ESSAYS IN MICROECONOMIC MODELLING OF THE BURDEN OF CHOICE

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Abstract

This thesis in applied microeconomics theory looks at three situations where economic agents have to make a decision under constraints that present a major burden. The burden can manifest itself in different ways. The first chapter presents a model of the market for top-level football players. The second chapter presents a new perspective on exploding offers and how agents can deal with them. The third and last chapter deals with how agents can overcome choice overload by using products' characteristics. In each case, I will put forward a theoretical model to represent each burden faced by economic agents, attempt to solve the model and show the properties of the equilibrium solution in terms of welfare. The research aims at showing that the impact of choice overload on agents' welfare can be mitigated, provided the correct conditions are met.

Football teams recruiting football players can avoid going into deficit if the market is sufficiently homogeneous compared to the cost of players. Exploding offers will be as efficient as open offers provided the lifetime of exploding offers is above a critical value. Finally, agents can use a choice among acceptable utility heuristic to perform efficient choices that satisfy WARP over a subset of the menu.

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When I look back, I find it hard to believe that a journey from a lowly and frightened undergraduate who left his home and his hometown of Lyon at the age of 17 would end up in the United Kingdom as a researcher in training. This journey is the consequence of one single choice I made, to ditch a seat I had in Lycée la Martinière Monplaisir to study engineering and instead go to Paris to do a dual degree in Political Sciences and Physics. I wanted to change the world. To save my country. Economics has been a revelation. They also made me despair because you cannot escape rationality in the long run. My western society. Western societies in general, are collapsing. You can't escape Prisoner's Dilemma's logic. The notion of common good and cooperation is fading. The only thing that keeps me going right now, is the firm knowledge that one can beat the Prisoner's Dilemma. All you need to do is to convince your people that they play an infinitely repeated game. How to do this? One lifetime of research is probably not enough to answer this question. It is still worth trying. In the meantime, here is my small contribution to economics.

Remerciements

Je tiens à remercier mes superviseurs de thèse, Hervé et Takashi pour leur patience. Sans eux, le présent document n'aurait pu voir le jour. J'étends aussi ces remerciements au Dr Patrick Harless qui a supervisé ma première année de thèse. Ses commentaires m'ont beaucoup aidé et m'ont donné la confiance nécessaire pour continuer. Mes remerciements à Sidartha Gordon sans lequel je n'aurais jamais mis les pieds à Glasgow.

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Statement of the author

I declare that, except where explicit reference is made to the contribution of others, that this thesis is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Introduction

Since the beginnings of economics, the act of choosing, from the arbitrages an agent has to make when shopping groceries to the ranking lists of high schools a student wants to apply to, is at the heart of the discipline. In addition, when strategic interactions start to intervene, the problems agents are facing are becoming increasingly complicated. This thesis in applied microeconomics theory looks at three cases where agents face a choice burden. The burden can come from a menu overloaded with alternatives or the multiple ramifications a choice can have, most of those not being under the agent's control. In each case, I will put forward a theoretical model to represent the specific burden of choice faced by economic agents, attempt to solve the model and show the properties of the equilibrium solution in terms of welfare. The research aims at showing that the impact of choice overload on agents' welfare can be mitigated, provided the correct conditions be met. Each chapter functions independently as they all cover different choice burdens and cover different gaps in the literature.

The first chapter presents a model that captures why football teams, especially the small ones, go through the process of choosing which players to hire become indebted and how to prevent this undesirable outcome from happening. The market for top-level football players is complex because of the abundance of players with many different characteristics on-field and off the playing field. Because of this high heterogeneity, high-level athletes are only partial substitutes. Teams not only have to make arbitrage between players of different qualities but also have to consider whether to compete or not to sign in the very best players, adding a second layer of complexity to the burden of choice. The model presented in chapter 1 unifies two sides of the theoretical literature on sports economics thought incompatible. The first family of models pictures North American sports leagues and feature a limited pool of players that teams have to share. The second family of models is more recent and caters to European sports leagues where the pool of players is assumed to be unlimited. The model of chapter 1 combines these two assumptions into a one-time simultaneous game. Teams now have the option to compete to hire high-quality players from a limited pool or to rely on an unlimited pool of lower quality players. I solve for the unique Nash equilibrium in pure strategies of this game and derive the equilibrium spending of both teams as well as the team composition and earnings. The model presented can predict the behaviour of teams in real life. The model predicts competitive unravelling without relying on multiple period setups: a small team that behaves as a profit maximizer can reduce its spending on superstar players when facing a utility-maximizing team even when superstar players' quality increases. Moreover, analysis of the teams' profit reveals the existence of non-trivial equilibrium where teams managed prudently can end up indebted while an opposing team owned by a sugar daddy with risky management can keep profiting. This undesirable situation arises when the cost of low-quality players is too high while the high quality/superstar players are too dominant on the field. Finally, the model shows that policies aiming at reducing the dominance of superstar players can help us avoid this undesirable equilibrium and allow both teams to make a positive profit.

The second chapter presents another type of difficult choice an agent has to make: dealing with exploding offers. Most of the literature on exploding offers features models with imperfect information regarding the applicants' quality but no capacity constraint problems. The main interaction explored by the literature is the arbitrage between hiring an applicant early with imperfect information, or waiting to get a perfect signal with the risk of a rival moving early and poaching an applicant. The model presented in chapter two on the other hand presents a model with no asymmetry of information regarding applicants' quality but instead presents a capacity constraint. It plugs a hole in the literature because many institutions use exploding offers when recruiting yet do not acquire extra information on applicants when waiting. Unlike the rest of the literature, the burden of choice rests solely on the applicants' shoulders. The model presented is a two-sided many-to-one matching market involving two recruiters (firms, universities etc.) who hire applicants (students, vacancies etc.) using exploding offers to streamline the overall recruitment process. Applicants can be of two types: high quality and low quality. Their preferences over the recruiters are identical and without loss of generality, recruiter one is preferred over recruiter two. Recruiter 1 does not have enough capacity to host all the applicants of high type and both recruiters have a combined capacity that is inferior to the overall applicant capacity. The Nash equilibrium of the game is unique and involves a cut-off period. The cut off period depends on the exogenous parameters of the model: the applicants' relative preferences between the two recruiters as well as the capacity constraint of both recruiters. At the equilibrium, before the cut off period, high-quality applicants let offers from the undesirable recruiter (recruiter 2) explode. After the cut off period, high-quality applicants always accept the exploding offer from recruiter 2 right before it expires and give up their chance at being hired by the high-quality recruiter. Low-quality applicants always accept an offer from the low-quality recruiter right before it expires. The model shows that exploding offers will not cause a loss of utility for anyone compared to a benchmark equilibrium with open offers, provided the exploding offer has a lifetime strictly longer than the cut off period. Where past literature has shown that the use of exploding offers yields to sub-optimal matching because they enable the matching market to unravel, I show that exploding offers can be used to smooth applicant processing and not lead to a loss of welfare.

The last chapter deals with how agents can overcome choice overload by using products' characteristics. It is a summary of recent theory about rational sequential choices and other multi-step models combined with some literature surveys from other disciplines like marketing and psychology. The goal is to overcome a problem facing the theoretical literature on revealed preferences. The chapter presents a new model that blends the sequential nature of the choice procedure with empirical findings showing that agents are unhappy with choice overload and try to filter a large menu, using the product's observable characteristics. In this model, an agent has to choose one alternative among a menu of varying sizes. The choice procedure is sequential with multiple steps and introduces an acceptability constraint instead of a simple filter like in the rational sequential choice models. Acceptability is an alternative to filtering. It states that the agents will discard an alternative based on its observable characteristics rather than the presence or absence of some other alternative in the menu. While the model presented in chapter three is not fully characterized, the chapter concludes with a lab experiment protocol to assess the empirical validity of the insights contained in the model.

Chapter 1

What kind of player to hire? The burden of European football teams

Following the football financial crisis in the late 2000s and the empirical studies on that phenomenon, economists of sports tried to model European sports leagues with clubs that act as utility-maximizer. This was a significant departure from earlier literature. In early theoretical sports economics, clubs were considered either as pure profit maximizers, like in Northern America as per El-Hodiri and Quirk (1971) [11], Fort and Quirk (1995) [13], Grossmann and Dietl (2009) [20] and Szymanski and Késenne (2004) [41]) or as pure win maximisers like Késenne (2006) [22] or Vrooman (2007) [44]. In this new generation of league models, the utility function of clubs was a weighted sum of profits and percentage of games won. Each team had one strategic variable: a total amount of talent (i.e players) to recruit from an infinite and non-competitive pool. They have been used to explain why teams backed by sugar daddies overspend (Lang et al. 2010) [24], the impact of revenue sharing on competitive balance and affinity for winning (Dietl et al. 2011) [8] or the impact of UEFA's financial fair play (Sass 2016) [35] on the long term competitive balance of a league.

However, the past literature featured some hidden hypotheses that are incompatible with empirical observations. The first one is the linearity of the wage structure of the players which contradicts Lucifora et al. (2003) [25]. This landmark paper has empirically shown in the data that players' wage at the right tail of the income distribution is not a linear function of their performance metrics. So far, no league model has managed to integrate this empirical fact. The second and most important hypothesis is the lack of an upper bound on the total talent a team can acquire. This is a legacy hypothesis from past literature. Talent used to be bounded in models like El-Hodiri and Quirk (1971) [11] and this assumption was relaxed in later models like Lang et al. 2010 [24] to account for the unlimited talent pool of European sports leagues. In the current league models, wealthy team owners can simply buy as much talent as they want to outperform their opponents. However, while the supply of people willing to play football for a living is infinite, one cannot create top-level players out of thin air nor field an infinite number of low rated players during a game.

This chapter presents a way to overcome these limitations while keeping some properties of the previous literature. It features two teams that act as utility-maximizer with limited squad capacity. Both teams can recruit two different types of players: superstar players whose supply is limited and regular players whose supply is infinite. The market for superstars resembles an all-pay auction or contest (e.g, Tullock (1983) [5]).

The model presented is similar to Arbatskaya and Mialon(2010) [3] multi-armed contest but differs critically: their model of a multi-arm contest is a combination of Tullock contests, one for each arm. Consequently putting no effort in one "arm" drives one's probability of success to zero. In the present model, only one "arm" is a Tullock contest while the other "arm" is non-competitive. Moreover, in this chapter, a squad capacity for each team dictates the participation in the non-competitive "arm". This means that putting zero effort in the competitive arm does not equate to a zero probability of success. This has major consequences on the equilibrium of the model and the profitability of teams. Overall, it is more consistent with how the football industry operates.

We show the heterogeneity between superstars and regular players has a major influence on the equilibrium. A change of this heterogeneity can have a strong effect on competitive balance, the spending of each team on players and can cause money-maximizing teams to stop competing for the championship. We also study the profitability of both teams at the equilibrium and identify some key cases. One of these cases is a mutually profitable coexistence between a money maximising team and a utility maximising team. The existence of such a cohabitation does not depend on the affinity for winning.

The model can be of interest to sports policymakers as it reveals the important role of labour market heterogeneity as a determinant of team spending and competitive balance. It explains the convexity of the wages of players observed and the observations of Schubert (2014) [37] on accounting quality of football clubs following the implementation of the Financial Fair Play. The framework can be applied to situations outside sports. For example, the model can describe the market for economics university professors, where superstars are academics with top journal publications.

The rest of the chapter is organized as follows. Section 1.1 presents the model. Section 1.2 develops the equilibrium solution. The team profitability is analysed in section 1.3. The conclusion summarises the main results and presents some possible extensions to this framework.

1.1 The model

Teams i=$\{1, 2\}$ complete for a share of the market of a sport league valued at 1[*]. Each team receives a share of equal to the percentage of games won during the season[†]. Firstly, each team recruits a mass of players, normalised to 1. These players may be superstars, available from an infinitely divisible mass of 1, or regular players, available in unlimited mass. To hire superstars, each team bids $x_i \in \mathbb{R}_{++}$. Team 1 after bidding x_1 will acquire a mass $q_1 = \frac{x_1}{x_1+x_2}$ of superstars while team 2 will receive its complement q_2 such that $q_1 + q_2 = 1$. Regular players are available at a constant cost of $c < 1$ fill the remaining roster mass of team i $(1 - q_i)$. Because the league market value is normalised to 1, the cost parameter c shall be interpreted as a relative cost of filling a team with only regular players, compared to the maximum revenues that can be obtained from the league market. The upper bound on c is a sanity check. Filling a full team with regular player should not cost more money than one is capable to earn through the championship.

Secondly, teams compete to sell merchandise to fans of the league as well as winning games during the league. Sales are increased by the quantity of superstars in the team and the market size of the superstar's reach $m > 1$. mq_1 is the money raised at the end of the season by selling merchandising to football fans. $w_1(q_1; q_2)$ is the money earned at the end of the season by winning games. We designate the function $w_1(q_1; q_2)$ as the Contest Success Function (C.S.F). Team 2 will receive the complements mq_2 and $w_2(q_1; q_2)$ such that $w_1(q_1; q_2) + w_2(q_1; q_2) = 1$. Team 1's payoff $U_1(x_1; x_2)$ depends on its share of superstars hired, proportion of games won and spending on

[*]The normalisation can be relaxed but does change the prediction and results of the chapter.

[†]This is a general assumption that could be relaxed by splitting a fraction of the common pot equally between league members and allocating the remaining fraction proportionally to the number of games won. However, relaxing this assumption does not yield any new research results.

superstar and regular players.

$$U_1(x_1, x_2) = mq_1 + (\gamma_1 + 1)w_1(q_1; q_2) - x_1 - c(1 - q_1)$$

where $\gamma_i \geq 0$[‡] represents the relative weight the team places on winning. The utility gained by winning matches can be split in two parts: the monetary part the depends on the size of the league audience (normalised to 1) and the "joy" of winning reflected in the payoff function by γ. In this chapter we use a linear C.S.F. We show in annex A.1 that this linear C.S.F under the parametric restrictions of the model is equivalent to using a logit C.S.F. with a variable change[§] The C.S.F of both teams is given below:

$$w_1(q_1, q_2) = \frac{1}{2} + \frac{d}{2}(q_1 - q_2)$$
$$w_2(q_1, q_2) = \frac{1}{2} + \frac{d}{2}(q_2 - q_1)$$

where $d \in [0; 1]$ measures the dominance of the superstars players on the field compared to the regular players. When $d = 0$, superstar players and regular players are perfect substitutes on the field. When $d = 1$, only superstar players can influence the outcome of a game.

To facilitate interpretation, let $x_{1,2} = x_1 + x_2$ and $\Delta W = w_1 - w_2$. $x_{1,2}$ is the total spending on superstars, the unit cost of superstar players. $\Delta W = w_1 - w_2$ measures the competitive balance in the league. Perfect competitive

[‡]The number 1 in $\gamma_1 + 1$ comes from the league market size which is normalised

[§]Using a logit function as a C.S.F is a standard assumption as in Andreff (2009) [1], Dietl et al. (2011) [8], Lang et al. (2010) [24] or Rottenberg (1956) [34]. Thus this approach is equivalent to the one used in past literature.

balance is achieved when $\Delta W = 0$. This measure of competitive balance is retained because it is the simplest one to compute for this model intead of w_1/w_2.

A Nash equilibrium of this model consists of a pair $(x_1; x_2)$ such that both team are best responding to each other's spending on superstars.

In reduced form the utility functions become:

$$
\begin{aligned}
U_1(x_1; x_2) &= \frac{x_1}{x_1 + x_2}(m + c + (\gamma_1 + 1)d) - x_1 + \frac{(\gamma_1 + 1)(1 - d)}{2} - c \\
U_2(x_1; x_2) &= \frac{x_2}{x_1 + x_2}(m + c + (\gamma_2 + 1)d) - x_2 + \frac{(\gamma_2 + 1)(1 - d)}{2} - c
\end{aligned}
$$

Proposition 1. The game has a unique Nash equilibrium in pure strategies. At the equilibrium both teams will spend a positive amount of money to recruit superstars. At equilibrium,

$$
\begin{aligned}
x_1^* &= \frac{(m + c + (\gamma_1 + 1)d)^2(m + c + (\gamma_2 + 1)d)}{(2m + 2c + (\gamma_1 + \gamma_2 + 2)d)^2} \\
x_2^* &= \frac{(m + c + (\gamma_1 + 1)d)(m + c + (\gamma_2 + 1)d)^2}{(2m + 2c + (\gamma_1 + \gamma_2 + 2)d)^2} \\
x_{1,2}^* = x_1^* + x_2^* &= \frac{(m + c + (\gamma_1 + 1)d)(m + c + (\gamma_2 + 1)d)}{(2m + 2c + (\gamma_1 + \gamma_2 + 2)d)} \\
\Delta W &= d \times \frac{d(\gamma_1 - \gamma_2)}{d(\gamma_1 + \gamma_2 + 2) + 2(1 + c)}
\end{aligned}
$$

The total spending on regular players are constant at c. The money spent by each team on regular players is given by:

$$
\begin{aligned}
x_1^R &= (1 - x_1^*)c \\
x_2^R &= (1 - x_2^*)c
\end{aligned}
$$

Proof : see annex annex A.2 for algebra and A.3 for uniqueness

This proposition shows that the market for superstars is stable. Every team no matter how small will place a bid to acquire superstar talent. This is consistent with the literature on investment in local talent by small teams (e.g Brandes et al. (2008) [4]). Although local superstars may not qualify as global superstars they drive teams' gate revenues by mere popularity (represented by the parameter m in the model) thus still quality as superstarsin the framework. The non linear wage structures stems from the contest structure. Tullock contests are known for the non-linear best responses of the contestants¶. Finding a contest model that keeps the predictions of the current literature while generating linear best response is not the aim of this chapter but such a research is not ruled out in the future.

1.2 Properties of the solution

The first property of the equilibrium of the model is a plausible conclusion. It is an inequality result between the price of superstars $x_{1,2}$ and the price of regular players c.

Proposition 2. For any value of γ_i $i \in \{1; 2\}$ and dominance factor d, $x^*_{1,2} > c$.

Proof : See annex A.4.

The intuition behind this result is straightforward : superstars are at least as good as the regular players on the field and they also help selling merchandising. It is logical that their unit price is higher than the regular players. The comparative statics of the solution are presented next.

Proposition 3. Comparative statics summary Throughout this section we will assume that $\gamma_1 < \gamma_2$ without loss of generality.

¶A phenomenon that is also observed empirically. Wärneryd (2018) [45] is an example

Parameter \ variables	$\frac{\partial x_1^*}{\cdots}$	$\frac{\partial x_2^*}{\cdots}$	$\frac{\partial x_{1,2}^*}{\cdots}$	Competitive balance
$\frac{\cdots}{\partial \gamma_1}$	>0	<0	>0	Shifts in favour of team 1
$\frac{\cdots}{\partial c}$	>0	>0	>0	Improves
$\frac{\cdots}{\partial d}$	variable	>0	>0	Deteriorates
$\frac{\cdots}{\partial m}$	>0	>0	>0	Improves

Figure 1.1: Summary of comparative statics of the solution

- $x_{1,2}^*$ is increased in each parameter γ because it makes the superstar players more attractive for at least one team.
- The impact of c on competitive balance may look surprising. Making regular players more costly lowers the available budget of both teams to spend on superstars, thus increasing lowering the absolute value of the competitive balance indicator, making the league overall more balanced ($\Delta W = 0$ indicates a perfectly balanced league). Moreover, making the regular players more expensive makes the superstars comparatively more attractive. This is an incentive to increase spending on superstars.
- The behaviour of x_1^*, x_2^* and $x_{1,2}^*$ w.r.t c confirms the intuition of an increased competition on the superstar market.
- The behaviour of competitive balance ΔW is more nuanced.
- The behaviour of ΔW w.r.t either γ is expected : increasing the affinity of team 1 for winning shifts the competitive balance in favour of team 1. The same holds for team 2.
- The market reach of superstars m behaves the same way as the reservation cost of regular players c
- Increasing the field dominance d of superstars always worsen the com-

petitive balance as the absolute value of ΔW increases. An increase of the dominance of the superstars has a more more ambiguous when it comes to individual team spending. As said before, in comparative statics, it is assumed that team 1 is the underdog without loss of generality. There are two possible scenarii:

- In the first case, both teams may increase spending on superstars. Although competitive balance deteriorates, the underdog still tries to keep up with the dominant team.
- In the second case, the dominant team increases its spending but the underdog decreases its spending. Both changes, will deteriorate the competitive balance even more than in case 1.

Effects of increasing the field dominance on individual spending and competitive balance The first main result of the chapter is presented below. It is about the existence of counter intuitive behaviour regarding superstar players:

Theorem 1. There exist a critical value $\bar{\gamma_1}$ such that $\partial x_1 / \partial d < 0$ iff

$$0 \leq \gamma_1 < \frac{m+c}{d} - 1$$
$$\gamma_1 < \bar{\gamma_1} < \gamma_2$$

The exact value of $\bar{\gamma_1}$ can be computed. The exact value is given in annex A.6.

Proof : see annex A.6.

Theorem 1 shows the existence of non trivial cases where making the superstars more attractive for everyone by increasing their field dominance d causes one of the team to lower its spending on superstars. The economic intuition behind this result is detailed below:

The first inequality in theorem 1 ensures that team 1 is is not too "win oriented". If team 1 is money maximising (i.e $\gamma_1 = 0$), it will always satisfy the first inequality. It is assumed without loss of generality that team 2 is strictly more "win oriented" than team 1. If the dominance parameter d increases then team 2 will systematically increase its spending on superstars, reflecting their increased importance when it comes to securing victory on the field.

The increased spending of team 2 on superstar players is creating an externality for team 1. If d increases, then team 1 now has to spend more money than before to secure the same share of superstar players. This makes regular players more attractive in comparison to superstars. While regular players are getting less efficient on the field compared to superstars, their cost remain constant at c. If γ_2 is sufficiently high (i.e it is above the threshold $\bar{\gamma_1}$), then the marginal utility that team 1 is getting from its superstar dips below the marginal cost of hiring this fraction of superstars. Consequently, team 1 will reduce its spending on superstars and will substitute them with regular players. This explains why when the two inequalities of theorem 1 hold, $\partial x_1/\partial d < 0$.

1.3 Analysis of the profit of the teams

This section aims at determining the domain of the exogenous parameters where both team end up making money by participating in the league. For the entire section, the parameter d is assumed to be strictly positive. $d = 0$

is a trivial case not worth covering here. For each team we define a profit function Π_i:

$$\Pi_1 = mq_1 + w_1(q_1; q_2) - x_1 - c(1 - q_1)$$
$$\Pi_2 = mq_2 + w_2(q_1; q_2) - x_2 - c(1 - q_2)$$

Let $e_1 = m + c + (1 + \gamma_1)d$ and $e_2 = m + c + (1 + \gamma_2)d$. The rest of the analysis will be performed for team 1. The conclusions will hold for team 2. Substituting the equilibrium values of q_1, q_2, x_1 and x_2 we obtain :

$$\Pi_1 = m\frac{e_1}{e_1 + e_2} + 1/2 + \frac{d}{2}\frac{e_1 - e_2}{e_1 + e_2} - \frac{e_1^2 e_2}{(e_1 + e_2)^2} - c\frac{e_2}{e_1 + e_2}$$

We can analyse the signs of this profit function for different values of γ, c and d. Solving for $\Pi_1 = 0$ is equivalent to solve for:

$$0 = e_1^2\left(\frac{1-d}{2} - c - d\gamma_2\right) + e_1 e_2(1 + m - c) + e_2^2\left(\frac{1-d}{2} - c\right)$$

This equation will not be solved completely for γ, d and c. Instead, let us focus on specific cases with interesting properties.

Case A: Small leagues becoming non-viable

The first case of interest that the model can describe is the impact of the Bosman Ruling on the smaller football leagues, like Belgium, Austria and others. By lifting the limitations on the movement of players, the Bosman ruling (and the subsequent rulings by the ECJ) has effectively put teams from different leagues in competition with each other for all types of players. This can drive up the price of regular players.

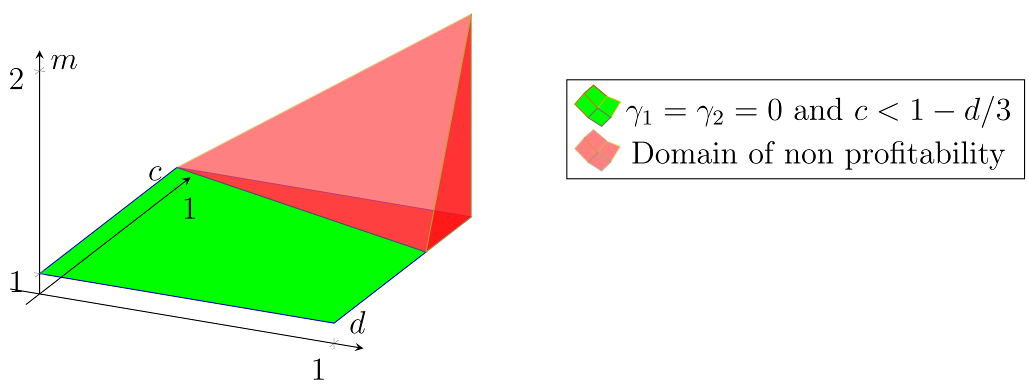
The following analysis presents a set of non trivial cases where a league with a pair of money maximising teams can both end up losing money. If both teams are money maximising, $\gamma_1 = \gamma_2 = 0$ then $e_1 = e_2 = m + c + d$. The equation $\Pi_1 = 0$ becomes:

$$0 = (m + c + d)^2(1 - 2c - d + 1 + m - c)$$
$$0 = 1 - 2c - d + 1 + m - c$$
$$m = 3c + d - 2$$

Using this result we can present the following proposition:

Proposition 4. There exist a class of equilibria where two money maximising teams are in deficit. This situation arises iff:
$\gamma_1 = \gamma_2 = 0$ and $m < 3c + d - 2$

Figure 1.2: Profitability domain for a league with two symmetric teams



The map above describes a profitability zone for a league with two money-maximising teams. The area in red represents a domain where the dominance coefficient d and the reservation cost of regular players c is too high compared to the market reach of superstars for the teams to make profits. While no team has any interest in overspending, the combination of a reservation cost that is too high and a market too small prevents both teams from being profitable. In this area, the league should not be viable and both teams should drop out of it.

Proposition 4 tells us that a league made of money maximising team is viable when the incentives to fight for superstars are reduced and hiring regular players is not too expensive compared to the league market size.

One of the effects of the Bosman ruling can be integrated into the model as an exogenous increase of the parameter c (cost of regular players) which can push a small league from a profitable zone to a non-profitable one. The model being one period cannot represent the exodus of talent from a poorer league to a richer one between two periods. While we do not see empirically leagues collapsing we do observe that teams in smaller leagues rely on the selling of promising players to wealthy teams in larger leagues to stay afloat, indicating a lack of resources. The Covid19 pandemic has exposed this phenomenon in bare light.

We remind the reader that c is a relative cost. Thus, $c = 1$ implies that filling an entire team with regular players would cost as much as half the total revenues that can be generated from the league's market. This is an extreme assumption.

One can argue that teams could hire players of a lower quality than regulars (amateurs for example) for a cheaper price. However, because European leagues are open, a team is required to maintain a minimum investment in players if it wants to keep its place in the league. Empirical observations showed that teams prefer to run deficits rather than be downgraded.

Case B: The superstar price inflation

A very salient effect of the post Bosman era is the inflation of spending on superstar players by major teams across Europe. An other salient effect is the exodus of talent from poorer leagues but the present model does not cover it. Previous papers on the topic have modeled such a phenomenon with predictable results. The present model makes a similar prediction and thus ties itself with the existing literature. For Case B we assume the league has two major teams that are competitive and value victory equally. These could be two sugar daddies wanting to win at all cost.

In this case $\gamma_1 = \gamma_2 = \gamma$ thus $e_1 = e_2 = m + c + d(1+\gamma)$. The equation $\Pi_1 = 0$ becomes:

$$\begin{aligned} 0 &= (m + c + d(1+\gamma))^2(1 - 2c - d + 2 - c - d\gamma) \\ 0 &= 2 - 3c - d - d\gamma + m \\ \gamma &= \frac{2 + m - 3c}{d} - 1 \end{aligned}$$

Proposition 5. A league with symmetric teams is profitable iff $c \leq \frac{2+m-d}{3}$ and $\gamma_1 = \gamma_2 \leq \frac{2+m-3c}{d} - 1$

Proposition 5 tells us the league is viable for a pair of symmetric teams if gamma is not above a specific threshold. The figure below gives a clearer view of the domain of profitability.

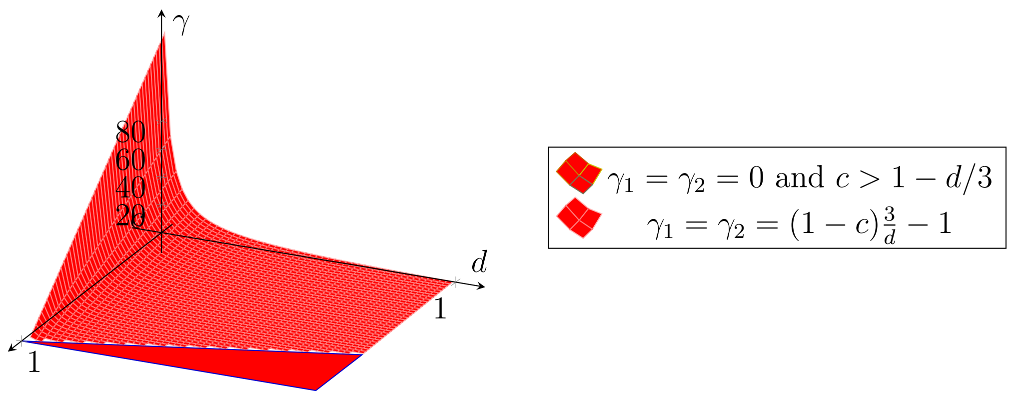


Figure 1.3: Profitability domain for a league with two symmetric teams (m=1)

The graph above is an extension of the one in Case A. This case applies to leagues that include several teams that have a very strong affinity for winning. The Spanish Liga and the English premier league are good examples. The plane described by $\gamma = 0$ represents case A with $m = 1$. The red surface represents the profitability frontier : if the coordinates of the point (d, c, γ) are above the red surface, both team lose money at the end of the season. The area marked with a triangle is the part of the surface that coincides with the plane $\gamma = 0$.

The figure shows that in an environment with highly dominant superstars (d is closer to 1), the parameter γ does not need to be very high to put the teams into deficit. The impact of the regular players' cost c is less pronounced than the impact of d on the teams' finances.

All the parameter combinations of γ, d, c that are above the red surface make the league not profitable. Below is the same figure with $m = 5$:

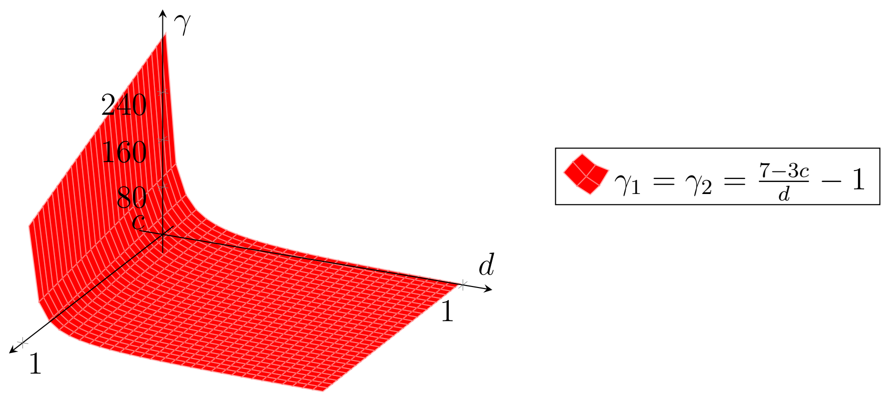


Figure 1.4: Profitability domain for a league with two symmetric teams (m=5)

For the case where $m = 5$ the domain of profitability has the same appearance as the case where $m = 1$. If superstars have a lot of impact on the playing field (d is close to one) then the team's affinity for victory γ does not need to be very high to put both teams into deficit.

This result is in line with past literature on sugar daddies. A fierce competition between teams that operate under a soft budget constraint is likely to result in an unbalanced budget as teams overbid on superstar players.

Case C: Long term stability of competitive imbalance

Case C is the most interesting of the three and the major contribution of this chapter. It is best suited for leagues with one single dominant team like the German Bundesliga (Munich) and the French ligue 1 (Paris in the late 2010 or Lyon in the early 2000).

Without loss of generality let team 1 be the utility maximizer (i.e $\gamma_1 > 0$) and team 2 be the profit maximizer (i.e $\gamma_2 = 0$). Thus $e_1 = (1+\gamma_1)d + m + c$ and $e_2 = m + d + c$. The equations profit equation $\Pi_1 = 0$ and $\Pi_2 = 0$

become:

$$\Pi_1 = 0 \Leftrightarrow 0 = e_1^2\left(\frac{1-d}{2} - c\right) + e_1e_2(1+m-c) + e_2^2\left(\frac{1-d}{2} - c\right)$$
$$\Pi_2 = 0 \Leftrightarrow 0 = e_2^2\left(\frac{1-d}{2} - c - d\gamma_1\right) + e_1e_2(1+m-c) + e_1^2\left(\frac{1-d}{2} - c\right)$$

With m, d and c fixed we can consider these two equations as polynomials of γ_1. With simple constraints we can ensure both of these are positive. The family of equilibrium presented below can be interpreted as mutually profitable bullying.

Theorem 2. If $\gamma_1 = 0$ and $c < \frac{1-d}{2}$ then the league is profitable for both teams for any value of m, γ_1 and d.

Proof: See annex A.7.

Theorem 2 shows that a league containing one money maximising team and one non money maximising team can be sustainable for everyone as long as the regular players are neither too expensive nor too lagging behind the superstars. The parameters m and Γ_1 have little impact on the overall profitability of the team. What matters is the relation between how important the superstars are compared to the regular players on the field and the cost of the regular players.

The intuition behind this result is rather simple. On one hand, the money maximising team is unlikely to win many championships it can win enough games to secure enough funding to finance the purchase of regular players. On the other hand, the utility maximising team on the other hand will not spend too much on superstar players because the money maximising team is not willing to overbid.

Concluding remarks

Our model sheds a new light on why sport teams (football teams in particular) can end up in deficit despite acting as a profit maximizer. Previous literature showed that sugar daddies and weak budget constraint could lead to unprofitable overbidding over players. Yet real life showed that any team in a football league, not just the ones owned by sugar daddies, can end up being in debt. The model keeps the predictions of past literature and adds a way to explain the generalised tendency of football teams to lose money.

While a profit maximising team can choose not invest a lot of effort on the market for superstars, it still has to spend money to fill up its roster with regular players (eleven and some spares in the case of football). If these regular players cannot compete properly with superstars, thus not securing the necessary funds to cover the expanses, my model predicts that such a profit maximising team can end up in deficit.

These predictions can be considered bleak, yet they contain a silver lining: While the model focuses on the heterogeneity of the player market to explain team profitability, the dominance parameter d measuring said heterogeneity could be impacted by the structure of the competition. While superstar players may have a marginally better performance on the field compared to regular player this small difference can be compounded into a large advantage by the format of a league or a tournament. A single elimination tournament like the FA cup/Coupe de France/Copa del Rey will see many more upsets than a tournament with a group phase like the European Champions League. This reinforced dominance could be translated as an increase of the d parameter in the model. Consequently, any policy aiming at preserving the competitive balance must account for the labour market otherwise, such policies may exacerbate the disparity among teams or worse put their budget in the red.

Finally, the present model could be extended by introducing home market heterogeneity for both teams. An other possible extension is to change the C.S.F or the mechanism to allocate superstars by applying the results of Ron Siegel (2010) [40]. Alternatively, the model could be applied to other fields where economic agents compete but face a capacity constraint that has to be met. The market for university teachers could be such an example. Universities have to recruit a minimum number of researchers to ensure the education of the enrolled students. In this example, the superstars would be researchers who have published on a top academic journal and the other researchers would be the regular players.

This chapters provided a concrete example of a choice burden: the hiring of players by football teams. Because of the sheer volume of players to choose from and the strategic interactions between teams with different objectives, choosing how to hire is a non trivial matter. As proved in this chapter, this burden can result in highly sub-optimal equilibrium as observed in the post Bosman era of football. Yet there is still hope. A shrewd policy maker can ensure mutual profitability in the long run. Let us move on to an other type of choice burden: exploding offers.

Chapter 2

To accept or to gamble? The burden of exploding offers

Matching markets are widespread and the literature associated with them is quite broad. However, most of the literature focused on algorithmic resolution of the matching problem. Deferred acceptance Gale and Shapley (1962) and top trading cycles Shapley, Lloyd; Scarf, Herbert (1974) [39] being some of the most well-known algorithms. In the majority of the papers, economists assumed that either the market or some planner acquires the relevant information from the players in the matching game and then provides an allocation for everyone at the same time. Players receive an unconditional offer or they do not receive anything at all.

Yet, in the real world there are many decentralised matching markets (consulting firms filling multiple vacancies, universities recruiting master and PhD students for example) that operate using exploding offers. Exploding offers are offers with a time limit. A player in a multi-period matching market has only a limited number of period to formally accept an exploding offer and exit the game. If a player does not accept an offer after a set number of period, the offer expires and becomes no longer valid. The exploding offers introduce a new problem for economists: they may unravel the market and force the players to issue sub-optimal offers early in the game instead of waiting for the last period to achieve the most efficient matching.

Niederle and Roth (2009) [31] have identified some markets at risk of unravelling that use exploding offers. A well know example is the market for Gastroenterology fellowship [30] in the US. Most of these market involve applicants whose quality in uncertain but can be discovered if given time. Pan (2018) [32] focused on two-period matching games with imperfect information where the quality of players of at least one side is unknown. Moreover, one side of the market (firms for example) can make strategic decision on whether to issue exploding offers earlier or later.

Yet these are not the only type of markets that feature exploding offers. The German DoSV (Grenet et al. 2019) [18] for matching high school graduates to universities is a multi-period process that is partially decentralised and has exploding offers. Large consulting corporations (Accenture, BCG, PwC etc.) use exploding offers as a way to streamline their recruitment. In these cases, the quality of the applicant is easy to assess but logistical constraints still make the use of exploding offers a necessity to prevent congestion and the formation of long waiting queues. How do exploding offers affect the quality of the matching market and who is impacted? Does the length of an exploding offer has an impact as well?

In this second chapter, I will present a model to study the impact of exploding offers where all players know in advance their qualities and the firm/university side of the market is bottlenecked by the capacity of the latter. Unlike previous literature, that uses a noisy signal to generate uncertainty, the present model uses a time-consuming process coupled with capacity constraints to generate uncertainty. Firm/universities will not be able to issue early exploding offers strategically because of serialised treatment of applicants similar to Roth and Xing (1997) [33]. It is a many-to-one matching model where there are two types of students to allocate to two universities. Unlike previous literature, the model can have any number of periods and accommodates the use of exploding offers of any length. I show that the use of exploding offers with long duration can help streamline the recruitment process without leading to a loss of utility compared to an equilibrium with an open offer.

I find that, assuming students know their quality perfectly; exploding offers length has little to no impact on the composition of the top quality form/university. Moreover, long lasting exploding offers benefit high quality applicants and lower quality firm/university while harming lower quality applicants, leading to a more positive associating matching.

The rest of the chapter will be organised as follows. Section 2.1 will present the model. Section 2.2 is a benchmark case where the universities can only send open offers. Section 2.3 presents the equilibrium solution with exploding offers of any length while section 2.4 analyses the welfare of applicants and firms/universities. Finally, section 2.5 extends the base model by introducing heterogeneous preferences for the high quality students.

Throughout the chapter, I will use a student/university terminology but one could apply the model to an applicant/firm setup as well.

2.1 The model

A population of students is to be matched to three universities A (very desirable), B (less desirable) and C (undesirable) through a multi-period procedure. There are two types of students called α (high quality) and β (low quality).

There are N_α students of type α and N_β students of type β in total where $N_\alpha, N_\beta \in \mathbb{N}$ and $N_\alpha \leq N_\beta$. Let $f = \frac{N_\alpha}{N_\alpha + N_\beta}$ be the ratio of students of type α inside the total student population. By construction $f \leq 1/2$. Students are assumed to know their type. The value of the fraction f is common knowledge.

Universities A and B have limited capacities (respectively c_A and c_B) but university C is considered so large it can accommodate all the students regardless of their type.

Matching procedure in detail : All students send their dossiers to apply to all universities. In period 0 university C presents all students with an unconditional offer that never expires. C should be considered an outside option that is always available as a last resort.

To streamline their recruitment process, both universities A and B will spread the processing of all the students' dossiers they receive over multiple periods. Let $T \geq 2$ be the number of periods needed by the universities to process all the received dossiers. It is assumed that one period in this model corresponds to at least one day and at most one week.

Because the procedure is done in a finite number of periods and universities A and B process dossiers independently we can divide the population of type α and β students into T^2 different states $(i,j) \in \{1;\ldots;T\}^2$. i is the period when the student will be contacted by university A and j is the period when the student will be contacted by university B. For example, a student of type α in state $(3;1)$ will have his dossier processed by university A in period 3 and processed from university B in period 1. There are exactly $n_\alpha = N_\alpha/T^2$ (resp. $n_\beta = N_\beta/T^2$) students of type α (resp. β) in a single state (i,j).*
Students have no way before the game starts to determinate the state they will find themselves in. Throughout the procedure, students will discover the state they are in by receiving answers from both universities.

At the beginning of each period t university A processes all the dossiers of α students in the states $(t;x)$ $\forall x \in \{1;\ldots;T\}$, and then processes all the dossiers of the β students in the same states once it has received the answers of the α students whose dossiers has been processed. A university always knows the type (α or β) of each student it interacts with but cannot discriminate between the different states $(t;x)$ $\forall x \in \{1;\ldots;T\}$. In other words you always know the quality of every single applicant but you do not know how an individual applicant interacted with the competing university.

At the beginning of each period t, university B processes all the dossiers in states $(y;t)$ $\forall y \in \{1;\ldots;T\}$ in a similar fashion (α first then β). Like university A, university B always knows the type of each student it interacts with but cannot discriminate between the states $(y;t)$ $\forall y \in \{1;\ldots;T\}$.

*Removing this divisibility assumption would bring technical complications without improving the chapter's message

In the benchmark model, universities will only issue open offers. Later models where universities will only be able to issue exploding offers of a varying lifetime, will be compared to the benchmark.

Players' payoff The utility function of all universities is the same and is common knowledge. It depends on the type of student they are matched with at the end of the procedure:

$$\mathcal{U}_A = \mathcal{U}_B = \mathcal{U}_C = \sum \text{Students of type } \beta + V_\alpha \sum \text{Students of type } \alpha$$

Where $V_\alpha > 1$ is the premium utility universities get by hiring α students. The utility function $\mathcal{U}_s$ of all the students is the same. It depends on the university the student is matched with at the end of the procedure:

$$\mathcal{U}_s = \begin{cases} 0 \text{ if matched with } C \\ 1 \text{ if matched with } B \\ V_A > 1 \text{ if matched with } A \end{cases}$$

Type of students and capacity constraints : To avoid trivial cases, restrictions on the capacity of both universities A and B will be placed and link these capacity constraints to the number of students of both types. The capacity of university A is such that $(T-1)Tn_\alpha < c_A < N_\alpha$. The upper bound on the capacity implies that not all students of type α will be able to enroll in university A (the desirable one). The capacity of university B is such that $T(T-1)n_\beta + Tn_\alpha \leq c_B \leq N_\beta$. The lower bound ensures the matching procedures detailed below will not be interrupted early and the upper bound eliminate trivial equilibria where each student of type α and

β has a guaranteed place in either A or B. The capacity constraint of both universities are common knowledge.[†]

Actions of the universities. After processing the dossier of a student the university learns its type and can choose to either:

- Present the student with an unconditional but exploding offer of duration $d \geq 0$. The notation for playing this strategy is O_t^A for university A (resp. O_t^B for university B) and $t \in \{1; \ldots; T\}$ (resp. $t \in \{1; \ldots; T\}$) is the time period when the offer is issued.
- Reject the student. The notation for playing this strategy is N_t^A for university A (resp. N_t^B for university B) and $t \in \{1; \ldots; T\}$ (resp. $t \in \{1; \ldots; T\}$) is the time period when the student is notified of his/her rejection.

The parameter $d \geq 0$ is exogenous and common knowledge. If $d \geq T-1$ the offers will be called "opened" as they cannot expire before the end of the procedure.

Actions of the students. At each period a student can thus receive a response from either university A or B or both of them or none of them. As soon as a student receives one offer or more, the student can:

- Accept one of the offers (s)he received, "enroll" in the corresponding university and exit the procedure. The notation for playing this strategy is E_t^X where $X \in \{A; B\}$ is the university whose offer has been accepted and $t \in \{1; \ldots; T\}$ is the time period when the offer is issued.
- Wait an extra period to see if a better offer comes up later. The notation for playing this strategy is W_t where $1 \leq t \leq T$ is the time period when

[†]The inequality itself always holds since $n_\alpha \leq n_\beta$

the student decides to wait. A student can play W as long as an offer is still valid.

At the end of period T, all offers that have not been accepted automatically expire. Students with no offer from A or B automatically enrolls in C at the end of period T. The students cannot observe the interactions between universities and other students throughout the matching procedure, nor can they observe the number of available spots left in any university during the procedure. Universities always know the type (α or β) of students they are interacting with. However, each university has no way of knowing the interactions of a given student with the competing university. When a student exits the procedure, all universities are informed.

Strategic restrictions A few guiding principles restrict the strategies of the players of this game.

- Students of the same type cannot be distinguished from one another. If a university at a given period has more dossiers of the same type to process than it has available capacity, then the university must sends offers randomly to the students of said type[‡].
- No backtracking : A university cannot renege an offer made to a student nor transform a rejection into an admission. Students who reject an offer or let an exploding offer go cannot re-apply nor enroll in the university they rejected.
- All offers have to be honored : A university cannot send offers to more students than it has available capacity.

[‡]Example : University A has 4 seats available in period T-1 but has 7 dossiers of type α and 8 dossiers of type β processed. University A will send an offer to 4 α students randomly chosen among the 7. All dossiers processed in the following period will be automatically rejected.

Extensive representation : To give the reader a visual representation of the game played by students, an extensive representation of a case where $T = 3$, $N_\alpha = 4$ and $N_\beta = 5$ is shown below . In this specific case, there are 9 different states, thus each student will play one game out of nine possible different games. Three of these games are represented below. To help the reader get a better grasp of the timing when the actions are played. Payoff notation:

- The cell at the end of each branch is a payoff and is noted as $(x; y; z)$.
- The first number $x \in \{0; 1; V_A\}$ is the student's payoff. The value depends on the university the student enrols in.
- The second number $y \in \{0; 1; V_\alpha\}$ is the extra payoff university A get from the specific student. If the student enrols in university B or C then $y = 0$. If the student enrols in A and is of type α, $y = V_\alpha$. Finally if the student enrols in A and is of type β then $y = 1$.
- The third number $z \in \{0; 1; V_\alpha\}$ is the extra payoff university B get from the specific student. If the student enrols in university A or C then $z = 0$. If the student enrols in B and is of type α, $z = V_\alpha$. Finally if the student enrols in B and is of type β then $z = 1$.

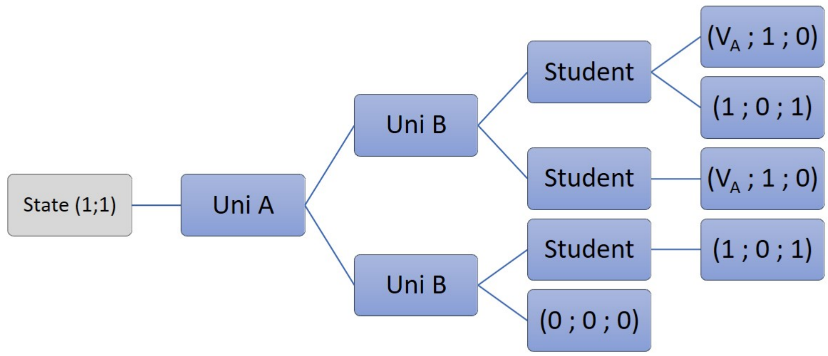
In period 1, university A will play the game $(1; 1)$ 4 times with a different α student each time and 5 times with a different β student each time. The same will happen with games $(1; 2)$ and $(1; 3)$. A is not able to to differentiate between the games $(1; 1)$, $(1; 2)$ and $(1; 3)$ (A does not know the interactions between a specific student and university B) but can perfectly discriminate games played with an α from games played with a β.

Respectively, university B will play the game $(1; 1)$ 4 times with an α student and 5 times with a β student. The same will happen with games $(2; 1)$ and

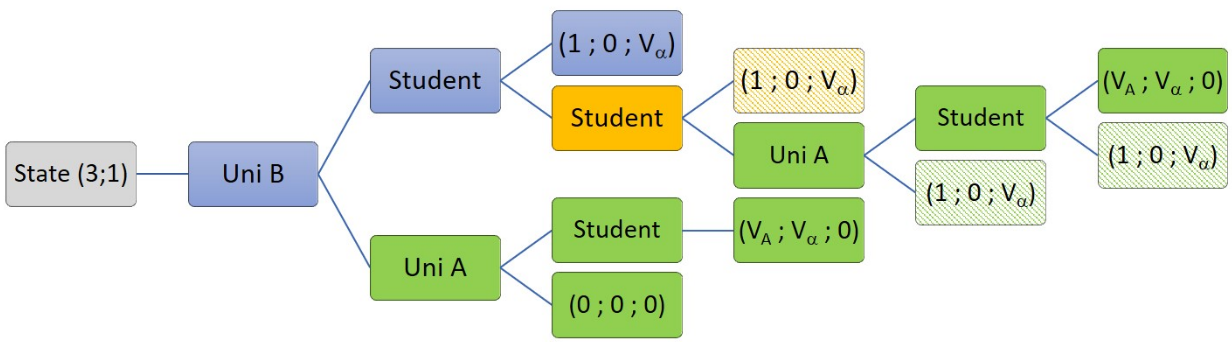
(3; 1). B is not able to to differentiate between the games (1; 1), (2; 1) and (3; 1) but can perfectly discriminate games played with an α from games played with a β.

In period 2, university A will play the game (2; 1) 4 times with an α student and 5 times with a β student. The same will happen with games (2; 2) and (2; 3). A is not able to to differentiate between the games (2; 1), (2; 2) and (2; 3) but still can perfectly discriminate games played with an α from games played with a β. Moreover, A will automatically know if a student in game (2; 1) has enrolled in B and exited the matching procedure. Extend the reasoning to other periods and universities.

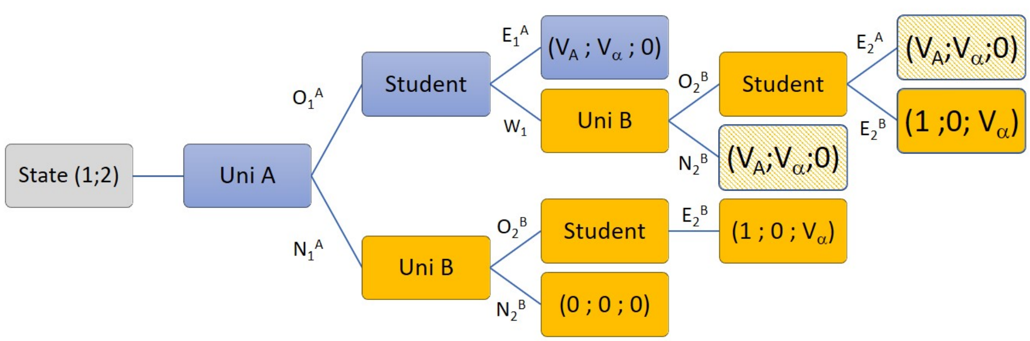
In the trees below, blue cells are played in period 1, orange cells in period 2, green cells in period 3. Cells with a hatching patters may be rendered unavailable (i.e replaced by a payoff of (0; 0; 0)) if the duration of the exploding offer is short enough.



(a) The game played by a β student in state (1;1)



(b) The game played by an α student in state (3;1)



(c) The game played by an α student in state (1;2)

Figure 2.1: The game played by students depending on their state and type

Specific notations

Receiving an offer from A at the last period. Because this specific event will come up very often when solving for an equilibrium it deserves a special notation. Let Ω_A be the event "A student of type α receives an offer from university A in period 3".

Letting an exploding offers expire. Some students may have an incentive to let an exploding offer expire and remain in the matching procedure, hoping to receive an offer from a better university. Given the students' utility function, only an exploding offer from B can be realistically let go. The event "letting an exploding offer from B expire at time t" will be noted W_t^*.

Welfare The aggregated utility of the whole student population will be noted W_s. The aggregated utility of universities A and B will be noted W_u.

Unraveling In the past literature on exploding offers, unraveling happens when firms issue exploding offers early instead of waiting to get the complete information about applicants. In this model, universities cannot strategically time their offers. However, α students can choose to enroll in the less desirable university B early instead of waiting for an answer from A. The matching procedure will "unravel at period t" if such an outcome happens at period t.

Equilibrium concept

Throughout the chapter we will be looking for the behaviour of α students in a sub-game perfect Nash equilibrium. The equilibrium exists since the game has a finite number of players each having a finite number of strategies.

2.2 Equilibrium solution and properties

Before comparing the impact of exploding offers duration let us start with a very general result that will be used as a bedrock for the following ones:

Property 1. If the students know their type perfectly, then university A will never in any equilibrium play O_t^A when encountering the dossier of a student of type β unless the number of dossier of α students left to process is lower than the available capacity of university A.

Proof : See annex B.1.1.

This result ensures that University A will not send an offer to a student of type β unless it has no other choice. This will not only restrict the set of equilibria but also allow us to write down the expected payoff of α students in a way that is easy to manipulate.

The equilibrium with $d \geq T - 1$ (open offers): In the case where d is large enough such that the offers can be considered opened, the equilibrium of the game is very straightforward.

- Every period $i \in \{1; \ldots ; T\}$, when processing the dossier of a type α student, university A will play O_i^A as long as it is not at full capacity. Once A is full, A only plays N_i^A. As long as A has not treated all the dossiers of type α students, it will play N_i^A when processing the dossier of a type β student, and play O^A once all the α students are treated.

- Each period, University B plays O_i^B for all students regardless of type if not already full. Afterwards, B plays N_i^B.

- When receiving offers from B in period i, students of type α play W_t $t \geq i$ until they have the opportunity to play O_i^A. At the last period, the students play E_3^B if they have not received an offer from A.

- Students of type β play E^B.

Proof : see annex B.1.2.

Property 2. The only equilibrium that maximises W_s and W_u for all values of V_A and c_A is the one where universities issue open offers.

Proof : see annex B.1.3.
This simple result allows the use of the equilibrium with open offers as a welfare benchmark. Open offers allows the market to correctly allocate the maximum number of type α students to the best university (A) and give the second best option to the α students who could not fit into A.

2.3 Exploding offers and student behaviour

In this section, universities will only issue exploding offers with duration $d < T - 1$ (a.k.a true exploding offers). The equilibrium of the model will be presented by the following three statements. The first one presents the equilibrium's structure, the second one shows the uniqueness of one key descriptor of the equilibrium and the third one deals with equilibrium uniqueness.

Theorem 3. For all equilibrium strategy profiles $\mathcal{S}$ there exist a unique $T^* \in \{1; \ldots ; T\}$ such that :

- If $t < T^*$ then all α students who face the choice between playing E_t^B and W_t^* will play W_t^*.
- If $t > T^*$ then all α students who face the choice between playing E_t^B and W_t^* will play E_t^B.
- α students may only mix between E_t^B and W_t^* if and only if $t = T^*$

Moreover at the equilibrium all students play E_t^A as soon as they have the opportunity to do so. All β students that received an offer from university B will play W_t unless they are facing the choice between W_t^* and E_t^B. In this case they play E_t^B.

University A plays O_t^A whenever it encounters an α student or if it encounters a β student and has more capacity left than there are dossiers of α students left to process. University B plays O_t^B all the time.

Proof : see annex B.2.1.

Proposition 6. Let $\mathcal{S}$ and $\mathcal{S}'$ be two equilibrium strategy profiles. Then $T^* = T^{*\prime}$

Proof : see annex B.2.2.

This simple result is a key step in proving the equilibrium uniqueness. If the game had two equilibria or more, the critical period of all of these would be the same.

Theorem 4. The equilibrium of the game is unique. Moreover, if $V_A \geq 2$, then $T^* = T$ or $T^* = T - 1$

Proof : see annex B.2.3.

This theorem wraps up section 2.3 by showing that the game presented in this chapter has only one equilibrium that will follow the structure presented by theorem 3. The proof of uniqueness relies on proposition 6. The equilibrium uniqueness allows comparative statics on both students and universities welfare.

2.3.1 Properties of the solution

Property 3. If $V_A > \frac{T^2 n_\alpha - C_A}{T n_\alpha}$ then $T^* = T$

Proof: $\mathbb{P}(\Omega_A)$ is bounded from below by the lowest probability for an α student to get an offer from A. The lowest probability is reached when the number of open seat for A is minimized and the number of applicants for these seats is maximised. This probability is equal to $\frac{T^2 n_\alpha - C_A}{T n_\alpha}$. If $V_A < \frac{T^2 n_\alpha - C_A}{T n_\alpha}$ then $\mathcal{U}(E^B_{T-1}) = 1 > \mathbb{E}(W^*_{T-1})$ so there is an incentive to at least randomise between enrolling in B or gambling for a seat in A.

This property sets a lower bound above which the market will not unravel as students will never accept an early exploding offer before the last period of the game.

Property 4. If $T > 6$ then $T^* > 1$.

Proof : see annex B.2.4

This property shows that once the game has a sufficiently high number of periods, students will never accept an exploding offer in the first period of the game (a.k.a the market will never fully unravel).

Property 5. Let $\Delta C_A = T^2 n_\alpha - C_A$ be the difference between the capacity of A and the total number of α students. Let T be the number of period of the matching game. The difference between the critical period T^* and T is constrained by:

$$(T - T^*) \leq \frac{1 + \sqrt{1 + 8\Delta C_A / n_\alpha}}{2}$$

Proof : see annex B.2.5.

Theorem 5 is a complement to property 4 that acts as an empirical check to see if the initial hypothesis holds. Given the capacity constraint of university A one can find a maximum difference between the critical period T^* and the maximum number of period T. If an α students plays E^B_t when $t < T^*$ then either the student is acting irrationally or the student does not know his/her type.

2.4 Student and university welfare

Since the equilibrium of the game is unique and follows a specific structure we can analyse the ex-ante welfare of both universities and both groups of students.

Theorem 5. The ex-post utility of university B is bounded from below by C_B. The ex-post utility of university A is bounded from below by $2C_A - (T - T^*)n_\alpha$. Both universities are always filled to capacity.

This result is quite straightforward and shows there is always a modicum of positive assortative matching even with exploding offers. The worst case scenario for B happens when all the α students who let their offer from B expire fail to get an offer from A. In this case A is filled to capacity with alpha students thus B can recruit from the entire pool of β students.

The worst case scenario for A is an extreme event where all the α students who could randomise end up enrolling in B. The welfare of B is maximised. A still has access to the pool of β students to fill its remaining seats.

Theorem 6. Let $\mathcal{S}$ be the equilibrium of a game with parameters T, c_A, c_B, V_A, n_α, n_β and an exploding offer of duration $d = 0$. Let T^* be the critical period associated with equilibrium $\mathcal{S}$.

Let $\mathcal{S}'$ be the equilibrium of the game with parameters T, c_A, c_B, V_A, n_α, n_β and an exploding offer of duration $d' \geq 1$. Let T'^* be the critical period associated with the equilibrium $\mathcal{S}'$.

If $d' \leq T^* - 1$ then $T^* = T'^*$ and if there is a randomisation, $p = p'$.

If $d' > T^* - 1$ then $T'^* = d$ and the equilibrium must be played in pure strategies.

The proof can be found in B.3.1. Below is a graphical illustration for the case when $T = 7$. We will assume w.l.o.g that $T^* = 4$

(1;7)	(2;7)	(3;7)	(4;7)	(5;7)	(6;7)	(7;7)
(1;6)	(2;6)	(3;6)	(4;6)	(5;6)	(6;6)	(7;6)
(1;5)	(2;5)	(3;5)	(4;5)	(5;5)	(6;5)	(7;5)
(1;4)	(2;4)	(3;4)	(4;4)	(5;4)	(6;4)	(7;4)
(1;3)	(2;3)	(3;3)	(4;3)	(5;3)	(6;3)	(7;3)
(1;2)	(2;2)	(3;2)	(4;2)	(5;2)	(6;2)	(7;2)
(1;1)	(2;1)	(3;1)	(4;1)	(5;1)	(6;1)	(7;1)

Figure 2.2: Base equilibrium $\mathcal{S}$ with $d = 0$

In the graph above, the α students in the green cell (state $(7;7)$) will compete for a seat in A for sure as their offer from B has not expired. α students in the yellow cells enroll in B just before the offer expires. Students in the blue cell may randomise with probability p in period 4. If the length of the exploding offer increases to 1, the illustration changes to the one below:

(1;7)	(2;7)	(3;7)	(4;7)	(5;7)	(6;7)	(7;7)
(1;6)	(2;6)	(3;6)	(4;6)	(5;6)	(6;6)	(7;6)
(1;5)	(2;5)	(3;5)	(4;5)	(5;5)	(6;5)	(7;5)
(1;4)	(2;4)	(3;4)	(4;4)	(5;4)	(6;4)	(7;4)
(1;3)	(2;3)	(3;3)	(4;3)	(5;3)	(6;3)	(7;3)
(1;2)	(2;2)	(3;2)	(4;2)	(5;2)	(6;2)	(7;2)
(1;1)	(2;1)	(3;1)	(4;1)	(5;1)	(6;1)	(7;1)

Figure 2.3: New equilibrium $\mathcal{S}'$ with $d = 1$

In this case the critical period is the same but students in a different state will randomise. Graphically, one row of students in red (let their offer from B expire to compete for a seat in A) became a row of green (compete for a seat in A while the offer from B is still up). Below is an other illustration when $d = 3 = T^* - 1$:

(1;7)	(2;7)	(3;7)	(4;7)	(5;7)	(6;7)	(7;7)
(1;6)	(2;6)	(3;6)	(4;6)	(5;6)	(6;6)	(7;6)
(1;5)	(2;5)	(3;5)	(4;5)	(5;5)	(6;5)	(7;5)
(1;4)	(2;4)	(3;4)	(4;4)	(5;4)	(6;4)	(7;4)
(1;3)	(2;3)	(3;3)	(4;3)	(5;3)	(6;3)	(7;3)
(1;2)	(2;2)	(3;2)	(4;2)	(5;2)	(6;2)	(7;2)
(1;1)	(2;1)	(3;1)	(4;1)	(5;1)	(6;1)	(7;1)

Figure 2.4: Base equilibrium $\mathcal{S}''$ with $d = 3$

If the duration of the exploding offers exceeds the critical period, the row of students who could randomise is converted into a row of students who still benefit from an offer from B. The rest of the students who will still have a choice to make between W^* and E^B will systematically pick the latter: the competition for the remaining seats in A is too fierce.

(1;7)	(2;7)	(3;7)	(4;7)	(5;7)	(6;7)	(7;7)
(1;6)	(2;6)	(3;6)	(4;6)	(5;6)	(6;6)	(7;6)
(1;5)	(2;5)	(3;5)	(4;5)	(5;5)	(6;5)	(7;5)
(1;4)	(2;4)	(3;4)	(4;4)	(5;4)	(6;4)	(7;4)
(1;3)	(2;3)	(3;3)	(4;3)	(5;3)	(6;3)	(7;3)
(1;2)	(2;2)	(3;2)	(4;2)	(5;2)	(6;2)	(7;2)
(1;1)	(2;1)	(3;1)	(4;1)	(5;1)	(6;1)	(7;1)

Figure 2.5: Base equilibrium $\mathcal{S}'''$ with $d = 4$

Theorem 6 demonstrate how small of an impact long lasting exploding offers have on the welfare of both students and universities. The only significant change in terms of welfare happens when d goes from 3 to 4: the welfare of A, B and α students is maximised and the welfare of β is now minimised.

2.5 Exploding offers with heterogeneous preferences

Now that we understand the behaviour of a model featuring students with homogeneous preferences let us see what happens when heterogeneous preferences are introduced. In this section we will modify the base model by splitting up the α students into q subgroups α_1 to α_q. There is a total of $T^2 n_{\alpha_q}$ students of type α_q for all q such that $\sum_{i=1}^{q} n_{\alpha_i} \leq n_\beta$. The students of type α_i $\forall i$ are identical to each other in every way except their preferences. The utility of α_i students is the following :

$$\forall\ i\ \mathcal{U}_{\alpha_i} = \begin{cases} 0 \text{ if matched with } C \\ 1 \text{ if matched with } B \\ V_{\alpha_i} > 1 \text{ if matched with } A \end{cases}$$

The utility of being unmatched is -∞ like everyone else. The valuations V_{α_i} are such that $1 < V_{\alpha_1} < V_{\alpha_2} < \ldots < V_{\alpha_q}$. The β students have a valuation $V_\beta = V_{\alpha_1}$

Universities cannot distinguish between any of the subgroups of α students before, during or after the recruitment process and receive the same utility from enrolling any of them. The capacity constraint of university A is now $(T-1)T(\sum_{i=1}^{q} n_{\alpha_i}) < c_A < T^2(\sum_{i=1}^{q} n_{\alpha_i})$ while the capacity constraint of university B[§] is $T(T-1)n_\beta + T(n_\alpha + n_\gamma) \leq c_B \leq T^2 n_\beta$.

The matching procedure is mostly unchanged. At the beginning of each period university A processes $T(\sum_{i=1}^{q} n_{\alpha_i} + n_\beta)$ dossiers and university B processes $T(\sum_{i=1}^{q} n_{\alpha_i} + n_\beta)$ dossiers of students of type α_i and β.

Theorem 7. If a strategy profile $\mathcal{S}$ is an equilibrium of the model with ex-

[§]These constraints will always hold since $n_\alpha + n_\gamma \leq n_\beta$

ploding offers then there exist q critical periods $T_1^* \leq T_2^* \leq \ldots \leq T_q$ such that $\forall i \in \{1; \ldots; q\}$:

- $T_i^* \in \{1; \ldots; T\}$
- If $t < T_i^*$ then all α_i students who face the choice between playing E_t^B and W_t^* will play W_t^*.
- If $t > T_i^*$ then all α_i students who face the choice between playing E_t^B and W_t^* will play E_t^B.
- α_i students may only mix between E_t^B and W_t^* if and only if $t = T_i^*$. If they don't mix they play W_t^*

Only one subgroup α_i of students may play mixed strategies in $\mathcal{S}$.

The proof can be found in B.4.1

Corollary: Eliciting relative preferences of students If one sees two α students and one plays E_t^B while the other plays W_{t+i}^* $i > 0$ then they belong to two different subgroups and the former has a lower valuation V_α than the latter.

This result enables a researcher to partially extract the relative valuation of α students by looking at their behaviour when facing the choice between letting an exploding offer go or taking it. It can be crossed referenced with a survey before or after the game has occurred to assess if the student has a rational behaviour.

Concluding remarks

In this chapter I have presented a multi-period model to assess the impact of exploding offers of different length on the welfare of the players in a two sided matching market with complete information about the quality of the players.

I conclude that there provided the students do not value the high quality university too much compared to the low quality one, using exploding offers with sufficiently long duration will result in the same outcome as open offers while still allowing universities to spread the workload. If the exploding offers are too short, the low quality universities may not be able to hire all the high-quality applicants that failed to get a place in the high quality institution. The equilibrium of such a model has a precise structure that makes it easy to identify. The incentives driving the equilibrium are similar to a Stackelberg oligopoly where the players who tie their hands early are able to keep some of the competition at bay. Applicants can use this strategy as they are perfectly aware of their own type.

Finally, introducing heterogeneous preferences to the model can enable economists to partially elicit the relative preferences of the high quality applicants based on the period at which they decide to go with the low quality firm/university.

Chapter 3

What should I choose? Choice overload, an overview and a suggestion

Literature review

The canonical model of consumers' rational choice behaviour; stating that economic agents choose among alternatives using a (1) complete, (2) transitive and (3) acyclic preference ranking is widely taught and used, thanks to its simplicity and applicability. This set of assumptions has lead to the utility representation theorem and the formulation of the weak axiom of revealed preferences (WARP). Yet, despite its easy applicability, the model of rational choice is violated on multiple occasions (e.g Gross (1995) [19] or Echenique et al. (2011) [10]). Decoy and attraction effects are commonly observed violations of rational choice theory. The causes of these anomalies has been widely studied and include decoy effect and choice overload among others. The model presented here features an extremely simplified heuristic that does not involve decision thresholds. Instead, alternatives are classified as desirable or undesirable for a given menus sizes limiting drastically the number of cases to consider.

Anchor and decoy effect   It is now solidly established that decisions makers are influenced by past choices and the first information available to them. Contrary to what the rational choice theory suggests, the irrelevant alternatives may have an impact on the end choice by consumers. Anchoring is a term used in psychology to cover a large number of effects induced by stimuli on decision-makers. Early offers and raw pieces of opinion heard first can create a bias in the decision-maker's response. Although given enough time, a decision-maker will progressively adjust his/her response toward the optimal point. Kahneman and Tversky 1974 [42] popularised the concept and insisted on the importance of framing in the formation of preferences in their 1986 [43] paper. An exhaustive review by Furnham and Boo in 2011 [14] underlines the ambiguous nature of anchoring. While Kahneman and Tversky have empirically shown the detrimental effect of anchors on choice efficiency, others like Goldstein and Gigerenzer (2003 and 2009) [16] have demonstrated the beneficial uses of such heuristics. In the right environment, these heuristics allow decision makers to make an accurate decision with minimal effort. McElroy and Dowd (2007) [28] conducted experiments showing that anchoring happens in all domains of knowledge. There is no reason to believe these effects are not present when a consumer chooses a product from any given menu.

Choice overload   Choice overload is often put forward as an explanation as to why consumers rely on imperfect heuristics to make their choice. The topic of choice overload is one that has been extensively studied by both economists, marketing specialists and psychologists.

Mogilner et al. (2008) [29] noted that the participation rate of workers in retirement plans in the US decreases when the number of options to choose from is ten or more. Iyengar and Lepper (2000) [21] set up a jam tasting experiment and concluded that consumers' satisfaction was higher when the number of jams available for testing was lower (6) rather than higher (24) even though more were inclined to participate in the test when the sample was larger. Schwartz 2004 [38] described this situation as a choice paradox. Consumers are attracted to large menus but end up dissatisfied with their final choice. Meta-analyses conducted by Scheibehenne et al. (2010) [36] and Chernev et al. (2015) [6] reach similar conclusions.

While these studies identify a perverse effect of large menus on consumer satisfaction, others offered opposite conclusions such as Greenleaf and Lehman (1995) [17] who argue that consumers have a tendency to delay their purchasing if they are not certain that the menu they face is representative of the entire range of options available. It appears that some consumers (or decision markers, we will use both terms interchangeably) can be attracted by a varied menu that is more likely to include a utility maximising alternative, many others may be less likely to commit to a choice or even completely discouraged by the volume of alternatives to choose from.

This apparent contradiction was resolved by Mogilner et al. 2008 [29] Their studies show that consumers who had a good heuristic to sort the items on the menu and choose rapidly are the ones preferring large menus, while those with little prior information and/or no choice heuristic, suffered from choice overload. The concept of screening categories is central to their work.

One must remember that choice overload is a mental state and thus cannot be directly observed by an economist. All the previous studies relied on proxies and external indicators such as satisfaction indexes, confidence level and ex-post regret.

Theoretical representation of choice heuristic The empirical research on choice overload and other choice-bias motivated theorists to model choice heuristics. This implied giving up the classical Weak Axiom of Revealed Preferences (WARP) for an even weaker property. Recent literature on the matter features many different choice models. All rely on either a weakening of the warp or a modification of the choice procedure itself. Instead of applying a simple preference relation on a given menu, recent choice models either involve the usage of more than one preference relation or use a multiple steps choice procedure. An application of multiple preference relations is found in Yildiz and Dogan [9] who use a set of pros and cons preference relations to describe how agents make choices.

An alternative approach in the literature is to use a two-stage choice procedure. In the first stage, the agent will filter alternatives from the initial menu. In the second stage, the agent will apply a complete, transitive and acyclic preference relation to the filtered menu obtained in stage one. An example of such a procedure is found in Mariotti and Manzini (2012) [27]. They present a procedure where agents eliminate in a menu all the alternatives that possess inferior categories then apply a single preference ranking on the filtered menu. However, the characterisation of choice functions is heavily based on the filtering procedure itself. Nonetheless, it is possible to make a characterisation of some two steps choice procedure, by using a weakening of the WARP (WWARP) formulated by Mariotti and Manzini or using the concept of route consistency as in Apesteguia and Ballester (2013) [2].

Another weakening of the WARP was introduced by Eliaz and Ok (2006) [12]. The Weak Axiom of Non-Revealed Inferiority states rationalises the choices made by agents who do not have a complete transitive and reflexive preference relation. WARNI allows incompleteness and rationality to coexist.

Finally, Xavier Gabaix (2014) [15] introduced the concept of sparsity in choice. His concept of a choice procedure leans on the hypothesis that agents simplify reality by willingly omitting information. In his work, there is a trade-off because the information will help make better choices but acquiring such information is costly for the decision-maker.

Although modelling non-rational choices has been an ordeal for economists, psychologists and marketing researchers, it is still possible to conduct welfare analysis and identify utility functions. In particular, Dalton and Ghosal (2018) [7] managed to fully characterise data of behavioural agents and suggest welfare benchmarks for public policies.

All of the mentioned papers bring critically important insight into how agents formulate their choice among menus. However, the procedures put forward feature one key element that binds most if not all of this literature. It is the presence of an underlying preference order that is exogenous and fixed throughout the procedure. This underlying preference order has sometimes been associated with one or multiple partial orderings (Mariotti and Manzini (2012) [27]) yet these orders exist independently. Such an implicit hypothesis is highly unlikely since agents filter products and formulate their preferences using the same information they have at their disposal. Moreover, any model using menu-based filtering runs into a problem of over-identification. When a menu grows its set of subsets grows exponentially faster, leading to an overabundance of special cases to manage for both the agent and the economist.

This chapter aims to present a new multiple steps choice procedure that can incorporate both the anchor effect, the impact of a good heuristic and rationality in one unified model. The procedure models the behaviour of agents that have to choose one alternative among many without the possibility of trying any. The agents are expected to face a choice overload and thus may not have the possibility of fully analysing every alternative in the menu. This will lead to a violation of the WARP and the Weak WARP mentioned above. It should be considered as a variant of the Choice through Attribute Filters (CAF) model of Kimya (2018) [23]. Kimya also uses the attributes of the alternatives but does not consider menu sizes or anchors. While his model includes constraints on how the filtering evolves as the menu changes, it still runs into a problem of excess variants of filters. Moreover, unlike previous work, the agent only takes into account the menu size and not the content of the menu when performing the filtering. Thus the number of special cases to manage grows linearly instead of exponentially. A problem the presented model aims to solve.

The rest of the chapter is organised as follows. Section 3.1 will present a simple example to illustrate the motivations and intuitions behind the choice procedure. It is entirely optional and readers familiar with the topic may want to skip directly to the next section. Section 3.2 presents the model formally. Section 3.3 some interesting properties found. Section 3.4 presents a protocol of a lab experiment to empirically verify the existence of the procedure as well as some concluding remarks.

3.1 A simple example

Let us assume Alice needs to replace her old car that is now broken. Alice will have to choose one car over an enormous menu. She can buy a new car or a used one. The price of cars can vary by a factor of one hundred if not more. Cars come in many shapes, sizes and colours. Some cars are better to use mostly in urban areas while others are suitable to travel over long distances etc. Because Alice faces such an overload of alternatives, she will want to create a shortlist of alternatives that are suitable to her needs. To do such filtering, she can look at the characteristics of the cars and sort them into different categories (similarly to Categorise then Choose). For example, a car can be in the categories "Colour: blue", "maximum passengers: 7" and "Price $\leq$ 50000 \$" among others.

Yet because the menu of cars is so large, she cannot filter over all the categories at once. With a very large menu, she will look first at a very small number of categories. When looking at a hundred cars, before looking at the material of the passenger seats, Alice will first look at the price of the car, its colour and maybe the maximum number of passengers. She will eliminate some alternatives in the initial menu based on these categories only before looking at any other categories.

Moreover, to avoid eliminating a car Alice would have liked to buy, she will not discard immediately all the cars that belong to an inferior category. Just because Alice prefers to purchase a blue car does not mean she will never choose a grey one as long as any blue one is present in the menu. She will immediately discard red cars from the menu for she loathes this colour. In other words, Alice has a preference ranking of the different car colours. She has a favourite colour, colours that she finds acceptable and colours she dislikes so much that she considers a car with such a colour "unacceptable" and discards it from the menu. One can apply the same reasoning to any category. Alice prefers cheap cars to expensive ones (all other things equal) yet some cars are so expensive that they are out of her budget and will be discarded immediately.

When choosing her new car, Alice will start with a large menu. She will look at a limited number of categories of each car in the menu and discard any alternative that has at least one "unacceptable" criteria. Alice will eliminate all the red cars, even the cheap ones and all the over-expensive ones, even if they are blue.

Alice now dealing with a smaller menu will have the opportunity to look at more categories. She will eliminate any car that is too large to fit in her garage etc. As the menu shrinks further at each step Alice will be able to analyse in detail the remaining items until she cannot eliminate anything anymore. At this stage, she will apply a preference ranking to the remaining alternatives. We can make simple assumptions regarding Alice's preferences regarding individual categories. Cheap is preferred over expensive, blue is preferred over grey. Alice can rank each category individually. However, when Alice will make a final decision she will look at multiple categories. Because we assume that any menu Alice will deal with is finite (although possibly large), continuous variables like price and size can be mapped into discrete categories.

However, while Alice prefers blue cars to grey cars and cheap cars to expensive ones, she may prefer cheap grey cars to expensive blue ones or the opposite. We do not know the impact of aggregating categories. We can assume that the ranking Alice will use when making a decision will depend on what categories she is looking at. If the elimination process is efficient and Alice ends up with a very small-curated menu, she can make almost perfect comparisons between the cars by looking at almost all the descriptive characteristics of the items in the menu. If the elimination process does not allow her to eliminate many alternatives, she will have to ignore some categories and this can change her ranking.

This process of eliminating unacceptable alternatives will be designated in this chapter as Choice Among Acceptable Alternatives (CAAA).

3.2 A choice heuristic based on menu size

Let us consider one consumer who has to make a choice from a set of alternatives $A = \{a_1;\ \ldots\ ;\ a_n\}$. Any subset $m \subseteq A$ is a menu.

Let $U_{|A|} \subseteq U_{|A|-1} \subseteq \ldots \subseteq U_1 \subseteq A$ a sequence of nested sets included in A. Alternatives a_j in U_i are called "unacceptable". The whole sequence of U_i is designated as U.

Let us define the Δ operator between two sets A and B. $A\Delta B$ is a subset of A where all of the elements in set A that are also in set B are removed. In mathematical terms :

$$A\Delta B = A - A \cap B$$

Let $f(U)$ be a filter function such that:

$$\begin{aligned} f :& 2^A \times U \to 2^A \times U \\ & (m; U) \mapsto (m\Delta U_{|m|}; U) \end{aligned}$$

Where $m\Delta U_{|m|} = m - m \cap U_{|m|}$.

We define f^∞ the infinite recursive application of f. The recursion stops as soon as any f^∞ returns image identical to its pre-image.

For $i \in \{2; \ldots ; n\}$ let $\succ_i$ a sequence of preference relations defined over the set A that are complete, transitive and acyclic. The set of these n-1 preferences relations is $\succ$. Let c_i be the choice function defined using $\succ_i$. Naturally $\forall i$ c_i satisfies WARP.

Let 0 be a special alternative called the "outside option". The outside option 0 is defined such that $\forall i;\ 0 \succ_i a \Leftrightarrow a \in U_i$, $0 \prec_i a \Leftrightarrow a \notin U_i$ and $c_i(\emptyset) = 0$.

We define a Choice Among Acceptable Alternatives function $C(U, \succ, A)$ as a composition of two sub-functions such that:

$$C(U, \succ, A)(m) = c_{|f^{\infty}(U)(m)|} \circ f^{\infty}(U)(m)$$

Note that the consumer does not have the opportunity to test the alternatives before choosing and never owned any of them. The latter condition is to get rid of any status-quo bias. No learning takes place. To be able to make an informed decision it is assumed that the consumer looks at the descriptive characteristics of each alternative in m.

However, the consumer has limited rationality. When dealing with a large menu the consumer cannot look at all the descriptive characteristics of every alternatives and only focus on a subset of these. The smaller the menu, the higher number of characteristic the consumer can scrutinize. This is the reason we are using nested sets of unacceptable alternatives. the smaller the menu the more opportunity the consumer has to find out that an alternative is unacceptable.

The set of multiple preference relations used in the CAAA function models the propensity of the consumer to change its ranking of alternatives as new information is acquired. For very large menus, a limited number of criteria is used to compare alternatives. With a smaller menu, new criteria come into play potentially leading to preference reversals.

The inclusion of the outside option stems from a sanity check requirement. If the agent faces a menu with only unacceptable it cannot chose any of them. The outside option represents this ability of not choosing anything.

Example of a choice function that is CAAA

Let us consider a choice function C that satisfies Independence from Irrelevant Alternative Swapping. Meaning, the following property holds :
$C(\{a_0; a_1; ...; a_m\}) = a_0$ and $C(\{a_0; a_{m+1}; ...; a_{2m}\}) = a_0$
$\Rightarrow \forall b_1; ...; b_m \in \{a_1; ...; a_{2m}\}$, $C(\{a_0; b_1; ...; b_m\}) = a_0$
IIAS is a simple weakening of the well know independence from irrelevant alternatives that only holds when the reshuffling of irrelevant alternatives preserves the cardinality of the initial menu. Any choice function C satisfying this property is also a CAAA.
Proof: Assume $\forall n,\ U_n = \emptyset$, then the filtering function becomes the identity function and we are left with a simple choice function which is driven by a preference relation dependant on the initial menu cardinality.

A choice function satisfying IIAS (and by extension CAAA) can violate the Weak Warp defined by Mariotti and Manzini (2007) [26].

3.3 Properties of CAAA functions

Reconstructing CTC with CAAA

Property 6. Let $C(U, \succ, A)$ be a CAAA function such that all the preference relations $\succ_i \in \succ$ are identical. C satisfies weak warp and expansion.

Proof: See Annex C.1

This property presents CAAA choice functions as an alternative way to formalize the Categorise and Choose choice functions as in Mariotti and Manzini (2012). The contrapositive of this property is suggesting that a violation of the weak warp can be attributed to a preference reversal happening somewhere.

CAAA and revealed preference

In this subsection we discuss about how the consumer's preferences and elimination procedure are. Unlike standard theory, in a CAAA function one can get changing preference rankings on top of alternative becoming unacceptable along the filtration process. Let us start with examples demonstrating the difficulty of this:

Example 1: Let $A = \{a_1, a_2, a_3, a_4\}$; $U_4 = \emptyset; U_3 = U_2 = U_1 = \{a_1; a_3; a_4\}$ and $C(U, \succ, A)$ a CAAA function such that:

- $C(U, \succ, A)(A) = a_1$
- $C(U, \succ, A)(m) = a_2$ if $m \subset A$ and $a_2 \in m$
- $C(U, \succ, A)(m) = 0$ if $m \subset A$ and $a_2 \notin m$

There is a great number of $\succ$ that can adequately represent C. Both $a_1 \succ_4 a_2 \succ_4 a_3 \succ_4 a_4$ and $a_1 \succ_4 a_4 \succ_4 a_3 \succ_4 a_2$ are valid and non trivial.

Example 2: Let $A = \{a_1, a_2\}$; $a_1 \succ_2 a_2$ and $C(U, \succ, A)$ a CAAA function such that:

- $C(U, \succ, A)(A) = a_1$
- $C(U, \succ, A)(a_1) = a_1$
- $C(U, \succ, A)(a_2) = 0$

Then the sequences $U_2 = \emptyset, U_1 = \{a_2\}$ and $U_2 = U_1 = \{a_2\}$ can generate the choice function above.

Property 7. Let $C(U, \succ, A)$ be a CAAA function. Let $m, m' \subset A$ such that $a = m \Delta m'$ and $a' = m' \Delta m$. If $C(m) \neq C(m')$ and $C(m), C(m') \notin \{a, a'\}$

then $\exists U_i \in U$ such that without loss of generality $a \in U_i$ and $a' \notin U_i$

Proof: See Annex C.2

This property allow to detect (more) undesirable alternatives by simply observing the choices of the consumer.

Representing a CAAA function

The following section aims at presenting a tool to represent any CAAA function. this tool may prove useful to elicit properties.

Definition 1. Rational filter graph

Let $A = \{a_1; \ldots; a_n\}$ be a set of alternatives. Let $U_n \subseteq U_{n-1} \subseteq \ldots \subseteq U_1 \subseteq A$. Let G(V, E) be a graph where $m \in V = 2^A$. The graph G is a rational filter iif

$$E = \{\{m; m'\}|\ m \neq m'; m' = m\Delta U_{|m|}\}$$

A rational filter graph will be noted as G(A, U) where A is the set of alternatives from which the vertices are constructed and U the sequence of sets from which the edges are constructed.

The vertices of a rational filter graph are the elements of the set of subsets of A i.e all the possible menus one can generate from A, including the empty set. The edges of this graph are defined such that if two menus are linked together it means:

1. One menu/vertex has at least one element that belong to a set of undesirable alternatives.
2. The other menu/vertex is the first one deprived of such undesirable elements.

The graph is the physical representation of the filtering process from any initial menu to a final menu that contains only alternative the agent finds acceptable under the information constraint.

One can notice that the graph has no circle and one vertex can be linked to at most one vertex of lower cardinality. For more details on why see Annex C.3.

Lemma 1. Let G(V;E) be a rational filter graph. Let T be the set of canonical trees of G. Let $T_q \in T$ a canonical tree of G, let $m \in T_q$ be a menu with the smallest cardinality of G. m is unique and the root of this tree.

Proof: See Annex C.4

Lemma 1 allows to uniquely define a root of each canonical tree of G. Moreover, one can index each canonical tree of a rational filter graph by using the cardinality of the root the tree. this indexing will enable us to construct sets of canonical trees with interesting properties.

Definition 2. A choice function c satisfies the Partial Weak Axiom of Revealed Preferences (Partial WARP) over a set M if for any pair of alternatives a and a', if a is revealed preferred to a' for any menu in M then a' cannot be revealed preferred to a for any menu in M.

When WARP applies to any menu containing a and a', P.WARP only applies to the menus in a set M. This weakening of the warp has little value on its own since any choice function satisfies Partial Warp (use a collection of singletons). However, it will be useful in property 8

Definition 3. Le G(A, U) be a rational filter graph. Let T be the set of canonical trees of G. Let Π be a partition of 2^A. The elements of Π are denoted as Π_i, $i = 1; \ldots ; |A|$. If $\forall m \in 2^A$, $m \in \Pi_i \Leftrightarrow m \in T_i \in T$ such that the root of T_i has a cardinality of i then Π is a indexed partition of 2^A.

This definitions introduces a way of grouping multiple trees of a rational filter graph based on the root of each tree.

Property 8. I choice function $C(U, \succ, A)$ satisfies CAAA then it can be represented by a rational filter graph G(U,A) such that :

- C is invariant along canonical tree of G
- C satisfies Partial WARP over each elements of the indexed partition of 2^A

Proof: See Annex C.5

3.4 Experimental protocol proposal

While not characterised in this chapter, we believe nonetheless that the presented framework has an experimental interest. We present the following experimental protocol.

This is a two steps lab experiment designed to test the validity of the CAAA framework. The three intuitions bundled in the CAAA model (anchor effect, choice overload and choice heuristic/two-steps choice) have either been tested separately before or, in the case of the two steps choice, are fully characterised.

The experiment is divided in two steps:

- One is a survey about the subjects individual preferences over single characteristics of a type of mundane object (a watch, a computer mouse etc.)
- The other is a sequence of timed choices in a large menu of the same mundane objects. The objective of the timing is to place a time constraint upon participants. This paired with sufficiently large menus (10

or more items) will force them to rely on heuristics to make a decision.

To control for the influence of step one, subjects will be randomly allocated in two groups. The first group will do the survey first then the timed choice while the second group will do the timed choice first followed by the survey. It will be important to check if there is a statistical difference between the rationality of group one and two. We expect people in group one to have prepared a more robust heuristic

The first step would aim at discovering for each subject the set of relevant characteristic that will be used in the heuristic later on. A Likert scale can be used to infer individual preferences over each descriptive element of the object. Additional rank ordering question can be added to understand how a subject weight two different characteristics together.

It is important to design the choice part of the experiment in a way that removes then anchor. The item chosen by a subject in the first menu should be removed from the second one and so on. If not done, subjects could lazily choose the same alternative over and over again to avoid looking for information.

Concluding remarks

In this chapter we have presented an overview of the various causes of violations of the classical weak axiom of revealed preferences. The main culprits being anchoring effect, choice overload, and use of a more or less inadequate choice heuristic. While a large part of the theoretical literature has dealt with the choice heuristics and the consideration set, we still believe there is a gap to close. When analysing different alternative, agents use filters. However, the information used when filtering (that can be modeled as a set of partial orders) should also be used to form the final preference ranking (be it complete or not). When aggregating the partial orders the agent should display a consistent behaviour. The proposed CAAA choice function tries to formalise this main idea. However, because it is not fully characterised, it needs empirical evidence to justify its existence.

Should the relevance be confirmed empirically the next challenge would be to figure out how agent aggregate the descriptive characteristics of good (which can be described as partial orders) and how different types of menus can influence the aggregation process. There is a branch of literature on algorithms that is dealing with partial orders aggregation. While there are results, the algorithms are complex and appear to require a very large number of operation if the rankings are precise.

Conclusion

In this thesis, I looked at three applications of the burden of choice. I have put the first two applications under the lens of applied microeconomics theory: which football player a team should hire and should one accept an early exploding offer or try his/her luck? I showed that under the right circumstances, the negative consequences of the burden of choice in both cases could be negated. In the last chapter, I have proposed an early theoretical framework to be tested empirically and to be used as the basis for future research be it applied or theoretical.

Matching football players with teams is a well know and well-documented challenge. Since the 70s, economists have attempted to model this two-sided matching market with a relative level of success. While modelling Northern American sports leagues has proved straightforward thanks to the fact they act as monopolies with a limited pool of players to hire, the same cannot be said of European sports leagues. Unlike Northern American sports leagues, European sports leagues are opened and the market for players can be considered as an infinite pool of talent. The post-Bosman-ruling era of football has seen the rise of various phenomena that have defied standard economic thinking. The rise of the superstar players' wages, the takeover of teams by rich people behaving, as sugar daddies and the failing of small teams both financially and sportively are puzzling. The model presented in chapter 1 improves on existing literature and captures all of these effects by combining the main features of the two families of sports leagues models. The model allows the teams to hire two different types of players: the superstars available in limited quantities as in North American league models, and regular players who are infinitely available as in classic European leagues models. Depending on the parametric assumptions made, the equilibrium of the model can explain the various phenomena observed in modern football. When two sugar daddies compete for victory, the superstars' wages explode and lead the teams to run persistent deficits. When a money-maximising team cohabit in the same league with sugar daddies, the money maximising team can end up in deficit when the regular players are too costly and the superstars are too dominant. The intuition behind this equilibrium is simple: the sugar daddy can spend more money to hire the very dominant superstar players and choke out the profit maximising team who cannot win enough games to cover the cost of regular players. The model predicts that a wise policymaker can alter a league to make it more unpredictable and allow both teams to be profitable in the end. In future research, the model could be expanded to a multiple period framework. However, for the solution of such a model to be tractable, one has to find a new way to model the hiring of players. The Tullock framework used in the theoretical sports economics literature since the 2000s features too many non-linear best replies to be easily used in a multi-period model.

Exploding offers have been used for a long time by many institutions. From universities to private consulting firms, many institutions rely on these types of offers to make their recruitment process easier. However, the literature on exploding offers is scarce. The main concern of economists so far has been the unravelling of a matching market featuring exploding offers when the information about the quality of the applicants is uncertain and partially revealed over time. The models are two-periods and there is a match between the number of applicants and the capacities of the institutions that are recruiting. The main dilemma faced by all the players is between hiring early an applicant whose quality is not well known and waiting to acquire more information at the risk of being poached or missing a spot. In chapter 2 of this thesis, I explored a completely new side of exploding offers. I presented a two-sided matching market with n-periods instead of 2 and where the information about applicants is perfect but capacities do not match. Two institutions are recruiting using exploding offers, a high-quality one and a low-quality one. The many applicants are of two types, high-quality and low-quality. Unlike in the rest of the literature, the quality of each applicant is common knowledge. There are more high-quality applicants than the high-quality institution can hire. And there are more applicants in total than the combined capacities of the two institutions. In this model, the dilemma is faced only by the high-quality applicants who have to choose between enrolling in the low-quality institution before the exploding offer expires or playing "double or nothing" by letting the offer from the low-quality institution explode and hoping to be hired by the high-quality institution. The equilibrium of this model predicts a partial unravelling when the capacity of the high-quality institution is too limited and the relative preferences for the high-quality institution are too small. However, provided the exploding offer has a sufficiently long lifetime, the equilibrium allows all players to achieve the same welfare outcome as the benchmark equilibrium with open offers. This is a new insight. The future of the model of chapter two is applied.

Chapter 3 looks at a burden that has been documented by economists and researchers in other disciplines: choice overload. There are many meta-analyses regarding choice overload and all point out the same conclusions: consumers are unhappier and less satisfied when having to choose among a very large menu of alternatives, especially if these alternatives are very similar to one another. Choice overload is described as a situation where an economic agent's cognitive capabilities are not enough to solve a choice problem. Economists and others usually assume this appears when the agent has to choose one (or a small number) of alternatives from a very large menu. This inability to fully assess the value of every single alternative of a menu can lead the agents to use simplifying routines or "two-step thinking" as described by Tversky and Kahneman [43]. This two-step thinking is also thought to be behind two of the most common violations of the Weak Axiom of Revealed Preferences: Anchor effect and decoy effect. Economists have attempted to model the effects of choice overload using multiple-step choice models, the first step being a filtering step, while the second step is the effective application of a preference relation over a filtered menu leading to a final choice. These models usually introduce a weaker version of the warp but miss either the anchoring or the decoy effect or have an identification problem. In chapter 3 I introduced a model that attempts at solving the identification problem. The model is a two steps choice model. It introduces the notion of an acceptable alternative as a replacement for the partial order filter. Initial (large) menus are purged of alternatives that are deemed "unacceptable" by the agent. Unlike filters, the set of unacceptable alternatives is not an ordering and increases in cardinality as the initial menu shrinks. This notion of unacceptability is much easier to elicit empirically than a partial ordering. While there have already been some major results in choice theory, this field of research is still in need of a model that can capture all the violations of classical rational choice theory while retaining empirical identification properties using the revealed preferences of consumers combined with the available information on observable characteristics of all the alternatives in a menu. Chapter 3's model is not fully characterised and instead, I have put forward an experimental protocol proposal for future research.

Freedom is the ability to make meaningful choices and experience the full consequences of them, good or bad, without externalities on others. This thesis in applied microeconomic theory has looked at instances where making a choice is particularly difficult and can lead to perfectly rational yet perfectly undesirable outcomes. By modelling these examples of difficult choices and studying the solutions, I have shown that one can replace undesirable equilibrium with desirable ones.

Appendix A

Proofs of chapter 1

A.1 Proof of equivalence between the linear and logit C.S.F

The superstar players are worth at least as much on the field than regular players. A team whose roster includes only regular players has a talent normalised to one on the field, while a team that has only superstar in its roster is worth $t \geq 1$ on the field.

If a team has a mixed composition its talent on the field is worth $tq_1 + (1 - q_1)$. The first term of the sum is the talent coming from superstar players. The second term is the talent coming from the regular players. Since by design $q_1 + q_2 = 1$ and both teams have a total roster size of 1, when injecting this formula into the classic logit C.S.F we obtain:

$$w_1(q_1; q_2) = \frac{tq_1 + (1 - q_1)}{tq_1 + (1 - q_1) + tq_2 + (1 - q_2)}$$
$$w_1(q_1; q_2) = \frac{tq_1 + (1 - q_1)}{t + 1}$$

Let us perform a variable change. Let $d = \frac{t-1}{t+1}$ be a remapping of the parameter t. Thus $t = \frac{1+d}{1-d}$. Substituting this into the C.S.F yields

$$
\begin{aligned}
w_1(q_1; q_2) &= \frac{\frac{1+d}{1-d} q_1 + (1 - q_1)}{\frac{1+d}{1-d} + 1} \\
w_1(q_1; q_2) &= \frac{(1+d)q_1 + (1-d)q_2}{(1+d) + (1-d)} \\
w_1(q_1; q_2) &= \frac{d(q_1 - q_2) + q_1 + q_2}{2} \\
w_1(q_1; q_2) &= 1/2 + \frac{d}{2}(q_1 - q_2)
\end{aligned}
$$

One ends up with a linear expression for the C.S.F. It can be further refined to completely eliminate the variable q_2:

$$
\begin{aligned}
w_1(q_1; q_2) &= 1/2 + \frac{d}{2}(q_1 - q_2) \\
w_1(q_1; q_2) &= \frac{1-d}{2} + dq_1
\end{aligned}
$$

A.2 Algebra behind proposition 1

In equilibrium, the teams' objective function satisfy the two following FOCs:

$$
\begin{aligned}
\frac{\partial U_1}{\partial x_1} &= 0 \\
\frac{\partial U_2}{\partial x_2} &= 0
\end{aligned}
$$

Rewriting

$$m + c + d(\gamma_1 + 1) = \frac{(x_1 + x_2)^2}{x_2} \tag{A.1}$$
$$m + c + d(\gamma_2 + 1) = \frac{(x_1 + x_2)^2}{x_1} \tag{A.2}$$

Dividing (A.1) by (A.2) yields:

$$\frac{x_1}{x_2} = \frac{m + c + d(\gamma_1 + 1)}{m + c + d(\gamma_2 + 1)}$$

Let $\mathcal{R} = \frac{m+c+d(\gamma_1+1)}{m+c+d(\gamma_2+1)}$. Substituting for x_1 the first equation becomes

$$1 + c + d(\gamma_1 + 1) = \frac{(x_2)^2(1 + \mathcal{R})^2}{x_2}$$
$$x_2 = \frac{(m + c + d(\gamma_1 + 1))(m + c + d(\gamma_2 + 1))^2}{(m + c + d(\gamma_1 + 1) + m + c + d(\gamma_2 + 1))^2}$$

Then also $x_1 = \frac{(m+c+d(\gamma_1+1))^2(m+c+d(\gamma_2+1))}{(m+c+d(\gamma_1+1)+m+c+d(\gamma_2+1))^2}$ and $x_1 + x_2 = x_{1,2} = \frac{(m+c+d(\gamma_1+1))(m+c+d(\gamma_2+1))}{(m+c+d(\gamma_1+1)+m+c+d(\gamma_2+1))}$

A.3 Proof of uniqueness of equilibrium

Note that :

$$\frac{\partial^2 U_i}{\partial x_i^2} = (1 + c + (\gamma_i + 1)d)x_j \frac{-2}{(x_1 + x_2)^3} < 0$$
$$i \in \{1; 2\},\ i \neq j$$

The utility functions of both team are concave w.r.t their strategic variable. The equilibrium exists and is unique. Furthermore there is no equilibrium where one team or more does not recruit superstars: Suppose the existence of an equilibrium where no team spend anything ($x_i = x_j = 0$). In this case either team has a profitable deviation : increase the spending by ε and recruit the totality of the superstars instead of only half of them.

Suppose the existence of an equilibrium where one team does not want to spend anything on superstars ($x_i = 0$ w.l.o.g.). The the opposing team has a profitable deviation : reduce the spending by ε and keep the totality of the superstars.

A.4 Proof of proposition 2

In this section we would like to remind the reader that the constraints $c < 1$ and $m > 1$ must hold all the time. Let $K_1 = (\gamma_1 + 1)d + m$ and $K_2 = (\gamma_2 + 1)d + m$From Proposition 1, $x_{1,2} = \frac{(K_1+c)(K_2+c)}{K_1+K_2+2c}$. Since $1 \geq c$, $K_1K_2 \geq c^2$ and $x_{1,2} \geq c$

$$\begin{aligned}\frac{(K_1 + c)(K_2 + c)}{K_1 + K_2 + 2c} &\geq c \\ \Leftrightarrow K_1K_2 + c^2 + c(K_1 + K_2) &\geq 2c^2 + c(K_1 + K_2) \\ \Leftrightarrow K_1K_2 &\geq c^2\end{aligned}$$

By construction $K_1 \geq 1$, $K_2 \geq 1$ and $c \leq 1$. Thus $x_{1,2} \geq c$ at all times.

For all the remaining proofs, let $\Gamma_i = 1 + \gamma_i$, $M = m + c$, $f_i = (M + d\Gamma_i)$ $i \in \{1, 2\}$

A.5 Proofs of the signs of the derivatives of x_1 w.r.t c

Let $f_i' = \frac{\partial f_i}{\partial c}$ $i \in \{1,2\}$:

$$\frac{\partial x_1}{\partial c} = \frac{f_1}{(f_1+f_2)^3}\left(2f_2^2 + f_1^2 - f_1 f_2\right)$$
$$\frac{\partial x_1}{\partial c} = \frac{f_1}{(f_1+f_2)^3}\left((f_1-f_2)^2 + f_1 f_2 + f_2^2\right)$$

Since f_1 and f_2 are strictly positive quantities, $\frac{\partial x_1}{\partial c} > 0$

A.6 Proof of theorem 1

Let $f_i' = \frac{\partial f_i}{\partial d}$ $i \in \{1,2\}$. Computing $\frac{\partial x_1^*}{\partial d}$:

$$\frac{\partial x_1}{\partial d} = \frac{(2f_1 f_1' f_2 + f_1^2 f_2')(f_1+f_2)^2 - f_1^2 f_2 \times 2(f_1+f_2)(f_1'+f_2')}{(f_1+f_2)^4}$$

$$\frac{\partial x_1}{\partial d} = \frac{f_1}{(f_1+f_2)^3} \times \left[2f_1' f_2^2 + f_1^2 f_2' - f_1 f_2 f_2'\right]$$

Now factoring,

$$\begin{aligned}\left[2f_1' f_2^2 + f_1^2 f_2' - f_1 f_2 f_2'\right] &= 2\Gamma_1(M+d\Gamma_2)^2 + (M+d\Gamma_1)^2\Gamma_2 - \Gamma_2(M+d\Gamma_1)(M+d\Gamma_2) \\ &= 2\Gamma_1 M^2 + d^2\Gamma_1\Gamma_2^2 + 5Md\Gamma_1\Gamma_2 + d^2\Gamma_1^2\Gamma_2 - Md\Gamma_2^2 \\ &= \Gamma_2^2 d(\Gamma_1 d - M) + \Gamma_2\Gamma_1 d(f_1 + 4M) + 2\Gamma_1 M^2\end{aligned}$$

This is a polynomial of Γ_2 (and by extension γ_2) of degree 2. It is positive if $\Gamma_1 d \geq M$ or $\Gamma_1 \geq \Gamma_2$ but it may be negative if $\Gamma_1 < \Gamma_2$ and $\Gamma_1 d < M$.

The discriminant is positive because $(\Gamma_1 d - M) < 0$ so the polynomial has real roots, one positive (designated as $\bar{\Gamma_1}$) and one negative which violates the assumptions. The exact expression of the root is $\bar{\gamma_1} = \bar{\Gamma_1} - 1 = \frac{\Gamma_1 d(f_1+4M)+\sqrt{\Delta_1}}{2d(M-(\gamma_1+1)d)} - 1$ where $\Delta_1 = (5Md(\gamma_1+1) + d(\gamma_1+1)^2)^2 + 8d(M-(\gamma_1+1)d)(\gamma_1+1)M^2$.

To see that $\bar{\gamma_1} > \gamma_1$:

$$\bar{\Gamma_1} = \frac{\Gamma_1 d(f_1+4M) + \sqrt{\Delta_1}}{2d(M - \Gamma_1 d)}$$

A few observations $\sqrt{\Delta_1} \geq \Gamma_1 d(f_1+4M)$ and $M \geq (M - \Gamma_1 d) > 0$. We can use these to get a lower bound for $\bar{\Gamma_2}$,

$$\begin{aligned} \bar{\Gamma_1} &\geq \frac{2(5Md\Gamma_1 + d\Gamma_1^2)}{2d\Gamma_1} \\ \bar{\Gamma_1} &\geq \frac{(5Md\Gamma_1 + d\Gamma_1^2)}{d\Gamma_1} > \Gamma_1 \end{aligned}$$

Conclusion : $\bar{\Gamma_1} - 1 > \Gamma_1 - 1 \Leftrightarrow \bar{\gamma_1} > \gamma_1$. The solution respects the constraints set in place.

A.7 Proof of theorem 2

Let us analyse the two following equations separately:

$$\begin{aligned} 0 &= e_1^2\left(\frac{1-d}{2} - c\right) + e_1 e_2(1+m-c) + e_2^2\left(\frac{1-d}{2} - c\right) \\ 0 &= e_2^2\left(\frac{1-d}{2} - c - d\gamma_1\right) + e_1 e_2(1+m-c) + e_1^2\left(\frac{1-d}{2} - c\right) \end{aligned}$$

The first polynomial is always positive as long as $c \leq (1-d)/2$ since both $e_1 > 0$ and $e_2 > 0$. The second polynomial can be developed with respect to $d\gamma_1$:

$$
\begin{aligned}
0 = (d\gamma_1)^2 \left(\frac{1-d}{2} - c\right) + d\gamma_1 \left(e_2(1+c-c) - e_2^2 + 2e_2 \left(\frac{1-d}{2} - c\right)\right) \\
+ e_2^2 \left(2\left(\frac{1-d}{2} - c\right) + 1 + m - c\right) \\
0 = (d\gamma_1)^2 \left(\frac{1-d}{2} - c\right) + d\gamma_1 e_2 \left(4\left(\frac{1-d}{2} - c\right)\right) \\
+ e_2^2 \left(2\left(\frac{1-d}{2} - c\right) + 1 + m - c\right)
\end{aligned}
$$

Since $c \leq 1$ and $e_2 > 0$ the equation above never holds, meaning that as long as $c \leq (1-d)/2$, team 2 is always profitable. $\square$

Appendix B

Proofs of chapter 2

B.1 Proofs for section 2.2

B.1.1 Proof of theorem 1

The proof will be by contradiction. Let us assume without loss of generality that there exists an equilibrium in which university A plays at least once O_t^A when facing a β student with probability one. We will show that an α student can profitably deviate to take the place of a β student.

1. University A is certain to get all the α students who are in state $(x; y)$ such that $y \in \{1; \ldots; T\}$ and $x + d \leq y$ where d is the duration of an exploding offer is there is one*. All the students (α or β) in these states will be contacted by university A before or at the same time as university B.

2. The number of students in these specific states is $\sum_{t=1}^{t=T} \text{Max}\{(t + d); T\}q > (1 + T) * T/2 * 2n_\alpha > T^2 n_\alpha > c_A$. Thus University A is ensure to be filled to capacity with students. University A will have

*Recall : $d = +\infty$ is the offer is open spare students to send offers to in the final period T.

3. Because we are in a trembling hand setup, university A weakly prefers an equilibrium in which the capacity is filled as late as possible to equilibria in which capacity is filled earlier. This allows to "catch" any α student deviating and applying to A instead of accepting an offer from B. As such in no equilibrium will university A be filled before period T.

4. Type α students know that A will always play O^A when encountering them unless the university is already full. As shown earlier, A is never full before the last period. Because the game is structured in such a way that β students are always processed after the α, an α student who deviates will always receive an offer from A.

5. Thus this is not an equilibrium. □

B.1.2 Proof of the equilibrium with open offers.

The proof is very intuitive. Students will never lose the opportunity to play E^B since the offers are opened. As such there is no risk to play W for students until the last period. However, $N_\alpha > C_A$ thus β students are fully aware they have zero chances to receive an offer from A and thus enroll in B immediately. As for α students they know they cannot be turned down by B as B will have enough spare capacity to host all the α who will not receive an offer from A because of capacity constraints. They can safely play W and hope a better opportunity shows up without taking any risk.

University A can confidently turn down all the β students, knowing no α student will bail out and enroll in B before receiving an answer from A first. □

B.1.3 Proof of theorem 2

The maximum combined utility for the universities A and B with an equilibrium involving open offers is $2c_A + 2(T^2 n_\alpha - c_A) + (c_B - ((T^2 n_\alpha - c_A))$. University A is filled to capacity with α students and university B enrolls the leftover α students then fills the spare capacity up with β students.
The rest of this proof will be by contradiction. Let us assume there exist an equilibrium with exploding offers that gives the maximum combined utility presented above.

- The equilibrium cannot involve mixed strategies, as this will cause ex-post mismatches with non-zero probabilities.
- Assume the existence of an equilibrium with exploding offers the gives this level of utility. Such an equilibrium require that $(T^2 n_\alpha - c_A)$ α students play E_t^B while the others wait for an offer from A. One can increase V_A to an arbitrarily large number such that the expected payoff of playing W_t^* is larger than playing E_t^B. At least one α will deviate. This alpha will have a positive probability to be rejected. University B will be forced to recruit a β student, lowering its utility. □

B.2 Proofs for the model with exploding offers

B.2.1 Proof of theorem 3

The proof has three steps: First we derive the formula of the expected payoff for α students who decide to let an offer from B expire. Then we show that this payoff is decreasing as the matching procedure goes on. Lastly, we use a proof by contradiction to show that the value of T^* is unique.

Expected payoff of students α

Let $k \in \{1; \ldots; T\}$. At the equilibrium, the expected payoff of α students who plays W^*_{T-k} is equal to :

$$\mathbb{E}(\mathcal{U}(W^*_{T-k})) = \sum_{i=1}^{k-1} \left(\frac{1}{k} V_A \right) + \frac{1}{k} \mathbb{P}(\Omega_A) V_A$$

Proof of step B.2.1 An α student will only play W^*_{T-k} is (s)he has received an offer from B at an earlier period (that we will call y). Because the student is still waiting for a response from A, the student does not know his/her state perfectly. The student can be in any state $(x; y)$ where $T-k = y+d < x \leq T$. Because of theorem 1, in every potential state except state $(T; y)$ the student is assured to receive an offer from A.

α students are uniformly distributed among all possible states thus the probability to be in a specific state $(x; y)$ where $T - k < x \leq T$ is $\frac{1}{k}$ $\square$.

The expected payoff of α students is decreasing

The expected payoff of playing W^*_t is decreasing in t.
Proof : Let $1 < j < k < T$. Both j and k are integers.

$$\begin{aligned}
\mathbb{E}(\mathcal{U}(W^*_{T-k})) &> \mathbb{E}(\mathcal{U}(W^*_{T-j})) \\
\sum_{i=1}^{k-1} \left(\frac{1}{k} V_A \right) + \frac{1}{k} \mathbb{P}(\Omega_A) V_A &> \sum_{i=1}^{j-1} \left(\frac{1}{j} V_A \right) + \frac{1}{j} \mathbb{P}(\Omega_A) V_A \\
\frac{k-1}{k} + \frac{1}{k} \mathbb{P}(\Omega_A) &> \frac{j-1}{j} + \frac{1}{j} \mathbb{P}(\Omega_A) \\
1 - \frac{1}{k}(1 - \mathbb{P}(\Omega_A)) &> 1 - \frac{1}{j}(1 - \mathbb{P}(\Omega_A)) \\
k &> j \quad \square
\end{aligned}$$

At any equilibrium the probability of the event Ω_A is the same for all students. Thus at any equilibrium whenever a student of type α plays W_t^* then all the α students facing a choice between $E_{t'}^B$ and $W_{t'}^*$ for all $t' < t$ will play the later strategy. Following the same logic if E_t^B is played by an alpha student instead of W_t^*, then it will be played by all α for every future time period. $\square$

B.2.2 Proof of proposition 6

Proof by contradiction. Let us assume $T^* \neq T^{*\prime}$. Without loss of generality let $T^* < T^{*\prime}$. In equilibrium $\mathcal{S}'$, all α students who have to choose between $E_{T^*}^B$ and $W_{T^*}^*$ will play the latter as per Theorem 3. Their expected utility is greater than 1. In equilibrium $\mathcal{S}$, the α students who have to choose between $E_{T^*}^B$ and $W_{T^*}^*$ will either mix or only play $W_{T^*}^*$. By definition, in equilibrium $\mathcal{S}$, $\mathcal{U}(E_{T^*}^B) = 1 = \mathbb{E}(\mathcal{U}(W_{T^*}^*))$.

However, because the strategy profile $\mathcal{S}'$ is an equilibrium the payoff of playing $W_{T^*}^*$ is strictly greater than 1 even when playing the strategy profile $\mathcal{S}$. Thus in $\mathcal{S}$ all α students can deviate and only play $W_{T^*}^*$ and force the other players to play equilibrium $\mathcal{S}'$. Thus the strategy profile $\mathcal{S}$ is not an equilibrium. $\square$

B.2.3 Proof of proposition 4

The proof is in three steps :

- The first focuses on the behaviour of α students when students have a sufficiently high valuation of university A ($V_A \geq 2$).
- The second step extends the reasoning to the general case.
- The third step focuses on the behaviour of β students.

Step 1: The simple case when $V_A \geq 2$

Starting with $V_A \geq 2$ greatly simplifies the equilibrium structure. Elements of the proof for the simple case will be re-used for the general case. For the simple case I will first show that the critical period can only the last period or the penultimate one.

Lemma: If $V_A \geq 2$, then $T^* = T$ or $t^* = T - 1$

Proof: As per lemma B.2.1, the payoff of playing W^*_{T-2} is equal to

$$\mathbb{E}(\mathcal{U}(W^*_{T-2})) = \left(\frac{1}{2}V_A + \frac{1}{2}\mathbb{P}(\Omega_A)V_A\right)$$

Since $V_A \geq 2$ then $\frac{1}{2}V_A \geq 1$ thus $\mathbb{E}(\mathcal{U}(W^*_{T-2})) > \mathcal{U}(E^B_{T^*})$. Because of property B.2.1, the reasoning can be extended to every period $T - k$ where $k \geq 2$. Thus $\forall t \leq T-2 :\ \mathbb{E}(\mathcal{U}(W^*_t)) > \mathcal{U}(E^B_t)$. As such the critical period can only be T or $T - 1$. $\square$.

If the critical period is the last period then all α students will let their offers from B expire. The equilibrium becomes trivial (and unique). As per theorem 3 at most one group of student will randomise strategies. Moreover as per lemma B.2.3 if students randomise, they will only do it during period $T - 1$. All students play the same randomization between W_{T^*} with probability p and $E^B_{T^*}$ with probability $(1 - p)$. It goes without saying that if the equilibrium involves mixed strategies then when $p = 0$, $\mathbb{E}(\mathcal{U}(W^*_{T^*})) > 1$ and if $p = 1$, $\mathbb{E}(\mathcal{U}(W^*_{T^*})) < 1$.

Lemma: $\mathbb{E}(\mathcal{U}(W^*_{T^*}))$ is a strictly decreasing function of p

Proof: If randomising happens in period T-1, the number of students still applying to university A (named A_A) at the last period is a random variable follows a binomial distribution $\mathcal{B}(n_\alpha; p)+\kappa$ where κ is a constant that includes the number of students who let offers from B expire in previous periods as well as the students who have an offer from B that has not expired yet. Let Δ_{C_A} the spare capacity of university A at the beginning of the last period. It is a fixed number if $V_A \geq 2$. The probability of a student of type α to receive an offer from A at the last period given the number of remaining applicants is:

$$\mathbb{P}(\Omega_A|A_A) = \text{Min}\left\{\frac{\Delta_{C_A}}{A_A}; 1\right\}$$

$\mathbb{P}(\Omega_A|A_A)$ is a strictly decreasing function of A_A. Because students are mixing, by definition $n_\alpha + \kappa > \Delta_{C_A}$. If all the randomising students decide to play $W^*_{T^*}$ the probability of each getting an offer from A cannot be one.

The expected utility of playing $W^*_{T^*}$ is:

$$\mathbb{E}(\mathcal{U}(W^*_{T^*})|p) = V_A \sum_{k=1}^{n_\alpha} \binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k} \mathbb{P}(\Omega_A|\kappa + k) \tag{B.1}$$

Where $\binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k}$ is the probability distribution function of a random variable following the binomial distribution $\mathcal{B}(n_\alpha; p)$. By definition $\sum_{k=1}^{n_\alpha} \binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k} = 1$ and $\frac{\partial}{\partial p} \sum_{k=1}^{n_\alpha} \binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k} = 0$.

$\forall\ 0 < p < 1$ there exist a constant k^* such that :

$$\forall k \leq k^* \ ; \ \frac{\partial}{\partial p}\left[\binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k}\right] < 0$$

$$\forall k \geq k^* \ ; \ \frac{\partial}{\partial p}\left[\binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k}\right] > 0$$

It follows that:

$$\forall k \leq k^* \ ; \ \frac{\partial}{\partial p}\left[\sum_{k=1}^{k^*}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\right] < 0$$
$$\frac{\partial}{\partial p}\left[\sum_{k=1}^{k^*}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\mathbb{P}(O_T^A|\kappa+k)\right] \leq \mathbb{P}(O_T^A|\kappa+k^*)\frac{\partial}{\partial p}\left[\sum_{k=1}^{k^*}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\right] < 0$$

and

$$\forall k \geq k^* \ ; \ \frac{\partial}{\partial p}\left[\sum_{k=k^*}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\right] > 0$$
$$0 < \frac{\partial}{\partial p}\left[\sum_{k=k^*}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\mathbb{P}(O_T^A|\kappa+k)\right] < \mathbb{P}(O_T^A|\kappa+k^*)\frac{\partial}{\partial p}\left[\sum_{k=k^*}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\right]$$

When combining the two parts of the sum of partial derivatives one obtains:

$$\frac{\partial}{\partial p}\left[\sum_{k=1}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\mathbb{P}(O_T^A|\kappa+k)\right] < \mathbb{P}(O_T^A|\kappa+k)\frac{\partial}{\partial p}\left[\sum_{k=1}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\right]$$
$$\frac{\partial}{\partial p}\left[\sum_{k=1}^{n_\alpha}\binom{n_\alpha}{k}p^k(1-p)^{n_\alpha-k}\mathbb{P}(O_T^A|\kappa+k)\right] < 0$$

Thus $\mathbb{E}(\mathcal{U}(W_{T^*}^*))$ is a strictly decreasing function of p. Thus the equation $\mathbb{E}(\mathcal{U}(W_{T^*}^*)) = 1$ has a unique solution for p. As such there can be only one equilibrium in which a group of α students mix W^* with probability p and $E_{T^*}^B$ with probability $(1-p)$ if $V_A \geq 2$.

Step 2: The general case with any value of V_A

When relaxing the values of V_A the critical period T^* may be lower than $T-1$. If it is not, the step 1 proof applies. If T^* is lower than $T-1$ but the equilibrium is played in pure strategies then it is unique (trivial). If T^* is lower than $T-1$ and the equilibrium is played in mixed strategies then α students randomise between W^* with probability p and $E_{T^*}^B$ with probability

$(1-p)$. In this case the expected utility of letting an offer from B expire at time T^* becomes:

$$\mathbb{E}(\mathcal{U}(W^*_{T^*})) = \sum_{i=1}^{T-T^*-1}\left(\frac{1}{T-T^*}V_A\right) + \frac{1}{T-T^*}\mathbb{P}(\Omega_A)V_A$$

Let Δ_{C_A} the spare capacity of university A at the beginning of the last period. It is not a fixed number anymore as there may be some students who will randomise in period T^* who will receive an offer from A before the last period. The probability of a student of type α to receive an offer from A at the last period given the number of remaining applicants and the spare capacity of A is:

$$\mathbb{P}(\Omega_A|\Delta_{C_A};\ A_A) = \text{Min}\left\{\frac{\Delta_{C_A}}{A_A};1\right\}$$

where

$$\Delta_{C_A} \sim \eta - \mathcal{B}((T-T^*-1)n_\alpha;p)$$
$$A_A \sim \mathcal{B}(n_\alpha;p) + \kappa$$

η is a constant and represents the leftover capacity of A at time T^* minus the number of α students who will be contacted by A before the last period and have either let their offer from B expire before T^* or will still have an offer from B that has not expired yet. κ is a constant that includes the number of students who will be contacted by A during the last period and who let offers from B expire in previous periods as well as the students who have an offer from B that has not expired yet.

Lemma: $\mathbb{P}(\Omega_A)(p)$ is a strictly decreasing function of p Let us write $\mathbb{P}(\Omega_A)$ as a function of two randomisation parameters p and ρ:

$$\begin{cases} \mathbb{P}(\Omega_A)(p;\rho) = \left[\sum_{k=1}^{n_\alpha} \binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k} \mathbb{E}(\mathbb{P}(O_T^A | \kappa + k)) \right] \\ \mathbb{E}(\mathbb{P}(O_T^A|\kappa+k)) = \left[\sum_{i=1}^{(T-T^*-1)n_\alpha} \binom{(T-T^*-1)n_\alpha}{i} \rho^i (1-\rho)^{(T-T^*-1)n_\alpha - i} \mathbb{P}(O_T^A | \eta - i;\ \kappa + k) \right] \end{cases}$$

Let $0 < p < p' < 1$ and $0 < \rho < \rho' < 1$ without loss of generality. From lemma B.2.3, $\mathbb{E}(\mathbb{P}(O_T^A|\kappa+k))$ is a weakly decreasing function of ρ. Moreover, $\mathbb{E}(\mathbb{P}(O_T^A|\kappa+k)) \geq \mathbb{E}(\mathbb{P}(O_T^A|\kappa+k'))$ while $\mathbb{P}(\Omega_A)(p;\rho)$ is a strictly decreasing function of p. It follows that:

$$\mathbb{P}(\Omega_A)(p;\rho) \geq \mathbb{P}(\Omega_A)(p;\rho') > \mathbb{P}(\Omega_A)(p';\rho')$$

To conclude all one has to do it to equate p and ρ to get that $\mathbb{P}(\Omega_A)(p)$ is a strictly decreasing function of p. Thus $\mathbb{E}(\mathcal{U}(W_{T^*}^*))$ is a strictly decreasing function of p and the equation $\mathbb{E}(\mathcal{U}(W_{T^*}^*)) = 1$ has a unique solution. and the equilibrium in mixed strategies is unique.

Step 3: β students never play W^*

Proving the uniqueness of strategy for β students is much more straightforward. If α students play using only pure strategies, then the probability of A being full is 1, thus βs immediately accept the offer from B as they have a zero probability of ever receiving an offer from A. The same reasoning applies for equilibrium where α play with mixed strategies but the probability of A being full is 1.

If there is a non-zero probability of A ending up not full (because C_A and V_A are too small), we need to check if β students have a profitable deviation by playing W^* instead of E_t^B. The expected utility of a β student of playing

W^*_{T-1} (the penultimate period).

$$\mathbb{E}(\mathcal{U}(W^*_{T-1})) = V_A \sum_{k=1}^{n_\alpha} \binom{n_\alpha}{k} p^k (1-p)^{n_\alpha - k} \mathbb{P}(O^A_T | \kappa + k \ ; \ \beta)$$

Notice that A only send an offer to a β if there is some spare capacity left after going through all the α applicants. The maximum spare capacity available for β students in the last period is n_α which occurs when all the α students randomising between W^*_{T-1} and E^B_{T-1} pick E^B_{T-1}.[†] Moreover, the β student who played W^*_{T-1} is in competition with all the β students in state $(T;T)$. As such : $\forall k \in \{0; \ \ldots \ n_\alpha\}$ $\mathbb{P}(O^A_T|\kappa + k \ ; \ \beta) < (\mathbb{P}(O^A_T|\kappa + k)$. Thus playing W^*_{T-1} as a β student at time $T-1$ will yield a strictly lower payoff than playing W^*_{T-1} as an α student i.e $\mathbb{P}(O^A_T|\beta) < \mathbb{P}(\Omega_A)$.

Now let us extend to a more general case of playing W^*_t $\forall t$

$$\mathbb{E}(\mathcal{U}(W^*_t)) = \sum_{i=1}^{T-t-1} \left(\frac{1}{T-t} V_A \mathbb{P}(O^A_{T-t+i}|\beta) \right) + \frac{1}{T-t} \mathbb{P}(O^A_T|\beta) V_A$$

Notice that if $T - t + i < T^*$ then $\mathbb{P}(O^A_{T-t+i}|\beta) = 0$. Before the critical period T^* all the α students play W^* and A knows it. As such there is no reason to send an offer to a β student. The β students can only hope to receive an offer from A once the critical period T^* is reached, and if a higher than expected number of alpha students randomise in favour of E^B_t. Thus $\mathbb{P}(O^A_t|\beta) < 1$. Knowing that $\mathbb{P}(O^A_T|\beta) < \mathbb{P}(\Omega_A)$ we can conclude that playing W^*_t $\forall t \geq T^*$ as a β student at time $T-1$ will yield a strictly lower payoff than playing W^*_t $\forall t \geq T^*$ as an α student.

To conclude, in en equilibrium where α students play mixed strategies in any

[†] In the event that α students start randomising before time $T-1$ any spare capacity would be immediately filled with β students in earlier periods

period where α students mix between E_t^B and W_t^*, β students prefer to play E_t^B. In any period where all alpha students play W_t^*, the expected utility of playing W_t^* for βs is even lower than when alpha students start mixing. As such β students never play W_t^* $\forall t$.

□

B.2.4 Proof of property 4

$\forall t > T^*$ α student plays W_t^* and thus will never end up with university A. The number n_{miss} of α students who will never apply to A is an algebraic series:

$$\begin{aligned} if:\ T^* < T \\ n_{miss} = \sum_{i=1}^{T-T^*} (i-1)n_\alpha \\ n_{miss} = \frac{T-T^*}{2}(T-T^*-1)n_\alpha \end{aligned}$$

Because in any equilibrium the probability of A being fully filled with α students cannot be zero, this implies that the number of missing A students cannot bring the number of A applicants below C_A. Since $(T-1)Tn_\alpha < C_A < T^2 n_\alpha$, n_{miss} is bounded from above by Tn_α. Notice that if $T = 6$ and $T^* = 2$, $n_{miss} = 6n_\alpha$. So the game with 6 periods does not fully unravel. Increasing the number of period T by one while keeping $T^* = 2$ violates the inequality as well. By recurrence, the partial unraveling holds for all values of T. □

B.2.5 Proof of theorem 5

This is a generalization of theorem 4 and parts of its proof (B.2.4). Instead of using the upper bound for n_{miss}, the true difference ΔC_A between the number of α and the capacity of A is used.

$\forall t > T^* \ \alpha$ student plays W_t^* and thus will never end up with university A. The number n_{miss} of α students who will never apply to A is an algebraic series:

$$\begin{aligned} if:\ T^* &< T \\ n_{miss} &= \sum_{i=1}^{T-T^*} (i-1)n_\alpha \\ n_{miss} &= \frac{T-T^*}{2}(T-T^*-1)n_\alpha \end{aligned}$$

Because in any equilibrium the probability of A being fully filled with α students cannot be zero, this implies that the number of missing A students cannot bring the number of A applicants below C_A. Thus:

$$\begin{aligned} n_{miss} &\leq \Delta C_A \\ \frac{T-T^*}{2}(T-T^*-1)n_\alpha &\leq \Delta C_A \\ (T-T^*)^2 - (T-T^*) - 2\Delta C_A/n_\alpha &\leq 0 \\ \iff \frac{1-\sqrt{1+8\Delta C_A/n_\alpha}}{2} &\leq (T-T^*) \leq \frac{1+\sqrt{1+8\Delta C_A/n_\alpha}}{2} \end{aligned}$$

Since $0 < T^* \leq T$ we can integrate these constraints back into the inequality above:

$$(T-T^*) \leq \mathrm{Min}\left\{\frac{1+\sqrt{1+8\Delta C_A/n_\alpha}}{2}; T\right\}$$

$\square$

B.3 Proofs for the players' welfare

B.3.1 Proof of theorem 6

The proof is by construction. Let $\mathcal{S}$ be the equilibrium of a game with parameters T, c_A, c_B, V_A, n_α, n_β and an exploding offer of duration $d = 0$. Let T^* be the critical period associated with equilibrium $\mathcal{S}$.

The notations κ and η from the proof of theorem 4 will be reused in this proof. In equilibrium $\mathcal{S}$ the number η of α students who will enroll in B before the last period is $\sum_{i=1}^{T-T^*}(i-1)n_\alpha$. In equilibrium $\mathcal{S}$ there are κ students of type α who will be competing for sure for a seat in A in the last period. These are alpha students in state $(T;T)$, all the ones in states $(T;x)$ $\forall x < T^*$ and either all the α students from state $(T;T^*)$ (equilibrium is played in pure strategies) or a random number of them. If the number of α students is random this implies a randomisation with probability p such that $\mathcal{U}(E^B_{T-1}) = 1 = \mathbb{E}(W^*_{T-1})$ where $\mathbb{E}(W^*_{T-1})$ is a function of κ and η.

Let $\mathcal{S}'$ be an equilibrium candidate for the game with parameters T, c_A, c_B, V_A, n_α, n_β and an exploding offer of duration $d' \geq 1$. If $T'^* = T^*$, then there are η' students of type α who will enroll in B before the last period. $\eta' = \sum_{i=1+d}^{T-T^*-d}(i-d-1)n_\alpha$. Notice that $\eta = \eta'$.

Following the same logic, there are κ' students of type α who will be competing for sure for a seat in A in the last period. These are all the α students in state $(T;x)$ $\forall x \geq T-d$, all the ones in states $(T;y)$ $\forall y < T^* - d$ and either all the α students from state $(T;T^*-d)$ (equilibrium is played in pure strategies) or a random number of them.

This means that in the equilibrium candidate $\mathcal{S}'$ in period $T'^{*} = T^{*}$, the α students face the exact same randomization problem than in $\mathcal{S}$. Thus they behave identically. $\square$

B.4 Proofs of the extended model

B.4.1 Proof of theorem 7

It is a straightforward reuse of the proof of theorem 3. Since $\mathbb{P}(\Omega_A)$ is the same for all α_i students the behaviour of each individual α_i subgroup of students is similarly structured:

- Let the offer from B expire before a critical period
- Enroll in B instead of waiting for an answer from A after the critical period.
- Either let the offer expire or randomise during the critical period.

Because the utility of each subgroup of α_i is different the critical periods may be different. However, because of lemma B.2.1, the critical periods will be ordered. $\square$

B.4.2 Proof of the corollary

Let there be two α students named i and j. Let us assume without loss of generality that student i played E_t^B while student j played W_{t+k}^{*} with $k > 0$. Either:

1. Students i and j have the same sub-type
2. Student i has a higher sub-type than student j

3. Student j has a higher sub-type than student i

If both students have the same subtype then this is a contradiction as once at least one student of a give subtype has played E^B then all students of the same subtype in a later period must play E^B as well.

If student i has a higher subtype than student j then at every point in the game $\mathbb{E}(\mathcal{U}_{\alpha_i}(W_t^*)) > \mathbb{E}(\mathcal{U}_{\alpha_j}(W_t^*))$. Thus if the student i plays E_t^B then all students that have the same subtype as student j will play E^B at time t. As stated above, if at least one student of a given sub-type plays E^B all the students of the same sub-type must play E^B in every subsequent period. We reach a contradiction.

The only option left is student j has a higher sub-type than student i. Which is possible. You simply need to have a V_{α_i} sufficiently high such that student i prefers to enrol while student j prefers to let the offer explode. $\square$

Appendix C

Proofs of chapter 3

C.1 Proof of property 6

The proof is straightforward : Take any menu between the small one and the big one. It will be filtered to a terminal menu. Either the terminal menu is between the small and the big menu => The chosen alternative is still acceptable and is preferred to anything else. Or, the terminal menu is smaller than the small one and possibly disjoint. Then it means some alternatives in the small menu were eliminated i.e unacceptable. However, if they are unacceptable they must have been eliminated from the smaller menu as well. Thus the chosen alternative is still acceptable at this terminal menu.

Proof of the expansion : A is chosen in the two menus thus is acceptable. It will still be acceptable in the union of the two menus. The joined menu will be filtered to a terminal menu. Apply the same reasoning as above to get the proof.

C.2 Proof of property 7

The proof is by contradiction. Let us assume a preference reversal generated by a swap of two irrelevant alternative that are both identically unacceptable or both acceptable all the time. Then the terminal root of these two nodes has the same cardinality and deduct that there cannot be a preference reversal.

C.3 Proof of the graph shape

Let us assume there exist a rational filter graph $G(V; E)$ that has at least one circle. This circle either:

- Has two vertices/menus of the same cardinality connected by an edge. This contradicts the assumption that the graph is a rational filter.
- Does not have two vertices/menus connected by an edge.

In the second case let us select one of the vertex/menu m in the cycle with the highest cardinality without loss of generality. This vertex has an edge with an other vertex $m' \subset m$. Because both m and m' are in a circle, there is an other path linking m and m'. Thus $\exists m'' \subset m$ such that $\{m; m''\} \in E$. This vertex does not comply with the definition of the vertices of a rational filter graph.

Since no rational filter graph can have a circle. therefore, it is a forest.

C.4 Proof of lemma 1

Let us assume the menu with the smallest cardinality is not unique. In this case $\exists m' \in T$ such that there is a path between m and m'. This path has to go through a vertex n such that $|n| > |m|$. n will have a link to at least two vertices/menus that link to smaller menus in contradiction to the definition of a rational filter graph.

C.5 Proof of property 8

$\Rightarrow$ Let A be a set of alternatives. Let $C : 2^A \rightarrow A \cup 0$ be a choice function satisfying CAAA. Thus $C = c \circ f$ as in section 3.2. Let G be a rational filter tree constructed using the sets U_i provided by the filter functions associated with the choice function C. It is possible to construct such a tree since $\forall i < j;\ U_j \subseteq U_i$.

Let $m; m' \in 2^A$ two different menus in the same tree T of G with m being the root of this tree. Thus $m \subset m'$. Either $\{m, m'\} \in E$ or there exist a path between m and m'. In either case $f(m) = f(m') = m$. Therefore $C(m) = c(f(m)) = c(m)$ and $C(m') = c(f(m')) = c(m)$. So C satisfy invariance along a tree.

Let Π_i an element of the homogeneous tree partition of G. Either Π_i includes a single tree, in this case the image set of Π_i by C is a single element of A and C trivially satisfies P.WARP over Π_i. Or Π_i includes more than one tree. In this case let m and m' be the root of two trees in Π_i. $|m| = |m'|$ since they are roots of two trees in the same element of an homogeneous tree partition. Thus $C(m) = c_{|m|}(m)$ and $C(m') = c_{|m|}(m')$. Because $\succ_{|m|}$ is a complete transitive and acyclic ordering, by extension any menus in the trees of m and m' will satisfy WARP since C is constant over a single tree. Thus C satisfies WRAP over all elements in Π_i.

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