profit maximization. This makes maintenance scheduling of electricity facilities more challenging. Considering the broad array of assets for which utilities are responsible – transformers, transmission lines, generators, protective devices – and the risks associated with asset failure, a reliable maintenance strategy is a priority for the electricity industry. As a consequence, it is the task of the Independent System Operator (ISO) of the network to ensure that the maintenance scheduling programs proposed by utilities do not conflict with each other and put at the same time at risk the reliable supply of electricity to the final consumers at the minimum cost.

This thesis focuses on preventive maintenance scheduling of generating units and transmission facilities, the most important parts of electric power systems, in a

market environment. Preventive maintenance is perhaps the single largest controllable cost for utilities and a factor that determines the adequate and secure supply of electricity to final consumers. In that sense, the thesis tackles the maintenance scheduling problem to allow utilities to make profits, ensure equipment availability and up-time, improve the adequate supply of the electricity service, and most importantly, reduce the system operation costs. While the maintenance scheduling problem is dealt considering the impact of renewable energy and its effect on market prices, the proposed solution does not consider voltage and frequency stability issues.

1.2 Drivers for Better Maintenance Scheduling Techniques

The electrical industry is one of the largest and most capital-intensive sectors of the economy of every country. What is more, the electric power generation and transmission industry is changing rapidly. A shift in ownership has occurred from regulated utilities to competitive suppliers. Furthermore, cleaner and more fuel-efficient power generation technologies are becoming available and their impact on the energy supply needs to be considered. In that sense, the present research has been defined considering the following motivations.

1.2.1 Deregulated electricity market

The restructuring of the electric power industry has resulted in market-based approaches for unbundling a multitude of services provided by self-interested entities such as many GENCOs and DISCOs, few TRANSCO, an ISO. As these entities operate in a deregulated electricity environment, market competition has created additional challenges for them to choose proper operational planning procedures for scheduling their equipment maintenance [2]. The choice must consider coordination between long-term and short-term maintenance schedules. It must also take into account complex cost-revenue trade-offs related to evaluating the impact of maintenance outages on the finances of electricity utilities and in the secure and adequate operation of the system.

There is an increasing need to apply mathematical numerical methods to coordinate the maintenance of electricity facilities belonging to different electricity companies, which are competing in an electricity market. To do so, complete datasets related to electricity assets and companies' policies are needed, representing a huge amount of information. As electricity utilities are separated and have different interests, this information may be unattainable or quite difficult to acquire. What is more, electricity utilities may be unwilling to share or report their sensitive information as it is not strategically correct to do so, but the information may be important for safekeeping the secure supply of electricity to final consumers.

Therefore, mathematical multi-objective and classical optimization techniques based on decomposition methods are advantageous to develop acceptable maintenance strategies and support the ISO on deciding the best maintenance schedule for the whole system. This is possible because of the dramatic improvements in computing technology in recent times, that have allowed power engineers to solve large-scale complex problems efficiently. In this way, a maintenance scheduling strategy can be developed for the whole electrical system so that end users can enjoy a reliable and cheap supply of electrical energy.

1.2.2 Power system adequacy and security

Many countries are facing huge costs associated with electricity supply disruptions, which have a great impact on the Loss of Load Probability (LOLP) or Energy not Supplied (ENS) indicators, as well as the level of investment needed to maintain and upgrade their electricity assets.

The reliability of a power system can be interpreted as satisfying two major functions: adequacy and security. A system is adequate when the amount of capacity of resources to meet the peak demand is available at all times. On the other side, a system is secure when the system can withstand changes or contingencies on a daily and hourly basis.

To ensure the security and adequacy of a power system, electricity generation and transmission assets must be kept working properly. Because of the expected near-term retirement of many aging generators and transmission lines, many utilities in the world face a significant need for continuous maintenance of existing facilities and the replacement or overhaul of ageing ones. Large electricity utilities may invest significantly in refurbishing and replacing existing assets [3].

For instance, according to the U.S Department of Energy, it is estimated that power quality disturbances and outages cost the economy from US\$25 to US\$180 billion annually. These costs could soar if they become more frequent or of longer duration, especially due to the replacement or maintenance of old equipment [4]. It is estimated that the U.S. network worth almost US\$ 876 billion, thus modernizing it would cost hundreds of billions of US\$. With nearly 75% of transmission lines and transformers are 25 years or older, outdated infrastructure is frequently cited as the biggest threat to the US energy supply. The American Society of Civil Engineers (ASCE) estimated in 2011 that maintaining this infrastructure would require \$673 billion of new investment by 2020 [5].

Furthermore, The Telegraph reported that coal power plant breakdowns and low wind power outputs force National Grid Company (NGC) in the UK to pay for dozens of businesses to reduce their energy usage [6]. Britain was forced to rely on last resort measures to keep the lights on for the first time on the 4 of April 2018 after coal power plants broke down and wind farms produced less than 1% of the required electricity. NGC blamed the power crunch on “multiple plant breakdowns”. Several aging coal-fired power plants had unexpected maintenance issues and temporarily shut down, reducing available supplies.

What is more, the International Electrotechnical Commission’s (IEC) Strategic Assessment of Power Networks White Paper [7], has surveyed electricity networks in many countries to get a snapshot of the age of their assets. The results are shown in Figure 1.1.

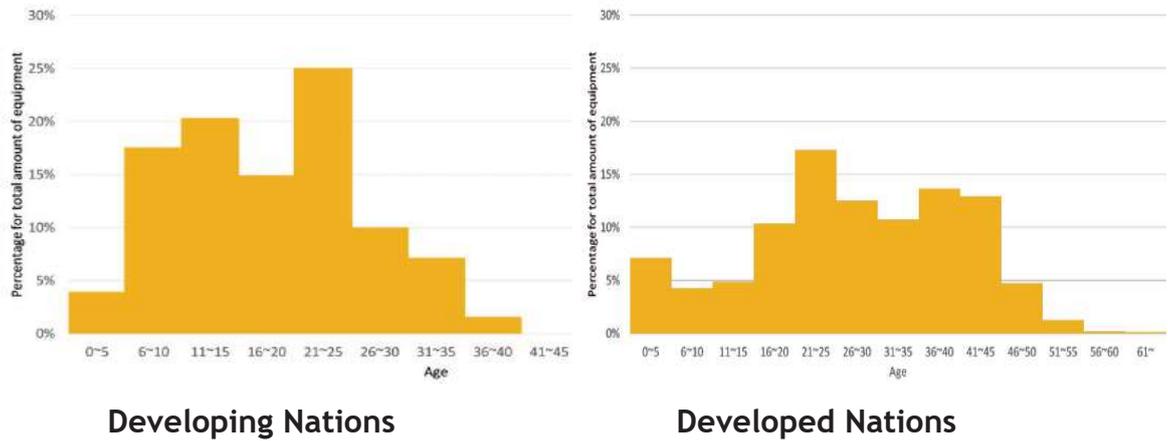


Figure 1.1 Typical electricity asset's age around the world (years) [5].

As it can be seen, 25% and 17% of electrical equipment are between 21-25 years old in developing and developed nations, respectively. Furthermore, much of the electrical equipment is more than 25 years old. Considering that transmission lines and electrical generators life span are around 25 to 30 years, it possible to say that electricity facilities around the world are reaching their useful life and will need overhauls, more frequent maintenance and even complete replacement. As a result, a new framework needs to be developed so that these activities can be done while preserving the security and adequacy of the system.

1.2.3 Impact of renewable energy

Many networks in the world have been designed primarily for transmitting electricity from distant large conventional power plants, like coal and natural gas, to load centres. However, today new technologies are making this approach to electricity transmission increasingly outdated. Electrical energy is now being produced by many types of renewable sources, like solar and wind, and the grid needs to adapt to accommodate them. This is especially crucial as renewable energy continues to boom and solar and wind power grow rapidly across the world. If current power systems is lack behind, countries could run under the risk of over-relying on conventional sources of energy rather than taking the advantage of renewable energy sources that meet energy demand with cleaner energy. This involves taking parts of the network out of service for replacement and overhaul works, that require time and may affect the reliability of the energy supply.

On the other side, renewable energies also have a significant impact on energy security due to the stochastic nature of their electricity generation. Since the recent developments of renewables worldwide are extremely rapid, it is imperative to analyse how the variability of their generation can impact the energy security to supply the demand, especially considering situations when several electricity facilities are going through maintenance coincidentally.

The benefits of renewables are significant. They can diversify the energy mix and the sources of supply, localize energy production, and reduce import requirements and costs. Renewables have less complex supply chains and are fuel-free technologies that reduce CO2 emissions and long-term price volatility. According to [8], the growth of renewable energy in Europe since 2005 has led to the reduction of natural gas domestic production and imports from foreign markets as shown in Figure 1.2. This trend is expected to continue in the future.

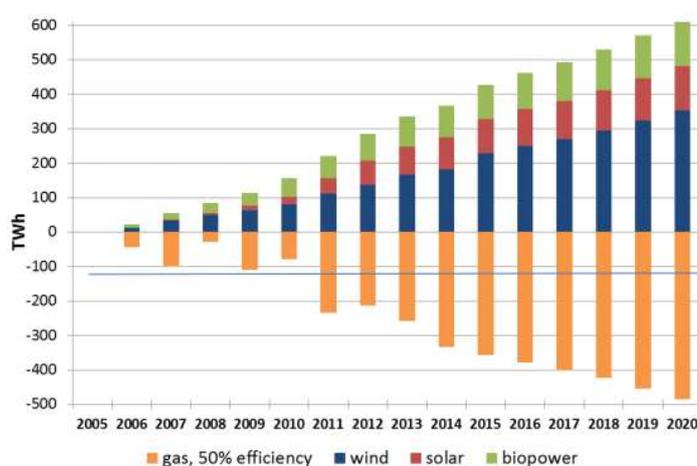


Figure 1.2 Production of renewable energy and natural gas in OECD Europe (2005-2020).

However, the dependency on large shares of renewable energy sources implies a paradigm shift for the secure supply of electricity. The classical risks associated with fossil fuels are replaced by risks related to the availability of natural resources such as water, biomass, wind, and sunlight. For instance, hydropower faces annual variations in precipitations, and it is exposed to the risk of droughts. The fluctuating availability of wind and sunlight leads to more rapid and pronounced swings in the electricity produced from these sources and a challenging demand balance of the whole system. The rapid rise of renewable generation sources is adding new challenges for maintaining system reliability and

in the required adaptation of the existing electrical infrastructure to these situations. This last issue may demand long periods of overhaul and replacement of electricity assets, which will have a critical impact on the reliability of the system.

1.3 Objectives

A new framework is required to coordinate long-term maintenance activities among different utilities and the ISO in deregulated electricity markets, where different types of generation technologies are being used, especially renewables which generate clean energy with a great variability characteristic. This coordination of the multiple interactions among the utilities and the ISO is critical to avoid that scheduled outages of the elements of the network and/or unexpected low level of generation from renewable energy sources put the reliable supply of electricity to consumers and sensitive loads at risk.

1.3.1 Overall aim

The overall aim of this project is to develop a singular approach to solve the long-term generation and transmission maintenance scheduling (GTMS) problem in a deregulated electricity market environment, by combining in a novel way classical optimization techniques with Multi-Objective Evolutionary Algorithms (MOEA) and that considers a profitable operation of GENCOs in the market, allows the ISO an adequate operation of the grid at the minimum cost, is robust against renewable energy uncertainties and permits the identification of the best maintenance schedule (MS) considering the electricity industry present practices.

1.3.2 Specific objectives

The specific objectives to achieve the above aim are the following:

- a) To understand and combine different classical and multi-objective optimization techniques to develop a generation and transmission maintenance scheduling tool or model.

- b) To determine the competitive relationship between the objectives of GENCOs and the ISO in a deregulated power system while determining feasible maintenance scheduling solutions for generators and transmission facilities.
- c) To evaluate the impact the generation and transmission maintenance schedules have over electricity prices in a deregulated electricity market.
- d) To consider the stochastic nature of wind generation and water inflows of reservoirs of hydroelectrical units during the development of the maintenance scheduling tool.
- e) To incorporate current electricity utilities practices in the maintenance scheduling tool to make it attractive to be used in the industry context.

1.4 Thesis outline

This introduction chapter is followed by six additional chapters, an appendix and reference sections.

In Chapter 2, a literature review is presented, where different single and multi-objective optimization techniques used, in the academia and in the electricity industry, to tackle different generation and transmission maintenance scheduling (GTMS) problems are described and analysed.

Chapter 3 presents the mathematical formulation of the generation and transmission maintenance scheduling problem for the present research project, identifying the variables involved and describing the objective functions and constraints considered for three different scenarios.

The newly proposed methodologies for solving the GTMS problem in the three scenarios is presented in Chapter 4, putting emphasis on the codification of the maintenance schedules (MS) solutions in the context of genetic algorithms, and on the strategy and code designed to solve the problem.

Chapter 5 presents information regarding the three scenarios defined. The test networks, generation units' technical characteristics, demand values, electricity facilities maintenance requirements and the parameters of the classical and multi-

objective optimization techniques used to evaluate the methodologies to solve the GTMS problem are described in this chapter.

In Chapter 6, an analysis of the numerical results obtained from applying the proposed methodologies to solve the maintenance scheduling problem in the different power system scenarios are shown. A description of the conflicting relationship among the objectives of various players in the electricity market, the methodology effectiveness for finding feasible non-dominated MS solutions, and the effect of these solutions in electricity prices is done.

Finally, Chapter 7 present a summary of the research project findings, highlighting its contributions, and suggesting future work to be done in the field of maintenance scheduling of electricity facilities.

CHAPTER 2

LITERATURE REVIEW

This chapter begins with a discussion on the definition of maintenance and the various types of maintenance strategies. This is followed by a description of the electricity industry organization and typical activities carried out in a deregulated market environment. Next, the general modeling considerations for the GTMS problem in typical power systems problems are presented. A more substantial literature review follows on multi-objective optimization techniques, including objective functions' normalization methods, constraints handling techniques, and genetic operators found in the literature. The chapter closes with a review of GTMS solution approaches applied so far in the literature and the industry, including the use of classical and multi-objective evolutionary algorithms (MOEAs) optimization methods.

2.1 Maintenance

Maintenance of engineering systems involves planned and unplanned actions carried out to keep, repair, overhaul or replace a facility to restore its operation to an acceptable condition [9]. The objective of maintenance is to ensure engineering systems availability, efficiency, and operation quality at a low cost to keep system lifespan, safety, low energy consumption, and to avoid the adverse effects of breakdown [10]. In electricity industry terms, poorly maintained generators and transmission facilities may lead to random breakdowns causing unavailability, loss of revenues, and electricity service interruption.

There are a variety of classifications of maintenance in the literature according to strategic plans and philosophies [11]. The main types of maintenance involve an unplanned breakdown and planned preventive, condition-based and reliability-centered maintenance as shown in Figure 2.1 [12]:

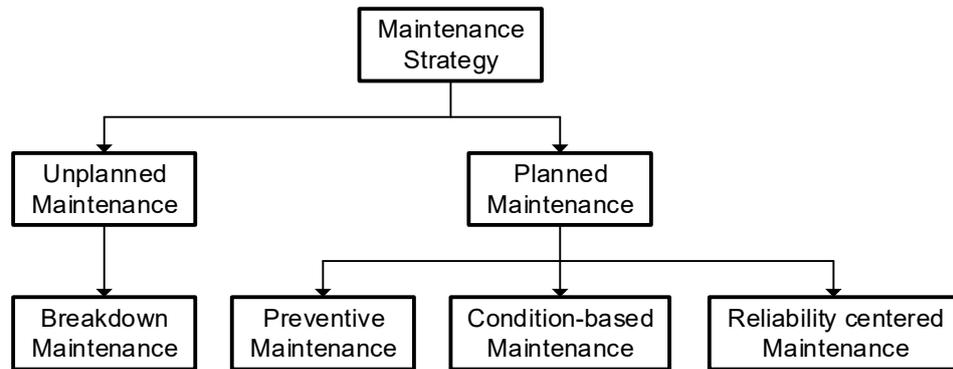


Figure 2.1 Classification of different types of maintenance activities [13] [11].

Unplanned breakdown or corrective maintenance takes place only after breakdowns have occurred or whenever a failure takes place. Corrective maintenance philosophy follows a “break and fix” approach and maintains the engineering systems or facilities by exception. However, this results in unscheduled downtimes and an increase in maintenance work effort. At the same time, it is very costly in terms of execution, it shortens the lifespan of equipment and, under hazardous system operation conditions, unpredicted failures that generate breakdown can be fatal.

To avoid these issues, planned or preventive scheduled maintenance is designed to maintain electricity facilities at scheduled intervals throughout their lifespan. Maintenance is planned, according to manufactures’ recommendations, utilities’ operation and financial goals and planning requirements of the systems where the facilities belong too. Maintenance activities are dependent on the time when they start and on their duration. Depending on the size and technical characteristics of the facilities, and the complexity of the maintenance works, preventive maintenance may require many days, weeks, or months [13]. While this type of maintenance keeps and increases the lifespan of electricity facilities, it represents an important cost for utilities and presents a more planned-focused approach.

To prevent these, it becomes necessary to predict the failure of components inside a facility sufficiently in advance, so that the performance of equipment can be optimized and enhanced, and maintenance expenditure can be reduced. These are the objectives of condition-based maintenance, which are fulfilled by constant facility monitoring to identify upcoming failures so maintenance can be proactively scheduled when it is needed and not before. To predict the failure

into the future, the maintenance system constantly monitors the facility symptoms, analyses their trends, and makes a decision as to the possible existence, location, cause, and severity of the fault. This condition monitoring feature triggers maintenance within a long enough time before failure, so maintenance works can be finished before the asset fails or its performance falls below an optimal level. To do this, an expensive but reliable means of facility condition monitoring and diagnosis system, capable of handling uncertain data and incomplete information, is used [13]. Even though the condition monitoring maintenance facility-focused approach improves the lifespan and operation of a facility, it involves unpredictable maintenance periods and cannot detect easily fatigue or uniform wear failures.

Finally, reliability-centered maintenance is a concept of maintenance planning that ensures maintenance works are performed in an efficient, cost-effective, reliable, and safe manner. Its successful implementation leads to an increase in cost-effectiveness, reliability, facility uptime, and a greater understanding of the level of risk that the facility owner is managing [11].

Besides the type of philosophy is used, maintenance demands some understanding of the structure and operation of the facility and of the general concepts of diagnosis and maintenance works. What is more, maintenance activities should be carried out according to a strategic plan, which must be consistent with the vision, mission, and objectives of the utility, owner of the engineering facilities. In that sense, a maintenance strategic plan considers strategic choices made concerning organization structure, maintenance methodologies, supporting systems, and outsourcing. Once selections are done, yearly long and medium-term plans are designed considering utilities' goals achievement, and maintenance tools and workforce availability. These are followed by a short-term plan where maintenance activities are scheduled for implementation followed by performance measurements for continuous feedback improvement, as shown in Figure 2.2 [14].

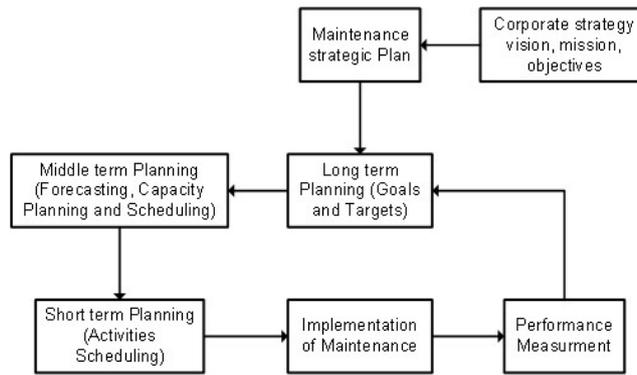


Figure 2.2 Maintenance Strategy Plan [14].

To be more precise, long-term maintenance planning involve periods from 5 to 10 years and aims to identify how the maintenance strategy will align itself with the goals and targets of the utility which owns the electricity facilities. Middle-term maintenance planning cover a period from 1 to 5 years and includes the use of forecasting and planning tools used to understand how external factors can affect the achievement of the goals and when maintenance should be scheduled. The short-term maintenance planning stresses the strong dependency between maintenance resource management and scheduling, and how their interaction influences the achievement of the maintenance goals in monthly, weekly, or daily basis. In the implementation stage, the highly technical and intense labour nature of maintenance activities is recognized. Therefore, strategic decisions related to the organization and scheduling of maintenance works, planning of maintenance resources, and availability of workforce can be made effectively. In that way, the unavailability of the facilities due to maintenance can have a minimum effect on the utilities' operation and a measurement of the maintenance results can be performed to evaluate the maintenance strategy. It is important to recognize that each step in the maintenance strategy plan represents an input to the next step. The performance of the maintenance is used as input to the long-term planning to adjust, if necessary, the strategy of the utilities to achieve their goals so that maintenance activities are viewed as an investment, rather than a costly activity.

Maintenance scheduling plays a very important part in electrical power systems since operation and planning activities are directly affected by it [15, 17]. In power systems, major components like generators and transmission lines, require periodical maintenance. The main decision variables in the generation and transmission maintenance scheduling (GTMS) problem are usually the availability

and operation status of electricity facilities and power generation outputs [9]. This dissertation deals specifically with planned preventative maintenance during medium-term planning, to prevent electricity facilities failure or breakdown. The costs of electricity facilities' downtime resulting from avoidable outages can reach values ten or more times greater than the ones corresponding to their maintenance costs [13]. Still, preventive maintenance makes electricity utilities incur in considerable expenses and its coordination in a market environment presents a challenge to the ISO when planning and operating the electricity system. It requires top equipment, skilled labour, constant stocking of replacement parts and, more importantly, coordination mechanisms among all the utilities participating in the electricity market [13].

2.2 Power Systems Problems

The electricity industry is focused on three main activities, namely generation, transmission, and distribution [15, 16]. Traditionally, the industry has been organized in two general structured manners. Figure 2.3a illustrates the first model which corresponds to the traditional monopoly, where the utility integrates all the activities mentioned above. The flow of electrical energy goes from the monopolistic utility to consumers, who pay for the electricity supply. On the other side, Figure 2.3b shows a deregulated power system, with many generation companies (GENCOs) competing to sell energy to many distribution companies (DISCOs) using the network of few transmission companies (TRANSCOs). The economic transactions and interactions among these players take place in a wholesale electricity market. An independent system operator (ISO) oversees the secure operation of the system by coordinating the generation activities of GENCOs with the evolution of the demand at a minimum cost. At the same time, DISCOs or retailers buy electrical energy on the wholesale market and resell it to consumers who do not wish to participate in the market directly. In this scenario, electrical energy flows from GENCOs to the consumers through TRANSCOs and DISCOs electrical infrastructure. Payments made by consumers go to DISCOs or retailers, who in turn pay for the energy they buy from GENCOs in a wholesale electricity market [13].

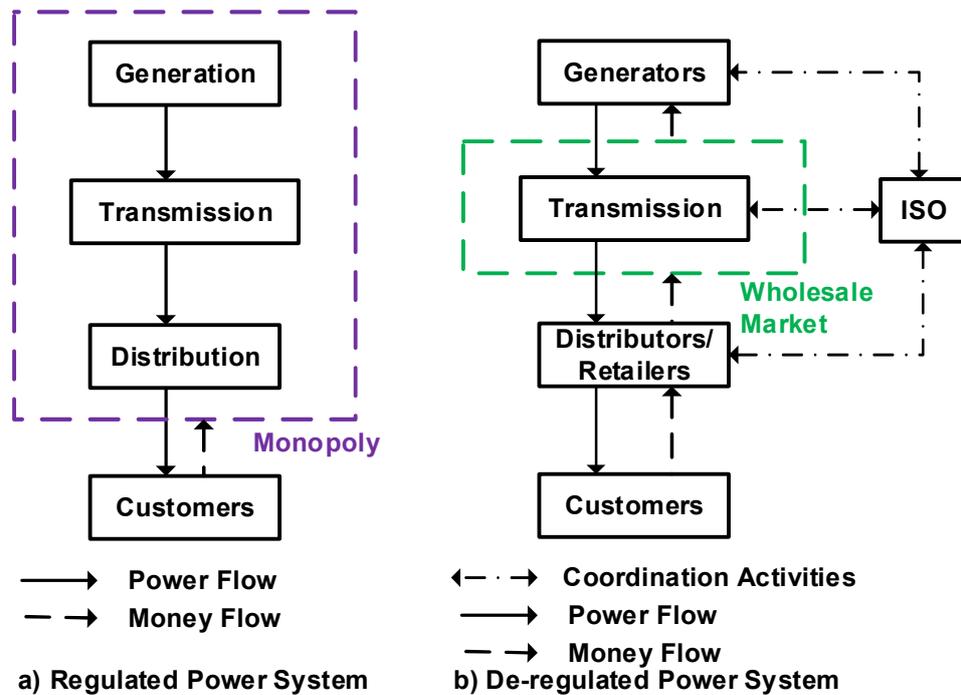


Figure 2.3 Regulated vs deregulated power systems.

Many countries like the United Kingdom, the United States of America, Canada, Australia, New Zealand, Chile, Argentina, Bolivia, Peru, Colombia, and the Scandinavian region, have deregulated electricity industries. South Africa still has a regulated power system that is solely controlled by the parastatal electricity company [17].

At the same time, each electricity industry type has a different impact in terms of generation and transmission maintenance scheduling planning used. In a deregulated power system, where independent GENCOs and TRANSCOs have self-interested objectives, maintenance of generation and transmission facilities need to be coordinated due to systems constraints like network congestion, losses, energy demand, and system reliability requirements; to find optimal or feasible maintenance schedules [15]. The functional separation of transmission and generation creates operation and planning problems related to scheduling of maintenance works. Additional bottlenecks arise in transmission maintenance as more challenging and complex unit commitment and dispatch problems may need to be solved to comply with the constraints of the system in terms of congestion and losses. Hence, maintenance schedules must be coordinated by the ISO to ensure reliable and economical electricity service. In a deregulated electricity industry environment, the ISO should cancel or reschedule planned transmission

and generation outages and solve conflicts between the GENCOs and TRANSCOs regarding maintenance schedules so that final consumers can enjoy a cheap and reliable supply of electricity at all time.

2.2.1 The Unit Commitment Problem

Unit commitment (UC) optimization is among the most important and critical problems in planning power industry facilities outages. It seeks to determine the optimal schedule and a production level for the available generating units over an interval of time, subject to a given load and system constraints [18]. To “commit” a unit can be interpreted as turning a generation unit on or bringing it into operation so that it can deliver power to the network [19]. The reason for not simply committing all the available power generating units, to ensure full satisfaction of the demand, is the exorbitant cost of keeping a power generating unit online unnecessarily.

Usually, the objectives of the UC problem include minimizing operating costs, minimizing emissions, or maximizing the demand satisfaction capability [20]. The objective of minimizing operating costs typically consists of minimizing production, maintenance, start-up, and shut-down costs [20,19]. Classical optimization methods that have been used to solve the UC problem include extensive enumeration, heuristic priority list (merit order) scheduling, dynamic programming (DP), Lagrangian relaxation and the branch-and-bound (B&B) methods.

The priority listing method initially arranges the generating units based on the lowest variable operation cost. The predetermined list is then used for UC such that the system load is always satisfied during the analysis [20, 21]. The application of the method is straightforward, but as the number of units increases in the system, it becomes more difficult to apply.

Dynamic programming is the earliest optimization-based method to be applied to solve the UC problem. It breaks into stages the problem so that the searching process of the optimal solution can be done either in a forward or backward

direction, meaning that the search can start at the beginning or at the end of the period of analysis. Its essence is for the total running cost of supplying a certain amount of MW of load with Nt generating units to be a minimum, the load on MW carried by unit j must be such that the remaining load on MW is carried by the remaining units also at minimum cost. It has the advantage of being able to solve problems of a variety of sizes and easily modified to model characteristics of specific electricity facilities [18], [20]. Dynamic programming is relatively easy to add constraints that affect system operation at a given time. Its main disadvantages are the requirement to limit the commitments considered at any time and its suboptimal treatment of minimum up and downtime constraints and time-dependent startup costs [21].

In [22] an approach for solving the UC problem based on the B&B method is presented, which incorporates all time-dependent constraints and does not require a priority ordering of units. To use it, the objective function and the constraints of the UC problem must be defined, stating the linear and the integer variables, as shown in expressions (2.1), (2.2), and (2.3):

$$F = \text{Min} \sum_{t=1}^T \sum_{j=1}^{Nt} c_j^g (V_{j,t} P_j^{g,\text{min}} + P_{j,t}^g) \quad (2.1)$$

Subject to the following load balance equality constraint:

$$\sum_{j=1}^{Nt} (V_{j,t} P_j^{g,\text{min}} + P_{j,t}^g) = \sum_{n=1}^N P_{n,t}^d + f_t^{\text{loss}}(P_{j,t}^g) \quad ; \quad \forall t \quad (2.2a)$$

And subject to inequality constraints related generation capacity limits and integer variables:

$$0 \leq P_{j,t}^g \leq P_j^{g,\text{max}} - P_j^{g,\text{min}} \quad ; \quad \forall t, \forall j \quad (2.2b)$$

$$P_{j,t}^g - V_{j,t} (P_j^{g,\text{max}} - P_j^{g,\text{min}}) \leq 0 \quad ; \quad \forall t, \forall j \quad (2.2c)$$

$$-P_{j,t}^g - W_{j,t} (P_j^{g,\text{max}} - P_j^{g,\text{min}}) \leq 0 \quad ; \quad \forall t, \forall j \quad (2.2d)$$

$$-V_{j,t} + W_{j,t} \leq 0 \quad ; \quad \forall t, \forall j \quad (2.2e)$$

Where:

$P_{j,t}^g$	Variable power output of thermal unit j (MW).
$P_{n,t}^d$	Power Demand at node n at time t (MW).
$P_j^{g,max}$	Maximum Capacity of thermal unit j (MW).
$P_j^{g,min}$	Fixed minimum Capacity of thermal unit j (MW).
$f_{l,t}^{loss}(P_{j,t}^g)$	Network power losses at time t as a function of generators power output (MW).
N	Number of nodes in the system.
Nt	Number of Thermal Units in the system.
$V_{j,t}, W_{j,t}$	Availability binary variables of thermal unit j at time t [0, 1].
T	Period under analysis.

Since the present dissertation addresses the GTMS problem in the medium-term, constraints related to start-up, downtime and ramping limits generation units' constraints need to be considered. The advantage of the B&B approach is that it solves efficiently large size mixed integer-linear programming problems (MILP). The GTMS problem should ideally be solved in conjunction with the UC problem, but it is often solved independently, in which case the GTMS problem's solutions are used to formulate availability constraints in the UC problem [15].

2.2.2 The Economic Dispatch Problem

The Economic Dispatch (ED) problem seeks to determine the optimal output from the available generation units to meet the expected electricity demand at the lowest possible cost, subject to various electrical system constraints. Several techniques are available to solve the ED problem. They depend on the characteristics of the power system, the nature of the generation technologies, and the assumptions made before solving the problem. Important aspects of this optimization problem are the generation cost of individual units, which are not proportional to the generation level of the corresponding units, how the power systems are geographically spread out, the transmission losses, the demand patterns, the generation units' availability, commitment, and the power flows within the system. As a matter of fact, the ED problem is typically modeled as a subproblem of the UC problem [21].

In that sense, the objective function of the ED optimization problem can be expressed as the minimization of total fuel cost over a given period, so that units' power outputs become the variables of the following optimization problem [23]:

$$F = \text{Min} \sum_{t=1}^T \sum_{j=1}^{N_t} F_{j,t} = \text{Min} \sum_{t=1}^T \sum_{j=1}^{N_t} [C_{0j} + C_{1j}P_{j,t}^g + C_{2j}(P_{j,t}^g)^2] \quad (2.3)$$

Subject to equality constraints related to the energy balance in the system:

$$\sum_{j=1}^{N_t} P_{j,t}^g = \sum_{n=1}^N P_{n,t}^d + f_t^{\text{loss}}(P_{j,t}^g) \quad ; \quad \forall t \quad (2.4a)$$

And subject to inequality constraints related to generation capacity limits:

$$P_j^{g,\text{min}} \leq P_{j,t}^g \leq P_j^{g,\text{max}} \quad ; \quad \forall t \quad (2.4b)$$

Where:

$P_{j,t}^g$ Unit j power output at time t (MW)

C_{0j}, C_{1j}, C_{2j} Generation cost quadratic curve coefficients of unit j in (US\$/h), (US\$/MWh) and (US\$/MW² h) respectively.

In terms of the GTMS problem, the following optimization techniques were considered, studied, and used during the elaboration of the research project.

a) Lambda-Iteration Method for single node system

In a Lambda Iteration Method (LIM), lambda λ is a Lagrange multiplier variable introduced while solving the constrained ED optimization problem by linearizing equation (2.3). This is done by formulating a Lagrange function as follows:

$$L = \sum_{j=1}^{N_t} F_{j,t} + \lambda (\sum_{j=1}^{N_t} P_{j,t}^g - f_t^{\text{loss}}(P_{j,t}^g) - \sum_{n=1}^N P_{n,t}^d) \quad ; \quad \forall t \quad (2.5)$$

The derivatives of the function with respect to each power output $P_{j,t}^g$ and λ results on a system of equations known as optimality conditions of the problem [21]:

$$\frac{\partial L}{\partial P_{j,t}^g} = \frac{\partial F_j}{\partial P_{j,t}^g} + \lambda \left(1 - \frac{\partial f_t^{loss}(P_{j,t}^g)}{\partial P_{j,t}^g} \right) = 0 \quad ; \quad \forall j \quad (2.6a)$$

$$\frac{\partial L}{\partial \lambda} = \sum_{j=1}^{Nt} P_{j,t}^g - f_t^{loss}(P_{j,t}^g) - P_{n,t}^d = 0 \quad (2.6b)$$

To account for transmission losses, the losses coefficients of a network are used, which relate the power output of each generation unit with the power losses in the system [24]:

$$f_t^{loss}(P_{j,t}^g) = \sum_{j=1}^{Nt} \sum_{k=1}^{Nt} P_{j,t}^g B_{j,k} P_{k,t}^g + \sum_{j=1}^{Nt} P_{j,t}^g B_{oj} + B_{oo} \quad ; \quad \forall t \quad (2.7)$$

Where:

$B_{j,k}, B_{oj}, B_{oo}$ Transmission network loss coefficients

The optimal dispatch is reached when the incremental costs of running each unit are equal:

$$\frac{\partial F_1}{\partial P_{1,t}^g} + \lambda \frac{\partial f_t^{loss}(P_{1,t}^g)}{\partial P_{1,t}^g} = \frac{\partial F_2}{\partial P_{2,t}^g} + \lambda \frac{\partial f_t^{loss}(P_{2,t}^g)}{\partial P_{2,t}^g} = \dots = \frac{\partial F_j}{\partial P_{j,t}^g} + \lambda \frac{\partial f_t^{loss}(P_{j,t}^g)}{\partial P_{j,t}^g} = \lambda \quad ; \quad \forall t \quad (2.8)$$

So that:

$$\lambda = \frac{\partial F_j}{\partial P_{j,t}^g} \left(\frac{1}{1 - \frac{\partial f_t^{loss}(P_{j,t}^g)}{\partial P_{j,t}^g}} \right) \quad (2.9a)$$

$$\frac{\partial f_t^{loss}}{\partial P_{1,t}^g} = 2 \sum_{k=1}^{Nt} P_{1,t}^g B_{j,k} + B_{o,j} \quad (2.9b)$$

Therefore, to solve an ED problem the above system of equations must be solved sequentially, applying the LIM algorithm detailed in Figure 2.4 as follows [21]. First initial values $P_{j,t}^g$ of generation outputs are assumed so that the demand of the system is met. Then, the incremental losses corresponding to each generation output $\frac{\partial f_t^{loss}}{\partial P_{j,t}^g}$ are found. Next, the value of power losses is updated and the new values of λ and generation outputs $P_{j,t}^g$ are determined using the set of equations corresponding to the optimality conditions. The old and new values of power

outputs are compared with each generator and, if they are less than a tolerance value ϵ , the process ends; otherwise, it is repeated until the condition is fulfilled.

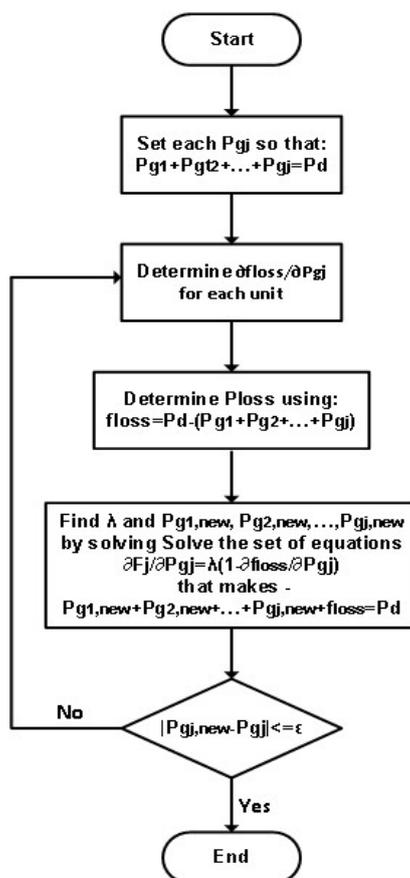


Figure 2.4 Lambda Iteration Algorithm considering power losses [21].

The main advantages of this technique are that it is easy to implement, and the results can be found in few iterations. However, the technique is not suitable when considering constraints related to network congestion and hydro-generators with water storage capacity.

b) Linear Programming for thermal ED with network constraints and losses

When dealing with network constraints, an ED becomes an optimal power flow problem which is solved using linear programming techniques related to Dual-Simplex method. In such a problem, the objective function and the respective constraints are formulated as linear functions. In case of the objective function, the new linear formulation used is shown in equation (2.10):

$$F_t = \text{Min} \sum_{t=1}^T \sum_{j=1}^{Nt} c_j^g P_{j,t}^g \quad (2.10)$$

Where:

c_j^g Unit j marginal generation cost (US\$/MWh)

The demand balance and the generation capacity limit constraints can still be represented as shown before in equation (2.4a) and inequality (2.4b) respectively. Furthermore, to add the transmission lines capacity limits, the following set of constraints are added to the problem:

$$[f]_t = [\beta]_t \left[[Sg][P^g]_t - \left([P^d]_t + \frac{1}{2} [St][r][f^2]_t \right) \right] \leq [\bar{f}] \quad ; \quad \forall t \quad (2.11)$$

Where:

$[f]_t$ Vector of transmission lines power flows at time t (MW).

$[\beta]$ Sensitivity matrix relating the increase of power flows in transmission facilities with a marginal increase of power injection in a particular node.

$[P^d]_t$ Vector of nodal demand at time t (MW).

$[P^g]_t$ Vector of power output of thermal generation units at time t (MW).

$[Sg]$ Incidence matrix that relates nodes and generators in the system.

$[St]$ Incidence matrix relating connections with their respective nodes.

$[r]$ Vector of resistances of transmission lines and transformer (pu).

To consider the losses quadratic function in the problem, two approaches can be considered. The first one, involves determining power losses in a line in terms of the power flowing through it from one of its nodes to the other. In that sense, by making the necessary simplifications to the power flow equations to have a DC-flow [25, 26], power losses $f_{l,t}^{loss}$ in a particular line l can be expressed as shown in equation (2.12):

$$f_{l,t}^{loss} = r_l (f_{l,t})^2 \quad ; \quad \forall t \quad (2.12)$$

Where:

$f_{l,t}$ Power flow in line l at time t (MW).

r_l Resistance of line l (pu).

From this equation, it is possible to represent the power losses component of equation (2.4a) as additional nodal loads, with a magnitude of half the value of the power loss of a line that connects two nodes, as shown in Figure 2.5.

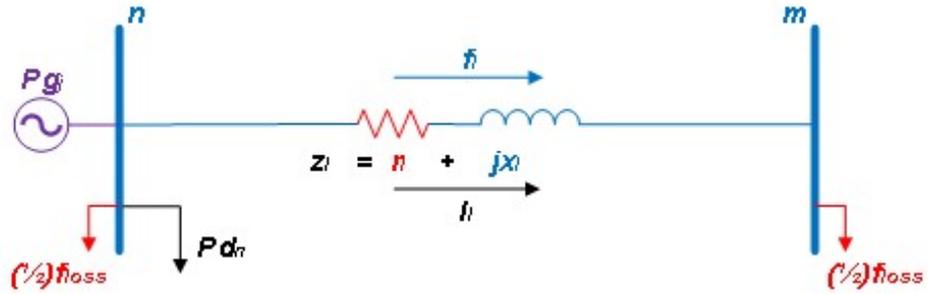


Figure 2.5 2-Transmission facility's power flow with losses.

$$[f^{loss}]_t = \frac{1}{2}[\emptyset][f^2]_t = \frac{1}{2}[|St|][r][f^2]_t \quad ; \quad \forall t \quad (2.13)$$

Where:

$[f^{loss}]_t$ = Vector of transmission losses assigned as additional loads to each node at time t (MW).

$[\emptyset]$ = Power Losses Auxiliary Matrix.

Therefore, to consider power losses in the power flow calculation, a DC flow is performed first to determine the initial power flows in lines and the power losses as well. Then, power losses are included as additional loads to the respective nodes in the network and a new and final DC flow is performed under this new condition [27, 28].

A second way to tackle the losses problems involves using successive linearization around a point in the equality constraint (2.4a) related to the quadratic power loss function [27]. The power losses' quadratic function can be linearized using Taylor expansion as follows [29]:

$$[f^{loss}(Pg)]_t = [f^{loss*}]_t + \left[\frac{\partial f^{loss}(Pg)}{\partial Pg} \right] [Sg]([Pg]_t - [Pg*]_t) \quad ; \quad \forall t \quad (2.14)$$

Deriving the power losses on expression (2.13) with respect to the power output results in equation (2.15):

$$\left[\frac{\partial f^{loss}(P^g)}{\partial P^g} \right] = \frac{1}{2} [\Phi] 2 [f^*]_t \left[\frac{\partial f}{\partial P^g} \right] = [\Phi] [f^*]_t [\beta]_t = [\varphi]_t \quad (2.15)$$

Where:

$[\varphi]_t$ = Nodal power losses sensitivity matrix at time t .

$[f^*]_t$ = Initial power flow vector on each line at time t (MW).

By manipulating equations (2.15) and (2.14), it is possible to rewrite the constraint in equation (2.4a) as follows:

$$\sum_{j=1}^{Nt} P_{j,t}^g + \sum_{i=1}^{Nh} P_{i,t}^h = \sum_{n=1}^N P_{n,t}^d + \sum_{n=1}^N \left\{ [f^{loss^*}]_t + [\varphi]_t [Sg] ([P^g]_t - [P^{g^*}]_t) \right\}; \forall t \quad (2.16)$$

In the same way, transmission line capacity limits constraint in (2.11) can be re-expressed as follows:

$$[\beta] \left[[Sg] [P^g]_t - ([P^d]_t + f^{loss^*}) + [\varphi]_t [Sg] ([P^g]_t - [P^{g^*}]_t) \right] \leq [|\bar{f}|]; \forall t \quad (2.17)$$

This last set of inequalities represent the linear cuts that are added to the optimization problem after each iteration until a condition related to a tolerance value ε is met [29]:

$$[\varphi]_t [Sg] ([P^g]_t - [P^{g^*}]_t) < \varepsilon \quad ; \quad \forall t \quad (2.18)$$

c) Bender Decomposition for hydro-thermal ED with network constraints and losses

The objective of the hydro-thermal power dispatch is to schedule the water turbinated by cascade hydroelectric units with water storage regulation capacity in their reservoirs so that the minimum operation cost is achieved during a period under analysis. This involves solving an ED problem in different instants of time, which are coupled together by water reservoir equilibrium constraints [29]. The

decision tree shown in Figure 2.6 represents the possible consequences of the use of water in time.

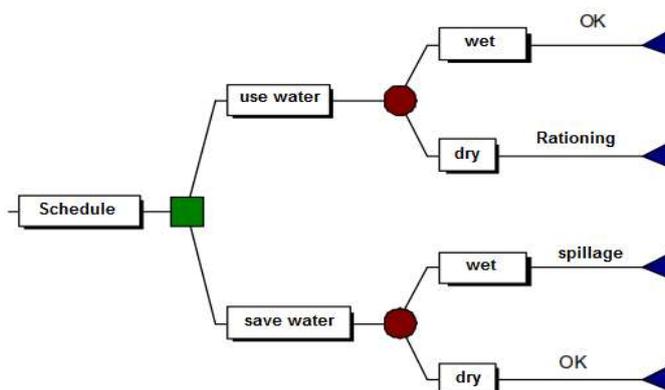


Figure 2.6 Decision tree of a hydro-thermal power system [29].

The ISO has the choice to use the water today to generate electricity to achieve a reduction of the thermal generation costs or to keep the water for future use. If the decision is to use the water in the present time and expect high water inflows in the reservoirs in the future due to a wet season, then the operation is efficient. However, if the water inflows are very low due to a dry season, enough water will not be able to be stored in the reservoirs, so more and expensive thermal generation might be used. Moreover, if the present decision is to keep water for its future use by using today more thermal generation, and if the future water inflows are high, then spillage of water or waste of stored energy will happen. However, if there are very low water inflows in the future, the water storage in the dams could be used to generate cheaper electricity in the future and even prevent the rationing of electricity.

In that sense, hydrogeneration has an opportunity cost of water associated to the thermal generation displaced in time. As it is seen in Figure 2.7, if a larger amount of water is used during the present stage the immediate cost (ICF), associated to thermal generation, and the water stored for the future decrease, increasing the future operation cost or cost to go function (FCF), and vice versa. Then an optimum decision in terms of operation cost of the system must be found by minimizing the sum of the immediate cost and cost to go functions during the planning horizon.

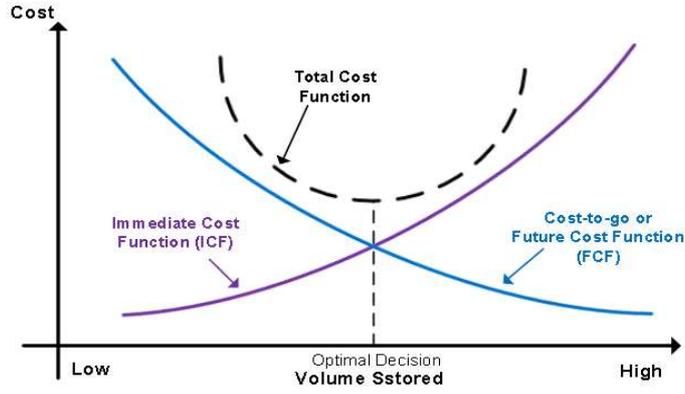


Figure 2.7 Immediate and cost-to-go functions in a hydro-thermal power system at a given time.

To include all these aspects in the hydro-thermal ED problem, the objective function on expression (2.19) can be re-written as:

$$F_t = \text{Min} \sum_{j=1}^{Nt} c_j P_{j,t}^g + \alpha(v_{t+1}) \quad ; \forall t \quad (2.19)$$

At the same time, the optimization problem needs to be complemented by adding the hydroelectrical generation variables and new constraints related to reservoirs water balance and hydroelectrical units' capacity limits into the formulation as follows:

$$\sum_{j=1}^{Nt} P_{j,t}^g + \sum_{i=1}^{Nh} P_{i,t}^h = \sum_{n=1}^N P_{n,t}^d + \sum_{n=1}^N f_{n,t}^{\text{loss}}(P_{j,t}^g, P_{i,t}^h) \quad ; \forall t \quad (2.20a)$$

$$v_{e,t} + \sum_{b=1}^{Nb} \left[\sum_{i \in \Phi_e}^{Nh} u_{i,b,t} - \sum_{i \in \Theta_e}^{Nh} u_{i,b,t} \right] + s_{e,t} - \sum_{i \in \Psi_e}^{Ne} s_{e,t} = v_{e,t}^o + a_{e,t} ; \forall e, \forall t \quad (2.20b)$$

$$[\beta]_t \left[[Sg][P^g P^h]_t - \left([P^d]_t + [f^{\text{loss}*}]_t + [\varphi][Sg]([P^g P^h]_t - [P^{g*} P^{h*}]_t) \right) \right] \leq [|\bar{f}|];$$

$$\forall l, \forall t \quad (2.20c)$$

$$P_j^{g,\text{min}} \leq P_{j,t}^g \leq P_j^{g,\text{max}} \quad ; \quad \forall j, \forall t \quad (2.20d)$$

$$P_i^{h,\text{min}} \leq P_{i,t}^h \leq P_i^{h,\text{max}} \quad ; \quad \forall i, \forall t \quad (2.20e)$$

$$u_i^{\text{min}} \leq u_{i,t} \leq u_i^{\text{max}} \quad ; \quad \forall i, \forall t \quad (2.20f)$$

$$v_e^{\text{min}} \leq v_{e,t} \leq v_e^{\text{max}} \quad ; \quad \forall e, \forall t \quad (2.20g)$$

$$0 \leq s_{e,t} \leq \infty \quad ; \quad \forall e, \forall t \quad (2.20h)$$

$$P_{i,t}^h = u_{i,t} \rho_i \quad ; \quad \forall i, \forall t \quad (2.20i)$$

Where:

- $P_{i,t}^h$ Hydro Generation of unit i at time t (MW).
 $v_{e,t}$ Final volume of reservoir e at time t (Hm³).
 $v_{e,t}^o$ Initial volume of reservoir e at time t (Hm³).
 $a_{e,t}$ Water inflow in reservoir e at time t (Hm³).
 $u_{i,t}$ Hydro unit i water discharge at time t (Hm³).
 $s_{e,t}$ Water spilled of reservoir e at time t (Hm³).
 Φ_e Set of hydro units connected to reservoir e (Hm³).
 Θ_e Set of hydro units upstream to reservoir e (Hm³).
 Ψ_e Set of reservoirs upstream to reservoir e (Hm³).
 $\alpha(v_{t+1})$ Cost to go Function in the future for using water in the present (US\$).
 ρ_i Productivity factor of hydro unit i (MW/Hm³).
 $[P^h]_t$ Vector of power output of hydro generation units (MW).
 v_e^{min} Minimum Volume of Reservoir e (Hm³).
 v_e^{max} Maximum Volume of Reservoir e (Hm³).
 u_i^{min} Minimum volume of water turbinated by hydroelectrical unit i (Hm³).
 u_i^{max} Maximum volume of water turbinated by hydroelectrical unit i (Hm³).

The formulation of this ED problem takes into account hydroelectrical units with water storage capacity only, ignoring pumped-storage or run-of-river hydro generators who are not consider at all during the present analysis.

To tackle this coupled in-time ED problem, two classical optimization approaches are used the most in the literature. The first involves the use of dynamic programming. It requires the period under analysis to be divided into intervals denominated stages. Then, using a recursive calculation technique, the best decision for each stage is found according to a defined objective function [16, 30]. The optimality of each decision is based on Bellman Optimality Principle, which implies that an optimal policy has the property that whatever the initial stage and initial decision are, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision [21].

To apply these concepts, the hydro-thermal ED problem is broken up into small subproblems, each one corresponding to a period or stage. The optimal decision

of the current stage depends on a series of decisions related to water used for hydro generation in future stages. The future decisions depend strongly on the current decision, and both have a direct impact on the overall total operation cost. At the same time, the water levels in each reservoir are discretized in states and all possible combinations of hydro-thermal power outputs that render a particular final water volume in each reservoir at each state are analyzed. This discretization results in a linear piecewise approximation to the real cost-to-go function. Using this approach, the ED problem is solved from the last decision stage using a recursion process. The optimal solution in a stage balances the decision on that stage with future stages. Consequently, the immediate operation costs are the thermal generation costs necessary to meet the load in a stage. Part of the demand is supplied by hydro generation according to future operation decisions, and the remaining is met using thermal generation. The disadvantage of dynamic programming is that each reservoir water level at each stage needs to be discretized in states. Thus, as the number of reservoirs increase, the number of possible states and variables in the problem increase as well. These, in turn, increases the computational calculation effort to solve the ED problem exponentially [31].

To tackle this “curse of dimensionality” problem associated with dynamic programming, a dual dynamic approach can be used. This approach is based on the observation that the cost-to-go function can be represented as a piecewise linear function, which does not need the discretization of the water levels in the reservoirs. The slope of the cost-to-go function around a given point corresponds to the expected water cost, which is given by the simplex multipliers associated with, the water balance constraints stated in (2.20b). In that sense, first, a forward iteration is made without any consideration of the amount of water being used. After reaching the last stage, a backward iteration is done, adding piecewise linear cost-to-go cuts, relative to each reservoir, as new constraints to the optimization problem. These cuts are shown in expression (2.21) [32].

$$\alpha(v_{t+1}) + \vartheta_{e,t+1} v_{e,t} \geq F_{t+1} + \vartheta_{e,t+1} v_{e,t}^* \quad ; \forall t, \forall e \quad (2.21)$$

Again, a new forward iteration is done with the new constraints or cuts added during the last backward iteration. In that way, for each stage, cost-to-go functions are created for each reservoir as shown in Figure 2.8 [29].

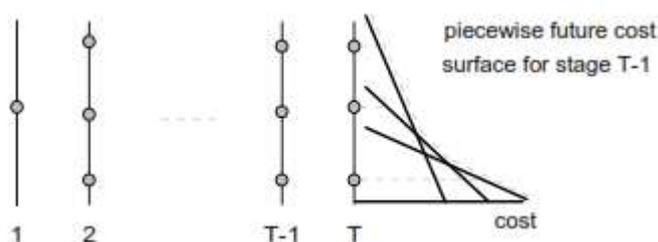


Figure 2.8 Calculation of the piecewise cost-to-go function on stage $T - 1$ [29].

The process ends when the total cost of the economic dispatch of the last forward iteration is equal to the total operation cost determined during the last backward iteration. The main advantage of this approach is that it allows many reservoirs to be considered in the system without the need of increasing the computation effort during the calculation. However, when dealing with water inflows to reservoirs, stochastic variables are added to the problem. Thus, when planning the system operation to solve a hydro-thermal ED problem, a set of historical water inflows to reservoirs are needed to apply forecasting methods and determine an expected total cost of operation of the system during a given period of time.

2.3 Stochastic Nature of Renewable Energy

Long-term hydro-thermal-wind operation planning is, by nature, a stochastic problem as the incremental water inflows to reservoirs and the wind speeds are uncertain. This leads to the use of Stochastic Dynamic Programming (SDP) or Stochastic Dual Dynamic Programming (SDDP) techniques to solve the ED problem. In either case, the way literature deals with these uncertainties is broad.

2.3.1 Water Inflows Forecast Methods

The inflow to the reservoir is seasonal and depends on the variation in the rainfall pattern, due to climate change. Specific tools and techniques from different

disciplines are required to face the stochastic nature of inflows and making decisions and real-time reservoir operation rules for existing or proposed projects. Forecasting of water inflow to a new reservoir and modeling of the historical behavior of an existing reservoir can be accomplished through different simulation models.

For instance, in [30] a set of monthly historical water inflows in a reservoir are adjusted to a normal distribution. The probability of each water inflow scenario is determined and used to find the expected cost-to-go function in each state during the dynamic programming calculation. In [31], a Hull algorithm is used to solve the SDP for each water inflow scenario and an expected cost-to-go function is found as well. In the same way, [33] uses Monte Carlo analysis to sample the different water inflow scenarios when calculating the cost-to-go function.

On the other side, forecasting modeling approaches are used to make hydrological models to provide input data to the SDDP or SDP. For example, [34] uses the Hydrologiska Byråns Vattenbalansavdelning (HBV) hydrology model to optimize the use of the hydro resource over time, given a current reservoir volume, fuel prices, consumption, and the expected water inflow its uncertainty. Due to the autocorrelation effect on the long-term storage of hydro resources and of snow accumulation, the effect of the initial hydrological condition can be seen well into the next year of the forecasted period. On the other hand, the parameters that characterize the water inflows like mean, standard deviation, asymmetry, and temporal correlation, usually present a periodical behavior during a year. That is why in [35], these sequences are analyzed using periodical autoregressive models (ARP) of order 1, so that all the present and past correlation information among water inflows are held in the correlation with the previous period of analysis. In this way, the autocorrelation is reduced exponentially in the model as the number of linear autoregressive periods of analysis increases. However, a potential drawback of using a periodic model in this application is that the model often requires the use of a substantial number of parameters.

In [36], an Artificial Neural Network (ANN) approach for forecasting long-term reservoir inflow using monthly inflow available data and applying a Levenberg-Marquardt Back Propagation (LMBP) algorithm has been used to develop a model

for the Sultan Mahmud hydropower reservoir in Malaysia. In developing the ANN model, different networks with different numbers of neuron hidden layers were tested. To evaluate the accuracy of the proposed model, the Mean Squared Error (MSE) and the Correlation Coefficient (CC) were employed. The results show that in the end, the network could forecast the testing data set with great accuracy. However, an important disadvantage with this approach is the need for training of the neural network and the testing that needs to be done to check results.

In [37], a Monte-Carlo approach was applied to study the water inflow impact on the performance of both single and multi-reservoir systems. In doing so, artificial statistics for the monthly inflow time series of each reservoir system and the probability distributions, quantitative reliability, vulnerability, and resiliency standards were analyzed in different simulation and optimization models considering dams in the Karum river in Iran. The results of the operation criteria analysis indicated that, for the operation of the whole system, the best quantitative reliability, vulnerability, and resiliency values were in the optimized single-reservoir model, and the best time reliability value was in the optimized multi-reservoir model. Moreover, the inflow uncertainty had a minimum and maximum impacts on the reliability and resiliency criterias respectively.

The main impacts of climate change have been addressed globally over the last decades. As far as water resources are concerned, a warmer climate would accelerate the hydrological cycle and alter the intensity and timing of rainfall. The resulting decline in rainfall may reduce net recharge and can affect water levels. A decrease in winter precipitation could reduce the total seasonal precipitation and can impose greater water stress [38]. In that sense, [39] considers these issues by applying a first-order stationary Markov model, which assumes a stationary mean, standard deviation, and lag 1 correlation for the historical and generated water inflow data for the simulation of annual stream flows and reservoir inflows. Seasonal models are introduced, in which the seasons can be any intra-annual period variation and the inherent periodicity adds on stationarity in the data. The model is tested in the Karappara stream flow and Idamalayar Reservoir inflow in the Kerala state in India. The results show that the first-order Markov model addressing the non-stationarity is a better alternative recommended for this situation since it models reservoir management problems

and optimizes water usage through SDP. This model can be effectively used in hydrological design to define the capacity of a reservoir and in simulations of the performance of a reservoir by using several sequences of generated data.

2.3.2 Wind Speeds Forecast Methods

Many methods for predicting wind speeds in the short term have been published in the past. Probability models are proposed in the literature for addressing the natural variability of wind resources which include air and terrain characteristics, mean, cut-in and cut-out wind velocities, Weibull probability distribution function parameters to predict wind velocities in a period.

For example, in [40] an hour-by-hour optimum schedule of hydro-thermal-wind power (HTWGS) stations is used to attain the least emission of pollutants from thermal plants and a reduced generation cost of thermal and wind plants for a 24-hour period, while satisfying the system constraints. The paper presents a detailed framework of the HTWGS problem and proposes a modified particle swarm optimization (MPSO) algorithm for finding a solution. Appropriate mathematical models are used to represent the water discharge, generation cost, and pollutant emission of the power stations incorporated in the system. Statistical analysis is performed by adjusting wind speeds historical data to a Weibull probability density function.

In the same way, the study in [41] presents a framework to assess the wind resource potential of a wind turbine using uncertainty analysis. It uses probability models for tackling the natural variability of wind resources that include air density, mean wind velocity and associated Weibull parameters, surface roughness exponent, and error for prediction of long-term wind velocity based on the measure-correlate-predict method that depends in a Monte-Carlo based numerical procedure. The model can effectively evaluate the expected annual energy production for different averaging periods and confidence intervals. However, the main issue is that the level of uncertainty in the assessment of wind energy potential can be reduced with detailed wind data, which is sometimes difficult to

come by. They may be included, as characteristics of individual wind resources found from short-term measurements in site, in terms of probability parameters.

In the same way, [42] presents a comparison between two methods used for estimating the annual energy production (AEP) for a 60 MW installed power wind farm based on real data recorded during the year 2014 in the Nord-East Region of Romania. The first method supposes a direct estimation of the energy production that does not imply any statistical distribution assumptions, being based on recorded data and the wind turbines power curve; while the second one is based on the Weibull distribution of the wind speed, considering linear and quadratic models to model the power curves. For this study, wind speeds were measured using a meteorological station. The wind speed values corresponding to the hub heights of the considered turbines were determined based on the power law expression for the variation of wind speed with height. For the AEP evaluation, polynomial, linear and quadratic mathematical models are considered. It is shown that the AEP determined with the polynomial model is the closest to the real values and is considered as a reference. What is more, for choosing the proper wind turbine type for a given location, both approaches consider not only the turbine hub height, rated power, cut-in, and cut-off wind speeds or the shape of the turbine power curve, but also if the wind speeds are mostly distributed within the cut-in and rated wind speeds limits. That is why the main disadvantage of these methods lies when they deal with low wind speeds since the appropriate mathematical model need to be chosen beforehand to reduce forecasting errors.

2.4 Electricity Markets

The main characteristic of most electricity industries today is a free market on the sale and purchase of electrical energy, for which the desegregation of the industry in generation, transmission, distribution, and retail independent parts is a necessary condition. The activities of transmission and distribution are run by TRANSCOs and DISCOS respectively as natural monopolies and, as such, regulation must come into place to set fair tariffs. The activities of generation and retail parts are open to free-market practices in a competitive environment, as long as GENCOs and retailers have open access to the transmission and distribution

networks. An adequate operation of an electricity market requires the following conditions to be fulfilled [16]:

- Economic efficiency of the electricity industry.
- Self - sustainability of the electricity industry to guarantee the expansion of the system.
- Operation of the system with a high degree of reliability according to the quality of the electricity service required by society.
- Universal access to electricity services to people in a society.

To fulfill these market requirements, any electricity industry at least needs to have an ISO and a Regulator. Besides the ISO already mentioned responsibilities regarding the operation of the electricity system according to quality and reliability requirements and to the use of all the energetic resources to operate the system at a minimum cost, it additionally very often plans the operation of the system in the medium and short terms and manages the electricity market determining the economic transactions among market participants and hiring ancillary services for the safe and economic operation of the system [43]. Planning is a crucial activity that involves forecasting the future demand of the system, defining the expansion of the network, pointing out the potential generation projects, and determining the maintenance schedules of electricity facilities that do not affect the supply of electricity to final consumers.

On the other side, Regulator has the attributions to fix the prices for the use of services that are captured in a natural monopoly, set the quality standards of the electricity service, stimulates the economic efficiency of the market, and promote the universalization of the electricity service. It also controls the evolution of prices in the market preserving economic equilibrium and fairness.

The fundamental principle of economic efficiency in an electricity market is valuing the production of electricity using marginal costs or the approximate incremental cost of supplying and additional MW of electricity demand. [44]. The way the power flows and losses in a transmission network are handled when dealing with network constraints allows the identification of two main

methodologies to remunerate generation and transmission activities and to charge distribution companies and retailers in the market. Besides the economic concepts used and the way transmission network is represented, these methodologies correspond to when the a-priori or a-posteriori solving of economic dispatch and unit commitment problems take place with respect to the real-time operation of the system. Since the present dissertation deals with maintenance schedule planning, the a-priori electricity market model is analyzed.

In this approach, the energy prices are calculated before system operation in real-time has been made, considering forecasted demand and water inflows, the availability of generation and transmission facilities, volume of water in reservoirs, the potential renewable energy generated, the variable generation costs of GENCOs. The model works as follows:

First, a slack bus in the system is defined as a reference node that is used to determine the effect of power losses and the system marginal cost. Next, assumptions are made in the power flow equations so that a DC optimal power flow can be used to solve the ED problem with the objective function stated in equation (2.19) and the constraints formulated in equations (2.20). No consideration is given to reactive power flows and nodal voltages to simplify the calculations. Later, the dual multipliers related to constraints (2.20a) and (2.20c) are determined and correspond to the system marginal cost and transmission line congestion cost, respectively. Dual multipliers are outputs of the linear programming technique and reflect the sensitivity of the optimal solution towards a marginal relaxation of the constraints the multipliers are related to. In that sense, dual multiplier π_t represents the increase of the system operation cost for a marginal increase in the demand of the system at any node. In the same way, dual multiplier $\mu_{l,t}$ represents the decrease of the system operation cost for a marginal increase of the maximum capacity of transmission line l . As such, these parameters have a direct impact on the electricity prices of the nodes interconnected by transmission lines. Since a marginal increase in the load involves a marginal increase in losses of the system, the system marginal price is affected by the penalty factors on each node of the system and, as a result, nodal electricity spot prices can be expressed as follows [45]:

$$[\tau]_t = [\pi]_t[NF]_t + [\mu]_t^T[\beta]_t \quad (2.22)$$

Where:

$[\tau]_t$ Vector of nodal energy prices at time t (US\$/MWh)

$[\pi]_t$ System marginal cost vector at time t (US\$/MWh)

$[NF]_t$ Vector of nodal penalty factors at time t

$[\mu]_t$ Congestion cost vector at time t (US\$/MWh)

$[\beta]_t$ Sensibility Matrix at time t

The penalty loss factor in the first term of expression (2.22) represents the effect that transmission losses have in the electricity prices. It is determined in terms of the following equation [46]:

$$NF_{n,t} = 1 + \frac{\partial f^{loss}}{\partial P_n^d} = 1 + 2 \sum_{l=1}^{Nl} r_{lf_{l,t}} \beta_{nl} \quad ; \forall t \quad (2.23)$$

Where:

$NF_{n,t}$ Nodal Penalty Factor in node n at time t .

β_{nl} Sensibility Matrix row corresponding to node n .

Economic transactions among GENCOs and DISCOs in a system are determined in terms of the way the network constraints are handled by the ISO while settling the market [43]. In general, in a market with nodal electricity prices, the amount of money that DISCOs expend buying electricity at nodal prices is more than the money GENCOs receive from selling their electricity generated at those nodal prices. In that sense, two main approaches are defined to settle the economic transactions in the market.

2.4.1 Management of the market with marginal price setting

When managing the market under this methodology, the way the optimal power flow problem is handled is important to define how the economic transactions will take place. First, unconstrained UC and ED problems considering losses are solved sequentially to determine the merit order generation dispatch, the unconstrained system operation cost F_t^u at a given time t and the nodal electricity spot prices

$[\lambda]_t$. The GENCOs' revenues R_t^u and the DISCO's or retailers' payments E_t^u are determined by multiplying the generated or consumed energy by the electricity spot price of the node through which they are connected to the system. Due to the power losses, the amount of money paid by DISCOs is greater than the amount of money received by GENCOs. This difference, or merchandising surplus MS_t , correspond to the revenue of the TRANSCO [43]:

$$MS_t = E_t^u - R_t^u \quad ; \forall t \quad (2.24)$$

Next, the system marginal cost is identified from the dual multiplier $\pi_{b,t}$ corresponding to the ED problem equality constraint of equation (2.22) related to demand balance. Then, transmission line capacities are considered, and UC and ED problems are solved again and the constrained system operation cost F_t^c determined. If there are congested lines operating, their associated dual multipliers $\mu_{l,t}$ will have an impact in the nodal electricity spot prices. As a result, an extra cost or uplift U_t is generated for keeping the system under an optimal and secure operation:

$$U_t = F_t^c - F_t^u \quad ; \forall t \quad (2.25)$$

This cost is allocated to each DISCO or retailer in a proportion γ to its electricity demand and added to its already defined payments under unconstrained system conditions to determine adjusted payments [44]:

$$E_t = E_t^u + \gamma U_t \quad ; \forall t \quad (2.26)$$

In the same way, GENCOs revenues are adjusted by identifying the units that, because of the constrained network, were forced to generate extra or less energy in comparison to the unconstrained system scenario [44]. An extra revenue ΔR_t is defined by charging the forced energy at a rate equal to the generation marginal cost of the corresponding units and it is added to their already found revenues:

$$R_t = R_t^u + \Delta R_t \quad ; \forall t, \forall j \quad (2.27)$$

Finally, to check that the allocation of economic resources among utilities in the electricity market has been done correctly, the final balance of the transactions must equal zero [44]:

$$Balance_t = E_t^u - R_t - MS_t + U_t = 0 \quad ; \forall t \quad (2.28)$$

The main drawback with this market transaction approach is that for any congestion, the uplift determined is shared by all loads in proportion of their values, no matter if they are or not responsible for network congestions in any part of the network.

2.4.2 Management of the market with nodal electricity prices setting

In this approach, constrained UC and ED problems are solved and the nodal electricity prices are determined. GENCOs are remunerated and DISCOs pay for electricity according to an electricity spot price $p_{n,t}$ at node n , through which they are connected to the network:

$$R_t = \sum_{j \in n}^{Nt} P_{j,t}^g \tau_{n,t} + \sum_{i \in n}^{Nh} P_{i,t}^h \tau_{n,t} \quad ; \forall t \quad (2.29), \quad E_t = \sum_{n=1}^N P_{n,t}^d \tau_{n,t} \quad ; \forall t \quad (2.30)$$

Since the nodal electricity prices can be decomposed as shown in equation (2.22), the merchandising surplus MS_t or revenue of the TRANSCO can be found as shown in equation (2.31) [43]:

$$MS_t = \sum_{j \in n}^{Nt} P_{j,t}^g \pi_t N F_{n,t} + \sum_{i \in n}^{Nh} P_{i,t}^h \pi_t N F_{n,t} - \sum_{n=1}^N P_{n,t}^d \pi_t N F_{n,t} \quad ; \forall t \quad (2.31)$$

The congestion cost or uplift can be also determined by finding the amount of money that DISCOs are charged by this concept:

$$U_t = \sum_{j \in n}^{Nt} P_{j,t}^g (\mu_{l,t} \beta_{l,t})_{n,t} + \sum_{i \in n}^{Nh} P_{i,t}^h (\mu_{l,t} \beta_{l,t})_{n,t} - \sum_{n=1}^N P_{n,t}^d (\mu_{l,t} \beta_{l,t})_{n,t} \quad ; \forall t \quad (2.32)$$

Finally, to check that the allocation of economic resources in the electricity market is done correctly, the final transactions' balance must equal zero [43]:

$$Balance_t = E_t - R_t - (MS_t + U_t) = 0 \quad ; \forall t \quad (2.33)$$

It is important to realize that the uplift cost collected from the consumer payments is not allocated to the TRANSCO, any GENCO, and neither to the ISO since this will generate a perverse incentive in the market to make extra income from congestions. This surplus is collected separately so that it can be invested in transmission facilities to solve congestions problems.

2.5 Multi-Objective Optimization Problems (MOOP)

2.5.1 Definition

A MOOP represents an optimization problem that has two or more conflicting objective functions, that need to be optimized simultaneously, and their corresponding set of constraints. As such, it can be represented mathematically as show in equations (2.34) and (2.35) [47]:

$$\text{Min/Max } f_m(x_i) \quad ; \quad m = 1, 2, 3, \dots, M \quad (2.34)$$

$$\text{subject to: } g_j(x_i) \geq 0 \quad ; \quad j = 1, 2, 3, \dots, p \quad (2.35a)$$

$$h_j(x_i) = 0 \quad ; \quad j = p + 1, \dots, J \quad (2.35b)$$

$$x_i^{\min} \leq x_i \leq x_i^{\max} \quad ; \quad i = 1, 2, 3, \dots, I \quad (2.35c)$$

Where:

f_m Objective function m of decision variable x_i

$g_j(x_i)$ Inequality constraint j as a function of decision variable i

$h_j(x_i)$ Equality constraint j as a function of decision variable i

x_i Decision Variable i

x_i^{\max} Maximum Value of decision variable i

x_i^{\min} Minimum Value of decision variable i

M Total Number of Objective Functions

J Total Number of Inequality and Equality Constraints

I Total Number of Decision Variables

The solution space S_s of the problem is defined as all the possible values that the decision variables can have within their stated limits:

$$S_s = \{x_i \in \mathbb{R}: x_i^{min} \leq x_i \leq x_i^{max}; \quad i = 1, 2, 3, \dots, I\} \quad (2.36a)$$

The feasible solutions space vector X is defined as the set of values of the decision variables that fulfil all the constraints stated in the optimization problem:

$$X = \left\{ \begin{array}{l} x_i \in \mathbb{R}: g_j(x_i) \geq 0 \quad ; \quad j = 1, 2, 3, \dots, p \\ \quad \quad \quad h_j(x_i) = 0 \quad ; \quad j = p + 1, \dots, J \\ x_i^{min} \leq x_i \leq x_i^{max} \quad ; \quad i = 1, 2, 3, \dots, I \end{array} \right\} \quad (2.36b)$$

Thus, the different objective functions values inside vector $F(X)$ in a MOOP will depend on the values of each decision variable inside a feasible solution set:

$$f(X) = \{f_1(X), f_2(X), f_3(X), \dots, f_m(X)\} \quad (2.36c)$$

Since the objective functions inside vector $f(X)$ are conflicting with each other, a MOOP does not have just one single optimal solution, but a set of feasible solutions that represent the trade-offs or good compromises among the different objective functions.

2.5.2 Pareto Optimal Set

For a given set of feasible solutions $X_1 \in \mathbb{R}^I$, the objective vector $F(X_1) = \{f_1(X_1), \dots, f_m(X_1)\}$ is Pareto optimal if there is no vector $X_2 \in \mathbb{R}^I$ such that **[49]**:

- $f_m(X_2) \leq f_m(X_1)$ for all $m = 1, 2, 3, \dots, M$
- there exists at least one $m \in \{1, 2, 3, \dots, M\}$ with $f_m(X_2) < f_m(X_1)$

The concept states that the solution of a MOOP is Pareto-optimal if there exists no other feasible solution which would decrease and objective without causing an increase in at least one other objective **[48]**. The set of all solutions with Pareto-optimal objective vectors is called the Pareto-optimal set.

2.5.3 Non-Domination Definition

Any MOOP works with a population of Np candidate solutions or individuals:

$$X = \{X_1, X_2, X_3, \dots, X_n \dots X_{Np}\} \quad (2.36d)$$

If there are two feasible solutions vectors X_1 and $X_2 \in \mathbb{R}^I$ in a population of Np individuals, then X_1 dominates X_2 , $X_2 \succ X_1$, if the next conditions are met [49]:

- $f_m(X_1) \leq f_m(X_2)$ for all $m = 1, 2, 3, \dots, M$
- there exists at least one $m \in \{1, 2, 3, \dots, M\}$ with $f_m(X_1) < f_m(X_2)$

Solution X_1 is non-dominated if all its objectives are better than those of X_2 or when at least one objective of X_1 is not worse than any objective of X_2 [48]. The first front generated by this procedure is the Pareto-optimal set. Following this criteria, further classifications or ranking of other solutions can be performed considering non-domination among their objective functions, so that feasible solutions can be split into further sets or ranks classifying them from the best Pareto-set to the worst. Figure 2.9 illustrates these concepts for a MOOP with two minimization conflicting objective functions and two decision variables.

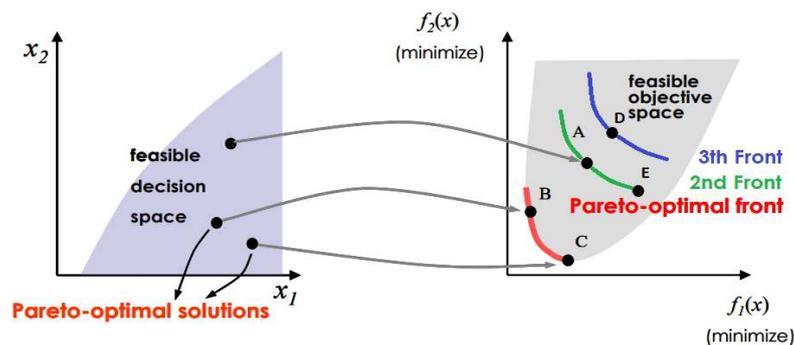


Figure 2.9 Pareto Optimal front of a MOOP with two objective functions and two decision variables.

The plot of the objective functions whose non-dominated solutions are in the Pareto-optimal set is called the Pareto-optimal front. The Pareto-optimal set contains the solutions of a MOOP and gives the decision-maker a picture of the

nature and the characteristics of the problem to undertake a decision-making action in the future to choose the best solution.

2.5.4 Approaches to solve a MOOP

The approach used to solve a MOOP largely depends on the type of variables and the convexity of the problem itself. In the case of the present thesis, the GTMS optimization problem is non-convex in nature. With this kind of problem, it becomes increasingly important to escape the local optima, in search of the global optima, where classical solution techniques easily get stuck. Therefore, certain metaheuristic multi-objective Evolutionary Algorithms (MOEA) techniques became a suitable approach to solve the problem.

These approaches work following the principle of operation of Genetic Algorithms (GA) [50]. A GA is a metaheuristic inspired by the process of natural selection that belongs to the Evolutionary Algorithms family (EA). It relies on genetic-inspired operators, such as mutation, crossover, and selection that are applied to every individual n inside a population of size Np so that they can evolve towards the Pareto-optimal set of solutions. The quality of a MOEA is evaluated in terms of the convergence of the solutions found towards the true Pareto-front and their diversity along that front.

According to [51], MOEAs are classified into different categories according to the elitism present in the methodology and when the preference of the objectives take place during the solution process, shown in Figure 2.10.

In MOEAs a-priori techniques, the decision-maker gives his preference to the objectives before the actual optimization takes place [52]. Some of the technique that fall into this category are Weight-based GA (WBGA), Goal Programming, Non-linear approaches, and Fuzzy Logic methods. WBGA scalarize the objective functions and maintains multiple weight vectors to find multiple Pareto-optimal solutions in one simulation run. It gives a weighted value to each objective function and accumulates them as a fitness value. Even though it is a simple approach, WBGA is useless against non-convex MOOP. Fuzzy logic techniques allow

the quantitative representation of multi-objective decision-making problems which have vague objectives and parameters. As such, they are well-suited to situations where alternatives must be assessed by using criteria that are subjective and of unequal importance. In goal programming (GP) approach, the objectives are formulated as goal criteria that the decision-maker wants each objective to possess. The criteria can be formulated considering objectives to be greater, less than or equal to a utopian solution. The GP problem is to find solutions whose criterion vector best compares with the utopian point.

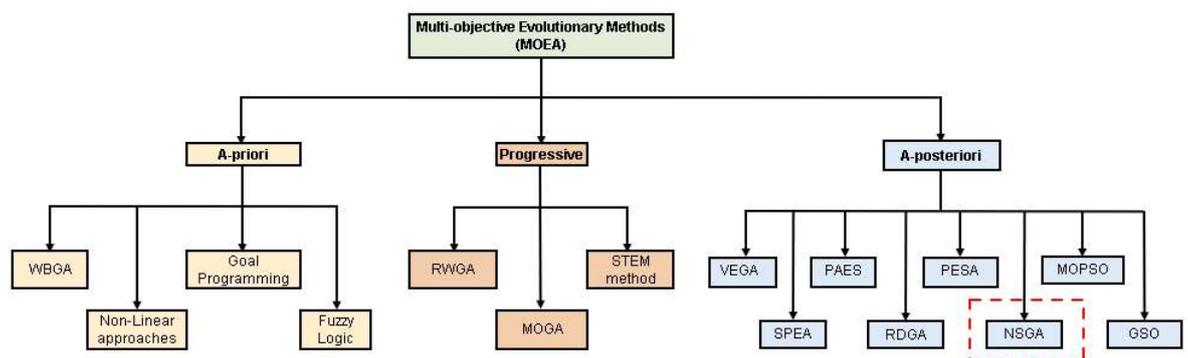


Figure 2.10 Multi-objective Evolutionary Algorithms Techniques.

Progressive methods allow the decision-maker to indicate the preferences for the objectives as the search moves and the decision-maker learns more about the problem [53]. Some techniques included in this category are the Random Weighted Genetic Algorithm (RWGA), STEM method, and Multi-Objective Genetic Algorithms (MOGAs). The STEM method is based on minimizing the Tchebychev distance from the ideal point to the criterion space. The parameters of the distance formula and the feasible space can be changed by a normalized weighting method based on the decision-maker's preferences in the previous solution. The method allows the decision-maker to recognize good solutions and the relative importance of the objectives. On the other side, RWGA uses the weights to build a single objective function. The weights are changed frequently during running time to allow the searching directions to be changed to sweep over the whole solution space. To do so, random numbers are generated to build the weights of the objectives and the solutions, found through changing directions, are collected in to build the Pareto-optimal set. Multi-objective genetic algorithms (MOGAs) rank solutions according to non-domination criteria to determine average fitness. These fitness values are scaled with a niche count computed using a sharing

parameter that preserves diversity of solutions. Application of non-dominated concepts and implementation of a sharing function to assign fitness values to the objectives are two important aspects on these algorithms [54].

The main advantages of these methods are that they are computational lighter than other alternatives. Additionally, they involve a learning process where the decision-maker gets a better understanding of the problem. Since the decision-maker is actively involved in the search it is likely that he accepts the final solution. However, their main disadvantages are the degree of effort required from the decision-maker during the entire search process. Moreover, the solution depends on his preference and if he changes his preferences or if a new decision-maker comes into play then the process must be restarted.

In a-posteriori methods, first the search space is scanned and then the Pareto optimal set of solutions is identified and presented to the decision-maker who, using a decision-making tool, can choose the best solution. Some of the well-known MOEAs in this category are the following. Vector Evaluated Genetic Algorithm (VEGA) consists of a simple GA with a modified selection mechanism that, in each iteration, generates a number of sub-populations by performing a proportional selection according to each objective function [52]. These sub-populations are shuffled together to obtain a new population, on which the GA would apply the crossover and mutation operators in the usual way. In the same way, Pareto Archived Evolution Strategy (PAES) uses a one-to-one evolution strategy (a single parent generates a single offspring) together with a historical histogram-like archive that records all the nondominated solutions previously found to use it as a comparison set. Furthermore, the Strength Pareto Evolutionary Algorithm (SPEA) [55] uses a secondary population to store the non-dominated solutions and cluster mechanism to ensure diversity. The improved version of this method, called SPEA 2 [56], incorporates a fine-grained fitness assignment strategy, nearest neighbor density estimation technique and enhanced archive truncation method to obtain the non-dominated solutions. The Pareto Envelope-based Selection Algorithm (PESA) uses features from both SPEA and PAES. The difference is attributed on how PESA integrates selection and diversity using a hyper grid-based crowding scheme. It employs a smaller internal population and larger external population. Whereas the external population stores the existing Pareto front approximation,

the internal population comprises new candidates competing for inclusion in the external archive [57]. In the same manner, Rank-Density Based Genetic Algorithm (RDGA) uses a cell-based density approach to convert a general bi-objective problem into a bi-objective optimization problem with the objectives to minimize the individual rank value and density of the population [58]. Multi-objective Particle Swarm Optimization extends the traditional particle swarm optimization (PSO) to deal with multi-objective optimization problems. It uses the concept of Pareto dominance to determine the flight direction of a particle maintaining previously found nondominated vectors in a global repository that is later used by other particles to guide their own flight [59]. Non-dominated Sorting Genetic Algorithm (NSGA) includes a non-dominated sorting methodology and manages to obtain a set of different solutions with objective function vector values that, after each iteration, get closer to the Pareto optimal front [49]. NSGA II is a first improvement of NSGA. Its computational complexity is lower, includes a fast non-dominated sorting and an elitist component in its methodology, and incorporates a sharing parameter to find spread solutions along a front [60]. A final improvement named NSGA III works well with problems that have four or more objective functions. It uses a set of reference points to maintain the diversity of the Pareto points during the search. This results in a very even distribution of Pareto points across the objective space. NSGA III can maintain better coverage of Pareto solutions by using a reference point mechanism [61], [62]. Group Search Optimizer with Multiple Procedures (GSOMP) is inspired by the producer-scrounger animal behavior model, which assumes that group members search either for “finding” (producer) or for “joining” (scrounger) opportunities. Based on this framework, animal scanning mechanisms, are employed in a sense to design optimum searching strategies for solving continuous optimization problems [63]. The main advantages of all these methods are that the solutions are independent of the decision-maker preferences and that the process of optimization is performed only once, so that Pareto-optimal set does not change if the problem description remains unchanged. However, these methods need large number of computations to be performed so they are computationally expensive. After a set of Pareto-optimal solutions is found the decision-maker need to use decision-making tools to choose the best solution. Table 2.1 presents a summary of the characteristics of the MOEAs described above and makes a further classification according to the presence of elitism in their structure [64].

Table 2.1 Characteristics of the main Multi-objective Evolutionary Algorithms

No	Type	Method	Fitness Function	Diversity Mechanism	External Population	Advantages	Disadvantages
1	Non - Elitist	VEGA	Sub-populations are formed and evaluated in terms of different objectives.	None	No	First MOEA with straightforward implementation.	Fitness is in terms of one objective, convergence results to individual set of champion solutions.
2		MOGA	Pareto Ranking based on number of individuals dominating other.	Fitness sharing by niching in objective space is parameter dependant .	No	Simple extension of single objective GA with easy fitness assignment scheme.	Slow Convergence due to different fitness assignment in individuals inside the same front.
3		WBGA	Weighted average of normalized objectives	Niching and weighting values are parameter dependant .	No	Simple extension of single objective GA	Difficulties in dealing with nonconvex objective functions , requires appropriate selection of weight vectors.
4		NPGA	No fitness assignment is made, tournament selection decides individuals who pass the next generation.	Parameter dependant niche count used as tiebreaker in tournament selection.	No	No need for specifying fitness values for each solution.	Problems related to parameter dependency with niche mechanism and subpopulation size for tournament selection.
5		NSGA	Ranking based in non-dominated and population size.	Fitness sharing by niching with parameter dependency	No	Fast convergence due to a fair assignment of fitness based on non-dominated sets.	Problems related to niche size parameter, high computation complexity.
6		GSOMP	A population individual, located in the most promising area, leads and directs the search for the optimal solution.	Niching is performed using average distances and standard deviation among differently crowded areas.	Yes	Improved exploitation of better areas based on information of individuals better positioned in the search space.	Slow convergence and easy to get trapped in local optima .
7	Elitist	RWGA	Weighted average of normalized objectives.	Randomly assigned weights and replacement of population with external individuals	Yes	Simple extension of single objective GA.	Difficulties in solving problems with nonconvex solution space .
8		DPGA	Fitness is assigned according to distance from individuals to the elite set.	None	Yes	Convergence and diversity among solutions is achieved with the fitness function formulation.	Since the elite population size is not restricted, the computational complexity increases as iteration progresses.
9		SPEA	Ranking based on the external archive of non-dominated solutions.	Free parameter-dependency clustering technique to truncate external population.	Yes	Adequate pressure to force population to the optimal front while keeping diversity.	Complex clustering algorithm and dependency on the size of the external population .
10		SPEA 2	Fine-grained fitness assignment strategy which incorporates density information.	Alternative truncation method is used to replace the clustering technique.	Yes	Individuals of external population participate in the mating selection process.	Computationally expensive fitness and density calculation.
11		PAES	No fitness assignment made, non-domination criteria decides individuals that pass to the next generation.	Hypercube division of objective space made to identify less crowded areas.	Yes	Direct control done through mutation in the diversity mechanism allows better handling of problems with non-uniform distributed solutions.	Hypercube size dependency. Side-length of a hypercubes depend on previous knowledge of maximum and minimum values of objective functions .
12		NSGA II	Ranking based in non-dominated and crowding distance.	Crowding distance based on the Euclidean distance between neighbouring individuals.	No	Free niching parameter tuning and good convergence properties for problems with non-uniform dense and diverse solutions.	As number of objective functions increase, crowding distance becomes computationally expensive.
13		NSGA III	Ranking based in non-dominated sorting and niching operator	Niching operator based on reference points and distance from these points to solutions.	No	Capacity to handle MOOP with more than 4 objective functions and non-convex solutions .	The number of reference points is related to the size of the population. If better results are expected, high number of reference points should be provided, with an increase in the computation effort.

Elitism plays a major role in the performance of a MOEA. An elite-preserving operator in a MOEAs favours the best solutions by giving them the chance to be directly carried over the next generation of solutions. In this way, good solutions found in early iterations are never lost unless better solutions are discovered. This improves the convergence and diversity in the solutions found by the MOEA.

Because the nature of the multi-objective GTMS problem to be solved, the a-posteriori techniques analysed and used during the research project are NSGA II and NSGA III because they can handle many objectives, present a widely spread of solutions along the Pareto-optimum front and combined with classical optimization techniques, can approach to the set of non-dominated solutions in a faster and efficient way.

2.5.5 Non-dominated Sorting Genetic Algorithm II (NSGA II)

NSGA II carries out non-dominated sorting of a combined parent and child population. Then, starting from the first front or the best non-dominated solutions, each front is accepted to the next generation until all available slots are filled, which enhances the convergence properties towards the Pareto-optimal front. What is more, NSGA II employs a parameter-less diversity preservation mechanism, named crowding distance, to guarantee diversity and spread of solutions without the use of sharing parameters. It estimates the density of solutions in the objective space to guide the selection process towards a uniformly spread Pareto-front. A general NSGA II procedure is implemented by following the next steps [60]:

1. A random parent initial population P_t , of size Np , is created.
2. For every individual n in P_t , the fitness function of each optimization problem in the MOOP is evaluated in terms of its objective function and constraints.
3. Using the fitness function values, the population is sorted into different fronts using a non-domination criterion. The first front is a completely non-dominant Pareto-set in the current population. Each individual inside a front is assigned a rank value based on the front in which it belongs to. Individuals in first front are given a rank of 1, individuals in second are assigned a rank of 2 and so on.

Additionally, crowding distance is calculated for every individual in the same front, so that they can be sorted. Large crowding distance values result in better diversity in the population, thus an individual with the highest crowding distance in a front is ranked first, then an individual with the second highest crowding distance in a front is ranked second and so on.

4. In each iteration t an offspring population Q_t , of size Np , is created using binary tournament selection, crossover, and mutation operators.
5. Again, for each new individual inside Q_t the fitness function for all optimization problems is evaluated.
6. A new mating pool R_t , of size $2Np$, is created by combining the parent population P_t and the offspring population Q_t .
7. The combined population R_t is sorted according to non-domination criteria to identify all non-dominated fronts (Fr_1, Fr_2, \dots, Fr_n). Additionally, crowding distance is calculated to measure how close an individual is to its neighbours to rank it inside a given front. Thus, the fitness of each individual is given by its rank inside the population and its crowding distance inside a front.
8. A new parent population P_{t+1} , of size Np , is created by selecting the best Np individuals from the sorted R_t pool. The rest of the individuals are rejected because of their high rank values or their low crowding distance values.
9. The whole process is repeated from step 5 until a maximum number of iterations is reached.

The NSGA II flow chart and diagram with its principle of operation are shown in Figures 2.11 and 2.12.

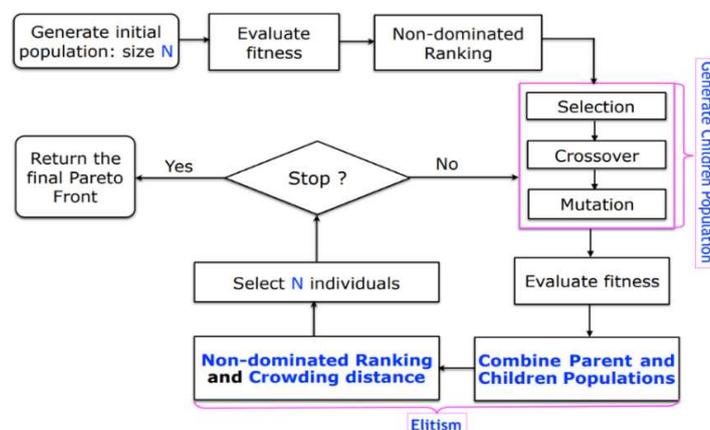


Figure 2.11 Non-dominated Sorting Genetic Algorithm II (NSGA II).

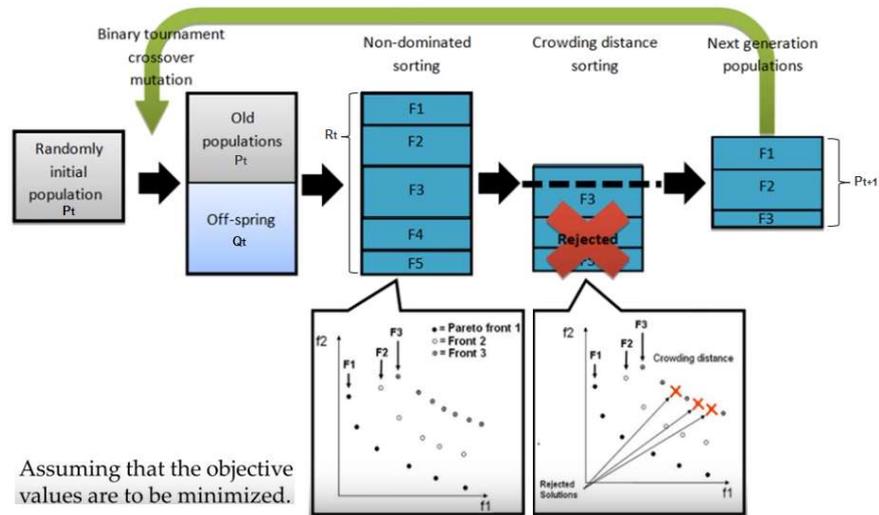


Figure 2.12 Basic Procedure of the Non-dominated Sorting Genetic Algorithm II (NSGA II) [51]

For NSGA II to work properly, the different selection techniques, crossover and mutation operators and non-dominated sorting and diversity mechanisms must be carefully studied and wisely chosen.

a) Crowding Distance determination

A comparison operator is needed to determine which of the individuals, inside a same front Fr_i , are favored. In [60] this operator is defined in terms of the density of individuals surrounding a particular one inside a front. This density is found by calculating the crowding distance. The crowding distance $CD(X_1)$ of an individual $X_1 \in P_t$ to its neighbors inside a front Fr_i is a measure of the size of the largest cuboid containing X_1 and no other solution of the population, as shown in Figure 2.13. The size of the cuboid is dependent to the objective function values of the individuals involved in the process.

The calculation of the crowding distance for $X_1 \in \mathbb{R}^I$ in a front Fr_i is done by sorting the individuals inside a set H according to objective function $f_m(X)$. This is repeated for all objective functions in the MOOP so that every individual will have two neighbors with respect to each objective function. The crowding distance of X_1 is found by adding up the differences between the objective values of the left and the right neighbors with respect to each of the objective functions.

The algorithm to determine the crowding distance (CD) for every individual in a population is shown in Figure 2.14.

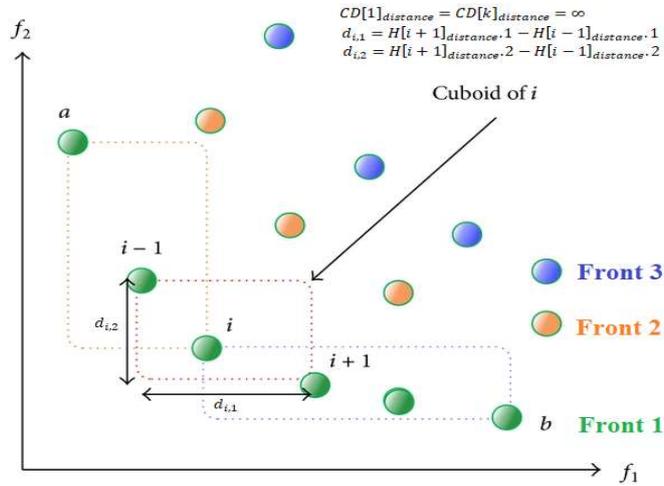


Figure 2.13 Crowding Distance Calculation.

```

k = |Fr|
for each i ∈ Fr
    CD[i] = 0
    for each m ∈ M
        H = sort(F, m) // Sort H with respect to each objective value
        CD[1] = CD[k] = ∞ // Solutions at the boundary are favored
        for each i = 2 to k - 1
            CD[i] = CD[i] + ( (H[i+1].m - H[i-1].m) / (f_m^max - f_m^min) )
    
```

Figure 2.14 Crowding Distance Algorithm.

Bear in mind that $H[i]$ is the i -th element in H and $H[i].m$ is the value of objective function f_m of the i th individual in H . The first and the last individuals in the sorted set H are given a crowding distance value of ∞ . Individuals with high crowding distance values are located in less-crowded regions and, as such they are favored in the sorting process. In this way, individuals of the same rank but with high crowding distance values have more chances of being selected to become parents and produce offspring individuals.

b) Selection mechanism

The selection mechanism determines which individuals are chosen for reproduction and how many offspring each selected individual will produce. The

main principle of selection is that the better is an individual, the higher is its chance of being a parent. An individual is better than other based in non-dominated criteria, rank and crowding distance values, with respect to the individual towards whom it is compared.

In [65] proportional roulette wheel selection mechanism is discussed and analyzed. In this mechanism, individuals are selected with a probability that is directly proportional to their fitness values. The probability of selecting a parent is represented by spinning a roulette wheel. Since the size of the different areas in the wheel are proportional to the parent's fitness, individuals with the largest fitness have more probability of being selected. This mechanism tends to give more weight to individuals with high fitness (super-individuals), which sometime makes the GA convergence early in the process to a local minimum. On the other side, in a rank-based roulette wheel selection mechanism the probability of an individual being selected is based on its fitness rank relative to the entire population [65]. This mechanism first sort individuals in the population according to their fitness and then grants them a rank according to a mapping fitness function. The size of the different areas in the spinning wheel is defined by these ranks. Hence, this mechanism maintains a constant pressure in the evolutionary search introducing a uniform scaling across the population and is not influenced by super-individuals. The mapping function is important for the performance of this selection scheme.

Reference [60] introduces the Binary Tournament Selection mechanism in the NSGA II procedure. Tournament selection is probably the most popular selection method due to its efficiency and simple implementation. It adapts well to solve a MOOP since it considers the ranking process of every individual in terms of its non-dominated status inside a population (front and crowding distance).

In tournament selection N_p individuals are selected randomly from a large population to compete against each other. The individual with the highest rank wins and will be included in a set of Parent individuals to produce the next generation population (offspring). In this sense, the non-dominated definition stated in section 2.4.3 can be complemented with these concepts as follows. If

there are two feasible individuals X_1 and $X_2 \in \mathbb{R}^l$ in a MOO, individual X_1 wins a tournament against X_2 if any of the following conditions are true [60]:

- Individual X_1 has a better rank: $X_2 \succ X_1$
- If they have the same rank but solution X_1 has a better crowding distance than solution X_2 : $CD(X_1) > CD(X_2)$

The number of individuals competing in each tournament, referred as tournament size, is usually set to 2 (binary tournament). This mechanism gives a chance to all individuals to be selected preserving diversity, although keeping diversity may degrade the convergence speed. Figure 2.13 illustrates the mechanism of tournament selection.

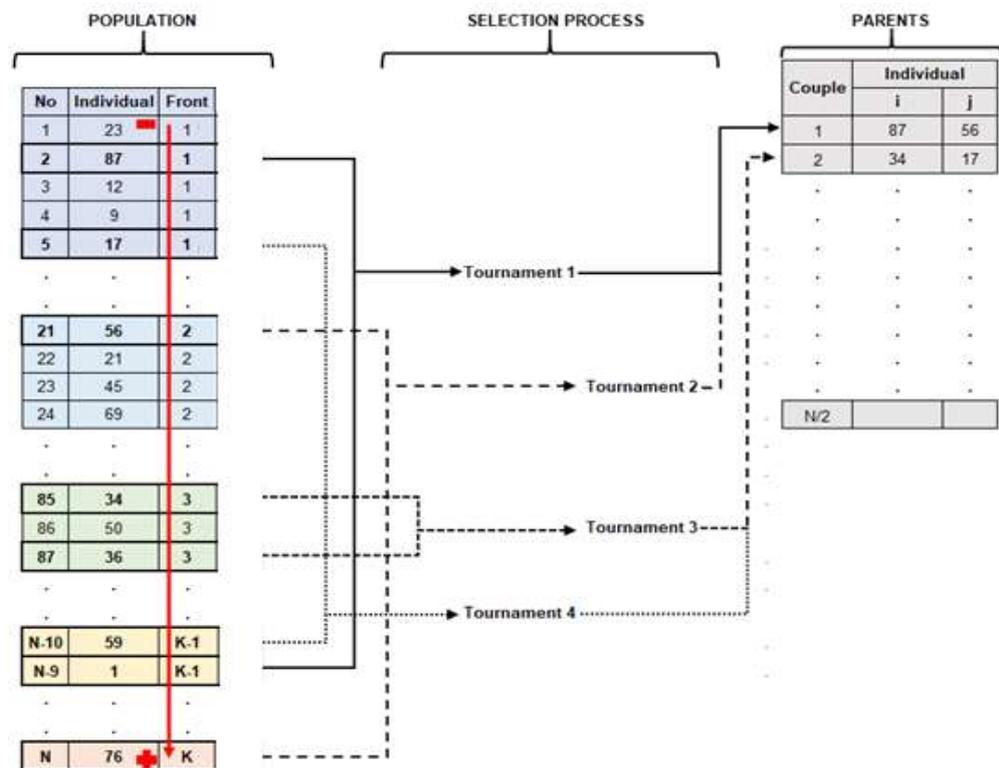


Figure 2.13 Binary Tournament Selection Process.

The tournament selection has several advantages like low time computation efficiency, low susceptibility to takeover by super-individuals and no requirement for fitness scaling [66]. Thus, this method was chosen as the selection operator for the development of the research project.

2.5.6 Non-dominated Sorting Genetic Algorithm III (NSGA III)

As it has been seen, NSGA II uses elitism to create a diverse Pareto-optimal front, has low computational complexity and does not require external parameters to be set before the simulation. Still, as any other MOEA, NSGA II faces drawbacks that compromises its convergence and diversity performances. For instance, if the number of individuals in the first non-dominated front becomes greater than N_p during the evolutionary process, a cycle of generation of Pareto-optimal and non-optimal solutions starts slowing convergence and degrading efficiency. In the same way, elite solutions are emphasized due to the tournament selection and recombination operator. This may lead to a loss of diversity, which generates and excessive selection pressure in a certain region of the search space. Finally, NSGA II uses a parameter-less diversity preservation mechanism, the crowding distance, that guarantees the diversity of solutions without the use of sharing parameters. As the number of objective functions in the MOOP increases beyond 4, it becomes complicated increasingly computational-expensive to determine the crowding distance. With many objective functions to be solved, individuals of a population are likely to be widely separated from each other requiring NSGA II to be complemented with special recombination and niching operators to find offspring solutions near to parent solutions [61].

These aspects are tackled by NSGA III, which is an improved version of NSGA II. The fundamental difference between NSGA II and NSGA III lies in the way the niche-preservation operation is performed. A general NSGA III procedure can be implemented by following the next steps [61, 62]:

1. A random parent initial population P_t , of size N_p , is created.
2. A set H of evenly distributed reference points r_p is generated on in the hyper-plane space defined by the objective functions of the problem.
3. For every individual n in P_t , the fitness function of each optimization problem in the MOOP is evaluated in terms of its objective function and constraints. This is done by applying normalization process determining ideal and extreme solutions, which are used to find a normalised hyper-plane.

4. Non-dominated sorting is applied to divide the population into different non-dominated fronts (Fr_1, Fr_2, \dots, Fr_n).
5. An offspring population Q_t , of size Np , is created using binary tournament selection, crossover, and mutation operators.
6. Parent population P_t and offspring population Q_t are combined to find a new mating set R_t , of size $2Np$. Again, fitness function is determined for individuals inside this new population and non-dominated sorting is applied to determine individuals' fronts.
7. A new population S_t is constructed by selecting individuals of different non-dominated levels one at a time, starting from Fr_1 , until the size of S_t is equal to Np or for the first time becomes larger than N . The first front is a completely non-dominant Pareto-set in the current population.
8. Individuals in $S_t \setminus Fr_1$ are chosen for P_{t+1} and the remaining population individuals are chosen from F_l by the diversity maintenance operator.
9. The perpendicular distance d from each individual in S_t to every reference line w (joining the origin with a reference point r_p) is calculated. Every individual in S_t is then associated with a reference point having the minimum perpendicular distance.
10. A niche-preservation operation is used to select members from F_l to the next generation. It works by identifying the reference points associated with solutions in F_l front that have the minimum number of solutions associated ρ_j . Then, depending on if any solution in the front F_l is associated to these reference points, a solution is added to P_{t+1} to increase the spread of the solutions. This step is repeated until the remaining population individuals of P_{t+1} are filled so that this set has the best Np individuals.
11. The whole process is repeated from step 5 until a maximum number of iterations is reached.

The NSGA III flowchart with the steps described so far is shown in Figure 2.16. As it can be seen, NSGA III employs elitism by storing all nondominated solutions discovered so far, beginning from the initial population, enhancing the convergence properties towards the Pareto-optimal set. NSGA III estimates the density of solutions in the objective space to guide the selection process towards a uniformly spread Pareto-front across a pre-defined set of reference points.

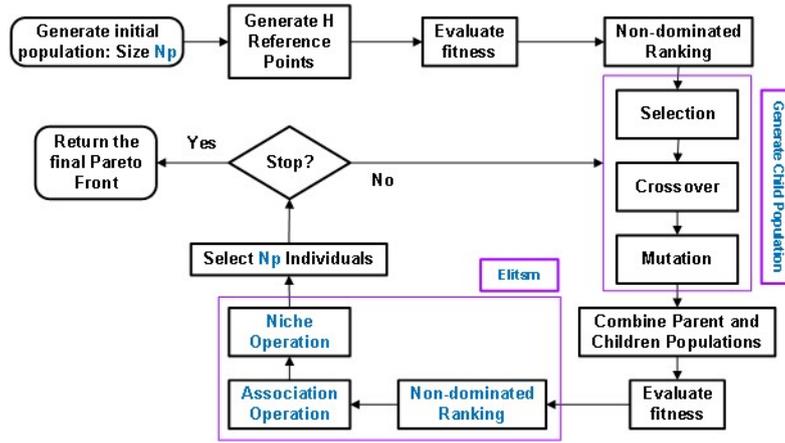


Figure 2.16 Non-dominated Sorting Genetic Algorithm III (NSGA III).

a) Reference Points

NSGA III uses a predefined set of reference points to ensure diversity in the obtained solutions. These points can be either predefined in a structured manner or supplied directly by the user. Usually, the solution of the MOOP that wants to be solved is not known beforehand, so there is no sense in pre-defining a preferred set of reference points. In that sense, a methodology that allows the definition of reference points in a structured fashion needs to be adopted.

For the present research project, the approach stated in [67] is used, which places points on a normalized hyper-plane, a M dimensional unit simplex, that is evenly distributed in the space, equally inclined to all objective axes and has an intercept of one on each axis. If p divisions are considered along each objective, the total number of reference points H in a M -objective problem is given by:

$$H = \binom{M + p - 1}{p} \quad (2.37a)$$

Figure 2.17 shows the distribution of 15 reference points created in a normalized hyperplane for a MOOP with 3 objective functions and 4 divisions. It is interesting to note that NSGA III not only emphasize non-dominated solutions, but also individuals in other fronts which are in some sense associated with each of these reference points. Since the reference points are evenly distributed in the normalized hyperplane, it is expected that each Pareto individual found is associated to a single reference point and, as a result, the front will be widely

distributed across the search space. In that sense, it is suggested that the number of reference points created is equal to the number of individuals in the population:

$$H \approx Np \quad (2.37b)$$

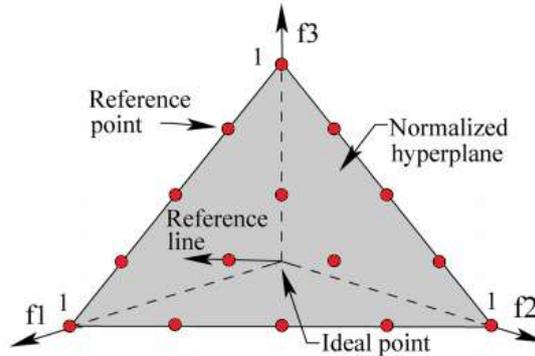


Figure 2.17 Reference points in a 3-dimensional normalized hyperplane with 4 divisions [61].

b) Association Operator

The NSGA III procedure requires to associate each population member with a reference point. For that purpose, a reference line corresponding to each reference point on the hyperplane is defined by joining the reference point with the origin. Then, the perpendicular distance from every individual to each of the reference lines is calculated as depicted in Figure 2.18. The reference points whose reference line is closest to an individual in the normalized objective space is considered associated with that individual [61].

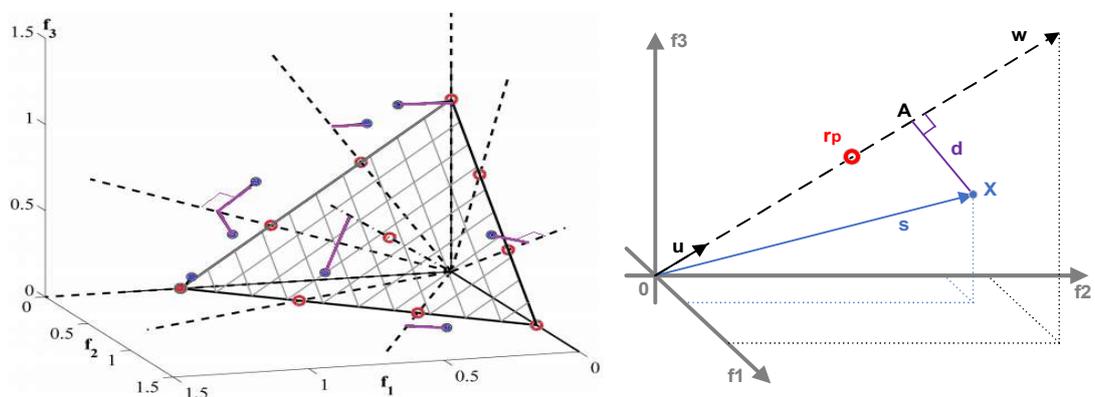


Figure 2.18 Association of Individuals with reference points.

c) Niching Mechanism

A comparison operator is needed to determine which of the individuals inside a same front F_i , are favored to pass to the next generation to have a well spread Pareto front. In NSGA II, a crowding distance operator is defined in terms of the density of individuals surrounding a particular solution inside a front [60]. As it has already be seen, as the number of objective functions in a MOOP gets bigger than 4, it becomes difficult to compute the crowding distance of every individual [61]. This is because the identification of neighboring solutions demands a huge computational effort and any attempt to make computations faster, results in a poor spread of solutions along the Pareto-optimal front [60].

To overcome this issue, NSGA III algorithm uses a niche preservation operator that works as follows [61]. After the non-domination sorting is applied to the combined offspring and parent population ($R_t = P_t \cup Q_t$), the combined population is divided into different non-dominated fronts. Then a new population S_t is constructed by selecting individuals of different non-dominated fronts one at a time, starting from the first front F_1 until the size of S_t is equal to Np or for the first time becomes larger than Np . If the first scenario happens, then the new population is equal to the combined set ($P_{t+1} = S_t$). However, if the second scenario is the case, then the last front F_l is identified and excluded from S_t .

Later, the number of individuals in S_t/F_l associated with each reference point is counted and stored in a niche count ρ_{r_p} . The reference point r_p with the minimum ρ_{r_p} is identified. In case there are more than one of such points, one of them is selected randomly. Under these circumstances two scenarios can happen [61]:

- If $\rho_{r_p} = 0$, there are no associated individuals of P_{t+1} to the reference point r_p . This implies two cases. First, there are one or more point from the F_l front that are associated with r_p . In this case the one with the shortest perpendicular distance from the reference line is added to P_{t+1} and the counter ρ_{r_p} is added by one. Second, front F_l does not have any member associated with the reference point r_p , so this reference point is excluded from further considerations.

- If $\rho_{r_p} > 0$, already one member associated with the reference point exists in S_t/F_t , the point with the minimum distance to the reference point r_p is chosen from front F_t , if it exists, and added to P_{t+1} . The count ρ_{r_p} is then increased by one.

After the niche counts are updated, the procedure is repeated until all vacant population slots of P_{t+1} are filled.

2.5.7 Fitness Function

The fitness function corresponding to every objective function in the MOOP must be determined according to the requirements of NSGA II and NSGA III and considering the different constraints involved in the problem.

a) Constrain Handling Techniques

The evaluation of the fitness of each objective function is a major component in a MOEA to assign a measure of the quality of every individual inside a population, no matter if it represents a feasible or infeasible solution. What is more, at one point in the evolutionary process any MOEA will necessary produce unfeasible individuals that must be evaluated somehow. This evaluation process implies using a methodology to handle constraints in a MOOP and proceed to determine its non-dominated status for comparison.

Consider a search space S consisting of two disjoint subsets of feasible and unfeasible subspaces, Ω and Z , respectively as shown in Figure 2.19. When solving a MOOP the MOEA searches for a feasible global optimum. During the search process it must deal with many feasible and unfeasible individuals in the search space [68]. This action is not easy because the evaluation of the fitness of every individual in a population must consider the impact of the constraints and objective functions in the fitness value. In this way, a proper comparison between individuals in the non-dominated process can be achieved. For instance, finding and comparing the fitness values of two feasible solutions x_1 and x_3 in Figure 2.19 can be done evaluating their objective function and applying non-dominated

definitions. However, finding and comparing fitness values of the two unfeasible solutions y_3 and y_4 requires another approach.

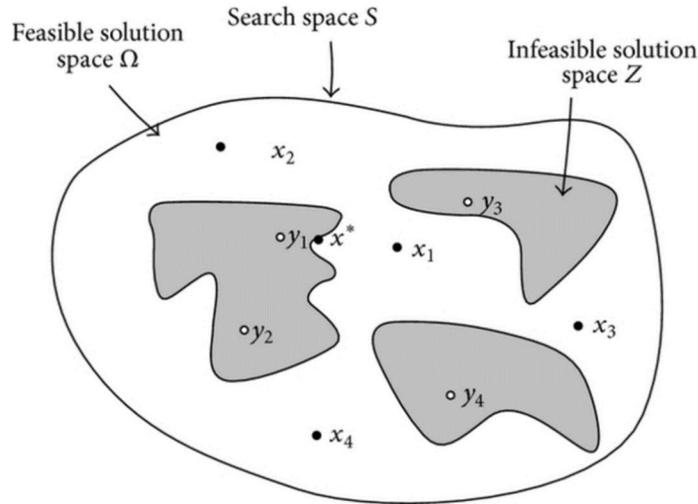


Figure 2.19 MOEA's search space for finding a feasible optimal solution.

It is important to define a mechanism to deal with unfeasible individuals to find their fitness and compare them using non-dominated concepts. Many techniques are available to deal with handling constraints in a MOOP. They combine evolutionary computation techniques with deterministic procedures for numerical optimization and correspond to the following groups:

- **Methods based on penalty functions**

Reference [68] describes several methodologies to penalize infeasible individuals using static, dynamic and adaptive penalty factors. Under these approaches, an infeasible solutions space vector X is defined as the set of decision variables whose values do not comply with at least one of the constraints formulated in the optimization problem:

$$X = \left\{ \begin{array}{ll} x_i \in \mathbb{R}: g_j(x_i) < 0 & ; j = 1, 2, 3, \dots, p \\ h_j(x_i) \neq 0 & ; j = p + 1, \dots, J \\ x_i^{\min} \leq x_i \leq x_i^{\max} & ; i = 1, 2, 3, \dots, I \end{array} \right\} \quad (2.38a)$$

The constraint violation $C(X)$ for each constraint j is calculated as follows:

$$c_j(X) = \begin{cases} g_j(x_i) + h_j(x_i); & \text{if } g_j(x_i) < 0 \text{ or } h_j(x_i) \neq 0 \\ 0 & ; \text{otherwise} \end{cases} \quad (2.38b)$$

All constraint violations are added together to get the overall constraint violation:

$$C(X) = \sum_{j=1}^J c_j(X) \quad (2.38c)$$

This constraint violation is multiplied by each penalty parameter R_m , and the product is added to each objective function value $f_m(X)$ to determine the fitness function $F_m(X)$ value:

$$F_m(X) = f_m(X) + R_m C(X) \quad (2.38d)$$

As a result, the different fitness functions inside vector $F(X)$, associated to their corresponding objective functions, depend on the values of each decision variable inside an infeasible set of solutions:

$$F(X) = \{F_1(X), F_2(X), F_3(X), \dots, F_m(X)\} \quad (2.38e)$$

The fitness function values consider the effect of the magnitude of the constraints violated on the objective functions. This effect is amplified in every fitness function using penalty factors. These factors are usually chosen by the decision-maker and the quality of the MOOP's solutions depend strongly on this choice.

Moreover, [69] proposes a constraint-handling technique for EA using a violation factor. The method works for convex and non-convex MOOPs, does not requires any tuning of a penalty parameter to be performed and keeps the fitness value of each individual equal to its objective function.

- **Search for feasible solutions Methods**

GENOCOP constrain handling technique uses a variation operator which builds linear combination of feasible solutions to preserve their feasibility [70]. In the same way, GENOCOP II is based on dynamic penalties, divides constraints into fours subsets (linear and nonlinear equations and inequalities) and can distinguish between linear and nonlinear constraints [68]. In a complementary manner, GENOCOP III constraint handling mechanism uses GENOCOP in one of its two populations while the other population stores feasible solutions [71]. A special

operator is designed to convert solutions from the first population, which just satisfies linear constraints, into fully feasible solutions.

In the same way, in [72] several handling techniques are also described like ϵ -constrained and stochastics ranking methods. In the first one, a user-defined parameter controls the criterion used for comparison of infeasible solutions based on their sum of constraint violation or on their objective function value. The second one transforms a constrained optimization problem into an unconstrained one. Based on the sum of constraint violation, it uses a relaxation of a limit value to consider a solution feasible; and a lexicographical ordering mechanism in which the minimization of the sum of constraint violation precedes the minimization of the objective function of the problem.

- **Hybrid methods**

In reference [73] a constrain handling technique is proposed that makes pair-wise comparison using a tournament selection operator to devise a penalty function approach that does not require any tuning. A comparison between feasible and unfeasible individuals is developed and the dominance between individuals is set by their feasibility and the magnitude of constrain violation. Dominance-based constraint handling techniques incorporate constraints into the fitness function of a genetic algorithm [74]. The approach does not require the use of a penalty function nor a niching approach to maintain diversity in the population. It determines the dominance status of individuals based on their feasibility and constraint violation magnitude. Constraint-handling method based on Pareto-optimality concept, designed for multi-objective and multi-constraint optimization problems, introduces the idea of non-dominance concept from the objective function space to the constraint function space [75]. What is more, [76] proposes a two-phase approach for solving constraint optimization problems. In the first phase, the objective function is completely disregarded, and the search effort is directed towards finding a single feasible solution. In the second phase, the problem is treated as a bi-objective optimization problem, turning the constraint optimization into a two-objective optimization. The two resulting objectives are the original objective function and the constraint violation degree. In the same way, Self-adaptive Constraint Handling (SACH) technique uses adaptive penalty functions and distance measures of individuals to modify the

objective functions, so that they can be used in the non-dominance sorting [77]. The Adaptive Trade-off Model (ATM) for constrained evolutionary optimization considers the evaluation of infeasible solutions when the population contains only infeasible individuals, balancing of feasible and infeasible solutions and the selection of feasible solutions when the population is composed of feasible individuals only [78]. Finally, an accelerating ATM is presented in [79] which aims to accelerate the ATM search process by using the shrinking space technique, so that the method can converge faster to competitive results without the loss of quality and precision.

b) Objective Functions and Constraints Normalization

The magnitudes and units of the objective functions and constraint violations in a MOOP are very different, making it difficult to compare them. What is more, some objective functions may need to be maximized, while others minimized. Thus, for a better analysis and fair application of constraint handling techniques, it is necessary to transform objective functions and constraints in such a way that all have similar order of magnitudes and to make all objective functions either be maximized or minimized [80]. To do this, it becomes necessary to normalize objective functions and constraints with respect to certain inner parameters, which usually are the maximum and minimum values, or nadir point, of the objectives search space. In the same way, applying duality concepts, maximization objective functions may be transformed into minimization ones multiplying them by -1 and vice-versa. Normalization play an important role in ensuring the consistency of optimal solutions with the preferences expressed by the decision-maker while handling constraints. Ensuring the use of the correct normalization methods plays a crucial factor in finding solutions near and spread along the Pareto-optimal front in a constrained MOOP.

In terms of the NSGA II, the normalization of objective functions and constraints is made using the concepts stated in [77]. Normalization takes place by making all objectives to be minimized and comparing them against their maximum and minimum values, while constraints violation normalization is carried out considering the maximum value of each constraint violation.

Due to the nature of NSGA III, the normalization process of objective functions requires the determination of M extreme points to form a M -dimension hyperplane with positive intercepts in the objective function axis. For instance, in [61] an adaptive normalization of population is considered that needs the computation of the minimum values of each objective function so that a translated objective function is found. An Argument Scalarization Function (ASF) is applied to these objective functions to find extreme points and form the hyper-plane. The intercepts of the hyperplane with the objective function axis are computed and the objective function are normalized considering the intercepts as nadir points. The problem with this approach is that the ASF used in the normalization procedure sometime fails to identify the solutions with the weak Pareto-dominance relation.

To overcome this issue, [60] proposes a modified ASF that considers an additional term in the function and allows the convergence of extreme points better. In the same way, [83] proposes a taxonomy of six different metamodeling ASF approaches for multiple and many-objective optimization problems. The paper describes a parameterized ASF procedure which involves the ASF formulation considering the impact of constraint violation, so the global optimum of the entire population can be targeted during the search.

Additional issues arise with the methodology regarding the calculation of extreme points of the hyper-plane. After identifying the extreme points, the hyper-plane may not be formed due to a degeneration problem related to negative intercepts and non-existence of a unique hyperplane due to linearly dependent extreme points [84]. In order to overcome these difficulties, [81] and [82] propose the use of the maximum values of each objective function as intercepting points in the normalization process.

c) Selected normalization and constraint handling techniques

Due to the non-convex and mixed-integer nature of the GTMS problem of the research project, the normalization procedures stated in [77] is used in combination with NSGA II, while the normalization process stated in [79] [81] and the adjusted ASF stated in [83] are used to obtain well-constructed hyperplanes,

whose intercepts can be used to normalize objective functions in NSGA III. In terms of constraint handling techniques, the SACH in [77] and the ATM in [80] and [79] are selected to deal with infeasible solutions found during the evolutionary process of NSGA II and NSGA III, respectively. With these methods in place, it is expected that infeasible solutions will be processed so that after each iteration, they can approach the feasible space of solutions as shown in Figure 2.20.

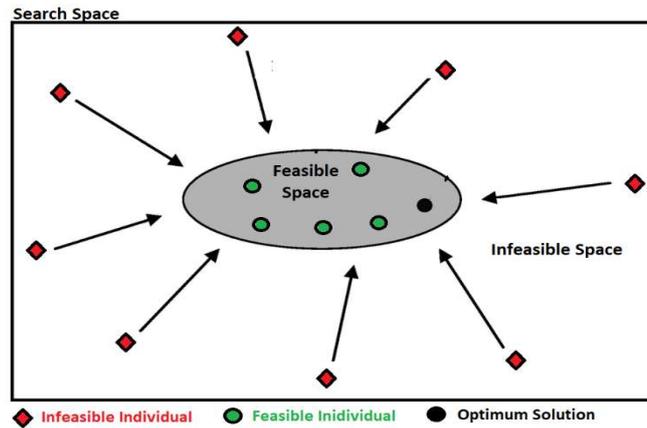


Figure 2.20 Search process of SACH and ATM-ASF Normalization and Constraint handling Techniques.

- **SACH Normalization and Constraint handling Technique**

The SACH technique considers that all objective functions in a MOOP need to be minimized. It exploits the information infeasible individuals carry by defining two components in a fitness function: a distance measure and an adaptive penalty function. To apply this mechanism, first consider a vector or set X of feasible and infeasible individuals inside a population of Np candidate solutions, whose decision variables define their respective objective function values. The minimum and maximum values of each objective function in the population are found using the following expressions:

$$f_m^{min} = \text{Min}_n f_m(X_n) \quad (2.39a);$$

$$f_m^{max} = \text{Max}_n f_m(X_n) \quad (2.39b)$$

Then, the normalization of objective functions takes place as follows:

$$\hat{f}_m(X_n) = \frac{f_m(X_n) - f_m^{min}}{f_m^{max} - f_m^{min}} \quad (2.39c)$$

Where: $\hat{f}_m(X_n)$ = Normalized value of objective function m

The normalized constraint violation of each infeasible individual is determined as the sum of ratios of the magnitude of every constraint violation and their maximum values, divided by the number of constraints present in the MOOP:

$$\hat{C}(X_n) = \frac{1}{J} \sum_{j=1}^J \frac{c_j(\tilde{X}_n)}{c_j^{max}} \quad (2.40a)$$

So that:

$$c_j(X_n) = \begin{cases} \text{Max}(0, g_j(X_n)) & ; \quad 1 \leq j \leq p \\ \text{Max}(0, |h_j(X_n) - \delta|) & ; p + 1 \leq j \leq J \end{cases} \quad (2.40b) ; \quad c_j^{max} = \text{Max}_j c_j(X_n) \quad (2.40c)$$

The term δ is a predefined very small tolerance value used to relax the equality constraints. For any feasible decision variable vector X , the constrain violation is $C_j(X) = 0$. Next, the distance measure $d_m(X_n)$ for each objective function dimension is determined using the following expression:

$$d_m(X_n) = \begin{cases} \hat{C}(X_n) & ; \text{ if } \varphi = 0 \\ \sqrt{\hat{f}_m(X_n) + \hat{C}(X_n)} & ; \text{ otherwise} \end{cases} \quad (2.41a)$$

So that:

$$\varphi = \frac{\text{Number of feasible individuals}}{\text{Population Size}} \quad (2.41b)$$

Notice that if there are no feasible individuals in the population, the distance measure values are equal to the constraint violation of the individual. In this way, individuals with small constraint violation dominate individuals with high constraint violation. On the other hand, if there is more than one feasible solution in the population, then feasible and infeasible individuals' distance values corresponding to a given objective function dimension m , will favor individuals closer to the origin in the m space dimension [77]. Apart from the penalty imposed to feasible and infeasible individuals using the distance measure concepts, two additional penalties factors are added to the normalization process as follows:

$$p_m(X_n) = (1 - \varphi)Y(X_n) + \varphi Z_m(X_n) \quad (2.42a)$$

So that:

$$Y(X_n) = \begin{cases} 0 & ; \text{if } \varphi = 0 \\ \hat{C}(X_n) & ; \text{otherwise} \end{cases} \quad (2.42b); \quad Z_m(X_n) = \begin{cases} 0 & ; \text{if } X_n \in Xf \\ \hat{f}_m(X_n) & ; \text{if } X_n \in Xi \end{cases} \quad (2.42c)$$

Where:

$p_m(X_n)$ = Adaptive penalty function of objective m

Xf = Subset of feasible solutions inside set X

Xi = Subset of infeasible solutions inside set X

Notice from this penalty function that, when the feasibility ratio φ of the population is small the penalty factor $Y(X_n)$ have more impact than the second penalty factor $Z_m(X_n)$. The former is formulated to have a large value for individuals with large amount of constraint violation. Hence, the amount of feasible and infeasible individuals present in the population will provide different penalty values favoring feasible individuals with low objective function values and penalizing infeasible individuals with large objective function values. The fitness function is formulated as the sum of distance measure and the adaptive penalty function of every objective function m :

$$F_m(X_n) = d_m(X_n) + p_m(X_n) \quad (2.43)$$

This fitness function formulation is very flexible and allows the utilization of infeasible and feasible individuals in a population efficiently. Notice that when the population is full of infeasible individuals, their constraint violation will be equal to the distance measure, and they will be compared based only on their constraint violation. On the other hand, if the population has mixed feasible and infeasible individuals, then individuals with low objective functions or constraint values are preferred. Similarly, if two individuals have similar distance measure values, then the feasibility ratio and the adaptive penalty function determine the dominant individual favouring individuals closer to the feasible search space. Finally in a population full of feasible individuals, individuals are compared based on their objective functions alone.

- **ATM/ASF Normalization and Constraint handling Technique**

To take advantage of the ASF normalization process described above, in this thesis it has been adapted to the nature of the ATM constraint handling technique. To do so, the constrain violation $C_j(X_n)$ of each individual X_n on the j th inequality or equality constraint are calculated using again equations (2.40b) and (2.40d) next:

$$\hat{C}_j(X_n) = \begin{cases} 0 & ; n \in Xf \\ \frac{c_j(X_n) - \text{Min}_{j \in Z_2} c_j(X_n)}{\text{Max}_{j \in Z_2} c_j(X_n) - \text{Min}_{j \in Z_2} c_j(X_n)} & ; n \in Xi \end{cases} \quad (2.40d)$$

During the search of feasible solutions, ATM divides the evolutionary process into 3 stages defined by the value of the feasibility proportion φ of the current population, stated in equation (2.41b). All these stages occur as population inside NSGA III evolves towards a set of optimum solutions:

- **The infeasible stage ($\varphi = 0$):** There is no feasible solution in the current population.
- **The infeasible scenario ($0 < \varphi < 1$):** Both infeasible and feasible solutions exist in the current population. The population is divided into feasible and unfeasible sets as seen in equations (2.40):

$$Xf = \{x_i \in X | \hat{C}(X_n) = 0\} \quad (2.41a)$$

$$Xi = \{x_i \in X | \hat{C}(X_n) > 0\} \quad (2.41b)$$

- **The feasible scenario ($\varphi = 1$):** Only feasible solutions exist in the current population.

The objective functions of every individual are adjusted in accordance with the stage of the evolutionary process as follows:

$$f_m^l(X_n) = f_m(X_n) \quad (2.42a)$$

$$f_m^l(X_n) = \begin{cases} f_m(X_n) & ; \text{if } n \in Xf \\ \text{Max}\{\varphi * f_m^{\min} + (1 - \varphi) * f_m^{\max}, f_m(X_n)\} & ; \text{if } n \in Xi \end{cases} \quad (2.42b)$$

So that:

$$f_m^{\max} = \text{Max}_{n \in Xf} \{f_m(X_n)\} ; \quad (2.42c) \quad , \quad f_m^{\min} = \text{Min}_{n \in Xf} \{f_m(X_n)\} ; \quad (2.42d)$$

Then, the extreme point of each objective function axis is identified using the adaptive achievement scalarization function (ASF) shown in equation (2.42), which allows the objective functions to be scalarized:

$$ASF(X_n, w)_{j,m} = \text{Max}_j \left\{ \frac{1}{w_{i,j}} * (f'_m(X_n) - f'_m)^* \right\} \quad (2.43a)$$

Where:

$w_{i,j}$ Axis direction on the m th objective axis and it satisfies $w_{i,j} = 0$ if $\hat{i} \neq \hat{j}$ and $w_{i,j} = 1$ if $\hat{i} = \hat{j}$.

f'_m Ideal solution or minimum value of the of the m th adjusted objective function.

Using equation (2.43), the ASF is then adapted by adding the normalized constraint violation $\hat{C}(X_n)$ to identify solutions with weak Pareto dominance relation and to identify easily global optimum in the search space.

$$ASF_{m,j}(X_n, w) = \begin{cases} ASF_{m,j} & ; \text{ if } n \in Xf \\ \text{Max}_j \{ ASF_{m,j} \} + \hat{C}(X_n) & ; \text{ if } n \in Xi \end{cases} \quad (2.43b)$$

Then, the M -extreme points along the M -objectives axes are found and are used to constitute a M -dimensional linear hyperplane. Later, the intercepts of the hyperplane along each of the M -objective axis are found using equation (2.44):

$$\begin{pmatrix} 1/a_1 \\ 1/a_2 \\ \vdots \\ 1/a_M \end{pmatrix} = Z^{-1}E^t \quad (2.44)$$

Where :

Z $M \times M$ matrix with the coordinates of extreme points.

E (1 1 1 ... 1) vector with a length of M .

a_m Intercept of hyperplane with m th objective axis.

Next, the objective functions of every individual are normalized using the following expression:

$$\hat{f}_m(X_n) = \frac{f'_m(X_n) - f'_m{}^*}{f'_m{}^{nad} - f'_m{}^*} = \frac{f'_m(X_n) - f'_m{}^*}{a_m - f'_m{}^*} \quad (2.45)$$

If any of the two degenerate cases described before appear during the normalization process, then the maximum value $f'_m{}^{max}$ is assigned to each objective function as the nadir point $f'_{m,j}{}^{nad}$. Finally, the fitness function of is determined by adding the normalized objective functions and constraint violation of every individual in the population:

$$F_m(X_n) = \hat{f}_m(X_n) + \hat{C}(X_n) \quad (2.46)$$

The proposed normalization process aims to adapt the ASF concepts to be used in constrained MOOP environment, handles in a better way individuals in the population with weak Pareto-dominance relation and use the different tradeoff stages of the ATM constraint handling technique in the search process of feasible solutions, by giving infeasible individuals with less constraint violation the chance to pass to the next generation. This last aspect is crucial, since after each generation infeasible individuals are pushed towards the feasible space so that, at one point during the evolutionary process, NSGA III only deals with feasible individuals. See Figure 2.20.

2.5.8 Non-dominated sorting

The selection of the adequate non-dominated sorting algorithm to be used with NSGA II and NSGA III is important to allow the creation of solutions as close as possible to the true Pareto Front and to improve the efficiency of the overall process. Many-nondominated algorithms are available in the literature, each of them with their own characteristics.

For instance, Kung algorithm is the most efficient for determining the non-dominated solutions in a population [85]. It first sorts the population in descending order in accordance with the first objective function. Thereafter, the population is recursively halved in a top T front and a bottom B front subpopulations. As top half is better in objectives, so the bottom half is checked for domination with top half. The solution of B , which is not dominated by solution of T , is combined with members of T to form a merged population M . This method

is computationally better and gives results in less time. However, if other fronts, beside the first one, need to be obtained, then the process must be repeated after extracting the individuals of the first front from the population.

On the other side, in [48] a non-dominated sorting algorithm is described, which classifies the Np individuals in a population P into fronts $\{Fr_1, Fr_2, \dots, Fr_n\}$, inside which there is a set of individuals that do not dominate each other. The algorithm starts by computing the first non-dominated front and subtracting the elements of this front to the original population P . Then, the second front is determined in a similar manner and again, the solutions of this front subtracted to the new population. The computation of the rest of the fronts are done sequentially until no further elements exists in the population that requires classification. This algorithm demands the evaluation of each single solution while defining a front, making it computationally expensive and time consuming.

In [60] a fast non-dominated sorting algorithm is proposed. In this approach the divide and conquer strategy is used by determining the number of individuals which dominate and the set of individuals that are dominated by a given individual. The algorithm first finds and store, for an individual X_1 of a population P_t , the number $n(X_1)$ of individuals that dominate X_1 and the set of individuals $S(X_1)$ dominated by X_1 . This implies a direct comparison between X_1 with all other individuals that belong to P_t . The first front Fr_1 corresponds to the individuals that do not have any other individual dominating them ($n(X_1) = 0$). Then, an individual X_2 inside $S(X_1)$, which belongs to every individual inside the last front created Fr_l , is selected and compared with the rest of individuals that do not belong to any front and added to a new set Q if it is a dominating individual. In this case, its counter $n(X_2)$ is reduced by one and if $n(X_2) = 0$, then X_2 is added to a new front Fr_i . The process is repeated until all individuals are incorporated into a front as shown in Figure 2.21.

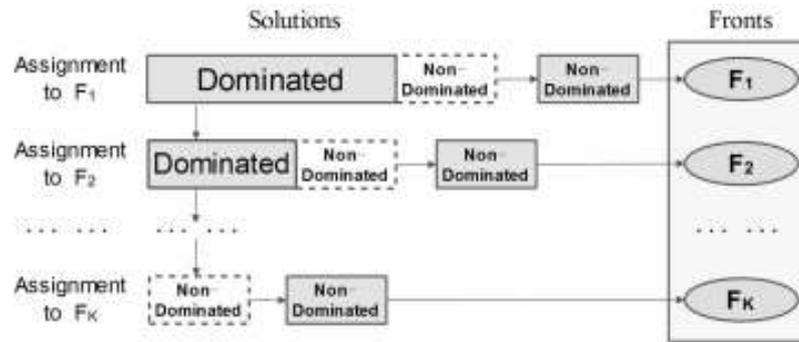


Figure 2.21 Fast Non-dominated Sorting Algorithm [60].

Even though this approach needs more storage requirements in comparison with the already discussed ones, it is the more efficient, fast, and less computational expensive, making it suitable to be used inside NSGA II and NSGA III. However, to apply effectively the fast non-dominated sorting algorithm, its procedure is adjusted so that it considers the constraint domination principle stated in [61] and [73]; and it incorporates the characteristics of the ATM constraint handling technique stated in [78].

With respect to the first aspect, fast constrained non-dominated concept must be defined. A constrained solution X_1 is said to constraint dominate another solution X_2 , if any of the following conditions is true [62]:

- a) If X_1 is feasible and X_2 is infeasible.
- b) If X_1 and X_2 are infeasible and X_1 has a smaller constraint violation value $C(X_1)$.
- c) If X_1 and X_2 are feasible and X_1 dominates X_2 with the usual domination principle explained before.

In terms of the second aspect, the sorting mechanism must be able to cope with infeasible and mixed feasible-infeasible populations during the evolutionary process. To do so, the feasibility ratio φ is used as follows to direct the non-dominated sorting procedure during the search process of NSGA II and NSGA III:

- a) **Infeasible Population ($\varphi = 0$):** The constrained MOOP with M objective functions is converted into an unconstrained problem with $M + 1$ objectives, by adding the constraint violation as an additional objective function. Then

the nondominated sorting is applied, and half of the individuals with less constraint violation (Part I) in the first front are chosen and removed from the population, since their constraint violation are less than those of the remaining individuals (Part II). This domination and sorting procedures are repeated with the remaining of the population so that half of the population with less constraint violation is removed. The process is repeated until a certain number of removed individuals is achieved so that they can be used to generate a new population using genetic operators. See Figure 2.22.

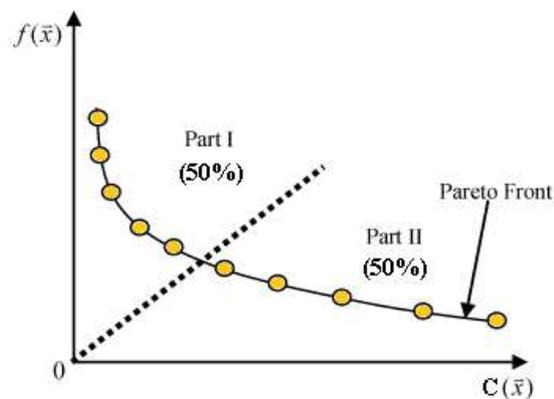


Figure 2.22 Hierarchical nondominated individual selection scheme [78].

- b) **Mixed Feasible/Infeasible Population ($\varphi > 0$):** The constrained nondominated sorting algorithm is applied in the MOOP with M objective functions as explained earlier.

2.5.9 Genetic Operators

The process of generation of news individuals from selected parents is done by crossover and mutation operators which combine and mutate gens randomly from selected parent individuals to generate new child individuals or offspring. The selection of the suitable genetic operators is of vital importance to generate better offspring solutions during the evolutionary process.

a) Crossover Operators

A basic strategy used in MOEAs is to create the best offspring by crossing the best selected parents. The mating of parents occurs if a random number $u \in [0,1]$ is

below a given crossover probability p_c with typical values of $p_c \approx 0.75 \dots 0.85$. Various crossover techniques are available in literature to get the optimum solution fast in few iterations as described in [86]. The selection of the best crossover operator has great impact on the performance of a MOEA to avoid premature convergence.

For instance, one-point crossover is the simplest crossover technique available [86]. It uses the single point fragmentation of the parent individuals and then combines them at the crossover point to create two offspring or children solutions. The crossover point is chosen randomly. K -point crossover uses the same principle with the difference that instead of choosing one crossing point, to provide a greater combination of parents, it randomly selects K crossover points [87]. The parent's gens in between certain crossover points are exchanged between the parents while others remain, so that two parents can produce two children individuals. Masked Crossover (MX) uses binary arrays of vectors (mask) to direct the exchange of gens between parents [88]. The only values in a mask array are $1s$ or $0s$ and are generated randomly. Two parent individuals can generate two new individuals and the gens in the children individuals depend in the positions (locus) of the $1s$ and the $0s$ in the mask relative to the location of the parent gens. Statistics-based adaptive non-uniform crossover operator (SANUX) makes use of statistical information implicitly contained in a population to explicitly guide the crossover operation [89]. It uses statistical information of the allele distribution in the current population to adjust the swapping probability for each gene locus adaptively during the evolutionary process. Partially-matched crossover (PMX) randomly chooses two crossover points $XP1$ and $XP2$ which break the two parents in three sections [90]. In the first and the third sections the sequences of first parent are copied to the first child, while the second section of the first child is formed by the genes of the second parent. The inverse situation happens for the gens of the second child. Blend Crossover (BLX-a) is designed to work with real variables [91]. It creates new offspring with a given crossover probability. This occurs by sampling a new value in the range limited by the minimum and maximum values of the gen locus of each parent.

SBX operator uses a probability distribution around two parents to create two child individuals, which is similar to the probability of creating children individuals in

crossover operators used in binary-coded Gas [92]. SBX puts stress on generating offspring near the parents and is defined in terms of the one-point crossover properties which are:

- **Average property:** The average value of a decision variables is the same before and after the crossover operation.
- **Spread Factor Property:** The relation between parents and children is defined by the spread factor β that, using equation (2.47), can be defined as:

$$\beta = \left| \frac{x_i^{2,t+1} - x_i^{1,t+1}}{x_i^{2,t} - x_i^{1,t}} \right| \quad (2.47)$$

Where:

$x_i^{2,t+1}$ = Decision variable i belonging to child individual number 2

$x_i^{1,t+1}$ = Decision variable i belonging to child individual number 1

$x_i^{2,t}$ = Decision variable i belonging to parent individual number 2

$x_i^{1,t}$ = Decision variable i belonging to parent individual number 1

This factor property states that probability of occurrence of $\beta = 1$ is more likely than any other β value.

These properties imply that the characteristics of the offspring is related to the values of the parents and of the spread factor β which combines two parents $x_i^{1,t}$ and $x_i^{2,t}$, such that $x_i^{2,t} > x_i^{1,t}$, to produce two new offspring, $x_i^{1,t+1}$ and $x_i^{2,t+1}$ [86]. They guaranty that the difference between children individuals is proportional to the difference between parent individuals. They also favor near parent individuals be chosen as children rather than individuals distant from the parents in the solution space [91]. When children are enclosed by the parent individuals, the spread of the children individuals is smaller than that of the parent individuals ($\beta < 1$). Since children replace parent individuals, the crossover operator seems to have a contracting effect respect to the parent individuals. What is more, when the children enclose the parent individuals, the absolute difference is more than that of the parent points. In this situation the crossover operator has the effect of expanding the children individuals respect to their parent individuals ($\beta > 1$).

Furthermore, if parents and children individuals are equal, then a stationary crossover situation arises ($\beta = 1$) [92]. See Figure 2.23.

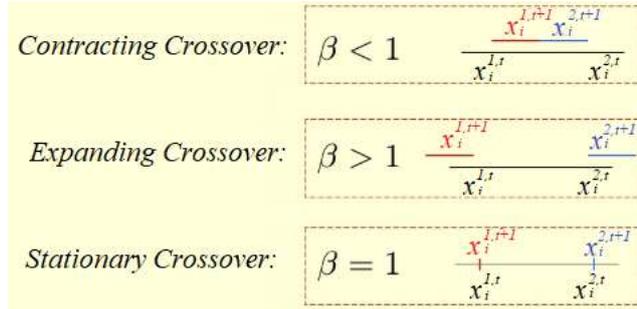


Figure 2.23 Spread Factor Relation between parent and children population.

To determine the probability of occurrence of a contracting, expanding or stationary crossover operator, the spread factor β' is adjusted to the probability density function shown in equation (2.48):

$$P(\beta) = \begin{cases} \frac{1}{2}(\eta_c + 1)\beta^{\eta_c} & ; \quad \text{if } \beta \leq 1; \\ \frac{1}{2}(\eta_c + 1)\frac{1}{\beta^{\eta_c+2}} & ; \quad \text{otherwise} \end{cases} \quad (2.48)$$

Figure 2.24 shows this probability distribution for 3 different values of distribution index η_c , which is a non-negative real number. A large value of η_c gives a higher probability for creating near parent solutions while a small value allows distant children solutions to be created.

Using the above equations, parameter β_q is found equating the area under the probability distribution to a generated random number $u \in [0,1]$ as follows:

$$\beta_q = \begin{cases} (u\alpha)^{\frac{1}{\eta_c+1}} & ; \quad \text{if } u \leq \frac{1}{\alpha} \\ \left(\frac{1}{2-u\alpha}\right)^{\frac{1}{\eta_c+1}} & ; \quad \text{otherwise} \end{cases} \quad (2.49)$$

So that:

$$\alpha = 2 - \beta_p^{-(\eta_c+1)} \quad (2.50a); \quad \beta_p = 1 + \frac{2}{x_i^{2,t} - x_i^{1,t}} \text{Min}(x_i^{1,t} - x_i^{\min}, x_i^{\max} - x_i^{2,t}) \quad (2.50b)$$

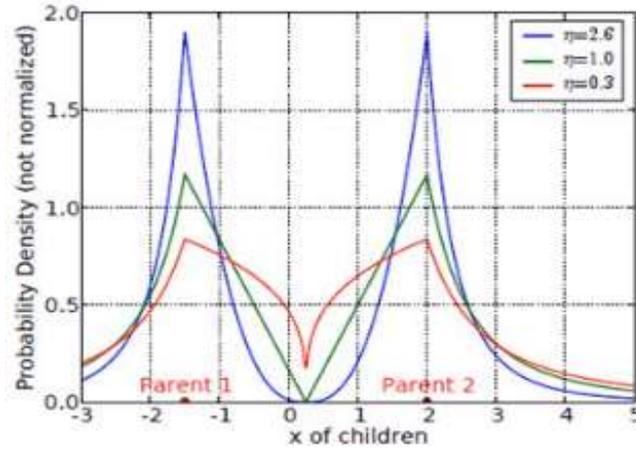


Figure 2.24 Probability density distribution function for SBX crossover operator.

Parameters α and β_p considers the maximum x_i^{max} and minimum x_i^{min} values that a decision variable x_i^t can have during the evolution process. In this way it is guarantee that the generated children individuals $x_i^{1,t+1}$ and $x_i^{2,t+1}$ are between these limits. Finally, the children individuals are calculated as follows:

$$x_i^{1,t+1} = \frac{1}{2} [(x_i^{2,t} + x_i^{1,t}) - \beta_q (x_i^{2,t} - x_i^{1,t})] \quad (2.51a)$$

$$x_i^{2,t+1} = \frac{1}{2} [(x_i^{2,t} + x_i^{1,t}) + \beta_q (x_i^{2,t} - x_i^{1,t})] \quad (2.51b)$$

If there is a situation when $x_i^{2,t} < x_i^{1,t}$, then a simple modification of the above equations can be made to determine offspring individuals.

SBX is found to be particularly useful in problems having multiple optimal solutions with a narrow global basin and where the lower and upper bounds of the global optimum are not known a priori [92]. It generates an important perturbation to the parent individuals which increases the exploitation of new solutions in the search space. However, a major drawback of the methodology is that the distance between child and parent individuals depends in the value of the distribution index η_c . This parameter has a direct effect in controlling the spread of offspring individuals. Since the search process of optimal solutions depends on the diversity of child individuals, an appropriate tuning of η_c becomes an important task.

To overcome this issue, [93] suggests a self-adaptive procedure of updating the η parameter by using the extension-contraction concept. If the created child

individual is better than the participating parent individual, the child individual is extended further in the hope of creating even better solutions. On the other hand, if a worse solution is created, a contraction is performed. Either task will result in an updated of η_c , so that the newly created extended or contracted offspring individual has an identical probability of creation with an updated η_c' . The methodology allows to direct the creation of new individuals towards feasible spaces by assigning a particular η_c' to every individual created. However, it requires an additional step related to evaluating when a child is better than a parent individual, which involve the use of non-dominated criteria. What is more, it requires the definition a priori of the magnitude of how better or worse a child individual is with respect to the parent individuals. The definition of this parameter is vital to update the value of the distribution index to η_c' .

On the other hand, [94] proposes the use of a Self-Adaptive Mechanism (SAM) to exploit and optimize the balance between exploration and exploitation during the evolutionary process of solutions. This “explore first and exploit later” approach is addressed through the automatic and dynamic adjustment of the distribution index η_c of the SBX operator. It is applied by implementing a modified diversity running performance metric defined in [95] and [96] to dynamically assess the diversity performance of the generated solution sets. The steps to calculate the diversity are as follows:

1. Divide the M -dimension hyperplane into p grids, according to stablished maximum and minimum limits, equal to the ratio of the population size Np and the number of objective functions M . The diversity performance metric depends on whether each grid contains a solution or not. The best diversity performance is achieved if all grids contain at least one solution point.
2. For each grid, a diversity array h is calculated using the following expression:

$$h(i) = \begin{cases} 1, & \text{if grid contains a solution} \\ 0, & \text{otherwise} \end{cases} \quad (2.52)$$

3. Assign a value $m()$ to each grid i considering its neighbouring grid's h value in the diversity array using the mapping table shown in Table 2.2.

Table 2.2 Mapping table to assign a value to $m()$ [95].

$h(i-1)$	$h(i)$	$h(i+1)$	$m(h(i-1), h(i), h(i+1))$
0	0	0	0.00
0	0	1	0.50
1	0	0	0.50
0	1	1	0.67
1	1	0	0.67
0	1	0	0.75
1	0	1	0.75
1	1	1	1.00

4. For each objective, a diversity measure d_m is found by averaging the $m()$ values of each grid using equation (2.53). This calculation is illustrated in Figure 2.25.

$$d_m = \frac{\sum_{i=1}^p m(h(i-1), h(i), h(i+1))}{p} \quad (2.53)$$

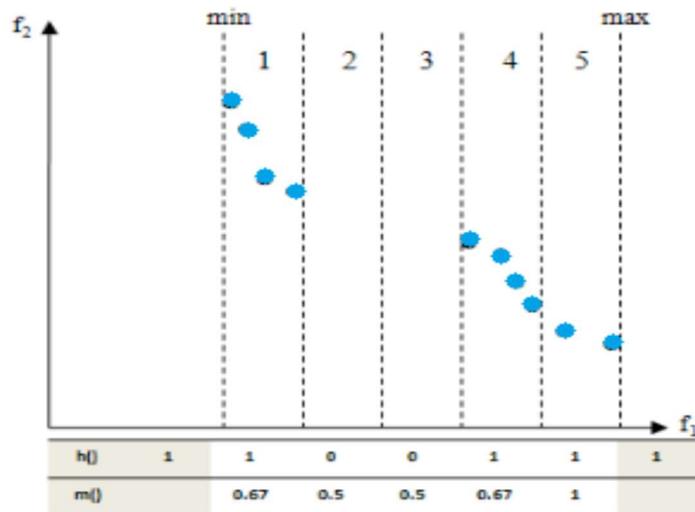


Figure 2.25 Illustration of the computing process of d_m .

5. An overall diversity performance metric D_M is calculated by averaging the diversity measure of all objective spaces using expression (2.54):

$$D_M = \frac{\sum_{m=1}^M d_m}{M} \quad (2.54)$$

6. An updated η_c' is determined based on the D_M value by defining a close value range (CVR), that corresponds to the range of values of β located between 0.9 and 1.1. In this sense, the probability of any β falling into the CVR equals to the diversity performance metric D_M as shown in Figure 2.26.

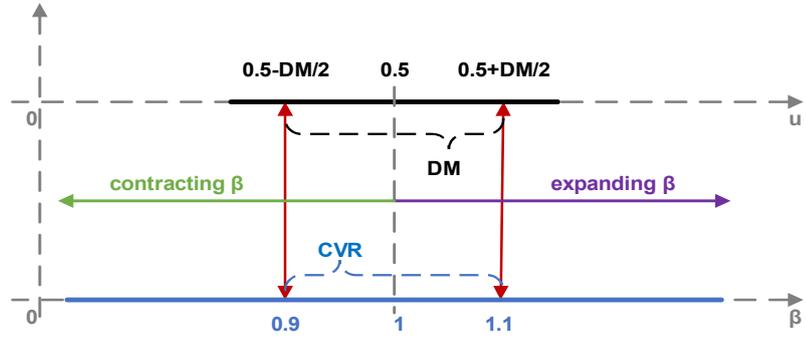


Figure 2.26 Mapping between β and u values in SBX-SAM operator.

To introduce the SAM into the SBX procedure, two values of u are determined by mapping the extreme values of β as shown in Figure 2.26. These values are introduced in equation (2.55) to determine two values of the distribution index (η_{c1}', η_{c2}') , which are averaged to update the distribution index to η_c' .

$$\eta_c' = \begin{cases} \frac{\log 2u}{\log \beta} - 1, & u \leq 0.5; \\ -\left(1 + \frac{\log 2(1-u)}{\log \beta}\right), & u > 0.5 \end{cases} \quad (2.55)$$

Random initialized populations cause poor diversity performance at the beginning of the evolution of the population; and consequently, lowers the probability of β falling into CVR. In a later stage, DM stabilizes at a relatively high value and the exploitation phase starts as the probability of β in between CVR is higher [94].

As it can be seen, SAM can alleviate the tedious process of parameter tuning which is time-consuming and involves the need to know beforehand the nature of the solution of the problem. What is more, it allows the SBX operator to become parameter-free, so that an initial value of η_c is set at the beginning of the evolutionary process. To take advantage of these benefits, the SBX-SAM is adapted to be used in the NSGA III environment by making the number of grids in each axis of the hyperplane space equal to the number of partitions defined during the generation of the set of reference points.

b) Mutation Operators

The purpose of mutation operation is to change the genes of the offspring and increase the search space and diversity of the population. A mutation operation happens if a random number $u \in [0,1]$ is below a given mutation probability p_m , with typical values of $p_m = 0.01 \dots 0.05$. This process does not occur often but enables a MOEA to escape local optimal solutions to avoid premature convergence.

Different mutation approaches have been proposed depending on the type of variables used in a MOOP. Reference [70] describes a random-mutation operator, where a child individual is chosen randomly from a given population and its value is randomly mutated to a new value inside given variable limits. What is more, publication [97] describes mirror-mutation and binary bit-flipping mutation operators which are similar in nature. The mirror mutator replaces a gene with its mirror value at the middle point of the boundary interval for the gene, whereas in the bit-flip mutation it remains unchanged. Furthermore, in [98] a Gaussian-mutation operator for real-variables is described, where it changes the value of a random gen of an individual to a neighbouring value using a normal probability distribution function.

An elitist polynomial mutation operator is described in [64] and [98]. In this type of mutation mechanism, the probability of creating an individual near to its parent is higher than the probability of creating an individual distant from its parent. This probability distribution is adjusted to a polynomial function of the type:

$$P(\delta) = \frac{1}{2}(\eta_m + 1)(1 - \delta)^{\eta_m} \quad (2.56)$$

The shape of the probability distribution is controlled by an external non-negative parameter η_m , called the distribution index for mutation, that determines how close the child value is respect to the parent's value as shown in Figure 2.27.

Using the above equation, it is possible to find the values of δ_q by equating the area under the probability distribution function to the values of a generated random number $u \in [0,1]$ as follows:

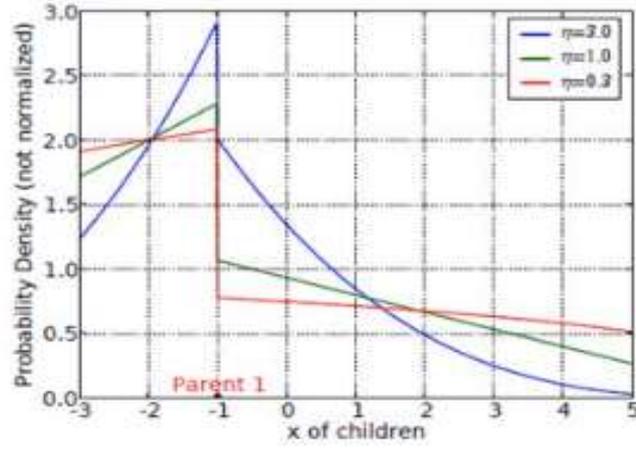


Figure 2.27 Probability density distribution function of Polynomial mutation operator.

$$\delta_q = \begin{cases} [2u + (1 - 2u)(1 - \delta)^{\eta_{m+1}}]^{\frac{1}{\eta_{m+1}}} - 1 & ; \quad \text{if } u \leq 0.5; \\ 1 - [2(1 - u) + 2(u - 0.5)(1 - \delta)^{\eta_{m+1}}]^{\frac{1}{\eta_{m+1}}} & ; \quad \text{otherwise} \end{cases} \quad (2.57a)$$

So that:

$$\delta = \frac{\min(x_i^{1,t} - x_i^{\min}, x_i^{\max} - x_i^{1,t})}{x_i^{\max} - x_i^{\min}} \quad (2.57b)$$

The value of δ is adjusted to consider the maximum x_i^{\max} and minimum x_i^{\min} values that a decision variable x_i^t can have during the evolution process. In this way it is guaranteed that the mutated child value $x_i^{1,t+1}$ is between these limits [99], [100]. After defining a random number u and calculating δ_q , the mutated child value is calculated using equation (2.58):

$$x_i^{1,t+1} = x_i^{1,t} + \delta_q(x_i^{\max} - x_i^{\min}) \quad (2.58)$$

Notice that δ_q represents the magnitude of the perturbation applied to the parent individual to generate a mutated child individual.

Polynomial mutation operator allows big jumps in the search space of the decision variable [99], allowing the optimization process better chances of escaping from local optima. However, it relies heavily in the mutation distribution index η_m , whose value will determine the magnitude of the perturbation that will define the mutated child solution. Thus, a time-consuming tuning process is needed to find

the adequate value of η_m , and of the mutation probability p_m as well, to obtain satisfactory results.

To overcome this situation, [101] and [112] propose a self-adaptive polynomial mutation operator that allows to update the values of p_m and η_m as iteration progress using equations (2.59a) and (2.59b):

$$p_m = \frac{1}{Np} + \frac{t}{t_{max}} \left(1 - \frac{1}{Np}\right) \quad (2.59a) ; \quad \eta_m = 0.5 + \frac{t}{10} \quad (2.59b)$$

Where: t = current iteration or generation of individuals.

t_{max} = Maximum number of iterations.

Np = Number of individuals in the population.

The scheme is designed to exploit the relatively diverse population in the initial generations, at the same time enabling good exploration of the decision space by keeping a lower value of η_m to have a high perturbation. During later generations, a higher p_m and a large η_m values ensure an effective local exploration [102].

The self-adaptive polynomial mutation operator is chosen to be used in the research project since it does not require any parameter tuning and at the same time it works well when solving constrained optimization problems.

2.5.10 Multi-criteria Decision-Making Tools

MOEAs, like NSGA II and NSGA III, have demonstrated that are capable of finding multiple and diverse near Pareto-optimal solutions at the end of the simulation run. It is natural to ask how a decision-maker chooses a particular solution from the obtained set of non-dominated solutions. This depends on the multi-criteria decision-making tools used and their characteristics.

Multi-criteria decision making refers to making choice of the best alternative from a finite set of decision solution alternatives in a MOOP with conflicting objectives [104]. Multi-criteria decision making involves the following activities:

- Determination of a set of solutions ranked according to their fitness value.
- Applications of a criteria to quantify the importance each objective inside the problem will have in the final optimal solution.
- Use a Multi-criteria decision-making tool to find the optimal final solution.

Multi-criteria decision-making tools help a decision-maker to select the best solution or, in the context of the project MOOP, best individual. Several multi-criteria tools are described in reference [105]. Their application depends on the nature of the MOOP, its internal consistency and transparency, its implementation, and data requirements.

The research focusses on the Technique for Order Preference by Similarity to the Ideal Solution method (TOPSIS) described in [103] since it can work with a-posteriori MOEAs. TOPSIS selects the individual (solution) with its objective function's values closest or farthest from the best or worst individuals, respectively. TOPSIS is based on information regarding the values of the objective functions and the weights the decision-maker assigns to each of them.

TOPSIS is a simple, rational, and comprehensible method, suitable for MOOP with many objective function and individuals and especially where objective functions values are available for decision making [103]. A detailed description on how it operates is presented in Appendix 1 of the present document. What is more, it is computationally efficient and accounts for both the best and worst objective functions in a population, to measure the relative performance of every individual in a simple mathematical way [104]. However, for using TOPSIS the weights of each objective function must be defined before-hand, so the determination of the best individual depends largely on the judgment made by the decision-maker when assigning these weights.

2.6 Generation and Transmission Maintenance Scheduling Models

The existing literature, in the academy and in the industry, tackles the optimal maintenance scheduling problem identifying the best optimization techniques, the type of market environment and considering the goals of the industry.

2.6.1 Academic approach to maintenance scheduling problems

In a deregulated market environment, electricity utilities compete to maximize their profits, minimize costs, and improve their competitiveness; hence their objectives conflict each other. Since electricity prices in the market are highly volatile, different generation and transmission MS arrangements will have particular financial impacts in every company.

Under these circumstances, GENCOs are unlikely to share sensitive information that may be of use for the competition. However, for planning purposes, the ISO may need to know at least what the GENCOs and TRANSCOs' maintenance scheduling strategies are in the medium-term to verify that a reliable operation of the system can be performed. In that sense, it is necessary that the ISO coordinates the MS proposed by the electricity utilities to maintain a secure and reliable operation of a power system to supply the demand at a minimum cost. There are many methodologies that can assist the ISO to perform this task, and the literature has classified them based on the facilities to be maintained and the optimization techniques used.

a) Maintenance of generation units

Publication [106] propose a new market mechanism for regulating the coordination of maintenance schedules developed by GENCOs for their thermal generation units, to keep the security and reliability of the power system operation. The proposal involves GENCOs submitting their MS and their willingness to pay to keep that schedule intact to the ISO. In turn, ISO analyse the proposal and, using the Loss of Load Probability (LOLP) index, checks the security of the system. In case the MS submitted by the GENCOs could lead to insecure system operation, then the ISO modifies the MS of the units belonging to the GENCOs with less willingness to pay informed. The mechanism puts emphasis in the security of a system with few units but, without considering GENCO's profits.

In the same manner, [107] presents an approach for formulating the maintenance scheduling problem of thermal power units in a competitive market using mixed

integer linear programming. It incorporates long-term bilateral contracts with defined profiles of traded power and price, and weekly forecasted market prices for market auction. In the proposed maintenance scheduling formulation, GENCO's aim is to maximize profit through negotiating in the market its bilateral contracts. The GENCO is responsible for performing necessary maintenance of its power units to sustain its position in the market. The maintenance periods of time for power units are scheduled either by one GENCOs only or by coordination between profit-seeking GENCOs and a reliability-concerned ISO.

Paper [108] proposes the scheduling of generation equipment maintenances under an economic criterion, considering reliability in the form of constraints. The maintenance scheduling objective is to maximize the profit of a GENCO participating in the market subject to keeping the reliability of the system. Additionally, a penalty component for GENCOs is added to the objective function to consider the energy not supplied (ENS) in the market. To replace the generation equipment capacity under maintenance, a reserve margin is defined in the model as a benchmark. The choice of this reserve is done by the ISO when making power balances.

In the same manner, reference [109] presents a dynamic programming methodology for finding the optimum preventive generation maintenance schedule of a GENCO in open power systems. The objective function for the GENCO is to sell electricity as much as possible, according to the market price forecast. Various constraints such as generation capacity, duration of maintenance and maintenance crew are considered. Furthermore, a new formulation for solving the GMS problem is proposed in [110], considering electricity prices, demand forecasts, generation capacity and maintenance resources constraints. It minimizes and maximizes GENCOs costs and profits respectively, to identify the best MS of conventional units of a GENCO. The approaches discussed above work efficiently with few units and do not consider system operation costs.

Publication [111] proposes an approach that minimizes the unit maintenance costs using integer programming method. If a unit is scheduled for maintenance early, a part of the investment made in the previous maintenance is forgone as it was meant for a longer duration of operation of the unit. On the other side,

deferring maintenance of the unit beyond the maximum period involves extra expenses for maintenance caused by partial or full damage of machines. The proposed method seeks a trade-off among the two, but without considering network constraints.

Reference [112] describes a method for calculating the Loss of Load Expectation (LOLE) and the expected ENS in a hydro-thermal power system. It converts the initial load curve into a thermal load curve, which enables the separation of thermal and hydro unit's generation in the system to facilitate both the reliability evaluations and the maintenance scheduling. The method uses a heuristic algorithm that seeks to level the risk of ENS in different periods, pursuing minimum risk in the least favorable period. Even though the reliability of the system is kept considering two reliability indexes, the proposed method assumes that the hydro energy and capacity are known and fixed for each period.

Paper [113] presents a GA methodology for finding the optimum preventive maintenance schedule of thermal generation units of a GENCO in a restructured power system. The problem is separated into two objective functions. One related to the maximization of profits, which is solved by the GENCO to obtain an initial schedule. This schedule is sent to the ISO who in turn solves a second objective function which is to maximize the net reserve of the system. If this problem is infeasible, the ISO sends a restriction to the GENCO who must add it to its objective function to determine a new generation schedule. In the same way, reference [114] presents results of modeling and solving a maintenance scheduling problem of generating units in a competitive environment. The objective function for the GENCO is to sell electricity as much as possible, according to market clearing prices forecasted. On the other side, the objective function of the ISO is to maximize the reserve capacity of the system in each period; provided the energy purchased cost is smaller than a predetermined amount when the units of GENCOs are out for maintenance. GA for finding the optimum preventive MS of generating units is used. Both references present mechanisms that keep the system reliability, without addressing the operation cost of the system.

Two approaches based on NSGA II and Group Search Optimizer are used in [115] and [116] respectively to solve the GMS problem. The GENCOs profits, the reliability and the total operation cost of the system are optimized simultaneously. A new variable encoding technique is used to represent the generator's maintenance, online and start-up status. The Pareto-optimal solutions generated by both approaches represent a set of unit's MS and show the conflicting nature of the objective functions considered. Each MS generated is associated to a single reliability index but to more than one system operation cost, increasing unnecessary the solutions in the feasible space. Thus, many solutions must be processed to identify the best generation MS.

b) Maintenance of transmission facilities

Publication [117] present a decomposition approach based on duality theory for line maintenance scheduling in the short term with transmission and voltage constraints. The formulation consists of a master program and sub-problems with two independent programming. In the master problem, maintenance problem is solved and in the subproblems, transmission and voltage problems are solved independently. Since canceling a transaction or purchasing reactive power relates to the loss of revenue, the trade-off between maintenance cost and revenue loss are optimized in the proposed method. While introducing the loss of revenue as one of the objective functions, the proposed method is flexible enough to accommodate various pricing objectives and methods.

Paper [118] presents a methodology for the evaluation of requested transmission outages, viewed as a constrained optimization problem. In this work the objective function is to minimize the rescheduling of a set of outages demanded by one or more utilities. The ISO is supposed to attend the proposed schedules. However, the optimal schedule should guarantee that system operation constraints are satisfied. Outages that lead to constraint violations are rescheduled so that these violations are eliminated, considering that the deviation of the initial schedule should be minimized. This approach uses a single objective function that only consider transmission outages priorities.

c) Maintenance of generation units and transmission facilities

Paper [119] presents a procedure to determine the maintenance scheduling of generation and transmission facilities in an electrical system. The maintenance schedules are determined by each GENCO and TRANSCO and sent to the ISO who collects them and determines which of these maintenance activities can proceed as requested without violating the system reliability. The ISO establishes the load point reliability criteria used to assess the adequacy of the system and load points when specific facilities are removed from service. The main issue with this approach is in defining the adequate load point reliability criteria.

Publication [120] proposes a methodology for the development of a scheduling technique for preventive maintenance of generating units and lines, with the inclusion of transmission constraints and forced outage rates, over a specified operation period. The main objective is to maximize the profit of the GENCOs and maintain the system reliability during the period of analysis using Benders Decomposition Method. The methodology proposed defines a master problem for the GENCO perspective, which involves the minimization of loss of profit due to a generation unit going into maintenance. In the same way, a master problem is defined for a TRANSCO which involves the minimization of maintenance costs. Both master problems are solved subject to constraints related to the duration of the maintenance and the simultaneity of electrical facilities maintenances. Both master problems are binary-integer problems. The proposed maintenance schedules are sent to the ISO who test the reliability violations in the network using subproblems that minimize the energy not supplied subject to the generation and transmission lines capacity and the energy balance in each node of the system. This is done by including the GENCO schedule as a constraint to the TRANSCO algorithm or vice versa. If any violation is detected, then the TRANSCO schedule is modified to get the final schedule. Even though the coordination mechanism works well, the approach of this publication has been evaluated in a small test system.

Publication [121] proposes to solve the maintenance scheduling problem using Bender Decomposition Method. At the first stage, a master problem is solved to determine a solution for maintenance schedule decision variables. In the second stage, sub-problems are solved to minimize operating costs while satisfying

network constraints and generators' forced outages. Benders cuts based on the solution of the sub-problem are introduced to the master problem for improving the existing solution. The iterative procedure continues until an optimal or near-optimal solution is found. Paper [122] proposes to consider transmission line maintenance scheduling and line capacity limits along with generation and line outages. It decomposes the global generator/transmission scheduling problem into a master problem and sub-problems using Benders' decomposition. In this case, a master problem is solved to minimize the maintenance costs to determine a first maintenance schedule. In the second stage, sub-problems are solved to minimize operating costs while satisfying the network constraints. Finally, publication [123] again uses Benders Decomposition Method to solve the maintenance problem, but this time including network constraints, fuel and emission constrain of generation units in the long term. Bear in mind that these decomposition mechanisms tackle the GTMS problem as a single objective optimization problem.

2.6.2 Industrial approach to Generation and Transmission Maintenance Scheduling

The electricity industry in many parts of the world tackles the GMS problem using classical optimization techniques combined with complex coordination mechanisms among GENCOs and the ISO, to find a unique MS that minimizes the operation cost of the system while keeping high the reliability and security of the network.

For example, the regulation in the electricity sector of Bolivia [124] and Ecuador [125], requires GENCOs and TRANSCOs to send their MS proposals to the ISO, who analyzes them and produce a unique MS, making sure that the reliability of the system is above a certain margin. If it is not, then the ISO requires the GENCOs to modify their MS accordingly until a final MS is agreed on.

In Brazil [126] [127], the coordination process described earlier happens as well but for each of the four subsystems defined inside the national interconnected network. At the same time, the ISO determines programmed unavailability factors for each unit according to the proposed MS which have a direct impact on the

capacity electricity prices and the remuneration of GENCOs. If there is a problem with the reliability, the ISO coordinates with the GENCOs involved in the problem a new MS using the programmed unavailability factors as negotiation tool.

In Chile [26], the GTMS problem is defined considering the cost of faults in the system as well, that may occur while certain facilities are on maintenance. In the UK [128], National Grid Electricity Transmission (NGET) receives the MS proposals of GENCOs to draw up a GMS draft plan and notify them about it, highlighting the reasons why certain electricity facilities outages have been modified. If any utility is unhappy with the outcome, it can contact NGET, explain its concerns and discuss a new solution. All these industry practices result in a unique MS that may not render the minimum operation cost or maximum reliability of the system.

CHAPTER 3

FORMULATION OF THE MAINTENANCE SCHEDULING PROBLEM

A power system maintenance scheduling problem is usually defined in terms of its integer and real variables and its non-convex nature [15]. For a given number of generating units and transmission lines that need maintenance along a horizon of time there are real variables, associated with the power output of generation units, and binary variables, corresponding to the availability of electricity facilities.

These variables need to be correctly identified and treated to formulate the conflicting objective or goals that the Generation Companies (GENCO) and the Independent System Operator (ISO) have in the market. At the same time, system operation requirements, Transmission Companies (TRANSCO) maintenance policies, market transactions among GENCOs and Distribution Companies (DISCO), and the renewable energy variability need to be added to the formulation as constraints.

In this way, the objective functions and constraints formulation will define the GTMS problem, so that it can be modeled correctly and, with the aid of three power system scenarios with different network configurations and generation technologies, understood accordingly.

3.1 Single-node thermal system scenario

This scenario is composed of many thermal units connected to a unique node to supply the demand of electricity of a single load. Under this system scenario, the formulation is performed to solve a generation maintenance scheduling (GMS) problem, considering the associated binary and real variables and the objectives

of the GENCOs and the Independent System Operator (ISO) in a competitive electricity market.

3.1.1 Variable Representation

The GMS problem deals with mixed integer-real variables corresponding to the maintenance statuses and outputs of generation units. To describe them, the following variable representation techniques are used.

a) Maintenance Variables

Generation unit j is assigned a maintenance variable or maintenance starting time x_j^g , found considering the allowed earliest s_j^g and latest k_j^g maintenance starting times and maintenance duration d_j^g :

$$x_j^g = \begin{cases} u'; & \text{if } d_j^g > 0 \text{ so that } s_j^g \leq w \leq k_j^g - d_j^g + 1 \\ 0; & \text{if } d_j^g = 0 \end{cases} \quad (3.1)$$

where u' is a random generated integer number.

If the maintenance duration of unit j is greater than zero, a random integer number u' is generated and the unit maintenance starting time x_j^g is found; otherwise, the implication is that the unit is not scheduled for maintenance at any time during the whole period of analysis.

With this representation, the outage duration and continuity of maintenance period constraints are both automatically satisfied, reducing the number of effective constraints and the complexity of the GMS problem.

b) Availability Variables

The intermediate availability variables $X_{j,t}^g$ represent the maintenance status of generation unit j at any instant of time t and depend on the value of the maintenance variables, as shown in equation (3.2):

$$X_{j,t}^g = \begin{cases} 0 & ; \text{ if } x_j^g \leq t \text{ and } x_j^g + d_j^g - 1 \geq t \\ 1 & ; \text{ otherwise} \end{cases} \quad (3.2)$$

The availability variables are binary in nature and define the maintenance status of generators. If unit j 's availability variable is zero at any time t , then the unit is on maintenance at that time, while if it is one the unit is available for generation. The use of these variables eliminates the presence of binary variables directly involved in the evolutionary part of the model, reducing the problem's complexity.

c) Generation Variables

The thermal generation variables $P_{j,b,t}^g$ represent the power output of generation unit j to supply the demand during subperiod b at any time t . This variable is expressed as a real number whose value lays between zero and the maximum generation capacity and in accordance with the value of its corresponding availability variable.

3.1.2 Problem Formulation

After defining the variables of the multi-objective GMS problem, it is possible to translate the goals of the GENCOs and the ISO in the market in a mathematical form, formulating the objective functions and constraints involved to clearly show the conflicting relationship among these goals and the system's requirements that need to be met.

a) Objective Functions

The three conflicting objective functions considered in the GMS problem are the following:

System Adequacy: The ISO objective in a deregulated electricity market is to maximize the power system's reliability. To introduce this aspect into the model,

an adequacy index F_1 is used which is defined as the average value of the relation between the net and the gross reserves in the system as follows:

$$F_1 = \text{Max} \frac{1}{T} \sum_{t=1}^T \left[\frac{\sum_{j=1}^{Ng} P_j^{g,max} X_{j,t}^g - P_t^{d,max}}{\sum_{j=1}^{Ng} P_j^{g,max} - P_t^{d,max}} \right] \quad (3.3)$$

Where:

$X_{j,t}^g$ Thermal unit j binary intermediate availability variable at time t .

Ng Number of thermal generation units.

T Duration of the period under analysis.

The adequacy index depends only on the availability variables of generation units, which in turn define the reserve in the system. The net reserve is the difference between the available installed capacity and the maximum demand $P_t^{d,max}$, while the gross reserve corresponds to the difference between the system's total installed capacity and the demand already mentioned [115].

System Total Operation Cost: The ISO aims to minimize the system total operation costs F_2 , that are composed of fuel and maintenance costs incurred by GENCOs to operate their units to meet the demand:

$$F_2 = \text{Min} \sum_{t=1}^T \sum_{j=1}^{Ng} \left\{ \sum_{b=1}^{Nb} \left[C_{0j} + C_{1j}(P_{j,b,t}^g) + C_{2j}(P_{j,b,t}^g)^2 \right] c_j^f T_{b,t} + C_{j,t}^g (1 - X_{j,t}^g) \right\} \quad (3.4)$$

Where:

$P_{j,b,t}^g$ Power output of unit j in sub-period b at time t (MW).

c_j^f Fuel price for unit j (US\$/MBtu).

C_{0j}, C_{1j}, C_{2j} Generation cost curve coefficients of unit j .

$C_{j,t}^g$ Maintenance cost of unit j at time t (US\$).

Nb Number of sub-periods considered.

$T_{b,t}$ Duration of sub-period b in time t (Hrs).

The variable generation costs are the result of the product of the quadratic input-output generation curves and their respective fuel price. What is more, these costs

depend on the duration of the sub-periods defined in every instant of time analyzed. On the other side, the fixed maintenance costs depend on the availability variables of generation units alone.

GENCOs' Profits: In a restructured electricity market, each GENCO tries to maximize its own profit F_G by generating electricity with the thermal units they own in periods of high demand and prices. In that sense, these objective functions are represented as shown in equation (3.5):

$$F_G = \text{Max} \sum_{t=1}^T \sum_{j \in \Phi_G}^{Ng} \left\{ \sum_{b=1}^{Nb} \left[\tau_{b,t} P_{j,b,t}^g - \left(C_{0j} + C_{1j}(P_{j,b,t}^g) + C_{2j}(P_{j,b,t}^g)^2 \right) c_j^f \right] T_{b,t} - C_{j,t}^g (1 - X_{j,t}^g) \right\} \quad (3.5)$$

Where:

$\tau_{b,t}$ Electricity price in subperiod b at time t (US\$/MWh).

Φ_G Set of generation units owned by GENCO G .

This expression is formulated for each GENCO G and depends on its units' generation output and variable operation and fixed maintenance costs. In the same way, the GENCO's profits depend on external factors like the duration of the sub-periods at every instant of time analysed, and the electricity prices fixed for this scenario.

b) Constraints

Once the objective functions have been defined, it is important to formulate the constraints that represent the systems' requirements that need to be met when solving the GMS problem. These constraints are the following:

Minimum Reserve: Inequation (3.6) ensures the net reserve of the system remains above a specified minimum reserve limit R^{min} at any time t :

$$\sum_{j=1}^{Nt} P_j^{g,max} X_{j,t}^g - P_t^{d,max} \geq R^{min} ; \quad \forall t \quad (3.6)$$

This constraint ensures that there is always enough generation spare capacity to meet the maximum demand $P_t^{d,max}$ at any time during the analysis.

Minimum number of available units: Due to system reserve requirements or to GENCOs maintenance limited resources at any given time t , a limited number of units N^{min} may need to be available for generation. This constraint is represented by inequation (3.7):

$$\sum_{j=1}^{Ng} X_{j,t}^g \geq N^{min} \quad ; \quad \forall t \quad (3.7)$$

Generators' capacity limits: The power output $P_{j,b,t}^g$ of thermal generators during subperiod b at time t must be kept within certain capacity limits and in accordance with their corresponding availability status. Therefore, this constraint is formulated as follows:

$$0 \leq P_{j,b,t}^g \leq P_j^{g,max} X_{j,t}^g \quad ; \quad \forall t, \forall b \quad (3.8)$$

Since no minimum generation capacity is defined in the single-node system scenario, no unit commitment (UC) problem need to be considered.

Power Balance: To ensure that the energy demand $P_{b,t}^d$ during subperiod b at time t is met by the power output of the most efficient units available, the following constraint in (3.9) is formulated:

$$\sum_{j=1}^{Nt} P_{j,b,t}^g = P_{b,t}^d \quad ; \quad \forall t, \forall b \quad (3.9)$$

where $P_{b,t}^d$ is the electricity demand in subperiod b at time t (MW)

Notice that in the formulation of the GMS problem, every time t under analysis is divided into subperiods b to model the daily demand variations more accurately.

3.2 Multi-node thermal system scenario

This scenario presents thermal generation units connected to network with many transmission lines and transformers, through different nodes. Under this scenario, transmission congestion and losses are incorporated into the analysis to solve a generation and transmission maintenance scheduling (GTMS) problem in a competitive electricity market environment.

3.2.1 Variable Representation

The GTMS problem deals with mixed integer-real variables corresponding to the maintenance status of generators and transmission lines and the generation units' output. To model it, the next variable representation techniques are used:

a) Maintenance Variables

The generation and transmission maintenance variables x_j^g and y_l are defined considering the duration and starting time of the maintenance of unit j and transmission line l as follows:

$$x_j^g = \begin{cases} s_j^g + \text{round}[u(k_j^g - d_j^g + 1 - s_j^g)]; & \text{if } d_j^g > 0 \\ 0 & ; \text{if } d_j^g = 0 \end{cases} \quad (3.10a)$$

$$y_l = \begin{cases} s_l + \text{round}[u(k_l - d_l + 1 - s_l)] & ; \text{if } d_l > 0 \\ 0 & ; \text{if } d_l = 0 \end{cases} \quad (3.10b)$$

Where:

x_j^g Starting time of maintenance of thermal unit j .

s_j^g Earliest start of maintenance of thermal unit j .

k_j^g Latest start of maintenance of thermal unit j .

d_j^g Duration of maintenance of thermal unit j .

y_l Starting time of maintenance of line l .

s_l Earliest start of maintenance of line l .

k_l Latest start of maintenance of line l .

d_l Duration of maintenance of line l .

u Random Number between [0, 1].

With this representation, again the outage duration and continuity of maintenance period constraints are both automatically satisfied, reducing the number of effective constraints and the complexity of the problem.

b) Availability Variables

The intermediate availability variables $X_{j,t}^g$ and $Y_{l,t}$ represents the maintenance status of thermal unit j and transmission line l at any time t respectively, and depend on the values of the maintenance variables defined earlier:

$$X_{j,t}^g = \begin{cases} 1 ; & \text{if } s_j^g < x_j^g \text{ or } x_j^g > k_j^g \\ 0 ; & \text{if } s_j^g \leq x_j^g \leq s_j^g + d_j^g - 1 \end{cases} \quad (3.11a)$$

$$Y_{l,t} = \begin{cases} 1 ; & \text{if } s_l < y_l \text{ or } y_l > k_l \\ 0 ; & \text{if } s_l \leq y_l \leq s_l + d_l - 1 \end{cases} \quad (3.11b)$$

As mentioned before, the availability variables are binary, and their values indicate if a unit or transmission line is unavailable due to maintenance or not at a certain time t . They limit the generators output and lines transmission capacity in the system at any time t and eliminate the presence of binary variables directly involved in the model, reducing the complexity of the problem.

c) Generation Variables

The generation variables $P_{j,b,t}^g$ represent the power output of generation unit j to supply the demand during subperiod b at any time t . This variable is expressed as a real number whose value lays between the minimum and the maximum generation capacity and in accordance with the value of its corresponding availability variable.

3.2.2 Problem Formulation

Now that the variables of the multi-objective GTMS problem have been defined, it is necessary to translate the goals of the GENCOs and the ISO in the market in mathematical terms. To do so, the objective functions and constraints involved

are formulated to clearly show the conflicting relationship among utilities' goals and the system's requirements and limits that must be met and respected.

a) Objective Functions

The conflicting objective functions considered in the model address the conflicting relationship between GENCOs and the ISO in a competitive electricity market and are the following:

System Adequacy: As stated before, one of the ISO's priorities in a deregulated electricity market is to maximize the system reliability. Again, an adequacy index is represented as the average value of the relation between the net and the gross reserve in the system as already shown in equation (3.3).

System Total Operation Cost: A further priority of the ISO is to minimize the system total operation cost. It consists of fuel and maintenance costs incurred by GENCOs to operate their units, and TRANSCO's maintenance cost to keep their transmission facilities operating. Thus, this objective is formulated in this scenario as follows:

$$F_2 = \sum_{t=1}^T \sum_{b=1}^{Nb} \{ \sum_{j=1}^{Nt} [c_{j,b}^g P_{j,b,t}^g T(t, b) + C_{j,t}^g (1 - X_{j,t}^g)] + \sum_{l=1}^{Nl} C_{l,t} (1 - Y_{l,t}) \} \quad (3.12)$$

Where:

F_2 System Total Operation Cost (US\$)

$P_{j,b,t}^g$ Generation of thermal unit j during subperiod b at time t (MW)

$Y_{l,t}$ Binary intermediate variable stating the availability status of line l at t

$c_{j,b}^g$ Marginal Generation Cost of unit j (US\$/MWh)

$C_{j,t}^g$ Maintenance Cost of unit j at time t (US\$)

$C_{l,t}$ Maintenance Cost of line l at time t (US\$)

Nb Number of subperiods considered

Nl Number of transmission lines

$T(t, b)$ Duration of subperiod b at time t (hrs)

The marginal generation cost of each unit is determined applying marginal theory concepts to their corresponding quadratic cost function [21]. What is more, the duration of the sub-periods defined at every instant of time have a huge impact on the magnitude of the operation cost. Furthermore, the fixed generation and transmission maintenance costs depend on their respective availability variables.

GENCOs' Profits: As in the previous power system scenario, each GENCO tries to maximize its own profits F_G by generating electricity, with the units it owns, when electricity prices are attractive. In that sense, these objective functions are formulated again as shown in equation (3.5).

This expression is formulated for each GENCO G and depends on generators' power outputs, operation and maintenance costs, and the nodal electricity prices. These prices are determined using dual-multiplier obtained when solving the economic dispatch (ED) problem, considering a pool-based electricity market, transmission congestion, and losses effects [45]:

$$[\tau]_{b,t} = [\pi]_{b,t}[NF]_{b,t} + [\mu]_{b,t}^T[\beta]_t \quad (3.13)$$

Where:

- $[\tau]_{b,t}$ Vector of Nodal Energy Prices during subperiod b at time t (US\$/MWh).
- $[\pi]_{b,t}$ Vector of System Marginal Cost during subperiod b at time t (US\$/MWh).
- $[NF]_{b,t}$ Vector of Nodal Penalty Factors during subperiod b at time t (US\$/MWh).
- $[\mu]_{b,t}$ Vector of transmission lines congestion cost during subperiod b at time t (US\$/MWh).
- $[\beta]_t$ Sensibility Matrix at time t .

The penalty loss factor in the first term of expression (3.13) reflects the marginal increase of power losses in all the system due to an increase in the demand in node n . It is determined in terms of the following equation [44]:

$$NF_{n,b,t} = 1 + \frac{\partial P_{loss}}{\partial P d_n} = 1 + 2 \sum_{l=1}^{NL} r_{lf_{l,t,b}} \beta_{nl} \quad ; \quad \forall b, \forall t \quad (3.14)$$

Where:

- $NF_{n,b,t}$ Node n penalty factor in subperiod b at time t .

- r_l Resistance of transmission line l (pu).
- $f_{l,t,b}$ Line l real power flow in subperiod b at time t (MW).
- β_{nl} Sensibility Matrix row corresponding to node n .

With nodal prices defined in this way, Distribution Companies (DISCOs) payments and GENCOs revenues are determined, and the market is settled. Since transmission is a natural monopoly, a unique TRANSCO is considered, and its revenues are calculated too.

b) Constraints

The constraints considered, related to the objective functions of the GTMS problem, involve the electricity facilities capacities, the system generation reserve, and the electricity demand balance at any instant of time. These constraints are formulated as follows:

Minimum Reserve: Again, using equation (3.6) ensures the net reserve of the system remains above a specified minimum reserve limit R^{min} at any time t . It guarantees that enough generation capacity is available to meet the demand.

Minimum number of available units: In the same way, at any given time t certain number of units N^{min} must be available for generation. This situation is addressed by using the constraint already formulated in expression (3.7).

Generators' capacity limits: The power output of generators must be kept within its minimum and maximum capacity limits and in accordance with their corresponding availability status. Thus, this constraint is formulated for the present system scenario as follows:

$$P_j^{g,min} X_{j,t}^g \leq P_{j,b,t}^g \leq P_j^{g,max} X_{j,t}^g \quad ; \quad \forall t, \forall b \quad (3.15)$$

Since $P_j^{g,min}$ is the minimum capacity of unit j (MW), then a UC problem needs to be solved to determine the optimal power output of each generation unit.

Transmission lines' capacity limits: The capacity of transmission lines and transformers must be kept within certain capacity limits and in accordance with their corresponding availability status. These constraints are expressed follows:

$$f_l^{min} Y_{l,t} \leq f_{l,b,t} \leq f_l^{max} Y_{l,t} \quad ; \quad \forall t, \forall b \quad (3.16)$$

where $f_{l,b,t}$ is the power flow in transmission line l in sub-period b at time t (MW)

The power flow calculation needs to be performed while solving the ED problem and using a DC-power flow technique to avoid any nodal voltage level and reactive power flows considerations.

Power Balance: To ensure that the electricity demand $P_{n,b,t}^d$ and the respective power losses $f_{l,b,t}^{loss}$ during subperiod b at time t is met by the power output of the most efficient units available, the following constraint is formulated:

$$\sum_{j=1}^{Ng} P_{j,b,t}^g = \sum_{n=1}^N P_{n,b,t}^d + \sum_{l=1}^{Nl} f_{l,b,t}^{loss} \quad ; \forall t, \forall b \quad (3.17)$$

Where:

$f_{l,b,t}^{loss}$	Line l losses in subperiod b at time t (MW)
$P_{n,b,t}^d$	System electricity demand at time t for high block demand (MW)
N	Total number of nodes in the system

Power losses in each transmission line depend on the resistance and the power flow in the line, as shown in equation (3.18) [27]:

$$f_{l,b,t}^{loss} = r_l f_{l,b,t}^2 \quad ; \quad \forall t, \forall b \quad (3.18)$$

The quadratic relationship between losses and power flows makes it difficult for transmission losses to be added directly to the GTMS formulation. Thus, for the present scenario, transmission losses in a line are found after a first DC flow is performed. They are then halved and added as additional loads to the nodes that the line connects, so that new ED and DC flow can be solved later to obtain final results [27].

3.3 Multi-node wind-hydro-thermal system scenario

This scenario presents thermal generators and hydroelectrical units, with water reservoir capacity, connected to a network of transmission facilities through different nodes. Pumped storage or run-of-river hydro units are neglected. In this system scenario, the GTMS problem is formulated in an electricity market environment considering transmission congestion and losses effect, and embedded wind generation. The variable representation techniques used, and the objective functions and constraints considered for this scenario are described next.

3.3.1 Variable Representation

The wind-hydro-thermal GTMS problem deals with mixed integer-real and stochastic variables that correspond to the maintenance statuses of electricity facilities and the generation output of hydro, thermal, and wind generation. The following variable representation techniques are used to describe them:

a) Maintenance Variables

In the same way that happened for thermal generators and transmission facilities in equations (3.10a) and (3.10b), maintenance variables are defined for hydroelectrical units considering the allowed earliest s_i^h and latest k_i^h maintenance starting times and their duration d_i^h , as shown next:

$$x_i^h = \begin{cases} s_i^h + \text{round}[u(k_i^h - d_i^h + 1 - s_i^h)]; & \text{if } d_i^h > 0 \\ 0 & ; \text{if } d_i^h = 0 \end{cases} \quad (3.10c)$$

Where:

- x_i^h Starting time of maintenance of hydroelectrical unit i
- s_i^h Earliest start of maintenance of hydroelectrical unit i
- k_i^h Latest start of maintenance of hydroelectrical unit i
- d_i^h Duration of maintenance of hydroelectrical unit i

This representation guarantees that constraints related to outage duration and continuity of maintenance period are both automatically satisfied, reducing the number of constraints and the complexity of the problem.

b) Availability Variables

To the already described intermediate availability variables $X_{j,t}^g$ and $Y_{l,t}$ in section 3.2.1, the availability variable $X_{i,t}^h$ is added to the problem which represents the maintenance status of hydroelectrical unit i at any time t . Similarly with equations (3.11 a and b), equation (3.11c) show how this variable depends on the unit's maintenance parameters:

$$X_{i,t}^h = \begin{cases} 1 ; & \text{if } s_i^h < x_i^h \text{ or } x_i^h > k_i^h \\ 0 ; & \text{if } s_i^h \leq x_i^h \leq s_i^h + d_i^h - 1 \end{cases} \quad (3.11c)$$

These new binary availability variables define when hydro units are on maintenance and limit the hydroelectrical generators' power outputs, eliminating the presence of binary variables directly involved in the evolutionary part of the model, reducing its complexity.

c) Generation Variables

The generation variables $P_{j,b,t}^g$ and $P_{i,b,t}^h$ are expressed as real numbers that represent the power output of thermal and hydro generators, respectively. Their values lay between minimum and maximum capacities and in depend on the values of their corresponding availability variables.

d) Stochastic Variables

Water inflows to reservoirs and wind speeds variables have a strong stochastic component that affects the GTMS problem. The uncertainty these variables hold has a huge impact on the results of the problem and on the decision-maker's ruling on the best solution. Is important to take this uncertainty into account to

determine electricity facilities MS that consider possible water inflows or wind speeds insufficiencies during the period of analysis.

In that sense, Monte Carlo analysis is used to determine the expected water inflow scenarios during the period under analysis, adjusting the historical water inflow data to a logarithmic-normal distribution and sampling it to determine the expected water inflows to reservoirs [129] [3]. Since one of the aims of the research is to use classical and metaheuristic techniques to determine electricity facilities MS, by solving the water inflow forecast problem separately, time is saved, use of extra computational resources is avoided, and an expected system operation cost can be determined for each MS found.

In terms of wind generation, the wind energy potential is estimated adopting the approach from [130] by first sorting the historical wind speeds data and adjusting them to a Weibull distribution using least squares method. In this way, the Weibull distribution probability function parameters are found. Then the probabilities of different values of wind velocity are determined. Next, a probability distribution function is built and sampled using Monte-Carlo analysis to determine the expected wind velocity. Finally, these results are used to determine the expected energy output from a wind farm, which is subtracted from the energy demand of the system. In this way, the wind energy estimation problem is tackled separately from the main problem and used as input to adjust the demand of the system.

3.3.2 Problem Formulation

Since the variables of the multi-objective GTMS problem have been already described, it is possible to translate the goals of several GENCOs, who own different thermal or hydroelectrical units, and the ISO in the electricity market in mathematical terms. The formulation must clearly show the conflicting relationship among these goals and the system's requirements that must be met.

a) Objective Functions

The three conflicting goals or objective functions considered in the wind-hydro-thermal GTMS model are the following:

System Adequacy: It is imperative for the ISO to keep the system reliability of the system during operation. This aspect is introduced into the model using an adequacy index F_1 or average relation between the system net and gross reserve:

$$F_1 = \frac{1}{T} \sum_{t=1}^T \left[\frac{\sum_{j=1}^{Nt} P_j^{g,max} X_{j,t}^g + \sum_{i=1}^{Nh} P_i^{h,max} X_{i,t}^h - \sum_{n=1}^N P_{n,t}^{d,max}}{\sum_{j=1}^{Ng} P_j^{g,max} + \sum_{i=1}^{Nh} P_i^{h,max} - \sum_{n=1}^N P_{n,t}^{d,max}} \right] \quad (3.19)$$

Where:

$X_{i,t}^h$ Hydro unit i binary intermediate availability variable at time t

Nh Number of hydroelectrical generation units

The adequacy index depends only on the availability variables of hydroelectrical and thermal generation units, which in turn define the reserve in the system. As stated before, the net reserve is the difference between the available installed capacity and the maximum demand, while the gross reserve represents the difference between the total installed capacity and the demand mentioned [115].

System Total Operation Cost: An additional objective of the ISO is to minimize the system total operation costs F_2 , composed of fuel and maintenance costs incurred by electricity utilities to operate their facilities, and the spilled water opportunity cost (3.20):

$$F_2 = \sum_{t=1}^T \left\{ \sum_{b=1}^{Nb} \left[\sum_{j=1}^{Nt} \left(C_{j,b}^g P_{j,b,t}^g T_{b,t} + C_{j,t}^g (1 - X_{j,t}^g) \right) \right] + \sum_{i=1}^{Nh} \left(C_{i,t}^h (1 - X_{i,t}^h) \right) + \sum_{l=1}^{Nl} C_{l,t} (1 - Y_{l,t}) + \sum_{e=1}^{Ne} c_{e,t}^s s_{e,t} \right\} \quad (3.20)$$

Where:

$C_{i,t}^h$ Maintenance cost of hydroelectrical unit i at time t (US\$)

$c_{e,t}^s$ Water Spilled cost in reservoir e at time t (US\$/Hm3)

$s_{e,t}$ Water spilled from reservoir e at time t (US\$)

Ne Number of reservoirs in the system

GENCOs' Profits: In a market environment, every GENCO tries to maximize its profits F_G by producing electricity, with the hydro or thermal units it owns, and taking advantage of electricity prices in the market. Therefore, these objective functions are represented as follows:

$$F_G = \sum_{t=1}^T \left\{ \sum_{b=1}^{Nb} \left[\left(\sum_{\substack{j=1 \\ j \in \Phi_G \\ j \in \Phi_n}}^{Ng} P_{j,b,t}^g (\tau_{n,b,t} - VC_{j,b,t}^g) + \sum_{\substack{i=1 \\ i \in \Phi_G \\ i \in \Phi_n}}^{Nh} P_{i,b,t}^h \tau_{n,b,t} \right) T_{b,t} \right] - FC_{j,t}^g - \right. \\ \left. FC_{i,t}^h - C_{j,t}^g (1 - X_{j,t}^g) - C_{i,t}^h (1 - X_{i,t}^h) \right\} \quad (3.21)$$

Where:

$P_{i,b,t}^h$ Generation of hydro unit i during subperiod b at time t (MW)

$VC_{j,b,t}^g$ Thermal unit j variable cost at subperiod b (US\$/MWh)

$FC_{j,t}^g$ Fixed cost of thermal unit j at time t (US\$)

$FC_{i,t}^h$ Fixed cost of hydro unit i at time t (US\$)

$\tau_{n,b,t}$ Node n energy price on subperiod b at time t (US\$/MWh)

Φ_G Set of generation units owned by GENCO G

Φ_n Set of generation units connected to node n

This expression is formulated for each GENCO G and depends on the units' generation outputs, their maintenance costs, and the nodal electricity prices. These prices are determined using dual multipliers obtained by solving the ED problem, as described in equations (3.13) and (3.14).

With nodal prices defined, DISCOs' payments and GENCO's revenues are found and, under this system scenario, a locational-price based electricity market is settled. Since transmission is a natural monopoly, again a unique TRANSCO is considered, and its revenues are calculated as well.

b) Constraints

The constraints considered in this system scenario involve the electricity facilities' capacities, the system's reserve, the expected wind energy generation, and the supply of the demand at any time. These constraints are formulated as follows:

Minimum Reserve: It is important to consider the contribution of hydroelectric capacity to ensure that the net reserve of the system remains above a specified minimum limit R^{min} at any time t :

$$\sum_{j=1}^{Ng} P_j^{g,max} X_{j,t}^g + \sum_{i=1}^{Nh} P_i^{h,max} X_{i,t}^h - \sum_{n=1}^N P_{n,t}^{d,max} \geq R^{min} \quad ; \forall t \quad (3.22)$$

This constraint ensures that there is always enough hydro-thermal spare capacity to meet the maximum demand at any time during the analysis.

Minimum number of available units: Due to reserve requirements and GENCOs limited maintenance resources, a minimum number of thermal and hydro units N^{min} must be available during the period of analysis as shown in expression (3.23):

$$\sum_{j=1}^{Ng} X_{j,t}^g + \sum_{i=1}^{Nh} X_{i,t}^h \geq N^{min} \quad ; \quad \forall t \quad (3.23)$$

Generators' capacity limits: The power outputs $P_{i,b,t}^h$ and $P_{j,b,t}^g$ of hydro and thermal generators respectively, must be kept within capacity limits and in accordance with their corresponding availability status. Thus, the respective constraints are formulated as follows:

$$P_j^{g,min} \cdot X_{j,t}^g \leq P_{j,t,b}^g \leq P_j^{g,max} \cdot X_{j,t}^g (1 - SR_{b,t}) \quad ; \forall t, \forall b \quad (3.24a)$$

$$P_i^{h,min} \cdot X_{i,t}^h \leq P_{i,t,b}^h \leq P_i^{h,max} \cdot X_{i,t}^h (1 - SR_{b,t}) \quad ; \forall t, \forall b \quad (3.24b)$$

Under the present system scenario, the generators' maximum capacity limit is reduced by the application of factor $SR_{b,t}$, that corresponds to the percentage of spinning reserve during subperiod b and time t .

Transmission lines' capacity limits: Power flows in lines must be kept below maximum capacity limits at any time in accordance with their corresponding availability status as already described in expression (3.16):

Under this scenario, power flows are calculated solving the ED problem and a dynamic piecewise DC lossy-load flow after losses have been linearized [133].

Reactive power flows and nodal voltages have not received any consideration to simplify the calculations.

Power Balance: To ensure that the electricity demand and the respective power losses during subperiod b at time t are met by the power outputs of the hydro and thermal units available, the power balance constraint in (3.25) is used:

$$\sum_{j=1}^{Nt} P_{j,t,b}^g + \sum_{i=1}^{Nh} P_{i,t,b}^h = \sum_{n=1}^N [P_{n,b,t}^d - E\{P_{n,b,t}^r\}] + \sum_{l=1}^{Nl} f_{l,b,t}^{loss} \quad ; \forall t, \forall b \quad (3.25)$$

where $E\{P_{n,b,t}^r\}$ is the expected wind generation in node n during subperiod b at time t (MW).

Energy demand is adjusted in accordance with the expected embedded wind energy generated. Power losses $f_{l,b,t}^{loss}$ in each line at any time, found using (3.18), are halved and each half is added up as extra load to the nodes their lines connect [27]. A dynamic piecewise DC lossy-load flow is applied to add linearized cuts to the optimization problem until losses converge to a single value [133].

Water Balance: The water balance on the reservoirs is ensured by adding the following constraint for every reservoir present in the GTMS problem:

$$v_{e,t} + \sum_{b=1}^{Nb} \left[\sum_{\substack{i=1 \\ i \in \Phi_e}}^{Nh} u_{i,b,t} - \sum_{\substack{i=1 \\ i \in \Theta_e}}^{Nh} u_{i,b,t} \right] + s_{e,t} - \sum_{\substack{e=1 \\ i \in \Psi_e}}^{Ne} s_{e,t} = v_{e,t}^o + E\{a_{e,t}\}; \forall e, \forall t \quad (3.26)$$

Where:

$v_{e,t}$ Final volume of reservoir e at time t (Hm³)

$v_{e,t}^o$ Initial volume of reservoir e at time t (Hm³)

$E\{a_{e,t}\}$ Expected water inflow in reservoir e at time t (Hm³)

$u_{i,b,t}$ Hydro unit i water discharge in subperiod b at time t (Hm³)

Φ_e Set of hydro units connected to reservoir e (Hm³)

Θ_e Set of hydro units upstream to reservoir e (Hm³)

Ψ_e Set of reservoirs upstream to reservoir e (Hm³)

The relation between the water discharge and the power output of a hydro generation unit is assumed to be linear:

$$P_{i,b,t}^h = \rho_i u_{i,b,t} \quad ; \quad \forall i, \forall b, \forall t \quad (3.27)$$

where ρ_i is the productivity factor of hydro unit i (MW/Hm³)

Constraints related to the water discharge, reservoir and water spilled limits are also added to the problem:

$$v_e^{\min} \leq v_{e,t} \leq v_e^{\max} \quad ; \quad \forall t \quad (3.28a) \quad ; \quad 0 \leq s_{e,t} \quad ; \quad \forall t \quad (3.28b)$$

$$u_i^{\min} \cdot X_{i,t}^h \leq u_{i,b,t} \leq u_i^{\max} \cdot X_{i,t}^h (1 - SR_{b,t}) \quad ; \quad \forall t, \forall b \quad (3.28c)$$

CHAPTER 4

SOLUTION METHODOLOGY

The mixed-integer and no-convex characteristics of the generation and transmission maintenance schedule (GTMS) optimization problem, as well as its objective functions and constraints, demand the development of new solution methodologies. These new approaches must handle the characteristics of the variables involved in the problem, exploit the features of classical and multi-objective evolutionary algorithms (MOEA) optimization tools, and consider present electricity industry practices, to offer an alternative way to tackle the maintenance schedules (MS) and determine a set of feasible solutions.

In that sense, this chapter describes the proposed models' methodologies for solving the maintenance scheduling problem on the three power system scenarios described in the last chapter. The general principle that lies within the models proposed is to take advantage in a novel way of classical and MOEA optimization techniques to develop a singular approach to solve the multi-objective GTMS problem, considering the conflicting relationship among the goals of the utilities in the market, the effect of the MS on electricity prices, the stochastic nature of wind generation and the standards used by the electricity industry today.

4.1 Single node thermal system scenario

In this scenario, a hybrid NSGA II/Lambda Iteration Method (LIM) model is proposed to find non-dominated maintenance schedule (MS) solutions for the generation maintenance scheduling (GMS) problem. It involves an iterative algorithm that combines in a novel way NSGA II and LIM features. It allows sequentially tackling the problem, simulating the rational behaviour of GENCOs and the ISO in the market. The model's methodology is described next.

First, an initial population of Np individuals or maintenance schedule scenarios is generated randomly. The codification of individuals (chromosomes) is done so that

the decision variables (gens) correspond to the starting time of maintenance x_j^g of each generation unit j as shown in Figure 4.1 and stated in equation (3.1). The length of an individual is equal to the number of generators under analysis.

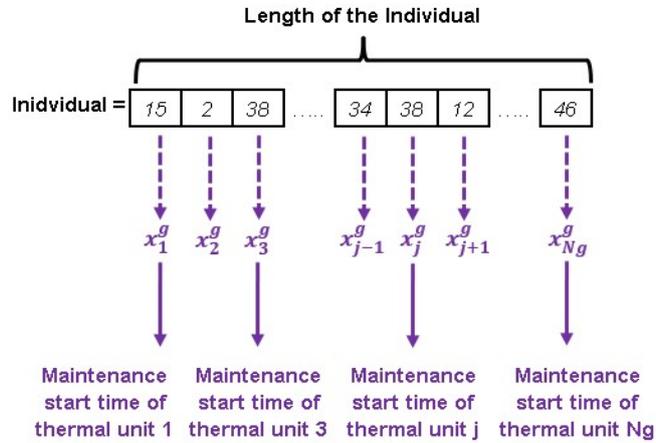


Figure 4.1. Codification of individuals in the Hybrid NSGA II/Lambda Iteration Method (LIM) model

Second, a fitness value corresponding to each objective function in the problem is calculated so that the performance of every individual is evaluated. To do this, the intermediate availability variables $X_{j,t}^g$ are determined using equation (3.2), so that every individual is evaluated in terms of the system adequacy index F_1 objective function. At the same time, with the availability variables and the demand of electricity at a given time, the LIM is used to determine the optimal generators' outputs $P_{j,b,t}^g$ that minimize the system generation cost, an aspect of importance for the ISO. To do so, each unit's cost function is considered. Since the generation cost optimization problem implies dealing with the linear equality constraint shown in equation (3.9), marginal theory is used to calculate the marginal generation cost of each unit [21]. These costs and the demand of electricity $P_{b,t}^d$ are introduced to the LIM to determine each unit's generation output. These generation outputs are in turn used to evaluate the fitness of individuals in terms of total system operation cost F_2 . Since the electricity prices in this scenario are given, this information and the generators' outputs are used to evaluate the GENCO's profit F_G .

While evaluating the fitness values corresponding to the objective functions, feasible and infeasible individuals are identified. Infeasible individuals are tackled

with the Self-Adaptive Constraint Handling (SACH) technique [77]. The flexible penalty function developed in the SACH provides different penalty values favoring feasible and infeasible individuals with good objective function and small constraint violation values, respectively, while penalizing infeasible individuals with unfavorable objective function values. In this way, SACH deals with infeasible individuals by pushing them after each iteration more and more towards the search space of feasible solutions.

Next, individuals in the population are sorted based on their objective functions' fitness using the non-domination criteria for feasible and infeasible individuals and the concepts stated in [49] and [48]. A fast non-dominated algorithm is used, so that every individual is assigned a rank that is equal to its non-dominated front, starting with front 1 as the best one, front 2 as the second-best, and so on. Then, the individuals inside the same front are ranked again using the crowding distance (CD) diversity mechanism [60]. The best individual inside a front has the highest CD value, the next best has the next highest CD value, and so on.

Then, individuals inside the sorted population are selected randomly according to their rank as parents using the binary tournament selection mechanism described in Chapter 2 and in [60]. Genetic operators are applied to the selected individuals to generate children individuals in a new population. The operators used are the Simulated Binary Crossover (SBX) [92] and Polynomial Mutation operators [132], both adapted to deal with integer variables.

Thereafter, the new and old populations are combined into a single population of size $2Np$. This combined population is again evaluated and sorted using non-dominated and crowding distance criteria already mentioned in Chapter 2 and in [60]. Since parent and children individuals are combined in a single set, elitism is ensured in the model.

Finally, the first Np top individuals in the sorted combined population are chosen to form a new population of parent individuals and TOPSIS is used as the decision-making tool to find the best individual inside this population [103]. The process repeats until a stopping criterion, based on the maximum number of iterations, is met. The flowchart of the model proposed is shown in Figure 4.2.

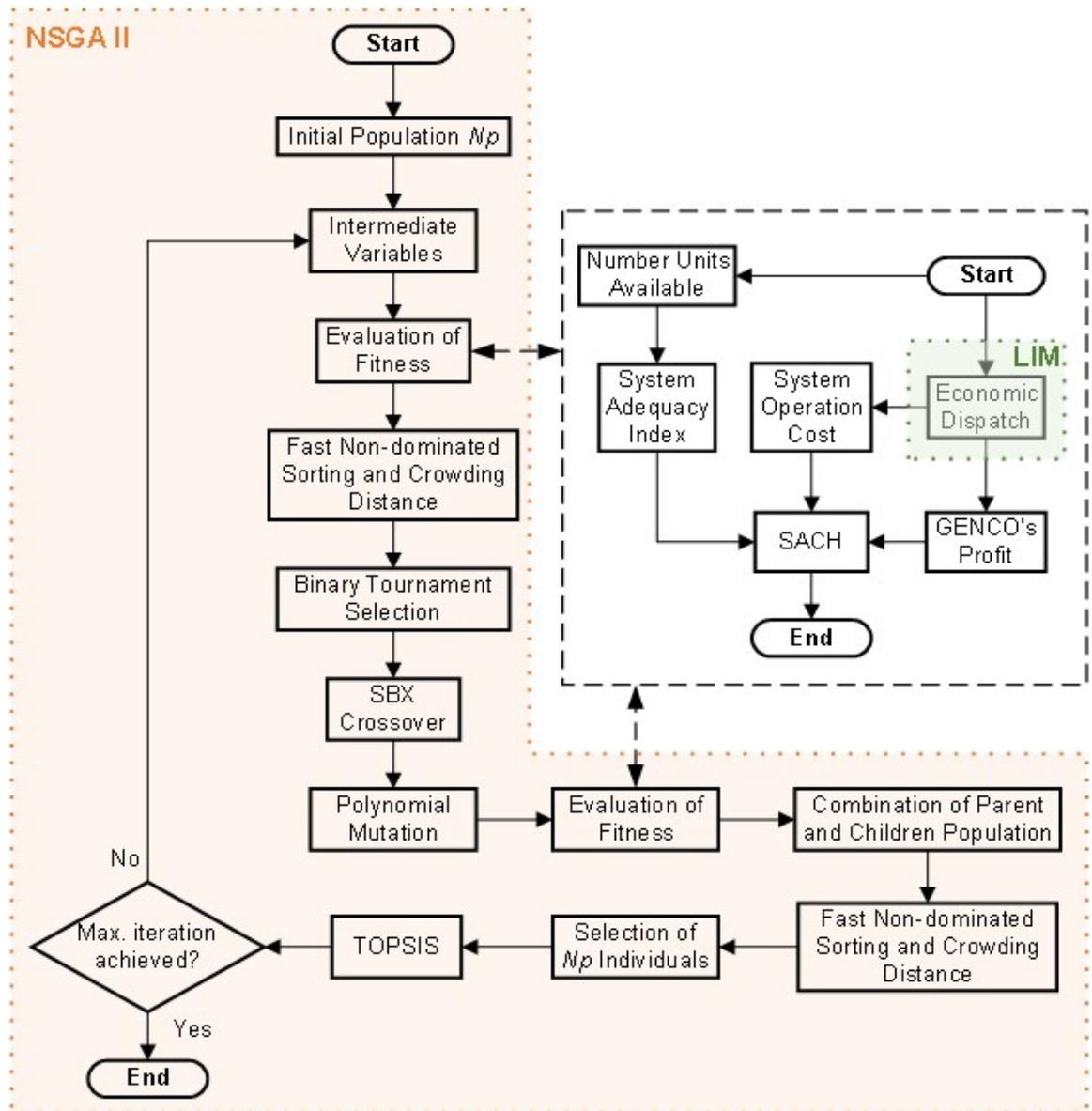


Figure 4.2 Hybrid NSGA II/LIM model flowchart to solve the GMS problem in a single node system.

4.2 Multi-node thermal system scenario

The GTMS scheduling problem in this scenario is solved by combining in a novel way NSGA III and the Dual Simplex (DS) approaches to develop a hybrid NSGA III/DS model that works in a system with only thermal units. The model's methodology involves an iterative algorithm that allows tackling this multi-objective problem in a sequential manner, simulating the rational behavior of GENCOs and of the ISO in a single marginal cost pool-based electricity market, to obtain a set of non-dominated MS solutions, from which the best can be chosen. The proposed model works as follows.

First, an initial population of Np individuals or MS scenarios is created randomly. The codification of individuals is done by generating random numbers u between 0 and 1 to determine the maintenance starting time variables x_j^g and y_l , corresponding to generators and transmission lines respectively, by using equations (3.10a-b) as depicted in Figure 4.3. The length of an individual is equal to the number of generators and transmission lines in the analysis.

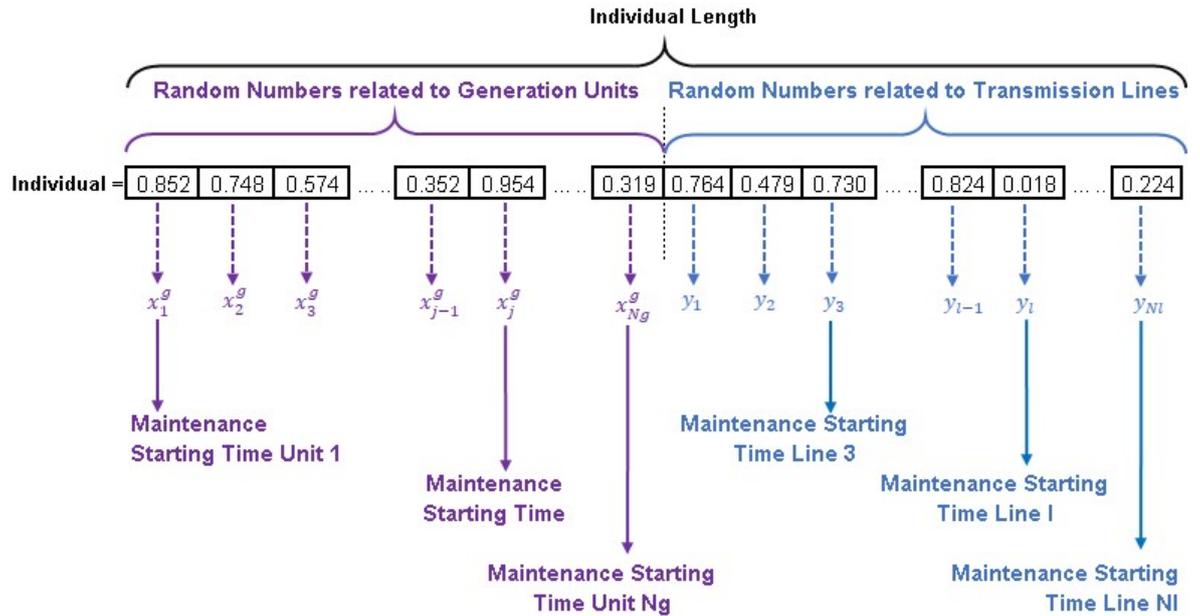


Figure 4.3 Codification of individuals in the hybrid NSGA III/Dual-Simplex (DS) model

Second, the reference points needed by the NSGA III to achieve diversity in the obtained solutions are generated according to [67] as described in Chapter 2. Since there is no information about the possible shape of the solution front, reference points are generated uniformly distributed across the search space.

Then, a fitness value corresponding to each objective function in the problem is found to evaluate the performance of every individual. To do so, generators and transmission lines intermediate availability variables $X_{j,t}^g$ and $Y_{l,t}$ are determined using equations (3.11a-b), so that every individual is examined in terms of the system adequacy index objective function F_1 . On the other hand, generators input-output curves and marginal theory are used to determine generation marginal costs, which will be used to bid in the market [21]. With these costs, the

demand of electricity, the availability variables, and the network characteristics, a unit commitment (UC) problem is solved. The results obtained and the DS method are used to solve a single economic dispatch (ED) to find generators' outputs $P_{j,b,t}^g$ and the system operation cost F_2 . Bear in mind that the operation cost minimization goal is related to the constraint shown in equation (3.17) and is an aspect of importance for the ISO. To calculate transmission losses $f_{l,b,t}^{loss}$, a lossy direct current (DC) load-flow is used where each line power loss found is halved, and each half is added up as extra load to the ends of each line [133] [27].

Next, power flows $f_{l,b,t}$, transmission losses penalty factors, congestion surpluses and nodal electricity prices $\tau_{n,b,t}$ are determined using DS' dual multipliers. With this information, DISCOs' payments and GENCOs' and TRANSCO's revenues are calculated and the economic transactions in the market are settled. These results in turn are used to find GENCO's profit objective function F_G . Notice that generation outputs $P_{j,b,t}^g$ and power flows depend on the values of generators and transmission lines availability variables.

While evaluating the fitness values related to the objective functions, feasible and infeasible individuals are identified using the Adaptive Trade-off Model (ATM) constraint handling technique [78]. ATM tackles the evaluation of solutions when the population is full and partially composed of infeasible individuals by privileging infeasible individuals with less constraint violations and by defining a feasibility proportion that has a direct impact on the fitness function calculation. In that way, the technique drives infeasible individuals slowly towards a feasible search space after each iteration. Then, a normalization process takes place by identifying the hyperplane's extreme points in each objective function axis using an adaptive achievement scalarization function (ASF) adjusted to handle solution's constraint violations values [81] [82] [84] as described in Chapter 2.

Later, individuals in the population are sorted based on their objective functions' fitness using the non-domination criteria for feasible and infeasible individuals and the concepts stated in [49] and [48]. A fast non-dominated algorithm is applied, so every individual is assigned a rank that is equal to its non-dominated front, starting with front 1 as the best one, front 2 as the second best and so on.

After that, individuals inside the sorted population are selected randomly according to their rank as parents using a binary tournament selection mechanism [60]. Genetic operators are applied to the selected individuals to generate children individuals in a new population. The operators used are the Self Adaptive Simulated Binary Crossover (SBX) and Polynomial Mutation operators [92] [132]. In the former operator, a self-adaptive mechanism [94] complements SBX to achieve an “explore first and exploit later” capability during the evolutionary process of solutions, to dynamically adjust the distribution index η_c through a diversity running performance metric defined in [95] and [96]. With respect to the later operator, the self-adaptive property described in [101] and [102] is introduced in the mutation operator to allow the update of the values of the probability of mutation p_m and the distribution index η_m as the iteration progress.

Thereafter, the parent and child populations are combined into a single population of size $2Np$. This combined population is again evaluated and sorted using non-dominated concepts already mentioned in [61]. Since parent and children individuals are combined in a single set, elitism is ensured in the model. From this, a new population is constructed by selecting individuals of different non-dominated fronts (one at a time), starting from the first front until the size of the new population is equal or for the first time becomes larger than Np .

Subsequently, the association operator is applied by defining a reference line to each reference point in the normalized hyperplane by joining the reference point with the origin [61]. Perpendicular distances from every individual to each reference line are calculated and the reference point whose reference line is closest to an individual is considered associated with that individual. In the same manner, the niching preservation operator is used. It highlights iteratively solutions nearest to the reference line of each reference point, by updating a niche count repeatedly until all vacant population slots of the new population are filled and its size reaches a value of Np [62].

Finally, TOPSIS is used to assist the decision-maker (ISO) to identify the best individual inside the population [103]. The process repeats until a stopping

criterion, based on the maximum number of iterations, is met. The flow chart of the model is shown in Figure 4.4.

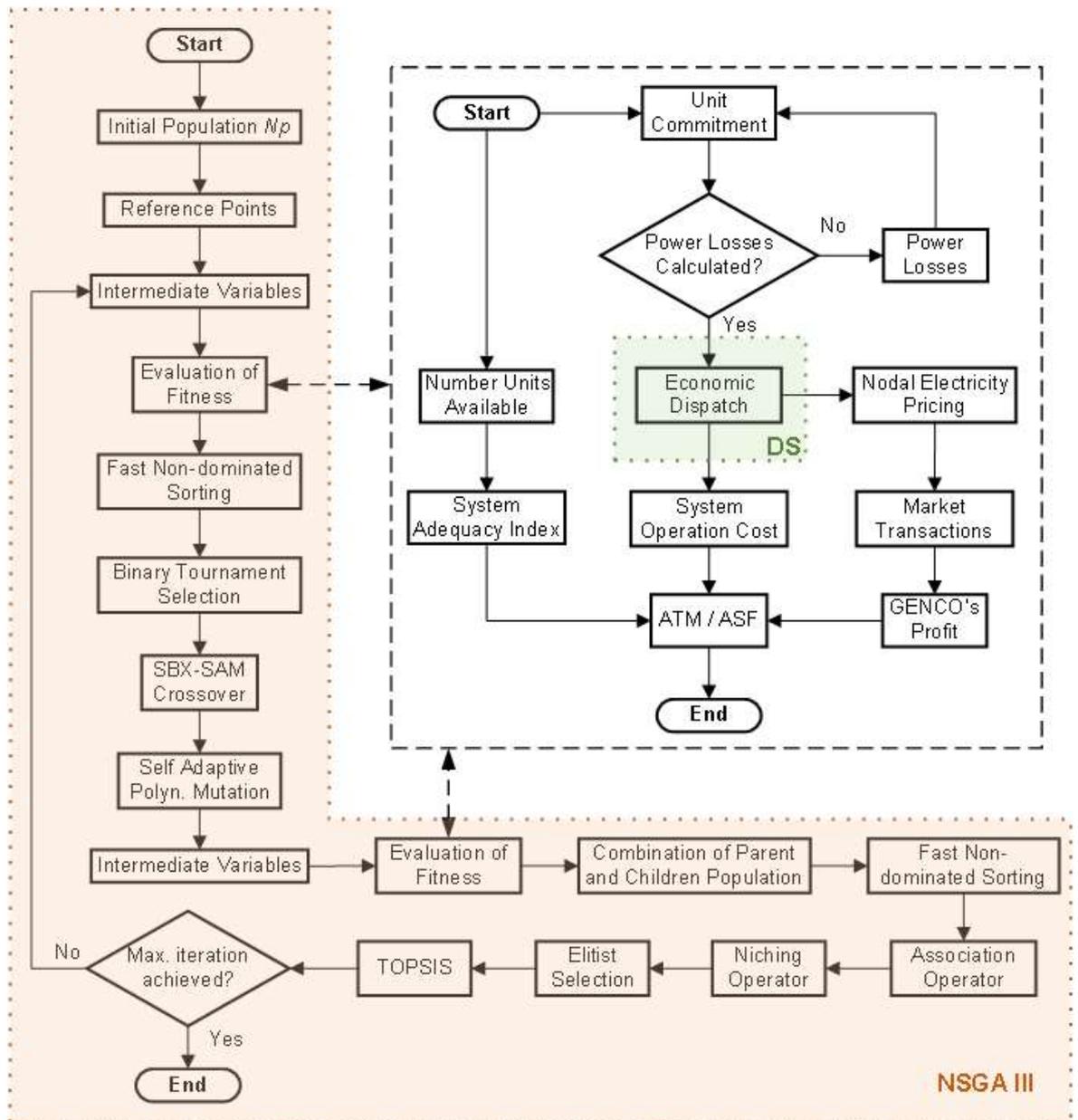


Figure 4.4 Hybrid NSGA III/Dual Simplex model flowchart to solve the GTMS problem considering network losses and congestion.

4.3 Multi-node wind-hydro-thermal system scenario

In this scenario, a hybrid NSGA III/Dual Dynamic model is proposed to solve the wind-hydro-thermal GTMS problem, combining in a novel way NSGA III and the Dual Dynamic Programming (DD) features. The model's methodology involves an iterative algorithm that allows tackling this multi-objective problem in a

sequential manner considering the characteristics of the variables involved, transmission facilities' constraint and losses, stochastic nature of renewable generation, and simulating the rational behavior of GENCOs and of the ISO in a locational price-based electricity market. The results expected correspond to a set of non-dominated MS solutions, which have a direct impact on the electricity prices in the market, that can be used by the ISO to identify the best. The model has a resemblance with the NSGA III/Dual Simplex model discussed before but is different in the following parts.

An initial population of Np individuals or MS scenarios is generated randomly. Individual's codification is done so that decision variables correspond to random numbers u between 0 and 1 as shown in Figure 4.5. With these numbers, the starting time of maintenance x_j^g , x_i^h and y_l variables, corresponding to thermal and hydro generators and transmission facilities respectively, are determined using equations (3.10a-c). The length of every individual is equal to the number of generators and transmission lines analyzed.

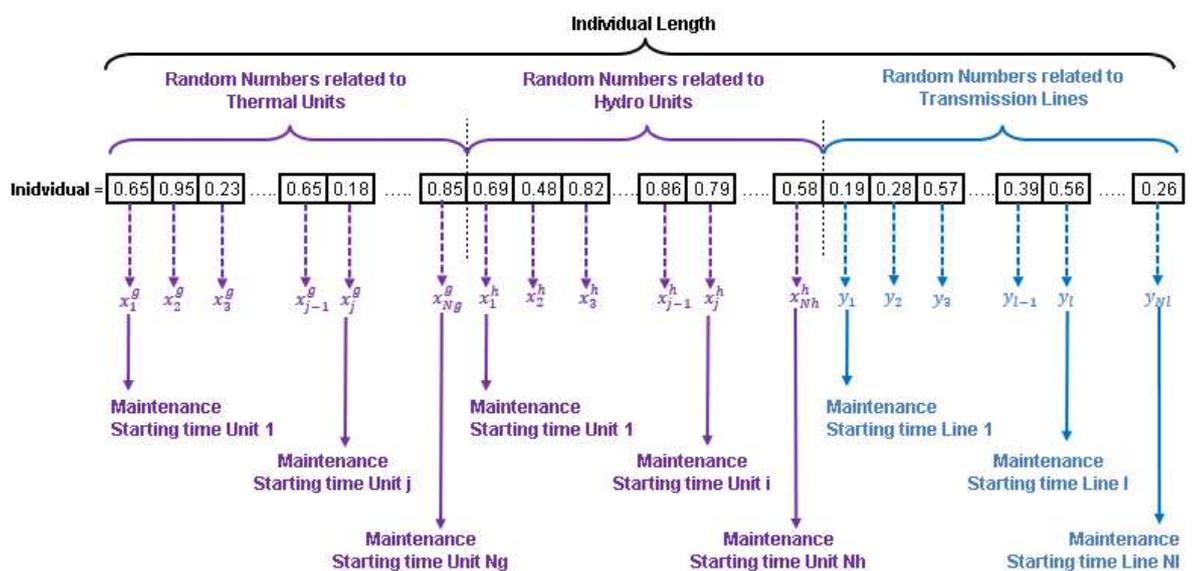


Figure 4.5 Codification of individuals in the hybrid NSGA III/Dual-Dynamic (DD) model.

A fitness value corresponding to each objective function in the problem is found so that the performance of every individual is evaluated. To do so, the reservoirs' water cost-to-go function is determined for a period greater than the GTMS time span using dynamic dual programming and considering no MS at all. The water

inflows and wind velocity forecasts are generated using Monte Carlo analysis and adjusting historical data to Logarithmic-Normal and Weibull distributions respectively [42] [134]. The intermediate availability variables $X_{j,t}^g$, $X_{i,t}^h$ and $Y_{l,t}$, corresponding to thermal and hydro units and transmission lines respectively, are determined using equations (3.11a-c) so that every individual is examined in terms of the system adequacy index objective function. On the other hand, linearly adjusted thermal generators heat rates for different power outputs, fuel prices, and marginal theory are used to determine generation costs (bidding prices) of each unit in the market [21]. With these costs, the reservoir's water cost-to-go function, the contribution of renewable energy, the demand of electricity, the availability variables, and the network characteristics, a UC problem is solved. The results are used as inputs to tackle an ED problem using the DD method to determine the thermal, and hydro generators power outputs $P_{j,b,t}^g$ and $P_{i,b,t}^h$, the water turbinated $u_{i,b,t}$ by hydro units and the water volume at reservoir $v_{e,t}$ that minimize the system operation cost F_2 , aspect of importance for the ISO. These two problems are solved iteratively, at any given time inside a period of analysis, with a dynamic piecewise DC lossy-load flow algorithm, where each line power loss $f_{l,b,t}^{loss}$ is halved and each half is updated and added up iteratively as extra load to each end of the lines until convergence is met [133] [27].

Next, power flows $f_{l,b,t}$, transmission losses penalty factors, congestion surpluses, and nodal electricity prices $\tau_{n,b,t}$ are determined. With this information, DISCOs' payments and GENCOs' and TRANSCO's revenues are calculated, the market economic transactions settled and GENCOs' profits objective function F_G found.

The rest of the methodology related to the NSGA III part is the same as in the model described before. For the fitness function determination, the ATM is used to tackle infeasible individuals in the population to make them approach the feasible region [78]. The normalization process of individuals is performed building a hyperplane with the ASF adjusted to handle infeasible individuals [81] [82] [83]. Fast non-dominated algorithm is used to sort individuals according to their fitness values [49] [48]. Self-adaptive SBX and Polynomial mutation operators are used to determine a child population [60] [92] [132] [94]. The advantage of these operators is that they do not require any parameter tuning to

be performed [95] [96] [101] [102]. Parent and child populations are combined and individuals inside are sorted using fast non-dominated sorting algorithm [61]. The association operator and the niching mechanism are used to generate a more spread set of solutions [61] [62].

Finally, TOPSIS is also applied to determine to assist the decision-maker (ISO) to identify the best MS solution [103]. The iterative process halts when a stopping condition is met. The flowchart of the model is shown in Figure 4.6.

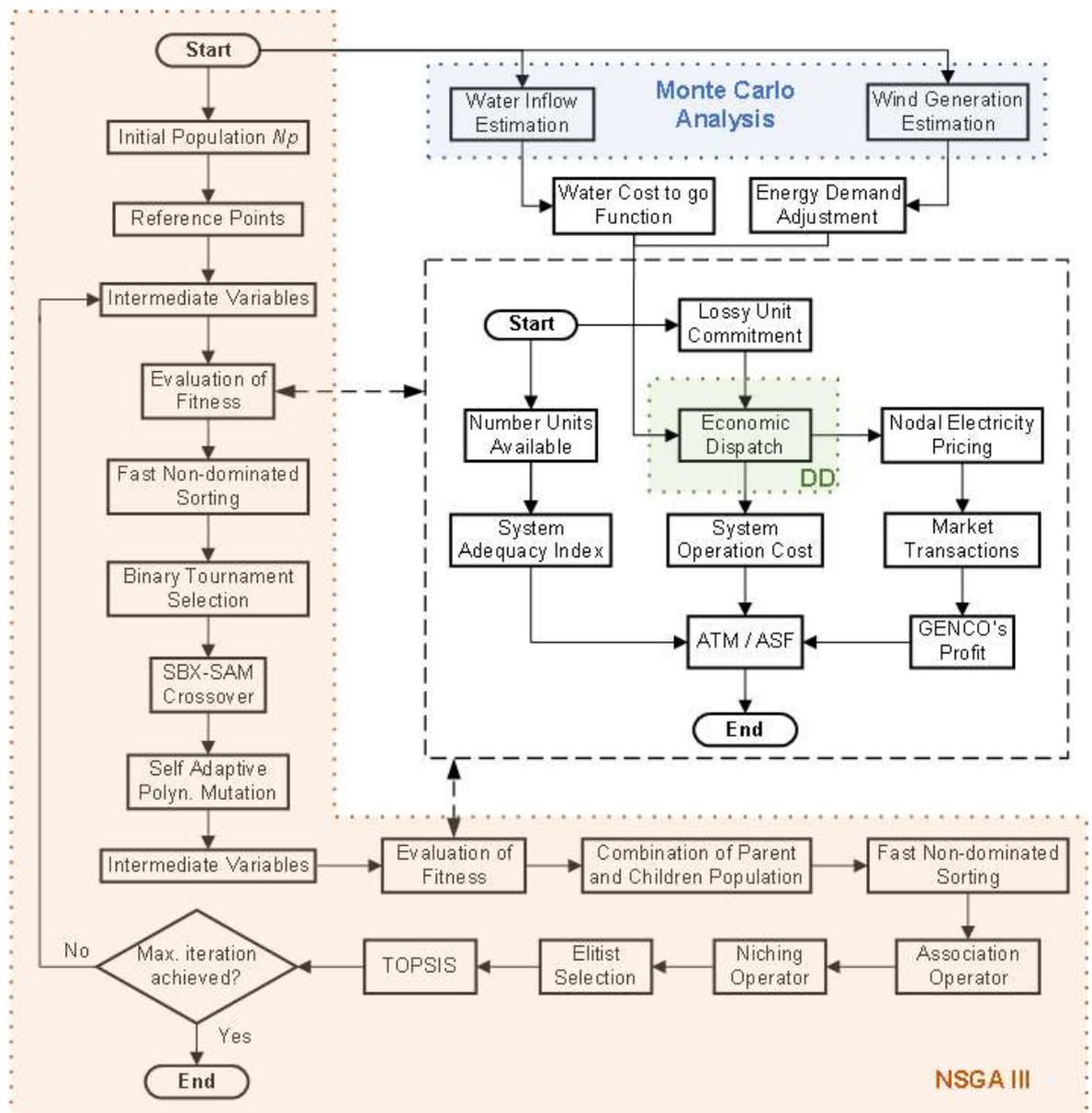


Figure 4.6 Hybrid NSGA III/Dual Dynamic model flowchart to solve the wind-hydro-thermal GTMS problem considering network losses and congestion.

4.4 Methodology of the models proposed to solve the GTMS problem

The GTMS problem is very large, contains mixed integer-real variables, and is non-convex in nature. The models developed and proposed to solve the problem in the three power systems scenarios defined during the medium term, require the following input data or information:

- Thermal generators' capacity limits, cost functions, fuel costs, and efficiency.
- Hydro generators' capacity limits and reservoir's hydroelectric configuration and water storage limits.
- Wind farms' technical characteristics and capacity limits.
- Network information relating to number and type of nodes used, transmission facilities capacity limits, and electrical characteristics.
- Maintenances starting times allowed and duration corresponding to generation units and transmission facilities.
- Identification of the type of electricity market to be considered and generators' marginal costs and DISCOs' electricity demand to be supplied.
- Water inflows hydrology and wind speeds historical data.

This information is feed into the novel hybrid MOEA-classical optimization models proposed. The models adapt the best features of these optimization techniques to process this information and tackle the GTMS problem considering the coordination mechanism that must exist between ISO and GENCOs in a market environment. Furthermore, Monte Carlo analysis is used when renewable energy is added to the problem. The main outputs that are expected from the proposed models are the following:

- A feasible non-dominated set of MS solutions for generation and transmission facilities from which the decision-maker (ISO) can choose the best.
- Generation and transmission MSs that guaranty an adequate supply of electricity at a minimum cost, which are the goals of the ISO in the market.
- Energy costs reduction for final consumers and cost savings for GENCOs profit maximization.
- MS's effect on electricity prices and on GENCOs' revenues in the market.

Finally, the methodology explained so far requires the proposed models to pass a validation stage. Due to the nature and size of the problem, the validation process encompasses the partial use of information corresponding to real-life power systems and an exhaustive analysis of the results obtained, considering especially the conflicting relationship among the objectives of the different utilities in the market. Figure 4.7 presents the methodology of the models developed to solve the GTMS problem in a market environment.

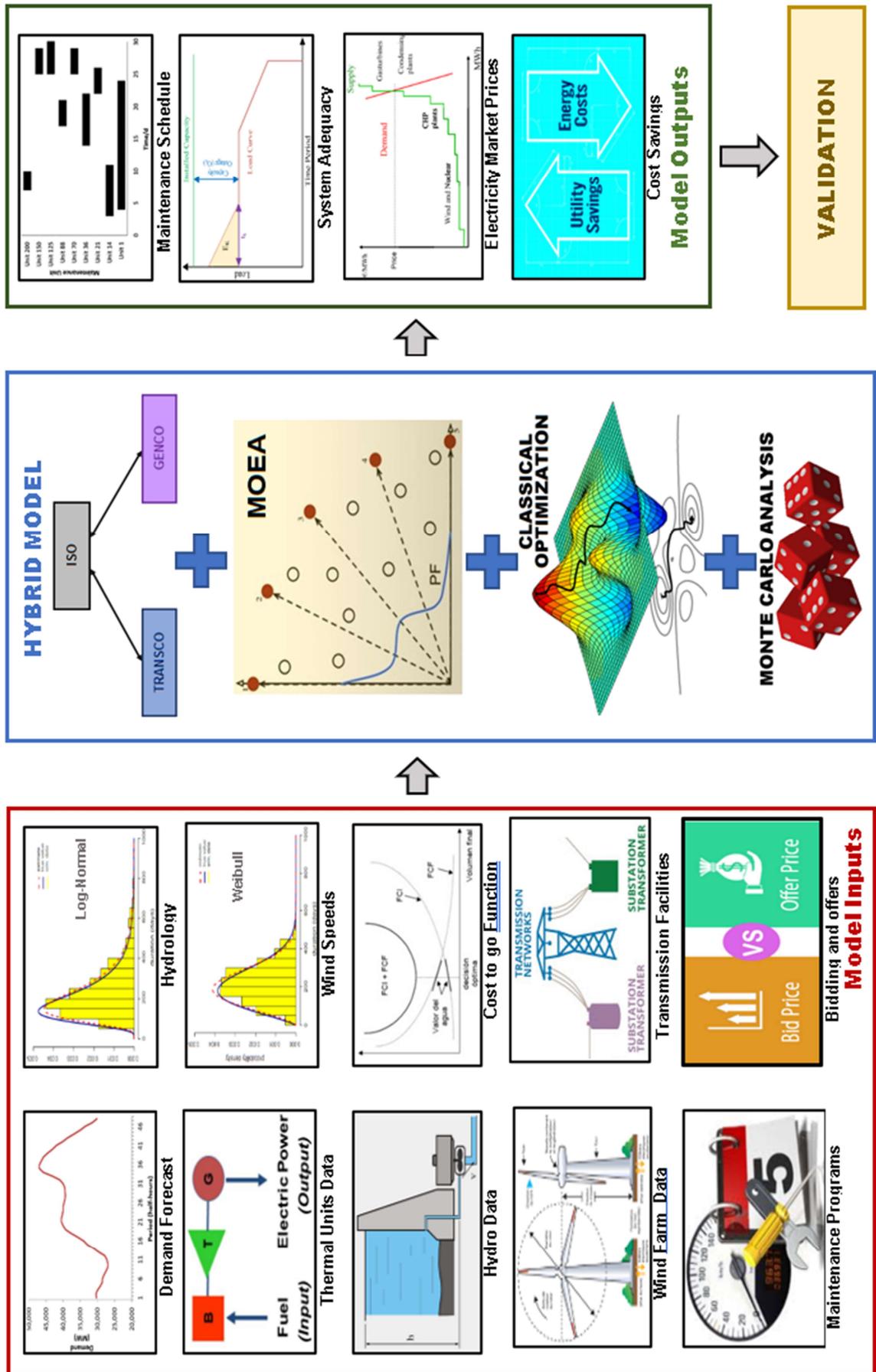


Figure 4.7 Methodology of the models proposed to solve the GTMS problem.

CHAPTER 5

CASE STUDY DATA

To showcase the workability of the novel hybrid Multi-Objective Evolutionary Algorithms (MOEAs) and classical optimization models proposed in Chapter 4 to solve the generation and transmission maintenance scheduling (GTMS) problem, the technical characteristics of the three power system scenarios defined are presented in this chapter.

These scenarios include data corresponding to well-known IEEE-benchmark networks established in the literature. Additionally, the third scenario uses real-life technical data of wind and hydroelectrical generation units and reservoir water storage limits belonging to Bolivian electricity utilities. At the same time, the parameters required by the MOEAs and the classical optimization techniques in the different scenarios are provided in this chapter.

5.1 Single node thermal system scenario

The novel hybrid NSGA II/LIM model proposed to solve the generation maintenance schedule (GMS) is applied on a single node test system using the IEEE-RTS 24 bus test network's 32 generators fired by coal, natural gas, diesel, and nuclear fuel [15]. Table 5.1 shows their technical characteristics, the GENCO they belong to, the type of fuel they use, annual maintenance costs, and the allowed starting times and duration of their maintenance.

Three GENCOs are defined in the analysis so that three profit maximization objective functions are considered in the problem. The period under analysis is of 52 weeks. The maximum annual demand considered is 2,850 MW. Table 5.2 shows the percentage of the peak demand used to find the demand in each week.

Table 5.1 Technical characteristics of generation units [15].

Name	Node	Size (MW)		Fossil Fuel Type	Fuel Price \$/Mbtu	Output Curve (MBtu/h)			Maintenance Cost		Maintenance (weeks)		GENCO	
		Min	Max			C2 (MBtu/MW ² h)	C1 (MBtu/MWh)	C0 (MBtu/h)	Fixed US\$/Yr	Variable US\$/kW/Yr.	Window Earliest Latest	Duration		
1	15	0.0	12.0	Natural Gas	2.3	0.0106	12.5957	6.6059	10.00	5.00	1	52	2	2
2	15	0.0	12.0	Natural Gas	2.3	0.0106	12.5957	6.6059	10.00	5.00	1	52	2	2
3	15	0.0	12.0	Natural Gas	2.3	0.0106	12.5957	6.6059	10.00	5.00	1	52	2	2
4	15	0.0	12.0	Natural Gas	2.3	0.0106	12.5957	6.6059	10.00	5.00	1	52	2	2
5	15	0.0	12.0	Natural Gas	2.3	0.0106	12.5957	6.6059	10.00	5.00	1	52	2	2
6	1	0.0	20.0	Diesel	3.0	0.0232	8.7480	9.5668	0.30	5.00	1	52	3	2
7	1	0.0	20.0	Diesel	3.0	0.0232	8.7480	9.5668	0.30	5.00	1	52	3	2
8	2	0.0	20.0	Diesel	3.0	0.0232	8.7480	9.5668	0.30	5.00	1	52	3	2
9	2	0.0	20.0	Diesel	3.0	0.0232	8.7480	9.5668	0.30	5.00	1	52	3	2
10	1	0.0	76.0	Coal	1.2	0.0107	14.8500	72.9167	10.00	0.90	1	52	4	2
11	1	0.0	76.0	Coal	1.2	0.0107	14.8500	72.9167	10.00	0.90	1	52	4	2
12	2	0.0	76.0	Coal	1.2	0.0107	14.8500	67.0833	10.00	0.90	1	52	4	3
13	2	0.0	76.0	Coal	1.2	0.0107	14.8500	67.0833	10.00	0.90	1	52	4	3
14	7	0.0	100.0	Natural Gas	2.3	0.0042	7.0391	39.7826	8.50	0.80	1	52	5	3
15	7	0.0	100.0	Natural Gas	2.3	0.0042	7.0391	39.7826	8.50	0.80	1	52	5	3
16	7	0.0	100.0	Natural Gas	2.3	0.0042	7.0391	39.7826	8.50	0.80	1	52	5	3
17	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
18	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
19	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
20	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
21	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
22	22	0.0	50.0	Natural Gas	2.3	0.0029	7.5652	25.8109	8.00	0.75	1	52	4	1
23	15	0.0	155.0	Coal	1.2	0.0040	8.9417	112.5000	7.00	0.80	1	52	6	1
24	16	0.0	155.0	Coal	1.2	0.0040	8.9583	112.5000	7.00	0.80	1	52	6	1
25	23	0.0	155.0	Coal	1.2	0.0040	8.9583	112.5000	7.00	0.80	1	52	6	2
26	23	0.0	155.0	Coal	1.2	0.0040	8.9583	112.5000	7.00	0.80	1	52	6	2
27	13	0.0	197.0	Natural Gas	2.3	0.0070	6.4565	70.7609	5.00	0.70	1	52	7	1
28	13	0.0	197.0	Natural Gas	2.3	0.0070	6.4565	70.7609	5.00	0.70	1	52	7	1
29	13	0.0	197.0	Natural Gas	2.3	0.0070	6.4565	70.7609	5.00	0.70	1	52	7	1
30	23	0.0	350.0	Coal	1.2	0.0025	8.9667	161.8750	4.50	0.70	1	52	8	1
31	21	0.0	400.0	Nuclear	0.6	0.0018	13.8983	334.1693	5.00	0.30	1	52	9	2
32	18	0.0	400.0	Nuclear	0.6	0.0018	13.8983	334.1693	5.00	0.30	1	52	9	3
Total		0.0	3,405.0											3

Table 5.2 Percentage of the annual maximum demand during each week [15].

Week	Annual Demand Percen. (%)						
1	90.5%	14	76.5%	27	88.4%	40	81.8%
2	90.0%	15	78.5%	28	86.4%	41	83.6%
3	88.5%	16	81.7%	29	84.3%	42	86.5%
4	87.5%	17	84.1%	30	82.5%	43	89.1%
5	86.1%	18	85.8%	31	80.9%	44	91.5%
6	84.3%	19	87.6%	32	79.7%	45	93.4%
7	83.2%	20	88.9%	33	78.9%	46	95.1%
8	80.8%	21	90.9%	34	78.7%	47	96.5%
9	78.4%	22	91.4%	35	78.4%	48	97.6%
10	77.5%	23	91.5%	36	79.0%	49	98.5%
11	76.1%	24	91.4%	37	79.5%	50	99.5%
12	75.5%	25	90.6%	38	80.1%	51	100.0%
13	75.1%	26	89.6%	39	81.1%	52	99.2%

The daily load curves of each day of a week have been divided into three load blocks or subperiods corresponding to high, medium, and low demand. The duration and the percentage of the weekly demand on each block are shown in Table 5.3 and the respective energy demand in each week is shown in Figure 5.1.

Table 5.3 Demand block duration and percentage for weekly demand.

Demand Block	Duration (hrs.)	Weekly Demand Percentage	Weekly Electricity Prices Percent.
High	5.0	100.0%	100.0%
Medium	12.0	92.0%	95.0%
Low	7.0	85.0%	85.0%
Total	24.0		

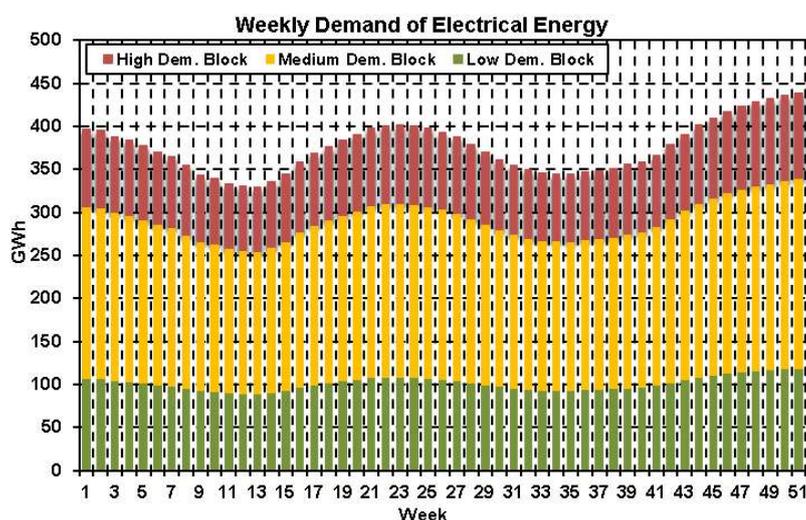


Figure 5.1 Demand of electrical energy per week (GWh) [15].

In a similar way, the weekly electricity price scenario used for the period under analysis is shown in Figure 5.2. This scenario is developed, using average wholesale electricity prices in Bolivia as a reference [135], so that the changes in prices reflect variations in electricity demand. Prices are usually highest when total demand is high because more expensive generation units are needed to meet the demand. Electricity prices for each subperiod of demand are found applying the percentages shown in Table 5.3 to the average Bolivian electricity prices used.

With respect to the constraints, a minimum reserve margin of 12% [136] of the annual peak demand is considered. Furthermore, the minimum number of available generators during the whole year of analysis is set to 14 units. Additionally, since a single bus is used, no transmission losses were considered.

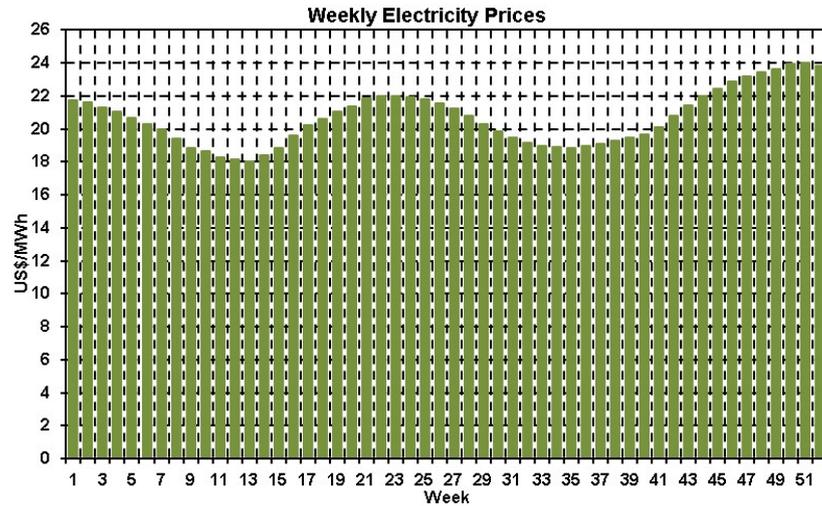


Figure 5.2 Weekly average electricity price scenario (US\$/MWh).

The parameter values related to the hybrid model proposed used to solve the GMS problem are shown in Table 5.4. Even though a large number of iterations is used to ensure better convergence of the NSGA-II, the convergence rate is expected to decline significantly at the end of the iteration process achieving only minor improvements with an additional computational budget [137]. Since the number of objective functions has a strong effect on the effectiveness of the NSGA II, the size of the population is set to 200 so that a quarter of the initial population generated belongs to the best non-dominated front as recommended in [138]. The crossover and mutation indexes values are defined to achieve a better space exploration capability on the NSGA II algorithm, by allowing it to generate offspring solutions distant from parent solutions [139]. The crossover probability indexes are defined so that most individuals in a parent population undergo a crossover process to produce child individuals with better fitness function values. What is more, the mutation probability is defined so that child solutions after each iteration can escape early convergence and achieve diversity. The values of both probabilities are set as suggested in [140] to enable an enhanced exploration capability of new solutions. On the other hand, the values of the LIM limits are set so that the marginal costs of all the generation units considered in the analysis are in the range of the maximum and minimum lambda values [21].

Table 5.4 Parameters defined for the hybrid NSGA II-LIM proposed.

No	Concept	Value
1 NSGA II		
	Size Initial Population:	$N_p = 200$
	Maximum Iteration:	$I_{max} = 800$
2 SBX Crossover Operator		
	Crossover probability:	$p_c = 80.0\%$
	Distribution Index:	$\eta_c = 3.5$
3 Polynomial Mutation Operator		
	Mutation probability:	$p_m = 5.0\%$
	Distribution Index:	$\eta_m = 1.5$
4 Lambda Iteration method		
	Maximum Lambda:	$\lambda_{max} = 150$
	Minimum Lambda:	$\lambda_{min} = 1$

Finally, to use TOPSIS, the weighting factors shown in Table 5.5 were assigned to each objective function of the problem. These weighting factors were defined in terms of any Electricity Regulation Agency (ERA) attitude towards the interests of the system. From the ERA perspective, the priority is to guarantee a reliable supply of electricity to consumers. The next priority is to deliver this electricity at a minimum cost possible. Finally, it is important to allow that each GENCO has an equal opportunity of making profits. With these weights defined and regulated in this way, then the decision-maker (ISO) can apply the hybrid model to find the best generation MS.

Table 5.5 TOPSIS weighting factors assigned to the objective function.

No	Objective Function			Weigth
	Type	Concept	Interested Party	
1	Maximization	Company Profits	GENCO 1	19.0%
2	Maximization	Company Profits	GENCO 2	19.0%
3	Maximization	Company Profits	GENCO 3	19.0%
4	Minimization	Operation Cost	ISO	20.0%
5	Maximization	System Reliability	ISO	23.0%
Total				100.0%

5.2 Multi-node thermal system scenario

The hybrid NSGA III/Dual Simplex model proposed is applied on the IEEE-RTS 24 bus test network with 32 generators and 38 transmission connections shown in Figure 5.3 [15]. Table 5.6 shows generators' technical characteristics, the GENCO they belong to, annual operation and maintenance costs, and the periods and duration of their maintenance.

Again, three GENCOs are defined in the problem to have three profit maximization objective functions. The period under analysis is of 52 weeks. The maximum annual demand considered is 2,850 MW. Table 5.2 shows the percentage values of the peak demand used to find the demand in each week.

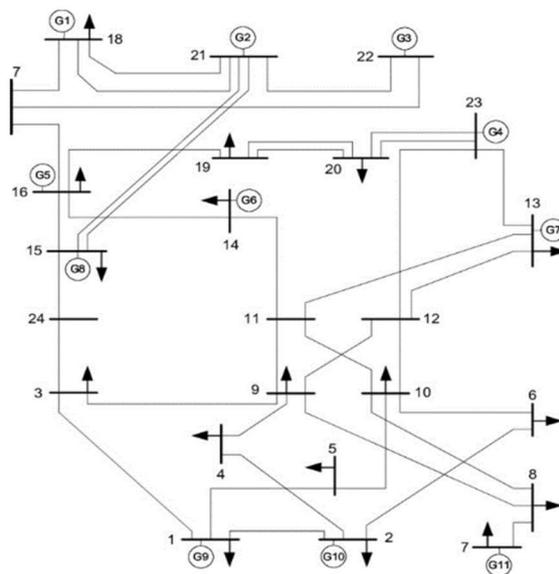


Figure 5.3 IEEE-RTS 24-Bus Test Network [15].

Table 5.6 Technical characteristics of generation units.

Name	Node	Size (MW)		Fossil Fuel Type	Heat Rate (Btu/kWh)	Output Curve (US\$/h)			Maintenance Cost		Maintenance (weeks)			GENCO
		Min	Max			a (US\$/MW ² h)	b (US\$/MWh)	c (US\$/h)	Fixed \$/kW/Yr	Variable \$/MWh	Earliest	Latest	Duration	
1	15	2.4	12.0	Gas Natural	13,219	0.02433	28.9700	15.1935	10.00	5.00	1	52	2	2
2	15	2.4	12.0	Gas Natural	13,219	0.02433	28.9700	15.1935	10.00	5.00	1	52	2	2
3	15	2.4	12.0	Gas Natural	13,219	0.02433	28.9700	15.1935	10.00	5.00	1	52	2	2
4	15	2.4	12.0	Gas Natural	13,219	0.02433	28.9700	15.1935	10.00	5.00	1	52	2	2
5	15	2.4	12.0	Gas Natural	13,219	0.02433	28.9700	15.1935	10.00	5.00	1	52	2	2
6	1	4.0	20.0	Diesel	9,859	0.06966	26.2440	28.7003	0.30	5.00	1	52	3	2
7	1	4.0	20.0	Diesel	9,859	0.06966	26.2440	28.7003	0.30	5.00	1	52	3	2
8	2	4.0	20.0	Diesel	9,859	0.06966	26.2440	28.7003	0.30	5.00	1	52	3	2
9	2	4.0	20.0	Diesel	9,859	0.06966	26.2440	28.7003	0.30	5.00	1	52	3	2
10	1	15.2	76.0	Coal	17,107	0.01280	17.8200	87.5000	10.00	0.90	1	52	4	2
11	1	15.2	76.0	Coal	17,107	0.01280	17.8200	87.5000	10.00	0.90	1	52	4	2
12	2	15.2	76.0	Coal	17,107	0.01280	17.8200	80.5000	10.00	0.90	1	52	4	3
13	2	15.2	76.0	Coal	17,107	0.01280	17.8200	80.5000	10.00	0.90	1	52	4	3
14	7	25.0	100.0	Natural Gas	8,089	0.00960	16.1900	91.5000	8.50	0.80	1	52	5	3
15	7	25.0	100.0	Natural Gas	8,089	0.00960	16.1900	91.5000	8.50	0.80	1	52	5	3
16	7	25.0	100.0	Natural Gas	8,089	0.00960	16.1900	91.5000	8.50	0.80	1	52	5	3
17	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
18	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
19	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
20	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
21	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
22	22	10.0	50.0	Natural Gas	8,800	0.00658	17.4000	59.3650	8.00	0.75	1	52	4	1
23	15	54.3	155.0	Coal	10,500	0.00481	10.7300	135.0000	7.00	0.80	1	52	6	1
24	16	54.3	155.0	Coal	10,500	0.00481	10.75000	135.0000	7.00	0.80	1	52	6	1
25	23	54.3	155.0	Coal	10,500	0.00481	10.75000	135.0000	7.00	0.80	1	52	6	2
26	23	54.3	155.0	Coal	10,500	0.00481	10.75000	135.0000	7.00	0.80	1	52	6	2
27	13	69.0	197.0	Natural Gas	8,348	0.01610	14.8500	162.7500	5.00	0.70	1	52	7	1
28	13	69.0	197.0	Natural Gas	8,348	0.01610	14.8500	162.7500	5.00	0.70	1	52	7	1
29	13	69.0	197.0	Natural Gas	8,348	0.01610	14.8500	162.7500	5.00	0.70	1	52	7	1
30	23	140.0	350.0	Coal	10,200	0.00300	10.7600	194.2500	4.50	0.70	1	52	8	1
31	21	100.0	400.0	Nuclear	12,751	0.00106	8.3390	200.5016	5.00	0.30	1	52	9	2
32	18	100.0	400.0	Nuclear	12,751	0.00106	8.3390	200.5016	5.00	0.30	1	52	9	3
Total		987.7	3,405.0											3

The daily load curves of each day of a week have been divided into three subperiods or demand blocks corresponding to high, medium, and low demand levels. The duration and the percentage of the weekly demand in each subperiod are shown in Table 5.7 and the corresponding weekly energy demand is the same shown in Figure 5.1.

Table 5.7 Demand subperiod duration and percentage for weekly demand.

Demand Block	Duration (hrs.)	Weekly Demand Percen. (%)
High	5.0	100.0%
Medium	12.0	92.0%
Low	7.0	85.0%

On the other side, the weekly electricity prices for each demand subperiod are determined by simulating a marginal price-based electricity market using the dual multipliers obtained after solving an economic dispatch (ED) problem. The system marginal price is found, and a marginal unit and node are identified using the following rules [141]:

- The marginal unit corresponds to the last dispatched unit in an unconstrained power dispatch.
- A unit forced to generate energy due to network congestion cannot be the marginal unit.
- A unit cannot be marginal if the electricity price of the node through which is connected to the system is different from its marginal generation cost.

The test network technical characteristics are shown in Table 5.8. Their maintenance cost considered are 2,500 US\$/miles.week for transmission lines and 40.5 US\$/MVA.week for power transformers [142].

With respect to the constraints, a minimum reserve margin of 12% of the annual peak demand was assumed [136]. Furthermore, the minimum number of available generators during the whole year of analysis was set to 14 units.

Table 5.8 Technical characteristics of transmission facilities.

No	Name	Node		Impedance		Length (miles)	Capacity (MW)	Maintenance			TRANSCO
		Conection		r (pu/line)	x (pu/line)			Window (week)		Duration (weeks)	
		i	j					Earliest	Latest		
1	LIN 1-2	1	2	0.0026	0.0139	3.0	193.00	1	52	2	1
2	LIN 1-3	1	3	0.0546	0.2112	55.0	208.00	1	52	3	1
3	LIN 1-5	1	5	0.0218	0.0845	22.0	208.00	1	52	0	1
4	LIN 2-4	2	4	0.0328	0.1267	33.0	208.00	1	52	2	1
5	LIN 2-6	2	6	0.0497	0.192	50.0	208.00	1	52	0	1
6	LIN 3-9	3	9	0.0308	0.119	31.0	208.00	1	52	2	1
7	TRA 3-24	3	24	0.0023	0.0839	0.0	510.00	1	52	1	1
8	LIN 4-9	4	9	0.0268	0.1037	27.0	208.00	1	52	0	1
9	LIN 5-10	5	10	0.0228	0.0883	23.0	208.00	1	52	2	1
10	LIN 6-10	6	10	0.0139	0.0605	16.0	193.00	1	52	0	1
11	LIN 7-8	7	8	0.0159	0.0614	16.0	208.00	1	52	0	1
12	LIN 8-9	8	9	0.0427	0.1651	43.0	208.00	1	52	3	1
13	LIN8-10	8	10	0.0427	0.1651	43.0	208.00	1	52	3	1
14	TRA 9-11	9	11	0.0023	0.0839	0.0	510.00	1	52	1	1
15	TRA 9-12	9	12	0.0023	0.0839	0.0	510.00	1	52	1	1
16	TRA 10-11	10	11	0.0023	0.0839	0.0	510.00	1	52	1	1
17	TRA 10-12	10	12	0.0023	0.0839	0.0	510.00	1	52	1	1
18	LIN 11-13	11	13	0.0061	0.0476	33.0	600.00	1	52	2	1
19	LIN 11-14	11	14	0.0054	0.0418	29.0	600.00	1	52	2	1
20	LIN 12-13	12	13	0.0061	0.0476	33.0	600.00	1	52	2	1
21	LIN 12-23	12	23	0.0124	0.0966	67.0	600.00	1	52	0	1
22	LIN 13-23	13	23	0.0111	0.0865	60.0	600.00	1	52	0	1
23	LIN 14-16	14	16	0.005	0.0389	27.0	600.00	1	52	0	1
24	LIN 15-16	15	16	0.0022	0.0173	12.0	600.00	1	52	2	1
25	LIN 15-21 A	15	21	0.0063	0.0490	34.0	600.00	1	52	2	1
26	LIN 15-21 B	15	21	0.0063	0.0490	34.0	600.00	1	52	2	1
27	LIN 15-24	15	24	0.0067	0.0519	36.0	600.00	1	52	0	1
28	LIN 16-17	16	17	0.0033	0.0259	18.0	600.00	1	52	0	1
29	LIN 16-19	16	19	0.003	0.0231	16.0	600.00	1	52	0	1
30	LIN 17-22	17	22	0.0135	0.1053	73.0	600.00	1	52	3	1
31	LIN 18-21 A	18	21	0.0033	0.0259	18.0	600.00	1	52	2	1
32	LIN 18-21 B	18	21	0.0033	0.0259	18.0	600.00	1	52	2	1
33	LIN 19-20 A	19	20	0.0051	0.0396	27.5	600.00	1	52	2	1
34	LIN 19-20 B	19	20	0.0051	0.0396	27.5	600.00	1	52	2	1
35	LIN 20-23 A	20	23	0.0028	0.0216	15.0	600.00	1	52	2	1
36	LIN 20-23 B	20	23	0.0028	0.0216	15.0	600.00	1	52	2	1
37	LIN 21-22	21	22	0.0087	0.0678	47.0	600.00	1	52	3	1
38	LIN 17-18	17	18	0.0018	0.0144	10.0	600.00	1	52	0	1

The parameter values related to the hybrid model proposed to solve the GMS problem are shown in Table 5.9. The number of objective functions has a huge effect on the proportion of non-dominated solutions present in an initial population with a certain number of individuals. In that sense, the population size is chosen so that a third of the individuals created in the initial population belong to the first non-dominated front, as recommended in [138]. Furthermore, the number of iterations used ensures convergence of the NSGA III avoiding any unnecessary computational effort [137]. Following the reference points creation technique, the number of partitions defined is a little bit greater than the number of objective functions in the problem. In this way, it is expected that every solution found will be associated with a particular reference point created [61,67]. The crossover and mutation probability distribution indexes defined

correspond to initial values and is expected that, with the self-adaptive capabilities introduced to these operators in this scenario, they will change according to the necessities of the evolutionary process. The number of grids defined for the diversity metric (DM) of the self-adaptive simulated binary crossover (SBX) operator value is equal to the number of partitions used to generate reference points. In such a manner, it is possible to couple the self-adaptive SBX operator with the NSGA III algorithm more effectively. On the other side, the crossover probability is defined so that most individuals in a parent population undergo a crossover process to produce child individuals with better fitness function values [140]. The mutation probability value is defined as a number equal or greater than the inverse of the number of variables or the length of the individuals in the problem [132].

Table 5.9 Parameters of the hybrid NSGA III-Dual Simplex model proposed.

No	Concept	Value	Concept	Value
1	NSGA III			
	Size of Population:	Np = 200	Number of Partitions:	p = 6
	Maximum Iteration:	Imax = 500	Number of Obj. Functions:	M = 5
	Number Ref. Points:	H = 210		
2	SBX-SAM Crossover Operator			
	Crossover Probability:	pc = 80.0%	Number of Grids:	G = 6
	Initial Distribution Index:	ηc = 10		
3	Self Adaptive Polynomial Mutation Operator			
	Mutation Probability:	pm = 5.0%		
	Initial Distribution Index:	ηm = 0.6		

Finally, TOPSIS is used as a decision-making tool again, to assist the decision-maker (ISO) to choose the best generation and transmission MS solution. The weighting factors shown again in Table 5.5 are determined from the perspective of the ERA as explained before.

5.3 Multi-node wind-hydro-thermal system scenario

The novel hybrid NSGA III/Dual Dynamic (DD) model is applied on the IEEE-RTS 24 bus test network already shown in Figure 5.3 [3]. Table 5.10 and Table 5.11 show thermal and hydro unit's technical characteristics respectively, the GENCO they belong to, their operation and maintenance costs, and the periods and duration of their maintenance.

Table 5.10 Technical characteristics of thermal generation units.

Name	Node	Size (MW)		Fossil Fuel Type	Fuel Price (US\$/Mbtu)	Rendimiento Termico (Btu/kWh)			Operation Cost (US\$/MWh)		Maintenance Cost (\$/kW/yr.)	Generation Cost (US\$/MWh)			Maintenance (weeks)			GENCO
		Min	Max			50%	75%	100%	Fixed (US\$/kW/yr)	Variable (US\$/MWh)		90%	85%	82%	Window		Duration	
															Earliest	Latest		
GEN1	15	2.4	12.0	Gas Natural	2.30	12,900	11,900	12,000	4.50	0.90	10.00	28.45	27.00	25.55	1	52	2	2
GEN2	15	2.4	12.0	Gas Natural	2.30	12,900	11,900	12,000	4.50	0.90	10.00	28.45	27.00	25.55	1	52	2	2
GEN3	15	2.4	12.0	Gas Natural	2.30	12,900	11,900	12,000	4.50	0.90	10.00	28.45	27.00	25.55	1	52	2	2
GEN4	15	2.4	12.0	Gas Natural	2.30	12,900	11,900	12,000	4.50	0.90	10.00	28.45	27.00	25.55	1	52	2	2
GEN5	15	2.4	12.0	Gas Natural	2.30	12,900	11,900	12,000	4.50	0.90	10.00	28.45	27.00	25.55	1	52	2	2
GEN6	1	4.0	20.0	Diesel	3.00	12,000	15,000	14,500	0.14	5.00	0.30	48.86	45.78	42.70	1	52	3	2
GEN7	1	4.0	20.0	Diesel	3.00	12,000	15,000	14,500	0.14	5.00	0.30	48.86	45.78	42.70	1	52	3	2
GEN8	2	4.0	20.0	Diesel	3.00	12,000	15,000	14,500	0.14	5.00	0.30	48.86	45.78	42.70	1	52	3	2
GEN9	2	4.0	20.0	Diesel	3.00	12,000	15,000	14,500	0.14	5.00	0.30	48.86	45.78	42.70	1	52	3	2
GEN10	1	15.2	76.0	Coal	1.20	12,900	11,900	12,000	4.73	0.90	10.50	15.27	14.49	13.71	1	52	4	1
GEN11	1	15.2	76.0	Coal	1.20	12,900	11,900	12,000	4.73	0.90	10.50	15.27	14.49	13.71	1	52	4	2
GEN12	2	15.2	76.0	Coal	1.20	12,900	11,900	12,000	4.73	0.90	10.50	15.27	14.49	13.71	1	52	4	3
GEN13	2	15.2	76.0	Coal	1.20	12,900	11,900	12,000	4.73	0.90	10.50	15.27	14.49	13.71	1	52	4	3
GEN14	7	25.0	100.0	Gas Natural	2.30	10,600	10,100	10,000	3.83	0.80	8.50	23.87	22.64	21.41	1	52	5	3
GEN15	7	25.0	100.0	Gas Natural	2.30	10,600	10,100	10,000	3.83	0.80	8.50	23.87	22.64	21.41	1	52	5	3
GEN16	7	25.0	100.0	Gas Natural	2.30	10,600	10,100	10,000	3.83	0.80	8.50	23.87	22.64	21.41	1	52	5	3
GEN17	15	54.3	155.0	Coal	1.20	10,100	9,800	9,700	3.15	0.80	7.00	12.48	11.82	11.16	1	52	6	1
GEN18	16	54.3	155.0	Coal	1.20	10,100	9,800	9,700	3.15	0.80	7.00	12.48	11.82	11.16	1	52	6	1
GEN19	23	54.3	155.0	Coal	1.20	10,100	9,800	9,700	3.15	0.80	7.00	12.48	11.82	11.16	1	52	6	2
GEN20	23	54.3	155.0	Coal	1.20	10,100	9,800	9,700	3.15	0.80	7.00	12.48	11.82	11.16	1	52	6	2
GEN21	13	69.0	197.0	Gas Natural	2.30	9,850	9,840	9,600	2.25	0.70	5.00	22.94	21.72	20.50	1	52	7	1
GEN22	13	69.0	197.0	Gas Natural	2.30	9,850	9,840	9,600	2.25	0.70	5.00	22.94	21.72	20.50	1	52	7	1
GEN23	13	69.0	197.0	Gas Natural	2.30	9,850	9,840	9,600	2.25	0.70	5.00	22.94	21.72	20.50	1	52	7	1
GEN24	23	140.0	350.0	Coal	1.20	9,600	9,500	9,500	2.03	0.70	4.50	12.10	11.44	10.78	1	52	8	1
GEN25	21	100.0	400.0	Nuclear	0.60	10,825	10,170	10,000	2.25	0.30	5.00	6.33	6.01	5.69	1	52	9	2
GEN26	18	100.0	400.0	Nuclear	0.60	10,825	10,170	10,000	2.25	0.30	5.00	6.33	6.01	5.69	1	52	9	3
Total		927.7	3,105.0															3

Table 5.11 Technical characteristics of hydro generation units [135].

Name	Node	Capacity (MW)		Turbinate Volume (m3/s)	Production Factor (MW/m3/s)	Reservoir Location		Operation Cost		Maintenance Cost (\$/kW/yr)	Type of Turbine	Maintenance			GENCO
		Min	Max			Up-stream	Down-stream	Fixed (\$/kW/yr)	Variable (US\$/MWh)			Window (week)		Duration (weeks)	
												Earliest	Latest		
YAN1	22	25.40	50.81	5.80	11.60	4.38	2	5.00	0.00	7.00	Francis	1	52	3	2
CHJ1	22	19.25	39.60	3.50	7.20	5.50	1	4.50	0.00	6.30	Francis	1	52	3	2
COR12	22	6.00	26.00	1.20	5.20	5.00	3	2.00	0.00	2.80	Pelton	1	52	1	1
COR34	22	6.00	26.00	1.20	5.20	5.00	3	2.00	0.00	2.80	Pelton	1	52	1	1
COR5	22	3.00	13.00	0.60	2.60	5.00	3	1.00	0.00	1.40	Pelton	1	52	1	1
SIS12	22	4.00	35.00	0.57	5.00	7.00	4	4.00	0.00	5.60	Pelton	1	52	1	1
SIS34	22	4.00	35.00	0.57	5.00	7.00	4	4.00	0.00	5.60	Pelton	1	52	1	1
SIS5	22	5.00	21.00	0.71	3.00	7.00	4	2.00	0.00	2.80	Pelton	1	52	1	1
SJO1	22	5.00	28.00	2.00	11.20	2.50	5	3.00	0.00	4.20	Pelton	1	52	2	1
SJO2	22	5.00	28.00	2.00	11.20	2.50	5	3.00	0.00	4.20	Pelton	1	52	2	1

What is more, Table 5.12 and Figure 5.4 present the reservoirs' capacity and their hydroelectrical configuration, respectively. As it can be seen, the initial water volumes in the reservoirs at the beginning of the simulation are stated. What is more, there are 2 groups of cascade dams to which 10 hydroelectrical generators are connected. Spillage costs are calculated for each reservoir based on the location of the dams in the hydroelectric configuration. Upstream reservoirs built along a river have a greater spillage cost since spillage water represents greater potential energy lost [135]. Table 5.13 shows the technical characteristics of the wind turbines considered in the system. Information related to hydro units, their reservoirs, and wind turbines technical data correspond to two GENCOs operating in the Bolivian Electricity Market [135].

Table 5.12 Technical characteristics of water reservoirs [135].

No	Capacity Limits (Hm ³)		Initial volume (Hm ³)	Upstream Reservoir	Spillage Cost (US\$/Hm ³)
	Maximum	Minimum			
1	5.600	0.030	4.50		147,494.3
2	0.110	0.060	0.10	1	82,107.1
3	145.000	7.400	140.00		612,071.4
4	0.120	0.040	0.10	3	388,142.9
5	0.170	0.004	0.12	4	0.0

Figure 5.4 Reservoir's hydroelectric configuration [135].

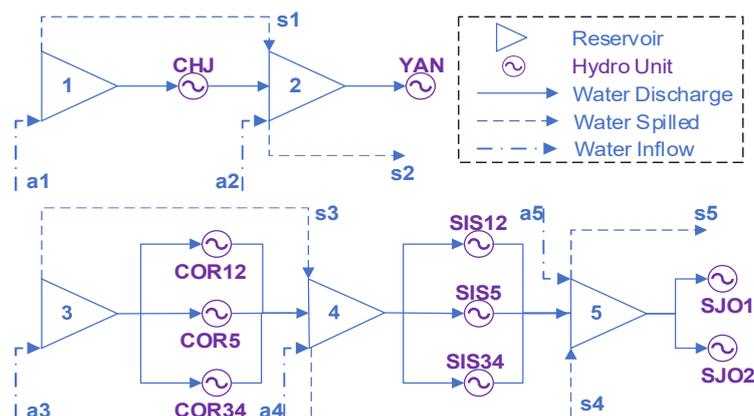


Table 5.13 Wind farms technical characteristics [135].

No	Number of Units	Capacity per Unit (MW)	Speed (m/s)			Rotor Blade Diameter (m)	Hub High (m)	Node
			Cut-in	Cut-out	Rated			
1	8	3.0	2.5	28.0	20.0	82.00	78.00	5
2	2	1.5	3.0	22.0	13.0	77.00	65.00	4

In terms of water inflows, historical data of weekly water inflows in the Bolivian reservoirs corresponding to 40 years obtained in [135] are used to determine the water cost-to-go function of the system for a period of 3 years. In the same way, average hourly wind speeds in Bolivian wind farms for each week of the past 10 years, described in [135], were used to determine the potential renewable energy generated for the year under analysis. All these data are detailed in Appendix 2.

Three profit maximization objective functions related to the three GENCOs in the problem are defined. The period under analysis is again 52 weeks. The annual peak demand used, and its weekly percentages are the same as in the previous scenario and are shown in Table 5.2.

The daily load curves have been divided into three subperiods corresponding to high, medium, and low demand levels. The duration, percentage of maximum demand and the generation capacity kept as spinning reserve on each subperiod

are shown in Table 5.14. The spinning reserve is allocated among generators so that more generation capacity is available to supply the peak demand and reduce generation costs as much as possible [135]. The weekly energy demand, adjusted with the contribution of wind generation, is shown in Figure 5.5. This potential wind contribution is found by applying Monte Carlo analysis to the wind farms and wind speeds data specified in the last chapter. It reached a total of 36.40 GWh for the year under analysis.

Table 5.14 Demand subperiod duration and percentage for weekly demand..

Demand Block	Time (hrs)	Weekly Demand Percen. (%)	Gen. Capacity kept as Spinning Reserve (%)
High	5.0	100.0%	10.00%
Medium	12.0	92.0%	15.00%
Low	7.0	85.0%	18.00%
Total	24.0		

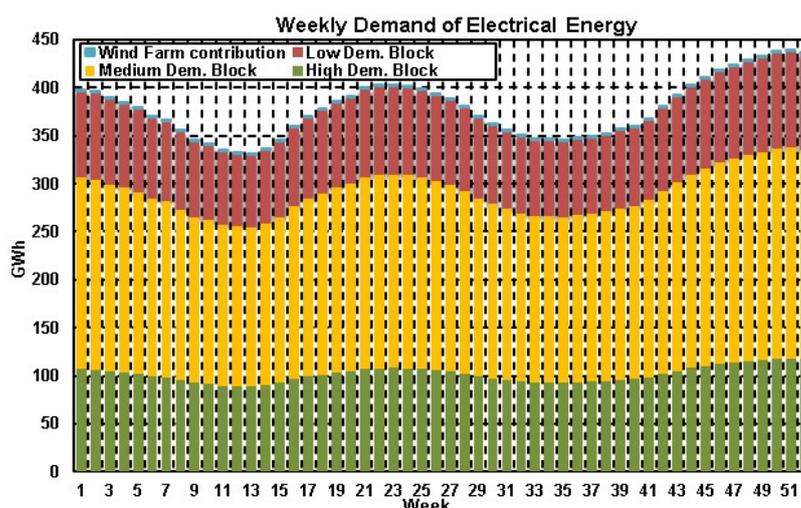


Figure 5.5 Demand of electrical energy per week (GWh)

Additionally, the weekly electricity prices for each demand subperiod were determined using the dual multipliers obtained from the solution of the ED problem and defining node 18 as the slack bus, so penalty factors can be calculated using that node as a reference bus.

Network technical characteristics are the same as the ones shown in Table 5.8. In the same way, the transmission costs used are again 2,500 US\$/miles.week for lines and 40.5 US\$/MVA.week for power transformers [142].

In terms of the problem constraints, a minimum reserve margin of 12% of the annual peak demand was assumed [3] and a minimum number of 18 available units was set. The NSGA III parameter values used to solve the wind-hydro-thermal GTMS problem are shown in Table 5.9. Finally, TOPSIS is used to assist in the decision-making about the best generation and transmission MS solution, and its weighting factors are determined as already explained in Table 5.5.

CHAPTER 6

NUMERICAL RESULTS

This chapter is devoted to the presentation of the results obtained from applying the hybrid classical and multi-objective evolutionary algorithm (MOEAs) novel models, formulated, and described in Chapters 3 and 4 respectively, to solve the generation and transmission maintenance scheduling (GTMS) problem in the three power systems scenarios, whose characteristics were stated in Chapter 5.

First, the evolution of the generation and transmission maintenance schedule (MS) solutions towards a feasible non-dominated set of solutions is shown. Graphical representations consisting of two-dimensional charts showing the conflicting relationship between the different objective functions of the best non-dominated front of MS solutions are presented and analyzed. At the same time, the evolution of the values of the objective functions corresponding to the best solutions to the GTMS problem is depicted. Finally, the best generation and transmission MS solution for each power system scenario is identified and an analysis of its power characteristics is made.

The simulations consider a period of analysis of 52 week, each week consisting of 3 subperiods designed to model more accurately the demand. These simulations, performed with the proposed hybrid NSGA II/LIM, NSGA III/Dual Simplex (DS) and NSGA III/Dual Dynamic (DD) models, were all conducted in MATLAB R2019b in an Intel® Core™ i7-8700 CPU 3.20GHz processor.

6.1 Single node thermal system scenario

The simulation of the proposed hybrid NSGA II/Lambda Iteration Method (LIM) model used to determine a set of non-dominated MS solutions in a single node thermal system scenario returns the results show in Appendix 3 and described below.

6.1.1 Feasibility of MS solutions

All the MS solutions generated by the NSGA II/LIM model in the final population are feasible and comply with the constraints formulated. The model allows the initial population to evolve after each iteration towards a feasible non-dominated set of solutions as shown in Figure 6.1.

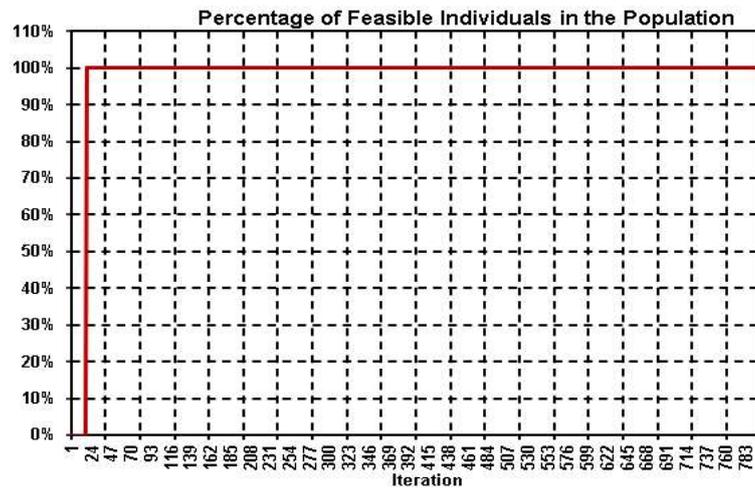


Figure 6.1 Percentage of feasible MS (individuals) in the population.

As it can be seen in Figure 6.1, with the Self-Adaptive Constraint Handling (SACH) technique used, the model allows the infeasible solutions in the initial population to evolve after each iteration towards a feasible and non-dominated set of MS solutions.

The NSGA II/LIM model returned a total of 155 different non-dominated MS solutions at the end of the iteration process. The maximum and minimum values of their objective functions are shown in Table 6.1.

Table 6.1 Maximum and minimum values of the objective functions of the non-dominated MS solutions.

Values	Adequacy Index	System Operation Cost (MMUS\$)	Profits (MMUS\$)		
			GENCO 1	GENCO 2	GENCO 3
Maximum	59.58%	276.10	35.36	41.61	29.91
Minimum	56.85%	275.87	33.69	39.92	28.90

Using these values, the normalized objective functions were found so that they could be plotted in the value path chart shown in Figure 6.2.

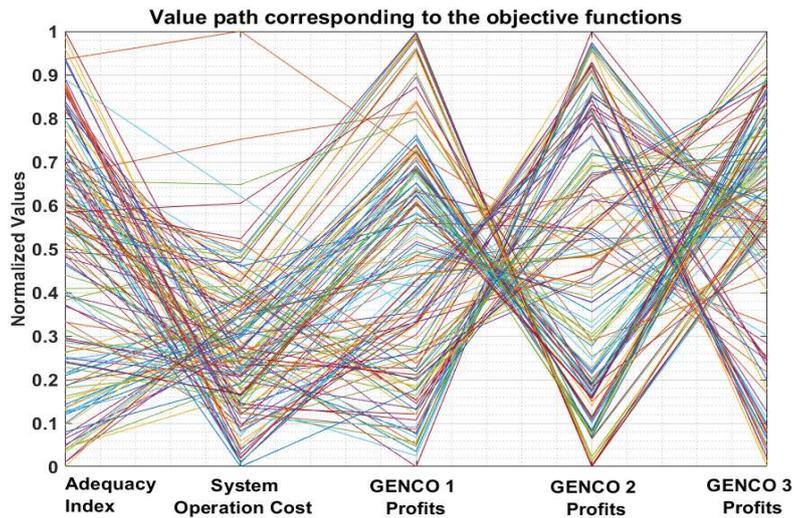


Figure 6.2 Value path chart corresponding to the non-dominated MS solutions.

From the figure, it can be observed that the model spreads the solutions found all over the entire adequacy index and the GENCOs' profits vectors. At the same time, there is a strong “zig-zag” effect among the objective functions, showing the conflicting relationship among the utilities' goals, more especially between the adequacy index and the system total operation cost.

6.1.2 Objective function's relationship

The feasible non-dominated MS results are spread across a five-dimensional space whose axes correspond to the system's adequacy index, total operation cost, and the profits of three GENCOs. To present and interpret the results, a set of two-dimensional plots with their non-dominated solutions identified are shown in Figures 6.3-6.5.

The non-dominated fronts of the three objective functions related to profit maximization of the GENCOs are shown in Figures 6.3a-d. The figure reflects the competitive relationship between the three companies. The relation is strong between GENCO 3 and the other two companies. This is because GENCO 3 owns some of the most efficient low-cost generation units in the system which are usually dispatched as a priority by the ISO to meet the energy demand. However, the competitive relation between GENCOs 1 and 2 is even stronger because both utilities own units with similar generation costs. When any of the generators of one of these companies are unavailable, the units of the other company can be

dispatched to cope with the loss of capacity and supply the demand. Figure 6.3d also shows the three-dimensional shape of the non-dominated front of profits of the three GENCOs when plotted together.

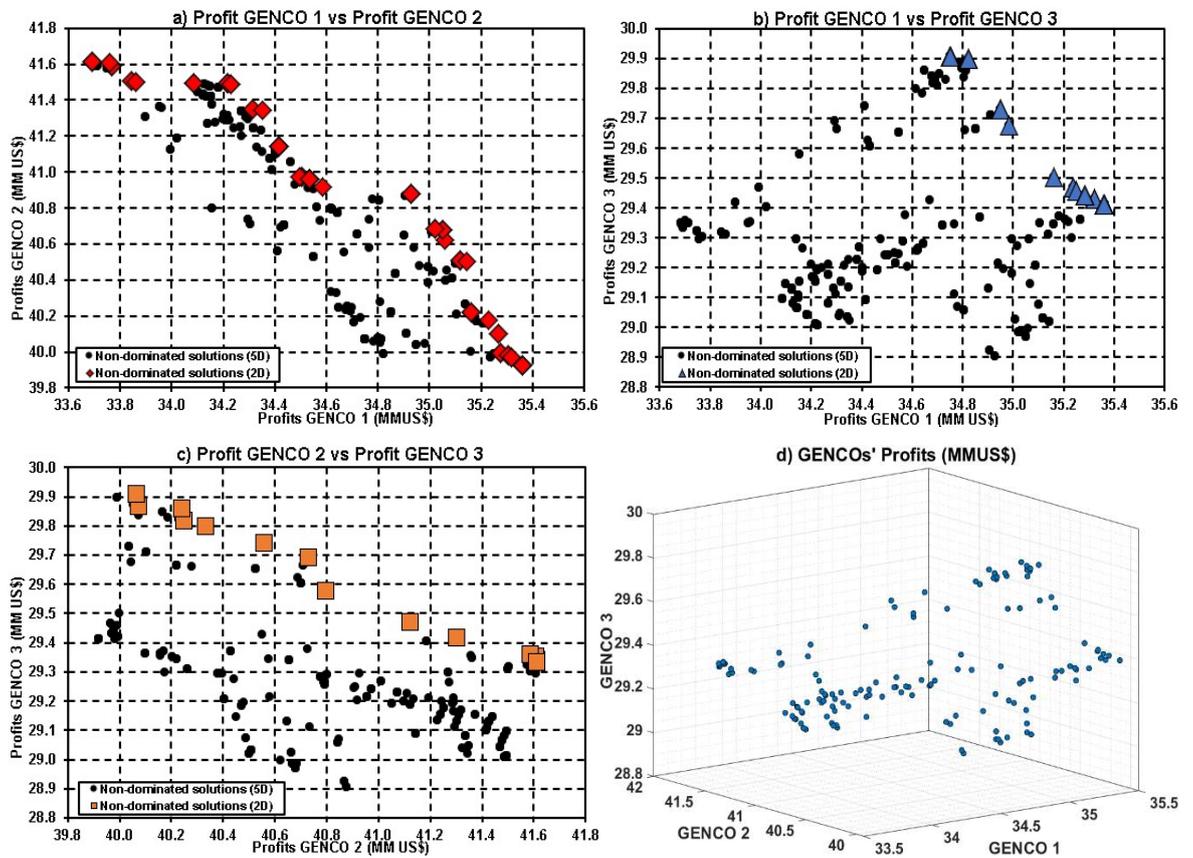


Figure 6.3 Non-dominated fronts for GENCOs profits objective functions (MMUS\$).

Figure 6.4 presents the non-dominated fronts related to the profits of each GENCO and the system total operation cost. Notice that when the GENCO's profits increase, the system operation cost increases as well. GENCO 3 and GENCO 2 owns the most efficient units in the system that generate electricity to supply the base load demand. This represents a cost for the system that can only increase if other less efficient units of these GENCOs are dispatched when generators of the other companies are unavailable due to maintenance. On the other side, GENCO 2 owns many of the less efficient units that get dispatched if other more efficient units in the system are unavailable due to maintenance. When this happens, the total operation cost of the system increases. Furthermore, since GENCO 3 and GENCO 1 have a strong competing relationship, when units of one of these GENCOs are on maintenance some units of the other GENCO are dispatched by the ISO, making

system operation cost less expensive. Figure 6.4d also shows the shape of the 3D-space front when the non-dominated profits of GENCOs 1 and 3 and the total operation cost of the system are plotted together.

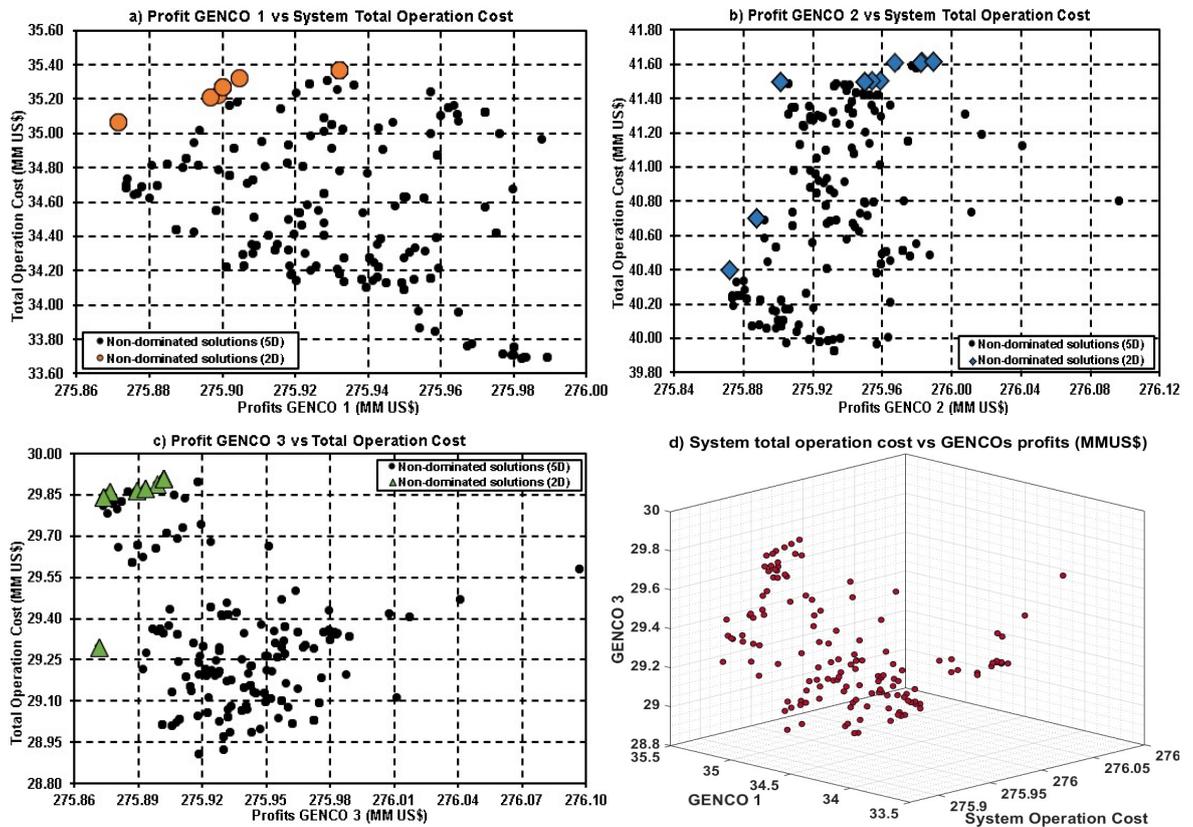


Figure 6.4 Non-dominated front for GENCO's profit and system total operation cost (MMUS\$)

Figure 6.5a-d shows the non-dominated fronts related to the profits of each GENCO, the system total operation cost, and the system adequacy index. Most of the inefficient units of GENCO 2 have small capacity and hardly ever get dispatched by the ISO, even when more efficient and larger capacity units are on maintenance, so when these units are unavailable the utility experience a small reduction in its profits. In such a case, the ISO manages to compensate the adequacy index of the system by making available generators of the other companies to supply the high subperiod demand. The same is true for GENCO 3 because of the strong competitive relation between this utility and GENCO 2. In case of GENCO 1, this situation is more pronounced since its efficient and large capacity units, which usually get dispatched, are crucial to increase its profits and the system adequacy index. Figure 6.5d also shows that the system total operation cost increases as the adequacy index increases as well and vice versa. This is

because of the necessity of the ISO to have many units available, especially during high demand periods, to keep the reserve margin but running more expensive generation units.

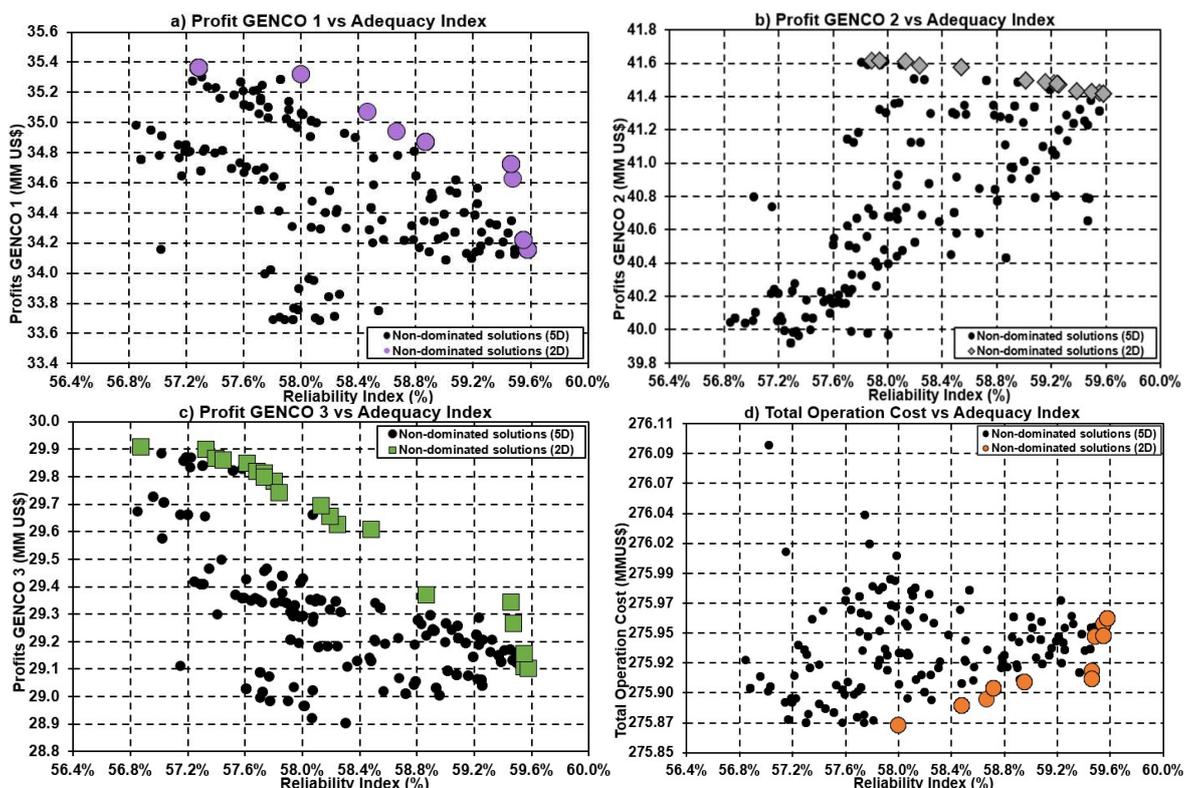


Figure 6.5 Non-dominated fronts for GENCO's profit (MMUS\$) and system adequacy index

6.1.3 Evolution of the best MS during simulation

Every individual inside these non-dominated sets represents a possible MS feasible solution. With the proposed hybrid model, each of these scenarios found have a minimum system total operation cost associated with a single adequacy index.

The model works in accordance with the GMS practices of the electricity industry, related to keeping adequacy but ensuring minimization of total operation costs, with the addition that it provides the ISO with a set of feasible non-dominated MS, among which the best can be chosen. Since the TOPSIS decision-making tool has been applied after each iteration, the evolution of the objective functions related to the best generation MS scenario was tracked during the simulation and is shown in Figure 6.6. After each iteration, the hybrid model tries to improve the system

adequacy index and reduce the total operation cost since these two objective functions have a greater weighting factor assigned. In the case of GENCO's profits, the model attempts to increase them without compromising the system adequacy index. At the same time, the competitive relationship between GENCO 2 and 3 is evident. As iteration progress, for every best MS solution found, an increase in the profits of one of these utilities represents a profit decrease in the other.

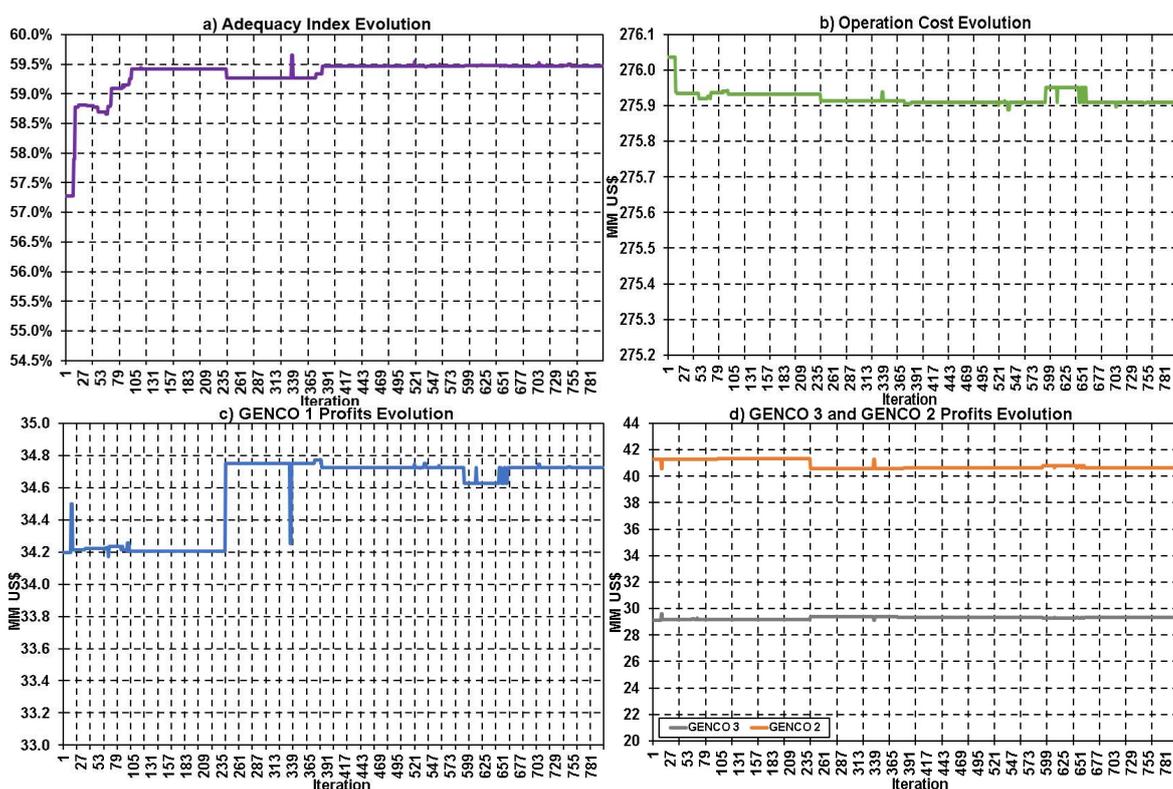


Figure 6.6 Evolution of the objective functions in each iteration.

Interesting to note is how, from one iteration to another, the best solution found changes by improving one objective function at the expense of another. This generates some oscillations on the MS best solution's objective functions values during the evolutionary process and shows that the model keeps finding better new MS solutions during the iterations and adding them to the final set.

6.1.4 Best Generation MS solution

The best generation MS scenario generated by the model is shown in Figure 6.7. Notice that at any time at least one of the most efficient units belonging to the GENCOs is available for generating electricity. These generators are nuclear, and coal-fired, and of large capacity. What is more, these units are scheduled for

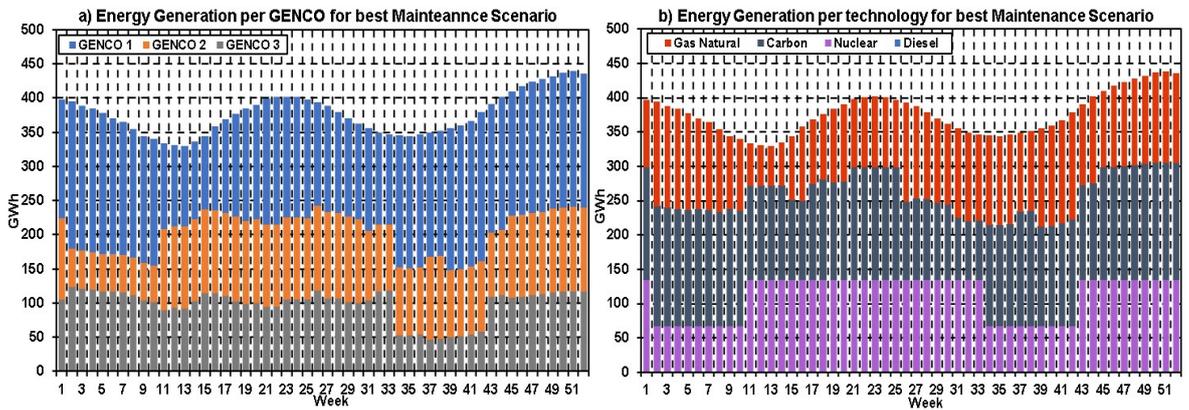


Figure 6.8 Energy generated by GENCOs and by type of fuel used.

Figure 6.9 shows the reserve margin of the system and the number of units available related to the best MS scenario found by the model. At the start, middle, and the end of the period of analysis there is a greater number of generation units available since during these weeks the demand is high, and more units are needed to keep a high adequacy index. Notice that a minimum of 25 units are available at week 12 and the reserve margin reaches a minimum value of 347.6.6 MW at week 9. Both values comply with the constraints stated for the GMS problem. Finally, it must be mentioned that the best MS scenario resulted in an adequacy index of 59.5%, a system total operation cost of MMUS\$ 275.91, and profits of MMUS\$ 34.72, MMUS\$ 40.66 and MMUS\$ 29.34 for GENCOs 1, 2, and 3, respectively.

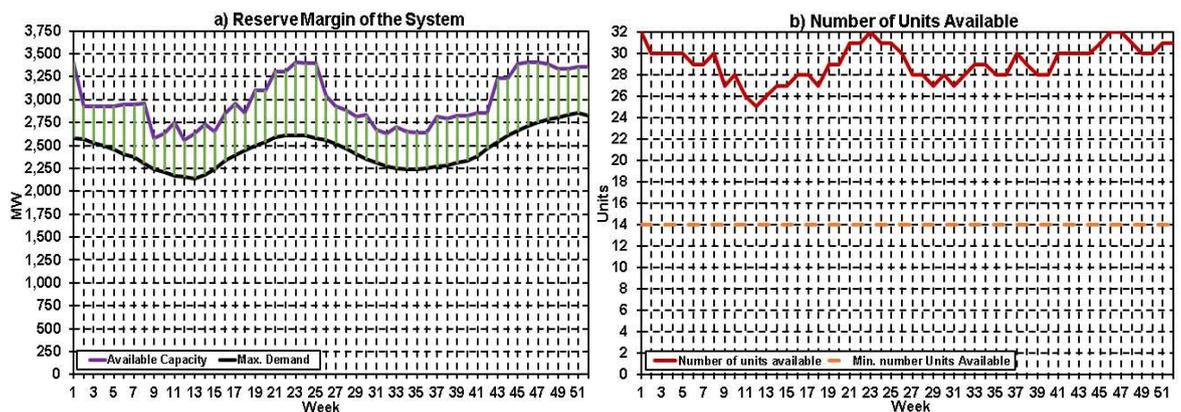


Figure 6.9 Reserve margin and number of units available in the system.

6.2 Multi-node thermal system scenario

The simulation of the proposed hybrid NSGA III/Dual Simplex (DS) model used to solve the GTMS problem to find a set of non-dominated MS solutions in a multi-node thermal system, gives the results shown in Appendix 4 and described below.

6.2.1 Feasibility of MS solutions

The results obtained show that all the solutions or MS scenarios (individuals) in the final population become part of a feasible set. As shown in Figure 6.10, the combined action of the adaptive trade-off model (ATM) constraint handling technique and the achievement scalarization function (ASF) normalization process allows the infeasible initial population to evolve after each iteration gradually towards a feasible and non-dominated set of MS solutions. From iteration 80 onwards, the number of feasible individuals in the population increases dramatically and reaches at the end of the simulation with a proportion of 100%.

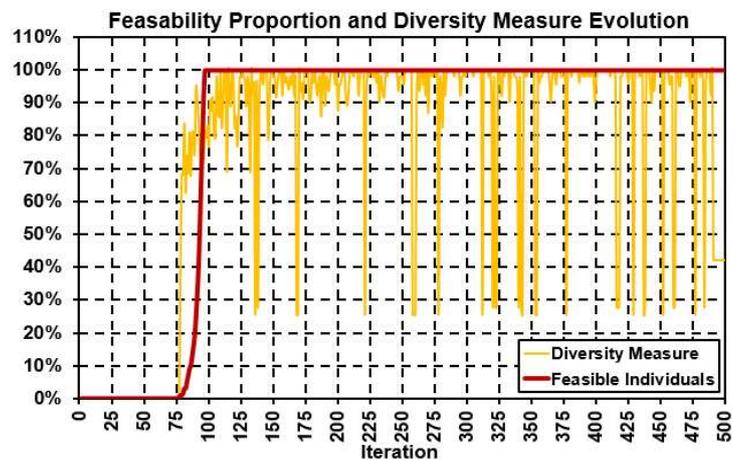


Figure 6.10 Percentage of feasible MS (individuals) in the population and diversity measure (DM) evolution.

What is more, the figure also presents the evolution of the Diversity Measure (DM) of the SBX-SAM operator from a low to a high and almost constant value near 100%. This is because at the beginning of the evolution process the population of MS is diverse making the low crossover distribution index values spread across a wide range. As evolution progresses, feasible solutions are found, the diversity of the population stabilizes, and DM gradually reaches a higher and constant value. At this stage, on occasions, DM is perturbed due to the mutation operator effect. Since no additional feasible solutions are found in the search space, the exploration phase of SBX-SAM ends, while the exploitation phase begins, making the value of the crossover distribution index reach high values located in a small range.

In this scenario, the NSGA III/DS model returned a total of 231 different non-dominated MS solutions at the end of the iteration process. The maximum and minimum values of their objective functions are shown in Table 6.2.

Table 6.2 Maximum and minimum values of the objective functions of the non-dominated MS solutions.

Values	Adequacy Index	System Operation Cost (MMUS\$)	Profits (MMUS\$)		
			GENCO 1	GENCO 2	GENCO 3
Maximum	59.79%	276.36	36.57	41.17	30.20
Minimum	56.70%	275.87	35.03	40.00	28.71

With this values, the normalized objective functions were found so that they could be plotted in the value path chart shown in Figure 6.11.

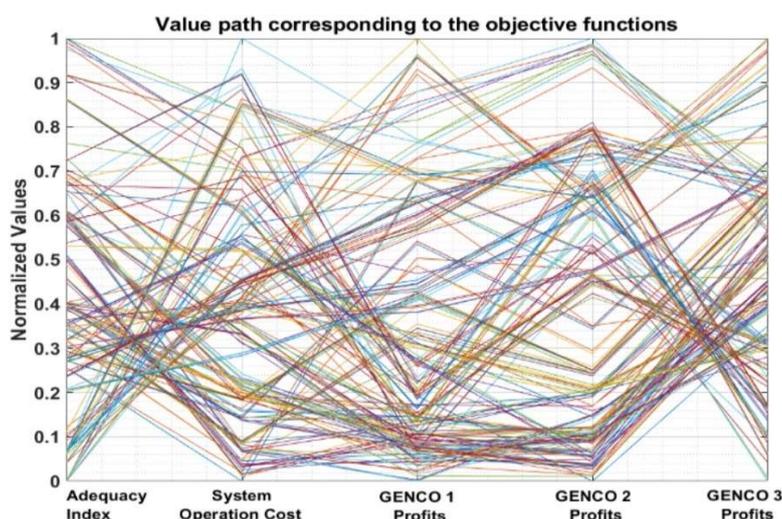


Figure 6.11 Value path chart corresponding to the non-dominated MS solutions.

From the figure, it can be observed that the model provides solutions spread more evenly over all the objectives functions vectors. Is important to realize that there is a strong “zig-zag” effect among the objective functions as well, showing the conflicting relationship among all the utilities’ goals in the market.

6.2.2 Objective function’s relationship

The feasible non-dominated MS results are spread across a five-dimensional (5D) space whose axis correspond to the systems adequacy index and total operation cost and the profits of three GENCOs. To represent better the results, a set of

two-dimensional (2D) plots with their non-dominated solutions identified are shown in Figures 6.12-6.14.

The non-dominated fronts of the three objective functions related to profit maximization of the GENCOs are shown in Figure 6.12, which reflects the competitive relationship between these utilities. This relation is weak between GENCO 1 and the other two companies. This is because units of GENCO 1 are usually the marginal units of the system and end up defining the electricity marginal cost of the system. The strong correlation between this cost and the demand has an important impact in electricity prices and revenues of all companies. This puts this GENCO in a strong position in the market, allowing it to make considerable profits and set prices. However, the competitive relation between GENCOs 3 and 2 is stronger because both utilities own units with similar generation costs. Then, when any of the units of one of these companies are on maintenance, the units of the other company are dispatched by the ISO immediately to cope with the loss of capacity in the system and supply the demand. Figure 6.12d also shows the 3D shape of the non-dominated front of profits of the three GENCOs when plotted together.

Figure 6.13 presents the non-dominated fronts related to the profits of each GENCO and the system total operation cost. It shows that when the profits of GENCO 3 increase, the system total operation cost increases as well, and vice versa. The same is true for the relationship between total generation costs with GENCO 2 profits and, to a lesser extent, with GENCO 1's profits too. GENCO 2 and GENCO 3 own the most efficient units of the system, which when on maintenance causes the system operation cost to increase. This situation represents a loss of profits for these GENCOs, which is compensated by the revenues made from selling energy generated with less efficient units at a higher price. On the other side, GENCO 1 owns many of the less efficient units in the system. When these are available, they get dispatched, and some become marginal units and fix the electricity prices in the system. As a result, the total operation cost of the system varies according to the demand fluctuations. Furthermore, Figure 6.13d also shows the shape of a non-dominated front in a 3D-space when the non-dominated profits of GENCOs 1 and 2 and the total operation cost of the system are plotted together.

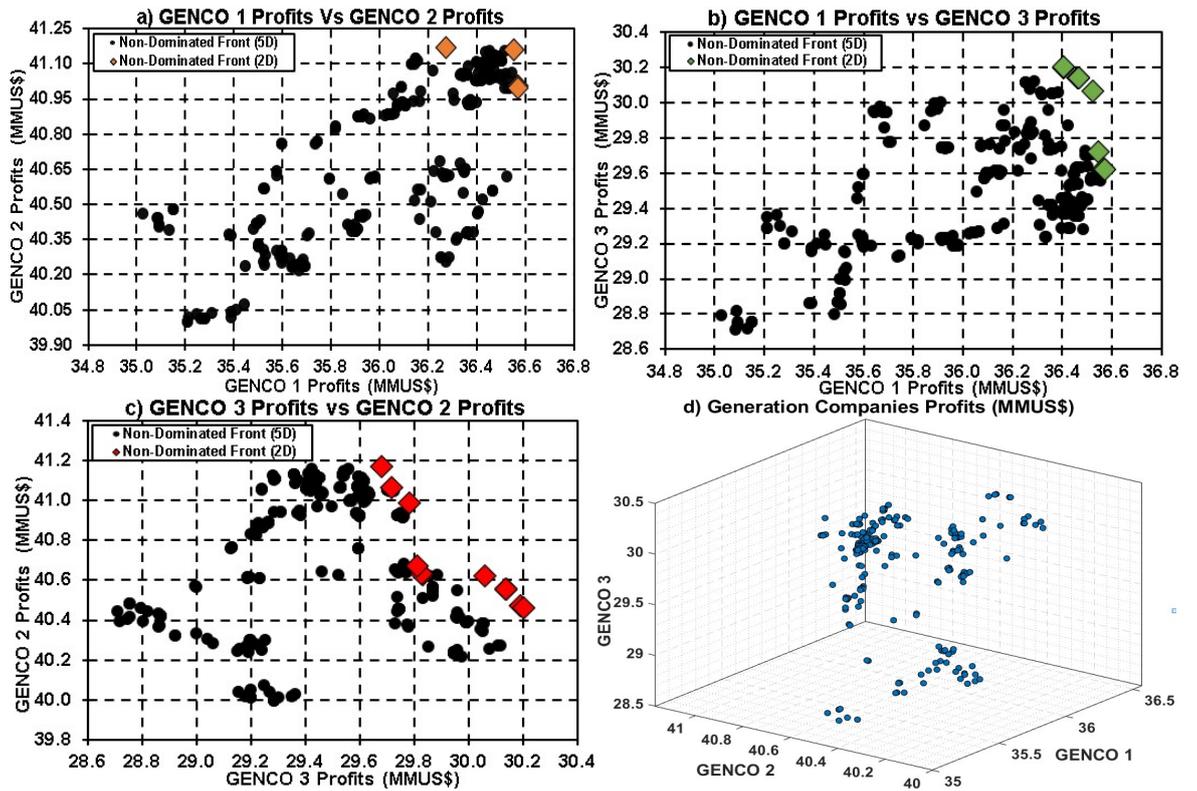


Figure 6.12 Non-dominated fronts for GENCOs profits objective functions (MMUS\$).

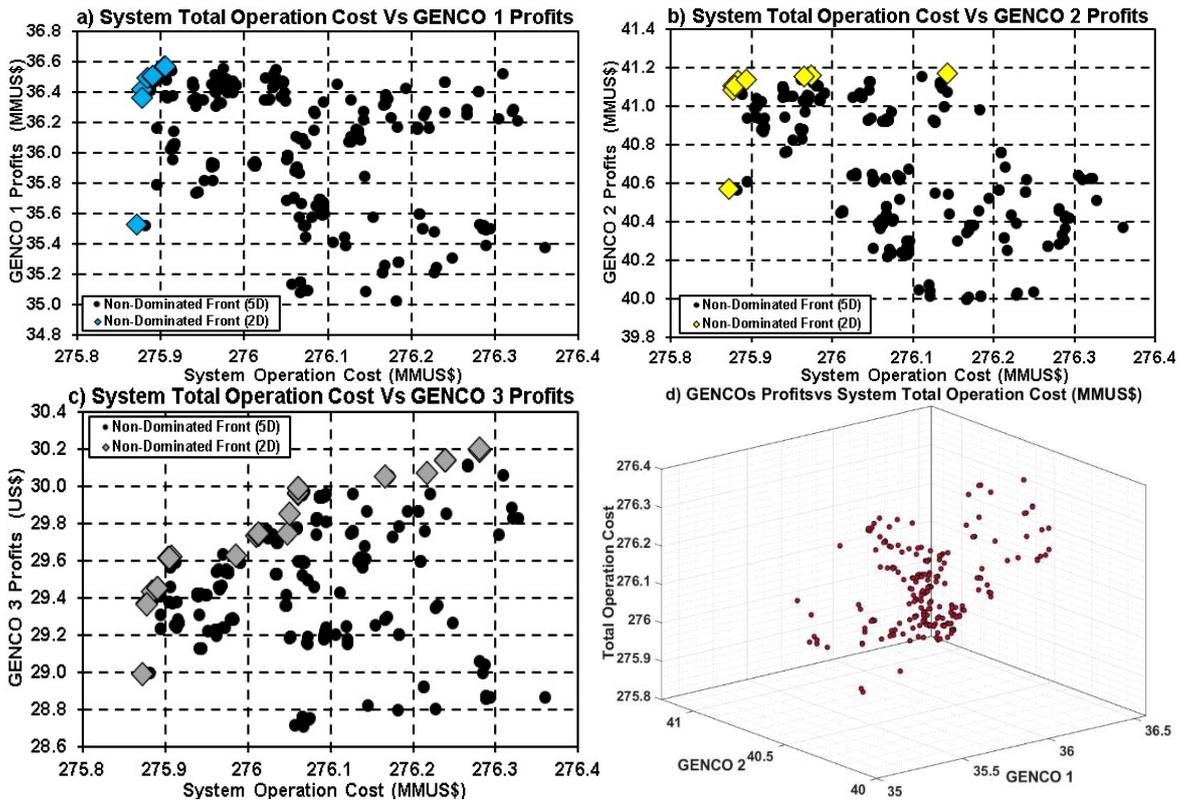


Figure 6.13 Non-dominated front for GENCO's profit and system total operation cost (MMUS\$).

Figure 6.14 shows the non-dominated fronts related to the profits of each GENCO and the system total operation cost and the system adequacy index. In the case of GENCO 1, its cheapest carbon unit is most profitable and has a large capacity. When this unit is unavailable, other less expensive and larger units get dispatched so that the system adequacy index increases while the company profits decrease. This conflicting relationship is a little bit more notorious between GENCO's 2 and 3 profits with the adequacy index. When efficient units of any of these utilities are on maintenance, the available units of the other company are dispatched, increasing the reliability of the system, and reducing the profits of the utility with its units unavailable. This situation is stressed for GENCO 2 since it owns the least efficient units in the system that hardly ever get dispatched. Thus, when its most efficient high-capacity unit is on maintenance, GENCO 2 experiences a great loss of profits; the ISO compensates for the loss of capacity by keeping available GENCO 2's less efficient units to improve the adequacy index.

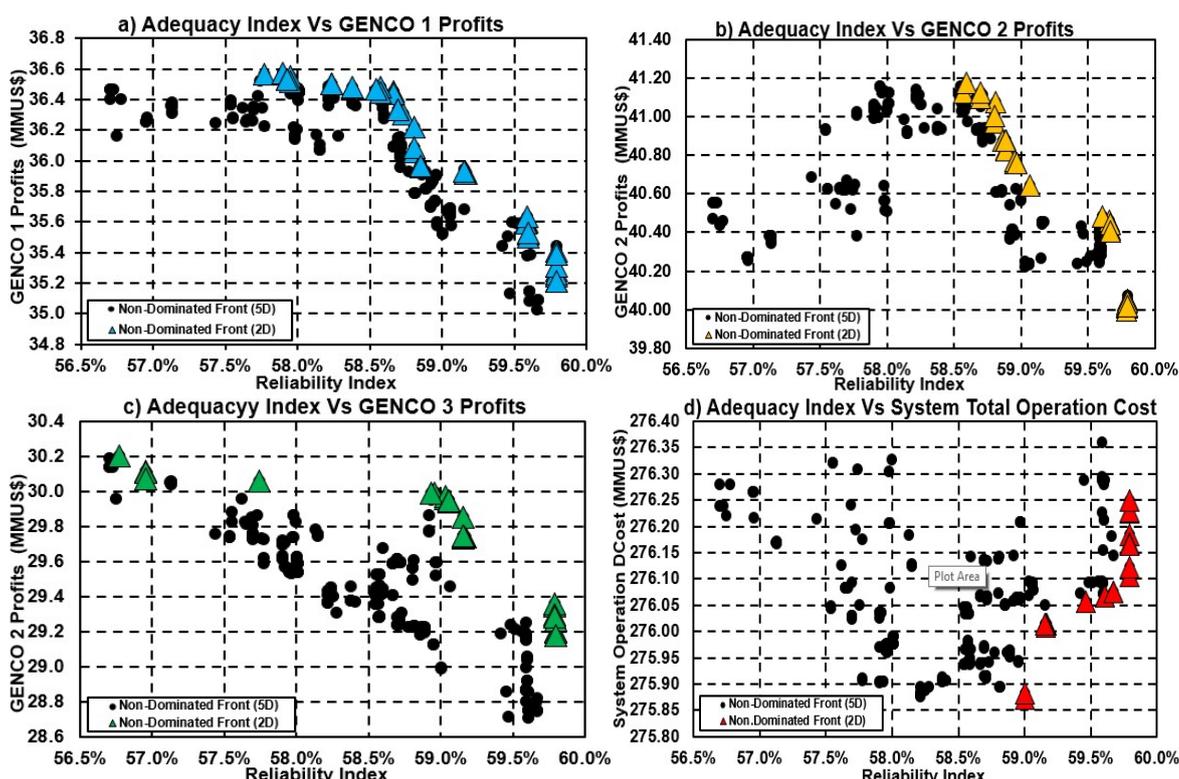


Figure 6.14 Non-dominated fronts for GENCO's profit (MMUS\$) and system adequacy index

Figure 6.14d shows that the system total operation cost decreases as the adequacy index of the system decreases too. This strongly depends on the fluctuations of the demand. The system total operation cost is higher at periods of high demand

when more available units are needed to keep the system reserve margin above the minimum defined. On the other side, this cost is lower for periods of low demand, when fewer units are needed to keep the reserve margin and the adequacy index.

6.2.3 Evolution of the Best MSs during simulation

Every individual inside the non-dominated set of feasible solutions represents a possible MS scenario. Using the proposed hybrid model, MS solutions are developed so that each scenario has its own single minimum system total operation cost, which does not happen in [115] and [116], related to an adequacy index.

This aspect works in accordance with the maintenance scheduling philosophy of the electricity industry, with the difference that the model allows the ISO not to have one but a set of feasible non-dominated MSs at disposal, among which it can choose the best. Since the TOPSIS decision-making tool has been applied after each iteration, the evolution of the objective functions related to the best GTMS scenario was tracked during the simulation and is shown in Figure 6.15.

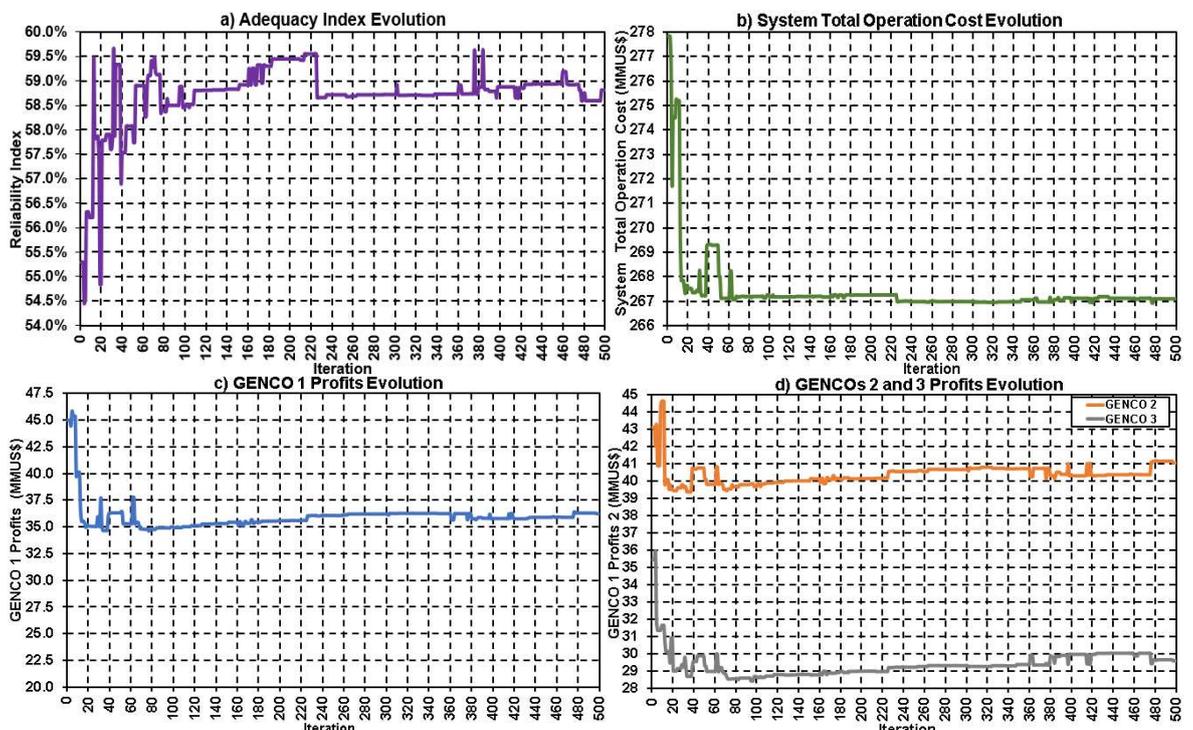


Figure 6.15 Evolution of the objective functions in each iteration.

Moreover, Figure 6.18 shows the energy generated by each unit in the system, per GENCO and per technology used, and Figure 6.19 shows the transmission lines load profile for high subperiod demand conditions; all corresponding to the best MS scenario found by the hybrid model. The figure shows that from weeks 13 to 21, GENCO 3 is not generating energy at all since its most profitable and efficient unit, powered by nuclear fuel and connected to the system through node 18, is on maintenance.

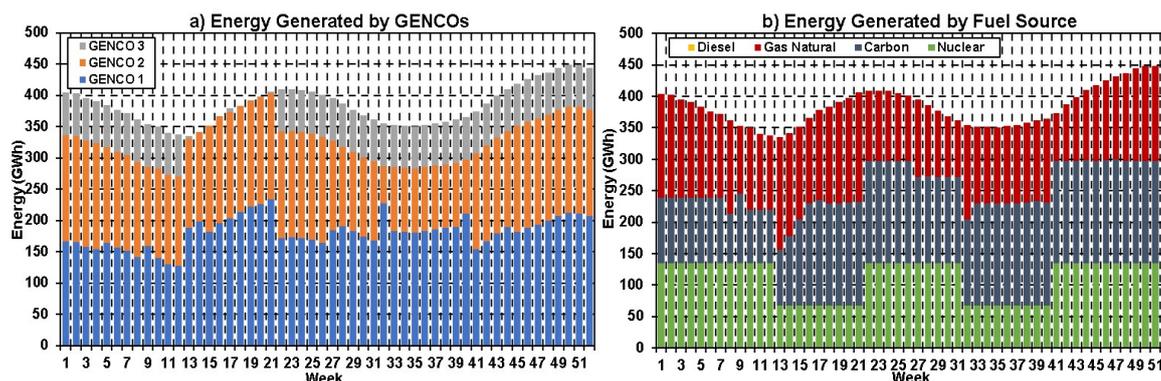


Figure 6.18 Energy generated by GENCOs and by fuel source.

At the same time, Figure 6.19 shows that at week 17, when line LIN 5-10 goes into maintenance, line LIN 6-10 gets congested due to the extra power that it needs to transport. At that week, line LIN 18-21B is on maintenance, increasing the load of its parallel counterpart LIN 18-21A. Notice that these two lines go into maintenance at weeks 16 and 35 respectively, when the demand is low. In the same way, the two parallel lines LIN 15-21 A and LIN 15-21 B go into maintenance at different weeks when the demand of the system is decreasing and starting to increase, respectively. Furthermore, natural gas-fired unit GEN-15, connected to the system through node 7, goes into maintenance at the same weeks when maintenance of line LIN 8-9 takes place which make sense since this line is critical for injection of the energy generated by this unit to the system. Furthermore, maintenance of transformers take place at different weeks with low demand levels, making it possible the transfer of energy from different parts of the system. All these results suggest that the hybrid model manage to develop a certain degree of coordination among generators and transmission MS and between them with the system energy demand.

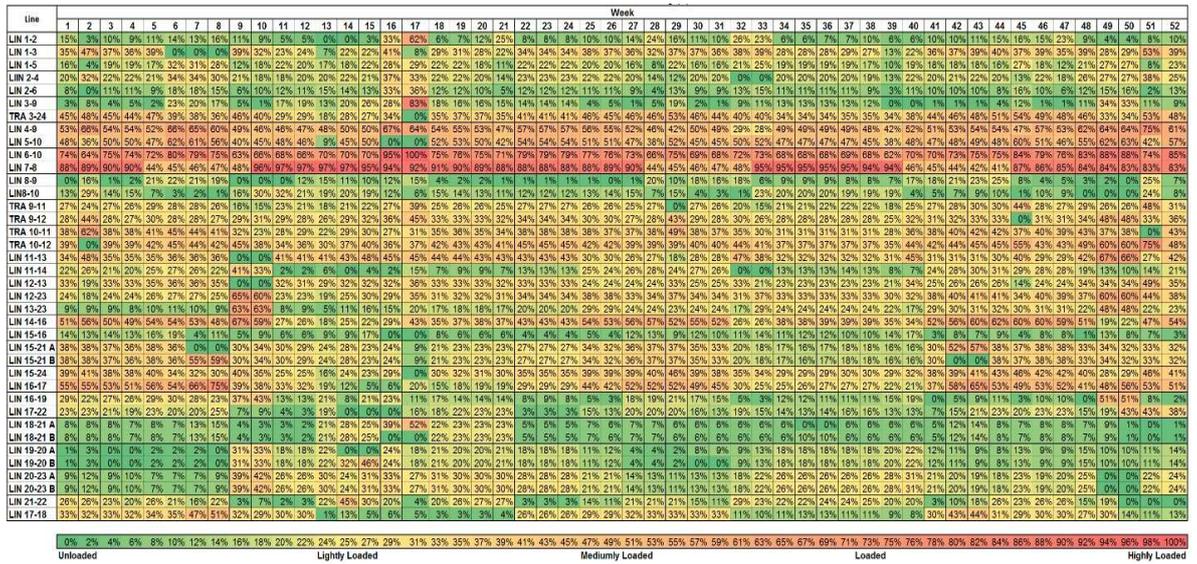


Figure 6.21 shows the average system marginal cost, and nodal energy prices, economic transactions on the electricity market, and GENCOs weekly profits for the best MS scenario. The average system marginal cost is not only affected by the demand fluctuation, but also by the MS of transmission lines and generation units. When efficient units are on maintenance, more inefficient units are required to supply the demand, which increases the system marginal cost. The average nodal energy prices have a strong correlation with this marginal cost.

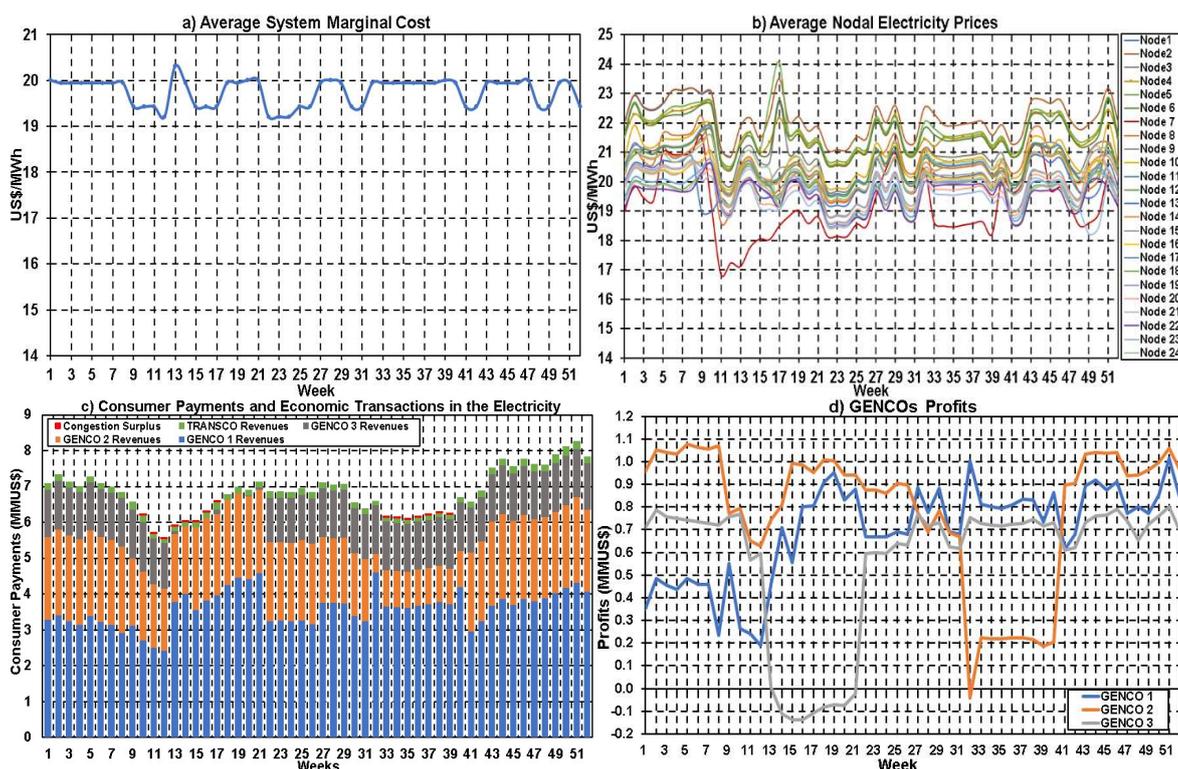


Figure 6.21 Average system marginal cost and electricity nodal prices, market economic transactions and GENCOs’ profits.

On the other hand, in terms of the economic transactions, most of the payments made by end consumers are allocated to GENCOs as revenues in accordance with the amount of energy produced. A small proportion represents the merchandising surplus that arises due to losses in the system and is allocated to the TRANSCO as revenue. Few congestion cases have been detected in the best MS result. They correspond to line LIN 6-10 during high block demand due to maintenance works; and line LIN 7-8 during medium and low block demand periods, since it injects energy generated by efficient gas-fired units directly to the system. The amount of congestion surplus is very small and is not allocated to any company at all.

Finally, it should be mentioned that the best MS scenario resulted in an adequacy index of 58.81%, a total system operation cost of MMUS\$ 276.14 and profits of MMUS\$ 36.22, MMUS\$ 41.07, and MMUS\$ 29.61 for GENCOs 1, 2, and 3 respectively.

6.3 Multi-node wind-hydro-thermal system scenario

The simulation of the proposed hybrid NSGA III/Dual Dynamic (DD) model used to find the set of non-dominated MS solutions in a multi-node hydro-thermal system operating in a competitive electricity market, presents the solutions shown in Appendix 5 and described next.

6.3.1 Feasibility of MS Solutions

The results obtained show that all the solutions or MS scenarios in the final population become feasible. The model allows the initial population to evolve in each iteration towards a feasible and non-dominated set of solutions, as seen in Figure 6.22. From iteration 50 onwards, the number of feasible individuals in the population increases rapidly reaching a value of 100%.

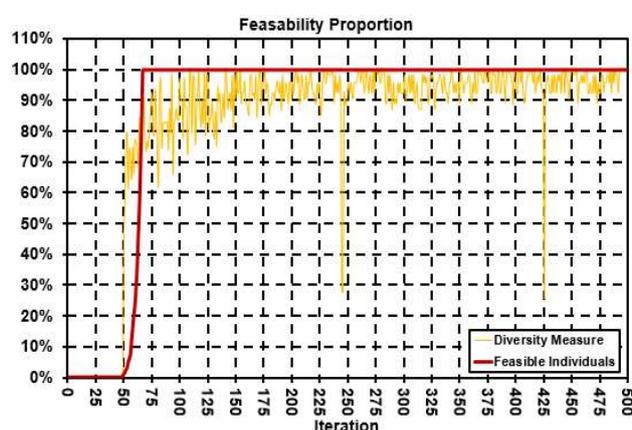


Figure 6.22 Percentage of feasible MS (individuals) in the population and diversity measure (DM) evolution.

The figure also shows the evolution of the DM of the SBX-SAM operator, from a low to a high value that approaches 100%. The reason for this is that at the beginning of the evolution the population of MS is diverse making low crossover distribution indexes spread across a wide range. As evolution progresses and feasible solutions

are found, the diversity of the population stabilizes, and DM gradually reaches a higher and constant value. On occasions, DM is perturbed due to the mutation operator effect, generating some spikes. When no additional feasible solutions are found in the search space, the exploration phase of SBX-SAM ends and the exploitation phase begins, making high a crossover distribution index located in a small range.

The NSGA III/DD model in this scenario returned a total of 249 different non-dominated MS solutions after the iteration process was completed. The maximum and minimum values of their objective functions are shown in Table 6.3.

Table 6.3 Maximum and minimum values of the objective functions of the non-dominated MS solutions.

Values	Adequacy Index	System Operation Cost (MMUS\$)	Profits (MMUS\$)		
			GENCO 1	GENCO 2	GENCO 3
Maximum	61.5%	323.34	260.72	154.19	130.17
Minimum	58.3%	318.63	228.02	128.82	111.66

These values were used to find the normalized objective functions so that they could be plotted in the value path chart shown in Figure 6.23.

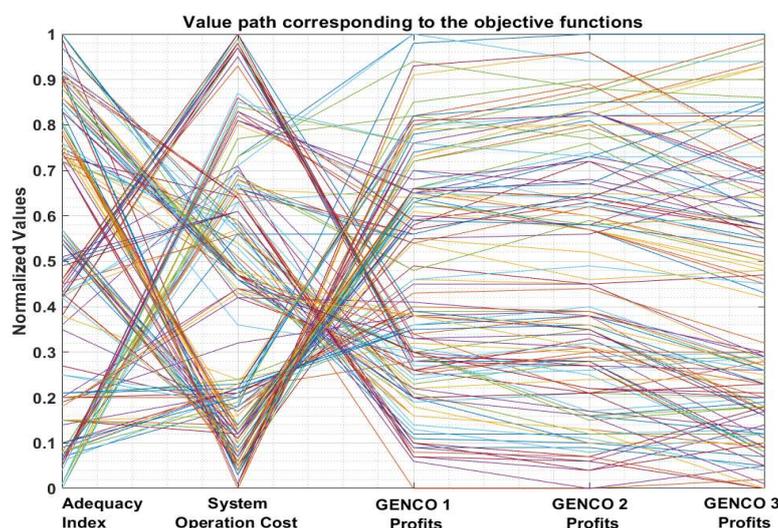


Figure 6.23 Value path chart corresponding to the non-dominated MS solutions.

From the figure, it can be observed that the model provides solutions spread evenly all over the objectives functions vectors. There is a strong ‘‘zig-zag’’ effect among the adequacy index, the system total operation cost and the profits of

GENCO 1, showing the conflicting relationship among these objective functions. However, this relationship is less competitive among the GENCOs since it is very sensitive to generators' ownership and the price setting mechanism used.

6.3.2 Objective Function's Relationship

The feasible non-dominated MS results are spread across a five-dimensional (5D) space whose axis correspond to the systems adequacy index and total operation cost and the profits of three GENCOs. To represent the results, a set of two-dimensional (2D) plots with their non-dominated solutions identified are shown in Figures 6.24-6.26.

The non-dominated fronts of the three objective functions related to profit maximization of the GENCOs are shown in Figure 6.24a-d. It reflects the degree of the competitive relationship among the GENCOs. This relation is weak between GENCO 1 and the other two companies because its generators define the system marginal cost. The strong correlation between this cost and the demand has a huge impact in electricity prices and revenues of all companies. This puts this GENCO in a strong position in the market, allowing it to make considerable profits. However, the competitive relation between GENCOs 3 and 2 is a little bit stronger because both utilities own units with similar efficiencies. When units of one of these companies are on maintenance, the units of the other company are dispatched by the ISO immediately to cope with the loss of capacity. Thus, the conflicting relationship among the profits of the GENCOs depends strongly on what generators each GENCO owns. Figure 6.24d also shows a 3D shape of the non-dominated front of GENCOs profits when plotted together.

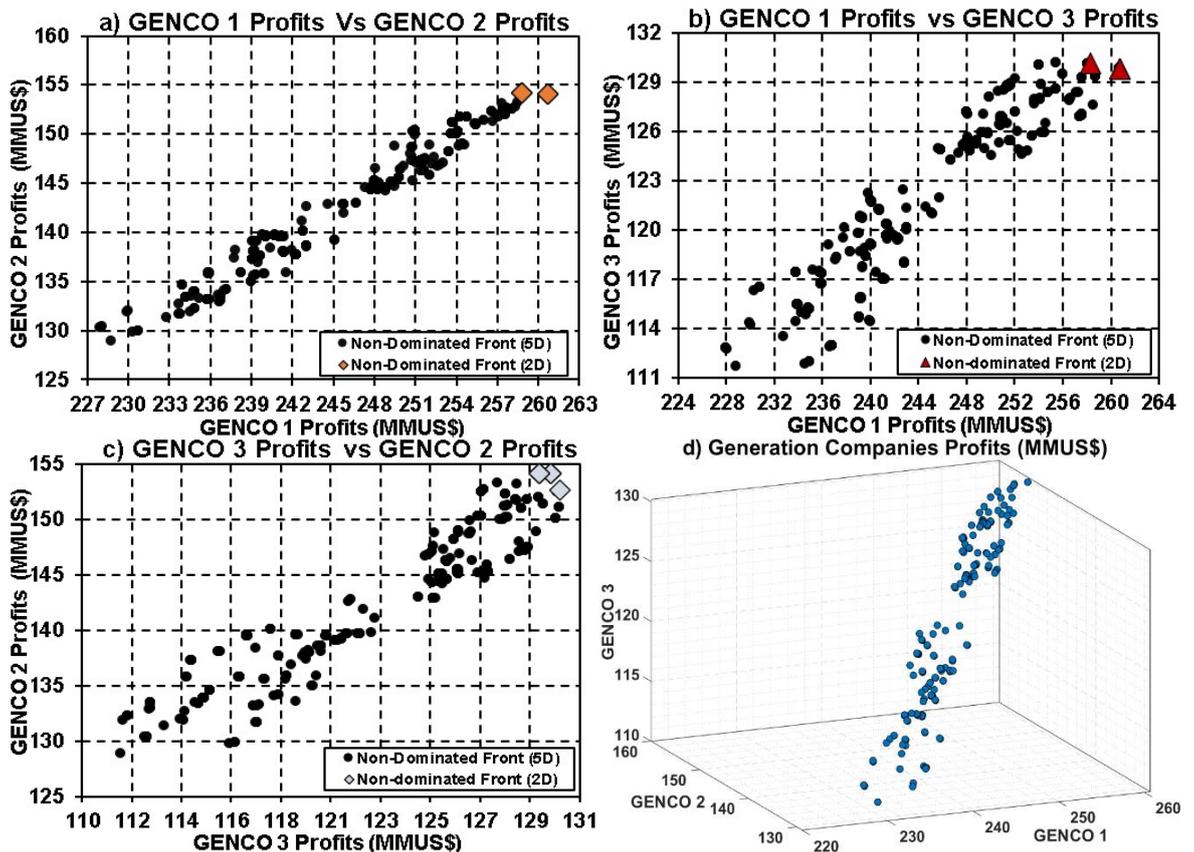


Figure 6.24 Non-dominated fronts for GENCOs profits objective functions (MMUS\$).

Figure 6.25a-d presents the non-dominated fronts related to the profits of each GENCO and the system total operation cost. It shows that when the profits of the three GENCOs increase, the system total operation cost increases as well, and vice versa. GENCO 2 and GENCO 3 own the most efficient units of the system that, when on maintenance, cause the system operation cost to increase. This situation represents a loss of profits for these GENCOs, which is compensated by the revenues made from selling energy generated with less efficient units at a higher price. What is more, GENCO 1 and 2 own hydroelectric units with high water storage capacity. When these units are on maintenance, the system operation cost increases sharply. To cover this unavailable capacity, thermal units of these companies are dispatched increasing their profits. Figure 6.25d shows the 3D-shape of the non-dominated front of GENCO 1 and 2 profits and system total operation cost plotted together.

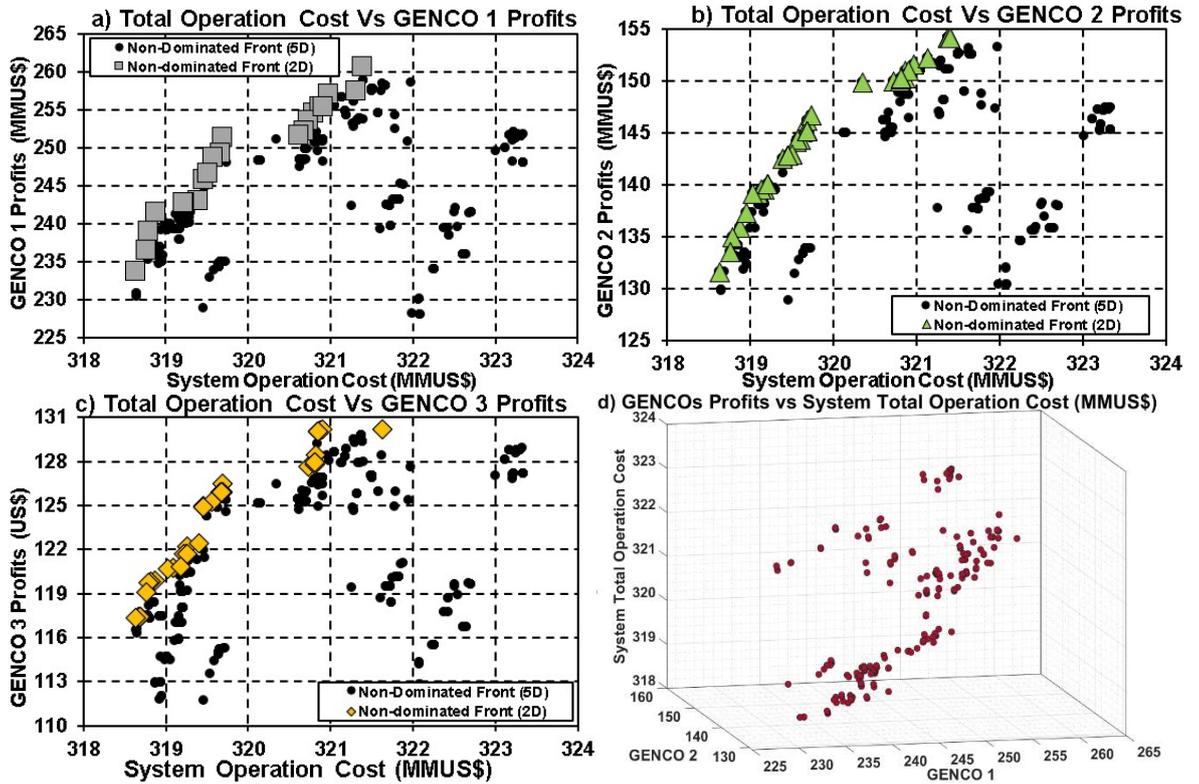


Figure 6.25 Non-dominated front for GENCO's profit and system total operation cost (MMUS\$).

Figure 6.26a-d shows the non-dominated fronts related to the profits of each GENCO and the system total operation cost with respect to the system adequacy index. Large units are the most profitable for GENCOs because of their high efficiency. When these units are unavailable, other less expensive and smaller units get dispatched, so that the system adequacy index increases while the company profits decrease. This conflicting relationship is a little bit more notorious between GENCO's 2 and 3 profits with the adequacy index since these companies own the most efficient units of the system. This situation is stressed for GENCO 2 since it owns the less efficient high-capacity unit that hardly ever get dispatched. Thus, when its most efficient high-capacity unit is on maintenance, a great loss of profits is experienced by GENCO 2, the ISO compensates for the loss of capacity by keeping available GENCO 2 less efficient units to improve the adequacy index.

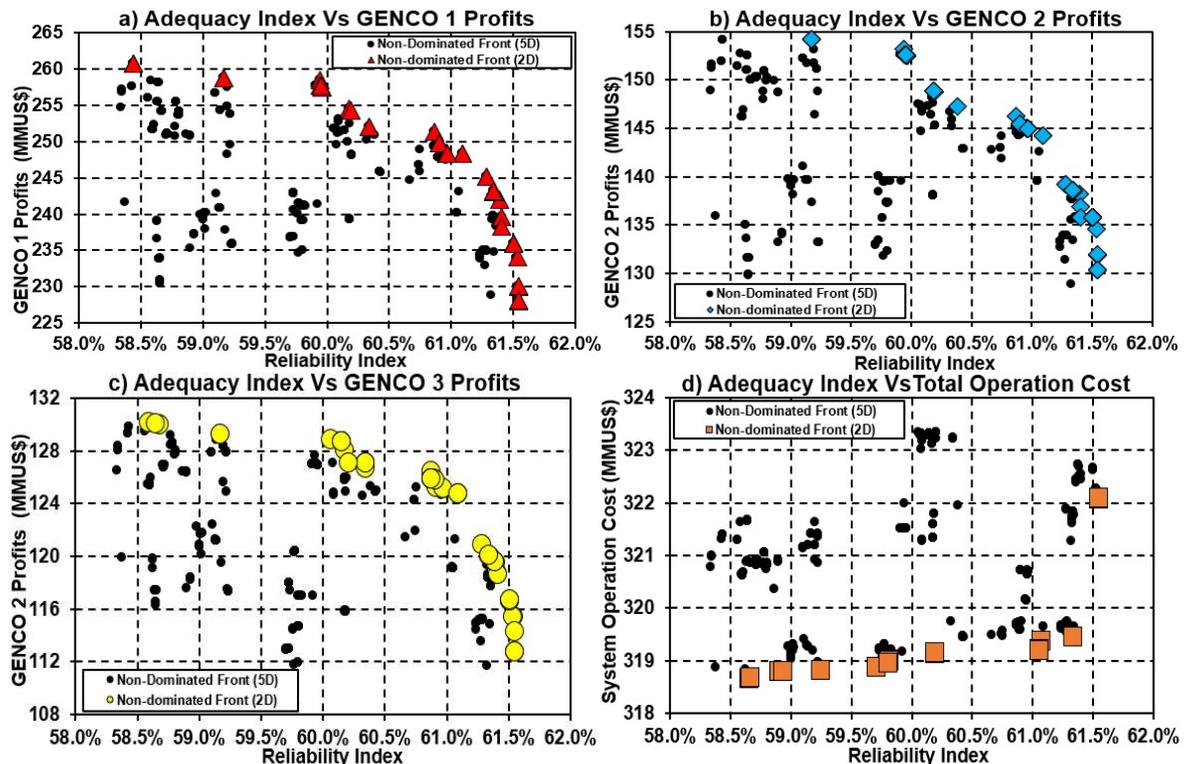


Figure 6.26 Non-dominated fronts for GENCO's profit (MMUS\$) and system adequacy index.

Figure 6.26d shows that the system total operation cost decreases as the adequacy index of the system decreases too. This strongly depends on the fluctuations of the demand of the system. The system operation cost is higher at periods of high demand when more available units are needed to keep the system reserve margin above the minimum and vice versa.

6.3.3 Evolution of the best MS solutions during simulation

Every individual inside the non-dominated set of feasible solutions represents a possible MS scenario. Using the proposed hybrid model, MS solutions are developed so that each scenario has a single minimum system total operation cost associated with an adequacy index, which does not happen in [115] and [116]. This aspect works in accordance with the maintenance scheduling philosophy of the electricity industry described in Chapter 2, with the difference that the model allows the ISO not to have one but a set of feasible non-dominated MS at disposal, among which it can choose the best in accordance with the weighting factors assigned to the objective functions by the ERA. Since the TOPSIS decision-making tool has been

applied after each iteration, the evolution of the objective functions related to the best GTMS scenario was tracked during the simulation as shown in Figure 6.27.

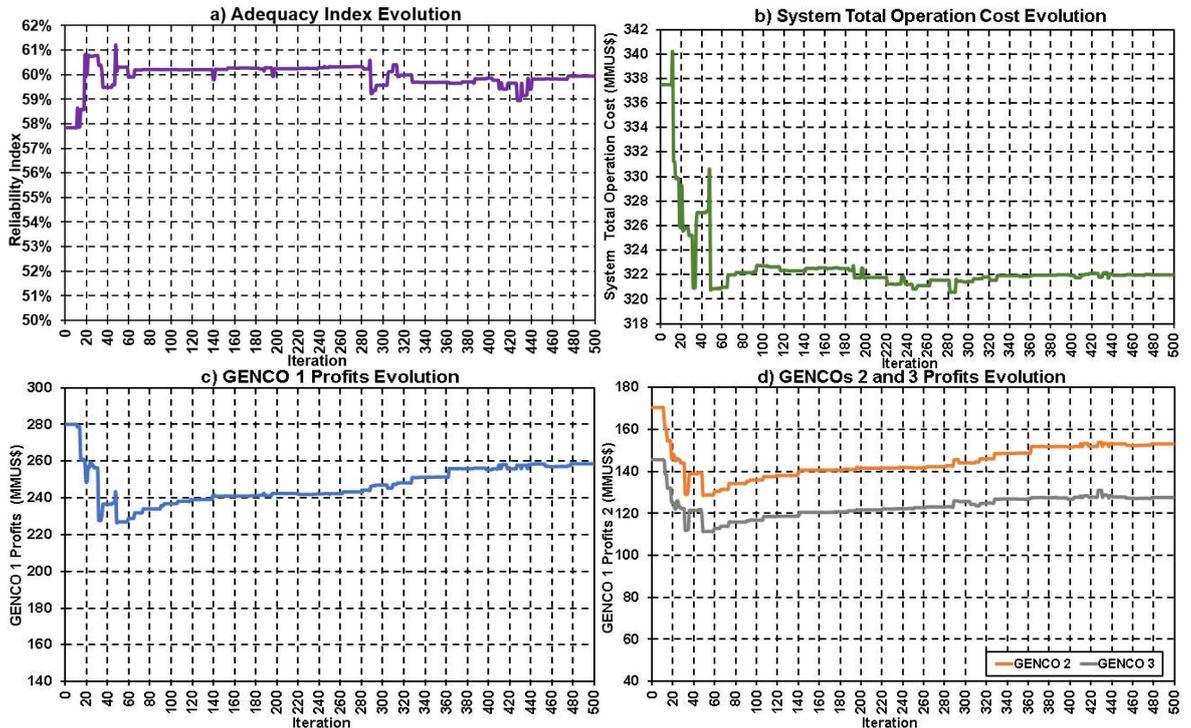


Figure 6.27 Evolution of the objective functions in each iteration.

This figure shows that after every iteration, the hybrid NSGA III/DD model tries to increase the system adequacy and reduce the system total operation cost because these two objective functions have a great weighting factor assigned. In the case of GENCO's profits, the model manages to increase them without compromising seriously the adequacy index. From one iteration to another, the best MS found changes by improving one objective function at the expense of another. This produces oscillations on the best solution's objective functions values during the evolutionary process, implying that the model keeps finding better new MS solutions during the iterations and adding them to the final set.

6.3.4 Best generation and transmission MS solution

The best wind-hydro-thermal generation and transmission MS scenario generated by the proposed model is shown in Figure 6.28. Notice that parallel lines become unavailable for maintenance one at a time in different weeks when the demand of the system is low. Furthermore, maintenance of most transformers takes place

maintenance, carbon-fired units take their place in the market. Figure 6.29c also shows the reservoirs' final volume at the end of each week. The MS of hydro units, especially the ones fed by reservoirs 1 and 3 which have significant water storage capacity, ensures enough water is stored for generating hydroelectrical energy during periods of high demand.

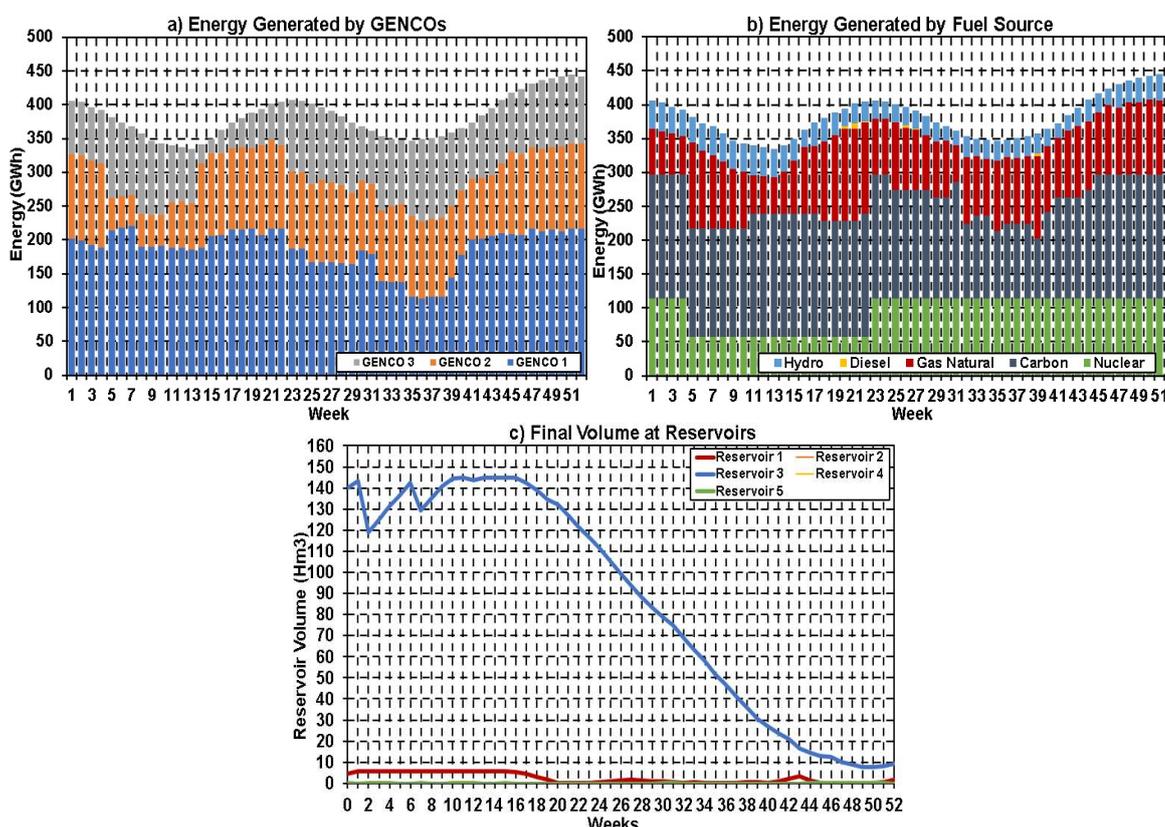


Figure 6.29 Energy generated by GENCOs, fuel source and Final Volume in Reservoirs for the best MS scenario.

Figure 6.30 presents the transmission lines load profile for high subperiod demand conditions. It is interesting to note that no congestion cases were detected during the simulation at any demand block scenario. Line LIN 7-8 becomes slightly loaded at weeks when units GEN 14, 15 and, 16 are available for generation and injecting their energy to the rest of the network, especially when the demand is low. In the same manner, line LIN 8-10 becomes loaded when unit GEN 19 generates energy, after ending its maintenance, to feed the loads nearby node 10. All the results described till now, suggest that the hybrid model develops a degree of coordination between generators and transmission MS and among them with reservoirs water storage and system demand.

Line	Week																																																			
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
LN-1.2	8%	8%	8%	7%	5%	1%	1%	2%	2%	1%	1%	8%	4%	4%	4%	3%	3%	20%	10%	10%	10%	0%	0%	0%	4%	4%	5%	0%	10%	9%	7%	0%	0%	24%	23%	4%	4%	4%	3%	13%	13%	10%	10%	8%	1%	2%	0%	0%	0%	6%	11%	6%
LN-1.3	10%	11%	10%	9%	3%	4%	4%	2%	5%	1%	3%	23%	1%	1%	1%	2%	2%	4%	3%	2%	1%	0%	0%	2%	1%	2%	3%	7%	0%	2%	1%	2%	1%	1%	1%	1%	2%	1%	13%	12%	12%	0%	0%	0%	7%	7%	7%	8%	7%	8%		
LN-1.5	22%	23%	23%	23%	20%	18%	18%	22%	20%	21%	20%	32%	24%	23%	22%	20%	18%	8%	11%	12%	11%	20%	19%	18%	20%	20%	22%	21%	10%	12%	13%	7%	33%	14%	12%	21%	21%	22%	12%	13%	10%	9%	12%	11%	10%	10%	17%	16%	12%	10%		
LN-2.4	12%	12%	13%	13%	3%	12%	12%	14%	14%	13%	8%	11%	15%	15%	14%	13%	12%	3%	2%	4%	9%	11%	11%	11%	11%	20%	17%	10%	5%	6%	7%	0%	11%	0%	0%	14%	14%	14%	17%	0%	0%	3%	4%	19%	8%	6%	10%	9%	9%	17%	9%	
LN-2.6	18%	19%	18%	14%	12%	10%	10%	2%	5%	2%	43%	43%	4%	5%	3%	4%	10%	14%	13%	14%	16%	17%	17%	16%	24%	21%	15%	12%	13%	14%	2%	7%	12%	11%	18%	18%	21%	15%	11%	13%	0%	12%	12%	15%	16%	16%	16%	12%	16%			
LN-3.8	13%	10%	16%	14%	12%	10%	10%	2%	5%	2%	43%	43%	4%	5%	3%	4%	10%	14%	13%	14%	16%	17%	17%	16%	24%	21%	15%	12%	13%	14%	2%	7%	12%	11%	18%	18%	21%	15%	11%	13%	0%	12%	12%	15%	16%	16%	16%	12%	16%			
TRA-3.24	41%	43%	42%	40%	26%	24%	24%	27%	24%	24%	27%	19%	25%	20%	30%	27%	27%	29%	27%	27%	27%	27%	30%	35%	31%	30%	30%	34%	35%	37%	42%	36%	34%	32%	23%	20%	25%	30%	30%	40%	37%	30%	41%	38%	39%	39%	38%	35%	35%			
LN-4.0	20%	20%	18%	18%	26%	18%	18%	15%	14%	14%	19%	15%	12%	12%	14%	16%	17%	27%	26%	27%	23%	21%	21%	21%	21%	14%	20%	24%	23%	22%	36%	30%	21%	22%	13%	13%	14%	11%	20%	26%	26%	28%	13%	25%	26%	24%	25%	26%	26%	18%	26%	
LN-5.10	8%	7%	6%	7%	3%	11%	10%	6%	7%	5%	4%	14%	7%	3%	4%	7%	8%	20%	20%	18%	13%	12%	9%	11%	1%	0%	0%	16%	14%	11%	17%	9%	10%	11%	3%	2%	3%	0%	10%	10%	17%	18%	14%	14%	15%	14%	15%	14%	17%	16%		
LN-6.10	40%	44%	45%	42%	32%	42%	42%	38%	37%	32%	42%	33%	32%	30%	40%	42%	52%	52%	50%	40%	40%	47%	47%	40%	31%	41%	40%	46%	45%	42%	65%	48%	42%	41%	37%	37%	38%	35%	32%	43%	40%	50%	53%	50%	52%	51%	50%	54%	50%	54%		
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on maintenance on weeks 20 and 26 respectively, the electricity prices spike due to the need for thermal generators to cope with the demand. The same effect is noticed in week 39 when natural gas-fired units, which usually set the system marginal cost, go into maintenance.

Furthermore, figure 6.32b shows that GENCOs' profits have a strong correlation with electricity prices in the market. The conflicting relationship between the profits on GENCO 1 and the other companies' profits can still be distinguished. Still, this relationship strongly depends on the ownership of the generation units. On the other hand, figure 6.32c shows the economic transactions on the electricity market. Notice that most of the money consumers pay is allocated to GENCOs as revenues according to energy produced. A small proportion represents the merchandising surplus arising due to losses in the system and allocated to the TRANSCO as revenue. No congestion surplus is detected in this scenario.

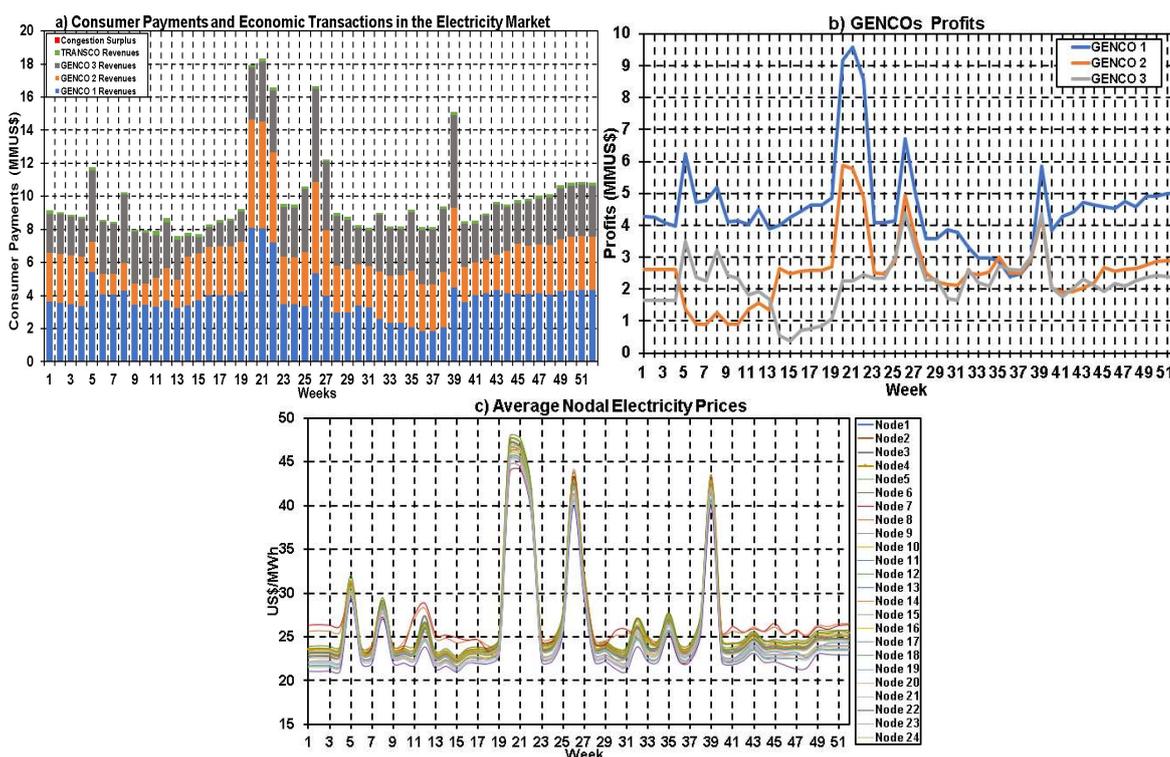


Figure 6.32 Average electricity nodal prices, market economic transactions and GENCOs' profits.

Finally, the best MS scenario presents an adequacy index of 58.46%, a total system operation cost of MMUS\$324.77, and profits of MMUS\$216.67, MMUS\$150.80, and MMUS\$188.52 for GENCOs 1, 2, and 3 respectively.

6.4 Summary of results

The results obtained so far can be summarized as follows. Table 6.4 presents the energy balance corresponding to the best MS solution found by the hybrid models in each scenario. Power losses increase the energy output of generators. Losses in the multi-node wind-hydro-thermal scenario are lower because of embedded wind generation in the demand. The average electricity prices are higher for multi-node scenarios. What is more, the locational-price electricity market assumed in this system scenario produces higher electricity prices than the marginal price-based electricity market used in the multi-node thermal system scenario. This is because of the setting-price rules established in section 5.2 to define the marginal unit of the system.

Table 6.4 Energy balance for the best MS solution found on each scenario.

No	Parameters	System scenarios		
		Single node thermal	Multi-node thermal	Multi-node wind-hydro-thermal
1	Generation (GWh)			
	Thermal	19,616.51	20,013.94	18,189.83
	Hydro	0.00	0.00	1,684.57
	Wind	0.00	0.00	36.40
	Total	19,616.51	20,013.94	19,910.80
2	Demand (GWh)	19,616.51	19,616.51	19,616.51
3	Losses (GWh)	0.00	397.43	294.28
4	Balance (GWh)	0.00	0.00	0.00
5	Average Price (US\$/MWh)	19.22	20.43	25.95

Furthermore, Table 6.5 presents the objective function's values corresponding to the best MS solutions found for the three scenarios. The higher profits obtained by GENCOs are due to the higher electricity prices determined in the market. The system total operation cost is almost the same for the first two scenarios. This is because the technical characteristics used for the thermal units were almost the same in both scenarios. However, due to water management and the use of slightly different maintenance costs and generator efficiencies, this cost is different in the third scenario. On the other side, adequacy index values are quite similar in the three scenarios, but still, the third scenario presents the lowest.

Table 6.5 Values of objective functions relative to the best MS solution for each scenario.

No	System scenarios	Adequacy Index (%)	System Operation Cost (MMUS\$)	Profits (MMUS\$)		
				GENCO 1	GENCO 2	GENCO 3
1	Single node thermal	59.50%	275.91	34.72	40.66	29.34
2	Multi-node thermal	58.81%	276.14	36.22	41.07	29.61
3	Multi-node wind-hydro-thermal	58.46%	324.77	216.67	150.80	188.52

What is more, Table 6.6 shows the parameters and number of variables used when solving the GTMS problems on the three scenarios. The number of iterations and the population size is greater for the single-node thermal system scenario compared to the other cases. This is because NSGA II finds it more challenging to find non-dominated solutions when the number of objective functions is greater and equal to 5, as explained in Chapter 2. In other words, the more variables and objective functions involved, the greater the size of the population and of the number of iterations [55]. Both multi-node system scenarios present crossover and mutation reproduction operators with self-adaptive capabilities. In that sense, the distribution indexes and the mutation probability stated in the table correspond to final values. These parameter values are low at the beginning to achieve a strong exploration effect during the search of the solutions at the beginning of the simulation. As iteration progresses, they are adapted reaching high values that enable the operators to apply an exploitation effect on the MS solutions found during the last part of the simulation, to find additional nearby solutions in the feasible space.

Table 6.6 Parameters of the hybrid models used.

No	Item	System scenarios		
		Single-node thermal	Multi-node thermal	Multi-node wind-hydro-thermal
1	Algorithms			
	MOEA	NSGA II	NSGA III	NSGA III
	Number Variables	32	70	70
	Classical	Lambda Iter.	Dual Simplex	Dual Dynamic
	Num. Variables/week	96	288	294
2	Population Size	200	200	200
3	SBX Crossover Op.			
	Final Distribution Index (η_c)	3.5	24.5	38.1
	Probability (pc)	80.0%	80.0%	80.0%
4	Polynomial Mutation Op.			
	Final Distribution Index (η_m)	1.5	50.5	50.5
	Final Probability (pm)	5.0%	100.0%	100.0%
5	Max. Iterations	800	500	500
6	Observation	-	Self-adaptive reproduction operators	Self-adaptive reproduction operators

Finally, in the three scenarios, the hybrid models manage to find a feasible set of non-dominated generation and transmission MS solutions. All these MS solutions comply with the required level of reserve stated in the problems and present acceptable adequacy indexes. What is more, the operation cost of the system for each MS solution found corresponds to the minimum and is associated with a single adequacy index. As iteration progresses, the objective functions values of the best MS solutions get better, especially the adequacy index and the system total operation cost. This is very helpful for the ISO for making medium-term planning since it can use this set of generation and transmission MS solutions as a portfolio of options, from which a new MS can be selected in case any GENCO or TRANSCO request a modification of a previous MS proposed by the ISO, due to technical or economic reasons.

CHAPTER 7

CONCLUSIONS AND RECOMENDATIONS

A summary of the research reported in this thesis was presented in the first section of this chapter. This was followed by a brief appraisal of the research contributions and conclusions. The chapter closes with several suggestions for future work.

7.1 Dissertation Summary

The first chapter of this thesis provided relevant background about the electricity industry and stated how important it is to handle electricity facilities maintenance from the perspective of the ISO, the electricity utilities, and final consumers. At the same time, drivers for better maintenance techniques for the industry were described in the context of electricity markets, adequacy of the electricity supply, and the impact of renewable energies. Then, a description of the objectives of the present research project was presented followed by a brief explanation of the structure of this thesis.

Chapter 2 opened with an introduction to the concepts of maintenance in terms of the electricity industry, the types, philosophies of maintenance, and their impact on electricity utilities. Next, a description of the electricity industry organization and of the different classical optimization techniques considered to solve power systems problems related to the GTMS was detailed, especially in the context of economic dispatch, unit commitment and electricity markets. Later, a group of Multi-Objective Evolutionary Algorithms (MOEAs) was described and the benefits of NSGA II and NSGA III to solve the GTMS problem were analyzed, and their characteristics were stated. Then, a brief description of how the industry and the academia have tackled the GTMS problem was presented, outlining the benefits and drawbacks of the approaches identified.

The three power system scenarios to solve the GTMS problem were described in Chapter 3. The description included the corresponding mathematical formulations of the objective functions and constraints considered. In addition, a description of how the objective functions selected conflict with each other was made. What is more, the chapter described how the decision variables, both real and integer, were treated and handled to make the GTMS problem much easier to solve mathematically and computationally.

In Chapter 4, the methodologies to solve the GTMS problem in the three power systems test case scenarios were defined. The way classical and MOEAs optimization techniques were put together in a single algorithm to determine a set of non-dominated MS solutions and the decision-making tools used to select the best solution was described. At the same time, the genetic operators used and how their parameters affect the search for feasible solutions were stated. What is more, the benefits of introducing self-adaptive capabilities to these operators are explained, among which the most important is avoiding any tuning of the reproduction parameters in the hybrid models developed. Finally, the constraint handling and the objective function normalization techniques used were established and their characteristics and efficacy clearly stated.

Chapter 5 contains a comprehensive description of the data pertaining to the three power system scenarios used in this thesis to test the effectiveness of the proposed GTMS hybrid models and fulfill the objectives of the PhD project. Important to mention is that wind and hydro generation units' data corresponds to information related to electricity utilities operating in the power industry of Bolivia.

The penultimate chapter of this thesis, Chapter 6, presented a description of the maintenance schedule (MS) solutions obtained when applying the hybrid models to solve the GTMS problem in the three power system scenarios. An analysis of the conflicting relationship among the different objective functions in each scenario was done, as well as a depiction of how the hybrid model managed to find feasible solutions as iterations progressed. The best generation and transmission MS solutions found for each scenario were shown and an analysis of their power characteristics and effect on electricity prices was performed, stressing the fact that all constraints in these solutions were completely satisfied. Finally, a

summary of the results and the characteristics of the hybrid models used to solve the multi-objective GTMS problem in each power system scenario was made, stating how they can be useful for the Independent System Operator (ISO) in the industry during its planning activities.

7.2 Appraisal of research contributions

A novel hybrid NSGA II/Lambda Iteration Method (LIM) approach has been developed to solve the generation maintenance scheduling (GMS) problem in a single-node electricity market environment, considering the nature and type of the variables involved, the objective functions related to GENCOs profits, system's total operation cost and adequacy and constraints corresponding to the availability of units, reserve margin, and demand balance. What is more, a novel hybrid NSGA III/Dual Simplex (DS) approach has been developed too, to solve the multi-objective GTMS problem in a multi-node competitive electricity market, taking into account the type of variables involved and the same objective functions and constraints described before. Additionally, this model considers network losses and constraints, and the effect of the MS solutions found in the electricity prices. In the same way, a novel hybrid NSGA III/Dual Dynamic (DD) model has been developed to solve the multi-objective GTMS problem in a multi-node electricity market, considering the type of the variables involved and the objective functions and constraints already explained. Furthermore, the model examines the effect of wind energy and water reservoir management in time, corresponding to cascade hydroelectrical units, in the MS solutions found. Thus, as it can be seen, the novel models have been upgraded as the system and the maintenance scheduling problem got more complicated.

The results show that the novel models developed generate a well spread non-dominated set of MS solutions whose characteristics comply with the reasonable behaviors of GENCOs and the ISO in the electricity market. The competitive relationships among GENCOs and the conflicting relationships between GENCOs and the ISO in the market are illustrated and clearly described. Furthermore, the results display a degree of coordination between transmission and generation MS and among them and the electricity demand. What is more, electricity prices are

determined considering the effect of the MS solutions found and their impact on the profits of the GENCOs. As iterations progress, the individuals evolve to a set full of feasible MS solutions, whose objective functions show that the best results, identified using TOPSIS, improve during the simulation and manage to converge. What is more, the sets of feasible solutions found by these hybrid models present generation and transmission MS scenarios with a reserve margin above the minimum specified and supplying electricity at a minimum cost. Additionally, for every adequacy index found there is a single operation cost associated, reducing the number of solutions in the feasible space.

In that sense, considering all the above mentioned, six main contributions are made in this thesis, which are described and elucidated in this section:

- **Contribution 1: An extensive literature review related to classical and MOEAs optimization techniques to solve the GTMS problems.**

Even though the most recent literature review in [9] presents an important literature on solving the GTMS problem in deregulated electricity markets, Chapter 2 complements it by including optimization models that use MOEAs. This is shown in Table 2.1 where the advantages and disadvantages of all these metaheuristic approaches are identified and analyzed. In the same way, a description of the current electricity industry practices carried out in some parts of the world to tackle the GTMS scheduling problem is done, highlighting the need to ensure in every MS a reliable supply of the demand at a minimum cost.

- **Contribution 2: Three novel hybrid classical-MOEA models for determining the best feasible non-dominated MS solutions for the GTMS problem.**

An important contribution of this thesis is the novel combination of classical and MOEAs optimization features to find the best feasible non-dominated MS solutions of the GTMS problem in a deregulated electricity market considering transmission losses, congestion, and the effect of wind energy and hydro generation corresponding to cascade units with water storage capacity. The way the availability variables were handled made it possible to reduce the

size and complexity of the problem. What is more, the objective functions and constraints formulated deal with the most important aspects an ISO must face when planning electricity facilities MS. The system reliability is formulated with a proposed adequacy index which measures the adequacy or relation between the net and the gross reserve margins. The system total operation cost to be minimized consists of the fuel and generation and transmission maintenance costs. GENCOs profits in the market environment were calculated by subtracting their revenues, made selling energy in the market, from the generation costs. Notice that by combining classical and MOEAs techniques in a novel way, it is possible to find generation and transmission MS solutions that operate at a minimum cost, fulfill the system adequacy requirements, and consider GENCOs financial health. There is no other approach in the literature that deals with the GTMS scheduling problem using such novel hybrid models.

- **Contribution 3: Analyzing the effect of generation and transmission MS solutions in the electricity prices in the market.**

Chapter 4 shows that the hybrid NSGA III/DD and NSGA III/DS models designed to tackle the GTMS problem in the multi-node system scenarios, considering transmission congestion and losses, were also able to determine the electricity nodal prices in the MS solutions, which in turn are used to determine the Distribution Companies' (DISCOs) charges and the GENCOs' revenues during the period of analysis. This is important since the profits of GENCOs are determined using those revenues and, as a consequence, the determination of the best non-dominated MS solution is strongly dependent on the effect that they have on the electricity prices in the system. Most approaches to solve the GTMS problem that consider GENCOs profit maximization as one objective function, described in Chapter 2, use electricity prices scenarios developed externally to be used as inputs to their models.

- **Contribution 4: Considering water management in reservoirs of wind-hydro-thermal systems.**

The hybrid NSGA III/DD model described in Chapter 4 is designed to tackle the GTMS problem in a wind-hydro-thermal multi-node system, considering integer and real variables related to thermal units' generation and availability, water storage in reservoirs, water turbinated by hydroelectric generators, and system cost-to go functions related to the use of the water in the future and its economic impact in the present. At the same time, the model allows tackling the maintenance scheduling problem in multi-reservoirs systems, taking advantage of the decomposition properties of the DD optimization technique.

- **Contribution 5: Industry practices to tackle the GTMS problem considered in the models.**

A further contribution of this thesis is the incorporation of current maintenance practices in the industry to the three hybrid models proposed and developed. As stated in Chapter 2, most of the multi-objective optimization techniques proposed to tackle the GTMS problem in the academia deal with maximizing system reliability or minimizing operation cost. However, Independent System Operators (ISO) in the electricity industry are concerned with keeping high reliability and minimizing the operation cost of the system at the same time, while determining the best generation and transmission MS scenarios for a given period. Even though the financial health of utilities is of importance to the system, the main concern of the ISO is most of the time focused on ensuring a reliable and cheap supply of electricity, under any MS scenario. The hybrid models developed provide the ISO not with one, but with a set of non-dominated feasible MS solutions. What is more, this set can be used by the ISO as a portfolio of MS scenarios among which the best can be selected using TOPSIS. This decision-making tool can be adjusted in such a way that its weighting factors reflect the priorities of any Electricity Regulation Agency (ERA) towards a system. At the same time, if any GENCO or TRANSCO has an observation with the MS selected, the ISO can choose another MS scenario from the portfolio as good as the previous one in terms of operation cost and adequacy index.

- **Contribution 6:** Three papers written describing the results obtained from applying the novel hybrid models to tackle the GTMS problem corresponding to each scenario.

During the elaboration of the research project, three papers were written: The first, titled “*Hybrid NSGA II/Lambda-Iteration Approach to Generation Maintenance Scheduling*”, has been sent to MPDI journal and presents the results obtained from solving the GMS problem in a single-node thermal power system. The second paper, titled “*Hybrid NSGA III/Dual Simplex Approach to Generation and Transmission Maintenance Scheduling*”, has been presented to the International Journal of Electrical Power and Energy Systems and shows the results got from solving the GTMS problem in a multi-node thermoelectric power system. The third paper, titled “*Hybrid NSGA III/Dual Simplex Approach to Hydro-Thermal Generation and Transmission Maintenance Scheduling*”, has been sent to the IEEE PES Journal and presents the results obtained by solving the GTMS in a multi-node wind-hydro-thermal power system considering hydro generation from cascade units with water storage reservoirs. All these publications are still under review.

Finally, the present research project does not intend to tackle a maintenance scheduling problem with a particular MOEA, as previous contributions described in Chapter 2 have already done. The main contribution of the research lies in the novel combination of MOEAs and classical optimization techniques to obtain a set of non-dominated MS solutions for a quite difficult problem, with almost intractable characteristics, as the GTMS problem. This set of feasible solutions can be used by planners and decision-makers in a deregulated electricity industry to identify the best MS or use the set as a negotiation tool to agree with utilities on a final MS solution.

7.3 Recommendations for future work

Six suggestions are made in this final section in respect of possible future work following the research findings reported in this thesis.

a) Incorporating stochastic reliability measures in a GTMS model formulation.

An important aspect not included in the scope of this thesis is the risk of generators and transmission facilities failures. Including a more explicit measure in the GTMS problem regarding the risk of failure of power system components could be an interesting avenue of future investigation. Many such measures exist in the GTMS literature, such as the loss of load probability (LOLP) or energy not served (ENS). For example, it is recommended to incorporate the risk of generators and transmission line unplanned failure as an objective function to be minimized. Some of these measures could also consider the prediction errors of the demand forecast, not the focus of this project.

b) Including fuel and emission constraints into the GTMS models.

As mentioned in Chapter 2, there are also some GMS model formulations that incorporate fuel consumption and emission constraints. Adding these constraints may increase the problem realism in terms of estimating fuel consumption and greenhouse gasses emitted to the atmosphere by thermal units. However, this addition should be weighed up against the considerable increase in computing time required to find feasible MS solutions by solving the GTMS problem with the hybrid models.

c) Improving the computing time of the algorithms designed.

As it could be inferred, as the number of variables increases in the scenarios developed, the hybrid models took more time to find the best non-dominated MS solutions to the GTMS problem. The main reason for this, is the time expended by MATLAB to solve the system operation cost minimization optimization problem, and the time required by NSGA II and NSGA III to find non-dominated solutions in a GTMS problem with five objective functions. This means that there are two issues related to computing time and scalability of the problem that need to be tackled to improve the hybrid models proposed. In that sense, first, it is suggested to use other optimization software/language, like GAMS and C++, which have better performance and require less RAM capacity to solve optimization problems. What is more, it is recommended to use improved versions of the NSGA III

technique, like the ones stated in [143], [144], and [145]. These NSGA III versions have better non-dominated algorithms and upgraded operators that achieve diverse results closest to the best non-dominated solutions space more efficiently. These operators maintain an elite population archive to preserve previously generated elite solutions that would probably be eliminated by a normal NSGA-III's selection procedure and can be applied to many-objective test problems with 3 to 15 objective functions. Another alternative way forward is to use parallel computing techniques in NSGA III, where fitness evaluations and the optimization process can be very time-consuming. A typical approach is to use “divide-and-conquer” strategy and clustering techniques to parallelize NSGA III and improve the speed of convergence [147]. Finally, to tackle the scalability issue, a cooperative coevolution framework capable of optimizing large-scale GTMS problems can be used. Coevolution is the process of reciprocal genetic change in one species, or group, in response to another. This framework uses a “divide-and-conquer” approach as well, to split the decision variables into subpopulations of smaller size so that each of these subpopulations can be optimized separately with a MOEA, as NSGA II or NSGA III [148].

d) Including GENCOs maintenance resources in the GTMS

GENCOs and TRANSCO can assign limited resources to carry out maintenance works in their facilities. It would make the GTMS problem more realistic if additional constraints related to crew, tools, and spare parts availability, and parallel maintenance works requirements for certain electricity facilities could be included in the formulation. If the effect of the costs of these constraints is considered, GENCOs' profits can be severely affected. For instance, adopting these constraints may lead to instances, especially during periods of high demand and prices, when most generators are operating, and idle crew or maintenance resources not being used may generate a cost to the GENCOs, even when no maintenance is taking place.

e) Employing other stochastic techniques to deal with renewable energy variations.

Other stochastic techniques might perform better than Monte Carlo simulation to predict wind speeds and water inflow to reservoirs in hydroelectric power stations considering historical data available. As stated in Chapter 2, auto-regressive models and neural network techniques are available in the literature to deal with these stochastic variables. The main challenge though is to include these techniques directly to the model, so that the effect of water inflows and wind speeds forecasts may be evaluated in the MS solutions found. This will make the GTMS models better able to deal with real-life power systems that operate in a deregulated electricity market environment, with the advantage of considering varying renewable energy resources.

f) Carrying out sensitivity analysis and statistical tests on the results.

Even though the two hybrid models proposed to solve the GTMS problem in the two multi-node power system scenarios are free of reproduction operator's parameter tuning due to their self-adaptive capabilities, still statistical and sensitivity analysis needs to be carried out on the results after many runs of the models under different size of population and number of iterations. For that purpose, statistical tools, like the running and performance metrics detailed in [95], [64], [60], [61], and [62], can be used to evaluate the quality of the non-dominated MS solutions considering convergence and diversity criteria.

APPENDIX 1

TECHNIQUE FOR ORDER PREFERENCE BY SIMILARITY TO THE IDEAL SOLUTION METHOD (TOPSIS)

Multi-criteria decision making refers to making choice of the best alternative from a finite set of decision solution alternatives in a multi-objective optimization problem (MOOP) with conflicting objectives [104]. Multi-criteria decision making involves the following activities:

- Determination of a set of optimal solutions ranked according to their fitness value.
- Applications of a criteria to quantify the importance each objective inside the problem will have in the final optimal solution.
- Use a Multi-criteria decision-making tool to find the optimal final solution.

Multi-criteria decision-making tools help a decision-maker to select the best solution or individual after solving a MOOP. Several multi-criteria tools are described in reference [105]. Their application depends on the nature of the MOOP, their internal consistency and transparency, their implementation characteristics, and data requirements.

The present thesis focuses on the Technique for Order Preference by Similarity to the Ideal Solution method (TOPSIS), described in [103], since it can work with a-posteriori MOEAs. TOPSIS selects the individual (solution) with its objective function's values closest and farthest from the best and worst solutions, respectively. TOPSIS is based on information regarding the values of the objective functions and the weights the decision-maker assigns to each of them. The TOPSIS procedure is performed in six stages as follows:

- a) For every individual in a population, normalize its objective functions using equation (A1.1) and store the results in a Normalized decision matrix.

$$\hat{f}_{n,m} = \frac{f_{n,m}}{\sqrt{\sum_{n=1}^{Np} f_{p,n}^2}} \quad (A1.1)$$

Where: $\hat{f}_{n,m}$ Normalized objective function m of individual n .

$f_{n,m}$ Objective function m of individual n .

b) Determine that weighted normalised matrix by applying the previously defined weight factors to every objective function.

$$v_{n,m} = \hat{f}_{n,m} * w_m \quad (A1.2)$$

Such that: $\sum_{m=1}^{No} w_m = 1$ (A1.3)

Where: $v_{n,m}$ Weighted normalized objective function m of individual n .

w_m Weight of objective function m .

c) Find the best ideal and worst ideal objective function. To do this, consider the type of each objective function in the problem. If the objective function is to be maximized, then the ideal best and the ideal worst corresponds to the maximum and minimum value of the objective function in the population, respectively. The inverse happens if the objective function is to be minimized.

$$v_m^+ = \begin{cases} \text{Max}(v_{1,m}, v_{2,m}, v_{3,m}, \dots, v_{n,m}, \dots, v_{Np,m}) & ; \text{ if } m: \text{Maximization} \\ \text{Min}(v_{1,m}, v_{2,m}, v_{3,m}, \dots, v_{n,m}, \dots, v_{Np,m}) & ; \text{ if } m: \text{Minimization} \end{cases} \quad (A1.4)$$

$$v_m^- = \begin{cases} \text{Min}(v_{1,m}, v_{2,m}, v_{3,m}, \dots, v_{n,m}, \dots, v_{Np,m}) & ; \text{ if } m: \text{Maximization} \\ \text{Max}(v_{1,m}, v_{2,m}, v_{3,m}, \dots, v_{n,m}, \dots, v_{Np,m}) & ; \text{ if } m: \text{Minimization} \end{cases} \quad (A1.5)$$

Where: v_m^+ Best ideal solution corresponding to objective function m .

v_m^- Worst ideal solution corresponding to objective function m .

d) For every individual, measure the Euclidean distances from the best ideal and worst ideal solution to each of the individual's objective function values using the equation shown below:

$$D_n^+ = \sqrt{\sum_{m=1}^{No} (v_{n,m} - v_m^+)^2} \quad ; \quad n = 1, 2, \dots, Np \quad (A1.6)$$

$$D_n^- = \sqrt{\sum_{m=1}^{No} (v_{n,m} - v_m^-)^2} \quad ; \quad n = 1, 2, \dots, Np \quad (A1.7)$$

Where: D_n^+ Distance from best ideal solution corresponding to individual n .

D_n^- Distance from worst ideal solution corresponding to individual n .

e) Calculate the performance score for every individual in a population, using the distances calculated in the last step using equation (A1.8):

$$P_n = \frac{D_n^-}{D_n^+ + D_n^-} \quad (A1.8)$$

where P_n is the performance score of individual n .

f) Rank all individuals in the population according to its P_n value from the highest value to the lowest. The best result corresponds to the individual with the highest P_n , the next best to the individual with the next highest P_n value and so on.

TOPSIS is a simple, rational, and comprehensible method, suitable for MOOP with many objective function and individuals and especially where objective functions values are available for decision making [103]. What is more, it is computationally efficient and accounts for both the best and worst objective functions in a population to measure the relative performance for every individual in a simple mathematical way [146]. However, for using TOPSIS the weights of each objective function must be defined before-hand, so the determination of the best individual depends largely on the judgment made by the decision-maker when assigning these weights.

Table A2.2 Average weekly wind speed for wind farm 1 (m/s) [135].

Year	Hour	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
2010	1	4.4	2.8	3.7	4.7	5.7	6.7	7.7	8.7	9.7	10.7	11.7	12.7	13.7	14.7	15.7	16.7	17.7	18.7	19.7	20.7	21.7	22.7	23.7	24.7	25.7	26.7	27.7	28.7	29.7	30.7	31.7	32.7	33.7	34.7	35.7	36.7	37.7	38.7	39.7	40.7	41.7	42.7	43.7	44.7	45.7	46.7	47.7	48.7	49.7	50.7	51.7	52.7																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
	2	5.2	3.0	3.1	3.9	4.7	5.5	6.3	7.1	7.9	8.7	9.5	10.3	11.1	11.9	12.7	13.5	14.3	15.1	15.9	16.7	17.5	18.3	19.1	19.9	20.7	21.5	22.3	23.1	23.9	24.7	25.5	26.3	27.1	27.9	28.7	29.5	30.3	31.1	31.9	32.7	33.5	34.3	35.1	35.9	36.7	37.5	38.3	39.1	39.9	40.7	41.5	42.3	43.1	43.9	44.7	45.5	46.3	47.1	47.9	48.7	49.5	50.3	51.1	51.9	52.7																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																														
	3	3.8	3.0	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5	15.0	15.5	16.0	16.5	17.0	17.5	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	24.5	25.0	25.5	26.0	26.5	27.0	27.5	28.0	28.5	29.0	29.5	30.0	30.5	31.0	31.5	32.0	32.5	33.0	33.5	34.0	34.5	35.0	35.5	36.0	36.5	37.0	37.5	38.0	38.5	39.0	39.5	40.0	40.5	41.0	41.5	42.0	42.5	43.0	43.5	44.0	44.5	45.0	45.5	46.0	46.5	47.0	47.5	48.0	48.5	49.0	49.5	50.0	50.5	51.0	51.5	52.0	52.5	53.0	53.5	54.0	54.5	55.0	55.5	56.0	56.5	57.0	57.5	58.0	58.5	59.0	59.5	60.0	60.5	61.0	61.5	62.0	62.5	63.0	63.5	64.0	64.5	65.0	65.5	66.0	66.5	67.0	67.5	68.0	68.5	69.0	69.5	70.0	70.5	71.0	71.5	72.0	72.5	73.0	73.5	74.0	74.5	75.0	75.5	76.0	76.5	77.0	77.5	78.0	78.5	79.0	79.5	80.0	80.5	81.0	81.5	82.0	82.5	83.0	83.5	84.0	84.5	85.0	85.5	86.0	86.5	87.0	87.5	88.0	88.5	89.0	89.5	90.0	90.5	91.0	91.5	92.0	92.5	93.0	93.5	94.0	94.5	95.0	95.5	96.0	96.5	97.0	97.5	98.0	98.5	99.0	99.5	100.0	100.5	101.0	101.5	102.0	102.5	103.0	103.5	104.0	104.5	105.0	105.5	106.0	106.5	107.0	107.5	108.0	108.5	109.0	109.5	110.0	110.5	111.0	111.5	112.0	112.5	113.0	113.5	114.0	114.5	115.0	115.5	116.0	116.5	117.0	117.5	118.0	118.5	119.0	119.5	120.0	120.5	121.0	121.5	122.0	122.5	123.0	123.5	124.0	124.5	125.0	125.5	126.0	126.5	127.0	127.5	128.0	128.5	129.0	129.5	130.0	130.5	131.0	131.5	132.0	132.5	133.0	133.5	134.0	134.5	135.0	135.5	136.0	136.5	137.0	137.5	138.0	138.5	139.0	139.5	140.0	140.5	141.0	141.5	142.0	142.5	143.0	143.5	144.0	144.5	145.0	145.5	146.0	146.5	147.0	147.5	148.0	148.5	149.0	149.5	150.0	150.5	151.0	151.5	152.0	152.5	153.0	153.5	154.0	154.5	155.0	155.5	156.0	156.5	157.0	157.5	158.0	158.5	159.0	159.5	160.0	160.5	161.0	161.5	162.0	162.5	163.0	163.5	164.0	164.5	165.0	165.5	166.0	166.5	167.0	167.5	168.0	168.5	169.0	169.5	170.0	170.5	171.0	171.5	172.0	172.5	173.0	173.5	174.0	174.5	175.0	175.5	176.0	176.5	177.0	177.5	178.0	178.5	179.0	179.5	180.0	180.5	181.0	181.5	182.0	182.5	183.0	183.5	184.0	184.5	185.0	185.5	186.0	186.5	187.0	187.5	188.0	188.5	189.0	189.5	190.0	190.5	191.0	191.5	192.0	192.5	193.0	193.5	194.0	194.5	195.0	195.5	196.0	196.5	197.0	197.5	198.0	198.5	199.0	199.5	200.0	200.5	201.0	201.5	202.0	202.5	203.0	203.5	204.0	204.5	205.0	205.5	206.0	206.5	207.0	207.5	208.0	208.5	209.0	209.5	210.0	210.5	211.0	211.5	212.0	212.5	213.0	213.5	214.0	214.5	215.0	215.5	216.0	216.5	217.0	217.5	218.0	218.5	219.0	219.5	220.0	220.5	221.0	221.5	222.0	222.5	223.0	223.5	224.0	224.5	225.0	225.5	226.0	226.5	227.0	227.5	228.0	228.5	229.0	229.5	230.0	230.5	231.0	231.5	232.0	232.5	233.0	233.5	234.0	234.5	235.0	235.5	236.0	236.5	237.0	237.5	238.0	238.5	239.0	239.5	240.0	240.5	241.0	241.5	242.0	242.5	243.0	243.5	244.0	244.5	245.0	245.5	246.0	246.5	247.0	247.5	248.0	248.5	249.0	249.5	250.0	250.5	251.0	251.5	252.0	252.5	253.0	253.5	254.0	254.5	255.0	255.5	256.0	256.5	257.0	257.5	258.0	258.5	259.0	259.5	260.0	260.5	261.0	261.5	262.0	262.5	263.0	263.5	264.0	264.5	265.0	265.5	266.0	266.5	267.0	267.5	268.0	268.5	269.0	269.5	270.0	270.5	271.0	271.5	272.0	272.5	273.0	273.5	274.0	274.5	275.0	275.5	276.0	276.5	277.0	277.5	278.0	278.5	279.0	279.5	280.0	280.5	281.0	281.5	282.0	282.5	283.0	283.5	284.0	284.5	285.0	285.5	286.0	286.5	287.0	287.5	288.0	288.5	289.0	289.5	290.0	290.5	291.0	291.5	292.0	292.5	293.0	293.5	294.0	294.5	295.0	295.5	296.0	296.5	297.0	297.5	298.0	298.5	299.0	299.5	300.0	300.5	301.0	301.5	302.0	302.5	303.0	303.5	304.0	304.5	305.0	305.5	306.0	306.5	307.0	307.5	308.0	308.5	309.0	309.5	310.0	310.5	311.0	311.5	312.0	312.5	313.0	313.5	314.0	314.5	315.0	315.5	316.0	316.5	317.0	317.5	318.0	318.5	319.0	319.5	320.0	320.5	321.0	321.5	322.0	322.5	323.0	323.5	324.0	324.5	325.0	325.5	326.0	326.5	327.0	327.5	328.0	328.5	329.0	329.5	330.0	330.5	331.0	331.5	332.0	332.5	333.0	333.5	334.0	334.5	335.0	335.5	336.0	336.5	337.0	337.5	338.0	338.5	339.0	339.5	340.0	340.5	341.0	341.5	342.0	342.5	343.0	343.5	344.0	344.5	345.0	345.5	346.0	346.5	347.0	347.5	348.0	348.5	349.0	349.5	350.0	350.5	351.0	351.5	352.0	352.5	353.0	353.5	354.0	354.5	355.0	355.5	356.0	356.5	357.0	357.5	358.0	358.5	359.0	359.5	360.0	360.5	361.0	361.5	362.0	362.5	363.0	363.5	364.0	364.5	365.0	365.5	366.0	366.5	367.0	367.5	368.0	368.5	369.0	369.5	370.0	370.5	371.0	371.5	372.0	372.5	373.0	373.5	374.0	374.5	375.0	375.5	376.0	376.5	377.0	377.5	378.0	378.5	379.0	379.5	380.0	380.5	381.0	381.5	382.0	382.5	383.0	383.5	384.0	384.5	385.0	385.5	386.0	386.5	387.0	387.5	388.0	388.5	389.0	389.5	390.0	390.5	391.0	391.5	392.0	392.5	393.0	393.5	394.0	394.5	395.0	395.5	396.0	396.5	397.0	397.5	398.0	398.5	399.0	399.5	400.0	400.5	401.0	401.5	402.0	402.5	403.0	403.5	404.0	404.5	405.0	405.5	406.0	406.5	407.0	407.5	408.0	408.5	409.0	409.5	410.0	410.5	411.0	411.5	412.0	412.5	413.0	413.5	414.0	414.5	415.0	415.5	416.0	416.5	417.0	417.5	418.0	418.5	419.0	419.5	420.0	420.5	421.0	421.5	422.0	422.5	423.0	423.5	424.0	424.5	425.0	425.5	426.0	426.5	427.0	427.5	428.0	428.5	429.0	429.5	430.0	430.5	431.0	431.5	432.0	432.5	433.0	433.5	434.0	434.5	435.0	435.5	436.0	436.5	437.0	437.5	438.0	438.5	439.0	439.5	440.0	440.5	441.0	441.5	442.0	442.5	443.0	443.5	444.0	444.5	445.0	445.5	446.0	446.5	447.0	447.5	448.0	448.5	449.0	449.5	450.0	450.5	451.0	451.5	452.0	452.5	453.0	453.5	454.0	454.5	455.0	455.5	456.0	456.5	457.0	457.5	458.0	458.5	459.0	459.5	460.0	460.5	461.0	461.5	462.0	462.5	463.0	463.5	464.0	464.5	465.0	465.5	466.0	466.5	467.0	467.5	468.0	468.5	469.0	469.5	470.0	470.5	471.0	471.5	472.0	472.5	473.0	473.5	474.0	474.5	475.0	475.5	476.0	476.5	477.0	477.5	478.0	478.5	479.0	479.5	480.0	480.5	481.0	481.5	482.0	482.5	483.0	483.5	484.0	484.5	485.0	485.5	486.0	486.5	487.0	487.5	488.0	488.5	489.0	489.5	490.0	490.5	491.0	491.5	492.0	492.5	493.0	493.5	494.0	494.5	495.0	495.5	496.0	496.5	497.0	497.5	498.0	498.5	499.0	499.5	500.0	500.5	501.0	501.5	502.0	502.5	503.0	503.5	504.0	504.5	505.0	505.5	506.0	506.5	507.0	507.5	508.0	508.5	509.0	509.5	510.0	510.5	511.0	511.5	512.0	512.5	513.0	513.5	514.0	514.5	515.0	515.5	516.0	516.5	517.0	517.5	518.0	518.5	519.0	519.5	520.0	520.5	521.0	521.5	522.0	522.5	523.0	523.5	524.0	524.5	525.0	525.5	526.0	526.5	527.0	527.5	528.0	528.5	529.0	529.5	530.0	530.5	531.0	531.5	532.0	532.5	533.0	533.5	534.0	534.5	535.0	535.5	536.0	536.5	537.0	537.5	538.0	538.5	539.0	539.5	540.0	540.5	541.0	541.5	542.0	542.5	543.0	543.5	544.0	544.5	545.0	545.5	546.0	546.5	547.0	547.5	548.0	548.5	549.0	549.5	550.0	550.5	551.0	551.5	552.0	552.5	553.0	553.5

Table A2.3 Weekly hydrological series for reservoir 2 (m3/s) [135].

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1979	7.71	9.98	14.57	11.98	12.49	8.42	5.89	8.44	8.13	12.26	14.17	7.44	8.43	3.36	5.19	5.08	2.54	1.94	1.94	2.25	1.67	1.54	1.29	1.25	1.07	1.09	1.05	0.92	0.89	0.75	0.69	0.71	0.44	0.42	0.42	0.47	0.97	1.92	1.39	0.65	6.30	5.66	1.92	4.55	5.34	11.57	10.15	7.85	6.99	8.63	11.99	
1980	4.84	6.41	9.95	8.86	5.56	12.24	6.16	6.87	9.23	6.57	7.98	7.3	6.41	4.32	5.05	2.60	2.51	1.61	1.54	1.39	1.32	1.24	1.13	1.03	0.85	0.86	1.18	0.92	0.82	0.54	1.42	1.47	0.53	1.20	0.71	0.39	0.49	1.68	0.63	1.06	1.44	3.70	5.70	1.50	0.61	0.55	1.69	2.51	1.91	2.29	4.63	6.48
1981	4.87	7.82	9.31	6.89	6.70	11.60	9.18	9.51	5.94	5.48	6.58	4.84	4.63	4.80	2.41	2.50	1.63	1.44	1.52	1.23	1.36	1.05	1.00	1.74	3.19	0.89	0.70	0.65	0.56	0.49	1.12	2.85	0.89	1.18	1.32	0.40	1.11	1.67	0.78	0.49	0.47	0.90	1.54	7.45	5.22	6.54	1.86	4.64	8.05	3.96	3.89	4.46
1982	9.88	8.83	12.25	8.13	6.46	8.87	8.07	9.45	6.42	7.69	7.56	5.31	9.12	8.44	4.79	2.84	2.37	1.87	1.56	1.46	1.34	1.24	1.20	1.31	0.95	0.83	0.81	0.76	0.67	0.67	0.57	0.49	0.64	0.33	0.35	0.40	2.44	2.56	0.45	1.35	1.19	5.08	3.13	0.89	1.58	2.50	6.14	9.59	5.96	7.83	6.63	5.40
1983	5.33	3.75	5.76	5.43	4.15	10.86	10.33	6.57	7.75	4.99	5.89	5.57	4.96	3.63	3.04	2.78	2.62	1.55	1.24	1.46	1.27	1.15	1.08	1.04	1.32	0.82	0.62	0.61	0.71	4.77	2.43	2.52	0.85	1.19	1.37	1.02	1.01	3.69	3.11	2.82	0.58	5.66	1.91	2.58	1.67	2.02	4.38	7.30	4.32	9.14	8.04	5.50
1984	7.54	15.76	18.64	19.01	19.21	10.52	5.58	10.47	8.90	8.22	7.69	6.18	7.19	8.41	4.08	4.82	3.31	2.29	1.89	1.73	1.64	1.57	1.35	1.79	1.20	1.36	1.07	0.79	0.94	2.96	1.17	0.58	0.69	0.45	1.39	1.09	1.16	2.01	0.57	0.38	0.81	1.41	1.49	0.55	11.52	3.02	3.81	9.14	3.86	6.75	5.94	7.58
1985	6.14	7.65	8.27	9.29	9.50	11.37	7.50	6.55	9.56	9.88	7.49	7.73	5.31	3.27	2.09	2.08	3.64	1.78	1.46	3.16	1.48	1.29	1.15	1.02	0.97	0.76	0.85	0.99	1.96	1.40	0.89	0.58	0.39	0.38	0.39	0.34	0.66	0.97	0.47	0.58	1.19	0.43	0.54	0.51	4.80	3.08	6.49	9.97	5.18	4.01	5.09	11.76
1986	7.00	6.72	6.62	7.43	7.86	14.78	12.40	9.45	11.36	9.46	10.95	6.47	9.75	6.77	4.46	3.39	2.21	1.92	1.75	1.66	1.50	1.39	1.36	1.18	1.04	0.97	0.97	0.93	1.06	1.50	1.73	1.79	0.50	0.55	1.01	1.91	2.39	1.61	2.00	1.30	1.42	1.55	2.08	0.71	0.87	3.26	2.42	2.18	3.33	5.95	8.47	7.04
1987	5.49	6.25	9.32	16.15	6.45	12.63	11.76	7.32	8.25	6.60	6.02	4.80	5.90	3.70	8.23	4.56	2.15	1.83	2.26	1.63	3.33	2.15	3.26	1.23	0.77	0.98	0.83	1.44	1.75	1.11	0.69	0.47	0.44	0.43	0.45	0.56	2.25	1.52	1.43	1.65	1.85	3.87	0.49	1.99	4.99	10.04	5.80	5.94	5.93	6.69	5.36	
1988	5.86	8.29	7.05	8.17	7.49	12.34	7.64	8.23	8.51	9.10	11.08	7.30	4.75	6.29	9.02	3.10	1.92	6.95	4.91	2.30	1.53	1.50	1.32	1.33	1.12	1.04	0.95	1.21	0.70	0.69	0.66	0.65	1.19	1.69	0.83	0.81	0.88	0.42	0.88	1.64	2.23	2.69	0.70	1.48	4.42	3.53	5.88	4.13	4.59	8.44	8.49	7.79
1989	7.35	8.08	6.06	6.43	8.89	11.20	10.69	14.68	8.70	11.02	7.63	7.36	7.01	3.38	3.34	9.10	6.15	2.51	1.93	2.05	2.07	2.11	1.36	1.44	1.11	1.09	1.03	0.83	0.99	0.66	1.34	2.50	0.63	0.44	0.68	3.10	1.87	2.49	0.84	0.68	0.39	0.38	3.02	0.79	0.42	1.20	2.58	3.09	3.13	6.84	5.95	
1990	3.38	12.32	8.42	5.52	6.80	9.34	8.40	5.87	9.15	8.99	5.81	4.61	7.16	3.04	3.09	4.18	4.03	3.11	2.62	1.89	1.26	3.39	2.08	1.91	1.03	1.08	0.81	0.71	0.67	0.70	0.76	0.79	0.98	2.08	0.88	0.46	1.05	0.78	0.47	0.62	1.51	2.32	3.10	1.42	7.83	5.54	4.18	3.83	4.37	4.34	8.49	7.85
1991	7.10	7.31	12.83	7.07	8.19	16.79	13.00	12.55	7.62	6.45	7.21	8.31	7.02	5.97	6.50	2.20	2.81	2.03	1.95	1.63	1.49	1.43	1.27	1.47	1.02	1.04	0.82	0.81	0.71	0.73	0.77	1.45	0.49	0.41	0.82	0.45	0.57	1.54	1.37	2.11	0.40	0.94	1.78	2.99	7.12	2.31	4.83	6.11	3.80	5.60	10.35	6.32
1992	6.12	8.59	6.80	7.33	5.37	10.50	11.36	6.51	7.49	5.99	7.52	6.56	6.25	7.32	4.15	2.61	1.99	2.39	1.79	1.35	1.23	1.23	1.06	1.08	0.85	0.82	0.70	0.65	0.60	1.62	0.85	0.54	0.47	1.19	1.65	1.75	0.37	0.52	0.92	3.32	2.00	1.94	1.07	0.56	7.46	4.97	2.74	0.90	1.81	4.35	4.78	6.36
1993	7.05	7.46	5.95	6.52	10.07	7.14	6.17	8.05	6.06	5.99	11.55	6.11	4.86	5.58	6.14	2.39	2.14	4.33	2.84	2.19	1.79	1.24	1.12	1.06	0.94	0.87	0.87	0.92	0.87	0.60	1.09	0.74	1.68	0.82	1.18	0.51	0.34	1.48	0.74	1.63	2.42	1.07	1.44	8.98	4.36	2.14	8.35	4.86	3.84	7.59	8.48	6.63
1994	7.98	5.71	6.52	7.37	7.17	10.15	12.26	10.94	8.23	7.26	5.82	5.95	7.72	7.39	2.97	4.63	8.66	4.00	2.92	1.92	1.84	1.46	1.27	1.20	1.06	1.08	1.27	1.40	2.69	0.87	0.62	0.64	0.60	1.46	1.84	0.44	1.94	2.41	0.64	0.33	0.53	2.87	2.25	6.76	6.05	0.69	7.09	3.38	5.36	7.14	5.41	9.53
1995	8.27	11.44	9.74	10.82	7.91	6.80	4.71	7.94	12.55	8.34	6.22	6.37	8.16	7.01	3.13	2.28	1.32	2.48	2.74	2.36	1.56	1.24	1.17	1.09	0.94	0.87	0.91	0.98	0.65	0.97	1.80	0.68	0.42	0.77	0.50	0.48	1.52	0.64	1.50	6.23	1.14	2.20	3.94	1.60	0.77	4.03	7.30	3.22	2.55	3.83	5.85	5.67
1996	6.11	7.34	9.71	9.11	8.62	6.76	8.11	7.61	11.97	9.43	5.77	4.93	7.90	5.43	2.27	4.76	3.54	2.46	3.72	2.07	1.62	1.32	1.27	1.43	0.99	0.89	1.51	0.91	0.62	0.83	1.10	0.59	0.47	4.01	1.61	0.69	2.06	1.68	0.68	0.44	1.01	2.20	3.50	0.73	7.70	7.63	5.24	8.86	4.38	9.98	6.00	5.27
1997	5.36	6.59	9.71	8.93	10.64	5.24	6.62	7.30	10.34	11.09	7.06	9.95	5.56	2.50	1.99	3.92	2.43	3.15	2.56	2.05	1.35	1.16	1.08	0.99	0.90	0.83	0.74	1.24	0.71	1.00	1.53	0.65	0.78	3.61	2.21	0.96	3.35	1.05	0.82	0.95	0.54	1.35	1.84	2.94	1.22	2.74	2.37	3.28	5.30	6.06	6.15	8.93
1998	4.00	6.61	8.62	5.80	5.65	10.62	10.90	8.82	6.34	5.90	5.38	4.60	7.50	3.62	1.88	4.31	4.29	2.01	1.39	1.23	1.15	1.06	1.03	1.22	0.83	1.70	1.54	1.10	0.89	0.54	0.48	0.72	0.44	0.45	1.56	0.60	1.15	0.62	1.12	2.65	4.66	2.38	2.59	1.69	0.73	0.67	4.56	9.62	3.93	8.54	6.12	5.87
1999	6.21	7.91	9.64	8.24	5.79	10.15	8.34	5.51	16.52	7.39	5.64	8.20	8.86	3.62	9.38	4.25	2.39	2.33	2.14	2.61	2.29	2.00	1.77	3.37	1.72	1.99	1.58	1.34	0.89	1.14	1.30	0.93	1.97	1.23	0.53	0.85	1.04	0.46	0.64	0.63	2.74	2.24	1.15	1.33	5.07	2.58	1.11	1.66	3.12	4.92	4.81	5.79
2000	7.38	11.35	6.42	4.36	15.12	6.50	5.91	12.11	9.65	14.15	12.15	7.70	7.05	3.14	4.09	3.64	3.39	3.30	3.56	1.74	1.45	1.80	3.57	3.20	2.96	1.24	0.92	0.88	1.10	0.83	0.73	1.40	1.93	2.01	4.38	4.62	0.94	0.78	1.70	2.99	1.89	0.93	1.88	0.95	1.96	1.59	0.62	0.85	1.29	2.69	3.56	10.29
2001	8.75	8.13	11.79	7.81	6.12	6.65	10.50	15.66	13.71	9.21	9.85	5.67	8.73	5.39	5.45	2.95	2.27	5.62	4.58	3.11	1.81	1.47	1.43	1.33	1.18	1.14	1.14	0.85	0.74	1.16	1.41	1.06	1.05	3.69	0.66	1.87	1.33	2.38	0.94	1.96	2.51	3.07	1.11	1.04	1.63	3.59	4.10	6.07	4.16	3.17	3.11	5.32
2002	6.70	5.59	4.53	6.90	5.11	6.21	6.34	9.90	14.08	13.80	8.72	5.88	5.26	8.83	5.18	5.24	3.18	3.73	1.85	1.45	1.39	1.23	2.53	1.91	1.31	3.12	1.60	1.49	1.32	1.58	1.31	1.27	1.09	1.29	1.30	1.33	1.79	1.30	2.34	2.41	2.11	2.23	4.60	3.07	2.64	4.32	4.34	5.54	4.64	3.96	3.49	
2003	3.61	5.63	9.11	12.63	7.10	9.50	6.54	12.96	12.63	14.29	9.23	9.06	6.21	5.69	4.10	3.35	1.75	1.77	1.93	1.69	1.78	1.23	1.19	1.02	0.94	0.84	0.76	0.72	0.73	1.10	1.05	1.06																				

Table A2.4 Weekly hydrological series for reservoir 1 (m3/s) [135].

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1979	9.99	11.73	16.35	15.04	14.16	10.90	7.73	10.55	11.55	14.03	16.36	9.54	9.40	4.20	2.44	2.22	1.37	1.01	0.97	1.00	0.77	0.69	0.59	0.56	0.49	0.49	0.66	0.61	0.56	0.41	0.33	0.41	0.20	0.20	0.21	0.25	0.43	1.02	1.66	1.27	0.60	2.31	2.50	1.90	3.10	4.01	7.10	9.33	8.84	8.70	10.88	14.88
1980	7.53	6.51	9.12	8.94	5.90	12.82	8.22	8.18	10.79	7.79	8.45	8.22	7.16	4.23	3.29	1.83	1.49	0.88	0.81	0.65	0.65	0.61	0.51	0.49	0.38	0.42	0.96	0.78	0.60	0.27	0.55	0.46	0.24	0.39	0.20	0.30	0.46	1.48	0.87	1.28	2.57	3.20	5.51	3.31	1.49	0.97	2.57	4.68	2.72	2.33	4.55	6.64
1981	6.36	10.66	13.24	10.53	9.14	10.36	9.17	9.19	6.28	5.80	6.76	5.18	4.71	4.35	1.95	1.23	0.82	0.68	0.78	0.56	0.62	0.42	0.39	0.49	0.42	0.30	0.28	0.28	0.22	0.18	0.38	0.40	0.26	0.39	0.22	0.13	0.91	1.27	1.02	0.58	0.44	0.41	1.34	0.46	4.22	3.67	5.11	3.00	6.68	4.22	3.94	8.16
1982	9.95	9.27	11.59	8.56	6.35	10.75	11.57	13.79	9.93	8.01	7.48	5.19	8.51	7.46	3.84	1.99	1.58	1.24	0.87	0.74	0.62	0.55	0.57	0.90	0.46	0.37	0.40	0.40	0.34	0.33	0.28	0.32	0.41	0.15	0.18	0.31	2.14	2.64	0.94	1.02	0.91	2.80	2.21	1.01	1.09	2.83	5.66	11.88	8.88	9.01	9.34	6.92
1983	6.19	4.63	6.11	6.18	4.90	16.12	18.44	13.27	12.75	6.20	4.88	4.42	4.08	3.21	2.03	1.52	2.18	1.25	0.62	1.19	0.94	0.70	0.62	0.74	0.72	0.55	0.33	0.40	0.51	2.58	1.82	1.57	0.78	0.73	0.65	0.54	0.67	1.96	2.27	2.08	0.59	2.25	1.33	1.15	1.21	1.52	3.06	9.24	6.45	10.21	10.01	7.15
1984	9.62	21.44	27.15	28.51	26.55	16.37	8.58	13.35	12.42	10.61	9.63	7.58	8.68	8.29	3.83	3.56	2.61	1.53	1.06	0.93	0.87	0.80	0.71	0.87	0.63	0.71	0.48	0.40	0.39	0.45	0.32	0.39	0.45	0.23	1.01	0.38	0.76	1.04	0.54	0.34	1.21	1.57	1.70	0.84	2.75	1.40	0.83	3.41	2.76	7.00	7.87	9.43
1985	7.51	8.05	8.50	8.40	8.35	7.71	5.64	4.61	6.62	10.06	8.95	8.46	6.08	3.97	1.69	1.41	3.97	1.88	0.84	0.80	0.51	0.48	0.44	0.41	0.39	0.31	0.29	0.27	0.28	0.20	0.25	0.72	0.15	0.21	0.22	0.15	0.66	0.98	0.19	0.46	1.17	0.44	0.40	0.28	1.55	1.27	2.05	4.78	5.14	4.67	6.17	13.40
1986	10.06	8.47	7.92	9.52	7.20	8.60	6.96	7.87	8.38	9.80	6.46	8.73	6.29	2.96	1.66	1.16	0.80	0.69	0.84	0.57	0.49	0.49	0.43	0.39	0.36	0.55	0.65	0.80	0.82	2.50	3.50	1.02	0.71	1.66	2.00	3.41	2.75	3.14	2.20	3.05	2.71	3.34	1.66	1.51	4.38	3.76	3.64	3.78	6.08	9.30	8.48	
1987	6.66	7.17	10.12	17.38	9.41	16.91	18.65	12.71	11.95	5.79	3.65	2.70	3.05	2.75	2.84	1.62	0.95	0.84	1.02	0.69	1.17	0.56	0.43	0.39	0.36	0.31	0.28	0.30	0.51	0.30	0.68	0.25	0.17	0.19	0.14	0.17	0.26	0.69	0.53	0.56	2.15	2.07	4.32	1.23	1.05	3.86	5.79	5.09	5.39	5.95	7.02	5.85
1988	6.16	8.92	8.31	9.00	8.05	14.49	11.71	11.55	11.69	10.39	12.10	8.46	5.43	4.97	5.58	2.45	1.14	2.90	2.88	1.43	0.77	0.63	0.55	0.67	0.46	0.46	0.39	0.60	0.42	0.27	0.27	0.44	0.63	0.68	0.27	0.42	0.66	0.28	0.91	1.82	2.95	2.50	0.92	1.47	2.80	2.55	3.33	3.61	4.55	8.89	10.05	9.42
1989	8.51	8.79	7.09	6.85	8.37	9.99	10.40	13.97	9.68	12.14	9.60	8.12	7.90	3.75	2.07	6.44	6.05	2.59	1.12	0.85	0.77	0.66	0.56	0.65	0.47	0.48	0.51	0.37	0.53	0.28	0.84	1.22	0.47	0.19	0.42	2.20	1.82	2.28	1.04	1.26	1.22	0.75	0.39	5.14	2.24	1.50	2.69	6.48	4.83	6.93	8.25	7.56
1990	4.69	13.39	11.49	7.54	7.67	11.08	11.29	8.63	11.04	9.69	6.44	4.55	6.54	3.14	1.84	2.39	3.40	2.13	1.71	1.02	0.65	0.87	0.49	0.49	0.44	0.40	0.36	0.31	0.31	0.40	0.47	0.62	0.46	0.62	0.28	0.32	0.82	0.16	0.38	0.53	0.93	1.69	3.30	2.18	3.23	3.25	2.36	2.57	3.41	4.12	8.28	8.60
1991	7.98	8.43	13.72	9.18	7.89	11.73	10.78	10.30	7.10	5.92	6.47	7.24	6.74	5.13	4.37	1.57	1.82	1.14	0.74	0.60	0.54	0.49	0.45	0.44	0.37	0.35	0.32	0.43	0.32	0.37	0.36	0.45	0.18	0.18	0.53	0.27	0.30	1.27	1.30	2.30	0.74	0.64	0.91	2.86	5.21	2.36	2.52	4.05	4.07	5.14	11.8	8.21
1992	7.36	10.41	8.89	8.74	6.56	12.19	15.09	9.98	9.41	6.18	6.47	5.69	5.55	5.36	2.87	1.60	1.15	1.10	0.86	0.59	0.53	0.52	0.47	0.65	0.37	0.44	0.34	0.37	0.37	1.64	0.51	0.35	0.22	0.53	0.49	0.40	0.15	0.45	0.75	3.72	2.99	1.88	1.06	0.64	3.30	3.63	2.66	4.98	1.28	4.15	5.30	7.14
1993	4.89	9.38	7.72	8.03	11.89	11.66	10.94	13.84	10.86	10.42	13.11	8.21	5.74	5.09	3.98	1.92	1.50	1.91	1.51	0.96	0.68	0.58	0.52	0.49	0.43	0.39	0.44	0.58	0.49	0.27	0.49	0.28	0.70	0.33	0.51	0.31	0.17	0.96	0.54	1.43	2.32	1.09	1.16	8.37	6.78	3.22	6.06	4.98	3.98	7.19	9.29	7.87
1994	8.67	6.85	7.13	7.92	7.30	8.70	10.77	10.47	8.27	8.42	7.05	6.60	9.08	7.60	2.94	3.08	6.90	3.97	1.52	0.81	0.75	0.60	0.53	0.51	0.44	0.42	0.46	0.42	0.55	0.26	0.24	0.42	0.30	1.09	0.50	0.17	1.64	2.33	0.92	0.24	1.00	5.22	5.54	5.44	3.94	1.00	1.03	0.91	3.86	7.13	6.51	10.19
1995	9.09	10.65	9.37	10.00	8.82	10.99	8.51	14.29	22.73	11.74	6.54	5.52	7.17	6.50	2.99	1.56	2.60	2.00	1.19	0.81	0.66	0.59	0.55	0.50	0.44	0.40	0.39	0.40	0.29	0.31	0.31	0.32	0.22	0.54	0.24	0.28	0.68	0.20	0.98	6.25	2.86	2.26	4.35	2.94	1.26	5.84	4.92	2.28	2.70	4.38	7.42	7.63
1996	8.04	9.42	11.94	11.83	10.43	9.25	11.45	11.37	15.58	11.41	6.61	4.88	7.31	5.59	2.10	3.04	2.82	1.52	1.74	1.02	0.79	0.59	0.53	0.53	0.43	0.39	0.81	0.51	0.28	0.40	0.41	0.36	0.23	1.89	0.91	0.76	2.93	2.75	1.36	0.52	2.14	3.05	5.10	1.94	5.01	5.84	4.70	6.69	4.92	9.90	7.62	5.93
1997	5.81	6.68	9.65	9.28	10.80	7.01	8.53	9.95	13.56	12.95	8.91	10.20	6.49	3.73	1.32	2.47	1.75	1.82	1.52	1.11	0.72	0.55	0.49	0.45	0.41	0.37	0.34	0.59	0.31	0.34	0.42	0.24	0.38	1.07	0.44	0.38	0.28	1.56	1.35	1.28	0.67	0.62	0.83	2.57	1.15	1.77	1.43	2.30	4.32	6.08	6.75	9.28
1998	5.62	8.21	10.81	8.27	6.58	10.32	12.27	10.69	7.94	7.98	7.47	6.12	10.13	5.81	1.92	3.02	3.97	1.93	0.89	0.65	0.58	0.53	0.52	0.68	0.44	0.87	0.42	0.42	0.31	0.24	0.21	0.79	0.27	0.25	0.84	0.14	0.79	0.23	0.84	1.02	2.62	1.80	1.33	1.47	0.79	0.61	2.93	7.25	4.65	9.10	7.81	7.00
1999	7.36	9.10	11.18	9.94	6.52	7.37	6.67	4.72	12.39	9.36	7.07	9.12	11.06	7.16	5.63	2.53	1.40	1.16	1.14	1.02	0.95	0.70	0.57	1.47	1.03	1.06	0.74	1.82	0.77	0.80	0.45	0.38	0.59	0.25	0.18	0.73	0.55	0.12	0.47	0.36	1.31	1.11	0.81	1.04	2.93	1.95	0.73	0.78	2.47	7.1	5.48	6.48
2000	4.54	12.89	8.93	5.78	16.13	9.78	6.47	12.93	11.80	16.25	14.90	9.83	8.01	3.39	2.63	2.88	2.72	2.86	2.65	1.24	0.73	1.14	2.01	2.58	2.27	0.85	0.47	0.37	0.69	0.38	0.38	1.12	0.76	1.38	3.59	4.85	1.66	0.72	0.70	2.21	1.13	0.30	0.99	0.13	1.04	0.76	0.28	0.58	0.60	1.09	1.94	10.58
2001	11.37	10.89	13.85	9.95	7.50	7.83	11.93	17.91	17.01	11.80	11.34	6.70	8.51	6.07	4.76	2.59	1.64	3.81	3.72	2.26	1.21	0.70	0.69	0.65	0.51	0.57	0.65	0.38	0.32	0.81	0.54	0.71	0.81	2.55	0.73	1.13	0.76	1.46	0.65	1.57	1.08	1.62	0.70	0.60	0.71	1.93	3.48	5.63	4.81	3.60	2.42	5.00
2002	7.25	6.04	4.17	6.66	5.35	8.41	9.66	11.39	16.33	17.28	11.84	7.24	5.29	4.03	2.84	3.93	4.54	2.65	1.73	1.03	0.64	0.61	0.50	1.46	1.05	1.00	1.28	0.94	1.00	0.88	1.23	1.37	1.55	1.95	1.22	2.09	1.59	3.73	2.36	6.16	2.21	1.61	4.21	4.12	5.16	3.70	3.51	2.61				
2003	2.89	5.31	10.39	17.88	8.11	10.01	6.59	12.77	12.62	15.79	10.51	10.33	5.68	4.86	2.96	1.94	1																																			

Table A2.5 Weekly hydrological series for reservoir 3 (m3/s) [135].

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1979	15.14	15.20	18.08	13.47	23.44	18.60	19.00	17.46	17.30	20.10	31.94	17.06	21.50	12.14	7.02	7.10	11.02	4.78	3.73	2.61	2.35	2.07	2.63	1.77	1.07	0.93	1.30	1.70	0.87	0.55	0.50	0.54	0.56	0.37	0.40	0.49	0.39	0.83	0.82	2.90	2.90	5.11	2.18	1.84	3.50	3.25	4.04	6.07	5.63	9.69	15.25	21.28
1980	17.16	15.72	21.02	18.53	14.12	9.70	9.24	10.06	8.66	16.26	16.02	18.02	22.42	9.81	5.26	3.85	2.64	4.42	2.33	1.71	1.12	0.90	0.75	0.74	1.03	0.65	1.22	0.66	0.46	0.46	0.32	0.67	2.03	0.85	4.10	1.95	1.56	1.44	1.05	2.99	4.46	4.11	6.60	5.59	3.08	1.46	2.23	1.85	3.11	6.01	3.01	4.21
1981	9.21	38.79	21.02	20.58	17.93	35.09	36.22	18.95	17.66	16.70	12.73	13.25	8.53	8.94	10.05	5.70	3.87	3.00	2.25	1.85	1.50	1.25	1.07	1.00	1.07	0.64	0.63	0.52	0.51	0.37	0.31	1.70	1.30	0.45	1.06	1.92	0.97	1.87	1.76	2.99	6.55	4.37	5.39	2.99	6.05	8.77	7.24	7.83	8.11	7.31	9.57	13.62
1982	14.81	19.70	23.16	19.72	10.95	11.42	10.62	17.82	26.22	30.33	14.10	14.85	15.97	11.45	13.27	7.03	7.13	4.69	3.30	2.71	2.60	1.88	1.59	1.33	1.70	0.95	0.88	0.76	0.62	0.63	0.61	0.36	0.41	0.28	0.25	0.37	0.16	0.55	2.05	0.46	3.81	1.91	5.96	5.10	9.03	8.56	16.55	11.16	13.46	9.32	11.79	11.80
1983	9.49	17.33	14.60	11.14	8.62	8.38	14.01	13.37	14.19	23.39	26.41	13.70	6.98	9.46	7.48	6.04	4.03	3.21	3.14	5.37	2.46	1.80	1.52	1.54	1.06	0.96	0.90	0.79	0.87	2.09	1.79	0.61	0.77	0.46	1.79	3.31	3.16	4.84	5.34	5.34	1.91	2.40	3.17	5.08	5.89	4.14	7.12	5.59	7.74	3.83	3.72	2.67
1984	17.94	25.07	27.26	32.86	26.76	26.69	17.30	30.28	39.32	17.96	21.28	30.85	21.77	13.56	7.66	5.70	12.43	5.89	3.86	3.67	2.77	2.28	1.95	1.57	1.32	1.13	0.96	0.93	0.72	0.61	0.63	1.29	0.65	0.90	0.35	0.38	0.34	0.29	0.83	0.63	1.52	2.46	1.31	2.01	5.70	5.25	9.22	12.52	9.28	10.90	10.25	10.60
1985	12.51	16.35	14.17	13.19	10.75	15.83	22.23	14.08	14.91	14.27	12.21	11.32	14.15	11.93	10.13	6.50	9.65	6.07	3.90	2.73	2.41	2.52	1.83	1.37	1.71	1.10	0.81	0.64	0.58	0.53	0.45	0.78	0.51	0.50	0.86	0.77	1.11	8.14	3.13	1.14	2.64	1.53	0.96	2.12	4.90	11.37	5.57	4.63	2.63	5.18	5.70	5.40
1986	12.86	9.42	12.03	9.38	6.58	10.42	17.83	12.29	20.64	15.69	32.99	33.13	11.79	6.59	9.44	6.95	6.52	4.80	3.29	3.56	3.14	2.02	1.49	1.13	1.01	1.03	0.89	0.60	0.99	0.61	0.41	0.51	0.47	0.77	0.36	0.61	1.20	1.19	1.84	0.57	0.32	1.21	1.99	3.85	3.30	2.58	6.60	9.44	4.51	20.08	15.33	12.16
1987	10.59	17.76	21.97	14.62	13.26	8.52	10.47	13.17	7.52	14.64	7.52	6.72	4.38	1.92	2.93	1.87	1.47	1.70	3.68	2.34	1.02	0.61	0.60	0.45	0.33	0.58	2.84	2.57	0.79	0.61	1.46	0.84	0.30	0.25	1.10	0.63	0.22	0.07	0.06	0.08	9.31	15.03	5.84	3.98	6.08	3.53	4.07	15.12	22.07	8.52	7.14	6.33
1988	5.48	3.76	17.49	35.04	20.74	16.64	22.20	13.69	7.98	11.62	21.62	16.54	22.96	26.85	11.78	10.23	8.93	12.87	6.46	4.22	3.89	2.98	2.40	2.17	1.61	1.34	1.22	1.04	0.86	0.72	0.60	0.84	0.53	0.39	0.32	0.33	0.73	0.86	0.80	0.37	3.95	1.05	1.29	4.44	2.19	1.26	2.54	3.10	6.20	3.07	2.11	6.72
1989	13.34	11.63	9.15	20.79	11.93	13.36	8.08	12.99	12.32	20.35	10.62	11.21	10.12	11.13	7.38	13.71	8.67	7.95	5.06	3.61	2.50	2.44	1.88	1.94	1.18	0.83	1.22	1.02	0.54	0.48	0.77	0.80	0.39	0.36	1.42	1.00	4.26	1.63	3.72	1.25	1.98	1.79	1.46	0.71	0.76	0.76	1.52	2.83	6.18	8.74	10.60	7.48
1990	12.00	15.57	20.68	37.85	21.04	16.87	10.77	9.87	12.59	8.53	10.14	8.57	5.50	5.60	6.01	3.02	2.44	2.14	1.66	3.35	1.88	2.11	3.12	2.95	1.37	1.06	0.78	0.70	1.67	0.91	0.56	0.45	0.57	0.79	1.22	0.90	1.06	0.63	0.99	1.02	2.14	12.93	8.45	8.88	14.19	17.39	11.61	8.31	18.20	16.06	9.86	13.42
1991	20.26	19.11	14.27	12.87	12.44	11.52	43.46	24.81	34.01	16.38	18.77	15.43	9.20	6.08	6.01	5.65	3.72	2.91	2.45	1.79	1.46	1.22	1.38	1.16	0.95	0.71	0.60	0.50	0.46	0.57	0.30	0.38	0.32	0.21	0.34	1.01	2.90	0.79	0.39	1.86	1.82	2.52	1.61	1.94	5.45	6.61	5.29	8.22	6.43	6.47	4.09	13.59
1992	17.71	23.97	22.59	14.53	8.28	9.81	8.99	18.43	17.50	15.15	12.62	7.39	4.72	4.89	5.52	2.66	2.91	1.81	1.22	1.15	0.83	1.94	2.71	1.23	0.87	4.77	17.58	2.65	4.40	1.95	1.53	1.58	1.42	1.07	1.26	1.76	3.58	3.00	3.45	1.58	6.56	6.87	5.64	3.70	2.94	4.05	4.27	6.88	12.65	7.34	4.67	8.18
1993	25.11	18.40	23.92	28.74	19.86	17.20	13.75	9.33	10.84	19.56	11.29	12.06	10.74	7.24	6.14	7.35	5.07	3.22	2.91	2.98	2.35	1.92	1.72	1.47	1.24	1.21	4.50	1.75	0.84	0.86	0.71	4.12	1.87	3.25	2.91	3.64	3.59	3.12	2.94	4.04	2.66	2.61	3.53	6.59	5.42	15.49	7.89	5.27	10.67	12.93	14.77	11.61
1994	18.11	12.23	13.99	25.92	16.07	21.18	34.76	24.64	25.83	9.64	8.69	7.69	10.09	6.99	9.77	7.80	7.62	4.79	3.67	2.47	1.81	1.53	1.18	1.49	0.94	0.93	0.81	0.41	0.50	0.70	0.42	0.27	0.30	0.49	0.36	0.49	0.34	0.08	1.36	2.28	1.20	1.13	2.02	5.62	8.19	11.17	11.29	5.27	7.66	10.50	15.49	9.18
1995	10.38	28.71	22.65	12.80	14.33	12.47	14.51	20.01	23.38	12.21	11.54	15.06	15.47	7.01	4.65	4.09	3.15	2.39	1.88	1.67	1.30	1.08	0.91	0.77	0.65	0.65	0.56	0.45	0.39	0.32	0.26	0.22	0.93	0.41	0.21	0.13	4.97	3.36	1.08	1.98	2.94	5.69	4.45	4.36	1.89	2.91	7.04	4.58	4.82	4.44	3.22	7.50
1996	10.88	12.99	27.47	24.93	29.21	18.30	15.25	11.36	16.44	19.32	8.76	11.32	12.92	14.39	8.28	5.47	4.22	3.12	2.40	1.70	1.78	1.31	1.14	1.08	1.44	0.85	0.81	1.06	0.72	0.57	0.45	0.58	0.71	0.42	4.67	2.28	1.09	0.99	2.01	0.60	0.91	0.79	3.30	1.11	4.82	10.81	7.08	4.52	6.64	12.55	9.14	9.41
1997	14.10	11.85	16.23	16.57	15.77	22.04	29.91	28.57	18.88	15.60	17.40	14.10	17.15	7.62	7.56	5.42	3.84	2.94	2.23	1.95	2.76	1.78	1.22	0.96	0.92	0.74	0.63	0.33	0.77	0.42	0.44	0.56	0.49	0.39	0.23	0.16	1.68	3.15	4.67	3.48	8.13	9.60	6.84	15.72	10.72	10.49	7.89	9.12	7.85	12.17	25.64	21.54
1998	26.12	27.17	11.98	13.81	22.73	35.91	14.58	29.73	20.43	15.80	17.24	15.86	18.95	17.52	9.63	6.71	9.24	5.15	3.68	3.15	2.44	2.06	1.73	1.69	1.70	2.34	1.20	1.41	0.85	0.65	0.52	0.56	0.90	1.47	0.59	2.20	1.02	1.41	1.03	2.40	3.52	4.00	6.29	9.62	6.65	7.90	9.81	8.74	8.97	8.97	7.88	5.43
1999	15.41	13.32	11.44	19.84	30.51	43.44	42.72	18.81	15.68	30.09	47.39	40.47	47.87	23.83	23.02	11.47	8.39	7.04	4.31	4.31	2.72	2.29	1.44	1.33	0.36	0.59	0.19	0.25	1.02	0.34	0.54	0.26	0.00	0.00	0.00	0.07	1.53	2.86	5.42	7.03	4.48	5.16	3.38	2.65	6.04	3.47	5.71	2.93	8.80	7.72	3.62	4.87
2000	8.03	14.02	24.48	31.73	34.82	11.11	12.46	20.30	15.83	30.32	20.24	11.30	7.31	5.81	4.64	3.41	2.40	2.11	1.30	1.40	1.50	0.90	2.49	1.78	1.59	0.81	0.93	0.76	0.34	0.54	0.31	0.46	0.18	0.53	2.93	0.69	3.99	2.43	1.75	0.86	3.58	2.76	4.06	4.64	4.97	3.26	2.88	2.63	9.56	11.39	6.91	16.10
2001	19.76	38.03	45.64	33.15	41.99	23.62	32.44	20.50	27.76	23.50	14.24	17.05	16.71	9.80	8.79	5.96	4.40	4.81	3.16	2.65	2.00	1.57	1.40	2.01	1.52	0.80	1.37	0.62	0.66	2.28	1.00	0.34	0.27	1.45	0.63	1.27	0.83	1.41	1.08	3.19	3.30	2.47	1.65	7.91	3.42	8.71	3.05	3.75	3.29	3.60	6.48	7.49
2002	8.22	15.33	4.47	4.10	13.18	22.16	19.67	37.26	30.07	28.66	28.44	19.86	10.74	12.73	7.07	7.95	13.14	9.02	7.34	5.18	3.34	2.93	1.50	1.75	1.55	1.13	5.48	2.36	1.77	1.12	1.80	1.77	1.17	1.10	0.75	0.49	0.83	2.49	0.90	0.70	0.86	2.37										

Table A2.6 Weekly hydrological series for reservoir 4 (m3/s) [135].

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52
1979	1.76	1.73	1.60	1.40	2.20	1.28	1.40	1.27	1.26	1.37	1.54	1.06	1.15	0.71	0.62	0.67	0.68	0.34	0.30	0.27	0.26	0.19	0.16	0.13	0.17	0.14	0.14	0.18	0.15	0.21	0.08	0.12	0.13	0.11	0.11	0.10	0.15	0.12	0.12	0.39	0.43	0.47	0.35	0.35	0.35	0.46	0.52	0.43	0.68	0.85	1.09	2.97
1980	1.39	0.92	0.84	0.83	1.16	0.80	0.79	0.65	0.56	1.25	1.01	1.14	1.68	0.72	0.57	0.50	0.47	0.22	0.24	0.20	0.15	0.12	0.18	0.12	0.11	0.17	0.15	0.11	0.11	0.11	0.21	0.27	0.18	0.23	0.20	0.19	0.22	0.11	0.20	0.50	0.56	0.44	0.52	0.60	0.43	0.34	0.38	0.35	0.39	0.39	0.35	0.36
1981	0.81	1.81	1.21	1.19	1.19	1.84	2.75	2.03	1.52	1.26	1.17	1.09	0.88	0.59	0.70	0.64	0.52	0.40	0.34	0.28	0.23	0.20	0.13	0.23	0.20	0.19	0.14	0.19	0.13	0.15	0.11	0.47	0.24	0.25	0.30	0.36	0.27	0.34	0.27	0.49	0.89	0.55	0.54	0.51	0.62	0.69	0.69	0.59	0.72	0.69	0.65	1.21
1982	1.32	1.47	1.62	1.00	0.87	0.87	1.17	1.10	3.96	1.46	1.29	2.19	1.65	1.86	1.53	0.78	0.58	0.32	0.22	0.24	0.17	0.28	0.13	0.22	0.12	0.20	0.15	0.15	0.14	0.16	0.23	0.14	0.21	0.16	0.15	0.20	0.15	0.24	0.14	0.42	0.38	0.50	0.57	0.59	0.63	0.95	1.37	0.97	0.88	0.73	0.81	0.88
1983	0.70	0.68	0.71	0.56	0.51	0.66	0.86	0.83	1.45	1.22	1.45	0.74	0.84	0.65	0.53	0.54	0.48	0.26	0.35	0.31	0.19	0.20	0.20	0.17	0.22	0.23	0.16	0.17	0.25	0.54	0.18	0.27	0.38	0.25	0.53	0.36	0.44	0.30	0.48	0.48	0.47	0.43	0.40	0.44	0.45	0.40	0.43	0.57	0.62	0.47	0.44	0.47
1984	1.92	1.96	2.58	3.27	4.12	2.25	2.06	3.52	3.29	1.54	3.01	3.03	3.15	0.98	0.76	0.84	0.76	0.36	0.33	0.29	0.27	0.25	0.21	0.13	0.19	0.17	0.15	0.21	0.15	0.14	0.16	0.34	0.14	0.16	0.12	0.15	0.12	0.08	0.13	0.41	0.39	0.41	0.34	0.76	0.68	0.87	1.23	0.86	0.63	0.28	0.28	0.93
1985	0.14	0.02	1.00	0.99	0.73	0.72	0.80	0.64	0.64	0.26	0.62	0.51	0.14	0.01	0.02	0.24	0.68	0.49	0.32	0.37	0.19	0.06	0.18	0.11	0.19	0.15	0.11	0.16	0.12	0.15	0.12	0.05	0.09	0.14	0.13	0.13	0.24	0.47	0.39	0.33	0.30	0.22	0.18	0.40	0.32	1.04	0.56	0.61	0.36	0.47	0.42	0.73
1986	1.00	0.82	0.72	0.65	0.72	0.60	1.35	1.28	1.86	1.67	0.77	0.18	0.23	0.04	0.65	0.05	0.46	0.41	0.46	0.41	0.27	0.26	0.24	0.18	0.11	0.15	0.10	0.15	0.25	0.14	0.14	0.06	0.03	0.14	0.38	0.11	0.33	0.39	0.34	0.27	0.57	0.21	0.44	0.25	0.68	0.35	0.40	0.93	0.54	0.54	1.42	1.16
1987	0.87	2.01	1.04	0.22	0.96	0.25	0.03	0.25	0.76	0.86	0.55	0.51	0.45	0.49	0.39	0.49	0.41	0.26	0.40	0.32	0.59	0.39	0.49	0.02	0.13	0.15	0.12	0.07	0.10	0.16	0.23	0.07	0.11	0.06	0.16	0.26	0.21	0.23	0.13	0.21	0.47	0.88	1.41	1.04	0.95	0.89	0.54	0.50	0.74	1.03	1.14	0.59
1988	0.51	0.99	1.65	1.27	1.73	0.73	0.06	0.13	0.20	0.19	0.12	0.68	0.03	0.14	0.63	0.14	0.64	0.79	0.39	0.50	0.27	0.26	0.24	0.17	0.08	0.40	0.09	0.15	0.12	0.09	0.06	0.16	0.11	0.09	0.13	0.05	0.10	0.28	0.44	0.17	0.29	0.14	0.23	0.40	0.18	0.18	0.27	0.50	0.36	0.22	0.31	1.15
1989	0.70	0.48	0.66	1.28	0.68	0.48	0.55	0.79	0.55	1.09	0.64	0.51	0.77	0.66	0.62	1.10	0.68	0.40	0.32	0.24	0.21	0.21	0.18	0.08	0.16	0.17	0.15	0.11	0.12	0.11	0.07	0.18	0.10	0.11	0.08	0.13	0.41	0.46	0.26	0.35	0.37	0.34	0.32	0.31	0.30	0.28	0.36	0.34	0.55	0.75	0.47	0.52
1990	0.52	0.93	1.39	2.15	1.53	1.27	0.53	0.71	0.69	0.56	0.84	0.23	0.51	0.64	0.47	0.60	0.38	0.12	0.16	0.23	0.27	0.18	0.46	0.15	0.15	0.14	0.11	0.14	0.25	0.12	0.18	0.15	0.22	0.54	0.32	0.26	0.23	0.25	0.16	0.32	0.45	0.63	0.64	0.55	1.28	2.00	0.84	0.95	1.47	1.84	0.73	0.84
1991	1.27	1.13	1.06	0.70	1.01	0.93	1.74	1.78	1.28	1.06	1.60	1.28	0.77	0.68	0.54	0.51	0.41	0.32	0.37	0.24	0.37	0.11	0.21	0.19	0.26	0.18	0.25	0.21	0.47	0.22	0.26	0.22	0.14	0.15	0.17	0.22	0.22	0.09	0.14	0.35	0.26	0.30	0.28	0.26	0.35	0.58	0.57	0.94	0.54	0.62	0.52	1.09
1992	1.76	1.37	1.93	1.19	0.72	0.54	0.57	1.13	1.46	1.20	0.91	0.56	0.47	0.47	0.47	0.34	0.35	0.26	0.25	0.51	0.20	0.24	0.30	0.17	0.17	0.08	0.26	0.54	0.48	0.34	0.38	0.29	0.33	0.49	0.35	0.29	0.50	0.60	0.74	0.54	0.40	1.13	0.97	0.64	0.55	0.52	0.81	0.65	0.80	0.95	0.48	0.77
1993	1.72	1.26	1.32	2.50	1.59	1.29	0.86	0.87	1.07	1.52	0.78	1.07	0.79	0.75	0.61	0.83	0.39	0.42	0.16	0.33	0.31	0.30	0.25	0.02	0.28	0.29	0.44	0.33	0.31	0.32	0.35	0.62	0.51	0.44	0.37	0.42	0.40	0.40	0.40	0.59	0.44	0.62	0.89	0.85	1.12	1.50	0.92	0.98	1.05	1.10	0.81	0.99
1994	1.12	0.69	0.95	1.92	1.25	1.85	2.71	2.43	1.25	0.80	0.83	0.69	0.93	0.42	0.99	0.85	0.74	0.44	0.50	0.43	0.34	0.32	0.33	0.32	0.30	0.28	0.28	0.17	0.24	0.21	0.10	0.16	0.14	0.12	0.10	0.16	0.10	0.20	0.19	0.19	0.13	0.16	0.44	0.38	0.92	1.09	0.80	0.63	0.61	0.93	1.16	0.74
1995	0.56	2.25	1.51	0.84	1.11	1.19	0.94	1.69	1.88	1.04	1.33	1.36	1.21	0.76	0.52	0.48	0.37	0.36	0.29	0.28	0.25	0.21	0.18	0.20	0.21	0.29	0.17	0.23	0.32	0.34	0.26	0.27	0.23	0.26	0.18	0.17	0.39	0.37	0.26	0.32	0.32	0.55	0.75	0.64	0.41	0.41	0.76	0.53	0.50	0.41	0.41	0.57
1996	0.59	0.81	1.60	1.97	2.18	1.78	1.49	1.38	1.52	1.56	1.03	1.39	1.27	1.60	0.94	0.78	0.62	0.53	0.46	0.44	0.36	0.35	0.33	0.38	0.30	0.19	0.31	0.19	0.21	0.20	0.23	0.14	0.17	0.16	0.64	0.31	0.26	0.29	0.25	0.25	0.52	0.40	0.53	0.45	1.02	1.09	1.00	0.89	0.81	1.67	1.26	1.57
1997	1.43	1.06	1.14	1.61	1.36	1.87	1.96	3.05	1.91	1.66	1.26	1.14	1.38	0.86	0.75	0.51	0.39	0.35	0.25	0.30	0.29	0.23	0.20	0.15	0.19	0.17	0.18	0.24	0.24	0.17	0.23	0.20	0.22	0.24	0.25	0.22	0.25	0.37	0.46	0.46	0.55	1.33	0.85	1.87	1.61	1.04	0.86	0.77	0.77	0.85	1.73	2.92
1998	1.86	2.87	1.72	0.99	2.30	2.46	2.47	2.74	2.31	1.63	1.24	1.57	1.27	2.61	1.30	0.82	0.92	0.71	0.49	0.39	0.35	0.32	0.25	0.22	0.23	0.33	0.26	0.30	0.31	0.30	0.28	0.19	0.27	0.28	0.28	0.31	0.28	0.22	0.17	0.12	0.27	0.31	0.31	0.83	0.71	0.85	1.23	0.93	0.95	1.35	0.85	1.17
1999	1.44	1.22	1.67	1.86	2.02	2.74	2.73	1.33	1.17	1.87	2.22	1.94	2.20	1.48	2.06	1.08	0.97	0.82	0.63	0.57	0.45	0.40	0.39	0.34	0.33	0.33	0.26	0.47	0.34	0.33	0.27	0.25	0.25	0.23	0.27	0.17	0.26	0.30	0.60	0.30	0.24	0.23	0.25	0.30	0.57	0.33	0.51	0.49	1.17	1.07	0.73	0.63
2000	0.70	0.90	1.91	2.36	2.93	1.45	1.19	1.44	1.48	2.13	1.21	1.06	0.77	0.68	0.57	0.52	0.42	0.38	0.30	0.20	0.24	0.22	0.29	0.30	0.24	0.22	0.22	0.17	0.19	0.14	0.18	0.18	0.23	0.13	0.38	0.26	0.25	0.35	0.33	0.31	0.66	0.55	0.78	0.71	0.87	0.65	0.79	0.62	1.21	1.16	1.41	1.45
2001	1.43	2.76	2.44	1.62	2.41	1.77	2.19	1.67	1.75	1.51	1.05	1.98	2.10	1.08	0.90	0.55	0.60	0.38	0.27	0.19	0.16	0.28	0.09	0.05	0.05	0.03	0.02	0.00	0.09	0.05	0.02	0.03	0.08	0.08	0.26	0.13	0.08	0.18	0.42	0.38	0.35	0.36	1.27	1.07	1.49	0.79	0.92	0.72	0.67	0.94	0.76	
2002	0.77	1.09	0.59	0.62	2.10	1.84	1.80	2.49	2.19	2.50	2.56	1.60	1.02	1.02	0.62	0.71	1.13	0.83	0.78	0.48	0.34	0.27	0.21	0.20	0.17	0.07	0.34	0.21	0.19	0.24	0.16	0.12	0.13	0.14	0.18	0.13	0.18	0.49	0.27	0.22	0.19	0.30	0.37	0.66	0.52	0.39	0.42	1.27	1.65	0.82	0.76	1.30
2003	1.09	1.07	2.24	2.50	1.32	1.06	1.01	1.94	1.22	2.51	1.54	1.15	1.65	1.88	1.06	0.42	0.31	0.25	0.23	0.16	0.16	0.13	0.23	0.14	0.15	0.10	0.12	0.09	0.11	0.14	0.19	0.17	0.15	0.12	0.13	0.16	0.19	0.														

APPENDIX 3

RESULTS FOR THE THERMAL SINGLE-NODE SYSTEM SCENARIO

Table A3.1 Generation maintenance schedule non-dominated solutions (starting week).

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	GEN 27	GEN 28	GEN 29	GEN 30	GEN 31	GEN 32
1	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
2	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
3	35	33	24	39	6	48	49	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
4	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
5	35	33	24	39	6	48	26	27	43	2	48	29	11	18	27	14	28	11	16	17	49	25	11	15	31	38	19	9	32	26	5	34
6	35	33	14	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
7	35	26	24	39	6	50	38	27	43	2	9	29	12	18	45	46	28	11	6	17	49	25	11	15	31	40	12	9	32	26	3	34
8	35	26	24	39	6	48	24	27	43	2	48	29	11	8	10	11	28	23	44	17	49	25	11	15	43	38	19	12	32	26	1	34
9	35	33	24	39	43	48	38	27	22	2	48	29	11	18	27	8	28	11	44	17	49	25	11	15	31	38	17	7	32	26	8	34
10	35	26	24	39	6	48	38	27	43	2	49	29	11	21	20	14	28	11	6	11	23	25	11	15	20	42	13	9	32	26	1	34
11	35	33	24	39	6	48	38	27	43	2	48	29	11	18	21	3	28	28	44	17	49	25	11	15	31	37	15	9	32	26	6	34
12	35	26	24	39	6	48	38	27	43	2	49	29	11	21	20	14	28	11	6	11	23	25	11	15	31	42	14	9	32	26	2	34
13	3	26	24	39	6	48	38	27	43	2	6	47	11	20	18	14	28	11	6	29	49	25	11	15	43	40	12	9	32	26	1	34
14	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	2	28	11	44	6	49	25	11	15	31	37	15	9	32	26	6	34
15	35	26	24	39	6	48	38	27	43	12	48	29	11	20	10	11	28	31	44	17	49	25	11	15	43	38	19	9	32	26	1	34
16	35	33	24	39	6	48	38	13	43	2	48	29	11	18	22	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
17	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	3	28	11	44	17	49	25	11	15	31	37	15	10	32	26	6	34
18	47	33	24	39	6	48	38	27	43	2	14	29	11	18	27	14	28	14	34	17	49	25	11	15	31	38	16	9	32	26	5	34
19	35	26	24	39	6	50	38	27	43	2	9	29	12	18	27	46	28	11	6	17	49	25	11	15	31	40	13	9	32	26	2	34
20	35	26	24	39	6	48	38	27	43	12	16	29	11	8	10	11	13	31	44	17	49	25	11	15	47	38	19	9	32	26	2	34
21	35	26	24	39	6	48	38	27	43	11	49	29	11	21	20	14	28	11	6	10	49	42	11	15	31	43	13	9	32	26	1	34
22	35	26	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	2	34
23	35	26	24	39	6	48	38	27	43	2	48	29	11	8	10	13	25	11	44	17	49	25	14	15	12	38	19	9	32	26	2	34
24	35	26	24	39	6	48	38	27	43	2	48	29	11	8	10	11	28	31	44	17	49	25	11	15	43	38	19	9	32	26	2	34
25	35	33	24	39	6	48	38	27	43	2	14	29	9	18	27	14	28	11	44	17	49	25	11	15	31	38	16	9	32	26	5	34
26	35	33	24	39	6	48	38	27	43	2	48	29	4	18	21	3	28	21	44	17	49	25	11	15	31	37	15	9	32	26	6	34
27	3	26	24	39	6	48	38	27	43	2	9	10	11	20	18	14	28	11	6	29	49	25	11	15	43	40	10	9	32	26	1	34
28	35	26	24	39	6	48	38	27	43	2	49	29	11	8	10	14	24	11	44	17	49	25	11	15	31	38	19	9	32	26	2	34
29	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	45	28	11	44	17	49	25	11	15	31	38	19	6	32	26	4	34
30	3	26	24	25	6	48	38	27	43	2	9	47	11	20	18	14	28	11	6	29	49	25	11	15	43	40	13	9	32	26	1	34
31	35	26	24	39	6	48	38	27	43	12	48	29	11	8	10	11	13	31	44	17	49	25	11	15	43	38	19	9	32	26	1	34
32	35	26	24	39	6	48	38	27	43	2	49	29	11	21	20	14	28	11	6	10	20	25	6	15	31	43	13	9	32	26	1	34
33	35	26	24	39	6	48	38	27	43	2	49	29	11	21	20	14	28	11	6	10	20	25	6	15	31	43	13	9	32	26	2	34
34	35	33	24	39	6	48	38	27	43	7	48	29	11	18	27	14	28	11	44	17	44	25	7	15	31	38	15	9	32	26	6	34
35	35	26	24	39	6	50	38	27	43	2	9	29	12	18	45	46	28	11	6	17	49	25	11	15	31	40	13	9	32	26	1	34
36	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	3	28	11	44	17	49	25	11	17	31	37	15	10	32	26	6	34
37	35	26	24	39	6	48	38	27	43	2	48	29	11	8	10	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	1	34
38	35	26	24	39	6	48	38	27	43	2	9	29	11	18	27	14	28	11	6	17	49	25	11	15	31	40	12	9	32	26	1	34
39	35	33	24	39	6	48	38	27	22	2	48	29	11	18	27	14	28	11	44	17	44	25	44	15	31	38	15	9	32	26	6	34
40	35	26	24	39	6	48	38	27	43	2	49	29	11	18	27	14	28	11	6	17	49	25	10	15	31	40	13	9	32	26	4	34
41	35	33	24	39	43	48	38	27	22	2	48	29	11	18	27	8	28	11	44	17	44	25	44	15	31	38	17	7	32	26	8	34
42	35	26	24	39	6	48	39	27	47	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	4	34
43	35	26	24	39	6	48	38	27	43	2	9	29	11	18	27	46	28	11	6	17	49	25	11	15	31	40	13	9	32	26	1	34
44	35	26	24	39	6	48	38	27	48	12	48	29	5	31	10	11	28	31	44	17	49	25	11	15	43	38	19	9	32	26	1	34
45	35	26	24	39	6	48	38	27	43	12	48	29	30	8	18	11	13	31	44	17	49	25	11	15	43	39	19	9	32	26	1	34
46	35	26	24	39	6	48	38	27	43	2	48	29	11	8	10	14	25	11	44	17	49	25	14	15	31	38	19	9	32	26	2	34
47	35	33	24	39	6	48	38	27	22	2	48	29	11	18	27	8	28	11	44	17	44	25	44	15	31	38	14	7	32	26	7	34
48	47	33	24	39	6	48	38	27	43	2	14	29	11	18	27	14	28	14	34	17	49	25	11	15	31	38	19	9	32	26	4	34
49	35	26	24	39	6	50	38	27	43	2	9	29	12	18	27	46	28	11	6	17	49	25	11	15	31	40	12	9	32	26	3	34
50	3	26	24	39	6	48	38	27	43	2	9	47	11	20	27	22	28	11	6	31	49	9	11	15	43	40	14	9	32	26	2	34
51	35	33	24	39	43	48	38	27	22	2	48	29	11	18	27	8	28	11	44	17	44	25	44	15	31	38	17	7	32	26	2	34
52	35	26	24	39	11	48	38	27	43	2	9																					

Table A3.1 (Cont.) Generation maintenance schedule non-dominated solutions (starting week).

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	GEN 27	GEN 28	GEN 29	GEN 30	GEN 31	GEN 32
101	47	33	24	39	6	48	38	27	43	2	14	29	11	18	27	14	28	14	34	17	49	25	11	15	31	38	16	9	32	26	4	34
102	3	26	24	39	6	41	38	27	43	2	6	47	11	20	18	14	28	11	6	29	49	25	11	15	43	40	12	9	32	26	2	34
103	35	26	24	39	6	48	38	27	43	2	49	29	11	21	20	14	28	11	6	11	23	25	11	15	20	42	13	9	32	26	2	34
104	35	26	24	39	6	48	38	27	43	2	49	29	11	18	27	14	28	11	6	17	49	25	11	15	31	40	13	9	32	26	3	34
105	35	33	24	39	43	48	38	27	22	2	48	29	23	18	27	8	28	11	44	17	44	25	44	15	31	38	17	8	32	26	8	34
106	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	21	44	17	44	25	7	15	31	38	15	9	32	26	6	34
107	3	26	24	39	6	41	38	27	43	2	6	47	11	20	18	14	28	11	6	29	49	25	10	15	43	40	12	9	32	26	2	34
108	35	26	24	39	6	50	38	16	43	2	9	29	12	18	45	46	28	11	6	17	49	25	11	15	31	40	12	9	32	26	2	34
109	35	26	24	39	6	48	38	27	43	1	9	29	11	18	27	14	28	11	6	17	49	25	11	15	31	37	12	9	32	26	2	34
110	47	33	24	39	6	48	38	27	43	2	14	29	11	21	27	14	28	14	34	17	49	25	11	15	31	39	16	9	32	26	5	34
111	3	26	24	39	6	48	38	27	43	2	6	47	11	20	18	14	28	11	6	29	49	25	11	15	43	40	12	9	32	26	2	34
112	35	26	24	39	6	48	38	27	43	12	48	29	11	20	10	11	28	31	44	17	49	25	11	15	43	38	19	9	32	26	2	34
113	47	33	24	39	6	48	38	27	43	2	14	29	11	21	27	14	28	14	34	17	49	25	11	15	31	39	16	9	32	26	5	34
114	35	26	24	39	6	48	38	27	43	2	9	29	11	18	27	46	28	11	6	17	49	25	11	15	31	40	14	9	32	26	1	34
115	35	33	24	39	43	48	38	27	22	2	48	29	11	18	24	8	5	11	44	17	44	25	44	15	31	38	17	7	32	26	7	34
116	35	26	24	39	6	48	38	27	43	2	48	29	11	9	10	11	28	31	44	17	49	25	11	15	43	35	19	9	32	26	2	34
117	3	26	24	39	6	48	38	27	43	2	9	47	11	20	27	22	28	10	6	31	49	6	11	15	43	30	13	9	32	26	2	34
118	3	26	24	39	6	48	38	27	43	2	9	10	11	20	18	14	43	11	6	29	49	25	11	15	43	40	12	9	32	26	1	34
119	35	26	30	39	6	50	38	27	43	2	9	29	12	18	43	46	28	11	6	17	49	25	11	15	31	40	13	9	32	26	2	34
120	35	26	24	39	6	48	24	27	43	2	48	29	11	8	10	11	28	31	44	17	49	25	11	15	43	38	19	12	32	26	2	34
121	35	33	24	39	6	48	38	27	22	2	48	29	11	18	27	14	3	11	44	17	44	25	44	15	31	38	15	9	32	26	5	34
122	35	26	24	39	6	48	38	27	43	2	9	29	11	18	27	9	28	11	6	17	49	15	11	15	31	39	12	9	32	26	2	34
123	35	26	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	40	13	9	32	26	4	34
124	3	26	24	39	6	48	38	27	43	2	9	10	11	20	18	14	28	11	6	29	49	25	11	15	43	40	12	9	32	26	1	34
125	35	33	24	39	6	48	38	27	43	2	14	29	9	18	27	14	28	11	44	17	49	25	11	15	31	37	16	9	32	26	4	34
126	35	33	24	39	6	48	38	27	22	2	48	29	11	18	27	8	28	11	44	17	44	25	44	15	31	39	15	7	32	26	5	34
127	35	26	24	39	6	48	38	27	43	12	48	29	11	8	10	11	13	31	44	17	49	25	11	15	43	38	19	9	32	26	2	34
128	35	26	24	39	6	48	38	27	43	2	9	46	11	18	27	14	28	7	6	17	49	25	11	15	31	37	12	9	32	26	2	34
129	35	26	24	39	6	50	38	27	43	2	4	29	12	18	45	46	28	11	6	17	49	25	11	15	29	40	13	9	32	26	2	34
130	35	26	24	39	6	48	38	27	43	2	49	29	11	18	27	14	28	11	6	17	49	25	11	15	31	40	13	9	32	26	2	34
131	35	26	24	39	6	48	38	27	43	2	9	29	11	18	27	14	28	11	6	17	49	25	11	15	31	37	12	9	32	26	2	34
132	35	26	24	39	6	48	38	27	43	2	48	29	11	9	10	11	28	31	44	17	49	3	11	15	43	35	19	9	32	26	2	34
133	35	26	24	39	6	50	38	27	43	2	9	29	12	18	45	46	28	11	6	17	49	25	11	16	31	40	12	9	32	26	2	34
134	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	2	28	11	44	6	49	25	11	15	31	37	15	9	32	26	6	34
135	35	26	24	39	6	50	38	27	43	2	9	29	12	18	27	46	28	11	6	17	49	25	11	15	31	40	13	9	32	26	3	34
136	35	33	24	39	6	48	38	27	43	2	48	7	11	18	27	14	28	11	31	17	44	25	7	15	31	38	15	9	32	26	6	34
137	35	33	24	39	6	48	38	27	43	2	48	29	11	18	21	3	28	28	44	17	49	25	11	15	31	37	15	9	32	26	5	34
138	35	33	24	39	6	48	38	27	43	2	48	29	11	18	21	3	28	28	44	17	49	25	11	15	31	37	15	9	32	26	6	34
139	35	26	24	39	6	48	38	27	43	2	45	29	11	8	10	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	2	34
140	35	33	24	39	6	48	38	27	43	2	48	7	11	18	27	14	28	11	31	17	44	25	7	15	31	38	15	10	32	26	6	34
141	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	2	28	11	44	6	49	25	11	15	31	37	15	9	32	26	5	34
142	35	26	24	39	6	48	24	27	43	2	48	29	11	8	10	11	28	23	44	17	49	25	11	15	43	38	19	12	32	26	2	34
143	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	3	28	4	44	17	49	25	11	15	31	37	15	9	32	26	6	34
144	40	26	24	39	6	48	38	46	43	12	48	29	30	8	19	11	13	31	44	17	49	25	11	15	43	39	19	9	32	26	2	34
145	35	33	24	39	6	48	38	13	43	2	48	29	11	18	22	14	28	11	44	17	49	25	11	15	31	38	19	9	32	26	5	34
146	35	26	24	39	6	50	38	27	43	2	9	29	12	18	45	46	44	11	6	17	49	25	20	15	31	40	12	9	32	26	2	34
147	35	33	24	39	6	48	38	27	43	2	48	46	11	18	21	3	28	28	44	17	49	25	11	15	31	37	14	9	32	26	5	34
148	35	26	24	39	6	50	38	27	43	2	9	29	12	18	45	46	28	11	6	17	49	25	11	15	31	40	12	9	32	26	1	34
149	35	26	24	39	6	48	38	27	35	2	48	29	11	18	27	14	28	11	44	17	49	25	11	15	31	38	12	9	32	26	3	34
150	35	26	24	39	6	48	24	27	43	2	48	29	11	8	10	11	28	23	44	17	49	25	11	15	43	38	19	12	32	26	1	34
151	35	26	24	39	6	48	38	27	43	2	48	29	11	8	10	11	28	31	44	17	49	25	11	15	43	38	19	9	32	26	1	34
152	35	33	24	39	6	48	38	27	43	2	48	29	11	18	27	14	28	11	44													

Table A3.2 Objective Functions' values for non-dominated solutions

Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)		
			GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3
1	58.31%	275,922,897.8	34,298,363.8	41,295,919.5	29,110,206.9	53	58.30%	275,918,393.6	34,927,413.3	40,877,064.9	28,904,406.4	105	57.94%	275,989,361.5	33,691,724.9	41,613,092.8	29,333,187.6
2	58.82%	275,918,900.3	34,169,306.5	41,275,702.4	29,263,449.9	54	57.78%	276,017,582.7	34,022,008.2	41,184,644.2	29,403,162.0	106	59.19%	275,939,743.1	34,100,306.2	41,440,615.0	29,146,695.4
3	58.50%	275,924,429.5	34,201,420.8	41,291,842.6	29,209,664.8	55	57.40%	275,889,376.1	34,799,148.7	40,073,678.8	29,865,183.9	107	57.69%	275,899,097.6	35,218,172.1	40,156,479.0	29,353,558.0
4	58.57%	275,918,683.0	34,222,592.5	41,291,513.6	29,194,567.4	56	59.39%	275,945,094.5	34,123,705.0	41,430,248.6	29,128,318.0	108	58.01%	275,930,306.5	35,050,326.2	40,678,751.8	28,967,913.7
5	58.78%	275,925,991.9	34,224,643.5	41,287,421.7	29,189,302.3	57	59.31%	275,959,479.3	34,213,696.9	41,289,869.2	29,164,330.7	109	59.45%	275,949,994.3	34,623,868.4	40,791,645.1	29,261,813.7
6	57.60%	275,979,748.5	34,669,683.9	40,551,016.5	29,426,903.7	58	57.75%	276,041,063.8	33,995,557.5	41,122,679.9	29,468,027.1	110	59.47%	275,915,129.7	34,348,657.0	41,230,089.2	29,133,489.3
7	58.30%	275,918,393.6	34,927,413.3	40,877,064.9	28,904,406.4	59	57.85%	275,980,295.8	33,707,273.9	41,591,503.3	29,348,291.9	111	57.60%	275,896,956.9	35,209,417.9	40,159,740.1	29,361,185.2
8	56.88%	275,902,002.0	34,752,174.7	40,067,089.6	29,906,073.8	60	59.45%	275,933,495.7	34,268,161.4	41,251,821.1	29,173,866.2	112	57.32%	275,880,850.3	34,809,128.0	40,278,684.6	29,658,724.4
9	58.13%	275,982,259.0	33,685,629.5	41,609,606.0	29,349,859.9	61	58.07%	275,930,286.5	34,907,490.5	40,867,504.7	28,922,008.9	113	59.36%	275,914,561.6	34,319,481.0	41,240,824.8	29,152,488.0
10	57.29%	275,932,335.5	35,361,917.8	39,921,979.6	29,411,108.9	62	58.50%	275,923,335.7	34,583,849.0	40,918,660.4	29,201,473.2	114	57.97%	275,987,823.8	34,964,003.7	40,480,056.1	29,195,462.5
11	58.95%	275,906,096.0	34,228,143.1	41,486,173.7	29,006,906.6	63	58.01%	275,930,306.5	35,050,326.2	40,678,751.8	28,967,913.7	115	57.95%	275,968,522.6	33,768,543.6	41,588,812.9	29,301,465.8
12	58.07%	275,893,776.8	35,017,472.4	40,442,784.1	29,273,320.6	64	57.31%	275,929,017.8	35,303,754.4	39,983,556.6	29,410,970.5	116	57.81%	275,875,917.7	34,642,207.0	40,327,485.4	29,781,779.0
13	57.31%	275,929,017.8	35,303,754.4	39,983,556.6	29,410,970.5	65	57.85%	275,919,689.6	34,410,664.1	40,557,910.9	29,739,011.3	117	57.91%	275,927,938.8	35,087,324.6	40,405,083.2	29,206,955.4
14	59.25%	275,932,295.1	34,182,304.2	41,471,783.4	29,040,984.5	66	57.33%	275,918,057.4	34,822,483.8	39,990,414.2	29,896,383.2	118	57.73%	275,931,632.5	35,249,606.9	39,989,372.1	29,456,683.1
15	57.03%	275,903,232.1	34,912,890.2	40,102,325.5	29,708,932.7	67	58.23%	275,977,278.5	33,714,318.9	41,587,098.3	29,348,649.2	119	58.07%	275,928,190.7	35,009,012.1	40,665,451.5	29,024,652.4
16	58.48%	275,905,749.2	34,288,240.1	41,303,658.9	29,129,700.7	68	58.10%	275,979,632.4	33,702,292.3	41,588,035.3	29,357,402.6	120	57.30%	275,873,802.9	34,679,505.0	40,232,994.8	29,841,043.2
17	59.22%	275,941,368.2	34,140,703.3	41,479,301.3	29,065,939.5	69	58.89%	275,920,448.9	34,141,264.2	41,269,336.5	29,296,309.4	121	58.05%	275,953,956.3	33,960,695.7	41,358,731.1	29,353,988.1
18	59.55%	275,942,986.7	34,216,332.3	41,313,739.7	29,154,283.6	70	57.53%	275,904,301.9	35,182,228.9	40,169,985.4	29,370,798.6	122	59.46%	275,908,439.0	34,722,365.1	40,655,872.6	29,340,606.9
19	58.38%	275,944,272.7	34,902,442.9	40,649,375.0	29,131,253.6	71	59.00%	275,958,793.3	34,389,480.9	41,011,052.5	29,267,978.4	123	58.90%	275,918,357.6	34,494,564.5	40,974,750.4	29,239,677.4
20	57.62%	275,906,853.6	34,707,561.5	40,165,533.8	29,847,393.3	72	57.19%	275,893,510.4	34,808,198.2	40,052,970.3	29,672,689.9	124	57.85%	275,924,222.0	35,283,638.9	39,977,923.9	29,441,607.5
21	57.71%	275,902,138.4	35,159,214.6	40,221,363.2	29,344,638.0	73	68.09%	275,964,837.4	33,952,609.8	41,361,160.4	29,348,920.8	125	69.26%	275,960,138.0	34,270,972.7	41,196,869.8	29,209,341.6
22	57.86%	275,947,687.6	34,575,347.3	40,727,199.9	29,377,118.4	74	57.92%	275,916,062.8	35,139,770.9	40,261,088.7	29,310,402.8	126	57.98%	276,007,968.6	33,899,545.0	41,303,163.2	29,416,672.2
23	58.13%	275,908,485.9	34,295,741.8	40,733,030.5	29,690,021.4	75	59.14%	275,927,893.9	34,404,460.3	41,097,742.3	29,197,247.0	127	57.74%	275,873,884.7	34,699,977.3	40,243,468.3	29,810,050.1
24	57.51%	275,882,306.7	34,693,275.9	40,228,089.9	29,823,700.0	76	58.91%	275,938,689.8	34,531,722.6	40,908,663.9	29,248,256.6	128	59.08%	275,955,442.5	34,620,150.8	40,794,304.8	29,257,436.3
25	59.41%	275,931,744.0	34,205,238.7	41,320,728.4	29,169,613.5	77	56.85%	275,924,335.6	34,982,541.3	40,044,541.2	29,675,900.5	129	57.71%	275,947,077.4	35,059,279.5	40,624,027.6	28,996,898.2
26	58.72%	275,901,326.3	34,218,661.4	41,494,929.6	29,012,430.4	78	57.77%	275,943,126.7	35,029,872.8	40,669,647.2	28,984,655.8	130	58.51%	275,939,836.9	34,766,596.0	40,577,712.7	29,343,194.1
27	58.00%	275,904,951.5	35,319,843.3	39,971,849.4	29,430,657.5	79	57.81%	275,983,446.1	33,693,773.6	41,607,084.3	29,343,055.3	131	59.48%	275,950,613.2	34,625,171.1	40,788,771.2	29,262,770.7
28	58.25%	275,892,247.3	34,423,447.3	40,688,141.2	29,623,481.8	80	59.49%	275,942,537.9	34,156,115.9	41,375,574.4	29,153,101.3	132	57.74%	275,880,120.8	34,617,029.9	40,333,779.0	29,796,467.8
29	57.70%	275,975,160.4	34,417,606.4	41,144,872.7	29,089,691.5	81	57.35%	275,957,205.9	35,237,049.1	39,966,534.3	29,466,533.8	133	57.90%	275,933,339.5	35,023,301.4	40,686,217.1	28,984,450.3
30	57.24%	275,936,305.3	35,276,541.0	39,993,992.4	29,420,474.6	82	58.19%	275,958,563.2	33,845,861.5	41,506,001.0	29,316,971.5	134	59.25%	275,932,295.1	34,182,304.2	41,471,783.4	29,040,984.5
31	57.45%	275,885,088.5	34,814,899.6	40,068,348.8	29,859,040.0	83	59.00%	275,941,651.6	34,242,508.7	41,244,662.1	29,198,567.0	135	58.67%	275,932,359.8	34,779,530.0	40,847,688.2	29,067,746.3
32	57.64%	275,964,758.6	35,106,648.5	40,205,592.5	29,350,345.7	84	57.29%	275,932,335.5	35,361,917.8	39,921,979.6	29,411,108.9	136	59.58%	275,957,407.1	34,151,854.5	41,417,390.3	29,100,712.2
33	57.93%	275,957,297.9	34,994,871.7	40,380,447.6	29,294,732.2	85	57.03%	275,903,232.1	34,912,890.2	40,102,325.5	29,708,932.7	137	58.77%	275,918,185.3	34,315,785.7	41,348,482.9	29,044,867.7
34	59.49%	275,949,439.9	34,125,300.2	41,426,675.6	29,125,953.7	86	58.67%	275,892,441.7	34,940,914.6	40,580,030.2	29,213,919.1	138	58.95%	275,906,096.0	34,228,143.1	41,486,173.7	29,006,906.6
35	57.60%	275,972,291.3	35,118,062.5	40,507,776.2	29,029,169.2	87	58.08%	275,927,956.9	34,477,334.7	40,929,101.6	29,292,946.2	139	58.49%	275,887,398.2	34,434,818.3	40,700,910.2	29,604,207.5
36	59.01%	275,949,879.2	34,085,483.7	41,495,844.9	29,096,100.7	88	57.69%	275,878,172.4	34,684,226.4	40,248,798.4	29,816,194.0	140	59.55%	275,952,599.1	34,148,893.5	41,420,109.4	29,105,735.8
37	58.20%	275,898,602.0	34,549,740.6	40,525,790.7	29,653,197.4	89	58.24%	275,912,641.5	34,403,814.5	41,125,423.7	29,185,462.7	141	59.07%	275,940,633.1	34,270,077.6	41,335,400.2	29,081,259.6
38	58.87%	275,959,157.3	34,869,294.0	40,430,306.5	29,368,572.2	90	57.94%	275,955,789.4	34,310,329.6	41,324,836.9	29,036,383.5	142	57.17%	275,876,694.6	34,647,400.3	40,243,443.6	29,859,794.1
39	58.27%	275,954,171.9	33,862,463.0	41,501,333.6	29,309,419.7	91	57.57%	275,874,180.4	34,735,405.9	40,189,144.1	29,828,621.0	143	59.16%	275,933,528.9	34,129,840.8	41,485,567.3	29,078,410.9
40	59.09%	275,921,409.4	34,533,929.1	40,957,572.0	29,214,432.7	92	58.92%	275,908,835.2	34,509,362.2	40,973,174.1	29,239,962.6	144	57.15%	275,890,103.2	34,853,772.0	40,219,229.6	29,664,202.5
41	58.13%	275,982,259.0	33,685,629.5	41,609,606.0	29,349,859.9	93	57.77%	275,960,194.2	35,103,324.2	40,489,458.5	29,074,328.4	145	58.48%	275,905,749.2	34,288,240.1	41,303,658.9	29,129,700.7
42	58.17%	275,912,641.5	34,403,814.5	41,125,423.7	29,185,462.7	94	59.23%	275,972,320.2	34,565,910.0	40,800,628.4	29,288,464.1	146	57.15%	276,011,457.4	34,767,515.6	40,736,003.6	29,112,340.3
43	58.11%	275,976,098.1	34,996,689.0	40,473,216.3	29,181,348.8	95	57.01%	275,899,110.2	34,784,279.5	40,056,640.7	29,887,322.8	147	58.56%	275,908,086.0	34,353,388.2	41,345,514.8	29,020,334.9
44	57.22%	275,911,878.8	34,804,187.2	40,076,902.4	29,834,426.1	96	58.46%	275,964,922.7	35,068,697.8	40,449,694.1	29,144,033.6	148	57.72%	275,962,310.0	35,144,638.3	40,502,758.7	29,017,589.6
45	56.96%	275,910,899.6	34,949,735.0	40,038,433.9	29,728,226.0	97	57.88%	275,982,922.3	33,690,611.1	41,613,074.1	29,340,749.1	149	59.04%	275,926,563.6	34,549,534.5	40,906,923.4	29,242,299.6
46	58.07%	275,951,549.8	34,303,545.9	40,710,169.5	29,662,061.9	98	57.98%	275,967,406.1	33,757,889.5	41,609,135.							

APPENDIX 4

RESULTS FOR THE MULTI-NODE THERMAL SYSTEM SCENARIO

Table A4.1 Non-dominated generation maintenance schedules solutions (starting week).

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	GEN 27	GEN 28	GEN 29	GEN 30	GEN 31	GEN 32
1	17	11	13	31	5	18	22	23	10	5	9	14	8	38	6	29	15	35	39	9	29	39	10	8	25	9	32	40	24	1	31	13
2	17	11	13	31	5	18	22	23	10	5	9	14	8	38	6	29	15	35	39	9	29	39	10	8	25	9	32	40	24	1	31	13
3	13	11	14	31	8	17	19	23	10	1	9	14	8	40	5	28	15	35	39	9	27	37	9	8	26	9	32	41	25	1	32	15
4	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
5	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
6	18	7	14	31	8	19	16	32	15	1	11	14	5	43	5	23	15	35	39	10	28	39	10	8	30	9	32	43	24	2	31	14
7	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	26	9	33	41	25	1	32	14
8	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	26	9	33	41	25	1	32	14
9	13	11	14	31	8	18	19	23	10	1	9	14	9	40	7	28	15	36	37	9	27	37	9	8	26	9	32	41	25	1	32	13
10	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	14	34	39	9	23	39	10	8	26	9	33	41	25	1	32	14
11	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
12	17	12	13	31	5	17	22	24	10	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
13	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	26	9	33	41	25	1	32	14
14	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
15	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	13
16	18	11	14	31	8	18	19	24	10	1	9	14	8	40	5	28	10	33	38	9	27	39	10	8	27	9	33	41	25	1	32	13
17	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	24	9	40	38	26	5	31	13
18	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
19	18	11	14	31	9	17	15	23	10	1	9	14	9	40	5	29	15	35	39	9	27	39	9	8	26	9	33	41	25	1	32	13
20	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	22	39	10	8	26	9	33	41	25	1	32	15
21	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	26	9	33	41	25	1	32	14
22	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	40	10	8	26	9	33	41	25	1	32	14
23	17	12	13	31	5	18	22	23	10	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	26	9	32	41	23	1	32	13
24	23	2	14	29	6	14	26	32	5	2	7	14	1	37	2	28	18	31	42	10	16	36	11	8	24	9	40	38	26	5	31	13
25	18	11	14	31	9	18	15	23	14	1	9	14	8	40	5	28	15	33	39	9	22	39	10	8	26	9	33	41	25	1	32	14
26	17	11	13	31	5	18	22	23	10	5	9	14	8	40	6	29	15	35	39	9	29	39	10	8	25	9	32	40	24	1	31	13
27	15	5	10	29	11	14	18	34	5	1	7	14	5	30	4	27	14	33	40	10	21	42	11	8	25	9	32	39	24	5	31	13
28	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	35	39	9	29	39	10	8	26	9	33	41	25	1	32	14
29	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	25	1	32	14
30	15	5	14	30	11	14	18	33	5	1	7	14	5	30	4	27	15	33	40	10	20	42	11	8	25	9	32	39	24	5	31	13
31	18	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	35	39	9	27	39	9	8	26	9	33	41	25	1	32	13
32	17	12	13	31	5	17	22	24	10	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
33	17	11	13	31	5	18	22	23	10	5	9	14	8	40	6	29	15	35	39	9	29	39	10	8	25	9	32	40	23	1	31	13
34	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	22	39	10	8	26	9	33	41	25	1	32	15
35	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	22	39	10	8	26	9	33	41	25	1	32	15
36	22	5	14	29	6	14	27	37	5	7	14	1	37	2	28	18	31	42	10	22	35	11	8	24	9	40	38	26	5	31	13	
37	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	22	39	10	8	26	9	33	41	25	1	32	15
38	23	2	14	29	6	14	26	34	5	3	7	14	1	37	2	28	18	31	42	10	16	36	11	8	24	9	40	38	26	5	31	13
39	17	12	13	31	5	17	22	23	11	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	31	13
40	18	7	16	31	9	19	16	32	15	1	10	14	5	43	5	24	15	35	39	9	28	39	10	8	30	9	32	43	24	1	33	14
41	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	21	39	10	8	26	9	33	41	24	1	32	15
42	17	12	13	31	5	17	22	24	10	5	9	14	8	40	5	28	14	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
43	17	12	13	31	5	17	22	23	10	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	26	9	32	41	24	1	32	13
44	18	11	14	31	8	20	19	24	10	1	9	14	8	40	5	28	12	33	39	9	27	37	10	8	27	9	33	41	25	1	32	13
45	17	11	13	31	5	18	22	23	10	5	9	14	8	40	6	29	15	35	39	9	29	39	10	8	25	9	32	40	24	1	31	13
46	19	9	14	31	7	19	19	32	10	1	11	15	5	43	5	26	15	33	40	9	24	37	10	8	24	9	32	43	24	1	30	15
47	17	11	14	31	9	17	15	23	10	1	9	14	8	40	5	28	15	34	39	9	22	39	10	8	26	9	33	41	26	1	32	14
48	17	12	13	31	4	17	22	23	11	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	31	13
49	18	11	14	31	9	19	19	24	9	1	9	14	8	40	5	28	15	33	38	9	22	40	10	8	26	9	32	41	24	1	32	14
50	17	12	13	31	4	17	22	23	11	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	31	13
51	25	12	15	27	12	17	15	35	11	1	7	14	5	28	3	22	14	28	41	9	40	41	11	8	36	9	30	38	24	5	31	13
52	18	11	14	31	9	17	21	23	9	1	9	14	9	40	5	29	15	35	38	9	27	39	9	8	26	9	33	41	25	1	32	13
53	25	12	15	27	12	17	15	35	11	1	7	14	5	28	3																	

Table A4.1 (Cont.) Non-dominated generation maintenance schedules solutions (starting week).

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	GEN 27	GEN 28	GEN 29	GEN 30	GEN 31	GEN 32
111	18	11	14	31	9	19	16	24	9	1	9	14	8	40	5	28	15	32	38	9	22	40	10	8	26	9	29	41	24	1	32	14
112	25	11	15	29	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	39	24	5	31	13
113	18	11	13	31	5	17	22	24	10	5	9	14	7	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
114	17	12	13	31	5	17	22	24	10	5	9	14	8	40	5	28	15	35	40	9	29	39	10	8	25	9	32	41	24	1	32	13
115	18	11	13	31	5	17	22	24	10	5	9	14	7	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
116	18	11	13	31	5	17	22	23	10	5	9	14	7	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
117	19	5	11	31	6	20	19	32	10	1	11	14	3	43	5	26	15	31	40	10	24	37	10	8	24	9	32	40	24	1	30	14
118	18	11	14	31	9	19	19	24	9	1	10	14	8	40	5	28	15	33	38	9	22	40	10	8	26	9	29	41	24	1	32	14
119	24	7	14	28	9	9	29	34	8	1	7	14	5	33	3	25	15	31	40	12	28	40	11	8	28	9	40	34	24	5	31	13
120	18	7	14	31	8	20	16	32	15	2	11	16	5	43	5	25	15	35	39	10	28	39	10	8	30	9	32	43	24	2	31	14
121	18	11	14	31	9	18	19	23	10	1	9	14	7	40	5	28	15	33	38	9	22	39	10	8	26	9	33	41	23	1	32	14
122	18	7	16	31	9	19	16	32	15	1	10	14	5	43	5	24	15	35	39	9	29	39	10	8	30	9	32	43	24	2	33	14
123	25	11	15	29	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	43	41	11	8	36	9	30	39	24	5	31	13
124	18	11	14	31	9	19	19	24	9	1	9	14	8	40	5	28	15	33	38	9	22	40	10	8	26	9	29	43	24	1	32	14
125	25	12	14	29	12	14	15	34	9	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	38	24	5	31	13
126	18	11	14	31	8	20	16	32	15	2	11	16	5	43	5	24	15	35	39	10	28	39	10	8	30	9	32	43	24	2	31	14
127	18	7	14	31	8	20	16	32	15	2	11	14	5	43	5	24	15	35	39	10	28	39	10	8	30	9	32	43	24	1	31	14
128	24	3	14	29	7	15	27	35	5	1	7	15	2	37	2	28	18	30	42	10	22	36	11	8	24	9	40	36	26	5	31	13
129	24	4	14	29	7	15	27	35	5	1	7	15	2	37	2	28	18	31	42	10	22	36	11	8	24	9	40	36	26	5	31	13
130	22	2	14	29	7	14	27	35	5	1	7	14	1	37	2	28	18	31	42	10	22	36	11	8	24	9	40	36	26	5	31	13
131	26	8	14	30	7	12	24	29	8	3	7	14	5	46	5	28	15	33	38	10	20	35	11	7	24	9	40	39	25	5	31	13
132	18	11	13	31	5	17	22	23	10	5	9	14	8	40	5	28	15	34	39	9	29	39	10	8	25	9	32	41	24	1	32	13
133	19	9	14	31	7	19	19	32	10	1	11	15	5	43	5	26	15	33	39	9	24	37	10	8	24	9	32	43	24	1	30	15
134	18	11	14	31	8	18	19	24	10	1	9	14	8	40	5	28	10	33	38	9	27	39	10	8	26	9	33	41	24	1	32	13
135	24	7	14	28	9	9	29	34	8	1	7	14	5	33	3	25	15	32	40	12	27	40	11	8	28	9	40	35	24	5	31	13
136	17	6	16	31	7	20	20	32	10	1	11	14	4	43	4	26	15	28	40	9	40	41	11	8	36	9	30	38	24	5	31	13
137	18	6	12	32	6	20	15	33	12	2	11	7	5	40	5	25	15	30	37	10	23	40	11	8	26	10	32	39	24	3	31	14
138	18	11	14	31	9	18	19	24	10	1	9	14	8	40	5	28	14	33	39	9	22	39	10	8	26	9	33	41	25	1	32	15
139	18	6	12	32	6	20	15	33	12	2	11	7	5	40	5	25	15	30	37	10	23	40	11	8	26	10	32	39	24	3	31	14
140	18	10	14	30	5	20	19	34	10	1	12	14	5	43	5	28	15	31	39	9	27	39	10	8	24	9	32	42	24	2	31	14
141	18	11	14	31	8	20	19	24	10	1	9	14	8	40	5	28	12	33	39	9	24	37	10	8	27	9	33	41	24	1	32	15
142	25	12	15	29	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	39	24	5	31	13
143	25	11	15	29	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	39	24	5	31	13
144	17	12	13	31	6	17	22	24	10	5	9	14	8	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
145	18	11	14	31	8	20	19	24	10	1	9	14	8	40	5	28	11	33	39	9	27	39	10	8	26	9	33	41	25	1	32	13
146	15	5	10	29	11	14	18	33	5	1	7	14	5	30	4	27	15	33	40	10	21	42	11	8	24	9	40	38	26	5	31	13
147	14	17	14	31	7	22	20	32	10	1	11	14	4	43	5	26	15	33	39	9	24	37	10	8	24	9	32	41	25	2	31	15
148	15	5	10	29	11	14	18	33	5	1	7	14	5	30	4	27	15	33	40	10	21	42	11	8	24	9	40	38	26	5	31	13
149	18	12	14	31	9	18	19	23	10	1	9	14	7	40	5	28	15	33	39	9	22	39	10	8	26	9	33	41	23	1	32	14
150	18	11	13	31	5	17	22	23	10	5	9	14	7	40	5	28	15	35	39	9	29	39	10	8	25	9	32	41	24	1	32	13
151	25	12	14	29	12	14	15	34	9	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	38	24	5	31	13
152	18	11	14	31	8	18	18	24	10	1	9	14	8	40	5	28	11	33	39	9	24	37	10	8	26	9	32	43	24	2	31	13
153	25	11	15	26	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	39	24	5	31	13
154	15	5	11	31	7	20	19	32	10	1	11	15	5	43	5	26	15	31	40	9	25	37	10	8	24	9	32	40	24	2	30	14
155	18	11	14	31	9	18	16	23	9	1	9	14	7	40	5	28	15	33	38	9	22	39	10	8	26	9	33	41	23	1	32	14
156	23	2	14	29	6	14	26	34	5	3	7	14	1	37	2	28	18	31	42	10	21	36	11	8	24	9	40	38	26	5	31	13
157	17	12	13	31	5	17	22	23	10	5	9	14	8	40	5	28	15	34	39	9	29	39	10	8	25	9	32	41	24	1	32	13
158	24	2	14	29	7	15	27	35	5	1	7	15	2	37	2	28	18	33	42	10	22	36	11	8	24	9	40	36	26	5	31	13
159	19	9	14	31	7	21	18	32	10	1	11	15	5	43	5	26	15	33	40	9	24	37	10	8	24	9	32	43	24	1	30	15
160	19	9	14	31	7	21	18	32	10	1	11	15	5	43	5	26	15	33	40	9	24	37	10	8	24	9	32	43	24	1	30	15
161	17	11	14	31	8	18	18	24	10	1	9	14	8	40	5	28	11	33	39	9	24	37	10	8	26	9	32	41	24	1	31	13
162	25	11	15	26	12	18	16	34	12	1	7	14	4	27	4	22	15	28	40	9	40	41	11	8	36	9	30	39	24	5	31	13
163	19	9	14	31	7	20	19	32	10	1	11	15	5	43	5	26	15	3														

Table A4.2 Non-dominated transmission maintenance schedules solutions (starting week).

Individual	LIN 1-2	LIN 1-3	LIN 1-5	LIN 2-4	LIN 2-6	LIN 3-9	LIN 3-24	LIN 4-9	LIN 5-10	LIN 6-10	LIN 7-8	LIN 8-9	LIN 8-10	LIN 9-11	LIN 9-12	LIN 10-11	LIN 10-12	LIN 11-13	LIN 11-14	LIN 12-13	LIN 12-23	LIN 13-23	LIN 14-16	LIN 15-16	LIN 15-21	LIN 21-B	LIN 15-24	LIN 16-17	LIN 16-19	LIN 17-22	LIN 18-21	LIN 18-21	LIN 19-20	LIN 19-20	LIN 20-23	LIN 20-23	LIN 21-22	LIN 17-18
1	14	6	0	32	0	40	17	0	16	0	0	7	50	32	45	45	2	17	32	10	0	0	17	7	38	0	0	17	35	22	15	30	49	47	49	0		
2	14	6	0	32	0	40	17	0	16	0	0	7	50	32	45	47	2	17	32	10	0	0	17	7	38	0	0	17	35	22	15	30	49	47	49	0		
3	14	5	0	32	0	40	19	0	16	0	0	7	50	32	46	48	4	17	32	10	0	0	18	7	42	0	0	17	35	15	30	29	49	50	0			
4	13	6	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	15	37	12	14	30	29	50	50	0		
5	13	6	0	33	0	40	17	0	16	0	0	5	50	30	48	51	2	16	32	10	0	0	16	7	41	0	0	17	37	13	14	30	29	47	48	0		
6	15	2	0	30	0	42	17	0	16	0	0	7	49	30	50	48	7	16	25	10	0	0	16	5	43	0	0	17	35	24	14	27	50	49	48	0		
7	13	6	0	33	0	40	15	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	47	50	0		
8	13	6	0	33	0	40	17	0	16	0	0	5	50	30	46	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	50	50	0		
9	14	5	0	32	0	40	19	0	16	0	0	7	50	32	46	48	4	17	32	10	0	0	18	7	42	0	0	17	35	15	30	29	49	50	0			
10	13	6	0	33	0	40	17	0	15	0	0	5	50	30	45	51	2	16	33	10	0	0	16	7	41	0	0	18	39	13	14	30	29	46	50	0		
11	14	3	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	33	10	0	0	16	7	41	0	0	17	37	13	14	30	29	45	50	0		
12	13	5	0	34	0	40	17	0	16	0	0	7	50	31	46	48	2	17	32	10	0	0	17	7	42	0	0	17	34	22	15	28	32	47	47	0		
13	13	6	0	33	0	40	15	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	50	50	0		
14	13	6	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	50	50	0		
15	13	5	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	33	12	0	0	16	7	41	0	0	17	37	13	14	30	29	45	50	0		
16	13	6	0	32	0	39	17	0	16	0	0	9	48	29	45	51	2	9	32	9	0	0	16	7	42	0	0	14	35	16	14	30	49	49	50	0		
17	13	6	0	33	0	40	15	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	16	37	12	14	30	28	49	50	0		
18	13	6	0	33	0	40	17	0	15	0	0	5	50	30	45	51	2	16	33	10	0	0	16	7	41	0	0	17	39	13	14	30	29	46	50	0		
19	14	5	0	32	0	40	17	0	16	0	0	7	47	32	46	48	3	17	32	10	0	0	16	7	42	0	0	17	35	16	15	30	49	46	50	0		
20	13	6	0	32	0	41	17	0	16	0	0	5	48	28	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	16	14	31	49	49	49	0		
21	13	6	0	33	0	40	15	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	28	49	50	0		
22	13	6	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	49	50	0		
23	13	5	0	29	0	40	17	0	16	0	0	7	50	32	44	50	2	17	32	10	0	0	17	7	42	0	0	17	35	22	15	28	49	47	49	0		
24	16	8	0	30	0	44	8	0	16	0	0	3	42	30	46	45	6	22	33	3	0	0	23	5	41	0	0	26	36	21	14	33	40	46	49	0		
25	13	6	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	12	14	30	29	50	50	0		
26	14	6	0	32	0	40	17	0	16	0	0	7	50	32	45	47	1	17	32	10	0	0	17	7	38	0	0	17	37	22	15	30	49	48	50	0		
27	15	6	0	32	0	46	11	0	11	0	0	11	46	35	47	52	4	16	25	6	0	0	16	5	45	0	0	27	35	21	15	35	49	50	50	0		
28	13	6	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	33	12	0	0	16	7	41	0	0	17	37	13	14	30	27	46	50	0		
29	13	3	0	33	0	40	17	0	16	0	0	5	50	30	45	51	3	16	33	10	0	0	16	7	41	0	0	17	37	14	14	30	29	45	50	0		
30	15	6	0	32	0	46	11	0	11	0	0	8	46	35	47	52	4	15	26	6	0	0	16	5	49	0	0	27	36	21	15	35	49	50	50	0		
31	14	5	0	33	0	40	17	0	16	0	0	7	47	32	46	48	3	17	32	10	0	0	18	7	42	0	0	17	35	16	15	30	49	46	49	0		
32	14	5	0	34	0	40	17	0	16	0	0	7	50	31	46	48	2	17	32	10	0	0	17	7	42	0	0	17	34	24	15	28	32	47	48	0		
33	14	6	0	32	0	40	17	0	16	0	0	8	50	32	45	47	1	17	32	10	0	0	17	7	38	0	0	17	35	22	15	30	49	48	49	0		
34	13	6	0	32	0	41	17	0	16	0	0	5	48	28	45	51	2	16	32	10	0	0	17	7	42	0	0	17	37	16	14	31	49	49	50	0		
35	13	6	0	32	0	40	17	0	16	0	0	6	48	28	45	51	2	16	32	10	0	0	17	7	42	0	0	17	39	16	14	31	49	46	50	0		
36	13	8	0	30	0	44	8	0	16	0	0	2	49	31	47	45	5	22	33	1	0	0	22	4	42	0	0	26	36	21	11	30	34	45	48	0		
37	13	6	0	32	0	40	17	0	16	0	0	5	48	29	45	51	2	16	32	10	0	0	17	7	42	0	0	17	38	16	14	31	49	46	50	0		
38	16	8	0	30	0	44	8	0	16	0	0	4	42	30	46	45	6	22	33	3	0	0	23	4	41	0	0	26	36	20	14	32	40	49	50	0		
39	14	6	0	32	0	40	17	0	16	0	0	7	50	32	45	48	2	17	32	10	0	0	17	7	38	0	0	17	35	22	14	28	49	47	50	0		
40	15	6	0	32	0	41	17	0	16	0	0	7	30	30	50	49	7	16	25	10	0	0	16	5	43	0	0	17	35	24	14	27	50	49	48	0		
41	13	6	0	32	0	40	17	0	16	0	0	6	48	28	45	51	2	16	32	10	0	0	17	7	42	0	0	17	39	16	14	31	49	46	50	0		
42	14	5	0	34	0	40	17	0	16	0	0	7	50	31	46	48	2	17	32	10	0	0	17	7	42	0	0	17	34	24	15	28	32	47	48	0		
43	13	5	0	29	0	40	17	0	16	0	0	7	50	32	44	50	2	17	32	10	0	0	17	7	42	0	0	17	35	22	15	28	49	47	50	0		
44	13	6	0	32	0	39	17	0	14	0	0	9	48	30	45	49	2	9	32	9	0	0	16	7	42	0	0	14	36	16	14	30	49	48	48	0		
45	14	6	0	32	0	40	16	0	16	0	0	7	50	32	45	47	1	17	32	10	0	0	17	7	38	0	0	17	37	22	15	30	48	49	50	0		
46	15	5	0	33	0	41	17	0	16	0	0	7	49	31	46	51	2	16	32	9	0	0	24	7	35	0	0	17	31	26	14	27	49	50	47	0		
47	13	3	0	33	0	40	17	0	16	0	0	5	50	30	45	51	2	16	33	12	0	0	16	7	41	0	0	17	37	13	14	30	27	46	50	0		
48	15	6	0	32	0	40	17	0	16	0	0	7																										

Table A4.2 (Cont.) Non-dominated transmission maintenance schedules solutions (starting week).

Individual	LIN 1-2	LIN 1-3	LIN 1-5	LIN 2-4	LIN 2-6	LIN 3-9	TRA 1-3	LIN 4-9	LIN 5-10	LIN 6-10	LIN 7-8	LIN 8-9	LIN 9-11	TRA 10-11	TRA 10-12	LIN 11-13	LIN 12-13	LIN 13-23	LIN 13-23	LIN 14-15	LIN 15-21	LIN 15-21	LIN 15-24	LIN 16-17	LIN 16-19	LIN 17-22	LIN 18-21	LIN 18-21	LIN 19-20	LIN 20-23	LIN 20-23	LIN 21-22	LIN 21-22	LIN 21-28	
111	13	6	0	32	0	42	17	0	16	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	16	14	30	48	50	49	0
112	7	4	0	29	0	47	8	0	15	0	1	47	34	47	42	6	2	30	6	0	0	16	2	41	0	0	17	37	22	11	35	34	44	49	0
113	14	5	0	32	0	40	17	0	16	0	12	50	32	44	48	3	17	32	11	0	0	17	7	44	0	0	17	35	22	17	30	50	49	50	0
114	14	5	0	34	0	40	17	0	16	0	7	50	31	46	48	2	17	32	10	0	0	17	7	44	0	0	17	34	22	15	28	32	47	48	0
115	14	5	0	32	0	40	17	0	16	0	12	50	32	44	48	3	17	32	11	0	0	17	7	44	0	0	17	35	22	17	30	50	49	50	0
116	14	5	0	32	0	40	17	0	16	0	11	50	32	44	48	2	17	32	11	0	0	17	7	44	0	0	17	35	22	15	31	49	48	50	0
117	13	5	0	32	0	41	17	0	16	0	7	47	32	46	51	2	16	32	9	0	0	24	7	35	0	0	17	32	26	14	25	49	45	47	0
118	13	6	0	35	0	42	17	0	16	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	16	14	30	48	50	49	0
119	15	11	0	30	0	48	8	0	18	0	7	49	43	46	51	3	22	36	6	0	0	21	6	41	0	0	26	35	35	15	35	49	47	49	0
120	14	2	0	30	0	42	17	0	16	0	8	49	30	51	48	7	16	25	10	0	0	16	5	43	0	0	16	35	24	13	27	50	48	50	0
121	13	6	0	32	0	40	17	0	16	0	5	50	31	45	51	2	16	32	11	0	0	18	7	42	0	0	17	38	16	14	35	49	50	49	0
122	15	2	0	30	0	42	17	0	16	0	7	50	30	50	49	7	16	25	10	0	0	16	5	43	0	0	17	35	24	13	27	50	49	48	0
123	7	4	0	29	0	47	8	0	15	0	2	47	34	47	42	6	1	30	6	0	0	16	2	41	0	0	24	37	22	11	35	34	44	49	0
124	13	6	0	32	0	42	17	0	16	0	5	50	30	45	51	2	16	32	10	0	0	16	7	42	0	0	17	37	15	14	30	48	51	49	0
125	11	4	0	29	0	46	8	0	16	0	2	48	34	46	42	3	1	30	6	0	0	17	2	41	0	0	24	37	22	11	35	50	47	48	0
126	14	2	0	30	0	42	17	0	16	0	8	49	30	51	48	7	16	25	10	0	0	16	5	43	0	0	16	35	24	13	27	50	48	50	0
127	14	2	0	31	0	42	18	0	16	0	8	49	30	51	48	7	16	25	10	0	0	16	5	43	0	0	17	35	24	13	27	50	50	49	0
128	13	13	0	28	0	44	8	0	16	0	2	49	29	46	45	6	22	32	3	0	0	22	1	43	0	0	26	35	16	12	29	31	49	49	0
129	13	13	0	28	0	44	8	0	16	0	2	49	29	45	46	4	22	32	3	0	0	22	1	43	0	0	26	35	26	18	29	50	48	50	0
130	13	8	0	30	0	44	8	0	16	0	2	49	31	47	45	5	22	33	1	0	0	22	4	42	0	0	26	36	21	11	30	34	45	50	0
131	19	9	0	32	0	44	8	0	13	0	6	45	32	48	51	7	23	32	6	0	0	22	8	39	0	0	22	40	27	8	35	49	47	50	0
132	14	5	0	32	0	40	17	0	16	0	12	50	32	44	48	2	17	32	11	0	0	17	7	44	0	0	17	35	22	15	30	46	48	50	0
133	15	5	0	33	0	41	17	0	16	0	7	49	31	46	51	2	16	32	9	0	0	24	7	35	0	0	17	31	26	14	27	49	50	47	0
134	13	6	0	32	0	39	17	0	16	0	9	48	29	45	51	2	9	32	9	0	0	16	7	42	0	0	14	35	16	14	30	49	49	50	0
135	15	11	0	30	0	49	8	0	18	0	7	49	45	46	51	3	22	36	6	0	0	21	6	41	0	0	26	35	35	15	34	49	46	50	0
136	15	5	0	33	0	43	18	0	16	0	7	49	31	44	50	2	16	32	9	0	0	24	3	35	0	0	17	32	27	16	33	49	50	50	0
137	13	5	0	30	0	44	18	0	14	0	8	49	29	46	51	2	15	25	6	0	0	17	3	43	0	0	17	36	21	17	31	31	46	47	0
138	13	6	0	32	0	41	17	0	16	0	5	48	28	45	51	2	16	32	10	0	0	18	7	42	0	0	17	37	17	14	31	49	49	49	0
139	13	5	0	30	0	44	18	0	14	0	8	49	29	46	51	3	15	25	6	0	0	17	3	43	0	0	17	36	21	17	31	31	50	50	0
140	13	5	0	30	0	39	17	0	16	0	7	50	31	47	51	2	17	32	9	0	0	24	6	34	0	0	17	37	16	12	27	30	47	48	0
141	13	6	0	32	0	39	17	0	14	0	9	48	30	45	49	2	9	32	9	0	0	16	7	42	0	0	14	35	16	14	30	49	49	50	0
142	7	4	0	29	0	47	8	0	15	0	2	47	34	47	42	6	1	30	6	0	0	16	2	41	0	0	24	36	22	11	35	34	45	49	0
143	11	4	0	29	0	46	8	0	15	0	2	47	33	47	40	6	1	31	6	0	0	16	2	41	0	0	24	36	22	10	35	34	45	49	0
144	14	2	0	34	0	40	17	0	16	0	7	50	31	46	48	2	17	32	10	0	0	17	7	44	0	0	17	34	22	14	30	49	49	50	0
145	13	6	0	32	0	39	17	0	14	0	9	48	30	45	49	2	9	32	9	0	0	16	7	42	0	0	14	35	16	14	30	49	49	50	0
146	15	6	0	32	0	46	11	0	11	0	11	46	35	47	52	4	16	26	6	0	0	16	5	45	0	0	27	35	21	15	35	50	50	50	0
147	15	5	0	33	0	41	18	0	16	0	7	49	31	46	50	2	14	32	9	0	0	24	7	35	0	0	17	31	27	16	28	45	49	50	0
148	15	6	0	32	0	46	11	0	11	0	11	46	35	47	52	4	16	25	6	0	0	16	5	45	0	0	27	35	21	15	35	49	50	50	0
149	13	6	0	32	0	40	17	0	16	0	5	50	31	45	51	2	16	32	11	0	0	17	7	42	0	0	17	37	16	14	30	49	50	50	0
150	13	5	0	32	0	40	17	0	16	0	11	50	32	44	48	2	17	32	11	0	0	17	7	44	0	0	17	35	22	15	31	49	48	50	0
151	11	4	0	29	0	46	8	0	16	0	2	48	34	46	42	3	1	30	6	0	0	16	2	41	0	0	24	37	22	11	35	51	50	50	0
152	13	6	0	32	0	39	18	0	16	0	5	48	30	45	51	2	9	32	10	0	0	15	7	42	0	0	14	35	16	14	30	49	50	50	0
153	7	4	0	29	0	46	8	0	15	0	2	47	34	47	42	6	1	30	6	0	0	16	2	41	0	0	24	36	22	11	35	34	45	49	0
154	15	5	0	33	0	41	17	0	16	0	7	47	31	46	51	2	16	32	9	0	0	23	7	35	0	0	17	32	25	14	27	49	49	50	0
155	13	6	0	32	0	40	17	0	16	0	5	50	31	45	51	2	16	32	11	0	0	18	7	42	0	0	17	37	16	14	35	49	50	50	0
156	8	0	30	44	8	0	16	0	16	0	4	42	30	46	45	7	22	33	3	0	0	23	4	41	0	0	26	36	21	14	32	40	47	49	0
157	14	5	0	33	0	44	17	0	16	0	7	50	31	46	48	2	17	32	10	0	0	17	7	44	0	0	17	34	22	15	28	32	47	48	0
158	13	13	0	28	0	44	8	0	16	0	2	49	29	46	45	6	23	33	3	0	0	22	4	42	0	0	26	40	17	15	30	50	49	49	0
159	15	5	0	33	0	41	17	0	16	0	7	49	31	46	51	2	16	32	9</																

Table A4.3 Objective Functions' values for non-dominated solutions.

Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)		
			GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3
1	59.00%	275,871,847.4	35,527,541.9	40,570,088.2	28,993,243.1	59	58.97%	276,209,302.8	35,598,058.7	40,759,211.8	29,595,864.2	117	57.70%	276,029,220.0	36,348,686.0	40,641,746.6	29,739,095.7	175	57.70%	276,094,295.4	36,332,696.8	40,672,966.2	29,810,209.5
2	59.00%	275,881,828.0	35,522,011.9	40,567,456.7	28,999,239.5	60	58.71%	275,914,140.2	36,037,314.5	40,884,636.6	29,262,435.9	118	58.01%	275,990,483.2	36,447,497.7	41,067,148.9	29,592,263.8	176	59.60%	276,213,293.2	35,501,641.9	40,319,876.4	28,920,466.4
3	58.81%	275,894,741.4	35,791,599.9	40,611,401.7	29,232,456.2	61	58.57%	275,980,233.9	36,398,651.0	41,107,047.4	29,287,372.5	119	59.67%	276,075,358.2	35,093,401.7	40,414,473.0	28,752,896.0	177	58.85%	276,051,739.3	35,963,769.3	40,613,111.1	29,188,009.0
4	58.23%	275,887,386.5	36,442,935.3	41,087,836.0	29,439,966.9	62	57.92%	275,905,974.2	36,513,845.5	40,997,389.0	29,570,682.1	120	57.77%	276,175,059.4	36,230,387.4	40,382,263.8	29,730,550.2	178	57.66%	276,083,264.3	36,273,033.5	40,619,966.1	29,830,933.0
5	58.38%	275,906,442.9	36,478,994.6	41,042,917.1	29,459,869.1	63	58.58%	276,033,506.6	36,427,964.1	41,062,586.7	29,526,181.0	121	57.96%	275,976,198.3	36,449,231.1	41,125,704.3	29,537,859.7	179	59.16%	276,013,072.5	35,917,124.9	40,452,957.3	29,744,122.1
6	57.74%	276,309,506.5	36,522,657.6	40,621,473.4	30,061,921.6	64	58.59%	275,942,547.6	36,358,650.1	41,052,699.5	29,421,449.9	122	57.69%	276,240,140.9	36,263,844.3	40,620,491.6	29,856,042.6	180	59.79%	276,248,664.7	35,310,874.9	40,038,601.2	29,268,125.5
7	58.21%	275,877,782.0	36,417,729.8	41,105,030.6	29,371,684.3	65	59.46%	276,056,665.4	35,133,246.6	40,391,892.1	28,717,672.5	123	59.49%	276,094,151.1	35,597,723.2	40,250,104.1	29,238,793.0	181	59.79%	276,165,892.7	35,210,903.7	39,997,047.4	29,286,804.0
8	58.21%	275,884,089.7	36,489,989.8	41,132,276.1	29,435,639.6	66	58.71%	275,911,884.5	36,022,719.3	40,879,243.4	29,255,465.2	124	57.54%	276,044,733.2	36,365,605.4	40,928,271.2	29,739,477.0	182	56.96%	276,216,747.6	36,271,789.4	40,254,173.0	30,076,104.4
9	58.81%	275,894,741.4	35,792,558.4	40,611,401.7	29,232,456.2	67	58.13%	276,182,735.7	36,168,307.7	40,983,931.6	29,784,069.7	125	59.79%	276,106,690.0	35,410,103.4	40,049,450.6	29,201,153.3	183	57.96%	275,976,532.6	36,463,183.7	41,127,705.2	29,539,523.0
10	58.28%	275,894,724.2	36,362,968.1	40,941,434.6	29,309,753.7	68	58.71%	275,914,473.7	36,051,433.2	40,886,631.6	29,264,316.2	126	57.73%	276,193,513.2	36,425,164.4	40,523,362.1	29,868,857.3	184	57.98%	275,976,901.1	36,463,705.6	41,127,701.1	29,539,523.0
11	58.40%	275,906,200.6	36,361,026.8	40,938,168.4	29,371,395.2	69	58.93%	276,061,483.7	35,896,610.3	41,045,659.7	29,964,942.9	127	57.55%	276,321,518.8	36,283,918.2	40,625,426.8	29,830,772.3	185	59.06%	276,089,891.9	35,679,576.1	40,236,630.5	29,945,012.4
12	58.57%	275,969,010.2	36,417,219.5	41,035,715.6	29,465,100.2	70	58.89%	275,962,113.8	35,915,839.3	40,877,003.1	29,219,992.2	128	59.16%	276,013,072.5	35,916,742.9	40,452,957.3	29,744,122.1	186	59.06%	276,087,058.3	35,642,938.1	40,237,753.1	29,944,866.2
13	58.21%	275,879,292.9	36,402,340.2	41,103,215.5	29,369,285.3	71	59.06%	276,080,314.7	35,575,109.5	40,644,316.7	29,459,205.8	129	58.92%	276,059,100.3	35,700,165.5	40,367,647.2	29,777,173.4	187	59.79%	276,121,343.0	35,390,089.7	40,037,521.7	29,177,887.6
14	58.23%	275,890,454.8	36,502,123.0	41,113,949.5	29,451,675.0	72	58.55%	275,938,773.8	36,467,134.9	41,103,281.4	29,419,880.2	130	58.93%	276,061,483.7	35,884,364.1	40,388,417.1	29,991,463.7	188	59.16%	276,013,406.1	35,929,415.7	40,455,228.1	29,746,002.4
15	58.73%	275,941,793.9	36,308,077.2	40,943,538.9	29,309,292.4	73	58.66%	276,181,700.8	35,025,113.8	40,458,681.0	28,795,020.0	131	58.96%	276,065,814.3	35,576,142.4	40,625,797.1	29,519,950.1	189	58.01%	275,976,902.2	36,464,111.9	41,126,363.7	29,540,841.4
16	58.81%	276,142,454.0	36,220,677.3	41,071,991.6	29,612,188.3	74	58.55%	275,938,266.5	36,455,241.9	41,101,154.8	29,417,419.1	132	58.57%	275,978,406.4	36,484,089.4	41,124,364.6	29,282,485.9	190	59.67%	276,144,943.9	35,089,857.0	40,443,007.4	28,821,016.8
17	58.21%	275,877,364.8	36,363,709.9	41,087,921.3	29,363,663.4	75	59.17%	276,013,072.5	35,916,742.9	40,452,957.3	29,744,122.1	133	56.73%	276,239,259.9	36,466,729.0	40,556,715.1	30,139,758.3	191	57.76%	276,049,695.0	36,344,567.8	40,647,204.4	29,734,198.1
18	58.25%	275,888,120.4	36,344,332.7	41,060,971.8	29,403,609.9	76	58.54%	275,938,266.5	36,441,973.0	41,101,154.8	29,417,419.1	134	58.60%	276,142,202.5	36,274,023.6	41,170,933.6	29,679,643.5	192	59.60%	276,280,973.3	35,529,804.4	40,284,251.7	29,061,317.9
19	58.69%	275,915,797.5	36,140,186.7	40,939,892.5	29,286,518.1	77	58.95%	275,941,540.7	35,733,780.2	40,761,252.6	29,126,044.9	135	59.61%	276,066,405.5	35,150,492.5	40,480,794.3	28,755,661.4	193	58.57%	275,983,265.9	36,424,141.3	41,105,416.2	29,285,804.1
20	57.90%	276,035,455.4	36,504,690.5	41,053,089.2	29,700,811.2	78	58.58%	276,034,579.3	36,432,987.1	41,065,881.5	29,529,076.0	136	56.95%	276,266,306.7	36,253,962.0	40,272,470.7	30,110,312.2	194	58.15%	276,127,008.6	36,072,121.5	40,915,742.7	29,756,990.6
21	58.21%	275,879,745.2	36,405,307.5	41,105,424.2	29,371,525.1	79	58.89%	275,951,969.6	35,818,916.6	40,825,030.0	29,220,798.7	137	58.86%	276,050,306.3	35,958,662.0	40,609,852.8	29,185,502.4	195	57.98%	276,083,544.6	36,145,986.2	40,516,171.3	29,739,951.7
22	58.21%	275,894,198.4	36,475,749.2	41,137,923.3	29,409,901.9	80	57.65%	276,082,996.3	36,273,015.3	40,627,851.1	29,818,064.3	138	57.90%	276,035,340.3	36,518,572.5	41,058,487.1	29,704,438.6	196	59.06%	276,089,780.5	35,679,777.0	40,236,551.1	29,945,312.3
23	58.54%	275,938,266.5	36,455,241.9	41,101,154.8	29,417,419.1	81	58.66%	275,939,964.8	36,437,919.3	41,092,279.3	29,412,208.1	139	58.85%	276,051,297.4	35,980,035.0	40,615,364.3	29,189,220.0	197	57.13%	276,166,715.4	36,314,450.9	40,346,994.1	30,054,366.7
24	59.03%	276,094,697.6	35,676,154.7	40,249,081.0	29,958,630.7	82	58.57%	275,965,765.3	36,396,307.4	41,027,448.7	29,454,752.0	140	57.64%	276,084,239.0	36,257,697.2	40,631,853.8	29,626,783.1	198	59.67%	276,074,415.0	35,991,879.8	40,406,456.0	28,745,174.1
25	58.24%	275,890,792.6	36,504,155.7	41,113,949.5	29,451,675.0	83	57.90%	275,904,958.6	36,571,305.8	40,997,086.9	29,618,838.3	141	58.80%	276,139,175.1	36,088,054.9	41,000,919.8	29,566,305.3	199	58.72%	276,061,377.1	36,106,093.1	40,921,941.0	29,599,271.8
26	58.89%	275,961,174.0	35,922,421.9	40,875,901.6	29,228,433.8	84	57.62%	276,126,979.6	36,345,673.3	40,548,195.3	29,958,673.5	142	59.42%	276,073,240.5	35,446,829.8	40,237,543.6	29,191,228.5	200	58.00%	275,985,780.8	36,396,703.0	41,030,700.8	29,614,096.0
27	59.61%	276,288,791.5	35,387,857.8	40,365,786.8	28,859,943.8	85	58.89%	275,962,644.4	35,928,217.6	40,877,051.9	29,225,700.3	143	59.59%	276,072,901.8	35,525,178.2	40,243,434.1	29,151,834.8	201	57.70%	276,029,778.2	36,352,309.2	40,652,088.8	29,728,872.7
28	58.25%	275,888,120.4	36,408,397.7	41,060,971.8	29,403,609.9	86	59.59%	276,293,579.4	35,498,619.4	40,418,586.9	28,867,633.7	144	58.58%	276,034,913.3	36,446,939.7	41,067,882.4	29,530,967.5	202	58.00%	276,327,161.8	36,210,214.9	40,511,461.6	29,831,619.2
29	58.38%	275,914,257.0	36,376,855.9	40,927,652.9	29,381,637.3	87	58.89%	275,963,870.9	35,915,900.3	40,877,143.9	29,224,719.8	145	58.70%	276,135,739.3	36,131,451.4	41,098,570.8	29,615,052.6	203	57.55%	276,320,143.6	36,274,231.5	40,625,016.2	29,885,508.0
30	59.60%	276,286,968.3	35,523,484.8	40,308,287.6	29,041,835.1	88	56.96%	276,066,322.5	36,286,376.7	40,272,470.7	30,117,237.0	146	59.59%	276,360,356.8	35,379,113.0	40,373,233.9	28,864,043.0	204	58.96%	276,063,666.2	35,909,154.7	40,395,592.3	29,999,866.4
31	58.71%	275,914,501.9	35,958,510.9	40,867,513.6	29,243,398.8	89	58.86%	275,961,174.0	35,909,873.5	40,875,901.6	29,228,433.8	147	57.13%	276,169,569.7	36,362,478.5	40,382,686.6	30,052,071.8	205	59.79%	276,119,481.1	35,443,237.0	40,075,306.6	29,248,249.5
32	58.57%	275,965,871.6	36,410,600.4	41,027,437.2	29,455,682.4	90	59.56%	276,094,697.6	35,599,956.3	40,285,091.1	29,203,940.2	148	59.59%	276,288,580.0	35,494,239.6	40,420,089.1	29,707,134.9	206	59.59%	276,227,644.3	35,479,973.4	40,393,969.2	28,802,149.9
33	58.77%	275,960,768.9	35,934,289.9	40,886,201.4	29,228,684.5	91	57.93%	275,905,966.8	36,533,404.8	40,997,389.0	29,564,011.0	149	57.95%	275,972,675.9	36,437,848.1	41,148,837.2	29,540,320.0	207	57.90%	276,025,141.2	36,492,970.9	41,045,587.1	29,724,642.0
34	57.90%	276,037,278.0	36,546,662.9	41,062,858.2	29,717,160.4	92	57.13%	276,189,312.6	36,319,485.4	40,357,925.9	30,042,704.9	150	58.55%	276,046,108.2	36,471,026.9	41,083,840.6	29,418,566.5	208	58.00%	275,983,834.7	36,453,081.5	41,033,134.3	29,623,354.3
35	57.90%</																						

APPENDIX 5

RESULTS FOR THE MULTI-NODE WIND-HYDRO-THERMAL SYSTEM SCENARIO

Table A5.1 Non-dominated generation maintenance schedule solutions (starting week)

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	YAN	CHJ	COR	COR	SIS	SIS	SIS	SJO	SJO	
1	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	27	35	7	39	32	9	25	32	5	15	42	21	46	16	46	1	35	16	3	42
2	48	23	37	17	25	43	13	24	30	40	31	8	35	39	4	40	28	35	7	38	31	9	25	32	5	14	41	27	46	5	44	5	21	13	4	44
3	50	23	37	18	27	42	13	24	30	40	31	9	35	39	5	40	28	34	10	38	31	9	25	32	5	14	40	29	45	5	50	6	21	14	4	44
4	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	14	41	27	46	5	47	5	21	13	4	44
5	48	23	37	18	25	43	13	24	30	40	31	8	35	39	4	40	28	34	6	38	31	9	25	32	5	14	41	27	46	5	47	5	21	13	4	44
6	47	27	37	18	27	43	15	46	31	40	31	18	35	39	26	47	25	34	5	39	30	8	23	32	5	14	41	26	46	16	42	5	20	16	3	44
7	46	26	33	18	20	45	18	46	30	40	31	9	35	39	13	40	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41
8	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	41	24	46	16	46	2	35	16	3	42
9	41	29	33	17	20	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	41	24	46	16	46	2	35	16	3	42
10	41	29	33	17	20	46	19	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	46	16	46	1	35	13	4	42
11	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42
12	46	26	33	18	20	45	18	46	30	40	31	9	35	39	13	46	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41
13	47	28	36	18	28	43	18	45	31	40	30	18	35	39	26	48	25	34	5	39	32	8	23	32	5	14	41	25	46	16	42	5	20	16	3	44
14	45	29	37	20	28	44	13	44	32	40	28	10	33	39	12	46	25	37	5	43	32	8	24	32	2	15	39	24	46	20	42	6	29	16	9	43
15	48	28	37	20	28	41	13	46	32	40	28	10	35	39	26	47	25	35	5	39	32	8	24	32	2	15	40	24	46	18	41	2	29	16	9	43
16	48	25	33	17	20	47	19	45	30	42	31	16	35	39	14	48	29	35	8	39	32	8	25	32	3	16	49	24	48	18	48	4	35	13	1	41
17	47	25	33	18	20	47	19	44	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	7	16	49	24	48	18	43	4	35	15	2	41
18	48	28	36	18	27	43	15	46	31	40	29	18	32	39	26	48	25	35	5	39	32	8	23	32	5	14	41	25	46	16	42	5	20	16	3	44
19	48	25	33	18	20	47	19	45	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	3	16	49	24	48	18	47	4	35	13	1	41
20	48	26	33	18	20	47	19	42	30	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	43	24	48	18	40	4	35	13	1	41
21	48	27	34	18	20	46	19	45	29	42	31	17	35	39	12	46	29	35	8	39	32	8	25	32	1	16	49	24	48	17	44	4	35	17	1	41
22	48	25	33	17	20	47	19	45	30	42	31	16	35	39	14	48	29	35	8	39	32	7	25	32	3	16	48	24	48	18	48	4	35	15	1	41
23	48	26	29	18	20	47	19	45	30	42	31	17	35	39	14	44	29	35	8	39	32	8	25	32	1	16	49	24	48	14	44	5	35	15	1	41
24	44	29	33	18	20	47	17	48	29	42	31	9	35	39	16	40	29	35	7	39	32	9	25	32	5	14	41	24	44	16	46	4	35	16	4	44
25	48	23	37	17	25	43	13	24	30	40	31	8	35	39	4	40	28	35	7	38	31	9	25	32	5	14	41	27	46	5	44	5	21	13	4	44
26	50	23	37	18	27	42	13	24	30	40	30	9	35	39	5	40	28	34	10	38	31	9	25	32	5	14	41	29	45	5	46	6	21	14	2	44
27	48	23	37	17	25	43	13	24	30	40	31	8	35	39	4	40	28	35	7	38	31	9	25	32	5	14	41	27	46	5	44	5	20	13	4	44
28	50	23	37	18	27	42	13	25	30	40	31	9	35	39	4	40	28	34	7	38	31	9	25	32	5	14	41	29	46	5	50	6	21	13	4	44
29	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	48	25	34	6	39	32	8	23	31	5	14	41	25	46	16	43	5	20	16	3	44
30	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	48	25	34	6	39	32	8	23	31	5	14	41	25	46	16	43	5	20	16	3	44
31	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	42	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42
32	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42
33	41	29	33	17	20	46	17	44	30	40	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	46	16	46	1	35	13	3	42
34	48	20	36	18	27	49	13	45	29	47	28	7	34	38	26	47	26	36	9	38	31	9	26	32	6	15	40	24	49	10	47	4	27	16	2	43
35	46	27	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	32	8	25	32	7	16	49	25	43	10	44	4	35	13	5	41
36	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	48	25	34	5	39	32	8	23	31	5	14	41	25	46	16	43	5	20	16	3	44
37	49	29	37	20	28	44	13	46	30	40	29	10	33	39	27	46	25	35	5	42	32	8	24	32	2	15	40	24	46	19	42	5	29	16	9	43
38	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	35	5	39	32	8	24	32	2	15	40	24	46	20	41	1	29	16	11	43
39	48	26	33	18	20	47	21	45	30	42	31	17	35	39	14	45	29	36	8	40	32	8	25	32	1	16	49	24	48	18	44	4	35	17	2	41
40	47	25	33	18	20	47	19	44	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	7	16	49	24	48	18	43	4	35	15	2	41
41	47	25	33	18	20	47	19	44	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	7	16	49	24	48	18	43	4	35	15	2	41
42	48	27	33	17	20	47	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41
43	48	26	33	18	20	47	19	42	30	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	40	4	35	13	1	41
44	48	26	33	18	20	47	19	42	30	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	46	24	48	18	40	4	35	13	1	41
45	48	27	34	18	20	46	19	45	29	42	31	17	35	39	13	48	29	35	8	39	32	8	25	32	1	16	49	24	48	17	44	4	35	17	1	41
46	50	28	34	17	15	47	19	40	33	41	28	9	35	39	13	40	29	35	7	39	32	9	25	32	5	14	42	24	46	16	43	1	33	14	5	44
47	50</																																			

Table A5.1 (Cont.) Non-dominated generation maintenance schedules solutions
(starting week).

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	GEN 27	GEN 28	GEN 29	GEN 30	YAN 1	CHJ 1	COR 12	COR 34	COR 45	SIS 12	SIS 34	SIS 45	SJO 1	SJO 2
84	48	25	33	17	20	47	19	45	30	42	31	16	35	39	14	48	29	35	8	39	32	8	25	32	3	16	47	24	48	18	48	4	35	13	1	41				
85	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	34	5	39	32	8	24	32	2	15	40	24	46	20	41	2	29	16	9	43				
86	48	28	34	17	15	47	15	40	33	41	28	9	35	39	13	40	29	35	7	39	32	9	25	32	5	14	42	24	46	16	43	1	33	14	5	44				
87	50	23	37	18	27	42	13	24	30	40	30	9	35	39	5	40	28	34	10	38	31	9	25	32	5	14	41	29	45	5	46	6	21	14	2	44				
88	50	23	37	18	27	42	13	25	30	40	31	9	35	39	4	40	28	34	7	38	30	9	25	32	5	14	41	29	46	5	50	6	21	14	4	44				
89	47	28	36	18	28	43	14	46	31	40	29	18	35	39	26	48	25	34	6	39	32	8	23	31	5	14	41	25	46	16	43	5	20	16	3	44				
90	47	28	36	18	28	43	14	44	31	40	29	18	35	39	26	48	25	34	6	39	32	8	24	32	5	14	41	25	46	16	43	5	20	16	3	44				
91	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	41	24	46	16	46	2	35	16	3	42				
92	41	29	33	17	19	46	15	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42				
93	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42				
94	48	20	36	18	27	49	13	45	30	47	28	7	34	38	26	47	26	34	9	38	31	9	26	32	6	15	40	24	49	10	47	3	27	16	2	40				
95	46	26	33	18	20	45	18	46	30	40	31	9	35	39	13	46	29	35	8	39	32	8	25	32	5	16	47	25	43	10	44	4	35	13	5	41				
96	49	29	37	20	28	44	14	46	30	40	29	10	33	39	27	46	25	35	5	42	32	8	24	32	2	15	40	24	46	19	42	5	29	16	9	43				
97	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	34	5	39	32	8	25	32	2	15	40	24	46	20	41	2	29	16	9	43				
98	48	26	33	18	20	47	19	45	30	42	31	17	35	39	14	45	29	36	8	40	32	8	25	32	1	16	49	24	48	18	44	4	35	17	2	41				
99	47	25	33	18	20	47	19	45	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	7	16	49	24	48	18	43	4	35	15	2	41				
100	48	27	33	17	20	47	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41				
101	48	26	33	18	20	47	19	42	28	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	40	4	35	13	1	41				
102	48	25	33	17	20	47	19	45	30	42	31	16	35	39	14	48	29	35	8	39	32	8	25	32	3	16	48	24	48	18	48	4	35	15	1	41				
103	48	29	34	17	20	47	19	45	30	42	31	17	35	39	14	47	29	35	8	39	32	8	25	32	1	16	49	25	48	18	44	4	35	13	1	41				
104	46	29	33	18	20	47	17	48	29	42	31	9	35	39	16	40	29	35	7	39	31	9	25	32	5	14	41	24	44	16	46	4	35	16	4	44				
105	50	23	37	18	27	42	13	24	30	40	31	9	35	39	5	40	28	34	10	38	31	9	25	32	5	14	40	29	45	5	50	6	19	14	4	44				
106	48	23	37	17	25	43	13	24	30	40	31	8	35	39	4	40	28	34	7	38	31	9	25	32	5	14	41	27	46	5	44	5	21	13	4	44				
107	47	28	36	17	27	43	15	46	31	40	29	18	32	39	26	48	25	35	5	39	32	8	23	32	5	14	41	25	46	16	42	5	20	16	3	44				
108	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	42	32	9	25	32	5	15	42	24	46	16	46	5	35	16	3	42				
109	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	3	35	16	3	42				
110	41	29	33	17	19	46	17	44	30	40	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42				
111	48	20	36	18	27	49	13	45	30	47	28	7	34	38	26	47	26	34	9	38	31	9	26	32	6	15	40	24	49	10	47	4	27	16	2	40				
112	43	26	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41				
113	49	29	37	20	28	44	13	46	30	40	29	10	33	39	27	46	25	35	5	42	32	8	24	32	2	15	40	24	46	19	42	5	29	16	9	43				
114	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	35	5	39	32	8	24	32	2	15	40	24	46	20	41	1	29	16	11	43				
115	48	26	33	18	20	47	19	45	30	42	31	17	35	39	14	45	29	36	8	40	32	8	25	32	1	16	49	24	48	18	44	4	35	17	2	41				
116	48	27	33	17	20	47	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41				
117	48	27	34	18	20	46	19	45	29	42	31	17	35	39	12	48	29	35	8	39	32	8	25	32	1	16	49	24	48	17	44	4	35	17	1	41				
118	48	25	33	17	20	47	19	45	30	42	31	16	35	39	14	48	29	35	8	39	32	8	25	32	3	16	47	24	48	18	48	4	35	13	1	41				
119	48	29	34	17	20	47	19	45	30	42	31	17	35	39	14	47	29	35	8	39	32	8	25	32	1	16	49	25	48	18	44	4	35	13	1	41				
120	50	28	34	17	15	47	15	40	33	41	28	9	35	39	13	40	29	35	7	39	32	9	25	32	5	14	42	24	46	16	43	1	33	14	5	44				
121	50	23	37	18	27	42	13	25	30	40	31	9	35	39	4	40	28	34	7	38	31	9	25	32	5	14	41	29	46	5	50	6	21	14	4	44				
122	48	28	36	18	27	43	15	44	31	40	29	18	32	39	26	48	25	34	5	39	32	8	23	32	5	14	41	25	46	16	41	5	20	16	3	45				
123	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	42	32	9	25	32	5	15	42	24	46	16	46	2	35	16	3	42				
124	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	45	16	46	3	35	16	3	42				
125	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42				
126	48	20	36	18	27	49	13	44	30	47	28	7	34	38	26	47	26	34	9	38	31	9	26	32	6	15	40	24	49	10	47	4	27	16	2	40				
127	43	26	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41				
128	49	29	37	20	28	44	13	46	33	40	29	10	33	39	27	46	25	35	5	42	32	8	24	32	2	15	40	24	46	19	42	5	29	16	9	43				
129	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	34	5	39	32	8	25	32	2	15	40	24	46	20										

**Table A5.1 (Cont.) Non-dominated generation maintenance schedules solutions
(starting week).**

Individual	GEN 1	GEN 2	GEN 3	GEN 4	GEN 5	GEN 6	GEN 7	GEN 8	GEN 9	GEN 10	GEN 11	GEN 12	GEN 13	GEN 14	GEN 15	GEN 16	GEN 17	GEN 18	GEN 19	GEN 20	GEN 21	GEN 22	GEN 23	GEN 24	GEN 25	GEN 26	YAN 1	CHJ 1	COR 12	COR 34	COR 5	SIS 12	SIS 34	SIS 5	SJO 15	SJO 2
167	48	26	29	17	20	49	19	45	30	39	31	17	35	44	14	44	29	35	8	39	32	8	25	32	1	16	49	24	48	16	44	5	35	15	1	41
168	48	28	34	19	14	47	16	40	33	41	28	9	34	39	14	40	29	35	7	39	32	9	25	32	5	14	42	24	46	16	43	1	33	14	4	44
169	50	23	35	18	27	42	13	24	30	40	31	9	35	39	5	40	28	34	10	38	31	9	25	32	5	14	40	29	45	5	50	6	19	14	4	44
170	48	23	37	18	25	43	13	24	30	40	31	8	35	39	4	40	28	34	7	38	31	9	25	32	5	14	41	27	46	5	47	5	21	13	4	44
171	47	28	36	17	27	43	15	46	31	40	29	18	32	39	26	48	25	35	5	39	32	8	23	32	5	14	41	25	46	16	42	5	20	16	3	44
172	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	48	25	34	6	39	32	8	23	32	5	14	41	25	46	16	43	5	20	16	3	44
173	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	35	5	39	32	8	24	32	5	15	40	24	46	20	41	2	29	16	9	43
174	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	3	35	16	3	42
175	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42
176	46	26	33	18	20	45	18	46	30	40	31	9	35	39	13	45	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	43
177	48	29	37	20	28	44	13	46	32	40	28	10	33	39	26	46	25	37	5	39	32	8	24	32	2	15	40	24	46	20	42	6	29	16	9	43
178	48	28	36	20	28	41	13	46	29	40	29	10	35	39	26	47	25	34	5	39	32	8	25	32	2	15	40	24	46	20	41	2	29	16	9	43
179	48	27	33	17	20	46	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41
180	48	26	33	18	20	47	19	42	30	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	40	4	35	13	1	41
181	48	25	33	17	20	47	19	45	30	42	31	17	35	40	14	48	29	35	8	39	32	8	25	32	3	16	48	24	48	18	48	4	35	15	1	41
182	48	29	37	20	28	44	13	46	33	40	28	10	33	39	26	46	25	35	5	40	32	8	24	32	2	15	40	24	46	20	42	5	29	16	9	43
183	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	46	25	34	6	39	32	8	23	32	5	14	41	25	46	16	43	5	20	16	3	44
184	49	29	37	20	28	44	13	46	33	40	28	10	33	39	28	46	25	35	5	39	32	8	25	32	2	15	40	24	46	20	42	5	29	16	9	43
185	41	29	33	17	19	46	18	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	24	46	16	46	3	35	16	3	42
186	40	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	46	16	46	1	35	13	3	42
187	47	26	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41
188	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	27	35	7	39	32	9	25	32	5	15	43	21	46	16	46	1	35	16	3	42
189	47	25	33	18	20	47	19	44	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	7	16	49	24	46	18	43	4	35	15	2	41
190	48	23	37	17	25	43	13	24	30	40	31	8	35	39	4	40	28	35	7	38	31	9	25	32	5	14	41	27	46	5	44	5	21	15	4	44
191	48	29	34	17	20	47	19	45	31	42	31	17	35	39	14	47	29	35	8	39	32	8	25	32	1	16	49	25	48	18	44	4	35	13	1	41
192	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42
193	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42
194	48	23	37	18	25	43	13	24	30	40	31	8	35	39	4	40	28	34	6	38	31	9	25	32	5	14	41	27	46	5	47	5	21	13	4	44
195	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	40	32	9	25	32	5	15	42	24	46	16	46	3	35	16	3	42
196	46	26	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	32	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41
197	48	26	33	18	20	47	19	42	30	40	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	40	4	35	13	1	41
198	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	41	24	46	16	46	2	35	16	4	42
199	48	28	36	20	28	41	13	46	29	40	28	10	35	39	26	47	25	34	5	39	32	8	25	32	2	15	40	24	46	20	41	2	29	16	9	43
200	47	28	36	18	28	43	18	45	31	40	30	18	35	39	26	48	25	34	5	39	32	8	23	32	5	14	41	25	46	16	42	5	20	16	3	44
201	50	28	34	17	15	47	15	40	33	41	28	9	35	39	13	40	29	35	7	39	31	9	25	32	5	14	42	24	46	16	43	1	33	14	5	44
202	48	28	36	18	28	43	14	44	31	40	29	18	35	39	26	48	28	34	6	39	32	8	24	32	5	14	41	25	46	16	41	5	20	16	3	44
203	46	29	33	18	20	47	17	48	29	42	31	9	35	39	16	40	29	35	7	39	31	9	25	32	5	14	41	24	44	16	46	4	35	16	4	44
204	41	29	33	17	19	46	17	44	30	41	31	6	35	39	13	40	29	35	6	39	32	9	25	32	5	15	42	25	46	16	46	3	35	16	3	42
205	43	26	33	18	20	45	18	46	30	40	31	9	35	39	13	47	29	35	8	39	31	8	25	32	5	16	48	25	43	10	44	4	35	13	5	41
206	49	29	37	23	28	44	13	46	33	40	29	10	33	39	27	46	25	35	5	42	32	8	24	32	2	15	40	24	46	19	42	5	29	16	9	43
207	47	28	36	18	28	43	14	45	31	40	29	18	35	39	26	48	25	34	6	39	32	8	23	32	5	14	41	25	46	16	43	5	20	16	1	44
208	48	27	33	17	20	47	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41
209	47	28	36	18	28	43	18	45	31	40	30	18	35	39	26	48	25	34	5	39	32	8	23	32	5	14	41	25	46	16	42	5	19	16	3	44
210	41	29	33	17	20	46	17	44	30	41	31	6	35	39	13	40	29	35	7	39	32	9	25	32	5	15	41	24	45	16	46	1	35	13	3	42
211	48	27	33	17	20	47	19	42	30	42	31	17	35	39	14	48	29	35	8	39	32	8	25	32	1	16	45	24	48	18	37	4	35	13	1	41
212	48	28	36	20	28	41	13	46	32	40	29	10	35	39	26	47	25	34	5	39	32	8	24	32	2	15	40	24	46	20	41	2	29	16	9	42
213	48																																			

Table A5.2 Non-dominated transmission maintenance schedules solutions (starting week).

Individual	LIN 1-2	LIN 1-3	LIN 1-5	LIN 2-4	LIN 2-6	LIN 3-9	TR A 3-	LIN 4-9	LIN 5-6	LIN 7-8	LIN 8-9	LIN 8-A	TR A 9-	TRA 10-	LIN 10-11	LIN 11-14	LIN 12-13	LIN 13-23	LIN 14-16	LIN 15-16	LIN 15-24	LIN 16-17	LIN 16-19	LIN 17-22	LIN 18-21	LIN 21-18	LIN 21-20	LIN 20-23	LIN 20-23	LIN 21-	LIN 17-18					
1	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	8	40	0	0	2	27	0	0	0	35	37	22	15	29	0	45	48	48	0
2	31	50	0	35	0	24	15	0	23	0	11	14	51	52	32	8	50	13	36	0	0	2	24	0	0	0	35	33	22	15	29	0	45	48	48	0
3	31	50	0	35	0	25	15	0	25	0	12	15	51	52	32	3	48	14	39	0	0	3	22	0	0	0	35	33	20	14	29	0	39	48	47	0
4	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	47	0
5	32	50	0	35	0	24	13	0	23	0	11	14	51	52	32	7	50	13	37	0	0	2	24	0	0	0	35	31	22	15	29	0	45	48	48	0
6	32	44	0	40	0	26	12	0	27	0	9	18	51	48	30	5	46	5	40	0	0	1	19	0	0	0	35	31	9	15	29	0	46	48	46	0
7	24	46	0	34	0	23	15	0	28	0	11	15	49	49	36	4	48	1	40	0	0	2	19	0	0	0	35	38	9	15	29	0	38	48	47	0
8	32	45	0	35	0	18	15	0	25	0	11	14	48	49	33	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	47	0
9	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	47	0
10	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	7	37	0	0	2	27	0	0	0	35	37	8	16	32	0	41	49	46	0
11	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	46	0
12	24	46	0	33	0	23	15	0	28	0	11	15	49	49	36	4	48	1	40	0	0	2	19	0	0	0	35	38	9	15	29	0	40	48	47	0
13	32	44	0	40	0	26	12	0	27	0	8	18	51	49	30	6	47	7	40	0	0	1	19	0	0	0	35	31	9	15	29	0	46	48	47	0
14	32	50	0	36	0	24	15	0	27	0	11	15	46	49	34	5	50	6	42	0	0	9	20	0	0	0	35	33	8	16	32	0	41	50	48	0
15	32	50	0	34	0	24	15	0	27	0	11	14	47	49	31	5	49	7	42	0	0	9	21	0	0	0	35	33	11	16	29	0	42	48	47	0
16	23	47	0	31	0	23	20	0	27	0	12	18	51	49	32	5	48	2	40	0	0	2	22	0	0	0	35	38	10	14	29	0	46	49	47	0
17	21	47	0	31	0	25	20	0	28	0	12	18	51	49	32	4	48	2	40	0	0	2	22	0	0	0	35	38	10	14	29	0	41	48	47	0
18	32	44	0	40	0	26	12	0	27	0	9	18	51	44	33	5	48	7	40	0	0	2	19	0	0	0	35	31	12	15	30	0	46	48	46	0
19	23	47	0	31	0	23	20	0	27	0	12	18	51	49	32	5	48	2	40	0	0	2	22	0	0	0	35	38	10	13	29	0	38	48	48	0
20	21	45	0	27	0	23	17	0	27	0	11	14	48	49	31	6	48	3	39	0	0	3	22	0	0	0	35	36	10	15	29	0	41	49	47	0
21	21	43	0	27	0	23	17	0	27	0	14	18	51	49	31	6	48	1	39	0	0	2	22	0	0	0	33	38	10	15	27	0	40	47	47	0
22	23	47	0	31	0	23	20	0	27	0	12	18	51	49	32	5	48	2	40	0	0	3	22	0	0	0	35	38	10	14	29	0	46	49	47	0
23	21	45	0	27	0	23	16	0	27	0	12	18	52	49	32	6	48	3	40	0	0	2	24	0	0	0	35	38	10	15	29	0	41	51	47	0
24	32	43	0	35	0	17	15	0	25	0	11	14	48	52	31	6	48	9	41	0	0	2	26	0	0	0	31	37	10	16	27	0	44	49	47	0
25	31	50	0	35	0	24	15	0	23	0	11	14	51	52	32	7	50	13	36	0	0	2	24	0	0	0	35	33	22	15	29	0	45	48	48	0
26	31	50	0	36	0	24	15	0	25	0	12	15	51	52	32	3	48	16	39	0	0	2	21	0	0	0	35	35	20	15	29	0	39	48	47	0
27	31	50	0	35	0	24	15	0	23	0	11	14	51	52	32	8	50	13	36	0	0	2	24	0	0	0	35	33	22	15	29	0	45	48	48	0
28	30	49	0	35	0	24	12	0	22	0	10	14	51	52	33	4	50	13	39	0	0	2	22	0	0	0	35	33	20	16	29	0	42	50	48	0
29	32	44	0	40	0	26	12	0	27	0	9	18	51	48	30	5	46	7	40	0	0	1	19	0	0	0	35	31	9	15	29	0	46	49	46	0
30	32	44	0	40	0	26	12	0	27	0	9	18	51	48	30	5	46	7	40	0	0	1	19	0	0	0	35	31	9	15	29	0	46	49	46	0
31	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	46	0
32	32	45	0	35	0	18	15	0	25	0	11	14	48	49	33	6	48	8	40	0	0	2	27	0	0	0	35	37	8	16	32	0	41	50	47	0
33	32	45	0	35	0	18	15	0	25	0	11	14	48	49	32	5	48	7	40	0	0	1	27	0	0	0	35	37	7	16	32	0	41	50	47	0
34	30	47	0	38	0	23	14	0	28	0	8	12	50	45	32	5	41	9	41	0	0	4	23	0	0	0	35	33	10	13	29	0	42	49	46	0
35	24	46	0	34	0	23	15	0	28	0	12	15	49	49	36	4	48	1	40	0	0	2	19	0	0	0	35	38	9	16	29	0	41	48	47	0
36	32	44	0	40	0	26	12	0	28	0	8	18	51	49	30	6	47	7	40	0	0	1	19	0	0	0	35	31	9	15	29	0	46	49	47	0
37	32	50	0	37	0	24	15	0	27	0	11	15	46	49	35	5	50	6	44	0	0	9	20	0	0	0	35	33	8	16	23	0	41	49	47	0
38	32	48	0	34	0	24	15	0	27	0	11	14	45	49	31	5	49	6	42	0	0	10	20	0	0	0	35	32	10	16	29	0	40	47	47	0
39	21	48	0	27	0	23	14	0	27	0	12	18	52	49	32	5	48	3	40	0	0	2	24	0	0	0	35	38	11	15	29	0	40	48	46	0
40	21	47	0	31	0	25	20	0	28	0	12	18	51	49	32	5	48	2	40	0	0	2	22	0	0	0	35	38	10	14	29	0	39	48	47	0
41	21	47	0	31	0	25	20	0	28	0	12	18	51	49	32	4	48	2	40	0	0	2	22	0	0	0	35	38	10	14	29	0	41	48	47	0
42	21	45	0	27	0	23	17	0	27	0	12	16	50	48	38	6	48	4	39	0	0	3	22	0	0	0	35	40	10	15	29	0	40	49	48	0
43	21	45	0	27	0	23	17	0	27	0	12	18	51	48	40	6	48	3	39	0	0	2	22	0	0	0	35	40	10	15	29	0	41	49	47	0
44	21	45	0	27	0	23	17	0	27	0	12	18	51	48	38	6	48	4	39	0	0	2	22	0	0	0	35	40	10	15	29	0	41	49	47	0
45	21	43	0	27	0	23	17	0	27	0	14	18	51	49	31	6	48	1	39	0	0	2	22	0	0	0	33	38	10	15	27	0	40	47	47	0
46	32	49	0	35	0	18	15	0	25	0	11	16	47	49	32	3	47	10	39	0	0	2	27	0	0	0	35	37	11	15	27	0	40	48	47	0
47	31	50	0	35	0	25	15	0	25	0	12	15	51	52	32	3	48	14	39	0	0	3	22	0	0	0	35	33	20	14	29	0	39	48	47	0
48	30	49	0	35	0	24	12	0	22	0	10	15	52	52	32	4	50	13	39	0	0	2	28	0	0											

**Table A5.2 (Cont.) Non-dominated transmission maintenance schedules solutions
(starting week).**

Individual	LIN 1-2	LIN 1-3	LIN 1-5	LIN 2-4	LIN 2-6	LIN 3-9	TRA 3-24	LIN 4-9	LIN 5-10	LIN 6-10	LIN 7-8	LIN 8-9	LIN 8-10	TRA 9-11	TRA 9-12	TRA 10-11	TRA 10-12	LIN 11-13	LIN 11-14	LIN 12-13	LIN 12-23	LIN 13-23	LIN 14-16	LIN 15-16	LIN 21 A	LIN 21 B	LIN 15-24	LIN 16-17	LIN 16-19	LIN 17-22	LIN 18 A	LIN 18 B	LIN 19 A	LIN 19 B	LIN 20 A	LIN 20 B	LIN 23 A	LIN 23 B	LIN 21-22	LIN 17-18
229	31	50	0	36	0	24	15	0	25	0	0	12	15	51	52	32	3	48	14	39	0	0	0	2	21	0	0	0	35	35	20	15	29	0	45	48	48	0		
230	31	50	0	36	0	24	15	0	25	0	0	12	15	51	52	32	3	48	8	39	0	0	0	2	21	0	0	0	35	35	20	15	29	0	39	48	47	0		
231	24	47	0	35	0	23	15	0	28	0	0	11	15	51	49	36	4	48	1	40	0	0	0	2	19	0	0	0	35	38	9	15	29	0	38	48	47	0		
232	32	44	0	40	0	26	12	0	27	0	0	9	18	51	48	31	5	46	7	40	0	0	0	1	19	0	0	0	35	31	9	15	29	0	46	49	46	0		
233	32	45	0	35	0	18	15	0	25	0	0	11	14	48	49	32	6	48	7	40	0	0	0	5	27	0	0	0	35	37	7	16	32	0	41	50	47	0		
234	31	44	0	40	0	27	12	0	27	0	0	9	18	51	48	30	5	46	7	40	0	0	0	1	19	0	0	0	35	31	9	15	29	0	46	49	47	0		
235	32	50	0	37	0	24	15	0	27	0	0	11	15	46	49	35	5	50	6	44	0	0	0	9	20	0	0	0	35	33	8	16	23	0	41	49	47	0		
236	32	50	0	36	0	24	15	0	27	0	0	11	15	46	49	35	5	50	6	42	0	0	0	9	20	0	0	0	35	33	8	16	23	0	41	48	47	0		
237	21	48	0	27	0	23	14	0	27	0	0	12	18	52	49	32	5	48	3	40	0	0	0	2	24	0	0	0	35	38	11	15	29	0	40	48	47	0		
238	31	44	0	40	0	26	12	0	27	0	0	9	18	51	48	30	5	46	7	40	0	0	0	1	19	0	0	0	35	31	9	15	29	0	46	50	47	0		
239	32	50	0	34	0	24	15	0	27	0	0	11	15	47	49	31	5	49	4	42	0	0	0	9	20	0	0	0	35	33	10	16	27	0	41	48	47	0		
240	32	50	0	37	0	24	15	0	27	0	0	11	15	46	49	35	5	50	6	44	0	0	0	9	20	0	0	0	35	33	8	16	23	0	41	47	47	0		
241	32	49	0	35	0	18	15	0	25	0	0	11	16	47	49	32	3	47	10	39	0	0	0	2	27	0	0	0	35	37	11	15	27	0	40	48	46	0		
242	31	50	0	34	0	24	15	0	27	0	0	11	14	47	49	31	5	49	6	42	0	0	0	9	20	0	0	0	35	33	10	16	29	0	42	47	46	0		
243	21	46	0	27	0	23	17	0	27	0	0	12	16	51	48	38	6	48	4	39	0	0	0	3	22	0	0	0	35	40	10	15	29	0	40	49	48	0		
244	30	47	0	38	0	23	14	0	28	0	0	7	12	50	45	32	5	41	9	41	0	0	0	4	24	0	0	0	35	33	10	13	29	0	43	49	46	0		
245	23	47	0	31	0	23	20	0	27	0	0	12	18	51	49	32	5	48	2	40	0	0	0	2	22	0	0	0	35	38	10	14	29	0	46	49	47	0		
246	32	46	0	35	0	18	15	0	25	0	0	11	16	47	48	32	5	47	10	39	0	0	0	1	27	0	0	0	35	40	11	17	29	0	44	47	47	0		
247	32	44	0	40	0	26	12	0	27	0	0	9	18	51	44	33	5	48	7	40	0	0	0	2	19	0	0	0	35	31	12	13	29	0	46	48	48	0		
248	32	45	0	35	0	18	15	0	25	0	0	11	14	48	49	32	6	48	7	40	0	0	0	1	27	0	0	0	35	37	7	16	32	0	41	50	47	0		
249	30	49	0	35	0	24	12	0	24	0	0	10	14	52	52	33	4	49	13	39	0	0	0	2	22	0	0	0	35	33	20	16	29	0	42	48	47	0		

Note. - Individual highlighted corresponds to the best maintenance schedule found using TOPSIS.

Table A5.3 Objective Functions' values for non-dominated solutions.

Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Adequacy Index	System Operation Cost (US\$)	Profits (US\$)			Individual	Reliability Index	System Operation Cost (US\$)	Profits (US\$)		
			GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3				GENCO 1	GENCO 2	GENCO 3
1	61.33%	319.465,101.4	228,825,218.9	128,821,854.9	111,657,215.2	63	59.20%	321,628,593.1	257,394,625.0	153,908,117.2	128,362,204.0	125	60.96%	320,685,073.8	248,333,984.3	144,982,112.1	125,182,998.3	187	60.07%	323,283,467.3	251,692,537.0	147,400,600.6	128,754,153.7
2	61.33%	321,683,281.6	242,395,419.5	137,697,941.8	119,424,210.4	64	58.43%	321,301,463.0	257,636,050.9	151,908,950.4	129,282,013.3	126	60.19%	321,794,470.5	254,228,400.2	148,775,683.8	125,914,046.3	188	60.75%	319,464,255.5	245,802,730.4	141,867,298.5	121,904,523.9
3	61.54%	323,638,838.0	234,003,066.2	134,565,350.9	115,419,327.5	65	61.05%	319,269,517.8	240,096,185.0	139,521,744.9	119,129,406.6	127	60.09%	323,182,157.5	251,564,505.8	147,161,938.4	128,727,095.9	189	59.96%	321,510,813.0	257,422,000.0	152,480,698.0	126,883,448.5
4	61.09%	319,638,838.3	248,290,539.3	144,277,792.2	124,818,107.8	66	61.36%	322,386,989.0	239,412,671.3	135,571,929.8	117,689,862.5	128	58.66%	320,688,585.0	233,709,888.3	137,862,000.0	117,417,942.7	190	61.33%	321,259,653.0	242,312,008.0	137,670,695.0	119,360,221.1
5	61.28%	321,856,968.7	245,186,196.8	139,183,668.6	120,972,720.0	67	61.50%	322,615,931.9	235,963,061.5	135,779,272.5	116,665,217.4	129	59.14%	319,259,258.8	240,768,211.0	139,644,573.2	121,181,685.2	191	58.65%	321,675,068.0	258,152,209.3	152,475,189.1	130,126,107.3
6	59.76%	319,001,802.3	240,041,877.5	135,712,369.3	114,400,547.4	68	61.55%	322,083,464.2	228,037,652.5	130,301,364.7	112,780,060.0	130	58.90%	320,914,278.0	250,768,233.1	148,660,358.3	126,397,524.5	192	60.96%	320,644,152.1	248,390,688.6	144,990,576.0	125,113,569.2
7	60.67%	319,483,384.1	244,642,420.1	142,736,052.6	121,396,734.2	69	61.34%	321,798,848.0	243,110,982.1	138,562,891.8	120,065,235.7	131	58.72%	320,861,479.2	251,053,991.0	150,270,620.0	126,799,418.6	193	60.90%	320,722,080.1	249,836,363.0	145,485,489.8	125,895,550.6
8	60.90%	319,573,756.9	247,838,211.1	144,288,989.4	125,104,535.7	70	59.81%	319,009,125.8	239,116,796.3	137,240,132.4	114,674,180.9	132	58.67%	320,841,941.8	254,079,060.6	150,014,021.0	130,043,970.6	194	61.28%	321,886,667.1	245,124,833.0	139,181,709.1	121,006,535.5
9	60.87%	319,688,731.9	251,415,872.9	146,218,391.2	126,496,122.4	71	59.82%	319,128,352.5	241,228,024.7	139,491,611.3	116,961,560.4	133	59.10%	321,146,819.7	256,624,773.2	152,234,626.9	127,901,025.4	195	60.75%	319,561,689.7	248,877,418.4	144,153,156.5	125,241,922.7
10	60.95%	320,128,773.5	248,298,032.4	144,953,177.6	125,096,063.6	72	58.65%	318,659,980.6	230,825,321.7	129,855,163.1	116,504,968.3	134	58.35%	320,988,192.2	256,731,824.7	151,242,170.7	127,989,659.1	196	60.08%	323,322,964.5	251,630,978.7	147,375,643.9	128,824,993.6
11	60.39%	321,950,592.1	250,819,317.5	147,281,874.2	125,302,846.3	73	60.87%	319,661,657.4	249,413,134.7	145,105,012.8	125,885,825.9	135	61.29%	319,731,504.4	234,866,536.3	133,838,842.6	115,211,428.0	197	58.81%	320,816,127.5	254,163,419.0	150,172,690.1	127,912,794.0
12	60.15%	323,314,221.8	251,458,285.3	147,243,105.0	128,764,654.6	74	60.94%	320,160,658.8	248,319,513.5	144,951,145.5	125,084,723.4	136	61.34%	321,744,171.1	239,705,417.4	137,991,218.5	118,351,307.9	198	60.90%	319,570,239.2	247,790,016.0	144,289,161.2	125,146,522.6
13	59.73%	319,205,426.6	242,846,583.0	140,041,371.1	117,974,084.2	75	60.09%	321,278,162.6	253,109,981.8	147,001,010.2	124,812,768.2	137	61.41%	322,548,799.4	239,544,338.6	136,874,824.0	118,847,480.5	199	59.11%	319,396,575.4	242,745,220.8	141,061,903.9	122,389,789.6
14	58.38%	318,873,441.2	241,637,970.5	135,880,760.3	119,915,915.7	76	60.15%	323,273,134.8	251,493,830.1	147,254,908.6	128,713,030.6	138	59.81%	318,965,624.5	234,927,874.3	132,239,212.5	111,939,855.4	200	59.73%	319,206,704.2	242,847,357.5	140,041,688.8	117,974,314.1
15	58.98%	319,262,930.4	239,895,189.5	139,715,600.7	122,237,385.1	77	59.73%	319,217,059.7	242,859,984.3	140,399,085.2	117,979,350.8	139	59.81%	319,164,453.1	241,140,177.8	139,467,724.8	116,974,224.2	201	61.28%	319,543,069.9	232,847,014.3	131,327,757.3	113,498,558.8
16	59.22%	320,857,404.6	249,520,854.8	148,764,943.8	124,900,292.3	78	58.63%	318,621,539.4	239,014,278.0	134,945,797.4	119,782,472.4	140	59.24%	318,821,806.4	235,969,944.4	135,131,160.5	117,286,341.4	202	60.19%	319,169,363.0	239,305,988.7	138,074,750.0	115,885,090.8
17	59.96%	321,509,153.7	257,430,532.1	152,481,308.6	126,892,006.7	79	59.00%	319,104,559.4	238,227,599.4	138,994,903.6	120,759,808.4	141	60.88%	319,676,014.2	248,934,111.7	144,514,193.1	125,428,387.0	203	61.05%	319,196,434.4	240,161,321.9	139,530,367.5	119,054,653.0
18	59.94%	321,981,426.8	258,552,236.6	153,216,468.2	127,583,196.4	80	60.32%	319,740,398.0	250,164,169.3	146,642,754.0	124,529,362.1	142	60.97%	320,698,432.2	242,352,700.6	144,993,733.9	125,188,254.9	204	60.88%	319,722,584.5	249,319,562.1	145,088,758.9	125,895,210.8
19	58.34%	320,782,007.2	254,646,138.7	148,838,021.7	126,471,187.8	81	59.91%	321,514,678.3	257,678,001.1	152,616,776.8	126,992,988.1	143	60.18%	321,335,128.3	253,497,870.8	148,135,362.7	123,312,632.0	205	60.08%	323,015,190.9	249,033,606.7	148,648,777.0	127,023,861.1
20	58.87%	320,346,146.7	250,986,123.6	149,838,448.5	126,408,471.1	82	58.71%	320,849,787.0	250,880,551.1	150,241,656.0	126,859,134.3	144	60.15%	323,271,834.3	251,503,306.7	147,243,815.1	128,719,434.7	206	58.65%	318,652,288.8	230,822,151.1	129,855,602.4	116,500,966.1
21	58.78%	320,844,789.7	252,084,691.0	148,801,947.8	129,192,416.3	83	58.78%	320,871,427.7	250,869,736.5	147,945,500.1	125,459,295.7	145	58.63%	318,785,766.9	239,067,712.8	134,949,351.8	119,479,446.8	207	59.80%	319,204,226.4	241,122,269.9	139,467,483.2	117,981,361.3
22	59.14%	321,198,697.0	254,297,195.8	151,741,239.3	128,828,299.8	84	59.22%	321,367,628.2	253,793,192.5	151,117,432.1	127,883,431.7	146	59.02%	319,180,302.2	237,860,587.0	138,081,463.8	120,141,916.2	208	58.72%	320,861,252.6	250,949,385.0	150,231,828.1	126,759,355.9
23	58.44%	321,380,670.8	260,722,432.9	154,095,839.2	129,791,641.7	85	58.35%	320,962,778.0	257,176,058.1	151,632,905.1	128,362,063.2	147	58.60%	320,648,647.1	251,666,822.8	146,190,093.7	123,824,636.6	209	59.74%	319,172,814.7	240,486,017.0	138,964,624.4	117,381,561.7
24	61.07%	319,390,646.1	243,089,092.4	142,536,847.3	121,277,259.0	86	61.29%	319,689,452.1	234,899,310.0	133,899,250.9	115,171,075.7	148	58.72%	320,812,115.2	251,008,843.8	150,235,455.6	126,716,398.9	210	60.96%	320,716,364.2	248,373,242.9	144,996,957.8	125,322,925.0
25	61.33%	321,726,887.6	242,352,124.3	137,697,218.7	119,451,356.2	87	61.50%	322,656,680.2	235,920,303.4	136,708,250.0	116,703,920.2	149	58.61%	320,666,051.2	252,266,866.8	146,855,876.1	126,946,262.9	211	58.72%	320,914,773.3	251,001,990.9	150,262,154.8	126,889,368.0
26	61.50%	322,643,642.7	235,893,139.6	135,788,860.7	116,663,724.6	88	61.36%	322,431,174.2	239,372,789.5	135,571,350.0	117,733,101.4	150	59.20%	321,187,909.1	254,788,911.3	151,724,693.3	128,320,837.4	212	59.03%	319,240,819.1	240,105,744.4	130,620,093.4	121,709,115.3
27	61.33%	321,616,258.0	239,225,288.2	135,519,706.0	118,617,318.2	89	59.80%	319,023,676.0	239,109,809.1	137,251,640.1	114,675,035.2	151	59.17%	321,400,902.8	258,755,964.9	154,190,517.7	123,250,510.0	213	58.64%	320,900,448.8	255,455,497.8	150,992,459.8	130,166,055.4
28	61.39%	322,520,891.2	242,024,692.9	138,156,715.3	119,596,771.9	90	60.19%	319,507,634.4	239,241,972.4	138,043,087.3	115,809,941.3	152	61.05%	319,220,560.5	240,108,609.7	139,521,998.0	119,079,952.6	214	59.23%	318,971,055.2	235,804,241.5	133,127,109.3	117,428,781.0
29	59.81%	319,022,122.2	239,106,839.9	137,251,378.5	114,670,298.9	91	60.90%	319,603,823.9	247,798,228.5	144,294,605.5	125,131,662.5	153	61.41%	322,442,517.7	238,338,313.4	135,798,938.7	118,653,508.4	215	61.54%	322,081,925.4	240,056,231.4	131,936,961.3	114,716,571.4
30	59.83%	319,199,134.5	241,118,960.8	139,469,892.1	117,007,258.3	92	60.88%	319,694,678.7	249,332,484.8	145,076,124.4	125,903,802.7	154	61.39%	322,685,675.4	241,327,280.2	137,970,620.2	119,639,048.3	216	61.35%	319,643,945.2	234,665,898.0	133,386,072.4	111,816,725.5
31	60.43%	319,449,536.0	245,849,584.8	142,797,257.2	124,862,170.5	93	60.96%	320,636,686.0	248,400,886.8	144,993,351.4	125,118,009.3	155	59.76%	319,065,163.9	239,983,076.4	135,728,241.7	114,450,761.9	217	60.10%	323,268,619.9	251,286,960.4	147,136,985.8	128,582,037.5
32	60.87%	319,682,323.8	249,391,574.0	145,102,553.1	125,904,980.0	94	60.18%	321,332,826.5	253,503,358.1	148,139,301.3	125,735,390.5	156	59.81%	319,164,080.0	241,140,136.9	139,467,721.0	116,974,027.4	218	59.01%	319,220,719.2	240,158,044.4	139,624,517.6	121,682,954.6
33	60.96%	320,685,716.7	248,350,770.6	144,995,382.7	125,192,057.7	95	60.17%	323,125,816.2	249,915,969.2	146,323,064.4	128,069,540.0	157	59.77%	319,264,045.2	241,443,685.0	139,450,328.8	120,311,985.1	219	59.02%	319,254,434.2	240,119,777.4	139,620,452	

Table A5.8 Weekly electricity nodal prices for best maintenance schedule solution (US\$/MWh).

BLOCK	NODE	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52						
HIGH	1	24.6	24.7	24.6	24.6	50.5	24.5	24.5	50.5	24.5	24.5	24.7	25.6	24.7	24.6	23.5	24.5	24.7	24.9	29.9	51.0	50.3	50.1	25.5	25.4	30.1	50.6	51.0	25.5	25.8	24.7	25.0	26.0	25.3	25.8	30.4	25.0	25.0	29.8	51.0	25.7	25.7	25.7	30.7	25.5	25.7	25.6	25.5	25.6	30.5	30.5	30.6	30.5						
	2	24.7	24.7	24.6	24.6	50.5	24.5	24.5	50.5	24.5	24.6	24.7	25.6	24.7	24.6	23.6	24.5	24.7	25.0	29.9	51.1	50.4	50.1	25.6	25.5	30.1	50.7	51.1	25.5	25.8	24.7	25.0	26.0	25.6	25.8	30.5	25.0	25.1	29.9	50.9	25.6	25.7	25.7	30.7	25.5	25.7	25.6	25.5	25.6	30.5	30.6	30.6	30.6						
	3	24.3	24.2	24.2	24.2	50.6	24.8	24.7	50.8	24.8	24.8	24.8	26.9	24.8	24.7	23.6	24.7	24.8	24.7	29.7	50.8	50.5	50.5	25.2	25.2	30.0	51.2	51.3	25.2	25.3	24.2	24.3	25.3	25.1	25.2	30.0	25.1	25.1	29.9	51.3	25.2	25.2	25.2	30.0	25.2	25.2	25.2	25.2	25.3	30.1	30.1	30.2	30.1						
	4	25.1	25.2	25.1	25.1	51.1	25.0	24.9	51.5	25.0	25.1	25.2	26.0	25.3	25.1	24.1	25.0	25.1	25.2	30.1	51.5	51.1	51.0	26.0	25.9	30.6	51.2	52.3	25.9	26.0	25.0	25.3	26.4	26.3	26.1	30.8	25.5	25.6	30.5	52.2	26.4	26.4	25.9	30.9	26.1	26.1	25.9	25.9	26.0	31.0	31.0	31.0	31.0						
	5	25.1	25.1	25.1	25.0	51.6	24.9	24.8	51.4	25.0	25.0	25.2	25.9	25.2	25.0	24.0	24.9	25.1	25.1	30.1	51.4	50.9	50.8	25.9	25.8	30.5	51.9	52.3	26.1	26.0	24.9	25.3	26.1	25.9	26.0	30.7	25.5	25.5	30.3	52.0	26.0	26.0	25.9	30.9	25.8	26.1	25.9	25.9	25.9	30.9	31.0	31.0	31.0						
	6	25.4	25.5	25.4	25.4	52.5	25.2	25.2	52.2	25.3	25.3	25.3	28.7	27.6	27.4	26.6	26.9	26.3	26.1	24.2	27.7	47.7	47.8	47.8	26.8	26.7	32.5	52.4	26.3	26.7	26.8	26.4	26.7	26.3	31.1	26.0	25.9	30.8	52.0	26.3	26.4	26.2	31.2	26.1	26.4	26.2	26.2	26.3	31.3	31.4	31.3	31.3							
	7	25.3	25.3	25.3	25.3	52.5	25.2	25.2	52.2	25.3	25.3	25.3	28.7	27.6	27.4	26.6	26.9	26.3	26.1	24.2	27.7	47.7	47.8	47.8	26.8	26.7	32.5	52.4	26.3	26.7	26.8	26.4	26.7	26.3	31.1	26.0	25.9	30.8	52.0	26.3	26.4	26.2	31.2	26.1	26.4	26.2	26.2	26.3	31.3	31.4	31.3	31.3							
	8	27.5	27.5	27.4	27.4	50.5	24.6	24.8	50.4	24.3	24.8	28.5	27.3	27.0	26.4	26.2	26.2	26.2	25.1	29.1	50.1	50.2	50.2	26.7	26.4	30.1	53.6	53.5	26.6	26.5	27.0	27.4	25.3	25.7	26.4	29.9	25.1	25.2	29.8	53.1	27.7	27.8	26.8	31.3	27.0	27.8	27.1	27.1	26.8	31.7	31.8	32.0	31.8						
	9	24.7	24.7	24.7	24.7	49.9	24.5	24.5	50.8	24.7	24.7	24.9	25.6	25.0	24.8	23.8	24.6	24.7	24.5	29.3	50.2	50.0	49.9	25.5	25.4	30.0	51.6	51.6	25.4	25.4	24.4	24.8	25.5	25.4	25.6	30.2	25.2	25.3	30.1	51.7	25.6	25.6	25.3	30.1	25.7	25.5	25.3	25.4	25.4	30.2	30.3	30.7	30.3						
	10	24.8	24.9	24.9	24.8	51.4	24.7	24.6	51.1	24.8	24.8	25.2	25.6	25.1	24.9	23.9	24.7	24.8	24.6	29.5	50.5	50.2	50.2	25.7	25.6	30.2	51.9	51.9	25.5	25.5	24.6	25.0	25.7	26.0	25.8	30.4	25.4	25.4	30.2	52.0	25.7	25.8	25.5	30.3	25.4	25.7	25.5	25.5	25.5	30.5	30.5	30.4	30.5						
	11	24.6	24.6	24.6	24.6	50.4	24.5	24.5	50.9	24.7	24.7	24.9	25.4	25.0	24.8	23.7	24.5	24.7	24.5	29.3	50.2	50.0	49.9	25.5	25.4	30.0	51.5	51.5	25.3	25.3	24.4	24.8	25.5	25.2	25.5	30.2	25.2	25.2	30.1	51.6	25.4	25.4	25.3	30.1	25.4	25.4	25.3	25.3	25.3	30.4	30.2	30.1	30.2						
	12	24.5	24.6	24.6	24.5	49.5	24.4	24.4	50.6	24.6	24.6	24.8	25.3	24.9	24.7	23.6	24.4	24.6	24.3	29.1	49.9	49.7	49.7	25.4	25.3	29.9	51.4	51.4	25.2	25.3	24.3	24.7	25.5	25.5	25.5	30.1	25.2	25.2	30.0	51.6	25.6	25.6	25.2	30.0	25.0	25.4	25.2	25.2	25.2	25.1	29.9	30.1	30.3	30.1					
	13	24.0	24.1	24.1	24.1	48.9	24.0	23.9	49.0	24.0	24.0	24.3	24.5	24.9	24.5	24.3	23.1	23.9	24.0	23.8	28.5	48.9	48.7	48.6	25.1	25.0	29.5	30.1	30.1	24.9	24.9	23.4	24.4	25.2	25.1	25.2	25.2	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9	24.9					
	14	24.2	24.3	24.3	24.2	50.3	24.5	24.5	50.3	24.6	24.6	24.7	25.0	24.8	24.6	23.5	24.5	24.6	24.4	29.4	29.5	50.2	50.0	49.9	25.2	25.1	29.7	51.1	51.1	25.1	25.1	24.3	25.2	25.0	25.2	29.9	25.0	25.1	29.9	51.3	25.1	25.1	25.0	29.8	25.1	25.1	25.0	25.1	25.1	25.1	25.1	25.1	25.1	25.1	25.1	25.1			
	15	23.4	23.4	23.4	23.4	49.4	24.2	24.2	49.5	24.2	24.2	24.2	24.2	24.2	24.1	23.0	24.1	24.2	24.0	28.8	49.4	49.1	49.1	24.4	24.4	24.1	49.9	49.9	24.5	24.5	23.5	23.4	24.4	24.4	24.1	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4					
	16	23.5	23.6	23.6	23.5	49.4	24.1	24.2	49.5	24.2	24.2	24.2	24.2	24.1	23.0	24.0	24.1	24.0	28.7	49.3	49.1	49.1	24.4	24.4	24.1	49.9	49.9	24.4	24.4	23.5	23.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5	24.5				
	17	23.1	23.1	23.1	23.1	48.9	23.9	23.9	49.0	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	28.5	48.9	48.8	48.8	24.0	24.0	28.6	49.1	49.1	24.0	24.0	23.1	23.2	24.1	24.0	24.0	28.7	24.0	24.0	28.6	49.2	24.0	24.0	28.6	24.0	24.0	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	
	18	22.9	22.9	22.9	22.9	48.9	23.9	23.9	48.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	28.5	48.9	48.9	48.9	23.9	23.9	28.5	48.9	48.9	23.9	23.9	22.9	22.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	
	19	23.6	23.6	23.6	23.6	49.2	24.1	24.1	49.6	24.2	24.2	24.1	24.3	24.2	24.0	23.0	23.9	24.0	23.9	28.6	49.1	48.9	48.9	24.5	24.5	29.1	49.9	49.9	24.4	24.4	23.5	23.7	24.8	24.8	24.8	29.5	24.7	24.7	29.4	50.7	24.6	24.5	24.5	29.2	24.5	24.5	24.4	24.5	24.4	24.5	24.4	24.5	24.4	24.5	24.4	24.5	24.4	24.5	
	20	23.4	23.5	23.5	23.4	48.7	23.7	23.7	48.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	28.6	48.8	48.8	48.8	24.4	24.4	28.6	49.0	49.0	24.4	24.4	23.5	23.6	24.8	24.8	24.8	29.5	24.8	24.8	29.4	50.8	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4	24.4		
	21	22.9	22.8	22.8	22.9	48.9	23.9	23.9	48.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	28.5	48.8	48.8	48.8	24.0	24.0	28.6	49.1	49.1	24.0	24.0	23.1	23.2	24.1	24.0	24.0	28.7	24.0	24.0	28.6	49.2	24.0	24.0	28.6	24.0	24.0	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9
	22	22.3	22.3	22.3	22.3	47.2	23.2	23.2	47.5	23.2	23.2	23.2	23.2	23.2	23.2	23.1	23.0	23.1	23.1	27.5	47.3	47.3	47.3	23.2	23.2	27.8	47.9	47.9	23.2	23.2	21.8	21.8	22.7	23.4	23.2	27.7	23.3	23.2	27.7	47.5	23.2	23.4	23.4	27.9	23.3	23.3	23.2	23.2	23.2	23.2	23.2	23.2	23.2	23.2	23.2	23.2	23.2		
	23	23.3	23.3	23.3	23.3	48.3	23.7	23.6	48.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	23.9	28.6	48.9	48.8	48.8	24.0	24.0	28.6	49.1	49.1	24.1	24.1	23.0	23.4	24.8	24.7	24.8	29.4	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	24.1	
	24	24.1	24.0	24.0	24.1	50.3	24.6	24.6	50.5	24.6	24.6	24.7	24.2	24.7	24.5	23.5	24.5	24.7	24.5	29.4	50.4	50.1	50.1	25.0	25.0	29.7	50.9	50.9	25.0	25.1	24.0	24.1	25.0	24.9	25.0	29.7	24.9	24.9	29.7	50.9	25.0	25.0	25.0	29.8	25.0	2													

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