THE YIELD OF MILD STEEL

(With particular reference to the effect of size of specimen)

by

JOHN L. M. MORRISON, B.Sc.

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INTRODUCTION.

Many theories have been proposed to provide a criterion for the failure by yield or rupture of material subjected to stress. The idea that the elastic limit is reached when the maximum principal stress, regardless of the other principal stresses, equals the elastic limit in simple tension, was generally accepted until the 19th century. Mariotte, however, and later Poncelet and St. Venant, held that the maximum strain was the true criterion of failure. The experimental investigation of Guest[1] on thin tubes is sufficient to disprove these hypotheses as applied to the elastic limit or yield of ductile materials. The theory advanced by Beltrami, and supported by Haigh, that the strain energy per unit volume stored in a material before reaching the plastic state is constant, has been disproved by Bridgman and by Cook. Cook[2] has shewn that the application of a high hydrostatic pressure, although lowering the elastic limit, has no effect on the yield point of mild steel, and that the criterion for yield must be the constancy of some function of the principal shear stresses.

The importance of shear stress in determining failure was first pointed out by Coulomb. Tresca's extrusion tests on the flow of metals gave support to his assumptions, and Guest, in the first important experimental research referred to above, clearly demonstrated that the yield of ductile materials was intimately associated with the intensity of the maximum shear stress. Subsequent experimental work has in general confirmed this conclusion, but there is still disagreement as to the possible influence of the accompanying stresses. The most important of the hypotheses which refer to shear stress as the fundamental criterion are

(a) That the maximum shear stress determines failure, irrespective of the other stresses. This hypothesis is frequently associated with the name of Guest; it would, however, be more accurate to attribute to him the second, namely

(b) That the intensity of the accompanying normal

Stress affects the limiting value of the shear stress. In his original paper, Guest (1900) concluded that the normal stress had some effect. This hypothesis was later (1900 & 1914) strongly advocated by Mohr, with whose name it is usually associated. It may be noted that, to account for the angle of shearing of brittle materials in compression tests, Perry (1897) assumed "a resistance to shearing of the nature of friction".

(c) That yield depends on all the principal shear stresses. Von Mises suggested that in order to avoid the mathematical discontinuities given by the shear stress theory, an equation representing a continuous surface and involving all three principal stresses might provide the criterion; later, Hencky shewed that this equation represents the condition of maximum shear strain energy.

Since a considerable amount of experimental work was completed before the third of these theories was formulated, it is of interest to reconsider this early work in order to see if it can be used to discriminate between them. It seems appropriate to examine in some detail Guest's original tests.[1] Those of his experiments which particularly concern the subject of the present paper were conducted on nine steel tubes, a total of 101 tests having been made in tension, torsion, combined tension and torsion, and either tension or torsion with internal pressure. In order to give the theories of limiting shear stress and limiting shear strain energy an equal representation, the "equivalent tensile stress" has been calculated for each test according to both theories. Thus if S_1, S_2, S_3 are the principal stresses (where $S_1 > S_2 > S_3$), according to the shear stress theory the equivalent tensile stress to cause failure is given by $Sg = 2(S_1 - S_3)$, while according to the shear strain energy theory the equivalent tensile stress is given by Sm, where $2Sm^2 = (S_1 - S_2)^2 + (S_2 - S_3)^2 + (S_3 - S_1)^2$. The average values of Sg and Sm for each tube have been calculated, and the mean percentage variation between Sg and Sm for each individual test and these average values are shewn in Table 1. Two tests on tube No. 2 where the figures are inconsistent or unsatisfactory have been omitted.

TABLE 1.

Tube No.	No. of tests carried out.	Mean variation percent according to shear stress.	Mean variation per-cent according to shear strain energy.
1	6	14.6	16.6
2	10	3.3	6.5
3	6	2.8	8.4
4	9	4.0	2.7
5	10	3.3	5.2
6	12	5.3	6.7
7	20	3.2	2.7
8	20	3.9	3.1
9	6	5.1	5.2

It is immediately obvious from these figures that Guest's experiments cannot be expected to furnish definite evidence to discriminate between the two theories. This method of representing the results is not, however, entirely satisfactory, since it gives no indication of a possible progressive trend in the variation; in addition, therefore, the results of all the tests involving biaxial stress have been plotted on the same diagram. Again to present the case in as fair a manner as possible this has been done twice, each test being in effect compared with the mean of the equivalent tensile stress according to each theory as before (Figs. 1 & 2) It may be noted that while the previous conclusion, that these results are not sufficient to descriminate between the theories, is substantiated, there is some evidence that the shear stress at yield is higher in torsion than in tension. Guest expressed this fact in the tentative equation $(S_1 - S_3) + \lambda(S_1 + S_3) = q$ = shear stress in torsion, the value found for λ being 0.04, and stated that a compression test "would be of much assistance" in checking the validity of this linear relationship. It will be seen that this suggestion is almost identical with one made by Mohr.

It is impossible in a single paper of this type to deal in similar detail with the work of many investigators. A few of the most important researches will therefore be briefly mentioned.

Scoble[3] carried out tests on yield in flexure and torsion of round bars $\frac{3}{4}$ inch in diameter, one test only being made on each bar. He found that the maximum shear stress was nearly constant and might be taken as the determinant to a first approximation. He considered the suggestion made by Guest and others, that a force corresponding to friction due to the force normal to the plane of the greatest shear might be responsible for the variations found, but he concluded that this hypothesis would not explain his results, and that the true reason was lack of isotropy in the material. It may be added that his results lend no support to the shear strain energy theory.

Mason[4] tested thin tubes, which he had annealed under conditions calculated to avoid excessive scaling, under tension, compression, internal and external pressure, and various combinations of these. The principal conclusion reached was that, for annealed material, the maximum shear stress at the yield point in compression, and the yield point stress in pure shear were in approximate agreement, the mean difference being about 3 per cent; and therefore that to a first approximation the value of the shear stress was independent of any normal stress on the planes of the slide.

Smith[5] considered critically the work of earlier investigators⊕ and pointed out some of the possible sources of error in their work. In his own experiments on 1-inch diameter specimens he took great care to measure the true stress, as opposed to the mean stress, at yield. He concluded that in the case of mild steel the shear stress law as applied to tension, compression and torsion was exact. In his results from a .09 per cent carbon steel the average variation from the mean of 20 tests was 2 per cent.

Cook and Robertson[6] tested thick tubes under internal pressure, and compared the results with direct tension. The internal diameter of the tubes was $\frac{5}{8}$ inch, and the ratio of external to internal diameter varied from 1.35 to 3.65. The shear stress at yield in the tubes was approximately constant,

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⊕ Guest, Hancock, Scoble, Turner.

and about 20 per cent in excess of that in the tension tests. It may be noted that though these results agree with neither the shear stress, nor the shear strain energy theory, their general character has been confirmed by Cook in another paper which will be considered later.

Seeley and Putnam[7] tested solid and hollow specimens in torsion, and solid specimens in tension and compression. The shear stress at yield in the former ranged from 1.1 to 1.3 times the shear stress in direct loading. While hardly conclusive, the results are more nearly in accordance with the hypothesis of shear strain energy than that of shear stress.

Roš and Eichinger[8] tested hollow specimens under tension, compression, torsion, and internal pressure. Their results agree almost exactly with the shear strain energy theory.

Cook[9] investigated the effect of the intermediate principal stress by testing thick tubes under internal pressure with an additional axial load. These tests shewed that this stress was without effect on the shear stress at yield, a result that is in accordance with the shear stress theory, but entirely at variance with that of shear strain energy. It was, however, found that the (uniform) shear stress in a direct tension test was lower than in the case of combined stress, where a stress gradient existed.

An earlier research by Cook[10] is too comprehensive to be summarized at this point, and will be referred to later. It need only be noted that neither the simple theory of shear stress, nor the theory of shear strain energy, provides a satisfactory explanation of his results.

While the work of Lode,[11] and the confirmation it has received from Taylor and Quinney,[12] are not directly applicable to the present argument, it should be remembered that they have shewn that, for a ductile material which has reached the plastic state, the shear strain energy law holds.

It will be appreciated that the chief criticism from the present point of view which can be applied to the earlier experimental work (e.g. that of Guest) referred to above, is that the standard of accuracy attained is insufficient to allow the criterion of yield to be ascertained with certainty. This statement is not to be taken as depreciatory. It must be remembered that the object of those experiments was to compare the validity of the theories of principal stress, principal strain, and shear stress, and that the standard of accuracy was sufficient to demonstrate conclusively the superiority of the shear stress theory. In view of the divergent conclusions quoted, it seemed to the author that a series of tests could with advantage be made, in which by paying scrupulous attention to detail and, for instance, by making use of recent developments in heat treatment, a still higher standard might be reached, and more evidence on the nature of yield under complex stress distribution be obtained.

MATERIALS AND HEAT TREATMENT.

Earlier work in the engineering department of the University of Bristol by Professor A. Robertson and the late A. J. Newport had indicated the importance of using a material which was as nearly uniform as possible, and of heat treating it after machining to remove the stresses set up in the surface by cold working. The steel used throughout the present tests was supplied by Messrs. The Park Gate Iron & Steel Co. in the "as rolled" condition in the form of ½ inch and 2 inch bars to the following specification

C	Si	S	P	Mn
0.21	.085	.023	.016	0.68

The preliminary check consisted in taking sulphur prints from the cross section, and, in the case of the large bars, in making 1 millimetre diameter ball Brinell tests across this section. The prints shewed no segregation, and the hardness number was constant. Standard commercial tensile tests at a low rate of strain, requiring 4 minutes to reach yield, and 40 minutes to break, on 0.282 inch diameter specimens gave the results shewn in table 2, and the corresponding autographic diagrams are reproduced in Fig.3.

TABLE 2.

Heat Treatment	Specimen No.	(Upper)Yield Stress, tons/sq. in.	Lower(Plastic) Yield Stress tons/sq. in.	Ultimate Stress tons/sq.in.	Extension on gauge length 1 inch $(4\sqrt{a})$, per cent.
None ("as received")	1	25.2	20.3	30.1	41
	2	26.4	20.2	30.2	42
"Normalised" ($\frac{3}{8}$" diameter cooled in air)	1	24.7	20.2	30.3	42
	2	24.9	19.8	30.4	42

As will be seen from the results to be given later, this material was found to be extremely uniform and reliable. Tests on material from various positions in the cross-section of the large bar gave no appreciable variation in strength, and no differences could be found, after normalisation, between the two bars in spite of the difference in size. The tension stress/strain diagram was straight up to the yield point. In Fig. 4 (a) a microphotograph (x40) of a longitudinal section is given to shew the degree of banding of the pearlite; it will be seen that this is very slight, a fact which is reflected in the results from longitudinal and transverse tests.

With a few exceptions which are stated, all the tests were carried out on material which had received the following treatment. The bars were sawn into suitable lengths, and normalised to remove any possible initial stresses. The specimens were then machined, finished by polishing with fine emery cloth until the scratches were hardly visible to the naked eye, and heated in a vacuum furnace to a temperature of 905°C for ten minutes. The vacuum tube containing the specimens was withdrawn from the furnace and allowed to cool. The vacuum attained was measured by means of a spark discharge, and in no case did the pressure in the tube rise to .001 mm of mercury. A typical heating and cooling curve is given in Fig. 5. It was unfortunately difficult to control the furnace perfectly, since there was a considerable time lag between adjusting the current and the settling down of the temperature, with the result that some minor variations in results which are otherwise inexplicable have to be ascribed to this cause. The combined tension and torsion tests to be described may be instanced as an illustration. Identical specimens were treated in two batches. The yield stress for those from the second batch was about 1.5 per cent greater than for corresponding tests from the first batch (c.f. table 4), but when the ratio between each individual test and the mean tensile stress for the same batch is plotted on one diagram (c.f. Fig. 14) the resulting curve is satisfactorily smooth. As the difference found between these two batches covers the whole range of variation found in all the tests, it may be concluded that heat-treatment is not responsible for a difference of more than ± 1 per cent in the results. Moreover, the effect of this slight variation has been as far as possible eliminated by comparing all the results on a basis of check specimens which were included in all the heat treatments.

In some preliminary experiments, the heat treatment was carried out in a "home-made" vacuum furnace. The design, which was completed before the difficulties of achieving a high vacuum at the required temperature were fully realised, incorporated a fairly long narrow bore tube to lead away the occluded gases, and the pressure, measured at the pump, was of the order of .05 mm of mercury. Although the specimens treated in this furnace were found on removal to be quite bright, test results suggested, and micrographical examination confirmed, that the surface was decarburised. The remaining microphotographs (x200) in Fig. 4 illustrate this point. (b) shews a section of a thin tube of low carbon steel normalised in the home-made furnace between two plates of untreated material; (c), (d) and (e) are different sections of a thin tube normalised in the high-vacuum furnace, etched to shew the crystal size and the pearlite content, and (f) is a section from the centre of a large mass of the same material, normalised at the same time. It will be seen that whereas in (b) decarburisation is nearly complete, there is no evidence of any decarburisation in the other photographs.

A photograph of the high-vacuum furnace is given in Fig. 6. Since it is in some respects unique, a short description may be of interest. The silica vacuum tube, 5 inches bore x 35 inches in length, is exhausted by two Apiezon oil condensation pumps and a rotary pump in series, the pipe leading the gases from the silica tube to the first pump having a bore of 2¼ inches. This equipment is capable of maintaining a vacuum pressure of .001 inch of mercury while a reasonable charge is being heated to 900°C in three hours, the pressure being measured at two points, between the oil pumps and before the rotary pump, by applying an e.m.f. capable of producing a half-inch spark in the atmosphere to a column of low pressure air 4 inches in length. The nature of the discharge indicates the pressure. The heating unit is a 6-kilowatt resistance furnace. The whole of this equipment was supplied by Messrs. The Metropolitan Vickers Electrical Co. Ltd.

The rest of the apparatus was constructed in the workshops of the University engineering department to the author's designs. A vertical mounting for the vacuum tube and its contents was adopted, in order that the specimens might have no bending stresses at high temperatures, and to facilitate the withdrawal of the furnace. The vacuum tube and furnace are counterbalanced and can be raised or lowered independently. A stainless steel tube closed at the lower end and running down the centre of the vacuum tube serves to support the specimens and to introduce a thermocouple at any desired depth. It was found that the temperature attained a sensibly uniform maximum value over a length of 10 inches. It was also found possible to estimate the rapidity of the heat transmission from the silica vacuum tube to the specimens by placing a second thermocouple between the silica tube and the furnace. At low temperatures the readings of the two thermocouples were widely different; above 400°C, however, the gap narrowed rapidly, and five minutes after the maximum reading of the outer couple had been reached the difference had been reduced to 2°C. As the inner couple was protected from the radiation from the hot walls by the specimens, the stainless steel tube, and its own silica sheath, it may reasonably be assumed that the temperatures of the specimens and the inner thermocouple were the same.

TECHNIQUE OF TESTING.

The experiments to be described in the present paper involve tests in tension, compression, flexure, torsion and combined tension and torsion. Since these experiments were carried out, not in the order which might appear logical when the results are known, but as seemed most expedient at the time, it will be convenient to describe first the technique adopted for each of these types of test, then to give the results of the various tests, and afterwards to discuss the relation of these results with each other and to those of other experimenters.

(a) Tension.

In certain preliminary experiments it was found that in spite of every care in machining and heat treating apparently identical specimens, the results of tension tests were distinctly less consistent than those of torsion tests. The variation from the mean in tension was of the order of ± 5 per cent, whereas in torsion it seldom exceeded ± 1 per cent, and was as a rule well within those limits. It was therefore apparent that the material was consistent in quality and that the error must lie in the method of carrying out tension tests. The shackles[13] used were designed to apply an axial load to the specimen through screwed ends, but since no other error could be found, it was decided that despite the precautions taken, there must still be an appreciable eccentricity of loading. The testing machine was therefore dismantled, and a specimen fixed in the shackles by the means illustrated in Fig. 12(a), where a plug screwed into the remote end of the adaptor is separated from the specimen by a hard steel ball resting in the centre on which the specimen has been turned. The whole unit, consisting of shackle, specimen and shackle was then mounted on centres in a lathe. It was found that with the ordinary standard of accuracy in cutting the screwed ends of the specimens the slackness in the thread was quite sufficient to allow the specimen to take up and retain under load an eccentricity in the shackles which would account for the variation in results. Since, therefore, short of making each specimen a "perfect" fit, true axiality of load could not be assured, it was decided to accept the inevitable eccentricity, and to measure the strain at three positions round the circumference of the specimen by means of three extensometers. From the readings of these extensometers the position and magnitude of the maximum strain can be calculated. A convenient method of doing this is described in the appendix 1. Since the steel used obeys Hooke's law within the limits of experimental error right up to the yield point (see Figs. 13,16, and 17) the maximum stress can be inferred directly.

The apparatus used is shewn, mounted on a tension specimen ·5644 inch in diameter, in Fig. 7. It consists of three separate Martens type mirror extensometers, of 1-inch gauge length, which are held at 120° from each other on the specimen under test by means of two thin rubber bands. This design was adopted since it was desired to apply the apparatus to as wide a range of size of specimen as possible; in practice it has been used satisfactorily on specimens of .282 inch diameter upwards. The setting up presents no insuperable difficulty, but of course takes a considerable time.

In order to check the accuracy obtainable with this triple extensometer it was fitted to a specimen in the testing machine, and readings were taken while the stress was raised to about 80 per cent of the yield stress, and reduced to 10 per cent of the yield stress. This load was sufficient to ensure that the specimen would not move in the shackles. The extensometers were then interchanged and readings taken over the same range of stress. This procedure was repeated twice more, so that the final positions were the same as in the first loading. Each member of the four sets of strain readings was within ± 1 per cent of its mean, while the average strain reading from the three extensometers in each position was within ± ½ per cent of the mean average. It was therefore considered that the accuracy obtainable was sufficient to justify a correction in the yield stress as measured.

It may be noted that if the points of application of the load at the ends of the specimen lie on opposite sides of the axis the eccentricity as measured by the triple extensometer will be lower than the true eccentricity at some cross-section. It is therefore important, even when such an extensometer is used, to keep the load as nearly axial as possible. This point will be referred to later in connection with compression tests.

The tension tests included in the present paper were made on specimens ranging from .282 inch diameter to 1 inch diameter. An attempt was also made to test specimens of .1 and .125 inch diameter. For these sizes the triple extensometer was, of course, unsuitable, and one unit was mounted to face in the three 120° directions in succession, the load being raised to about 80 per cent of the probable yield and reduced to a value just sufficient to hold the specimen while the extensometer was being reset. It was hoped in this way to obtain some estimate of the eccentricity of loading. Unfortunately, however, the strain readings obtained during the first application of the load could not be repeated during subsequent applications. This phenomenon was never found with any of the larger specimens, the load on which had often, as is explained later, to be relieved many times during a test in combined tension and torsion. The author believes that this is a point of some importance, and it will be referred to later, in the discussion of the results given by some thin tubes. Since it was not found possible with the available apparatus to correct the yield stress reading for these small specimens, the results have been omitted.

In a few early tests on 1 inch diameter specimens no suitable axial loading shackles were available. These tests were therefore carried out, using the spherically seated shackles supplied by the makers of the testing machine. It was found that, although the spherical bearings were carefully greased before each test, the eccentricity of the load was so great that in an extreme case one side of the specimen was, in the early stages of loading, put into compression.

One further difficulty in the securing of accurate results remains to be discussed. It is well known that a stress concentration appears at any change in section of a material under stress. If therefore the transition from the parallel part of be lower than the true eccentricity at some cross-section. It is therefore important, even when such an extensometer is used, to keep the load as nearly axial as possible. This point will be referred to later in connection with compression tests.

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One further difficulty in the securing of accurate results remains to be discussed. It is well known that a stress concentration appears at any change in section of a material under stress. If therefore the transition from the parallel part of a test piece to the enlarged ends is insufficiently gradual, the stress concentration at the bottom of the fillet may cause yield to occur at this point before the stress in the parallel part reaches the true yield stress. Even if an extensometer is used, the recorded yield stress will be low, since in general, due to the stress drop, yielding spreads rapidly through the rest of the test piece. In the case of one early test in which the eccentricity of loading was larger than usual, the spreading of the yield was sufficiently delayed to provide definite evidence of yield occurring in this manner. The parallel part, $1\frac{1}{2}$ inches in length, of a 1 inch diameter specimen was joined to the ends, which were screwed with a $1\frac{1}{4}$ inch gas thread, by a fillet of radius $1\frac{3}{4}$ inches, i.e. $3\frac{1}{2}$ times the radius of the specimen. The yield stress of the material (c.f. table 7) was later found to be 22·84 tons per square inch. Up to an average stress of 20 tons per square inch the specimen behaved in a normal manner, the eccentricity as deduced from the readings of the triple extensometer being at this stage equivalent to a maximum stress of 21·46 tons per square inch. When the average stress was increased to 20.5 tons per square inch the maximum strain as measured by one unit "C" of the extensometer was actually reduced instead of being increased, but the increased reading on the other extensometers A and B shewed that the whole of the material within the gauge length was still in the elastic region. For approximately ten minutes the specimen withstood the applied load, while the extensometers shewed that the strain at the side C was first gradually reduced and then built up again to a value greater than the previous maximum. The beam of the testing machine then dropped suddenly, when all the extensometers shewed a large extension of the gauge length.

A reasonable explanation of these phenomena would seem to be as follows. The maximum stress within the gauge length did not exceed 22 tons per square inch, but it is known that the fillet would cause a stress concentration. The maximum stress at the bottom of the fillet outside the gauge length must have reached the yield stress of 22·84 tons per square inch, so causing yield to take place in a limited volume of material and reducing the effective eccentricity. As the yielding spread over the cross section at the bottom of the fillet the original eccentricity was restored, and the yielding area spread along the specimen under a stress less than the true yield stress. It may therefore be tentatively deduced that the stress concentration was of the order of 4 per cent, and it may be noted that in the absence of the triple extensometer no indication would have been given that yield had first occurred outside the gauge length.

Some concentration of stress is unavoidable in a specimen of finite length. In order to minimise the difficulty, the parallel part of the specimen used in the later tests was joined to the ends by a transition curve, instead of the usual radius. The curve chosen for the large specimens is shewn in Fig. 8. In practice, this profile was obtained by removing the tool rest feed screw from a lathe, and guiding the tool by means of a roller held by dead weight against a hand-shaped former. The small specimens were finished individually to a similar shape by machining continually decreasing steps from shoulder to parallel, and afterwards removing the steps by hand file and emery cloth. Since all the specimens machined in this way gave consistent results irrespective of size, whereas a few preliminary tests on 1-inch specimens with the more usual radius gave slightly lower results, as has been described, the author believes that the residual error is very small. Additional confirmation may be found in the fact that the tension-torsion ratio for ·2822 inch diameter specimens was about 1 per cent lower in the combined stress experiments first described where the fillet radius used was $\frac{5}{8}$ inch than in the subsequent experiments, in all of which a transition curve was used.

The effect of rate of strain will be more fully discussed in connection with the flexure tests. It is, however, of some importance in all the tests. In an earlier paper[14] the author has shewn that in a similar mild steel the yield stress in tension approaches a minimum when the rate of strain is of the order of 10^{-6} per second. In all the tests, other than flexure, discussed in the present paper the rate of strain was reduced to zero while the extensometer was read, and also at very small increments in load when the yield point was being approached. In spite of this care it is doubtful if the absolute minimum value has been attained in every case, since it is not rare for a specimen to withstand a load for 20-30 seconds, and then suddenly to yield.

(b) Compression.

The same problems of ensuring axiality of loading and minimising stress concentrations apply to compression tests. As will be shewn later, the size of the specimen does not affect the tension results; there is consequently no reason to suppose that there will be a size effect in compression. It was therefore originally intended to test compression specimens 0.5644 inches in diameter only, this size being chosen to suit the testing machine. The method adopted was to machine the test piece to the same profile as in the tension tests, the ends being squared in the lathe and rubbed down on fine emery cloth on a surface table to ensure flatness. Each end was turned to $\frac{7}{8}$ inch diameter, and covered by a half ball made by grinding a $\frac{7}{8}$-inch hard steel [redacted] ball, approximate concentricity being ensured by a short length of tubing bored to a good push fit on both. Hard steel [redacted] rollers were then inserted in the compression plates of the testing machine, and a load greater than that required to test the specimen applied to a whole $\frac{7}{8}$-inch ball between the rollers. In this way two minute depressions were formed just sufficient to centre the specimen in the machine, with a radius of curvature larger than that of the half ball. The triple extensometer was used to measure the eccentricity of loading.

It was later found desirable to determine the strength of the material when subjected to a principal stress at right angles to the direction of rolling. Since the maximum length of a transverse specimen was limited by the size of the bar to less than 2 inches, the extensometer could not be accommodated on a tension test piece. It was therefore decided to use compression specimens of this size. The apparatus illustrated in Fig. 9 was available. It consists of a block of metal bored from end to end to take two plungers which have hardened inner ends and are fitted with half balls at the outer ends, to transmit the load applied by the compression plates of the testing machine. The specimen is inserted between the plungers in a 2 inch gap in the block. As the plungers are a good fit in the block, they serve to ensure that slackness in the head of the machine will not cause a large eccentricity in the loading. The method used for the tests described in the present paper was to insert the specimen, complete with its own half-ball ends and fitted with the triple extensometer, between the plungers. The readings of the extensometer (c.f. Fig. 17 curves (a) to (d).) shew that this gave satisfactory results.

In view, however, of the fact that other experimenters have used copper packing in compression tests, it is of interest to refer to a preliminary test in which, in order to avoid the practical difficulty of inserting the specimen with ball ends in the limited gap, pads of annealed copper were inserted between the flat ends of the specimen and of the plungers. The extensometer curves are shewn in Fig. 17 (e). It will be seen that the eccentricity of loading was very large, and that before the yield point of the steel was approached, the plastic flow of the copper caused the stress to be redistributed in such a way that it is quite useless to attempt to calculate the maximum value.

It will be appreciated that, in view of the emphasis which has in the present paper been laid on the technique of tension and compression tests, and the errors which arise through neglect of the precautions described, much earlier work is, in the opinion of the author, invalidated by this [redacted] consideration. It is therefore of importance, in order to justify this opinion, to shew how large these errors may be.

If in a tension or compression test the load P is eccentric by an amount "e", the maximum fibre stress s is given by

$$s = \frac{P}{A}\left(1 + \frac{ey}{K^2}\right)$$

where A is the cross-sectional area, k its radius of gyration, and y the distance from the centre to the outer fibre. The ratio of the true to the average (measured) stress is therefore $(1 + \frac{ey}{K^2}) : 1$, or for a solid circular specimen, $(1 + \frac{4e}{r}) : 1$. It is instructive to note the numerical value of the eccentricity which will entail an error of say, 5 per cent. The case of a few representative sizes is shown in the following table:-

TABLE 3.

Diameter of specimen	1	0.5	0.25	0.125
eccentricity of load	.0062	.0031	.0016	.0008

The danger of assuming without corroboration that axial loading has been attained is evident.

In the tests described in the present paper, the corrections ranged from 1.4 per cent to, in an extreme case, 12 per cent, and the errors which would, in the absence of the triple extensometer, have been recorded, existed in spite of the axial loading devices used. It cannot be assumed that other tests, in which the screwed ends of the specimens, for instance, may have been more accurately machined, were subject to such a large error, but the possibility cannot be ignored.

The frequent assumption that spherically seated shackles provide axial loading can easily be shown to be extremely dangerous, (c.f. the test cited above, where one side of a 1-inch tension specimen was actually put into compression at a low load). The load may be applied, as far as the shackles are concerned, through any point in the friction sphere. If for the sake of argument it is assumed that the radius of curvature of the spherical surfaces is ten times the radius of the specimen, and that the coefficient of greasy friction is 0.15, it will be seen that the shackles are capable of applying a load through an axis which lies right outside the specimen.

It is interesting to attempt to deduce the effect of different values of the eccentricity on the results of a tension test on a material which has a drop of stress at yield. If the eccentricity is very small, the stress distribution over the cross-section will be nearly uniform. When, therefore, a small highly stressed area yields, the increase in the load on the rest of the cross-section will be sufficient to raise the stress above the critical value. The whole area will then yield, and there will be a sudden decrease in the load which the specimen can sustain. As the eccentricity increases, so the average stress at which yield starts and the rapidity with which yielding spreads over the cross-section will decrease. This secondary effect of less sudden drop is distinctly noticeable even over the range of eccentricity found. With still larger values of the eccentricity, a point may be reached at which the stress increment due to the yielding of a small area will not be sufficient to cause yield in the remainder of the section. Two effects will result. The stress-strain line as ordinarily measured will begin to curve, and the specimen will bend into a position in which the eccentricity is reduced. With increase in load, the area which has yielded will spread slowly across the whole cross-section. The average stress, therefore, may never rise appreciably above that corresponding to plastic yield. The whole phenomenon of stress drop may be completely masked, the stress-strain line curving gradually into the horizontal plastic strain line.

There is therefore, a strong prima facie case for the contention that the measured upper yield stress in a test where insufficient attention has been paid to axiality of loading may have any value between that for the true upper yield stress and the lower or plastic yield stress, and that a fictitious elastic limit depending on the value of the eccentricity may be found.

(c) Flexure.

The apparatus used for flexure tests is shewn diagrammatically in Fig. 10. The objective in its design was the application to the test piece of a uniform bending moment without torque or axial constraint.

The test specimen S is held in a master specimen or weighbar M, which in turn is held in a stiff shaft A. A is mounted in two self-aligning ball bearings B_1, B_2. B_2 is free to roll on a flat plate, B_1 either on the plate or a parallel plate immediately above it. The bending moment is applied to S by means of the orthodox arrangement shewn. In order to accommodate the range of sizes required two sets of apparatus were constructed to the same general design.

The use of long steel tapes in the load applying device allows the specimen longitudinal freedom. If the ball-bearings used throughout were entirely frictionless a pure bending moment of exactly equal magnitude would be applied to S and to M. That B_1 and B_2 ~~are~~ were satisfactory was checked by giving a slight lateral displacement to the specimen, when the whole apparatus oscillated freely; in addition, the weight of one mirror when unbalanced was easily sufficient to turn A, M, and S before the loading beam was attached.

The friction in the pulley bearings was ascertained experimentally, and a correction applied to the results. Since this correction is less than 0.5 per cent, the apparatus was considered satisfactory. That this correction is of the correct order of magnitude was again checked when the master specimen was calibrated under increasing and decreasing load by dead weights.

In the flexure tests, the rate of strain is a matter of considerable importance. Until an outer fibre stress corresponding to the tensile yield stress is reached, conditions are exactly similar to those in a tension test. When, however, the bending moment is such that this value is exceeded, the stress in the outer fibre may correspond to the plastic yield stress in tension, and it has been shewn in the paper[14] already referred to that the value of this stress does not reach a minimum value even at the lowest (10^{-6} per second) rate of strain used. It is therefore to be expected that the specimen will show signs of "creep" at some time after the yield stress has been reached but considerably before the maximum value of the bending moment is attained. This was, in fact, the case. There is therefore some uncertainty as to the precise shape of the bending moment/strain curve. If straining is restarted before the creep is exhausted the minimum value of the bending moment will not have been reached; if the specimen is left too long, the outer fibres will have begun to "strain harden", and the subsequent curve may be distorted.

Two methods were adopted to overcome this difficulty. In the first series of flexure tests intermittent strain was used, and the periods of rest allowed were such that the creep was apparently almost exhausted. The last reading taken was assumed to be a point on the moment/strain curve. In the second, a very low rate of continuous strain (about 10^{-7} per second on the outer fibres) was adopted, and readings taken at frequent intervals over a period of some hours until the maximum value of the bending moment had been exceeded.

It will be seen from the curves that the results from the two methods are not widely different.

(d) Torsion.

The torsion machine used for specimens up to 0.3 inch diameter is illustrated in Fig. 11. It was designed by the late A. J. Newport, B.Sc., a colleague on the staff of the University of Bristol Engineering Department, and it was made in the departmental workshop. The torque is applied, by hand or motor, through a double reduction worm gear to the specimen under test and an interchangeable master specimen in tandem with thise test piece. The ends of the specimen are locked in cross pieces, to each of which the torque is transmitted through two balls which have lateral freedom and which are the only points of contact between the cross pieces and the machine. Since the specimen is free to move, only a pure torque can be applied. Mirrors on the test piece are used to measure the angle of twist, and mirrors on the master specimen, which was calibrated by dead weights, to measure the torque. Similar grips were fitted to an existing torsion testing machine for larger specimens.

With the material used, there is a large reduction in torque at yield. The conditions with regard to rate of strain are therefore, similar to those for tension tests. Strain was stopped while readings were being taken, and at very small increments of torque when yield was approached.

(e) Combined tension and torsion. X Insert sentence on p. 22

The testing machine[13] used is fitted with axial loading shackles of the type in which a hard steel ball rests on a hard steel plate, this being the only point of contact between the shackle and the rest of the machine. It is therefore evident that if the specimen under test is locked in the shackles and a torque applied between the shackles, the only reactions which can oppose this torque are those due to the torque in the specimen and the friction between the balls and plates. The only problem is therefore, to ensure that the frictional constraint is zero.

If it could be ensured that the connection between specimen and shackle be rigid, it would be sufficient to apply any desired torque before the axial load, since the friction couple depends on the reaction between ball and plate. In practice, however, the locking was effected by the means described in discussing the axiality of the tension load. When the axial load was applied, therefore, any minute axial movement between specimen and shackle might allow the friction couple to reduce the true torque on the specimen.

Two methods were adopted to ensure that this friction couple should not reach a value sufficient to invalidate the results. In the first place, preliminary tests were carried out in which (a) a torque was applied and then the axial load increased until yield took place; (b) the same torque was applied, an axial load of 95 per cent of the previous load was applied and removed, and again increased up to yield; and (c) the same torque was applied, and an axial load was applied and removed in increasingly large amounts, the increments being very small as yield was approached. The effect of this treatment may be considered as follows. If at the first application of the load the locking device tends to slip, a certain relative rotation between the specimen ends of the shackles will take place, and a frictional couple will be set up at the remote ends. When the load is removed, this frictional couple will cease to exist and the whole shackle will straighten up in a position more rigid than the original. This process will be repeated until the lock is so tight that the movement under load may be considered as negligible. The results are shewn (points X,Y,Z) in Fig. 14.

It will be seen that the repeated application and removal of the load does seem to be effective in eliminating the frictional couple. In the second place, a ball thrust bearing was inserted between the lower shackle and the straining head, so that the maximum frictional couple was definitely limited to that due to the friction of the race.

The triple extensometer referred to in the tension tests ~~were~~ was fitted to the specimen after the torque had been applied.

A diagrammatic sketch of the apparatus used to apply the torque is given in Fig. 12.

insert at foot of p. 20

RESULTS.

1. Tests in combined tension and torsion.

In an isotropic material, all possible ratios of the principal shear stresses can be obtained by subjecting a test piece to combined tension and torsion. The original object of the experiments was therefore, to apply to specimens of mild steel pure tension, pure torsion, and combined tension and torsion. The type of specimen chosen was .2822 inch in diameter, with screwed ends which could be used to apply the tension load or locked in adaptors to apply the torque. This size was selected for several reasons. It was suited both to a tension testing machine and to a torsion testing machine which were available, and with which a considerable amount of experience had already been gained. The specimens could be machined from the ½ inch diameter material, and sufficient could be accommodated in the vacuum furnace to carry out all the required tests in one or two batches, so that the effect of varying heat treatment could be eliminated.

If in a test in combined tension and torsion p is the tensile stress due to the applied direct load and q is the shear stress due to the applied torque the principal stresses are $\frac{p}{2} + \sqrt{(\frac{p}{2})^2 + q^2}$, o, and $\frac{p}{2} - \sqrt{(\frac{p}{2})^2 + q^2}$. If the criterion of yield is shear stress, yield should therefore occur when

$p^2 + 4q^2 = f^2$, where f is the maximum principal stress in pure tension. If the criterion is shear strain energy, yield should occur when $p^2 + 3q^2 = f^2$. In particular the ratio of the shear stress at yield in pure torsion to the principal stress at yield in pure tension should have the value $\frac{1}{2}$ or $\frac{1}{\sqrt{3}}$(0.577) respectively.

The first eleven tests were carried out in pure tension and pure torsion. The stress/strain and torque/angle curves are given in Fig. 13, and the results in table 4, (specimens A 1 - 5 and B 1 - 6). Fig. 13 brings out clearly the degree of constancy attained. Whilst the uncorrected tension results shew an overall variation of 11 per cent, after correction the greatest divergence from the mean is under 2 per cent, and the greatest divergence from the mean of the torsion results is less than 1 per cent.

In these eleven tests, the shear/tensile stress ratio was found to be 0.578 and 0.579 for the two heat treatment batches. The closeness of agreement between these experimental results and the second of the theoretical figures could hardly be regarded as other than fortuitous, but it seemed to point unmistakeably to shear strain energy as the criterion of failure. The programme was therefore completed with the remainder of the specimens. These results also are given in table 4, (specimens A 6 - 8 and B 7 - 9) and the stress/strain curves for the application of the direct load in Fig. 13.

The results of all these tests (along with two preliminary tests, points X and Y, already referred to) have been plotted on one diagram, Fig. 14, on which the ellipses $p^2 + 4q^2 = f^2$ and $p^2 + 3q^2 = f^2$ have also been drawn. The agreement between the results and the predictions of the shear strain energy hypothesis is remarkably good.

It was therefore decided to repeat these experiments on material in the "as received" condition; that is, without any heat treatment. The specimens were cut from one end of a new bar, and numbered consecutively. As will be seen from the tabulated results (table 5) the end specimen gave a result which was much

TABLE 4.

Heat Treatment	Specimen number	Type of Test	Stresses at Yield. Tensile (uncorrected) tons per sq.in.	Tensile (corrected) tons per sq.in.	Shear tons per sq. in.	p/f	q/f
A	1	tension	21.84	23.04	-	1.018	-
	2		21.00	22.38	-	0.989	-
	3		22.00	22.47	-	0.993	-
	4	torsion	-	-	13.02	-	.575
	5		-	-	13.14	-	.580
	6	Combined tension and torsion	16.90	17.91	8.15	.791	.360
	7		8.56	8.96	12.16	.396	.537
	8		12.40	13.33	10.56	.589	.467
B	1	tension	20.40	22.80	-	.993	-
	2		21.84	22.72	-	.989	-
	3		22.64	23.40	-	1.019	-
	4	torsion	-	-	13.40	-	.583
	5		-	-	13.26	-	.577
	6		-	-	13.25	-	.576
	7	Combined tension and torsion	4.47	4.75	12.97	.207	.565
	8		19.90	20.91	4.95	.910	.216
	9		19.90	21.32	4.95	.928	.216

f = mean tensile yield stress (heat treatment A) = 22.63 tons per square inch.

f = mean tensile yield stress (heat treatment B) = 22.97 tons per square inch.

TABLE 5.

Heat Treatment	Specimen number	Type of Test	Stresses at Yield.			p/f	q/f
			Tensile (uncorrected) tons per sq.in.	Tensile (corrected) tons per sq.in.	Shear tons per sq. in.		
None	1	Tension	26.70	27.80	-		-
	5		27.50	29.28	-	0.972	-
	9		29.60	30.94	-	1.025	-
	10		28.36	30.37	-	1.006	-
	2	Torsion	-	-	17.5	-	.580
	4		-	-	17.3	-	.574
	7	Combined tension and torsion	17.50	18.03	13.92	.598	.462
	8		11.25	11.64	15.95	.386	.529
	11		26.69	27.70	6.63	.918	.220
	12		23.39	24.15	10.68	.800	.354

f = mean tensile yield stress,

(neglecting specimen no. 1) = 30.20 tons per square inch.

too low compared with any of the others. It has therefore been neglected in drawing the curve shewn in Fig. 15. The other points once again lie on a curve which is indistinguishable from that predicted by the shear strain energy hypothesis.

2. Size effect in torsion: First series of experiments.

Two facts, however, gave rise to doubt as to whether these results could be accepted at their face value. Firstly, in some early experiments which are not included in this paper, it had been found that there was some discrepancy between the values of the shear stress at yield in pure torsion as given by .2822 inch and smaller specimens. At the time this discrepancy had been attributed to differences in heat treatment, either surface decarburisation or unequal rates of cooling, or surface cold-working due to machining, since the good vacuum furnace was not then available. Secondly, in describing tests on thick tubes under internal pressure, Cook has shewn that a size effect undoubtedly exists, and suggests that it should also appear in torsion and flexure.

It was, therefore, decided to investigate this possible size effect. The ½-inch bars which had been used for the tests in combined tension and torsion were obviously not suitable for such experiments. Two 2-inch bars of the same steel were, however, available. As a preliminary experiment to check the uniformity of the material three .125-inch torsion specimens were cut from the core, mean radius, and outside of a normalised part of the bar, three .282-inch specimens, one from the core and two from the outside, and one 1-inch specimen from the middle of the bar. The .125-inch specimens gave a shear stress at yield of 14.2, 14.5 and 14.3 tons per square inch respectively, the .282-inch specimens 13.55, and 13.4 and 13.9 tons per square inch respectively, and the 1-inch specimen 12.14 tons per square inch. It was therefore concluded that the material was satisfactorily uniform.

In order to make certain that the size effect indicated by these results was due neither to rate of cooling (in the earlier experiments) nor to surface cold working, two further experiments were made. In the first, five of the above specimens were placed in the furnace and annealed. Since the time taken for the temperature to fall from 900°C to 600°C was $2\frac{1}{2}$ hours, the mass of the specimen could hardly affect the rate of cooling. The results obtained on retesting were:-

TABLE 6.

Diameter, ins.	1	:282	.125
q, tons/in^2	10.78	11.33 11.22	12.20 12.22

In the second, six new specimens, three tension and three torsion, were heat treated in the standard way. The results are shewn in the table below:-

TABLE 7.

diameter, ins.	Tension f tons/in.^{2}	Torsion q tons/in.^{2}	$\frac{q}{f}$ mean.
1	22.94	12.05	0.527
.564	22.93	12.39	0.542
.282	22.65	13.07	0.572
Mean	22.84	-	-

These experiments indicated not only that the size effect in torsion was genuine, but also that the apparent confirmation of the shear strain energy theory in the combined stress experiments was entirely fortuitous, and due to the size of specimen chosen.

It is, however, obvious that none of the accepted theories of failure can account, in a material which obeys Hook's law, for a size effect in torsion. It therefore follows, if the above results be accepted, that either an entirely new hypothesis must be formulated, or some modification made in an existing one.

Two directions in which an explanation may be sought are:-

(1) If the surface and interior of a specimen differ, then, since the shear stress in torsion varies from zero at the centre to a maximum at the surface, whereas the stress in tension is (approximately) uniform over a cross section, the simple

relationship between tension and torsion predicted by any accepted theory will be upset. Moreover, the ~~balance~~ ratio between the torque borne by the surface and interior will be affected by the size of the specimen.[10]

(2) If material at a lower stress in close proximity to a part which is on the point of yielding can in any way support the latter, then for the same reasons as in (1), a simple theory of yield will require modification, and a size effect may be expected.

A further element of uncertainty which exists when, for instance, torsion tests are compared with tension tests, arises from the fact that the planes of maximum shear stress are not the same. The ratio of the shear stress at yield in torsion to the tensile yield stress will therefore be affected by any departure from isotropy in the material.

The subsequent experiments were therefore directed to an investigation of these points. It seemed that a series of flexure tests might throw some light on the size effect in torsion. Since, however, flexure introduces a compressive stress, it is essential to confirm that yield under tension and compression are in fact identical. The balance of evidence from accurate published work supports such an assumption, but there is a certain weight of evidence which opposes it. It has already been remarked that longitudinal specimens only could be tested in tension. With the introduction of compression tests, however, it became possible, by testing specimens cut at right angles to the axis of the bar, to check directly the degree of isotropy of the material.

3. Comparison between tests in tension and compression.

Three tension specimens and three compression specimens, 0.5644 inch in diameter, were made, and heat treated together. They were cut from adjacent parts of the bar and machined to the same profile, so that apart from the enlarged ends they were identical in every respect. The results are given in table 8, and the stress/strain curves in Fig. 16. The mean yield stress in compression was 1.3 per cent less than the mean yield stress in

TABLE 8.

Type of Test	Average stress at yield, tons per sq. in.	Correction for eccentricity per cent.	Upper Yield stress, tons per sq. in.	Plastic Yield stress, tons per sq. in.	E, tons per sq.in.
Tension	22.08	5.88	23.38	< 16.0	13,560
"	21.50	7.12	23.03	15.2	13,500
"	22.21	4.52	23.25	-	13,500
Mean			23.22		13,520
Compression	22.78	1.76	23.18	16.0	13,600
"	21.62	5.37	22.78	-	13,560
"	22.32	1.69	22.70	15.0	13,620
Mean			22.90		13,590

tension, a variation which is within the limits of experimental error, since the variation between the individual specimens and the mean was 0.8 per cent for tension and 1.2 per cent for compression. The values of the modulus of elasticity and the lower or plastic yield stress were also identical. It may be noted that the testing machine used in this instance was not ideal for measuring the plastic yield stress, since it was not of the type in which the load is automatically relieved. The least load at which creep on the extensometers could be seen was taken as indicating plastic conditions.

4. Determination of degree of isotropy of material by means of longitudinal and transverse compression tests.

The size of the transverse specimens was limited by the diameter of the bar. It was therefore difficult to obtain a satisfactory transition curve for the specimen profile while providing a parallel part long enough to accommodate the extensometer, and a compromise was effected by machining the specimens to a diameter of 0.35 inch with a parallel part $\frac{3}{4}$ inch in length. The ends of the extensometer therefore, rested on the specimen at a section where the diameter was five thousandths of an inch greater than the minimum. In order to eliminate any differences arising from this shape of specimen, two transverse specimens and two identical longitudinal specimens were made and heat treated together.

The stress/strain diagrams from these tests are shewn in Fig. 17, curves (a) to (d), and the results are given in table 9. The two longitudinal specimens gave values for the yield stress which agree with the previous values obtained, and the transverse specimens gave values differing by approximately $\pm$ 1 per cent from the same mean. As this is well within the limits of experimental error, it may be concluded that the material is, for the present purpose, isotropic. The plastic yield stress measured was slightly higher for the longitudinal specimens than for the transverse specimens, but as stated in discussing the compression tests, the method of measuring this stress is not above suspicion.

5. Flexure tests: Size effect in flexure.

Six flexure specimens, ranging in diameter from 0.5 inch to 0.1 inch, were made, and heat treated along with other specimens to be described later, and two tension specimens to check the heat treatment. The largest of these flexure specimens was tested first with intermittent loading, and the test was stopped when the first deviation from straightness (at an apparent stress 3.3 per cent greater than the tensile yield stress) of the moment/angle curve was found. The smaller specimens gave a less abrupt departure from the straight moment/angle curve, and the tests were carried further, though not, in all cases, to the stage at which bending continued at a reduced value of the moment. The first test has therefore not been included in the results. The curves for the other tests have been drawn in Fig. 18. It will be seen that in general there is no sudden change in the gradient of the curves as the moment is increased; it is therefore impossible to tabulate the value of the stress at which yield occurred, but it is apparent that the shape of the curves is affected by the size of the specimen.

The difficulty experienced with regard to rate of strain in flexure has already been discussed. A second batch of flexure specimens with check tension specimens was therefore made and heat treated later. All the tests in this second series were made under conditions of continuous strain, and continued for some time after the maximum value of the bending moment had been attained. The curves have been plotted on the same diagram (Fig. 18) and it will be seen that they agree satisfactorily with those for intermittent loading.

6. Direct investigation of surface layer: tests on thin tubes.

If the size effect is due to a surface layer which differs from the interior of a specimen, it should be possible (since, as will be pointed out, the layer must have a thickness of at least five thousandths of an inch to account for the observed facts) to discover the properties of this layer by tests on thin tubes. Two such tubes, therefore, were made, $1\frac{1}{8}$ inch in internal diameter, and .006 and .015 inch in thickness respectively.

They were heat treated along with the flexure specimens and the check tension specimens, and tested in tension. The stress/strain curves are shewn in Fig. 16. The thinner of these tubes was eccentric, the wall thickness varying from .00530 inch to .00835 inch, with the result that the load was applied along an axis which did not coincide with the centroid of the cross-section. The readings of the extensometer units therefore varied widely, and owing to the nature of the mean stress/strain curve, no satisfactory correction for the yield stress can be made. If the value of the eccentricity below a mean stress of 10 tons per square inch be assumed to be constant, the calculated yield stress is 16.9 tons per square inch. This, however, overestimates considerably the true stress, since the stress strain relationship for the material is far from linear. It is safe only to give the extreme limits for the yield stress as 13.5 and 16.9 tons per square inch, as compared with 22.7 tons per square inch for the solid material. The thicker of the tubes was much more nearly concentric, and the yield stress found, which was not subject to the same ambiguity, was 19.8 tons per square inch.

It was thought that these figures might be due to decarburisation of the material. The thinner tube was therefore sectioned and examined. The result of this examination is shewn in the microphotographs in Fig. 4, and has already been discussed. There is no indication of decarburisation. Further discussion of these results may be deferred until the next section of the paper.

7. Size effect in torsion: further experiments.

Six torsion specimens varying in size from 0.5 to 0.1 inch in diameter were heat treated along with the flexure specimens, tubes, and tension specimens. This range of sizes repeats part of that previously covered, and extends it in the direction of smaller specimens. The torque/angle curves for all these torsion specimens are shewn in Fig. 19, and the results are given in table 10. The results for all the torsion tests have also been plotted on one diagram (Fig.20) which shews the variation of the ratio of the shear stress at yield in torsion to to the principal stress at yield in tension.

TABLE 10.

f = tensile yield stress (from check specimens)
= 22.70 tons per sq. in.

Diameter of specimen, ins.	q = apparent shear stress at yield tons per sq. in.	q/f
0.1018	13.83	0.609
0.1091	13.80	0.608
0.1215	13.80	0.608
0.1280	13.78	0.607
0.2782	12.96	0.571
0.4729	12.36	0.545

DISCUSSION OF RESULTS.

The compression tests confirm that the elastic and yield phenomena in tension and compression are identical. It therefore follows that when discussing flexure tests no special consideration need be given to the side of the specimen which is in compression. Further, it is evident that the normal stress on the planes of maximum shear has no measurable effect on the yield stress. The absence of this normal stress in a torsion test cannot therefore be adduced to account for any part of the increase in shear stress at yield in torsion as compared with tension.

The results of the transverse tests remove any uncertainties regarding the effective isotropy of the material.

The tests on thin tubes are of considerable interest. The yield stress attained by the thinner tube is little more than half that of a solid specimen, and the character of the yield is entirely different. It corresponds, in fact, more nearly to that which would be expected from a single-crystal specimen of pure iron. From the microphotograph (Fig. 4) it will be seen that the wall thickness and the crystal size are of the same order of magnitude, and that lines of shear stress can easily be drawn through the few ferrite crystals in the wall without encountering more than a very small proportion of pearlite. It seems therefore not unreasonable that the strain phenomena should be of this character. Further, this test offers strong confirmatory evidence that the drop in stress at yield in polycrystalline iron or steel is intimately associated with the interactions of the crystals. This point will be discussed more fully in connection with the flexure tests. The thicker of the tubes, as might be expected, gave a result intermediate between that of the thin tube and a solid specimen. Not only is the stress-strain diagram much more nearly straight, but also the characteristic drop of stress at yield is beginning to appear, and the plastic yield stress is much higher. These results may be considered to throw some light on the behaviour of the .1 inch diameter tension specimens. It has already been remarked that, as distinct from larger tension specimens, they tended to give a stress-strain curve on first loading which could not, in subsequent loadings, be repeated. The immediate conclusion to be drawn from these tests is that the skin, isolated from the body of the material, is certainly no better able to withstand stresses than the interior. It may be argued that when supported by the interior it may possess different properties, but there would seem to be no justification for such a hypothesis.

The flexure tests shew that the size of the specimen affects not only the value of the stress at which yield starts, but also the shape of the moment / angle curves after yield. If the skin material were to behave as in the test on the thin tube, the stress distribution would be affected by the curvature of the stress-strain line. The moment-angle lines would therefore not be perfectly straight, and the curvature would increase as the specimen diameter was reduced. This effect is, as will be seen, just noticeable. But such a cause could not account for the increase in apparent yield stress with reduction in specimen size; on the contrary, the apparent stress would be reduced. The explanation must therefore lie in the fact that understressed material is present. Nor, on consideration, does this seem unreasonable. In order to present the argument fully it will be necessary to describe in some detail what the author believes to be the mechanism of yield.

In the case of a specimen which consists of a single crystal of iron, it is known that slip manifests itself at a low value of the stress, and that yield takes place when the "resolved shear stress" (that is, the shear stress resolved along any plane through one of the possible slip directions) reaches a critical value. The apparent yield stress in a tension test will therefore depend on the crystal orientation. There is no obvious reason for a sudden large diminution in the stress required to sustain slip after it has begun, nor, in fact, is such a diminution found. Considering now the case of a test on two single-crystal specimens in parallel, which are independent except in so far that the strains are equal, it will be seen that the stresses will be divided between the crystals (in some ratio depending on their elastic moduli) until one of them reaches yield. Any further increase in load will increase the stress in the second crystal only; yield of the two crystals will therefore be reached at some stress value intermediate between those which would be found for the crystals separately. Again no drop in stress at yield will be found.

If however, the two crystals oriented at random together form a single specimen, they may behave quite differently. *no paragraph* Up to the point at which the slip in each crystal is less than the interatomic distance, little interference between the crystals need be expected. When the value of the load at which yield of the separate crystals would occur has been attained, each crystal is "willing" to slip in one or more of a few strictly limited directions, which will not in general be the same for the two crystals. It may be postulated that the two crystals cannot separate. Slip in the crystals must therefore be accompanied by some atomic readjustment in the neighbourhood of the crystal boundary. The type of readjustment cannot be predicted from this argument; it might for instance involve only a severe distortion of the lattice, or a complicated sliding of fragments of the crystals, or a dislocation and rotation of fragments of the crystals, or even a "melting" of the edge of the crystals, in the sense that some atoms might be so severely distorted in the lattice that they could no longer be considered as belonging to it, but rather as constituting material in an amorphous state. But it is clear that the forces required to initiate any such readjustment will be greater than those required to cause slip in the separate crystals. In the average test specimen the cross-section contains many thousands of crystals, and at the boundary of each some such readjustment must occur before yield of the specimen as a whole occurs. It seems to the author evident that the force to initiate yield will be higher than the sum of the forces required to initiate yield in the separate crystals.

The question discussed above has been investigated experimentally by Gough and Wood[15] by means of X-ray diffraction, and the results may be considered to supply the evidence required to particularise the nature of the readjustment. In a static tensile test on normalised mild steel, Gough and Wood found that up to the limit of proportionality no change occurred in the crystal structure. Between the elastic limit and the yield point an occasional grain was found to have been dislocated, while a small amount of "crystallites" (i.e. crystals of size 10^{-4} to 10^{-5} cm) had been produced. At the yield point an abrupt change was shewn. Every crystal was dislocated, and the production of crystallites was greatly increased. At fracture the specimen consisted of randomly oriented crystallites, each of which had suffered distortion of the crystal lattice. The author therefore accepts the view that yielding of polycrystalline iron or steel consists in internal slipping of the crystals with consequent production of crystallites mainly in the neighbourhood of the original crystal boundaries.

The further yielding of the material remains to be considered. If the material is originally in the normalised condition it may be assumed that the crystals are initially unstressed, and that when the breakdown at yield is approached the stress distribution throughout a cross-section, though not the resolved shear stress on each crystal, is reasonably uniform. It is difficult to conceive of a movement at yield which will leave the stress distribution unaltered; it is therefore probable that immediately the first considerable slip takes place the stress on some of the crystals will be greatly increased. Moreover, it seems probable that the crystals will be in a condition of internal strain, that is, some of the atoms in a given plane in the lattice may have taken a "glide jump", leaving the remaining atoms in the plane in an unstable condition. For both reasons yielding may continue under a load considerably smaller than that necessary to initiate yield.

As yielding progresses the structure of the material will degenerate further and the proportion of randomly oriented crystallites to the material arranged in a regular lattice formation will increase. Slipping over the planes of what is left of the original crystals will therefore be more and more restricted and the force required to cause further slipping will increase until the material finally reaches the stage of containing crystallites only. It is then virtually in the state of cold worked material. The ultimate breaking stress, calculated on the final area, should therefore be similar whatever the original condition of the material. Although this deduction is not strictly related to the yielding of mild steel it is one which admits of verification. Accordingly six previous tests were used, and four new specimens were made, specifically to check this conclusion. The previous tests are those detailed in the normal manner in table 2, together with two specimens, one ~~normalised,~~ heat treated in vacuo and one as received, which had been stressed to a stage beyond yield in combined torsion and tension under a small direct load and a large torque. For the new tests, firstly, a bar of the same material in the "as received" condition was pulled in a testing machine until it fractured. From the overstrained parts two pieces were cut of sufficient length to provide .282 inch diameter specimens, and of these pieces one only was normalised before machining. Secondly, two similar lengths were cut from a bar of bright-rolled mild steel, and of these one also was normalised before machining. In this way ten tests altogether were made on material in a variety of states of overstrain. The autographic diagrams from these tests are reproduced in Fig. 3, and the true stresses are given in table 11. If the very wide divergencies shewn in the autographic diagrams are considered together with the comparatively trifling differences

TABLE 11.

Material	Treatment	At Maximum Load			At Break.		
		Load tons	Dia. ins.	Stress tons/in.	Load tons	Dia. ins.	Stress tons/in
'Park Gate' 0.2% C steel	as received	1.88	.250	38.2	1.35	.163	64.5
	as received	1.885	.250	38.3	1.35	.163	64.5
	normalised	1.89	.250	38.4	1.35	.163	64.5
	normalised	1.90	.250	38.6	1.35	.163	64.5
	as rec'd. overstrained in combined ten. and torsion	2.13	.257	40.9	1.415	.167	64.5
	heat treated in vacuo. overstrained in combined tension and torsion.	1.86	.255	36.4	1.36	.165	63.5
	as received, overstrained in tension.	2.46	.281	39.6	1.73	.183	65.7
	as received, overstrained in tension, normalised.	1.90	.2405	41.8	1.415	.168	63.7
'Bright Rolled' mild steel	as received	2.46	.280	39.8	1.63	.174	68.5
	normalised	1.80	.247	37.5	1.23	.155	65.0

in the stresses, it will be seen that the deduction is satisfactorily near the truth.

In the case of a flexure test the part of the above argument which refers to the early stages of yielding will apply to conditions at the outer fibre. Since, however, yield cannot occur in an individual crystal surrounded by unyielding material but only in a number of crystals which occupy a sufficient thickness to permit of the complicated readjustment which must take place before movement can occur, it is unreasonable to expect to find yield before a stress equal to the yield stress in uniform tension is applied to a depth of this magnitude. The depth cannot, of course, be estimated from an argument as rough and ready as the above, but it seems reasonable to suppose that it might be of the order of a few crystal diameters.

The exact nature of the stress distribution in this layer of the material is uncertain. Two conditions which may be considered to represent the limiting cases have been considered, the first in which the stress varies as the distance from the neutral axis, reaching a value in excess of the tensile yield stress before the stress drop occurs, the second in which the stress throughout the layer is uniform and equal to the yield stress. It is probable that the true distribution is intermediate between these assumptions.

The dotted curves in Fig. 18 have actually been drawn on the second assumption, that the stress distribution is that shewn in Fig. 29, that is, that a layer of thickness .005 inch immediately beyond the elastic core is able to withstand the yield stress only, the stress outside this layer falling to the plastic yield stress. The figure .005 inch, about three or four crystal diameters, was chosen to suit the experimental curves. If the first assumption is made, a slightly thinner layer gives curves which are indistinguishable from those shewn.

Until a certain stage in the bending is reached the calculated moment has a continuously increasing value; the whole length of the specimen should therefore take part in the bending.

Beyond that limit it will be apparent that yield will be self-propagating through the core of the material. The bending moment should therefore fall to the value corresponding to that for a specimen wholly in the plastically overstrained condition. The agreement between the calculated and experimental curves is remarkable, both as regards magnitude and shape.

If now the torsion results are examined in the light of the flexure tests, it will be seen that part at least of the size effect is directly explicable on similar grounds. The stress distribution at yield will be similar to that in the early stages of flexure, a layer of material which can withstand the yield stress surrounding the elastic core. It is unnecessary to consider the later stage when the stress in the outer fibres is reduced, since the torque which the specimen can sustain is then also reduced, yield spreading locally through the core. If, therefore, when yield is reached, r_1 is the radius of the elastic core, R being the radius of the specimen, the torque T required to cause yield is given by

$$T = \int_0^{r_1} 2\pi r\,dr\,\frac{r}{r_1} q_c r + \int_{r_1}^{R} 2\pi r\,dr\,q_c r = \frac{\pi}{6}\left[4R^3 - r_1^3\right] q_c$$

where q_c is the shear yield stress. The value of $\frac{q_a}{f}$, where q_a is the apparent shear stress derived from the usual formula $T = \frac{\pi}{2}R^3 q_a$, has already been found experimentally for a range of specimens of diameters 0.1 to 1 inch. It follows that the shear/tensile stress ratio when allowance is made for the reinforcing effect of the understressed material is given by $\frac{q_c}{f} = \frac{q_a}{f} \cdot \frac{q_c}{q_a}$

$$\text{or} \quad \frac{q_c}{f} = \frac{q_a}{f} \cdot \frac{6\pi R^3}{2\pi\left[4R^3 - r_1^3\right]} = \frac{q_a}{f} \cdot \frac{3}{4 - \left(\frac{r_1}{R}\right)^3}$$

While there is little justification for assuming that exactly the same thickness "t" of highly stressed material as in flexure is required before yield can take place, it is probable that "t" will be of the same order of magnitude. The values of $\frac{q_c}{f}$

have therefore been calculated on the assumption that "t" is the same, .005 inch, as in flexure, and also, for comparison .010 inch. These values, along with those for $\frac{q_a}{f}$, are shewn for a few representative sizes of specimens in Table 12.

It will be seen that the size

TABLE 12.

diameter of specimen, inches	1	0.5	0.25	0.125	0.100
$\frac{q_a}{f}$ from experimental curve, Fig.20.	.527	.545	.577	.604	.613
$\frac{q_a}{f}$ assuming t = .005 inch	.522	.535	.555	.562	.562
$\frac{q_a}{f}$ assuming t = .010 inch	.517	.525	.537	.532	.527

effect could be entirely attributed to the argument detailed above on the assumption that "t" had the value .010 inch, since the residual variation in q_c/f cannot be said to lie outside the limits of experimental error. On the other hand, if .005 inch is taken as a more probable value of "t", an appreciable size effect remains to be explained. In either case the deduced value of q_c/f is appreciably greater than 0.5, the figure predicted by a straightforward application of the shear stress theory.

There remains to be considered a possible reason for such a discrepancy, namely, the effect of random orientation of the crystals on the resolved shear stresses induced by torsion and by tension. The question has been investigated in detail by Cox and Sopwith[16] for the case in which the crystals may be individually considered, that is, where the interactions of the crystals which are willing to slip may be neglected, and it seems probable that such interactions would not invalidate the argument. The conclusion reached in this investigation is that "if elastic failure of polycrystalline specimens is determined entirely by the maximum shear stress developed locally, the ratio of the yield in torsion to the yield in tension should be 0.500, whereas if elastic failure is determined by the complete yield of all the crystals of the specimen, the value of this ratio should be about 0.577". It is further shewn that intermediate values of this ratio are to be expected if yield of a torsion specimen depends on yield of all the crystals in the surface. While the values calculated are based on assumptions which may not be in strict correlation with actual conditions in the material, they offer a reasonable explanation of the ratios found in the present experiments. It may be argued further that since, according to the theory of yield advanced above, the ratio of the amount of material involved in the breakdown of a torsion specimen to the whole material increases as the specimen diameter is reduced, some size effect is to be expected from this cause also. That is to say, if the values for q_c/f in table 11 deduced directly from consideration of the flexure tests be accepted, they may be entirely accounted for by the effects of random crystal orientation.

CORRELATION OF OTHER RESEARCHES.

Little need be added, with reference to the early experimental work during the years 1900 to 1909, to the remarks made in the introduction. The tests were in general made on large specimens in which the size effect noted in the present paper would not be apparent. It should, however, be pointed out that Smith was the first investigator to adopt the technique forced upon the author of accepting and measuring the inevitable eccentricity in a tension or compression test. For this reason his direct stresses (tension and compression) were higher than for instance, Guest's, in relation to his pure torsion results, and he was led to the conclusion that the normal stress on the plane of maximum shear was without effect.

A number of investigators have found that the yield stress determined by the failure of a beam in flexure is considerably higher than that given by the material in a tension test. The discrepancy in the case of mild steel was first pointed out by Kennedy.[17] Kennedy's tests have been analysed by Muir and Binnie[18] who have given an explanation of the results by shewing that for a material which has no drop in stress at yield the value of the bending moment corresponding to a large deflection of a beam is much greater than that at which the yield stress is attained on the outer fibres, and that the experimental moment/angle curves for such a material can be predicted by a calculation similar to that detailed in the appendix 2. Muir's argument is based on the assumption that wedges of plastically overstrained material extend from the surface to the elastic core, the rest of the outer fibres being still in the elastic condition, but this assumption does not affect the calculation, since the stress in these fibres is not reduced.

Flexure tests by Muir and Binnie giving similar results have been used to support their contention that the drop in stress at yield in a tension test is fictitious, and that the large reduction in the total load required to increase the extension of a test piece after yielding has begun is entirely due to the piecemeal character of the stretching. The author cannot agree with this interpretation. In order to make the argument clear two diagrams, Figs. 21 and 22 are given. The first is a reproduction of a diagram given in the paper cited, the second shews the author's flexure tests on the two largest specimens, together with certain calculated moment/angle curves. The calculated curves represent, (A) that predicted by the tension and compression test data allowing for the reinforcement due to the understressed material as detailed in appendix 2, a refinement which makes very little difference on this size of specimen: if this reinforcement is neglected the calculated curve A coincides almost perfectly with the lower of the experimental curves; (B) that for a material which yields at the same stress but has no stress drop, (C) that for a material which yields without stress drop at the value found for the lower or plastic yield stress, and (D) that for a material which yields without stress drop at a value of 0.75 times the yield stress found. Inspection of the second diagram makes clear that curve A represents the experimental facts much more closely than any of the other curves, and that curve B would give a value of the bending moment at large curvatures far above that actually attained.

It may also be seen that of the other curves D gives an approximation to the experimental curve which is reasonably good, but which exhibits divergences which are not without significance. The experimental curve rises above D in the early stages of the yielding, and later falls below it. The stress drop found for the steel used by the author is perhaps unusually large (33 per cent of the yield stress); with a smaller value of the drop the difference in the shape of the curves would be less noticeable.

The same characteristics are to be seen in the curves in Fig. 21. It therefore seems (to the author) possible that the tensile yield stress found by Muir and Binnie was lower than the true yield stress, and that the neglected stress drop is responsible for bringing the experimental curve so close to the calculated curve.

A further point of some interest may be discussed here. Muir and Binnie lay considerable stress on the fact that yield in a tension test is of a piecemeal character, and that in a flexure test wedges of plastically overstrained material penetrate the beam from the outer fibres. These facts are advanced to refute the stress drop theory. It seems to the author, on the other hand, that they provide a strong argument in its favour. The phenomena associated with a tension test have already been discussed; it need only be added that a drop of stress associated with yield will obviously tend to localise its action, provided that the extension of the specimen causes the load to be reduced. This is always the case, since in the absence of special means to keep the load in balance, the beam of the testing machine drops suddenly until supported by the frame of the machine.

The case of a flexure test may now be considered. As the bending moment increases the curvature increases in direct proportion until, as has been described, the yield stress has been applied to a depth of a few crystal diameters measured from the outer surface. The outer surface is now able to yield, and the curvature increases more rapidly than before in relation to the bending moment. This is shewn by the curving of the moment/angle line. A wedge of overstrained material will not yet, however, penetrate deeply, since even when the stress drop is taken into account such a penetration would correspond to a higher value of the bending moment than has yet been applied. As the bending moment increases further, the rate of change of curvature increases still more rapidly and the beam soon reaches the condition at which increase in curvature is accompanied by no increased resistance to the applied bending moment. At this point, therefore, a wedge of overstrained material will penetrate to a considerable depth, and the moment which the beam can resist will fall, with a rapidity which depends on the change in the plastic yield stress with rate of strain, and on the difference between the maximum bending moment which has been applied and that corresponding to the beam when wholly in the plastically overstrained condition, to the latter value of the bending moment.

The reliance placed in the foregoing description, in the absence of tests designed to investigate this point, must rest on the evidence of the moment/angle curves. In the author's opinion, the series which is shewn in Fig. 18 provides that evidence as conclusively as could be expected. The maximum divergence between the calculated and experimental values of the bending moment for any value of the curvature is of the order of 3 per cent; moreover, the shape of the curves and the value of the maximum bending moment before it falls to that corresponding to plastic conditions give additional corroboration. In this connection it should perhaps be restated that the depth of material which, it is assumed, is necessary to allow yielding, was chosen to suit the experimental curves.

It should be noted that the curve reproduced from Muir and Binnie's paper refers to a rectangular specimen, the author's to circular specimens, in which the propagation of the wedges would be slightly more delayed.

Seeley and Putnam[7] carried out a large number of tests on six grades of steel in tension, compression and torsion. Among their conclusions the following may be noted. "The correct value of the elastic shearing strength of steel as measured by the proportional limit or the useful limit point is from fifty-five to sixty-five hundredths (0.55 to 0.65) of the elastic tensile strength and may be taken with reasonable accuracy as six-tenths (0.6) of the elastic tensile strength. The maximum shear theory of the failure of elastic action of ductile steel (sometimes called Guest's law) is, therefore, not an accurate statement of the law of elastic breakdown, since Guest's law assumes that the elastic shearing strength is one-half (0.5) of the elastic tensile strength. The maximum shear theory as expressed by Guest's law, however, is of much use in obtaining approximate results". Since, however, spherically seated shackles were relied upon to provide axial loading, and the form of tension specimen was not one to avoid stress concentration, this conclusion can hardly be accepted.

The conclusions of M. Ros and A. Eichinger[8] have already been briefly mentioned. These were based on tests (in tension, compression, torsion, internal pressure, and a combination of these) on tubes 0.1 inch thick "durch Kaltbearbeitung hergestellt". The results are in very good agreement with the shear strain energy theory. But again the values obtained for the yield stress in tension and compression are open to doubt. In the first place, no attempt seems to have been made to ensure axiality of load. In the second place, the tension specimens had a solid end, and a 1 inch diameter bore 10 inches in length. It may therefore be doubted, especially as no special machining methods are mentioned, whether the tubes themselves were truly concentric. In the third place, the comparatively massive ends of the specimens were joined to the tubular test piece by a radius of 0.1 inch. That such a sudden change of section would cause a grave stress concentration cannot be doubted, and the effect would be more serious in tension or compression than in the other modes of testing. For those reasons it is the opinion of the author that the results cannot be accepted as conclusive proof that failure at yield is in accordance with the shear strain energy theory.

The work of Professor Cook[10] has already been referred to in connection with the effect of a surface layer which differs in properties from the interior of the material. In the paper which suggests this possible explanation of the differences between the yield stresses in uniform and non-uniform stress distribution the results of tests in tension, torsion, flexure, and internal pressure are recorded.

The first point which must be noted is that tension tests were carried out on specimens heat treated in vacuo cut in the longitudinal and transverse directions from $1\frac{3}{8}$ inch diameter bars. Axial loading shackles were, of course, used, and the parallel part of the specimens was joined to the screwed ends by a fillet of radius approximately three times the diameter. The results are summarized in the table below:-

TABLE 13.

Steel	Direction	Upper Yield Stress.	Plastic Yield Stress.	Ratio upper/ plastic Yield Stress.
A (.13%C)	Longitudinal Transverse	20.8 19.5	15.0 15.3	0.72 0.78
B (.13%C)	Longitudinal Transverse	25.3 22.3	17.1 15.0	0.67 0.67
C (.21%C)	Longitudinal Transverse	21.6 18.9	15.5 14.8	0.72 0.78

It will be seen that in every case the transverse specimens give a smaller value for the upper yield stress. On the other hand, the author's tests to investigate the same point shewed no difference between the strength in the two directions. Four possible explanations for this discrepancy are:

(1) The steels used by Cook may have had a more "banded" structure than those used by the author. That is, the results may be accepted at their face value.

(2) The longitudinal specimens were taken from the material between the centre and surface of the bar. The transverse specimens on the other hand, necessarily included material from the centre. Any segregation in the bar might cause a difference between the results.

(3) The difficulties experienced in avoiding surface decarburisation during heat treatment have been discussed in an early part of this paper. The high speed vacuum pumps which the author found necessary were not available when Cook's experiments were carried out; it is therefore probable that the surface was to some extent decarburised. But the transverse specimens were of necessity small, 3/16 inch diameter compared with 5/16 inch diameter longitudinal specimens. The effect of a constant depth of decarburisation would be greater in the smaller specimens.

(4) As stated above, the transverse specimens were smaller in diameter; they were also shorter than the longitudinal specimens. Both conditions would tend to aggravate any error due to eccentricity of loading.

The influence of (4) may be estimated by considering the value of the plastic yield stress, on which a small eccentricity would have no effect. It will be seen that in the case of steel B, the ratio of upper to plastic yield stress is constant; it would therefore appear that the transverse yield stress is probably correct. In the case of steels A and C, on the other hand, the ratio is different. If therefore the measured transverse upper yield stress is neglected, and the value estimated by multiplying the longitudinal upper yield stress by the ratio of the plastic stresses two new values are obtained which are, in the opinion of the author, more likely to be correct. It is, of course, equally possible to assume, since the longitudinal and transverse plastic yield stresses are not widely different, that for these steels (A and C) the longitudinal upper yield stress is sufficiently accurate as a measure of the transverse strength; the resultant differences are not great.

The flexure tests were carried out on one size of specimen only, 0.2 inch in diameter. They are therefore in this respect directly comparable with certain of the flexure tests described in the present paper. But in another respect, that of rate of strain, the conditions were slightly different. Although the rate is not directly stated, it may be inferred that the time taken to reach yield was of the order of 30 minutes. This represents a rate of strain of about 10^{-6} per second, that is, ten times the rate adopted in the present tests. A fairly definite yield was observed at an apparent stress on the average 13 per cent greater than the yield stress in direct tension. As has already been remarked, the nature of the moment/angle curves in flexure obtained in the present tests (Fig. 18) makes it impossible to give a definite value to the yield stress, but it is sufficiently obvious that the apparent yield stress for a 0.2 inch diameter specimen is appreciably higher (about 8 per cent) than the yield stress in tension. In order to provide a better comparison, Cook's moment/angle curve for the "B" steel in which the ratio of upper to plastic yield stress has the same value as in the author's material, has been plotted on the same diagram (Fig. 23) as the corresponding present test. It will be seen that the results are, within the limits of experimental error, similar. If it might be assumed that Cook's tensile yield stress was slightly low, the curves would agree even more closely. The influence of surface decarburisation, however, would be in the opposite direction.

The torsion tests were carried out on the same specimens, 0.2 inch in diameter, and the ratio of the apparent shear stress in torsion to the direct stress in tension was 0.585, 0.585, and 0.564 for steels A, B and C respectively. The tensile stress used in calculating these figures has (as suggested by Cook) been taken as the mean of the longitudinal and transverse stresses, the transverse value chosen being found as described above. As in the case of flexure, eccentricity in the tension tests would cause these ratios to be increased, appreciable surface decarburisation would cause them to be decreased. It will be seen from the curve in Fig. 20 that the value of this ratio found by the author for a 0.2 inch diameter specimen is 0.587, a figure in very satisfactory agreement with those found by Cook. It may also be noted that the ratio is slightly lower in the case of steel C than in the other steels.

Apparatus for tests under internal pressure was not available to the author. In view of the satisfactory nature of the agreement between the tests just referred to and the author's flexure and torsion tests, however, it may be permissible to assume that the materials used are sufficiently similar to justify the acceptance of Cook's experiments as a basis for further argument and explanation.

Cook shewed clearly that a size effect was manifested in the failure of thick tubes under internal pressure, and that this effect was primarily dependent on the internal diameter of the tube. In Fig. 24 the author has replotted Cook's results on a slightly different basis. The tensile yield stress with which the results are compared is the value adjusted as explained in the paragraphs dealing with the transverse tests, and the curves drawn shew the variation of apparent shear stress qa at yield with internal diameter. It will be noticed, if these curves are compared with the original, that the differences between the three steels are much less marked, a fact which suggests that the transverse yield stress correction is valid. As in the case of the torsion results the curve for steel C is distinctly below those for A and B, suggesting that the tensile stress is, in comparison, high. The second diagram, Fig. 25, shews the effect of the assumption that a layer of depth .005 inch of highly stressed material (as in the case of the author's flexure tests) is required for the initiation of yield. As was pointed out in discussing the calculated moment/angle curves for flexure, it makes very little difference whether it be assumed that the stress in this layer continues to increase ~~to~~

to a value greater than the tensile shear yield stress in proportion to the strain, or that the stress remains constant and equal to the yield stress. In the former case the shear stress q_b at any radius 'b' is given by the ordinary thick cylinder stress equations as

$$q_b = P.\frac{a_1^2}{a_1^2 - a_0^2} \cdot \frac{a_0^2}{b^2}$$ where P is the internal pressure at yield, and a_1 and a_0 are the external and internal radii of the tube respectively. In the latter case[19] the stress q_c (the "calculated yield stress") is given by $\frac{P}{q_c} = 1 + 2\log\frac{b}{a_0} - \frac{b^2}{a_1^2}$. In table 14 the values of q_a, q_b, and q_c are given, in Fig. 25 the variation of q_c with the internal diameter of the tube is shewn.

Cook[20] has recently published another series of carefully conducted tests on thick tubes of similar material, the results of which are shewn in Fig. 26. The author has carried out similar calculations to determine q_b and q_c, and the results are given in table 15. The deduced values of q_c are plotted in Fig. 27 to a base of internal diameter of tube. It will be seen, if these figures (24 to 27) are compared, that the ratio of shear stress to tensile stress is lower in the later series. This, however, is hardly a fair comparison, since the tensile stress for the later tests is a value from longitudinal tests, whereas in the earlier tests a transverse yield stress was determined. Moreover, the tubes used in the later tests did not all receive an identical heat treatment, that for the larger tubes offering a possible opportunity for the formation of larger crystals and for a greater degree of decarburisation. Nevertheless it will be seen that the assumption of a layer of depth .005 inch able to withstand the yield stress does offer an explanation, to a first approximation at least, of the size effect; if a slightly thicker layer be assumed for the later tests, the size effect can be entirely accounted for. Alternatively, the explanation of the residual size effect and the discrepancy between the values of .500 as predicted by the shear stress theory and from .53 to .55 as given roughly by the earlier tests for the ratio of shear stress at failure to tensile yield stress may, it is suggested, be found in the effect of random crystal orientation.

It should be pointed out that the explanation proposed

TABLE 14.

Tests by Prof. G. Cook on thick cylinders under internal Pressure (Phil. Trans. Roy. Soc. A Vol.230 p.135)

Steel	Internal Dia. ins.	$\frac{qa}{f}$	$\frac{qb}{f}$	$\frac{qc}{f}$
A	.312	.576 (mean)	.540 (mean)	.545 (mean)
	.222	.609 "	.556 "	.561 "
	.127	.675 "	.580 "	.587 "
B	.437	.572	.545	.547
	.375	.567	.536	.539
	.312	.586	.550	.550
	.250	.599	.552	.554
	.187	.622	.558	.560
	.125	.653	.560	.566
C	.437	.554	.528	.52
	.375	.554	.523	.524
	.312	.554	.519	.519
	.250	.564	.520	.521
	.187	.597	.536	.540
	.125	.612	.524	.531

TABLE 15.

Tests by Prof. G. Cook on thick cylinders under internal pressure (Inst. of Engineers and Shipbuilders in Scotland, Paper No. 986)

Dia. ratio	Internal Diameter, inches	$\frac{qa}{f}$	$\frac{qb}{f}$	$\frac{qc}{f}$
2:1	.500	.509	.492	.490
	.437	.520	.496	.497
	.375	.526	.498	.500
	.312	.541	.507	.510
	.257	.553	.512	.514
	.187	.582	.523	.526
	.125	.594	.508	.516
3:1	.437	.504	.480	.482
	.375	.508	.480	.482
	.312	.530	.497	.500
	.250	.540	.497	.500
	.187	.567	.510	.515
	.125	.605	.518	.524

by Cook for the size effect approximates very closely in effect to that suggested above. He assumed that yield first occurred, not at the surface where the stress was a maximum, but at some depth "d" where the material was, by virtue of its position in the mass, less able to withstand the applied stresses. The value of "d" found necessary to account for the experimental results from steel A was between .0131 and .0208 inch.

CONCLUSIONS.

1. Combined tension and torsion.

The series of tests carried out in combined tension and torsion provide apparently conclusive evidence that the criterion of yield in mild steel is the shear strain energy per unit volume stored in the material. The subsequent tests refute this evidence, shewing that the apparent confirmation is fortuitous, and entirely due to the choice of the size of specimen.

2. Direct Stress.

(a) The maximum principal stress at yield is the same in tension and compression. It therefore follows that the normal stress on the planes of the maximum shear stress is without measurable effect.

(b) Whilst the stress/strain line for tensile tests on solid specimens .282 inch or more in diameter is straight, that for small specimens and thin tubes shews an appreciable curvature during the first application of the load, the yield stress is lower than for solid material, and the drop of stress at yield is greatly reduced as the thickness of the specimen approaches in magnitude the diameter of a crystal. The inference drawn is that the character of the yielding of mild steel is a result of the polycrystalline nature of the material.

3. Flexure.

For the steel used, the moment/angle curve is sensibly straight until an apparent stress greater that the tensile yield stress is reached on the outer fibres; it then rises to a maximum, and falls. The values of the bending moment at which there is a departure from linearity, of the maximum moment attained, and of the angle of curvature at which the reduction in moment starts, all depend on the size of the specimen tested.

The shape of the moment/angle curves can be explained by assuming that the core of the material is elastic, that a layer of thickness .005 inch immediately beyond the elastic region is able to withstand the yield stress, and that outside this layer the stress falls to the plastic yield stress.

4. Torsion.

The apparent shear stress at yield in torsion depends upon the size of the specimen. A part of this size effect can be explained by assuming a stress distribution similar to that suggested for flexure. A possible explanation of the residual size effect and of the difference between the shear stress at yield in torsion, when allowance has been made for the stress gradient, and the shear stress at yield in uniform tension is, as shown by Cox and Sopwith, provided by consideration of the effect of random crystal orientation.

5. Criterion of Yield.

Before discussing the criterion of yield it is essential to define the term "yield" in the light of the test results.

The present tests to investigate the strength of a material involve the determination of the relationship between the forces acting on a specimen and the deformation which it undergoes. This relationship may be represented by a curve, which has certain general features in common in all the tests. When the load is small, the resulting deformation is proportional to it; the curve is therefore straight. As the load increases, the deformation increases more rapidly. This departure from linearity, the onset of which is referred to as the limit of proportionality, or, less accurately, as the elastic limit, is small for the specimens used in the present tests, and in the case of the larger specimens too small to be measured with the apparatus used. When the load is further increased the deformation increases still more rapidly. In the tests under uniform stress and in torsion the specimen deforms at this stage in a catastrophic manner, the deformation increasing under a reduced load. In accordance with the usual practice, this condition has been referred to throughout the paper as the "yield" of the specimen. In the tests on thick ~~fibres~~ tubes under internal pressure, to which references have been made, the deformation is not catastrophic but the change of slope of the curve is sufficiently abrupt to define without ambiguity the yield of the specimen. In the case of the flexure tests, however, the slope of the curve does not even change suddenly, but continuously as the load increases. The term "yield" should not, in the author's opinion, be applied to such a flexure specimen.

It may, therefore, be accepted that the "yield" of a specimen as a whole is sufficiently defined for those tests to which the term is applicable. But any attempt to define "yield of the material" introduces further difficulties. There could be little doubt on general grounds that the existence of a limit of proportionality implied a "yielding" or slipping at this stage of some elements of the material: the X-ray experiments of Gough and Wood confirm this hypothesis. But quite apart from the difficulties, which with the apparatus in general use are insuperable, of locating accurately the limit of proportionality, it is beyond question that this condition is not the one which is ever referred to in the use of the phrase "yielding of a material", nor is it a condition of any practical importance. It is equally absurd to attempt to define yield of a material as the stage at which all the elements of the material have "yielded" or slipped, since in a test under non-uniform stress this condition may not arise until long after yield of the specimen has been passed. The conclusion is therefore inevitable that "yield of a material" must refer to some intermediate stage at which an indeterminate number of elements have yielded, sufficient to cause a sudden change in the rate of deformation of the specimen under test. Since this amounts to a restatement of the definition of "yield of a specimen" it must be accepted that the term "yield of a material" is by itself meaningless.

But the present tests show that, to a first approximation, specimens of mild steel yield, in cases to which the term applies, whatever the nature of the applied stresses, when a shear stress of constant magnitude is applied to a thickness of material of the order of three or four crystal diameters ~~or more~~.

In this sense, therefore, the criterion of yield in mild steel is the intensity of the maximum shear stress.

ACKNOWLEDGEMENTS.

The author wishes to acknowledge with gratitude the constant help and encouragement which have been given to him throughout the prosecution of this research, by Professor Andrew Robertson, D.Sc., M. Inst. C. E., M. I. Mech. E., and to thank Messrs. The Park Gate Iron & Steel Co. Ltd., who provided the steel used.

The whole of the work was carried out in the Engineering Department of the University of Bristol.

Appendix I.

To find, from the readings of ~~these~~ three extensometers set at 120° relative to each other round a cylindrical test piece of circular section, the maximum strain along any line parallel to the axis round the circumference.

In Fig. 28, let the circle on DE as diameter represent a plan view of the cross section of the test piece, and let D and E represent the positions at which the strain is minimum and maximum respectively. Let A, B, C be the position of the extensometer units, and let the measured strains at A, B and C be $\Delta + 0, \Delta + \delta_1, \Delta + \delta_2$, respectively. In the elevation let ST represent a plane parallel to the fixed ends of the extensometers and SR the plane through the other ends O, P′, Q′ of the extensometers. Then with the notation shown, the maximum strain exceeds the average strain $(\Delta + \frac{\delta_1 + \delta_2}{3})$ by an amount $h/2$.

$$\text{But} \quad \frac{h}{ST} = \frac{\delta_1}{OQ} = \frac{\delta_2}{OP}$$

where $OP = a \sin\alpha$

$OQ = a \sin\alpha + a \sin(60° - \alpha) = \frac{1}{2} a \sin\alpha + \frac{\sqrt{3}}{2} a \cos\alpha$

$ST = \frac{2a}{\sqrt{3}}$

therefore $h = \frac{2a}{\sqrt{3}} \cdot \frac{1}{a \sin\alpha} \cdot \delta_2 = \frac{2}{3} \cdot \sqrt{3}\,\delta_2 \operatorname{Cosec}\alpha$ .... (1)

$$\frac{\delta_1}{\delta_2} = \frac{OQ}{OP} = \frac{1}{2} + \frac{\sqrt{3}}{2} \operatorname{Cot}\alpha$$

therefore $\sqrt{3}\,\delta_2 \operatorname{Cosec}\alpha = 2\sqrt{\delta_1^2 - \delta_1\delta_2 + \delta_2^2}$ .... (2)

therefore $\frac{h}{2} = \frac{2}{3}\sqrt{\delta_1^2 - \delta_1\delta_2 + \delta_2^2}$

therefore Maximum strain = average strain + $\frac{2}{3}\sqrt{\delta_1^2 - \delta_1\delta_2 + \delta_2^2}$

Appendix 2.

To find the moment/strain curve for a flexure specimen of circular section, assuming the stress distribution after yield shewn in Fig. **29**; that is, assuming that the core of the material is elastic, that a layer of thickness "t" immediately beyond the elastic region is able to withstand the yield stress, and that outside this layer the stress falls to the plastic yield stress. The ratio of the plastic yield stress to the upper yield stress f, as found from tension and compression tests, is two-thirds. The calculation given is a general case, of which two particular cases are (a) that in which "t" = 0; i.e. where there is no reinforcement due to the understressed material and (b) that in which "t" is greater than the radius of the specimen; i.e. where there is no reduction in stress after yield.

The total moment of resistance M is the sum of the moments due to the elastic core and to the regions beyond the elastic core if sustaining a stress f, less the reduction in moment due to the stress drop.

$$\text{i.e. } M = 2\int_0^a x\, dy \cdot y \cdot \frac{y}{a} \cdot f + 2A_1 \bar{y}_1 f - 2A_2 \bar{y}_2 \cdot \frac{f}{3}$$

where A_1 = area for which $r > y > a$

A_2 = " " " $r > y > (a + t)$

$\bar{y}_1$ = distance of centroid of A_1 from XX

$\bar{y}_2$ = " " " " A_2 " XX.

$$\tfrac{1}{2}M = \int_\beta^{2\pi} 4 \cdot \frac{fr^3}{\cos\beta} \cdot \sin^2\theta \cos^2\theta\, d\theta + A_1 \cdot \frac{2r^3 \sin^3\beta}{3A_1} \cdot f - \frac{1}{3} \cdot A_2 \cdot \frac{2r^3 \sin^3\alpha}{3A_2} \cdot f$$

where $y = r\cos\theta$

$$= \frac{fr^3}{4\cos\beta}\left[\theta - \frac{\sin 4\theta}{4}\right]_\beta^{\pi/2} + \frac{2}{3} \cdot f\, r^3 \sin^3\beta - \frac{2}{9} f\, r^3 \sin^3\alpha.$$

$$M = \frac{fr^3}{8\cos\beta}(2\pi - 4\beta + \sin 4\beta) + fr^3\left(\frac{4}{3}\sin^3\beta - \frac{4}{9}\sin^3\alpha\right)$$

But bending moment when yield stress is attained on outer

$$\text{fibres} = M_y = \frac{\pi}{4} f\, r^3$$

$$\frac{M}{M_y} = \frac{4}{\pi}\left(\frac{2\pi - 4\beta + \sin 4\beta}{8\cos\beta} + \frac{4}{3}\sin^3\beta - \frac{4}{9}\sin^3\alpha\right)$$

For particular cases as above

(a) $\alpha = \beta$

(b) $\alpha = 0$

For values of the strain 'e' corresponding to M

$$\frac{\text{strain}}{\text{strain at yield}} = \frac{e}{e_y} = \frac{r}{a} = \sec \beta$$

By giving values to β, the corresponding values of $\frac{M}{M_y}$ and of $\frac{e}{e_y}$ can therefore be calculated for any given value of 't'. The results shewn in the following table have been calculated for the flexure specimens tested, on the assumption that 't' = ·005 inch.

TABLE. 16

Degrees	0	10	20	25	30	35	40	45	50	55	60	65	70	75	80
$\frac{M}{M_y}$ for $\frac{t}{r} > 1$	1.000	1.0153	1.0624	1.0980	1.141	1.191	1.247	1.307	1.370	1.432	1.494	1.550	1.600	1.641	1.670
" $\frac{t}{r}$ = 0.1	1.000	1.015	1.062	1.098	1.131	1.156	1.178	1.190	1.200	1.204	1.204	-	-	-	-
= .087	1.000	1.015	1.062	1.097	1.125	1.148	1.166	1.180	1.189	1.193	1.194	1.190	-	-	-
= .071	1.000	1.015	1.062	1.093	1.117	1.137	1.154	1.167	1.176	1.180	1.181	1.178	-	-	-
= .050	1.000	1.015	1.060	1.084	1.105	1.123	1.138	1.149	1.158	1.162	1.164	1.161	-	-	-
= .0354	1.000	1.015	1.056	1.076	1.095	1.112	1.126	1.137	1.145	1.150	1.153	1.152	-	-	-
= .0333	1.000	1.015	1.055	1.075	1.093	1.110	1.124	1.135	1.144	1.148	1.149	1.151	1.150	-	-
= .025	1.000	1.015	1.052	1.071	1.088	1.104	1.118	1.128	1.137	1.142	1.145	1.146	1.145	-	-
= .020	1.000	1.015	1.049	1.068	1.085	1.100	1.113	1.124	1.133	1.138	1.143	1.143	1.142	-	-
= 0	1.000	1.012	1.040	1.055	1.070	1.084	1.097	1.107	1.116	1.121	1.126	1.129	1.130	1.131	1.131
$\frac{e}{e_y}$	1.000	1.0154	1.064	1.103	1.155	1.221	1.305	1.414	1.556	1.743	2.000	2.366	2.924	3.864	5.76

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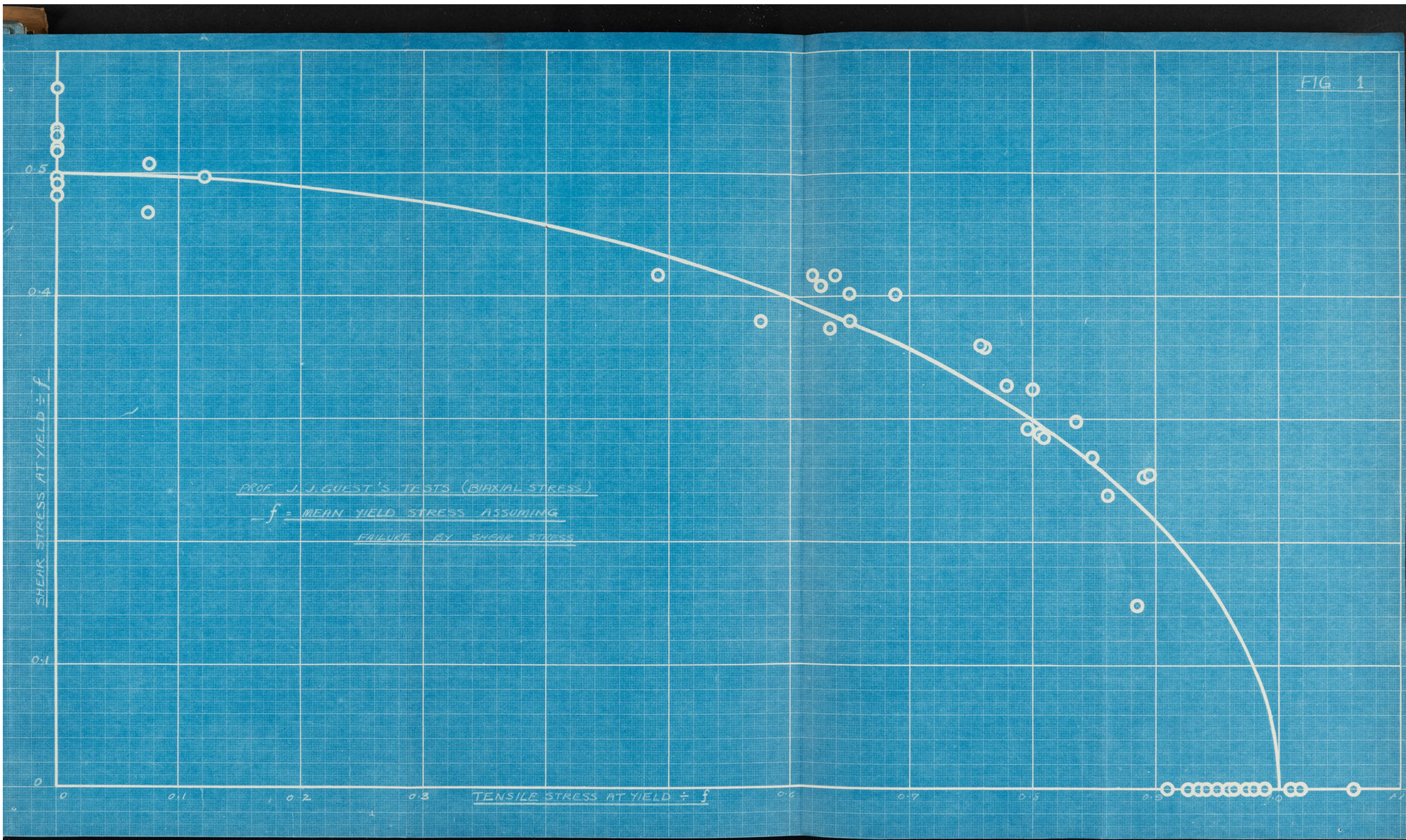

PROF. J. J. GUEST'S TESTS (BIAXIAL STRESS)
f = MEAN YIELD STRESS ASSUMING FAILURE BY SHEAR STRESS
SHEAR STRESS AT YIELD ÷ f
TENSILE STRESS AT YIELD ÷ f
0·5
0·4
0·1
0
0·1
0·2
0·3
0·6
0·7
0·8

FIG 1

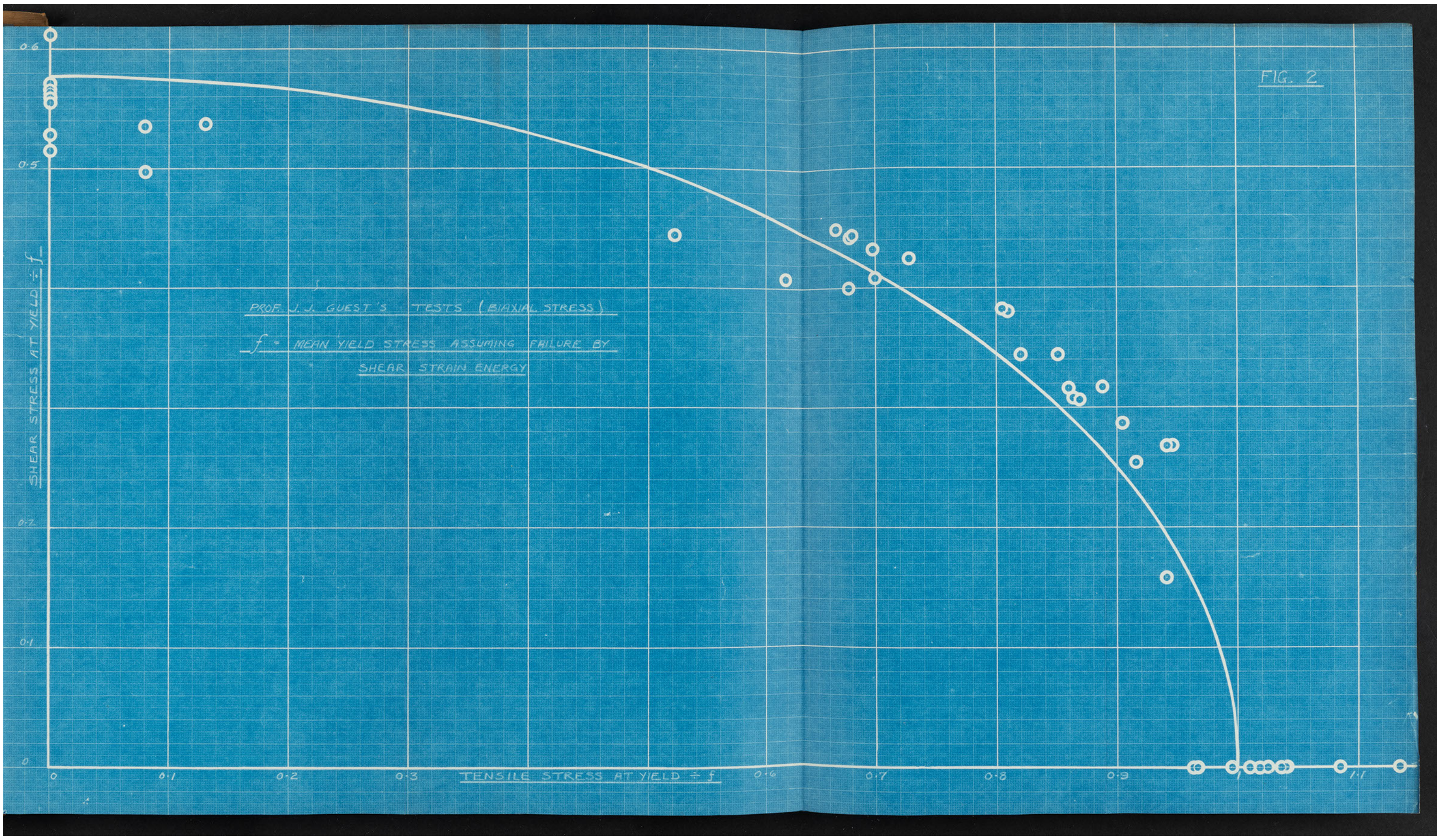
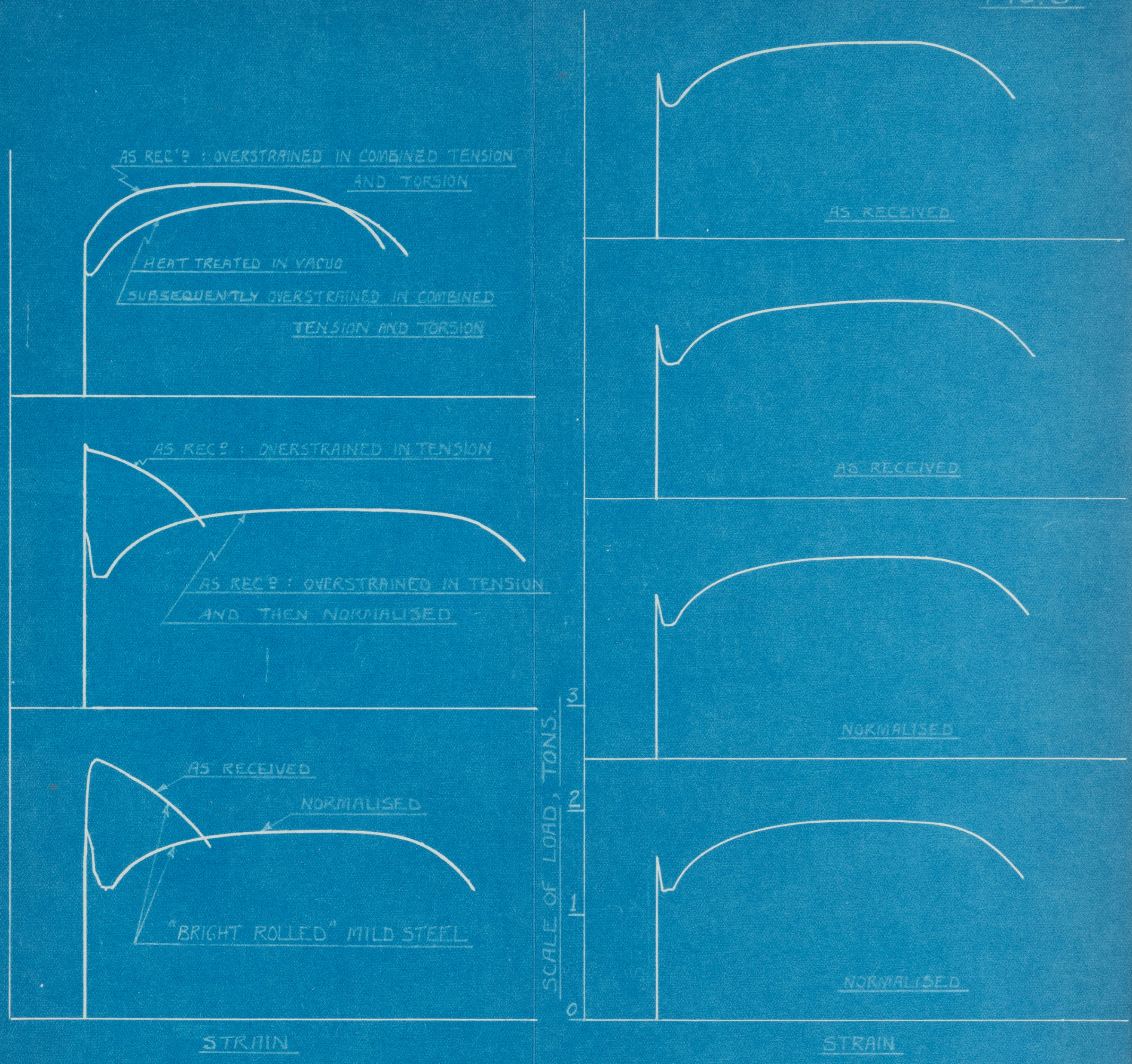


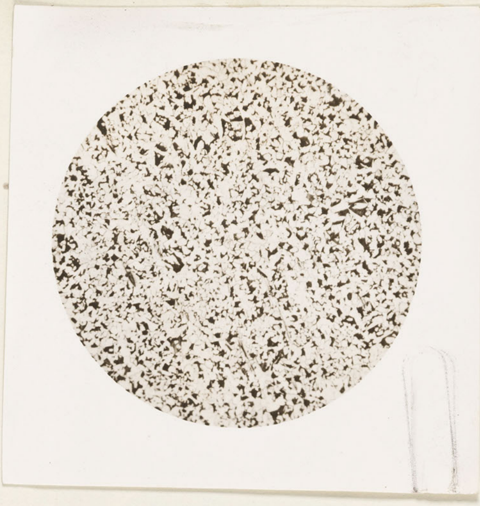
FIG. 2
PROF. J. J. GUEST'S TESTS (BIAXIAL STRESS)
f = MEAN YIELD STRESS ASSUMING FAILURE BY SHEAR STRAIN ENERGY
SHEAR STRESS AT YIELD ÷ f
TENSILE STRESS AT YIELD ÷ f
0·6
0·5
0·2
0·1
0
0·1
0·2
0·3
0·6
0·7
0·8
0·9
1
1·1

FIG. 3.

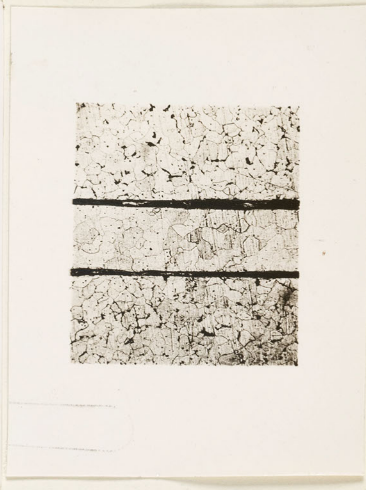


AUTOGRAPHIC DIAGRAMS FOR TESTS IN TENSION

FIG. 4.



(a)



(b)

(c)

(d)

(e)

(f)

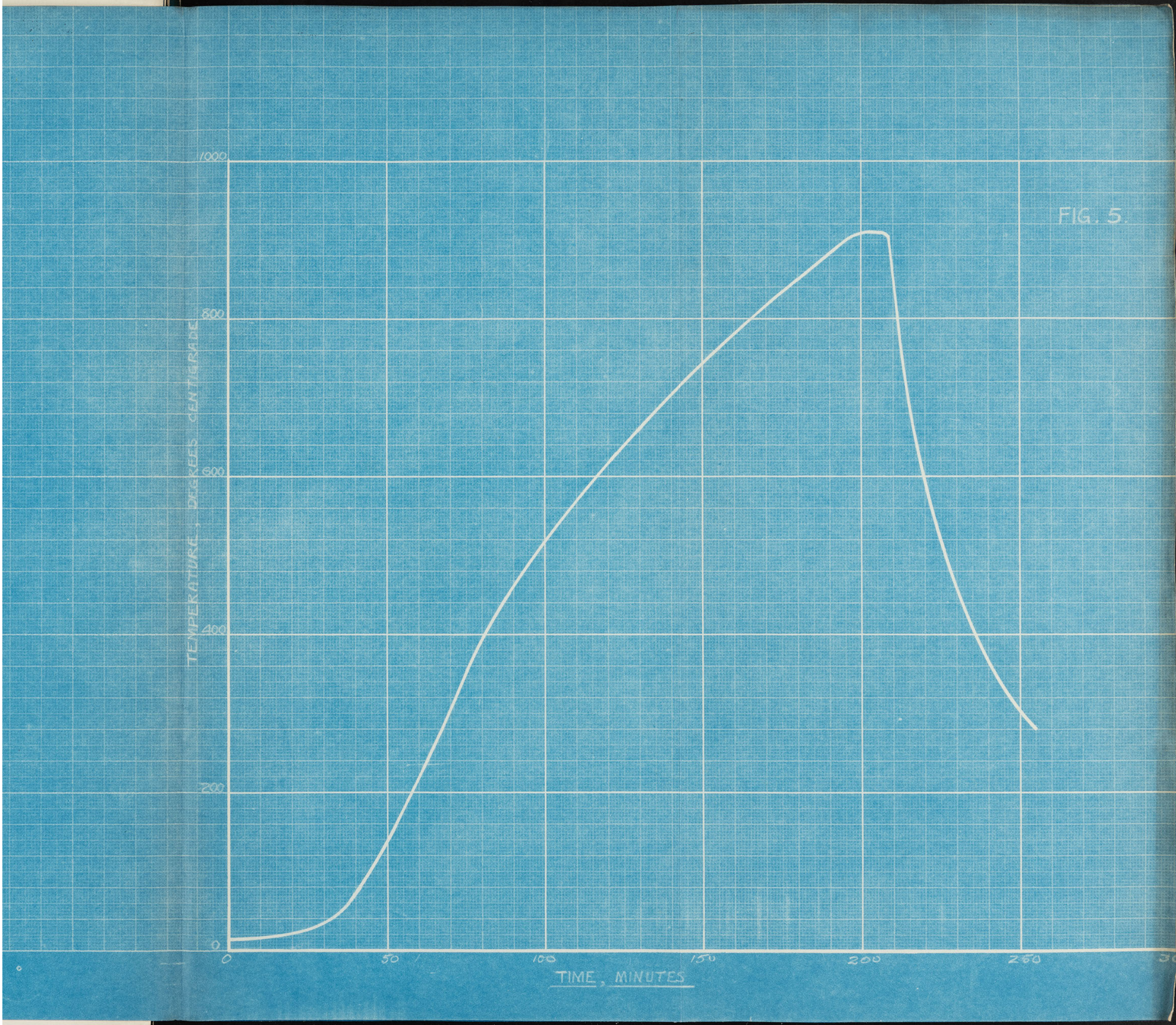

TEMPERATURE, DEGREES CENTIGRADE
1000
800
600
400
200
0
0
50
100
150
200
250
TIME, MINUTES

FIG. 5.

FIGS. 6 & 7.

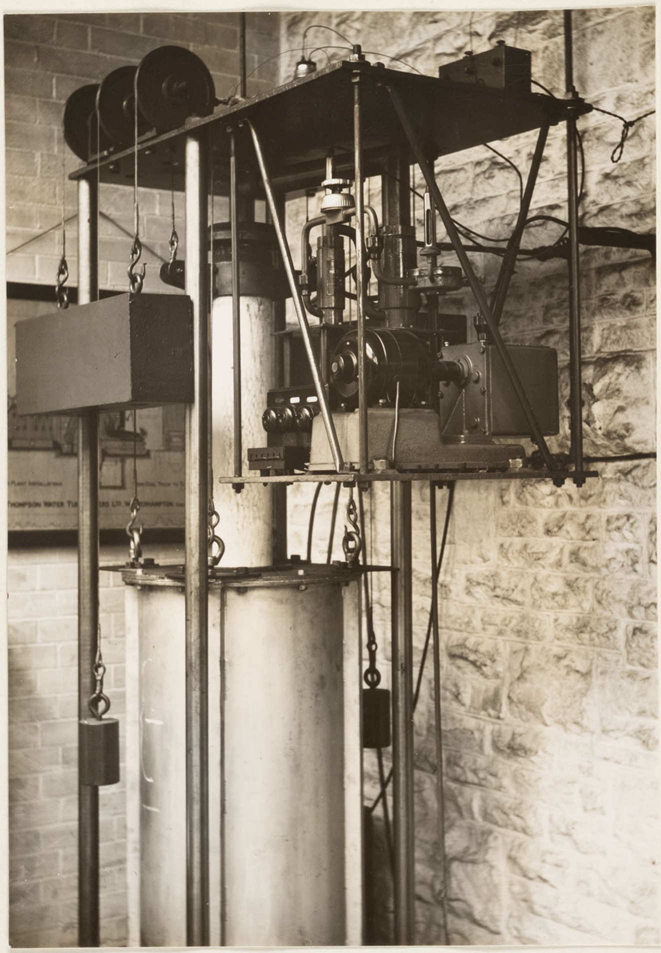


FIG. 6

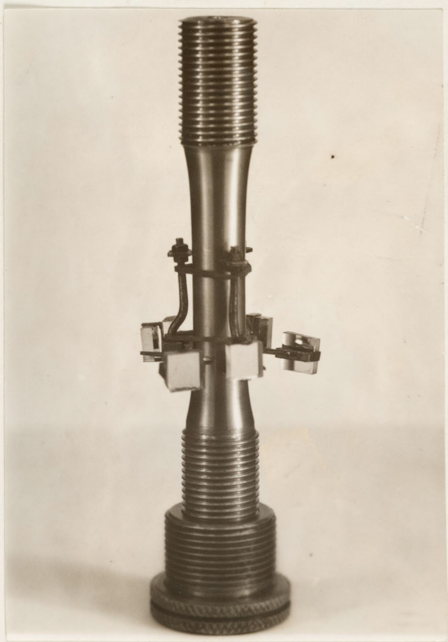
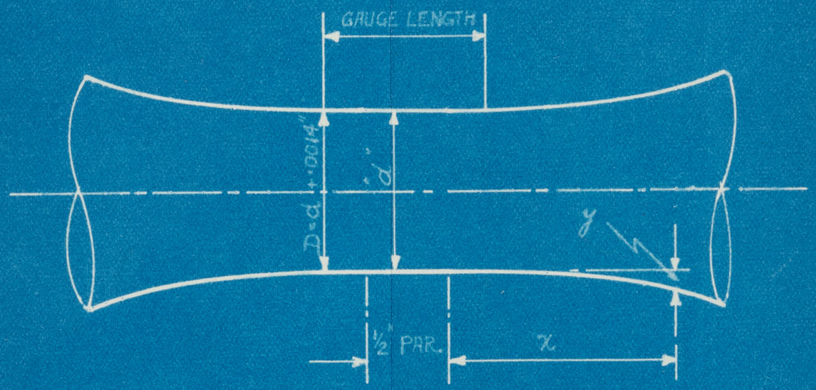


FIG. 7.

FIG 8.



EQUATION TO TRANSITION CURVE:

$$y = 0{\cdot}044\, x^3$$

PROFILE ADOPTED FOR LARGE SPECIMENS
TO MINIMISE STRESS CONCENTRATION

FIGS. 9 & 11

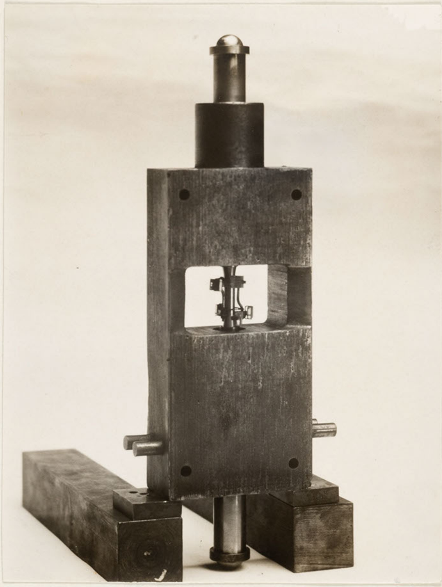


FIG 9

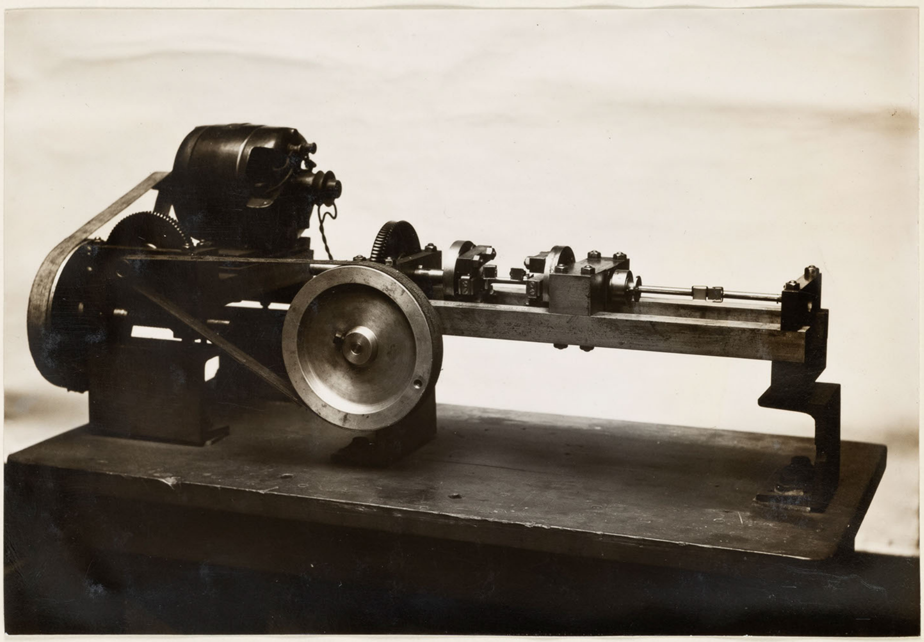
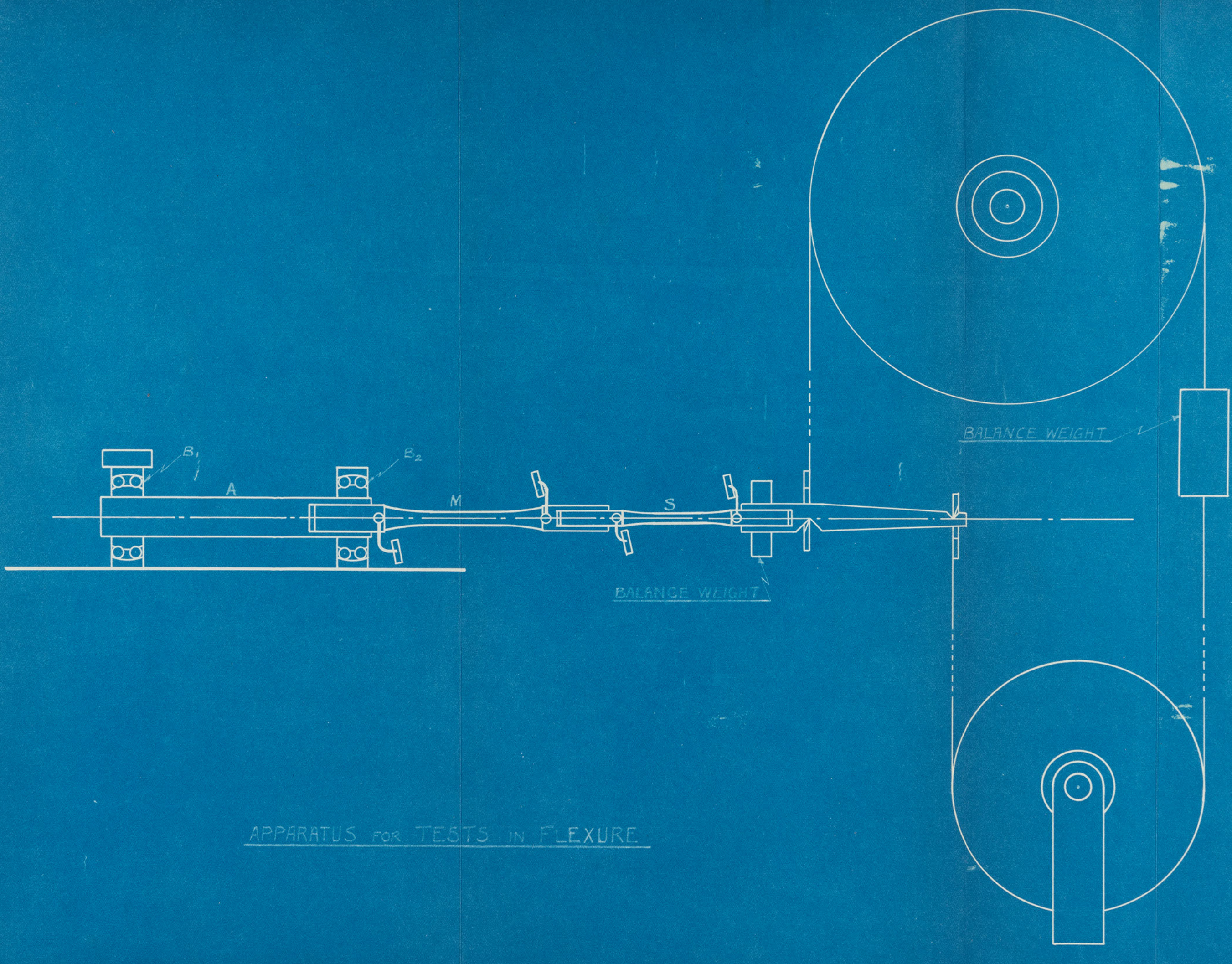


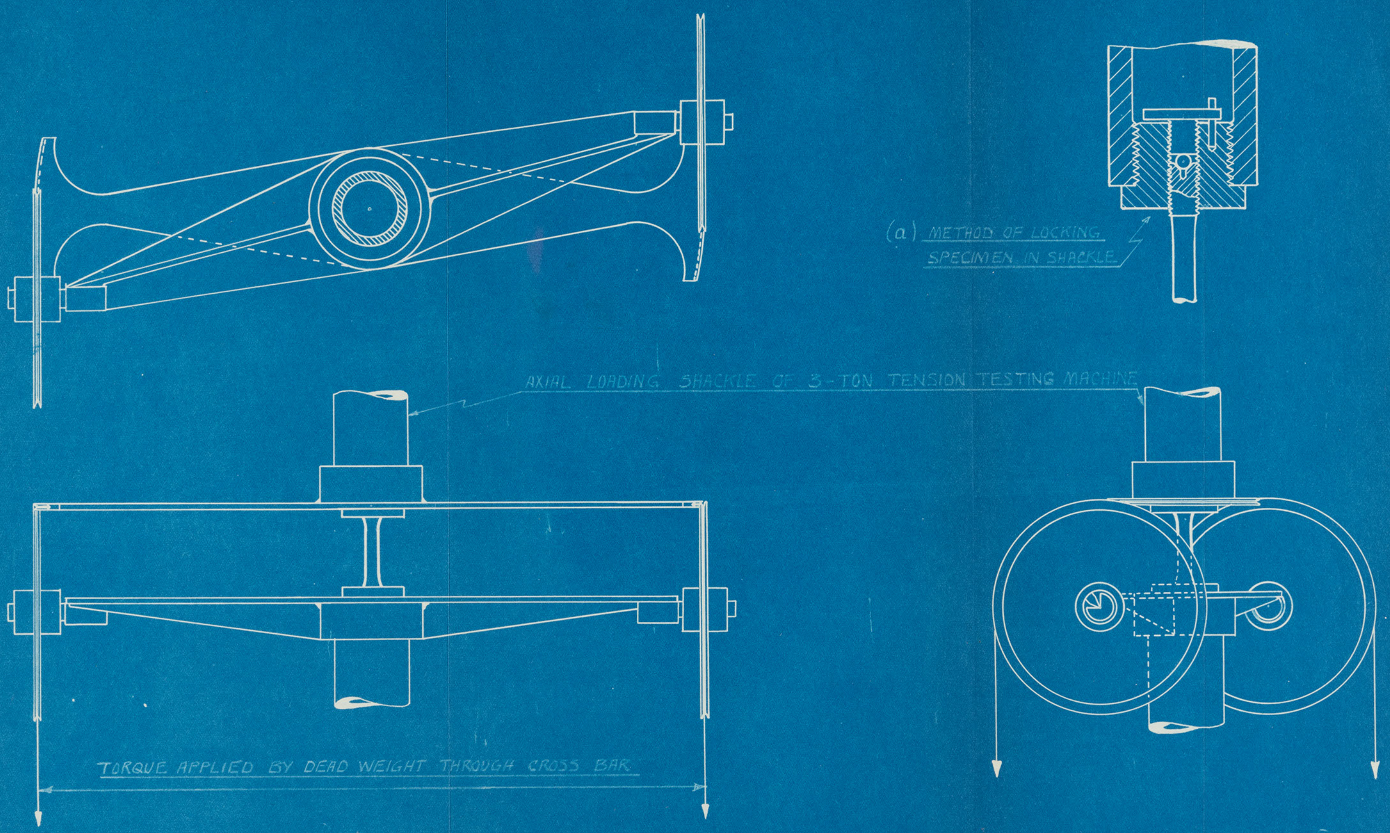
FIG 11.

FIG. 10



APPARATUS FOR TESTS IN FLEXURE

FIG. 12.



APPARATUS FOR TESTS IN COMBINED TENSION AND TORSION

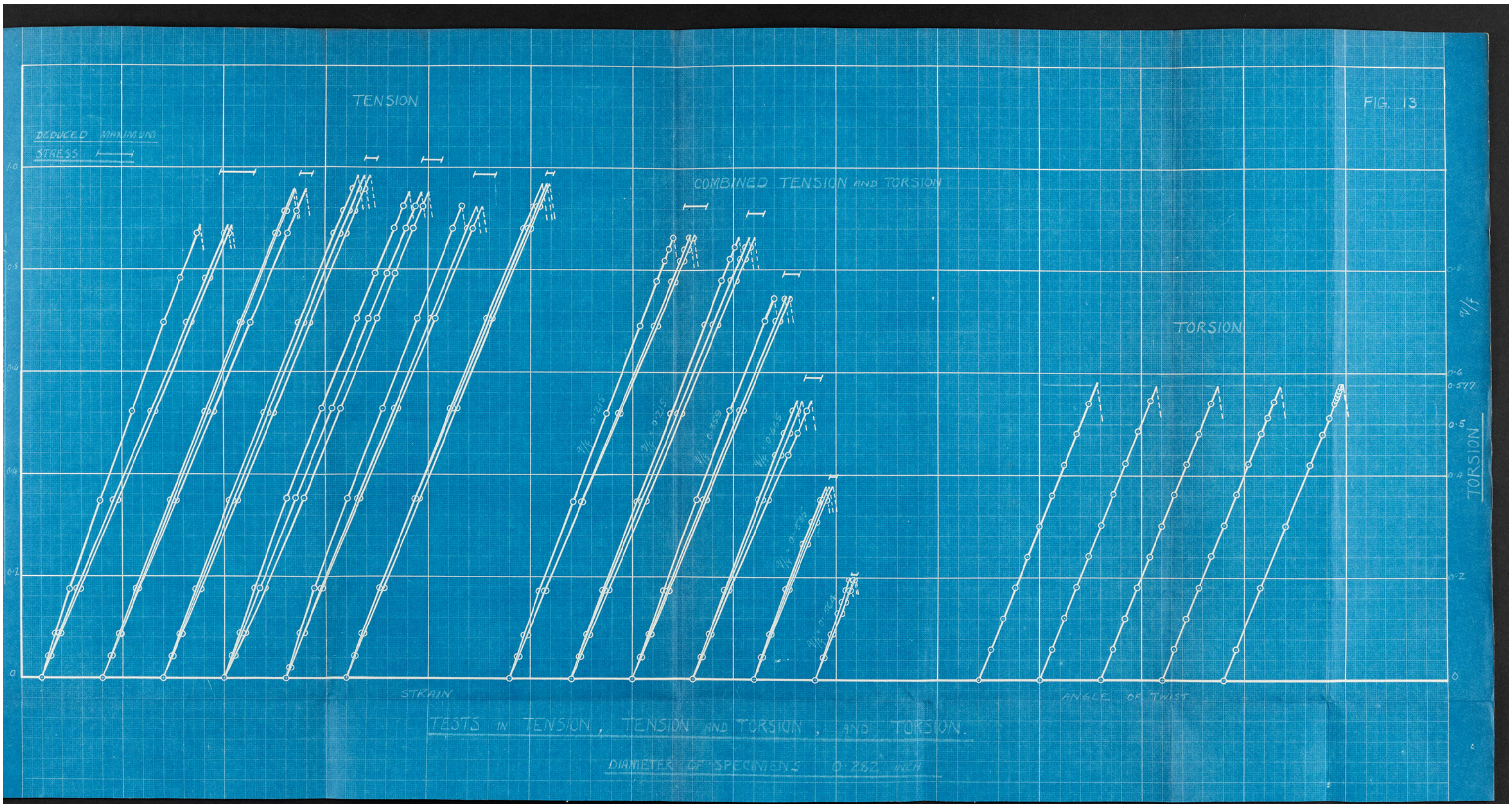
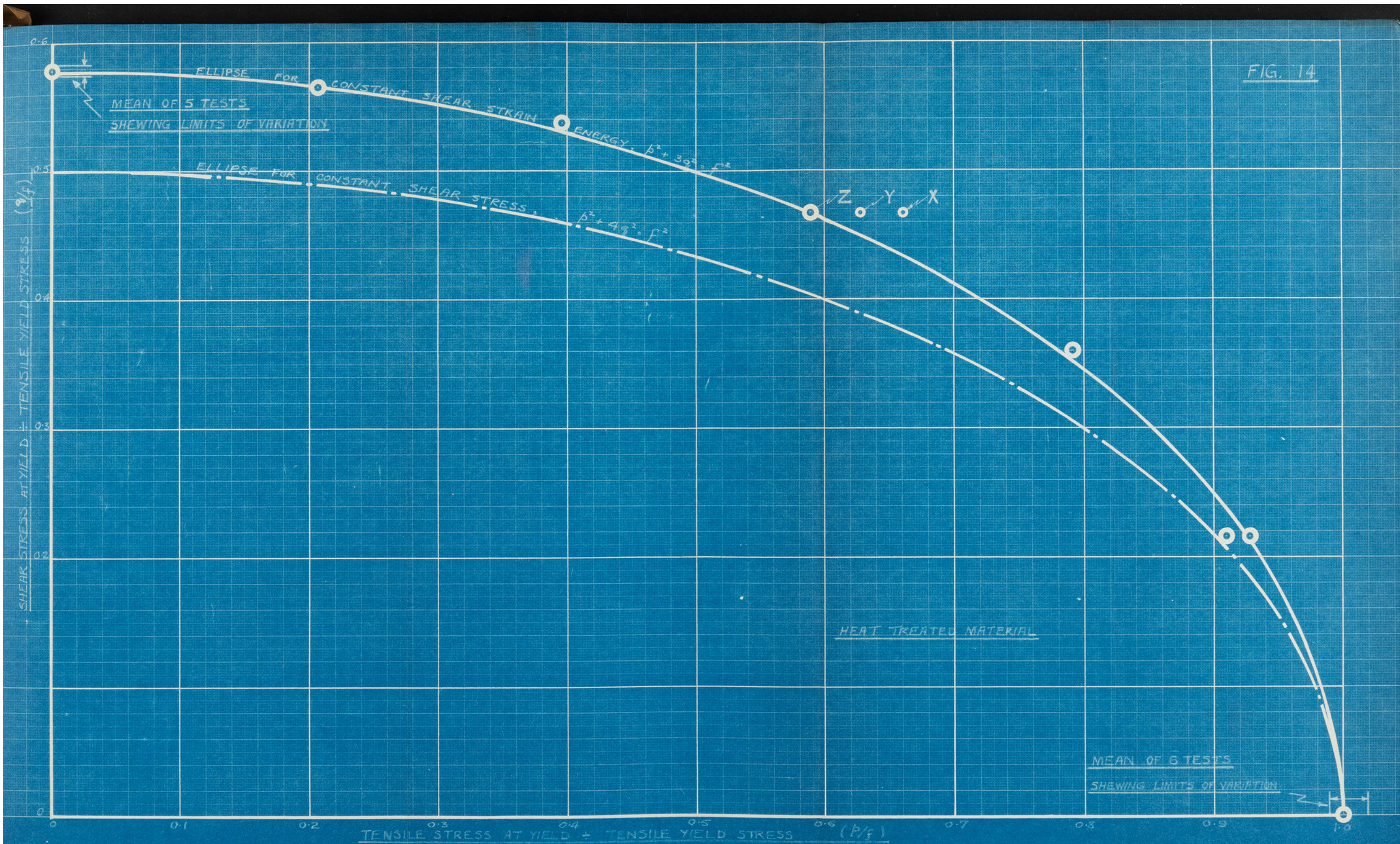
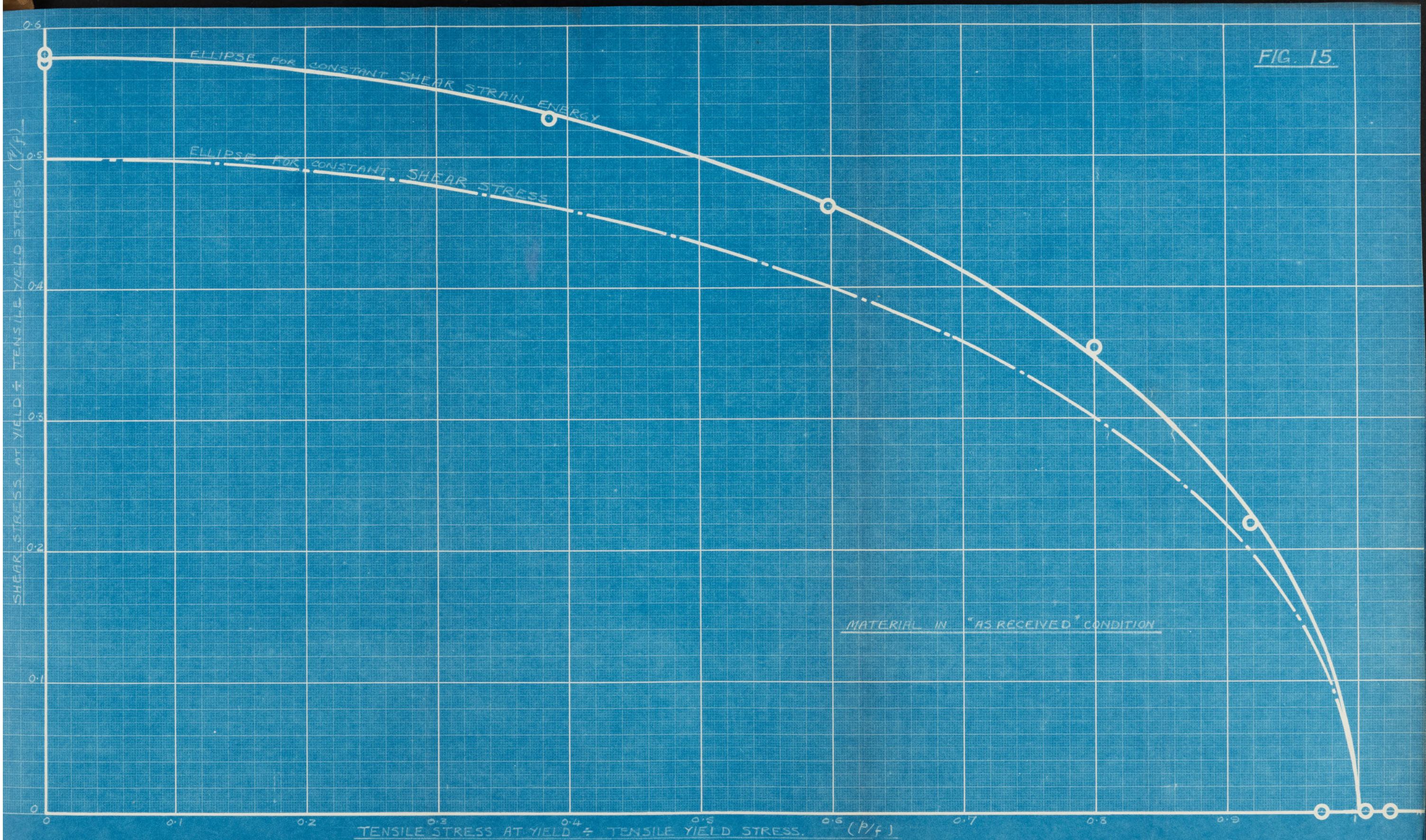


FIG. 13
TENSION
DEDUCED MAXIMUM
STRESS
COMBINED TENSION AND TORSION
TORSION
STRAIN
ANGLE OF TWIST
TESTS IN TENSION, TENSION AND TORSION, AND TORSION.
DIAMETER OF SPECIMENS 0·282 INCH

FIG. 14



TESTS IN COMBINED TENSION AND TORSION, SHEWING APPARENT CONFIRMATION OF SHEAR STRAIN ENERGY THEORY



TESTS IN COMBINED TENSION AND TORSION, SHEWING APPARENT CONFIRMATION OF SHEAR STRAIN ENERGY THEORY

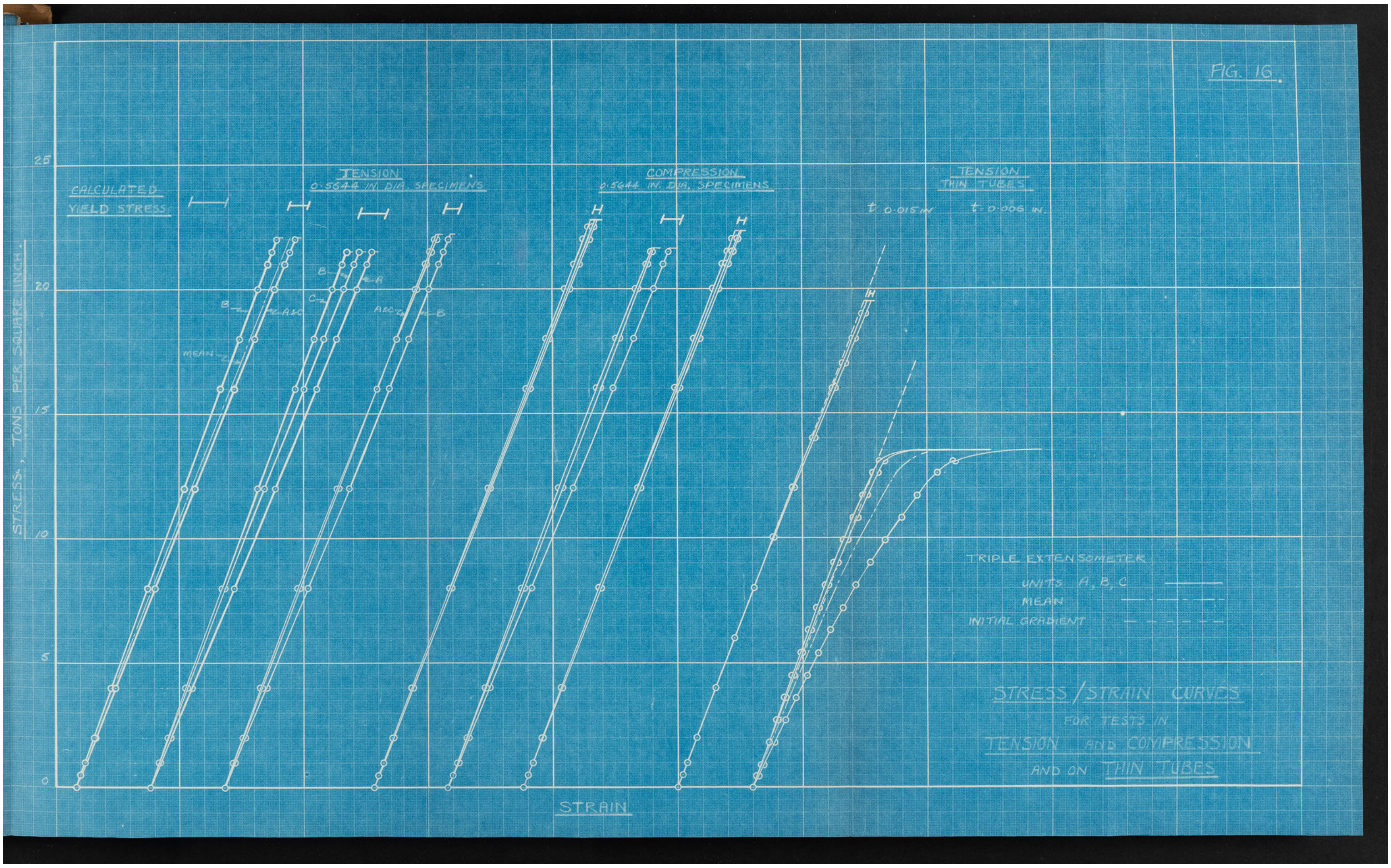
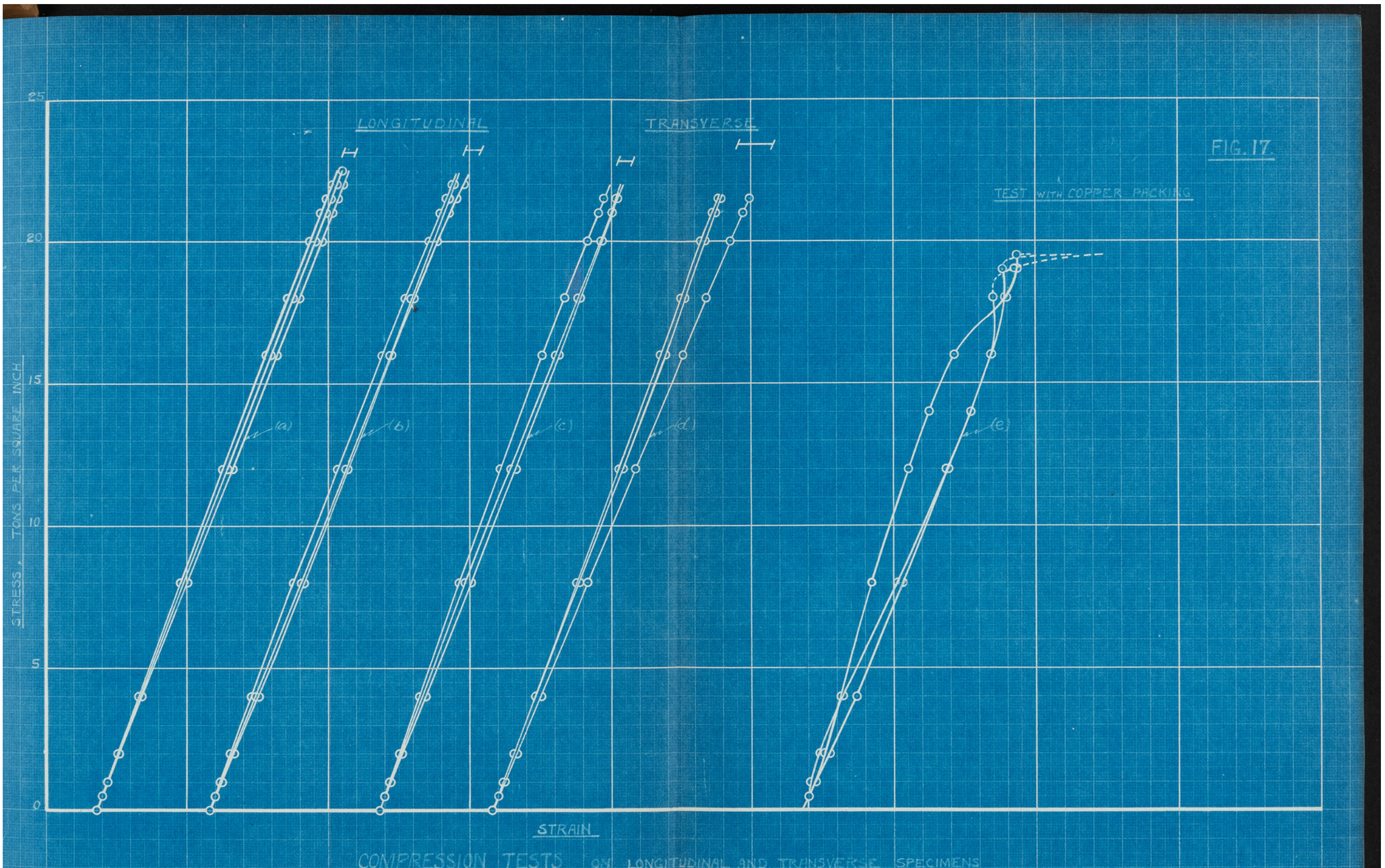


FIG. 16.

STRESS / STRAIN CURVES FOR TESTS IN TENSION AND COMPRESSION AND ON THIN TUBES



COMPRESSION TESTS ON LONGITUDINAL AND TRANSVERSE SPECIMENS

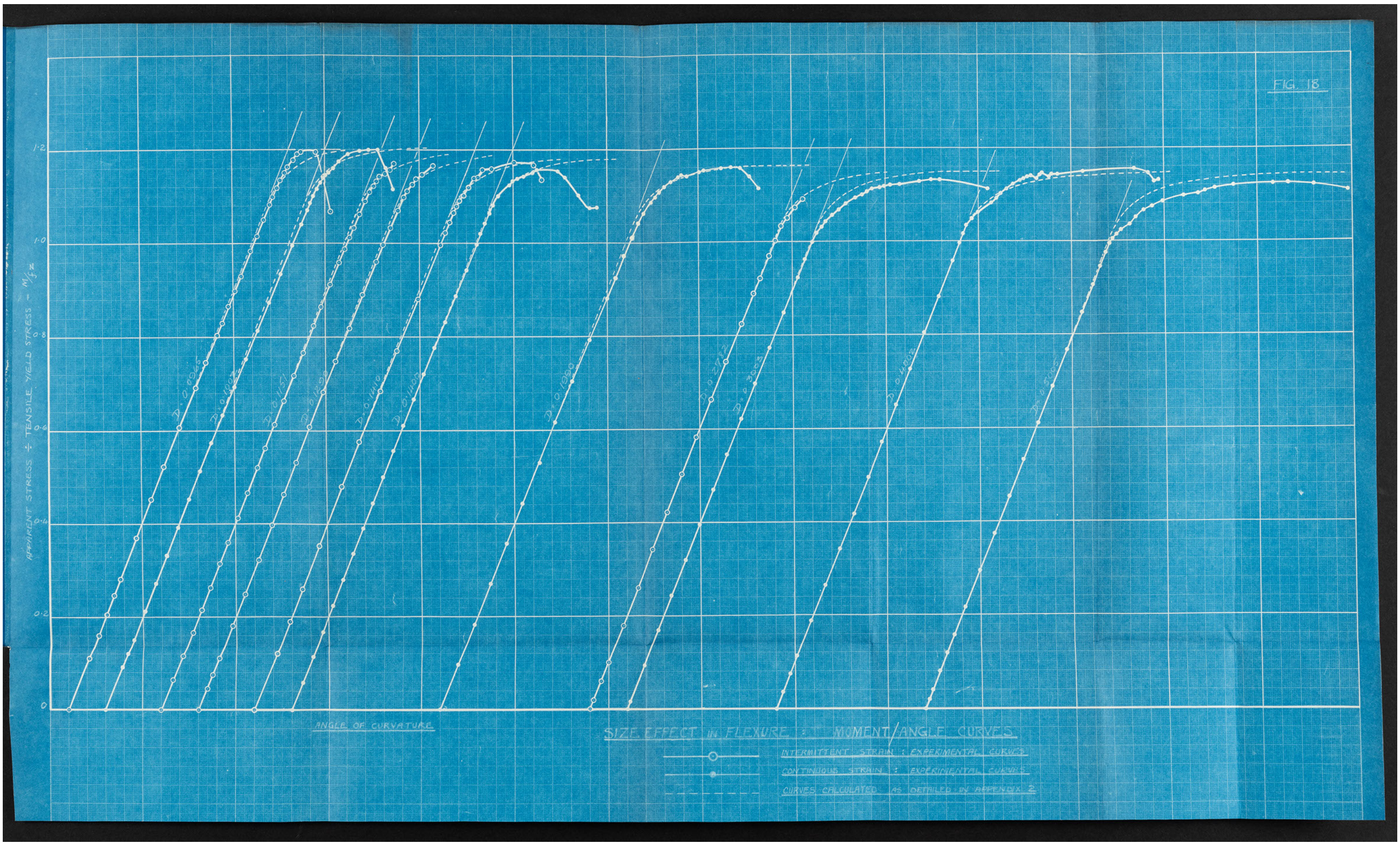
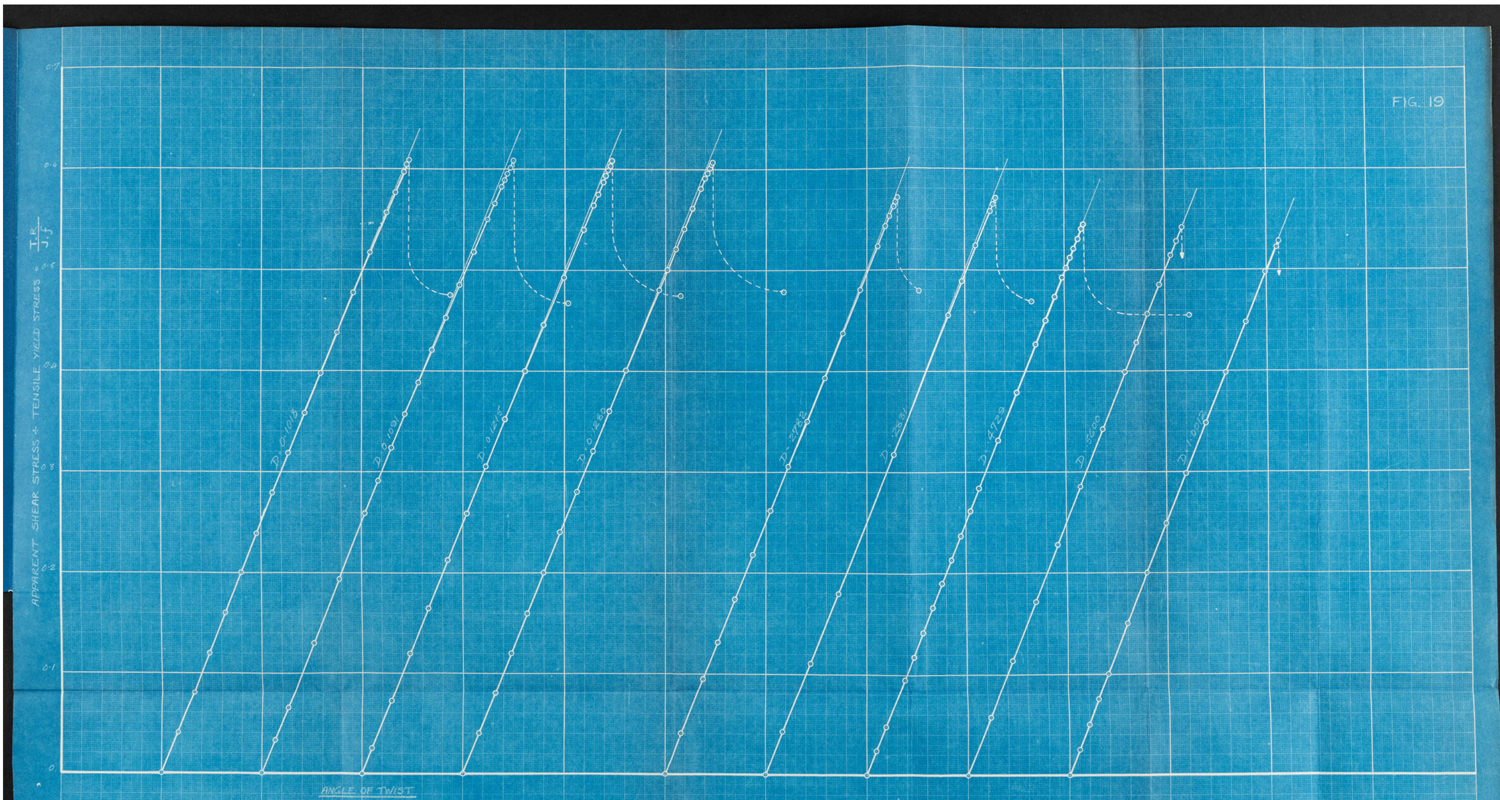


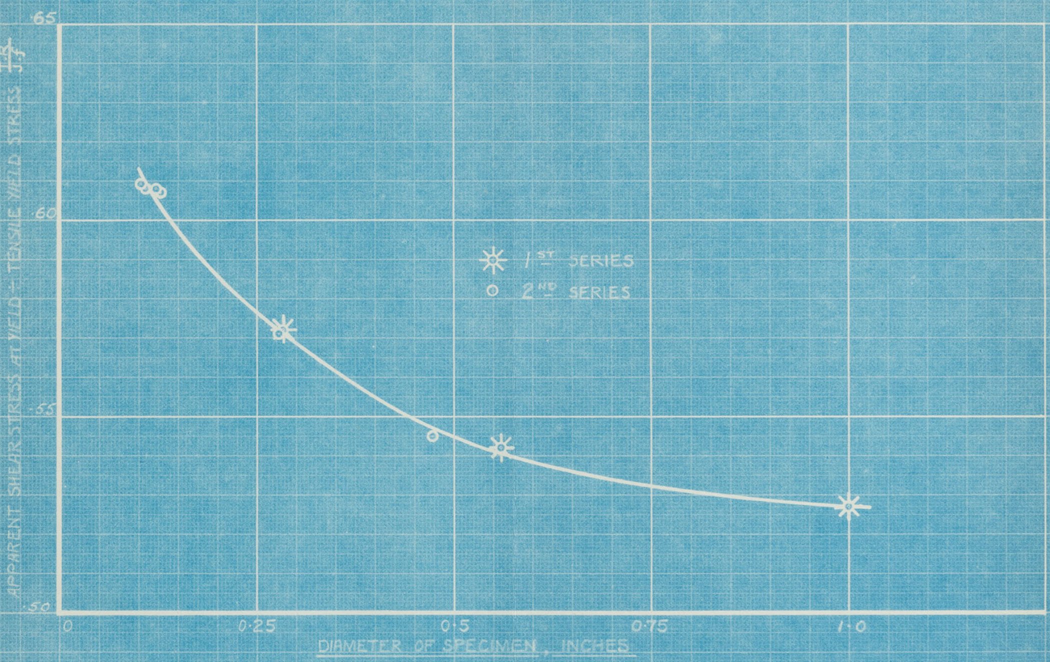
FIG. 18

SIZE EFFECT IN FLEXURE : MOMENT/ANGLE CURVES



SIZE EFFECT IN TORSION : TORQUE / ANGLE CURVES

FIG. 20



SIZE EFFECT IN TORSION

FIGS. 21 & 22

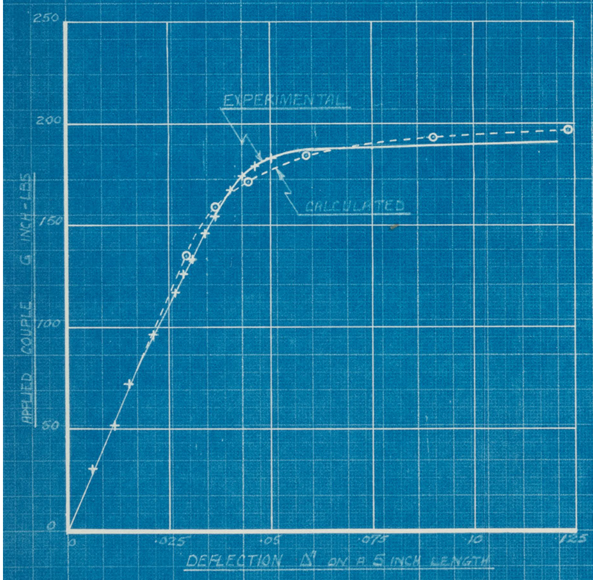


FIG. 21 : REPRODUCTION OF FLEXURE TEST FROM PAPER BY PROF. MUIR & D. BINNIE

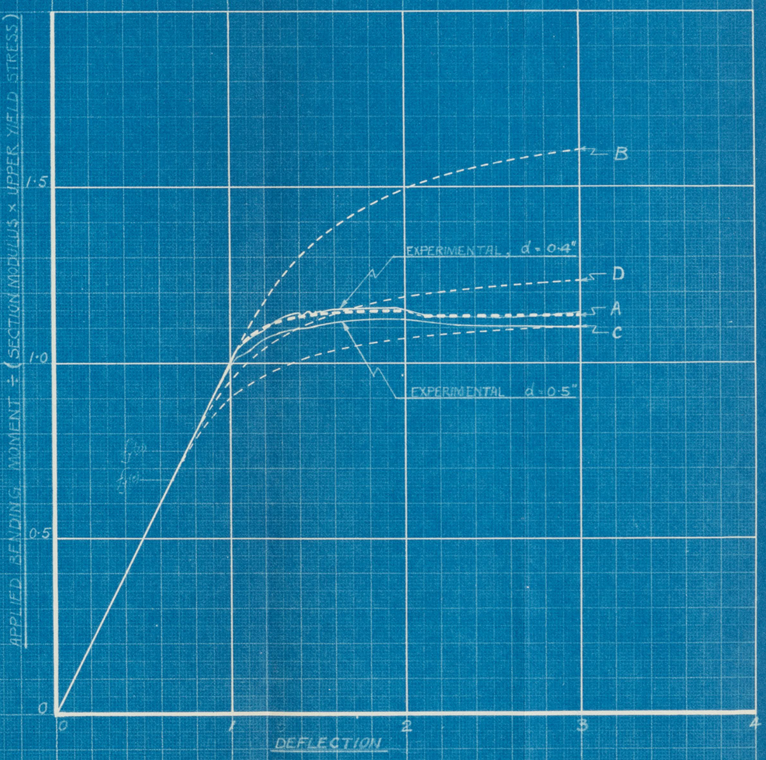
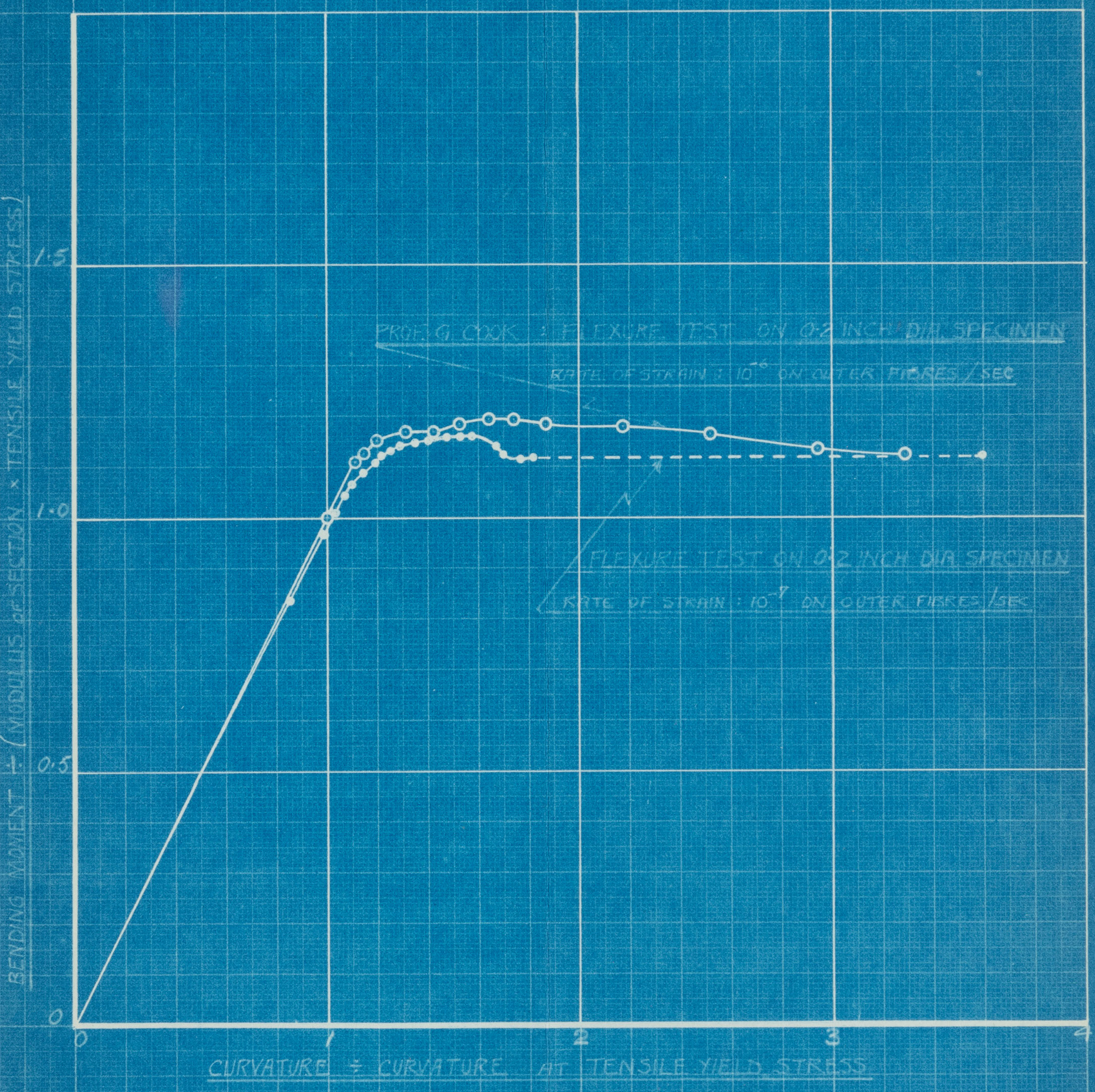


FIG. 22: CURVES FOR FLEXURE TESTS WITH VARIOUS CALCULATED CURVES

FIG. 23



PROF. G. COOK : FLEXURE TEST ON 0·2 INCH DIA. SPECIMEN
RATE OF STRAIN : 10^{-6} ON OUTER FIBRES / SEC
FLEXURE TEST ON 0·2 INCH DIA. SPECIMEN
RATE OF STRAIN : 10^{-7} ON OUTER FIBRES / SEC
BENDING MOMENT ÷ (MODULUS OF SECTION × TENSILE YIELD STRESS)
CURVATURE ÷ CURVATURE AT TENSILE YIELD STRESS
0
0·5
1·0
1·5
0
1
2
3
4

FIGS. 24 to 27

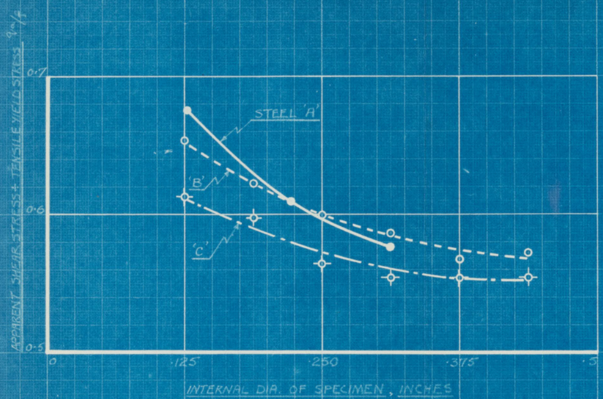


FIG. 24

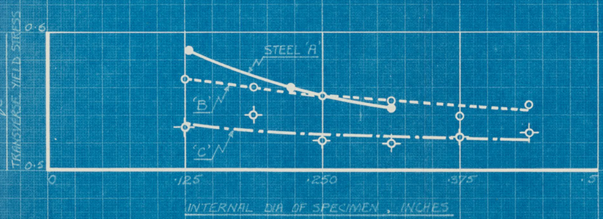


FIG. 25

THICK TUBES UNDER INTERNAL PRESSURE: TESTS BY PROF. G. COOK

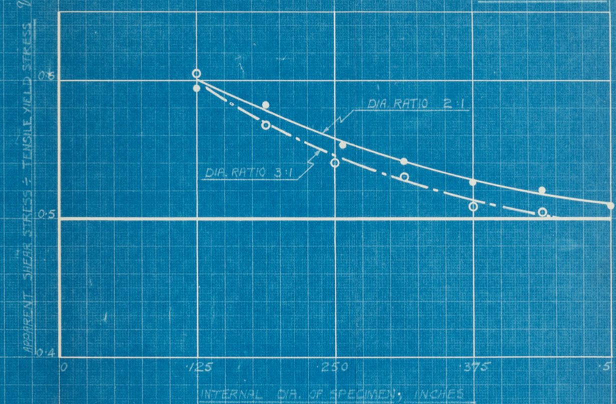


FIG. 26

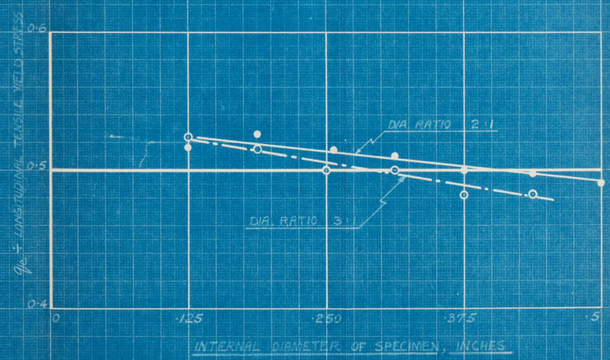


FIG. 27

FIG. 28

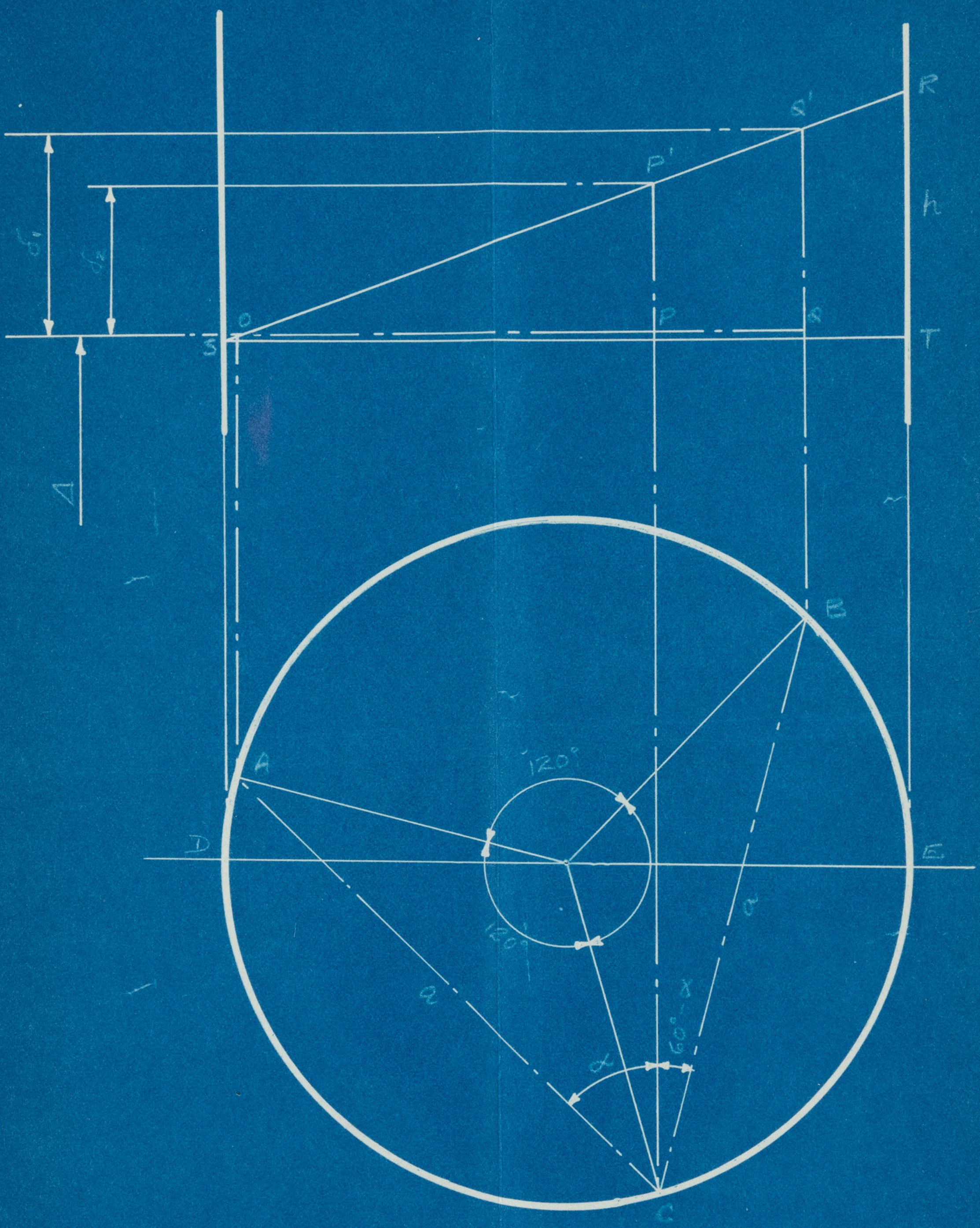

R
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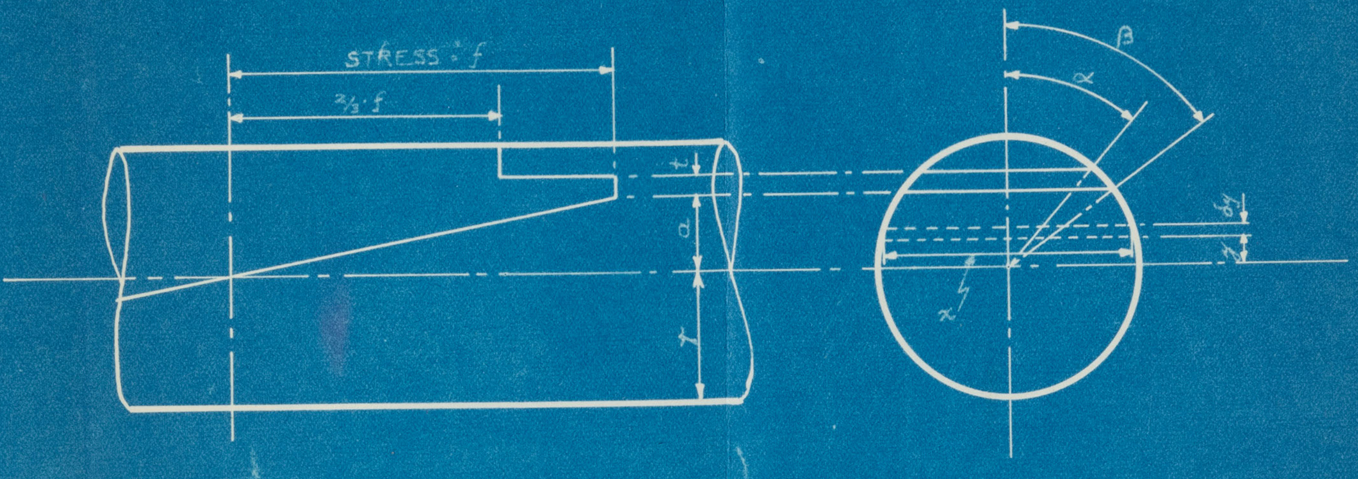
FIG 29



Reprinted from The Engineer *June 22nd, 1934.*

A Three-Ton Testing Machine.*

By JOHN L. M. MORRISON, B.Sc.

IN a paper included in the First Report of the Steel Structures Research Committee, Professor Andrew Robertson drew attention to the desirability of a modification of the usual testing procedure if accurate yield point measurements were required. He showed that small turned specimens tested in axial loading shackles gave data which were more reliable than those provided by the usual large specimens tested in wedge grips, and suggested that a small 3-ton machine would be sufficient for most purposes. Professor Robertson invited the present author to design a small machine provided with an autographic recorder for tension tests. The type selected was a spring-controlled compound lever machine, with variable speed motor for straining the specimen. In order to eliminate many of the complications which are necessary in a machine which has to be used for a variety of purposes, it was decided to adopt a specimen standardised as to diameter, length, and end fitting.

The machine described was made in the workshop of the Merchant Venturers Technical College, Bristol. If a number of similar machines were to be made under normal production conditions, the design could easily be modified, when the cost of the machine would be comparatively small. The completed machine is shown in Fig. 1, and the general arrangement—with the exception of the load-indicating dial—in Fig. 2.

Straining Gear.

A small electric motor applies the load through a double worm reduction gear, of overall ratio 1680 to 1, and a square-threaded screw of five threads per inch, on which the second worm wheel turns. During a test the square-threaded screw is prevented from turning by a feather in the bush through which it passes; consequently, rotation of the worm wheel

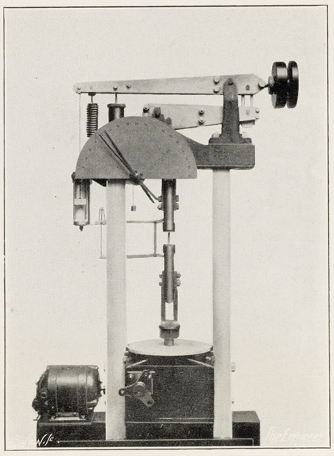


FIG. 1

pulls the screw downwards. In order to change the specimen, however, a locking dowel pin is lifted out of a large disc to which the bush is attached, and the disc is turned by hand. The screw can, therefore, be moved rapidly up or down to accommodate the new specimen, and to take up any slackness, so that the load is applied immediately the motor starts. In this way considerable time may be saved without the necessity of changing the gearing, and without danger of applying a sudden load to the specimen. Attached by a swivel joint to the square-threaded screw is the lower of two axial loading shackles. These shackles are designed on similar lines to those described by Cook and Robertson in *Engineering* (1911, Vol. 92, p. 786), and consist in principle of a hard steel ball, accurately concentric with the centre line of the specimen, and free to roll on a flat, ground, hard steel plate. Sufficient clearance is provided in this shackle to remove and replace an unbroken specimen without altering the position of the square-threaded screw; the disc adjustment, therefore, is used mainly to compensate for the elongation of a broken specimen.

Measurement of Load.

The load is measured by the compound lever spring balance at the top of the machine. The ends of the tension spring used to measure the load are screwed into small spherical seatings in V-notched machined hooks, which fit over knife edges in the upper lever and frame; various springs, which can be changed in a few seconds, are provided, so that, if desired, the whole of the scale may be used and accurate results obtained with materials having a low ultimate stress. Since the energy stored in the spring is suddenly released when the specimen breaks, a dashpot, mounted on the frame of the machine, is attached to the upper lever close to the spring. The speed of return of the lever is very large compared with its speed during a test, and a solid loose-fitting piston in a slightly tapered oil-filled cylinder is completely satisfactory. A counterbalance weight on the upper lever keeps the spring under slight tension.

The load is indicated on the dial and recorded

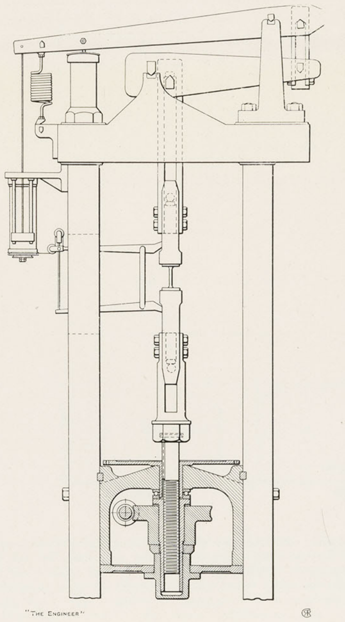


FIG. 2

by the autographic mechanism. A steel tape from the upper lever passes round a pulley on the shaft carrying the pointer, and a small weight from a second pulley keeps a constant tension in the steel tape. The dial is provided also with a free pointer to indicate the maximum load. The pencil carrier for the autographic recorder—Fig. 3—slides on vertical guides, and is attached to the upper lever by a long rod; the pencil is mounted on a piece of thin spring steel, and can be held off the drum by a small hook.

Measurement of Elongation.

In order to avoid the difficulties introduced by the lateral contraction which accompanies elongation, it was decided to use as a strain gauge the "scissors" mechanism shown in Fig. 3. The ends of the scissors are fitted with steel balls which rest in the centres on which the specimen has been turned. The outer ends are coupled by a push-rod which rests in the lower arm and passes through a swivelling rod in the upper arm. This rod projects into a horizontal slot in a bracket on the frame of the machine, so providing support for the outer end of the device without imposing lateral constraint on the shackles, and allowing the specimen to move down as a whole when the load is applied, without affecting the motion of the drum. An adjustable spring in the drum provides sufficient tension in the cord to hold the whole mechanism together. The strain is measured over the whole length of the specimen instead of the more usual 1in. gauge length, but since the part which stretches is limited to a length little greater than the $1\frac{1}{4}$in. parallel section, the two elongations are directly comparable. In Table I. the elongations of twelve

Table I.

Specimen number.	Gauge extension.	Overall extension.	Gauge extn. / overall extn.
A 10	0·38	0·43	0·88
A 11	0·41	0·48	0·85
A 12	0·40	0·44	0·91
A 13	0·41	0·47	0·87
A 14	0·415	0·47	0·88
A 15	0·41	0·46	0·89
A 16	0·415	0·47	0·88
A 17	0·415	0·47	0·88
A 19	0·39	0·45	0·87
A 20	0·41	0·47	0·87
A 22	0·41	0·46	0·89
A 23	0·405	0·45	0·90

consecutive specimens as measured on 1in. gauge length and overall are compared; the ratio is practically constant in spite of the fact that the specimens were turned individually; had they been made to a jig, the figures would probably have been even closer. For the mild steel specimens used, the extension on a 1in. gauge length was 0·88 times the overall extension.

Test Results.

The following tests illustrate the usefulness and accuracy of the machine. In Table II. are given

Table II.

Specimen number.	Yield stress, tons/in.²	Plastic yield stress, tons/in.²	Ultimate stress, tons/in.²
A 90	23·2	17·8	29·2
A 91	23·0	17·8	29·2
A 92	23·4	17·8	29·4
A 93	24·5	19·5	30·2
A 94	24·6	19·5	30·5
A 95	24·8	19·6	30·4
A 96	—	20·9	30·9
A 97	—	20·8	30·8
A 98	—	20·6	30·8
A 99	23·0	16·5	28·3
A 100	22·6	16·5	28·3
A 101	22·6	16·8	28·4

results from twelve specimens, cut from a length of aircraft standard 0·2 per cent. carbon steel, and normalised after machining. They were tested under four different conditions, and in each case the maximum variation between the three individual specimens is less than ± 1 per cent. The autographic records, reproduced for one set in figures 4*a*, *b*, and *c*, are indistinguishable in appearance. In Figs. 5*a* and 5*b* are shown typical diagrams for normalised 0·2 and 0·6 per cent. carbon steels.

Table III. and Figs. 6*a*, *b*, and *c* refer to specimens cut from the same 24in. by $7\frac{1}{2}$in. R.S.J. section for which results obtained both in works tests and in tests on similar small machined specimens have already been quoted.* When allowance is made for the lack of uniformity of the material and the rates of strain adopted, it will be seen that the figures for the tests already carried out with extensometers and those for the tests in this machine are in satisfactory agreement. In addition, the character of the autographic diagram shows at a glance the condition of

Table III.

Specimen number.	Yield stress.			Plastic yield stress.		Ultimate stress.		
	Works test.	M.V.T.C. test.	Present test.	M.V.T.C. test.	Present test.	Works test.	M.V.T.C. test.	Present test.
J.D. 1	17·2	17·0*	21·4	—	17·9	32·5	31·7*	32·0
Ditto		18·0†	18·7	15·9†	16·7		32·5†	32·5
J.D. 2	16·2	14·0*	14·9	—	14·7	32·0	30·2*	31·9
Ditto		14·0†	14·6	13·1†	14·5		31·3†	31·8
J.D. 3	18·2	15·0*	18·8	—	16·9	32·5	30·0*	32·1
Ditto		16·6†	16·6	14·7†	15·7		31·5†	32·2
J.D. 4	19·0	14·0*	16·2	—	15·5	32·6	29·5*	30·5
Ditto		15·0†	16·5	13·4†	15·1		30·4†	30·4
J.D. 5	19·4	16·0*	17·0	—	16·1	33·2	30·0*	32·2
Ditto		16·8†	17·1	14·8†	15·8		31·5†	31·8

* Large specimen (flat). All works tests on large specimens.

† Small specimen (turned). All present tests on small specimens.

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* Communicated by the Director of Building Research, Department of Scientific and Industrial Research.

* Steel Structures Research Committee, First Report of "Materials" Panel.

the material, where an ordinary test would give no indication of the reason for abnormal results. Thus the three diagrams reproduced are for specimens taken from (*a*) the tip of the flange, (*b*) the root, and (*c*) the web of the section.

Sir A. B. W. Kennedy and Professor Ashcroft have used autographic recorders, and Professor Dalby† has demonstrated exhaustively the value of accurate autographic diagrams in the testing of materials, with particular reference to the effects of heat treatment and cold working. Figs. 4 and 7 show the type of diagram which can be obtained from this machine. They have as their chief interest their value for teach-

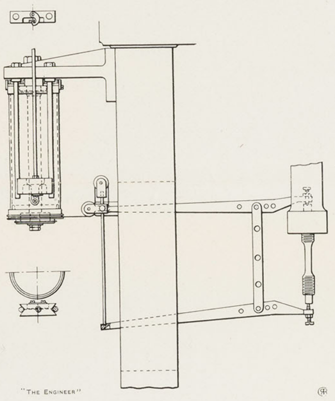


FIG. 3

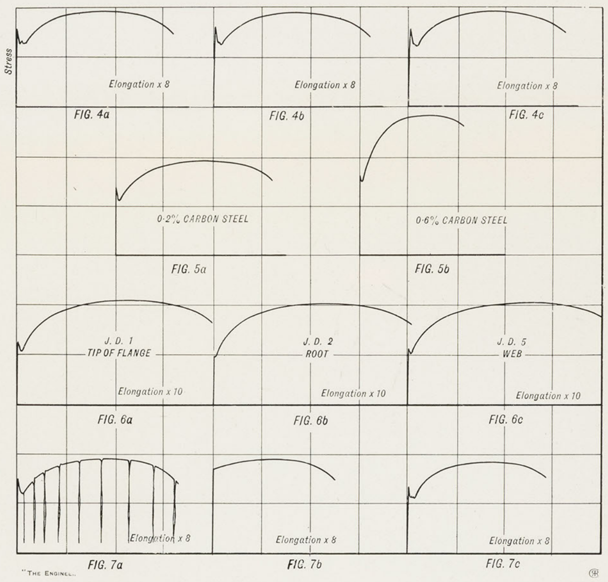
ing purposes. Six similar specimens have been tested, the first three (Fig. 4) normally. The fourth test was interrupted several times and the load reduced almost to zero (Fig. 7*a*); the fifth specimen was overstrained to about 10 per cent. elongation before being tested (Fig. 7*b*); and the sixth (Fig. 7*c*) overstrained in exactly the same way, but renormalised before test. When allowance is made for the reduced cross-sectional area the last diagram is almost identical with the original tests. Similar tests can readily be devised to demonstrate graphically, for instance, the results of pauses in loading, of different heat treatments before or after overstrain, or the characteristics of ductile and brittle materials.

Machining of Specimens.

The machine at present has been designed simply for turned specimens, since for accurate results such specimens are essential. The screwed end type of specimen was chosen for convenience, since it was desirable to be able to make specimens from small-diameter material. In the case of commercial testing, it would probably be more satisfactory to use collar ends in order to provide more room for an inspector's type marks on the test piece. In special cases where, for example, an accurate yield value is not needed, or where it is desirable that each of a number of students should carry out a complete test for himself, the cost of machining the specimens may not be justified by the additional accuracy. In these cases it would be an easy matter to design wedge grips to replace temporarily the screwed adaptors in the shackles.

Use of Large or Small Specimens.

The use of small specimens may be criticised on the grounds that they do not represent the general quality of the material. It may be suggested, on the other hand, that since even a normal "large" specimen is small compared with a full-sized member of a structure, and since these small specimens are very large compared with the crystal size of the material, the advantage in using large specimens is fictitious. This is emphasised in the diagrams, Fig. 6, which are fairly representative of the usual conditions in a rolled section. The material is not in a uniform state, and the larger the specimen the more does the true condition tend to be masked by the average condition of



the part from which the specimen has been cut. Even if the full section is tested, it is more than doubtful if the results are reliable, since the values obtained are an average for the section and unless under actual conditions the member is subjected to exactly the same type of loading the stress distribution may be quite different. The least unsatisfactory method of testing therefore would seem to be to select small specimens representative of the different material in the section.

If a reliable value for the yield stress is to be obtained, it is quite certain that present testing practice must be considerably modified. The tests must be made at a much lower speed, preferably in a machine of small inertia, mechanically driven at a constant rate. In a test department where large numbers of tests are conducted, this must inevitably mean more testing machines, and here the small specimen offers a very considerable economy, particularly if, as the author would recommend, each machine is designed as practically a single-purpose machine.

Compression Tests.

As was stated at the outset, the present machine was designed simply for tension tests. It is possible, however, that in the future compression tests may also be included in some specifications. It would not be difficult to alter the machine to allow compression diagrams to be obtained in an exactly similar manner. The axial loading shackles would be replaced by tie rods passing each other to apply a compression load to a specimen between them. In order to obtain axial loading, the compression specimen would be mounted concentrically between two coaxial plungers sliding in a frame supported from one of the side columns.‡ A slight modification of the scissors device would also be required, but the machine is otherwise suitable for compression testing.

Acknowledgments.

The construction of the machine was made possible by a grant to Professor Andrew Robertson, D.Sc., M. Inst. C.E., M.I. Mech. E., by the Building Research Station, and the author acknowledges with thanks this assistance. He wishes also to express his indebtedness to Professor Robertson for his constant interest and help in the work.

† "Strength and Structure of Steel," W. E. Dalby (Arnold).

‡ See description of compression testing apparatus in I.C.E. Selected Paper No. 28, "The Strength of Struts," by Professor Andrew Robertson.

Reprinted from The Engineer, 24th August, 1934

The Influence of Rate of Strain in Tension Tests.

By JOHN L. M. MORRISON, B.Sc.

IN the commercial testing of steel the three criteria of quality which have in the past received most attention are the ultimate tensile stress, the ductility as measured by the elongation and reduction of area of the test piece, and the ability to withstand without fracture bending tests. In very many instances, however, these properties have little direct bearing on the effective strength of a structure. It is obvious that, apart from local yielding which may relieve high local stresses, it is extremely undesirable that the stress in even the tension members of a structure should rise to the yield point of the material, since though fracture may not occur, the large consequent permanent deflections are likely to cause trouble in other parts of the structure. In the case of struts and beams the yield stress is even more immediately important, since it, and not the ultimate stress, determines the failure load.

It must, of course, be recognised that by far the greater part of commercial testing of mild steel is carried out with a view to ensuring the uniformity and general reliability of the product, the tacit assumption being that if the tensile strength and elongation reach a certain standard the steel is also satisfactory in every other respect. With the huge accumulation of experience which is behind the manufacture and use of mild steel, this attitude may be a justifiable one; there is, however, a growing tendency on the part of users to ask for steel with a high and definite yield point. In the case of high-tensile steels the position is rendered more acute by the comparatively slight experience of them which manufacturers and users have, and by the fact that the yield point of such steels is often more difficult to determine than is the yield point of mild steel.

The importance of the yield phenomena in the case of struts was pointed out in 1887 by Marshall,[1] who concluded that their compressive strength within a considerable range was governed by the elastic limit of the material. The results of Buchanan,[2] Jensen,[3] Howard,[4] and Robertson[5] all go to prove that the yield stress of the material determines the strength of the vast majority of practical columns. The Special Committee on Steel Columns and Struts of the American Society of Civil Engineers[6] concluded in their final report (1933) that "The ultimate strength of such columns" (hinged ends and eccentric loads) "agrees fairly well with the theoretical strength based upon the yield point of the material . . . " and that "The yield point strength of the material may be used in a formula for ultimate strength." That the same conclusion has been accepted in Great Britain is indicated by the statement in the first report of the "Struts" panel of the Steel Structures Research Committee of the Department of Scientific and Industrial Research that "it is now generally agreed that for the maximum allowable stress in a column the yield strength of the material is the significant factor."

Measurement of Yield.

If, then, it is agreed that the yield stress of the material is of paramount importance, it is obvious that the definition of yield must be standardised, and the method of measuring it above suspicion. There is no doubt, however, that the present position in these respects is unsatisfactory. In the report of the American Society of Civil Engineers Committee, already referred to, it is stated: "It is evident that the yield point, as recorded by the ordinary com-

1 "Trans.," Amer. Soc. Civ. Eng., Vol. XVII.
2 *Eng. News*, Dec. 26th, 1907.
3 *Engineering*, April 3rd, 1908, p. 433.
4 "Proc.," Amer. Soc. for Testing Materials, Vol. VIII.
5 Brit. Ass. Report, 1913; Inst. Civ. Eng. Selected Paper No. 28.
6 "Trans.," Amer. Soc. Civ. Eng., 1918 and 1933.

mercial tests, even when the machine is run at comparatively low speeds, does not give the correct index of the strength of the material."

The first serious difficulty presents itself in the securing of uniform stress distribution over the cross section of the test piece. It may be accepted that the use of wedge grips or even of spherically seated shackles is inadmissable since the yield stress obtained with these methods of gripping the specimen may lie, fortuitously, anywhere between the true yield and the plastic or lower yield. By plastic yield stress is meant the reduced, practically uniform, stress at which the material continues to stretch after the yield point has been reached, and it will be evident that since in the case of an eccentrically loaded tension test piece yield at the highly stressed side will tend to straighten it and to equalise the stress distribution, yield of the test piece as a whole will not be observed until the average stress is at least equal to the plastic yield stress. In the present tests axial loading shackles, similar in principle to those designed by Robertson and Cook, and described in *Engineering* (1911, Vol. 2, page 786), were used. It need hardly be stated that if any advantage is to be derived from the use of such shackles, the test piece itself must be machined; the pattern adopted was the British Standard 0.282in. diameter specimen with screwed ends, and the whole machining of the parallel part and of the screwed ends was carried out on centres in a lathe.

The second difficulty lies in the definition of, and method of measuring, the yield stress. There are two cases here which must be considered separately. If the material under test gives a distinct drop of stress at yield and a large plastic extension, a carefully conducted test, in which the speed is kept low and axial loading shackles are used, will give a reliable value for the yield if this is determined by the usual drop-of-beam method, or even by dividers. But failure to observe any one of these conditions renders the results unreliable. If even a moderate rate of loading is used the effect of the inertia of the testing machine may be considerable, while it is only too easy for the operator to overshoot the load. Even the interpretation of the terms "floating beam" and "drop of beam" may lead to a considerable difference —as much as 2 tons per square in.—in the result of a test. If, on the other hand, the material gives a stress-strain diagram in which the elastic portion curves round gradually till the plastic state is reached, identification of the yield point by any of the usual methods is quite impossible. The only reliable method is the use of an accurate extensometer; even then it may be difficult to justify the choice of any particular figure unless the yield be taken as that point at which the slope of the tangent to the stress-strain curve has a definite fraction of its original value, and in this case it would probably be better not to use the word "yield" for the result, but some other term, such as the "useful limit." The tests described in this paper were all conducted on material giving a definite drop of stress, and in order to obtain the yield a special form of autographic recorder of stress and strain was employed.

When satisfactory conditions for the determination of the yield point and for axial loading of the test piece have been obtained, there still remains the uncertainty with which these experiments are primarily concerned. The yield stress, according to the British Engineering Standards Association, is to be measured "when the load is increased at a moderately fast rate," a vaguenesss which is inseparable from testing as conducted at present. In a report of the American Society for Testing Materials Yield Research Committee, constituted in 1926 in order to investigate the significance of the yield point, its range in commercial structural steel, and the commercial testing procedure, it is stated that "increasing the speed of loading" (*i.e.*, movement of head with 10in. gauge length) "from 0.05in. to 2.0in. per minute caused the following average increase in the yield point of the bar steels tested:—Carbon steel, 9 per cent.; nickel steel, 7 per cent.; silicon steel, 12 per cent."

Such figures point to the desirability of defining more accurately the speed of testing. These tests were carried out in five laboratories, but in all cases wedge grips were used, and the range of speeds was not very great. Four methods of determining the yield point were used—drop of beam, scaling, multiplying dividers, and a strain gauge. In the last method yield was taken to be "the lowest load at which there was visible acceleration in the rate of motion of the dial hand on the gauge." Of these different methods, "the multiplying dividers gave the most reliable results."

Testing Machines.

It was therefore thought desirable to carry out a series of tests over as large a range of speeds as possible, using axial loading shackles, and determining the yield point autographically. Two machines were used, one for very low to moderate speeds, reaching yield in from 12 hours to 16 seconds, and the other for the range 16 seconds to about one-twentieth of a second. In the first machine[7] the applied load is transmitted through the specimen, which is held in axial loading shackles, and a double lever, to a spring which measures the load. The load and extension of the specimen are recorded autographically, but if the rate of strain in this machine is increased much beyond the limit chosen, the errors due to the inertia of the moving parts become appreciable. Another testing machine[8] was therefore used

[7] Described in The Engineer, Vol. 157, p. 626 (June 22nd, 1934).

[8] Described in Aeronautical Research Committee "Reports and Memoranda," No. 1161 (Robertson and Newport).

for higher speeds. In this second machine the load is applied through the same axial loading shackles to the specimen and to a tube arranged in tandem and provided with an extensometer. The load is measured by the extension of this tube, which causes the mirror of an extensometer to rotate. A beam of light is reflected successively from a mirror fixed to the frame of the extensometer, and from the moving mirror, and focussed on a revolving drum on which a photographic film is mounted. Movement of the extensometer as a whole does not affect the position of the reflected beam on the revolving drum. Since the speed of the drum is known, the rate of strain is easily calculable. In the slower machine, the times to reach the yield point and the breaking point are measured by a stop watch.

Each machine is driven through gearing by an electric motor, so that the rates of strain up to yield and from yield to break are each sensibly constant. They are not identical, since the elasticity of the machine is negligible compared with the plastic extension though not compared with the elastic extension of the test piece. A wide speed range on each motor and external gearing are used to obtain the different rates of strain.

Materials and Heat Treatment.

Two steels were used for these tests, one an aircraft standard (2 S 21), 0.2 per cent. carbon steel, and the second an open-hearth steel made by the Park Gate Company, Ltd., to the specification C 0.58 per cent., Si 0.16 per cent., S 0.037 per cent., P 0.033 per cent., Mn 0.54 per cent. The material was received in the form of ½in. black bar, 16ft. long. It was cut into test bar lengths, and normalised to eliminate any possible cold working effects. The test pieces were then machined, and again heat treated by being kept in a vacuum furnace for a quarter of an hour at 910 deg. Cent. in the case of the 0.2 per cent. carbon steel, and 860 deg. Cent. in the case of the 0.6 per cent carbon steel. The vacuum tube was removed from the furnace to cool, but the test pieces could not, of course, be taken out of the tube. As a result, the heat treatment tends to make the steel slightly softer than ordinary normalising, but also tends to promote uniformity.

Results.

(*a*) *Stress*.—The numerical results of the tests are summarised in Tables I and II, and typical load-extension diagrams for the low-speed machine and load-time diagrams for the high-speed machine are reproduced in Figs. 1 to 3.

The curves, Figs. 4 and 5, give the results of the whole series of tests on both types of steel. Each point represents the average of all the specimens tested at a particular speed. The tabulated values show the extent of the variation between individual specimens. The curves are plotted to a base of the logarithm of the strain per second, as it is, of course, impossible otherwise to accommodate the time-scale variation, and the approximate times for a test piece to reach yield and to be broken at various rates of strain are indicated.

While all the stress values tend to rise with increased speed of testing, the form of the stress/rate-of-strain curves differs. The value of the yield stress seems to approach asymptotically at low speeds a definite minimum value, and rises very rapidly at the other end of the rate-of-strain scale. The plastic yield and ultimate stresses, on the other hand, suggest that the material after yield behaves like an extremely viscous fluid. They do not increase so rapidly at high speeds, but they give little indication of approaching a fixed minimum value at the lowest speeds used. Thus in the case of the 0.6 per cent. carbon steel the ultimate stress in the 20-hour test is 1 ton per square inch less than in the 9-minute test, while in the case of the 0.2 per cent. carbon steel the values at the same rates of strain differ by 1.2 tons per square inch. In the first complete series the slowest tests on low carbon steel took about 30 hours ; a second series covering again the lower end of the range was therefore carried out to check the absence of a definite minimum stress. In this case the slowest tests took between four and five days, but the new results, though not coinciding exactly with the first, probably because of different heat treatment, gave a very similar shape for the stress/rate-of-strain curve.

Over the range of speeds used—that is, from about 4 days to 0.8 second for the complete test at approximately uniform rate of strain—the increases in the case of the 0.2 per cent. carbon steel were:

Yield stress	6.2 tons per square inch,	or 27 per cent.
Plastic yield stress	6.6 tons per square inch,	or 40 per cent.
Ultimate stress ...	4.4 tons per square inch,	or 15 per cent.

while in the case of the 0.6 per cent. carbon steel, in tests lasting from 20 hours to 0.4 second, the increases were:

Yield stress	7.9 tons per square inch,	or 33 per cent.
Plastic yield stress	6.7 tons per square inch,	or 30 per cent.
Ultimate stress ...	4.4 tons per square inch,	or 10 per cent.

(*b*) *Elongation*.—The rate of testing has surprisingly little influence on the elongation. The results are shown in column 7 of Tables I and II. It will be noticed that the elongation is given for only a few of the tests in the high-speed machine. The stress extensometer in this machine was apt to be thrown out of adjustment when the specimen broke ; the tests, except in the case of the highest speeds, were therefore, stopped when the load began to fall off appreciably after the maximum. The figures obtained, however, cover the whole range of speeds used, and it will be seen that the variation between individual specimens is greater than any effect found due to rate of testing.

Commercial Testing.

In view of these results, it may not be out of place to put forward some suggestions as to commercial testing procedure, with particular reference to the present experiments.

In the case of tests which are required to determine only the ultimate stress and the elongation, there is one point which calls for comment. If a steel is produced to a 5-ton limit—for instance, the ordinary 28—33-ton mild steel—then a factor which can produce a change of 4.4 tons per square inch in the ultimate stress should not be left to the discretion of the operator. It is not suggested, of course, that such a large variation will occur in practice, especially since the majority of operators will, after a short time, tend unconsciously to standardise their methods; but it is apparent that in a border line case the acceptance of steel might depend on the speed of testing, and that tests under works conditions cannot be directly compared with tests under laboratory conditions. The remedy would seem to lie in the use either of a mechanically operated machine driven from a constant-speed motor, or a hydraulic machine with a needle valve controlling the rate of flow of the fluid to the cylinder of the testing machine.

In the case of tests which are to include the yield stress this constant-speed feature is even more desirable, since the speed variation which certainly does occur in practice between different machines at the same works, and between works and laboratory tests, may be responsible for at least a 10 per cent. difference in the results, but the other points which have already been discussed must also be provided for. There are three main requirements: (1) The use of a machine of small effective inertia ; (2) axial loading shackles, and (3) a more satisfactory definition of the yield point. The importance of keeping the effective inertia of the machine to a minimum value is well illustrated by a simple experiment. If a specimen is lightly loaded in a large single-lever machine, and fitted with an extensometer, moving the beam up and down an inch of two is quite sufficient to alter the stress in the specimen by several tons per square inch, while it is easy to over-strain the specimen from almost no load to beyond the yield point simply by pulling the beam from the top to the bottom stop. But at the speeds in common use in commercial testing it is quite impossible to keep the beam floating steadily midway between the stops, and a vibration of an inch or two when the specimen is approaching yield may completely mask the yield phenomena. In the case of mild steel the effect would probably be to give the plastic yield stress instead of the true yield stress, an error which is on the safe side when estimating the quality of the steel. In the case of a high tensile steel, however, where the plastic region is much less extensive, sometimes almost non-existent, the error is more serious. If, as is sometimes the case, the extension at yield is not sufficient to allow the beam to drop decidedly, the yield is taken as the stress at which two clearly distinguishable lines can be scribed with dividers, using an 8in. gauge length. There can be no doubt that the state corresponding to this definition is not, as intended, somewhere in the plastic yield region, but at a considerably later state in the extension, with the result that the yield stress reported bears little relation to the true yield. The necessity for axial loading shackles has been stressed several times, but it may once again be emphasised that the use of wedge grips in such a test helps still further to make the transition from the elastic to plastic states less definite.

If, then, yield determinations are to be included in the test results of any steel, the following are some of the conditions which are desirable to secure reasonable accuracy. In the case of a steel with a well-marked yield point—that is, with a definite drop of stress at yield under constant rate of strain :—

(1) The use of a machine with small inertia.

(2) The standardisation of a constant rate of strain during the test.

(3) The use of axial loading shackles and machined specimens.

(4) Yield to be measured by drop of beam or flicker of pointer, and, preferably, in doubtful cases, to be checked by an autographic record.

In the case of a material with a poorly defined yield point, it is preferable to discontinue the use of the term " yield stress." In order to determine the stress term suitable as a basis for design two methods are possible. The first, as used in the testing of certain Admiralty and aircraft materials, is to apply a " proof stress " to the specimen, and to measure the permanent extension produced. The second, originally proposed by Johnson, and advocated by the Steel Columns Committee of the American Society of Civil Engineers[9], is to give the name " Useful Limit " to the point where the gradient of the stress-strain curve is reduced to one-half of its original value. For steels this value appears to be appropriate, though for others materials a different fraction may be found to be more satisfactory Both methods involve the use of an accurate extensometer ; there is little to choose between them as regards saving in time, though the advantage lies with the proof stress. The determination of the U.L.P., however, gives considerably more information as to the state of the material and its quality, and in the author's opinion is the preferable test.

Acknowledgment.

The author wishes to express his indebtedness to Professor Andrew Robertson, D.Sc., M. Inst. C.E., M.I.Mech.E., for his constant help and encouragement in the work, which was carried out in the Merchant Venturer's Technical College, Bristol.

[9] " Trans.," Am. Soc. Civ. Eng. Vol., LXXXIII. p. 1618

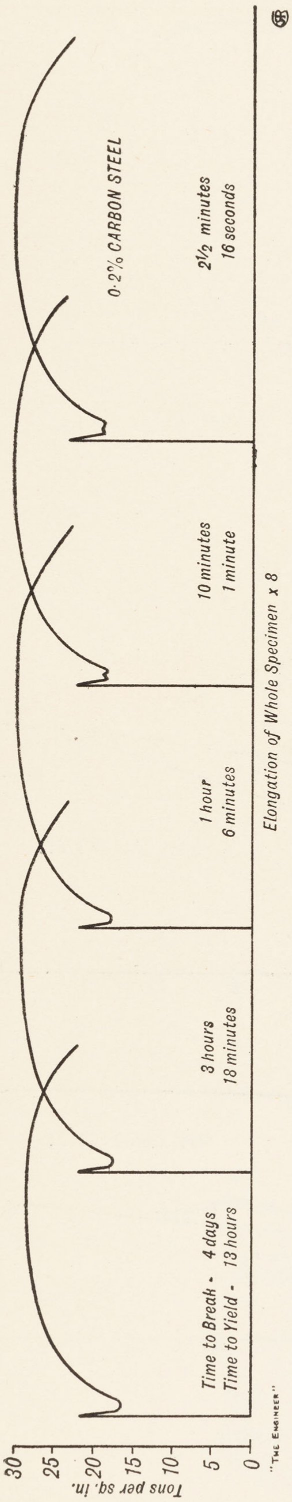


Fig. 1—Load-Extension Diagrams for Various Rates of Loading.

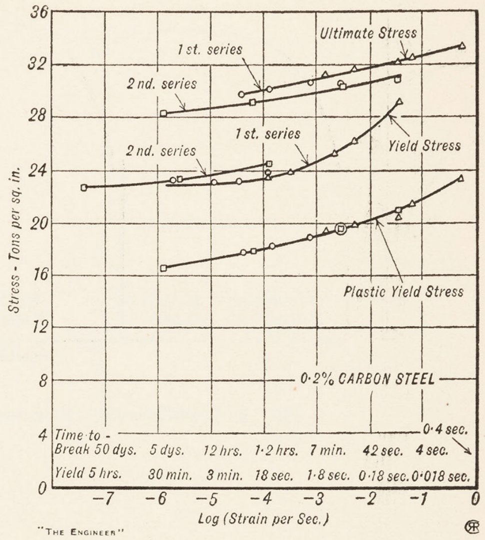
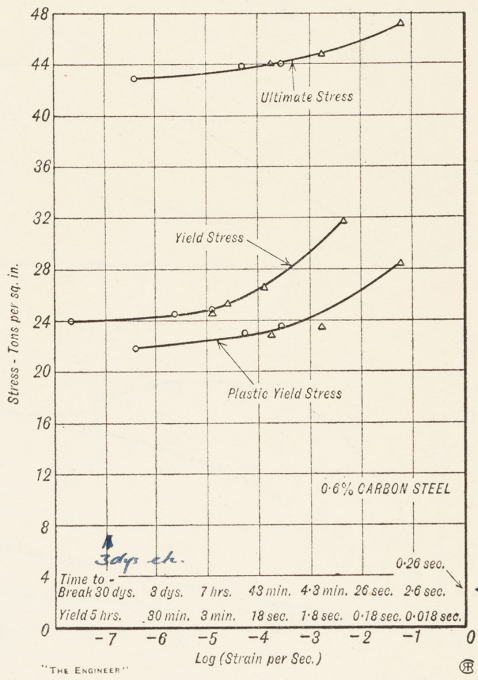


Fig. 4—Variation of Yield and Ultimate Stress with Rate of Strain.



SCALES SHOULD BE MOVED ONE SQUARE TO LEFT.

Fig. 5—Variation of Yield and Ultimate Stress with Rate of Strain.

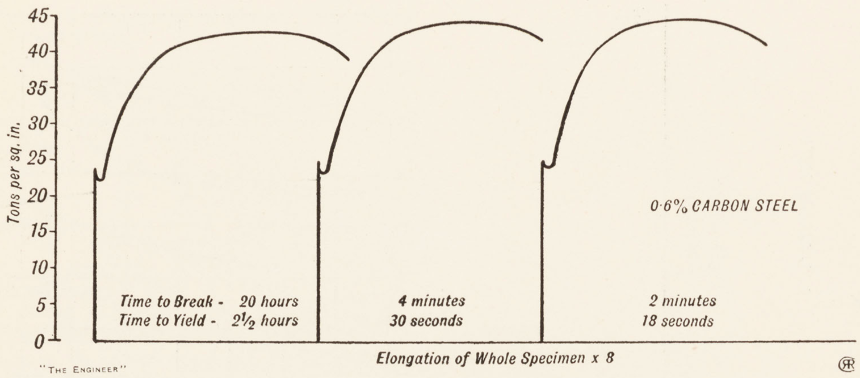


Fig. 2—Load-Extension Diagrams for Various Rates of Loading.

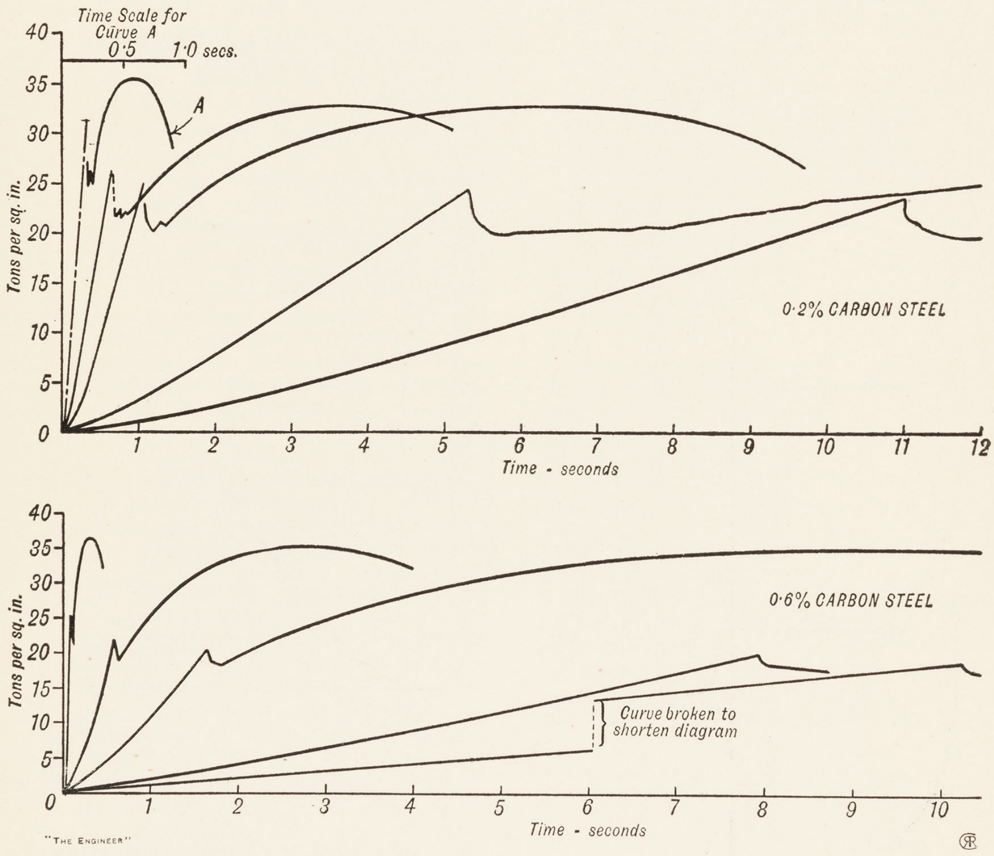


Fig. 3—Load-Time Diagrams for Rapid Rates of Loading.

TABLE I.—RESULTS OF TESTS ON 0.2 PER CENT. C. STEEL.

Tests in Low-speed Machine.

Specimen No.	Time to yield (sec.).	Time to break (sec.).	Yield stress (ton/in.^{2})	Plastic yield stress (ton/in.^{2})	Ultimate stress (ton/in.^{2})	Elongation (in.)
A 71	1,000	11,000	23.8	17.8	29.6	0.41
A 89			22.7	17.8	30.2	0.42
Mean			23.2	17.8	29.9	0.42
A 69	170	3,000	22.9	18.2	30.2	0.44
A 70			22.7	18.2	30.2	0.45
A 86			24.0	18.6	30.1	0.42
A 87			22.2	18.1	29.6	0.40
Mean			23.0	18.3	30.0	0.43
A 67	55	570	23.7	19.2	31.0	0.44
A 68			22.6	18.7	31.2	0.44
A 84			22.7	18.6	30.4	0.43
A 85			23.5	19.0	30.4	0.43
Mean			23.1	18.9	30.7	0.435
A 65	16	150	23.8	19.4	31.4	0.44
A 66			22.0	19.4	29.8	0.44
A 82			25.4	20.0	30.8	0.41
A 83			24.2	19.4	30.9	0.44
Mean			23.8	19.5	30.7	0.43
Repeat Tests in Low-speed Machine.						
A 99	48,000	360,000	23.0	16.6	28.3	0.38
B 1			22.6	16.6	28.3	0.36
B 2			22.6	16.8	28.4	0.43
Mean			22.7	16.6	28.3	0.39
A 90	750	7,000	23.2	19.8	29.3	0.40
A 91			23.0	17.8	29.2	0.41
A 92			23.5	17.8	29.4	0.42
Mean			23.2	17.8	29.3	0.41
A 93	15	140	24.5	19.5	30.2	0.45
A 94			24.6	19.5	30.5	0.41
A 95			24.7	19.7	30.4	0.43
Mean			24.6	19.6	30.4	0.43
A 96	—	13	Speed	20.9	30.9	0.39
A 97			too	20.8	30.8	0.38
A 98			high	20.7	30.8	0.40
Mean				20.8	30.8	0.39
Tests in High-speed Machine.						
A 54	16	240	23.8	20.2	31.7	—
A 55			23.2	19.3	31.5	—
A 56			23.2	19.3	31.5	—
A 80			23.2	19.4	30.9	—
A 81			23.7	19.0	30.7	—
Mean			23.5	19.4	31.3	—
A 57	5.5	80	24.2	20.1	32.0	—
A 58			24.3	19.9	31.8	—
A 78			24.0	19.9	31.4	—
A 79			23.0	19.4	31.4	—
Mean			23.9	19.8	31.6	—
A 59	0.85	12	25.3	21.3	32.6	—
A 60			24.8	20.0	32.6	—
A 76			24.8	20.1	31.8	—
A 77			25.8	19.8	31.8	—
Mean			25.2	20.3	32.2	—
A 61	0.35	6	26.9	21.6	32.9	—
A 62			26.2	22.4	32.8	—
A 74			26.1	20.6	32.6	—
A 75			25.8	20.7	31.8	—
Mean			26.2	21.3	32.5	—
A 63	0.052	0.8	29.9	23.5	33.4	0.42
A 64			29.8	23.2	33.8	0.41
A 72			28.3	22.7	32.9	0.44
A 73			28.6	24.0	33.6	0.41
Mean			29.1	23.3	33.4	0.42

TABLE II.—RESULTS OF TESTS ON 0.6 PER CENT. C. STEEL.

Tests in Low-speed Machine.

Specimen No.	Time to yield (sec.).	Time to break (sec.).	Yield stress (ton/in.^{2})	Plastic yield stress (ton/in.^{2})	Ultimate stress (ton/in.^{2})	Elongation (in.)
C 6	9,000	70,000	23.4	21.9	43.3	0.235
C 23			24.5	22.0	42.7	0.27
C 41			24.2	21.9	43.0	0.295
Mean			24.0	21.9	43.0	0.267
C 5	80	550	24.6	22.8	45.0	0.24
C 22			24.5	23.4	44.3	0.27
C 38			24.9	22.8	44.3	0.30
C 39			23.7	23.0	41.8	0.26
C 40			24.8	23.0	41.6	0.27
Mean			24.5	23.0	44.0	0.268
C 2	18	100	24.8	23.5	45.2	0.24
C 3			24.8	23.6	45.4	0.25
C 19			24.3	23.0	41.9	0.28
C 20			25.9	23.6	44.3	0.28
C 21			24.8	24.0	44.5	0.27
C 37			24.3	23.5	43.3	0.26
Mean			24.8	23.5	44.1	0.263
Tests in High-speed Machine.						
C 7	17	150	24.5	22.4	44.0	
C 8			24.5	23.0	44.3	—
C 24			25.4	23.0	44.2	—
C 25			24.8	23.2	43.9	—
C 42			23.8	22.9	44.3	—
C 43			24.3	23.0	44.4	—
Mean			24.6	22.9	44.2	—
C 9	8	120	25.1	22.9	44.4	—
C 26			25.8	23.4	43.9	—
C 44			25.1	22.9	43.9	—
Mean			25.3	23.1	44.1	—
C 10	1.5	15	26.4	23.4	45.2	—
C 12			26.1	22.8	45.1	—
C 27			27.3	23.5	44.3	—
C 28			25.8	24.0	44.6	—
C 45			26.7	23.8	45.2	—
C 47			27.3	23.7	45.1	—
Mean			26.6	23.5	44.9	—
C 13	0.04	0.4	30.9	28.9	44.6	0.23
C 14			31.5	28.6	48.1	0.24
C 31			32.5	28.0	47.4	0.28
C 32			32.9	29.8	47.8	0.29
C 50			31.5	28.2	46.7	0.26
C 51			32.3	29.6	48.0	0.26
Mean			31.9	28.6	47.4	0.26

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WELBECSON PRESS LTD.
London, W.11