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Essays on Housing and Buy to Let Markets

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Abstract

This thesis provides an in-depth discussion of housing markets and their effects on the volatility of aggregate economy. This thesis consists of three major theoretical models and one empirical model. Chapter one gives an introduction to the housing markets and what the literature suggests about their effects on the wealth of households, as well as the contribution and structure of the thesis are discussed.

Chapter two examines the role of collateral constraints in a baseline model with no Buy to Let (BTL) sector in the economy. Using a core framework of Dynamic Stochastic General Equilibrium (DSGE) models, I found that when houses act as collateral, change in house prices have a substantial effect on the consumption of agents who are collaterally constrained. In particular, Borrowers are better off when the house prices are subjected to an increase, compared to that of agents who are not collaterally constrained. Furthermore, when amount of lending is based on income, I found that labour dynamics play a huge role and Borrowers are subjected to a substitution effect with a positive technology shock in housing market and this leads to an increase in labour and net income of Borrowers and in turn an increase in the their housing is observed.

In light of the Bank of England paper by Baptista et al. (2016) and then consultation by the Financial Policy Committee (FPC) on the risks of BTL markets on the economy, chapter three examines the role of BTL markets on the volatility of house prices using a DSGE framework with Dixit Stiglitz Lite Utility. As the policy is expected to operate at business cycle frequency, I built a DSGE model rather than an OLG model. The results from the model indicate that by altering the size of BTL markets using downpayment ratio as a macro prudential policy has a very little effect on the volatility of house prices as opposed to the agent based model Baptista et al(2016). Such results indicate that there is a chance for some serious supply constraints in the economy especially in the urban areas. However, changes in size of BTL markets do have a substantial effect on the volatility of an aggregate economy.

This leads us to chapter four, which investigates the effects of a rich set of shocks including news shocks on my model economy with agents subjected to CES utility. I use this framework to analyze which policy out of the Macro prudential of Downpayment ratio and Monetary policy is better to curb the volatility of the aggregate economy. The results indicate that labour markets play a pivotal role in most of the dynamics and the volatility stemmed from the Monetary policy shock is substantially high on housing market compared to that of the consumption goods market. I also found that Monetary policy is the only effective policy which affects the Hand to Mouth agents optimal choices. However, Macro

prudential policy is more effective in both the Borrowers and Savers volatilities of choice variables. With a News shock in Monetary policy, I tend to observe that the economy volatility is only affected by the Monetary policy with no affect from Macro Prudential policy.

Finally in chapter five, I take my baseline models augmented with habit formation: one without housing and the other one with housing, to data. Using the 1980 to 2020 quarterly data of U.S and employing the Bayesian Estimation techniques, I have estimated some key parameters and compared the estimated parameters between the two baseline models. Results show that most of the estimates are in line with the literature. Inclusion of housing has substantially increased the effect of the monetary policy and it's persistence in the economy. Due to the inclusion of collateral constraint, agents are subjected a better wealth effects and this leads to them reacting less to the change in wages. Habits have slightly increased with the inclusion of Borrowers in the economy as they are constrained and to account for the aggregate consumption levels, Borrowers tend to form higher internal habits in consumption.

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Declaration

“I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.”

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1 Introduction

Housing markets have always been subjected to debate among the economists around the globe. We have seen cross country house price growth in the long run especially after the mid twentieth century. On the other hand, these markets have been subjected to quite a lot of booms and busts in house prices during the short and medium runs. On top of that, housing wealth is perceived to have both wealth effects and collateral effects on agents during the life cycle. In this section, I will go through a brief literature review on housing markets, behavior of house prices in the long run, house price movements, housing crisis during short run and the wealth effects of the such housing markets.

1.1 An Introduction of Housing Market

The long run trend of the house prices has always been on a rise since the world war II see e.g., Knoll, Schularick and Steger (2017). “U.K. average house prices have risen by over 160 percent in real terms since the middle of 1996. However, the home ownership in U.K. remains around its lowest level for a generation. Among political leaders, policymakers and commentators there is a broad consensus that these problems are largely down to one failing: decades of undersupply of housing ” Mulheirn (2019). This has led to a strong empirical literature on the supply constraints of housing and it’s impact on the house prices. Smith, Barton A., (1976), suggested that the location premiums had a big impact on the supply of private housing and such premiums have substantially effected the land prices especially in the urban areas. Hilber and Vermeulen (2014) suggested that in England from 1974 to 2008, supply of housing in the urban areas has been adversely effected by the scarcity of developable land in such areas. The paper also shows that such constraints on supply are worse during the periods of boom in house prices than that of the bust periods. Contrary to the effects of supply constraints, Mulheirn (2019) also suggested that during the Great Moderation period the rise in house prices in the U.K. is primarily caused by the changing economic fundamentals which includes the combination of rental markets and the capital costs as well as to a lesser extent by the easy accessibility of credit and low mortgage interest rates. However, in the U.S. Duca et al. (2011) has shown that a substantial decrease in the lending standards from around 2002 is consistent with a large increase in the house price growth. Literature has suggested a variety of reasons for such long run growth in the house prices. Knoll, Schularick and Steger (2017) suggested that the increase in land prices are the reason for the trajectory of the global house prices. Regulations on the land use are also seen as the reason for such trajectory in Boston, U.S.A. according to Davis and Heathcote (2007). According to Kohler and van der Merwe (2015), a strong rise in population growth explained such rise in the house prices.

On the other hand, over the last couple of decades, economists have realized that housing markets play a very prominent role in the volatility of the aggregate economy in the short run see eg., Cerutti et al. (2015) document 85 housing booms across 53 countries between 1970 and 2012. Davis and Heathcote (2007) show that, the volatility in the residential market is more than twice of the volatility in the non residential market in the U.S. The subprime mortgage crisis in 2008 leads to one of the most depressing periods which has affected not only the U.S but worldwide leading to a Global Financial Crisis. As a result, several stricter financial restrictions were imposed on the high street banks and other mortgage lenders. The scars of the 2007-2008 global financial crisis are still visible to-date in some of the countries' housing markets. According to some of the real estate agents, these stricter financial impositions on mortgage lenders have led to a shortage in housing supply which in turn made housing unaffordable to first time buyers. Arguably, this led to most of the younger millennials in the U.S. being renters. Housing markets have always been on top of the policy makers list especially after the Global Financial Crisis in 2008. Similar to that of agriculture markets, literature shows that housing markets have been subjected to booms and busts see e.g., Meen (2001), Cameron, Muellbauer and Murphy (2006). Muellbauer and Murphy (1997) documented an econometric model which analyzed the volatile behavior of the house prices in the UK from 1957 to 1994. In particular, the paper suggested the financial deregulation in the 1980s and the presence of transaction costs lead to nonlinearities in the aggregate demand of housing. There has also been a significant literature on empirical and econometric evidence on the housing crisis. In fact, I have seen such housing crisis from 1878 after Glasgow suffered a bank failure which lead to major housing consequences see e.g. Cairncross (1934). Elbourne (2008) as well as Bjørnland and Jacobsen (2013) have also shown the role of house prices in the monetary policy transmission mechanism using structural Vector AutoRegression (VAR) models. Sinai and Souleles (2005) has found empirically that from a volatility perspective, households who rent by purchasing the housing services from the spot markets are still subjected to rent uncertainty and with an increase in such uncertainty leads to increase in probability of home ownership and house prices do increase relative to the rental prices.

Greenwood and Hercowitz (1991) find that the comparatively, residential capital stock is substantially higher than that of the business capital. In a life cycle model, houses are typically the largest component of household wealth and according to the life cycle hypothesis, changes in prices of houses have wealth effects on the consumption of agents see e.g., Carroll et al (2006), Muellbauer and Murphy (1990) and Skinner (1994). Case, Quigley, and Shiller (2003) have clearly shown a significant statistical relation between housing wealth and consumption of the households during the great moderation periods. "Between 1983 and 1992, the real estate assets of elderly households remain constant. However, the probability of owing debt against the real estate has increased"stated by Poterba and Samwick (1997). "A regression estimate by Flavin and Yamashita (1998) suggested that there has been a strong correlation between housing wealth and stock holdings among the households". In fact Fratantoni (1997) has shown an increase in housing wealth have made households to hold on to safe assets rather than the risky ones as housing leads to a mortgage commitment. Excluding the home equity effects, from a geographically linked micro data Aladangady (2017) has shown that an increase in house prices will increase the living costs adversely effect the consumption of households. However, from the same paper, the increase in house prices also increase the collateral value of the households and this in turn positively effect the consumption of the households. From U.K's perspective, Campbell and Cocco (2007) have observed that a change in house prices had a significant effect on elder homeowners and an insignificant effect on the young renters. On the other hand, contradictory to the wealth effects of housing on consumption, Iacoviello (2004) argued that an increase in house prices will lead the liquidity constrained households to borrow more and in turn that leads to a higher consumption. This arguably leads us to an another effect of housing wealth. Houses also act as the key collateral for bank lending and play a central role for long-run trends in wealth-to-income ratios and the size of the financial sector as an increase in house prices will make households to borrow more against their collateral value see e.g., , Aoki et al (2001), Aladangady (2017), Aron and Muellbauer (2006) and Piketty and Zucman (2014). Another paper by Muellbauer and Murphy (1990) argues that due to the liquidity constraint of housing, the impact of housing wealth on consumption is more down to the structural and institutional reasons and thus vary quite a lot between countries and over time. Using the Danish panel data, Leth-Petersen (2010) have shown that the credit market reform which gave an increased access to the availability of loans gave access for house owners to use housing equity as collateral for increased consumption. The literature clearly shows the importance of housing wealth. In particular most of the households assets is taken by housing wealth. Developments in the housing market are significant drivers of economic dynamics: they affect the household consumption patterns, increase income inequality, affect credit growth. In the chapter 3 of my thesis, I will try to look at the impact of the Buy to Let markets and what the literature say about such markets.

1.2 Contribution and Structure

Monopolistic competition, price rigidities and non neutrality of monetary policy have been the basic building blocks of a New Keynesian model. These models helped us to gain insights on the short run effects of business cycles and monetary policy interactions. We have seen a substantial literature in such models see e.g. Goodfriend and King (1997), Rotemberg and Woodford (1997), Clarida et al. (1999), Woodford (2003). However, the role of housing goods have not been analyzed by most of these models in such frameworks. A growing empirical and econometric evidence shows a need for economy to be modeled in two sectors consumption and housing markets. Most of the existing literature on the housing and Macroeconomic dynamics employs Overlapping Generation (OLG) models and focuses on the household consumption and self-insurance against the idiosyncratic income shocks in the presence of borrowing constraints, see e.g. Fernandez-Villaverde and Krueger (2011), Kiyotaki, Michaelides and Nikolov (2011), Gary-Bobo and Nur (2015). However, very few of these models model rental markets and their influence on the whole housing sector in a New Keynesian framework. Two of the most notable models with rental markets are Gervais (2002) and Iacoviello and Pavan (2011). Also, the rental market, volatility of housing prices and the macro prudential policy are modeled in a recent Bank of England working paper Baptista, Farmer, Hinterschweiger, Low, Tang and Uluc (2016). However, they apply an agent-based approach, which lacks micro foundations and relies on ad hoc behavioral assumptions for economic agents. As the main aim of the model is to develop a policy relevant model, which allows studying macro prudential policies designed to curb the macroeconomic volatility. As the policy is expected to operate at business cycle frequency, I will build a DSGE model rather than an OLG model. I develop a canonical model which can be thought of as a Hybrid of Iacoviello and Pavan (2011) which accounts for the modeling of the housing rental markets, but I intend to introduce more detailed modeling of the rental market sector with buy-to-let lending. Arguably, as the fiscal policy implementation which involves lengthy legislative decision making is much debatable and subject to criticism from the public, I focus on the monetary and macro prudential policies in analyzing such markets.

In light of the motivation to understand the Buy to Let markets, this thesis contribute to the analysis of collateral and income constraints in the housing markets, effects of the Buy to let markets and their impact on the economic stability. The aim of the chapter 2 and chapter 3 is to estimate the role of collateral and income constraints on agents' choice variables in the economy and further validate if the Buy-to-Let market affects the volatility of the house prices with a Cobb-Douglas Utility in the General Equilibrium framework. Chapter 2 shows that as discussed in the empirical literature, the model shows that there has been a positive impact on the consumption of agents whenever houses act as collaterals. This I haven't seen when agents are especially income constrained. Agents labour decisions play a huge role when they are income constrained and the role of income and substitution effects play a crucial part in explaining the household optimal choices. From the chapter 3, I have also shown that in my General Equilibrium model, by altering the size of BTL sector from altering the downpayment ratio doesn't have much impact on the volatility of house prices as opposed to partial equilibrium agent based model by Baptista et al(2016). One of the main reasons I believe is that there can be some substantial volatility of relative house prices stemming from the supply side constraints in the economy such as the land for construction being a fixed factor. Chapter 4 focuses on developing a canonical "sectoral heterogenous" agent model with housing rental markets and their implications on economic stability. As volatile markets are deemed to be highly risky for economic stability, I analyzed an important policy question of whether macro-prudential policy or monetary policy is more effective in curbing the volatility of the economy. The model aims to understand how buy-to-let markets and housing wealth evolve for all the agents with positive productivity shocks and news shocks in both the housing and consumption firm sectors. The model shows that the labour markets play a pivotal role in most of the dynamics. I have also found that the volatility stemmed from the Monetary policy shock is substantially high on housing output compared to that of the consumption goods output. The final chapter takes the previous theoretical model to data. This 5th chapter develops and estimates a DSGE model of non housing and housing elements with nominal rigidities and habit formation using Bayesian methods. With to the inclusion of collateral constraint and housing in the model, agents are subjected to a better wealth effects and this leads to an increase in the inverse Frisch elasticity. Habits have slightly increased with the inclusion of Borrowers in the economy as they are constrained and to account for the aggregate consumption levels, Borrowers tend to form higher internal habits in consumption.

2 New Keynesian Model with Owned Housing and Collateral Constraints

2.1 Introduction

The basic New Keynesian framework with housing helps us to analyze and answer several important questions. As we know that the New Keynesian Framework in contrast to the real counterparts, is generally built on an economic environment where not all markets are perfectly competitive and hence arise a need for the introduction of wage or price rigidity mechanisms in the market. I assume there are two firm sectors in the economy, one sector produces the consumption goods and the other one housing goods. In each firm sector, there are continuum of firms who produce the intermediate goods and are monopolistic competitive. These intermediate firms are subjected to downward sloping demand curve and are the price setters in nature which is the cause of price stickiness embedded into the business cycle models. There are final good firms as well who aggregate all the intermediate goods and are price takers in nature. We can introduce this price stickiness by assuming that the firms are subjected to quadratic costs when they change prices Rotemberg, J., (1982) or by assuming that in each period, firms are subjected to a fixed probability of being allowed to change the prices Calvo (1983). In my chapters, I assume the rotemberg quadratic costs in implementing the price stickiness. Also I have assumed that the intermediate firms in the housing markets are subjected to flexible prices (this assumption is to take into the consideration of negotiations involved in the housing markets), whereas I have accommodated price rigidity using quadratic costs in the consumption goods market.

There has been an extensive literature on collateral effects of the houses which shows that an appreciation in house prices will positively effect the consumption of constrained households. However, most of it is an empirical one and to my knowledge, I have seen a little literature on income constraint on the housing wealth when housing is the most prominent or one of the very few investment channels in the economy. Greenwood and Hercowitz (1991), Benhabib, Rogerson, and Wright (1991), Davis and Heathcote (2007) and Fisher (2007) are some of the parers in literature which shows the housing and non housing goods in the economy. This chapter is an attempt and a good starting point to model a New Keynesian framework with micro foundations and to understand the effects of income and collateral constraints with housing sector and consumption sector in the economy. A few of interesting results have emerged from this chapter . Conforming to the conventional results in the literature, houses acting as collaterals have a positive effect on the consumption of agents when the house prices appreciated. When the same agents are constrained by the income as collateral, the labour dynamics in particular the substitution effect and the income effect of agents played a prominent role.

Using a core framework of Dynamic Stochastic General Equilibrium (DSGE) models, I found that when houses act as collateral, change in house prices have a substantial effect on the consumption of agents who are collaterally constrained. In particular, Borrowers are better off when the house prices are subjected to an increase, compared to that of agents who are not collaterally constrained. Furthermore, when amount of lending is based on income, I found that labour dynamics play a huge role and Borrowers are subjected to a substitution effect with a positive technology shock in housing market and this leads to an increase in labour and net income of Borrowers and in turn an increase in the their housing is observed.

2.2 The Model with Collateral Constraint

I now develop a simple New Keynesian model with two sectors in the economy in which one of the sectors (housing market) follow a flexible price setting and the other (consumption goods market) is accommodating a Sticky price setting. I will develop a model with sectoral heterogeneity and hence the economy is composed of two different representative households. They are named Borrowers and Savers, of measure ω and $1-\omega$ respectively. These types of households differ in time preference factor, with Borrowers being more impatient than Savers. Borrowers are also constrained in such a way the the houses and it's value act as collateral. From the firm's perspective, I assume to have two sectors in the production economy which comprises of housing and consumption Non Durable goods. In each of these sectors a perfectly competitive final good producer purchases intermediate goods. These intermediate goods firms in the consumption sector are monopolistically competitive and incurs a quadratic costs when they change the price and hence act as the vehicle for nominal rigidity. On the other hand, the intermediate goods firms in the housing sector follow a flexible prices as there is a possibility of negotiations in such markets. The economy is populated by a continuum of households in the interval (0; 1).

2.2.1 Borrowers

Borrowers are the focus of my model where I have owned house Borrowers in the model. These households can borrow under collateral normally I assumed that houses act as their collateral, borrow money from the banks in terms of loans to invest in their owned housing. To minimize the complexity of the model, I assume that these loans from banks are in turn supplied by the Savers in the economy. These households budget constraint can be thought of as follows:

$$P_{c,t}C_{b,t} + Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) + R_{t-1,o}D_{t-1,o} = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} \tag{1}$$

where C_t^b denotes the consumption of the final consumer service from the Borrowers sector, $P_{c,t}$ is the given price of the consumption goods in terms of money and the money is numeraire, Q_t^h price of the house at time t, $H_{b,t}$ denotes services from the stock of the final house at the end of period 't' and also depreciates at the rate δ, $D_{t,o}$ one period nominal debt from the bank at the end of period t provided to the owned housing Borrowers sector , $R_{t-1,o}$ is nominal debt lending rate for the loan. All profits are expropriated by the government and redistributed as transfers T_{bt}. The Borrowers consumption expenditure and the the interest rate they pay for their liabilities will be equal to the wages they get for the labour provided and the loan amount from the financial intermediaries. In particular all the expenditures and investment form the Borrowers will be equal to their gains.

I build a class of models with housing in the utility functions. I have taken such flavor from Monacelli (2008). Borrowers gain utility from the under roof housing services $H_{b,t}$, consumption service $C_{b,t}$ and gets a disutility from the labour $N_{b,t}$. The utility of these households is assumed to be a Constant Elasticity of Substitution function with a nested additively non separable Cobb Douglas between consumption and housing inputs (Dixit-Stiglitz lite) and is as follows:

$$\sum_{t=0}^{\infty}\beta^t\left(\frac{1}{1-\sigma}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}\left(N_{b,t}\right)^{1+\phi}\right) \tag{2}$$

where, as always, the momentary utility function is assumed to be strictly concave, twice continuously differentiable, and to satisfy the Inada conditions. $\alpha$is the share of consumption goods in the composite Cobb Douglas consumption bundle which includes housing and consumption goods. I have assumed such additively non separable function between consumption and housing "Greenwood–Hercowitz–Huffman preferences nested in a CES utility" so as to mimic the conventional effects of housing wealth on the consumption of agents especially when the agents are collaterally constrained. σ is the inverse of the inter temporal elasticity which measures the growth rate in a consumption bundle with respect to a change in the interest rate. ϕ being the inverse elasticity of labour (Frisch Elasticity) which measures the responsiveness of labour with respect to wage rate. For these class of models I have used the Macro estimate of Frisch Elasticity to be around 2-4 taken from the paper by William B. Peterman (2015).

I also assume the Borrowers are under some constraints, where the maximum amount of the combined loan and repayment on the loans they borrow should be only less than or equal to the fraction of the house which I assume will be determined by the central bank. The collateral constraint for the sector can be thought of as following:

$$R_{t,o}D_t^o \leq (1-\chi)Q_t^h H_{b,t} \tag{3}$$

At the start of the period, borrower households problem is to choose an optimal plan of consumption $C_{b,t}$, labour $N_{b,t}$, housing of Borrowers $H_{b,t}$ and the demand for loans $D_{t,o}$ by maximizing (2) subject to (1) and the collateral constraint (3) taking the interest rate set by banks as given. Consider a binding borrowing constraint and forming the Lagrangian L, with ξ_t, Ψ_t being the Lagrange multipliers for budget constraint and collateral constraint respectively we have:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \left(\frac{1}{1-\sigma} \left((C_{b,t})^{\alpha} (H_{b,t})^{1-\alpha} \right)^{1-\sigma} - \frac{1}{1+\phi} (N_{b,t})^{1+\phi} \right) \\ +\xi_t \left(\begin{array}{c} N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} \\ -P_{c,t}C_{b,t} - Q_t^h (H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}D_{t-1,o} \end{array} \right) \\ +\Psi_t \left((1-\chi)Q_t^h H_{b,t} - R_{t,o}D_{t,o} \right) \end{array} \right) \tag{4}$$

Due to the additively non separability between consumption and housing in the consumption bundle of the utility function, the marginal effect of utility with respect to consumption is indeed affected by the housing of the agents and vice versa. The corresponding optimal conditions for $x_t = \frac{X_t}{P_{c,t}}$ are as follows:

We obtain the consumption leisure decision, the interpretation of this equation is that the marginal rate of substitution between leisure $(1-N)$ and consumption is to equate with the relative price of leisure i.e., the wages. This equation also gives us the average marginal productivity of labour. However, with an assumption of monopolistically competitive intermediate firms, the average marginal productivity of labour will be less than that of the social planner's problem and hence leads to a distortion. In particular, a presence of markup in the firms leads to an allocation distortion in the economy. We can also see here that due to the non additively separable utility, the marginal effect of utility with respect to consumption is indeed affected by the housing of the agents and vice versa.

$$\left(N_{b,t}\right)^{\phi}=w_{b,t}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma} \tag{5}$$

$$\begin{aligned}0=&\left(\left(1-\alpha\right)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right)\\&-\left[P_{c,t}\xi_t\right]q_t^h+\left[P_{c,t}\Psi_t\right](1-\chi)q_t^h+\beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)\end{aligned} \tag{6}$$

Inter temporal decisions:

$$0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,o}\frac{1}{1+\pi_{t+1}} \tag{7}$$

Lagrange Multiplier / the shadow price of consumption. This shows us the increase in the utility of Borrowers when they have a bit more of wealth. It is the shadow value of consumption as this increase in the wealth will lead to an increase in the consumption of Borrowers:

$$\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}=P_{c,t}\xi_t \tag{8}$$

Budget Constraint :

$$0=N_{b,t}w_{b,t}+d_{t,0}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}d_{t-1,o}\frac{1}{1+\pi_t} \tag{9}$$

Collateral Constraint:

$$0=(1-\chi)q_t^hH_{b,t}-R_{t,o}d_{t,o} \tag{10}$$

2.2.2 Savers

On the other hand, Savers in the economy consume, they supply their labour and also save. They supply funds to the financial intermediaries and let the banks to circulate their money in terms of Credit.

I also assume the discount factor for this sector is high than the previous sector's and I denote it with θ as this sector of agents tend to save money.

$$0.99=\theta>\beta=0.985$$

Budget Constraint of this sector will be

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} \tag{11}$$

The deposits from the working representatives of the saver's household are one-period bonds that pay with the return $R_{t-1,s}$ from t − 1 to t. Let $B_{s,t}$ be the debt the saver's household acquires, all profits are expropriated by the government and redistributed as transfers T_{st}.

The Savers households gain utility from the under roof housing services $H_{s,t}$, consumption service $C_{s,t}$ and the leisure $N_{s,t}$. The typical utility for these households is as follows:

The Saver's household discounted Utility is as :

$$E_0 \sum_{t=0}^{\infty} \theta^t \left(\frac{1}{1-\sigma} \left((C_{s,t})^{\alpha} (H_{s,t})^{1-\alpha} \right)^{1-\sigma} - \frac{1}{1+\phi} (N_{s,t})^{1+\phi} \right) \tag{12}$$

The Savers households problem is to choose $C_{s,t}, N_{s,t}, H_{s,t}, B_{s,t}$ by maximizing (12) subject to (11) . Forming the Lagrangian L , we have:

Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \theta^t U^s (C_{s,t}, N_{s,t}, H_{s,t}) + \lambda_t [N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} - P_{c,t}C_{s,t} - Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - B_{s,t}] \tag{13}$$

the corresponding optimal conditions for Savers For $x_t = \frac{X_t}{P_{c,t}}$:

Consumption leisure decision:

$$\frac{(N_{s,t})^{\phi}}{\left((C_{s,t})^{\alpha} (H_{s,t})^{1-\alpha} \right)^{-\sigma} \alpha (C_{s,t})^{\alpha-1} (H_{s,t})^{1-\alpha}} = w_t^s \tag{14}$$

$$0 = \left((1-\alpha) (H_{s,t})^{-\alpha} (C_{s,t})^{\alpha} \left((C_{s,t})^{\alpha} (H_{s,t})^{1-\alpha} \right)^{-\sigma} \right) - [P_{c,t}\lambda_t]\, q_t^h + \theta \left([P_{c,t+1}\lambda_{t+1}]\, q_{t+1}^h (1-\delta) \right) \tag{15}$$

Inter temporal decisions:

$$P_{c,t}\lambda_t = \theta \left[P_{c,t+1}\lambda_{t+1} \right] \frac{R_{t,s}}{1+\pi_{t+1}} \tag{16}$$

Lagrange Multiplier

$$P_{c,t}\lambda_t = \left((C_{s,t})^{\alpha} (H_{s,t})^{1-\alpha} \right)^{-\sigma} \alpha (C_{s,t})^{\alpha-1} (H_{s,t})^{1-\alpha} \tag{17}$$

Budget Constraint :

$$0 = N_{s,t}w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - b_{s,t} \tag{18}$$

2.2.3 Firms

I have two sectors in the production economy which comprises of housing and consumption Non Durable goods. In each of these sectors a perfectly competitive final good producer purchases intermediate goods. so is the labour from both the agents in the economy $N_{b,t}, N_{s,t}$ is divided into two sectors one works for the housing sector and the other works for the Non Durable consumption goods sector which I will denote $N^h_{t,b}, N^h_{t,s}$ for housing and $N^c_{b,t}, N^c_{s,t}$ for consumption for Borrowers and Savers respectively. For the better traction of the model, I have opted out the introduction of capital in the economy.

Final side of the firms

I model a final side of the goods sector as stand in aggregate firm which follows a Constant Elasticity of Substitution (CES) production technology to aggregate the intermediate products. These firms buy all the produced intermediate goods and aggregate into a bundle according to the following function:

$$Y_t^j = \left(\int y_t^j(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{19}$$

Where j can take either a housing sector $(j = h)$ or it can take a consumption goods sector perspective $(j = c)$. Y_t^j denotes the final goods of the respective firm sector and $y_t^j(i)$ denotes the i^{th} intermediate input in the j^{th} firm sector. ε governs the elasticity of substitution between the intermediate goods. As $\varepsilon \to \infty$, all the intermediate goods will be perfectly substitutable and to assume that all the firms are producing I assume ε to be finite. Also I assume that all the intermediate goods produced will be aggregated into the final product. We also have to be considerate to the assumption that these final good firms are in a perfectly competitive market.

The above final good firm gathers all the intermediate goods and make it into a single product and sells to the agents. They are the price takers in the economy. These firms maximize the profits of the final good firms which is :

$$P_t^j Y_t^j - \int P_t^j(i) . y_t^j(i) \, di \tag{20}$$

In particular, they maximize the profit function (20) with respect to the production technology (19) and the only choice variables for such firms side is to choose the demand for the intermediate goods $y_t^j(i)$ and the supply of the bundled final goods Y_t^j taking the prices of the final good P_t^j and the intermediate goods $P_t^j(i)$ as given. The optimal output decision of these perfectly competitive final good firms turns out to be

$$y_t^j(i) = Y_t^j \left(\frac{P_t^j(i)}{P_t^j} \right)^{-\varepsilon} \tag{21}$$

The above equation shows us that the i^{th} intermediate relative demand $y_t^j(i)$ is directly proportional to the aggregate output in the j^{th} sector Y_t^j and is a function of the relative price $\left(\frac{P_t^j(i)}{P_t^j} \right)$.

Substituting the demand of the intermediate goods (21) back into the CES production technology aggregate function (19) gives us the expression for an aggregate price level, which is a function of prices from the intermediate inputs:

$$P_t^i = \left(\int P_t^j(i)^{(1-\varepsilon)} di \right)^{\frac{1}{(1-\varepsilon)}} \tag{22}$$

Intermediate Housing firms

From the real world data we can safely assume that the housing prices unlike the consumption goods prices are taken as flexible prices due to the inclusion of negotiations in the contract. Also for a fact that the labour is segregated into two sectors , we can say that output is a function of two different types of labour from two sectors which denotes the number of hours worked for the housing sector. Profit maximization problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. I deal with each of these problems separately.

Employment in the intermediate housing firms Consider a continuum $i \in [0,1]$ of firms in which each of them produce a differentiated good subjected to the same technology. The production function is dependent on the labour from Borrowers and Savers in the housing sector taken as:

$$y_t^h(i) = Z_{ht} N_{b,t}^h(i)^{\nu} N_{s,t}^h(i)^{1-\nu} \tag{23}$$

Where $N_{b,t}^h(i)$ and $N_{s,t}^h(i)$ represent the labour from the Borrowers and Savers in the intermediate housing goods respectively. All intermediate firms are subjected to Z_{ht}, level of technology and evolve exogenously over time according to the AR(1) process. Also note that the firm's problem is static as opposed to the households and the level of technology is same for all the intermediate firms and hence no subscript of i.

These intermediate goods firms are subjected to perfectly competitive labour markets taking the real wage rate. However, they always act to minimize the costs.

$$\min_{N_{s,t}^h(i), N_{b,t}^h(i))} W_{b,t} N_{b,t}^h(i) + W_{s,t} N_{s,t}^h(i). \tag{24}$$

Now the cost minimization problem of the intermediate good producer is to choose the labour from both the Borrowers and Savers $N_{b,t}^{h}(i), N_{s,t}^{h}(i)$ by minimizing (23) subject to (24) and will go down to as :

Lagrangian:

$$L = W_{b,t}N_{b,t}^{h}(i) + W_{s,t}N_{s,t}^{h}(i) - P_{ct}\eta_t\left(Z_{ht}N_{b,t}^{h}(i)^{\nu}N_{s,t}^{h}(i)^{1-\nu} - y_t^h(i)\right) \tag{25}$$

The Lagrange multiplier $P_{ct}\eta_t$ interpretation is that the change in costs when the intermediate firm produce an extra unit of output. This is nothing but the marginal costs incurred to the intermediate firms.

By rearranging, we obtain the labour demand equations for both Borrowers and Savers in the housing sector $N_{b,t}^{h}$ and $N_{s,t}^{h}$:

$$N_{b,t}^{h} = \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h \tag{26}$$

$$N_{s,t}^{h} = \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h \tag{27}$$

The above equations clearly shows us that the labour demand from the firms are directly proportional to the output the housing firms can produce Y_t^h and indirectly proportional to the level of technology in the firm sector Z_{ht}. The interpretation is that to conform to the same amount of output the firms need less labour due to the increase in the marginal productivity of the firms.

Price setting in the intermediate housing firms Firms choose prices to maximize expected profit for the obtained labour, as discussed earlier, this housing production sector follows the flexible prices due to the involvement of negotiations. Firms will discount profits s periods into the future by $m_{t,s}$, where $m_{t,s} = \theta^s \frac{U'(C_{s,t+s})}{U'(C_{s,t})}$ which is the stochastic discount factor. Also note that we have considered the stochastic discount factor from the Savers perspective as most of the firms is assumed to be owned by the Savers.

$$\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s}\left(y_t^h(i)\,Q_t^h(i) - W_{s,t}N_{s,t}^{h}(i) - W_{b,t}N_{b,t}^{h}(i)\right) \tag{28}$$

Substituting the labour demand function we obtained above (26) and (27) into the expected profit (28), we have :

$$y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\right) = \left(y_t^h(i)\,Q_t^h(i) - y_{ht}(i)\,MC_t\right) \tag{29}$$

$$\eta_t = \frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu} \tag{30}$$

where the marginal costs is equal to the Lagrange multiplier $MC_t = \eta_t P_{ct}$. Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms which also implies that the intermediate-good producing firms each have the same real marginal costs of production.

The firm's decision problem boils down to choosing the intermediate good prices in the housing sector $Q^h_t(i)$ in order to maximize the expected profit equation (29) subject to the demand function (21) gives us the equation for the aggregate house prices:

$$\frac{Q^h_t}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\frac{1}{Z_{ht}}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu} \tag{31}$$

Consumption goods Firms

The Profit optimization problem is standard as the housing goods sector. A firm chooses employment and prices to maximize profit subject to the production constraint in the consumption goods sector. The intermediate consumption goods labour choice remains the same as that of the intermediate housing goods firms with labour from both Borrowers and Savers in the consumption goods sector. Firm i in the intermediate consumption goods sector minimizes nominal cost subject to the production constraint

The cost minimization problem of the intermediate good producer by choosing labour $N^c_{b,t}(i), N^c_{s,t}(i)$ will go down to as :

$$L = W_{s,t}N^c_{s,t}(i) + W_{b,t}N^c_{b,t}(i) - P_{ct}\zeta_t\left(Z_{ct}N^c_{b,t}(i)^{\nu}N^c_{s,t}(i)^{1-\nu} - y^c_t(i)\right) \tag{32}$$

From where we obtain the similar equations for the aggregate demand for the two kinds of labour in the intermediate consumption goods sector:

$$N^c_{b,t} = \frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y^c_t \tag{33}$$

$$N^c_{s,t} = \frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y^c_t \tag{34}$$

Price setting in the intermediate consumption good firms Because the final good producers are price takers and the Intermediate input goods into the final goods are imperfect substitutes, the intermediate good producers have market power and can set their prices. Here I assume the intermediate good producers use the rotemberg price model Rotemberg (1982) to set their prices and we will look into it below. Firms choose prices to maximize expected profit:

$$\max_{\{p_{ct}^{*}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left(y_t^c(i) P_{ct}(i) - W_{s,t} N_{s,t}^c(i) - W_{b,t} N_{b,t}^c(i) \right) \tag{35}$$

$$= \left(y_t^c(i) P_{ct}(i) - y_t^c(i) MC_s \right) \tag{36}$$

where $MC_t = \zeta_t P_t$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation

$$\max_{\{p_s^{*}(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left(y_t^c(i) P_{ct}(i) - y_t^c(i) MC_s \right) \tag{37}$$

Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices.

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_{b,t}}{\nu} \right)^{\nu} \left(\frac{w_{s,t}}{1-\nu} \right)^{1-\nu} \tag{38}$$

where $MC_s = \zeta_t P_{ct}$. Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 y_t^c(i)$. This yields us :

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(y_t^c(i) \frac{P_{ct}(i)}{P_{ct}} - y_t^c(i) \frac{MC_t}{P_{ct}} \right) - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 y_t^c(i) \right] \tag{39}$$

subject to Intermediate goods demand equation :

$$y_t^c(i) = Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \tag{40}$$

As the marginal costs are firm independent and the firms facing the same quadratic costs in the Rotemberg scenario, firms will choose the same optimal relative prices. The first order condition leads us to the equation for the aggregate inflation (New Keynesian Phillips Curve):

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega} \zeta_t + E_t \left[\theta_t m_{t+1} \left[(\pi_{t+1}) \frac{Y_{t+1}^c}{Y_t^c} (1+\pi_{t+1}) \right] \right] = \left(\pi_t (1+\pi_t) - \frac{1}{2} (\pi_t)^2 \varepsilon \right) \tag{41}$$

Where aggregate price level gross inflation is denoted by $(1+\pi_t)$. Firms will discount profits s periods into the future by $m_{t,s}$, where $m_{t,s} = \theta^s \frac{U'(C_{s,t+s})}{U'(C_{s,t})}$ which is the stochastic discount factor. Also note that we have considered the stochastic discount factor from the Savers perspective as most of the firms is assumed to be owned by the Savers.

2.2.4 Profits of firms and Government Transfers

To close the model, we need to aggregate the real profits from the firms and has to be distributed among the households in terms of transfers. Aggregate inter-period nominal profit is the total output from both the firm sectors from which the wages of the agents are taken away. Please note because of the quadratic costs involved in the rigid intermediate consumption goods firms, that has to be taken away as well. This leads to:

$$\tilde{\Pi}_t = Y_t^c P_{ct} - W_{s,t} N_{s,t}^c - W_{b,t} N_{b,t}^c - \frac{\Omega}{2} \pi_t^2 Y_t^c P_{ct} + Y_t^h Q_t^h - W_{s,t} N_{s,t}^h - W_{b,t} N_{b,t}^h \tag{42}$$

I assume that the profit is 100 percent taxed by the government and redistributed according to the following rule:

$$t_{bt} = (1-x) \frac{\tilde{\Pi}_t}{P_{ct}} \tag{43}$$

$$t_{st} = x \frac{\tilde{\Pi}_t}{P_{ct}} \tag{44}$$

Substituting the budget constraints leads to:

Transfer to the Borrowers in terms of dividends from the profits of the firms

$$t_{bt} = (1-x) \left(Y_t^c - w_{s,t} N_{s,t}^c - w_{b,t} N_{b,t}^c - \frac{\Omega}{2} \pi_t^2 Y_t^c + Y_t^h q_t^h - w_{s,t} N_{s,t}^h - w_{b,t} N_{b,t}^h \right) \tag{45}$$

Transfer to the Savers in terms of dividends from the profits of the firms

$$t_{st} = x \left(Y_t^c - w_{s,t} N_{s,t}^c - w_{b,t} N_{b,t}^c - \frac{\Omega}{2} \pi_t^2 Y_t^c + Y_t^h q_t^h - w_{s,t} N_{s,t}^h - w_{b,t} N_{b,t}^h \right) \tag{46}$$

2.2.5 Market Clearing conditions:

In equilibrium I have the following resource constraints:

The demand and supply of loans in the equilibrium are the same.

$$B_{s,t} = D_{t,o} \tag{47}$$

The aggregate labour of the agents are the sum of labour to the intermediate housing and consumption good firms.

$$N_{b,t} = N_{b,t}^h + N_{b,t}^c \tag{48}$$

$$N_{s,t} = N^h_{s,t} + N^c_{s,t} \tag{49}$$

The central bank sets the interest rate by Taylor rule:

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r} \tag{50}$$

The total output in the economy is equal to the consumption of both housing goods and non durable consumption goods from the households including the depreciation of housing and the quadratic costs incurred in the rigid intermediate consumption good firms.

$$Y^c_t + Y^h_t q^h_t = C_{b,t} + C_{s,t} + q^h_t (H_{b,t} - (1-\delta)H_{b,t-1}) + q^h_t (H_{s,t} - (1-\delta)H_{s,t-1}) + \frac{\Omega}{2}\pi^2_{ct} Y^c_t \tag{51}$$

2.2.6 Private Sector Equilibrium

Please Refer to the Appendix for the whole Private Sector Equilibrium for this model.

2.2.7 Results

Transmission Mechanism Of The Impulse Response Functions in flexible markets:

The share of durables which is not used as collateral (χ) is set to 0.10. The quadratic costs incurred in changing prices is set as Rotemberg parameter (Ω) is set to 0.75 in the consumption goods sector assuming that the consumption goods sector and housing goods sector are both flexible. The discount factors are set at Savers' discount rate (θ) as 0.99 and Borrowers' discount rate (β) as 0.985. I set the Quarterly house depreciation rate (δ) as 0.01 and share of consumables in consumption basket (α) as 0.84. With other parameters at share of profits paid to Savers (x) as 0.9, elasticity of substitution (ε) as 11, Inter temporal elasticity (σ) at 2, Inverse Elasticity of labour (ϕ) at 1/3 and Scaling of labour in production function (ν) at 0.6.

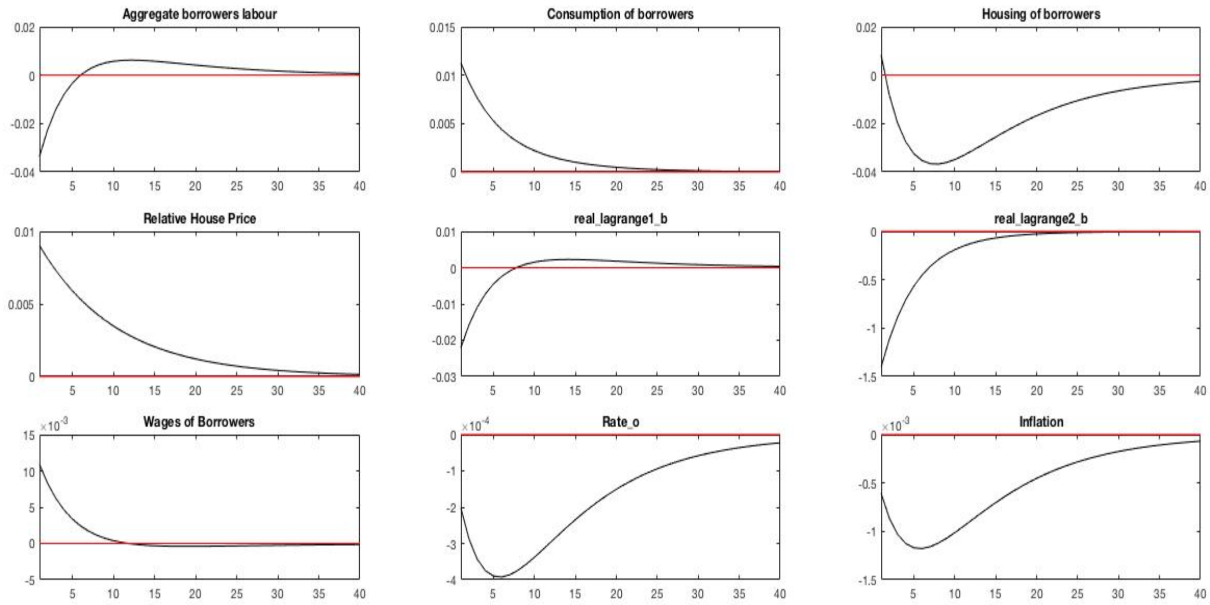
In the following, I will consider impulse responses to increase in productivity of the consumption goods at date 't'. I will look at the shock impact on model variables when the shock realized in time 't'.

The technology shocks follow an AR(1) process in the housing goods sector:

$$\ln\left(Z^h_t\right) = \rho_h \ln\left(Z^h_{t-1}\right) + e_h$$

and an AR(1) process in the consumption goods sector:;

Figure 1: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in consumption sector I



$$\ln\left(Z_t^c\right) = \rho_c \ln\left(Z_{t-1}^c\right) + e_c$$

where e_c, e_h are i.i.d processes with variances of σ_c and σ_h respectively which are both calibrated as 0.009^2. The shock persistence in both the sectors is ρ_h and $\rho_c$is taken as 0.3.

Effect of productivity shock in Consumption good firms:

The temporary increase in the consumption good production technology directly translates into higher output in consumption goods sector Y_c as the output is directly proportional to the technology shock, as we will see later although the income effect dominates in the Borrowers sector resulting in a decrease in percentage rise of Borrowers labour in the consumption firms sector N_b^c, the effect of increase in technology shock Z_{ct} and substitution effect in Savers sector dominates their impact on the output resulting in an increase in output change Y_c.

$$y_t^c(i) = Z_{ct} N_{s,t}^c(i)^{1-\nu} N_{b,t}^c(i)^{\nu}$$

From the consumption goods firm's perspective, marginal productivities of labour from both the labour sectors have increased, they want to have more of both labour inputs, pushing up the real wages of Borrowers w_b and that of Savers w_s to induce households to supply more labour and this results in moving of

Figure 2: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in consumption sector II

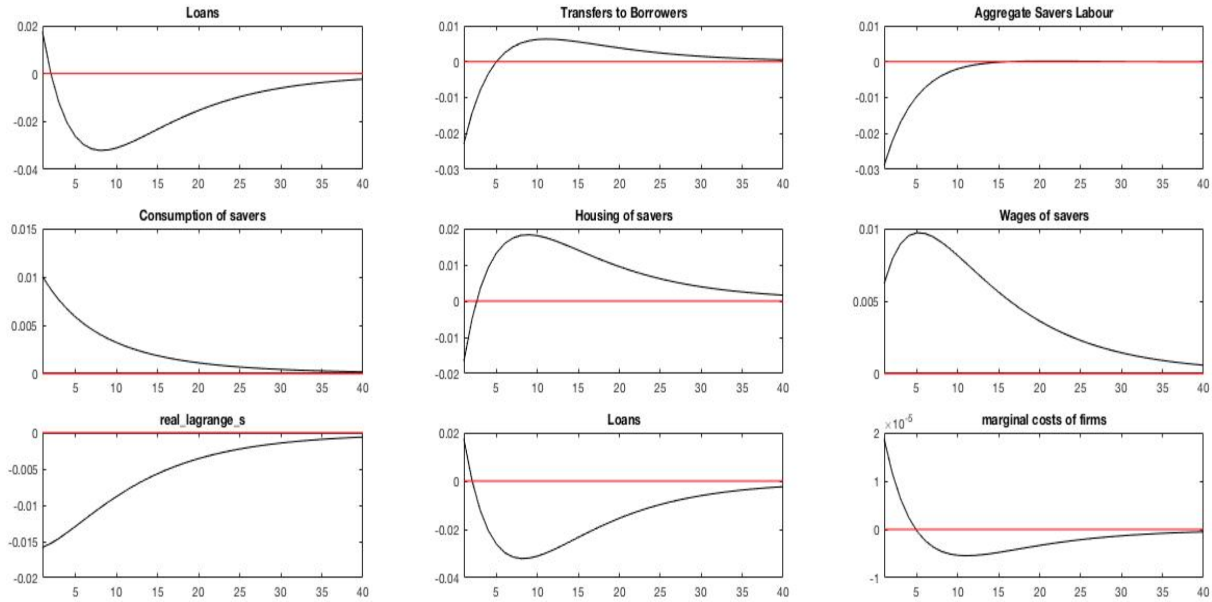
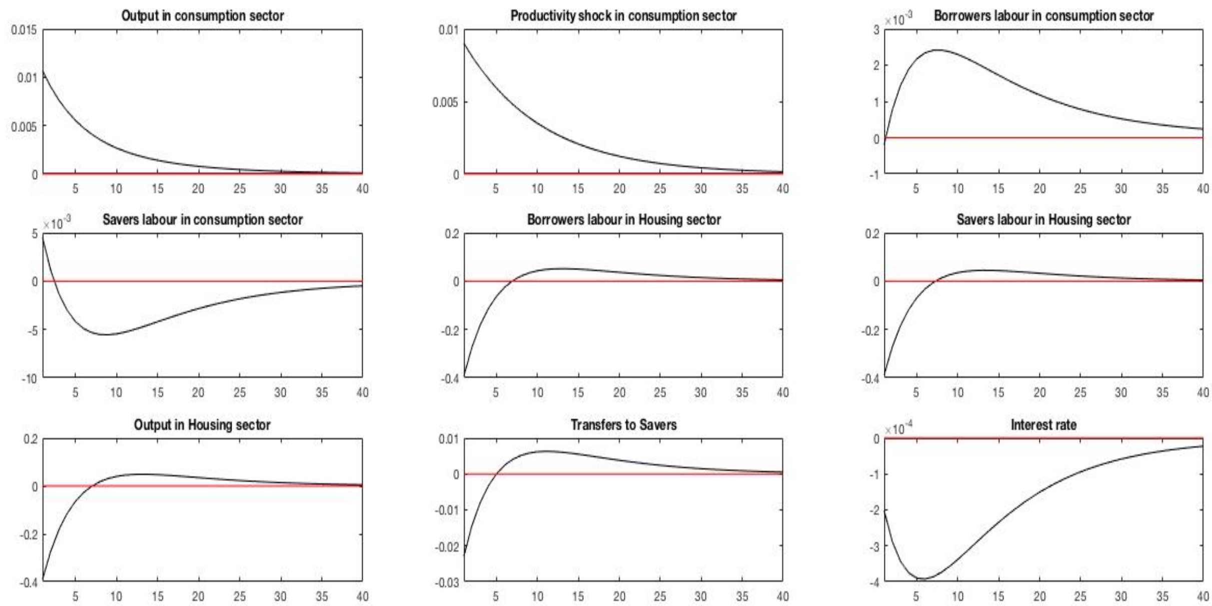


Figure 3: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in consumption sector III



the labour from housing goods sector to consumption goods sector which results in decrease of Borrowers labour in the housing sector N_b^h and a decrease in Savers labour in the housing sector N_s^h lowering the output in housing goods sector Y_h .

$$y_t^h(i) = N_{b,t}^h(i)^v N_{s,t}^h(i)^{1-v}$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in both sector's wages but with different magnitude whereas rise in wages of Borrowers is more than that of the Savers $w_b > w_s$. Even from the steady state values, the same result holds. From the below equation, we see that wages of Borrowers $W_{b,t}$ are directly proportional to the output $y_t^c(i)$ and marginal cost $P_{ct}\zeta_t$ and inversely proportional to number of hours worked $N_{b,t}^c(i)$. With higher output, higher marginal cost of firms and income effect of Borrowers, wages of Borrowers increases.

$$W_{b,t} - P_{ct}\zeta_t \nu \frac{y_t^c(i)}{N_{b,t}^c(i)} = 0$$

From the below equation, we see that wages of Savers $W_{s,t}$ are directly proportional to the consumption sector output $y_t^c(i)$ and marginal cost from firm's side $P_{ct}\zeta_t$ and inversely proportional to number of hours worked by Savers in the consumption goods sector $N_{s,t}^c(i)$. With higher output, higher marginal cost of firms and substitution effect of Savers, wages of Savers $W_{s,t}$ increases but with less in magnitude than that of Borrowers.

$$W_{s,t} - P_{ct}\zeta_t(1-\nu)\frac{y_t^c(i)}{N_{s,t}^c(i)} = 0$$

From households perspective,an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In my model, the Income effect clearly dominates the substitution effect for the Borrowers sector. For the considered parameter Scaling of labour in production function $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Borrowers sector N_b^c is inversely proportional to the productivity shock in the consumption sector Z_c and wages of Borrowers w_b. A positive productivity shock in consumption sector and an increase in the wages of Borrowers dominates the rise in wages of Saversw_s and thus prompted the decrease in the labour of Borrowers in the consumption sector N_b^c resulting in Income effect.

$$N_{b,t}^c = \frac{1}{Z_{ct}} \frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_t^c$$

Where as the substitution effect dominates the income effect in the Savers sector. As wages increase, workers will want to work more, and will increase consumption. Here we have substitution effect on utility between consumption and leisure. The relative price of leisure went down (wages are the opportunity cost of leisure), so consumption increases and hours of work increase. For the considered parameter $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Savers sector N_s^c is inversely proportional to the productivity shock Z_c and wages of Savers w_s , hence as increase in wages of Borrowers w_b dominates the rise in productivity shock and rise in wages of Savers w_s and thus prompted the increase in N_s^c resulting in substitution effect.

$$N_{s,t}^c = \frac{1}{Z_{ct}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} Y_t^c$$

As wages increase, workers from both the sectors will want to increase consumption, resulting in a rise of consumption of both Borrowers and Savers C_s and C_b . Also we can observe from the consumption goods output equation, that as output Y^c is directly proportional to the consumption of goods from both the agents C_b and C_s, with increase in output directly translates to an increase in consumption.

$$Y^c = C_b + C_s + \frac{\Omega}{2}\pi^2 Y^c$$

Households know that the shock is temporary and they smooth their consumption. Also the magnitude of rise in wages is both sectors are different and with higher increase in wages for the Borrowers sector, additional income of Borrowers is not consumed completely but part of it is saved, i.e. invested into the housing stock H_b resulting in an increase in the housing stock of Borrowers H_b. Increase in consumption is not that big though.

$$P_{c,t}C_{b,t} + Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) + R_{t-1,o}D_{t-1,o} = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t}$$

On the other hand, as Borrowers tend to invest in housing, combining with the decrease in the output of the housing sector Y^h, the price of the houses Q^h increases resulting in Savers to issue more of loans B_s and consume more C_s and investing in their housing stock H_s slightly decreases which shows that the demand of the housing stock is mostly from the Borrowers sector. The Savers sector tend to divert some of the housing stock towards the increase in issuing loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

On aggregate, leisure from both Savers and Borrowers $1-N_b$ and $1-N_s$ increase i.e., labour from agents in the economy N_b, N_s decreases which shows that Income effect dominates the substitution effect. We can observe from the following that number of hours worked N_b, N_s are directly proportional to wages $w_b$and w_s and the respective Lagrange multipliers in both the sectors.

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$(N_{s,t})^{\phi} = w_{s,t}\left[P_{c,t}\lambda_t\right]$$

The increase in productivity of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, fall in inflation πin the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation decreases, the interest rate tend to decrease.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate decreases, Borrowers tend to obtain more loans B_s and increase their consumption. They tend to increase the investment in housing stock which further increases their collateral value and tend to have less dependency around the income of the Borrowers and consume more. As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment in housing stock towards issuing more loans and hence we see a decrease in H_s. As the Savers tend to get more returns in terms of loans in volume, they tend to increase their consumption and an increase in consumption from both the sectors make firms to produce more of consumption goods which in turn make firms to hire more of labour with better wages. As the productivity in consumption good increases with the shock, more of wages and more of labour demand from the both the sectors in consumption good production side results in income effect from Borrowers side, where Borrowers tend to decrease their labour supply and have more leisure as wages increase and substitution effect in Savers side, where Savers tend to work more and increase their labour supply as the wages increase. The increase in wages in the consumption goods side results in labour moving to consumption goods production from the housing goods production sector resulting in decrease of housing goods output.

As the Borrowers are constrained by the collateral constraint, a change in relative house price play a huge role in the interpretation of the system.

$$R_{t,o}D_t^o \leq (1-\chi)Q_t^h H_{b,t}$$

Even with an increase in the relative house price, demand for loans from the Borrowers have increased and in turn we see an increase in the housing of Borrowers.

Figure 4: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in housing sector I

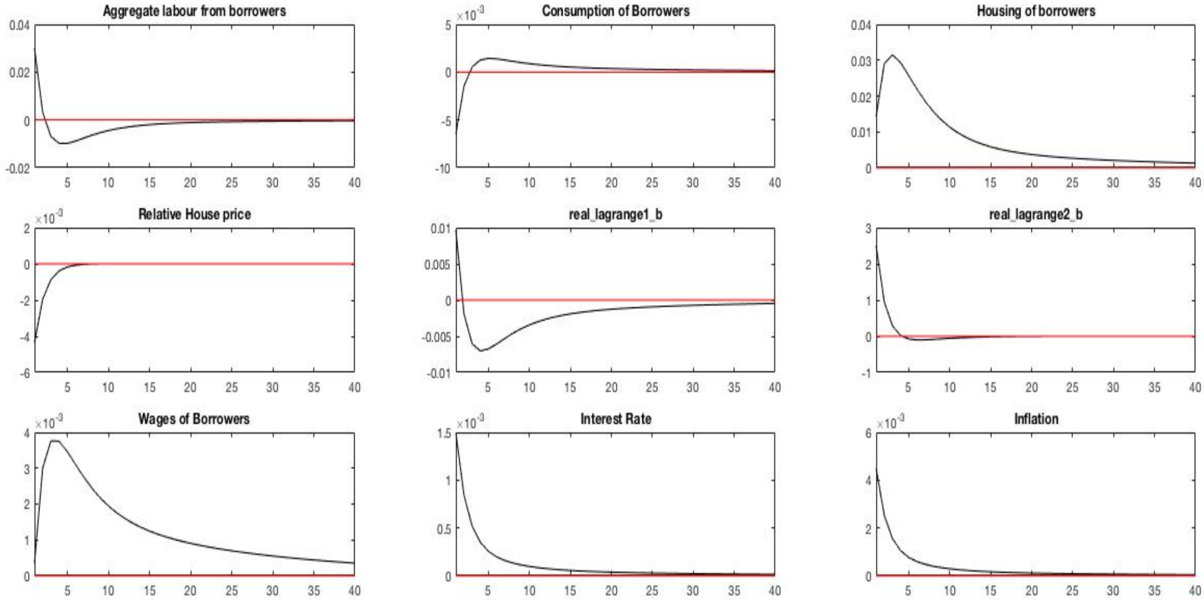


Figure 5: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in housing sector II

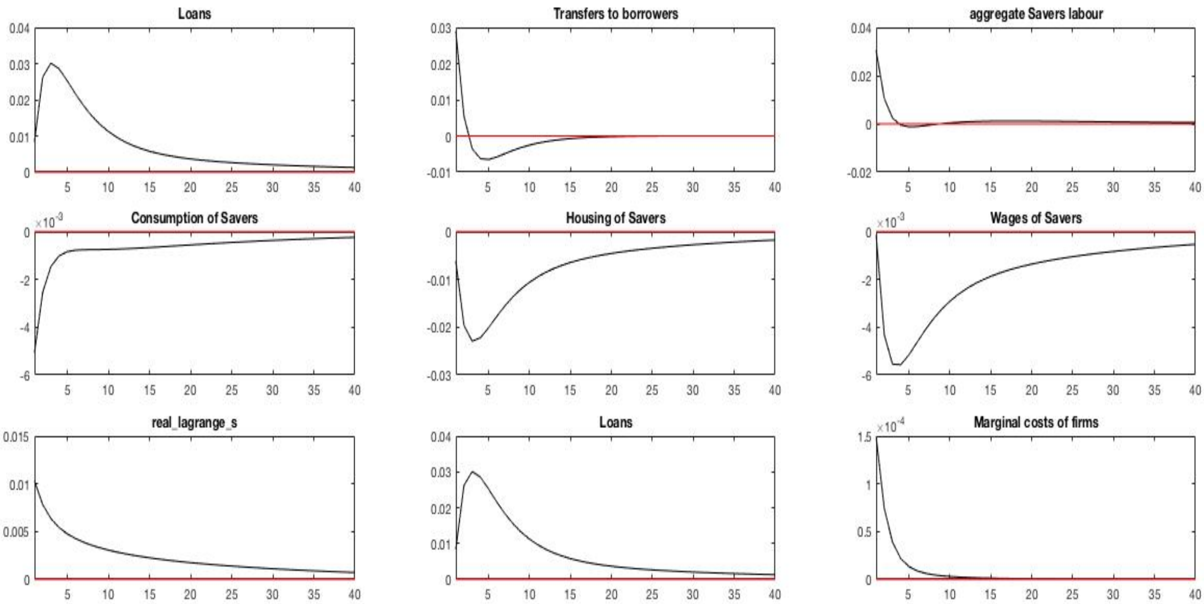
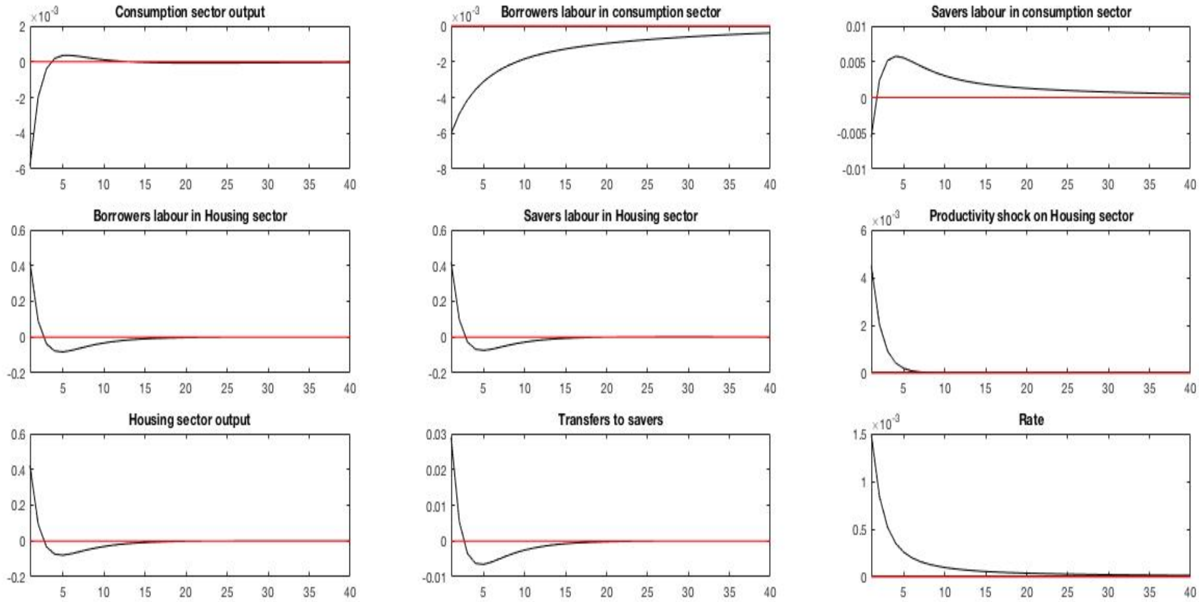


Figure 6: Collateral Constraint Model: Impulse Response Functions with a positive technology shock in housing sector III



Effect of productivity shock in Housing good firms:

The temporary increase in the housing good production technology directly translates into higher output in housing goods sector Y_h as the output is directly proportional to the technology shock Z_{ht},

$$y_t^h(i) = Z_{ht} N_{s,t}^h(i)^{1-v} N_{b,t}^h(i)^v$$

From the housing goods firm's perspective, marginal productivities of labour has increased, they want to have more of labour input, N_b^h, N_s^h and this results in moving of the labour from consumption goods sector to housing goods sector which results in decrease of labour from both Borrowers and Savers in the consumption goods sector N_s^c, N_b^c

$$y_t^c(i) = N_{b,t}^c(i)^v N_{s,t}^c(i)^{1-v}$$

As I assume that the wages are same across the two sectors of production, there is a rise in Borrowers sector's wages and fall in saver's wages with different magnitude . From the below equation, we see that wages of Borrowers are directly proportional to the output $y_t^h(i)$ and marginal cost of the firms $P_{ct}\eta_t$ and inversely proportional to number of hours worked in the housing sector $N_{b,t}^h(i)$. An increase in the housing sector's output, increase in marginal costs dominate the increase in labour supply from the Borrowers to the housing sector. This resulted in a slight increase in wages of the Borrowers $w_{b,t}$ and the Borrowers substitution effect in the housing sector.

$$W_{b,t}-P_{ct}\eta_t\nu\frac{y_t^h(i)}{N_{b,t}^h(i)}=0$$

From the below equation, we see that wages of Savers $W_{s,t}$ are directly proportional to the housing sector output $y_t^h(i)$ and marginal cost from firm's side $P_{ct}\zeta_t$ and inversely proportional to number of hours worked by Savers in the housing goods sector $N_{s,t}^h(i)$. Even with higher output, higher marginal cost of firms, the substantial increase in number of hours worked by Savers in the housing goods sector resulted in wages of Savers $W_{s,t}$ to decrease but with less in magnitude.

$$W_{s,t}-P_{ct}\eta_t(1-\nu)\frac{y_t^h(i)}{N_{s,t}^h(i)}=0$$

From households perspective, In my model with productivity shock in housing good firms, wages increase in Borrowers sector. For the considered parameter $\nu=0.6$, we can see that the number of hours worked for consumption good firms in Borrowers sector N_b^h is inversely proportional to the productivity shock in the housing sector Z_h and wages of Borrowers w_b and directly proportional to the output in the housing sector Y_t^h and wages of Savers w_s. Hence as increase in output dominates the increase in wages of Borrowers and the rise in productivity shock in the housing sector Z_h and prompted the increase in the labour of Borrowers in the housing sector N_b^h. In fact, the substitution effect dominates the income effect in the Borrowers sector with an increase in real wage in Borrowers sector. As wages increase, Borrowers will want to work more.

$$N_{b,t}^h=\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h$$

On the other hand, from a saver's perspective. For the considered parameter $\nu=0.6$, we can see that the number of hours worked for consumption good firms in Savers sector N_s^h is directly proportional to the wages of Borrowers $w_b$as well as output in the housing sector Y^h. However, number of hours worked for consumption good firms in Savers sector N_s^h is inversely proportional to the technology shock Z_h and the wages of Savers w_s , hence an increase in the housing sector output Y_h, an increase in the Borrowers wages and a decrease in the wages of the Savers in the housing sector dominates the rise in technology shock Z_h and thus prompted the increase in N_s^h .

$$N_{s,t}^h=\frac{1}{Z_{h,t}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h$$

As wages decrease in Savers sector , workers from this sector will want to decrease consumption, resulting in a fall of consumption of Savers C_s and on the other hand although wages in Borrowers sector increase, the decrease in output of consumption goods sector make Borrowers to invest in housing more and hence a decrease in C_b can be seen. Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with decrease in output directly translates to an decrease in consumption.

$$Y^c = C_b + C_s + \frac{\Omega}{2}\pi^2 Y^c$$

On the other hand, as the output in housing sector increases, this results in an increase in the consumption of housing services from Borrowers sector leading to an increase in H_b . But as the income to the Savers decrease, they tend to decrease their consumption and also they decrease their housing stock which in turn divert the funds from Savers to loans and this resulted in an increase in loans B_s. This results in an increase of housing stock from Borrowers sector and decrease in the housing stock of Savers as they divert most of their deposits into loans due to an increase in demand for loans from the Borrowers sector.

$$Y_t^h = (H_{b,t} - (1-\delta)H_{b,t-1}) + (H_{s,t} - (1-\delta)H_{s,t-1})$$

Households know that the shock is temporary and they smooth their consumption. As the wages for the Borrowers sector increase, with a tendency to consume less of consumption goods and more of housing goods resulting in an increase in the housing stock of Borrowers.

$$P_{c,t}C_{b,t} + Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) + R_{t-1,o}D_{t-1,o} = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t}$$

On the other hand, as the wages for the Savers sector decrease, Savers tend to consume less of consumption goods and less of housing goods resulting in an decrease in the housing stock of Savers. However an increase in supply of loans from the Savers.

Combining with the increase in the output of the housing sector, the price of the houses q decreases resulting in Savers to issue more of loans B_s as we see they also consume less C_s and investing in their housing stock H_s decreases which shows that the demand of the housing stock is mostly from the Borrowers sectors. The Savers sector tend to divert some of the housing stock to loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

On aggregate, leisure $1 - N_b$ and $1 - N_s$ decreases i.e., labour N_b, N_s increases.

The increase in productivity of housing good firms enhances to supply more of the housing goods and less of consumption goods with the same inputs and due to the monopolistic competition, firms increase the price of the consumption goods for better marginal profits and as a result, rise in inflation πin the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation increases, the interest rate is tend to increase.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}$$

The effect in the economy tend to persist in a cyclical effect. Even though the interest rate increases, Borrowers tend to obtain more loans, increase their housing stock and decrease their consumption as houses are the only vehicle for Borrowers to invest. Borrowers tend to increase the investment in housing stock with a decrease in the house prices. As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment from housing stock and hence we see an decrease in H_s. As the Savers tend to decrease their investment in housing, they tend to decrease their consumption and an decrease in consumption from both the sectors make firms to produce less of consumption goods which in turn make firms to hire less of labour for consumption goods sector and tend to hire more for housing goods sector.

Rigid consumption and flexible housing markets compared to both flexible markets :

I have repeated the exercise as above with a rigid consumption market and flexible housing market by calibrating the Rotemberg parameter (Ω) in the consumption sector set to 100 and decreasing the Frisch elasticity to “2”. When the system is exposed to a productivity shock in consumption sector, I have seen a very similar results to that of flexible markets in both consumption and housing sectors. In particular as the agents are only constrained by the collateral constraint, I have seen very similar results with a rigid consumption market apart from a drastic increase in the marginal cost.

On the other hand, when the system is subjected to a productivity shock in housing good firms, I have seen the wages of both Borrowers and Savers have increased compared to that of both flex price markets as the increase in productivity from the housing firms negates the quadratic costs from the consumption good firms. The positive productivity shock in housing good firms will increase the marginal productivity of such firms leading to an increase in wages. Considering that the wages are same across both the firm sectors, the consumption goods sector will need to maintain their production and will increase their labour intake from both the agents which result in an increase in labour from both Savers and Borrowers in the consumption goods sector. As wages are perceived to be same among the two firm sectors, labour tend to move from housing sector (which goes through an increase in productivity) to the consumption sector (rigid market with quadratic costs). This results in a decrease in the housing sector labour from both the agents. This movement of agents labour to the consumption goods sector leads to housing firms having more profits and that in turn leads to a decrease in the relative house prices. This leads to an increase in the number of loans and a decrease in the Interest rate. This leads to fairly similar Borrowers housing as the decrease in collateral value negates the decrease in interest rate who are primarily constrained by the collateral.

$$R_{t,o}D_t^o \leq (1-\chi)Q_t^h H_{b,t}$$

We could see a relative dip in the housing output and an increase in the consumption output due to the labour movement and a decrease in the housing of Borrowers. As the Savers are not collateral constrained, a decrease in the interest rate and housing output in turn leads to a decrease in the saver’s housing. This resulted in a substantial increase in the consumption of both Borrowers and Savers.

2.3 The Model with amount of lending based on income

I now have developed a similar model as above but with an amount of lending based on income instead of the house acting as a collateral constraint. I assume this model follows the same structure as the previous one and the only difference will be the collateral constraint of the Borrowers. Savers consumes, supply labour inelastically and also save. They supply funds to the financial intermediaries and let the banks to circulate their money in terms of Credit. The Savers households gain utility from the under roof housing services , consumption service and the leisure.

Borrowers budget constraint and utility can be thought of as the same as above model. However, the biggest difference in the collateral constraint is that instead of house acting as collateral the financial intermediaries provide loans to the Borrowers taking income from labour as the collateral. In particular, I assume the Borrowers are under some constraints, where the maximum amount of the combined loan and repayment on the loans they borrow should be only less than or equal to the fraction of the wages which I assume will be determined by the central bank. Savers do have a higher discount factor to that of the Borrowers. I also assume that the these fractions are sector dependent. The value of collateral is defined so that only the constant proportion of wages value $\mu N_{b,t} W_{b,t}$ can be used as collateral.

The collateral constraint for the sector can be thought of as follows:

$$R_{t,o} D_t^o \leq \mu N_{b,t} W_{b,t} \tag{52}$$

The Borrowers households problem is to choose consumption, aggregate supply of labour, demand for the loans and the housing of Borrowers by maximizing unaltered Borrowers utility (2) subject to their un altered budget constraint (1) and the new income collateral constraints (51) and that leads us to the optimality conditions for Borrowers. Let us look at the key differences in optimal choices of Borrowers when they are subjected to the housing collateral constraint and the income collateral constraint and how they affect the Borrowers choices. In fact, the Inter temporal decision and the budget constraint will remain the same with different collateral constraints and hence the Lagrange Multiplier / the shadow price of consumption $P_{c,t}\xi_t$ also remains the same. On top of the differences in the collateral constraint, we have a big change in the consumption leisure decision of Borrowers optimality conditions.

Consumption leisure decision: This equation also gives us the marginal utility of labour in the Borrowers sector. Compared to that of the housing collateral constraint, the marginal utility of leisure goes up ceteris paribus in the income collateral constraint due to the inclusion of the Lagrange multiplier on the constraint $w_{b,t}\mu P_{c,t}\Psi_t$. This clearly shows us that the labour dynamics play a huge role in the borrowing and in turn holdings of the house from Borrowers and they would want to work less and can lead to an Income effect in the Borrowers. However, we will see the transmission mechanism in the later section to analyze how such amount of lending based on income affects the Borrowers labour decisions and in turn the housing of Borrowers.

Housing collateral:

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}P_{c,t}\xi_t \tag{53}$$

Income collateral

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t + \mu P_{c,t}\Psi_t\right] \tag{54}$$

2.3.1 Private Sector Equilibrium

Please Refer to the Appendix for the whole Private Sector Equilibrium for this model.

2.3.2 Results

Transmission Mechanism Of The Impulse Response Functions in flexible consumption and housing markets:

The share of durables which is not used as collateral (χ) is set to 0.10. The quadratic costs incurred in changing prices is set as Rotemberg parameter (Ω) is set to 0.75 in the consumption goods sector assuming that the consumption goods sector and housing goods sector are both flexible. The discount factors are set at Savers' discount rate (θ) as 0.99 and Borrowers' discount rate (β) as 0.985. I set the Quarterly house depreciation rate (δ) as 0.01 and share of consumables in consumption basket (α) as 0.84. With other parameters at share of profits paid to Savers (x) as 0.9, elasticity of substitution (ε) as 11, Inverse Inter temporal elasticity (σ) at 2, Inverse Elasticity of labour (ϕ) at 1/3 and Scaling of labour in production function (ν) at 0.6. The parameter on the amount of lending based on income which is the number of time period's salary acting as collateral is taken as 12.

Effect of productivity shock in Consumption good firms:

The temporary increase in the consumption good production technology directly translates into higher output in consumption goods sector as the output is directly proportional to the technology shock, as we will see later although the income effect dominates in the Borrowers sector and Savers sector resulting in a decrease in percentage rise of Borrowers labour and Savers labour in the consumption firms sector, the effect of increase in technology shock dominates it's impact on the output resulting in an increase in output change Y_c.

From the consumption goods firm's perspective, marginal productivities of labour from both the labour sectors have increased, they want to have more of both labour inputs, pushing up the real wages of Borrow-

Figure 7: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in consumption sector I

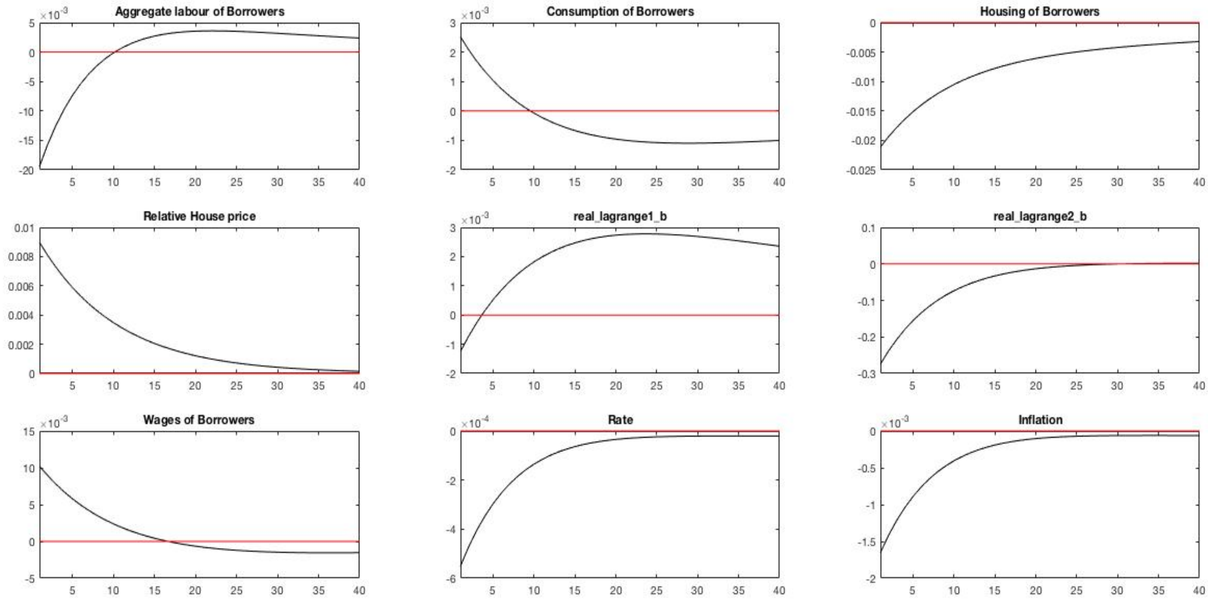


Figure 8: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in consumption sector II

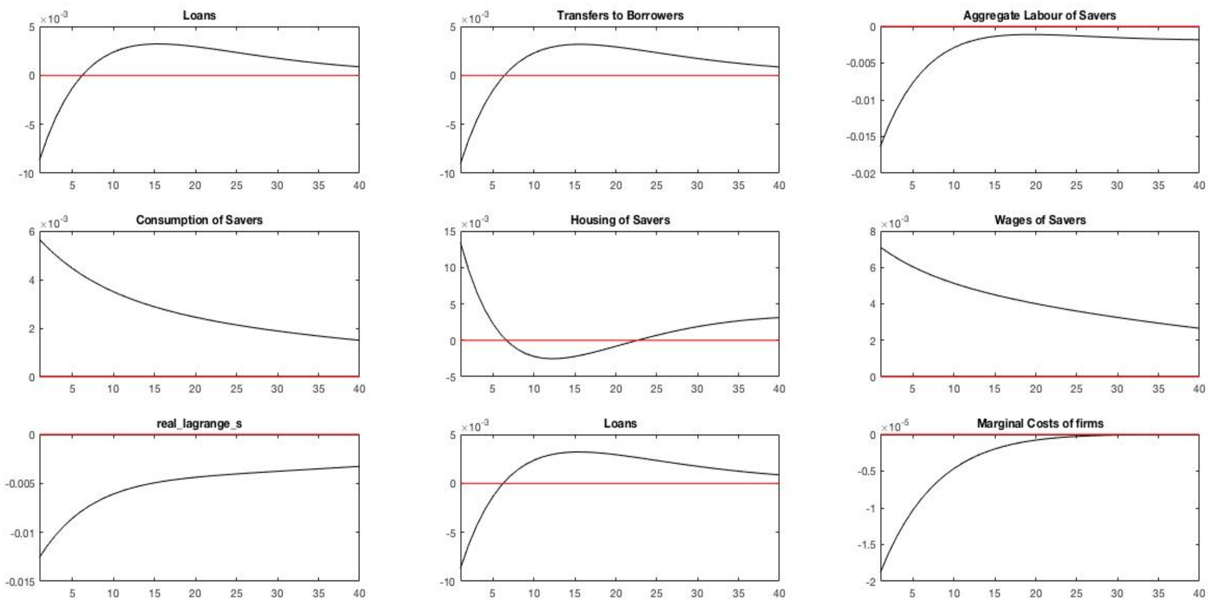
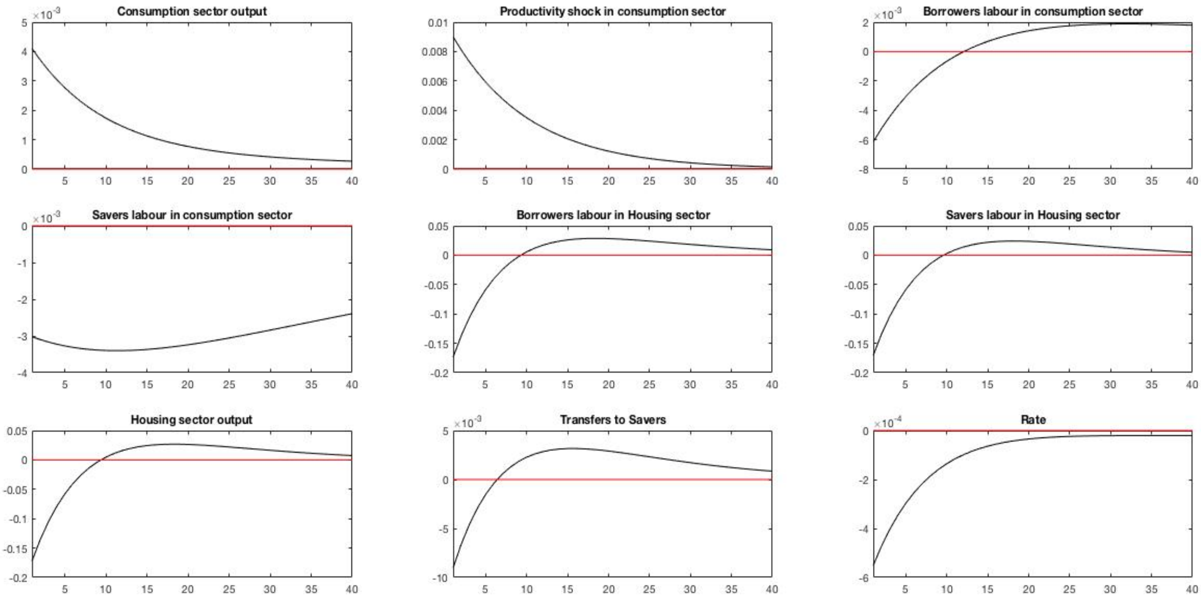


Figure 9: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in consumption sector III



ers and that of Savers to induce households to supply more labour and this results in moving of the labour from housing goods sector to consumption goods sector which results in decrease of Borrowers labour in the housing sector and a decrease in Savers labour in the housing sector lowering the output in housing goods sector.

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in both sector's wages but with different magnitude whereas rise in wages of Borrowers is more than that of the Savers . Even from the steady state values, the same result holds.

We also see that wages of Borrowers are directly proportional to the output and marginal cost and inversely proportional to number of hours worked. With higher consumption sector output and income effect of Borrowers dominating the decrease in marginal cost of firms, results in wages of Borrowers to increase.

On the other hand, I have seen a similar pattern for wages of Savers. With higher output, dominating the decrease in marginal cost of firms, results in wages of Savers to increase in my model, the Income effect clearly dominates the substitution effect for the Borrowers sector as well as the savings sector. For the considered parameter Scaling of labour in production function $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Borrowers sector is inversely proportional to the productivity shock in the consumption sector and wages of Borrowers . A positive productivity shock in consumption sector and an increase in the wages of Borrowers dominates the rise in wages of Savers and thus prompted the decrease in the labour of Borrowers in the consumption sector resulting in Income effect.

So is the income effect dominates the substitution effect in the Savers sector. As wages increase, workers will want to work more, and will increase consumption. For the considered parameter $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Savers sector is inversely proportional to the productivity shock and wages of Savers. A positive productivity shock in consumption sector and an increase in the wages of Borrowers dominates the rise in wages of Savers and thus prompted the decrease in the labour of Borrowers in the consumption sector resulting in **Income effect**

As wages increase, workers from both the sectors will want to increase consumption, resulting in a rise of consumption of both Borrowers and Savers. Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods from both the agents, with increase in output directly translates to an increase in consumption.

Households know that the shock is temporary and they smooth their consumption. Also the magnitude of rise in wages in both sectors are different and with higher increase in wages for the Borrowers sector, additional income of Borrowers is mostly consumed. In fact, considering the increase in wages, the substantial decrease in labour hours of the Borrowers have adversely affected the amount of lending based on income leading to a decrease in the housing of Borrowers. The decrease in the supply of housing also **adversely affected the Borrowers holding of houses**. This in turn leads to a decrease in the loans demanded from the Borrowers.

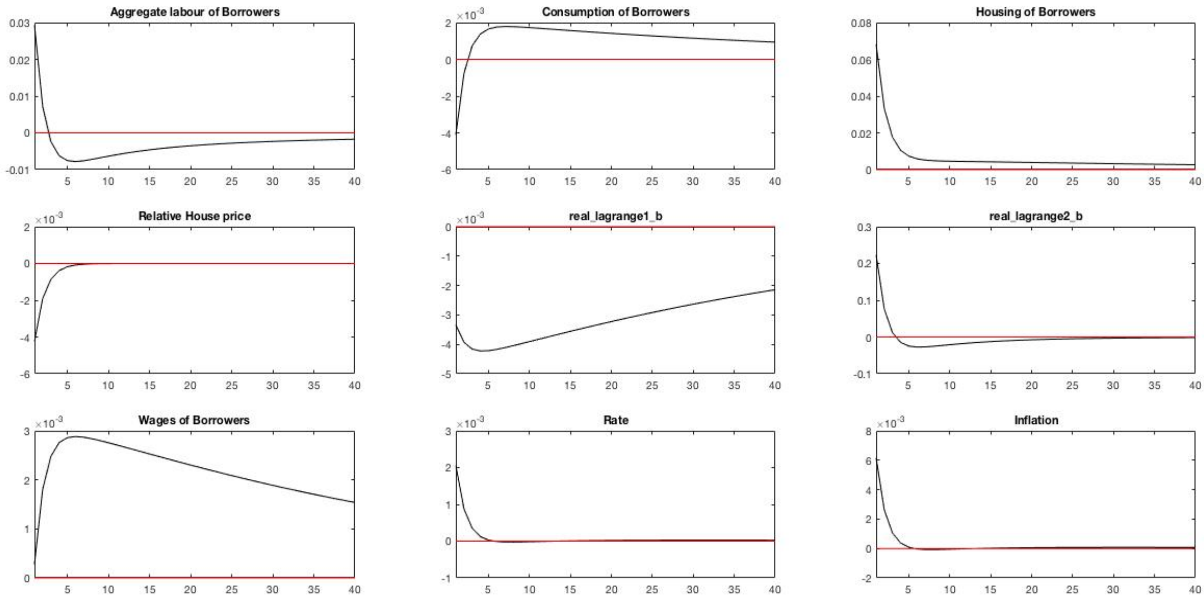
On the other hand, as Borrowers tend to consume more, combining with the decrease in the output of the housing sector , the price of the houses increases resulting in Savers to issue less of loans and consume more and investing in their housing stock substantially increases which shows that the demand of the **housing stock is mostly from the Savers sector**. The Savers sector tend to be better off in both consumption and housing.

On aggregate, leisure from both Savers and Borrowers increase i.e., labour from agents in the economy decreases which shows that Income effect dominates the substitution effect. We can observe from that number of hours worked are directly proportional to wages and the respective Lagrange multipliers in both the sectors.

The increase in productivity of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, fall in inflation in the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation decreases, the interest rate tend to decrease as well.

The effect in the economy tend to persist in a cyclical effect. Even though interest rate decreases, Borrowers tend to obtain less of loans B_s and increase their consumption as the decrease in labour hours from the Borrowers affect the amount of lending based on income adversely. Borrowers tend to decrease their investment in housing stock which further increases their tendency to consume more. As the demand for loans decrease, Savers tend to supply less of deposits as they tend to divert some of their investment to housing stock and hence we see an increase in the housing stock of Savers. As the Savers are not constrained by the amount of lending based on income, even though the returns in terms of loans in volume goes down , Savers tend to increase their consumption and an increase in consumption from both the agents make firms to produce more of consumption goods which in turn make firms to hire labour with better wages. As the productivity in consumption good increases with the shock, more of wages and more of labour demand

Figure 10: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in housing sector I



from the both the sectors in consumption good production side results in income effect from Borrowers and Savers side, where both the agents tend to decrease their labour supply and have more leisure as wages increase. The increase in wages in the consumption goods side results in labour moving to consumption goods production from the housing goods production sector resulting in decrease of labour in the housing goods sector and a decrease in housing goods output.

As the Borrowers are constrained by the amount of lending based on income, labour dynamics play a huge role in the interpretation of the system.

$$R_{t,o}D_t^o \leq \mu N_{b,t}W_{b,t}$$

For a Frisch elasticity of '3', agents are relatively more responsive to labour supply with a change in wage rates. As the income effect dominates the substitution effect, an increase in the wages of Borrowers leads to a substantial decrease in the aggregate labour from them. This results in the decrease of demand in loans from these agents as they are income constrained and in turn we see a decrease in the housing stock from the Borrowers.

Effect of productivity shock in Housing good firms:

The temporary increase in the housing good production technology directly translates into higher output in housing goods sector as the output is directly proportional to the technology shock.

From the housing goods firm's perspective, marginal productivities of labour has increased, they want to

Figure 11: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in housing sector II

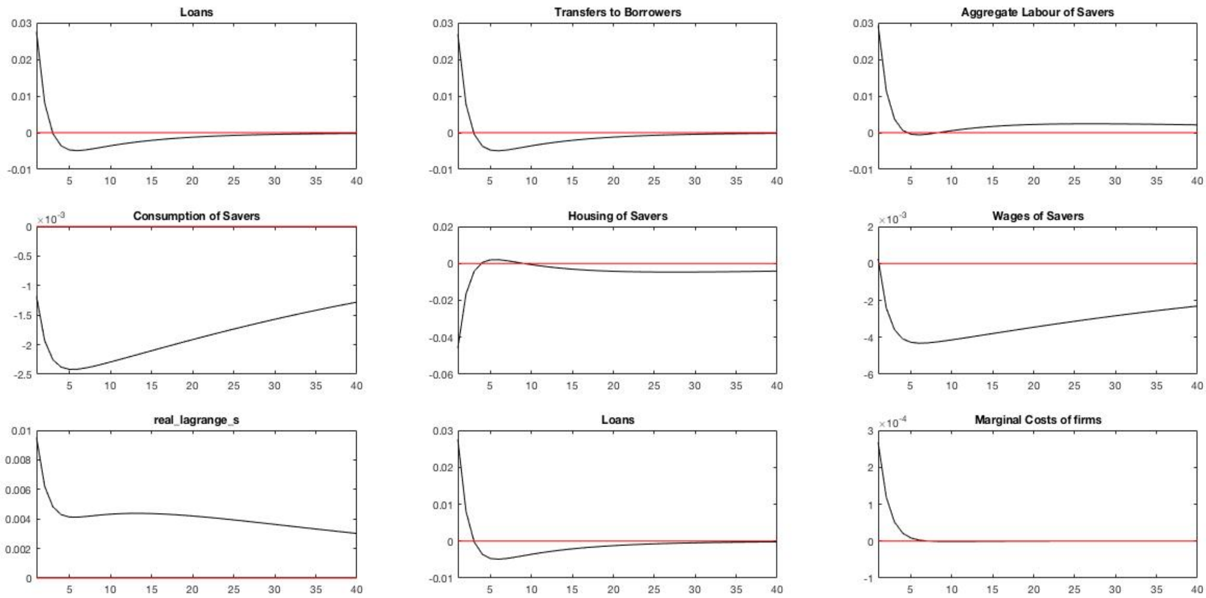
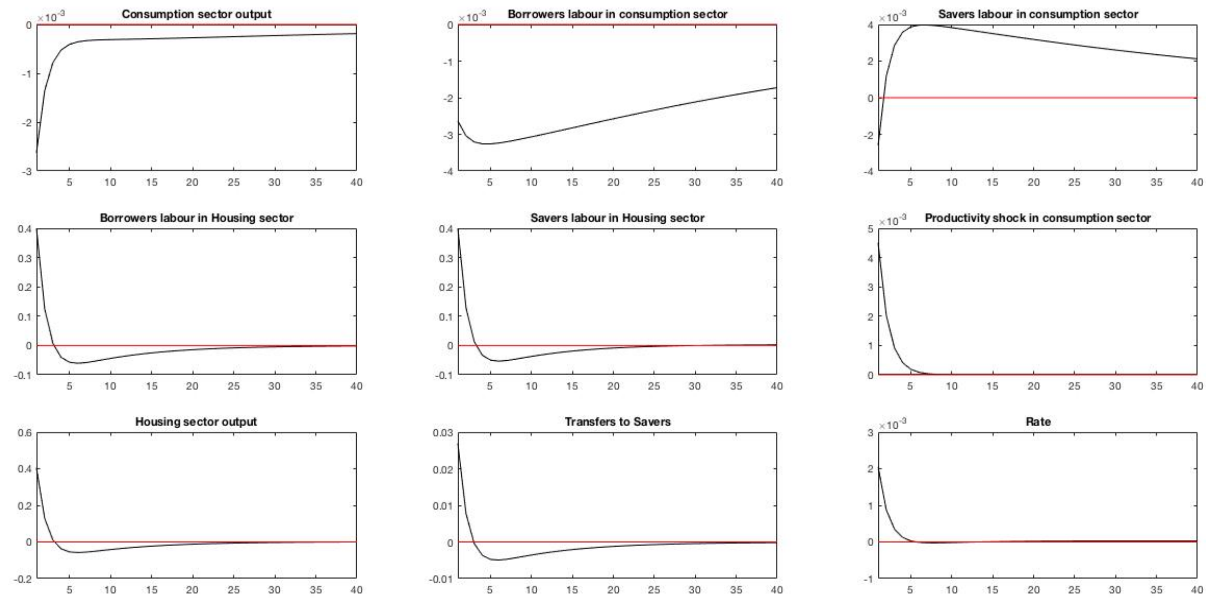


Figure 12: Amount of lending based on income Model: Impulse Response Functions with a positive technology shock in housing sector III



have more of labour input from both the agents, and this results in moving of the labour from consumption goods sector to housing goods sector which results in decrease of labour from both Borrowers and Savers in the consumption goods sector

As we assume that the wages are same across the two sectors of production, t**here is a rise in Borrowers and Savers wages although with different magnitudes.** We know that wages of Borrowers are directly proportional to the output and marginal cost of the firms and inversely proportional to number of hours worked in the housing sector. An increase in the housing sector's output, increase in marginal costs dominate the increase in labour supply from the Borrowers to the housing sector. This resulted in the Borrowers substitution effect in the housing sector and an increase in wages of the Borrowers.

On the other hand, the wages of Savers are directly proportional to the housing sector output and marginal cost from firm's side and inversely proportional to number of hours worked by Savers in the housing goods sector. With higher output, higher marginal cost of firms resulted in **wages of Savers to increase** even though there is an increase in number of hours worked by Savers in the housing goods sector. This shows that the Savers are subjected to substitution effect.

From households perspective, In my model with productivity shock in housing good firms, wages increase in both the agents. For the considered parameter $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Borrowers sector is inversely proportional to the productivity shock in the housing sector and wages of Borrowers and directly proportional to the output in the housing sector and wages of Savers. Hence as increase in output dominates the increase in wages of Borrowers and the rise in productivity shock in the housing sector and prompted the increase in the labour of Borrowers in the housing sector. In fact, the substitution effect dominates the income effect in the Borrowers sector with an increase in real wage in Borrowers sector. As wages increase, Borrowers will want to work more.

On the other hand, from a saver's perspective. For the considered parameter $\nu = 0.6$, we can see that the number of hours worked for consumption good firms in Savers sector is directly proportional to the wages of Borrowers as well as output in the housing sector . However, number of hours worked for consumption good firms in Savers sector is inversely proportional to the technology shock and the wages of Savers. The substitution effect dominates the income effect in the Savers sector as well with an increase in real wage in Savers sector. As wages increase, Savers will want to work more.

Even though the wages **increase** in Savers sector , workers from this sector will want to decrease consumption, resulting in a fall of consumption of Savers C_s and on the other hand although wages in Borrowers sector increase, the decrease in output of consumption goods sector make Borrowers to invest in housing more and hence a decrease in C_b can be seen. Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with decrease in output directly translates to an decrease in consumption.

On the other hand, as the output in housing sector increases, this results in an increase in the consumption of housing services from Borrowers sector leading to an increase in H_b. The amount of lending based on income accompanied by the decrease in house prices also increase the demand of housing from the Borrowers. Due to this substantial increase in the Borrowers housing, even though the income of the Savers increase, they tend to decrease their consumption and also they decrease their housing stock which in turn divert the funds from Savers to loans and this resulted in an increase in loans. This results in an increase of housing stock from Borrowers sector and decrease in the housing stock of Savers as they divert most of their deposits into loans due to an increase in demand for loans from the Borrowers sector.

Households know that the shock is temporary and they smooth their consumption. As the wages for the Borrowers sector increase, with a tendency to consume less of consumption goods and more of housing goods resulting in an increase in the housing stock of Borrowers.

On the other hand, as the wages for the Savers sector increase, Savers tend to consume less of consumption goods and less of housing goods resulting in an decrease in the housing stock of Savers. However an increase in supply of loans from the Savers.

Combining with the increase in the output of the housing sector, the price of the houses decreases resulting in Savers to issue more of loans as we see they also consume less and investing in their housing stock decreases which shows that the demand of the housing stock is mostly from the Borrowers sectors. The Savers sector tend to divert some of the housing stock to loans.

On aggregate, leisure $1-N_b$ and $1-N_s$ decreases i.e., labour N_b, N_s increases.

The increase in productivity of housing good firms enhances to supply more of the housing goods and less of consumption goods with the same inputs and due to the monopolistic competition, firms increase the price of the consumption goods for better marginal profits and as a result, rise in inflation πin the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation increases, the interest rate is tend to increase.

The effect in the economy tend to persist in a cyclical effect. Even though the interest rate increases, Borrowers tend to obtain more loans due to the decrease in house prices and increase in wages which will effect the amount of lending based on income positively, this in turn increase their housing stock and decrease their consumption as houses are the only vehicle for Borrowers to invest. Borrowers tend to increase the investment in housing stock with a decrease in the house prices. As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment from housing stock and hence we see an decrease in H_s. As the Savers tend to decrease their investment in housing, they tend to decrease their consumption and an decrease in consumption from both the sectors make firms to produce less of consumption goods which in turn make firms to hire less of labour for consumption goods sector and tend to hire more for housing goods sector.

A positive productivity shock in housing sector leads to a substitution effect from Borrowers, an increase in the wages leads to an increase in the labour from them and this in fact will turn to more demand for loans from the Borrowers and in turn an increase in the housing of Borrowers.

$$R_{t,o}D_t^o \leq \mu N_{b,t} W_{b,t}$$

Rigid consumption and flexible housing markets compared to both flexible markets :

I have repeated the exercise as above with a rigid consumption market and flexible housing market by calibrating the Rotemberg parameter (Ω) in the consumption sector set to 100 and decreasing the Frisch elasticity to “2”. Because the Borrowers are constrained by the income, we expect to see a drastic changes in the labour decisions and wages of the agents in the economy. When the system is exposed to a productivity shock in consumption sector, we have seen a very similar results to that of flexible markets in both consumption and housing sectors. Even though with a positive productivity shock in consumption good firms and increase in marginal productivity of such firms, due to the quadratic costs in the production function, consumption good firms could make less profits whenever there is a change in price and hence we could see a slight decrease in the wages of both the agents. This leads to both Borrowers and Savers labour relatively going down in consumption sector. In fact, the Savers labour have gone down significantly in the consumption sector and there is an increase in the housing sector labour from both agents. Compared to flexible markets in both sectors, rigid consumption and flexible housing markets also leads to a decrease in consumption sector output and the consumption in both Savers and Borrowers. However, due to the increase in the labour to the housing sector, we could see a relative increase in the housing sector output. Both Savers and Borrowers aggregate labour goes up slightly and this effects the amount of lending based on income of the Borrowers and in turn leads to an increase in the housing of Borrowers and a slight decrease in housing of Savers.

On the other hand, when the system is subjected to a productivity shock in housing good firms, we have seen the wages of both Borrowers and Savers have increased compared to that of both flex price markets as the increase in productivity from the housing firms negates the quadratic costs from the consumption good firms. The positive productivity shock in housing good firms will increase the marginal productivity of such firms leading to an increase in wages. Considering that the wages are same across both the firm sectors, the consumption goods sector will need to maintain their production and will increase their labour intake from both the agents which result in an increase in labour from both Savers and Borrowers in the consumption goods sector. As wages are perceived to be same among the two firm sectors, labour tend to move from housing sector (which goes through an increase in productivity) to the consumption sector (rigid market with quadratic costs). This results in a decrease in the housing sector labour from both the agents. However, the aggregate labour of Borrowers and the aggregate labour of Savers have substantially decreased which resulted in a substantial decrease of Borrowers housing who are primarily constrained by the income and an increase in the saver’s housing who are not income constrained.

$$R_{t,o}D_t^o \leq \mu N_{b,t}W_{b,t}$$

We could see a relative dip in the housing output and an increase in the consumption output due to the labour movement and a decrease in the housing of Borrowers. This resulted in a substantial increase in the consumption of both Borrowers and Savers. Relative to the flexible prices in both the markets, Interest rate and the amount of loans have decreased in a market with rigid consumption firms and flexible housing firms. So is the house price has decreased. This leads to a decrease in the aggregate demand for the housing from the Borrowers and in turn decreases the housing output. The decrease in the loans have also made Savers to invest in housing

2.4 Conclusions

There are very interesting results we have seen from this chapter. As discussed in the empirical literature Aladangady (2017), houses acting as collaterals have a positive effect on the consumption of agents when the house prices appreciated as opposed to agents constrained on income as collateral in which the substitution effect and the income effect of agents played a prominent role. In fact, with agents subjected to amount of lending based on income we have seen that the agents income or substitution effect played a crucial role in the holding of housing stock.

We have seen the mathematical interpretation of the productivity shocks in the above sections. let us conclude this chapter by looking at the differences between the collateral and amount of lending based on incomes when the system is subjected to productivity shocks in both firms sectors. One of the main results when the agents are subjected to amount of lending based on income is that,

$$R_{t,o}D_t^o \leq (1-\chi)Q_t^h H_{b,t}$$

With a positive productivity shock in the consumption sector, the income effect dominated the substitution effect for all the agents in the economy and especially Borrowers oversee a decrease in aggregate demand for loans and in turn a decrease in the stock of the housing from them. Whereas Savers are not constrained and this leads to and increase in the housing fo Savers even with a decrease in the labour from Savers and income effect dominating. On the other hand, with a positive productivity shock in the housing sector, the substitution effect dominated in all the agents as Borrowers oversee an increase in wages and labour and in turn the demand for the loans increase and ultimately an increase in the housing from them. However, as the Savers are not constrained by the income, an increase in the wages leads to a decrease in the housing stock from them.

As the Borrowers are constrained by the collateral constraint, a change in relative house price play a huge role in the interpretation of the system.

$$R_{t,o}D_t^o \leq (1-\chi)Q_t^h H_{b,t}$$

Even though we know the labour dynamics do play a role even when agents are collaterally constrained, the effect is lower compared to that of the amount of lending based on income.As opposed to the result when agents are subjected to amount of lending based on income, with a positive productivity shock in the consumption sector, we have seen a decrease in the housing output and an increase in the relative house price and this in turn increases the demand for loans from the Borrowers and ultimately we see a slight increase in the housing of Borrowers. This also made Borrowers better off in terms of consumption as the collateral value of houses has a positive effect on the consumption of Borrowers. As Savers are not constrained on the relative house price, with an increase in loans, the housing stock of Savers have decreased and the funds are diverted to an increase in consumption from them. On the other hand, with a positive productivity shock in the housing sector, we could see an interesting result of an increase in the housing stock of Borrowers even with a decrease in the relative house prices. Borrowers have given up on the consumption and increased their holdings of the houses as the relative low prices make houses more affordable to the them. Again we could see that houses acting as collateral has direct consumption effects on the agents.

3 Buy to Let Markets in DSGE Framework with Dixit Stiglitz Lite Utility and Collateral Constraint

3.1 Introduction

Recently, both in the UK and internationally, there were considerable public debates as to how developments in housing markets pose risks to financial stability and how best to address such risks. Quantitatively, risks can be great. The second home ownership in the western countries has substantially increased during the recent years (Dijst et al., 2005). There is a strong evidence in the literature that Buy to Let markets do have a contribution on the higher house price growth during the years see e.g., Gibb and Nygaard, (2005) and Hickman et al, (2007). The National Housing and Planning Advice Unit (NHPAU) in 2008 has shown that after the introduction of Buy to Let mortgages in 1996, 7 percent of the increase in the house prices in the year 2007 is attributed to the Buy to Let Mortgages. During the Great Recession of 2007 to 2009, Wallace, A. and Rugg, J. (2014) shown that the possessions in the Buy to Let market in the U.K had significantly increased whereas the possessions of the assets had steadily reduced in the rest of the residential market. Adelino. M , Schoar. A and Severino. F. (2018) also suggested that during the boom period of the house prices in the U.S., the fraction of the second homes and the speculative investment properties have significantly increased especially in the areas where the house prices have increased rapidly.

However, the biggest concern of Buy to Let market among the governments and economists is it's effect on the stability of the financial markets. The Private Rented Sector (PRS) or Build to Rent sector in the UK has grown substantially in recent years and accounts for approximately 27 percent of all UK households in 2017 which has increased from 13 percent by the end of financial period 2007. "Office for National Statistics (ONS) (2018)". There is an increasing number of investors who are buying up properties for the purposes of buy-to-let benefitting from both value rises in their assets and from rising rents. Buy-to-let mortgage lending is a significant share of both the flow of residential mortgage lending and stock of mortgage lending on lenders' balance sheets. Recent statistics from the consultation by the Financial Policy Committee (FPC) show that outstanding buy-to-let mortgages represent 16 percent of the total stock of residential mortgage by value, up from 12 percent of the stock in 2008 and 4 percent of the stock in 2002. Jordà, Schularick, and Taylor (2016) documented the rise in the share of mortgages on banks' balance sheets which resulted from a sharp rise in the mortgage lending to households. The paper also shows that the lending booms in the real estate markets are followed by deeper recessions with slower recoveries. The recent developments show that, Buy-to-Let markets and the Buy-to- Let Mortgage finance, if continues to grow rapidly, can be a potential threat to the financial stability of the UK. The 2017 government consultation on this issue has granted powers of direction in the Buy-to-Let market to the Financial Policy Committee of the Bank of England. The Bank of England will be allowed to regulate two financial instruments Loan to Value Ratios (LTV) and the Interest Coverage Ratios (ICRs) in respect of buy-to-let lending. The Financial Policy Committee has stated that a power of direction to limit high Loan to Value Ratios lending and low Interest Coverage Ratios lending must also be available to be applied to buy-to-let lending extended by regulated entities. It is predicted that the above tools would act to reduce the supply of credit. A Loan to Value Ratio limit acts by directly limiting the exposure of individual lenders and the system as a whole to credit risk, thus reducing the total number of defaults. High Loan to Value Ratios could also reduce total losses for a given number of defaults. An Interest Coverage Ratio limit helps to reduce the likelihood of a borrower needing to draw on other income sources to meet mortgage repayments if interest rates rise, maintenance costs increase, or rental income falls. LTV and Interest Coverage Ratios limits could also reduce the scale of the amplification channel. Using an agent based model, Bank of England working paper 619 by Baptista, Farmer, Hinterschweiger, Low, Tang and Uluc (2016) suggested that size of the buy to let markets is the main vehicle for increase in the house rice volatility. In particular, "The results suggest that an increase in the size of the buy-to-let sector may amplify house price cycles and increase house price volatility". This paper being one of the main motivation in understanding the Buy to Let markets, In this chapter of my thesis, using a Dynamic Stochastic General Equilibrium (DSGE) framework, I will analyze whether we can confirm to such results in my model.

3.2 The Model

Specifically, my model will include three different types of household agents. First, there will be 'hand-to-mouth' households who work and rent the houses; second, there will be rich households which are able to smooth their consumption over time, they are Savers in this and provide funds to the other households, who can borrow; and third, there will be Borrowers who are credit-constrained but still can borrow under suitable collateral. The model will include a central bank who will be responsible for monetary and macro-prudential policy aiming to reduce house price volatility. I also include two types of firms which will produce consumption goods and housing goods. The Borrowers play the main role in this model, and the policy will be designed to affect the incentives of the Borrowers.

3.2.1 Hand-to-mouth workers

Hand to Mouth workers are completely credit constrained, they do not have suitable collateral for borrowing from the bankers. Each of these households will consume, supply labour and rents a house and gains utility from the house they rent, and consume. I assume these households couldn't save and the budget constraint can be as follows:

Budget Constraint (Nominal terms):

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t} \tag{55}$$

where Q_t^r rental price of the house,$P_{c,t}$ is the given price of the consumption goods, H_t^r is the rented house, C_t^p consumption of the representative household, N_t^p production labour from Hand-to-Mouth workers. W_t^p represent the wage rate. These households will earn the wages from providing the labour and will spend all of it on consumption and rent. All profits are expropriated by the government and redistributed as transfers T_{pt} to this sector of agents.

A typical Hand-to-Mouth household consumes both the consumption services and the housing services. I also assume that the wage depends only on the type of labour, not on the type of firm, which allows the labour to move freely across the firms. The utility of these households is assumed to be a Constant Elasticity of Substitution function with a nested additively non separable Cobb Douglas between consumption and housing inputs and is as follows:

Utility :

$$\max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \gamma^t U\left(C_t^p, N_t^p, H_t^r\right)$$

whereC_t^pdenotes the consumption of the final consumer service from the Hand-to-Mouth Workers, H_t^r denotes the housing services from which these households incur the utility, typically assumed to be the roof under which the household survives. N_t^pdenotes the combined labour supplied by these households for the two types of firms due to the assumption that the wage depends only on the type of labour, not on the type of firm.

The Hand-to-Mouth worker will typically maximize the utility as follows:

$$\max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \beta^t U\left(C_t^p, N_t^p, H_t^r\right) = \max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma}\left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}\left(N_t^p\right)^{1+\phi}\right) \tag{56}$$

subject to nominal budget constraint

$$P_{c,t} C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t} \tag{57}$$

The Hand-to-Mouth household problem is to choose N_t^p, H_t^r . Forming the Lagrangian L, with λ_t being the Lagrange multiplier. we have:

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma}\left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}\left(N_t^p\right)^{1+\phi}\right)$$

$$C_t^p = \frac{N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r}{P_{c,t}}$$

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma}\left(\left(\frac{1}{P_{c,t}}\right)^{\alpha}\left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}\left(N_t^p\right)^{1+\phi}\right)$$

leading to the optimal conditions:

We obtain the consumption leisure decision, Inter temporal choice and optimal consumption. These agents will only consume as they are assumed to not save:

$$\left(N_t^p\right)^{\phi} = \left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{-\sigma} \alpha \left(C_t^p\right)^{\alpha-1}\left(H_t^r\right)^{1-\alpha} w_t^p \tag{58}$$

$$0 = \left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{-\sigma}\left[-\alpha\left(C_t^p\right)^{\alpha-1}\left(H_t^r\right)^{1-\alpha} q_t^r + (1-\alpha)\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{-\alpha}\right] \tag{59}$$

$$C_t^p = N_t^p w_t^p + t_{p,t} - H_t^r q_t^r \tag{60}$$

3.2.2 Borrowers

Borrowers are the central focus of my model where I have two types of Borrowers: Buy-to-Let Borrowers and owned house Borrowers.These households can borrow under collateral, normally I assumed that houses act as their collateral, borrow money from the banks in terms of loans to invest in the either owned housing or Buy-to-Let housing which is determined endogenously by the agents in their decision choice. The housing can also be divisible and part of the house is owner occupied and also other part of it is rented to other sector of hand-to-mouth households. These households budget constraint can be thought of as follows:

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,d}D_{t-1}^d + Q_t^h H_{b,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{b,t} - H_{b,t-1}}{H_{b,t-1}}\right]^2 = H_t^r Q_t^r + N_t^b W_t^b + D_t^d + T_{b,t} \tag{61}$$

where C_t^bdenotes the consumption of the final consumer service from the Borrowers sector, $P_{c,t}$ is the given price of the consumption goods, Q_t^h price of the house at time t, H_t^r rented house to the Borrowers sector, Q_t^r being the rental rate of the let house at time t, H_t^b total house , rented and owner occupied and also depreciates at the rate δ, D_t^d one period nominal debt from the bank at the end of period t provided to the housing Borrowers sector and $R_{t-1,d}$ is nominal debt lending rate of loan. In particular all the expenditures and investment form the Borrowers will be equal to their gains. All profits are expropriated by the government and redistributed as transfers T_{bt} to this sector of agents.

Let us denote $H_{t,o}$Owner occupied part of the house, which is given by to the total housing stock of Borrowers - BTL house :

$$H_t^o = H_t^b - H_t^r;$$

I introduce quadratic costs while Borrowers agents invest in housing if agent's housing investment this period is substantially differ with the housing stock from last period. I introduced investment quadratic costs for further validation of the results. However, for simplicity in this work I presented the results with the parameter on the quadratic costs taken to be zero which completely negates the following

$$Q_t^h H_{b,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{b,t} - H_{b,t-1}}{H_{b,t-1}}\right]^2 \tag{62}$$

These households gain utility from the under roof housing services , consumption service C_t^b and the leisure $1 - N_t^b$. The typical utility for these households is as follows:

Utility

$$\max_{C_t^b, N_t^b, H_t^b, H_t^r, D_t^o, D_t^r} E_0 \beta^t \sum_{t=0}^{\infty}\left(\frac{1}{1-\sigma}\left(\left(C_t^b\right)^{\alpha}\left(H_t^b - H_t^r\right)^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}\left(N_t^b\right)^{1+\phi}\right)$$

Note that I include owner-occupied housing $H_t^o = H_t^b - H_t^r$ in utility, not the rental house, as I treat utility of housing as being under the roof.

I also assume the Borrowers are under some collateral constraints, where the maximum amount of the combined loan and repayment on the loans they borrow should be only less than or equal to the fraction of the house which I assume will be determined by the central bank. The collateral constraint can be thought of as follows:

$$R_{t,b}D_t^b \leq \mu Q_t^h H_t^b \tag{63}$$

$\mu = (1 - downpayment)$ means the central bank imposes a constraint on the maximum credit amount which this sector of households can get. Hence these are not fully credit unconstrained.

The Borrowers households problem is to choose $C_t^b, N_t^b, H_t^b, H_t^r, D_t^o, D_t^r$ by maximizing Utility subject to budget constraint and the collateral constraint .

Forming the Lagrangian L, with ξ_t, Ψ_t being the Lagrange multipliers we have:

Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} U\left(C_t^b, N_t^b, H_t^b - H_t^r\right) \\ +\xi_t \left(\begin{array}{c} H_t^r Q_t^r + N_t^b W_t^b + D_t^b + T_{b,t} \\ -P_{c,t}C_t^b - Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) - R_{t-1,b}D_{t-1}^b - Q_t^h H_{b,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{b,t} - H_{b,t-1}}{H_{b,t-1}}\right]^2 \end{array} \right) \\ +\Psi_t\left(\mu Q_t^h H_t^b - R_{t,b}D_t^b\right) \end{array} \right)$$

The optimal conditions for Borrowers are as follows:

We obtain the similar optimality conditions of Borrowers to that of the previous models apart from the inclusion of under the roof housing in the utility and the rents in the budget constraint. In particular, we see here that due to the non additively separable utility, the marginal effect of utility with respect to consumption is indeed affected by the housing of the agents and vice versa.

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right] \tag{64}$$

$$\begin{aligned} 0 = &\left((1-\alpha)\left(H_{b,t} - H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t} - H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right) \\ &- \left[P_{c,t}\xi_t\right]\left(q_t^h + q_t^h.\chi\left[\frac{H_{b,t} - H_{b,t-1}}{H_{b,t-1}}\right]\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h \\ &+ \beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta) - q_{t+1}^h\frac{\chi}{2}\left[\frac{H_{b,t+1} - H_{b,t}}{H_{b,t}}\right]^2 + q_{t+1}^h\chi\left[\frac{H_{b,t+1} - H_{b,t}}{H_{b,t}}\right]\frac{H_{b,t+1}}{H_{b,t}}\right) \end{aligned} \tag{65}$$

$$0=\left(\left(1-\alpha\right)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_{t}\right]q_{t}^{r} \tag{66}$$

$$\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}=P_{c,t}\xi_{t} \tag{67}$$

$$0=\left[P_{c,t}\xi_{t}\right]-\left[P_{c,t}\Psi_{t}\right]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}} \tag{68}$$

$$0=H_{t}^{r}q_{t}^{r}+N_{b,t}w_{b,t}+d_{t,d}+t_{b,t}-C_{b,t}-q_{t}^{h}(H_{b,t}-(1-\delta)H_{b,t-1}) \tag{69}$$

$$-R_{t-1,d}d_{t-1,d}\frac{1}{1+\pi_{t}}-q_{t}^{h}H_{b,t-1}\cdot\frac{\chi}{2}\left[\frac{H_{b,t}-H_{b,t-1}}{H_{b,t-1}}\right]^{2} \tag{70}$$

$$0=\mu q_{t}^{h}H_{b,t}-R_{t,d}d_{t,d} \tag{71}$$

3.2.3 Savers

Savers consumes, supply labour inelastically and also save. They supply funds to the financial intermediaries out of their savings and let the banks to circulate their money in terms of Credit.

I also assume the discount factor for this sector is high than the previous sector's and I denote it with θ as this sector of agents tend to save money.

$$0.99=\theta>\beta=0.985$$

Budget Constraint of this sector will be

$$P_{c,t}C_{s,t}+Q_{t}^{h}(H_{s,t}-(1-\delta)H_{s,t-1})+B_{s,t}+Q_{t}^{h}H_{s,t-1}\cdot\frac{\chi}{2}\left[\frac{H_{s,t}-H_{s,t-1}}{H_{s,t-1}}\right]^{2}=N_{s,t}W_{s,t}+R_{t-1,s}B_{s,t-1}+T_{s,t} \tag{72}$$

The deposits from the working representatives of the saver's household are one-period bonds that pay with the return $R_{t-1,s}$ from t − 1 to t. Let $B_{s,t}$ be the debt the saver's household acquires, all profits are expropriated by the government and redistributed as transfers T_{st} to this sector of agents. Again, for simplicity in this work I presented the results with the parameter on the quadratic costs taken to be zero which completely negates the following

$$Q_t^h H_{s,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{s,t}-H_{s,t-1}}{H_{s,t-1}}\right]^2$$

The Savers households gain utility from the under roof housing services $H_{s,t}$, consumption service $C_{s,t}$ and the leisure $N_{s,t}$. The typical utility for these households is as follows:

The Saver's household discounted Utility is as :

$$\max_{C_t^s, N_t^s, H_t^s, B_t^s} E_0 \sum_{t=0}^{\infty} \theta^t \left(\frac{1}{1-\sigma}\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{1-\sigma} - \frac{1}{1+\phi}(N_{s,t})^{1+\phi}\right)$$

The Savers households problem is to choose $C_{s,t}, N_{s,t}, H_{s,t}, B_{s,t}$ by maximizing utility subject to budget constraint . Forming the Lagrangian L, we have:

Lagrangian

$$\begin{aligned} L = {} & E_0 \sum_{t=0}^{\infty} \theta^t U^s (C_{s,t}, N_{s,t}, H_{s,t}) \\ & + \lambda_t [N_{s,t} W_{s,t} + R_{t-1,s} B_{s,t-1} + T_{s,t} - P_{c,t} C_{s,t} - Q_t^h (H_{s,t} - (1-\delta) H_{s,t-1}) - B_{s,t} \\ & - Q_t^h H_{s,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{s,t}-H_{s,t-1}}{H_{s,t-1}}\right]^2] \end{aligned}$$

leading to the Savers optimal conditions :

$$\frac{(N_{s,t})^{\phi}}{\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma} \alpha (C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}} = w_t^s \tag{73}$$

$$0 = \left((1-\alpha)(H_{s,t})^{-\alpha}(C_{s,t})^{\alpha}\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\right) - [P_{c,t}\lambda_t]\left(q_t^h + q_t^h \cdot \chi\left[\frac{H_{s,t}-H_{s,t-1}}{H_{s,t-1}}\right]\right) \tag{74}$$

$$+ \theta [P_{c,t+1}\lambda_{t+1}]\left(q_{t+1}^h(1-\delta) - q_{t+1}^h \frac{\chi}{2}\left[\frac{H_{s,t+1}-H_{s,t}}{H_{s,t}}\right]^2 + q_{t+1}^h \chi \left[\frac{H_{s,t+1}-H_{s,t}}{H_{s,t}}\right]\frac{H_{s,t+1}}{H_{s,t}}\right) \tag{75}$$

$$P_{c,t}\lambda_t = \left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma} \alpha (C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha} \tag{76}$$

$$P_{c,t}\lambda_t = \theta [P_{c,t+1}\lambda_{t+1}] \frac{R_{t,s}}{1+\pi_{t+1}} \tag{77}$$

$$0 = N_{s,t}w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - b_{s,t} - q_t^h H_{s,t-1}.\frac{\chi}{2}\left[\frac{H_{s,t} - H_{s,t-1}}{H_{s,t-1}}\right]^2 \quad (78)$$

3.2.4 Firms

I have two sectors in the production economy one of which produces housing services and the other consumption goods. In these two sectors a competitive final good producer demand and purchase $y_t^j(i)$ units of intermediate goods. where j represents the firm's production sector whereas in here j is housing h and Non durable consumption goods c. Also Each type of labour $N_{t,p}, N_{t,b}, N_{t,s}$ works for both housing sector and consumption sector which I will denote by $N_{p,t}^h, N_{b,t}^h, N_{s,t}^h$ for housing and $N_{p,t}^c, N_{b,t}^c, N_{s,t}^c$ for consumption at time t from all the three types of households. All the intermediate goods firms will hire labour from the perfectly competitive market.

Intermediate Housing firms

All of the final goods producers will take the intermediate goods as inputs to produce the final goods and these intermediate firms will provide the input goods. For tractability Let's assume the Intermediate firms will be of unit mass.

House prices follow a flexible prices unlike consumption prices due to the chance of negotiations in house prices and hence, I assume housing service prices follow a flexible prices. These housing intermediate firms will pay wages to the labour provided by the three sector of households and the employment equation is as follows:

Employment in the intermediate housing firms

$$\min_{N_{s,t}^h(i), N_{b,t}^h(i), N_{p,t}^h(i)} W_t^p N_{p,t}^h(i) + W_t^b N_{b,t}^h(i) + W_t^s N_{s,t}^h(i). \quad (79)$$

The firm tries to minimize their production costs of intermediate goods equation by choosing the values for the hire of labour provided by the households, $N_{s,t}^h(i), N_{b,t}^h(i), N_{p,t}^h(i)$ subject to the production constraint

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu} \quad (80)$$

Where Z_{ht} is the firm's sector specific technology shock, note that this shock is aggregate shock to the sector rather than firm specific and forming the Lagrangian L, with η_t being the Lagrange multiplier we have:

$$L = W_t^s N_{s,t}^h(i) + W_t^b N_{b,t}^h(i) + W_t^p N_{p,t}^h(i) - P_{ct}\eta_t \left(Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v} - y_t^h(i) \right)$$

Optimality conditions gives us the labour demand equations for all the three agents in the housing sector:

$$N_{p,t}^h = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^{v-1} Y_t^h \tag{81}$$

$$N_{b,t}^h = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^{u-1} \left(\frac{w_t^p}{v} \right)^{v} Y_t^h \tag{82}$$

$$N_{s,t}^h = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(-u-v)} \left(\frac{w_t^b}{u} \right)^{u} \left(\frac{w_t^p}{v} \right)^{v} Y_t^h \tag{83}$$

Price setting in the intermediate housing firms Firms choose prices to maximize expected profit for the obtained labour, as discussed earlier, this housing production sector follows the flexible prices due to the involvement of negotiations. Firms will discount profits s periods into the future by $m_{t,s}$, where $m_{t,s} = \theta^s \frac{U'(C_{s,t+s})}{U'(C_{s,t})}$ which is the stochastic discount factor. Also note that we have considered the stochastic discount factor from the Savers perspective as most of the firms is assumed to be owned by the Savers.

$$\max_{\{Q_s^h(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left(Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{1-\varepsilon} - \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} MC_t \right) \tag{84}$$

Substituting the labour demand. We have the Lagrange multiplier as:

$$\eta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^{u} \left(\frac{w_t^p}{v} \right)^{v} \tag{85}$$

The firm's decision problem boils down to choosing the intermediate good prices in the housing sector $Q_t^h(i)$ in order to maximize the expected profit equation (83) subject to the demand function (21) gives us the equation for the aggregate house prices:

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^{u} \left(\frac{w_t^p}{v} \right)^{v} \tag{86}$$

Intermediate Consumption goods Firms

The Profit optimization problem is standard as the housing goods sector. A firm chooses employment and prices to maximize profit subject to the production constraint in the consumption goods sector. The intermediate consumption goods labour choice remains the same as that of the intermediate housing goods firms with labour from hand to mouth agents Borrowers and Savers in the consumption goods sector. Firm i in the intermediate consumption goods sector minimizes nominal cost subject to the production constraint

$$\max_{\left\{N_{p,t}^{c}(i),N_{b,t}^{c}(i),N_{s,t}^{c}(i),p_{s}^{*}(i)\right\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} m_{t,s}\left(y_{t}^{c}(i)\,P_{ct}(i)-W_{t}^{p}N_{p,t}(i)+W_{t}^{b}N_{b,t}(i)+W_{t}^{s}N_{s,t}(i)\right).$$

subject to the production constraint

$$y_{t}^{c}(i)=Z_{ct}N_{s,t}(i)^{v}N_{b,t}(i)^{u}N_{r,t}(i)^{1-u-v},$$

The firm tries to minimize their production costs of intermediate goods equation by choosing the values for the hire of labour provided by the households, $N_{p,t}^{c}(i), N_{b,t}^{c}(i), N_{s,t}^{c}(i)$subject to the production constraint

The final good firm's demand equation of the input goods is as follows:

$$y_{t}^{c}(i)=Y_{t}^{c}\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon},$$

and price rigidity which I assume to follow the Rotemberg price setting scenario in which firms face a quadratic costs in changing the goods price.

Profit maximization problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. I deal with each of these problems separately. I use subscript 'p' do denote sector producing perishable goods. Both output and employment have this index.

Employment

Consumption good prices follow a Sticky prices unlike housing prices and hence, I assume housing service prices follow a Rotemberg model of prices, where as the house good prices are completely flexible. These consumption good intermediate firms will pay wages to the labour provided by the three sector of households and the employment equation is as follows:

Firm i minimizes nominal cost:

$$\min_{N_{s,t}(i),N_{b,t}(i),N_{p,t}(i)} W_{t}^{s}N_{s,t}^{c}(i)+W_{t}^{b}N_{b,t}^{c}(i)+W_{t}^{p}N_{p,t}^{c}(i).$$

subject to the production constraint

$$y_{t}^{c}(i)=Z_{ct}N_{p,t}^{c}(i)^{v}N_{b,t}^{c}(i)^{u}N_{s,t}^{c}(i)^{1-u-v}$$

optimality conditions yield us the solution for the Lagrange multiplier:

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v \tag{87}$$

and the labour demand equations for all the three household sectors:

$$N_{p,t}^c = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^{v-1} Y_t^c \tag{88}$$

$$N_{b,t}^c = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^{u-1} \left(\frac{w_t^p}{v} \right)^v Y_t^c \tag{89}$$

$$N_{s,t}^c = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v Y_t^c \tag{90}$$

The above equations clearly shows us that the labour demand from the firms are directly proportional to the output the consumption goods firms can produce Y_t^c and indirectly proportional to the level of technology in the firm sector Z_{ct}.

Price setting Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices.

$$\zeta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

where $MC_s = \zeta_t P_{ct}$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1\right)^2 y_t^c(i)$. The firm discounts future profits by the gross real interest rate between today and future dates (stochastic discount factor),

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

This yields us :

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(y_t^c(i) P_{ct}(i) - y_t^c(i) MC_t \right) - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 y_t^c(i) \right]$$

subject to Intermediate goods demand equation :

$$y_t^c(i) = Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon}$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

FOC w.r.t $\widetilde{P_{ct}}$ yields us the New Keynesian Phillips curve equation:

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + E_t \left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]} \left[(\pi_{t+1}) \frac{Y_{t+1}^c}{Y_t^c} (1+\pi_{t+1}) \right] \right] = \left(\pi_t (1+\pi_t) - \frac{1}{2} (\pi_t)^2 \varepsilon \right) \tag{91}$$

where the stochastic discount factor is given by:

$$m_{t,t+1} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right] \tag{92}$$

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

3.2.5 Profits of firms and Government Transfers

To close the model, we need to aggregate the real profits from the firms and has to be distributed among the households in terms of transfers. Aggregate inter-period nominal profit is the total output from both the firm sectors from which the wages of the agents are taken away. Please note because of the quadratic costs involved in the rigid intermediate consumption goods firms, that has to be taken away as well. This leads to:

$$\tilde{\Pi}_t = Y_t^c P_{ct} - W_{s,t}N_{s,t}^c - W_{b,t}N_{b,t}^c - W_{p,t}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c P_{ct} + Y_t^h Q_t^h - W_{s,t}N_{s,t}^h - W_{b,t}N_{b,t}^h - W_{p,t}N_{p,t}^h$$

I assume that the profit is 100 percent taxed by the government and redistributed according to the following rule:

$$t_{bt} = (1-x-y)\frac{\tilde{\Pi}_t}{P_{ct}}$$

$$t_{st} = x\frac{\tilde{\Pi}_t}{P_{ct}}$$

$$t_{pt} = y\frac{\tilde{\Pi}_t}{P_{ct}}$$

3.2.6 Financial Intermediaries:

Considering these are owned by Savers, as of now, the role of financial intermediaries in this model will be minimal, they just pool the deposits from the Savers and provide loans to Borrowers. For simplicity, I assume they pay the same interest rate to depositors as they charge from the Borrowers:

$$R_{t,o} = R_{t,s} \tag{93}$$

3.2.7 Market Clearing

In equilibrium we have the following resource constraints:

The aggregate labour of the agents are the sum of labour to the intermediate housing and consumption good firms.

$$\begin{aligned} N_t^p &= N_{p,t}^h + N_{p,t}^c & (94)\\ N_t^b &= N_{b,t}^h + N_{b,t}^c & (95)\\ N_t^s &= N_{s,t}^h + N_{s,t}^c & (96) \end{aligned}$$

The demand and supply of loans in the equilibrium are the same.

$$B_{s,t} = D_{t,d} \tag{97}$$

The central bank sets the interest rate by Taylor rule:

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r} \tag{98}$$

The total output in the economy is equal to the consumption of both housing goods and non durable consumption goods from the households including the depreciation of housing, investment quadratic costs for all the three agents and the quadratic costs incurred in the rigid intermediate consumption good firms.

$$\begin{aligned} Y_t^c + Y_t^h q_t^h = {} & C_{b,t} + C_{s,t} + C_{p,t} + q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) + q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) \\ & + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c + q_t^h H_{b,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{b,t} - H_{b,t-1}}{H_{b,t-1}}\right]^2 + q_t^h H_{s,t-1} \cdot \frac{\chi}{2}\left[\frac{H_{s,t} - H_{s,t-1}}{H_{s,t-1}}\right]^2 \end{aligned}$$

3.2.8 Private Sector Equilibrium

Please Refer to the Appendix for the whole Private Sector Equilibrium for this model.

3.3 Calibration:

The share of durables which is used as collateral (μ) is set to 0.90 "Kovacs and Moran (2019)". The discount factors are set at Savers' discount rate (θ) as 0.99 "Iacoviello and Pavan (2011)" and Borrowers' discount rate (β) as 0.98 "Kovacs and Moran (2019)". I set the Quarterly house depreciation rate (δ) as 0.01 "Iacoviello and Neri (2010)" and share of consumables in consumption basket (α) as 0.84. With other parameters at share of profits paid to Savers (x) as 0.7, share of profits paid to hand to mouth workers (y) as 0.1, if I assume that the average markup equals 10 percent, then this implies ε = 11, Inverse Intertemporal elasticity (σ) at 2 for Savers, Borrowers and hand to mouth guys respectively ""Iacoviello (2004)", Inverse Elasticity of labour (ϕ) at 1/3 "William B Peterman (2014)" and Scaling of hand to mouth labour in production function (ν) at 0.1, scaling of Borrowers labour in production function (u) at 0.2 and Scaling of Savers labour in production function (u) at 0.7 "The Labour Market Story: Skills for the Future (July 2014)", Taylor rule parameter as 1.1.

For the above parameters taken, the model also provides an estimate of percentage consumption of housing goods and consumption goods by different sectors of agents which closely replicates "Kaplan, Violante, Weidner (2014)" which is illustrated in the below table :

Agents Consumption	Consumption in Steady State	Total	Percentage
Consumption of Hand to Mouth Agents	0.0978		9.2 percent
Consumption of Borrowers	0.1988		18.8 percent
Consumption of Savers	0.7669		72 percent
		1.0635	
Rental housing	0.9157		8.3 percent
Housing of Borrowers	2.7767		25.1 percent
Housing of Savers	7.3403		66.6 percent
		11.0327	

3.4 Steady State Statics:

The calibrated model is used to perform a series of comparative statics exercises to investigate the impact of the size of the rental/ BTL sector on different decision rules of the agents. It is also used to qualitatively assess the effect of macro-prudential policy which regulates Downpayment ratio.

	BTL		BTL	Change with respect to Down payment ratio
	Down payment = 10 percent		Down payment = 25 percent	
Consumption of hand-to-mouth	0.09783		0.09780	Decrease because of increase in rents
Consumption of Borrowers	0.199		0.204	Increase because of Divert some of increase in rents to consumption but not to housing
Consumption of Savers	0.767		0.763	Decrease, due to decrease in deposits
BTL Property	0.915		0.882	Decreases due to decrease in Borrowers housing
Housing of Borrowers	2.78		2.72	Decrease, due to increase in Down payment
Housing of Savers	7.34		7.30	Decrease, due to decrease in deposits and the income it produces
Wages of Borrowers	0.0897		0.093	Increase, due to decrease in labour supply
Wages of hand-to-mouth	0.0260		0.0259	Decrease, due to increase in labour supply
Wages of Savers	0.931		0.923	Decrease, due to increase in labour supply

	BTL		BTL	Change with respect to Down payment ratio
Deposits	2.47		2.022	Decrease, due to less demand from Borrowers as increase in down payment
hand-to-mouth Labour	4.069		4.09	
Borrowers Labour	2.362		2.29	Decrease, as Borrowers has no incentive to work and to invest in housing
Savers Labour	0.796		0.803	
Consumption goods output	1.0635		1.0641	Increase, labour tend to move from Housing sector to consumption sector
Housing goods output	0.1012		0.1002	Decrease. Due to less demand for housing
Rental Price	0.0204		0.0211	Increase,Can be due to less supply of BTL

3.5 The Baseline Model Results

The technology shocks follow an AR(1) process in the housing goods sector:

$$\ln\left(Z_t^h\right) = \rho_h \ln\left(Z_{t-1}^h\right) + e_h$$

and an AR(1) process in the consumption goods sector:;

$$\ln\left(Z_t^c\right) = \rho_c \ln\left(Z_{t-1}^c\right) + e_c$$

where e_c, e_h are i.i.d processes with variances of σ_c and σ_h respectively which are both calibrated as 0.009^2. The shock persistence in both the sectors is ρ_h and $\rho_c$is taken as 0.3.

3.5.1 Effect of productivity shock in Consumption good firms:

In the following, I will consider impulse responses to increase in productivity of the consumption goods at date t. The temporary increase in the consumption good production technology directly translates into higher output in consumption goods sector Y_c as the output is directly proportional to the technology shock, as we will see later although the income effect dominates in all the agent sectors, resulting in a decrease in percentage rise of labour from all the three agents to the consumption goods sector N_b^c, N_p^c,N_s^c, the substantial increase in productivity of labour combined with the effect of technology shock dominates their impact on the output resulting in an increase in output change. Also note that the income effect dominates in the agents, especially with a positive technology shock in the consumption goods sector as the above equations of labour demand clearly shows us that the labour demand from the firms are directly proportional to the output the consumption goods firms can produce Y_t^c and indirectly proportional to the level of technology in the firm sector Z_{ct}.

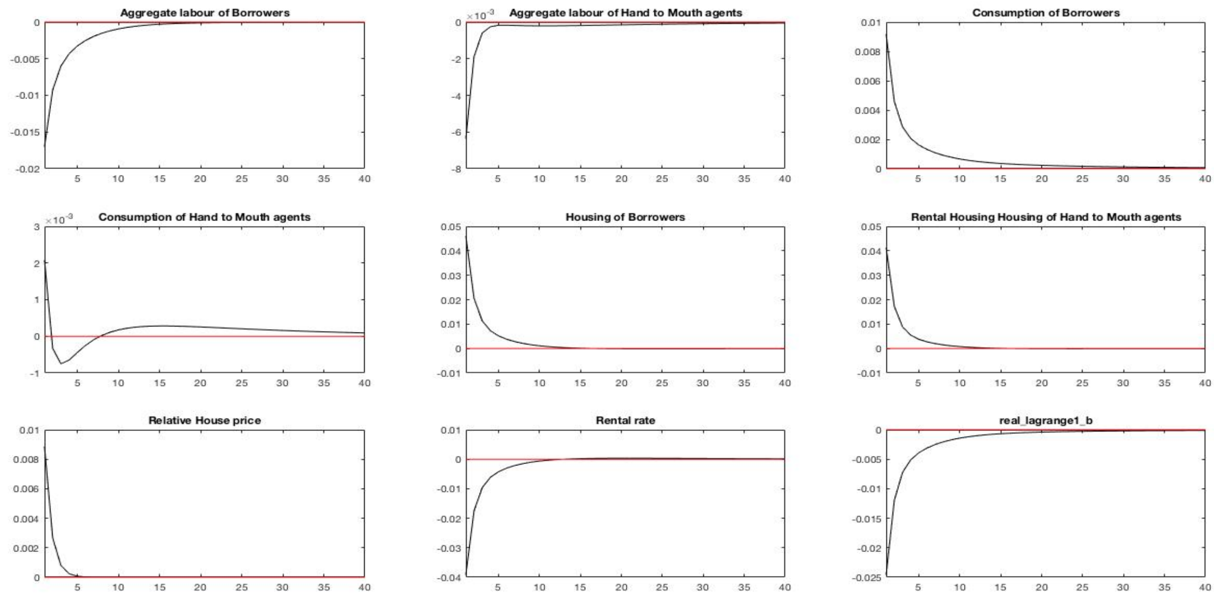
$$y_t^c(i) = Z_{ct} N_{p,t}(i)^{v} N_{b,t}(i)^{u} N_{s,t}(i)^{1-u-v},$$

From the consumption goods firm's perspective, marginal productivities of labour from all of the labour sectors have increased, they want to have more of all labour inputs, pushing up the real wage w_p, w_b, w_s to induce households in moving of the labour from housing good sector to consumption goods sector which results in decrease of N_b^h, N_s^h and N_p^h lowering the output in housing goods sector Y_h .

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^{v} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-v}$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in all of the sector's wages . From the below equation, we see that wages of Borrowers, Savers and hand to mouth guys are directly proportional to the output and marginal cost and inversely proportional to number of hours worked respectively for their sectors. Although there is an income effect from the agents, with higher output, higher marginal cost of firms dominates the income effect of agents, which result in wages of all the sectors to increase.

Figure 13: Baseline BTL Model: Impulse Response Functions with a positive technology shock in consumption sector I



$$W_{b,t} - P_{ct}\zeta_t u \frac{y_t^c(i)}{N_{b,t}^c(i)} = 0$$

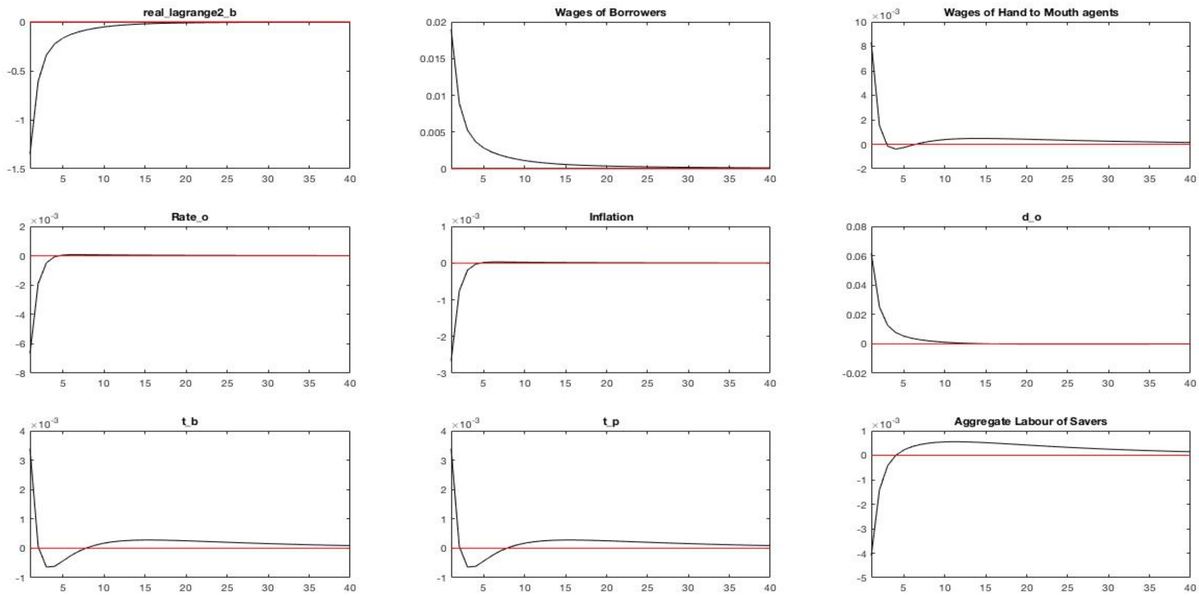
$$W_{p,t} - P_{ct}\zeta_t \nu \frac{y_t^c(i)}{N_{p,t}^c(i)} = 0$$

$$W_{s,t} - P_{ct}\zeta_t (1-u-\nu) \frac{y_t^c(i)}{N_{s,t}^c(i)} = 0$$

From households perspective, an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In my model, the Income effect clearly dominates the substitution effect for all the three sectors. We can see that the number of hours worked for consumption good firms in all sector N_b^c,N_p^c,N_s^c is inversely proportional to Z_c and directly proportional to w_b, w_p, w_s respectively , In my model, the Income effect clearly dominates the substitution effect for all the sectors.

$$\begin{aligned}
N_{p,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1} Y_t^c \\
N_{b,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} Y_t^c \\
N_{s,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} Y_t^c
\end{aligned}$$

Figure 14: Baseline BTL Model: Impulse Response Functions with a positive technology shock in consumption sector II



On aggregate, leisure $1-N_b$, $1-N_s$, $1-N_p$ increase i.e., labour N_b, N_s, N_p decreases which shows that Income effect dominates the substitution effect. We can observe from the following that number of hours worked are directly proportional to wages and the respective Lagrange multipliers in Borrowers and Savers sectors. where as the number of hours worked are directly proportional to wages, consumption and rental housing in poor workers.

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$\left(N_{s,t}\right)^{\phi} = w_{s,t}\left[P_{c,t}\lambda_t\right]$$

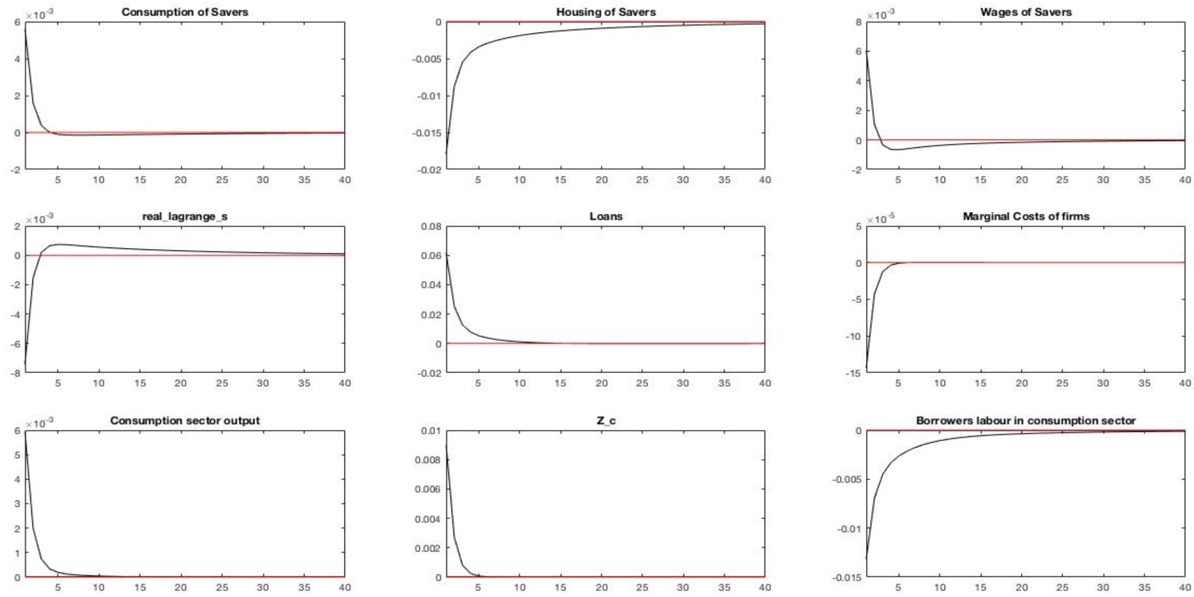
$$\left(N_t^p\right)^{\phi} = \left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_t^p\right)^{\alpha-1}\left(H_t^r\right)^{1-\alpha}w_t^p$$

As wages increase, workers from all the sectors will want to increase consumption, resulting in a rise of C_s and C_b and C_p . Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with increase in output directly translates to an increase in consumption of two sectors.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

Households know that the shock is temporary and they smooth their consumption. Also the magnitude of rise in wages is different from Borrowers and poor workers and with higher increase in wages for the

Figure 15: Baseline BTL Model: Impulse Response Functions with a positive technology shock in consumption sector III



Borrowers sector, additional income of Borrowers is not consumed completely but part of it is saved, i.e. invested into the housing stock H_b resulting in an increase in the housing stock of Borrowers which in turn also results in the increase in the BTL stock.

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

On the other hand, as Borrowers tend to invest in housing, combining with the decrease in the output of the housing sector, the price of the houses q_h increases resulting in Savers to issue more of loans B_s and consume more C_s and investing in their housing stock H_s decreases which shows that the demand of the housing stock is mostly from the Borrowers sector. The Savers sector tend to divert some of the housing stock towards the increase in issuing loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

Whereas, hand to mouth guys cannot invest in housing, combining with the increase of investment in housing sector from the Borrowers, Buy to let houses H_r have increased and the hence price of the rental rate q_r decreases due to more supply of BTL houses. As the wages of hand to mouth guys increasing and due to a very high income effect with the rents q_r decreasing results in an increase in consumption of this sector .

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

The increase in productivity of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, fall in inflation πin the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation decreases, the interest rate tend to decrease.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate decreases, Borrowers tend to obtain more loans B_s and increase their consumption. Borrowers tend to increase the investment in housing stock which further increases their collateral value and tend to have less dependency on the income of the Borrowers and consume more.

The increase in investment from Borrowers will in turn affect the Buy-to-let market which in turn increases the rental properties which increase in H_r. As the rental properties increases, this will reduce the rental price $q_r$and as a result, poor workers tend to have more for consumption and $C_p$increases. As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment in housing stock towards issuing more loans and hence we see a decrease in H_s.

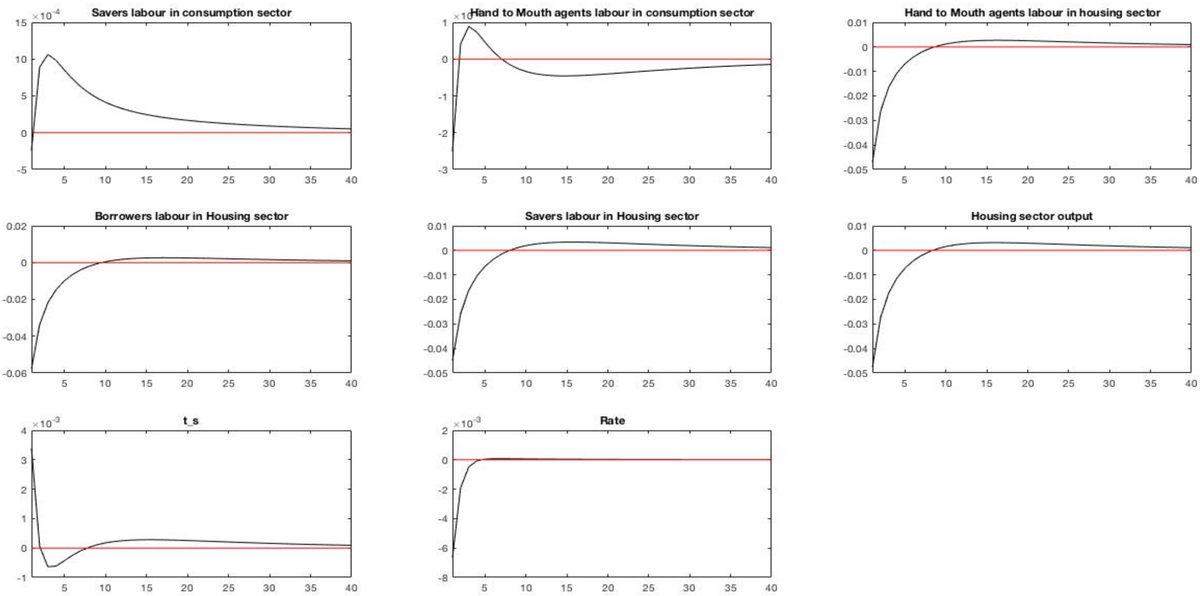
As the Savers tend to get more returns in terms of loans in volume, they tend to increase their consumption and an increase in consumption from all the sectors make firms to produce more of consumption goods which in turn make firms to hire with better wages. As the productivity in consumption good increases with the shock, more of wages and more of labour demand from the both the sectors in consumption good production side results in income effect from all sides, where all sectors tend to decrease their labour supply and have more leisure as wages increase.

The increase in wages in the consumption goods side results in labour moving to consumption goods production from the housing goods production sector resulting in decrease of housing goods output.

Interpretation:

We observe that the labour markets play a pivotal role in most of the dynamics. An innovation Z_c to consumption goods sector, translates to an increase in consumption output Y_c and a decrease in housing output Y_h which results in an increase in house prices q_h and decrease in consumption good prices (rigid market). The increase in house prices reduce the affordability of constrained agents (Borrowers) and Borrowers have no incentive to produce more of their labour and tend to see a decrease in supply of labour N_b. As the supply of labour from Borrowers decrease, wages of Borrowers w_b tend to increase, and with the effect of consumption good prices decrease, the disposable income of these agents would increase. This increase in disposable income will be translated to a very slight increase in consumption and also housing by the Borrowers as H_b increases and this in turn increase the demand for the loans B_s. As we observe, literally all the increase in borrower's housing is moved to BTL as the personal consumption of housing by the Borrowers remain constant. The increase in BTL will tend to decrease the rental rate as the q_r decreases. As the hand-to-mouth guys spending on rents decrease, they have more of disposable income and hence

Figure 16: Baseline BTL Model: Impulse Response Functions with a positive technology shock in consumption sector IV



they consume more. The increase in demand for loans is met by Savers who followed a similar trend to Borrowers in the labour markets, this results in a decrease in the rate returns R_s on the loans and a decrease in the housing of Savers. Overall the disposable income increase for all the three agents and are better off in consumption for a positive consumption shock. Borrowers tend to invest in housing and Savers tend to supply more loans with decrease in housing. I have also seen a similar trend in the change in variables from the collateral constraint model in the previous chapter. However, with the inclusion of Hand to mouth agents (Renters) in the economy, the housing stock of Borrowers has substantially increased at a cost of slight decrease in their consumption.

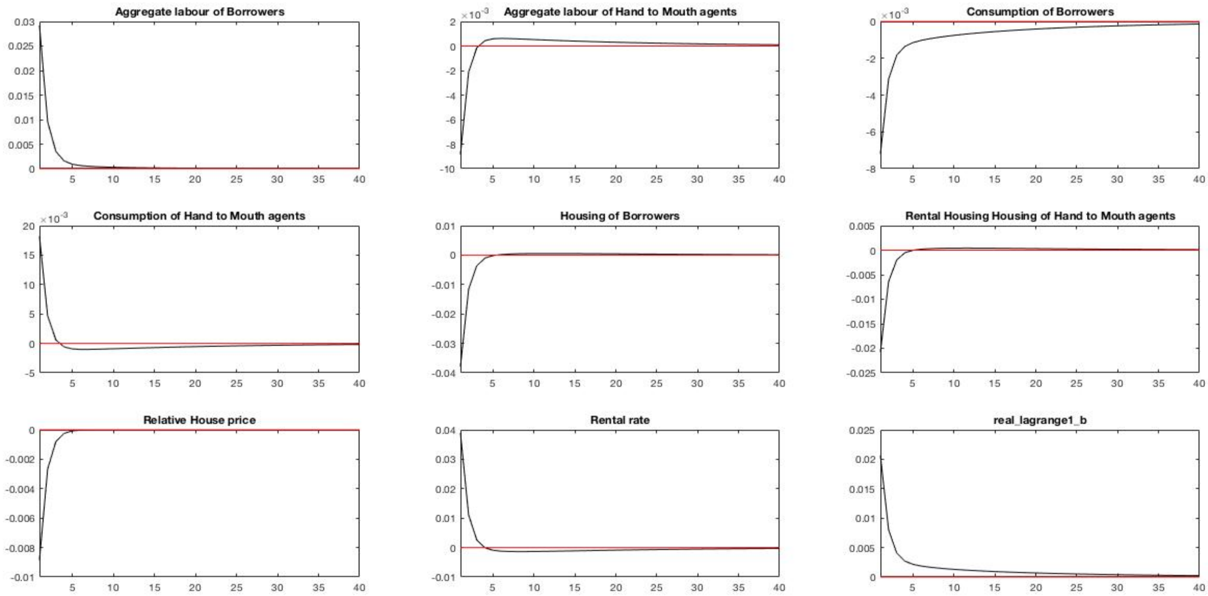
3.5.2 Effect of productivity shock in Housing good firms:

The temporary increase in the housing good production technology directly translates into higher output in housing goods sector Y_h as the output is directly proportional to the technology shock,

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v}$$

From the housing goods firm's perspective, marginal productivities of labour from the Savers labour sector has increased, they want to have more of labour input which induce to supply more labour N_b^h, N_s^h, N_p^h and this results in moving of the labour from consumption good sector to housing goods sector which results in decrease of N_s^c, N_p^c but we can see an increase in N_b^c as Borrowers would see a decrease in wages and to compensate tend to go with supply of more labour. However the magnitude of percentage decrease of

Figure 17: Baseline BTL Model: Impulse Response Functions with a positive technology shock in housing sector I



N_s^c, N_p^c is higher than that of increase in N_b^cwhich results in a decrease of y_c

$$y_t^c(i) = Z_{ct} N_{s,t}(i)^{\nu} N_{b,t}(i)^{u} N_{r,t}(i)^{1-u-v}$$

As I assume that the wages are same across the two sectors of production, there is a rise in hand to mouth sector's wages and fall in Savers and borrower's wages with different magnitude .

From the below equation, we see that wages of poor guys are directly proportional to the output and marginal cost and inversely proportional to number of hours worked. With higher output, higher marginal cost of firms and substitution effect of poor workers, wages of poor workers increase. In this case, the output effect dominates the labour supply effect.

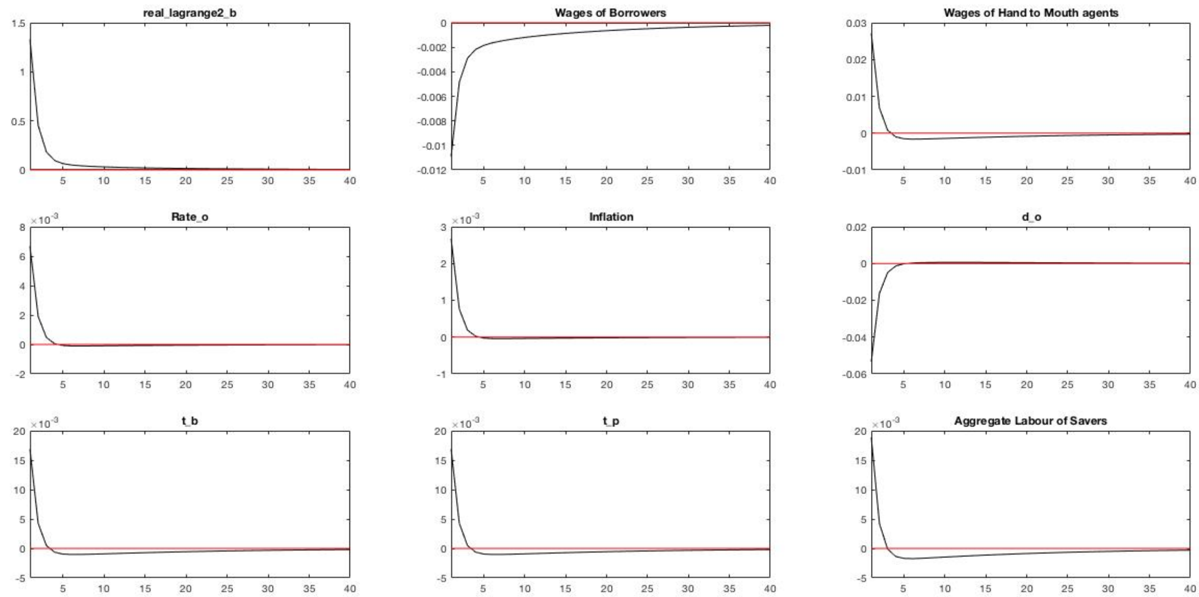
$$W_t^p - P_{ct}\eta_t \nu \frac{y_t^h(i)}{N_{p,t}^h(i)} = 0$$

From the below equation, we see that wages of Borrowers are directly proportional to the output and marginal cost and inversely proportional to number of hours worked. Although with higher output, due to the labour supply effect dominates the output, wages of Borrowers decreases. This is also the result of the fact that as Borrowers tend to supply more labour to take advantage of the housing boom. (However, as we will see later Borrowers could not afford much housing) . The wages of the Borrowers tend to decrease.

$$W_{b,t} - P_{ct}\eta_t u \frac{y_t^h(i)}{N_{b,t}^h(i)} = 0$$

From the below equation, we see that wages of Savers are directly proportional to the output and

Figure 18: Baseline BTL Model: Impulse Response Functions with a positive technology shock in housing sector II



marginal cost and inversely proportional to number of hours worked. Although with higher output, due to the labour supply effect dominates the output, wages of Savers decrease.

$$W_{s,t} - P_{ct}\eta_t (1-u-\nu)\frac{y_t^h(i)}{N_{s,t}^h(i)} = 0$$

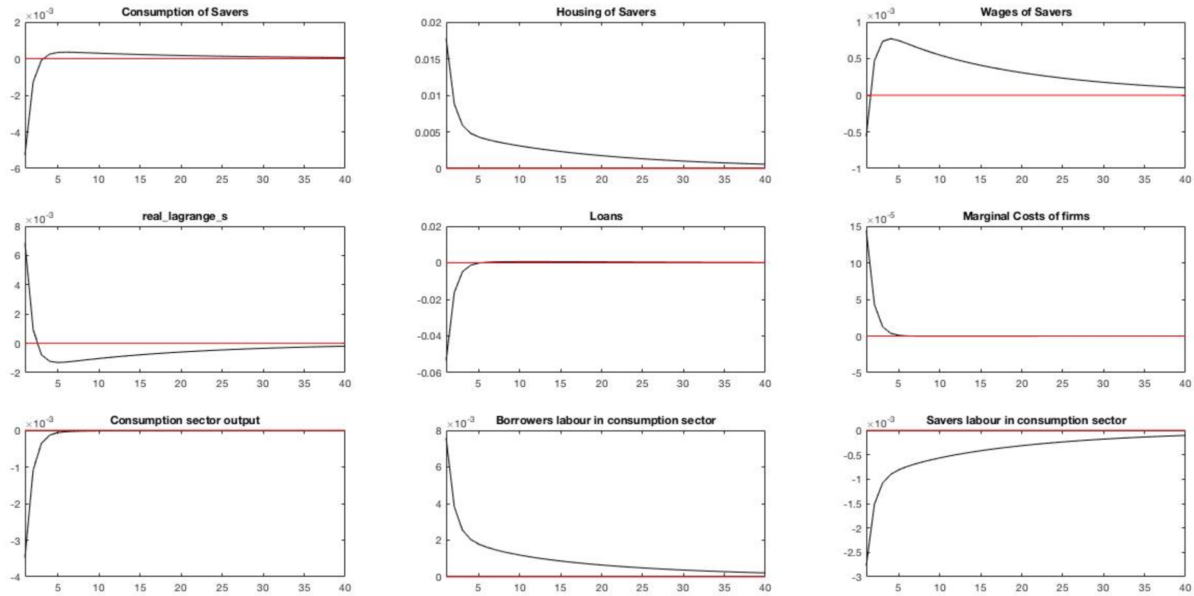
From households perspective, In my model with productivity shock in housing good firms, wages decrease in Borrowers sector. For the considered parameter "scaling of poor workers labour in production"$\nu = 0.1$ and "scaling of Borrowers labour in production" $u = 0.2$, we can see that the number of hours worked for consumption good firms in Borrowers sector N_b^h is inversely proportional to Z_h and w_b and directly proportional to the output w_p, w_s, hence as increase in output w_p and decrease in wages of Borrowers dominates the rise in Z_h and decrease in $w_s$prompted the increase in N_b^h .

$$N_{b,t}^h = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h$$

Where as due to the income effect in the Savers sector with decrease in real wage in Savers sector. As wages decrease, for the considered parameter "scaling of poor workers labour in production"$\nu = 0.1$ and "scaling of Borrowers labour in production" $u = 0.2$, , we can see that the number of hours worked for consumption good firms in Borrowers sector N_s^h is inversely proportional to Z_h and w_s , hence as increase in w_p and Y_h with decrease in wages of Savers dominates the rise in two variable Z_h and decrease in w_b prompted the increase in N_s^h

$$N_{s,t}^h = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h$$

Figure 19: Baseline BTL Model: Impulse Response Functions with a positive technology shock in housing sector III



Here, the substitution effect dominates the income effect in the poor workers sector with increase in real wage in this sector. As wages increase, workers will want to work more. Here we have substitution effect on utility between renting housing stock and leisure.We can see that the number of hours worked for housing good firms in this sector N_p^h is inversely proportional to Z_h, as increase in Y_h, w_pdominates the rise in variable Z_h and a decrease in w_b, w_s prompted the increase in N_s^h resulting in substitution effect.

$$N_{p,t}^h = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^{v-1} Y_t^h$$

As wages decrease in Borrowers sector , workers from this sector will want to decrease consumption, resulting in a fall of C_b and on the other hand as wages in Savers sector decrease, the decrease in output of consumption goods sector make Savers to invest in housing more and hence a decrease in C_s can be seen. On the other hand, although the rental rates have increased, the wages of the poor workers increase dominates the rental rates which results in an increase in consumption from poor workers. Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with decrease in output directly translates to an decrease in consumption of two sectors and an increase in poor guys.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2} \pi_{ct}^2 Y_t^c$$

On the other hand, as the output in housing sector increases, this results in a decrease of relative price of houses, although we see a slight increase in rental rates, the decrease in relative house prices affect the

Borrowers more due to the collateral constraint and housing services from Borrowers sector decreases. This result in an increase in the rental rates as well. Also as the income to the Borrowers decrease, they tend to decrease their consumption and also they decrease their housing stock which in turn results in a decrease in the Buy to Let houses H_r and also in loans B_s. This results in an increase of housing stock from Savers sector as they divert most of their deposits into housing due to a decrease in demand for loans from the Borrowers sector.

$$Y_t^h = (H_{b,t} - (1-\delta)H_{b,t-1}) + (H_{s,t} - (1-\delta)H_{s,t-1})$$

Households know that the shock is temporary and they smooth their consumption. As the wages for the Borrowers sector decrease, they tend to consume less of consumption goods and less of housing goods resulting in an decrease in the housing stock of Borrowers. This in turn will decrease the housing stock for Buy-to-Let properties and result in a decrease of H_r

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

Also as the BTL properties decreases and an increase in wages of the poor guys will increase the demand of the BTL properties which in turn will increase the price of the rents q_r due to constrained supply from the Borrowers.

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

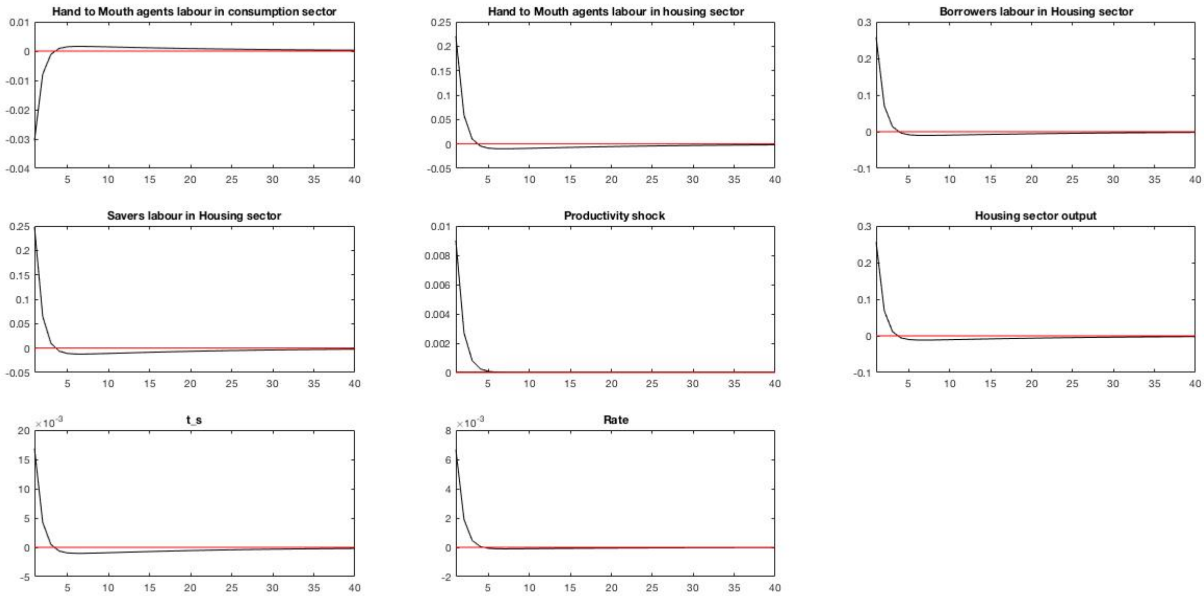
On the other hand, combining with the increase in the output of the housing sector, the price of the houses q_h decreases resulting in Savers to issue less of loans B_s and as we see they also consume less C_s and investing in their housing stock H_s increases which shows that the demand of the housing stock is mostly from the Savers sectors. The Savers sector tend to divert some of the loans to housing stock.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

On aggregate, leisure $1-N_b$ and $1-N_s$ decreases i.e., labour N_b, N_s increases whereas for the poor workers $1-N_p$ increases i.e., labour N_p decreases.

The increase in productivity of housing good firms enhances to supply more of the housing goods and less of consumption goods with the same inputs and due to the monopolistic competition, firms increase the price of the consumption goods for better marginal profits and as a result, rise in inflation πin the economy occurs. The central bank sets the interest rate according to the Taylor rule and as the inflation increases, the interest rate is tend to increase.

Figure 20: Baseline BTL Model: Impulse Response Functions with a positive technology shock in housing sector IV



$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate increases, Borrowers tend to obtain less loans B_s and decrease their consumption. They tend to decrease the investment in housing stock which further decrease their collateral value and also the BTL properties. As the demand for loans decrease, Savers tend to supply less of deposits as they tend to divert some of their investment in housing stock and hence we see an increase in H_s. As the Savers tend to invest more in housing, they tend to decrease their consumption and an decrease in consumption from both the sectors make firms to produce less of consumption goods although an increase in consumption from one sector(Poor guys) which in turn make firms to hire more labour for housing goods sector.

Interpretation:

On top of the labour markets role, collateral constraint also pays a huge role in the housing shock dynamics. Hence we don't see a quite mirrored results of the consumption shock. An innovation Z_h to housing sector, translates to an increase in housing output Y_h and a decrease in consumption goods output Y_c which results in an decrease in house prices q_h and increase in consumption good prices (rigid market). As the consumption goods market has some price rigidity, the increase wouldn't be that high. The Hand-to-Mouth agents have no incentive to provide more labour in this scenario as N_p decreases which result in an increase in wages w_p of these agents. As the house price decreases, the collateral value of the Borrowers would decrease and in combination of an increase in interest rate, this makes the Borrowers to decrease their housing stock which is purely based on the collateral constraint. This results in Borrowers decreasing their housing stock even though the house prices decrease. On the other hand, Savers increase their housing stock to compensate the increase in housing output and also decrease in loans make them divert some of the funds to housing. As the housing stock of Borrowers decrease, the BTL stock also decreases. However this time unlike the consumption shock, Borrowers have to take a penalty on their own housing consumption as there will be an increase in demand of housing which also account for the rental income from the Hand to mouth guys but decrease in their own investment of housing stock. An increase in wages of the Hand-to-Mouth agents and decrease in supply of BTL will result in an increase in consumption of Hand-to-Mouth agents who are better off in consumption and a bit worse off in rental housing with a positive housing shock. As the collateral constraint plays a huge role in the housing decision choice of Borrowers, to compensate for the wealth loss from the BTL, Borrowers tend to produce more labour which again reduce the wages of Borrowers in a very slight order. With a positive housing shock, Borrowers are a bit worse off and reduce their BTL holdings. The Savers sector tend to move with the Borrowers sector in terms of labour market which results in a very slight decrease of consumption, however their holding of houses increase drastically. Overall, with a positive housing shock, the BTL holdings have gone down. As opposed to the previous model of non BTL collateral constraint, with an increase in the housing sector productivity, the effect of decrease in price of a house is substantial as Borrowers decrease their housing substantially.

3.6 Volatility

3.6.1 Down Payment Ratio:

As Baptista et al(2016) (Agent based model) suggests, I also would like to validate if Buy-to-Let market effects the volatility of the house prices in General Equilibrium framework. I try to find the volatility of house prices with Down payment ratio as the policy tool with the benchmark share of household agents. The downpayment ratio can act as a policy tool in constraining BTL as we see that the changes in Borrowers housing stock is mostly passed to the changes in BTL as they try to maintain a constant share of personal housing consumption.

Table 2: Std. deviation of house prices

	BTL		BTL	BTL
	Down payment ratio = 0.10		Down payment ratio = 0.25	Down payment ratio = 0.5
Std. Deviation of House prices	0.0126		0.0126	0.0126

With adjustment costs, I see no change in volatility of relative house prices which contradicts the claim by Baptista et al(2016) (Agent based model). Altering the size of BTL sector from altering the downpayment ratio doesn't have much impact on the volatility of house prices in the General Equilibrium model as opposed to partial Equilibrium Agent based model by Baptista et al(2016).

3.7 Conclusions

I conclude this work with the following results. As discussed in the previous chapter, the housing wealth has a positive effect on the consumption of Borrowers when the relative house price increases. However, we have also seen effect of labour dynamics on the households decisions. An increase in housing supply from an technology shock in the housing sector doesn't necessarily translates to an increase in the BTL sector. In fact, the housing stock of Borrowers as a whole decreases although the prices of the houses decrease with an increase in supply of houses. This is mainly driven by the collateral constraint where the Borrowers collateral value would decrease and these agents wouldn't want to invest in housing. However, Savers are the ones who will be having more of houses in this situation as I assume the hand-to-mouth agents can't afford and don't invest in housing. The second interesting result is that by altering the size of BTL sector with altering the downpayment ratio doesn't have much impact on the volatility of house prices as opposed to the partial equilibrium Agent-based model by Baptista et al(2016). I have also repeated the exercise with a rigid housing markets and found the similar result. I believe, several reasons can explain the low volatility of relative house prices with respect to the size of BTL markets. One of the main reasons I believe is that there can be some substantial volatility of relative house prices stemming from the supply side constraints in the economy. In particular, the land for construction being a fixed factor. This leads us to an important question, whether constraining BTL markets is an optimal policy to only places where there is a supply constraint such as urban places? my model can be expanded to assess the effects of even more policy options. Further research into incorporating a rigidity on housing side in greater detail and the analysis of increase in stamp duty on BTL markets in the rural housing markets could introduce us to some new results.

4 Buy to Let Markets with CES Utility and Policy Analysis

4.1 Introduction

From the results of the previous chapter, even though we have observed that the size of BTL hasn't had much impact on the relative house prices. In this chapter, I would also like to assess and analyze an important policy question of whether macro prudential policy which regulates downpayment ratio or the Monetary policy which uses short term interest rate is more effective for curbing the volatility of important variables in the economy as volatile markets are deemed to be highly risky for economic stability. First, we find that with news shocks hitting the economy, Monetary policy is more effective but with only productivity shocks we see Macro prudential policy is more effective. I also analyze the economy with a news shocks and especially a news shock on rental returns and how hand to mouth agents and Borrowers behave would be an interesting scenario. I will also look at how the agents behave with other different expectation shocks to the model. In particular, I will augment the model with expectation shocks on rental returns of Borrowers, expectation shocks on technology in both housing and consumption goods sectors and an expectation shock on Monetary policy. The structure of this model is similar to that of the one in chapter 3. However, I look from a perspective of how agents behave with a CES utility between consumption and housing choices which I have taken from literature on housing models: see e.g., Sefton and Miles (2018). As I assume the labour to be additively separable in the utility function, the utility stemming from the consumption bundle which includes both housing and consumption goods plays an important role in the analysis. With an assumption of Cobb Douglas utility between consumption and housing in chapter 3, consumption and housing goods are subjected to an unit Intra-temporal Marginal Rate of Substitution and hence the proportional response to the change in housing goods is same as a change in non housing consumption goods. However, such assumption might be at odds with the real world scenario and the model can be too restrictive. To counter such limitation, I have assumed the additively separable CES utility between housing and consumption. Intra-temporal elasticity of substitution between housing services and non housing consumption is represented by a parameter. For high values of such parameter; agents are willing to substitute consumption and housing goods within each period. The two goods become perfect substitutes as the parameter tends to infinity and perfect complements as the parameter tends to zero. Same as my previous models, the model will include three different types of household agents. As opposed to the previous models, I will look at an additively separable utility function.

First, there are poor 'hand-to-mouth' households who work and rent the houses; second, there are rich households who are able to smooth their consumption over time, they are Savers who provide funds to the other households, who can borrow and lastly there will be Borrowers who are credit-constrained but still can borrow under suitable collateral. The model includes financial intermediaries (FI) which has a minimal role of pooling the loans from the Savers and providing them to the Borrowers. In addition, there will be a Central Bank and the central bank will be responsible for monetary and macro-prudential policy aiming to reduce house price volatility. There are two types of firms which produce consumption goods and housing goods. Both the firm sectors are subjected to price rigidities in different magnitudes. Interactions between Borrowers and Central Bank policies (Both Monetary and Macro-Prudential policies) will have to play the main role in this model, and the policies will be designed to affect the incentives of the Borrowers. In addition to the changes in utility and the degree of price rigidities, in this chapter, I look into News shocks and also how the policies interact with each other. I will also analyze the affects and interactions of both Monetary and Macro prudential policies for a range of realistic policy parameters.

4.2 The Model

The model has a very similar structure of Sectoral heterogeneity as previous models. There are three different types of household agents. First, there are poor 'hand-to-mouth' households who work and rent the houses; second, there are rich households who are able to smooth their consumption over time, they are Savers in this and provide funds to the other households, who can borrow. Lastly there will be Borrowers who are credit-constrained but still can borrow under suitable collateral. The model includes financial intermediaries (FI) which has a minimal role of pooling the loans from the Savers and providing them to the Borrowers. In addition, there is a Central Bank and is responsible for monetary and macro-prudential policy aiming to reduce house price volatility. There are two types of firms which produce consumption goods and housing goods. I also assume both the firm sectors are subjected to price rigidities. The policy will be designed to affect the incentives of the Borrowers.

4.2.1 Hand-to-mouth workers

Hand to Mouth workers are completely credit constrained, they do not have suitable collateral for borrowing from the bankers. Each of these households will consume, supply labour and rents a house and gains utility from the house they rent, and consumption. I assume these households couldn't save and the budget constraint can be as follows:

Budget Constraint (Nominal terms):

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t} \tag{99}$$

where Q_t^r rental price of the house, $P_{c,t}$ is the given price of the consumption goods, H_t^r rented house, C_t^p consumption of the representative household, N_t^p production labour from Hand-to-Mouth workers. W_t^prepresent the wage rate. These households will earn the income from providing the labour to both the sectors of firms and will spend all of it on consumption and rent.

A typical Hand-to-Mouth household consumes both the consumption services and the housing services. I also assume that the wage depends only on the type of labour, not on the type of firm. Unlike the previous version of the models where we have a elasticity of substitution of 1 (Cobb Douglas Utility), the utility of these households follows a Constant Elasticity of Substitution between housing and consumption in this version of model:

Utility :

$$\sum_{t=0}^{\infty} \gamma^t U\left(C_t^p, N_t^p, H_t^r\right)$$

$$\sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma} \left[\left(aC_t^{p\,1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^p\right)^{1+\phi} \right) \tag{100}$$

where C_t^p denotes the consumption of the final consumer service from the Hand-to-Mouth Workers, H_t^r denotes the housing services from which these households incur the utility, typically assumed to be the roof under which the household survives. This roof under housing services would be the rented house for hand to mouth agents. N_t^p denotes the combined labour supplied by these households for the two types of firms due to the assumption that the wage depends only on the type of labour, not on the type of firm. Where ρ is the Intra-temporal elasticity of substitution between housing and consumption goods and α reacts the degree of inter-temporal substitutability.

The Hand-to-Mouth households maximize their Utility (100) given the prices by choosing N_t^p, H_t^r .

Solving the above maximization problem gives us the labour supply of Hand to mouth agents in both the firm sectors

$$\begin{aligned}\left(N_t^p\right)^{\phi} &= \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\ &\frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{-\frac{1}{\rho}} w_t^p \end{aligned} \tag{101}$$

Also the solution provides us with the housing demand (the rented house demand from the hand to mouth agents) of this sector

$$\begin{aligned} 0 = &\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\ &\frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\ &\left[-a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{\left(-\frac{1}{\rho}\right)} q_t^r + (1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right] \end{aligned} \tag{102}$$

At last as these agents have no power over savings whatever is left in their budget goes to the consumption which is given as

$$C_t^p = N_t^p w_t^p + t_{p,t} - H_t^r q_t^r \tag{103}$$

4.2.2 Borrowers

Borrowers are the central focus of the model. These agents acquire loans from the Financial Intermediaries and invest in two types of housing: Buy-to-Let housing and Owned housing. I assume these Borrowers choose their share of Buy to Let housing endogenously also depending on the demand from the Hand to

Mouth agents. These households can borrow under collateral normally I assumed that houses act as their collateral, borrow money from the banks in terms of loans to invest in the either owned housing or Buy-to-Let housing.

I assume the housing is a divisible and part of the house is owner occupied and also other part of it is rented to other sector of hand-to-mouth households. These households budget constraint can be thought of as follows:

Budget Constraint (Nominal terms):

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,d}D_{t-1}^d = H_t^r Q_t^r + N_t^b W_t^b + D_t^d + T_{b,t} \tag{104}$$

where C_t^b denotes the consumption of the final consumer service from the Borrowers sector, $P_{c,t}$ is the given price of the consumption goods, Q_t^h price of the house at time t, H_t^r rented house to the Borrowers sector, Q_t^r being the rental rate of the let house at time t, H_t^b total house , rented and owner occupied and also depreciates at the rate δ, D_t^d one period nominal debt from the bank at the end of period t provided to the housing of Borrowers sector and $R_{t-1,d}$ is nominal debt lending rate on loan demanded at time period 't-1' D_{t-1}^d. In particular all the expenditures and investment from the Borrowers will be equal to their gains.

Let us denote $H_{t,o}$ Owner occupied part of the house:

$$H_t^o = H_t^b - H_t^r;$$

As I assumed all households gain utility from the under roof housing services $H_t^o = H_t^b - H_t^r$ is the Owner occupied housing of Borrowers, consumption service C_t^b and the leisure $1 - N_t^b$.

The typical utility for these households follows a CES function similar to that of the Hand to Mouth agents and is as follows:

Utility:

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t^b, N_t^b, H_t^b - H_t^r\right)$$

$$E_0\beta^t \sum_{t=0}^{\infty} \left(\frac{1}{1-\sigma} \left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi}\left(N_t^b\right)^{1+\phi} \right) \tag{105}$$

Note that I include owner-occupied housing $H_t^o = H_t^b - H_t^r$ in utility, not the rental house, as we treat utility of housing as being under the roof.

I also assume the Borrowers are under some collateral constraints, where at any time period 't' the maximum amount of the combined loan and repayment on the loans the Borrowers promised to pay in the following period should be only less than or equal to the fraction of the expected future house value which I assume will be determined by the central bank. The collateral constraint can be thought of as follows:

$$R_{t,b}D_t^b \leq \mu Q_t^h H_t^b \tag{106}$$

$\mu = (1 - downpayment)$ means the central bank imposes a constraint on the maximum credit amount which this sector of households can get. Hence these are not fully credit unconstrained.

The Borrowers households problem is to choose $C_t^b, N_t^b, H_t^b, H_t^r, D_t^o, D_t^r$ by maximizing Utility (108) subject to Budget Constraint (107) and the the collateral constraint (109) :

Solving the above maximization problem gives us the labour supply of Borrowers in both the firm sectors

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right] \tag{107}$$

where

$$\begin{aligned}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\ \frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(C_t^b\right)^{-\frac{1}{\rho}} \\ = P_{c,t}\xi_t\end{aligned} \tag{108}$$

Also the solution provides us with the Rented housing supply specifically the endogenous choice of the Buy to Let sector and the other optimal conditions are as follows:

$$\begin{aligned}0 = &\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\ &\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b - H_t^r\right)^{-\frac{1}{\rho}}\right) \\ &- \left[P_{c,t}\xi_t\right]\left(q_t^h\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h \\ &+ \beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)\end{aligned} \tag{109}$$

$$0 = \left[\left(a{C_t^b}^{1-\frac{1}{\rho}} + (1-a) \left(H_t^b - H_t^r \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$

$$\frac{\rho}{\rho - 1} \left[\left(a{C_t^b}^{1-\frac{1}{\rho}} + (1-a) \left(H_t^b - H_t^r \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] \left((1-a) \left(1 - \frac{1}{\rho} \right) \left(H_t^b - H_t^r \right)^{-\frac{1}{\rho}} \right) - \left[P_{c,t} \xi_t \right] q_t^r \tag{110}$$

$$0 = \left[P_{c,t} \xi_t \right] - \left[P_{c,t} \Psi_t \right] R_{t,o} - \beta \left[P_{c,t+1} \xi_{t+1} \right] R_{t,d} \frac{1}{1 + \pi_{t+1}} \tag{111}$$

$$0 = H_t^r q_t^r + N_{b,t} w_{b,t} + d_{t,d} + t_{b,t} - C_{b,t} - q_t^h (H_{b,t} - (1-\delta) H_{b,t-1}) - R_{t-1,d} d_{t-1,d} \frac{1}{1 + \pi_t} \tag{112}$$

$$0 = \mu q_t^h H_{b,t} - R_{t,d} d_{t,d} \tag{113}$$

4.2.3 Savers

Savers consumes, supply labour inelastically and also save. They supply funds to the financial intermediaries and let the banks to circulate their money in terms of Credit.

I also assume the discount factor for this sector is high than the previous sector's and I denote it with θ as this sector of agents tend to save money.

$$0.99 = \theta > \beta = 0.985$$

Budget Constraint of this sector will be

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} \tag{114}$$

$(H_{s,t} - (1-\delta)H_{s,t-1})$ is the investment in housing from the Savers sector

The deposits from the working representatives of the saver's household are one-period bonds that pay with the return $R_{t-1,s}$ from t – 1 to t. Let $B_{s,t}$ be the debt the saver's household acquires, all profits are expropriated by the government and redistributed as transfers T_{st}.

The Savers households gain utility from the under roof housing services $H_{s,t}$, consumption service $C_{s,t}$ and the leisure $N_{s,t}$. The typical utility for these households is as follows:

The Saver's household discounted Utility is as :

$$\sum_{t=0}^{\infty} \theta^t U^s\left(C_{s,t}, N_{s,t}, H_{s,t}\right)$$

$$= \sum_{t=0}^{\infty} \theta^t \left(\frac{1}{1-\sigma} \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)H_t^{s\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^s\right)^{1+\phi} \right) \tag{115}$$

The Savers households problem is to choose $C_{s,t}, N_{s,t}, H_{s,t}, B_{s,t}$ by maximizing CES Utility (115) subject to Budget Constraint (114) .

Solving the above maximization problem gives us the labour supply of Savers in both the firm sectors

$$\frac{\left(N_{s,t}\right)^{\phi}}{\left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)H_t^{s\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma} \frac{\rho}{\rho-1} \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a\left(1-\frac{1}{\rho}\right)\left(C_t^s\right)^{-\frac{1}{\rho}}} = w_t^s \tag{116}$$

The housing demand from the Savers will also be obtained as

$$
\begin{aligned}
0 = & \left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
& (1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^s\right)^{-\frac{1}{\rho}} \\
& -\left[P_{c,t}\lambda_t\right]\left(q_t^h\right) \\
& +\theta\left[P_{c,t+1}\lambda_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)
\end{aligned} \tag{117}
$$

where

$$
\begin{aligned}
P_{c,t}\lambda_t = & \left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
& \frac{\rho}{\rho-1}\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(C_t^s\right)^{-\frac{1}{\rho}}
\end{aligned} \tag{118}
$$

The Interest Rate can be obtained as :

$$
P_{c,t}\lambda_t = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}} \tag{119}
$$

$$
0 = N_{s,t}w_{s,t}+t_{st}+b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t}-C_{s,t}-q_t^h(H_{s,t}-(1-\delta)H_{s,t-1})-b_{s,t}
$$

4.2.4 Firms

I have two sectors in the production economy one of which produces housing services and the other consumption goods. In one of the sectors (consumption sector) a competitive final good producer demand and purchase $y_t^c(i)$ units of intermediate goods. where c represents the firm's production sector whereas in here is Non durable consumption goods c. Also Each type of labour $N_{t,p}, N_{t,b}, N_{t,s}$ works for both housing sector and consumption sector which I will denote by $N_{p,t}^h, N_{b,t}^h, N_{s,t}^h$ for housing and $N_{p,t}^c, N_{b,t}^c, N_{s,t}^c$ for consumption at time t from all the three types of households. All the intermediate goods firms will hire labour from the perfectly competitive market. I model a final side of the goods sector as stand in aggregate firm which follows a Constant Elasticity of Substitution (CES) production technology to aggregate the intermediate products. Please refer to the Appendix for the Final goods section. Profit optimization problem is standard as we have seen in the above chapter with three types of labour. Profit maximization problem can be split into two separate problems: intermediate firms choose labour to minimize cost intra-temporally and choose prices of these intermediate goods to maximize future profit subjected to the production constraint. These intermediate goods firms are subjected to perfectly competitive labour markets taking the real wage rate. However, they always act to minimize the costs.

Intermediate Consumption firms

. A firm chooses employment and prices to maximize profit:

$$\max_{\left\{N^c_{p,t}(i),N^c_{b,t}(i),N^c_{s,t}(i),p^*_s(i)\right\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} m_{t,s}\left(y^c_t(i)\,P_{ct}(i) - W^p_t N_{p,t}(i) + W^b_t N_{b,t}(i) + W^s_t N_{s,t}(i)\right). \tag{120}$$

subject to the production constraint

$$y^c_t(i) = Z_{ct} N_{s,t}(i)^{\nu} N_{b,t}(i)^{u} N_{r,t}(i)^{1-u-\nu}, \tag{121}$$

The final good firm's demand equation of the input goods is as follows:

$$y^c_t(i) = Y^c_t \left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}, \tag{122}$$

and price rigidity which I assume to follow the Rotemberg price setting scenario in which firms face a quadratic costs in changing the goods price.

Profit maximization problem can be split into two separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. We deal with each of these problems separately and we obtain the Phillips curve for the consumption goods sector as:

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + \theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y^c_{t+1}}{Y^c_t}(1+\pi_{t+1})\right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2\varepsilon\right) \tag{123}$$

where

$$m_{t,t+1} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right] \tag{124}$$

$$\frac{1}{R_{t,s}} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right] \tag{125}$$

Intermediate Housing firms

I follow a housing market where the sector is split into two sub sectors: one for final goods which are not perfect competitive firms as in consumption goods sector. However, I assume the final goods sector follows a monopolistically competitive approach. These monopolistic firms who produce final houses doesn't involve labour and capital as for the intermediate housing goods sector which follows a sticky prices and involve labour from all the three sectors of households

Profit optimization problem is standard. A firm chooses employment and prices to maximize profit:

$$\max_{\left\{N^c_{p,t}(i),N^c_{b,t}(i),N^c_{s,t}(i),p^*_s(i)\right\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} m_{t,s}\left(y^h_t(i)\,Q^h_t(i) - W^s_t N^h_{s,t}(i) - W^b_t N^h_{b,t}(i) - W^p_t N^h_{p,t}(i)\right). \tag{126}$$

subject to the production constraint

$$y^h_t(i) = Z_{ht} N^h_{p,t}(i)^{\nu} N^h_{b,t}(i)^{u} N^h_{s,t}(i)^{1-u-v} \tag{127}$$

The final good firm's demand equation of the input goods is as follows:

$$y^h_t(i) = Y^h_t \left(\frac{Q^h_t(i)}{Q^h_t}\right)^{-\varepsilon}, \tag{128}$$

and price rigidity which I assume to follow the Rotemberg price setting scenario in which firms face a quadratic costs in changing the goods price.

Profit maximization problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. We deal with each of these problems separately and will lead us to the Phillips Curve for the housing sector prices: :

$$\frac{(1-\varepsilon)}{\Omega_h} + \frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q^h_t} + E_t\left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{h,t+1})\frac{Y^h_{t+1}}{Y^h_t}(1+\pi_{h,t+1})\right]\right] = \left(\pi_{h,t}(1+\pi_{h,t}) - \frac{1}{2}(\pi_{h,t})^2\varepsilon\right) \tag{129}$$

where

$$m_{t,t+1} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right] \tag{130}$$

$$\frac{1}{R_{t,s}} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right] \tag{131}$$

4.2.5 Profits of firms and Government Transfers

Aggregate inter-period nominal profit is

$$\tilde{\Pi}_t = Y_t^c P_{ct} - W_{s,t}N_{s,t}^c - W_{b,t}N_{b,t}^c - W_{p,t}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c P_{ct} + Y_t^h Q_t^h - W_{s,t}N_{s,t}^h - W_{b,t}N_{b,t}^h - W_{p,t}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h$$

I assume that the profit is 100 percent taxed by the government and redistributed according to the following rule:

$$t_{bt} = (1-x-y)\frac{\tilde{\Pi}_t}{P_{ct}} \tag{132}$$

$$t_{st} = x\frac{\tilde{\Pi}_t}{P_{ct}} \tag{133}$$

$$t_{pt} = y\frac{\tilde{\Pi}_t}{P_{ct}} \tag{134}$$

Leading us to:

$$\begin{aligned} Y_t^c P_{ct} - W_{s,t}N_{s,t}^c - W_{b,t}N_{b,t}^c - W_{p,t}N_{p,t}^c + Y_t^h Q_t^h - \frac{\Omega}{2}\pi_t^2 Y_t^c P_{ct} - W_{s,t}N_{s,t}^h - W_{b,t}N_{b,t}^h - W_{p,t}N_{p,t}^h &= \tilde{\Pi}_t \\ &= P_{ct}t_{bt} + P_{ct}t_{st} + P_{ct}t_{pt} \end{aligned}$$

4.2.6 Financial Intermediaries

Considering these are owned by Savers,The role of financial intermediaries in this model will be minimal, they just pool the deposits from the Savers and provide loans to Borrowers. They pay the same interest rate to depositors as they charge from the Borrowers:

$$R_{t,o} = R_{t,s} \tag{135}$$

4.2.7 Market Clearing

In equilibrium we have the following resource constraints:

The aggregate labour of the agents are the sum of labour to the intermediate housing and consumption good firms.

$$N_t^p = N_{p,t}^h + N_{p,t}^c \tag{136}$$

$$N_t^b = N_{b,t}^h + N_{b,t}^c \tag{137}$$

$$N_t^s = N_{s,t}^h + N_{s,t}^c \tag{138}$$

The demand and supply of loans in the equilibrium are the same.

$$B_{s,t} = D_{t,d} \tag{139}$$

The central bank sets the interest rate by Taylor rule:

$$\frac{R_{t,o}}{R_o} = \left(\left((1+\pi_{h,t})^{cpih}(1+\pi_t)^{1-cpih}\right)\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r} \tag{140}$$

The central bank also sets the Macro prudential policy which regulates down payment ratio by feeding on the output gap:

$$\frac{\mu}{\bar{\mu}} = \left(\frac{Y_t}{Y}\right)^{\gamma_\mu} \tag{141}$$

Where $\bar{\mu}$ is the steady state Downpayment ratio.

We can Aggregate the Budget Constraints to yield

-

$$0 = N_{s,t}w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - b_{s,t}$$

$$\begin{aligned} Y_t^c + Y_t^h q_t^h = {} & C_{b,t} + C_{s,t} + C_{p,t} + q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) + q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) \\ & + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c + \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h \end{aligned} \tag{142}$$

Definitions:

$$\frac{q_t^h}{q_{t-1}^h} = \frac{1+\pi_{h,t}}{1+\pi_t}$$

$$\begin{aligned} q_t^h &: = \frac{Q_t^h}{P_{c,t}} \\ 1+\pi_t^H &: = \frac{Q_t^h}{Q_{t-1}^h} \\ 1+\pi_t^C &: = \frac{P_{c,t}}{P_{c,t-1}} \end{aligned}$$

so that

$$\frac{q_t^h}{q_{t-1}^h} = \frac{Q_t^h}{Q_{t-1}^h}\frac{P_{c,t-1}}{P_{c,t}} = \frac{1+\pi_t^H}{1+\pi_t^C} \tag{143}$$

and log-linearized:

$$\hat{\pi}_t^H - \hat{\pi}_t^C = \hat{q}_t^h - \hat{q}_{t-1}^h$$

or

$$\hat{q}_t^h = \hat{\pi}_t^H - \hat{\pi}_t^C + \hat{q}_{t-1}^h$$

Here $\hat{q}_{t-1}^h$ is predetermined variable. Now relative house price does not jump on its own, but is determined as a ratio of two inflations.

4.2.8 Private Sector Equilibrium

Please Refer to the Appendix for the whole Private Sector Equilibrium for this model.

4.3 Calibration

The share of durables which is used as collateral (μ) is set to 0.90 "Kovacs and Moran (2019)". The discount factors are set at Savers' discount rate (θ) as 0.99 "Iacoviello and Pavan (2013) " and Borrowers' discount rate (β) as 0.98 "Kovacs and Moran (2019) ". I set the quarterly house depreciation rate (δ) as 0.01 "Iacoviello and Neri (2010) " and share of consumables in consumption basket (α) as 0.84.

With other parameters at share of profits paid to Savers (x) as 0.7, share of profits paid to hand to mouth workers (y) as 0.1, if I assume that the average markup equals 10 percent, then this implies ε = 11 ("Prof.Richard Dennis Notes"), inverse inter-temporal elasticty (σ) at 2 for Savers, Borrowers and hand to mouth guys respectively "Iacoviello (2005)", Inverse Elasticity of labour (ϕ) at 1/3 "William B Peterman (2014) " and

Scaling of hand to mouth labour in production function (ν) at 0.1, scaling of Borrowers labour in production function (u) at 0.2 and Scaling of Savers labour in production function (u) at 0.7 "The Labour Market Story: Skills for the Future (July 2014)", Taylor rule parameter as 1.1. The Intra-temporal elasticity of substitution between housing and consumption goods is taken to be 0.8.

For the above parameters taken, the model also provides an estimate of percentage consumption of housing goods and consumption goods by different sectors of agents which closely replicates "Kaplan, Violante, Weidner (2014)" which is illustrated in the below table :

Agents Consumption	Consumption in Steady State	Total	Approx. Percentage
Consumption of Hand to Mouth Agents	0.1091		9.5 percent
Consumption of Borrowers	0.2208		19.1 percent
Consumption of Savers	0.8241		71.4 percent
		1.154	
Rental housing	0.6518		8.5 percent
Housing of Borrowers	1.9708		25.8 percent
Housing of Savers	5.0208		65.7 percent
		7.6434	

Table 3: Calibration

4.4 Results

4.4.1 Productivity Shocks

In the following, I will consider impulse responses to increase in productivity of the both consumption and housing goods at date t.

The technology shocks follow an AR(1) process in the housing goods sector:

$$\ln\left(Z_t^h\right) = \rho_h \ln\left(Z_{t-1}^h\right) + e_h$$

and an AR(1) process in the consumption goods sector:;

$$\ln\left(Z_t^c\right) = \rho_c \ln\left(Z_{t-1}^c\right) + e_c$$

where e_c, e_h are i.i.d processes with variances of σ_c and σ_h respectively which are both calibrated as 0.009^2. The shock persistence in both the sectors is ρ_h and $\rho_c$is taken as 0.3.

4.4.2 Effect of productivity shock in Consumption good firms

The temporary increase in the consumption good production technology directly translates into higher output in consumption goods sector Y_c as the output is directly proportional to the technology shock, also as we see from the private sector equilibrium, the shock enters the system in the marginal cost of producing consumption goods as a denominator and hence a positive consumption shock will decrease the marginal cost of producing a consumption good:

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)} \left(\frac{w_t^b}{u}\right)^u \left(\frac{w_t^p}{v}\right)^v$$

as $MC_s = \zeta_t P_{ct}$, we see a decrease in the marginal costs and a decrease in ζ_t

As the marginal costs decrease in consumption sector of the economy, the effect of “Technological advancement and price rigidity” of this sector makes the firms from this sector hire less labour and we see a decrease in Borrowers, Savers and Hand to mouth agents labour in consumption sector

$$y_t^c(i) = Z_{ct} N_{p,t}^c(i)^v N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v}$$

From the consumption goods firm's perspective, marginal productivities of labour from all of the labour sectors have increased, they want to have more of all labour inputs, pushing up the real wages of all the households to supply more labour.

However, the effect of price rigidity has a strong affect here and firms cut their labour inputs demand and this results in moving of the labour from consumption good sector to housing goods sector (which have relatively flexible prices and houses acting as assets) which results in increase of (all types of agents labour in housing sector) and increasing the output in housing goods sector Y_h . The result of moving labour from consumption sector to housing sector is again from a two fold affect of flexible price market for housing and houses acting as assets.

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v}$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in all of the sector's wages .

From households perspective, an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In my model, from the consumption sector's perspective, the Income effect clearly dominates the substitution effect for all the three agents and from housing sector's perspective the substitution effect dominates the income effect again in all the three types of agents.

On aggregate, leisure $1-N_b$, $1-N_s$, $1-N_p$ increase i.e., labour of all the three households N_b, N_s, N_p decreases which shows that Income effect dominates the substitution effect on aggregate. As wages increase, workers from all the sectors will want to increase consumption, resulting in a rise of C_s and C_b and C_s . Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with increase in output directly translates to an increase in consumption of all the household agents.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

Households know that the shock is temporary and they smooth their consumption. Also the magnitude of rise in consumption is different from Borrowers and hand to mouth workers and with lower increase in consumption for the Borrowers sector, additional income of Borrowers is saved, i.e. invested into the housing stock H_b resulting in an increase in the housing stock of Borrowers which in turn also results in the increase in the BTL stock.

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

On the other hand, as Borrowers tend to invest in housing, the price of the houses q_h increases resulting in Savers to issue more of loans B_s and consume more C_s and investing in their housing stock H_s decreases which shows that the demand of the housing stock is mostly from the Borrowers sector. The Savers sector tend to divert some of the housing stock towards the increase in issuing loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

Whereas, hand to mouth guys cannot invest in housing, combining with the increase of investment in housing sector from the Borrowers, Buy to let houses H_r have increased and the hence price of the rental rate q_r decreases due to more supply of BTL houses. As the wages of hand to mouth guys increasing and due to a decrease in rental price results in a substantial increase in consumption of this sector.

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

The increase in productivity of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, fall in inflation π in the economy occurs. However, as the housing sector is flexible we see an increase in housing inflation π_h as the demand from the houses increases from the Borrowers sector. The central bank sets the interest rate according to the Taylor rule and as the inflation decreases, the interest rate tend to decrease.

$$\frac{R_{t,o}}{R_o} = \left(\left((1+\pi_{h,t})^{cpih}(1+\pi_t)^{1-cpih}\right)\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r} \tag{144}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate decreases, Borrowers tend to obtain more loans B_s and increase their consumption. Borrowers tend to increase the investment in housing stock which further increases their collateral value. The increase in investment from Borrowers will in-turn affect the Buy-to-let market which in-turn increases the rental properties which increase in H_r. As the rental properties increases, this will reduce the rental price q_r and as a result, poor workers tend to have more for consumption and C_p increases. As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment in housing stock towards issuing more loans and hence we see a decrease in H_s. As the Savers tend to get more returns in terms of loans in volume, they tend to increase their consumption and we see an increase in consumption from all the sectors.

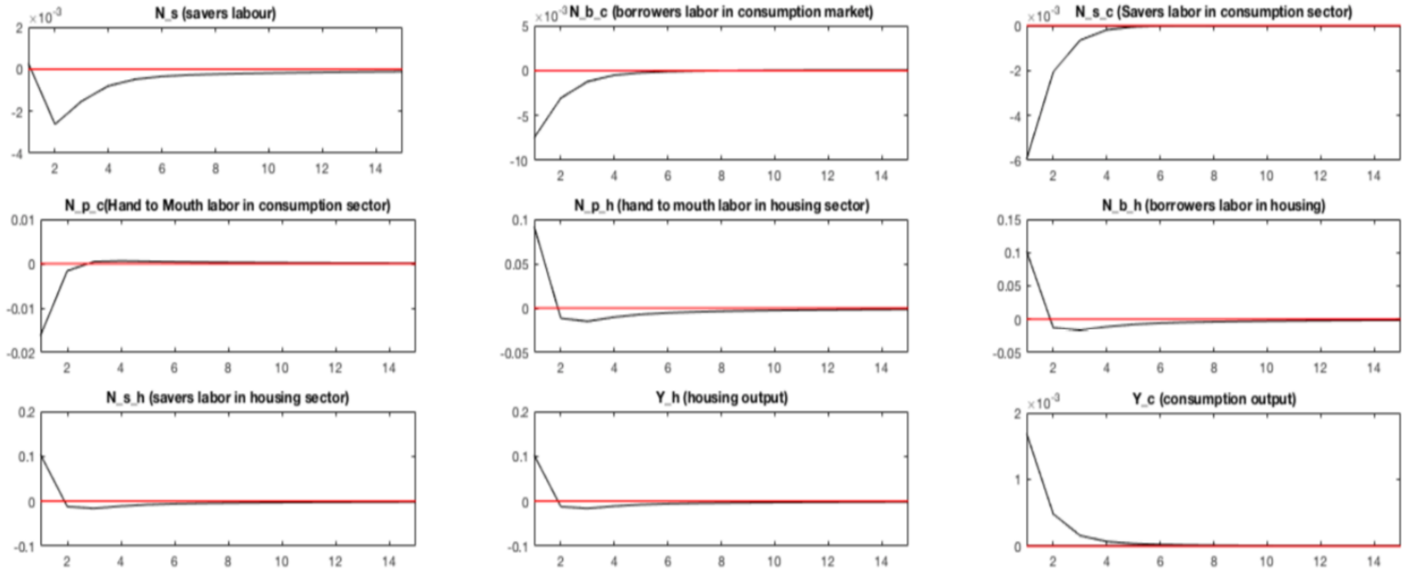


Figure 21: IRFs for Positive Consumption Sector Productivity Shock

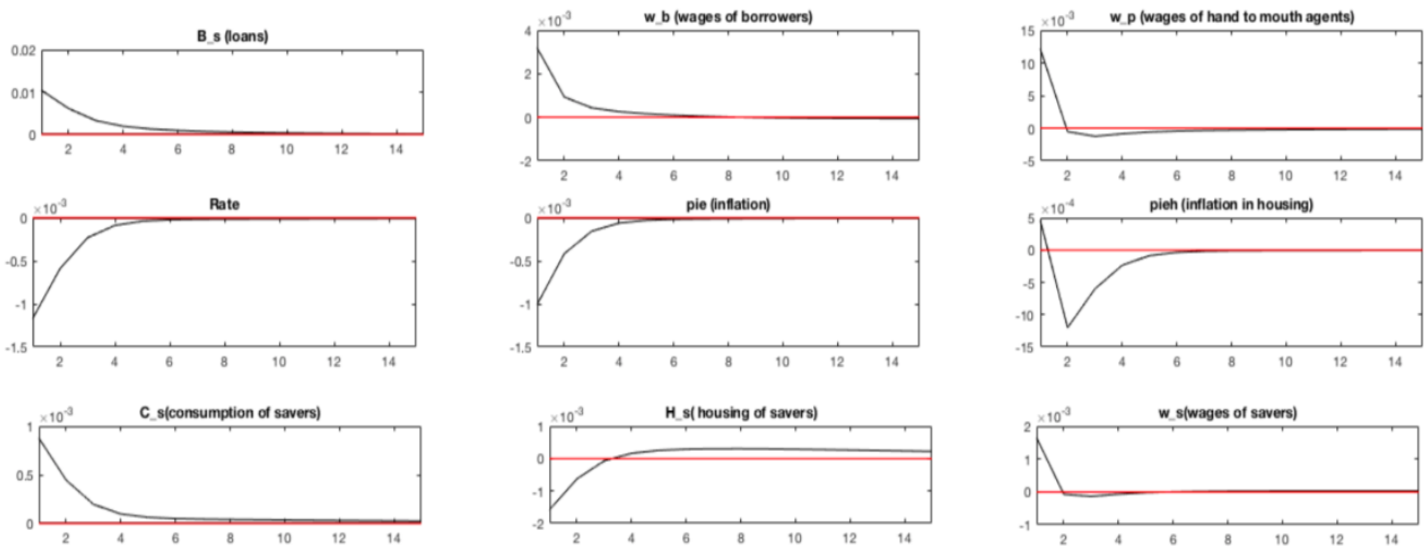


Figure 22: IRFs for Positive Consumption Sector Productivity Shock

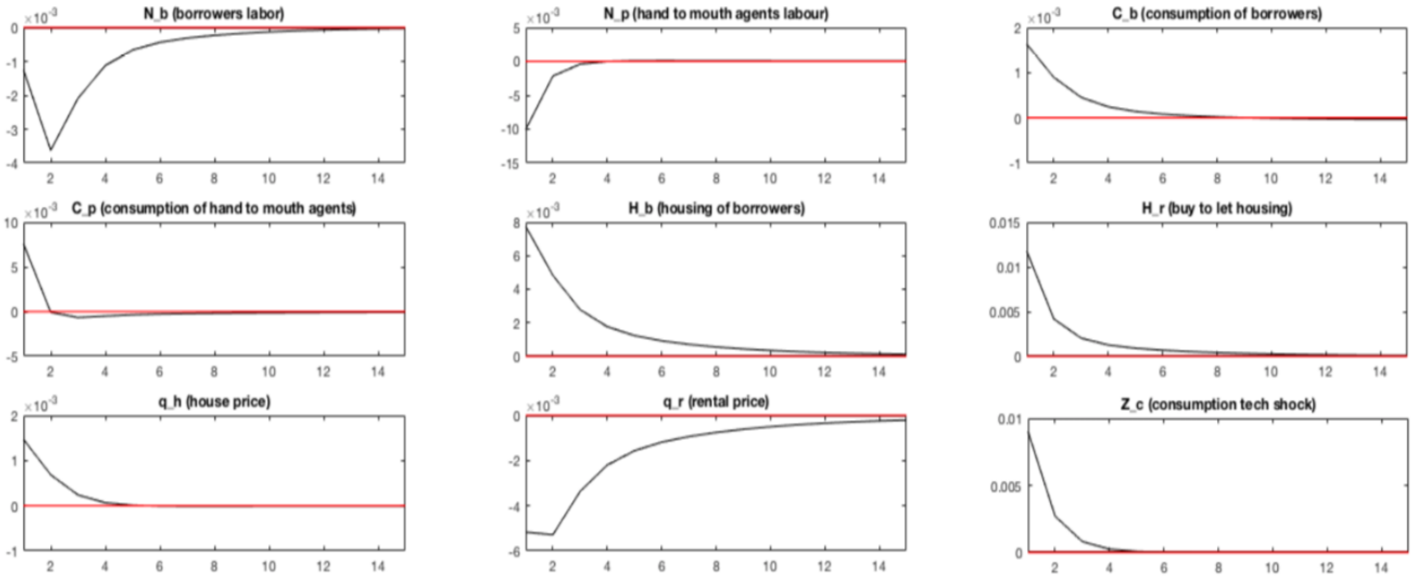


Figure 23: IRFs for Positive Consumption Sector Productivity Shock

Interpretation:

Firms Decisions:

We observe that the labour markets play a pivotal role in most of the dynamics. An innovation Z_{ct} to consumption goods sector, translates to an increase in consumption output Yc and as wages are same across firms, price rigidity in consumption sector helps in the movement of labour from consumption sector to housing sector which in turn increases the housing output Y_h.

Households Decisions:

Because of the increase in wages, we see an increase in disposable income for all the household agents. From Borrowers perspective an increase in disposable income can be spent on consumption goods or on housing, as the role of collateral constraint kicks-in, these agents tend to increase their investment in housing which acts as booth collateral and a means to get returns(rents from BTL). This will be translated to a slight increase in consumption and the demand for housing by the Borrowers H_b increases which results in an increase in house prices q_h even though we see an increase in housing output Y_h and this in turn increase the demand for the loans Bs. From Savers perspective, again the increase in disposable income can be spent on consumption goods, housing or on providing loans. As the demand of housing from the Borrowers increase, there will be an increase in demand for loans and this in turn will make Savers to invest in providing loans as this is their only means of returns (No BTL from Savers sector and No collateral constraints for these agents). hence the Savers will increase their consumption and loans but decrease their housing investment. This again results in a decrease in the rate returns (Rate). From Hand to mouth agents perspective, as they cannot invest in housing they consume more of this disposable income (C_p increases) and also the affect of reduction in rental rate q_r amplifies the increase in consumption from these agents. Overall the disposable income increase for all the three agents and are better off in consumption for a positive consumption shock. As opposed to the previous chapters, rigid consumption and flexible housing goods market played a huge role in the increase of housing output with respect to positive consumption shock. Borrowers tend to invest in housing and Savers tend to supply more loans with decrease in housing from them.

4.4.3 Effect of productivity shock in Housing firms

The temporary increase in the housing good production technology directly translates into higher output in housing goods sector Y_h as the output is directly proportional to the technology shock.

From the housing goods firm's perspective, marginal productivities of labour from all of the labour sectors have increased, they want to have more of all labour inputs, pushing up the real wage w_p, w_b, w_s to induce households to supply more labour. we see a increase in Borrowers, Savers and Hand to mouth agents labour in housing sector (N_b^h,N_s^h and N_p^h increases): This result of increase in labour is quite opposite to the one in the shock to consumption sector where we see a decrease in agent labour, this is mainly because of several important factors:

1) Flexible prices in housing sector and rigid prices in consumption sector: Flexible prices in the market allows agents to consume more of housing services than the rigid markets and this results in an incentive for households to supply more labour in housing market.

2) Collateral Constraint: houses act as collaterals for Borrowers in the economy and also they indirectly affect Savers who provide loans to the Borrowers.

3) Houses as assets: Borrowers perceive houses as assets where they get returns from BTL market.

The above reasons and increase in marginal productivities make agents to supply more labour in housing market and hence the housing output Y_h increases:

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu}$$

From the same reasons above, there is an incentive for Borrowers and Savers to supply more of labour and to have more of disposable income so that they can invest in houses and loans respectively. Hence these two agents provide even more labour to the consumption firms sector which results in an increase in labour of Borrowers and Savers in consumption firms sector (N_b^c,N_s^c increases). However, Hand to mouth agents doesn't have any incentives they tend to reduce their labour in the consumption sector which result in a decrease of (N_p^c). Since the rise in labour from Borrowers and Savers dominates the decrease in labour from hand to mouth agents, the consumption output Y_c increases:

$$y_t^c(i) = Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^{u} N_{s,t}^c(i)^{1-u-\nu},$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in all of the sector's wages .

From households perspective, the Substitution effect clearly dominates the income effect for Borrowers and Savers however we also observe that the Income effect dominates in Hand to mouth agents (which are down to the reasons mentioned above) and from housing sector's perspective the substitution effect dominates the income effect again in all the three types of agents.

On aggregate, leisure $1-N_b$, $1-N_s$, increase i.e., labour of Borrowers and Savers increases and the labour from hand to mouth agents decrease. As wages increase, hand to mouth agents want to increase consumption, resulting in a rise of Cp. However because the Borrowers and Savers and affected by the housing market they tend to decrease their consumption. This decrease in consumption from Borrowers and Savers is mainly by the fact that As Borrowers observe a positive technology shock to the housing market they expect the output to go up and the prices of the houses to go down which decrease their collateral value. This decrease in collateral value puts off the Borrowers to invest in their own housing H_b decreases (As housing as collateral plays a role in here) however they tend to increase investment in BTL H_r increases (As BTL is considered more as an Asset with Rents as Returns)

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

As Borrowers invest less in housing their demand for loans decrease B_s decrease. this directly affects the Savers who provide loans to Borrowers and this results in Savers to increase their housing stock (H_S Increase) with more disposable income they acquire from increase in wages. However as the loans decrease their aggregate returns on loans decrease and an increase in housing will hamper the consumption of these agents slightly.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

On the other hand, as Borrowers see the demand for BTL increases from the Hand to mouth agents disposable income increase. The Rental price increase and this even make the Borrowers to increase only BTL houses.

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

The increase in productivity of housing firms enhances to supply more of the housing goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the houses for better marginal profits and as a result, fall in housing inflation π_h in the economy occurs. However, as the consumption sector is rigid we see an increase in inflation π as the demand decreases from the houses of the Borrowers sector. The central bank sets the interest rate according to the Taylor rule and as the inflation increases, the interest rate tend to increase.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate Increase, Borrowers tend to obtain less loans B_s and decrease their consumption. Borrowers tend to decrease the investment in housing stock which further decreases their collateral value. Borrowers will in-turn affect the Buy-to-let market by increase investment in BL houses as they see them as assets with returns. As the demand for rental properties increase substantially, this will increase the rental price q_r and as a result, Hand to mouth workers tend to have more housing and are better off. As the demand for loans decrease, Savers tend to supply less of deposits as they tend to divert some of their investment towards housing stock and hence we see a increase in H_s. As the Savers tend to get less aggregate returns in terms of loans in volume, they tend to decrease their consumption and we see an decrease in consumption from Borrowers and Savers but an increase in consumption from Hand to mouth agents.

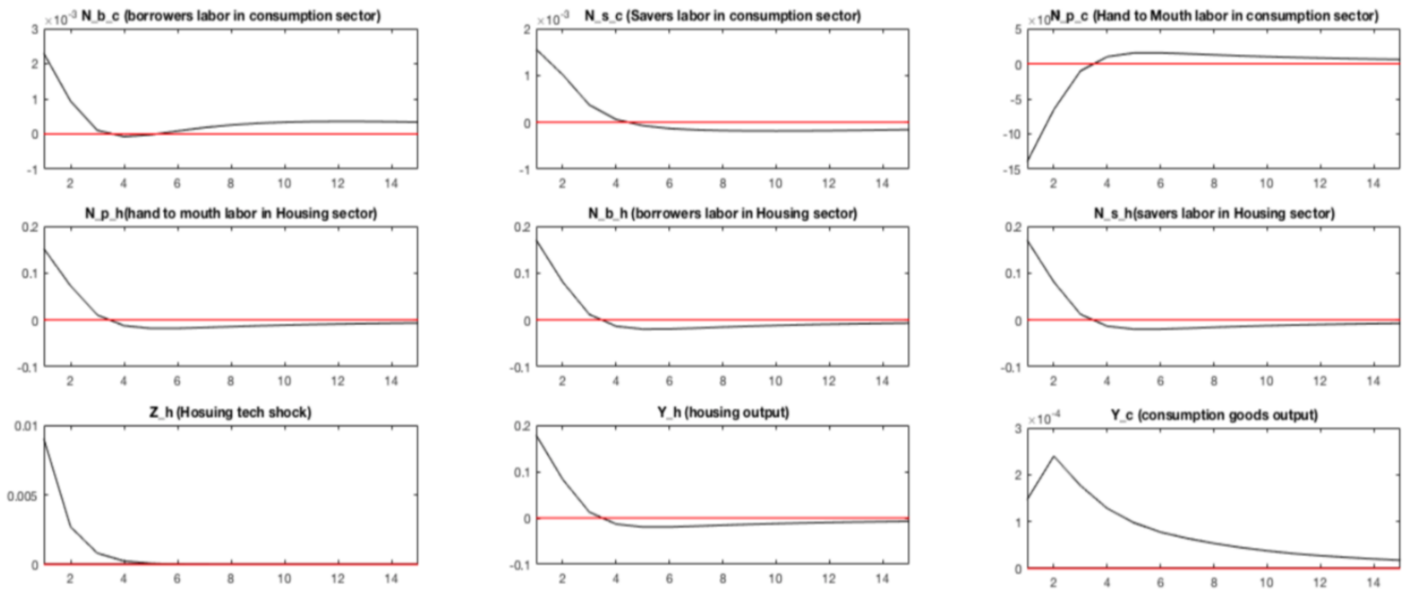


Figure 24: IRFs for Positive Housing Sector Productivity Shock

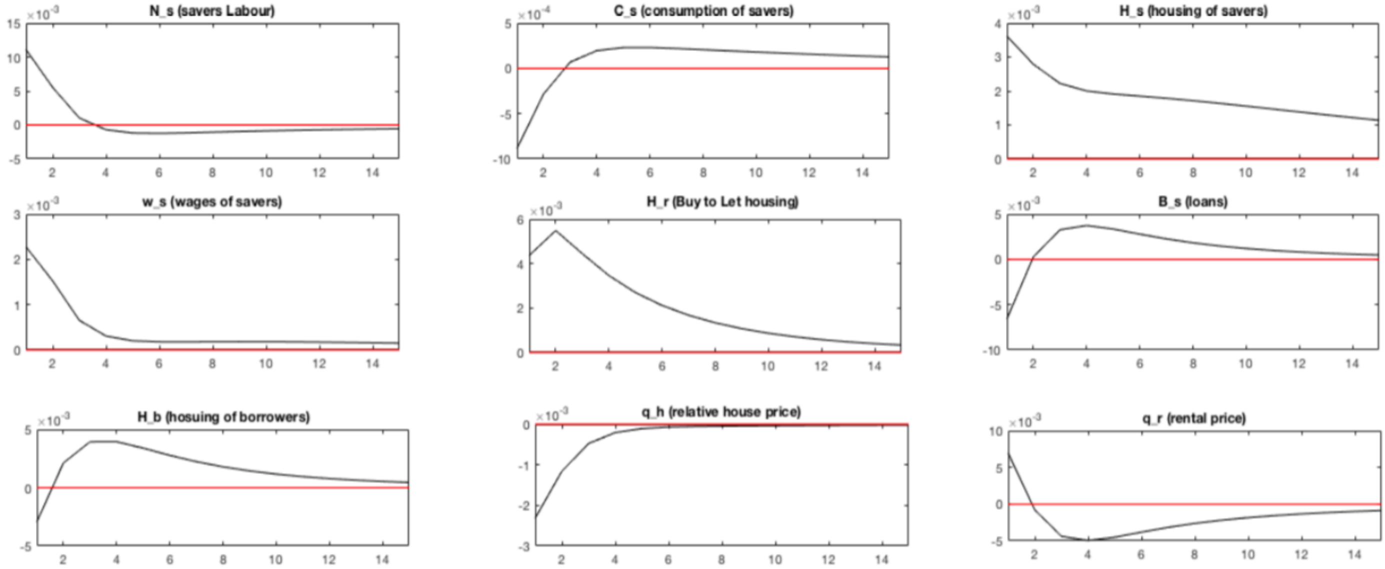


Figure 25: IRFs for Positive Housing Sector Productivity Shock

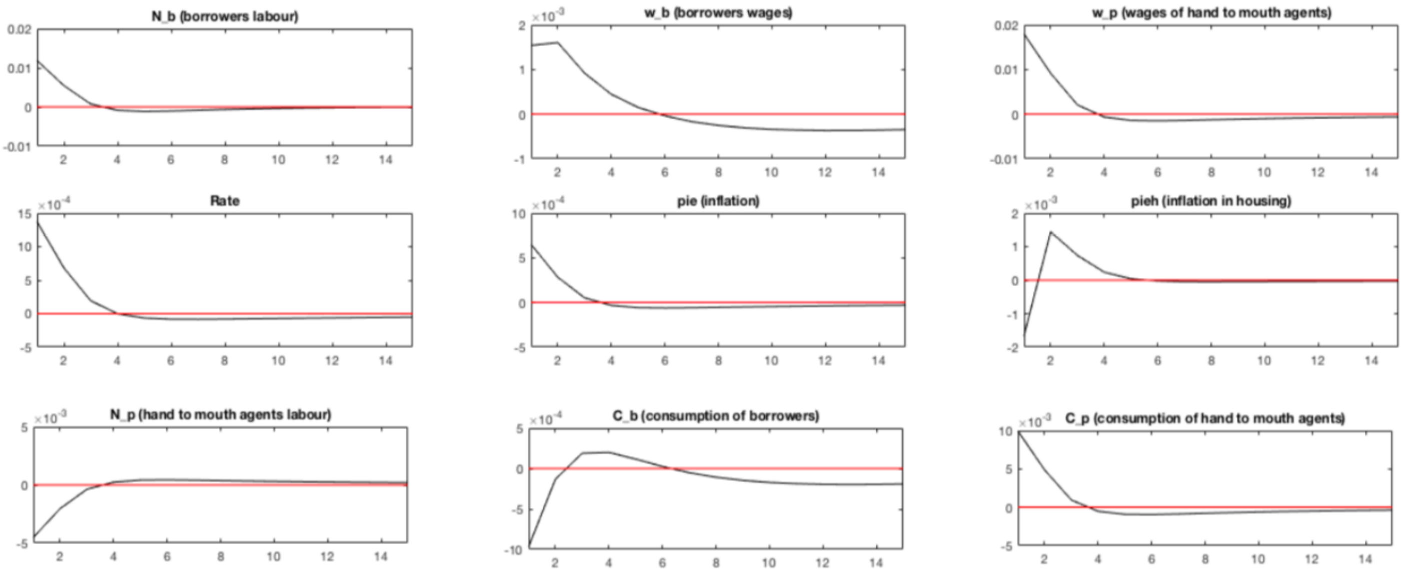


Figure 26: IRFs for Positive Housing Sector Productivity Shock

Interpretation:

Firms Decisions:

We observe that the labour markets play a pivotal role in most of the dynamics. An innovation Z_{ht} to housing goods sector, translates to an increase in housing output Yh and as wages are same across firms, housing market affect on Borrowers helps in the movement of certain labour (Savers and Borrowers) from housing sector to consumption sector which in turn increases the consumption output Yh.

Households Decisions:

Because of the increase in wages, we see an increase in disposable income for all the household agents. From Borrowers perspective an increase in disposable income can be spent on consumption goods or on housing, as the role of collateral constraint kicks-in, these agents tend to decrease their investment in housing as they see a decrease in price of housing from increase in houses output. Houses act as both collateral and a means to get returns(rents from BTL). This will be translated to a slight increase in BTL housing investment, however they decrease their consumption of goods and consumption of personal housing by the Borrowers H_b decreases which results in an increase in Rental prices q_r and this in turn decrease the demand for the loans Bs. From Savers perspective, again the increase in disposable income can be spent on consumption goods, housing or on providing loans. As the demand of housing from the Borrowers decrease, there will be an decrease in demand for loans and this in turn will make Savers to invest in housing . Hence the Savers will decrease their consumption and loans but increase their housing investment. This again results in a increase in the rate returns (Rate). From Hand to mouth agents perspective, as they cannot invest in housing they consume more of this disposable income (C_p increases) and also they affect of the demand for BTL amplifies and this increase in demand from these agents amplifies the rental price. Overall the disposable income increase for all the three agents. However, when the shock was just realized Hand to mouth agents are better off in consumption and housing and the other two agents are worse off.

4.4.4 Cost Push Shocks

I give a temporary Inverse shock ("I gave a positive shock to the ε elasticity of substitution between intermediate goods in both consumption and housing goods sector): The temporary increase in the elasticity of substitution between intermediate goods in-turn make the price of the final goods decrease as the substitution between intermediate goods increase and this directly translates into a decrease in Inflation of consumption goods π and a decrease in Inflation of Housing goods sector π_h.

The inverse cost push shocks follow an AR(1) process in the housing goods sector:

$$\ln\left(cp_t^h\right) = \rho_{cph}\ln\left(cp_{t-1}^h\right) + e_{cph}$$

and an AR(1) process in the consumption goods sector:;

$$\ln\left(cp_t^c\right) = \rho_{cpc}\ln\left(cp_{t-1}^c\right) + e_{cpc}$$

where e_{cpc}, e_{cph} are i.i.d processes with variances of σ_{cpc} and σ_{cph} respectively which are both calibrated as 0.005^2. The shock persistence in both the sectors is ρ_{cph} and ρ_{cpc} is taken as 0.3.

4.4.5 Effect of Inverse cost push shock in Consumption good firms

As we see from the private sector equilibrium, the shock enters the system in the Phillips Curve of the consumption goods sector as a inverse relation and hence a positive cost push shock will decrease the inflation of the consumption goods prices:

$$\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{t+1})\frac{Y^c_{t+1}}{Y^c_t}(1+\pi_{t+1})\right]\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)$$

As the Interest rate set by the central bank is feed by the inflation change, we see the interest rate also goes down as the Inflation goes down: $Rate_o$ decreases.

$$\frac{R_{t,o}}{R_o}=(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

The decrease in Interest rates will result in a direct increase in demand of loans from Borrowers. As loans d_o increase, the housing of Borrowers also tend to increase as we know houses act as collaterals and as assets from the Borrowers perspective. We will also observe that most of the increase in Borrowers housing (Not all of Borrowers housing) is driven by the increase in Buy to Let housing from Borrowers: This results in an increase of H_b and H_r.

$$P_{c,t}C^b_t+Q^h_t(H^b_t-(1-\delta)H^b_{t-1})+R_{t-1,o}D^o_{t-1}=H^r_tQ^r_t+N^b_tW^b_t+D^o_t+T_{b,t}$$

As the choice of housing is a control variable from Borrowers perspective, an Increase in demand for the housing from the Borrowers sector will lead to an increase in the price of houses q_h, however, this increase in relative house price is not substantial which is in line with the increase in personal consumption of Borrowers housing which again is not substantial. This increase in prices of housing again results in the increase in supply of houses: Y_h increases very slightly and starts decreasing from then on (Decreasing after the shock is realized between time period 1 and 2) as the labour moves from housing to consumption goods sector.

As the share of the Hand to Mouth agents in the economy is a constant parameter, an increase in Buy to Let housing in the economy will reduce the rental price and we see a decrease in q_r. As we know a decrease in relative price of one good has both substitution effect and Income effect, from the Substitution effect perspective, the decrease in the nominal inflation leads to the decrease in relative price of the consumption goods and this makes the consumption goods relatively cheaper for agents to consume and all the agents will increase their consumption goods usage: C_p, C_b, C_s increase.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

From Income effect perspective, a decrease in consumption goods price lead to agents having more of relative income and As houses act as collaterals and assets for Borrowers, Borrowers tend to increase their consumption of housing as well:H_b increases (As I assume both consumption and houses are Normal Goods).

From Savers perspective, the increase in demand for loans d_o from Borrowers result in a increase in supply of loans from Savers and leads to a decrease in housing of Savers: H_s decreases as they divert their funds to providing loans rather than Invest in housing (housing doesn't act as an Asset with returns for Savers as I assume they don't invest in Buy to Let Market and also they don't act as collaterals for Savers).

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

The decrease in Inflation of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, Increase in production of consumption goods in the economy occurs: Y_c Increases. Consumption good firms want to hire more labour as a result The consumption goods sector firms increase the wages in the economy: W_b, W_s, W_p increases.

From households perspective, an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In my model, from the consumption sector's perspective, the Income effect clearly dominates the substitution effect for the Borrowers (Because of the fact that Borrowers have even more disposable income from the Buy to Let assets and also increase in collateral Value of houses) and the substitution effect dominates the income effect in Savers and Hand to Mouth agents. As I assume that the labour is free to move between the firm sectors, we see an increase in labour supply from Savers and Hand to Mouth agents to the consumption sector: $N_{s,c}, N_{p,c}$ increases (Because of Substitution effect). However, we see a decrease in the labour supply of Borrowers in the consumption sector as a result of Income effect.

We see a similar effect in housing sector firms. However, we see a very slight increase in Supply of labour from Savers and Hand to Mouth agents as opposed to the substantial increase in labour from the same agents in consumption goods sector. We also see a dominant income effect in Borrowers agents from the housing firms perspective. This leads to On aggregate, leisure $1-N_s$, $1-N_p$ decreases in Savers and Hand to Mouth agents i.e., labour of Savers and Hand to Mouth agent households N_s, N_p increases which shows that Income effect dominates the substitution effect on Hand to Mouth agents and Savers. Leisure $1-N_b$ increases in Borrowers i.e., labour of Borrowers N_b decreases which shows that Substitution effect dominates the Income effect on Borrowers. Also the effect is cyclical As wages increase, workers from all the sectors will want to increase consumption, resulting in a rise of Cs and Cb and Cp . Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with increase in output directly translates to an increase in consumption of all the household agents.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

We can observe that the main reason for the slight Substitution effect of Savers and Hand to Mouth agents in housing sector firms even though I give a temporary Inverse shock (Inflation in consumption goods decrease) is from the fact that the housing sector is relatively flexible price sector and as there are no quadratic costs involved in housing sector. Agent can have more of housing consumption with flex prices: resulting in an slight increase in Y_h.

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu}$$

As the demand for housing substantially increase from the Buy to Let perspective, with the effect of very slight increase in housing output and a substantial increase in Demand from the Borrowers, the result is an increase in house prices q_h and an increase in house price Inflation :π_h increases.

Overall the disposable income increase for all the three agents increase and are better off in consumption for a Inverse cost push shock on Inflation in consumption goods. Borrowers tend to invest in housing and Savers tend to supply more loans with decrease in housing.

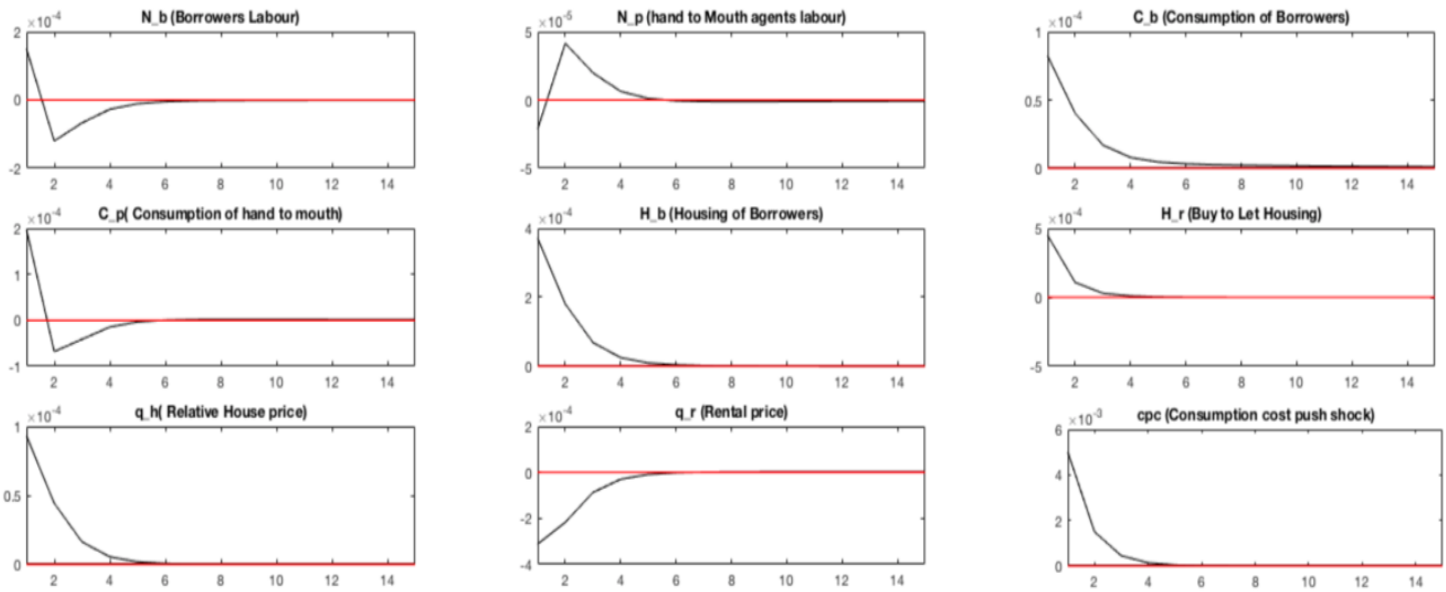


Figure 27: IRFs for Inverse cost push shock in Consumption good firms

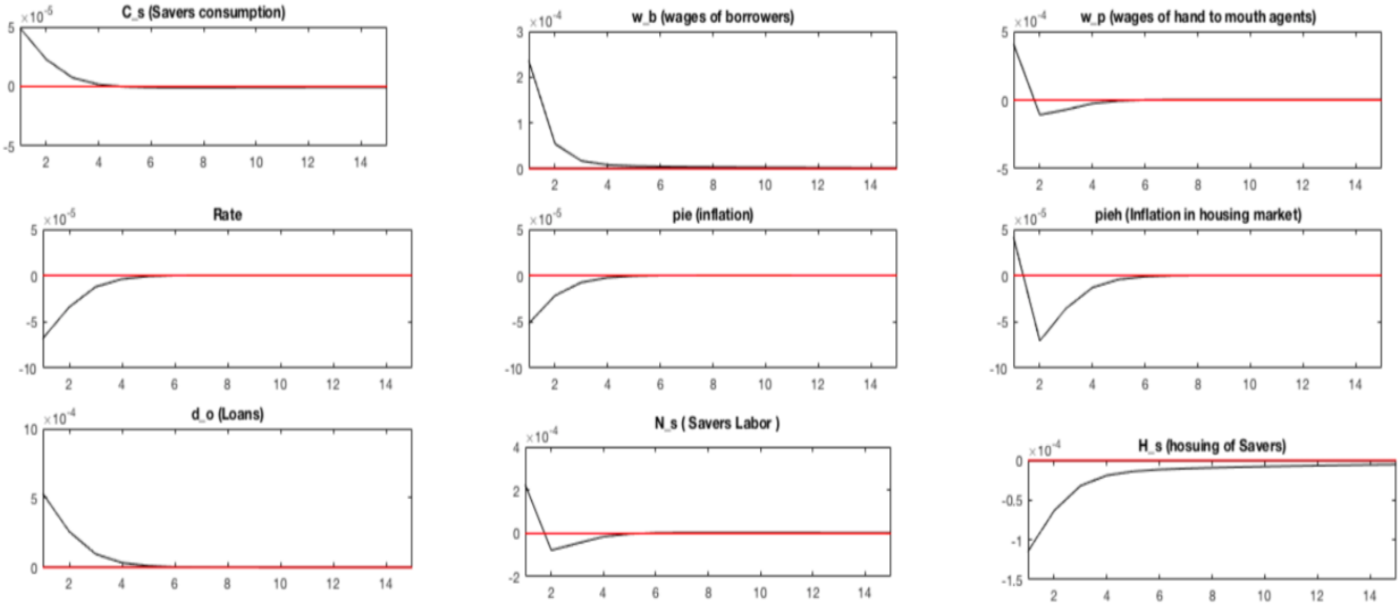


Figure 28: IRFs for Inverse cost push shock in Consumption good firms

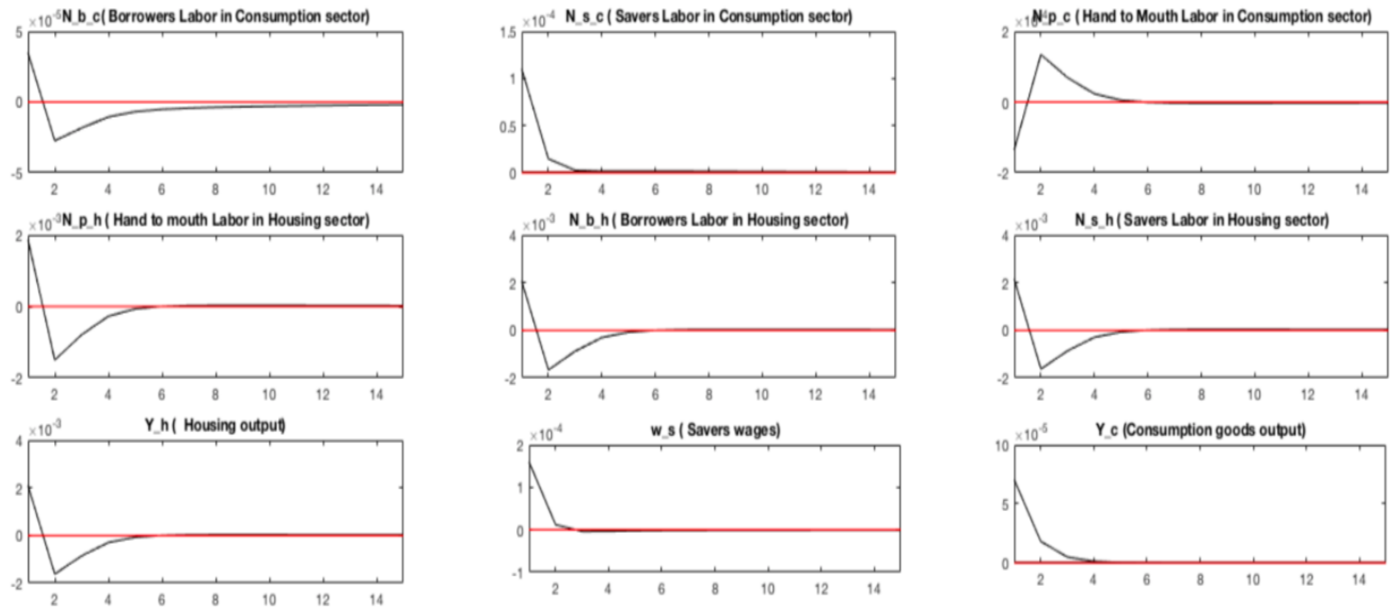


Figure 29: IRFs for Inverse cost push shock in Consumption good firms

4.4.6 Effect of Inverse cost push shock in Housing good firms

I give a temporary Inverse shock ("I gave a positive shock to the ε elasticity of substitution between intermediate goods in housing sector): The temporary increase in the elasticity of substitution between intermediate goods in-turn make the price of the final goods decrease as the substitution between intermediate goods increase and this directly translates into a decrease in Inflation of housing goods π_h, also as we see from the private sector equilibrium, the shock enters the system in the Phillips Curve of the housing goods sector as a inverse relation and hence a positive cost push shock will decrease the inflation of the house prices:

$$\frac{(1-\varepsilon)}{\Omega_h}+\frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q_t^h}+E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[\left(\pi_{h,t+1}\right)\frac{Y_{t+1}^h}{Y_t^h}(1+\pi_{h,t+1})\right]\right]=\left(\pi_{h,t}(1+\pi_{h,t})-\frac{1}{2}\left(\pi_{h,t}\right)^2\varepsilon\right)$$

The decrease in Inflation of housing good firms enhances to supply more of the housing goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the housing goods for better marginal profits and as a result, Increase in production of housing goods in the economy occurs: Y_h Increases. Housing good firms want to hire more labour as a result The housing goods sector firms increase the wages in the economy: $W_b, W_{s,} W_p$ increases.

From households perspective, an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In the model, from the consumption sector's perspective, the Income effect clearly dominates the substitution effect for the Hand to Mouth Agents and the substitution effect dominates the income effect in Savers and Borrowers. As I assume that the labour is free to move between the firm sectors, we see an increase in labour supply from all three agents to the housing sector: $N_{s,c}, N_{b,c}$ increases (Because of Substitution effect). However, we see a decrease in the labour supply of Hand to Mouth agents in the consumption sector as a result of Income effect.

We see a bit different effect in housing sector firms (which is mainly driven by the fact that housing sector is relatively flex price system). We see a very substantial increase in Supply of labour from Borrowers,Savers and Hand to Mouth agents as opposed to that of the Cost Push shock on the consumption goods sector. Which is basically we see a dominant income effect in all three of the agents from the housing firms perspective: $N_{s,h}, N_{b,h}, N_{p,h}$ increases.

$$y_t^h(i)=Z_{ht}N_{p,t}^h(i)^{\nu}N_{b,t}^h(i)^{u}N_{s,t}^h(i)^{1-u-\nu}$$

This leads to On aggregate, leisure $1-N_s$, $1-N_b$ decreases in Savers and Borrowers i.e., labour of Savers and Borrowers households N_s, N_b increases which shows that Income effect dominates the substitution effect on Borrowers and Savers. Leisure $1-N_p$ increases in Hand to Mouth agents i.e., labour of Hand to Mouth agents N_p decreases which shows that Substitution effect dominates the Income effect on Hand to Mouth agents.

We can observe that the main reason for the slight Substitution effect of Savers and Borrowers in consumption sector firms even though I give a temporary Inverse shock (Inflation in housing goods decrease) is from the fact that the consumption sector is relatively rigid price sector and as there are quadratic costs involved in consumption sector. Agent will have more of consumption goods consumption with rigid prices: resulting in a slight decrease in Y_c. And also the increase in inflation in consumption goods result in Borrowers and Savers to consume less of consumption goods C_b, C_s decreases. Borrowers consumption decreases as they get less disposable income now as they reduce their Buy to Let holdings and also their house collateral value goes down drastically, from Savers perspective they reduce their consumption as they now provide less loans to Borrowers and they miss out on the returns from these loans. However from the hand to Mouth agents perspective, we see an increase in consumption of goods as they have more disposable income from the increase in wages W_p and tend to live in smaller houses which is a result from the decrease in supply of Buy to Let from the Borrowers.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

This decrease in Total consumption output is accompanied by the increase in the price of consumption goods which result in an increase in the consumption goods inflation: πincreases.

As the Interest rate set by the central bank is feed by the inflation, we see the interest rate also goes up as the Inflation goes up: $Rate_o$ increases.

$$mc : 34 : \frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r}$$

The increase in Interest rates will result in a direct decrease in demand of loans from Borrowers. As loans d_o decrease, the housing of Borrowers also tend to decrease as we know houses act as collaterals and as assets from the Borrowers perspective. We will also observe that most of the decrease in Borrowers housing (Not all of Borrowers housing) is driven by the decrease in personal consumption of housing from Borrowers (As they get some returns from increase in Rental return which we will see later, they don't want to decrease a lot of their Buy to Let holdings): This results in an decrease of H_b and H_r.

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

As the choice of housing is a control variable from Borrowers perspective, a decrease in demand for the housing from the Borrowers sector will lead to an decrease in the price of houses q_h, which again results in a decrease in the collateral value of the Borrowers and this is the main reason for the substantial decrease in the personal consumption of housing from the Borrowers $H_b - H_r$ compared to the slight decrease in Buy to Let holdings from Borrowers H_r .

As the share of the Hand to Mouth agents in the economy is a constant parameter, an decrease in Buy to Let housing in the economy will increase the rental price and we see an increase in q_r. As we know a decrease in relative price of one good has both substitution effect and Income effect, from the Substitution effect perspective, the decrease in the housing inflation leads to the decrease in relative price of the housing goods (which is q_h decreasing) and this makes the housing goods relatively cheaper for agents to consume and as houses act as collaterals this is a detrimental affect on Borrowers and the Borrowers will decrease their consumption of housing usage: H_b, H_r decrease.

From Savers perspective, the decrease in demand for loans d_o from Borrowers result in a decrease in supply of loans from Savers and leads to a increase in housing of Savers: H_s increases as they divert their funds to invest in housing rather than providing loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

This tend to work in a cyclical effect As the demand for housing decrease from the Buy to Let perspective, with the effect of very substantial increase in housing output and a substantial decrease in Demand of consumption of housing from the Borrowers, the result is an decrease in house prices q_h and an decrease in house price Inflation :π_h decreases.

Overall the disposable income increase for all the three agents increase and Borrowers and Savers are worse off in consumption for a Inverse cost push shock on Inflation in housing goods. Hand to mouth agents are better off in consumption. Borrowers tend to invest less in housing and Savers tend to supply less loans with increase in housing.

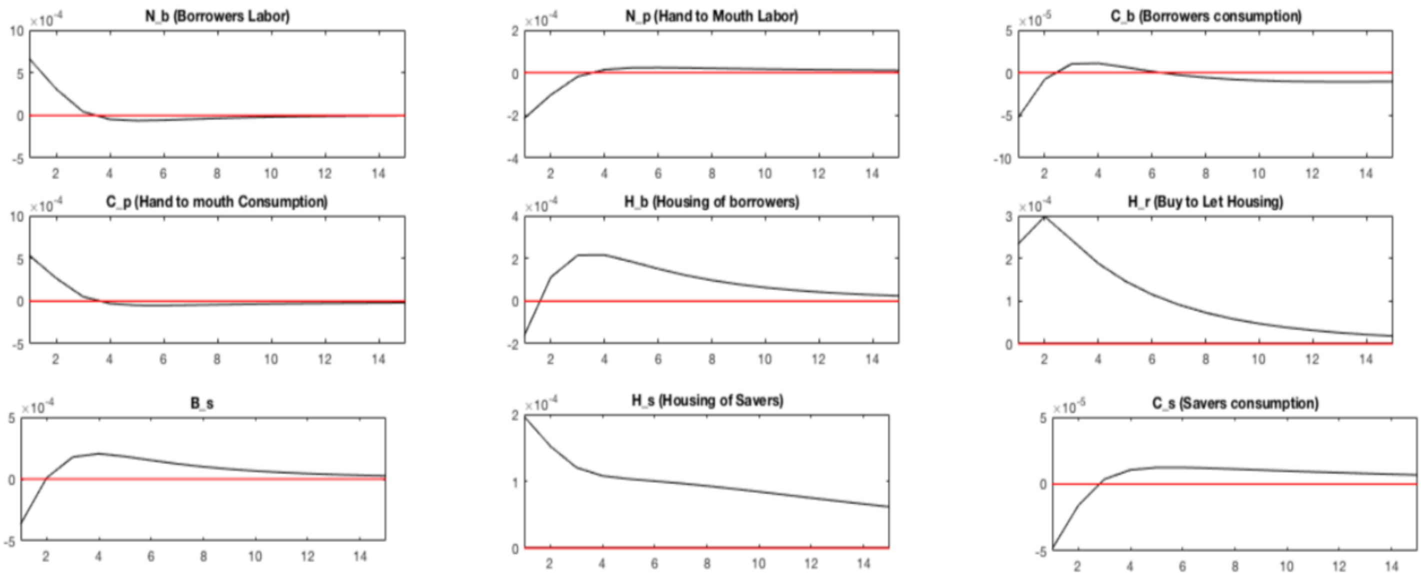


Figure 30: IRFs for Inverse cost push shock in Housing good firms

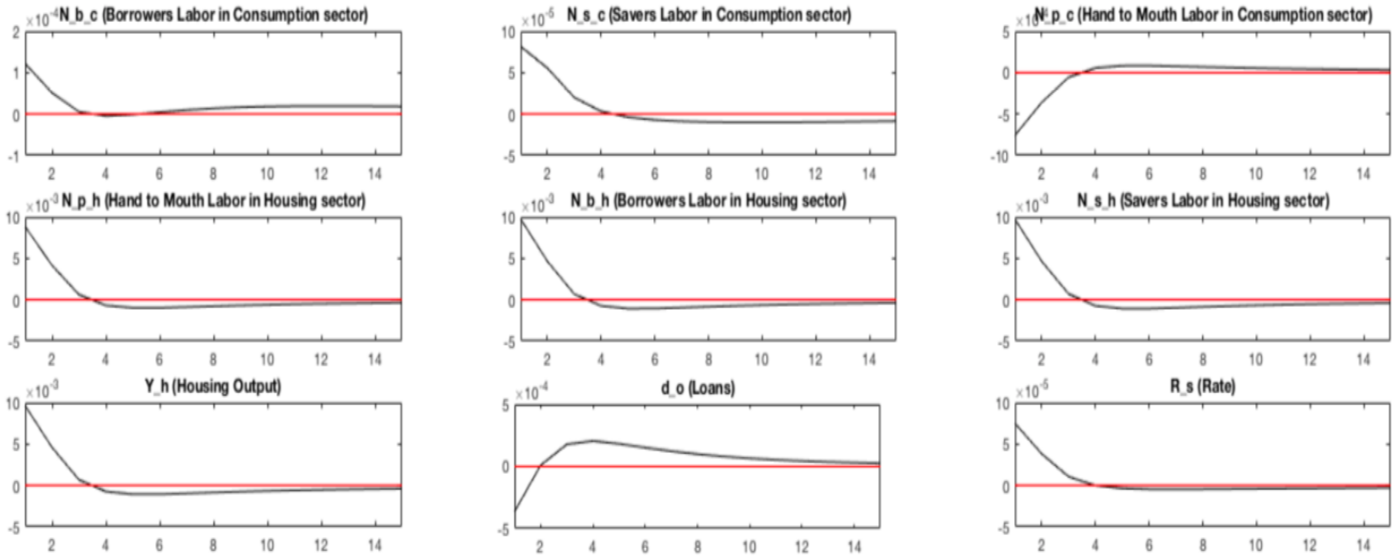


Figure 31: IRFs for Inverse cost push shock in Housing good firms

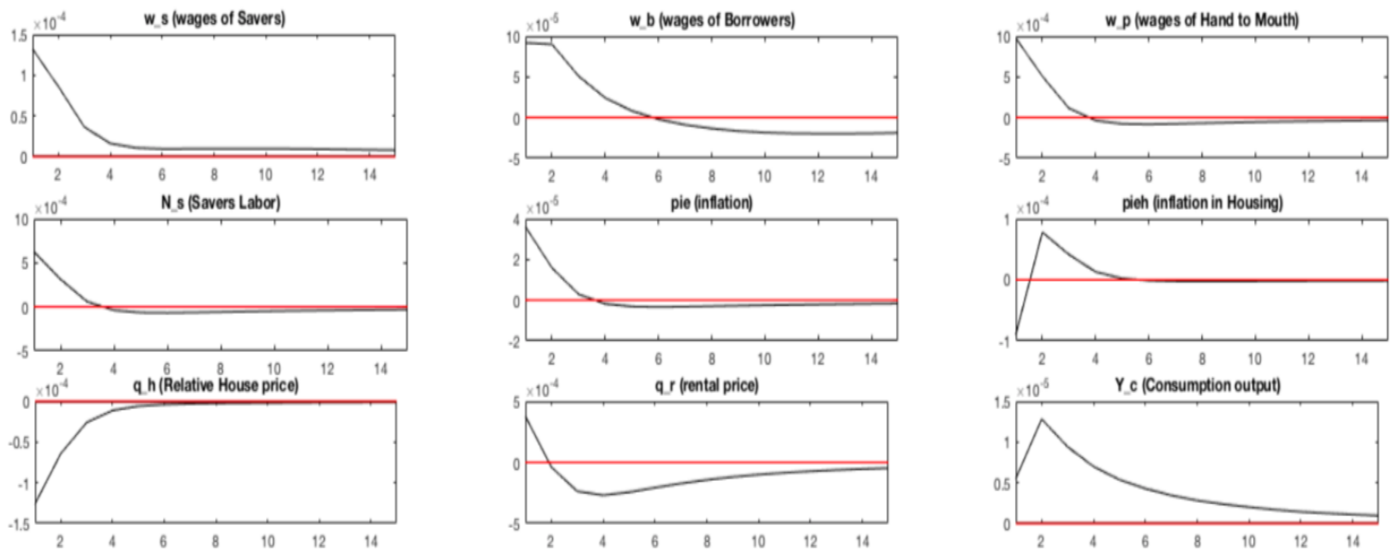


Figure 32: IRFs for Inverse cost push shock in Housing good firms

4.4.7 News (Expectation) Shocks :

4.4.8 Effect of Expectation shock on Rental returns:

I give an Expected Shock Fujiwara, Ippei and Kang, Heedon, (2008) of 5 time periods to the Rental Returns (q_r) i.e., the agents (Renters/Hand to Mouth Agents) receive clear signals about the rental rate they face in the future i.e., after 5 time periods. Renters anticipate that the rents are going to Decrease (From an external scenario such as Brexit) in the future. This however gives a signal to Borrowers that the rents might decrease. The implication of such expected shocks can be interpreted before the Shock has been realized and after the shock is realized.

The Expectation shock on the rental returns follow an AR process in the rental sector:

$$\ln(s_5) = \rho_s \ln(s_{5-1}) + e_s$$

$$\ln(s_4) = \ln(s_{5-1})$$

$$\ln(s_3) = \ln(s_{4-1})$$

$$\ln(s_2) = \ln(s_{3-1})$$

$$\ln(s_1) = \ln(s_{2-1})$$

$$\ln(s) = \ln(s_{1-1})$$

where e_s is an i.i.d processes with variances of σ_s which is calibrated to 0.00203^2. The shock persistence ρ_s is assumed to take high value as 0.9.

Anticipation (Before the Shock is realized) (Time period 1 to 5):

Agents Housing Decisions:

Agents in the Economy gets a clear signal that the Rents from the Buy to Let sector are going to go down after 5 time periods. Houses act as assets for the Borrowers, where they get returns from the Rents. Borrowers anticipate the decrease in rents (which is a decrease in their return on houses) and hence they also decrease their investment in Buy to Let sector which leads to a decrease in H_r which decreases in a almost similar rate from time period t = 1 to time period t = 5.

A decrease in demand for Buy to Let housing from Borrowers will in turn decrease the total housing from Borrowers sector H_b as substantial part of housing from Borrowers is in Buy to Let housing. This decrease is substantial decrease from t = 3 to t = 5 as the house prices also decrease a lot in this time period especially agents realize this decline in house prices substantially from time period t = 3 (from this time period house prices is below the steady state level).

We can see this substantial decrease from t = 3 as this is the time period from which all the agents start to react to the anticipatory shock (which happens at t = 5) strongly. This also results in a decrease in supply of housing. This results in a substantial decrease of Y_h especially from t = 3 (we see a drastic drop in housing output from time period = 3). A decrease in housing supply is also attributed to a slight increase in relative house price q_h (only in time period t = 1 and t = 2). However, this increase is very low and only for 2 time periods.

If we observe the Personal consumption of housing from Borrowers in t = 1, $H_b - H_r$, this has increased slightly (in the order of $1.36 * 10^{-5}$) which is the result of houses acting as collaterals. Houses being a collateral and a slight increase in price will make Borrowers to increase their personal consumption of housing in time period t = 1 and t = 2. This personal housing of Borrowers again drastically drop from time period t = 3 once the house price start falling.

As we also know that the fraction of Hand to Mouth agents in the economy is a constant parameter (0.1), a decrease in Buy to Let housing from Borrowers would increase the demand from Renters and this results in an increase in the rental price q_r until the shock is realized.

Also from Savers perspective, the decrease in demand of housing from Borrowers lead to a decrease in demand for loans B_s and we also see a drastic decrease in loan demand from Borrowers from time period t = 3, this result leads us to see a slight decrease in Savers housing at t = 0,1 and gradually increase their housing from there on to a substantial increase from time period t = 3 as a result we see an increase in their personal consumption of Savers housing H_s from t = 3.

Agents Labour Decisions:

As there is a less supply of housing from the firms (Y_h decreases), all the agents would like to shift their labour away from housing sector as I assume labour can move freely between the sectors which results in a decrease of N_sh, N_bh, N_ph. Again we will see a gradual decrease in labour from all three sectors in housing sector in time periods t = 0,1 and t = 2. However, we will see a drastic decrease in the same labour from time t = 3. This again results in a gradual decrease of housing output until t = 2 and a drastic decrease in housing out from t = 3 to the time until the shock is realized.

As the house prices are more in t = 1 and t = 2, Borrowers have an incentive to work more and invest in their own personal housing. This results in an increase of Borrowers labour in consumption goods sector N_{bc} and this labour supply starts falling from time t = 3 when the house prices start falling.

From the Hand to mouth agents perspective, to account for the increase in rents until the shock is realized they want to produce more labour and this results in an increase in N_{pc}.

Savers tend to have an income effect until time period t = 3 as the house price is positive until t = 2 and the demand for loans from Borrowers is not bad because of personal consumption of housing from Borrowers is positive. As the Savers supply less of labour until t = 3, even though the the consumption output starts above from the steady state, it tend to decrease gradually in period t = 1 and t = 2 (Y_c has increased but starts decreasing in t = 1 and t = 2 and again increase from t = 3 until the shock is realized).

On aggregate, we see a gradual increase in labour from Hand to mouth guys N_p in t = 1 and t = 2 and a drastic increase in labour from the same agents from t = 3 to the time until the shock is realized, this results in a decrease in wages of these agents w_p decreases gradually until t = 2 and drastically from t = 3.

From Borrowers perspective, we see a gradual decrease in N_b in time period t = 1 and t = 2 (Which is a result of anticipation of decrease in returns) and drastic decrease from t = 3 to the time the shock is realized (which is a result of anticipation of decrease in Returns on housing and also decrease in collateral value from time period t = 3). Even though we see the wages of Borrowers decrease in t = 0, decrease in N_b results in a gradual increase of wages for Borrowers in t = 1 and t = 2 and a substantial increase in wages of Borrowers from t = 3.

From Savers perspective, we see a gradual decrease in N_s in time t = 1 and t = 2 but we see a drastic decrease from t = 3 to the time where the shock is realized (which is a result of gradual decrease in demand for loans until t = 2 and a drastic decrease in loans from t = 3). This results in an gradual increase in wages of Savers from t = 1 and t = 2 and a drastic increase from t = 3 to the point where the shock is realized.

Agents Consumption Decisions:

From Borrowers perspective, they tend to invest their disposable income in their personal consumption of housing until the house prices start to fall "after t = 3" because of the collateral affect and this results in a decrease if consumption in time period t = 0,1,2 and 3. Once the house prices start falling, these agents tend to invest less in housing both in Buy to Let and personal consumption of housing and start having more of consumption which results in an increase of consumption after t = 3 C_b increases after t = 3. Overall the consumption of Hand to mouth agents decreased from steady state at time t = 0.

From hand to Mouth agents perspective, Because of the gradual decrease in supply of Buy to Let housing in time periods t = 0,1,2 and a substantial decrease of Supply of Buy to Let after t = 3, we see a similar pattern of consumption of hand to mouth which decreases gradually until t = 3 and substantially decrease in consumption after t = 3 until the shock is realized. Overall the consumption of Borrowers decreased when they get the news at t = 0.

From Savers perspective, we see an increase in Savers consumption C_s at t = 0 and slowly decreases until t = 3 (When the demand for loans is not bad as the personal housing consumption of Borrowers is still positive) and then the consumption of Savers decreases gradually. Overall The consumption of Savers has increased from steady state at t = 0.

Inflation and Interest Rate:

At t = 0, we see a decrease in consumption from Borrowers and Hand to Mouth agents (C_b and C_p decrease), even though we see a slight increase in consumption from Savers C_s. The increase in consumption goods output and the affect of decrease in consumption from Borrowers and Hand to Mouth agents dominates the affect of slight increase in consumption from Savers and this results in a decrease of inflation πat period t = 0 when the agents get to know the news shock. This trend in decrease of inflation will be seen until time t = 3 where the consumption of Borrowers start increasing (As the house prices increase they tend to divert some of their income to consumption) and this results in an increase in Inflation after time t = 3. The Central bank sets the interest rate according to changes in inflation, we will see a decrease in Interest Rate at t = 0 and a decreasing trend until t = 3 (Which is same as the Inflation) and from then (after t = 3) we will see an increase in interest rate until the shock is realized.

Coming to the housing Inflation π_h, at t = 0, we see a decrease in housing Inflation as the demand for housing from all the sectors of agents (Borrowers aggregate housing and Savers housing) decrease. This trend continues until time period t = 3. Once the Relative house price starts falling q_h, the housing Inflation π_h increases which is completely driven by the Savers Investment in housing increases.

Moment the Shock is fully realized (Time period 6):

The Shock will take it's full affect from period 6. In period 6, we would see a decrease in the rental rate of houses (q_r decreases) as anticipated by the renters. This results in a decrease of Buy to Let Investment from Borrowers (H_r decreases) and also results in a decrease of total housing of Borrowers H_b. The decrease in demand of housing from Borrowers sector results in a decrease in relative house price q_h. The decrease in relative price results in a decrease in total housing output Y_h.

From Agents labour Perspective, as expected we see a decrease in housing sector labour from all the agents N_sh, N_bh, N_ph decreases. We also see an increase in consumption sector labour from Savers N_{sc} and Hand to Mouth agents N_{pc} whereas we see a decrease in consumption sector labour in Borrowers sector N_{bc} (There is no incentive for Borrowers to supply more labour as Both the returns on housing and collateral value of housing decreases). On aggregate, we see an increase in labour from hand to mouth agents as the rents were high until the time period 5 and they gradually decrease their labour supply from t = 6, this results in a decrease in wage rate w_p for Hand to Mouth agents. On the other hand Savers and Borrowers on aggregate decrease their labour N_b, N_s decrease (As expected as there is no incentive for them to supply more labour), this results in an increase in Wage rate for both these agents: W_s, W_b increases. Overall we see an increase in disposable income for Borrowers (As they don't invest in housing and with better wages) and Savers (As they see a reduction in Supply of loans and with better wages) and this results in an increase in consumption for both these agents C_s and C_b increases. However, we see a reduction in disposable income for Hand to mouth agents mainly driven by the decrease in wages of these agents, this results in a decrease in consumption from these hand to mouth agents: C_p decreases.

The movement of labour from housing sector to consumption sector from all the three agents ensure that we see a substantial increase in consumption goods output Y_c and a decrease in housing goods output Y_h. With this substantial increase in consumption goods output and decrease in consumption from hand to mouth agents we see a decrease in inflation π even though the consumption from Borrowers and Savers increase slightly.

Also as the demand for housing decreases from Borrowers, the demand for loans B_s decrease and also result in a decrease in Interest rate $Rate_o$ from the Savers to encourage the Borrowers to take more loans. From the Central Bank perspective, the decrease in inflation will reduce the Interest rate $Rate_o$ and The effect in the economy tend to persist in a cyclical effect. Inflation in housing π_h also decreases as we see a substantial decrease in consumption of housing services from Borrowers even though we see a slight increase in Savers housing and a slight decrease in the housing output.

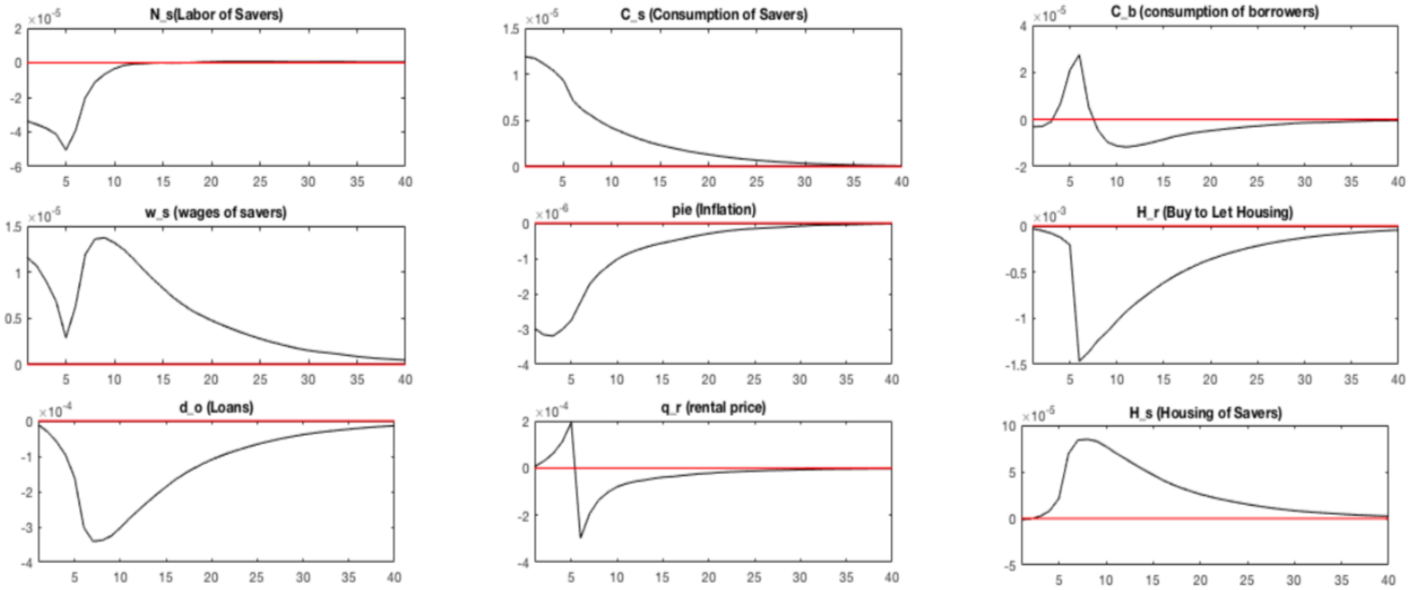


Figure 33: IRFs for News shock on rental Returns

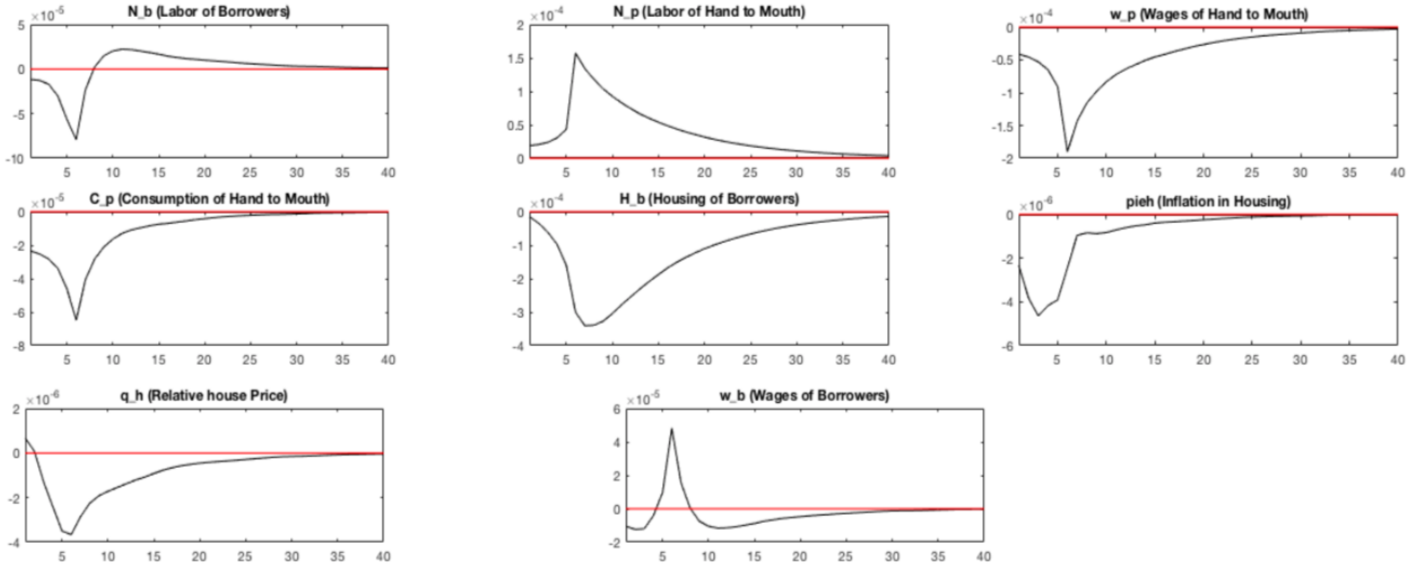


Figure 34: IRFs for News shock on rental Returns

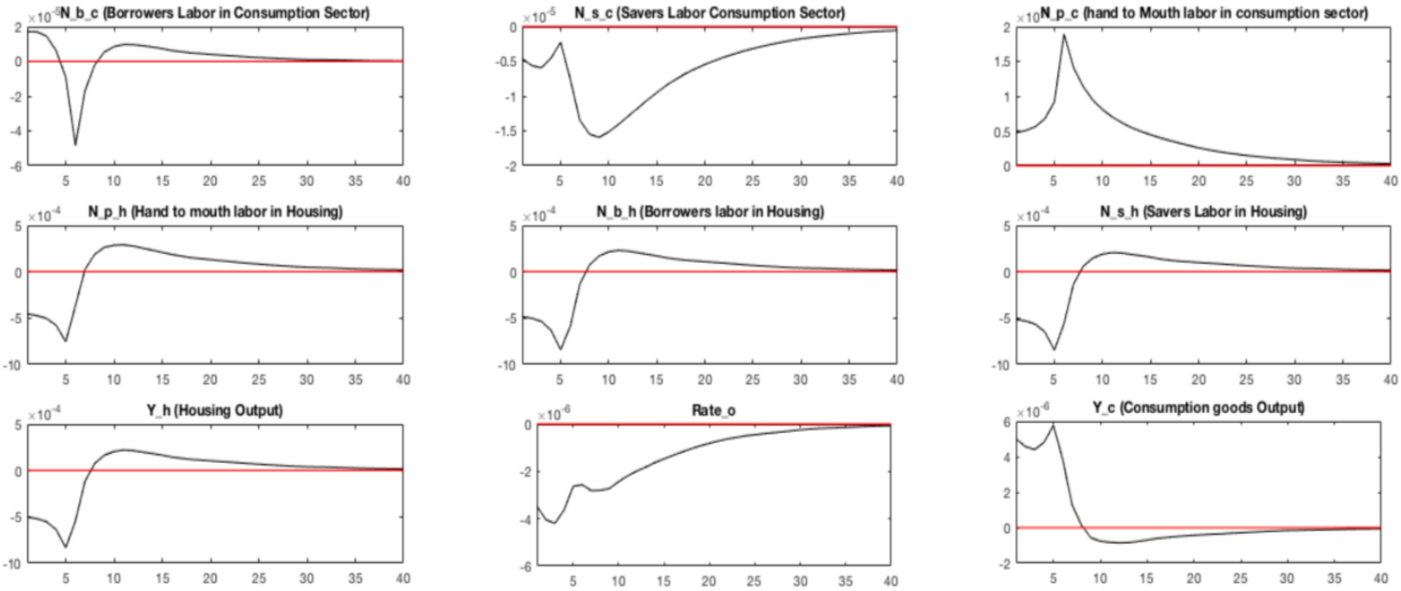


Figure 35: IRFs for News shock on rental Returns

4.4.9 Effect of Expectation technology shock on Consumption good Firms

I give an Expected Shock of 5 time periods to the productivity of consumption good firms i.e., the agents receive clear signals about the productivity of consumption good firms they face in the future i.e., after 5 time periods. Agents anticipate that the productivity of consumption good firms is going to increase in the future. The implication of such expected shocks can be interpreted before the Shock has been realized and after the shock is realized.

The Expectation shock on the both the technology shocks follow a similar AR process in both the consumption and housing sector to that of the news shock on the expected rental returns in the rental sector. The i.i.d processes in both the housing and consumption goods sectors are with variances calibrated to 0.009^2. The shock persistences for both the shocks are assumed to take a value of 0.3.

Anticipation (Before the Shock is realized) (Time period 1 to 5):

Firms Decisions:

Firms in the Economy gets a clear signal that the production of consumption goods (consumption good output Y_c) increases after 5 time periods. This doesn't results in the firms to hire less labour now and more once the shock is realized because the consumption good firms are rigid and firms take this into consideration because the agents need not respond quickly to the positive technology shock in consumption sector and also the fact that the housing sector firms are flexible. consumption goods firms also increase their wages to all the agents as a result of this anticipation of increase in productivity in the near future and want to have more labour by the time shock hits the economy. All the agents would like to shift their labour away from housing sector as I assume labour can move freely between the sectors and the demand for labour in housing firms is lower now in t = 1, which results in a slight decrease of N_sh, N_bh, N_ph (until time t = 2). As agents come close to the realization of shock the flexible housing market plays a role and they again increase their labour supply to the housing firms as the shock realized (From t = 3). On aggregate, the labour supply of Savers and Hand to Mouth agents increase at t = 0, but the aggregate labour of Borrowers decrease (As a result of the implication from housing market which we will see a bit later). This increase in labour from Hand to mouth agents, Savers and an increase in wages will leave these agents will better disposable income. The substantial increase in wages for the Borrowers also lead to a better disposable income for Borrowers.

Agents Consumption Decisions:

In the early stages of the news shock, agents in the Economy gets a signal that the production of consumption (housing output Y_c) increases for sure after 5 time periods, all the agents in the economy have a better disposable incomes. The result of the anticipation of increase in consumption goods output and the increase in disposable income leads to an increase in consumption for all the three agents at period t = 1.

Agents Housing Decisions:

In the early stages of the news shock, agents in the Economy gets a signal that the production of consumption (housing output Y_c) increases after 5 time periods as a result of labour moving from housing sector to consumption sector. houses act as collateral for the Borrowers. Borrowers see the decrease in the productivity of housing market in the early stages of anticipation shock and this results in a increase in the relative prices of houses and hence they also increase their investment in housing: H_b increases. Buy to Let sector also increases: H_r increases. From period t = 2 the total housing of Borrowers start decreasing even though the prices of houses increase. This again is quite opposite result to the Expected tech shock on housing because Borrowers purchasing power of houses decrease with increase in price and they form an anticipation that house prices even go up as the shock hits the economy and this leads to an increase in personal consumption of housing $(H_b - H_r)$which acts as collateral and decrease the Buy to Let housing (H_r).

As stated before, as the agents near the time t = 5, the affect of increase in house prices will encourage the Borrowers to buy a bit more houses as they get more collateral value from having a big house. This decrease in house purchasing power of Borrowers will be negated by the effect of increase in collateral value and this in-turn will increase the personal consumption of houses from Borrowers $H_b - H_r$ and decrease the demand for Buy to Let housing from Borrowers: H_r starts decreasing from period t = 2.

We can see this increase in Buy to Let holdings from Borrowers from t = 4 as this is the time period from which all the agents start to react to the anticipatory shock (which happens at t = 5) strongly. This also results in a increase in supply of housing. This results in a slight increase of Y_h especially at t = 4 (we see a increase in housing output from time period = 4). A increase in housing demand from Borrowers is substantial and is also attributed to a substantial increase in relative house price q_h (the collateral affect is in place here).

As we also know that the fraction of Hand to Mouth agents in the economy is a constant parameter (0.1), an increase in Buy to Let housing from Borrowers would result in a decrease in the rental price q_r until the shock is realized.

Also from Savers perspective, the increase in demand of housing from Borrowers lead to a increase in demand for loans B_s and we also see a drastic increase in loan demand from Borrowers from time period t = 3, this result leads us to see a substantial decrease in Savers housing from time period t = 3 as a result we see an decrease in their personal consumption of Savers housing H_s from t = 3.

Agents Labour Decisions:

Agents in the economy anticipate that a temporary increase in the consumption goods production technology directly translates into higher output in consumption goods sector Y_c. As there is a less supply of housing from the firms (Y_h decreases before the shock is realized), all the agents would like to shift their labour away from housing sector as I assume labour can move freely between the sectors which results in a decrease of N_sh, N_bh, N_ph. Again we will see a gradual decrease in labour from all three sectors in housing sector in time period t = 6 when the shock is realized.

As the house prices are increasing, Borrowers have more disposable income and a higher collateral value, this results in a decrease in the supply of labour from these agents N_{bc} decreases.

From the Hand to mouth agents perspective, to account for the increase in consumption until the shock is realized they want to produce more labour and this results in a slight increase in N_{pc}.

Savers tend to have an substitution effect as the demand for loans from Borrowers have increased because of increase in overall consumption of housing from Borrowers.

On aggregate, we see a substitution effect in Hand to Mouth agents and Savers with increase in wage rate and income effect in Borrowers with increase in wage rate in the early stages of the expected shock. This results in an increase in aggregate labour from Savers and Hand to Mouth agents and a decrease in aggregate labour from Borrowers.

Inflation and Interest Rate:

At t = 0, as we see an increase in consumption from all of the three agents (C_b,C_S and C_p increases), the substantial increase in consumption goods output results in a decrease of inflation πat period t = 0 when the agents get to know the news shock. The Central bank sets the interest rate according to changes in output and follows the trend of inflation after period t = 1.

Coming to the housing Inflation π_h, at t = 0, we see a increase in housing Inflation as the demand for housing from Borrowers substantially increase at t = 0.

Moment the Shock is fully realized (Time period 6):

The temporary increase in the consumption good production technology directly translates into higher output in consumption goods sector Y_c as the output is directly proportional to the technology shock, also as we see from the private sector equilibrium, the shock enters the system in the marginal cost of producing consumption goods as a denominator and hence a positive consumption shock will decrease the marginal cost of producing a consumption good:

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

as $MC_s = \zeta_t P_{ct}$, we see a decrease in the marginal costs and a decrease in ζ_t

As the marginal costs decrease in consumption sector of the economy, the effect of "Technological advancement and price rigidity" of this sector makes the firms from this sector hire less labour and we see a decrease in Borrowers, Savers and Hand to mouth agents labour in consumption sector (N_b^c,N_s^c and N_p^c decreases)

$$y_t^c(i) = Z_{ct} N_{p,t}^c(i)^v N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v},$$

From the consumption goods firm's perspective, marginal productivities of labour from all of the labour sectors have increased, they want to have more of all labour inputs, pushing up the real wage w_p, w_b, w_s to induce households to supply more labour.

However, the effect of price rigidity has a strong affect here and firms cut their labour inputs demand and this results in moving of the labour from consumption good sector to housing goods sector (which have flexible prices and houses acting as assets) which results in increase of (all types of agents labour in housing sector) $N_b^h$and N_s^h N_p^h and increasing the output in housing goods sector Y_h . The result of moving labour from consumption sector to housing sector is again from a two fold affect of flexible price market for housing and houses acting as assets.

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v}$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in all of the sector's wages .

From households perspective, an increase in the real wage rate has two effects: an income and a substitution effect. Because of the higher income the agent wants to work less and instead enjoy a higher amount of leisure. This is the income effect. On the other hand, a higher real wage leads to a substitution of leisure with labour. This is the substitution effect. In the model, from the consumption sector's perspective, the Income effect clearly dominates the substitution effect for all the three agents and from housing sector's perspective the substitution effect dominates the income effect again in all the three types of agents.

On aggregate, leisure $1-N_b$, $1-N_s$, $1-N_p$ increase i.e., labour of all the three households N_b, N_s, N_p decreases which shows that Income effect dominates the substitution effect on aggregate. As wages increase, workers from all the sectors will want to increase consumption, resulting in a rise of Cs and Cb and Cp . Also we can observe from the consumption goods output equation, that as output is directly proportional to the consumption of goods, with increase in output directly translates to an increase in consumption of all the household agents.

$$Y_t^c = C_{b,t} + C_{s,t} + C_{p,t} + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

Households know that the shock is temporary and they smooth their consumption. Also the magnitude of rise in consumption is different from Borrowers and hand to mouth workers and with lower increase in consumption for the Borrowers sector, additional income of Borrowers is saved, i.e. invested into the housing stock H_b resulting in an increase in the housing stock of Borrowers which in turn also results in the increase in the BTL stock.

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

On the other hand, as Borrowers tend to invest in housing, combining with the decrease in the output of the housing sector, the price of the houses q_h increases resulting in Savers to issue more of loans B_s and consume more C_s and investing in their housing stock H_s decreases which shows that the demand of the housing stock is mostly from the Borrowers sector. The Savers sector tend to divert some of the housing stock towards the increase in issuing loans.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

Whereas, hand to mouth guys cannot invest in housing, combining with the increase of investment in housing sector from the Borrowers, Buy to let houses H_r have increased and the hence price of the rental rate q_r decreases due to more supply of BTL houses. As the wages of hand to mouth guys increasing and due to a decrease in rental price results in a substantial increase in consumption of this sector.

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

The increase in productivity of consumption good firms enhances to supply more of the consumption goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the consumption goods for better marginal profits and as a result, fall in inflation πin the economy occurs. However, even though the housing sector is flexible we see a decrease in housing inflation π_h as the demand from the houses increases substantially from the Borrowers sector at t = 6. The central bank sets the interest rate according to the Taylor rule and as the inflation decreases, the interest rate tend to decrease.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate decreases, Borrowers tend to obtain more loans B_s and increase their consumption. Borrowers tend to increase the investment in housing stock which further increases their collateral value. The increase in investment from Borrowers will in-turn affect the Buy-to-let market which in-turn increases the rental properties which increase in H_r. As the rental properties increases, this will reduce the rental price q_r and as a result, poor workers tend to have more for consumption and C_p increases.

As the demand for loans increase, Savers tend to supply more of deposits as they tend to divert some of their investment in housing stock towards issuing more loans and hence we see a decrease in H_s. As the Savers tend to get more returns in terms of loans in volume, they tend to increase their consumption and we see an increase in consumption from all the sectors.

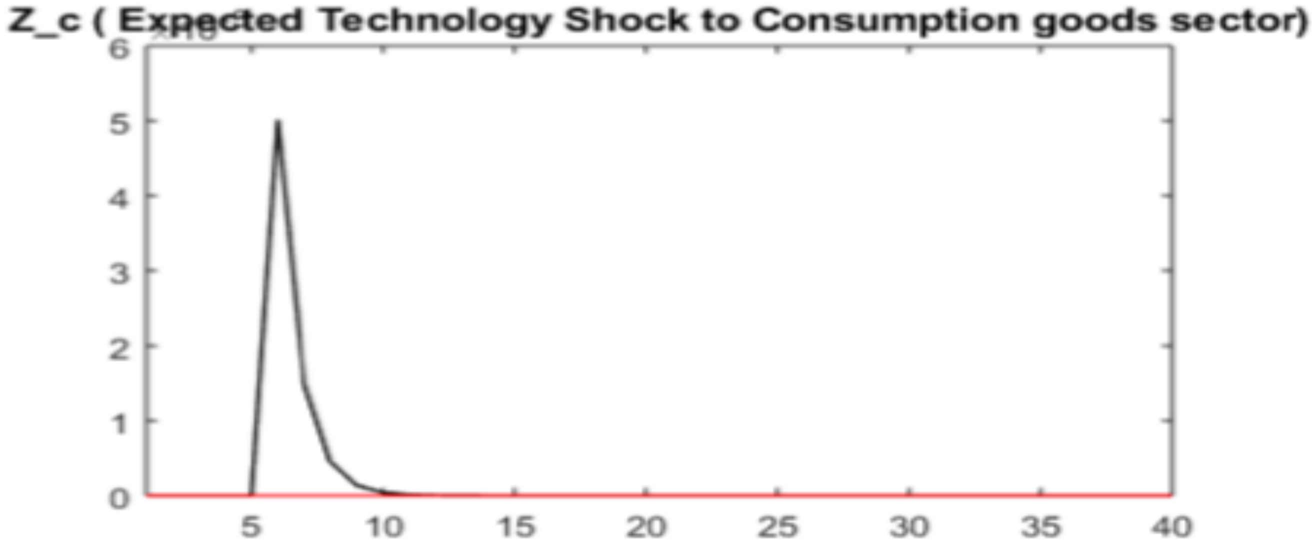


Figure 36: IRFs for News shock on Consumption good Firms Productivity

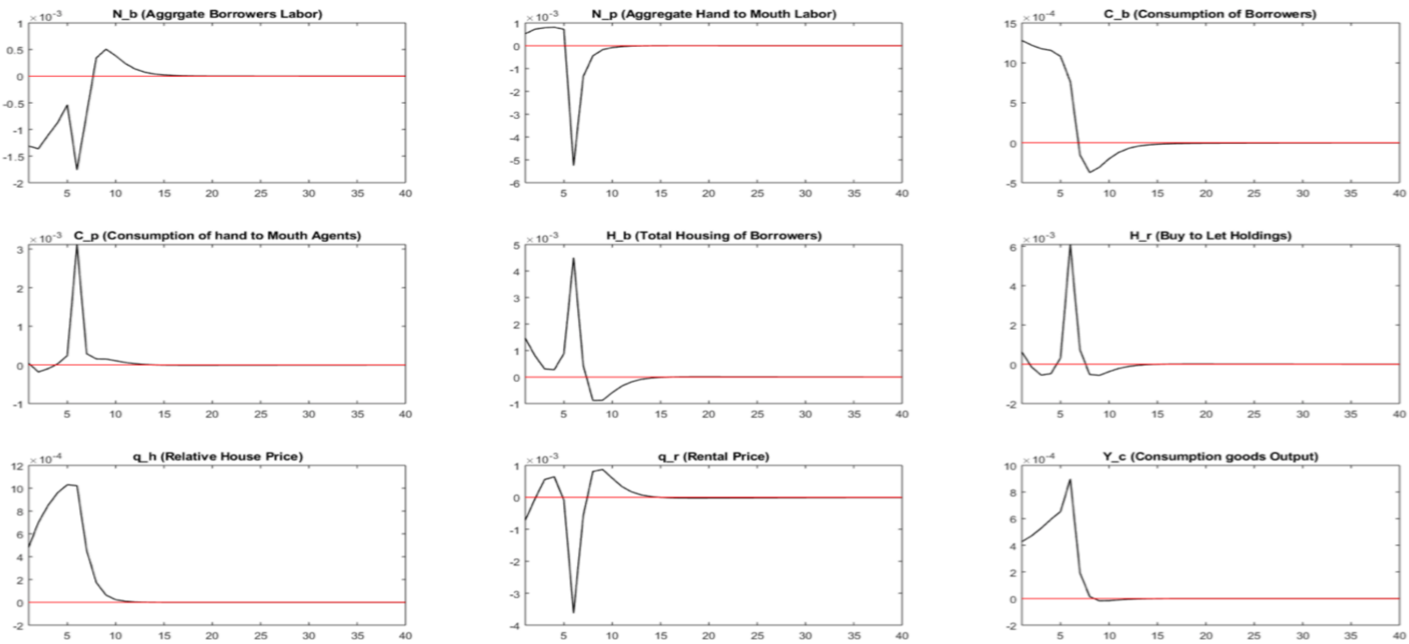


Figure 37: IRFs for News shock on Consumption good Firms Productivity

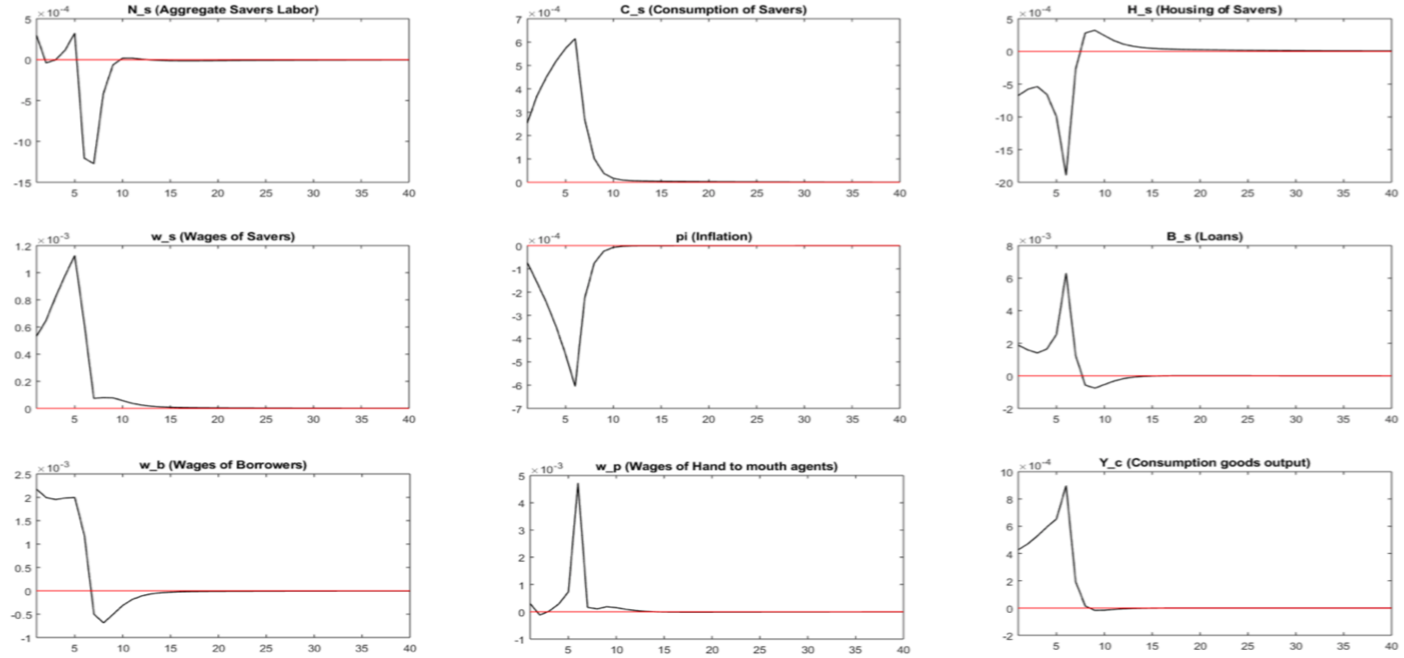


Figure 38: IRFs for News shock on Consumption good Firms Productivity

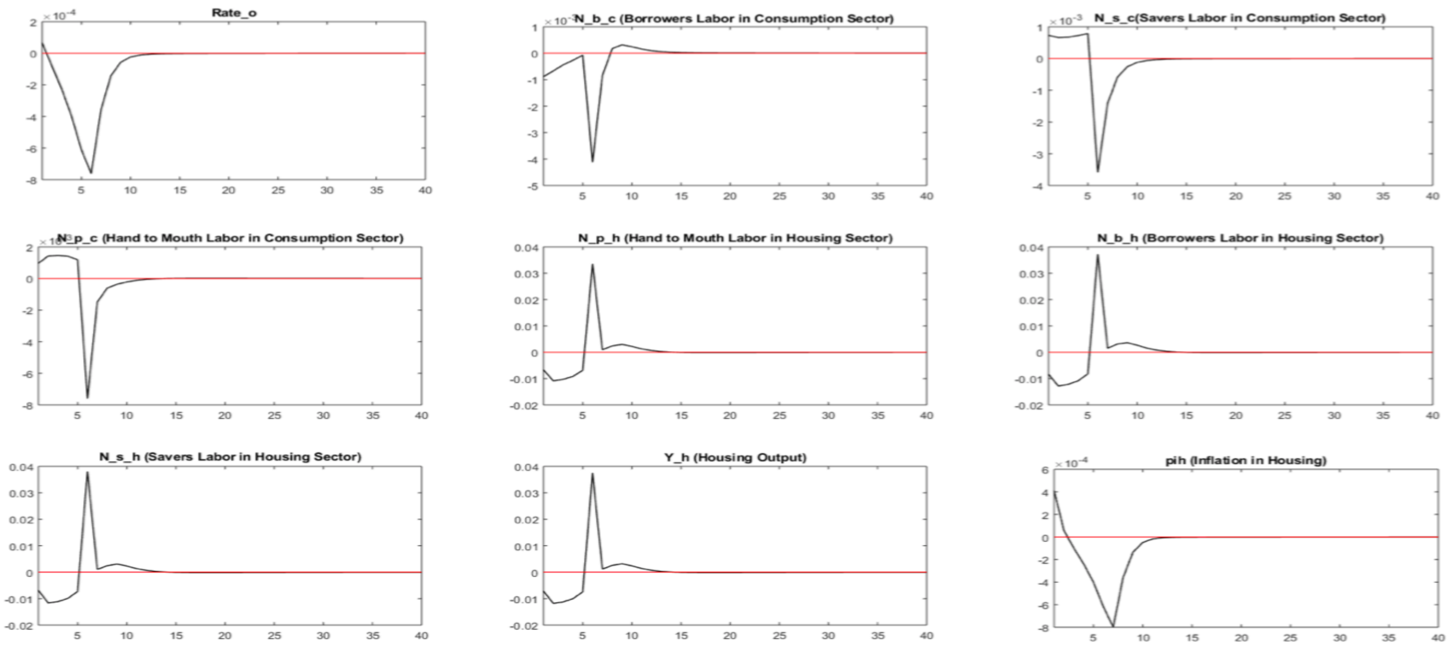


Figure 39: IRFs for News shock on Consumption good Firms Productivity

Interpretation:

Firms Decisions:

We observe that the labour markets play a pivotal role in most of the dynamics. An innovation Z_{ct} to consumption goods sector, translates to an increase in consumption output Yc and as wages are same across firms, price rigidity in consumption sector helps in the movement of labour from consumption sector to housing sector which in turn increases the housing output Yh.

Households Decisions:

Because of the increase in wages, we see an increase in disposable income for all the household agents. From Borrowers perspective an increase in disposable income can be spent on consumption goods or on housing, as the role of collateral constraint kicks-in, these agents tend to increase their investment in housing which acts as booth collateral and a means to get returns(rents from BTL). This will be translated to a slight increase in consumption and the demand for housing by the Borrowers H_b increases which results in an increase in house prices q_h even though we see an increase in housing output Y_h and this in turn increase the demand for the loans Bs. From Savers perspective, again the increase in disposable income can be spent on consumption goods, housing or on providing loans. As the demand of housing from the Borrowers increase, there will be an increase in demand for loans and this in turn will make Savers to invest in providing loans as this is their only means of returns (No BTL from Savers sector and No collateral constraints for these agents). hence the Savers will increase their consumption and loans but decrease their housing investment. This again results in a decrease in the rate returns (Rate). From Hand to mouth agents perspective, as they cannot invest in housing they consume more of this disposable income (C_p increases) and also the affect of reduction in rental rate q_r amplifies the increase in consumption from these agents. Overall the disposable income increase for all the three agents and are better off in consumption for a positive consumption shock. Borrowers tend to invest in housing and Savers tend to supply more loans with decrease in housing.

4.4.10 Effect of Expectation technology shock on Housing Firms

I give an Expected Shock of 5 time periods to the productivity of housing firms i.e., the agents receive clear signals about the productivity of housing firms they face in the future i.e., after 5 time periods. Agents anticipate that the productivity of housing firms is going to increase in the future. The implication of such expected shocks can be interpreted before the Shock has been realized and after the shock is realized.

Anticipation (Before the Shock is realized) (Time period 1 to 5):

Firms Decisions:

Firms in the Economy gets a clear signal that the production of houses (housing output Y_h) increases after 5 time periods. This results in the firms to hire less labour now and more once the shock is realized as they can have more marginal output. All the agents would like to shift their labour away from housing sector as I assume labour can move freely between the sectors and the demand for labour in housing firms is lower now, which results in a decrease of N_sh, N_bh, N_ph (Even though we see a decrease in consumption output Y_c, which shows that the agents have income effect especially after period t = 3). Housing firms also decrease their wages as a result of this anticipation of increase in the near future (This result is quite opposite to that of the Expected technology shock on consumption good firms as consumption good firms are rigid and firms take this into consideration because the agents need not respond quickly to the positive technology shock in consumption sector). This decrease in labour and decrease in wages will affect the households to have less of disposable income. This decrease in disposable income also results in a decrease in consumption from all the agents C_p, C_b, C_s decreases.

Agents Housing Decisions:

Agents in the Economy gets a clear signal that the production of houses (housing output Y_h) increases after 5 time periods. houses act as collateral for the Borrowers. Borrowers anticipate the increase in the productivity of housing market results in a decrease in the relative prices of houses and hence they also decrease their investment in housing: H_b decreases. Buy to Let sector also decreases: H_r decreases which decreases at an almost similar rate from time period t = 1 to time period t = 4 and when the agents near the shock realization period they fall their consumption of housing drastically at period t = 4. However, as the agents near the time t = 5, the affect of decrease in house prices will encourage the Borrowers to buy a bit more houses as they get utility from living in a big house (Also they anticipate the collateral value of housing going to go down even further and they can provide more Buy to Let houses in future with better returns by decreasing their personal consumption of houses when the value of house / value of the collateral is very low). This increase in house purchasing power of Borrowers will negate the effect of decrease in collateral value and this in-turn will increase the personal consumption of houses from Borrowers $H_b - H_r$ and decrease the demand for Buy to Let housing from Borrowers: H_r decreases. This difference between personal consumption of housing and the Buy to Let is high when the shock is about to be realized (at t = 5) as the house prices also decrease a lot in this time period. We can see this substantial decrease in Buy to Let holdings from Borrowers from t = 4 as this is the time period from which all the agents start to react to the anticipatory shock (which happens at t = 5) strongly. This also results in a decrease in supply of housing. This results in a slight decrease of Y_h especially at t = 4 (we see a drop in housing output from time period = 4). A decrease in housing demand from Borrowers is substantial and is also attributed to a substantial decrease in relative house price q_h. If we observe the Personal consumption of housing from Borrowers in t = 5, $H_b - H_r$, this has increased (in the order of $1.3 * 10^{-4}$) which is the result of increase in purchasing power of houses from Borrowers . As we also know that the fraction of Hand to Mouth agents in the economy is a constant parameter (0.1), a decrease in Buy to Let housing from Borrowers would increase the demand from Renters and this results in an increase in the rental price q_r until the shock is realized. Also from Savers perspective, the decrease in demand of housing from Borrowers lead to a decrease in demand for loans B_s and we also see a drastic decrease in loan demand from Borrowers from time period t = 4, this result leads us to see a substantial increase in Savers housing from time period t = 4 as a result we see an increase in their personal consumption of Savers housing H_s from t = 4.

Agents Labour Decisions:

Agents in the economy anticipate that a temporary increase in the housing good production technology directly translates into higher output in housing goods sector Y_h. As there is a less supply of housing from the firms (Y_h decreases before the shock is realized), all the agents would like to shift their labour away from housing sector as I assume labour can move freely between the sectors which results in a decrease of N_sh, N_bh, N_ph (Even though we see a decrease in consumption output Y_c, which shows that the agents have income effect especially after period t = 3). Again we will see a gradual decrease in labour from all three sectors in housing sector in time period t = 4. However, we will see a drastic decrease in the same labour from time t = 4. This again results in a gradual decrease of housing output until t = 4 and a drastic decrease in housing out from t = 4 to the time until the shock is realized. As the house prices are falling, Borrowers have an incentive to work more and invest in their own personal housing (This result is quite opposite to the one we see when I give a shock to the Returns: Because the purchasing power of houses from Borrowers dominates the decrease in collateral value as the Increase in housing output from the Productivity shock is substantial in this case) . This results in an increase of Borrowers labour in consumption goods sector N_{bc} and this labour supply starts falling from time t = 4. From the Hand to mouth agents perspective, to account for the increase in rents until the shock is realized they want to produce more labour and this results in a slight increase in N_{pc}. Savers tend to have an income effect until time period t = 4 as the demand for loans from Borrowers is not bad because of personal consumption of housing from Borrowers is positive. As the Savers supply less of labour until the shock is realized, the consumption output Y_c decreases (Even though we see a slight increase in labour supply from Borrowers and Hand to Mouth agents). On aggregate, we see a gradual decrease in labour from Hand to mouth guys N_p until t = 3 and an increase in labour from the same agents from t = 4 to the time until the shock is realized (to counteract the rise in rental rate). From Borrowers perspective, we see an increase in their supply of aggregate labour once these agents get the news shock (to counter the loss in Rental returns from the Buy to Let sector) and a gradual decrease in N_b until time period t = 4 (Which is a result of anticipation of decrease in house prices) and drastic decrease from t = 4 to the time the shock is realized (which is a result of anticipation of decrease in Returns on housing and also decrease in collateral value from time period t = 3). From Savers perspective, we see a gradual decrease in N_s until time t = 4 and we see a drastic decrease from t = 4 to the time where the shock is realized (which is a result of gradual decrease in demand for loans until t = 4 and a drastic decrease in loans from t = 4).

Agents Consumption Decisions:

From Borrowers perspective, they tend to have less of their disposable income from firms decisions and this results in their personal consumption of housing to decrease: C_b decreases which is the same case with other agents: C_s, C_P decrease.

Inflation and Interest Rate:

At t = 0, even though we see a decrease in consumption from all of the three agents (C_b, C_S and C_p decrease), the substantial decrease in consumption goods output results in a decrease of inflation πat period t = 0 when the agents get to know the news shock. The Central bank sets the interest rate according to changes in inflation. Coming to the housing Inflation π_h, at t = 0, we see a decrease in housing Inflation as the demand for housing from Borrowers substantially decrease.

Moment the Shock is fully realized (Time period 6):

The temporary increase in the housing good production technology directly translates into higher output in housing goods sector Y_h as the output is directly proportional to the technology shock. From the housing goods firm's perspective, marginal productivities of labour from all of the labour sectors have increased, they want to have more of all labour inputs, pushing up the real wage w_p, w_b, w_s to induce households to supply more labour. we see a increase in Borrowers, Savers and Hand to mouth agents labour in housing sector (N_b^h,N_s^h and N_p^h increases): This result of increase in labour is quite opposite to the one in the shock to consumption sector where we see a decrease in agent labour, this is mainly because of several important factors:

1) Flexible prices in housing sector and rigid prices in consumption sector: Flexible prices in the market allows agents to consume more of housing services than the rigid markets and this results in an incentive for households to supply more labour in housing market.

2) Collateral Constraint: houses act as collaterals for Borrowers in the economy and also they indirectly affect Savers who provide loans to the Borrowers.

3) Houses as assets: Borrowers perceive houses as assets where they get returns from BTL market.

The above reasons and increase in marginal productivities make agents to supply more labour in housing market and hence the housing output Y_h increases:

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu}$$

From the same reasons above, there is an incentive for Borrowers and Savers to supply more of labour and to have more of disposable income so that they can invest in houses and loans respectively. Hence these two agents provide even more labour to the consumption firms sector which results in an increase in labour of Borrowers and Savers in consumption firms sector (N_b^c,N_s^c increases). However, Hand to mouth agents doesn't have any incentives they tend to reduce their labour in the consumption sector which result in a decrease of (N_p^c). Since the rise in labour from Borrowers and Savers dominates the decrease in labour from hand to mouth agents, the consumption output Y_c increases:

$$y_t^c(i) = Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^{u} N_{s,t}^c(i)^{1-u-\nu},$$

As I assume that the wages are same across the two sectors of production, marginal productivities of labour have increased, translating to rise in all of the sector's wages . From households perspective, the Substitution effect clearly dominates the income effect for Borrowers and Savers However we also observe that the Income effect dominates in Hand to mouth agents (which are down to the reasons mentioned above) and from housing sector's perspective the substitution effect dominates the income effect again in all the three types of agents. On aggregate, leisure $1-N_b$, $1-N_s$, increase i.e., labour of Borrowers and Savers increases and the labour from hand to mouth agents decrease. As wages increase, Hand to mouth agents want to increase consumption, resulting in a rise of Cp. However because the Borrowers and Savers are affected by the housing market they Savers to decrease their consumption and Borrowers tend to increase their consumption. This decrease in consumption from Savers is mainly by the fact that As Borrowers observe a positive technology shock to the housing market they expect the output to go up and the prices of the houses to go down which decrease their collateral value. This decrease in collateral value puts off the Borrowers to invest in their overall housing H_b decreases (As housing as collateral plays a role in here) even though they tend to increase investment in BTL H_r increases (As BTL is considered more as an Asset with Rents as Returns) and the demand for the loans decrease. This results in an increase in the consumption of Borrowers and a decrease in the consumption of Savers

$$P_{c,t}C_t^b + Q_t^h(H_t^b - (1-\delta)H_{t-1}^b) + R_{t-1,o}D_{t-1}^o = H_t^r Q_t^r + N_t^b W_t^b + D_t^o + T_{b,t}$$

As Borrowers invest less in housing their demand for loans decrease B_s decrease. this directly affects the Savers who provide loans to Borrowers and this results in Savers to increase their housing stock substantially (H_S Increase) with more disposable income they acquire from increase in wages. However as the Loans decrease their aggregate returns on loans decrease and an increase in housing will hamper the consumption of these agents slightly.

$$P_{c,t}C_{s,t} + Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) + B_{s,t} = N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t}$$

On the other hand, as Borrowers see the demand for BTL increases from the Hand to mouth agents disposable income increase. The Rental price increase and this even make the Borrowers to increase only BTL houses.

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

The increase in productivity of housing firms enhances to supply more of the housing goods with the same inputs and due to the monopolistic competition, firms can decrease the price of the houses for better marginal profits and as a result, I expect to see a fall in housing inflation π_h in the economy but this won't be the case as the demand of housing from the Savers is quite high at t = 6, and the demand from the loans from Borrowers is quite low: This results in a slight increase in housing inflation π_h. However, as the consumption sector is rigid we see an increase in inflation π as the demand from the consumption goods increase from both the Borrowers and Hand to Mouth sector. The central bank sets the interest rate according to the Taylor rule and as the inflation increases, the interest rate tend to increase.

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}$$

The effect in the economy tend to persist in a cyclical effect. As interest rate Increase, Borrowers tend to obtain less loans B_s and increase their consumption. Borrowers tend to decrease the investment in housing stock which further decreases their collateral value. Borrowers will in-turn affect the Buy-to-let market by increase investment in BTL houses as they see them as assets with returns. As the demand for rental properties increase substantially, this will increase the rental price q_r and as a result, Hand to mouth workers tend to have more housing and are better off. As the demand for loans decrease, Savers tend to supply less of deposits as they tend to divert some of their investment towards housing stock which increases substantially and hence we see a increase in H_s. As the Savers tend to get less aggregate returns in terms of loans in volume, they tend to decrease their consumption and we see an decrease in consumption from Borrowers and Savers but an increase in consumption from Hand to mouth agents.

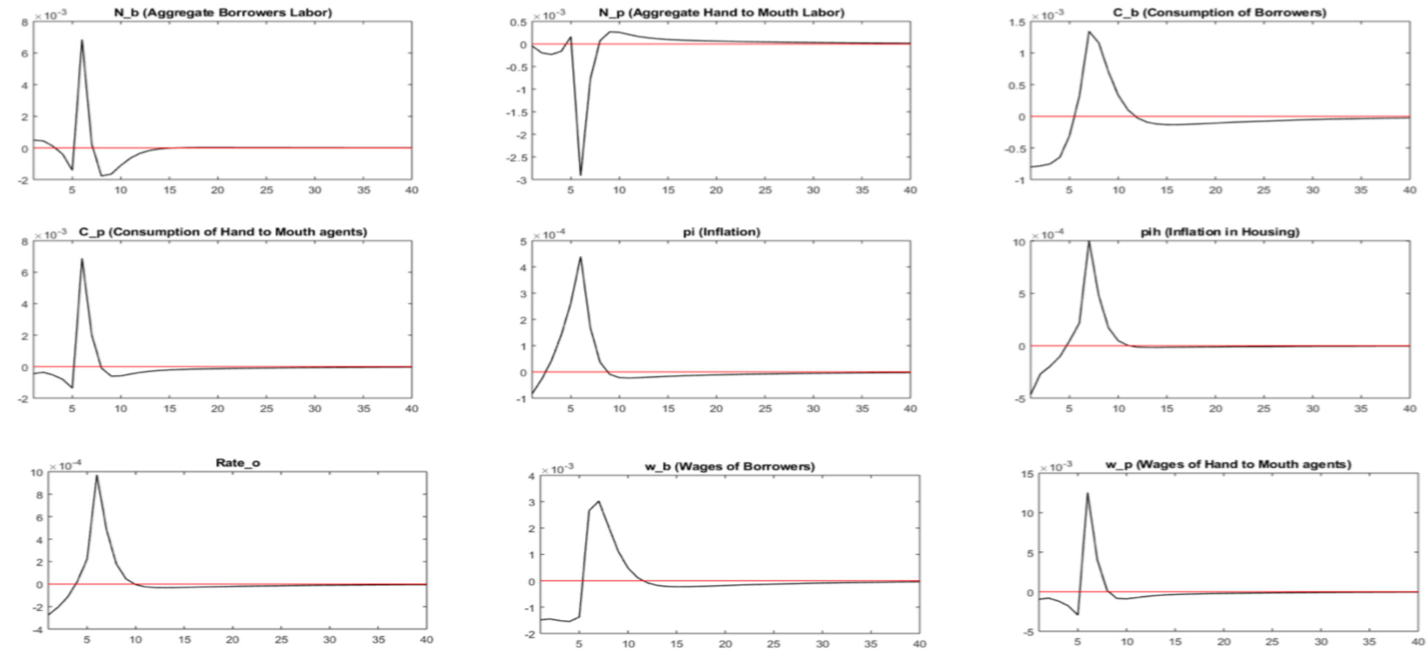


Figure 40: IRFs for News shock on Housing good Firms Productivity

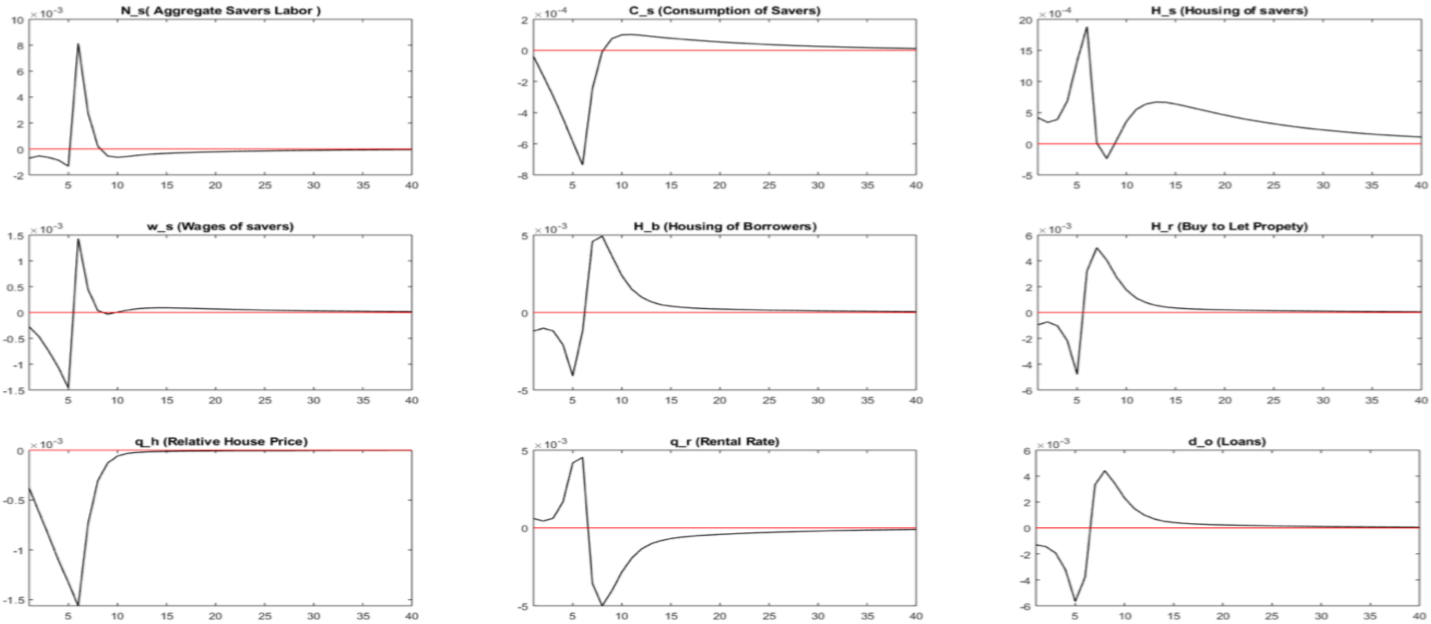


Figure 41: IRFs for News shock on Housing good Firms Productivity

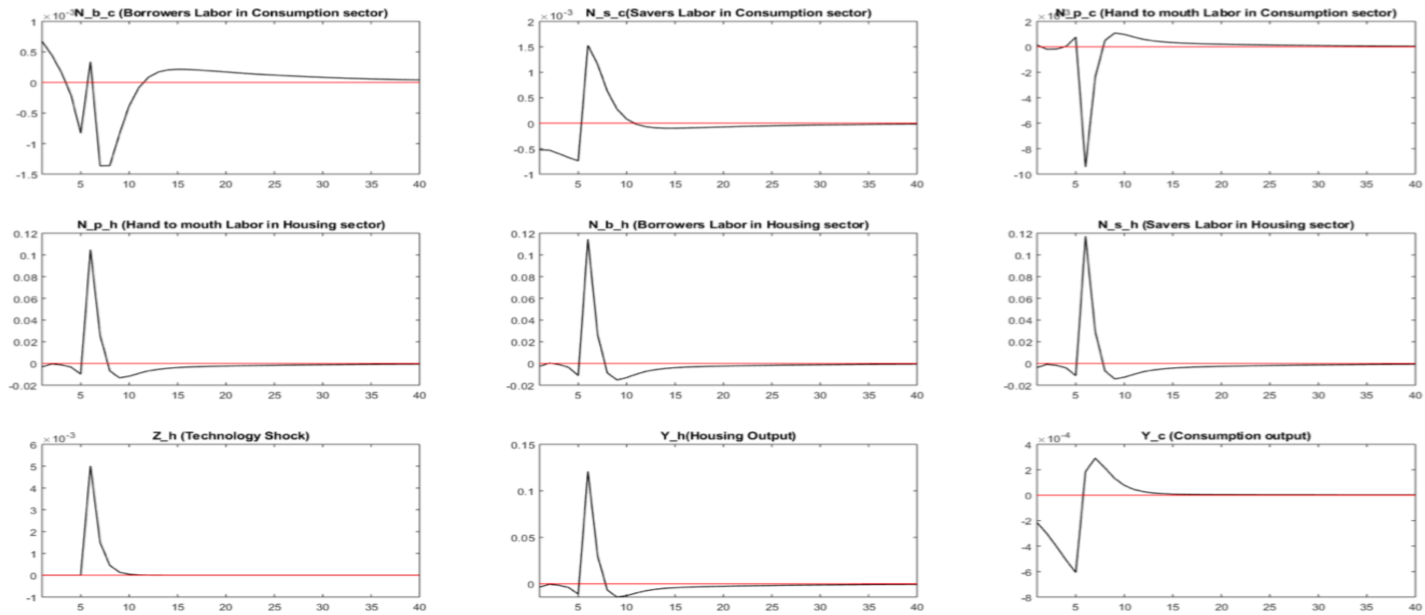


Figure 42: IRFs for News shock on Housing good Firms Productivity

Interpretation after the shock is realized:

Firms Decisions:

We observe that the labour markets play a pivotal role in most of the dynamics. An innovation Z_{ht} to housing goods sector, translates to an increase in housing output Y_h and as wages are same across firms, housing market affect on Borrowers helps in the movement of certain labour (Savers and Borrowers) from housing sector to consumption sector which in turn increases the consumption output Y_h.

Households Decisions:

Because of the increase in wages, we see an increase in disposable income for all the household agents. From Borrowers perspective an increase in disposable income can be spent on consumption goods or on housing, as the role of collateral constraint kicks-in, these agents tend to decrease their investment in housing as they see a decrease in price of housing from increase in houses output. Houses act as both collateral and a means to get returns (rents from BTL). This will be translated to a slight increase in BTL housing investment, and they increase their consumption of goods and consumption of personal housing by the Borrowers H_b decreases which results in an increase in Rental prices q_r and this in turn decrease the demand for the loans Bs. From Savers perspective, again the increase in disposable income can be spent on consumption goods, housing or on providing loans. As the demand of housing from the Borrowers decrease, there will be an decrease in demand for loans and this in turn will make Savers to invest in housing . Hence the Savers will decrease their consumption and loans but increase their housing investment substantially. This again results in a increase in the rate returns (Rate). From Hand to mouth agents perspective, as they cannot invest in housing they consume more of this disposable income (C_p increases) and also they affect of the demand for BTL amplifies and this increase in demand from these agents amplifies the rental price. Overall the disposable income increase for all the three agents. However, when the shock was just realized Hand to mouth agents are better off in consumption and housing.

4.4.11 Effect of Expectation shock on Monetary Policy:

The Expectation shock on the monetary policy follow a similar AR process in that of the news shock on the expected rental returns in the rental sector. The i.i.d processes in the news shock is calibrated to a variance of 0.005^2. The shock persistences for such shock is assumed to take a high value of 0.9, similar to that of the shock persistence on the rental return .

Monetary policy being one of the primary policy instruments for the central bank will have a very diverse set of effects on agents in the economy. In this section, I give an Expected Shock of 5 time periods to the Monetary policy i.e., the agents especially the Borrowers and Savers receive clear signals about the interest rate they face in the future i.e., after 5 time periods. Agents in the economy anticipate that the interest rates are going to increase in the future (a contractionary monetary policy). The implication of such expected shocks can be interpreted before the Shock has been realized and after the shock is realized.

Agents in the Economy gets a clear signal that the interest rate from the central bank is going to increase after 5 time periods. The first impact of the news of increase in interest rates will lead Borrowers to get more loans at the time t = 0. This resulted in a substantial increase in the number of loans from time period t = 1 to t = 5. Houses act as assets for the Borrowers, where they get returns from the Rents. Borrowers anticipate this increase in the interest rates from central bank in 5 time periods and hence they obtain more loans today and also increase their investment in their house stock. This also leads to an increase in the Buy to Let sector H_r which increases in an almost similar rate from time period t = 1 to time period t = 5. We also see due to the increase in the demand for loans, Savers divert most of their income in providing loans and decrease their stock of housing. The housing output is completely determined by the changes in demand of housing from the agents. As the Borrowers realize this at time period t = 1, they tend to increase their housing stock. Due to the relatively sharp increase in the Borrowers housing after time t = 1, house prices do follow a similar trend where the relative house price slightly increased and starts decreasing with a sharp rise in the Borrowers demand for housing. We can also see from a perspective that the housing demand from Borrowers and Savers negates each other and due to the relative sharp increase in the housing of Borrowers at time period t = 2 compared to that of an relative decrease in Savers housing, we tend to observe a U shaped housing output from time period t = 1 to t = 5. Due to an increase in the BTL sector, the rental rate has sharply declined during this period and the relative house prices are a little sluggish to respond to the monetary shock. As I assume the labour moves between the firm sectors freely, we tend to see a similar pattern of U shaped in the agents labour in the housing sector as it is in the housing output until the shock is realized. Due to the substantial increase in the housing from the Borrowers, the consumption goods sector observes a decrease in the output as well as the agents labour in the intermediate consumption firms. The aggregate labour of Borrowers and Savers also follow such U shaped increase and decrease. This clearly shows that due to the houses acting as collateral, the monetary shock will effect the interest rate and in turn the housing of the agents. Due to the positive consumption effect of the increase in house prices, consumption of Borrowers and Savers also increase and decrease between time periods t = 1 and t = 5. There has been a literature recently on the effects of monetary news in the economy. It shows that monetary shocks are responsible for around 5 percent to 30 percent of volatility in the output. The model shows a very similar results when the economy is subjected to the monetary shock. However, we could clearly see that the effect is more prominent in the housing sector compared to that of the consumption sector output. We have to also consider that houses acting as collateral will amplify such effect on the volatility of housing output.

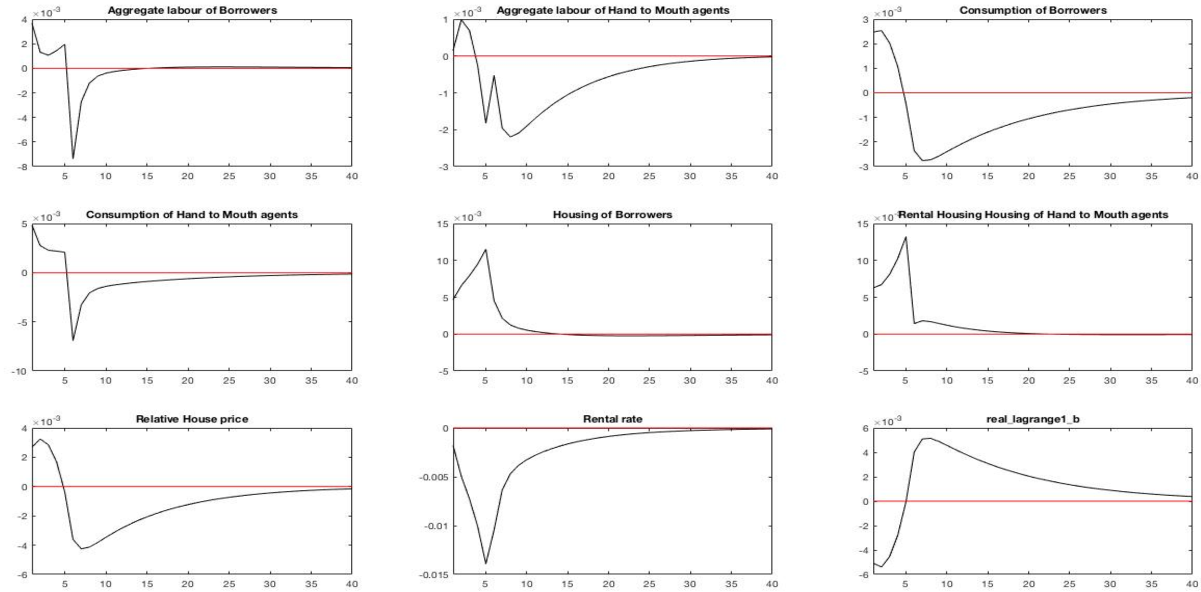


Figure 43: IRFs for News shock on Monetary Policy

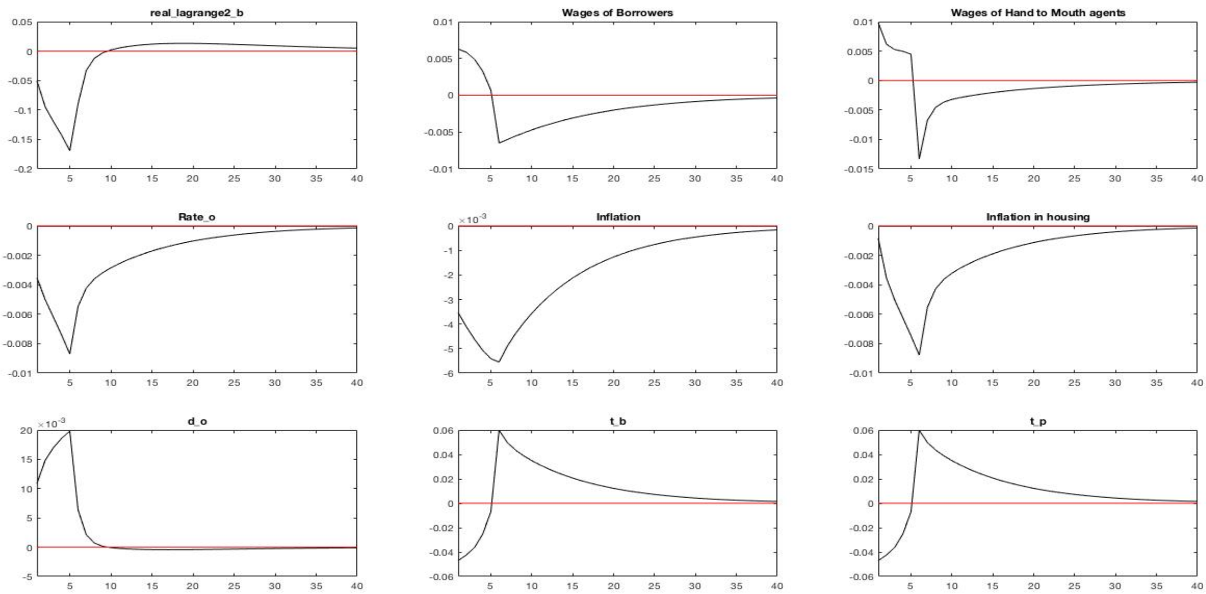


Figure 44: IRFs for News shock on Monetary Policy

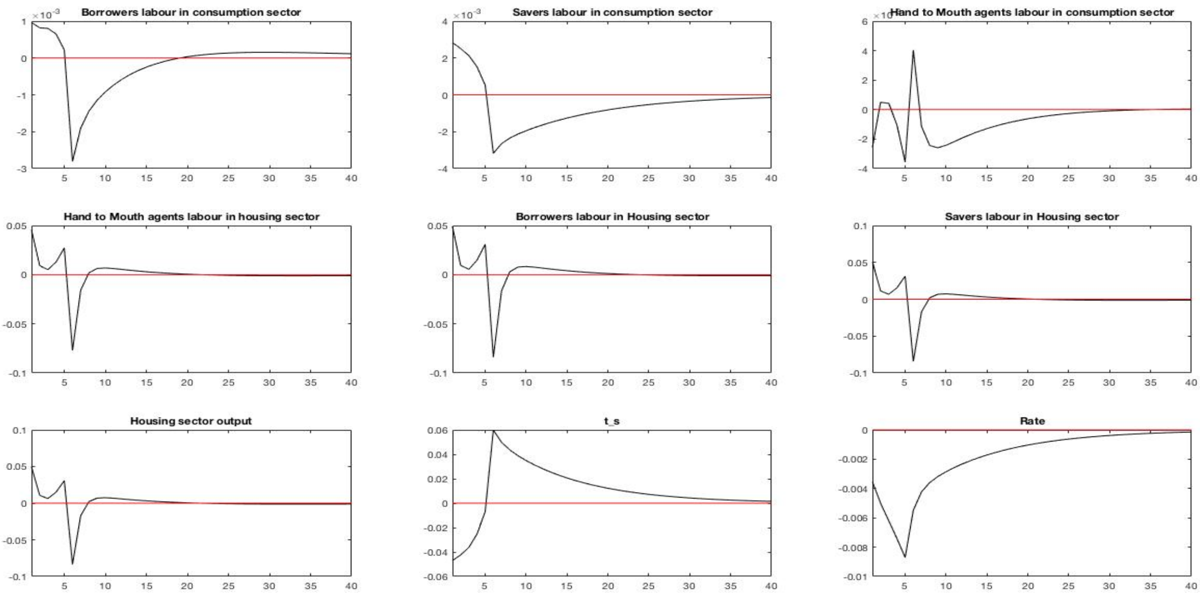


Figure 46: IRFs for News shock on Monetary Policy

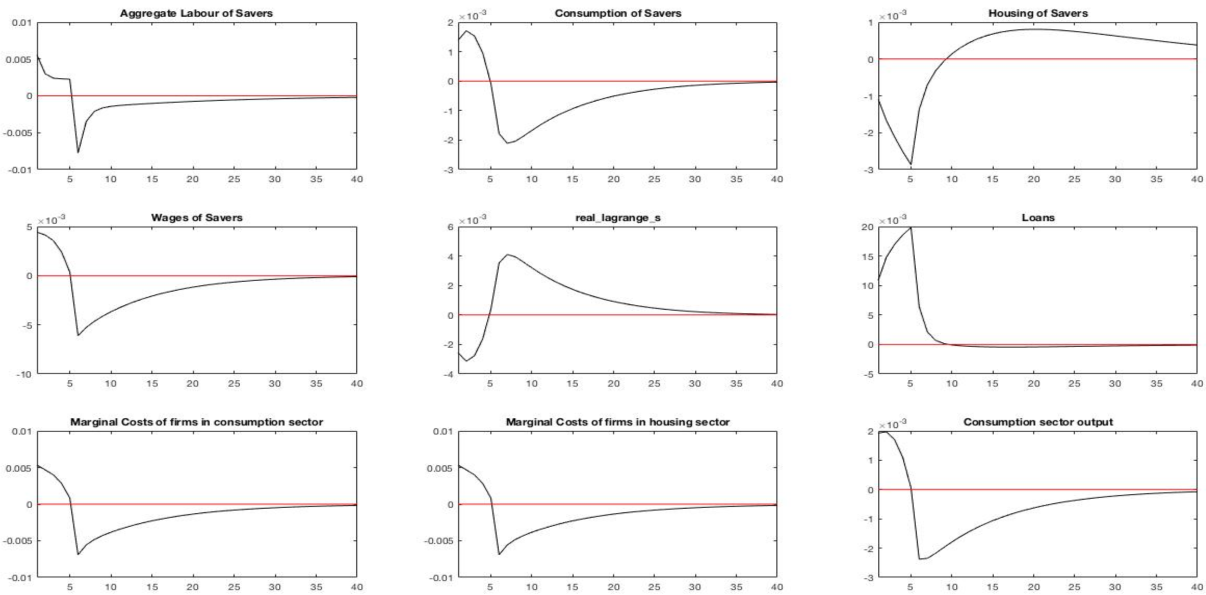


Figure 45: IRFs for News shock on Monetary Policy

4.4.12 Policy Analysis

Now I look at the policy analysis for the above model with all the above mentioned shocks augmented with an expected Monetary policy shock. As I have discussed before, the main tools of the central government in the model relies on both the Macro Prudential policy which regulates Downpayment Ratio and the Monetary policy. In this section of the analysis, I will look at how some important choice variables' volatilities behave with different affordable range of policy values in the model with expected shocks. Specifically, I will look at how the policy changes within a specific range affect the volatility of some of the important variables and I can also obtain which policy is more effective for curbing the volatility in that particular variable.

For this analysis, I will start by choosing the range of values for both the policies that the central bank could afford. As I assume that the central bank sets the interest rate by the Taylor rule

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r}$$

I also assume that the Central bank take the values of ϕ_π which is the inflation coefficient of the Taylor rule in between [1.5 , 3.5].

Also the Central bank chooses the Macro prudential policy which regulates Downpayment ratio by choosing the value of μin the Borrower's collateral constraint.

$$R_{t,b}D_t^b \leq \mu Q_t^h H_t^b$$

I also assume the affordable range of values for the downpayment ratio to be in between [0.6 , 0.9] which is like a downpayment ratio can be in between 40 percent to 10 percent of the house Value. Once I pinned down the range of values, we can now look at the volatilities of different choice variables. I do this by plotting the contours which contains the isolines of the volatilities of all the variables for combination of policy values in the range mentioned above. In precise, I plot the contours of the volatilities on a XY grid where X represent the range of values that the Downpayment ratio can take and Y represent the range of Taylor Parameters that the central Bank could afford. Again I would like to emphasize that I could curb the volatility of the housing economy by looking at the specific range of policy values that the central bank could afford. Overall, I try to analyze how the agents in the economy and markets (both housing and consumption goods) behave when we have different set of Monetary policies and Macro-prudential Policies (I consider Downpayment Ratio in housing as a MacroPrudential Policy). In particular I would like to analyze how volatile the markets and agents decisions are with respect to different combinations of the Downpayment ratios [10 percent to 40 percent] and Inflation coefficients in the Taylor Rule [1.5 to 3.5].

I can always look at this analysis in the following ways

- Which policy (Monetary or Macro Prudential) is better to curb the volatility of this variable around the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points.

I would like to emphasize the expected monetary shock as this shock accounts for at least 50 percent - 80 percent of the volatility I tend to observe for almost all the variables. This also results in Monetary policy being more effective than the Macro prudential policy in the analysis.

Consumption

From agents consumption perspective, all the agents tend to smooth their consumption and a change in Macro Prudential policy (Downpayment ratio) is inefficient in curbing the volatility of the consumption of all the agents around the Benchmark case. However, we tend to see that the Monetary policy of Taylor rule parameter tend to be very effective in curbing the volatility of consumption in all the agents especially in Savers. As Savers have higher discount factors, and being not constrained they tend to smooth their consumption over time with a very minimal effect of change in Macro prudential policy. On relative comparison level we also see, even though the Macro Prudential policy is Inefficient in all the agents consumption volatility, we tend to see that the volatility of Hand to mouth agents' consumption is more affected by the change in Downpayment ratio than that of the Borrowers consumption and Savers are the least affected by the change in this policy. This analysis shows that again most of the volatilities in consumption of all the agents stem from Expected monetary shock and hence Monetary policy tends to be more effective.

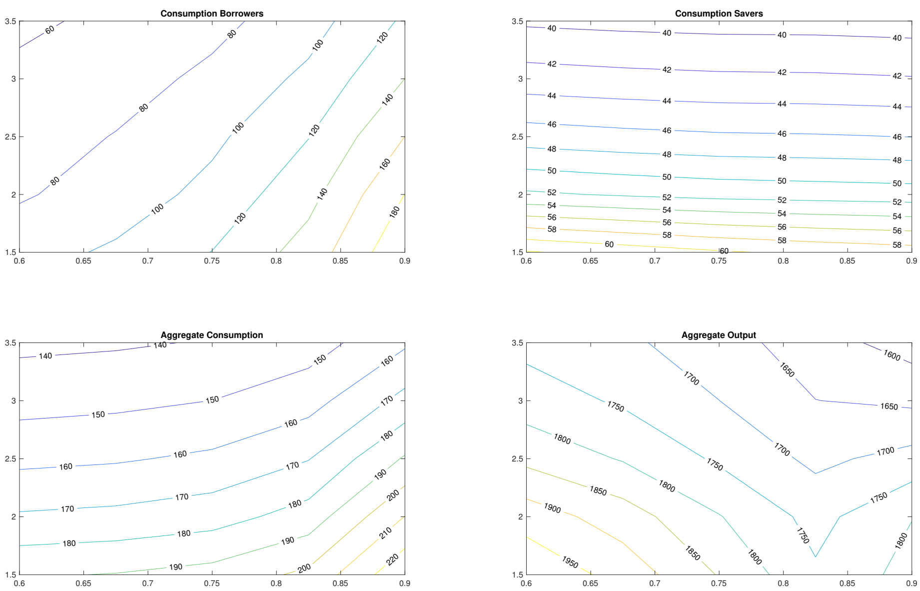


Figure 47: Volatility of Consumption choice variables

[1]

[1]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Consumption of Hand to Mouth Agents:

From households consumption perspective, I can assess which policy is effective in curbing the volatility of the consumption of Hand to Mouth agents around the Benchmark Case. To do this, I look at the Absolute change in Downpayment Ratio (Macro Prudential policy) per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, and this absolute change in Downpayment ratio turns out to be 0.77 per 0.1 points change in Taylor Rule parameter which shows that Monetary Policy is effective for curbing the volatility of the consumption of Hand to Mouth agents around the Benchmark case .

Consumption of Borrowers:

From Borrowers consumption perspective, to asses which policy is effective in curbing the volatility of the consumption of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.73 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is effective for curbing the volatility of the consumption of Borrowers around the Benchmark case .

Consumption of Savers:

From Savers consumption perspective, the volatility of these agents' consumption doesn't seem to alter with a change in Downpayment ratio. On the other hand, when I increase the Inflation coefficient in the Taylor rule (Which is equivalent to loosening the Taylor Rule/ Decreasing the interest rate), the volatility of consumption of the Savers have gone down which shows that these agents are better off in terms of consumption with loose Monetary Policy. However in Savers case, the volatility of their consumption is independent of Macro prudential policy as these agents have higher discount factor than the Borrowers, they always smooth their consumption even when they see a change in Downpayment ratio. As we see in this case, Taylor rule is the only effective policy to alter their consumption volatility.

Labour

Housing sectors labour tends to be more volatile than most of the other variables I discuss. With different expectation shocks in the model, agents tend to adjust their labour decisions more frequently than any other variables. Especially the expected productivity shocks in both the sectors and the expected monetary shock has huge impact on the volatility of labour. As I discussed before, houses acting as collaterals and housing market being relatively more flexible than the consumption goods market, we tend to see higher volatilities in the housing sector's labour of all the three agents. If we closely observe the variance decomposition, most of the volatility in housing labour from agents stem from the Expected productivity shock in housing sector. We have also seen some interesting result in volatility of the housing labour for all the three agents: We tend to see a 'V' shaped contours which shows that within the range of affordable Macro Prudential policy values , there is a specific subset of policy values (downpayment ratio of [0.8, 0.85]) where the volatility tends to be minimum and outside this sub range, the volatilities tend to increase. I assume this result is due to the fact that an expected housing productivity shock (housing productivity shock has most of the variance decomposition in housing sector's labour of all agents) result in an increase in housing stock and a decrease in house prices. This leads to two way affect in agents especially Borrowers who drive the housing economy: As houses act as collaterals, the decrease in house prices has two contradicting affects:

- Too little of downpayment ratio (around 10 percent) augmented with low house prices will make the houses easily feasible to Borrowers and as a result these agents tend to supply more of labour to increase the volatility of Borrowers housing sector labour
- Too much of downpayment ratio augmented with low house prices also has a similar affect on the Borrowers: Houses acting as collaterals with low house prices, increase in downpayment ratio will make the Borrowers to work more to invest in housing and this agin results in an increase in volatility of Borrowers labour in housing sector.

As housing markets are flexible markets, we tend to see a very similar results from Hand to Mouth agents and Savers in housing sectors' labour. Most of the volatility in the housing sector is driven by the productivity shock in that sector and Borrowers being the main drivers of housing sector economy. As consumption goods sector is rigid, we don't see such results in the labour of Savers and Hand to Mouth agents. On Aggregate labour for all agents, we tend to see the Macro prudential Policy is not as effective as the Monetary policy. However, relatively Borrowers are the guys who have relatively high effect with change in Downpayment ratio as Borrowers are the driving force of housing economy and houses acting as collaterals. Then Hand to mouth agents' aggregate labour is slightly affected by the downpayment ratio and Savers are the the agents with high discount factors and are least affected by change in Macro Prudential policy.

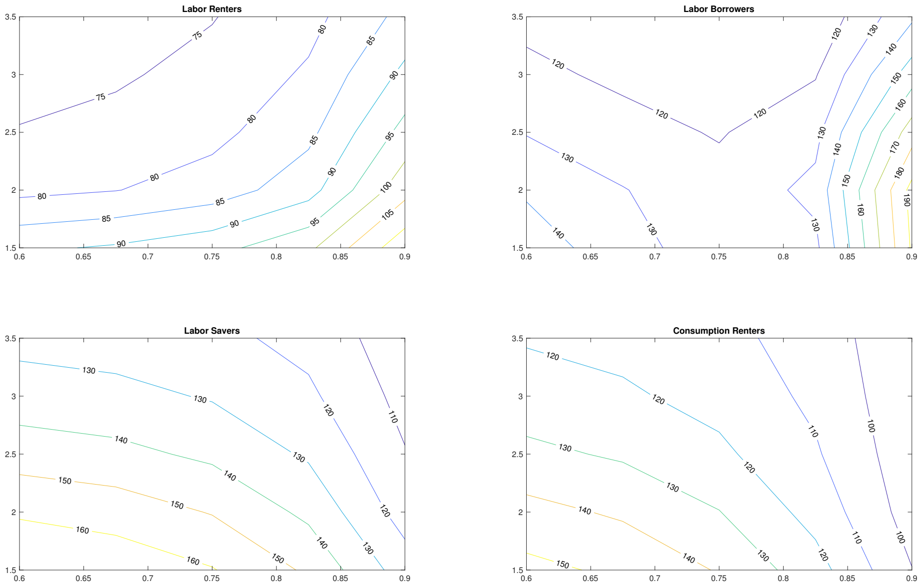


Figure 48: Volatility of labour choice variables

[2]

Labour of Hand to Mouth Agents:

From households labour perspective, we can assess which policy is effective in curbing the volatility of the consumption of Hand to Mouth agents around the Benchmark Case. To do this, I look at the Absolute change in Downpayment Ratio (Macro Prudential policy)which turns out to be 0.67 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is effective for curbing the volatility of the consumption of Hand to Mouth agents around the Benchmark case.

Labour of Borrowers:

From Borrowers labour perspective, I tend to see Macro prudential policy has a very minimal effect on the volatility of Borrowers labour. Even the effect of macro prudential policy is minimal, this effect is higher than that on the Hand to mouth agents labour. To asses which policy is effective in curbing the volatility of the consumption of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.51 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy

[2]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

is effective for curbing the volatility of the labour of Borrowers around the Benchmark case. Aggregate Borrowers labour also has a similar trend of 'V' shaped contours like that in housing sectors' labour. This is due to the fact that Borrowers are the most affected agents of all the three agents with a change in Downpayment ratio as houses act as collateral for these agents. We also see that the Macro prudential policy would be very effective at Lower levels of downpayment ratio around 10 percent-15 percent.

Labour of Savers:

From Savers labour perspective, we also tend to see Macro prudential policy has a minimal effect (in fact the least of all the three agents) on volatility of Savers labour. As we see in this case, Savers having high discount factors, Taylor rule is the an even more effective policy than Macro Prudential policy to alter their consumption volatility than that of other agents.

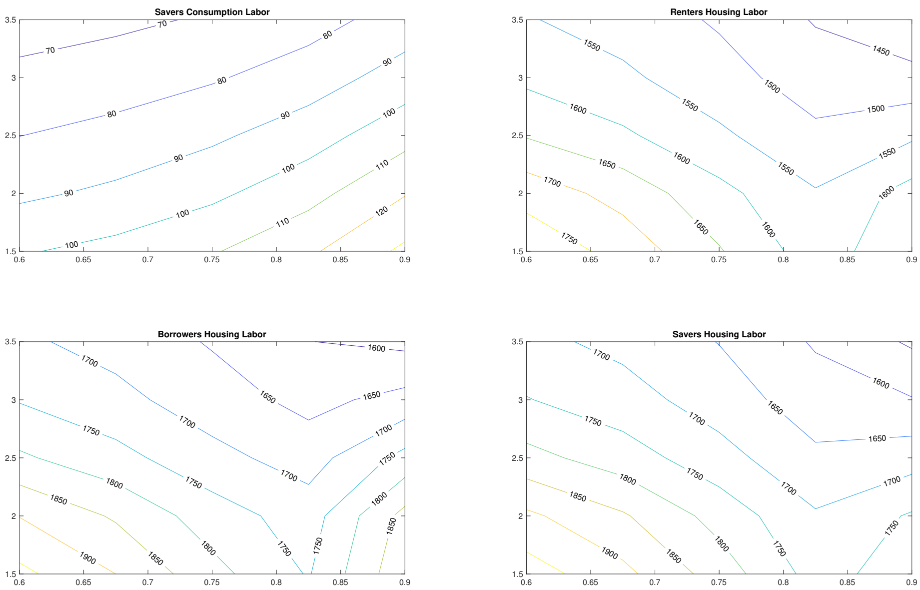


Figure 49: Volatility of labour choice variables in each sector

3

[3]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Housing

As we have seen above, the households labour volatilities are high and especially in housing sector, we tend to see a 'V' shaped contours which also results in a similar 'V' shaped contours for housing output. However, that is not the case for housing of agents. We don't see a 'V' shaped patterns in housing stocks and the volatilities are not as high as that of labour. This shows that agents are adjusting their labour more rapidly than that of consumption and housing where they want to smooth their consumption.

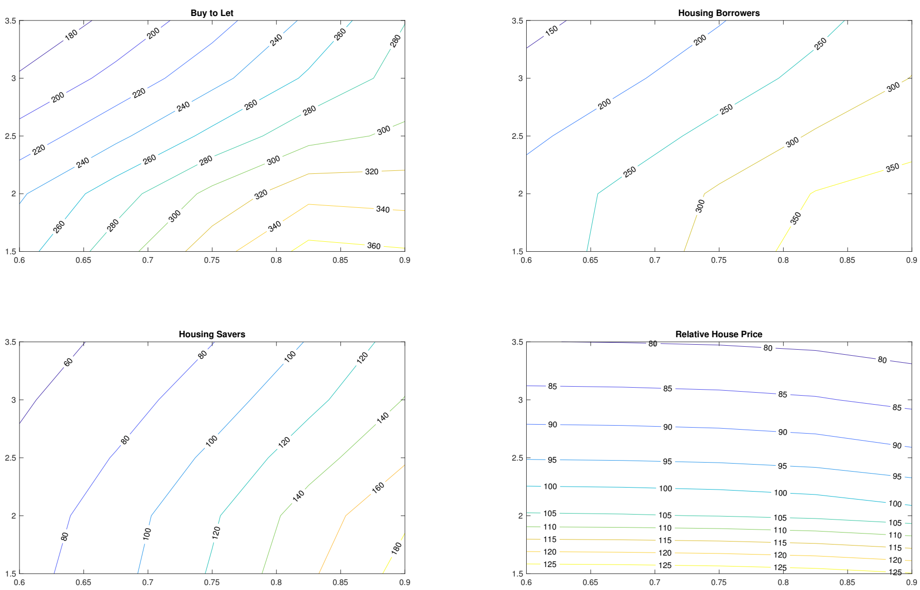


Figure 50: Volatility of Housing choice variables

4

Buy to Let Housing:

From Borrowers Buy to Let houses perspective, we can assess which policy is effective in curbing the volatility of the Buy to Let housing stock around the Benchmark Case. To do this, I look at the Absolute change in Downpayment Ratio (Macro Prudential policy)which turns out to be 2.23 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is highly effective for curbing the volatility of the housing of Hand to Mouth agents around the Benchmark case.

[4]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Borrowers Housing:

From Borrowers Aggregate housing perspective, we tend to see Macro prudential policy has a very minimal effect on the volatility of Borrowers housing and this effect is also lower than that on the Hand to mouth agents labour. This shows that the Macro prudential policy has some effect on the housing of Borrowers where they live in $H_b - Hr$. To asses which policy is effective in curbing the volatility of the Total housing of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 1.79 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is still highly effective (but not as highly effective as in Buy to Let housing) for curbing the volatility of the housing of Borrowers around the Benchmark case.

Savers Housing:

From Savers perspective, we also tend to see Macro prudential policy has a low effect (However, highest of all the three agents) on volatility of Savers labour. To asses which policy is effective in curbing the volatility of the Total housing of Savers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.92 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is still highly effective (but not as highly effective as in Buy to Let housing and for Borrowers) for curbing the volatility of the consumption of Borrowers around the Benchmark case. As we have looked at some of the choice variables of agents, now we look at volatilities of some of the other important variables such as Inflation, Relative house prices and Rental prices.

Prices and Inflation

As we see agents adjust their labour more rapidly than that of consumption and housing services, we tend to see a very stable volatilities in the economy with respect to the macro prudential policy (downpayment ratio). As we have an expected monetary policy shock in the model. We expect to see Monetary policy is very effective compared to that of Macro prudential policy in curbing the volatilities of prices and Inflation of both housing and consumption sector and of course the volatility of interest rate. In fact for Interest rate, relative house prices and Inflation in both housing and consumption sector, Macro prudential policy has no effect or very minimal effect in curbing the volatilities. However, for rental prices volatility we tend to see a very low effect of macro prudential policy, even though, Monetary policy is still effective in curbing the volatility of Rental prices around the bench mark case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 2.08 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case. Almost for all the variables including wages of all the agents, we have seen Monetary policy to be more affective than Macro prudential policy. This is to be expected as I have analyzed the system with Expected shocks and with an expected monetary shock.

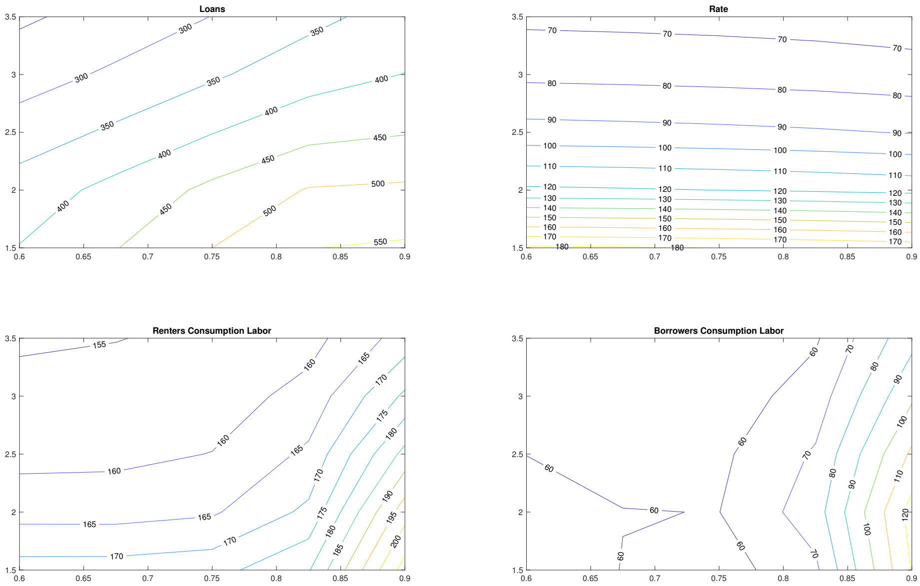


Figure 51: Volatility of loans

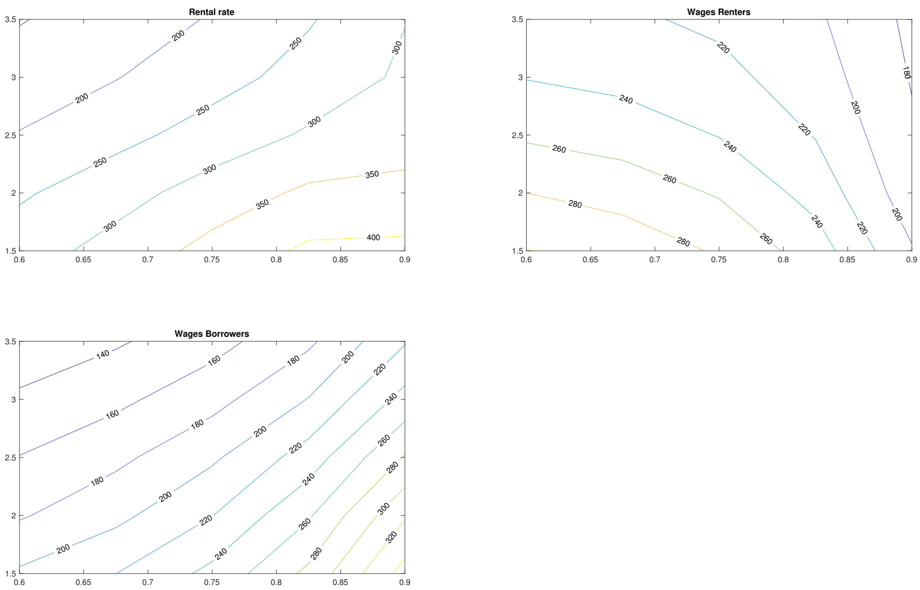


Figure 52: Volatility of rent and wages

[5]

[5]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

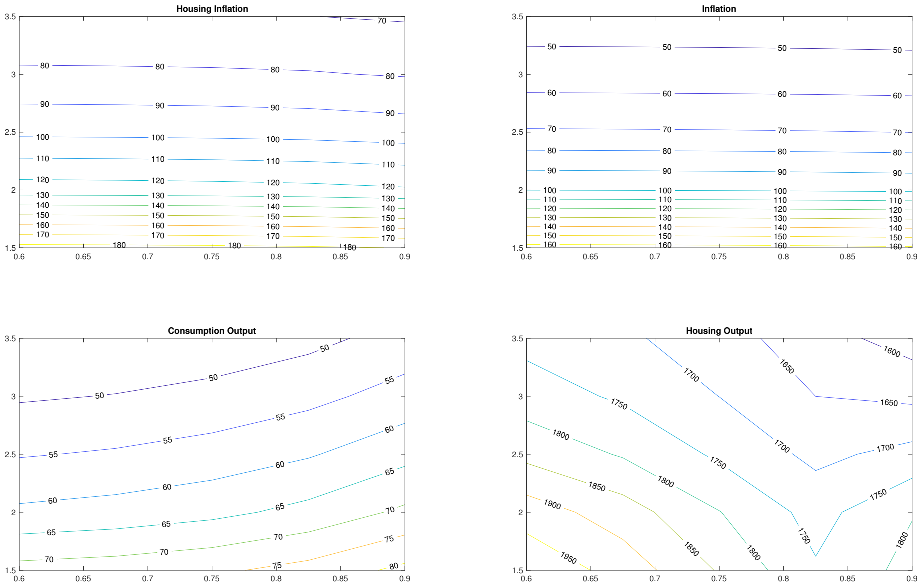


Figure 53: Volatility of Inflation

6

4.4.13 Policy Analysis with Productivity Shocks

As we have seen in the previous section, Monetary policy is effective with almost all the variables and most of the percentage of volatilities in all the variables stem from Expected Monetary shock. To rule out the effects of expected shocks now we look at the policy analysis with only Technology shocks and cost push shocks. In this analysis, I have also accounted for the steady state changes with a change in Monetary policy by assessing the effect of policy on the aggregate output. This analysis can give us some good insights which we will look now.

With no expected shocks including the Rental returns shock, we tend to see most of the volatility of variables stem from both the productivity shocks. In this analysis, I try to look at

- Which shock contributes most to the volatility of the choice variable around the benchmark case?
- Which policy (Monetary or Macro Prudential) is better to curb the volatility of this variable around the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points?

[6]When the system is subjected to expectation shocks on Rental returns, Monetary policy and technology shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Borrowers

If we look at the overall picture of consumption, Borrowers consumption is more volatile than that of Savers and Hand to Mouth agents. Most of the volatility in Borrowers consumption stems from the housing Productivity tech shock, specifically around 70 percent of the Borrowers consumption volatility decomposition stems from a positive productivity shock in housing sector. This is a result of Borrowers being the main drivers of the housing economy. However, we do not see a similar trend in Aggregate housing stock volatility of Borrowers H_b. Most of the volatility around 64 percent of volatility in housing stock of Borrowers stem from a positive technology shock in consumption sector. This shows that the Borrowers are frequently adjusting their housing choices with greater magnitude than consumption choices when a positive consumption technology shock happens and frequently adjusting their consumption choices with greater magnitude than housing choices with a positive housing technology shock. This makes sense as for a case with positive housing productivity shock, as explained in the Technology shocks section, as Borrowers observe a positive technology shock to the housing market they expect the output to go up and the prices of the houses to go down which decrease their collateral value. This decrease in collateral value puts off the Borrowers to invest in their own housing H_b decreases (As housing as collateral plays a role in here) however they tend to increase investment in BTL H_r increases (As BTL is considered more as an Asset with Rents as Returns), these two opposing factors in housing when the prices are low makes Borrowers to observe less volatility in housing holdings, when a positive housing productivity shock occurs.

As for a positive consumption technology shock, the effect of price rigidity in consumption sector has a strong affect here and firms cut their labour inputs demand and this results in moving of the Borrowers labour from consumption good sector to housing goods sector (which have relatively flexible prices and houses acting as assets) which results in increase of Borrowers labour in housing sector and increasing the output in housing goods sector Y_h (We also observe most of the volatility in housing output around 80 percent stems from a positive technology shock in consumption sector). Borrowers additional income and is invested into the housing stock H_b resulting in an increase in the housing stock of Borrowers which in turn also results in the increase in the BTL stock. This increase in stock of housing from Borrowers tend to increase the volatility of housing from Borrowers. From labour choices perspective, Borrowers labour volatility tends to stem from both the shocks in a relatively equal proportions. This is also a result of houses acting as collaterals and assets with returns, Borrowers tend to adjust their labour choices more rapidly no matter which shock they are experiencing. However, It is also important to mention that this volatility of Borrowers labour is relatively higher than that of both Savers and Hand to Mouth agents around the benchmark case. As expected, housing firms being more flexible and Borrowers being indifferent between the shocks, volatility of wages for Borrowers tend to be higher in Positive housing technology shock (around 62 percent of the total volatility) rather than in the positive consumption technology shock. Let's look at the second question, which policy is effective in curbing the volatility of the Borrowers choices given the system is around the Benchmark case:

Consumption of Borrowers:

From Borrowers consumption perspective, to asses which policy is effective in curbing the volatility of the consumption of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.016 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential Policy is effective for curbing the volatility of the consumption of Borrowers around the Benchmark case. This result is completely different to the one we found in the above section with expected Monetary shock.

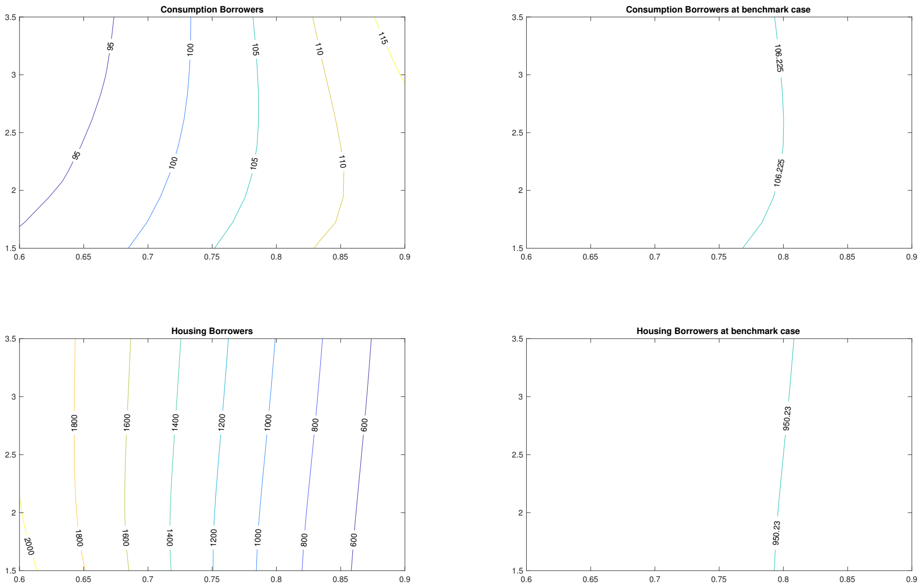


Figure 54: Volatility of Borrowers Consumption and Housing choices

[7]

Borrowers Housing:

From Borrowers Aggregate housing perspective, to asses which policy is effective in curbing the volatility of the Total housing of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.091 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that again Macro Prudential Policy is still highly effective for curbing the volatility of the housing of Borrowers around the Benchmark case.

[7]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Labour of Borrowers:

From Borrowers labour perspective, to asses which policy is effective in curbing the volatility of the labour of Borrowers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 1.0714 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is effective for curbing the volatility of the labour of Borrowers around the Benchmark case. Aggregate Borrowers labour also has a similar trend of 'V' shaped contours like that in housing sectors' labour. This is due to the fact that Borrowers are the most affected agents of all the three agents with a change in Downpayment ratio as houses act as collateral for these agents.

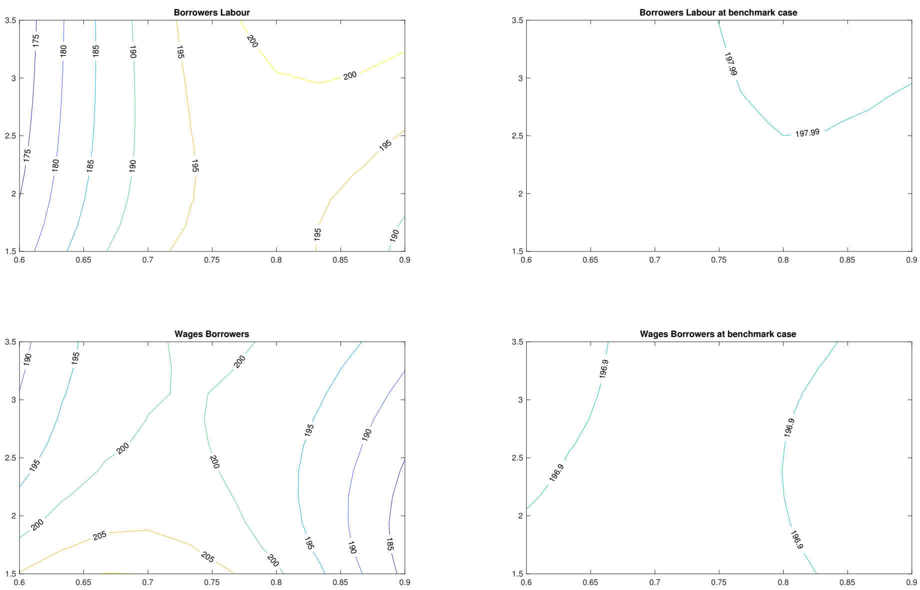


Figure 55: Volatility of Borrowers Labour and Wages

8

Wages of Borrowers:

From wages perspective, we observe an absolute change in Downpayment ratio to be 0.086 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential policy which regulates Downpayment ratio is effective for curbing the volatility of the labour of Borrowers around the Benchmark case.

[8]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Volatility	Effective Policy to curb the volatility
Consumption of Borrowers	Macro Prudential Policy
Borrowers Labour	Monetary policy
Aggregate Housing of Borrowers	Macro Prudential Policy
Wages of Borrowers	Macro Prudential Policy

Table 4: Effective Policy for Borrowers

Hand to Mouth Agents

Hand to mouth agents consumption is more volatile than that of Savers and less volatile than that of Borrowers. Most of the volatility in Hand to Mouth agents' consumption stems from the consumption goods Productivity tech shock, specifically around 64 percent of the Hand to Mouth agent's consumption volatility decomposition stems from a positive productivity shock in consumption goods sector. We also see a similar trend in Buy to Let housing stock volatility H_r. Most of the volatility around 65 percent of volatility in Buy to Let housing stock stem from a positive technology shock in consumption sector. This shows that most of the volatility in the Hand to Mouth agent choice variables stems from a positive technology shock in consumption goods sector. For a case with positive housing productivity shock, as explained in the Technology shocks section, there is an incentive for Borrowers and Savers to supply more of labour and to have more of disposable income so that they can invest in houses and loans respectively. Hence these two agents provide even more labour to the consumption firms sector which results in an increase in labour of Borrowers and Savers. However, Hand to mouth agents doesn't have any incentives, they tend to reduce their labour in the consumption sector which result in a relatively low volatility of Hand to mouth agents labour, which in turn result in a relatively low volatilities of Hand to mouth agents' consumption and wages with a positive housing technology shock. Let's look at the analysis of which policy is effective in curbing the volatility of the Hand to Mouth agents choices given the system is around the Benchmark case:

Consumption of Hand to Mouth agents:

From Hand to mouth agents' consumption perspective, to asses which policy is effective in curbing the volatility of the consumption of hand to mouth agents around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.085 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential Policy is effective for curbing the volatility of the consumption of Hand to Mouth agents around the Benchmark case. This result is completely different to the one we found in the above section with expected Monetary shock.

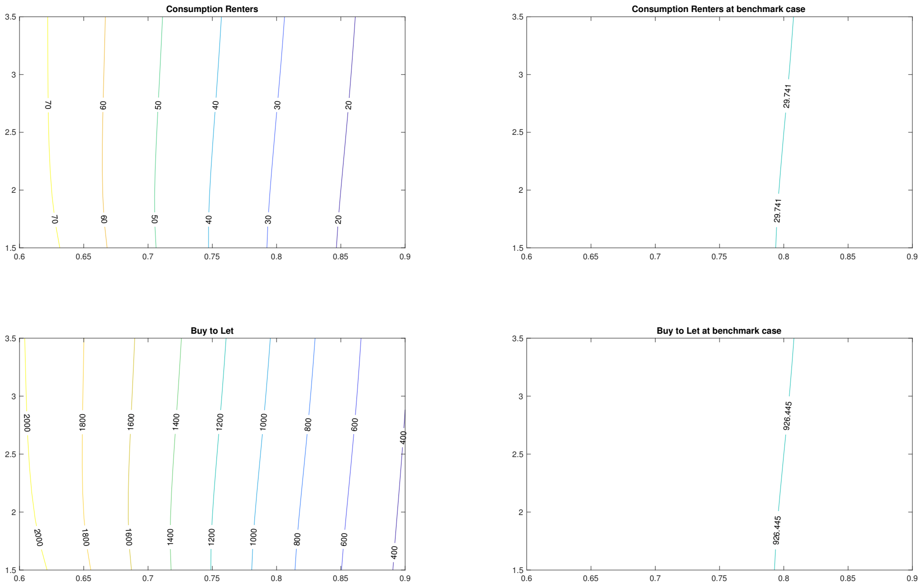


Figure 56: Volatility of Hand to Mouth agents Consumption and Housing choices

[9]

Buy to Let Housing:

From Buy to let housing perspective, to asses which policy is effective in curbing the volatility of the Buy to let housing around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.09 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that again Macro Prudential Policy is slightly more effective for curbing the volatility of the housing of Hand to mouth agents around the Benchmark case.

Labour of Hand to mouth agents:

From Hand to mouth agents' labour perspective, to asses which policy is effective in curbing the volatility of the labour of Hand to mouth agents' around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.09 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which still shows that the Macro prudential Policy is effective for curbing the volatility of the labour of Hand to mouth agents around the Benchmark case. This result is different to that of the volatility of Borrower's labour. Overall, Macro

[9] When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

prudential policy is more effective than the Monetary policy in curbing the volatility of these hand to mouth agents. As these agents are completely credit constrained, Macro prudential policy being more effective makes sense.

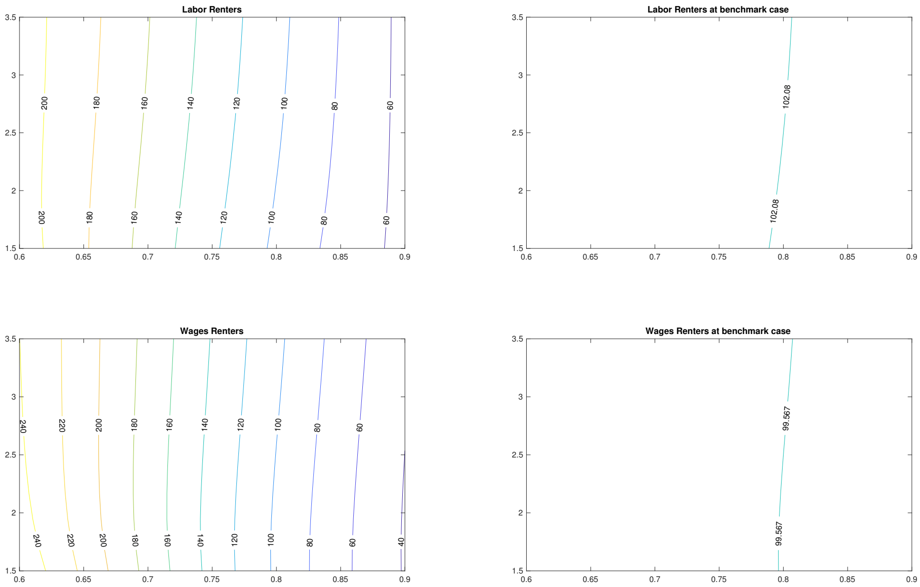


Figure 57: Volatility of Hand to Mouth agents Labour and Wages

[10]

Wages of Hand to mouth agents:

From wages perspective, we observe an absolute change in Downpayment ratio to be 0.074 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential policy which regulates Downpayment ratio is effective for curbing the volatility of the labour of Borrowers around the Benchmark case.

[10] When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Volatility	Effective Policy to curb the volatility
Consumption of Hand to Mouth Agents	Macro Prudential Policy
Hand to Mouth Agents Labour	Macro Prudential Policy
Buy to Let Housing stock	Macro Prudential Policy
Wages of Hand to Mouth Agents	Macro Prudential Policy

Table 5: Effective Policy for hand to Mouth Agents

Savers

As Savers have high discount factors and are not collaterally constrained, these agents consumption is the least volatile among all the three agents. Their consumption volatility tends to stem from both the shocks in a relatively equal proportions. This is also a result of Savers having high discount factors and tend to smooth their consumption no matter which shock they are experiencing. However, we do not see a similar trend in Aggregate housing stock volatility of Savers H_s. Most of the volatility around 63 percent of volatility in housing stock of Savers stem from a positive technology shock in consumption sector. This result is very similar to that of Borrowers housing volatility. This shows that the Savers are frequently adjusting their housing choices with greater magnitude when a positive consumption technology shock happens. As for a positive consumption technology shock, the effect of price rigidity in consumption sector has a strong affect here and firms cut their labour inputs demand and this results in moving of the Savers labour from consumption good sector to housing goods sector (which have relatively flexible prices and houses acting as assets) which results in increase of Savers labour in housing sector and increasing the output in housing goods sector Y_h (We also observe most of the volatility in housing output around 80 percent stems from a positive technology shock in consumption sector). From labour choices perspective, housing firms being more flexible than the consumption goods sector most of the Savers labour volatility tends to stem from positive housing technology shock.

Let's look at the analysis of which policy is effective in curbing the volatility of the Savers choices given the system is around the Benchmark case:

Consumption of Savers:

From Savers consumption perspective, to asses which policy is effective in curbing the volatility of the consumption of Savers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.38 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is effective for curbing the volatility of the consumption of Savers around the Benchmark case. This result is the same as the one we found in the above section with expected Monetary shock.

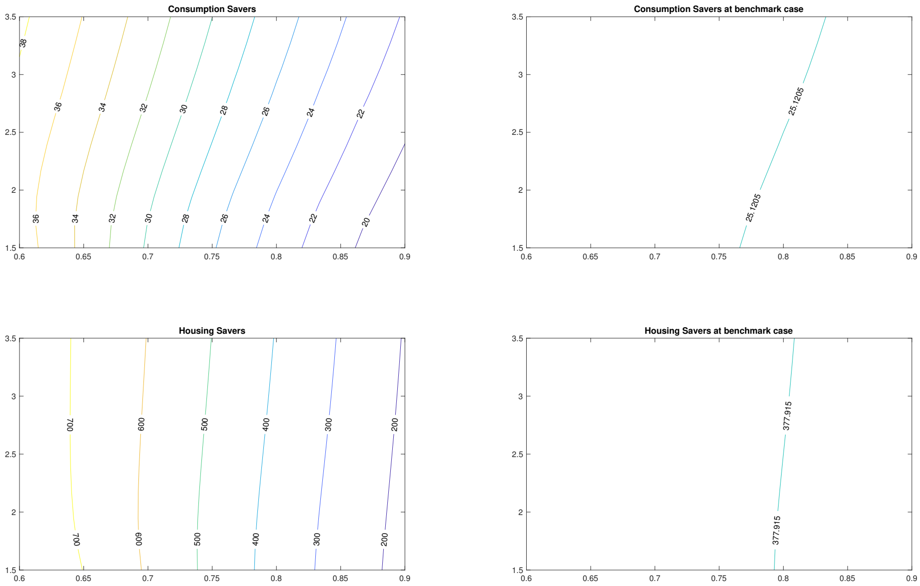


Figure 58: Volatility of Savers Consumption and Housing choices

[11]

Savers Housing:

From Savers housing perspective, to asses which policy is effective in curbing the volatility of the Total housing of Savers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.094 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that again Macro Prudential Policy is still effective for curbing the volatility of the housing of Savers around the Benchmark case.

Labour of Savers:

From Savers labour perspective, to asses which policy is effective in curbing the volatility of the labour of Savers around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.94 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary Policy is effective for curbing the volatility of the labour of Savers around the Benchmark case. In terms of curbing the volatilities of the Savers choices, Monetary policy seems to be more effective than the macro prudential policy. As Borrowers

[11] When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

and Savers are the agents who provide and acquire loans, compared to hand to mouth agents, Borrowers and Savers are more affected by the Monetary policy.

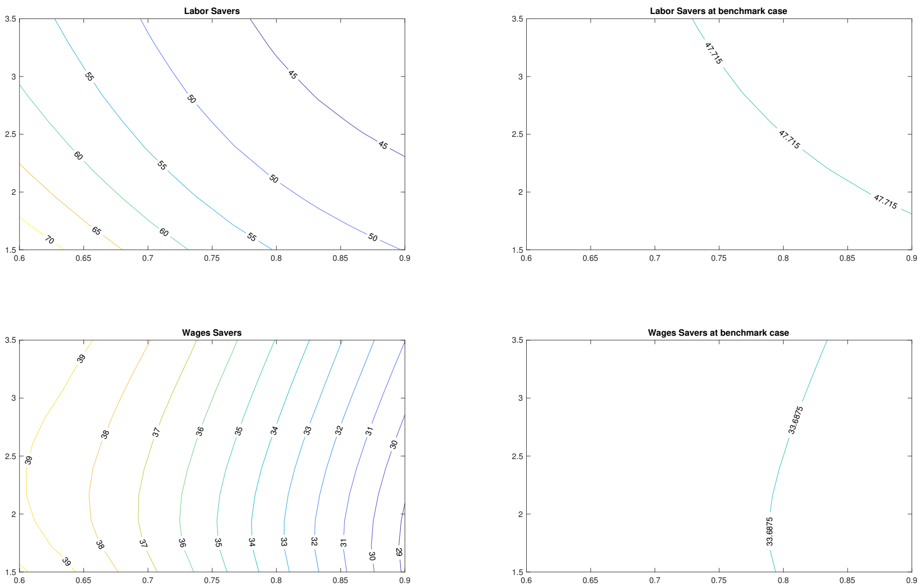


Figure 59: Volatility of Savers Labour and Wages

[12]

Wages of Savers:

From wages perspective, we observe an absolute change in Downpayment ratio to be 0.28 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that agin Monetary policy is effective for curbing the volatility of the labour of Savers around the Benchmark case.

Volatility	Effective Policy to curb the volatility
Consumption of Savers	Monetary policy
Savers Labour	Monetary policy
Housing stock of Savers	Macro Prudential Policy
Wages of Savers	Monetary policy

Table 6: Effective Policy for Savers

[12]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

4.4.14 Prices and Inflation

Unlike the ineffectiveness of macro prudential policy in the previous model with augmented expected shocks of Monetary policy and Rental returns, a Buy to Let market with technology shocks tend to have Macro prudential policy which regulates Downpayment ratio as a better policy and effective in curbing the volatility of most of the choice variables of the agents. Now we will look at the other important variables of prices and Inflation below. With positive consumption technology shock, we have seen that volatility of Aggregate Borrowers housing stock and Buy to Let housing stock tends to be higher. This high volatilities in housing stock also leads to the the rental rate having most fraction of the volatility from the consumption technology shock (Around 65 percent). The rents seems to be very highly volatile with higher downpayment ratios. To asses which policy is effective in curbing the volatility of the rents around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.09 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential Policy is effective for curbing the volatility of the housing of Savers around the Benchmark case. Looking at the Inflation's volatility. we tend to see the inflation volatility stems from both the shocks in a equal proportions. However, the housing inflation tends to stem most of it's volatility from housing productivity shock. Let's look at which policy is better to curb the Inflation around the benchmark case: as expected we tend to observe that the Monetary policy will be more effective in altering the volatility of inflation around the steady state. As for housing inflation, Macro prudential policy doesn't seems to have any affect on changing the volatility around the Benchmark model.

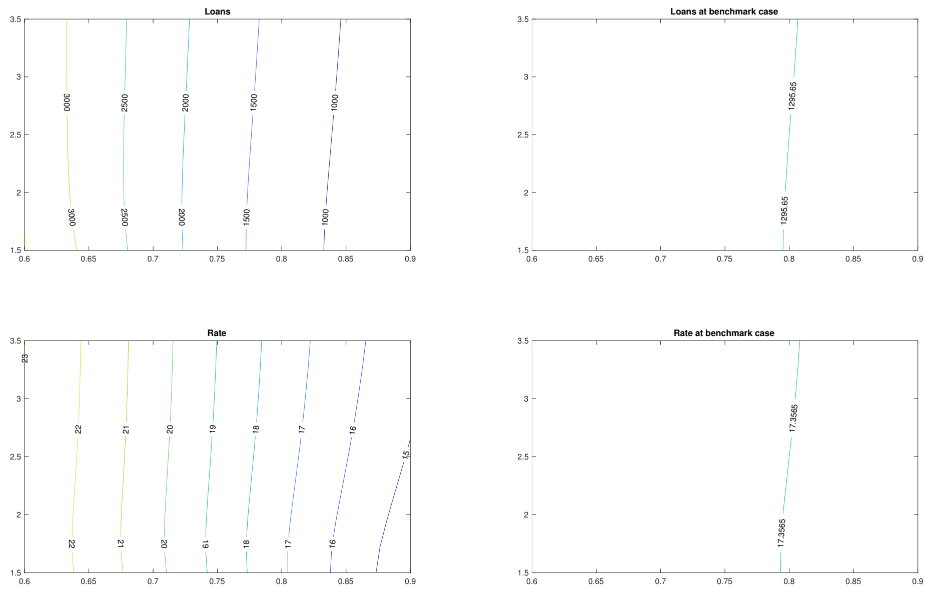


Figure 60: Volatility of Loans and Interest Rate

[13]

[13]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors. Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

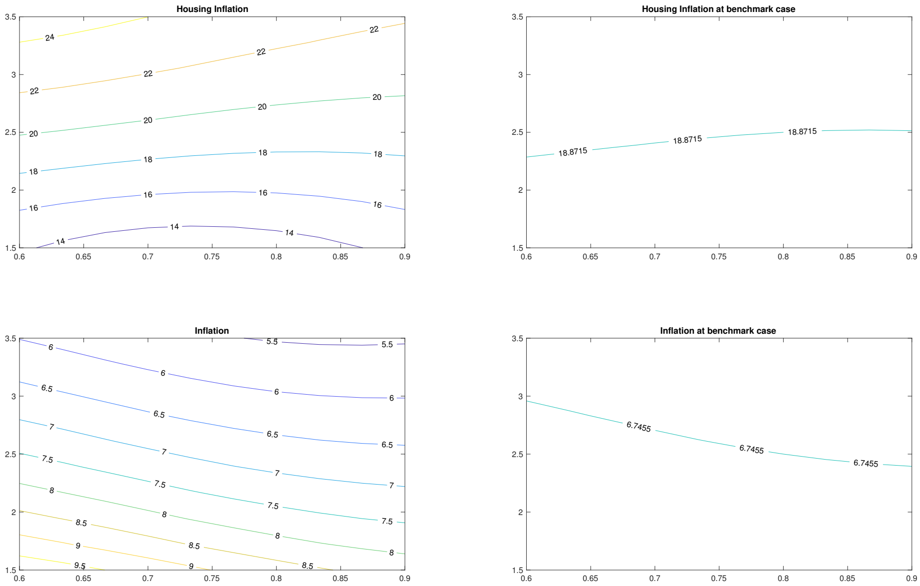


Figure 61: Volatility of Inflation

14

Volatility	Effective Policy to curb the volatility
Loans	Macro Prudential Policy
Interest Rate	Monetary policy
Inflation	Monetary policy
Housing Inflation	Monetary policy

Table 7: Effective Policy to curb Loans and Inflation

The interesting result in this case is Monetary policy is very effective in curbing the volatility of Relative house price volatility even with the case of augmented monetary policy. Even though we see Macro prudential policy to be very effective in some of the choice variables of Hand to mouth agents and Borrowers, Monetary policy is very effective in terms of curbing the house price volatility. To asses which policy is effective in curbing the volatility of the house prices around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 14.59 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Monetary policy is the highly effective and Macro Prudential Policy is highly ineffective for curbing the volatility of the housing of Savers around the Benchmark case.

[14]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Another interesting result is as for the interest rate we see almost all of it's volatility (around 93 percent) stems from a housing productivity shock. That is when we tend to see a surge in housing production. The interest rates tend to be highly volatile. and in terms to curbing the volatility of interest rate, Monetary policy tends to be slightly more effective than that of the Macro prudential policy. From loans perspective, to asses which policy is effective in curbing the volatility of the loans around the Benchmark Case, the Absolute change in Downpayment Ratio (Macro Prudential policy) turns out to be 0.074 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that Macro Prudential Policy is effective for curbing the volatility of the loans around the Benchmark case.

However, in terms of volatilities of both consumption and housing output, Monetary policy tends to be more effective than that of the macro prudential policy around the benchmark case.

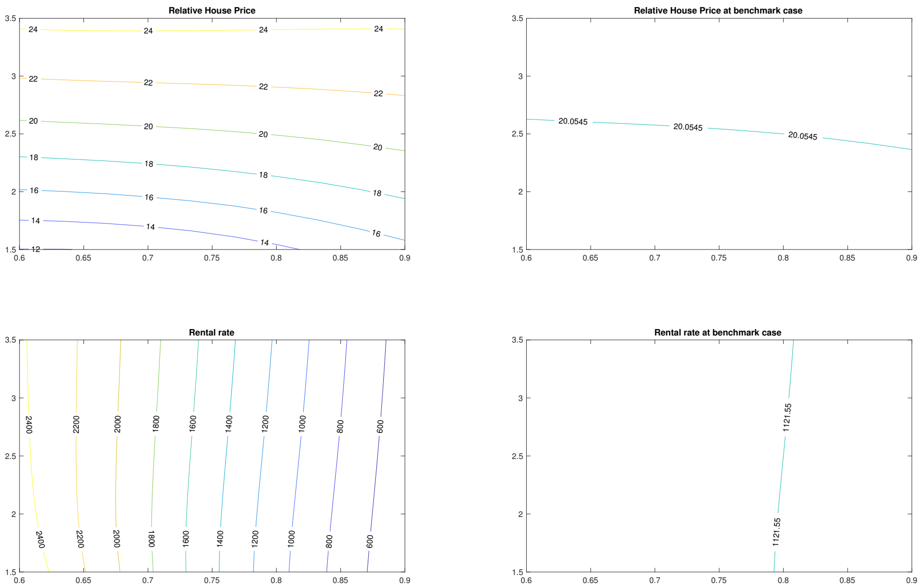


Figure 62: Volatility of Rents and House prices

[15]

[15]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

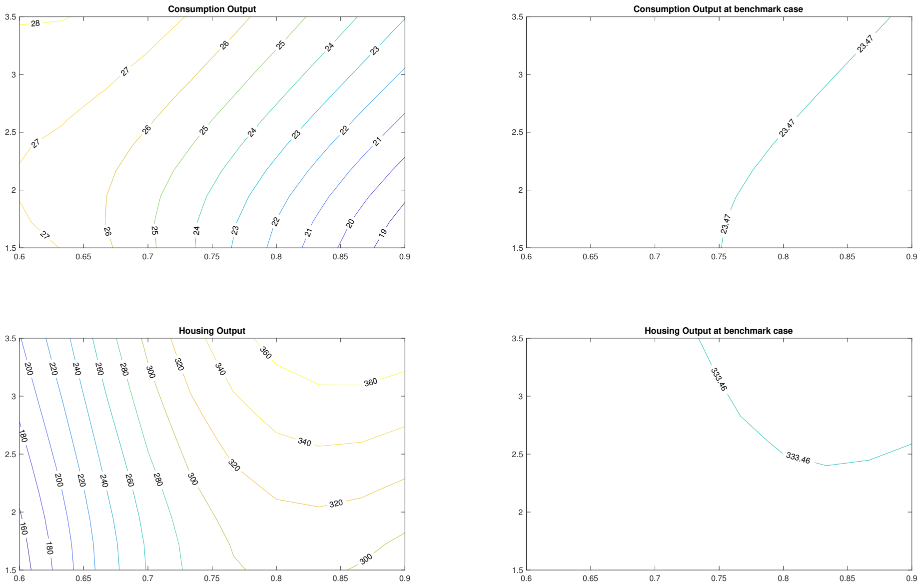


Figure 63: Volatility of Outputs

[16]

Volatility	Effective Policy to curb the volatility
Rents	Macro Prudential Policy
Relative House price	Monetary policy
Housing output	Monetary policy
Consumption output	Monetary policy

Table 8: Effective Policy to curb Prices and Output

4.4.15 Welfare Analysis

We have seen some of the important variables and their volatiles in the above sections. now let's look at the volatility of welfare of all the agents with only productivity shocks. To calculate the welfare, I have included the Bellman equation of welfare for all the agents in the economy; see Bellman (1952). In particular, the welfare is measured simply as present discounted value of the flow of utility for all the three agents of hand to mouth, Borrowers and Savers respectively.

$$W_t^p = E_t \left\{ \sum_{i=0}^{\infty} \beta^i U\left(C_t^p, (H_t^r), N_t^p\right) \right\}$$

[16]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

$$W_t^c = E_t \left\{ \sum_{i=0}^{\infty} \beta^i U\left(C_t^c, \left(H_t^b - H_t^r\right), N_t^b\right) \right\}$$

$$W_t^s = E_t \left\{ \sum_{i=0}^{\infty} \theta^i U\left(C_t^s, (H_t^s), N_t^s\right) \right\}$$

The recursive equation on welfare is then used to calculate the unconditional welfare of the each respective representative household. As in the first order approximation of our model, the steady state values and the expected values of the variables are similar. It is imperative for us to use the second order approximation of our model to get the welfare analysis. A higher order approximation can also come in handy when wanting to evaluate the effects of policy rule coefficients (e.g. a Taylor rule for monetary policy) which have no effect on the steady state. The set of recursive Bellman equations can be written as follows (see also Sims et al (2014)):

$$W_t^p = U\left(C_t^p, (H_t^r), N_t^p\right) + \beta E_t \left\{W_{t+1}^p\right\} \tag{145}$$

$$W_t^b = U\left(C_t^b, \left(H_t^b - H_t^r\right), N_t^b\right) + \beta E_t \left\{W_{t+1}^b\right\} \tag{146}$$

$$W_t^s = U\left(C_t^s, (H_t^s), N_t^s\right) + \theta E_t \left\{W_{t+1}^s\right\} \tag{147}$$

For a benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points, the volatility of welfare calculated using the above Bellman equation are as 5.6, 46.33 and 16.4 respectively for Hand to mouth agents, Borrowers and Savers. This shows that Borrowers who are the main stake of Buy to Let markets welfare has been subjected to the most volatility under the productivity shocks.

Borrowers

Let's start with Borrowers as these are the main focus point of the economy. From Borrowers perspective, we see the volatility of their welfare at the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points is about 46.33. Their welfare is highly volatile compared to that of other two agents. Another important result is with a constant Taylor rule parameter of 2.5 (Ceteris paribus) Borrowers volatility is high and tends to increase within a specific range of downpayment ratio of [0.79 0.82] and the volatility decreases outside this range of downpayment ratio. Specifically we tend to see a peak welfare volatility within a downpayment ratio range of [0.79 0.82]. We can attribute such kind of results to housing acting as collaterals and as assets. We can say that Borrowers tend to invest a lot in housing with a decrease in consumption within this range of Downpayment ratio as the few important factors of housing counteract with each other perfectly within this range.

- Flexible prices in housing sector and rigid prices in consumption sector: Flexible prices in the market allows agents to consume more of housing services than the rigid markets and this results in an incentive for households to supply more labour in housing market. Which again we expect to see most of the Borrowers welfare volatility stemming from positive housing technology shock which turns out to be around 65 percent.

- Collateral Constraint: Houses act as collaterals for Borrowers in the economy and they indirectly affect Savers who provide loans to the Borrowers. With a positive housing technology shock and a very high downpayment ratio, Borrowers do not want to invest in housing and hence their volatility in welfare decreases, as agents tend to smooth their consumption of perishable goods, housing services and labour. On the other hand,

- Houses also act as assets: Borrowers perceive houses as assets where they get returns from BTL market. A positive housing shock and a low downpayment ratio again leads to a low volatility of Borrowers welfare as they can have more returns from the Returns in assets

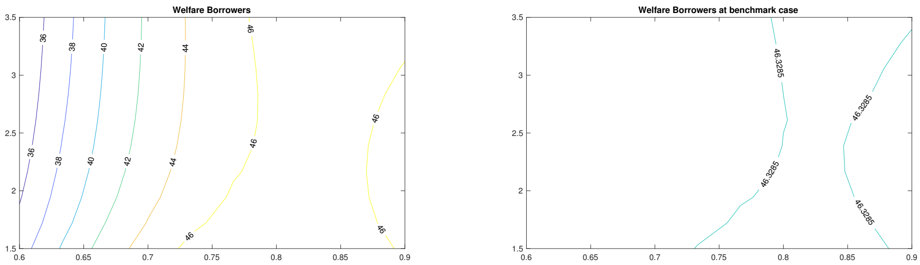


Figure 64: Volatility of Borrowers Welfare

[17]

In terms of curbing the volatilities of the Borrowers Welfare, Monetary policy seems to be more effective than the macro prudential policy. We observe an absolute change in Downpayment ratio to be 0.29 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that agin Monetary policy is effective for curbing the volatility of the Borrowers welfare around the Benchmark case.

[17]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Savers

From Savers perspective, we see the volatility of their welfare at the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points is about 16.4. Their welfare is highly volatile compared to that of Hand to mouth agents and less volatile than that of Borrowers. We tend to see a very similar results to that of Borrowers welfare, with a constant Taylor rule parameter of 2.5 (Ceteris paribus) Borrowers volatility is high and tends to increase within a specific range of downpayment ratio and the volatility decreases outside this range of downpayment ratio. Specifically we tend to see a peak welfare volatility within a downpayment ratio range. In terms of curbing the volatilities of the Borrowers Welfare, Monetary policy seems to be more effective than the macro prudential policy. We observe an absolute change in Downpayment ratio to be 1.35 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that agin Monetary policy is very highly effective for curbing the volatility of the Savers welfare around the Benchmark case.

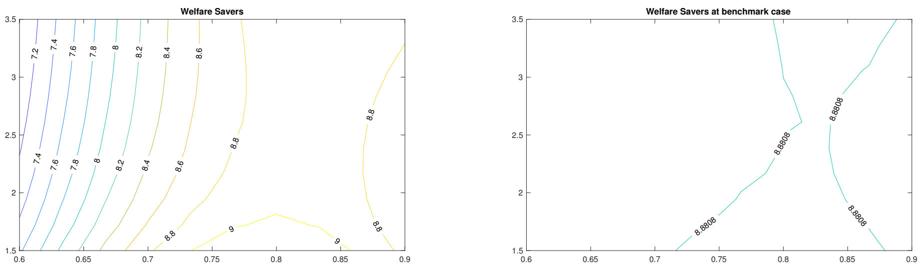


Figure 65: Volatility of Savers Welfare

18

[18]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

Hand to Mouth Agents

From hand to mouth agents perspective, we see the volatility of their welfare at the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points is about 5.6. Their welfare is the least volatile of all the agents. We do not tend to see a similar results to that of Borrowers and Savers welfare, with a constant Taylor rule parameter of 2.5 (Ceteris paribus) Borrowers volatility is high and tends to increase with a n increase in downpayment ratio. From hand to mouth agents perspective, having a low downpayment ratio is good for curbing their welfare volatility. In terms of curbing the volatilities of the Hand to mouth agents Welfare, Monetary policy seems to be slightly more effective than the macro prudential policy. In fact, both the policies are equally efficient in affecting the volatility of hand to mouth agents which shows that Hand to mouth agents are indifferent. We observe an absolute change in Downpayment ratio to be 0.13 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case, which shows that agin Monetary policy is slightly highly effective for curbing the volatility of the hand to mouth agents welfare around the Benchmark case.

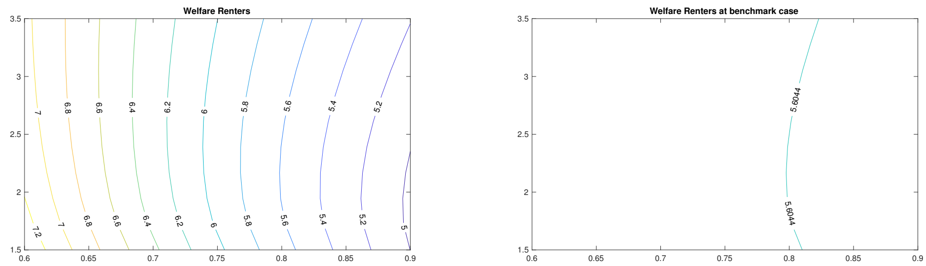


Figure 66: Volatility of Hand to Mouth agents Welfare

[19]

[19]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.

Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

4.4.16 Aggregate Welfare

The volatility of agents aggregate welfare at the benchmark case of downpayment ratio being 20 percent of the house value and the Taylor parameter on Inflation being 2.5 points is about 16.4, most of it stemming from Borrowers. In terms of curbing the volatilities of the Aggregate Welfare, Macro prudential policy seems to be highly effective than the monetary policy. We observe an absolute change in Downpayment ratio to be 0.06 per 0.1 point change in Taylor Rule parameter (Monetary Policy) to remain in the same contour of volatility at the benchmark case. One interesting result is most of the volatilities for all the agents welfare stems from a positive housing productivity technology shock.

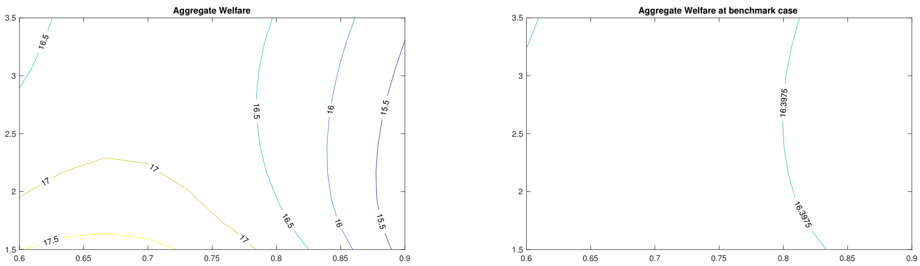


Figure 67: Volatility of Aggregate Welfare

[20]

4.5 Conclusion

This chapter is devoted to analyze how the markets behave in perspective of a CES Utility between consumption and housing choices. It aims to understand how the Buy to let markets and housing wealth evolve for all the agents with positive productivity shocks in both the firm sectors. On top of that I also analyze interesting scenarios of how the markets behave with Expectation shocks on Rental returns and productivity shocks and how agents behave when the news is out, before the shock is realized and after the shock is

[20]When the system is subjected to only technology shocks and Inverse cost push shocks in both consumption and housing sectors.
Considering the Inflation Coefficient on Taylor rule on Y axis and (1-Downpayment ratio) on X axis.

realized. I use this framework to analyze which policy out of the Macro prudential and Monetary policy is better to curb the volatility of the economic variables. I have also found that the volatility stemmed from the Monetary policy shock is substantially high (Std. Deviation of the housing output is 0.1067) on housing output compared to that of the consumption goods output (Std. Deviation of the consumption goods output is 0.0070). I find that the labour markets play a pivotal role in most of the dynamics. With a positive technology shock in consumption goods sector, disposable income increase for all the three agents and are better off in consumption, Borrowers tend to invest in housing and Savers tend to supply more loans with decrease in housing stock of Savers. With a positive technology shock in housing goods sector, disposable income increase for all the three agents. However, when the shock was just realized Hand to mouth agents are better off in consumption and housing and the other two agents are worse off. I also find that houses acting as collateral and also source of returns has two opposing affects in Borrowers choices of housing. Borrowers are the driving force in the housing markets. I also find that in curbing the volatility of Hand to Mouth agents markets, Monetary policy is the only effective policy and Macro prudential policy which regulates Downpayment ratio has no effect over their choice variables. However, Macro prudential policy is more effective in both the Borrowers and Savers volatilities of choice variables. With a News shock in Monetary policy, we tend to observe that the economy volatility is only affected by the Monetary policy with no affect from Macro Prudential policy.

5 An Empirical Bayesian Estimation of Housing Markets

5.1 Introduction

In this chapter, I estimate the model using Bayesian techniques. The model is slightly modified to improve its empirical properties. In particular, I introduce habit persistence and inflation inertia. Constantinides et al. (1990), Abel (1990) and Boldrin et al., 2001 suggested that adding habits to a real business cycle framework can help in explaining the joint behavior of consumption and asset prices. There has been a lot of literature on the housing and non housing multi sector models, Greenwood and Hercowitz (1991), Benhabib, Rogerson, and Wright (1991), Davis and Heathcote (2007) and Fisher (2007) are such examples. However, these models are subjected to calibrated parameters and a very few in the literature have estimated those parameters with such multi sector economies see eg., Lee, Song (2014) who analyzes the role of housing in the Korean business cycles in an empirical model. Funkea, Kirkbyb and Mihaylovski (2018) has incorporated the multi sector economy of housing and using DSGE models and Bayesian estimation shown that the monetary policy has large spillover effects on house prices in New Zealand. Other example of incorporating multi sector housing economy and empirical techniques is Eric C.Y. Ng (2015), in the context of housing market in China, he suggested that housing preference shocks drove more than one-third of the volatility of housing prices and about 12 to 30 percent of the house price variance is explained by the monetary shocks in his empirical model. However, one of the most prominent and the motivation of this chapter is Iacoviello and Neri (2010) in which the authors have estimated the model with US housing markets with technology and monetary shock. They also empirically suggested the similar result which we have obtained in the last chapter that monetary factors played a huge role in the housing output cycles. Similar to that of Iacoviello and Neri (2010) the analysis combines key elements of developing a multi-sector structure with housing and non housing goods, nominal rigidities in the consumption firms sector and a rich set of shocks which include the sectoral heterogenous cost push and technology shocks to both consumption and housing goods. I have also included the monetary shock, habit persistence shock and the rental returns shock in the analysis.

5.2 The Theoretical Model with Internal Habit Persistence

Even though there is a small literature on no evidence of habit formation especially with panel data from households Dynan (2000), most of the literature argues for habit formation in the agents utility and this helps in the better fit of models. Christiano, Eichenbaum and Evans (2005) suggested that agents in the DSGE models should be subjected to habit formation in their utility and this causes the model to be able to generate a hump-shaped response of consumption to various shocks. This is due to the reasoning that changes in the consumption become costly for agents due to being subjected to habit formation in consumption, thereby inducing smoothness in such dynamics see Kano & Nason, 2014. Smets and Wouters (2007) and Adolfson et. al. (2007) also confirm that introduction of habit formation in a DSGE model is clearly favored by the data. In fact there is quite a lot of literature on the importance of habit formation in explaining the data see. eg., Aylin Seckin(2000), In this section, I will be having a very similar structure of sectoral heterogeneity as the previous models. In particular, I would like to estimate a DSGE model when agents are subjected to a CES Utility which addresses the additive separability between consumption basket and labour choices. Same as above models, the model will include three different types of household agents. First, there will be poor 'hand-to-mouth' households who work and rent the houses; second, there will be rich households who are able to smooth their consumption over time, they are Savers in this and provide funds to the other households, who can borrow and lastly there will be Borrowers who are credit-constrained but still can borrow under suitable collateral. However, all of these hand to mouth agents, Borrowers and Savers are subjected to internal habits. As suggested by Boldrin et al., 2001, such agents with internal Habit formation (as opposed to the external habit formation) and a favorable risk aversion resulted in agents not subjected to "catching-up-with-the-joneses" phenomenon. Such internal habit persistence requires the utility function to be time separable and hence the reason for the habit formation in the consumption basket of the agent's utility. The model will include financial intermediaries (FI) which has a minimal role of pooling the loans from the Savers and providing them to the Borrowers. In addition, there will be a Central Bank and the central bank will be responsible for monetary and macro-prudential policy aiming to reduce house price volatility. I also include two types of firms which will produce consumption goods and housing goods.

5.2.1 Hand-to-mouth workers

Hand to Mouth workers are completely credit constrained, they do not have suitable collateral for borrowing from the bankers. Each of these households will consume, supply labour and rents a house and gains utility from the house they rent, and consumption. I assume these households couldn't save and the budget constraint can be as follows:

Budget Constraint (Nominal terms):

$$P_{c,t}C_t^p + H_t^r Q_t^r = N_t^p W_t^p + T_{p,t}$$

where Q_t^r rental price of the house,$P_{c,t}$ is the given price of the consumption goods, H_t^r rented house, C_t^p consumption of the representative household, N_t^p production labour from Hand-to-Mouth workers. W_t^p represent the wage rate. These households will earn the labour from providing the labour and will spend all of it on consumption and rent.

A typical Hand-to-Mouth household consumes both the consumption services and the housing services. I also assume that the wage depends only on the type of labour, not on the type of firm. The utility of these households is as follows:

Utility :

$$\max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \gamma^t U\left(X_t^p, N_t^p, \frac{H_t^r}{A_t}\right)$$

where X_t^p is the habit subjected consumption basket for the agents, H_t^r denotes the housing services from which these households incur the utility, typically assumed to be the roof under which the household survives. This roof under housing services would be the rented house for hand to mouth agents. N_t^p denotes the combined labour supplied by these households for the two types of firms due to the assumption that the wage depends only on the type of labour, not on the type of firm. Where ρ is the elasticity of substitution between housing and consumption goods and α reacts the degree of inter-temporal substitutability.

Such agents are also subjected to internal habit formation only in the consumption of these agents as the literature suggests that there is no evidence of habit formation in the housing from the agents Iacoviello and Neri (2010).

$$X_t^{c,p} = \left(\frac{C_t^p}{A_t} - \omega \frac{C_{t-1}^{Aggp}}{A_{t-1}} \right).$$

where C_t^p denotes the consumption of hand to mouth agents in time period "t" and C_{t-1}^{Aggp} is the aggregate consumption of all the other households in the same sector of agents and ω is the habit parameter which characterizes the household preference by a habit formation. At the start of the period, hand to mouth households problem is to choose an optimal plan of consumption C_t^p, labour N_t^p, housing of hand to mouth agents which is essentially a rented housing H_t^r.

$$\max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \beta^t U\left(C_t^p, N_t^p, H_t^r\right) = \max_{N_t^p, H_t^r} \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma} \left[\left(a X_t^{p\,1-\frac{1}{\rho}} + (1-a) \frac{H_t^r}{A_t}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^p\right)^{1+\phi} \right)$$

where

$$X_t^{c,p} = \left(\frac{C_t^p}{A_t} - \omega \frac{C_{t-1}^{Aggp}}{A_{t-1}} \right).$$

subject to nominal budget constraint
for $\tilde{x}_t = \frac{X_t}{A_t}$, we have the budget constraint to be:

$$P_{c,t} \tilde{C_t^p} + Q_t^r \tilde{H_t^r} = N_t^p \tilde{W_t^p} + \tilde{T_{p,t}}$$

By maximizing the utility subject to the budget constraint of the hand to mouth agents leads us to the new optimal conditions. We obtain the consumption leisure decision, Inter temporal choice and optimal consumption. These agents will only consume as they are assumed to not save.

consumption leisure decision

$$\begin{aligned} \left(N_t^p\right)^{\phi} = & \left[\left(a \left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_t^r}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma} \\ & \frac{\rho}{\rho-1} \left[\left(a \left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_t^r}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a \left(1 - \frac{1}{\rho}\right) \left(\tilde{C_t^p}\right)^{-\frac{1}{\rho}} \tilde{w_t^p} \end{aligned} \tag{148}$$

Inter temporal choice and

$$0=\left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left[-a\left(1-\frac{1}{\rho}\right)\left(\tilde{C_t^p}\right)^{\left(-\frac{1}{\rho}\right)}q_t^r+(1-a)\left(1-\frac{1}{\rho}\right)\tilde{H}_t^{r-\frac{1}{\rho}}\right] \tag{149}$$

optimal consumption

$$\tilde{C_t^p}=N_t^p\tilde{w_t^p}+\tilde{t_{p,t}}-\tilde{H}_t^r q_t^r \tag{150}$$

5.2.2 Borrowers

Borrowers are the central focus of the model where I have two types of Borrowers: Buy-to-Let Borrowers and Owned house Borrowers.These households can borrow under collateral normally I assumed that houses act as their collateral, borrow money from the banks in terms of loans to invest in the either owned housing or Buy-to-Let housing. The housing can also be divisible and part of the house is owner occupied and also other part of it is rented to other sector of hand-to-mouth households. These households Budget constraint can be thought of as follows:

$$P_{c,t}C_t^b+Q_t^h(H_t^b-(1-\delta)H_{t-1}^b)+R_{t-1,d}D_{t-1}^d=H_t^rQ_t^r+N_t^bW_t^b+D_t^d+T_{b,t}$$

where C_t^b denotes the consumption of the final consumer service from the Borrowers sector, $P_{c,t}$ is the given price of the consumption goods, Q_t^h price of the house at time t, H_t^r rented house to the Borrowers sector, Q_t^r being the rental rate of the let house at time t, H_t^b total house , rented and owner occupied and also depreciates at the rate δ, D_t^d one period nominal debt from the bank at the end of period t provided to the housing Borrowers sector and $R_{t-1,d}$ is nominal debt lending rate of loan. In particular all the expenditures and investment form the Borrowers will be equal to their gains.

Let us denote $H_{t,o}$Owner occupied part of the house:

$$H_t^o=H_t^b-H_t^r;$$

for $\tilde{x}_t=\frac{X_t}{A_t}$, we have the budget constraint to be:

$$P_{c,t}\tilde{C_t^b}+Q_t^h(\tilde{H_t^b}-(1-\delta)H_{t-1}^{\tilde{b}}\frac{1}{z_t})+R_{t-1,d}D_{t-1}^{\tilde{d}}\frac{1}{z_t}=\tilde{H}_t^rQ_t^r+N_t^b\tilde{W_t^b}+\tilde{D_t^d}+\tilde{T_{b,t}}$$

where the technology growth rate is given as:

$$z_t = \frac{A_t}{A_{t-1}}$$

These households gain utility from the under roof housing services $H_t^o = H_t^b - H_t^r$, consumption service C_t^b and the leisure $1 - N_t^b$. The typical utility for these households is as follows:

Utility

$$\max_{C_t^b, N_t^b, H_t^b, H_t^r, D_t^o} E_0 \beta^t \sum_{t=0}^{\infty} \left(\frac{1}{1-\sigma} \left[\left(a X_t^{c,b 1-\frac{1}{\rho}} + (1-a) \left(\tilde{H}_t^b - \tilde{H}_t^r \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^b \right)^{1+\phi} \right)$$

where

$$X_t^{c,b} = \left(\tilde{C}_t^b - \omega C_{t-1}^{\tilde{p}} \right).$$

Note that I include owner-occupied housing $H_t^o = H_t^b - H_t^r$ in utility, not the rental house, as I treat utility of housing as being under the roof.

I also assume the Borrowers are under some collateral constraints, where the maximum amount of the combined loan and repayment on the loans they borrow should be only less than or equal to the fraction of the house which I assume will be determined by the central bank. I also assume that the these fractions are sector dependent. In particular the fractions of houses will differ between the Buy-to-Let and Owned housing Borrowers sector. The collateral constraint can be thought of as follows:

$$R_{t,b} D_t^b \leq \mu Q_t^h H_t^b$$

for $\tilde{x}_t = \frac{X_t}{A_t}$, we have the collateral constraint to be:

$$R_{t,b} \tilde{D}_t^b \leq \mu Q_t^h \tilde{H}_t^b$$

$\mu = (1 - downpayment)$ means the central bank imposes a constraint on the maximum credit amount which this sector of households can get. Hence these are not fully credit unconstrained.

The Borrowers households problem is to choose $C_t^b, N_t^b, H_t^b, H_t^r, D_t^o$ by maximizing Utility subject to the borrowing constraint and the collateral constraints. Consider a binding borrowing and collateral constraints and forming the Lagrangian L, with ξ_t, Ψ_t being the Lagrange multipliers for budget constraint and collateral constraint respectively we have:

Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \left(\frac{1}{1-\sigma} \left[\left(a X_t^{c,b\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_t^b} - \tilde{H_t^r} \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^b \right)^{1+\phi} \right) \\ +\xi_t \left(\tilde{H_t^r} Q_t^r + N_t^b \tilde{W_t^b} + \tilde{D_t^d} + \tilde{T_{b,t}} - P_{c,t} \tilde{C_t^b} - Q_t^h (\tilde{H_t^b} - (1-\delta) H_{t-1}^{\tilde{b}} \frac{1}{z_t}) - R_{t-1,d} D_{t-1}^{\tilde{d}} \frac{1}{z_t} \right) \\ +\Psi_t \left(\mu Q_t^h \tilde{H_t^b} - R_{t,b} \tilde{D_t^b} \right) \end{array} \right)$$

where the Borrowers are subjected to the internal habit formation as well.

$$X_t^{c,b} = \left(\tilde{C_t^b} - \omega C_{t-1}^{\tilde{b}} \right).$$

The for $x_t = \frac{X_t}{P_{c,t}}$ the optimal conditions are as follows:

We obtain the consumption leisure decision, the interpretation of this equation is that the marginal rate of substitution between leisure $(1-N)$ and consumption is to equate with the relative price of leisure i.e., the wages. This equation also gives us the average marginal productivity of labour. However, with an assumption of monopolistically competitive intermediate firms, the average marginal productivity of labour will be less than that of the social planner's problem and hence leads to a distortion. In particular, a presence of markup in the firms leads to an allocation distortion in the economy. This optimality condition is no different to that of the previous model apart from the fact that instead of just the consumption of Borrowers, the consumption basket subjected to the internal habit formation is now affecting the labour decision which makes this as a consumption basket with internal habit formation and the leisure decision optimality condition.

$$\begin{aligned} \left(N_{b,t} \right)^{\phi} = \tilde{w_{b,t}} & \left[\left(a X_t^{c,b\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_t^b} - \tilde{H_t^r} \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma} \\ \frac{\rho}{\rho-1} & \left[\left(a X_t^{c,b\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_t^b} - \tilde{H_t^r} \right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a \left(1 - \frac{1}{\rho} \right) \left(X_t^{c,b} \right)^{-\frac{1}{\rho}} \end{aligned} \tag{151}$$

We also tend to see similar optimality conditions as in the previous models apart from the habit formation.

$$
\begin{aligned}
0 = &\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
&\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{-\frac{1}{\rho}}\right) \\
&- [P_{c,t}\xi_t]\left(q_t^h\right) + [P_{c,t}\Psi_t]\,\mu q_t^h \\
&+ \frac{1}{z_{t+1}}\beta\,[P_{c,t+1}\xi_{t+1}]\left(q_{t+1}^h(1-\delta)\right)
\end{aligned}
\tag{152}
$$

$$
\begin{aligned}
0 = &\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
&\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{-\frac{1}{\rho}}\right) - [P_{c,t}\xi_t]\,q_t^r
\end{aligned}
\tag{153}
$$

$$
\begin{aligned}
&\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b} - \tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
&a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,b}\right)^{-\frac{1}{\rho}} = P_{c,t}\xi_t
\end{aligned}
\tag{154}
$$

Inter temporal choices

$$
0 = [P_{c,t}\xi_t] - [P_{c,t}\Psi_t]\,R_{t,o} - \beta\,[P_{c,t+1}\xi_{t+1}]\,R_{t,d}\frac{1}{1+\pi_{t+1}}\frac{1}{z_{t+1}} \tag{155}
$$

Budget Constraint

$$
0 = \tilde{H_t^r}q_t^r + N_{b,t}\tilde{w_{b,t}} + \tilde{d_{t,d}} + \tilde{t_{b,t}} - \tilde{C_{b,t}} - q_t^h(\tilde{H_t^b} - (1-\delta)H_{t-1}^{\tilde{b}}\frac{1}{z_t}) - R_{t-1,d}\tilde{d_{t-1,d}}\frac{1}{1+\pi_t}\frac{1}{z_t} \tag{156}
$$

Collateral Constraint

$$
0 = \mu q_t^h\tilde{H_{b,t}} - R_{t,d}\tilde{d_{t,d}} \tag{157}
$$

where the technology growth rate is given as:

$$
z_t = \frac{A_t}{A_{t-1}}
$$

and $1+\pi_t$ is the gross inflation which we obtained from the basic Inflation definition:

$\%\pi_t = \%$change in the price levels between two periods$=\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{P_t}{P_{t-1}} = 1 + \pi_t$

5.2.3 Savers

Savers consumes, supply labour inelastically and also save. They supply funds to the financial intermediaries and let the banks to circulate their money in terms of Credit.

I also assume the discount factor for this sector is high than the previous sector's and I denote it with θ as this sector of agents tend to save money.

$$0.99 = \theta > \beta = 0.985$$

for $\tilde{x}_t = \frac{X_t}{A_t}$, we have the budget constraint of Savers as:

$$P_{c,t}\tilde{C}_{s,t} + Q_t^h(\tilde{H}_{s,t} - (1-\delta)\tilde{H_{s,t-1}}\frac{1}{z_t}) + \tilde{B_{s,t}} = N_{s,t}\tilde{W_{s,t}} + R_{t-1,s}\tilde{B_{s,t-1}}\frac{1}{z_t} + \tilde{T_{s,t}}$$

The deposits from the working representatives of the saver's household are one-period bonds that pay with the return $R_{t-1,s}$ from t − 1 to t. Let $B_{s,t}$ be the debt the saver's household acquires, all profits are expropriated by the government and redistributed as transfers T_{st}. The Savers households gain utility from the under roof housing services $H_{s,t}$, consumption service $C_{s,t}$ and the leisure $N_{s,t}$. The typical utility for these households is as follows:

The Saver's household discounted Utility is as :

$$\max_{C_{s,t},N_{s,t},H_{s,t},B_{s,t}} E_0 \sum_{t=0}^{\infty} \theta^t U^s \left(X_t^{c,s}, N_{s,t}, H_{s,t}\right)$$

$$= \max_{C_t^s,N_t^s,H_t^s,B_t^s} E_0 \sum_{t=0}^{\infty} \theta^t \left(\frac{1}{1-\sigma} \left[\left(a X_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H}_t^{s\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^s\right)^{1+\phi} \right)$$

where

$$X_t^{c,s} = \left(\tilde{C}_t^s - \omega \tilde{C_{t-1}^{Aggs}} \right).$$

C_{t-1}^{Aggs} is the aggregate consumption of all the households in the same sector of agents

The Savers households problem is to choose $C_{s,t}, N_{s,t}, H_{s,t}, B_{s,t}$ by maximizing utility subject to the budget constraint . Forming the Lagrangian L, we have:

Lagrangian

$$L = E_0 \sum_{t=0}^{\infty} \theta^t U^s \left(X_t^{c,s}, N_{s,t}, \tilde{H_{s,t}}\right) + \lambda_t [N_{s,t}\tilde{W_{s,t}} + R_{t-1,s}\tilde{B_{s,t-1}}\frac{1}{z_t} + \tilde{T_{s,t}} - \tilde{P_{c,t}}\tilde{C_{s,t}} - Q_t^h(\tilde{H_{s,t}} - (1-\delta)\tilde{H_{s,t-1}}\frac{1}{z_t}) - \tilde{B_{s,t}}]$$

For $x_t = \frac{X_t}{P_{c,t}}$, solving the above maximization problem gives us the labour supply of Savers in both the firm sectors:

$$(N_{s,t})^{\phi} = \tilde{w_t^s} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$
$$\frac{\rho}{\rho-1} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a \left(1 - \frac{1}{\rho}\right) \left(X_t^{c,s}\right)^{-\frac{1}{\rho}} \tag{158}$$

The housing demand from the Savers will also be obtained as

$$0 =$$
$$\left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$
$$\frac{\rho}{\rho-1} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] (1-a) \left(1 - \frac{1}{\rho}\right) \left(\tilde{H_{s,t}}\right)^{-\frac{1}{\rho}}$$
$$- [P_{c,t}\lambda_t] \left(q_t^h\right) + \theta\, [P_{c,t+1}\lambda_{t+1}] \left(q_{t+1}^h (1-\delta) \frac{1}{z_{t+1}}\right) \tag{159}$$

Lagrange Multiplier

$$P_{c,t}\lambda_t = \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$
$$\frac{\rho}{\rho-1} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a) \left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a \left(1 - \frac{1}{\rho}\right) \left(X_t^{c,s}\right)^{-\frac{1}{\rho}} \tag{160}$$

Inter temporal decisions from the Savers:

$$P_{c,t}\lambda_t = \theta\, [P_{c,t+1}\lambda_{t+1}] \frac{R_{t,s}}{1+\pi_{t+1}} \frac{1}{z_{t+1}} \tag{161}$$

and the budget constraint:

$$0 = N_{s,t}\tilde{w_{s,t}} + \tilde{t_{st}} + \tilde{b_{s,t-1}} \frac{R_{t-1,s}}{1+\pi_t} \frac{1}{z_t} - \tilde{C_{s,t}} - q_t^h (\tilde{H_{s,t}} - (1-\delta)\tilde{H_{s,t-1}} \frac{1}{z_t}) - \tilde{b_{s,t}} \tag{162}$$

where the technology growth rate is given as:

$$z_t = \frac{A_t}{A_{t-1}}$$

5.2.4 Firms

I have two sectors in the production economy one of which produces housing services and the other consumption goods. In one of the sectors (consumption sector) a competitive final good producer demand and purchase $y_t^c(i)$ units of intermediate goods. where c represents the firm's production sector whereas in here is Non durable consumption goods c. Also Each type of labour $N_{t,p}, N_{t,b}, N_{t,s}$ works for both housing sector and consumption sector which I will denote by $N_{p,t}^h, N_{b,t}^h, N_{s,t}^h$ for housing and $N_{p,t}^c, N_{b,t}^c, N_{s,t}^c$ for consumption at time t from all the three types of households. All the intermediate goods firms will hire labour from the perfectly competitive market.

5.2.5 Final Housing goods Side of the Production Economy

I model the production of the final good of housing via a single stand in aggregate firm that produces according to the production technology:

$$Y_t^h = \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Where Y_t^h denotes the final goods of housing sector, $y_t^h(i)$ denotes the ith intermediate input, ε is the elasticity of substitution between the intermediate goods. Notice that as $\varepsilon \to \infty$, the intermediate input goods become perfectly substitutable and to avoid this I assume it to be finite.

The final good firms acquires the intermediate goods from the intermediate produces and make it into a single final product and sells it to the households in the economy. Also the final producers doesn't have any control over the prices, and they take the prices $Q_t^h, Q_t^h(i)$ as given. Hence, the decision problem for the final good producer in the housing goods side is to choose the demand for theY_t^h and intermediate goods$y_t^h(i)$ to maximize the profit which is given by :

$$Q_t^h.Y_t^h - \int_0^1 Q_t^h(i) y_t^h(i).di$$

where Q_t^h is the price of the final housing good and
$Q_t^h(i)$ is the price of the ith intermediate good
taking both the prices as given from the final goods firm perspective,

$$Q_t^h.Y_t^h - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

subject to the

$$Y_t^h = \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

which gives us:

$$Q_t^h. \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

The final good firms will chose $\left\{ y_t^h(i) \right\}_{i=0}^{1}$,forming the Lagrangian equation with the above equation gives us:

$$\Pi_{h,t}^f = Q_t^h. \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

This equation gives us the demand for each intermediate input as follows:

$$y_t^h(i) = \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} Y_t^h$$

substituting the demanded intermediate goods from the above $y_t^h(i)$ back into the profit function of final goods gives us

$$\left(\int_0^1 \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{1-\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon-1}} = 1$$

The above equation has two implications

1) By substituting into the profits equation gives us the profits as zero. i.e., the profits of the final goods are equal to zero.

2) By re arranging, we have

$$Q_t^h = \left[\int_0^1 \left(Q_t^h(i) \right)^{1-\varepsilon} di \right]^{\frac{1}{\varepsilon-1}}$$

Which says that the prices of the final good follow the same aggregation rule and is a function of the intermediate good prices.

5.2.6 Final Consumption Goods Side of the Production Economy

I model the production of the final good of consumption good via a single stand in aggregate firm that produces according to the production technology:

$$Y_t^c = \left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Where Y_t^c denotes the final goods of consumption goods sector, $y_t^c(i)$ denotes the ith intermediate input, ε is the elasticity of substitution between the intermediate goods. Notice that as $\varepsilon \to \infty$, the intermediate input goods become perfectly substitutable and to avoid this I assume it to be finite.

The final good firms acquires the intermediate goods from the intermediate produces and make it into a single final product and sells it to the households in the economy. Also the final producers doesn't have any control over the prices, and they take the prices $P_t^c, P_t^c(i)$as given. Hence, the decision problem for the final good producer in the consumption goods side is to choose the demand for the Y_t^c and intermediate goods$y_t^c(i)$ to maximize the profit which is given by :

$$P_t^c.Y_t^c - \int_0^1 P_t^c(i)\, y_t^c(i).di$$

where P_t^c is the price of the final consumption good and
$P_t^c(i)$ is the price of the ith intermediate good
taking both the prices as given from the final goods firm perspective, leads us to the equation which gives us the demand for each intermediate input as follows:

$$y_t^c i) = \left(\frac{P_t^c(i)}{P_t^c} \right)^{-\varepsilon} Y_t^c$$

substituting the demanded intermediate goods from the above $y_t^c(i)$ back into the profit function of final goods implies that this consumption goods sector has the same two implications as for the final good producers in housing sector. i.e., the profits of the final goods in the consumption sector are equal to zero and the prices of the final good follow the same aggregation rule and is a function of the intermediate good prices in the consumption goods sector.

5.2.7 Intermediate Consumption firms

Profit optimization problem is standard. A firm chooses employment and prices to maximize profit:

$$\max_{\left\{N_{p,t}^c(i), N_{b,t}^c(i), N_{s,t}^c(i), p_s^*(i)\right\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} m_{t,s} \left(y_t^c(i) P_{ct}(i) - W_t^p N_{p,t}(i) + W_t^b N_{b,t}(i) + W_t^s N_{s,t}(i) \right).$$

subject to the production constraint

$$y_t^c(i) = A_t Z_{ct} N_{s,t}(i)^v N_{b,t}(i)^u N_{r,t}(i)^{1-u-v},$$

The firm tries to minimize their production costs of intermediate goods equation (43) by choosing the values for the hire of labour provided by the households, $N_{p,t}^c(i), N_{b,t}^c(i), N_{s,t}^c(i)$subject to the production constraint (44)

The final good firm's demand equation of the input goods is as follows:

$$y_t^c(i) = Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon},$$

and price rigidity which I assume to follow the Rotemberg price setting scenario in which firms face a quadratic costs in changing the goods price.

Profit maximization problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. I deal with each of these problems separately. I use subscript 'p' do denote sector producing perishable goods. Both output and employment have this index.

Employment Consumption good prices follow a Sticky prices unlike housing prices and hence, I assume housing service prices follow a Rotemberg model of prices, where as the house good prices are completely flexible. These consumption good intermediate firms will pay wages to the labour provided by the three sector of households and the employment equation is as follows:

Firm i minimizes nominal cost:

$$\min_{N_{s,t}(i), N_{b,t}(i), N_{p,t}(i)} W_t^s N_{s,t}^c(i) + W_t^b N_{b,t}^c(i) + W_t^p N_{p,t}^c(i).$$

subject to the production constraint

$$y_t^c(i) = A_t Z_{ct} N_{p,t}^c(i)^v N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v}$$

The optimality conditions will yield us the solution for the Lagrange multiplier:

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(1-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^u \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^v \tag{163}$$

Substitute back the Lagrange multiplier back yields us the labour demand equations for all the three household sectors and Aggregation yields (I denote $y_t(i) = Y_t \int \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$ are as follows:

$$N_{p,t}^{c} = \frac{1}{Z_{ct}}\left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v}\right)^{(1-u-v)}\left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u}\right)^{u}\left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v}\right)^{v-1}\frac{Y_t^c}{A_t} \tag{164}$$

$$N_{b,t}^{c} = \frac{1}{Z_{ct}}\left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v}\right)^{(1-u-v)}\left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u}\right)^{u-1}\left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v}\right)^{v}\frac{Y_t^c}{A_t} \tag{165}$$

$$N_{s,t}^{c} = \frac{1}{Z_{ct}}\left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v}\right)^{(-u-v)}\left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u}\right)^{u}\left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v}\right)^{v}\frac{Y_t^c}{A_t} \tag{166}$$

Price setting Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices. We also know the Lagrange multiplier to be:

$$\zeta_t = \frac{1}{Z_{ct}}\left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v}\right)^{(1-u-v)}\left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u}\right)^{u}\left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v}\right)^{v}$$

where $MC_s = \zeta_t P_{ct}$. Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 y_t^c(i)$. The firm discounts future profits by the gross real interest rate between today and future dates(stochastic discount factor),

$$\frac{1}{R_{t,s}} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]$$

This yields us :

$$V(i) = E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(y_t^c(i)P_{ct}(i) - y_{ct}(i)MC_t\right) - \frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 y_t^c(i)\right]$$

subject to Intermediate goods demand equation :

$$y_t^c(i) = Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

Let $\frac{P_{ct}(i)}{P_{ct}} = \widetilde{P_{ct}}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

where π_t is the Gross inflation in the aggregate price level of the consumption goods side.

$$\max_{\left\{\widetilde{P_{ct}}\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

I can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = \zeta_t P_{ct}$ which implies the relative price $\widetilde{P_{ct}}$ is equal to 1.

By optimizing, the FOCs gives us the New Keynesian Phillips curve in the consumption goods sector:

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + E_t \left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]} \left[(\pi_{t+1}) z_{t+1} \frac{\widetilde{Y_{t+1}^c}}{\widetilde{Y_t^c}} (1+\pi_{t+1}) \right] \right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2 \varepsilon \right) \tag{167}$$

where

the stochastic discount factor is:

$$m_{t,t+1} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

and the technology growth rate is given as:

$$z_t = \frac{A_t}{A_{t-1}}$$

5.2.8 Intermediate Housing firms

I follow a housing market where the sector is split into two sub sectors: one for final goods which are not perfect competitive firms as in consumption goods sector. However, I assume the final goods sector follows a monopolistically competitive approach. These monopolistic firms who produce final houses doesn't involve labour and capital as for the intermediate housing goods sector which follows a sticky prices and involve labour from all the three sectors of households

Profit optimization problem is standard. A firm chooses employment and prices to maximize profit:

$$\max_{\left\{N_{p,t}^{c}(i),N_{b,t}^{c}(i),N_{s,t}^{c}(i),p_{s}^{*}(i)\right\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} m_{t,s}\left(y_{t}^{h}(i)\,Q_{t}^{h}(i)-W_{t}^{s}N_{s,t}^{h}(i)-W_{t}^{b}N_{b,t}^{h}(i)-W_{t}^{p}N_{p,t}^{h}(i)\right).$$

subject to the production constraint

$$y_{t}^{h}(i)=A_{t}Z_{ht}N_{p,t}^{h}(i)^{v}N_{b,t}^{h}(i)^{u}N_{s,t}^{h}(i)^{1-u-v}$$

The firm tries to minimize their production costs of intermediate goods equation (43) by choosing the values for the hire of labour provided by the households, $N_{p,t}^{h}(i),N_{b,t}^{h}(i),N_{s,t}^{h}(i)$subject to the production constraint

The final good firm's demand equation of the input goods is as follows:

$$y_{t}^{h}(i)=Y_{t}^{h}\left(\frac{Q_{t}^{h}(i)}{Q_{t}^{h}}\right)^{-\varepsilon},$$

and price rigidity which I assume to follow the Rotemberg price setting scenario in which firms face a quadratic costs in changing the goods price.

Profit maximization problem can be split into to separate problems: choose labour to minimize cost intra-temporally and choose prices to maximize future profit. I deal with each of these problems separately. I use subscript 'p' to denote sector producing perishable goods. Both output and employment have this index.

Employment Housing good prices follow a Sticky prices like consumption prices and hence, I assume housing service prices follow a Rotemberg model of prices. These housing good intermediate firms will pay wages to the labour provided by the three sector of households and the employment equation is as follows:

Firm i minimizes nominal cost:

$$\min_{N_{s,t}^{h}(i),N_{b,t}^{h}(i),N_{p,t}^{h}(i)} W_{t}^{p}N_{p,t}^{h}(i)+W_{t}^{b}N_{b,t}^{h}(i)+W_{t}^{s}N_{s,t}^{h}(i).$$

The firm tries to minimize their production costs of intermediate goods equation (27) by choosing the values for the hire of labour provided by the households, $N_{s,t}^{h}(i),N_{b,t}^{h}(i),N_{p,t}^{h}(i)$subject to the production constraint (28)

$$y_{t}^{h}(i)=A_{t}Z_{ht}N_{p,t}^{h}(i)^{v}N_{b,t}^{h}(i)^{u}N_{s,t}^{h}(i)^{1-u-v}$$

Where $Z_{ht}$is the firm's sector specific technology shock, note that this shock is aggregate shock to the sector rather than firm specific and forming the Lagrangian L, with η_t being the Lagrange multiplier we have:

$$L = W_t^s N_{s,t}^h(i) + W_t^b N_{b,t}^h(i) + W_t^p N_{p,t}^h(i) - P_{ct}\eta_t \left(A_t Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v} - y_t^h(i)\right)$$

the optimality conditions will yield the solution for the Lagrange multiplier in the housing sector:

$$\eta_t = \frac{1}{Z_{ht}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(1-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^u \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^v \tag{168}$$

Substitute back the Lagrange multiplier back yields us the labour demand equations for all the three household sectors. Now that we have all the labour demanded from intermediate goods . Aggregating the labour demand equations through the intermediate firms will give us the aggregate labour for all the three sectors of households using the aggregation rule .

$$\Xi_t^h = \int \Xi_t^h(i)\, di$$

yields us

$$N_{p,t}^h = \int N_{p,t}(i)\, di = \frac{1}{Z_{ht}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(1-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^u \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^{v-1} \frac{Y_t^h}{A_t} \tag{169}$$

$$N_{b,t}^h = \int N_{b,t}(i)\, di = \frac{1}{Z_{ht}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(1-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^{u-1} \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^{v} \frac{Y_t^h}{A_t} \tag{170}$$

$$N_{s,t}^h = \int N_{s,t}(i)\, di = \frac{1}{Z_{ht}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^{u} \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^{v} \frac{Y_t^h}{A_t} \tag{171}$$

Price setting Substituting the labour demand from the above profit function. We have:

$$\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} \left(y_t^h(i)\, Q_t^h(i) - W_t^s N_{s,t}^h(i) - W_t^b N_{b,t}^h(i) - W_t^p N_{p,t}^h(i)\right)$$

$$= y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\right) = \left(y_t^h(i)\, Q_t^h(i) - y_{ht}(i)\, MC_t\right)$$

Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices. We also know the Lagrange multiplier in the housing sector to be:

$$\eta_t = \frac{1}{Z_{ht}} \left(\frac{\left(\frac{w_t^s}{A_t}\right)}{1-u-v} \right)^{(1-u-v)} \left(\frac{\left(\frac{w_t^b}{A_t}\right)}{u} \right)^{u} \left(\frac{\left(\frac{w_t^p}{A_t}\right)}{v} \right)^{v}$$

where $MC_s = \eta_t P_{ct}$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{Q_t^h(i)}{Q_{t-1}^h(i)} - 1\right)^2 y_t^h(i)$. The firm discounts future profits by the gross real interest rate between today and future dates(stochastic discount factor),

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{Q_t^h(i)}{Q_t^h} - \frac{\eta_t P_{ct}}{Q_t^h} \right) Y_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} - \frac{\Omega_h}{2} \left(\frac{Q_t^h(i)}{Q_{t-1}^h(i)} - 1 \right)^2 \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} Y_t^h \right]$$

Let $\frac{Q_t^h(i)}{Q_t^h} = \widetilde{Q_t^h}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{Q_t^h} - \frac{\eta_t}{q_t^h} \right) Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} - \frac{\Omega_h}{2} \left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}} - 1 \right)^2 Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} \right]$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = \eta_t P_{ct}$which implies the relative price $\widetilde{Q_t^h}$ is equal to 1. FOC w.r.t $\widetilde{Q_t^h}$and dividing the optimality equation with A_t on both sides leads us to the New Keynesian Phillips curve in the housing sector:

$$\frac{(1-\varepsilon)}{\Omega_h} + \frac{\varepsilon}{\Omega_h} \frac{\eta_t}{q_t^h} + E_t \left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]} \left[(\pi_{h,t+1}) z_{t+1} \frac{\tilde{Y}_{t+1}^h}{\tilde{Y}_t^h} (1+\pi_{h,t+1}) \right] \right] = \left(\pi_{h,t}(1+\pi_{h,t}) - \frac{1}{2} (\pi_{h,t})^2 \varepsilon \right) \tag{172}$$

where

the stochastic discount factor is

$$m_{t,t+1} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$
$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

where $\pi_{h,t}$ is the Gross inflation of housing in the aggregate price level of the housing goods side. The technology growth rate is given as:

$$z_t = \frac{A_t}{A_{t-1}}$$

5.2.9 Profits of firms and Government Transfers

To close the model, I need to aggregate the real profits from the firms and has to be distributed among the households in terms of transfers. Aggregate inter-period nominal profit is the total output from both the firm sectors from which the wages of the agents are taken away. Please note because of the quadratic costs involved in the rigid intermediate consumption goods firms, that has to be taken away as well. This leads to:

$$\tilde{\Pi}_t = Y_t^c P_{ct} - W_{s,t}N_{s,t}^c - W_{b,t}N_{b,t}^c - W_{p,t}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c P_{ct} + Y_t^h Q_t^h - W_{s,t}N_{s,t}^h - W_{b,t}N_{b,t}^h - W_{p,t}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h$$

I assume that the profit is 100 percent taxed by the government and redistributed to the Borrowers, Savers and hand to mouth agents according to the following rule :

$$t_{bt} = (1-x-y)\frac{\tilde{\Pi}_t}{P_{ct}}$$

$$t_{st} = x\frac{\tilde{\Pi}_t}{P_{ct}}$$
$$t_{pt} = y\frac{\tilde{\Pi}_t}{P_{ct}}$$

For $\tilde{x}_t = \frac{X_t}{A_t}$, transfer to the Borrowers in terms of dividends from the profits of the firms

$$\tilde{t_{b,t}} = (1-x-y) \left(\tilde{Y_t^c} - \tilde{w_{s,t}}N_{s,t}^c - \tilde{w_{b,t}}N_{b,t}^c - \tilde{w_{p,t}}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2\tilde{Y_t^c} + \tilde{Y_t^h}q_t^h - \tilde{w_{s,t}}N_{s,t}^h - \tilde{w_{b,t}}N_{b,t}^h - \tilde{w_{p,t}}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2\tilde{Y_t^h} \right) \tag{173}$$

Transfer to the Savers in terms of dividends from the profits of the firms

$$\tilde{t_{st}} = x\left(\tilde{Y_t^c} - \tilde{w_{s,t}}N_{s,t}^c - \tilde{w_{b,t}}N_{b,t}^c - \tilde{w_{p,t}}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2\tilde{Y_t^c} + \tilde{Y_t^h}q_t^h - \tilde{w_{s,t}}N_{s,t}^h - \tilde{w_{b,t}}N_{b,t}^h - \tilde{w_{p,t}}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2\tilde{Y_t^h}\right) \tag{174}$$

Transfer to the hand to mouth agents in terms of dividends from the profits of the firms

$$\tilde{t_{pt}} = y\left(\tilde{Y_t^c} - \tilde{w_{s,t}}N_{s,t}^c - \tilde{w_{b,t}}N_{b,t}^c - \tilde{w_{p,t}}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2\tilde{Y_t^c} + \tilde{Y_t^h}q_t^h - \tilde{w_{s,t}}N_{s,t}^h - \tilde{w_{b,t}}N_{b,t}^h - \tilde{w_{p,t}}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2\tilde{Y_t^h}\right) \tag{175}$$

5.2.10 Financial Intermediaries:

Considering these are owned by Savers,The role of financial intermediaries in this model will be minimal, they just pool the deposits from the Savers and provide loans to Borrowers. They pay the same interest rate to depositors as they charge from the Borrowers:

$$R_{t,o} = R_{t,s} \tag{176}$$

5.2.11 Aggregation:

As I am working with representative agent models, I have a single representative agent in the model from each sector and this results in $C_{t-1}^{Aggx} = C_{t-1}^{x}$

$$C_{t-1}^{Aggp} = C_{t-1}^{p} \tag{177}$$

$$C_{t-1}^{Aggb} = C_{t-1}^{b} \tag{178}$$

$$C_{t-1}^{Aggs} = C_{t-1}^{s} \tag{179}$$

5.2.12 Market Clearing

In equilibrium we have the following resource constraints:

The aggregate labour of the agents are the sum of labour to the intermediate housing and consumption good firms.

$$N_t^p = N_{p,t}^h + N_{p,t}^c \tag{180}$$
$$N_t^b = N_{b,t}^h + N_{b,t}^c \tag{181}$$
$$N_t^s = N_{s,t}^h + N_{s,t}^c \tag{182}$$

The demand and supply of loans in the equilibrium are the same.

$$\tilde{B_{s,t}} = \tilde{D_{t,d}} \tag{183}$$

The central bank sets the interest rate by Taylor rule:

$$\frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_r} \tag{184}$$

The central bank also sets the Macro prudential policy which regulates down payment ratio by feeding on the output gap:

$$\frac{\mu}{\bar{\mu}} = \left(\frac{Y_t}{Y}\right)^{\gamma_\mu} \tag{185}$$

The total output in the economy is equal to the consumption of both housing goods and non durable consumption goods from the households including the depreciation of housing and the quadratic costs incurred in the rigid intermediate consumption good firms.

$$\begin{aligned} \tilde{Y_t^c} + \tilde{Y_t^h} q_t^h = \tilde{C_{b,t}} + \tilde{C_{s,t}} + \tilde{C_{p,t}} + q_t^h(\tilde{H_{b,t}} - (1-\delta)\tilde{H_{b,t-1}}\frac{1}{z_t}) + q_t^h(\tilde{H_{s,t}} - (1-\delta)\tilde{H_{s,t-1}}\frac{1}{z_t}) \\ + \frac{\Omega}{2}\pi_{ct}^2 Y_t^c + \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h \end{aligned} \tag{186}$$

Also the definitions:

$$\begin{aligned} q_t^h &:= \frac{Q_t^h}{P_{c,t}} \\ 1+\pi_t^H &:= \frac{Q_t^h}{Q_{t-1}^h} \\ 1+\pi_t^C &:= \frac{P_{c,t}}{P_{c,t-1}} \end{aligned}$$

5.2.13 Private Sector Equilibrium

Please Refer to the Appendix for the whole Private Sector Equilibrium for this model.

5.3 Parameter Estimates

In the light of the previous model with internal habit formation, due to the constraints in obtaining the rental data for the bayesian estimation of the above model, I will consider estimating parameters of two sub models: one with baseline model with no housing and the other baseline model with housing. Regarding these models, I will estimate some of the parameters and compare their values when Borrowers and housing is introduced. Until now, for the calibrated parameters, I have solved the above DSGE models using perturbation methods from the reduced form of the model equations in terms of a state space representation. With such reduced form of state space, unobserved variables and the dynamics around the steady state I have calculated the approximate theoretical moments and as well. However, the main focus of this chapter is using Bayesian techniques to estimate the important parameters of the model with internal habit formation. To facilitate this, we need to transform the data and model into the suitable form for computing the likelihood function. In particular, we could estimate the parameters of the model using some observed variables, an inclusion of non structural errors and measurement equations and we could bring these DSGE models with reduced form of state space representation to data. I follow a Bayesian estimation strategy of Metropolis Hastings based up on the Markov Chain Monte Carlo (MCMC) methods which is built around a DSGE model derived likelihood function.

5.4 Baseline Model with no Housing:

To understand and for the better tractability of the the estimation procedure, I have ignored the housing sector of the above model for now. Essentially, I assumed to have only Savers, consumption good firms and no collateral constraints in the economy. In this section I outline a basic model with single household, consumption goods sector, a New Keynesian Phillips curve and a rich set of shocks. Such system is as follows:

Lagrange Multiplier of households:

$$P_{c,t}\lambda_t = \left[\left(X_t^{c,s}\right)\right]^{-\sigma}$$

Inter temporal decision of households:

$$P_{c,t}\lambda_t = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}\frac{1}{z_{t+1}}$$

Aggregate output:

$$Y_t^c = \tilde{C_{s,t}} + \frac{\Omega}{2}\pi_{ct}^2\tilde{Y_t^c}$$

New Keynesian Phillips curve:

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{t+1})z_{t+1}\frac{\widetilde{Y_{t+1}^c}}{\widetilde{Y_t^c}}(1+\pi_{t+1})\right]\right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2\varepsilon\right)$$

Monetary Policy:

$$\frac{R_{t,o}}{R_o} = ((1+\pi_t))^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

Internal Habit Formation:

$$X_t^{c,s} = \left(\frac{C_t^s}{A_t} - \omega\frac{C_{t-1}^{Aggs}}{A_{t-1}}\right).$$

Marginal costs incurred by Intermediate firms:

$$\zeta_t = \frac{1}{Z_{ct}}\frac{\tilde{w_t^s}}{Z_{ct}}$$

Leisure consumption choice of households:

$$s:7:(N_s)^\phi = \tilde{w^s}\left[(X^{c,s})\right]^{-\sigma}$$

Labour Decision:

$$N_{s,t}^c = \frac{\tilde{Y_t^c}}{Z_{ct}}$$

where the technology growth rate is given as:

$$z_t = \frac{A_t}{A_{t-1}}$$

Data:

This baseline model with no housing in the economy uses data from the U.S. This model is estimated using the seasonally adjusted quarterly data of the U.S. from period 1980 Quarter 1 to 2020 Quarter 1. I have taken three observed variables in the form of output, inflation and nominal interest rate. All this data is downloaded from the FRED, the inflation is calculated from the change in the implicit price deflator and all the data has been de-trended. To exactly correspond the actual observed variable to that of the model variable, I have added the equations which address this issue. We know that due to the perturbation techniques we used in solving the DSGE models in particular with Taylor approximating around the steady state, the output in the model is typically stationary. However, that of the observed output is non stationary, and as we need to correspond these observed and model output, I have augmented the model variable with the change in technology z_t. The inflation and interest rate however are stationary as observable and model variable and hence I have an observation equation for output in the model. In Particular, I have:

$$Y_{obs} = Y_t^c - Y_{t-1}^c + z$$

The model is augmented with the technology shock in the consumption sector which follows an AR(1) process:

$$\ln(Z_t^c) = \rho_c \ln\left(Z_{t-1}^c\right) + e_c$$

and an AR(1) process in the cost push shock to the New Keynesian Phillips Curve in the consumption goods sector:;

$$\ln(cp_t^c) = \rho_{cpc} \ln\left(cp_{t-1}^c\right) + e_{cpc}$$

I have also a monetary shock to the Taylor Rule:

$$\ln(ms_t) = \rho_{ms} \ln(ms_{t-1}) + e_{ms}$$

where e_c, e_{cpc}, e_{ms} are i.i.d processes with variances of σ_c, σ_{cpc} and σ_{ms} respectively and $\rho_c, \rho_{cpc}, \rho_{ms}$ are the shock persistence of the three shocks. Overall there are three shocks in the model as well as three observable variables and that will take care of the identification issue.

Calibration:

As some of the parameters are better estimated with a specific set of observed variables and some of the others are very difficult to estimate, I have fixed few of the parameters in my baseline model. I try to be consistent with the literature in this regard. I have calibrated the household's discount factor which is set at Savers' discount rate (θ) as 0.99 "Iacoviello and Pavan (2013) " . If we assume that the average markup equals 10 percent, then this implies the elasticity of substitution between the intermediate goods from the firms sector is ε = 11.

Prior Distributions:

I have taken the distributions of the priors and I estimate the posterior distributions of such parameters using the Metropolis Hasting methods; augmenting baseline no housing model with the shocks in the form of technology shock in consumption sector, monetary policy shock and a cost push shock in the consumption sector. As we know, the assumed distributions of priors play a huge role in the estimation of DSGE models

Table 9: Priors

Parameter	Prior Density	Prior Mean	Prior Standard Deviation
Std.dev technology shock	Inv.Gamma	0.001	0.01
Std.dev cost push shock	Inv.Gamma	0.001	0.01
Std.dev monetary shock	Inv.Gamma	0.001	0.01
Persistence technology shock	Beta	0.5	0.15
Persistence cost push shock	Beta	0.5	0.15
Persistence monetary shock	Beta	0.5	0.15
Habits	Beta	0.5	0.05
Inverse Frisch	Normal	2.5	0.25
Inverse Inter temporal elasticity of substitution	Normal	2.5	0.25
Taylor coefficient inflation	Gamma	2	0.05
Taylor coefficient output growth	Gamma	0.5	0.15
Interest rate smoothing	Beta	0.7	0.05

see An and Schorfheide (2007), I try to be consistent with the previous literature in terms of the prior distributions. For the standard errors of technology, monetary and cost push shocks I have always used the Inverse gamma distributions as the prior distributions with a prior mean of 0.001 and the prior standard deviation to be 0.01: Iacoviello and Neri (2010). For the persistence in all of the three shocks, I have followed in the footsteps of Smets, and Wouters (2007) and taken beta distributions as the prior distributions with a prior mean of 0.5 and a prior standard deviation of 0.15. In accordance with the paper by Rabanal (2018), I set the prior distribution of internal habit formation parameter to be a beta distribution with a prior mean of 0.5 and the prior standard deviation of 0.05; also the prior distribution of Inverse Frisch elasticity of labour ϕ follow a normal distribution with a mean of 2.5 and a standard deviation of 0.25. I chose the prior for Inverse inter temporal elasticity of substitution σ as a normal distribution with a prior mean of 2.5 and a standard deviation of 0.25: Chen, X., Kirsanova, T. and Leith, C. (2017). The coefficient of output ϕ_r and the coefficient of inflation in the Taylor rule ϕ_π of the model will also be estimated and the prior distributions for both these parameters are taken as gamma distributions. The prior mean on the Taylor Coefficient of inflation is taken to be 2.0 with a standard deviation of 0.05 and the prior mean on the Taylor Coefficient of output is taken to be 0.5 with a standard deviation of 0.15: Paez-Farrell (2015). The monetary policy smoothing and parameter on forward looking inflation in log linearized Hybrid NKPC is assumed to follow a beta prior distribution with a prior mean of 0.7 and 0.5; standard deviation of 0.05 and 0.15 respectively. The table below provides the overview for the priors I have taken.

Posterior Distributions:

I found a very similar posterior distributions in line with most of the literature. We tend to see the estimate of the Inverse Frisch elasticity of labour as 3, which is in accordance with William B. Peterman (2016). The estimate of habits show that households show a moderate degree of habit formation in the consumption. We could see that compared to that of an output deviation, U.S. had aggressively reacted to the inflation deviation in terms of policy reaction. It also shows a very low persistence in the interest rate where as relatively the persistence in technology and cost push shock are higher. The rest of the estimates are mostly in line with the literature. In order to check the stability of the parameters posterior distributions, graphically

Table 10: Posterior

Parameter	Prior Mean	Posterior Mean	Posterior 90 percent HPD
Std.dev technology shock	0.001	0.0063	[0.0057, 0.0070]
Std.dev cost push shock	0.001	0.0020	[0.0017, 0.0024]
Std.dev monetary shock	0.001	0.0074	[0.0061, 0.0087]
Persistence technology shock	0.5	0.4367	[0.3546, 0.5182]
Persistence cost push shock	0.5	0.9074	[0.8766, 0.9386]
Persistence monetary shock	0.5	0.2764	[0.1480, 0.4030]
Habits	0.5	0.5978	[0.5115, 0.6828]
Inverse Frisch	2.5	3.0699	[2.7055, 3.4242]
Inverse Inter temporal elasticity of substitution	2.5	2.0127	[1.6026, 2.4244]
Taylor coefficient inflation	2	2.0531	[1.9710, 2.1346]
Taylor coefficient output growth	0.5	0.1936	[0.0989, 0.2827]
Interest rate smoothing	0.7	0.5130	[0.4379, 0.5922]

I have assessed the convergence as described in Brooks and Gelman (1998). The following table presents the mean posterior estimates and associated 90 percent high probability densities of the posterior distributions of the parameters for the considered baseline model with no housing.

5.5 Baseline Model with Housing and no BTL:

In light of the above results, now I have tried to incorporate housing into the above model and estimate the parameters. In particular, I have Borrowers who are collaterally constrained as well as Savers in the economy. There are two sectors of firms from both housing and consumption goods. This models is solved and we will have the following private sector equilibrium:

Borrowers First order conditions are given as:

$$\left(N_{b,t}\right)^{\phi} = \tilde{w_{b,t}}\left[P_{c,t}\xi_t\right]$$

$$\begin{aligned}
0 = & \left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
& \frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_t^b}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H_t^b}\right)^{-\frac{1}{\rho}}\right) \\
& -\left[P_{c,t}\xi_t\right]\left(q_t^h\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h \\
& + \frac{1}{z_{t+1}}\beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)
\end{aligned}$$

$$\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$

$$\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,b}\right)^{-\frac{1}{\rho}}=$$

$$P_{c,t}\xi_t$$

$$0=[P_{c,t}\xi_t]-[P_{c,t}\Psi_t]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}}\frac{1}{z_{t+1}}$$

$$0=N_{b,t}\tilde{w_{b,t}}+\tilde{d_{t,d}}+\tilde{t_{b,t}}-\tilde{C_{b,t}}-q_t^h(\tilde{H_t^b}-(1-\delta)\tilde{H_{t-1}^b}\frac{1}{z_t})-R_{t-1,d}\tilde{d_{t-1,d}}\frac{1}{1+\pi_t}\frac{1}{z_t}$$

$$0=\mu q_t^h\tilde{H_{b,t}}-R_{t,d}\tilde{d_{t,d}}$$

Savers optimality conditions are as follows:

$$(N_{s,t})^{\phi}=\tilde{w_t^s}\left[P_{c,t}\lambda_t\right]$$

$$\begin{aligned}
0=&\\
&\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\\
&\frac{\rho}{\rho-1}\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right](1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H_{s,t}}\right)^{-\frac{1}{\rho}}-\left[P_{c,t}\lambda_t\right]\left(q_t^h\right)\\
&+\theta\left[P_{c,t+1}\lambda_{t+1}\right]\left(q_{t+1}^h(1-\delta)\frac{1}{z_{t+1}}\right)
\end{aligned}$$

$$\begin{aligned}
P_{c,t}\lambda_t=&\\
&\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\\
&\frac{\rho}{\rho-1}\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,s}\right)^{-\frac{1}{\rho}}
\end{aligned}$$

$$P_{c,t}\lambda_t=\theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}\frac{1}{z_{t+1}}$$

The two New Keynesian Phillips curves for both the consumption and housing goods sector respectively:

$$\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta+E_t\left[\theta\frac{[P_c\lambda]}{[P_c\lambda]}\left[(\pi)\frac{\widetilde{Y^c}}{\widetilde{Y^c}}(1+\pi)\right]\right]=\left(\pi(1+\pi)-\frac{1}{2}(\pi)^2\varepsilon\right)$$

$$\frac{(1-\varepsilon)}{\Omega_h}+\frac{\varepsilon}{\Omega_h}\frac{\eta}{q^h}+E_t\left[\theta\frac{[P_c\lambda]}{[P_c\lambda]}\left[(\pi_h)z\frac{\tilde{Y^h}}{\tilde{Y^h}}(1+\pi_h)\right]\right]=\left(\pi_h(1+\pi_h)-\frac{1}{2}(\pi_h)^2\varepsilon\right)$$

Interest Rate rule:

$$\frac{R_{t,o}}{R_o}=\left(\left((1+\pi_{h,t})^{cpih}(1+\pi_t)^{1-cpih}\right)\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

Internal Habit formation in both the agents:

$$X^{c,s}=\left(\frac{C^s}{A}-\omega\frac{C^{Aggs}}{A}\right).$$

$$X^{c,b}=\left(\frac{C^b}{A}-\omega\frac{C^{Aggs}}{A}\right).$$

Marginal costs in both the firms sectors:

$$\zeta=\left(\frac{\tilde{w_t^b}}{\nu}\right)^{\nu}\left(\frac{\tilde{w_t^s}}{1-\nu}\right)^{1-\nu}$$

$$\eta=\left(\frac{\tilde{w_t^b}}{\nu}\right)^{\nu}\left(\frac{\tilde{w_t^s}}{1-\nu}\right)^{1-\nu}$$

Aggregate output in consumption goods sector:

$$\tilde{Y^c}=\tilde{C_s}+\tilde{C_b}+\frac{\Omega}{2}\pi_c^2\tilde{Y^c}$$

Labour demand conditions:

$$N_b^c=\left(\frac{\tilde{w^b}}{\nu}\right)^{\nu-1}\left(\frac{\tilde{w^s}}{1-\nu}\right)^{1-\nu}\tilde{Y^c}$$

$$N_s^c=\left(\frac{\tilde{w^b}}{\nu}\right)^{\nu}\left(\frac{\tilde{w^s}}{1-\nu}\right)^{-\nu}\tilde{Y^c}$$

$$N_b^h=\left(\frac{\tilde{w^b}}{\nu}\right)^{\nu-1}\left(\frac{\tilde{w^s}}{1-\nu}\right)^{1-\nu}\tilde{Y^h}$$

$$N_s^h=\left(\frac{\tilde{w^b}}{\nu}\right)^{\nu}\left(\frac{\tilde{w^s}}{1-\nu}\right)^{-\nu}\tilde{Y^h}$$

Market Equilibrium conditions:

$$N^b=N_b^h+N_b^c$$

$$N^s = N^h_s + N^c_s$$

Aggregate output in housing goods sector:

$$\tilde{Y^h} = (\tilde{H_b} - (1-\delta)\tilde{H_b}) + (\tilde{H_s} - (1-\delta)\tilde{H_s})$$
$$+ \frac{\Omega_h}{2}\pi_h^2\tilde{Y^h}$$

Data:

This baseline model with Borrowers housing and collateral constraint but with no BTL in the economy uses data from the U.S. This model is estimated using the seasonally adjusted quarterly data of the U.S. from period 1980 Quarter 1 to 2020 Quarter 1. I have taken four observed variables in the form of output, inflation in consumption sector, inflation in the housing sector and nominal interest rate. All this data is downloaded from the FRED, the inflation in the consumption sector is calculated from the change in the implicit price deflator as well as the inflation in housing sector is calculated from the change in the House Price Index taken from All-Transactions House Price Index for the United States, Index 1980:Q1=100, Quarterly, Seasonally Adjusted and all the data has been de-trended. To exactly correspond the actual observed variable to that of the model variable, I have added the equations which address this issue. We know that due to the perturbation techniques we used in solving the DSGE models in particular with Taylor approximating around the steady state, the output in the model is typically stationary. However, that of the observed output is non stationary, and as we need to correspond these observed and model output, I have augmented the model variable with the change in technology z_t. The inflation and interest rate however are stationary as observable and model variable and hence I have an observation equation for output in the model. In Particular, I have:

$$Y_{obs} = Y^c_t - Y^c_{t-1} + z$$

The model is augmented with the technology shock in the firms sector which follows an AR(1) process and an AR(1) process in the cost push shock to the New Keynesian Phillips Curve in the consumption goods sector. I have also a monetary shock to the Taylor Rule. As discussed above, to estimate a set of parameters the number of shocks in the economy should be at least the same as the number of observed variables. Overall in the model, there are four observable variables and that leads us to have at least four shocks and exact four shocks will take care of the identification issue. On top of the three shocks mentioned above, the model is augmented with a shock on the downpayment ratio, cost push shock in housing sector.

Calibration:

As some of the parameters are better estimated with a specific set of observed variables and some of the others are very difficult to estimate, I have fixed few of the parameters in the baseline model. In particular, parameters such as the downpayment ratio μ is notoriously hard to estimate and need to have the data of

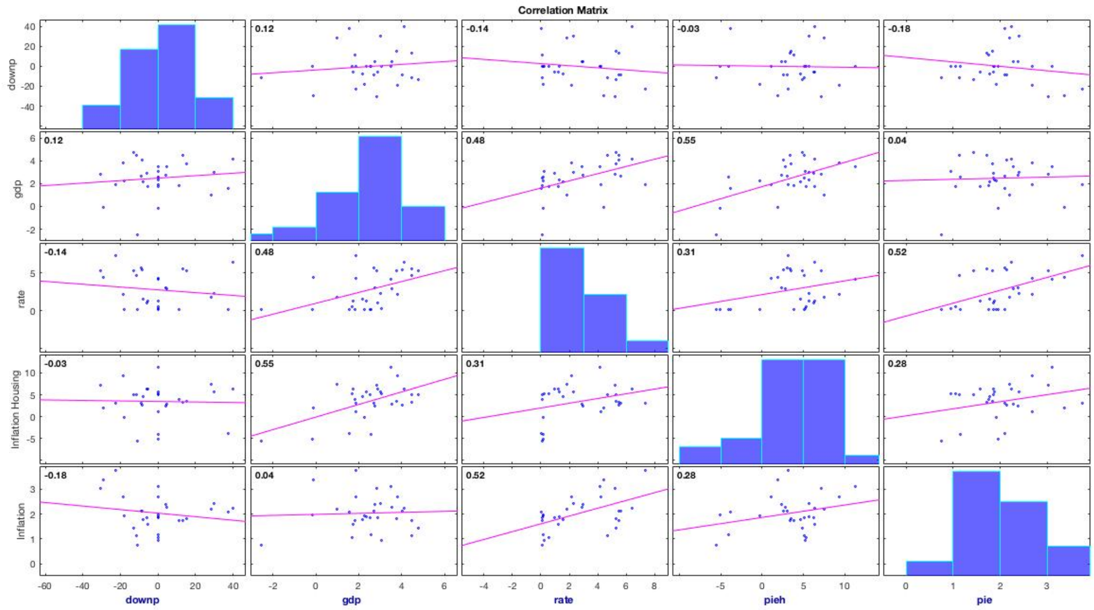


Figure 68: Correlation

observed variables such as housing share in the household debt see Iacoviello and Neri (2010). However, I in this section will analyze on which factors does the downpayment ratio will be rely on. To find a relationship between downpayment ratio and the rest of the macroeconomic variables, I ran a regression of annual U.S data from 1989 to 2019 taking median downpayment ratio in percentage terms as the dependent variable and aggregate output, interest rate, inflation in consumption goods sector and inflation in the housing goods sector as the independent variables. From the data, I found (We can see from the below correlation charts) that there is a positive correlation between downpayment ratio and aggregate output and a slight negative correlation (or approximately non correlation) between downpayment ratio and inflation in housing and consumption goods sectors as well as interest rates. This clearly shows us that the downpayment ratio is mostly determined by the structural variables within the financial intermediaries and is difficult to estimate from the data.

I try to be consistent with the literature for the rest of the calibrated parameters. I have calibrated the household's discount factor which is set at Savers' discount rate (θ) as 0.99 "Iacoviello and Pavan (2013) " . If I assume that the average markup equals 10 percent, then this implies the elasticity of substitution between the intermediate goods from the firms sector is $\varepsilon = 11$. The share of durables which is used as collateral (μ) is set to 0.90 "Kovacs and Moran (2019)". The discount factor of Borrowers is set at (β) of 0.98 "Kovacs and Moran (2019) ". I set the Quarterly house depreciation rate (δ) as 0.01 "Iacoviello and Neri (2010) ". With other parameters at share of profits paid to Savers (x) as 0.7, share of profits paid to hand to mouth workers (y) as 0.1.

Prior Distributions:

I have taken the distributions of the priors and I estimate the posterior distributions of such parameters using the Metropolis Hasting methods; augmenting baseline no housing model with the shocks in the form of technology shock in consumption sector, monetary policy shock, downpayment ratio shock and a cost push shock in the consumption sector. As we know, the assumed distributions of priors play a huge role in the estimation of DSGE models see : An and Schorfheide (2007), I try to be consistent with the previous literature in terms of the prior distributions. For the standard errors of technology, monetary, downpayment and cost push shocks I have always used the Inverse gamma distributions as the prior distributions with a prior mean of 0.001 and the prior standard deviation to be 0.01: Iacoviello and Neri (2010). For the persistence in all of the three shocks, I have followed in the footsteps of Smets, and Wouters (2007) and taken beta distributions as the prior distributions with a prior mean of 0.5 and a prior standard deviation of 0.05. In accordance with the paper by Rabanal (2018), I set the prior distribution of internal habit formation parameter to be a beta distribution with a prior mean of 0.5 and the prior standard deviation of 0.05; also the prior distribution of Inverse Frisch elasticity of labour ϕ follow a normal distribution with a mean of 2.5 and a standard deviation of 0.25. I chose the prior for Inverse inter temporal elasticity of substitution σ as a normal distribution with a prior mean of 2.5 and a standard deviation of 0.25: Chen, X., Kirsanova, T. and Leith, C. (2017). The coefficient of output ϕ_r and the coefficient of inflation in the Taylor rule ϕ_π of the model will also be estimated and the prior distributions for both these parameters are taken as gamma distributions. The prior mean on the Taylor Coefficient of inflation is taken to be 2.0 with a standard deviation of 0.05 and the prior mean on the Taylor Coefficient of output is taken to be 0.5 with a standard deviation of 0.15: Paez-Farrell (2015). The monetary policy smoothing and parameter on forward looking inflation in log linearized Hybrid NKPC is assumed to follow a beta prior distribution with a prior mean of 0.7 and 0.5; standard deviation of 0.05 and 0.15 respectively. Even though we have seen that the estimation of downpayment ratio is notoriously difficult to estimate unless with the financial intermediaries structural data and the households housing share in their debt, I have also tried to estimate downpayment ratio from the data by assuming that the macro prudential policy which regulates downpayment ratio is similar in flavor to that of the Taylor rule with a policy smoothing component. In particular, I have endogenized the downpayment ratio by following rule:

$$\frac{\mu_t}{\bar{\mu}} = \left(\left(\left(1+\pi_{h,t}\right)^{cpih}\left(1+\pi_t\right)^{1-cpih}\right)\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\left(\frac{R_t}{R}\right)^{\phi_r}$$

Where μ_t denotes the downpayment ratio and $\pi_{h,t}.\pi_t$ denotes the inflation in housing and consumption goods respectively and Y_t denotes the aggregate output as well as R_t denotes the interest rate. For the downpayment ratio, I have taken the prior distribution to be beta and the prior mean and standard deviation are assumed to be 0.5 and 0.15 respectively. This prior mean and standard deviation is assumed in the lines that the aggregate of the population will be have an average of 50 percent downpayment. The downpayment ratio coefficient of aggregate output ϕ_y are assumed to follow a normal distribution and to assess the non-correlation I obtained in the above section between downpayment ratio and output, interest rate and inflations in both consumption and housing sectors, I have assumed the prior mean to take a value of zero and the prior standard deviation to take a value of 0.15. On top of that, I have also tried to estimate the share of Borrowers in the economy: νassuming such parameter takes a prior distribution of beta with a prior mean of 20 percent Borrowers in the economy and a prior standard deviation of 5 percent change see Jappelli (1990) who estimates 20 percent of the population to be constrained.

The table below provides the overview for the priors I have taken.

Table 11: Priors

Parameter	Prior Density		Prior Mean	Prior Std Deviation
Std.dev technology shock	Inv.Gamma	σ_z	0.001	0.01
Std.dev cost push shock	Inv.Gamma	σ_{cpc}	0.001	0.01
Std.dev monetary shock	Inv.Gamma	σ_{ms}	0.001	0.01
Std.dev downpayment ratio shock	Inv.Gamma	σ_{smu}	0.001	0.01
Persistence technology shock	Beta	ρ_z	0.5	0.05
Persistence cost push shock	Beta	ρ_{cpc}	0.5	0.05
Persistence monetary shock	Beta	ρ_{ms}	0.5	0.05
Persistence downpayment ratio shock	Beta	ρ_{smu}	0.5	0.05
Habits	Beta	ω	0.5	0.05
Inverse Frisch	Normal	ϕ	2.5	0.25
Inverse Inter temporal elasticity of substitution	Normal	σ	2.5	0.25
Taylor coefficient inflation	Gamma	ϕ_π	2	0.05
Taylor coefficient output growth	Gamma	ϕ_r	0.5	0.15
Interest rate smoothing	Beta	ρ_r	0.7	0.05
Downpayment ratio coefficient inflation	Normal	ϕ_π	0	0.15
Downpayment ratio coefficient output growth	Normal	ϕ_y	0	0.15
Downpayment ratio coefficient housing inflation	Normal	$cpih$	0	0.15
Downpayment ratio coefficient interest rate	Normal	ϕ_r	0	0.15
Share of Borrowers in the economy	Beta	ν	0.2	0.05
Downpayment ratio smoothing	Beta	μ_r	0.5	0.15

Posterior Distributions:

I found a very similar posterior distributions in line with most of the literature as well as the previous model without housing. We tend to see the estimate of the Inverse Frisch elasticity of labour as 4 compared to that of a value of 3 in the previous model, which is in accordance with William B. Peterman (2016). This clearly shows that the elasticity of labour supply with respect to wages has gone down: as agents have a additional wealth effect from inclusion of housing and borrowing constraint in the model. The Borrowers can smooth their consumption with inclusion of collateral constraint and the Savers can be better off by supplying the credit in the economy. This leads to a decrease in agents labour elasticity. The estimate of habits show that households show a moderate degree of habit formation in the consumption as well. However, this internal habit formation of consumption is higher when there are inclusions of Borrowers and housing in the economy. As opposed to the previous model with only single sector of households in the economy, I believe this is the result of the inclusion of Borrowers in the economy who tend to form higher level of habit formations to smooth out their consumption in order to match the persistence in consumption of the data. This results in an increase in the estimation of the aggregate habit formations. We could see that in the context of monetary policy reaction, U.S. had aggressively reacted to the inflation deviation compared to that of an output deviation. However with the inclusion of collateral constraint on housing in the economy we expect the central banks policy response to slightly increase with both inflation and output deviations and we see such results. As the literature clearly shows that there is a strong impact of monetary policy on the housing markets, compared to that of a no housing estimation model, an inclusion of the housing markets has substantially increased the persistence and the standard deviation of the monetary shock. The rest of

Table 12: Posterior

Parameter	Prior Mean	Posterior Mean	Posterior 90 percent HPD
Std.dev technology shock	0.001	0.0077	[0.0069, 0.0085]
Std.dev cost push shock	0.001	0.0013	[0.0011, 0.0014]
Std.dev monetary shock	0.001	0.0104	[0.0094, 0.0115]
Std.dev downpayment ratio shock	0.001	0.0017	[0.0002, 0.0044]
Persistence technology shock	0.5	0.3891	[0.3472, 0.4283]
Persistence cost push shock	0.5	0.3493	[0.3213, 0.3678]
Persistence monetary shock	0.5	0.3691	[0.3433, 0.3984]
Persistence downpayment ratio shock	0.5	0.4939	[0.4731, 0.5188]
Habits	0.5	0.6575	[0.6379, 0.6784]
Inverse Frisch	2.5	3.9419	[3.8223, 4.0482]
Inverse Inter temporal elasticity of substitution	2.5	1.9702	[1.8742, 2.0441]
Taylor coefficient inflation	2	2.0858	[2.0385, 2.1299]
Taylor coefficient output growth	0.5	0.6953	[0.6814, 0.7083]
Interest rate smoothing	0.7	0.5415	[0.5164, 0.5641]
Downpayment ratio coefficient inflation	0	-0.3222	[-0.4082, -0.2360]
Downpayment ratio coefficient output growth	0	0.23736	[-0.53553, 0.79224]
Downpayment ratio coefficient housing inflation	0	-0.017029	[-0.50349, 0.44639]
Downpayment ratio coefficient interest rate	0	-0.1976	[-0.2508, -0.1630]
Share of Borrowers in the economy	0.2	0.1557	[0.1382, 0.1732]
Downpayment ratio smoothing	0.5	0.8785	[0.8011, 0.9549]

the estimates including the interest rate smoothing are mostly in line with the literature. However, in this model, due to the inclusion of Borrowers in the economy, I have tried to estimate the share of Borrowers in the economy: νassuming such parameter takes a prior distribution of beta with a prior mean of 20 percent Borrowers in the economy and a prior standard deviation of 5 percent change see Jappelli (1990) who estimates 20 percent of the population to be constrained. I found the data shows that there are around 16 percent of Borrowers in the economy with a maximum range of around 18 percent in the U.S from 1980 Q1 to 2020 Q1. As discussed in the above sections, I have also endogenized the downpayment ratio and estimated the several parameters on the Taylor type macro prudential downpayment rule. As seen from the data correlation chart in figure 68, I have found the estimates to be negative for all the downpayment ratio coefficients of aggregate housing inflation, consumption goods inflation and interest rate. As evident from the model, we could see a positive downpayment ratio coefficient on output growth and this shows that financial intermediaries base their downpayment ratio slightly on the aggregate output in the economy. However, I clearly believe that the structural decisions of the financial intermediaries will highly affect the downpayment ratio. In order to check the stability of the parameters posterior distributions, graphically I have assessed the convergence as described in Brooks and Gelman (1998).

The following table presents the mean posterior estimates for some of the important parameters to compare with the previous model and their associated 90 percent high probability densities of the posterior distributions of the parameters for the considered baseline model with housing in the economy.

5.6 Conclusion:

From the estimation of the two baseline models with and without housing, we found that the most of the estimates are in line with the literature. Inclusion of housing has substantially increased the effect of the monetary policy and it's persistence in the economy. Due to the inclusion of collateral constraint, agents are subjected a better wealth effects and this leads to them reacting less to the change in wages. This we could see from an increase in the inverse Frisch elasticity from 3 to 4. From the regression analysis, we have found out that the downpayment ratio is slightly positively correlated with the aggregate output and negatively correlated with the interest rate as well as the inflation in both consumption and housing sector. Similar results we tend to see in the estimation of such parameters using the model with housing. Habits have slightly increased with the inclusion of Borrowers in the economy as they are constrained and to account for the aggregate consumption levels, Borrowers tend to form higher internal habits in consumption. Overall the estimates suggest the increased wealth effect of housing and collateral constraints on agents. The average share of Borrowers is estimated to be around 16 percentage in the economy from 1980 to 2021 in the U.S.

6 Concluding Remarks and Further Research

This thesis sheds light on the role of housing markets and Buy to Let markets in affecting the volatility of the economy. However, there is a scope to further strengthen the evidence following this thesis. Chapter two examines the role of collateral constraints in a baseline model with no Buy to Let (BTL) sector in the economy. We found that when houses act as collateral, change in house prices have a substantial effect on the consumption of agents who are borrowing constrained. Furthermore, when amount of lending is based on income, we found that labour dynamics play a quantitatively important role. Borrowers are subjected to a substitution effect with a positive technology shock in housing market and this leads to an increase in net income of Borrowers. In turn this leads to an increase in the their housing of the Borrowers.

Chapter three examines the role of BTL markets on the volatility of house prices using a DSGE framework with Dixit Stiglitz Lite Utility. The results from the model indicate that by altering the size of BTL markets using downpayment ratio as a macro prudential policy has a very little effect on the volatility of house prices as opposed to the agent based model Baptista et al(2016).

This leads us to chapter four, which investigates the effects of a rich set of shocks including news shocks on the model economy with agents subjected to CES utility. The results indicate that labour markets plays an important role in most of the dynamics and the volatility stemmed from the Monetary policy shock is substantially high on housing market compared to that of the consumption goods market. We also found that Monetary policy is the only effective policy which affects the Hand to Mouth agents optimal choices. However, Macro prudential policy is more effective in both the Borrowers and Savers volatilities of choice variables.

Finally in chapter five, I take the baseline models augmented with habit formation: one without housing and the other one with housing, to data using Bayesian estimation techniques. The results show that the inclusion of housing has substantially increased the effect of the monetary policy and it's persistence in the economy. Habits have slightly increased with the inclusion of Borrowers in the economy as they are constrained and to account for the aggregate consumption levels, Borrowers tend to form higher internal habits in consumption.

For the possible further research on this thesis, it would be interesting to see how an increase in the house prices during the pandemic affects the consumption patterns of the agents in the economy. It would also be interesting to investigate the role of BTL markets in the rural areas if any. Further research can be helpful by introducing the segregation of Borrowers as first time Borrowers and second house Borrowers who are subjected to different interest rates. Such models can give us better insights to tackle BTL markets volatility on the house prices and the volatility of rents stemming from such markets. As the U.K government has increased the stamp duty for second home buyers, introduction of the Fiscal policy in the model can give us better insights on the volatility of BTL markets and to investigate which policy is better in curbing such volatilities in the economy. It would also be interesting to estimate a model with BTL markets in it for different countries where the data for rental markets and the share of housing in the household's debt is available. As such remarks are only meant to be suggestive, digging more in detail into the fiscal implications and structural determinants of the shocks with BTL markets is an important topic for future research.

A Appendix

A.1 The Model with Collateral Constraint

A.1.1 FOCs for Borrowers:

The Borrowers households will have the efficiency conditions for the Lagrangian as follows:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left(\begin{array}{c} \left(\frac{1}{1-\sigma} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{1-\sigma} - \frac{1}{1+\phi} \left(N_{b,t}\right)^{1+\phi} \right) \\ +\xi_t \left(\begin{array}{c} N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} \\ -P_{c,t}C_{b,t} - Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}D_{t-1,o} \end{array} \right) \\ +\Psi_t \left((1-\chi)Q_t^h H_{b,t} - R_{t,o}D_{t,o} \right) \end{array} \right)$$

FOCs:

$$\frac{\partial L}{\partial N_{b,t}} = U_{N_t}\left(C_{b,t}, N_{b,t}, H_{b,t}\right) + \xi_t W_{b,t} = -\left(N_{b,t}\right)^{\phi} + \xi_t W_{b,t}$$

$$\frac{\partial L}{\partial H_{b,t}} = \beta^t \left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha} \left(C_{b,t}\right)^{\alpha} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \right) - \xi_t Q_t^h + \Psi_t(1-\chi)Q_t^h \right) + \beta^{t+1}\xi_{t+1}Q_{t+1}^h(1-\delta)$$

$$\frac{\partial L}{\partial C_{b,t}} = \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_{b,t}\right)^{\alpha-1} \left(H_{b,t}\right)^{1-\alpha} - \xi_t P_{c,t}$$

$$\frac{\partial L}{\partial D_{t,o}} = \xi_t - \Psi_t R_{t,o} - \beta \xi_{t+1} R_{t,o}$$

$$\frac{\partial L}{\partial \xi_t} = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} - P_{c,t}C_{b,t} - Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}D_{t-1,o}$$

$$\frac{\partial L}{\partial \Psi_t} = (1-\chi)Q_t^h H_{b,t} - R_{t,o}D_{t,o}$$

$$\frac{\left(N_{b,t}\right)^{\phi}}{\xi_t} = W_{b,t}$$

$$0 = \left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha} \left(C_{b,t}\right)^{\alpha} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \right) - \xi_t Q_t^h + \Psi_t(1-\chi)Q_t^h \right) + \beta\xi_{t+1}Q_{t+1}^h(1-\delta)$$

$$\frac{\left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_{b,t}\right)^{\alpha-1} \left(H_{b,t}\right)^{1-\alpha}}{P_{c,t}} = \xi_t$$

$$0 = \xi_t - \Psi_t R_{t,o} - \beta \xi_{t+1} R_{t,o}$$

$$0 = N_{b,t} W_{b,t} + D_{t,o} + T_{b,t} - P_{c,t} C_{b,t} - Q_t^h (H_{b,t} - (1-\delta) H_{b,t-1}) - R_{t-1,o} D_{t-1,o}$$

$$0 = (1-\chi) Q_t^h H_{b,t} - R_{t,o} D_{t,o}$$

Representing all the FOCs In Real Terms:

$$\left(N_{b,t}\right)^{\phi} = \frac{W_{b,t}}{P_{c,t}} \left[P_{c,t} \xi_t\right]$$

$$0 = \left(\left((1-\alpha) \left(H_{b,t}\right)^{-\alpha} \left(C_{b,t}\right)^{\alpha} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \right) - \left[P_{c,t} \xi_t\right] \frac{Q_t^h}{P_{c,t}} + \left[P_{c,t} \Psi_t\right] (1-\chi) \frac{Q_t^h}{P_{c,t}} \right) + \beta \left[P_{c,t+1} \xi_{t+1}\right] \frac{Q_{t+1}^h}{P_{c,t+1}} (1-\delta)$$

$$\left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_{b,t}\right)^{\alpha-1} \left(H_{b,t}\right)^{1-\alpha} = P_{c,t} \xi_t$$

$$0 = \frac{\left[P_{c,t} \xi_t\right]}{P_{c,t}} - \left[P_{c,t} \Psi_t\right] \frac{R_{t,o}}{P_{c,t}} - \beta \left[P_{c,t+1} \xi_{t+1}\right] \frac{R_{t,o}}{P_{c,t}} \frac{1}{1+\pi_{t+1}}$$

$$0 = N_{b,t} \frac{W_{b,t}}{P_{c,t}} + \frac{D_{t,o}}{P_{c,t}} + \frac{T_{b,t}}{P_{c,t}} - C_{b,t} - \frac{Q_t^h}{P_{c,t}} (H_{b,t} - (1-\delta) H_{b,t-1}) - R_{t-1,o} \frac{D_{t-1,o}}{P_{c,t-1}} \frac{1}{1+\pi_t}$$

$$0 = (1-\chi) \frac{Q_t^h}{P_{c,t}} H_{b,t} - R_{t,o} \frac{D_{t,o}}{P_{c,t}}$$

for $x_t = \frac{X_t}{P_{c,t}}$ are:

$$\left(N_{b,t}\right)^{\phi} = w_{b,t} \left[P_{c,t} \xi_t\right]$$

$$0 = \left(\left((1-\alpha) \left(H_{b,t}\right)^{-\alpha} \left(C_{b,t}\right)^{\alpha} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} \right) - \left[P_{c,t} \xi_t\right] q_t^h + \left[P_{c,t} \Psi_t\right] (1-\chi) q_t^h \right) + \beta \left[P_{c,t+1} \xi_{t+1}\right] q_{t+1}^h (1-\delta)$$

$$\alpha \left(C_{b,t}\right)^{\alpha-1} \left(H_{b,t}\right)^{1-\alpha} \left(\left(C_{b,t}\right)^{\alpha} \left(H_{b,t}\right)^{1-\alpha} \right)^{-\sigma} = P_{c,t} \xi_t$$

$$0 = \left[P_{c,t} \xi_t\right] - \left[P_{c,t} \Psi_t\right] R_{t,o} - \beta \left[P_{c,t+1} \xi_{t+1}\right] R_{t,o} \frac{1}{1+\pi_{t+1}}$$

$$0 = N_{b,t}w_{b,t} + d_{t,0} + t_{b,t} - C_{b,t} - q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}d_{t-1,o}\frac{1}{1+\pi_t}$$

$$0 = (1-\chi)q_t^h H_{b,t} - R_{t,o}d_{t,o}$$

A.1.2 FOCs for Savers:

The Savers households will have the efficiency conditions for the Lagrangian as follows:

$$L = E_0\sum_{t=0}^{\infty}\theta^t U^s\left(C_{s,t},N_{s,t},H_{s,t}\right) + \lambda_t[N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} - P_{c,t}C_{s,t} - Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - B_{s,t}]$$

FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial N_t^s} &= U_{N_t}\left(C_{s,t},N_{s,t},H_{s,t}\right) + \lambda_t W_{s,t} \\
\frac{\partial L}{\partial H_t^s} &= \theta^t\left(U_{H_t}\left(C_{s,t},N_{s,t},H_{s,t}\right) - \lambda_t Q_t^h\right) + \theta^{t+1}\left(\lambda_{t+1}Q_{t+1}^h(1-\delta)\right) \\
\frac{\partial L}{\partial C_t^s} &= U_{C_t}\left(C_{s,t},N_{s,t},H_{s,t}\right) - \lambda_t P_{c,t} \\
\frac{\partial L}{\partial B_t^s} &= -\theta^t\lambda_t + \theta^{t+1}\lambda_{t+1}R_{t,s} \\
\frac{\partial L}{\partial \lambda_t} &= N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} - P_{c,t}C_{s,t} - Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - B_{s,t}
\end{aligned}$$

Representing all the FOCs In Real Terms:

$$\frac{-U_{N_t}\left(C_{s,t},N_{s,t},H_{s,t}\right)}{U_{C_t}\left(C_{s,t},N_{s,t},H_{s,t}\right)} = \frac{W_{s,t}}{P_{c,t}}$$

$$0 = U_{H_t}\left(C_{s,t},N_{s,t},H_{s,t}\right) - [P_{c,t}\lambda_t]\frac{Q_t^h}{P_{c,t}} + \theta\left([P_{c,t+1}\lambda_{t+1}]\frac{Q_{t+1}^h}{P_{c,t+1}}(1-\delta)\right)$$

$$P_{c,t}\lambda_t = U_{C_t}\left(C_{s,t},N_{s,t},H_{s,t}\right)$$

$$P_{c,t}\lambda_t\frac{1}{P_{c,t}} = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}\frac{1}{P_{c,t}}$$

$$0 = N_{s,t}\frac{W_{s,t}}{P_{c,t}} + \frac{T_{s,t}}{P_{c,t}} + \frac{B_{s,t-1}}{P_{c,t-1}}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - \frac{Q_t^h}{P_{c,t}}(H_{s,t} - (1-\delta)H_{s,t-1}) - \frac{B_{s,t}}{P_{c,t}}$$

For $x_t = \frac{X_t}{P_{c,t}}$:

$$\frac{(N_{s,t})^{\phi}}{\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\alpha(C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}}=w_t^s$$

$$0=\left((1-\alpha)(H_{s,t})^{-\alpha}(C_{s,t})^{\alpha}\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\right)-[P_{c,t}\lambda_t]\,q_t^h+\theta\left([P_{c,t+1}\lambda_{t+1}]\,q_{t+1}^h(1-\delta)\right)$$

$$P_{c,t}\lambda_t=\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\alpha(C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}$$

$$P_{c,t}\lambda_t=\theta\,[P_{c,t+1}\lambda_{t+1}]\,\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$0=N_{s,t}w_{s,t}+t_{st}+b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t}-C_{s,t}-q_t^h(H_{s,t}-(1-\delta)H_{s,t-1})-b_{s,t}$$

A.1.3 Housing Firms:

Employment Cost Minimization problem:

$$L=W_{b,t}N_{b,t}^h(i)+W_{s,t}N_{s,t}^h(i)-P_{ct}\eta_t\left(Z_{ht}N_{b,t}^h(i)^{v}N_{s,t}^h(i)^{1-v}-y_t^h(i)\right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{b,t}^h(i)} &= W_{b,t}-P_{ct}\eta_tZ_{ht}\nu N_{b,t}^h(i)^{v-1}N_{s,t}^h(i)^{1-v}=W_{b,t}-P_{ct}\eta_t\nu\frac{y_t^h(i)}{N_{b,t}^h(i)}=0\\
\frac{\partial L}{\partial N_{s,t}^h(i)} &= W_{s,t}-P_{ct}\eta_tZ_{ht}(1-\nu)N_{b,t}^h(i)^{v}N_{s,t}^h(i)^{-v}=W_{s,t}-P_{ct}\eta_t(1-\nu)\frac{y_t^h(i)}{N_{s,t}^h(i)}=0
\end{aligned}$$

$$\frac{\partial L}{\partial P_{ct}\eta_t}=Z_{ht}N_{b,t}^h(i)^{v}N_{s,t}^h(i)^{1-v}-y_t^h(i)=0$$

for $w_{jt}=\frac{W_{jt}}{P_t}$, FOCs are :

$$\begin{aligned}
w_{b,t}N_{b,t}^h(i) &= \nu\eta_t y_t^h(i)\\
w_{s,t}N_{s,t}^h(i) &= (1-\nu)\,\eta_t y_t^h(i)
\end{aligned}$$

It also follows that η_t can be derived as:

$$\begin{aligned}
w_{b,t}-\eta_tZ_{ht}\nu N_{b,t}^h(i)^{v-1}N_{s,t}^h(i)^{1-v} &= 0\\
w_{s,t}-\eta_tZ_{ht}(1-\nu)N_{b,t}^h(i)^{v}N_{s,t}^h(i)^{-v} &= 0
\end{aligned}$$

$$\left(\frac{w_{b,t}}{\eta_t Z_{ht}\nu}\right)^{\frac{1}{\nu-1}} = \frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}$$
$$\left(\frac{w_{s,t}}{\eta_t Z_{ht}(1-\nu)}\right)^{\frac{1}{\nu}} = \frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}$$

$$\eta_t = \frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

substitute η_t into the FOCs from the intermediate firm's cost minimization problem gives us the demand for two different labour inputs in this sector:

$$N_{b,t}^h(i) = y_t^h(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}$$
$$N_{s,t}^h(i) = y_t^h(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}$$

Aggregation yields (I denote $\Delta_t = \int\left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$

$$\begin{aligned}
N_{b,t}^h &= \int N_{b,t}^h(i)\,di = \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\int y_t^h(i)\,di \\
&= \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h \\
&= \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h \\
N_{s,t}^h &= \int N_{s,t}^h(i)\,di = \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}\int y_t^h(i)\,di \\
&= \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h \\
&= \frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h
\end{aligned}$$

Price setting:

$$N_{b,t}^h(i) = y_t^h(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}$$
$$N_{s,t}^h(i) = y_t^h(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}$$

$$
\begin{aligned}
&\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} \left(y_t^h(i)\, Q_t^h(i) - W_{s,t} N_{s,t}^h(i) - W_{b,t} N_{b,t}^h(i) \right) \\
= \;& y_t^h(i)\, Q_t^h(i) - W_{s,t} \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} y_t^h(i) \\
& - W_{b,t} \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} y_t^h(i) \\
= \;& y_t^h(i) \left(Q_t^h(i) - \eta_t P_{ct} \left(\frac{W_{b,t}}{P_{ct}} \left(\frac{w_{b,t}}{\nu} \right)^{-1} + \frac{W_{s,t}}{P_{ct}} \left(\frac{w_{s,t}}{1-\nu} \right)^{(-1)} \right) \right) \\
= \;& y_t^h(i) \left(Q_t^h(i) - \eta_t P_{ct} \right) = \left(y_t^h(i)\, Q_t^h(i) - y_{ht}(i)\, MC_t \right)
\end{aligned}
$$

$$
\eta_t = \frac{1}{Z_{ht}} \left(\frac{w_{b,t}}{\nu} \right)^{\nu} \left(\frac{w_{s,t}}{1-\nu} \right)^{1-\nu}
$$

where $MC_t = \eta_t P_{ct}$ .Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms which also implies that the intermediate-good producing firms each have the same

real marginal costs of production. . So we come to familiar formulation

$$
\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s} \left(y_t^h(i)\, Q_t^h(i) - y_t^h(i)\, MC_s \right)
$$

subject to

$$
y_t^h(i) = Y_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon}
$$

Substitute demand

$$
\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left(Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{1-\varepsilon} - \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} MC_t \right)
$$

FOCs

$$
0 = \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left((1-\varepsilon)\, Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} + \varepsilon \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon-1} MC_t \right)
$$

$$
\left(\frac{Q_t^h(i)}{Q_t^h} \right) \frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}
$$

Aggregate

$$
\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}
$$

substitute and we obtain the equation for the house prices:

$$
\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \left(\frac{w_{b,t}}{\nu} \right)^{\nu} \frac{1}{Z_{ht}} \left(\frac{w_{s,t}}{1-\nu} \right)^{1-\nu}
$$

A.1.4 Consumption good Firms:

Cost Minimization:

The cost minimization problem of the intermediate good producer by choosing labour $N^c_{b,t}(i), N^c_{s,t}(i)$ will go down to as :

$$L = W_{s,t}N^c_{s,t}(i) + W_{b,t}N^c_{b,t}(i) - P_{ct}\zeta_t\left(Z_{ct}N^c_{b,t}(i)^{\nu}N^c_{s,t}(i)^{1-\nu} - y^c_t(i)\right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N^c_{b,t}(i)} &= W_{b,t} - P_{ct}\zeta_t Z_{ct}\nu N^c_{b,t}(i)^{\nu-1}N^c_{s,t}(i)^{1-\nu} = W_{b,t} - P_{ct}\zeta_t\nu\frac{y^c_t(i)}{N^c_{b,t}(i)} = 0 \\
\frac{\partial L}{\partial N^c_{s,t}(i)} &= W_{s,t} - P_{ct}\zeta_t Z_{ct}(1-\nu)N^c_{b,t}(i)^{\nu}N^c_{s,t}(i)^{-\nu} = W_{s,t} - P_{ct}\zeta_t(1-\nu)\frac{y^c_t(i)}{N^c_{s,t}(i)} = 0
\end{aligned}$$

From where

$$\begin{aligned}
W_{b,t}N^c_{b,t}(i) &= P_{ct}\zeta_t\nu y^c_t(i) \\
W_{s,t}N^c_{s,t}(i) &= P_{ct}\zeta_t(1-\nu)y^c_t(i)
\end{aligned}$$

$$\begin{aligned}
w_{b,t}N^c_{b,t}(i) &= \zeta_t\nu y^c_t(i) \\
w_{s,t}N^c_{s,t}(i) &= \zeta_t(1-\nu)y^c_t(i)
\end{aligned}$$

It also follows that ζ_t can be derived as:

$$\begin{aligned}
w_{b,t} &= \zeta_t\nu Z_{ct}N^c_{b,t}(i)^{\nu-1}N^c_{s,t}(i)^{1-\nu} \\
w_{s,t} &= \zeta_t(1-\nu)Z_{ct}N^c_{b,t}(i)^{\nu}N^c_{s,t}(i)^{-\nu}
\end{aligned}$$

$$\begin{aligned}
\left(\frac{w_{b,t}}{\zeta_t Z_{ct}\nu}\right)^{\frac{1}{\nu-1}} &= \frac{N^c_{b,t}(i)}{N^c_{s,t}(i)} \\
\left(\frac{w_{s,t}}{\zeta_t Z_{ct}(1-\nu)}\right)^{\frac{1}{\nu}} &= \frac{N^c_{b,t}(i)}{N^c_{s,t}(i)}
\end{aligned}$$

$$\zeta_t = \frac{1}{Z_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

substitute ζ_t into the FOCs from the intermediate firm's cost minimization problem gives us the demand for two different labour inputs in this sector:

$$
\begin{aligned}
N_{b,t}^{c}(i) &= y_t^c(i)\frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\
N_{s,t}^{c}(i) &= y_t^c(i)\frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}
\end{aligned}
$$

Aggregation yields (we denote $\Delta_t = \int\left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$

$$
\begin{aligned}
N_{b,t}^{c} &= \int N_{b,t}^{c}(i)\,di = \frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\int y_t^c(i)\,di \\
&= \frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\int Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon} di \\
&= \frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^c \\
N_{s,t}^{c} &= \int N_{ct}(i)\,di = \frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}\int y_t^c(i)\,di \\
&= \frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}\int Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon} di \\
&= \frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^c
\end{aligned}
$$

Price Setting:

Firms choose prices to maximize expected profit:

$$
\begin{aligned}
&\max_{\{p_{ct}^*(i)\}_{s=t}^{\infty}} \mathscr{E}_t\sum_{s=t}^{\infty} Q_{t,s}\left(y_t^c(i)P_{ct}(i) - W_{s,t}N_{s,t}^c(i) - W_{b,t}N_{b,t}^c(i)\right) \\
=\;& \left(y_t^c(i)P_{ct}(i) - W_t^s\frac{1}{Z_{ct}}\frac{(w_{b,t})^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}y_t^c(i) - W_t^b\frac{1}{Z_{ct}}\frac{(w_{b,t})^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}y_t^c(i)\right) \\
=\;& \left(y_t^c(i)\left(P_{ct}(i) - Z_{ct}\zeta_t P_{ct}\left(\frac{W_{b,t}}{P_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{-1} + \frac{W_{s,t}}{P_{ct}}\left(\frac{w_{s,t}}{1-\nu}\right)^{(-1)}\right)\right)\right) \\
=\;& \left(y_t^c(i)P_{ct}(i) - y_t^c(i)MC_s\right)
\end{aligned}
$$

where $MC_t = \zeta_t P_t$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation

$$
\max_{\{p_s^*(i)\}_{s=t}^{\infty}} \mathscr{E}_t\sum_{s=t}^{\infty} Q_{t,s}\left(y_t^c(i)P_{ct}(i) - y_t^c(i)MC_s\right)
$$

Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices.

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_{b,t}}{\nu}\right)^{\nu} \left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

where $MC_s = \zeta_t P_{ct}$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1\right)^2 y_t^c(i)$. This yields us :

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(y_t^c(i) \frac{P_{ct}(i)}{P_{ct}} - y_t^c(i) \frac{MC_t}{P_{ct}} \right) - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 y_t^c(i) \right]$$

subject to Intermediate goods demand equation :

$$y_t^c(i) = Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon}$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

Let $\frac{P_{ct}(i)}{P_{ct}} = \widetilde{P_{ct}}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

where $1+\pi_t$is the Gross inflation in the aggregate price level of the consumption goods side.

$$\max_{\left\{\widetilde{P_{ct}}\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

FOC w.r.t $\widetilde{P_{ct}}$:

$$0 = \frac{\partial}{\partial \widetilde{P_{ct}}} \left(E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right] \right)$$

$$= (1-\varepsilon) \left(\widetilde{P_{ct}} \right)^{-\varepsilon} Y_t^c + \varepsilon \left(\widetilde{P_{ct}} \right)^{-\varepsilon-1} \zeta_t Y_t^c$$

$$-\Omega\left(\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)\frac{(1+\pi_t)}{\widetilde{P_{ct-1}}}\left(\widetilde{P_{ct}}\right)^{-\varepsilon}-\frac{1}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2\varepsilon\left(\widetilde{P_{ct}}\right)^{-\varepsilon-1}\right)Y_t^c$$

$$+\Omega\theta m_{t+1}\left[\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}}-1\right)Y_{t+1}^c\left(\widetilde{P_{ct+1}}\right)^{-\varepsilon}\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}^2}\right)\right]$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = Z_{ct}\zeta_t P_{ct}$which implies the relative price $\widetilde{P_{ct}}$is equal to 1. substitute and we obtain the equation for the aggregate inflation:

$$0=(1-\varepsilon)Y_t^c+\varepsilon\zeta_t Y_t^c-\Omega Y_t^c\left((\pi_t)(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)+\Omega\theta m_{t,t+1}\left[(\pi_{t+1})Y_{t+1}^c(1+\pi_{t+1})\right]$$

$$\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+\theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)\to\pi_t$$

$$\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+E_t\left[\theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right]\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)$$

A.1.5 Private Sector Equilibrium

$$b:1:\left(N_{b,t}\right)^{\phi}=w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$b:2:0=$$
$$\left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_t\right]q_t^h+\left[P_{c,t}\Psi_t\right](1-\chi)q_t^h\right)$$
$$+\beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)$$

$$b:3:\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}=P_{c,t}\xi_t$$

$$b:4:0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,o}\frac{1}{1+\pi_{t+1}}$$

$$b:5:0=N_{b,t}w_{b,t}+d_{t,0}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}d_{t-1,o}\frac{1}{1+\pi_t}$$

$$b:6:0=(1-\chi)q_t^hH_{b,t}-R_{t,o}d_{t,o}$$

$$s:7:(N_{s,t})^{\phi}=w_{s,t}\left[P_{c,t}\lambda_t\right]$$

$$s:8:0=\left((1-\alpha)\left(H_{s,t}\right)^{-\alpha}\left(C_{s,t}\right)^{\alpha}\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\lambda_t\right]q_t^h+\theta\left(\left[P_{c,t+1}\lambda_{t+1}\right]q_{t+1}^h(1-\delta)\right)$$

$$s:9:P_{c,t}\lambda_t=\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}$$

$$s:10:P_{c,t}\lambda_t=\theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$s:11:Y_t^c=C_{b,t}+C_{s,t}+\frac{\Omega}{2}\pi_t^2Y_t^c$$

$$\begin{aligned}
fc &: 12:\frac{(1-\varepsilon)+\varepsilon\zeta_t}{\Omega}+\theta_t\frac{\left[P_{c,t+1}\lambda_{t+1}\right]}{\left[P_{c,t}\lambda_t\right]}\left[\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})(\pi_{t+1})\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)\\
fc &: 13:N_{b,t}^c=\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^c\\
fc &: 14:N_{s,t}^c=\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^c
\end{aligned}$$

$$\begin{aligned}
fh &: 15:q_t^h=-\frac{\varepsilon}{(1-\varepsilon)}\frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}\\
fh &: 16:N_{b,t}^h=\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h\\
fh &: 17:N_{s,t}^h=\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h
\end{aligned}$$

$$\begin{aligned}
mc &: 18:B_{s,t}=D_{t,o}\\
mc &: 19:N_t^b=N_{b,t}^h+N_{b,t}^c\\
mc &: 20:N_t^s=N_{s,t}^h+N_{s,t}^c
\end{aligned}$$

$$
\begin{aligned}
21: t_{bt} &= (1-x)\left(Y_t^c - w_{s,t}N_{s,t}^c - w_{b,t}N_{b,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c + Y_t^h q_t^h - w_{s,t}N_{s,t}^h - w_{b,t}N_{b,t}^h\right) \\
22: t_{st} &= x\left(Y_t^c - w_{s,t}N_{s,t}^c - w_{b,t}N_{b,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c + Y_t^h q_t^h - w_{s,t}N_{s,t}^h - w_{b,t}N_{b,t}^h\right)
\end{aligned}
$$

$$
23 \quad : \quad Y_t^h = (H_{b,t} - (1-\delta)H_{b,t-1}) + (H_{s,t} - (1-\delta)H_{s,t-1})
$$

$$
24: \frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}
$$

$$
25: \zeta_t = \frac{1}{Z_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}
$$

$$
26: R_{t,o} = R_{t,s}
$$

where

$$
\begin{aligned}
m_{t,t+1} &= \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right] \\
\frac{1}{R_{t,s}} &= \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]
\end{aligned}
$$

A.2 The Model with amount of lending based on income

A.2.1 FOCs for Borrowers:

$$
\begin{aligned}
\frac{\partial L}{\partial N_{b,t}} &= U_{N_t}\left(C_{b,t}, N_{b,t}, H_{b,t}\right) + \xi_t W_{b,t} + \Psi_t \mu W_{b,t} \\
&= -\left(N_{b,t}\right)^{\phi} + \xi_t W_{b,t} + \Psi_t \mu W_{b,t}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial H_{b,t}} = \beta^t &\left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \xi_t Q_t^h\right) \\
&+ \beta^{t+1}\xi_{t+1}Q_{t+1}^h(1-\delta)
\end{aligned}
$$

$$
\frac{\partial L}{\partial C_{b,t}} = \left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha} - \xi_t P_{c,t}
$$

$$
\frac{\partial L}{\partial D_{t,o}} = \xi_t - \Psi_t R_{t,o} - \beta\xi_{t+1}R_{t,o}
$$

$$
\frac{\partial L}{\partial \xi_t} = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} - P_{c,t}C_{b,t} - Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}D_{t-1,o}
$$

$$
\frac{\partial L}{\partial \Psi_t} = \mu N_{b,t}W_{b,t} - R_{t,o}D_{t,o}
$$

$$
\frac{\left(N_{b,t}\right)^{\phi}}{\xi_t + \Psi_t\mu} = W_{b,t}
$$

$$
\begin{aligned}
0 = &\left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \xi_t Q_t^h\right) \\
&+ \beta\xi_{t+1}Q_{t+1}^h(1-\delta)
\end{aligned}
$$

$$
\frac{\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}}{P_{c,t}} = \xi_t
$$

$$
0 = \xi_t - \Psi_t R_{t,o} - \beta\xi_{t+1}R_{t,o}
$$

$$
0 = N_{b,t}W_{b,t} + D_{t,o} + T_{b,t} - P_{c,t}C_{b,t} - Q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}D_{t-1,o}
$$

$$
0 = \mu N_{b,t}W_{b,t} - R_{t,o}D_{t,o}
$$

Representing all the FOCs In Real Terms:

$$\left(N_{b,t}\right)^{\phi}=\frac{W_{b,t}}{P_{c,t}}\left[P_{c,t}\xi_t+P_{c,t}\Psi_t\mu\right]$$

$$0=\left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_t\right]\frac{Q_t^h}{P_{c,t}}\right)$$
$$+\beta\left[P_{c,t+1}\xi_{t+1}\right]\frac{Q_{t+1}^h}{P_{c,t+1}}(1-\delta)$$

$$\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}=P_{c,t}\xi_t$$

$$0=\frac{\left[P_{c,t}\xi_t\right]}{P_{c,t}}-\left[P_{c,t}\Psi_t\right]\frac{R_{t,o}}{P_{c,t}}-\beta\left[P_{c,t+1}\xi_{t+1}\right]\frac{R_{t,o}}{P_{c,t}}\frac{1}{1+\pi_{t+1}}$$

$$0=N_{b,t}\frac{W_{b,t}}{P_{c,t}}+\frac{D_{t,o}}{P_{c,t}}+\frac{T_{b,t}}{P_{c,t}}-C_{b,t}-\frac{Q_t^h}{P_{c,t}}(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}\frac{D_{t-1,o}}{P_{c,t-1}}\frac{1}{1+\pi_t}$$

$$0=\mu N_{b,t}\frac{W_{b,t}}{P_{c,t}}-R_{t,o}\frac{D_{t,o}}{P_{c,t}}$$

For $x_t=\frac{X_t}{P_{c,t}}$:

$$\left(N_{b,t}\right)^{\phi}=w_{b,t}\left[P_{c,t}\xi_t+\mu P_{c,t}\Psi_t\right]$$

$$0=\left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_t\right]q_t^h\right)+\beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)$$

$$\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}=P_{c,t}\xi_t$$

$$0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,o}\frac{1}{1+\pi_{t+1}}$$

$$0=N_{b,t}w_{b,t}+d_{t,0}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}d_{t-1,o}\frac{1}{1+\pi_t}$$

$$0=\mu N_{b,t}w_{b,t}-R_{t,o}d_{t,o}$$

A.2.2 FOCs for Savers:

FOCs:

$$
\begin{aligned}
\frac{\partial L}{\partial N_t^s} &= U_{N_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) + \lambda_t W_{s,t} \\
\frac{\partial L}{\partial H_t^s} &= \theta^t \left(U_{H_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t Q_t^h\right) + \theta^{t+1}\left(\lambda_{t+1} Q_{t+1}^h (1-\delta)\right) \\
\frac{\partial L}{\partial C_t^s} &= U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t P_{c,t} \\
\frac{\partial L}{\partial B_t^s} &= -\theta^t \lambda_t + \theta^{t+1} \lambda_{t+1} R_{t,s} \\
\frac{\partial L}{\partial \lambda_t} &= N_{s,t} W_{s,t} + R_{t-1,s} B_{s,t-1} + T_{s,t} - P_{c,t} C_{s,t} - Q_t^h (H_{s,t} - (1-\delta) H_{s,t-1}) - B_{s,t}
\end{aligned}
$$

Representing all the FOCs In Real Terms:

$$\frac{-U_{N_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)}{U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)} = \frac{W_{s,t}}{P_{c,t}}$$

$$0 = U_{H_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \left[P_{c,t}\lambda_t\right] \frac{Q_t^h}{P_{c,t}} + \theta \left(\left[P_{c,t+1}\lambda_{t+1}\right] \frac{Q_{t+1}^h}{P_{c,t+1}} (1-\delta)\right)$$

$$P_{c,t}\lambda_t = U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)$$

$$P_{c,t}\lambda_t \frac{1}{P_{c,t}} = \theta \left[P_{c,t+1}\lambda_{t+1}\right] \frac{R_{t,s}}{1+\pi_{t+1}} \frac{1}{P_{c,t}}$$

$$0 = N_{s,t}\frac{W_{s,t}}{P_{c,t}} + \frac{T_{s,t}}{P_{c,t}} + \frac{B_{s,t-1}}{P_{c,t-1}} \frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - \frac{Q_t^h}{P_{c,t}} (H_{s,t} - (1-\delta) H_{s,t-1}) - \frac{B_{s,t}}{P_{c,t}}$$

For $x_t = \frac{X_t}{P_{c,t}}$:

$$\frac{\left(N_{s,t}\right)^{\phi}}{\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma} \alpha \left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}} = w_t^s$$

$$0 = \left((1-\alpha)\left(H_{s,t}\right)^{-\alpha}\left(C_{s,t}\right)^{\alpha}\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \left[P_{c,t}\lambda_t\right] q_t^h + \theta \left(\left[P_{c,t+1}\lambda_{t+1}\right] q_{t+1}^h (1-\delta)\right)$$

$$P_{c,t}\lambda_t = \left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma} \alpha \left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}$$

$$P_{c,t}\lambda_t = \theta \left[P_{c,t+1}\lambda_{t+1}\right] \frac{R_{t,s}}{1+\pi_{t+1}}$$

$$0 = N_{s,t} w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h (H_{s,t} - (1-\delta) H_{s,t-1}) - b_{s,t}$$

A.2.3 Housing Firms:

Employment Cost Minimization problem:

$$L = W_{b,t}N_{b,t}^{h}(i) + W_{s,t}N_{s,t}^{h}(i) - P_{ct}\eta_t\left(Z_{ht}N_{b,t}^{h}(i)^{\nu}N_{s,t}^{h}(i)^{1-\nu} - y_t^{h}(i)\right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{b,t}^{h}(i)} &= W_{b,t} - P_{ct}\eta_t Z_{ht}\nu N_{b,t}^{h}(i)^{\nu-1}N_{s,t}^{h}(i)^{1-\nu} = W_{b,t} - P_{ct}\eta_t\nu\frac{y_t^{h}(i)}{N_{b,t}^{h}(i)} = 0 \\
\frac{\partial L}{\partial N_{s,t}^{h}(i)} &= W_{s,t} - P_{ct}\eta_t Z_{ht}(1-\nu)N_{b,t}^{h}(i)^{\nu}N_{s,t}^{h}(i)^{-\nu} = W_{s,t} - P_{ct}\eta_t(1-\nu)\frac{y_t^{h}(i)}{N_{s,t}^{h}(i)} = 0
\end{aligned}$$

$$\frac{\partial L}{\partial P_{ct}\eta_t} = Z_{ht}N_{b,t}^{h}(i)^{\nu}N_{s,t}^{h}(i)^{1-\nu} - y_t^{h}(i) = 0$$

for $w_{jt} = \frac{W_{jt}}{P_t}$, FOCs are :

$$\begin{aligned}
w_{b,t}N_{b,t}^{h}(i) &= \nu\eta_t y_t^{h}(i) \\
w_{s,t}N_{s,t}^{h}(i) &= (1-\nu)\eta_t y_t^{h}(i)
\end{aligned}$$

It also follows that η_t can be derived as:

$$\begin{aligned}
w_{b,t} - \eta_t Z_{ht}\nu N_{b,t}^{h}(i)^{\nu-1}N_{s,t}^{h}(i)^{1-\nu} &= 0 \\
w_{s,t} - \eta_t Z_{ht}(1-\nu)N_{b,t}^{h}(i)^{\nu}N_{s,t}^{h}(i)^{-\nu} &= 0
\end{aligned}$$

$$\begin{aligned}
\left(\frac{w_{b,t}}{\eta_t Z_{ht}\nu}\right)^{\frac{1}{\nu-1}} &= \frac{N_{b,t}^{h}(i)}{N_{s,t}^{h}(i)} \\
\left(\frac{w_{s,t}}{\eta_t Z_{ht}(1-\nu)}\right)^{\frac{1}{\nu}} &= \frac{N_{b,t}^{h}(i)}{N_{s,t}^{h}(i)}
\end{aligned}$$

$$\eta_t = \frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

substitute η_t into the FOCs from the intermediate firm's cost minimization problem gives us the demand for two different labour inputs in this sector:

$$\begin{aligned}
N_{b,t}^{h}(i) &= y_t^{h}(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\
N_{s,t}^{h}(i) &= y_t^{h}(i)\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}
\end{aligned}$$

Aggregation yields (we denote $\Delta_t = \int \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$

$$
\begin{aligned}
N_{b,t}^{h} &= \int N_{b,t}^{h}(i)\,di = \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \int y_t^h(i)\,di \\
&= \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_t^h \\
&= \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} Y_t^h \\
N_{s,t}^{h} &= \int N_{s,t}^{h}(i)\,di = \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} \int y_t^h(i)\,di \\
&= \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} Y_t^h \\
&= \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} Y_t^h
\end{aligned}
$$

Price setting:

$$
\begin{aligned}
N_{b,t}^{h}(i) &= y_t^h(i) \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} \\
N_{s,t}^{h}(i) &= y_t^h(i) \frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}
\end{aligned}
$$

$$
\begin{aligned}
&\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} \left(y_t^h(i)\,Q_t^h(i) - W_{s,t}N_{s,t}^h(i) - W_{b,t}N_{b,t}^h(i)\right) \\
&= y_t^h(i)\,Q_t^h(i) - W_{s,t}\frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}} \frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}} y_t^h(i) - W_{b,t}\frac{1}{Z_{ht}} \frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}} \frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}} y_t^h(i) \\
&= y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\left(\frac{W_{b,t}}{P_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{-1} + \frac{W_{s,t}}{P_{ct}}\left(\frac{w_{s,t}}{1-\nu}\right)^{(-1)}\right)\right) \\
&= y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\right) = \left(y_t^h(i)\,Q_t^h(i) - y_{ht}(i)\,MC_t\right)
\end{aligned}
$$

$$
\eta_t = \frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}
$$

where $MC_t = \eta_t P_{ct}$ .Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms which also implies that the intermediate-good producing firms each have the same

real marginal costs of production. . So we come to familiar formulation

$$
\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s}\left(y_t^h(i)\,Q_t^h(i) - y_t^h(i)\,MC_s\right)
$$

subject to

$$y_t^h(i) = Y_t^h \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}$$

Substitute demand

$$\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left(Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{1-\varepsilon} - \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon} MC_t \right)$$

FOCs

$$0 = \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left((1-\varepsilon) Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon} + \varepsilon \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon-1} MC_t \right)$$

$$\left(\frac{Q_t^h(i)}{Q_t^h}\right) \frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}$$

Aggregate

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}$$

substitute and we obtain the equation for the house prices:

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \left(\frac{w_{b,t}}{\nu}\right)^{\nu} \frac{1}{Z_{ht}} \left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

A.2.4 Consumption good Firms:

Cost Minimization:

The cost minimization problem of the intermediate good producer by choosing labour $N_{b,t}^c(i), N_{s,t}^c(i)$ will go down to as :

$$L = W_{s,t} N_{s,t}^c(i) + W_{b,t} N_{b,t}^c(i) - P_{ct}\zeta_t \left(Z_{ct} N_{b,t}^c(i)^{\nu} N_{s,t}^c(i)^{1-\nu} - y_t^c(i) \right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{b,t}^c(i)} &= W_{b,t} - P_{ct}\zeta_t Z_{ct} \nu N_{b,t}^c(i)^{\nu-1} N_{s,t}^c(i)^{1-\nu} = W_{b,t} - P_{ct}\zeta_t \nu \frac{y_t^c(i)}{N_{b,t}^c(i)} = 0 \\
\frac{\partial L}{\partial N_{s,t}^c(i)} &= W_{s,t} - P_{ct}\zeta_t Z_{ct} (1-\nu) N_{b,t}^c(i)^{\nu} N_{s,t}^c(i)^{-\nu} = W_{s,t} - P_{ct}\zeta_t (1-\nu) \frac{y_t^c(i)}{N_{s,t}^c(i)} = 0
\end{aligned}$$

From where

$$\begin{aligned}
W_{b,t} N_{b,t}^c(i) &= P_{ct}\zeta_t \nu y_t^c(i) \\
W_{s,t} N_{s,t}^c(i) &= P_{ct}\zeta_t (1-\nu) y_t^c(i)
\end{aligned}$$

$$
\begin{aligned}
w_{b,t}N^{c}_{b,t}\left(i\right) &= \zeta_{t}\nu y^{c}_{t}\left(i\right) \\
w_{s,t}N^{c}_{s,t}\left(i\right) &= \zeta_{t}\left(1-\nu\right)y^{c}_{t}\left(i\right)
\end{aligned}
$$

It also follows that ζ_t can be derived as:

$$
\begin{aligned}
w_{b,t} &= \zeta_{t}\nu Z_{ct}N^{c}_{b,t}\left(i\right)^{\nu-1}N^{c}_{s,t}\left(i\right)^{1-\nu} \\
w_{s,t} &= \zeta_{t}\left(1-\nu\right)Z_{ct}N^{c}_{b,t}\left(i\right)^{\nu}N^{c}_{s,t}\left(i\right)^{-\nu}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{w_{b,t}}{\zeta_{t}Z_{ct}\nu}\right)^{\frac{1}{\nu-1}} &= \frac{N^{c}_{b,t}\left(i\right)}{N^{c}_{s,t}\left(i\right)} \\
\left(\frac{w_{s,t}}{\zeta_{t}Z_{ct}\left(1-\nu\right)}\right)^{\frac{1}{\nu}} &= \frac{N^{c}_{b,t}\left(i\right)}{N^{c}_{s,t}\left(i\right)}
\end{aligned}
$$

$$
\zeta_{t}=\frac{1}{Z_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}
$$

substitute ζ_t into the FOCs from the intermediate firm's cost minimization problem gives us the demand for two different labour inputs in this sector:

$$
\begin{aligned}
N^{c}_{b,t}\left(i\right) &= y^{c}_{t}\left(i\right)\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\left(\nu-1\right)}}{\nu^{\left(\nu-1\right)}}\frac{\left(w_{s,t}\right)^{\left(1-\nu\right)}}{\left(1-\nu\right)^{\left(1-\nu\right)}} \\
N^{c}_{s,t}\left(i\right) &= y^{c}_{t}\left(i\right)\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{\left(-\nu\right)}}{\left(1-\nu\right)^{\left(-\nu\right)}}
\end{aligned}
$$

Aggregation yields (we denote $\Delta_t = \int \left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$

$$
\begin{aligned}
N^c_{b,t} &= \int N^c_{b,t}(i)\,di = \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\int y^c_t(i)\,di \\
&= \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}\int Y^c_t\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon} di \\
&= \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y^c_t \\
N^c_{s,t} &= \int N_{ct}(i)\,di = \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}\int y^c_t(i)\,di \\
&= \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}\int Y^c_t\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon} di \\
&= \frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y^c_t
\end{aligned}
$$

Price Setting:

Firms choose prices to maximize expected profit:

$$
\begin{aligned}
&\max_{\{p^*_{ct}(i)\}^\infty_{s=t}} \mathscr{E}_t\sum_{s=t}^{\infty} Q_{t,s}\left(y^c_t(i)P_{ct}(i) - W_{s,t}N^c_{s,t}(i) - W_{b,t}N^c_{b,t}(i)\right) \\
=& \left(y^c_t(i)P_{ct}(i) - W^s_t\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{\left(w_{s,t}\right)^{(-\nu)}}{(1-\nu)^{(-\nu)}}y^c_t(i) - W^b_t\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{\left(w_{s,t}\right)^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}y^c_t(i)\right) \\
=& \left(y^c_t(i)\left(P_{ct}(i) - Z_{ct}\zeta_t P_{ct}\left(\frac{W_{b,t}}{P_{ct}}\left(\frac{w_{b,t}}{v}\right)^{-1} + \frac{W_{s,t}}{P_{ct}}\left(\frac{w_{s,t}}{1-v}\right)^{(-1)}\right)\right)\right) \\
=& \left(y^c_t(i)P_{ct}(i) - y^c_t(i)MC_s\right)
\end{aligned}
$$

where $MC_t = \zeta_t P_t$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation

$$\max_{\{p^*_s(i)\}^\infty_{s=t}} \mathscr{E}_t\sum_{s=t}^{\infty} Q_{t,s}\left(y^c_t(i)P_{ct}(i) - y^c_t(i)MC_s\right)$$

Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices.

$$\zeta_t = \frac{1}{Z_{ct}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

where $MC_s = \zeta_t P_{ct}$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1\right)^2 y^c_t(i)$. This yields us :

$$V(i)=E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(y_t^c(i)\frac{P_{ct}(i)}{P_{ct}}-y_t^c(i)\frac{MC_t}{P_{ct}}\right)-\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 y_t^c(i)\right]$$

subject to Intermediate goods demand equation :

$$y_t^c(i)=Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i)=E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\frac{P_{ct}(i)}{P_{ct}}-\zeta_t\right)Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}-\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}\right]$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i)=E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\frac{P_{ct}(i)}{P_{ct}}-\zeta_t\right)Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}-\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 Y_t^c\left(\frac{p_{ct}(i)}{P_{ct}}\right)^{-\varepsilon}\right]$$

Let $\frac{P_{ct}(i)}{P_{ct}}=\widetilde{P_{ct}}$

$$V(i)=E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\widetilde{P_{ct}}-\zeta_t\right)Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}-\frac{\Omega}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2 Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}\right]$$

where $1+\pi_t$is the Gross inflation in the aggregate price level of the consumption goods side.

$$\max_{\left\{\widetilde{P_{ct}}\right\}_{s=t}^{\infty}} E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\widetilde{P_{ct}}-\zeta_t\right)Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}-\frac{\Omega}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2 Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}\right]$$

FOC w.r.t $\widetilde{P_{ct}}$:

$$0=\frac{\partial}{\partial\widetilde{P_{ct}}}\left(E_t\sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\widetilde{P_{ct}}-\zeta_t\right)Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}-\frac{\Omega}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2 Y_t^c\left(\widetilde{P_{ct}}\right)^{-\varepsilon}\right]\right)$$

$$=(1-\varepsilon)\left(\widetilde{P_{ct}}\right)^{-\varepsilon}Y_t^c+\varepsilon\left(\widetilde{P_{ct}}\right)^{-\varepsilon-1}\zeta_t Y_t^c$$

$$-\Omega\left(\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)\frac{(1+\pi_t)}{\widetilde{P_{ct-1}}}\left(\widetilde{P_{ct}}\right)^{-\varepsilon}-\frac{1}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2\varepsilon\left(\widetilde{P_{ct}}\right)^{-\varepsilon-1}\right)Y_t^c$$

$$+\Omega\theta m_{t+1}\left[\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}}-1\right)Y_{t+1}^c\left(\widetilde{P_{ct+1}}\right)^{-\varepsilon}\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}^2}\right)\right]$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s=Z_{ct}\zeta_t P_{ct}$which implies the relative price $\widetilde{P_{ct}}$is equal to 1.

substitute and we obtain the equation for the aggregate inflation:

$$0 = (1-\varepsilon)Y_t^c + \varepsilon\zeta_t Y_t^c - \Omega Y_t^c\left((\pi_t)(1+\pi_t) - \frac{1}{2}(\pi_t)^2\varepsilon\right) + \Omega\theta m_{t,t+1}\left[(\pi_{t+1})Y_{t+1}^c(1+\pi_{t+1})\right]$$

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + \theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2\varepsilon\right)$$

$$\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + E_t\left[\theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right]\right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2\varepsilon\right)$$

A.2.5 Private Sector Equilibrium

$$b:1:\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t + \mu P_{c,t}\Psi_t\right]$$

$$b:2:0 = \left(\left((1-\alpha)\left(H_{b,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \left[P_{c,t}\xi_t\right]q_t^h\right) + \beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)$$

$$b:3:\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}\right)^{1-\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}\right)^{1-\alpha}\right)^{-\sigma} = P_{c,t}\xi_t$$

$$b:4:0 = \left[P_{c,t}\xi_t\right] - \left[P_{c,t}\Psi_t\right]R_{t,o} - \beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,o}\frac{1}{1+\pi_{t+1}}$$

$$b:5:0 = N_{b,t}w_{b,t} + d_{t,0} + t_{b,t} - C_{b,t} - q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,o}d_{t-1,o}\frac{1}{1+\pi_t}$$

$$b:6:0 = \mu N_{b,t}w_{b,t} - R_{t,o}d_{t,o}$$

$$s:7:(N_{s,t})^{\phi} = w_{s,t}\left[P_{c,t}\lambda_t\right]$$

$$s:8:0 = \left((1-\alpha)(H_{s,t})^{-\alpha}(C_{s,t})^{\alpha}\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\right) - \left[P_{c,t}\lambda_t\right]q_t^h + \theta\left(\left[P_{c,t+1}\lambda_{t+1}\right]q_{t+1}^h(1-\delta)\right)$$

$$s:9:P_{c,t}\lambda_t=\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\alpha(C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}$$

$$s:10:P_{c,t}\lambda_t=\theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$s:11:Y_t^c=C_{b,t}+C_{s,t}+\frac{\Omega}{2}\pi_t^2Y_t^c$$

$$\begin{aligned}
fc &: 12:\frac{(1-\varepsilon)+\varepsilon\zeta_t}{\Omega}+\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})(\pi_{t+1})\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)\\
fc &: 13:N_{b,t}^c=\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^c\\
fc &: 14:N_{s,t}^c=\frac{1}{Z_{ct}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^c
\end{aligned}$$

$$\begin{aligned}
fh &: 15:q_t^h=-\frac{\varepsilon}{(1-\varepsilon)}\frac{1}{Z_{ht}}\left(\frac{w_{b,t}}{\nu}\right)^{\nu}\left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}\\
fh &: 16:N_{b,t}^h=\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{(\nu-1)}}{\nu^{(\nu-1)}}\frac{(w_{s,t})^{(1-\nu)}}{(1-\nu)^{(1-\nu)}}Y_t^h\\
fh &: 17:N_{s,t}^h=\frac{1}{Z_{ht}}\frac{\left(w_{b,t}\right)^{\nu}}{\nu^{\nu}}\frac{(w_{s,t})^{(-\nu)}}{(1-\nu)^{(-\nu)}}Y_t^h
\end{aligned}$$

$$\begin{aligned}
mc &: 18:B_{s,t}=D_{t,o}\\
mc &: 19:N_t^b=N_{b,t}^h+N_{b,t}^c\\
mc &: 20:N_t^s=N_{s,t}^h+N_{s,t}^c
\end{aligned}$$

$$\begin{aligned}
21:t_{bt} &= (1-x)\left(Y_t^c-w_{s,t}N_{s,t}^c-w_{b,t}N_{b,t}^c-\frac{\Omega}{2}\pi_t^2Y_t^c+Y_t^hq_t^h-w_{s,t}N_{s,t}^h-w_{b,t}N_{b,t}^h\right)\\
22:t_{st} &= x\left(Y_t^c-w_{s,t}N_{s,t}^c-w_{b,t}N_{b,t}^c-\frac{\Omega}{2}\pi_t^2Y_t^c+Y_t^hq_t^h-w_{s,t}N_{s,t}^h-w_{b,t}N_{b,t}^h\right)
\end{aligned}$$

$$23 \quad : \quad Y_t^h = (H_{b,t} - (1-\delta)H_{b,t-1}) + (H_{s,t} - (1-\delta)H_{s,t-1})$$

$$24 : \frac{R_{t,o}}{R_o} = (1+\pi_t)^{\phi_\pi}$$

$$25 : \zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_{b,t}}{\nu}\right)^{\nu} \left(\frac{w_{s,t}}{1-\nu}\right)^{1-\nu}$$

$$26 : R_{t,o} = R_{t,s}$$

where

$$m_{t,t+1} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]$$

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]$$

A.3 Buy to Let Markets in DSGE Framework with Dixit Stiglitz Lite Utility and Collateral Constraint

A.3.1 Hand-to-mouth workers

The Hand-to-Mouth household problem is to choose N_t^p, H_t^r by maximizing the utility subject to the budget constraint. Forming the Lagrangian L, with λ_t being the Lagrange multiplier. we have:. The Hand-to-Mouth households will have the efficiency conditions for the Lagrangian as follows:

FOCs:

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma} \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^p\right)^{1+\phi} \right)$$

$$C_t^p = \frac{N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r}{P_{c,t}}$$

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma} \left(\left(\frac{1}{P_{c,t}} \right)^{\alpha} \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^p\right)^{1+\phi} \right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_t^p} &= \\
& \frac{1-\sigma}{1-\sigma} \left(\left(\frac{1}{P_{c,t}} \right)^{\alpha} \left(\left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right) \right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \\
& \alpha \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right)^{\alpha-1} \left(\frac{1}{P_{c,t}} \right)^{\alpha} \left(H_t^r\right)^{1-\alpha} W_t^p - \left(N_t^p\right)^{\phi} \\
\frac{\partial L}{\partial H_t^r} &= \\
& \frac{1-\sigma}{1-\sigma} \left(\left(\frac{1}{P_{c,t}} \right)^{\alpha} \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \\
& \Bigg[-Q_t^r \alpha \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} \left(\frac{1}{P_{c,t}} \right)^{\alpha} + (1-\alpha) \left(\left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r\right) \right)^{\alpha} \left(H_t^r\right)^{-\alpha} \Bigg(\frac{1}{P_{c,}} \\
0 &= N_t^p W_t^p + T_{p,t} - P_{c,t} C_t^p - H_t^r Q_t^r \to C_t^p
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial N_t^p} &= \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_t^p\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} \frac{W_t^p}{P_{c,t}} - \left(N_t^p\right)^{\phi} \\
\frac{\partial L}{\partial H_t^r} &= 0 = \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \left[-\alpha \left(C_t^p\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} q_t^r + (1-\alpha) \left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{-\alpha} \right] \\
P_{c,t} C_t^p &= N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r
\end{aligned}$$

$$\begin{aligned}
\left(N_t^p\right)^{\phi} &= \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_t^p\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} w_t^p \\
& 0 = \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \left[-\alpha \left(C_t^p\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} q_t^r + (1-\alpha) \left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{-\alpha} \right] \\
C_t^p &= N_t^p w_t^p + t_{p,t} - H_t^r q_t^r
\end{aligned}$$

A.3.2 Borrowers

The Borrowers households problem is to choose $C_t^b, N_t^b, H_t^b, H_t^r, D_t^o, D_t^r$ by maximizing utility subject to the budget constraint and the collateral constraint. Forming the Lagrangian L, with ξ_t, Ψ_t being the Lagrange multipliers we have:The Borrowers households will have the efficiency conditions for the Lagrangian as follows:

FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial N_t^b} &= U_{N_t^b}\left(C_t^b, N_t^b, H_t^b - H_t^r\right) + \xi_t W_t^b = -\left(N_t^b\right)^{\phi} + \xi_t W_t^b \\
\frac{\partial L}{\partial H_t^b} &= \beta^t\left(U_{H_t^b}\left(C_t^b, N_t^b, H_t^b - H_t^r\right) - \xi_t Q_t^h + \Psi_t \mu Q_t^h\right) + \beta^{t+1}\xi_{t+1} Q_{t+1}^h (1-\delta) \\
&= \left(\left(\left(C_t^b\right)^{\alpha}\left(H_t^b - H_t^r\right)^{1-\alpha}\right)^{-\sigma}(1-\alpha)\left(H_t^b - H_t^r\right)^{-\alpha}\left(C_t^b\right)^{\alpha}\right) - \xi_t Q_t^h + \Psi_t \mu Q_t^h + \beta \xi_{t+1} Q_{t+1}^h (1-\delta) \\
\frac{\partial L}{\partial H_t^r} &= U_{H_t^r}\left(C_t^b, N_t^b, H_t^b - H_t^r\right) + \xi_t Q_t^r - \Psi_t \mu Q_t^h : \\
&= -\left(\left(\left(C_t^b\right)^{\alpha}\left(H_t^b - H_t^r\right)^{1-\alpha}\right)^{-\sigma}(1-\alpha)\left(H_t^b - H_t^r\right)^{-\alpha}\left(C_t^b\right)^{\alpha}\right) + \xi_t Q_t^r \\
\frac{\partial L}{\partial C_t^b} &= U_{C_t^b}\left(C_t^b, N_t^b, H_t^b - H_t^r\right) - \xi_t P_{c,t} = \alpha\left(C_t^b\right)^{\alpha-1}\left(H_t^b - H_t^r\right)^{1-\alpha}\left(\left(C_t^b\right)^{\alpha}\left(H_t^b - H_t^r\right)^{1-\alpha}\right)^{-\sigma} - \xi_t P_{c,t} \\
\frac{\partial L}{\partial D_t^b} &= \xi_t - \Psi_t R_{t,b} - \beta \xi_{t+1} R_{t,b} \\
\frac{\partial L}{\partial \xi_t} &= P_{c,t} C_t^b + Q_t^h (H_t^b - (1-\delta) H_{t-1}^b) + R_{t-1,b} D_{t-1}^b - H_t^r Q_t^r - N_t^b W_t^b - D_t^b - T_{b,t} \\
\frac{\partial L}{\partial \Psi_t} &= \mu Q_t^h H_t^b - R_{t,b} D_t^b
\end{aligned}$$

Simplify

$$\frac{\left(N_{b,t}\right)^{\phi}}{\xi_t} = W_{b,t}$$

$$0 = \left(\left((1-\alpha)\left(H_{b,t} - H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t} - H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \xi_t Q_t^h + \Psi_t \mu Q_t^h\right) + \beta \xi_{t+1} Q_{t+1}^h (1-\delta)$$

$$0=-\left(\left(1-\alpha\right)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)+\xi_t Q_t^r$$

$$\frac{\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}}{P_{c,t}}=\xi_t$$

$$0=\xi_t-\Psi_t R_{t,b}-\beta\xi_{t+1}R_{t,b}$$

$$0=H_t^r Q_t^r+N_t^b W_t^b+D_t^o+T_{b,t}-P_{c,t}C_{b,t}-Q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}D_{t-1}^o$$

$$0=\mu Q_t^h H_t^b-R_{t,o}D_t^o$$

Representing all the FOCs In Real Terms:

$$\left(N_{b,t}\right)^{\phi}=\frac{W_{b,t}}{P_{c,t}}\left[P_{c,t}\xi_t\right]$$

$$\begin{aligned}0=&\left(\left(1-\alpha\right)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)\\&-\left[P_{c,t}\xi_t\right]\frac{Q_t^h}{P_{c,t}}+\left[P_{c,t}\Psi_t\right]\mu\frac{Q_t^h}{P_{c,t}}\\&+\beta\left[P_{c,t+1}\xi_{t+1}\right]\frac{Q_{t+1}^h}{P_{c,t+1}}(1-\delta)\end{aligned}$$

$$\begin{aligned}0=&-\left(\left(1-\alpha\right)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)\\&+\left[P_{c,t}\xi_t\right]\frac{Q_t^r}{P_{c,t}}\end{aligned}$$

$$\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}=P_{c,t}\xi_t$$

$$0=\frac{\left[P_{c,t}\xi_t\right]}{P_{c,t}}-\left[P_{c,t}\Psi_t\right]\frac{R_{t,d}}{P_{c,t}}-\beta\left[P_{c,t+1}\xi_{t+1}\right]\frac{R_{t,d}}{P_{c,t}}\frac{1}{1+\pi_{t+1}}$$

$$0=H_t^r\frac{Q_t^r}{P_{c,t}}+N_{b,t}\frac{W_{b,t}}{P_{c,t}}+\frac{D_{t,d}}{P_{c,t}}+\frac{T_{b,t}}{P_{c,t}}-C_{b,t}-\frac{Q_t^h}{P_{c,t}}(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,o}\frac{D_{t-1,d}}{P_{c,t-1}}\frac{1}{1+\pi_t}$$

$$0=\mu\frac{Q_t^h}{P_{c,t}}H_{b,t}-R_{t,d}\frac{D_{t,d}}{P_{c,t}}$$

for $x_t=\frac{X_t}{P_{c,t}}$ are:

$$\left(N_{b,t}\right)^{\phi}=w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$0=\left(\left((1-\alpha)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_t\right]q_t^h+\left[P_{c,t}\Psi_t\right]\mu q_t^h\right) \\ +\beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)$$

$$0=\left((1-\alpha)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\right)-\left[P_{c,t}\xi_t\right]q_t^r$$

$$\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}=P_{c,t}\xi_t$$

$$0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}}$$

$$0=H_t^r q_t^r+N_{b,t}w_{b,t}+d_{t,d}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,d}d_{t-1,d}\frac{1}{1+\pi_t}$$

$$0=\mu q_t^h H_{b,t}-R_{t,d}d_{t,d}$$

A.3.3 Savers

The Savers households problem is to choose $C_{s,t},N_{s,t},H_{s,t},B_{s,t}$ by maximizing the utility function subject to their budget constraint . Forming the Lagrangian L, we have:. The Savers households will have the efficiency conditions for the Lagrangian as follows:

FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial N_t^s} &= U_{N_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) + \lambda_t W_{s,t} \\
\frac{\partial L}{\partial H_t^s} &= \theta^t\left(U_{H_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t Q_t^h\right) + \theta^{t+1}\left(\lambda_{t+1} Q_{t+1}^h (1-\delta)\right) \\
\frac{\partial L}{\partial C_t^s} &= U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t P_{c,t} \\
\frac{\partial L}{\partial B_t^s} &= -\theta^t \lambda_t + \theta^{t+1}\lambda_{t+1} R_{t,s} \\
\frac{\partial L}{\partial \lambda_t} &= N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} - P_{c,t}C_{s,t} - Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - B_{s,t}
\end{aligned}$$

Representing all the FOCs In Real Terms:

$$\frac{-U_{N_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)}{U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)} = \frac{W_{s,t}}{P_{c,t}}$$

$$0 = U_{H_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \left[P_{c,t}\lambda_t\right]\frac{Q_t^h}{P_{c,t}} + \theta\left(\left[P_{c,t+1}\lambda_{t+1}\right]\frac{Q_{t+1}^h}{P_{c,t+1}}(1-\delta)\right)$$

$$P_{c,t}\lambda_t = U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right)$$

$$P_{c,t}\lambda_t \frac{1}{P_{c,t}} = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}\frac{1}{P_{c,t}}$$

$$0 = N_{s,t}\frac{W_{s,t}}{P_{c,t}} + \frac{T_{s,t}}{P_{c,t}} + \frac{B_{s,t-1}}{P_{c,t-1}}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - \frac{Q_t^h}{P_{c,t}}(H_{s,t} - (1-\delta)H_{s,t-1}) - \frac{B_{s,t}}{P_{c,t}}$$

For $x_t = \frac{X_t}{P_{c,t}}$:

$$\frac{\left(N_{s,t}\right)^{\phi}}{\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}} = w_t^s$$

$$0 = \left((1-\alpha)\left(H_{s,t}\right)^{-\alpha}\left(C_{s,t}\right)^{\alpha}\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\right) - \left[P_{c,t}\lambda_t\right]q_t^h + \theta\left(\left[P_{c,t+1}\lambda_{t+1}\right]q_{t+1}^h(1-\delta)\right)$$

$$P_{c,t}\lambda_t = \left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}$$

$$P_{c,t}\lambda_t = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$0 = N_{s,t}w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - b_{s,t}$$

the corresponding optimal conditions for savers For $x_t = \frac{X_t}{P_{c,t}}$:

$$\frac{(N_{s,t})^{\phi}}{\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\alpha(C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}} = w_t^s$$

$$0 = \left((1-\alpha)(H_{s,t})^{-\alpha}(C_{s,t})^{\alpha}\left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\right) - [P_{c,t}\lambda_t]\, q_t^h + \theta\left([P_{c,t+1}\lambda_{t+1}]\, q_{t+1}^h(1-\delta)\right)$$

$$P_{c,t}\lambda_t = \left((C_{s,t})^{\alpha}(H_{s,t})^{1-\alpha}\right)^{-\sigma}\alpha(C_{s,t})^{\alpha-1}(H_{s,t})^{1-\alpha}$$

$$P_{c,t}\lambda_t = \theta\,[P_{c,t+1}\lambda_{t+1}]\,\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$0 = N_{s,t}w_{s,t} + t_{st} + b_{s,t-1}\frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - b_{s,t}$$

A.3.4 Firms

Final Housing goods Side of the Production Economy

The final good firms will chose $\left\{y_t^h(i)\right\}_{i=0}^{1}$,forming the Lagrangian equation with the above equation gives us:

$$\Pi_{h,t}^f = Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 Q_t^h(i)\,y_t^h(i).di$$

$$\frac{\partial \Pi_{h,t}^f}{\partial y_t^h(i)} = \left[\frac{\varepsilon}{\varepsilon-1}.Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\dfrac{1}{\varepsilon-1}}.\frac{\varepsilon-1}{\varepsilon}.y_t^h(i)^{\dfrac{-1}{\varepsilon}}\right] - Q_t^h(i)$$

$$0 = \left[\frac{\varepsilon}{\varepsilon-1}.Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}\,di\right]^{\dfrac{1}{\varepsilon-1}}.\frac{\varepsilon-1}{\varepsilon}.y_t^h(i)^{\dfrac{-1}{\varepsilon}}\right] - Q_t^h(i)$$

$$0 = Q_t^h.Y_t^{h\dfrac{1}{\varepsilon}}.y_t^h(i)^{\dfrac{-1}{\varepsilon}} - Q_t^h(i)$$

Divide by Q_t^h gives

$$0 = Y_t^{h\dfrac{1}{\varepsilon}}.y_t^h(i)^{\dfrac{-1}{\varepsilon}} - \frac{Q_t^h(i)}{Q_t^h}$$

$$0=\left(\frac{y_t^h(i)}{Y_t^h}\right)^{\dfrac{-1}{\varepsilon}}-\frac{Q_t^h(i)}{Q_t^h}$$

$$\left(\frac{y_t^h(i)}{Y_t^h}\right)^{\dfrac{-1}{\varepsilon}}=\frac{Q_t^h(i)}{Q_t^h}$$

$$\frac{y_t^h(i)}{Y_t^h}=\left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}$$

This equation gives us the demand for each intermediate input as follows:

$$y_t^h(i)=\left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}Y_t^h$$

substituting the demanded intermediate goods from the above $y_t^h(i)$ back into the profit function of final goods producer:

$$Q_t^h.Y_t^h-\int_0^1 Q_t^h(i)\,y_t^h(i).di$$

$$Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}-\int_0^1 Q_t^h(i)\,y_t^h(i).di$$

gives us

$$Q_t^h.\left[\int_0^1\left(\left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}Y_t^h\right)^{\frac{\varepsilon-1}{\varepsilon}}di\right]^{\frac{\varepsilon}{\varepsilon-1}}-\int_0^1 Q_t^h(i)\left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}Y_t^h.di$$

$$Q_t^h.\left[\int_0^1\left((\frac{Q_t^h(i)}{Q_t^h})^{1-\varepsilon}.Y_t^{h\dfrac{\varepsilon-1}{\varepsilon}}\right)di\right]^{\dfrac{\varepsilon}{\varepsilon-1}}-\int_0^1 Q_t^h.(\frac{Q_t^h(i)}{Q_t^h})^{1-\varepsilon}.Y_t^h.di$$

$$Q_t^h.Y_t^h\left[\int_0^1\left((\frac{Q_t^h(i)}{Q_t^h})^{1-\varepsilon}\right)di\right]\left(\left[\int_0^1(\frac{Q_t^h(i)}{Q_t^h})^{1-\varepsilon}\right]^{\dfrac{1}{\varepsilon-1}}-1\right)$$

substituting the demanded intermediate goods $y_t^h(i)$

$$y_t^h(i)=\left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon}.Y_t^h$$

back into the production function of the final good producer :

$$Y_t^h = \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

gives us

$$Y_t^h = \left[\int_0^1 \left(\left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} Y_t^h \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

which implies

$$\left(\int_0^1 \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{1-\varepsilon} di \right)^{\dfrac{\varepsilon}{\varepsilon-1}} = 1$$

Final Consumption Goods Side of the Production Economy

The final good firms will chose $\{y_t^c(i)\}_{i=0}^1$,forming the Lagrangian equation with the above equation gives us:

$$\Pi_{c,t}^f = P_t^c . \left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t^c(i) y_t^c(i).di$$

$$\frac{\partial \Pi_{c,t}^f}{\partial y_t^c(i)} = \left[\frac{\varepsilon}{\varepsilon-1} .P_t^c . \left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\dfrac{1}{\varepsilon-1}} . \frac{\varepsilon-1}{\varepsilon} .y_t^c(i)^{\dfrac{-1}{\varepsilon}} \right] - P_t^c(i)$$

$$0 = \left[\frac{\varepsilon}{\varepsilon-1} .P_t^c . \left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\dfrac{1}{\varepsilon-1}} . \frac{\varepsilon-1}{\varepsilon} .y_t^c(i)^{\dfrac{-1}{\varepsilon}} \right] - P_t^c(i)$$

$$0 = P_t^c .Y_t^{c\dfrac{1}{\varepsilon}} .y_t^c(i)^{\dfrac{-1}{\varepsilon}} - P_t^c(i)$$

Divide by P_t^h gives

$$0 = Y_t^{c\dfrac{1}{\varepsilon}} .y_t^c(i)^{\dfrac{-1}{\varepsilon}} - \frac{P_t^c(i)}{P_t^c}$$

$$0 = \left(\frac{y_t^c(i)}{Y_t^c}\right)^{\dfrac{-1}{\varepsilon}} - \frac{P_t^c(i)}{P_t^c}$$

$$\left(\frac{y_t^c(i)}{Y_t^c}\right)^{\dfrac{-1}{\varepsilon}} = \frac{Q_t^c(i)}{Q_t^c}$$

$$\frac{y_t^c(i)}{Y_t^c} = \left(\frac{P_t^c(i)}{P_t^c}\right)^{-\varepsilon}$$

This equation gives us the demand for each intermediate input as follows:

$$y_t^c i) = \left(\frac{P_t^c(i)}{P_t^c}\right)^{-\varepsilon} Y_t^c$$

substituting the demanded intermediate goods from the above $y_t^c(i)$ back into the profit function of final goods producer:

$$P_t^c.Y_t^c - \int_0^1 P_t^c(i)\, y_t^c(i).di$$

$$P_t^c.\left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t^c(i)\, y_t^c(i).di$$

gives us

$$P_t^c.\left[\int_0^1 \left(\left(\frac{P_t^c(i)}{P_t^c}\right)^{-\varepsilon} Y_t^c\right)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t^c(i)\left(\frac{P_t^c(i)}{P_t^c}\right)^{-\varepsilon} Y_t^c.di$$

$$P_t^c.\left[\int_0^1 \left((\frac{P_t^c(i)}{P_t^c})^{1-\varepsilon}.Y_t^c{}^{\dfrac{\varepsilon-1}{\varepsilon}}\right) di\right]^{\dfrac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t^c.(\frac{P_t^c(i)}{P_t^c})^{1-\varepsilon}.Y_t^c.di$$

$$P_t^c.Y_t^c\left[\int_0^1 \left((\frac{P_t^c(i)}{P_t^c})^{1-\varepsilon}\right) di\right]\left(\left[\int_0^1 (\frac{P_t^c(i)}{P_t^c})^{1-\varepsilon}\right]^{\dfrac{1}{\varepsilon-1}} - 1\right)$$

substituting the demanded intermediate goods $y_t^c(i)$

$$y_t^c(i) = \left(\frac{P_t^c(i)}{P_t^c}\right)^{-\varepsilon}.Y_t^c$$

back into the production function of the final good producer :

$$Y_t^c = \left[\int_0^1 y_t^c(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

gives us

$$Y_t^c = \left[\int_0^1 \left(\left(\frac{P_t^c(i)}{P_t^c} \right)^{-\varepsilon} Y_t^c \right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

which implies

$$\left(\int_0^1 \left(\frac{P_t^c(i)}{P_t^c} \right)^{1-\varepsilon} di \right)^{\dfrac{\varepsilon}{\varepsilon-1}} = 1$$

A.3.5 Intermediate Housing firms

Write down the Lagrangian:

$$L = W_t^s N_{s,t}^h(i) + W_t^b N_{b,t}^h(i) + W_t^p N_{p,t}^h(i) - P_{ct}\eta_t \left(Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu} - y_t^h(i) \right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{p,t}^h(i)} &= W_t^p - P_{ct}\eta_t \nu Z_{ht} N_{p,t}^h(i)^{\nu-1} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu} = W_t^p - P_{ct}\eta_t \nu \frac{y_t^h(i)}{N_{p,t}^h(i)} = 0 \\
\frac{\partial L}{\partial N_{b,t}^h(i)} &= W_t^b - P_{ct}\eta_t u Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u-1} N_{s,t}^h(i)^{1-u-\nu} = W_t^b - P_{ct}\eta_t u \frac{y_t^h(i)}{N_{b,t}^h(i)} = 0 \\
\frac{\partial L}{\partial N_{s,t}^h(i)} &= W_t^s - P_{ct}\eta_t (1-u-\nu) Z_{ht} N_{p,t}^h(i)^{\nu} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{1-u-\nu-1} = W_t^s - P_{ct}\eta_t (1-u-\nu) \frac{y_t^h(i)}{N_{s,t}^h(i)} = 0
\end{aligned}$$

from where

$$\begin{aligned}
W_t^p N_{p,t}^h(i) &= P_{ct}\eta_t \nu y_t^h(i) \\
W_t^b N_{b,t}^h(i) &= P_{ct}\eta_t u y_t^h(i) \\
W_t^s N_{s,t}^h(i) &= P_{ct}\eta_t (1-u-\nu) y_t^h(i)
\end{aligned}$$

$$\begin{aligned}
w_t^p N_{p,t}^h(i) &= \eta_t \nu y_t^h(i) \\
w_t^b N_{b,t}^h(i) &= \eta_t u y_t^h(i) \\
w_t^s N_{s,t}^h(i) &= \eta_t (1-u-\nu) y_t^h(i)
\end{aligned}$$

Substitute the production function into the above equations, from which we get :

$$
\begin{aligned}
w_t^p &= \eta_t v Z_{ht} N_{p,t}^h(i)^{v-1} N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v} \\
w_t^b &= \eta_t u Z_{ht} N_{p,t}^h(i)^{v} N_{b,t}^h(i)^{u-1} N_{s,t}^h(i)^{1-u-v} \\
w_t^s &= \eta_t (1-u-v) Z_{ht} N_{p,t}^h(i)^{v} N_{b,t}^h(i)^{u} N_{s,t}^h(i)^{-u-v}
\end{aligned}
$$

$$
\begin{aligned}
\frac{w_t^p}{\eta_t v Z_{ht}} &= \left(\frac{N_{p,t}^h(i)}{N_{s,t}^h(i)}\right)^{v-1} \left(\frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}\right)^{u} \\
\frac{w_t^b}{\eta_t u Z_{ht}} &= \left(\frac{N_{p,t}^h(i)}{N_{s,t}^h(i)}\right)^{v} \left(\frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}\right)^{u-1} \\
\frac{w_t^s}{\eta_t (1-u-v) Z_{ht}} &= \left(\frac{N_{p,t}^h(i)}{N_{s,t}^h(i)}\right)^{v} \left(\frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}\right)^{u}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{w_t^p}{\eta_t v Z_{ht}} \left(\frac{N_{p,t}^h(i)}{N_{s,t}^h(i)}\right)^{1-v}\right)^{\frac{1}{u}} &= \left(\frac{N_{b,t}^h(i)}{N_{s,t}^h(i)}\right) \\
\left(\frac{w_t^b}{\eta_t u Z_{ht}} \left(\frac{w_t^p}{\eta_t v Z_{ht}}\right)^{\frac{1}{u}(1-u)}\right)^{\frac{1}{(1-v)\frac{1}{u}(u-1)+v}} &= \left(\frac{N_{p,t}^h(i)}{N_{s,t}^h(i)}\right) \\
\frac{w_t^s}{\eta_t (1-u-v) Z_{ht}} &= \frac{w_t^p}{\eta_t v Z_{ht}} \left(\left(\frac{w_t^b}{\eta_t u Z_{ht}} \left(\frac{w_t^p}{\eta_t v Z_{ht}}\right)^{\frac{1}{u}(1-u)}\right)^{\frac{1}{(1-v)\frac{1}{u}(u-1)+v}}\right)
\end{aligned}
$$

to yield the solution for the Lagrange multiplier:

$$
\eta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)} \left(\frac{w_t^b}{u}\right)^{u} \left(\frac{w_t^p}{v}\right)^{v}
$$

Substitute back the Lagrange multiplier back yields us the labour demand equations for all the three household sectors and are as follows:

$$
\begin{aligned}
N_{p,t}^h(i) &= \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)} \left(\frac{w_t^b}{u}\right)^{u} \left(\frac{w_t^p}{v}\right)^{v-1} y_t^h(i) \\
N_{b,t}^h(i) &= \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)} \left(\frac{w_t^b}{u}\right)^{u-1} \left(\frac{w_t^p}{v}\right)^{v} y_t^h(i) \\
N_{s,t}^h(i) &= \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)} \left(\frac{w_t^b}{u}\right)^{u} \left(\frac{w_t^p}{v}\right)^{v} y_t^h(i)
\end{aligned}
$$

Now that we have all the labour demanded from intermediate goods . Aggregating the labour demand equations through the intermediate firms will give us the aggregate labour for all the three sectors of households using the aggregation rule .

$$\Xi_t^h = \int \Xi_t^h(i)\,di$$

yields us

$$\begin{aligned}
N_{p,t}^h &= \int N_{p,t}(i)\,di = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1} Y_t^h \to N_{p,t}^h \\
N_{b,t}^h &= \int N_{b,t}(i)\,di = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h \to N_{b,t}^h \\
N_{s,t}^h &= \int N_{s,t}(i)\,di = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h \to N_{s,t}^h
\end{aligned}$$

Substituting back the aggregate labour demands into the aggregate Production function gives us:

$$\begin{aligned}
Y_{ht} = Z_{ht}N_{p,t}^{h,v}N_{b,t}^{h,u}N_{s,t}^{h,1-u-v} = Z_{ht}&\left(\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1} Y_t^h\right)^{v} \\
&\left(\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h\right)^{u} \\
&\left(\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h\right)^{1-u-v}
\end{aligned}$$

Prices Substituting the labour demand from the above profit function. We have:

$$\begin{aligned}
N_{p,t}^h(i) &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1} y_t^h(i) \\
N_{b,t}^h(i) &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} y_t^h(i) \\
N_{s,t}^h(i) &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} y_t^h(i)
\end{aligned}$$

$$\begin{aligned}
&\max_{\{Q_s^*(i)\}_{s=t}^{\infty}} \left(y_t^h(i)\,Q_t^h(i) - W_t^s N_{s,t}^h(i) - W_t^b N_{b,t}^h(i) - W_t^p N_{p,t}^h(i)\right) \\
=\;& y_t^h(i)\,Q_t^h(i) - W_t^s\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v} y_t^h(i) \\
& -W_t^b\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} y_t^h(i) \\
& -W_t^p\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1} y_t^h(i) \\
=\;& y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\left(\frac{W_t^p}{P_{ct}}\left(\frac{w_t^p}{v}\right)^{-1} + \frac{W_t^b}{P_{ct}}\left(\frac{w_t^b}{u}\right)^{-1} + \frac{W_t^s}{P_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(-1)}\right)\right) \\
=\;& y_t^h(i)\left(Q_t^h(i) - \eta_t P_{ct}\right) = \left(y_t^h(i)\,Q_t^h(i) - y_{ht}(i)\,MC_t\right)
\end{aligned}$$

As From , we have the Lagrange multiplier as:

$$\eta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

Substitute demand of input goods and choosing the values of the prices by the Intermediate good producers gives us:

$$\max_{\left\{Q_s^h(i)\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left(Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{1-\varepsilon} - \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} MC_t \right)$$

FOCs w.r.t $Q_t^h(i)$

$$0 = \sum_{s=t}^{\infty} m_{t,s} Y_t^h \left((1-\varepsilon) Q_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} + \varepsilon \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon-1} MC_t \right)$$

$$\left(\frac{Q_t^h(i)}{Q_t^h} \right) \frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}$$

By Aggregating the prices of intermediate goods, we have:

$$\frac{Q_t^h(i)}{Q_t^h} = 1$$

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{MC_t}{P_{ct}}$$

substitute $MC_t = \eta_t P_{ct}$,

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \eta_t = -\frac{\varepsilon}{(1-\varepsilon)} \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

$$\frac{Q_t^h}{P_{ct}} = -\frac{\varepsilon}{(1-\varepsilon)} \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

A.3.6 Intermediate Consumption goods Firms

Write down the Lagrangian:

$$L = W_t^s N_{s,t}^c(i) + W_t^b N_{b,t}^c(i) + W_t^p N_{p,t}^c(i) - P_{ct} \zeta_t \left(Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} - y_t^c(i) \right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{p,t}^c(i)} &= W_t^p - P_{ct} \zeta_t \nu Z_{ct} N_{p,t}^c(i)^{\nu-1} N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} = W_t^p - P_{ct} \zeta_t \nu \frac{y_t^c(i)}{N_{p,t}^c(i)} = 0 \\
\frac{\partial L}{\partial N_{b,t}^c(i)} &= W_t^b - P_{ct} \zeta_t u Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^{u-1} N_{s,t}^c(i)^{1-u-v} = W_t^b - P_{ct} \zeta_t u \frac{y_t^c(i)}{N_{b,t}^c(i)} = 0 \\
\frac{\partial L}{\partial N_{s,t}^c(i)} &= W_t^s - P_{ct} \zeta_t (1-u-v) Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v-1} = W_t^s - P_{ct} \zeta_t (1-u-v) \frac{y_t^c(i)}{N_{s,t}^c(i)} = 0
\end{aligned}$$

From where

$$
\begin{aligned}
W_t^p N_{p,t}^c(i) &= P_{ct}\zeta_t \nu y_t^c(i) \\
W_t^b N_{b,t}^c(i) &= P_{ct}\zeta_t u y_t^c(i) \\
W_t^s N_{s,t}^c(i) &= P_{ct}\zeta_t (1-u-v) y_t^c(i)
\end{aligned}
$$

$$
\begin{aligned}
w_t^p N_{p,t}^c(i) &= \zeta_t \nu y_t^c(i) \\
w_t^b N_{b,t}^c(i) &= \zeta_t u y_t^c(i) \\
w_t^s N_{s,t}^c(i) &= \zeta_t (1-u-v) y_t^c(i)
\end{aligned}
$$

Substitute the production function into the above equations, from which we get :

$$
\begin{aligned}
w_t^p &= \zeta_t \nu Z_{ct} N_{p,t}^c(i)^{\nu-1} N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} \\
w_t^b &= \zeta_t u Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^{u-1} N_{s,t}^c(i)^{1-u-v} \\
w_t^s &= \zeta_t (1-u-v) Z_{ct} N_{p,t}^c(i)^{\nu} N_{b,t}^c(i)^u N_{s,t}^c(i)^{-u-v}
\end{aligned}
$$

$$
\begin{aligned}
\frac{w_t^p}{\zeta_t \nu Z_{ct}} &= \left(\frac{N_{p,t}^c(i)}{N_{s,t}^c(i)}\right)^{\nu-1}\left(\frac{N_{b,t}^c(i)}{N_{s,t}^c(i)}\right)^u \\
\frac{w_t^b}{\zeta_t u Z_{ct}} &= \left(\frac{N_{p,t}^c(i)}{N_{s,t}^c(i)}\right)^{\nu}\left(\frac{N_{b,t}^c(i)}{N_{s,t}^c(i)}\right)^{u-1} \\
\frac{w_t^s}{\zeta_t (1-u-v) Z_{ct}} &= \left(\frac{N_{p,t}^c(i)}{N_{s,t}^c(i)}\right)^{\nu}\left(\frac{N_{b,t}^c(i)}{N_{s,t}^c(i)}\right)^u
\end{aligned}
$$

:

$$
\begin{aligned}
\left(\frac{w_t^p}{\zeta_t \nu Z_{ct}}\left(\frac{N_{p,t}^c(i)}{N_{s,t}^c(i)}\right)^{1-\nu}\right)^{\frac{1}{u}} &= \left(\frac{N_{b,t}^c(i)}{N_{s,t}^c(i)}\right) \\
\left(\frac{w_t^b}{\zeta_t u Z_{ct}}\left(\frac{w_t^p}{\eta_t \nu Z_{ct}}\right)^{\frac{1}{u}(1-u)}\right)^{\frac{1}{(1-v)\frac{1}{u}(u-1)+v}} &= \left(\frac{N_{p,t}^c(i)}{N_{s,t}^c(i)}\right) \\
\frac{w_t^s}{\zeta_t (1-u-v) Z_{ct}} &= \frac{w_t^p}{\zeta_t \nu Z_{ct}}\left(\left(\frac{w_t^b}{\zeta_t u Z_{ct}}\left(\frac{w_t^p}{\zeta_t \nu Z_{ct}}\right)^{\frac{1}{u}(1-u)}\right)^{\frac{1}{(1-v)\frac{1}{u}(u-1)+v}}\right)
\end{aligned}
$$

to yield the solution for the Lagrange multiplier:

$$
\zeta_t = \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^v
$$

Substitute back the Lagrange multiplier back yields us the labour demand equations for all the three household sectors and are as follows:

$$
\begin{aligned}
N^c_{p,t}(i) &= \frac{P_{ct}\zeta_t \nu y^c_t(i)}{W^p_t} = \frac{\nu y^c_t(i)}{w^p_t}\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^v \\
&= y^c_t(i)\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^{v-1} \\
N^c_{b,t}(i) &= \frac{P_{ct}\zeta_t u y^c_t(i)}{W^b_t} = \frac{u y^c_t(i)}{w^b_t}\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^v \\
&= y^c_t(i)\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^{u-1}\left(\frac{w^p_t}{v}\right)^{v} \\
N^c_{s,t}(i) &= \frac{P_{ct}\zeta_t (1-u-v) y^c_t(i)}{W^s_t} = \frac{(1-u-v) y^c_t(i)}{w^s_t}\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^v \\
&= y^c_t(i)\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(-u-v)}\left(\frac{w^b_t}{u}\right)^{u}\left(\frac{w^p_t}{v}\right)^{v}
\end{aligned}
$$

[It will be obvious soon why

$$
\zeta_t = \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^v = mc_t
$$

]

Aggregating the labour demand equations through the intermediate firms will give us the aggregate labour for all the three sectors of households using the aggregation rule . Aggregation yields (we denote $\Delta_t = \int\left(\frac{p_t(i)}{P_t}\right)^{-\varepsilon} di$

$$
\begin{aligned}
N_{p,t} &= \int N_{p,t}(i)\,di = \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^{v-1}\int y^c_t(i)\,di \\
&= \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^{v-1} Y^c_t \\
N_{b,t} &= \int N_{b,t}(i)\,di = \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^{u-1}\left(\frac{w^p_t}{v}\right)^{v}\int y^c_t(i)\,di \\
&= \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^{u-1}\left(\frac{w^p_t}{v}\right)^{v} Y^c_t \\
N_{s,t} &= \int N_{s,t}(i) = \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(-u-v)}\left(\frac{w^b_t}{u}\right)^{u}\left(\frac{w^p_t}{v}\right)^{v}\int y^c_t(i)\,di \\
&= \frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(-u-v)}\left(\frac{w^b_t}{u}\right)^{u}\left(\frac{w^p_t}{v}\right)^{v} Y^c_t
\end{aligned}
$$

$$
\begin{aligned}
N_{p,t}^{c} &= \frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v-1}Y_{t}^{c}\\
N_{b,t}^{c} &= \frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u-1}\left(\frac{w_{t}^{p}}{v}\right)^{v}Y_{t}^{c}\\
N_{s,t}^{c} &= \frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v}Y_{t}^{c}
\end{aligned}
$$

Substituting back the aggregate labour demands into the aggregate Production function gives us:

$$
Y_{t}^{c}=Z_{ct}N_{p,t}^{c,v}N_{b,t}^{c,u}N_{s,t}^{c,1-u-v}=Z_{ct}\left(\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v-1}Y_{ct}\right)^{v}
$$
$$
\left(\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u-1}\left(\frac{w_{t}^{p}}{v}\right)^{v}Y_{ct}\right)^{u}
$$
$$
\left(\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v}Y_{ct}\right)^{1-u-v}
$$

Price setting First let's see whether the Lagrange multiplier will be equal to the Lagrange multiplier in nominal terms.

$$
\begin{aligned}
N_{p,t}^{c}(i) &= y_{t}^{c}(i)\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v-1}\\
N_{b,t}^{c}(i) &= y_{t}^{c}(i)\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u-1}\left(\frac{w_{t}^{p}}{v}\right)^{v}\\
N_{s,t}^{c}(i) &= y_{t}^{c}(i)\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v}
\end{aligned}
$$

$$
\begin{aligned}
&\max_{\{p_{s}^{*}(i)\}_{s=t}^{\infty}}\left(y_{t}^{c}(i)P_{ct}(i)-W_{t}^{s}N_{s,t}^{c}(i)-W_{t}^{b}N_{b,t}^{c}(i)-W_{t}^{p}N_{p,t}^{c}(i)\right)\\
= {}& y_{t}^{c}(i)P_{ct}(i)-W_{t}^{s}\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v}y_{t}^{c}(i)\\
&-W_{t}^{b}\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u-1}\left(\frac{w_{t}^{p}}{v}\right)^{v}y_{t}^{c}(i)\\
&-W_{t}^{p}\frac{1}{Z_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_{t}^{b}}{u}\right)^{u}\left(\frac{w_{t}^{p}}{v}\right)^{v-1}y_{t}^{c}(i)\\
= {}& y_{t}^{c}(i)\left(P_{ct}(i)-\zeta_{t}P_{ct}\left(\frac{W_{t}^{p}}{P_{ct}}\left(\frac{w_{t}^{p}}{v}\right)^{-1}+\frac{W_{t}^{b}}{P_{ct}}\left(\frac{w_{t}^{b}}{u}\right)^{-1}+\frac{W_{t}^{s}}{P_{ct}}\left(\frac{w_{t}^{s}}{1-u-v}\right)^{(-1)}\right)\right)\\
= {}& y_{t}^{c}(i)\left(P_{ct}(i)-\zeta_{t}P_{ct}\right)=\left(y_{t}^{c}(i)P_{ct}(i)-y_{ct}(i)MC_{t}\right)
\end{aligned}
$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\phi}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

Let $\frac{P_{ct}(i)}{P_{ct}} = \widetilde{P_{ct}}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\phi}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

where π_t is the Gross inflation in the aggregate price level of the consumption goods side.

$$\max_{\left\{\widetilde{P_{ct}}\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\phi}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

FOC w.r.t $\widetilde{P_{ct}}$:

$$\begin{aligned}
0 &= \frac{\partial}{\partial \widetilde{P_{ct}}} \left(E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\phi}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right] \right) \\
&= (1-\varepsilon) \left(\widetilde{P_{ct}} \right)^{-\varepsilon} Y_t^c + \varepsilon \left(\widetilde{P_{ct}} \right)^{-\varepsilon-1} \zeta_t Y_t^c \\
&\quad -\phi \left(\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right) \frac{(1+\pi_t)}{\widetilde{P_{ct-1}}} \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{1}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 \varepsilon \left(\widetilde{P_{ct}} \right)^{-\varepsilon-1} \right) Y_t^c \\
&\quad +\phi_t \theta m_{t+1} \left[\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}} - 1 \right) Y_{t+1}^c \left(\widetilde{P_{ct+1}} \right)^{-\varepsilon} \left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}^2} \right) \right]
\end{aligned}$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = \zeta_t P_{ct}$ which implies the relative price $\widetilde{P_{ct}}$ is equal to 1.

$$0 = (1-\varepsilon) Y_t^c + \varepsilon \zeta_t Y_t^c - \phi \left((\pi_t)(1+\pi_t) - \frac{1}{2} (\pi_t)^2 \varepsilon \right) Y_t^c + \phi_t \theta m_{t+1} \left[(\pi_{t+1}) Y_{t+1}^c (1+\pi_{t+1}) \right]$$

$$\frac{(1-\varepsilon)}{\phi} + \frac{\varepsilon}{\phi} \zeta_t + \theta_t m_{t+1} \left[(\pi_{t+1}) \frac{Y_{t+1}^c}{Y_t^c} (1+\pi_{t+1}) \right] = \left(\pi_t (1+\pi_t) - \frac{1}{2} (\pi_t)^2 \varepsilon \right) Y_t^c$$

A.3.7 Private Sector Equilibrium

$$p:1: \left(N_t^p\right)^{\phi} = \left(\left(C_t^p\right)^{\alpha} \left(H_t^r\right)^{1-\alpha} \right)^{-\sigma} \alpha \left(C_t^p\right)^{\alpha-1} \left(H_t^r\right)^{1-\alpha} w_t^p$$

$$p:2:0=\left(\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{1-\alpha}\right)^{-\sigma}\left[-\alpha\left(C_t^p\right)^{\alpha-1}\left(H_t^r\right)^{1-\alpha}q_t^r+(1-\alpha)\left(C_t^p\right)^{\alpha}\left(H_t^r\right)^{-\alpha}\right]$$

$$p:3:C_t^p=N_t^p w_t^p+t_{p,t}-H_t^r q_t^r$$

$$b:4:\left(N_{b,t}\right)^{\phi}=w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$\begin{aligned}b:5:0=(1-\alpha)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\\ -\left[P_{c,t}\xi_t\right]q_t^h+\left[P_{c,t}\Psi_t\right]\mu q_t^h+\beta\left[P_{c,t+1}\xi_{t+1}\right]q_{t+1}^h(1-\delta)\end{aligned}$$

$$\to b:6:0=(1-\alpha)\left(H_{b,t}-H_{r,t}\right)^{-\alpha}\left(C_{b,t}\right)^{\alpha}\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}-\left[P_{c,t}\xi_t\right]q_t^r$$

$$b:7:\left(\left(C_{b,t}\right)^{\alpha}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{b,t}\right)^{\alpha-1}\left(H_{b,t}-H_{r,t}\right)^{1-\alpha}=P_{c,t}\xi_t$$

$$b:8:0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,d}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}}$$

$$b:9:0=H_t^r q_t^r+N_{b,t}w_{b,t}+d_{t,d}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,d}d_{t-1,d}\frac{1}{1+\pi_t}$$

$$b:10:0=\mu q_t^h H_{b,t}-R_{t,d}d_{t,d}$$

$$s:11:\frac{\left(N_{s,t}\right)^{\phi}}{\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}}=w_t^s$$

$$s:12:0=(1-\alpha)\left(H_{s,t}\right)^{-\alpha}\left(C_{s,t}\right)^{\alpha}\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}-\left[P_{c,t}\lambda_t\right]q_t^h+\theta\left[P_{c,t+1}\lambda_{t+1}\right]q_{t+1}^h(1-\delta)$$

$$s:13:P_{c,t}\lambda_t=\left(\left(C_{s,t}\right)^{\alpha}\left(H_{s,t}\right)^{1-\alpha}\right)^{-\sigma}\alpha\left(C_{s,t}\right)^{\alpha-1}\left(H_{s,t}\right)^{1-\alpha}$$

$$s:14:P_{c,t}\lambda_t=\theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$s:15:Y_t^c=C_{b,t}+C_{s,t}+C_{p,t}+\frac{\Omega}{2}\pi_{ct}^2Y_t^c$$

$$\begin{aligned}
fh:16:N_{p,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1}Y_t^h\\
fh:17:N_{b,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v}Y_t^h\\
fh:18:N_{s,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v}Y_t^h
\end{aligned}$$

$$fh:19:\frac{Q_t^h}{P_{ct}}=-\frac{\varepsilon}{(1-\varepsilon)}\frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v}$$

$$\begin{aligned}
fc:20:N_{p,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1}Y_t^c\\
fc:21:N_{b,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v}Y_t^c\\
fc:22:N_{s,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v}Y_t^c
\end{aligned}$$

$$fc:23:\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{t+1})\frac{Y^c_{t+1}}{Y^c_t}(1+\pi_{t+1})\right]\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)$$

$$\begin{aligned}
24:t_{bt} &= (1-x-y)\left(Y^c_t-w_{s,t}N^c_{s,t}-w_{b,t}N^c_{b,t}-w_{p,t}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2Y^c_t+Y^h_tq^h_t-w_{s,t}N^h_{s,t}-w_{b,t}N^h_{b,t}-w_{p,t}N^h_{p,t}\right)\\
25:t_{st} &= x\left(Y^c_t-w_{s,t}N^c_{s,t}-w_{b,t}N^c_{b,t}-w_{p,t}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2Y^c_t+Y^h_tq^h_t-w_{s,t}N^h_{s,t}-w_{b,t}N^h_{b,t}-w_{p,t}N^h_{p,t}\right)
\end{aligned}$$

$$26:t_{pt}=y\left(Y^c_t-w_{s,t}N^c_{s,t}-w_{b,t}N^c_{b,t}-w_{p,t}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2Y^c_t+Y^h_tq^h_t-w_{s,t}N^h_{s,t}-w_{b,t}N^h_{b,t}-w_{p,t}N^h_{p,t}\right)$$

$$27:Y^h_t=(H_{b,t}-(1-\delta)H_{b,t-1})+(H_{s,t}-(1-\delta)H_{s,t-1})$$

$$\begin{aligned}
mc:28:N^p_t &= N^h_{p,t}+N^c_{p,t}\\
mc:29:N^b_t &= N^h_{b,t}+N^c_{b,t}\\
mc:30:N^s_t &= N^h_{s,t}+N^c_{s,t}
\end{aligned}$$

$$mc:31:B_{s,t}=D_{t,o}$$

$$mc:32:R_{t,o}=R_{t,s}$$

$$mc:33:\frac{R_{t,o}}{R_o}=(1+\pi_t)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

$$34:\zeta_t=\frac{1}{Z_{ct}}\left(\frac{w^s_t}{1-u-v}\right)^{(1-u-v)}\left(\frac{w^b_t}{u}\right)^u\left(\frac{w^p_t}{v}\right)^v$$

A.4 Buy to Let Markets with CES Utility and Policy Analysis

A.4.1 Hand-to-mouth workers

The Hand-to-Mouth household problem is to choose N_t^p, H_t^r by maximizing the utility subject to the budget constraint. Forming the Lagrangian L, with λ_t being the Lagrange multiplier. we have:

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t \left(\frac{1}{1-\sigma} \left[\left(a C_t^{p\,1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{1-\sigma} - \frac{1}{1+\phi} \left(N_t^p \right)^{1+\phi} \right)$$

$$C_t^p = \frac{N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r}{P_{c,t}}$$

$$L = E_0 \sum_{t=0}^{\infty} \gamma^t$$
$$\left(\frac{1}{1-\sigma} \left[\left(\left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} a \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{(1-\sigma)} - \frac{1}{1+\phi} \left(N_t^p \right)^{1+\phi} \right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_t^p} &= \frac{1-\sigma}{1-\sigma} \\
& \left[\left(\left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} a \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma} \\
& \frac{1}{1-\frac{1}{\rho}} \left[\left(\left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} a \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] \\
& a \left(1 - \frac{1}{\rho} \right) \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{-\frac{1}{\rho}} \left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} W_t^p - \left(N_t^p \right)^{\phi} \\
\frac{\partial L}{\partial H_t^r} &= \frac{1-\sigma}{1-\sigma} \left[\left(\left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} a \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma} \\
& \frac{1}{1-\frac{1}{\rho}} \left[\left(\left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} a \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{1-\frac{1}{\rho}} + (1-a) H_t^{r\,1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] \\
& \left[a \left(1 - \frac{1}{\rho} \right) \left(N_t^p W_t^p + T_{p,t} - H_t^r Q_t^r \right)^{-\frac{1}{\rho}} \left(\frac{1}{P_{c,t}} \right)^{1-\frac{1}{\rho}} \left(-Q_t^r \right) + (1-a) \left(1 - \frac{1}{\rho} \right) H_t^{r-\frac{1}{\rho}} \right] \\
0 &= N_t^p W_t^p + T_{p,t} - P_{c,t} C_t^p - H_t^r Q_t^r
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial L}{\partial N_t^p} &= \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
&\quad \frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
&\quad a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{-\frac{1}{\rho}}\frac{W_t^p}{P_{c,t}}-\left(N_t^p\right)^{\phi} \\
\frac{\partial L}{\partial H_t^r} &= 0=\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
&\quad \frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
&\quad \left[-a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{\left(-\frac{1}{\rho}\right)}q_t^r+(1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right] \\
P_{c,t}C_t^p &= N_t^pW_t^p+T_{p,t}-H_t^rQ_t^r
\end{aligned}
$$

$$
\begin{aligned}
\left(N_t^p\right)^{\phi} &= \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \\
&\quad \frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
&\quad a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{-\frac{1}{\rho}}w_t^p \\
&\quad 0=\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}}+(1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
&\quad \left[-a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{\left(-\frac{1}{\rho}\right)}q_t^r+(1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right] \\
C_t^p &= N_t^pw_t^p+t_{p,t}-H_t^rq_t^r
\end{aligned}
$$

A.4.2 Borrowers

The Borrowers households problem is to choose $C_t^b, N_t^b, H_t^b, H_t^r, D_t^o, D_t^r$ by maximizing utility subject to the budget constraint and the collateral constraint. Forming the Lagrangian L, with ξ_t, Ψ_t being the Lagrange multipliers we have:

Lagrangian

$$
L=E_0\sum_{t=0}^{\infty}\beta^t\left(\begin{array}{c} U\left(C_t^b,N_t^b,H_t^b-H_t^r\right) \\ +\xi_t\left(\begin{array}{c} H_t^rQ_t^r+N_t^bW_t^b+D_t^b+T_{b,t} \\ -P_{c,t}C_t^b-Q_t^h(H_t^b-(1-\delta)H_{t-1}^b)-R_{t-1,b}D_{t-1}^b \end{array}\right) \\ +\Psi_t\left(\mu Q_t^hH_t^b-R_{t,b}D_t^b\right) \end{array}\right)
$$

$$\max_{C_t^b,N_t^b,H_t^b,H_t^r,D_t^o,D_t^r} E_0\beta^t \sum_{t=0}^{\infty}\left(\frac{1}{1-\sigma}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{1-\sigma}-\frac{1}{1+\phi}\left(N_t^b\right)^{1+\phi}\right)$$

The optimal conditions are as follows:

FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial N_t^b} &= U_{N_t^b}\left(C_t^b,N_t^b,H_t^b-H_t^r\right)+\xi_t W_t^b=-\left(N_t^b\right)^{\phi}+\xi_t W_t^b\\
\frac{\partial L}{\partial H_t^b} &= \beta^t\left(U_{H_t^b}\left(C_t^b,N_t^b,H_t^b-H_t^r\right)-\xi_t\left(Q_t^h\right)+\Psi_t\mu Q_t^h\right)\\
&\quad +\beta^{t+1}\xi_{t+1}\left(Q_{t+1}^h(1-\delta)\right)\\
&=\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\\
&\quad \frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b-H_t^r\right)^{-\frac{1}{\rho}}\right)-\xi_t\left(Q_t^h\right)\\
&\quad +\Psi_t\mu Q_t^h+\beta\xi_{t+1}\left(Q_{t+1}^h(1-\delta)\right)\\
\frac{\partial L}{\partial H_t^r} &= U_{H_t^r}\left(C_t^b,N_t^b,H_t^b-H_t^r\right)+\xi_t Q_t^r=-\\
&\quad \left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\\
&\quad \left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b-H_t^r\right)^{-\frac{1}{\rho}}\right)+\xi_t Q_t^r\\
\frac{\partial L}{\partial C_t^b} &= U_{C_t^b}\left(C_t^b,N_t^b,H_t^b-H_t^r\right)-\xi_t P_{c,t}=\\
&\quad \left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\\
&\quad \frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(C_t^b\right)^{-\frac{1}{\rho}}-\xi_t P_{c,t}\\
\frac{\partial L}{\partial D_t^b} &= \xi_t-\Psi_t R_{t,b}-\beta\xi_{t+1}R_{t,b}\\
\frac{\partial L}{\partial \xi_t} &= P_{c,t}C_t^b+Q_t^h(H_t^b-(1-\delta)H_{t-1}^b)+R_{t-1,b}D_{t-1}^b-H_t^r Q_t^r-N_t^b W_t^b-D_t^b-T_{b,t}\\
\frac{\partial L}{\partial \Psi_t} &= \mu Q_t^h H_t^b-R_{t,b}D_t^b
\end{aligned}$$

Simplify to yield

$$\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right]$$

$$0 = \left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b - H_t^r\right)^{-\frac{1}{\rho}}\right) - \left[P_{c,t}\xi_t\right]\left(q_t^h\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h$$
$$+\beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)$$

$$0 = \left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$
$$\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b - H_t^r\right)^{-\frac{1}{\rho}}\right) - \left[P_{c,t}\xi_t\right]q_t^r$$

$$\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$a\left(1-\frac{1}{\rho}\right)\left(C_t^b\right)^{-\frac{1}{\rho}}$$
$$= P_{c,t}\xi_t$$

$$0 = \left[P_{c,t}\xi_t\right] - \left[P_{c,t}\Psi_t\right]R_{t,o} - \beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}}$$

$$0 = H_t^r q_t^r + N_{b,t}w_{b,t} + d_{t,d} + t_{b,t} - C_{b,t} - q_t^h(H_{b,t} - (1-\delta)H_{b,t-1}) - R_{t-1,d}d_{t-1,d}\frac{1}{1+\pi_t}$$

$$0 = \mu q_t^h H_{b,t} - R_{t,d}d_{t,d}$$

A.4.3 Savers

The Savers households problem is to choose $C_{s,t}, N_{s,t}, H_{s,t}, B_{s,t}$ by maximizing the utility function subject to their budget constraint . Forming the Lagrangian L, we have:

Lagrangian

$$L = E_0\sum_{t=0}^{\infty}\theta^t U^s\left(C_{s,t}, N_{s,t}, H_{s,t}\right) + \lambda_t\left[N_{s,t}W_{s,t} + R_{t-1,s}B_{s,t-1} + T_{s,t} - P_{c,t}C_{s,t} - Q_t^h(H_{s,t} - (1-\delta)H_{s,t-1}) - B_{s,t}\right]$$

FOCs:

$$\begin{aligned}
\frac{\partial L}{\partial N_t^s} &= U_{N_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) + \lambda_t W_{s,t} \\
\frac{\partial L}{\partial H_t^s} &= \theta^t \left(U_{H_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t \left(Q_t^h\right)\right) \\
&\quad + \theta^{t+1} \lambda_{t+1} \left(Q_{t+1}^h (1-\delta)\right) \\
\frac{\partial L}{\partial C_t^s} &= U_{C_t}\left(C_{s,t}, N_{s,t}, H_{s,t}\right) - \lambda_t P_{c,t} \\
\frac{\partial L}{\partial B_t^s} &= -\theta^t \lambda_t + \theta^{t+1} \lambda_{t+1} R_{t,s} \\
\frac{\partial L}{\partial \lambda_t} &= N_{s,t} W_{s,t} + R_{t-1,s} B_{s,t-1} + T_{s,t} - P_{c,t} C_{s,t} - Q_t^h (H_{s,t} - (1-\delta) H_{s,t-1}) - B_{s,t}
\end{aligned}$$

the corresponding optimal conditions for Savers For $x_t = \frac{X_t}{P_{c,t}}$:

$$\frac{(N_{s,t})^{\phi}}{\left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a) H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1} \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a \left(1-\frac{1}{\rho}\right) \left(C_t^s\right)^{-\frac{1}{\rho}}} = w_t^s \tag{187}$$

$$\begin{aligned}
0 = & \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a) H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1} \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] \\
& (1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^s\right)^{-\frac{1}{\rho}} - \left[P_{c,t}\lambda_t\right]\left(q_t^h\right) \\
& + \theta\left[P_{c,t+1}\lambda_{t+1}\right]\left(q_{t+1}^h (1-\delta)\right)
\end{aligned}$$

$$P_{c,t}\lambda_t = \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a) H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1} \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a \left(1-\frac{1}{\rho}\right) \left(C_t^s\right)^{-\frac{1}{\rho}}$$

$$P_{c,t}\lambda_t = \theta\left[P_{c,t+1}\lambda_{t+1}\right] \frac{R_{t,s}}{1+\pi_{t+1}}$$

$$0 = N_{s,t} w_{s,t} + t_{st} + b_{s,t-1} \frac{R_{t-1,s}}{1+\pi_t} - C_{s,t} - q_t^h (H_{s,t} - (1-\delta) H_{s,t-1}) - b_{s,t}$$

A.4.4 Final Housing goods Side of the Production Economy

We model the production of the final good of Housing via a single stand in aggregate firm that produces according to the production technology:

$$Y_t^h = \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Where Y_t^h denotes the final goods of Housing sector, $y_t^h(i)$ denotes the ith intermediate input, ε is the elasticity of substitution between the intermediate goods. Notice that as $\varepsilon \to \infty$, the intermediate input goods become perfectly substitutable and to avoid this we assume it to be finite.

The final good firms acquires the intermediate goods from the intermediate produces and make it into a single final product and sells it to the households in the economy. Also the final producers doesn't have any control over the prices, and they take the prices $Q_t^h, Q_t^h(i)$as given. Hence, the decision problem for the final good producer in the housing goods side is to choose the demand for theY_t^h and intermediate goods$y_t^h(i)$ to maximize the profit which is given by :

$$Q_t^h.Y_t^h - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

where Q_t^h is the price of the final Housing good and
$Q_t^h(i)$ is the price of the ith intermediate good
taking both the prices as given from the final goods firm perspective,

$$Q_t^h.Y_t^h - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

subject to the

$$Y_t^h = \left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

which gives us:

$$Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

The final good firms will chose $\left\{y_t^h(i)\right\}_{i=0}^1$,forming the Lagrangian equation with the above equation gives us:

$$\Pi_{h,t}^f = Q_t^h.\left[\int_0^1 y_t^h(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 Q_t^h(i)\, y_t^h(i).di$$

This equation gives us the demand for each intermediate input as follows:

$$y_t^h(i) = \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{-\varepsilon} Y_t^h$$

substituting the demanded intermediate goods from the above $y_t^h(i)$ back into the profit function of final goods gives us

$$\left(\int_0^1 \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{1-\varepsilon} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1$$

The above Equation has two implications

1) by substituting the value of above equation in the profits equation gives us the profits as zero. i.e., the profits of the final goods are equal to zero.

2)

$$\left(\int_0^1 \left(\frac{Q_t^h(i)}{Q_t^h}\right)^{1-\varepsilon} di\right)^{\frac{\varepsilon}{\varepsilon-1}} = 1$$

implies

$$\left(Q_t^h\right)^{\varepsilon} \left[\int_0^1 (Q_t^h(i))^{1-\varepsilon} di\right]^{\frac{\varepsilon}{\varepsilon-1}} = 1$$

$$\left(Q_t^h\right)^{-\varepsilon} = \left[\int_0^1 (Q_t^h(i))^{1-\varepsilon} di\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

$$Q_t^h = \left[\int_0^1 (Q_t^h(i))^{1-\varepsilon} di\right]^{\frac{1}{\varepsilon-1}}$$

Which says that the prices of the final good follow the same aggregation rule and is a function of the intermediate good prices.

A.4.5 Final Consumption Goods Side of the Production Economy

Final Consumption goods side follow the same as Final Housing side .

A.4.6 Intermediate Consumption firms

Employment Firm i minimizes nominal cost:

$$\min_{N_{s,t}(i),N_{b,t}(i),N_{p,t}(i)} W_t^s N_{s,t}^c(i) + W_t^b N_{b,t}^c(i) + W_t^p N_{p,t}^c(i).$$

subject to the production constraint

$$y_t^c(i) = Z_{ct} N_{p,t}^c(i)^\nu N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v}$$

Write down the Lagrangian:

$$L = W_t^s N_{s,t}^c(i) + W_t^b N_{b,t}^c(i) + W_t^p N_{p,t}^c(i) - P_{ct}\zeta_t\left(Z_{ct} N_{p,t}^c(i)^\nu N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} - y_t^c(i)\right)$$

Write down the Lagrangian:

$$L = W_t^s N_{s,t}^c(i) + W_t^b N_{b,t}^c(i) + W_t^p N_{p,t}^c(i) - P_{ct}\zeta_t\left(Z_{ct} N_{p,t}^c(i)^\nu N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} - y_t^c(i)\right)$$

$$\begin{aligned}
\frac{\partial L}{\partial N_{p,t}^c(i)} &= \\
& W_t^p - P_{ct}\zeta_t \nu Z_{ct} N_{p,t}^c(i)^{\nu-1} N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v} \\
& = W_t^p - P_{ct}\zeta_t \nu \frac{y_t^c(i)}{N_{p,t}^c(i)} = 0 \\
\frac{\partial L}{\partial N_{b,t}^c(i)} & \\
&= W_t^b - P_{ct}\zeta_t u Z_{ct} N_{p,t}^c(i)^\nu N_{b,t}^c(i)^{u-1} N_{s,t}^c(i)^{1-u-v} \\
& = W_t^b - P_{ct}\zeta_t u \frac{y_t^c(i)}{N_{b,t}^c(i)} = 0 \\
\frac{\partial L}{\partial N_{s,t}^c(i)} & \\
&= W_t^s - P_{ct}\zeta_t (1-u-v) Z_{ct} N_{p,t}^c(i)^\nu N_{b,t}^c(i)^u N_{s,t}^c(i)^{1-u-v-1} \\
& = W_t^s - P_{ct}\zeta_t (1-u-v) \frac{y_t^c(i)}{N_{s,t}^c(i)} = 0
\end{aligned}$$

From where

$$\begin{aligned}
W_t^p N_{p,t}^c(i) &= P_{ct}\zeta_t \nu y_t^c(i) \\
W_t^b N_{b,t}^c(i) &= P_{ct}\zeta_t u y_t^c(i) \\
W_t^s N_{s,t}^c(i) &= P_{ct}\zeta_t (1-u-v) y_t^c(i)
\end{aligned}$$

$$\begin{aligned}
w_t^p N_{p,t}^c(i) &= \zeta_t \nu y_t^c(i) \\
w_t^b N_{b,t}^c(i) &= \zeta_t u y_t^c(i) \\
w_t^s N_{s,t}^c(i) &= \zeta_t (1-u-v) y_t^c(i)
\end{aligned}$$

Substitute the production function into the above equations, from which we get the solution for the

Lagrange multiplier:

$$\zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

Price setting Firms choose prices to maximize expected profit and let's assume the firms follow the Rotemberg price setting where there incurs a quadratic costs in changing prices.

$$\zeta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

where $MC_s = \zeta_t P_{ct}$ Note that wages here do not depend on index i, as labour of each type is assumed to be perfectly mobile and so wages of particular type are equalized across all firms. So we come to familiar formulation of setting prices in Rotemberg setting where the quadratic cost is taken as $\frac{\Omega}{2}\left(\frac{P_{ct}(i)}{P_{ct-1}(i)}-1\right)^2 y_t^c(i)$. The firm discounts future profits by the gross real interest rate between today and future dates(stochastic discount factor),

$$\frac{1}{R_{t,s}} = \theta \left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t} \right]$$

This yields us :

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[(y_t^c(i) P_{ct}(i) - y_t^c(i) MC_t) - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 y_t^c(i) \right]$$

subject to Intermediate goods demand equation :

$$y_t^c(i) = Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon}$$

The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{P_{ct}(i)}{P_{ct}} - \zeta_t \right) Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{P_{ct}(i)}{P_{ct-1}(i)} - 1 \right)^2 Y_t^c \left(\frac{p_{ct}(i)}{P_{ct}} \right)^{-\varepsilon} \right]$$

Let $\frac{P_{ct}(i)}{P_{ct}} = \widetilde{P_{ct}}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

where π_t is the Gross inflation in the aggregate price level of the consumption goods side.

$$\max_{\{\widetilde{P_{ct}}\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{P_{ct}} - \zeta_t \right) Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} - \frac{\Omega}{2} \left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}} - 1 \right)^2 Y_t^c \left(\widetilde{P_{ct}} \right)^{-\varepsilon} \right]$$

FOC w.r.t $\widetilde{P_{ct}}$:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial \widetilde{P_{ct}}}\left(E_t \sum_{s=t}^{\infty} \theta^s m_{t,s}\left[\left(\widetilde{P_{ct}}-\zeta_t\right) Y_t^c \left(\widetilde{P_{ct}}\right)^{-\varepsilon} - \frac{\Omega}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2 Y_t^c \left(\widetilde{P_{ct}}\right)^{-\varepsilon}\right]\right) \\
&= (1-\varepsilon)\left(\widetilde{P_{ct}}\right)^{-\varepsilon} Y_t^c + \varepsilon \left(\widetilde{P_{ct}}\right)^{-\varepsilon-1} \zeta_t Y_t^c \\
&\quad -\Omega\left(\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)\frac{(1+\pi_t)}{\widetilde{P_{ct-1}}}\left(\widetilde{P_{ct}}\right)^{-\varepsilon} - \frac{1}{2}\left(\frac{\widetilde{P_{ct}}(1+\pi_t)}{\widetilde{P_{ct-1}}}-1\right)^2 \varepsilon \left(\widetilde{P_{ct}}\right)^{-\varepsilon-1}\right) Y_t^c \\
&\quad +\Omega \theta m_{t+1}\left[\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}}-1\right) Y_{t+1}^c \left(\widetilde{P_{ct+1}}\right)^{-\varepsilon}\left(\frac{\widetilde{P_{ct+1}}(1+\pi_{t+1})}{\widetilde{P_{ct}}^{2}}\right)\right]
\end{aligned}
$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = \zeta_t P_{ct}$which implies the relative price $\widetilde{P_{ct}}$is equal to 1.

$$
0 = (1-\varepsilon) Y_t^c + \varepsilon \zeta_t Y_t^c - \Omega\left((\pi_t)(1+\pi_t) - \frac{1}{2}(\pi_t)^2 \varepsilon\right) Y_t^c + \Omega_t \theta m_{t+1}\left[(\pi_{t+1}) Y_{t+1}^c (1+\pi_{t+1})\right]
$$

$$
\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + \theta_t m_{t+1}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right] = \left(\pi_t(1+\pi_t) - \frac{1}{2}(\pi_t)^2 \varepsilon\right)
$$

FOC w.r.t $\widetilde{P_{ct}}$: We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s = \zeta_t P_{ct}$which implies the relative price $\widetilde{P_{ct}}$is equal to 1.

$$
\frac{(1-\varepsilon)}{\Omega} + \frac{\varepsilon}{\Omega}\zeta_t + E_t\left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{t+1})\frac{Y_{t+1}^c}{Y_t^c}(1+\pi_{t+1})\right]\right] = \left(\pi_t(1+\pi) - \frac{1}{2}(\pi_t)^2 \varepsilon\right)
$$

where

$$
m_{t,t+1} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]
$$

$$
\frac{1}{R_{t,s}} = \theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]
$$

A.4.7 Intermediate Housing firms

Employment Housing good prices follow a Sticky prices like consumption prices and hence, we assume housing service prices follow a Rotemberg model of prices. These Housing good intermediate firms will pay wages to the labour provided by the three sector of households and the employment equation is as follows:

Firm i minimizes nominal cost:

$$\min_{N_{s,t}^{h}(i), N_{b,t}^{h}(i), N_{p,t}^{h}(i)} W_t^p N_{p,t}^h(i) + W_t^b N_{b,t}^h(i) + W_t^s N_{s,t}^h(i).$$

The firm tries to minimize their production costs of intermediate goods equation (27) by choosing the values for the hire of labour provided by the households, $N_{s,t}^h(i), N_{b,t}^h(i), N_{p,t}^h(i)$subject to the production constraint (28)

$$y_t^h(i) = Z_{ht} N_{p,t}^h(i)^v N_{b,t}^h(i)^u N_{s,t}^h(i)^{1-u-v}$$

Where $Z_{ht}$is the firm's sector specific technology shock, note that this shock is aggregate shock to the sector rather than firm specific and forming the Lagrangian L, with η_t being the Lagrange multiplier we have:

Write down the Lagrangian and solve to get the solution for the Lagrange multiplier:

$$\eta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

Price setting The problem for the optimal prices setting at time t can, equivalently, be written as

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\frac{Q_t^h(i)}{Q_t^h} - \frac{\eta_t P_{ct}}{Q_t^h} \right) Y_t^h \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} - \frac{\Omega_h}{2} \left(\frac{Q_t^h(i)}{Q_{t-1}^h(i)} - 1 \right)^2 \left(\frac{Q_t^h(i)}{Q_t^h} \right)^{-\varepsilon} Y_t^h \right]$$

Let $\frac{Q_t^h(i)}{Q_t^h} = \widetilde{Q_t^h}$

$$V(i) = E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{Q_t^h} - \frac{\eta_t}{q_t^h} \right) Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} - \frac{\Omega_h}{2} \left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}} - 1 \right)^2 Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} \right]$$

where $\pi_{h,t}$ is the Gross inflation of housing in the aggregate price level of the Housing goods side.

$$\max_{\left\{\widetilde{Q_t^h}\right\}_{s=t}^{\infty}} E_t \sum_{s=t}^{\infty} \theta^s m_{t,s} \left[\left(\widetilde{Q_t^h} - \frac{\eta_t}{q_t^h} \right) Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} - \frac{\Omega_h}{2} \left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}} - 1 \right)^2 Y_t^h \left(\widetilde{Q_t^h} \right)^{-\varepsilon} \right]$$

FOC w.r.t $\widetilde{Q_t^h}$:

$$\begin{aligned}
0 &= \frac{\partial}{\partial \widetilde{Q_t^h}}\left(E_t \sum_{s=t}^{\infty}\theta^s m_{t,s}\left[\left(\widetilde{Q_t^h}-\frac{\eta_t}{q_t^h}\right)Y_t^h\left(\widetilde{Q_t^h}\right)^{-\varepsilon}-\frac{\Omega_h}{2}\left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}}-1\right)^2 Y_t^h\left(\widetilde{Q_t^h}\right)^{-\varepsilon}\right]\right)\\
&= (1-\varepsilon)\left(\widetilde{Q_t^h}\right)^{-\varepsilon}Y_t^h+\varepsilon\left(\widetilde{Q_t^h}\right)^{-\varepsilon-1}\frac{\eta_t}{q_t^h}Y_t^h\\
&\quad -\Omega_h\left(\left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}}-1\right)\frac{(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}}\left(\widetilde{Q_t^h}\right)^{-\varepsilon}-\frac{1}{2}\left(\frac{\widetilde{Q_t^h}(1+\pi_{h,t})}{\widetilde{Q_{t-1}^h}}-1\right)^2\varepsilon\left(\widetilde{Q_t^h}\right)^{-\varepsilon-1}\right)Y_t^h\\
&\quad +\Omega_h\theta m_{t+1}\left[\left(\frac{\widetilde{Q_{t+1}^h}(1+\pi_{h,t+1})}{\widetilde{Q_t^h}}-1\right)Y_{t+1}^h\left(\widetilde{Q_{t+1}^h}\right)^{-\varepsilon}\left(\frac{\widetilde{Q_{t+1}^h}(1+\pi_{h,t+1})}{\widetilde{Q_t^h}^2}\right)\right]
\end{aligned}$$

We can safely say that all the firms will chose the same optimal price which is the relative price in our case due to the Rotemberg scenario assumption that all firms are identical in changing prices and also the same marginal cost which is firm independent $MC_s=\eta_t P_{ct}$which implies the relative price $\widetilde{Q_t^h}$ is equal to 1.

$$0=(1-\varepsilon)Y_t^h+\varepsilon\frac{\eta_t}{q_t^h}Y_t^h-\Omega_h\left(\left(\pi_{h,t}\right)(1+\pi_{h,t})-\frac{1}{2}\left(\pi_{h,t}\right)^2\varepsilon\right)Y_t^h+\Omega_h\theta m_{t+1}\left[\left(\pi_{h,t+1}\right)Y_{t+1}^h(1+\pi_{h,t+1})\right]$$

$$\frac{(1-\varepsilon)}{\Omega_h}+\frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q_t^h}+\theta_t m_{t+1}\left[\left(\pi_{h,t+1}\right)\frac{Y_{t+1}^h}{Y_t^h}(1+\pi_{h,t+1})\right]=\left(\pi_{h,t}(1+\pi_{h,t})-\frac{1}{2}\left(\pi_{h,t}\right)^2\varepsilon\right)$$

$$\frac{(1-\varepsilon)}{\Omega_h}+\frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q_t^h}+E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[\left(\pi_{h,t+1}\right)\frac{Y_{t+1}^h}{Y_t^h}(1+\pi_{h,t+1})\right]\right]=\left(\pi_{h,t}(1+\pi_{h,t})-\frac{1}{2}\left(\pi_{h,t}\right)^2\varepsilon\right) \quad (188)$$

where

$$m_{t,t+1}=\theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]$$

$$\frac{1}{R_{t,s}}=\theta\left[\frac{P_{c,t+1}\lambda_{t+1}}{P_{c,t}\lambda_t}\right]$$

A.4.8 Profits of firms and Government Transfers

Aggregate inter-period nominal profit is

$$\tilde{\Pi}_t=Y_t^cP_{ct}-W_{s,t}N_{s,t}^c-W_{b,t}N_{b,t}^c-W_{p,t}N_{p,t}^c-\frac{\Omega}{2}\pi_t^2Y_t^cP_{ct}+Y_t^hQ_t^h-W_{s,t}N_{s,t}^h-W_{b,t}N_{b,t}^h-W_{p,t}N_{p,t}^h-\frac{\Omega_h}{2}\pi_{h,t}^2Y_t^h$$

We assume that the profit is 100 percent taxed by the government and redistributed according to the following rule:

$$t_{bt} = (1-x-y)\frac{\tilde{\Pi}_t}{P_{ct}}$$

$$t_{st} = x\frac{\tilde{\Pi}_t}{P_{ct}}$$

$$t_{pt} = y\frac{\tilde{\Pi}_t}{P_{ct}}$$

A.4.9 Private Sector Equilibrium

$$p:1:\left(N_t^p\right)^{\phi} = \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$
$$\frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{-\frac{1}{\rho}} w_t^p$$

$$p:2:0 = \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left[-a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{\left(-\frac{1}{\rho}\right)} q_t^r + (1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right]$$

$$p:3:C_t^p = N_t^p w_t^p + t_{p,t} - H_t^r q_t^r$$

$$b:4:\left(N_{b,t}\right)^{\phi} = w_{b,t}\left[P_{c,t}\xi_t\right]$$

$b:5:$

$$0 = \left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$
$$\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}} + (1-a)\left(H_t^b - H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b - H_t^r\right)^{-\frac{1}{\rho}}\right)$$
$$-\left[P_{c,t}\xi_t\right]\left(q_t^h\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h$$
$$+\beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)$$

$$b:6:0=\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left((1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^b-H_t^r\right)^{-\frac{1}{\rho}}\right)-\left[P_{c,t}\xi_t\right]q_t^r$$

$$b:7:\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{b\,1-\frac{1}{\rho}}+(1-a)\left(H_t^b-H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$a\left(1-\frac{1}{\rho}\right)\left(C_t^b\right)^{-\frac{1}{\rho}}$$
$$=P_{c,t}\xi_t$$

$$b:8:0=\left[P_{c,t}\xi_t\right]-\left[P_{c,t}\Psi_t\right]R_{t,d}-\beta\left[P_{c,t+1}\xi_{t+1}\right]R_{t,d}\frac{1}{1+\pi_{t+1}}$$

$$b:9:0=H_t^r q_t^r+N_{b,t}w_{b,t}+d_{t,d}+t_{b,t}-C_{b,t}-q_t^h(H_{b,t}-(1-\delta)H_{b,t-1})-R_{t-1,d}d_{t-1,d}\frac{1}{1+\pi_t}$$

$$b:10:0=\mu q_t^h H_{b,t}-R_{t,d}d_{t,d}$$

$$s:11:\frac{(N_{s,t})^{\phi}}{\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(C_t^s\right)^{-\frac{1}{\rho}}}$$
$$=w_t^s$$

$$s:12:0=\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}\left[\left(aC_t^{s\,1-\frac{1}{\rho}}+(1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$(1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^s\right)^{-\frac{1}{\rho}}-\left[P_{c,t}\lambda_t\right]\left(q_t^h\right)$$
$$+\theta\left[P_{c,t+1}\lambda_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)$$

$$s:13:P_{c,t}\lambda_t = \left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)H_t^{s\,1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$

$$\frac{\rho}{\rho-1}\left[\left(aC_t^{s\,1-\frac{1}{\rho}} + (1-a)\left(H_t^s\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(C_t^s\right)^{-\frac{1}{\rho}}$$

$$s:14:P_{c,t}\lambda_t = \theta\left[P_{c,t+1}\lambda_{t+1}\right]\frac{R_{t,s}}{1+\pi_{t+1}}$$

$$s:15:Y_t^c = C_{b,t}+C_{s,t}+C_{p,t}+\frac{\Omega}{2}\pi_{ct}^2 Y_t^c$$

$$\begin{aligned}
fh:16:N_{p,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1}Y_t^h\\
fh:17:N_{b,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v}Y_t^h\\
fh:18:N_{s,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v}Y_t^h
\end{aligned}$$

$$def:19:\frac{q_t^h}{q_{t-1}^h} = \frac{1+\pi_{h,t}}{1+\pi_t}$$

$$fh:20:\frac{(1-\varepsilon)}{\Omega_h}+\frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q_t^h}+E_t\left[\theta_t\frac{\left[P_{c,t+1}\lambda_{t+1}\right]}{\left[P_{c,t}\lambda_t\right]}\left[\left(\pi_{h,t+1}\right)\frac{Y_{t+1}^h}{Y_t^h}\left(1+\pi_{h,t+1}\right)\right]\right] = \left(\pi_{h,t}\left(1+\pi_{h,t}\right)-\frac{1}{2}\left(\pi_{h,t}\right)^2\varepsilon\right)$$

$$\begin{aligned}
fc:21:N_{p,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v-1}Y_t^c\\
fc:22:N_{b,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v}Y_t^c\\
fc:23:N_{s,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^{u}\left(\frac{w_t^p}{v}\right)^{v}Y_t^c
\end{aligned}$$

$$fc:24:\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+E_t\left[\theta_t\frac{\left[P_{c,t+1}\lambda_{t+1}\right]}{\left[P_{c,t}\lambda_t\right]}\left[\left(\pi_{t+1}\right)\frac{Y_{t+1}^c}{Y_t^c}\left(1+\pi_{t+1}\right)\right]\right] = \left(\pi_t\left(1+\pi_t\right)-\frac{1}{2}\left(\pi_t\right)^2\varepsilon\right)$$

$$
\begin{aligned}
25 : t_{bt} &= (1-x-y)\widetilde{\Pi} \\
26 : t_{st} &= x\widetilde{\Pi}
\end{aligned}
$$

$$27 : t_{pt} = y\widetilde{\Pi}$$

$$\widetilde{\Pi} = \left(Y_t^c - w_{s,t}N_{s,t}^c - w_{b,t}N_{b,t}^c - w_{p,t}N_{p,t}^c - \frac{\Omega}{2}\pi_t^2 Y_t^c + Y_t^h q_t^h - w_{s,t}N_{s,t}^h - w_{b,t}N_{b,t}^h - w_{p,t}N_{p,t}^h - \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h \right)$$

$$
\begin{aligned}
28 : Y_t^h = {} & (H_{b,t} - (1-\delta)H_{b,t-1}) + (H_{s,t} - (1-\delta)H_{s,t-1}) \\
& + \frac{\Omega_h}{2}\pi_{h,t}^2 Y_t^h \to Y_t^{h2}
\end{aligned}
$$

$$
\begin{aligned}
mc : 29 : N_t^p &= N_{p,t}^h + N_{p,t}^c \\
mc : 30 : N_t^b &= N_{b,t}^h + N_{b,t}^c \\
mc : 31 : N_t^s &= N_{s,t}^h + N_{s,t}^c
\end{aligned}
$$

$$mc : 32 : B_{s,t} = D_{t,o}$$

$$mc : 33 : R_{t,o} = R_{t,s}$$

$$mc : 34 : \frac{R_{t,o}}{R_o} = \left(\left((1+\pi_{h,t})^{cpih} (1+\pi_t)^{1-cpih} \right) \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_r}$$

$$35 : \zeta_t = \frac{1}{Z_{ct}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

$$36 : \eta_t = \frac{1}{Z_{ht}} \left(\frac{w_t^s}{1-u-v} \right)^{(1-u-v)} \left(\frac{w_t^b}{u} \right)^u \left(\frac{w_t^p}{v} \right)^v$$

A.5 News Shocks

A.5.1 Effect of Expectation shock on Rental returns:

The Expectation shock on the rental returns follow an AR process in the rental sector:

$$\ln(s_5) = \rho_s \ln(s_{5-1}) + e_s$$

$$\ln(s_4) = \ln(s_{5-1})$$

$$\ln(s_3) = \ln(s_{4-1})$$

$$\ln(s_2) = \ln(s_{3-1})$$

$$\ln(s_1) = \ln(s_{2-1})$$

$$\ln(s) = \ln(s_{1-1})$$

where e_s is an i.i.d processes with variances of σ_s which is calibrated to 0.00203^2. The shock persistence ρ_s is assumed to take high value as 0.9. The shock is augmented to the rental sector of the hand to mouth agents to mimic a news effect of decrease in rental returns for the borrowers sector. The shock on rental returns 's' enters the model in the following equations of the Hand to mouth agents:

$$p:2:0 = \left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}\left[\left(a\left(C_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(H_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left[-a\left(1-\frac{1}{\rho}\right)\left(C_t^p\right)^{\left(-\frac{1}{\rho}\right)} q_t^r * s + (1-a)\left(1-\frac{1}{\rho}\right)\left(H_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right]$$

$$p:3: C_t^p = N_t^p w_t^p + t_{p,t} - H_t^r q_t^r * s$$

A.5.2 Effect of Expectation technology shock on Consumption good Firms

The Expectation shock on the technology shock on Consumption good Firms follow an AR process in the Consumption goods sector:

$$\ln(Z_5^c) = \rho_s \ln\left(Z_{5-1}^c\right) + e_{z^c}$$

$$\ln(Z_4^c) = \ln\left(Z_{5-1}^c\right)$$

$$\ln\left(Z_3^c\right) = \ln\left(Z_{4-1}^c\right)$$

$$\ln\left(Z_2^c\right) = \ln\left(Z_{3-1}^c\right)$$

$$\ln\left(Z_1^c\right) = \ln\left(Z_{2-1}^c\right)$$

$$\ln\left(Z^c\right) = \ln\left(Z_{1-1}^c\right)$$

where e_{Z^c} is an i.i.d processes with variances of σ_{Z^c} which is calibrated to 0.00203^2. The shock persistence ρ_{Z^c} is assumed to take high value as 0.9. The shock 'Z^c' enters the model in the following equations of the consumption goods sector:

$$\begin{aligned}
fc:21:N_{p,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v-1}Y_t^c \\
fc:22:N_{b,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v}Y_t^c \\
fc:23:N_{s,t}^c &= \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v}Y_t^c
\end{aligned}$$

$$35:\zeta_t = \frac{1}{Z_{ct}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v}$$

A.5.3 Effect of Expectation technology shock on Housing good Firms

The Expectation shock on the technology shock on Housing good Firms follow an AR process in the Housing goods sector:

$$\ln\left(Z_5^h\right) = \rho_s \ln\left(Z_{5-1}^h\right) + e_{z^h}$$

$$\ln\left(Z_4^h\right) = \ln\left(Z_{5-1}^h\right)$$

$$\ln\left(Z_3^h\right) = \ln\left(Z_{4-1}^h\right)$$

$$\ln\left(Z_2^h\right) = \ln\left(Z_{3-1}^h\right)$$

$$\ln\left(Z_1^h\right) = \ln\left(Z_{2-1}^h\right)$$

$$\ln\left(Z^h\right) = \ln\left(Z_{1-1}^h\right)$$

where e_{Z^h} is an i.i.d processes with variances of σ_{Z^h} which is calibrated to 0.00203^2. The shock persistence ρ_{Z^h} is assumed to take high value as 0.9. The shock 'Z^h' enters the model in the following equations of the housing goods sector:

$$\begin{aligned}
fh:16:N_{p,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v-1} Y_t^h \\
fh:17:N_{b,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^{u-1}\left(\frac{w_t^p}{v}\right)^{v} Y_t^h \\
fh:18:N_{s,t}^h &= \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v} Y_t^h
\end{aligned}$$

$$36:\eta_t = \frac{1}{Z_{ht}}\left(\frac{w_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{w_t^b}{u}\right)^u\left(\frac{w_t^p}{v}\right)^{v}$$

A.5.4 Effect of Expectation shock on Monetary Policy:

The Expectation shock on the Monetary follow an AR process :

$$\ln(ms_5) = \rho_s \ln(ms_{5-1}) + e_{ms}$$

$$\ln(ms_4) = \ln(ms_{5-1})$$

$$\ln(ms_3) = \ln(ms_{4-1})$$

$$\ln(ms_2) = \ln(ms_{3-1})$$

$$\ln(ms_1) = \ln(ms_{2-1})$$

$$\ln(ms) = \ln(ms_{1-1})$$

where e_{ms} is an i.i.d processes with variances of σ_{ms} which is calibrated to 0.00203^2. The shock persistence ρ_{ms} is assumed to take high value as 0.9. The shock on monetary policy '*ms*' enters the model in the following equations of the Monetary policy:

$$mc:34:\frac{R_{t,o}}{R_o}=\left(\left(\left(1+\pi_{h,t}\right)^{cpih}\left(1+\pi_t\right)^{1-cpih}\right)\right)^{\phi_\pi}*ms*\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

A.6 An Empirical Bayesian Estimation of BTL Markets

A.6.1 Private Sector Equilibrium of Theoretical Model with Internal Habit Persistence

$$\left(N_t^p\right) = \left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}$$
$$\left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(\tilde{C}_t^p\right)^{-\frac{1}{\rho}} \tilde{w}_t^p$$

$$0 = \left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}\left[\left(a\left(X_t^p\right)^{1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left[-a\left(1-\frac{1}{\rho}\right)\left(\tilde{C}_t^p\right)^{\left(-\frac{1}{\rho}\right)} q_t^r + (1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H}_t^r\right)^{\left(-\frac{1}{\rho}\right)}\right]$$

$$\tilde{C}_t^p = N_t^p \tilde{w}_t^p + \tilde{t_{p,t}} - \tilde{H}_t^r q_t^r$$

$$\left(N_{b,t}\right)^{\phi} = \tilde{w_{b,t}}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^b - \tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$
$$\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^b - \tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right] a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,b}\right)^{-\frac{1}{\rho}}$$

$$0 = \left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^b - \tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma} \frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}} + (1-a)\left(\tilde{H}_t^b - \tilde{H}_t^r\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]$$
$$\left((1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H}_t^b - \tilde{H}_t^r\right)^{-\frac{1}{\rho}}\right) - \left[P_{c,t}\xi_t A_t\right]\left(q_t^h\right) + \left[P_{c,t}\Psi_t\right]\mu q_t^h$$
$$+ \beta\left[P_{c,t+1}\xi_{t+1}\right]\left(q_{t+1}^h(1-\delta)\right)$$

$$0=\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$

$$\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]\left((1-a)\left(1-\frac{1}{\rho}\right)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{-\frac{1}{\rho}}\right)$$

$$-\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}\frac{\rho}{\rho-1}$$

$$\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,b}\right)^{-\frac{1}{\rho}}q_t^r$$

$$\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$

$$\frac{\rho}{\rho-1}\left[\left(aX_t^{c,b1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_t^b}-\tilde{H_t^r}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,b}\right)^{-\frac{1}{\rho}}$$

$$=P_{c,t}\xi_t$$

$$0=[P_{c,t}\xi_t]-[P_{c,t}\Psi_t]R_{t,o}-\beta\left[P_{c,t+1}\xi_{t+1}\right]\frac{1}{z_{t+1}}R_{t,d}\frac{1}{1+\pi_{t+1}}$$

$$0=\tilde{H_t^r}q_t^r+N_{b,t}\tilde{w_{b,t}}+\tilde{d_{t,d}}+\tilde{t_{b,t}}-\tilde{C_{b,t}}-q_t^h(\tilde{H_t^b}-(1-\delta)\tilde{H_{t-1}^b}\frac{1}{z_t})-R_{t-1,d}\tilde{d_{t-1,d}}\frac{1}{z_t}\frac{1}{1+\pi_t}$$

$$0=\mu q_t^h\tilde{H_t^b}-R_{t,d}\tilde{d_{t,d}}$$

$$(N_{s,t})^{\phi}=$$

$$\tilde{w_t^s}\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{1-\frac{1}{\rho}}}\right]^{-\sigma}$$

$$\frac{\rho}{\rho-1}\left[\left(aX_t^{c,s1-\frac{1}{\rho}}+(1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}}\right)^{\frac{1}{\rho-1}}\right]a\left(1-\frac{1}{\rho}\right)\left(X_t^{c,s}\right)^{-\frac{1}{\rho}}$$

$$0 = \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$

$$\frac{\rho}{\rho-1} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] (1-a) \left(1-\frac{1}{\rho}\right) \left(\tilde{H_{s,t}}\right)^{-\frac{1}{\rho}} - [P_{c,t}\lambda_t] \left(q_t^h\right)$$

$$+ \theta \left[P_{c,t+1}\lambda_{t+1}\right] \left(q_{t+1}^h (1-\delta) \frac{1}{z_{t+1}} \right)$$

$$P_{c,t}\lambda_t = \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\tilde{H_{s,t}}^{1-\frac{1}{\rho}} \right)^{\frac{1}{1-\frac{1}{\rho}}} \right]^{-\sigma}$$

$$\frac{\rho}{\rho-1} \left[\left(aX_t^{c,s\,1-\frac{1}{\rho}} + (1-a)\left(\tilde{H_{s,t}}\right)^{1-\frac{1}{\rho}} \right)^{\frac{1}{\rho-1}} \right] a \left(1-\frac{1}{\rho}\right) \left(X_t^{c,s}\right)^{-\frac{1}{\rho}}$$

$$P_{c,t}\lambda_t = \theta \left[P_{c,t+1}\lambda_{t+1}\right] \frac{R_{t,s}}{1+\pi_{t+1}} \frac{1}{z_{t+1}}$$

$$\tilde{Y_t^c} = \tilde{C_{b,t}} + \tilde{C_{s,t}} + \tilde{C_{p,t}} + \frac{\Omega}{2}\pi_{ct}^2 \tilde{Y_t^c}$$

$$\begin{aligned}
N_{p,t}^h &= \frac{1}{Z_{ht}} \left(\frac{\tilde{w_t^s}}{1-u-v} \right)^{(1-u-v)} \left(\frac{\tilde{w_t^b}}{u} \right)^{u} \left(\frac{\tilde{w_t^p}}{v} \right)^{v-1} \tilde{Y_t^h} \\
N_{b,t}^h &= \frac{1}{Z_{ht}} \left(\frac{\tilde{w_t^s}}{1-u-v} \right)^{(1-u-v)} \left(\frac{\tilde{w_t^b}}{u} \right)^{u-1} \left(\frac{\tilde{w_t^p}}{v} \right)^{v} \tilde{Y_t^h} \\
N_{s,t}^h &= \frac{1}{Z_{ht}} \left(\frac{\tilde{w_t^s}}{1-u-v} \right)^{(-u-v)} \left(\frac{\tilde{w_t^b}}{u} \right)^{u} \left(\frac{\tilde{w_t^p}}{v} \right)^{v} \tilde{Y_t^h}
\end{aligned}$$

$$\frac{q_t^h}{q_{t-1}^h} = \frac{1+\pi_{h,t}}{1+\pi_t}$$

$$\frac{(1-\varepsilon)}{\Omega_h} + \frac{\varepsilon}{\Omega_h}\frac{\eta_t}{q_t^h} + E_t \left[\theta_t \frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]} \left[\left(\pi_{h,t+1}\right) z_{t+1} \frac{\widetilde{Y_{t+1}^h}}{\widetilde{Y_t^h}} (1+\pi_{h,t+1}) \right] \right] = \left(\pi_{h,t}(1+\pi_{h,t}) - \frac{1}{2}\left(\pi_{h,t}\right)^2 \varepsilon \right)$$

$$
\begin{aligned}
N^c_{p,t} &= \frac{1}{Z_{ct}}\left(\frac{\tilde{w^s_t}}{1-u-v}\right)^{(1-u-v)}\left(\frac{\tilde{w^b_t}}{u}\right)^{u}\left(\frac{\tilde{w^p_t}}{v}\right)^{v-1}\tilde{Y^c_t}\\
N^c_{b,t} &= \frac{1}{Z_{ct}}\left(\frac{\tilde{w^s_t}}{1-u-v}\right)^{(1-u-v)}\left(\frac{\tilde{w^b_t}}{u}\right)^{u-1}\left(\frac{\tilde{w^p_t}}{v}\right)^{v}\tilde{Y^c_t}\\
N^c_{s,t} &= \frac{1}{Z_{ct}}\left(\frac{\tilde{w^s_t}}{1-u-v}\right)^{(-u-v)}\left(\frac{\tilde{w^b_t}}{u}\right)^{u}\left(\frac{\tilde{w^p_t}}{v}\right)^{v}\tilde{Y^c_t}
\end{aligned}
$$

$$
fc:24:\frac{(1-\varepsilon)}{\Omega}+\frac{\varepsilon}{\Omega}\zeta_t+E_t\left[\theta_t\frac{[P_{c,t+1}\lambda_{t+1}]}{[P_{c,t}\lambda_t]}\left[(\pi_{t+1})\,z_{t+1}\frac{\widetilde{Y^c_{t+1}}}{\widetilde{Y^c_t}}(1+\pi_{t+1})\right]\right]=\left(\pi_t(1+\pi_t)-\frac{1}{2}(\pi_t)^2\varepsilon\right)
$$

$$
\begin{aligned}
25:\ \tfrac{t_{b,t}}{A_t} &= (1-x-y)\\
&\left(\tilde{Y^c_t}-\tilde{w_{s,t}}N^c_{s,t}-\tilde{w_{b,t}}N^c_{b,t}-\tilde{w_{p,t}}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2\tilde{Y^c_t}+\tilde{Y^h_t}q^h_t-\tilde{w_{s,t}}N^h_{s,t}-\tilde{w_{b,t}}N^h_{b,t}-\tilde{w_{p,t}}N^h_{p,t}-\frac{\Omega_h}{2}\pi^2_{h,t}\tilde{Y^h_t}\right)\\
26:\frac{t_{st}}{A_t} &= x\\
&\left(\tilde{Y^c_t}-\tilde{w_{s,t}}N^c_{s,t}-\tilde{w_{b,t}}N^c_{b,t}-\tilde{w_{p,t}}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2\tilde{Y^c_t}+\tilde{Y^h_t}q^h_t-\tilde{w_{s,t}}N^h_{s,t}-\tilde{w_{b,t}}N^h_{b,t}-\tilde{w_{p,t}}N^h_{p,t}-\frac{\Omega_h}{2}\pi^2_{h,t}\tilde{Y^h_t}\right)
\end{aligned}
$$

$$
27:\frac{t_{pt}}{A_t}=y\left(\tilde{Y^c_t}-\tilde{w_{s,t}}N^c_{s,t}-\tilde{w_{b,t}}N^c_{b,t}-\tilde{w_{p,t}}N^c_{p,t}-\frac{\Omega}{2}\pi_t^2\tilde{Y^c_t}+\tilde{Y^h_t}q^h_t-\tilde{w_{s,t}}N^h_{s,t}-\tilde{w_{b,t}}N^h_{b,t}-\tilde{w_{p,t}}N^h_{p,t}-\frac{\Omega_h}{2}\pi^2_{h,t}\tilde{Y^h_t}\right)
$$

$$
\begin{aligned}
28:\tilde{Y^h_t} &= (\tilde{H_{b,t}}-(1-\delta)\tilde{H_{b,t-1}}\frac{1}{z_t})+(\tilde{H_{s,t}}-(1-\delta)\tilde{H_{s,t-1}}\frac{1}{z_t})\\
&+\frac{\Omega_h}{2}\pi^2_{h,t}\tilde{Y^h_t}\rightarrow Y^{h2}_t
\end{aligned}
$$

$$
\begin{aligned}
mc:29:N^p_t &= N^h_{p,t}+N^c_{p,t}\rightarrow N^p_t\\
mc:30:N^b_t &= N^h_{b,t}+N^c_{b,t}\rightarrow N^b_t\\
mc:31:N^s_t &= N^h_{s,t}+N^c_{s,t}\rightarrow N^s_t
\end{aligned}
$$

$$
mc:32:\tilde{B_{s,t}}=\tilde{D_{t,o}}\rightarrow b^s_t
$$

$$mc:33:R_{t,o}=R_{t,s}\rightarrow R_{t,s}$$

$$mc:34:\frac{R_{t,o}}{R_o}=\left(\left(\left(1+\pi_{h,t}\right)^{cpih}\left(1+\pi_t\right)^{1-cpih}\right)\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_r}$$

$$35:\zeta_t=\frac{1}{Z_{ct}}\left(\frac{\tilde{w}_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{\tilde{w}_t^b}{u}\right)^u\left(\frac{\tilde{w}_t^p}{v}\right)^v$$

$$36:\eta_t=\frac{1}{Z_{ht}}\left(\frac{\tilde{w}_t^s}{1-u-v}\right)^{(1-u-v)}\left(\frac{\tilde{w}_t^b}{u}\right)^u\left(\frac{\tilde{w}_t^p}{v}\right)^v$$

A.6.2 Baseline Model with no Housing:

Observable Variables	Definition
Output (Y_t)	Real Gross Domestic Product, Percent Change from Preceding Period, Quarterly, Seasonally Adjusted Annual Rate
Consumption Inflation (π_t)	The Inflation in consumption sector is calculated from the change in the implicit price deflator Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted
Nominal interest rate (r_t)	Interest Rates and Price Indexes; Effective Federal Funds Rate (Percent), Level, Percent, Quarterly, Not Seasonally Adjusted (Seasonally adjusted manually)

Table 13: Observable Variables without Housing Market

Observable Variables	Data Source
Output (Y_t)	https://fred.stlouisfed.org/series/A191RL1Q225SBEA, February 12, 2021.
Consumption Inflation (π_t)	https://fred.stlouisfed.org/series/GDPDEF, November 24, 2021.
Nominal interest rate (r_t)	https://fred.stlouisfed.org/series/BOGZ1FL072052006Q, November 24, 2021.

Table 14: Observable Variables without Housing Market Data Source

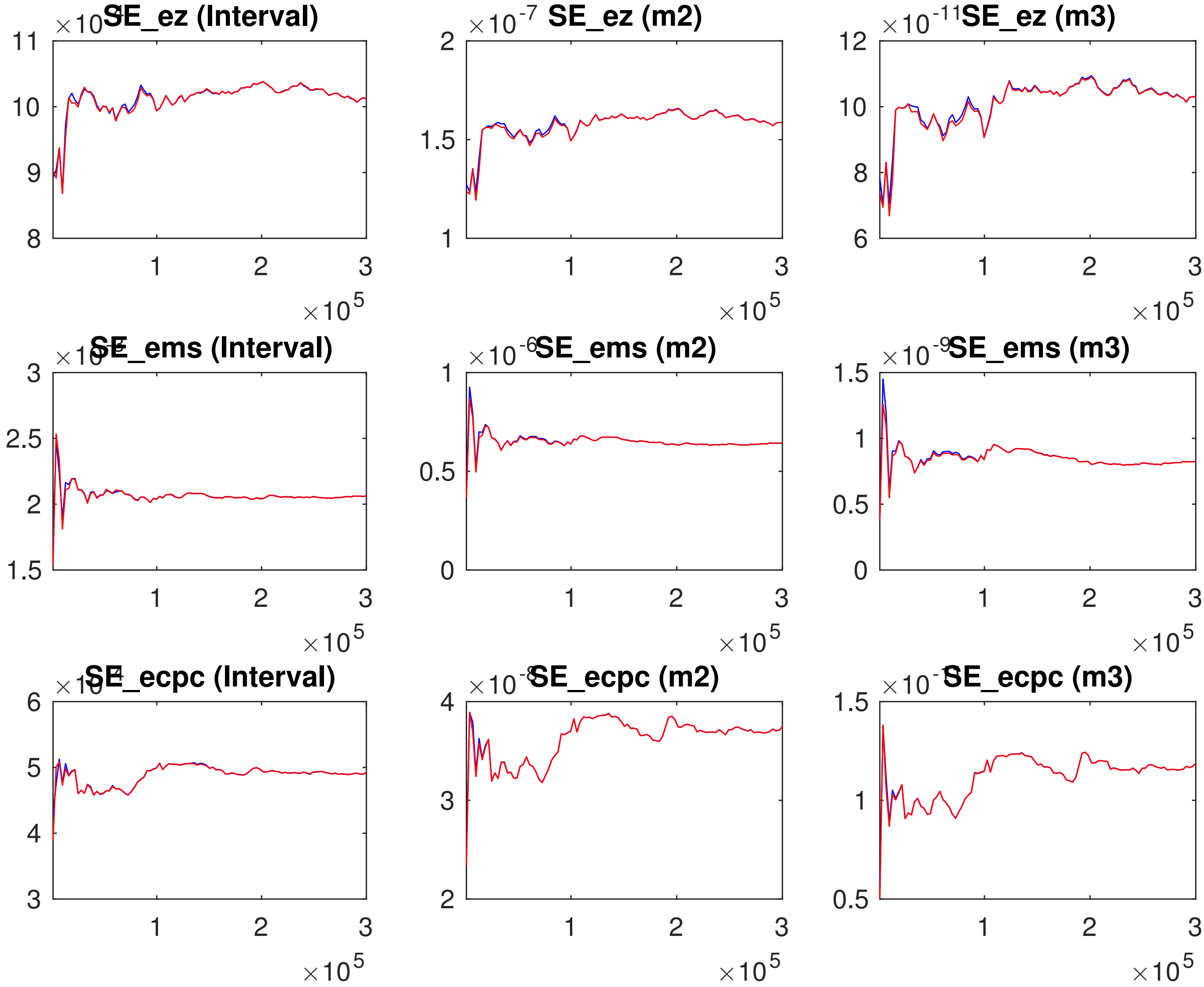


Figure 69: Diagnostics of No Housing Model

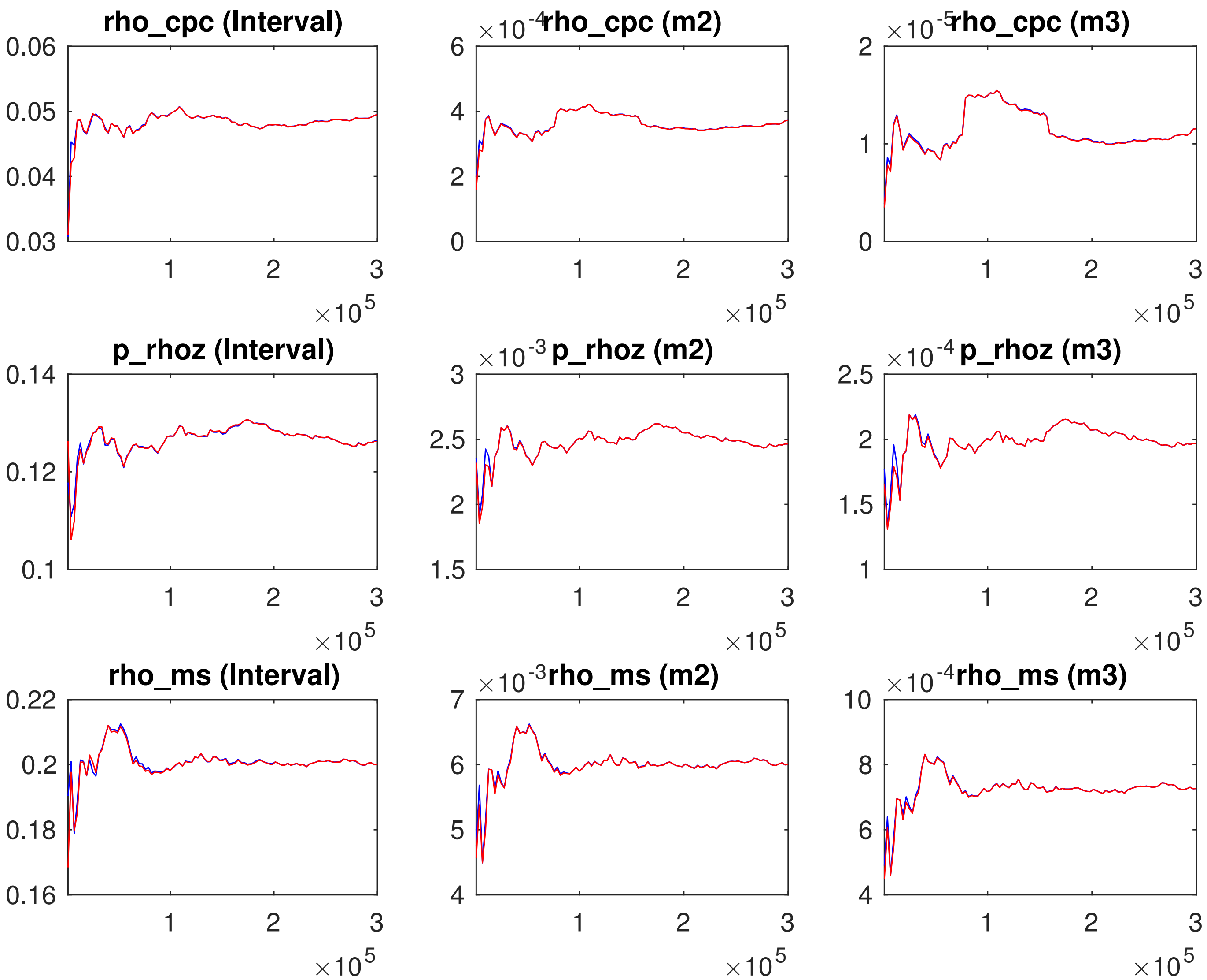


Figure 70: Diagnostics of No Housing Model

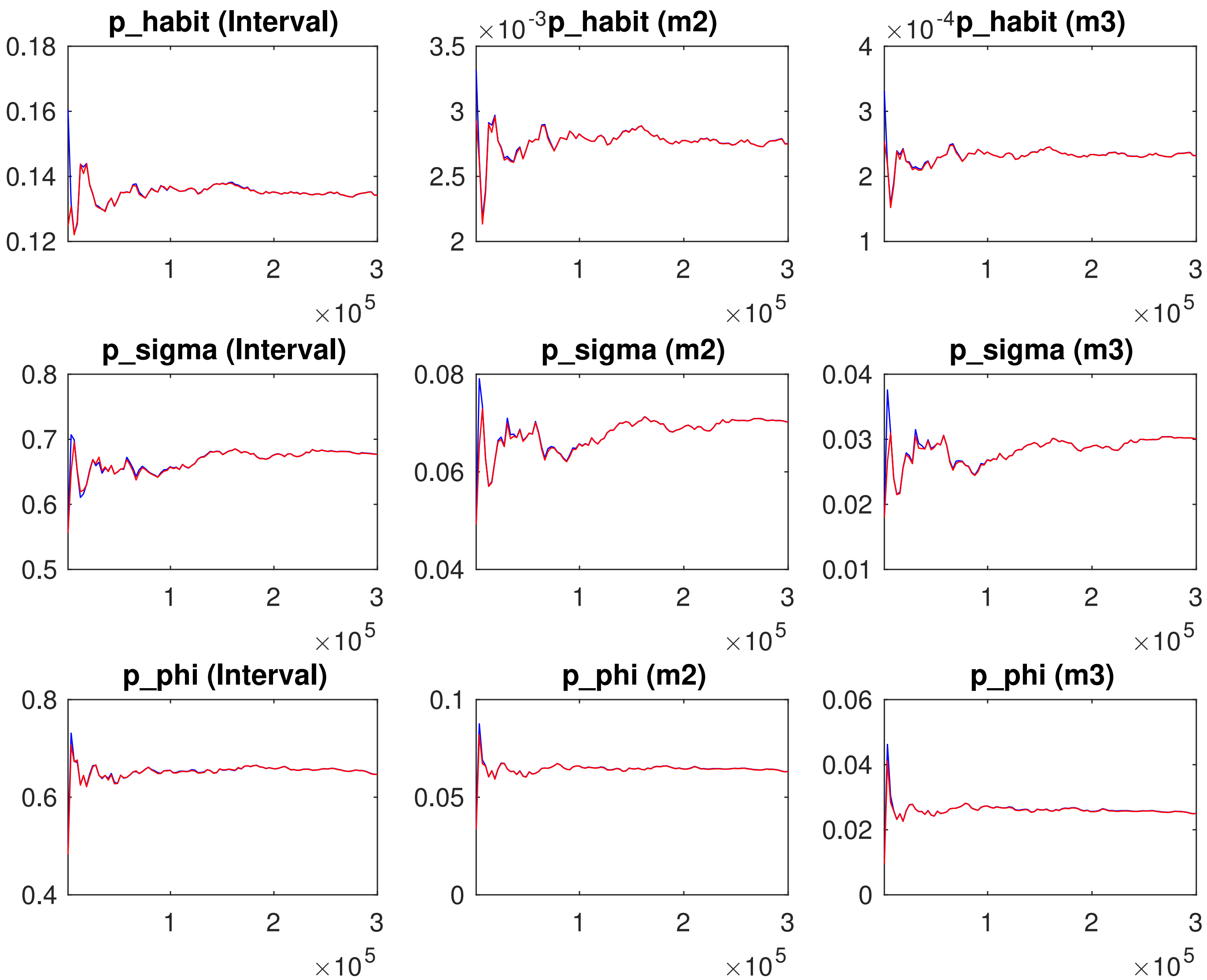


Figure 71: Diagnostics of No Housing Model

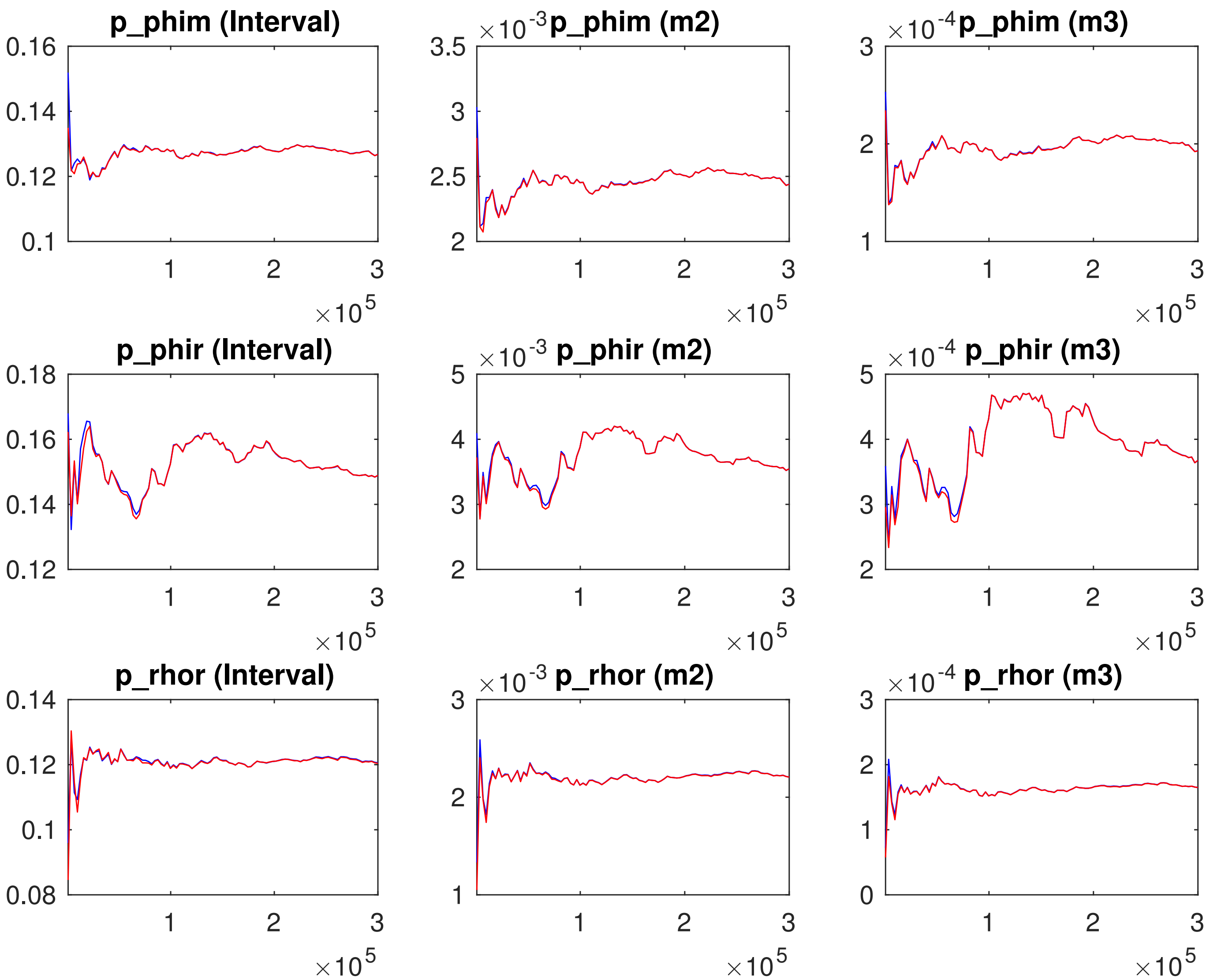


Figure 72: Diagnostics of No Housing Model

A.6.3 Baseline Model with Housing:

Observable Variables	**Definition**
Output (Y_t)	Real Gross Domestic Product, Percent Change from Preceding Period, Quarterly, Seasonally Adjusted Annual Rate
Consumption Inflation (π_t)	The Inflation in consumption sector is calculated from the change in the implicit price deflator Gross Domestic Product: Implicit Price Deflator, Index 2012=100, Quarterly, Seasonally Adjusted
Nominal interest rate (r_t)	Interest Rates and Price Indexes; Effective Federal Funds Rate (Percent), Level, Percent, Quarterly, Not Seasonally Adjusted (Seasonally adjusted manually)
Housing Inflation $\left(\pi_t^h\right)$	The Inflation in housing sector is calculated from the change in the All-Transactions House Price Index for the United States, Index 1980:Q1=100, Quarterly, Seasonally Adjusted

Table 15: Observable Variables with Housing market

Observable Variables	**Data Source**
Output (Y_t)	https://fred.stlouisfed.org/series/A191RL1Q225SBEA, February 12, 2021.
Consumption Inflation (π_t)	https://fred.stlouisfed.org/series/GDPDEF, February 24, 2021.
Nominal interest rate (r_t)	https://fred.stlouisfed.org/series/BOGZ1FL072052006Q, November 24, 2021.
Housing Inflation $\left(\pi_t^h\right)$	https://fred.stlouisfed.org/series/USSTHPI, February 11, 2021.

Table 16: Observable Variables with Housing market Data Source

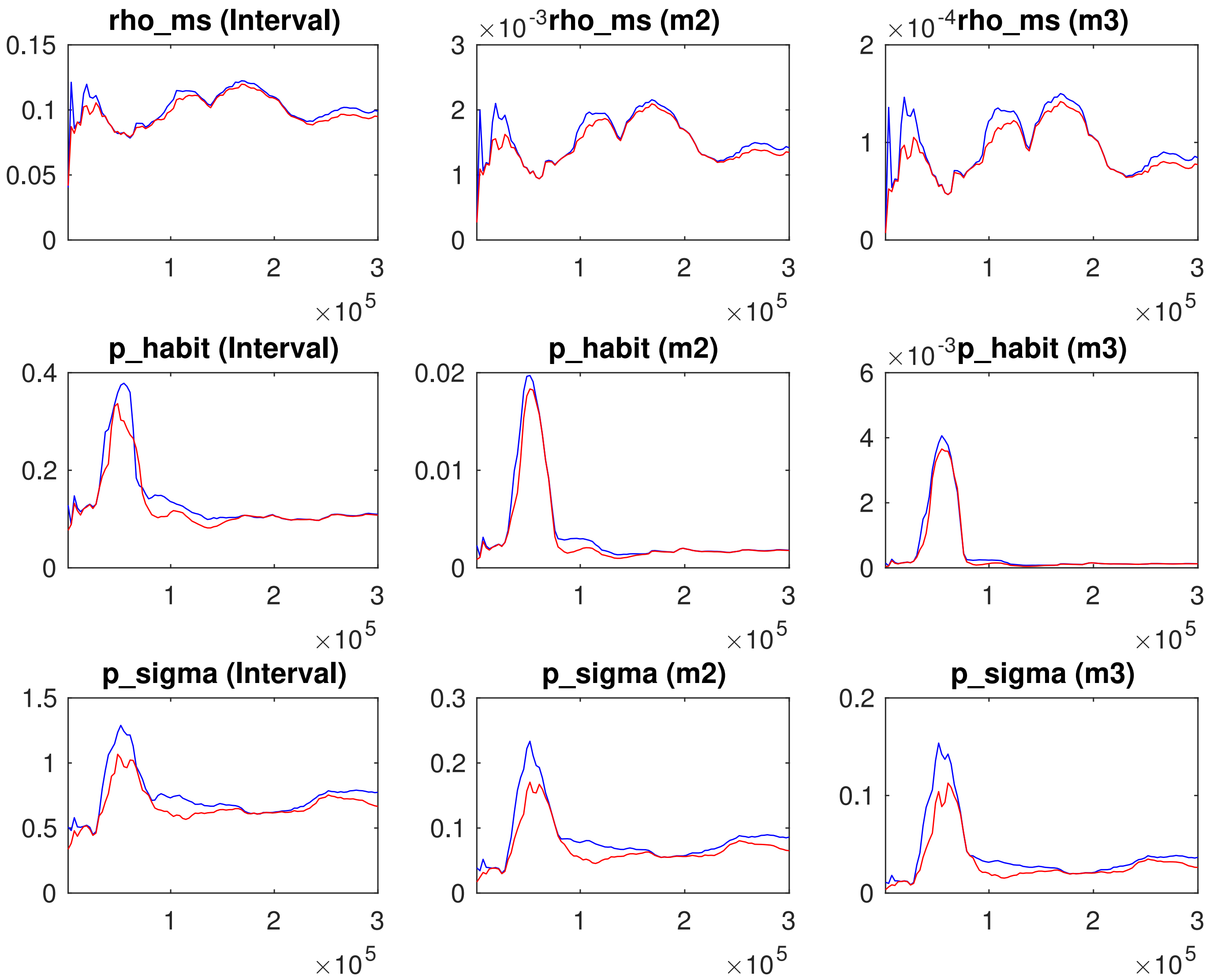


Figure 73: Diagnostics of Housing Model

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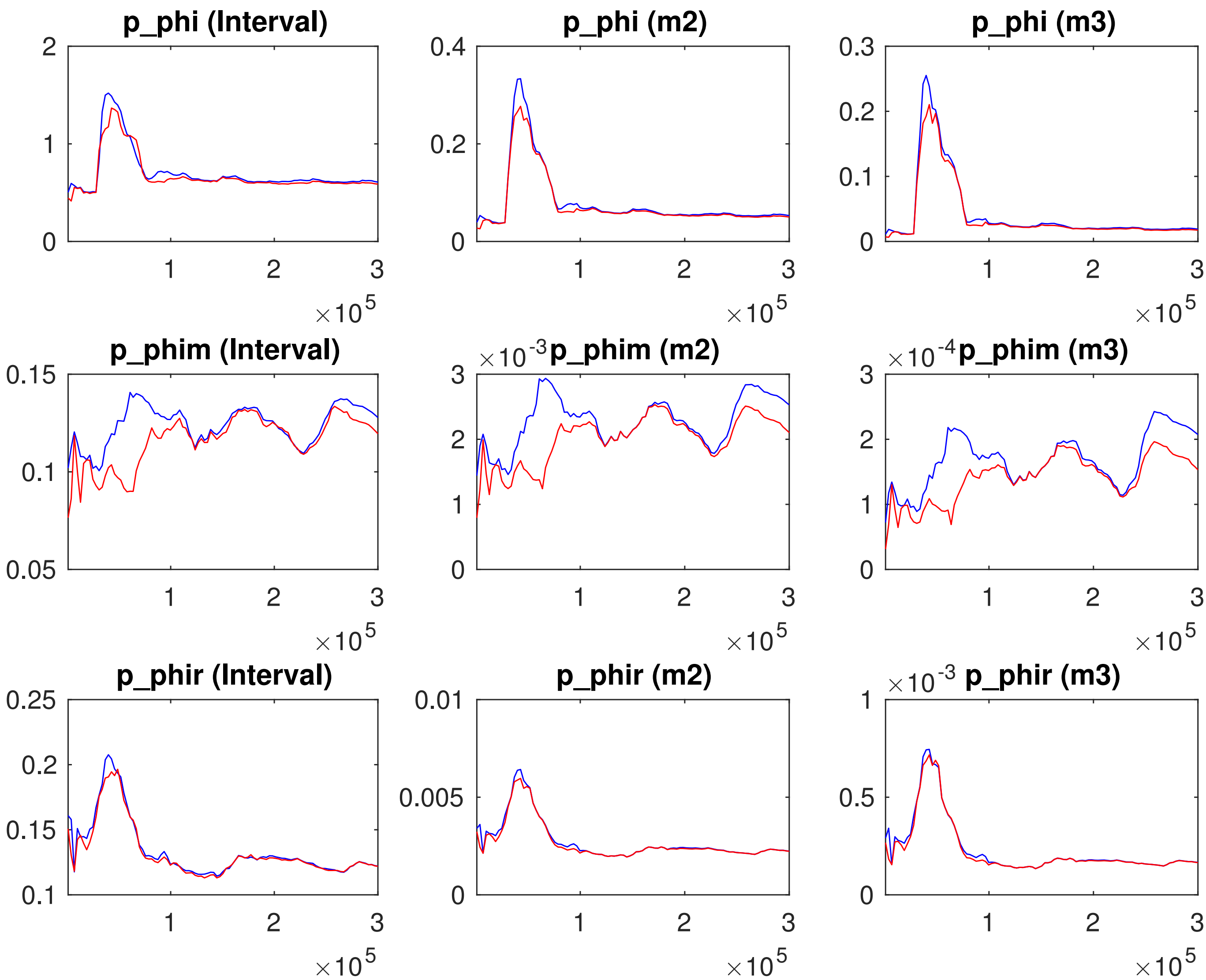

Figure 74: Diagnostics of Housing Model

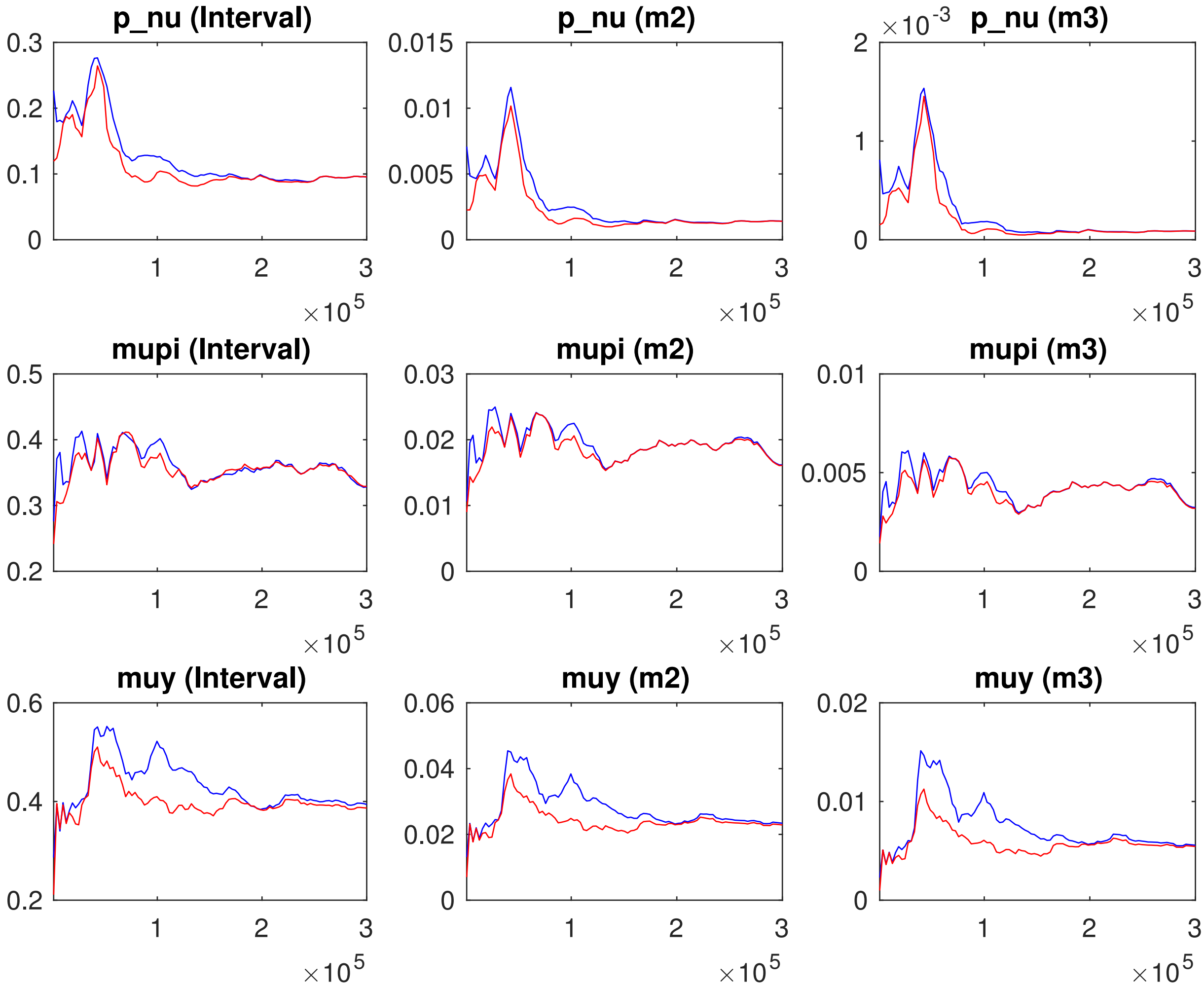

Figure 75: Diagnostics of Housing Model

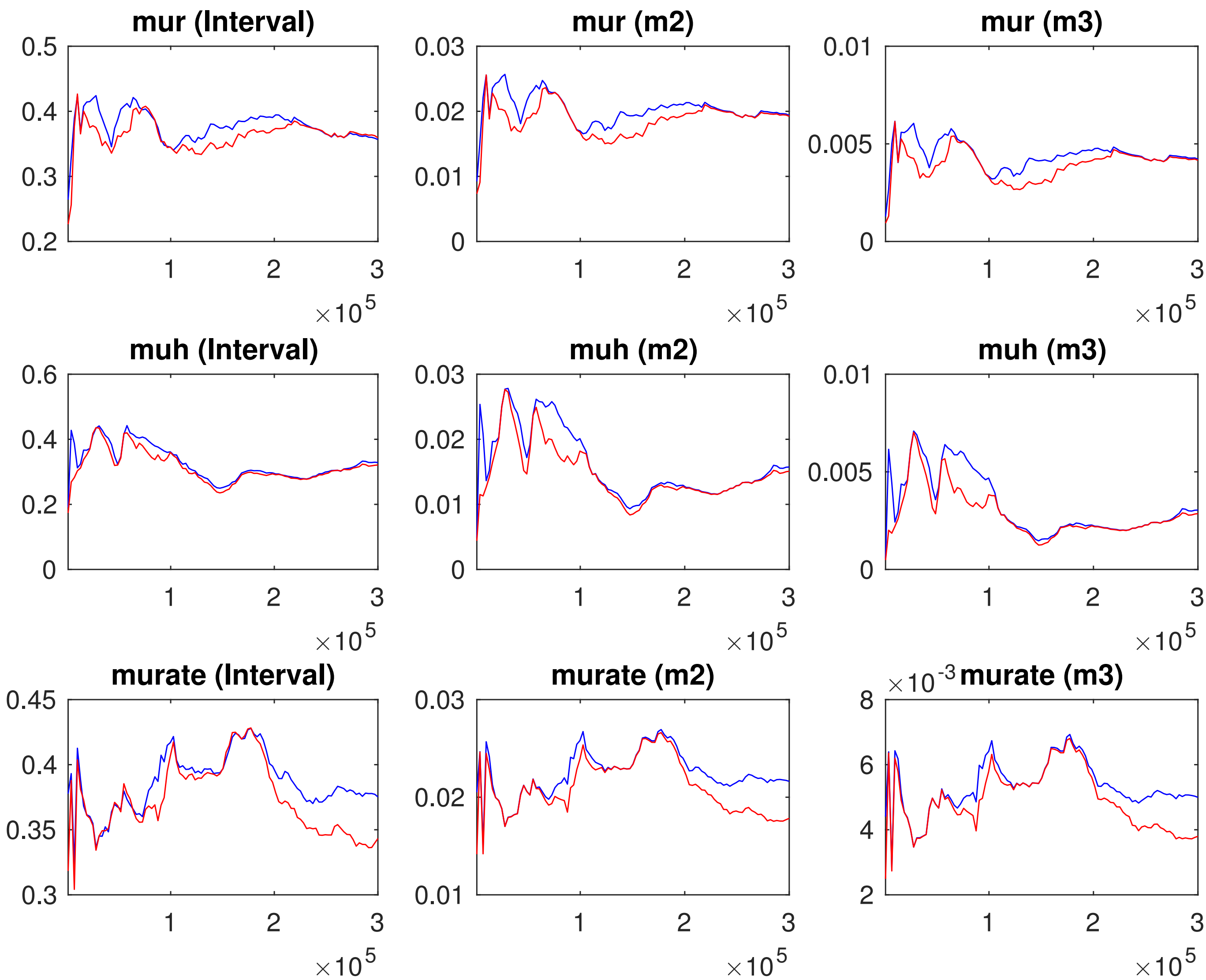

Figure 76: Diagnostics of Housing Model

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