# An Empirical Essay in Forex Market

by

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# Abstract

The Forex market is the largest financial market globally, with daily trading volumes of about 6.6 trillion U.S. dollars per day in April 2019 (Bank for International Settlements, (BIS) 2019). The forex market always attracts the view of many researchers. This doctoral thesis particularly pays attention to some popular forex market questions.

The first chapter provides a comprehensive review of recent developments of forex market researches. The first section of the chapter briefly introduces the currencies-specific pricing factors and the forex market portfolio management. The second part gives an introduction to the risk forecasting model in the financial market. Finally, I will illustrate the concept of the forward premium puzzle and interest rate parity in the last part and review the literature on the forward premium puzzle.

In the second chapter, I focus on the prevalent forex factors' tail dependence. Lustig et al. (2011) have introduced some forex factors, the 'dollar risk factor' (DOL) and the 'carry trade factor' (HML), and shown that they can price carry trade portfolios in the cross-section. This new result is helpful in the academic literature on cross-sectional asset pricing, risk management, and portfolio optimization. Those factors are also widely used in the industry. This thesis tests the relevance of four popular currency-specific factors contributing to a diversified forex portfolio and risk management. I show that modeling non-linear dependency across the factors is essential and adds value to a forex portfolio.

Then, I concentrate on the risk forecasting of the forex factor portfolios, which I discussed in the previous chapter. Risk forecasting is a popular research question in the financial market. Value at Risk (VaR) is the standard measure of risk, which is the common risk measure when forecasting the risk. Meanwhile, the expected shortfall (ES) is the conditional expectation of exceeding beyond the VaR. I will apply the Copula to improve the risk forecast model of Patton et al. (2019) to forecast the VaR and ES of factors portfolio returns. I apply the goodness-of-fit test and the Diebold-Mariano tests to evaluate the performance of the forecasting models. The results show that the Copula multivariate dynamic forecasting models have their benefit when estimating the future risk.

In the last chapter, I present a model, following Burnside et al. (2009), which apply the adverse selection problem between a market maker and trader rationalized, to discuss the negative covariance between the forward premium and spot rate change. I first apply the unique order flow data set to estimate Burnside et al. (2009) model. Then, I creatively discuss the bond and spot exchange transactions to explain the excess return of carry trade. According to the estimation results, I could conclude that the adverse selection could generate the forward premium puzzle.

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# Dedication

I dedicate this dissertation to my parents Hongwei Li and Leping Zhao, for their constant support and unconditional love.

# Declaration

I declare that this paper is the result of my work and has not been submitted for any other degree at the University of Glasgow or any other institution, except that it explicitly mentions the contributions of others.

Signature:

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# Introduction

There are various popular research questions in the forex market area, such as carry trade, forward premium, the pricing factor, market efficiency, etc. In this thesis, I focus on the forex market to discuss recent exciting questions. In this thesis, I focus on three crucial forex market research questions: i) whether the joint distribution among the forex factors is significant asymmetry; ii) whether applying the Copula model can add the forecasting ability of risk or not; iii) whether the adverse selection can explain the forward premium puzzle or not.

I discuss the first question in chapter 2. The motivations of this chapter are shown below. Firstly, the factor portfolios are correlated with each other linearly and non-linearly in the forex market. For instance, the HML factor and VAL factor in the forex market have an endogenous relationship. HML factor is related to the interest rate, while the VAL factor is related to the country's price level. The interest rate has a strong correlation with the price level. Secondly, the non-linear relationship between those risk factors has still not been studied in detail. In comparison, the popularity of the forex factors has grown exponentially, not only in academic literature but also among practitioners. Very little of this topic has been done from the perspective of forex asset allocation. Finally, there is a distinct difference between stock market factors and forex market factors. Although the forex market is the largest financial market with the highest daily trading volumes, it has a relatively much smaller number of commodities than the stock market. This leads to the different factors' similar picking, which may cause a higher correlation between the forex factors.

I have several research objects to test the benefit from modeling the dependence structure of the currency factors. First, I will apply the threshold correlation to show the evidence of the non-linearity among the factors. Then, I apply the Copula to model the tail dependence and show more evidence of the asymmetry among the factors' joint distribution. Finally, I apply the Copula model in the real-time investment to test the economic value of the forex portfolio. Chapter 2 has three main contributions. First, the non-linear relationship among the currency pricing factors is still a gap as I discussed above. I provide a comprehensive study of the dependence structure among the forex market's pricing factors. I show strong evidence that estimating the tail dependence of the factors could help me avoid underestimating the risk and adding economic value in the portfolios. Second, I show the empirical evidence that adding the asymmetry could improve the forex portfolios' performance, compared with the linear correlation assumption. Note that the linear correlations among the forex factors are higher than the equity factors. Finally, I test the new investment strategies in forex assets, which provide a new view to forecasting the exchange rate.

As for the second research question, I provide a detailed discussion in chapter 3 . This question is motivated by the below reasons. Patton et al. (2019) show the new GAS risk forecasting model with a new risk measure. Some authors apply the benefit of the Copula in risk forecasting. Applying the Copula to extend the GAS risk forecasting model from the univariate model to the multivariate model is interesting. Second, in portfolio management, risk could come from a multi-asset structure. The univariate risk forecasting model may ignore the joint risk between assets, which would lead to the underestimation of the risk. Hence, I combine the Copula and risk forecasting models to forecast the risk of the portfolios in the multivariate model. What's more, I also want to test the non-linear tail dependence in the risk forecasting area.

The objects of the second research question are as follow: i) apply the univariate model from Patton et al. (2019) to forecast the risk of forex factor portfolios; ii) apply the Copula model to improve the model from univariate model to multivariate model; iii) compare the new model with the model from Patton et al. (2019) by the goodness-of-fit test and the Diebold-Mariano tests. The results of the performance confirm the benefit of the Copula model in risk management. Moreover, the new risk forecasting model also has a significant contribution for the industry when they want to estimate the future risk of the portfolios.

The two reasons for researching the third question are discussed below. First, Burnside et al. (2009) apply a microstructure approach to understanding the forward premium puzzle. However, they only discuss the model and the reason for the forward premium puzzle without estimates of the model. I apply the order flow data, which appreciates the adverse selection model, to find the estimations of the parameters. I try to find the reason for the negative correlation between forward premium and change of spot rate. Second, I add the bond market in the microstructure model, which includes three different markets (forward market, spot market, and bond market). Using adverse selection, I apply the linkage among these markets to discuss the UIP and CIP in the new version.

In chapter 4, I first apply the simple regression to show the evidence of the forward premium puzzle. Then, I apply two methods to estimate Burnside et al. (2009) microstructure model from the order flow data. Furthermore, I also add a new market in the microstructure model and increase the number of participants to improve the model. I estimate the microstructure model from Burnside et al. (2009) from the actual market order flow data, which gives strong support and evidence to Burnside et al. (2009) on forward premium puzzle explanation. Moreover, I also discuss the main reason for the failure of the interest rate parity, which contributes to the forex market research.

This thesis will be structured as follows: chapter 1 discusses the relative literature of the research questions in this thesis; chapter 2 introduces the first question and apply the empirical method to show the importance of the tail dependence among the forex factors; chapter 3 focuses on the second research question and propose a new multi-dimension risk forecasting model; the forward premium puzzle would be discussed in chapter 4; chapter 5 provides the main conclusion of the thesis.

# Chapter 1

# Literature Review of Forex Market

In this chapter, I discuss the empirical literature related to the research questions in this thesis. A brief introduction of the forex market's prevalent risk factors is given in the first section. I review the risk forecasting model of the financial market in the next section. The studies of the forward premium puzzle are shown in the last section.

## 1.1 Forex Market Asset Pricing and Portfolio Management

Finding new pricing factors to explain the excess return in financial markets is a popular research question. These factors can help investors to create mimicking portfolios and adjust the risk exposure of their portfolios. Many factors have been discussed in the forex market, such as DOL(dollar risk factor), HML (high minus low factor or carry trade factor), VOL (volatility factor), SKW (skewness factor), MOM (momentum factor), and VAL (value factor). These factors are created in the portfolio returns format, which is homogeneous with the regression of the excess return. My research essentially focuses on the portfolio analysis of the dependence structure of the forex factors. In this section, I first introduce prevalent factors in the forex market. Then, I discuss the empirical literature of the portfolios analyses in the forex market.

### 1.1.1 Risk Factors in the Foreign Exchange Market

The risk factor could be treated as the specific risk exposure of the investors in the financial market. The risk factors are often nearly orthogonal since the risk factors often have a single kind of risk exposure. Hence, the authors apply the risk factors to explain the excess returns in the forex market in linear regression (see Fama and French (1993)). As the largest financial market, it is interesting to find the currency-specific risk factors which could help the currency investors to adjust the specific risk exposure and explain the excess return. There are many factors in the forex market toward different risks.

Historically, research investigating the risk factors associated with the forex market focused on applying the heterogeneity in exposure to the risk, which can explain the carry trade returns. Lustig et al. (2011) try to separate the risk premium of the currency market into two main factors: country-level risk and global risk factors (carry trade risk). To build upon these factors, Lustig et al. (2011) propose the currency portfolios sorted by forward discounts. The country's specific risk factor could be treated as a level factor. Lustig et al. (2011) form the home risk premium by averaging the cross-section of foreign currency excess returns. As for the global risk factor, they argue that this is a slope factor, which can also be treated as the carry trade risk premium. The HML factor could be calculated from the return of high-yielding currencies without the return of low-yielding ones. Lustig et al. (2011) apply those two risk factors to explain the excess returns from carry trade. About two-thirds of the cross-sectional variation could be explained by these factors when they allow for time variation in the betas of individual currencies.

In contrast, Menkhoff et al. (2012b) propose a new factor to explain the excess return of the momentum strategy. Following Jegadeesh and Titman (1993), they argue for a new factor (momentum factor) in the forex market. They indicate that the forex market is more liquid with larger transaction volumes and lower transaction costs than the stock market. Therefore, the forex market will have large amounts of excess returns from momentum strategies. They find that momentum strategy returns are affected by the transaction costs. The higher transaction costs and large turnovers of the momentum portfolio would lead to lower profitability. However, transaction costs cannot fully account for economic momentum returns. The forex market is different from the stock market. However, momentum factor has similar properties in those two markets (stock market and forex market). The reason is that the momentum profits of different asset classes still have common roots. Both stock and forex markets have a similar explanation for the high excess return of the momentum factors, which is the high transaction cost. The practical barriers that limit the deployment of arbitrage capital can also explain the momentum persists. Menkhoff et al. (2012b) find those currency momentum strategies are risky because their returns are not robust in the short term, and their exposures have to bear basic investment risks, which are reflected by the characteristics of the currencies.

Ang et al. (2006) propose the volatility factor in the equity market. Following Ang et al. (2006), Menkhoff et al. (2012a) test the relationship between the returns of currency portfolios with the linear framework and the sensitivity of excess returns to global forex volatility risks. They find that there is a negative correlation between high-interest-rate currencies and innovations in global foreign exchange volatility. Therefore, when low-interest-rate currencies are related to the unexpectedly high volatility. In other words, arbitrage trading performs particularly poorly during market shocks, so that it can rationalize its high returns from the perspective of standard asset pricing. Menkhoff et al. (2012a) show that the excess returns of arbitrage trading compensation for the risks during the time-varying.

To explain the excess return on carry trade in another way, Burnside et al. (2008) focus on the skewness realized at the individual currency level. Following Burnside et al. (2008), Rafferty (2012) extends the individual currency to cross-section focusing on the global currency skewness factor. Rafferty (2012) points out that the asymmetry in the distribution of excess currency returns is important when considering the forex risk premium. Currency portfolios that have a bad return require a lower risk premium. Conversely, the expected return of a currency portfolio with good returns during this period is negative because investors are prepared to use it as a hedge.

Asness et al. (2013) test the excess return of MOM and VAL factors in eight different asset market, including the forex market. Kroencke et al. (2014) argue that the economically large and significant benefits of diversification are from style-based management of the forex component of international investments carry trade, momentum, and value strategies. Researching on the risk factors in the forex market, Kroencke et al. (2014) propose a detailed calculation of factor VAL (value strategy), which depends on the price level of consumer goods expressed in national currency.

In comparison, my thesis does not focus on proposing a new forex factor to explain the excess return and risk of the currencies. I apply the existing important forex factors above to estimate the joint distribution among the factors, which could help me avoid underestimating the risk when we apply the factors to adjust the specific risk.

### 1.1.2 Portfolio Analysis and Dependence Structure in the Forex Market

Fama and French (1993) apply the factors to explain the expected stock returns and bond returns. To build the risk factor as the inputs in the regression, they use mimicking to create portfolios that replicate each factor. Fama and French (1993) found three factors that could explain the cross-section of stock returns, which include a wide range of market premiums (Market factor), the spread between small and large market value stocks (HML factor), and the relationship between value and growth (Value factor). Although the risk factors have been proven to explain the excess returns in equity cross-sections, there is still a non-linear correlation between those factors that would affect the regression's explanatory power. Christoffersen and Langlois (2013) use Copula to focus on the non-linear joint distribution to measure the co-movement across popular equity factors. They believe that the factor portfolios are valuable and popular because they are nearly orthogonal to each other. Hence, the factor portfolios can be applied to explain the cross-section returns in the equity market. According to their low linear correlations, the regression would be more precise when researchers apply these popular factor portfolios to price the market return. Although it is beneficial to have orthogonal factors, it is still dangerous to focus only on linear correlations. Christoffersen and Langlois (2013) show that the dependence structure of factors is empirically important. It will lead to an underestimation of extreme risks and sub-optimal portfolio allocation when the investors ignore the non-linear correlation between the factors.

Christoffersen and Langlois (2013) apply market, size, and value pricing factors, following Fama and French (1993), to test the dependence structure. Furthermore, the momentum factor, argued by Jegadeesh and Titman (1993), is also included by Christoffersen and Langlois (2013). Therefore, Christoffersen and Langlois (2013) focus on the tail dependence of four famous and orthogonal equity market pricing factors by modeling their joint distribution. First, they apply the threshold correlation to verify the non-linear correlation existing between the four orthogonal factors. Then, they apply the Copula to model the joint distribution and test the Copula model performance by investing in the real market. Christoffersen and Langlois (2013) prove a non-linear correlation between the various factors, and they use the Copula implied by the multi-dimensional skewed t distribution for modeling. The Copula can model the asymmetric non-linear dependence across the equity factors while retaining the relatively moderate linear correlation found in the factors return data. By contrast, I focus on the forex market non-linear correlation between the currency-specific factors.

The Copula model has been widely used to estimate the dependence structure among the currencies. Patton (2006b) applies the conditional Copula to show that the mark-dollar and yen-dollar exchange rates have a higher correlation when depreciating than appreciating. Central banks can induce this asymmetry to exchange rate movements. Scotti and Benediktsdottir (2009) follow Patton (2006b) to focus on the tail dependence between currencies using similar Copula models. Furthermore, they discuss the effect of the business cycle and interest rate differentials on the dependence structures. Hurd et al. (2007) use the marginal distribution, implied by the option function, to model the joint distribution of the euro against the dollar and the dollar against the pound. These marginal distributions satisfy the triangular no-arbitrage through the Copula (a non-parametric dependent function of the Bernstein Copula) condition. Bouyé and Salmon (2009) derive the implicit (non-linear) form of conditional quantile relations of currencies (dollar-yen, dollar-sterling, and dollar-DM), which can be achieved by assuming an arbitrary distribution of margins. Albulescu et al. (2018) use daily data from 1999 to 2014 to study the bivariate dependence structure of four international currencies (Euro, British Pound, Canadian Dollar, and Japanese Yen) against the US dollar. They find that the tail dependence is positive or negative, which depends on the changes over time. Cubillos-Rocha et al. (2019) study the exchange rate dependence of seven countries from four different regions of the world and find evidence of currency exchange rate contagion. This thesis proposes a study on the dependence structure of currency-specific currencies.

Many researchers find the dependence structure between the forex market and other markets (e.g. the stock market). Wang et al. (2013) propose a dependence conversion correlation model to test the tail dependence between the stock and forex market. They believe that it may not be appropriate to analyze cross-market linkages within a time-varying Copula framework. Aloui et al. (2013) use the Copula-GARCH method to estimate the joint distribution between oil prices and the US dollar exchange rate. During the period 2000-2011, they find that almost all pairs of oil exchange rates under consideration have significant and symmetrical dependence. They find that the increase in oil prices is positively correlated with the depreciation of the dollar. Based on the dynamic Copula method, Wang et al. (2014) propose a method based on time-varying correlation networks to study the dynamics of forex networks. Mensi et al. (2017) study the Middle East, North Africa and other developing and developed countries with different terms dependence structures between oil and forex market currencies. The Copula results show strong evidence that the time-varying and highly average (tail) dependence between oil yields and the forex market is significant in the short and medium time frames. Kumar et al. (2019) use the Copula model of dependence conversion to study the structure of the dependence relationship between BRICS stocks and forex markets. In comparison, I just estimate the non-linear correlation of the factors in the forex market.

Some researchers focus on improving the Copula model. Liu et al. (2017) propose a new time-varying optimal Copula (TVOC). They find evidence that the dependence structure between different markets (including the forex market) will change over time, while emergencies are usually the main reason for sudden changes in the dependence structure. Karmakar (2017) investigates the dependence structure and estimated portfolio risk based on the Indian forex market data. He applied AR-t-GARCH-EVT model to the marginal distribution of each currency in the five currency return series. Wang et al. (2020) use the dynamic hybrid Copula extreme theory (DMC-EVT) to propose a new method for studying financial contagion and contagion channels. This method helps them to clarify the complex and dynamic tail dependence. I follow Christoffersen et al. (2013) to apply the DCC-Copula model to estimate the joint distribution.

My research is also related to Barroso and Santa-Clara (2015b), although its main contribution is on carry trade and momentum in optimizing the equity market. They find that optimized portfolio performance could be explained by carry, momentum, and reversal. Barroso and Santa-Clara (2015b) simply measure the long-short portfolio based on the previous six months' realized volatility and target the constantly volatile strategy. Adjusting the portfolio to have constant volatility over a while is a more natural way to implement this strategy, rather than maintaining a constant number of long and short positions with varying volatility. This is widely accepted in the industry and practice, and it is more common to target ex-ante volatility than to use a constant leverage ratio. After managing the momentum, the Sharpe ratio has a dramatic increase. Furthermore, the risk has been managed at a low level during the crash period. Barroso and Santa-Clara (2015b) also explain excess returns and risk hedging when managing risk, with realized variances in momentum. They illustrate this because the specific risk is more persistent and predictable than the market component. They explain the excess return and risk of currencies by the momentum strategy. However, the specific factors with specific risks also have non-linear correlations, which may have a higher risk than we expected. Hence it is necessary to study the joint distribution of the currency factors.

Jordà and Taylor (2012) show that the different strategies would be negatively impacted by the 2008 financial crisis, including the carry trade strategy. They test the carry trade momentum strategies of the exchange rate and indicated that this strategy could generate positive returns in both in-sample and out-of-sample. Jordà and Taylor (2012) indicated that the simple carry trade has been profitable for a long time. However, out-of-sample analysis shows that the rate of return during the financial crisis time became negative. The global financial crisis has yielded many investment strategies, and arbitrage trading is not excepted. From this perspective, the problem with arbitrage trading seems to be eased at least. I also show similar finding in this thesis. The investment strategies based on the Copula models are also influenced by the financial crisis. However, they can still outperform the simple diversified strategy.

### 1.1.3 Research Gap of the Tail Dependence of Forex Factors

Researchers often focus on discovering new factors to explain the excess return in the asset pricing research field. There are different factors, for instance, which could price returns in the forex market, such as DOL, HML, DVOL, SKW, VAL, and MOM (Lustig et al. (2011); Menkhoff et al. (2012b); Rafferty (2012); Kroencke et al. (2014), and Menkhoff et al. (2012a)). These factors are prevalent and widely applied in risk exposure management in the forex market. However, the literature focuses on applying those factors to explain the excess return linearly. The non-linearity and joint distribution among the pricing factors would lead to underestimating the risk when applying factor portfolios. The non-linear relationship between those risk factors has still not been studied in detail.

In the past decades, numerous studies discuss the dependence structure and portfolio management in the forex market. The dependence structure in the forex market is a popular research question (see, for example, Patton (2006b); Scotti and Benediktsdottir (2009); Hurd et al. (2007); Bouyé and Salmon (2009); Wang et al. (2013); Aloui et al. (2013); Liu et al. (2017); Karmakar (2017); Mensi et al. (2017); Albulescu et al. (2018); Cubillos-Rocha et al. (2019); Kumar et al. (2019); Wang et al. (2020)). Some authors have discussed pricing factors in the forex market in the portfolio management (see Barroso and Santa-Clara (2015b); Jordà and Taylor (2012)). Surprisingly, the joint distribution of the pricing factors in the forex market with portfolio management still has a research gap.

Overall, Copula has been extensively used in the forex market to model the joint distribution among currencies or currency and other kinds of the asset. However it is still a gap to apply the Copula to model the tail dependence of the popular currency-specific factors. It is necessary and exciting to find the non-linear relationship among the forex factors, which is essential for asset pricing in the forex area. I try to apply the Copula to fill this gap in chapter 2.

## 1.2 Forex Market Risk Forecasting

Risk is an integral part of portfolio management. Markowitz (1952) proposes variance, which is the deviation from the mean of the return distribution, to measure risk. Standard deviation is the square root of the variance, which is often treated as the portfolio's volatility in the financial market. Considering the risk when estimate the utility of the investor, several new risk measures have been proposed to evaluate the risk in different situations. Morgan (1994) introduces a new risk measure, value at risk, which shows the loss in a certain period with a given probability. Embrechts et al. (1999) introduce a concept with the k-expected shortfall or k-tail mean. Following this concept, Uryasev (2000) proposes the conditional value at risk, the expected value of the losses exceeding the VaR. Acerbi and Tasche (2002) indicate that the expected shortfall (ES) coincides with that of the conditional value at risk in the case of continuous random variables. With the risk measures proposed by the existing discourse, authors begin to apply different models to forecast the future risk measures discussed above. In this section, I discuss the risk forecasting model in the forex market. First, I give a short introduction to the univariate risk forecasting model. Then, I discuss the multivariate risk forecasting model.

### 1.2.1 Univariate Risk Forecasting Model

Some authors focus on volatility forecasting in the financial market. West et al. (1993) applied a unit function to forecast the volatility by the maximum utility for the portfolios with five different currencies. They applied the univariate models to estimate the volatility of each currency using the mean-variance utility, following Markowitz (1952). The models include homoscedastic GARCH autoregressive and non-parametric models. The GARCH model has the best performance among these univariate models. Noh et al. (1994) applied the ARCH and GARCH models to forecast the volatility in the option market. They also find the benefit of GARCH when applying the GARCH model to forecast volatility. Lopez (2001) converts the volatility forecasts into probability forecasts of related events and evaluates them using selected scoring rules and calibration tests. He proposes a framework for volatility forecast univariate model evaluation based on probability scoring rules. González-Rivera et al. (2004) analyze the ability to forecast the different volatility models' performance in the stock market. They evaluate 15 volatility models with daily S&P500 index data through out-of-sample forecasting. They evaluate the performance based on two economic loss functions (an option pricing formula and a utility function) and two statistical loss functions (a goodness-of-fit and a prediction likelihood function based on VaR calculations). As for the option pricing formula, they apply the difference between the actual price and the estimated price to get the forecasting performance. The utility loss function is based on the investors' utility to estimate the volatility. In order to benefit from the fact that more number of models gives a higher probability of finding excellent predictive capabilities, González-Rivera et al. (2004) implement the White (2000) plausibility check. They claim that the preferred model largely depends on the loss function. The results show that simple models can be implemented when pricing options, such as the exponentially weighted moving average (EWMA) proposed by risk-metrics and any GARCH models. The asymmetric GARCH model has the best performance when applying the utility loss function as the evaluation rule.

Hartz et al. (2006) base on bootstrap and bias correction steps to propose a re-sampling method to improve the VaR prediction ability of the ordinary GARCH model. This model could restore the simplicity of the ordinary AR(1)-GARCH(1,1) model and overcome its shortcomings in VaR prediction. Hartz et al. (2006) propose this data-driven method and use a re-sampling to correct the apparent trend of the model underestimating VaR. Although the re-sampling method is usually digitally intensive, its implementation is straightforward. Besides, the re-sampling model is faster to estimate than several more complex models. It is also significantly numerically reliable on account of avoiding the intervention of advanced numerical methods. The results show that there is no need to give up the simple normal-GARCH model. Polanski and Stoja (2010) propose a simple method to predict the value at risk. They use the Gram-Charlier Expansion (GCE) to extend the standard normal distribution. They show that it provides an accurate and reliable estimate of the realized VaR by using GCE method when compared with other VaR prediction models.

Furthermore, Chen et al. (2012) indicate a parametric method to estimate and forecast the VaR and expected shortfall (ES) of the series of financial returns. They model the volatility process and capture the leverage effect by applying the GJR-GARCH model. The model could capture the potential skewness and tails by assuming an asymmetric form of the distribution. In addition, they allow the shape parameters in this distribution, which can help them model dynamics in higher moments. They applied the Markov Chain Monte Carlo (MCMC) sampling to estimate the process. This MCMC combines the Metropolis-Hastings (MH) algorithm and Gaussian distribution. Compared with the single Gaussian proposed MH method, the simulation study emphasizes the precision of the estimation and improves the inference. The proposed model performed better than other popular models by applying standard and non-standard tests. By contrast, Gerlach et al. (2011) apply the Bayesian time-varying quantile to forecast the VaR. They base on the Skewed-Laplace distribution to aim at the general quantile regression problem by Bayesian solution. Applying the Markov chain Monte Carlo sampling scheme, Gerlach et al. (2011) estimate the parameters of the model. Simulation research shows that its estimation accuracy is higher when compared to standard numerical optimization methods. The proposed model has the best performance in their empirical study with ten major stock markets. The model is better in predicting VaR over a two-year period, when compared to other models. Lucas and Zhang (2016) provide a simple method that uses a recursive update scheme to model the time changes of volatility and other high-order moments. They update the parameters by applying the score of the predicted distribution. This method can make the parameters dynamically and automatically adapt to the non-normal data set and improve the stationary of subsequent estimates. The new method, which embeds some early extensions of the exponentially weighted moving average (EWMA), can be easily extended to higher dimensions and alternative forecast distributions. This method is suitable for value-at-risk predictions with student t distribution and skewed t distributions during the time-varying. They show the benefit of the new method when predicting the volatility in the stock and forex market. Patton et al. (2019) also propose a new dynamic forecasting model with loss function from Fissler (2017), which based on a GAS framework of Creal et al. (2013). They compare ten different univariate models with four popular stock indexes with the goodness of fit test. They show evidence that the GAS risk forecasting models give a better performance.

### 1.2.2 Multivariate Risk Forecasting Model

Compared with the univariate model, the multivariate forecasting model considers the joint distribution of different assets in the portfolios, considering the linear and non-linear correlation. In this subsection, I discuss the relevant literature on the multivariate risk forecasting model.

Brooks and Persand (2003) explore many statistical models to predict the daily volatility of several critical financial time series in the UK. They evaluate the performance of those linear and GARCH-type volatility forecasting models with forecasts derived from multivariate methods in the out-of-sample data set. Forecasts are evaluated using traditional indicators (such as a mean-square error). They find that the evaluation of the forecasting models is sensitive to their methods. McAleer and Da Veiga (2008) use the univariate and multivariate conditional volatility models to compare the performance of the single index and portfolio in predicting the value-at-risk threshold. The predicted VaR shows that the single index model will lead to too many violations (usually serial-related violations), while the portfolio model will lead to fewer violations. Chiriac and Voev (2011) propose a method for dynamic modeling and predicting the realized covariance matrix based on the score integration process. This method allows a flexible dependency model and automatically guarantees the certainty of the prediction. Santos et al. (2013) compare multivariate and univariate GARCH models to predict the VaR of the portfolio. They conduct a comprehensive study of this issue using both simulated data and actual data, to consider diversified portfolios that include different kinds of assets. In addition, they apply statistical tests to evaluate the models. They conclude that the multivariate model is better than the univariate model on the out-of-sample basis.

The Copula model has been applied in the risk forecasting model, which could help to structure joint distribution in the multivariate models. Müller and Righi (2018) evaluate the performance of multivariate models to forecast value at risk (VaR), expectation value at risk (EvaR), and Expected Shortfall (ES) in different scenarios created by the Monte Carlo simulations. The models they use include historical simulation, dynamic conditional correlation-GARCH. They also apply different Copula models, including regular Copulas, vine Copulas, and nested Archimedes Copulas (NAC). They show that the regular and vine Copulas demonstrated better performance in the Gaussian distribution assumption, and NAC performs the best in Student's t distribution assumptions. Badaye and Narsoo (2020) apply a multiplicative component GARCH model for each return series and use correlation functions to model the dependence structure to predict VaR and ES.

### 1.2.3 Research Gap of the Multivariate Risk Forecast Model

The univariate model plays a vital role in forecasting risk in the financial market. However, the univariate model may underestimate the risk when the assets correlate. It is clear why the multivariate forecasting model has become more popular as time goes by. Some authors apply the benefit of the Copula in risk forecasting. Patton et al. (2019) show the a new GAS risk forecasting model with new risk measure, which I discussed in subsection 1.2.1. This model applies the new risk measure and catches the dynamics of the risk of the variable. It is interesting to apply the Copula to extend the GAS risk forecasting model from the univariate model to the multivariate model to improve the risk forecasting ability of the models. The Copula model has its benefit when managing the risk in the multivariate structure. The new model I proposed provides better performance when forecasting the risk of portfolios.

## 1.3 Forward Premium Puzzle and Uncovered Interest Rate Parity

Fama (1984) finds that forward rates contain a time-varying premium. He applies linear regression to discuss the relationship between the rate of depreciation and the forward premium. The results show a negative correlation between those items, which violate the basic expectation in the forex market. Bilson (1980) also mentions the negative relationship between the forward premium and spot rate changes. This puzzle is also confirmed by many other studies (see Hansen and Hodrick (1983); Cumby and Obstfeld (1984); Hsieh (1984); and Hodrick and Srivastava (1986)). The regression discussed by the researchers is shown below:

$$s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1} \tag{1.1}$$

where $s_{t+1}$ denotes the spot rate changes, while $f_t$ is the forward premium. The literature shows that the slope $\beta$ in this regression often shows a negative estimation. The reason why the forward premium puzzle is popular is that this puzzle violates the primary hypothesis of the forex market (uncovered interest rate parity (UIP) and covered interest rate parity (CIP)). UIP indicates that people could apply the difference of the interest rates between countries to explain the spot rate changes, while the CIP argues that the difference of the interest rates between countries could be implied in the forward premium.

When UIP and CIP both hold, the forward rate should be the unbiased estimation of the spot rate on the maturity date. However, the correlation between the rate of depreciation and forward premium is negative. This question attracts several researchers to find an explanation for the forward premium puzzle. In this section, I discuss the empirical literature on the forward premium puzzle. The first subsection focuses on the discussion regarding whether the forward premium is a puzzle or not. The different explanations for the forward premium puzzle are shown in the second subsection.

### 1.3.1 Forward Premium is a Puzzle

Many authors argue that the forward premium puzzle is not an anomaly. Baillie and Bollerslev (2000) argue that the forward premium is not a severely biased estimation of the spot exchange rate since the spot exchange rate has persistent volatility in the daily period. They show that the monthly data could let the forward premium convert to the unity's true value (unbiased estimation) at a significantly slow rate. Roll and Yan (2000) also believe that the forward premium is not a puzzle. They apply a simple model which can fit the data and indicate that the forward exchange rate is still an unbiased predictor of the expected value of the spot exchange rate. The results of their study show that this problem arises because the spot exchange rate changes and the forward premium follow an almost non-stationary time series process. They document these attributes with an extended sample and explain why they look like a puzzle. Gospodinov (2009) argues that the forward premium is not a puzzle in a new way since the non-linear relationship between the forward premium and rate of depreciation is still positive. He decomposes spot and forward exchange rates into non-linear trend components (permanent) and fixed components(temporary). When predicting the permanent (temporary) component of the corresponding forward-spot exchange rate, he checks the unbiased of the permanent (temporary) component of the forward rate. He also finds a robust non-linear co-trend relationship between forward and future spot exchange rates. In the long run, the assumption of no remote exchange rate can be maintained.

In contrast, many authors believe that the forward premium is still a puzzle and show evidence to support this. Bansal and Dahlquist (1999) try to find the difference in the risk premium between developed and emerging economies. They find that the differences between various economies are related to GDP per capital, average inflation rate, and inflation rate. Furthermore, they show the evidence regarding the forward premium puzzles from developed and emerging economies. Snaith et al. (2013) illustrate the horizon effect on the forward premium puzzle. They estimate the Fama's regressions with monthly data for short, medium, and long-term horizons for the five most essential currencies against the US dollar from 1980 to 2006. The results show that the puzzle could always exist except in a three-year horizon. Boudoukh et al. (2016) propose a new method by using the lagged forward interest rate differentials to place the spot interest rate differentials in the regression, which can deepen the puzzle. They show that exchange rate changes depend on the difference in interest rates and the exchange rate deviation. At the same time, the exchange rate deviation is implied between the current exchange rate and purchasing power parity.

### 1.3.2 Explanation of Forward Premium Puzzle

In the past few decades, many researchers discuss the forward premium puzzle in a different view. McCallum (1994) argues that the UIP has the difference from forward exchange rates as the unbiased estimation of future spot rates. Meredith and Ma (2002) think that the forward premium puzzle can be explained by the risk premium or expectations errors. Extending McCallum (1994) model, Meredith and Ma (2002) show that the exchange rate and monetary policy could affect the correlation between forward premium and the deviations from rational expectations.

Some authors try to explain the puzzle from statistical regression. Researchers try to explain the anomaly directly from the regression since the forward premium puzzle has been settled by the unexpected slope estimation between forward premium and changes in the spot rate. Bansal (1997) argues that expected exchange rate changes are negatively correlated with interest rate differences between countries, which means that interest rate parity is not found in the data that is violated. He provides new evidence that the undiscovered violations of interest rate parity (UIP and CIP) and their economic impact depend on the signs of interest rate differentials. They establish a framework to explain the confusing relationship between expected exchange rate changes and interest rate differences. Chakraborty and Evans (2008) apply the discounted perpetual learning to explain the forward premium puzzle. In their opinion, perpetual learning can explain the mystery of the long-term premium by copying other data features.

Some studies focus on a specific model to explain the forward premium anomaly. Lucas Jr (1982) proposes a model set in an infinitely lived two-country to price the currency. Following Lucas Jr (1982), many authors (Hodrick (1989); Backus et al. (1993); Bansal et al. (1995); Bekaert (1996)) apply this equilibrium model to explain the forward premium puzzle. Hodrick (1989) uses the maximization equilibrium model to predict how changes in the conditional variance of monetary policy, government spending, and income growth affect the risk premium and induce conditional fluctuations in the exchange rate. He argues that the exogenous conditional variances could explain the changes in the currency exchange rates. In contrast, Backus et al. (1993) test the large standard deviations of the forward and spot rates using the equilibrium model. They think the forward premium could be explained when an agent's preferences exhibit habit persistence. Bansal et al. (1995) develop a new method for estimating the equilibrium model with the weekly U.S.-German currency market data. They apply the model to structure non-linearity when explaining the anomaly. The forward premium is explained by the variable velocity, durability, and habit persistence in the equilibrium model from Bekaert (1996). The term structure model is also applied to explain the forward premium puzzle (Backus et al., 1994). Moreover, Burnside et al. (2009) propose a microstructure model for the forex market to help and understand the forward premium puzzle. They apply adverse selection problems between participants in forex markets to explain the puzzle. They argue that there are three risk-neutral participants: market maker, informed traders, and uninformed traders. Uninformed traders can only have public information, while informed traders can get a signal of private information. Through the microstructure model, they find the reason for the negative correlation between forward premium and rate of depreciation and explain the forward premium puzzle. Burnside et al. (2011b) provide an explanation for the problem of forward premiums in the forex market based on investor overconfidence. In this model, overconfident people may overreact to information, which leads to an overestimation of the forward exchange rate than the spot exchange rate at the beginning time. The overreaction can lead to higher inflation. When the agent observes a signal of higher future inflation, they will give a higher forward premium in the exchange rate market. This model explains the size of the deviation of forward premium and the joint behavior between the forward exchange rate and the spot exchange rate. Yu (2013) explains the puzzle from a sentiment-based model where agents overestimate or underestimate economic growth rates. In his model, the domestic interest rates are higher than foreign interest rates when the perceived domestic country consumption growth is higher than the perceived foreign country consumption growth. At the same time, econometricians expect the value of the local currency to increase. In the short term, a model with investor misunderstandings can explain the issue of forward premiums. In addition, these misunderstandings can help reduce the correlation between consumption growth and exchange rate growth. He provides empirical evidence to support this emotion-based interpretation mechanism. Djeutem (2014) tries to apply the model miss-specification to explain the forward premium puzzle. He argues that the model uncertainty premium could explain the observed excess returns in the forex market.

Some authors explain the forward premium puzzle by the forward risk premium. Tai (2003) study the risk and volatility of the Asia-Pacific region forex market over time understand whether currency risk can be a potential source of the risk premium and explain the anomaly of the forward premium. He applies the international CAPM and asymmetric multivariate GARCH to study the risk premium. Tai (2003) argues that non-linearity of forward risk premium is a new concept that needs to be explained. The estimation results show that the specific term structure model can explain the puzzling empirical evidence. Lustig and Verdelhan (2007) try to explain the forward premium puzzle from a new perspective. They try to use the overall consumption growth risk to explain the failure of the interest rate parity. In their study, the domestic investors obtain positive excess returns in high-interest-rate and negative excess returns in low-interest-rate currency. High-interest-rate currencies depreciate on average and low-interest-rate currencies will appreciate when domestic consumption growth is low. Hence, domestic investors could apply the low-interest-rate currencies to hedge against total domestic consumption growth risk. By contrast, Burnside (2007) argues that the forward premium is still a puzzle against Lustig and Verdelhan (2007). He re-tests the model by Lustig and Verdelhan (2007) and provides evidence to support his claim. Londono and Zhou (2017) also explain the forward premium puzzle through the risk premium method. They provide new empirical evidence that the world currencies and US stock variance risk premium has the predictive power for the appreciation of the US dollar, especially in the four-month and one-month data set.

### 1.3.3 Research Gap of the Forward Premium Puzzle

The forward premium is still a puzzle, according to Bansal and Dahlquist (1999), Snaith et al. (2013), and Boudoukh et al. (2016). Robust evidence shows that the anomaly still exists. Hence, the explanation of forward premium is a widespread research concern in the forex market.

Researchers still try to find an explanation of the forward premium puzzle. There is no widely accepted explanation for the puzzle. Burnside et al. (2009) creatively propose a microstructure model on adverse selection to explain the forward premium puzzle, an exciting and creative approach. However, they do not have any empirical tests on the model. I am motivated by this and have tried to apply the order flow data to estimate Burnside et al. (2009) model. The empirical analysis of the microstructure model will strongly support to the explanation from Burnside et al. (2009). Furthermore, I will add the bond market in the microstructure model to discuss the UIP in the adverse selection theory.

## 1.4 Conclusion

In this chapter, I discussed the relevant literature of the research questions. I find the apparent gap in these areas and the large contribution of my research questions.

In the first section, I discuss asset pricing models with risk factors and portfolio analysis, as they are discussed in the existing literature. I find that the factor portfolio dependence structure still has a gap that is discussed in the equity market. I try to fill this gap by discussing the prevalent factors of portfolios tail dependence to find the non-linear relationship among the forex factors in chapter 2.

The literature about risk management in the forex market shows that many researchers focus on the univariate risk forecasting model of the forex portfolios. Similarly, several studies focus on the multivariate risk forecasting model in the forex market. Barroso and Santa-Clara (2015a) predict the portfolio's risk to improve performance and find that the specific risk is more predictable. I propose a new multivariate forecasting model based on the model of Patton et al. (2019) in chapter 3.

The forward premium puzzle points out that exchange rate changes cannot make up for the differences in interest rates, thus providing considerable profits for arbitrage transactions. For decades, a large amount of literature has been studying the issue of the forward premium anomaly. However, researchers have not yet agreed on what influences the forward premium. I follow Burnside et al. (2009) model and conduct an empirical study to check the model in the real market by the unique order flow data. Moreover, I consider the UIP in the microstructure model by adding the bond market.

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# Chapter 2

# The Joint Distribution of Forex Market Factors in Copula-based Model

This chapter will discuss the tail dependence of the popular factor portfolios in the forex market. I test the relevance of these factors in contributing to a diversified forex portfolio. Surprisingly, very little has been done on this vital issue. In contrast to the existing literature, I first consider an extensive and detailed study to investigate the effect of introducing asymmetry and time-varying effects among the factors. I then measure their economic value to a forex portfolio in terms of forex investment allocation and risk management. I show that modeling non-linear dependency is essential and adds economic value to a forex portfolio.[1]

[1]Note that many research results are applied in the working paper (Cerrato et al., 2020).

## 2.1 Introduction

Currency anomalies are difficult to fit in a stochastic discount factor (SDF) model with traditional risk factors or consumption growth (e.g., Burnside et al. (2010), Burnside (2011b) and Lustig et al. (2011)) which led researchers to construct currency market-specific pricing factors.

Lustig et al. (2011) propose the dollar risk factor (DOL) and the carry trade factor (HML). The DOL factor is the cross-sectional average of all currency excess returns. The HML factor is the return of high interest rate currencies minus the return of low interest rate currencies. Menkhoff et al. (2012a) propose the currency volatility innovation factors, which is the cross-sectional average of volatility innovations of all currencies. Della Corte et al. (2016) and Della Corte et al. (2021) introduce the currency volatility risk premium. They find currencies that are cheap to insurance (by using currency options) provide higher returns. Asness et al. (2013), Menkhoff et al. (2017), and Kroencke et al. (2014) discussed the currency value strategy (VAL), which is the return difference between overvalued currencies and undervalued currencies. Whether the currency is over-or-under valued depends on the consumer price index (CPI) in a foreign country to the US. Menkhoff et al. (2012b) find that the excess return of currency momentum strategies (MOM), in cross-section, is impressive. Burnside et al. (2011a) find that the currency momentum is not correlated with other currency factors. Among others, the forex factors cited above have become pervasive in the literature. An SDF model that employs a DOL and another currency-specific factor capture substantial cross-sectional carry trade returns.

The DOL factor and HML factor which proposed by Lustig et al. (2011) are widely discussed in the forex market researches. The momentum strategy and value strategy are popular in equity market. Then, I follow the Menkhoff et al. (2012a) and Kroencke et al. (2014) to focus on the momentum factor and value factor in the forex market. I choose to model those prevalent factors (with highest cited and applied in the academic area) tail dependence structure.

Factor investing has been widely studied in the equity market. It involves allocating portfolio weights to known 'factors', such as the market, value, size, and momentum. Given that a composite set of currency factors has been established, some straightforward questions arise i) What is the correlation structure among currency factors? ii) How should investors choose among currency factors in forming portfolios, or put differently; what is the economic value of the factor investing in the currency market? However, little attention has been paid to using the currency factors in portfolio optimization and risk management.

I try to fill this gap by focusing on the four most popular currency factors, namely, DOL, HML, VAL, and MOM. Using factors, instead of individual currencies, as the basic unit informing optimal currency portfolios provides two advantages. At first, the country-specific risk could be averaged out. Secondly, factors are rebalanced every month, so it has stable risk property over time. In contrast, the risk property of individual currencies could change with the economic fundamentals or government policies.

Modeling the correlation structure is of great importance, especially for currency factors. Unlike equity factors which are nearly orthogonal to each other, the currency factors are correlated with each other inherently. Due to the limited investable universe, factors could source correlations from the same picking of currencies. Menkhoff et al. (2012a) show that the mimicking portfolio of the currency volatility factor loads similarity to a carry trade strategy. The HML factor and VAL factor could also be correlated. Because CPI and interest rates, the sorting variables of factor HML and VAL, are highly correlated.

I value the idea of Christoffersen and Langlois (2013) and Arnott et al. (2019), who suggest investors should not ignore the tail behavior and non-normality in factor investing. This paper employs the dynamic conditional correlation Copula (DCC-Copula) model with normal, student's t, and skewed t kernels. I show that the quantile-quantile plot and threshold correlations suggest non-normality and non-linear correlation structures among currency factors. Thus, the DCC-Copula with skewed t kernel fits the data best in terms of the log-likelihood.

Based on the DCC-Copula model, I build optimal currency portfolios with 24 years of weekly out-of-sample returns. Under the setting of a constant relative risk aversion (CRRA) utility investor, I find the significant economic value of the model informing optimal currency portfolios. I consider two benchmark models: i) the orthogonal model, which ignores the correlation structure ii) the normal model, which assumes the linear correlation. The DCC-Copula with skewed t kernel outperforms two benchmark models in terms of Sharpe ratios, and certainty equivalents. This result is robust across different levels of risk aversion, sub-sample of developed or developing currencies, and even more robust when transaction costs are considered.

For risk management, I forecast the value-at-risk (Var) and expected shortfalls (ES). The DCC-Copula model still shows robustness compared with the benchmark models. I apply the Diebold-Mariano tests, following Patton et al. (2019), to rank the performance of the models. The DCC-Copula with skewed t consistently ranks the best among the models in different scenarios. Modeling the asymmetry and the dynamic correlation could improve the ability to forecast the risk measures.

To my limited knowledge, I am the first to investigate the correlation structure and construct optimal currency portfolios at the factor level. Previous literature tends to perform a similar analysis at the individual currency level. For example, Patton (2006b) first introduces the Copula model to discuss the tail dependence for mark-dollar and yen-dollar exchange rates. Bouyé and Salmon (2009) derive the implicit (always non-linear) form of conditional quantile relations of dollar-yen, dollar-sterling, and dollar-DM. It has important practical and academic meanings. A closely related study is Barroso and Santa-Clara (2015a), who form optimal currency portfolios and detect relevant variables using portfolio policies (Brandt et al., 2009). They show that carry, momentum and value work better than fundamentals informing the optimal portfolio. My paper extends Barroso and Santa-Clara (2015a) how that detailed investigations for factor-level correlation structure are considered.

My paper is related to the literature on factor investing in general. Christoffersen and Langlois (2013) apply the Copula model to invest the market factor, size factor, value factor, and momentum factor in the out-of-sample data set and show that correlations of the factors in the equity market are not orthogonal. Arnott et al. (2019) also discuss the correlations between the factors in the equity market when applying the factor returns to adjust the portfolios.

My paper is also related to the literature that uses Copula models to manage the tail behavior financial time series joint distributions. Patton (2006b) first uses normal Copula and student's t Copula to model the bivariate distribution of individual currencies. Patton (2006b) find that, compared with the normal Copula, the student's t could manage the kurtosis of the variables. Christoffersen et al. (2012) propose the constant and dynamic Copula models to focus on the multivariate joint distribution, which I follow in this paper. The skewed t Copula model proposed by Christoffersen et al. (2012) could model the asymmetry, including more features of the variables. Furthermore, the dynamic conditional correlation Copula model could also manage the time-varying correlations among the variables.

The paper is organized as follows: section 2.2 describes my data; section 2.3 introduces the univariate modeling and pairwise correlation analysis among forex factors; section 2.4 presents the joint distribution modeling of forex factors by using Copula models; section 2.5 introduces the economic implication of the Copula model by constructing optimal portfolios for risk-averse investor; the skewed t t factor model would be discussed in the next section; the actual return investment results and CBD would be shown in section 7,8; section 10 illustrates the performance of the portfolios during the financial crisis time; 2.10 provides the conclusion.

## 2.2 Data

I use weekly forward and spot rates from January 1, 1989, to March 20, 2020, for 31 active trading currencies[2]. The data are all from DATASTEAM. The excess return of carry trade is calculated using the term $t$ log forward rate less the term $t+1$ log spot rate for each currency. I now discuss how the currency factors ( DOL, HML, MOM, and VAL) have been constructed. The DOL factor is simply the mean of the 31 currencies' excess return. This is what I denote as the dollar risk factor. In constructing the HML factor, I follow Lustig et al. (2014), sort the currency returns from lowest to highest based on the forward premium and allocate them into five portfolios. The HML factor is the difference between the mean returns of the fifth portfolio (the largest forward premium) and the first portfolio (the smallest forward premium). I denote it as the carry trade factor. For the momentum (MOM) factor, I follow Menkhoff et al. (2012b) and use the previous 6-week formation period and 1-week holding period to sort the currencies into five portfolios based on their lagged returns. The MOM factor is the difference between the mean returns of the lowest lagged return portfolio and the

[2]The list of the currencies: 10 important developed countries currencies (AUDUSD, CADUSD, CHFUSD, DKKUSD, EURUSD, GBPUSD, JPYUSD, NOKUSD, NZDUSD, and SEKUSD); 8 important emerging countries currencies (CZKUSD, HUFUSD, ILSUSD, ISKUSD, PLNUSD, RUBUSD, TRYUSD, and ZARUSD); 6 Asian currencies (HKDUSD, KRWUSD, MYRUSD, PHPUSD, SGDUSD, and THBUSD); 5 Latin America currencies (BRLUSD, CLPUSD, COPUSD, MXNUSD, and PENUSD); 2 Middle East currencies (JODUSD and KWDUSD).

highest lagged return portfolio. Finally, I compute the VAL factor following (Kroencke et al., 2014)

$$Q_{j,t} = \frac{S_{j,t} P_{j,t}}{P^*_{j,t}} \tag{2.1}$$

where $P_{j,t}$ denotes the price level of consumer goods in country $j$ at term $t$, and $P^*_{j,t}$ the corresponding foreign price level (here is USD). While the $Q_{j,t}$ denotes the real exchange rate of country $j$ at term $t$.

$$F_{VAL,j,t} = \left( \frac{Q_{j,t-3}}{Q_{j,t-13}} - 1 \right) (-1) \tag{2.2}$$

The VAL factor can be calculated using the average real exchange rate over 3 and 13 weeks. I then sort the currency returns from lowest to highest based on the VAL factor and allocate them into five portfolios to obtain the VAL portfolio.

## 2.3 Currency Market Factors Dependence Structure

### 2.3.1 Descriptive Statistics

This section presents some descriptive statistics for the 4 portfolios (DOL, HML, MOM, and VAL). Figure 2.1 graphs the time series plot of the factors. There is a clear presence of a volatility cluster during the 2008 financial crisis period, and this seems to be more evident for MOM and VAL factors.

[Figure 2.1 of time series plot of factor values is about here]

I report the descriptive statistics in Table 2.1. I include the annualized sample mean, the Newey-West standard error adjusted test statistics, the annualized standard deviation, the skewness, kurtosis,[3] autocorrelation coefficient and linear correlation matrix. The annualized mean return is the highest for the carry trade factor HML and is negative for the DOL factor (close to zero). Interestingly, for the full data set, all factors show excess kurtosis, and the skewness is negative for all factors but positive for VAL. The excess return shows the fat tail of 4 factors, which indicates the importance of the tail dependence of forex factors.

The second panel shows the autocorrelation coefficients. Most of the factors, apart from the DOL, have strong second-order and third-order autocorrelation. This autocorrelation could induce the non-normality shape observed in Figure 2.1.

[3] I multiply the weekly average by 52 to annualized the factor value.

I report the sample linear correlation matrix in the last panel. There are significant pairs of correlations among all factors. I observe a negative correlation between MOM and DOL. This result is also documented in equity momentum studies, (see, for example, Daniel and Moskowitz (2016)). Following Trichet (2010), I set the financial crisis from 2007 to 2009. I surprisingly find that correlation between HML and MOM is positive, and HML and VAL is negative. Since most studies have reported a negative correlation between HML and MOM factors, I further investigate this issue and split the full sample into two parts, including the 2008 financial crisis and one not including it. Table 2.2 shows the results. The financial crisis does not seem to be causing that result (see Table 2.1). However, when I split the sample into developed and developing countries, I find a clear difference for correlations of HML and MOM or HML and VAL. In developed countries, HML and MOM have the expected negative correlation while the HML and VAL are positively correlated. For developing countries, HML and VAL are also positively correlated.

[Table 2.1 factor descriptive stats table is about here]

Further evidence of non-normality can be seen in Table 2.1, while Figure 2.2 complements and supports that evidence. Factors' empirical quantiles diverge significantly from a normal distribution.

[Figure 2.2QQ plot is about here]

Although in a simple form, the empirical evidence above seems to support my key objective: correlation among forex factors is not captured by a normal distribution. In the following section, I shall investigate this issue in more detail and investigate its implications.

### 2.3.2 Modeling Dependence Among the Forex Factors

In this section, I conduct a more detailed analysis of the dependence among the forex factors. I model the dependence structure for each pair of currency factors using threshold correlations or quantile dependence, as in Christoffersen and Langlois (2013). [4] The idea here is to characterize the dependence of two variables in the joint lower or joint upper tails, respectively. Unlike linear correlation, this approach involves modeling the asymmetric dependence structure between extreme events, which is appropriate in the presence of skew and excess kurtosis observed in Table 1. I define the threshold correlation $\overline{\rho}_{i,j}(u)$ for any two factors $i$ and $j$ as follows:

$$\overline{\rho}_{i,j}(u) = \begin{cases} corr(r_i, r_j | r_i < F_i^{-1}(u), r_j < F_j^{-1}(u)) & when\, u \leq 0.5 \\ corr(r_i, r_j | r_i \geq F_i^{-1}(u), r_j \geq F_j^{-1}(u)) & when\, u \geq 0.5 \end{cases} \tag{2.3}$$

[4] The same method was used by Longin and Solnik (2001), Ang and Chen (2002), Ang and Bekaert (2002), and Patton (2004).

where $u$ is a threshold between 0 and 1, and $F_i^{-1}(u)$ is the empirical quantile function of the univariate distribution of $r_i$.

Figure 2.3 shows, on the left, the scatter plot of two factors. Alongside I plot the empirical threshold correlation against the threshold $u$ for the same pair of factors.[5] As a comparison, I assume that the theoretical threshold correlation, given the factors pairs, follows a bivariate normal distribution (see the dashed line). For bivariate normal distributions, the threshold correlation will be symmetric around 0.5 and will gradually approach 0. Figure 2.3 shows that the bivariate normal assumption does not hold, as I observe increasing correlations in extreme events. The empirical correlations show a significant degree of asymmetry, especially in the tail. Correlations among factors appear to be, in general, positive and large [6]. my results show that assuming linear dependency among the factors will lead to underestimating portfolio risk in extreme event scenarios, and so diversification, in this case, will not work in reducing the overall risk exposure. The empirical results above are essential and new as they shed new light on the literature (see, for example, Kroencke et al. (2014)) and show that dependence among the forex factors is very significant.

[Figure 2.3 Threshold correlation graph about here]

### 2.3.3 Univariate Modeling

The empirical results in Table 2.1 also show that autocorrelation could be an essential issue for factors' returns. In Figure 2.4, the autocorrelation function is plotted by a dashed line for all the factors up to 100 lags, a 95% confidence boundary included. Financial time series are generally subject to heteroscedasticity and volatility clustering. I plot the autocorrelation function for the absolute value of the factors on the same graph. I find a strong and persistent serial correlation.

[Figure 2.4 autocorrelation graph about here]

I model the dynamics of factors by using a univariate autoregressive-non-linear generalized autoregressive conditional heteroscedasiticity (AR-NGARCH) process. To focus on the tail dependence of the factors, I choose the AR(1) process. The conditional mean is estimated by an AR(1) process as follows:

$$r_{j,t} = \phi_{0,j} + \phi_{i,j} r_{j,t-1} + \sigma_{j,t}\epsilon_{j,t} \tag{2.4}$$

where $r_{j,t}$is the factor value of factor $j$ at time $t$. The conditional volatility is governed by an NGARCH (Engle and Ng, 1993)

[5] I follow Christoffersen and Langlois (2013), who compute the threshold correlation when at least 20 pairs of values are available.

[6] Although these results appear rather interesting and worthy of further investigation, this is not the objective of this paper, and I leave this question for future research.

$$\sigma_{j,t}^2 = \omega_j + \beta_j \sigma_{j,t-1}^2 + \alpha_j \sigma_{j,t-1}^2 (\epsilon_{j,t-1} - \theta_j)^2 \quad (2.5)$$

The NGARCH model allows for the asymmetric influence of past return innovations $\epsilon_{j,t-1}$. Since financial time series generally show a 'leverage effect', an unexpected drop in return may have a bigger impact on conditional volatility than an unexpected increase (i.e. $\theta_j$is positive). Under this circumstance, the NGARCH model is expected to mitigate the skewness and excess kurtosis. I use the maximum likelihood method under the assumption of i.i.d. normal innovations of $\epsilon_{j,t}$.

[Table 2.3 Estimation table of normal residuals about here]

Table 2.3 reports the coefficient estimates and diagnostic tests under the normal assumption for $\epsilon_{j,t}$. In the first panel, I report the estimated coefficients and standard errors of an AR(1)-NGARCH model $\phi_0$, $\phi_1$, $\alpha$, $\beta$, and $\theta$. The parameters ($\phi_0$) are all significant except for the DOL. Most parameters of the NGARCH model are also significant. The coefficient $\theta$ of the VAL and the MOM factors have large positive values, which are statistically significant, while the DOL factors have insignificant negative $\theta$. The log-likelihoods are all significant and positive.

The divergence between model skewness/kurtosis points towards strong non-normality of $\epsilon_j$. To better model the factor dynamics, I employ the skewed t distribution of Hansen (1994) for error term $\epsilon_{j,t}$, where the coefficients $\kappa_j$ and $\upsilon_j$ govern the skewness and the kurtosis. I use the maximum likelihood method under the assumption of skewed t distribution of $\epsilon_{j,t}$ to estimate the AR(1)-NGARCH model. The results are reported in Table 2.4, which shows that the kurtosis parameters ($\upsilon$) are all significant and the skewness factors ($\kappa$) of HML are not significant.[7]

[Table 2.4 Estimation table of skewed t residuals about here]

Figure 2.5 graphs the autocorrelation function for the residual and its absolute value. The serial correlation in absolute value is highly reduced after adjusting the skewness and excess kurtosis by assuming a normal distribution, the serial correlation in absolute value is highly reduced. Figure 2.6 is the QQ plot of the residuals from skewed t AR(1)-NGARCH. When comparing these results with Figure 2.2, I see that most of the skewness and kurtosis have been modeled after using the AR(1)-NGARCH.

[autocorrelation graph of residual series about here]

[QQ plot of residuals about here]

[7]By comparing the significance for the whole AR(1)-NGARCH model in Table 2.4, I find that the AR(1)-NGARCH model with the normal distribution fits the data well.

## 2.4 Modeling Asymmetry Among the Forex Factors

The empirical evidence above supports the presence of non-normality and asymmetry in the threshold correlation. To account for these features, I use Copula models as in Patton (2006b). I use this methodology as it is a flexible framework to characterize multivariate distributions. The joint probability density function $f_t(r_{1,t+1},...,r_{N,t+1})$ of the $N$ forex pricing factors can be decomposed as follow:

$$f_t(r_{1,t+1},...,r_{N,t+1}) = c_t(\eta_{1,t+1},...,\eta_{N,t+1})\prod_{j=1}^{N} f_{j,t}(r_{j,t+1}), \tag{2.6}$$

where $f_{j,t}(r_{j,t+1})$ is the univariate marginal probability density function for factor $j$ and time $t$; $c_t(\eta_{1,t+1},...,\eta_{N,t+1})$ is the conditional density Copula function; $\eta_{j,t+1}$ is the marginal probability density for factor $j$.

$$\eta_{j,t+1} = F_{j,t}(r_{j,t+1}) \equiv \int_{-\infty}^{r_{j,t+1}} f_{j,t}(r)dr \tag{2.7}$$

I follow the univariate skewed t AR(1)-NGARCH model given in section 2.3.3. The $F_{j,t}$ is the cumulative distribution function (CDF) of the skewed t distribution of Hansen (1994).

### 2.4.1 Copula Models

Patton (2006b) discusses the flexibility of Copula models and shows that this methodology can capture observed empirical facts in the forex market. For example, the correlation structure for currencies against the US Dollar is stronger when the currency depreciates than when it appreciates. Therefore, in my case, Copula models help me estimate the factors' joint dynamic distribution.

I shall introduce the Copula model in this section. The most common functional forms of Copula models in financial time series are normal Copula and student t Copula. However, these two Copula models can only generate symmetric multivariate distributions and fail to account for the asymmetry in threshold correlations that I have empirically shown above for the factors. Copulas from the Archimedean family (The Clayton, the Gumbel, and Joe-Clayton specifications) can be used for asymmetric bivariate distributions, but they are not easily generalized to high dimensional cases.

Demarta and McNeil (2005) propose the skewed t distribution and the skewed t Copula, which have been widely used in financial modeling.[8] The skewed t distribution belongs

[8]The skewed t Copula is used by Christoffersen et al. (2012) for the analysis of international equity

to the multivariate normal variance mixtures class. In this thesis, I mainly follow the Christoffersen et al. (2012) Copula models. An $N$-dimensional skewed t random variable X has the following representation:

$$X = \sqrt{W}Z + \lambda W \tag{2.8}$$

where $W$ follows an inverse Gamma $IG(v/2, v/2)$ distribution; $Z$ is a $N$-dimensional normal distribution with mean 0 and correlation matrix $\Psi$; $\lambda$ is a $N \times 1$ asymmetry parameter vector. The multivariate probability density function of the skewed t distribution is:

$$f_t(r; v, \lambda, \Psi) = \frac{2^{\frac{2-(v+N)}{2}} K_{\frac{v+N}{2}}\left(\sqrt{(v + z^{*\top}\Psi^{-1}z^*)\lambda^\top\Psi^{-1}\lambda}\right) e^{z^{*\top}\Psi^{-1}\lambda}}{\Gamma\left(\frac{v}{2}\right)(\pi v)^{\frac{N}{2}} \mid \Psi \mid^{\frac{1}{2}} \left(\sqrt{(v + z^{*\top}\Psi^{-1}z^*)\lambda^\top\Psi^{-1}\lambda}\right)^{-\frac{v+N}{2}} \left(1 + \frac{z^{*\top}\Psi^{-1}z^*}{v}\right)^{\frac{v+N}{2}}} \tag{2.9}$$

The Copula density function derived from the above probability density function of skewed t Copula is:

$$\begin{aligned} c_t(\eta; \lambda, v, \Psi) \quad = \quad & \frac{2^{\frac{(v-2)(N-1)}{2}} K_{\frac{v+N}{2}}\left(\sqrt{(v + z^{*\top}\Psi^{-1}z^*)\lambda^\top\Psi^{-1}\lambda}\right) e^{z^{*\top}\Psi^{-1}\lambda}}{\Gamma\left(\frac{v}{2}\right)^{1-N} \mid \Psi \mid^{\frac{1}{2}} \left(\sqrt{(v + z^{*\top}\Psi^{-1}z^*)\lambda^\top\Psi^{-1}\lambda}\right)^{-\frac{v+N}{2}} \left(1 + \frac{z^{*\top}\Psi^{-1}z^*}{v}\right)^{\frac{v+N}{2}}} \\ & \times \prod_{j=1}^{N} \frac{\left(\sqrt{\left(v + \left(z_j^*\right)^2\right)\lambda_j^2}\right)^{-\frac{v+1}{2}} \left(1 + \frac{\left(z_j^*\right)^2}{v}\right)^{\frac{v+1}{2}}}{K_{\frac{v+1}{2}}\left(\sqrt{\left(v + \left(z_j^*\right)^2\right)\lambda_j^2}\right) e^{z_j^*\lambda_j}} \end{aligned} \tag{2.10}$$

where K(·) denotes the modified Bessel function of the second kind, and $z^* = t_{\lambda,v}^{-1}(\eta_i)$ denotes the Copula shocks where $t_{\lambda,v}(\eta_i)$ is the univariate skewed t distribution:

$$t_{\lambda,v}(\eta_i) = \int_{-\infty}^{\eta_i} \frac{2^{1-\frac{v+1}{2}} K_{\frac{v+1}{2}}\left(\sqrt{(v + x^2)\lambda_i^2}\right) e^{x\lambda_i}}{\Gamma\left(\frac{v}{2}\right)\sqrt{\pi v}\left(\sqrt{(v + x^2)\lambda_i^2}\right)^{-\frac{v+1}{2}} \left(1 + \frac{x^2}{v}\right)^{\frac{v+1}{2}}} dx \tag{2.11}$$

However, a closed-form solution for skewed t quantile function is not available. I use simulation to define the quantile function and employ 1,000,000 replications of equation 2.8.

diversification and Christoffersen and Langlois (2013) for equity market factor modeling. Cerrato et al. (2017a) use this model for joint credit risk analysis of UK banks. Cerrato et al. (2017b) model the higher-order components of equity portfolios.

I also apply the normal and student t Copula in this thesis to compare with the skewed t Copula model as the benchmark. The Copula density function of student t Copula is:

$$c_t(\eta; \lambda, v, \Psi) = \frac{\mathit{\Gamma}\left(\frac{v+N}{2}\right)}{|\Psi|^{\frac{1}{2}}\,\mathit{\Gamma}\left(\frac{v}{2}\right)}\left(\frac{\mathit{\Gamma}\left(\frac{v}{2}\right)}{\mathit{\Gamma}\left(\frac{v+1}{2}\right)}\right)\frac{\left(1+\frac{1}{v}z^{*T}\Psi^{-1}z^{*}\right)^{-\frac{v+N}{2}}}{\prod_{j=1}^{N}\left(1+\frac{z_j^{*2}}{v}\right)^{-\frac{v+N}{2}}}$$

where $z^* = t_v^{-1}(\eta_i)$ and $t_v(\eta_i)$ denotes the univariate student t distribution:

$$t_v(\eta_i) = \int_{-\infty}^{\eta_i} \frac{\mathit{\Gamma}\left(\frac{v+1}{2}\right)}{\sqrt{\pi v}\,\mathit{\Gamma}\left(\frac{v}{2}\right)}\left(1+\frac{x^2}{v}\right)^{-\frac{v+N}{2}} dx$$

The normal Copula model is further nested as $v \to \infty$. Then, I could write the Copula density function of normal Copula is:

$$c(\eta; \Psi) = \frac{1}{|\Psi|^{\frac{1}{2}}} e^{-\frac{1}{2}z^{*T}\left(\mathit{\Psi}^{-1}-I_N\right)z^{*}}$$

where $z^* = \Phi^{-1}(\eta_i)$ and $\Phi(\eta_i)$ denotes the univariate normal distribution:

$$\Phi(\eta_i) = \int_{-\infty}^{\eta_i} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx$$

The univariate density function of student t and normal have the closed-form, which is different from skewed t. Hence, the Copula model results would be more stable than the skewed t Copula. The contour plots of those three Copula have been shown in the figure 2.9.

### 2.4.2 Modeling Dynamic Dependence Among the Forex Factors

Another interesting feature of the results above is that correlations change over time, and in this section, I discuss how I account for this feature. The difference between the dynamic model and the constant model is whether the correlation of factors is constant or not. Following Christoffersen et al. (2012) and Christoffersen and Langlois (2013), I use Engle (2002a)'s dynamic conditional correlation (DCC), where the correlation matrix dynamic is generated as 2.12

$$Q_t = Q(1-\beta_c-\alpha_c) + \beta_c Q_{t-1} + \alpha_c z_{t-1} z_{t-t}^T \tag{2.12}$$

In the case of $N$ pricing factors, $Q_t$ is a $N \times N$ positive semi-definite matrix for time $t$; $\alpha_c$ and $\beta_c$ are scalars; $z_t$is a $N \times 1$ row vector of standardized residuals with $j$th entry $z_{j,t} = F_c^{-1}(\eta_{j,t})$, where $F_c^{-1}$ is the inverse CDF from Copula estimation; $Q$ is a constant matrix which is a full-sample correlation matrix. The dynamic conditional correlation between factor $i$ and $j$ for time $t$ is defined as

$$\Psi_{ij,t} = \frac{Q_{ij,t}}{\sqrt{Q_{ii,t}Q_{jj,t}}} \tag{2.13}$$

Coefficient $\beta_c$ and $\alpha_c$ are estimated to allow the dynamic correlation. Note that the dynamic Copula mean-reverts to the full sample correlation matrix $Q$. The estimates of coefficient $\beta_c$and $\alpha_c$ are showed in Table 2.5.

### 2.4.3 Estimation Method

I use a composite log-likelihood estimation inspired by Engle et al. (2009) and Christoffersen et al. (2012), which have been discussed in the first chapter.[9] The composite likelihood function in my case is defined as :

$$CL(\theta) = \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j>i} \ln c_t\left(\eta_{i,t}, \eta_{j,t}; \theta_{i,j}\right) \tag{2.14}$$

where $\theta$ is the parameter set; $c_t(\eta_{i,t}, \eta_{j,t}; \theta_{i,j})$ is the bivariate Copula distribution of factor pair $i$ and $j$. I maximize the composite log-likelihood function $CL(\theta)$ to get the Copula coefficient estimates $\theta_{i,j}$ for each factor pair. I then average $\theta_{i,j}$ to obtain an estimator of the parameter set $\theta$. The standard errors are based on Engle et al. (2009). Following Christoffersen et al. (2012), all the Copula models are estimated by this method. I also report the parameter estimates from maximizing the conventional likelihood function and parameter standard error based on Chen and Fan (2006) in the Appendix.

### 2.4.4 Empirical Results of the Dynamic Model

The first panel of Table 2.5 shows the composite likelihood estimates for static/dynamic parameters of normal, student t, and skewed t Copula. The degree of freedom $\nu$

[9]Engle et al. (2009) find that the traditional likelihood method yields biased estimates in the large-scale DCC model.

and most of the skewness parameter $\lambda$ in skewed t Copula are all significant. This is consistent with the non-normal and asymmetric dependence of currency factors. For the constant Copula models, the full sample correlation estimates are reported. For dynamic Copula models, I report DCC parameter estimates $\alpha_c, \beta_c$ and long-term mean-reverting correlation matrix $Q$ as in equation 2.12. I follow Patton (2004) to apply the bootstrap method to calculate the standard deviations of the parameters. The estimates of $Q$ are about the same as for the full sample correlation of the static Copula models. DCC parameters $\alpha_c$ and $\beta_c$are significant in all three models. This result supports the time-varying correlation.

In the lower panel of Table 2.5, I report the model diagnostic statistics. I report the log-likelihood and the PLR test statistic test. The dynamic Copula models display the best fit. This is consistent with the presence of time-varying correlation and asymmetric dependence. Following Chen and Fan (2006), I perform the pseudo-likelihood ratio (PLR) test to show that the skewed t Copula model outperforms the student t Copula. The null hypothesis is that the asymmetry parameters ($\lambda$) in the skewed t Copula are all zero. The pseudo-likelihood ratio (PLR) test cannot reject the null hypothesis. Thus, the skewed t Copula models are significantly asymmetric and different from the student t Copula.

[Table 2.5 Copula results about here]

Figure 2.7 shows the dynamic correlation implied by the skewed t dynamic Copula during the period from January 1 1989, to March 20 2020. I consider the most difficult, the financial crisis. The correlations of pairs HML&VAL and HML&MOM move around the value of $Q$ (in equation 2.12). During 2008, all pairs of correlations fluctuate considerably. The financial crisis hugely impacted the forex market, invalidating models.

[Figure 2.7 Dynamic correlations of residuals about here]

To reinforce my empirical results pointing towards non-normality and checking their robustness, in Figure 2.8 I plot the empirical threshold correlation of residuals $z^*$from the AR-NGARCH model along with the standard bivariate normal implied threshold correlation, student t Copula, and skewed t Copula implied threshold correlations. It is evident that the empirical threshold correlations are far from a bivariate normal distribution. In what follows, I rely on the skewed t Copula to model the dependency structure across factors in the forex market.

[Figure 2.8Threshold Correlations for Factor Residuals and Copula Models]

## 2.5 Economic Implication

The empirical evidence above suggests that the forex factors have significant time-varying asymmetric dependence. What is the economic cost for a forex trader to ignore this dependence structure? In the following section, I shall consider two examples: forex portfolio management and forex portfolio risk management. I shall assess the economic value of considering this type of dependence structure in a forex portfolio. As in Kroencke et al. (2014), I use a real-time strategy. I show first that once I implement an optimized forex strategy and consider asymmetry and time-varying in the dependence structure, the benefit in terms of utility for the investor is primarily improved and second, that portfolio Value at Risk and Expected Short-Fall are highly reduced. I compare a battery of models accounting for different dependence structures. For portfolio analysis, I assume that at each time t, investors allocate their wealth, based on the weighting vector $w_t$, across the 4 currency factors to maximize their expected utility. I compare the return characteristics of alternative strategies by using different dependence structure models and an extensive real-time out-of-sample analysis.

### 2.5.1 The Investor's Optimization Problem

I assume that investors follow a constant relative risk aversion (CRRA) utility function:

$$U(\gamma) = \begin{cases} (1-\gamma)^{-1}\left(P_0\left(1+w_t^\top r_{t+1}\right)^{1-\gamma}\right) & if\, \gamma \neq 1 \\ log\left(P_0\left(1+w_t^\top r_{t+1}\right)\right) & if\, \gamma = 1 \end{cases} \tag{2.15}$$

where $P_0$ is the initial wealth which I set at \$1 here, $r_t$is the vector 4 currency factor returns at time $t$, $w_t$ is the weighting vector, $\gamma$ denotes the degree of relative risk aversion (RRA). I consider 3 levels of RRA: $\gamma = 3, 7, 10$. The weighting vector for each time t is obtained by maximizing the expected utility function given different assumptions for the factors' joint distribution.

$$w_t^* \equiv \underset{w \in W}{arg\,max}\, E_{\hat{f}_{t+1}}\left(U\left(1+w_t^\top r_{t+1}\right)\right) \tag{2.16}$$

I assume that investors face investment constraints in that the risk exposure to any single factor and the four factors in total is less than \$1. Thus the weighting matrix $w = \left\{(w_1, w_2, w_3, w_4) \in [-1,1]^4 : \mid w_1 \mid + \mid w_2 \mid + \mid w_3 \mid + \mid w_4 \mid \leq 1\right\}$. Due to the complexity of the joint distribution $f_{t+1}\left(r_{t+1}\right)$, solution for $w_t$ is generally not given analytically. I solved 2.16 by simulating 10,000 Monte Carlo replications for the four factors using a multivariate distribution $f_{t+1}\left(r_{t+1}\right)$.

### 2.5.2 Forex Portfolio

My weekly investment strategy is implemented in two stages: the first stage consists of modeling the dependence structure or joint distribution for the expected return $f_{t+1}(r_{t+1})$; the second stage involves the estimation of the factor weighting vector by maximizing the investors' utility function 2.16 given the estimated joint distribution in the first stage. To begin with, I estimate the skewed t AR-NGARCH model (equation 2.42.5) for the four factors using the previous data sample. After that, I estimate the dependence structure among the four residuals from the AR-NGARCH by using Copula models.[10] Each time $t$, the expected factor return for factor $j$ is generated by equation 2.17:

$$r_{j,t+1} = \phi_{0,j} + \phi_{1,j} r_{j,t} + \sigma_{j,t+1}\epsilon_{j,t+1} \tag{2.17}$$

where $\phi_{o,j}$ and $\phi_{1,j}$ are the AR coefficients; $\sigma_{j,t+1}$ is the 1-step-ahead forecasted conditional volatility in the NGARCH model; $\epsilon_{j,t+1}$ is simulated from the joint distribution function which is characterized by the Copula model. Note that the parameter estimates in the AR-NGARCH and Copula models are updated once a year using the whole previous data sample. The factor correlation is updated weekly for dynamic Copula models, where DCC is used to model the time-varying correlation coefficient. I start my investment on April 1, 1994, giving me an investment period of over 25 years.

In the second stage, I use the simulated 10,000 draws from $f_{t+1}(r_{t+1})$ to value the integral in 2.16. Thus maximizing 2.16 is equivalent to maximizing 2.18

$$w_t^* \equiv \underset{w\in W}{arg\,max} n^{-1} \sum_{i=1}^{n} U^*\left(R_{t+1,i}^*(w)\right) \tag{2.18}$$

where

$$R_{t+1,i}(w) = 1 + w_t^\top r_{t+1} \tag{2.19}$$

$$\varepsilon = 2.2204 \times 10^{-16}$$

and

$$R_{t+1,i}^*(w) = \begin{cases} R_{t+1,i}(w) & if\, R_{t+1,i}(w) > \varepsilon \\ 2\varepsilon\left(1 - \frac{1}{1+e^{R_{t,i}(w)-\varepsilon}}\right) & if\, R_{t+1,i}(w) \leq \varepsilon \end{cases} \tag{2.20}$$

[10] I also used a multivariate standard normal distribution as a benchmark for comparison with Copula models.

$$\bar{U} = n^{-1} \sum_{i=1}^{n} U\left(R^{*}_{t,i}\left(w^{*}_{t-1}\right)\right) \quad (2.21)$$

$$U^{*}\left(R_{t+1,i}\left(w\right)\right) = \frac{100}{\mid \bar{U} \mid} U\left(R_{t+1,i}\left(w\right)\right) \quad (2.22)$$

The cut-off $2.2204 \times 10^{-16}$ was chosen as the machine epsilon. I use the function $U^{*}$ instead of $U$ directly since the numerical maximization routine does not work well with extremely small or large values. The $\bar{U}$does not affect the ranking of alternatives, and the 100 value is the reverting mean of the $\bar{U}$.

By maximizing equation 2.18, I obtain the optimal weighting vector $w_t$ for time $t$. Each time t, investors liquidate the previous position and re-balance their portfolios according to $w_t$.

### 2.5.3 Performance of Different Strategies

[The real-time investment results about here]

The empirical results based on an extensive battery of dependence structure models are reported in Table 2.6. I consider three levels for the RRA, namely $\gamma = 3$ in Panel A, $\gamma = 7$ in Panel B, and $\gamma = 10$ in Panel C. I follow Christoffersen and Langlois (2013); Patton (2004). As the value of $\gamma$ increases the risk-averse level would also increase and the turnover would decrease. The portfolio mean, volatility, skewness and kurtosis of returns for the 5 different models are given in Table 2.6. Following Christoffersen and Langlois (2013), I use the average return of the previous two years as the expected return of the factors. This helps me to focus on the impact of higher moments on portfolio selection. I start with the complete data set (i.e. developed and developing countries).

To find out whether richer models lead to better performance by generating a better trading signal, I also report the average turnover:

$$Average\, turnover\,(\%) = \frac{100}{4T} \sum_{t=1}^{T} \sum_{i=1}^{4} \mid w_{i,t} - w_{i,t-1} \mid \quad (2.23)$$

The estimates are all around 12%-21%, depending on risk aversion. These values are similar within each of the panels. This indicates that the improvement in realized utility across the models is not driven by the difference in trading turnover.

I compute the certainty equivalent (CE) of the average realized utility for each strategy as follows:

$$CE = U^{-1}\left(\frac{1}{T}\sum_{t=1}^{T}\frac{(1+r_{p,t})^{1-\gamma}}{1-\gamma}\right) = \left(\frac{1}{T}\sum_{t=1}^{T}(1+r_{p,t})^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{2.24}$$

where $U^{-1}$ is the inverse of the utility function and where

$$r_{p,t} = w_{t-1}^{\top} r_t \tag{2.25}$$

are the out-of-sample portfolio returns.

I use the multivariate standard normal model as my benchmark. For completeness, I also include the performance of a simple diversified portfolio. There is clear empirical evidence that asymmetry is economically relevant (i.e. the skewed t Copula outperforms the other models). Thus, by considering asymmetry, one can add value to a forex portfolio. The equally weighted forex strategy produces a very different performance from the Copula strategies.

### 2.5.4 Transaction Costs

Transaction costs can significantly reduce the performance of a trading strategy. There is empirical evidence (Menkhoff et al., 2012b), for example, that the performance of a momentum strategy is highly reduced after considering transaction costs. In Table 2.7, I consider transaction costs to check the robustness of the results presented in the previous table. To compute the cost, I follow Barroso and Santa-Clara (2015a) and write it as:

$$c_{i,t} = \frac{F_{i,t,t+1}^{ask} - F_{i,t,t+1}^{bid}}{F_{i,t,t+1}^{ask} + F_{i,t,t+1}^{bid}} \tag{2.26}$$

where $c_{i,t}$ is the transaction cost of currency $i$ at time $t$. $F_{i,t,t+1}^{ask}$ and $F_{i,t,t+1}^{bid}$ denote the bid and ask price of the forward exchange rate of currency $i$ at time $t$. To convert currency transaction costs into factor transaction costs, I use the same method and parameters to calculate the factor transaction cost by simply changing currency excess return to the currency transaction cost.

I consider transaction costs for combined strategies and not a 'stand-alone' strategy as it may well be that when I consider transaction costs for a momentum strategy, the cost offsets the return for that strategy, but when it is combined with other strategies (for example carry trade) the higher profit of this combined strategy offsets the transaction costs. Clearly, transaction costs are essential. However, overall the main results remain unchanged.

### 2.5.5 Performance in Developed and Developing Countries

In the following section, I split the data into developing and developed countries. I do this for several reasons: first, I aim to check whether my results are driven by country-specific factors affecting the exchange rates. Second, the benefits are known of diversifying forex portfolios by including developing countries' exchange rates. The table shows that the p-values reject the null hypothesis for the developed countries only at the 10% significance level. Thus, the rejection is weaker than in the previous tables. The annualized mean return is, generally higher for the t-skew Copula model, while annualized volatility and skewness stay unchanged. The large negative skew may signal the presence of crash risk. As before, if I consider an investor with a relative risk aversion of 3, they would know gain now 0.011%, this is 1.15bp per month if using the skew t-Copula instead of my benchmark model.

[Table 2.8 Out of sample investment in developed countries currencies about here]

[Table 2.9 Out of sample investment in developing countries currencies about here]

The results for developing countries also point towards an economic gain from using a skew t-Copula as opposed to my benchmark one, but in general, they are weaker than the ones presented for all the countries: the benefit for my investor from using a skew t-Copula model, in this case, is only 1.94bp per month. There is an economic benefit in diversifying a forex portfolio between developed and developing markets. The annualized mean return for developing countries is higher than the one for developed countries, but annualized volatility is also higher. Overall, the CE measure for developing countries is the highest. The equal-weighted strategy always has the worst performance; see Table 2.8, 2.9.

### 2.5.6 Test the Contribution for the Portfolios from the Factors

[Table 2.10 Average weights in the portfolios about here]

I apply the sum of the absolute weight of each factor in the portfolios to find the most crucial factor in different circumstances. Table 2.10 shows the average absolute weights of different portfolios. The HML factor always has the highest weight, while the DOL factor reaches the second place. It illustrates that the carry trade factor still plays an important role in the factor portfolios in the forex market. The high returns and Sharpe ratio of the portfolios mainly depend on the HML factor.

## 2.6 Skewed t-t Factor Model in Investment

I also apply the skewed t-t factor Copula model from Oh and Patton (2018), which could overcome the lack of data for high-dimension applications. When I consider a vector of N variables $Y$, with the joint distribution $F_y$, and Copula $C$ with marginal distribution $F_i$:

$$[Y_1, \ldots\ldots, Y_N] \equiv Y \sim F_y = C\,[F_1, \ldots\ldots, F_N]$$

As for the latent factor structure, I base on the $N + K$ latent variables:

$$X_i = \sum_{k=1}^{K} \beta_{ik} Z_k + \varepsilon_i$$

Consequently,

$$[X_1, \ldots\ldots, X_N]' \equiv X = BZ + \varepsilon$$

where $\varepsilon_i$ follow the student t distribution with the degree of freedom $\gamma_\varepsilon$,$\varepsilon_i \sim iid\, F_\varepsilon\,(\gamma_\varepsilon)$. $Z_k \sim inid F_{zk}(\gamma_k)$ indicate that the $Z_k$ follow the univariate skewed t distribution.

Then,

$$X \sim F_x = C\,(G_1(\theta), ..., G_N(\theta); \theta)$$

where $\theta \equiv [vec(B)', \gamma_\varepsilon', \gamma_1', ..., \gamma_K']'$. The Copula of the latent variables $X$, denoted $C(\theta)$, is used as the model for the Copula of the observable variables $Y$. Skewed t-t factor Copula use the maximum log-likelihood to estimate the $\theta$. After I get the $\theta$ of the skewed t-t factor distribution, I could start to simulate the excess return in the investment step. Then, I will get an estimated return from forecasting, which would help me to set the weights to maximize the utility of portfolios. The main difference between the skewed t-t factor model and multi-factor model with skewed t distributions is the assumption of distributions of the residuals.

[Table 2.11 Out of sample investment results with skewed t-t factor model about here]

The skewed t-t factor model with the average returns from the factors is shown in the table. I set the skewed t Copula model as the benchmark to compare the performance between those two models. Table 2.11 shows the average return portfolios results. The annualized return of the skewed t-t factor model is much lower than the skewed t Copula model. I could indicate that managing the dependence structure could help increase the return while the univariate model cannot perform very well.

## 2.7 Real Return Investment Results of Dynamic Copula Models

I discussed the forex portfolio with the average factors' return which helps me to focus on the tail dependence of the forex factors. However, I am also interested in the performance of the Copula models in the real market. Thus, I apply the actual return of the factors of those four factors to compare the return and ratios in this section.

[Table 2.14 Out of sample investment with real factor return about here]

Table 2.14 shows the actual investment with the real return. The skewed t Copula model still has the highest annualized return among the different portfolios when the $\gamma$ in levels 3 and 7. The annualized return in panel 3(level 10) is lower than the normal benchmark. Hence, the real return would affect the stationary of the skewed t performance since the simulation when defining the quantile function.

## 2.8 CDB Application in Ranking Portfolios

Conditional diversification benefit (CDB) could measure the diversification benefits, which considers higher-order moments and non-linear dependence. I followed Christoffersen et al. (2012) to calculate the CDB of each portfolio as follows.

Firstly, I need to calculate the expected shortfall $ES$:

$$ES_t^q\left(R_{i,t}\right) = -E\left[R_{i,t} \mid R_{i,t} \leq F_{i,t}^{-1}\left(q\right)\right]$$

where $R_{i,t}$ is the return of factors $i$ at term $t$, $F_{i,t}^{-1}\left(q\right)$ denotes the inverse cumulative distribution function and $q$ is the probability which I set 5% here. $ES_t^q\left(R_{i,t}\right)$ denotes the expected shortfall of factors $i$ at term $t$ with the percentile $q$.

$$ES_t^q\left(w_t\right) \leq \sum_{i=1}^{N} w_{i,t} ES_t^q\left(R_{i,t}\right) \; for\, all\, w_t$$

where $ES_t^q\left(w_t\right)$ is the expected shortfall for the portfolios with the weight $w_t$. Hence, I could set the upper bound as

$$\overline{ES_t^q}\left(w_t\right) \equiv \sum_{i=1}^{N} w_{i,t} ES_t^q\left(R_{i,t}\right)$$

as for the lower bound, I set the value at risk of the portfolio with the weight $w_t$:

$$\underline{ES_t^q}\left(w_t\right) \equiv -F_{i,t}^{-1}\left(w_t, q\right)$$

where the $F_{i,t}^{-1}\left(w_t, q\right)$ denotes the inverse cumulative distribution function of the portfolio with the weight $w_t$.

Consequently, I could calculate the CDB for the portfolio from the functions above:

$$CDB_t\left(w_t, q\right) \equiv \frac{\overline{ES_t^q}\left(w_t\right) - ES_t^q\left(w_t\right)}{\overline{ES_t^q}\left(w_t\right) - \underline{ES_t^q}\left(w_t\right)}$$

CDB could help to test the ability of models to reduce the tail risk.

[Table 2.12 Dynamic Copula models' CDB with real return portfolios about here]

[Table 2.13 Dynamic Copula models' CDB with average return portfolios about here]

Table 2.12 and 2.13 illustrate the descriptive statistics of CDB in different portfolios. There is no doubt that the simple diversified could manage more risk since the Copula models focus on the utility. To get a higher utility of constant relative risk aversion, the model with a low level of risk aversion could focus on the higher return. The CDB would increase when the risk aversion level increase.

## 2.9 Model Performance in Financial Crisis Time

I use weekly forward and spot rates from January 1, 1989, to March 20, 2020, for 31 active trading currencies, which is long-term data set. This data set includes the critical period, the 2008 financial crisis. During the financial crisis, the exchange rate volatility would increase dramatically. Hence, many of the strategies cannot perform well during this period. Following Trichet (2010), I will test each portfolio during in financial crisis time from Sep 2007 to Sep 2009, finding the best model to manage the risk in the extreme event.

[Table 2.15 Out of sample investment with real factor return in crisis time about here]

[Table 2.16 Dynamic Copula models' CDB with real return in crisis time about here]

Table 2.15 and 2.16 show the performance and CDB of the portfolios. The skewed t Copula always has the highest CDB and lowest loss during the financial crisis. This also indicates that the factors have asymmetric dependence. The Copula model could help increase the return and decrease the loss in different scenarios by managing the dependence structure of the factors.

## 2.10 Conclusion

This chapter focuses on the tail dependence structure of the vital currency-specific factors. I choose 4 currency factors that are most popular in the forex market, such as DOL, HML, MOM and VAL. First, I apply the threshold correlations to show the evidence of the non-linearity between the factors. Then, I use the AR-NGARCH and Copula (followingChristoffersen et al. (2012)) to model the joint distribution among the tails of the factors. I test the out-of-sample portfolio performance by certainty equivalent when considering the non-linearity to show the benefit in economic value during the period from April 1, 1994, to March 20, 2020. To check the performance in different scenarios, I evaluate the portfolio performance in other situations, such as transaction cost and real return. The data set includes the 2008 financial crisis period. Hence, I also compare the ability to avoid loss among the optimal models during the crisis circumstances. The results consistently show robustness.

There are several interesting findings. The linear correlations of the four factors are high and significant, which confirms the previous literature. Secondly, the results of threshold correlation indicate that the factors have a much higher correlation during the extreme event(dramatic increase and decrease). The main reason is that most of the factor portfolios are high minus low portfolios. Then, I show that the dependence structure among forex factors is more complex than has been considered in the literature. To evaluate the economic cost to a hedge fund of neglecting these modeling features, the results of forex portfolio management show that adding asymmetry and time-varying dependence among the factors improves portfolio performance. The robust benefit of managing the asymmetry of the factors shows that managing the tail dependence of the factor is necessary.

This chapter makes some remarkable contributions. First, I find significant evidence that the non-linear dependence of weekly monetary portfolio returns is much stronger than the traditional linear correlation coefficients imply. I focused on weekly returns and found that the asymmetrical Student t Copula can capture the asymmetries and dependencies of factors neatly. Importantly, it can produce strong asymmetric tail dependencies in almost unrelated factors. Second, I conduct an extensive and detailed study on the dependence structure among some of the most widely investigated forex factors in the literature that is also very relevant to the hedge fund industry when designing forex trading strategies, which is robust in different situations. Barroso and Santa-Clara (2015a) show a literature on the optimal currency portfolios. By contrast, I indicate a detailed investigations for factor-level correlation structure.

There are some benefits of implications from this chapter. My results are very relevant for the academic literature in this area as I shed some new light on the dependence structure across the popular forex factor. The currency-specific factors are often applied in the asset pricing to explain the specific risk features of the return. The joint distribution of the factors could help me understand the risk structure and forecast the risk more precisely. It is also relevant to the forex market investors (eg. hedge funds) when designing forex trading strategies. The investors use the currency factors to adjust the exposure to its specific risk exposure in the forex portfolio. The tail dependence structure could help the investors to avoid underestimating the risk of the forex portfolios.

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**Table 2.1** – Description Statistics of Weekly Factor Return

I report the mean, volatility, skewness, kurtosis and autocorrelation and cross-correlation for logged weekly return of four factors. The period of the sample is from January 1, 1989, to March 20, 2020. The significant correlation is marked by * and ** denoting the 5% and 1% levels.

| Sample Moments | DOL | HML | MOM | VAL |
|---|---|---|---|---|
| Annualized mean | -0.0033 | 0.1817 | 0.0790 | 0.0336 |
| Weekly mean | -0.0001 | 0.0035 | 0.0015 | 0.0006 |
| Annualized volatility | 0.0646 | 0.0826 | 0.0935 | 0.0964 |
| Weekly volatility | 0.0090 | 0.0114 | 0.0130 | 0.0134 |
| Skewness | -0.3476 | -0.3983 | -0.1451 | 0.2589 |
| Kurtosis | 4.7311 | 5.1602 | 6.9311 | 6.8570 |
| Autocorrelation | | | | |
| First-order | 0.0410 | -0.0055 | -0.0101 | 0.0923** |
| Second-order | 0.0354 | 0.0922** | 0.0640* | 0.1751** |
| Third-order | 0.0228 | 0.0781** | 0.0870** | 0.1436** |
| Cross Correlations | | | | |
| DOL | 1.0000 | 0.3120** | -0.0786* | -0.1596** |
| HML | 0.3120** | 1.0000 | 0.0522* | -0.2655** |
| MOM | -0.0786* | 0.0522* | 1.0000 | 0.5585** |
| VAL | -0.1596** | -0.2655** | 0.5585** | 1.0000 |

**Table 2.2** – Different Group and Period of Four Factors' Correlations

I present the different group and period correlations to understand the reason for the positive correlation between HML and MOM or negative correlation between HML and VAL. The first section presents the correlation of the group of developed country factors, while the second section shows the correlations from developing country factors. The last section is the cross-section data without the 2008 financial crisis.

| | developed countries | | | | developing countries | | | | without financial crisis data | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL |
| DOL | 1.0000 | 0.2770 | -0.1377 | -0.0580 | 1.0000 | 0.4045 | -0.0462 | -0.0011 | 1.0000 | 0.2918 | -0.0604 | -0.1755 |
| HML | 0.2770 | 1.0000 | -0.1582 | 0.0106 | 0.4045 | 1.0000 | 0.2248 | 0.3090 | 0.2918 | 1.0000 | 0.0718 | -0.3292 |
| MOM | -0.1377 | -0.1582 | 1.0000 | 0.1443 | -0.0462 | 0.2248 | 1.0000 | 0.3187 | -0.0604 | 0.0718 | 1.0000 | 0.4328 |
| VAL | -0.0580 | 0.0106 | 0.1443 | 1.0000 | -0.0011 | 0.3090 | 0.3187 | 1.0000 | -0.1755 | -0.3292 | 0.4328 | 1.0000 |

**Table 2.3** – Estimation Table of Normal Residuals

I report parameter estimates and model diagnostics for the AR-GARCH model with normal shocks. Standard errors which are in parentheses are calculated from the outer product of the gradient at the optimum parameter values. The model estimated is $r_{j,t} = \phi_{0,j} + \phi_{i,j} r_{j,t-1} + \sigma_{j,t}\epsilon_{j,t}$, where $\sigma^2_{j,t} = \omega_j + \beta_j \sigma^2_{j,t-1} + \alpha_j \sigma^2_{j,t-1}(\epsilon_{j,t-1} - \theta_j)^2$. Here $\omega$ is fixed by variance targeting, and variance persistence denotes the sum of parameters of the model.I also provide the p-value for Ljung-Box (L-B) tests of the residuals and absolute residuals by 20 lags. The empirical skewness and excess kurtosis of the residuals are compared to the model implied levels from the normal model.

| Parameter Estimates | DOL | HML | MOM | VAL |
|---|---|---|---|---|
| $\phi_0$ | -0.0003 | 0.0031 | 0.0012 | 0.0005 |
| | (0.0008) | (0.0003) | (0.0003) | (0.0003) |
| $\phi_1$ | 0.0435 | 0.0152 | 0.0118 | 0.1075 |
| | (0.0407) | (0.0306) | (0.0279) | (0.0315) |
| $\alpha$ | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | (0.0000) | (0.0000) | (0.0000) | (0.0000) |
| $\beta$ | 0.9330 | 0.9519 | 0.9106 | 0.0022 |
| | (0.5577) | (0.0189) | (0.0942) | (0.0023) |
| $\theta$ | -0.3675 | 0.0842 | 0.4922 | 0.1167 |
| | (0.2199) | (0.1203) | (0.1116) | (0.0160) |
| $\kappa$ | / | / | / | / |
| $\upsilon$ | / | / | / | / |
| | | | | |
| Diagnostics | | | | |
| Log-likelihood | 5131.1000 | 4714.6000 | 4572.5000 | 4451.7000 |
| Variance persistence | 0.9330 | 0.9519 | 0.9106 | 0.0022 |
| L-B(20) p-value | 0.1366 | 0.0000 | 0.0000 | 0.0000 |
| Absolute L-B(20) p-value | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Empirical skewness | -0.3305 | -0.3874 | -0.1572 | 0.1523 |
| Model skewness | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Empirical excess kurtosis | 4.7064 | 5.1739 | 7.0292 | 6.9138 |
| Model excess kurtosis | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

**Table 2.4** – Estimation Table of Skewed t Residuals

I report parameter estimates and model diagnostics for the AR-GARCH model with skewed t shocks. Standard errors which are in parentheses are calculated from the outer product of the gradient at the optimum parameter values. The model estimated is $r_{j,t} = \phi_{0,j} + \phi_{i,j} r_{j,t-1} + \sigma_{j,t}\epsilon_{j,t}$, where $\sigma^2_{j,t} = \omega_j + \beta_j \sigma^2_{j,t-1} + \alpha_j \sigma^2_{j,t-1}(\epsilon_{j,t-1} - \theta_j)^2$. Here $\omega$ is fixed by variance targeting, and variance persistence denotes the sum of parameters of the model.I also provide the p-value for Ljung-Box (L-B) tests of the residuals and absolute residuals by 20 lags. The empirical skewness and excess kurtosis of the residuals are compared to the model implied levels from the asymmetric model.

| Parameter Estimates | DOL | HML | MOM | VAL |
|---|---|---|---|---|
| $\phi_0$ | 0.0086 | 0.0187 | 0.0127 | 0.0010 |
| | (4.4267) | (39.8799) | (19.0416) | (0.0003) |
| $\phi_1$ | 0.1205 | -0.9963 | -0.3788 | 0.0842 |
| | (21.1115) | (626.7629) | (95.5201) | (0.0290) |
| $\alpha$ | 0.1420 | 0.1426 | 0.1357 | 0.0000 |
| | (99.8384) | (2238.7000) | (10.3699) | (0.0000) |
| $\beta$ | 0.0487 | 0.0495 | 0.0872 | 0.9278 |
| | (0.7801) | (1001.0000) | (16.2953) | (0.0310) |
| $\theta$ | 0.0980 | 0.1031 | 0.1537 | 0.0326 |
| | (6.9545) | (340.2772) | (17.5421) | (0.1055) |
| $\kappa$ | 0.6267 | 0.6386 | 0.7202 | 0.0294 |
| | (984.0096) | (4693.0000) | (124.6940) | (0.0393) |
| $\upsilon$ | 8.5314 | 9.4683 | 6.6711 | 8.6676 |
| | (313.4826) | (7898.4000) | (1234.2000) | (1.6343) |
| | | | | |
| Diagnostics | | | | |
| Log-likelihood | 3092.6000 | 2768.6000 | 2968.2000 | 4542.9000 |
| Variance persistence | 0.1907 | 0.1921 | 0.2229 | 0.9278 |
| L-B(20) p-value | 0.0188 | 0.0000 | 0.0000 | 0.0000 |
| Absolute L-B(20) p-value | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| Empirical skewness | -0.2970 | -0.1663 | 0.1166 | 0.1773 |
| Model skewness | 1.3041 | 1.2484 | 1.6589 | 0.0770 |
| Empirical excess kurtosis | 4.6803 | 4.1084 | 4.9599 | 6.8957 |
| Model excess kurtosis | 6.2522 | 5.7910 | 8.9601 | 4.2916 |

**Table 2.5** – Estimation Results for Copula Models with Composite Method

This table presents parameter estimates for the dependence models of the residuals from the NGARCH model for the period January 1, 1989, to March 20, 2020. All models are estimated by maximum likelihood. Standard errors (in parentheses) are computed using the methodology of Engle et al. (2009). The last line presents the pseudo-likelihood ratio test statistics. I followed Chen and Fan (2006) for the null hypothesis that the asymmetry parameters in skewed t Copula are all equal to 0. The * and ** denote the significant levels of 5% and 1%.

| | 4 factors | | | | | |
|---|---|---|---|---|---|---|
| | constant | | | dynamic | | |
| | normal | t | skewed t | normal | t | skewed t |
| $\upsilon$ | | 9.1563 | 7.2539 | | 6.0653 | 6.4937 |
| | | (2.2558) | (0.1277) | | (0.9437) | (0.2823) |
| $\lambda_{DOL}$ | | | -0.0009 | | | -0.0025 |
| | | | (0.0002) | | | (0.0013) |
| $\lambda_{HML}$ | | | 0.0015 | | | 0.0031 |
| | | | (0.0005) | | | (0.0007) |
| $\lambda_{MOM}$ | | | -0.0013 | | | -0.0022 |
| | | | (0.0162) | | | (0.0051) |
| $\lambda_{VAL}$ | | | -0.0095 | | | -0.0002 |
| | | | (0.0026) | | | (0.0106) |
| $\beta_c$ | | | | 0.8115 | 0.7951 | 0.8087 |
| | | | | (0.0338) | (0.0130) | (0.0212) |
| $\alpha_c$ | | | | 0.0247 | 0.0369 | 0.0304 |
| | | | | (0.0069) | (0.0016) | (0.0027) |
| $\rho$(DOL,HML) | 0.1615 | 0.1489 | 0.1474 | 0.2058 | 0.2131 | 0.2129 |
| $\rho$(DOL,MOM) | 0.0184 | 0.0403 | 0.0403 | 0.0180 | -0.0025 | -0.0005 |
| $\rho$(DOL,VAL) | -0.0531 | -0.0050 | -0.0036 | -0.0537 | -0.0990 | -0.0947 |
| $\rho$(HML,MOM) | 0.0814 | 0.0948 | 0.0946 | 0.1058 | 0.0800 | 0.0820 |
| $\rho$(HML,VAL) | -0.1598 | -0.1605 | -0.1584 | -0.1822 | -0.1904 | -0.1896 |
| $\rho$(MOM,VAL) | 0.5533 | 0.5597 | 0.5575 | 0.5544 | 0.5683 | 0.5676 |
| Model Properties | | | | | | |
| Correlation persistence | 0.0000 | 0.0000 | 0.0000 | 0.8362 | 0.8320 | 0.8391 |
| Log-likelihood | 334.9903 | 544.7528 | 551.9646 | 521.3643 | 602.8038 | 610.3774 |
| No. of parameters | 6.0000 | 7.0000 | 11.0000 | 8.0000 | 9.0000 | 13.0000 |
| Pseudo-likelihood | | | 15.2801** | | | 12.4748** |

**Table 2.6** – Out of Sample Investment Results

The period of the out-of-sample is from April 1, 1994, to March 20, 2020. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | dynamic correlation models | | | |
|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula |
| Panel A. $\gamma$=3 | | | | |
| Annualized mean(%) | 18.6041 | 18.5902 | 18.6048 | 18.8868 |
| Variance | 0.0166 | 0.0166 | 0.0166 | 0.0162 |
| Skewness | -0.1503 | -0.1505 | -0.1525 | -0.0346 |
| Kurtosis | 2.9245 | 2.9257 | 2.9320 | 2.5787 |
| Average turnover(%) | 1.8146 | 1.8802 | 1.9300 | 1.7100 |
| CE(basis point) | 35.6977 | 35.6709 | 35.6991 | 36.2452 |
| Annualized diff in CE(%) | – | – | – | 0.2847 |
| Panel B. $\gamma$=7 | | | | |
| Annualized mean(%) | 18.4487 | 18.4349 | 18.5316 | 18.6866 |
| Variance | 0.0167 | 0.0167 | 0.0167 | 0.0162 |
| Skewness | -0.1285 | -0.1299 | -0.1482 | 0.0049 |
| Kurtosis | 2.9503 | 2.9526 | 2.9547 | 2.6629 |
| Average turnover(%) | 2.1904 | 2.2436 | 2.2159 | 2.1294 |
| CE(basis point) | 35.2920 | 35.2649 | 35.4512 | 35.7602 |
| Annualized diff in CE(%) | – | – | – | 0.2435 |
| Panel C. $\gamma$=10 | | | | |
| Annualized mean(%) | 18.3096 | 18.3115 | 18.4622 | 18.6422 |
| Variance | 0.0168 | 0.0168 | 0.0168 | 0.0168 |
| Skewness | -0.1264 | -0.1283 | -0.1571 | -0.1413 |
| Kurtosis | 2.9185 | 2.9218 | 2.9117 | 2.8456 |
| Average turnover(%) | 2.2007 | 2.1922 | 2.2228 | 2.3522 |
| CE(basis point) | 34.9407 | 34.9443 | 35.2332 | 35.5814 |
| Annualized diff in CE(%) | – | – | – | 0.3332 |

**Table 2.7** – Out of Sample Investment with Transaction Cost

The period of the out-of-sample is from April 1, 1994, to March 20, 2020. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns with cost, the average turnover, as well as the certainty equivalent.

| | dynamic correlation models with cost | | | |
|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula |
| Panel A. $\gamma$=3 | | | | |
| Annualized mean(%) | 17.7378 | 17.7003 | 17.6939 | 18.0688 |
| Variance | 0.0173 | 0.0174 | 0.0174 | 0.0168 |
| Skewness | -0.2020 | -0.2010 | -0.2123 | -0.0574 |
| Kurtosis | 2.9719 | 2.9645 | 2.9875 | 2.6052 |
| Average turnover(%) | 1.8146 | 1.8802 | 1.9300 | 1.7100 |
| CE(basis point) | 34.0243 | 33.9519 | 33.9393 | 34.6662 |
| Annualized diff in CE(%) | – | – | – | 0.3338 |
| Panel B. $\gamma$=7 | | | | |
| Annualized mean(%) | 17.3506 | 17.3091 | 17.3994 | 17.5951 |
| Variance | 0.0175 | 0.0176 | 0.0175 | 0.0169 |
| Skewness | -0.1675 | -0.1728 | -0.1680 | -0.0093 |
| Kurtosis | 2.9482 | 2.9569 | 2.9267 | 2.7304 |
| Average turnover(%) | 2.1904 | 2.2436 | 2.2159 | 2.1294 |
| CE(basis point) | 33.1593 | 33.0783 | 33.2541 | 33.6448 |
| Annualized diff in CE(%) | – | – | – | 0.2525 |
| Panel C. $\gamma$=10 | | | | |
| Annualized mean(%) | 17.1896 | 17.1886 | 17.3188 | 17.4373 |
| Variance | 0.0177 | 0.0176 | 0.0176 | 0.0176 |
| Skewness | -0.1657 | -0.1645 | -0.1719 | -0.1557 |
| Kurtosis | 2.9228 | 2.9147 | 2.8781 | 2.8346 |
| Average turnover(%) | 2.2007 | 2.1922 | 2.2228 | 2.3522 |
| CE(basis point) | 32.7564 | 32.7547 | 33.0061 | 33.2354 |
| Annualized diff in CE(%) | – | – | – | 0.2491 |

**Table 2.8** – Out of Sample Investment in Developed Countries Currencies

The period of the out-of-sample is from April 1, 1994, to March 20, 2020.. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | developed countries models | | | |
|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula |
| Panel A. $\gamma$=3 | | | | |
| Annualized mean(%) | 16.5457 | 16.5447 | 16.5022 | 16.6840 |
| Variance | 0.0182 | 0.0182 | 0.0183 | 0.0182 |
| Skewness | -0.4936 | -0.4941 | -0.4906 | -0.5003 |
| Kurtosis | 2.8998 | 2.9021 | 2.8891 | 2.9011 |
| Average turnover(%) | 2.7972 | 2.8082 | 2.6592 | 2.5465 |
| CE(basis point) | 31.7229 | 31.7210 | 31.6389 | 31.9890 |
| Annualized diff in CE(%) | – | – | – | 0.1384 |
| Panel B. $\gamma$=7 | | | | |
| Annualized mean(%) | 16.2548 | 16.2608 | 16.1788 | 16.2690 |
| Variance | 0.0182 | 0.0182 | 0.0184 | 0.0182 |
| Skewness | -0.4429 | -0.4457 | -0.4464 | -0.4540 |
| Kurtosis | 2.8775 | 2.8837 | 2.8400 | 2.8702 |
| Average turnover(%) | 2.8179 | 2.7921 | 2.8059 | 2.3974 |
| CE(basis point) | 31.0357 | 31.0474 | 30.8865 | 31.0629 |
| Annualized diff in CE(%) | – | – | – | 0.0141 |
| Panel C. $\gamma$=10 | | | | |
| Annualized mean(%) | 16.1396 | 16.1490 | 16.0834 | 16.1621 |
| Variance | 0.0183 | 0.0183 | 0.0184 | 0.0184 |
| Skewness | -0.4256 | -0.4254 | -0.4229 | -0.4226 |
| Kurtosis | 2.8316 | 2.8289 | 2.7956 | 2.7986 |
| Average turnover(%) | 2.7410 | 2.7062 | 2.7327 | 2.4695 |
| CE(basis point) | 30.7151 | 30.7327 | 30.6034 | 30.7549 |
| Annualized diff in CE(%) | – | – | – | 0.0207 |

**Table 2.9** – Out of Sample Investment in Developing Countries Currencies

The period of the out-of-sample is from April 1, 1994, to March 20, 2020. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | developing countries models | | | |
|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula |
| Panel A. $\gamma$=3 | | | | |
| Annualized mean(%) | 23.8652 | 23.8711 | 23.8622 | 23.9173 |
| Variance | 0.0228 | 0.0228 | 0.0228 | 0.0228 |
| Skewness | -0.0466 | -0.0445 | -0.0464 | -0.0475 |
| Kurtosis | 2.7375 | 2.7381 | 2.7372 | 2.7331 |
| Average turnover(%) | 1.9524 | 1.9990 | 2.0320 | 1.8765 |
| CE(basis point) | 45.3976 | 45.7572 | 45.7398 | 45.8461 |
| Annualized diff in CE(%) | – | – | – | 0.2332 |
| Panel B. $\gamma$=7 | | | | |
| Annualized mean(%) | 23.7773 | 23.7775 | 23.7976 | 23.8410 |
| Variance | 0.0232 | 0.0232 | 0.0232 | 0.0231 |
| Skewness | -0.1144 | -0.1141 | -0.1158 | -0.1137 |
| Kurtosis | 2.7709 | 2.7700 | 2.7811 | 2.7852 |
| Average turnover(%) | 2.3941 | 2.3756 | 2.4624 | 2.3812 |
| CE(basis point) | 45.3650 | 45.3654 | 45.4049 | 45.4898 |
| Annualized diff in CE(%) | – | – | – | 0.0275 |
| Panel C. $\gamma$=10 | | | | |
| Annualized mean(%) | 23.9086 | 23.9090 | 23.9154 | 23.9951 |
| Variance | 0.0230 | 0.0231 | 0.0230 | 0.0230 |
| Skewness | -0.1484 | -0.1508 | -0.1537 | -0.1571 |
| Kurtosis | 2.9374 | 2.9403 | 2.9567 | 2.9740 |
| Average turnover(%) | 2.5953 | 2.6082 | 2.6819 | 2.6416 |
| CE(basis point) | 45.4693 | 45.4698 | 45.4829 | 45.6374 |
| Annualized diff in CE(%) | – | – | – | 0.0874 |

**Table 2.10** – Average Weights in the Portfolios

| | DOL | HML | MOM | VAL |
|---|---|---|---|---|
| Panel A. $\gamma$=3 | | | | |
| Normal distribution | 0.7745 | 0.8881 | 0.6784 | 0.6507 |
| Normal Copula | 0.7755 | 0.8869 | 0.6786 | 0.6510 |
| Student t Copula | 0.7757 | 0.8875 | 0.6783 | 0.6498 |
| Skewed t Copula | 0.7740 | 0.8846 | 0.6924 | 0.6388 |
| Panel B. $\gamma$=7 | | | | |
| Normal distribution | 0.7912 | 0.8741 | 0.6612 | 0.6333 |
| Normal Copula | 0.7910 | 0.8737 | 0.6599 | 0.6339 |
| Student t Copula | 0.7910 | 0.8718 | 0.6566 | 0.6368 |
| Skewed t Copula | 0.7892 | 0.8734 | 0.6520 | 0.6401 |
| Panel C. $\gamma$=10 | | | | |
| Normal distribution | 0.7958 | 0.8541 | 0.6564 | 0.6318 |
| Normal Copula | 0.7953 | 0.8543 | 0.6568 | 0.6324 |
| Student t Copula | 0.7949 | 0.8524 | 0.6515 | 0.6377 |
| Skewed t Copula | 0.8017 | 0.8524 | 0.6438 | 0.6371 |

**Table 2.11** – Out of Sample Investment Results with Skewed t-t Factor Model

The period of the out-of-sample is from April 1, 1994, to March 20, 2020. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | dynamic correlation models | |
|---|---|---|
| | skewed t-t factor | skewed t Copula |
| Panel A. $\gamma$=3 | | |
| Annualized mean(%) | 7.2701 | 18.8868 |
| Annualized volatility | 0.0109 | 0.0162 |
| Skewness | 0.3543 | -0.0346 |
| Kurtosis | 2.9389 | 2.5787 |
| Average turnover(%) | 1.3285 | 1.7100 |
| CE(basis point) | 13.9467 | 36.2452 |
| Annualized diff in CE(%) | | 0.2847 |
| Panel B. $\gamma$=7 | | |
| Annualized mean(%) | 7.5807 | 18.6866 |
| Annualized volatility | 0.0103 | 0.0162 |
| Skewness | 0.0381 | 0.0049 |
| Kurtosis | 2.7182 | 2.6629 |
| Average turnover(%) | 1.2185 | 2.1294 |
| CE(basis point) | 14.5476 | 35.7602 |
| Annualized diff in CE(%) | | 0.2435 |
| Panel C. $\gamma$=10 | | |
| Annualized mean(%) | 7.7675 | 18.6422 |
| Annualized volatility | 0.0101 | 0.0168 |
| Skewness | -0.0005 | -0.1413 |
| Kurtosis | 2.7872 | 2.8456 |
| Average turnover(%) | 1.2617 | 2.3522 |
| CE(basis point) | 14.9080 | 35.5814 |
| Annualized diff in CE(%) | | 0.3332 |

**Table 2.12** – Dynamic Copula Models' CDB with Real Return Portfolios

I report the 935 CDB results in 6 different portfolios, (normal distribution, normal Copula student t Copula, skewed t Copula and 4-factor model and 1/N portfolios). I present the the mean, volatility, skewness, kurtosis, maximum value, minimum value and median of different investment portfolios.

| | Normal Distribution | Normal Copula | Student t Copula | Skewed t Copula | Skewed t t-factor Copula |
|---|---|---|---|---|---|
| Panel A. $\gamma$=3 | | | | | |
| Mean | 0.8101 | 0.8147 | 0.8142 | 0.8109 | 0.8991 |
| Volatility | 0.0082 | 0.0088 | 0.0083 | 0.0105 | 0.0071 |
| Skewness | -1.2115 | -1.2029 | -1.2142 | -0.8449 | 0.3908 |
| Kurtosis | 4.0984 | 3.9413 | 4.0231 | 3.1013 | 2.0403 |
| Maximum | 0.8226 | 0.8273 | 0.8264 | 0.8257 | 0.9167 |
| Minimum | 0.7806 | 0.7843 | 0.7837 | 0.7756 | 0.8876 |
| Median | 0.8116 | 0.8166 | 0.8160 | 0.8138 | 0.8976 |
| Panel B. $\gamma$=7 | | | | | |
| Mean | 0.8291 | 0.8211 | 0.8229 | 0.8222 | 0.9072 |
| Volatility | 0.0111 | 0.0093 | 0.0090 | 0.0095 | 0.0117 |
| Skewness | -0.4863 | -1.0434 | -0.9397 | -0.9049 | 0.5844 |
| Kurtosis | 2.5511 | 3.7542 | 3.6266 | 2.8900 | 1.5461 |
| Maximum | 0.8473 | 0.8361 | 0.8381 | 0.8391 | 0.9283 |
| Minimum | 0.8003 | 0.7879 | 0.7904 | 0.7937 | 0.8931 |
| Median | 0.8311 | 0.8239 | 0.8253 | 0.8258 | 0.9002 |
| Panel C. $\gamma$=10 | | | | | |
| Mean | 0.8312 | 0.8302 | 0.8333 | 0.8145 | 0.9121 |
| Volatility | 0.0097 | 0.0106 | 0.0104 | 0.0170 | 0.0132 |
| Skewness | -0.4811 | -0.1808 | -0.3364 | -0.0774 | 0.6608 |
| Kurtosis | 2.7755 | 2.4969 | 2.6867 | 2.2808 | 1.6869 |
| Maximum | 0.8513 | 0.8519 | 0.8533 | 0.8440 | 0.9386 |
| Minimum | 0.8042 | 0.8039 | 0.8060 | 0.7701 | 0.8980 |
| Median | 0.8322 | 0.8278 | 0.8315 | 0.8121 | 0.9038 |

**Table 2.13** – Dynamic Copula Models' CDB with Average Return Portfolios

I report the 935 CDB results in 6 different portfolios, (normal distribution, normal Copula student t Copula, skewed t Copula and 4-factor model and 1/N portfolios). I present the the mean, volatility, skewness, kurtosis, maximum value, minimum value and median of different investment portfolios.

| | Normal Distribution | Normal Copula | Student t Copula | Skewed t Copula | Skewed t t-factor Copula |
|---|---|---|---|---|---|
| Panel A. $\gamma$=3 | | | | | |
| Mean | 0.8427 | 0.8417 | 0.8391 | 0.8336 | 0.9492 |
| Volatility | 0.0114 | 0.0124 | 0.0149 | 0.0176 | 0.0021 |
| Skewness | -1.2264 | -1.1052 | -1.0364 | -1.2727 | -0.0764 |
| Kurtosis | 4.4813 | 3.6557 | 2.8982 | 3.6421 | 3.5137 |
| Maximum | 0.8623 | 0.8630 | 0.8627 | 0.8617 | 0.9541 |
| Minimum | 0.8024 | 0.8020 | 0.7977 | 0.7790 | 0.9414 |
| Median | 0.8455 | 0.8458 | 0.8453 | 0.8410 | 0.9489 |
| Panel B. $\gamma$=7 | | | | | |
| Mean | 0.8455 | 0.8444 | 0.8459 | 0.8482 | 0.9512 |
| Volatility | 0.0103 | 0.0102 | 0.0095 | 0.0188 | 0.0016 |
| Skewness | -1.0532 | -0.9651 | -1.1169 | -0.1127 | -0.2401 |
| Kurtosis | 4.9370 | 4.5572 | 5.2915 | 1.3898 | 3.1291 |
| Maximum | 0.8659 | 0.8648 | 0.8671 | 0.8763 | 0.9551 |
| Minimum | 0.7999 | 0.8015 | 0.8021 | 0.8155 | 0.9462 |
| Median | 0.8466 | 0.8455 | 0.8476 | 0.8503 | 0.9513 |
| Panel C. $\gamma$=10 | | | | | |
| Mean | 0.8446 | 0.8458 | 0.8462 | 0.8352 | 0.9505 |
| Volatility | 0.0115 | 0.0113 | 0.0109 | 0.0107 | 0.0027 |
| Skewness | -0.7429 | -0.6991 | -0.7926 | 0.6382 | 0.2018 |
| Kurtosis | 3.9053 | 3.7957 | 4.2302 | 2.5016 | 2.5157 |
| Maximum | 0.8689 | 0.8703 | 0.8691 | 0.8611 | 0.9569 |
| Minimum | 0.7982 | 0.8004 | 0.8005 | 0.8157 | 0.9448 |
| Median | 0.8455 | 0.8464 | 0.8468 | 0.8336 | 0.9502 |

**Table 2.14** – Out of Sample Investment with Real Factor Return

The period of the out-of-sample is from April 1, 1994, to March 20, 2020 with real factor return. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | dynamic correlation models | | | | |
|---|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula | skewed t-t factor |
| Panel A. $\gamma$=3 | | | | | |
| Annualized mean(%) | 20.5759 | 20.6490 | 20.5266 | 21.4097 | 8.2480 |
| Annualized volatility | 0.1349 | 0.1347 | 0.1347 | 0.1342 | 0.0814 |
| Skewness | -0.1330 | -0.1310 | -0.1371 | -0.1811 | -0.1055 |
| Kurtosis | 5.2512 | 5.2013 | 5.2398 | 5.2074 | 8.9224 |
| Average turnover(%) | 12.6694 | 12.7369 | 12.7477 | 13.3181 | 5.9724 |
| CE(basis point) | 34.3149 | 34.4739 | 34.2387 | 35.9661 | 13.9490 |
| Panel B. $\gamma$=7 | | | | | |
| Annualized mean(%) | 20.1511 | 20.2459 | 20.2974 | 20.6030 | 7.4207 |
| Annualized volatility | 0.1329 | 0.1326 | 0.1323 | 0.1322 | 0.0764 |
| Skewness | -0.1041 | -0.1123 | -0.1142 | -0.1406 | -0.4694 |
| Kurtosis | 5.2496 | 5.2303 | 5.2578 | 5.2828 | 5.9895 |
| Average turnover(%) | 13.1357 | 13.2050 | 13.4268 | 13.3083 | 5.5600 |
| CE(basis point) | 26.7784 | 27.0009 | 27.1629 | 27.7475 | 10.2836 |
| Panel C. $\gamma$=10 | | | | | |
| Annualized mean(%) | 19.8030 | 19.9773 | 19.5762 | 18.5431 | 7.1728 |
| Annualized volatility | 0.1313 | 0.1311 | 0.1296 | 0.1312 | 0.0744 |
| Skewness | -0.1010 | -0.1069 | -0.1476 | -0.4420 | -0.5820 |
| Kurtosis | 5.3296 | 5.3304 | 5.3257 | 6.9387 | 6.0201 |
| Average turnover(%) | 13.4190 | 13.5396 | 13.7571 | 13.1763 | 5.3044 |
| CE(basis point) | 21.2692 | 21.6416 | 21.2195 | 18.3434 | 8.3427 |

**Table 2.15** – Out of Sample Investment with Real Factor Return in Crisis Time

The period of the out-of-sample is from Sep 21, 2007, to Sep 25, 2009 with real factor return. For each level of relative risk aversion, the performance of the three Copula models is compared to the benchmark normal distribution. Panels A,B and C show the results for a relative risk aversion coefficient of 3,7 and 10, respectively. I report the realized moments of the portfolio returns, the average turnover, as well as the certainty equivalent.

| | developing countries models | | | |
|---|---|---|---|---|
| | normal distribution | normal Copula | student t Copula | skewed t Copula |
| Panel A. $\gamma$=3 | | | | |
| Annualized mean(%) | -21.4929 | -21.6420 | -21.8330 | -20.7801 |
| Annualized volatility(%) | 18.4753 | 18.4760 | 18.4661 | 17.8681 |
| Skewness | -0.3367 | -0.3336 | -0.3311 | -0.5494 |
| Kurtosis | 5.8999 | 5.8962 | 5.9054 | 6.0267 |
| Average turnover(%) | 2.3999 | 2.4031 | 2.4213 | 2.4865 |
| Sharpe ratio | -1.1633 | -1.1714 | -1.1823 | -1.1630 |
| Panel B. $\gamma$=7 | | | | |
| Annualized mean(%) | -22.8137 | -22.7402 | -22.5431 | -20.4798 |
| Annualized volatility(%) | 18.4226 | 18.4274 | 18.3980 | 18.5446 |
| Skewness | -0.3408 | -0.3298 | -0.3380 | -0.3295 |
| Kurtosis | 5.8477 | 5.8799 | 5.8715 | 5.7346 |
| Average turnover(%) | 2.4271 | 2.4347 | 2.4427 | 2.5048 |
| Sharpe ratio | -1.2384 | -1.2340 | -1.2253 | -1.1044 |
| Panel C. $\gamma$=10 | | | | |
| Annualized mean(%) | -23.0604 | -22.4718 | -21.9915 | -20.2426 |
| Annualized volatility(%) | 18.3726 | 18.3445 | 18.2445 | 18.1748 |
| Skewness | -0.3624 | -0.3574 | -0.3495 | -0.2906 |
| Kurtosis | 5.8272 | 5.9069 | 5.9463 | 6.0975 |
| Average turnover(%) | 2.4204 | 2.4278 | 2.4111 | 2.4489 |
| Sharpe ratio | -1.2552 | -1.2250 | -1.2054 | -1.1138 |

**Table 2.16** – Dynamic Copula Models' CDB with Real Return in Crisis Time

I report the 935 CDB results in 6 different portfolios, (normal distribution, normal Copula student t Copula, skewed t Copula and 4-factor model and 1/N portfolios). I present the the mean, volatility, skewness, kurtosis, maximum value, minimum value and median of different investment portfolios.

| | Normal Distribution | Normal Copula | Student t Copula | Skewed t Copula |
|---|---|---|---|---|
| Panel A. $\gamma$=3 | | | | |
| Mean | 0.8141 | 0.8205 | 0.8193 | 0.8196 |
| Volatility | 0.0048 | 0.0038 | 0.0025 | 0.0054 |
| Skewness | -0.5077 | -0.6406 | -0.3701 | -0.8707 |
| Kurtosis | 1.6161 | 1.8738 | 1.6943 | 2.1351 |
| Maximum | 0.8200 | 0.8247 | 0.8226 | 0.8248 |
| Minimum | 0.8050 | 0.8130 | 0.8151 | 0.8084 |
| Median | | | | |
| Panel B. $\gamma$=7 | 0.8294 | 0.8218 | 0.8240 | 0.8252 |
| Mean | 0.0041 | 0.0041 | 0.0025 | 0.0028 |
| Volatility | -0.6282 | -0.5724 | -0.2877 | -0.2992 |
| Skewness | 1.7943 | 1.7391 | 1.6591 | 1.6788 |
| Kurtosis | 0.8340 | 0.8262 | 0.8272 | 0.8288 |
| Maximum | 0.8213 | 0.8138 | 0.8197 | 0.8202 |
| Minimum | | | | |
| Median | 0.8308 | 0.8270 | 0.8301 | 0.8343 |
| Panel C. $\gamma$=10 | 0.0032 | 0.0014 | 0.0015 | 0.0024 |
| Mean | -0.5066 | 0.2991 | 0.3158 | -0.2777 |
| Volatility | 1.6023 | 2.6541 | 2.4532 | 1.8480 |
| Skewness | 0.8350 | 0.8303 | 0.8337 | 0.8376 |
| Kurtosis | 0.8255 | 0.8235 | 0.8270 | 0.8300 |
| Maximum | 0.8689 | 0.8703 | 0.8691 | 0.8611 |
| Minimum | 0.7982 | 0.8004 | 0.8005 | 0.8157 |
| Median | 0.8455 | 0.8464 | 0.8468 | 0.8336 |

**Figure 2.1** – Time Series Plot for 4 Factors

The figure below illustrates the time series of weekly returns of each factor for the period January 1, 1989, to March 20, 2020.

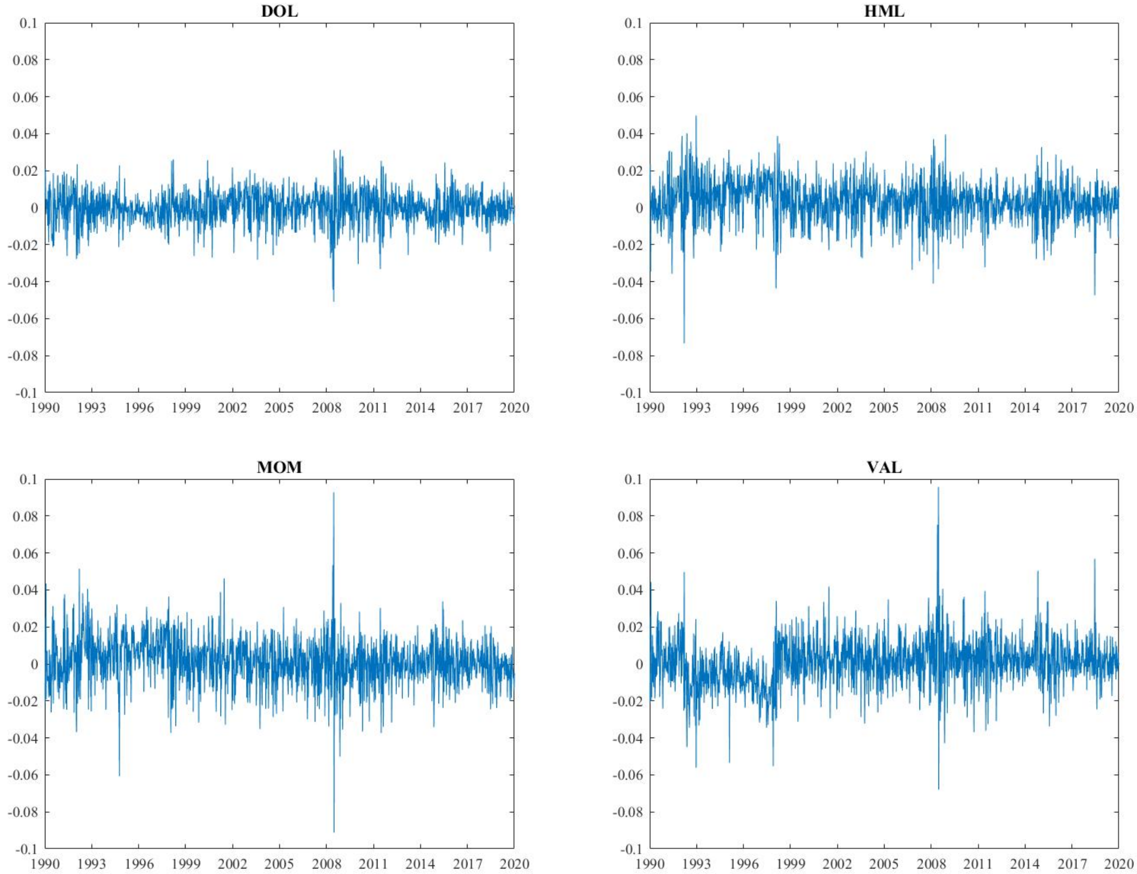


**Figure 2.2** – Quantile-Quantile Plots for 4 Factors

For each observation I scatter plot the empirical quantile on the vertical axis against the corresponding quantile from the standard normal distribution on the horizontal axis. If returns are normally distributed, then the data points will fall randomly around the 45° line ,which is marked by dashes.

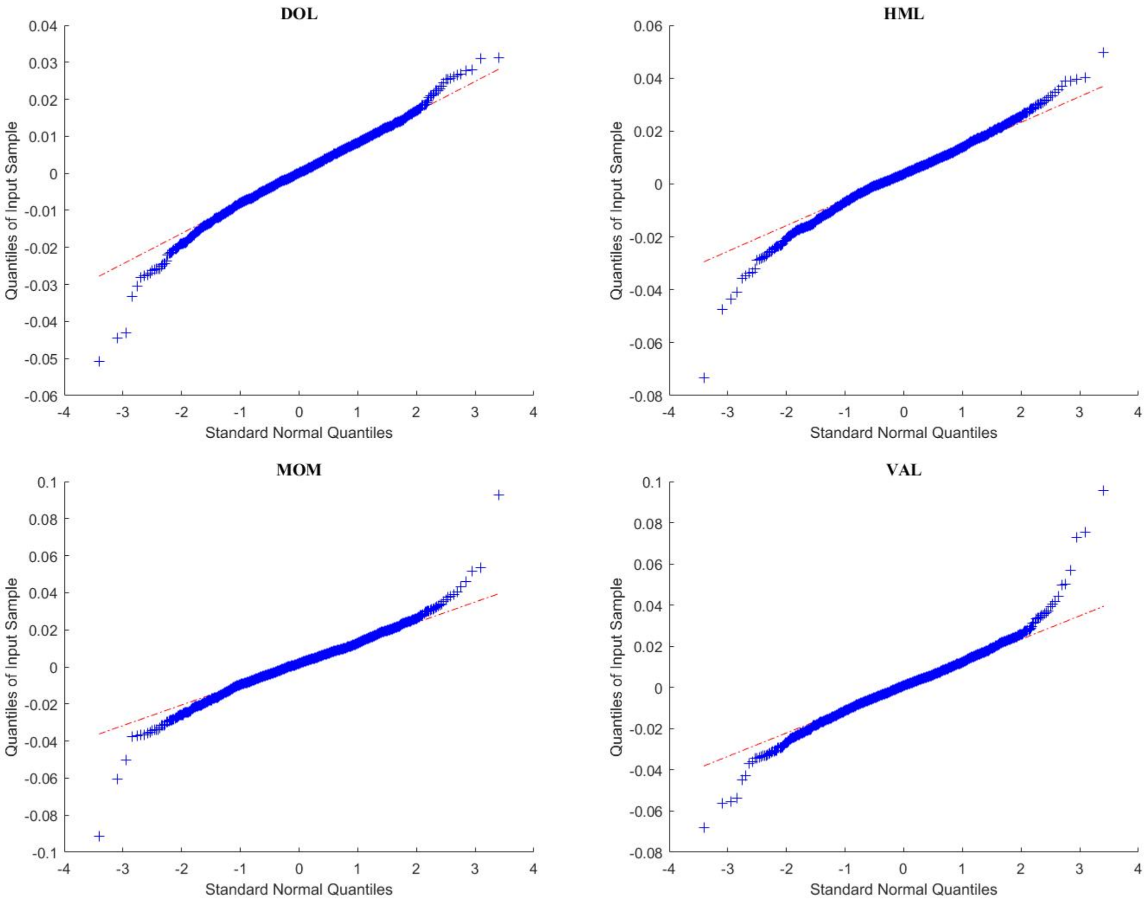


**Figure 2.3** – Threshold Correlation for 4 Factors

Figure 3 presents threshold correlations between the 4 factors . my sample consists of weekly returns from January 1, 1989, to March 20, 2020. The continuous line represents the correlations when both variables are below (above) a threshold when this threshold is below (above) the median. The dashed line represents the threshold function for a bivariate normal distribution using the linear correlation coefficient from the data.

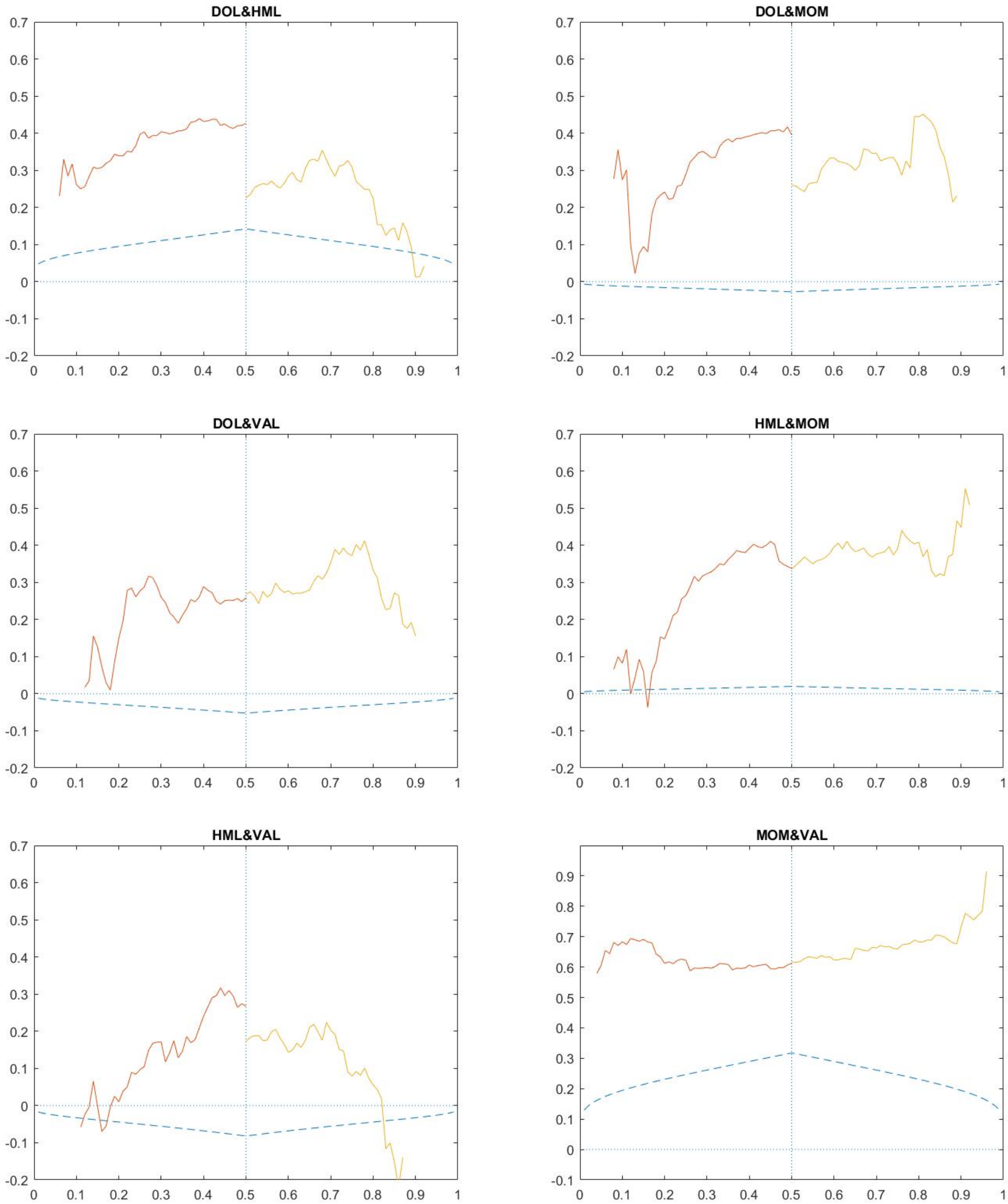


**Figure 2.4** – Autocorrelation 4 Factors and the Absolute Value of 4 Factors

autocorrelation of weekly returns (dashed line) and absolute returns (solid line) from January 1, 1989, to March 20, 2020. The horizontal dotted lines provide a 95 confidence interval around 0.

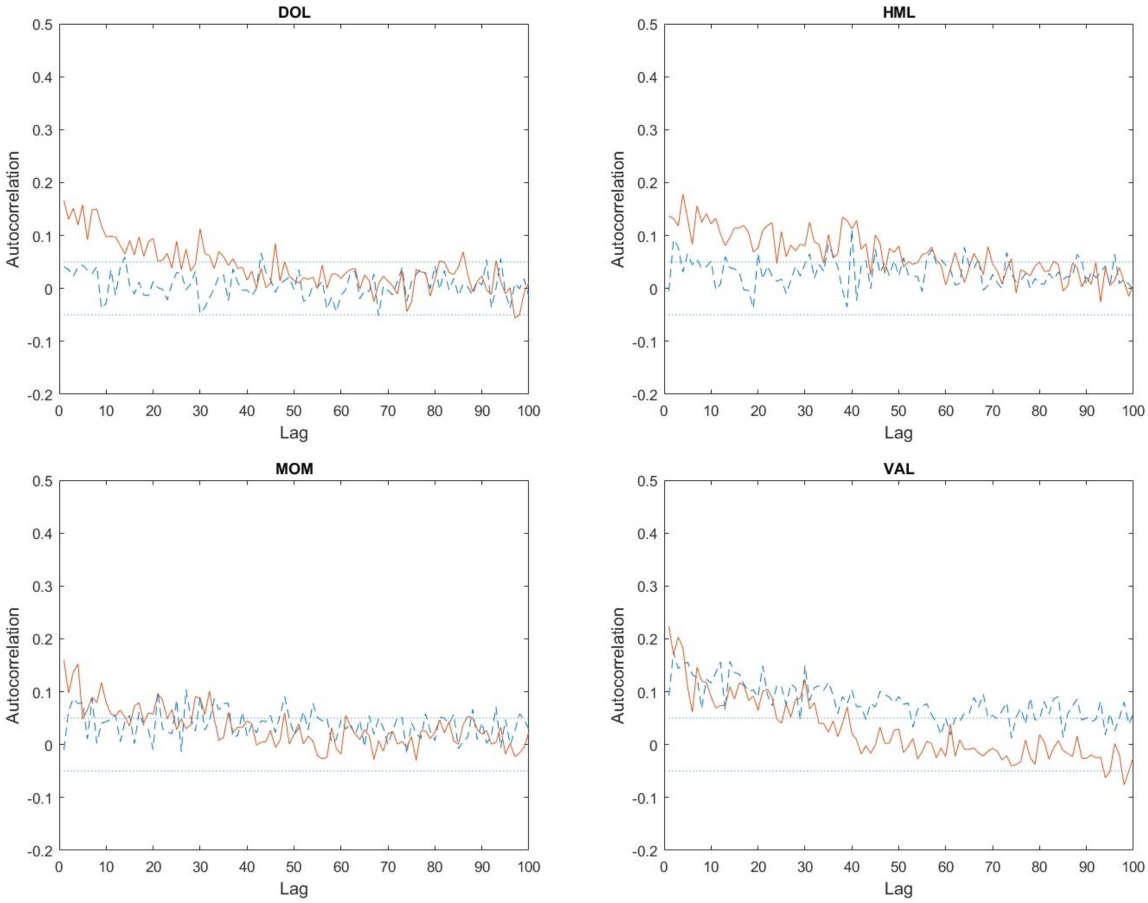


**Figure 2.5** – Autocorrelation Graph of Residual Series

autocorrelation of AR-FARCH residuals (dashed line) and absolute residuals (solid line) from January 1, 1989, to March 20, 2020. The horizontal dotted lines provide a 95 confidence interval around 0.

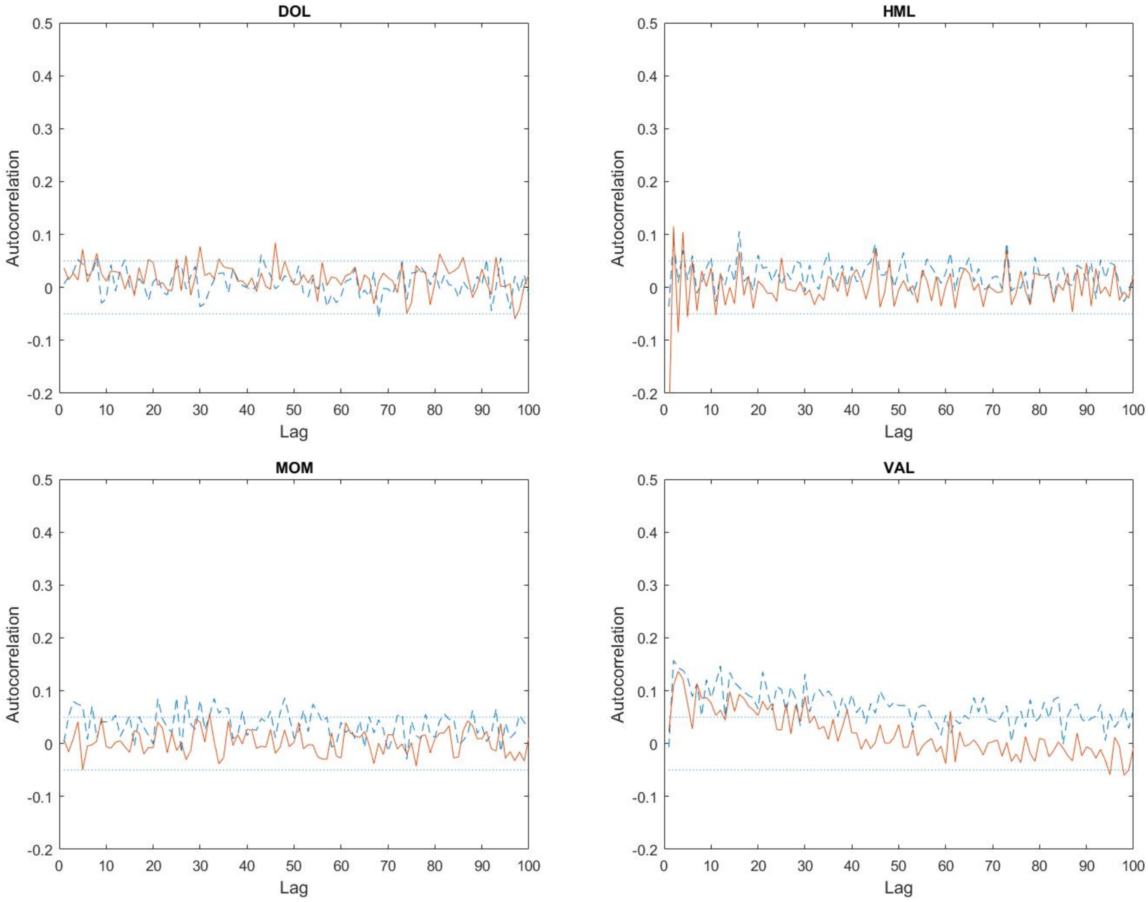


**Figure 2.6** – QQ Plot of Residuals Series

For each observation I scatter plot the empirical quantile on the vertical axis against the corresponding quantile from the skewed t distribution on the horizontal axis. If the AR-GARCH residuals adhere to the skewed t distribution, then the data points will fall on the 45º line, which is marked by dashes. The parameters for the skewed t distribution are from Table 3.

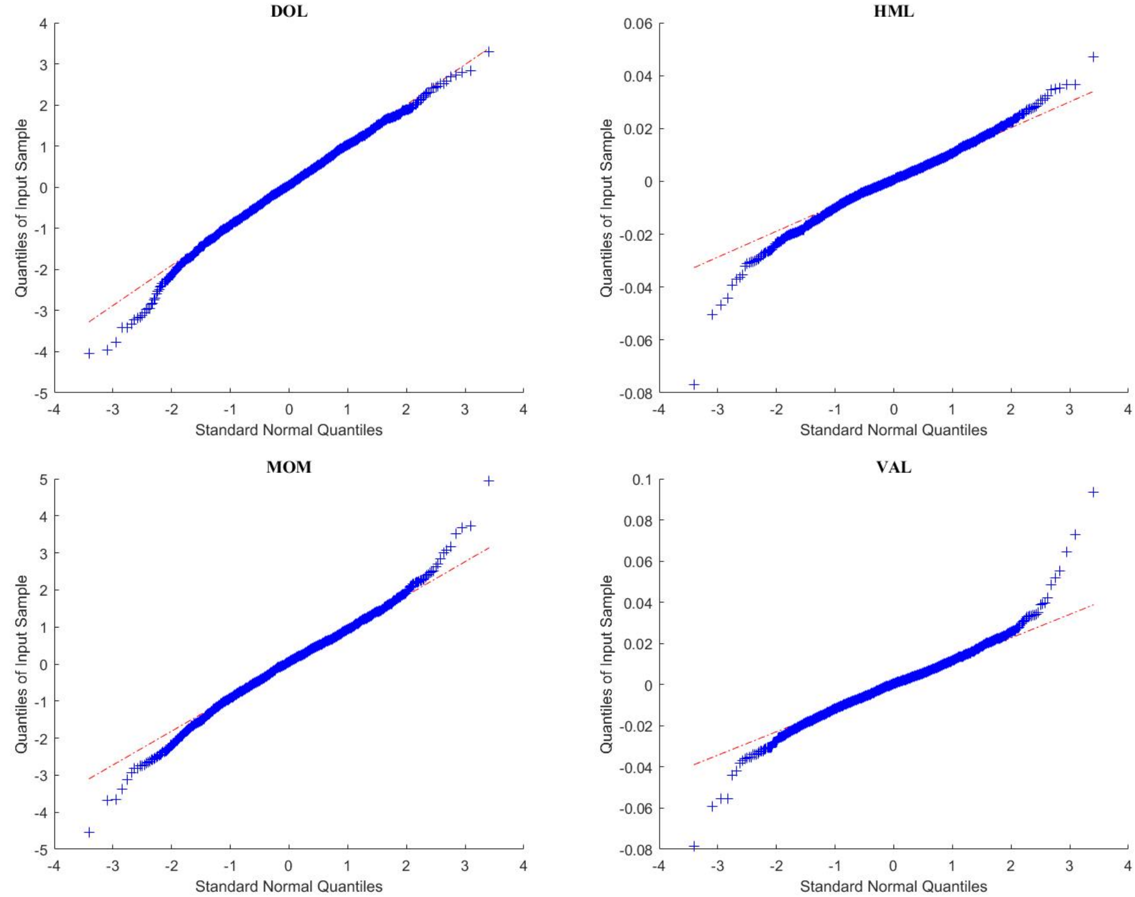


**Figure 2.7** – Skewed t Copula Dynamic Correlations with Composite Method

I report dynamic conditional Copula correlation for each pair of factors from January 1, 1989, to March 20, 2020. The correlations are obtained by estimating the dynamic skewed t Copula model on the factor return residuals from the AR-GARCH model. This sample is used in estimation of the models.

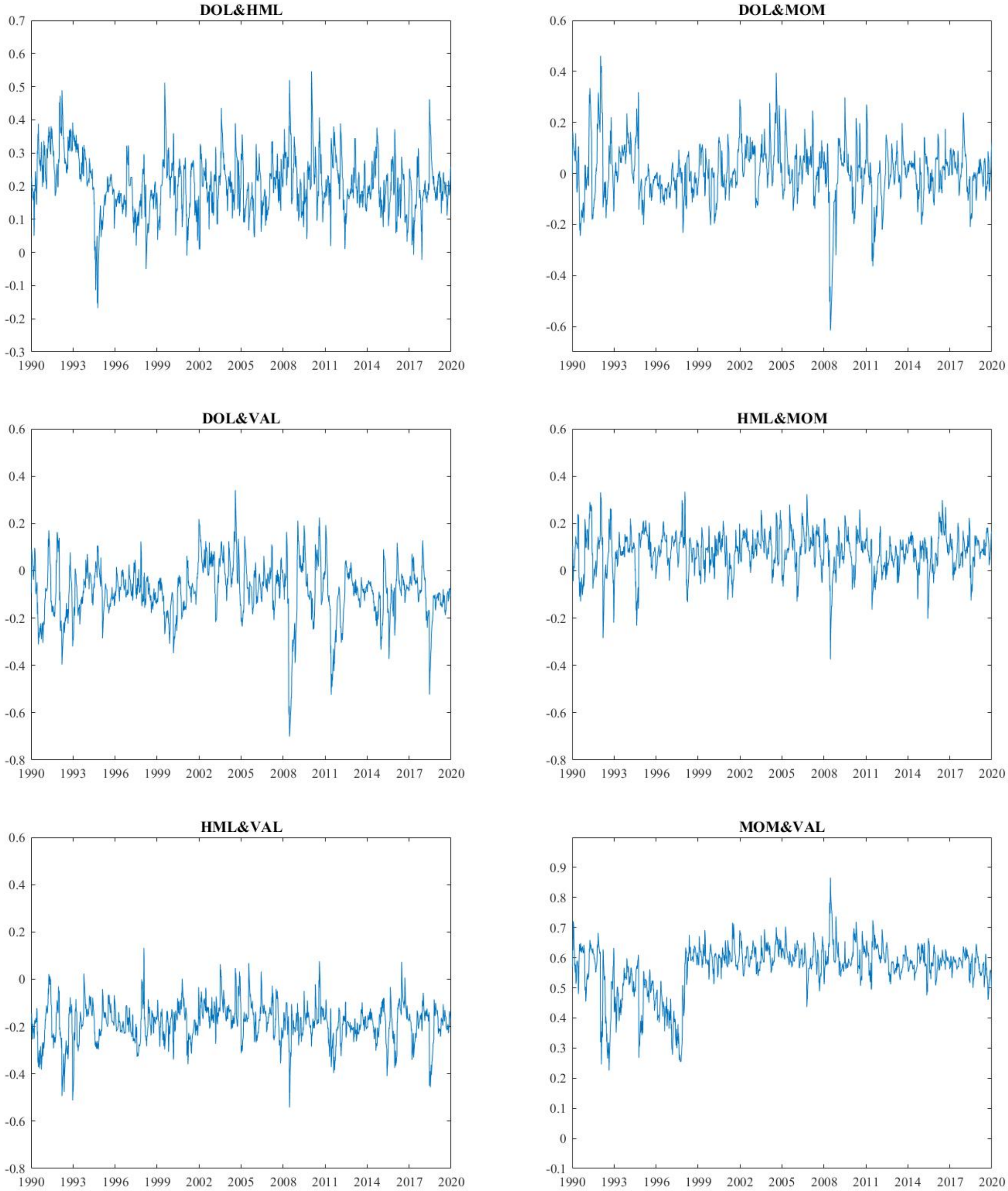


**Figure 2.8** – Threshold Correlations for Factor Residuals and Copula Models

I present threshold correlations computed on AR-GARCH residuals from January 1, 1989, to March 20, 2020. The thick continuous line represents the empirical correlation. The threshold correlation functions are computed for thresholds for which there are at least 24 data points available. I compared the empirical correlations to those implied by the normal Copula and the constant t and skewed t Copulas.

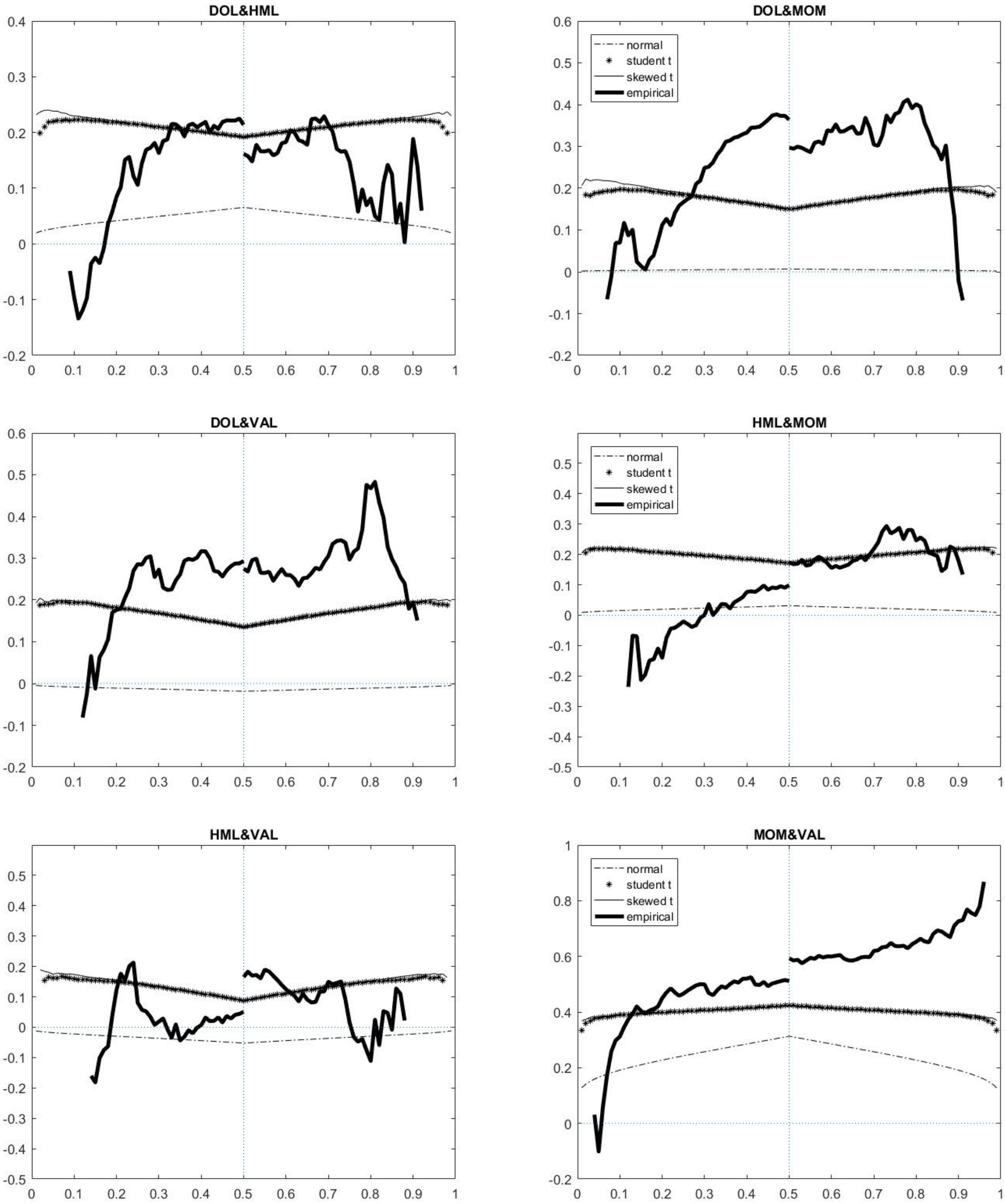


**Figure 2.9** – Contour Plots of Different Copula

I present the contour plots of the normal, student t and skewed t Copula models with different parameters.

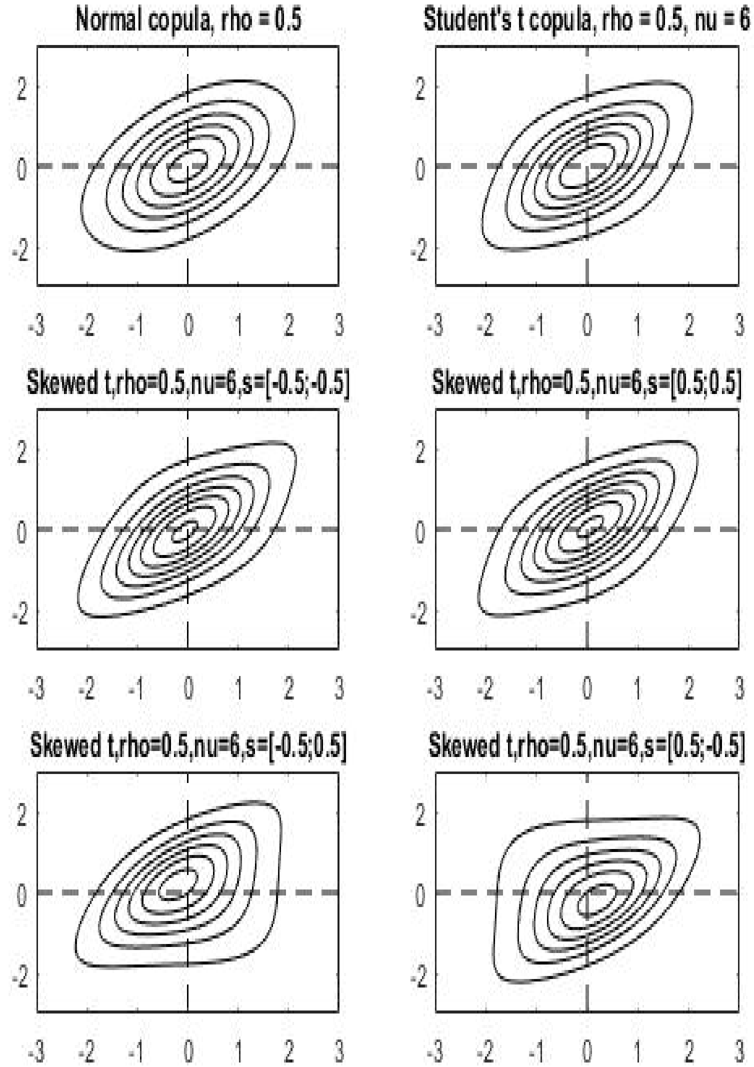
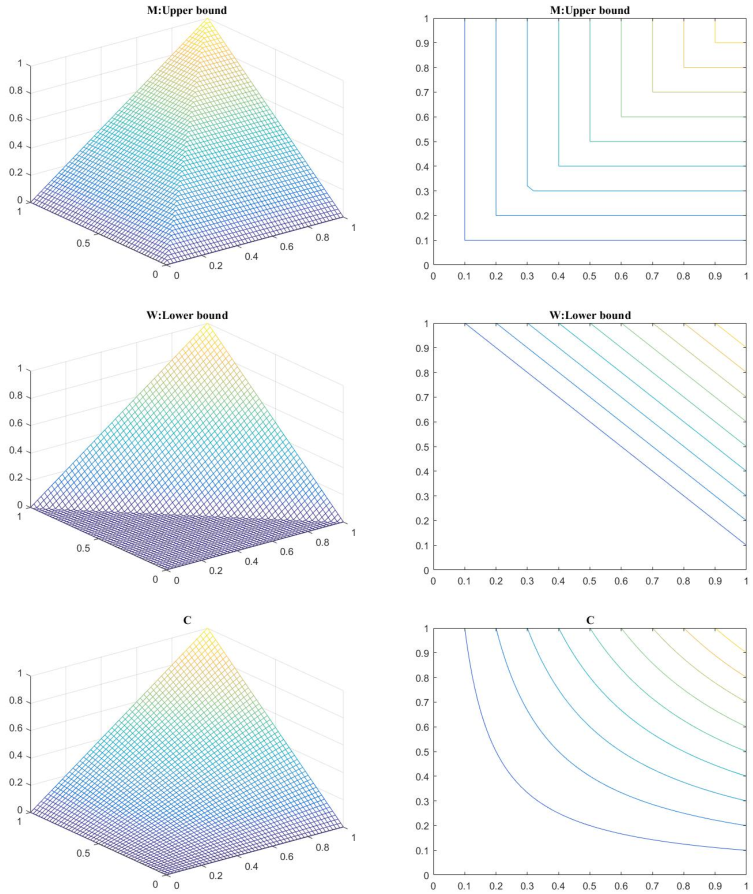


**Figure 2.10** – Fréchet-Hoeffding Bounds

I draw the Fréchet-Hoeffding lower and upper bounds and Copula model in this figure. M show the upper bond, while W show the lower bond. C indicate the distribution of Copula model.



# Appendix.2

## Appendix.A The Review of the Calculation of the Factors

In this section, I will discuss the calculation functions and method of the currency factors from the relevant literature.

I set the $s$ to denote the log value of the spot exchange return, while $f$ is the log value of the forward exchange rate. An appreciation of the home currency indicates an increase in the spot rate $s$ and forward rate $f$. I set the USD as the local currency. When I buy the foreign currency at time $t$ in the forward market and sell the foreign currency at time $t+1$ at the spot market, I could get an excess return from the currency market. Then, The log excess return $rx$ should as follow:

$$rx_{t+1} = f_t - s_{t+1}$$

The country's specific risk factor could be treated as a level factor. Lustig et al. (2011) form this home risk premium by average the cross-section foreign currency excess return, the DOL factor. As for the global risk factor, they argue this as the slope factor, which would also be treated as the carry trade risk premium. The HML factor, which is the return of high-yield currencies minus the return of low-yield currencies and they show that the factors can price carry trade portfolios in the cross-section. To build the HML factor, they apply the following method.

At the end of each period $t$, Lustig et al. (2011) put the excess returns of different currencies in six portfolios based on the forward premium (forward discount) $f - s$ observed at the end of period $t$ from low to high. The first portfolio includes the currencies' excess return with the lowest interest rate (smallest forward discount), while the sixth portfolio includes the currencies' excess return with the highest interest rate (largest forward discount). To benefit from the carry trade and build the HML factor, Lustig et al. (2011) assume that the investor shorts all foreign currencies in the first portfolio and buys all other foreign currencies in the last portfolios.

Compared with the creation of HML factor from Lustig et al. (2011), Menkhoff et al.

(2012b) build the MOM (momentum factor) in a similar way.

To create the momentum factor in each term, Menkhoff et al. (2012b) apply the lagged returns over $f$ (formation period) $h$ (holding period) to sort the currencies excess return. They set $f = 1, 3, 6, 9, 12$ and $h = 1, 3, 6, 9, 12$. They sort the currencies from a low lagging return to a high lagging return in six different portfolios, which is similar to the momentum portfolios in the literature of the equity market. However, interest rate differences (forward discounting) account for a large proportion of excess returns on currency investments. They also track the pure spot rate movements of the momentum portfolio. Then, they can check whether currency movements are driven mainly by interest rate spreads or spot rates. These portfolios are represented as $MOM_{f,h}$.

Kroencke et al. (2014) propose the detailed calculation of factor VAL, which depends on the price level of consumer goods expressed in national currency. The basic idea of the value strategy is going long in a currency that is thought to be below its fundamental value and shorting in a currency that is thought to be above its fundamental value. They first consider the real exchange rates as follow:

$$Q_{j,t} = \frac{S_{j,t}P_{j,t}}{P^*_{j,t}} \tag{2.27}$$

where $P_{j,t}$ denotes the price level of consumer goods expressed in national currency in country $j$, while $P^*_{j,t}$ is the corresponding U.S. dollar (domestic currency) price level. If the purchasing power parity (PPP) holds, then the above equation should be equal to 1. Therefore, currencies whose real exchange rates are (higher than) lower than the unified currency can be considered undervalued (overvalued). PPP is a strong assumption because the equilibrium real exchange rate can easily deviate from unification (the Harrod-Balassa-Samuelson effect). Therefore, in order to avoid the problem of defining the equilibrium real exchange rate, Kroencke et al. (2014) use value minus the five-year cumulative change in the real exchange rate as their condition variable as follow:

$$F_{VAL,j,t} = \left(\frac{Q_{j,t-3}}{Q_{j,t-60}} - 1\right)(-1) \tag{2.28}$$

where $Q_{j,t-60}$ denotes the real exchange rate over 5 years in the past, while the $Q_{j,t-3}$ is the real exchange rate of 3-month in the past. I apply this factor to build the VAL portfolio in the chapter.

# Appendix.B Copula Model Applications

## Introduction of Copula

Tail dependence could help researchers to focus on the non-linear relationship between variables. To discuss the joint distribution and the dependence structure of forex factors, I will apply the Copula to achieve this.

The word Copula is a Latin word, meaning "a link, tie, bond" (Cassell's Latin dictionary), which could also be decried as "the part of the proposition that connects the subject and the predicate" (English Oxford Dictionary). Fréchet (1951) firstly study the joint distribution functions, which could be regarded as the start of the Copula theory.

Sklar (1959)obtains the most profound results in this theory. Firstly, he used the word Copula in the mathematical or statistical sense in the theorem (now named after him), which describes the function of connecting one-dimensional distribution functions to form multivariate distribution functions. Nelsen (2007) defines it as a function that combines or couples multivariate distribution function with its one-dimensional marginal distribution function.

The ideal properties of Copula can be defined as follows (McNeil et al. (2015)). First, authors can separate the marginal distribution from the dependency structure according to the flexibility of Copula, without requiring them to come from the same joint distribution. Secondly, the Copula model helps researchers to use the Monte Carlo model flexibly and practically. Third, Copula can describe non-linear and tail dependence by quantile dependence, especially in the risk management environment. Fourth, authors can combine the more complex marginal model with various possible correlations and use Copula to study risk sensitivity to dependent norms.

These satisfying properties have indeed attracted many financial researchers. Cherubini et al. (2004) summarize the classical methods of financial dependence modeling, which are typically rooted in the static Copula theory. Embrechts et al. (2002) are very popular as a working paper in 1999, dealing with static (time-independent) cases and emphasizing Copula representation of random vector dependence.

In the existing literature, there are some important reviews on connection theory and application. Two basic comprehensive texts, Joe (1997) and Nelsen (2007), introduce the theoretical and mathematical evidence of dependency modeling in detail. Ang and Chen (2002) build on the basis of Longin and Solnik (2001) and Ang and Bekaert (2002), showing that the correlation between the domestic stock portfolio and the overall market is greater in falling markets than in rising markets. Cherubini et al. (2004) summarize the financial mathematical application models based on the Copula. Embrechts (2009) briefly reviews the review of financial and insurance companies, which also included a list of reference materials needed to establish linkages and comments on future developments in this area. In order to test the importance of univariate and multivariate asymmetry in the optimal allocation between small and large capital portfolios, Patton (2004) uses the rotating Gumbel Copula that produces asymmetric correlations. In addition, Hong et al. (2007) find that for investors who are dissatisfied with aversion preferences, adding asymmetric dependencies is very important for portfolio selection. Jaworski et al. (2010) conduct many investigations on different aspects of the interconnection model and conducted empirical research on the application of interconnection in the field of finance and insurance. McNeil et al. (2015) and its updated version discuss the quantitative risk management model based on the Copula.

## Important Properties of the Copula

In this section, I will introduce vital properties of the Copula model. First, I will discuss Sklar's Theorem. Then, the Survival Copula will be introduced. Finally, I will show the Fréchet-Hoeffding Bounds for Copula.

**Sklar's Theorem** Firstly, I will introduce the basic theorem of the Copula model. Let's consider the multivariate situations of joint distribution functions. Following Sklar (1959), I could set a vector random variable $X = [X_1, X_2, ..., X_n]'$ . The joint distribution function of the variable $X$ is $F$, while the variable $X_n$ has the distribution function $F_n$. Then, I could write the Copula $C$ model as:

$$F(x_1, ..., x_n) = C(F_1(x_1), F_2(x_2), ..., F_n(x_n)), \forall x \in \mathbb{R}^n \tag{2.29}$$

**Survival Copula** Then, I will introduce the application of Copula in survival questions. The probability of an individual surviving or surviving outside $x$ time is given by the survival function $\bar{F}(x) = P[X > x] = 1 - F(x)$. When considering a bivariate situation as $(X, Y)$ of random variables with joint distribution function $H$, the survival function with the joint situation would be $\bar{H}(x, y) = P[X > x, Y > y]$ . The margins of $\bar{H}$ are the functions $\bar{H}(x, -\infty)$ and $\bar{H}(-\infty, y)$. There, then, are two univariate survival functions for variable $X$ (distribution function $F$) and $Y$(distribution function $G$). It is interesting to find the relationship between the bivariate survival function and univariate survival function, when the Copula model of $X$ and $Y$ is $C$. Then $\bar{H}(x, y)$ could be written as:

$$\begin{aligned} \bar{H}(x, y) &= 1 - F(x) - G(y) + H(x, y) \\ &= \bar{F}(x) + \bar{G}(y) - 1 + C(F(x), G(y)) \\ &= \bar{F}(x) + \bar{G}(y) - 1 + C\left(1 - \bar{F}(x), 1 - \bar{G}(y)\right) \end{aligned} \tag{2.30}$$

**The Fréchet-Hoeffding Bounds for Joint Distribution Functions** To explain the form of the Copula, I will introduce the Fréchet-Hoeffding Bounds to show the up-bound and the low bound of the Copula model in the bivariate situation. The

Fréchet-Hoeffding bounds as universal bounds for Copula, i.e., for any Copula $C$ and for all $u$,$v$ in $I$,

$$W(u,v) = \max(u+v-1,0) \le C(u,v) \le \min(u,v) = M(u,v)$$

The figures show the Fréchet-Hoeffding Bounds in Figure 2.10.

[Figure 2.9 contour plots of different Copula about here]

[Figure 2.10 Fréchet-Hoeffding Bounds about here]

Let's consider the more general situation, if the joint distribution function of $X$ and $Y$ is $H$, while $X$ and $Y$ have the margins distribution functions $F$ and $G$.

$$\max(F(x)+G(y)-1,0) \le H(x,y) \le \min(F(x),G(y))$$

# Appendix.C The GARCH Model (GARCH$(p,q)$ Processes)

The autoregressive conditional heteroscedasticity (ARCH) model was introduced by Engle (1982), and GARCH (generalized ARCH) extension was proposed by Bollerslev (1986). The key concept in these models is conditional variance, which is based on past conditional variances. In the classical GARCH model, the conditional variance is expressed as a linear function of the square of the sequence's past values.

GARCH $(p,q)$ process could be defined when the two conditional moments exist and satisfy:

(i) $E(t|u, u<t) = 0, t \in Z$.

(ii) There exist constants ω , $\alpha_i, i = 1, ..., q$ and $\beta_j, j = 1, ..., p$ such that

$$\sigma_t^2 = Var(\epsilon_t \mid \epsilon_u, u<t) = \omega + \sum_{i=1}^{q} \alpha_{0i}\epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_{0i}\sigma_{t-j}^2, \tag{2.31}$$

If the second part coefficient $\beta_{0i} = 0$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_{0i}\epsilon_{t-i}^2$$

and the process is called an ARCH($q$) process.

# Chapter 3

# Risk Forecasting in Forex Market

In this chapter, I focus on the risk forecasting part of portfolio management. I apply the same factors in chapter 2 as the portfolios. The risk measures of each factor are forecasted by different models. To apply the benefit of Copula in the risk management part, I extend the univariate models of Patton et al. (2019) to multivariate forecasting models. Both univariate and multivariate forecasting models are discussed in this chapter. The results show that adding Copula in the forecasting model could indeed improve the performance.

## 3.1 Introduction

The risk could be perceived as the volatility of outcomes, such as equity, assets, and portfolios. Hence, a way to measure and manage risk is vital for financial researchers. The most prevalent risk measures, Value at Risk (VaR) and Expected shortfall (ES), could be simply identified as:

$$ES_t = E[Y_t \mid Y_t \leq VaR_t, \mathcal{F}_{t-1}] \tag{3.1}$$

where $VaR_t = F_t^{-1}(\alpha), for\ \alpha \in (0,1)$ and $Y_t \mid \mathcal{F}_{t-1} \sim F_t$

The risk measure development could be summarized as follows. Duffie and Pan (1997) describe some fundamental issues involved in measuring the market risks of financial enterprises and prescribe a list of tools that help enterprises bear financial risks. Taking advantage of the criterion that the probability of exceeding VaR in each period must be independent of all past information, Engle and Manganelli (2004) introduce a new test for the appropriateness of the model, namely the dynamic quantile test. Jorion (2006) and Christoffersen (2009) provide helpful overview methods for VAR calculations. Engle and Manganelli (2004) propose a regression method based on conditional quantile. Artzner et al. (1999) define the concept of consistent risk measurement and show that the expected shortfall (ES) is consistent. Taylor (2008) provides an econometric tool for the ES calculation. Basak and Shapiro (2001) study ES and find that when the loss is significant, loss caused by ES risk management tends to be lower than that caused by VaR risk management. In contrast, Cuoco et al. (2008) believe that as long as VaR and ES risk measures are recalculated, they will produce the same results. Cuoco et al. (2008) assume that returns usually follow some distributions, which could be applied to calculate the VaR and ES. Yamai and Yoshiba (2005) compare the VaR and ES from a practical view. Berkowitz and O'Brien (2002) and Alexander and Baptista (2006) study VaR from regulation.

To develop the risk measurement, Fissler (2017) indicates loss function which could be a new risk measure and can be explained as follows. $F$ is a class of distribution functions on R while $A_0 := x \in R^2 : x_1 \geq x_2$. Then, the loss function with the score $S$ could be written as:

$$\begin{aligned} S(x_1, x_2, y) = {} & (1\{y \leq x_1\} - \alpha) G_1(x_1) - 1\{y \leq x_1\} G_1(y) \\ & + G_2(x_2)\left(x_2 - x_1 + \frac{1}{\alpha}(1\{y \leq x_1\}(x_1 - y))\right) - \vartheta_2(x_2) + a(y) \end{aligned} \tag{3.2}$$

where $G_1$ denotes increasing and $G_2$ denotes the increasing and convex. I discuss the loss function of Fissler (2017) in detail, which has been simplified by Patton et al. (2019) in the next section.

As discussed in chapter 1, the Copula model has its benefits in the risk management part. The motivation for this chapter is to apply joint distribution in the risk management section. After I apply the Copula model to forecast the return in the real investment market in the chapter 2, it is also interesting to forecast the risk of the portfolios by applying Copula models. Patton et al. (2019) show a model which applies univariate distribution to estimate the future risk measures, which could be easily improved for the multivariate model using Copula. Furthermore, the dynamic forecast model of Patton et al. (2019) can also be combined with a Copula, considering that joint distribution by Copula has been proved to add economic value in real investment circumstances. I apply the loss function of Fissler (2017) to forecasting the multivariate risk. The forecasted risk could help me determine the potential loss in the future.

Forecasting risk in the finance field has been discussed by many researchers. González-Rivera et al. (2004) evaluate the performance of various volatility models of stock returns when forecasting the risk. In order to improve the value-at-risk (VaR) prediction ability of the ordinary GARCH model, Hartz et al. (2006) develop a resampling method based on bootstrapping and bias correction steps. McAleer and Da Veiga (2008) use univariate and multivariate conditional volatility models to estimate the performance of the single index and portfolio models in predicting the VaR threshold of the portfolio. Polanski and Stoja (2010) use the Gram-Charlier Expansion (GCE) to expand the standard normal distribution of the first four dynamic moments. Chen et al. (2012) indicate a parametric approach to forecast the VaR and expected shortfall (ES). Gerlach et al. (2011) apply the Bayesian time-varying quantile to forecast VaR. Lucas and Zhang (2016) provide a simple method that uses a recursive update scheme similar to the familiar Risk-metrics method to model time changes in volatility and other higher-order moments.

In this chapter, I discuss risk from the forecasting model. The motivation of this chapter is to apply the benefit of the Copula in risk management in multivariate models. I extend the model proposed by Patton et al. (2019) to capture joint dependence[1] across the factors, and forecast the VaR and ES of a forex portfolio. The univariate distribution models of Patton et al. (2019) are used as the benchmark. Patton et al. (2019) apply the dynamic semi-parametric model to forecast the VaR and ES, which is helpful in risk management. I follow their GAS dynamic models and add the Copula to extend them from a univariate model to a multivariate model. The results confirm my findings in the chapter2. The Copula model could help me to manage the tail risk and improve the risk forecasting model.

The chapter is structured as follows: The primary Fissler loss function is introduced in the first section. I discuss Copula application in the distribution forecasting models in the fourth section. The next section shows the new model, which combines the GAS forecasting model and Copula. The conclusion is in the last section.

[1]The joint distribution can help me find a non-linear relationship between different assets. This is helpful since the tail dependence structure can improve risk management.

## 3.2 Basic Model of Risk Measures (Fissler loss function)

In this chapter, the Fissler (2017) loss function is the risk measure applied in VaR and ES forecasting GAS model. Fissler (2017) point out that these variables meet the conditions, and show the scoring rules of VaR and ES. The loss function is shown below:

$$\begin{aligned} L_{FZ}(Y, v, e, G_1, G_2) &= \underset{(v,e)}{argmin} \left(1\{Y \leq v\} - \alpha\right) \left(G_1(v) - G_1(Y) + \frac{1}{\alpha} G_2(e)\, v\right) \\ &\quad - G_2(e) \left(\frac{1}{\alpha} 1\{Y \leq v\} Y - e\right) - \mathcal{G}_2(e) \end{aligned} \tag{3.3}$$

$(v, e)$ denotes the VaR and ES. $G_1$ and $G_2$ are the scoring function in the loss function, which needs to be defined in different circumstances.To simplify the loss function, Patton et al. (2019) assume that the loss differences from the loss function are homogeneous of degree zero. Thus, the $G_1$ and $G_2$ will be $G_1(x) = 0\, and\, G_2(x) = -1/x$. $G_2$ is the differential coefficient function of $\mathcal{G}_2$, $\mathcal{G}_2^{'} = G_2$.

To estimate risk measures by minimizing the loss function, I can get the VaR and ES as the function below:

$$(VaR_t, ES_t) = \underset{(v,e)}{argmin} \mathbb{E}_{t-1} \left[L_{FZ}(Y, v, e, G_1, G_2)\right] \tag{3.4}$$

Following the Patton et al. (2019), the loss function could be rewritten the as below form:

$$L_{FZ0}(Y, v, e; \alpha) = \frac{1}{\alpha e} 1\{Y \leq v\} (v - Y) + \frac{v}{e} + log(-e) - 1 \tag{3.5}$$

With the zero-loss function, the VaR and ES could be estimated through the function given below:

$$(VaR_t, ES_t) = \left(v\left(\mathbf{Z}_{t-1}; \theta\right), e\left(\mathbf{Z}_{t-1}; \theta\right)\right) \tag{3.6}$$

Following Patton et al. (2019), I could estimate the parameters of the loss function as below.

$$\hat{\theta}_T = \underset{\theta}{argmin} \frac{1}{T} \sum_{t-1}^{T} L_{FZ0}(Y, v(\mathbf{Z}_{t-1}; \theta), e(\mathbf{Z}_{t-1}; \theta); \alpha) \tag{3.7}$$

The estimates could be gotten from minimizing the average loss from the FZ loss function. In the appendix, I introduce the 9 different univariate forecasting models from Patton et al. (2019). I also introduce the evaluation method: goodness-of-fit test and Diebold-Mariano tests in the appendix.

## 3.3 The Multi-forecasting Models

In portfolio management, risk could come from a multi-asset structure. The univariate risk forecasting model may ignore the joint risk between assets, which would lead to the underestimation of the risk. Hence, I combine the Copula and risk forecasting models to forecast the risk of the portfolios in the multivariate model. I apply the Copula to consider the joint distribution of four factors. Then, the univariate distribution models could be expanded to Copula multivariate models. The model is discussed below in detail. As an in-sample, I still apply the first 750 terms to estimate the parameters of Copula models. The out-of-sample period is from 11 March 2005 to 20 March 2020.

### 3.3.1 Copula Application in Multi-distribution Models

In this section, I build a model based on the univariate model of Patton et al. (2019), using joint distribution.

I apply joint distribution of asset returns, mean, and variance to forecast the VaR and ES. Using the standardized residual, I apply GARCH dynamics for the conditional mean and variance to build my models. The Copula forecasting model is:

$$Y_t = \mu_t + \sigma_t \eta_t \tag{3.8}$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma \eta_{t-1}^2 \tag{3.9}$$

$$\eta_t \sim c_t \, copula \, distribution \tag{3.10}$$

I also use the NGARCH model discussed below, to consider leverage effects[2] in the

[2]An unexpected drop in return may have a bigger impact on conditional volatility than an unexpected increase (i.e. $\theta_j$is positive)

portfolios:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\sigma_{t-1}^2(\eta_{t-1} - \theta)^2 \tag{3.11}$$

$Y_t$ denotes the portfolio's return, where $\mu_t$ is specified to the ARMA model and $\sigma_t^2$ is specified to the GARCH model. $F_\eta(0, 1)$ denotes the distribution of $\eta_t$. Given $F_\eta$, the forecasting of VaR and ES can be estimated as:

$$v_t = \mu_t + a\sigma_t, \; where\, a = c_t^{-1}(\alpha) \tag{3.12}$$

$$e_t = \mu_t + b\sigma_t, \; where\, b = \mathbb{E}\left[\eta_t \mid \eta_t \le a\right] \tag{3.13}$$

Here $\mu_t$ is the mean value of $Y_t$ and the $\sigma_t$is estimated by using the GARCH or NGARCH models[3]. Hence, I obtained new forecasting values using the Copula model. As for the univariate models, I applied the same GARCH model to estimate the parameters. The main difference between the univariate model and the Copula model is in the estimation of parameters $(a, b)$. In the Copula multivariate model, the parameters $(a, b)$ are calculated from the CDF of the joint distribution. I tested nine different Copula models in this section. The first three models are the univariate distribution models from Patton et al. (2019), which is considered the benchmark. The next three models are dynamic Copula with GARCH models. The last three models are dynamic Copula with NGARCH models.

I consider three choices for $F_\eta$ to describe the distributions of $\eta_t$:

$$\eta_t \sim Normal\, copula \tag{3.14}$$

$$\eta_t \sim Student\, t\, copula \tag{3.15}$$

$$\eta_t \sim Skewed\, t\, copula \tag{3.16}$$

Monte Carlo simulations are used to estimate the parameters $(a, b)$. I used simulations to define the quantile function and employed 1,000,000 replications using the equation below. Thereafter, I sorted the replications to obtain the quantile value $\alpha$.

[3]I test the forecasted model with both linear and non-linear GARCH models to observe the differences between these assumptions and structures

$$X = Z \tag{3.17}$$

where $Z$ is the multi-normal distribution simulated by the correlation matrix from the normal Copula.

$$X = \sqrt{W} Z \tag{3.18}$$

where $W$ follows an inverse Gamma $IG(v/2, v/2)$ distribution and $v$ is from the student t Copula model.

$$X = \sqrt{W} Z + \lambda W \tag{3.19}$$

where $\lambda$ denotes a $N \times 1$ asymmetry parameter vector, and $(v, \lambda)$ are all from the skewed t Copula model.

### 3.3.2 Comparison Results

The results can be shown in two main ways: figures and tables. The figures will show the forecasted risk measures performance, while the tables show the goodness-of-fit test and Diebold-Mariano tests I introduced in the appendix.

[Figure 3.6 NGARCH dynamic Copula ES about here]

In figure 3.6, I show the forecasting of the ES from dynamic Copula models. This figure plots the expected ES of four different factors using Copula multivariate distribution models. The blue line shows the normal distribution model. The red line is the results of the student t distribution model, while the yellow line denotes the results of the skewed t distribution model. The graph shows that there has been a similar path with the univariate distribution models. The normal Copula model obviously forecasts a lower risk and lower average loss in table 3.7. What stands out in figure 3.6 is that the normal Copula model could be better than the student t and the skewed t Copula models since the lower average risk from.

[Table 3.7 Average loss and goodness fit test of Copula forecasting models is about here]

Table 3.7 shows the multivariate forecasting results of four different factor portfolio returns. The multi NGARCH normal Copula model (NGCH-n-dcc) reaches the lowest average loss in factor DOL and VAL, while the GARCH normal Copula model (GCH-n-dcc) has the best performance in HML and MOM according to the average loss.

Almost all multi Copula models could pass the goodness-of-fit test for VaR and ES, except for the NGARCH normal Copula model in DOL.

[Table 3.8 Average loss and goodness fit test of Copula forecasting models of developed countries data set is about here]

[Table 3.9 Average loss and goodness fit test of Copula forecasting models of developing countries data set is about here]

Tables 3.8 and 3.9 give the forecasting results for the factors in developed and developing countries. I discussed the performances of the factors portfolios in chapter 2. The developed countries are considered with the higher risk. It would be interesting to discuss the performance of those models under different circumstances. What can be seen in table 3.8 is that the NGARCH multi Copula models have obviously more outstanding performance than other models in HML, MOM, and VAL with a low p-value which passes all the circumstances. In the developing factors portfolios, NGARCH multi Copula models always reach the lowest average loss and the best performance. Overall, the NGARCH models outperform other models in developed and developing countries' factors. To check the average loss statistically, I also show the Diebold-Mariano tests of the whole data set factors.

[Table 3.12 DOL combine models comparison is about here]

[Table 3.13 HML combine models comparison is about here]

[Table 3.14 MOM combine models comparison is about here]

[Table 3.15 VAL combine models comparison is about here]

The t-statistics from Diebold-Mariano tests comparing the average losses results could be found in table 3.12, 3.13, 3.14 and 3.15 in the first nine rows and columns. The positive value indicates how the row model statistically under-performs the column model. The best model in DOL and VAL factors is the univariate empirical distribution model. The GARCH student t model has the best performance in the MOM factor, while the GARCH skewed t model performs better than other models in the HML factor. The Diebold-Mariano tests illustrate that the multi-Copula models have their benefit in forecasting the risk measures. In the next section, I try to combine the Copula and GAS forecasting models.

## 3.4 Combination Between the GAS Forecasting Model and Copula

In this section, I try to combine the joint distribution and the GAS forecasting models. The new model is discussed in the subsection 3.4.1. Then, I compare the performances of the combined model and the multivariate Copula model.

### 3.4.1 The Combined Model

In the GAS model, the parameter $(a, b)$ is estimated[4]. However, I can estimate the parameter $(a, b)$ from the Copula model. In this section, I apply the estimates from the Copula in the GAS model to combine both models to improve multivariate GAS model. Because the two-factor GAS model is not consistent with the Copula model, I only extend GARCH-FZ, one-factor, and hybrid model.

Let's consider the extension of the hybrid model as an example. The returns of the factors $Y_t$ include two parts, $\kappa_t$ and $\eta_t$. In the univariate GAS model from Patton et al. (2019), $\eta_t$ follow the independent individual normal distribution. The $(a, b)$ are two of the parameters which need to be estimated. In the combined model, $\eta_t$ can follow the Copula joint distribution instead of independent individual distribution. Then, $(a, b)$ become the exogenous variables when estimating the GAS model, which could be calculated from the Copula models.

$$Y_t = \exp\{\kappa_t\}\,\eta_t \tag{3.20}$$

$$\eta_t \sim c_t\, copula\, distribution \tag{3.21}$$

$$v_t = a\exp\{\kappa_t\} \tag{3.22}$$

$$e_t = b\exp\{\kappa_t\} \tag{3.23}$$

where $\kappa_t$ could be identified as:

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma H_{t-1}^{-1}s_{t-1} + \delta\log\left|Y_{t-1}\right| \tag{3.24}$$

Then, the score and the Hansen item is still as follow:

$$s_t \equiv \frac{\partial L_{FZ0}\left(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\}; \alpha\right)}{\partial\kappa} = -\frac{1}{e_t}\left(\frac{1}{\alpha}\mathbf{1}\{Y_t \le v_t\}\,Y_t - e_t\right) \tag{3.25}$$

[4]The definition of parameter $(a, b)$ are shown in the section 3.6

$$I_t = \frac{\partial^2 \mathbb{E}_{t-1}\left[L_{FZ0}\left(Y_t, a\exp\left\{\kappa_t\right\}, b\exp\left\{\kappa_t\right\}; \alpha\right)\right]}{\partial \kappa_t^2} = \frac{\alpha - k_\alpha a_\alpha}{\alpha} \tag{3.26}$$

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma\frac{1}{b\exp\left\{\kappa_t\right\}}\left(\frac{1}{\alpha}\mathbf{1}\left\{Y_{t-1} \leq a\exp\left\{\kappa_{t-1}\right\}\right\}Y_{t-1} - b\exp\left\{\kappa_{t-1}\right\}\right) + \delta\log\left|Y_{t-1}\right| \tag{3.27}$$

There are three parameters in this model $(\beta, \gamma, \delta)$, and the Copula GAS model of one-factor and the GARCH-FZ that are analogously combined by setting the $(a, b)$ as exogenous parameters. The $(a, b)$ could be estimated from the Copula model in the joint distribution, which is a multivariate model.

### 3.4.2 Results of Combined Models

[Figure 3.7 Combination between Copula and GAS model about here]

Figure plots the expected ES of four different factors of distribution models. The blue line shows the GARCH-FZ GAS model. The red line shows the results of the one-factor GAS model, while the yellow line denotes the results of the hybrid model. Through Figure 3.7, I can see that the combined model of one-factor and hybrid models are not robust because of their underestimation of the smoothed forecasts), while the GARCH-FZ model shows good practice results. The one-factor and hybrid model forecasting have been smoothed and under-optimized since the model is not stable and losing these details when forecasting.

[Table 3.10 Average loss and goodness fit test of Copula forecasting models with combined models is about here]

The average loss of combined models is close to the multi-Copula forecasting models. I can see that the Hybrid model cannot pass the goodness-of-fit test for the HML, MOM, and VAL factors. The one-factor combined model often reaches the lowest average loss among all the combined models. However, this result conflicts with the opinion from Figure 3.7. Hence, I still show the t-statistics from Diebold-Mariano tests comparing the average losses of the combined models and Copula models. According to Tables 3.12, 3.13, 3.14 and 3.15, the GARCH-FZ combined model shows the best forecast results for MOM and VAL factors among 13 different models. The univariate empirical model has the best performance, while the GARCH skewed t multivariate model gives the best forecasting. An improvement could be shown in the performance of the GARCH-FZ combined model by combining the Copula and GAS forecasting models. I conclude that the Copula can indeed improve the ability of forecasting risk.

## 3.5 The Risk of Factor Portfolios

As I discussed in chapter 2, the four factor portfolios are the popular risk factor in the forex market. It is interesting to forecast the risk of those pricing factors during the long-term time series. DOL is a market risk for US investors. HML denotes the factor which focuses on the risk of carry trade. MOM factor illustrates the risk from momentum strategies in the forex market, while the VAL factor shows the value risk (price level of consumer goods expressed in national currency) from trading different 'value' currencies.

When I check the goodness-of-fit test of different models, it was surprising to see that the univariate model cannot pass in the DOL factor, while it can pass in the other factors. Market risk is hard to forecast compared to other risks. This phenomenon could also be found in the results from Copula multivariate models for developed and developing factors. The DOL factor denotes the risk related to the US dollar, which is the most essential currency in the forex market. Consequently, the DOL factor plays a similar role in the market factor in the forex market. It is a little hard to forecast the risk of DOL compared to other factors, especially in the univariate model.

Through the figures of univariate models, the difference in forecasting risk measures from the four factors is clear. Figure 3.2 shows the expected ES of the four factors. All the factors receive a massive increase in the risk on account of the 2008 financial crisis. The volatility of the HML risk is the highest among the four factors. The extreme event of VAL and MOM factors follow a similar path, which confirms the high threshold correlation I discussed in chapter 2. The Copula multivariate models have a similar path for those factor portfolios since they follow the joint distribution from Copula. Through the results, considering the risk in the joint distribution could indeed improve the forecasting ability.

## 3.6 Conclusion

In this chapter, I am motivated by the benefit of using Copula in risk management. I follow Patton et al. (2019) to concentrate on risk forecasting questions. I first test the univariate model of Patton et al. (2019). Then, I add the Copula to the distribution models to improve this from univariate to multivariate models. Finally, I combine the Copula and the GAS forecasting models. I apply the goodness-of-fit test and Diebold-Mariano tests to evaluate the performance of the risk forecasting models. I still apply the currency specific factors portfolios as the data. The out-of-sample period is from 11 March 2005 to March 20, 2020.

There are some interesting findings. First, the results show that the distribution Copula forecasting model could outperform the univariate model when forecasting the risk. This confirms the Copula model benefit in the risk management area. Secondly, I test the risk distribution of the forex market and indicate that adding asymmetry and time-varying dependence among the factors improves risk management. My new risk forecasting model consistently ranks the best. The skewed t GAS GARCH model has the best performance according to the average rank among the 4 factor portfolios risk forecasting. I also compare the risk forecasting model performance in each specific currency factor portfolio. The features of different factors could be found in the risk forecasting results.

I make some contributions in the risk management area. First, I extend the univariate risk forecasting model in Patton et al. (2019) to the multivariate model by Copula. The univariate risk forecasting model may ignore the joint risk between assets, which would lead to the underestimation of the risk. The multivariate model could consider the joint distribution among the factor portfolios in the tail dependence. Then, I apply the Copula model to re-estimate the joint distribution among the forex factors, which shows strong support to the findings from the previous chapter. The results confirm the asymmetry in the tail dependence of the fore factors.

The new model I proposed has some implications in both academic and industry areas. My risk forecast model could estimate the future Value at Risk and Expected shortfall of the assets and portfolios. It is helpful for investors or banks to forecasting the risk of their portfolios and assets or liabilities. Furthermore, This chapter extends the application of Copula in the risk management area.

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**Table 3.1** – Average Loss and Goodness-of-fit Test of univariate Model

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, 1994, to March 20, 2020 of the developed countries data set, for 9 different forecasting models. The first three rows correspond to the rolling window forecast, the next three rows correspond to distribution forecasts, the last four rows give the forecasting results from GAS model. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Rank of test | | | | Good of fit-VaR | | | | Good of fit-ES | | | | average rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | among the 4 factors |
| RW-26 | 7 | 9 | 9 | 9 | **0.0110** | 0.0000 | 0.0290 | **0.1940** | 0.0040 | 0.0000 | 0.0260 | 0.0850 | 10.75 |
| RW-52 | 8 | 8 | 8 | 7 | 0.0050 | 0.0420 | **0.1030** | **0.2540** | 0.0100 | 0.0000 | **0.1860** | **0.6520** | 7.75 |
| RW-104 | 5 | 7 | 7 | 8 | **0.3210** | **0.1050** | **0.2700** | 0.0000 | 0.0320 | 0.0770 | **0.3360** | 0.0000 | 6.75 |
| GCH-n | 4 | 4 | 4 | 4 | **0.8960** | **0.2980** | 0.0000 | 0.0000 | **0.5820** | 0.0500 | 0.0000 | 0.0000 | 4 |
| GCH-skt | 3 | 2 | 3 | 3 | **0.8430** | **0.3880** | 0.0030 | 0.0000 | **0.6100** | 0.0910 | 0.0000 | 0.0000 | 2.75 |
| GCH-edf | 2 | 1 | 1 | 1 | **0.8430** | 0.0000 | 0.0000 | 0.0000 | **0.5970** | 0.0000 | 0.0000 | 0.0000 | 1.25 |
| FZ1F | 6 | 6 | 6 | 6 | 0.0020 | 0.0160 | 0.0000 | 0.0000 | 0.0260 | **0.6960** | 0.0000 | 0.0000 | 6 |
| GCH-FZ | 1 | 3 | 2 | 2 | **0.7030** | **0.3410** | 0.0000 | 0.0000 | **0.5840** | 0.0960 | 0.0000 | 0.0000 | 2 |
| Hybrid | 9 | 5 | 5 | 5 | **0.1460** | **0.3060** | 0.0000 | 0.0000 | **0.1920** | **0.9600** | 0.0000 | 0.0000 | 6 |

**Table 3.2** – DOL Factors' Diebold-Mariano t-statistics on Average Out-of-sample Loss Differences

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 9 different forecasting models of DOL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to rolling window forecasts, the next three rows correspond to GARCH forecasts based on different models for the standardized residuals, and the last four rows correspond to GAS models.

| . | RW-26 | RW-52 | RW-104 | GCH-n | GCH-skt | GCH-edf | FZ1F | GCH-FZ | Hybrid |
|---|---|---|---|---|---|---|---|---|---|
| RW-26 | NaN | -0.8010 | 0.3520 | 1.4740 | 1.5870 | 1.6040 | 0.3470 | 1.5840 | -0.3470 |
| RW-52 | 0.8010 | NaN | 1.5040 | 3.8350 | 3.7510 | 3.6840 | 1.2040 | 3.4460 | -0.0190 |
| RW-104 | -0.3520 | -1.5040 | NaN | 1.4740 | 1.4410 | 1.4080 | -0.0360 | 1.4830 | -1.2130 |
| GCH-n | -1.4740 | -3.8350 | -1.4740 | NaN | 0.4000 | 0.3420 | -1.5550 | 0.4950 | -1.4980 |
| GCH-skt | -1.5870 | -3.7510 | -1.4410 | -0.4000 | NaN | 0.1320 | -1.5390 | 0.4540 | -1.4570 |
| GCH-edf | -1.6040 | -3.6840 | -1.4080 | -0.3420 | -0.1320 | NaN | -1.5070 | 0.4310 | -1.4370 |
| FZ1F | -0.3470 | -1.2040 | 0.0360 | 1.5550 | 1.5390 | 1.5070 | NaN | 1.7130 | -1.0980 |
| GCH-FZ | -1.5840 | -3.4460 | -1.4830 | -0.4950 | -0.4540 | -0.4310 | -1.7130 | NaN | -1.4850 |
| Hybrid | 0.3470 | 0.0190 | 1.2130 | 1.4980 | 1.4570 | 1.4370 | 1.0980 | 1.4850 | NaN |

**Table 3.3** – HML Factors' Diebold-Mariano t-statistics on Average Out-of-sample Loss Differences

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 9 different forecasting models of HML factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to rolling window forecasts, the next three rows correspond to GARCH forecasts based on different models for the standardized residuals, and the last four rows correspond to GAS models.

| . | RW-26 | RW-52 | RW-104 | GCH-n | GCH-skt | GCH-edf | FZ1F | GCH-FZ | Hybrid |
|---|---|---|---|---|---|---|---|---|---|
| RW-26 | NaN | 0.6620 | 0.8940 | 1.5390 | 2.0070 | 2.0390 | 1.2820 | 1.8280 | 1.3880 |
| RW-52 | -0.6620 | NaN | 0.6020 | 0.9790 | 1.4470 | 1.4810 | 0.7960 | 1.3270 | 0.8530 |
| RW-104 | -0.8940 | -0.6020 | NaN | 0.6760 | 1.2790 | 1.3260 | 0.3870 | 1.1830 | 0.6060 |
| GCH-n | -1.5390 | -0.9790 | -0.6760 | NaN | 1.8340 | 1.7520 | -0.5710 | 1.2240 | -0.0390 |
| GCH-skt | -2.0070 | -1.4470 | -1.2790 | -1.8340 | NaN | 0.9660 | -1.6830 | -0.1240 | -1.1710 |
| GCH-edf | -2.0390 | -1.4810 | -1.3260 | -1.7520 | -0.9660 | NaN | -1.8100 | -0.3640 | -1.3220 |
| FZ1F | -1.2820 | -0.7960 | -0.3870 | 0.5710 | 1.6830 | 1.8100 | NaN | 1.5400 | 0.7160 |
| GCH-FZ | -1.8280 | -1.3270 | -1.1830 | -1.2240 | 0.1240 | 0.3640 | -1.5400 | NaN | -1.3230 |
| Hybrid | -1.3880 | -0.8530 | -0.6060 | 0.0390 | 1.1710 | 1.3220 | -0.7160 | 1.3230 | NaN |

**Table 3.4** – MOM Factors' Diebold-Mariano t-statistics on Average Out-of-sample Loss Differences

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 9 different forecasting models of MOM factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to rolling window forecasts, the next three rows correspond to GARCH forecasts based on different models for the standardized residuals, and the last four rows correspond to GAS models.

| . | RW-26 | RW-52 | RW-104 | GCH-n | GCH-skt | GCH-edf | FZ1F | GCH-FZ | Hybrid |
|---|---|---|---|---|---|---|---|---|---|
| RW-26 | NaN | 0.8170 | 1.3570 | 2.0770 | 2.3610 | 2.4460 | 0.3090 | 2.4560 | 1.0200 |
| RW-52 | -0.8170 | NaN | 1.0250 | 1.5040 | 1.7930 | 1.8880 | -0.1080 | 1.8990 | 0.5730 |
| RW-104 | -1.3570 | -1.0250 | NaN | 1.2290 | 1.6570 | 1.8130 | -0.7140 | 1.8620 | 0.1370 |
| GCH-n | -2.0770 | -1.5040 | -1.2290 | NaN | 2.2620 | 2.1150 | -1.6070 | 1.6040 | -1.0130 |
| GCH-skt | -2.3610 | -1.7930 | -1.6570 | -2.2620 | NaN | 1.6420 | -1.9550 | 0.7800 | -1.4170 |
| GCH-edf | -2.4460 | -1.8880 | -1.8130 | -2.1150 | -1.6420 | NaN | -2.1160 | -0.0470 | -1.5990 |
| FZ1F | -0.3090 | 0.1080 | 0.7140 | 1.6070 | 1.9550 | 2.1160 | NaN | 2.2810 | 3.0550 |
| GCH-FZ | -2.4560 | -1.8990 | -1.8620 | -1.6040 | -0.7800 | 0.0470 | -2.2810 | NaN | -1.7320 |
| Hybrid | -1.0200 | -0.5730 | -0.1370 | 1.0130 | 1.4170 | 1.5990 | -3.0550 | 1.7320 | NaN |

**Table 3.5** – VAL Factors' Diebold-Mariano t-statistics on Average Out-of-sample Loss Differences

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 9 different forecasting models of VAL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to rolling window forecasts, the next three rows correspond to GARCH forecasts based on different models for the standardized residuals, and the last four rows correspond to GAS models.

| . | RW-26 | RW-52 | RW-104 | GCH-n | GCH-skt | GCH-edf | FZ1F | GCH-FZ | Hybrid |
|---|---|---|---|---|---|---|---|---|---|
| RW-26 | NaN | 1.7120 | 0.8970 | 1.0260 | 1.0640 | 1.3070 | 0.9260 | 1.3240 | 0.9530 |
| RW-52 | -1.7120 | NaN | -0.2270 | 0.3750 | 0.4170 | 0.6920 | -0.0740 | 0.6880 | 0.0440 |
| RW-104 | -0.8970 | 0.2270 | NaN | 0.5420 | 0.5900 | 0.8990 | 0.1260 | 0.8890 | 0.2200 |
| GCH-n | -1.0260 | -0.3750 | -0.5420 | NaN | 5.6360 | 6.3090 | -0.5800 | 3.8810 | -0.6050 |
| GCH-skt | -1.0640 | -0.4170 | -0.5900 | -5.6360 | NaN | 6.3050 | -0.6370 | 3.3510 | -0.6780 |
| GCH-edf | -1.3070 | -0.6920 | -0.8990 | -6.3090 | -6.3050 | NaN | -1.0070 | -0.1610 | -1.1530 |
| FZ1F | -0.9260 | 0.0740 | -0.1260 | 0.5800 | 0.6370 | 1.0070 | NaN | 1.0170 | 0.2800 |
| GCH-FZ | -1.3240 | -0.6880 | -0.8890 | -3.8810 | -3.3510 | 0.1610 | -1.0170 | NaN | -1.1740 |
| Hybrid | -0.9530 | -0.0440 | -0.2200 | 0.6050 | 0.6780 | 1.1530 | -0.2800 | 1.1740 | NaN |

**Table 3.6** – Rank of Diebold-Mariano Tests and Goodness-of-fit Test of Copula Forecasting Models

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from January 7 1994 to January 5 2018, for 9 different forecasting models. The first five rows correspond to dynamic Copula forecasts, the last five rows give the forecasting results from NAGARCH dynamic Copula models.The left panel show the rank of Diebold-Mariano tests. The detail of Diebold-Mariano tests are shown in the appendix.The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold. The last column show the average rank of the four factors. The panel.A shows the test results of the whole data set. The panel.B present the results of the developed countries data set, while panel.C give the results of the developing countries data set.

| | Rank from test | | | | Good of fit-VaR | | | | Good of fit-ES | | | | Average rank of the models |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Panel.A cross section | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | Amongst 4 factors |
| GCH-n-dcc | 1 | 8 | 10 | 3 | **0.9110** | 0.0000 | 0.0000 | 0.0000 | **0.6270** | 0.0000 | 0.0000 | 0.0000 | 5.5 |
| GCH-t-dcc | 4 | 2 | 1 | 5 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0070 | 0.0000 | 0.0000 | 0.0000 | 3 |
| GCH-skt-dcc | 5 | 1 | 2 | 4 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0060 | 0.0000 | 0.0000 | 0.0000 | 3 |
| GCH-bm-dcc | 2 | 10 | 9 | 1 | **0.9110** | 0.0000 | 0.0000 | 0.0000 | **0.5830** | 0.0000 | 0.0000 | 0.0000 | 5.5 |
| GCH-orth-dcc | 3 | 9 | 8 | 2 | **0.9100** | 0.0000 | 0.0000 | 0.0000 | **0.5800** | 0.0000 | 0.0000 | 0.0000 | 5.5 |
| NGCH-n-dcc | 9 | 6 | 7 | 9 | 0.0920 | 0.0010 | 0.0000 | 0.0100 | 0.0750 | 0.0050 | 0.0000 | 0.0000 | 7.75 |
| NGCH-t-dcc | 6 | 4 | 4 | 6 | 0.0750 | 0.0010 | 0.0000 | 0.0170 | 0.0880 | 0.0030 | 0.0000 | 0.0000 | 5 |
| NGCH-skt-dcc | 7 | 3 | 3 | 7 | 0.0810 | 0.0010 | 0.0000 | 0.0100 | 0.0730 | 0.0030 | 0.0000 | 0.0000 | 5 |
| NGCH-bm-dcc | 8 | 5 | 5 | 8 | 0.0920 | 0.0010 | 0.0000 | 0.0100 | 0.0750 | 0.0040 | 0.0000 | 0.0000 | 6.5 |
| NGCH-orth-dcc | 10 | 7 | 6 | 10 | **0.1190** | 0.0010 | 0.0000 | 0.0100 | 0.0830 | 0.0050 | 0.0000 | 0.0000 | 8.25 |
| Panel.B developed countries | | | | | | | | | | | | | |
| GCH-n-dcc | 3 | 5 | 4 | 2 | **0.1340** | 0.0000 | 0.0000 | **0.9100** | **0.1400** | 0.0000 | 0.0000 | **0.8890** | 3.5 |
| GCH-t-dcc | 4 | 2 | 2 | 5 | 0.0610 | 0.0000 | 0.0000 | **0.2820** | 0.0370 | 0.0000 | 0.0000 | **0.5750** | 3.25 |
| GCH-skt-dcc | 5 | 1 | 1 | 1 | 0.0050 | 0.0000 | 0.0000 | **0.5480** | 0.0020 | 0.0000 | 0.0000 | **0.8010** | 2 |
| GCH-bm-dcc | 1 | 3 | 5 | 4 | **0.1340** | 0.0000 | 0.0000 | **0.8940** | **0.1400** | 0.0000 | 0.0000 | **0.8400** | 3.25 |
| GCH-orth-dcc | 2 | 4 | 3 | 3 | **0.1340** | 0.0000 | 0.0000 | **0.9100** | **0.1440** | 0.0000 | 0.0000 | **0.8870** | 3 |
| NGCH-n-dcc | 8 | 6 | 8 | 8 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 7.5 |
| NGCH-t-dcc | 6 | 9 | 7 | 10 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8 |
| NGCH-skt-dcc | 7 | 10 | 6 | 9 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8 |
| NGCH-bm-dcc | 9 | 8 | 9 | 7 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.25 |
| NGCH-orth-dcc | 10 | 7 | 10 | 6 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 8.25 |
| Panel.B developing countries | | | | | | | | | | | | | |
| GCH-n-dcc | 4 | 10 | 7 | 6 | **0.8240** | 0.0000 | 0.0000 | 0.0780 | **0.7400** | 0.0020 | 0.0000 | 0.0060 | 6.75 |
| GCH-t-dcc | 3 | 7 | 4 | 4 | **0.6530** | 0.0180 | 0.0040 | **0.2460** | **0.6680** | 0.0230 | 0.0000 | 0.0490 | 4.5 |
| GCH-skt-dcc | 6 | 6 | 3 | 5 | 0.0010 | 0.0420 | 0.0000 | **0.1440** | 0.0390 | 0.0370 | 0.0000 | 0.0240 | 5 |
| GCH-bm-dcc | 5 | 8 | 6 | 7 | **0.8240** | 0.0000 | 0.0010 | 0.0390 | **0.7410** | 0.0020 | 0.0000 | 0.0030 | 6.5 |
| GCH-orth-dcc | 7 | 9 | 5 | 8 | **0.8240** | 0.0000 | 0.0010 | 0.0390 | **0.7400** | 0.0020 | 0.0000 | 0.0030 | 7.25 |
| NGCH-n-dcc | 10 | 1 | 9 | 10 | 0.0000 | **0.1330** | 0.0000 | 0.0000 | **0.5890** | 0.0800 | **0.5470** | **0.2500** | 7.5 |
| NGCH-t-dcc | 1 | 4 | 8 | 2 | 0.0000 | **0.2950** | 0.0000 | 0.0000 | **0.6560** | **0.5880** | **0.7550** | 0.0670 | 3.75 |
| NGCH-skt-dcc | 2 | 5 | 2 | 1 | 0.0000 | **0.2950** | 0.0000 | 0.0000 | **0.7800** | **0.5680** | **0.6530** | **0.7130** | 2.5 |
| NGCH-bm-dcc | 8 | 3 | 1 | 9 | 0.0000 | **0.1330** | 0.0000 | 0.0000 | **0.7980** | 0.0810 | **0.7340** | **0.2570** | 5.25 |
| NGCH-orth-dcc | 9 | 2 | 10 | 3 | 0.0000 | **0.1330** | 0.0000 | 0.0000 | **0.7860** | 0.0780 | **0.5500** | **0.3560** | 6 |

**Table 3.7** – Average Loss and Goodness-of-fit Test of Copula Forecasting Models

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, to March 20, 2020, for nine different forecasting models. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the last three rows give the forecasting results from NAGARCH dynamic Copula models. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Avg loss | | | | Good of fit-VaR | | | | Good of fit-ES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL |
| GCH-n-simple | -3.9150 | -3.7830 | -3.6070 | -3.6800 | **0.8960** | **0.2980** | 0.0000 | 0.0000 | **0.5820** | 0.0500 | 0.0000 | 0.0000 |
| GCH-skt-simple | -3.9180 | -3.8160 | -3.6290 | -3.6840 | **0.8430** | **0.3880** | 0.0030 | 0.0000 | **0.6100** | 0.0910 | 0.0000 | 0.0000 |
| GCH-emp-simple | -3.9190 | -3.8180 | -3.6370 | -3.7090 | **0.8430** | 0.0000 | 0.0000 | 0.0000 | **0.5970** | 0.0000 | 0.0000 | 0.0000 |
| GCH-n-dcc | -3.9150 | -3.6180 | -3.4830 | -3.7940 | **0.9110** | 0.0000 | 0.0000 | 0.0000 | **0.6250** | 0.0000 | 0.0000 | 0.0000 |
| GCH-t-dcc | -3.8960 | -3.7940 | -3.6250 | -3.6880 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0070 | 0.0000 | 0.0000 | 0.0000 |
| GCH-skt-dcc | -3.8940 | -3.7980 | -3.6210 | -3.7050 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0080 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-n-dcc | -3.8290 | -3.7450 | -3.5160 | -3.3520 | **0.1050** | 0.0010 | 0.0000 | 0.0100 | 0.0790 | 0.0050 | 0.0000 | 0.0000 |
| NGCH-t-dcc | -3.8410 | -3.7560 | -3.5440 | -3.4110 | 0.0750 | 0.0010 | 0.0000 | 0.0170 | 0.0880 | 0.0030 | 0.0000 | 0.0000 |
| NGCH-skt-dcc | -3.8390 | -3.7590 | -3.5500 | -3.3990 | 0.0960 | 0.0010 | 0.0000 | 0.0170 | 0.0960 | 0.0030 | 0.0000 | 0.0000 |

**Table 3.8** – Average Loss and Goodness-of-fit Test of Copula Forecasting Models of Developed Countries Dataset

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, to March 20, 2020 of the developed countries data set, for nine different forecasting models. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the last three rows give the forecasting results from NAGARCH dynamic Copula models. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Avg loss | | | | Good of fit-VaR | | | | Good of fit-ES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL |
| GCH-n-simple | -3.5760 | -2.8620 | -2.8670 | -3.1640 | **0.3140** | 0.0000 | 0.0000 | 0.0000 | **0.1710** | 0.0000 | 0.0000 | 0.0000 |
| GCH-skt-simple | -3.5460 | -3.0900 | -2.9750 | -3.1470 | 0.0190 | 0.0000 | 0.0000 | 0.0000 | 0.0260 | 0.0000 | 0.0000 | 0.0000 |
| GCH-emp-simple | -3.5630 | -3.0280 | -2.9710 | -3.0190 | **0.1340** | 0.0000 | 0.0000 | 0.0000 | 0.0800 | 0.0000 | 0.0000 | 0.0000 |
| GCH-n-dcc | -3.5770 | -2.8650 | -2.8710 | -3.1540 | **0.3640** | 0.0000 | 0.0000 | 0.0000 | **0.1950** | 0.0000 | 0.0000 | 0.0000 |
| GCH-t-dcc | -3.6000 | -2.9780 | -2.9640 | -3.2600 | **0.3850** | 0.0000 | 0.0000 | 0.0000 | **0.2990** | 0.0000 | 0.0000 | 0.0000 |
| GCH-skt-dcc | -3.6060 | -3.0240 | -3.0130 | -3.2040 | **0.4180** | 0.0010 | 0.0000 | 0.0000 | **0.3590** | 0.0010 | 0.0000 | 0.0000 |
| NGCH-n-dcc | -3.5480 | -0.7130 | -1.9240 | -2.0360 | 0.0260 | 0.0000 | 0.0000 | 0.0000 | 0.0060 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-t-dcc | -3.5660 | -0.9930 | -2.0110 | -2.1710 | 0.0290 | 0.0000 | 0.0000 | 0.0000 | 0.0090 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-skt-dcc | -3.5550 | -1.1350 | -2.0990 | -2.0810 | 0.0260 | 0.0000 | 0.0000 | 0.0000 | 0.0070 | 0.0000 | 0.0000 | 0.0000 |

**Table 3.9** – Average Loss and Goodness-of-fit Test of Copula Forecasting Models of Developing Countries Dataset

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, to March 20, 2020 of the developing countries data set, for nine different forecasting models. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the last three rows give the forecasting results from NAGARCH dynamic Copula models. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Avg loss | | | | Good of fit-VaR | | | | Good of fit-ES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL |
| GCH-n-simple | -3.8760 | -3.3600 | -3.3900 | -3.4980 | **0.1450** | 0.0000 | 0.0000 | 0.0080 | 0.0620 | 0.0000 | 0.0000 | 0.0040 |
| GCH-skt-simple | -3.9320 | -3.4720 | -3.4170 | -3.5060 | **0.6000** | 0.0000 | 0.0000 | 0.0020 | **0.5220** | 0.0010 | 0.0000 | 0.0080 |
| GCH-emp-simple | -3.9420 | -3.5680 | -3.4310 | -3.5570 | **0.9230** | 0.0030 | 0.0000 | 0.0720 | **0.7170** | 0.0050 | 0.0000 | **0.3430** |
| GCH-n-dcc | -3.8770 | -3.3650 | -3.3950 | -3.5000 | **0.1450** | 0.0000 | 0.0000 | 0.0080 | **0.0640** | 0.0000 | 0.0000 | 0.0040 |
| GCH-t-dcc | -3.9100 | -3.4640 | -3.4400 | -3.5310 | **0.2460** | 0.0000 | 0.0000 | 0.0720 | **0.1740** | 0.0010 | 0.0000 | 0.0410 |
| GCH-skt-dcc | -3.9110 | -3.2090 | -3.4490 | -3.5540 | **0.3370** | 0.0000 | 0.0000 | **0.8470** | **0.2240** | 0.0000 | 0.0000 | **0.5210** |
| NGCH-n-dcc | -3.7530 | -1.4520 | -1.4670 | -2.5500 | 0.0030 | 0.0000 | 0.0000 | 0.0000 | 0.0040 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-t-dcc | -3.7840 | -1.7870 | -1.6820 | -2.6600 | 0.0100 | 0.0000 | 0.0000 | 0.0000 | 0.0110 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-skt-dcc | -3.7880 | -1.8510 | -1.7090 | -2.6720 | 0.0100 | 0.0000 | 0.0000 | 0.0000 | 0.0110 | 0.0000 | 0.0000 | 0.0000 |

**Table 3.10** – Average Loss and Goodness-of-fit Test of Copula Forecasting Models with Combine Models

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, to March 20, 2020 of the developing countries data set, for 12 different forecasting models. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Avg loss | | | | Good of fit-VaR | | | | Good of fit-ES | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL |
| GCH-n-simple | -3.9150 | -3.7830 | -3.6070 | -3.6800 | **0.8960** | **0.2980** | 0.0000 | 0.0000 | **0.5820** | 0.0500 | 0.0000 | 0.0000 |
| GCH-skt-simple | -3.9180 | -3.8160 | -3.6290 | -3.6840 | **0.8430** | **0.3880** | 0.0030 | 0.0000 | **0.6100** | 0.0910 | 0.0000 | 0.0000 |
| GCH-emp-simple | -3.9190 | -3.8180 | -3.6370 | -3.7090 | **0.8430** | 0.0000 | 0.0000 | 0.0000 | **0.5970** | 0.0000 | 0.0000 | 0.0000 |
| GCH-n-dcc | -3.9150 | -3.6180 | -3.4830 | -3.7940 | **0.9110** | 0.0000 | 0.0000 | 0.0000 | **0.6250** | 0.0000 | 0.0000 | 0.0000 |
| GCH-t-dcc | -3.8960 | -3.7940 | -3.6250 | -3.6880 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0070 | 0.0000 | 0.0000 | 0.0000 |
| GCH-skt-dcc | -3.8940 | -3.7980 | -3.6210 | -3.7050 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0080 | 0.0000 | 0.0000 | 0.0000 |
| NGCH-n-dcc | -3.8290 | -3.7450 | -3.5160 | -3.3520 | **0.1050** | 0.0010 | 0.0000 | 0.0100 | 0.0790 | 0.0050 | 0.0000 | 0.0000 |
| NGCH-t-dcc | -3.8410 | -3.7560 | -3.5440 | -3.4110 | 0.0750 | 0.0010 | 0.0000 | 0.0170 | 0.0880 | 0.0030 | 0.0000 | 0.0000 |
| NGCH-skt-dcc | -3.8390 | -3.7590 | -3.5500 | -3.3990 | 0.0960 | 0.0010 | 0.0000 | 0.0170 | 0.0960 | 0.0030 | 0.0000 | 0.0000 |
| GARCH-fz | -3.9120 | -3.7910 | -3.6470 | -3.8320 | **0.7830** | 0.0000 | 0.0000 | 0.0000 | **0.5830** | 0.0000 | 0.0000 | 0.0000 |
| One-factor GAS | -3.8450 | -3.5820 | -3.3640 | -3.6140 | 0.0030 | 0.0010 | 0.0000 | 0.0360 | 0.0020 | 0.0000 | 0.0000 | 0.0040 |
| Hybrid GAS | -3.8960 | -3.7830 | -3.5920 | -3.7400 | 0.0320 | **0.1470** | **0.1980** | **0.1000** | 0.0400 | **0.1160** | **0.4730** | **0.2420** |

**Table 3.11** – Average Loss and Goodness-of-fit Test of Copula Forecasting Models with Combine Models

The left-hand panel of this table presents the average losses, using the FZ loss function following Fissler (2017), for four factors return series, over the out-of-sample period from March 11, 2005, to March 20, 2020 of the developing countries data set, for 12 different forecasting models. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models. The middle and right-hand panels of this table present p-values from goodness-of-fit tests of the VaR and ES forecasts respectively. The results of good fit test which do not pass at the 10% level in bold.

| | Rank of test | | | | Good of fit-VaR | | | | Good of fit-ES | | | | average rank |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | DOL | HML | MOM | VAL | among the 4 factors |
| GCH-n-simple | 3 | 10 | 10 | 4 | **0.8960** | **0.2980** | 0.0000 | 0.0000 | **0.5820** | 0.0500 | 0.0000 | 0.0000 | 6.75 |
| GCH-skt-simple | 2 | 9 | 6 | 3 | **0.8430** | **0.3880** | 0.0030 | 0.0000 | **0.6100** | 0.0910 | 0.0000 | 0.0000 | 5 |
| GCH-emp-simple | 1 | 8 | 5 | 2 | **0.8430** | 0.0000 | 0.0000 | 0.0000 | **0.5970** | 0.0000 | 0.0000 | 0.0000 | 4 |
| GCH-n-dcc | 4 | 11 | 11 | 5 | **0.9110** | 0.0000 | 0.0000 | 0.0000 | **0.6250** | 0.0000 | 0.0000 | 0.0000 | 7.75 |
| GCH-t-dcc | 6 | 2 | 2 | 8 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0070 | 0.0000 | 0.0000 | 0.0000 | 4.5 |
| GCH-skt-dcc | 8 | 1 | 3 | 7 | 0.0040 | 0.0000 | 0.0060 | 0.0000 | 0.0080 | 0.0000 | 0.0000 | 0.0000 | 4.75 |
| NGCH-n-dcc | 12 | 7 | 9 | 12 | **0.1050** | 0.0010 | 0.0000 | 0.0100 | 0.0790 | 0.0050 | 0.0000 | 0.0000 | 10 |
| NGCH-t-dcc | 10 | 6 | 7 | 10 | 0.0750 | 0.0010 | 0.0000 | 0.0170 | 0.0880 | 0.0030 | 0.0000 | 0.0000 | 8.25 |
| NGCH-skt-dcc | 11 | 5 | 8 | 11 | 0.0960 | 0.0010 | 0.0000 | 0.0170 | 0.0960 | 0.0030 | 0.0000 | 0.0000 | 8.75 |
| GARCH-fz | 5 | 3 | 1 | 1 | **0.7830** | 0.0000 | 0.0000 | 0.0000 | **0.5830** | 0.0000 | 0.0000 | 0.0000 | 2.5 |
| One-factor GAS | 9 | 12 | 12 | 9 | 0.0030 | 0.0010 | 0.0000 | 0.0360 | 0.0020 | 0.0000 | 0.0000 | 0.0040 | 10.5 |
| Hybrid GAS | 7 | 4 | 4 | 6 | 0.0320 | **0.1470** | **0.1980** | **0.1000** | 0.0400 | **0.1160** | **0.4730** | **0.2420** | 5.25 |

**Table 3.12** – DOL Combine Models Comparison

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 12 different forecasting models of DOL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models.

| . | GCH-n-simple | GCH-skt-simple | GCH-emp-simple | GCH-n-dcc | GCH-t-dcc | GCH-skt-dcc | NGCH-n-dcc | NGCH-t-dcc | NGCH-skt-dcc | GARCH-fz | One-factor GAS | Hybrid GAS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GCH-n-simple | NaN | 0.4000 | 0.3420 | -0.4200 | -0.5780 | -0.6170 | -1.1670 | -1.1980 | -1.1580 | -0.4380 | -0.8870 | -0.8570 |
| GCH-skt-simple | -0.4000 | NaN | 0.1320 | -0.4380 | -0.8960 | -0.9460 | -1.1430 | -1.1840 | -1.1400 | -0.7890 | -0.9820 | -1.1880 |
| GCH-emp-simple | -0.3420 | -0.1320 | NaN | -0.3690 | -0.9990 | -1.0540 | -1.1270 | -1.1680 | -1.1240 | -0.6820 | -1.0010 | -1.2320 |
| GCH-n-dcc | 0.4200 | 0.4380 | 0.3690 | NaN | -0.5790 | -0.6180 | -1.1630 | -1.1940 | -1.1540 | -0.4280 | -0.8860 | -0.8570 |
| GCH-t-dcc | 0.5780 | 0.8960 | 0.9990 | 0.5790 | NaN | -4.4860 | -0.6980 | -0.6770 | -0.6610 | 0.5250 | -0.7800 | -0.0030 |
| GCH-skt-dcc | 0.6170 | 0.9460 | 1.0540 | 0.6180 | 4.4860 | NaN | -0.6840 | -0.6600 | -0.6450 | 0.5670 | -0.7590 | 0.0470 |
| NGCH-n-dcc | 1.1670 | 1.1430 | 1.1270 | 1.1630 | 0.6980 | 0.6840 | NaN | 0.7120 | 0.9450 | 1.1650 | 0.1110 | 0.8640 |
| NGCH-t-dcc | 1.1980 | 1.1840 | 1.1680 | 1.1940 | 0.6770 | 0.6600 | -0.7120 | NaN | -0.3330 | 1.2120 | 0.0300 | 0.8640 |
| NGCH-skt-dcc | 1.1580 | 1.1400 | 1.1240 | 1.1540 | 0.6610 | 0.6450 | -0.9450 | 0.3330 | NaN | 1.1630 | 0.0450 | 0.8340 |
| GARCH-fz | 0.4380 | 0.7890 | 0.6820 | 0.4280 | -0.5250 | -0.5670 | -1.1650 | -1.2120 | -1.1630 | NaN | -0.8260 | -0.9140 |
| One-factor GAS | 0.8870 | 0.9820 | 1.0010 | 0.8860 | 0.7800 | 0.7590 | -0.1110 | -0.0300 | -0.0450 | 0.8260 | NaN | 0.6270 |
| Hybrid GAS | 0.8570 | 1.1880 | 1.2320 | 0.8570 | 0.0030 | -0.0470 | -0.8640 | -0.8640 | -0.8340 | 0.9140 | -0.6270 | NaN |

**Table 3.13** – HML Combine Models Comparison

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 12 different forecasting models of HML factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models.

| . | GCH-n-simple | GCH-skt-simple | GCH-emp-simple | GCH-n-dcc | GCH-t-dcc | GCH-skt-dcc | NGCH-n-dcc | NGCH-t-dcc | NGCH-skt-dcc | GARCH-fz | One-factor GAS | Hybrid GAS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GCH-n-simple | NaN | 4.1560 | 4.0990 | -4.7990 | 3.3380 | 3.2090 | 1.4690 | 1.5240 | 1.5160 | 1.8860 | -0.6190 | 2.6230 |
| GCH-skt-simple | -4.1560 | NaN | 3.0280 | -4.2010 | 2.6210 | 2.4740 | 0.3560 | 0.5130 | 0.5460 | 1.0580 | -2.1630 | 1.3220 |
| GCH-emp-simple | -4.0990 | -3.0280 | NaN | -4.1440 | 2.5430 | 2.3990 | 0.2450 | 0.4110 | 0.4490 | 0.9750 | -2.2950 | 1.1960 |
| GCH-n-dcc | 4.7990 | 4.2010 | 4.1440 | NaN | 3.3860 | 3.2580 | 1.5420 | 1.5900 | 1.5800 | 1.9410 | -0.5020 | 2.7030 |
| GCH-t-dcc | -3.3380 | -2.6210 | -2.5430 | -3.3860 | NaN | 1.2260 | -1.0110 | -0.7810 | -0.7000 | -0.0730 | -3.1400 | -0.2700 |
| GCH-skt-dcc | -3.2090 | -2.4740 | -2.3990 | -3.2580 | -1.2260 | NaN | -1.1260 | -0.9060 | -0.8250 | -0.2140 | -3.1800 | -0.3880 |
| NGCH-n-dcc | -1.4690 | -0.3560 | -0.2450 | -1.5420 | 1.0110 | 1.1260 | NaN | 0.9850 | 0.8600 | 0.9590 | -1.7220 | 0.6660 |
| NGCH-t-dcc | -1.5240 | -0.5130 | -0.4110 | -1.5900 | 0.7810 | 0.9060 | -0.9850 | NaN | 0.5970 | 0.8560 | -1.8360 | 0.4510 |
| NGCH-skt-dcc | -1.5160 | -0.5460 | -0.4490 | -1.5800 | 0.7000 | 0.8250 | -0.8600 | -0.5970 | NaN | 0.8350 | -1.8590 | 0.3840 |
| GARCH-fz | -1.8860 | -1.0580 | -0.9750 | -1.9410 | 0.0730 | 0.2140 | -0.9590 | -0.8560 | -0.8350 | NaN | -2.4450 | -0.1470 |
| One-factor GAS | 0.6190 | 2.1630 | 2.2950 | 0.5020 | 3.1400 | 3.1800 | 1.7220 | 1.8360 | 1.8590 | 2.4450 | NaN | 2.5080 |
| Hybrid GAS | -2.6230 | -1.3220 | -1.1960 | -2.7030 | 0.2700 | 0.3880 | -0.6660 | -0.4510 | -0.3840 | 0.1470 | -2.5080 | NaN |

**Table 3.14** – MOM Combine Models Comparison

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 12 different forecasting models of MOM factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models.

| . | GCH-n-simple | GCH-skt-simple | GCH-emp-simple | GCH-n-dcc | GCH-t-dcc | GCH-skt-dcc | NGCH-n-dcc | NGCH-t-dcc | NGCH-skt-dcc | GARCH-fz | One-factor GAS | Hybrid GAS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GCH-n-simple | NaN | 5.0250 | 4.9550 | -5.5430 | 4.0780 | 4.2250 | 0.3860 | 0.7830 | 0.8720 | 2.7360 | -1.8190 | 1.8050 |
| GCH-skt-simple | -5.0250 | NaN | 4.3970 | -5.0560 | 3.4760 | 3.6370 | -0.4740 | -0.1000 | -0.0080 | 2.0750 | -3.1090 | 0.8410 |
| GCH-emp-simple | -4.9550 | -4.3970 | NaN | -4.9780 | 3.0170 | 3.1960 | -0.8540 | -0.5100 | -0.4220 | 1.6820 | -3.7240 | 0.3290 |
| GCH-n-dcc | 5.5430 | 5.0560 | 4.9780 | NaN | 4.1100 | 4.2560 | 0.4380 | 0.8350 | 0.9240 | 2.7720 | -1.7450 | 1.8580 |
| GCH-t-dcc | -4.0780 | -3.4760 | -3.0170 | -4.1100 | NaN | -1.7430 | -1.5460 | -1.2970 | -1.2270 | 0.7990 | -5.0960 | -0.8510 |
| GCH-skt-dcc | -4.2250 | -3.6370 | -3.1960 | -4.2560 | 1.7430 | NaN | -1.4900 | -1.2290 | -1.1570 | 0.8830 | -4.9610 | -0.7380 |
| NGCH-n-dcc | -0.3860 | 0.4740 | 0.8540 | -0.4380 | 1.5460 | 1.4900 | NaN | 2.3670 | 2.2980 | 1.7170 | -1.7570 | 0.9110 |
| NGCH-t-dcc | -0.7830 | 0.1000 | 0.5100 | -0.8350 | 1.2970 | 1.2290 | -2.3670 | NaN | 1.9980 | 1.5490 | -2.2320 | 0.6360 |
| NGCH-skt-dcc | -0.8720 | 0.0080 | 0.4220 | -0.9240 | 1.2270 | 1.1570 | -2.2980 | -1.9980 | NaN | 1.5050 | -2.3450 | 0.5650 |
| GARCH-fz | -2.7360 | -2.0750 | -1.6820 | -2.7720 | -0.7990 | -0.8830 | -1.7170 | -1.5490 | -1.5050 | NaN | -5.5800 | -1.7210 |
| One-factor GAS | 1.8190 | 3.1090 | 3.7240 | 1.7450 | 5.0960 | 4.9610 | 1.7570 | 2.2320 | 2.3450 | 5.5800 | NaN | 4.7220 |
| Hybrid GAS | -1.8050 | -0.8410 | -0.3290 | -1.8580 | 0.8510 | 0.7380 | -0.9110 | -0.6360 | -0.5650 | 1.7210 | -4.7220 | NaN |

**Table 3.15** – VAL Combine Models Comparison

This table presents t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017), over the out-of-sample period from March 11, 2005, to March 20, 2020, for 12 different forecasting models of VAL factor returns. A positive value indicates that the row model has higher average loss than the column model. Values greater than 1.96 in absolute value indicate that the average loss difference is significantly different from zero at the 95% confidence level. Values along the main diagonal are all identically zero and are omitted for interpretability. The first three rows correspond to the univariate distribution forecast, the next three rows correspond to dynamic Copula forecasts, the next three rows give the forecasting results from NAGARCH dynamic Copula models, the last three rows correspond to the Copula GAS forecasting models.

| . | GCH-n-simple | GCH-skt-simple | GCH-emp-simple | GCH-n-dcc | GCH-t-dcc | GCH-skt-dcc | NGCH-n-dcc | NGCH-t-dcc | NGCH-skt-dcc | GARCH-fz | One-factor GAS | Hybrid GAS |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| GCH-n-simple | NaN | 4.3470 | 4.9350 | -6.7620 | -10.5370 | -10.0160 | -2.7380 | -2.5640 | -2.6010 | 2.4880 | -2.0910 | -1.3030 |
| GCH-skt-simple | -4.3470 | NaN | 4.9450 | -5.3220 | -10.4040 | -9.8780 | -2.7640 | -2.5910 | -2.6280 | 2.3490 | -2.1330 | -1.3820 |
| GCH-emp-simple | -4.9350 | -4.9450 | NaN | -5.0830 | -9.3810 | -8.8630 | -2.9310 | -2.7690 | -2.8030 | 1.1660 | -2.4050 | -1.8890 |
| GCH-n-dcc | 6.7620 | 5.3220 | 5.0830 | NaN | -10.6250 | -10.1010 | -2.7230 | -2.5480 | -2.5850 | 2.5910 | -2.0660 | -1.2550 |
| GCH-t-dcc | 10.5370 | 10.4040 | 9.3810 | 10.6250 | NaN | 13.8360 | -1.9970 | -1.7710 | -1.8210 | 6.7990 | -0.8030 | 1.1250 |
| GCH-skt-dcc | 10.0160 | 9.8780 | 8.8630 | 10.1010 | -13.8360 | NaN | -2.1060 | -1.8880 | -1.9360 | 6.2610 | -0.9940 | 0.7670 |
| NGCH-n-dcc | 2.7380 | 2.7640 | 2.9310 | 2.7230 | 1.9970 | 2.1060 | NaN | 4.6820 | 4.8240 | 3.1080 | 1.7390 | 2.5570 |
| NGCH-t-dcc | 2.5640 | 2.5910 | 2.7690 | 2.5480 | 1.7710 | 1.8880 | -4.6820 | NaN | -4.1500 | 2.9540 | 1.4350 | 2.3440 |
| NGCH-skt-dcc | 2.6010 | 2.6280 | 2.8030 | 2.5850 | 1.8210 | 1.9360 | -4.8240 | 4.1500 | NaN | 2.9860 | 1.5040 | 2.3910 |
| GARCH-fz | -2.4880 | -2.3490 | -1.1660 | -2.5910 | -6.7990 | -6.2610 | -3.1080 | -2.9540 | -2.9860 | NaN | -2.6600 | -2.0930 |
| One-factor GAS | 2.0910 | 2.1330 | 2.4050 | 2.0660 | 0.8030 | 0.9940 | -1.7390 | -1.4350 | -1.5040 | 2.6600 | NaN | 2.0960 |
| Hybrid GAS | 1.3030 | 1.3820 | 1.8890 | 1.2550 | -1.1250 | -0.7670 | -2.5570 | -2.3440 | -2.3910 | 2.0930 | -2.0960 | NaN |

**Figure 3.1** – Univariate Model of Rolling Window Expected ES

This figure plots the expected ES of four different factors of rolling window models. The blue line show the rolling window with 26 observations (6 month). The red line is the results of 52 observations rolling window model, while the yellow line denotes the results of 104 observations rolling window model.

**Figure 3.2** – Univariate Model of Distribution Models Expected ES

This figure plots the expected ES of four different factors of distribution models. The blue line show the normal distribution model. The red line is the results of skewed t distribution model, while the yellow line denotes the results of empirical distribution model.

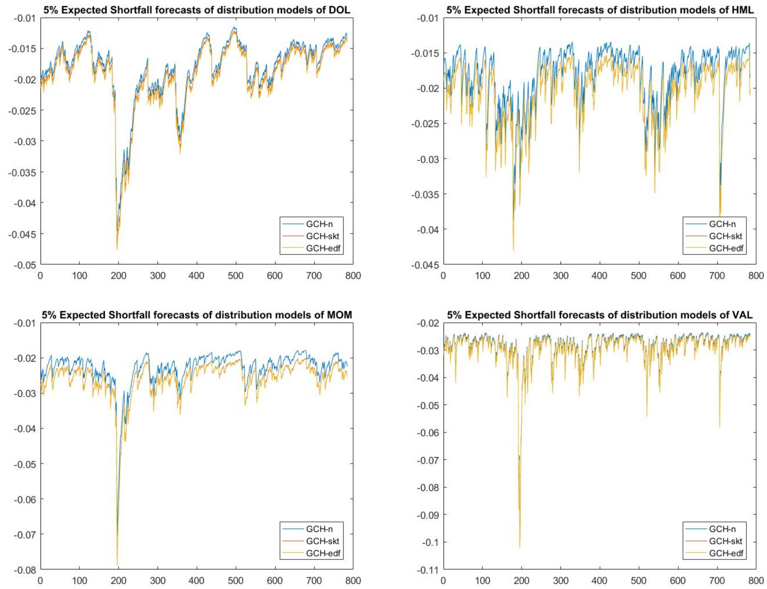


**Figure 3.3** – Univariate Model of GAS Models Expected ES

This figure plots the expected ES of four different factors of distribution models. The blue line show the two-factor GAS model. The red line is the results of one-factor GAS model, while the yellow line denotes the results of GARCH-FZ GAS model. The purple line show the expected ES of hybrid model.

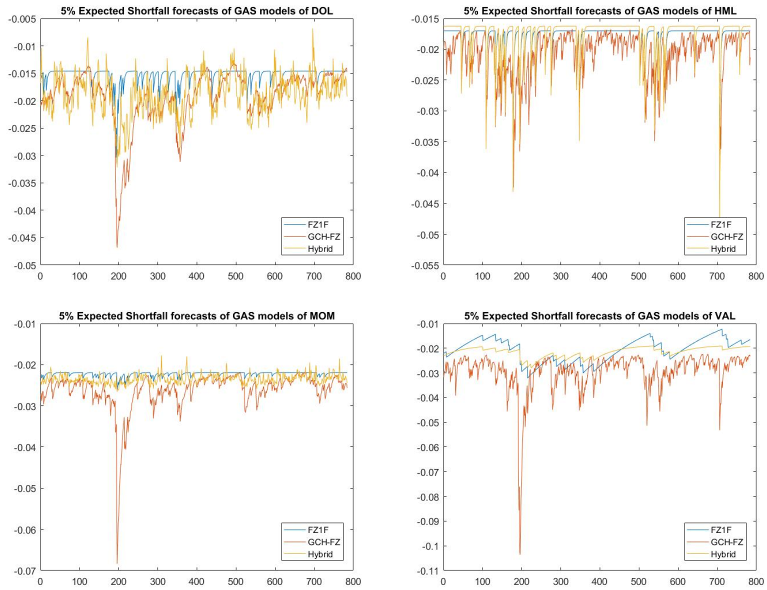


**Figure 3.4** – Univariate Model of Whole Sample Forecasting

This figure plots the estimated 5% Value-at-Risk (VaR) and Expected Shortfall (ES) for daily returns on the four different factors, over the period April 1, 1994, to March 20, 2020. The estimates are based on a one-factor GAS model, a empirical distribution model, and a rolling window using 26 observations.

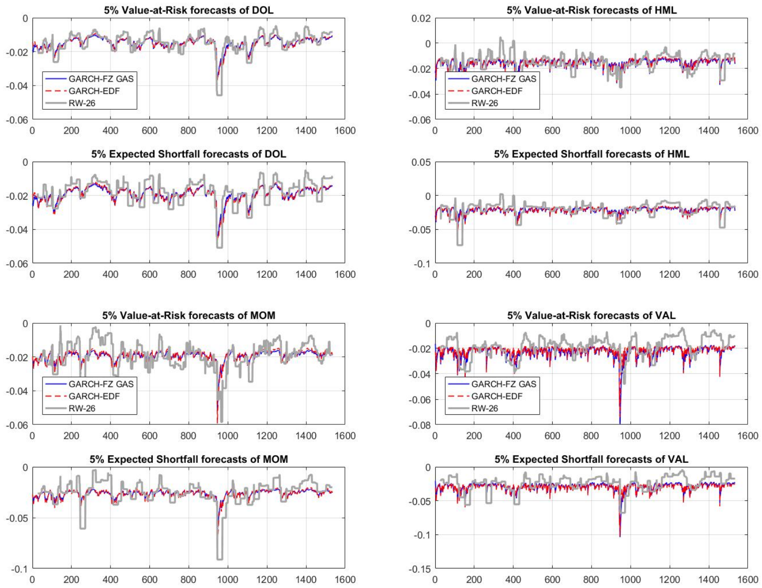


**Figure 3.5** – Univariate Model During the Financial Crisis

This figure plots the estimated 5% Value-at-Risk (VaR) and Expected Shortfall (ES) for daily returns on the four different factors, over the period January Sep 21, 2007, to March 20, 2020. The estimates are based on a one-factor GAS model, a empirical distribution model, and a rolling window using 26 observations.

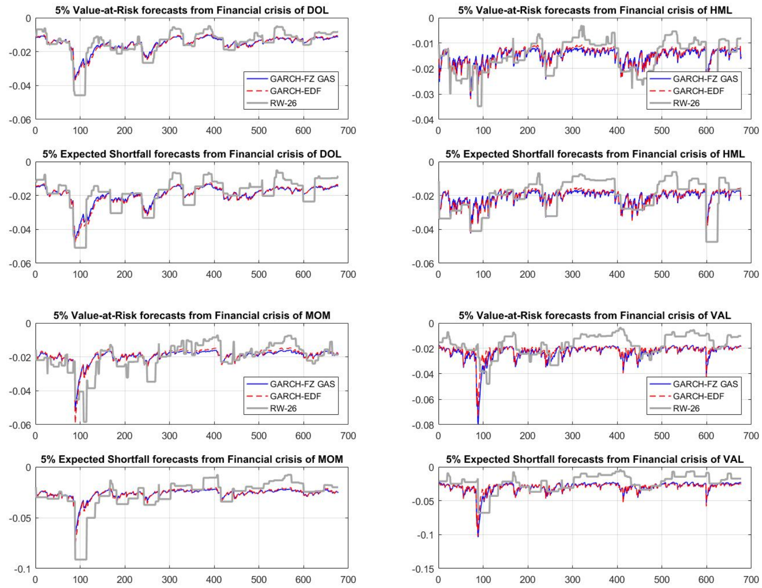


**Figure 3.6** – Multivariate NGARCH Dynamic Copula Expected ES

This figure plots the expected ES of four different factors of Copula multivariate distribution models. The blue line show the normal distribution model. The red line is the results of student t distribution model, while the yellow line denotes the results of skewed t distribution model.

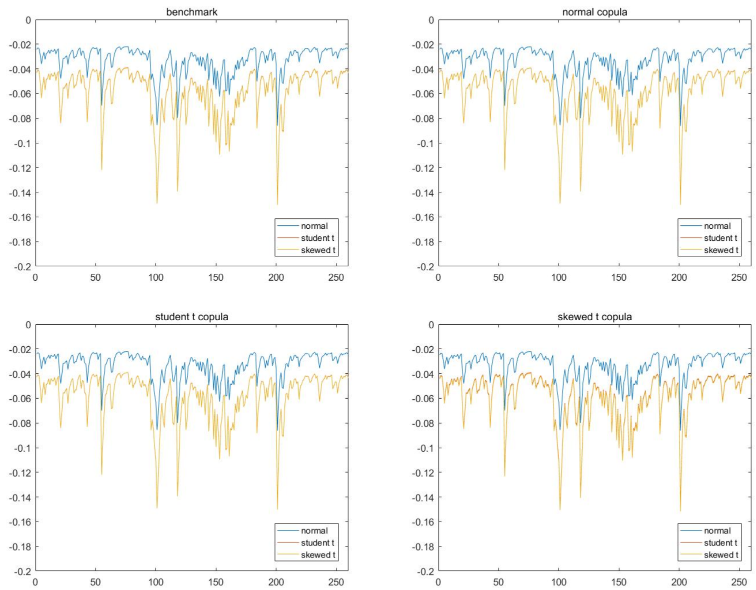
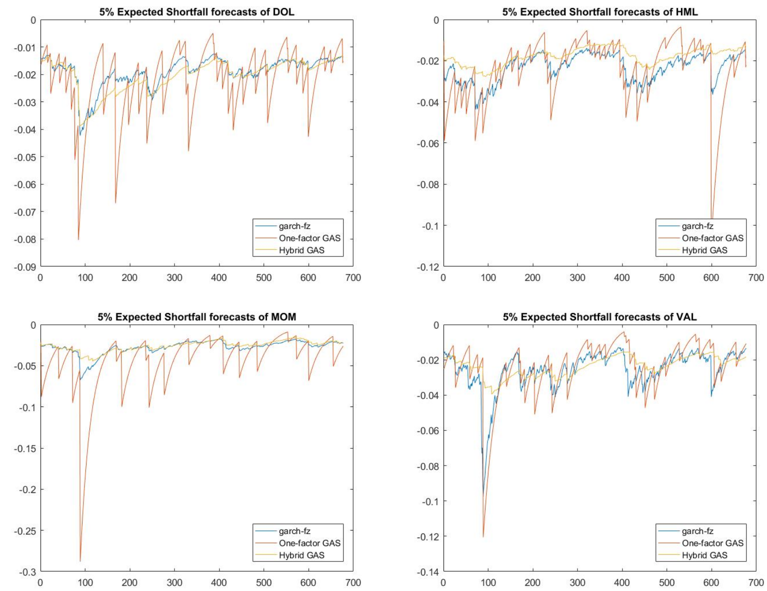


**Figure 3.7** – Multivariate Copula GAS Model Expected ES

This figure plots the expected ES of four different factors of distribution models. The blue line show the GARCH-FZ GAS model. The red line is the results of one-factor GAS model, while the yellow line denotes the results of hybrid model.



# Appendix.3 Univariate Model from Patton

## The Univariate Model of Patton et al. (2019)

In this section, I follow the univariate models of Patton et al. (2019) to forecast the portfolio risk. First, I introduce 9 different models by Patton et al. (2019). The first six models are the rolling window and distribution forecasting models. The loss functions I discussed in section 3.2 are applied in the GAS forecasting model with the last four models. I then explain the method of estimation of those GAS models. The data set is the four factor portfolios from the chapter 2. I set the first 750 weekly observations as the in-sample data set. Hence, the rest of the data set is treated as out-of-sample data. In this section, I use univariate models to forecast the out-of-sample risk measures. The figures of the forecasted risk measures are shown after the introduction of each model.

### Rolling Window Forecasting

The first model is the rolling window forecasting model. The past rolling window applies the historical period to find the forecasting estimates of risk measures (VaR and ES). The shorter the period that I choose, the more sensitive the estimates would be to the recent values. The simplest model of forecasting is the rolling window estimate, shown as follows.

$$\widehat{VaR}_t = \widehat{Quantile}\left\{Y_s\right\}_{s=t-m}^{t-1} \tag{3.28}$$

$$\widehat{ES}_t = \frac{1}{\alpha m}\sum_{s=t-m}^{t-1} Y_s 1\left\{Y_s \leq \widehat{VaR}_s\right\} \tag{3.29}$$

where $\widehat{Quantile}\left\{Y_s\right\}_{s=t-m}^{t-1}$ denotes the quantile of $Y_s$ during the period $s \in [t-m, t-1]$. As for the window size, I choose 26, 52, and 104 weeks, corresponding to half-year, one-year, and two-years of weekly return observations, respectively. For the forecasting of the VaR and ES, I apply the data in the window to calculate the risk measure of the next term based on the equation above.

[Figure3.1 univariate model of rolling window about here]

Figure 3.1 shows the forecasted ES of four different factors through rolling window models. The blue line shows the rolling window with observations of the 26-week (6-month) window. The red line shows the results of observations of the 52-week (one-year) rolling window model, while the yellow line denotes the results of observations of the 104-week (two-year) rolling window model. The smaller rolling window would be more sensitive to the risk changes in the market, while the larger rolling window would be more hysteretic. The blue line (26-week) could monitor extreme changes, while the other two models are smoothed and hysteretic. The graph shows that there has been a sharp decrease at the 200 terms, which was affected by the 2008 financial crisis, especially in DOL, MOM, and VAL factors.

## Simple Distribution Forecasting

Secondly, Patton et al. (2019) choose more challenging competitor models to forecast the VaR and ES of the portfolios' returns. They based on the ARMA-GARCH dynamics for the conditional mean and variance to build my models, using the assumption for the standardized residual distributions. The models are shown as follows:

$$Y_t = \mu_t + \sigma_t \eta_t \tag{3.30}$$

$$\eta_t \sim iid\, F_\eta(0,1) \tag{3.31}$$

The $Y_t$ denotes the factor returns whose risk measure needs to be forecasted. $\mu_t$ is mean of the $Y_t$ and $\sigma_t^2$ is specified to the volatility of the $Y_t$. $F_\eta(0,1)$ denotes the distribution of $\eta_t$(which could have a different distribution, eg. normal, student t and skewed t distribution). Given the $F_\eta$, the forecasting of VaR and ES can be estimated as:

$$v_t = \mu_t + a\sigma_t,\ where\, a = F_\eta^{-1}(\alpha) \tag{3.32}$$

$$e_t = \mu_t + b\sigma_t,\ where\, b = \mathbb{E}\left[\eta_t \mid \eta_t \leq a\right] \tag{3.33}$$

where $v_t$ denotes the VaR estimates at term $t$, and $e_t$ is the estimates of ES at term

$t$. The parameters $(a, b)$ can be calculated from the distributions I discussed in the equation above.

They use three choices for $F_\eta$ to describe the distributions of $\eta_t$:

$$\eta_t \sim iid\, N(0,1) \tag{3.34}$$

$$\eta_t \sim iid\, empirical\ distribution \tag{3.35}$$

$$\eta_t \sim iid\, Skew\, t(0,1,\nu,\lambda) \tag{3.36}$$

Except for the common distribution and normal distribution, they also use the skewed t distribution, following Hansen (1994). Furthermore, they apply the empirical distribution function (EDF) as a non-parameter alternative to estimate the distribution of $\eta_t$. By using the distribution for forecasting the VaR and ES, they apply four different ways to achieve.

$$(VaR_t^\alpha, ES_t^\alpha) = (a_\alpha, b_\alpha)\,\sigma_t \tag{3.37}$$

$$a_\alpha = \Phi^{-1}(\alpha) \tag{3.38}$$

$$b_\alpha = -\phi\left(\Phi^{-1}(\alpha)\right)/\alpha \tag{3.39}$$

The $\Phi$ denotes the CDF of the distribution and $\phi$ is the PDF of the standardized normal distribution. As the estimate of the risk measures of distribution, I forecast as follows. First, I get the estimates of GARCH model. Then, the $\sigma_t^2$ can be estimated through the GRACH model. I use the CDF to calculate the parameters $(a_\alpha, b_\alpha)$ and, then, forecast the $(VaR_t^\alpha, ES_t^\alpha)$ by equation 3.32 and 3.33. Then, the model 4,5,6 have been discussed.

[Figure3.2 univariate model of distribution models about here]

As for the distribution models in figure 3.2, the forecasted ES from the distribution models are rougher compared to the rolling window results since the $\sigma_t$ of the GARCH model led to the movement of the VaR and ES in distribution models. The blue line shows the normal distribution model. The red line shows the results of the skewed t distribution model, while the yellow line denotes the results of the empirical distribution model. The normal distribution model forecasts the lower risk (smaller predicted ES), while the empirical model and skewed t model could forecast similar risks during the extreme event.

## GARCH GAS Model

Patton et al. (2019) combine the GARCH and GAS models to estimate the VaR and ES as follow. The main difference between the GARCH GAS model and the distribution model is that the parameters $(a, b)$ need to be estimated in the GARCH GAS model. Hence, the GARCH-GAS model does not focus on the distribution of the $\eta_t$. The parameters needed in the GARCH GAS model are $(\beta, \gamma, a, b)$, while the parameter $\omega$ is set to zero. The GARCH model is shown below:

$$Y_t = \sigma_t \eta_t, \eta_t \sim iid\, F_\eta(0, 1) \tag{3.40}$$

$$\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \gamma Y_{t-1}^2 \tag{3.41}$$

However, as I discussed before, the parameter $(a, b)$ are not from the distribution of the as the previous model. Then, the VaR and ES could be estimated as follow:

$$v_t = a\sigma_t,\ where\, a = F_\eta^{-1}(\alpha) \tag{3.42}$$

$$e_t = b\sigma_t,\ where\, b = \mathbb{E}\left[\eta_t \mid \eta_t \leq a\right] \tag{3.43}$$

When estimating the $(a, b)$, the VaR and ES would be forecasted at each term.

## The One-factor GAS Model for ES and VaR

Then, I will introduce the more complex GAS model of Patton et al. (2019). In this situation, a new parameter $\kappa_t$ has been added in the GAS model, which drives the risk measures VaR and ES. The one-factor model is shown as follow:

$$Y_t = \exp\left\{\kappa_t\right\} \eta_t \tag{3.44}$$

$$\eta_t \sim iid\, F_\eta(0, 1) \tag{3.45}$$

The VaR and ES could be calculated from $\kappa_t$:

$$v_t = a \exp\{\kappa_t\} \tag{3.46}$$

$$e_t = b \exp\{\kappa_t\} \tag{3.47}$$

where $\kappa_t$ is:

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma H_{t-1}^{-1} s_{t-1} \tag{3.48}$$

$\kappa_t$ denotes the log-volatility. The $\kappa_t$ is related with two part: the AR(1) process $\kappa_{t-1}$ and the forcing variable, $H_{t-1}^{-1} s_{t-1}$. The second part could be written as below:

$$s_t \equiv \frac{\partial L_{FZ0}\left(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\}; \alpha\right)}{\partial \kappa_t} = -\frac{1}{e_t}\left(\frac{1}{\alpha}\mathbf{1}\{Y_t \le v_t\} Y_t - e_t\right) \tag{3.49}$$

$$I_t = \frac{\partial^2 \mathbb{E}_{t-1}\left[L_{FZ0}\left(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\}; \alpha\right)\right]}{\partial \kappa_t^2} = \frac{\alpha - k_\alpha a_\alpha}{\alpha} \tag{3.50}$$

where $k_\alpha$ is a negative constant item while $k_\alpha \in (0, 1)$. From the equations above, I could rewrite the $\kappa_t$ as below:

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma\frac{1}{b\exp\{\kappa_t\}}\left(\frac{1}{\alpha}\mathbf{1}\{Y_{t-1} \le a\exp\{\kappa_{t-1}\}\} Y_{t-1} - b\exp\{\kappa_{t-1}\}\right) \tag{3.51}$$

Hence, the parameters in the one-factor model are $(\beta, \gamma, a, b)$. However, the main difference between GARCH GAS model and the one-factor model is from the volatility item.

## The Hybrid GAS Model for ES and VaR

Then, Patton et al. (2019) combine the one-factor model and the GARCH model together to create the hybrid GAS model as follow. They specify:

$$Y_t = \exp\{\kappa_t\}\eta_t \tag{3.52}$$

$$\eta_t \sim iid\, F_\eta(0,1) \tag{3.53}$$

$$v_t = a \exp\{\kappa_t\} \tag{3.54}$$

$$e_t = b \exp\{\kappa_t\} \tag{3.55}$$

where $\kappa_t$ is:

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma H_{t-1}^{-1} s_{t-1} + \delta \log\left|Y_{t-1}\right| \tag{3.56}$$

They find that the score and Hessian is still the same as the one-factor model:

$$s_t \equiv \frac{\partial L_{FZ0}\left(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\}; \alpha\right)}{\partial \kappa} = -\frac{1}{e_t}\left(\frac{1}{\alpha}\mathbf{1}\{Y_t \leq v_t\} Y_t - e_t\right) \tag{3.57}$$

$$I_t = \frac{\partial^2 \mathbb{E}_{t-1}\left[L_{FZ0}\left(Y_t, a\exp\{\kappa_t\}, b\exp\{\kappa_t\}; \alpha\right)\right]}{\partial \kappa_t^2} = \frac{\alpha - k_\alpha a_\alpha}{\alpha} \tag{3.58}$$

$$\kappa_t = \omega + \beta\kappa_{t-1} + \gamma \frac{1}{b\exp\{\kappa_t\}}\left(\frac{1}{\alpha}\mathbf{1}\{Y_{t-1} \leq a\exp\{\kappa_{t-1}\}\} Y_{t-1} - b\exp\{\kappa_{t-1}\}\right) + \delta\log\left|Y_{t-1}\right| \tag{3.59}$$

$\kappa_t$ denotes the log-volatility. There are five parameters in the hybrid GAS model $(\beta, \gamma, \delta, a, b)$, and I estimate them using FZ loss minimization. The difference between the hybrid model and one-factor mode is that the hybrid model adds a new parameter-$\delta$, and its new part in the $\kappa_t$ which adds the direct effect from $Y_{t-1}$.

## Estimation of GAS Models for ES and VaR

I now introduce the ways to estimate parameters in the GAS model in detail. First, I will introduce the asymptotic theory of Patton et al. (2019) during the minimizing to estimate the risk measures. I set the factors returns as $(Y_1, \ldots, Y_T)$ the significant level of VaR and ES is constant $\alpha = 0.05$. I will apply the GAS model to forecast the risk measure of the factor returns. Suppose the factor portfolio returns have the distribution $\mathcal{F}_{t-1}$, the distribution function is $F_t(\cdot \mid \mathcal{F}_{t-1})$. $v_1(\theta_0)$ and $e_1(\theta_0)$ denotes the value of VaR and ES at the initial time. I have $\mathcal{F}_{t-1} = \sigma\left(Y_{t-1}, \mathbf{X}_{t-1}, \ldots, Y_1, \mathbf{X}_1\right)$. where $X_t$ denotes the exogenous variables. Then, the VaR and ES could be written as:

$$\begin{bmatrix} VaR_\alpha\left(Y_t \mid \mathcal{F}_{t-1}\right) \\ ES_\alpha\left(Y_t \mid \mathcal{F}_{t-1}\right) \end{bmatrix} = \begin{bmatrix} v\left(Y_{t-1}, \mathbf{X}_{t-1}, \ldots, Y_1, \mathbf{X}_1; \theta^0\right) \\ e\left(Y_{t-1}, \mathbf{X}_{t-1}, \ldots, Y_1, \mathbf{X}_1; \theta^0\right) \end{bmatrix} = \begin{bmatrix} v_t\left(\theta^0\right) \\ e_t\left(\theta^0\right) \end{bmatrix} \tag{3.60}$$

The unknown parameters are estimated as:

$$\hat{\theta}_T \equiv \arg\min_{\theta\in\Theta} L_T\left(\theta\right) \tag{3.61}$$

$$L_T\left(\theta\right) = \frac{1}{T}\sum_{t=1}^{T} L_{FZ0}\left(Y_t, v_t\left(\theta\right), e_t\left(\theta\right); \alpha\right) \tag{3.62}$$

and the FZ loss function defined in equation 3.3.

I now show an example of the hybrid GAS model for the estimation. In the first step, I set the start value of the parameters and calculate the VaR and ES at Term 1. Then, I get the $\kappa_t$ and expected loss for term 1. The equation 3.59 is used to estimate the $\kappa_t$ for the next term. The VaR and ES are calculated from the $\kappa_t$, while the expected loss is computed consequently. Following these steps, I get the forecasts of VaR, ES, $\kappa_t$, and the expected loss for each term. In the minimizing step, the mean of the FZ loss from each term is minimized to obtain the estimation of the parameters $(\beta, \gamma, \delta, a, b)$ using the optimization.

[Figure3.3 univariate model of GAS models about here]

The figure 3.3 shows the GAS model results. The red line represents the results of the one-factor GAS model, while the yellow line denotes the results of the GARCH-FZ GAS model. The purple line represents the expected ES of the hybrid model. Since the one-factor and hybrid models often have unstable forecasting risk measures which have under-optimization estimations, the under-optimization estimations of the parameters may lead to a dire prediction of VaR and ES. I then indicate that the GARCH-FZ model has the most robust performance among the four GAS models.

# Comparison with VaR and ES of Different Methods in Out-of-sample Forecasting

In this section, I compare the 9 univariate models from Patton et al. (2019), introduced in section 3.6. I draw the forecasted risk measures of different kinds of models for comparison. I also show the goodness-of-fit-test and Diebold-Mariano tests to compare the performance of these models. Both average loss in the goodness-of-fit-test table and Diebold-Mariano tests are the methods to rank the performance of models. The figures and tables are shown and discussed below.

## Comparison in Figures

[Figure3.4 univariate model of whole sample forecasting about here]

[Figure3.5 univariate model during the financial crisis about here]

I draw the figures to compare the forecasted risk measure between different kinds of models in figures 3.4 and 3.5. Figure 3.4 shows the forecasting ES of the whole period, while the figure 3.5 shows the forecasting performance during the period of the financial crisis. I chose the 26-week rolling window model, empirical distribution model, and GARCH-FZ GAS model from each kind of forecasting model. The blue line shows the GARCH-FZ GAS model. The red dashed line shows the results of the empirical distribution model, while the grey line denotes the results of the rolling window model with a 26-week period. What can be seen clearly in figures 3.4 and 3.5 is that the three models follow a similar path. However, the rolling window model still has the most hysteretic performance since the forecasted risk measures are smoothed by the rolling window. The empirical distribution model and GARCH-FZ GAS mode show similar details of forecasting since the risk measures of these two models are almost superposition, as they are both based on the same volatility $\sigma_t$ parameters from the GARCH model.

## Ranking from Different Test

I followed the goodness-of-fit-test and Diebold-Mariano tests of Patton et al. (2019). The two tests are discussed below. The idea to test the good fit in forecasting is that the VaR and ES should have the correct specifications as follows:

$$\mathbb{E}_{t-1}\begin{bmatrix}\partial L_{FZ0}\left(Y_t, v_t, e_t; \alpha\right)/\partial v_t \\ \partial L_{FZ0}\left(Y_t, v_t, e_t; \alpha\right)/\partial e_t\end{bmatrix} = 0 \tag{3.63}$$

The equation 3.63 imply $\mathbb{E}_{t-1}\left[\lambda_{v,t}\right] = \mathbb{E}_{t-1}\left[\lambda_{e,t}\right] = 0$. Then, $(\lambda_{v,t}, \lambda_{e,t})$ is the generalized residual of the equation 3.63. I, then, standardize the $(\lambda_{v,t}, \lambda_{e,t})$ to concentrate on the impact of the serial correlation in these measures as follow:

$$\begin{aligned}\lambda_{v,t}^{s} \equiv \tfrac{\lambda_{v,t}}{v_t} = 1\left\{Y_t \leq v_t\right\} - \alpha \\ \lambda_{e,t}^{s} \equiv \tfrac{\lambda_{e,t}}{e_t} = \tfrac{1}{\alpha}1\left\{Y_t \leq v_t\right\}\tfrac{Y_t}{e_t} - 1\end{aligned} \tag{3.64}$$

Consequently, these standardized and generalized residuals also have the expected value zero, $\mathbb{E}_{t-1}\left[\lambda^s_{v,t}\right] = \mathbb{E}_{t-1}\left[\lambda^s_{e,t}\right] = 0$ which is similar as $(\lambda_{v,t}, \lambda_{e,t})$. Patton et al. (2019) adopt the dynamic quantile (DQ) testing approach by simple regressions of these generalized residuals at each term $t$.

$$\begin{aligned} \lambda^s_{v,t} &= a_0 + a_1\lambda^s_{v,t-1} + a_2 v_t + u_{v,t} \\ \lambda^s_{e,t} &= b_0 + b_1\lambda^s_{e,t-1} + b_2 e_t + u_{e,t} \end{aligned} \tag{3.65}$$

Following Patton et al. (2019), I set the hypothesis that all parameters in these regressions are zero, which is different from the usual two-sided alternative. The VaR and ES are tested separately.

[Table 3.1 Average loss and goodness fit test of univariate model is about here]

I apply the goodness-of-fit-test to estimate the performance of different models. Table 3.1 shows the average loss, using the FZ loss function following Fissler (2017), and p-values from goodness-of-fit tests of the VaR and ES forecasts. The hybrid GAS model (Hybrid) gets the lowest average loss among all the univariate models in the DOL factor's risk measure. The 26-week rolling window model (RW-26) reaches the lowest average loss in the HML and MOM factors' risk measures, while the 104-week rolling window model (RW-104) has the lowest average loss in the VAL factor's risk measure. Table 3.1 presents p-values from goodness-of-fit tests of the VaR and ES forecasts, respectively. The results of the goodness-of-fit test, which does not pass at the 10% level in bold. The DOL factor's results do not perform well since the goodness-of-fit test of many models cannot pass, while the one-factor GAS model (FZ1Z) and the 54-week rolling window model (RW-54)could pass the goodness-of-fit test. Although the average loss could partly indicate the best performance, the ranking from the average loss is not robust. For instance, the hybrid GAS model (Hybrid) has good performance in the DOL factor but cannot pass the goodness-of-fit test for both VaR and ES. Hence, I apply the Diebold-Mariano test to show the t-statistics ranking in the four tables below:

Through the results of the goodness-of-fit test, many different models may pass the goodness-of-fit test in some factors while failing in other factors, which may increase the difficulty of ranking them. The t-statistics from Diebold-Mariano tests aim to rank the average loss in a statistical way. I follow Patton et al. (2019) to calculate the t-statistics of average loss, as follows.

[Table 3.2 DOL factors' Diebold-Mariano t-statistics on average out-of-sample loss differences is about here]

[Table 3.3 HML factors' Diebold-Mariano t-statistics on average out-of-sample loss differences is about here]

[Table 3.4 MOM factors' Diebold-Mariano t-statistics on average out-of-sample loss differences is about here]

[Table 3.5 VAL factors' Diebold-Mariano t-statistics on average out-of-sample loss differences is about here]

I show the t-statistics from Diebold-Mariano tests comparing the average losses, using the loss function from Fissler (2017) over an out-of-sample period from 11 March, 2005, to 20 March, 2020, for ten different forecasting models of four factors' returns in Tables 3.2, 3.3, 3.4 and 3.5. The tests are conducted as row model minus column model. Consequently, a positive number means that the column model has a better performance than the row model. For instance, in Table 3.2, the GCH-FZ GAS (10th column) shows the positive t-statistics for all rows. This indicates that the GCH-FZ GAS model shows the best performance and stability during forecasting, while the GCH-empirical distribution model has the best performance in the other three factors. Overall, the GCH-empirical has the best performance among the 9 univariate models when forecasting the risk of four factors.

# Chapter 4

# Forward Premium Puzzle and Uncovered Interest Parity (UIP)

This chapter focuses on a famous puzzle in the forex market: the forward premium puzzle. Following Burnside et al. (2009), I argue that adverse selection problems between participants in foreign exchange markets can explain this forward premium puzzle. I present a model in which adverse selection problems between market makers and traders rationalized a negative covariance between the forward premium and spot rate changes. I first apply the unique order flow data set to test Burnside et al. (2009) model. Then, I creatively discuss the transaction between the bond and spot exchange markets to explain the excess return of carry trade. This chapter is accepted by the European Financial Management Ph.D. seminar[1].

[1]This paper can be found at https://www.efmaefm.org/0EFMAMEETINGS/EFMA%20ANNUAL%20MEET Leeds/An%20Explanation%20of%20Forward%20Premium%20Puzzle.pdf.

## 4.1 Introduction

UIP illustrates that if investors in the forex market are risk-neutral and form expectations rationally, there will be no opportunity to carry trade. According to empirical research, Menkhoff et al. (2012a) find that changes in exchange rates will not compensate for interest rate differences, which means the UIP could hardly appear in the currency market. The forward premium puzzle is a general topic in modern international finance. Fama (1984) first argues that carry trades strategy could get a positive profit, which violates the UIP. This is called 'forward premium puzzle' by the researchers.

During the last few decades, many researchers documented this puzzle, such as Hansen (1982), Fama (1984). Engel (1984) and Fama (1984) try to apply the existence of the time-varying risk premium to explain the puzzle. Bansal and Dahlquist (1999) find the difference in the risk premium between developed and emerging economies. Londono and Zhou (2017) and Gospodinov (2009) also try to explain the forward premium puzzle through the risk premium method. The risk premium are the primary way to discuss the forward premium puzzle. However, Burnside et al. (2009) argue that adverse selection problems between the investors and the market maker in foreign exchange markets can explain the forward premium puzzle. In this paper, I base the adverse selection model on the one suggested by Burnside et al. (2009) to find the main reason for the unresolved forward premium puzzle in international finance. [2]

However, how could the customer earn profit with carry trade? The process in carry trade to get the payoff is discussed below. Let's set US. dollar (USD) as the domestic currency. $i_t^*$ denotes interest rate on risk-less foreign denominated securities and $i_t$ denotes interest rate on risk-less domestic denominated securities.Then, the payoff for a investor to borrow one USD, in order to lend the foreign currency, is:

$$(1 + i_t^*) \frac{S_{t+1}}{S_t} - (1 + i_t) \tag{4.1}$$

where $S_t$ is the spot exchange rate (USD per FCUs). Then, the payoff of the carry trade is:

$$Z_{t+1} = sign\,(i_t^* - i_t) \left[ (1 + i_t^*) \frac{S_{t+1}}{S_t} - (1 + i_t) \right] \tag{4.2}$$

Where the $Z_{t+1}$ is the payoff in the next period. In period $t$, the investor has 1 USD and wants to change it into foreign currency, which has the interest rate $i_t^*$. Then, the investor will get $1/S_t$ foreign currency. During the period $t + 1$, the foreign currency would be $(1+i_t^*)/S_t$ and by changing it into USD the investor would get $(1+i_t^*)*S_t+1/S_t$ USD at last. The investor would just get $(1 + i_t)$ USD if he only let the 1 dollar in the domestic country. If the investor applies the difference between two investments, the $Z_{t+1}$ would be the payoff of the carry trade strategy. When the USD, and the foreign

[2]The evolution of microstructure model in the forex market has been discussed in the appendix.A.

currency violate the UIP, the $Z_{t+1}$ would not be zero and the carry trade can get the excess return. The carry trade could apply the forward rate and spot rate to arbitrage. The payoff of the carry trade between the forward rate and spot rate is:

$$Z_{t+1} = sign\left(F_t - S_t\right)\left[\left(1 + i_t\right)\frac{\left(F_t - S_{t+1}\right)}{F_t}\right] \tag{4.3}$$

This equation is similar to the equation of interest. When forward premium and discount appear, a carry trade opportunity will arise. For the exchange rate carry trade, the method would be a little different. The investor could sell a forward contract at period $t$, and change into USDs in period $t+1$. Then the 1 USD would $(1+i_t) * S_t + 1/F_t$. While the investor would still get $(1 + i_t)$ USDs if he only let the 1 dollar without any operations. The payoff to this carry trade would be $\frac{(1+i_t)}{F_t}\left(F_t - S_{t+1}\right)$.

Covered interest rate parity (CIP) could be written as:

$$\frac{1 + i_t}{1 + i_t^*} = \frac{F_t}{S_t} \tag{4.4}$$

It could also write as $\left(1 + i_t^*\right)\frac{S_{t+1}}{S_t} = \left(1 - i_t\right)$, which means the investment for the forward contract of foreign currency would get the same payoff as the investment in domestic currency. When UIP holds, the two-way carry trade has the same payoff. While the UIP only holds when the carry trade is not profitable:

$$E\left(Z_{t+1}\right) = E\left(sign\left(i_t^* - i_t\right)\left[\left(1 + i_t^*\right)\frac{S_{t+1}}{S_t} - \left(1 + i_t\right)\right]\right) = 0 \tag{4.5}$$

However, carry trade could have a positive profit many times. When CIP and UIP both hold, the forward exchange rate would be the unbiased forecaster of the expected value of the spot rate, i.e. $F_t = E(S_{t+1})$.

The forward premium puzzle can be called the negative correlation between the change of spot exchange rate and the forward premium. If the forward premium exists, the domestic currency will depreciate (appreciate) when the nominal interest rate decreases (increases). Consequently, people could apply a simple strategy to earn a profit called carry trade: buy currencies with a higher nominal interest rate and sell the currency with a lower nominal interest rate. Hence, when the carry trade could get excess returns, it could indicate that the forward premium exists.

In this paper, I use the unique order flow data set to test the significance of the model proposed by Burnside et al. (2009). The generalized method of moments (GMM) helps me find the estimations of those parameters.

There are two main methods to apply order flow data. First, I make a switch (switch method) when I apply the spot rate market order flow, which is different from the assumption of Burnside et al. (2009). I discuss the switch in section 4.5. Second, I choose to inverse (inverse method) the model from the forward rate to the spot rate, from the model proposed by Burnside et al. (2009). I discuss the issue of the forward premiums puzzle from a new perspective. My analysis highlights the problem of adverse selection between market makers and investors. To isolate the effects of adverse selection, I use a simple model completely abstracted from considering risk. My model is based on the microstructure method developed by Burnside et al. (2009). I assume that the forward exchange rate follows an exogenous stochastic process with empirically realistic time series characteristics. My goal explains the forward premium puzzle in the microstructure method.

The basic structure of my model is as follows: Two main types of risk-neutral traders (informed traders and uninformed traders) and risk-neutral market makers. Informed traders have more information about exchange rate changes than market makers and uninformed traders with a signal. Uninformed traders and market makers have the same public information. Uninformed traders follow the rules of behavioral trading: when the pound is expected to appreciate (depreciate), they are more likely to buy (sell) the pound forward.

The appearance of informed brokers brings about the problem of unfavorable choices for market makers. Market-makers do not know when they receive an order, whether it is from an informed trader or an uninformed trader. He can only give different prices for buy and sell orders and determine these prices according to whether he wants the pound to rise or fall. My main result is that adverse selection can solve the forward premium puzzle. Specifically, consider a researcher using data generated by my model to perform a regression analysis of exchange rate changes in the forward premium. Under the condition of maintaining regularity, the researcher estimates that the slope coefficient $\beta$ is negative. This result can be obtained regardless of whether they use the forward exchange rate of the bid (the trader can sell the forward exchange rate of the seller to the market maker) and the forward exchange rate of the asking price (long-term price). The long-term exchange rate that a trader can buy from a market maker is the average asking price and the buying price.

Under normal circumstances, following Burnside et al. (2009), agents are required to predict exchange rates based on public information, and interest rate information is less than the private information available to informed traders. There is another explanation for this normality. As long as it is difficult to predict exchange rates using public information and well-informed traders make positive expected profits, then the forward premium puzzle must exist. The main feature of my model is that the adverse selection problem faced by market makers is more severe when brokers trade based on public information signals. Understand why it's useful to pay attention to asking about prices. Suppose, on the basis of public information, that sterling will depreciate, an uninformed trader might sell sterling. So if a market-maker receives a buy order, most likely from an informed trader, it is expected that sterling will appreciate. Thus, market makers offer high purchase orders, and hence high forward rates. When sterling depreciates, forward premiums (as measured by asking price) are higher on average. Thus, the model captures the negative correlation that defines the difficulty of the forward premium.

There are two motivations for writing this article concerning the forward premium puzzle. First, Burnside et al. (2009) apply a microstructure approach to understanding the forward premium puzzle. However, they only discuss the model and the reason for the forward premium puzzle without estimates of the model. I apply the order flow data, which appreciates the adverse selection model. After getting an estimation of the parameters, I try to find the reason for the negative correlation between forward premium and change of spot rate. Second, I add the bond market in the microstructure model, which includes forward market, spot market, and bond market. Using adverse selection, I apply the linkage among these markets to discuss the UIP and CIP in the new version.

This chapter is structured as follows: the order flow data and the exchange rate data is introduced in the first section; the simple regression between forward premium and the change of spot rate is discussed in the section 4.3; then, the original model from Burnside et al. (2009) through the exchange rate data set is then estimated; the switch method is discussed in the section 4.5 and 4.6; and the inverse method is introduced in section 4.7. The main results are discussed in the last section.

## 4.2 Data

I used the order flow data from one of the top forex dealers with 12 different pairs of currencies from 2nd Nov 2001 to 23rd Dec 2012. This data set includes exchanges like EURUSD, USDJPY, EURJPY, GBPUSD, EURGBP, USDCHF, EURCHF, AUDUSD, NZDUSD, USDCAD, EURSEK, and EURNOK. The order flow data set has four different investors: asset managers, corporates, hedge funds, and private clients of spot exchange currencies. I assume that asset managers and hedge funds are informed investors in the forex market, while the corporates and private clients are uninformed traders. The data set also includes bid, ask, and the average rate for both forward and spot rates of these 12 exchange currencies from DATASTREAM within the same period. There is an obvious limitation of the data set. The order flow data is an outdated data set that ends in 2012, which cannot estimate the recent market. I cannot update the data set since the orderflow data is unobtainable.

## 4.3 Simple Regression

In this section, I test the UIP with simple regression. I apply a simple exploratory model between the forward premium and the change of spot rate as below:

$$s_{t+1} = \alpha + \beta f_t + \varepsilon_{t+1} \tag{4.6}$$

[Table 4.1Different period regression is about here]

[Table 4.2Different period excess return of carry trade strategy is about here]

where $\alpha$ denotes the intercept and $\beta$ is the slope of the model. The aim of doing a simple regression is to test the uncovered interest rate parity in the market. When UIP holds, the $\alpha$ should equal zero and $\beta$ should equal 1. Then, following Richard and Shu (2000), I apply the Wald F-test with the null hypothesis $H_0 \{\alpha = 0, \beta = 1\}$. The results are given in Table 4.1 with three different periods - before the financial crisis, during the financial crisis, and after the financial crisis. Following Trichet (2010), I set the financial crisis from 2007 to 2009. The financial crisis does not affect the relationship between the forward premium and the change of spot rate for these 12 exchanges in Table 4.1. Furthermore, it is clear that the value of beta is always close to zero with no significance, and the p-value of the F-test is zero in every case. The results indicate that the UIP cannot hold for these exchanges during these periods. Consequently, I find the reason for the failure of UIP by adding the influence from the interest rate in the model in the sections that follow.

I also calculate the annualized mean of the forward premium, rate of depreciation, and excess return for the same period. The excess return of the same exchange has different signs during different periods.

## 4.4 Burnside et al. (2009) Model

Burnside et al. (2009) model assumes that the spot exchange rate follows an exogenous stochastic process, while the forward rate is from the interaction among informed traders, uninformed traders, and market makers. Burnside et al. (2009) attempt to apply the adverse selection model to explain the forward premium puzzle. I apply the standard exchange data to estimate the parameters as shown in table 4.3. Firstly, I will introduce the model from Burnside et al. (2009).

The stochastic process for growth rate of the spot exchange rate was given by:

$$\frac{S_{t+1} - S_t}{S_t} = \phi_t + \varepsilon_{t+1} + \omega_{t+1}. \tag{4.7}$$

They let $S_t$ be the spot exchange rate expressed as foreign currency units (FCUs) per British pound FCU/USD.

The variable $\phi_t$ denotes the public information which represent the section predicted by the investor at time $t$. All traders could observe $\phi_t$ at the beginning of time $t$,. For simplicity they assumed that this variable is i.i.d. and obeys:

$$\phi_t = \begin{cases} \phi & \text{with probability } 1/2, \\ -\phi & \text{with probability } 1/2, \end{cases} \tag{4.8}$$

where $\phi > 0$.

The variable $\varepsilon_{t+1}$ is private information at time $t$, none of the agents could observe it directly. But only informed traders could receive advance signals about its value. This variable is i.i.d. and obeys:

$$\varepsilon_{t+1} = \begin{cases} \varepsilon & \text{with probability } 1/2, \\ -\varepsilon & \text{with probability } 1/2, \end{cases} \tag{4.9}$$

where $\varepsilon > 0$.

Finally, none of the agents in the model could observe the value of $\omega_{t+1}$at time $t$. The presence of this shock allows the model to generate an exchange rate volatility that is not tied to either private or public information. The variable $\omega_{t+1}$ is i.i.d., mean zero, and has variance $\sigma_\omega^2$. The three information $\phi_t$, $\varepsilon_{t+1}$, and $\omega_{t+1}$ are mutually orthogonal in the model.

If the market maker was selling the pound forward then his profit (in FCUs), and if $\phi_t = \phi$, the market maker's profit from selling one pound forward, $\pi_{t+1}^m$, is:

$$\pi_{t+1}^m = F_t^a(\phi) - S_{t+1}. \tag{4.10}$$

Here, $\pi_{t+1}^m$ is denominated in FCUs. Since the market maker's expected profit is zero, it follows that:

$$E(\pi_{t+1}^m|\text{buy}, \phi) = F_t^a(\phi) - E(S_{t+1}|\text{buy}, \phi) = 0. \tag{4.11}$$

Using equation (4.7) I have:

$$F_t^a(\phi) = S_t\left[1 + \phi + E(\varepsilon_{t+1}|\text{buy}, \phi)\right]. \tag{4.12}$$

Following the Bayesian rule, I evaluate the expectation of the market maker of $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right)(-\varepsilon) \tag{4.13}$$

The function given below is implied in the Bayesian rule:

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \phi\right)} \tag{4.14}$$

When they compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right)$, they need to consider the informed and uninformed traders separately. When $\phi_t = \phi$, uninformed traders would buy the pound forward. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) = 1 - \alpha + \alpha q \tag{4.15}$$

$$Pr\left(buy \mid \phi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.16}$$

They also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right)$ in a similar way, and it follows that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) = 1 - \alpha + \alpha\left(1 - q\right) \tag{4.17}$$

They use equations 4.15, 4.16 and 4.17 to get the equation below:

$$Pr\left(buy \mid \phi\right) = \left(1 - \alpha + \alpha q\right)\frac{1}{2} + \left[1 - \alpha + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.18}$$

Equations 4.15, 4.18 and 4.14 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{1 - \alpha\left(1 - q\right)}{2 - \alpha} \tag{4.19}$$

Since

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = 1 - Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) \tag{4.20}$$

They have

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = \frac{1 - \alpha q}{2 - \alpha} \tag{4.21}$$

By substituting equations 4.19, 4.21 and 4.13, I obtain

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon \tag{4.22}$$

They obtain from the equation above that

$$F_t^a\left(\phi\right) = S_t\left[1 + \phi + \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon\right] \tag{4.23}$$

Hence, this is the ask forward rate with positive public information.$F_t^a\left(\phi\right)$ would be influenced by the value of $\phi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the other three situations as given below[3]:

[3]The complete derivative process is shown in the Appendix. C.

$$
\begin{cases}
F_t^a(\phi_t) = \begin{cases} S_t\left[1+\phi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\ \phi_t=\phi, \\ S_t\left[1-\phi+(2q-1)\varepsilon\right] & if\ \phi_t=-\phi, \end{cases} \\
F_t^b(\phi_t) = \begin{cases} S_t\left[1+\phi-(2q-1)\varepsilon\right] & if\ \phi_t=\phi, \\ S_t\left[1-\phi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\ \phi_t=-\phi. \end{cases}
\end{cases}
\tag{4.24}
$$

[Table 4.3GMM model-1 results with normal forex data is about here]

Table 4.3 shows the GMM model estimations with the five moments (model-1). [4] The J-test examines the moments of the GMM models' reasonability. Because the P-values of the J test are all larger than 10%, I indicate that the moments in this model are all appropriate. The public information parameter $\varphi$, private information parameter $\varepsilon$, and probability $q$ are all significant in each currency. $\alpha$ also has a low standard error with several exchanges with no significance. The value of the public information parameter $\varphi$ is much lower than that of the private information parameter $\varepsilon$, which indicates that public information has lesser effects on the exchange rate. Although the model has a good estimation, the value of $\alpha$ is still much higher than the assumption with almost 30 estimations. However, I cannot add more parameters and comments in this situation because of the lack of data. Hence, I use the unique order flow data, which includes four different kinds of investors' order flows of the spot exchange rate. Since the data set is the order flows of the spot exchange rate. I need to switch the model computing assumption, which will be discussed in the next section.

## 4.5 Exchange Rate in the Forward and Spot Market(switch method)

In this section, I introduce the switch method. The motivation to apply the spot rate order flow data lacks data to estimate the microstructure model. The switch model could help me to apply the spot rate order flow data logically. The main difference between the basic model and switch method is that I first consider the expected value of the spot rate at term $t+1$ instead of the forward rate.

Given the setup from Burnside et al. (2009), I thought about how the forward rate should be determined. I thought of traders arriving in the forward market and making transactions with a market maker. If the trader wanted to buy (sell) a pound forward, the market maker would take the opposite position by selling (buying) a pound forward. The basic idea in the model was to have the market maker be risk-neutral and set the forward rates (the ask and the bid rate) so that the expected profit implicit in the market maker's position should be zero. This occurs when the forward rates for the market maker selling (buying) at time $t$ is equal to the spot rates market maker buying (selling) at time $t+1$[5]. Hence, I can switch from setting the forward rates (the ask

[4] The GMM model has been introduced in appendix.B.

[5] The equation4.26 illustrates this assumption

and the bid rate) directly to estimating the expectation of spot rates at time $t+1$ in the spot market. I assume that the market maker could also observe the order flow in the spot market at time $t$. I apply the order flow of the spot rates to forecast the spot rates at time $t+1$. Then, the spot rate order flow data could be applied in estimating the parameters. The model looks similar to the Burnside et al. (2009) model.

If the market maker was selling the pound forward then his profit (in FCUs), and if $\phi_t = \phi$, the market maker's profit from selling one pound forward, $\pi^m_{t+1}$, is:

$$\pi^m_{t+1} = F^a_t(\phi) - S_{t+1}. \tag{4.25}$$

Here, $\pi^m_{t+1}$ is denominated in FCUs. Since the market maker's expected profit is zero, it follows that:

$$E(\pi^m_{t+1}|\text{buy}, \phi) = F^a_t(\phi) - E(S_{t+1}|\text{buy}, \phi) = 0. \tag{4.26}$$

Using equation 4.7 I have:

$$F^a_t(\phi) = S_t\left[1 + \phi + E(\varepsilon_{t+1}|\text{buy}, \phi)\right]. \tag{4.27}$$

Note that I apply the expected value of $S_{t+1}$ which is in the spot market, to estimate the value of $F^a_t(\phi)$ which is in the forward market. I have the order flow data for the spot market, which could help me in estimating the expected value of $S_{t+1}$, since the value of $S_{t+1}$ could be affected by the spot market order flow. As I discussed earlier, the spot rate would follow the exogenous stochastic process. The value of $\varepsilon_{t+1}, \omega_{t+1}$ cannot be observed by any participant in the spot market. Only informed traders receive advance signals about $\varepsilon_{t+1}$ value. Hence, I apply the order flow of the spot rate market to estimate the value of the expectation of $S_{t+1}$, which is consequently equal to the value of $F^a_t(\phi)$.

Following the Bayesian rule, I evaluate the expectation of the market maker of $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right)(-\varepsilon) \tag{4.28}$$

The function given below is implied in the Bayesian rule:

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \phi\right)} \tag{4.29}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right)$, I need to consider the informed and uninformed traders separately. When $\phi_t = \phi$, uninformed traders would buy the pound forward. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) = 1 - \alpha + \alpha q \tag{4.30}$$

$$Pr\left(buy \mid \phi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.31}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right)$ in a similar way, and it follows that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) = 1 - \alpha + \alpha\left(1 - q\right) \tag{4.32}$$

I use equations 4.30, 4.31 and 4.32 to get the equation below:

$$Pr\left(buy \mid \phi\right) = \left(1 - \alpha + \alpha q\right)\frac{1}{2} + \left[1 - \alpha + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.33}$$

Equations 4.30, 4.33 and 4.29 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{1 - \alpha\left(1 - q\right)}{2 - \alpha} \tag{4.34}$$

Since

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = 1 - Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) \tag{4.35}$$

I have

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = \frac{1 - \alpha q}{2 - \alpha} \tag{4.36}$$

By substituting equations 4.34, 4.36 and 4.28, I obtain

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon \tag{4.37}$$

I obtain from the equation above that

$$F_t^a\left(\phi\right) = S_t\left[1 + \phi + \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon\right] \tag{4.38}$$

Hence, this is the ask forward rate with positive public information.$F_t^a\left(\phi\right)$ would be influenced by the value of $\phi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the other three situations as given below[6]:

$$\begin{cases} F_t^a(\phi_t) = \begin{cases} S_t\left[1+\phi+(2q-1)\varepsilon\alpha/(2-\alpha)\right] & if\ \phi_t=\phi, \\ S_t\left[1-\phi+(2q-1)\varepsilon\right] & if\ \phi_t=-\phi, \end{cases} \\ F_t^b(\phi_t) = \begin{cases} S_t\left[1+\phi-(2q-1)\varepsilon\right] & if\ \phi_t=\phi, \\ S_t\left[1-\phi-(2q-1)\varepsilon\alpha/(2-\alpha)\right] & if\ \phi_t=-\phi. \end{cases} \end{cases} \tag{4.39}$$

Substituting equations from 4.39, I could obtain

$$\ln\left[F_t^a(\phi_t)/F_t^b(\phi_t)\right] \cong \frac{2}{2-\alpha}(2q-1)\varepsilon \tag{4.40}$$

This equation indicates that the bid-ask spread independent of the public information $\phi_t$.

When I consider the regression, like given below, I get the equation as

$$\frac{S_{t+1}-S_t}{S_t} = a+\beta\frac{F_t-S_t}{S_t}+\xi_{t+1} \tag{4.41}$$

Let $\frac{S_{t+1}-S_t}{S_t} = \delta_{t+1} = \phi_t+\varepsilon_{t+1}+\omega_{t+1}$,and $\frac{F_t-S_t}{S_t} = f_t$.

the slope $\beta$ could also be computed as follows:

$$plim\hat{\beta} = \frac{cov(\delta_{t+1}, f_t)}{var(f_t)} \tag{4.42}$$

I rewrite the model of $f_t$ as

$$f_t \;=\; \begin{cases} \phi-(2q-1)(1-\alpha)/(2-\alpha)\varepsilon & if \phi_t=\phi \\ -\phi+(2q-1)(1-\alpha)/(2-\alpha)\varepsilon & if \phi_t=-\phi \end{cases} \tag{4.43}$$

then I could compute the variance and covariance of $\delta_{t+1}$ and $f_t$ , and let:

$$\theta = (2q-1)(1-\alpha)/(2-\alpha) \tag{4.44}$$

[6]The complete derivative process is shown in the Appendix. C.

$$var\left(f_t\right)=\frac{1}{2}\left(\phi-\theta\varepsilon\right)^2+\frac{1}{2}\left(-\phi+\theta\varepsilon\right)^2=\left(\phi-\theta\varepsilon\right)^2 \tag{4.45}$$

$$cov\left(\delta_{t+1},f_t\right)=\frac{1}{2}\phi\left(\phi-\theta\varepsilon\right)+\frac{1}{2}\left(-\phi\right)\left(-\phi+\theta\varepsilon\right)=\phi\left(\phi-\theta\varepsilon\right) \tag{4.46}$$

I then get

$$plin\hat{\beta}=\frac{\phi}{\phi-\theta\varepsilon} \tag{4.47}$$

and consequently,

$$plim\hat{\beta}=\frac{\phi}{\phi-\left(1-\alpha\right)\left(2q-1\right)\varepsilon/\left(2-\alpha\right)} \tag{4.48}$$

If

$$\phi<\frac{1-\alpha}{2-\alpha}\left(2q-1\right)\varepsilon \tag{4.49}$$

then $plim\hat{\beta}<0$. I show the estimates of the slope of different currencies from the results of GMM estimation later.

From equations 4.39, I could obtain

$$\frac{F_t^a\left(\phi_t\right)-S_t}{S_t}=\frac{F_t^b\left(\phi_t\right)-S_t}{S_t}+\frac{2}{2-\alpha}\left(2q-1\right)\varepsilon \tag{4.50}$$

It is clear that whether I use ask, bid, or the average forward rate in the regression, the effect would be only on the intercept $\alpha$.

When I set the parameter $q=\frac{1}{2}$(all the traders are uninformed), the ask and bid forward rate would be the same, then

$$\frac{F_t^a\left(\phi_t\right)-S_t}{S_t}=\frac{F_t^b\left(\phi_t\right)-S_t}{S_t}=\phi_t \tag{4.51}$$

When I set the $\alpha=1$ (all traders are informed), the forward premium would be

$$\frac{F_t^a(\phi_t) - S_t}{S_t} = \phi_t + (2q - 1)\,\varepsilon, \tag{4.52}$$

$$\frac{F_t^b(\phi_t) - S_t}{S_t} = \phi_t - (2q - 1)\,\varepsilon. \tag{4.53}$$

When I set $q = 1, \varepsilon_{t+1} = \varepsilon, \phi_t = -\phi$, the ask forward rate would be

$$F_t^a(-\phi) = S_t(1 - \phi + \varepsilon) \tag{4.54}$$

and I obtain the following from the equation, 4.37 with $q = 1, \varepsilon_{t+1} = \varepsilon$

$$E(\varepsilon_{t+1} \mid buy, \phi) = \frac{\alpha}{2-\alpha}\varepsilon < \varepsilon \tag{4.55}$$

Consequently, the forward rate would be

$$F_t^a(\phi) = S_t\left(1 + \phi + \frac{\alpha}{2-\alpha}\varepsilon\right) \tag{4.56}$$

Comparing equation 4.53, 4.55 and 4.49, I have

$$F_t^a(\phi) < F_t^a(-\phi) \tag{4.57}$$

The forward market rate would follow the $plim\hat{\beta} < 0$

Basically, the rest of the paper works out convenient expressions for $F_t^a(\phi)$, $F_t^b(\phi)$, $F_t^a(-\phi)$, $F_t^b(-\phi)$ by evaluating objects like $E(\varepsilon_{t+1}|\text{buy}, \phi)$ given the differing information sets of traders and market makers, and whether the transaction was a buy or sell of the pound.

[Table 4.4 GMM model-2 is about here]

[Table 4.5 GMM model-3 is about here]

Tables 4.4 and 4.5 show the results of the estimation of the parameters, while the estimations are appropriate with a good p-value of the J-test. Table 4.4 (model-2) is the result of the basic model of Burnside et al. (2009), which is the same as model-1.

The parameters are almost significant.[7] The public information parameter $\phi$ has a low value compared to the private information parameter $\varepsilon$. The correct probability of informed traders $q$ is always higher than 50%, consistent with my assumption. The informed trader $\alpha$ remains at a low value indicating that informed traders have a low proportion in the market, which has a much lower and more reasonable value than model-1. Table 4.5 (model-3) shows the model with a new parameter $v$. [8] The results show that most uninformed traders choose to believe public information. The estimates of parameter $v$ range from 71% to 100%, which is consistent with the assumption.

[Table 4.10 The slope and profit of the traders is about here]

[Table 4.24 Characteristics of the data is about here]

[Table 4.10 The slope and profit of the traders is about here]

Table 4.10 gives the slope and expected returns of informed traders. When the slope is negative, the forward premium puzzle will exist. Negative slopes from the regression in Table 4.24 are EURUSD, USDJPY, USDCHF, EURCHF, EURSEK, and EURNOK. I can see from the slope that estimates are negative for currencies for the basic model are EURUSD, USDJPY, EURJPY, USDCHF and USDCAD. All these currencies have positive informed trader's returns, which could explain the forward premium puzzle. In the next section, I consider the relationship between the spot exchange market and the bond market.

## 4.6 The Spot Market and Bond Market(Add UIP in the Model)

Burnside et al. (2009) mentioned the interest rate effect in their microstructure model using public information. They argue the interest rate could be included in the public information, which could be observed by all participants in the market. However, I think the interest rate should be discussed separately since the UIP and CIP assumption. Hence, I add the bond market in the microstructure model to discuss the relationship between those three markets (forward market, spot market, and bond market).

When considering the spot market and bond market, I still set the exchange rate as FCU/USD. The stochastic process for the growth rate of the spot exchange rate would be changed to:

$$S_{t+1} = S_t \frac{1 + i_t}{1 + i_t^*} \left(1 + \varphi_t + \varepsilon_{t+1} + \omega_{t+1}\right). \qquad (4.58)$$

[7]The detailed model and the moments of the GMM methods have been given in appendix. E.

[8]The derivation of the model with the parameter $v$ has been given in the appendix.D.

Note that equation 4.58 is different from the assumption in section4.5. I consider the UIP when setting the stochastic process.

The variable $\varphi_t$ represents the change in the exchange Rate, that is predictable on the basis of time $t$ public information. However, the difference between the $\varphi_t$ and $\phi_t$ is the $\varphi_t$ excludes the effect from the interest rate. From the UIP, I strip the interest influence from the $\phi_t$, which means the $\varphi_t$ denotes public information without the interest rate effect. Hence, the $\phi > \varphi$. At the beginning of time $t$, all traders observe $\varphi_t$. For simplicity I assumed that this variable is i.i.d., and obeys:

$$\varphi_t = \begin{cases} \varphi & \text{with probability } 1/2, \\ -\varphi & \text{with probability } 1/2, \end{cases} \tag{4.59}$$

where $\varphi > 0$.

Then, let me consider the UIP in traders' transactions. Assume the foreign country has a higher interest rate than the US interest rate. Then, the trader wants to exercise the carry trade strategy, between the foreign country and the US, in the spot exchange rate market and bond market. Imagine that at time $t$ the traders have $S_t/(1+i_t)$ USD which would be worth $S_t$ USD in the US bond market at time $t+1$. To get a higher interest rate in the foreign bond market, he could also convert it to $1/(1+i_t)$ FCUs in the spot market by using USD to buy FCU at spot rate $S_t$, and then earn at the interest rate $i_t^*$ in the foreign bond market. This means the traders have $\frac{1+i_t^*}{1+i_t}$ FCUs at $t+1$. Hence, the terminated value of the trader in the foreign bond market at time $t+1$ is $S_{t+1}\frac{1+i_t^*}{1+i_t}$ USD, when he sells the FCUs to USDs. According to the UIP, the trader should have the same value with $S_t$ and $S_{t+1}\frac{1+i_t^*}{1+i_t}$ at time $t+1$, and the profit from carry trade in this circumstance should be zero:

$$\tilde{\pi}_{t+1}^m = S_{t+1}\frac{1+i_t^*}{1+i_t} - S_t^a \tag{4.60}$$

I then consider the market maker who should enter in the opposite party, in this circumstance. Hence the profit of market maker should be equal to:

$$\tilde{\pi}_{t+1}^m = S_t^a - S_{t+1}\frac{1+i_t^*}{1+i_t} \tag{4.61}$$

But notice that if I modified equation 4.58 to say that

$$S_{t+1} = S_t\frac{1+i_t}{1+i_t^*}\left(1+\varphi_t+\varepsilon_{t+1}+\omega_{t+1}\right) \tag{4.62}$$

then you would have just an equally useful expression. I would then have

$$\tilde{\pi}_{t+1}^m = S_t^a - S_t\left(1+\varphi_t+\varepsilon_{t+1}+\omega_{t+1}\right) \tag{4.63}$$

If UIP hold, the expected value of $\tilde{\pi}_{t+1}^m$ should be zero. The effect of the interest rate would have been counteracted. The ask exchange spot rate, then, should be equal to

$$S_t^a = S_t\left(1 + \varphi_t + E\left(\varepsilon_{t+1}\right)\right) \tag{4.64}$$

The difference between $S_t^a$ and $F_t^a$ is the difference between $\varphi_t$ and $\phi_t$.

No agents could observe the fact $\varepsilon_{t+1}$, however, public information $\varphi_t$ is available for all participants in the market. Hence I have 4 different spot rate $S_t^a(\varphi)$, $S_t^a(-\varphi)$, $S_t^b(\varphi)$, and $S_t^b(-\varphi)$.

When $\varphi_t = \varphi$, the market maker would get the profit from selling one pound spot, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m = S_t^a(\varphi) - S_t\left(1 + \varphi_t + \varepsilon_{t+1} + \omega_{t+1}\right) \tag{4.65}$$

The expected profit of the market maker should be zero, hence

$$E\left(\pi_{t+1}^m \mid buy, \varphi\right) = S_t^a(\varphi) - E\left(S_{t+1} \mid buy, \varphi\right) = 0 \tag{4.66}$$

By applying the equation above, I get the equation given below

$$S_t^a\left(\varphi\right) = S_t\left[1 + \varphi + E\left(\varepsilon_{t+1} \mid buy, \varphi\right)\right] \tag{4.67}$$

Following the Bayesian rule, I evaluate the expectations of the market maker from $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right)(-\varepsilon) \tag{4.68}$$

The function given below is implied in the Bayesian rule:

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \varphi\right)} \tag{4.69}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right)$, I need to consider informed and uninformed traders separately. When $\varphi_t = \varphi$, uninformed traders would buy the pound spot. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound spot with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) = 1 - \alpha + \alpha q \tag{4.70}$$

$$Pr\left(buy \mid \varphi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.71}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right)$ by the similar way, and it follow that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) = 1 - \alpha + \alpha\left(1 - q\right) \tag{4.72}$$

I use equations 4.70, 4.71 and 4.72 to get the equation below:

$$Pr\left(buy \mid \varphi\right) = \left(1 - \alpha + \alpha q\right)\frac{1}{2} + \left[1 - \alpha + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.73}$$

Equations 4.70, 4.73 and 4.69 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{1 - \alpha\left(1 - q\right)}{2 - \alpha} \tag{4.74}$$

Since

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right) = 1 - Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) \tag{4.75}$$

I have

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right) = \frac{1 - \alpha q}{2 - \alpha} \tag{4.76}$$

By substituting equations 4.74, 4.76 and 4.68, I obtain

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon \tag{4.77}$$

I obtain from equation4.67

$$S_t^a\left(\varphi\right) = S_t\left[1 + \varphi + \frac{\alpha}{2 - \alpha}(2q - 1)\varepsilon\right] \tag{4.78}$$

Hence, this is the ask spot rate with positive public information. $S_t^a(\varphi)$ would be influenced by the value of $\varphi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the other three situations as below:

$$\begin{cases} S_t^a(\varphi_t) = \begin{cases} S_t\left[1+\varphi+(2q-1)\varepsilon\alpha/(2-\alpha)\right] & if\,\varphi_t=\varphi, \\ S_t\left[1-\varphi+(2q-1)\varepsilon\right] & if\,\varphi_t=-\varphi, \end{cases} \\ S_t^b(\varphi_t) = \begin{cases} S_t\left[1+\varphi-(2q-1)\varepsilon\right] & if\,\varphi_t=\varphi, \\ S_t\left[1-\varphi-(2q-1)\varepsilon\alpha/(2-\alpha)\right] & if\,\varphi_t=-\varphi. \end{cases} \end{cases} \quad (4.79)$$

Since I consider the effects of interest rate on the bond market and spot market transaction, I have separated the interest rate information from public information. Because I have already considered the impact from the interest rate to the spot exchange rate, I do not consider the order flow affected by the interest rate information. The UIP cannot be found because informed traders get a positive profit from the UIP transaction.

If the UIP holds between the bond market and spot market, no one could get a positive profit through carry trade, and the expected profit should be equal to zero.

When I found that the interest rate effect has been counteracted, I realized that the interest rate mainly influences the exchange rate of the forward and term t+1 spot rate, which is consistent with the UIP and CIP. Furthermore, the order flow is also a factor, which has been proven in the literature.

[Table 4.6 GMM model-4 is about here]

[Table 4.7 GMM model-5 is about here]

Tables 4.6(model-4) and 4.7(model-5) show the results of the estimation of the parameters. The public information parameter $\varphi$ without interest rate information is much lower than the public information parameter $\phi$. For instance, the parameter $\phi$ of EURUSD in model-2 and model-3 is 0.0003, while the parameter $\varphi$ is 0.0001. This is because I got rid of this effect of the interest rate. These results could indicate that interest rate information plays an important role in public information.

### 4.6.1 Overestimate of the Effect of the Uninformed Traders

Let me consider a stricter circumstance. According to the UIP and CIP, I know that the interest rate greatly influences the exchange rate. The interest rate is also known as public information, which can be observed at time t. Hence the interest rate information is a part of the public information $\phi$. When I add the UIP in the stochastic process for the growth rate of the spot exchange rate, I need to eliminate the effect of interest information from public information. Then, I have a new public information parameter $\varphi$. However, I ignore the uninformed traders, who follow the interest rate and whose order flow has already been reflected in the UIP. Hence, I add a new parameter $\varrho$ in this section to get rid of the order flow of the interest rate.

$$S_t^a(\varphi) = S_t\left[1 + \varphi + E\left(\varepsilon_{t+1} \mid buy, \varphi\right)\right] \tag{4.80}$$

Following the Bayesian rule, I evaluate the expectation of the market maker from $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right)(-\varepsilon) \tag{4.81}$$

The function given below is implied in the Bayesian rule:

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \varphi\right)} \tag{4.82}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right)$, I need to consider the informed and uninformed traders separately. When $\varphi_t = \varphi$, uninformed traders would buy the pound spot. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound spot with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$. I set $\varrho$ as the uninformed traders who follow the interest rate.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) = 1 - \alpha - \varrho + \alpha q \tag{4.83}$$

$$Pr\left(buy \mid \varphi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.84}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right)$ by the similar way, and it follow that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) = 1 - \alpha - \varrho + \alpha\left(1 - q\right) \tag{4.85}$$

I use equations 4.83, 4.84 and 4.85 to get the equation below:

$$Pr\left(buy \mid \varphi\right) = \left(1 - \alpha - \varrho + \alpha q\right)\frac{1}{2} + \left[1 - \alpha - \varrho + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \varrho - \frac{\alpha}{2} \tag{4.86}$$

Equations 4.83, 4.86 and 4.82 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{1 - \varrho - \alpha\left(1 - q\right)}{2 - 2\varrho - \alpha} \tag{4.87}$$

Since

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right) = 1 - Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) \tag{4.88}$$

I have

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right) = \frac{1 - \varrho - \alpha q}{2 - 2\varrho - \alpha} \tag{4.89}$$

By substituting equations 4.87, 4.89 and 4.81, I obtain

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = \frac{\alpha}{2 - 2\varrho - \alpha}(2q - 1)\varepsilon \tag{4.90}$$

I obtain from equation 4.80

$$S_t^a\left(\varphi\right) = S_t\left[1 + \varphi + \frac{\alpha}{2 - 2\varrho - \alpha}(2q - 1)\varepsilon\right] \tag{4.91}$$

Hence, this is the ask spot rate with positive public information. $S_t^a\left(\varphi\right)$ would be influenced by the value of $\varphi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the other three situations as shown below:

$$\begin{cases} S_t^a\left(\varphi_t\right) = \begin{cases} S_t\left[1 + \varphi + (2q - 1)\varepsilon\alpha/\left(2 - 2\varrho - \alpha\right)\right] & if\, \varphi_t = \varphi, \\ S_t\left[1 - \varphi + (2q - 1)\varepsilon\right] & if\, \varphi_t = -\varphi, \end{cases} \\ S_t^b\left(\varphi_t\right) = \begin{cases} S_t\left[1 + \varphi - (2q - 1)\varepsilon\right] & if\, \varphi_t = \varphi, \\ S_t\left[1 - \varphi - (2q - 1)\varepsilon\alpha/\left(2 - 2\varrho - \alpha\right)\right] & if\, \varphi_t = -\varphi. \end{cases} \end{cases} \tag{4.92}$$

[Table 4.8 GMM model-6 is about here]

Table 4.8 shows the results of the estimation of the parameters. The parameter $\varrho$ has an extensive range from 0.1530 to 0.9372, with non-significant. It seems that the setting of moments has some problems. Hence, I discuss the parameter $\varrho$ later in model-7, which is estimated in the next subsection.

[Table 4.9 Comparison of public information in different models is about here]

Table 4.9 (model-6) illustrates the comparative difference in public information parameters between the three models. It is not surprising to see that the public information parameter has the lowest value by getting rid of the full effect of the interest rate.

$$\frac{S_t^a(\varphi_t) - S_t}{S_t} = \frac{S_t^b(\varphi_t) - S_t}{S_t} + \frac{2 - 2\varrho}{2 - 2\varrho - \alpha}(2q-1)\varepsilon \tag{4.93}$$

$$\ln\left[S_t^a(\varphi_t)/S_t^b(\varphi_t)\right] \cong \frac{2}{2 - 2\varrho - \alpha}(2q-1)\varepsilon \tag{4.94}$$

I can see that the bid-ask spread of the spot rate is not affected by public information since all the participants in the market know this. The higher the proportion of informed traders, the more correct probability for the signal and the more important private information that would increase the bid-ask spread. If more uninformed traders follow the interest rate information, the bid-ask spread will increase. In the next subsection, I consider the three markets together.

### 4.6.2 Discussion of the Spot Rate and Forward Rate

If the CIP holds, equation 4.61 and 4.25 will help me to obtain:

$$S_{t+1} = S_t^a(\varphi_t)\frac{1+i_t}{1+i_t^*} = F_t^a(\phi_t) \tag{4.95}$$

Then, I get the equations below:

$$\begin{array}{c}
\frac{1+i_t}{1+i_t^*}S_t\left[1+\varphi+(2q-1)\varepsilon\alpha/\left(2-2\varrho-\alpha\right)\right] = S_t\left[1+\phi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right]\ if\ \varphi_t=\varphi, \phi_t=\phi \\
\frac{1+i_t}{1+i_t^*}S_t\left[1-\varphi+(2q-1)\varepsilon\right] = S_t\left[1-\phi+(2q-1)\varepsilon\right]\ if\ \varphi_t=-\varphi, \phi_t=-\phi \\
\frac{1+i_t}{1+i_t^*}S_t\left[1+\varphi-(2q-1)\varepsilon\right] = S_t\left[1-\phi+(2q-1)\varepsilon\right]\ if\ \varphi_t=\varphi, \phi_t=\phi \\
\frac{1+i_t}{1+i_t^*}S_t\left[1-\varphi-(2q-1)\varepsilon\alpha/\left(2-2\varrho-\alpha\right)\right] = S_t\left[1-\phi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right]\ if\ \varphi_t=-\varphi, \phi_t=-\phi
\end{array} \tag{4.96}$$

I eliminate $S_t$ for both side, and get:

$$\frac{1+i_t}{1+i_t^*}\left[1+\varphi+(2q-1)\varepsilon\alpha/\left(2-2\varrho-\alpha\right)\right]=\left[1+\phi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right]\ if\ \varphi_t=\varphi,\phi_t=\phi$$
$$\frac{1+i_t}{1+i_t^*}\left[1-\varphi+(2q-1)\varepsilon\right]=\left[1-\phi+(2q-1)\varepsilon\right]\ if\ \varphi_t=-\varphi,\phi_t=-\phi$$
$$\frac{1+i_t}{1+i_t^*}\left[1+\varphi-(2q-1)\varepsilon\right]=\left[1-\phi+(2q-1)\varepsilon\right]\ if\ \varphi_t=\varphi,\phi_t=\phi$$
$$\frac{1+i_t}{1+i_t^*}\left[1-\varphi-(2q-1)\varepsilon\alpha/\left(2-2\varrho-\alpha\right)\right]=\left[1-\phi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right]\ if\ \varphi_t=-\varphi,\phi_t=-\phi \tag{4.97}$$

[Table 4.11 GMM model-7 is about here]

I found that the parameter $q, \alpha, \varepsilon$ should be the same in sections 4.5 and 4.6.1 through the equation 4.97. Then, I estimate the moments in those two sections together to see the precise value of two different public information $\phi$ and $\varphi$. Table 4.11(model-7) shows the results of the estimates. I see that the $\phi$ is still higher than the $\varphi$. It is interesting to estimate all the moments among the three different markets together. The results show that the moments are more appropriate than model-6. The value of the parameter $\varrho$ will be more stable and significant with a value of around 20%. It indicates that around 20% of uninformed traders follow public information. The parameter $\varphi$ is always smaller than the parameter $\phi$, which is evidence for the effect from the interest rate.

Once equation 4.95 holds, the difference between $S_t^a(\varphi_t)$ and $F_t^a(\phi_t)$ is the interest rate. From this, I can indicate that the exchange rate could be affected by both interest rate and order flow. In the next section, I apply the inverse method to estimate the models. The CIP and UIP should hold when the exchange rates are adjusted by the order flow.

## 4.7 Apply the Forward Rate as the Exogenous Variable(inverse method)

In this section, I try to apply the order flow data to estimate the adverse selection model as discussed in section 4.4, the basic model and data cannot explain the forward premium puzzle well. Hence, I apply the unique order flow data to estimate the model with more parameters with the inverse method. However, the order flow data is for the spot exchange rate instead of the forward exchange rate, which needs adjustments to fit the model. I then inverse the position for term $t$ forward rate $F_t$ and term $t+1$ spot rate $S_{t+1}$, because the order flow data is for the spot exchange rate.

$$\frac{F_t - S_t}{S_t} = \varphi_t + \varepsilon_{t+1} + \omega_{t+1} \tag{4.98}$$

where $\varphi_t$ is the public information influence at term t, which could be observed by all traders. I assume the influence would be positive or negative in the same probability.

$$\varphi_t = \begin{cases} \varphi \, with \, probability & 1/2, \\ -\varphi \, with \, probability & 1/2. \end{cases} \tag{4.99}$$

where $\varepsilon_{t+1}$ is not observed directly at time t, which can be observed by informed investors as a signal $\zeta_t \in \{\varepsilon, -\varepsilon\}$.

$$\varepsilon_{t+1} = \begin{cases} \varepsilon \, with \, probability & 1/2, \\ -\varepsilon \, with \, probability & 1/2. \end{cases} \tag{4.100}$$

the value of the influence from public and private information $(\varphi, \varepsilon)$ are both positive. Finally, $\omega_{t+1}$ denotes the information which no agents in the market would observe, while variable $\omega_{t+1}$ independently and identically follow the normal distribution with a mean zero and variance $\sigma_\omega^2$. The three information parameters $(\varphi_t, \varepsilon_{t+1}, \omega_{t+1})$ in the model are mutually orthogonal.

At the beginning of term $t$, informed traders could observe the private information signal $\zeta_t$ which could be positive or negative. The probability for informed traders to get the right private information signal is $q$. Hence, I have the function given below:

$$Pr\left(\zeta_t = \varepsilon \mid \varepsilon_{t+1} = \varepsilon\right) = Pr\left(\zeta_t = -\varepsilon \mid \varepsilon_{t+1} = -\varepsilon\right) = q > \frac{1}{2} \tag{4.101}$$

I assume the informed traders could use a better technique to get $\zeta_t$ or use a unique method to access private information signal with the correct probability $q$, which is higher than half.

No agents could observe the fact $\varepsilon_{t+1}$, however, public information $\varphi_t$ is available for all participants in the market. Hence I could have 4 different spot rate $S^a_{t+1}(\varphi)$, $S^a_{t+1}(-\varphi)$, $S^b_{t+1}(\varphi)$, and $S^b_{t+1}(-\varphi)$.

When $\varphi_t = \varphi$, the profit the market maker would get from selling one pound spot, $\pi^m_{t+1}$, is

$$\pi^m_{t+1} = S^a_{t+1}(\varphi) - F_t \tag{4.102}$$

The expected profit of the market maker should be zero, hence

$$E\left(\pi^m_{t+1} \mid buy, \varphi\right) = S^a_{t+1}(\varphi) - E\left(F_t \mid buy, \varphi\right) = 0 \tag{4.103}$$

By applying equation 4.98, I obtain the following equation:

$$S^a_{t+1}(\varphi) = S_t\left[1 + \varphi + E\left(\varepsilon_{t+1} \mid buy, \varphi\right)\right] \tag{4.104}$$

Following the Bayesian rule, I evaluate the expectations of the market maker of $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right)(-\varepsilon) \tag{4.105}$$

The function given below is implied in the Bayesian rule,

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \varphi\right)} \tag{4.106}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right)$, I need to consider the informed and uninformed traders separately. When $\varphi_t = \varphi$, uninformed traders would buy the pound spot. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound spot with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) = 1 - \alpha + \alpha q \tag{4.107}$$

$$Pr\left(buy \mid \varphi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.108}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right)$ by the similar way, and it follow that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi\right) = 1 - \alpha + \alpha\left(1 - q\right) \tag{4.109}$$

I use equations 4.107, 4.108 and 4.109 to obtain the equation below:

$$Pr\left(buy \mid \varphi\right) = \left(1 - \alpha + \alpha q\right)\frac{1}{2} + \left[1 - \alpha + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.110}$$

Equations 4.107, 4.110 and 4.106 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{1 - \alpha\left(1 - q\right)}{2 - \alpha} \tag{4.111}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\varphi\right) \tag{4.112}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi\right)=\frac{1-\alpha q}{2-\alpha} \tag{4.113}$$

By substituting equations 4.111, 4.113 and 4.105, I obtain

$$E\left(\varepsilon_{t+1} \mid buy,\varphi\right)=\frac{\alpha}{2-\alpha}(2q-1)\varepsilon \tag{4.114}$$

I obtain from equation 4.104

$$S_{t+1}^{a}\left(\varphi\right)=S_t\left[1+\varphi+\frac{\alpha}{2-\alpha}(2q-1)\varepsilon\right] \tag{4.115}$$

Hence, this is the ask spot rate with positive public information. $S_t^a\left(\varphi\right)$ would be influenced by the value of $\varphi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar ways, I could deduce the other three situations as below

$$\begin{cases} S_{t+1}^{a}\left(\varphi_t\right)=\begin{cases} S_t\left[1+\varphi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\,\varphi_t=\varphi, \\ S_t\left[1-\varphi+(2q-1)\varepsilon\right] & if\,\varphi_t=-\varphi, \end{cases} \\ S_{t+1}^{b}\left(\varphi_t\right)=\begin{cases} S_t\left[1+\varphi-(2q-1)\varepsilon\right] & if\,\varphi_t=\varphi, \\ S_t\left[1-\varphi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\,\varphi_t=-\varphi. \end{cases} \end{cases} \tag{4.116}$$

Uninformed traders have the right to choose to follow public information or not, and I give the parameter $v$ to denote the proportion of uninformed traders.

When I compute the $Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\varphi\right)$, I need to consider the informed and uninformed traders separately. When $\varphi_t=\varphi$, uninformed traders would buy the pound spot. When $\varepsilon_{t+1}=\varepsilon$, informed traders would buy the pound spot with probability $q$, the signal $\zeta_t=\varepsilon=\varepsilon_{t+1}$. As I have said earlier, I assume asset managers and hedge funds are informed traders. However, asset managers tend to be risk-averse and hedge funds tend to be risk-seeking. When the informed traders receive the private information, they may not follow the signal $\zeta_t$ to invest since the risk for following $\zeta_t$ is high. Hence, I add a parameter $h$ to the model from (Burnside et al., 2009), which denotes the actual proportion of informed traders who choose to follow the private signal $\zeta_t$.

Although the asset manager and hedge fund are informed traders, many of them still do not have the skill to process the information to get the signal, and then choose whether to follow the public information or choose to believe neither the public information nor the signal they get.

With this new assumption, I obtain the new $plim\hat{\beta}$ as

$$plim\hat{\beta} = \frac{\varphi}{\varphi - \alpha\left(z-1\right)\left(2q*h-1\right)\varepsilon/z\left(2-z\right)} \tag{4.117}$$

where $z = 2v\left(1-\alpha\right)+\alpha$, $v$ is the probability of data are generated by a version of this model.

No agents could obverse the fact $\varepsilon_{t+1}$, however, public information $\varphi_t$ is available for all participants in the market. Hence I could have 4 different forward rate $F_t^a(\varphi)$, $F_t^a(-\varphi)$, $F_t^b(\varphi)$, and $F_t^b(-\varphi)$.

When $\varphi_t = \varphi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m = S_{t+1}^a\left(\varphi\right) - F_t \tag{4.118}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^m \mid buy, \varphi\right) = S_{t+1}^a(\varphi) - E\left(F_t \mid buy, \varphi\right) = 0 \tag{4.119}$$

By applying the equation 2.3, I obtain the below equation:

$$S_{t+1}^a\left(\varphi\right) = S_t\left[1+\varphi+E\left(\varepsilon_{t+1} \mid buy, \varphi\right)\right] \tag{4.120}$$

Following the Bayesian rule, I evaluate the expectations of the market maker of $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \varphi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right)\left(\varepsilon\right) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi\right)\left(-\varepsilon\right) \tag{4.121}$$

The following functions are implied by the Bayesian rule,

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \varphi\right)} \tag{4.122}$$

$$Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\varphi\right)=\left(1-\alpha\right)*v+\alpha hq \tag{4.123}$$

$$Pr\left(buy \mid \varphi\right)=Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\varphi\right)Pr\left(\varepsilon_{t+1}=\varepsilon\right)+Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\varphi\right)Pr\left(\varepsilon_{t+1}=-\varepsilon\right) \tag{4.124}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\varphi\right)$ by the similar way, and it follow that:

$$Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\varphi\right)=(1-\alpha)v+\alpha\left(1-hq\right) \tag{4.125}$$

I use equations 4.123, 4.124 and 4.125 to arrive at the equation below:

$$Pr\left(buy \mid \varphi\right)=v(1-\alpha)+\frac{\alpha}{2} \tag{4.126}$$

Equations 4.123, 4.126 and 4.122 imply that

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\varphi\right)=\frac{\left(1-\alpha\right)v+\alpha hq}{2v\left(1-\alpha\right)+\alpha} \tag{4.127}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\varphi\right) \tag{4.128}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi\right)=\frac{\left(1-\alpha\right)v+\alpha(1-hq)}{2v\left(1-\alpha\right)+\alpha} \tag{4.129}$$

By substituting equations 4.127, 4.129 and 4.121, I obtain

$$E\left(\varepsilon_{t+1} \mid buy,\varphi\right)=\frac{\left(2qh-1\right)\alpha\varepsilon}{2v\left(1-\alpha\right)+\alpha} \tag{4.130}$$

I obtain from equation 4.120

$$S^{a}_{t+1}\left(\varphi\right)=S_t\left[1+\varphi+\frac{\left(2qh-1\right)\alpha}{2v\left(1-\alpha\right)+\alpha}\varepsilon\right] \tag{4.131}$$

Hence, this is the ask spot rate with positive public information. $S^a_{t+1}(\varphi)$ would be influenced by the value of $\varphi$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the other three situations as given below:

$$\begin{cases} S^a_{t+1}(\varphi_t) = \begin{cases} S_t\left[1+\varphi+\frac{(2qh-1)\alpha}{2v(1-\alpha)+\alpha}\varepsilon\right] & if\ \varphi_t=\varphi, \\ S_t\left[1-\varphi+\frac{\alpha(2qh-1)}{2(1-\alpha)(1-v)+\alpha}\varepsilon\right] & if\ \varphi_t=-\varphi, \end{cases} \\ S^b_{t+1}(\varphi_t) = \begin{cases} S_t\left[1+\varphi-\frac{\alpha(2qh-1)}{2-2(1-\alpha h)v-\alpha}\varepsilon\right] & if\ \varphi_t=\varphi, \\ S_t\left[1-\varphi-\frac{\alpha(2qh-1)}{2v-(2v-1)\alpha}\varepsilon\right] & if\ \varphi_t=-\varphi. \end{cases} \end{cases} \tag{4.132}$$

To access the sign of public information $\varphi$, I sum up the net order flow of corporate and private investors to represent the sign of public information. I assume that the parameter $v > 0.5$, and the sign from the sum of corporate and private investors order flow should be the sign of public information. I assume that the net order flow from the sum of asset managers and hedged funds could represent the order flow from informed traders. Consequently, the proportion of informed traders $\alpha$ could be computed by using the net order flow from the sum of asset managers and hedged funds and the gross order flow. As for informed traders who would buy the pound forward with probability $q$, I apply the sign of net order flow from the sum of asset managers and hedged funds, compared to the spot rate change. If the signs are both positive, which means the informed traders successfully accessed the private information, I set the value to $q = 1$. Otherwise, I set it to $q = 0$. The estimate of $q$ should equal the mean value over the whole time series.

[Table 4.12 Inversion model-8 results with new parameter $v$ is about here]

[Table 4.13 Inversion model-9 results with new parameter $h$ is about here]

Hence, I have two models here, model-8 which only includes parameter $v$ and model-9 which, includes parameters $v, h$. Tables 4.12 and 4.13 below shows the GMM estimation for the parameters with standard error in the parenthesis. Model-8 has similar estimates as model-3 since they have the same parameters. The interesting finding is that the private information $\varepsilon$ of model-8 is much higher than model-3. In Table 4.13, parameter $h$ ranges from 0.7101 to 0.8872. Interestingly, the value of $h$ is still half the distance to 1, which means not all informed traders follow the private information signal.

### 4.7.1 Uncovered Interest Rate Parity in Inverse Model Application

In this section, I apply the UIP to improve the model. Interest rates play an essential role in the forex market. I separate the interest rate information from public information and add a new participant who only follows the interest rate information. I still focus on the explanation of the forward premium puzzle in this section. First, I assume the market only has one kind of investor called UIP arbitragers. The arbitragers would find arbitrage opportunities from the misprizing for spot rate and forward rate against the UIP. Hence, the UIP could be held in this situation. I assume that the forward exchange rate follows an exogenous stochastic process. Spot rates are determined by the interaction between competitive market makers, informed and uninformed traders. I get the function as:

$$\frac{F_t}{S_t} = 1 + i_t = \frac{1 + i_d}{1 + i_f} = 1 + \varphi \tag{4.133}$$

In this assumption, public information is the only information available to all investors, and all investors are rational arbitragers. Hence the ask and bid spot rate would be:

$$\begin{cases} S^a_{t+1}(\varphi_t) = & S_t[1+i] = S_t[1+\varphi] \\ S^b_{t+1}(\varphi_t) = & S_t[1-i] = S_t[1-\varphi] \end{cases} \tag{4.134}$$

[Table 4.14 Inversion model-10 results with only UIP is about here]

Table 4.14 shows the results from the GMM model (model-10) in the first situation. This model is an extreme event that I only consider for the interest rate effect. The parameter $\varphi$ only includes the interest rate effect. The value of $\varphi$ still remains low except for several exchanges, such as EURGBP, USDCAD, EURSEK, and EURNOK. Clearly, the model needs to consider the more complex circumstances. This situation is too strict and deviates from the actual. Hence, I again add the uninformed investors and informed investors into the model.

I still apply UIP to estimate the forward rate; however, there is some other information that would affect the forward rate, which would lead to a deviation from the forward rate (from the UIP). Hence, I set the forward rate equal to:

$$\frac{F_t}{S_t} = \frac{1 + i_d}{1 + i_f} + \varphi_t + \varepsilon_{t+1} + \omega_{t+1} = 1 + i_t + \varphi_t + \varepsilon_{t+1} + \omega_{t+1} \tag{4.135}$$

$$i_t = \begin{cases} i \, with \, probability & 1/2, \\ -i \, with \, probability & 1/2. \end{cases} \tag{4.136}$$

where $i_t$ is the interest rate information which have no relationship with public information influence at term $t$, which could be observed by all participants. I assume the influence would be positive or negative in the same probability.

$$\varphi_t = \begin{cases} \varphi \, with \, probability & 1/2, \\ -\varphi \, with \, probability & 1/2. \end{cases} \tag{4.137}$$

where $\varphi_t$ is the public information which has no relationship with interest rate influence at term $t$, which could be observed by all participants. I assume that the influence would be positive or negative in the same probability.

$$\varepsilon_{t+1} = \begin{cases} \varepsilon \, with \, probability & 1/2, \\ -\varepsilon \, with \, probability & 1/2. \end{cases} \tag{4.138}$$

where $\varepsilon_{t+1}$ is not observed directly at time $t$ which could be observed by informed investors as a signal $\zeta_t \in \{\varepsilon, -\varepsilon\}$.

the value of the influence from public and private information $(\varphi, \varepsilon)$ are both positive. Finally, $\omega_{t+1}$ denotes the information which no agents in the market would observe, while variable $\omega_{t+1}$ independently and identically follows the normal distribution with the mean zero and variance $\sigma_\omega^2$. The four information parameters $(i_t, \varphi_t, \varepsilon_{t+1}, \omega_{t+1})$ in the model are mutually orthogonal.

At the beginning of term $t$, informed traders could observe the private information signal $\zeta_t$ which could be positive or negative. The probability for informed traders to get the correct private information is $q$. Hence I have the function as below:

$$Pr\left(\zeta_t = \varepsilon \mid \varepsilon_{t+1} = \varepsilon\right) = Pr\left(\zeta_t = -\varepsilon \mid \varepsilon_{t+1} = -\varepsilon\right) = q > \frac{1}{2} \tag{4.139}$$

I assume the informed traders could use the better techniques to get $\zeta_t$ or use a special method to access private information signal with the correct probability $q$, which is higher than half.

No agents could obverse the fact $\varepsilon_{t+1}$, however, interest rate information $i_t$ and public information $\varphi_t$ is available for all participants in the market. Hence I could have 8 different spot rate $S^a_{t+1}(i, \varphi)$, $S^a_{t+1}(i, -\varphi)$, $S^b_{t+1}(i, \varphi)$, and $S^b_{t+1}(i, -\varphi).S^a_{t+1}(-i, \varphi)$, $S^a_{t+1}(-i, -\varphi)$, $S^b_{t+1}(-i, \varphi)$, and $S^b_{t+1}(-i, -\varphi)$.

When $\varphi_t = \varphi \, and \, i_t = i$, the market maker would get the profit from selling one pound spot, $\pi^m_{t+1}$, is

$$\pi^m_{t+1} = F_t - S^a_{t+1}(\varphi, i) \tag{4.140}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi^m_{t+1} \mid buy, \varphi, i\right) = E\left(F_t \mid buy, \varphi, i\right) - S^a_{t+1}(\varphi, i) = 0 \tag{4.141}$$

By applying the equation 4.135, I obtain the below equation:

$$S^a_{t+1}\left(\varphi, i\right) = S_t\left[1 + i + \varphi + E\left(\varepsilon_{t+1} \mid buy, \varphi, i\right)\right] \tag{4.142}$$

Following the Bayesian rule, I evaluate the expectation of market maker of $\varepsilon_{t+1}$, based on his information set:

$$E\left(\varepsilon_{t+1} \mid buy, \varphi, i\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi, i\right)\left(\varepsilon\right) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \varphi, i\right)\left(-\varepsilon\right) \tag{4.143}$$

The following function is implied in the Bayesian rule:

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \varphi, i\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi, i\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \varphi, i\right)} \tag{4.144}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi, i\right)$, I need to consider the informed and uninformed traders separately. When $i_t = i$, UIP arbitragers would buy the pound spot. When $\varphi_t = \varphi$, uninformed traders would buy the pound spot. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound spot with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi, i\right) = \gamma + \left(1 - \alpha - \gamma\right) + \alpha q \tag{4.145}$$

$$\begin{aligned} Pr\left(buy \mid \varphi, i\right) = {} & Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \varphi, i\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) \\ & + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi, i\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \end{aligned} \tag{4.146}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi, i\right)$ by the similar way, and it follow that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \varphi, i\right) = \gamma + \left(1 - \alpha - \gamma\right) + \alpha\left(1 - q\right) \tag{4.147}$$

I use equations 4.145, 4.146 and 4.147 to get the equation below:

$$Pr\left(buy \mid \varphi, i\right) = \left(\gamma + \left(1 - \alpha - \gamma\right) + \alpha q\right)\frac{1}{2} + \left[\gamma + \left(1 - \alpha - \gamma\right) + \alpha\left(1 - q\right)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.148}$$

Equations 4.145, 4.148 and 4.144 imply that

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\varphi,i\right)=\frac{1-\alpha\left(1-q\right)}{2-\alpha} \tag{4.149}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi,i\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\varphi\right) \tag{4.150}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\varphi,i\right)=\frac{1-\alpha q}{2-\alpha} \tag{4.151}$$

By substituting equations 4.149, 4.151 and 4.143, I obtain

$$E\left(\varepsilon_{t+1} \mid buy,\varphi,i\right)=\frac{\alpha}{2-\alpha}(2q-1)\varepsilon \tag{4.152}$$

I obtain from equation 4.142

$$S^{a}_{t+1}\left(\varphi,i\right)=S_t\left[1+i+\varphi+\frac{\alpha}{2-\alpha}(2q-1)\varepsilon\right] \tag{4.153}$$

Hence, this is the ask spot rate with positive public information. $S^{a}_{t}\left(\varphi,i\right)$ would be influenced by the value of $\varphi$, the value of $i$, the proportion of informed traders $\alpha$, the probability for the signal $\zeta_t$ is correct and the value of private information $\varepsilon$. Applying similar methods, I can derive the seven other situations as shown below (model-11):

$$\begin{cases} S^{a}_{t+1}\left(\varphi_t,i_t\right)=\begin{cases} S_t\left[1+i+\varphi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\,\varphi_t=\varphi, i_t=i\\ S_t\left[1+i-\varphi+(2q-1)\varepsilon\alpha/\left(2\gamma+\alpha\right)\right] & if\,\varphi_t=-\varphi, i_t=i\\ S_t\left[1-i+\varphi+(2q-1)\varepsilon\alpha/\left(2-\alpha-2\gamma\right)\right] & if\,\varphi_t=\varphi, i_t=-i\\ S_t\left[1-i-\varphi+(2q-1)\varepsilon\right] & if\,\varphi_t=-\varphi, i_t=-i \end{cases}\\ S^{b}_{t+1}\left(\varphi_t,i_t\right)=\begin{cases} S_t\left[1+i+\varphi-(2q-1)\varepsilon\right] & if\,\varphi_t=\varphi, i_t=i\\ S_t\left[1+i-\varphi-(2q-1)\varepsilon\alpha/\left(2-\alpha-2\gamma\right)\right] & if\,\varphi_t=-\varphi, i_t=i\\ S_t\left[1-i+\varphi-(2q-1)\varepsilon\alpha/\left(2\gamma+\alpha\right)\right] & if\,\varphi_t=\varphi, i_t=-i\\ S_t\left[1-i-\varphi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\,\varphi_t=-\varphi.i_t=-i \end{cases} \end{cases} \tag{4.154}$$

$$\frac{F_t-S_t}{S_t}=a+\beta\frac{S_{t+1}-S_t}{S_t}+\xi_{t+1} \tag{4.155}$$

To have a clear and straightforward way to estimate the change of spot rate and forward premium in my model, I exchange the positions of these two items.

To understand the forward premium puzzle, I need to estimate the value of the $plim\hat{\beta}$ and the expected profit of informed traders $\pi_i^e$. The negative value of the $plim\hat{\beta}$ and positive expected profit of informed traders $\pi_i^e$ would be consistent with the forward premium puzzle.

The values shown in the parentheses are the standard error of the estimations. The value of private information is larger than the value of public information. The value of $q$ is always larger than 50%. Alpha has a low percentage but not close to zero. Gamma also is not significantly high, which indicates that uninformed traders play an essential role in the market. The P-value of the J-test shows that the moments are all correct and reasonable.

[Table 4.15 Inversion model-11 results with three kinds of investors with new parameter $i$ is about here]

[Table 4.16 Inversion model-12 results with three kinds of investors and parameter $v$ is about here]

[Table 4.17 Inversion model-13 results with three kinds of investors and parameter $h$ is about here]

I set the $i$ as the parameter in table 4.15 (model-11), 4.16(model-12) and 4.17(model-13). As the number of parameters increases, the significance of the model decreases. The difference between the inverse method and the switch method when adding the interest rate effect is that I set the interest rate as the information factor in the inverse model. Model-11, model-12, and model-13 are based on model-8 and model-9, with the interest rate effect. Parameter $(\gamma, i)$ illustrates the interest effect in these models. The results show that the interest rate indeed influences the exchange rate significantly.

### 4.7.2 Different Agents in the Forex Market

[Table 4.18 Inversion model-14 results with three kinds of investors with new parameter $i$ is about here]

[Table 4.19 Inversion model-15 results with three kinds of investors and parameter $v$ is about here]

[Table 4.20 Inversion model-16 results with three kinds of investors and parameter $h$ is about here]

In this section, I add more participants to the model. I already added the interest rate effect in the previous section. However, I used only one market participant (UIP arbitragers). There is still one market participant in carry trade strategy, who is also related to the interest rate effects that I have not considered. I add the new market participant 'carry traders' in the model with tables 4.18 (model-14) 4.19(model-15) 4.20(model-16). Carry traders would hold the opposite order against uninformed traders, who follow interest rate information. It is surprising to see that carry traders and interest rate followers have a similar proportion.

[Table 4.21 Inversion model-17 results with 4 kinds of investors and parameter $h$ is about here]

I consider four different kinds of investors: asset managers, corporates, hedge funds and private clients. I follow the existing literature and set asset managers and hedge funds as informed traders, corporates and private clients as uninformed traders. Nevertheless, the adverse selection between asset managers and hedge funds is different. Asset managers tend to be more risk-averse, while hedge funds would be more risk-seeking. Hence, those two agents would make a different choice when they face the private information signal. Asset managers who make h proportion and choose to follow the signal, while $1-h$ may choose to not react to the signal since the signal has $1-q$ probability to be wrong. On the other hand, hedge funds would follow the signal because they are risk-seeking. Following this idea, I added two new agents in the model as $A$ (asset managers) and $H$ (hedge funds), to replace the informed traders. The results are given in Table 4.21(model-17). The informed traders in both these types show reasonably low values.

[Table 4.22 Inversion model-18 results with three kinds of investors with new parameter $i$ is about here]

When I add carry traders, I also need to consider the order flow from them. If the interest rate gives a buy signal to arbitragers, carry traders would receive a co-instantaneous sell signal. This leads to opposite orders towards the interest rate, which would decrease the effect from interest rate information. Hence, I try to add an item $\gamma - \theta$ on the interest rate information $i$ to get the fact effect after counteracting between arbitragers and carry traders. Consequently, I arrive at a new table 4.22 (model-18) . The difference between model-18 and model-14 is small. The assumption of the model becomes closer to reality, which would lead to the precise estimation.

### 4.7.3 Derivation and Value of $\hat{\beta}$ and $\pi$

Following Burnside et al. (2009), I estimate value of of $\hat{\beta}$ and $\pi$ to understand the forward premium puzzle. I use the model-11 as an example to show the derivation. When the value of $\hat{\beta}$ is negative and the informed traders could still get a positive profit, I indicate that the puzzle exist in the market. I then apply my estimations from each model to calculate the value of $\hat{\beta}$ and $\pi$. Let $\delta_{t+1} = (S_{t+1} - S_t)/S_t$ and $f_t = (F_t - S_t)/S_t$ and I could consider a regression from equation 4.155,

$$f_t = a + \beta\delta_{t+1} + \xi_{t+1} \tag{4.156}$$

In the inverse circumstance, the $f_t$ could be calculated as:

$$f_t = i_t + \varphi_t + \varepsilon_{t+1} + \omega_{t+1} \tag{4.157}$$

Then, I can rewrite the spot exchange rate as below:

$$\delta_{t+1}(i_t, \varphi_t) = \begin{cases} i + \varphi - \frac{(1-\alpha)(2q-1)\varepsilon}{2-\alpha} = 0 & \varphi_t = \varphi, i_t = i \\ i - \varphi + \frac{(1-2\gamma-\alpha)(2q-1)\varepsilon\alpha}{(2\gamma+\alpha)(2-2\gamma-\alpha)} = 0 & \varphi_t = -\varphi, i_t = i \\ -i + \varphi - \frac{(1-2\gamma-\alpha)(2q-1)\varepsilon\alpha}{(2\gamma+\alpha)(2-2\gamma-\alpha)} = 0 & \varphi_t = \varphi, i_t = -i \\ -i - \varphi + \frac{(1-\alpha)(2q-1)\varepsilon}{2-\alpha} = 0 & \varphi_t = -\varphi, i_t = -i \end{cases} \tag{4.158}$$

Let's set two new parameters to make the model more clear,

$$\theta_1 = \frac{(1-\alpha)(2q-1)\varepsilon}{2-\alpha} \tag{4.159}$$

$$\theta_2 = \frac{(1-2\gamma-\alpha)(2q-1)\varepsilon\alpha}{(2\gamma+\alpha)(2-2\gamma-\alpha)} \tag{4.160}$$

The slope coefficient in the regression has the following property:

$$plim\hat{\beta} = \frac{cov(\delta_{t+1}, f_t)}{var\,(\delta_{t+1})} \tag{4.161}$$

Through the equation 4.158, I could calculate the $cov(\delta_{t+1}, f_t)$ and $var\,(\delta_{t+1})$ as below:

$$var\,(\delta_{t+1}) = \frac{1}{2}(i + \varphi - \theta_1\varepsilon)^2 + \frac{1}{2}(i - \varphi + \theta_2\varepsilon)^2 \tag{4.162}$$

$$cov(\delta_{t+1}, f_t) = \frac{1}{2}(i + \varphi - \theta_1\varepsilon)(i + \varphi) + \frac{1}{2}(i - \varphi + \theta_2\varepsilon)(i - \varphi) \tag{4.163}$$

Then, the $plim\hat{\beta}$ should equal to:

$$plim\hat{\beta} = \frac{\frac{1}{2}\left(i+\varphi-\theta_1\varepsilon\right)\left(i+\varphi\right)+\frac{1}{2}\left(i-\varphi+\theta_2\varepsilon\right)\left(i-\varphi\right)}{\frac{1}{2}\left(i+\varphi-\theta_1\varepsilon\right)^2+\frac{1}{2}\left(i-\varphi+\theta_2\varepsilon\right)^2} \tag{4.164}$$

and the expected profit of informed traders should be:

$$\pi_i^e = \left(\frac{1}{4}\left(1-\frac{\alpha}{2-\alpha}\right)+\frac{1}{4}\left(1-\frac{\alpha}{2\gamma+\alpha}\right)+\frac{1}{4}\left(1-\frac{\alpha}{2-\alpha-2\gamma}\right)\right)(2q-1)\varepsilon S_t \tag{4.165}$$

[Table 4.23 $plim\hat{\beta}$ and expected return of informed traders $\pi_i^e$ is about here]

Table 4.23 illustrate the value of $plim\hat{\beta}$, and the expected profit of informed $\pi_i^e$ calculated by the parameters from the microstructure models. The odd rows are the value of $\beta$ and the even rows show the value of $\pi$. I find that most estimations of $\beta$ were close to one. I discussed the regression value of $\beta$ in section 3. The value of $\beta$ was far from one, and the F-test rejected the null hypothesis with a low p-value.

I check the equation below:

$$plim\hat{\beta} = \frac{\varphi}{\varphi-\left(1-\alpha\right)\left(2q-1\right)\varepsilon/\left(2-\alpha\right)} \tag{4.166}$$

when the item $\left(1-\alpha\right)\left(2q-1\right)\varepsilon/\left(2-\alpha\right)$ close to zero, the function would change to $plim\hat{\beta} = \frac{\varphi}{\varphi-0} = 1$. I also discuss the regression value of $\beta$ in a complex situation in model-10 as below. I set

$$\begin{cases} a1 = 2\left(\gamma+\left(1-\alpha-\gamma-\theta\right)v\right)+\alpha \\ a2 = 2\left(\gamma+\left(1-\alpha-\gamma-\theta\right)\left(1-v\right)\right)+\alpha \\ a3 = 2\theta+2\left(1-\alpha-\gamma-\theta\right)v+\alpha \\ a4 = 2\theta+2\left(1-\alpha-\gamma-\theta\right)\left(1-v\right)+\alpha \end{cases} \tag{4.167}$$

I get

$$\begin{cases} x1 = (2qh-1)\varepsilon\alpha(a1-1)/(a1(2-a1)) \\ x2 = (2qh-1)\varepsilon\alpha(a2-1)/(a2(2-a2)) \\ x3 = (2qh-1)\varepsilon\alpha(a3-1)/(a3(2-a3)) \\ x4 = (2qh-1)\varepsilon\alpha(a4-1)/(a4(2-a4)) \end{cases} \tag{4.168}$$

I apply the above equations to calculate the value of $\beta$ as

$$
\begin{aligned}
\hat{\beta} &= \frac{cov(\delta_{t+1}, f_t)}{var\,(\delta_{t+1})} \\
&= \frac{(0.25(i+\varphi-x1)(i+\varphi)+0.25(i-\varphi+x2)(i-\varphi)+0.25(-i+\varphi-x3)(-i+\varphi)+0.25(-i-\varphi+x4)(-i-\varphi))}{(0.25(i+\varphi-x1)^2+0.25(i-\varphi+x2)^2+0.25(-i+\varphi-x3)^2+0.25(-i-\varphi+x4)^2)}
\end{aligned} \tag{4.169}
$$

Since items $x1, x2, x3, x4$ have the $\varepsilon$ on the numerator. When $\varepsilon$ closes to zero, the value of item $x1, x2, x3, x4$ would also be close to zero. Then, the value of $\beta$ could be rewritten as:

$$
\begin{aligned}
\hat{\beta} &= \frac{(0.25(i+\varphi)^2+0.25(i-\varphi)^2+0.25(-i+\varphi)^2+0.25(-i-\varphi)^2)}{(0.25(i+\varphi)^2+0.25(i-\varphi)^2+0.25(-i+\varphi)^2+0.25(-i-\varphi)^2)} \\
&= \frac{(0.5(i+\varphi)^2+0.5(i-\varphi)^2)}{(0.5(i+\varphi)^2+0.5(i-\varphi)^2)} \\
&= 1
\end{aligned} \tag{4.170}
$$

Although the model change is complex and the participants change from 2 to 4, the value of $\beta$ still depends on the value of $\varepsilon$. Hence, when the $\varepsilon$ is significantly small, the market could hold the UIP and CIP. I also state that the failure of UIP and CIP is caused by private information significantly influencing the price. The results show that the case with $\beta$, which is close to one also has the small $\varepsilon$. Furthermore, the interest rate information and other public information could be observed by anyone and will not lead to the failure of CIP and UIP.

## 4.8 Conclusion

This chapter presents a model in which adverse selection problems between market makers and traders rationalize a negative covariance between the forward premium and spot rate changes. First, I apply the simple regression to show the evidence of the forward premium puzzle. Then, I estimate the original model from Burnside et al. (2009) by the common forex data. When I want to extend the model to a complex situation, I find that the common forex data cannot support the estimation of the complex models. Hence, I choose to apply the order flow data to estimate the complex models. I apply the unique order flow data in two ways: switch to consider the spot rate at term $t + 1$, or the inverse of the forward rate as the exogenous value.

The findings are shown below. In the first method (switch method), UIP exists between the forward and spot rates after adjusting the order flow. The informed traders always have positive profit, in line with my hypothesis. When increasing the number of parameters, the models become much closer to reality. Most of the models could have significant estimations with reasonable values. Adding uncovered interest rate parity helps me to explain the failure of UIP. The results show that the main reason for the failure of UIP and CIP is the effect of private information. Overall, the adverse selection could generate the forward premium puzzle.

This chapter has several contributions. First, I estimate the value of the adverse selection micro model from Burnside et al. (2009), which gives the real estimation of the parameters. This could help me to test whether the micro model has the same performance with the assumptions. The results indicate that the adverse selection could logically explain the forward premium puzzle. Second, I add more participants and new information parameters in the adverse selection model to extend the model closer to reality.

This chapter has some policy implications in the government and forecasting area. The main implication of the microstructure model is that I can explain the various classic forward premium puzzles, basing on the assumptions about information friction. Furthermore, this model could be applied to estimate the market efficiency by estimating the exchange rate's private information effect. The significant effect compares with the public information indicates the inefficiency of the forex market.

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**Table 4.1** – Different Period Regression

I present the estimation of the forward premium and the depreciation of spot rate. $\alpha$ is the intercept of the regression while the $\beta$ denotes the slope of the regression. The F-statistics is for the Wald test of null hypothesis: $H_0\{\alpha = 0, \beta = 1\}$. The $p - value$ are showed in the parenthesis for both estimations and F-test.

| | 2005 Aug-2007 July | | | 2007 Aug-2009 July | | | 2009 Aug-2011 July | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\alpha$ | $\beta$ | F | $\alpha$ | $\beta$ | F | $\alpha$ | $\beta$ | F |
| EURUSD | -0.0004 | 0.0001 | 2.0077E+06 | 0.0001 | 0.0008 | 7.9161E+05 | 0.0001 | 0.0001 | 4.8277E+06 |
| | (0.0000) | (0.8413) | (0.0000) | (0.0000) | (0.4986) | (0.0000) | (0.0000) | (0.9019) | (0.0000) |
| USDJPY | 0.0009 | 0.0012 | 2.7207E+06 | 0.0004 | 0.0010 | 4.4037E+05 | 0.0001 | 0.0003 | 2.5986E+07 |
| | (0.0000) | (0.0585) | (0.0000) | (0.0000) | (0.4977) | (0.0000) | (0.0000) | (0.1437) | (0.0000) |
| EURJPY | 0.0006 | 0.0004 | 1.4196E+06 | 0.0005 | 0.0013 | 8.3853E+05 | 0.0001 | 0.0001 | 7.7708E+06 |
| | (0.0000) | (0.6190) | (0.0000) | (0.0000) | (0.2317) | (0.0000) | (0.0000) | (0.7804) | (0.0000) |
| GBPUSD | 0.0000 | 0.0012 | 2.0234E+06 | -0.0003 | 0.0017 | 1.2311E+06 | 0.0000 | 0.0019 | 2.1529E+05 |
| | (0.4610) | (0.09890 | (0.0000) | (0.0000) | (0.0647) | (0.0000) | (0.3591) | (0.3907) | (0.0000) |
| EURGBP | -0.0004 | 0.0007 | 2.0863E+06 | 0.0004 | 0.0017 | 9.7960E+05 | 0.0001 | 0.0013 | 9.4002E+05 |
| | (0.0000) | (0.3325) | (0.0000) | (0.0000) | (0.0943) | (0.0000) | (0.0071) | (0.2205) | (0.0000) |
| USDCHF | 0.0007 | 0.0002 | 2.8440E+06 | 0.0002 | 0.0005 | 6.5731E+05 | 0.0001 | -0.0007 | 1.0877E+07 |
| | (0.0000) | (0.7732) | (0.0000) | (0.0000) | (0.6953) | (0.0000) | (0.0000) | (0.0206) | (0.0000) |
| EURCHF | 0.0003 | -0.0017 | 2.0535E+06 | 0.0003 | -0.0010 | 4.0210E+05 | 0.0001 | -0.0004 | 4.9270E+06 |
| | (0.0000) | (0.0169) | (0.0000) | (0.0000) | (0.5320) | (0.0000) | (0.0000) | (0.3894) | (0.0000) |
| AUDUSD | -0.0002 | 0.0006 | 5.5651E+06 | -0.0006 | 0.0008 | 1.9321E+06 | -0.0008 | 0.0002 | 4.2543E+06 |
| | (0.0000) | (0.1888) | (0.0000) | (0.0000) | (0.2705) | (0.0000) | (0.0000) | (0.6533) | (0.0000) |
| NZDUSD | -0.0005 | 0.0003 | 4.4851E+06 | -0.0008 | 0.0022 | 1.2181E+06 | -0.0005 | 0.0000 | 2.2254E+07 |
| | (0.0000) | (0.4630) | (0.0000) | (0.0000) | (0.0176) | (0.0000) | (0.0000) | (0.9366) | (0.0000) |
| USDCAD | 0.0002 | 0.0001 | 7.5141E+06 | -0.0001 | 0.0008 | 1.8961E+06 | -0.0001 | 0.0001 | 3.5316E+06 |
| | (0.0000) | (0.8919) | (0.0000) | (0.0000) | (0.2769) | (0.0000) | (0.0000) | (0.8175) | (0.0000) |
| EURSEK | 0.0001 | 0.0004 | 5.6658E+06 | -0.0001 | -0.0111 | 4.5844E+04 | -0.0001 | 0.0015 | 8.2797E+05 |
| | (0.0000) | (0.3571) | (0.0000) | (0.1770) | (0.0207) | (0.0000) | (0.0000) | (0.1843) | (0.0000) |
| EURNOK | 0.0000 | -0.0010 | 2.5381E+06 | -0.0003 | -0.0060 | 3.3389E+05 | -0.0003 | 0.0003 | 4.4585E+06 |
| | (0.0000) | (0.1157) | (0.0000) | (0.0000) | (0.0008) | (0.0000) | (0.0000) | (0.5326) | (0.0000) |

**Table 4.2** – Different Period Excess Return of Carry Trade Strategy

I present the annualized mean value of forward premium, rate of depreciation and excess return of carry trade and the estimated value of $\hat{\beta}$.

| | 2005 Aug-2007 July | | | | 2007 Aug-2009 July | | | | 2009 Aug-2011 July | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | forward premium | rate of depreciation | excess return | $\hat{\beta}$ | forward premium | rate of depreciation | excess return | $\hat{\beta}$ | forward premium | rate of depreciation | excess return | $\hat{\beta}$ |
| EURUSD | -0.0189 | -0.0516 | 0.0308 | 0.0001 | 0.0064 | -0.0217 | 0.0275 | 0.0008 | 0.0026 | 0.0487 | -0.0160 | 0.0001 |
| USDJPY | 0.0477 | -0.0200 | 0.0748 | 0.0012 | 0.0192 | 0.1247 | -0.0893 | 0.0010 | 0.0026 | 0.0945 | -0.0857 | 0.0003 |
| EURJPY | 0.0288 | -0.0755 | 0.1057 | 0.0004 | 0.0256 | 0.1023 | -0.0619 | 0.0013 | 0.0052 | 0.1403 | -0.1017 | 0.0001 |
| GBPUSD | 0.0004 | 0.0473 | -0.0538 | 0.0012 | -0.0136 | -0.0921 | 0.0905 | 0.0017 | -0.0013 | -0.0225 | 0.0126 | 0.0019 |
| EURGBP | -0.0192 | -0.0877 | 0.0847 | 0.0007 | 0.0200 | 0.1066 | -0.0630 | 0.0017 | 0.0040 | 0.0864 | -0.0285 | 0.0013 |
| USDCHF | 0.0345 | 0.0247 | 0.0164 | 0.0002 | 0.0082 | 0.0808 | -0.0533 | 0.0005 | 0.0037 | 0.0749 | -0.0894 | -0.0007 |
| EURCHF | 0.0157 | -0.0325 | 0.0473 | -0.0017 | 0.0146 | 0.0454 | -0.0258 | -0.0010 | 0.0063 | 0.1143 | -0.1054 | -0.0004 |
| AUDUSD | -0.0095 | 0.0461 | -0.0443 | 0.0006 | -0.0329 | 0.0407 | -0.0515 | 0.0008 | -0.0419 | 0.0702 | -0.1474 | 0.0002 |
| NZDUSD | -0.0259 | 0.0119 | -0.0286 | 0.0003 | -0.0425 | 0.0215 | -0.0303 | 0.0022 | -0.0258 | 0.0563 | -0.1168 | 0.0000 |
| USDCAD | 0.0101 | 0.0653 | -0.0499 | 0.0001 | -0.0042 | -0.0046 | 0.0110 | 0.0008 | -0.0047 | 0.0256 | -0.0497 | 0.0001 |
| EURSEK | 0.0059 | 0.0035 | 0.0094 | 0.0004 | -0.0046 | -0.0356 | 0.0408 | -0.0111 | -0.0045 | 0.0441 | -0.0774 | 0.0015 |
| EURNOK | -0.0014 | 0.0008 | -0.0013 | -0.0010 | -0.0157 | -0.0446 | 0.0250 | -0.0060 | -0.0149 | 0.0501 | -0.0824 | 0.0003 |

**Table 4.3** – GMM model-1 Results with Normal Forex Data

I firstly test the original model from Burnside et al. (2009). The parameters of the basic model are $\varphi, \varepsilon, q, \alpha$. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The J-test is aim to test the moments of GMM model. If the $p - value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|
| EURUSD | 0.0002 | 0.0050 | 0.5806 | 0.4049 | 0.0705 | 0.7906 |
| | (0.0000) | (0.0008) | (0.0112) | (0.1443) | | |
| USDJPY | 0.0001 | 0.0050 | 0.5896 | 0.3727 | 0.1514 | 0.6972 |
| | (0.0000) | (0.0006) | (0.0141) | (0.1936) | | |
| EURJPY | 0.0001 | 0.0050 | 0.6045 | 0.3176 | 0.1037 | 0.7474 |
| | (0.0000) | (0.0008) | (0.0226) | (0.2381) | | |
| GBPUSD | 0.0004 | 0.0050 | 0.6561 | 0.3817 | 0.0309 | 0.8605 |
| | (0.0000) | (0.0008) | (0.0033) | (0.0412) | | |
| EURGBP | 0.0003 | 0.0050 | 0.5745 | 0.3160 | 0.1032 | 0.7480 |
| | (0.0000) | (0.0007) | (0.0118) | (0.1466) | | |
| USDCHF | 0.0002 | 0.0050 | 0.5853 | 0.3065 | 0.0906 | 0.7634 |
| | (0.0000) | (0.0007) | (0.0113) | (0.1327) | | |
| EURCHF | 0.0003 | 0.0050 | 0.5785 | 0.3058 | 0.1035 | 0.7476 |
| | (0.0000) | (0.0007) | (0.0106) | (0.0496) | | |
| AUDUSD | 0.0005 | 0.0050 | 0.6588 | 0.0971 | 0.0537 | 0.8167 |
| | (0.0000) | (0.0004) | (0.0068) | (0.0459) | | |
| NZDUSD | 0.0006 | 0.0050 | 0.6490 | 0.0523 | 0.1387 | 0.7096 |
| | (0.0000) | (0.0007) | (0.0132) | (0.0922) | | |
| USDCAD | 0.0002 | 0.0050 | 0.5795 | 0.2982 | 0.1147 | 0.7349 |
| | (0.0000) | (0.0006) | (0.0051) | (0.0547) | | |
| EURSEK | 0.0002 | 0.0050 | 0.6216 | 0.3025 | 0.0853 | 0.7703 |
| | (0.0000) | (0.0007) | (0.0098) | (0.0724) | | |
| EURNOK | 0.0004 | 0.0050 | 0.7278 | 0.2572 | 0.0624 | 0.8027 |
| | (0.0000) | (0.0008) | (0.0217) | (0.0496) | | |

**Table 4.4** – GMM Model-2

I firstly test the original model from Burnside et al. (2009). The parameters of the basic model are $\phi, \varepsilon, q, \alpha$. $\phi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The J-test is aim to test the moments of GMM model. If the $p - value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate. The standard error of the estimation are given in the parenthesis.

| | $\phi$ | $\varepsilon$ | $q$ | $\alpha$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|
| EURUSD | 0.0003 | 0.0050 | 0.6344 | 0.0648 | 0.0912 | 0.9990 |
| | (0.0000) | (0.0005) | (0.0120) | (0.0022) | | |
| USDJPY | 0.0004 | 0.0065 | 0.6783 | 0.0880 | 0.5953 | 0.9636 |
| | (0.0000) | (0.0005) | (0.0125) | (0.0026) | | |
| EURJPY | 0.0004 | 0.0054 | 0.6380 | 0.1052 | 0.1329 | 0.9979 |
| | (0.0000) | (0.0006) | (0.0142) | (0.0039) | | |
| GBPUSD | 0.0004 | 0.0072 | 0.5047 | 0.0860 | 0.0822 | 0.9992 |
| | (0.0000) | (0.0006) | (0.0012) | (0.0030) | | |
| EURGBP | 0.0003 | 0.0051 | 0.5325 | 0.1154 | 0.0431 | 0.9998 |
| | (0.0000) | (0.0011) | (0.0079) | (0.0036) | | |
| USDCHF | 0.0004 | 0.0079 | 0.6457 | 0.0840 | 0.2010 | 0.9953 |
| | (0.0000) | (0.0006) | (0.0099) | (0.0027) | | |
| EURCHF | 0.0005 | 0.0073 | 0.5274 | 0.1007 | 0.7307 | 0.9475 |
| | (0.0000) | (0.0004) | (0.0020) | (0.0028) | | |
| AUDUSD | 0.0005 | 0.0085 | 0.5002 | 0.1237 | 0.0858 | 0.9991 |
| | (0.0000) | (0.0008) | (0.0004) | (0.0048) | | |
| NZDUSD | 0.0005 | 0.0094 | 0.5001 | 0.1950 | 0.0774 | 0.9993 |
| | (0.0000) | (0.0007) | (0.0003) | (0.0072) | | |
| USDCAD | 0.0003 | 0.0050 | 0.5905 | 0.1495 | 0.0715 | 0.9994 |
| | (0.0000) | (0.0005) | (0.0087) | (0.0061) | | |
| EURSEK | 0.0004 | 0.0050 | 0.5423 | 0.2363 | 0.0574 | 0.9996 |
| | (0.0000) | (0.0005) | (0.0075) | (0.0067) | | |
| EURNOK | 0.0004 | 0.0053 | 0.5790 | 0.2776 | 0.0667 | 0.9995 |
| | (0.0000) | (0.0006) | (0.0086) | (0.0091) | | |

**Table 4.5** – GMM Model-3

I then test the model from Burnside et al. (2009). The parameters of the basic model are $\phi, \varepsilon, q, v, \alpha$. $\phi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The new parameter $v$ is the proportion of uninformed traders who choose to follow the public information. The J-test is aim to test the moments of GMM model. If the $p - value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate.The standard error of the estimation are given in the parenthesis.

| | $\phi$ | $\varepsilon$ | $q$ | $\alpha$ | $v$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|
| EURUSD | 0.0003 | 0.0051 | 0.6356 | 0.0657 | 0.8599 | 0.1018 | 0.9916 |
| | (0.0000) | (0.0007) | (0.0198) | (0.0024) | (0.0482) | | |
| USDJPY | 0.0004 | 0.0057 | 0.6706 | 0.0876 | 0.8642 | 0.6064 | 0.8950 |
| | (0.0000) | (0.0006) | (0.0201) | (0.0026) | (0.0366) | | |
| EURJPY | 0.0005 | 0.0070 | 0.6386 | 0.1049 | 0.8224 | 0.1314 | 0.9878 |
| | (0.0000) | (0.0008) | (0.0228) | (0.0039) | (0.0676) | | |
| GBPUSD | 0.0004 | 0.0072 | 0.5054 | 0.0860 | 0.9890 | 0.0823 | 0.9939 |
| | (0.0000) | (0.0006) | (0.0221) | (0.0030) | (0.2578) | | |
| EURGBP | 0.0003 | 0.0053 | 0.5332 | 0.1154 | 0.9871 | 0.0448 | 0.9975 |
| | (0.0000) | (0.0013) | (0.0210) | (0.0036) | (0.0614) | | |
| USDCHF | 0.0004 | 0.0064 | 0.6435 | 0.0847 | 0.8953 | 0.1933 | 0.9787 |
| | (0.0000) | (0.0007) | (0.0202) | (0.0028) | (0.0318) | | |
| EURCHF | 0.0005 | 0.0057 | 0.5307 | 0.1005 | 0.9800 | 0.7232 | 0.8677 |
| | (0.0000) | (0.0004) | (0.0214) | (0.0028) | (0.0577) | | |
| AUDUSD | 0.0005 | 0.0085 | 0.5006 | 0.1238 | 0.7107 | 0.0863 | 0.9934 |
| | (0.0001) | (0.0008) | (0.0263) | (0.0048) | (30.0332) | | |
| NZDUSD | 0.0005 | 0.0093 | 0.5003 | 0.1942 | 0.7132 | 0.0778 | 0.9944 |
| | (0.0001) | (0.0008) | (0.0240) | (0.0074) | (64.8890) | | |
| USDCAD | 0.0003 | 0.0053 | 0.6124 | 0.1507 | 0.8591 | 0.0569 | 0.9964 |
| | (0.0000) | (0.0006) | (0.0201) | (0.0061) | (0.0630) | | |
| EURSEK | 0.0004 | 0.0050 | 0.5423 | 0.2363 | 0.9966 | 0.0575 | 0.9964 |
| | (0.0000) | (0.0006) | (0.0243) | (0.0067) | (0.1179) | | |
| EURNOK | 0.0005 | 0.0050 | 0.5591 | 0.2746 | 0.9747 | 0.0699 | 0.9952 |
| | (0.0001) | (0.0006) | (0.0223) | (0.0091) | (0.1082) | | |

**Table 4.6** – GMM Model-4

I test the original model of spot and bond market. The parameters of the basic model are $\varphi, \varepsilon, q, \alpha$. $\varphi$ denotes the public information without the interest rate information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The J-test is aim to test the moments of GMM model. If the $p-value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|
| EURUSD | 0.0001 | 0.0010 | 0.6278 | 0.0675 | 0.0119 | 1.0000 |
| | (0.0000) | (0.0011) | (0.6676) | (0.0888) | | |
| USDJPY | 0.0000 | 0.0011 | 0.6307 | 0.1049 | 0.0515 | 0.9997 |
| | (0.0002) | (0.0027) | (0.5099) | (0.0855) | | |
| EURJPY | 0.0002 | 0.0037 | 0.5307 | 0.1139 | 0.0575 | 0.9996 |
| | (0.0001) | (0.0012) | (0.6535) | (0.0854) | | |
| GBPUSD | 0.0002 | 0.0021 | 0.5176 | 0.1008 | 0.0648 | 0.9995 |
| | (0.0002) | (0.0036) | (0.5061) | (0.1247) | | |
| EURGBP | 0.0001 | 0.0026 | 0.5070 | 0.1937 | 0.0416 | 0.9998 |
| | (0.0002) | (0.0025) | (0.6177) | (0.1466) | | |
| USDCHF | 0.0003 | 0.0047 | 0.5439 | 0.2327 | 0.0569 | 0.9996 |
| | (0.0003) | (0.0049) | (0.5536) | (0.2699) | | |
| EURCHF | 0.0005 | 0.0073 | 0.5274 | 0.1007 | 0.0627 | 0.9995 |
| | (0.0000) | (0.0004) | (0.0020) | (0.0028) | | |
| AUDUSD | 0.0005 | 0.0085 | 0.5002 | 0.1237 | 0.0623 | 0.9995 |
| | (0.0000) | (0.0008) | (0.0004) | (0.0048) | | |
| NZDUSD | 0.0005 | 0.0094 | 0.5001 | 0.1950 | 0.0644 | 0.9995 |
| | (0.0000) | (0.0007) | (0.0003) | (0.0072) | | |
| USDCAD | 0.0003 | 0.0050 | 0.5905 | 0.1495 | 0.0199 | 1.0000 |
| | (0.0000) | (0.0005) | (0.0087) | (0.0061) | | |
| EURSEK | 0.0004 | 0.0050 | 0.5423 | 0.2363 | 0.0321 | 0.9999 |
| | (0.0000) | (0.0005) | (0.0075) | (0.0067) | | |
| EURNOK | 0.0004 | 0.0053 | 0.5790 | 0.2776 | 0.0486 | 0.9997 |
| | (0.0000) | (0.0006) | (0.0086) | (0.0091) | | |

**Table 4.7** – GMM Model-5

I add the parameter $v$ based on the model-4. The parameters of the basic model are $\varphi, \varepsilon, q, v, \alpha$. $\varphi$ denotes the public information without the interest rate information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The new parameter $v$ is the proportion of uninformed traders who choose to follow the public information. The J-test is aim to test the moments of GMM model. If the $p-value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate.The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $v$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|
| EURUSD | 0.0001 | 0.0033 | 0.6482 | 0.0662 | 0.8288 | 0.0154 | 0.9995 |
| | (0.0008) | (0.0501) | (0.0191) | (0.0032) | (4.1505) | | |
| USDJPY | 0.0001 | 0.0010 | 0.6414 | 0.0865 | 0.9600 | 0.0560 | 0.9965 |
| | (0.0012) | (0.0844) | (0.0207) | (0.0029) | (8.0604) | | |
| EURJPY | 0.0000 | 0.0014 | 0.6306 | 0.1049 | 0.8258 | 0.0577 | 0.9964 |
| | (0.0021) | (0.0860) | (0.0231) | (0.0047) | (19.8534) | | |
| GBPUSD | 0.0002 | 0.0039 | 0.5070 | 0.0856 | 0.9929 | 0.0644 | 0.9957 |
| | (0.0000) | (0.0450) | (0.0274) | (0.0032) | (0.8077) | | |
| EURGBP | 0.0002 | 0.0040 | 0.5382 | 0.1142 | 0.9506 | 0.0448 | 0.9975 |
| | (0.0003) | (0.0650) | (0.0203) | (0.0034) | (2.0648) | | |
| USDCHF | 0.0002 | 0.0012 | 0.6536 | 0.0818 | 0.9656 | 0.0493 | 0.9971 |
| | (0.0009) | (0.0583) | (0.0185) | (0.0042) | (4.0667) | | |
| EURCHF | 0.0002 | 0.0024 | 0.5182 | 0.1008 | 0.9650 | 0.0629 | 0.9959 |
| | (0.0002) | (0.0650) | (0.0200) | (0.0039) | (2.9354) | | |
| AUDUSD | 0.0001 | 0.0035 | 0.5111 | 0.1246 | 0.9011 | 0.0634 | 0.9958 |
| | (0.0002) | (0.0877) | (0.0340) | (0.0056) | (5.9036) | | |
| NZDUSD | 0.0001 | 0.0030 | 0.5054 | 0.1938 | 0.9888 | 0.0644 | 0.9957 |
| | (0.0002) | (0.0862) | (0.0377) | (0.0080) | (5.65980) | | |
| USDCAD | 0.0001 | 0.0048 | 0.6186 | 0.1479 | 0.7625 | 0.0182 | 0.9993 |
| | (0.0024) | (0.0621) | (0.0219) | (0.0072) | (6.7040) | | |
| EURSEK | 0.0003 | 0.0050 | 0.5513 | 0.2313 | 0.9981 | 0.0294 | 0.9987 |
| | (0.0005) | (0.0299) | (0.0266) | (0.0063) | (1.0438) | | |
| EURNOK | 0.0003 | 0.0047 | 0.5780 | 0.2677 | 0.8422 | 0.0443 | 0.9976 |
| | (0.0028) | (0.0683) | (0.0228) | (0.0100) | (7.2889) | | |

**Table 4.8** – GMM Model-6

I consider the affect from the interest rate and add the new parameter $\varrho$. Since I only have the spot exchange currency order flow data, I set the forward rate as the endogenous variable. The parameters of the basic model are $\varphi, \varepsilon, q, v, \alpha, \varrho$. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The new parameter $v$ is the proportion of uninformed traders who choose to follow the public information. $\varrho$ is the uninformed traders who follow the interest rate. The J-test is aim to test the moments of GMM model. If the $p - value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate.The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $\varrho$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|
| EURUSD | 0.0001 | 0.0010 | 0.6280 | 0.0676 | 0.1530 | 0.0115 | 0.9997 |
| | (0.0090) | (0.0350) | (0.0190) | (0.0032) | (706.5261) | | |
| USDJPY | 0.0001 | 0.0011 | 0.6645 | 0.0886 | 0.3226 | 0.0516 | 0.9969 |
| | (0.0100) | (0.0304) | (0.0202) | (0.0028) | (266.8670) | | |
| EURJPY | 0.0000 | 0.0013 | 0.6178 | 0.1052 | 0.3581 | 0.0589 | 0.9963 |
| | (0.0088) | (0.0374) | (0.0235) | (0.0048) | (206.8867) | | |
| GBPUSD | 0.0002 | 0.0027 | 0.5096 | 0.0854 | 0.4978 | 0.0647 | 0.9957 |
| | (0.0009) | (0.0434) | (0.0267) | (0.0032) | (97.9335) | | |
| EURGBP | 0.0000 | 0.0019 | 0.5255 | 0.1129 | 0.8968 | 0.0409 | 0.9978 |
| | (0.0035) | (0.0696) | (0.0200) | (0.0033) | (3.0434) | | |
| USDCHF | 0.0001 | 0.0011 | 0.6552 | 0.0855 | 0.5724 | 0.0565 | 0.9965 |
| | (0.0094) | (0.0304) | (0.0187) | (0.0035) | (104.7801) | | |
| EURCHF | 0.0001 | 0.0011 | 0.5141 | 0.1008 | 0.9185 | 0.0624 | 0.9959 |
| | (0.0019) | (0.0642) | (0.0198) | (0.0039) | (2.9155) | | |
| AUDUSD | 0.0001 | 0.0032 | 0.5000 | 0.1251 | 0.9372 | 0.0611 | 0.9961 |
| | (0.0003) | (0.0879) | (0.0329) | (0.0056) | (0.1568) | | |
| NZDUSD | 0.0001 | 0.0024 | 0.5159 | 0.1921 | 0.4610 | 0.0706 | 0.9951 |
| | (0.0027) | (0.0776) | (0.0375) | (0.0079) | (87.7411) | | |
| USDCAD | 0.0002 | 0.0023 | 0.6172 | 0.1460 | 0.4988 | 0.0201 | 0.9992 |
| | (0.0098) | (0.0419) | (0.0208) | (0.0077) | (52.5869) | | |
| EURSEK | 0.0000 | 0.0049 | 0.5297 | 0.2301 | 0.7920 | 0.0208 | 0.9992 |
| | (0.0017) | (0.0294) | (0.0256) | (0.0064) | (0.9940) | | |
| EURNOK | 0.0002 | 0.0041 | 0.5538 | 0.2659 | 0.6820 | 0.0412 | 0.9978 |
| | (0.0086) | (0.0808) | (0.0221) | (0.0100) | (8.6987) | | |

**Table 4.9** – Comparison of Public Information in Different Models

I compare the public information parameters of different models in this table. While the parameter $\phi$ is for model-1, parameter $\varphi$ is for model-3 and model-5.

| | model-2 | model-4 | model-6 |
|---|---|---|---|
| EURUSD | 0.0003 | 0.0001 | 0.0001 |
| | (0.0000) | (0.0000) | (0.0090) |
| USDJPY | 0.0004 | 0.0000 | 0.0001 |
| | (0.0000) | (0.0002) | (0.0100) |
| EURJPY | 0.0004 | 0.0002 | 0.0000 |
| | (0.0000) | (0.0001) | (0.0088) |
| GBPUSD | 0.0004 | 0.0002 | 0.0002 |
| | (0.0000) | (0.0002) | (0.0009) |
| EURGBP | 0.0003 | 0.0001 | 0.0000 |
| | (0.0000) | (0.0002) | (0.0035) |
| USDCHF | 0.0004 | 0.0003 | 0.0001 |
| | (0.0000) | (0.0003) | (0.0094) |
| EURCHF | 0.0005 | 0.0005 | 0.0001 |
| | (0.0000) | (0.0000) | (0.0019) |
| AUDUSD | 0.0005 | 0.0005 | 0.0001 |
| | (0.0000) | (0.0000) | (0.0003) |
| NZDUSD | 0.0005 | 0.0005 | 0.0001 |
| | (0.0000) | (0.0000) | (0.0027) |
| USDCAD | 0.0003 | 0.0003 | 0.0002 |
| | (0.0000) | (0.0000) | (0.0098) |
| EURSEK | 0.0004 | 0.0004 | 0.0000 |
| | (0.0000) | (0.0000) | (0.0017) |
| EURNOK | 0.0004 | 0.0004 | 0.0002 |
| | (0.0000) | (0.0000) | (0.0086) |

**Table 4.10** – The Slope and Profit of the Traders

This table show the estimates of value of $plim\hat{\beta}$ and expected return of informed traders $\pi_i^e$ for other models.

| | | model-2 | model-3 |
|---|---|---|---|
| EURUSD | $plim\hat{\beta}$ | -0.7282 | 1.6531 |
| | $\pi_i^e$ | 0.0007 | 0.0012 |
| USDJPY | $plim\hat{\beta}$ | -0.5897 | 2.0151 |
| | $\pi_i^e$ | 0.0011 | 0.0016 |
| EURJPY | $plim\hat{\beta}$ | -1.6474 | 1.5987 |
| | $\pi_i^e$ | 0.0007 | 0.0016 |
| GBPUSD | $plim\hat{\beta}$ | 1.0827 | 1.0757 |
| | $\pi_i^e$ | 0.0000 | 0.0000 |
| EURGBP | $plim\hat{\beta}$ | 1.9617 | 1.7402 |
| | $\pi_i^e$ | 0.0002 | 0.0002 |
| USDCHF | $plim\hat{\beta}$ | -0.6450 | 2.1581 |
| | $\pi_i^e$ | 0.0011 | 0.0015 |
| EURCHF | $plim\hat{\beta}$ | 1.6655 | 1.3482 |
| | $\pi_i^e$ | 0.0002 | 0.0002 |
| AUDUSD | $plim\hat{\beta}$ | 1.0027 | 1.0011 |
| | $\pi_i^e$ | 0.0000 | 0.0000 |
| NZDUSD | $plim\hat{\beta}$ | 1.0011 | 1.0009 |
| | $\pi_i^e$ | 0.0000 | 0.0000 |
| USDCAD | $plim\hat{\beta}$ | -3.7622 | 2.6260 |
| | $\pi_i^e$ | 0.0004 | 0.0009 |
| EURSEK | $plim\hat{\beta}$ | 1.8190 | 1.7775 |
| | $\pi_i^e$ | 0.0002 | 0.0002 |
| EURNOK | $plim\hat{\beta}$ | 6.8196 | 1.8997 |
| | $\pi_i^e$ | 0.0004 | 0.0003 |

**Table 4.11** – GMM Model-7

I then test the forward, spot and bond market together. The parameters of the basic model are $\varphi, \varepsilon, q, v, \alpha, \varrho, \phi$. $\varphi$ denotes the public information without interest information, $\phi$ denotes the public information within interest information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The new parameter $v$ is the proportion of uninformed traders who choose to follow the public information.$\varrho$ is the uninformed traders who follow the interest rate. The J-test is aim to test the moments of GMM model. If the $p-value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate.The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $\varrho$ | $\phi$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0001 | 0.0013 | 0.6325 | 0.0668 | 0.2414 | 0.0002 | 0.0500 | 1.0000 |
| | (0.0000) | (0.0002) | (0.0168) | (0.0024) | (0.0889) | (0.0000) | | |
| USDJPY | 0.0001 | 0.0012 | 0.6646 | 0.0907 | 0.2135 | 0.0003 | 0.0563 | 1.0000 |
| | (0.0000) | (0.0001) | (0.0192) | (0.0026) | (0.1325) | (0.0000) | | |
| EURJPY | 0.0001 | 0.0012 | 0.6238 | 0.1041 | 0.2336 | 0.0003 | 0.0598 | 1.0000 |
| | (0.0000) | (0.0002) | (0.0207) | (0.0044) | (0.2519) | (0.0000) | | |
| GBPUSD | 0.0002 | 0.0013 | 0.5087 | 0.0869 | 0.0243 | 0.0003 | 0.0700 | 1.0000 |
| | (0.0000) | (0.0004) | (0.0054) | (0.0031) | (1.2225) | (0.0000) | | |
| EURGBP | 0.0002 | 0.0038 | 0.5204 | 0.1155 | 0.0031 | 0.0003 | 0.0494 | 1.0000 |
| | (0.0000) | (0.0009) | (0.0077) | (0.0033) | (0.4883) | (0.0000) | | |
| USDCHF | 0.0002 | 0.0013 | 0.6483 | 0.0865 | 0.2171 | 0.0003 | 0.0597 | 1.0000 |
| | (0.0000) | (0.0002) | (0.0176) | (0.0031) | (0.1210) | (0.0000) | | |
| EURCHF | 0.0002 | 0.0014 | 0.5262 | 0.1011 | 0.2605 | 0.0003 | 0.0651 | 1.0000 |
| | (0.0000) | (0.0003) | (0.0111) | (0.0035) | (0.6538) | (0.0000) | | |
| AUDUSD | 0.0001 | 0.0013 | 0.5014 | 0.1211 | 0.3202 | 0.0003 | 0.0666 | 1.0000 |
| | (0.0000) | (0.0007) | (0.0024) | (0.0048) | (57.3197) | (0.0000) | | |
| NZDUSD | 0.0001 | 0.0013 | 0.5010 | 0.1943 | 0.3024 | 0.0003 | 0.0667 | 1.0000 |
| | (0.0000) | (0.0007) | (0.0019) | (0.0073) | (38.3224) | (0.0000) | | |
| USDCAD | 0.0002 | 0.0039 | 0.5977 | 0.1558 | 0.1884 | 0.0003 | 0.0485 | 1.0000 |
| | (0.0000) | (0.0004) | (0.0110) | (0.0069) | (0.0514) | (0.0000) | | |
| EURSEK | 0.0002 | 0.0039 | 0.5218 | 0.2343 | 0.0000 | 0.0003 | 0.0663 | 1.0000 |
| | (0.0000) | (0.0005) | (0.0101) | (0.0059) | (0.1955) | (0.0000) | | |
| EURNOK | 0.0002 | 0.0036 | 0.5744 | 0.2685 | 0.1951 | 0.0003 | 0.0527 | 1.0000 |
| | (0.0000) | (0.0004) | (0.0121) | (0.0094) | (0.0667) | (0.0000) | | |

**Table 4.12** – Inversion Model-8 Results with New Parameter $v$

I then test the model from Burnside et al. (2009). Since I only have the spot exchange currency order flow data, I set the forward rate as the endogenous variable. The parameters of the basic model are $\varphi, \varepsilon, q, v, \alpha$. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The new parameter $v$ is the proportion of uninformed traders who choose to follow the public information. The J-test is aim to test the moments of GMM model. If the $p-value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate.The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.0582 | 0.6336 | 0.7363 | 0.0661 | 0.2433 | 0.9986 |
| | (0.0000) | (0.0087) | (0.0199) | (0.0173) | (0.0024) | | |
| USDJPY | 0.0002 | 0.0501 | 0.5052 | 1.0000 | 0.0772 | 2.0467 | 0.8426 |
| | (0.0000) | (0.2058) | (0.0213) | (0.0064) | (0.0027) | | |
| EURJPY | 0.0002 | 0.0500 | 0.5248 | 0.8890 | 0.0870 | 1.2108 | 0.9438 |
| | (0.0000) | (0.0484) | (0.0239) | (0.0101) | (0.0044) | | |
| GBPUSD | 0.0003 | 0.1508 | 0.5009 | 0.6884 | 0.1381 | 1.1182 | 0.9525 |
| | (0.0000) | (3.7279) | (0.0223) | (0.1643) | (0.0041) | | |
| EURGBP | 0.0009 | 0.1625 | 0.5023 | 0.7218 | 0.1175 | 0.1145 | 0.9998 |
| | (0.0000) | (1.4595) | (0.0208) | (0.1569) | (0.0035) | | |
| USDCHF | 0.0005 | 0.0568 | 0.6437 | 0.7449 | 0.0893 | 0.4687 | 0.9932 |
| | (0.0000) | (0.0078) | (0.0198) | (0.0164) | (0.0027) | | |
| EURCHF | 0.0004 | 0.1532 | 0.5140 | 0.8259 | 0.1025 | 0.3377 | 0.9969 |
| | (0.0000) | (0.2347) | (0.0214) | (0.0107) | (0.0029) | | |
| AUDUSD | 0.0005 | 0.0750 | 0.5004 | 0.9952 | 0.1210 | 0.0995 | 0.9998 |
| | (0.0000) | (4.3490) | (0.0254) | (0.1436) | (0.0048) | | |
| NZDUSD | 0.0005 | 0.1316 | 0.5002 | 0.7279 | 0.1953 | 0.0867 | 0.9999 |
| | (0.0000) | (11.8890) | (0.0211) | (1.2468) | (0.0068) | | |
| USDCAD | 0.0000 | 0.0512 | 0.5113 | 0.8057 | 0.1531 | 0.7142 | 0.9822 |
| | (0.0000) | (0.0954) | (0.0209) | (0.0272) | (0.0057) | | |
| EURSEK | 0.0004 | 0.1507 | 0.5009 | 0.6892 | 0.1411 | 3.6552 | 0.6000 |
| | (0.0000) | (4.0282) | (0.0241) | (0.3289) | (0.0074) | | |
| EURNOK | 0.0001 | 0.1726 | 0.5030 | 0.9971 | 0.2955 | 0.6475 | 0.9857 |
| | (0.0000) | (0.9268) | (0.0161) | (0.0081) | (0.0091) | | |

**Table 4.13** – Inversion Model-9 Results with New Parameter $h$

I then add a new parameter $h$. Since I only have the spot exchange currency order flow data, I set the forward rate as the endogenous variable. The parameters of the basic model are $\varphi, \varepsilon, q, v, \alpha$. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. The new parameter $h$ denotes the informed traders who choose to follow the private information signal. The J-test is aim to test the moments of GMM model. If the $p - value$ of the J-test is larger than the significant level (1%, 5% and 10%) I could indicate the moments are all appropriate. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | $h$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.0242 | 0.6373 | 0.7600 | 0.0639 | 0.7585 | 0.1852 | 0.9993 |
| | (0.0000) | (16.6096) | (0.0199) | (0.0247) | (0.0024) | (17.9234) | | |
| USDJPY | 0.0005 | 0.0196 | 0.6670 | 0.5052 | 0.0905 | 0.7589 | 0.3202 | 0.9972 |
| | (0.0000) | (154.3690) | (0.02000 | (0.1266) | (0.0027) | (73.1974) | | |
| EURJPY | 0.0001 | 0.0266 | 0.6689 | 0.9585 | 0.1044 | 0.7764 | 1.0275 | 0.9603 |
| | (0.0000) | (0.8521) | (0.0227) | (0.0089) | (0.0040) | (0.9255) | | |
| GBPUSD | 0.0006 | 0.0200 | 0.5011 | 0.7365 | 0.0893 | 0.8872 | 0.1096 | 0.9998 |
| | (0.0000) | (4.5369) | (0.0215) | (0.0220) | (0.0028) | (25.0584) | | |
| EURGBP | 0.0009 | 0.0179 | 0.5100 | 0.6238 | 0.1192 | 0.8854 | 0.1334 | 0.9997 |
| | (0.0000) | (11.5118) | (0.0206) | (0.0557) | (0.0035) | (61.1571) | | |
| USDCHF | 0.0005 | 0.0280 | 0.6534 | 0.5046 | 0.0902 | 0.7692 | 0.4781 | 0.9929 |
| | (0.0000) | (357.3861) | (0.0203) | (0.12550 | (0.0026) | (51.0413) | | |
| EURCHF | 0.0005 | 0.0196 | 0.5170 | 0.7784 | 0.0969 | 0.7897 | 0.4841 | 0.9927 |
| | (0.0000) | (2.4022) | (0.0214) | (0.0104) | (0.0028) | (21.7768) | | |
| AUDUSD | 0.0005 | 0.0240 | 0.5164 | 0.7376 | 0.1201 | 0.8417 | 0.1064 | 0.9998 |
| | (0.0000) | (6.4649) | (0.0264) | (0.0203) | (0.0037) | (34.0741) | | |
| NZDUSD | 0.0005 | 0.0291 | 0.5886 | 0.7330 | 0.1994 | 0.7101 | 0.1218 | 0.9997 |
| | (0.0000) | (4.0262) | (0.0227) | (0.0149) | (0.0044) | (19.2923) | | |
| USDCAD | 0.0004 | 0.0193 | 0.5956 | 0.7644 | 0.1420 | 0.7874 | 0.1673 | 0.9994 |
| | (0.0000) | (3.1899) | (0.0209) | (0.0160) | (0.0054) | (8.6125) | | |
| EURSEK | 0.0005 | 0.0270 | 0.5266 | 0.7383 | 0.2373 | 0.8164 | 0.1682 | 0.9994 |
| | (0.0000) | (2.4913) | (0.0238) | (0.0239) | (0.0064) | (12.3036) | | |
| EURNOK | 0.0004 | 0.0274 | 0.6121 | 0.9574 | 0.2780 | 0.8395 | 0.2730 | 0.9981 |
| | (0.0000) | (0.2481) | (0.0222) | (0.0139) | (0.0091) | (0.2035) | | |

**Table 4.14** – Inversion Model-10 Results with Only UIP

I only test the uncovered interest parity in my model with one parameter $\varphi$. I assume the market just have public information which is also interest rate information. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | J-test | P value of J-test |
|---|---|---|---|
| EURUSD | 0.0001<br>(0.0000) | 0.1498 | 0.9853 |
| USDJPY | 0.0001<br>(0.0000) | 0.0898 | 0.9930 |
| EURJPY | 0.0001<br>(0.0000) | 0.1484 | 0.9854 |
| GBPUSD | 0.0001<br>(0.0000) | 0.1109 | 0.9905 |
| EURGBP | 0.0485<br>(0.0059) | 0.0304 | 0.9986 |
| USDCHF | 0.0001<br>(0.0000) | 0.1154 | 0.9899 |
| EURCHF | 0.0001<br>(0.0000) | 0.1706 | 0.9822 |
| AUDUSD | 0.0001<br>(0.0000) | 0.2371 | 0.9714 |
| NZDUSD | 0.0001<br>(0.0000) | 0.0949 | 0.9924 |
| USDCAD | 0.0239<br>(0.0034) | 0.0179 | 0.9994 |
| EURSEK | 0.0085<br>(0.0012) | 0.0195 | 0.9993 |
| EURNOK | 0.0597<br>(0.0065) | 0.0380 | 0.9981 |

**Table 4.15** – Inversion Model-11 Results with Three Kinds of Investors with New Parameter $i$

I have 6 parameters in this model. I add new market participant in this model by parameter $\gamma$. I set the model-4 as the basic model and add the parameters back which I discuss in model-1. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $\gamma$ | $i$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.0945 | 0.6272 | 0.0657 | 0.3312 | 0.0061 | 0.0642 | 1.0000 |
| | (0.0002) | (0.0131) | (0.0164) | (0.0023) | (0.0500) | (0.0001) | | |
| USDJPY | 0.0005 | 0.0767 | 0.6711 | 0.0916 | 0.3640 | 0.0013 | 0.0677 | 1.0000 |
| | (0.0000) | (0.0086) | (0.0188) | (0.0026) | (0.0568) | (0.0000) | | |
| EURJPY | 0.0005 | 0.0834 | 0.6239 | 0.1034 | 0.3077 | 0.0005 | 0.0631 | 1.0000 |
| | (0.0000) | (0.0140) | (0.0205) | (0.0044) | (0.0353) | (0.0000) | | |
| GBPUSD | 0.0005 | 0.2458 | 0.5002 | 0.0868 | 0.6967 | 0.0165 | 0.0623 | 1.0000 |
| | (0.0006) | (26.8039) | (0.0197) | (0.0029) | (0.2283) | (0.0003) | | |
| EURGBP | 0.0005 | 0.1586 | 0.5074 | 0.1183 | 0.5845 | 0.0002 | 0.0591 | 1.0000 |
| | (0.0000) | (0.4047) | (0.0189) | (0.0035) | (0.0361) | (0.0000) | | |
| USDCHF | 0.0005 | 0.0792 | 0.6518 | 0.0891 | 0.2984 | 0.0018 | 0.0627 | 1.0000 |
| | (0.0000) | (0.0088) | (0.0168) | (0.0033) | (0.0336) | (0.0000) | | |
| EURCHF | 0.0005 | 0.2045 | 0.5164 | 0.1021 | 0.3043 | 0.0004 | 0.0630 | 1.0000 |
| | (0.0000) | (0.2860) | (0.0229) | (0.0034) | (0.0295) | (0.0000) | | |
| AUDUSD | 0.0006 | 0.1085 | 0.5004 | 0.1213 | 0.1574 | 0.0101 | 0.0625 | 1.0000 |
| | (0.0004) | (5.9031) | (0.0235) | (0.0046) | (0.2746) | (0.0002) | | |
| NZDUSD | 0.0005 | 0.1823 | 0.5002 | 0.1974 | 0.1408 | 0.0052 | 0.0627 | 1.0000 |
| | (0.0002) | (20.9954) | (0.0214) | (0.0078) | (0.3792) | (0.0001) | | |
| USDCAD | 0.0007 | 0.4114 | 0.5321 | 0.0000 | 0.0000 | 0.0005 | 2.0412 | 0.9960 |
| | (0.0000) | (0.2410) | (0.0184) | (0.0065) | (3136.7605) | (0.0000) | | |
| EURSEK | 0.0005 | 0.1634 | 0.5032 | 0.2383 | 0.2545 | 0.0108 | 0.0611 | 1.0000 |
| | (0.0004) | (1.1719) | (0.0232) | (0.0058) | (0.0443) | (0.0003) | | |
| EURNOK | 0.0005 | 0.1218 | 0.5535 | 0.2667 | 0.2830 | 0.0034 | 0.0623 | 1.0000 |
| | (0.0001) | (0.0499) | (0.0215) | (0.0089) | (0.0489) | (0.0001) | | |

**Table 4.16** – Inversion Model-12 Results with Three Kinds of Investors and Parameter $v$

I have 7 parameters in this model. I set the model-4 as the basic model and add the parameters back which I discuss in model-2.$\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | $\gamma$ | $i$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.2188 | 0.6423 | 0.7522 | 0.0658 | 0.3242 | 0.0056 | 0.0628 | 1.0000 |
| | (0.0002) | (0.0274) | (0.0164) | (0.0124) | (0.0023) | (0.0200) | (0.0001) | | |
| USDJPY | 0.0004 | 0.3396 | 0.5681 | 0.7924 | 0.0935 | 0.0960 | 0.0010 | 0.1778 | 1.0000 |
| | (0.0000) | (0.0949) | (0.0185) | (0.0097) | (0.0026) | (0.0199) | (0.0000) | | |
| EURJPY | 0.0005 | 0.1780 | 0.6247 | 0.7437 | 0.1032 | 0.2642 | 0.0004 | 0.0631 | 1.0000 |
| | (0.0000) | (0.0301) | (0.0205) | (0.0168) | (0.0044) | (0.0339) | (0.0000) | | |
| GBPUSD | 0.0008 | 0.0548 | 0.5010 | 0.5135 | 0.0869 | 0.0141 | 0.0167 | 0.0641 | 1.0000 |
| | (0.0006) | (1.0718) | (0.0197) | (2.6669) | (0.0028) | (7.3702) | (0.0003) | | |
| EURGBP | 0.0005 | 0.1994 | 0.5062 | 0.7562 | 0.1179 | 0.2956 | 0.0002 | 0.0629 | 1.0000 |
| | (0.0000) | (0.6088) | (0.0189) | (0.1789) | (0.0035) | (0.4126) | (0.0000) | | |
| USDCHF | 0.0005 | 0.1470 | 0.6843 | 0.7254 | 0.0912 | 0.3313 | 0.0015 | 0.0718 | 1.0000 |
| | (0.0000) | (0.0152) | (0.0169) | (0.0244) | (0.0033) | (0.0768) | (0.0000) | | |
| EURCHF | 0.0005 | 0.2575 | 0.5135 | 0.7438 | 0.1021 | 0.3511 | 0.0004 | 0.0630 | 1.0000 |
| | (0.0000) | (0.4309) | (0.0224) | (0.0348) | (0.0034) | (0.1009) | (0.0000) | | |
| AUDUSD | 0.0006 | 0.1946 | 0.5003 | 0.7373 | 0.1207 | 0.1419 | 0.0102 | 0.0627 | 1.0000 |
| | (0.0004) | (18.8731) | (0.0246) | (22.2224) | (0.0043) | (21.2324) | (0.0002) | | |
| NZDUSD | 0.0005 | 0.1566 | 0.5003 | 0.6963 | 0.1996 | 0.2702 | 0.0052 | 0.0631 | 1.0000 |
| | (0.0002) | (12.9608) | (0.0215) | (1.5601) | (0.0079) | (4.2699) | (0.0001) | | |
| USDCAD | 0.0005 | 0.2670 | 0.5774 | 0.7610 | 0.1520 | 0.2645 | 0.0005 | 0.0704 | 1.0000 |
| | (0.0000) | (0.0592) | (0.0173) | (0.0145) | (0.0066) | (0.0521) | (0.0000) | | |
| EURSEK | 0.0005 | 0.1604 | 0.5032 | 0.7207 | 0.2378 | 0.3476 | 0.0108 | 0.0612 | 1.0000 |
| | (0.0004) | (1.1570) | (0.0231) | (0.0408) | (0.0059) | (0.1514) | (0.0003) | | |
| EURNOK | 0.0005 | 0.1372 | 0.5652 | 0.7425 | 0.2611 | 0.2584 | 0.0034 | 0.0671 | 1.0000 |
| | (0.0001) | (0.0451) | (0.0213) | (0.0121) | (0.0098) | (0.0364) | (0.0001) | | |

**Table 4.17** – Inversion Model-13 Results with Three Kinds of Investors and Parameter $h$

I have 8 parameters in this model. I set the model-4 as the basic model and add the parameters back which I discuss in model-3.$\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. The parameter $h$ denotes the informed traders who choose to follow the private information signal. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | $\gamma$ | $h$ | $i$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0027 | 0.3612 | 0.6507 | 0.7677 | 0.0663 | 0.3029 | 0.7923 | 0.0053 | 0.0630 | 1.0000 |
| | (0.0002) | (4187.0431) | (0.0171) | (0.2527) | (0.0023) | (0.5948) | (277.5363) | (0.0001) | | |
| USDJPY | 0.0000 | 0.2149 | 0.5083 | 0.5439 | 0.0792 | 0.1109 | 1.0000 | 0.0000 | 6.6965 | 0.5697 |
| | (0.0000) | (8.5502E+04) | (0.0186) | (0.5379) | (0.0026) | (1.5144) | (6471.1762) | (0.0000) | | |
| EURJPY | 0.0029 | 0.2758 | 0.6270 | 0.7600 | 0.1032 | 0.2745 | 0.8054 | 0.0004 | 0.0627 | 1.0000 |
| | (0.0001) | (6.7283E+05) | (0.0211) | (0.7948) | (0.0042) | (1.6687) | (1.9452E+04) | (0.0001) | | |
| GBPUSD | 0.0049 | 0.4508 | 0.5122 | 0.6991 | 0.1297 | 0.2961 | 0.9359 | 0.0168 | 0.4091 | 0.9999 |
| | (0.0005) | (162.9241) | (0.0199) | (0.0713) | (0.0033) | (0.1628) | (14.5682) | (0.0002) | | |
| EURGBP | 0.0030 | 0.0749 | 0.5106 | 0.7495 | 0.1169 | 0.2547 | 0.6667 | 0.0002 | 0.0604 | 1.0000 |
| | (0.0001) | (495.7177) | (0.0189) | (0.1241) | (0.0035) | (0.2231) | (2069.8324) | (0.0001) | | |
| USDCHF | 0.0001 | 0.0270 | 0.6373 | 0.5019 | 0.0015 | 0.9950 | 0.9170 | 0.0001 | 5.8036 | 0.6692 |
| | (0.0000) | (451.4853) | (0.0174) | (0.1376) | (0.0028) | (0.8389) | (2215.4144) | (0.0000) | | |
| EURCHF | 0.0027 | 0.0616 | 0.5106 | 0.5000 | 0.0974 | 0.0488 | 0.5107 | 0.0000 | 1.0104 | 0.9982 |
| | (0.0000) | (742.6417) | (0.0222) | (0.0869) | (0.0032) | (0.1168) | (5643.0737) | (0.0000) | | |
| AUDUSD | 0.0032 | 0.1376 | 0.5623 | 0.5014 | 0.1231 | 0.0859 | 0.7132 | 0.0096 | 0.1774 | 1.0000 |
| | (0.0002) | (2.4185E+06) | (0.0226) | (0.0904) | (0.0046) | (0.1786) | (3.0930E+06) | (0.0001) | | |
| NZDUSD | 0.0037 | 0.4157 | 0.6889 | 0.7332 | 0.2100 | 0.1827 | 0.6979 | 0.0050 | 0.2006 | 1.0000 |
| | (0.0001) | (3.5135E+07) | (0.0223) | (0.1116) | (0.0086) | (0.1178) | (2.3510E+06) | (0.0001) | | |
| USDCAD | 0.0027 | 0.0396 | 0.6039 | 0.5234 | 0.1422 | 0.0369 | 0.7887 | 0.0001 | 0.0715 | 1.0000 |
| | (0.0001) | (2338.1697) | (0.0169) | (0.7803) | (0.0066) | (1.3722) | (2321.2356) | (0.0001) | | |
| EURSEK | 0.0033 | 0.2543 | 0.6122 | 0.6896 | 0.2376 | 0.3002 | 0.7771 | 0.0110 | 0.1053 | 1.0000 |
| | (0.0003) | (233.4782) | (0.0234) | (0.0501) | (0.0057) | (0.1079) | (36.3772) | (0.0002) | | |
| EURNOK | 0.0029 | 0.3100 | 0.5942 | 0.6175 | 0.2834 | 0.0081 | 0.8246 | 0.0026 | 0.1037 | 1.0000 |
| | (0.0001) | (1024.6304) | (0.0220) | (0.1523) | (0.0096) | (0.0847) | (55.5704) | (0.0001) | | |

**Table 4.18** – Inversion Model-14 Results with Three Kinds of Investors with New Parameter $i$

I have 7 parameters in this model. I add new market participant in this model by parameter $\theta$. I set the model-4 as the basic model and add the parameters back which I discuss in model-1.$\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. $\theta$ is the proportion of the traders who follow the carry trade strategy. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $\gamma$ | $i$ | $\theta$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.2205 | 0.6416 | 0.0657 | 0.2741 | 0.0053 | 0.2548 | 0.0628 | 1.0000 |
| | (0.0001) | (0.0283) | (0.0165) | (0.0023) | (0.0133) | (0.0001) | (0.0137) | | |
| USDJPY | 0.0003 | 0.3384 | 0.5844 | 0.0961 | 0.2227 | 0.0010 | 0.4959 | 0.2072 | 1.0000 |
| | (0.0000) | (0.0775) | (0.0188) | (0.0026) | (0.0042) | (0.0000) | (0.0613) | | |
| EURJPY | 0.0005 | 0.1320 | 0.6785 | 0.1056 | 0.2654 | 0.0004 | 0.2677 | 0.0773 | 1.0000 |
| | (0.0000) | (0.0165) | (0.0205) | (0.0044) | (0.0107) | (0.0000) | (0.0185) | | |
| GBPUSD | 0.0005 | 0.0994 | 0.5006 | 0.0855 | 0.3972 | 0.0167 | 0.2838 | 0.0658 | 1.0000 |
| | (0.0006) | (3.1927) | (0.0197) | (0.0029) | (2.9329) | (0.0003) | (0.9364) | | |
| EURGBP | 0.0005 | 0.1377 | 0.5098 | 0.1178 | 0.2799 | 0.0002 | 0.2746 | 0.0592 | 1.0000 |
| | (0.0000) | (0.2629) | (0.0189) | (0.0035) | (0.0342) | (0.0000) | (0.0317) | | |
| USDCHF | 0.0006 | 0.2501 | 0.6039 | 0.0851 | 0.6925 | 0.0015 | 0.1641 | 0.2140 | 1.0000 |
| | (0.0000) | (0.0419) | (0.0167) | (0.0032) | (0.0127) | (0.0000) | (0.0040) | | |
| EURCHF | 0.0005 | 0.1890 | 0.5196 | 0.1020 | 0.2342 | 0.0004 | 0.2295 | 0.0632 | 1.0000 |
| | (0.0000) | (0.2230) | (0.0229) | (0.0034) | (0.0076) | (0.0000) | (0.0188) | | |
| AUDUSD | 0.0006 | 0.2309 | 0.5002 | 0.1207 | 0.2242 | 0.0101 | 0.2991 | 0.0627 | 1.0000 |
| | (0.0004) | (28.5098) | (0.0238) | (0.0044) | (23.9091) | (0.0002) | (41.3294) | | |
| NZDUSD | 0.0009 | 0.1474 | 0.5679 | 0.0000 | 0.2718 | 0.0051 | 0.4133 | 2.1577 | 0.9950 |
| | (0.0002) | (28.5323) | (0.0214) | (0.0068) | (19.4888) | (0.0001) | (50.1916) | | |
| USDCAD | 0.0005 | 0.1656 | 0.6092 | 0.1428 | 0.2651 | 0.0004 | 0.2674 | 0.0624 | 1.0000 |
| | (0.0000) | (0.0264) | (0.0170) | (0.0068) | (0.0156) | (0.0000) | (0.0265) | | |
| EURSEK | 0.0005 | 0.1597 | 0.5034 | 0.2383 | 0.2517 | 0.0108 | 0.2499 | 0.0611 | 1.0000 |
| | (0.0004) | (1.0891) | (0.0231) | (0.0058) | (0.0369) | (0.0003) | (0.0383) | | |
| EURNOK | 0.0006 | 0.1280 | 0.5337 | 0.2193 | 0.1455 | 0.0012 | 0.1343 | 0.1448 | 1.0000 |
| | (0.0000) | (0.0825) | (0.0215) | (0.0101) | (0.0096) | (0.0000) | (0.0115) | | |

**Table 4.19** – Inversion Model-15 Results with Three Kinds of Investors and Parameter $v$

I have 8 parameters in this model.$\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. $\theta$ is the proportion of the traders who follow the carry trade strategy. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | $\gamma$ | $i$ | $\theta$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0004 | 0.1513 | 0.7080 | 0.7185 | 0.0687 | 0.2277 | 0.0053 | 0.2409 | 0.0972 | 1.0000 |
| | (0.0001) | (0.0139) | (0.0170) | (0.0627) | (0.0024) | (0.0828) | (0.0001) | (0.0298) | | |
| USDJPY | 0.0005 | 0.2267 | 0.6391 | 0.6345 | 0.0914 | 0.3234 | 0.0010 | 0.2433 | 0.0730 | 1.0000 |
| | (0.0000) | (0.0335) | (0.0192) | (0.1186) | (0.0027) | (0.0309) | (0.0000) | (0.0253) | | |
| EURJPY | 0.0005 | 0.2007 | 0.6265 | 0.7549 | 0.1032 | 0.2621 | 0.0004 | 0.2549 | 0.0630 | 1.0000 |
| | (0.0000) | (0.0352) | (0.0206) | (0.0499) | (0.0044) | (0.1111) | (0.0000) | (0.0943) | | |
| GBPUSD | 0.0005 | 0.0775 | 0.5005 | 0.6795 | 0.0872 | 0.5283 | 0.0167 | 0.1206 | 0.0636 | 1.0000 |
| | (0.0006) | (3.4442) | (0.0197) | (6.2926) | (0.0032) | (13.9695) | (0.0003) | (3.8201) | | |
| EURGBP | 0.0005 | 0.1643 | 0.5081 | 0.6872 | 0.1175 | 0.2652 | 0.0002 | 0.2762 | 0.0607 | 1.0000 |
| | (0.0000) | (0.3799) | (0.0188) | (0.3745) | (0.0035) | (0.2671) | (0.0000) | (0.2936) | | |
| USDCHF | 0.0005 | 0.3879 | 0.5768 | 0.7036 | 0.0867 | 0.1813 | 0.0014 | 0.0903 | 0.1462 | 1.0000 |
| | (0.0000) | (0.0870) | (0.0168) | (0.0208) | (0.0033) | (0.0676) | (0.0000) | (0.0455) | | |
| EURCHF | 0.0005 | 0.1740 | 0.5215 | 0.7540 | 0.1019 | 0.2522 | 0.0004 | 0.2447 | 0.0631 | 1.0000 |
| | (0.0000) | (0.1837) | (0.0229) | (0.0712) | (0.0034) | (0.0477) | (0.0000) | (0.0340) | | |
| AUDUSD | 0.0005 | 0.1732 | 0.5003 | 0.6430 | 0.1209 | 0.3048 | 0.0101 | 0.0710 | 0.0626 | 1.0000 |
| | (0.0004) | (12.9789) | (0.0232) | (6.5917) | (0.0048) | (34.8342) | (0.0002) | (6.8605) | | |
| NZDUSD | 0.0005 | 0.1659 | 0.5003 | 0.7297 | 0.1996 | 0.2344 | 0.0052 | 0.2403 | 0.0632 | 1.0000 |
| | (0.0002) | (14.3896) | (0.0218) | (6.4478) | (0.0080) | (9.2405) | (0.0001) | (8.2974) | | |
| USDCAD | 0.0004 | 0.1549 | 0.6154 | 0.7235 | 0.1425 | 0.1117 | 0.0004 | 0.2009 | 0.0680 | 1.0000 |
| | (0.0000) | (0.0247) | (0.0179) | (0.0571) | (0.0068) | (0.0901) | (0.0000) | (0.0505) | | |
| EURSEK | 0.0007 | 0.1206 | 0.5020 | 0.5499 | 0.2440 | 0.2663 | 0.0003 | 0.3042 | 6.5235 | 0.6866 |
| | (0.0000) | (1.3954) | (0.0235) | (3.8725) | (0.0059) | (4.7548) | (0.0000) | (1.8301) | | |
| EURNOK | 0.0006 | 0.1370 | 0.5266 | 0.7692 | 0.2364 | 0.5261 | 0.0011 | 0.0749 | 0.1788 | 1.0000 |
| | (0.0000) | (0.1190) | (0.0218) | (0.0881) | (0.0105) | (0.1039) | (0.0000) | (0.0393) | | |

**Table 4.20** – Inversion Model-16 Results with Three Kinds of Investors and Parameter $h$

I have 9 parameters in this model. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. $\theta$ is the proportion of the traders who follow the carry trade strategy. $h$ denotes the informed traders who choose to follow the private information signal. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $\alpha$ | $\gamma$ | $i$ | $\theta$ | $h$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0001 | 0.4363 | 0.5763 | 0.7420 | 0.0911 | 0.1419 | 0.0010 | 0.7569 | 0.8680 | 3.4539 | 0.9027 |
| | (0.0000) | (1.0675E+05) | (0.0172) | (4276.3968) | (0.0023) | (98.8367) | (0.0000) | (80.0123) | (98.9311) | | |
| USDJPY | 0.0026 | 0.4069 | 0.6681 | 0.9164 | 0.0905 | 0.0565 | 0.0010 | 0.0526 | 0.7499 | 0.0610 | 1.0000 |
| | (0.0001) | (1141.7909) | (0.0199) | (1.8236) | (0.0028) | (0.9678) | (0.0001) | (0.9015) | (4.2653) | | |
| EURJPY | 0.0029 | 0.2560 | 0.6255 | 0.6576 | 0.1036 | 0.2863 | 0.0004 | 0.3382 | 0.7900 | 0.0628 | 1.0000 |
| | (0.0001) | (1.4544E+04) | (0.0204) | (3.0065) | (0.0045) | (0.8021) | (0.0001) | (0.6662) | (528.2677) | | |
| GBPUSD | 0.0049 | 0.3213 | 0.6814 | 0.8039 | 0.1005 | 0.2530 | 0.0165 | 0.3121 | 0.6506 | 0.2918 | 1.0000 |
| | (0.0004) | (3552.9456) | (0.0202) | (0.1719) | (0.0032) | (0.1090) | (0.0003) | (0.0411) | (920.2485) | | |
| EURGBP | 0.0031 | 0.2813 | 0.5258 | 0.7890 | 0.1172 | 0.2738 | 0.0002 | 0.2719 | 0.8404 | 0.0605 | 1.0000 |
| | (0.0001) | (1.1148E+04) | (0.0189) | (0.1608) | (0.0036) | (0.2075) | (0.0001) | (0.1877) | (4383.3404) | | |
| USDCHF | 0.0027 | 0.3693 | 0.6446 | 0.7568 | 0.0926 | 0.2375 | 0.0014 | 0.2502 | 0.7696 | 0.0676 | 1.0000 |
| | (0.0001) | (9501.9713) | (0.0181) | (0.9126) | (0.0031) | (1.1225) | (0.0001) | (1.2149) | (158.2888) | | |
| EURCHF | 0.0027 | 0.0645 | 0.5734 | 0.9708 | 0.1224 | 0.0888 | 0.0003 | 0.1132 | 0.6388 | 0.4495 | 0.9999 |
| | (0.0000) | (163.4989) | (0.0231) | (0.0399) | (0.0033) | (0.0462) | (0.0001) | (0.0517) | (591.5039) | | |
| AUDUSD | 0.0047 | 0.2544 | 0.5669 | 0.7272 | 0.1212 | 0.2465 | 0.0104 | 0.3038 | 0.7538 | 0.0804 | 1.0000 |
| | (0.0003) | (504.2045) | (0.0246) | (0.2880) | (0.0048) | (0.1379) | (0.0002) | (0.1280) | (253.9973) | | |
| NZDUSD | 0.0038 | 0.2726 | 0.6021 | 0.7539 | 0.2054 | 0.2285 | 0.0052 | 0.2483 | 0.7580 | 0.1022 | 1.0000 |
| | (0.0001) | (933.2531) | (0.0220) | (0.1512) | (0.0080) | (0.1068) | (0.0001) | (0.0841) | (247.9677) | | |
| USDCAD | 0.0032 | 0.0591 | 0.6187 | 0.8670 | 0.1101 | 0.1812 | 0.0008 | 0.0280 | 0.7131 | 0.3666 | 1.0000 |
| | (0.0001) | (61.7136) | (0.0172) | (0.1700) | (0.0064) | (0.2712) | (0.0001) | (0.0478) | (99.3052) | | |
| EURSEK | 0.0031 | 0.2975 | 0.5289 | 0.7728 | 0.2371 | 0.2141 | 0.0109 | 0.2666 | 0.8815 | 0.0647 | 1.0000 |
| | (0.0003) | (1.1059E+04) | (0.0235) | (0.1776) | (0.0060) | (0.1262) | (0.0003) | (0.0707) | (2372.1182) | | |
| EURNOK | 0.0036 | 0.1684 | 0.6321 | 0.7896 | 0.2609 | 0.2439 | 0.0033 | 0.2226 | 0.7461 | 0.1013 | 1.0000 |
| | (0.0002) | (4893.9867) | (0.0222) | (0.3163) | (0.0100) | (0.1972) | (0.0002) | (0.1314) | (1304.5137) | | |

**Table 4.21** – Inversion Model-17 Results with 4 Kinds of Investors and Parameter $h$

I have 10 parameters in this model. I add market participant in this model by parameter $A\&H$. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. The parameter $v$ is the proportion of uninformed traders who choose to follow the public information. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. $\theta$ is the proportion of the traders who follow the carry trade strategy. $h$ denotes the informed traders who choose to follow the private information signal. $A$ denotes the proportion of the asset managers, while $H$ is the proportion of the Hedge funds in the forex market.The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $v$ | $A$ | $\gamma$ | $i$ | $\theta$ | $h$ | $H$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0026 | 0.2486 | 0.6421 | 0.7552 | 0.0333 | 0.2545 | 0.0051 | 0.2662 | 0.7539 | 0.0346 | 0.0608 | 1.0000 |
| | (0.0001) | (9464.8552) | (0.0170) | (0.0313) | (0.0016) | (0.0260) | (0.0002) | (0.0274) | (7813.6715) | (0.0017) | | |
| USDJPY | 0.0023 | 0.0684 | 0.7797 | 0.6526 | 0.0457 | 0.3575 | 0.0010 | 0.2239 | 0.9629 | 0.0482 | 0.1780 | 1.0000 |
| | (0.0021) | (1.0573E+07) | (0.2381) | (13.8205) | (0.0266) | (18.7038) | (0.0008) | (11.9080) | (1.0819E+08) | (0.0066) | | |
| EURJPY | 0.0030 | 0.4961 | 0.6600 | 0.7500 | 0.0544 | 0.2486 | 0.0004 | 0.2479 | 0.5976 | 0.0508 | 0.0699 | 1.0000 |
| | (0.0001) | (4.7609E+05) | (0.0268) | (1.8111) | (0.0031) | (1.5039) | (0.0000) | (1.5058) | (6.3681E+04) | (0.0026) | | |
| GBPUSD | 0.0040 | 0.4948 | 0.5212 | 0.7605 | 0.0684 | 0.2862 | 0.0135 | 0.2447 | 0.8741 | 0.0210 | 0.6834 | 0.9996 |
| | (0.0001) | (147.2703) | (0.0196) | (0.0019) | (0.0023) | (0.0041) | (0.0003) | (0.0021) | (21.6412) | (0.0017) | | |
| EURGBP | 0.0029 | 0.2964 | 0.5457 | 0.7544 | 0.0731 | 0.2384 | 0.0002 | 0.2439 | 0.7375 | 0.0464 | 0.0686 | 1.0000 |
| | (0.0001) | (2.0714E+04) | (0.0189) | (0.4302) | (0.0030) | (0.3517) | (0.0000) | (0.3567) | (8784.0724) | (0.0020) | | |
| USDCHF | 0.0026 | 0.4644 | 0.6787 | 0.7407 | 0.0420 | 0.2595 | 0.0014 | 0.2487 | 0.5627 | 0.0495 | 0.0713 | 1.0000 |
| | (0.0001) | (1.2269E+04) | (0.0176) | (0.1831) | (0.0019) | (0.1586) | (0.0000) | (0.1558) | (3584.5175) | (0.0027) | | |
| EURCHF | 0.0029 | 0.3918 | 0.5220 | 0.7422 | 0.0464 | 0.2633 | 0.0004 | 0.2663 | 0.7837 | 0.0557 | 0.0632 | 1.0000 |
| | (0.0000) | (2.3321E+04) | (0.0234) | (0.2005) | (0.0025) | (0.1656) | (0.0000) | (0.1649) | (7350.6893) | (0.0021) | | |
| AUDUSD | 0.0043 | 0.4049 | 0.5137 | 0.7555 | 0.0771 | 0.2595 | 0.0093 | 0.2290 | 0.8368 | 0.0773 | 0.1529 | 1.0000 |
| | (0.0003) | (7.7876E+05) | (0.0369) | (4.6168) | (0.0179) | (3.5909) | (0.0005) | (3.2514) | (2.1092E+05) | (0.0045) | | |
| NZDUSD | 0.0039 | 0.3552 | 0.5063 | 0.7528 | 0.0934 | 0.2208 | 0.0051 | 0.2442 | 0.8454 | 0.1044 | 0.0628 | 1.0000 |
| | (0.0001) | (1.7128E+04) | (0.0220) | (0.2544) | (0.0041) | (0.1740) | (0.0001) | (0.1857) | (6183.6195) | (0.0061) | | |
| USDCAD | 0.0027 | 0.3719 | 0.5969 | 0.7249 | 0.0739 | 0.2441 | 0.0004 | 0.2346 | 0.7151 | 0.0770 | 0.0561 | 1.0000 |
| | (0.0002) | (1.3065E+04) | (0.0195) | (0.3789) | (0.0042) | (0.3319) | (0.0000) | (0.3227) | (1640.0261) | (0.0048) | | |
| EURSEK | 0.0001 | 0.4824 | 0.5485 | 0.5001 | 0.0602 | 0.5677 | 0.0000 | 0.0000 | 0.7319 | 0.1372 | 6.1581 | 0.6295 |
| | (0.0000) | (170.0294) | (0.0243) | (0.0342) | (0.0047) | (0.4584) | (0.0000) | (0.0207) | (7.7624) | (0.0050) | | |
| EURNOK | 0.0035 | 0.2971 | 0.5509 | 0.7544 | 0.1443 | 0.2089 | 0.0031 | 0.2138 | 0.7880 | 0.1276 | 0.0619 | 1.0000 |
| | (0.0002) | (1.3606E+06) | (0.0276) | (4.7164) | (0.0290) | (2.9519) | (0.0002) | (3.0252) | (1.7321E+05) | (0.0156) | | |

**Table 4.22** – Inversion model-18 Results with Three Kinds of Investors with New Parameter $i$

I consider the carry trade influence here. I have 7 parameters in this model. $\varphi$ denotes the public information, while $\varepsilon$ is the private information. $q$ is the probability of the informed traders have the right private information signal, and $\alpha$ is the proportion of the informed trader in the market. $\gamma$ is the proportion of the traders who only focus on the opportunities for UIP arbitrage. $i$ denotes the interest rate information. The standard error of the estimation are given in the parenthesis.

| | $\varphi$ | $\varepsilon$ | $q$ | $\alpha$ | $\gamma$ | $i$ | $\theta$ | J-test | P value of J-test |
|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0005 | 0.2397 | 0.6069 | 0.0732 | 0.2086 | 0.0118 | 0.2119 | 0.1955 | 1.0000 |
| | (0.0000) | (0.0643) | (0.0178) | (0.0022) | (0.0573) | (0.0083) | (0.0573) | | |
| USDJPY | 0.0005 | 0.1747 | 0.6761 | 0.0934 | 0.6103 | 0.0001 | 0.2345 | 0.1829 | 1.0000 |
| | (0.0000) | (0.0204) | (0.0192) | (0.0026) | (0.0371) | (0.0000) | (0.0164) | | |
| EURJPY | 0.0005 | 0.1907 | 0.6247 | 0.1065 | 0.2755 | 0.0037 | 0.2779 | 0.0670 | 1.0000 |
| | (0.0000) | (0.0346) | (0.0208) | (0.0044) | (0.0177) | (0.0032) | (0.0175) | | |
| GBPUSD | 0.0005 | 0.0992 | 0.5001 | 0.0870 | 0.0287 | 0.0227 | 0.0292 | 0.0630 | 1.0000 |
| | (0.0000) | (14.6173) | (0.0189) | (0.0031) | (2.3655) | (0.0052) | (2.3657) | | |
| EURGBP | 0.0005 | 0.2010 | 0.5065 | 0.1178 | 0.2499 | 0.0169 | 0.2491 | 0.0700 | 1.0000 |
| | (0.0000) | (0.5858) | (0.0189) | (0.0035) | (0.0378) | (0.0119) | (0.0370) | | |
| USDCHF | 0.0005 | 0.2734 | 0.6101 | 0.0871 | 0.2642 | 0.0132 | 0.2661 | 0.0842 | 1.0000 |
| | (0.0000) | (0.0435) | (0.0168) | (0.0033) | (0.0045) | (0.0050) | (0.0046) | | |
| EURCHF | 0.0005 | 0.1924 | 0.5182 | 0.1023 | 0.2509 | 0.0014 | 0.2511 | 0.0636 | 1.0000 |
| | (0.0000) | (0.2397) | (0.0225) | (0.0032) | (0.0074) | (0.0038) | (0.0081) | | |
| AUDUSD | 0.0005 | 0.1621 | 0.5004 | 0.1208 | 0.2735 | 0.0143 | 0.2740 | 0.0631 | 1.0000 |
| | (0.0000) | (8.8054) | (0.0207) | (0.0047) | (6.3138) | (0.0051) | (6.3140) | | |
| NZDUSD | 0.0005 | 0.1545 | 0.5003 | 0.1992 | 0.2606 | 0.0075 | 0.2614 | 0.0632 | 1.0000 |
| | (0.0000) | (12.5479) | (0.0217) | (0.0075) | (1.1452) | (0.0049) | (1.1452) | | |
| USDCAD | 0.0005 | 0.1929 | 0.5921 | 0.1485 | 0.2601 | 0.0145 | 0.2610 | 0.0704 | 1.0000 |
| | (0.0000) | (0.0372) | (0.0177) | (0.0069) | (0.0049) | (0.0073) | (0.0050) | | |
| EURSEK | 0.0005 | 0.1670 | 0.5007 | 0.2369 | 0.2607 | 0.0154 | 0.2599 | 0.0627 | 1.0000 |
| | (0.0000) | (5.2796) | (0.0233) | (0.0056) | (0.0938) | (0.0059) | (0.0938) | | |
| EURNOK | 0.0006 | 0.1341 | 0.5277 | 0.2691 | 0.1154 | 0.0081 | 0.1162 | 0.0639 | 1.0000 |
| | (0.0000) | (0.1037) | (0.0212) | (0.0095) | (0.0182) | (0.0063) | (0.0181) | | |

## Table 4.23 – $Plim\hat{\beta}$ and Expected Return of Informed Traders $\pi_i^e$

Since model-4 just have one parameter $\varphi$ and only one investor type UIP arbitrager, I only estimate the value of $plim\hat{\beta}$ and expected return of informed traders $\pi_i^e$ for other models.

| | | model-7 | model-8 | model-9 | model-11 | model-12 | model-13 | model-14 | model-15 | model-16 | model-17 | model-18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| EURUSD | $plim\hat{\beta}$ | -1.3352 | -22.1746 | 0.9356 | 0.5576 | 0.7713 | 0.9908 | 0.8770 | 0.8941 | 0.9757 | 0.9929 | 0.0372 |
| | $\pi_i^e$ | 0.0002 | 0.0143 | -0.0007 | 0.0226 | 0.0568 | 0.0102 | 0.0575 | 0.0590 | 0.0002 | 0.0004 | 0.0462 |
| USDJPY | $plim\hat{\beta}$ | 1.0011 | -11.6236 | 1.0004 | -0.1672 | 0.1140 | 0.0020 | 0.1698 | 0.3027 | 1.0005 | 0.9444 | 0.0387 |
| | $\pi_i^e$ | 0.0000 | 0.0003 | 0.0002 | 0.0241 | 0.0405 | 0.0033 | 0.0509 | 0.0577 | 0.0007 | 0.0002 | 0.0548 |
| EURJPY | $plim\hat{\beta}$ | -0.6861 | -3.3000 | -1.1085 | -0.1196 | 0.0542 | 0.9960 | 0.0894 | 0.1828 | 0.9793 | 0.9738 | 0.0608 |
| | $\pi_i^e$ | 0.0004 | 0.0020 | 0.0007 | 0.0187 | 0.0387 | 0.0024 | 0.0414 | 0.0459 | -0.0027 | 0.0009 | 0.0418 |
| GBPUSD | $plim\hat{\beta}$ | -3.1801 | 1.0563 | 0.8400 | 1.0008 | 1.0000 | 0.9950 | 1.0000 | 1.0000 | 1.0227 | 0.9985 | 0.9997 |
| | $\pi_i^e$ | 0.0005 | 0.0002 | -0.0020 | 0.0001 | 0.0001 | -0.0157 | 0.0001 | 0.0001 | -0.0332 | -0.0009 | 0.0000 |
| EURGBP | $plim\hat{\beta}$ | -3.3843 | 1.0489 | 0.9483 | 0.2693 | 0.8931 | 0.7702 | 0.9513 | 1.0234 | 0.8109 | 0.9557 | 0.9261 |
| | $\pi_i^e$ | 0.0006 | 0.0006 | -0.0015 | 0.0021 | 0.0021 | -0.0204 | 0.0024 | 0.0024 | -0.0292 | -0.0006 | 0.0022 |
| USDCHF | $plim\hat{\beta}$ | 1.0002 | -2.0377 | 1.0002 | -0.1951 | 0.2073 | 0.0360 | 0.1779 | 0.4859 | 0.9947 | 0.9449 | 0.0415 |
| | $\pi_i^e$ | 0.0000 | 0.0145 | 0.0001 | 0.0220 | 0.0479 | 0.0039 | 0.0458 | 0.0542 | -0.0027 | 0.0012 | 0.0541 |
| EURCHF | $plim\hat{\beta}$ | -2.7717 | -25.2575 | 0.6855 | -0.3539 | 0.6206 | 0.9973 | 0.7184 | 0.9343 | 0.6294 | 0.9791 | 0.6789 |
| | $\pi_i^e$ | 0.0006 | 0.0036 | -0.0031 | 0.0061 | 0.0060 | -0.0266 | 0.0065 | 0.0068 | -0.0143 | -0.0007 | 0.0061 |
| AUDUSD | $plim\hat{\beta}$ | -2.4229 | 1.0623 | 0.7419 | 1.0014 | 1.0000 | 0.9992 | 1.0000 | 1.0000 | 1.0236 | 0.9957 | 0.9999 |
| | $\pi_i^e$ | 0.0007 | 0.0000 | -0.0027 | 0.0001 | 0.0001 | -0.0239 | 0.0001 | 0.0001 | -0.0330 | -0.0010 | 0.0001 |
| NZDUSD | $plim\hat{\beta}$ | -6.5353 | 1.0101 | 0.5452 | 1.0023 | 1.0000 | 0.9492 | 1.0000 | 0.9996 | 0.9926 | 0.9862 | 0.9999 |
| | $\pi_i^e$ | 0.0006 | 0.0000 | -0.0037 | 0.0001 | 0.0001 | -0.0121 | 0.0200 | 0.0001 | -0.0193 | -0.0011 | 0.0001 |
| USDCAD | $plim\hat{\beta}$ | -1.6051 | -0.5699 | 0.8211 | -0.0863 | 0.0370 | 1.0000 | 0.0937 | 0.1819 | 0.9596 | 0.9911 | 0.0594 |
| | $\pi_i^e$ | 0.0006 | 0.0009 | -0.0010 | 0.0264 | 0.0336 | -0.0016 | 0.0304 | 0.0310 | -0.0059 | 0.0004 | 0.0296 |
| EURSEK | $plim\hat{\beta}$ | -0.5988 | 1.0396 | 0.5861 | 1.0171 | 0.9999 | 0.9873 | 1.0000 | 1.0136 | 1.0354 | 0.0082 | 0.9989 |
| | $\pi_i^e$ | 0.0004 | 0.0002 | -0.0027 | 0.0008 | 0.0007 | -0.0090 | 0.0008 | 0.0004 | -0.0158 | 0.0002 | 0.0002 |
| EURNOK | $plim\hat{\beta}$ | -0.8882 | -0.1492 | 2.2555 | 0.6168 | 0.7192 | 0.9940 | 0.5228 | 0.5446 | 0.9827 | 0.9963 | 0.1524 |
| | $\pi_i^e$ | 0.0008 | 0.0004 | 0.0004 | 0.0099 | 0.0125 | -0.0044 | 0.0061 | 0.0051 | -0.0072 | -0.0004 | 0.0048 |

**Table 4.24** – Characteristics of The Data

| | Median bid-ask spread | Std of bid-ask spread | Std of Rate of depreciation | Std of Forward premium | Ratio | regression $\hat{\beta}$ | $MSE_S/MSE_F$ | $MSE_S - MSE_F$ | estimate of $MSE_S - MSE_F$ |
|---|---|---|---|---|---|---|---|---|---|
| EURUSD | 0.0002 | 0.0001 | 0.0143 | 0.0002 | 0.0172 | -0.0002 | 0.9992 | 0.0000 | 0.0002 |
| USDJPY | 0.0000 | 0.0000 | 0.0146 | 0.0003 | 0.0224 | -0.0001 | 1.0015 | 0.0000 | 0.0002 |
| EURJPY | 0.0000 | 0.0000 | 0.0173 | 0.0002 | 0.0139 | 0.0000 | 0.9996 | 0.0000 | 0.0003 |
| GBPUSD | 0.0004 | 0.0001 | 0.0142 | 0.0003 | 0.0183 | 0.0009 | 1.0006 | 0.0000 | 0.0002 |
| EURGBP | 0.0002 | 0.0001 | 0.0262 | 0.0004 | 0.0171 | 0.0005 | 1.0003 | 0.0000 | 0.0007 |
| USDCHF | 0.0005 | 0.0001 | 0.0158 | 0.0002 | 0.0156 | -0.0005 | 1.0009 | 0.0000 | 0.0003 |
| EURCHF | 0.0006 | 0.0002 | 0.0094 | 0.0001 | 0.0150 | -0.0016 | 0.9983 | 0.0000 | 0.0001 |
| AUDUSD | 0.0000 | 0.0001 | 0.0198 | 0.0003 | 0.0127 | 0.0003 | 0.9951 | 0.0000 | 0.0004 |
| NZDUSD | 0.0000 | 0.0000 | 0.0204 | 0.0002 | 0.0101 | 0.0008 | 0.9958 | 0.0000 | 0.0004 |
| USDCAD | 0.0005 | 0.0002 | 0.0142 | 0.0002 | 0.0130 | 0.0004 | 0.9999 | 0.0000 | 0.0002 |
| EURSEK | 0.0001 | 0.0000 | 0.0095 | 0.0003 | 0.0367 | -0.0053 | 0.9880 | 0.0000 | 0.0001 |
| EURNOK | 0.0001 | 0.0000 | 0.0107 | 0.0003 | 0.0239 | -0.0020 | 0.9944 | 0.0000 | 0.0001 |
| AVERAGE | 0.0002 | 0.0001 | 0.0155 | 0.0003 | 0.0180 | -0.0006 | 0.9978 | 0.0000 | 0.0003 |

# Appendix.4

## Appendix.A The Evolution of microstructure Model in the Forex Market

Frankel et al. (1997) illustrate the development and motivation of researchers in applying microstructure models in the foreign exchange market. Since implementing a wide-ranging floating exchange rate system in 1973, exchange rate economics has made considerable progress.

The asset market exchange rate approach has produced many models that have proven helpful for explaining and quantifying exchange rate changes. The first characteristic of these models is that they are macro models. In other words, they are highly aggregated. They try to capture and determine all the determinants that affect foreign exchange demand and supply, including those outside the foreign exchange market. Considering this macro approach, the focus of the model is on financial asset markets, so the focus is on the behavior of agents in these asset markets. There is a tension between comprehensive macro policies and emphasis on asset market dynamics.

Why study the microstructure of the foreign exchange market? Interest in the workings of foreign exchange markets stems, at least in part, from problems revealed by macro models of asset markets. The first is the initial contradiction between model and reality. As mentioned earlier, the model does not include asset transactions. On the contrary, one of the most important empirical facts of the foreign exchange market is that there is a lot of trading every day. This inconsistency raises the question of whether the standard model's inability to account for foreign exchange trading volumes is a sign of a more severe problem, which may prevent researchers from successfully explaining empirical phenomena that other researchers have focused on. These empirical phenomena include abnormal return behavior in the foreign exchange market, the difficulty of predicting exchange rates in the short term and the inability to explain exchange rate fluctuations after the fact. Macro models do not explain these phenomena satisfactorily.

Frankel et al. (1997) can naturally solve these empirical problems of the macro exchange rate model. These problems stem from the assumption of equilibrium in the asset market. From the perspective of the microstructure of the foreign exchange market, the description of the current microstructure of the foreign exchange market can provide a more satisfactory explanation for the microstructure of the foreign exchange market.

Sapp (2002) shows that certain banks are price leaders and their offers are ahead of others (Deutsche Bank and Chemical Companies), so they are actually price leaders. This result is supported by other empirical studies. So in these models, private information seems to be very relevant. Ito et al. (1998) found that volatility doubled after the introduction of midday trading. In the absence of any public information, this increase in volatility is likely due to customer order flow, so there must be at least some information content to some extent. Evans and Lyons (2005) developed a general equilibrium model which assumes that dealers adjust their quotes by adjusting their views of fundamentals based on the signals they receive from customer order flows. They looked at the disaggregated data over a long period of time but only looked at EUR/USD against currencies. The report's results show that the classification and aggregation models (based on user type and location (non-US or US)) improve the predictive power, but they do not mention any characteristics of each end-user segmentation.

The generalized moment method (GMM) was first introduced into econometric literature by Hansen (1982). It is widely used in economic and financial analysis data. The development of large-scale statistical inference technology based on GMM estimator stimulated and stimulated this interest. These are widely used in microeconomics, finance, agricultural economics, environmental economics, and labor economics. GMM has been applied to time series, cross-section, and panel data. Input in this book. These are the most widely used areas of GMM, and therefore have the most remarkable development impact. In the researches focus on the exchange rate, many authors choose the GMM model, see Hansen and Hodrick (1980), Melino and Turnbull (1990), Modjtahedi (1991), Bekaert and Hodrick (1992), Cumby and Huizinga (1992), Backus et al. (1993), İmrohoroğlu (1994), Dumas and Solnik (1995), Bekaert and Hodrick (2001), Groen and Kleibergen (2003).

Some authors apply the GMM model to deal with the microstructure in finance, such as Madhavan and Smidt (1993), Huang and Stoll (1997), Madhavan et al. (1997), Grammig and Wellner (2002). In the next section, I will introduce the GMM model.

# Appendix.B GMM Models

I will give a sample of GMM model in practice, following Hall et al. (2005). Let's show an example of the GMM model. Consider there is a Population Moment Condition. Let $\theta_0$ be a vector of unknown parameters which are to be estimated, $v_t$ be a vector of random variables and $f(.)$ a vector of functions, then a population moment condition takes the form

$$E[f(v_t, \theta_0)] = 0 \tag{4.171}$$

for all $t$.

When I want to give an example of the population moment, the researchers suggest estimating the parameter vector through the value implied by the corresponding sampling moment. I can abstract in generality and focus only on specific members, namely the normal distribution. This distribution depends on just two parameters: the population mean, $\mu_0$ , and the population variance,$\sigma_0^2$ . These two parameters satisfy the population moment conditions

$$E[v_t] - \mu_0 = 0 \tag{4.172}$$

$$E[v_t^2] - (\sigma_0^2 + \mu_0^2) = 0 \tag{4.173}$$

In the first step to create a GMM model, I need to find the moments related to the parameters which need to be estimated in the GMM model. In this circumstance, the moment condition can be obtained by putting

$$f(v_t, \theta) = \begin{bmatrix} v_t - \mu_0 \\ v_t^2 - (\sigma_0^2 + \mu_0^2) \end{bmatrix}$$

where $\theta_0 = (\mu_0, \sigma_0^2)$ .

Just as in Minimum Chi-Square, GMM involves choosing parameter estimators to minimize a quadratic form in a weighting matrix, $W_T$, and the sample moment $T^{-1} \sum_{t=1}^{T} f(v_t, \theta)$.

Generalized Method of Moments could help to estimate the value of the parameters based on the below equation, which minimizes :

$$Q_T(\theta) = T^{-1} \sum_{t=1}^{T} f(v_t, \theta)' W_T T^{-1} \sum_{t=1}^{T} f(v_t, \theta) \tag{4.174}$$

where $W_T$ is a positive semi-definite matrix that may depend on the data but converges in probability to a positive definite matrix of constants. The restrictions on the weighting matrix are required to ensure that $Q_T(\theta)$ is a meaningful measure of distance. Notice that the positive semi-definiteness of $W_T$ ensures both that $Q_T(\theta) \geq 0$ for any $\theta$ , and also that $Q_T(\hat{\theta}_T) = 0$ if $T^{-1} \sum_{t=1}^{T} f(v_t, \theta)$. Hansen (1982) referrers to the estimator as Generalized Method of Moments, and that is the name by which the method is known in econometric.

# Appendix.C The Basic Model of Section 4.5

Follow the previous literature, I assume the hedge funds and asset managers are informed investors. The spot rate $S_{t+1}$ at term $t + 1$ include the public information for term $t$ and private information for term $t+1$. Hence the relationship between $S_{t+1}$ and $S_t$ would be as below:

$$\frac{S_{t+1} - S_t}{S_t} = \phi_t + \varepsilon_{t+1} + \omega_{t+1} \tag{4.175}$$

where $\varphi_t$ is the public information influence at term $t$, which could be observed by all participants. I assume the influence would be positive or negative in the same probability.

$$\phi_t = \begin{cases} \phi \, with \, probability & 1/2, \\ -\phi \, with \, probability & 1/2. \end{cases} \tag{4.176}$$

where $\varepsilon_{t+1}$ is not observed directly at time $t$ which could be observed by informed investors as a signal $\zeta_t \in \{\varepsilon, -\varepsilon\}$.

$$\varepsilon_{t+1} = \begin{cases} \varepsilon \, with \, probability & 1/2, \\ -\varepsilon \, with \, probability & 1/2. \end{cases} \tag{4.177}$$

the value of the influence from public and private information $(\phi, \varepsilon)$ are both positive. Finally, $\omega_{t+1}$ denotes the information which no agents in the market would observe, while variable $\omega_{t+1}$ independently and identically follows the normal distribution with the mean zero and variance $\sigma_\omega^2$. The three information parameters $(\phi_t, \varepsilon_{t+1}, \omega_{t+1})$ in the model are mutually orthogonal.

At the beginning of term $t$, informed traders could observe the private information signal $\zeta_t$ which could be positive or negative. The probability for informed traders getting the correct private information is $q$. Hence I have the function as below:

$$Pr\left(\zeta_t = \varepsilon \mid \varepsilon_{t+1} = \varepsilon\right) = Pr\left(\zeta_t = -\varepsilon \mid \varepsilon_{t+1} = -\varepsilon\right) = q > \frac{1}{2} \tag{4.178}$$

I assume the informed traders could use a better technique to get $\zeta_t$ or use the special method to access private information signal with the correct probability $q$, which is higher than half.

No agents could observe the fact $\varepsilon_{t+1}$, however, public information $\varphi_t$ is available for all participants in the market. Hence I could have 4 different forward rate $F_t^a(\phi)$, $F_t^a(-\phi)$, $F_t^b(\phi)$, and $F_t^b(-\phi)$.

When $\phi_t = \phi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^{m} = F_t^a(\phi) - S_{t+1} \tag{4.179}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^{m} \mid buy, \phi\right) = F_t^a(\phi) - E\left(S_{t+1} \mid buy, \phi\right) = 0 \tag{4.180}$$

By applying the equation 4.175, I get the below equation:

$$F_t^a(\phi) = S_t\left[1 + \phi + E\left(\varepsilon_{t+1} \mid buy, \phi\right)\right] \tag{4.181}$$

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right)(\varepsilon) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right)(-\varepsilon) \tag{4.182}$$

the below function is implied in Bayesian rule,

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid \phi\right)} \tag{4.183}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right)$, I need to consider the informed and uninformed traders separately. When $\phi_t = \phi$, uninformed traders would buy the pound forward. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) = 1 - \alpha + \alpha q \tag{4.184}$$

$$\begin{aligned} Pr\left(buy \mid \phi\right) = {} & Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) \\ & + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \end{aligned} \tag{4.185}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right)$ in similar way, and it follows that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, \phi\right) = 1 - \alpha + \alpha\left(1 - q\right) \tag{4.186}$$

I use equations 4.184, 4.185 and 4.186 to obtain the equation below:

$$Pr\left(buy \mid \phi\right) = \left(1-\alpha+\alpha q\right)\frac{1}{2} + \left[1-\alpha+\alpha\left(1-q\right)\right]\frac{1}{2} = 1-\frac{\alpha}{2} \tag{4.187}$$

Equations 4.184, 4.187 and 4.183 imply that

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) = \frac{1-\alpha\left(1-q\right)}{2-\alpha} \tag{4.188}$$

Since

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = 1 - Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right) \tag{4.189}$$

I have

$$Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right) = \frac{1-\alpha q}{2-\alpha} \tag{4.190}$$

By substituting equations 4.188, 4.190 and 4.182, I obtain

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = \frac{\alpha}{2-\alpha}(2q-1)\varepsilon \tag{4.191}$$

I obtain from equation 4.181

$$F_t^a\left(\phi\right) = S_t\left[1+\phi+\frac{\alpha}{2-\alpha}(2q-1)\varepsilon\right] \tag{4.192}$$

The ask rate when the public signal is negative. $F_t^a\left(-\phi\right)$ is equal to the market maker's expectation of $S_{t+1}$ conditional on having received a buy order and on $\phi_t = -\phi$

Follow the Bayesian rule, I could evaluate the expectation of market maker of $\varepsilon_{t+1}$, based on his information set:

When $\phi_t = -\phi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m = F_t^a\left(-\phi\right) - S_{t+1} \tag{4.193}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^{m} \mid buy, -\phi\right) = F_t^a\left(-\phi\right) - E\left(S_{t+1} \mid buy, -\phi\right) = 0 \tag{4.194}$$

applying the equation 4.175, I could get the below equation:

$$F_t^a\left(-\phi\right) = S_t\left[1 - \phi + E\left(\varepsilon_{t+1} \mid buy, -\phi\right)\right] \tag{4.195}$$

$$E\left(\varepsilon_{t+1} \mid buy, -\phi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, -\phi\right)\left(\varepsilon\right) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, -\phi\right)\left(-\varepsilon\right) \tag{4.196}$$

the below function is implied in Bayesian rule,

$$Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, -\phi\right) = \frac{Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, -\phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right)}{Pr\left(buy \mid -\phi\right)} \tag{4.197}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, -\phi\right)$, I need to consider the informed and uninformed traders separately. When $\phi_t = -\phi$, uninformed traders would sell the pound forward. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, -\phi\right) = \alpha q \tag{4.198}$$

$$Pr\left(buy \mid -\phi\right) = Pr\left(buy \mid \varepsilon_{t+1} = \varepsilon, -\phi\right) Pr\left(\varepsilon_{t+1} = \varepsilon\right) + Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, -\phi\right) Pr\left(\varepsilon_{t+1} = -\varepsilon\right) \tag{4.199}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, -\phi\right)$ in similar way, and it follows that:

$$Pr\left(buy \mid \varepsilon_{t+1} = -\varepsilon, -\phi\right) = \alpha\left(1 - q\right) \tag{4.200}$$

I could use equations 4.198, 4.199 and 4.200 to get the equation below:

$$Pr\left(buy \mid -\phi\right) = \left(\alpha q\right)\frac{1}{2} + \left[\alpha\left(1 - q\right)\right]\frac{1}{2} = \frac{\alpha}{2} \tag{4.201}$$

Equations 4.198, 4.201 and 4.197 imply

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,-\phi\right)=q \tag{4.202}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,-\phi\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,-\phi\right) \tag{4.203}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,-\phi\right)=1-q \tag{4.204}$$

By substituting equations 4.202, 4.204 and 4.196, I obtain

$$E\left(\varepsilon_{t+1} \mid buy,-\phi\right)=(2q-1)\,\varepsilon \tag{4.205}$$

I obtain from equation 4.195

$$F_t^a\left(-\phi\right)=S_t\left[1-\phi+\frac{\alpha}{2-\alpha}(2q-1)\varepsilon\right] \tag{4.206}$$

The bid rate when the public signal is positive.$F_t^b\left(\phi\right)$ is equal to the market maker's expectation of $S_{t+1}$ conditional on having received a buy order and on $\phi_t=\phi$

Follow the Bayesian rule, I could evaluate the expectation of market maker of $\varepsilon_{t+1}$, based on his information set:

When $\phi_t=\phi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m=F_t^b\left(\phi\right)-S_{t+1} \tag{4.207}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^m \mid sell,\phi\right)=F_t^b\left(\phi\right)-E\left(S_{t+1} \mid sell,\phi\right)=0 \tag{4.208}$$

By applying the equation 4.175, I obtain the below equation:

$$F_t^b(\phi) = S_t\left[1 + \phi + E(\varepsilon_{t+1} \mid sell, \phi)\right] \tag{4.209}$$

$$E(\varepsilon_{t+1} \mid sell, \phi) = Pr(\varepsilon_{t+1} = \varepsilon \mid sell, \phi)(\varepsilon) + Pr(\varepsilon_{t+1} = -\varepsilon \mid sell, \phi)(-\varepsilon) \tag{4.210}$$

the below function is implied in Bayesian rule,

$$Pr(\varepsilon_{t+1} = \varepsilon \mid sell, \phi) = \frac{Pr(sell \mid \varepsilon_{t+1} = \varepsilon, \phi)\, Pr(\varepsilon_{t+1} = \varepsilon)}{Pr(sell \mid \phi)} \tag{4.211}$$

When I compute the $Pr(sell \mid \varepsilon_{t+1} = \varepsilon, \phi)$, I need to consider the informed and uninformed traders separately. When $\phi_t = \phi$, uninformed traders would buy the pound forward. When $\varepsilon_{t+1} = \varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t = \varepsilon = \varepsilon_{t+1}$.

$$Pr(sell \mid \varepsilon_{t+1} = \varepsilon, \phi) = 1 - \alpha q \tag{4.212}$$

$$Pr(sell \mid \phi) = Pr(sell \mid \varepsilon_{t+1} = \varepsilon, \phi)\, Pr(\varepsilon_{t+1} = \varepsilon) + Pr(sell \mid \varepsilon_{t+1} = -\varepsilon, \phi)\, Pr(\varepsilon_{t+1} = -\varepsilon) \tag{4.213}$$

I also need to compute $Pr(sell \mid \varepsilon_{t+1} = -\varepsilon, \phi)$ in similar way, and it follows that:

$$Pr(sell \mid \varepsilon_{t+1} = -\varepsilon, \phi) = 1 - \alpha(1 - q) \tag{4.214}$$

I use equations 4.212, 4.213 and 4.214 to obtain the equation below:

$$Pr(sell \mid \phi) = (1 - \alpha q)\frac{1}{2} + \left[1 - \alpha(1 - q)\right]\frac{1}{2} = 1 - \frac{\alpha}{2} \tag{4.215}$$

Equations 4.212, 4.215 and 4.211 imply that

$$Pr(\varepsilon_{t+1} = \varepsilon \mid sell, \phi) = 1 - q \tag{4.216}$$

Since

$$Pr(\varepsilon_{t+1} = -\varepsilon \mid sell, \phi) = 1 - Pr(\varepsilon_{t+1} = \varepsilon \mid sell, \phi) \tag{4.217}$$

I have

$$Pr(\varepsilon_{t+1} = -\varepsilon \mid sell, \phi) = q \tag{4.218}$$

By substituting equations 4.216, 4.218 and 4.210, I obtain

$$E(\varepsilon_{t+1} \mid sell, \phi) = -(2q-1)\varepsilon \tag{4.219}$$

I obtain from equation 4.209

$$F_t^b(\phi) = S_t[1 + \phi - (2q-1)\varepsilon] \tag{4.220}$$

The bid rate when the public signal is negative. $F_t^a(-\phi)$ is equal to the market maker's expectation of $S_{t+1}$ conditional on having received a buy order and on $\phi_t = -\phi$

Follow the Bayesian rule, I could evaluate the expectation of market maker of $\varepsilon_{t+1}$, based on his information set:

When $\phi_t = -\phi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m = F_t^b(-\phi) - S_{t+1} \tag{4.221}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^m \mid sell, -\phi\right) = F_t^b(-\phi) - E(S_{t+1} \mid sell, -\phi) = 0 \tag{4.222}$$

By applying the equation 4.175, I get the below equation:

$$F_t^b(-\phi) = S_t[1 - \phi + E(\varepsilon_{t+1} \mid sell, -\phi)] \tag{4.223}$$

$$E(\varepsilon_{t+1} \mid sell, -\phi) = Pr(\varepsilon_{t+1} = \varepsilon \mid sell, -\phi)(\varepsilon) + Pr(\varepsilon_{t+1} = -\varepsilon \mid sell, -\phi)(-\varepsilon) \tag{4.224}$$

the below function is implied in Bayesian rule,

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid sell,-\phi\right)=\frac{Pr\left(sell \mid \varepsilon_{t+1}=\varepsilon,-\phi\right)Pr\left(\varepsilon_{t+1}=\varepsilon\right)}{Pr\left(sell \mid -\phi\right)} \tag{4.225}$$

When I compute the $Pr\left(sell \mid \varepsilon_{t+1}=\varepsilon,-\phi\right)$, I need to consider the informed and uninformed traders separately. When $\phi_t=-\phi$, uninformed traders would sell the pound forward. When $\varepsilon_{t+1}=\varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t=\varepsilon=\varepsilon_{t+1}$.

$$Pr\left(sell \mid \varepsilon_{t+1}=\varepsilon,-\phi\right)=1-\alpha q \tag{4.226}$$

$$Pr\left(sell \mid -\phi\right)=Pr\left(sell \mid \varepsilon_{t+1}=\varepsilon,-\phi\right)Pr\left(\varepsilon_{t+1}=\varepsilon\right)+Pr\left(sell \mid \varepsilon_{t+1}=-\varepsilon,-\phi\right)Pr\left(\varepsilon_{t+1}=-\varepsilon\right) \tag{4.227}$$

I also need to compute $Pr\left(sell \mid \varepsilon_{t+1}=-\varepsilon,-\phi\right)$ by the similar way, and it follow that:

$$Pr\left(sell \mid \varepsilon_{t+1}=-\varepsilon,-\phi\right)=1-\alpha\left(1-q\right) \tag{4.228}$$

I use equations 4.226, 4.227 and 4.228 to obtain the equation below:

$$Pr\left(sell \mid -\phi\right)=\left(1-\alpha q\right)\frac{1}{2}+\left[1-\alpha\left(1-q\right)\right]\frac{1}{2}=1-\frac{\alpha}{2} \tag{4.229}$$

Equations 4.226, 4.229 and 4.225 imply that

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid sell,-\phi\right)=\frac{1-\alpha\left(1-q\right)}{2-\alpha} \tag{4.230}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid sell,-\phi\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid sell,-\phi\right) \tag{4.231}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid sell,-\phi\right)=\frac{1-\alpha q}{2-\alpha} \tag{4.232}$$

By substituting equations 4.230, 4.232 and 4.224, I obtain

$$E\left(\varepsilon_{t+1} \mid sell, -\phi\right) = -\frac{\alpha}{2-\alpha}(2q-1)\varepsilon \tag{4.233}$$

I obtain from equation 4.223

$$F_t^b\left(-\phi\right) = S_t\left[1 - \phi - \frac{\alpha}{2-\alpha}(2q-1)\varepsilon\right] \tag{4.234}$$

I then have forward exchange rate as:

$$\begin{cases} F_t^a\left(\phi_t\right) = \begin{cases} S_t\left[1+\phi+(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\ \phi_t = \phi, \\ S_t\left[1-\phi+(2q-1)\varepsilon\right] & if\ \phi_t = -\phi, \end{cases} \\ F_t^b\left(\phi_t\right) = \begin{cases} S_t\left[1+\phi-(2q-1)\varepsilon\right] & if\ \phi_t = \phi, \\ S_t\left[1-\phi-(2q-1)\varepsilon\alpha/\left(2-\alpha\right)\right] & if\ \phi_t = -\phi. \end{cases} \end{cases} \tag{4.235}$$

# Appendix.D Simple Derivation of the Model with the Parameter $v$

When $\phi_t = \phi$, the market maker would get the profit from selling one pound forward, $\pi_{t+1}^m$, is

$$\pi_{t+1}^m = F_t^a\left(\phi\right) - S_{t+1} \tag{4.236}$$

The expected profit of market maker should be zero, hence

$$E\left(\pi_{t+1}^m \mid buy, \phi\right) = F_t^a\left(\phi\right) - E\left(S_{t+1} \mid buy, \phi\right) = 0 \tag{4.237}$$

By applying the equation 4.175, I get the below equation:

$$F_t^a\left(\phi\right) = S_t\left[1 + \phi + E\left(\varepsilon_{t+1} \mid buy, \phi\right)\right] \tag{4.238}$$

$$E\left(\varepsilon_{t+1} \mid buy, \phi\right) = Pr\left(\varepsilon_{t+1} = \varepsilon \mid buy, \phi\right)\left(\varepsilon\right) + Pr\left(\varepsilon_{t+1} = -\varepsilon \mid buy, \phi\right)\left(-\varepsilon\right) \tag{4.239}$$

the below function is implied in Bayesian rule,

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\phi\right)=\frac{Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\phi\right)Pr\left(\varepsilon_{t+1}=\varepsilon\right)}{Pr\left(buy \mid \phi\right)} \tag{4.240}$$

When I compute the $Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\phi\right)$, I need to consider the informed and uninformed traders separately. When $\phi_t=\phi$, uninformed traders would buy the pound forward. When $\varepsilon_{t+1}=\varepsilon$, informed traders would buy the pound forward with probability $q$, the signal $\zeta_t=\varepsilon=\varepsilon_{t+1}$. Parameter $v$ is the proportion of uninformed traders who choose to follow the public information.

$$Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\phi\right)=(1-\alpha)v+\alpha q \tag{4.241}$$

$$Pr\left(buy \mid \phi\right)=Pr\left(buy \mid \varepsilon_{t+1}=\varepsilon,\phi\right)Pr\left(\varepsilon_{t+1}=\varepsilon\right)+Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\phi\right)Pr\left(\varepsilon_{t+1}=-\varepsilon\right) \tag{4.242}$$

I also need to compute $Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\phi\right)$ in similar way, and it follows that:

$$Pr\left(buy \mid \varepsilon_{t+1}=-\varepsilon,\phi\right)=(1-\alpha)v+\alpha\left(1-q\right) \tag{4.243}$$

I use equations 4.241, 4.242 and 4.243 to get the equation below:

$$Pr\left(buy \mid \phi\right)=\left((1-\alpha)v+\alpha q\right)\frac{1}{2}+\left[(1-\alpha)v+\alpha\left(1-q\right)\right]\frac{1}{2}=(1-\alpha)v+\frac{\alpha}{2} \tag{4.244}$$

Equations 4.241, 4.244 and 4.240 imply that

$$Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\phi\right)=\frac{(1-\alpha)v+\alpha q}{(1-\alpha)2v+\alpha} \tag{4.245}$$

Since

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\phi\right)=1-Pr\left(\varepsilon_{t+1}=\varepsilon \mid buy,\phi\right) \tag{4.246}$$

I have

$$Pr\left(\varepsilon_{t+1}=-\varepsilon \mid buy,\phi\right)=\frac{(1-\alpha)v+\alpha-\alpha q}{(1-\alpha)2v+\alpha} \tag{4.247}$$

By substituting equations 4.245, 4.247 and 4.239, I obtain

$$E\left(\varepsilon_{t+1} \mid buy,\phi\right)=\frac{\alpha}{(1-\alpha)2v+\alpha}(2q-1)\varepsilon \tag{4.248}$$

I obtain from equation 4.238

$$F_t^a(\phi)=S_t\left[1+\phi+\frac{\alpha}{(1-\alpha)2v+\alpha}(2q-1)\varepsilon\right] \tag{4.249}$$

The derivation in other situations will follow a similar path in Appendix.C.

# Appendix.E Sample of Different Model Values from the Order Flow and Moments of GMM Models

The GMM moments of spot rate price (model-1).

$$\begin{aligned}
\ln\left[F_t^a(\varphi_t)/F_t^b(\varphi_t)\right]-\frac{2(2q-1)\varepsilon}{(2-\alpha)}&=0\\
\frac{F_t-S_t}{S_t}-a-\beta\frac{S_{t+1}-S_t}{S_t}&=0\\
\frac{F_t-S_t}{S_t}-\varphi_t-\varepsilon_{t+1}&=0\\
\frac{F_t^a(\varphi_t)-S_t}{S_t}+\frac{F_t^b(\varphi_t)-S_t}{S_t}-2\varphi&=0\\
\frac{F_t-S_{t+1}}{S_t}-(1-\alpha/((2v(1-\alpha)+\alpha)(2-(2v(1-\alpha)+\alpha))))(2q-1)\varepsilon&=0
\end{aligned}$$

The GMM moments of spot rate price (model-8).

$$\ln\left[S^a_{t+1}(\varphi_t)/S^b_{t+1}(\varphi_t)\right] - \frac{2(2q-1)\varepsilon}{((2v(1-\alpha)+\alpha)(2-(2v(1-\alpha)+\alpha))} = 0$$
$$\frac{F_t - S_t}{S_t} - a - \beta\frac{S_{t+1}-S_t}{S_t} = 0$$
$$\frac{F_t - S_t}{S_t} - \varphi_t - \varepsilon_{t+1} = 0$$
$$q - \hat{q} = 0$$
$$\alpha - \hat{\alpha} = 0$$
$$\frac{S^a_{t+1}(\varphi_t) - S_t}{S_t} + \frac{S^b_{t+1}(\varphi_t) - S_t}{S_t} - 2\varphi = 0$$
$$S^a_{t+1}(\varphi_t) * \delta + (1-\delta) * S^b_{t+1}(\varphi_t) - S_{t+1} = 0$$
$$\frac{F_t - S_{t+1}}{S_t} - (1-\alpha/((2v(1-\alpha)+\alpha)(2-(2v(1-\alpha)+\alpha))))(2q-1)\varepsilon = 0$$

The GMM moments of spot rate price (model-9).

$$\ln\left[S^a_{t+1}(\varphi_t)/S^b_{t+1}(\varphi_t)\right] - \frac{2(2qh-1)\varepsilon}{((2v(1-\alpha)+\alpha)(2-(2v(1-\alpha)+\alpha))} = 0$$
$$\frac{F_t - S_t}{S_t} - a - \beta\frac{S_{t+1}-S_t}{S_t} = 0$$
$$\frac{F_t - S_t}{S_t} - \varphi_t - \varepsilon_{t+1} = 0$$
$$q - \hat{q} = 0$$
$$\alpha - \hat{\alpha} = 0$$
$$\frac{S^a_{t+1}(\varphi_t) - S_t}{S_t} + \frac{S^b_{t+1}(\varphi_t) - S_t}{S_t} - 2\varphi = 0$$
$$S^a_{t+1}(\varphi_t) * \delta + (1-\delta) * S^b_{t+1}(\varphi_t) - S_{t+1} = 0$$
$$\frac{F_t - S_{t+1}}{S_t} - (1-\alpha/((2v(1-\alpha)+\alpha)(2-(2v(1-\alpha)+\alpha))))(2qh-1)\varepsilon = 0$$
$$S^a_{t+1}(\varphi_t) - S^a_{t+1} = 0$$
$$S^b_{t+1}(\varphi_t) - S^b_{t+1} = 0$$

The GMM moments of spot rate price (model-10).

$$\frac{F_t - S_t}{S_t} - \varphi = 0$$
$$S^a_{t+1}(\varphi_t) - S_t[1+\varphi] = 0$$
$$S^b_{t+1}(\varphi_t) - S_t[1-\varphi] = 0$$
$$\frac{1+i_d}{1+i_f} - 1 - \varphi = 0$$

The GMM moments of spot rate price (model-11).

$$
\begin{aligned}
\frac{F_t - S_t}{S_t} - \varphi - i &= 0 \varphi_t = \varphi, i_t = i \\
\frac{F_t - S_t}{S_t} + \varphi - i &= 0 \varphi_t = -\varphi, i_t = i \\
\frac{F_t - S_t}{S_t} - \varphi + i &= 0 \varphi_t = \varphi, i_t = -i \\
\frac{F_t - S_t}{S_t} + \varphi + i &= 0 \varphi_t = -\varphi, i_t = -i \\
q - q &\hat{=} 0 \\
\alpha - \hat{\alpha} &= 0 \\
\beta - plim\hat{\beta} &= 0 \\
S_{t+1}^{a}\left(\varphi_t, i_t\right) - S_t\left[1 + i + \varphi + (2q-1)\varepsilon\alpha / \left(2 - \alpha\right)\right] &= 0 \varphi_t = \varphi, i_t = i \\
S_{t+1}^{a}\left(\varphi_t, i_t\right) - S_t\left[1 + i - \varphi + (2q-1)\varepsilon\alpha / \left(2\gamma + \alpha\right)\right] &= 0 \varphi_t = -\varphi, i_t = i \\
S_{t+1}^{a}\left(\varphi_t, i_t\right) - S_t\left[1 - i + \varphi + (2q-1)\varepsilon\alpha / \left(2 - \alpha - 2\gamma\right)\right] &= 0 \varphi_t = \varphi, i_t = -i \\
S_{t+1}^{a}\left(\varphi_t, i_t\right) - S_t\left[1 - i - \varphi + (2q-1)\varepsilon\right] &= 0 \varphi_t = -\varphi, i_t = -i \\
S_{t+1}^{b}\left(\varphi_t, i_t\right) - S_t\left[1 + i + \varphi - (2q-1)\varepsilon\right] &= 0 \varphi_t = \varphi, i_t = i \\
S_{t+1}^{b}\left(\varphi_t, i_t\right) - S_t\left[1 + i - \varphi - (2q-1)\varepsilon / \left(2 - \alpha - 2\gamma\right)\right] &= 0 \varphi_t = -\varphi, i_t = i \\
S_{t+1}^{b}\left(\varphi_t, i_t\right) - S_t\left[1 - i + \varphi - (2q-1)\varepsilon / \left(2\gamma + \alpha\right)\right] &= 0 \varphi_t = \varphi, i_t = -i \\
S_{t+1}^{b}\left(\varphi_t, i_t\right) - S_t\left[1 - i - \varphi - (2q-1)\varepsilon\alpha / \left(2 - \alpha\right)\right] &= 0 \varphi_t = -\varphi, i_t = -i
\end{aligned}
$$

The model and GMM moments of spot rate price with the parameter $v$ (model-12).

$$
\begin{cases}
S_{t+1}^{a}\left(\varphi_t\right) = \begin{cases}
S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2q-1)\varepsilon\alpha}{2(\gamma+(1-\alpha-\gamma)v)+\alpha}\right] = 0 & \varphi_t = \varphi, i_t = i \\
S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2q-1)\varepsilon\alpha}{2(\gamma+(1-\alpha-\gamma)(1-v))+\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = i \\
S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2q-1)\varepsilon\alpha}{2(1-\alpha-\gamma)v+\alpha}\right] = 0 & \varphi_t = \varphi, i_t = -i \\
S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2q-1)\varepsilon\alpha}{2(1-\alpha-\gamma)(1-v)+\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = -i
\end{cases} \\
S_{t+1}^{b}\left(\varphi_t\right) = \begin{cases}
S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2q-1)\varepsilon\alpha}{2-2(\gamma+(1-\alpha-\gamma)v)-\alpha}\right] = 0 & \varphi_t = \varphi, i_t = i \\
S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2q-1)\varepsilon\alpha}{2-2(\gamma+(1-\alpha-\gamma)(1-v))-\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = i \\
S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2q-1)\varepsilon\alpha}{2-2(1-\alpha-\gamma)v-\alpha}\right] = 0 & \varphi_t = \varphi, i_t = -i \\
S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2q-1)\varepsilon\alpha}{2-2(1-\alpha-\gamma)(1-v)-\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = -i
\end{cases}
\end{cases} \tag{4.250}
$$

$$\frac{F_t - S_t}{S_t} - \varphi_t - \varepsilon_{t+1} = 0 \varphi_t = \varphi, \varepsilon_t = \varepsilon$$
$$\frac{F_t - S_t}{S_t} - \varphi_t + \varepsilon_{t+1} = 0 \varphi_t = \varphi, \varepsilon_t = -\varepsilon$$
$$\frac{F_t - S_t}{S_t} + \varphi_t - \varepsilon_{t+1} = 0 \varphi_t = -\varphi, \varepsilon_t = \varepsilon$$
$$\frac{F_t - S_t}{S_t} + \varphi_t + \varepsilon_{t+1} = 0 \varphi_t = -\varphi, \varepsilon_t = -\varepsilon$$
$$q - q \hat{=} 0$$
$$\alpha - \hat{\alpha} = 0$$
$$\beta - plim\hat{\beta} = 0$$
$$S^a_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2q-1)\varepsilon\alpha}{2(\gamma + (1-\alpha-\gamma)v) + \alpha}\right] = 0 \varphi_t = \varphi, i_t = i$$
$$S^a_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2q-1)\varepsilon\alpha}{2(\gamma + (1-\alpha-\gamma)(1-v)) + \alpha}\right] = 0 \varphi_t = -\varphi, i_t = i$$
$$S^a_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2q-1)\varepsilon\alpha}{2(1-\alpha-\gamma)v + \alpha}\right] = 0 \varphi_t = \varphi, i_t = -i$$
$$S^a_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2q-1)\varepsilon\alpha}{2(1-\alpha-\gamma)(1-v) + \alpha}\right] = 0 \varphi_t = -\varphi, i_t = -i$$
$$S^b_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2q-1)\varepsilon\alpha}{2 - 2(\gamma + (1-\alpha-\gamma)v) - \alpha}\right] = 0 \varphi_t = \varphi, i_t = i$$
$$S^b_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2q-1)\varepsilon\alpha}{2 - 2(\gamma + (1-\alpha-\gamma)(1-v)) - \alpha}\right] = 0 \varphi_t = -\varphi, i_t = i$$
$$S^b_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2q-1)\varepsilon\alpha}{2 - 2(1-\alpha-\gamma)v - \alpha}\right] = 0 \varphi_t = \varphi, i_t = -i$$
$$S^b_{t+1}(\varphi_t) - S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2q-1)\varepsilon\alpha}{2 - 2(1-\alpha-\gamma)(1-v) - \alpha}\right] = 0 \varphi_t = -\varphi, i_t = -i$$

The model and GMM moments of spot rate price with the parameter $h$ (model-13).

$$\begin{cases} S^a_{t+1}(\varphi_t) = \begin{cases} S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(\gamma+(1-\alpha-\gamma)v)+\alpha}\right] = 0 & \varphi_t = \varphi, i_t = i \\ S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(\gamma+(1-\alpha-\gamma)(1-v))+\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = i \\ S_t\left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(1-\alpha-\gamma)v+\alpha}\right] = 0 & \varphi_t = \varphi, i_t = -i \\ S_t\left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(1-\alpha-\gamma)(1-v)+\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = -i \end{cases} \\ S^b_{t+1}(\varphi_t) = \begin{cases} S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2qh-1)\varepsilon\alpha}{2-2(\gamma+(1-\alpha-\gamma)v)-\alpha}\right] = 0 & \varphi_t = \varphi, i_t = i \\ S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2qh-1)\varepsilon\alpha}{2-2(\gamma+(1-\alpha-\gamma)(1-v))-\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = i \\ S_t\left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2qh-1)\varepsilon\alpha}{2-2(1-\alpha-\gamma)v-\alpha}\right] = 0 & \varphi_t = \varphi, i_t = -i \\ S_t\left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2qh-1)\varepsilon\alpha}{2-2(1-\alpha-\gamma)(1-v)-\alpha}\right] = 0 & \varphi_t = -\varphi, i_t = -i \end{cases} \end{cases} \tag{4.251}$$

$$\frac{F_t - S_t}{S_t} - \varphi_t - \varepsilon_{t+1} = 0 \varphi_t = \varphi, \varepsilon_t = \varepsilon$$
$$\frac{F_t - S_t}{S_t} - \varphi_t + \varepsilon_{t+1} = 0 \varphi_t = \varphi, \varepsilon_t = -\varepsilon$$
$$\frac{F_t - S_t}{S_t} + \varphi_t - \varepsilon_{t+1} = 0 \varphi_t = -\varphi, \varepsilon_t = \varepsilon$$
$$\frac{F_t - S_t}{S_t} + \varphi_t + \varepsilon_{t+1} = 0 \varphi_t = -\varphi, \varepsilon_t = -\varepsilon$$
$$q - q \hat{=} 0$$
$$\alpha - \hat{\alpha} = 0$$
$$\beta - plim\hat{\beta} = 0$$
$$S_{t+1}^{a}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(\gamma + (1-\alpha-\gamma)v) + \alpha}\right] = 0 \varphi_t = \varphi, i_t = i$$
$$S_{t+1}^{a}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(\gamma + (1-\alpha-\gamma)(1-v)) + \alpha}\right] = 0 \varphi_t = -\varphi, i_t = i$$
$$S_{t+1}^{a}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} + \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(1-\alpha-\gamma)v + \alpha}\right] = 0 \varphi_t = \varphi, i_t = -i$$
$$S_{t+1}^{a}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} - \varphi + \frac{(2qh-1)\varepsilon\alpha}{2(1-\alpha-\gamma)(1-v) + \alpha}\right] = 0 \varphi_t = -\varphi, i_t = -i$$
$$S_{t+1}^{b}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2qh-1)\varepsilon\alpha}{2 - 2(\gamma + (1-\alpha-\gamma)v) - \alpha}\right] = 0 \varphi_t = \varphi, i_t = i$$
$$S_{t+1}^{b}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2qh-1)\varepsilon\alpha}{2 - 2(\gamma + (1-\alpha-\gamma)(1-v)) - \alpha}\right] = 0 \varphi_t = -\varphi, i_t = i$$
$$S_{t+1}^{b}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} + \varphi - \frac{(2qh-1)\varepsilon\alpha}{2 - 2(1-\alpha-\gamma)v - \alpha}\right] = 0 \varphi_t = \varphi, i_t = -i$$
$$S_{t+1}^{b}(\varphi_t) - S_t \left[\frac{1+i_d}{1+i_f} - \varphi - \frac{(2qh-1)\varepsilon\alpha}{2 - 2(1-\alpha-\gamma)(1-v) - \alpha}\right] = 0 \varphi_t = -\varphi, i_t = -i$$

# Chapter 5

# Conclusions and Future Research

This thesis focus on popular research questions. I focus on three important forex market research questions: the tail dependence of the factors, the forecasting ability of Copula models, and adverse selection with the forward premium puzzle.

The chapter2 focuses on the tail dependence structure of important currency-specific factors. I apply the Copula to model the four most popular currency factors in the forex market. First, I use threshold correlation to show evidence of nonlinearity between factors. I then use AR-NGARCH and Copula (following Christoffersen et al. (2012)) to model the joint distribution between the tails of the factors. Considering nonlinearity, I test the performance of the out-of-sample portfolio to show the economic value benefit from April 1, 1994 to March 20, 2020. This chapter contains several exciting findings. The linear correlations of the factors are high and significant, confirming previous literature. Secondly, the threshold correlation analysis results show that the correlation of each factor is significantly increased (sharply increased or decreased) when extreme events occur. Finally, I show that the dependency structure between foreign exchange factors is much more complex than has been considered in the literature and that asymmetry and time dependence are very correlated. In order to assess the economic costs of hedge funds ignoring these modeling features, the results of foreign exchange portfolio management show that increasing asymmetry and time-varying dependence between factors can improve portfolio performance. The robust benefits of asymmetric management factors show that tail dependence of management factors is necessary. This chapter has some remarkable contributions. First, I find significant evidence that the non-linear dependence of weekly currency portfolio returns is much stronger than traditional linear correlation coefficients would imply. Importantly, it can produce strong asymmetric tail dependence in almost unrelated factors. Second, in designing forex trading strategies, I have conducted extensive and detailed research into the dependency structure between some of the most extensively investigated forex factors in the literature, which are also very relevant to the hedge fund industry and are robust under different circumstances. The policy implications of this chapter have some advantages. My results are very relevant to the academic literature in this area because I have some new insights into the dependency structure of popular foreign exchange factors. Currency-specific factors are often applied to asset pricing to explain the returns of specific risk characteristics. The combined distribution of these factors can help me understand the risk structure and predict the risk more accurately. When designing foreign exchange trading strategies, investors use currency factors to adjust for specific exposures in their foreign exchange portfolios. Tail-dependent structures can help investors avoid underestimating the risk of a foreign exchange portfolio. There are several important challenges for future research. Firstly, this paper only studies the four-factor model. It would be interesting to extend my analysis beyond the 4-factor model. It is also interesting to investigate which economic variables drive variances, correlations, and asymmetries in the forex market.

In the next chapter, I am inspired by the benefits of using Copula in risk management. I followed Patton et al. (2019) to focus on the problem of risk prediction. I first examine the univariate model of Patton et al. (2019). Then, I improve the distribution model from a univariate model to a multivariate model by adding contacts. Finally, the Copula is combined with the GAS prediction model. I use the goodness-of-it test and Diebold-Mariano test to evaluate the performance of the risk prediction model. There are some interesting findings. First, the results show that the distributed correlation model is better than the univariate model in risk prediction. This confirms the benefits of the multivariate model in the field of risk management. Second, I examine the distribution of risk in foreign exchange markets and show that increased asymmetry and time-varying dependence between factors improve risk management. My new risk prediction model is always the best. The skewed t GAS GARCH model performed best according to the average ranking of the risk predictions of the four factors. I also compare the performance of risk forecasting models for each particular currency combination. The characteristics of different factors can be found in the risk prediction results. I make some contributions in the field of risk management. First, I extend Patton et al. (2019) univariate risk prediction model to multivariate model through Copula. Then, I use Copula model to re-estimate the joint distribution between foreign exchange factors, which strongly supports the research results of the previous chapter. The results confirm the asymmetry of the tail dependence of the factors. The new model proposed in this paper has specific theoretical and practical significance. My risk prediction model can estimate the future VaR and ES of assets and portfolios. It is helpful for investors or banks to predict the risks of their portfolios and assets or liabilities. The asymptotic theory presented in this chapter helps to consider a significant extension of the model presented here. The interesting extension is to take advantage of exogenous information in the model. In the proposed model, one might expect information from options markets, high-frequency data, or news bulletins to help predict VaR and ES.

In the last chapter, I present a model in which the adverse selection problem between market makers and traders to explain the negative covariance between forward premium and spot rate changes. First, I use simple regression to prove the evidence for the forward premium puzzle. I then estimate the original model of Burnside et al. (2009) using commonly used foreign exchange data (forward and spot exchange rate). When I want to extend the model to a complex situation, I find that ordinary foreign exchange data did not support the estimates of the complex model. Therefore, I choose to apply order flow data to estimate complex models. I apply this unique order flow data in two different ways: converting to the spot rate that considers the $t+1$ term or the reciprocal of the forward rate as the exogenous value. The results are shown below. Informed traders always have positive profits, which is consistent with my hypothesis. When the number of parameters is increased, the model becomes closer to reality. Most models have reasonable estimates. The results show that the influence of private information is the main reason for the failure of UIP and CIP. This chapter has several contributions. First, I estimate the adverse selection micro-model of Burnside et al. (2009), which gives a true estimate of the parameters. This helps me test whether the micro model has the same performance as the hypothesis. The results show that adverse selection can reasonably explain the premium puzzle. Secondly, more participants and new information parameters are added to the adverse selection model to make it closer to reality. This chapter has some policy significance in the government and forecasting field. The main implication of the micro-structural model is that I can explain various classical exchange rate puzzles based on the assumption of information friction. In addition, the model can estimate market efficiency by estimating the impact of private information on exchange rates. Compared with public information, the influence of private information is larger, indicating that the foreign exchange market is inefficient. Macroeconomists generally believe that asset markets are risk-neutral. Empirically, this assumption is problematic. Future studies could attempt to extend the model by applying risk-aversion and risk-seeking assumptions. Further work on this issue would be beneficial to assess the feasibility of the proposed solution to the forward premium puzzle. In addition, it will be interesting to apply the data order flow data to test the performance of the model during the pandemic period.

Overall, this thesis provides a comprehensive study on several essential forex market research questions. I apply the Copula to model the tail dependence of the forex factors and propose a new model based on Copula to forecast the risk of the factor portfolios. The last chapter provides an empirical study to test a microstructure model and shows an explanation of forward premium puzzle.

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