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Investigating the Effectiveness of Using GeoGebra Software on Students' Mathematical Proficiency

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A Thesis Submitted in Fulfilment of the Requirements for the Degree of Doctor of Philosophy (PhD)

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Abstract

The main aim of this study is to investigate the effectiveness of using GeoGebra software (GGS)-based pedagogy on students' mathematical proficiency. The National Research Centre's mathematical proficiency strands provide a framework for this study. It aims to compare the results of a mathematical proficiency test completed by two groups of students, namely an experimental group taught using GGS-based pedagogy and a control group taught using a traditional approach. All the mathematical proficiency test items are drawn from the international examination, Trends in International Mathematics and Science Study (TIMSS). The study also seeks to ascertain teachers' views regarding learning and teaching mathematics using GGS.

This research used a mixed method approach. The study deployed four research instruments: three mathematical proficiency tests (MPTs) and a productive deposition questionnaire with students, and interviews and written responses with teachers. This study used non-equivalent group (pre-test and post-test) measures, conducted in two stages. In addition to the MPTs in each stage, which aimed to measure the first four strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning), a questionnaire was administered twice with each group, before and after the tests. These questionnaires aimed to measure the fifth strand of mathematical proficiency (productive disposition). Also, in this study, a group of teachers underwent a Professional Development Course (PDC) for five days on using GGS effectively in classrooms, as well as three rounds of semi-structured interviews: before, during, and after the PDC. The main aim of these interviews was to explore teachers' perceptions of using GGS in the classroom and its effect on students' mathematical proficiency. They were also involved in a group discussion on the fifth day to discuss and write about their experiences using GGS after the PDC (Teachers' Written Responses).

The study found that the impact of GGS-based pedagogy was unstable and varied across differing strands of mathematics proficiency. For example, the results revealed the use of GGS-based pedagogy had a significant effect on students' overall mathematics achievement, procedural fluency, and productive disposition in two units (Numbers and Geometry). In contrast, for strategic competence and adaptive reasoning, using GGS-based pedagogy appeared to have no influence over and above the traditional approach. Regarding the conceptual understanding strand, the results showed that the effect of using GGS-based pedagogy differed from one unit to another. In the case of the first unit (Number), using a

GGS-based pedagogy had a substantial effect on students' understanding of concepts. In contrast, in the second unit (Geometry), the results revealed that using GGS-based pedagogy did not influence students' uptake of the concepts. The results of qualitative analysis showed that the mathematics teachers had positive views of the effectiveness of using GGS as a tool for teaching and learning mathematics. This positive views are presented in five themes: GGS as time/effort saver; GGS's representational capacity in enhancing teaching; GGS as an effective learning tool; GGS as a facilitator of student engagement; and GGS as a supporter of mathematics skills.

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I want to express my heartfelt gratitude to all of my colleagues and friends in the United Kingdom for the wonderful collaboration we have had, and I wish them all the best.

Declaration

I hereby declare that, except where explicit reference is made, the work presented in this thesis is entirely my own and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signature:

Bakri Mohammed Awaji

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List of Abbreviations

ACARA	Australian Curriculum, Assessment and Reporting Authority
AR	Adaptive reasoning
C3D	Cabri 3D
CAS	Computer Algebra System
CCSS	Common Core State Standard
CU	Conceptual understanding
DGS	Dynamic Geometry Software
DMS	Dynamic Mathematics Software
ECSME	Excellence Research Centre of Science and Mathematics Education
ETEC	Education and Training Evaluation Commission
FG	Future Gate
GGS	GeoGebra Software
GSP	Geometer's Sketchpad
KSA	Kingdom of Saudi Arabia
MMR	Mixed method research
MoE	Ministry of Education
MoFA	Ministry of Foreign Affairs
MPT	Mathematical proficiency test
NAEP	National Assessment of Educational Progress
NCSM	National Council of Supervisors of Mathematics
NRC	National Research Council
PD	Productive disposition
PDC	Professional development course
PF	Procedural fluency
PSSM	Principles and Standards for School Mathematics
SC	Strategic competence
TEHC	Tatweer Education Holding Company
TIMSS	Trends in International Mathematics and Science Study

Chapter 1 Introduction

1.1 Introduction

We currently live in an age of technology, in which technological innovations affect every aspect of our daily lives, including the ways in which we teach and learn (Prodromou, 2014). Technology has changed our lives to such a degree that the term “digital illiteracy” has been coined to refer to individuals who do not use digital technologies. It is claimed that the use of technology improves the educational process, as it enables students to be innovative learners and to learn more independently. Teachers should therefore be creative and adjust their teaching approach from individualized instruction to modes that require cooperation and that enable the whole school to become a collaborative environment (Wasserman & Millgram, 2005; Cachia et al., 2010).

According to Heid (1997), one of the consequences of the trend towards teaching in a cooperative way is the development and dissemination of various technological tools designed to be used within education. Since these tools are merely variations of the smartphones and tablets already commonly used by children in their leisure time, students should be permitted to bring new technologies into the classroom. The modern society in which children live and learn is awash with innovative technologies and digital trends and as a consequence technology holds no fears for them (Hohenwarter et al., 2008; Chiu & Churchill, 2016). Many previous studies have explored the effectiveness of new strategies on teaching and learning and fresh approaches that support effective teaching and learning have been designed and developed. As Bwalya (2019) observed, the integration of technology in education has positive effects and other researchers have noted that it plays a key role in modifying the classroom setting, transforming schools, and encouraging more significant and outcome-oriented learning.

More specifically, Wenglinsky (1998) argued that the use of technology in both teaching and learning mathematics can improve thinking skills, while Mistretta (2005) found that it also raises levels of achievement and effectiveness. According to the National Council for Teachers of Mathematics (NCTM, 2000), technological tools are not only of major importance in teaching mathematics, but also affect the type of mathematics taught and improve students’ learning. There are many software programs available that encourage students to become independent learners and to approach subjects in a creative and engaged way. Mathematics is the gateway to technology and the software that can be harnessed to teach and learn the subject gives students the knowledge they need to enter and take part in the world of high technology (Zulnaidi & Zakaria, 2012). As Furner and Marinas (2007) have argued, educators have a duty to prepare students to excel in a world that relies on and values mathematics, science, and technology.

According to Bwalya (2019), the vast body of research produced in recent years regarding the process of teaching and learning has highlighted innovative technological methods aimed at ensuring that both teachers and students benefit from helpful new techniques. For example, Dynamic Mathematics Software (DMS) is an excellent resource for teaching mathematics (Xistouri & Pitta-Pantazi, 2013). One such application, GeoGebra software (GGS), an open-source DMS that focuses on geometry, has been integrated into mathematics courses and has attracted the attention of researchers and teachers alike, since it offers a revolutionary approach to teaching and learning mathematics. This particular software has the features of DMS, Computer Algebra Systems (CAS) and spreadsheets within one package (Hohenwarter et al., 2009). It offers students a virtual environment in which they can view concurrently an algebraic component, such as an equation or a coordinate, and the corresponding geometric features of an object (Preiner, 2008). GGS was designed by Markus Hohenwarter on the basis of specific mathematics criteria and was developed further by programmers from Florida Atlantic University using an approach that allows students to gain a detailed understanding of mathematical facts and theories in a practical manner, inspiring independent learning (Alkhateeb & Al-Duwairi, 2019).

Understanding mathematical ideas is crucial when learning mathematics. According to the NCTM (2000), such understanding requires nurturing and expansion. Zulnaidi and Oktavika (2016) suggest that if students want to gain a comprehensive understanding of mathematical concepts, they should not solely memorize formulae. These scholars found a strong link between technology and student learning, particularly concerning the use of GGS. Hence, the use of technology in general and GGS specifically may lead to improved outcomes for students' learning of mathematics.

This study reflects trends that seek to integrate technology, such as GGS, in the teaching and learning of mathematics. GGS is free and easily accessible dynamic software that is compatible with various platforms, such as iPhones, laptops and personal computers. In addition, its website includes numerous resources that offer a wide range of pre-prepared support for teachers.

This study primarily focuses on the impact of using GGS on students' performance in mathematics, taking the National Research Centre (NRC, 2001) mathematical proficiency strands as a framework to capture the performance. Kilpatrick et al. (2001) define mathematical proficiency as consisting of five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. They describe these five strands as follows (Kilpatrick et al., 2001, p. 116):

- Conceptual understanding – understanding of mathematical concepts, operations, and relations.
- Procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence – the ability to formulate, represent, and solve mathematical problems.
- Adaptive reasoning – the capacity for logical thought, reflection, explanation, and justification.
- Productive disposition – a habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

1.2 Significance of the study

The strategic use of technology strengthens mathematics teaching and learning in a balanced mathematics programme (Dick & Hollebrands, 2011). According to NCTM (2015), teachers and students should have regular access to technologies that promote mathematical sense-making, reasoning, problem-solving, and communication. Efficient teachers optimize the use of technology to improve their students' comprehension, to enhance their interest in the subject and to increase their mathematics skills, delivering superior access to mathematics for all students when they use technology strategically (NCTM, 2015).

Numerous studies (Hohenwarter & Fuchs, 2004; Hohenwarter & Lavicza, 2007; Hohenwarter & Preiner, 2007; Hohenwarter et al., 2008; Diković, 2009; Hohenwarter et al., 2009; Mehanovic, 2011; Zulnaidi & Zakaria, 2012; Aydos, 2015; Gono, 2016; Gafoor & Sarabi, 2016) have assessed the effects of using technology on students' mathematics performance. The majority of these studies and other studies discussed in the literature, to some extent, report positively on the effect of GGS on the teaching and learning of mathematics. These studies have mainly focused on one aspect of mathematics learning, such as learners’ achievement, or the effects of using GGS on skills such as understanding, fluency, problem-solving, and reasoning, or students' attitudes towards the subject.

In contrast to other studies in the field, this research did not solely investigate the effect of using GGS on one aspect of learning mathematics in terms of students' achievement, but rather evaluated the effects of using GGS on different aspects of mathematics learning at the same time. These aspects are related to the NRC’s mathematical proficiency strands, i.e. understanding of concepts, fluency in doing mathematics tasks, reasoning, and students’ attitudes towards mathematics. Focusing on different aspects of mathematics learning can alert teachers to the multifaceted nature of learning the subject (Kilpatrick et al., 2001). This recognizes that success in an overall measure of mathematics achievement or any specific skill, such as understanding concepts or fluency in doing tasks, does not always mean that students are successful in mathematics. Thus, it is essential to know how the students are progressing in different aspects of mathematics learning at the same time to help them improve their learning.

In addition, this study is significant in being conducted in the context of the Kingdom of Saudi Arabia (KSA). Although the use of technology in the KSA is widespread, it is less the case for teaching mathematics, where its introduction has encountered certain difficulties. For example, Al-rwaise (2011) argued that it is not acceptable practice to use technology in mathematics classes. Moreover, teachers in the country lack the experience and willingness to use technology in the mathematics classroom and this, together with their dependence on traditional teaching methods, constitutes additional obstacles to its implementation (Al-Enazy & AL-Mosaad, 2018). This research therefore seeks to promote the integration of technology in Saudi Arabian middle schools and to enhance students’ skills and teachers’ practices in classrooms by offering a training opportunity for Grade 8 mathematics teachers, based on the design and implementation of a Professional Development Course (PDC) to help them master their use of GGS in the classroom. It also aims to provide detailed information concerning teachers’ views of learning mathematics using GGS.

1.3 Aims and research questions

This study has two main aims:

- To investigate the effectiveness of using GGS-based pedagogy to develop students’ mathematical proficiency.

- To ascertain teachers' views regarding the learning and teaching of mathematics using GGS.

To address these research aims, the study employed the following research questions:

Research question A (RQA): Does the use of GGS-based pedagogy have a significant effect on students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition) and achievement compared to the traditional teaching approach?

Research question B (RQB): What are the mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics?

1.4 Overall research design and methods

To achieve the project aims, the study used mixed methods as the principal research methodology. The study used a quasi-experimental approach to investigate the impact of GGS-based pedagogy on the mathematical proficiency of students in Grade 8 (11–14 years), comparing the results of two Grade 8 units (Numbers and Geometry) taught based on the traditional method versus the GGS-based pedagogy. The teachers attended a PDC to be shown how to use GGS in their classrooms. The study used two sets of research instruments. The first set, for students, comprised three mathematical proficiency tests (MPTs), conducted both before and after the intervention in which some students were taught using GGS-based pedagogy, and a productive disposition questionnaire. The second set of instruments, for teachers, comprised interviews and teacher written responses. Quantitative methods were used to assess the proficiency test scores and the responses to the productive disposition questionnaire, whereas a qualitative approach was used to explore the teachers' interviews and their written responses.

1.5 Structure and organization of the thesis

This thesis is structured in eight chapters, including this introductory chapter, as detailed in Table 1-1.

Table 1-1 Structure of the thesis

Chapter	Description
Chapter 1: Introduction	This chapter presents a brief background and introduction to the study, discussing the significance of the study and the research problem. It also provides a brief description of the methodology and methods used in the study, together with the research questions and aims, and the structure of the thesis.
Chapter 2: Saudi Context	This chapter presents the Saudi Arabian context in which the data collection took place. The chapter has five sub-sections: an introduction to the country of Saudi Arabia, the education system in Saudi Arabia, educational developments in the country, Saudi Vision 2030, and the use of technology in Saudi Arabia.
Chapter 3: Conceptual Framework	This chapter discusses the concept of mathematical proficiency and consists of three main sections. The first focuses on the history of mathematical proficiency and competency frameworks, while the second addresses the NRC strands. The last part discussed using of the NRC's mathematical proficiency strands in this study
Chapter 4: Literature Review	This chapter reviews the previous literature relevant to the study, namely the use of technology in teaching mathematics, the role of terachers and PDC in the use of technology, DMS, GGS, the potential effectiveness of using GGS in students' mathematical learning, the views of teachers regarding the use of GGS, and literature concerning mathematical proficiency. It also defines the research gap.
Chapter 5: Methodology and Methods	This chapter presents the research methodology employed to address the research questions, the methods used to conduct data collection and data analysis, and the research instruments and research procedures employed in the study.
Chapter 6: Student-Related Findings	This chapter presents the results of the two student interventions.
Chapter 7: Teacher-Related Findings	This chapter presents the findings of the interviews with teachers and the teachers' written responses.
Chapter 8: Discussion	This chapter discusses the findings of the study presented in Chapters 6 and 7 in relation to the literature reviewed in Chapter 4 to address the research questions.
Chapter 9: Conclusion	This chapter provides the conclusions, together with a discussion of the study's contribution to the field, its implications and limitations, and suggestions and recommendations for further research.

Chapter 2 Saudi Arabian Context

2.1 Introduction

This chapter presents the Saudi Arabian context in which the data collection process took place. The chapter has five sections. The first describes the KSA, including key demographics, such as population information, borders, economy, culture and religion. The second section illustrates the education system in the KSA and explains the oversight of the Ministry of Education (MoE) over all educational stages. The third section presents developments related to education and describes chief initiatives, such as the role of King Abdullah bin Abdulaziz in the Development of Education, the Development of the Mathematics and Science Curricula Project, the Excellence Research Centre for Science and Mathematics Education, and the Education and Training Evaluation Commission. The fourth section presents the Saudi Vision 2030, incorporating education and mathematics learning. The final section illustrates the status of the use of technology in the KSA.

2.2 The Kingdom of Saudi Arabia

Established in 1902 by King Abdulazeez Al-Saud, the Kingdom of Saudi Arabia, the official name for which was first declared by Abdulazeez Ibn Abdurrahman Al-Saud in 1932, is an independent, Muslim and Arab monarchy. Stretching from the Arabian Gulf in the east to the Red Sea in the west, the KSA, as the largest country in the Middle East, occupies 80% of the Arabian Peninsula, making it similar in size to the entirety of Western Europe (Ministry of Foreign Affairs (MoFA), 2017). The KSA covers some 2,250,000 square kilometres (868,730 square miles) and is situated to the south-west of Asia, with the United Arab Emirates, Qatar and Bahrain to the east. The country also has borders with Kuwait, Iraq and Jordan to the north and Yemen and Oman to the south. According to the General Authority of Statistics (GASTAT, 2019), the population of the KSA is 34.2 million (72% Saudi and 28% of other origin). The country's culture is founded on Islam. The KSA, home to approximately 25% of global oil reserves, stands at the crossroads of international trade and has done so for centuries (MoFA, 2017). In December 2005, Saudi Arabia joined the World Trade Organisation as its 149th member and began to open up its economy to the world (MoFA, 2017).

Culture and religion

The KSA is a constitutional monarchy, based on both the Holy Quran (Koran) and Sharia Law. The monarch acts as the head of government and the Council of Ministers, who represent the executive and administrative arms of the state (Al-Salloom, 1989). Saudi Arabia's Islamic background places education as a religious obligation, with the state taking responsibility for it. Every citizen must, in turn, respect their duty to themselves, their community, and Islam.

Under Islamic law girls and boys were traditionally taught in separate school buildings. In 2019–2020, this situation changed and girls and boys can now study together, in the same building, during the first three years of primary school, when they are taught by female teachers (MoE, 2020a).

2.3 Saudi educational system

In 1925, Saudi Arabia established the Directorate of Knowledge to supervise four primary schools, thereby creating a fledgling educational system which eventually saw the Directorate overseeing 323 schools (MoE, 2020a). Currently, the KSA is developing every field and sector and as in other countries, strong emphasis is placed on the importance of general and higher education levels. Those working in the educational field have experienced numerous changes, such as privatization, financing, foreign competition, and greater attention paid to the labour market (MoE, 2020a). Staff in the educational sector oversee the expansion of their remit and ensure the systematic evaluation of every change (MoE, 2020a). In addition, due to local and global changes, staff need to familiarize themselves with implementing and developing schemes, and setting up organizations (MoE, 2020a).

According to the Saudi Ministry of Finance (2020), the Saudi government spent over US$50 billion (approximately 25% of the national budget for 2020) on its educational system and human resources. The Ministry of Finance (2020) aims to expand the Saudi education system by establishing new schools and universities.

2.3.1 Ministry of Education

According Oyaid (2009), the MoE, which dates back to the 1950s under different names, has several objectives relevant to this study:

1. To define and expand the criteria relating to teaching qualifications and upgrade educational competencies.
2. To improve the quality of education.
3. To improve educational outcomes to meet the needs of the community and development goals.
4. To invest in information communication technology (ICT).

The MoE currently has responsibility for all schooling from ages 3 to 18, across the four main educational stages up to and including university. The following sections will consider these stages in detail.

2.3.2 Stages of education

General education

In Saudi Arabia, education is divided into general and higher education, both of which include public and private educational institutions. The former stage is mandatory and free as the government takes responsibility for supplying the teaching environment, textbooks, transport, and facilities. The first stage in the education system is pre-primary, catering for children under six years; it is non-compulsory and takes place in public and community nurseries. The primary stage lasts for six years, and children are accepted when they are five and a half or six years old. It is necessary to have a certificate of completion to go on to middle school. Middle school lasts for three years and requires students to gain a certificate to progress to secondary school, where they will spend another three years. Secondary schools include vocational schools and students have to pass all three years to gain a certificate which will allow them to apply to university. The government provides schools for students who have disabilities or those who are outstanding (gifted students).

Higher education

The majority of provinces in Saudi Arabia have access to the country's 30 universities, which can take large numbers of students. The universities, situated in different areas, fall under the supervision of the MoE while also enjoying a high degree of administrative and academic autonomy. The MoE oversees all Saudi students who study abroad in order to gain the skills needed in KSA. It cooperates with the universities and focuses on scientific research, which represents a key element of scientific and cultural progress, and the promotion and support of which remains one of the primary responsibilities of universities in the KSA. Undergraduates who are Saudi citizens can study for free (MoE, 2020a). In addition to the

30 public universities, the country also currently has 14 private universities and 29 private colleges (MoE, 2020a).

2.4 Development of Education in Saudi Arabia

Education in the KSA has passed through multiple stages. Developments in education have kept pace with processes of construction and progress elsewhere in the Kingdom. The state pays considerable attention to education, with plans in place to ensure it can keep up with the world's most up-to-date educational systems.

Despite the many achievements witnessed within the Kingdom's education sector, it currently faces various challenges, such as globalization and worldwide competition. These challenges have given it the impetus to develop, which has created a need for new skills and integrated knowledge within the state's general economy. This requires a new vision developed through schooling, the educational curricula that depend on it, and competent and responsible teachers (Najlaa, 2019). In addition, with emerging indicators suggesting that the educational system is performing poorly, many specialists have stated the need for development projects that will enable Saudi institutions to compete with educational systems worldwide (Brahim, 2014). Therefore, numerous projects have been initiated to develop education in the Kingdom, such as Tatweer (Tatweer, 2020), meaning "reform" in Arabic, intended to play a strategic role in supporting the MoE and the private sector in raising the level of education.

2.4.1 Important projects and bodies in Saudi mathematics and science education

Science and Mathematics Curriculum Development Project (SMCDP)

The Saudi Science and Mathematics Curriculum Development Project (SMCDP) represents one of the most promising projects in the region and aims to ensure the comprehensive development of science and mathematics education. It does so by drawing on the translation and adaptation of international educational materials that have helped improve education. The project is based on adapting and translating international science and mathematics curricula (Macmillan/McGraw-Hill) at all levels of education (Alshayee & Abalhameed, 2011). Furthermore, the project aims to benefit from global experience and help developed countries foster a generation capable of solving and preventing societal problems (Alshayee & Abalhameed, 2011). According to the MoE (2006, p. 19), the project aims to achieve the development of curricula that encourage active learning in light of recent global changes and research.

The new curricula for mathematics focus on solving problems through key steps, skills and strategies, placing attention on higher-order thinking, communication, the diversification of learning and the consideration of individual differences (Rafea & Al-oeshiq, 2010). Macmillan/McGraw-Hill Education in the United States (US) published the relevant textbooks that form part of the mathematics curriculum established for the NCTM publication "A Quest for Coherence" (NCTM, 2006). According to McGraw-Hill (2012, cited in Alanazi, 2016), the series aims to assist teachers in helping students become proficient in mathematics and thus meet NCTM-2000 standards. The series also aims "to reflect both the findings from key research on mathematics instruction, instructional best practices and curricular focal points" (NCTM, 2006, p. 2).

However, there are differences between the translated series and the original series. For example, activities and exercises aiming to increase thinking skills have been deleted, crucial parts of some lessons have been omitted, and others have been merged with the Arab Bureau of Education for the Gulf States (2012). AL-Shaalan's (2013) study confirms this, revealing differences between the original and the adapted versions of the Grade 5 primary book.

The Excellence Research Centre for Science and Mathematics Education (ECSME)

The Excellence Research Centre for Science and Mathematics Education (ECSME) has funded and encouraged researchers to continue working in this field. ECSME was established in 2007 at the University of King Saud and was funded by the MoE for five years (ECSME, 2020). The centre provides training courses and seminars and conducts research for various purposes: (i) to develop science and mathematics education based on scientific research through initiatives emanating from the centre itself or through participation in local and international events; (ii) to provide professional development opportunities for researchers in science and mathematics education; (iii) to develop community partnerships, represented in research work and advisory services in the field of science and mathematics education for the concerned authorities, including institutions, governments, and private agencies. The centre's aims include assessing scientific research priorities, undertaken national projects and research, and the production and dissemination of scientific knowledge through partnerships in the fields of mathematics and science education. It houses research groups working on teacher professional development, assessment, teaching and learning, and curriculum analysis.

The Education and Training Evaluation Commission (ETEC)

The Education and Training Evaluation Commission (ETEC), established in 2018 as a legal entity with financial and administrative autonomy, has links to the President of the Council of Ministers. Its roles include evaluating, assessing and accrediting qualifications in the fields of education and training in the public and private sectors. The aim of this undertaking includes enhancing their quality and efficiency, increasing their contribution in serving the economy and promoting national development in line with Saudi Vision 2030 (ETEC, 2020). This body shows its importance by setting national standards for evaluating training and general education curricula.

2.5 Saudi Vision 2030

The Saudi Vision 2030, which was recently released, serves as a roadmap for growth and economic activity in the KSA, and aims to make the country a global leader in all fields (Saudi Vision 2030, 2020). It focuses on three pillars: a vibrant community, a prosperous economy, and an ambitious nation, as well as optimizing the country’s fundamental strengths to assist people in achieving their goals (Saudi Vision 2030, 2020).

2.5.1 Education in the Saudi Vision 2030

To build on the promise of the next generation, the KSA places a premium on educational growth and promotion (Saudi Vision 2030, 2020). Through the Vision 2030, via various techniques and methods, the Kingdom demonstrates its desire to achieve excellence and progress in the development of education. Human capital development is one such initiative, seeking to increase the output of education and training programmes at all levels, from early learning to lifelong education and training (Saudi Vision 2030, 2020). It is intended to reach an international level through keeping pace with education, qualifications, and training programmes (Saudi Vision 2030, 2020). One of its goals includes having students achieve advanced results compared to their international peers and obtain advanced classifications aligned with global indicators of educational achievement (AL-yamy, 2019).

The teaching and learning of mathematics will play an important role in achieving the goals of Vision 2030 in the educational domain (AL-yamy, 2019). Saudi Arabia participated in the Trends in International Mathematics and Science Study (TIMSS), evaluating mathematics and science education for fourth- and eighth-grade students in 2003, 2007, 2011, 2015, and 2019, as shown in Table 2-1.

Table 2-1 TIMSS scores for Saudi Arabia

Year of participation	Grade	Saudi Average	International Average
2003	8	331	466
2007	8	329	500
2011	4 & 8	394	500
2015	4 8	383 368	500
2019	4 8	398 394	500

Table 2-1 shows that Saudi students' scores have been significantly lower than the international average. These scores do not reflect the efforts and funds the country has spent on improving the education system. In addition, the country participated in the 2018 PISA study and scored an average of 373, which is low compared to the average international score of 450.

2.6 Using technology in Saudi Arabian education

The Saudi education sector is facing increasing demands for the effective integration of technology in educational practice. The government is addressing this through substantial funding for schemes that provide technology to schools and keep the curriculum updated. The MoE has tried to provide the majority of schools in the country with technological equipment and computer labs. Moreover, the MoE has employed several strategies governing the use of technology in education:

1. Circulars to schools focusing on the need to use different computer applications in the classroom alongside scheduled books.
2. Provision of Internet services to schools as approximately 915 of institutions have access to the Internet and most have computer labs so students can access various scientific sources through the Web, exchange information, and solve problems.
3. Paying attention to the process of digital education, as part of which the implementation of several agreements signed with technological institutions began on the ground.
4. Encouraging e-learning in open universities and the possibility of distance learning for students who cannot attend in person. (Olayan, 2019)

In addition, the Ministry supported the Development of the Mathematics and Science Curricula Project, a prominent aspect of which includes the adoption of the curriculum to integrate technology into mathematics lessons and requires training to improve teachers' skills. The project is aligned with principles such as integrating technology in the curriculum to cultivate effective teaching methods and activate exploratory learning skills based on multiple approaches (Al-rwaise, 2011). Vision 2030 emphasizes the role of information technology and its use in the educational process, including the diversification of social media channels and tools to create an educated, autonomous and independent generation able to take on leadership roles in the future. Strategies also include training in technology use and integrating technology in educational curricula (Saudi Vision 2030, 2020).

However, using technology in teaching mathematics presents difficulties. For example, Al-rwaise (2011) found that teachers demonstrate less-than-acceptable levels of technology use in mathematics classes. Furthermore, Al-Enazy and AL-Mosaad (2018) identified issues that affected teachers' use of technology in their classes, such as lack of experience and knowhow. In addition, teachers show a lack of willingness to change their current instruction patterns, being rather dependent on traditional methods of teaching mathematics (Al-Enazy & AL-Mosaad, 2018).

2.6.1 E-learning opportunities – Future Gate (FG)

The Saudi MoE has taken a keen interest in e-learning and created Future Gate (FG) in cooperation with the Tatweer Company for Education Technologies (TCET) to shift towards digital learning. This represents one of the initiatives undertaken by the MoE as part of the 2020 national transformation stage to realize Vision 2030. The aim here is to initiate a move towards a digital environment, promoting educational strategies and self-learning opportunities and providing a student-centred educational environment based on the application of modern educational methods that underscore the positive use of technology (MoE, 2020b). The MoE places students the centre of its endeavour to create a new educational environment reliant on technology to communicate knowledge and improve educational outcomes, based on the ability to develop teachers' scientific and educational capabilities (MoE, 2020b).

Thus, this research is aligned with the Saudi Vision 2030 in two respects. The first concerns employing technology in teaching and learning mathematics as this study used GGS. The second relates to the use of released TIMSS items, meaning that if students perform well in the tests, the use of GGS can be recommended to enhance learners' performance on TIMSS.

Chapter 3 Conceptual framework

3.1 Introduction

This chapter articulates the meaning of mathematical proficiency. It consists of three main sections. The first section focuses on the historical development of mathematical proficiency and competency frameworks. The second part examines the five-strand framework implemented by the NRC, illustrating its definition in relation to all of its five strands, as well as determining the relationship between them. The last section presents why and how this study uses the NRC's mathematical proficiency framework.

3.2 Historical development of mathematical proficiency

There has been a considerable transformation in the methods used for the teaching and learning of mathematics over recent decades. This transformation has taken place to adapt to the changes brought about by modern lifestyles, including aspects such as the methods used for basic counting operations and finding solutions for complex tasks. Brownell (2007) argued that the learning of mathematics should involve more than simply the practice of mechanical skills, rather that all mathematical components taught (including ideas, principles, generalizations, relationships and skills) should (1) fulfil extra-territorial purposes and (2) promote their employment in situations separate from the learning context. The significance of this is that students need to be proficient in mathematics rather than just being able to undertake various procedures without understanding the steps, and underlying concepts. According to Schoenfeld (2007), the best mathematical tasks follow certain principles: (i) ensuring that students use multiple solutions in their answers to exam questions; (ii) ensuring that students generate mathematical explanations; (iii) ensuring that students actively participate in high-level thinking and logic. These characteristics are fundamentally embodied in the mathematical proficiency that students may demonstrate in their responses (Cragg & Gilmore, 2014; Greenes, 2014).

The teaching of mathematics should not only seek to develop numeracy, but also enhance individual thinking and the acquisition of skills, in particular by developing various aspects of a student's abilities (i.e. logical thinking and methods of resolving real-world problems). However, this can be unclear to non-specialists in mathematics, as well as children, who are now learning mathematics in ways that differ considerably from those experienced by their parents and grandparents (Kilpatrick et al., 2001). Potentially, the most vital and exciting driver of teaching and learning mathematics is the need for mathematics in our lives.

Since the beginning of the 21st century, proficiency in mathematics has provided the foundation of a new educational model within several educational systems, thus reflecting a need to respond to changes in both society and economies (Ropohl et al., 2018). Furthermore, mathematical proficiency is now deemed an important aim for both students and teachers of mathematics at all educational levels (Haines et al., 2007). Indeed, students should achieve a level of mathematical competence in numeracy capable of inspiring them to become active members of the community (Evans, 2007).

3.2.1 Defining mathematical proficiency

Mathematical proficiency is currently central to various fields of mathematical application; however, the term itself remains somewhat vague (El Asame & Wakrim, 2018). Kilpatrick (2014, p. 85) stated that "the concept of competence is one of the most elusive concepts in the educational literature".

A number of studies have sought to undertake the challenge of determining an appropriate definition of competence or proficiency. El Asame and Wakrim (2018) stated that knowledge and technical skills are not sufficient to form the principal components of mathematical proficiency, which requires mastery of additional skills necessary for successful functioning within society (i.e. communication and language). Moreover, there are many theoretical approaches to the objective of learning mathematics, but the meaning of "proficiency" tends to differ between individuals and there is no single unified conceptual framework (Weinert, 2001). According to Kilpatrick (2011, p. 10):

> …mathematical proficiency … we defined as composed of five interwoven strands that were to be developed simultaneously. We did not want the question, "Is it skill, or is it understanding?" We said: "It is skill, it is understanding, and it is more than either of those."

In this context, the NRC committee aimed to go beyond the skill–understanding dichotomy by applying the concept of mathematical proficiency. Bartell et al. (2013) defined mathematical proficiency as the development of mathematical skills and knowledge. Kilpatrick et al. (2001) described mathematical proficiency as comprising knowledge, skill, ability, and facility in mathematics. Groves (2012) considered mathematical proficiency to be the capability of working in mathematics with understanding, calculating, applying, thinking, and engaging. From these descriptions of mathematical proficiency, it is possible to conclude that it denotes learning different skills in mathematics. Thus, to say that students are proficient in mathematics, they should have a sufficient level in various mathematics skills, such as understanding, computing, problem solving, and thinking mathematically, all at once.

3.2.2 Policy trends in mathematical proficiency

Over the previous six decades, Bloom's (1956) taxonomy has proven the most popular of the many frameworks related to mathematical proficiency and competency. The Taxonomy of Educational Objectives (Bloom, 1956) is considered the progenitor of competency in mathematics education (Kilpatrick, 2014). Despite initially attracting little attention, it subsequently became widely recognized and cited, eventually being translated into over 22 languages (Krathwohl, 2002). Bloom's (1956) taxonomy lists six cognitive domains: (1) knowledge; (2) comprehension; (3) application; (4) analysis; (5) synthesis; (6) evaluation. This framework progresses from the simple to the complex, and finally to abstract concepts. However, this taxonomy has been criticized by a number of educators in mathematics, including Hans Freudenthal and Chris Ormell, as being ill-suited to the subject due to it neglecting content in favour of process (Kilpatrick, 2014).

The NCTM in the US has paid considerable attention to improving students' learning of mathematics. The NCTM asserts that mathematical proficiency should open doors to productive futures, seeking to change the belief that mathematics can only be of specific use to a small number of individuals (NCTM, n.d.). The NCTM further opines that all students should be given the opportunity to learn mathematics and have the necessary support to develop a deep understanding of and be proficient in the subject (NCTM, n.d.). The NCTM created the highly influential Curriculum and Evaluation Standards for School Mathematics (1989), which focused on five primary goals vital to the attainment of all students between kindergarten and the 12th grade, these being: (1) that they learn to value mathematics; (2) that they become confident in their ability to do mathematics; (3) that they become mathematical problem solvers; (4) that they learn to communicate mathematically; (5) that they learn to reason mathematically (NCTM, 1989, p. 5). These standards spawned the "wars of mathematics" in the US during the 1990s, in which the opposing sides held different views concerning how to come to grips with the subject, in particular the requirements for effective teaching (Niss et al., 2017). The NCTM subsequently revised its standards, publishing a new edition in 2000 entitled "Principles and Standards for School Mathematics" (NCTM, 2000). The new framework included Six Principles for School Mathematics: (1) equity; (2)

curriculum; (3) teaching; (4) learning; (5) assessment; (6) technology. It also included the following five Content Standards: (1) number and operations; (2) algebra; (3) geometry; (4) measurement and data analysis; (5) probability. Moreover, in addition to described the standards for content, an additional five process standards were also established as essential for gaining and applying content knowledge and categorizing information according to the following five elements: (1) problem solving; (2) reasoning and proof; (3) communication; (4) connections; (5) representation.

The framework provides a vision for mathematics learning based on high performance among all students in schools. The NCTM vision of mathematics learning and education identifies an integrated process in which students investigate mathematical ideas in a social environment, such that communication involving mathematical reasoning is an important part of learning and students confidently participate in complicated mathematical tasks carefully selected by their teachers (Collins, 2011). This suggests a more complex definition of mathematical proficiency than merely the capability to calculate fluently and use mathematical expressions (Collins, 2011). Thus, the NCTM's principles and standards provide suggestions and guidelines for what students need to learn and how teaching practices might be implemented effectively in class, rather than being specifically a proficiency framework.

Furthermore, in 2000, the National Assessment of Educational Progress (NAEP) in the US introduced a framework influenced by the NCTM. As with previous mathematics assessments in 1990, 1992, and 1996, the NAEP 2000 mathematics assessment set out a framework based on the domains of mathematical abilities and mathematical power. It also described such mathematical abilities as underlying the nature of knowledge and processes, reflecting the following three main strands: (1) conceptual understanding; (2) procedural knowledge; (3) a combination of both of these for the process of problem solving (Braswell et al., 2001). The second domain in the framework (i.e. mathematical power) reflected the processes emphasized as major goals of the mathematics curriculum, including the ability of students to: (1) reason mathematically; (2) communicate successfully; (3) connect concepts and skills across mathematical strands, or between mathematics and other curricular areas (Braswell et al., 2001). This assessment framework is, to some extent, similar to the NRC framework, as discussed in 3.3, because it uses three of the NRC strands concerning the nature of knowledge and mathematical reasoning related to process; however, it does not include the productive disposition strand. According to Kilpatrick et al. (2001, p. 118):

> This Framework has some similarities with the one used in recent mathematics assessments by the National Assessment of Educational Progress (NAEP), which features three mathematical abilities (conceptual understanding, procedural knowledge, and problem solving) and includes additional specifications for reasoning, connections, and communication.

Adding It Up: Helping Children Learn Mathematics (2001) was published by the US NRC and pertains to the teaching of school mathematics from pre-kindergarten to the eighth grade. It describes the term "mathematical proficiency" and specifies the following five interwoven strands: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence; (4) adaptive reasoning; (5) productive disposition (Kilpatrick et al., 2001). The US RAND Mathematics Study Panel (2003) produced a similar definition (Loewenberg, 2003). However, Kilpatrick et al. (2001, p. 116) suggested that no single term could completely capture all aspects of competence in mathematics (i.e. capability, knowledge, and skill), stating "we have selected mathematical proficiency to capture what we believe is necessary for anyone to learn mathematics successfully".

Furthermore, Kilpatrick et al. (2001) considered that mathematical proficiency included the following five components or strands:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations.
- Procedural fluency: skills in conducting procedures flexibly, accurately, efficiently, and appropriately.
- Strategic competence: the ability to formulate, represent and solve mathematical problems.
- Adaptive reasoning: the capacity for logical thought, reflection, explanation and justification.
- Productive disposition: the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

According to Kilpatrick et al. (2001), that these five strands are interwoven and interdependent, operating simultaneously in the development of proficiency in the domain of mathematics, as illustrated in Figure 3-1.

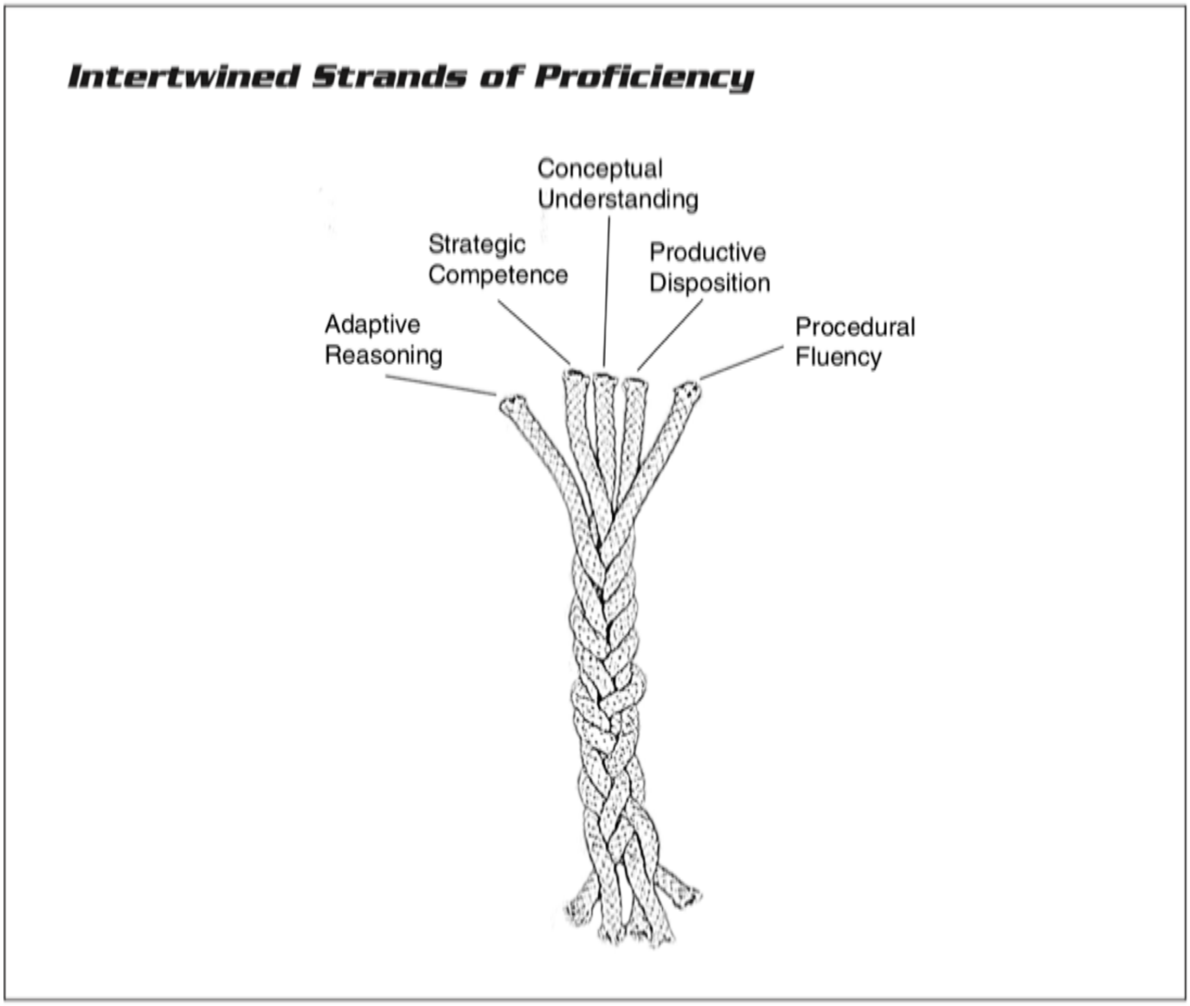


Figure 3-1 The intertwined strands of mathematical proficiency (Kilpatrick et al., 2001)

In addition, the US National Governors Association Centre for Best Practices and the Council of Chief State School Officers (NGA and CSSO, 2010) drew on the NRC's (2001) five strands of proficiency and the NCTM's (2000) five process standards to create eight mathematical practices described in the Common Core State Standards for Mathematics (CCSS) (Allsopp et al., 2017). The CCSS classify the eight elements of mathematical proficiency as being the ability to: (1) make sense of mathematics problems and continue in solving them; (2) reason quantitatively and abstractly; (3) construct viable arguments and analysis the reasoning of others; (4) model with mathematics; (5) use suitable tools strategically; (6) attend to precision; (7) look for, and make use of, structure; (8) look for and express consistency in repeated reasoning (NGA, 2010).

In contrast, the new Australian Curriculum: Mathematics (F–10) (2013) employed the first four of Kilpatrick et al.'s (2001) proficiency strands. These emphasize the breadth and depth of the mathematical competencies students are required to obtain by means of study, alongside the following content strands (Groves, 2012): (1) understanding; (2) fluency; (3) problem solving; (4) reasoning (Australian Curriculum, Assessment and Reporting

Authority [ACARA], 2013). The Australian Curriculum describes proficiency in mathematics as follows:

> Mathematics includes proficiencies that focus on developing increasingly sophisticated and refined mathematical understanding, fluency, reasoning and problem-solving skills. These proficiencies enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently. (ACARA, 2013, mathematical proficiency, paragraph 1)

The curriculum also includes three goals describing the behaviour required of students when engaged in learning, as well as when using the content of the mathematics curriculum. The aim is to ensure that student learning and independence remain at the core of mathematics teaching, enabling students to respond to both familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems in an efficient manner (ACARA, 2013).

To some extent, the above competencies and proficiencies view the learning of mathematics in the classroom as demanding more than simply teaching mathematical ideas that are subsequently used to perform a specific task. Rather, they can be seen to emphasize that the acquisition of mathematical competency requires a focus on improving different aspects of students' skills. Kilpatrick (2014) noted that proficiency frameworks are intended to establish that the learning of mathematics consists of more than gaining facts and that implementation is more than conducting well-rehearsed procedures. This view represents an attempt to present a more nuanced description of a field requiring the development of a variety of proficiencies.

3.3 NRC mathematical proficiency strands

3.3.1 Conceptual understanding

Conceptual understanding refers to "an integrated and functional grasp of mathematical ideas" (Kilpatrick et al., 2001, p. 118). This strand is a key element for the learning of mathematics as in the absence of sufficient understanding students may be unable to attain the requisite mathematical skills. Conceptual understanding enables students to distinguish more than simply remote facts and procedures, also recognizing the significance of specific mathematical knowledge and the contexts in which it can prove beneficial (Kilpatrick et al., 2001). While it is important to acquire the ability to understand a mathematical concept, the most important aspect is how and when each student presents his/her understanding of a specific idea. Geary (1995) noted that although tutors frequently seek evidence of students' conceptual understanding to describe the connections between concepts and representations, this may not always be straightforward. It needs to be recognized that students frequently gain an awareness of a concept before acquiring the ability to verbalize such understanding.

The concept of understanding is not one that is new in mathematical frameworks. For example, conceptual understanding in the NAEP (1996) mathematical power definition framework reflects the ability of students to employ careful application of the concepts of definitions, relationships, or representations. In addition, ACARA (2013) stated that the development of students' conceptual understanding involves the development of considerable flexible and transferable mathematical concepts and facts, followed by attempting to make strong connections between related concepts, then gradually applying familiar concepts to develop new ideas. This therefore enables students to develop an understanding of the relationship between the "why" and the "how" of mathematics. In addition, it reflects students' understanding of concepts and the development of strong relationships between mathematical ideas, along with the ability to subsequently transfer such knowledge to new conditions and apply it to new frameworks (Kilpatrick et al., 2001; Sale et al., 2002; Chinnappan & Forrester, 2014; Awofala, 2017; Mills, 2019).

Example of conceptual understanding (Kilpatrick et al., 2001, p. 119)

…suppose students are adding fractional quantities of different sizes, say $\frac{1}{3}+\frac{2}{5}$. They might draw a picture or use concrete materials of various kinds to show the addition. They might also represent the number sentence $\frac{1}{3}+\frac{2}{5}=?$ as a story. They might turn to the number line, representing each fraction by a segment and adding the fractions by joining the segments. By renaming the fractions so that they have the same denominator, the students might arrive at a common measure for the fractions, determine the sum, and see its magnitude on the number line. By operating on these different representations, students are likely to use different solution methods. This variation allows students to discuss the similarities and differences of the representations, the advantages of each, and how they must be connected if they are to yield the same answer.

Conceptual understanding is an important component of acquiring mathematical skills, focusing on the understanding of relevant aspects of mathematical concepts and their connections (Nguyen et al., 2016). Furthermore, it is also essential when it comes to other mathematical proficiency strands. For example, Hiebert and Lefevre (1986) pointed out that conceptual understanding constitutes a consideration of the underlying relationships demonstrating why a procedure can prove effective. This explains the existence of a relationship between conceptual and procedural knowledge, although exactly how they are associated and contribute to mathematics proficiency remains a matter of discussion (Schneider et al., 2010).

There are a number of benefits to emphasizing conceptual understanding when teaching mathematics, including ensuring that students are less likely to forget concepts than procedures and that once acquired, conceptual understanding can be used to reconstruct a procedure they may otherwise have forgotten (Mills, 2019). Therefore, conceptual understanding, once established, is able to sit alongside procedural knowledge (Rittle-Johnson et al., 2001). Although conceptual understanding is required to enable students to select appropriate procedures for the solving of mathematical problems, it can subsequently be intertwined with procedural knowledge, with the combination being accepted as more powerful than either used singly (Wong & Evans, 2007).

3.3.2 Procedural fluency

Procedural fluency refers to the "knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently" (Kilpatrick et al., 2001, p. 121), exemplified as follows:

Students need to be efficient and accurate in performing basic computations with whole numbers (6 + 7, 17 – 9, 8 × 4, and so on) without always having to refer to tables or other aids.

Fluency in mathematics implies the ability of students to make flexible decisions between methods and strategies capable of assisting them solve mathematical tasks, including first understanding and explaining their approaches and second being able to effectively produce accurate answers (Leinwand, 2014). Procedural fluency has been described in relation to many contexts and frameworks. For example, the ACARA stated that:

> Students develop skills in selecting suitable procedures; […] students are fluent in efficiently estimating answers, knowing effective ways to answer questions, selecting correct methods and approximations, remembering concepts and using

> information on a regular basis, and manipulating expressions and formulas to find solutions. (ACARA, 2013, procedural fluency, paragraph 3)

According to NAEP (1994), students demonstrate procedural fluency in mathematics when they are able correctly to select and apply the appropriate procedures. In addition, students often reflect their procedural fluency when connecting an algorithmic process to a given problem, i.e. employing this process correctly and communicating the outcomes of the algorithm in the framework of the relevant setting (NAEP, 1994).

Procedural fluency can be consisted more than an attempt to memorize facts or undertake procedures, as well as understanding and being able to use one formula for a given situation. It builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NCTM, 2000; Kilpatrick et al., 2001; CCSS, 2010; NCTM, 2015). An important component of learning mathematics is the completion of tasks; however, it is even more important to undertake such tasks in the light of appropriate understanding. Kilpatrick et al. (2001) stated that it is vital for computational procedures to be: (i) useful; (ii) used accurately; (iii) result in the correct answers. This indicates that accuracy and efficiency are improved with practice, which can also result in students maintaining fluency, as well as being able to operate procedures in a flexible manner. Research has suggested that when students memorize and practise procedures in the absence of understanding, they demonstrate less enthusiasm for acquiring the meaning and reasoning behind the procedures (Hiebert, 1999). Sufficient procedural fluency skills are vital for enhancing students' mathematical understanding. Kilpatrick et al. (2001) suggested that without adequate fluency skills, students will have trouble developing their understanding of mathematical concepts or solving problems.

Example of procedural fluency (Kilpatrick et al., 2001, p. 122)

Students should be able to use a variety of mental strategies to multiply by 10, 20, or 300 (or any power of 10 or multiple of 10). Also, students should be able to perform such operations as finding the sum of 199 and 67 or the product of 4 and 26 by using quick mental strategies rather than relying on paper and pencil.

Students need practice in incorporating concepts and procedures to develop procedural fluidity and expand on familiar procedures as they construct their own informal strategies and procedures (National Council of Teachers of Mathematics, 2014). In addition, they require opportunities to validate both commonly used procedures and informal strategies to first support and justify their ranges of proper procedures and second strengthen their understanding and skills by means of practice (NCTM, 2015). One method of improving mathematical attainment is for students to increase their fluency, i.e. their capability to answer tasks automatically and with accuracy (Hinton et al., 2014).

It has been acknowledged that there is a close connection between conceptual understanding and procedural fluency, with a deep understanding leading to a fluent procedure, which in turn increases students' understanding (Booth et al., 2013). Moreover, issues experienced in each of these can result in difficulties with the other. Therefore, when students encounter any difficulties in understanding a mathematical concept, this can also lead to difficulties in completing any related tasks. For this reason, studying students' procedural fluency when making mistakes can assist teachers in identifying their students' issues (including insights into any confusion), aiding in the development of subsequent stages in teaching (Booth et al., 2013).

3.3.3 Strategic competence

Kilpatrick et al. (2001, p. 124) described strategic competence as follows:

> Strategic competence refers to the ability to formulate mathematical problems, represent them, and solve them. This strand is similar to what has been called problem solving and problem formulation in the literature of mathematics education and cognitive science.

Kilpatrick et al. (2001, p. 125) give the following example of strategic competence:

- At ARCO, gas sells for $1.13 per gallon.

This is 5 cents less per gallon than gas at Chevron. How much does 5 gallons of gas cost at Chevron?

A more proficient approach is to construct a problem model that is, a mental model of the situation described in the problem. A problem model is not a visual picture per se; rather, it is any form of mental representation that maintains the structural relations among the variables in the problem. One way to understand the first two sentences, for example, might be for a student to envision a number line and locate each cost per gallon on it to solve the problem.

Mathematical problem solving consists of engaging in a mathematical task for which a solution or steps have not previously been determined (NCTM, 2000). Schoenfeld (1983, p.

41) considered that “problem is only a problem (as mathematicians use the word) if you don’t know how to go about solving it”. Thus, a mathematical problem that has no “surprises” in store and can be solved easily by routine or familiar procedures can be considered an exercise (Mcdonald, 2017). Interestingly, Schoenfeld (1985) offered an alternative view of problem solving as consisting of a specific connection between the learner and the mathematical task offering him/her any difficulties. Polya (1981, p. 9) presented problem solving as a feature of human effort, stating that:

> Solving a problem means finding a way out of a difficulty, a way around an obstacle, attaining an aim which was not immediately attainable. Solving problems is the specific achievement of intelligence, and intelligence is the specific gift of mankind: problem solving can be regarded as the most characteristically human activity.

Cai and Lester (2005, p. 221) viewed problem solving as an activity leading to the learner engaging in a diversity of cognitive activities, each of which requires a different aspect of knowledge and skill, some of which are not routine. Also, NEAP (1996) pointed out that students illustrate problem solving in mathematics through six steps: (i) identifying and formulating problems; (ii) evaluating the reliability of data; (iii) using methods, software, and models; (iv) creating, expanding, and changing procedures; (v) using logic in new settings; (vi) assessing the rationality and accuracy of solutions.

A considerable number of curriculum frameworks view problem solving as a significant component of mathematics. In Singapore, for example, problem solving forms the principal approach to primary and secondary mathematics teaching and therefore lies at the heart of both the learning and teaching framework. The Singapore MoE (2007, p. 3) stated that “problem solving is central to mathematics learning. It involves the acquisition and application of mathematical concepts in a wide range of situations, including non-routine, open-ended and real-world problems”. In Australia, ACARA developed the Australian Curriculum to provide teachers, parents, students, and the community with a clear understanding of the content of students’ learning, viewing problem solving as a main component of acquiring mathematical skills (ACARA, 2013). Thus, students gain the ability to: (i) make choices; (ii) perceive, formulate, design, and discuss problem situations; (iii) effectively communicate solutions (ACARA, 2013). Students are seen as being able to formulate and resolve problems when:

- using mathematics to describe new or concrete circumstances;
- designing investigations and preparing their responses;

- applying their current strategies;
- finding solutions;
- verifying their answers as reasonable. (ACARA, 2013)

Stonewater (2005) described problem solving and inquiry learning as being the most effective means of providing students with the required skills and attitudes in mathematics. Furthermore, Hung (2006) noted that as well as obtaining information to improve problem solving skills, students should be able to understand where, when, and how to apply such information.

3.3.4 Adaptive reasoning

According to Kilpatrick et al. (2001, p. 129), adaptive reasoning refers to "the capacity to think logically about the relationships among concepts and situations. Such reasoning is correct and valid, stems from careful consideration of alternatives, and includes knowledge of how to justify the conclusions". Thus, in mathematics, adaptive reasoning is considered to be the glue holding everything together (Kilpatrick et al., 2001). ACARA (2013, reasoning, paragraph 1) described adaptive reasoning as follows:

> Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices.

To exemplify reasoning (Kilpatrick et al., 2001, p. 139):

A multiple-choice problem in which students were asked to estimate $\frac{12}{13} + \frac{7}{8}$.

The choices were 1, 2, 19, and 21. Simply observing that $\frac{12}{13}$, $\frac{7}{8}$ are numbers less than one and that the sum of two numbers less than one is less than two would have made it apparent that 19 and 21 were unreasonable answers.

Mathematical reasoning is a thinking process which focuses on making sense of mathematical ideas and concepts essential to procedures (Bieda et al., 2014). In addition, it is assumed to provide assertions and the ability to grasp assumptions (Boesen et al., 2010). Newton (2010) viewed adaptive reasoning as a key component of mathematical competence. Students using reasoning may be able to think logically about mathematics and thus they can clarify and defend their approach. It can therefore be seen as one of the basic mathematical proficiencies students need to acquire when learning mathematics (Rokhima et al., 2019).

Students of mathematics are required to use adaptive reasoning during mathematical argumentation, a procedure that includes the creation and justification of mathematical claims (Bieda et al., 2014). Boesen et al. (2010) stated that argumentation consists of validation, which is the aspect of reasoning aimed at convincing others (or oneself) that such reasoning is appropriate. Students' reasoning may help them to note patterns, structures or regularities, connecting them with both real-world situations and symbolic objects. This includes considering whether patterns, structures or regularities can be viewed as accidental or seen to occur for a reason, one that they are subsequently able to inference and prove (NCTM, 2000).

The meaning of the adaptive reasoning thus is not simply a means of justifying work or proving a succession of facts. Kilpatrick et al. (2001) stated that adaptive reasoning includes a far broader informal explanation and justification, as well as inductive and intuitive cognitive reasoning based on pattern, analogy, and metaphor. There are two kinds of reasoning in mathematics, inductive and deductive (Kilpatrick et al., 2001; Boesen et al., 2010; Bieda et al., 2014; Wibowo, 2016). Inductive reasoning entails a move from specifics and a particular example to make a general claim. Deductive reasoning, in contrast, concerns the drawing of conclusions from correct statements or evidence, so this type of reasoning often employs deductively proven hypotheses or formulae (Wibowo, 2016).

It is essential for teachers of mathematics to offer their students opportunities to improve their reasoning and for them to use instruments to measure students' reasoning skills. Many studies have concluded that most aspects of the learning environment (i.e. teaching, textbooks and teacher-made algorithmic procedures) fail to offer sufficient opportunities for students to learn different types of reasoning (Bergqvist, 2007). It could thus be the teacher's approach that acts to impair students' reasoning abilities during the learning of mathematics. For example, educators who focus heavily on procedural and mechanical issues (i.e. lecture-based learning and presenting mathematical concepts) are often informative, but students may be trained to complete their work without gaining any in-depth knowledge. Therefore, enhancing students' mathematical reasoning requires more than undertaking procedures, instead including activities that involve students in a variety of thinking and sense-making processes (Mata-Pereira & da Ponte, 2017).

3.3.5 Productive disposition

Productive disposition refers to "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (Kilpatrick et al., 2001, p. 131). Siegfried (2012) stated that a productive disposition ensures students do not view mathematics as a set of arbitrary rules to be memorized, but rather they have an overview that allows them to recognize mathematics as a system of connected conceptions that can be understood as a result of diligent study. As noted by Siegfried (2012), this strand is completely separate from the other four strands in that it includes issues such as an individual's affect, beliefs, and identity, whereas the other strands focus primarily on cognitive processes. Indeed, Philipp and Siegfried (2015) stated that productive disposition changes the concept of mathematical proficiency, conferring an affective dimension not generally included in the definition of mathematical understanding.

As articulated by Kilpatrick et al. (2001), the first aspect of a productive disposition concerns an ability to recognize sense in mathematics. This is significant because learners need to view mathematics as understandable rather than arbitrary in order to develop the other four strands of mathematical proficiency (Kilpatrick et al., 2001). The predisposition to see sense in mathematics concerns a fundamental belief about the nature of mathematics (Siegfried, 2012). Thus, students' perception of mathematics could be a major factor influencing their learning, performance, and abilities. In addition, students' mathematical abilities are significantly related to their beliefs concerning the importance of mathematics, as well as their own ability, along with self-regulation (Suthar et al., 2013; Gafoor & Sarabi, 2016). This raises the issue of which factors can be seen to construct students' beliefs about mathematics. The NAEP (1996) study found that 54% of fourth-grade students and 40% of eighth-grade students believed mathematics to consist primarily of rules requiring memorization (Kilpatrick et al., 2001), with numbers, rules, and formulae to be learnt by rote (Gafoor & Sarabi, 2016). In contrast, students who view mathematics as a technique of thinking learn by doing tasks aimed at developing interactive understanding (Crawford et al., 1994). Thus, if teachers wish to change the attitudes of their students towards the subject, they should also believe that mathematics always makes sense and they need to see that students' conceptions may entail deep mathematical complexity, even when it comes to elementary mathematics (Siegfried, 2012).

The last aspect of the productive disposition considered by Kilpatrick et al. (2001) is seeing oneself as an effective learner and doer of mathematics. Siegfried (2012) highlighted that to be actively engaged, students need to be confident that they are capable of learning mathematics and making progress in their ability to perform mathematical tasks, which then assists them in developing further strands of mathematical proficiency. When individuals see themselves as having the capacity to learn mathematics and use it to solve problems, they are able to continue to develop their procedural fluency, alongside their adaptive reasoning skills (Kilpatrick et al., 2001). Graven et al. (2013, p. 29) pointed out that:

> In the sense that productive disposition involves "seeing oneself" as an effective learner and doer of mathematics, and that dispositions commonly refer to a habitual tendency to act in a certain way, they relate to learner ways of participating in mathematical learning situations.

Another component of Kilpatrick et al.'s (2001) productive disposition is self-efficacy, which is defined as "people's judgements of their capabilities to organise and execute courses of action required to attain designated types of performances" (Bandura, 1986, p. 391). A sense of self-efficacy arises as an individual interprets information from the following four foundations: (i) mastering experience; (ii) vicarious experiences; (iii) social persuasions; (iv) a student's emotional and physiological state (Bandura et al., 1999). Bandura (1994) suggested that when faced with a difficult task, individuals who doubt their capacity have a tendency to give up, while those with high self-efficacy exert additional effort when they are challenged. Self-efficacy can make a significant contribution to the extension of enthusiasm (Tschannen-Moran & Hoy, 2001). Thus, self-efficacy may play a crucial role in enhancing students' productive disposition as defined by Kilpatrick et al. (2001).

A review of various studies focusing on the learning of mathematics shows that self-efficacy exerts a significantly positive influence on students' performance and achievement (Zimmerman, 2000; Taşdemir, 2016; Ozkal, 2019). Self-efficacy is considered to represent students' confidence about completing a variety of mathematical tasks, from understanding the concepts to solving problems (May, 2009). A high level of self-efficacy in relation to mathematics can inspire positive learning outcomes, with good results increasing the motivation felt by students to continue the learning process and achieve good learning outcomes (Masitoh & Fitriyani, 2018). Thus, learners should be encouraged to develop high levels of self-efficacy as this supports the success of the mathematical learning process, ultimately enhancing the students' achievement (Masitoh & Fitriyani, 2018). Indeed, mathematical self-efficacy can exert a robust influence on mathematical attainment (Fast et al., 2015).

From the studies above, it is apparent that students' beliefs about themselves form a significant component informing the mathematical learning process. Students' motivation to study mathematics tends to be influenced by their attitudes and beliefs, along with their levels of confidence and anxiety. For example, students with a productive disposition are more likely to develop skills and steps that encourage working towards an answer through tasks than students who do not develop this strand.

Thus, students with positive attitudes are more likely to be motivated to think mathematically and so gain an understanding of the content of lessons, as well as to expend additional effort, than those students holding negative attitudes towards the content of the same course (Kargar et al., 2010). In addition, those with a high level of confidence in their mathematical skills tend to consider their efforts worthwhile (Galbraith & Haines, 2000). This leads them to address complex topics in an efficient manner, as well as encouraging them to feel comfortable about studying mathematics as a subject, confident that they will achieve good results (Galbraith & Haines, 2000). However, students demonstrating low levels of confidence tend to experience anxiety when exposed to new materials, expecting to have difficulties with all areas of mathematics. This can lead them to perform poorly and subsequently view mathematics as their most problematic subject (Galbraith & Haines, 2000). A crucial objective when developing functions and learning methods in relation to mathematics has been to develop students' willingness to engage and confidence in their ability to approach challenging areas, as well as to create positive attitudes towards the subject concerned (NCTM, 2000).

3.3.6 Relationships between mathematical proficiency strands

The five strands of mathematical proficiency are interdependent (Kilpatrick et al., 2001) and intertwined, enhancing each other in establishing mathematical proficiency (Syukriani et al., 2016) . According to Kilpatrick et al. (2001, p. 133), "The five strands are interconnected and must work together if students are to learn successfully. Learning is not an all-or-none phenomenon, and as it proceeds, each strand of mathematical proficiency should be developed in synchrony with the others".

An example of this is that mathematical proficiency relies on developing both conceptual understanding and procedural knowledge, following broad agreement that conceptual knowledge frequently both leads and supports procedural fluency (Rittle-Johnson et al., 2015). NCTM (2014) employed a conceptual and procedural viewpoint when drawing up their principle that procedural fluency both follows and builds on a foundation of conceptual understanding, while conceptual understanding establishes the foundation for procedural fluency and is therefore necessary for its development. However, whether the relationship is bi-directional or unidirectional is contested (Rittle-Johnson et al., 2015). Evidence has shown that the relationship between conceptual understanding and procedural fluency is frequently bi-directional, with the enhancement of procedural fluency often supporting improvements in conceptual understanding, and vice versa (Rittle-Johnson et al., 2015). Some studies in the mathematical domain have argued that the development of conceptual understanding and procedural fluency in mathematics is often iterative, with one type of knowledge supporting gains in another, and this in turn supporting a further form of knowledge development; for example, conceptual understanding can assist in the construction, selection, and appropriate execution of procedures (Rittle-Johnson & Schneider, 2015). Furthermore, procedural fluency can assist students develop and extend their understanding of mathematical concepts, particularly when a task is designed to clarify the underlying concepts (Rittle-Johnson & Schneider, 2015).

In addition, there is a strong relationship between conceptual understanding, procedural fluency, and strategic competence, with Kilpatrick et al. (2001, p. 127) stating that "there are mutually supportive relations between strategic competence and both conceptual understanding and procedural fluency, …". Johnson and Christensen (2014) concluded that both conceptual understanding and procedural fluency are crucial for successful instruction in mathematics. Flexibility in terms of procedure involves knowledge of multiple approaches and a propensity to select the most effective solution based on specific characteristics (McDonald, 2017). Existing research, exemplified by these studies, thus argues the existence of a strong relationship between strategic competence, conceptual understanding, and procedural fluency.

Problem-solving skills can benefit students even in the absence of mathematical proficiency (Wibowo, 2016). According to Kilpatrick et al. (2001, p. 127):

> The development of strategies for solving non-routine problems depends on understanding the quantities involved in the problems and their relationships, as well as on fluency in solving routine problems. Similarly, developing competence in solving non-routine problems provides a context and motivation for learning to solve routine problems and for understanding concepts such as given, unknown, condition, and solution.

Adaptive reasoning is also related to other strands of mathematical proficiency. Students in possession of excellent adaptive reasoning and strategic competence have demonstrated their ability to acquire mathematical proficiency (Syukriani et al., 2016). According to Ball and Bass (2003), mathematical adaptive reasoning forms a basic skill in mathematics and is therefore obligatory for the understanding of mathematical concepts, followed by the use of mathematical ideas and procedures in a way that is both fluent and flexible (Wibowo, 2016). The capacity of reasoning to create a new connection with a previously developed relationship enables students to experience the application of reasoning abilities in the learning of mathematics in a more meaningful manner (Wibowo, 2016). Kilpatrick et al. (2001, p.127) stated that:

> Adaptive reasoning interacts with the other strands of proficiency, particularly during problem solving. Learners draw on their strategic competence to formulate and represent a problem, using heuristic approaches that may provide a solution strategy, but adaptive reasoning must take over when they are determining the legitimacy of a proposed strategy.

Adaptive reasoning is a cognitive activity that uses mathematical concepts, facts, and procedures to resolve mathematical problems (Syukriani et al., 2016). This cognitive activity in adaptive reasoning can be detected by means of mental activity: (i) logically labelling the relationship between concepts and problems; (ii) identifying appropriate procedures and methods to explain a problem logically; (iii) familiarizing concepts, facts, and procedures, along with the problem, by means of logical justification (Kilpatrick et al., 2001; Suh, 2008; Ostler, 2011; Herbert, 2014; Loong, 2014). Success in learning mathematics can be seen not only in the ability of students to arrive at solutions, but also as a skill of logical thinking that enables the delivery of both an explanation and justification of the results (Syukriani et al., 2016). Furthermore, students with good adaptive reasoning also have the ability to assess whether a solution is correct and then provide a logical justification (Kilpatrick et al., 2001; Ostler, 2011) .

3.4 The use of the NRC’s mathematical proficiency strands in this study

This section presents two important considerations regarding the use of NRC’s mathematical proficiency strands as a framework in this study: why I chose to use this framework and the how this framework has been implemented in the study.

3.4.1 The choice of the NRC mathematical proficiency strands

This study used the NRC mathematical proficiency strands as a framework to assess the students' performance in mathematics. Kilpatrick et al. (2001) described mathematical proficiency as a set of knowledge, skills, abilities, and beliefs grounded on an association of research in cognitive psychology and mathematics education. Various proficiency frameworks were presented earlier (see 3.2.2). These frameworks are exclusively focused on knowledge of mathematical content, similar to the first four NRC mathematics proficiency strands. The NRC's mathematical proficiency strands, to some extent, share certain crucial parts with other frameworks. For example, they share three parts of the NEAP mathematics framework: conceptual understanding, procedural knowledge, and a combination of these two for problem solving. They also have two of the NCTM process standards in common: problem solving, and reasoning and proof. However, the NRC mathematical proficiency strands differ from other frameworks by focusing on the students' disposition. The disposition strand is not related specifically to mathematical awareness but rather to people's behaviours and values concerning mathematics and how strongly they associate with being a learner and doer of mathematics (Siegfried, 2012). This strand is crucial for students to continue to work and engage in mathematics tasks. Kilpatrick (2011, p. 11) explains the reason for considering this strand as part of mathematical proficiency, although it does not belong to mathematics knowledge, as follows: "We can't have something called mathematical proficiency if students are turned off by mathematics. Proficiency has to have some affective component". The NRC mathematical proficiency framework covers all the skills that students need to be proficient in mathematics in addition to the disposition strand, which is an important component in learning mathematics, but rather ignored in other frameworks.

> For instance, a learner who has a productive disposition (e.g., recognizes the challenge of solving a problem as an opportunity for learning and so perseveres rather than accept a road block and gives up on the problem) is more likely to develop skills that promote working toward solutions for a broad range of problem types. (Adams & LaFramenta, 2013, p. 8)

Also, it is applicable to all mathematics topics, so it is suitable for teaching or assisting in all mathematics lessons. This means that these strands are appropriate for use in learning mathematics proficiently and enhancing students' performance in different topics. According to Kilpatrick et al. (2001), the five mathematical proficiency strands can readily be applied to other mathematics domains, such as measurement, probability, statistics, and geometry. ACARA (2010) described the relationship between mathematic content and proficiency strands. The mathematic content explains the “what” that is to be learnt and taught, while the mathematical proficiency strands illustrate the “how” (manner) of developing or exploring content (ACARA, 2010). Thus, the mathematical proficiency strands explain how students cooperate with the content, namely how the mathematical content is enacted via mathematical activities. As a result, the strands can easily be implemented with regard to different content and levels of mathematics.

Some curricular policy documents and other frameworks, such as the Australian Curriculum: Mathematics (F–10) (2013) and the CCSS (2010), have been influenced by the mathematical proficiency strands. The Australian Curriculum includes proficiencies that emphasize the development of increasingly complex and sophisticated mathematical understanding, fluency, reasoning and problem-solving skills (ACARA, 2013). These five proficiency strands empower students to respond to usual and unusual situations by engaging mathematical strategies to create informed judgments and solve tasks efficiently (ACARA, 2013). In the same vein, the CCSS (2010) has been influenced by the NRC mathematical proficiency strands. According to Adams and LaFramenta (2013, p. 8):

> The strands of mathematical proficiency are reflected in the CCSSM content standards (e.g., focus on students’ development of procedural fluency) and in the mathematical practices (e.g., focus on students modelling mathematics through conceptual understanding of mathematical concepts). The connection between the strands and the aim of the CCSSM is even more apparent given the evident focus on the learner of mathematics as an active participant in the learning process.

The fact that this framework has been used in a leading country, such as Australia, based on many years of research to reform and improve its mathematics curriculum, and also that the strands have had a clear effect on other frameworks and standards such as the CCSS, provides a clear indication that this framework is strong enough to use and be implemented in teaching and learning mathematics and is worth all the attention paid to it.

Kilpatrick et al. (2001) stressed that the five strands are intertwined and interdependent in the development of mathematical proficiency. They can be used to provide a single overall measure of students’ performance, or be looked at in greater depth, with each strand applied separately to evaluate students’ performance. This attribute distinguishes this framework from others and can help identify the students’ weaknesses and areas to be worked on, thus leading to improvements in students’ overall mathematical proficiency.

The title of the NRC framework, referring to mathematical proficiency, distinguishes it and makes it stand out from other frameworks, reflecting what students need in learning mathematics. The NRC committee discussed different terms and phrases that might be employed to characterize what was meant by “successful mathematics learning” and ended up with the term mathematical proficiency (Kilpatrick, 2011). According to ACARA (2010), the term “working mathematically” can be applied to define applications or activities related to mathematics, but does not provide a description of standards or expectations of those activities. Hence, it was proposed to use the term mathematical poficiency based on the recommendations in Adding it Up (Kilpatrick et al., 2001) in the the national mathematics curriculum (ACARA, 2013). The title of the NRC framework has played a crucial role in the impact of its use; for example, saying that a study is focused on improving students’ mathematics proficiency may be preferable to saying that it focuses on students’ performance in mathematics based on (x) assessment framework or (x) standards.

To sum up, this study adopted mathematical proficiency to assess the students' performance for several reasons. The first is that the NRC mathematical proficiency strands differ from other frameworks by focusing on the students' productive disposition, which is an important component for mathematics learning. The second is that this framework is suitable for all mathematics topics and it is easy to apply with any type of mathematical content. The third is that some policies and curricula have been influenced by this framework (e.g. ACARA and CCSS), leading to increased confidence regarding its use in this study. The last one is that by its very name it is distinguished from other frameworks because it reflects what students need in learning mathematics.

3.4.2 The influence of the NRC mathematical proficiency strands in this study

This study has benefited from the NRC framework in three different respects: the design of the teacher's guide in the two modified units, assessing the students' performance in mathematics, and analysing the findings concerning the student dimensions in the research.

The design of the teacher's guide aimed to improve the students' mathematical proficiency using GGS. The lessons in the two units aim to focus on the strands in different respects. First, there is a table before each activity containing information on each step, such as the time of the activity, how to implement it and the strands targeted. The guide includes a concise overview of mathematical proficiency and the related strands and these are also discussed in the teacher guide, together with the definition of mathematical proficiency strands in Grade 8 (for further detail, see 5.5.2).

Another aspect influenced by the five NRC mathematical proficiency strands was the design of the three mathematical proficiency tests conducted in this study to assess the students' performance in mathematics. These three tests investigated the effects of using the GGS-based pedagogy on students' overall mathematical proficiency and more specifically, performance related to the first four strands: conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. Also, a questionnaire was used to assess the students' productive disposition (for more information, see 5.4.2).

Also, this framework affected the analysis of findings concerning the students’ performance, which is presented according to the mathematical proficiency strands. First, the effects of the GGS-based pedagogy on the students' overall mathematical proficiency are presented, followed by the effects on each mathematical proficiency strand (for further information, see Chapter 6 on student-related findings).

Chapter 4 Literature Review

4.1 Introduction

This study investigates the effectiveness of using GGS to develop students' mathematical proficiency. First, it compares the results of two groups of students in a mathematical proficiency test, with the experimental group taught through GGS and the control by means of a traditional approach. Second, it discusses teachers' perceptions of teaching mathematics using GGS. The chapter reviews the literature concerning the specific area of study and comprises seven sections. The first section focuses briefly on using technology in mathematics. The second section discusses the role of teachers and professional development courses in the use of technology. The third examines the application of a Digital Geometry System (DGS) in the mathematics field, including a specific example of one DGS and providing more information on GGS (which is used in this study). The fourth section discusses the literature concerning the potential effectiveness of GGS in fostering student achievement and in relation to each of the five mathematical proficiency strands. The fifth section presents an exploration of studies concentrating on the perceptions of teachers with respect to employing GGS for the teaching and learning of mathematics. The sixth section reviews the literature concerning some aspects of research into mathematical proficiency. Then, the last section points out the research gap, which this study aims to address.

4.2 Importance of using technology in the teaching and learning of mathematics

In today's era of advanced technology, education systems are under increasing pressure to remain up to date. As a result of the advance of world economies, international connections, rapid technological changes, the explosion of information available, national security issues, and several other factors, those responsible for developing educational curricula are being forced to consider how new, innovative teaching approaches can be applied (Wells, 2019). Technology is now a fundamental feature of our daily lives and has thus become increasingly important in modern education (Glaubke, 2007; McCarrick & Katy, 2007). Students are eager to implement facets of their out-of-school activities related to computer games, smartphones, mp3 players, social networks, and various other technologies in the classroom (Gutnick et al., 2011; Rideout et al., 2011).

Modern learning in the 21st century has seen significant transformations across the world. This is mostly due to the need to incorporate 21st-century skills, including decision-making skills, problem-solving skills, innovative thinking skills, information and technological skills, and the ability to come up with new, innovative ideas. All such skills are relevant to education settings and can help students to overcome everyday issues in a systematic way (Jackson et al., 2012). The traditional chalk and talk approach simply requires students to memorize formulae and arithmetic strategies that are written on the board and in student practice books. This is considered an old-fashioned approach that is no longer suited to the needs of students in the 21st century (Saltrick et al., 2011; Jackson et al., 2012), giving rise to many changes in educational approaches and pedagogies in recent years. In mathematics, such changes may address difficulties in learning as many students consider the subject difficult and complicated, one involving language, space, and quantity (Sarma & Ahmed, 2013). It is perhaps a subject particularly prone to educator–pupil misunderstanding. When a teacher draws symbols on the board in a traditional approach, the meaning and conclusions are clear to the teacher, but students may find them very difficult to understand (Sarma & Ahmed, 2013).

The application of technology in the teaching of mathematics is a topic that is receiving a great deal of research attention. It is outlined in the NCTM (2000) Technology Principle that technology is fundamental to the teaching of mathematics, that it influences the type of mathematics taught and how it is taught, and ultimately improves students' learning of the subject. Educational technology has been defined by Robinson et al. (2008) as study and ethical practices that enable learning to occur and enhance performance through the implementation of effective technological resources and processes. Through the implementation of technology in education, students can learn to develop the critical, creative and innovative thinking skills needed to resolve everyday issues (Way & Beardon, 2003; Jackson et al., 2012). The use of technology is relatively new in the field of mathematics education. However, it can be used by all students regardless of their level. To ensure that teaching is effective, the integration of technology in teaching must be carefully planned and managed (Hutkemri & Nordin, 2011). What is more, technology can give students a higher quality of education and equip them with the skills required to solve problems (Meyen et al., 2006). Certain technologies can enable students to see and touch information, which can aid their understanding and enhance their learning capacity. It is widely believed that technology will enhance education, making learning and teaching approaches more relevant, attractive, and meaningful to students. Ultimately, this will enhance students' understanding of educational subjects (Karasavvidis & Kollias, 2014).

A recent study by Emaikwu et al. (2015) showed that technology plays a significant role in changing the classroom setting and restructuring schools to create more meaningful education processes and results-driven learning. The same researchers also found that computers, projectors and videos are the technologies most commonly used in schools. The teaching of mathematics can be significantly enhanced through the use of computers. With regard to the teaching of mathematics, two different systems can be used, namely CAS and Dynamic Geometry Systems (DGSs). CASs (such as Wolform Mathematica, LiveMaths) can help students develop and enhance their computational skills, as well as aiding them in learning, visualizing and practising mathematical concepts. Furthermore, such programs help teachers to create teaching materials, enhance communication with students, and even support distance education (Majewski, 1999). However, a DGS is more concerned with the relationships between points, lines, shapes, angles and various other geometric concepts, for example simulating straight-edge and compass constructions in Euclidean geometry. Moreover, in such systems the geometric relationship underpinning a specific construction can be preserved whilst moving a part of the construction. Students are able to move constructions, alter their size or direction, and even rotate them, without changing the underlying axiom or theorem. It is possible to compute angle measurements, segment lengths, and polygon area automatically.

The use of technology in mathematics classes nevertheless faces certain internal and external obstacles. External barriers include getting access to the technology and having the time, support, and professional development (PD) training to use it effectively. Internal hurdles stem from the teachers' lack of confidence in embracing technology and using it in the classroom. Nikolopoulou and Gialamas (2015) found that teachers' use of technology was limited by inadequate technical and administrative support, finance, equipment, and free access to the equipment already in place. If there is not enough equipment and no computer lab and the internet connection is poor, Hur et al. (2016) argue that it will be difficult to integrate technology in the classroom. Mukama and Andersson (2008) conducted a study focused on IT literacy and concluded that if teachers are to benefit from new tools, such tools must work with school-based curricula and the teachers must have the right training and professional development to critically assess the technology.

In a study examining barriers to technology use, Ertmer et al. (2012) identified that teachers' attitudes and faith in the advantages of integrating technology in their practice were more important in tackling resource and access limitations than any other factor. The authors stated that while external barriers clearly did have to be removed, it was more important to focus on developing the knowledge and skills needed to transform and modify teachers' attitudes and beliefs concerning technology in order for it to be integrated successfully in the classroom. Indeed, Mueller et al. (2008) found that teachers' hesitancy about integrating technology often demonstrated their lack of confidence in their own abilities to use it correctly, so they would benefit from practice sessions and training in general. In addition, not every student believes that computer-assisted instruction can help them learn or has a place in their education. D'Souza and Wood (2003) study showed that many students would opt to use pen and paper as they found traditional techniques reliable and convenient and tended to mistrust software, worrying that their work would be erased, etc. In short, both teachers and students have encountered difficulties in using technological tools and establishing how to work with them in their teaching and learning (Desimone, 2009).

Even though the use of technologies may encounter some barriers (Desimone, 2009) , it has recently become important in education (Glaubke, 2007; McCarrick & Katy, 2007). Students and teachers are becoming more familiar with technological applications, which might make their use in class easier than before. Also, the use of technology in teaching and learning mathematics to some extent seems beneficial and promising (Fabian et al., 2016; Higgins et al., 2019) . Indeed, the teaching of mathematics can potentially be enhanced significantly through the use of new technologies (Higgins et al., 2019). More information about role of teachers and professional development courses in the use of technology in section 4.3. Also, one such technology is DMS, addressed in greater detail in section 4.4.

4.3 The role of teachers and professional development courses in the use of technology

Teachers play a key role in introducing and using technology in the classroom and hence it is essential to analyse their contribution to the integration of technology in the learning process if this is to be carried out effectively (Ferguson, 1997). Wenglinsky (2001, cited in Gilakjani and Branch, 2017) points out that technology use, in isolation, does not have a major impact on learners' achievement and it is teachers who make it a useful tool. Evaluating the usefulness of technology, the NCTM (2015) states that effective teachers maximize the ability of technology to develop the understanding of students, encourage their interest in the subject and make them more proficient in mathematics. Consequently, what matters is not the fact that teachers use technology when teaching mathematics, but the way in which they integrate it in classwork. It has been found that many students and teachers alike use technology for storing data, carrying out calculations, and illustrating static materials, but these types of usage are extremely unlikely to meet the aims cited above (Coban et al., 2001; Ertmer, 2005). In this regard, the NCTM (2000) points out that creating a technological environment could assist teachers in modifying their instructional and teaching approach effectively to meet the needs of their students. Integrating technological tools in standard teaching practice will lead to creative opportunities to support the students' learning and bolster their ability to acquire mathematical knowledge and the skills it offers (Hohenwarter et al., 2008).

Fishman and Davis (2006) and Zhu (2010) agree that teachers are an important factor in applying educational technologies and as more technology is integrated in educational practices the role of teachers and classroom activities will change accordingly. Zhu (2010) notes that teachers have multiple roles: they are experts, figures of authority, role models for their students, facilitators, and delegators. As experts, teachers must have a comprehensive understanding of the subject they teach and be a source of knowledge for the learners. Similarly, as figures of authority, they must know their area and establish the rules the students need to follow. As role models, teachers' words, how they act and what they do in class will influence the learners, who will strive to emulate them and in the process develop their own knowledge and abilities. Teachers are also facilitators and provide guidance and support for learners, showing them how to absorb new information and skills on the basis of what they have already learned and making the learning process itself motivating and positive. Teachers are delegators and hand out the tasks the learners must complete (Gilakjani and Branch, 2017). Finally, Williams et al. (2000) state that teachers must have the ability and knowledge to implement ICT in both teaching and learning.

In today’s environment, it is becoming increasingly important to ensure teachers have the skills to use technology in the classroom, both effectively and efficiently. Many studies have focused on what teachers need to know if they are to integrate technology successfully in their practice (NCTM, 2000, Hughes, 2005, Mishra and Koeler ,2006). Mishra and Koeler (2006) coined the term "technological pedagogical content knowledge" to describe the framework of what teachers need to know if they are going to be effective in integrating technology in classroom activities. Knowing how to use technology has to be connected to content and quality mathematics teaching requires the teacher to understand how technology relates to both pedagogy and mathematics (Hughes, 2005).

Mathematics teachers must help their students overcome any problems they face in the field. Thus, teachers need to have a clear view of the processes of teaching and learning in the classroom if they are to be able to assist their students in developing proficiency in mathematics (Rosli and Aliwee, 2021). Each time the curriculum changes, teachers have to adapt their teaching approach and techniques. As a result, teachers should be offered continuous PD and training, so that they can keep up to date with new tools and skills and fully master how to use technology in their practice. Abuhmaid (2011) argues that pre-service education is not adequate for preparing teachers for a lifetime of teaching and therefore they must be offered CPD and support. So, teacher training courses, whether carried out pre- or in-service, will help inexperienced and anxious teachers embrace technology and demonstrate new ways of implementing ICT as part of their work to those teachers who are keen to use ICT in the classroom (Abuhmaid, 2011).

The last few decades have seen greater attention being paid to teachers' PD (Rosli and Aliwee, 2021). According to Auletto and Stein (2019), there are two major factors involved: teachers must be willing to undertake PD and the PD on offer must be appropriate to the educational system. Teaching skills can only be upgraded through effective PD programmes which focus on classroom practice (Rosli and Aliwee, 2021). As well as improving teachers' skills, PD programmes significantly expand teachers' subject knowledge (Gee and Whaley, 2016). In addition, PD programmes require teachers to upgrade their skills and their knowledge and understanding of the current educational system. Qualified teachers shape the next generation, improve student outcomes, and foster character growth and individual development (Bruckmaier et al.,2016). Darling-Hammond, Hyler and Gardner (2017) stress the importance of teacher PD, because it not only helps in teachers' personal development, but also enhances their knowledge and practice. Darling-Hammond et al. (2017) assert that mathematics teachers' PD can best be upgraded when the teachers' needs and wishes in relation to providing classroom learning are taken into account. A number of studies have identified certain factors which enhance the likelihood of PD programmes being successful. Research has established that teachers need to be involved in the PD, to be committed and engaged in the activities, to have a positive attitude before and after undertaking PD, and to be driven and able to implement what they have learned in both teaching and learning (Rosli and Aliwee, 2021).

Hence, teachers play a crucial role in integrating technology into classrooms, and they must have the knowledge and experiences to integrate technology into classrooms. In addition, good use of technology in the class may lead to creative opportunities to support the students' learning and bolster their ability to acquire mathematical knowledge and the skills it offers. So, teachers must be trained and provide training courses to improve their use of technology.

This study used PD to improve and enrich teachers' use of GGS in their lessons. GGS is a tool that can be used to stimulate learners and to encourage their ability to think mathematically by using an enquiry-based approach (Kul, 2013). A number of studies have found positive responses among teachers who use GeoGebra in terms of the benefits it offers to teachers and students alike (Baltaci et al., 2015; Zakaria and Lee, 2012).

4.4 Dynamic Mathematics Software (DMS)

The key purpose of DMS is to enhance students' mathematics performance by immersing them in mathematical activities (Gutiérrez et al., 1999). DGSs, as part of DMS, are regarded by Hoyles and Noss (2003) as important pedagogic tools that are designed specifically to enhance the exploration of mathematical domains and to create a setting conducive to experimenting with geometrical objects. This is because they enable the generation of experimental environments in which collaborative learning and investigation are encouraged. Several studies have explored DMS (e.g. Artigue, 2002; Ruthven & Hennessy, 2002) and have found that such software can be an extremely effective tool in mathematics education.

This research used the DMS GeoGebra, assessing the impact of using the software on students' mathematics proficiency. As technological advancements have been made, there has been an increase in the development of software providing effective visual learning environments for students. As GGS includes both symbolic and visualization features (e.g. the direct coding of equations and coordinates), it is regarded as a CAS. It is also a form of DGS as it involves activities relating to points, segments, lines, and conic segments and enables the dynamic relationships between the concepts to be identified, the latter being a key GGS feature that can be accessed through CAS and DGS. There are many benefits in using DGS (and GGS specifically) in mathematics classes and thus they are important tools for both teachers and students. Such benefits include instant feedback, quick execution of activities, and opportunities for multiple representations. These are further detailed below.

4.4.1 Instant feedback

The use of technology in teaching of mathematics has many advantages that cannot be offered through traditional classroom teaching approaches. DMS has the potential to provide quick and reliable feedback. This feedback is impartial and non-judgemental. It is important that students feel motivated to engage in learning activities within the digital learning environment. Through instant feedback, students can receive responses to their input immediately after completing an activity (Olsson, 2015). However, Olsson (2015) also warns that computer-generated feedback may be insufficient and even unhelpful for some students. Both Clements (2000) and Olsson (2015) suggested that working in real time, students feel more motivated to create and test new ideas.

Feedback plays a fundamental role in interactive learning software (Fest, 2010). In cognitive learning theory, it is stated that for software to be effective, it must provide students with additional information about their performance (Gono, 2016). Gono (2016) asserted that constructivist learning theory is based on the principle that a user must be able to control the time and level of feedback received. In addition, the feedback system must be able to review a variety of potential solutions and techniques that can be applied in an open learning environment (Fest, 2010). A computer can provide extensive feedback to any student at any time, whereas a human teacher would not be able to do this. Thus, technology can significantly help the learning process. For example, by receiving immediate feedback, students can learn and memorize modulus functions at their own pace based on the common features of the graphs presented to them on the computer screen (Olsson, 2015; Gono, 2016). Gono (2016) also added that instant feedback can enable students to take control of their learning, which is a key aspect of constructivism.

Fest (2010) identified three key types of interactive feedback, as follows:

1. Timed feedback: feedback may be provided straight away following user interaction or as required by the student. It may be provided following a completed session or at the end of the entire learning process.
2. Information feedback: "verification feedback" and "elaboration feedback" are the two most significant types of feedback. While “verification feedback” simply informs students of whether or not their answer was correct, “elaboration feedback” provides the correct solution alongside a detailed explanation. Sometimes, it can be more beneficial to display only partial solutions. Thus, there are multiple levels for this feedback dimension.
3. Presentation feedback. Finally, feedback can be presented to students in various ways. It may be provided visually or acoustically. In terms of the former, it may be presented in graphical (iconic) or textual form. The latter may be presented in an animated or static form.

GGS (the software employed in this research) provides efficient and effective feedback for GGS exercises containing a feedback icon. Olsson (2015) highlighted the key benefits of

GGS, stating that it is not only effective for exploring mathematical patterns, but also contains advanced tools that can be used to incorporate feedback in learning materials.

4.4.2 Quick execution of activities

Using DGS, students can engage in and complete several tasks and questions in a short period of time. It is more time-efficient than conventional teaching approaches. It can also help teachers to carry out their lessons more quickly and with less effort. Gono (2016) explained that DMS allows learners to use many different techniques when attempting to solve a mathematical problem. It allows them to observe patterns and to make and justify generalizations. He also asserted that, in contrast to the heavily time-consuming traditional learning methods (i.e. pencil-and-paper environment), studying arithmetic and algebraic algorithms through DMS gives students more time to learn and understand topics, and to develop reasoning and application. It thus equips learners with the tools needed to take control of their learning (Gono, 2016). The CAS part of the GGS enables students to make calculations quickly and accurately. The student only has to enter the equation or task in the input section, press enter and wait for the result.

Horzum and Ünlü (2017) found that pre-service mathematics teachers believed GGS could save them significant time and effort by allowing them to create drawings on the computer. Moreover, Hohenwarter and Preiner (2007) asserted that GGS is the most commonly used software in the teaching of mathematics due to its features and advantages (e.g. affordances and time-saving benefits). The software can help teachers prepare classwork in advance. They can then give the pre-prepared activities to students in the class. Hohenwarter et al. (2008, p. 3) also stated that:

> Teachers could also create self-contained dynamic worksheets prior to the lesson […] Being able to customize the user interface of the integrated interactive applets (e.g. showing or hiding the algebra window, reducing the number of available tools, displaying the toolbar help), teachers can decide beforehand how much freedom or guidance they want to provide for their students and which features and tools should be available for their students.

In addition, GGS can be used freely in both online and offline modes without any licencing issues (Zulnaidi et al., 2020), allowing students and teachers to work in different locations and at different times. Moreover, it can facilitate the production of more accurate shapes and better pictures than afforded in traditional teaching and learning environments very quickly.

The software can thus provide opportunities for students to tackle mathematical problems quickly and effectively.

4.4.3 Capacity for multiple representations

Multiple representations play a fundamental role in the teaching and learning of mathematics and have received a great deal of research attention over the last decades. In mathematics education and science, the need for students to use multiple representations has been widely recognized (Gono, 2016). This is reflected in the increased interest in the development of free technology that facilitates different representations of mathematical concepts. Also, this is primarily due to a recommendation made by the NCTM that multiple representations should be used during mathematics education. Moreover, the NCTM's 2001 yearbook discussed the issue of representation in school mathematics:

> Different representations of problems serve as different lenses through which students interpret the problems and the solutions. If students are to become mathematically powerful, they must be flexible enough to approach situations in a variety of ways and recognize the relationships among different points of view. (NCTM ,1989, p. 84)

Here, there is an emphasis on the need for the application of digital technologies to make representations more accessible for students. DGS provides a multiple representation environment that enhances students' comprehension of mathematics. In addition, DGS makes it easier for students to grasp mathematical ideas, representations, and interrelationships than traditional teaching techniques (Pierce et al., 2011). The system helps students in a computer-based environment simulate, model, and verify mathematical relationships (Olivero & Robutti, 2007). Furthermore, DGS creates efficiencies for educators in representing and illustrating mathematical concepts, especially in comparison to typical paper-and-pencil methods (Laborde, 2007).

Strong correlations have been found between the use of multiple representations and improved mathematics learning among students. In particular, multiple representations have been found to improve students' understanding of key mathematical concepts. Indeed, it is not possible for one representation to describe a mathematical concept completely and thus multiple external representations can significantly help learners (Duval, 2006). Elia et al. (2008) too noted that multiple representations can help students to understand concepts far more effectively than a single representation. They also enhance students' understanding of more advanced topics, such as multivariate calculus (McGee & Moore-Russo, 2015).

Özgün-Koca (2001) pointed out that the application of multiple representations can have significant advantages for meaningful algebra learning. In addition, it has been found that multiple representations displayed on one screen can help enhance students' knowledge development and depth of understanding (Bayazit & Aksoy, 2007). Further research by Duncan (2010) concluded that multiple representations are able to stimulate investigations and aid students in enhancing their learning. The findings of both Özgün-Koca (2001) and Pitt (2004) highlighted the effectiveness of multiple representations in the teaching of college-level algebra. Both studies found that through multiple representations students learned how to make associations between different types of representational modes. Another important study on the topic was that of Hwang et al. (2007), who found that the use of multiple representations in mathematics plays a fundamental role in the solving of mathematic problems. According to Bayazit (2011), most students rely on the symbolic representation of conjugation and their capacity to alternate between iconic and algebraic conjugation symbols is crucial in understanding conjugation as a whole. Therefore, it is possible that the multiple representations offered by DGS improves the teaching and understanding of mathematics.

Nonetheless, several drawbacks have been identified concerning the use of multiple representations through DGS in the field of mathematics education. Those who advocate for constructivist teaching approaches assert that the process of dynamically linking representations through technology only allows students a passive role in their own learning (Ainsworth, 1991). Moreover, in Yerushalmy's (1991) study, a mere 12% of students were able to solve problems involving both numeral and visual representations despite having extensively studied functions through multiple representations. For the majority of questions, students responded using just one representation. Duval (2006) discovered that when the roles of representations in source registers and target registers were reversed during multiple representation conversion tasks, this significantly changes the problem that students were asked. It was found that students could easily present equations in graph form, but were unable to give the correct answers when registers were reversed (i.e. identifying the correct algebraic function from information in the graph).

As previously stated, there are several different types of DMS. In this research, GGS was used. CASs (including Mathematica, Maple) and DGS (including Geometer's Sketchpad(GSP), Cabri 3D Geometry(C3D)) are highly effective technological tools that can be used in mathematics education. GGS, however, is very user friendly and can help students find connections between geometric, algebraic, and numerical representations. The software provides opportunities to cover important mathematical topics, including geometry, algebra, calculus, and arithmetic. GGS allows users to create multiple representations to enhance students' understanding. This is also beneficial for teachers. The software also allows students to move between visual mathematics options on various screens and dynamically change the objects. Gono (2016) noted that on-screen images can be made larger and functions can be adapted and even colour coded. GGS is discussed in more detail in 4.3.4.

4.4.4 GeoGebra Software (GGS)

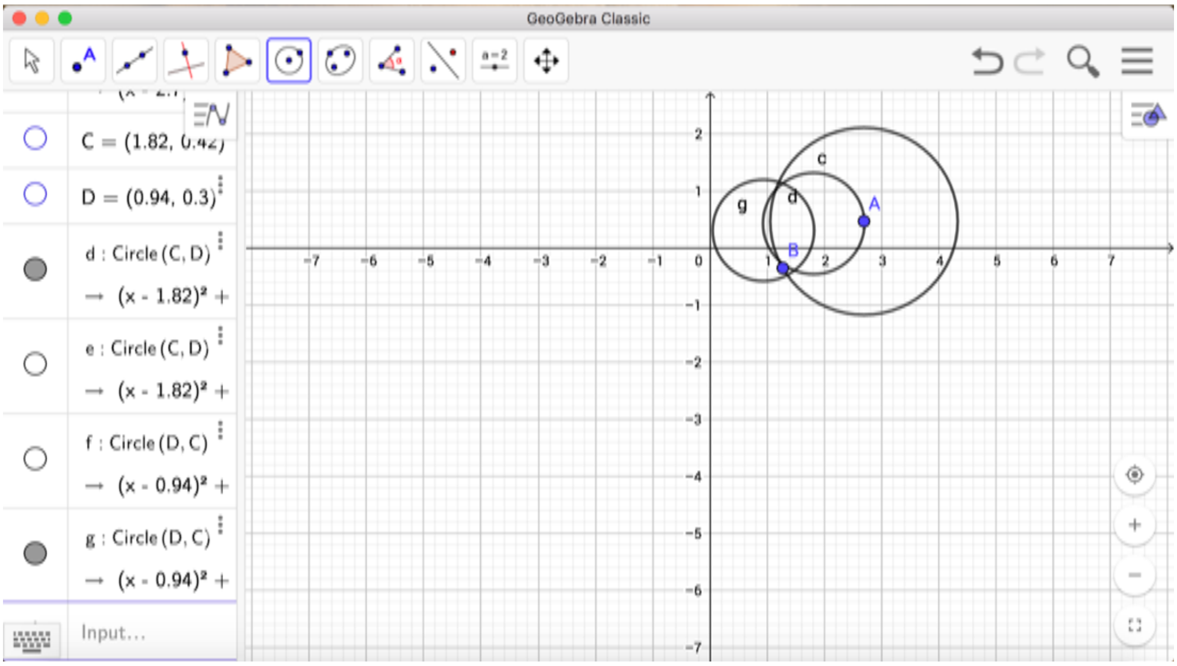
In 2001, Markus Hohenwarter began research on GGS for his master's thesis, developing a software program for constructing and manipulating dynamic geometry and algebra in the plane (Hohenwarter & Fuchs, 2004; Hohenwarter & Lavicza, 2007). In 2002, after releasing GGS on the Internet, Hohenwarter was suddenly contacted by several teachers who expressed their interest in using GGS in their classrooms (Hohenwarter & Lavicza, 2007). Since it was officially launched in 2006, GGS has also gained increasing international attention in the field of mathematics education (Hohenwarter & Jones, 2007; Hohenwarter & Lavicza, 2007; Hohenwarter & Preiner, 2007; Chrysanthou, 2008; Preiner, 2008; Diković, 2009; Mehdiyev, 2009). The main goal during software development was to create a complex, agile program that incorporated geometry, algebra, and calculus (spreadsheet and computer algebra extensions are soon to be added to the program), handled separately by other software, in one easy-to-use platform (Hohenwarter & Lavicza, 2007; Hohenwarter et al., 2009).

4.3.4.1 Overview of GGS

GGS is a dynamic, advanced mathematics program that integrates geometry, algebra, spreadsheets, graphing, statistics, and calculus in one user-friendly platform for all levels of education. The name GeoGebra comes from merging geometry and algebra (Hohenwarter & Fuchs, 2004). According to its official website (geogebra.org, 2020), the use of GGS has grown rapidly as millions now use it across all regions. Furthermore, it is a leading provider of innovative mathematics applications, promoting science, technology, engineering, and maths (STEM) education, and education technologies around the world (geogebra.org, 2020). The program was developed to combine features of dynamic geometry applications such as C3D, GSP and CASs such as Instant Derive and Maple in a user-friendly, integrated system for learning and teaching mathematics (Hohenwarter et al., 2009; Saha et al., 2010).

GGS is free, with no license requirements or restrictions on permissions associated with its use. It can be freely downloaded from *www.geogebra.org* and installed on a personal computer or laptop. It is also offered as an iPad application and for all smartphone devices. Arguably the best aspect of this platform is that it is always available and functional even without an Internet connection. Thus, it can be used by teachers in their classrooms to teach and improve student learning anytime and anywhere. GGS has several windows displays and it has the function of representing every point and every input in an algebra frame. GGS allows free movement between displays and changes any algebraic representation shift automatically. According to Chrysanthou (2008), GGS provides good visualization of changes in geometry and algebra relationships, and is in fact more accurate than using paper and pencil (see Figure 4-1). Diković (2009) described some features of using GGS:

- GGS is easier to use than a graph calculator.
- GGS provides a user-friendly interface, menus in different languages, commands, and support.
- Students can customize their creations easily by making changes in the interface.
- GGS was created to help students gain a better understanding of mathematics.
- The input of algebra enables the user to generate new objects or change existing ones through the command line. The worksheet files can be published as webpages with ease.



○ C = (1.82, 0.42)

○ D = (0.94, 0.3)

● d : Circle(C, D)
→ (x - 1.82)² +

○ e : Circle(C, D)
→ (x - 1.82)² +

○ f : Circle(D, C)
→ (x - 0.94)² +

● g : Circle(D, C)
→ (x - 0.94)² +

Figure 4-1 Snapshot from GeoGebra software

4.3.4.2 Integrating GGS in classrooms

Many platforms are currently being used to teach and learn mathematics besides GGS, such as GSP, Mathematica, Logo, Excel, and so on. Among these, GGS is arguably one of the most popular. According to Belgheis and Kamalludeen (2018), among the state-of-the-art educational resources, GGS is attracting the most interest from students, mathematics educators, and researchers for its ability to improve mathematics pedagogy. It also has features for dynamic geometry applications, computer algebra, and spreadsheets, which can also be used to teach 2D or 3D geometry, algebra, statistics, calculus, and other aspects of mathematics, all in one integrated platform (Antohe, 2009; Hohenwarter et al., 2009; Rincon, 2009; Zulnaidi & Zakaria, 2012).

Diković (2009) highlighted some advantages of using GGS:

- GGS promotes mathematics learning for students, supporting multiple presentations, creative experimentation, directed learning, and discovery.
- GGS encourages teachers to employ and test mathematics visualization technology; mathematics research; on-site or remote immersive mathematics classes; mathematics and its applications, among others.

Nonetheless, Diković (2009) also pointed out some deficiencies in using GGS, such as the following:

- Students with no previous programming experience will likely struggle to insert algebraic commands in the input box. Even though basic commands are not hard to understand and learn, students may feel frustrated, confused, or embarrassed.
- Some methodological methods, such as independent research and experimentation, are not necessarily appropriate for all students.
- From a technical perspective, GGS does not have an in-built animation extension. For future models, including GGS animation modules will be an essential technical feature.
- Future releases of GGS should provide more symbolic functionality of computer algebra systems, further strengthening the scope for complex mathematical analyses and 3D extensions.

Furthermore, studies addressed issues with the use of GGS in classrooms. For example, Erkek and Işıksal-Bostan (2015) encountered issues in the use of GGS, such as not all students having the same level of proficiency in using the system, resulting in skewed class participation. Another disadvantage lies in the fact that some tasks may not align with GGS usage. Hence, students who receive feedback from GGS might take a broad view easily without reasoning. Thus, students may become easily persuaded by the general validity of a conjecture through continuous transformations of the DGS object. Also, Mehanovic (2011) found that the introduction and the integration of GGS proved challenging and complex for both students and teachers. Implementation of the software in mathematical activities complicated the didactic situation, resulting in a noticeable distinction between the learning and the software (Mehanovic, 2011). Regarding the mathematics teachers' work with GGS, research has revealed that they encountered three types of barriers preventing them from using the software's full didactic ability in their mathematics teaching:

- Technical: The teacher cannot run the program in the expected manner.
- Epistemological: The teacher is not aware of GGS's didactic ability and how to use it in a way that encourages the learning of integrals by students.
- Didactic: The teacher is not aware of the complexities of technology-based environments and is not satisfied with his/her ability to implement the software integration process in his/her teaching without external assistance and support. (Mehanovic, 2011)

Moreover, the use of another DGS may not prove less effective than the use of GGS in learning and teaching mathematics. For example, Meng and Sa (2013) and Güven and Kos (2008) found that GSP is a useful geometry tool for teachers since it offers an atmosphere in which students can explore geometric relationships and create and test conjectures. Such characteristics give students the chance to explore, create, and learn three-dimensional geometry since the software shows specific measurements, such as angle, length, and surface area, on the screen (Güven & Kosa, 2008). GSP helps students construct accurate figures and handle them intuitively (Oldknow et al., 2010). GSP also allows learners to create mental models that consider geometric shapes and their characteristics. Satterfield (2001) considered that GSP is also ideal for easy adaptation and also serves teachers as a show device so that students can still understand the visual components of geometry if the teacher has limited computer measurement.

Another DGS that may prove helpful in teaching and learning mathematics is the C3D application. C3D is a learning tool applicable in teaching and learning geometry that makes it easy for students to understand mathematical problems (Muhammad et al., 2017). C3D can enable visualization to understand geometry, making it useful for learning and teaching the subject. For instance, students can imagine 3D forms with the aid of C3D software, thus finding a way to solve problems (Kepceoglu, 2010). The system is exciting for learners and makes it easier for students to grasp lines and angles (Nurlaelah, 2018). Thus, the primary reasons for the popularity of GGS in comparison to GSP and C3D include the fact that it is free and covers the majority of mathematics topics, while GSP and C3D are solely designed to focus on geometry topics.

4.5 Potential effects of the use of GGS on students' performance in mathematics

The use of technology in everyday teaching practice has been shown to have significant benefits for both teachers and students as far as the development of mathematical knowledge and skills is concerned, but also in consolidating them over time (Hohenwarter et al., 2008). The core advantage of technological applications in mathematics education is the time saved, which can then be reallocated to higher-order skill acquisition and application, such as reasoning, reflection, and problem solving. Studies have also found that students are able to acquire additional ambient skills (such as lateral thinking) in classroom environments with consistent application of technology (Hollebrands, 2007). A central aspect here is the use of technology to visualize mathematical concepts and data. Other studies have found that data and object visualization foster deeper and more long-lasting comprehension among students (Hohenwarter et al., 2009).

GGS has been successfully deployed in classrooms, with a heavy emphasis on demonstration and modelling, as well as classroom-based group work. GGS allows students to gain an improved understanding of abstract concepts and correlate various mathematical concepts with their personal experiences through visualization and complex modelling (Zulnaidi et al., 2020). It has been demonstrated to be particularly effective in teaching calculus (Hohenwarter et al., 2008), as well as other visualization-heavy topics such as geometry and algebra (Antohe 2009; Haciomeroglu et al., 2009). In particular, a relatively recent study showed that student comprehension across geometry, algebra, and calculus was improved using GGS (Rincon, 2009).

In what follows, six sub-sections present the potential effects of the use of GGS on students' performance in different areas of mathematical learning. The first sub-section presents the potential effects of GGS use on students' achievement compared to other teaching and learning approaches. The other five sub-sections present the potential effects of the use of GGS on students' conceptual understanding, procedural fluency, strategic competence, and reasoning, and on their attitudes towards learning mathematics. These five parts cover the NRC mathematical proficiency strands, which constitute the framework for this study.

4.5.1 Potential effects of the use of GGS on mathematics achievement

There is a burgeoning literature on the role of technology in teaching mathematics (e.g. see Fabian et al., 2016; Higgins et al., 2019). Some studies have explored the specific use of GGS as a teaching tool in mathematics education both at primary and secondary level (Zengin et al., 2012; Doktoroğlu, 2013; Martinez, 2017; Alkhateeb & Al-Duwairi, 2019).

One early study found that students working with GGS scored higher than a control group undertaking geometry without it (Zengin et al., 2012). The study used a quasi-experimental approach, in order to determine the impact of dynamic mathematics software on student achievement in the teaching of trigonometry. This study lasted for five weeks, which is considered good enough to present more differences between the two approaches used in the study. However, the study has not shown how they chose their samples, and there is no information about how students engaged with GGS. Also, a recent study by Martinez (2017) presented promising findings in terms of the overall benefits of GGS use on student test scores in mathematics at middle-school level, but cautioned the need for more research and the implementation of the software in combination with other technological applications in high-school classrooms (e.g. tablets or laptops).

More recently still, research has tested the efficacy of GGS and other mobile applications (e.g. GSP) for teaching geometry and found enhanced student outcomes, including comprehension of basic concepts (Alkhateeb & Al-Duwairi, 2019). The study also found that GGS had a greater impact on student learning than GSP.

Doktoroğlu (2013) found mixed results in the use of GGS. He investigated the benefits of teaching linear equations using GGS with seventh grade students in comparison to their usual instruction, finding that the use of the Cartesian coordinate system and linear relations using GGS failed to demonstrate any significant results. The study also indicated that teaching the graphing of linear equations using GGS had a significantly positive influence on the achievement of seventh-grade students. It should be noted that this intervention study took place outside the students' usual school hours, which could have resulted in the participants choosing to not take the study particularly seriously. Also, they had already experienced using GGS, which may therefore not have made any significant difference to their learning ability as measured in the study.

Overall, research has found that student engagement with mathematics – specifically with abstract concepts and objects – is enhanced using GGS (Rincon, 2009). Again, the main benefit is understood to be enhanced engagement through visualization (pictures, images, graphics) boosting conceptual knowledge by helping students see problems in concrete terms to which they can relate. Thus, teaching with the aid of GGS can potentially take the most abstract mathematical topics and help relate them to aspects of students' lives to enhance engagement and ultimately learning.

4.5.2 Potential effects of the use of GGS on conceptual understanding

Conceptual understanding plays an important role in mathematics education. Using GGS in teaching and learning mathematical concepts can aid learners to order mathematical concepts that are connected with learning in school and outside it (Furner & Marinas, 2013). As Hohenwarter and Jones (2007) noted over a decade ago, GGS brings together the algebraic, graphical and database formats of mathematical objects. Thus, using GGS fosters students' conceptual understanding of intended mathematics topics (Ocal, 2017). The use of GGS also aids teachers, who are able to present information more quickly and in a straightforward manner, and assists in testing students' conceptual knowledge or providing feedback in a more effective way (Furner & Marinas, 2013).

GGS is also part of a new trend in activity-based learning in mathematical education, an approach that emphasizes "real world" application of basic mathematics concepts and ideas (Zulnaidi & Zamri, 2017). In this regard, GGS is said to enhance the learning environment through dynamic application and the use of the technology to undertake and test complex models of real-world issues and challenges, such as design or complex problem solving, topics that are often left until higher high-school classes or even university (Bu et al., 2011).

The visualization and use of engaging graphic interfaces and step-by-step learning and problem solving boosts student engagement and performance with GGS (Hohenwarter et al., 2008; Rincon, 2009; Zulnaidi & Zamri, 2017). Thus, GGS helps students clearly understand abstract concepts and then relate them to mathematical functions more readily. Since GGS makes it possible to deliver mathematical concepts through animation-based activities built into the software, teachers are also very enthusiastic about its application.

Similarly, studies have emphasized the practical dimensions of GGS use, enabling students to "picture" functional algebra in ways that are concrete, rather than abstract and remote (Antohe, 2009; Zulnaidi & Zamri, 2017). One additional gain is the ability to assess student solutions online or remotely, thus boosting students' interest in mathematics and their cognitive abilities (Furner & Marinas, 2013).

A significant amount of research has focused on GGS and related software in the teaching of geometry. For example, back in the early 2010s, Kutluca (2013) found that using GGS had a significant positive effect on students' understanding of the Van Hiele geometry model. In the same year, a widely cited study looked at how the software could be used to play around with circles and other spherical shapes among primary students (Shadaan & Leong, 2013). The research showed that students in the experimental group outperformed those in the control group and also indicated a strong interest in using GGS for other facets of mathematics education, including other aspects of geometry. More recently, research from Delhi University found that mathematics courses in which GGS and related software were used across the high-school curriculum improved understanding of concepts in comparison to conventional methods (Jelatu & Ganesha, 2018).

4.5.3 Potential effects of the use of GGS on procedural fluency

In their seminal text, *Conceptual and Procedural Knowledge: The Case of Mathematics*, Hiebert and Lefevre (1986) divided procedural knowledge in mathematics into two categories, namely: 1) knowledge about the language (the symbols used to express the form) and 2) knowledge of the rules or algorithms used in mathematical problem solving. Procedural knowledge is actually a basic skill and it is crucial that every student masters it by the end of primary school (Zulnaidi & Zamri, 2017). This is where technology comes in, given that teachers are able to convey basic mathematical skills in a way that is straightforward but also highly effective, especially at lower grades (Zulnaidi & Zamri, 2017).

Given the importance of procedural fluency and procedural knowledge in mathematics, they have been the subject of much research. In terms of GGS, several scholars have tested the efficacy of the software in this particular domain (e.g. Zulnaidi & Zakaria, 2012; Hutkemri, 2014; Dijanić & Trupčević, 2017; Ocal, 2017; Zulnaidi & Zamri, 2017). All but one of these studies found that GGS can boost students' procedural fluency in mathematics overall, which is quite a positive result. For example, Zulnaidi and Zamri (2017) employed a quasi-experiment approach on 345 students, divided into two groups, to investigate the influence of GGS on procedural knowledge related to functions. Students in the experimental groups were found to have higher levels of mathematical procedural knowledge in comparison to those learning mathematics through conventional methods. Hutkemri and Zakaria (2014) noted that GGS boosts students' conceptual and procedural knowledge of the all-important limit function, which teachers often find difficult to convey effectively. Dijanić and Trupčević (2017) studied the effect of digital textbooks consisting of GeoGebra interactive applets on students' conceptual and procedural knowledge in mathematics. The results can be considered reliable and significant, due to the number of students (703 students) and schools (12 schools) involved. The study found that students' procedural fluency to be definitely and statistically significantly boosted by the use of the software, although they could not conclude why this was so.

One study found that the efficacy of the software was still unproven in the field of mathematics (Ocal, 2017). The research investigated the use of GGS in classrooms to teach mathematics in terms of differential calculus and tested for enhanced outcomes in conceptual and procedural knowledge. While Ocal (2017) did find that student performance on test scores in applied mathematics was higher among those using GGS than those not, this was not reflected in tests for conceptual and procedural knowledge.

4.5.4 Potential effects of the use of GGS on problem solving (strategic competence)

According to the NCTM (2008), it is crucial to design mathematical interventions in the classroom that engage and motivate students, especially in the domains of problem solving and conceptual comprehension. This builds on prior research that emphasizes the role of technology in reducing the cognitive load associated with a given problem set by providing a ready basis for acquiring the problem schema (Limjap, 2002). In this vein, Velikova and Petkova's (2019) recent study showed that new learning technologies emphasizing interactive mathematics offer a range of platforms and applications, as well as modelling mechanisms, that can aid student problem solving at the university level.

In this context, DGS – of which GGS is a leading example – offers a range of innovative interface options that can make it much easier for students and teachers alike to engage with the complex mathematics involved in digital work. According to Christou et al. (2005), the DGS interface also provides students with the opportunity to use visual reasoning in mathematics and helps, via the dragging function, to generalize problems. The authors note how DGS, as a mediation tool, encouraged students to use it in problem-solving and posing the processes of conjecturing, modelling, analysing and generalising (Christou et al., 2005). The "dynamic geometry environment" within the software thus supports the relationship between the construction of new problems and the usage of problem-solving strategies and high-level problem solving. A key function here is the experimental environment that such software provides, enabling instructors and learners to adjust their models in a highly effective and efficient way compared to the traditional paper-and-pencil approach, which was costly and naturally very, very time consuming (Marrades & Gutiérrez, 2000; Straesser, 2002). The use of GGS has a real impact on developing non-routine problem solving; the GGS learning method provides an easy way for students to understand mathematics problems that they find in daily life (Ramadhani & Narpila, 2018). Kim and Md-Ali (2017) found that the use of GGS has the potential to enhance students' knowledge and skills, particularly when it comes to supporting the use of problem-based learning for 21st century learning.

Bu and Haciomeroglu (2011) discussed strategies to assist in mathematics problem solving using GGS, including: (1) exploration and planning; (2) comprehension; (3) showing results; (4) guessing and checking; (5) managing complexity. The study found that understanding dependency and managing complexity strategies were applied more effectively using GGS, even though traditional instruction methods could also be effective. The authors concluded that a dynamic mathematics learning environment, such as offered by GGS, simply has more technological tools and functions available for teachers and students alike to bed down their knowledge and keep their reasoning and higher level thinking skills at the top end of the range.

4.5.5 Potential effects of the use of GGS on reasoning

Mathematical reasoning represents a thinking process with a focus on making sense of mathematical ideas and concepts that are essential to procedures (Bieda et al., 2014). The importance of mathematical reasoning was laid out in a policy paper by the NCTM over a decade ago (NCTM, 2009). The White Paper established crucially that reasoning in mathematics must be applied in real-world contexts, beyond the decontextualized problem sets in classrooms where conclusions are deduced logically, based on assumptions and definitions, in a structured way. The paper thus emphasized the role of "creative informality" in the process:

> Using technology to display multiple representations of the same problem can aid in making connections… When technology allows multiple representations to be linked dynamically, it can provide new opportunities for students to take mathematically meaningful actions and immediately see mathematically meaningful consequences fertile ground for sense-making and reasoning activities. (NCTM, 2009, p. 14)

These premises have been tested in terms of the efficacy of new information technologies such as computer software and web-based applications (O'Donnell, 2006; Higgins et al., 2019). An early and pioneering study in the field found that metacognitive feedback from a computer-based program can enhance student reasoning in mathematics over and above simple result-based feedback (Kramarski & Zeichner, 2001). While the study is now nearly 20 years old, it builds on formative work that overall mathematical proficiency is driven ultimately by students' mathematical procedural and conceptual reasoning. Very recently, Abdurrahman et al. (2020) underscored the role played by electronic support tools in boosting mathematical reasoning and thus advancing students' mathematical procedural and conceptual understanding.

GGS has been proven in the studies already cited, but also in others, to be a powerful tool for promoting a range of real-world problem-solving and critical thinking skills (Bitter & Pierson, 2002; Wiske et al., 2006; Nat et al., 2011; Dogan, 2018). One recent study found that students' mental processes can go into hectic overdrive when they use GGS to build, explore, and observe geometrical characteristics in the mathematics classroom (Dogan, 2018) and this research built on a couple of earlier studies (Diković, 2009; Shadaan & Leong, 2013).

Another analytical dimension has concerned the issue of creative thinking and interactive applications like GGS. One recent study explored how the use of scenario-based software tools to structure problem set constructs facilitated an enhanced set of outcomes among students in a challenging context (Olsson, 2017). Specifically, Olsson's study found that GGS promotes three key traits one would want to see any young person develop early in life, namely teamwork, innovative mathematical reasoning, and high-order problem-solving skills. GGS could then be used in a dynamic, iterative way to test the given outputs (i.e. to verify or falsify a given set of hypotheses). GGS's dynamic, iterative functionality is the underlying driver of its use as a tool for creative thinking and problem set management. However, GGS does require – as the study found – a clear set of foundational instructions, which means it cannot be used as a stand-alone tool.

4.5.6 Potential effects of the use of GGS on students' attitudes

As has been discussed at length and highlighted recently by Huscroft et al. (2019), technology is now central to teaching and learning in mathematics at the primary and secondary levels. The impact of technology is, however, mediated by student-level factors, among which student attitudes are crucial. Research has shown that student attitudes play an outsized role in influencing learning outcomes (McMillan, 2011; Rimland, 2013). Here, the benefits of instructional technology have been shown to depend on how teachers and students engage and the “usability” factor looms large in this context (Mo, 2011). More concretely, as Higgins et al. (2019) have shown, the use of technology in the classroom affects students’ behaviours (such that students are motivated to approach learning proactively), emotions (in terms of their engagement and motivation), and mental engagement (the cognitive investment needed to comprehend content). Instructors who integrate technology effectively are able to boost all three dimensions of engagement to enhance collaboration and ensure students engage in the learning process (Higgins et al., 2019).

GGS has also been tested recently in terms of its impact on student attitudes or motivation with regard to mathematical problem solving and mathematics learning overall (Murni et al., 2017). The authors found that when GGS is used in the context of “discovery learning”, students resolved issues in the problems addressed in a timely and effective manner, drawing on the software’s instant answer mechanism. The research also noted that GGS can be applied with a much wider range of topics than covered by the current study, potentially even beyond mathematics to include economics, finance, and theology.

One promising recent piece of research has considered the role of GGS in enhancing overall mathematical thinking, conceptual understanding, and cognitive development in school-aged children (Aydos, 2015). The study investigated the impact of a GGS-supported environment for teaching limits and continuity on students’ conceptual knowledge and attitudes towards learning mathematics by means of technology. The experimental group was found to have a higher level of achievement than the control cohort in an open-ended test of basic mathematics concepts. In terms of student attitudes to mathematics education overall, Aydos (2015) found that GGS-related applications could boost calculus and geometry outcomes through improved motivation. This study’s sample consists of 34 students from the unique high school for gifted and talented students in Turkey, so its results may differ if used with the normal students. This is because of this school its environment differs from the others school and the type of students also, not as the other students. Rosyid and Umbara (2019) undertook an in-depth study across the entire US state of Missouri in high school-level mathematics. The results revealed that the students predominantly demonstrated positive attitudes, as shown by 66.59% being in favour of the implementation of the GGS-assisted Missouri Mathematics Project, a testing program of all sophomore high-school students in the state.

Finally, the software has not only been tested in Western contexts or classrooms in advanced industrial societies. For example, one recent study sought to test the impact of GGS on student outcomes in geometrical transformations and attitudes towards learning mathematics in high schools in Zambia (Bwalya, 2019). Bwalya (2019) found a statistically significant improvement in student results from the application of a course of six months. The impact of GGS thus translates in a cross-cultural context.

Even though some studies have found that the use of GGS does not significantly enhance results, and its effects do not differ from those achieved using conventional methods (e.g. Doktoroğlu , 2013 ; Ocal, 2017, most studies in the previous six sections make it possible to conclude that the use of GGS in the teaching and learning of mathematics has a positive effect on students' performance, more so than the traditional approach. However, not all the studies reviewed considered every NRC strand, rather focusing on the students' achievement in general or on specific strands. In contrast, this study focuses on all strands at the same time and thus provides a clear view of the effects of using GGS.

4.6 Mathematics teachers' perceptions of using GGS

There are a number of factors that influence the effective use of technology in the mathematics classroom. Teachers play a significant role as it is for them to decide (as with any other teaching process) if a technical tool can successfully be employed (NCTM, 2000). Research has been undertaken into several aspects of mathematics teachers' perceptions of using GGS. Many researchers have examined this subject with different kinds of teachers, including in-service teachers and pre-service teachers. Bu et al. (2013) and Hutkemri and Nordin (2011) focused on the views of in-service teachers, while Doruk et al. (2013) and Zengin (2017) examined the views of pre-service mathematics teachers. All of these studies presented positive perceptions regarding the use of GGS in mathematics classes. For example, Bu et al. (2013) found that mathematics benefited to a considerable degree from the use of GGS in a number of areas, including: (1) personal mathematical research; (2) improving attitudes towards mathematics and mathematics' education; (3) pedagogical contemplation, i.e. the benefits of interaction between mathematics and teachers and students. Also, Hutkemri and Nordin (2011) found that the respondents demonstrated an interest in using GGS to explain mathematical concepts and procedures. Overall, the participants found it easy to use, including navigating the software, i.e. it was apparent that the respondents only required a short time to master the software. Doruk et al. (2013) found that the majority of the participants viewed GGS as a beneficial tool when it came to the teaching of mathematics. In addition, they stated that GGS facilitates more effective teaching of mathematics and can help to overcome difficulties in a way that surpasses traditional approaches. In contrast, some cited a number of limitations, including: (1) difficulties in transcribing mathematical expressions into GGS; (2) GGS being over-responsive, with minor details leading to errors; (3) the latest 3D edition being difficult to use; (4) a lack of capacity to teach simple concepts; (5) the initial thrill of using GGS disappearing over time and it then becoming tedious to use once enjoyment has dissipated (Doruk et al., 2013).

4.7 Aspects of research into mathematical proficiency

Numerous studies have focused on mathematical proficiency, including those that evaluate the concept of proficiency as a whole unit, and those that employ one or more strands. Indeed, a significant body of research has studied the separate strands of proficiency without employing the term, using instead the following expressions: (1) understanding or knowledge; (2) procedure or fluency; (3) problem solving; (4) reasoning and disposition. Some researchers have sought to classify the objectives of developing mathematical competencies to assist in fostering mathematical proficiency, or have investigated the effect of different variables on mathematical competence, including focusing on a specific strand. For example, Groves (2012) sought to address the issue of the kinds of classroom practice that can provide opportunities for the development of such proficiencies in elementary schools. This is a significant issue, in particular due to the opportunity it affords to emphasize various forms of practice and class activities capable of enhancing each strand of mathematical proficiency. The study drew on data from a number of projects, together with existing literature, to offer illustrative examples. Groves (2012) argued the need to establish complex changes in teaching pedagogy to develop the full set of proficiencies, concluding that class discussion provides opportunities for students to develop their conceptual understanding, alongside their strategic competence and adaptive reasoning.

Studies have evaluated the effect of additional variables on mathematical proficiency, namely approaches to teaching and integrating technology in classrooms. Samuelsson (2010) investigated the influence of two methods, traditional and problem-solving, on the respective achievements of students who experienced them. The Samuelsson (2010) study was similar to the above research in terms of its aims and its investigation into the effectiveness different teaching approaches on students’ mathematical proficiency, along with the use of the NRC’s mathematical proficiency strands. However, it differed when it came to the type of teaching approach employed, including the instruments used to measure mathematical proficiency.

The study used the NRC's mathematical proficiency strands to assess performance and yielded several interesting results, including that children of either sex performed equally well when taught using similar methods. The evaluation of procedural fluency revealed no significant differences between teaching methods, while students' performance improved significantly when teachers used a problem-based approach, especially with regard to conceptual understanding, strategic competence, and adaptive reasoning. However, the study also determined that the traditional approach tended to improve productivity.

Awofala (2017) examined mathematical proficiency in relation to gender and mathematical performance. The study used the NRC's strands to assess performance and revealed significantly high mathematical proficiency among the students. In addition, the study found a potential relationship between the mathematical proficiency strands and performance of senior secondary school students. However, no differences in mathematical proficiency were found in relation to gender. This study had certain limitations, such as not using qualitative analysis to evaluate the students' responses, thus limiting the scope of the work. The study sample was limited in range due to solely focusing on elitist schools with better teaching environments, thereby potentially limiting generalization of the results due to a lack of input from non-elitist institutions.

Kinnari (2010) investigated the proficiency of first-year engineering students based on the NRC's five strands of mathematical proficiency. The study identified a lack of mathematical proficiency in three of the strands: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence. The study used an electronic questionnaire to measure the elementary algebra and numeracy. However, the use of online questionnaire imposes constraints on the evaluation of results, such as the way of evaluating student's capacity to formulate and solve problems, the way of assisting the effective and flexible use of mathematical concepts and procedures. In addition, Wu (2008) assessed three strands from the NRC framework: (1) conceptual understanding; (2) procedural fluency; (3) strategic competence. The study used the Model, Strategy Application (MSA) approach with 491 Chinese sixth-grade students. The findings indicated that although the students had higher procedural fluency in comparison to their conceptual understanding and ability to address word problems in real-world applications, they lacked a deep understanding of fractions and decimals.

In contrast, studies examining the impact of technology on competence were conducted by Dijanić and Trupčević (2017), McDonough and Tra (2017), and Chang et al. (2015). Dijanić and Trupčević (2017) assessed the impact of using digital textbooks consisting of GGS interactive apps on students' conceptual and procedural knowledge in mathematics. The study sample consisted of fifteen teachers of mathematics and 703 of their students from the sixth, seventh and eighth grades from twelve schools participating in the research. The researchers employed both an experimental and a control group, teaching the experimental group by means of a model of computer-guided discovery learning and the control group by traditional teaching methods. The results demonstrated a number of statistically significant differences between the two groups, in favour of the experimental group. The model used in this study is based on the theory of constructivism, Pólya's heuristic strategy and Schoenfeld's problem-solving model. So, there is no evidence that the differences between the two groups are related to these elements or because of the use of GGS. McDonough and Tra (2017) explored the impact of computer-based tutorials on high-school mathematics proficiency by means of the High School Proficiency Exam (HSPE) in mathematics. The study made no mention of the NRC's mathematical proficiency framework or its strands, which form the main elements of this research. However, it revealed evidence of increased mathematical proficiency related to tutorial participation when using computer-aided learning.

Chang et al. (2015) studied the impact of a learning game known as "The Math App" on the mathematics proficiency of middle-school students over a nine-week period. The researchers focused on 306 students from grades six to eight, attending two rural schools in southwest Virginia, US. The results revealed that the group following the game intervention presented evidence of higher mathematical proficiency than those in the paper-and-pencil group. However, it should be noted that this study did not explain in detail the kinds of proficiency assessed (i.e. mathematical achievement or ability), nor did it refer to the NRC's mathematical proficiency framework or its strands, or any form of proficiency framework.

Finally, a number of studies have been implemented to analyse mathematical proficiency and its associated strands. For example, Khairani and Nordin (2011) assessed the three strands of conceptual understanding, along with procedural fluency and strategic competence, with several students aged 14, obtaining empirical evidence of how the mathematics competency constructs contributed to ability in each strand.

In addition, Groth (2017) analysed qualitative classroom data from a series of video-recorded lessons, describing how a pair of prospective teachers employed the five strands to analyse and reflect upon their own teaching. Mentoring the pair on using the five-strand framework for qualitative classroom data analysis assisted the author in developing and gradually refining a protocol to lead the process. The protocol assists teachers in taking advantage of the numerous sources of qualitative data that are available in practically every classroom on a daily basis. It was anticipated that the insights gained through this process would help others understand the dynamics involved in employing the five-strand framework and consider the class data. Suh (2008) emphasised those classroom practices promoting mathematical proficiency for all students. She found that the focus on the five strands of mathematical proficiency led to a noticeable change in her class, in particular a transformation in her students' attitude towards mathematics. She added that processes promoting the development of the five strands of mathematical proficiency included: (1) problem solving; (2) reasoning and proof; (3) communication; (4) connections; and (5) representation.

4.8 Research gap

As discussed in this chapter, the use of technology in teaching and learning is fundamental to keep educational systems up to date. The literature shows the importance of using technology in mathematics, especially DMS. Indeed, Artigue (2002) and Ruthven and Hennessy (2002) claimed that the use of DMS can be an extremely efficient tool in mathematics learning. Gono (2016) suggested that it is essential to address matters such as the general effects of GGS on learning mathematics, the range of methods available for GGS and its use to improve understanding of mathematical ideas. Moreover, longitudinal research is needed to assess its impact on students' mathematical achievement and attainment over time. Numerous studies have found that the use of GGS especially has a crucial impact on students' learning and performance in mathematics (i.e. Hohenwarter & Fuchs, 2004; Hohenwarter & Lavicza, 2007; Zengin et al., 2012; Doktoroğlu, 2013; Martinez, 2017; Alkhateeb & Al-Duwairi, 2019). Studies in the field have covered different aspects of mathematics learning, such as students' overall achievement in mathematics, learning and understanding of concepts, fluency in doing mathematics tasks, solving routine and non-routine problems, reasoning, and students' attitudes towards mathematics. Also, some studies have covered more than strands of mathematical proficiency (e.g. Hutkemri, 2014; Dijanić & Trupčević, 2017; Ocal, 2017; Zulnaidi & Zamri, 2017), investigating the effects of using GGS on students' procedural and conceptual knowledge. Moreover, Murni et al. (2017) examined the impact of GGS on students' problem-solving ability and their attitudes towards mathematics. Khairani and Nordin (2011) assessed three strands, namely conceptual understanding, procedural fluency, and strategic competence, obtaining empirical evidence of how mathematics competency constructs contributed to ability in each strand.

However, there is still a lack of research on the effects of using GGS on all five mathematical proficiency strands simultaneously. Hence, this study will add to the field by investigating the impact of using GGS on mathematics from two perspectives: (i) investigating the effect on students' overall mathematics achievement; (ii) examining in greater depth to determine the effects on each of the NRC's mathematical proficiency strands. Also, most research that has studied the impact of using GGS on one or more strands of mathematical proficiency has solely used quantitative methods. This study also adds to the field by combining quantitative and qualitative methods, employing quantitative methods to investigate the effectiveness of using GGS on students' mathematical proficiency and qualitative methods to explore the teachers' perceptions of the use of GGS in teaching and learning mathematics. The reason for focusing on the five strands is to investigate the effect of GGS on what students need to learn mathematics effectively. Also, the five strands are – to some extent – measurable, making them amenable to examination. Thus, the research may capture the whole effect of the use of GGS on most of the mathematics learning processes, rather than just focusing on one aspect. Today, students need to understand the mathematics they are learning; that is, the strands of learning are interdependent and interwoven (Khairani & Nordin, 2011). According to Schoenfeld (2007, p. 63):

> Students who experience skills-focused instruction tend to master the relevant skills, but do not do well on tests of problem solving and conceptual understanding [while] students who study more broad-based curricula tend to do reasonably well on tests of skills …[and] do much better … on assessments of conceptual understanding and problem solving.

Also, according to Groth (2017), the focus on these five strands of mathematical proficiency helps teachers develop essential habits in terms of basing daily instructional decisions on reflections of their students' strengths and requirements. In addition, focusing on these skills helps improve them in and of itself. Thus, this study aims to focus on the five strands potentially providing a holistic view of the effects of using GGS on students' learning in each strand and on overall student achievement.

Chapter 5 Methodology and Methods

5.1 Introduction

The main aim of this chapter is to illustrate the research methodology, methods, procedures, and instruments deployed in this study. According to the American Educational Research Association (2019), Bassey (1999), and Mertler and Charles (2005), educational research is defined as a scientific field of study that studies and examines all learning and education procedures, as well as the human attributes that shape educational products, such as organizations, interactions, and institutions, with the aim of improving educational practices. It has five main objectives: exploration, description, explanation, prediction, and influence (Johnson & Christensen, 2014). William (2005) and Johnson and Christensen (2014) outlined the steps required when conducting such research:

- Identifying and clarifying the specific problem
- Reviewing information
- Formulating research questions
- Collecting data
- Analysing the data
- Drawing conclusions

The main aim of this study is to investigate the effectiveness of using GGS-based pedagogy in developing students' mathematical proficiency. This investigation was conducted by comparing the results of two groups of students in mathematical proficiency tests (MPTs), with the experimental group taught using GGS-based pedagogy and the control group taught using a traditional approach. All of the MPT items were drawn from the TIMSS international assessment. Also, the study aimed to ascertain teachers' perceptions of learning and teaching mathematics using GGS. Consequently, to achieve the above research aims, the research addressed two main questions:

Research Question A (RQA): Does the use of GGS-based pedagogy have a significant effect on students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition) and achievement compared to the traditional teaching approach?

Research Question B (RQB): What are the mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics?

5.2 Research methodology

According to Jonker and Pennink (2010), a methodology is a map or a domain, while a method refers to a set of steps to travel between two places on the map. A methodology is a model, guide, or frame based on which research is conducted within a particular paradigm. O'Leary (2004) defines methodology as the framework which is associated with a specific set of paradigmatic assumptions that researchers use to conduct their research. Usually, researchers state that they are conducting "qualitative" or "quantitative" research, rather than "positivist" or "interpretive" research, because methodologies are closer to research practice than the philosophical concepts found in paradigms (Sarantakos, 2005). Johnson and Christensen (2008) pointed out that there are three major approaches in educational research: quantitative, qualitative, and a mix of the two (commonly called mixed method research). The research methodology used in this study was mixed method.

5.2.1 Mixed method approach

Recently, mixed method research (MMR) has emerged as a comprehensive approach that has taken its place alongside qualitative and quantitative research methods and has been recognized as the third dominant research approach (Johnson et al., 2007; Johnson & Christensen, 2014). In the social, behavioural and human sciences, MMR started with methodologists and researchers who believed that both quantitative and qualitative methods and viewpoints were useful as they addressed their research questions (Johnson et al., 2007). MMR came about as a result of a pragmatic approach, which constitutes the theoretical framework (Mackenzie & Knipe, 2006; Johnson et al., 2007; Bazeley, 2009). Johnson et al. (2007, p. 123) analysed 19 definitions of MMR and concluded as follows:

> Mixed methods research is the type of research in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches (e.g., use of qualitative and quantitative viewpoints, data collection, analysis, inference techniques) for the broad purposes of breadth and depth of understanding and corroboration.

This research used a mixed method approach as according to Tashakkori and Creswell (2007), it leads to strength in educational research. Johnson and Christensen (2014) pointed out that the principle of MMR is that researchers strategically and sympathetically combine qualitative and quantitative research methods, approaches and procedures in a way that produces an overall design with complementary strengths and non-overlapping weaknesses.

Tashakkori and Creswell (2007) suggested that strong research questions and objectives lead to strongly mixed methods and that when a study explores a mix of research questions with interconnecting quantitative and qualitative components or aspects, the end product of the project will also include mixed methods. In addition, MMR often leads to a larger amount of in-depth information responding to the research questions and consequently a holistic view of the outcomes, giving readers more confidence in the results and the conclusions drawn from the study. O'Cathain et al. (2010) also indicated that MMR confers reliability and confidence in the results and thus the study conclusions.

Mixing quantitative and qualitative approaches is vital for two purposes:

- To study the context of the research and thus to understand the success or otherwise of interventions in real-world applications.
- To gather a large amount of in-depth information addressing the research questions, leading to a more holistic view of the research outcomes. (Albright et al., 2013)

According to Bazeley (2018), MMR contributes to building better knowledge and deeper inferences through:

- increasing confidence in results that are supported by multiple sources of evidence;
- designing better instruments and samples;
- increasing the depth or breadth of a study;
- providing a more complete or comprehensive understanding of the topic;
- initiating fresh insights through contradiction and paradox. (p. 12)

It was therefore decided that a mixed method approach was the most appropriate for the design of this study, as it the best match for the two research questions outlined above. The first question is about measuring the effectiveness of the GGS-based pedagogy on students' mathematical proficiency through MPTs and a questionnaire, drawing on quantitative data and analysis; the second is about how the process of teaching and learning mathematics changes when using GGS-based pedagogy, based on the opinions, thoughts, and feelings of teachers and therefore drawing on qualitative data. The reason for using the two research approaches in this study was to endeavour to reach conclusions based on a holistic view of the outcomes and to ensure confidence in those outcomes. Using one approach in this study, for example quantitative, would only provide conclusions based on numbers and statistical tests, namely the effects of using GGS on the students' performance in the MPTs. This would neglect the important issue of what mathematics teachers think about this kind of software.

Equally, if the study were just carried out based on what mathematics teachers think about GGS, it would ignore how the GGS affects the students' learning of mathematics. Thus, the use of the two methods was important in this study to provide a more detailed view of the teaching and learning process, collect rich data to address the two research questions, and allow a coherent picture of the efficacy of using GGS in mathematics learning. However, MMR has a number of limitations, including the need for large-scale data collection and the time required to analyse both quantitative and qualitative data (Alshehri, 2021). Furthermore, the researcher must be well-versed in both quantitative and qualitative research methods (Creswell, 2009).

Numerous typologies have been developed to classify and identify different types of mixed method strategies and mixed methodologies can be conducted in various ways. The two main factors that drive the design of a study are the way in which different methods are prioritized and organized (Creswell, 2009). According to Teddlie and Tashakkori (2009), the purpose of typologies is to be comprehensive but not exhaustive, as designs develop according to circumstances and objectives. However, "flexibility in both the design and conduct of mixed methods research is required" (Bazeley, 2018, p. 23). As Weiss et al. (2005, p. 61, cited in Bazeley, 2018) reported:

> We learned to impose structure on our mixed-methods process but also to be pragmatic and tolerate complexities … we learned that mixed-methods approaches could only be rough guides and that intentional designs might have to give way to real-world problems of data availability and deadlines. Accordingly, we developed a sense of our mixed-methods work as a dynamic hands-on process, guided only very generally by mixed-methods analytic models.

One of the most promoted method-oriented designs is that of Creswell and Clark (2018), which categorizes four types of mixed methods designs, illustrated in Figures 5-1 to 5-4.

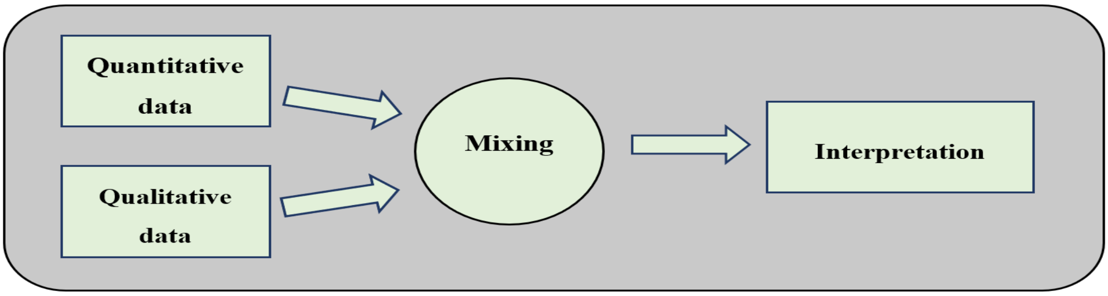


Figure 5-1 Convergent parallel design

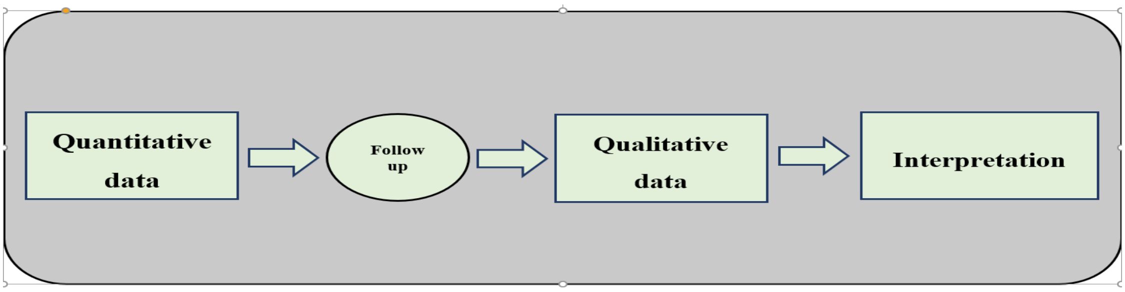


Figure 5-2 Explanatory sequential design

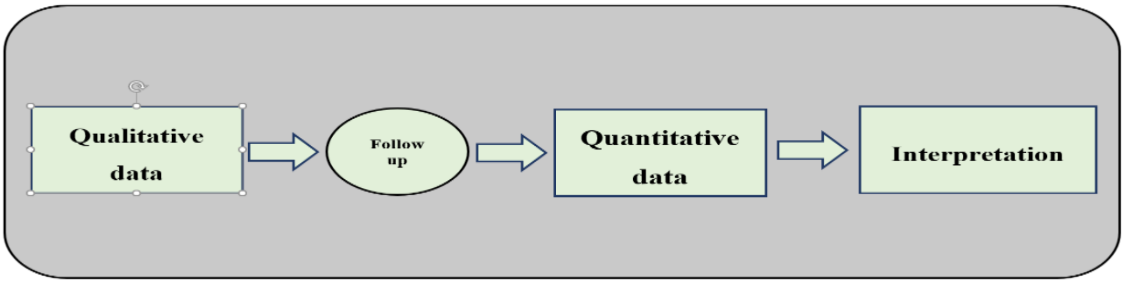


Figure 5-3 Exploratory sequential design

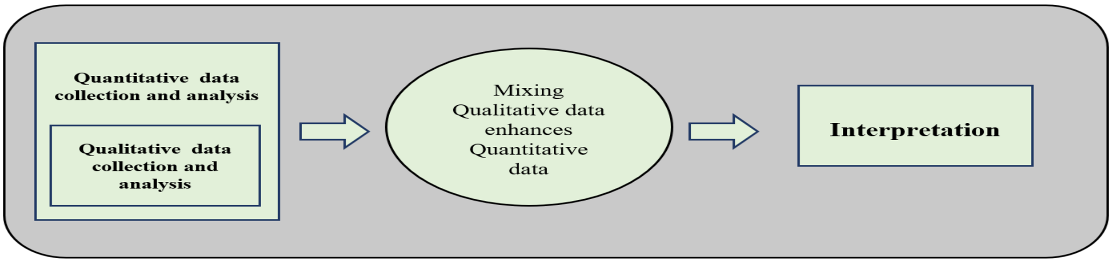


Figure 5-4 Embedded design

The typology that fits the design used in this study is embedded design (see Figure 5-4), sometimes known as a concurrent or nested design. According to Creswell (2009), the embedded design is used by researchers when they need to incorporate qualitative or quantitative data to answer research questions within a mostly quantitative or qualitative approach. The two kinds of data are collected simultaneously, but with one type of data playing a supportive role to the other form of data (Creswell, 2012). In this study, the quantitative data related to the effectiveness of the use of GGS-based pedagogy in enhancing the students' mathematical proficiency played the main role, while the qualitative data, collected simultaneously, were used to support and understand the quantitative data. This was part of a quasi-experimental design, an example of an embedded design, exemplified by Creswell (2012, p. 545):

> During the quantitative experiment, the researcher may collect qualitative data to examine how participants in the treatment condition are experiencing the

> intervention. Also, the researcher may collect qualitative data either before or after the experiment to help support the experimental study.

Creswell's description is quite similar to the design of this study in terms of the way in which data were collected in the field and the importance of each kind of data for the study and thus it is apparent that the embedded design is the appropriate typology for this study.

5.2.2 Research design

The research design for the study comprised two main parts, the first relating to students (Figure 5-5) and the second to teachers (Figure 5-6). With regard to the student data collection and analysis, a quasi-experimental design played a significant role as a quantitative approach. In the design of a true experiment, the experimental group and control group are treated identically in all respects other than the independent variable that is manipulated, so these designs are well suited to causal inference (Frey, 2018). However, such experiments often cannot be carried out for a variety of ethical, realistic, legal, or political reasons, especially in educational studies (Frey, 2018). Instead, a quasi-experimental design can be used, particularly as commonly a random assignment to groups is not practical or possible (Creswell, 2013; Johnson & Christensen, 2014). According to Coolican (2018), the term "quasi-experimental" describes well-controlled research designs that are "almost" experiments, but lack one or more primary features of true experiments, such as the random allocation of participants to conditions and the independent variable being absolutely under the influence of the experimenter. Quasi-experimental design is commonly used to investigate causes, such as the ways in which a new approach or teaching method affects students' outcomes, and is thus ideal for this study. In educational research, this type of design is especially helpful in addressing evaluation questions regarding the impact and effectiveness of programmes (Gribbons et al., 1997; Creswell, 2014; Johnson & Christensen, 2014). According to Frey (2018), the use of a quasi-experimental design reduces the threat to ecological validity and allows for greater generalization; it can also be readily repeated in various settings without strict supervision. However, the correlational nature and lack of randomization in this design causes a threat to internal validity, so researchers who use this design attempt to exclude unrelated explanations by pairing groups and doing statistical research, showing that the results are primarily due to the intervention (Frey, 2018).

According to Creswell (2014) and Johnson and Christensen (2014), there are three types of quasi-experimental design, namely non-equivalent group (post-test) only, equivalent group (pre-test and post-test), and a time series. This study used the non-equivalent group design (pre-test and post-test) to measure the students' mathematical proficiency before and after the intervention study. This type of quasi-experimental design ensures that the two classes have the same level of mathematical proficiency before starting the intervention (Creswell, 2013). Thus, any differences between the two groups after the intervention may be attributed to the use of GGS-based pedagogy.

In this study, the quasi-experimental design was conducted in two stages, corresponding to the two taught units. In the first stage, the experimental group and control group were taught one unit and the groups were then reversed for the teaching of the second unit so that the experimental group becomes the control group and the control group became the experimental group. The purpose of this switch was to ensure that both groups were treated equally for ethical reasons. In addition to the MPTs, which aimed to measure the first four strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning), a questionnaire was administered twice to each group, before and after the intervention. These questionnaires aimed to measure the fifth strand of mathematical proficiency (productive disposition) and included three scales. The quasi-experimental research design is presented in Figure 5-5

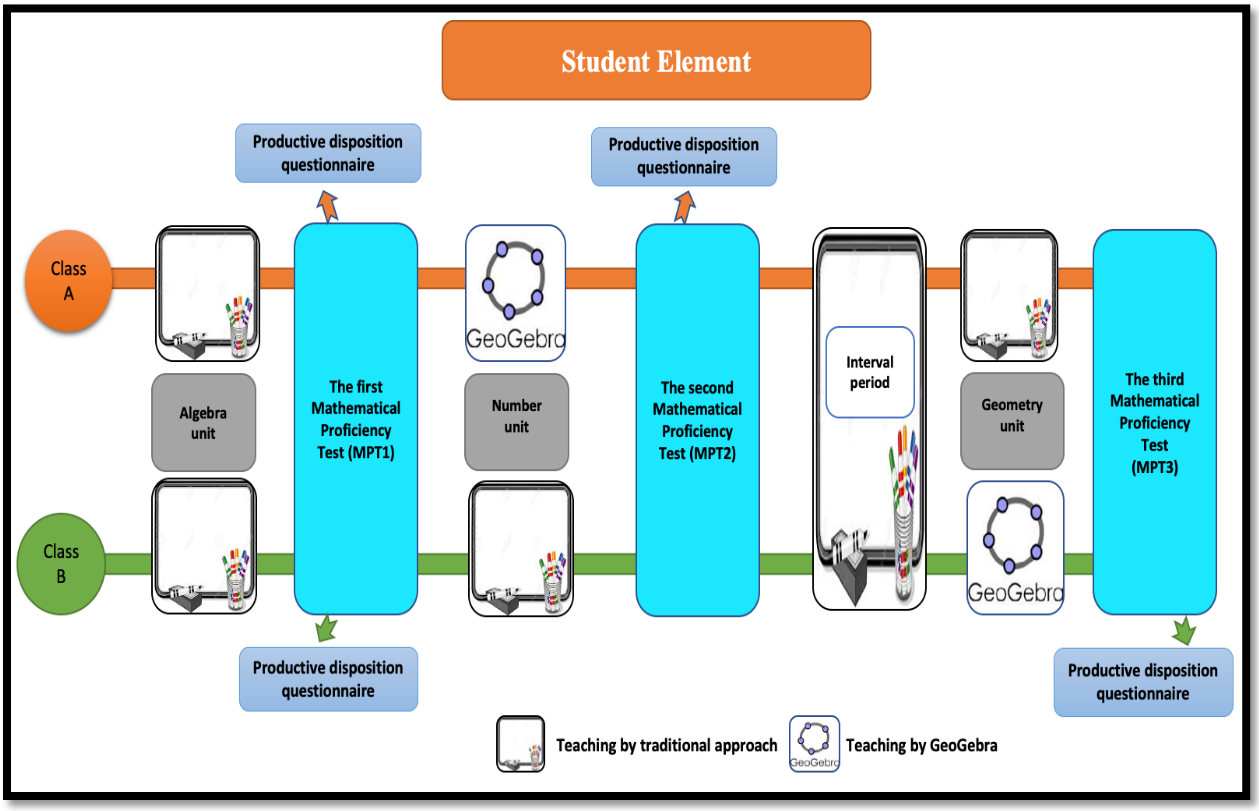


Figure 5-5 Research design (student element)

In the part of the study relating to teachers, a group of teachers underwent a PDC for five days on using GGS effectively in the classroom, as well as three rounds of semi-structured interviews: before, during, and after the PDC. The main aim of these interviews was to explore teachers' perceptions of using GGS in the classroom. They were also involved in a group discussion on the fifth day to discuss and write about their experiences of using GGS after the PDC (Teachers' Written Responses).

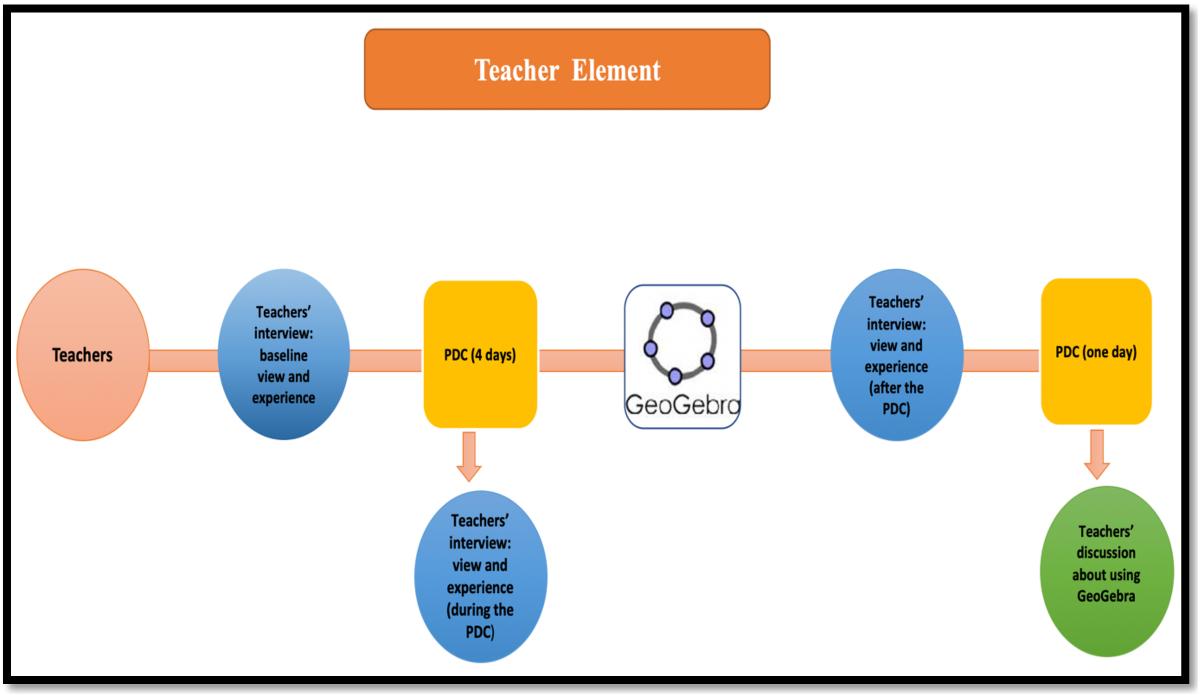


Figure 5-6 Research design (teacher element)

5.2.3 Research procedure

The data collection procedure comprised two main elements shown in Figures 5-5 and 5-6:

- The process regarding the student element consisted of an intervention study and a productive disposition questionnaire.
- The process regarding the teacher element consisted of interviews before, during, and after the PDC and the teachers' written responses from the last day of the PDC.

Student element of the data-gathering process

The process that the students underwent entailed four sub-stages.

Stage One

This stage consisted of training teacher Ali (pseudonym used to preserve anonymity), student training, administering the student questionnaires and the first MPT (based on the content from the Algebra unit).

After teacher Ali had been chosen for the intervention study based on the online questionnaire, I met him three times before starting the intervention with the students. Although he had some previous experience of using GGS, I provided additional training on using GGS individually for two days to revise his previous experience and cover some new skills that were part of the PDC (because the intervention began before the PDC took place).

Then, the productive disposition questionnaire was administered with both groups of students to measure the three scales with regard to the student's productive dispositions. The first MPT was also implemented with the two Grade 8 classes under the condition of a 35-minute time limit. In all, 37 students participated in the first stage of the intervention, 19 in Group A, the experimental group, and 18 students in Group B, the control group.

Then, teacher Ali trained the students in the primary use of GGS over two lessons (90 minutes in total). In the first lesson, the students were initially put in groups to gain an understanding of the primary use of GGS; in the second lesson, they had a chance to use GGS individually. Teacher Ali also attended the PDC with me.

Stage Two

This stage consisted of the second MPT (based on the content from the Numbers unit) and student questionnaires.

Teacher Ali started teaching Group A using GGS-based pedagogy, whereas Group B was taught using a traditional approach. This lasted for five weeks and then the second MPT was administered to both groups. At the end of this stage, the experimental Group A completed a productive disposition questionnaire to measure their disposition after they had studied one unit using GGS.

Stage Three

In this stage, Groups A and B were both taught using the traditional approach. This stage was considered an interval period to give students a chance to eliminate any carry-over effects from the previous stage that might affect their results and performance in the next stage.

Stage Four

This final stage in the student process consisted of the third MPT (based on the content from the Geometry unit) and administering the student questionnaire to assess their productive disposition.

In this stage, the two groups were swapped around, so that Group B was taught using the GGS-based pedagogy, while Group A was taught using the traditional approach. Thus Group B, treated in the first round of the intervention as the control group, became the experimental group in this round, and vice versa for Group A. This stage lasted three weeks, shorter than the first unit because there were fewer lessons in the unit, and then the third MPT was administered to both groups. At the end of this stage, experimental Group B completed the productive disposition questionnaire to measure their disposition after they had studied the unit using GGS.

Teacher element of the data-gathering process

The teacher element process comprised five sub-stages:

1. First round of interviews: six Grade 8 mathematics teachers were interviewed before they became involved in the PDC, including teacher Ali. This interview was designed to identify teachers' perceptions of the use of technology in classrooms and whether or not they had ever used GGS before.

2. The PDC was implemented over four days of training with 15 teachers, including teacher Ali.

3. A second round of interviews was conducted with the same six teachers as in the first round on the third and fourth days of the PDC. These interviews asked about the teachers' perceptions of GGS, and how they would describe their new experiences and their expectations of using the new software, GGS.

4. A third round of interviews was conducted with the six teachers again two weeks following the end of the PDC, aimed at understanding teachers' perceptions of GGS after they had been using it in their classrooms, how they would describe their experiences, and what they had found from using GGS.

5. On the last day of the four-week PDC, a meeting was held with all teachers in which they had a group discussion to share and write about their experiences of GGS (Teachers' Written Responses).

5.3 Research population and sampling

5.3.1 Research population

The research population, sometimes called a target population, is a set of all elements; it is a large group of people from among whom samples are selected (Johnson & Christensen, 2008; Coolican, 2018). The population of this study comprised all Grade 8 students and teachers in the city of Jazan, in the south-west of the KSA.

5.3.2 Research sample and participants

In this study, there were two types of participants, namely mathematics teachers (all male) and students (all male). To recruit teachers for the PDC (all teachers in this study), an online questionnaire was conducted with all teachers of mathematics at Grade 8 in Jazan. The online questionnaire was conducted with the help of the Mathematics Department in the General Department of Education in Jazan (the role of the Mathematics Department here was solely to facilitate conducting the online questionnaire with all Grade 8 mathematics teachers). The first 15 teachers who responded were chosen randomly from among those meeting the inclusion criteria. The questionnaire also included a section that asked teachers if they would volunteer to participate in the interviews and from those who volunteered, the first six teachers were chosen. All data provided by the teachers at this stage who were not involved later were deleted immediately after selection of the study participants. Teacher Ali was also recruited as a volunteer from among the PDC participants.

In this study, there were no power relations between myself as the researcher and the participants that might have affected the results. All participants were informed that their participation was voluntary and they were free to withdraw at any time during the data collection period without giving any reason. They were informed that this would not affect the evaluation of their work in any way or have any other detrimental effects on their relationships with others in the school.

Table 5-1 Summary of process of selecting the research sample and participants

Steps	Participants
First step	An online questionnaire was conducted by all teachers who were teaching mathematics at Grade 8 in Jazan
Second step	The first 15 teachers who responded to the online questionnaire were randomly chosen from among those meeting the inclusion criteria. Some of them volunteered for PDC only and some of them volunteered to participate in the PDC and interviews.
Third step	Six teachers were selected for the interviews (including teacher Ali) from the 15 teachers who had volunteered to participate in the interviews.
Fourth step	The teacher was selected for the intervention study (teacher Ali) from the six who volunteered to participate in the interviews (the school had to have a computer lab)

The student sample was obtained as follows:

1. The school was chosen with the assistance of teacher Ali, who volunteered to teach students using the GGS-based pedagogy; some other teachers volunteered to participate in the quasi-experimental part, but this school was chosen because it had a computer lab which was used during the study. The school had 236 students and was located in an urban district in a small city, Samtah, situated in the south-west of KSA. It was a middle and secondary school with 12 classes, 6 at each of the 2 levels. Each grade had two classes at the middle level (Grades 7, 8, and 9) and two classes at the secondary level (Grades 10, 11, and 12). The two classes in Grade 8 were used in the quasi-experimental approach. The school had four mathematics teachers, two at secondary level and two at middle-school level, including teacher Ali.
2. The sample then comprised the two Grade 8 classes, with a total of 37 students aged 11–14 years (Group A: 19 students; Group B: 18 students).

5.4 Research instruments

5.4.1 Mathematical proficiency tests (MPTs)

This study deployed four research instruments. The two conducted with the students were the MPTs and the productive disposition questionnaire; the two with the teachers comprised interviews and written responses. The three MPTs followed the same structure for different topics (Algebra, Number, and Geometry) and addressed the same underlying proficiency areas in each case (see Appendix 1). They were employed to measure the constructs for all strands (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning). Each instrument is described in greater detail in the following paragraphs.

First mathematical proficiency test (MPT)

This test was undertaken before the start of the intervention, this being one of the essential components in the non-equivalent group (pre-test and post-test) design to ensure that the study results are not biased; significant differences at the beginning can lead to significant differences at the end (Johnson & Christensen, 2014). The main aim of this test was thus to mitigate any change of bias and ensure the level of prior knowledge of each group was at the same level. Using pre-tests avoids specific threats to validity (Harris et al., 2006). This test was made up of TIMSS items focused on algebra, corresponding to the unit that both classes had just completed prior to the test.

Second and third mathematical proficiency tests (MPTs)

Two MPTs were conducted with both groups after they were taught different units using GGS-based pedagogy. These tests aimed to indicate any differences between group scores in the MPTs in general and in each strand of the NRC's mathematical proficiency framework. These tests were conducted twice: once after finishing the unit on Numbers and second after completing the unit on Geometry.

Mathematical proficiency test (MPT) structure

Each MPT consisted of 14 mathematics questions, which were a combination of multiple-choice items and short answer questions. The tests consisted of four items measuring the conceptual understanding strand, four items covering the procedural fluency strand, three items to examine the strategic competence strand, and three items which assessed adaptive reasoning. The first two strands covered 60% of the test, 30% each, while the second two strands covered 40%, 20% each (Table 5-2).

Table 5-2 Mathematical proficiency strand percentage coverage

Mathematical proficiency test items (% coverage)			
	Strand	**Numbers of items**	**Percentage**
1	Conceptual understanding	4	30%
2	Procedural fluency	4	30%
3	Strategic competence	3	20 %
4	Adaptive reasoning	3	20%

All mathematics items in the three tests were drawn from the released items of the TIMSS from 2003, 2007, 2011, and 2015. The TIMSS framework applies three knowledge domains, knowing, applying, and reasoning (Martin et al., 2016). The first domain includes the facts, concepts, and procedures students need to understand, the second focuses on the skill of students in applying conceptual understanding to solve problems or answer questions, and the third domain, reasoning, goes beyond the solution of routine problems to encompass unfamiliar situations and complex contexts (Martin et al., 2016). The first domain is similar to the first strands of mathematical proficiency articulated by the NRC, so its items were used to assess the students' conceptual understanding. The third, reasoning, is similar to the fourth strand, i.e. adaptive reasoning, so its items were used to assess the students' productive reasoning.

A change was made with regard to the second domain of the TIMSS framework, used to assess the second and third strands of mathematical proficiency, i.e. fluency and strategic competence. The second domain of the TIMSS, applying, already incorporates these two parts: the first concerns applying mathematical knowledge of facts, skills, and procedures used to assess the students' procedural fluency in this study. The second part relates to solving a problem, used to assess strategic competence in this study (see Table 5-3). Although TIMSS tests are international assessments and have high levels of reliability and validity, I also tested the validity and reliability of the tests to confirm their applicability in this study.

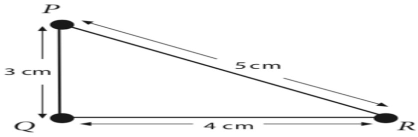
Table 5-3 Use of TIMSS domains in this study

	TIMSS cognitive domains	Mathematical proficiency strand
1	Knowing	Conceptual understanding
2	Applying	Procedural fluency Strategic competence
3	Reasoning	Adaptive reasoning

In the Table (5-4) some examples shows how each strands was measure in the three MPTs

Table 5-4 Examples from the MPTs for each mathematical proficiency strands

<table>
<tr><th></th><th>Strand</th><th>MPT</th><th>Example</th></tr>
<tr><td>1</td><td>Conceptual understanding</td><td>MPT2</td><td>$\frac{4}{100}+\frac{3}{1000}=$
A. 0.043 B. 0.1043 C. 0.403 D. 0.43</td></tr>
<tr><td colspan="4">The question may seem like a question testing procedural understanding in the first instance. However, as students try to eliminate the wrong choices, they need to think about 4 in the Hundredths place and 3 in the Thousandths place, whereas there should be zero tenths. Therefore, anyone choosing an answer other than 0.043 can be considered as not thinking about the conceptual meaning of place value for decimals. Only thinking about the conceptual meaning would allow one to decide on the correct places of 4 and 3.</td></tr>
<tr><td>2</td><td>Procedural fluency</td><td>MPT2</td><td>- Which shows a correct method for finding $\frac{1}{3}-\frac{1}{4}$?
A. $\frac{1-1}{3-4}$
B. $\frac{1}{4-3}$
C. $\frac{3-4}{3\times4}$
D. $\frac{4-3}{3\times4}$</td></tr>
<tr><td colspan="4">The students attempting to answer this question need to run the common denominator algorithm and match it with one of the given choices. Since the given choices summarize the application of common denominator, choosing the right answer [(4-3)/(3x4)] give the investigator an indication of student's fluent use of a procedure, finding the common denominator to solve a fraction subtraction question. Therefore, this question is thought to measure students' procedural fluency.</td></tr>
</table>

<table>
<tr><td>3</td><td>Strategic competence</td><td>MPT2</td><td>Georgia wants to send letters to 12 of her friends. Half of the letters will need 1 page each and the other half will need 2 pages each.
How many pages will be needed altogether?</td></tr>
<tr><td colspan="4">This task asses the students' ability to answer a non-routine mathematics problem. The students need to distinguish between the first part of the problem and the second part. In the first part, they need to identify how many pages are required for half of Georgia’s friends, which is 6 (she needs 1 page for each friend). She then needs 12 pages for the second half of her friends (2 pages each). Then, the students need to calculate the sum of the two results to find the total number of pages. This question aims to measure students' strategic competence. Since students cannot answer this question directly, also, students need to engage in a diversity of mathematics operations, which requires a different aspect of knowledge and skill, some of which are not routine tasks.</td></tr>
<tr><td>4</td><td>Adaptive reasoning</td><td>MPT3</td><td>
Which of these is the reason that triangle PQR is a right triangle?
A. $3^2 + 4^2 = 25$ B. 5 < 3 +4 C. 3 + 4 = 12 - 5
D. 3 > 5 - 4</td></tr>
<tr><td colspan="4">This task encourages students to think mathematically and find the right relationship between the measures of the sides and angles of the triangle. In this triangle, the side measures are 3, 4, and 5. The students need to present the mathematical reason for the PQR being a right-angle triangle by applying Pythagoras' theorem: “If the squares of the two shorter sides add up to the square of the hypotenuse, the triangle contains a right angle”. So, students need to choose the right answer, which is A. $3^2 + 4^2 = 25$. So, students need to present their mathematics reasoning by choosing the correct answer that makes the triangle is right. This question differs from the conceptual understanding because students here need to select the reason that makes this shape a right triangle.</td></tr>
</table>

Test marking

The full score for each test is 20 points, with a range of scores for each question from 0 to 2. A correct answer was given either 1 or 2 points, depending on the question. Standard questions were marked out of 1, whereas more complex questions, requiring more than one step, were marked out of 2. Partially correct answers were marked between 0 and 1 for those questions with maximum score of 1, and between 0 and 2 for those with a maximum score of 2. All tests were marked and revised by two different teachers (not teacher Ali).

Test validity

Validating instruments is an essential step before starting to collect data. It is important to ensure that the instrument used actually measures what it purports to measure (Kimberlinet al., 2008; Gravetter & Forzano, 2018).

There are three types of validity: face validity, concurrent validity, and consistency. Face validity was used in this study to validate all the three tests. Hardesty and Bearden (2004) defined face validity as the expert judge's degree of responses to which items of measurement in the research are appropriate to the targeted aims concerning construct and assessment. All three tests, after I had prepared and translated them from English to Arabic, were presented to eight arbitrators, including five mathematics teachers, for them to validate, check, and give their opinion about the tests in terms of:

- the clarity of the test items;
- the appropriateness and relevance of the content of the items;
- any adjustments and observations they considered appropriate;
- the distribution of marks for the test items;
- the relevance of the questions and items in relation to each mathematical proficiency strand.

All their comments and recommendations were considered before preparation of the final version of the three MPTs. For example, the translation of some items was changed to make the items more understandable for the students. Also, the marks allowed for some questions were changed from one to two.

Test reliability

Reliability is a second element that determines the quality of research instruments and is defined as "the extent to which test scores are free from measurement error" (Kimberlin et al., 2008; Gravetter & Forzano, 2018). Reliability is a measure of internal consistency or the stability of tools in measuring specific ideas (Jackson, 2015). According to Creswell (2002), there are various types of reliability, for instance test-retest reliability, alternate forms reliability, internal consistency and inter-rater reliability. In this study, test-retest reliability and internal consistency were used to determine the level of reliability of the MPTs.

Test-retest reliability

Test-retest entails repeating the same test with the same student under the same conditions after an appropriate period. According to Gravetter and Forzano (2018), test-retest reliability is established by comparing the two test scores for the same individual, then calculating the correlation between the scores of the two tests. Thus, to measure the reliability of the MPTs, each test was administered to 15 Grade 8 students as a pilot sample. These students were located in a different school from the school in the intervention study. Then, after two weeks, the two tests were conducted again under identical conditions. Finally, the correlations between the two sets of scores were calculated using the Pearson correlation coefficient. The level of reliability in the two tests (MPT in Numbers and Geometry) were acceptable, with scores of 0.89 and 0.90 respectively.

Internal consistency for mathematical proficiency strands

"Internal consistency describes the extent to which all the items in a test measure the same concept or construct and hence it is connected to the inter-relatedness of the items within the test" (Tavakol & Dennick, 2011, p. 53). The internal consistency scale is one of two common measures used as an indicator of consistency, most appropriately measured using Cronbach's alpha coefficient (Pallant, 2016). Although TIMSS is an international exam and is supposed to have a high level of validity and quality, measuring the internal consistency was necessary for this study because the items were drawn from different TIMSS exams across a number of years and this, to some extent, could affect the reliability of the tests.

Table 5-5 Cronbach's alpha values for MPTs

Internal Consistency: Cronbach's Alpha Values		
Strand	MPT2	MPT3
Conceptual understanding	.742	718
Procedural fluency	.825	.789
Strategic competence	6.18	.537
Adaptive reasoning	.570	.643
Total	.762	.759

Table 5-5 illustrates that the internal consistency levels were acceptable in most strands of MPT2, except for reasoning, which was 0.57. Also, they were acceptable in most strands of MPT3, except for strategic competence, which was 0.54. There are two possible reasons for the reduced internal consistency related to strategic competence and adaptive reasoning. First, both strands were measured using three items, fewer than the first two strands, and Cronbach's alpha is sensitive to some items; thus, in a short test, the scale will typically be low (Pallant, 2016). Second, students in Saudi Arabia seem to experience some difficulty in answering such types of question, judging by the country's results in the TIMSS in 2011, 2015, and 2019. However, overall, the internal consistency was acceptable for both the second and third MPTs.

5.4.2 Productive disposition questionnaire

Researchers commonly use questionnaires to explore the thoughts, attitudes, feelings, beliefs, values, personality, perceptions, and behavioural intentions of study participants. To identify the effect of using a GGS-based pedagogy on the fifth strand of mathematical proficiency (productive disposition), which relates to the first research question, three questionnaires based on items from TIMSS 2015 were used with each class as a productive disposition questionnaire, delivered both before and after teaching them using GGS (see Appendix 2).

The questionnaires aimed to measure students' self-perceptions and attitudes towards learning, the main components being: "Students like learning mathematics"; "Students' confidence in mathematics", and "Students value mathematics" (Martin et al., 2016). Each part contained nine items, measured on a 4-point Likert-type scale (0 = disagree a lot, 1 = disagree, 2 = agree, 3 = agree a lot). According to Cohen (1988), Likert-type scales measure a range of pre-defined answers to a statement or a specified question, and are used to indicate stance. The standard score is given between 0 and 3. Some of the scale items were reverse coded; for instance, for "Mathematics makes me nervous", "disagree a lot" was scored 3, whereas "agree a lot" was scored 0 (see Table 5-6).

Table 5-6 Likert-type scale responses in questionnaires

Likert Scale	**Description (normal)**	**Description (reversed)**
0	Disagree a lot	Agree a lot
1	Disagree	Agree
2	Agree	Disagree
3	Agree a lot	Disagree a lot

Validity and reliability of questionnaires

According to Martin et al. (2016), as evidence that the questionnaire scales would provide comparable measurement across countries, reliability coefficients were computed for each scale for every country and benchmarked for participants, and principal components analysis of the scale items was conducted. The Cronbach's alpha coefficients were generally at an acceptable level, with almost all above 0.7 and many above 0.8 (Table 5-7).

Table 5-7 Cronbach's alpha coefficients for scales (Source: Martin et al., 2016)

Scale	Cronbach's alpha
Students like learning mathematics	0.92
Students' confidence in mathematics	0.75
Students value mathematics	0.89
Total	0.85

After I had translated all three scales from English to Arabic, they were presented to eight arbitrators to validate, check, and give their opinion about the clarity of scale items and if they found any issues with the clarity of each statement.

5.4.3 Teacher interviews

Interviews can be characterized as a subjective investigative method which entails conducting conversations with a small number of respondents to investigate their viewpoints on a specific issue, programme or situation (Powney & Watts, 1984). An interviewer or researcher manages a discussion with a number of people, with the substance of the conversation being recorded, analysed and reported (Powney & Watts, 1984). It is a very influential instrument for obtaining qualitative data (Punch, 1998; Walliman, 2001). Interviews help the researcher to obtain rich, in-depth, detailed qualitative data allowing a deeper understanding of participants' experiences and they give participants the opportunity to describe their experiences and whatever meaning they make of those experiences (Rubin & Rubin, 2011).

There are three main types of interview, namely structured, unstructured and semi-structured, all of which can be conducted either individually (one-to-one) or in a group (Fontana & Frey, 2000; Dawson, 2019). All three types can be conducted face-to-face, by telephone, or by email (Meho, 2006; Walliman, 2006). Conducting one-to-one interviews is the most time-consuming and costly approach; however, it is popular in educational research (Creswell, 2002).

In this study, one-to-one, semi-structured interviews were conducted. Using semi-structured interviews gives participants the freedom to say what they want (Stringer, 2004) and also provides a degree of focus that unstructured interviews do not possess. Wilson (2013) pointed out that semi-structured interviews help to collect systematic information about the central topic, while also allowing some further exploration as new problems or topics occur. Six Grade 8 mathematics teachers, including the teacher in the intervention study, were invited to participate in the interviews before, during and after the PDC. The core aim of the interviews was to answer RQB, and to understand the teachers' views of the use of technology for teaching and learning mathematics in general, their perceptions of the changes using GGS may have caused in the learning and teaching of mathematics in their classes in terms of each strand of mathematical proficiency, and their perceptions of learning and teaching mathematics using GGS (see Appendix 3).

5.4.4 Teachers' group written responses

The element of data collection gathering teachers' written responses took place on the last day of the PDC, four weeks after the four-day workshop on using GGS. After teachers had completed the four-day course, they returned to their schools with the teacher guide and most should have started to use GGS. When they returned for the final day of the PDC, they were seated randomly in three groups of five. The discussion lasted approximately two to two-and-a-half hours. All the participants were teaching mathematics to Grade 8 students at that time. They came from different schools and had different amounts of experience, as shown in Table 5-8.

Table 5-8 Mathematics teachers' experience

No. of Teachers	Years of Experience
5	0–5
4	6–10
5	11–15
1	> 15

I gave them three sets of questions, related to the research questions, in three phases (see Appendix 3). They had time to discuss and share their opinions in their groups, then they nominated one person in each group to write down their collective answers. They were told that their answers should not be positive or negative, but to write what they thought and what they had found from their use of GGS. I was not involved in any of the group discussions, but rather took the role of a passive observer. When they were finished, I asked each group to share their answers with the other two groups. This step was done orally by choosing one individual from each group to present the group's answers, the aim being to share the three groups' experiences and knowledge.

5.5 Teacher training: Professional development course (PDC)

This study was designed to investigate the effectiveness of using GGS in enhancing students' mathematical proficiency. This required training teachers to use GGS in their lessons. Fifteen Grade 8 mathematics teachers attended a PDC for five days, each with his own laptop. The General Department of Education in Jazan offered a training room for this course.

The PDC had six main objectives:

1. To introduce GGS and familiarize mathematics teachers with its use in the teaching and learning of mathematics.
2. To enhance mathematics teachers' use of GGS technology by training them to use it effectively in their classrooms.
3. To provide new material (Teachers' Guide) to help them to use GGS in their classrooms effectively.
4. To help teachers design lessons.
5. To discuss with mathematics teachers their experiences after they had returned to their schools and used GGS in their classes.

5.5.1 Format of the PDC

The PDC was based on a tutorial book entitled "Learn GGS Classic", taken from the GGS website (https://www.GGS.org/m/XUv5mXTm), aimed at covering many aspects of mathematics over the five days. The course was designed to help teachers use GGS effectively in their teaching and provide a comprehensive introduction to the software. The PDC covered all aspects of mathematics required by Grade 8 teachers and consisted of 20 workshops which covered the primary use of GGS, algebra and the CAS, the two kinds of geometry (2D and 3D), spreadsheets and probabilities (see Table 5-9). The central role of the training course was to give an opportunity for teachers to investigate the program by themselves in each part of the PDC. Professional development course content

Table 5-9 Professional development course content

Day	Workshop	Activity	Time
First day	1	Basic use of GGS	60 minutes
	2	Exercises	80 minutes
	3	2D	60 minutes
	4	Exercises	80 minutes
Second day	1	3D geometry	60 minutes
	2	Algebra and CAS	80 minutes
	3	Spreadsheets (e.g. calculating functions) and probabilities	60 minutes
	4	Exercises	80 minutes
Third day	1, 2, 3, 4	Designing lessons	240 minutes
Fourth day	1, 2, 3, 4	Teachers' presentation and mathematical proficiency strands	240 minutes
Fifth day	1, 2, 3, 4	Discussion after 4 weeks of using GGS in classrooms	240 minutes
Five days	20 workshops		22 hours

First day

On the first day, teachers were divided into four groups, each comprising three to four teachers, for two workshops. The first workshop aimed to provide an introduction to the GGS and its features, as well as brief information about myself as the researcher and the research project. Then, the rules, system, and contents of the PDC were explained, and what was expected from them by the end of the course. In this workshop, the teachers learned how to download the program from the GG website, how to choose the best version for their devices, how to install it and how to open it for the first time. Then, I gave them a chance to explore the GG interface and the main menu by themselves; after that, they discussed the main menu and its contents.

I then introduced the construction tools and gave them a chance to explore the tools by themselves and discuss them, initially in their groups and then between groups. This strategy aimed to give the teachers more time using GGS, to let them get used to its environment and raise any issues they might encounter in using GGS for the first time. Some previously prepared examples of GGS were shown to the teachers. The final part of the workshop was about the toolbox and how to save any work in the GGS.

The second workshop focused on 2D geometry and covered the drawing tools, followed by an activity to draw a simple shape using the tools. After that, they drew different geometric shapes, including a basic parallelogram, square, and circumcircle of a triangle. In each activity, they worked individually with a time limit and at the end of each activity, I presented a solution to the activity. Finally, they learned how to use the Show/Hide object tool (used to show or hide any object), the navigation bar (presenting all steps used in drawing any GG exercise, step-by-step) and the construction protocol (which presents all tools used in drawing any GG exercise).

Second day

On the second day, there were three workshops. The first was on 3D geometry and covered the 3D graphics tools and how to use them. The training activities started by constructing a basic tower (combining a cylinder and a cone in a number of construction steps), and a basic parallelepiped construction. The workshop then taught teachers how to customize the 3D Graphics view. In each activity, they worked individually with a time limit and at the end of each activity, I presented a solution. The second workshop was about the CAS and covered the following items: the CAS tools and how to use them; how to enter algebraic input into the CAS view; how to use CAS commands; how to manipulate equations; tips and tricks for CAS input. The third workshop was about spreadsheets and covered the spreadsheet tools and how to use them, as well as how to input data and refer to cells.

Third day

The workshops were slightly different on the third day because the aim here was to help teachers to use the GGS effectively in the classroom. The first workshop was designed to help teachers register on the GG website and make their files. Then they learned how to download material from the website and how to edit it to be suitable for their lessons. Some advanced GG exercises were presented to the teachers and given to them with model steps to help them to do it again. The aim here was to build teacher confidence in using GGS. At the end of the day, all participants were asked to prepare some GG activities, which they would present to the group the following day, complete with constructer steps.

Fourth day

This was the final day of instructor-led training and consisted of two workshops. In the first, the teachers presented their work to the other teachers using the data show projector. The second workshop was about the mathematical proficiency strands, covering information about mathematical proficiency, the associated strands, and the meaning of mathematical proficiency strands in Grade 8. The aim was to give the teachers information about the mathematical proficiency strands before they returned on the last day of the PDC, to help involve them in the discussion. At the end of the day, I gave them a brief description of the teachers' guide and its contents if they wanted to use it in their teaching because it had many pre-prepared GG exercises which were ready to use.

5.5.2 Teachers' guide

I designed a teachers' guide for the Numbers and Geometry units in order to provide a comprehensive understanding for the teacher of each step of the lesson. This guide consisted of information about each part of the lesson and unit, starting with the mathematical proficiency strand and ending with homework. It was designed to support the teaching of the Numbers and Geometry units using GGS. These two units were modified and adjusted in line with the aims of the research. The main changes in these two units were in the warm-up and progression stages. These two stages followed the same procedures in the lessons, but they differed in terms of content. The traditional method in the two stages purely focused on direct input from the teacher, whereas the GGS-based pedagogy gave students the flexibility of working in a dynamic environment (GGS), both individually and together with peers. The guide gave teachers information about each lesson in general and about each part of the lesson in detail, such as lesson steps (Figure 5-7) and implementation tables, providing details of each step, such as the time of the activity, how to implement it and the targeted strands (Table 5-10 and 5-11).

The contents of the guide comprised a brief description of mathematical proficiency and associated strands, the unit's aims, lesson table, expected results from the unit, requirements of the unit, unit lesson stages, and the GGS exercises. The guide also provided a brief description of mathematical proficiency and the associated strands, as well as the meaning of the mathematical proficiency strands in Grade 8. The aim here was to improve teachers' understanding of each mathematical strand, helping them when they started the intervention study.

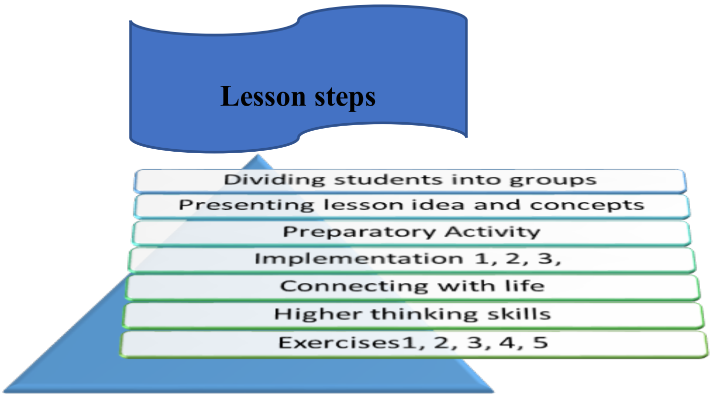


Figure 5-7 Lesson steps

In the teachers' guide there were two types of table: the first was the lesson table, which showed information on lesson ideas, lesson concepts, the proposed teaching approach, homework, and the previous skills required (Table 5-10); the second was an activity table that detailed each lesson step, providing information about timing, teaching aim (targeted strand), activity aim, and procedures (Table 5-11).

Table 5-10 Lesson table

Converting fractions to decimals	
Lesson idea	Converting fractions and mixed numbers to decimal numbers and rational decimals to fractions
Concepts	Rational numbers, terminated numbers, repeated numbers
Proposed teaching instrument	Students' book, PCs or laptops, GGS
Proposed teaching approach	The discovery-based approach, cooperative learning, learning with peers
Homework	Last exercise from each exercise stage
Previous skills required	Whole numbers and their operations, integers and their operations

Table 5-11 Activity table

Converting fractions to decimals	
Time	15 minutes
Teaching aim	Conceptual understanding, and Procedural fluency
Activity aim	Supporting students' procedural fluency by converting fractions and mixed numbers to decimals and supporting students' conceptual understanding by identifying repeated and terminated numbers.
Procedure	Giving students the two following exercises on converting fractions and mixed numbers to decimals through GGS, then allowing students to discuss the method of transforming fractions and mixed numbers to decimals in groups with the whole class. Finally, check students' understanding by asking them to complete some exercises in their groups and individually.

5.6 Mathematics lesson (time, structure, teachers' guide)

5.6.1 Time

Students in Grade 8 have five mathematics lessons per week, generally one lesson per day, each lasting 45 minutes. The total is 225 minutes per week.

5.6.2 Structure

In this study, there were two types of lesson approach: the first was the traditional approach and the second was the experimental approach, in which students were taught using GGS-based pedagogy.

Traditional approach

The traditional approach always followed the mathematics Grade 8 teacher textbook structure. This book is adapted from the McGraw-Hill book series which is offered by the MoE in the KSA for all students in all grades (from 1 to 12). It has four main steps: preparation, teaching, training, and assessment.

- Preparation: The lesson starts with preparation exercises which aim to revise a student's previous knowledge and connect it with new concepts in the lesson. In this section, students try to answer questions either by themselves or by getting help from their peers or the teacher.

- Teaching: In this step, the teacher-centred segment, the teacher introduces and explains the lesson concepts and elements to students. Each concept has some examples (mathematics tasks) to introduce it to the students and enhance their understanding when the teacher explains the concepts.
- Training: After the teacher finishes the teaching step, he asks students to answer some new questions regarding this concept or element individually. This is called *check your progress* and comes after each concept and element. Some students answer the questions alone and some get some help from their teacher. Then the teacher presents the model answers on the board. The teaching and training steps may be repeated more than once, depending on the number of elements in each lesson.
- Assessment: After finishing all the lesson concepts and covering all elements, the teacher moves on to the exercises, known in the textbook as *check your understanding*. In this segment, the teacher tries to choose some exercises that are similar to the examples given in the lesson by way of an assessment. The teacher asks each student to complete the chosen tasks. Some students answer them alone and some get some help from their teacher. Then, at the end of the lesson, the teacher presents model answers on the board before giving out the homework.

GGS-based approach

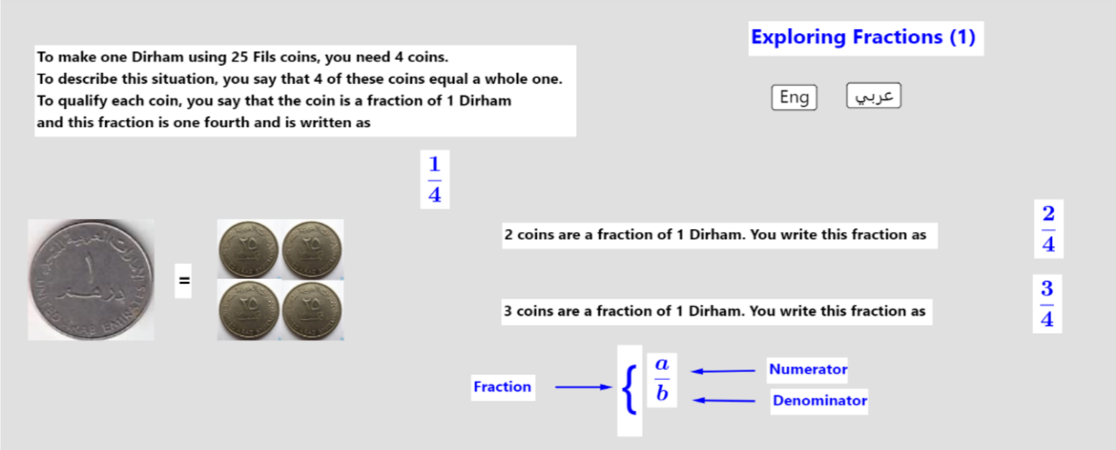
The main aim of this study was to investigate the effectiveness of using GGS-based pedagogy in enhancing students' mathematical proficiency, defined according to the NRC strands, compared to the traditional approach. The lesson structure in the intervention study differed from the traditional approach in that there were extra steps to integrate GGS, giving students more space to explore concepts by themselves through the new software. This approach also differed in its aims, because it tried to improve each strand of mathematical proficiency by focusing on each strand in the lesson stages. The GGS part of the lesson consisted of some GG exercises as preparation exercises and for each concept and element in the lesson. I had prepared some of these exercises and some of them were taken from the GGS website, which is a free open source for learners and mathematics teachers. These GGS exercises were used by students individually in the computer lab at the school (in most lessons, each student worked on the GGS tasks individually, but in some lessons, there were students who needed to work with another student because of some technical issues, so they worked in pairs). The two additional parts were a connecting with life stage, which focused on enhancing strategic competence, and a higher-order thinking skills stage, which focused on enhancing students' adaptive reasoning. Teaching using GGS in this study consisted of five stages.

- **Warm-up**: The lesson started with the GGS preparation exercises, which aimed to review a student's previous knowledge and connect it with new concepts in the lesson. In this stage, the teacher asked his students to open the exercise which was already prepared for this section and try to explore and understand the main idea of the exercise. Then he gave them some discussion questions regarding the exercise. Students discussed these questions in small groups (peer-to-peer) and with the whole class with the aim of enhancing their adaptive reasoning by presenting their thinking and ideas to their colleagues. They then completed some new mathematics tasks in the group and individually (Figure 5-8).

Preparation

1. What are Rational Numbers?

Time	**10 minutes**
Teaching aim	Conceptual understanding
Activity aim	Support students 'conceptual understanding by presenting the concept of rational numbers and linking them to smaller groups of numbers, which are used to build fractions, and identifying repeated and terminated numbers.
Procedure	Giving the students the following exercises about fractions via the GeoGebra software, then allowing the students to discuss the meaning of rational numbers in groups and as a class.



Exploring Fractions (1)

To make one Dirham using 25 Fils coins, you need 4 coins.
To describe this situation, you say that 4 of these coins equal a whole one.
To qualify each coin, you say that the coin is a fraction of 1 Dirham
and this fraction is one fourth and is written as

$\frac{1}{4}$

2 coins are a fraction of 1 Dirham. You write this fraction as $\frac{2}{4}$

3 coins are a fraction of 1 Dirham. You write this fraction as $\frac{3}{4}$

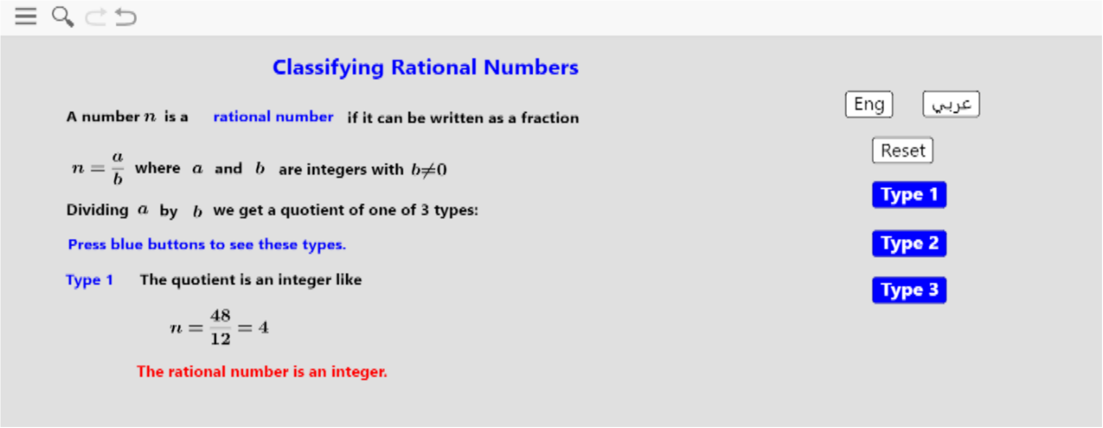
Figure 5-8 Warm-up stage

- **Progression**: This was the main stage in the use of GGS. Lessons could have one implementation step or more steps, depending on the numbers of elements and concepts in each lesson. Every single implementation step focused on a specific mathematical proficiency strand, and then the other strands if possible. The teacher knew about the targeted strands from the teachers' guide. This stage also started with GGS exercises which aimed to introduce the new concept or element to students. The number of GGS exercises depended on how many elements were in the lesson. In this stage, the teacher asked the students to open the exercise which was already prepared for this section and try to explore and understand the main idea of the exercise. From this exercise, students should have a whole or partial understanding of the targeted concept. Each GGS exercise was followed by a group discussion before the teacher gave out some discussion questions regarding the exercise and the new concept. Students discussed these questions in their group with the aim of enhancing their adaptive reasoning by presenting their thinking and ideas to their colleagues. Then the teacher presented the examples from the student's book (Figure 5-9).

Implementation 1

1. What are Rational Numbers?

Time	**10 minutes**
Teaching aim	Conceptual understanding
Activity aim	Support students 'conceptual understanding by presenting the concept of rational numbers and linking them to smaller groups of numbers, which are used to build fractions, and identifying repeated and terminated numbers.
Procedure	Giving the students the following exercises about fractions via the GeoGebra software, then allowing the students to discuss the meaning of rational numbers in groups and as a class.



Classifying Rational Numbers

A number n is a rational number if it can be written as a fraction

$n = \frac{a}{b}$ where a and b are integers with $b \neq 0$

Dividing a by b we get a quotient of one of 3 types:

Press blue buttons to see these types.

Type 1 The quotient is an integer like

$$n = \frac{48}{12} = 4$$

The rational number is an integer.

1. **Following the exercise above, discuss the following questions with your group:**
   - **What do rational number look like? Can you define a rational number?**
   - **How can you write it? Give examples?**

Figure 5-9 Progression stage

- **Training**: This stage was similar to the training stage in the traditional approach. After the students had finished the implementation stage, the teacher asked them to answer two new questions regarding the concept or element in their groups, then answer one question individually. This is known as *check your progress* and it came after each concept and element. Some students answer it individually and some got some help from their teacher. Then the teacher presented a model answer on the board.

  Note: The implementation and training stages could be repeated more than once in a lesson, depending on the numbers of elements in each lesson.

- **Assessment**: This stage aimed to assess students' understanding and differed from the traditional approach because all the exercises in the "*check your understanding*" phase were organized according to the related mathematical proficiency strand. Therefore, there were four types of exercise: the first focused on conceptual understanding, the second on procedural fluency, the third on strategic competence, and the fourth on adaptive reasoning. Students in this stage tried to answer as much as they could by themselves, then checked their answer with their peers. Then, at the end of the lesson, the teacher presented a model answer on the board before setting homework that covered each mathematical proficiency strand.

The main differences between the two teaching approaches (traditional approach, and GGS based-pedagogy) are presented in table 5-12:

Table 5-12 Differences between the two teaching approaches (traditional approach, and GGS based-pedagogy)

	Traditional approach	**GGS-based pedagogy**
Content of the lesson	Same content	Same content
Numbers of stages in the lesson	Four stages	Four stages
Warm-up stage	Using exercises in the students' book	Students had the chance to work on targeted mathematical ideas via assigned GGS tasks before starting the lesson. The goal in this step was to start from where students were and build on that as the class progressed. This ensured a clear sequence of activities in the classes and linked them with the students' prior knowledge and experiences.
Progression stage	In the traditional teaching approach, All new ideas in the lesson were introduced with several examples and directly explained to students in the progression stage (teacher-centred part).	The teaching continued with GGS tasks which aimed to introduce the new concept or idea to students. In this stage, students had the chance to become involved in targeted mathematical activities in the lesson via assigned GGS tasks. Note that the number of tasks in this stage depended on the number of new ideas or concepts to be introduced in the lesson.
Discussion	Discussions do not take place.	Each GGS task was followed by a discussion in small groups and with the whole class for which the teacher set some discussion questions regarding the new concept
Training stage	From the students' book (check your progress)	From the students' book (check your progress)
	From the students' book (check your understanding)	From the students' book, but categorized according to the mathematical proficiency strands
Homework	Exercise selected randomly from the students' book	Exercises cover all the mathematical proficiency strands (four exercises)

5.7 Quantitative data analysis

In this study, quantitative analysis was used to address the first research question. The aim was to evaluate the impact of the use of GGS on the students' mathematical proficiency and the first four strands in the MPTs, as well as the fifth strand through the student questionnaires. The analytic procedures used in this research are discussed in the following sections. The results are presented in Chapter 6.

5.7.1 Descriptive statistics

In this study, descriptive statistics were used where appropriate. Means and standard deviations (SDs) were used to give a measure of the central tendency and the spread of the data respectively.

5.7.2 Test of normal distribution

When using a statistical test that assumes normal distribution, it is important to assess the normal distribution of the data before starting formal statistical analysis to avoid drawing erroneous inferences and arriving at wrong conclusions (Das et al., 2016). According to Laerd Statistics (n.d.) "When analysing differences between groups using parametric tests (e.g., the independent-samples t-test, one-way ANOVA), a common assumption in all these tests is that the dependent variable is approximately normally distributed for each group of the independent variable (N.B., the 'groups' of an independent variable are also referred to as 'categories' or 'levels')". There are two comprehensive methods for measuring normal distribution: using numerical procedures (e.g. statistical tests) or using graphical techniques (e.g. visual inspection of graphs). There are many statistical tests of the distribution which aim to determine whether or not the sample data are normally distributed. However, it is usually enough to use one test to assess the normality of distribution (Ghasemi & Zahediasl, 2012; Howell, 2012; Cardinal & Aitken, 2013; Field, 2013).

The Shapiro–Wilk test (Shapiro & Wilk, 1965) is recommended as the optimal choice for checking the normal distribution of sample data based on tests (Ghasemi & Zahediasl, 2012; Howell, 2012; Cardinal & Aitken, 2013; Field, 2013) and was thus deployed in this study to investigate the distribution of the three MPTs and the mathematical proficiency strands. The null hypothesis states that the population is normally distributed and the alternative hypothesis is that it is not normally distributed. If the p-value is less than the confidence level (0.05), the null hypothesis is rejected and it is concluded that the data are not normally distributed. The normality of distribution was tested for the MPTs in the first phase, then the mathematical proficiency strands. The Shapiro–Wilk test was repeated three times (for each of the three MPTs) (see Appendix 4).

5.7.3 Homogeneity of variance

Some tests assume homogeneity of variance, i.e. that the population variances of two or more samples are considered equal. Levene's (1960) test examines this assumption. In testing the homogeneity of variance, the null hypothesis is $H_0 : \sigma^2{}_1 = \sigma^2{}_2$, whereas the alternative hypothesis is $H_1 : \sigma^2{}_1 \neq \sigma^2{}_2$. If the result of the test is significant ($p < 0.05$), the assumption is violated (Ghasemi & Zahediasl, 2012; Howell, 2012; Cardinal & Aitken, 2013; Field, 2013). In this, study the assumptions of homogeneity of variance between groups were tested in the three MPTs using Levene's test (see Appendix 4).

5.7.4 Inferential statistics

To test the hypothesis for each of the research questions, inferential statistics were used. All analyses were performed in IBM SPSS v 25 and at a 5% ($p = 0.05$) level of significance. It is usually accepted that this level indicates the significance of findings and the scholar can be "reasonably confident" in rejecting the null hypothesis (Coolican, 2018). At this level of significance, there is just a 5% margin of the investigator committing a Type I error, in the case of rejecting the null hypothesis when it is true.

5.7.5 Parametric and non-parametric tests

Generally, there are two types of inferential statistical analysis: parametric tests and non-parametric tests. Parametric tests are usually preferable because they have superior statistical power over non-parametric alternatives (Coolican, 2018). Thus, parametric analysis is more likely to detect an impact if it occurs and it gives researchers more confidence in their statistical analysis (Caulfield & Hill, 2018). Parametric tests also allow more complicated analyses to be conducted; for instance, it is possible to control for the effects of a confounding variable. Non-parametric tests are less effective than parametric tests and they normally require a greater sample size to obtain the same power as parametric tests in identifying a discrepancy between groups where it occurs (Sullivan & Artino, 2013).

Each type works best under different conditions; parametric tests work when the data follow a specified distribution (usually a normal distribution), while non-parametric tests work when the data cannot be assumed to come from a given distribution, for example in the case of ordinal data. However, there are some statistical assumptions that should be considered in parametric testing, for example:

- Data must be continuous.
- The sample should be taken from a population that is normally distributed.

The samples being compared must be drawn from populations with the same variance (otherwise known as homogeneity of variance) (Brace et al., 2012, p. 12).

Initial checks were conducted to check whether or not the sample data met these criteria prior to the analysis being carried out (see Appendix 4), and if any of the above assumptions were violated, non-parametric tests were used instead of parametric tests. For example, in the overall MPT, the three tests met these assumptions, so the parametric t-test was used to determine the differences between the two groups each time. However, with regard to the mathematical proficiency strands, the scores did not meet the normal distribution assumptions, so the Mann–Whitney U test was employed to investigate the differences between the two groups in each strand.

5.7.6 Choice of statistical analyses

Independent t-test

The t-test is used to compare the means of two groups and is an example of a parametric test that works on normally distributed scale data (Gerald, 2018). It is used when there are two groups or two sets of data, and for the purposes of this study, could be used to determine the impact of teaching approaches and methods on the students' performance (Pallant, 2016; Gerald, 2018). As noted by McMillan and Schumacher (2010), the t-test refers to an inferential statistical procedure for determining the probability level of rejecting the null hypothesis that two means are the same. There are three types of t-test, namely the independent samples t-test, the dependent samples t-test, and the one sample t-test. The one sample t-test is used to compare the mean of a sample to a predefined value. The dependent or paired sample t-test is used to compare the means of two conditions in which the same participants contributed to the study. The independent or unpaired sample t-test is used to compare the means of two groups of participants.

In this study, an independent samples t-test were used three times (in the three MPTs) to determine whether there were any significant differences in the student's mathematical proficiency between the means of the two groups (control and experimental). All assumptions of the independent samples t-test were considered and calculated at the beginning of the data analysis procedure.

Mann-Whitney U test (Wilcoxon rank sum test)

The Mann-Whitney U test was developed by Wilcoxon in 1945 and is beneficial in terms of comparing the location of two independent samples (Salkind, 2012). It is often considered the non-parametric equivalent of the independent t-test (MacFarland & Yates, 2016) in that both tests have similar purposes, because they are both used to determine if there are statistically significant differences between two groups (MacFarland & Yates, 2016; Pallant, 2016). However, the Mann–Whitney U test is used with non-parametric data, whereas the independent t-test is used with parametric distributions (MacFarland & Yates, 2016). According to Salkind (2012), the Mann–Whitney U test rests on two fundamental assumptions, i.e. that the scores are independent and that they come from a continuous probability distribution.

In this study, where the assumptions of the t-test were violated, the non-parametric Mann–Whitney U test was used as an alternative to the independent samples t-test (Pallant, 2016). The Wilcoxon signed rank test was also used to test differences in productive disposition within the groups before and after the intervention study.

Effect size

Effect size is "simply a way of quantifying the size of the difference between two groups. It is easy to calculate, readily understood and can be applied to any measured outcome in Education or Social Science" (Coe, 2002, p. 1). It is defined as a quantitative reflection of the measure of some phenomenon that is used for the determination of addressing an enquiry of interest (Kelley & Preacher, 2012). According to Rodriguez (2011), there are two types of effect size measures: the first concerns measures of correlation or relationship, and the second measures differences in standardized or relative means.

To measure the effect of the GGS-based pedagogy on the students' mathematical proficiency in this study, the effect size was calculated using Cohen's d:

> Cohen's d is a measure of effect size in standard deviation units that is very commonly used as a basis to estimate the sample sizes required to ensure statistical power for a two-sample problem and as the data for conducting a meta-analysis. (Rodriguez, 2011, p. 3)

According to McLeod (2019), Cohen's d is an appropriate effect size measure for making a comparison between two means. It could be used, for example, in reporting t-test and analysis of variance (ANOVA) results. Table 5-13 provides the magnitudes of d = 0.01 to 2.0 and the associated descriptions.

Table 5-13 Description of Cohen's d values (source: Sawilowsky, 2009)

Effect size	D
Very small	0.01
Small	0.20
Medium	0.50
Large	0.80
Very large	1.20
Huge	2.0

5.8 Qualitative data analysis

Qualitative data in this research were collected using semi-structured interviews, and the written responses from the teachers, and the data were then subjected to thematic analysis. Thematic analysis is “a method for identifying, analysing and reporting patterns (themes) within data” (Braun & Clarke, 2006, p. 79) and is a basic skill in qualitative analysis (Holloway & Todres, 2003). It is also considered a useful tool within grounded theory and sociocultural theory for locating and identifying themes within these frameworks (Boyatzis, 1998). A theme could present something significant about the data related to the research question and analysing data through themes gives the opportunity to interpret the data in light of social and personal considerations (Creswell, 2002). However, it is difficult to demarcate thematic analysis strategies or name the steps clearly. According to Braun and Clarke (2006, p. 81):

> Thematic analysis can be an essentialist or realist method, which reports experiences, meanings and the reality of participants, or it can be a constructionist method, which examines the ways in which events, realities, meanings, experiences and so on are the effects of a range of discourses operating within society. It can also be a “contextualist” method, sitting between the two poles of essentialism and constructionism, and characterised by theories, such as critical realism (e.g., Willig, 1999), which acknowledge the ways individuals make meaning of their experience, and, in turn, the ways the broader social context impinges on those meanings, while retaining focus on the material and other limits of “reality”.

Clarke and Braun (2013), in contrast to some authors who demarcate thematic analysis as a phenomenological method, classify it as an analytic method, which means that it is not matched with a specific epistemological or theoretical viewpoint. The thematic analysis is a very flexible process (Clarke & Braun, 2013) aimed at identifying significant or interesting trends in data and then using them to address the study topic or make a point about an issue (Maguire & Delahunt, 2017). An outstanding thematic analysis does more than merely summarize the data; it characterizes and makes sense of them (Maguire & Delahunt, 2017). Themes within data can be revealed using either an inductive or a deductive method. The inductive ("bottom-up) method draws themes only from the data themselves, without necessarily engaging in *a priori* analysis of the literature (Braun & Clarke, 2006, 2013). In contrast, the deductive method is determined by the researcher's analytical or theoretical interest in the area (Braun & Clarke, 2006, 2013). Within each of these two methods, there are additionally two ways of classifying themes: semantic (explicit) or latent (interpretive) (Boyatzis, 1998). Semantic classification is used to identify data with an obvious meaning, whereas for a latent, or interpretive classification, the analyst goes behind the explicit data (Braun & Clarke, 2006). This study identified themes inductively from the teachers' interviews and written responses, doing so at the semantic level.

5.8.1 Data analysis process

In this study, the teacher interviews, their written responses, and the student focus group data were analysed in six stages, as suggested by Braun and Clarke (2006), with the help of NVivo, version 12 (see Figure 5-10).

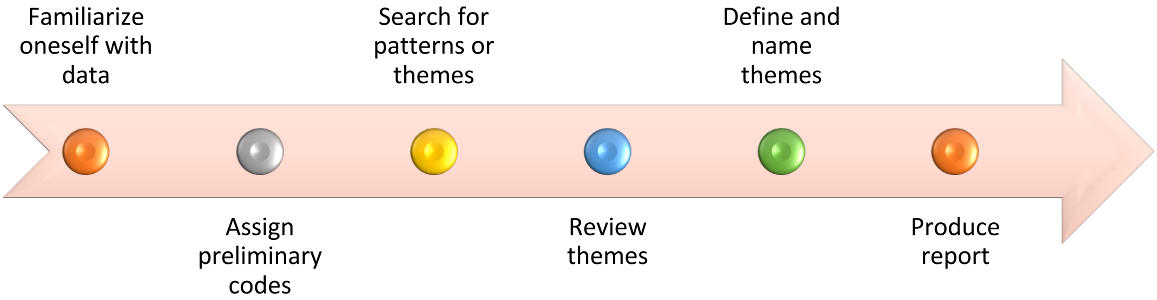


Figure 5-10 The six phases of thematic analysis

- **First step**

  After the data were collected in Arabic, I transcribed and translated them into English. I then started to familiarize myself with the data, which were input in the NVivo software to run a query about initial codes before searching for commonly repeated words.

- **Second step**

  I started gathering the initial codes. All coding and gathering was carried out manually in the NVivo software and all codes that were similar or related were collected in one file.

- **Third step**

  In this step, I sorted the codes into themes.

- **Fourth step**

  In this step, all the themes were reviewed. I read all the themes to ensure that a coherent pattern was formed and that all parts of every theme had meaningful relationships with each other.

- **Fifth step**

  The fifth step was to define each theme, relating the data analysed within each theme and then determining what aspects of the data were included in each theme.

- **Sixth step**

  In this step, the final report was produced, ensuring sufficient evidence was provided for each theme.

5.9 Ethical considerations

This study was carried out subject to the ethical procedures of the University of Glasgow. The College of Social Sciences Research Ethics Committee reviewed and approved it. Plain language statements were given to all participants (teachers and students) and student carers to illustrate the aims and details of the study and the procedures with respect to collecting the data.

Data were gathered and coded by letters rather than the names of participants, and once the data have achieved their purpose, they will be destroyed. Access to computer files is available by password only, and after analysis and any publications arising from the data have been completed, the data will be destroyed securely. Assurances regarding confidentiality will be strictly adhered to unless evidence of wrongdoing or potential harm is uncovered, and the participants will be made aware that in such cases the university may be obliged to contact relevant statutory bodies/agencies. All data used in the thesis/publications arising will be reviewed by the supervisors/co-authors to ensure that any identifying information is removed, so that the risk of identifying participants is minimized.

Both participant teachers and students were informed that their participation was voluntary and that they were free to withdraw at any time without giving any reason during the data collection period. They were informed that this would not affect the evaluation of their work in any way or have any other detrimental effects in terms of their relationship with others in the school. Also, they were informed that all interviews would be audio-taped and gave their consent for audio recording to be used.

The teachers and students who participated in this study were given a consent form to sign before starting the study. Students' consent forms were to be signed by their parents or carers. They were given the consent forms after they had been provided with a plain language statement (PLS) explaining the study. Moreover, the my email was provided at the end of the consent form in case they needed more information about the study. The PLS aimed to ensure that all participants understood the aims of the research and what it was about, how they would participate and be assessed, and how long the study would last (see Appendix 5).

The quasi-experimental setup was a significant aspect of the research design which ensured that both groups were treated equally. This was done by exchanging the group roles in the two taught units. In the first unit, class A was the experimental group and class B was the control group, whereas in the next unit, class B was the experimental group and class A was the control group.

5.10 Permissions required

Four sets of permissions were required prior to starting the data collection for this study.

1. Approval ethics form (see Appendix 6).
2. Permission letter from the General Department of Education in the Jazan region. This is normally required under the education policy in Saudi Arabia before conducting any research. It requires the researcher to contact the research department in the administration and attach the study instruments for approval before they can be implemented in schools. Permission was granted on 18/05/2018 (see Appendix 7).
3. Permission letter from the selected school to implement the intervention study. The letter was essential for the General Department of Education in the Jazan region to ensure that the implementation would not have any effect on the smooth running of the school during the study period. The school accepted these assurances and permission was duly granted before starting the study (see Appendix 8).

4. A letter of permission from the International Association for the Evaluation of Educational Achievement (IEA) to use released TIMSS items and student scales. According to the IEA website (TIMSS, 2015):

> TIMSS and PIRLS are registered trademarks of IEA. Use of these trademarks without permission of IEA by others may constitute trademark infringement. Furthermore, the website and its contents, together with all online and printed publications and released items by TIMSS, PIRLS, and IEA are and will remain the copyright of IEA. All publications and released items by TIMSS, PIRLS, and IEA, as well as translations thereof, are for non-commercial, educational, and research purposes only. Prior notice is required when using IEA data sources for assessments or learning materials. IEA reserves the right to refuse copy deemed inappropriate or not properly sourced.

Hence, it was important to seek permission from the IEA before using its material. The inquiry was sent by email after completing the permission request form required from anyone seeking permission to reuse, reproduce, or translate IEA material. Permission was granted for the use of released TIMSS items and questionnaires (see Appendix 9).

5.11 Conclusion

This chapter has presented the research methodology, methods, procedures, and instruments used in this study. All the research procedures were implemented under the ethical guidance employed by the University of Glasgow. Mixed methods were used as the principal research methodology. A quasi-experimental approach was employed to investigate the effectiveness of using GGS-based pedagogy in enhancing students' mathematical proficiency. Two units (Numbers and Geometry) were taught according to either the traditional approach or based on GGS-based pedagogy, both at Grade 8 level. With regard to the participating teachers, a PDC was conducted to show them how to use GGS in their classrooms.

Overall, four types of research instruments were used: for students, three MPTs and a productive disposition questionnaire; with teachers, interviews and written responses. To analyse the data, quantitative methods were used to examine the proficiency test scores and the results of the productive disposition questionnaire, whereas a qualitative approach was used to analyse the teachers' interview data and their written responses.

All sections above have been described in detail in this chapter. The next two chapters present the results of the study.

Chapter 6 Student-Related Findings

6.1 Introduction

This research investigated the effect of using GGS-based pedagogy on students' mathematical proficiency. It also sought to determine teachers' perceptions of teaching mathematics using the GGS-based pedagogy. To achieve these research aims, different instruments were employed. Three mathematical proficiency tests (MPTs) were conducted to measure the students' achievement and performance in the first four NRC mathematical proficiency strands, namely conceptual understanding (CU), procedural fluency (PF), strategic competence (SC), and adaptive reasoning (AR). To identify the effect of using the GGS-based pedagogy on the fifth strand of mathematical proficiency, productive disposition (PD), a questionnaire consisting of three parts was conducted with each class before and after teaching the students using GGS-based pedagogy.

Three statistical tests were employed to analyse the quasi-experimental quantitative study data: an independent samples t-test, the Mann–Whitney U test, and the Wilcoxon signed rank test. The findings demonstrated that the use of the GGS-based pedagogy significantly improved the students' overall mathematical achievement in the second MPT (Numbers), and the third MPT (Geometry), and showed that the approach was more effective than traditional teaching methods. However, this was only the case in terms of the students' mathematics achievement in certain strands of mathematical proficiency. In particular, in the Numbers unit, the experimental group demonstrated better performance than the control group in the mathematical achievement test overall and in the CU, PF and PD strands, but not in the SC or AR strands. For the Geometry unit, the students in the experimental group demonstrated better performance in the mathematical proficiency test overall than those in the control group, and in the PF and PD strands in particular, while they demonstrated no significant improvement in the CU, SC or AR strands.

6.2 Mathematical proficiency tests (MPTs)

To assess the students' mathematical proficiency, MPTs were employed on three occasions: before commencing the quasi-experimental study (the first MPT), and after teaching the students using the GGS-based pedagogy and traditional approach for two units, with one test completed after each unit (the second and third MPTs). The main aim of the MPT at the first time point was to investigate whether any prior differences existed between the two groups in terms of their mathematical achievement and the mathematical proficiency strands. The tests were also used to assess any differences between the two groups in terms of their mathematical achievement and the mathematical proficiency strands after being taught using GGS-based pedagogy. In addition to the mathematical proficiency tests, the four mathematical proficiency strands, namely CA, PF, SC, and AR, were considered on each occasion as sub-tests. The final strand, PD, was measured using a separate questionnaire.

The aim of using the MPTs was to answer the first research question, which is as follows:

Research Question A (RQA): Is there a significant effect from using a GGS-based pedagogy on students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition), and their mathematics achievement compared with using the traditional teaching approach?

The independent samples t-test was employed three times to compare the groups in the three MPTs as the main test, except when the data did not meet the assumptions of normal distribution and homogeneity of variance, in which case the Mann–Whitney U test was used instead.

For the fifth strand, in which the productive disposition questionnaire was used, the Wilcoxon signed rank test and the Mann–Whitney U test were employed. The Mann–Whitney U test was used to test the differences between the groups before the intervention study, while the Wilcoxon signed rank test was used to examine the differences within the groups before and after the intervention study. These tests are non-parametric and were used because the Likert scale employed in the questionnaire constitutes an ordinal scale of measurement for ordering categories (McCrum-Gardner, 2008). Moreover, as noted by Pallant (2016, p. 213), "'non-parametric techniques are ideal for use when you have data that is measured on nominal (categorical) and ordinal (ranked) scales".

The effect size was also calculated after each statistical test and showed significant results in terms of measuring the magnitude of the effect of the GGS-based pedagogy on students' performance. In the case of the t-tests, this was calculated as Eta squared (Pallant, 2016, p. 243):

$$\text{Eta squared} = \frac{t^2}{t^2 + (N1 + N2 - 2)} .$$

For the Mann–Whitney U and Wilcoxon signed rank tests, the effect size was calculated as the Cohen's d (Cohen, 1988) (r = z / square root of N where N = total number of cases (Pallant, 2016)).

6.2.1 Test of normal distribution

It is critical to assess whether the data obtained meets the test requirements before commencing certain kinds of formal statistical analysis to avoid drawing erroneous inferences and wrong conclusions (Das et al., 2016). In this study, the Shapiro–Wilk test (Shapiro & Wilk, 1965) was employed to examine the normal distribution of responses for the MPTs and for the mathematical proficiency strands. The test revealed that the overall MPT scores were approximately normally distributed for the two classes ($p > 0.05$), as detailed in Table 6-1.

Table 6-1 Normal distribution test for the three MPTs

Shapiro–Wilk test				
	Class	Statistic	df	Sig.
First MPT	Class A	.934	19	.203
	Class B	.935	18	.239
Second MPT	Class A	.974	19	.855
	Class B	.970	18	.789
Third MPT	Class A	.928	19	.157
	Class B	.947	18	.374

Also, for the three MPTs, the Z values for skewness and kurtosis were within (-1.96) and (+1.96), and through the visual examination of the histogram normal Q_Q plots and box plots, it was clear that the data were approximately normally distributed. However, the test showed that the CU, PF, SC, and AR scores were not approximately normally distributed ($p < 0.05$) (see Appendix 4).

A further assumption of the independent samples t-test is the homogeneity of variance. This means that the test assumes that the population variances of two or more samples are equal. Levene's (1960) test examines this assumption. Levene's test was applied to compare the two groups and produced an overall result of $p > 0.05$ for the MPTs (first MPT $p = 0.585$; second MPT $p = 0.560$; third MPT $p = 0.091$). Therefore, according to these results, the null hypothesis was accepted and the variances of the two groups were deemed to be equal in the three tests (for all tables, see Appendix 4).

In all three cases (first, second, and third MPTs), the test scores achieved the assumptions of the independent samples t-test: normal distribution and homogeneity of variance. Therefore, the independent samples t-test was used to estimate the effect of the GGS-based pedagogy on performance. However, not all the mathematical proficiency strand scores met the normal distribution assumption and thus the Mann–Whitney U test was used to estimate the differences between the groups in the three cases (first, second, and third MPTs) for the first four mathematical proficiency strands.

6.3 First MPT

A pre-test is one of the critical elements of a quasi-experimental study design, ensuring that the study results are not biased, because if there is a significant difference between the groups initially, this might be sufficient to explain any differences observed after the intervention (Johnson & Christensen, 2014). The main aim of this test in the case of this study was to ensure that the level of prior knowledge in each group was on a par. Moreover, using pre-tests helps a researcher to avoid some of the threats to a study's validity, such as bias (Harris et al., 2006). It was therefore employed here to ensure confidence that any differences between the students in the control and experimental conditions were due to the teaching intervention. The test focused on the Algebra unit. Both the control and the experimental groups had already completed this unit before the test.

The data obtained the tests showed that there were no statistically significant differences between the groups at baseline either in terms of their overall mathematical achievement (t-test) or in any of the strands of mathematical proficiency (Mann-Whitney U tests) (see Appendix 4). This provided increased confidence that any differences in the tests after the intervention study would result from the intervention, namely teaching the students using the GGS-based pedagogy.

6.4 Effect of GGS-based pedagogy on mathematics proficiency

To test the effect of the teaching intervention, the second and third MPTs were conducted, assessing the impact on the first four strands of mathematical proficiency, CU, PF, SC, and AR. In addition, the fifth strand, PD, was assessed through a questionnaire. This section presents the results, first in relation to overall mathematical achievement, based on the MPT results for Strands 1–4, and then broken down by strand, across all five strands. Overall, the findings provided evidence of a positive effect of the use of GGS-based pedagogy on mathematical proficiency, with this effect driven primarily by a positive effect on the PF and PD strands.

The effect of the GGS-based pedagogy was assessed in terms of its overall effect on mathematical achievement in the MPTs by examining the total scores of the students in the two tests. Because of the cross-over design of the study, both groups had the opportunity to experience the GGS-based pedagogy and there are therefore two sets of results for this section. Class A served as the experimental group for the Numbers unit, whereas Class B served as the experimental group for the Geometry unit.

This section presents the results, first in relation to the second MPT (Numbers unit) and second in relation to the third MPT (Geometry unit).

6.4.1 Effect of GGS-based pedagogy on mathematical proficiency in the Numbers unit

This section presents the effects of using GGS-based pedagogy in two parts, the first concerning the students' overall mathematics achievement and the second addressing the mathematical proficiency strands (1–5), highlighting the differences between the experimental group (Class A) and control group (Class B).

Impact on overall mathematics achievement (Numbers unit)

Table 6-2 provides the results of the independent samples t-test examining the differences in overall mathematical proficiency achievement between Class A (experimental) and Class B (control) following the intervention using the GGS-based pedagogy.

Table 6-2 Results of the independent samples t-test for the Numbers unit

Second MPT (Numbers unit)				
Group	Mean	SD	t-value	P-value (2-tailed)
Experimental N = 19	9.76	3.05	3.454	0.001
Control N = 18	6.17	3.28		

The analysis revealed that for the Numbers unit there was a significant difference in the scores between the two groups (Table 6-2) in favour of the experimental group (Class A). The effect size (Eta squared) was 0.25, which is considered to be large. Therefore, the use of GGS-based pedagogy had a substantial effect on improving the students' mathematical achievement.

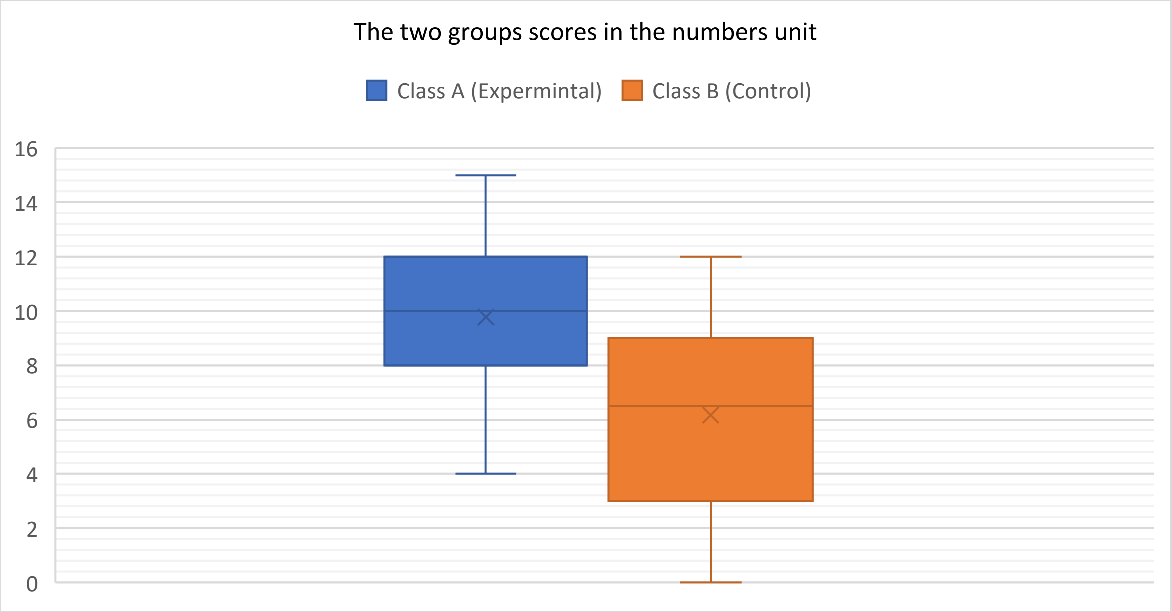


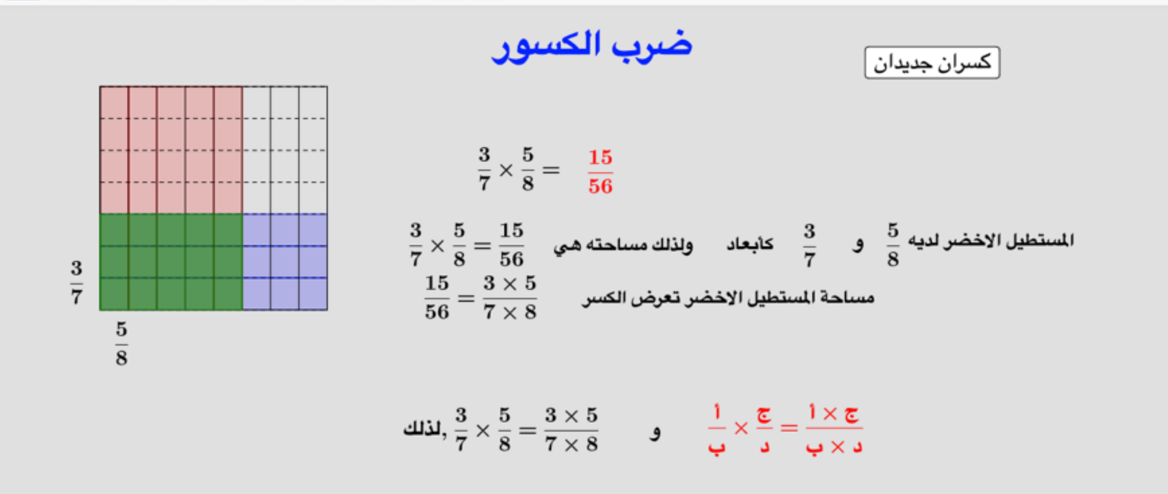
Figure 6-1 Box-and-whisker plots of the overall MPT values for the Numbers unit

Figure 6-1 shows that the difference in overall performance between the two groups applies to both actual scores and mean scores, as well as the range. For example, Class A's scores range between 4 and 15, whereas Class B's scores range between 0 and 12. Comparing the interquartile ranges also reveals a difference (8–12 for Class A and 3–9 for Class B).

Since the two groups had similar results on their initial test, the differences found can be attributed to the use of GGS-based pedagogy. In other words, the GGS-based pedagogy seems to lead to higher student attainment than traditional teaching. The main difference between the two approaches seems to result from the warm-up and progression stages in GGS-supported classes. These two stages follow the same procedure in the lesson, but they are different in terms of content. The warm-up stage aims to revise content that the students have already studied but that they need in the lesson to understanding the new concepts. In the progression stage, they explore new ideas and concepts. In the traditional method, the two stages purely focus on direct input from the teacher, whereas the GGS-based pedagogy gives students the flexibility to work in a dynamic environment (GGS), both individually and together with peers, and to reflect on their work (in most lessons, each student works on the GGS tasks individually; however, in some lessons, there may be students who need to work with another student because of some technical issues, so they work in pairs). The work in the GGS environment is further detailed below.

In the warm-up stage, students had the chance to work on targeted mathematical ideas via assigned GGS tasks before starting the lesson. The goal in this step was to start from where students were and build on that as the class progressed. This ensured a clear sequence of activities in the classes and linked them with the students' prior knowledge and experiences. The teacher initially introduced each GGS task to the students and explained how it worked, then he let students work by themselves to explore the targeted concepts or ideas through the GGS tasks.

For example, in the lesson on Division of Rational Numbers, students were given the exercise shown in Figure 6-2 during the warm-up stage. The main objective of this activity was to relate the students' knowledge of multiplying fractions to dividing rational numbers.



ضرب الكسور

$\frac{3}{7}\times\frac{5}{8}=\frac{15}{56}$

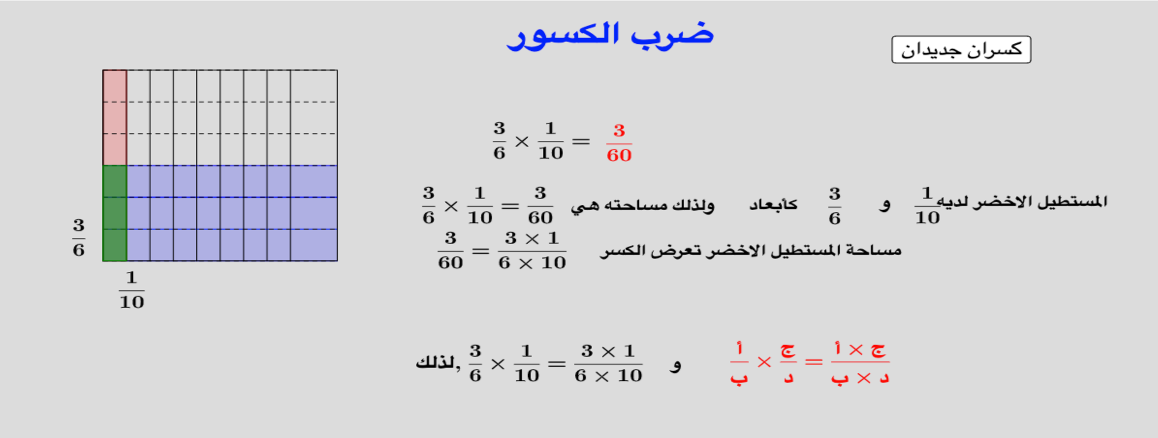
المستطيل الاخضر لديه $\frac{5}{8}$ و $\frac{3}{7}$ كابعاد ولذلك مساحته هي $\frac{3}{7}\times\frac{5}{8}=\frac{15}{56}$

مساحة المستطيل الاخضر تعرض الكسر $\frac{15}{56}=\frac{3\times 5}{7\times 8}$

لذلك, $\frac{3}{7}\times\frac{5}{8}=\frac{3\times 5}{7\times 8}$ و $\frac{\text{أ}}{\text{ب}}\times\frac{\text{ج}}{\text{د}}=\frac{\text{ج}\times\text{أ}}{\text{د}\times\text{ب}}$

Figure 6-2 Screenshot from the GGS exercise for multiplying fractions (1)

In this exercise, students were given a rectangle with sides measured in rational numbers. For example, the pictorial representation given in Figure 6-2 shows a green rectangle with sides 3/7 and 5/8 and the green area represents $\frac{3}{7}\text{x}\frac{5}{8}$. Note that the larger rectangle is vertically divided into 7 equal parts and horizontally divided into 8 equal parts. Hence, the green area corresponds to 3 groups of 5 parts that are of the size 15/56. The results for this calculation given next to the diagram are linked to the rectangular area in green. Students can find the result by calculating the number of green units (3 groups of 5) giving 15, and think about the size of each green unit as 1/56 as there are 56 units in total in the whole rectangle. The same idea is revisited when students change the fractions using the "new fractions" button. There are two such examples given in Figures 6-3 and 6-4 [($\frac{3}{6}\text{x}\frac{1}{10}$) and ($\frac{2}{4}\text{x}\frac{7}{9}$)]:



ضرب الكسور

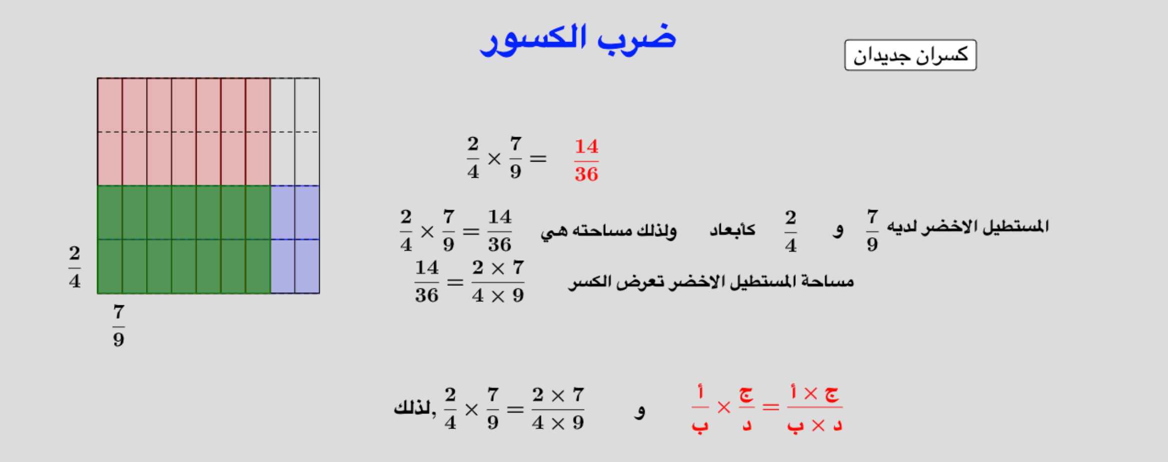
$\frac{3}{6}\times\frac{1}{10}=\frac{3}{60}$

المستطيل الاخضر لديه $\frac{1}{10}$ و $\frac{3}{6}$ كأبعاد ولذلك مساحته هي $\frac{3}{6}\times\frac{1}{10}=\frac{3}{60}$

مساحة المستطيل الاخضر تعرض الكسر $\frac{3}{60}=\frac{3\times 1}{6\times 10}$

$\frac{أ}{ب}\times\frac{ج}{د}=\frac{ج\times أ}{د\times ب}$ و $\frac{3}{6}\times\frac{1}{10}=\frac{3\times 1}{6\times 10}$,لذلك

Figure 6-3 Screenshot from the GGS exercise for multiplying fractions (2)



ضرب الكسور

$\frac{2}{4}\times\frac{7}{9}=\frac{14}{36}$

المستطيل الاخضر لديه $\frac{7}{9}$ و $\frac{2}{4}$ كأبعاد ولذلك مساحته هي $\frac{2}{4}\times\frac{7}{9}=\frac{14}{36}$

مساحة المستطيل الاخضر تعرض الكسر $\frac{14}{36}=\frac{2\times 7}{4\times 9}$

$\frac{أ}{ب}\times\frac{ج}{د}=\frac{ج\times أ}{د\times ب}$ و $\frac{2}{4}\times\frac{7}{9}=\frac{2\times 7}{4\times 9}$,لذلك

Figure 6-4 Screenshot from the GGS exercise for multiplying fractions (3)

Making changes to the fractions and observing the changes in the diagrams synchronously allowed students to see the differences in the results of given problems.

After assigning the tasks, each student was asked to work alone (or in a pair) for a while in the warm-up stage and continue to work on the main idea for multiplying fractions. Then the students were allowed to discuss the meaning of multiplying fractions and share their reasoning and understanding with the class (the same style was also used later in the progression stage). This warm-up stage helped students understand the meaning of multiplying fractions based on their work in the GGS environment and their discussions with peers. Such tasks seemed to help students to gain an in-depth understanding of multiplying fractions to some extent. The dynamic change in the diagram, in tandem with the change in the fractions chosen, may have improved the students' understanding.

Another example from the warm-up stage is Adding and Subtracting Like Fractions (Figure 6-5). The purpose of this activity was to help students relate their knowledge of adding and subtracting integer numbers to adding and subtracting like fractions.

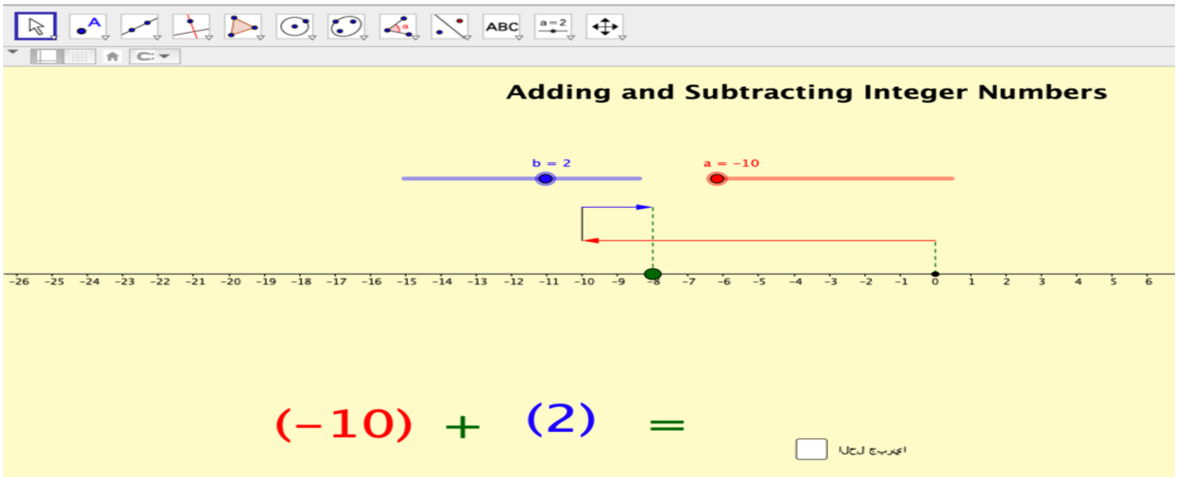


Figure 6-5 Screenshot from the GGS exercise for adding integer numbers (1)

In this exercise, students were given the number line with two sliders and a button to check their answers. The task in Figure 6-5 represents the operation of [(-10) + (2)]. The two numbers in this exercise (-10, 2) are represented by the red and blue arrows. The red arrow (number -10) shows 10 steps to the left from the origin (because of the negative sign) and the blue arrow shows two steps to the right from -10 (because of the positive sign). The result of this operation was represented by the green point on -8, which is the number of steps between zero and the green point. The same idea is revisited when students change the two numbers using the two sliders.

Students also had the chance to test their own understanding by hiding the result using the "hide/show" button; this gave them a chance to find out the correct answer independently and get feedback from the GGS by then revealing the correct answer. Such capability helps students to work with confidence and overcome any fear of making mistakes as they have the chance to check their answers by themselves without the teacher.

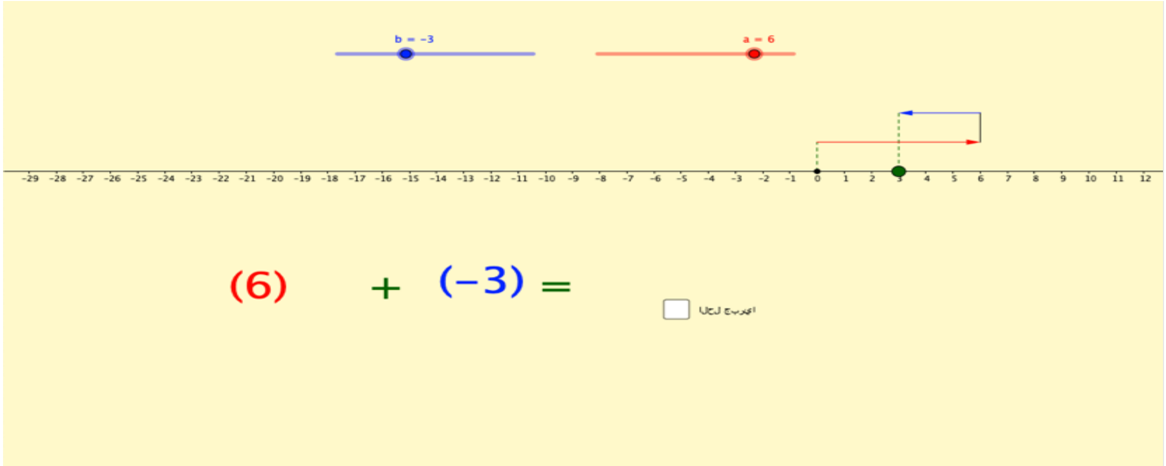


Figure 6-6 Screenshot from the GGS exercise for adding integer numbers (2)

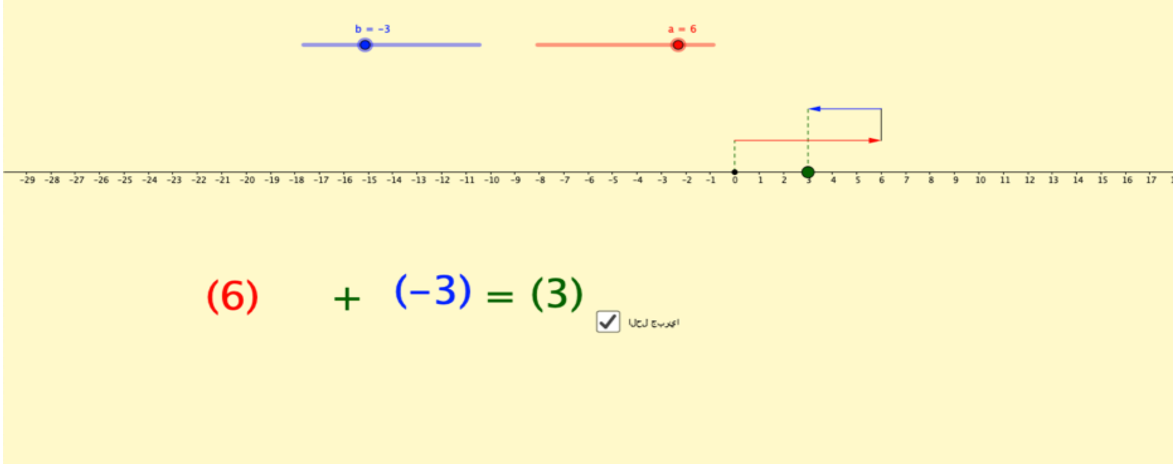
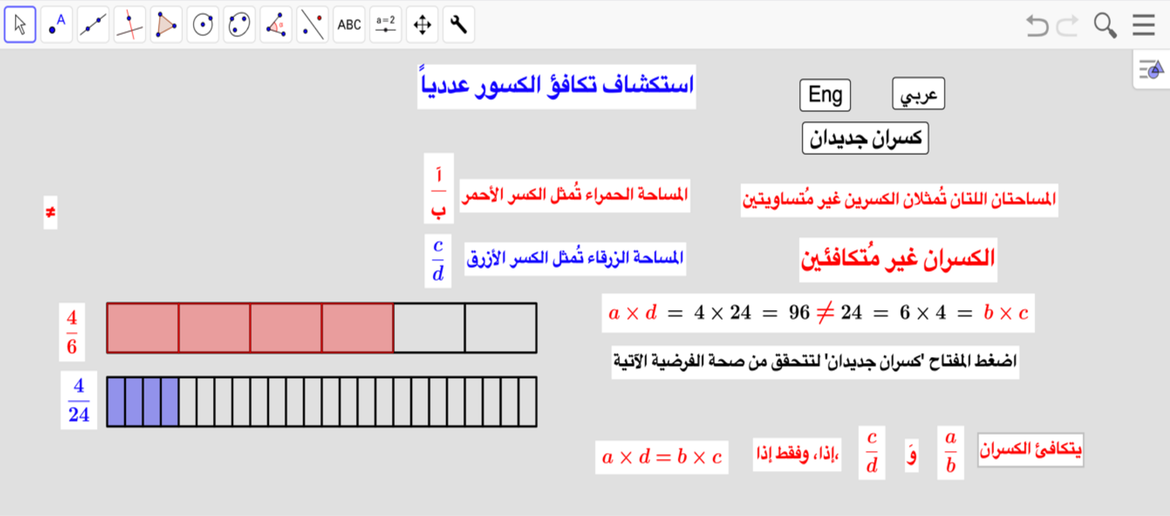


Figure 6-7 Screenshot from the GGS exercise for adding integer numbers (3)

In the traditional teaching approach, all new ideas in the lesson were introduced with several examples and directly explained to students in the progression stage (teacher-centred part). In contrast, in the progression stage of the GGS-based pedagogy, the teaching continued with GGS tasks which aimed to introduce the new concept or idea to students. In this stage, students had the chance to become involved in targeted mathematical activities in the lesson via assigned GGS tasks. Note that the number of tasks in this stage depended on the number of new ideas or concepts to be introduced in the lesson. From these tasks, students may have been able to obtain a whole or partial understanding of the target concept or idea. Each GGS task was followed by a discussion in small groups and with the whole class for which the teacher set some discussion questions regarding the new concept. For example, in the lesson on Comparing and Ordering Rational Numbers, students started learning how to compare two fractions with similar or different denominators using the exercise shown in Figure 6-8.



استكشاف تكافؤ الكسور عددياً

$\frac{أ}{ب}$ المساحة الحمراء تُمثل الكسر الأحمر

$\frac{c}{d}$ المساحة الزرقاء تُمثل الكسر الأزرق

المساحتان اللتان تُمثلان الكسرين غير مُتساويتين

الكسران غير مُتكافئين

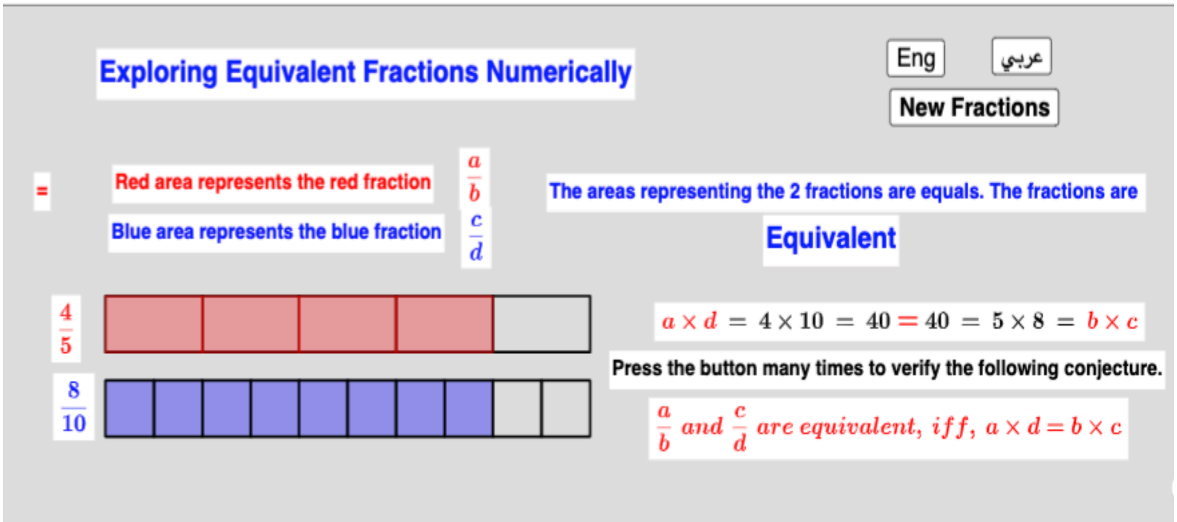
$a \times d = 4 \times 24 = 96 \neq 24 = 6 \times 4 = b \times c$

اضغط المفتاح 'كسران جديدان' لتتحقق من صحة الفرضية الآتية

يتكافئ الكسران $\frac{a}{b}$ وَ $\frac{c}{d}$ إذا، وفقط إذا $a \times d = b \times c$

Figure 6-8 Screenshot from the GGS exercise for comparing fractions (1)

In this exercise, each fraction has a different colour and shape. The first fraction given is represented in red, whereas the second fraction is represented in blue below it. Students were able to change the two fractions by pressing the “new fraction” button. Each time they pressed this button, a new fraction was randomly generated by the GGS, with a different shape and comments on whether the fractions were equivalent or not and why.



Exploring Equivalent Fractions Numerically

Red area represents the red fraction

Blue area represents the blue fraction

The areas representing the 2 fractions are equals. The fractions are

Equivalent

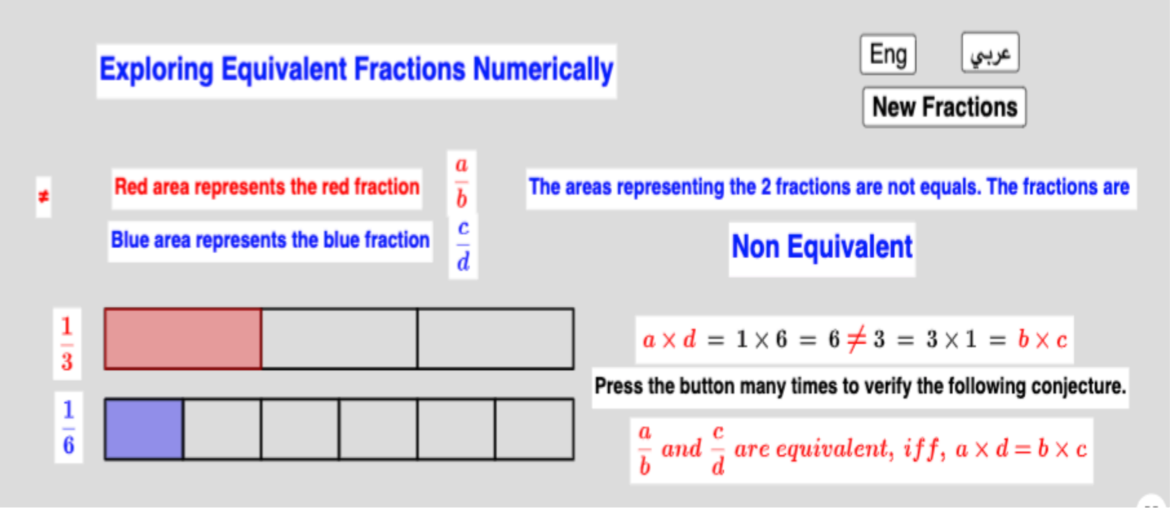
$a \times d = 4 \times 10 = 40 = 40 = 5 \times 8 = b \times c$

Press the button many times to verify the following conjecture.

$\frac{a}{b}$ and $\frac{c}{d}$ are equivalent, iff, $a \times d = b \times c$

Figure 6-9 Screenshot from the GGS exercise for comparing fractions (2)

In this exercise, the students learned how to compare two fractions by comparing the coloured shape associated with each one, in which each fraction represented the same area. If the two shapes cover the same area, the two fractions are equal; if they do not cover the same area, they are not equal (Figure 6-10). Such comparisons allow students to recognize the differences or similarities between two fractions.



Exploring Equivalent Fractions Numerically

Red area represents the red fraction

Blue area represents the blue fraction

The areas representing the 2 fractions are not equals. The fractions are

Non Equivalent

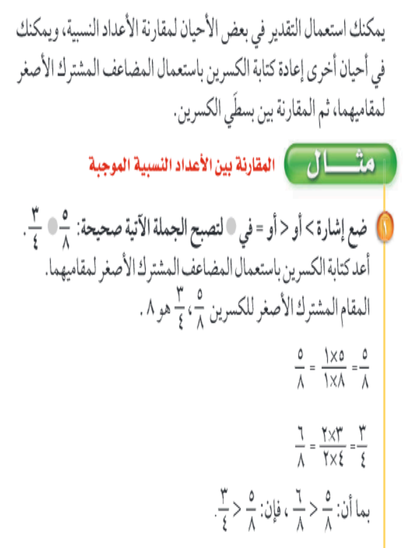
$a \times d = 1 \times 6 = 6 \neq 3 = 3 \times 1 = b \times c$

Press the button many times to verify the following conjecture.

$\frac{a}{b}$ and $\frac{c}{d}$ are equivalent, iff, $a \times d = b \times c$

Figure 6-10 Screenshot from the GGS exercise for comparing fractions (3)

In the student textbook that is used in the traditional teaching approach, this part of the lesson is communicated using the following sentences and example, which the teacher presents to students to teach them how to compare two fractions.



يمكنك استعمال التقدير في بعض الأحيان لمقارنة الأعداد النسبية، ويمكنك في أحيان أخرى إعادة كتابة الكسرين باستعمال المضاعف المشترك الأصغر لمقاميهما، ثم المقارنة بين بسطَي الكسرين.

مثـــال المقارنة بين الأعداد النسبية الموجبة

① ضع إشارة > أو < أو = في ● لتصبح الجملة الآتية صحيحة: $\frac{٥}{٨}$ ● $\frac{٣}{٤}$.

أعد كتابة الكسرين باستعمال المضاعف المشترك الأصغر لمقاميهما.

المقام المشترك الأصغر للكسرين $\frac{٥}{٨}$، $\frac{٣}{٤}$ هو ٨ .

$\frac{٥}{٨} = \frac{١\times٥}{١\times٨} = \frac{٥}{٨}$

$\frac{٣}{٤} = \frac{٢\times٣}{٢\times٤} = \frac{٦}{٨}$

بما أن: $\frac{٥}{٨} < \frac{٦}{٨}$ ، فإن: $\frac{٥}{٨} < \frac{٣}{٤}$.

You can compare the rational numbers by using the estimation, or you can compare these two fractions by rewriting them again by the use of the least common multiple for their denominators, then compare their numerators.

EXAMPLE

1. Put < or > or = signal between the two fractions to make this mathematics sentence is correct: $\frac{3}{4}$ $\frac{5}{8}$

Rewrite the two fractions again by the use of the least common multiple for their denominators which is for $\frac{3}{4}$,$\frac{5}{8}$ is 8.

$\frac{5}{8} = \frac{5*1}{8*1} = \frac{5}{8}$

$\frac{3}{4} = \frac{3*2}{4*2} = \frac{6}{8}$

Then , because of $\frac{5}{8} < \frac{6}{8}$ this mean that $\frac{3}{4} < \frac{5}{8}$

Figure 6-11 Example of comparing fractions from the Grade 8 student's book (p. 8) in Saudi Arabia (own translation)

The difference between the GGS-based pedagogy and the traditional approach is that in contrast to the teacher-centred approach, the GGS-based pedagogy enables students to try the given tasks many times, relating the comparison of fractions to the software-produced diagram, which helps them to recognize the differences between the two fractions. In addition, after each section in the GGS-based pedagogy, students discuss with each other what they have understood and then independently try to answer some sample questions from the textbook. The differences between the two approaches in this stage may have affected the experimental group, causing them to perform better than the control group on overall mathematics achievement in this unit.

To sum up, from the examples above, which present the main differences between the two approaches (traditional vs. GGS-based pedagogy), it seems that the differences between the performance in the two groups resulted from those between the two approaches. When using the GGS-based pedagogy, students had the chance to connect concepts with pictorial and symbolic representations in multiple cycles for each task, which saved time and allowed them to make extended efforts. Also, working independently and receiving instant feedback from the software might have helped students to overcome the fear of failing, giving them a better opportunity to engage in the lesson and also to discuss what they had learned each time with their peers. The students in the experimental group had the benefit of the GGS feature which helps make abstract concepts more concrete and visible. The two groups both completed the same textbook exercises after each lesson.

The other aim of the intervention study was to assess the effects of the GGS-based pedagogy on achievement in the NRC mathematical proficiency strands individually.

Impact on strands of mathematical proficiency for the Numbers topic

In terms of the effect of using the GGS-based pedagogy on achievement in the first four individual mathematical proficiency strands (CU, PF, SC, AR, and PD), a positive effect was found for Class A (the experimental group) in the first two strands, CU, and PF, and in the fifth strand, PD.

Strands 1–4: CU, PF, SC, and AR

The first four mathematical proficiency strands for the Numbers unit were evaluated using the Mann–Whitney U test (Table 6-3), which revealed a significant difference between the two groups in certain strands. In particular, in CU and PF, there was a significant difference between the experimental group and the control group, whereas there was no significant difference for other proficiency strands (p > 0.05), as seen in Table 6-3.

Table 6-3 Mann–Whitney U test for the proficiency strands in the Numbers unit

Mann–Whitney U test	
Strand	P-value (2-tailed)
CU	.046
PF	.003
SC	.233
AR	.210

In the first two strands (CU, PF), there were statistically significant differences between the two groups. These results regarding the CU strand may reflect that the GGS-based pedagogy enabled the students to understand the lessons in the Numbers unit better than the traditional teaching approach. The experimental group may have gained greater understanding of the unit's concepts as a result of the GGS features presenting the concept in the form of a pictorial representation or model. It may be that exposure to the GGS's dynamic features, and its representational capacity improved the students' understanding of the target concepts. There was also a significant difference between the two groups in favour of the experimental group in the PF strand, demonstrating that the GGS-based pedagogy helped students improve their fluency in this unit. This may be due to certain opportunities which the GGS provides for students, such as the possibility of trying to complete the task many times over a short period with minimal effort, which helps them to enhance their skills in completing mathematics tasks fluently.

With regard to the SC and AR strands, there was no statistical difference between the two groups. There are at least two possible explanations for this. First, despite the positive effect of the GGS-based pedagogy on overall mathematics achievement and the CU, PF, and PD strands specifically, its effect on the SC and AR strands was either null or not sufficiently strong to identify differences between the two groups. Alternatively, it could be that these two strands are more complex and challenging than the other strands and require higher cognitive abilities and mathematics skills. Hence, from these results it can be concluded that neither approaches made an appreciable difference to achievement between the two groups in terms of SC and AR.

Strand 5: PD

The fifth mathematical proficiency strand, PD, was measured using a questionnaire completed by the students. Overall, the findings provided evidence that the use of the GGS-based pedagogy improved the PD of the students in the experimental group compared with their PD before the intervention. The differences in the students' PD were assessed using the Wilcoxon signed rank test.

Table 6-4 Results of the Wilcoxon signed rank test for Class A's PD overall

Wilcoxon signed rank test for the self-report questionnaire							
			N	**Mean rank**	**Sum of ranks**	**Z**	**Assumption of significance (Assump. Sig.) (2-tailed)**
Overall PD	Before–After	Negative ranks	0	.00	00	-3.824	0.000
		Positive ranks	19	10.00	190		
		Ties	0				
		Total	19				

The analysis revealed the responses to the productive disposition questionnaire from all Class A students after the intervention were better than those before, as detailed in Table 6-4. Moreover, the effect size was large, based on the Cohen's d value of 0.62 (Cohen, 1988). Therefore, the use of the GGS-based pedagogy improved the students' productive disposition towards mathematics. In addition, in the three sub-scales of the questionnaire, the students' responses after the intervention were better than before (Table 6-5).

Table 6-5 Wilcoxon signed rank test for the PD sub-scales

Wilcoxon signed rank test results for productive disposition sub-scales	
Sub-scale	**P-value (2-tailed)**
Students' confidence in mathematics	.000272
Students like learning mathematics	.000130
Students value mathematics	.000130

These results in the overall questionnaire and sub-scales for productive disposition suggest that the use of GGS-based pedagogy had a positive impact in enhancing students' PD. These changes in the students' PD might have resulted from the new atmosphere that the GGS provided to students and its dynamic features. Also, the students had opportunities to work with the software independently and experience it themselves, which may have been interesting and enjoyable for them. Moreover, the use of diagram modelling and interactive exercises offers better opportunities for students to engage with modified unit exercises, with the capability to try many times. In addition, the feedback feature in the exercises allows students to check their answers without fear of failing. Since none of these are present in the traditional approach, these features and opportunities may be important in helping students develop their productive disposition.

6.4.2 Mathematics proficiency in the Geometry unit

To determine whether or not the improved results in the previous section were due to the effects of the GGS-based pedagogy, a second intervention study was conducted using a different unit and switching the roles of the two groups (Class B as the experimental group and Class A as the control). The second unit was taught after an interim period of four weeks to eliminate any effects from the first round of the study.

Overall mathematics achievement (Geometry unit)

In line with the cross-over design of this study, the third MPT related to the Geometry unit was implemented by switching the groups, such that the experimental group became Class B and the control group was Class A. The differences in mathematical achievement between the two groups after this second intervention were examined using an independent samples t-test (see Table 6-6).

Table 6-6 Results of the independent samples t-test for the Geometry unit

Third MPT (Geometry unit)					
Group	Mean	SD	t-value	df	P-value (2-tailed)
Experimental N = 18	9.78	1.93	-2.656	35	0.012
Control N = 19	7.56	2.97			

The analysis revealed that for the Geometry unit there was a significant difference in the scores between the two groups in favour of the experimental group (Table 6-7). The effect size (Eta squared) value of 0.16 is considered to be large. Therefore, the use of the GGS-based pedagogy appeared to have had a substantial effect on improving the students' mathematical achievement on the Geometry unit, relative to the traditional approach.

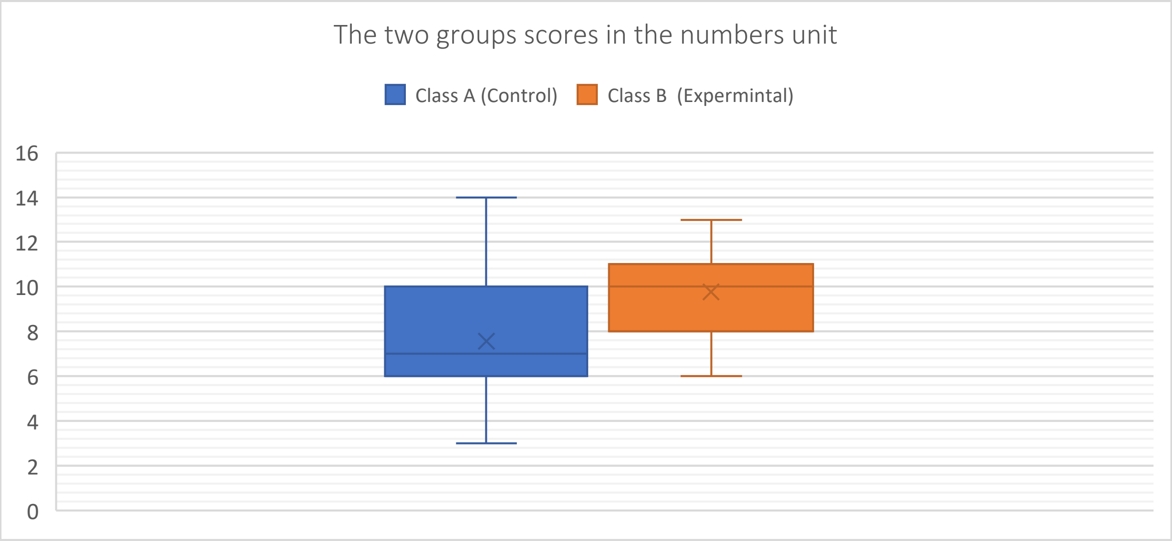


Figure 6-12 Box-and-whisker plots of the overall MPT values for the Geometry unit

Figure 6-12 shows that the difference in overall performance between the two groups applies to both actual scores and mean scores, as well as the range. For example, Class B's scores range from 6 to 13, whereas Class A's scores range from 2.5 to 14. Comparing the interquartile ranges also reveals a difference in favour of Class B (6–10 for A and 8–11 for B).

The statistical results for the second round of the intervention study revealed differences in the performance between the two groups in the MPT in favour of the experimental group, indicating that the use of GGS can again be considered an effective piece of learning software that enhances students' performance in mathematics. This suggests that features of the GGS interactive software, such as dynamic shapes, 2D and 3D shapes, and modelling, helped the students in the experimental group perform better than those in the control group.

As mentioned in relation to the first unit, the two classes initially achieved similar results in the baseline test and the difference noted above can therefore be attributed to the application of GGS-based pedagogy. That is, the GGS-based pedagogy had a greater positive impact on the students' attainment than the traditional teaching approach. As previously noted, the main difference between the two approaches seems to be related to the warm-up and progression stages of the GGS-based pedagogy. The traditional method focused on direct teacher input, whereas the GGS-based pedagogy gave students the flexibility of working in a dynamic environment (GGS) alone or together with peers and then reflecting on their work. The work conducted in the GGS environment is further detailed below.

As previously explained for the Numbers unit, the first difference between the GGS-based pedagogy and the traditional teaching approach concerned the warm-up stage, in which students were given the chance to become involved in targeted mathematical ideas via assigned GGS tasks before starting the lesson. For example, in the lesson on "Relationships between Angles and Lines", the students were given the exercise presented in Figure 6-13 during the warm-up stage. The main objective of this activity was to revise students' knowledge of angle types.

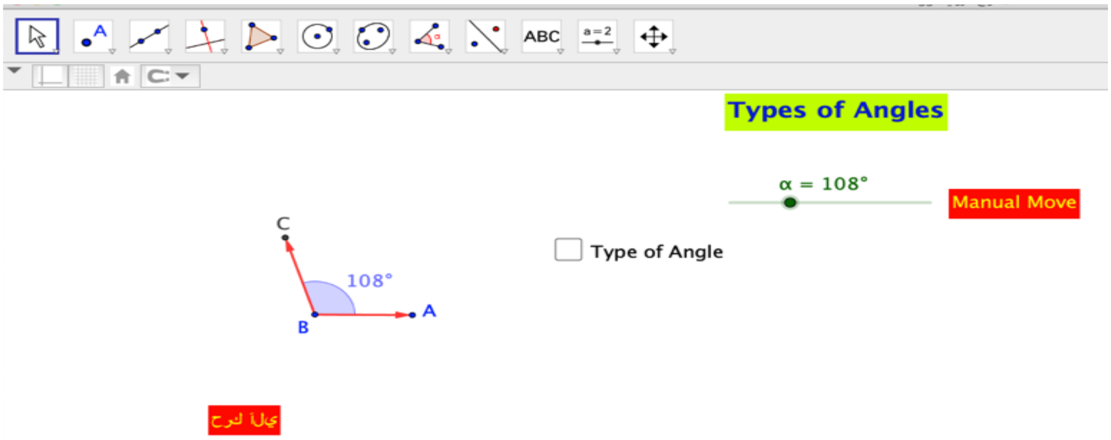


Figure 6-13 Screenshot from the GGS exercise for angle types (1)

In this exercise, students were presented with the angle ABC and could use the slider to change the size of the angle from 0 ° to 360 °. The students were able to change the angle size many times using the slider. They could see the differences each time and guess the type of each new angle and they were also able to check their answer by clicking the "Type of Angle" button.

During this stage, an additional GGS task sought to check students' understanding of the types of angles without seeing the angle size (Figure 6-14).

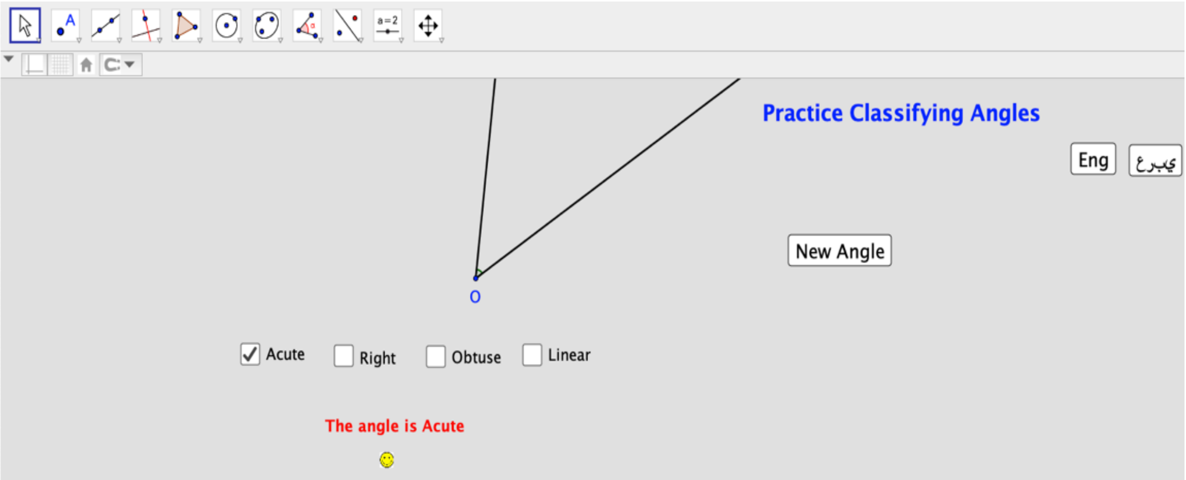
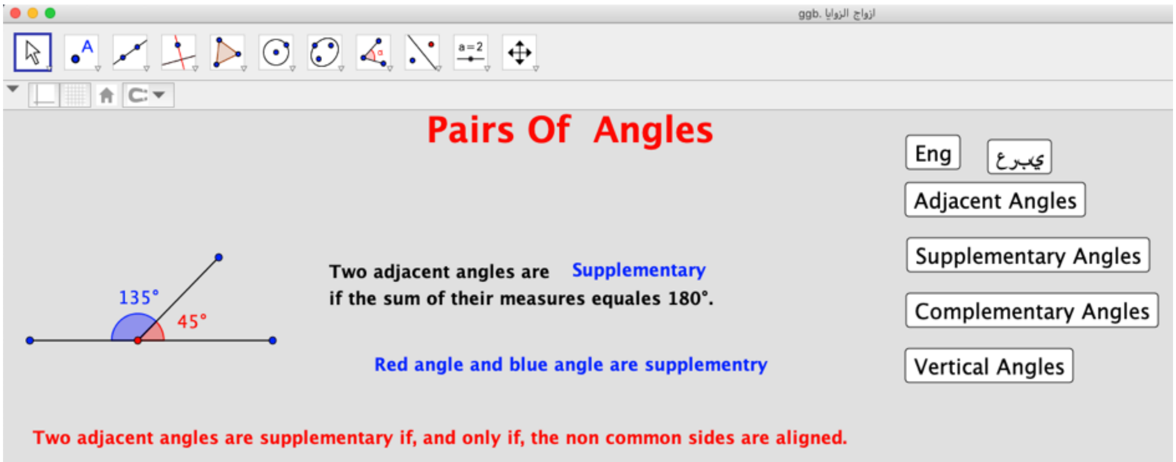


Figure 6-14 Screenshot from the GGS exercise for angle types (3)

In this task, the students were given an angle with four choices and had to choose the correct angle type from the choices given. They received instant feedback regarding whether their choice was correct or not. In the task presented in Figure 6-14, the students were given an acute angle, so they had to select the first box on the left. If a student chose an incorrect answer, they would receive instant feedback to warn them that their choice was incorrect. The students were then allowed to discuss the types of angles and share their understanding in groups.

The second difference between the GGS-based pedagogy and the traditional approach was in the progression stage. In this stage, students had the chance to become involved in targeted mathematical ideas in the lesson via assigned GGS tasks. Each GGS task is followed by a discussion, with the teacher setting some discussion questions regarding the new concept. For example, in the lesson on "Relationships between Angles and Lines", students were given the exercise shown in Figure 6-15. The aim of this task was to teach students about the relationships between different pairs of angles (e.g. supplementary, adjacent).



Pairs Of Angles

Two adjacent angles are Supplementary
if the sum of their measures equales 180°.

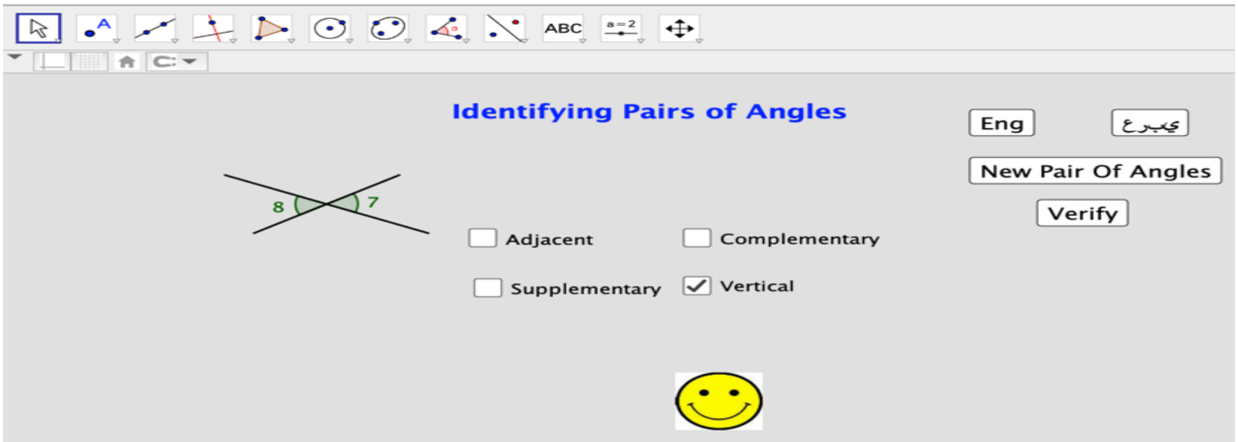
Red angle and blue angle are supplementry

Two adjacent angles are supplementary if, and only if, the non common sides are aligned.

Figure 6-15 Screenshot from the GGS exercise for pairs of angles (1)

In this task, students were given four choices for the four types of angle pairs – adjacent, supplementary, complementary, and vertical. When they chose any angle type from the four choices, the shapes of the chosen angles appeared on the screen along with their sizes and an explanation of the angle type. For example, when students chose supplementary angles, the shape of supplementary angles appeared on the left with the explanation "Two adjacent angles are called supplementary angles if the sum of their measures equals 180 °". This was repeated for all other types of angle.

After students had completed this task, they were given the exercise shown in Figure 6-16 to check their understanding by themselves. This task consisted of four choices. The students had to select the correct name for the two angles shown and they were able to check their answers by clicking the “Verify” button. This process was repeated for each pair of angles (see Figure 6-16).



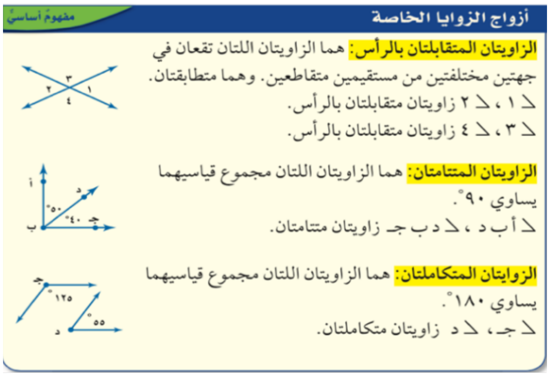
Identifying Pairs of Angles

☐ Adjacent ☐ Complementary

☐ Supplementary ☑ Vertical

Figure 6-16 Screenshot from the GGS exercise for pairs of angles (2)

In contrast, in the students’ textbook used in the traditional teaching approach, this part of the lesson was communicated using sentences that the teacher presented to students to teach them the types of angle pairs, as shown in Figure 6-17.



أزواج الزوايا الخاصة

الزاويتان المتقابلتان بالرأس: هما الزاويتان اللتان تقعان في جهتين مختلفتين من مستقيمين متقاطعين. وهما متطابقتان.
∠ ١ ، ∠ ٢ زاويتان متقابلتان بالرأس.
∠ ٣ ، ∠ ٤ زاويتان متقابلتان بالرأس.

الزاويتان المتتامتان: هما الزاويتان اللتان مجموع قياسيهما يساوي ٩٠°.
∠ أ ب د ، ∠ د ب جـ زاويتان متتامتان.

الزوايتان المتكاملتان: هما الزاويتان اللتان مجموع قياسيهما يساوي ١٨٠°.
∠ جـ، ∠ د زاويتان متكاملتان.

Special pairs of angles (Basic concept)

Vertical angles: are the angles opposite each other when two lines cross, and they are the same size.

Complementary angles: are two angles that add up to 90°.

Supplementary angles: are two angles that add up to 180°.

Figure 6-17 Translated example sentences on angle pairs taken from the Saudi Arabian Grade 8 student textbook (p. 76) (own translation)

Another example of the progression stage is the lesson on “Polygons and Angles”, shown in Figure 6-18. The task presented aimed to determine the difference between two types of polygon: regular polygons and irregular polygons.

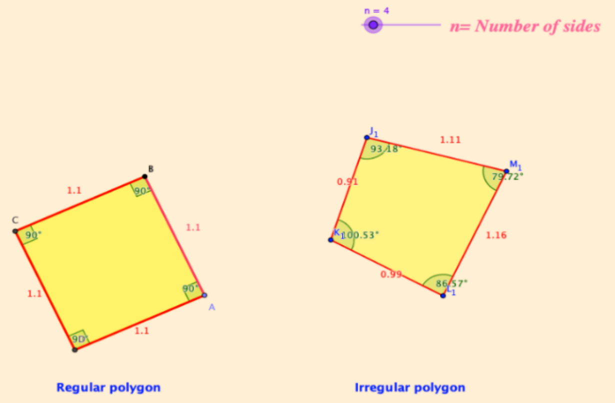


Figure 6-18 Difference between regular polygons and irregular polygons

In this task, students were given two polygons with a slider to change the number of sides. When students changed the number of sides using the slider, they were able to see the difference between the sides and angles of the two shapes. The regular polygon kept the same-sized angles and same length for all its sides, whereas the irregular polygon had different-sized angles and different side lengths. In this way, students could understand that a regular polygon has all equal sides and all equal angles and is different from an irregular polygon. The students then took part in a small-group discussion and whole class discussion about what they understood from the task.

As presented so far, the main difference between the two approaches (GGS-based and traditional) was the way in which the ideas were presented to students. In the traditional teaching approach, students were given just one or two sentences to explain each idea, whereas the GGS-based pedagogy with its representational capacity and flexibility helped students experience and try out each task targeting a concept for themselves. This may have had a positive effect on the students' understanding of angles. Furthermore, the students were given the ability to check their understanding of the different tasks and they could try to complete the exercise many times in a short amount of time and receive immediate feedback. Finally, they were then able to explain their understanding to the class in the next step. All these features could have affected students' understanding and attainment with regard to the Geometry unit.

To sum up, the examples presented suggest reasons for the differences observed in the performance of the two groups as a result of the two approaches. With the GGS-based pedagogy, students had the chance to connect geometry concepts with dynamic shapes in multiple cycles for each task, which saved time and allowed them to try more examples. Also, working independently and receiving instant feedback from the software helped them overcome any fear of failure and gave them a better chance to engage in the lesson, with the opportunity to discuss what they had learned each time with their peers. In addition, the use of GGS-based pedagogy generated better understanding, since dragging and modifying geometric objects seemed to provide the students with useful visual representations and more hands-on experience of the geometric concepts. Hence, the differences between the two approaches may have affected the experimental group, causing them to perform better than the control group regarding overall mathematical achievement in this unit.

The other aim of the intervention was to determine the effects of GGS-based pedagogy on mathematical proficiency strands. This refers to whether the differences between the two approaches had an impact on the improvement in students' mathematical proficiency, discussed in the following section.

Strands of mathematical proficiency for the Geometry unit

In terms of the effects of using GGS-based pedagogy on the mathematical proficiency strands in the Geometry unit, this was achieved by identifying the differences between the two groups regarding the first four strands (CU, PF, SC, and AR) after the intervention. For the fifth strand, PD, the effect of using the GGS-based pedagogy was examined based on the responses of Class B (experimental group) before and after the intervention. The results are detailed below.

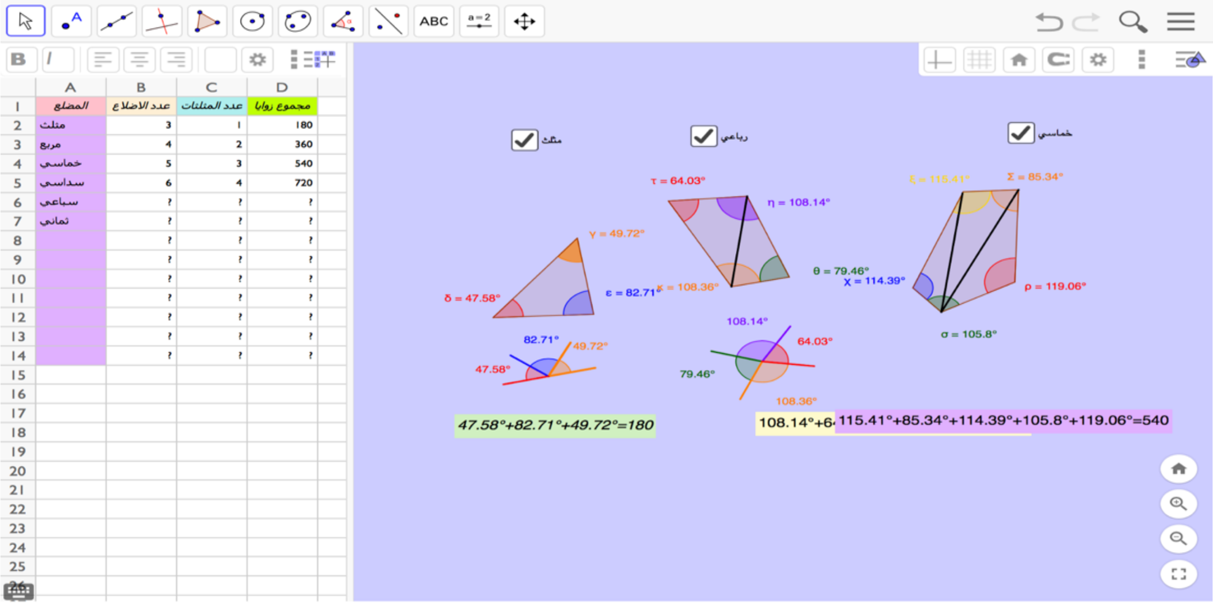
Strands 1–4: CU, PF, SC, and AR

The Mann–Whitney U tests conducted to achievement in the mathematical proficiency strands for the Geometry unit revealed significant differences between the two groups in some strands, but not others. For example, for the PF strand, there was a significant difference between the experimental group (Class B), and the control group (Class A) (see Table 6-7).

Table 6-7 Mann–Whitney U test for the proficiency strands in the Geometry unit

Mann–Whitney U test	
Strand	P-value (2-tailed)
CU	0.134
PF	0.006
SC	0.327
AR	0.343

Table 6-7 shows that for the Geometry unit, among the effects of the GGS-based pedagogy in the first four strands, only PF was significant. That is, the GGS-based pedagogy led to a greater improvement in PF for the experimental group relative to the control group. The differences between the two groups can perhaps be attributed to the use of the GGS tasks, with their unique features and flexibility. When completing the GGS tasks, the students were able to attempt the tasks repeatedly, which explains the improvement in their fluency. In addition, the students in the experimental group might have benefited from working independently in the classroom and from the supportive functionality provided by the majority of the GGS activities in this unit. This effect was analogous to that observed for the same strand in the first intervention (the Numbers topic), confirming that the application of the GGS-based pedagogy positively affected the PF strand. The fluency skills related to the two units share the same meaning, namely that the students need to be flexible when performing calculations and executing procedures to ensure they find the correct answer to the problems. For example, in the lesson on “Relationships between Angles and Lines” (Figure 6-5), the focus is on angles (primarily geometric content), but the student needs to perform some calculations (e.g. checking the angles add up to 90 °) to determine the correct category of angles. Another example can be taken from the lesson on “Polygons and Angles” (Figure 6-9) in which the students focused on the relationship between the number of triangles present in a given polygon, summing the angles. For this task, students needed to calculate the number of triangles in each polygon and then multiply them by 180 to establish the total sum of the interior angles for the polygon, as illustrated in Figure 6-19.



	A	B	C	D
1	المضلع	عدد الاضلاع	عدد المثلثات	مجموع زوايا
2	مثلث	3	1	180
3	مربع	4	2	360
4	خماسي	5	3	540
5	سداسي	6	4	720
6	سباعي	؟	؟	؟
7	ثماني	؟	؟	؟
8		؟	؟	؟
9		؟	؟	؟
10		؟	؟	؟
11		؟	؟	؟
12		؟	؟	؟
13		؟	؟	؟
14		؟	؟	؟
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Figure 6-19 The total sum of the interior angles for the polygon

From the examples above, it can be concluded that despite the differing content of the Geometry and Numbers units, the procedural fluency used in the Numbers unit did not different greatly in this respect from that in the Geometry unit. Hence, the features distinguishing the experimental and control groups in the Numbers unit differentiate in the same way in the Geometry unit (e.g. multiple tries in a short time, feedback, flexibility, and independent work in class).

Regarding the CU strand, although a positive impact from the GGS-based pedagogy was observed for this strand in the Numbers unit, this was not the case in the second round of intervention in the Geometry unit. Thus, despite the advantages of using GGS overall, there was apparently no significant difference between the two groups for the Geometry unit. These results might have arisen because of the teaching time allocated to each of the units. The Numbers unit was allocated more time than the Geometry unit and the students engaged in additional GGS tasks for the Numbers unit. The other difference between the two units concerned the ways in which the topics were introduced to the students. Although the two units employed similar procedures and structures, when completing the first unit on Numbers the students might have been impressed by the novelty of the approach to teaching, in particular the shift from an abstract form to one with more vibrant pictorial representations and shapes with colours and options for dynamic moves. In contrast, the method of introducing the geometry concepts was relatively similar to the traditional approach in terms of the use of shapes and diagrams. For example, in the lesson on Comparing and Ordering Rational Numbers (Figure 6-6), the traditional numeric and abstract approach requires students to compare two fractions by making calculations, whereas in the GGS-based pedagogy, they learn visually through concrete engagement with graphical representations. This suggests that the effects of the GGS-based pedagogy were not sufficient to lead to a difference between the two groups for the Geometry unit, unlike for the Numbers unit.

In addition, there were no differences between the two groups for the SC and AR strands, which mirrors the results for the first round and confirms that the GGS-based pedagogy had no significant effect on SC and AR in the two units covered by the intervention study. Thus, the traditional approach and the GGS-based pedagogy had the same effect on both strands. This could be because these two strands are more complicated than the others, requiring more and higher cognitive skills to answer the questions assigned for these two parts. Moreover, to some extent, there was no difference between the two groups in the CU strand in this unit, which could be an essential component when solving mathematics problems, so this may have led to the same performance in these two strands. A further explanation may be that the students in the experimental groups did not use GGS in the MPT, so they had the same chance as the students in the control group. Hence, from the two results of the second stage of the intervention study, the effects of the use of GGS were not sufficient to lead to a difference between the two groups when they engaged in answering questions without using the GGS.

Strand 5: PD

The fifth strand of mathematical proficiency, PD, was measured using a questionnaire. Overall, the findings of the questionnaire for the experimental group (Class B) provided good evidence that using the GGS-based pedagogy improved the students' PD compared to their PD before the intervention. The differences indicated by the use of the GGS-based pedagogy on the students' PD were assessed using the Wilcoxon signed rank test.

Table 6-8 Wilcoxon signed ranks test results for Class B's PD overall

			N	Mean rank	Sum of ranks	Z	Assumption of Significance (Assump . Sig.) (2-tailed)
Overall PD	After–before	Negative ranks	0	.00	00	-3.725	.000195
		Positive ranks	18	9.50	171		
		Ties	0				
		Total	18				

The analysis revealed the responses to the productive disposition questionnaire of all Class B students after the intervention study were better than those before, as detailed in Table 6-8. Moreover, the effect size, Cohen's d (Cohen, 1988) 0.62, is considered to be large. Therefore, the use of the GGS-based pedagogy improved the students' PD values concerning mathematics. In addition, in the three sub-scales of the questionnaire, the students' responses after the intervention were better than before (see Table 6-9).

Table 6-9 Wilcoxon signed rank test for the PD questionnaire sub-scales

Wilcoxon signed rank test for the productive disposition questionnaire sub-scales	
Sub-scale	P-value (2-tailed)
Students' confidence in mathematics	.001
Students like learning mathematics	.000
Students value mathematics	.000

Thus, one positive effect in this unit was observed for the PD strand, with use of the GGS-based pedagogy positively affecting students' PD. These changes in the students' PD may have resulted from the new atmosphere that the GGS provided to students and from its dynamic features. Also, the students were given the opportunity to work with the software independently and experience it themselves, which they may have found interesting and enjoyable. Also, the use of shapes and modelling and interactive exercises offered better opportunities for students to engage with modified unit exercises, with the capability to try each task many times. In addition, the feedback feature in the exercises allowed students to check their answers without fear of failing. In addition, the students may have been influenced by the novelty of the approach; that is, their responses may have been influenced by the fact that the approach was new to them and they were impressed by its features. These results are similar to those obtained for the Numbers unit and confirm that the use of GGS-based pedagogy had a positive effect on students' PD.

6.5 Comparing the effect of the GGS-based pedagogy between the two units

In the two-phase intervention study, both units were introduced to the students using the same procedures, and they both differed from the traditional approach in two stages of the lesson sequence – the warm-up stage and the progression stage. With regard to overall mathematical achievement, the use of the GGS-based pedagogy had a substantial effect on improving the students' mathematical achievement in the two units. However, regarding mathematical proficiency strands, the GGS-based pedagogy also appeared to be more effective than the traditional approach in the Numbers unit for three strands (CU, PF, and AR), whereas in the Geometry unit it was more effective for two strands only (PF and AR). In the first unit, the intervention study lasted for five weeks and students were engaged in 21 GGS tasks. In the second unit, the intervention study lasted for three weeks and students were engaged in 18 GGS tasks. This was because the Geometry unit had fewer and shorter lessons than the Numbers unit. Hence, the differences between the two units may to some extent be due to the length of each intervention and the number of the GGS tasks. Also, teaching Geometry using GGS-based pedagogy may have made less of a difference than the teaching of the Numbers unit since the GGS tasks and traditional teaching were closer to each other, both using pictures and diagrams in a similar way. In contrast, in the Numbers unit, the traditional approach only uses numbers whereas the GGS approach supports students with a variety of representations and dynamic features.

6.6 Chapter summary

This chapter has presented and analysed the quantitative data collected from the two rounds of the intervention study conducted with Grade 8 students. Data were collected regarding mathematical proficiency for two units: the Numbers unit and the Geometry unit. Each test included five mathematical proficiency strands as sub-tests. Regarding overall mathematical achievement, the use of GGS-based pedagogy had a substantial effect on improving the students' mathematical achievement in the two units relative to the traditional approach.

Generally, it is possible to conclude that the impact of GGS-based pedagogy is not consistent and varies across differing strands of mathematics proficiency, as evidenced through the two rounds of intervention. For example, regarding the five mathematical proficiency strands (CU, PF, SC, AR, and PD), in the first unit (Numbers), the effect of using the GGS-based pedagogy on mathematical proficiency was significant for three strands (CU, PF, and PD), whereas there was no evidence of an effect on the SC or AR strands. Meanwhile, in the second unit (Geometry), the effect of using the GGS-based pedagogy on the strands of mathematical proficiency was significant for only two strands, PF and PD, and there was no evidence of an effect on the CU, SC, or AR strands.

Therefore, the results indicated that GGS-based pedagogy had a significant positive impact on overall mathematical proficiency achievement and the PF and PD strands in both units. With regard to the CU strand, the effect was only significant in the first unit but was not significant in the second unit. Finally, no effect was observed for the SC or AR strands in either unit.

Chapter 7 Teacher-Related Findings

7.1 Introduction

The main aim of this research was to investigate the effectiveness of a GGS-based pedagogy on students' mathematical proficiency. It also sought to explore teachers' views of teaching mathematics using the software. Different instruments were employed to achieve these research aims. MPTs and PD questionnaire were employed to measure students' achievement within five mathematical proficiency strands. These were all discussed in detail in the previous chapter.

In addition – and supplementary to the student data – 15 teachers were invited to participate in a professional development course (PDC). Initially, six teachers were invited to complete a semi-structured interview in three phases: before, during, and after the PDC. After completing four days of the PDC, the teachers returned to their schools with new experience and materials in hand. Finally, all 15 teachers reported back after four weeks, on the fifth and final day of the PDC, to share and discuss their experience of using GGS in the classroom (Teacher Group Written Responses). The findings obtained from the interviews with the teachers, and from the teacher group written responses sought to answer the second research question:

RQB: What are the mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics?

The aim of this chapter is to present the findings obtained from the two sources of data, the teacher interviews and the teacher group written responses. After analysing the collected data using thematic analysis in NVivo version 12 (see Chapter 5), five themes emerged: GGS as a time/effort saver; GGS's representational capacity in enhancing teaching; GGS as an effective learning tool; GGS as a facilitator of student engagement; GGS as a supporter of mathematics skills. Each theme is discussed in detail in the following five sections.

7.2 GGS as a time/effort saver

This section presents the mathematics teachers' views expressed during the teacher interviews and teacher group written responses regarding the use of GGS in terms of saving time and effort in mathematics lessons. The teachers considered GGS to be both a time- and effort-saving tool when used in mathematics classes and they believed that it was of benefit to both teachers and students. As will be justified with teachers' quotes in the following paragraphs, for teachers, GGS is a time-saver and effort-saver since it:

a. allows better zooming in on student reasoning and better management of instructional activities,
b. allows better learning opportunities.

What follows is an analysis articulating each of these points.

The participant teachers believed that the use of GGS enabled them to conduct a greater number of activities during classes and gave them opportunities to focus on several ideas rather than one within a class period. The teachers also talked about the use of GGS in terms of examining their students' thinking and performance. Teacher AL explained "Changing my approach to using the GGS was a good opportunity for me. Now, I have time to examine my students' thinking when they use the software to begin thinking about the other shapes, and the differences between them". Similarly, Teacher AW observed:

> Traditionally, we spent a lot of time drawing more than one shape, and this took up time in the lesson, and the drawings [produced] may not be as they should be. However, when we use GGS, it is better, more attractive, and very useful for helping students to understand, and it does not take [so much] time.

In this regard, GGS was considered to be a time/effort-saver in terms of giving the teacher the opportunity to use more appropriate representations and more effectively and zoom in on student reasoning. Also, the new tool helped them to do what they were doing in their previous approach in less time and with better quality in the final outputs of drawing or shapes. This enhanced students' understanding in the lesson.

Moreover, the teachers reported that the use of GGS gave them flexibility in managing the instructional activities since they could do more activities in less time and use extra time to hone in on student work and discussion. For example, Teacher AL explained:

> …The next step is asking students to discuss what they found with their colleagues, in pairs, then in groups. After that, I investigate the students' understanding by giving them exercises. Here, GGS lets me use student-centred learning in the beginning, then I use pair work, and finally I use group work. All these activities, in addition to the exercises, take less than 20 minutes. Then I use the rest of the class to give the

> students more practice at different levels, depending on each student's progress in the lesson.

Also, in terms of saving effort, one teacher connected the use of GGS with the number of students in the class. As Teacher Y explained, "It saves time, facilitates information, and is a useful tool in classes which have a large number of students". In the teacher group written responses, the same message was also conveyed as Group 1 explained:

> GGS is a good tool to use in a class with a large number of students. It can be used to present shapes and concepts to students through projectors or by encouraging students to work in groups on the GGS.

Also, Group 2 pointed out:

> GGS helps us to reduce the numbers of geometry instruments used in the classroom to explain the lesson and concept to students. Normally we need to carry materials from one class to another, but now we do not need it anymore, and so we use the GGS features to draw and present what we want for students.

Such remarks from teachers (either individually or as a group) suggest that GGS helps teachers to manage crowded classes better with respect to presenting lessons and work with students in more effective ways in terms of time and effort. Also, it helped them expend less effort in drawing and presenting shapes for students because they did not need to use the traditional geometry set to draw and they did not need to carry it from one class to another.

The participant teachers also believed that by saving time and effort, the use of GGS provided better learning opportunities. For example, Teacher MO explained "GGS leads to more effective learning; it leads to learning maths concepts quickly, and facilitates [understanding of] difficult questions, and means that students can try again, because it saves their time". Teacher AW added "It makes teaching more effective, because students become more interactive, they understand quickly, then they use the rest of the time for practising and discussing with others". Teacher MO noted that "Ease, speed, and proficiency are the most important factors that made me go for the use of GGS". This was also supported by teacher group written responses in which Group 1 responded that "The use of GGS helps to save time in lessons, students in the lesson also understand concepts and idea quickly, and they do not spend as long a time understanding the lesson as usual". Moreover, Group 1 added "When students use the software to answer, they use more time to think and less time to write, which saves their time and is better than using paper-and-pen environment". As a result, the time-saving feature of GGS provides better learning opportunities for students.

In summary, as evidenced by the above quotations, the participant teachers considered GGS useful in terms of its time/effort-saving capacities. They considered this important since GGS allowed greater focus on student reasoning, helped them to do what they were doing in their previous approach but in less time and better quality in terms of the final outputs (drawing or shapes), and manage instructional activities more effectively, as well as provide better learning opportunities. Furthermore, it helped the teachers in the effective use of mathematics instruments in their classes and introducing more than one idea in a class period.

7.3 GGS's representational capacity in enhancing teaching

This section presents the mathematics teachers' views expressed during the teacher interviews and teacher group written responses about the effectiveness of the representational capacity of GGS in the teaching of mathematical concepts. The teachers considered that the capability of GGS in presenting concepts in the form of models using pictures and simulations, and connecting them with real life, enhanced the teaching and learning of these concepts. As demonstrated by quotes from the participant teachers in the following sections, GGS has a representational capacity (e.g. use of examples, models, representations) that helps enrich the teaching of concepts that are difficult and/or abstract. Such capacity, moreover, helps students make comparisons between different concepts and investigate their relationships.

The participants found that the use of GGS in teaching mathematics concepts made them easier to explain. The teachers found better opportunities to apply the new concepts using the GGS exercises and the software also enabled the teachers to support these concepts with examples. They though that these opportunities made the meanings of these concepts clearer for students, which improved their understanding. For example, Group 1 stated that "GGS helps us to introduce mathematical concepts and ideas, which were difficult for us to explain following the typical approach, facilitating presentation to the students in an easier way". Also, Teacher AW explained:

> The difference is that the traditional way involved explaining abstract concepts to students that are not supported [by examples] that help them to understand and apply them. However, GGS is quite the opposite. It helps students to understand the

> abstract and difficult concepts and enhances their understanding of these concepts using the GGS exercises.

Also, teacher MO concurred, explaining "In the traditional approach, much effort was needed to achieve a full understanding of some concepts before using the program, but using the program helped the students to understand the concepts in a meaningful way". This also was supported by teacher group written responses, Group 2 stating "The program helps teachers to present mathematical concepts and lessons to students easily". Thus, GGS was considered to be a beneficial tool in teaching concepts since it provided a better environment in which to apply difficult and abstract concepts using examples and exercises, which led to enhanced students' learning and understanding of these concepts. As Teacher AW concluded above, it was previously challenging to support the learning of difficult and abstract concepts with exercises and Teacher MO concluded that students needed to expend more effort to understand concepts comprehensively in the traditional approach. Thus, they stated that the use of GGS in teaching concepts was better than their traditional approach.

The participant teachers considered GGS a helpful tool in offering better opportunities to present concepts through models and graphics and providing the ability to provide simulations connecting the concepts to real-life situations. In terms of this capability, teacher BA observed "Information and concepts are taught to students clearly and accurately by the models and graphics, and [it is possible to] explain the differences between the concepts at the same time". Also, Group 3 stated that "Connecting mathematical concepts with real life helps students to change their negative ideas and they think about them as something else, not only abstract ideas, because the use of models and simulations becomes more sensible". Moreover, teacher AL claimed that "...GGS and its features contribute to illuminating and simplifying these concepts for students. For example, for the sum of the angles of a triangle, GGS allows students to draw many triangles, and to calculate the sum each time".

Teachers BA and AL, as well as Group 3, considered the GGS tools and features provided scaffolding, such that the ways of presenting concepts made them easier to understand and clearer for students. For example, GGS allows students to see different representations all at once (e.g. drawing different representations of a triangle for summing angles) and interpret them (e.g. seeing that angle sum does not change). They also explained, as evident in their quotes above, that presenting such concepts via models and graphics helped to introduce them to their students and made them easier to understand. Also, they believed that these features helped students access any abstract ideas surrounding them so they are functional for students. They also noted that students were able to make comparisons between two different concepts at the same time, which may have helped them recognize the differences between them. This is because the GGS features makes it possible to present more than one concept, which makes the comparison task easier for students. Thus, the teachers considered the capabilities of the software helpful in teaching concepts since they supported students' visualization of concepts using models and graphics.

The teachers also believed that the use of GGS helped their students gain a better understanding of concepts, which led them then to connect concepts. Supporting this claim, Group 1 responded "When we use 3D features to presents concepts to students, this gives students an opportunity to see the shape from different perspectives and let them connect it to other concepts". Using the example of how students grasp the relationship between congruence and translation in geometry, Teacher AL explained:

> GGS permits students to discover the meaning of certain mathematical concepts. For example, in the translation lesson, the student finds that the idea of translation is to transfer the geometrical shape from one location to another. Then, if he calculates the measurements of the sides and angles in the original geometric shape (pre-image), and the translated shape (image), using translation, he will find it to be similar. The concept of translation preserves the measurements of the sides and angles without changes [when it moved from one location to another]. This reflects the concept of congruence, which means that two different shapes have the same side and angle measurements. This example helps students to recognize the relationship between the translation and the congruence concepts, and the integration of these two concepts.

Teacher AL above considered GGS a useful tool in teaching concepts, such as translation as a geometric transformation, because it helped students make good connections between different concepts. The connection in this example came about as a result of the dynamic feature of GGS allowing investigation of how and under what conditions the pre-image becomes the image and what properties are preserved (e.g. angles, side lengths) in such a transformation. The representational capacity of GGS in terms of presenting the changes in the angles and side lengths also helps students recognize the connection between the two concepts, translation and congruence.

Teachers also highlighted some examples of GGS providing models using pictures or simulations. Below are three quotes from different individual teachers and a group response talking about the capacity of GGS and its impact on teaching and learning.

> GGS helps students to learn concepts, and to compare between these concepts. In my class, I used pictorial representation [an exercise conducted using GGS in which a shape represents the fraction, and it changes when the student changes it, or introduces a different number, for example a third or two quarters] to explain the meaning of fractions. This was the first time I felt that all of my students understood what a fraction is from the outset. Using colours and shapes with the slider icons was very useful. [each shape is shaded with two different colours, representing the part and whole unit of each fraction, while the slider enables students to change the fraction numerous times]. (Teacher Y)

> I struggled for many years to present and explain the hidden parts in the 3D shapes, such as in pyramids and prisms. However, with GGS, I could present them from different viewpoints, and I could present the construction steps for the students, step by step, using the net icon. (Teacher Y)

> In my opinion, the choices that GGS offers in the classroom are the best way to enhance students' learning. It has different instruments and tools that gives me more opportunities for teaching students. The slider icon is the most powerful tool. It causes a change in the picture size and shapes in different ways. Also, the 3D view helps students to understand the concepts in the right way. (Teacher AW)

> The use of the GGS feature to control the 2D or 3D geometry shapes, and change their colours, or change their size by using the slider helps students to think about them and recognise the relationship between them. (Group 3)

What is common in all these quotes is that they all refer to different capabilities of the GGS. Using the slider feature of GGS, for example, is found to be useful in teaching different mathematical concepts, such as fractions or 3D figures. As Teacher Y and Group 3 explained, as students use the sliders, they can observe instant changes in the figures and think about the meaning of those changes. GGS also allows them to investigate given representations with different colours and dynamically change the angles. As Teacher AW explained, GGS provides opportunities to build 3D figures, investigate them using variety of viewpoints, and focus on nets. They claim that all these capabilities of GGS made their lessons easier for their students to understand and grasp concepts better.

To sum up, from the teachers' quotes above, the teachers believed that the capabilities of GGS enhanced the teaching and learning of concepts and made the process easier for both students and teachers. The teachers witnessed this capacity in different respects, such as offering more exercises and examples for students, presenting concepts via models and graphics, and connecting them with real life, all of which improved their students' understanding.

7.4 GGS as an effective learning tool

This section presents the mathematics teachers' views about the effectiveness of the use of GGS in teaching and learning mathematics. The participant teachers believed that GGS was an effective learning tool which supported students' independent learning and enabled more effective and longer lasting learning for students.

The participants pointed out that the use of GGS had a positive impact on facilitating students' learning of mathematics. For instance, it had a positive impact in increasing students' confidence when learning new concepts and lessons. Also, it was better suited to students with different ways of learning. Teacher MO explained that "… the program helps the learner to be a self-learner, and this enhances the learner's confidence". When he was asked about the differences between the use of GGS and his normal teaching approach, he added "[using GGS] facilitates [teaching of the] subjects to students … and enhances students' self-study abilities". Teacher AL also explained this, stating "The software is good for all types of students, visual [learners] could learn from the pictures, and auditory [learners] could learn from a discussion with others when they are working in pairs or groups". Also, he added "It is a useful tool which lets students learn and acquire skills through training and practice". In addition, Teacher Y observed "It [the software] has a crucial impact on facilitating learning; students learn quickly, and it helps students who are struggling with drawing by their hands, which makes learning easier".

These teachers thereby highlighted certain aspects of the ways in which the use of GGS as a learning tool facilitated the learning of mathematics for students. As seen in the quotes above, teacher AL highlighted the benefits of its use for teaching students with different preferred ways of learning; for example, those who focus on visual representations may find it easier to learn using GGS, due to its use of different shapes and colours, and the software is a good way of learning by doing, offering more chances for students to practise their learning. However, his observation regarding its benefits for promoting group and pair work was unclear, since such work could be organized without the use of GGS. He also observed that GGS offers a good environment for students to learn by themselves through practice, leading them to gain skills when they learn by doing. Moreover, teacher Y highlighted the fact that the use of GGS facilitated learning, as it enabled students to understand the subject matter of a lesson more quickly, and enabled them to draw shapes more easily than when using traditional methods. Thus, the teachers considered using GGS a facilitating learning tool since it was suitable for different learner approaches and enabled students to understand lessons and draw shapes more easily than before. Thus, the participants considered GGS an effective learning tool since it facilitated students' learning of mathematics.

The teachers also considered that the use of GGS made students more independent in their learning of mathematics. Teacher MA noted that "students are able to learn more effectively when they have the chance to use GGS by themselves", while teacher BA commented, "Surely, it will enhance learning, and make it more effective, because it is a new and an interesting environment, and new and interesting tools always make learning more effective". This inspired the teachers to let students to explore concepts and mathematical ideas using GGS, and teacher MO explained that:

> The more effective learning happens when students explore GGS by themselves, to learn a new concept, understand it, and try to make a connection with their previous knowledge and ideas. It makes teaching more effective, because students become more active, then they understand quickly.

All of these observations exemplified the fact that the teachers found GGS to be a tool that promoted effective learning, whether by facilitating students' understanding of ideas, or by enabling them to be more independent in their learning of mathematics.

Finally, the teachers believed that the use of GGS enhanced the permanence of their learning, explaining that self-study enabled the students to retain what they had learned for a longer duration than when using other methods. They believed that this occurred as a result of the students learning concepts and the relationships between them by themselves, which might provide a strong basis for their understanding, and they theorized that their students were able to recall information quickly and to retain it for longer because they were able to view it visually, or to explore it themselves. For example, teacher BA reported:

> Using GGS makes learning more permanent. For example, I asked my students about certain concepts one week after the time they studied these concepts, and they still remembered all the details. This completely differs from the previous approaches. In the past, when I need to recall any idea with students, I always needed to explain it again.

Moreover, teacher Y added that allowing students to work using the software and to learn by themselves made the learning more permanent, noting “It enhances student self-learning, which makes learning more permanent”. Hence, GGS was considered to be an effective learning tool which made student learning more permanent since it enabled students to retain what they learned for a longer duration than when using traditional methods.

In summary, the teachers reported that GGS was an effective learning tool, explaining that the use of the software by students facilitated effective and independent learning, which in turn promoted more permanent learning. This inspired the teachers to let students explore concepts and mathematical ideas using GGS. The teachers reported that the use of GGS in mathematics classes had a positive impact by enhancing the students’ self-learning, which built their confidence in working with the subject matter. Moreover, they argued that it made learning more permanent since it enabled the students to retain what they learned for a longer duration than when using other methods.

7.5 GGS as a facilitator of student engagement

This section presents the mathematics teachers’ views expressed during the teacher interviews and teacher group written responses regarding their students’ engagement in the classes in which they used GGS. The participant teachers believed that the use of GGS in their classes had a positive effect on their students’ in-class engagement. They found these differences in different aspects of students’ engagement, such as participation, dealing with mathematics tasks, and enjoyment of lessons. All these positive changes are supported by the teachers' quotes in what follows.

The teachers reported that their students’ behaviour in class changed and some reported observing differences in the degree of their students’ participation in mathematics lessons. For example, teacher AL explained “I feel that something is different in my classes; the students come to the class, and they are eager to come”. Teacher BA discussed the changes in his students’ in-class behaviour when they used GGS, explaining that they wanted to remain in the class for a longer time and to continue studying, observing:

> My students' behaviour, and desire for learning mathematics, has changed. I always found my students were getting bored in class, and I tried to change my approach from time to time to avoid that. When I started using GGS in my lesson, their reaction in class was completely different from their reaction to the conventional approach.

Some students may become bored in mathematics classes, especially if they do not enjoy the subject, or if they encounter difficulties understanding the lessons. As Teacher BA explained, he attempted to change his teaching approach to address this matter and observed that the newly introduced use of GGS altered his students' in-class behaviour, and he found that they learned more effectively than previously. He believed that this change occurred because of the use of GGS in the class, which led the students to behave in a way that was more conducive to learning than when using the traditional approach. As he explained, "By using GGS, the lesson provided a new and interesting approach" and the students "react[ed] better than [when using] the traditional approach".

The participants found that the use of GGS in the teaching and learning of mathematics improved the students' eagerness and desire to deal with mathematics tasks and questions. To support this, teacher AL noted:

> Yes, learning through GGS helps students to discover concepts and relationships on their own. This generates the desire to deal with mathematics, and to use it to solve the problems they face, and it removes all kinds of struggles and difficulties in learning mathematics.

In addition, teacher BA claimed that "the students now love mathematics lessons". As a result, "they work very hard, and they are keen to finish all their tasks". Similarly, in the teacher group written responses, Group 3 reported "Learning through GGS helps change the traditional atmosphere and environments in the class as they become interesting for students. So the students work better than before which helps to enhance their productivity". Also, Group 2 added "There is a big difference when students become interested in the class and find it more enjoyable. They like to work on the GGS and draw shapes; also, they continue work for a long time".

Teacher AL elaborated on this, explaining how his students' desire and eagerness to learn had changed as a result of using GGS, noting that the use of GGS had a positive impact on students' engagement to work in mathematics lessons. He linked this change to their solo in-class use of the software, which promoted their desire to work in class. The same teacher reported that the software's use in facilitating learning enabled the removal of certain difficulties which students encountered with mathematics topics, which subsequently enhanced their eagerness to learn. Teacher BA connected the students' feelings towards mathematics with their engagement in doing tasks. In the extract above, he expressed that through the use of the GGS, students developed positive feelings regarding the mathematics lessons, leading them to engage positively in working hard and trying to finish all their tasks.

From the teachers' quotes above, it can clearly be concluded that they believed that the use of GGS in mathematics lessons helped to enhance students’ engagement positively in class, which is apparent in various ways, including their willingness to answer questions, using GGS, and working hard to finish their tasks.

All of the teachers (either individually or as a group) agreed that the use of GGS made mathematics lessons more interesting and enjoyable for their students than their traditional classes. For example, regarding this idea, Group 1 pointed out “There are positive changes in students’ attitudes to mathematics lessons and more fully than before, they become interested in the class”. Also, Teacher Y reported “I see how excited they are, how they help each other, and spend all their time doing exercises using software”. He added “They like it very much, and they always seem happy in my classroom when I use GGS”. Teacher MO added, “Using the program is enjoyable, and this leads the students to enjoyment [of the lesson]”. In addition, Group 1 stated that “The use of the GGS in the classroom provides an interesting and magnetic lesson for students, the use of features such as the slider and animations make the lesson interesting lesson and captivating”.

The teachers thereby highlighted that the use of the dynamic software was interesting and enjoyable for their students: they were more actively engaged in the class, and the software enhanced their learning and understanding of mathematics, because they undertook many different activities in class. This interest in the lessons and activities may have had positive effects on the students’ understanding of the subject matter concerned, which in turn may have enhanced their self-confidence and self-efficacy. As Teacher BA observed:

> I think that when the class is interesting for students, their understanding will increase. Due to their increased understanding, students enjoy learning, and it gives them more chances to do more exercises, which helps students to improve their confidence and self-efficacy.

Thus, the use of GGS helped to provide interesting and enjoyable lessons. This was clear from the teachers' quotes above: in their view, GGS makes students interested, they enjoy learning using it and the use of the software improves students’ understanding.

Overall, the teachers agreed that the use of the software fostered improvements in their students’ engagement with mathematics lessons. They explained that these improvements occurred in various aspects of learning, for example noting the differences in their students' degree of participation in the lessons. Therefore, the use of GGS reportedly improved the students' eagerness to learn, and all of the teachers concurred that the use of GGS in mathematics lessons made the lessons more interesting and enjoyable for their students than previously.

7.6 GGS as a supporter of mathematics skills

This section presents the mathematics teachers' views expressed during the teacher interviews and teacher group written responses regarding the effectiveness of the use of GGS on students' consideration of their mistakes when they were doing mathematics exercises. The mathematics teachers considered GGS a helpful tool for students to consider and correct their mistakes, which might enhance their mathematics reasoning. However, one teacher claimed that he has not noticed any effects of GGS on his students.

The participant teachers viewed GGS as a helpful tool in enhancing students' abilities in recognizing their mistakes and correcting them when doing tasks. They argued that the opportunities GGS offers for students to correct their mistakes had a positive effect on students' efforts in answering more tasks on their own. They also noted that the skills of the students in undertaking such tasks increased, because they were able to check their answers using the software, without struggling as a result of their mistakes. Teacher BA explained:

> By using GGS, all the steps become clear for the students. The students understood it very well, and they tried to do it again. If I found any of my students were stuck on any steps, I asked them to look at the steps in the program, and to correct their mistakes by themselves. I think when the students correct their errors by themselves, it builds their knowledge, and gives them more skills to implement the procedures with flexibility, accuracy, and appropriateness in the next time.

Group 2 added:

> The use of the GGS helps to save time in lessons, which allows students to do more exercises, and helps them to learn from their mistakes. They think about it, because if they do more exercises, they might make more mistakes, so they will learn from their mistakes and then become more cautious about their mistakes.

The teachers above agreed that the use of GGS gave their students more opportunities to think about their mistakes when addressing the assigned tasks. They explained that the use of GGS, with the aids and feedback it provides, enhanced students' ways of engaging with exercises. The teachers thus believed that students were capable of answering tasks with the help of the GGS and therefore attempted to answer questions and to undertake further steps. In addition, the teachers found that GGS provided better opportunities for students to answer more questions in class. They believed that if students had the chance to engage in more practice and tasks, this might lead them to make some mistakes from which they could subsequently learn. Thus, the next time they would become cautious about these errors and try to avoid them. So, the teachers considered GGS supportive of students in answering mathematics exercises since it helped them correct their mistakes and provided helpful feedback when they encountered difficulties at any point.

The chances which GGS offers for students to consider and correct their mistakes may help them to enhance their mathematics reasoning. The participant teachers believed that the use of GGS improved their students' mathematics reasoning. For example, teacher BA stated "GGS enhances students' mathematical thinking, they learn to explain [their] steps and relationships". Also, teacher AL reported that "GGS gives students space to think critically and logically, and they understand concepts very well and try to write some questions around them". This argument is also backed up by the teacher group written responses. Group 1 pointed out that "GGS enhances students' mathematical thinking and reasoning, because it presents concepts and tasks which let students think more deeply before answering and share their ideas with teachers and their colleagues". Whereas Group 3 responded that "GGS has an ability that helps to connect mathematics concepts and ideas with real life which gives students a chance to think mathematically and creatively to connect these concepts and ideas with models".

From the teachers' quotes above, it can be concluded that they believed GGS had improved their students' reasoning. For teachers, this was clear in the students' ability to explain their steps and the relationships between these steps. Also, as Teacher B stated above, GGS gave students space to think mathematically; these improvements in their ability came as a result of the deep understanding of their lessons. Also, they thought that the visualization features of GGS helped present concepts and connect them to real life. This could have a crucial impact on improving students' reasoning and give them more space to think about and share their ideas inside the classroom. Thus, teachers considered GGS supportive of reasoning since it gave the students space to think mathematically, helped them gain better understanding of the lesson, and provided better visualization and connected lessons to real life.

However, teacher AW believed that the use of GGS did not affect his students' skills in mathematics, especially their fluency and problem solving, as he did not perceive a difference in their skills before and after using the software, explaining:

> I think no, I have not noticed any differences between the use of GGS and my previous approach. I know and I found how much this software is useful; however, I have not found any significant changes in my students when they are doing tasks or solving problems.

This was an interesting claim that contradicted the other mathematics teachers. The crucial difference between this teacher and the others was the way in which he employed GGS in his class. While most of the teachers explained that they used GGS as an in-class learning tool, teacher AW employed it as a demonstration tool to present the lessons to his students. This highlighted the fact that the way in which GGS is used in class may affect the teachers' views of its effectiveness, since while the teachers who used the software as a learning tool claimed that it affected their students' mathematics skills positively, the teacher who used it as a demonstration tool did not observe any improvement in his students' skills.

In summary, teachers believed that using GGS had an impact in supporting students to correct their errors and improved their reasoning skills. The teachers viewed GSS as supportive in this, since it helped the students avoid mistakes when they were doing exercises and provided helpful feedback when they encountered difficulties at any point. They also believed that GGS supported the students' reasoning because it gave them space to think mathematically and helped them to gain better understanding of the lesson.

7.7 Summary

The data resulted in five themes: GGS as a time/effort-saver; GGS's representational capacity in enhancing teaching; GGS as an effective learning tool; GGS as a facilitator of student engagement; GGS as a supporter of mathematics skills. These five themes will be explained briefly in the following paragraphs.

GGS as a time/effort-saver. The participant teachers found this function essential because GGS enabled a focus on students' logic since the new method enabled them to do what they were doing in their previous approach in less time and with more consistency in drawing or forming final outputs. Furthermore, it helped the teachers by limiting the use of mathematics instruments in class and allowed them to introduce more than one idea at the same time.

GGS's representational capacity in enhancing teaching. The participant teachers considered the representational capacity of GGS when teaching mathematics concepts to offer three different benefits in class. The first was that it made concepts easier to understand. The second was that it helped students recognize the meaning of concepts and mitigated their abstract nature, making them more concrete and functional for students through models and simulations, and connecting them with real life. The last was that these features helped students connect different concepts.

GGS as an effective learning tool. The participant teachers indicated that GGS was an effective learning tool as they demonstrated that the students' use of the program promoted successful learning, which in turn enabled more continuous learning. That inspired the teachers to encourage students to use GGS to explore concepts and mathematical ideas. The teachers reported that the use of GGS in mathematics classes had a positive impact in enhancing the students' independent learning, which built their confidence in working with the subject matter.

GGS as helpful for improving students' engagement in class. The teachers believed GGS was helpful in terms of engagement because they thought it provided interesting and enjoyable lessons, changing normal lessons to create a dynamic atmosphere. Therefore, the use of GGS reportedly improved the students' eagerness to learn, and all of the teachers concurred that the use of GGS in mathematics lessons made the lessons more interesting and enjoyable for their students than previously.

GGS as enhancing fluency and reasoning. According to the teachers, the use of GGS developed students' consideration of their errors when they were engaged in mathematics tasks and exercises. It also developed students' thinking by providing clearer examples of the relevance of mathematics for real life. However, one teacher claimed that GGS did not affect his students' skills in mathematics, especially their fluency and problem solving.

Chapter 8 Discussion

8.1 Introduction

The main aim of this research was to investigate the effectiveness of a GGS-based pedagogy as a tool for improving students' mathematical proficiency. It also sought to explore teachers' opinions about teaching mathematics using the software. This chapter discusses the study findings from the students' perspectives (quantitative instrument) and based on the results of the MPTs (Chapter 6: Student-Related Findings). The data were analysed descriptively utilizing SPSS software and drawing on key themes elicited from the collected data using qualitative instruments, including teachers' interviews and written responses (Chapter 7: Teacher-Related Findings), the data from which were analysed through thematic analysis. In this chapter, the findings are addressed and synthesized with reference to previous studies with a view to answering the operational research questions set out below:

Research Question A (RQA): Is there a significant effect of GGS-based pedagogy on students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition) and achievement compared to the traditional teaching approach?

Research Question B (RQB): What are the mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics?

Thus, in this chapter, the study findings will be discussed in two main sections; the first concerns the effects of GGS-based pedagogy on the students' mathematical proficiency and associated strands, and the second relates to the mathematics teachers' views in relation to adopting GGS-based pedagogy for teaching and learning mathematics.

8.2 Main findings

This section briefly presents the main findings from the intervention study and from the teacher responses.

8.2.1 Student-related findings

The findings demonstrated that the use of a GGS-based pedagogy significantly improved students' mathematical proficiency in the second and third MPTs (for the Numbers and the

Geometry topics) and thus had advantages over traditional teaching methods. This improvement, however, occurred only for students' mathematics achievement in specific strands of mathematical proficiency. In particular, in the Numbers topic, the experimental group demonstrated better performance than the control group in the mathematics achievement test overall and in the conceptual understanding and procedural fluency strands, but not the strategic competence or adaptive reasoning strands. Also, for the Geometry topic, students in the experimental group demonstrated better performance than the control group overall in the MPT and in the procedural fluency strand in particular. They did not show any significant improvement in the conceptual understanding, strategic competence, or adaptive reasoning strands though.

Furthermore, there is robust evidence that using a GGS-based pedagogy improves students' productive disposition across the three scales: "students like learning mathematics", "students' confidence in mathematics", and "students value mathematics". More details about these findings will be discussed in the following sections.

8.2.2 Teacher-related findings

The data resulted in five themes: GGS as a time/effort-saver; GGS's representational capacity in enhancing teaching; GGS as an effective learning tool; GGS as a facilitator of student engagement; GGS as a supporter of mathematics skills. These five themes will be explained briefly in the following paragraphs.

GGS as a time/effort-saver. The participants considered the savings in time an effort important since GGS allowed a greater focus on student reasoning. The new tool helped them do what they were doing in their previous approach in less time and with more consistency in drawing or forming final outputs. Furthermore, it helped the teachers by limiting the use of mathematics instruments in class and enabled them to introduce more than one idea at the same time.

GGS's representational capacity in enhancing teaching. The participant teachers considered the representational capacity of GGS of value when teaching mathematics concepts as it offered three different benefits in class. The first was that it made concepts easier to understand. The second was that it helped students to recognize the meaning of concepts and made them functional for students through models and simulation and connecting them with real life. The last was that these features helped students connect different concepts.

GGS as an effective learning tool. The participant teachers indicated that GGS was an effective learning tool as they demonstrated that the students' use of the program promoted successful learning, which in turn enabled more permanent learning for students. That inspired the teachers to encourage students to use GGS to explore concepts and mathematical ideas. The teachers reported that the use of GGS in mathematics classes had a positive impact in enhancing the students' independent learning, which built their confidence in working with the subject matter.

GGS as helpful for improving students' engagement in class. The teachers believed GSS enhanced student engagement because they thought it provided interesting and enjoyable lessons, changing normal lessons to create a dynamic atmosphere. Therefore, the use of GGS reportedly improved the students' eagerness to learn and all of the teachers concurred that the use of GGS in mathematics lessons made the lessons more interesting and enjoyable for their students than was previously the case.

GGS as enhancing fluency and reasoning. According to the teachers, GGS developed the students' consideration of their errors when they were engaged in mathematics tasks and exercises. It also developed students' thinking by providing clearer examples of the relevance of mathematics for real life.

8.3 Discussion of findings for RQA

This section discusses the students' overall mathematics achievement based on the results of the two MPTs. In addition, it examines the results concerning each aspect of mathematical proficiency.

The findings reveal that the GGS-based pedagogy had a substantial effect in terms of improving students' mathematical outcomes in two areas compared to the traditional approach: Numbers and Geometry. The results imply that the use of a GGS-based pedagogy enhanced the students' performance in mathematics beyond what would have been achievable following a traditional approach. The qualitative analysis details positive responses from mathematics teachers with regard to their perceptions of the GGS-based pedagogy, highlighting perceived benefits to students, but not referring to data detailing overall mathematics achievement.

The quantitative results presented here are aligned with the findings reported in previous research (e.g. Zengin et al., 2012; Martinez, 2017; Alkhateeb & Al-Duwairi, 2019). However, they are in contrast with Saha et al.'s (2010) study, as the control and experimental groups showed no statistical differences in terms of the pre- and post-tests. Nevertheless, the group that employed GGS in their study had an insignificantly higher score relative to the other group with regard to visual-spatial capacity. In the case of Doktoroğlu's (2013) study, no statistically significant evidence of improvement in test scores for the Cartesian coordinate system or linear relations was reported, although the sample group showed improvements in mathematics learning overall. Doktoroğlu's (2013) study assigned three hours to each aspect, which may not have been sufficient to present differences between the two groups. This study lasted longer, five weeks for the first unit and three weeks for the second, which may explain why the students in the experimental group achieved notably better results than those in the control group. Therefore, the period of time spent using GGS in class resulted in a difference between the two groups and between the results for the two units taught. Based on the students' results for the two units, the results for the first unit (Numbers) were better than those for the second unit (Geometry). Therefore, arguably the time spent teaching and the students' use of GGS to some extent affected the students' mathematics performance. It is also possible that the differences in the students' performance might depend on the lesson content. For example, the use of a GGS-based pedagogy appeared to make less difference when teaching the Geometry unit, since the GGS tasks and traditional teaching were similar, using both pictures and diagrams. However, in the Numbers unit, the traditional approach uses only numbers, whereas the GGS approach supports learning with a variety of pictorial representations and dynamic features. This corresponds to results presented by Doktoroğlu (2013) who examined the different content of lessons for teaching the Cartesian coordinate system, linear relations and graphs for linear equations and found the differences between content were most marked for graphs for linear equations.

Utilising GGS for the teaching and learning of mathematics produces good achievement results and facilitates students' understanding. This is confirmed by the students' results from the two MPTs in which the two experimental groups outperformed the control groups. Moreover, the mathematics teachers explained that using GGS helped to improve students' learning and facilitated their comprehension of difficult and abstract concepts. These findings are supported by Alkhateeb and Al-Duwairi (2019), whose study examined the use of Sketchpad and GGS to enhance students' understanding of geometry concepts and confirmed a positive impact on achievement levels They observed that GGS had a greater impact on student learning than Sketchpad, despite both being DMS. They attribute this to the features of GGS, which include drawing orders that are easier to implement than those in Sketchpad. In addition, the users of GGS were skilled at dealing with and controlling drawing with accuracy and mastery because GGS integrates properties which include a distinct role in controlling drawing elements, thereby differing from Sketchpad. Hutkemri and Nordin (2011), Hohenwarter and Lavicza (2007), and Hohenwarter et al. (2009) similarly attributed GGS's success to the fact that it is a relatively easy-to-use platform.

Typically, the use of technology has positive affordances for teaching and learning mathematics. Certainly, evidence of the positive effects of technology use on students' achievements, specifically a GGS-based pedagogy are presented elsewhere. The broad conclusion to date has been that using technology instruction to support student achievement is objectively positive (Glassett & Schrum, 2009; McKenna, 2012; Pilli & Aksu, 2013; Yang & Tsai, 2013; Rosen & Beck-Hill, 2014). In addition, several meta-analyses have reported a moderate to significant impact on achievement (Slavin et al., 2008; Johnson & Rubin, 2011; Cheung & Slavin, 2013; Higgins et al., 2019). Thus, the findings of this study are not novel in that they reflect others detailing the positive effects arising from the use of technology on students' achievement and performance. However, in terms of reporting specific data regarding Saudi students' capacity to achieve comprehensive mathematics proficiency, it is unique. The findings show that GGS adoption enabled the Saudi students to perform and achieve better in both the units targeted, i.e. Numbers and Geometry. Also, this study used TIMSS items to measure the students' performance in terms of overall achievement and in each mathematical proficiency strand, this being a measure that is considered to have good reliability and is based on international criteria, giving greater confidence about the results of this study.

As mentioned earlier, despite the positive effect of the use of GGS in the areas examined, there is no guarantee that effects will be seen among all the students for all types of mathematics content. In addition, according to the findings, the effects of using GGS are more marked for certain mathematical proficiency strands than others, as will be covered in the following paragraphs.

The first mathematical proficiency strand considered was *conceptual understanding*. A sub-assessment was conducted as part of the mathematical proficiency assessment to assess this particular strand. Two differing outcomes were obtained. During the first phase of the intervention study, the experimental group achieved statistically significant improvements based on the results of statistical analysis for the Numbers unit. A possible explanation is that compared with traditional teaching methods, the material in the Numbers unit was easier for learners to comprehend through a GGS-based pedagogy. The pictorial model or representation of concepts for learners is a characteristic of GGS that could support enhanced comprehension of the unit among the experimental group. Furthermore, teaching and instruction in mathematics using GGS was identified as incorporating numerous positive variables according to the mathematics teachers interviewed for the qualitative component of the research. Ultimately, the teachers found the instruction and learning of ideas simpler due to the advantages of GGS.

Researchers, including Antohe (2009), Furner and Marinas (2013), Jelatu and Ganesha (2018), Ocal (2017), and Zulnaidi and Zamri (2017), have established that learners' understanding of mathematical concepts can be improved through the implementation of GGS, as is also supported by the data collected for this research. The comprehension of mathematical concepts by learners is fundamentally enhanced through the incorporation of aesthetic graphics and depictions within GGS. Abstract ideas can be comprehended in a straightforward manner and then linked to mathematical concepts through applying the GGS approach (Rincon, 2009). In this study, the students in the two experimental groups were taught mathematics concepts using GGS. Based on the differences between the two groups in the Numbers unit, it is apparent that GGS enabled the students to understand the lessons better than those taught using the traditional teaching approach. It is thought the experimental group improved their understanding of the concepts in the unit as a result of the GGS features, namely presenting information to the students in the form of a pictorial representation or model. It might also be that exposure to the GGS's dynamic features and its representational capacity improved the students' understanding of the targeted concepts.

In addition, GGS animations were available for use with the student tasks so that mathematical concepts could be explained more effectively by educators as part of in-class teaching. This is supported by the studies of Hutkemri and Zakaria (2012) and Zulnaidi and Zamri (2017), which observed that numerous mathematical ideas can be investigated by learners through interactive educational approaches facilitated through GGS. For example, in the first unit, instruction in fractions, related operations, and the characteristics of fractions were the consistent focus. The students engaged with the different tasks and used the interactive tasks and pictorial representations in GGS accordingly. The use of such representations is essential to improve students' understanding of concepts, as is apparent from the outcomes of this unit, which demonstrate that the students taught using GGS with multiple representations were more successful than the other students in the intervention study. The identification of crucial approaches to the teaching of operations and fractions was undertaken by Lamon (2020), who investigated how educators could effectively incorporate multiple representations. She suggested that the comprehension of fixed concepts and their associated calculations could be enhanced through the presentation of various fraction representations or models to learners.

Accordingly, on the basis of findings from previous studies and this research, learners' comprehension of mathematical subjects and their grasp of concepts could be facilitated by the adoption of GGS. Nevertheless, for the Geometry unit, which comprised the second phase of this intervention, there was no evidence that the GGS-based pedagogy had any beneficial effect, despite the Numbers unit affirming the advantages of this approach. Ultimately, for the Geometry unit, the data from the control and experimental groups reflected no significant variation, irrespective of the general benefits of adopting GGS and the control group learners' performance being surpassed by that of the experimental group learners overall. There could be various different reasons for the improvements in students' conceptual understanding in the Numbers unit and not the Geometry unit. For example, the use of more vibrant pictorial representations and shapes with colours and options for dynamic moves changed the method employed when introducing numbers concepts to students, helping to shift these concepts from an abstract to a concrete, visible form. In the Geometry unit, there were considerable benefits from using GGS in class, such as the option to use graphs, shapes, diagrams and dynamic affordances, but these did not help the students in the experimental group perform better than the control group. This could be because the method employed to introduce geometry concepts is to some extent similar to the traditional approach, which also uses shapes and diagrams. In contrast, the approaches used to introduce numbers concepts were completely new to the students, which may have impressed them and led them to perform better than the control group. Therefore, additional investigation is necessary to compare the effects.

How GGS-based pedagogy was used in this study and its effects differed across each of the NRC mathematical proficiency strands. In terms of procedural fluency, it emerged that the learners' fluency in relation to the two units was enhanced by the GGS-based pedagogy, with the performance on the procedural fluency strand among the experimental group differing significantly from that of the control group in both phases of the intervention. The learners gained unique opportunities through the use of GGS, for example the allocation of brief periods of time to work independently, and the ability to finish an activity at different times, which might explain the improvements in their fluent completion of mathematics tasks with GGS. Learners' proficiency in identifying errors and rectifying them during mathematics activities improved through the use of GGS according to the mathematics teachers interviewed as part of the qualitative research component. Greater independence when completing tasks was witnessed among the learners, with their capacity to rectify their own errors improving through opportunities arising from use of GGS. In addition, because the program enabled learners to verify their answers, the opportunities for completing these activities were also enhanced as the teachers clarified.

A beneficial impact on the *procedural fluency* of learners arising from GGS-based mathematics lessons may be posited based on the qualitative and quantitative results obtained. This reflects the findings from studies including those of Antohe (2009), Furner and Marinas (2013), Jelatu and Ganesha (2018), Ocal (2017), as well as Zulnaidi and Zamri (2017), which found learners' procedural fluency was enhanced by GGS.

The GGS tasks for the intervention study were prepared to apply instant feedback, which helped the students revise their answers by themselves and potentially leading them to avoid mistakes in the future. In addition, the learners' opportunities to repeat their activities were expanded upon when GGS was adopted for in-class teaching, resulting in reduced effort and time required for class-based activities. Hence, the research revealed learners' fluency might have be enhanced in relation to the majority of activities as a result of the provision of immediate feedback and tasks being repeated with less time and effort. These two affordances offer a chance for students to complete their calculations with as few mistakes as possible. As Kilpatrick et al. (2001) suggested, computational processes need to be employed correctly if they are to be beneficial and produce optimal outcomes. Accordingly, the capacity to undertake these processes fluidly and the promotion of fluency among learners are outcomes linked to repeated practice, and strengthened efficacy and precision. As Olsson (2015) and Gono (2016) have argued, instantaneous feedback enables learners to engage with the typical graph characteristics that appear on the computer screen in their own time, thus helping them understand modulus functions and commit them to memory.

Moreover, as one of the fundamental aspects of constructivist-based education, independent learning is also instilled by delivering immediate feedback (Gono, 2016). Indeed, learners' ongoing ability to complete activities is based on beneficial immediate feedback as verified in the teacher interviews, comprising the qualitative research component. The ability to learn from errors and improve and to undertake further activities with improved time efficiency during lessons are all advantages of GGS. Therefore, independent completion of activities by learners is encouraged. With fewer problems due to errors, along with the capacity to verify answers through the program, the teacher interviewees noted that learners' proficiency was enhanced when carrying out the GGS activities. Consequently, it is possible that the procedural fluency of learners is improved because GGS provides immediate feedback on the tasks carried out by learners.

Moreover, the positive effects of the adoption of GGS were apparent from the students' *productive disposition*, as the results show the use of the GGS developed the students' productivity across the two units. Furthermore, the results from the pre-intervention questionnaire were surpassed by the post-intervention results in all three sections of the learners' questionnaire. In addition, the learners' classroom participation was considered to have been shaped positively through the use of GGS by mathematics teachers, as noted during the interviews. Classroom enthusiasm, the resolution of mathematical activities, and the contributions and other features of learners' engagement were improved. Indeed, learners' productive disposition concerning mathematics can be strengthened via GGS according to much of the extant research (e.g. Aydos, 2015; Jatiariska et al., 2019).

The use of GGS as a tool for learning mathematics, especially as found in this study, can help enhance students' attitudes. One of the reasons for this is the multiple representation of each of the tasks, as the students were able to deal with different shapes and pictures in each task. In addition, the use of pictorial representation means the tasks for learning numbers and fractions differ from traditional methods, thereby improving motivation to learn. In addition, the flexible characteristics and fresh learning environment that GGS offers learners may be responsible for alterations in their productive disposition. Their enthusiasm and interest might also have been nurtured effectively through the ability to autonomously engage with and use the program themselves. According to Aydos (2015), Hohenwarter and Fuchs (2004), Hohenwarter and Jones (2007), and Hohenwarter and Lavicza (2009), the multiple representational characteristics of GGS are more in accordance with the learning approaches preferred by students over traditional approaches, and GGS has the ability to increase the tangibility and visual impact of subjects, leading to greater engagement and improved learning among students due to the use of GGS. Moreover, Jatiariska and Astawa (2019) suggested that GGS-oriented classrooms help the learners experience greater opportunities throughout the learning process to discover their skills and comprehend the resources made available through GGS. Furthermore, they can go through an inquiry procedure to develop their understanding and engage in learning tasks more willingly. Indeed, each activity could be attempted numerous times as part of the tasks in the amended unit, increasing the learners' potential for engagement through interactive modelling and the shape elements of the tasks. Their anxiety concerning potential failure could also have been alleviated by their ability to verify answers using the task feedback tool. A further element may have been the original nature of the GGS strategy, which could have affected learners by exciting them with the program's characteristics and its novelty. The learners' productive disposition was influenced positively by adopting the GGS-based pedagogy based on the research findings and this was reflected in the Numbers unit outcomes. Affective engagement and confidence in mathematics are two variables posited as being connected to the formulation of learners' positive perspectives regarding computer-assisted mathematics learning (Pierce et al., 2007; Barkatsas et al., 2009).

Learners' confidence when studying mathematics was strengthened by the adoption of GGS, as the research findings indicate. The initial implementation of the productive disposition questionnaire was used to determine this, with the further two questionnaire implementations showing that the two control groups' confidence was surpassed by that of the learners in the experimental group. Learners' self-confidence is broadly enhanced through the adoption of technology, as evidenced in this research. Learners' results might derive primarily from technological provision of instant rectification of mistakes, with the instant feedback provided by GGS enabling the correction of answers by learners. Improved knowledge acquisition and memorization by learners is facilitated by the provision of instant feedback. Furthermore, Brosvic et al. (2006), McDaniel et al. (2007), and Nussbaum (2007) provided evidence that learners' confidence is strengthened through engagement with digital games. Consequently, this is likely also to be the case with digital games integrating mathematics learning components. This study found the use of GGS helped improve students' self-efficacy, a result supported by Zetriuslita (2020), who reported that with regard to developing self-efficacy, traditional learning approaches are inferior to GSS approaches in the study of geometry. Furthermore, Liu et al. (2009) established that mathematics learning results can be positively influenced when self-efficacy is a pre-existing characteristic among learners. Self-discipline and self-efficacy are correlated, meaning the perspectives of learning and learning results are affected when both of these traits are possessed by learners, as this study supports.

In contrast, the quantitative findings revealed the adoption of the GGS-based pedagogy made no difference in the two NRC strands *strategic competence* and *adaptive reasoning*. Regarding strategic competence, the results showed no statistical difference between the two groups. This means that despite the positive effect of the GGS-based pedagogy on overall mathematics achievements and the conceptual understanding and procedural fluence strands, its effect on the strategic competence strand was either null or not sufficiently strong to detect differences between the two groups. Although in the qualitative part of the study, the views of mathematics teachers regarding the effect of the use of GGS in class on students' mathematics learning were positive, the students' ability to solve mathematics problems was not enhanced measurably. Hence, this research concludes there was no effect arising from the use of GGS on students' strategic competence.

The results of this study contrast with existing literature in the field, which found that using GGS in class is beneficial for improving students' strategic competence compared to a traditional approach (e.g. Bu & Haciomeroglu, 2011; Jacinto & Carreira, 2017; Kim & Md-Ali, 2017; Kustiawati et al., 2019). Furthermore, the results of this study differ from those reported by Velikova and Petkova (2019) and Villarreal and Borba (2010), who found that in terms of in-depth comprehension and appropriate approaches to problem formulation and resolution in mathematics, learners are advantaged by the GGS context.

In this study, one of the main aims of the intervention was to determine the extent to which using GGS to teach students would help them to answer and solve mathematics problems. Jacinto and Carreira (2017) focused on theoretical approaches concerned with the correlation of technological and mathematical proficiency for effective problem solution as a means to explore perspectives related to technology-focused problem solving by learners. Learners' mathematical proficiency and perspectives regarding the advantages of GGS were determined to be interlinked, providing a foundation for resolving mathematics problems. Indeed, in the literature overall, problem solving in mathematics appears to be successfully supported through the adoption of technology based on studies analysing technologies and problem solving in relation to GGS (see Meira, 1998; Marrades & Gutiérrez, 2000; Straesser, 2002; Sinclair, 2004; Constantinos Christou et al., 2005). However, the results of this study show no effects from using GGS on problem solving. This result may be due to the students being unable to use GGS during their examinations; thus, the students in the two experimental groups were not able to use GGS in the second and third MPTs as they completed the same test as the two control groups. Therefore, it could be concluded that using GGS in class to teach students might not help them solve mathematics problems in tests in which they cannot have access to the software. This point requires more investigation in further studies.

The effects of the use of GGS on students' *adaptive reasoning* were similar to the effects on strategic competence. The mathematics teachers in their interview responses and in their written responses claimed that using GGS was helpful for improving students' reasoning and their ability to explain their steps and the relationships between them. They considered GGS supportive of reasoning since it gave the students the space to think mathematically and gain a clearer understanding of lesson content, provided better visualization, and connected lessons to real life. However, in the two intervention study rounds, no differences were noted between the two groups in the adaptive reasoning strand, confirming the the GGS-based pedagogy had no significant effect on the adaptive reasoning of the study participants. There are two different aspects to address when considering the effects of the use of GGS on students' reasoning. The contrast between the teachers' views and the quantitative results arises from GGS use in class. Mathematics teachers might see improvements in students' reasoning when they are using GGS in class as it gives the students more opportunity to think through mathematics ideas in terms of different relationships. However, in neither round of the intervention study testing were the students able to use GGS, which may have reduced their reasoning ability.

Research by Diković (2009), Gunčaga and Majherová (2012), Hutkemri (2014), Khalil et al. (2017), and Shadaan and Leong (2013) produced results explaining the enhancement in reasoning and mathematical cognition of learners through the adoption of GGS, results from which this study's findings diverge. Diković (2009) pointed to how GGS techniques can foster more effective links through visual and symbolic representations with learners because certain arrays of activities and operations are available to learners within a given educational environment. For both units, various kinds of representation were made available to the learners, for example in the Geometry unit they could apply various geometrical shapes and in the Numbers unit they could use active line numbers for visual representation. Nevertheless, in terms of reasoning, the experimental groups did not surpass the control groups, despite the various active representations and tools available.

8.4 Discussion of findings regarding RQB

This section discusses the results of the study concerning Grade 8 mathematics teachers' views of the effect of using GGS on their students' learning ability. The teachers' interviews and the group written responses were analysed using thematic analysis, and the following five main themes emerged: 1) GGS as a time/effort-saver, 2) GGS's representational capacity for enhancing teaching, 3) GGS as an effective learning tool, 4) GGS as a facilitator of student engagement, and 5) GGS as a supporter of mathematics skills. Each theme is discussed in detail in this section.

The teachers considered GGS to be both a time- and effort-saving tool when used in their mathematics classes and believed it to be of benefit for both teachers and students. This echoes Gono's (2016) findings. He suggested that the use of DMS provides students with the opportunity to use many different techniques when attempting to solve a mathematical task. He also reported that as opposed to the heavily time-consuming traditional learning methods of pen-and-paper learning, studying arithmetic and algebraic algorithms using DMS gives students more time to learn and understand topics, to enhance their reasoning, and to comprehend application. It thus equips learners with the tools required to take control of their learning (Gono, 2016). This study found that GGS had many features of value to both teachers and students in the classroom, including the fact that the students were able to make many attempts at completing a task in less time than before. This concurs with the findings of Hohenwarter and Preiner (2007), who reported that GGS is the most commonly used software for the teaching of mathematics due to its features and advantages, such as its affordability and time-saving benefits.

Moreover, the software was found to help teachers perform better than when using the traditional approach, as they were able to focus on different aspects of the class concurrently. This is reflected the findings of the study conducted by Horzum and Ünlü (2017), who observed that pre-service mathematics teachers considered the use of GGS could save them a significant amount of time and effort by allowing them to create images on the computer. In addition, Hohenwarter and Preiner (2007) reported that the use of software can help teachers prepare classwork and activities for their students in advance of lessons. The teachers in this study also believed that the in-class use of GGS helped use mathematical instruments in their classes effectively and they were able to introduce more than one idea in a class period. To a degree, this is reflected in the findings of Hohenwarter et al. (2008), who suggested that the capacity to use the integrated interactive software’s user interface, for example hiding the algebra window, limiting the tools available, and displaying the help bar, allowed teachers to elect in advance how to help their students, how much help their students would receive, and which tools/features they would be allowed to use.

The mathematics teachers in this study not only considered GGS to be a time/effort-saver, but also reported that the capability of GGS to present concepts in the form of models, using images and simulations, and to connect them with real-life situations enhanced both the teaching and learning of the mathematics concepts concerned. This view is supported by many previous studies (e.g. Özgün-Koca, 2001; Duval, 2006; Elia et al., 2007; McGee & Moore-Russo, 2015), which found that the multiple representations made possible using GGS play a crucial role in improving students’ understanding of mathematical concepts.

The greater number of examples of a concept and representations thereof afforded by DMS may help to facilitate students' understanding of concepts. According to Duval (2006), it is not possible for one representation to describe a mathematical concept completely and thus multiple external representations can significantly help learners. This echoes the view of the teacher participants in this study, who believed that the use of GGS for teaching mathematical concepts made them easier to explain. They also reported that there were better opportunities to apply the new concepts using the GGS exercises than traditional teaching methods and GGS enabled them to support the concepts using examples. According to Elia et al. (2008), multiple representations can help students understand concepts far more effectively than single representations and Özgün-Koca (2001) highlighted that the application of multiple representations can have significant advantages for meaningful mathematics learning. In addition, Bayazit (2007) reported that multiple representations displayed on one screen can help enhance students' knowledge development and depth of understanding and Duncan (2010) found that multiple representations are able to stimulate investigation and help enhance students' learning. This view was also articulated by the mathematics teachers in this study, who reported that the capability of GGS in presenting concepts in the form of models using pictures and simulations, and connecting them with real life enhanced the teaching and learning of the concepts.

Furthermore, previous studies have found that the use of multiple representations in GGS helps students solve mathematical problems. For example, Özgün-Koca (2001) found that the use of multiple representations in mathematics plays a fundamental role in facilitating the solving of mathematical problems. The teacher participants in this study focused on the effects of GGS on their students' understanding of mathematical concepts and did not connect this with the solving of mathematical problems.

It is therefore apparent that the use of GGS helps to improve students' understanding of mathematical concepts. However, in terms of the results of the two student intervention studies employed, there was only a significant effect of the use of GGS in the Numbers unit and it was less apparent in the Geometry unit. Mathematical concepts are generally abstract and can only be made accessible to students through the use of representations (Duval, 2006). The teachers in this study believed that the representational capacity of GGS, namely the use of examples, models, and representations, helped enrich the teaching of concepts that are difficult and/or abstract. This capability also helped them to introduce more detail than using the traditional teaching approach. The use of GGS thus contributes to enhancing students' conceptual understanding and is a valuable tool for teaching concepts, eliminating/preventing misconceptions, developing thinking skills, helping perceive the details involved, understanding the sense of a subject, and helping to avoid rote learning (Horzum & Ünlü, 2017).

The use of multiple representations is an principal concept that is relevant to all aspects of mathematics (Dreher et al., 2016). This use of different representations helps to enhance students' understanding and connects concepts with real life. According to the teachers in this study, the use of GGS was helpful for teaching concepts as it offered a range of means of presenting mathematical concepts using models and graphics and had the ability to provide simulations connecting mathematical concepts to real-life situations. This reflects the findings of previous studies, such as those conducted by Ball (1993) and Malle (2004), who reported that the use of different representations in mathematics lessons helps to develop students' ability to connect symbolic-numerical images with pictorial representations, such as diagrams, sketches, and illustrations, and that content-related demonstrations, such as real-world situations, can play a vital role in the learning of fraction calculations.

This study also found that GGS was an effective learning tool supporting students' self-learning and facilitated more effective and permanent learning than traditional teaching methods, a finding that was unsurprising given the many features and characteristics of GGS. However, the results of the two rounds of intervention found that the use of GGS was not equally effective for all five NRC mathematical proficiency strands. Nevertheless, it may be possible to consider GGS an effective learning tool overall. The results of this study reflect those of certain previous studies, such as those conducted by Saha et al. (2010), Zulnaidi and Zakaria (2012), Martín-Caraballo (2015), and Zengin (2017).

All the teacher participants in the present study agreed that the use of GGS helped improve students' independent learning, which concurs with the findings of Murni et al. (2017), who reported that the use of GGS facilitates students' interactive exploration, thereby encouraging students to be actively involved in the learning process. Moreover, Jelatu and Ganesha (2018) suggested that the use of GGS makes mathematics learning exploratory and enables students to comprehend the association between the logic and visual representations of mathematical concepts instantly.

GGS was initially designed to combine the mathematical topics of geometry and algebra, and therefore its reported effectiveness for teaching these two topics is unsurprising, as is its recognition as an effective tool for enhancing mathematics teaching and specifically learning in this field (Shadaan & Eu, 2013). However, while the teacher participants in this study considered it to be an effective learning tool, as noted previously, its effectiveness in the Geometry unit was less than in the Numbers unit; therefore, its value may differ between individuals and the claim of some studies that the use of GGS is effective for teaching geometry may not always be the case. Nevertheless, this study found GGS was helpful for teaching and learning mathematics in general as the lesson process overall and individual procedures were found to be better than when using the traditional approach. The teacher participants also reported that GGS was helpful in enabling students to consider and correct their mistakes, which may enhance skills such as mathematics reasoning. This supports the findings of Dogan and Icel (2010), who observed that the use of GGS encouraged higher-order thinking, which had a positive effect on students' ability to retain knowledge for a longer period. However, while previous studies have found that the use of GGS also supports students' cognitive processes and enables them to develop their formal and logical reasoning, and cooperation and communication skills (Gunčaga & Majherová, 2012), the results in the reasoning strands of this study showed no improvements.

The use of new technologies in the classroom helps to alter the traditional atmosphere and general structure of the class. These changes can help enhance students' motivation and attitude in mathematics classes. This study found that the use of GGS was effective for improving students' classroom engagement as it offered better opportunities for students to engage and participate during lessons than traditional teaching. The in-class use of GGS meant that the students found the class more enjoyable and they were consequently more interested and engaged. This finding was reflected in the quantitative aspect of the study as the results from the productive disposition questionnaire suggested that the students' attitudes to mathematics improved as a result of adopting the use of GGS. This echoes the results of previous studies, such as that conducted by Doruk et al. (2013), who found that the majority of participants viewed GGS as a beneficial tool for teaching mathematics, particularly given that its use increased students' interest in the subject, ensured effective and enjoyable learning, increased students' enjoyment of mathematics, and enabled them to overcome their fear of the subject. In addition, in their in-depth examination of students' attitudes to the implementation of the GGS-assisted Missouri Mathematics Project, Rosyid and Umbara (2019) found that the students' attitudes were positive. This concurs with the results of the study conducted by Aydos (2015), which found that the use of technology, namely GGS, engendered an improvement in students' attitudes to learning mathematics and he concluded that GGS was an effective tool for teaching calculus to gifted and talented students.

According to the findings of both this study and previous research, mathematics teachers view GGS overall as an effective tool for enhancing students' positive attitudes to mathematics. However, not all previous studies agree, as while many report that most students express a positive attitude to the use of GGS for the teaching and learning of mathematics (Shadaan & Leong, 2013; Arbain & Shukor, 2015; Radović et al., 2018; Tuba et al., 2018), some have found that a proportion of students express negative perceptions of using GGS. For example, Shadaan and Leong (2013) found that some students reported a lack of confidence when using GGS, while Chua et al. (2017) found that some students continued to express uncertainty concerning its use, preferring traditional classroom methods to GGS.

Therefore, while both this study and some previous research found that the use of GGS engendered a positive effect on students' productive disposition, it is not possible to generalize this finding, due to the differing findings of other studies in the field.

Chapter 9 Conclusion

9.1 Main findings

The main research findings are presented and discussed here to answer the two research questions in relation to the literature reviewed in the previous chapter. This study investigated two issues, the first concerning the impact of using GGS-based pedagogy on students' mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, productive disposition) and achievement compared to the traditional teaching approach, and the second addressing the mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics.

Regarding the first issue, the findings of this study revealed various different aspects that contribute to answering this question. Some show a positive effect of GGS-based pedagogy on students' mathematics learning. Generally, in answering this research question, it is possible to state that the impact of GGS-based pedagogy is inconsistent and varies across differing strands of mathematics proficiency. These results are in line with Samuelsson (2010), who pointed out that there is no single approach that affects all strands of mathematical proficiency with the same impact.

In terms of overall *mathematical achievement*, the findings reveal that the use of GGS-based pedagogy has a significant effect on students' achievement in mathematics. This was apparent in the two rounds of the intervention, namely in the Numbers and Geometry units. The findings of this study are consistent with research undertaken by Doktoroğlu (2013), Rincon (2009), Zengin et al. (2012), and Martinez (2017). The differences in the performance of the two groups resulted from the differences between the two approaches. The use of GGS-based pedagogy offers students the chance to connect concepts with pictorial and symbolic representations and dynamic shapes in multiple cycles for each task. Bayazit (2011) suggested that most students rely on the symbolic representation of conjugation and that learners' capacity to alternate between iconic and algebraic conjugation symbols is crucial for gaining a comprehensive understanding of conjugation. The use of GGS helps students by saving time whilst enabling them to try more.

According to Hohenwarter and Preiner (2007), GGS is the most commonly implemented software in the teaching of mathematics. This is due to its features and advantages (for example, affordances and time-saving benefits). Also, working independently and receiving instant feedback from the software might have assisted students overcome their fear of failure while offering them a better chance of engaging in the lesson (Hohenwarter & Preiner, 2007). Olsson (2015) states that GGS is not only effective for exploring mathematical patterns, but also contains advanced tools that can be used to incorporate feedback into learning materials. By receiving immediate feedback, students can learn and memorize the modulus functions at their own pace based on the common features of the graphs presented to them on the computer screen (Olsson, 2015; Gono, 2016).

In terms of the differing mathematical proficiency strands, the apparent effects of using the GGS-based pedagogy vary. For example, regarding the conceptual understanding strand, the results showed that the effect of using GGS-based pedagogy differed from one unit to another. In the case of the first unit (Numbers), it was demonstrated that using a GGS-based pedagogy had a substantial effect on the students' understanding of concepts. The results of the study with regard to this unit correspond with those of Antohe (2009), Zulnaidi and Zamri (2017), and Jelatu and Ganesha (2018). They found that the use of GGS had a positive effect on students' understanding of concepts. In contrast, in the second unit (Geometry), the results revealed that using GGS-based pedagogy did not influence the students' uptake of concepts. However, this finding is contrary to previous studies, such as those conducted by Kutluca (2013) and Shadaan and Leong (2013). This might be because changing the method of introduction of numbers concepts from an abstract form to one with more vibrant pictorial representations and shapes with colours and options for dynamic moves was beneficial for students. However, the method of introducing geometrical concepts is to some extent comparable to the traditional approach with the use of shapes and diagrams.

With reference to the *procedural fluency* and *productive disposition* strands, the data showed that using a GGS-based pedagogy improved both aspects across the two units, i.e. Numbers and Geometry, when compared with the traditional approach. In the procedural fluency strand, the students benefited from the GGS tasks, with their unique features and flexibility. When completing the GGS tasks, the students were able to attempt the tasks repeatedly. This explains the improvement in their fluency. In addition, the students in the experimental group might have benefited from working independently in the classroom, and from the supportive functionality provided by the majority of the GGS activities in this unit. In addition, the feedback feature in the exercises allowed students to check their answer without fear of failure. This results are in line with the findings of a number of studies, such as Dijanić and Trupčević (2017), Hutkemri (2014), Ocal (2017), Zulnaidi and Zakaria (2012), and Zulnaidi and Zamri (2017). However, the findings do not support Ocal's (2017) study, which did not find any significant effects of using GGS on the students' procedural knowledge.

The changes in the students' productive disposition may have been due to several reasons, such as the change in atmosphere that the GGS provided students. They may have been influenced by the novelty of the approach; that is, their responses may have been influenced by the fact that the approach was new to them and they were impressed by its features. Also, the students may have found this interesting and enjoyable because they were given the opportunity to work with the software independently and experience it themselves. In addition, the use of interactive exercises offered better opportunities for students to engage with modified unit exercises with the capability of trying each task many times. The positive effects of GGS on the students' productive disposition are confirmed by the results of Bwalya, (2019), Rosyid and Umbara (2019), and Aydos (2015).

Concerning *strategic competence* and adaptive reasoning, using the GGS-based pedagogy appears to have had no influence compared to the traditional approach. This could be because these two strands are more complicated than the other strands, requiring more and higher cognitive skills to answer the questions assigned for these two parts. A further explanation may be that the students in the experimental groups could not use GGS during the mathematical proficiency tests, so they had the same chance as the students in the control group. The results related to the adaptive reasoning strand challenge some previous studies, such as Bitter and Pierson (2002), Wiske et al. (2006), Diković (2009), Nat et al. (2011), Shadaan and Eu (2013), Hutkemri (2014), and Dogan (2018), which found that the use of GGS helped improve students' reasoning.

Regarding the second issue, concerning mathematics teachers' views regarding the effectiveness of using GGS for teaching and learning mathematics, the mathematics teachers had positive perceptions of the effectiveness of using GGS as a tool for teaching and learning mathematics. These positive perceptions confirm the results of some previous studies (Hutkemri & Nordin, 2011; Bu et al., 2013; Doruk et al., 2013; Zengin, 2017). The participant teachers acknowledged the benefits for both teachers and students; in particular, they explained that it reduces the time and effort taken to communicate information. Compared to the traditional approach, they found that saving in time and effort helped them focus on their students' learning. They noted that the use of GGS afforded a better representational capacity that enhanced mathematics learning and teaching. They found that GGS helped them present concepts in the form of models, using pictures and simulations, and connecting them with real life, thus enhancing the teaching and learning of concepts. These multiple representation capacities enriched the teaching of concepts that are typically difficult and/or abstract. Such capacity also helped students make contrasts between different concepts and investigate their relationships.

The mathematics teachers viewed the use of GGS in class as leading to effective mathematics learning, supporting students' independent learning and facilitating more effective and permanent learning for students. In addition, the use of GGS in class was found to have had a positive effect on student engagement. They observed changes affecting different aspects of students' engagement, such as participation, approach to mathematics tasks, and enjoyment of lessons. In addition, they believed that GGS had a positive effect on students' consideration of their mistakes when completing mathematics exercises. The mathematics teachers explained that GGS was a helpful tool in assisting students to consider and correct their mistakes, which could in turn enhance their mathematics reasoning.

9.2 Contribution of the study

The contribution made by this study to the field of mathematics teaching and learning can be categorized as set out in the following paragraphs.

Contribution related to the analysis of the theoretical framework. The majority of the studies identified as investigating the effects of using GGS on students to some extent simply focused on one area, such as achievement, or additional aspects such as understanding, fluency, problem solving, reasoning, or attitudes. This study contributes to the literature by focusing on all these areas simultaneously, providing the opportunity to perceive all the effects arising from using GGS on multiple aspects of mathematics learning. This can help teachers identify their students' strengths and weaknesses during mathematics learning, which can lead to improved performance.

Methodological contribution. In previous research, the focus has primarily been on applying quantitative methods to investigate the effects of using GGS on students' learning, although some have employed a qualitative approach to do so. In contrast, this study contributes to the field by using a mixed method approach, seeking to combine students' results in mathematical proficiency tests and the mathematics' teachers views regarding the effect of using of GGS on the students' learning. In addition, using TIMSS items to test students' mathematical proficiency contributes to the field by employing a reliable and rigorous, internationally verified measure to lend greater confidence to the results. Also, the design of this study, especially the two interventions, is novel in the field. This contributes to the field by opening up the opportunity to investigate the effects of different factors repeatedly. The second round of intervention not only lends greater reliability and support to the quality of the results, but also ensures equality between the two groups study.

Knowledge contribution. The results reported in this study found the use of GGS helped improve students' performance in general based on measuring their achievement in two units (Numbers and Geometry). However, improvements were not found in all aspects of mathematics learning (understanding, fluency, problem solving, reasoning, and attitudes). Hence, these results clarify that it is important not just to concentrate on overall results as various aspects need to be considered independently. Therefore, this study contributes to the body of related research by cautioning that overall improved achievement does not necessarily indicate that the tool is beneficial across all aspects of mathematics learning.

In this study, comparing the results for the two units shows that the use of GGS was more effective for the Numbers unit than the Geometry unit, although its principal use relates to geometry content. This might be attributable to factors such as the duration of use of GGS in class and the similarity between GGS tasks and the traditional teaching approach in terms of applying pictures and diagrams. In contrast, in the Numbers unit, the traditional approach only employs numbers, whereas the GGS approach supports students with a variety of visual representations and dynamic features. Therefore, this study contributes to the field by indicating that the use of GGS could be more effective in some mathematics topic areas than others, and that its effectiveness may also be related to the length of use of GGS in the class and the type of content, as found in this study.

9.3 Impilications of the study

The results of this study and the instruments developed, including the PDC, the teachers' guide for the two units, and the three MPTs, may contribute to improve the teaching and learning of mathematics. This study may be useful to different stakeholders, specifically policymakers, researchers, and teachers in the field.

This study highlights the importance of technology integration and focusing on mathematical proficiency. According to the ACARA (2013), incorporating the five mathematical proficiency strands in the curriculum will ensure that student learning and independence remain at the forefront of teaching. Paying particular attention to the mathematical proficiency strands may enable students to deal with familiar and unfamiliar conditions by engaging mathematical approaches to choose the right operations and efficient solutions for mathematical problems. Therefore, policymakers may point to the importance of integration of GGS and the mathematical proficiency strands into teaching practices and curriculum development efforts when making calls to improve mathematics education in schools. Students need to understand mathematical concepts, compute fluently, be involved in analytical reasoning and communicate mathematically, as developing these skills is crucial for students preparing to challenge problems that arise in the classroom and in their lives. According to Loewenberg (2003), the new ambitions and expectations of this era mean that basic arithmetic skills are no longer sufficient for most adults. While number sense and computing proficiency are vital, other domains of mathematics skills and knowledge are becoming increasingly significant. Thus, new curricula should be developed with a focus on the mathematical proficiency strands. Also, technologies have become an important part of daily life, so policymakers should consider these tools to be part of the mathematics curriculum and call for opportunities for training so that mathematics teachers can improve their use of technology. A range of methods should be used to encourage teachers to apply technologies in their teaching and ensure equal learning opportunities for all students. This study designed a PDC which may help with the use of GGS in the teaching and learning of mathematics and thus implementing this course or something similar could be useful in the mathematics education field.

This study and its results and instruments may also be beneficial for researchers. For example, structuring assessments of performance on the mathematical proficiency strands through the three MPTs may help researchers investigate the effects of specific pedagogies or technology on students' mathematical proficiency. The three MPTs have a high level of credibility and validity. Because of these tests used TIMSS items, which is a measure that is considered to have good reliability and is based on international criteria, giving greater confidence about the results of this study. So, these ready MPTs may help researchers save effort and time when using them in their research. In addition, several aspects of the results of this study need to be followed up in further research, for example differences in the effects of the use of GGS at different levels of schooling. Also, the PDC in this study may help investigate the effects of dynamic software on teachers' knowledge and their practices in schools. Finally, researchers may use these study instruments and follow up on this research, observing how GGS changes students' learning in greater detail.

This study also provides many opportunities and benefits for mathematics teachers in terms of improving their practice. The teacher's guides and the structures of the lessons used in this study could be beneficial bases for them to improve their lessons. These tools can help teachers focus on the five strands of mathematics proficiency, assisting them in developing the critical habit of basing daily teaching decisions on reflections of their student's strengths and needs (Groth, 2017). Also, the PDC in this study may help to improve teachers' use of

GGS in class, not just in the two units used in this study but for mathematics content more widely. The mathematics teachers in this study mentioned various positive aspects of the use of GGS in their lessons. For example, GGS reduces the time and effort taken to communicate information and it affords a better representational capacity by presenting concepts in the form of models, using pictures and simulations, and connecting them with real life, thus enhancing the teaching and learning of concepts. So, the use of GGS in teaching and learning mathematics may help improve teachers' experiences of using technologies in the classroom.

9.4 Limitations of the study

This study contributes to the literature on the use of DGS by investigating the effectiveness of GGS in improving students' mathematical proficiency. Furthermore, it presents the views of mathematics teachers regarding the effects of using this software for teaching and learning. The primary limitation of this research relates to the number of participants in the study, namely investigating the effectiveness of GGS with two small groups of students. In addition, the study was conducted in only one school. Clearly, more reliable results could have been obtained had the study been conducted in more schools with larger groups of students. Also, the two student samples were not chosen randomly, which may make it difficult to generalize the results, generalization requiring random and laminated samples of significant size (William, 2005). The study is also limited to male students due to the gender segregation system in Saudi Arabia. Some studies have found that there are differences between males and females in terms of learning styles (Slater et al., 2007; Saadi, 2014). Thus, the results of this study are to some extent affected by difficulties of generalization.

Another key limitation is associated with the type of students who participated in the intervention study. This was conducted in public schools which charge no tuition fees. Lower socioeconomic status families are unable to afford the charges of private schools and this implies that the characteristics of private school students might differ from those in public education. In terms of students' mathematics performance and skills, there may also be differences, since there is a significant correlation between students' lower socioeconomic status and insufficient growth in learning (Bradley & Corwyn, 2002; Mullis et al., 2012).

Moreover, there were some limitations with regard to conducting the interviews with the mathematics teachers. The majority were unfamiliar with GGS. Thus, they may have been impressed by GGS and its features, and this might have affected their responses in the second and third rounds of interviews. In addition, if the participants had experience of GGS and had been using it for a long time, they might have had a clearer understanding of its uses in class and presented more information to their students. In this study, the participant teachers were trained for one week and then started to use GGS in their classes on returning to school. However, they possibly focused more on aspects of the use of GGS with which they were familiar and then reported positively regarding these features in the interviews.

Also, the teachers' written responses may have been affected by bias by virtue of being part of this study. The reason for this is that I trained them to use the GGS and gave them the "teachers' guide" to use in their lessons. Thus, when they responded to the written questions, they may have wished to present what they benefited from or had understood from the PDC and the teachers' guide. This does not necessarily mean that there was bias, but the possibility is presented here for the sake of transparency. Also, this way of collecting data from the participants may not be as useful as individual interviews, especially in terms of gaining in-depth data on using GGS. In this part of data collection, the mathematics teachers were sitting in groups, so their expression of their own personal ideas might have been affected by the ideas of others in the same group. Thus, they may not have expressed their honest and personal experiences about using GGS in their classes. Also, they might have been influenced by other teachers with more experience than them, or their ability to express their ideas clearly in the group.

9.5 Recommendations for future research

This study suggests there is excellent potential for learners to benefit when using GGS to teach mathematics to Grade 8 students. For example, it improves overall student achievement and enhances certain mathematical proficiency strands, such as fluency, as well as students' disposition towards mathematics. However, its effect on students' conceptual understanding was clear in the Numbers unit but not in the Geometry unit. Also, there were no effects on strategic competence or reasoning in either unit. Additional research is required to focus on all these strands within different school contexts simultaneously, over a more prolonged study period to establish how far this study's results can be supported or challenged. In addition, variables such as gender and socioeconomic background should be considered.

This study focused on the Numbers and Geometry topics and therefore there is scope for future studies focusing on different topic areas. These studies should not focus on one mathematics skill only, but need to consider overall mathematics achievement and the mathematics skills required to learn mathematics effectively. Such studies might also use the NRC’s mathematical proficiency strands as a framework, or alternative skills frameworks such as the TIMSS cognitive domains or the PISA assessment domains. In addition, the GGS to some extent covers all mathematics topic areas, so some studies comparing the effects of using GGS on different mathematics topics are vital to ascertain whether there are any disparities influencing its impact.

Finally, further research into teachers’ professional development enabling them to use GGS effectively in the classroom is needed. Moreover, a study investigating the effects of using GGS on students’ mathematical proficiency with mathematics teachers who are expert at using GGS is essential; this may bring out more details about its effects on the students' mathematical proficiency.

References

Abdurrahman, M. S., Abdullah, A. H., Osman, S., Hilmi, M. & Hamzah, B. (2020). Enhancing students' higher-order thinking skills (HOTS) through an inductive reasoning strategy using Geogebra. *International Journal of Emerging Technologies in Learning (IJET)*, 15(3), 156–179.

Abuhmaid, A. (2011). ICT Training Courses for Teacher Professional Development in Jordan. TOJET: The Turkish Online Journal of Educational Technology, 10(4), 195–210.

Adams, T. & LaFramenta, J. (2013). *Math hnow-how: Answers to your most persistent teaching issues, Grades 3-5*. CA: Corwin Press.

Ainsworth, S. (1991). The functions of multiple representations. *Computers & Education*, 33(2–3), 131–152.

Al-Enazy, A. & AL-Mosaad, A. (2018). The reality of the use of technology in teaching mathematics for the elementary stage in Arar City schools from the point of view of male and female teachers. *Arab Journalof Science and Research Publishing*, 23(2).

Al-rwaise, A. (2011). The reality of the use of technology in teaching mathematics from the viewpoint of its teachers for the intermediate stage in the Kingdom of Saudi Arabia. *Arabian Gulf Message*, 121(32), 15.

Al-Salloom, H. (1989). *The system and development of general education and its equivalent programmes in the Kingdom of Saudi Arabia*. Al-Mubta'ath , Saudi Arabian Cultural Mission.

AL-Shaalan, S. bint H. (2013). *Comparing the fifth grade math textbook with the original version (McGrawHill)*. Riyadh: College of Education, King Saud University.

AL-yamy, H. (2019). A future vision for the development of education in the Kingdom of Saudi Arabia in light of Vision 2030. *Arab Journalof Science and Research Publishing*, 26(2), 32–49.

Alanazi, M. (2016). *An investigation of developing teachers' understanding of using dialogic approach in Saudi primary mathematics classrooms* [unpublished doctoral thesis]. University of Manchester.

Albright, K., Gechter, K. & Kempe, A. (2013). Importance of mixed methods in pragmatic trials and dissemination and implementation research. *Academic Pediatrics*, 13(5), 400–407.

Alkhateeb, M. A. & Al-Duwairi, A. M. (2019). The effect of using mobile applications (GeoGebra and Sketchpad) on the students' achievement. *International Electronic Journal of Mathematics Education*, 14(3), 523–533.

Allsopp, D., Lovin, L. H. & van Ingen, S. (2017). Supporting mathematical proficiency. *TEACHING Exceptional Children*, 49(4), 273–283.

Alshayee, F. & Abalhameed, N. (2011). The development of the mathematics and science curricula project (hopes and challenges). *The 15th Scientific Conference*, 113–128.

Alshehri, O. A. (2021). *Examining the existing reality of using social media as e-learning tools at an Emerging University in Saudi Arabia from the viewpoint of tutors and students.* [Unpublished doctoral thesis]. University of Glasgow.

American Educational Research Association. (2019). *What is education research?* http://www.aera.net/About-AERA/What-is-Education-Research

Antohe, V. (2009). Limits of educational soft “GeoGebra” in a critical constructive review. *Annual Computer Science Series*, 47–54.

Arab Bureau of Education for the Gulf States (ABEGS). (2012). *Evaluation of the Gulf States math and science text book*. Riyadh: ABEGS Press.

Arbain, N. & Shukor, N. A. (2015). The effects of GeoGebra on students achievement. *Procedia - Social and Behavioral Sciences*, 172, 208–214.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7(3), 245–274.

Auletto, A., & Stein, K. C. (2019). Observable mathematical teaching expertise among upper elementary teachers: connections to student experiences and professional learning. Journal of Mathematics Teacher Education 2019 23:5, 23(5), 433–461.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2010). *Shape of the Australian Curriculum: Mathematics*.

Australian Curriculum, Assessment and Reporting Authority (ACARA). (2013). *Australian Curriculum: Mathematics proficiency strands*. https://www.acara.edu.au/curriculum/foundation-year-10/learning-areas-subjects/mathematics

Awofala, A. (2017). Assessing senior secondary school students’ mathematical proficiency as related to gender and performance in mathematics in Nigeria. *International Journal of Research in Education and Science*, 3(2), 488–502.

Aydos, M. (2015). *The ımpact of teaching mathematics with geogebra on the conceptual understanding of limits and contınuıty: The case of Turkish gifted and talented student*. [Unpublished doctoral thesis]. Bilkent University.

Baltaci, s., Yildiz, A., & Kosa, T. (2015). The potential of GeoGebra dynamic mathematics software in teaching analytic geometry: The opinion of pre-service mathematics teachers. Turkish Journal of Computer and Mathematics Education, 6(3), 483–505.

Bandura, A. & Ramachaudran, V. S. (1994). *Encyclopedia of human behavior* (4th ed.). NY: Academic Press.

Bandura, A. C. (1986). *Social foundations of thought and action.* NJ : Englewood Cliffs.

Bandura, A., Freeman, W. H. & Lightsey, R. (1999). *Self efficacy: Exercise of control*. NY: W. H. Freeman & Co.

Barkatsas, Tasos, A., Kasimatis, K. & Gialamas, V. (2009). Learning secondary mathematics with technology: Exploring the complex interrelationship between students’ attitudes, engagement, gender and achievement. *Computers & Education*, 52(3), 562–570.

Bayazit, I. (2011). Prospective teachers' inclination to single representation and their development of the function concept. *Educational Research and Reviews*, 6(5), 436–446.

Bayazit, I., & Aksoy, Y. (2007). *Connecting representations and mathematical ideas with the use of GeoGebra: The case of functions and equations.* [Unpublished doctoral thesis]. Erciyes University, Turkey.

Bazeley, P. (2009). Integrating data analyses in mixed methods research. *Journal of Mixed Methods Research*, *3*(3), 203–207.

Bazeley, P. (2018). *Integrating analyses in mixed methods research*. Thousands Oaks, CA: SAGEPublications Inc.

Belgheis, S. & Kamalludeen, R. (2018). The intention to use GeoGebra in the teaching of mathematics among Malaysian teachers. *Malaysian Online Journal of Educational Technology*, 6(1), 109–115.

Bergqvist, E. (2007). Types of reasoning required in university exams in mathematics. *Journal of Mathematical Behavior*, 26(4), 348–370.

Bieda, K., Ji, X., Drwencke, J. & Picard, A. (2014). Reasoning-and-proving opportunities in elementary mathematics textbooks. *International Journal of Educational Research*, 64, 71–80.

Bitter, G. & Pierson, M. (2002). *Using technology in the classroom*. Boston, MA: Allyn Bacon.

Bloom, B. (1956). *Bloom's taxonomy*. NY: McKay.

Boesen, J., Lithner, J. & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics*, 75(1), 89–105.

Booth, J., Lange, K., Koedinger, K. & Newton K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*, 25, 24–34.

Boyatzis, R. E. (1998). *Transforming qualitative information: Thematic analysis and code development*. London: SAGEPublications.

Brace, N., Snelgar, R. & Kemp, R. (2012). *SPSS for Psychologists* (5th ed.). NY: Macmillan International Higher Education.

Bradley, R. H. & Corwyn, R. F. (2002). Socioeconomic status and child development. *Annual Review of Psychology*, 53, 371–399.

Brahim, H. (2014). Developing education for sustainable development in the Kingdom of Saudi Arabia Saudi. *Risālat Al-Tarbiyah Wa-'Ilm Al-Nafs.*, 186(44), 1–32.

Braswell, J. S., Lutkus, A. D., Grigg, W. S., Santapau, S. L., Tay-Lim, B. T.-L. & Johnson, M. S. (2001). *The Nation's Report Card: Mathematics, 2000.*

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, 3(2), 77–101.

Braun, V. & Clarke, V. (2013). *Successful qualitative research: A practical guide for beginners*. London: SAGEPublications Ltd.

Brosvic, G. M., Dihoff, R. E., Epstein, M. L. & Cook, M. L. (2006). Feedback facilitates the acquisition and retention of numerical fact series by elementary school students with mathematics learning disabilities. *Psychological Record*, 56(1), 35–54.

Brownell, W. A. (2007). The progressive nature of learning mathematics. *The Mathematics Teacher –100 Years of Mathematics Teacher*, 100(26–34).

Bruckmaier, G., Krauss, S., Blum, W., & Leiss, D. (2016). Measuring mathematics teachers' professional competence by using video clips (COACTIV video). ZDM Mathematics Education, 48(1–2), 111–124.

Bu, L., Erhan, H. & Mumba, F. (2011). Mathematical problem solving in dynamic learning environments: The case of Geogebra. *The Second North American GeoGebra Conference, GeoGebra-NA 2011*, 20.

Bu, L., Mumba, F., Henson, H., & Wright, M. (2013). GeoGebra in professional development: The experience of rural inservice elementary school (K-8) teachers. *Mevlana International Journal of Education (MIJE)*, 3(3), 64–76.

Bwalya, D. (2019). Influence of Geogebra on students' achievement in geometric transformations and attitude towards learning mathematics with technology. *Leong & Noorbaizura*, 10(13).

Cachia, R., Ferrari, A., Ala-Mutka, K. & Punie, Y. (2010). *Creative learning and innovative teaching: Final report on the study on creativity and innovation in education in EU member states*. Seville

Cai, J. & Lester, F. K. (2005). Solution representations and pedagogical representations in Chinese and U.S. classrooms. *Journal of Mathematical Behavior*, 24(3–4), 221–237.

Cardinal, R. & Aitken, M. (2013). *ANOVA for the behavioral sciences researcher*. NY: Psychology Press.

Caulfield, L. & Hill, J. (2018). *Criminological skills and research for beginners: A student's guide* (2nd ed.). NY: Routledge.

Cemal Nat, M., Walker, S., Bacon, L., Dastbaz, M. & Flynn, R. (2011). *Impact of metacognitive awareness on learning in technology enhanced learning environments*. London.

Chang, M., Evans, M. A., Kim, S., Norton, A. & Samur, Y. (2015). Differential effects of learning games on mathematics proficiency. *Educational Media International*, 52(1), 47–57.

Cheng Meng, C. & Chap Sam, L. (2013). Developing pre-service teachers' technological pedagogical content knowledge for teaching mathematics with the Geometer's Sketchpad through lesson study. *Journal of Education and Learning*, 2(1), 1–8.

Cheung, A. C. & Slavin, R. E. (2013). The effectiveness of educational technology applications for enhancing mathematics achievement in K-12 classrooms: A meta-analysis. *Educational Research Review*, 9, 88–113.

Chinnappan, M. & Forrester, P. (2014). Generating procedural and conceptual knowledge of fractions by pre-service teachers. *Mathematics Education Research Journal*, 26(4), 871–896.

Chiu, T. K. F. & Churchill, D. (2016). Design of learning objects for concept learning: Effects of multimedia learning principles and an instructional approach. *Interactive Learning Environments*, 24(6), 1355–1370.

Chrysanthou, I. (2008). *The use of ICT in primary mathematics in Cyprus: The case of GeoGebra*. Cambridge: University of Cambridge.

Clarke, V. & Braun, V. (2013). Teaching thematic analysis: Overcoming challenges and developing strategies for effective learning. *The Psychologist*, 120–123.

Clements, D. H. (2000). From exercises and tasks to problems and projects: Unique contributions of computers to innovative mathematics education. *Journal of Mathematical Behavior*, 19(1), 9–47.

Coe, R. (2002). It's the effect size, Stupid. What effect size is and why it is important. *The British Educational Research Association Annual Conference*.

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences*. Hillsdale, NJ: L. Erlbaum Associates.

Collins, A. (2011). *Using classroom assessment to improve student learning: Math problems aligned with NCTM and Common Core State Standards.* Reston, VA: National Council of Teachers of Mathematics.

Common Core State Standards for Mathematics (CCSS). (2010). *Common Core State Standards for Mathematics.* Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State.

Constantinos C., Mousoulides, N., Pittalis, M. & Pitta-Pantazi, D. (2005). Problem solving and problem posing in a dynamic geometry environment. *The Montana Mathematics Enthusiast*, 2(2), 125–143.

Coolican, H. (2018). *Research methods and statistics in psychology* (6th ed.). NY: Psychology Press.

Cragg, L. & Gilmore, C. (2014). Skills underlying mathematics: The role of executive function in the development of mathematics proficiency. *Trends in Neuroscience and Education*, 3(2), 63–68.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1994). Conceptions of mathematics and how it is learned: The perspectives of students entering university. *Learning and Instruction*, 4(4), 331–345.

Cresswell, J. (2014). *Research design: Qualitative, quantitative, and mixed methods approaches* (4th ed.). Thousand Oaks, CA: SAGEPublications.

Creswell, J. (2002). *Educational research: Planning, conducting, and evaluating quantitative research*. Upper Saddle River, NJ: Prentice Hall.

Creswell, J. (2009). Mapping the field of mixed methods research. *Journal of Mixed Methods Research*, 3(2), 95–108.

Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). London: Pearson.

Creswell, J. W. & Plano Clark, V. L. (2018). *Designing and conducting mixed methods research* (3rd ed.). Thousand Oaks, CA: SAGEPublications, Inc.

D'Souza, S., & Wood, L. (2003). Secondary Students' Resistance Toward Incorporating Computer Technology into Mathematics Learning. Mathematics & Computer Education, 37(1), 284–293.

Darling-Hammond, L., Hyler, M. E., Gardner, M. (2017). (2017). Effective teacher professional development. Palo Alto, CA: Learning Policy Institute.

Das, K., Sheikh, B., Rahman, M., Das, K. R. & Rahmatullah Imon, A. H. M. (2016). A brief review of tests for normality. *American Journal of Theoretical and Applied Statistics*, 5(1), 5–12.

Dawson, C. (2019). *Introduction to research methods: A practical guide for anyone undertaking a research project*. Lodon: Little, Brown Book Group.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. Educational Researcher, 38(3), 181–199.

Dick, T. & Hollebrands, K. (2011). Focus in high school mathematics: Technology to support reasoning and sense making. In *Mathematical and Pedagogical Knowledge* (pp. xi–xvii). Reston, VA: National Council of Teachers of Mathematics.

Dijanić, Ž. & Trupčević, G. (2017). The impact of using GeoGebra interactive applets on conceptual and procedural knowledge. *The Sixth International Scientific Colloquium Mathematics and Children (Mathematics Education as a Science and a Profession)*, 1–15.

Diković, L. (2009). Applications GeoGebra into teaching some topics of mathematics at the college level. *Computer Science and Information Systems*, 6(2), 191–203.

Dogan, E. (2018). An application of the partial credit IRT model in identifying benchmarks for polytomous rating scale instruments. *Practical Assessment, Research, and Evaluation*, 23(7), 1–10.

Dogan, M. & Icel, R. (2010). The role of dynamic geometry software in the process of learning: GeoGebra example about triangles. *Journal of Human Sciences*, 8(1), 1441–1458.

Doktoroğlu, R. (2013). *The effects of teaching linear equations with dynamic mathematics software on seventh grade students' achievement.* Ankara: Middle East Technical University.

Doruk, B. K., Aktümen, M., Aytekin, M. & Cahit, A. (2013). Pre-service elementary mathematics teachers' opinions about using GeoGebra in mathematics education with reference to "teaching practices." *Teaching Mathematics and Its Applications*, 32, 140–157.

Dreher, A., Kuntze, S. & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *International Journal of Science and Mathematics Education*, 14(2), 363–382.

Duncan, A. G. (2010). Teachers' views on dynamically linked multiple representations, pedagogical practices and students' understanding of mathematics using TI-Nspire in Scottish secondary schools. *ZDM Mathematics Education*, 42(7), 763–774.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61(1–2), 103–131.

El Asame, M. & Wakrim, M. (2018). Towards a competency model: A review of the literature and the competency standards. *Education and Information Technologies*, 23(1), 225–236.

Elia, I., Panaoura, A., Eracleous, A. & Gagatsis, A. (2007). Relations between secondary pupils' conceptions about functions and problem solving in different representations. *International Journal of Science and Mathematics Education*, 5(3), 533–556.

Elia, I., Panaoura, A., Gagatsis, A., Gravvani, K. & Spyrou, P. (2008). Exploring different aspects of the understanding of function: Toward a four-facet model. *Canadian Journal of Science, Mathematics and Technology Education*, 8(1), 49–69.

Emaikwu, S. O., Iji, C.t O. & Abari, M. T. (2015). Effect of Geogebra on senior secondary school students' interest and achievement in statistics in Makurdi Local Government Area of Benue State, Nigeria. *IOSR Journal of Mathematics*, 11(3), 14–21.

Erkek, Ö. & Işıksal-Bostan, M. (2015). Is the use of GeoGebra advantageous in the process of argumentation? *CERME 9 – Ninth Congress of the European Society for Research in Mathematics Education*, 121–127.

Ertmer, P. A. (2005). Teacher pedagogical beliefs: The final frontier in our quest for technology integration? Educational Technology Research and Development, 53(4), 25–39.

Ertmer, P. A., Ottenbreit-Leftwich, A. T. ., Sadik, O., Sendurur, E., & and Sendurur, P. (2012). Teacher beliefs and technology integration practices: A critical relationship. Computers & Education, 59(2), 423–435.

Evans, D. (2007). Developing mathematical proficiency in the Australian context: Implications for students with learning difficulties. *Journal of Learning Disabilities*, 40(5), 420–426.

Excellence Research Center of Science and Mathematics Education (ECSME). (2020). *Excellence Research Center of Science and Mathematics Education*. https://ecsme.ksu.edu.sa/ar/node/1034

Fabian, K., Topping, K. J. & Barron, I. G. (2016). Mobile technology and mathematics: effects on students' attitudes, engagement, and achievement. *Journal of Computers in Education*, 3(1), 77–104.

Fast, L. A., Lewis, J., Bryant, M. J., Bocian, K. A., Cardullo, R. A., Rettig, M. & Hammond, K. A. (2015). Does math self-efficacy mediate the effect of the perceived classroom environment on standardized math test performance? *Journal of Educational Psychology*, 12(3), 729.

Ferguson, D. (2001). Technology in a Constructivist Classroom. Information Technology in Childhood Education Annual, 2001(1), 45–55.

Fest, A. (2010). Creating interactive user feedback in DGS using scripting interfaces. *Acta Didactica Napocensia*, 3(2), 79–88.

Field, A. (2013). *Discovering statistics using IBM SPSS statistics* (4th ed.). London: SAGEPublications Ltd.

Fishman, B., & Davis, E. (2006). Teacher learning research and the learning sciences. In R. K. Sawyer (Ed.), The Cambridge handbook of the learning sciences . (pp. 535–555). Cambridge: Cambridge University Press.

Fontana, A. & Frey, J. H. (2000). Interviewing: The art of science. Denzin & Lincoln (Eds.), Thousand Oaks, CA: Sage.

Frey, B. (2018). *The SAGEencyclopedia of educational research, measurement, and evaluation*. Thousand Oaks, CA: SAGEPublications, Inc.

Furner, J. M. & Marinas, C. A. (2007). Geometry sketching software for elementary children: Easy as 1, 2, 3. *Eurasia Journal of Mathematics, Science & Technology Education*, 3(1), 83–91.

Furner, J. M. & Marinas, C. A. (2013). Learning math concepts in your environment using photography and GeoGebra. *Electronic Proceedings of the Twenty-Fifth Annual International Conference on Technology in Collegiate Mathematics Boston, Massachusetts*, 209–2018.

Gafoor, A. & Sarabi, M. (2016). Influence of student beliefs about nature of school mathematics on their achievement at secondary kevel in Kerala. *Online Submission*, 4(2), 536–548.

Galbraith, P. & Haines, C. (2000). Conceptual mis(understandings) of beginning undergraduates. *International Journal of Mathematical Education in Science and Technology*, 31(5), 651–678.

Gau Bartell, T., Webel, C., Bowen, B. & Dyson, N. (2013). Prospective teacher learning: Recognizing evidence of conceptual understanding. *Journal of Mathematics Teacher Education*, 16(1), 57–79.

Geary, D. C. (1995). Reflections of evolution and culture in children’s cognition: Implications for mathematical development and instruction. *American Psychologist*, 50(1), 24–37.

Gee, D., & Whaley, J. (2016). Learning Together: Practice-Centred Professional Development to Enhance Mathematics Instruction. Mathematics Teacher Education and Development, 18(1), 87–99.

General Authority for Statistics (GASTAT). (2019). *Population by age groups, and gender*. Riyadh.

Geogebra.org. (2020). *About GeoGebra – GeoGebra*. https://www.geogebra.org/about

Gerald, B. (2018). A brief review of independent, dependent and one sample t-test. *International Journal of Applied Mathematics and Theoretical Physics*, 4(2), 50–54.

Ghasemi, A. & Zahediasl, S. (2012). Normality tests for statistical analysis: A guide for non-statisticians. *International Journal of Endocrinology and Metabolism*, 10(2), 486–489.

Gilakjani, A. P., & Branch, L. (2017). A Review of the Literature on the Integration of Technology into the Learning and Teaching of English Language Skills. International Journal of English Linguistics, 7(5). 95-106.

Glassett, K. & Schrum, L. (2009). Teacher beliefs and student achievement in technology-rich classroom environments. *International Journal of Technology in Teaching and Learning*, 5(2), 138–153.

Glaubke, C. R. (2007). *The effects of interactive media on preschoolers' learning: A review of the research and recommendations for the future*. Oakland, CA: Children Now.

Graven, M., Hewana, D. & Stott, D. (2013). The evolution of an instrument for researching young mathematical dispositions. *African Journal of Research in Mathematics, Science and Technology Education*, 17(1–2), 26–37.

Gravetter, F. & Forzano, L. (2018). *Research methods for the behavioral sciences*. Wadsworth: Cengage Learning

Greenes, C. (2014). Tasks to advance the learning of mathematics. *Journal of Mathematics Education at Teachers College*, 5(1), 1–7.

Gribbons, B. & Herman, J. (1997). True and quasi-experimental designs. *Practical Assessment, Research, and Evaluation*, 5(1), 14.

Groth, R. E. (2017). Classroom data analysis with the five strands of mathematical proficiency. *A Journal of Educational Strategies, Issues and Ideas*, 90(3), 103–109.

Groves, S. (2012). Developing mathematical proficiency. *Journal of Science and Mathematics Education in Southeast Asia*, 35(2), 119–145.

Gunčaga, J. & Majherová, J. (2012). GeoGebra as a motivational tool for teaching and learning in Slovakia. *North American GeoGebra Journal*, 1(1), 2162–3856.

Gutiérrez, A., Laborde, C., Noss, R. & Rakov, S. (1999). Tools and technologies. *European Research in Mathematics Education*, 1(3), 183–188.

Gutnick, A. L., Bernstein, L. & Levine, M. H. (2011). *Always connected:The new digital media habits of young children*. Joan Ganz Cooney Center at Sesame Workshop.

Güven, B. & Kosa, T. (2008). The effect of dynamic geometry software on student mathematics teachers' spatial visualization skills. *The Turkish Online Journal of Educational Technology – TOJET*, 7(4), 100–107.

Haciomeroglu, E., Bu, L., Schoen, R. & Hohenwarter, M. (2009). Learning to develop mathematics lessons with GeoGebra… *MSOR Connections*, 9(2), 24–26.

Haines, C., Galbraith, P., Blum, W. & Khan, S. (2007). *Mathematical modelling: education, engineering and economics – ICTMA 12*. Chichester: Horwood Publishing.

Hardesty, D. M. & Bearden, W. O. (2004). The use of expert judges in scale development: Implications for improving face validity of measures of unobservable constructs. *Journal of Business Research*, 57(2), 98–105.

Harris, A. D., McGregor, J. C., Perencevich, E. N., Furuno, J. P., Zhu, J., Peterson, D. E. & Finkelstein, J. (2006). The use and interpretation of quasi-experimental studies in medical informatics. *Journal of the American Medical Informatics Association*, 13(1), 16–23.

Heid, M. K. (1997). The technological revolution and the reform of school mathematics. *American Journal of Education*, 106(1), 5–61.

Herbert, S. (2014). A framework for teachers' knowledge of mathematical reasoning. *Mathematics Education Research Group of Australasia*, 702–705.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3–19.

Hiebert, J. & Lefevre, H. (1986). *Conceptual and procedural knowledge: The case of mathematics.* Hillsdale, NJ: Lawrence Erlbaum Associates

Higgins, K., Huscroft-D'angelo, J. & Crawford, L. (2019). Effects of technology in mathematics on achievement, motivation, and attitude: A meta-analysis. *Journal of Educational Computing Research*, 57(2), 283–319.

Hinton, V., Strozier, S. D. & Flores, M. M. (2014). Building mathematical fluency for students with disabilities or students at-risk for mathematics failure. *International Journal of Education in Mathematics*, 2(4), 257–265.

Hohenwarter, J., Hohenwarter, M. & Lavicza, Z. (2008). Introducing dynamic mathematics software to secondary school teachers: The case of GeoGebra. *Journal of Computers in Mathematics and Science Teaching*, 18(2), 135–146.

Hohenwarter, M. & Fuchs, K. (2004). Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. *Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference*, 1–6.

Hohenwarter, M. & Jones, K. (2007). Ways of linking geometry and algebra, the case of Geogebra. *Proceedings of the British Society for Research into Learning Mathematics*, 126–131.

Hohenwarter, M. & Lavicza, Z. (2007). Mathematics teacher development with ICT: towards an International GeoGebra Institute. *Proceedings of the British Society for Research Into Learning Mathematics*, 49–54.

Hohenwarter, M. & Preiner, J. (2007). Dynamic mathematics with GeoGebra. *Journal of Online Mathematics and Its Applications*, 1, 1488.

Hohenwarter, M., Hohenwarter, J., Kreis, Y. & Lu, Y. (2008). Teaching and learning calculus with free dynamic mathematics software GeoGebra. *11th International Congress on Mathematical Education.*

Hohenwarter, M., Jarvis, D. & Lavicza, Z. (2009). Linking geometry, algebra, and mathematics teachers: GeoGebra Software and the establishment of the International GeoGebra Institute. *International Journal for Technology in Mathematics Education*, 16(2), 83–86.

Hollebrands, K. F. (2007). The role of a dynamic software program for geometry in the strategies high school mathematics students employ. *Journal for Research in*

Mathematics Education, 38(2), 164–192.

Holloway, I. & Todres, L. (2003). The status of method: Flexibility, consistency and coherence. *Qualitative Research*, 3(3), 345–357.

Horzum, T. & Ünlü, M. (2017). Pre-service mathematics teachers' views about GeoGebra and its use. *Acta Didactica Napocensia*, 10(3), 77–90.

Howell, D. (2012). *Statistical methods for psychology* (8th ed.). Belmont, CA: Cengage Learning.

Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? In Bishop A.J., Clements M.A., Keitel C., Kilpatrick J., Leung F.K.S. (Eds.), *Second international handbook of mathematics education* (pp. 323–349). Dordrecht: Springer Netherlands.

Hughes, J. (2005). The role of teacher knowledge and learning experiences in forming technology-integrated pedagogy. Journal of Technology and Teacher Education, 13(2), 277–302.

Hung, W. (2006). The 3C3R Model: A conceptual framework for designing problems in PBL. *Interdisciplinary Journal of Problem-Based Learning*, 1(1), 54–76.

Hur, J., Shannon, D., & T S Wolf. (2016). An investigation of relationships between internal and external factors affecting technology integration in classrooms. Ournal of Digital Learning in Teacher Education, 32(3), 105–114.

Huscroft D. J., Higgins, K. & Crawford, L. (2019). Technology, attitude and mathematics: a descriptive examination of the literature spanning three decades. *International Journal of Technology Enhanced Learning*, 11(1), 36–61.

Hutkemri, E. Z. & Nordin, N. M. (2011). Employing Geogebra in the teaching of mathematics: The teachers' perspective. *Elixir Psychology*, 36, 3483–3485.

Hutkemri, H. & Zakaria, E. (2012). The effect of geogebra on students' conceptual and procedural knowledge of function. *Indian Journal of Science and Technology*, 5(12), 104–110.

Hutkemri, Z. E. (2014). Impact of using GeoGebra on students' conceptual and procedural knowledge of limit function. *Mediterranean Journal of Social Sciences*, 5(23), 873–881.

Hwang, W., Chen, N., Dung, J. & Yang, Y. (2007). Multiple representation skills and creativity effects on mathematical problem solving using a multimedia whiteboard system. *Journal of Educational Technology & Society*, 10(2), 191–212.

Jacinto, H. & Carreira, S. (2017). Mathematical problem solving with technology: The techno-mathematical fluency of a student-with-GeoGebra. *International Journal of Science and Mathematics Education*, 15(6), 1115–1136.

Jackson, S. (2015). *Research methods and statistics: A critical thinking approach* (5th ed.). Belmont, CA: Cengage Learning.

Jackson, T., Kehayes, J., Li, J., Perkins, D. & Stewart, V. (2012). *Teaching and learning 21st century skills: Lessons from the learning sciences*. New York.

Jatiariska, I. G. A., Sariyasa & Astawa, I. W. (2019). The influence of Knisley Mathematical Learning Model with Geogebra towards mathematical connection and mathematical disposition. *Journal of Physics: Conference Series*, 1503(1), 12013.

Jelatu, S. & Ganesha, S. (2018). Effect of GeoGebra-Aided REACT Strategy on understanding of geometry concepts. *International Journal of Instruction*, 11(4), 325–336.

Johnson, B, & Christensen, L. (2008). *Educational research: Quantitative, qualitative, and mixed approaches*(3rd Ed.). LA : SAGEPublications.

Johnson, B, & Christensen, L. (2014). *Educational research: Quantitative, qualitative, and mixed approaches* (5th ed.). London: SAGEPublications.

Johnson, D. A. & Rubin, S. (2011). Effectiveness of interactive computer-based instruction: A review of studies published between 1995 and 2007. *Journal of Organizational Behavior Management*, 31(1), 55–94.

Johnson, R. B., Onwuegbuzie, A. J. & Turner, L. A. (2007). Toward a definition of mixed methods research. *Journal of Mixed Methods Research,* 1(2), 112–133.

Jonker, J. & Pennink, B. (2010). *The essence of research methodology: A concise guide for Master and PhD students in management science*. Heidelberg: Springer.

Karasavvidis, I. & Kollias, V. (2014). Technology integration in the most favorable conditions: Findings from a professional development training program. In *Research on e-Learning and ICT in Education* (pp. 197–244). NY: Springer.

Kargar, M., Tarmizi, R. A. & Bayat, S. (2010). Relationship between mathematical thinking, mathematics anxiety and mathematics attitudes among university students. *Procedia Social and Behavioral Sciences*, 8, 537–542.

Kelley, K. & Preacher, K. (2012). On effect size estimation. *Psychological Methods*, 17(2), 137–152.

Khairani, A. Z. & Nordin, M. S. (2011). The development and construct validation of the mathematics proficiency test for 14-year-old students. *Asia Pacific Journal of Educators and Education*, 26(1), 33–50.

Khalil, M., Sultana, N. & Khalil, U. (2017). Exploration of mathematical thinking and its development through Geogebra. *Journal of Educational Research*, 20(1), 83–99.

Kilpatrick, J. (2011). Slouching toward a national curriculum. *Journal of Mathematics Education at Teachers College*, 2(1), 8–17.

Kilpatrick, J. (2014). Competency frameworks in mathematics education. In S. Lerman (Eds.), *Encyclopedia of mathematics education* (pp. 85–87). Dordrecht: Springer Netherlands.

Kilpatrick, J., Swafford, J. & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Kim, K. M. & Md-Ali, R. (2017). GeoGebra: Towards realizing 21st century learning in mathematics education. *Malaysian Journal of Learning and Instruction: Special Issues*, 2017, 93–115.

Kimberlin, C. L., Winterstein, G. & Winterstein, A. (2008). Validity and reliability of measurement instruments used in research. *American Journal of Health-System Pharmacy*, 65(23), 2276–2284.

Kinnari, H. (2010). A study of the mathematics proficiency. *In 1st International Workshop on Maths and ICT: Education, Research and Applications*, 35–39.

Kösa, T. & Karakuş, F. (2010). Using dynamic geometry software Cabri 3D for teaching analytic geometry. *Procedia – Social and Behavioral Sciences*, 2(2), 1385–1389.

Kramarski, B. & Zeichner, O. (2001). Educational media international using technology to enhance mathematical reasoning: Effects of feedback and self-regulation learning. *Educational Media International*, 38(2), 77–82.

Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into Practice*, 41(4), 212–218.

Kul, U. (2018). Influences of technology integrated professional development course on mathematics teachers *. European Journal of Educational Research, 7(2), 233–243.

Kustiawati, D., Kusumah, Y. S. & Herman, T. (2019). Using of GeoGebra to improve mathematical reasoning with the problem-solving method. *First International Conference on Technology and Educational Science.*

Kutluca, T. (2013). The effect of geometry instruction with dynamic geometry software; GeoGebra on Van Hiele geometry understanding levels of students. *Educational Research and Reviews*, 8(17), 1509–1518.

Laborde, C. (2007). The role and uses of technologies in mathematics classrooms: Between challenge and modus vivendi. *Canadian Journal of Science, Mathematics and Technology Education*, 7(1), 68–92.

Lamon, S. J. (2020). *Teaching fractions and ratios for understanding: Essential content knowledge* (4th ed.). NY: Routledge.

Leinwand, S. (2014). *Principles to actions: Ensuring mathematical success for all.* National Council of Teachers of Mathematics, Incorporated.

Limjap, A. A. (2002). *Issues on problem solving: Drawing implications for a techno-mathematics curriculum at the collegiate level.* Tanglaw

Ling Ling Chua, G., Amilin Tengah, K., Shahrill, M., Tan, A. & Leong, E. (2017). Analysing students' perspectives on geometry learning from the combination of Van Hiele phase-based instructions and Geogebra. *Proceeding of the 3rd International Conference on Education*, 3, 205–213.

Liu, X. & Koirala, H. (2009). The effect of mathematics self-efficacy on mathematics achievement of high school students. *NERA Conference Proceedings*, 30.

Loewenberg, D. (2003). *Mathematical proficiency for all students: Toward a strategic research and development program in mathematics education*. Santa Monica, CA: Rand Education.

Loong, E. (2014). A primary teacher's developing understanding of mathematical reasoning. *Curriculum in Focus: Research Guided Practice: Proceedings of the Mathematics*

Education Research Group of Australasia 2014 Annual Conference, 706–709.

MacFarland, T. W., & Yates, J. M. (2016). Mann–Whitney U Test. In *Introduction to nonparametric statistics for the biological sciences using R* (pp. 103–132). Cham: Springer International Publishing.

Mackenzie, S. & Knipe, N. (2006). Research dilemmas: Paradigms, methods and methodology. *Issues In Educational Research,* 16(2), 193–205.

Maguire, M. & Delahunt, B. (2017). Doing a thematic analysis: A practical, step-by-step guide for learning and teaching scholars. *All Ireland Journal of Higher Education*, 9(3), 3351-33514.

Majewski, M. (1999). Pitfalls and benefits of the use of technology in teaching mathematics. *The Asian Technology Conference in Mathematics*, 52–59.

Marrades, R. & Gutiérrez, Á. (2000). Proofs produced by secondary school students learning geometry in a dynamic computer environment. *Educational Studies in Mathematics*, 44(1–3), 87–125.

Martín-Caraballo, A. M. & Tenorio-Villalón, Á. F. (2015). Teaching numerical methods for non-linear equations with Geogebra-based activities. *International Electronic Journal of Mathematics Education*, 10(2), 53–56.

Martin, M. O., Mullis, I. V. S., Foy, P., & Hooper, M. (2016). *TIMSS 2015 International Results in Science*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/international-results/

Martin, M. O., Mullis, I. V. S., Hooper, M., Yin, L., Foy, P., & Palazzo, L. (2016). *Creating and Interpreting the TIMSS 2015 Context Questionnaire Scales*. In M. O. Martin, I. V. S. Mullis, & M. Hooper (Eds.), *Methods and Procedures in TIMSS 2015* (pp. 15.1-15.312). Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timss.bc.edu/publications/timss/2015-methods/chapter-15.html

Masitoh, L. F. & Fitriyani, H. (2018). Improving students' mathematics self-efficacy through problem based learning. *Malikussaleh Journal of Mathematics Learning (MJML)*, 1(1), 2620–6323.

Mata-Pereira, J. & da Ponte, J. P. (2017). Enhancing students' mathematical reasoning in the classroom: Teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, 96(2), 169–186.

May, D. (2009). *Mathematics self-efficacy and anxiety questionnaire* [Unpublished doctoral thesis]. University of Georgia.

McCarrick, K. & Li, X. (2007). Buried treasure: The impact of computer use on young children's social, cognitive, language development and motivation. *AACE Journal*, 15(1), 73–95.

McCrum-Gardner, E. (2008). Which is the correct statistical test to use? *British Journal of Oral and Maxillofacial Surgery*, 46(1), 38–41.

McDaniel, M. A., Roediger, H. L. & McDermott, K. B. (2007). Generalizing test-enhanced learning from the laboratory to the classroom. *Psychonomic Bulletin and Review*, 14(2), 200–206.

McDonald, P. A. (2017). *A study of Scottish teachers' beliefs about the interplay of problem solving and problem posing in mathematics education*. [Unpublished doctoral thesis]. University of Glasgow.

McDonough, I. K. & Tra, C. I. (2017). The impact of computer-based tutorials on high school math proficiency. *Empirical Economics*, 52(3), 1041–1063.

McGee, D. L. & Moore-Russo, D. (2015). Impact of explicit presentation of slopes in three dimensions on students' understanding of derivatives in multivariable calculus. *International Journal of Science and Mathematics Education*, 13(2), 357–384.

McKenna, C. (2012). There's an App for that: How two elementary classrooms used iPads to enhance student learning and achievement. *Education*, 2012(5), 136–142.

McLeod, S. (2019). What does effect size tell you? *Simplypsychology.org/effect-size*. https://www.simplypsychology.org/effect-size.html

McMillan, J. (2011). *Educational research: Fundamentals for the consumer* (6th ed.). Boston, MA: Pearson/Allyn and Bacon.

McMillan, J.H. & Schumacher, S, J. H. (2010). *Research in education: Evidence-based inquiry*. Boston, MA: Pearson.

Mehanovic, S. (2011). *The potential and challenges of the use of dynamic software in upper secondary mathematics: Students' and teachers' work with integrals in GeoGebra based environments*. Linköping: Linköping University Electronic Press.

Mehdiyev, R. (2009). *Exploring students' learning experiences when using a dynamic geometry software (DGS) tool in a geometry class at a secondary school in Azerbaijan* [Unpublished doctoral thesis]. Universiteit van Amsterdam.

Meho, L. I. (2006). E-mail interviewing in qualitative research: A methodological discussion. *Journal of the American Society for Information Science and Technology*, 57(10), 1284–1295.

Meira, L. (1998). Making sense of instructional devices: The emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education*, 29(2), 121–142.

Meyen, E., Poggio, J., Seok, S. & Smith, S. (2006). Equity for students with high-incidence disabilities in statewide assessments: A technology-based solution. *Focus on Exceptional Children*, 38(7), 1–8.

Mills, J. (2019). Making multiplication meaningful: Teaching for conceptual understanding. *Teachers and Curriculum*, 19(1), 17–25.

Ministry of Foreign Affairs. (2017). *About Saudi Arabia | Kingdom of Saudi Arabia*. Riyadh: Ministry of Foreign Affairs. https://www.mofa.gov.sa/sites/mofaen/aboutKingDom/Pages/KingdomGeography46466.aspx

Ministry of Education (MoE) (2006). *The Development of the Mathematics and Science Curricula Project*. Riyadh: Ministry of Education.

Ministry of Education (MoE) (2020a). *Ministry of Education*. Riyadh: Ministry of Education https://www.moe.gov.sa/en/pages/default.aspx

Ministry of Education (MoE) (2020b). *Future Gate*. Riyadh: Ministry of Education. https://fg.moe.gov.sa/faq

Ministry of Education in Singapore (MoE) (2007). Primary mathematics syllabus. Singapore: Author

Ministry of Finance (2020). *Pre-budget statement FY 2020 "Spending Efficiency. .. Private Sector Empowerment"*. Riyadh: Ministry of Finance.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. Teachers College Record, 108(6), 1017–1054.

Mistretta, R. (2005). Integrating technology into the mathematics classroom: The role of teacher preparation programs. *The Mathematics Educator*, 15(1), 18–24.

Mo, S. (2011). Evidence on instructional technology and student engagement in an auditing course. *Academy of Educational Leadership Journal*, 15(4), 149.

Mueller, J., Wood, E., Willoughby, T., Ross, C., & Specht, J. (2008). Identifying discriminating variables between teachers who fully integrate computers and teachers with limited integration. Computers & Education, 51(4), 1523–1537.

Muhammad, M., Purwanto, J. & Prisnaini, R. M. (2017). Improving students' spatial skill with learning cycle using Cabri 3D application in junior high school. *4th Asia Pacific Education Conference (AECON 2017)*, 328–332.

Mukama, E., & Andersson, S. B. (2008). Coping with change in ICT-based learning environments: newly qualified Rwandan teachers' reflections. Journal of Computer Assisted Learning, 24(2), 156–166.

Mullis, I.V., Martin, M.O., Foy, P. and Arora, A. (2012). *TIMSS 2011 international results in mathematics*. International Association for the Evaluation of Educational Achievement. Amsterdam.

Murni, V., Sariyasa, S. & Ardana, I. M. (2017). GeoGebra assist discovery learning model for problem solving ability and attitude toward mathematics. *Journal of Physics: Conference Series*, 895(1), 012049.

Najlaa. (2019). *Research about the Education Development in Saudi Arabia*. https://www.almrsal.com/post/774177

National Assessment Governing Board (NAGB). (1994). *Mathematics framework for the 1996 and 2000 National Assessment of Educational Progress*.

National Council of Teachers of Mathematic (NCTM). (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*.

National Council of Teachers of Mathematics (NCTM). (1989). *Principles and standards for school mathematics*. National Council for Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and Standards for School Mathematics*. National Council for Teachers of Mathematics.

National Council of Teachers of Mathematics (NCTM). (2009). *Focus in high school mathematics reasoning and sense making*. NCTM.

National Council of Teachers of Mathematics (NCTM). (2014). *Procedural fluency in mathematics*.

National Council of Teachers of Mathematics (NCTM). (2015). *Strategic use of technology in teaching and learning mathematics*.

National Council of Teachers of Mathematics (NCTM). (n.d.). *Executive Summary Principles and Standards for School Mathematics Overview*.

National Governors Association (2010). *Common core state standards.* Washington, DC.

Newton, N. (2010). *Developing adaptive reasoning in the math classroom: Part I | Dr. Nicki's Guided Math Blog*. https://guidedmath.wordpress.com/2010/09/28/developing-adaptive-reasoning-the-math-classroom-part-i/

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C. & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly*, 36, 550–560.

Nhamo Gono, E. (2016). *The contributions of interactive dynamic mathematics software in probing understanding of mathematical concepts: Case study on the use GeoGebra in learning the concept of modulus functions.* [Unpublished doctoral thesis]. University College London.

Nikolopoulou, K., & Gialamas, V. (2015). Barriers to the integration of computers in early childhood settings: Teachers' perceptions. Education and Information Technologies, 20(2), 285–301.

Niss, M., Bruder, R., Planas, N., Turner, R. & Villa-Ochoa, J. A. (2017). Conceptualisation of the role of competencies, knowing and knowledge in mathematics education research. *Proceedings of the 13th International Congress on Mathematical Education*, 235–248.

Nurlaelah, E. (2018). Improving junior high school students' spatial reasoning ability through model eliciting activities with Cabri 3D. *International Education Studies*, 11(1), 148–154.

Nussbaum, M. (2007). *Games, learning, and cognitive divide games, learning,collaboration and cognitive divide*. Escuela de Ingeniería

O'Cathain, A., Murphy, E. & Nicholl, N. (2010). Three techniques for integrating data in mixed methods studies. *British Medical Journal*, 341(7781), 1147–1150.

O'Donnell, A. (2006). *Introduction: Learning with technology.* Mahwah, NJ: Lawrence Erlbaum Associates.

O'Leary, Z. (2004). *The essential guide to doing research*. London: Sage.

Ocal, M. F. (2017). The effect of Geogebra on students' conceptual and procedural knowledge: The case of applications of derivative. *Higher Education Studies*, 7(2), 67–78.

Olayan, N. (2019). Use of modern technology in the educational process. *Journal of the*

College of Basic Education for Educational and Human Sciences, 42, 271–288.

Oldknow, A., Taylor, R. & Tetlow, L. (2010). *Teaching mathematics using ICT* (3rd ed.). London: Continuum.

Olivero, F. & Robutti, O. (2007). Measuring in dynamic geometry environments as a tool for conjecturing and proving. *International Journal of Computers for Mathematical Learning*, 12(2), 135–156.

Olsson, J. (2015). Students reasoning and utilization of feedback from software. *In 20th International Conference on Mathematical Views (MAVI-20)*, 139–150.

Olsson, J. (2017). *GeoGebra, enhancing creative mathematical reasoning* [Unpublished doctoral thesis]. Umeå universitet.

Ostler, E. (2011). Teaching adaptive and strategic reasoning through formula derivation: beyond formal semiotics. *International Journal of Mathematics Science Education Technomathematics Research Foundation*, 4(2), 16–26.

Oyaid, A. (2009). *Education policy in Saudi Arabia and its relation to secondary school teachers' ICT use, perceptions, and views of the future of ICT in education* [Unpublished doctoral thesis]. University of Exeter.

Özgün-Koca, S. (2001). *Computer-based representations in mathematics classrooms: The effects of multiple linked and semi-linked representations on students' learning of linear relationships* [Unpublished doctoral thesis]. The Ohio State University.

Ozkal, N. (2019). Relationships between self-efficacy beliefs, engagement and academic performance in math lessons. *Cypriot Journal of Educational Sciences*, 14(2), 190–200.

Pallant, J. (2016). *SPSS survival manual: A step by step guide to data analysis using IBM SPSS* (6th ed.). Berkshire: Open University Press.

Philipp, R. A. & Siegfried, J. M. (2015). Studying productive disposition: The early development of a construct. *Math Teacher Education*, 18(5), 489–499.

Pierce, R, Stacey, K. & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education*, 48(2), 285–300.

Pierce, R., Stacey, K., Wander, R. & Ball, L. (2011). The design of lessons using mathematics analysis software to support multiple representations in secondary school mathematics. *Technology, Pedagogy and Education*, 20(1), 95–112.

Pilli, O. & Aksu, M. (2013). The effects of computer-assisted instruction on the achievement, attitudes and retention of fourth grade mathematics students in North Cyprus. *Computers & Education*, 62, 62–71.

Pitts, V. (2004). *Representations of functions: An examination of pre-service mathematics teachers' knowledge of translations between algebraic and graphical representations* [Unpublished doctoral thesis]. University of Pittsburg.

Polya, G. (1981). *Mathematical discovery.* NY: Wiley.

Powney, J. & Watts, M. J. (1984). Reporting interviews: A code of good practice. *Research Intelligence*, 17, 164-168.

Preiner, J. (2008). *Introducing dynamic mathematics software to mathematics teachers: The case of GeoGebra* [Unpublished doctoral thesis]. University of Salzburg.

Prodromou, T. (2014). GeoGebra in Teaching and Learning Introductory Statistics. *Electronic Journal of Mathematics & Technology 8*, *8*(5), 363–376.

Punch, K. F. (1998). *Introduction to social research: Quantitative and qualitative approaches*. London: SAGEPublications.

Radović, S., Radojičić, M., Veljković, K. & Marić, M. (2018). Interactive learning environments examining the effects of Geogebra applets on mathematics learning using interactive mathematics textbook. *Interactive Learning Environments*, 28(1), 32–49.

Rafea, A. & Al-oeshiq, N. (2010). *The development of the mathematics and science curricula project – Adapting and translating an international series*. School of Eduaction, King Saud university, Riyadh.

Ramadhani, R. & Dahlya Narpila, S. (2018). Problem based learning method with Geogebra in mathematical learning. *International Journal of Engineering & Technology*, 7(2).

Rideout, V., Saphir, M., Tsang, V., & Bozdech, B. (2011). Zero to eight: Children's media use in America (pp. 1-44). San Francisco, CA: Common Sense Media.

Rimland, E. (2013). Assessing affective learning using a student response system. *portal: Libraries and the Academy*, 13(4), 385–401.

Rincon, L. (2009). *Designing dynamic and interactive applications using Geogebra software in the 6–12 mathematics curriculum* [Unpublished doctoral thesis]. Kean University.

Rittle-Johnson, B. & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. In Cohen, R. & Dowker, A. (Eds.), *Oxford handbook of numerical cognition* (pp. 1118–1134). Oxford: Oxford University Press.

Rittle-Johnson, B., Schneider, M., Star, J. R., Rittle-Johnson, B., Schneider, M. & Star, J. R. (2015). Not a one-way street: Bidirectional relations between procedural and conceptual knowledge of mathematics. *Educational Psychology Review*, 27(4), 587–597.

Rittle-Johnson, B., Siegler, R. S. & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, 93(2), 46–82.

Rodriguez, W. (2011). Effect size. In N. J. Salkind (Ed.), *Encyclopedia of measurement and statistics* (pp. 301-304). Thousand Oaks, CA: SAGEPublications, Inc.

Rokhima, W. A., Kusmayadi, T. A. & Fitriana, L. (2019). Mathematical reasoning of student in senior high school based on gender differences. *Journal of Physics: Conference Series PAPER*, 012092.

Ropohl, M., Nielsen, J. A., Olley, C., Rönnebeck, S. & Stables, K. (2018). The concept of competence and its relevance for science, technology and mathematics education. In xx (Eds.), *Transforming assessment* (pp. 3–25). Cham: Springer.

Rose Martinez, A. (2017). *The effects of using GeoGebra on student achievement in* *secondary mathematics* [Unpublished doctoral thesis]. California State University.

Rosen, Y.& Beck-Hill, D. (2014). Intertwining digital content and a one-to-one laptop environment in teaching and learning lessons from the Time To Know Program. *Journal of Research on Technology in Education*, 44(3), 225–241.

Rosli, R., & Aliwee. (2021). Professional Development of Mathematics Teacher: A Systematic Literature Review. Contemporary Educational Researches Journal, 11(2), 43–54.

Rosyid, A. & Umbara, U. (2019). Analysis of students' attitudes towards implementation of Geogebra-assisted Missouri mathematics project. *Journal of Physics: Conference Series*, 1265(1), 1–10.

Rubin, H. & Rubin, I. (2011). *Qualitative interviewing: The art of hearing data*. Thousand Oaks, CA: Sage.

Ruthven, K. & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics*, 49(1), 47–88.

Saadi, I. A. (2014). Gender and learning styles in Saudi Arabia Schools. *The Clute Institute International Academic Conference*. San Antonio, Texas, USA .

Saha, R. A., Ayub, A. F. M. & Tarmizi, R. A. (2010). The effects of GeoGebra on mathematics achievement: Enlightening coordinate geometry learning. *Procedia – Social and Behavioral Sciences*, 8, 686–693.

Sale, J., Lohfeld, L. & Brazil, K. (2002). Revisiting the quantitative-qualitative debate: Implications for mixed-methods research. *Quality and Quantity*, 36(1), 43–53.

Salkind, N. (2012). *Encyclopedia of measurement and statistics*. London: SAGEPublications, Inc.

Saltrick, S., Hadad, R., Pearson, M., Fadel, C., Regen, B.& & Wyan, J. (2011). 21st century skills map. In xx (Eds.), *Partnership for 21st century skills*. Washington: Partnership for 21st Century Skills.

Samuelsson, J. (2010). The impact of teaching approaches on students' mathematical proficiency in Sweden. *International Electronic Journal of Mathematics Education*, 5(2), 61–78.

Sarantakos, S. (2005). *Social research* (3rd ed.). London: Palgrave Macmillan.

Sarma, M. & Ahmed, M. (2013). A study on the difficulty of teaching and learning mathematics in under graduate level with special reference to Guwahati City. *International Journal of Soft Computing and Engineering*, 3(1), 409.

Satterfield, M. (2001). The Geometer's Sketchpad: Single-user package, version 3. *Mathematics Teaching in the Middle School*, 7(3), 197–208.

Saudi Vision 2030 (2020). *Saudi Vision 2030*. https://vision2030.gov.sa/en

Sawilowsky, S. S. (2009). New effect size rules of thumb. *Journal of Modern Applied Statistical Methods*, 8(2), 597–599.

Schneider, M., Stem, E. & Zurich, E. (2010). The developmental relations between conceptual and procedural knowledge: A multimethod approach. *Developmental Psychology*, 46(1), 178–192.

Schoenfeld, A. H. (1983). *Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography*. Washington D.C.: Mathematical Association of America.

Schoenfeld, A. H. (2007). What is mathematical proficiency and how it can be assessed? In Alan H. Schoenfeld (Ed.), *Assessing mathematical proficiency* (pp. 59–73). NY: Cambridge University Press.

Shadaan, P. & Kwan Eu, L. (2013). Effectiveness of using Geogebra on students' understanding in learning circles. *The Malaysian Online Journal of Educational Technology*, 1(4), 1–11.

Shapiro, S.S. & Wilk, M.B. (1965). An analysis of variance test for normality (complete samples). *Biometrika*, 52(3/4), 591–611.

Siegfried, J. (2012). *The hidden strand of mathematical proficiency: Defining and assessing for productive disposition in elementary school teachers' mathematical content knowledge* [Unpublished doctoral thesis]. UC San Diego.

Sinclair, M. (2004). Working with accurate representations: The case of preconstructed dynamic geometry sketches. *Journal of Computers in Mathematics and Science Teaching*, 23(2), 191–208.

Slater, J. A., Lujan, H. L. & DiCarlo, S. E. (2007). Does gender influence learning style preferences of first-year medical students? *American Journal of Physiology – Advances in Physiology Education*, 31(4), 336–342.

Slavin, R. E., Cheung, A., Groff, C. & Lake, C. (2008). Effective reading programs for middle and high schools: A best-evidence synthesis. *Reading Research Quarterly*, 43(3), 290–322.

Stonewater, J. K. (2005). Inquiry teaching and learning: The Best Math Class Study. *School Science and Mathematics*, 105(1), 36–47.

Straesser, R. (2002). Cabri-géomètre: Does dynamic geometry software (DGS) change geometry and its teaching and learning? *International Journal of Computers for Mathematical Learning*, 6(3), 319–333.

Stringer, E. T. (2004). *Action research in education*. Edinburgh: Pearson.

Suh, J. M. (2008). Tying it all together: Classroom practices that promote mathematical proficiency for all students. *Teaching Children Mathematics*, 143(3), 163–169.

Sullivan, G. M. & Artino, A. R. (2013). Analyzing and interpreting data from Likert-type scales. *Journal of Graduate Medical Education*, 5(4), 541–542.

Suthar, V., Khooharo, A. A. & Khooharo, A. (2013). Impact of students' mathematical beliefs and self-regulated learning on mathematics ability of university students. *European Academic Research*, 1(6), 1346–1360.

Syukriani, A., Juniati, D., Yuli, T. & Siswono, E. (2016). Adaptive reasoning and strategic competence in solving mathematical problem: A case study of male-field independent (FI) student. *Proceeding of 3rd International Conference on Research, Implementation and Education of Mathematics and Science*, 21–28.

Taşdemir, C. (2016). An examination of vocational school students' self-efficacy beliefs in mathematics and of their achievement levels. *Educational Research and Reviews,* 11(8), 804–811.

Tashakkori, A. & Creswell, J. W. (2007). Editorial: Exploring the nature of research questions in mixed methods research. *Journal of Mixed Methods Research*, 1(3), 207–211.

TATWEER (2020). *Tatweer Co. for Educational Services - Home Page*. https://t4edu.com/en

Tavakol, M. & Dennick, R. (2011). Making sense of Cronbach's alpha. *International Journal of Medical Education*, 2, 53–55.

Teddlie, C. & Tashakkori, A. (2009). *Foundations of mixed methods research: Integrating quantitative and qualitative approaches in the social and behavioral sciences*. London: Sage.

Testing for normality in SPSS statistics | Laerd Statistics Premium. (n.d.). https://statistics.laerd.com/premium/spss/tfn/testing-for-normality-in-spss.php

Tschannen-Moran, M. & Hoy, A. W. (2001). Teacher efficacy: Capturing an elusive construct. *Teaching and Teacher Education*, 17(7), 783–805.

Tuba Dikkartın Övez, F. & Kıyıcı, O. D. (2018). 6th grade students' views about mathematical teaching based on technology integration. *World Journal of Education*, 8(5), 160–171.

Velikova, E. & Petkova, M. (2019). Analysing students' creativity in integrating GeoGebra applets in solving geometrical problems. *Baltic Journal of Modern Computing*, 7(3), 419–429.

Villarreal, M. E. & Borba, M. C. (2010). Collectives of humans-with-media in mathematics education: Notebooks, blackboards, calculators, computers and ... notebooks throughout 100 years of ICMI. *International Journal on Mathematics Education*, 42(1), 49–62.

Walliman, N. (2001). *Research and the research problem. Your research project: A step by step guide for the first time researcher*. London: Sage.

Walliman, N. (2006). *Social research methods*. London: SAGEPublications Ltd.

Wasserman, E. & Millgram, Y. (2005). Changes in the approaches of teachers following computerization of schools. *Journal of Educational Computing Research*, 32(3), 241–264.

Way, J. & Beardon, T. (2003). *ICT and primary mathematics*. Berkshir: McGraw-Hill Education (UK).

Weinert, F. (2001). Concept of competence: A conceptual clarification. In D. S. Rychen & L. H. Salganik (Ed.), *Defining and selecting key competencies* (pp. 45–46). Seattle, WA:

Hogrefe & Huber.

Wells, J. (2019). STEM education: The potential of technology education. *Proceeding of The Mississippi Valley Conference in the 21st Century: Fifteen Years of Influence on Thought and Practice*, 195-229

Wenglinsky, H. (1998). *Does it compute? The relationship between educational technology and student achievement in mathematics.* Princeton, NJ: Educational Testing Service.

Wibowo, T. (2016). Adaptive reasoning junior high school students in mathematics problem solving. *Proceeding of 3rd International Conference on Research, Implementation, and Education of Mathematics and Science*, 239–244.

William, W. A. J. (2005). *Research methods in education* (8th ed.). Boston: Allyn & Bacon.

Williams, D., Coles, L., Wilson, K., Richardson, A., & Tuson, J. (2000). Teachers and ICT: Current use and future needs. British Journal of Educational Technology, 31(4), 307–320.

Wilson, C. (2013). *Interview techniques for UX practitioners: A user-centered design method*. Boston: Elsevier.

Wiske, M., Franz, K. & Breit, L. (2006). *Teaching for understanding with technology*. NY: Wiley.

Wong, M. & Evans, D. (2007). Improving basic multiplication fact recall for primary school students. *Mathematics Education Research Journal*, 19(1), 89–106.

Wu, Z. (2008). Using the MSA model to assess Chinese sixth graders' mathematics proficiency. *Journal of Mathematics Education*, 1(1), 74-95.

Xistouri, X. & Pitta-Pantazi, D. (2013). Using GeoGebra to develop primary school students' understanding of reflection. *North American GeoGebra Journal*, 2(1), 19–23.

Yang, D. C. & Tsai, Y. F. (2013). Promoting sixth graders' number sense and learning attitudes via technology-based environment. *Journal of Educational Technology & Society,* 13(4), 112–125.

Yerushalmy, M. (1991). Student perceptions of aspects of algebraic function using multiple representation software. *Journal of Computer Assisted Learning*, 7(1), 42–57.

Zakaria, E., & LS Lee. (2012). Teacher's perceptions toward the use of GeoGebra in the teaching and learning of Mathematics. Journal of Mathematics and Statistics, 8(2), 253–257.

Zengin, Y. (2017). The potential of GeoGebra software for providing mathematical communication in the light of pre-service teachers' views. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education*, 11(1), 101–127.

Zengin, Y., Furkan, H. & Kutluca, T. (2012). The effect of dynamic mathematics software geogebra on student achievement in teaching of trigonometry. *Procedia – Social and Behavioral Sciences*, 31, 183–187.

Zetriuslita, Z., Nofriyandi, N. & Istikomah, E. (2020). The effect GeoGebra-assisted direct instruction on students' self-efficacy and self-regulation. *Journal of Mathematics*

Education, 9(1), 41–48.

Zhu, C. (2010). Teacher roles and adoption of educational technology in the Chinese context. Journal for Educational Research Online, 2(2), 72–86.

Zimmerman, B. J. (2000). Self-efficacy: An essential motive to learn. *Contemporary Educational Psychology*, 25(1), 82–91.

Zulnaidi, H. & Oktavika, E. (2016). The effect of GeoGebra on students' misconception of limit function topic. *Jurnal Kurikulum & Pengajaran Asia Pasifik*, 6(1), 1–6.

Zulnaidi, H. & Zakaria, E. (2012). The effect of using GeoGebra on conceptual and procedural knowledge of high school mathematics students. *Asian Social Science*, 8(11).

Zulnaidi, H. & Zamri, S. N. A. S. (2017). The effectiveness of the Geogebra software: The intermediary role of procedural knowledge on students' conceptual knowledge and their achievement in mathematics. *Eurasia Journal of Mathematics, Science and Technology Education*, 13(6), 2155–2180.

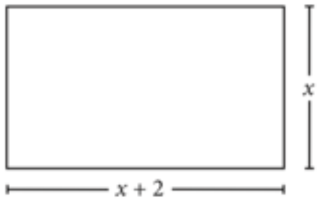
Zulnaidi, H., Oktavika, E. & Hidayat, R. (2020). Effect of use of GeoGebra on achievement of high school mathematics students. *Education and Information Technologies*, 54, 51–72.

Appendices

Appendix 1 **Mathematical proficiency tests**

Appendix 1-1 Mathematical proficiency tests (English)

Mathematical Proficiency Test 1 (Algebra)		
Understanding	1 k	The taxi company has a basic charge of 25 zeds and a charge of 0.2 zeds for each kilometre the taxi is driven. Which of these represents the cost in zeds to hire a taxi for a trip of n kilometres? A. $25 + 0.2n$ B. $25 \times 0.2n$ C. $0.2 \times (25 + n)$ D. $0.2 \times 25 + n$
	2 k	What does $xy + 1$ mean? A. Add 1 to y, then multiply by x. B. Multiply x and y by 1. C. Add x to y, then add 1. D. Multiply x by y, then add 1.
	3 k	There were m boys and n girls in a parade. Each person carried 2 balloons. Which of these expressions represents the total number of balloons that were carried in the parade? E. $2(m + n)$ F.$2 + (m + n)$ C. $2m+n$ D. $m + 2n$

<table>
<tr><td></td><td>4
k</td><td>K= 7 and i= 10 , i= 3
What is the value of P when P= $\frac{3ki}{g}$?</td></tr>
<tr><td>Fluency</td><td>1
A</td><td>What is the area of this rectangle?
A. $x^2 + 2$
B. $x^2 + 2x$
C. $2x + 2$
D. $4x + 4$

 </td></tr>
<tr><td></td><td>2
A</td><td>What is the sum of the three consecutive whole numbers with 2<i>n</i> as the middle number?
A. 6<i>n</i> + 3
B. 6<i>n</i>
C. 6<i>n</i> – 1
D. 6<i>n</i> – 3</td></tr>
<tr><td></td><td>3
A</td><td>$y = \frac{a+b}{c}$
$a = 8$, $b = 6$, and $c = 2$ What is the value of <i>y</i>?
A. 7
B. 10
C. 11
D. 14</td></tr>
<tr><td></td><td>4
A</td><td>If: x+y = 25
What is the value of
2x+2y+4?</td></tr>
<tr><td>Problem-Solving</td><td>1
A</td><td><table><tr><th>Bush height (cm)</th><th>Shadow length (m)</th></tr></table></td></tr>
</table>

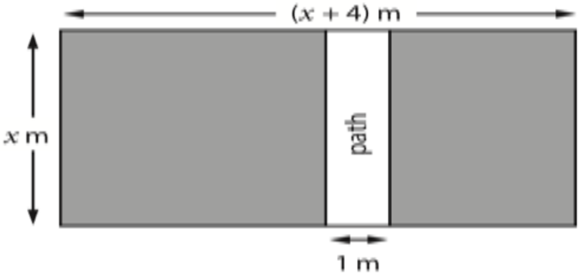
20	16
40	23
60	48
80	64

The table above shows the shadow lengths of four Bushes of different heights at 10 a.m. What is the shadow length at 10 a.m. of a bush that has height of 50 centimetres?

A. 36 cm
B. 38 cm
C. 40 cm
D. 42 cm

2

A



This is diagram of rectangular garden. The white area is rectangular path that is 1 meter wide. Which expression shows the area of the shaded portion of the garden in m^2?

A. $x^2 + 3x$
B. $x^2 + 4x$
C. $x^2 + 4x - 1$
D. $x^2 + 3x - 1$

3

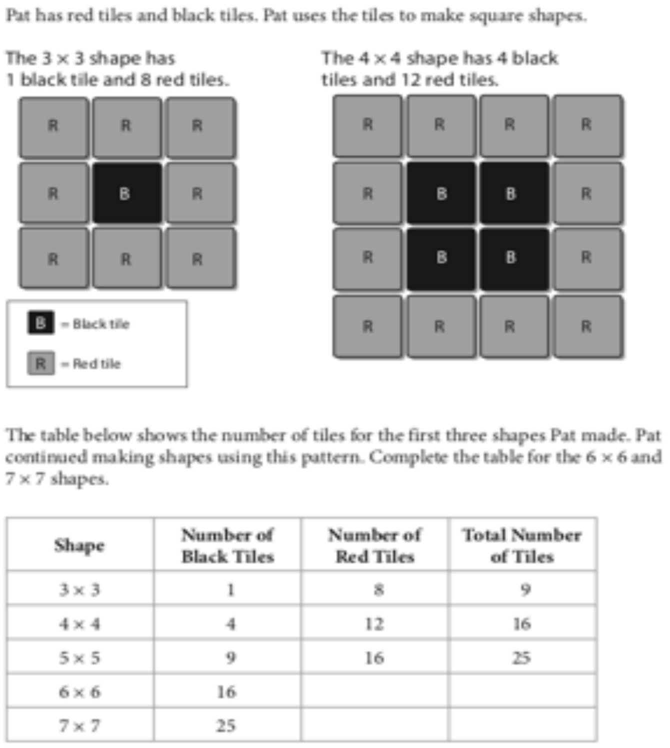
A

In Zedland, total shipping charges to ship an item are given by the equation y = 4x+30 where x is the weight in grams and y is the cost in zeds. If you have 150 zeds, how many grams can you ship?

A. 630
B. 150
C. 120
D. 30

Reasoning

1



Pat has red tiles and black tiles. Pat uses the tiles to make square shapes.

The 3 × 3 shape has 1 black tile and 8 red tiles.

The 4 × 4 shape has 4 black tiles and 12 red tiles.

B = Black tile

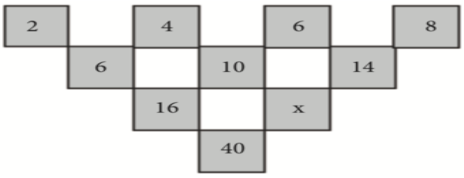
R = Red tile

The table below shows the number of tiles for the first three shapes Pat made. Pat continued making shapes using this pattern. Complete the table for the 6 × 6 and 7 × 7 shapes.

Shape	Number of Black Tiles	Number of Red Tiles	Total Number of Tiles
3 × 3	1	8	9
4 × 4	4	12	16
5 × 5	9	16	25
6 × 6	16		
7 × 7	25		

2

What is the value of x in this pattern?



3

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$$

What would term number 100 be?

Answer: ________________

Mathematical Proficiency Test 2 (Numbers)

Understanding

1

k

$\frac{4}{100}+\frac{3}{1000}=$

B. 0.043
C. 0.1043
D. 0.403
E. 0.43

2

k

- Which of these is the Best estimate of $\frac{7.21\times3.86}{10.09}$?

A. $\frac{7\times3}{10}$
B. $\frac{7\times4}{10}$
C. $\frac{7\times3}{11}$
D. $\frac{7\times4}{11}$

3

K

- Which of these shows how 36 can be expressed as a product of prime factors?

G. 6×6
H. 4×9
I. $4\times3\times3$
J. $2\times2\times3\times3$

4

k

- Which number is equal to $\frac{3}{5}$?

A. 0.8
B. 0.6
C. 0.53
D. 0.35

Fluency

1

A

- Look at this table

4^1	4^2	4^3	4^4	4^5	4^6
4	16	64	256	1024	4096

Use this table to express the value of 256×4096

A. 4^{10}
B. 4^{16}
C. 4^{20}
D. 4^{24}

2

A

- Which shows a correct method for finding $\frac{1}{3}-\frac{1}{4}$?

E. $\frac{1-1}{3-4}$

F. $\frac{1}{4-3}$

G. $\frac{3-4}{3\times4}$

H. $\frac{4-3}{3\times4}$

3

A

- The fraction $\frac{2}{14}$ and $\frac{\quad}{21}$ are equivalent.

What is the value of ☐

A. 3
B. 7
C. 11
D. 14

4

The table shows the number of students in the 7th and 8th grades in a given school.

Grade	**Number of students**
7	60
8	55

Complete the grade 8 row pictograph below to represent the number of students in each graph

One ☺ sents 10 students

Grade 7	☺ ☺ ☺ ☺ ☺ ☺
Grade 8	

Problem-Solving

1

A

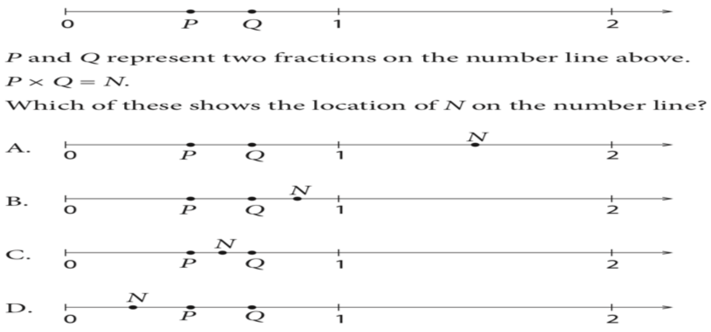
- Georgia wants to send letters to 12 of her friends. Half of the letters will need 1 page each and the other half will need 2 pages each. How many pages will be needed altogether?

2

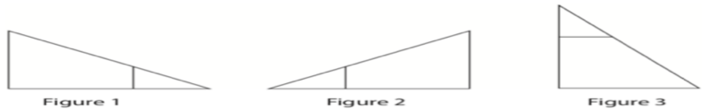
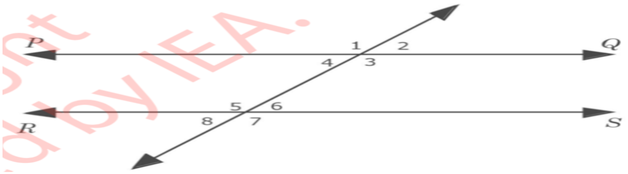
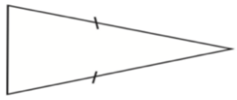
A

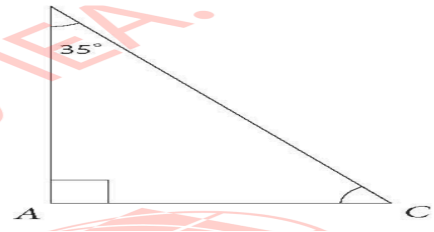
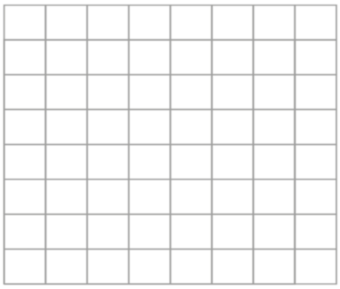
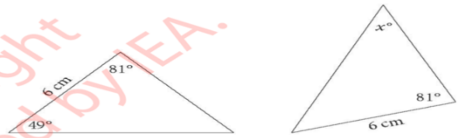
- One school trip there was 1 teacher for every 12 students. If 108 students went on the trip, how many teachers were on the trip?

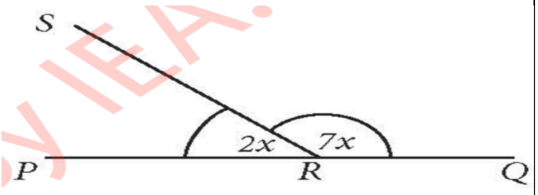
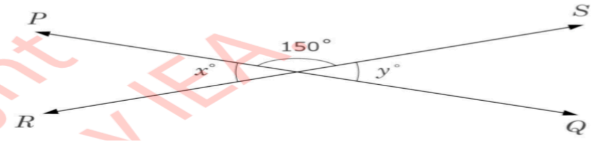
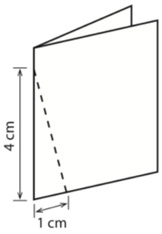
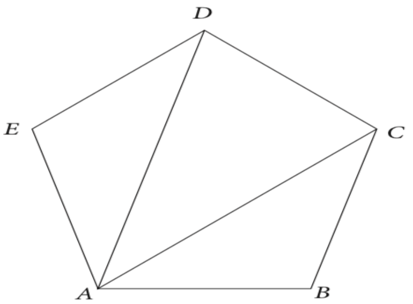
A. 7
B. 8
C. 9
D. 10

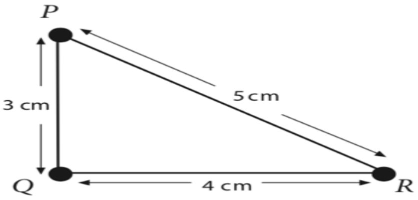
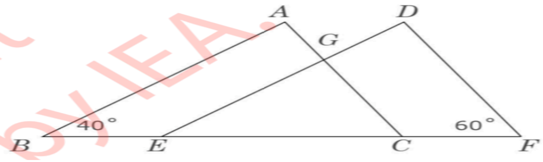
	3 A	- A workman cut off $\frac{1}{5}$ of a pipe. The piece he cut off was 3 meters long. How many meters long was the original pipe? A. 8 m B. 12 m C. 15 m D. 18 m
Reasoning	1	Here is a pattern: 3 –3 = 0 3 –2 = 1 3 –1 = 2 3 –0 = 3 What will the next line in the pattern be? Answer: _______________
	2	The scale on a map indicates that 1 centimetre on the map represents 4 kilometres on the land. The distance between two towns on the map is 8 centimetres. How many kilometres apart are the two towns? A. 2 B. 8 C. 16 D. 32
	3	 *P* and *Q* represent two fractions on the number line above. $P \times Q = N$. Which of these shows the location of *N* on the number line? A. B. C. D.

Mathematical Proficiency Test 3 (Geometry)

Understanding	1 k	 Which of these transformations, taken in order, can be used so that Figure 1 above becomes Figure 2 and then Figure 3? A. Reflection then translation B. Reflection and then $\frac{1}{4}$ turn rotation clockwise C. $\frac{1}{2}$ turn rotation and then translation D. $\frac{1}{4}$ turn rotation contraclockwise and then reflection
	2 k	In this figure, *PQ* and *RS* are parallel.  Of the following, which pair of angles has the sum of 180°? A. $\angle 5$ and $\angle 7$ B. $\angle 3$ and $\angle 6$ C. $\angle 1$ and $\angle 5$ D. $\angle 2$ and $\angle 8$
	3 k	Which shape has a line of symmetry? A.  B.  C.  D. 

	4 k	Which of these could be the measure of the area of a triangle ? A. 2 cm B. 3 m C. 5 cm^2 D. 8 cm^3
Fluency	1 A	What is the perimeter of a square which has an area of 100 square meters?
	2 A	 What is the measure of angle *C* in the triangle above? A. 45° B. 55° C. 65° D. 145°
	3 A	The length of side of each of small square represents 1 cm. Draw an isosceles triangle with a base of 4 cm and height of 5 cm. 
	4	The triangles shown are congruent. The measure of same of the sides and angles are given. What is the value of x?  A. 49

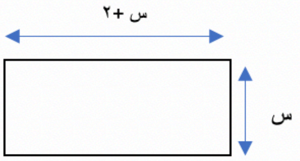
		B. 50 C. 60 D. 70
Problem-Solving	1 A	In this figure PQ is a straight line. What is the degree measure of angle PRS?  A. 10° B. 20° C. 40° D. 70°
	2 A	In the figure, *PQ* and *RS* are intersecting straight lines. What is the value of $x+y$?  A. 15 B. 30 C. 60 D. 180
	3 A	A piece of paper in the shape of a rectangle is folded in half as shown in the figure below. It is then cut along the dotted line, and the small piece that is cut is opened, what is the shape of the cut out figure? A. An isosceles triangle B. Two isosceles triangles C. A right triangle D. An equilateral triangle 
Reasoning	1	 What is the sum of all the interior angles of pentagon ABCDE?

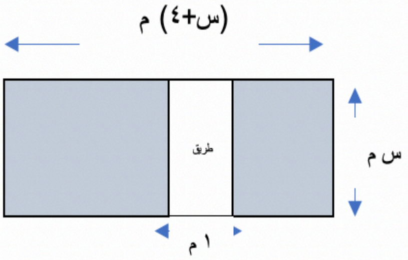
	2	 Which of these is the reason that triangles *PQR* is right angle triangle? A. $3^2 + 4^2 = 25$ B. $5 < 3 + 4$ C. $3 + 4 = 12 - 5$ D. $3 > 5 - 4$
	3	In this figure, triangles *ABC and DEF* are congruent with *BC=EF*  What is the measure of angle EGC? A. 20° B. 40° C. 60° D. 80°

Appendix 1-2 Mathematical proficiency tests (Arabic)

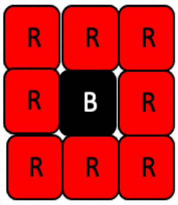
الاختبار القبلي (الجبر)

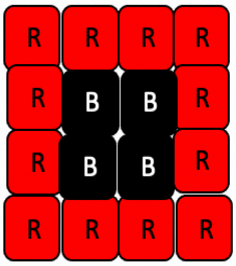
اسم الطالب : .... الفصل : ....

١	الاستيعاب المفاهيمي	١	- تحسب احدى شركات التاكسي اجرة المشوار الواحد كما يلي: ٢٥ ريال كمبلغ أساسي و ٠.٢ ريال لكل كيلو متر تتحركه السيارة. أي مما يلي يعبر عن التكلفة بالريالات لاستئجار سيارة لمشوار س من الكيلومترات؟ A. ٢٥ + ٠.٢ س B. ٢٥ × ٠.٢ س C. ٠.٢ × (٢٥ + س) D. ٠.٢ × ٢٥ + س
		٢	- ماذا تعني العبارة الرياضية التالية س ص +١ A. إضافة الى س، وضربه في ص B. ضرب س و ص في ١ C. إضافة س إلى ص، ثم إضافة ١ D. ضرب س في ص، ثم إضافة ١
		٣	- س = ٧ و ع = ١٠ و ل = ٣ ما قيمة ص عندما ص = $\frac{\text{٣ ع س}}{\text{ل}}$
		٤	- في احد العروض كان هناك عدد س من الأطفال الذكور وعدد ص من البنات، كل طفل يحمل بالونين، أي من العبارات التالية تمثل العدد الكلي للبالونات A. ٢ (س + ص) B. ٢+ (س +ص) C. ٢ س + ص D. س + ٢ص
٢	الطلاقة الإجرائية	١	ماهي مساحة المستطيل المجاور  A. ٢س +٢ B. $٢\text{س}^{٢}$ C. $\text{س}^{٢}$ + ٢س D. ٢س
		٢	ما هو مجموع ثلاثة أعداد متتالية اذا كان العدد الأوسط هو ٢ص ؟ A. ٦ص B. ٦ص +٣ C. ٦ص-١ D. ٦ص-٣
		٣	ص = $\frac{\text{ب+ج}}{\text{د}}$ اذا كانت: ب = ٨، ج= ٦ ، و د =٢ فما هي قيمة ص A. ٧ B. ١٠ C. ١١ D. ١٤
		٤	اذا كانت: س + ص = ٢٥ فماهي قيمة ٢ س + ٢ص +٤

<table>
<tr>
<td rowspan="3">٣</td>
<td rowspan="3">حل المشكلات</td>
<td>١</td>
<td>
<table>
<tr><th>طول المبنى بالمتر</th><th>طول الظل بالمتر</th></tr>
<tr><td>٢٠</td><td>١٦</td></tr>
<tr><td>٤٠</td><td>٢٤</td></tr>
<tr><td>٦٠</td><td>٤٨</td></tr>
<tr><td>٨٠</td><td>٦٤</td></tr>
</table>
يعرض الجدول أعلاه طول الظل لأربعة مباني مختلفة الاطوال عند الساعة العاشرة صباحا.
ما هو طول ظل مبنى اذا كان ارتفاعه ٥٠ مترا ؟
A. ٣٦ سم
B. ٣٨ سم
C. ٤٠ سم
D. ٤٢ سم
</td>
</tr>
<tr>
<td>٢</td>
<td>


الشكل المجاور يمثل حديقة مستطيلة الشكل. تمثل المساحة البيضاء في المنتصف طريق مستطيل الشكل عرضه مترا واحدا، أي من العبارات التالية يمثل المساحة المضللة بالمتر المربع ($م^{٢}$)؟
A. $س^{٢}$ + ٣س
B. $س^{٢}$+٤ س
C. $س^{٢}$+٤ س -١
D. $س^{٢}$ +٤س – ١
</td>
</tr>
<tr>
<td>٣</td>
<td>
في احدى شركات الشحن تحسب القيمة الإجمالية لشحن أي قطعة من خلال المعادلة التالية:
ص= ٤س + ٣ حيث س يمثل وزن القطعة بالجرام و ص القيمة الاجمالية للشحن. اذا كان لديك ١٥٠ ريالا، فكم عدد الجرامات التي يمكنك شحنها؟
A. ٦٣٠
B. ١٥٠
١٢٠
C. ٣٠
</td>
</tr>
<tr>
<td></td>
<td>التبرير المنطقي</td>
<td>١</td>
<td>
لدى علي مجموعتين من الاشكال، حمراء و سوداء اللون، استخدم علي هذه الاشكال لعمل مربع كبير
- يحتوي الشكل ٣x٣ على شكل اسود واحد و ثمانية اشكال حمراء.
يحتوي الشكل ٤x٤ على أربعة اشكال سوداء و اثنى عشرة شكلا احمراً.
</td>
</tr>
</table>

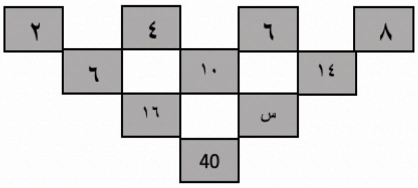




من خلال ما سبق اكمل الجدول التالي:

الشكل	عدد الاشكال السوداء	عدد الاشكال الحمراء	عدد الاشكال
٣x٣	١	٨	٩
٤x٤	٤	١٢	١٦
٥x٥	٩	١٦	٢٥
٦x٦	١٦		
٧x٧	٢٥		

٢ ما قيمة س في الشكل المجاور



٣ $\frac{١}{٢}$، $\frac{٢}{٣}$، $\frac{٣}{٤}$، $\frac{٤}{٥}$، $\frac{٥}{٦}$

كيف سيكون الكسر رقم ١٠٠؟

(اختبار وحدة الاعداد النسبية)

اسم الطالب :.................................الفصل :......................   الرقم :............

الفهم المفاهيمي

١

$\frac{٤}{١٠٠} + \frac{٣}{١٠٠٠}$

A. ٠.٠٤٣
B. ٠.١٠٤٣
C. ٠.٤٠٣
D. ٠.٤٣

٢

أي من الكسور التالية يمثل أفضل تقريب للكسر $\frac{٧,٢١\times٣,٨٦}{١٠,٠٩}$

A. $\frac{٧\times٣}{١٠}$
B. $\frac{٧\times٤}{١٠}$
C. $\frac{٧\times٣}{١١}$
D. $\frac{٨\times٤}{١١}$

٣

A. أي مما يلي يمثل العناصر الأولية للعدد ٣٦؟

A. ٣٦ × ١
B. ٤ × ٩
C. ٤ × ٣ × ٣
D. ٢ ×٢ × ٣ × ٣

٤

B. ما هو العدد المساوي ل $\frac{٣}{٥}$ ؟

A. ٠.٨
B. ٠.٦
C. ٠.٠٥٣.
D. ٠.٣٥

الطلاقة الاجرائية

١

$٤^{١}$	$٤^{٢}$	$٤^{٣}$	$٤^{٤}$	$٤^{٥}$	$٤^{٦}$
٤	١٦	٦٤	٢٥٦	١٠٢٤	٤٠٩٦

من خلال الجدول السابق، عبر عن قيمة ٢٥٦ × ٤٠٩٦

A. $٤^{١٠}$
B. $٤^{١٦}$
C. $٤^{٣}$
D. $٤^{٢٤}$

٢

أي من الكسور التالية يمثل الطريقة لطرح الكسرين $\frac{١}{٢}$ - $\frac{١}{٤}$ ؟

A. $\frac{1-1}{4\times3}$ B. $\frac{3-4}{4\times3}$

C. $\frac{1}{3-4}$ D. $\frac{4-3}{4\times3}$

٣
- الكسر $\frac{2}{١٤}$ و $\frac{\square}{٢١}$ متساويان . ماهي قيمة العدد في المربع الخالي؟

٦

A. ٣
B. ٧
C. ١١
D. ١٤

٤
- يظهر الجدول ادناه عدد الطلاب في الصف السابع وفي الصف الثامن في احدى المدارس :

الصف	عدد الطلاب
السابع	٦٠
الثامن	٥٥

اكمل الجدول الخاص بالصف الثامن من خلال عرض عدد الطلاب بواسطة الصورة ☺ . حيث تمثل كل صورة عشرة طلاب .

☹ = ١٠

الصف السابع	☺ ☺ ☺ ☺ ☺ ☺
الصف الثامن	

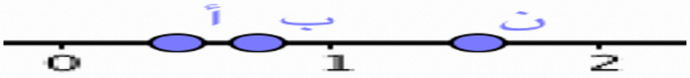
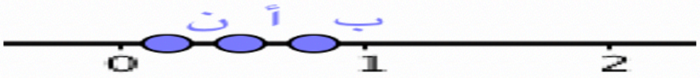
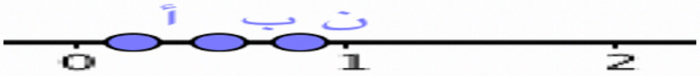
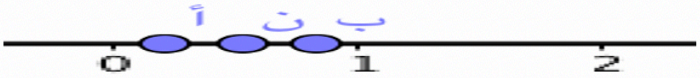
حل المشكلات

١
تريد هند ان ترسل ١٢ رسالة بالبريد الى صديقاتها، نصف هذه الرسائل تحتاج إلى طابع واحد فقط و النصف الاخر تحتاج إلى طابعين، كم عدد الطوابع التي تحتاجها هند ؟

٢
في رحلة لإحدى المدارس هناك معلم لكل ١٢ طالب، اذا ذهب ١٠٨ من الطلاب الى هذه الرحلة ، فكم عدد المعلمين في هذه الرحلة ؟

A. ٧
B. ٨
C. ٩
D. ١٠

٣
- قام عامل بقطع $\frac{١}{٥}$ من احد الانابيب، حيث كان طول القطعة التي تم اخذه تساوي ٣ متر .
فما هو طول الانبوب كاملا؟.

		A. ٨ م B. ١٢ م C. ١٥ م D. ١٨ م
التبرير المنطقي	١	أكمل النمط التالي: - ٣-٣ = ٠ ٣-٢ = ١ ٣- ١= ٢ ٣-٠ = ٣ ٣ - .... = ...
	٢	يشير المقياس على الخريطة إلى أن كل سنتيمترًا واحدًا على الخريطة يمثل 4 كيلومترات على الأرض. فاذا كانت المسافة بين مدينتين على الخريطة هي 8 سنتيمترات. كم عدد الكيلومترات بين تلك المدينتين؟
		 يمثل أ وب كسرين بين ٠ و١ على خط الاعداد أعلاه. بحيث: أ x ب = ن أي مما يلي يمثل قيمة ن على خط الاعداد. A.  B.  C.  D. 

(اختبار وحدة الهندسة)

اسم الطالب : .... الفصل :...................... الرقم :..............................

السؤال الأول

١ (درجتان)

شكل ٣

شكل ٢

شكل ١

- أي من العبارات التالية تمثل التحويلات المتخذة على الشكل ١ ليصبح الشكل رقم ٢ ثم يصبح الشكل رقم ٣؟

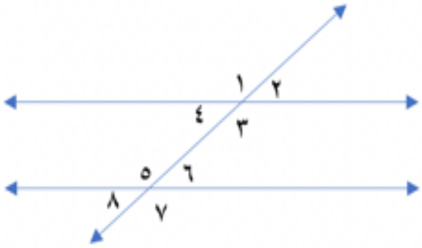
A. انعكاس ثم انسحاب

B. انعكاس ثم $\frac{١}{٤}$ دورة مع عقارب الساعة

C. $\frac{١}{٢}$ دورة ثم انسحاب

D. $\frac{١}{٤}$ دورة عكس عقارب الساعة ثم انعكاس

٢ (درجتان)



- أي من زوج من ازواج الزوايا السابقة مجموعهما يساوي ١٨٠ ؟

A. ∠٥ و ∠٧

B. ∠٣ و ∠٦

C. ∠١ و ∠٥

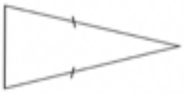
D. ∠٢ و ∠٨

٣ (درجة)

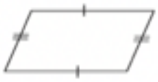
- أي من الاشكال التالية لها محور تناظر؟

A.

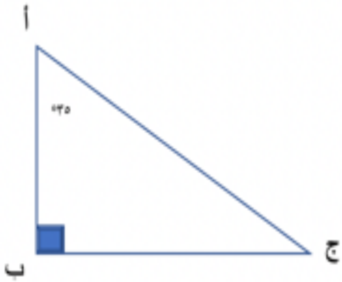
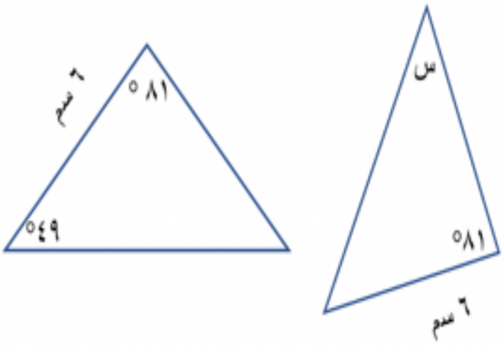
B.

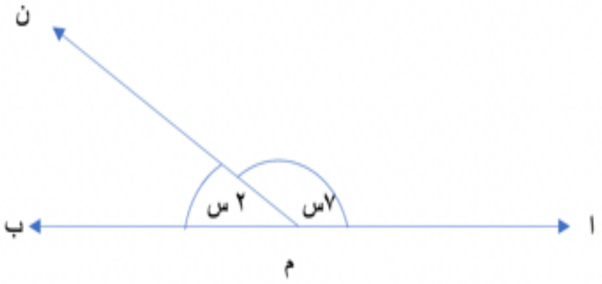
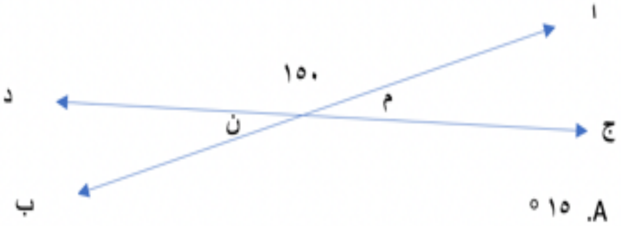
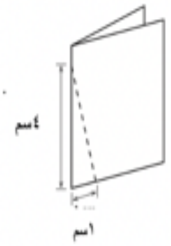


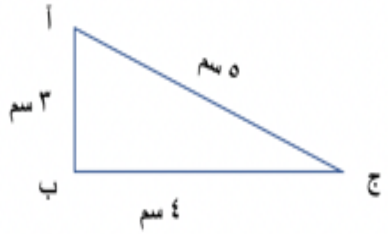
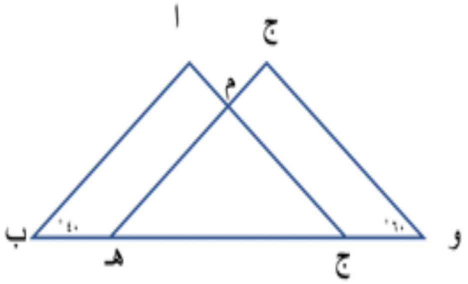
C.



D.

	٤ (درجة)	- أي مما يلي يمكن ان يكون مساحة مثلث؟ A. ٢ سم B. ٣ م C. ٦ سم^{٢} D. ٨ سم^{٣}
الطلاقة الاجرائية	١ (درجتان)	- ماهو قياس ضلع مربع والذي مساحته تساوي ١٠٠ سم^{٢} ؟
	٢ (درجة)	 - ماهي قياس زاوية ج في المثلث أعلاه؟ A. ٤٥ ° B. ٥٥ ° C. ٦٥ ° D. ١٤٥ °
	٣ (درجة)	- طول ضلع كل مربع صغير يساوي ١ سم ، ارسم مثلث متساوي الضلعين طوله قاعدته ٤ سم وارتفاعه ٥ سم . 
	٤ (درجتان)	- المثلثان المجاوران متطابقين، قياسات بعض الاضلاع والزوايا معطاة في الشكل . ماهي قيمة الزاوية س؟  A. ٤٩° B. ٥٠° C. ٦٠° D. ٧٠°

حل المشكلات	١ (درجتان)	A. في الشكل المجاور اب خط مستقيم، ما هو قياس الزاوية ب م ن ؟  A. ١٠ ° B. ٢٠ ° C. ٤٠ ° D. ٧٠ °
	٢ (درجة)	في الشكل المجاور اذا كان المستقيم اب و المستقيم ج د مستقيمان متقاطعان ، ماهي قيمة ن +م ؟  A. ١٥ ° B. ٣٠ ° C. ٦٠ ° D. ١٨٠ °
	٣ (درجة)	- تم طي ورقة من النصف كما في الشكل ادناه ، اذا تم قص الورقة بشكل مستقيم على طول الخط المنقط بعد ذلك تم اخذ الجزء المقصوص وفتحه ، ما هو شكل الورقة المقصوصة بعد فتحها؟ A. مثلث متطابق الضلعين B. مثلثات متطابقة الاضلاع C. مثلث قائم الزاوية D. مثلث متطابق الاضلاع 
التبرير المنطقي	١ (درجتان)	- ما هو مجموع قياسات الزوايا الداخلية للمضلع الخماسي ادناه ؟ 

	٢ (درجة)	 - ما هو السبب الذي يجعل المثلث ا ب ج مثلث قائم الزاوية؟ A. $٤^٢ + ٣^٢ = ٥^٢$ B. ٤+٣ > ٥ C. ٣+٤ = ١٢ -٥ D. ٣ > ٥-٤
	٣ (درجة)	 - المثلث ا ب ج و المثلث ج هـ و مثلثان متطابقين ، و أيضا ب ج = هـ و . ماهو قياس الزاوية م هـ ج A. ٢٠° B. ٤٠° C. ٦٠° D. ٨٠°

Appendix 2 Students' productive disposition questionnaire

Appendix 2-1 Students' productive disposition questionnaire (English)

1. **How much do you agree with these statements about learning mathematics?**

Fill in one answer on each line

	Statement	**Agree a lot**	**Agree a little**	**Disagree a little**	**Disagree a lot**
A	I enjoy learning mathematics				
B	I wish I did not have to study mathematics				
C	Mathematics is boring				
D	I learn many interesting things in mathematics				
E	I like mathematics				
F	I like any schoolwork that involves numbers				
G	I like to solve mathematics problems				
H	I look forward to mathematics class				
i	Mathematics is one of my favourite subjects				
Source: TIMSS 2015, student questionnaires, p. 13, Q17 "Students like learning mathematics" scale					

2. **How much do you agree with these statements about mathematics?**

Fill in one answer on each line

	Statement	**Agree a lot**	**Agree a little**	**Disagree a little**	**Disagree a lot**
A	I usually do well in mathematics				
B	Mathematics is more difficult for me than for many of my classmates				
C	Mathematics is not one of my strengths				
D	I learn things quickly in mathematics				
E	Mathematics makes me nervous				
F	I am good at working out difficult mathematics problems				
G	My teacher tells me I am good at mathematics				
H	Mathematics is harder for me than any other subject				
i	Mathematics makes me confused				
Source: TIMSS 2015, student questionnaire, p. 17, Q18 "Students confident in mathematics" scale					

3. How much do you agree with these statements about mathematics?

Fill in one answer on each line

	Statement	**Agree a lot**	**Agree a little**	**Disagree a little**	**Disagree a lot**
A	I think learning mathematics will help me in my daily life				
B	I need mathematics to learn other school subjects				
C	I need to do well in mathematics to get into the <university> of my choice				
D	I need to do well in mathematics to get the job I want				
E	I would like a job that involves using mathematics				
F	It is important to learn about mathematics to get ahead in the world				
G	Learning mathematics will give me more job opportunities when I am an adult				
H	My parents think that it is important that I do well in mathematics				
I	It is important to do well in mathematics				
Source: TIMSS 2015, student questionnaire, p.18, Q19 “Students value mathematics” scale					

Appendix 2-2 Students’ productive disposition questionnaire (Arabic)

نوع الاستبانة :

الفصل :　　المجموعة : ....　　رقم الطالب :.........

1. إلى اي مدى تتفق مع العبارات التاليه والمتعلقه بتعلمك للرياضيات؟

قم باختيار إجابة واحدة لكل عبارة

	العبارة	اتفق بشدة	اتفق	لا اتفق	لا اتفق بشدة
A	انا استمتع في تعلم الرياضيات				
B	أتمنى انني لا احتاج الى تعلم الرياضيات				
C	الرياضيات مملة				
D	انا اتعلم الكثير من الأشياء الممتعة في الرياضيات				
E	انا أحب الرياضيات				
F	أحب اي عمل مدرسي يحتوي على اعداد				
G	أحب حل المسائل الرياضية				
H	انا انتظر حصة الرياضيات بشغف				
i	الرياضيات من المواد المفضلة لدي				

2. إلى أي مدى تتفق مع العبارات التالية والمتعلقة بمادة الرياضيات؟

قم باختيار إجابة واحدة لكل عبارة

	العبارة	اتفق بشدة	اتفق	لا اتفق	لا اتفق بشدة
A	انا دائما احل بشكل جيد في الرياضيات				
B	الرياضيات صعبة علي اكثر من زملائي في الفصل				
C	الرياضيات ليست من نقاط قوتي				
D	انا اتعلم الأشياء بسرعة في الرياضيات				
E	الرياضيات تجعلني ابدو عصبيا				
F	انا احل المسائل الرياضية الصعبة بشكل جيد				
G	اخبرني معلمي انني جيد في الرياضيات				
H	الرياضيات اصعب عي من المواد الأخرى				
I	الرياضيات تحيرني				

2. إلى أي مدى تتفق مع العبارات التالية والمتعلقة بمادة الرياضيات؟

قم باختيار إجابة واحدة لكل عبارة

	العبارة	اتفق بشدة	اتفق	لا اتفق	لا اتفق بشدة
A	انا اعتقد ان الرياضيات تساعدني في حياتي اليومية				
B	احتاج الرياضيات لتعلم المواد الدراسية الأخرى				
C	احتاج الى الرياضيات لاختار الجامعة المناسبة لي				
D	احتاج لأكون جيدا في الرياضيات لأجد العمل المناسب لي				
E	أتمنى ان احصل على عمل يحتوي على الرياضيات				
F	من المهم تعلم الرياضيات للحصول على مستقبل جيد في هذا العالم				
G	تعلم الرياضيات يوفر لي كثير من فرص العمل				
H	يعتقد والدي انه من المهم ان أكون جيدا في الرياضيات				
I	من المهم ان أكون جيدا في الرياضيات				

Appendix 3 **Teachers' Interview Questions**

Appendix 3-1 Teachers' Interview Questions (English)

Teachers Interview I (before PDC)

1. Could you share with me some information about yourself?
2. Do you usually use technology when you are teaching mathematics? How? Why?
3. Do you use a particular type of technology? Can you give me an example?
4. Have you received any technical training on the use of technology in the classroom or outside the classroom? If so, what?
5. What do you think about using technology as a tool in mathematics teaching and learning?
6. What kind of changes do you think technology brings about when teaching and learning mathematics?
7. Based on your previous experiences, how do you feel about the difference technology makes when teaching and learning mathematics?
8. What do you know about GeoGebra software? Have you used it? If so how?
9. Do think it is useful for teaching and learning mathematics? in what ways? Give an example.

Teachers Interview II (During PDC)

1. What do you think about GeoGebra now?
2. Have you enjoyed using its features? What is the best feature in your opinion? Why?
3. What do you think about using GeoGebra for mathematics?
4. What kind of opportunities does GeoGebra provide teachers and students with?
5. What kind of changes do you think GeoGebra could make to teaching and learning in mathematics?
6. What do you think the effects of using GeoGebra are on teaching and learning mathematics in terms of facilitating learning?
7. What do you think the effects of using GeoGebra for teaching and learning mathematics are in terms of making teaching more effective? Please explain your answer.
8. What do you think the effects of using GeoGebra for teaching and learning mathematics are in terms of producing better drawings?
9. What do you think the effects of using GeoGebra are on teaching and learning mathematics to make learning more permanent?
10. What do you think the effects of using GeoGebra are on teaching and learning mathematics in terms of facilitating classroom management?

Teachers Interview III (After PDC)

- Have you used GeoGebra in your teaching? (Yes/ NO)
  IF YES

1. What are the differences between using GeoGebra and a traditional approach? Can you explain further?
2. Do you usually use GeoGebra as a demonstration tool, or do students use it as a learning tool, or both?
3. How confident do you feel when you using GeoGebra? Could you explain further?
4. How much do you think students enjoy using it?
5. Have you recognized any differences in student's engagement since you using GeoGebra when compared to before it was introduced? Could you explain further?
6. Do you think GeoGebra has affected students' conceptual understanding? If so, how?
7. Do you think GeoGebra has affected students' procedural fluency? If so, how?
8. Do you think GeoGebra has affected students' strategic competence? If so, how?
9. Do you think GeoGebra has affected students' adoptive reasoning? If so, how?
10. Do you think your students' dispositions towards mathematics have been changed by using GeoGebra? If so, how?
11. What do you think the effects of using GeoGebra on teaching and learning mathematics are in terms of facilitating learning?
12. What do you think the effects of using GeoGebra on teaching and learning mathematics are in terms of making teaching more effective? Please explain your answer.
13. What do you think the effects of using GeoGebra for teaching and learning mathematics are, in terms of producing better drawings?
14. What do you think the effects of using GeoGebra are on teaching and learning mathematics in terms of making learning more permanent?
15. What do you think the effects of using GeoGebra are on teaching and learning mathematics in terms of facilitating classroom management?

IF NO
Why you haven't used it?

Teachers' views of GeoGebra (last day of PDC)

1. **What changes does GeoGebra introduce to teaching, in relation to each strand of mathematical proficiency?**
    - What changes does GeoGebra introduce to mathematics teaching in relation to Conceptual Understanding?
    - What changes does GeoGebra introduce to mathematics teaching in relation to Procedural Fluency?
    - What changes does GeoGebra introduce to mathematics teaching in relation to Strategic Competence?
    - What changes does GeoGebra introduce to mathematics teaching in relation to Adoptive Reasoning?
    - What changes does GeoGebra introduce to mathematics teaching in relation to Productive Disposition?

2. **How does GeoGebra bring about changes in teaching and learning, in terms of each strand of mathematical proficiency?**
    - How does GeoGebra bring about changes in mathematics teaching in terms of Conceptual Understanding?
    - How does GeoGebra bring about changes in mathematics teaching in terms of Procedure Fluency?
    - How does GeoGebra bring about changes in mathematics teaching in terms of Strategic Competence?
    - How does GeoGebra bring about changes in mathematics teaching in terms of Adoptive Reasoning?
    - How does GeoGebra bring about changes in mathematics teaching in terms of Productive Disposition?

3. **What are your views about the effects on learning and teaching mathematics when using GeoGebra?**
    - What are your views about the effects on learning mathematics when using GeoGebra to facilitate learning?
    - What are your views about the effects on learning mathematics when using GeoGebra to make teaching more effective?
    - What are your views about the effects on learning mathematics when using GeoGebra to produce better drawings?
    - What are your views about the effects on learning mathematics when using GeoGebra to make learning more permanent?
    - What are your views about the effects on learning mathematics when using GeoGebra to facilitate classroom management?
    - What are your views about the effects on learning mathematics when using GeoGebra to provide an enjoyable environment?

Appendix 3-2 Teachers’ Interview Questions (Arabic)

أسئلة المقابلات مع المعلمين

I. المقابلات قبل البرنامج التدريبي

1. هل من الممكن ان تعرف لي بنفسك وخبراتك التعليمية؟
2. هل تستخدم التقنية باستمرار عند تدريسك لمادة الرياضيات؟ كيف، ولماذا؟
3. هل تستخدم أي برنامج محدد؟ هل من الممكن ان تعطي مثال؟
4. هل حصلت على دورات تدريبية خاصة باستخدام التقنية في الصف او خارج الصف؟ اذا نعم، هل من الممكن ان تعطي مثال؟
5. ما رأيك حول استخدام التقنية في تعلم وتعليم الرياضيات؟
6. ما نوع التغيير الذي تعتقد أن التكنولوجيا تحدثه عند استخدامها في تعلم وتعليم الرياضيات؟
7. بناء على تجاربك السابقة، ما رأيك حول الاختلافات التي تحدثها التكنولوجيا عند تدريس وتعلم الرياضيات؟
8. ماذا تعرف عن برنامج الجيوجبرا؟ هل استخدمته؟ كيف
9. هل تعتقد انه مفيد في تعلم وتعليم الرياضيات؟ كيف، ولماذا؟

II. المقابلات اثناء البرنامج التدريبي

1. ما رأيك في برنامج الجيوجبرا الان؟
2. هل استمتعت بخصائص البرنامج؟ ما هي أفضل خاصية اعجبت بها، ولماذا؟
3. ما رأيك حول استخدام الجيوجبرا في تعليم الرياضيات؟
4. ماهي الفرص التي يوفرها البرنامج للطلاب وللمعلمين اثناء تدريس الرياضيات؟
5. ما نوع التغيير الذي تعتقد أن الجيوجبرا يمكن ان يحدثه عند استخدامه في تعلم وتعليم الرياضيات؟
6. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث تسهيل التعلم؟
7. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث زيادة تأثير التعليم؟ ممكن ان تعطي شرح لإجابتك
8. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث عمل رسومات أفضل؟
9. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث بقاء اثر التعلم افضل؟
10. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث تسهيل إدارة الفصول الدراسية؟

III. المقابلات بعد البرنامج التدريبي

- **هل استخدمت برنامج الجيوجبرا خلال تدريسك لمادة الرياضيات؟ نعم، او لا**

إذا نعم فأجب عن الأسئلة

1. ما هو الفرق بين استخدام الجيوجبرا وطريقتك التقليدية؟ هل يمكن أن تشرح ذلك؟

2. هل عادة ما تستخدم الجيوجبرا كوسيلة تعليم، أم أن الطلاب اثناء تعلمهم، أو كلاهما؟

3. ما مدى الثقة التي تشعر بها عند استخدام الجيوجبرا؟ هل يمكن أن توضح أكثر؟

4. هل تعتقد أن الطلاب يستمتعون باستخدامها؟

5. هل هناك فرق في تفاعل الطلاب منذ استخدامك لـ الجيوجبرا مقارنة بما كان قبل استخدام البرنامج؟ هل يمكن أن تفسر أكثر؟

6. هل تعتقد أن الجيوجبرا قد أثر على الاستيعاب المفاهيمي لدى الطلاب؟ كيف؟

7. هل تعتقد أن الجيوجبرا قد أثر على الطلاقة الإجرائية للطلاب؟ كيف؟

8. هل تعتقد أن الجيوجبرا قد أثرت على الكفاءة الاستراتيجية للطلاب؟ كيف؟

9. هل تعتقد أن الجيوجبرا قد أثرت على التبرير المنطقي للطلاب؟ كيف؟

10. هل تعتقد أن الجيوجبرا قد أثرت على الرغبة المنتجة للطلاب تجاه الرياضيات؟ كيف؟

11. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث تسهيل التعلم؟
12. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث زيادة تأثير التعليم؟ ممكن ان تعطي شرح لإجابتك
13. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث عمل رسومات أفضل؟
14. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث بقاء اثر التعلم افضل؟
15. ما رأيك في تأثير استخدام الجيوجبرا على تعلم وتعليم الرياضيات من حيث تسهيل إدارة الفصول الدراسية؟

- إذا كان لا، لماذا لم تستخدمها؟

IV. المقابلات في اليوم الأخير من البرنامج التدريبي للمعلمين

1 .ما هي التغييرات التي يمكن ان يحدثها برنامج الجيوجبرا في تدريس فيما يتعلق بكل مكون من مكونات البراعة الرياضية؟

I. ما هي التغييرات التي يمكن ان يحدثها برنامج الجيوجبرا في تدريس الرياضيات فيما يتعلق بالاستيعاب المفاهيمي؟
II. ما هي التغييرات التي يمكن ان يحدثها برنامج في تدريس الرياضيات فيما يتعلق. بالطلاقة الإجرائية؟
III. ما هي التغييرات التي يمكن ان يحدثها برنامج الجيوجبرا في تدريس الرياضيات فيما يتعلق بالكفاءة الاستراتيجية؟
IV. ما هي التغييرات التي يمكن ان يحدثها برنامج الجيوجبرا في تدريس الرياضيات فيما يتعلق بالتبرير المنطقي؟
V. ما هي التغييرات التي يمكن ان يحدثها برنامج الجيوجبرا في تدريس الرياضيات فيما يتعلق بالرغبة المنتجة؟

2 .كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات ، من حيث كل مكون من مكونات الكفاءة الرياضية؟

I. كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات فيما يتعلق بالاستيعاب المفاهيمي؟
II. كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات فيما يتعلق. بالطلاقة الإجرائية؟
III. كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات فيما يتعلق بالكفاءة الاستراتيجية؟
IV. كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات فيما يتعلق بالتبرير المنطقي؟
V. كيف يمكن برنامج الجيوجبرا أن يحدث تغييرات في تدريس الرياضيات فيما يتعلق بالرغبة المنتجة؟

3 .ما هي وجهة نظرك حول تأثير استخدام برنامج الجيوجبرا في تدريس الرياضيات ؟

I. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث تسهيل التعلم؟
II. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث زيادة تأثير التعليم؟
III. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث عمل رسومات أفضل؟
IV. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث بقاء اثر التعلم افضل؟

V. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث تسهيل إدارة الفصول الدراسية؟

VI. ما هي وجهة نظرك حول تأثير استخدام الجيوجبرا على تعلم الرياضيات من حيث جعل الحصة الدراسية اثر متعة؟

Appendix 4 **Statistical tests**

Appendix 4-1 Statistical tests of Mathematical proficiency test 1 (MPT1) (Algebra)

1. Normality test

Normal Distribution Test of the MPT1

	Class	Kolmogorov-Smirnov^{a}			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.173	19	.138	.934	19	.203
	Class B	.169	18	.185	.935	18	.239

Normal Distribution Test of Procedural fluency of the MPT1

	Class	Kolmogorov-Smirnov^{a}			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.196	19	.054	.865	19	.012
	Class B	.238	18	.008	.929	18	.188

Normal Distribution Test of strategic competence of the MPT1

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.238	19	.006	.813	19	.002
	Class B	.286	18	.000	.819	18	.003

Normal Distribution Test of adaptive reasoning of the MPT1

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.195	19	.055	.912	19	.080
	Class B	.250	18	.004	.876	18	.022

2. T-test of MPT1

Independent Samples Test of the MPT1							
		Levene's Test for Equality of Variances		t-test for Equality of Means			
		F	Sig.	t	df	P-value (2-tailed)	Mean Difference
Scores	Equal variances assumed	.303	.585	.216	35	.830	.18056
	Equal variances not assumed			.217	34.989	.830	.18056

3. Mann-Whitney Test of the mathematical proficiency strands of the MPT1

Mann-Whitney U Test of conceptual understanding for MPT1

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	19.71	374.50
	Class B	18	18.25	328.50
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	157.500
Wilcoxon W	328.500
Z	-.435
Asymp. P-value (2-tailed)	.664
Exact P-value [2*(1-tailed Sig.)]	.685[b]

Mann-Whitney U Test of procedural fluency for MPT1

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	20.74	394.00
	Class B	18	17.17	309.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	138.000
Wilcoxon W	309.000
Z	-1.030
Asymp. P-value (2-tailed)	.303
Exact P-value [2*(1-tailed Sig.)]	.327[b]

Mann-Whitney U Test of strategic competence for MPT1				
Ranks				
	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	17.21	327.00
	Class B	18	20.89	376.00
	Total	37		

Test Statistics[a]	
	Scores
Mann-Whitney U	137.000
Wilcoxon W	327.000
Z	-1.091
Asymp. P-value (2-tailed)	.275
Exact P-value [2*(1-tailed Sig.)]	.313[b]

Mann-Whitney U Test of adaptive reasoning for MPT1

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	18.21	346.00
	Class B	18	19.83	357.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	156.000
Wilcoxon W	346.000
Z	-.467
Asymp. P-value (2-tailed)	.640
Exact P-value [2*(1-tailed Sig.)]	.663[b]

4. **Mann-Whitney U Test of productive disposition (before the intervention study)**

Mann-Whitney U Test of overall productive disposition

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	19.74	375.00
	Class B	18	18.22	328.00
	Total	37		

Test Statistics^a

	Scores
Mann-Whitney U	157.000
Wilcoxon W	328.000
Z	-.426
Asymp. P-value (2-tailed)	.670
Exact P-value [2*(1-tailed Sig.)]	.685^b

Mann-Whitney U Test of Students Like Learning Mathematics
(before the intervention study)

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	19.45	369.50
	Class B	18	18.53	333.50
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	162.500
Wilcoxon W	333.500
Z	-.259
Asymp. P-value (2-tailed)	.795
Exact P-value [2*(1-tailed Sig.)]	.799[b]

Mann-Whitney U Test of Students Confidence in Mathematics (before the intervention study)

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	19.68	374.00
	Class B	18	18.28	329.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	158.000
Wilcoxon W	329.000
Z	-.396
Asymp. P-value (2-tailed)	.692
Exact P-value [2*(1-tailed Sig.)]	.707[b]

Mann-Whitney U Test of Students Value Mathematics (before the intervention study)				
Ranks				
	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	20.32	386.00
	Class B	18	17.61	317.00
	Total	37		

Test Statistics	
	Scores
Mann-Whitney U	146.000
Wilcoxon W	317.000
Z	-.762
Asymp. P-value (2-tailed)	.446
Exact P-value [2*(1-tailed Sig.)]	.461^{b}

Appendix 4-2: Statistical tests of Mathematical Proficiency Test 2 (MPT2) (Numbers)

1. Normal Distribution Test

Normal Distribution Test for MPT2

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.110	19	.200*	.974	19	.855
	Class B	.140	18	.200*	.970	18	.789

Normal Distribution Test for the Conceptual Understanding strand for MPT2

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.261	19	.001	.884	19	.026
	Class B	.168	18	.192	.919	18	.124

Normal Distribution Test for the Procedural Fluency strand for MPT2

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.230	19	.009	.893	19	.037
	Class B	.294	18	.000	.853	18	.009

Normal Distribution Test for the Strategic Competence strand for MPT2

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.232	19	.008	.868	19	.013
	Class B	.222	18	.019	.860	18	.012

Normal Distribution Test for the Adaptive Reasoning strand for MPT2

	Class	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.331	19	.000	.738	19	.000
	Class B	.343	18	.000	.749	18	.000

2. T-test of Mathematical Proficiency Test 2 (MPT2)

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means			
		F	Sig.	t	df	P-value (2-tailed)	Mean Difference
Scores	Equal variances assumed	.346	.560	3.454	35	.001	3.59649
	Equal variances not assumed			3.448	34.463	.002	3.59649

3. Mann-Whitney U Test of MPT2 strands (Numbers)

Mann-Whitney U Test of Conceptual Understanding for MPT2

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	22.45	426.50
	Class B	18	15.36	276.50
	Total	37		

Test Statistics

	Scores
Mann-Whitney U	105.500
Wilcoxon W	276.500
Z	-2.034
Asymp. P-value (2-tailed)	.042
Exact P-value [2*(1-tailed Sig.)]	.046[b]

Mann-Whitney U Test of the Procedural Fluency strand for MPT2

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	24.11	458.00
	Class B	18	13.61	245.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	74.000
Wilcoxon W	245.000
Z	-3.074
Asymp. P-value (2-tailed)	.002
Exact P-value [2*(1-tailed Sig.)]	.003[b]

Mann-Whitney U Test of the Strategic Competence strand for MPT2

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	21.08	400.50
	Class B	18	16.81	302.50
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	131.500
Wilcoxon W	302.500
Z	-1.251
Asymp. P-value (2-tailed)	.211
Exact P-value [2*(1-tailed Sig.)]	.233[b]

Mann-Whitney U Test of the Adaptive Reasoning strand for MPT2

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	21.21	403.00
	Class B	18	16.67	300.00
	Total	37		

Test Statistics

	Scores
Mann-Whitney U	129.000
Wilcoxon W	300.000
Z	-1.372
Asymp. P-value (2-tailed)	.170
Exact P-value [2*(1-tailed Sig.)]	.210[b]

Appendix 4-3: Productive Disposition Questionnaire Class A

Overall productive disposition class A

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
ClassAafter - ClassAbefore	Negative Ranks	0[a]	.00	.00
	Positive Ranks	19[b]	10.00	190.00
	Ties	0[c]		
	Total	19		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter = ClassBbefore

Test Statistics[a]

	ClassAafter - ClassAbefore
Z	-3.824[b]
Asymp. P-value (2-tailed)	.000

Productive disposition questionnaire (Students Like Learning Mathematics) Class A

Ranks

		N	Mean Rank	Sum of Ranks
ClassAafter - ClassAbefore	Negative Ranks	0[a]	.00	.00
	Positive Ranks	19[b]	10.00	190.00
	Ties	0[c]		
	Total	19		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter = ClassBbefore

Test Statistics[a]

	Class A after – Class A before
Z	-3.826[b]
Asymp. P-value (2-tailed)	.000

Productive disposition questionnaire (Students’ Confidence in Mathematics) Class A

Ranks

		N	Mean Rank	Sum of Ranks
ClassAafter - ClassAbefore	Negative Ranks	1[a]	2.00	2.00
	Positive Ranks	17[b]	9.94	169.00
	Ties	1[c]		
	Total	19		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter = ClassBbefore

Test Statistics[a]

	ClassAafter – ClassAbefore
Z	-3.640[b]
Asymp. P-value (2-tailed)	.000

Productive disposition questionnaire (Students Value Mathematica) Class A

Ranks

		N	Mean Rank	Sum of Ranks
ClassAafter - ClassAbefore	Negative Ranks	0[a]	.00	.00
	Positive Ranks	19[b]	10.00	190.00
	Ties	0[c]		
	Total	19		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter = ClassBbefore

Test Statistics[a]

	ClassAafter - ClassAbefore
Z	-3.826[b]
Asymp. P-value (2-tailed)	.000

Appendix 4-4: Statistical tests of Mathematical proficiency test 3 (MPT3) (Geometry)

Normality test of MPT3

Tests of Normality

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.176	19	.123	.928	19	.157
	Class B	.213	18	.031	.947	18	.374

Normal Distribution Test of Conceptual Understanding for MPT3

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.197	19	.051	.881	19	.022
	Class B	.312	18	.000	.751	18	.000

Normal Distribution Test of Procedural Fluency for MPT3

		Kolmogorov-Smirnov^a			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.272	19	.001	.896	19	.040
	Class B	.261	18	.002	.889	18	.038

Normal Distribution Test of Strategic Competence for MPT3

		Kolmogorov-Smirnov^a			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.234	19	.007	.805	19	.001
	Class B	.260	18	.002	.807	18	.002

Normal Distribution Test of Adaptive Reasoning for MPT3

		Kolmogorov-Smirnov^a			Shapiro-Wilk		
	Class	Statistic	df	Sig.	Statistic	df	Sig.
Scores	Class A	.226	19	.012	.858	19	.009
	Class B	.248	18	.005	.816	18	.003

1. T-test of mathematical proficiency test 2 for MPT3

Independent Samples Test for the MPT3

		Levene's Test for Equality of Variances		t-test for Equality of Means			
		F	Sig.	t	df	P-value (2-tailed)	Mean Difference
Scores	Equal variances assumed	3.019	.091	-2.656	35	.012	-2.19883
	Equal variances not assumed			-2.686	31.063	.012	-2.19883

2. Mann-Whitney U Test

Mann-Whitney U Test of the Conceptual Understanding strand for MPT3

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	16.37	311.00
	Class B	18	21.78	392.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	121.000
Wilcoxon W	311.000
Z	-1.592
Asymp. P-value (2-tailed)	.111
Exact P-value [2*(1-tailed Sig.)]	.134[b]

Mann-Whitney U Test of the Procedural Fluency strand for MPT3

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	14.29	271.50
	Class B	18	23.97	431.50
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	81.500
Wilcoxon W	271.500
Z	-2.800
Asymp. P-value (2-tailed)	.005
Exact P-value [2*(1-tailed Sig.)]	.006[b]

Mann-Whitney U Test of the Strategic Competence strand for MPT3

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	17.26	328.00
	Class B	18	20.83	375.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	138.000
Wilcoxon W	328.000
Z	-1.073
Asymp. P-value (2-tailed)	.283
Exact P-value [2*(1-tailed Sig.)]	.327[b]

Mann-Whitney U Test of the Adaptive Reasoning strand for MPT3

Ranks

	Class	N	Mean Rank	Sum of Ranks
Scores	Class A	19	17.32	329.00
	Class B	18	20.78	374.00
	Total	37		

Test Statistics[a]

	Scores
Mann-Whitney U	139.000
Wilcoxon W	329.000
Z	-1.003
Asymp. P-value (2-tailed)	.316
Exact P-value [2*(1-tailed Sig.)]	.343[b]

Appendix 4-5: Productive disposition questionnaire Class B

Overall productive disposition Class B

Wilcoxon Signed Ranks Test

Ranks

		N	Mean Rank	Sum of Ranks
ClassBafter - ClassBbefore	Negative Ranks	0[a]	.00	.00
	Positive Ranks	18[b]	9.50	171.00
	Ties	0[c]		
	Total	18		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore

c. ClassBafter = ClassBbefore

Test Statistics[a]

	ClassBafter - ClassBbefore
Z	-3.725[b]
Asymp. P-value (2-tailed)	.000

Productive disposition questionnaire (Students Like Learning Mathematics) Class B

Ranks

		N	Mean Rank	Sum of Ranks
ClassBafter - ClassBbefore	Negative Ranks	2[a]	1.50	3.00
	Positive Ranks	16[b]	10.50	168.00
	Ties	0[c]		
	Total	18		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore

c. ClassBafter = ClassBbefore

Test Statistics[a]

	ClassBafter - ClassBbefore
Z	-3.595[b]
Asymp. P-value (2-tailed)	.000

Productive disposition questionnaire (Students' Confidence in Mathematics) Class B

Ranks				
		N	Mean Rank	Sum of Ranks
ClassBafter - ClassBbefore	Negative Ranks	1[a]	9.00	9.00
	Positive Ranks	17[b]	9.53	162.00
	Ties	0[c]		
	Total	18		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter =ClassBbefore

Test Statistics[a]	
	ClassBafter - ClassBbefore
Z	-3.336[b]
Asymp. P-value (2-tailed)	.001

Productive disposition questionnaire (Students Value Mathematics) Class B

Ranks

		N	Mean Rank	Sum of Ranks
ClassBafter - ClassBbefore	Negative Ranks	0[a]	.00	.00
	Positive Ranks	18[b]	9.50	171.00
	Ties	0[c]		
	Total	18		

a. ClassBafter < ClassBbefore b. ClassBafter > ClassBbefore c. ClassBafter =ClassBbefore

Test Statistics[a]

	ClassBafter - ClassBbefore
Z	-3.725[b]
Asymp. P-value (2-tailed)	.000

Appendix 5 **Plain language statement and consent forms**

Appendix 5-1. Plain Language Statement – Teachers

University of Glasgow

College of Social Sciences

Plain Language Statement – Teachers

Title of project and researcher details

Investigating the Effectiveness of Using GeoGebra on Students' Mathematical Proficiency

Researcher: Mr Bakri Awaji

Supervisors: Professor Victor Lally and Dr Rebecca Mancy

Course: PhD of Education

Invitation

You have been invited you to take part in this research study. Before you decide whether you would like to take a part it is important for you to understand why the research is being done and what it will involve.

Please take the time to read the following information carefully and discuss it with me if you wish. Please feel free to ask questions about anything about which you are unclear. If you would like to have more information, please contact me. Please take your time to consider whether you wish to take part.

The purpose of the study

The purpose of this study is to investigate the effectiveness of using GeoGebra on students' mathematical proficiency. Mathematical proficiency is defined by Kilpatrick, Swafford, and Findell (2001) as consisting of five stands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. To do this, I will elicit students' and teachers' views of learning mathematics using GeoGebra and identify how this software can change teacher practices in the classroom. The study will also determine any difficulties students encounter in answering pre-tests and post-tests.

Why have I been selected?

a. (All 15–20 teachers) for the professional development course (PDC).

You have been selected because you already filled in the online questionnaire and you already volunteered to participate in the professional development course (PDC).

b. (5-8 teachers from the 15 to 20 teachers above) for the interviews

You have been selected because you already filled in the online questionnaire and you already volunteered to participate in the interview with the researcher three times (before, during and after the (PDC).

c. One teacher from the (5–8 teachers above)

You have been selected because you already filled in the online questionnaire and you already volunteered to participate in the intervention study to teach students using GeoGebra software. In addition your school already has good technological facilities which is required in the implementation of this study

How participants' personal details will be kept confidential

Data will be gathered and coded by numbers or letters rather than the names of participants, and once the data has achieved its purpose, it will be destroyed. Access to computer files will be available by password only and, after analysis, the data will be destroyed in the presence of the researcher and the supervisors.

Please note that assurances on confidentiality will be strictly adhered to unless evidence of wrongdoing or potential harm is uncovered. In such cases the University may be obliged to contact relevant statutory bodies/agencies.

Do I have to take part?

It is up to you to decide whether or not to you wish to take part. If you decide to take part, you are still free to withdraw any time and you do not need to provide a reason. Please also be aware that if you decide not to be participated, this will not affect your evaluation in any way or have any other detrimental effect in respect of your relationship with others in the school.

What will happen to me if I take part?

a. (All 15–20 teachers)

You will be invited to participate in a professional development course for five days. Four days together and one day after 4 or 5 weeks. The course will be designed to help you using GeoGebra software in your classroom effectively.

b. (5–8 teachers from the 15 to 20 teachers above) for the interviews

Because you already volunteered to participate in the interview with the researcher three times (before, during and after the (PDC), you will invited to have three interviews with the researcher (before, during and after the (PDC). Each session will take approximately 20 to 40 minutes.

c. One teacher from the (5–8 teachers above)

Because you already volunteered to participate in the intervention study, you will teach students (two grade 8 classes) for two units, one using GeoGebra and one the normal way (the approach normally taken).This will take 6–8 weeks.

Organisation funding the research

Saudi Cultural Bureau

What will happen to the results of the research study?

All results will be part of the researcher's PhD thesis. In addition some parts of the results may be published as a journal article, conference paper, or a book. So, if requested, you will receive a copy of the result of this research study as well as research findings. You may also receive a copy of my thesis or any published item if you so requested it.

Who will review this study?

This project has been considered and approved by the College of Social Sciences Research Ethics Committee at the University of Glasgow, UK.

Contact for further information

For further information, please contact me (b.awaji.1@research.gla.ac.uk). Alternatively, you may contact my supervisors, Professor Victor Lally (victor.lally@glasgow.ac.uk, telephone: 0141 3303424) or Dr Rebecca Mancy (rebecca.mancy@glasgow.ac.uk, telephone: 0141 330 3560) at the University of Glasgow.

If you have any concerns regarding the conduct of this research project, you can contact the College of Social Sciences Ethics Officer, Dr Muir Houston, email: Muir.Houston@glasgow.ac.uk

Appendix 5-2: Plain Language Statement – Students

University of Glasgow

College of Social Sciences

Plain Language Statement – Students

Title of project and researcher details

Investigating the Effectiveness of Using GeoGebra on Students' Mathematical Proficiency

Researcher: Mr Bakri Awaji

Supervisors: Professor Victor Lally and Dr Rebecca Mancy

Course: PhD of Education

Invitation

I would like to invite you to take part in this research study. Before you decide whether you would like to take a part it is important for you to understand why the research is being done and what it will involve.

Please take the time to read the following information carefully and discuss it with me if you wish. Please feel free to ask questions about anything about which you are unclear. If you would like to have more information, please contact me. Please take your time to consider whether you wish to take part.

Do I have to take part?

It is up to you to decide whether or not to you wish to take part. If you decide to take part, you are still free to withdraw any time and you do not need to provide a reason.

Please also be aware that if you decide not to participate, this will not affect your school marks in any way or have any other detrimental effect in respect of your relationship with others in the school.

What will happen to me if I take part?

The purpose of this study is to investigate the effect on mathematics learning. So, if you decide to take part, you will take a pre-test before we start. Then your teacher will teach your class for two units, one unit using GeoGebra and one unit by the normal approach. You will also take two post-tests after each unit.

None of the three tests (pre- and two post-tests) will affect your grades at any point. You do not have to answer any questions that you don't want to.

Also, you may be selected or volunteer to participate in a focus group to describe your experiences and also in the focus group participation will not affect your grades. In addition, you do not have to answer any questions that you don't want to.

What will happen to the results of the research study?

All results will be a part of the researcher's PhD thesis. In addition some parts of the results may be published as a journal article, conference paper, or a book. So, if requested, you will receive a copy of the result of this research study as well as research findings. You may also receive a copy of my thesis or any published item if you so requested it.

How participants' personal details will be kept confidential

Data will be gathered and coded by numbers or letters rather than the names of participants, and once the data has achieved its purpose, it will be destroyed. Access to computer files will be available by password only and, after analysis, the data will be destroyed in the presence of the researcher and the supervisors.

Please note that assurances on confidentiality will be strictly adhered to unless evidence of wrongdoing or potential harm is uncovered. In such cases the University may be obliged to contact relevant statutory bodies/agencies.

Who will review this study?

This project has been considered and approved by the College of Social Sciences Research Ethics Committee at the University of Glasgow, UK

Contact for further information

For further information, please contact me. Alternatively, you may contact my supervisors, Professor Victor Lally (victor.lally@glasgow.ac.uk, telephone: 0141 3303424) or Dr Rebecca Mancy (rebecca.mancy@glasgow.ac.uk, telephone: 0141 330 3560) at the University of Glasgow.

If you have any concerns regarding the conduct of this research project, you can contact the College of Social Sciences Ethics Officer, Dr Muir Houston, email: Muir.Houston@glasgow.ac.uk

Appendix 5-3 : Consent Form/teachers

University of Glasgow

College of Social Sciences

Title of Project

Investigating the Effectiveness of Using GeoGebra on Students' Mathematical Proficiency

Name of Researcher: Bakri Awaji

Consent Form

1. I confirm that I have read and understand the Plain Language Statement for the above study and have had the opportunity to ask questions.
2. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving any reason.
3. I hereby consent to the interview being audio-taped.

Confidentiality/anonymity

I acknowledge that all participants will be referred to by pseudonym.

Dependent relationships

I acknowledge that there will be no effect on my grades/employment arising from my participation or non-participation in this research.

Data usage and storage

- The material will be treated as confidential and kept in secure storage at all times.
- The material will be destroyed once the project is complete.
- I agree to waive my copyright to any data collected as part of this project.

CONSENT

I agree to take part in this research study ☐

I do not agree to take part in this research study ☐

Name of Participant. ………………………….Signature ……………. Date ……………

Name of Researcher ………………………………Signature ………………………Date …………

End of consent form

For further information, please contact me Mr Bakri Awaji
, or my supervisors Prof Victor Lally at the University of Glasgow
(victor.lally@Glasgow.ac.uk , telephone: 0141 330 3036 , or Dr Rebecca Mancy at the University of Glasgow (rebecca.mancy@glasgow.ac.uk) , telephone: 0141 330 3560)

If you have any concerns regarding the conduct of this research project you can contact the College of Social Sciences Ethics Officer, Dr Muir Houston, email: Muir.Houston@glasgow.ac.uk

Appendix 5-4 : Consent Form/ students

College of Social Sciences

Title of Project

Investigating the Effectiveness of Using GeoGebra on Students' Mathematical Proficiency

Name of Researcher: Bakri Awaji

Consent Form

1. I confirm that I have read and understand the Plain Language Statement for the above study and have had the opportunity to ask questions.
2. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving any reason.
3. I hereby consent to the focus group being audio-taped (if I volunteered to take part).

Confidentiality/anonymity

I acknowledge that all participants will be referred to by pseudonym.

Dependent relationships

I acknowledge that there will be no effect on my grades/employment arising from my participation or non-participation in this research.

Data usage and storage

- The material will be treated as confidential and kept in secure storage at all times.
- The material will be destroyed once the project is complete.
- I agree to waive my copyright to any data collected as part of this project.

CONSENT

I agree to take part in this research study ☐

I do not agree to take part in this research study ☐

Name of Participant…………………………………… Signature ……………. Date ……………

Name of Parents of students or Carer ……………………Signature …. Date ………………………

Name of Researcher ………………Signature …………………………………Date ……………………

End of consent form

For further information, please contact me (b.awaji.1@research.gla.ac.uk). Alternatively, you may contact my supervisors, Prof. Victor Lally (victor.lally@glasgow.ac.uk, telephone: 0141 3303424) or Dr Rebecca Mancy (rebecca.mancy@glasgow.ac.uk, telephone: 0141 330 3560) at the University of Glasgow.

If you have any concerns regarding the conduct of this research project, you can contact the College of Social Sciences Ethics Officer, Dr Muir Houston, email: Muir.Houston@glasgow.ac.uk