Conventional, Unconventional, and Macro-Prudential Optimal Policy.

by

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Introduction

As a consequence of the Great Recession (2007-09), the standard New Keynesian model for analyzing optimal policy has changed from assuming frictionless financial markets to including financial rigidities. These changes render the new framework suitable for analyzing the interaction between macroeconomic policy and financial events.

In the present dissertation, I analyze optimal monetary, unconventional, and macro-prudential policy under commitment. I make use of a model with a banking sector that faces balance sheet constraints.

In order to emphasize the role played by monetary policy in containing financial instability, in the first chapter the sole policy instrument is the nominal interest rate. Then, I allow the central bank to make use of additional policy instruments. In the second chapter, the central bank can undertake purchases of private securities. Finally, the third chapter considers the optimal mix between monetary and prudential policy.

Chapter 1. In order to emphasize the role played by the monetary policy in containing financial instability, I assume that the sole policy instrument is the nominal interest rate. The main distortions in this economy are: the monopolistic competition, sticky prices, and the balance sheet constraint of banks. Sticky prices allow monetary policy to have real effects. This friction interacts with the financial distortions and create trade-offs for the central bank. If a financial shock hits, the gap between the actual and the efficient allocations widens. This fluctuation is costly and the central bank attempts to stabilize the financial market, but the cost is fluctuation in inflation. The main result of this chapter is that financial events matter. Stabilizing the financial sector is welfare improving, but with only one policy instrument the central bank cannot stabilize inflation and financial variables at the same time. A modified Taylor rule that consider a feedback parameter on the deviations of the cost of credit from its steady state level can implement the optimal policy.

However, in this framework there are more objectives than policy instruments. In the next step, I allow the central bank to use asset purchases of private securities and I deal with the optimal mix of conventional and unconventional monetary policy.

Chapter 2. In this chapter, I extend the model in chapter 1 in order to allow central bank to undertake direct lending to firms. Asset purchases is the unconventional policy instrument. In this framework, the central bank affect the price of credit (interest rate) and the provision of credit (lending in the private credit markets). The nominal interest rate influences the cost of credit. The credit intermediation by the central bank seeks to influence the availability of and the price of credit. Together, the conventional and unconventional policy can serve to stabilize inflation and the financial markets. The central bank can implement the optimal policy by means of two policy rules: the conventional Taylor rule which sets the nominal interest rate, and an asset purchases rule.

Unconventional monetary policy can give a hand to conventional policy in order to stabilize inflation and financial activity. However, if the central bank cannot access to unconventional means to stabilize the economy, monetary policy would still need support from other branches of policy in order to achieve price and financial stability. Even if the economy can be stabilized with monetary policy alone, the question is can it be stabilized more effectively with macro-prudential policies working alongside monetary policy? The model in chapter three is designed to answer this question.

Chapter 3. In this chapter, I consider the optimal policy mix between monetary and prudential policy. I make substantial modifications to the model used in chapters 1, and 2, in order to make it useful in assessing macro-prudential policies consistent with the evidence.

In the model, the banks face balance sheet constraints. They lend to households and firms. Agent are heterogeneous: firstly, they are poor or rich; secondly, the groups differ by their degree of patience; thirdly, as in the empirical evidence, the poorest contribute more to aggregate consumption than to the aggregate disposable income, I capture this by allowing the poor-borrowers to possess external habits, while the rich-savers possess internal habits in consumption.

The habits externality drives these agents to overconsume and to overborrow. Given that consumers with external habits overborrow from banks, there are motives to introducing reserve requirements as a prudential instrument.

The reserve requirement acts to reduce the overconsumption. The increase in the reserve requirement makes the credit more expensive and the central bank can stabilize the economy when the shocks hit.

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Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

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Chapter 1

Financial Frictions and Optimal Policy.

Abstract.

I analyze optimal monetary policy in a New Keynesian model with a banking sector that faces balance sheet constraints. Additionally, I consider monetary rules that can implement the optimal policy.

In the presence of financial frictions, inflation stabilization is not as desirable as it is in their absence. Rather, financial stability becomes an objective of policy. When shocks hit, the policy maker cannot simultaneously stabilize inflation and the financial sector. In the presence of the financial accelerator, the monetary policy faces a trade off between stabilizing the cost of credit, which contribute to keep a healthy financial sector, and stabilizing inflation.

The simple rule that implements the optimal policy shows a strong reaction to changes in the cost of credit. This rule is inertial and has a small feedback coefficient on inflation.

1.1 Introduction.

The Great Recession (2007-09) has renewed interest in analyzing the role of financial events on propagation and amplification of exogenous shocks. The disruption observed in the financial markets during the crisis shows that the credit markets play a crucial role in macroeconomic stability.

The conventional New Keynesian model assumes that financial markets work perfectly. For example, Christiano et al. (2005) and Smets and Wouters (2007) develop quantitative models with several nominal and real rigidities, but assume frictionless financial markets[1].

Economic modelling has advanced in the introduction of imperfect financial markets into the conventional framework for analyzing monetary policy. For example, a moral hazard problem in

[1]Some exceptions to this are BGG(1999) and Kiyotaki and Moore (1997). However, the two previous studies focus on the qualitative aspects of the financial frictions rather than analyzing the quantitative effects of such distortions.

Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) impedes the banks issuing an efficient amount of loans to non-financial firms. They analyze unconventional policy as conducted by the Federal Reserve in the past crisis. However, they abstract from optimal policy considerations. I fill in this gap.

Following the work of Gertler and Karadi (2011), in the current model, prices are sticky, there is monopolistic competition, and a banking sector facing balance sheet constraints. There is a moral hazard problem between savers and bankers. In particular, in every period the banks can divert a share of the funds available for lending. In order to prevent this, households impose an incentive constraint on the banks. This has the effect of tying the supply of credit to the value of the capital in the bank. In this context, a shock reducing the value of the banks' assets increases the cost of credit, which leads to a fall in investment and asset prices. By directly affecting banks equity, swings in asset prices affect the cost of credit and tend to amplify movements in investment. This creates and endogenous feedback loop between asset prices and real activity. I analyze optimal monetary policy in such circumstances.

Does there exist a trade-off for optimal monetary policy in the presence of financial frictions?

The financial frictions create inefficient activity and they place an additional constraint on optimal policy. The central bank has to engineer an optimal response that stabilizes the financial sector, and inflation. However, there is only one policy instrument available. Within this framework, a productivity shock, a cost-push shock, or a financial shock are inflationary and recessionary. When negative shocks hit, the balance sheet constraints of the banks tighten. As a consequence of this, the banks reduce the supply of credit. This starts a cycle in which the initial shock amplifies the reduction in investment and the increase in the cost of credit, multiplying the effect on real activity. An optimizing central bank would seek to prevent this situation.

The main result of this chapter is that the introduction of financial frictions creates a trade-off between inflation and financial stabilization [2]. If the central bank pursues inflation stabilization, it comes at the cost of increased financial disruption and large deviations from the efficient allocation. Along this line, the central bank cannot simultaneously achieve inflation and financial stability with only one policy instrument. If the only policy instrument available is the nominal interest rate, the optimal policy trade-offs financial and inflation fluctuation. In particular, it reacts to increases in the cost of credit. If the premium on capital deviates from its long-run average, the central bank aims to reduce inefficient fluctuations in output by making the credit cheaper by contributing to the appreciation of the assets held by the financial institutions.

Price stability is suboptimal because policymakers stabilize the financial markets in order to reduce inefficient fluctuation in output. For example, if inflation rises, the typical policy of in-

[2] In the benchmark New Keynesian model it is necessary to introduce a cost-push shock to generate a non-trivial policy trade-off (Woodford (2003)). Leith et al. (2015, 2012) show how in the New Keynesian model, the presence of deep habits in consumption serves to create interesting policy trade-offs. Ravenna and Walsh (2006) show that productivity shocks can create policy trade-offs if there is cost channel in which the firms' marginal costs depend directly on the nominal interest rate.

creasing the nominal interest rate to reduce the inflation pressures, elevates the banks' cost of funding. In this case, the banks would require a larger premium on their loans, which will in turn exacerbate the collapse in investment spending and real activity, increasing the deviation from the efficient allocation. Hence, the optimal policy consists in allowing a temporary deviation from price stability in exchange for a partial stabilization of the financial markets.

How should optimal policy be conducted in this economy? Monetary policy can affect all the parts of the financial sector. By changing the cost of credit, the central bank can affect the incentives for leveraging in the financial sector. When negative shocks arise and financial frictions are present, it is optimal to aggressively reduce interest rates in order to stabilize the financial sector. This policy reduces the cost of funding, revalues the financial assets, and protects the profitability of the banking sector. In contrast, in the absence of financial frictions, the monetary stance is not required to be as expansionary ; inflation stabilization is optimal in that economy.

How can the central bank implement this optimal policy?

The second result of this chapter is that a central bank can mimic the optimal policy if it reacts to changes in the financial conditions, such as the cost of credit for firms. Not reacting to financial events is welfare decreasing. The optimal implementation of policy delivers an inertial rule that has feedback coefficients on both inflation deviations and deviations of the premium on capital. In particular, if the cost of credit for firms increases, which normally happens in a bad times, the central bank should cut the interest rate to make the cost of funding cheaper. In this case, the feedback coefficient on inflation is smaller, while larger coefficients decrease welfare. It is optimal to set the coefficient on output fluctuations to zero. The inertial rule has advantages over the non-inertial. The introduction of the inertial component allows the central bank to commit itself to stabilize the financial markets in the short-run, while, if necessary, reversing its policy in the long-run in order to anchor inflation expectations and to achieve price-level control. This setting implements the optimal policy.

To answer the questions posed here, I use a New Keynesian model with a banking sector that faces balance sheet constraints, as in Gertler and Karadi (2011). In this economy, I analyze the optimal monetary commitment. In order to accurately compare the welfare across different policies, I follow the approach developed by Schmitt-Grohé and Uribe (2004).

The plan of the paper is as follows. In the next section I present the literature review. In the third section I present the model. The fourth section contains the benchmark calibration. Section five presents the problem faced by a benevolent social planner who seeks to maximize the social welfare. This efficient allocation serves to compare the results of optimal policy, which is contained in section six. The optimal implementation of policy is presented in the seventh section. The section after that presents robustness checks. And the ninth section concludes.

1.2 Related Literature.

The current paper can be related to the literature analyzing policy trade-offs in the presence of real frictions. For example, in models without financial frictions, the introduction of a cost-push shock can generate significant trade-offs for the policy maker (Woodford (2003)). Leith et al. (2012, 2015) show that the introduction of deep habits in the utility function of the representative consumer can generate a non-trivial optimal policy exercise. Ravenna and Walsh (2006) show that if firms' marginal costs depend directly on the nominal interest rate, the optimal policy is to allow inflation fluctuations.

However, the particular emphasis of the current paper is on the trade-offs faced by the policy maker in the presence financial frictions. There is a group of works analyzing the optimal monetary commitment in the presence of financial frictions. For example, using the cost-channel mechanism and a costly state verification, De Fiore and Tristani (2012) show that productivity shocks can generate a trade-off for monetary policy. In their framework, the optimal policy is to mitigate output fluctuations and to allow deviations of inflation from its long run level. The central banks trades off stability of inflation for stability of real activity.

In Carlstrom et al. (2010), borrowers are restricted to borrow at efficient rates because there is a constraint that ties the amount of loans to their collateral. They show that the central bank's loss function is partly a function of the tightness of the credit constraint, which they interpret as a risk premium. However, their model abstracts from capital accumulation, which in the current paper is relevant to introduce the financial friction.

Cúrdia and Woodford (2010) analyze optimal policy in an extended version of the New Keynesian model which incorporates household heterogeneity and financial frictions. Borrowers and savers discount future consumption at different rates, creating a positive wedge between borrowing and lending rates; the loans are costly to produce and this constrains the supply of credit. They conduct the optimal policy exercise using a linear-quadratic approach. In contrast, I conduct optimal policy in a medium size DSGE model with a banking sector facing balance sheet constraints. Similarly to the current paper, Cúrdia and Woodford (2010) consider the implementation of the optimal policy using Taylor rules. They also find that financial variables should be introduced into such rule.

Leduc and Natal (2015) also consider the optimal commitment in a model with financial frictions. The optimal monetary policy should lean against movements in asset prices and risk-premia. Their result is similar to one of the main conclusions in this paper. The optimal policy can be approximated by including a speed-limit rule that places a substantial weight on the growth of financial variables. In their model, the financial friction is on the borrowers side. In particular, they rely on the financial accelerator model by Bernanke, Gertler and Gilchrist (1999). In their framework, the demand for credit is constrained by entrepreneurs' net wealth. In contrast, in the current paper, financial frictions are on the supply side and the constrained agents are the banks, not the borrowers.

After the analysis of optimal policy in the presence of financial frictions, I deal with the issue of its optimal implementation. There is large literature researching the ability of simple rules to lean against the financial markets. For example, in Andres et al. (2010), borrowing is subject to collateral constraints and banks are monopolistically competitive. The optimal monetary commitment implies a short-run trade-off between output and inflation. A Taylor rule augmented with a feedback coefficient on the real-state prices implements the optimal policy.

Similarly, Gambacorta and Signoretti (2013) develop a DSGE with both a firm's balance sheet channel and a bank-lending channel. They assess whether Taylor rules augmented with asset prices and credit can improve upon a standard rule in terms of macroeconomic stabilization. If the central bank reacts to the financial variables, welfare is maximized. Inflation targeting and a standard Taylor rule are less effective in stabilizing fluctuations.

In a model with search and matching frictions in the credit market, Fujimoto et al. (2014) conclude that the optimal rule must maintain a balance between financial and real economic activity. By taking financial variables into account, monetary policy may contribute to financial stability. Notarprieto et al. (2015) analyze the implementation of optimal policy in a model with a housing sector. The social welfare-maximizing monetary policy rule features a reaction to house price variations. Similarly to the previous studies, I find that augmenting the conventional monetary rule to include financial elements is desirable. In particular, stabilizing the cost of credit increases welfare in the economy.

Kamber and Thoenissen (2012) show that the amplification of monetary shocks introduced by the feedback loop between financial and real events can be overturned by assuming a more canonical Taylor-type interest rate rule where the policy rate reacts to both inflation and the output gap. Output stabilization matters in this context and they find a case to reduce the inflation stabilization motive. The model they use for their analysis is similar to financial accelerator model by Bernanke, Gertler and Gilchrist (1999).

Finally, a group of authors find that there is no case to extend the conventional Taylor rules to include financial variables. For example, Gilchrist (2002) concludes that, although asset prices, and the economy as a whole, can exhibit large fluctuations in response to financial shocks, there is not a strong case for including asset prices in monetary policy rules. The reason, he argues, is that as asset channels are similar to aggregate demand channels, they tend to increase both output and inflation. Inflation targeting, therefore, yields most of the benefits of asset prices targeting. Faia and Monacelli (2007) study optimal Taylor-type rules in an economy with credit market imperfections. They conclude that for low values of the feedback coefficient in the policy rule, responding to a measure of assets is welfare improving. However, when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes. A strong anti-inflationary stance always attains the highest level of welfare.

In contrast to most of the literature presented above, I conduct an optimal policy exercise in a medium-size DSGE model in which the financial frictions affect the supply of credit, rather than demand. Similarly to most of them, I present the ability of simple rules to implement the optimal policy, which leans strongly against financial events. In the next section, I present my benchmark model for conducting this optimal policy analysis.

1.3 The Model.

The model I use for the analysis is a New Keynesian DSGE, similar to Christiano et al. (2005) and Smets and Wouters (2007), but modified by Gertler and Karadi (2011) to include financial intermediaries that face balance sheet constraints. Within this framework, an agency problem between borrowers and lenders limits the supply of credit. The number of loans that can be intermediated by the banking sector depends on the value of net wealth in this sector. A reduction in the value of this wealth has the effect of increasing the cost of credit. The increase in the cost of credit negatively affects investment. As a consequence, the economic activity decreases. The effects of the shock are amplified with respect to the case in which the financial friction is absent.

There are five groups of agents: households, financial intermediaries, non-financial producers, capital producers, and retailers.

1.3.1 Households.

Households choose consumption (C_t), labor (L_t), and debt $\left(D_{t+1}^h\right)$ in order to maximize their utility. Each household has a continuum of members. Within the household there is perfect consumption insurance. There are two types of agents inside each household. At each period, the fraction $(1-f)$ represents workers and (f) bankers. A household owns the banks managed by its members. The deposits of this household are in intermediaries they do not own.

The survival horizon of banks is finite. Introducing this finite horizon has the effect of ensuring that over time the banks do not reach the point where they can fund all the investment from their own capital. (θ) is the probability that a bank operates until the next period. This probability is independent of how long the agent has been a banker. The average survival length of a bank is $\left(\frac{1}{1-\theta}\right)$.

The relative share of workers and bankers is constant. Each period, the number of bankers leaving the industry is $(1-\theta)f$. The same number of workers become bankers. Households provide their new bankers with startup funds. When a bank leaves the industry its retained profits are returned in a lump-sum transfer to its owner.

Preferences.

To capture consumption dynamics, the utility function includes habits in consumption. The utility function for the representative household is:

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\frac{(C_\tau - hC_{\tau-1})^{1-\sigma}}{1-\sigma} - \chi \frac{L_\tau^{1+\varphi}}{1+\varphi} \right] \tag{1.1}$$

where (L_t) is labor. $0 < \beta < 1$, is the subjective discount factor. The parameter h measures the habit persistence in consumption. σ is the inverse of the intertemporal elasticity of substitution. χ is the weight of labor disutility. φ is the inverse of the Frisch elasticity of labor supply.

The budget constraint of the household is:

$$C_t = W_t L_t + \Pi_t + R_t \left[D_t + B_t^g \right] - \left[D_{t+1} + B_{t+1}^g \right] - T_t. \tag{1.2}$$

Deposits (D_{t+1}) and government bonds $\left(B_{t+1}^g\right)$ are short-term assets paying the same return in equilibrium. Following Gertler and Karadi (2011), this condition is imposed from the beginning. $\left[B_{t+1}^g + D_{t+1}\right]$ is the quantity of short-term riskless debt that the household acquires at period (t). The gross real return on those assets is (R_t). This return is paid from $(t-1)$ to (t).

Profits (Π_t) from financial and non-financial firms are net of the amount the household gives to its starting bankers at period (t). (T_t) are lump sum transfers from the government. The real wage (W_t) complements the household's budget constraint.

Optimality Conditions.

It is assumed that the intertemporal elasticity of substitution is unitary. The intertemporal maximization of (1.1) subject to the set of constraints of the form (1.2) implies the optimality conditions[3]:

Optimal labor supply:

$$\frac{\chi L_t^{\varphi}}{U_{ct}} = W_t. \tag{1.3}$$

Euler equation:

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{1.4}$$

where marginal utility of consumption (U_{ct}) is:

$$U_{ct} = E_t \left[\frac{1}{(C_t - hC_{t-1})} - h\beta \frac{1}{(C_{t+1} - hC_t)} \right] \tag{1.5}$$

and

$$\Lambda_{t,t+1} = \frac{U_{ct+1}}{U_{ct}}. \tag{1.6}$$

[3]Appendix A.1 contains the detailed derivations of these conditions.

1.3.2 Banks.

Balance Sheet.

The financial intermediary (j) receives deposits from households (D_{jt+1}). These deposits pay the short-term real interest (R_{t+1}) from (t) to $(t+1)$. These funds complement the accumulated wealth of banks (N_{jt}). Banks make use of these two sources of funds to make loans to producers. Loans pay the rate $\left(R_{t+1}^{k}\right)$ between (t) and $(t+1)$.

The quantity of assets that the bank holds is (S_{jt}). The relative price of the financial asset is (Q_t). In each period the total value of assets held by the representative bank is $(Q_t S_{jt})$. The value of the bank's liabilities plus capital is $(D_{jt+1} + N_{jt})$. The balance sheet of the representative bank is:

$$Q_t S_{jt} = D_{jt+1} + N_{jt}. \tag{1.7}$$

Evolution of Wealth.

A bank's net wealth evolves according to

$$N_{jt+1} = R_{t+1}^{k} Q_t S_{jt} - R_{t+1} D_{jt+1}, \tag{1.8}$$

which is the difference between the return on its assets $\left(R_{t+1}^{k} Q_t S_{jt}\right)$ and the cost of its liabilities $(R_{t+1} D_{jt+1})$. After solving (1.7) for deposits and inserting the result in (1.8), the evolution of wealth can be expressed as:

$$N_{jt+1} = \left[R_{t+1}^{k} - R_{t+1}\right] Q_t S_{jt} + R_{t+1} N_{jt}, \tag{1.9}$$

the term $\left[R_{t+1}^{k} - R_{t+1}\right]$ is the asset's premium over the riskless rate.

The banker will not fund a project with a return less than the cost of deposits. If the discount factor applied by the bank to assets between period (t) and $(t+i)$ is $\left[\beta^{i} \Lambda_{t,t+i}\right]$, then the next condition should apply for the bank to operate:

$$E_t \beta^{1+i} \Lambda_{t,t+1+i} \left[R_{t+1+i}^{k} - R_{t+1+i}\right] \geq 0 \tag{1.10}$$

in any period $(i \geq 0)$. In frictionless capital markets this relationship holds with equality. By contrast, when the financial frictions are present, this risk adjusted premium may be positive. The presence of a positive spread in equilibrium will translate into inefficiently low levels of capital and overall economic activity.

Bank Maximization Problem.

The problem of the bank is to maximize the expected value of its terminal wealth (V_{jt})

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1 - \theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i} \theta_k\right] \beta^{i+1} \Lambda_{t,t+1+i} (N_{jt+1+i}) \tag{1.11}$$

where

$$N_{jt+1+i} = \left[R_{t+1+i}^k - R_{t+1+i}\right] Q_{t+i} S_{jt+i} + R_{t+1+i} N_{jt+i}$$

The probability of survival of banks (θ_t) is subject to a random shock, which evolves as

$$\ln(\theta_t) = \rho_\theta \ln(\theta_{t-1}) + \varepsilon_{\theta_t}.$$

where (ε_{θ_t}) has mean zero and variance $(\sigma_{\theta t}^2)$.

There is a frictionless process of lending and borrowing between producers and banks. The possibility of making profits encourages the banker to remain in the industry as long as possible. In order to issue new loans, the bank borrows from households. Then, the bank uses its accumulated wealth and the deposits to issue loans to producers. It is assumed that banks face frictions in this process of borrowing from households. This friction reduces the ability of the bank to issue new loans.

In particular, every period, the bankers can divert a fraction (λ) of available funds. To avoid that the bank absconds with the funds, the household imposes an incentive constraint on the bank. The cost to the banker of diverting funds is that the households can force the bank to shut down and households can recover the fraction $(1 - \lambda)$ of assets. For the lender to be willing to supply funds to the banker, the following incentive constraint must be satisfied:

$$V_{jt} \geq \lambda Q_t S_{jt}. \tag{1.12}$$

The left-hand part (V_{jt}) is the expected present value of the bank's financial activity if it remains in the industry. This is what the bank would lose if it is forced to leave the industry. The term $(\lambda Q_t S_{jt})$ is what the bank would gain if it absconds with the funds. The bank assesses this trade-off and acts optimally. The bank would remain in the industry as long as the benefits from doing so covers the benefits from absconding with a share of assets.

The household would deposit in the bank only if the benefit for the bank of lending and borrowing is at least as large as the benefit for the bank from diverting funds. This contract limits the ability of the banking sector to raise funds from households. As a consequence, the banks have limits on the loans they can issue. This will impact the level of capital that firms can accumulate and the overall economic activity would be inefficiently low.

In the appendix A.1 it is shown that the conjectured solution to the banks maximization problem can be expressed as

$$V_{jt} = v_t Q_t S_{jt} + \eta_t N_{jt} \tag{1.13}$$

where

$$
\begin{aligned}
v_t &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R_{t+1}^k-R_{t+1}\right) \\
&+E_t\theta_{t+1}\beta\Lambda_{t,t+1}x_{t,t+1}v_{t+1}
\end{aligned} \tag{1.14}
$$

and

$$
\begin{aligned}
\eta_t &= E\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1} \\
&+E_t\theta_{t+1}\beta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}.
\end{aligned} \tag{1.15}
$$

The term (v_t) is the marginal expect return to the bank of increasing assets. (η_t) is the marginal expected return to the bank of increasing its accumulated wealth.

The term

$$x_{t,t+i}=\frac{Q_{t+i}S_{jt+i}}{Q_tS_{jt}} \tag{1.16}$$

is the gross growth of assets between period t and $t+i$. Over the same period, the net wealth of the banker has a gross growth of

$$z_{t,t+1}=\frac{N_{jt+i}}{N_{jt}}. \tag{1.17}$$

Leverage Ratio.

Substituting the conjectured solution (1.13) in the incentive constraint (1.12)

$$\nu_tQ_tS_{jt}+\eta_tN_{jt}\geq\lambda Q_tS_{jt}, \tag{1.18}$$

and solving for assets, the incentive constraints can be expressed as

$$\frac{Q_tS_{jt}}{N_{jt}}\geq\frac{\eta_t}{\lambda-\nu_t}. \tag{1.19}$$

Defining the leverage ratio in the banking sector (ϕ_t) as the maximum ratio of loans to net wealth $\left[\frac{Q_tS_{jt}}{N_{jt}}=\phi_t\right]$, then

$$\phi_t=\frac{\eta_t}{\lambda-\nu_t}. \tag{1.20}$$

Combining (1.19) and (1.20), it is possible to express the assets intermediated by the bank as

$$Q_tS_{jt}=\phi_tN_{jt}, \tag{1.21}$$

which is the leverage ratio times the bank's net wealth. The previous expression means that the maximum amount of loans issued by the representative bank is limited by the maximum leverage ratio tolerated by the household. This leverage ratio is a function of the diverting preference of the banks and the profitability of the banking industry. The maximum amount of loans is also restricted by the amount of accumulated wealth of the bank.

Substituting the leverage ratio in the evolution of wealth (eq. 1.9)

$$N_{jt+1} = \left\{ \left[R^k_{t+1} - R_{t+1} \right] \phi_t + R_{t+1} \right\} N_{jt}, \tag{1.22}$$

and using this in (1.16) and (1.17)

$$z_{t,t+1} = \left[R^k_{t+1} - R_{t+1} \right] \phi_t + R_{t+1}, \tag{1.23}$$

and the gross rate of assets can be written as

$$x_{t,t+1} = \frac{\phi_{t+1}}{\phi_t} z_{t,t+1}. \tag{1.24}$$

Evolution of Aggregate Leverage Ratio.

The components of the leverage ratio are the same for each bank. After aggregating (1.21),

$$Q_t S_t = \left[\frac{\eta_t}{\lambda - \nu_t} \right] N_t. \tag{1.25}$$

the overall demand for assets in the economy ($Q_t S_t$) can be written as a function of the leverage ratio and the accumulated wealth (N_t) in the banking sector

Evolution of Aggregate Net Wealth.

The evolution of aggregate wealth (N_t) is the sum of two components: the net worth of the existing banks (N_{et}), and the net wealth of the new banks (N_{nt})

$$N_t = N_{et} + N_{nt}. \tag{1.26}$$

The fraction of bankers (θ_{t-1}) at ($t-1$) survives until (t). Then, using the aggregation of (1.22),

$$N_{et} = \theta_{t-1} \left\{ \left[R^k_t - R_t \right] \phi_{t-1} + R_t \right\} N_{t-1}. \tag{1.27}$$

As outlined by Gertler and Karadi (2011), I assume that the newly entering bankers receive start-up funds from their respective households. It is assumed that these start-up funds are equal to a small fraction of the value of assets that exiting bankers had intermediated in their final operating period. The total value of assets of exiting bankers is $(1 - \theta_{t-1}) Q_t S_{t-1}$. It is assumed that each period the household transfers a fraction $\left[\frac{w}{1-\theta_{t-1}}\right]$ of those assets to its new bank. In aggregate $[N_{et} = wQ_t S_{t-1}]$. The evolution of aggregate wealth is

$$N_t = \theta_{t-1}\left\{\left[R_t^k - R_t\right]\phi_{t-1} + R_t\right\} N_{t-1} + wQ_t S_{t-1}. \tag{1.28}$$

1.3.3 Intermediate Goods Producers.

The goods produced in this competitive sector are sold to retailers. At the end of period (t) intermediate producers acquire (K_{t+1}) units of capital from capital goods producers. This capital is for use in the subsequent periods. At the end of period $(t+1)$ the firm has the option of reselling the undepreciated capital in the open market. There are no capital adjustment costs at the firm level.

To purchase capital, intermediate producers issue (S_t) claims for each unit of capital acquired (K_{t+1}). These contingent claims are acquired by the banks. The price of each claim is the same as of each unit of capital (Q_t). Then, the value of capital acquired is equal to the value of contingent claims

$$Q_t K_{t+1} = Q_t S_t. \tag{1.29}$$

Financial intermediation between banks and intermediate producers is frictionless. The claims (S_t) can be thought as perfectly state-contingent debt. Every period the producer pays the full return on capital to the bank.

Production of Intermediate Goods.

The production (Y_{mt}) in this sector is given by

$$Y_{mt} = A_t\left(U_t \xi_t K_t\right)^{\alpha} L_t^{1-\alpha} \tag{1.30}$$

where (A_t) is the total factor productivity, (K_t) the capital acquired in the previous period and used in this period. (L_t) is the labor demand and (U_t) the utilization rate. Following Gertler and Karadi (2011), the term (ξ_t) is an exogenous shock to the quality of capital. This shock can be interpreted as a sudden obsolescence on the capital[4] and provides an exogenous source of variation to the price of capital.

The relative price of the goods in this sector is (P_{mt}). In the appendix A.1, it is shown that from profits maximization in this sector:

Labor demand:

[4] Gertler et al. (2012) provide the microfoundations for this shock.

$$(1-\alpha)\, P_{mt} \frac{Y_{mt}}{L_t} = W_t.$$

Optimal utilization rate:

$$\alpha P_{mt} \frac{Y_{mt}}{U_t} = bU_t \xi_t K_t, \tag{1.31}$$

where depreciation of capital is a function of the utilization rate. It is assumed that depreciation takes the form

$$\delta_t = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta}, \tag{1.32}$$

where (ζ) is the elasticity of depreciation respect to utilization rate.

Rate of Return on Capital.

The firms in this sector are perfectly competitive and gain zero profits state by state. Each period, the firm pays to the bank the full return on capital. It is as if banks are the owners of the capital on the firm. The return on capital is the remainder of the profits after paying the wage bill. From the optimal conditions of the maximization problem of these firms, in appendix A.1 it is shown that the return to capital is:

$$R_t^k = \frac{1}{Q_{t-1}} \left\{ \alpha P_{mt} \frac{Y_{mt}}{K_t} + [Q_t - \delta_t]\, \xi_t \right\}. \tag{1.33}$$

1.3.4 Capital Producers.

Competitive capital producers purchase the depreciated capital from the intermediate producers at the end of the period (t). The capital is repaired and sold together with the new capital. The cost of repairing worn out capital is unity. The value of selling one unit of new capital is (Q_t). Investment adjustment cost are associated with the net investment (I_{nt}):

$$I_{nt} = I_t - \delta_t \xi_t K_t \tag{1.34}$$

where (I_t) is the total investment.

Each period the firm maximizes

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau} \left\{ (Q_t - 1)\, I_{nt} - \frac{\phi\_i}{2} \left(\frac{I_{n\tau} - I_{n\tau-1}}{I_{n\tau-1} + I_{ss}} \right)^2 (I_{n\tau} + I_{ss}) \right\}. \tag{1.35}$$

The investment adjustment costs, associated with the net flow of investment, are

$$\frac{\phi\_i}{2} \left(\frac{I_{n\tau} - I_{n\tau-1}}{I_{n\tau-1} + I_{ss}} \right)^2 (I_{n\tau} + I_{ss})$$

where (ϕ_i) is the inverse of the elasticity of net investment to the price of capital. Each of the firms in this sector chooses the same level of net investment. So, it is not necessary to index investment by firm. From this maximization problem the optimal price of capital

$$
\begin{aligned}
Q_t &= 1 + \frac{\phi\_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 \\
&+ \phi\_i \left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)\left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}\right) \\
&- E_t \beta \Lambda_{t,t+1} \phi\_i \left(\frac{I_{nt+1} - I_{nt}}{I_{nt} + I_{ss}}\right)\left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}}\right)^2 .
\end{aligned}
\tag{1.36}
$$

1.3.5 Retailers.

Final output is a composite of a continuum of differentiated retail goods. The only input of production is the intermediate good. Retailers purchase inputs from the intermediate producers and re-package it. The final product is aggregated according to

$$
Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.37}
$$

(Y_{ft}) is the output purchased to the retailer (f). (ε) is the elasticity of substitution across varieties.

Optimal Demand for Retailers.

As shown in the appendix A.1, from cost minimization, those purchasing the final good have an optimal demand for each variety equal to

$$
Y_{ft} = \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} Y_t \tag{1.38}
$$

which implies the optimal price index

$$
P_t = \left[\int_0^1 (P_{ft})^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} . \tag{1.39}
$$

Profit Maximization.

The only cost of production for the retailer is the price of the intermediate good. This cost is given by (P_{mt}) because it takes only one unit of intermediate good to produce one unit of the retail good. Each period, firms can adjust their price with probability $(1-\gamma)$. For the periods in which the firm is not able to set prices, it indexes it to the lagged rate of inflation.

In contrast to Gertler and Karadi (2011), I assume that this economy can be subject to a cost-push shock. In particular, the government imposes a distortionary tax on sales. Following

Chen et al. (2014), shocks to this tax, evolve according to

$$\ln\left(1-\tau_t\right) = \rho_\mu \ln\left(1-\tau_{t-1}\right) + \left(1-\rho_\mu\right)\ln\left(1-\tau\right) - \varepsilon_t^\mu \tag{1.40}$$

$\left(\varepsilon_t^\mu\right)$ is i.i.d. with mean zero and variance $\left(\sigma_t^2\right)$.

The firm's problem in this sector is to choose the optimal price $\left(P_t^*\right)$ to maximize its discounted expected profits:

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\left(1-\tau_t\right) \frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \left[\pi_{t+k-1}\right]^{\gamma^p} - P_{mt+i} \right] Y_{ft+i} \tag{1.41}$$

subject to

$$Y_{ft+i} = \left[\frac{P_t^*}{P_{t+i}}\right]^{-\varepsilon} Y_{t+i} \tag{1.42}$$

where π_t is the rate of inflation from $(t-i)$ to (t). And (γ^p) is a parameter with values $[0,1]$ and which measures the inflation indexation. The first order condition is

$$E_t \sum_{i=0}^{\infty} \gamma^i \beta^i \Lambda_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} \left[\pi_{t+k-1}\right]^{\gamma^p} - \frac{\varepsilon}{\varepsilon-1} P_{mt+i} \right] Y_{ft+i} = 0. \tag{1.43}$$

As shown in the appendix A.1, the optimal price, implied by the solution to the previous problem is:

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{F_t}{Z_t} \tag{1.44}$$

with

$$F_t = P_{mt}Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{-\gamma^\rho \varepsilon} \pi_{t+1}^{\varepsilon} F_{t+1} \tag{1.45}$$

and

$$Z_t = \left(1-\tau_t\right) Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{\gamma^\rho (1-\varepsilon)} \pi_{t+1}^{-(1-\varepsilon)} F_{t+1}. \tag{1.46}$$

Evolution of the price index.

Every period there is a share $(1-\gamma)$ of producers adjusting price optimally. The remaining (γ) simply index their price to the previous period inflation. Using the optimal price index (1.39) the evolution of the price index

$$P_t^{1-\varepsilon} = \left[\left(1-\gamma\right)\left(P_t^*\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho} P_{t-1}\right)^{1-\varepsilon} \right]. \tag{1.47}$$

Price Dispersion.

As shown in the appendix A.1, price dispersion is defined as

$$\Delta_t = \int_0^1 \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} df. \tag{1.48}$$

Using the law of movement of the price index and the definition of price dispersion this measure evolves according to:

$$\Delta_t = (1-\gamma)\left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^{\rho}}\pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma\left[\pi_{t-1}^{\gamma^{p}}\pi_t^{-1}\right]^{-\varepsilon}\Delta_{t-1}. \tag{1.49}$$

1.3.6 Government Budget Constraint.

The government spending, which evolves exogenously, (G_t) and the payments on the debt acquired previously $(R_t B_t^g)$ are financed with a tax on sales $(\tau_t Y_t)$, issue of new government bonds $\left(B_{t+1}^g\right)$, and using lump-sum taxation (T_t). The government's budget constraint is

$$T_t = G_t + R_t B_t^g - B_{t+1}^g - \tau_t Y_t \tag{1.50}$$

The initial level of debt (B_t^g) is zero. The lump-sum tax ensures that the debt of the government is stabilized over time and that its budget constraint is balanced, then

$$T_t = G_t - \tau_t Y_t. \tag{1.51}$$

where government consumption (G_t) is fixed at its steady state value (G). Regarding the steady state government spending to GDP ratio, $\left(\frac{G}{Y}\right)$ is 0.2, this is a conventional value, and between 1980-2010 the average was 19.8 percent (BEA NIPA table 1.1.10).

1.3.7 Aggregate Resource Constraint.

Consumption, government spending, total investment and the costs associated with the change in investment adjustment are the demand faced by the final producers. Then, the aggregate resource constraint is

$$Y_t = C_t + G_t + I_t + \frac{\phi\_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 (I_{nt} + I_{ss}). \tag{1.52}$$

1.3.8 Law of movement of Capital.

From the law of movement of capital

$$K_{t+1} = (1 - \delta_t)\, \xi_t K_t + I_t \tag{1.53}$$

and the definition of net investment

$$I_{nt} = I_t - \delta_t \xi_t K_t \tag{1.54}$$

capital evolves according to

$$K_{t+1} = \xi_t K_t + I_{nt}. \tag{1.55}$$

1.3.9 Monetary Policy.

Optimal policy is conducted in a Ramsey fashion. However, to gain some insights on the dynamics of this competitive economy, the following section presents the results of the model when the economy follows simple rules. I make use of the Fisher equation to relate nominal and real interest rates

$$i_t = E_t R_{t+1} \pi_{t+1}. \tag{1.56}$$

If monetary policy is not conducted in an optimal fashion, then simple rules are implemented by a central bank following a Taylor rule to set the nominal interest rate. That rule is

$$\frac{i_t}{i} = E_t \left[\frac{i_{t-1}}{i}\right]^{\kappa_R} \left[\frac{\pi_t}{\pi^*}\right]^{\kappa_\pi} \left[\frac{Y_t}{Y}\right]^{\kappa_Y} \varepsilon_{it}, \tag{1.57}$$

where (ε_{it}) is an exogenous monetary policy shock with mean zero and variance (σ_m^2). Eventually the policy maker can choose to smooth the interest rate, the size of this smoothing preference is controlled by (κ_R).

The set of all the equilibrium conditions is listed in the Appendix A.2.

1.4 Calibration.

The model is calibrated to a quarterly frequency. To calibrate the model I follow the work of Gertler and Karadi (2011), who in turn follow Primiceri et al. (2006). The habits parameter (h) is set to 0.815. The intertemporal elasticity of substitution (σ) is set to unity. The subjective discount factor $(\beta = 0.99)$ implies an annual real interest of 4.1 percent.

The inverse of the Frisch elasticity (φ) takes a value of 0.276. The weight of labor in the utility function is $(\chi = 3.4)$. The elasticity of capital in the production function (α) takes a value of 0.33. The depreciation in steady state is 2.5 percent per quarter and the elasticity of marginal depreciation to the utilization rate (ζ) takes a value of 7.2. The inverse of the elasticity of net investment to the price of capital $\left(\phi_{\_i}\right)$ is assumed to be 1.728.

The probability that a firm does not adjust its price this period ($\gamma = 0.779$) implies that a firm keeps its price for around 4 quarters. The size of the indexation of the price to the previous period inflation (γ^p) takes a value of 0.241.

The elasticity of substitution between varieties of goods is ($\varepsilon = 4.167$). The value of this coefficient is very low. This implies a very large monopolistic distortion. The results are robust to a more competitive economy. In the robustness section I make use of more standard values for this parameters. In particular, [$\varepsilon = 11$; $\varepsilon = 7$]. The coefficient measuring the reaction of the nominal interest rate to changes in inflation in the Taylor rule (κ_π) is 1.5 and the the coefficient on output deviations is (κ_Y) is 0.5. I assume that the smoothing parameter (κ_R) is zero. I assume that inflation in steady state is zero.

The persistence of the shock to productivity, the shock to the quality of capital, and the shock to government spending take the values $\rho_A = 0.95$, $\rho_\xi = 0.66$, and $\rho_g = 0.95$, respectively. The persistence of the shock to the probability of dying in the banking sector and to the cost-push shock are $\rho_\theta = 0.66$ and $\rho_\tau = 0.95$, respectively. The government spending (G) is one fifth of the total output.

Following the work of Gertler and Karadi (2011), the spread between the rate of return on capital and the riskless rate $\left(R^k_{t+1} - R_{t+1}\right)$ is 25 basis points quarterly, which implies and annual spread of 1 percentage point. The leverage ratio in steady state is assumed to be 4. And the transfers to starting banks (ω), is calibrated to match the leverage ratio. The value of this parameter is 0.0022. It is assumed that the average survival time of a bank is 40 quarters, which implies a probability (θ) equal to 0.9715. The previous values for the financial variables imply a share of diverting funds equal to (0.3815). Table 1 summarizes the value of the parameters.

In order to compare some of the results to more conventional analyses of monetary policy, I make use of a model without financial frictions. This model is the conventional DSGE. Table 1 presents the list of the parameters for both cases.

In the next section I present the problem faced by a benevolent social planner who seeks to maximize the social welfare in this economy. This problem is relevant because the social planer delivers the efficient allocation in this economy. The Ramsey planner would seek to mimic that allocation.

Table 1.1: List of Parameters

Parameter		DSGE	Financial
h	Habits in consumption	0.815	0.815
β	Subjective discount factor	0.99	0.99
χ	Disutility of labor	3.41	3.41
α	Capital share	0.33	0.33
φ	Inverse Frisch elasticity of labor supply	0.27	0.27
θ	Probability of survival banks	–	0.97
λ	Share of diverting loans	–	0.3815
ω	Transfer to starting banks	–	0.002
ϕ_i	Elasticity investment adjustment costs	1.72	1.72
ζ	Elasticity of marginal depreciation to utilization	7.2	7.2
γ	Share of firms no adjusting price	0.77	0.77
γ^p	Degree of price indexation	0.241	0.241
ε	Elasticity of substitution	4.1	4.1
ρ_a	Persistence coefficient technology shock	0.95	0.95
ρ_ξ	Persistence coefficient quality shock	0.66	0.66
ρ_θ	Persistence coefficient prob. of survival banks	–	0.66
ρ_i	Persistence coefficient monetary shock	0.75	0.75
ρ_μ	Persistence coefficient cost-push shock	0.93	0.93
σ_a	St. dev. shock to productivity	0.01	0.01
σ_ξ	St. dev. shock to quality of capital	0.01	0.01
σ_θ	St. dev. shock to survival probability	0.01	0.01
σ_μ	St. dev. cost-push shock	0.0647	0.0647
σ_i	St. dev. monetary policy shock	0.01	0.01
κ_π	Inflation coefficient. Taylor rule.	1.5	1.5
κ_Y	Output coefficient. Taylor rule.	0.5	0.5
κ_R	Smoothing parameter. Taylor rule.	0	0

Table 1.2: Variables in Steady State

Variable		DSGE	Financial
C	Consumption	0.5537	0.5375
R	Real interest rate (quarterly %)	1.01	1.01
L	Labor	0.3383	0.3333
P_m	Price intermediate production	0.76	0.76
Y	Output	0.8912	0.8488
R^k	Return on capital (quarterly %)	1.01	1.26
Q	Price of capital	1	1
K	Capital	6.3676	5.6616
Y_m	Intermediate production	0.8912	0.8488
U	Utilization of capital	1	1
I	Investment	0.1592	0.1415
G	Government spending	0.1782	0.1698
Δ	Price dispersion	1	1
π	Inflation	1	1
i	Nominal interest rate (quarterly %)	1.01	1.01
$Spread$	Premium on capital (basis points)	0	25
ϕ	Leverage ratio		4
N	Wealth of banks		1.4154
N_e	Wealth surviving banks		1.4028
v	Marginal return on bank assets		0.0037
η	Marginal return on bank wealth		1.5110

1.5 Social Planner's Allocation.

In order to have a benchmark against which I can compare the results of the optimal policy exercise, in this section, I describe and solve the problem faced by a social planner who seeks to maximize the utility of the consumer subject to the resource constraint, and the production technology.

This social planner maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\ln\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\varphi} L_t^{1+\varphi} \right], \tag{1.58}$$

subject to the production function

$$Y_t = A_t \left(U_t \xi_t K_t\right)^{\alpha} L_t^{1-\alpha}, \tag{1.59}$$

the evolution of depreciation

$$\delta_t = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta}, \tag{1.60}$$

net investment

$$I_{nt} = I_t - \delta_t \xi_t K_t, \tag{1.61}$$

the evolution of capital

$$K_{t+1} - \xi_t K_t = I_{nt}, \tag{1.62}$$

and the aggregate resource constraint

$$Y_t = C_t + G_t + I_t + \frac{\phi_i}{2} \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} - 1 \right)^2 \left(I_{nt} + I_{ss}\right). \tag{1.63}$$

The solution to this problem delivers the efficient allocations (denoted with $(*)$)[5]:

$$Y^* = \left(\frac{K^*}{L^*}\right)^{\alpha} L^* \tag{1.64}$$

with

$$U^* = \left\{ \frac{1-\beta\left[1-\delta\right]}{b\beta} \right\}^{\frac{1}{\zeta}} = 1 \tag{1.65}$$

$$\frac{K^*}{L^*} = \left[\frac{1-\beta\left[1-\delta\right]}{\alpha\beta} \right]^{\frac{1}{\alpha-1}} \tag{1.66}$$

$$L^* = \left\{ \frac{1-\beta h}{1-h} \frac{1-\alpha}{\chi} \left(\frac{K^*}{L^*}\right)^{\alpha} \left[\frac{C^*}{L^*}\right]^{-1} \right\}^{\frac{1}{1+\varphi}} \tag{1.67}$$

[5] The Appendix A.3 shows the detailed derivation of these values.

$$\frac{C^*}{L^*} = \left(\frac{K^*}{L^*}\right)^{\alpha} \left[1 - \overline{G}\right] - [\delta] \frac{K^*}{L^*}. \tag{1.68}$$

After using the optimal value for utilization ($U^* = 1$) the equations for output, labor, and capital can be written as

$$Y^* = \left[\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right]^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{-\alpha(1+\varphi)}{\alpha-1}} \left[1-\overline{G}\right] \\ -[\delta]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{1-2\alpha-\alpha\varphi}{\alpha-1}} \end{array}\right]^{-\frac{1}{1+\varphi}} \tag{1.69}$$

$$K^* = \left\{\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right\}^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[1-\overline{G}\right]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{1+\varphi}{1-\alpha}} \\ -[\delta]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{\alpha+\varphi}{1-\alpha}} \end{array}\right]^{-\frac{1}{1+\varphi}} \tag{1.70}$$

$$L^* = \left\{\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right\}^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[1-\overline{G}\right] \\ -[\delta]\left[\frac{\alpha\beta}{1-\beta[1-\delta]}\right] \end{array}\right]^{-\frac{1}{1+\varphi}}. \tag{1.71}$$

which are equations in terms of the deep parameters.

In the next section, I present and solve the Ramsey problem. I also present the main distortions of this economy. These distortions prevent the economy to achieve the efficient levels of activity. The presence of these distortions can open the door to the policy trade-offs.

1.6 Ramsey Policy and Distortions.

1.6.1 Distortions.

In this section, I present the main distortions associated with this economy. In particular: monopolistic competition and sticky prices, and a positive spread between the lending and deposit rate in the banking sector. The monopolistic competition and sticky prices are a conventional way to provide monetary policy with the ability to affect the real variables. The positive spread between the lending and the deposit rate in the banking sector is a distortion that allows the financial imperfections to affect the business cycle.

In order to have a better understanding of the effects of each of the previous distortions over the business cycle and stabilization policy, I make use of the proper subsidies just as a devices to switch individual distortions on or off in order to isolate their impact. 2

Monopolistic Competition.

In this section, I present the main distortions associated with this economy. From the optimal conditions of the competitive equilibrium, the labor market equilibrium, in steady state is given by

$$L^{\varphi+1}=\left[\frac{1-\alpha}{\chi}\right]U_cP_mY, \tag{1.72}$$

where the marginal utility of consumption (U_c) in steady state is

$$U_c=\left[\frac{1-\beta h}{C\left(1-h\right)}\right].$$

It is possible to express the labor market equilibrium as a function of the deep parameters in the economy and the capital-labor ratio as:

$$L=\left\{\frac{1-\beta h}{1-h}\left[\frac{1-\alpha}{\chi}\right]\left(\frac{K}{L}\right)^{\alpha}\left[\left[1-\overline{G}\right]\left(\frac{K}{L}\right)^{\alpha}-\frac{\delta K}{L}\right]^{-1}P_m\right\}^{\frac{1}{1+\varphi}}. \tag{1.73}$$

From the Social Planner's allocation, I know that the efficient level of labor is given by

$$L^{*}=\left\{\frac{1-\beta h}{1-h}\left[\frac{1-\alpha}{\chi}\right]\left(\frac{K^{*}}{L^{*}}\right)^{\alpha}\left[\left[1-\overline{G}\right]\left(\frac{K^{*}}{L^{*}}\right)^{\alpha}-\frac{\delta K^{*}}{L^{*}}\right]^{-1}\right\}^{\frac{1}{1+\varphi}} \tag{1.74}$$

(1.73) and (1.74) would be equal if the term (P_m) would be equal to unity in (1.73). The price of the intermediated goods (P_m) is different from unity and it is a function of the parameter governing the monopolistic competition in the economy. This has the effect of distorting the levels of economic activity in steady state. Assuming, for the time being, that no other distortions exist, it is possible to get the efficient level of economic activity if a subsidy in steady state eliminates this distortion.

In steady state, the relative price of intermediate goods is given by

$$P_m=\frac{\varepsilon-1}{\varepsilon},$$

which is different from unity. Subsidizing the sales of this good (τ^{mon}) allows me to write the equilibrium in the presence of this subsidy as:

$$1=\frac{\varepsilon}{\varepsilon-1}\frac{1}{1+\tau^{mon}}P_m \tag{1.75}$$

Then, the value of subsidy the that eliminates the distortion associated with monopolistic competition is equal to

$$1+\tau^{mon}=\frac{\varepsilon}{\varepsilon-1}. \tag{1.76}$$

Given the values of the parameters used to calibrate the model, the subsidy (τ^{mon}) is equal to 0.3158. I assume that the subsidies are financed using lump-sum taxes. This subsidy would deliver the efficient levels of the variables if no other distortion existed. However, the presence of the imperfect banking sector also contributes to distort the economy.

Positive spread.

In this model, the steady state value of the variables are also affected by the presence of a positive spread between the return on capital and the risk-free rate. From the optimal conditions of the social planner's allocation, the capital-labor ratio is

$$\frac{K^*}{L^*} = \left[\frac{R-[1-\delta]}{\alpha}\right]^{\frac{1}{\alpha-1}}. \tag{1.77}$$

In this financial model, the capital-labor ratio, once the subsidy on sales is present, is given by

$$\frac{K}{L} = \left[\frac{R^k-[1-\delta]}{\alpha P_m\left(1+\tau^{mon}\right)}\right]^{\frac{1}{\alpha-1}}. \tag{1.78}$$

In models without frictions, capital is expanded until the point in which the return on capital $\left(R^k\right)$ is equal to the real interest rate (R), which in turn equates the inverse of the households' subjective discount factor,

$$R^k = R = \frac{1}{\beta}. \tag{1.79}$$

However, in this model this is no longer possible because of the existence of a positive spread in equilibrium associated with the financial frictions. Hence,

$$R^k - R = Spread, \tag{1.80}$$

with $Spread > 0$. Substituting (1.80) in (1.78)

$$\frac{K}{L} = \left[\frac{R+Spread-[1-\delta]}{\alpha P_m\left(1+\tau\right)}\right]^{\frac{1}{\alpha-1}}. \tag{1.81}$$

Then, once the subsidy to the sales is in place $\left(P_m\left(1+\tau^{mon}\right)=1\right)$, the difference between (1.81) and (1.77) is due to the imperfect banking sector. A subsidy to the acquisition of capital $\left(\tau^{SP}\right)$ can eliminate the spread. In this case, the capital-labor relationship is:

$$\frac{K}{L} = \left[\frac{R+Spread+\tau^{SP}-[1-\delta]}{\alpha}\right]^{\frac{1}{\alpha-1}}. \tag{1.82}$$

The value of the subsidy that eliminates this distortion in equilibrium is

$$\tau^{SP} = -Spread \tag{1.83}$$

after substitution and using $\left(R=\frac{1}{\beta}\right)$

$$\frac{K}{L} = \left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}. \tag{1.84}$$

which is the efficient value of the variables when the two subsidies are implemented. When this subsidy to the return on capital is present, I can eliminate the financial distortion. In this case, I return to the conventional DSGE. The presence of the two subsidies delivers the efficient allocation.

1.6.2 Welfare Cost.

In this section, I present the measure of welfare used to analyze the welfare cost associated with each distortion. In order to accurately compare welfare, I follow the work of Schmitt-Grohé and Uribe (2004, 2007) and use a second-order approximation to the full model. I measure the welfare cost as the amount of consumption that agents in the Ramsey regime are willing to renounce in order to have the same welfare as in the alternative policy scenario. The level of welfare associated with the time-invariant stochastic allocation in the Ramsey policy conditional on a particular state of the economy in period zero is

$$V_0^R = E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t^R, L_t^R\right] \tag{1.85}$$

the variables $\left(C_t^R, L_t^R\right)$ are the contingent plans for consumption and labor under the Ramsey policy. Similarly, an implementable regime has conditional welfare equal to

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t^I, L_t^I\right] \tag{1.86}$$

As in Schmitt-Grohé and Uribe (2007), I assume that at time zero, the value of all the variables are equal to their non-stochastic Ramsey steady-state. Using this assumption helps to ensure that the economy starts from the same initial point under all the alternative regimes. If the consumption cost of following an alternative policy regime instead of the Ramsey policy on a particular state in period zero is represented by $\left[W^C\right]$ the cost of the alternative policy is implicitly defined by

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U\left[\left(1 - W^C\right) C_t^R, L_t^R\right]. \tag{1.87}$$

where $\left[W^C\right]$ is the fraction of consumption of the Ramsey regime that a household is willing to renounce in order to be indifferent between that regime and the alternative policy. Using the particular utility function

$$U = \ln\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\psi} L_t^{1+\psi},$$

solving equation (1.87) for $\left[W^C\right]$ and approximating to a second order, the cost of choosing an alternative policy is

$$W^C \approx \frac{1}{2}(1-\beta)\left[V_{\sigma\varepsilon\sigma\varepsilon}^R - V_{\sigma\varepsilon\sigma\varepsilon}^I\right]\sigma_\varepsilon^2. \tag{1.88}$$

Table 1.3: Steady-State and Distortions.

		Social Planner	Benchmark	DSGE τ^{SP}	τ^{Mon}	$\tau^{SP}+\tau^{Mon}$
		(1)	(2)	(3)	(4)	(5)
1.	Consumption	0.7700	0.5375	0.5537	0.7487	0.7700
2.	Investment	0.3203	0.1415	0.1592	0.2829	0.3203
3.	Labor	0.4520	0.3333	0.3383	0.4424	0.4520
4.	Capital	12.8126	5.6616	6.3676	11.3176	12.8126
5.	Government	0.2726	0.1698	0.1782	0.2579	0.2726
6.	Output	1.3628	0.8488	0.8912	1.2896	1.3628

The derivation of this measure of welfare cost is detailed in the Appendix A.4.

Welfare Costs of Each Distortion.

In this section, I present the welfare costs associated with each distortion in this model. By using the appropriate subsidies, it is possible to eliminate the identified distortions: monopolistic competition, financial frictions, or it is possible return to the flexible-price equilibrium.

Table 3 presents the values of selected variables in the steady state when different subsidies are in place. The first column shows the efficient case. The second column presents the benchmark case in the presence of the monopolistic and financial distortions

The utilization of a subsidy to the excess return on capital $\left(\tau^{SP}\right)$ delivers the allocation associated with the conventional DSGE. In the column (4), I make use of a subsidy to the sales in steady state $\left(\tau^{Mon}\right)$ in order to remove monopolistic competition. Hence, the financial frictions are the sole distortion.

As shown in table 3, the largest distortion in steady state is associated with the presence of monopolistic competition. Table 4 shows the welfare cost in the non-stochastic steady state associated with each of the cases described above. The cost is the percentage of the stream of consumption of the social planner's allocation that the agents would be willing to renounce in order to have the same welfare as in the alternative case.

The monopolistic competition implies a cost of 1.02 percent, respect to the efficient allocation (column (3)). The presence of the friction in the financial sector has a cost of 0.06 percent (last column). When the two frictions are present, the welfare cost increases to 1.6 percent, this is the benchmark case. Table 5 presents the conditional welfare cost when there is uncertainty in the economy.

The highest conditional welfare cost is observed in the benchmark economy (1.85%). When there is a subsidy to the excess return on capital $\left(\tau^{SP}\right)$, the conditional welfare cost is 1.26 percent (column (3)). This is the cost of monopolistic competition and sticky prices.

Table 1.4: Deterministic Welfare Cost

	Social Planner	Benchmark	DSGE or τ^{SP}	τ^{Mon}
	(1)	(2)	(3)	(4)
% SPA		1.6113	1.0216	0.0640

Table 1.5: Conditional Welfare (second order approximation)

	Social Planner	Benchmark	DSGE or τ^{SP}	τ^{Mon}	Flex Prices
	(1)	(2)	(3)	(4)	(5)
% SPA		1.8517	1.2567	0.5163	0.0484

When there is only a subsidy to monopolistic competition $\left(\tau^{Mon}\right)$, the conditional welfare cost is 0.52 percent (column (4)). This is the cost of the financial friction in the presence of uncertainty. In the deterministic case, the cost of the financial friction (0.064%) is a small fraction of the cost of monopolistic competition (1.02%). However, when uncertainty is present, the financial friction has a considerable welfare cost (0.52%).

1.6.3 Ramsey Policy.

In this section, I present the optimal monetary policy in the presence of financial frictions. The Ramsey planner seeks to maximize the welfare of the society subject to the competitive equilibrium conditions. I assume that the central bank is committed to follow the announced plan from a timeless perspective (Woodford (2003)). As in Schmitt-Grohé and Uribe (2005), I assume that at time (t) the Ramsey planner has been operating for an infinite number of periods.

The period (t) objective function of the Ramsey planner is the utility function

$$U_t = \ln\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\psi} L_t^{1+\psi}. \tag{1.89}$$

I assume that the discount factor of the Ramsey planner is equal to the subjective discount factor of households in the competitive economy (β). This policy maker maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t, L_t\right] \tag{1.90}$$

subject to the competitive equilibrium conditions.

Then, the central bank maximizes the welfare function (1.90) subject to the competitive equilibrium restrictions choosing at period (t) processes for the 30 endogenous variables U_{ct}, C_t, Λ_t,

L_t, v_t, x_t, η_t, z_t, ϕ_t, K_{t+1}, N_t, N_{et}, N_{nt}, R_{kt+1}, Y_{mt}, Q_t, δ_t, U_t, I_{nt}, P_{mt}, G_t, I_t, Y_t, Δ_t, F_t, Z_t, π_t^*, π_t, R_t, i_t and the 29 Lagrange multipliers.

The process for the shocks A_t, τ_t, ξ_t, θ_t, g_t are the same as those described in the competitive equilibrium. The values for the variables listed above are given dated $t<0$, and also the values of the Lagrange multipliers associated with the competitive equilibrium constraints are given at $t<0$. Then, as explained in Schmitt-Grohé and Uribe (2005), the structure of the optimality conditions associated with the Ramsey equilibrium are time-invariant.

The Appendix A.5 presents the Lagrangian for the optimal policy from a timeless perspective.

In the next section, I present the optimal monetary response when the economy is hit by: a shock to the quality of capital, a shock to productivity, and a cost-push shock.

Shock to the Quality of Capital.

In the presence of a feedback loop between real and financial activity, a decrease in the quality of capital leads to a fall in the asset prices, which decrease the value of the bank. A bank's net worth is less valuable. This tightens the balance sheet constraint and reduces the supply of credit. The decrease in the supply of loans makes credit more expensive, this is reflected in the higher premium on loans. In turn, this results in lower investment. The decrease in investment and output depresses asset prices, which then feeds back into reduced net worth and investment, creating a feedback loop between the financial and real variables. This is the financial accelerator, which propagates and amplifies shocks.

The effects of not acting optimally are clearly observed in the Figure 2, in which the monetary policy is set according to the conventional Taylor rule, in that case the volatility of inflation and the real and financial variables is higher than in the optimal case. The optimizing central bank would like to smooth this financial accelerator.

Figure 1.1: Optimal Policy. Shock to Quality of Capital.

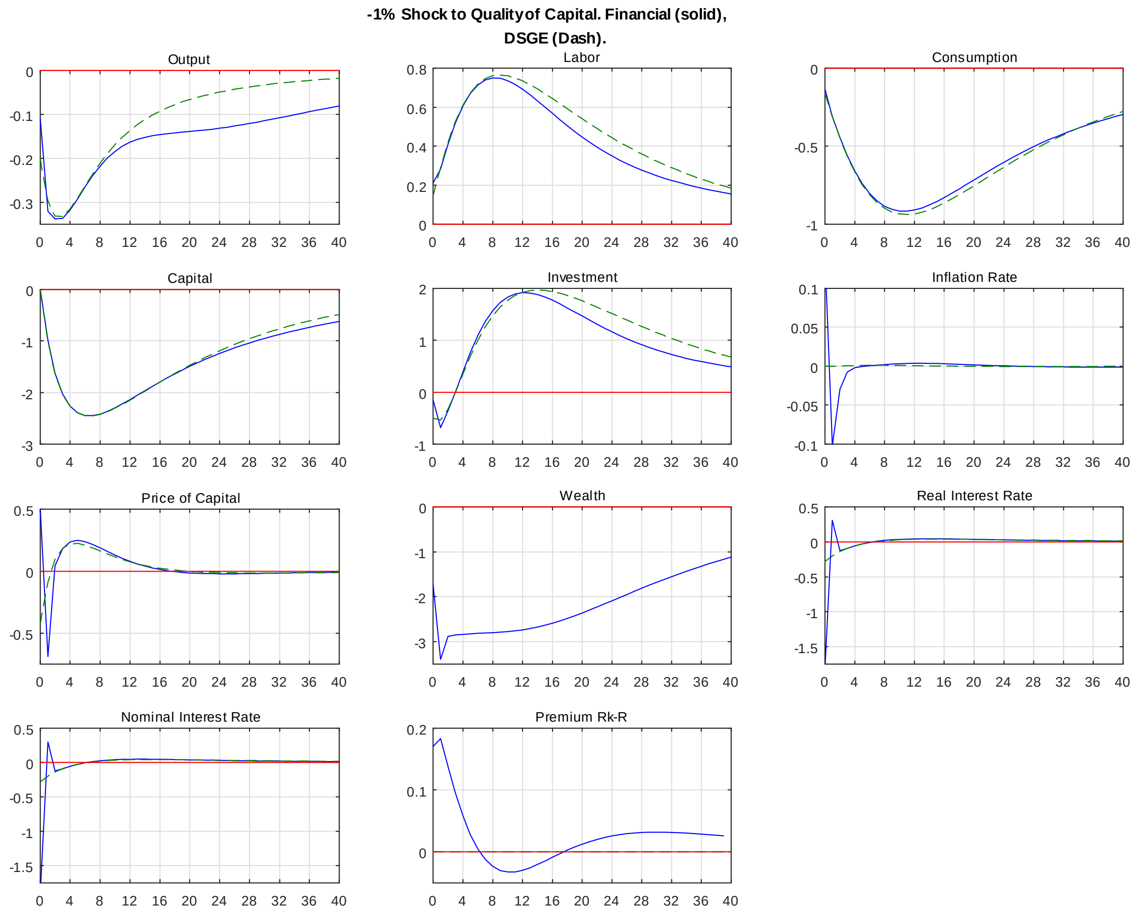

Figure 1. Optimal Policy. 1% Reduction in the Quality of Capital. Financial Model (solid) and DSGE (dashed).

In order to understand the relevance of the optimal policy in this economy with financial frictions, I conduct a series of comparisons. Firstly, figure 1 shows the optimal policy in this model with financial (solid lines) and compares the policy with that implemented in a model without financial frictions (dashed lines).

Secondly, figure 2 shows the impulse response to a decrease in the quality of capital when policy is implemented via the conventional Taylor rule. The solid lines show the model with financial frictions and the dashed lines the model without such frictions.

Finally, I present the gap variables in the figure 3. In order to define the gap variables, I follow the work of Leith et al. (2015). This gap is the difference between the actual value of the variable and the value that would be chosen by a benevolent social planner as a percentage of the value chosen by this planner. In other words, the gap is the difference between the optimal and the efficient response.

Figure 1 shows that when there is optimal commitment, consumption shows similar optimal responses when financial frictions are present and when the are absent. The greatest differences are observed in the inflation, investment, financial variables, and interest rates.

Compared to the optimal policy in the conventional DSGE (figure 1), monetary policy tends to induce a larger reduction in the nominal interest rate. In the absence of financial frictions, the optimal policy is to stabilize inflation. In contrast, in the presence of these frictions, monetary policy is more expansionary. This leads to an initial burst of inflation. On impact, the expansionary policy serves to increase the price of capital. This policy seeks to appreciate the bank's assets in order to reduce the tightening of the bank's balance sheet and foster an increase in the credit supply, reducing the inefficiency associated with the financial friction. As a consequence of this policy, the bank's net worth prevented from falling as much as it would in the case of implementing policy under the conventional Taylor rule (figure 2).

Figure 1.2: Taylor Rule. Shock to Quality of Capital.

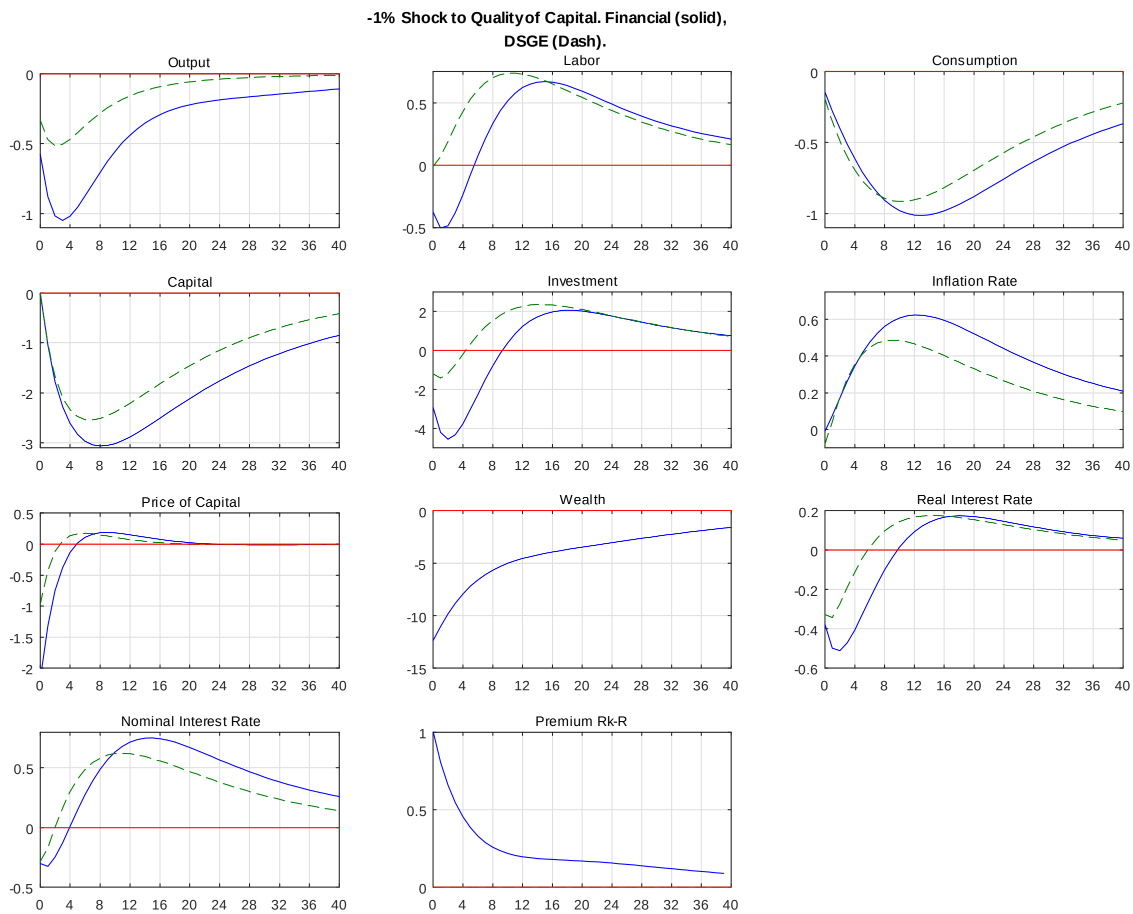

Figure 2. Taylor Rule. 1% Fall in the Quality of Capital. Financial Model (solid) and DSGE (dashed).

The optimal policy dampens the effects of the shock on the financial variables. For example, under simple rules, on impact, the net wealth of the banks falls 12.5 percent and the premium on capital jumps 100 basis points. At the deepest of the recession, output is 1 percent below equilibrium while investment is 5 percent below its long-term average. The conventional Taylor rule produces larger fluctuations in financial and real activity and it is very costly in terms of welfare (last row in table 6). In contrast, the optimal policy prevents this from happening.

The optimal policy stabilizes the financial sector at the cost of increased inflation. It is optimal to trade-off inflation for financial stability. The combination of monopolistic competition and financial frictions create a non-constant wedge between the flexible-price economy and the efficient allocation. The objective of a maximizing policy maker is to keep the economy as close as possible to the efficient allocation. This explains the initial reaction of the central bank; the strong reduction in the nominal interest rate keeps the economy as close as possible to its efficient allocation.

The central bank induces a reduction in the nominal rate, which has the effect of reducing the real rate, via the Fisher relation, and increasing inflation. The reduction in the real interest rate has two effects on the financial sector. Firstly, it appreciates the prices of the assets, by stimulating investment. Secondly, it reduces the cost of deposits for the banks. The central bank realizes that in order to stop the feedback loop between financial and real activity, it is necessary to protect the profitability of banks. If agents are content with the profitability of the banking sector, then the incentive constraint does not tighten. This avoids the inefficient jump in the premium on capital observed when the policy is conducted in a Taylor fashion.

In this way, the gap between the actual level of output and the efficient allocation remains as small as possible. If the central bank does not smooths the financial accelerator, the feedback loop between real and financial variables pushes the economy away from the efficient allocation. Inflation stabilization is suboptimal in the presence of financial frictions.

There is an additional aspect of the optimal policy which I highlight. The initial reduction of 1.5 percent in the real interest rate explains the initial increase in inflation of 15 basis points. This expansionary policy is reflected on the output. Compared to the conventional DSGE, output decreases only 0.1 percent, whereas in the DSGE the initial reduction is 0.2 percent. Given that the financial accelerator is procyclical, the optimal policy smooths its effects. However, in the subsequent periods the optimal policy is reversed.

For example, in the second period the central bank contracts the economy. The increase in the nominal and real interest rate deflate the price of the assets. The cost of credit increases in the second period and investment and net wealth falls more than in the initial period. This change in policy is explained by the desire of the central bank to achieve price-level control in the long-run.

In order to compensate for the initial increase in inflation it is optimal to reduce inflation in the next period and keep this deflation for the next three periods. After four periods, the inflation rate remains very close to its long-run equilibrium. This strategy of the central bank enables effective control of inflation in the long-run. In turn, the control of inflation in the long-run also generates price level control (once the inflation indexation has been removed) which is typical of the optimal commitment in models without financial frictions (Woodford (2003)). Even when financial frictions are present, price level control holds under commitment.

Hence, the optimal policy in the short-run is to stabilize the financial markets to prevent inefficient fluctuation of real activity. However, once the feedback loop between financial and real variable has been smoothed, the central bank can focuses on inflation control. The central bank takes advantage of its commitment technology. It commits to initially contribute to stabilize financial markets in the short-run, at the cost of an increase in inflation, while it stabilizes inflation in the long-run, reducing the attention to the financial markets. This policy maximizes the social welfare.

As a conclusion, the welfare maximizing policy is the one that protects the financial sector. The stability of the financial sector prevents undesired fluctuation of the real variables. The optimal policy trades-off financial stability for inflation. This trade-off does not exist in the absence of financial frictions. Only after stabilizing the financial sector, the central bank seeks price-level control in the long-run. Price-level control is a result of optimal policy under commitment in the benchmark New Keynesian model. This is robust to the introduction of financial frictions.

Trade-offs Faced by the Ramsey planner. In order to understand the trade-offs faced by the Ramsey planner in this economy, I make use of an additional policy instrument. Suppose that the Ramsey planner has access to an optimal subsidy that eliminates the financial distortion.

When this subsidy is in place if a shock reduces the quality of capital, the nominal interest rate decreases by a smaller amount. This stimulates the private spending and prevents deflation. In that case, the financial sector is stabilized using the subsidy to the return on capital. The nominal rate stabilizes inflation. This case is akin to the model without financial frictions (dashed lines in the figure 1).

Now, I remove the subsidy. Removing that subsidy implies that the response of the policy maker changes completely. The policy maker seeks a profitable banking sector and stable inflation. However, with only one instrument, the central bank has to renounce to stabilizing inflation in the short-run. The interest rate has to do the job of the subsidy. In this case, the optimal policy is more expansionary. In this way, the cost of capital remains as close as possible to its long-run level. In making all the banks content with their expected return on assets, the policy maker helps them to meet their balance sheet constraint, and the optimal policy switches off the financial accelerator. This policy protects the financial system and avoids contagion to the real economy. Financial health becomes a key objective of this policy maker. But, the policy maker cannot simultaneously stabilize inflation and the financial sector. This is the case shown by the solid lines in the figure 1.

Gap Variables. Figure 3 shows the gap between the response of the Ramsey policy and the benevolent Social Planner. The economy is hit by a negative shock to the quality of capital. The left panel shows the model with financial frictions. The right panel shows the model without financial frictions.

Figure 1.3: Gap Variables. Shock to Quality of Capital.

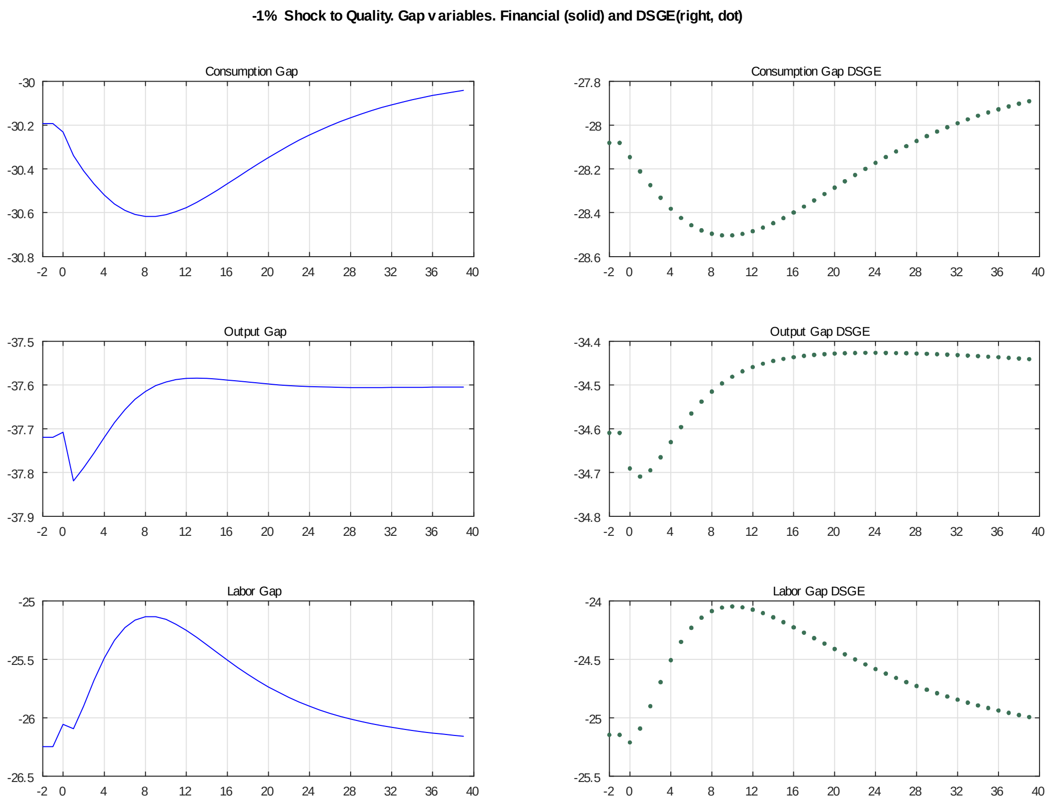

Figure 3. Gap Variables. 1% Reduction in the Quality of Capital. Financial Model (left) and DSGE (right). The gap is the difference between the actual level of the variable under the optimal policy and the efficient allocation as a percentage of the efficient allocation. A decrease in the output gap means that the economy is closer to the efficient allocation.

When the financial frictions are present, the optimal response is to reduce the nominal interest rate and increase inflation in the initial periods (figure 1). This is possible because there is an initial decrease in the output gap, the economy is closer to the efficient allocation, (left panel, figure 3). This initial decrease in the output gap contributes to the increase in the inflation. Because the financial accelerator is procyclical, the reduction in the gap smooths the feedback loop between real and financial variables. After this period, the output gap increases, which is associated with the fall in inflation in the second period. After these two periods the output gap decreases, which explains the smooth return of inflation to its long-run level and serve to stimulate the recovery of the financial variables.

This behavior is absent when the final markets are frictionless (right panel, figure 3). In this case, the optimal policy is to stabilize inflation. When the shock hits, the output gap increases and has an additional increase in the next period. After this, the output gap starts to close, in this way the central bank stabilizes inflation at its long-run level.

The figure 3 captures some of the contributions of financial frictions to the optimal policy. However, the economies in that figure feature monopolistic competition. Figure 4 shows the gap variables when there are financial frictions, but the monopolistic competition distortion has been removed. This isolates the effects of the financial friction.

In figure 4, a subsidy to the sales eliminates the distortion associated with the monopolistic competition (solid lines).

Figure 1.4: Gap Variables. Shock to Quality of Capital.

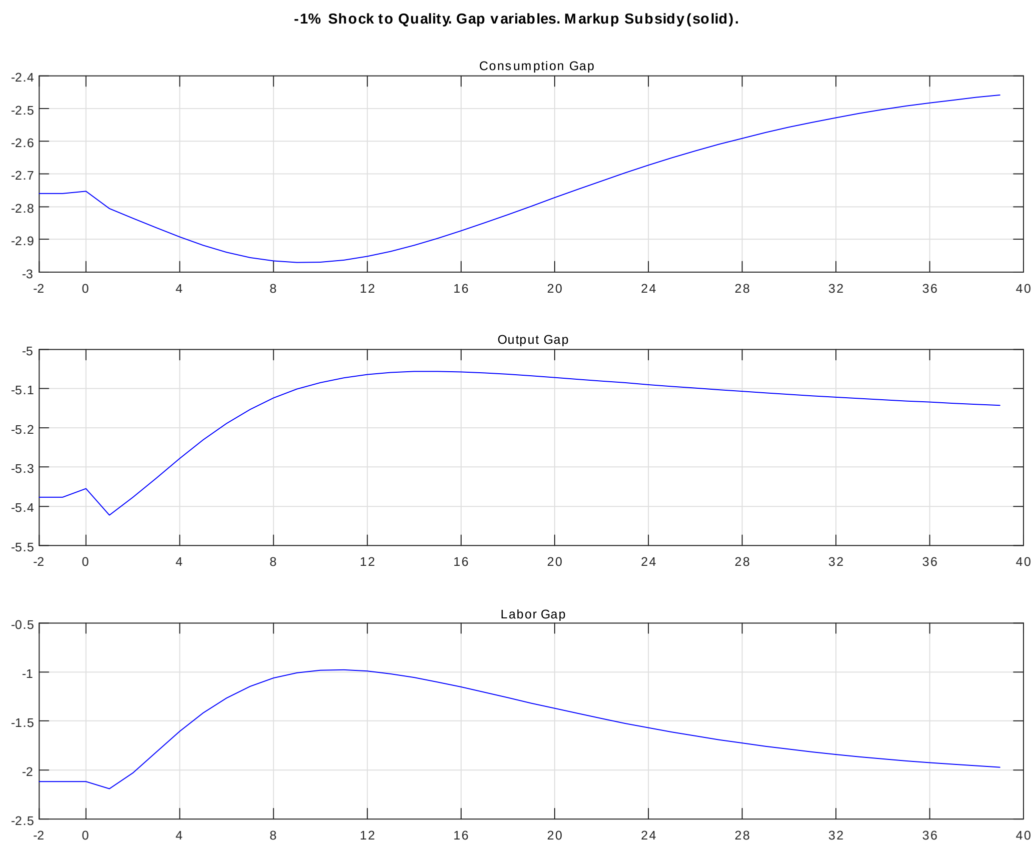

Figure 4. Gap Variables. 1% Reduction in the Quality of Capital. Financial Model (solid) . The solid lines show the gap between the economy without monopolistic competition and the efficient allocation. A decrease in the output gap means that the economy is closer to the efficient allocation.

When the financial friction is the sole distortion in steady state, the optimal response, after a shock to the quality of capital, implies a reduction in the output gap. In this way, the economy is closer to its efficient allocation and the effects of the financial accelerator are smoothed; the optimal monetary policy seeks to mimic the efficient response. This is why the central bank (figure 1) strongly reduces the interest rate in the benchmark case.

In the next section, I present the optimal policy when there is positive shock to productivity.

Shock to Productivity.

In the presence of a feedback loop between real and financial activity, an increase in productivity leads to higher asset prices, which revalues the bank's assets. This has the effect of loosening the balance sheet constraint and contributes to increase the supply of credit. The credit becomes cheaper, this is reflected in the fall of the premium on loans. In turn, this stimulates investment. The higher investment and output produce a boom in asset prices, which then feeds back into net worth and investment. This is the financial accelerator. Figure 5 shows the optimal response.

Figure 1.5: Optimal Policy. Shock to Productivity.

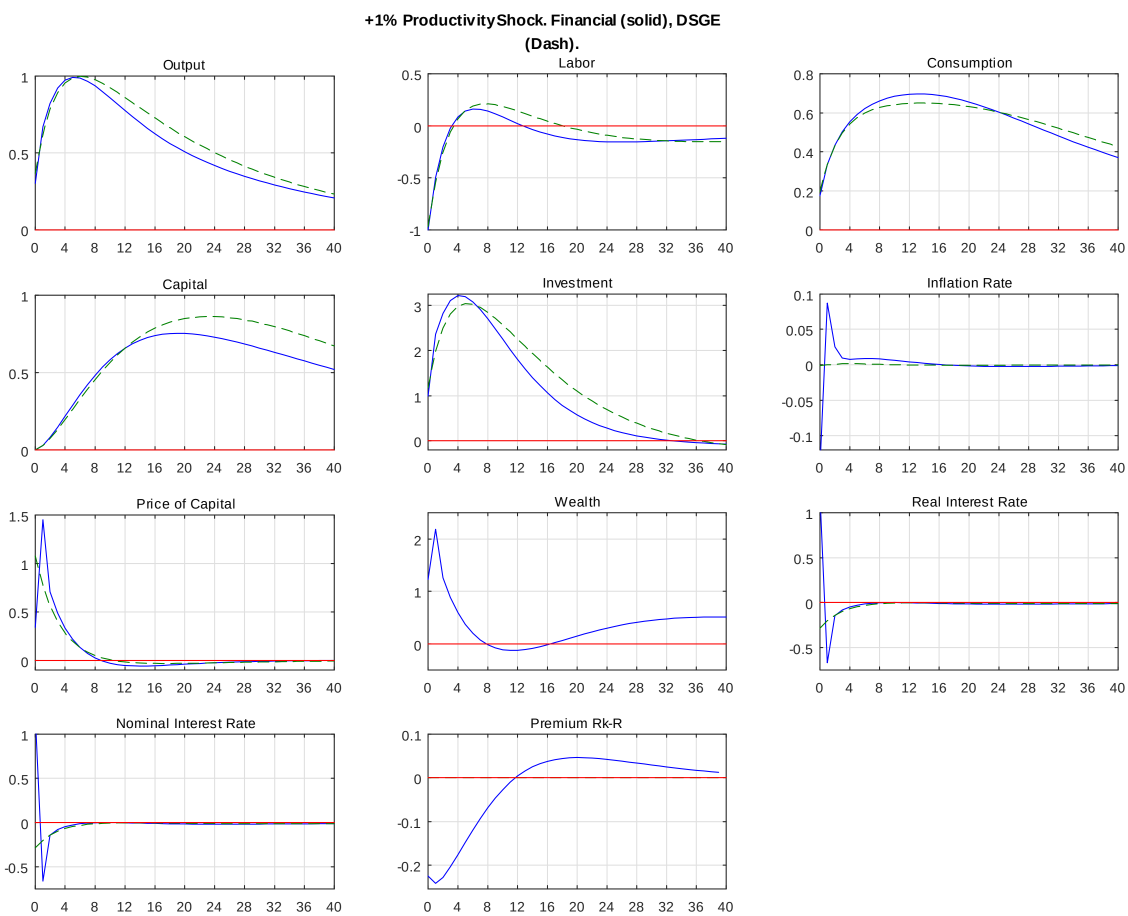

Figure 5. Optimal Policy. 1% Increase in Productivity. Financial Model (solid) and DSGE (dashed).

Compared to the optimal policy under the conventional DSGE, monetary policy tends to be contractionary. This leads to deflation. On impact, the tightening of policy serves to ameliorate the appreciation of the banks' assets. This serves to prevent an overexpansion in the supply of credit. Net worth is stopped to boom as it would be in the case of implementing policy under a Taylor rule in the presence of financial frictions (figure 6). This policy dampens the effects of the shock on the financial variables.

There is a procyclical relationship between output and the premium on capital when financial and real shocks hit the economy. Because this positive shock to productivity would reduce the premium on capital, banks could lend to non-financial firms at lower rates. This would increase investment. In order to prevent an overexpansion of investment, the central bank makes it more expensive for banks to fund new assets. This is the reason behind the increasing real interest rate in the first period.

This policy is effective at stabilizing the financial sector and preventing an overexpansion. But the cost is deflation. It is optimal to trade-off deflation for financial stability. In the case of optimal policy, less loans are granted to firms than in the case of policy implemented via simple rules (figure 6). This is so because the optimal response is to increase the nominal interest rate to keep the economy as close as possible to the efficient allocation. This prevents the boom observed under the Taylor rule.

In the presence of financial frictions, a one percent increase in productivity reduces inflation by 10 basis points. The central bank finds it optimal to undertake a monetary tightening in order to stabilize the financial markets. The monetary contraction reduces the boom in the financial sector, but at the cost of deflation. In contrast, when the financial frictions are absent, the optimal policy is expansionary, and inflation remains under control.

Figure 5 shows that in order to achieve price-level control, it is optimal for the central bank to undo its initial policy from the second period onwards. The central bank exploits the benefits of commitment in order to stabilize financial markets in the short-run and achieve price-level control in the long-run. The policy of the central bank turns expansionary in the second period, and this compensates for the initial deflation. As in the case of the shock to the quality of capital, the central bank deals initially with financial stability. Once this is achieved, the central bank can deal with inflation control. The central bank finds it optimal to keep the nominal interest rate below its long-run equilibrium for several periods. This compensates for the initial deflation.

In conclusion, when productivity shocks arise and financial frictions are present, the central banks exploits the benefits of commitment. In the short-run, it commits itself to financial stabilization. Once the inefficient fluctuation associated with the feedback loop between financial and real variables has been smoothed, it commits itself to price-level control in the long-run.

Figure 1.6: Taylor Rule. Shock to Productivity.

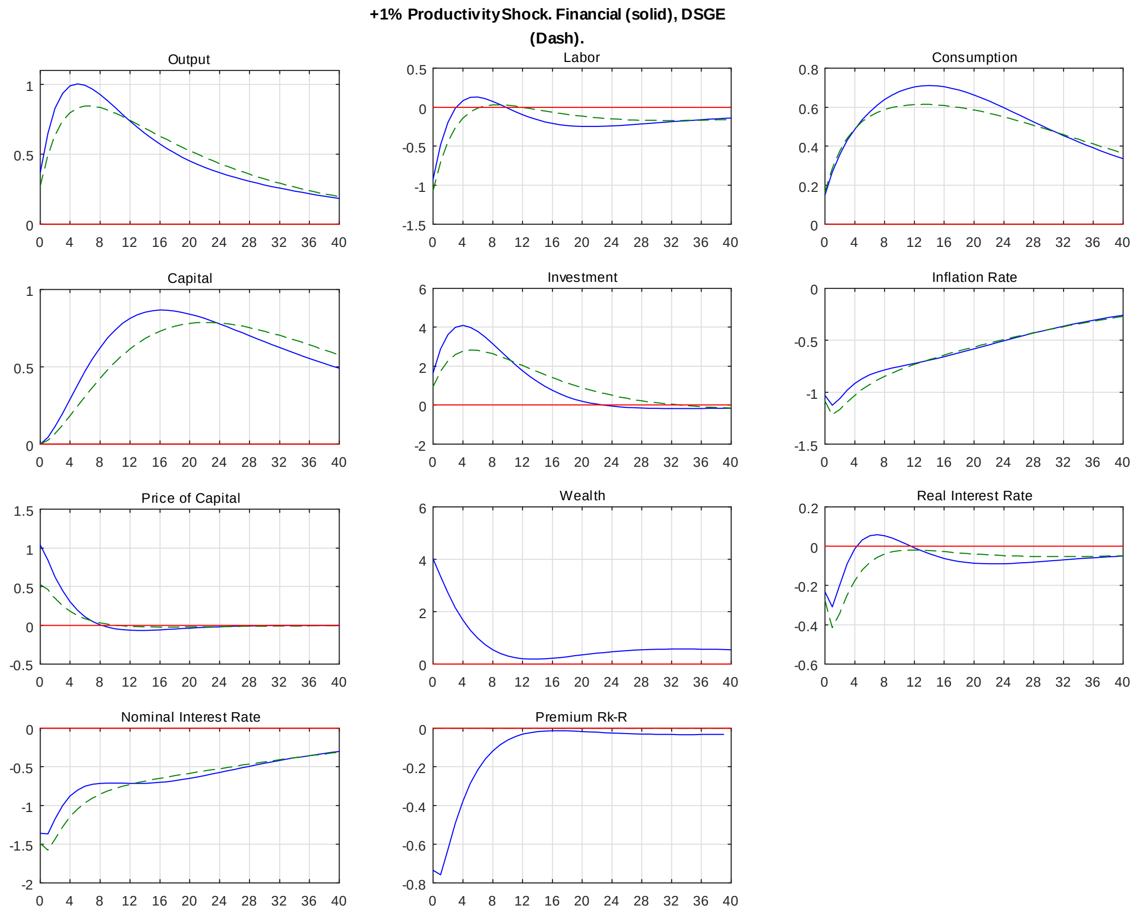

Figure 6. Taylor Rule. 1% Productivity Improvement. Financial Model (solid) and DSGE (dashed).

Cost-push shock.

In this section, I present the response of the Ramsey policy when there is a cost-push shock of one percent. Figure 7 shows the optimal response. As in the case of a shock to the quality of capital, the initial reaction needs to be very strong to prevent the starting of the financial accelerator and its contagion to the real sector. Indeed, the expansionary policy is very effective reducing the effects of this recessionary shock.

In the conventional DSGE, this shock is contractionary and inflationary and it creates a trade-off for policy. In the presence of financial imperfections, the trade-off remains. But given the presence of the financial accelerator, the trade-off is bigger. The initial reaction of the central bank is more inflationary than in the absence of financial frictions. This has the benefit of a milder recession, which prevents inefficient fluctuation of financial and real variables. In this case, the economy is kept as close as possible to the efficient allocation.

Figure 1.7: Optimal Policy. Cost-Push Shock.

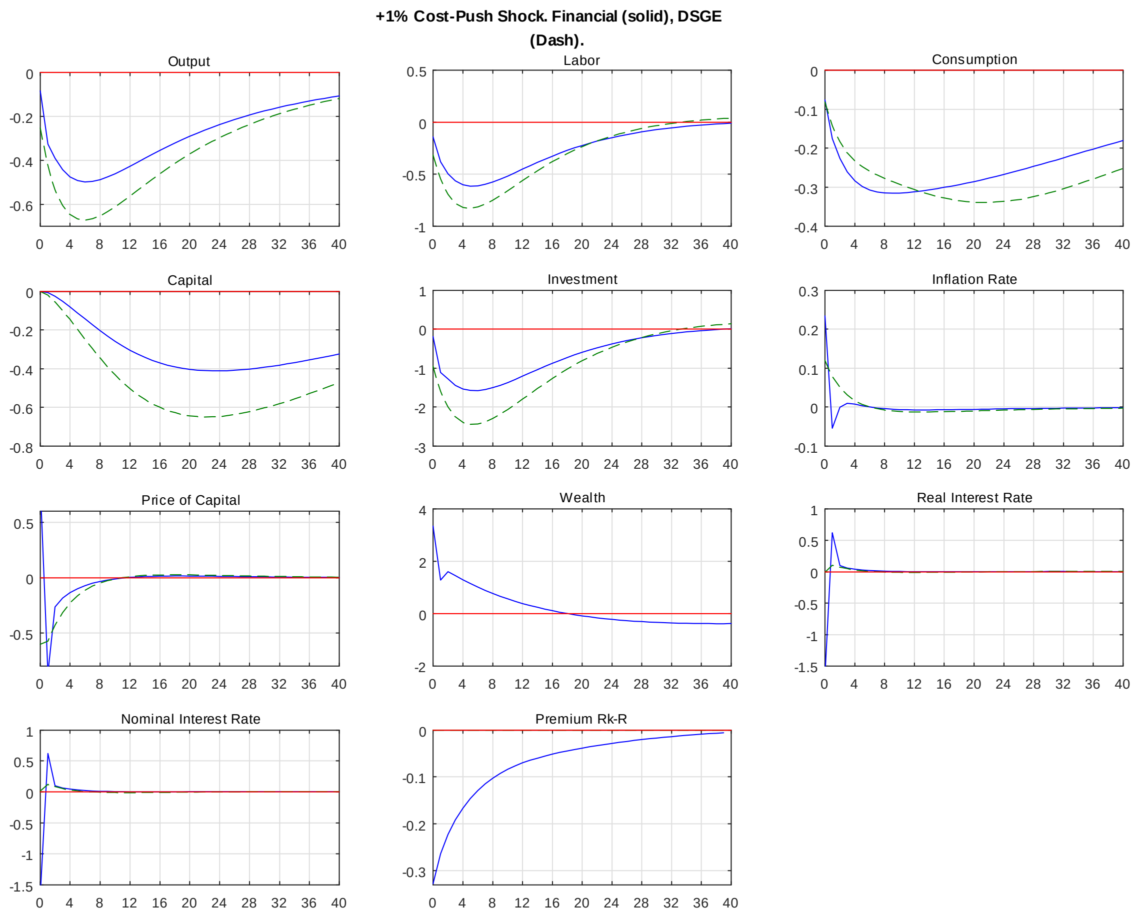

Figure 7. Optimal Policy. 1% Cost-Push Shock. Financial Model (solid) and DSGE (dashed).

In the next section, I deal with the implementation of optimal policy. In particular, I investigate whether the simple rules can implement optimal policy.

1.7 Implementation of Optimal Policy.

In this section, I deal with the implementation of optimal policy. I present the results of the welfare comparison across different regimes. Table 6 summarizes the main results. I restrict attention to policy rules that have the form

$$\begin{aligned} \ln\left(\frac{i_t}{i}\right) &= \kappa_R \ln\left(\frac{i_{t-1}}{i}\right) + (1-\kappa_R)\left\{ \begin{array}{c} \kappa_\pi \ln\left(\frac{\pi_{t-m}}{\pi}\right) + \kappa_Y \ln\left(\frac{Y_{t-m}}{Y}\right) \\ +\kappa_{SP}\left[\ln E_t\left(\frac{R^k_{t+1}}{R_{t+1}}\right) - \ln\left(\frac{R^k}{R}\right)\right] \end{array} \right\}, \qquad (1.91) \\ m &= -1, 0, 1, \end{aligned}$$

Table 1.6: Implementation of the Optimal Policy

Optimized		κ_π	κ_Y	κ_R	κ_{SP}	Cost
Not Reacting to Financial Events						
1.	Non-Inertial	2.325	0	0	0	0.0058
2.	Inertial	0.0505	0	0.95	0	0.0029
		$\kappa_{\pi-1}$	κ_{Y-1}	κ_R	κ_{SP}	Cost
3.	Backward	2.578	0	0	0	0.0052
		$\kappa_{\pi+1}$	κ_{Y+1}	κ_R	κ_{SP}	Cost
4.	Forward	2.017	0	0	0	0.0039
Reacting to Financial Events		κ_π	κ_Y	κ_R	κ_{SP}	Cost
5.	Non-Inertial	1.60	0	0	-0.78	0.0045
6.	Inertial	0.144	0	0.95	-0.1631	0.0022
Non-Optimized						
	Non-Inertial	κ_π	κ_Y	κ_R	κ_{SP}	Cost
7.	Taylor Simple	1.5	0	0	0	0.0105
8.	Taylor	1.5	0.5	0	0	0.0216
	Inertial					
9.	Taylor Simple	1.5	0	0.85	0	0.0078
10.	Taylor	1.5	0.5	0.85	0	0.0091

where (i_t) is the nominal interest rate and (i) is its long-run level, (π_t) is the inflation rate and (π) the long-run level of inflation. (Y_t) represents output and (Y) its steady-state level. (κ_π) is the policy coefficient on inflation deviations and (κ_Y) is the policy coefficient on output deviations. The index m can take three values 1,0, and -1. When m = 1, I refer to the interest rate rule as backward looking, when m = 0 as contemporaneous, and when i = -1 as forward looking.

Given that the optimal policy suggests stabilizing the financial variables, I explore the case in which the policy rule contains a coefficient (κ_{SP}) which measures the relevance of reacting to deviations of the premium on capital $E_t\left[\frac{R^k_{t+1}}{R_{t+1}}\right]$ respect to its average $\left[\frac{R^k}{R}\right]$.

1.7.1 Not Reacting to Financial Events.

Non-Inertial Rules.

The welfare cost represents the percentage of consumption that agents in the alternative policy scenario are loosing respect to the Ramsey regime. Optimized refers to a policy regime wherein the policy coefficients $[\kappa_\pi,\ \kappa_Y,\ \kappa_R,\ \kappa_{SP}]$ minimize the welfare cost. The search for policy coefficients was constrained to lie in the interval $[0,\ 3]$.

When financial frictions are present and the optimized rule contains only the policy coefficients $[\kappa_\pi,\ \kappa_Y]$, the welfare-maximizing rule has policy coefficients equal to $[2.325,\ 0]$. This is shown in

Table 1.7: Implementation of Optimal Policy (DSGE)

	Optimized		Taylor No Y	Taylor Rule
	$[\kappa_\pi,\ \kappa_Y]$	$[\kappa_\pi,\ \kappa_Y]$	$[\kappa_\pi,\ \kappa_Y]$	$[\kappa_\pi,\ \kappa_Y]$
	$[305, 0]$	$[3, 0]$	$[1.5, 0]$	$[1.5, 0.5]$
	(1)	(2)	(3)	(4)
Welfare cost (%)	0.00001	0.003	0.0044	0.0204

the first row of table 6. This policy costs 0.006 percent.

In order to have an understanding of the relevance of the coefficients in the previous rule, I compare these results to the case in which the financial frictions are absent. Table 7 presents the welfare costs of optimal rules in this case.

Initially, the search for the policy coefficients was restricted to lie in the interval [0, 3]. In this case, the policy coefficient on inflation takes the largest possible value and the cost is 0.003 percent (column (2)). This rule implements the optimal policy, which in the absence of financial frictions is akin to price stability.

When the upper bound on the search for optimal coefficient was removed, and it was allowed to take any non-negative value, the policy coefficient $[\kappa_\pi]$ takes a value of 305 (column(1)). This policy has a cost of 0.00001 percent. In this case, the policy coefficient $[\kappa_\pi]$ is large but finite. This reflects the desire of the optimizing policy maker to stabilize inflation in the absence of financial frictions.

One difference can be observed between the financial and non-financial models. In the conventional DSGE, the larger the value of the coefficient on inflation, the higher the welfare. Columns (1) and (2) in table 7, show this. However, this is not the case in the financial model. For example, when the financial frictions are present, a policy rule with coefficients [5, 0] implies a welfare cost of 0.0064 percent. In the presence of financial frictions, inflation stabilization is not as desirable as it is in their absence.

One similitude can be observed between these economies. In both cases, the coefficient $[\kappa_Y]$ is equal to zero. Indeed, the costs increase as the value of the coefficient $[\kappa_Y]$ increases. For example, the policy rule [1.5, 0], in the conventional model costs 0.0044 percent. But the conventional Taylor rule, fourth column in table 7, implies a welfare cost of 0.0204 percent when financial frictions are added.

Similarly, in the financial model, the rule [1.5, 0] has a cost of 0.0105 (row 7 in table 6). The conventional Taylor rule [1.5, 0.5] costs 0.0216 percent (last row in table 6). Schmitt-Grohé and Uribe (2007) show and explain the reasons behind the optimality of not responding to changes in output in a model without financial frictions. If the monetary rule contains a cyclical component, in the face of productivity or supply shocks, the economy would not be allowed to adjust efficiently.

This can create price dispersion, which in models with sticky prices is costly. After observing the results of the optimal rules, that result and explanation is robust to the presence of financial frictions.

Next, I show the effects of allowing an inertial term in the previous rules. After that section, I analyze whether a simple rule that reacts to financial variables, such as the cost of credit, can implement the optimal policy.

Inertial Rules.

In this section, I check the robustness of the previous results to the introduction of inertial policy rules. Woodford (2003, 2003b) and Sims (2013) show the advantages of introducing inertial components in the policy rules.

An inertial policy rule is a good approximation to the optimal policy under commitment. Reacting to an endogenous state variable serves the policy maker with the ability to exploit the expectational advantages of commitment. By having persistence in the rule, the central bank can anchor inflation expectations, which could in turn improve the current policy trade-offs faced when financial frictions are present.

Row 2 in table 6 shows the inertial rule. The fact that the optimized rule is inertial suggest that the central bank reacts more strongly to inflation in the long-run than in the short-run. This is observed also in the figures 1 and 5. The coefficient on the lagged value of the nominal interest rate takes the largest possible value. Reacting to contemporaneous inflation has a very small weight. It is optimal to not responding to output in this case. This rule welfare-dominates the non-inertial rule in the presence of financial frictions (row 1).

Backward Looking Rule.

The optimal backward looking rule also implies a zero reaction to the past level of output and to the past level of the nominal interest rate (row 3). When the nominal interest rate reacts to the past value of the variables, the cost are higher than in the contemporaneous or forward-looking rules.

Forward Looking Rule.

The forward looking rule, a rule that responds to expected inflation and the expected output deviations, also does a good job in approximating the welfare implied by the optimal commitment. This rule implies a strong reaction to future changes in inflation and zero reaction to future changes in output (row 4). When the rule is forward-looking the coefficient on the lagged value of the nominal rate is optimally driven to zero.

1.7.2 Reacting to the Financial Variables.

When the financial frictions are present, the optimized policy coefficient $[\kappa_\pi]$ takes a smaller value than in the model without these frictions. Inflation stabilization is not as desirable in this case as it would be in the absence of the financial frictions[6]. However, to what extent does reacting to changes in the cost of credit (the premium on capital) improve social welfare?

Rows 5 and 6 of table 6 provide an answer. If the monetary rule can react to changes in the spread between the return on capital and the risk-free rate, there are welfare gains. If, in addition, the rule is inertial, that rule is the welfare maximizing one (row 6).

Comparing rows 1, and 5, there is a cost-reduction of 0.0013 percent if monetary policy reacts to changes in the cost of credit. There are also welfare gains respect to the Taylor rule.

When the rule is inertial (row 6), the relevance of the feedback coefficient on the financial variable decreases but the welfare gain respect to the not reacting to changes in the cost of credit increases to 0.0036 percent (row 1 minus row 6). This is the welfare maximizing rule because making the policy rule history dependent serves to anchor inflation expectations. This allows the central bank to react to the financial events in the short-run and commit itself to increase the rate if necessary in the future. By exploiting the commitment technology, it is possible for the central bank to react to financial events in the short-run and inflation in the long-run.

1.7.3 A Summary of Optimal Implementation.

The welfare maximizing rule reduces the volatility in the financial markets. In the presence of financial frictions, inflation stabilization is not as desirable as it is in their absence.

A simple rule that reacts to changes in the cost of credit is able to implement the optimal policy. Making that rule history dependent allows the central bank to smooth financial volatility in the short-run and commit itself, if necessary, to revert its policy in the future to achieve price-level control. This rule mimics the Ramsey policy.

1.8 Robustness Checks.

1.8.1 Monopolistic Competition.

The main distortion in the model is due to the presence of monopolistic competition. The elasticity of substitution across goods (ε) governs the degree of monopolistic competition in steady state. The value of this parameter used in the benchmark calibration follows the estimation results of Primiceri et al. (2006). However, this implies a markup of around 30 percent, which is in the

[6]The determinacy properties are not altered by the introduction of the term reacting to changes in the spread on capital. The Taylor principle continues holding.

Table 1.8: Welfare comparison. Productivity shock

Optimized ($\varepsilon = 4.17$)	κ_π	κ_Y	κ_R	κ_{SP}	Cost
1. Non- Inertial	2.1883	0	0	0	0.000045
2. Inertial	0.3170	0	0.95	0	0.000031
No Optimized					
No Inertial					
3. Taylor Simple	1.5	0	0	0	0.00071
4. Taylor	1.5	0.5	0	0	0.00110
Inertial					
5. Taylor Simple	1.5	0	0.95	0	0.00040
6. Taylor	1.5	0.5	0.95	0	0.00100

upper bound of the conventional values. Hence, as a robustness check, I use a more competitive economy by increasing the value of (ε).

In the financial accelerator model, monopolistic competition is important because it results in a non-constant gap between the efficient and the natural allocation. And then, this can create a trade-off between inflation and financial stabilization.

In order to understand if the policy trade-offs remain in a more competitive economy, I derive optimal policy and its implementation for the case in which the economy is more competitive. For example, choosing a value of $\varepsilon = 11$, which would imply a markup of about 10 percent in steady state. The optimal policy is similar to that in the benchmark case. It is optimal to allow inflation to increase after a shock to productivity, a financial shock, or a markup shock. However, the size of the trade-off decreases. This is in line with Leduc and Natal (2015), who found in a model with a financial accelerator, similar to that in Bernanke, Gertler and Gilchrist (1999) that the policy trade-offs under monopolistic competition, price stickiness and financial frictions are increasing in the monopolistic competition.

1.8.2 Only Shocks to Productivity.

Optimized Rules.

When financial frictions are present and only productivity shocks are considered, the policy rule that implements the optimal policy, is inertial. These results are shown in table 8.

A central result is that when financial frictions are present, and the economy is subject only to a productivity shock, the coefficient on inflation is large, but it is several orders of magnitude smaller than when these frictions are absent. For example, in the conventional model, if there are only productivity shocks the welfare cost decreases in the size of the coefficient on inflation.

Figure 1.8: Welfare and Productivity Shocks

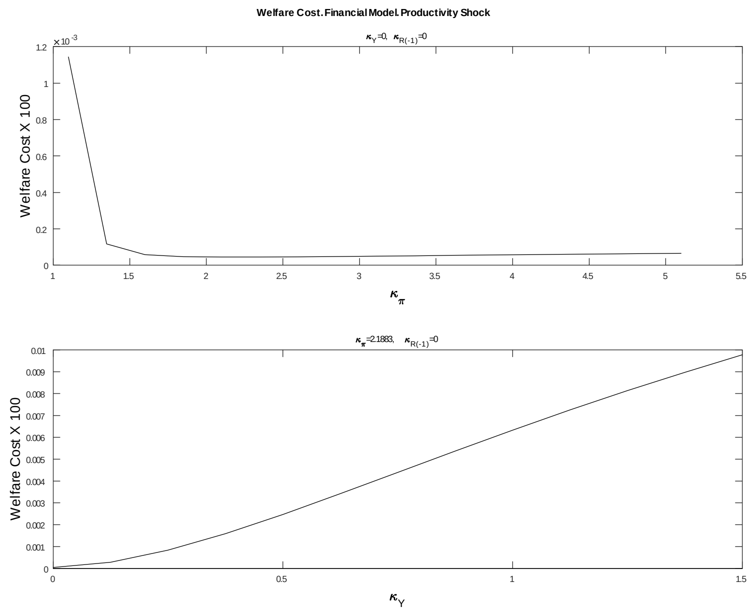

Figure 8. Welfare Cost in a Model with Financial Frictions and only Productivity Shocks. Top panel shows the welfare cost of increasing the feedback coefficient on inflation. Bottom panel shows the welfare cost of increasing the feedback coefficient on output.

Figure 1.9: Welfare and Productivity Shocks (DSGE)

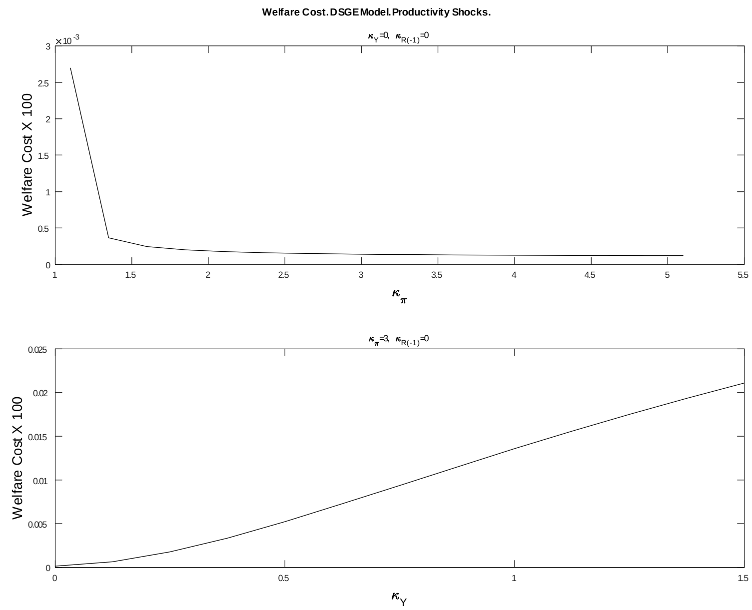

Figure 9. Welfare Cost in the DSGE Model and Only Shocks to Productivity. Top panel shows the welfare cost of increasing the feedback coefficient on inflation. Bottom panel shows the welfare cost of increasing the feedback coefficient on output.

Figure 8 shows that when financial frictions are present, welfare decreases if the weight of inflation in the policy rule is larger than 2.1883. Welfare costs are also increasing as the coefficient on output increases. Finally, figure 11 shows the same information as in figure 10 but for the conventional DSGE. In contrast, welfare costs are decreasing as the inflation coefficient increases. In this case, the policy rule would select a very large coefficient on inflation.

1.8.3 When all the shocks are present.

Figures 10 and 11 show the welfare cost if a shock to the quality of capital, a cost-push shock, and a shock to productivity are present. The top plot of figure 10 shows the welfare cost as the inflation coefficient increases in the presence of financial frictions. Figure 11 shows the case for the conventional model.

Figure 1.10: Welfare and Shocks

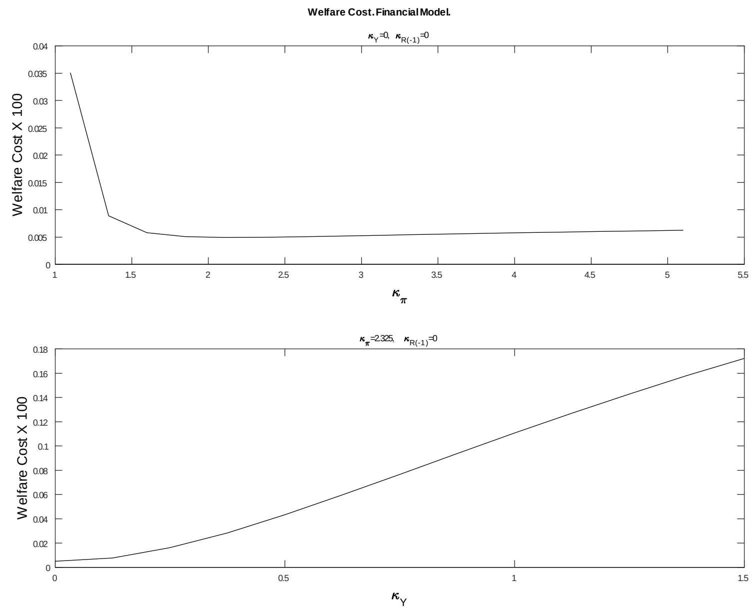

Figure 10. Welfare Cost in a Model with Financial Frictions and Various Shocks. The shocks are: a productivity shock, a shock to the quality of capital and a cost-push shock. Top panel shows the welfare cost of increasing the feedback coefficient on inflation. Bottom panel shows the welfare cost of increasing the feedback coefficient on output.

In figure 10, the analysis consider a set of shocks: a technology shock, a shock to the quality of capital and a cost-push shock. These shocks have been introduced previously in the text and the equation for them are contained in the quations 30, 31 and 34 in the Appendix A.5. In particular, the shock to the quality of capital follows Gertler and Karadi (2011) and it is introduced to mimic a financial crisis. The technology shock is a sudden decrease in the productivity in the economy, following previous literature, and the cost-push shock is a sudden increase in the markup of the firms.

The key difference between the two cases is that in the conventional model the cost decreases on the inflation coefficient (κ_π), while in the financial model there is a maximum value for this coefficient. In the conventional model, after a value of 5 the cost is closely flat. For small values of (κ_π) this cost decreases quickly as the inflation coefficient increases. In the financial model the cost is also decreasing for small values of (κ_π). But, this cost has a minimum at 2.325. For larger values the cost increases. These plots reflect the results of table 6. In both cases, welfare decreases as the coefficient on output increases.

Figure 1.11: Welfare and Shocks (DSGE)

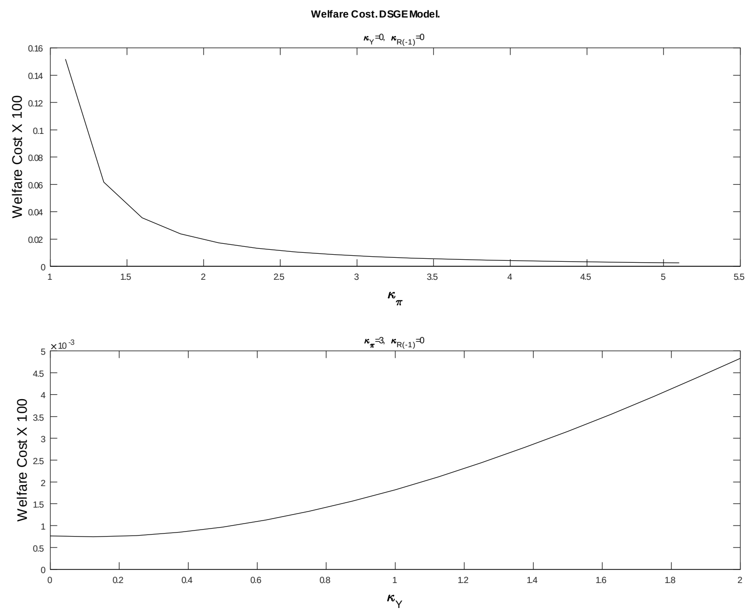

Figure 11. Welfare Cost in the DSGE Model and Various Shocks. The shocks are: a productivity shock, a shock to the quality of capital and a cost-push shock. Top panel shows the welfare cost of increasing the feedback coefficient on inflation. Bottom panel shows the welfare cost of increasing the feedback coefficient on output.

1.9 Conclusion.

In a standard New Keynesian model with a banking sector that faces balance sheet constraints, the optimal policy seeks to stabilize the financial markets by reducing the volatility of the cost of credit; a healthy financial sector is desirable. In this framework, there is a trade-off between inflation stabilization and financial stabilization. This holds if the economy is subject only to a productivity shock or if the economy becomes more efficient by reducing the monopolistic competition.

The implementation of optimal policy suggests stabilizing the spread between the return on capital and the risk-free rate. When a shock hits the economy, this policy suggests an aggressive reaction in the initial periods.

The simple rule that mimics optimal policy suggests a zero coefficient on changes in output, and a non-zero coefficient to changes in the premium on capital with respect to its long-run average. Stabilizing the financial sector enhances social welfare. In contrast, a strong anti-inflationary stance may be welfare decreasing. Additionally, inertial rules serve to anchor inflation expectations in the long-run, while stabilizing financial markets in the short-run, mimicking the optimal policy under commitment.

Chapter 2

Optimal Unconventional Policy.

Abstract.

I analyze optimal policy in a New Keynesian model with a banking sector that faces balance sheet constraints. The central bank now has access to two policy instruments: the nominal interest rate and credit easing through asset purchases. In addition, I consider the implementation of the optimal policy.

In the presence of financial frictions, if a negative shock hits the economy there are stabilization and welfare gains from allowing the central bank to participate in direct lending activities. In the benchmark case, these gains represent 0.3 percent the consumption of the Ramsey economy. The combination of the monetary rule and the asset purchase rule implement the optimal mix of policy.

2.1 Introduction.

As a consequence of the Great Recession, the understanding of the interactions between banking and monetary policy has become a central issue in policy design. The severity of the financial crisis has exposed the limitations of the conventional tools utilized by the central bank to stabilize the economy. Recent research has found that shocks to the financial sector are important drivers of business cycles. For example, Nolan and Thoenissen (2009), in a model with a feedback loop between financial and real events, find that financial shocks are the key drivers for output, investment, the external finance premium, the federal funds rate, and hours worked in the postwar U.S. economy.

Given the relevance of financial shocks as drivers of business cycles, and the limitations observed by conventional monetary policy in stabilizing the economy, it is relevant to analyze the optimality of equipping the central bank with additional policy instruments. This is the focus of this chapter.

If financial frictions are present, how should the central bank react to a negative financial shock? Are the conventional policy instruments enough to stabilize the economy? What additional policy tools should the central bank be provided with in order to stabilize the economy?

In this paper, I answer these questions. In a model with financial frictions, such as that presented in the previous chapter, I introduce an additional policy instrument: central bank's direct lending in the financial markets.

In this model, the banking sector faces endogenous financial constraints due to the presence of a moral hazard problem. In particular, in every period the bank can divert a share of the funds available for lending. The households impose a constraint on the banks' behavior in order to prevent this situation. This incentive constraint imposed by households to banks has the effect of restricting the supply of credit. If a shock damages the value of banks' assets, the cost of credit rises and this depresses investment spending, which contributes to amplify the disruption to the bank's balance sheet. The amplification effect can be such that the conventional policy is not enough to stabilize the economy. Hence, there is room for central bank's intermediation in the private markets, such as that observed during the Great Recession.

In this framework, if the central bank is allowed to act as intermediary in the private lending market, the central bank issues riskless bonds to households and utilizes these funds to issue loans to banks or to non-financial firms at the market interest rate. The advantages of the central bank in the financial markets relies on the assumption that it always honors its debts. However, it is less efficient at monitoring and choosing the right investment opportunities than the private sector. These efficiency costs limit the ability of the central bank to participate in financial markets.

If there is a shock that disrupts economic activity, the optimal unconventional policy suggests increasing the central bank's participation in private lending markets. In turn, this reduces the overall size of the economic disruption. Government direct lending reduces financial stress by increasing the total supply of funds intermediated in the economy. This decreases the cost of credit and stabilizes the premium over the riskless rate. The balance sheet of private banks is stabilized and the financial accelerator effect is muted. With two policy instruments, the central bank improves its policy trade-offs. Simple rules can implement the optimal mix of policy: the conventional Taylor rule is complemented by a rule that governs the central bank's lending as a function of the financial variables (in particular, the deviations in the cost of credit).

I focus on the Quantitative Easing program (QE 1) because it was the largest in magnitude of the three QE programs implemented by the FED[1]. In particular, the combined stimulus of purchasing Government Sponsored Enterprises (GSE) agency debt (1.2 percent), Mortgage Backed Securities (8.7 percent) and Treasuries (2.1 percent) amounted to 12 percent of the economy (Fawley and Neely (2013))[2].

The weight of evidence suggests that QE1 was more effective than QE2 or Operation Twist. Gertler and Karadi (2013) highlight different reasons for this: the stimulus under QE1 was larger

[1]Gagnon et al. (2011) estimate that between December 2008 and March 2010, the fed purchased 22 percent of the 7.7 trillion stock of longer-term agency debt, fixed-rate agency MBS and Treasury securities outstanding at the beginning of the LSAP.

[2]QE2, which mainly focused on the purchase of long-term government bonds represented 8.9 percent of GDP (4.2 percent in Treasuries and 4.7 percent in the maturity extension program). QE3 was the less important program and represented 1.1 percent of the economy.

than the stimulus under the following interventions; it included purchases of private assets and not just government securities; and, it was undertaken at the height of the crisis when financial markets and institutions were under maximum affliction.

It would be interesting, in future versions, to extend this analysis of optimal policy to the case in which the central bank purchase both private and public securities[3].

The plan of this paper is as follows: in the second section I present the model and introduce the unconventional policy instrument. The model in this chapter is similar to the model in the first chapter. Hence, in section two, I present only the parts of the model that must be amended in order to introduce the asset purchases. The third section presents the calibration of the model. In the fourth section, I present the optimal unconventional monetary policy exercise. In the fifth section, I present the implementation of the optimal mix of policy. The last section shows the literature review.

2.2 The Model.

This model is a DSGE similar to Christiano et al. (2005) and Smets and Wouters (2007) but modified in Gertler and Karadi (2011) to include financial intermediaries that face balance sheet constraints.

In this model, there are five groups of agents: households, financial intermediaries, non-financial producers, capital-goods producers, and retailers. The household sector contains two groups of agents: bankers and consumers. There is perfect risk-sharing within each household.

As a consequence of the incentive constraint imposed by the savers on the banks, these banks limit the supply of credit. The banks, which have a finite horizon to guarantee dividend payments, are owned by households, who transfer an amount of resource to the new bankers. The capital-good producers purchase, repair the worn out capital, and sell the repaired and new capital to the non-financial producers. The non-financial producers are required to borrow from the banks in order to fund their capital acquisition. In doing so, the non-financial producers issue state contingent claims on the returns of each unit of capital to the banks. Hence, borrowing and lending at this level is frictionless. Retailers purchase inputs from the non-financial firms and set the price of their products considering Calvo-type price stickiness.

The central bank has two policy instruments: the nominal interest rate, and asset purchases. Because the central bank is assumed to not be constrained, as the private banks are, if there is a shock which disrupts financial intermediation, the central bank can ameliorate the effects by purchasing private assets. These purchases increase the price of the asset, which revalues the bank's balance sheet, ameliorating the effects of the financial constraint. This, in turn, reduces

[3]In Gertler and Karadi (2013), the central bank can intermediate both public and private securities. The effects on the economy are stronger in the case of purchasing private securities. This result relies on the assumption that the financial friction is more pronounced in the private markets than in the government sector.

the cost of credit and investment spending and the overall level of activity can be stabilized.

Next, I present the sections of the model which need to be amended in order to introduce central bank direct lending. In particular, the unconventional policy affects the banking sector and the policy section in the model. All the remaining relationships are similar to the model presented in chapter 1. Appendix B.2 lists the full set of equilibrium conditions.

2.2.1 Private Financial Intermediation.

Banks.

Balance Sheet. The financial intermediary (j) receives deposits from households (D_{jt+1}). These deposits pay the short-term real interest (R_{t+1}) from (t) to $(t+1)$. These funds complement the accumulated wealth of banks (N_{jt}). The banks make use of these two sources of funds to issue loans to producers. Loans pay the rate $\left(R_{t+1}^{k}\right)$ between (t) and $(t+1)$.

The quantity of financial claims on intermediate producers that the bank holds is $\left(S_{jt}^{P}\right)$, its relative price is (Q_t). Then, each period the value of assets held by the representative bank is $\left(Q_t S_{jt}^{P}\right)$ and its total liabilities plus capital are $(D_{jt+1}+N_{jt})$. The balance sheet of the bank is:

$$Q_t S_{jt}^{P} = D_{jt+1} + N_{jt}. \tag{2.1}$$

Evolution of Wealth. Bank's net wealth evolves according to:

$$N_{jt+1} = R_{t+1}^{k} Q_t S_{jt}^{P} - R_{t+1} D_{jt+1}, \tag{2.2}$$

which is the difference between the return on its assets $\left(R_{t+1}^{k} Q_t S_{jt}^{P}\right)$ and the cost of its liabilities $(R_{t+1} D_{jt+1})$. Solving (2.1) for deposits and using (2.2), allows to express the evolution of wealth in terms of assets and the capital accumulated by the bank:

$$N_{jt+1} = \left[R_{t+1}^{k} - R_{t+1}\right] Q_t S_{jt}^{P} + R_{t+1} N_{jt}, \tag{2.3}$$

where the term $\left[R_{t+1}^{k} - R_{t+1}\right]$ is the asset's premium over the riskless rate.

The banker will not fund a project with a return lower than the cost of deposits. If the discount factor applied by the bank to assets between period (t) and $(t+i)$ is $\left[\beta^{i}\Lambda_{t,t+i}\right]$, then the next condition should apply for the bank to operate

$$E_t \beta^{1+i} \Lambda_{t,t+1+i} \left[R_{t+1+i}^{k} - R_{t+1+i}\right] \geq 0, \tag{2.4}$$

in any period $(i \geq 0)$. In frictionless capital markets this relationship holds with equality. When the financial frictions are present this risk-adjusted premium may be positive. The presence of a positive spread in equilibrium will translate into inefficiently low levels of capital.

When a shock arises, this spread can fluctuate. This fluctuation in the spread will translate into inefficient fluctuations of the real activity. If the shock damages the banks' balance sheet and if the central bank faces some constraint in its ability to reduce the nominal interest rate, there is some room for unconventional policy to stabilize this spread.

Incentive constraint. In order to issue loans to the producers, the bank uses its accumulated wealth and the funds raised by selling deposits to the savers. It is assumed that banks face frictions in this process of borrowing from households. This friction reduces the ability of the bank to issue new loans. Given this frictions, the commercial bank cannot fully exploit the arbitrage opportunities in the borrowing and lending markets. This explains the existence of the positive premium in equilibrium.

The friction exists because every period the bankers can divert a fraction (λ_t) of total deposits. When the households realize this, they force the bank to shut down and households can recover only the remaining $(1 - \lambda_t)$ fraction of deposits. The earnings to the bank of diverting funds are the total funds they can divert $\left(\lambda_t Q_t S_{jt}^P\right)$. In order to avoid that the banker absconds with these funds, the households impose an incentive constraint to the bankers:

$$V_{jt} \geq \lambda_t Q_t S_{jt}^P. \tag{2.5}$$

The term (V_{jt}) is the expected present value of the bank's financial activity if it remains in the industry; this is what the bank would lose if it is forced to leave the industry. The term $\left(\lambda_t Q_t S_{jt}^P\right)$ is what the banker would gain if it absconds with the funds. The bank assesses this trade-off and acts optimally. The bank would remain in the industry as long as the benefits from doing so cover the benefits from absconding with a share of assets.

The household would place deposits in the bank only if the benefit for the bank of lending and borrowing is at least as large as the benefit for the bank from diverting funds. In this way, the households ensure that the bank will not abscond with funds and households will confidently deposit in that bank.

Eventually, the share of diverting assets (λ_t) can be subject to an exogenous shock. An increase in this ratio can be interpreted as an increase in the probability of cheating, which increases the financial stress and precipitates a financial crisis. In particular, this exogenous shock evolves as

$$\ln \lambda_t = \rho_\lambda \ln \left[\lambda_{t-1}\right] + \varepsilon_{\lambda_t}, \tag{2.6}$$

where $(\varepsilon_{\lambda_t})$ has mean zero and variance $(\sigma_{\lambda t}^2)$.

Bank Maximization Problem. The possibility of making profits in the intermediation process acts as an incentive for the banker to remain in the industry as long as possible. The problem of the bank is to maximize the expected value of its terminal wealth (V_{jt})

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\, \theta^i \beta^{i+1} \Lambda_{t,t+1+i} \left(N_{jt+1+i} \right), \tag{2.7}$$

where

$$N_{jt+1+i} = \left[R^k_{t+1+i} - R_{t+1+i} \right] Q_{t+i} S^P_{jt+i} + R_{t+1+i} N_{jt+i}. \tag{2.8}$$

Optimal Conditions for Banks. In the appendix A.2 it is shown that the banks maximization problem can be expressed as

$$V_{jt} = v_t Q_t S^P_{jt} + \eta_t N_{jt}, \tag{2.9}$$

where

$$v_t = E_t (1-\theta)\, \beta \Lambda_{t,t+1} \left(R^k_{t+1} - R_{t+1} \right) + E_t \theta \beta \Lambda_{t,t+1} x_{t,t+1} v_{t+1}, \tag{2.10}$$

and

$$\eta_t = E (1-\theta)\, \beta \Lambda_{t,t+1} R_{t+1} + E_t \theta \beta \Lambda_{t,t+1} z_{t,t+1} \eta_{t+1}. \tag{2.11}$$

The term (v_t) is the marginal expected gain for the bank of increasing one unit of assets while keeping the net wealth without change. Similarly, (η_t) is the marginal expected return for the bank of increasing wealth in one unit while the assets remain constant.

The term

$$z_{t,t+1} = \frac{N_{t+1}}{N_t},$$

is the gross growth of private net wealth over the period (t) and $(t+1)$. Over the same period, the bank's assets have a gross growth of

$$x_{t,t+1} = \frac{Q_{t+1} S^P_{t+1}}{Q_t S^P_t}. \tag{2.12}$$

Leverage Ratio of Private Banks. In frictionless financial markets the bank would seek to increase assets until the point in which the discounted return on capital equates the discounted cost of deposits. The risk-adjusted premium would be zero. However, in this model, the incentive constraint prevents the bank to arrive to that point. After substituting the conjectured solution of the bank's problem (2.9) in the incentive constraint (2.5)

$$\nu_t Q_t S^P_{jt} + \eta_t N_{jt} \geq \lambda_t Q_t S^P_{jt}\ , \tag{2.13}$$

and solving for assets, the leverage ratio is

$$\frac{Q_t S_{jt}^P}{N_{jt}} = \frac{\eta_t}{\lambda_t - \nu_t}, \tag{2.14}$$

when this constraint holds with equality. The term $\left[\frac{Q_t S_{jt}^P}{N_{jt}}\right]$ is the leverage ratio of private banks (ϕ), which can be defined as the ratio of loans to bank's net wealth. Hence,

$$\phi_t = \frac{\eta_t}{\lambda_t - \nu_t}. \tag{2.15}$$

The leverage ratio (ϕ_t) is a function of the diverting rate (λ_t) and the profitability of the banking industry ($\upsilon_{t,}\ \eta_t$). If the profitability of banks increase, either by an increase in (ν_t), the profitability of each unit of assets, or in (η_t), the profitability of each unit of accumulated wealth, the bank can issue more loans without breaking the incentive constraint.

In contrast, if the diverting of funds increases (λ_t) the maximum tolerated leverage ratio decreases. This reduces the amount of loans issued by banks, in order to avoid breaking the incentive constraint, and increases the premium, increasing the cost of credit and precipitating a recession.

The evolution of bank's wealth (eq.(2.3)) can be expressed as:

$$N_{jt+1} = \left\{\left[R_{t+1}^k - R_{t+1}\right]\phi_t + R_{t+1}\right\} N_{jt}, \tag{2.16}$$

after the substitution of the leverage ratio.

Evolution of Aggregate Private Leverage Ratio. The components of the key variables in the leverage ratio are the same for each bank. Then, aggregating $\left[Q_t S_{jt}^P = \phi_t N_{jt}\right]$ over banks, the total assets intermediated by private banks $\left(Q_t S_t^P\right)$ and the bank's net wealth (N_t) are a function of the private leverage ratio

$$Q_t S_t^P = \phi N_t, \tag{2.17}$$

this means that the maximum amount of loans issued by the private banks $\left(Q_t S_t^P\right)$ is limited by the leverage ratio (ϕ_t) times the banks' capital (N_t). If a shock decreases the value of the bank's capital, the amount of loans issued by banks would decrease, precipitating a recession. In this context, there is room for central bank's direct lending.

Evolution of Aggregate Wealth of Private Banks. The evolution of private banks' aggregate wealth is the sum of two components. The wealth associated with the banks surviving until the previous period (N_{et}) plus the aggregate wealth of the starting banks (N_{nt}).

$$N_t = N_{et} + N_{nt}. \tag{2.18}$$

The fraction of bankers surviving at t is (θ). Then, using the aggregate expression of (2.16),

$$N_{et} = \theta \left\{ \left[R_t^k - R_t \right] \phi_{t-1} + R_t \right\} N_{t-1}. \tag{2.19}$$

The $(1-\theta) f$ new bankers receive starting funds proportional to the previous period intermediated assets. It is assumed that the new bankers receive a fraction $\left(\frac{w}{1-\theta}\right)$ of this assets to start business. Then evolution of aggregate wealth is

$$N_t = \theta \left\{ \left[R_t^k - R_t \right] \phi_{t-1} + R_t \right\} N_{t-1} + wQ_t S_{t-1}^P. \tag{2.20}$$

In the next section, I describe the central bank's direct lending activities.

2.2.2 Central Bank's Lending.

Share of Central Bank's Lending. Given the financial friction, and possibly some restrictions on the central bank to reduce the nominal interest rate, complementary policies may be useful to stabilize the economy. In particular, I assume that the central bank can implement direct lending to private firms[4].

When the central bank participates in the financial intermediation the value of total financial assets in the economy$[Q_t S_t]$, is

$$Q_t S_t = Q_t \left[S_t^P + S_t^G \right], \tag{2.21}$$

where $\left[Q_t S_t^P\right]$ represents the value of private assets and $\left[Q_t S_t^G\right]$ the value of assets intermediated by the government.

If the government is able to intermediate each period a fraction $\left[\kappa_t^{QE}\right]$ of total assets, then

$$\kappa_t^{QE} = \frac{Q_t S_t^G}{Q_t S_t}. \tag{2.22}$$

When $\kappa_t^{QE} = 0$, I return to the same case described in the first chapter, where only private intermediation and conventional monetary policy exist.

Funding the Central Bank's Lending. The assets intermediated by the government are funded by issuing riskless government bonds. Households purchase these risk-free bonds $\left[B_{t+1}^g\right]$ and in exchange they receive the rate $[R_{t+1}]$ between period (t) and $(t+1)$. The advantage of the government in this scenario is that it always honors its debts. Then, households rely on the government to purchase as much debt as the central bank requires. The financial friction exists only between households and private banks.

[4]The solution to the model is identical if, instead, I assume that the central bank lends to private banks at the market interest rate and then the private bank use these funds to lend to non-financial firms (the same overall balance sheet constraint arise) and hence, it does not matter to assume one or the other type of government's lending.

The amount of debt issued by the government equals the amount of assets the government lends to the non-financial firms (or to the banks)

$$Q_t S_t^G = B_{t+1}^g. \tag{2.23}$$

If the private banks could issue as many loans as demanded by non-financial firms (2.23) would be equal to zero, in frictionless financial markets, for instance. However, in periods of financial distress, given the financial friction, the central bank's intermediation could be non-zero. Hence, the central bank could fulfill the financial needs of non-financial markets. This would reduce the financial distress and would ameliorate the effects of recessions.

Return on Central Bank's Lending. When allowed to participate in the lending activities, the government exploits the arbitrage opportunities in the financial market. The government lends the funds obtained from households to the non-financial firms at the rate of return of capital observed in the market $\left[R_{t+1}^k\right]$. The profits for the central bank of its financial intermediation are equal to

$$\left[R_{t+1}^k - R_{t+1}\right] B_{t+1}^g, \tag{2.24}$$

which must be taken into account in the government's budget constraint.

I assume that the central bank is not balance-sheet constrained, as the banks are. It issues riskless government bonds and lend those funds to non-financial firms. However, the central bank is less efficient than private banks in regards to the financial activities. In particular, the government faces efficiency costs per unit of loan to non-financial firms. Those costs are $[\varsigma_t]$ and are related to the costs of evaluating and monitoring investment opportunities. I assume these costs are not present in private markets given that banks have specific knowledge of the market in which they are participating.

The efficiency costs of the central bank intermediation can increase if there are deviations respect to the share of assets held by the central bank in steady state. In particular, the cost function is:

$$\varsigma_t = \left\{ 1 + \frac{1}{2} \left[\frac{Q_t S_t^G}{Q_{t-1} S_{t-1}^G} - 1 \right]^2 \right\} \tau Q_t S_t^G. \tag{2.25}$$

2.2.3 Total Assets in the Economy.

The total amount of assets in the economy is:

$$Q_t S_t = Q_t S_t^P + Q_t S_t^G, \tag{2.26}$$

which can be written as

$$Q_t S_t = \phi_t N_t + \kappa_t^{QE} Q_t S_t, \tag{2.27}$$

after the substitution of (2.17) and (2.22). Alternatively,

$$Q_t S_t = \frac{1}{1-\kappa_t^{QE}} \phi_t N_t. \tag{2.28}$$

The previous relationship is the total amount of loans intermediated in the economy. The term $(Q_t S_t)$ represents the total needs of funds by the producers. In models in which only conventional policy is present, like that presented in the previous chapter, it is assumed that private banks alone satisfy these needs. In particular, that $\left(\kappa_t^{QE} = 0\right)$, and that the government decided not to participate in the financial markets.

However, when the government decides to participate $\left(\kappa_t^{QE} > 0\right)$, given a level of demand for loans, the supply of financial assets is now complemented by the government. This contributes to the reduction of the tightening of the balance sheet of the banks, which is translated into a reduction in the cost of credit.

2.2.4 Government Budget Constraint.

Government spending (G_t), the costs associated with the financial intermediation of the government (ς_t), the purchase of non-financial firm's assets, and the payments of the debt acquired previously $(R_t B_t^g)$ are financed with a tax to sales $(\tau_t Y_t)$, issue of new government bonds $\left(B_{t+1}^g\right)$, with the revenue from the participation in the credit markets $\left(R_t^k B_t^g\right)$ and using lump-sum taxation (T_t).

The government's budget constraint can be written as

$$G_t + \varsigma_t + R_t B_t^g = T_t + \tau_t Y_t + R_t^k B_t^g + B_{t+1}^g. \tag{2.29}$$

Government spending evolves exogenously. In particular

$$G_t = G g_t$$

where (G) is the steady state level of government spending $[G = 0.2Y]$. This spending can be subject to an exogenous shock (g_t) which evolves as

$$\ln(g_t) = \rho_g \ln(g_{t-1}) + \varepsilon_{gt} \tag{2.30}$$

where the (ε_{gt}) is an i.i.d. process with mean zero and variance $\left(\sigma_g^2\right)$.

Distortionary taxation is fixed at its steady state value and it serves only as a cost push shock which evolves exogenously as

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right) \ln(1-\tau) - \varepsilon_t^\mu, \tag{2.31}$$

(ε_t^{μ}) is i.i.d. with mean zero and variance $\left(\sigma_{\mu t}^2\right)$.

In addition, the government has access to lump-sum taxation which guarantees that the debt does not explode over time. Any gains or losses from the government's financial activity are returned lump-sum to the households. The financial participation of the government is fully funded with the issue of new debt $\left(B_{t+1}^g\right)$ which completely finances government intermediation $\left[\kappa_t^{QE} Q_t K_{t+1}\right]$.

The lump-sum taxes evolve as

$$T_t = G_t + \varsigma_t - \left[R_t^k - R_t\right] B_t^g - \tau_t Y_t - \kappa_t^{QE} Q_t K_{t+1}, \tag{2.32}$$

after substituting lump-sum taxes in the government's budget constraint

$$B_{t+1}^g = \kappa_t^{QE} Q_t K_{t+1}. \tag{2.33}$$

2.2.5 Aggregate Resource Constraint.

Consumption, government spending, and the efficiency costs associated with the government's financial intermediation, total investment and the costs associated with the net investment adjustment are the demand faced by the final product. Then, the aggregate resource constraint is

$$Y_t = C_t + I_t + \frac{\phi\_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 (I_{nt} + I_{ss}) + G_t + \varsigma_t. \tag{2.34}$$

2.2.6 Policy.

Monetary Policy.

The link between real and nominal interest rate (i_t) is given by the Fisher equation

$$i_t = E_t R_{t+1} \pi_{t+1}. \tag{2.35}$$

When the economy is analyzed utilizing simple rules, the central bank follows a Taylor rule to set the nominal interest rate; that rule is

$$\ln\left[\frac{i_t}{i}\right] = \kappa_R \ln\left[\frac{i_{t-1}}{i}\right] + (1 - \kappa_R)\left\{\kappa_\pi \ln\left[\frac{\pi_t}{\pi^*}\right] + \kappa_Y \ln\left[\frac{Y_t}{Y}\right]\right\} + \varepsilon_{it}, \tag{2.36}$$

where (ε_{it}) is an exogenous monetary policy shock. Eventually the policy maker can choose to smooth the interest rate; the size of this smoothing preference is controlled by (κ_R).

The Government's Financial Intermediation Policy.

When the policy is implemented via simple rules, given that the government has an additional instrument a rule for that policy must be introduced. In particular, I assume that the government's financial intermediation targets the deviation of the premium with respect to its steady state. This assumption relies on the fact that optimal policy suggests to stabilize this premium. In particular, the rule for the government's intermediation follows

$$\kappa_t^{QE} = \kappa_{QE} E_t \left[\log\left(\frac{R_t^k}{R_t}\right) - \log\left(\frac{R^k}{R}\right) \right]. \tag{2.37}$$

where the coefficient (κ_{QE}) reflects the importance the government assigns to the deviations of the premium from its steady state. The set of all the equilibrium conditions is listed in the Appendix B.2.

2.3 Calibration.

I calibrate the model to a quarterly frequency. the calibration section is divided into four subsections: in the first, I detail the choice of the non-financial parameters; this is followed by the calibration of the financial parameters; in the third subsection, I present the policy variables, where I show the calibration of the unconventional policy rule; the calibration of the efficiency costs is presented in the last subsection. Table 1 summarizes the benchmark calibration.

2.3.1 Non-Financial Sector.

Two main sources are used to calibrate the non-financial parameters: U.S. National Accounts and estimated models of the U.S. economy. In the non-financial sector, 4 parameters match historical averages of their empirical counterparts: the share of capital on national income (α), the subjective discount factor (β), the depreciation rate (δ) and the share of the government spending $\left(\frac{G}{Y}\right)$. The disutility of labor (χ), is calibrated to match the working hours. And the remaining 11 non-financial parameters are taken from the results of the estimated model in Primiceri et al. (2006).

Based on Historical Averages.

The share of capital on national income, the subjective discount factor, the depreciation rate and the relevance of government spending $\left[\alpha,\ \beta,\ \delta,\ \frac{G}{Y}\right]$ match the historical averages of their empirical counterparts.

The steady state share of capital to national income (α) is set to 0.33. This value for α approximates the observed compensation of employees as a share of national income ($1-\alpha$) (BEA NIPA table 1.12).

Table 2.1: List of Parameters

Consumers			Persistence of Shocks		
h	Habits in consumption	0.815	ρ_a	Technology	0.96
β	Subjective discount factor	0.9951	ρ_ξ	Quality of capital	0.89
χ	Disutility of labor	3.9401	ρ_g	Gov. spending	0.98
Financial Sector			ρ_λ	Diverting funds	0.98
θ	Probability of survival banks	0.9716	ρ_i	Monetary policy	0.58
λ	Share of assets diverted	0.3825	ρ_μ	Cost-push	0.854
ω	Transfer to starting banks	0.0035	St. dev. of Shocks		
Non-Financial Firms			σ_a	Technology	0.0068
α	Capital share	0.33	σ_ξ	Quality of capital	0.0037
φ	Inv. Frisch elasticity of labor supply	0.27	σ_g	Gov. spending	0.0065
ϕ_i	Elasticity inv. adj. costs	1.72	σ_λ	Diverting funds	0.0088
ζ	Elasticity of marginal dep.	7.2	σ_i	Monetary policy	0.0038
δ	Depreciation rate	0.025	σ_μ	Cost-push	0.00103
γ	Share of firms no adjusting price	0.77	Policy Variables		
γ^p	Degree of price indexation	0.241	κ_π	Inflation coefficient. Taylor rule	2.19
ε	Elasticity of substitution	6.0	κ_Y	Output coefficient. Taylor rule	0.20
			κ_R	Smoothing parameter. Taylor rule	0.39
			κ_{QE}	Feedback Parameter Asset Purchases	24.8
			τ_{QE}	Efficiency costs Asset Purchases	0.0025

The subjective discount factor $\beta = 0.995062$, matches an average real rate of interest rate of 2.0 percent (FRED). As seen in the work of Yun (1996), I assume that the firms can, without cost, index their prices to the steady-state rate of inflation. The depreciation rate (δ) takes the value 0.025, implying a 10 percent annual depreciation (BEA NIPA table 1.11). Regarding the steady state government spending to GDP ratio, $\left(\frac{G}{Y}\right)$ is 0.2, this is a conventional value (BEA NIPA table 1.1.10). The weight of the disutility of labor (χ), is 3.94. This value was calibrated assuming a steady state value of working hours of 0.33, a common value in the literature. The remaining parameters for the non-financial sector are taken from estimated studies. Next, I present the values for these parameters.

From Estimated Models.

Following Primiceri et al. (2006), I assume the following values for the non-financial parameters. The habits parameter (h) is set to 0.815. The inverse of the Frisch elasticity (φ) takes a value of 0.276. The elasticity of marginal depreciation to the utilization rate (ζ) takes a value of 7.2. The inverse of the elasticity of net investment to the price of capital (ϕ_i) is assumed to be 1.728.

The probability that a firm does not adjust its price this period ($\gamma = 0.779$) implies that a firm keeps its price for around 4 quarters. The size of the indexation of the price to the previous period inflation (γ^p) takes a value of 0.241.

The elasticity of substitution between varieties of goods (ε) is 6. In the estimated results by Taheri (2014), this elasticity takes a value close to 4.1, similar to that estimated by Primiceri et al. (2006) and utilized by Gertler and Karadi (2011). In Villa (2014), it is assumed that the elasticity would imply a gross markup of 1.25 in steady state. And in the majority of the literature using calibrated New Keynesian models this elasticity is between 6 and 11. When assuming that $\varepsilon = 6$, I am in the middle of the range of values used for this parameter in models with and without financial frictions[5].

Shocks.

Following Taheri (2014), the persistence of the shock to productivity, the shock to the quality of capital, the monetary shock and the shock to government spending take the values $\rho_A = 0.96, \rho_\xi = 0.89$, $\rho_i = 0.58$ and $\rho_g = 0.98$, respectively. The persistence of the shock to the share of diverting funds in the banking sector and to the cost-push shock are $\rho_\theta = 0.9782$ and $\rho_\tau = 0.85$, respectively. The parameters for the shocks to the diverting funds follow the work of Nolan and Thoenissen (2009), who present estimates for a shock to the contractual relations between borrowers and lenders, in a model of the financial accelerator. For the cost-push shock I follow Primiceri et al. (2006), who estimate this shock as a shock to the elasticity of substitution.

[5] As a robustness check I do exercises with different values of this elasticity. The effects of a more competitive economy imply larger values for the real variables in steady state. But the optimal policy does not change. These results are available upon request.

The standard deviation of the shock to productivity, the shock to the quality of capital, the monetary shock, and the shock to government spending take the values $\sigma_a = 0.0068, \sigma_\xi = 0.0037$, $\sigma_i = 0.0038$ and $\sigma_g = 0.0065$, respectively. These are the same values estimated by Taheri (2014). For the shock to the diverting funds $\sigma_\lambda = 0.0088$, following Nolan and Thoenissen (2009). And I take the estimates of Primiceri et al. (2006) for cost-push shock $\sigma_\mu = 0.00103$.

2.3.2 Financial Sector.

In this section, I explain my choice regarding the financial parameters. Firstly, I present the results of the choice of the three parameters that are particular to the financial sector. After that, I explain how I selected these values.

The survival probability of banks (θ), equals 0.9716 and matches a life interval of 35 quarters. The share of diverting funds (λ) is 0.3825, which is consistent with a spread of 100 basis points in equilibrium. The share of funds transferred to starting bankers (w) is 0.0035, after assuming a leverage ratio of 4 in the banking sector.

I explain now the evidence behind each of these values.

Survival Probability.

The survival probability of banks (θ) exists in the model to avoid banks increasing their size up to the point where they do not longer need to borrow from households. In this case, the incentive constraint would not bind.

The value of the survival probability in the banking industry is consistent with the share of diverting funds, and the share of starting funds implies that banks survive on average 35 quarters, $\theta = 0.9716$. This is in line with the work of Gertler and Karadi (2011). In their estimation exercises, Taheri (2014) and Villa (2014) assume that this parameter is fixed at this value before the estimation exercise.

Spread and Share of Diverting Funds.

The calibration implies a share of diverting funds (λ) of 0.3825. The share of diverting funds (λ) is set to target a spread of 100 basis points in steady state. This value of the spread reflects the historical spread between AAA-BAA corporate bonds for the U.S. economy between 1954 and 2015 (FRED).

The spread between the AAA bond and the 10 years Treasury bills is around 100 basis points. Finally, between 2005 and 2015, the average spread between government bonds and mortgages rates was in this neighborhood of 100 basis points (FRED). Assuming a spread of one percent in steady state therefore reflects the evidence on different measures of the spread.

Next, I present the evidence on the leverage ratio which serves to pin down the startup transfers to banks.

Leverage Ratio and Transfers to Banks.

The leverage ratio in steady state serves to pin down the startup transfer, (ω). The share of funds transferred to the starting bankers is $\omega = 0.0035$. This is the result of targeting a steady state leverage ratio of 4, in line with the leverage ratio observed in the U.S. financial sector.

Next, I present the different sources that register the possible values of the leverage ratio for the U.S. economy.

Banking Sector and Corporate Sector. In the previous crisis the leverage ratio of the banks in troubles were considerably higher than the average for the banking sector or for the corporate business sector. For example, during 2007, for the Big-5 investment banks that ratio was between 25 and 35 (Bhatia and Bayoumi (2012)). However, for commercial banks the leverage ratio (total assets to equity in the banks) had an average of 10.4 between 2000-2007, before the crisis (FRED). And the non-financial corporate sector showed a ratio of assets to equity of 2 (FRED).

To capture the overall leverage ratio in the economy, given that QE1 was designed to support the Government Sponsored Enterprises (GSE) and banks, I take the weighted average of the leverage ratio of these institutions. This weighted average showed a ratio of 4.05 on average between 2000 and 2007.

Financial Sector. As a robustness check I utilize data from OECD and FED. I utilize information on the U.S. financial corporations[6]. I construct the leverage indicator as the ratio of financial assets to shares and other equity. The average of this indicator between 2000 and 2013 was 3.75. And during 2007-2008, this average was 4.07.

Thus, the leverage ratio measured as a weighted average of commercial banks and non-financial corporations or measured for the overall financial corporations has observed a value of about 4.

2.3.3 Policy Variables.

The coefficients of the monetary policy rule take values from an estimated DSGE model with financial frictions in Taheri (2014). This estimated model also provides the information for the autorregresive coefficients in the shock processes and their variances.

I rely on empirical evidence associated with the overhead costs of the banking industry, and on the FED's share of assets held at the end of QE1 and its associated reduction in the spread, in order to calibrate the unconventional policy rule and the efficiency costs associated with it.

[6] Financial corporations include both public and private firms in the financial business. This includes monetary institutions, depository corporations, financial auxiliaries and insurance companies and pension funds. This is a broader indicator than just commercial banks.

Nominal Interest Rate Rule.

In order to calibrate the monetary rule, I follow the work of Taheri (2014): the coefficient measuring the reaction of the nominal interest rate to changes in inflation in the Taylor rule (κ_π) is 2.19; the feedback coefficient on output deviations (κ_Y) is 0.2; the smoothing parameter (κ_R) is 0.39. This is the benchmark calibration for the nominal interest rate.

Asset Purchases Rule.

In order to calibrate the parameters associated with the unconventional policy, I follow the next rule

$$\kappa_t^{QE} = \kappa_0 + \kappa_{QE} E_t \left[\log\left(\frac{R_{t+1}^k}{R_{t+1}}\right) - \log\left(\frac{R^k}{R}\right) \right], \tag{2.38}$$

where κ_0 is a constant and κ_{QE} is the feedback parameter on changes in the spread respect to its long-run level.

The benchmark calibration takes the values $\kappa_0 = 30.5$, and $\kappa_{QE} = 24.8$. In the appendix B.3, I explain the strategy followed to calibrate that rule.

Efficiency Costs.

Calibrating the efficiency costs is relevant for the welfare analysis and policy advice. For example, a fully optimal exercise, with zero efficiency costs, imply that the central bank will purchase all the assets in the economy. As the efficiency costs increase, the optimal asset holdings by the central bank decrease.

Once the asset purchases rule has been calibrated, I can calibrate the efficiency costs associated with that evidence. The strategy is to assume that the FED did not implemented a larger program because the efficiency costs prevent that to happen. Hence, in this section, I find numerically the value of the efficiency costs that imply that the rule with feedback coefficients equal to the benchmark case ($\kappa_0 = 30.5$, $\kappa_{QE} = 24.8$) was welfare maximizing. The efficiency cost used as benchmark in the calibration is $\tau = 0.0025$.

2.4 Ramsey Policy and Welfare.

2.4.1 Welfare Cost.

In this section, I present the measure utilized to calculate welfare costs across different policy regimes. In the most of the cases, I use the welfare associated with the Ramsey policy as a benchmark. Against the welfare implied by the Ramsey policy, I compare the welfare associated with following simple rules for conducting policy. The advantage of those alternative regimes is that they can be easily implemented by a central bank.

I measure the welfare cost as the amount of consumption that agents in the alternative regime are willing to accept in order to have the same welfare as in the benchmark economy.

To accurately compare welfare across different policies, I need to use a second-order approximation to the full model. Hence, I follow the work of Schmitt-Grohé and Uribe (2004, 2007).

Using the particular utility function

$$U = \ln \left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\psi} L_t^{1+\psi},$$

the consumption cost $\left[W^C\right]$, of choosing an alternative policy $\left(V^I_{\sigma\varepsilon\sigma\varepsilon}\right)$ relative to the time-invariant Ramsey policy $\left(V^R_{\sigma\varepsilon\sigma\varepsilon}\right)$, after approximating the result up to a second order, is:

$$W^C \approx \frac{1}{2}\left(1-\beta\right)\left[V^R_{\sigma\varepsilon\sigma\varepsilon} - V^I_{\sigma\varepsilon\sigma\varepsilon}\right]\sigma_\varepsilon^2. \tag{2.39}$$

The derivation of this measure of welfare cost is identical to the used in the Chapter 1 and is detailed in the Appendix A.4.

2.4.2 Optimal Policy.

The Ramsey planner seeks to maximize the welfare of the society subject to the competitive equilibrium conditions. I assume that the central bank is committed to following the announced plan from a timeless perspective. As in Schmitt-Grohé and Uribe (2005), I assume that at time (t) the Ramsey planner has been operating for an infinite number of periods.

The period (t) objective function of the Ramsey planner is the utility function

$$U_t = \ln \left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\psi} L_t^{1+\psi}. \tag{2.40}$$

I assume that the discount factor of the Ramsey planner is equal to the subjective discount factor of households in the competitive economy (β). This policy maker maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t, L_t\right] \tag{2.41}$$

subject to the competitive equilibrium conditions.

Then, the central bank maximizes the welfare function (2.41) subject to the competitive equilibrium restrictions choosing at period (t) processes for the 31 endogenous variables U_{ct}, C_t, Λ_t, L_t, v_t, x_t, η_t, z_t, ϕ_t, K_{t+1}, N_t, N_{et}, N_{nt}, R_{kt+1}, Y_{mt}, Q_t, δ_t, U_t, I_{nt}, P_{mt}, G_t, I_t, Y_t, Δ_t, F_t, Z_t, π_t^*, π_t, R_t, i_t, κ_t^{QE} and the 29 Lagrange multipliers associated with the 29 equilibrium conditions. The processes for the exogenous shocks are given. The values for the variables listed above are given dated t<0, and also the values of the Lagrange multipliers associated with the competitive equilibrium constraints are given at t<0. Then, as explained in Schmitt-Grohé and Uribe (2005), the structure of the optimality conditions associated with the Ramsey equilibrium are time-invariant.

The Appendix B.2 presents the Lagrangian for the optimal policy from a timeless perspective.

There are welfare gains to allowing the central bank to utilize unconventional means to stabilize the economy. The optimal level of the asset purchases depends heavily on the efficiency costs assumed. The fully optimal case with zero efficiency suggests that the central bank holds all the assets in the economy. There is a level of efficiency costs which prevent the central bank in participating at all in the financial markets.

2.4.3 Impulse-Response Analysis.

In order to have a better understanding of the role of the asset purchases, I present the impulse-response figures for an economy with flexible-prices and without policy, and for the same economy with flexible prices but with asset purchases as the policy instrument. In this economy, the monopolistic competition and the financial frictions are present.

When the central bank implements asset purchases, it holds 59.6 percent of assets, the optimal level of asset purchases given the calibrated welfare costs (Table 3).

2.4.4 Shock to the Quality of Capital.

Flexible Prices.

In this section, I present the response of this economy to a one percent decrease in the quality of capital. Figure 1 presents two cases: the decentralized economy in which no policy is available (crosses), and the optimal policy with asset purchases (circles).

This shock reduces the value of the bank's assets. This tightens the balance-sheet constraint. Without policy, the shock decreases the bank's net wealth in 4 percent (crosses). Given the endogenous leverage constraint, if there is a negative shock hitting banks' balance sheet, they would require a higher return on their assets in order to be able to issue new loans and to meet the incentive constraint imposed by households.

Figure 2.1: Optimal Asset Purchases. Flexible Prices

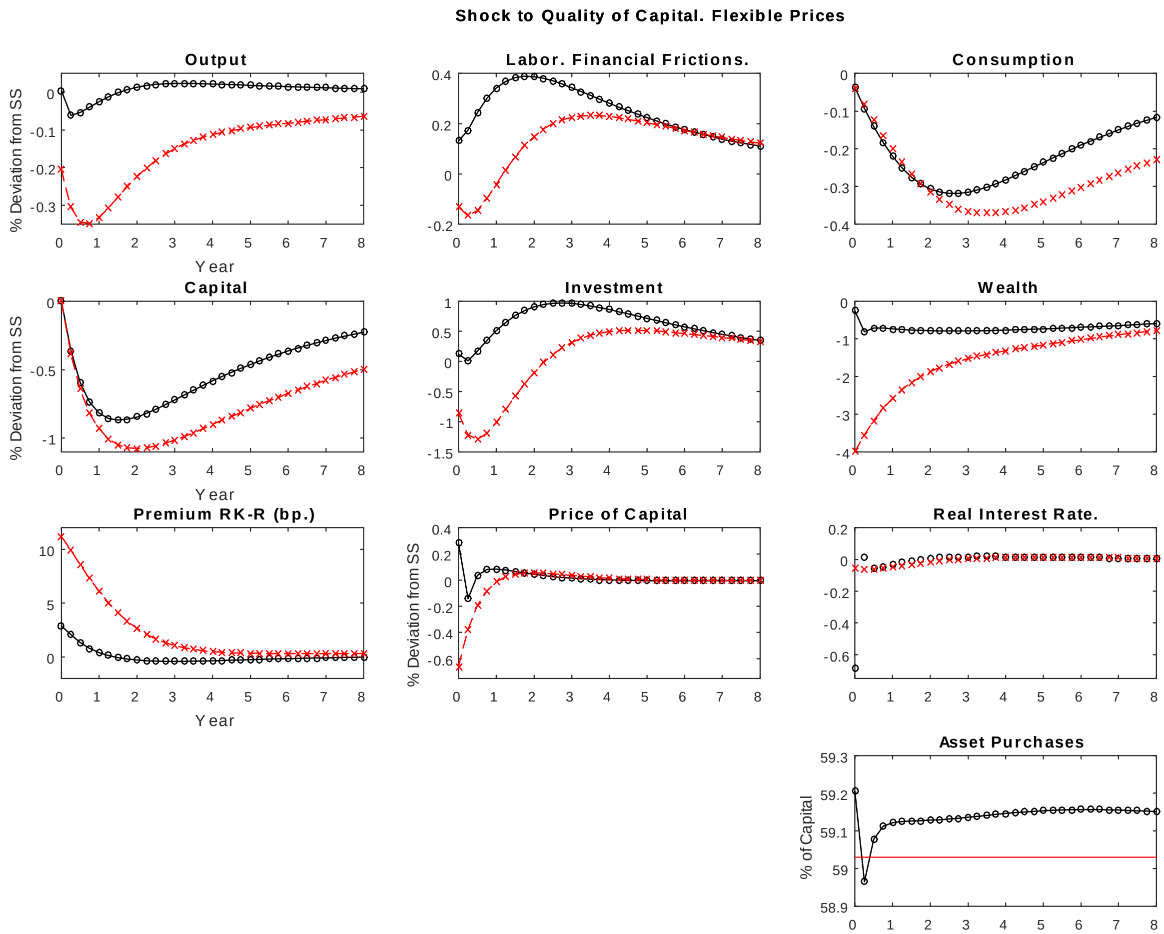

Figure 1. Optimal Asset Purchases. Flexible Prices. 1% Decrease in the Quality of Capital. Asset Purchases (circles), No Policy (crosses).

If the expected return on the assets increases, the banks can meet their balance-sheet constraints after the negative shock has hit the value of their accumulated wealth. However, the return on the banks' assets is at the same time the cost of credit for non-financial firms. Hence, a shock damaging the banks' balance sheet damages the investment spending through the increase in the cost of credit. In turn, the reduction in the investment spending reduces the demand for loans, depreciating the value of the bank's assets. Loans are the banks' assets. Hence, the balance sheet of the banks are damaged again. The feedback loop between the real and financial variables starts on. There is room for the unconventional policy to stabilize this economy.

As a consequence of the presence of the financial accelerator, the recession is deeper. Without implementing any kind of policy (crosses in the figure), this shock can produce a recession that is

7 times deeper than in the optimal policy case (circles in the figure).

Figure 2.2: Optimal Asset Purchases. Gap Variables

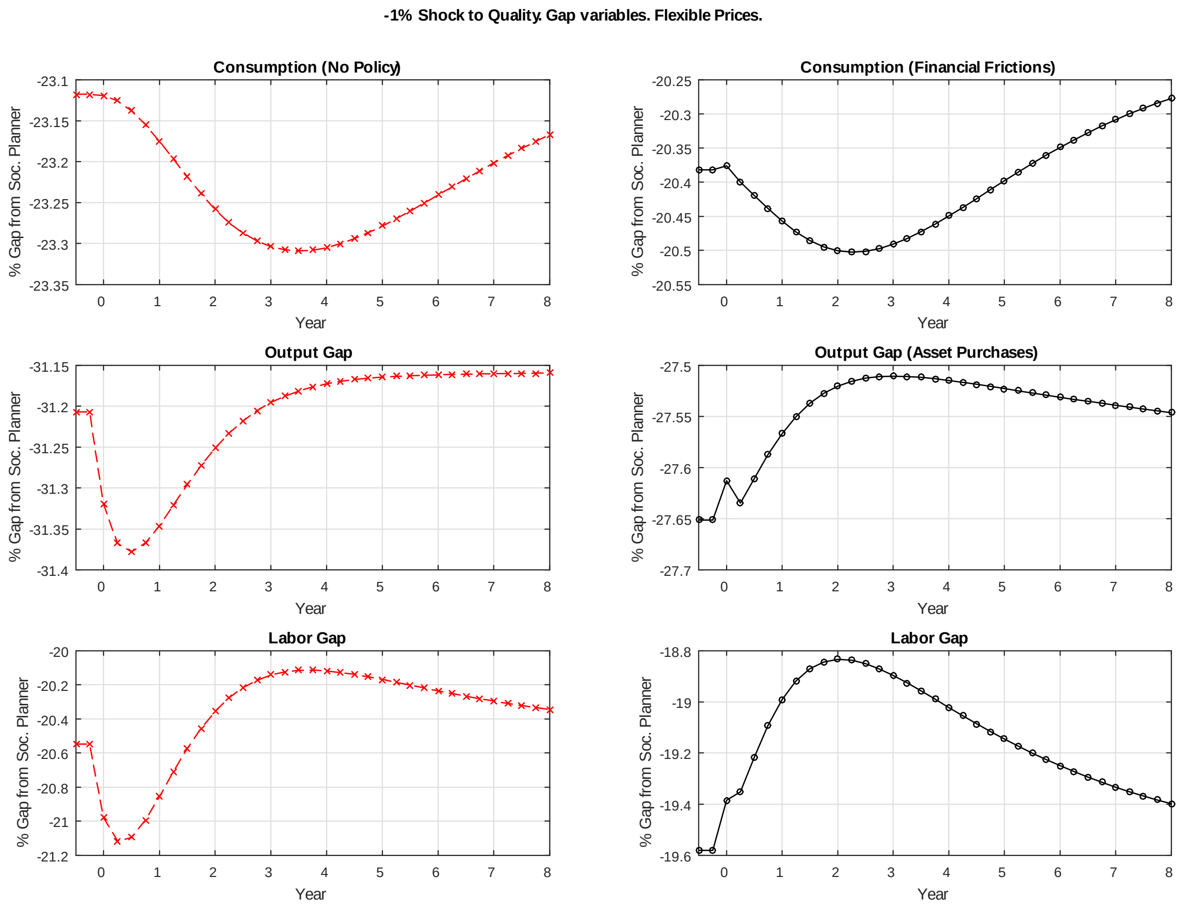

Figure 2. Gap Variables. Flexible Prices. 1% Decrease in the Quality of Capital. Asset Purchases (circles), No Policy (crosses). A decrease in this gap means that the economy is closer to the social planner's allocation; this is captured by a less negative value in this gap.

If the expected value of the bank decreases, as a consequence of the shock, the incentive constraint tightens and the banks are able to issue new loans only if there is an increase in the return on their assets. This starts on the feedback loop between financial and real variables.

The central bank can prevent the starting of the financial accelerator by purchasing assets. On impact, the asset purchases increase. The increase in the government's financial intermediation increases the demand for the firms' assets and appreciate the value of the banks. This policy increases the price of the assets of 0.25 percent on impact, and the real interest rate falls 0.7 percent, reflecting the increase in the price of the asset. Hence, the asset purchases have two effects: they revalue the bank's assets and they decrease the real interest rate, which makes the cost of deposits cheaper. Both effects protect the profitability of the banks. Without any policy intervention, the shock produces a jump on the spread (12 basis points), which is four times the increase in that variable in the presence of asset purchases (3 basis points). Hence, the optimal policy is to increase the financial intermediation in order to appreciate the value of banks' assets. Otherwise, the commercial banks find it optimal to raise the cost of funding for firms, exacerbating the negative effects of the shock.

Why does the central bank find it optimal to follow an aggressive expansionary policy in the first period and a contractionary policy in the second period?

The economy has two distortions: monopolistic competition, and the financial friction. Hence, the economy is far away from the efficient allocation. The maximizing policy-maker would like to push the economy toward the efficient allocation. However, in order to produce more, households have to work more. If the central bank pushes continuously the output gap above its initial level, more working hours are required to produce the additional output.

Figure 2 shows that the expansionary policy decreases the output gap on impact (right panel). This is supported by an increase in the working hours (a source of disutility), and the policy maker compensates the households with a decrease in the consumption gap (a source of utility), which is supported by the effect of the asset purchases on the real interest rate, balancing the sources of utility and disutility of the households. The central bank can do this because it is acting under commitment. Given that the central bank is acting under commitment, it can promise that the initial inflationary process will be reverted in the following periods, hence it is exploiting the expectational benefits of commitment. In particular, the central bank decreases the output gap this period, in order to prevent the starting of the financial accelerator, and it promises to reverse its policy in the next period (so that households do not overwork) in order to maximize the social welfare. This explains the behavior of the asset purchases. The central bank is balancing the households' sources of utility and disutility and exploiting the expectational benefits of commitment.

In addition to not increasing the working hours more than is desirable for households, the central bank is acting under its inefficiency in the intermediation of assets. The efficiency costs, which are increasing in the level of assets held by the central bank, prevent this institution from implementing a more expansionary policy. Overall, this policy smooths the financial accelerator and prevents the large deviation in the output gap observed in the absence of policy.

In the next section, I reintroduce the nominal inertia and the monetary policy in this model with optimal asset purchases.

Reintroducing the Sticky Prices.

Finally, in this section I introduce the nominal friction. Prices are sticky as in Calvo (1983) and the central bank can use the nominal interest rate and asset purchases as policy instruments when financial frictions are present.

Three sources of inefficiency exits in this case: monopolistic competition, the positive spread in equilibrium, and the inability of firms to adjust prices every period.

When financial frictions are present, and a shock to the quality of capital hits the economy, inflation increases (lines with stars in figure 3). This is not observed in the absence of these frictions (diamonds in figure 3); the central bank does not find it optimal to trade inflation for something else when the financial markets are frictionless. However, this not the case when the financial frictions are present. It is optimal to trade inflation for reductions in the output gap. Figure 3 shows the results.

With sticky prices and without financial frictions, fluctuating inflation is the dominant source of welfare loss. Hence, the central bank stabilizes inflation (blue diamonds in figure 3). In this case, the central bank has one policy instrument available, but only inflation represents a challenge for the central bank's objectives. Then, it is possible to successfully stabilize inflation. Inflation stabilization more than compensates for the increase in the output gap (central panel in figure 4).

With financial frictions and nominal rigidities, the financial sector represents an additional challenge for the central bank. With only the nominal interest rate available (this case is akin to the problem presented in chapter 1) the central bank finds it optimal to increase inflation in order to stabilize the economy, even when this is a source of welfare loss. This serves to stabilize the economy close to its efficient allocation, which compensates the households for the discomfort associated with the increase in inflation.

Figure 2.3: Optimal Asset Purchases. Shock to Quality of Capital

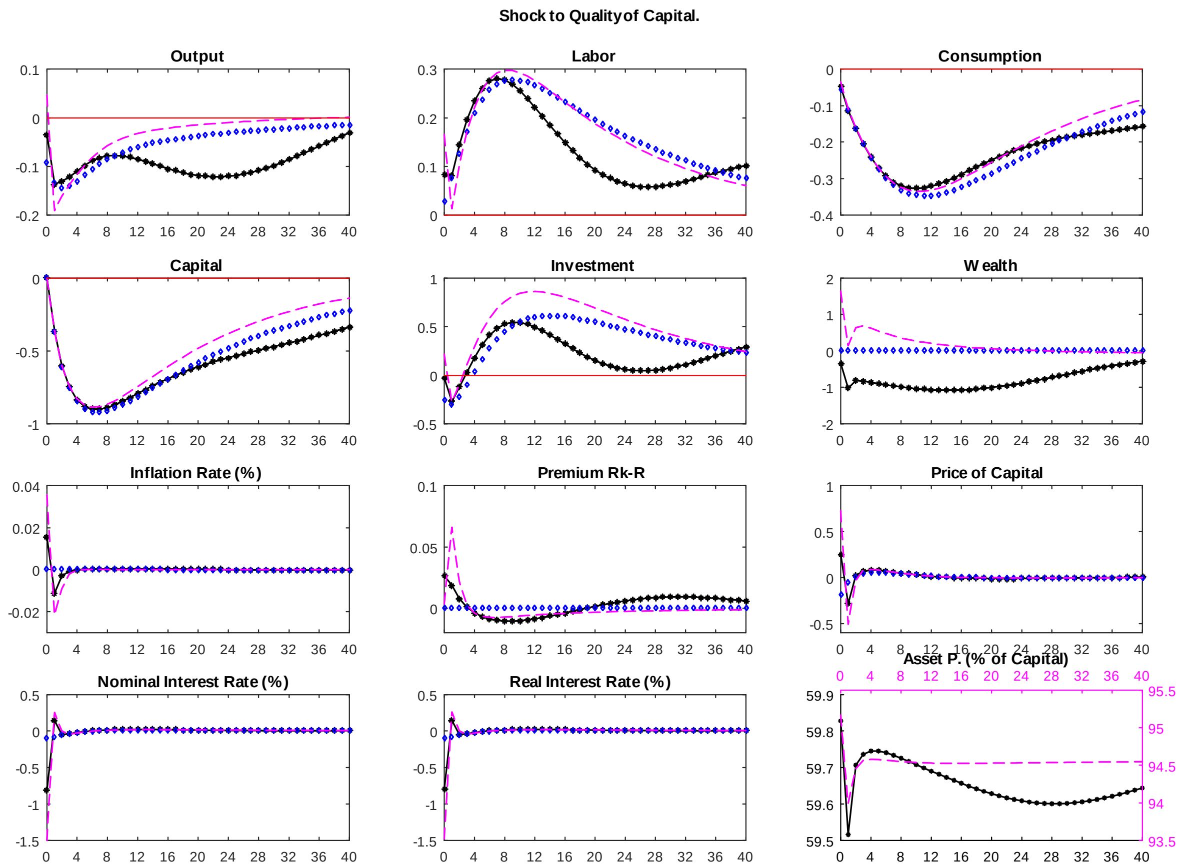

Figure 3. Optimal Policy. 1% Decrease in the Quality of Capital. Unconventional (Black stars), Zero Efficiency Costs (dashed), and Conventional DSGE (blue diamonds).

If the central bank has access to asset purchases, this shock remains inflationary. However, it is optimal to show a reduction in the output gap (left panel in figure 4). The movement in the nominal rate is similar in magnitude and direction to that when the central bank can utilize only the conventional instrument. However, the asset purchases serve to decrease the output gap in almost 1 basis point. In the presence of financial frictions, movements in the output gap become more relevant for the central bank because bad economic events encourage the feedback loop between real and financial events, which is damaging. This pattern of exchanging increases in inflation for reductions in the output gap are better observed as the central bank becomes more efficient (dashed lines in figure 3 and 4).

Figure 2.4: Unconventional Policy. Gap Variables

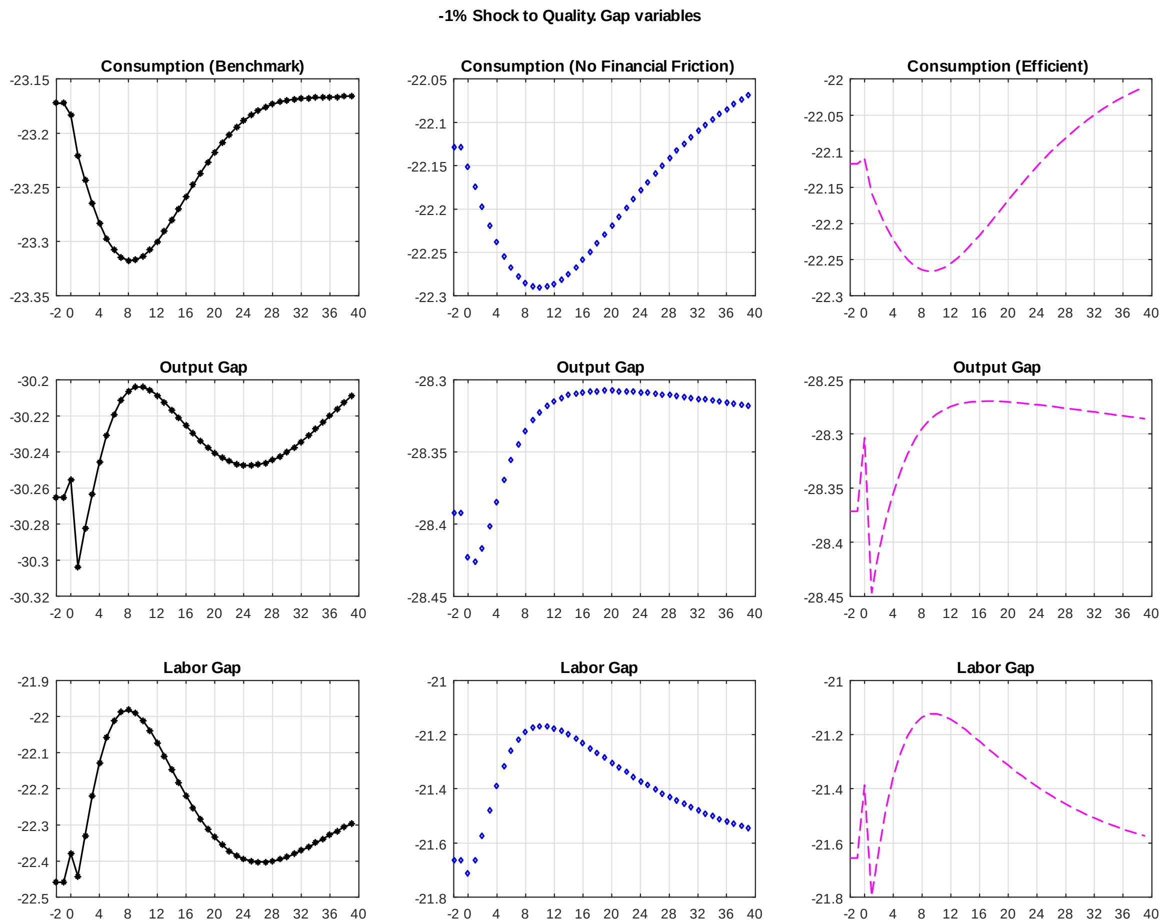

Figure 4. Gap Variables. 1% Decrease in the Quality of Capital. Unconventional (Black stars), Zero Efficiency Costs (dashed) , and Conventional DSGE (blue diamonds). A decrease in this gap means that the economy is closer to the social planner's allocation; this is captured by a less negative value in this gap.

As the central bank becomes more efficient in the lending activities, the output gap decreases even more. However, a larger increase in inflation is observed. The losses for suffering a higher inflation in this case, are compensated with the gains from a reduction in the output gap. In this efficient case, the output gap decreases 7 basis points on impact. In the first period, the central bank, trades-off the welfare losses from the inflation increase, in exchange for the reduction in the output gap. The central bank can use more intensively its policy instrument because it faces almost zero efficiency costs, hence it can support higher welfare costs associated with increasing the inflation because at simultaneously it can encourage a smaller output gap. When the unconventional instrument is efficient, the speed of convergence of output to its long-run level is faster than in the other cases. In this way, the central bank maximizes the social welfare.

Table 2.2: Welfare Cost (Percentage of S.P. Consumption)

	DSGE	Efficient	Benchmark	No Asset Purchases.
	(1)	(2)	(3)	(4)
Cost (%)	0.534	0.541	0.708	1.006

When the financial frictions are present, the expansionary monetary and unconventional policy decrease the output gap and the inflation increases. However, this arrangement is welfare increasing. With the two policy instruments, the central bank can control the increase in the cost of credit and reduce the negative effects on investment spending. The reduction in the nominal interest rate and the increase in the asset purchases increase the demand for capital. This, increases the price of capital and prevents a damage to the banks' balance sheet. If the central bank is highly efficient in the financial intermediation, the policy is largely expansionary on impact, decreasing considerably the output gap (dashed lines in figure 4).

What is the contribution of asset purchases to welfare? Table 5 shows that when the economy has financial frictions and only the nominal interest rate as policy instrument, the welfare cost is more than 1 percent of the stream of consumption of the social planner's allocation (last column). The introduction of the unconventional instrument is welfare increasing and the welfare cost decreases to 0.7 percent. Hence, the central bank's financial intermediation improves welfare by 0.3 percent.

In table 4, I did not subsidize the steady state in order to eliminate the distortion associated with the monopolistic competition, hence, the steady state of all the cases is affected by such distortion.

If the financial friction is completely eliminated and the central bank implements only conventional policy, monopolistic competition and sticky prices cost 0.53 percent of the stream of consumption of the social planner's allocation (first column). As the central bank becomes a more efficient intermediary, the welfare gains can mimic those of the case without financial frictions (second column).

In the next section, I present the optimal policy in this model after a negative shock to productivity.

2.4.5 Shock to Productivity.

In this section, I present the behavior of this economy after a decrease of one standard deviation in productivity. Figure 5 shows the flexible price case. The lines with circles show the optimal asset purchases. The red crosses show the non-policy case.

Flexible Prices.

This shock is recessionary, On impact, output falls 0.25 percent. However, in the presence of the optimal policy (circles), the recession is milder than in the absence of policy (crosses). This shock hits the value of the bank's assets and turns on the feedback loop between financial and real variables. Without policy, the shock decreases the equity in the banks by 6 percent . The commercial banks have to meet their balance sheet constraint. This induces an increase in the cost of credit. If there are no asset purchases, the spread increases 18 basis points, whereas the optimal asset purchases stop this increase.

Figure 2.5: Unconventional Policy. Shock to Productivity. Flexible Prices

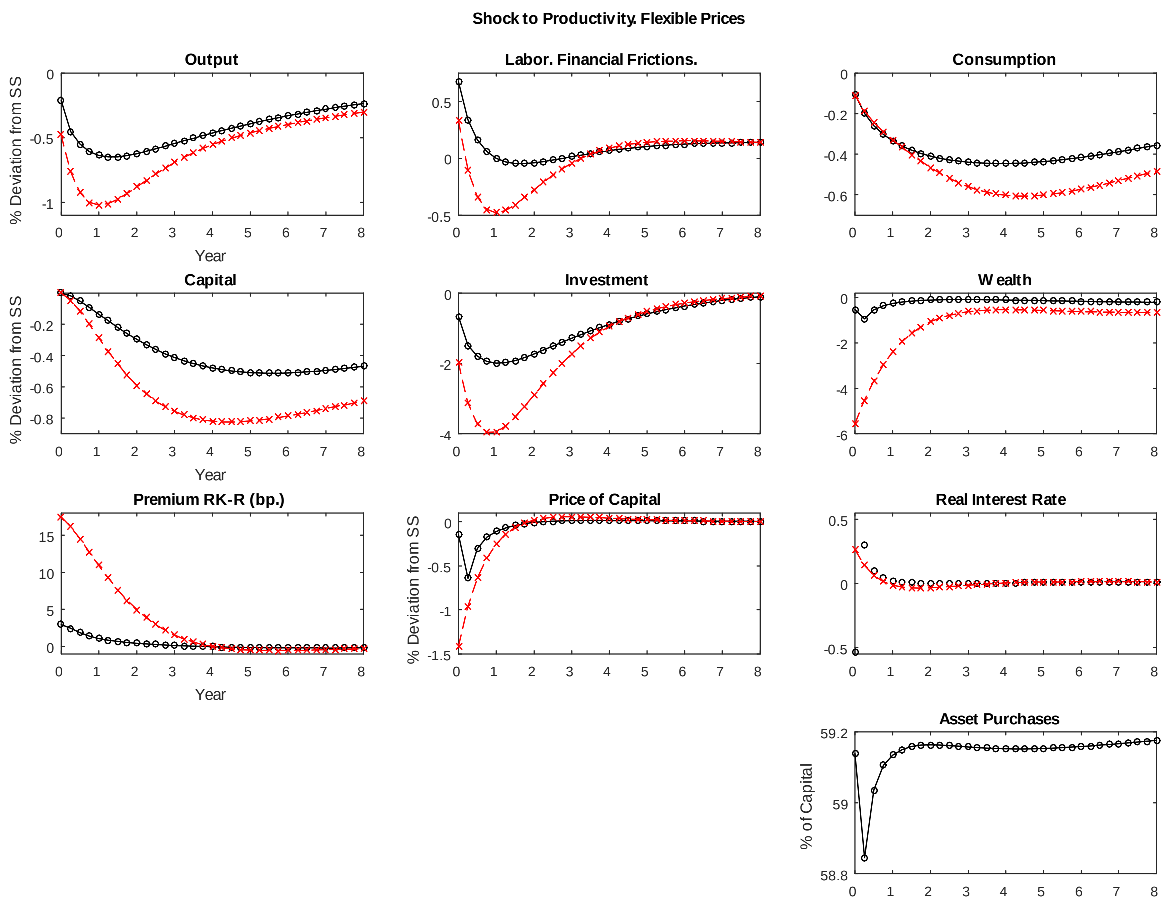

Figure 5. Optimal Asset Purchases. Flexible Prices. 1% Decrease in Productivity. Asset Purchases (circles), No Policy (crosses).

The asset purchases increase slightly in the first period, this stops the fall in the price of the assets. Without this policy, the price of the assets fall almost 1.5 percent, which is highly damaging to the financial health and produces a large increase in the output gap. In contrast, the asset purchases contribute to revalue the bank's capital (the price of the assets falls only 0.2 percent on impact), and the net wealth decreases only 1 percent on impact. However, this policy is less expansionary than if a shock to the quality of capital hits.

If a shock to productivity hits, the central bank increases its participation in the financial markets, and in the second period the policy turns contractionary, in order to not overstimulate the economy. The role of the central bank in this case is to stop the financial accelerator, but it commits itself to not doing more than that.

Figure 6 shows that it is optimal to allow the output gap to increase when the shock hits (right panel). However, the increase is not as large as in the absence of the unconventional policy. This is due to the fact that the central bank would like to prevent the instability associated with the financial accelerator, but it would not like to overstimulate the economy. This policy reduces the labor gap, which discomforts the households, and the central bank compensates these households with a mild reduction in the consumption gap. The central smooths the feedback loop between real and financial variables.

Figure 2.6: Unconventional Policy. Shock to Productivity. Gap Variables

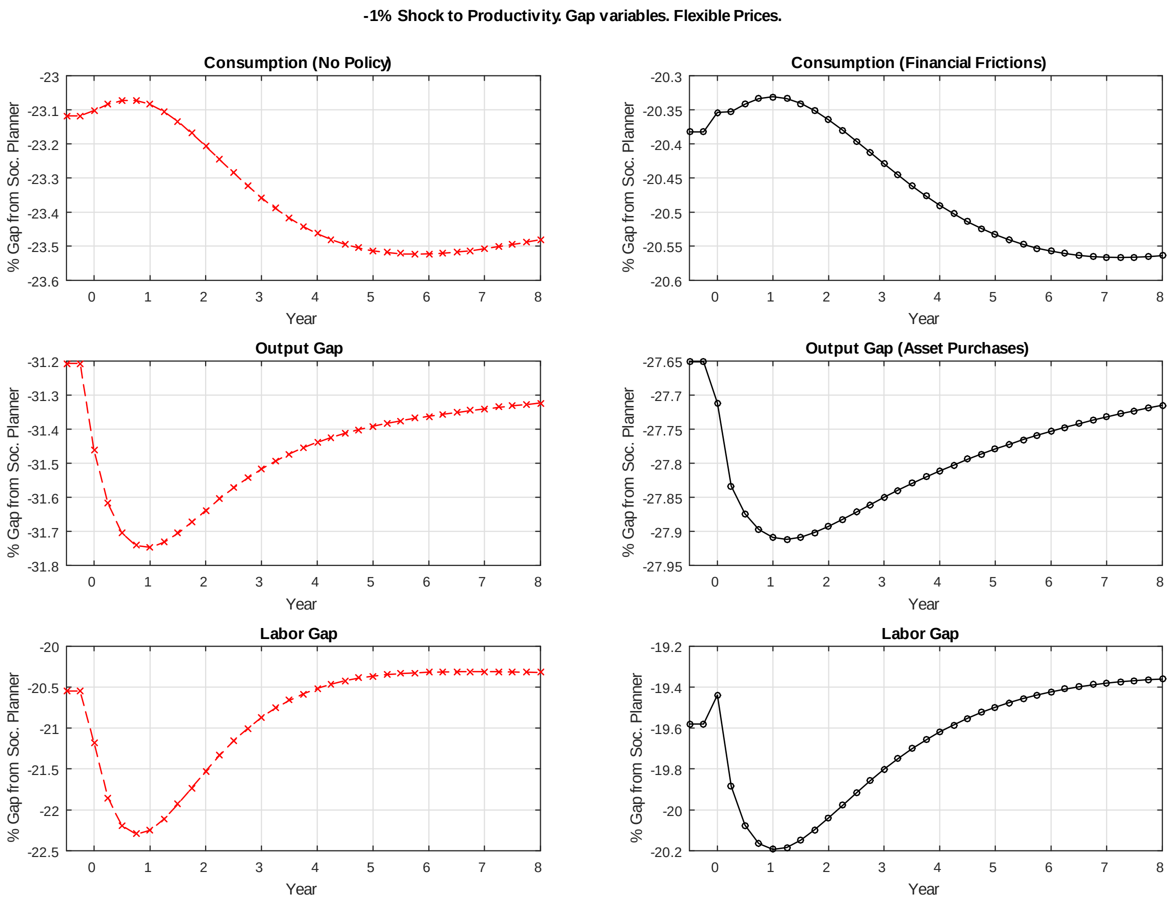

Figure 6. Gap Variables. Flexible Prices. 1% Decrease in Productivity Asset Purchases (circles), No Policy (crosses). A decrease in this gap means that the economy is closer to the social planner's allocation; this is captured by a less negative value in this gap.

Reintroducing Sticky Prices.

In this section, I reintroduce the nominal inertia and present the optimal mix of monetary and asset purchases policy. Figure 7 shows the results.

Figure 2.7: Unconventional Policy. Shock to Productivity. Sticky Prices

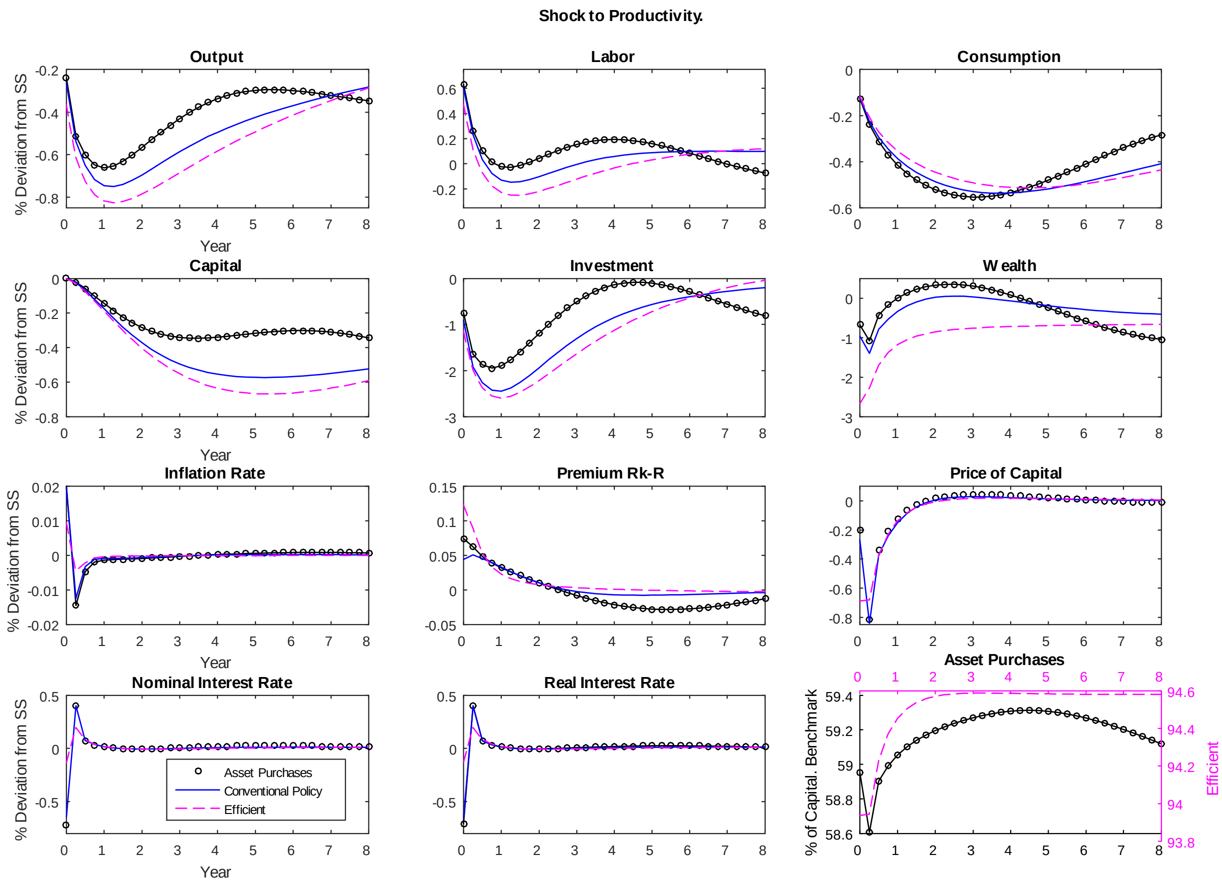

Figure 7. Optimal Policy. 1% Decrease in Productivity. Unconventional (Black stars), Zero Efficiency Costs (dashed), and Conventional DSGE (blue diamonds).

The benchmark case, in which conventional and unconventional policy is implemented, is shown by the black lines with stars. The model without financial frictions is shown by the blue diamonds, and the dashed lines show the efficient case.

On impact, output decreases 0.25 percent. To ameliorate the effects of this recessionary shock, the nominal interest rate decreases 72 basis points on impact (black stars). As a consequence, inflation increases 2 basis points. The asset purchases decrease slightly on impact and show a further decrease in the next period. After these two periods, the asset purchases recover and it remain above the long-run level for the next nine years. In the first period, the output gap increases 8 basis points in the benchmark case (left panel in figure 8). In this case, the central bank utilizes the widening in the output gap to ameliorate the inflation pressures. The central bank utilizes the nominal interest rate to reduce the cost of credit and to revalue the net wealth in the banks. This contributes to smoothing the financial accelerator. However, the central bank has to face a higher inflation rate. This increase in inflation is tamed by the contractionary unconventional policy. If the unconventional policy were more expansionary, inflation would increase even more.

The optimal monetary policy is expansionary, which creates inflation pressures. The contractionary unconventional policy seeks to ameliorate these pressures. The output gap widens (figure 8). In contrast, if the financial friction is eliminated, the nominal interest rate is contractionary and inflation is under control at all times. In this case, the output gap increases to help the central bank to control inflation.

Figure 2.8: Unconventional Policy. Shock to Productivity

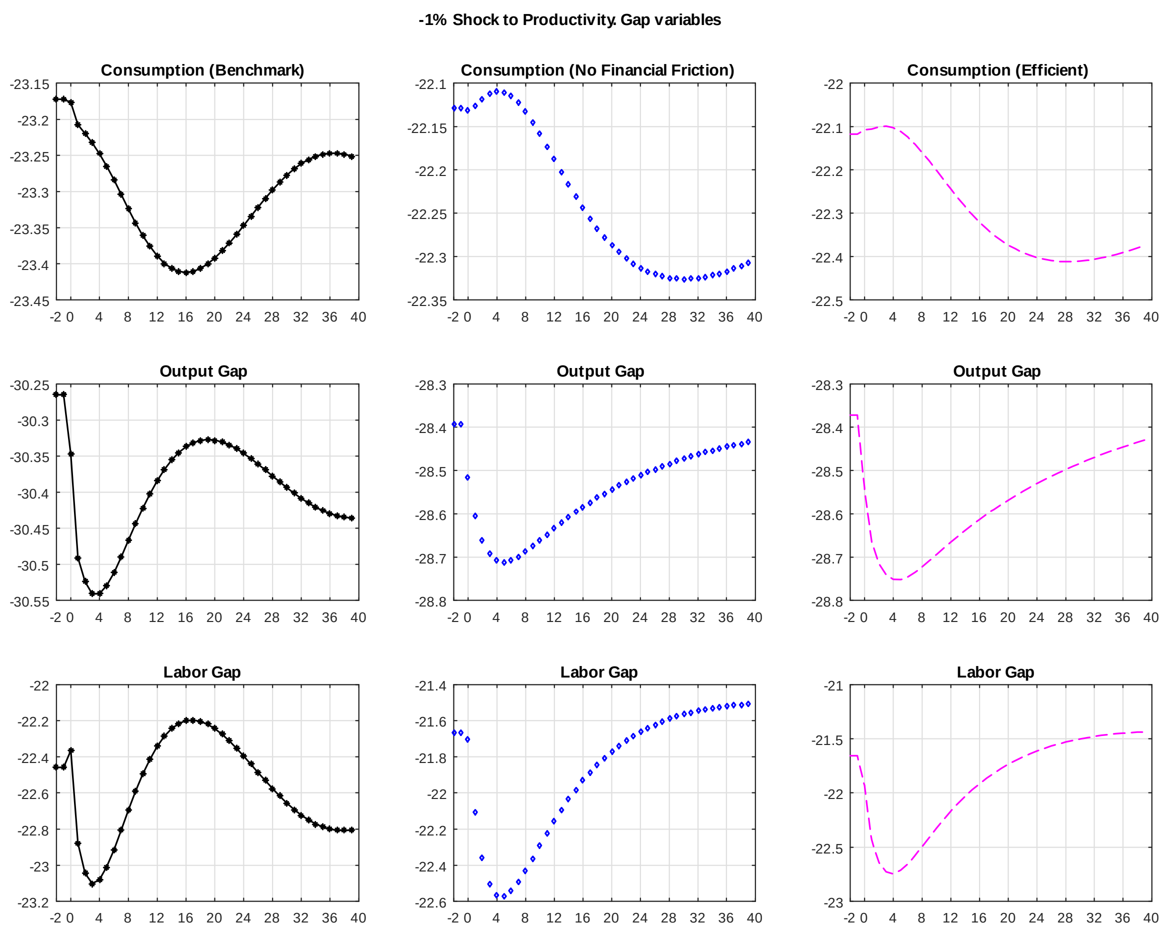

Figure 8. Gap Variables. 1% Decrease in the Productivity. Unconventional (Black stars), Zero Efficiency Costs (dashed) , and Conventional DSGE (blue diamonds). A decrease in this gap means that the economy is closer to the social planner's allocation; this is captured by a less negative value in this gap.

In the first period, the combination of expansionary conventional policy and contractionary unconventional policy decreases the price of capital, however, the decrease is not as large as it would be in the absence of the asset purchases. The decrease in the price of capital damages the balance sheet of banks by decreasing the value of their assets. The balance sheet of the banks is further weakened when the policy becomes more contractionary in the second period. As a consequence, the premium on capital increases, reflecting the weaker financial position of the financial intermediaries. The cost of credit increases, depressing investment and consumption. The mix of policy serves to control inflation from the second period onwards. But in the first period, the central bank smooths the effects of the financial accelerator.

After these two periods, the nominal interest rate returns to equilibrium. After the third period, and when the nominal interest rate is back at equilibrium, the central bank finds it optimal to increase its asset purchases. And these purchases remain above equilibrium for the next 9 years and they converge very slowly to equilibrium. Given the optimal mix of conventional and unconventional policy, consumption shows a faster return to equilibrium.

In this case, the central bank would not like to overstimulate the economy, in order to prevent a further rise in inflation. With a more efficient intermediation, it can stabilize inflation and prevent an inefficient level of real and financial activity. The central bank trade-offs improve as it has a more efficient instrument. In the next section, I analyze whether simple rules can implement this optimal policy commitment.

2.5 Implementation of Optimal Unconventional Policy.

In this section, I present the implementation of the optimal mix of policy. The implementation is via simply rules, which maximize the social welfare. The benchmark for comparison is the welfare implied by the Ramsey allocation under unconventional optimal policy. I measure the welfare cost as the amount of consumption that agents in the Ramsey regime are willing to renounce in order to have the same welfare as in the alternative policy scenario.

Table 5 summarizes the welfare results regarding different rules.

The nominal interest rate rule takes the form

$$\ln\left[\frac{i_t}{i}\right] = \alpha_R \ln\left[\frac{i_{t-1}}{i}\right] + (1-\alpha_R)\left\{\alpha_\pi \ln\left[\frac{\pi_t}{\pi^*}\right] + \alpha_Y \ln\left[\frac{Y_t}{Y}\right]\right\}, \tag{2.42}$$

and the rule governing the assets purchases takes the form

$$\kappa_t^{QE} = \kappa_{QE} E_t\left[\log\left(\frac{R_t^k}{R_t}\right) - \log\left(\frac{R^k}{R}\right)\right]. \tag{2.43}$$

2.5.1 Unrestricted Rules.

The numerical search for the policy coefficients was unrestricted and then restricted.

Row 2 in table 5 shows that the inflation coefficient (α_π) takes a large value (40.4), as does the

Table 2.3: Implementation of the Optimal Unconventional Policy

		Welfare Cost (%)	α_π	α_Y	α_R	κ_{QE}
1.	Taylor	0.4157	2.2	0.2	0.39	0
2.	Optimized. Unconstrained.	0.0951	40.4	0	0.85	196
3.	Optimized. Constrained	0.1269	5	0	0.95	25

feedback coefficient on the asset purchases ($\kappa_{QE} = 196$). The rule shows a degree of inertia (α_R) of 0.85. It is optimal to move the unconventional instrument when there are financial pressures. It is optimal that the central bank increases its asset purchases when the premium on capital is above its long-run average. The rule shows that not reacting to changes in real activity is optimal ($\alpha_Y = 0$). This is the welfare maximizing rule in the presence of financial friction and asset purchases. The welfare gains respect to the case where it is utilized only the Taylor rule are about 0.32 percent of consumption.

Given that the feedback coefficients in the welfare maximizing rule, described above, are larger than the coefficients commonly utilized in the simple rules, I restrict the interval for the search of the policy coefficients to take more conventional values. In the next section I explain the results.

2.5.2 Restricted Rules.

I impose the next restrictions on the value of the feedback coefficients: the inflation coefficient (α_π) cannot take values larger than 5. The coefficient (κ_{QE}) was restricted to a maximum of 25. This means that if the spread increases 1 basis point then the asset purchases increase by 0.25 percent of the stock of capital. The results are: the policy coefficients for inflation and asset purchases take the largest possible value; it continues being optimal not reacting to changes in output; and the rules show a high degree of inertia. Row 3 in table 6 summarizes these results.

Given that the difference in the cost between the restricted and the unrestricted case is small (0.03 percent of consumption), the restricted rule can serve as guidance for the optimal implementation of monetary and unconventional policy in the presence of financial frictions.

In the next section, I present the impulse-response functions of these optimal rules when the economy is hit by: a shock to the asset purchases, a monetary policy shock, a shock to the quality of capital, a cost-push shock, a shock to productivity, and a sudden increase in the diverting share of funds.

In particular, the monetary rule takes the form:

$$\ln\left[\frac{i_t}{i}\right] = \alpha_R \ln\left[\frac{i_{t-1}}{i}\right] + [1-\alpha_R]\left\{\alpha_\pi\left[\frac{\pi_t}{\pi^*}\right] + \alpha_Y\left[\frac{Y_t}{Y}\right]\right\}, \tag{2.44}$$

with the optimized coefficients $[\alpha_R = 0.95;\ \alpha_\pi = 5;\ \alpha_Y = 0]$. And the rule for assets purchases:

$$\kappa_t^{QE} = \kappa_{QE} E_t \left[\log\left(\frac{R_t^k}{R_t}\right) - \log\left(\frac{R^k}{R}\right) \right], \tag{2.45}$$

with $[\kappa_{QE} = 25]$.

The analysis compares two cases for each shock. The case in which the central bank can use both, the Taylor rule, and the asset purchase rule with the optimal coefficients $\alpha_\pi = 5$; $\alpha_Y = 0$; $\alpha_R = 0.95$; and, $\kappa_{QE} = 25$, and the case in which the central bank can use only the nominal interest rate. In this last case, the Taylor rule governing the changes in the nominal interest rate is calibrated as in the estimated model of Taheri (2014), with: $\alpha_\pi = 2.19$; $\alpha_Y = 0.2$; $\alpha_R = 0.39$; and, $\kappa_{QE} = 0$.

2.5.3 Impulse-Response Functions.

In this section, I present the impulse-response functions in this economy. The effects of financial shocks (for example, a shock to the quality of capital or a sudden increase in the moral hazard problem in the banking sector) can be curbed using the combination of conventional and unconventional monetary policy. Even in the face of non-financial shocks (for example a productivity shock), the central bank can improve welfare if it can use both conventional and unconventional instruments. In contrast, if only conventional instruments are utilized when the economy is subject to financial frictions, the economy cannot be completely stabilized. Finally, in the presence of a cost-push shock, the central bank can improve on its ability to stabilize inflation and the financial markets if it can implement conventional and unconventional policy.

Asset Purchases Shock.

Figure 9 presents the effects on the economy of increasing the asset purchases in 1 percent. I assume that this shock has no persistence.

This shock is expansionary. An increase in the assets purchases implies an increase of almost 1 basis point in output. The effect is small, however. As a consequence of the shock, consumption increases, but the increase is small, around one order of magnitude less than the increase in output. The behavior of labor mimics that of output.

The main effects of this shocks are observed on the financial sector. In particular, the increase in asset purchases reduces the premium by 10 basis points. This serves to stimulate investment spending which increases 4 basis points. This has a positive effect on the valuation of financial assets.

The price of financial assets increases almost 10 basis points on impact. The increase in the value of assets and the reduction in the cost of credit improves the balance sheet of the financial intermediaries. In consequence, the wealth in the banking sector increases by about 0.4 percent on impact. The shock has a small effect on the nominal and the real interest rate, which basically move in the same direction as inflation.

Overall, changes in this policy instrument affect mainly the financial sector. Its expansionary effects reduce the cost of credit, increasing investment spending. The increase in asset purchases increases the demand for the firms' assets. This, in turn, increases the asset's price and net wealth in the banking sector, reducing the pressure on the bank's balance sheet.

Figure 2.9: Simple Rules. Increase in Asset Purchases

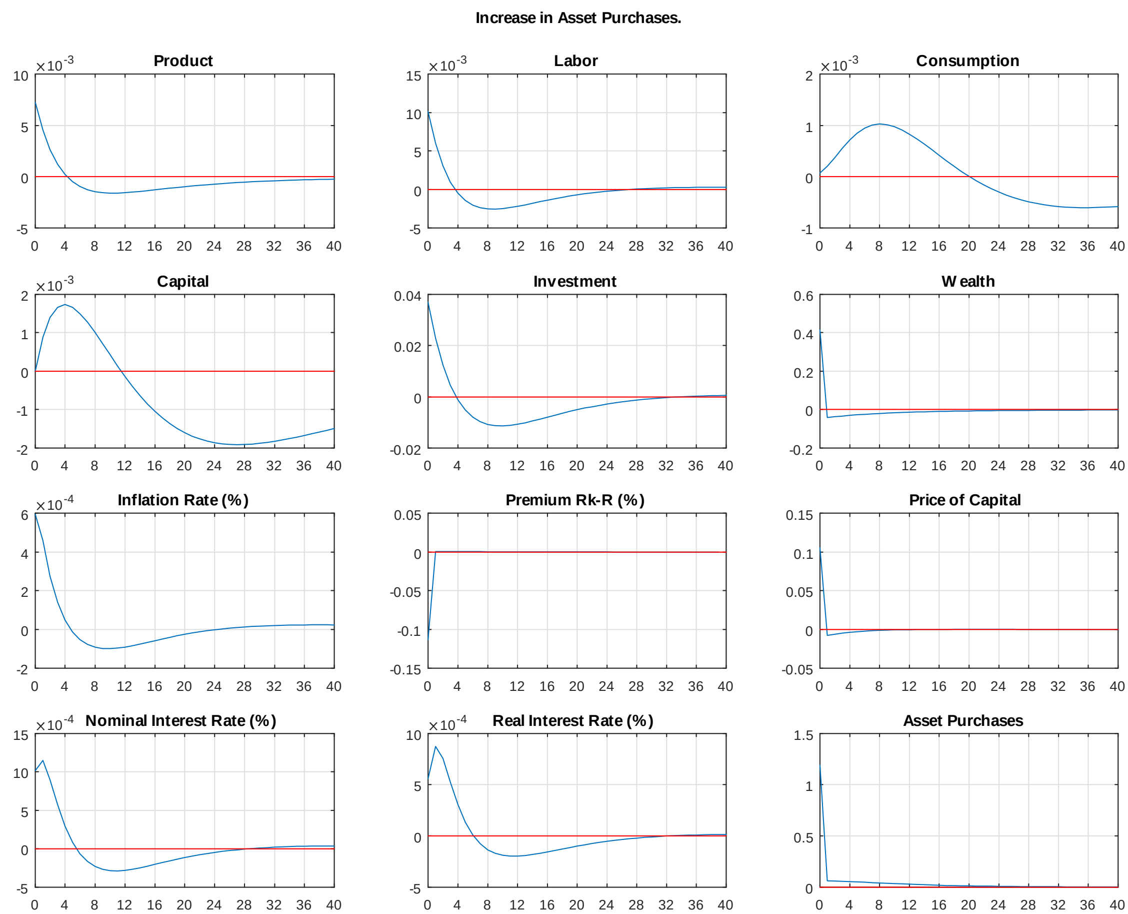

Figure 9. Implementation of Optimal Policy. 1% Increase in the Asset Purchases. Solid lines $\alpha_\pi = 5$, $\alpha_Y = 0$, $\alpha_R = 0.95$, $\kappa^{QE} = 25$.

Monetary Policy Shock.

In this section, I present the response of this economy to an increase of 25 basis points in the nominal interest rate. Figure 10 shows the results.

As a consequence of this shock, there is a reduction in the output and the inflation. The effects of this shock can be reduced if the unconventional policy is in place. Inflation behaves similarly in both models. However the deflation is slightly smaller in the unconventional policy case. The fall in output is ameliorated with respect to the conventional monetary policy case. When policy can be implemented through unconventional means, the largest reduction in output is 0.22 percent. When the central bank has access only to the conventional Taylor rule, the fall in output increases to 0.30 percent. The behavior of the premium on capital and its effects on the investment spending contribute to explain the differences observes in the reaction of output.

An increase in the central bank's nominal interest rate increases the cost of credit. This explains the increase in the risk premium and the fall in net wealth. The banking sector in this economy issues deposits to the households in order to finance the issuing of loans to the producers. The bank also makes use of its own accumulated wealth in order to issue new loans to the non-financial producers. The deposits in the banks pay the short-term real interest rate. And the opportunity cost of the accumulated net wealth is the also the real interest rate.

This monetary surprise increases the real interest rate, as long as the prices are sticky. The increase in the real interest rate translates into an increase in the cost of deposits. This reduces the profitability of banks. Because the banks must meet their balance sheet constraint, they require a larger premium for their loans. This increases the cost of credit. Because of the presence of financial frictions and the deterioration of the banks' balance sheet, the premium on assets increases and it remains above its steady state for more than four years.

Figure 2.10: Simple Rules. Monetary Policy Shock

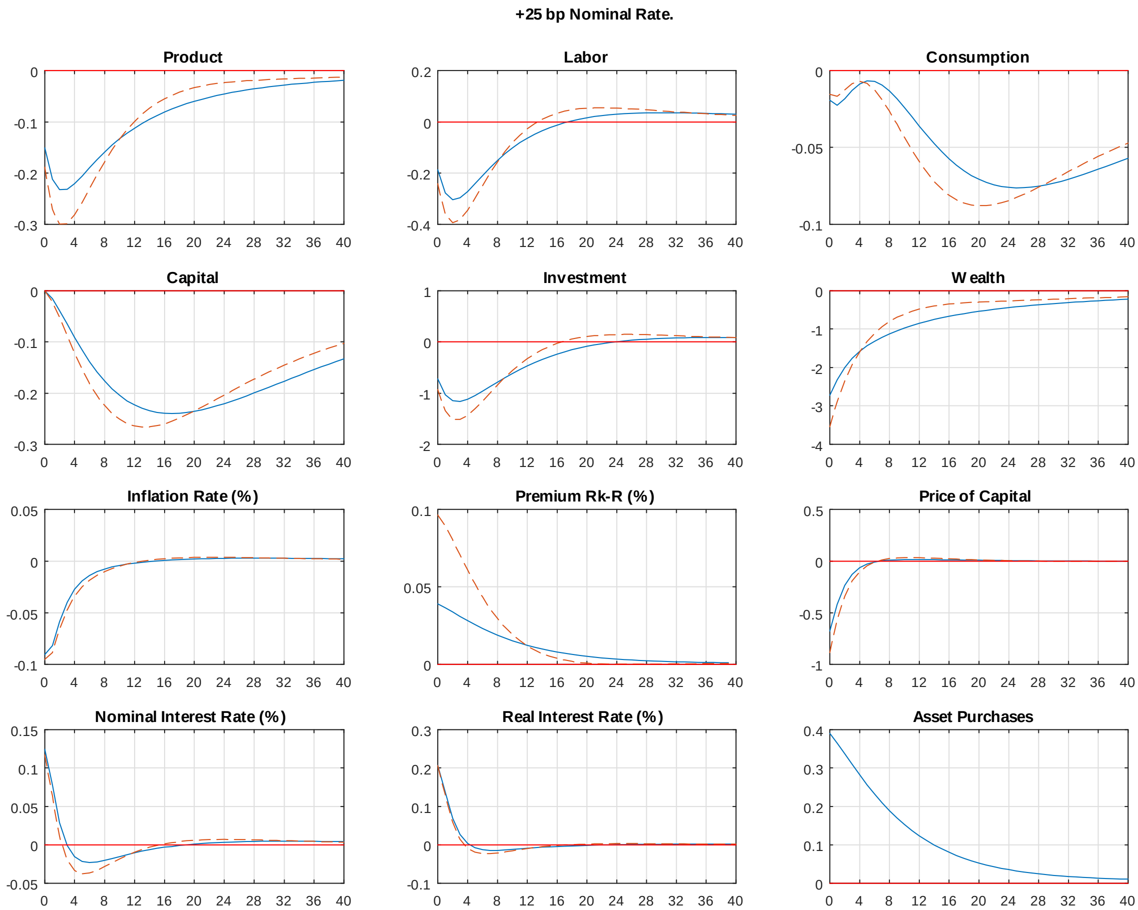

Figure 10. Implementation of Optimal Policy. 25 Basis Points Increase in the Nominal Interest Rate. Solid lines $\alpha_\pi = 5,\ \alpha_Y = 0,\ \alpha_R = 0.95,\ \kappa^{QE} = 25$. Dashed $\alpha_\pi = 2.19,\ \alpha_Y = 0.2,\ \alpha_R = 0.39,\ \kappa^{QE} = 0$.

The increase in the cost of credit reduces the investment spending. On impact, the investment spending falls 0.8 percent if the central bank simultaneously implements asset purchases and conventional monetary policy. If the central bank implements only the conventional monetary policy (dashed lines) the recession on investment is deeper because the cost of credit increases more.

The reduction in investment depresses the price of capital goods. This reduction in the value of assets activates the financial-accelerator mechanism. When the reduction in investment spending reduces the value of the assets, the net wealth of the banks decreases. This deteriorates the bank's balance sheet. In order to meet their incentive constraint, the banks require an increase in the return on assets. This, in turn, increases the costs of credit for non-financial firms, reducing the investment spending.

The slow recovery in production and investment is associated with this behavior of the premium on capital. However, if the central bank is also implementing the unconventional policy, the monetary shock is less recessionary because the asset purchases prevent a larger increase in the cost of credit. In this case, the net wealth of the banks decreases less and the bank's balance sheet tightens less than in the absence of the unconventional policy. The unconventional policy deals with this by increasing the share of assets in 0.5 percentage points. This policy is persistent and has its main effects on the cost of credit and the financial sector.

Inflation behaves similarly in both cases. This means that if the central bank has access to the two policy instruments, it can achieve a given level of inflation with a smaller fluctuation in output and in the financial variables. While the monetary instrument is contractionary, the unconventional instrument can be expansionary. This combination of policy can balance the financial and inflationary fluctuations. And this will improve the central bank's trade-offs. The monetary policy can deal with the inflationary effects of shocks, while the unconventional instrument serve to ameliorate the effects of that policy on the real and financial fluctuation.

Shock to the Quality of Capital.

The shock is a decrease of one percent in the quality of capital. Figure 11 presents the response of this economy when, in addition to the conventional Taylor rule, the unconventional policy is implemented (solid lines). The dashed lines show the case in which only the conventional monetary policy is implemented, via the Taylor rule.

Due to the presence of the financial accelerator, the initial fall in output is larger than if the financial frictions were not present. If the central bank can implement conventional and unconventional monetary policy, the recession is milder (solid lines) than if it has access only to the conventional Taylor rule (dashed lines). When the central bank has access to the two policy instruments, the recession in output, investment, and employment is almost one-half smaller than if the central bank has access only to the monetary policy.

Figure 2.11: Simple Rules. Shock to the Quality of Capital

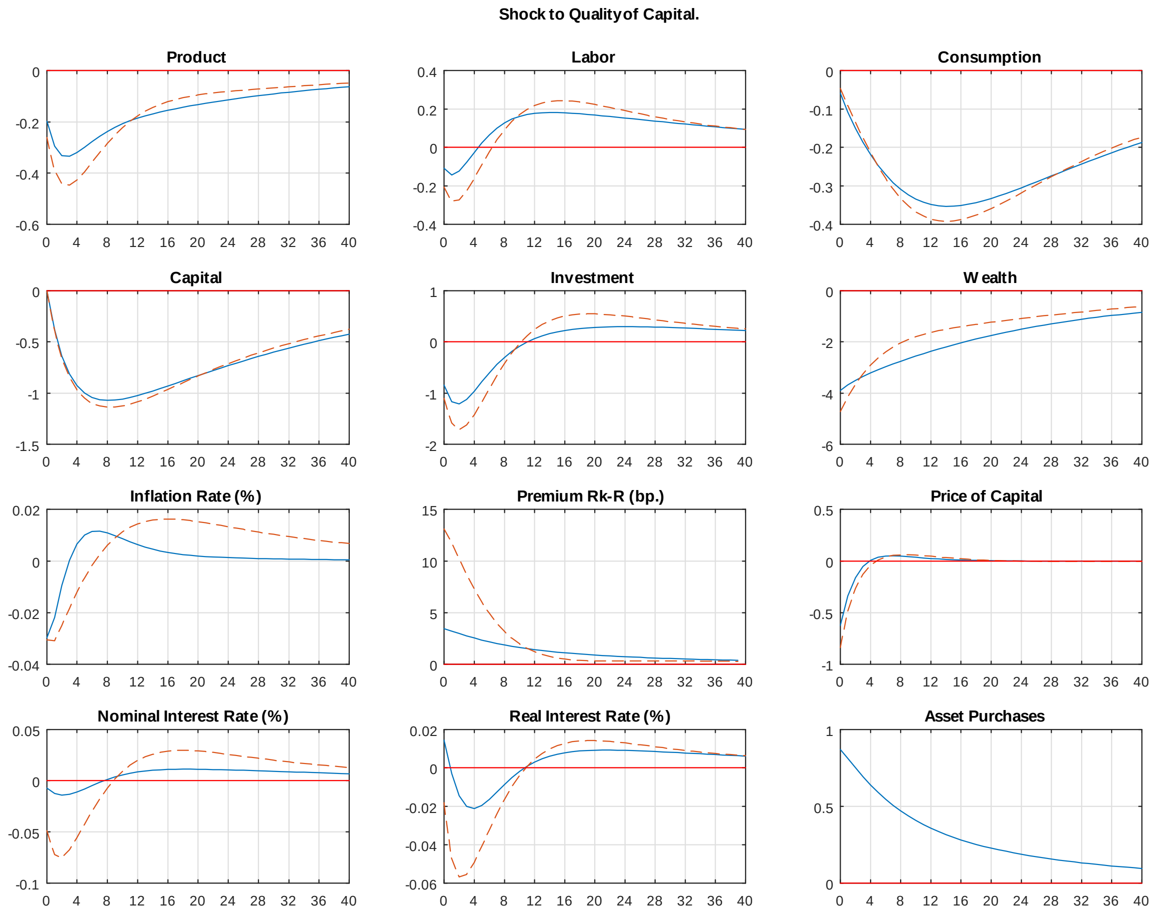

Figure 11. Implementation of Optimal Policy. 1% Decrease in the Quality of Capital. Solid lines $\alpha_\pi = 5$, $\alpha_Y = 0$, $\alpha_R = 0.95$, $\kappa^{QE} = 25$. Dashed $\alpha_\pi = 2.19$, $\alpha_Y = 0.2$, $\alpha_R = 0.39$, $\kappa^{QE} = 0$.

The mix of the unconventional and conventional policy is very effective in producing a less severe recession. With respect to the conventional Taylor rule, the economy contracts only 0.2 percent on impact and the deepest fall in output is 0.3 percent. When the central bank implements conventional policy the output falls up to 0.45 percent.

This optimal policy implies that employment remains very close to equilibrium on impact. In contrast, the conventional Taylor rule implies a reduction of 0.2 percent at the moment of the shock. The optimal policy is very effective at avoiding a large fall in equity and investment. The optimal policy mix implies less volatility in inflation than in the conventional case.

When the central bank utilizes the unconventional policy to stabilize the financial markets, it is possible to stimulate the economy by reducing the nominal interest rate to fight the initial deflation. However the gains in output stabilization are due to the unconventional instrument because the asset purchases stabilize the spread and the investment spending.

The unconventional policy implies that, on impact, the spread moves only 3 basis points. In the conventional case this spread increases 15 basis points. This unconventional policy stabilizes the financial sector, as suggested by the Ramsey exercise. In order to stabilize the financial markets, the central bank increases its participation in the private markets by almost 1 percentage point on impact and it is optimal to remain in the lending activities for a long period. Ten years after the shock the participation in the financial activities is close to equilibrium.

The reduction in consumption is quite similar when conventional and unconventional policy is utilized. However, the deepest fall in consumption is smaller when the unconventional policy is utilized. Capital accumulation shows a better schedule when the central bank can intermediate financial assets.

Investment spending is the component of aggregate demand which shows the greatest effects of the unconventional policy. This is so because the financial accelerator works mainly through its effects on investment spending. The unconventional policy has its main effects on switching this accelerator off.

Investment decreases 0.8 percent at the moment of the shock (it decreases 1.0 percent when only conventional policy is implemented). If it is not possible to implement unconventional policy, then, the largest fall in investment would be about 2 percent (two periods after the shock). When it is possible that the central bank purchase assets to reduce the financial distress, the largest fall in investment is 1 percent (two periods after the shock). The asset purchases serve to stabilize the investment spending. This stabilization of the spread and of the investment spending contributes to the revaluation of assets, improving the balance sheet of the financial intermediaries.

In order to explain the behavior in the real sector it is necessary to look at the financial variables. The shock initially reduces the value of assets of banks by reducing their quality. This loss in the value of assets tightens the credit conditions. The reduction in the value of assets reduces the value of the net wealth of banks, deteriorating the banks' balance sheet.

The deterioration of the banks' balance sheet increases the premium on capital. Given that the banks must meet their incentive constraint, when a shock negatively affects the value of assets, banks require a higher return on their assets. Because the rate of return on these assets is equivalent to the cost of credit for non-financial firms, the demand for investment decreases. Accordingly, the demand for loans falls. If only conventional policy is implemented the deepest fall in investment is equal to 1.7 percent. When the central bank can purchase private assets this reduction is equal to 1.2 percent.

This reduction in the demand for loans contributes to the reduction in the price of the financial assets which decreases again the value of net wealth accumulated. This contributes to the tightening of the financial constraint. The cost of credit increases and the demand for investment decreases. So, the initial exogenous shock reduces the value of assets. The second round effects generate an endogenous reduction in the value of banks' assets through the reduction in the demand for assets and the price of these assets. The financial accelerator is this procyclical feedback loop between financial and real events.

This effect can be reduced if the central bank utilizes an additional instrument to stabilize the financial sector. For example, when the central banks has access to assets purchases then the financial sector can be stabilized more efficiently. This is shown by the solid lines in the figure 11.

When only conventional monetary policy is implemented, the initial fall in the net worth is around 5 percent. However, when unconventional policy is also implemented, the reduction in the wealth of banks is only 4 percent. The fall in net wealth tightens the banks' balance sheet. This tightening of the financial conditions is stronger when the central bank has only one policy instrument.

The increase in the demand for private loans, as a consequence of the assets purchases, serves to stabilize the price of the financial assets and the wealth in the banking sector. The price of the financial assets decreases only 0.5 percent. If the central bank does not intermediate assets, the reduction in the price of assets would be about 1 percent.

The unconventional policy also serves to generate a smaller reduction in the nominal and real interest rate in order to stabilize the economy. The overall participation of the government in the lending market increases in 1 percentage point. The central bank withdraws from the financial markets slowly.

Overall, when financial frictions are present and financial shocks hit, if the central bank has access to conventional and unconventional instruments, its ability to stabilize the economy improves. The conventional instrument can stabilize inflation. The unconventional instrument serves to stabilize the financial markets. A better result in terms of output variation and inflation variation is achieved. This is reflected in the welfare gains with respect to the conventional Taylor rule.

Cost-Push Shock.

In this section, I consider the effects of an increase of one percent in the cost-push variable. Figure 12 shows the response of the economy to this shock.

The decrease in the output is larger when only the conventional monetary policy is implemented (dashed lines). This is explained by a larger decrease in investment. Even when non-financial shocks hit the economy, the presence of the unconventional policy improves the results in the real economy.

Consumption shows a deeper recession when only conventional policy is implemented. As a consequence of this shock, working hours decrease. 4 years after the shock, employment is close to its long-run equilibrium.

The recession decreases the demand for capital goods. In turn, the investment spending reduces and the price of the assets fall. As a consequence, net wealth decreases 1.5 percent. This reduction is almost 4 percent when only conventional policy is implemented (dashed lines). This deteriorates the banks' balance sheet and increases the premium on capital. The increase in the cost of credit is 3 times larger if the unconventional policy is not implemented and the reduction in the price of capital almost doubles.

At the moment of the shock, the premium between the return on capital and the risk-free rate moves around 10 basis points when there are no asset purchases. In the unconventional case this spread moves only 2 basis points. This is due to the increase in the assets purchases in 0.5 percentage points.

The effects of the increase in this cost of credit depress the investment spending. In turn, the fall in demand investment reduces the value of the assets. The devaluation of the financial asset contributes to the reduction in the value of equity in the banking sector. The financial-accelerator is present and the demand for capital goods can have a smaller reduction when the central bank utilizes its new policy instrument to stabilize the financial markets.

Figure 2.12: Simple Rules. Cost-Push Shock

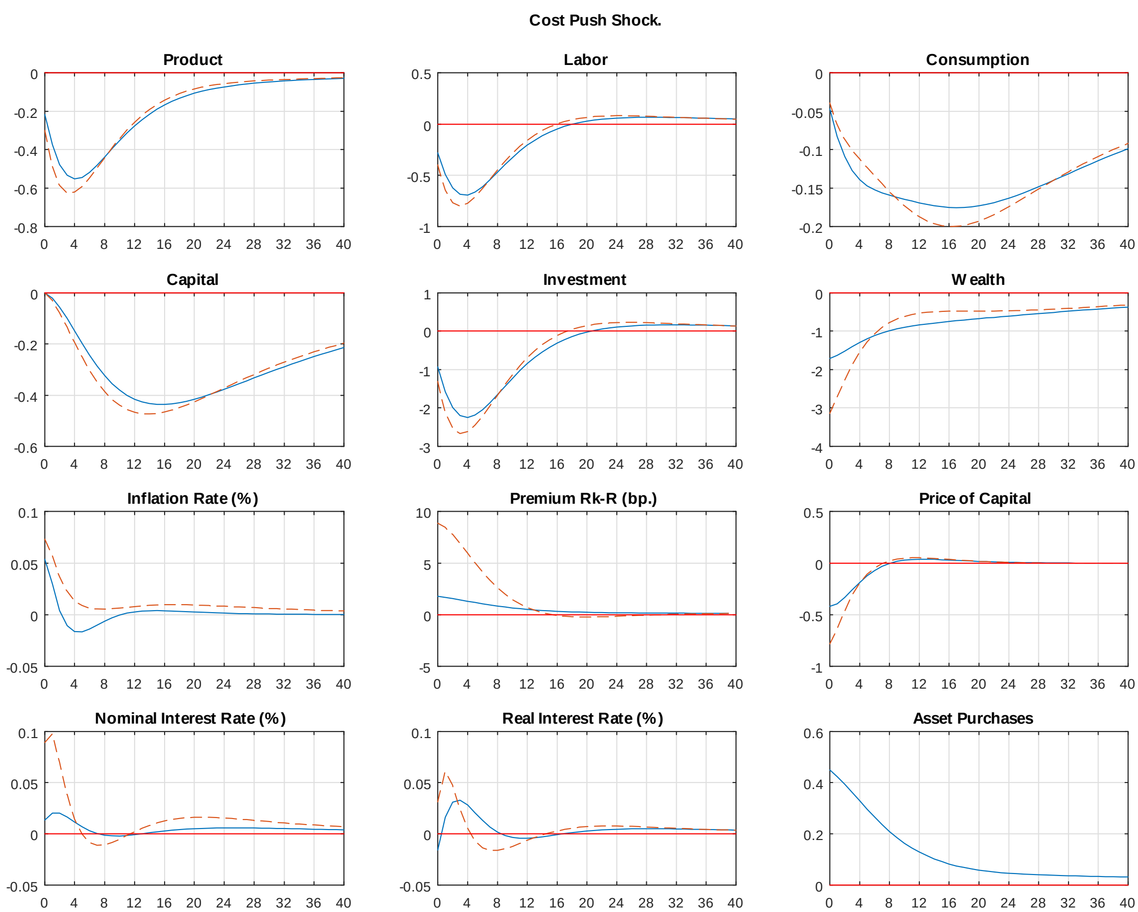

Figure 12. Implementation of Optimal Policy. 1% Cost-Push Shock. Solid lines $\alpha_\pi = 5$, $\alpha_Y = 0$, $\alpha_R = 0.95$, $\kappa^{QE} = 25$. Dashed $\alpha_\pi = 2.19$, $\alpha_Y = 0.2$, $\alpha_R = 0.39$, $\kappa^{QE} = 0$.

As a consequence of the cost-push shock, there is an increase in the inflation. This shock is recessionary and inflationary. However, the inflation increases less than in the conventional exercise. If this shock hits, the nominal interest rate is less contractionary if the central bank has access to asset purchases. The increase in the asset purchases serves to stabilize the financial and real activity, while the nominal interest rate deals with the inflationary consequences of this shock.

When the financial frictions are present, and a cost-push shock hits the economy, if the central bank can implement unconventional policy, the real and financial variables show a less volatile behavior. This contributes to the improvement in the welfare schedule for the consumers. With two policy instruments, the central bank improves the welfare of the society when financial shocks hit (for example, a shock to the quality of capital), but also when non-financial shocks hit (for example, when a cost-push shock hits).

Productivity shock.

Figure 13 shows the response of this economy when there is a decrease of one percent in productivity. This shock has the effect of increasing inflation. In order to depress the economy and to avoid additional increases in inflation, the central bank increases the nominal interest rate. The increase in the nominal rate is considerably smaller if the asset purchases are in place. If only conventional monetary policy is implemented (dash lines), the real interest rate increases and the aggregate spending decreases in order to tame the inflationary pressures. However, the required contraction in output, when the central bank has access to unconventional policy, is slightly smaller than when only conventional policy can be implemented. In this case, the monetary policy is less contractionary than in the absence of the unconventional policy, and the assets purchases are expansionary.

Consumption decreases initially 0.15 percent and its hump-shaped response has a minimum four years after the shock when this variable is 0.5 percent below its long-run equilibrium.

Figure 2.13: Simple Rules. Productivity Shock

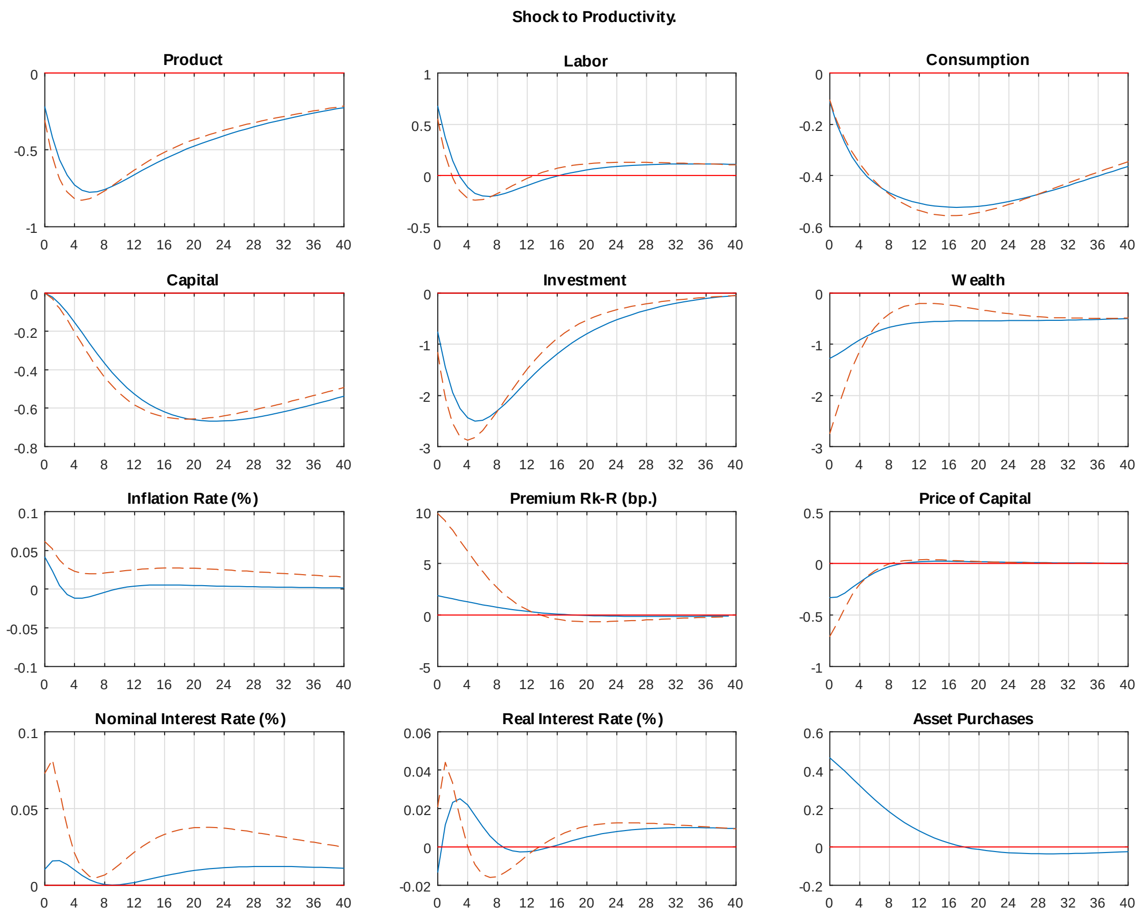

Figure 13. Implementation of Optimal Policy. 1% Decrease in Productivity. Solid lines $\alpha_\pi = 5$, $\alpha_Y = 0$, $\alpha_R = 0.95$, $\kappa^{QE} = 25$. Dashed $\alpha_\pi = 2.19$, $\alpha_Y = 0.2$, $\alpha_R = 0.39$, $\kappa^{QE} = 0$.

The increase in the real rate has effects on the real spending and on the financial sector. The cost of credit increases because of the change in the real rate but also because of the fluctuation in the value of the net wealth of banks. However, unconventional policy serves to stabilize the premium and it produces a smaller reduction in the value of the equity in the banking sector. For example, if only the conventional monetary policy is implemented, the net wealth of the banks falls 3 percent on impact and the premium on capital jumps 10 basis points. However, the implementation of the unconventional policy reduces the effects of this shock on the banks' balance sheet and net wealth falls only 1 percent, while the premium on capital increases only 2 basis points.

When the premium on capital increases, investment spending decreases and this feeds back into the financial sector. Because banks are less healthier, the cost of credit is increased and investment and capital accumulation fall. The presence of the financial sector generates a deeper recession in the economy if only conventional policy is implemented.

The central bank can achieve a better trade-off in terms of output and inflation stabilization if it has access to two policy instruments. The unconventional policy improves the trade-offs of the central bank in a competitive economy even if non-financial shocks hit the economy. The increase in the central bank's intermediation, in 0.5 percentage points reduces, the increase in the spread. The conventional monetary policy is less contractionary in this case, and together, the conventional and unconventional policy can create a smaller output contraction and they can reduce the inflationary pressures, increasing the social welfare.

Shock to the Share of Diverting Funds.

Figure 14 shows the response of the conventional and unconventional policy when a shock to the share of diverting funds hits the economy. In particular, this shock is a sudden increase of 1 standard deviation in the share of funds that banks can divert. A worsening of the agency problem is due to exogenous factors. This shock generates financial distress and a recession. The unconventional policy is very effective at stabilizing this shock.

The unconventional policy is very efficient at stabilizing the economy when this shock arises. Output decreases only 0.04 percent initially (solid lines). By contrast, the conventional policy implies a reduction of 0.1 percent in output (dash lines).

Inflation remains very close to its long-run level in both cases. As a consequence, the Taylor rule suggests a reduction in the nominal and real interest rates to stimulate spending, which is reflected in the initial increase in consumption. However, if the central bank has access to unconventional policy, the nominal interest rate is kept very close to its long-run level, the nominal interest rate is less expansionary in this case. The unconventional policy takes care of this financial shock. This unconventional policy is very efficient at stabilizing the financial activity. The increase in the assets purchases serves to keep under control the cost of credit and to avoid a disruption in the investment and the real sectors. On impact, the asset purchases increase in 0.5 percentage points. However, the unconventional policy is persistent. Ten years after this shock, the asset purchases have not returned to equilibrium.

There is an increase of 8 basis points in the premium when there are no unconventional policy. This is reflected in the less benevolent behavior of investment spending and net wealth.

Figure 2.14: Simple Rules. Financial Shock

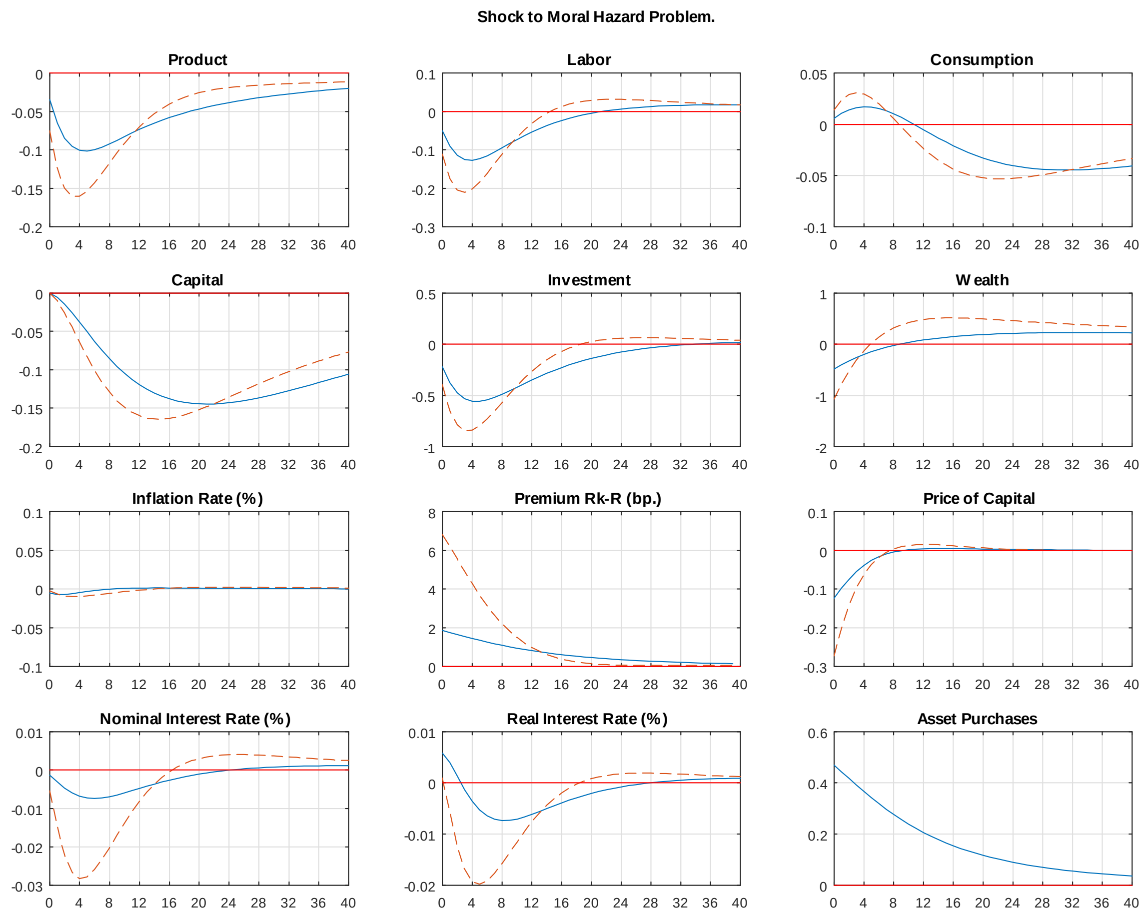

Figure 14. Implementation of Optimal Policy. 1% Increase in the Diverting Share of Funds. Solid lines $\alpha_\pi = 5,\ \alpha_Y = 0,\ \alpha_R = 0.95,\ \kappa^{QE} = 25$. Dashed $\alpha_\pi = 2.19,\ \alpha_Y = 0.2,\ \alpha_R = 0.39,\ \kappa^{QE} = 0$.

The unconventional policy is very efficient at facing financial shocks. It can generate a smaller recession and almost fully stabilize the financial crisis. The nominal interest rate is utilized to support inflation stabilization and to give a hand to the unconventional instrument to achieve a faster recovery. There are stabilization and welfare gains from allowing the central bank to participate in direct lending activities, even when the nominal interest rate is far from the zero lower bound constraint and in spite of the efficiency costs.

This means that unconventional asset purchases represent an efficient policy instrument to face different kind of shocks. The effects of financial and non financial shocks (for example, a shock to the quality of capital or a sudden increase in the moral hazard problem in the banking sector) can be curbed using the combination of the conventional and unconventional monetary policy. In contrast, if only conventional instruments are utilized when the economy is subject to financial frictions and financial crisis, the economy cannot be completely stabilized.

2.6 Related Literature.

There is a set of literature analyzing the impact of the Large Scale Asset Purchases Program (LSAP) on long-term rates. Gagnon et al. (2011), Khrishnamurthy and Vissing (2011), Christensen et al. (2011), and D'Amico and King (2010), present compelling evidence that the unconventional policy measures employed by the Federal Open Market Committee (FOMC) since the end of 2008 have significantly lowered longer-term Treasury yields. Joyce et al. (2012) also present a summary of the different studies which have empirically estimated the effects of the different LSAP in the U.S., U.K., and Europe.

IMF (2013 a) summarizes the wide spectrum of unconventional policies implemented around the world since the Great Recession. They show that for the U.S., the purchase of private securities through the QE1 program largely contributed to stabilize that economy.

Gilchrist et al. (2015) also support the view that unconventional policy reduced the costs of borrowing for firms and households. Lowering the term premia accounts for the most of the reduction in those borrowing costs. According to their results, unconventional asset purchases had effects not only on the government-bonds rates but also on the private assets such as Mortgage Backed Securities. In their results the unconventional policy does not affect inflation. The effect on the interest rates is completely due to the changes in the real rate.

However, the focus of the current paper is more in the line of the effects of unconventional policy using DSGE models. In this line of research Joyce et al. (2012), describe the channels of transmission of quantitative easing. They also explore the conditions in which credit easing is effective in promoting the real activity. They show that as long as the credit frictions are binding in equilibrium and agents are heterogeneous in their asset holdings, the asset purchases foster economic activity. If the constraints do not bind, then the asset purchases displace the private intermediation. Their main focus is on analyzing the portfolio rebalancing channel of asset purchases. Vayanos and Vila (2009) develop a model of preferred portfolio habitats, in which asset purchases are non-neutral and their effects work mainly through the portfolio rebalancing channel described in Joyce et al. (2012). In this line of research, Ellison and Tischbirek (2014), using short and long-term bonds, analyze the effectiveness of unconventional monetary policy. The analysis builds on a preferred habits assumption, which considers preferences of investors for specific asset maturities. Long and short-term government debt are not perfect substitutes. In this framework, they allow the central bank to arbitrage across the maturities. This provides a wider set of policy instruments to the central bank. This can lead to significant welfare gains with respect to the case where the central bank uses only one instrument.

Carlstrom et al. (2014) develop a model with segmented markets in which households can buy long-term asset through the banking sector. However, the banks cannot arbitrage the return differentials because the amount of deposits they can receive is limited by the net worth of the bank. In this case, asset purchases are non-neutral. A monetary policy that includes the term premium in a Taylor rule can dampen movements in the market segmentation distortion. In particular, welfare is improved if the rule can react to changes in the term premium.

Chen et al. (2012) analyze the effects of LSAP programs, in particular Quantitative Easing 2 (QE2). Their estimated model allows for government bonds market segmentation. In contrast to Cúrdia and Woodford (2010), Del Negro et al. (2011) and Gertler and Karadi (2011), who focus on private markets, the authors evaluate the effects of government bonds purchases. Their modelling strategy is to assume that households have heterogeneous preferences for assets of different maturities. In their model, the risk premium that arises in the model as a consequence of transaction costs is a positive function of the supply of long-term Treasury securities. If the government purchases assets, then the supply diminishes. This reduces the risk premium component of the interest rate. Hence, the rate of return decreases. If the central bank commits itself to keep the nominal interest rate at zero for at least four quarters, the effects of LSAP II are smaller than a 25 basis point cut in the nominal interest rates.

In contrast to them, I consider binding financial constraints in the supply-side of credit. Banks cannot extend additional credit because they face a leverage constraint, which always binds. In the current paper, central bank direct lending is welfare enhancing because it increase the supply of credit, which the commercial banks cannot. Similar to them, I also find that asset purchases are welfare improving.

Araújo et al. (2015) consider the effects of the central bank purchases of a risky asset. This policy complements conventional interest rate policy. In their model there are collateral constraints. The effects of asset purchases are: they relax the financial constraint, increase aggregate demand, and even achieve a Pareto improvement. They find that so long as there are some binding financial constraint, as exposed by Joyce et al. (2012), pure changes in the central bank's balance sheet, can affect asset prices, the allocation of resources and the general level of prices. The friction in their model arises because all privately issued financial claims must be collateralizable. This framework allows them to analyze how collateral constraint matter for the effects of both conventional and unconventional monetary policies. In contrast to them, I analyze asset purchases in a model in which the financial friction limits the supply, rather than demand of credit.

Correia et al. 2013, present a model in which the firms require to borrow in advance in order to produce. There is a financial friction limiting the ability of the firms to rise funds. In the presence of the Zero Lower Bound (ZLB) that constrains the nominal interest rate, subsidizing the interest rate on loans is the optimal policy. They assume a model without monopolistic competition and flexible prices. In this case the conventional monetary policy has little room to manoeuvre. In their model, direct lending by the central bank would be desirable when the banks are balance-sheet constrained.

Brendon et al. (2011) in a model with a housing sector and with collateral constraints as in

Iacoviello (2005), also analyze the optimal monetary and unconventional policy. They compare the benefits of commitment respect to discretion. The unconventional instrument is an asset purchases of private securities. This affects the spreads on private borrowing rates. As in the present paper, these purchases have the effect of increasing the price of the assets, contributing to directly relax the collateral constraint. The benefits of the unconventional instrument are magnified in the presence of the Zero Lower Bound (ZLB) or when the central bank cannot commit to its announced policy. In contrast, I do not deal with the ZLB, neither with discretionary policy. These can be avenues for future research.

Reis (2016) analyses the effects of quantitative easing (QE) on welfare. He concludes that the QE can ameliorate the welfare losses associated with the decrease in the aggregate demand and financial disruption associated with a fiscal crisis. Managing the central bank's balance sheet can serve to stabilize inflation and the real activity. However, his paper focuses more on the fiscal implications of QE, than on the interaction of monetary and unconventional issues. I do not deal with fiscal issues in this paper.

Gerali et al. (2010) study the relevance of credit supply factors in business cycle fluctuations. Their model contains an imperfectly competitive banking sector. The spread between the lending and borrowing rates depend on the capital to assets ratio and on the degree of stickiness of the interest rate. After estimating the model they find that shocks affecting the banking sector are able to explain the largest share of contraction of economic activity in the past recession. However they do not analyze unconventional policy measures.

In summary, the most of the empirical literature has found positive effects of the central bank's direct lending on stabilizing the economy. The DSGE literature find it optimal to include the unconventional instrument if there is some financial friction which limits the arbitrage opportunities in the economy. These frictions can take the form of preferred habits models, collateral constraints, or constraints to the supply of credit in the economy.

In particular, I follow the case in which there are limits to the supply of credit. In these circumstances, the commercial banks cannot increase the supply of funds because they face an endogenous leverage constraints. In bad times, these constraints tighten and the cost of credit increases, precipitating a severe recession. Hence, the central bank can intermediate assets in that market. I analyze optimal policy under commitment. Similarly to the most of the studies, I find that if the private intermediaries face financial stress and reduce the lending, the central bank's direct intermediation can contribute to smooth fluctuations in inflation and in real and financial variables. As some of the literature presented above, the optimal policy can be implemented via simple rules. The conventional Taylor rule is complemented by an asset purchase rule which has a feedback coefficient on the deviations of the cost of credit from its long-run average.

2.7 Conclusion

As a consequence of the Great Recession, the understanding of the interactions between banking and monetary policy has become a central issue in policy design. The severity of the financial crisis has exposed the limitations of the conventional tools utilized by the central bank to stabilize the economy. In this chapter, I analyzed whether, there are welfare gains of equipping the central bank with an additional policy instrument.

In an economy with financial frictions , the central bank now has access to two policy instruments: the nominal interest rate and credit easing through asset purchases. If a negative shock hits the economy there are stabilization and welfare gains from allowing the central bank to participate in direct lending activities. The trade offs for the central bank improve in the presence of monopolistic competition, financial frictions, and two policy instruments, respet to the case in which there is only monetary policy available.

The optimal policy can be implemented via simple rules. One rule sets optimally the nominal interest rate, while the other set the central bank policy financial intermediation. This arrangemente is welfare improving and the central bank improves its policy trade offs.

Chapter 3

Macro-prudential and Monetary Policy

Abstract.

I present an extension of a New Keynesian DSGE model with a banking sector to analyze macro-prudential policy.

The consumers are heterogeneous. One group of consumers has a utility function with internal habits (savers). The other group possesses external habits (borrowers).

Reserve requirements function as the prudential instrument. These reserves contribute to increase welfare, to reduce overborrowing, and alter the steady-state composition of the leverage ratio in the banking sector; the bank is more leveraged on its loans to firms and less leveraged on its loans to consumers than in the absence of this policy.

If a financial shock hits, the monetary contraction is reinforced by the prudential response. However, if a supply or a cost-push shock hit, the monetary and the prudential policy act in opposite directions. The main effect of the prudential policy is to curb the excessive borrowing associated with the inefficient consumption.

3.1 Introduction.

The Great Recession of 2007-09 emphasized the relevance of financial events for macroeconomic stability. Financial stability matters. It matters more than previously thought (Clarida (2012)). Inflation targeting is a key element of macroeconomic stability, but it may be insufficient to ensure financial stability; monetary policy might be supported with additional instruments that target for some financial stability concerns such as credit growth, leverage, or asset prices. In this paper, I analyze the optimal mix between monetary and macro-prudential policy.

There are a number of papers which have been designed to capture financial frictions following the financial crisis. In particular, the conventional New Keynesian framework for analyzing optimal policy (Christiano et al. (2005), Smets and Wouters (2007) and Woodford (2003)) has been adapted to consider financial imperfections (Cúrdia and Woodford (2010), Woodford (2012)

Christiano et al. (2013)). Typically, these more sophisticated models include financial frictions based on the financial accelerator (Bernanke, Gertler and Gilchrist (BGG(1999), henceforth) or some collateral constraint (Kiyotaki and Moore (1997)) or can include some of the previous frictions together with an imperfectly competitive (Gerali et al. (2010)) or balance-sheet constrained (Gertler and Kiyotaki (2010)) banking sector.

The new literature make it possible to consider financial intermediaries with a meaningful role in the transmission mechanism of monetary policy. These literatures can consider cases of under-borrowing such as we saw in the aftermath of the financial crisis and can consider the Quantitative Easing (QE) policies to alleviate the slowdown originating in the financial sector.

Additionally, some of these models have made clear that individual financial institutions do not internalize the effects of their behavior on society. This lack of internalization of the agents' behavior justifies the introduction of a regulatory authority, and it has raised the question of whether and how to optimally mix monetary and macro-prudential policy.

For example, Quint and Rabanal (2014), Gelain and Ilbas (2014) and Christiano et al. (2005) have extended the financial frictions literature to consider macroprudential policies. These models, based on the BGG(1999) financial accelerator mechanism or the Gertler and Kiyotaki (2010) banking model, imply an economy with underborrowing. However, when the underlying friction implies under -rather than over-borrowing then it is difficult to motivate the use of macroprudential instruments which will further depress borrowing. As a consequence, these papers tend to consider macroprudential policy as a stabilization tool outside of steady state.

Other papers succesfully introduce a tendency towards over-borrowing, which is a more natural environment in which to consider macro-prudential policies. Some examples are: Collard et al. (2017) in which the limited liability and deposit insurance creates a socially excessive level of borrowing. Here, prudential policies play a role in taming the excessive risk taking.

In current paper, we add to the literature by analyzing the optimal interaction between monetary and prudential policy in a model with a banking sector similar to that in Gertler and Karadi (2011). The main contribution of this paper lies in considering a model where there can be simultaneously over and under-borrowing in the economy. The financial friction limits borrowing by firms, while a consumption externality creates over-borrowing by some households. Interestingly, macro-prudential policy has a differential impact on these which is that gives the instrument its efficacy relative to conventional policy. We consider the ability of our model to explain both the run-up to the financial crisis and its aftermath and consider what role macroprudential policy could play in both preventing the crisis and offsetting the effects.

3.1.1 A Preview of the Model.

I depart from current models by introducing heterogeneity in households' patience, habits preferences and net wealth in a model with a frictional banking sector such as that by Gertler and Karadi

(2011). The first type of heterogeneity is common in the literature analyzing financial imperfections (for example, Cúrdia and Woodford (2010)). The second and third type of heterogeneity are less common in the studies referring to optimal macro-prudential policy.

Why is it necessary to introduce differences in households' patience? The difference in patience creates borrowers and savers in equilibrium. This can create a role for the financial intermediaries to lend to households. In this model, banks, facing a balance-sheet constraint, raise deposits from savers and issue loans to borrowers and firms.

Why is it necessary to introduce wealth heterogeneity? During the Great Recession, differences in households' net worth were and important driver of the dynamics of aggregate variables in the U.S. In particular, the wealth-poor households explained more than 20 percent of aggregate consumption; and their precautionary motives accelerated the fall in consumption (Krueger et al. (2016))[1]. Hence, the relevance of poor households in the U.S. make it appropriate to include them when modeling this economy. I consider these differences.

Broer et al. (2016) analyze the implications for monetary transmission in a New Keynesian model with two groups of households (workers and capitalists). Capitalists receive income only from their equity ownership. Workers obtain their income only from labor. Similarly, I use the equity on the firms as an approximation to net wealth. Differences in the ownership of this equity introduce wealth inequality in the model. In contrast to Broer et al. (2016), I allow the two groups of households to derive income from wealth and labor, and both groups smooth consumption[2].

Why is it relevant the habits heterogeneity? The heterogeneity in the habits preferences is relevant for the optimal policy. The current literature considering habits and financial frictions, choose between external or internal. And given that this modelling decision is not innocuous for optimal policy, the choice should be backed by some empirical evidence.

Why the choice of habits preferences is relevant for optimal policy? Because the externality introduced by external habits can contribute to overconsumption and overborrowing. Consider an economy with external habits, monopolistic competition, and banks' balance-sheet constraints. The first and third friction make the economy inefficiently small. However, the external habits make borrowing inefficiently large. Hence, in the presence of financial frictions, monopolistic competition and external habits and using a plausible calibration, the economy is inefficiently large (Leith et al. (2012), and Leith et al. (2015)).

In this context, prudential regulation can reduce overborrowing. This policy makes agents internalize the externalities which originated the inefficient borrowing. Reserve requirements, the prudential instrument, increase the cost of credit. Debtors have less incentives to borrow. Overborrowing and risk-taking decrease. The economy is less vulnerable to the amplification effects of

[1]Campbell and Mankiw (1989), Kaplan and Violante (2014), Kaplan et al. (2014), Krueger et al. (2016), and Gornemann et al. (2016) have shown the effects of heterogeneity on the macroeconomy.

[2]In Broer et al. (2016), the wealth-rich do not smooth consumption, their consumption is a function only of wealth. The capitalists (or wealth-rich) have a behavior similar to the hand-to-mouth consumers in Campbell and Mankiw (1989) or Galí et al. (2007); and closely resemble the wealthy hand-to-mouth consumers in Kaplan et al. (2014).

shocks. Macro-prudential policy is welfare increasing in this context.

With internal habits this is not the case. In this paper, the aim of macro-prudential policy is to curb overborrowing, with habits of internal kind, the level of lending is not changed by the internal habits. Hence, prudential policy would not act by reducing excessive financial activity.

What is the right modelling decision? To answer this, I separate the households in two groups. The external (habits) consumers are the wealth-poor households of the empirical evidence[3]. The rich group is captured in the model by the internal (habits) consumers. I allow the relative consumption between these two groups to define the relevance of habits, rather than selecting in an ad hoc way the type of habits.

Why is it relevant to consider the Ramsey-optimal prudential policy? Prudential policy can potentially alleviate part of the distortions associated with the frictions in the economy, in addition to its stabilization properties. If well, simple rules can be an approximation to how policy is implemented in central banks, much of the current literature analyzing the interaction between monetary and prudential policy consider only the coefficients entering the simple rules. That literature is putting aside one important benefit of prudential policy; its contribution to eliminate or reduce steady-state distortions, and this could lead to an under appreciation of its potential benefits. I consider both, the Ramsey-optimal contingent plans, and the ability of simple rules to mimic these plans.

Why does the current literature find it optimal to introduce a prudential instrument? Suppose there is a demand shock. In the boom, credit, real activity and the marginal cost of firms increase. Firms with the opportunity to adjust price would do so and inflation would rise. Risk-taking also increments. The Taylor rule and the prudential rule jointly determine a less volatile environment. The task for the central bank is well defined: the Taylor rule deals with inflation stabilization; whereas the prudential instrument deals with excessive risk-taking. The monetary rule calls for a contractionary setting. The prudential rule calls for a more astringent policy. Curbing the business cycle, due to the monetary tightening and the reinforcement by the prudential response, contributes to smooth inflation, output, and financial volatility. Welfare increases.

In these frameworks, there are relatively small welfare improvements because they take only the contributions of prudential policy to smooth fluctuations, missing the benefits associated with the reduction of the inefficiencies in the steady state (exceptions are Collard et al. (2017) and De Paoli and Paustian (2013)).

How much of the contribution of the prudential instrument to welfare is due to its contribution to inflation control and how much to its contribution to financial stability? Potentially, the strongest contribution of prudential policy in such environments could be its reinforcement of inflation control, which in models with sticky prices is a big contributor to social welfare. It is not clear if the policy mix implemented would still deliver welfare benefits in the absence of sticky

[3] As pointed out by Krueger et al. (2016), this group represents a large share of the population and consumption. They have access to financial markets but hold almost zero net wealth. Their precautionary motives contribute to explain the large drop in consumption during the Great Recession.

prices. Considering the Ramsey-optimal contingent plan could deliver a clearer answer. And, in contrast to most of the current literature, I do so.

To the best of my knowledge, no other paper has yet studied optimal macro-prudential policy under net wealth, habits heterogeneity[4], and an overborrowing problem associated with inefficient consumption. I fill in this gap.

I proceed now to present a brief review of related literature. The next section contains two subsections. The first presents the studies analyzing the interaction between monetary and prudential policy via simple rules. The second presents the studies which use Ramsey-optimal policy as their benchmark for analysis. The number of papers in this last section is less extensive; the current paper adds to this branch of literature.

3.2 Related Literature.

The financial frictions can be modeled on the demand (e.g. some limits to borrow associated with the value of collateral or default risk) or supply side of credit (e.g. an imperfect banking sector which limits the value of loans issued). The most of the current literature analyzing the welfare implications of prudential policy assume that prudential and monetary policy are implemented via simple rules.

Duncan and Nolan (2015) and Galati and Moessner (2013) provide extensive surveys on the literature concerning micro-prudential, macro-prudential policy, and its interaction with the other dimensions of government policy.

3.2.1 Macroprudential Policy as a Stabilization Tool.

As noted in the introduction, some of the current literature introduce macroprudential policy as a stabilization tool in an underborrowing framework. In particular, to the monopolistically competitive economy they add a frictional financial sector that further reducer the financial activity. For example: Kannan et al. (2012) follow Iacoviello (2005), they set a rule for monetary policy and another for the macro-prudential authority. Then, they optimally find the value of the feedback coefficients entering the policy rules, but they leave aside the steady-state considerations of such instrument. In such an economy the prudential instrument could work as a subsidy rather than a tax on borrowing in order to foster credit growth. Similarly, Unsal (2011) find that macro-prudential policy is welfare improving but they do not deal with steady state considerations.

In Angelini et al. (2012) conduct their welfare analysis based on an ad hoc loss function. When a shock hits the supply of credit, and produces financial distress, prudential policy is welfare

[4] Quint and Rabanal (2014) estimate a two countries model for the euro area. They consider savers and borrowers with external habis. Their degree of external habits be different, but they do not deal with the issue of considering different kind of habits.

improving. However, they do not consider introducing the prudential instrument as a way to solve the steady state financial problem.

Similarly to the current paper, Quint and Rabanal (2014) consider habits' heterogeneity, but of the same kind. Borrowers can have a different value of their external habits than savers. They optimize the coefficients in the policy rules. But, do not deal with the issue of the steady-state value of the prudential instrument.

Levine and Lima (2015) extend the model in Gertler and Karadi (2011) to introduce a subsidy to net wealth and a tax on lending. They optimize their policy rules and find that, leaning against financial markets is welfare increasing. Their policies rules work as a stabilization device rather than a device to correct the inefficiencies in steady state.

Leduc and Natal (2015) also pursue the optimal policy in a model in which entrepreneurs face credit constraint. Their demand for credit is limited by the value of the firm. This is another example of an economy with a problem of underborrowing in steady state. Hence, the prudential instrument can be use only as a stabilization device. The steady-state in this economy is highly distorted. The monopolistic competition distortion is reinforced by the external premium distortion. It is not clear in this study how reserve requirement, which have the effect of increasing the cost of credit and reinforce distortions, can contribute to eliminate the underborrowing distortions in steady state.

The number of paper that introduce prudential policy in an overborrowing framework is more limited. Next, I present some of these papers.

3.2.2 The Case of Ramsey Policy.

Clerc et al. (2015) analyze the effects of capital requirements on the steady-state welfare. Banks have incentives to overborrow because of the limited liability and deposit insurance. The macro-prudential instrument is welfare increasing because it reduces the bank's risk-taking. Collard et al. (2017) jointly determine optimally the nominal interest rate and bank-capital requirements. The financial distortion is due to limited liability and deposit insurance. This creates a socially excessive risk-taking by banks. The prudential instrument is a bank's capital requirement. The Ramsey planner sets both instruments to maximize social welfare. In contrast, I use reserve requirements as the prudential instrument.

De Paoli and Paustian (2013) analyze the welfare implications of cooperation between monetary and macro-prudential authorities. Firms can borrow only a certain fraction of their net worth, as in Carlstrom et al. (2010). The policy authority optimally sets a tax on borrowing to control leveraging. If faced with cost-push shocks, policy authorities should cooperate and commit to a given course of action. If monetary and macro-prudential tools are set independently and under discretion, they suggest that assigning conservative mandates (à la Rogoff [1985]) and having one of the authorities act as a leader can mitigate coordination problems.

There is a different branch of the literature which considers occasionally binding constraints. Jean and Korinek (2010 and 2016), Bianchi (2011), Benigno et al. (2013), are some examples. The benefits of such approach is that they are able to analyze optimal prudential policy in good and bad times. This modeling strategy allows the prudential authority to prevent risk-taking during good times. As a consequence, in bad times, the economy is in a better position to face shocks. In contrast, in the current paper, financial constraints are always binding.

Bianchi and Mendoza (2011) examine the effects of a credit externality in a model with collateral constraint, following Kiyotaki and Moore (1997). Borrowers do not internalize that their borrowing increases systemic risk. The prudential instrument depresses borrowing when the economy is growing, reducing the magnitude of the financial crisis if bad shocks occur. The social planner's allocation is the benchmark for comparison. However, they do not deal with the interaction of monetary and prudential policy.

Next, I present the benchmark model for analysis, which considers: heterogeneity in the households' preferences; the Ramsey-optimal plan; and, simple rules and their ability to mimic the results of the Ramsey plan.

3.3 The Model.

The model I use for the analysis is a DSGE similar to Christiano et al. (2005) and Smets and Wouters (2007) but modified in Gertler and Karadi (2011) to include financial intermediaries that face balance-sheet constraints. In this framework, an agency problem between borrowers and lenders limits the amount of funds that banks can lend. The amount of loans that can be intermediated by the banking sector depend on the value of the net wealth in this sector.

There are five groups of agents: households (savers and borrowers), banks, goods producers, capital-goods producers and retailers. I depart from the assumption of a representative consumer. In the model there will be two groups of consumers. They differ from each other in their patience, in the way they form habits in consumption, and in their net financial position. Next, I discuss the characteristics of households.

3.3.1 Households.

Households choose consumption (C_t), labor (L_t) and savings $\left(D_t^h\right)$ to maximize their utility. Each household has a continuum of members. Within the household there is perfect consumption insurance. There are two types of agents inside each household. At each period, the fraction $(1-f)$ represents workers and (f) bankers. A household owns the banks managed by its members. The deposits of this household, are in intermediaries that it does not own.

The survival horizon of banks is finite. Introducing this finite horizon has the effect of ensuring that over time the banks do not reach the point where they can fund all the investment from their own capital. (θ) is the probability that a bank operates until the next period. This probability is independent of how long the agent has been a banker. The average survival length of a bank is $\left(\frac{1}{1-\theta}\right)$.

The relative share of workers and bankers is constant. Each period, the number of bankers leaving the industry is $(1-\theta)\,f$. The same number of workers become bankers. Households provide their new bankers with starting funds. When a bank leaves the industry its retained profits are returned, lump-sum, to its owner.

Consumers.

There are two groups of consumers in the model. The impatient households derive utility from an external habits specification. The number of these consumers is (Υ). The representative consumer of the second group is a patient saver and has a utility function showing internal habits in consumption. There are $(1-\Upsilon)$ consumers in this group.

To ensure they both have the same quantity of labour in equilibrium, I assume that they have different attitudes toward work. In particular, the disutility of labour would be different.

Borrowers.

Following Leith et al. (2012) and Leith et al. (2015), I assume that this group of consumers form habits at the level of an aggregate consumption good. The habit-adjusted composite good of the household n is $(X_{n,t})$

$$X_t^n = C_{n,t}^{Ex} - h^{Ex} C_{t-1}^{Ex},$$

and the household n's consumption, $C_{n,t}^{Ex}$ is an aggregate of the continuum of goods $i \in [0,1]$,

$$C_{n,t}^{Ex} = \left(\int_0^1 \left[\left(C_{it}^{Ex} \right)^n \right]^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

(ε) is the elasticity of substitution between goods and C_{t-1}^{Ex} is the cross-sectional average of consumption.

The maximization problem of the representative consumer in this group can be expressed using the Lagrangian:

$$\mathcal{L} = E_t \left\{ \begin{array}{c} \sum\limits_{i=0}^{\infty} d_{t+s} \left(\beta^{Ext} \right)^i \left[\frac{1}{1-\sigma} \left[C_{n,t+i}^{Ex} - h^{Ex} C_{t+i-1}^{Ex} \right]^{1-\sigma} - \frac{\chi^{Ex}}{1+\varphi} \left(L_{n,t+i}^{Ex} \right)^{1+\varphi} \right] \\ +\lambda_{t+i}^{Ex} \left[W_{t+i} L_{n,t+i}^{Ex} + \Pi_{t+i}^{Ex} - T_{t+i}^{Ex} - R_{t+i}^{b} \left[q_{t-1+i} B_{t-1+i}^{Ex} \right] + \left[q_{t+i} B_{t+i}^{Ex} \right] - C_{n,t+i}^{Ex} \right] \end{array} \right\},$$

where, $L_{n,t}^{Ex}$ are the per capita working hours, Π_t^{Ex} and T_t^{Ex} are the profits and the lump-sum taxes of the external-habits consumers. W_t is the real wage.

This group of households is more impatient than the savers. This is captured by the subjective discount factor $\left(\beta^{Ext}\right)$. Then, households with external habits are allowed to borrow from the commercial banks. Given that borrowers are more impatient than savers $\beta^{Ext} < \beta$, where β is the discount factor of the savers. This assumption separates households in savers and borrowers. Every period they acquire loans with value of $\left(q_t B_t^{Ex}\right)$, where q_t is the price of one unit of per capita borrowing $\left(B_t^{Ex}\right)$. R_t^k is the real interest rate paid from period t-1 to t.

$\left(\frac{1}{\sigma}\right)$ is the intertemporal elasticity of substitution, $\left(h^{Ex}\right)$ measures the degree of external habits, $\left(\chi^{Ex}\right)$ is the weight of labour in the utility and (φ^{-1}) is the Frisch elasticity of labour supply.

The first order condition respect to consumption is:

$$E_t \lambda_t^{Ex} = d_t E_t \left[\frac{1}{\left[C_{n,t}^{Ex} - h^{Ex} C_{t-1}^{Ex}\right]^{\sigma}} \right]. \tag{3.1}$$

Respect to labour:

$$d_t \chi^{Ex} \left(L_{n,t}^{Ex}\right)^{\varphi} = \lambda_t^{Ex} W_t. \tag{3.2}$$

Respect to borrowing:

$$E_t \lambda_t^{Ex} q_t = \beta^{Ext} E_t \lambda_{t+1}^{Ex} q_t R_{t+1}^b. \tag{3.3}$$

And the budget constraints:

$$W_t L_{n,t}^{Ex} + \Pi_t^{Ex} - R_t^b \left[q_{t-1} B_{t-1}^{Ex}\right] + \left[q_t B_t^{Ex}\right] - T_t^{Ex} = C_{n,t}^{Ex}. \tag{3.4}$$

The marginal utility of consumption at period $(i = 0)$ can be expressed as:

$$U_{ct}^{Ex} = d_t E_t \left[\frac{1}{\left[C_{n,t}^{Ex} - h^{Ex} C_{t-1}^{Ex}\right]^{\sigma}} \right]. \tag{3.5}$$

Then, optimal labour supply is

$$\frac{\chi^{Ex} \left(L_{n,t}^{Ex}\right)^{\varphi}}{U_{ct}^{Ex}} = W_t. \tag{3.6}$$

And the consumption-saving decision

$$1 = \beta^{Ext} E_t \frac{d_{t+1}}{d_t} \frac{U_{ct+1}^{Ex}}{U_{ct}^{Ex}} R_{t+1}^b. \tag{3.7}$$

Savers.

The maximization problem of the savers can be expressed using the Lagrangian:

$$\mathcal{L} = E_t \left\{ \begin{array}{c} \sum_{i=0}^{\infty} d_{t+i}\beta^i \left[\frac{1}{1-\sigma}\left(C^I_{t+i} - h^{Int}C^I_{t+i-1}\right)^{1-\sigma} - \frac{\chi}{1+\varphi}\left(L^I_{t+i}\right)^{1+\varphi}\right] \\ +\lambda_{t+i}\left[W_{t+i}L^I_{t+i} + \Pi^I_{t+i} - T^I_{t+i} + R_{t-1+i}\left[D^I_{t-1+i}\right] - \left[D^I_{t+i}\right] - C^I_{t+i}\right] \end{array} \right\}$$

where, $\left(C^I_t\right)$ is the per capita consumption, L^I_t working hours, Π^I_t, T^I_t and D^I_t are the profits, the lump-sum taxes and the deposits of the internal-habits consumers. (β) is the subjective discount factor of this group. $\left(\chi^{Int}\right)$ is the weight of labour in the utility. All the other parameters and variables are identical to those of the external-habits consumers.

The first order conditions respect to consumption is:

$$\lambda_t = E_t\left[\frac{d_t}{\left(C^I_t - h^{Int}C^I_{t-1}\right)^{\sigma}} - d_{t+1}\frac{\beta h^{Int}}{\left(C^I_{t+1} - h^{Int}C^I_t\right)^{\sigma}}\right]. \tag{3.8}$$

Respect to labor:

$$d_t\chi^{Int}\left(L^I_t\right)^{\varphi} = E_t\lambda_t W_t. \tag{3.9}$$

Respect to savings:

$$E_t\lambda_t = \beta E_t\lambda_t R_t. \tag{3.10}$$

And the Budget constraints:

$$W_tL^I_t + \Pi^I_t + R_{t-1}\left[D^I_{t-1}\right] - \left[D^I_t\right] - T^I_t = C^I_t. \tag{3.11}$$

The marginal utility of consumption at period $(i = 0)$ can be expressed as:

$$U^I_{ct} = E_t\left[\frac{d_t}{\left(C^I_t - h^{Int}C^I_{t-1}\right)^{\sigma}} - \frac{d_{t+1}\beta h^{Int}}{\left(C^I_{t+1} - h^{Int}C^I_t\right)^{\sigma}}\right]. \tag{3.12}$$

Then, optimal labour supply is

$$\frac{d_t\chi^{Int}\left(L^I_t\right)^{\varphi}}{U^I_{ct}} = W_t. \tag{3.13}$$

And the consumption-saving decision

$$1 = E_t\beta\frac{U^I_{ct+1}}{U^I_{ct}}R_t.$$

3.3.2 Banks.

Banks assist in channeling funds from savers to borrowers. In addition, they engage in maturity transformation. They hold long-term assets, funded with short-term liabilities. Banks in the model aim to capture the entire banking sector: investment banks as well as commercial banks.

Balance Sheet and Evolution of Wealth.

The financial intermediary (j) receives deposits from households with internal habits (D_{jt}). These deposits pay the short-term real interest (R_t) from (t) to $(t+1)$. These funds complement the accumulated wealth of banks at the end of period t, (N_{jt}). The banks make use of these two sources of funds to make loans to producers and to the households with external habits.

The bank funds two type of assets: the loans granted to firms $\left(S_{jt}^f\right)$ and the loans granted to the households with external habits $\left(S_{jt}^{Ext} = \Upsilon B_{jt}^{Ex}\right)$, where (Υ) is the number of households with external habits funded by this bank. Loans to firms pay the rate $\left(R_{t+1}^k\right)$ between (t) and $(t+1)$. Loans to consumption pay the rate $\left(R_{t+1}^b\right)$ over the same period. The difference in the return on assets is due to the different ability of the bank to divert funds. The bank can divert a fraction (λ) of its assets on firms, but a fraction $(\Delta\lambda)$ of its consumer loans. In equilibrium, (Δ) is calibrated to match the observed difference in the lending rates. The relative price of loan to firms is (Q_t).

I assume that the only difference between the loans to consumption and the loans to firms is the different ability of the bank to divert each type of assets. Hence, following the work of Gertler et al. (2012), I normalize the units of loans to consumption so that each unit is a claim to the future returns on one unit of the capital in the firms held by the bank. Allow $\mathrm{Z}_t^f = \alpha \frac{P_{mt} Y_{mt}}{\xi_t K_t}$ be the flow of return at period (t) by one unit of bank's holding of firms assets[5]. The relative price of the loans to consumption is (q_t). Then, the interest rate on loans to consumption is given by

$$E_t R_{t+1}^b = E_t \frac{\xi_{t+1}}{q_t} \left[Z_{t+1}^f + q_{t+1} \left(1 - \delta_{t+1}\right) \right]. \tag{3.14}$$

The payoff to loans to consumption is adjusted by the physical depreciation and the quality of capital that underlies the bank's loans to firms. The general equilibrium of the model determines Z_t^f and q_t.

Each period the total value of assets held by the representative bank is:

$$Q_t S_{jt}^f + q_t S_{jt}^{Ext}.$$

Let (N_{jt}) be the value of equity that the intermediary (j) holds at the end of period (t). The balance sheet of the representative bank is

$$Q_t S_{jt}^f + q_t S_{jt}^{Ext} = N_{jt} + D_{jt}, \tag{3.15}$$

where the assets of this bank are funded with the deposits received from the households with internal habits (D_{jt}) and its accumulated wealth (N_{jt}). The total deposits received by the bank are

$$D_{jt} = (1 - \Upsilon) D_{jt}^I$$

[5] Z_t is determined optimally as the result of the maximization of the non-financial firms.

The bank's net wealth evolves as the difference between the return on its assets and the cost of funding them,

$$N_{jt} = R^k_{t-1} Q_{t-1} S^f_{jt-1} + R^b_{t-1} q_{t-1} S^{Ext}_{jt-1} - R_{t-1} D_{jt-1}, \tag{3.16}$$

which is the difference between the return on loans $\left(R^k_{t-1} Q_{t-1} S^f_{jt-1} + R^b_{t-1} q_{t-1} S^{Ext}_{jt-1}\right)$ and the cost of deposits $(R_t D_{jt-1})$.

The balance sheet of the banks eq.(3.15) can be inserted in the evolution of the banks wealth eq. (3.16), to obtain

$$N_{jt} = \left[R^k_{t-1} - R_{t-1}\right] Q_{t-1} S^f_{jt-1} + \left[R^b_{t-1} - R_{t-1}\right] q_{t-1} S^{Ext}_{jt-1} + R_{t-1} N_{jt-1}. \tag{3.17}$$

The term $\left[R^k_{t-1} - R_{t-1}\right]$ is the premium over the riskless rate for each unit of loans to firms. The private bank has a premium equal to $\left[R^b_{t-1} - R_{t-1}\right]$ over the riskless rate for each unit of loan to consumption. The term $[R_{t-1} N_{jt-1}]$ is the bank's benefit of retained earnings.

Incentive Constraint.

There is a frictionless process of lending and borrowing between producers and banks and between banks and borrowers. The possibility of making profits encourages the banker to remain in the industry as long as possible. When the risk-adjusted premium is positive, the intermediary would like to expand assets to exhaust the profit opportunities. However, an agency problem between banks and depositors reduce the ability of the bank to issue new loans.

In particular, every period the bankers can divert a fraction (λ) of loans to firms and a fraction $(\Delta\lambda)$ of loans to consumption. In order to avoid that the bank absconds with the funds, the households impose an incentive constraint to the bank. The cost to the banker of diverting funds is that the households can force the bank to shut down and households can recover the fraction of assets.

For the households to be willing to supply funds to the banker, the next incentive constraint must be satisfied:

$$V_{jt} \geq \lambda Q_t S^f_{jt} + \Delta\lambda q_t S^{Ext}_{jt}. \tag{3.18}$$

The left-hand part (V_{jt}) is the expected present value of the bank's financial activity if it remains in the industry. This is what the bank would lose if it is forced to leave the industry. The term $\lambda\left(Q_t S^f_{jt}\right)$ is what the bank would gain if it absconds the loans to firms, while $\left(\Delta\lambda q_t S^{Ext}_{jt}\right)$ is the gain if it absconds with the loans to consumption. The banks asses this trade-off and acts optimally. The bank would remain in the industry as long as the benefits from doing so cover the benefits from absconding with a share of funds.

The household would deposit in the bank only if the benefit for the bank of lending and borrowing is at least as large as the benefit for the bank from diverting funds. This contract limits the ability of the banking sector to raise funds from households. As a consequence, the banks have limits on the loans they can issue. This will impact the level of capital that firms can accumulate and the level of the variables in equilibrium.

Maximization of Profits.

Banks are not interested in funding projects with an expected discounted cost larger than its expected discounted return. A fraction $\left(\Upsilon_{Ext}^{ownership}\right)$ of the representative bank is owned by the households with external habits in consumption. The remaining $\left(1-\Upsilon_{Ext}^{ownership}\right)$ of profits of this bank belongs to households with internal habits.

The profits of the bank are returned to households. Hence, the relevant discount factor for those resources is the weighted average of the discount factor of each group of households. In particular, the resources are discounted at the rate

$$E_t\left[\Upsilon_{Ext}^{ownership}\beta^{Ext}\frac{U_{ct+1}^{Ex}}{U_{ct}^{Ex}}+\left(1-\Upsilon_{Ext}^{ownership}\right)\beta\frac{U_{ct+1}^{I}}{U_{ct}^{I}}\right] \tag{3.19}$$

between period (t) and $(t+1)$. Between period t and $(t+i)$ the discount factor is

$$E_t\Lambda_{t,t+1+i}=E_t\left\{\Upsilon_{Ext}^{ownership}\left(\beta^{Ext}\right)^{1+i}\frac{U_{ct+i}^{Ex}}{U_{ct}^{Ex}}+\left(1-\Upsilon_{Ext}^{ownership}\right)\beta^{1+i}\frac{U_{ct+i}^{I}}{U_{ct}^{I}}\right\}. \tag{3.20}$$

At the end of period t, the bank maximizes its expected discounted terminal wealth according to

$$V_{jt}=\max E_t\sum_{i=0}^{\infty}\left(1-\theta\right)\theta^i\Lambda_{t,t+1+i}\left(N_{jt+1+i}\right). \tag{3.21}$$

Substituting the evolution of wealth eq.(3.17)

$$V_{jt}=\max E_t\sum_{i=0}^{\infty}\left(1-\theta\right)\left[\theta^i\right]\Lambda_{t,t+1+i}\begin{pmatrix}\left[R_t^k-R_t\right]Q_tS_{jt}^f+\left[R_t^b-R_t\right]q_tS_{jt}^{Ext}\\+R_tN_{jt}\end{pmatrix} \tag{3.22}$$

The bank then maximizes eq. (3.22) subject to the incentive constraint (3.18). The conjectured solution is

$$V_{jt}=\nu_t^fQ_tS_{jt}^f+\nu_t^{Ext}q_tS_{jt}^{Ext}+\eta_tN_{jt} \tag{3.23}$$

The maximization produces the marginal benefit of issuing and additional loan to firms $\left(v_t^f\right)$

$$v_t^f=E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^k-R_t\right). \tag{3.24}$$

The marginal benefit of issuing and additional loan to households

$$v_t^{Ext} = E_t \Lambda_{t,t+1} \Omega_{t+1} \left[R_t^b - R_t \right]. \tag{3.25}$$

And the marginal profits of accumulating an additional unit of net worth (η_t)

$$\eta_t = (1-\theta) + \theta \left(\nu_{t+1}^f \phi_{t+1} + \eta_{t+1} \right). \tag{3.26}$$

The profit maximization of this bank implies that the marginal return on the two assets are equalized, this means that $\left[\nu_t^{Ext} = \Delta \nu_t^f \right]$. And

$$\Omega_{t+1} = (1-\theta) + \theta \left(\nu_{t+1}^f \phi_{t+1} + \eta_{t+1} \right), \tag{3.27}$$

is the bank's augmented discount factor.

Leverage Ratio.

From the maximization conditions of the bank, the optimal leverage ratio is given by

$$\frac{\eta_t}{\lambda - \nu_t^f} = \frac{Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext}}{N_{jt}}, \tag{3.28}$$

which can be written as

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t^f}.$$

The previous expression means that the maximum amount of loans issued by the representative bank is limited by the maximum leverage ratio tolerated by the household. This leverage ratio is function of the diverting preference of the banks and the profitability of the banking industry. The maximum amount of loans is also restricted by the amount of accumulated wealth of the bank (N_{jt}).

The components of the leverage ratio are the same for each bank. After aggregating (3.28),

$$Q_t S_t^f + \Delta q_t S_t^{Ext} = \left[\frac{\eta_t}{\lambda - \nu_t^f} \right] N_t. \tag{3.29}$$

the overall demand for assets in the economy $\left(Q_t S_t^f + \Delta q_t S_t^{Ext} \right)$ can be written as a function of the leverage ratio and the accumulated wealth (N_t) in the banking sector.

Evolution of Aggregate Net Wealth.

The evolution of aggregate wealth (N_t) is the sum of two components: the net worth of the existing banks (N_{et}), and the net wealth of the new banks (N_{nt})

$$N_t = N_{et} + N_{nt}. \tag{3.30}$$

The fraction of bankers (θ) at $(t-1)$ survives until (t). Then, using the aggregate expression of eq. (3.17) and the definition of leverage,

$$N_{et} = \theta \left[\left[R^k_{t-1} - R_{t-1} \right] \frac{Q_{t-1} S^f_{t-1}}{N_{t-1}} + \left[R^b_{t-1} - R_{t-1} \right] \frac{q_{t-1} S^{Ext}_{t-1}}{N_{t-1}} + R_t \right] N_{t-1}. \tag{3.31}$$

As in Gertler and Karadi (2013), I assume that the newly entering bankers receive start-up funds from the households. Because the bank is jointly owned, the total transfer received by the households is a weighted sum of the size of each group. It is assumed that this start-up funds are equal to a small fraction of the value of assets that exiting bankers had intermediated in their final operating period.

There are $(1-\theta)$ banks exiting every period. It is assumed that each period the households with external habits transfers the fraction $\left[\left(\frac{\Upsilon}{1-\theta} \right) w \right]$ of the intermediated assets in the previous period to their new bank. The household with internal habits transfer a fraction $\left[\left(\frac{1-\Upsilon}{1-\theta} \right) w \right]$. There are $(1-\theta)$ new banks this period. Hence, in the aggregate, the startup transfer to banks is $\left[N_{nt} = w \left(Q_t S^f_{t-1} + q_{t-1} \Delta S^{Ext}_{t-1} \right) \right]$.

The evolution of aggregate wealth is

$$N_t = \theta \left[\left[R^k_{t-1} - R_{t-1} \right] \frac{Q_{t-1} S^f_{t-1}}{N_{t-1}} + \left[R^b_{t-1} - R_{t-1} \right] \frac{q_{t-1} S^{Ext}_{t-1}}{N_{t-1}} + R_t \right] N_{t-1} + w \left(Q_t S^f_{t-1} + q_{t-1} \Delta S^{Ext}_{t-1} \right). \tag{3.32}$$

3.3.3 Non-financial Intermediate Producers Firms.

The production (Y_{mt}) in this sector is given by

$$Y_{mt} = A_t \left(U_t \xi K_t \right)^{\alpha} L_t^{1-\alpha} \tag{3.33}$$

The income for the firms is the value of its product ($P_{mt} Y_{mt}$) plus the income coming from the reselling the undepreciated capital. The costs are: the wage bill ($W_t L_t$), the return on the capital acquired in the previous period and paid in this $\left(R^k_t \right) Q_{t-1} K_t$ and assuming that cost of replacement of worn out capital is unit, the profits problem for the firm in this period is to choose (U_t) and (L_t) to maximize

$$P_{mt} Y_{mt} + \left[Q_t - \delta_t \right] \xi_t K_t - R^k_t Q_{t-1} K_t - W_t L_t \tag{3.34}$$

subject to eq.(3.33).

The labour demand is:

$$(1-\alpha) P_{mt} \frac{Y_{mt}}{L_t} = W_t, \tag{3.35}$$

the optimal utilization rate is:

$$\alpha P_{mt} \frac{Y_{mt}}{U_t} = b U_t^{\zeta} \xi_t K_t, \tag{3.36}$$

after assuming the depreciation function:

$$\delta_t = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta} \tag{3.37}$$

Return to Capital.

The return to capital is the remaining of the profits after paying the wage bill and the other costs of production. Substituting the optimal condition (3.35) in the profits equation (3.34) total profits should be zero as long as the firms pays all the return to capital to the banks

$$P_{mt} Y_{mt} + Q_t (1-\delta_t) \xi_t K_t - R_t^k Q_{t-1} K_t - (1-\alpha) P_{mt} Y_{mt} = 0$$

simplifying and solving for the return to capital

$$R_t^k = \left[\alpha P_{mt} \frac{Y_{mt}}{K_t} + Q_t (1-\delta_t) \xi_t \right] \frac{1}{Q_{t-1}} \tag{3.38}$$

where the value of the marginal productivity of capital is $\left[\alpha P_{mt} \frac{Y_{mt}}{K_t} \right]$.

3.3.4 Capital Producers.

Competitive capital producers purchase the depreciated capital to the intermediate producers at the end of the period (t). The capital is repaired and sold together with the new capital. The cost of repairing worn out capital is unity. The value of selling one unit of new capital is (Q_t). As in Gertler and Karadi (2011), investment adjustment cost are associated with the net investment (I_{nt}):

$$I_{nt} = I_t - \delta_t \xi_t K_t \tag{3.39}$$

where (I_t) is the total investment.

Each period the firm maximizes

$$\max E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left[(Q_\tau - 1) I_{n\tau} - \frac{\phi_i}{2} \left(\frac{I_{n\tau} - I_{n\tau-1}}{I_{n\tau-1} + I_{ss}} \right)^2 (I_{n\tau} + I_{ss}) \right]. \tag{3.40}$$

Because these firms are owned by the members of both households, the relevant discount factor is the same as for the banks

$$E_t \Lambda_{t,\tau} = E_t \left[\Upsilon_{Ext}^{ownership} \left(\beta^{Ext}\right)^{1+i} \frac{U_{ct+i}^{Ex}}{U_{ct}^{Ex}} + \left(1 - \Upsilon_{Ext}^{ownership}\right) \beta^{1+i} \frac{U_{ct+i}^{I}}{U_{ct}^{I}} \right]$$

The investment adjustment costs, associated with the net flow of investment, are

$$\frac{\phi_i}{2} \left(\frac{I_{n\tau} - I_{n\tau-1}}{I_{n\tau-1} + I_{ss}} \right)^2 (I_{n\tau} + I_{ss})$$

where (ϕ_i) is the inverse of the elasticity of net investment to the price of capital. Each of the firms in this sector chooses the same level of net investment. So, it is not necessary to index investment by firm. From this maximization problem the optimal price of capital

$$\begin{aligned} Q_t = 1 &+ \frac{\phi_i}{2} \left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}} \right)^2 \\ &+ \phi_i \left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}} \right) \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} \right) \\ &- E_t \beta \phi_i \Lambda_{t,t+1} \left(\frac{I_{nt+1} - I_{nt}}{I_{nt} + I_{ss}} \right) \left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}} \right)^2 . \end{aligned} \tag{3.41}$$

3.3.5 Retailers.

Final output is a composite of a continuum of differentiated retail goods. The only input of production is the intermediate good. Retailers purchase inputs from the intermediate producers and re-package it. Final product is aggregated according to

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.42}$$

(Y_{ft}) is the output purchased to the retailer (f). (ε) is the elasticity of substitution across varieties.

Optimal Demand for Retailers.

As shown in the appendix, from cost minimization, those purchasing the final good have an optimal demand for each variety equal to

$$Y_{ft} = \left[\frac{P_{ft}}{P_t} \right]^{-\varepsilon} Y_t \tag{3.43}$$

which implies the optimal price index

$$P_t = \left[\int_0^1 (P_{ft})^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}} . \tag{3.44}$$

Profit Maximization.

The only cost of production for the retailer is the price of the intermediate good, then this cost is given by (P_{mt}) because it takes only one unit of intermediate good to produce one unit of the retail good. Each period firms can adjust their price with probability $(1-\gamma)$. For the periods in which the firm is not able to set price it index it to the lagged rate of inflation.

In contrast to Gertler and Karadi (2011) I assume that this economy can be subject to a cost-push shock. In particular, the government imposes a distortionary tax on sales. Following Chen et al. (2014) shocks to this tax, evolve according to

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right)\ln(1-\tau) - \varepsilon_t^\mu \tag{3.45}$$

(ε_t^μ) is i.i.d. with mean zero and variance (σ_t^2).

The firm's problem in this sector is to choose the optimal price (P_t^*) to maximize its discounted expected profits:

$$\max E_t \sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i} \left[(1-\tau_t)\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} [\pi_{t+k-1}]^{\gamma^p} - P_{mt+i}\right] Y_{ft+i} \tag{3.46}$$

subject to

$$Y_{ft+i} = \left[\frac{P_t^*}{P_{t+i}}\right]^{-\varepsilon} Y_{t+i} \tag{3.47}$$

where π_t is the rate of inflation from $(t-i)$ to (t). And (γ^p) is a parameter with values $[0,1]$ and which measures the inflation indexation. The discount factor of those profits is the weighted by the size of each group of households.

The first order condition is

$$E_t \sum_{i=0}^{\infty} \gamma^i \Lambda_{t,t+i} \left[(1-\tau_t)\frac{P_t^*}{P_{t+i}} \prod_{k=1}^{i} [\pi_{t+k-1}]^{\gamma^p} - \frac{\varepsilon}{\varepsilon-1} P_{mt+i}\right] Y_{ft+i} = 0. \tag{3.48}$$

As shown in the appendix, the optimal price, implied by the solution to the previous problem is:

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1}\frac{F_t}{Z_t} \tag{3.49}$$

with

$$F_t = P_{mt} Y_t + E_t \gamma \Lambda_{t,t+1} \pi_t^{-\gamma^\rho \varepsilon} \pi_{t+1}^{\varepsilon} F_{t+1} \tag{3.50}$$

and

$$Z_t = (1-\tau_t) Y_t + E_t \gamma \Lambda_{t,t+1} \pi_t^{\gamma^\rho(1-\varepsilon)} \pi_{t+1}^{-(1-\varepsilon)} F_{t+1}. \tag{3.51}$$

Evolution of the price index.

Every period there is a share $(1-\gamma)$ of producers adjusting price optimally. The remaining (γ) simply index their price to the previous period inflation. Using the optimal price index, the evolution of the price index can be written as

$$P_t^{1-\varepsilon} = \left[(1-\gamma) (P_t^*)^{1-\varepsilon} + \gamma \left(\pi_{t-1}^{\gamma^\rho} P_{t-1} \right)^{1-\varepsilon} \right]. \tag{3.52}$$

Price Dispersion.

As shown in the appendix, price dispersion is equal to

$$\Delta_t = \int_0^1 \left[\frac{P_{ft}}{P_t} \right]^{-\varepsilon} df. \tag{3.53}$$

Using the law of movement of the price index and the definition of price dispersion this measure evolves according to:

$$\Delta_t = (1-\gamma) \left[\frac{1-\gamma \left(\pi_{t-1}^{\gamma^\rho} \pi_t^{-1} \right)^{1-\varepsilon}}{1-\gamma} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma \left[\pi_{t-1}^{\gamma^p} \pi_t^{-1} \right]^{-\varepsilon} \Delta_{t-1}. \tag{3.54}$$

3.3.6 Government Budget Constraint.

The government spending, which evolves exogenously, (G_t) and the payments on the debt acquired previously $\left(R_{t-1} B_{t-1}^g\right)$ are financed with a tax on sales $(\tau_t Y_t)$, issue of new government bonds (B_t^g) and using lump-sum taxation (T_t). The government's budget constraint is

$$T_t = G_t + R_{t-1} B_{t-1}^g - B_t^g - \tau_t Y_t \tag{3.55}$$

The initial level of debt $\left(B_{t-1}^g\right)$ is zero. The lump-sum tax ensures that the debt of the government is stabilized over time and that its budget constraint is balanced, then

$$T_t = G_t - \tau_t Y_t \tag{3.56}$$

where the government consumption (G_t) evolves exogenously

$$G_t = \frac{G}{Y} g_t.$$

with (g_t) equal to

$$\ln(g_t) = \rho_g \ln(g_{t-1}) + \varepsilon_{gt} \tag{3.57}$$

where the (ε_{gt}) is an i.i.d. process with mean zero and variance $\left(\sigma_g^2\right)$. The evolution of the distortionary taxation follows

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right)\ln(1-\tau) - \varepsilon_t^\mu.$$

3.4 Aggregation.

3.4.1 Labour Market.

Total labour demand is

$$L_t = \Upsilon L_t^{Ex} + (1-\Upsilon) L_t^I \tag{3.58}$$

Using the market wage in the per capita supply of labour of the households with external habits

$$L_t^{Ex} = \left[(1-\alpha)\frac{U_{ct}^{Ex}}{\chi^{Ex}}\frac{P_{mt}Y_{mt}}{L_t}\right]^{\frac{1}{\varphi}}. \tag{3.59}$$

And using the market wage in the per capita supply of labour of the households with internal habits

$$L_t^I = \left[(1-\alpha)\frac{U_{ct}^I}{\chi^{Int}}\frac{P_{mt}Y_{mt}}{L_t}\right]^{\frac{1}{\varphi}}. \tag{3.60}$$

Using equation 3.59 and 3.60 in 3.58 and solving for aggregate labour (L_t),

$$L_t = \left\{\left[(\Upsilon)\left(\frac{U_{ct}^{Ex}}{\chi^{Ex}}\right)^{\frac{1}{\varphi}} + (1-\Upsilon)\left(\frac{U_{ct}}{\chi}\right)^{\frac{1}{\varphi}}\right]\left[(1-\alpha)P_{mt}A_t\left(U_t\xi_t K_t\right)^\alpha\right]^{\frac{1}{\varphi}}\right\}^{\frac{\varphi}{\varphi+\alpha}}. \tag{3.61}$$

3.4.2 Financial Markets.

The firms equate issue one unit of security for each unit of capital. This capital can be used in production after it is affected by a exogenous shock (ξ_t). Then, total amount of loans issued to the firms is

$$Q_t S_t^f = Q_t \xi_t K_{t+1}, \tag{3.62}$$

the total amount of loans issued to the households is

$$q_t S_t^{Ext} = q_t \Upsilon B_t^{Ex}, \tag{3.63}$$

where (Υ) is the number of household with external habits. The total deposits in the economy are

$$\left[D_t = (1-\Upsilon)\, D_t^{Int}\right].$$

3.4.3 Aggregate Resource Constraint.

Consumption, government spending, total investment and the costs associated to the change in investment adjustment are the demand faced by the final producers. Then, the aggregate resource constraint is

$$Y_t = \Upsilon C_t^{Ex} + [1-\Upsilon]\, C_t^I + G_t + I_t + \frac{\phi_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 (I_{nt} + I_{ss})\,. \tag{3.64}$$

3.4.4 Law of movement of Capital.

From the law of movement of capital

$$K_{t+1} = (1-\delta_t)\,\xi_t K_t + I_t \tag{3.65}$$

and the definition of net investment

$$I_{nt} = I_t - \delta_t \xi_t K_t \tag{3.66}$$

the capital evolves according to

$$K_{t+1} = \xi_t K_t + I_{nt}. \tag{3.67}$$

3.4.5 Deposits.

The aggregate deposits in the economy are

$$D_t = [1-\Upsilon]\, D_t^I. \tag{3.68}$$

For the households with external habit, demand for borrowing is

$$\Upsilon B_{t-1}^{Ex} + E_t \sum_{i=0}^{\infty} \prod_{m=0}^{i} \left[\frac{1}{R_{t+m}^b}\right] \left[\Pi_{t+i}^{Ex} + W_{t+i} \Upsilon L_{t+i}^{Ex} - T_{t+i}^{Ex}\right] = E_t \sum_{i=0}^{\infty} \prod_{m=0}^{i} \left[\frac{1}{R_{t+m}^b}\right] \left[\Upsilon C_{t+i}^{Ex}\right]. \tag{3.69}$$

Aggregate transfers from the government are

$$T_t = G_t - \tau_t Y_t,$$

the households with external habits receive $\left[T_t^{Ex} = \left(\Upsilon_{Ext}^{transfer}\right) T_t\right]$, and the households with internal habits receive the remaining of the transfers $\left(T_t^I = \left(1 - \Upsilon^{transfer}\right) T_t\right)$, where $\left(\Upsilon_{Ext}^{transfers}\right)$ can be different from the population size.

Similarly, profits received by the households with external habits are $\left[\Pi_t^{Ext} = \left(\Upsilon_{Ext}^{ownership}\right) \Pi_t\right]$ and households with internal habits receive $\Pi_t^{Int} = \left(1 - \Upsilon_{Ext}^{ownership}\right) \Pi_t$, where Π_t are the total profits and $\left(\Upsilon_{Ext}^{ownership}\right)$ is the participation of the households with external habits in the firms.

3.5 Macro-Prudential Policy.

IMF (2013) defines systemic risk as the tendency of the financial sector to amplify adverse aggregate shock. This is the result of macro-financial feedback mechanisms that result in an over-exposure to such adverse aggregate shocks. In this model with endogenously constrained banking sector, the maximizing behavior of individual banks leads the aggregate economy to have an overexposure to positive and negative shocks. In particular, the leverage ratio, which is the result of optimization of individual financial intermediaries, serves as a multiplier of any shock hitting the net wealth of the sector.

The role of macro-prudential policy is to detain those feedback processes that drive the economy towards a vulnerable position. By reducing the procyclical feedback between asset prices and credit the macro-prudential policy can detain the volatility of the business cycle.

I follow the work of Claessens et al. (2013, 2014) in defining the reserve requirement (B_t^{rev}) as an instrument of macro-prudential policy. Kashyap and Stein (2012) suggest the use of this instrument as a Pigouvian tax in order to make financial institutions to internalize the effects of their activity on society[6]. This macro-prudential instrument can affect the effect of shocks ex ante, by making the banks to optimally choose a smaller leverage ratio. And can also affect the economy once the shock hits. In addition, the macro-prudential instrument serves as a tax on consumption making the agents to internalize the effects of their consumption externality on borrowing. The Macro-prudential regulator can change every period its macro-prudential instrument to make the economy converge to the regulator's objectives.

3.5.1 Reserve Requirements (RRs).

In the model, the bank has access to three assets. Loans to non-financial firms $(\mathrm{Q}_t \mathrm{S}_t^f)$, the loans to households $\left(q_t S_t^{Ext}\right)$ and Reserve Requirements (B_t^{rv}). To fund those assets the bank has two

[6] The definition of reserve requirements here matches the definition in IMF (2012 a) : Central banks can use variations in the level of RRs to affect broader credit conditions. When RRs are remunerated below the policy rate or are unremunerated, a variation in the level of the requirement imposes a tax on lending.

sources of funds: Deposits (D_t) and accumulated wealth (N_t). The bank can optimally choose the assets it would like to intermediate each period. This problem is subject to the incentive constraint imposed by households, to the cost of funding the deposits and to the reserve requirement imposed by the regulator.

Its balance sheet is:

$$Q_t S_t^f + q_t S_t^{Ext} + B_t^{rv} = N_t + D_t \tag{3.70}$$

These reserve requirements are assets which the bank holds but which cannot lend. They are held at the vaults of the central bank. These assets are short-term assets which can be used to face any adverse shock which decreases the ability of banks to raise deposits. Banks in the economy are highly vulnerable to financial shocks because they are specialist in transforming short-term liabilities into long-term assets. Then, there is a mismatch between the term structure of their lending and their borrowing. The reserves requirements aim to reduce this mismatch. The central bank has full control of this instrument.

In particular, the central bank can set a reserve requirement equal to a fraction of the deposits issue by the commercial bank

$$B_t^{rv} = [1 - \alpha_t^{res}]\, D_t \tag{3.71}$$

This restriction says that after the commercial bank have collected deposits D_t, the central bank force them to hold the fraction $(1 - \alpha_t)$ as reserves. These funds cannot be intermediated. If the total assets of the banks are $Q_t S_t^f + q_t S_t^{Ext}$, then, every period the banks can intermediate resources equal to

$$Q_t S_t^f + q_t S_t^{Ext} = N_t + \alpha_t^{res} D_t \tag{3.72}$$

where $0 \leq \alpha^{res} < 1$.

Over time, the net wealth of the bank evolves as the difference between the return on its assets and the cost of funding them, this includes the taxing and subsidizing

$$N_t = R_{t-1}^k Q_{t-1} S_{jt-1}^f + R_{t-1}^b q_{t-1} S_{jt-1}^{Ext} - R_{t-1} D_{t-1}. \tag{3.73}$$

Even when the commercial bank intermediates only $\alpha_t^{res} D_t$ of its new deposits, it has to pay the cost of raising the full amount of the deposits. Here, I try to capture the idea that the reserve requirement can act as tax on financial intermediation (IMF (2012 a)).

Solving eq. (3.72) for deposits and using eq. (3.73)

$$N_t = \left(R_{t-1}^k - \frac{R_{t-1}}{\alpha_{t-1}^{res}} \right) Q_{t-1} S_{t-1}^f + \left(R_{t-1}^b - \frac{R_{t-1}}{\alpha_{t-1}^{res}} \right) q_{t-1} S_{t-1}^{Ext} + \frac{R_{t-1}}{\alpha_{t-1}^{res}} N_{t-1}, \tag{3.74}$$

This provide incentives for the bank to accumulate more wealth.

The reserve ratio $(1-\alpha_t^{res})$ affects the accumulation of wealth. When the reserve ratio increases (α_t^{res}, smaller) each unit of accumulated wealth is more valuable $\left(\frac{R_t}{\alpha_t^{res}} N_t\right)$. The external cost of bank's funds has increased and the bank values more each additional unit of accumulated equity. At the same time, the increase in the reserve ratio decreases the premium over the risk free rate associated with each type of loan (S_{t-1}^f and S_{jt-1}^{Ext}). This can alter the bank's incentives to issue loans to the external consumers.

By affecting the profitability of assets, the central bank directly affects the commercial banks' incentives to expand their liabilities. With a positive value of reserves, the central bank incentives the financial intermediaries to accumulate more equity and to borrow less.

3.5.2 Maximization with Reserve Ratio.

The bank maximize its terminal wealth. At the end of period (t), a surviving bank has a probability of dying tomorrow equal to $(1-\theta)$. Then, at the end of period t, the bank maximizes its expected discounted terminal wealth according to

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\, \theta^i \Lambda_{t,t+1+i} \left(N_{jt+1+i}\right) \tag{3.75}$$

which takes into account the evolution of terminal wealth (3.74). This problem is subject to the incentive constraint imposed by households

$$V_{jt} \geq \lambda \left[Q_t S_t^f + \Delta q_t S_t^{Ext} \right]. \tag{3.76}$$

The maximization problem delivers the maximum amount of assets the bank can intermediate

$$Q_t S_t^f + q_t S_t^{Ext} = \frac{\eta_t}{\lambda - v_t^f} N_t, \tag{3.77}$$

where

$$\eta_t = E_t \Lambda_{t,t+1} \Omega_{t+1} \frac{R_t}{\alpha_t^{res}}, \tag{3.78}$$

$$v_t^f = E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^k - \frac{R_t}{\alpha_{t-1}^{res}} \right),$$

$$v_t^{Ext} = E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^b - \frac{R_t}{\alpha_t^{res}} \right), \tag{3.79}$$

with

$$\nu_t^{Ext} = \Delta \nu_t^f,$$

the bank's augmented discounts factor is

$$\Omega_{t+1} = (1-\theta) + \theta \left(\nu_{t+1}^{f} \phi_{t+1} + \eta_{t+1} \right),$$

and

$$\phi_t = \frac{\eta_t}{\lambda - v_t^f}$$

is the maximum leverage ratio chosen by firms, taking into account the macro-prudential policy $\left(\frac{1}{\alpha_t^{res}}\right)$ and the subsidies and taxes $\left(\tau_t^b, \tau_t^s\right)$. In the next analysis, the tax and subsidies would not be used out of equilibrium. If used, they would be used only to affect the steady state of this economy.

The macro-prudential authority can affect the optimal leverage ratio (3.77) by affecting its components $\left(\eta_t \text{ and } v_t^f\right)$, which is behind the multiplier effect of financial shocks over the real activity.

There is a trade-off when the central bank optimally selects this ratio. On one hand, a high value of reserves ratio decreases the leverage ratio. Total assets and total capital in the economy are a positive function of this leverage ratio. This decreases the overall value of the real activity in equilibrium. On the other hand, a smaller leverage ratio has the effect of a smaller multiplier effect of negative shocks.

This leverage ratio is increasing in the marginal value of and additional unit of bank's capital (η_t). An increase in the reserve ratio decreases α_t^{res}, the share of assets that banks can intermediate from the deposits, this increases the term $\left(\frac{1}{\alpha_t^{res}}\right)$ and has the effect of increasing the marginal value of accumulated capital in the bank (η_t), it is more profitable for banks to accumulate net wealth. The opportunity cost of borrowing from households has increased.

This has the effect of increasing (ϕ_t). Implicitly, households tolerate a higher leverage because the bank is using more of its own capital to make loans. This serves to discipline its activities. If the banks put more of their own resources at risk, they will be more careful in selecting the preferred investment options. This reserve ratio has the effect of forcing the bank to internalize the effect of its risk-taking behavior.

On the other hand, the increase in the reserve ratio, which decreases α_t^{res} and increases $\left(\frac{1}{\alpha_t^{res}}\right)$, has the effect of decreasing $\left(v_t^f\right)$, the marginal return on assets intermediated by the bank. This has the effect of decreasing the leverage ratio. Then, the choice of the optimal leverage ratio has two opposite effects. It can increase the leverage ratio by making the banks to increase the return on their own capital, but at the same time it reduces the leverage ratio due to the reduction on the return on the banks' assets.

In the next section, I discuss the strategy to calibrate the model.

3.6 Calibration.

I calibrate the model to a quarterly frequency. In the benchmark calibration, table 1, I set the parameters to hit the observed leverage in the U.S. financial sector.

In order to calibrate the parameters associated with the heterogeneous households, I present information from the Panel Study of Income Dynamics (PSID), and the Survey of Consumer Finances (SCF). The next step is to explain the strategy behind the calibration of the parameters associated with the heterogeneity of households. After this, I present the calibration of the parameters associated with the nonfinancial sector, the shocks, and the government. These parameters can be directly obtained from the historical averages of their empirical counterparts or from estimated studies of the U.S. economy (for example, Primiceri et al. (2006) and Taheri (2014)). The final section contains the calibration of the parameters associated with the financial sector. These parameters can be calibrated by targeting a set of observed ratios of macroeconomic variables.

The column (1) in table 1 shows the values of the parameters in the benchmark calibration. The column (2) shows the piece of evidence used to calibrate the model.

3.6.1 Poor and Rich Households.

Distribution of Net Wealth.

In this subsection, I present the evidence behind the existence of wealth-rich and wealth-poor households in the U.S. Krueger et al. (2016) analyze the effects of the distribution of wealth on consumption before and after the Great Recession. Table 2 presents information on the households' disposable income, consumption, age, and education for each quintile of wealth distribution.

The poorest households (the bottom 60 percent of the wealth distribution) hold less than 5 percent of net wealth[7]; they receive around 37 percent of the aggregate disposable income; they explain 40 percent of aggregate consumption; the poorest group is also the youngest and less educated group[8]. Hence, this group is relevant to capture the dynamics of the aggregate variables in the economy.

Savings, Leverage Ratio, Income, and Consumption.

In the U.S., the saving rate (savings to disposable income) has decreased since 1960. The top panel in figure 1 shows the saving rate between 1988 and 2015, as reported by the U.S. Bureau

[7] According to the information from the Panel Study of Income Dynamics (PSID), the poorest households hold less than 5% of net wealth for at least each survey's year in 1999-2013. Their demographic conditions do not change considerably in that period.

[8] Krueger et al. (2016) show that this group is able to smooth consumption. Hence, they cannot be associated with the Rule-of-Thumb (RoT) households (Campbell and Mankiw (1989), Galí et al. (2007)). RoT households do not smooth consumption. Hence, I use the characteristics of the wealth-poor households to calibrate the borrowers in the model.

Table 3.1: Benchmark Calibration

Model			Data	
Parameter		Value	Variable	Source of Data
		(1)	(2)	
Heterogeneity of Agents				
Υ	Population ext.	0.60	Wealth-poor (60%.)	Krueger et al. (2016)
Υ_{Ext}^{own}	Ownership ext.	0.05	Share of wealth (5%)	Krueger et al. (2016)
Υ_{Ext}^{trans}	Transfers to ext.	0.1449	Share of income (9.3%)	Broer et al. (2016)
χ^{Ex}	Dis. of labor ext.	15.7063	Working hours (1/3)	Assumption
χ^{Int}	Dis. of labor int.	2.6649	Working hours (1/3)	Assumption
h^{Ext}	Habits ext.	0.6401	Credit to consumers (61.3%)	FED (2016), Av. 2000-15
h^{Int}	Habits internal	0.8389	Consumption $\left(\frac{\text{borrower}}{Saver}=68\%\right)$	Krueger et al. (2016)
β^{Ext}	Disc. factor ext.	0.9911	Bank's lending rate (3.62%)	FED: Average 2006-15
Consumers				
$\frac{1}{\sigma}$	Inter. elast. of subs.	1	Conventional	Assumption
β	Discount factor savers	0.9951	Real rate (2.0%)	FED: Average 1980-2008
Financial Sector				
θ	Prob. of survival	0.9716	Life (35 quarters)	Gertler and Karadi (2011)
λ	Diverting loans (firms)	0.3785	Lending spread (100 bp.)	FED: Average 2005-15
$\Delta\lambda$	Diverting loans (consumption)	0.6043	Lending spread (162 bp.)	FED: Average 2006-16
ω	Transfer starting banks	0.0035	Leverage (4)	FED: Average 2000-07
Non-Financial Firms				
α	Capital share	0.33	Historical average	BEA: Average 1980-2010
φ	Inv. Frisch elasticity	0.27	Estimated	Primiceri et al. (2006)
ϕ_i	Elasticity inv. adj. costs	1.72	Estimated	Primiceri et al. (2006)
ζ	Elasticity of marginal dep.	7.2	Estimated	Primiceri et al. (2006)
δ	Depreciation rate	0.025	Historical average	BEA: Average 1980-2010
γ	Firms no adjusting price	0.77	Estimated	Primiceri et al. (2006)
γ^p	Price indexation	0.241	Estimated	Primiceri et al. (2006)
ε	Elasticity of substitution	6.0	Conventional	Assumption
Shocks				
ρ_a	Persistence technology	0.96	Estimated	Taheri (2014)
ρ_ξ	Persistence quality	0.89	Estimated	Taheri (2014)
ρ_μ	Persistence cost-push	0.854	Estimated	Primiceri et al. (2006)
σ_a	St. dev. productivity	0.0068	Estimated	Taheri (2014)
σ_ξ	St. dev. quality	0.0037	Estimated	Taheri (2014)
σ_μ	St. dev. cost-push	0.00103	Estimated	Primiceri et al. (2006)

Table 3.2: Socioeconomic Characteristics of U.S. Population

Table 2.					
Net Wealth		Disp. Income	Consumption	Head's (Years)	
Quintile	% held			Age	Education
Q1	-0.9	8.7	11.3	39.2	12
Q2	0.8	11.2	12.4	40.3	12
Q3	4.4	16.7	16.8	42.3	12.4
Q4	13.0	22.1	22.4	46.2	12.7
Q5	82.7	41.2	37.2	48.8	13.9
Bottom 60%	4.3	36.6	40.5		
Top 40%	95.7	63.3	59.6		
Relative Poor/Rich		0.578	0.679		

Source: Elaborated with Table 1 and 2 in Krueger et al. (2016)

of Economic Analysis (BEA). The richest group has had a higher saving rate than the national average, and much higher than the poorest, since 1988 (middle panel).

The contribution to this ratio has been different across poor and rich households[9]. In particular, between 1988 and 2012, the U.S. saving rate decreased 0.2 percentage points (top panel, figure 1); the richest contributed with 0.8 percentage points to the change in this rate (bottom panel), while the poorest explained a negative contribution of 1 percentage point to that change.

Before the Great Recession, between 2000 and 2006, the richest increased their saving ratio by 2.2 percentage points (middle panel), from 7.3 to 9.5 percent, while the poorest decreased it in 1.1 percentage point.

[9] I take information from the Survey of Consumer Finances (SCF) in order to calculate the contribution to this rate by the bottom 60, and the top 40 percent of the net wealth distribution.

Figure 3.1: Savings to Disposable Income

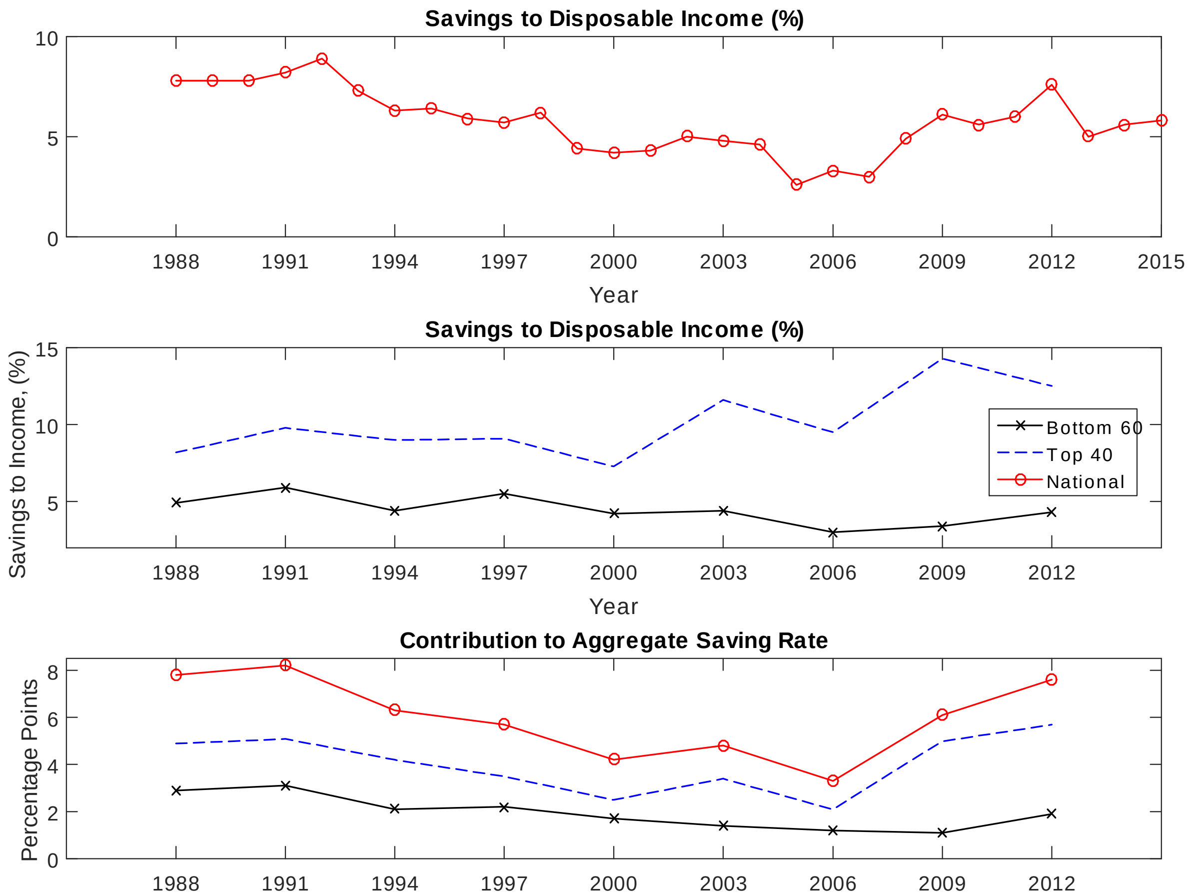

Figure 1. Savings to Disposable Income. Top Panel: National (Red Circles). Middle Panel: Savings to Disposable Income per Group. Bottom 60 (Black Stars) and Top 40 (Dash). Bottom Panel: Contribution to The Saving Rate. The National Measures Takes Information From BEA, NIPA Tables. The Remaining Information is From the SCF.

At the beginning of 2000, the debt of the poor households represented 110 percent of their income (figure 2, middle panel), before the start of the Great Recession in 2003-2006, this ratio had increased by 17 percentage points. This group was decreasing its saving ratio, and, at the same time, increasing its debt before the crisis. In contrast, the richest decreased their leverage ratios in 2003-2006, and contributed to the increase in the saving ratio.

If the leverage ratio is measured respect to the value of the assets, the poorest increased this ratio between 2000 and 2006 in 70 percentage points[10]. This may suggest that these households

[10]The S&P/Case-Shiller 20-City Composite Home Price Index in 2006 was 92.6 percent above the 2000 level, and the Dow Jones Industrial Average index had increased 15.5 percent over the same period. Hence, the increase in the leverage ratio respect to assets cannot be related to the decrease in the value of these assets.

were not taking on debt in order to expand their assets; potentially, they may have increased debt in order to finance consumption.

Figure 3.2: Leverage Ratio

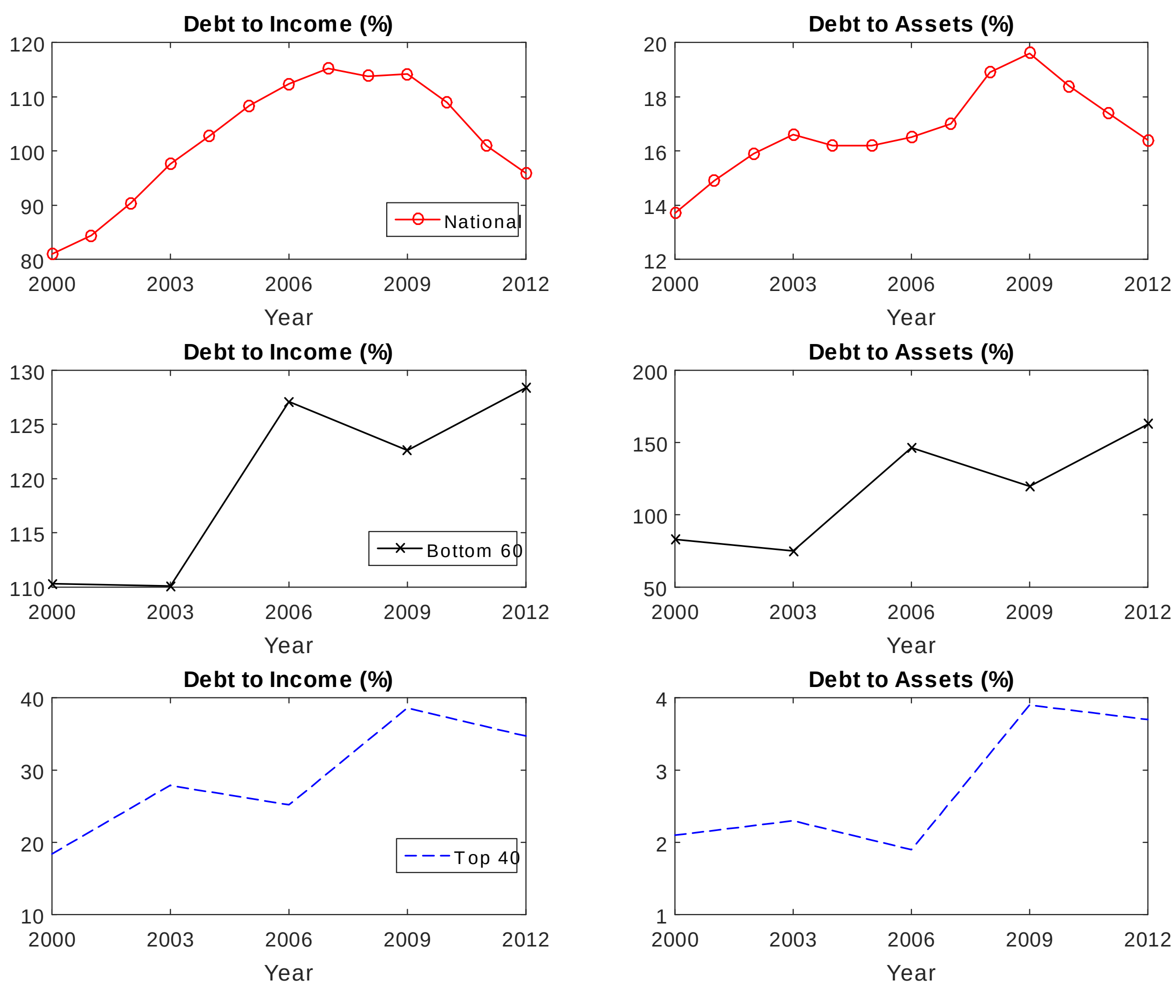

Figure 2. Leverage Ratio. Ratio of Total Debt to Total Income and Ratio of Total Debt to Total Assets. Bottom 60 (Black Stars) and Top 40 (Dash). The National Measures Take Information From the Federal Reserve Z.1 Financial Accounts and BEA, NIPA Tables. The Remaining Information is From the SCF.

Indeed, Mian and Sufi (2010, 2011), using information on homeowners credit files, show that between 2002 and 2007, the household sector doubled its debt. They find no evidence that home equity-based borrowing is used to pay down credit car balances. They find little evidence that borrowing in response to increased house prices is used to purchase new homes or investment properties. This is suggestive that households were increasing their leverage ratio mainly to increase consumption, and not to increase their assets. These characteristics were stronger for younger homeowners. In the evidence presented in table 2, the youngest population are the poorest. Mian and Sufi (2011), suggest that the characteristics of the homeowners who borrowed aggressively against the rising values of their houses, may proxy for individuals with self-control problems, as in the work of Laibson (1997).

Figure 3.3: Aggregate Consumption vs. Aggregate Disposable Income

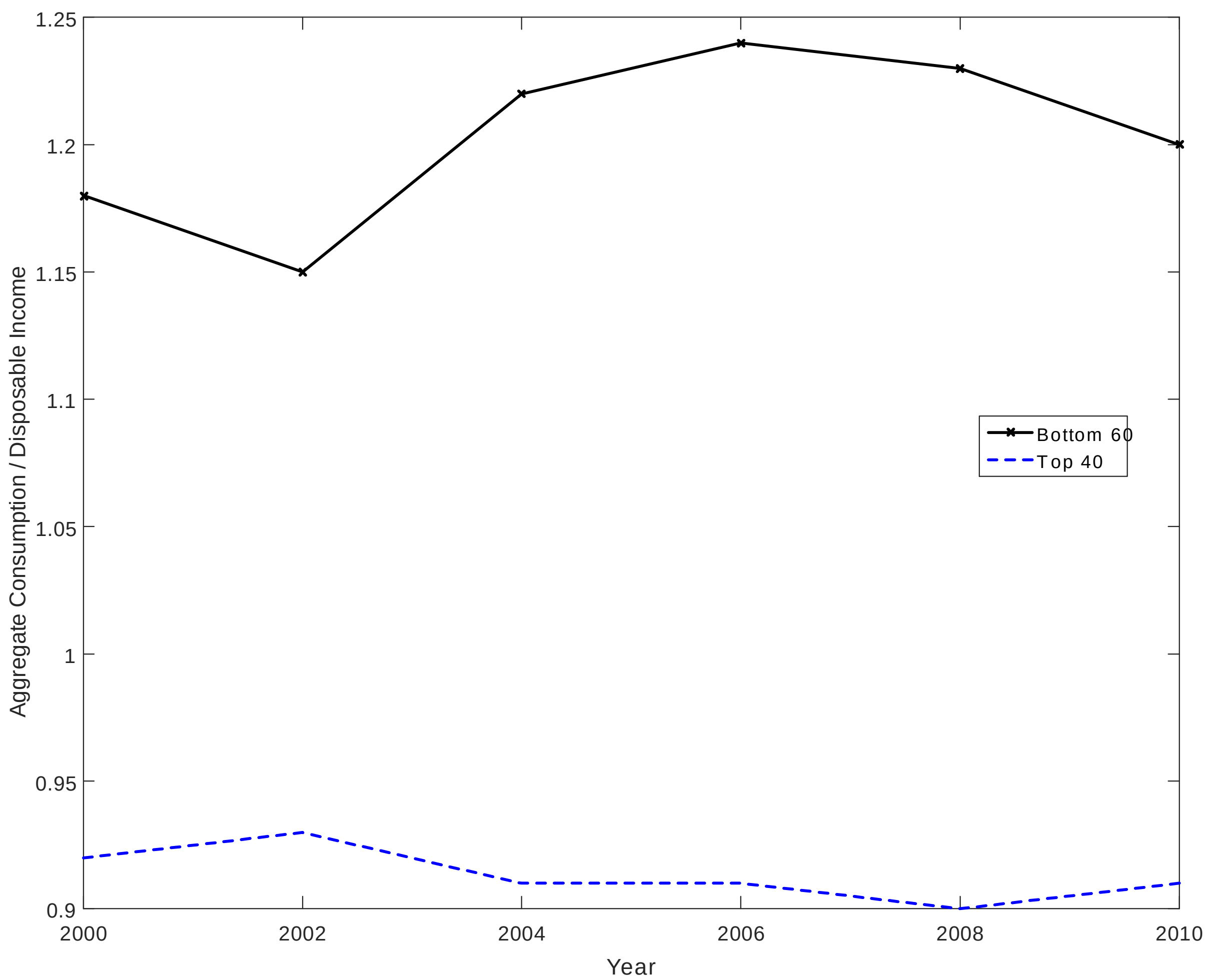

Figure 3. Contribution to Aggregate Consumption Relative to The Contribution to Disposable Income. With information From Panel Study of Income Dynamics (PSID).

Figure 3 shows the relative contribution of the poorest group to consumption as a ratio of their relative contribution to disposable income. Before the Great Recession, the wealth-poor households were contributing more to consumption than to disposable income, and this ratio peaked in 2006; this coincides with the peak of the leverage ratio of these households (both respect to income and to assets). This may put these households in a vulnerable position at the onset of the crisis.

They were consuming too much and hiring too much debt, but their income and assets were not increasing at the same speed.

In conclusion, the poorest households have decreased their contribution to national savings over the last 25 years (figure 2). This reduction was more pronounced in the years leading to the crisis. They are a group that contributes more to aggregate consumption than to aggregate income (figure 3) and they have increasing levels of debt (figure 2). Their leverage ratios have increased considerably since 2000, showing the largest increases in 2003-2006. The leverage ratio of this group has increased even more if measured in terms of their assets rather than their income, which suggests that they did not hire debt to increase assets, rather they increased consumption.

This evidence may suggest that the poorest group took advantage of the of shift in monetary policy from 2003 to 2005 when the Federal Reserve held the interest rate lower than the two previous decades (Taylor (2014)). Potentially, this relaxation of policy could have exacerbated the consumption externality of this group, which is then reflected in their increasing leverage ratios and lowering saving rates. These households were highly vulnerable at the onset of the crisis. Macro-prudential policy could have remedied this.

In this paper, the overconsumption of the wealth-poor households is captured by allowing them to show external habits in consumption. This externality makes these households to show a pattern of consumption, relative to the other consumers, who hold internal habits, similar to that observed in the data. And macro-prudential policy may play a role in fixing this overconsumption and overborrowing pattern.

3.6.2 Households Heterogeneity.

There are eight parameters associated with households' heterogeneity: the population of borrowers[11] (Υ); the borrowers' ownership of the businesses in the economy $\left(\Upsilon_{Ext}^{ownership}\right)$; the share of government transfers received by the borrowers $\left(\Upsilon_{Ext}^{transfers}\right)$; the disutility of labor of the borrowers $\left(\chi^{Ex}\right)$ and savers $\left(\chi^{Int}\right)$; the external $\left(h^{Ext}\right)$ and internal $\left(h^{Int}\right)$ habits; and the subjective discount factor of the impatient households $\left(\beta^{Ext}\right)$. Next, I present the evidence used to calibrate these parameters.

Column (1) in table 1 shows the values of the parameters in the benchmark calibration. Column (2) shows the piece of evidence used in the calibration.

The model's population of borrowers (Υ) is 60 percent. This is the share of wealth-poor agents in the U.S. economy. In the model, the wealth-poor households are the borrowers.

The wealth-poor households hold around 5% of the total net wealth (Krueger et al. (2016)). I capture this fact by assuming that the share of assets held by the borrowers $\left(\Upsilon_{Ext}^{ownership}\right)$ in the model is 5%. The savers own the remaining 95% of the net wealth.

In order to calibrate the transfers received by the households $\left(\Upsilon_{Ext}^{transfers}\right)$, I take data on the

[11] I use indistincly the terms borrowers, externals, or wealth-poor agents to refer to the same group.

Table 3.3: Wealth Percentile and Labor Income (SCF, 2004)

Percentile	0-5	5-20	20-40	40-60	60-80	80-95	95-100
Labor Income	92	83	91	89	89	81	55
Financial Income	1	1	2	5	6	14	41
Transfers	7	16	8	6	5	6	3

Source: Taken from Broer et al. (2016)

Table 3.4: Credit to Firms and Households

Financial sector (% of total loans)			Source
1.	Loans to firms % of total loans	38.7	FED (2016), Average: 00-15
2.	Loans to consumption % of total loans	61.3	FED (2016), Average: 00-15
Banking sector			
3.	Loans to firms % of total loans	57.7	FED, Average: 00-15
4.	Loans to consumption % of total loans	42.3	FED, Average: 00-15

relevance of such income respect to the total income of the poor households. Using the Survey of Consumer Finances (2004), Broer et al. (2016) shows the components of household income for different percentiles of the wealth distribution. This information is copied in Table 3. I calibrate the share of transfer so that the wealth-poor households derive on average 9.3% of their income from these transfers, as in the empirical evidence. The calibration indicates that poor households receive 14.49% of the total government transfers. Hence, $\Upsilon_{Ext}^{trans} = 0.1449$.

The disutility of labor of borrowers $\left(\chi^{Ext}\right)$ is 15.7063. And for the internal households $\left(\chi^{Int}\right)$ is 2.6649. These values were calibrated after assuming a steady-state value of working hours of 0.33, as usual in the literature.

The external habits $\left(h^{Ext}\right)$ takes a value of 0.6401. This value was calibrated to match the observed share of loans granted by the financial sector to households. On average, between 2000 and 2015, the ratio of loans granted by the financial sector to households was 61.3% (table 4, second row)[12]. The benchmark calibration matches this evidence.

In order to calibrate the internal habits $\left(h^{Int}\right)$, I match the relative consumption between poor

[12]The information on loans received by households from the financial sector is taken from the Z.1 Financial Accounts of the US, 2016. In the data download program of the FED, the identifier L.214 (A) Loans, n.s.a.

Given that in the model the financial sector does not intermediate the government's assets, I use private sector's data. The total value of loans is the sum of loans to households, and corporate and noncorporate business in the data.

and rich households. This ratio was 68 percent in 2006 (last row in Table 1). The benchmark calibration matches this evidence and the value for the internal habits is 0.8389. This value is in line with the estimated by Primiceri et al. (2006).

The last parameter associated with the heterogeneity of households is the borrowers discount factor $\left(\beta^{Ext}\right)$. I present the results in the financial section.

Next, I present the calibration of the non-financial parameter, which can be obtained from historical averages of their empirical counterparts.

3.6.3 Non-Financial Sector.

Based on Historical Averages.

The share of capital on national income, the subjective discount factor of the savers, the depreciation rate, and the relevance of the government spending $\left[\alpha,\ \beta,\ \delta,\ \frac{G}{Y}\right]$ match the historical averages of their empirical counterparts. As in Yun (1996), I assume that the firms can, without cost, index their prices to the steady-state rate of inflation.

The steady-state share of capital on national income (α) is set to 0.33. The compensation of employees as a share of national income ($1-\alpha$) averaged 65.1 percent in 1980-2010. This value for α approximates that evidence (BEA, NIPA table 1.12).

During 1990-2008, the average real rate of interest was 2 percent (FRED). The subjective discount factor of the savers ($\beta = 0.995062$) implies an annual real interest of 2 percent, matching the evidence.

The depreciation rate (δ) takes the value 0.025 given the quarterly frequency of this calibration, implying a 10 percent annual depreciation. This value matches the historical average of consumption of fixed capital observed in the U.S. (BEA NIPA table 1.11).

And the steady state government spending to GDP ratio $\frac{G}{Y} = 0.2$, this is a conventional value for this variable and between 1980-2010 the average was 19.8 percent (BEA NIPA table 1.1.10).

The remaining parameters for the non-financial sector are taken from estimated studies. Next, I present the values for those parameters.

From Estimated Models.

Following Primiceri et al. (2006), I assume the next values for the parameters in the model's non-financial sector. The inverse of the Frisch elasticity (φ) is 0.276. The elasticity of marginal depreciation to the utilization rate (ζ) is 7.2. The inverse of the elasticity of net investment to the price of capital (ϕ_i) is 1.728.

The probability that a firm does not adjust its price this period ($\gamma = 0.779$) implies that a firm keeps its price for around 4 quarters. The size of the indexation of the price to the previous period inflation (γ^p) is 0.241.

The elasticity of substitution between varieties of goods (ε), is 6. In the estimated results by Taheri (2014) this elasticity takes a value close to 4.1, similar to that utilized by Gertler and Karadi (2011). In Villa (2014) it is assumed that the elasticity would imply a markup of 1.25 in steady state. In Primiceri et al. (2006) this elasticity is close to 5. And in the most of the literature using calibrated New Keynesian models this elasticity is between 8 and 11. When taking a value of $(\varepsilon = 6)$, I am in the middle of the range of values used for this parameter in models with and without financial frictions. Next, I proceed to explain the calibration of the shocks.

Shocks.

Following Taheri (2014), the persistence of the shock to productivity, and the shock to the quality of capital, take the values $\rho_A = 0.96$, and $\rho_\xi = 0.89$, respectively. The persistence of the cost-push shock is $\rho_\tau = 0.85$, as in Primiceri et al. (2006).

The standard deviation of the shock to productivity, and the shock to the quality of capital take the values $\sigma_a = 0.0068$, and $\sigma_\xi = 0.003$, respectively. Which are the values estimated by Taheri (2014). And I take the estimates of Primiceri et al. (2006) for cost-push shock, $\sigma_\mu = 0.00103$.

The next step is to present the calibration of the parameters associated with the heterogeneity of households.

3.6.4 Financial Sector.

In this section, I explain the calibration of the financial parameters. Five parameters are particular of the financial sector. The survival probability in the banking industry (θ), the share of diverting funds associated with the loans to firms (λ), the share of diverting funds associated with the loans to consumptions $(\Delta\lambda)$, the startup transfer to banks (w), and the discount factor of the impatient households $\left(\beta^{Ext}\right)$.

Survival Probability.

The survival probability of banks (θ) exists in the model to avoid that the banks increase their size up to the point where they do not need borrowing from households. In that case, the incentive constraint would not bind.

The value of the survival probability in the banking industry is set to imply an average life expectancy of banks equal to 35 quarters. Hence, $\theta = 0.9716$. This value is in line with the assumptions in related studies, for example Gertler and Karadi (2011).

In their estimation exercises Taheri (2014) and Villa (2014) assume that this parameter is fixed at that value before the estimation exercise.

Table 3.5: Leverage Ratio in the U.S.

Sector	Leverage	Source	Period
Financial sector.			
Financial sector. Benchmark.	4	FRED data	00-07
Financial sector.	3.75	OECD	00-13
Banking sector			
Commercial banks	10.4	FRED	00-07
Core Banking system.	8-13	Bhatia and Bayoumi (2012)	2007
Big-5 Investment banks	25-35	Bhatia and Bayoumi (2012)	2007
Corporate and non-corporate sector			
Non-financial corporate sector	2	FRED	00-07
Corporate and noncorporate sector	2	Gertler and Karadi (2011)	

Leverage Ratio.

The funds transferred to banks (w) take the value 0.0035, which implies a leverage ratio of 4. This value for the leverage ratio can be considered as low, but this is the leverage observed in a wide definition of the U.S. financial sector [13]. Next, I present the different sources that register the possible values of the leverage ratio for the U.S. economy.

Table 5 shows different sources and different leverage ratios in the U.S. During the past crisis the leverage ratio was not equal for all the institutions in the financial system. Institutions at the center of the crisis were highly leveraged. For example, during 2007, for the Big-5 investment banks that ratio was between 25 and 35 Bhatia and Bayoumi (2012). For the core banking system the ratio was between 8 and 13 Bhatia and Bayoumi (2012). For the corporate and non-corporate business the ratio was close to 2 Gertler and Karadi (2011).

Banking Sector and Corporate Sector. In the previous crisis the leverage ratio of the banks in troubles were considerably higher than the average for the banking sector or for the corporate business sector. For example, during 2007, for the Big-5 investment banks that ratio was between 25 and 35 (Bhatia and Bayoumi (2012)). However, for commercial banks the leverage ratio (total assets to equity in the banks) had an average of 10.4 between 2000-2007, before the crisis (FRED). And the non-financial corporate sector showed a ratio of assets to equity of 2 (FRED).

To capture the overall leverage ratio in the economy, given that QE1 was designed to support the Government Sponsored Enterprises (GSE) and banks, I take the weighted average of the leverage ratio of these institutions. This weighted average showed a ratio of 4.05 on average between 2000

[13] Financial corporations include both public and private firms in the financial business. This includes monetary institutions, depository corporations, financial auxiliaries and insurance companies and pension funds. This is a broader indicator than just commercial banks.

and 2007.

Financial Sector. As a robustness check I utilize data from OECD and FED. I utilize information on the U.S. financial corporations[14]. I construct the leverage indicator as the ratio of financial assets to shares and other equity. The average of this indicator between 2000 and 2013 was 3.75. And during 2007-2008, this average was 4.07.

Thus, the leverage ratio measured as a weighted average of commercial banks and non-financial corporations or measured for the overall financial corporations has observed a value of about 4. This is the value used in the benchmark calibration.

Spreads and Share of Diverting Funds.

The spreads in steady state serve to pin down the moral-hazard problem associated with each type of loan. The share of diverting funds associated with loans to firms (λ) is 0.3785, which is consistent with a spread of 100 basis points in equilibrium. This spread reflects the historical spread between AAA-BAA corporate bonds in the U.S. in 1954-2015 (FRED). Alternatively, the spread between the AAA bond and the 10 years Treasury bills is also in this neighborhood of 100 basis points.

In order to determine the moral-hazard parameter associated with loans to consumption, I need to determined the spread between the lending rate to consumption and the deposit rate in the bank. In the first step, I take the average lending rates for different categories of spending. In the next step, I take information from the Survey of Consumer Finances (SCF) to obtain the composition of debt. Finally, I construct a weighted interest rate which delivers the value for the discount factor of the impatient households $\left(\beta^{Ext}\right)$. This discount factor is equal to 0.9911 and implies an annual interest rate of 3.63 percent.

Table 6 shows the results. The first row shows the components of households' debt. I take the 4 main components and leave the other categories aside. The main component is the mortgage debt (87%). Then, I look at the lending rates for each of these concepts. The spread is calculated respect to government bonds with comparable maturity: the 30-years government bond for mortgages, the 1-year government bond for credit card, the 10-year government bonds for education, and the 5-year government bond for vehicles. Finally, I weight the spread according to its relevance on household's debt[15]. The first column shows weighted spread. This spread is 163 basis points and it serves to pin down the moral hazard problem of the loans associated with consumption. This moral hazard problem ($\Delta\lambda$) is equal to 0.6043.

[14] Financial corporations include both public and private firms in the financial business. This includes monetary institutions, depository corporations, financial auxiliaries and insurance companies and pension funds. This is a broader indicator than just commercial banks.

[15] The weighted spread is robust to use the swap rates rather than the government bonds.

Table 3.6: Households' Debt Composition

	Total*/	Mortgage	Credit Card	Education	Vehicles
% of Debt (2004-2013)	100	87	3	5	5
Lending Rate					
Average 2006-16 (%)		4.4	12.6	8.33	5.81
Spread (bps.)*		98	1,092	547	362
Weighted Spread	163	85	33	27	18

*/ I do not consider the concept Others Debts.

*Respect to term comparable U.S. Treasury Bond.

Source: Survey of Consumer Finances (SCF) and Federal Reserve.

3.7 Optimal Policy.

In the next section, I present the optimal policy for two cases. In the first case, I assume that the monetary and the prudential policy cooperate in order to maximize the social welfare, given by the weighted sum of the households utility function. In the second case, I assume that the policy authorities do not cooperate. In this case, the objective function of the policy authorities differ. In particular, the monetary authority sets the nominal interest rate in such a way that social welfare is maximized, whereas the prudential authority sets the reserve requirement in such a way that the inefficient borrowing is minimized.

3.7.1 Optimal Level of Reserves.

In this section, I present the effects of the reserve requirements on the steady state of this economy. Table 7 shows the value of selected variables in the monetary and prudential regime.

Mechanism.

In this section I explain how the macro-prudential instrument affects the economy. The reserve requirement serves as a tax on the part of borrowing associated with the inefficient consumption. By requiring a fraction of the deposits in the form of reserves, mandatory reserves can act as an implicit tax on financial intermediation; and by altering the cost of funding, they may be useful to reduce the volatility of credit.

The macro-prudential instrument affects the economy because the cost of higher reserve requirement is passed on in full to creditors in the form of higher lending rates $\left(R_t^k, R_t^b\right)$. Hence, affecting the rate at which the agents are willing to postpone consumption, reducing borrowing. An increase in the reserve ratio drives to a drop in demand for credit and deposits. Because credit is funded via deposits, in addition to the accumulated net wealth. An increase in reserves turns

Table 3.7: Variables in Steady State. Benchmark Calibration

	Variable. Financial sector.	Monetary	Macro-prudential	%Δ
1.	Reserve ratio (%, Quarterly)	0	0.71	
2.	Leverage ratio	4	3.93	-1.85
3.	% Loans to firms	38.7	49.1	10.4
4.	% Loans to consumption	61.3	50.9	-10.4
5.	Consumption. External	0.233	0.218	-6.4
6.	Consumption. Internal	0.338	0.316	-6.5
7.	Aggregate labor supply	0.33	0.32	-3.0
8.	Labor supply. External.	0.33	0.32	-3.0
9.	Labor supply. Internal.	0.33	0.32	-3.0
10.	Wage	1.621	1.501	-7.4
11.	Output	0.968	0.862	-11.0
12.	Capital	8.430	6.411	-23.9

deposits into a more expensive source of funding. And the cost is transmitted to the borrowers, increasing the rate they have to pay. This depresses credit growth. Then, the central bank can directly influence the consumption decision by changing the reserve requirement and reducing the consumption associated with the habits externality.

Consumption of the borrowers is linked to the real lending rate through the Euler Equation. An increase in the rate on lending makes consumption more expensive. Given the Euler equation, this induces households to reduce today consumption

$$U_{ct}^{Ex} = \beta^{Ext} E_t U_{ct+1}^{Ex} R_{t+1}^{b},$$

hence, the channel of transmission is through the Euler equation of the borrowers. Increasing the reserve requirement implies an increase in the lending rate $\left(R_t^b\right)$. In order to equilibrate this relationship, the marginal utility of consumption increases, this is possible due to a reduction in today's consumption. Hence, the reserve requirement affects the equation governing the consumption pattern. And this equation reflects the distortion associated with the the externality. Hence, the reserve requirement is acting on the same margin as the externality.

The reserve requirement increases the cost of credit. In particular, the spread on loans to firms and on loans to consumption increases in 50 basis points. This implies a reduction in the demand for credit.

The reserve requirement changes the portfolio composition of the banks in steady state. In particular, under the optimal reserve requirements, the commercial banks lend more to firms than in the absence of this requirement.

Table 3.8: Welfare Gains of Cooperative Monetary and Macro-Prudential Policy

	Relative to */	Monetary Policy
		(1)
1.	Steady State	1.6421
2.	Cyclical	0.4356

*/ In % of Consumption Equivalent (C.E.) of the cooperative economy.

The participation of loans to firms in the total amount of bank's loans increase in 10.4 percentage points in equilibrium. In contrast, the participation of the loans to consumption decrease by 10.4 percentage points. The level of consumer credit decrease 50.2 percent, while credit to firms falls only 24 percent.

The composition of the bank's portfolio in the non-stochastic steady state is determined by the demand for loans. There is a larger reduction in the loans to consumption than in the loans to firms because the demand for loans to consumption is more sensitive to changes in the interest rate than the demand for production. I associate this with the fact that impatient households depend heavily on loans in order to consume, and an increase in the cost of credit rapidly decreases their demand for loans.

However, the loans granted to firms are a function of the cost of credit and of the overall level of demand in the economy. Savers do not borrow to consume. Then, the increase in the cost of credit affects mainly the consumption of the borrowers. Firms, still have to produce in order to meet the demand for the savers. Hence, they still have to demand capital and this explains the fact that the loans to consumption fall faster than the loans to production.

The reduction in the level of credit has the effect of reducing the consumption of the borrowers in 6.4%. And it shrinks the level of output in 11%. Effectively, the macro-prudential policy in steady state works through the reduction of the socially inefficient levels of consumption, borrowing, output, and labor.

Table 8 shows the welfare gains, in terms of the stream of consumption of the cooperative Ramsey allocation, of setting the prudential instrument at its optimal level.

As observed in table 8, the prudential policy is welfare increasing in the steady state of the economy because it reduces the excessive working hours, consumption, and overborrowing. In the first row in table 8, the two steady states of the economy are compared: with and without the optimal level of reserve requirements. In steady state, the optimal level of reserves increases the welfare in the society in 1.6421 percent, respect to a case in which only the monetary policy is implemented.

The macro-prudential policy also has benefits out of steady state. I present the results in the second row of table 8. In this case, the economies start at the same steady state, in which the optimal level of reserves is 0.71 percent, and it compares the welfare benefits when the shocks hit and monetary and prudential policy are implemented. In this case, there is a welfare gain of 0.44 percent respect to the case in which only the monetary policy is implemented.

In the next section, I present the optimal prudential and monetary policy in the cooperative and non-cooperative equilibrium.

3.7.2 Cooperative Policy.

The monetary and prudential authorities set their policy instruments in a cooperative way. I assume that in this case, the Ramsey planner seeks to maximize the welfare of the society subject to the competitive equilibrium conditions. I assume that the central bank is committed to follow the announced plan from a timeless perspective. As in Schmitt-Grohé and Uribe (2005), and Woodford (2003), I assume that at time (t) the Ramsey planner has been operating for an infinite number of periods.

The period (t) objective function of the Ramsey planner is the weighted utility function of the consumers. I follow the work of Monacelli (2008), Mendicino and Pescatori (2008), and Rubio and Carrasco (2014) and assume that the Ramsey planner weights the utility of each group by their population size and one minus their discount factor $[\left(1-\beta^{Ext}\right)$ and $(1-\beta)$, respectively$]$. The welfare function is then:

$$W_0^R = (\Upsilon)\left(1-\beta^{Ext}\right) V_0^{R,B} + (1-\Upsilon)(1-\beta) V_0^{R,S}, \tag{3.80}$$

where

$$V_0^{R,B} = \left\{ E_0 \sum_{t=0}^{\infty} \left(\beta^{Ext}\right)^t U\left[C_t^{R,B}, L_t^{R,B}\right] \right\}, \tag{3.81}$$

and

$$V_0^{R,S} = \left\{ E_0 \sum_{t=0}^{\infty} (\beta)^t U\left[C_t^{R,S}, L_t^{R,S}\right] \right\}, \tag{3.82}$$

represent the utility function of the borrowers and the savers, respectively. (Υ) is the population size and $\left(\beta^{Ext}, \beta\right)$ the subjective discount factor of the borrowers and savers, respectively.

The Ramsey planner selects optimally the nominal interest rate, and the reserve requirement in order to maximize $\left(W_0^R\right)$, subject to the competitive equilibrium conditions. See appendix D for the derivation of the measure of the welfare cost utilized to evaluate the different policy alternatives.

In the next section, I present the impulse-response function of the optimal policy.

3.7.3 Impulse-Response Functions.

Financial Shock.

This a 1 standard deviation increase in the quality of capital. Figure 4 shows the impulse-response functions associated with this shock. The solid lines shows the optimal monetary policy, while the blue circles show the optimal mix of monetary and macro-prudential policy.

The positive shock to the quality of capital induces a relaxation of the banks' balance sheet constraint. As a consequence of this relaxation, the banks can issue more loans. In turn, the relaxation of the bank's financial constraint reduces the cost of credit, and the firms and the consumers demand more loans. As a consequence of its cheaper cost, the credit to consumption and to firms increases. The increase in the demand for credit revalues the bank assets, which additionally relaxes their balance sheet constraint and allows them to lend more and at lower rates. There is a positive feedback loop between the real and the financial variables.

This financial accelerator amplifies and propagates the financial shocks. The improvement in the credit conditions lead to an increase in borrowing by the consumers with external habits. Neither consumers, nor banks internalize the effects of the externalities on the other agents in the economy. Given the existence of the positive spread, the commercial banks would like to issue as much loans as possible in order to accumulate as much wealth as possible. On the other side, households with external habits and higher impatience overborrow and overconsume.

Banks are content with this situation because they are increasing their profits, enriching their owners. However, this situation makes the economy more vulnerable to shocks. The presence of the endogenous financial constraint contributes to the amplification and propagation of shocks, and the presence of the external habits produce inefficient consumption and borrowing. The central bank would like to stop this situation. In particular, if the central bank can impose a cost on the behavior of banks or on the behavior of consumers, the overconsumption can be smoothed. For example, the increase in the nominal interest rate increases the cost for funding new loans.

Figure 3.4: Optimal Macro-Prudential Policy. Shock to Quality of Capital

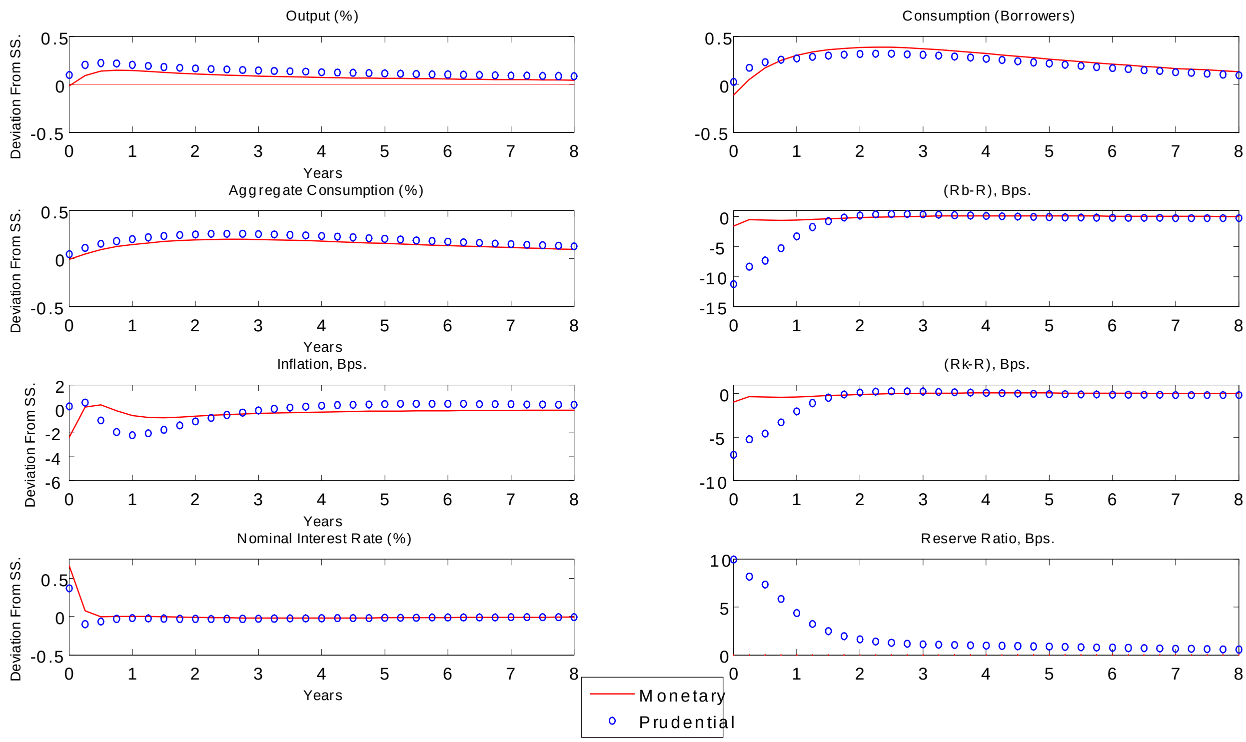

Figure 4. Optimal Policy. 1 sd. Increase in the Quality of Capital. Monetary Policy (Solid Lines), Macro-prudential Policy (Circles).

Hence, when the shock hits, the central bank increases the nominal interest rate, in order to increase the cost of funding for commercial banks. The monetary authority would like to curb an excessive increase in consumer credit in order to prevent exacerbating the externality associated with the borrowers' consumption and to stop the propagation associated with the financial accelerator.

The increase in the real interest rate makes the cost of funding new loans more expensive, banks fund the issuing of loans by paying the real interest rate on the deposits. The contractionary monetary policy reduces the bank's profits and their balance sheet constraint tightens. Because the availability of credit is limited by the value of the bank franchise, the reduction in the profitability of the banks serves to reduce the overborrowing associated with the excessive consumption of the borrowers. The initial increase in the nominal interest rate is large because the central bank would like to smooth the excessive credit growth. However, this creates a deflation, which is partially compensated in the second period. Given that the central bank has access to only one policy instrument, it is exploiting the expectational benefits of its commitment technology. The central banks commits itself to a contractionary policy in the first period in order to reduce the demand for credit, and in the second period it commits to an expansionary policy in order to compensate the agents for the initial contraction. In this way the central bank is maximizing its intertemporal trade-offs.

The monetary policy, in the absence of the macro-prudential instrument is more contractionary (solid lines). However, it is not enough to stop the change in inflation. The central bank increases the nominal interest rate in order to stop the credit growth. However, not all the increase in the credit is associated with the inefficient level of borrowing. The firms increase their borrowing in order to produce more and to satisfy the increase in the consumption of the savers and the borrowers. The increase in the consumption of the savers is not associated with the habits externality, but the tightening in the monetary policy affects the cost of credit for the firms, which are producing to satisfy also the consumption of the savers. Hence, the central bank accepts an increase in inflation in order to not damaging the level of production associated with the consumption of the savers.

If the central bank has access to the monetary, and the macro-prudential instrument (the reserve requirement), then the trade-offs for the central bank can be improved (circles in figure 1). On impact, there is an increase in the reserve requirement. This increase is equal to 10 basis points (the optimal level of the reserves in steady state is 0.71 percent).

If the central bank has access to the reserve ratio, the increase in this requirement induces a less contractionary monetary policy. The increase in the reserve requirement imposes and additional cost to the banks. Because these reserves are non-remunerated, if the commercial banks seek to increase the credit, they have to bear the additional cost associated with the issue of the new deposits. This has the effect of reducing their profits and makes their financial constraint more astringent.

The tightening of the bank's balance sheet, respect to the case in which the macro-prudential policy is absent, has the effect of reducing the bank's ability to issue new loans. The increase in the reserve ratio reduces the credit growth: if only the monetary policy is implemented, on impact, the credit increases 2.5 percent, while in the presence of the reserve requirement the initial increase in credit is only 1.7 percent.

The central bank utilizes its two instruments to curb the inflation changes and the credit growth. In this way, the central bank can reduce the welfare damage associated with the price dispersion in this economy with sticky prices, and the overborrowing associated with the habits externality.

The initial reaction of the monetary and prudential policy serves to reduce the expansion of credit and to keep inflation under control. This mix of policy also serves to stop the financial accelerator mechanism. Once the central bank has smoothed the effects of this mechanism, it would like to return the economy to equilibrium. The monetary policy is contractionary and then turns expansionary in the second period. The initial contraction is explained by the desire to avoid exacerbating the habits externality. But, the expansion in the second period is explained by the technology commitment and the Phillips curve. In particular, the central bank can promise to deliver inflation in this period and deflation in the next period. In this way, the central bank is exploiting the benefits of the commitment technology. The monetary instrument continues being expansionary for almost one year in order to achieve inflation and price level control in the long-run, which is the typical result of the commitment technology.

In summary, in the model, overconsumption is a source of inefficiency. The macro-prudential policy would like to reduce the overborrowing associated with the excessive consumption. Hence, macro-prudential makes the cost of credit more expensive for these borrowers. This depresses their consumption motives and the economy is better-off because it is less exposed to the multiplier effects of shocks, and inflation and the real variables reduce their volatility respect to the case in which only the monetary policy is present. When the prudential instrument is in place, both policies are contractionary, but the monetary policy is less contractionary than in the absence of the prudential instrument. The reserves ratio increases in order to make more expensive the cost of credit for consumers. The trade-offs for the central bank have improved.

Technology Shock.

This a 1 standard deviation increase in the productivity. Figure 5 shows the impulse-response functions associated with this shock. The solid lines shows the optimal monetary policy, while the blue circles show the optimal mix of monetary and macro-prudential policy.

Figure 3.5: Optimal Macro-Prudential Policy. Shock to Technology

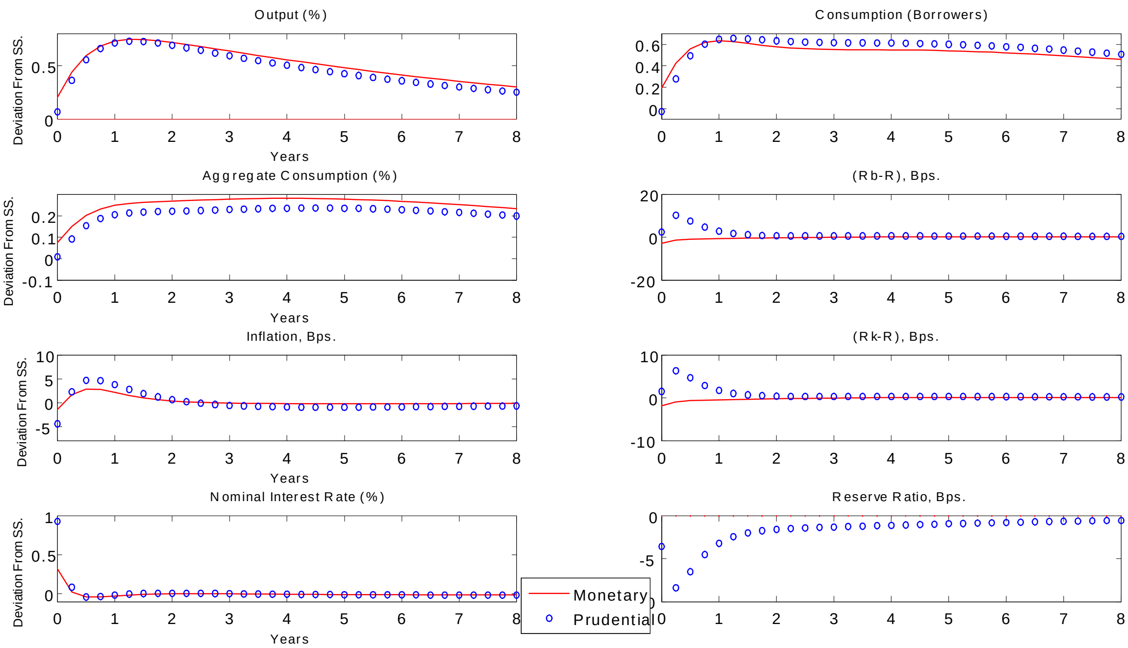

Figure 5. Optimal Policy. 1 sd Increase in Productivity. Monetary Policy (Solid Lines), and Macro-prudential Policy (Circles).

A positive shock to technology increases the return per unit of capital. The increase in the return on capital has the effect of increasing the cost of credit. The policy is expansionary and the central bank increases the nominal interest rate in order to prevent excessive increase in consumption. This policy creates an initial deflation.

The increase in the cost of credit, given that the return per unit of capital has increased, and reinforced by the contractionary monetary policy, reduces the demand for credit and the bank's balance sheet is damaged. As a consequence of the increase in the cost of credit, the issuing of credit falls. If the central bank has access to the reserve requirement, then the prudential policy turns expansionary in order to stop the reduction in credit associated with the contractionary monetary stance. The expansionary prudential policy contributes to reduce the inflation volatility and to increase the growth of consumption by reducing the cost of credit.

In this case, the monetary and the prudential policy act in opposite directions. However, their actions contribute to deliver less volatility in inflation and credit. The combination of the contractionary monetary policy and the expansionary prudential policy deliver a larger in consumption.

In the next section, I present the results of the implementation of the optimal policy. The implementation is via simple rules. In particular, I present two policy rules: one for the monetary policy and one for the prudential instrument. These rules have the characteristic of maximizing the social welfare.

3.7.4 Non-Cooperative Policy.

In order to analyze the effects of implementing monetary and prudential policy in a non-cooperative way, I extend the model to consider strategic games between these policy makers. The non-cooperative solution assumes that the monetary authority maximizes the social welfare, which is equal to the social welfare of the cooperative solution, by choosing the contingent path of the nominal interest rate, whereas the prudential authority deals with the overborrowing problem; in particular, it minimizes the loans granted to consumption by choosing a level of reserve requirement which makes the consumption efficient.

The objective of the prudential regulator is to minimize every period the deviations of the consumption of the borrowers $\left(C_t^{Ex}\right)$ respect to the case in which the habits externality is zero (C_t^*),

$$\min -\frac{1}{2}\left[C_t^{Ex} - C_t^*\right]^2 . \tag{3.83}$$

Table 9 shows the welfare cost of non-cooperative policies. The cost is measured respect to the cooperative case. When the shock hit, the non-cooperative policy implies a welfare cost of 0.34% of the stream of consumption of the cooperative allocation.

Shock to Productivity.

Figure 6 shows the combination of monetary and macro-prudential policy after a shock to productivity when the policy makers do not cooperate. The blue circles show the cooperative policy, whereas the dashed lines show the non-cooperative policy.

When the two policy makers cooperate, the monetary policy is contractionary, while the prudential policy is expansionary. If they do not cooperate, the prudential policy becomes more expansionary. When the policy makers are cooperating, the nominal interest rate increases almost one percent on impact and the prudential instrument move slightly. But if they do not cooperate, the central bank finds it optimal a smooth contractionary monetary policy, and the prudential authority implements an expansionary policy. The prudential policy in the non-cooperative case is considerably more expansionary than in the cooperative case, but the monetary policy is less contractionary.

The combination of monetary and prudential policy reduce inflation in almost 10 basis points on impact and the deflationary process last for two years. After the second period, the monetary policy is more contractionary than in the cooperative case, this explains the prolonged deflationary period. Initially, output and consumption grow more than in the cooperative case. However, after one year, the consumption and output show a smaller growth rate than in the cooperative case. In the non-cooperative case, inflation and the spread are more volatile than in the cooperative case.

Table 3.9: Welfare Cost of Non-Cooperative Policy

	Relative to	Cost % of C.E.
		(1)
1.	Cooperative Policy	0.3420

Figure 3.6: Non-Cooperative Policy. Shock to Quality of Capital

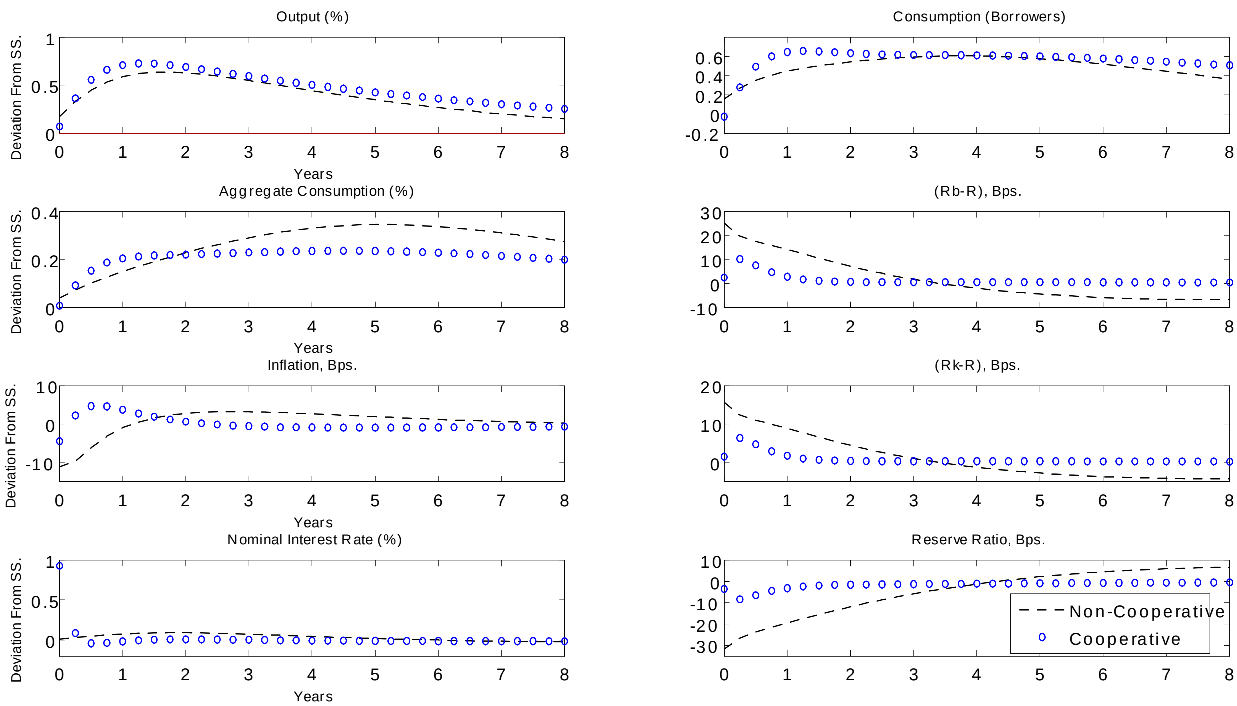

Figure 6. Non-Cooperative Policy. 1 sd Increase in Productivity. Cooperative Macro-prudential Policy (Circles), Non-Cooperative(Dashed).

Cost-Push Shock.

Figure 7 shows the monetary and prudential response after an increase of one standard deviation in the cost-push variable.

This shock is more contractionary and more inflationary when the authorities are not cooperating. Under the non-cooperative framework, the nominal interest rate is more expansionary than under cooperation. Given that the prudential regulator anticipates the more contractionary monetary setting, the prudential policy is considerably more expansionary. This provides incentives to the banks to lend more and consumption increases more than in the cooperative case.

The reduction in the reserve requirement increases the bank's profitability and they are able to issue more loans. The combination of monetary and prudential policy are not as effective in stabilizing inflation and output as they are when the policy makers are cooperating.

Figure 3.7: Non-Cooperative Policy. Cost-Push Shock

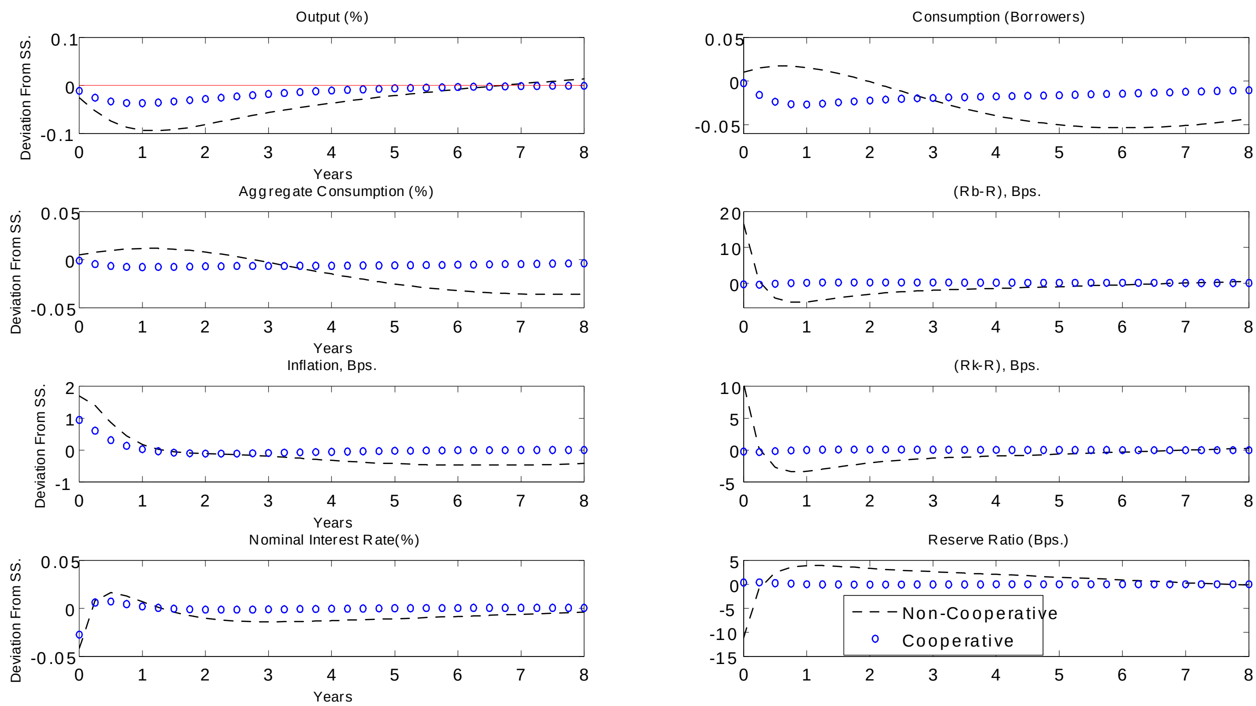

Figure 7. Non-Cooperative Policy. 1 sd Cost-Push Shock. Cooperative Macro-prudential Policy (Circles), Non-Cooperative(Dashed).

3.8 Conclusion.

Reserve requirements can work as a Pigouvian tax in an economy with an overborrowing problem. A positive level of this requirement increases the cost of credit. Under this regime agents internalize the effects of their consumption externality and reduce their demand for credit. Setting a positive level of this requirement is welfare increasing; the economy reduces its overall size and agents works less and reduce their borrowing. The optimal level of reserve requirement is 0.7% and this instrument can be used out of the steady state. When shocks hit the economy, the mix of prudential and monetary policy can stabilize inflation and reduce the excessive level of borrowing, contributing to increase welfare.

In contrast to some of the current literature, in this paper we set up a model in which some agents have an overborrowing problem, while other have an underborrowing problem. The prudential instrument is able to increase welfare by reducing overborrowing and making agents to internalize their consumption decisions.

Chapter 4

Conclusions

Along this disseration, I analyzed the optimal policy in the presence of financial frictions. This analysis was divided in three chapters. In the first chapter I analyze the optimal monetary policy recommendation in the presence of financial frictions and its implementation via simple rules. In the second chapter, I allow the central bank to have an additional policy instrument. The nominal interest rate is complemented with the possiblity of implementing financial intermediation. This new instrument seeks to mimic the policy implemente by the Federal Reserve of the United States in the past recession. Finally, the third chapter analyzed the interaction between macroprudential and monetary policy when there is an overborrowing problem.

In the chaper one, I utilize a standard New Keynesian model with a banking sector that faces balance sheet constraints, the optimal policy seeks to stabilize the financial markets by reducing the volatility of the cost of credit; a healthy financial sector is desirable. In this framework, there is a trade-off between inflation stabilization and financial stabilization. This holds if the economy is subject only to a productivity shock or if the economy becomes more efficient by reducing the monopolistic competition.

The implementation of optimal policy suggests stabilizing the spread between the return on capital and the risk-free rate. When a shock hits the economy, this policy suggests an aggressive reaction in the initial periods.

The simple rule that mimics optimal policy suggests a zero coefficient on changes in output, and a non-zero coefficient to changes in the premium on capital with respect to its long-run average. Stabilizing the financial sector enhances social welfare. In contrast, a strong anti-inflationary stance may be welfare decreasing. Additionally, inertial rules serve to anchor inflation expectations in the long-run, while stabilizing financial markets in the short-run, mimicking the optimal policy under commitment.

In the second chapter, the central bank can participate in the financial markets. As a consequence of the Great Recession, the understanding of the interactions between banking and monetary policy has become a central issue in policy design. The severity of the financial crisis has exposed the limitations of the conventional tools utilized by the central bank to stabilize the economy. In this chapter, I analyzed whether, there are welfare gains of equipping the central bank with an additional policy instrument.

In an economy with financial frictions , the central bank now has access to two policy instruments: the nominal interest rate and credit easing through asset purchases. If a negative shock hits the economy there are stabilization and welfare gains from allowing the central bank to participate in direct lending activities. The trade offs for the central bank improve in the presence of monopolistic competition, financial frictions, and two policy instruments, respet to the case in which there is only monetary policy available.

The optimal policy can be implemented via simple rules. One rule sets optimally the nominal interest rate, while the other set the central bank policy financial intermediation. This arrangemente is welfare improving and the central bank improves its policy trade offs.

The third chapter considers the optimal interaction between two aspects of policy: monetary and macroprudential policy in an economy in which there are incentives to overborrowing. In this economy, a consumption externality creates an inefficiently high level of credit. This can put the economy at a vulnerable position in case of a financial shock.

Reserve requirements can work as a Pigouvian tax in an economy with an overborrowing problem. A positive level of this requirement increases the cost of credit. Under this regime agents internalize the effects of their consumption externality and reduce their demand for credit. Setting a positive level of this requirement is welfare increasing; the economy reduces its overall size and agents works less and reduce their borrowing. The optimal level of reserve requirement is 0.7% and this instrument can be used out of the steady state. When shocks hit the economy, the mix of prudential and monetary policy can stabilize inflation and reduce the excessive level of borrowing, contributing to increase welfare.

In contrast to some of the current literature, in this paper we set up a model in which some agents have an overborrowing problem, while other have an underborrowing problem. The prudential instrument is able to increase welfare by reducing overborrowing and making agents to internalize their consumption decisions.

Appendix A

Appendix Chapter 1.

A.1 Derivation of Equations.

A.1.1 Households.

The maximization problem of the household can be expressed using the Lagrangian:

$$\mathcal{L} = E_t \left\{ \begin{array}{c} \sum_{i=0}^{\infty} \beta^i \left[\ln\left(C_{t+i} - hC_{t+i-1}\right) - \frac{\chi}{1+\varphi} L_{t+i}^{1+\varphi} \right] \\ +\lambda_{t+i} \left[W_{t+i} L_{t+i} + \Pi_{t+i} + T_{t+i} + R_{t+i} \left[D_{t+i} + B_{t+i}^{g} \right] - \left[D_{t+i+1} + B_{t+i+1}^{g} \right] - C_{t+i} \right] \end{array} \right\}$$

The first order conditions are:

Respect to Consumption:

$$E_t \lambda_{t+i} = E_t \left[\frac{1}{(C_{t+i} - hC_{t+i-1})} - h\beta \frac{1}{(C_{t+i+1} - hC_{t+i})} \right] \tag{A.1}$$

Respect to labor:

$$\chi E_t L_{t+i}^{\varphi} = E_t \lambda_{t+i} W_{t+i} \tag{A.2}$$

Respect to Savings:

$$E_t \lambda_{t+i} = E_t \lambda_{t+i+1} R_{t+i+1} \tag{A.3}$$

And the Budget constraints:

$$W_{t+i} L_{t+i} + \Pi_{t+i} + R_{t+i} \left[D_{t+i} + B_{t+i}^{g} \right] - \left[D_{t+i+1} + B_{t+i+1}^{g} \right] + T_{t+i} = C_{t+i} \tag{A.4}$$

The marginal utility of consumption at period $(i = 0)$ can be expressed as:

$$U_{ct} = E_t \left[\frac{1}{(C_t - hC_{t-1})} - h\beta \frac{1}{(C_{t+1} - hC_t)} \right] \tag{A.5}$$

Then, optimal labor supply is

$$\frac{\chi L_t^{\varphi}}{U_{ct}} = W_t \tag{A.6}$$

And the consumption-saving decision

$$1 = E_t \beta \frac{U_{ct+1}}{U_{ct}} R_{t+1}$$

It is defined

$$\Lambda_{t,t+i} = \frac{U_{ct+i}}{U_{ct}} \tag{A.7}$$

A.1.2 Financial Intermediaries.

incentive constraint and Maximization of Banks Final Wealth.

The bank is interested in maximizing its terminal net wealth (N_{jt+i}). It has a finite horizon and the probability of surviving from today to tomorrow is (θ_t). At the end of period t, the surviving bank maximizes its terminal wealth for the end of period $(t+1)$ on.

The bank's net wealth evolves as the difference between the return on its assets and the cost of funding them, eq.(1.9) in the main text

$$N_{jt+1} = \left[R_{t+1}^k - R_{t+1} \right] Q_t S_{jt} + R_{t+1} N_{jt}$$

and because the bank is not interested in funding projects with an expected discounted cost larger than its expected discounted return, the next condition should apply for the bank to operate

$$E_t \beta^{1+i} \Lambda_{t,t+1+i} \left[R_{t+1+i}^k - R_{t+1+i} \right] \geq 0 \tag{A.8}$$

in any period $(i \geq 0)$.

At the end of period (t), a surviving bank has a probability of dying tomorrow equal to $(1 - \theta_{t+1})$. If a bank survives that period with probability (θ_{t+1}), it will have a probability of leaving the industry in $(t+2)$ equal to $(1 - \theta_{t+2})\, \theta_{t+1}$. Banks surviving that period, with probability (θ_{t+2}), have a probability of leaving the industry in $(t+3)$ equal to $(1 - \theta_{t+3})\, \theta_{t+2}\theta_{t+1}$. So, the probability of dying in the period $(t+i)$ is $(1 - \theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i} \theta_k \right]$ for $(i \geq 0)$. Then, at the end of period t, the bank maximizes its expected discounted terminal wealth according to

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1 - \theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i} \theta_k \right] \beta^{i+1} \Lambda_{t,t+1+i} \left(N_{jt+1+i} \right) \tag{A.9}$$

which takes into account the evolution of terminal wealth (1.9), the discount factor $\left(\beta^{i+1}\Lambda_{t,t+1+i}\right)$ and the survival pattern. Substituting the evolution of wealth eq.(1.9)

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i}\theta_k\right] \beta^{i+1}\Lambda_{t,t+1+i} \begin{pmatrix} \left(R^k_{t+1+i} - R_{t+1+i}\right) Q_{t+i}S_{jt+i} \\ +R_{t+1+i}N_{jt+i} \end{pmatrix} \tag{A.10}$$

I can split the right-hand side of eq. $(A.10)$ in one term associated with total assets and other associated with the equity part. Then, the problem of the bank can be expressed as

$$V_{jt} = V^v_{jt} + V^\eta_{jt} \tag{A.11}$$

with

$$V^v_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i}\theta_k\right] \beta^{i+1}\Lambda_{t,t+1+i} \left(R^k_{t+1+i} - R_{t+1+i}\right) Q_{t+i}S_{jt+i} \tag{A.12}$$

and

$$V^\eta_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i}\theta_k\right] \beta^{i+1}\Lambda_{t,t+1+i} R_{t+1+i}N_{jt+i} \tag{A.13}$$

Assets.

Now, working with the assets part eq.$(A.12)$

$$V^v_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i}\theta_k\right] \beta^{i+1}\Lambda_{t,t+1+i} \left(R^k_{t+1+i} - R_{t+1+i}\right) Q_{t+i}S_{jt+i} \tag{A.14}$$

the update one period-ahead of the previous equation is

$$V^v_{jt+1} = \max E_t \sum_{i=0}^{\infty} (1-\theta_{t+2+i}) \left[\Pi_{k=t+2}^{t+1+i}\theta_k\right] \beta^{i+1}\Lambda_{t+1+i,t+2+i} \left(R^k_{t+2+i} - R_{t+2+i}\right) Q_{t+1+i}S_{jt+1+i} \tag{A.15}$$

Eq. $(A.14)$ can be expressed as

$$\begin{aligned} V^v_{jt} &= E_t (1-\theta_{t+1}) \beta^1 \Lambda_{t,t+1} \left(R^k_{t+1} - R_{t+1}\right) Q_t S_{jt} + \\ & \quad E_t \sum_{i=1}^{\infty} (1-\theta_{t+1+i}) \left[\Pi_{k=t+1}^{t+i}\theta_k\right] \beta^{i+1}\Lambda_{t,t+1+i} \left(R^k_{t+1+i} - R_{t+1+i}\right) Q_{t+i}S_{jt+i} \end{aligned}$$

and the second part of the right-hand side can be expressed as

$$
\begin{aligned}
V_{jt}^{v} &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R_{t+1}^{k}-R_{t+1}\right)Q_tS_{jt}+\\
&\quad E_t\left(\theta_{t+1}\right)\beta\Lambda_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta_{t+2+i}\right)\left[\Pi_{k=t+2}^{t+1+i}\theta_k\right]\beta^{i+1}\Lambda_{t+1+i,t+2+i}\left[\begin{array}{c}\left(R_{t+2+i}^{k}-R_{t+2+i}\right)\\ Q_{t+i+1}S_{jt+i+1}\end{array}\right]
\end{aligned}
$$

and using A.15

$$
\begin{aligned}
V_{jt}^{v} &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R_{t+1}^{k}-R_{t+1}\right)Q_tS_{jt}+\\
&\quad E_t\left(\theta_{t+1}\right)\beta\Lambda_{t,t+1}V_{jt+1}^{v}
\end{aligned}
$$

Multiplying by $\left(\frac{1}{Q_tS_{jt}}\right)$

$$
\frac{1}{Q_tS_{jt}}V_{jt}^{v}=\left(1-\theta_{t+1}\right)E_t\beta\Lambda_{t,t+1}\left(R_{t+1}^{k}-R_{t+1}\right)+E_t\theta_{t+1}\beta\Lambda_{t,t+1}\frac{1}{Q_tS_{jt}}V_{jt+1}^{v} \tag{A.16}
$$

I can use the definitions $\left(v_t=\frac{V_{jt}^{v}}{Q_tS_{jt}}\right)$ which implies $\left(v_{t+1}=\frac{V_{jt+1}^{v}}{Q_{t+1}S_{jt+1}}\right)$. Substituting this in the previous equation

$$
v_t=E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R_{t+1}^{k}-R_{t+1}\right)+E_t\theta_{t+1}\beta\Lambda_{t,t+1}\frac{1}{Q_tS_{jt}}v_{t+1}Q_{t+1}S_{jt+1}
$$

defining the gross growth of asset between period (t) and $(t+i)$ as

$$
x_{t,t+i}=\frac{Q_{t+i}S_{jt+i}}{Q_tS_{jt}}
$$

I arrive to

$$
v_t=E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R_{t+1}^{k}-R_{t+1}\right)+E_t\theta_{t+1}\beta\Lambda_{t,t+1}x_{t,t+1}v_{t+1} \tag{A.17}
$$

Equity.

Working with the net wealth part eq. $(A.13)$

$$
V_{jt}^{\eta}=\max E_t\sum_{i=0}^{\infty}\left(1-\theta_{t+1+i}\right)\left[\Pi_{k=t+1}^{t+i}\theta_k\right]\beta^{i+1}\Lambda_{t,t+1+i}R_{t+1+i}N_{jt+i}
$$

updating one period-ahead the previous equation

$$
V_{jt+1}^{\eta}=\max E_t\sum_{i=0}^{\infty}\left(1-\theta_{t+2+i}\right)\left[\Pi_{k=t+2}^{t+1+i}\theta_k\right]\beta^{i+1}\Lambda_{t+1+i,t+2+i}R_{t+1+i}N_{jt+i}
$$

I can separate $\left(V_{jt}^{\eta}\right)$ as

$$\begin{aligned} V_{jt}^{\eta} &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1}N_{jt} + \\ & E_t\sum_{i=1}^{\infty}\left(1-\theta_{t+1+i}\right)\left[\Pi_{k=t+1}^{t+i}\theta_k\right]\beta^{i+1}\Lambda_{t,t+1+i}\left(R_{t+1+i}\right)N_{jt+i} \end{aligned}$$

starting the summation from zero

$$\begin{aligned} V_{jt}^{\eta} &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1}N_{jt} + \\ & E_t\left(\theta_{t+1}\right)\beta\Lambda_{t,t+1}\sum_{i=0}^{\infty}\left(1-\theta_{t+2+i}\right)\left[\Pi_{k=t+2}^{t+1+i}\theta_k\right]\beta^{i+1}\Lambda_{t+1+i,t+2+i}\left(R_{t+2+i}\right)N_{jt+i+1} \end{aligned}$$

the term in the summation is the one period-ahead update of $\left(V_{jt}^{\eta}\right)$. Then

$$\begin{aligned} V_{jt}^{v} &= E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1}N_{jt} + \\ & E_t\left(\theta_{t+1}\right)\beta\Lambda_{t,t+1}V_{jt+1}^{v} \end{aligned}$$

I define now $\left(\eta_t = \frac{V_{jt}^{\eta}}{N_{jt}}\right)$ and $\left(\eta_{t+1} = \frac{V_{jt+1}^{\eta}}{N_{jt+1}}\right)$. Multiplying the previous equation by $\left(\frac{1}{N_{jt}}\right)$

$$V_{jt}^{\eta}\frac{1}{N_{jt}} = E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1} + E_t\theta_{t+1}\beta\Lambda_{t,t+1}\frac{N_{jt+1}}{N_{jt}}\eta_{t+1}$$

the gross rate of net wealth between period (t) and $(t+i)$ can be defined as

$$z_{t,t+i} = \frac{N_{jt+i}}{N_{jt}}$$

Then, the previous equation can be written as

$$\eta_t = E\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1} + E_t\theta_{t+1}\beta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1} \tag{A.18}$$

Equation $(A.11)$ is equal to

$$V_{jt} = \nu_t Q_t S_{jt} + \eta_t N_{jt} \tag{A.19}$$

Which is the conjectured solution to the banks problem.

A.1.3 Non-financial Intermediate Producers Firms.

The firm production function is

$$Y_{mt} = A_t \left(U_t \xi K_t\right)^{\alpha} L_t^{1-\alpha} \tag{A.20}$$

The income for the firms is the value of its product $(P_{mt}Y_{mt})$ plus the income coming from the reselling the undepreciated capital $(1-\delta_t)\,\xi_t K_t$.

The costs are: the wage bill $(W_t L_t)$, the return on the capital acquired in the previous period and paid in this $\left(R_t^k\right) Q_{t-1}K_t$ and assuming that cost of replacement of worn out capital is unit, the profits problem for the firm in this period is to choose (U_t) and (L_t) to maximize

$$P_{mt}Y_{mt} + \left[Q_t - \delta_t\right]\xi_t K_t - R_t^k Q_{t-1}K_t - W_t L_t \tag{A.21}$$

subject to eq.$(A.20)$. The first order condition respect to labor is

$$(1-\alpha)\,P_{mt}\frac{Y_{mt}}{L_t} = W_t \tag{A.22}$$

Respect to Utilization rate.

$$\alpha P_{mt}\frac{Y_{mt}}{U_t} = bU_t^{\zeta}\xi_t K_t \tag{A.23}$$

I am assuming the depreciation function:

$$\delta_t = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta} \tag{A.24}$$

$$\delta'\left(U_t\right) = bU_t^{\zeta}$$

Return to Capital.

The return to capital is the remaining of the profits after paying the wage bill and the other costs of production. Substituting the optimal condition $(A.22)$ in the profits equation $(A.21)$ total profits should be zero as long as the firms pays all the return to capital to the banks

$$P_{mt}Y_{mt} + \left[Q_t - \delta_t\right]\xi_t K_t - R_t^k Q_{t-1}K_t - (1-\alpha)\,P_{mt}Y_{mt} = 0$$

simplifying and solving for the return to capital

$$R_t^k = \left\{\alpha P_{mt}\frac{Y_{mt}}{K_t} + \left[Q_t - \delta_t\right]\xi_t\right\}\frac{1}{Q_{t-1}} \tag{A.25}$$

where the value of the marginal productivity of capital is

$$\alpha P_{mt}\frac{Y_{mt}}{K_t}$$

A.1.4 Capital Producers.

Each period the firms chooses the level of net investment to solve

$$\max E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_{t,\tau} \left\{ (Q_t - 1) I_{nt} - f\left(\frac{I_{n\tau} + I_{ss}}{I_{n\tau-1} + I_{ss}} \right) (I_{n\tau} + I_{ss}) \right\}$$

with

$$f\left(\frac{I_{n\tau} + I_{ss}}{I_{n\tau-1} + I_{ss}} \right) = \frac{\phi\_i}{2} \left(\frac{I_{n\tau} + I_{ss}}{I_{n\tau-1} + I_{ss}} - 1 \right)^2 \tag{A.26}$$

The first order condition respect to net investment is

$$\begin{aligned} Q_t &= 1 + \frac{\phi\_i}{2} \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} - 1 \right)^2 + \phi\_i \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} - 1 \right) \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} \right) \\ &\quad - \beta E_t \Lambda_{t,t+1} \phi\_i \left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}} - 1 \right) \left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}} \right)^2 \end{aligned}$$

A.1.5 Retailers.

Demand for Final Product.

Each of the consumers of the final good must minimize the cost of buying one unit of the composite good. This good is aggregated according to:

$$Y_t = \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A.27}$$

where (ε) is the elasticity of substitution between varieties.

Then, the minimization problem is

$$\mathcal{L}_t = \int_0^1 P_{ft} Y_{ft} df + \lambda_t \left\{ Y_t - \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{\varepsilon}{\varepsilon-1}} \right\}$$

The first order condition respect to (Y_{ft}) is

$$P_{ft} = \lambda_t \left[\int_0^1 Y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right]^{\frac{1}{\varepsilon-1}} Y_{ft}^{-\frac{1}{\varepsilon}} \tag{A.28}$$

Using the definition of the composite good

$$P_{ft} = \lambda_t Y_t^{\frac{1}{\varepsilon}} Y_{ft}^{-\frac{1}{\varepsilon}}$$

Solving for the demand of individual good (Y_{ft})

$$Y_{ft} = \left[\frac{P_{ft}}{\lambda_t}\right]^{-\varepsilon} Y_t \tag{A.29}$$

Substituting eq.$(A.29)$ in $(A.27)$

$$\lambda_t = \left[\int_0^1 (P_{ft})^{1-\varepsilon}\, df\right]^{\frac{1}{1-\varepsilon}} \tag{A.30}$$

The Lagrange multiplier can be though as the correct price index. Then,

$$\lambda_t = P_t \tag{A.31}$$

where the price index is defined as

$$P_t = \left[\int_0^1 (P_{ft})^{1-\varepsilon}\, df\right]^{\frac{1}{1-\varepsilon}}$$

Substituting eq.$(A.31)$ in $(A.29)$

$$Y_{ft} = \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} Y_t \tag{A.32}$$

Which is the optimal demand for the final good (f). And substituting this in the definition of spending

$$\int_0^1 P_{ft} Y_{ft} df = S_t \tag{A.33}$$

I can write the aggregate spending of the consumer of the final good as

$$Y_t P_t = \int_0^1 P_{ft} Y_{ft} df \tag{A.34}$$

Evolution of the price index.

From the previous section we know that the price index is equal to:

$$P_t = \left[\int_0^1 (P_{ft})^{1-\varepsilon}\, df\right]^{\frac{1}{1-\varepsilon}}$$

Given that the fraction $(1-\gamma)$ of the firms reoptimize price in period (t) and that a fraction (γ) is not able to reoptimize in this period, and that those firms not reoptimizing this period partially index (γ^p) their price to the past period inflation $\left(\pi_{t-1}^{\gamma^\rho}\right)$ and allowing for the optimal price to be (P_t^*), equation (1.39) can be written as:

$$P_t = \left[\int_0^{1-\gamma} \left(P_{ft}^*\right)^{1-\varepsilon} df + \int_{1-\gamma}^{\gamma} \left(\pi_{t-1}^{\gamma^\rho} P_{ft-1}\right)^{1-\varepsilon} df\right]^{\frac{1}{1-\varepsilon}} \tag{A.35}$$

then,

$$P_t^{1-\varepsilon} = (1-\gamma)\left(P_t^*\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho} P_{t-1}\right)^{1-\varepsilon} \tag{A.36}$$

Dividing $(A.36)$ by $\left(P_t^{1-\varepsilon}\right)$

$$1 = (1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon} \tag{A.37}$$

solving for the relative price

$$\frac{P_t^*}{P_t} = \left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon}}{(1-\gamma)}\right]^{\frac{1}{1-\varepsilon}} \tag{A.38}$$

Equation $(A.37)$ is the evolution of the optimal price.

A.1.6 Price setting.

Following Christiano et al. (2005), Ascari and Sbordone (2014) and Hornstein (2007), In each period there is a fixed probability $(1-\gamma)$ that a firm can reoptimize its price P_t^*. For those firms not reoptimizing this period they index their price to previous period inflation. This happens with a probability (γ). In this case

$$P_{it}^* = \pi_{t-1}^{\gamma^\rho} P_{it-1}$$

with the parameter $\rho \in [0,1]$ indicating the degree of indexation to previous period inflation. The problem is then

$$\max_{P_{it}^*} E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j}\gamma^j \left[(1-\tau_{t+j})\frac{P_{it}^*\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} Y_{it+j} - P_{mt+j}Y_{it+j}\right] \tag{A.39}$$

subject to the demand function

$$Y_{it+j} = \left(\frac{P_{it}^*\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j} \tag{A.40}$$

The cumulative inflation between period (t) and $(t+j)$ is

$$\Pi_{t,t+j} = \begin{array}{cc} 1 & \text{for } j=0 \\ \frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\frac{P_{t+3}}{P_{t+2}}\cdots\frac{P_{t+k}}{P_{t+k-1}} & \text{for } j \geq 1 \end{array}$$

$$\Pi_{t-1,t+j-1} = \begin{array}{cc} 1 & \text{for } j=0 \\ \frac{P_t}{P_{t-1}}\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}\cdots\frac{P_{t+k-1}}{P_{t+k-2}} & \text{for } j \geq 1 \end{array}$$

Substituting demand eq.$(A.40)$ in eq.$(A.39)$

$$\max_{P_{it}^*} E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left[\begin{array}{c} (1-\tau_{t+j}) \frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \\ -P_{mt+j} \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{array} \right] \tag{A.41}$$

which is equal to

$$\max_{P_{it}^*} E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left[\begin{array}{c} (1-\tau_{t+j}) \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} \\ -P_{mt+j} \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{array} \right] \tag{A.42}$$

the first order condition

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left[\begin{array}{c} (1-\varepsilon)(1-\tau_{t+j}) \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} \frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} Y_{t+j} \\ -(-\varepsilon) P_{mt+j} \left(\frac{P_{it}^* \Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon-1} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right) Y_{t+j} \end{array} \right] = 0 \tag{A.43}$$

simplifying

$$E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left[\begin{array}{c} P_{it}^{*-\varepsilon} (1-\tau_{t+j}) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} \\ -\frac{\varepsilon}{\varepsilon-1} P_{mt+j} P_{it}^{*-\varepsilon-1} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{array} \right] = 0 \tag{A.44}$$

solving for the optimal price

$$\begin{aligned} & P_{it}^* E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j (1-\tau_{t+j}) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} \\ = \quad & \frac{\varepsilon}{\varepsilon-1} E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j P_{mt+j} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{aligned} \tag{A.45}$$

dividing by $\left(\frac{P_t}{P_t}\right)$

$$\begin{aligned} & \frac{P_{it}^*}{P_t} P_t E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j (1-\tau_{t+j}) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j} \\ = \quad & \frac{\varepsilon}{\varepsilon-1} E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j P_{mt+j} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \end{aligned} \tag{A.46}$$

using

$$P_t = P_t^{1-\varepsilon+\varepsilon}$$

and solving for the optimal price

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j P_{mt+j} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left(1-\tau_{t+j}\right) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} \right)^{1-\varepsilon} Y_{t+j}} P_t^{-(1-\varepsilon)-\varepsilon} \tag{A.47}$$

introducing the price inside the parenthesis in the numerator and denominator

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j P_{mt+j} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} P_t \right)^{-\varepsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left(1-\tau_{t+j}\right) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{P_{t+j}} P_t \right)^{1-\varepsilon} Y_{t+j}} \tag{A.48}$$

using the definition of cumulative inflation

$$\frac{P_{it}^*}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j P_{mt+j} \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{\Pi_{t,t+j}} \right)^{-\varepsilon} Y_{t+j}}{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t,t+j} \gamma^j \left(1-\tau_{t+j}\right) \left(\frac{\Pi_{t-1,t+j-1}^{\gamma^\rho}}{\Pi_{t,t+j}} \right)^{1-\varepsilon} Y_{t+j}} \tag{A.49}$$

Which is the optimal relative price for the firm.

Evolution of Inflation.

The price index is eq.$(A.31)$

$$P_t = \left[\int_0^1 \left(P_{ft}\right)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}$$

Distributing between the $(1-\gamma)$ setting price optimally this period and this (γ) indexing their price to the previous period inflation

$$P_t = \left[\int_0^{1-\gamma} \left(P_{ft}^*\right)^{1-\varepsilon} df + \int_{1-\gamma}^1 \left(\pi_{t-1}^{\gamma^\rho} P_{ft-1} \right)^{1-\varepsilon} df \right]^{\frac{1}{1-\varepsilon}}$$

integrating across each group

$$P_t^{1-\varepsilon} = \left[(1-\gamma) \left(P_t^*\right)^{1-\varepsilon} + \gamma \left(\pi_{t-1}^{\gamma^\rho} P_{t-1} \right)^{1-\varepsilon} \right]$$

dividing by$\left(P_t^{1-\varepsilon}\right)$

$$\frac{P_t^{1-\varepsilon}}{P_t^{1-\varepsilon}} = \left[(1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}\right]$$

using inflation definition

$$1 = \left[(1-\gamma)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon}\right] \tag{A.50}$$

A.1.7 Price Dispersion.

At the firm level demand must be equal to the supply, then

$$Y_{mft} = \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} Y_t \tag{A.51}$$

From the intermediate good production

$$Y_{mft} = A_t\left(U_t\xi_t K_{ft}\right)^\alpha L_{ft}^{1-\alpha} \tag{A.52}$$

Aggregate labor is

$$L_t = \int_0^1 L_{ft} df \tag{A.53}$$

Aggregate effective capital is

$$K_t = \int_0^1 K_{ft} df \tag{A.54}$$

And aggregating eq.($A.51$)over all the firms and taking into account the definitions of aggregate variables

$$A_t\left(U_t\xi_t K_t\right)^\alpha L_t^{1-\alpha} = Y_t\int_0^1 \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} df \tag{A.55}$$

Defining price dispersion as

$$\Delta_t = \int_0^1 \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} df \tag{A.56}$$

Then, the aggregate resource constraint can be written as

$$Y_{mt} = Y_t\Delta_t \tag{A.57}$$

A.1.8 Evolution of Price Dispersion.

Price dispersion was defined as

$$\Delta_t = \int_0^1 \left[\frac{P_{ft}}{P_t}\right]^{-\varepsilon} df \tag{A.58}$$

Each period there is a fraction $(1-\gamma)$ choosing price optimally and (γ) indexing the price to the previous period inflation

$$\Delta_t = \int_0^{1-\gamma} \left[\frac{P_{ft}^*}{P_t}\right]^{-\varepsilon} df + \int_{1-\gamma}^1 \left[\pi_{t-1}^{\gamma^p} \frac{P_{ft-1}}{P_t}\right]^{-\varepsilon} df$$

multiplying inside the second integral by $\left(\frac{P_{t-1}}{P_{t-1}}\right)$

$$\Delta_t = \int_0^{1-\gamma} \left[\frac{P_{ft}^*}{P_t}\right]^{-\varepsilon} df + \int_{1-\gamma}^1 \left[\pi_{t-1}^{\gamma^p} \frac{P_{t-1}}{P_t} \frac{P_{ft-1}}{P_{t-1}}\right]^{-\varepsilon} df$$

integrating over the firms

$$\Delta_t = (1-\gamma) \left[\frac{P_t^*}{P_t}\right]^{-\varepsilon} + \gamma \left[\pi_{t-1}^{\gamma^p} \pi_t^{-1}\right]^{-\varepsilon} \Delta_{t-1} \tag{A.59}$$

from the price index, eq.$(A.38)$ I know

$$\left\{\left[1-\gamma\left(\pi_{t-1}^{\gamma^\rho} \pi_t^{-1}\right)^{1-\varepsilon}\right] \frac{1}{1-\gamma}\right\}^{\frac{1}{1-\varepsilon}} = \left(\frac{P_t^*}{P_t}\right)$$

substituting in $(A.59)$

$$\Delta_t = (1-\gamma) \left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^\rho} \pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma \left[\pi_{t-1}^{\gamma^p} \pi_t^{-1}\right]^{-\varepsilon} \Delta_{t-1} \tag{A.60}$$

A.2 Set of Equilibrium Conditions. Conventional Monetary Policy

List of variables (36).

$$U_{ct}, C_t, R_t, \Lambda_{t,t+1}, L_t, P_{mt}, Y_t$$

$$v_t, R^k_{t+1}, x_{t,t+1}, \phi_t, z_t, \eta_t,$$

$$Q_t, K_t, N_t, N_{et}, N_{nt},$$

$$Y_{mt}, U_t,$$

$$I_{nt}, \delta_t, I_t, G_t, \tau_t,$$

$$\xi_t, g_t, A_t, \theta_t,$$

$$\Delta_t, \pi_t, F_t, Z_t, \pi^*_t,$$

$$i_t, S_{pt}$$

Households.

1. Marginal Utility of Consumption (U_{ct})

$$U_{ct} = E_t \left[(C_t - hC_{t-1})^{-1} - \beta h \left(C_{t+1} - hC_t \right)^{-1} \right]$$

2. Euler Equation. Consumption Saving (C_t)

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = 1$$

3. Stochastic Discount Factor ($\Lambda_{t,t+1}$)

$$E_t \Lambda_{t,t+1} = E_t \frac{U_{ct+1}}{U_{ct}}$$

4. labor Market Equilibrium (L_t)

$$\frac{\chi}{(1-\alpha)} \frac{L_t^{\varphi+1}}{U_{ct}} = P_{mt} Y_{mt}$$

Banks.

5. Marginal Return on Bank's Assets (v_t)

$$v_t = E_t\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}\left(R^k_{t+1}-R_{t+1}\right)+E_t\theta_{t+1}\beta\Lambda_{t,t+1}x_{t,t+1}v_{t+1}$$

6. Gross Growth of Bank Assets ($x_{t,t+1}$)

$$x_{t,t+1} = E_t\frac{\phi_{t+1}}{\phi_t}z_{t,t+1}$$

7. Marginal Return on Bank's Wealth (η_t)

$$\eta_t = E\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1}+E_t\theta_{t+1}\beta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1}$$

8. Gross Growth of Bank Wealth ($z_{t,t+1}$)

$$z_{t,t+1} = E_t\left[\left(R^k_{t+1}-R_{t+1}\right)\phi_t+R_{t+1}\right]$$

9. Leverage Ratio (ϕ_t)

$$\phi_t = \frac{\eta_t}{\lambda - v_t}$$

10. Aggregate Capital (K_t)

$$Q_t K_{t+1} = \phi_t N_t$$

11. Net worth in the banking sector (N_t)

$$N_t = N_{et} + N_{nt}$$

12. Existing wealth (N_{et})

$$N_{et} = \theta_{t-1}\left[\left(R^k_t - R_t\right)\phi_{t-1}+R_t\right]N_{t-1}$$

13. Wealth of new banks (N_{nt})

$$N_{nt} = wQ_t\xi_t K_t$$

Intermediate Producers.

14. Return to capital (R_{kt})

$$R^k_t = \frac{\xi_t}{Q_{t-1}}\left[\alpha\frac{P_{mt}Y_{mt}}{\xi_t K_t}+Q_t-\delta_t\right]$$

15. Production of Intermediate goods (Y_{mt})

$$Y_{mt} = A_t\left(U_t\xi_t K_t\right)^\alpha L_t^{1-\alpha}$$

Capital Goods Producers.

16. Investment (Q_t)

$$
\begin{aligned}
Q_t &= 1 + \frac{\phi_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \phi_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} \\
&\quad - E_t\beta\Lambda_{t,t+1}\phi_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2
\end{aligned}
$$

17. Depreciation function (U_t)

$$\delta_t = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta}$$

18. Optimal Capacity Utilization (P_{mt})

$$\alpha P_{mt}\frac{Y_{mt}}{U_t} = bU_t^{\zeta}\xi_t K_t$$

19. Net Investment (δ_t)

$$I_{nt} = I_t - \delta_t \xi_t K_t$$

20. Law of movement of capital (I_{nt})

$$K_{t+1} = \xi_t K_t + I_{nt}$$

21. Exogenous government consumption (G_t)

$$G_t = Gg_t$$

22. Aggregate resources (I_t)

$$Y_t = C_t + G_t + I_t + \frac{\phi_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 (I_{nt}+I_{ss})$$

Retailers

23. Final Production (Y_t)

$$Y_{mt} = Y_t\Delta_t$$

24. Price Dispersion (Δ_t)

$$\Delta_t = (1-\gamma)\left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^{\rho}}\pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma\left[\pi_{t-1}^{\gamma^{p}}\pi_t^{-1}\right]^{-\varepsilon}\Delta_{t-1}$$

25. Optimal Price Choice (F_t)

$$F_t = P_{mt} Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{-\gamma^\rho \varepsilon} \pi_{t+1}^{\varepsilon} F_{t+1}$$

26. (Z_t)

$$Z_t = (1 - \tau_t) Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{\gamma^\rho (1-\varepsilon)} \pi_{t+1}^{-(1-\varepsilon)} Z_{t+1}$$

27. Optimal choice of price (π_t^*)

$$\pi_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{F_t}{Z_t} \pi_t$$

28. Evolution of inflation (π_t)

$$\pi_t^{1-\varepsilon} = \left[(1-\gamma) (\pi_t^*)^{1-\varepsilon} + \gamma \left(\pi_{t-1}^{\gamma^\rho} \right)^{1-\varepsilon} \right]$$

Policy and Exogenous Variables.

29. Fisher Equation (R_t)

$$i_t = E_t R_t \pi_{t+1}$$

30. Monetary policy (i_t)

$$\frac{i_t}{i} = E_t \left[\frac{i_{t-1}}{i} \right]^{\phi_R} \left[\frac{\pi_t}{\pi} \right]^{\phi_\pi} \left[\frac{Y_t}{Y} \right]^{\phi_Y} \varepsilon_{it}$$

31. Technology Shock

$$\ln A_t = \rho_a \ln A_{t-1} - \varepsilon_{at}$$

32. Capital Quality Shock (ξ_t)

$$\ln \xi_t = \rho_\varepsilon \ln \xi_{t-1} - \varepsilon_{\xi t}$$

33. Government Shock (g_t)

$$\ln g_t = \rho_g \ln g_{t-1} - \varepsilon_{gt}$$

34. Shock to the Probability of dying (θ_t)

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} - \varepsilon_{\theta t}$$

35. Markup shock (τ_t)

$$\ln\left(1-\tau_t\right)=\rho_\mu \ln\left(1-\tau_{t-1}\right)+\left(1-\rho_\mu\right)\ln\left(1-\tau\right)-\varepsilon_t^\mu$$

36. Premium (Sp_t)

$$S_{pt}=R_{kt}-R_t$$

Summary of Variables (36) **and Equations** (36).

A.3 Social Planner's Problem.

The Social Planner's Problem.

Introducing eq.(1.62) in eq.(1.63)

$$Y_t = C_t + G_t + I_t + \frac{\phi_i}{2}\left(\frac{K_{t+1} - K_t\left[1+\xi_t\right] + \xi_{t-1}K_{t-1}}{K_t - \xi_{t-1}K_{t-1} + I_{ss}}\right)^2 \left(K_{t+1} - \xi_t K_t + I_{ss}\right). \tag{A.61}$$

Combining eq.(1.62) with eq.(1.61) and introducing in eq.($A.61$)

$$\begin{aligned} Y_t &= C_t + G_t + K_{t+1} - \left(1-\delta_t\right)\xi_t K_t \\ &+ \frac{\phi_i}{2}\left(\frac{K_{t+1} - K_t\left[1+\xi_t\right] + \xi_{t-1}K_{t-1}}{K_t - \xi_{t-1}K_{t-1} + I_{ss}}\right)^2 \left(K_{t+1} - \xi_t K_t + I_{ss}\right). \end{aligned} \tag{A.62}$$

Now, substituting eq.(1.60) in eq.($A.62$)

$$\begin{aligned} Y_t &= C_t + G_t + K_{t+1} - \left[1 - \delta_c - \frac{b}{1+\zeta}U_t^{1+\zeta}\right]\xi_t K_t \\ &+ \frac{\phi_i}{2}\left(\frac{K_{t+1} - K_t\left[1+\xi_t\right] + \xi_{t-1}K_{t-1}}{K_t - \xi_{t-1}K_{t-1} + I_{ss}}\right)^2 \left(K_{t+1} - \xi_t K_t + I_{ss}\right). \end{aligned} \tag{A.63}$$

Finally, substituting eq.(1.59) in the previous equation

$$\begin{aligned} A_t\left(U_t\xi_t K_t\right)^\alpha L_t^{1-\alpha} &= C_t + G_t + K_{t+1} - \left[1 - \delta_c - \frac{b}{1+\zeta}U_t^{1+\zeta}\right]\xi_t K_t \\ &+ \frac{\phi_i}{2}\left(\frac{K_{t+1} - \left(1+\xi_t\right)K_t + \xi_{t-1}K_{t-1}}{K_t - \xi_{t-1}K_{t-1} + I_{ss}}\right)^2 \left(K_{t+1} - \xi_t K_t + I_{ss}\right). \end{aligned} \tag{A.64}$$

Then, the social planner chooses [C_t, L_t, U_t and K_{t+1}] to maximize the utility of the consumer eq.(1.58) subject to the restriction eq.($A.64$). The Lagrangian for the problem is

$$\begin{aligned} \mathcal{L} &= E_t\sum_{t=0}^{\infty}\beta^t\left[\ln\left(C_t - hC_{t-1}\right) - \frac{\chi}{1+\varphi}L_t^{1+\varphi}\right] \\ &+ E_t\sum_{t=0}^{\infty}\beta^t\lambda_t\left[\begin{array}{c} A_t\left(U_t\xi_t K_t\right)^\alpha L_t^{1-\alpha} \\ -C_t - G_t - K_{t+1} + \left[1-\delta_c - \frac{b}{1+\zeta}U_t^{1+\zeta}\right]\xi_t K_t \\ -\frac{\phi_i}{2}\left(\frac{K_{t+1} - K_t(1+\xi_t) + \xi_{t-1}K_{t-1}}{K_t - \xi_{t-1}K_{t-1} + I_{ss}}\right)^2\left(K_{t+1} - \xi_t K_t + I_{ss}\right). \end{array}\right] \end{aligned} \tag{A.65}$$

The first order conditions are:

$$C_t : \frac{1}{(C_t - hC_{t-1})} - \lambda_t - E_t\left(\frac{\beta h}{C_{t+1} - hC_t}\right) = 0 \tag{A.66}$$

$$L_t : -\chi L_t^{\varphi} + \lambda_t (1-\alpha) A_t (U_t \xi_t K_t)^{\alpha} L_t^{-\alpha} = 0 \tag{A.67}$$

$$U_t : \lambda_t \alpha A_t \frac{(U_t \xi_t K_t)^{\alpha} L_t^{1-\alpha}}{U_t} - b\lambda_t \xi_t K_t U_t^{\zeta} = 0 \tag{A.68}$$

$$\begin{aligned}
K_{t+1} \quad : \quad & 0 = -\lambda_t - \lambda_t \phi_i E_t \left(\frac{K_{t+1} - K_t (1+\xi_t) + \xi_{t-1} K_{t-1}}{K_t - \xi_{t-1} K_{t-1} + I_{ss}}\right) \times \\
& (K_{t+1} - \xi_t K_t + I_{ss}) \left(\frac{1}{K_t - \xi_{t-1} K_{t-1} + I_{ss}}\right) && \text{(A.69)} \\
& -\lambda_t \frac{\phi_i}{2} E_t \left(\frac{K_{t+1} - K_t (1+\xi_t) + \xi_{t-1} K_{t-1}}{K_t - \xi_{t-1} K_{t-1} + I_{ss}}\right)^2 \\
& +\beta E_t \lambda_{t+1} \alpha \frac{A_{t+1} \left(U_{t+1} \xi_{t+1} K_{t+1}\right)^{\alpha} L_{t+1}^{1-\alpha}}{K_{t+1}} + \beta E_t \lambda_{t+1} \left[1 - \delta_c - \frac{b}{1+\zeta} U_{t+1}^{1+\zeta}\right] \xi_{t+1} \\
& -\beta E_t \lambda_{t+1} \phi_i \left[\begin{array}{c}
\left(\frac{K_{t+2} - K_{t+1}\left(1+\xi_{t+1}\right) + \xi_t K_t}{K_{t+1} - \xi_t K_t + I_{ss}}\right) \times \\
\left(K_{t+2} - \xi_{t+1} K_{t+1} + I_{ss}\right) \times \\
\left(\frac{-\left(1+\xi_{t+1}\right)\left(K_{t+1} - \xi_t K_t + I_{ss}\right) - \left(K_{t+2} - K_{t+1}\left(1+\xi_{t+1}\right) + \xi_t K_t\right)}{\left(K_{t+1} - \xi_t K_t + I_{ss}\right)^2}\right)
\end{array}\right] \\
& +\beta E_t \lambda_{t+1} \frac{\phi_i}{2} \left(\frac{K_{t+2} - K_{t+1}\left(1+\xi_{t+1}\right) + \xi_t K_t}{K_{t+1} - \xi_t K_t + I_{ss}}\right)^2 \left(\xi_{t+1}\right) && \text{(A.70)}
\end{aligned}$$

and the resource constraint

$$\begin{aligned}
A_t (U_t \xi_t K_t)^{\alpha} L_t^{1-\alpha} \quad = \quad & C_t + G_t + E_t K_{t+1} - \left[1 - \delta_c - \frac{b}{1+\zeta} U_t^{1+\zeta}\right] \xi_t K_t && \text{(A.71)} \\
& + \frac{\phi_i}{2} E_t \left(\frac{K_{t+1} - (1+\xi_t) K_t + \xi_{t-1} K_{t-1}}{K_t - \xi_{t-1} K_{t-1} + I_{ss}}\right)^2 (K_{t+1} - \xi_t K_t + I_{ss}).
\end{aligned}$$

The Social Planner's Steady State.

In this section I present the steady state that faces the Social Planner's Allocation. This steady state is calculated when $[Y_{t+1} = Y_t = Y_{t-1} = Y^*]$ for each variable. Eq.$(A.66 - A.71)$ can, respectively, be written in steady state as

$$\lambda = \frac{[1 - \beta h]}{C (1-h)} \tag{A.72}$$

$$L = \left[\lambda \frac{(1-\alpha)}{\chi} (UK)^{\alpha}\right]^{\frac{1}{\varphi+\alpha}} \tag{A.73}$$

$$U = \left[\frac{\alpha}{b}\left(\frac{K}{L}\right)^{\alpha-1}\right]^{\frac{1}{1+\zeta-\alpha}} \tag{A.74}$$

$$\frac{K}{L} = \left[\frac{1-\beta\left[1-\delta\right]}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}\frac{1}{U} \tag{A.75}$$

and the resource constraint

$$C = (UK)^{\alpha} L^{1-\alpha} - G - \delta K \tag{A.76}$$

where the value of shock to the quality of capital and to productivity in steady state $[\xi = A = 1]$ and the depreciation in steady state

$$\delta = \delta_c + \frac{bU^{1+\zeta}}{1+\zeta}$$

were used.

Inserting eq.$(A.74)$ into $(A.75)$ and solving for the capital labor ratio

$$\frac{K^*}{L^*} = \left[\frac{1-\beta\left[1-\delta\right]}{\alpha\beta}\right]^{\frac{1}{\alpha-1}}. \tag{A.77}$$

The government spending was assumed as a fraction of the total output. That fraction is $\left[\overline{G}\right]$ then $\left[G = \overline{G}Y\right]$ with

$$Y^* = (K^*)^{\alpha} L^{*1-\alpha} \tag{A.78}$$

solving the resource constraint for $\left(\frac{C^*}{L^*}\right)$ and using the efficient rate $\left[\frac{K^*}{L^*}\right]$ eq.$(A.76)$ becomes

$$\frac{C^*}{L^*} = \left(\frac{K^*}{L^*}\right)^{\alpha}\left[1-\overline{G}\right] - \delta\frac{K^*}{L^*} \tag{A.79}$$

with

$$U^* = \left\{\frac{1-\beta\left[1-\delta\right]}{b\beta}\right\}^{\frac{1}{\zeta}} = 1 \tag{A.80}$$

after substituting $\left[\frac{K^*}{L^*}\right]$ in eq.$(A.74)$.

Substituting eq.$(A.72)$ in eq.$(A.73)$ and using the efficient values of the variables

$$L^* = \left\{\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\left(\frac{K^*}{L^*}\right)^{\alpha}\left[\frac{C^*}{L^*}\right]^{-1}\right\}^{\frac{1}{1+\varphi}}. \tag{A.81}$$

Finally, using the efficient values of $\left[\frac{K^*}{L^*}\right]$ and $[U^*]$

$$U^*\frac{K^*}{L^*} = \left\{\frac{1-\beta\left[1-\delta\right]}{\alpha\beta}\right\}^{\left[\frac{1}{\alpha-1}\right]}. \tag{A.82}$$

After using the optimal value for utilization $(U^* = 1)$ the equations for consumption, labor and capital can be written as

$$Y^* = \left[\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right]^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{-\alpha(1+\varphi)}{\alpha-1}} \left[1-\overline{G}\right] \\ -\left[\delta\right]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{1-2\alpha-\alpha\varphi}{\alpha-1}} \end{array}\right]^{-\frac{1}{1+\varphi}} \tag{A.83}$$

$$K^* = \left\{\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right\}^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[1-\overline{G}\right]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{1+\varphi}{1-\alpha}} \\ -\left[\delta\right]\left[\frac{1-\beta[1-\delta]}{\alpha\beta}\right]^{\frac{\alpha+\varphi}{1-\alpha}} \end{array}\right]^{-\frac{1}{1+\varphi}} \tag{A.84}$$

$$L^* = \left\{\frac{1-\beta h}{1-h}\frac{1-\alpha}{\chi}\right\}^{\frac{1}{1+\varphi}} \left[\begin{array}{c} \left[1-\overline{G}\right] \\ -\left[\delta\right]\left[\frac{\alpha\beta}{1-\beta[1-\delta]}\right] \end{array}\right]^{-\frac{1}{1+\varphi}} \tag{A.85}$$

which are equations in terms of the deep parameters.

A.4 Derivation of the Welfare Cost.

Following Schmitt-Grohé and Uribe (2007) I compare the welfare cost of each alternative policy relative to the time invariant equilibrium of the Ramsey policy. The welfare associated with the optimal Ramsey policy conditional on a particular state of the economy in period zero is

$$V_0^R = E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t^R, L_t^R\right] \tag{A.86}$$

and the welfare associated with an alternative implementable regime is

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U\left[C_t^I, L_t^I\right]. \tag{A.87}$$

If the consumption cost of following an alternative policy regime instead of the Ramsey policy on a particular state in period zero is represented by $\left[W^C\right]$ the cost of the alternative policy is implicitly defined by

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t U\left[\left(1 - W^C\right) C_t^R, L_t^R\right]. \tag{A.88}$$

where $\left[W^C\right]$ is the fraction of consumption of the Ramsey regime that a household is able to renounce in order to be indifferent between that regime and the alternative policy. As in Schmitt-Grohé and Uribe (2007), I assume that at time zero the variables of the economy equal their respective Ramsey steady state value.

Substituting the particular form of the utility function in $(A.88)$

$$V_0^I = E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left[\left(1 - W^C\right) C_t^R - h\left(1 - W^C\right) C_{t-1}^R\right] - \frac{\chi}{1+\psi} L_t^{R1+\psi}\right]. \tag{A.89}$$

Equation $[A.89]$ can be written

$$V_0^I = \frac{\ln\left(1 - W^C\right)}{1-\beta} + E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln\left[C_t^R - hC_{t-1}^R\right] - \frac{\chi}{1+\psi} L_t^{R1+\psi}\right]. \tag{A.90}$$

Solving this equation for the welfare cost

$$\frac{1}{(1-\beta)} \ln\left(1 - W^C\right) = V_0^I - V_0^R \tag{A.91}$$

which makes use of $[A.86]$. Noting that

$$\ln(1+x) \approx x,$$

the welfare cost $[A.91]$ can be written as

$$W^C = [1-\beta]\left[V_0^R - V_0^I\right], \tag{A.92}$$

which is the welfare cost function that is necessary to approximate up to second order in order to have accurate welfare comparisons across regimes.

Approximating the welfare cost up to second order.

Equation $(A.92)$ can be approximated up to second order around the deterministic Ramsey steady state $(x_0, \sigma_\varepsilon)$ with $[x_0 = x]$ and $\sigma_\varepsilon = 0$. Because in equilibrium V_0^R and V_0^I are functions of the initial state vector (x_0) and the parameter scaling the standard deviations of the shocks (σ_ε), the conditional welfare cost can be written as

$$W^C(x_0, \sigma_\varepsilon) = [1-\beta]\left[V_0^R(x_0, \sigma_\varepsilon) - V_0^I(x_0, \sigma_\varepsilon)\right] \tag{A.93}$$

Because I want to compare the welfare results using the same deterministic Ramsey steady state, only the first and second order derivatives of the cost respect to (σ_ε) have to be considered (see Schmitt-Grohé and Uribe (2007)). Following this, the second order approximation of the previous equation can be written in general terms as

$$W^C \approx W^C(x_0, \sigma_\epsilon) + W^C_{\sigma_\epsilon}(x_0, \sigma_\epsilon)\,\sigma_\epsilon + \frac{1}{2}W^C_{\sigma_\epsilon\sigma_\epsilon}(x_0, \sigma_\epsilon)\,\sigma_\epsilon^2. \tag{A.94}$$

Now, because all the regimes are approximated across the same deterministic Ramsey steady state, the constant term $\left[W^C(x_0, \sigma_\epsilon)\right]$ in eq. $(A.94)$ disappears in the comparison. This means that

$$W^C(x_0, \sigma_\epsilon) = 0.$$

The terms containing the first order approximation of the policy function $\left[W^C_{\sigma_\epsilon}(x_0, \sigma_\epsilon)\,\sigma_\epsilon\right]$ are zero. This is shown in Schmitt-Grohé and Uribe (2004). Up to a first order of approximation, the derivative of the policy function respect to the parameters scaling the variance of the shocks is zero. For this particular case

$$W^C_{\sigma_\epsilon}(x, 0)\,\sigma_\epsilon = [1-\beta]\left[V^R_{0\sigma\varepsilon}(x, 0) - V^I_{0\sigma\varepsilon}(x, 0)\right]\sigma_\epsilon = 0$$

The term containing the second order approximation is

$$W^C_{\sigma\varepsilon\sigma\varepsilon}(x, 0) = \frac{1}{2}[1-\beta]\left[V^R_{0\sigma\varepsilon\sigma\varepsilon}(x, 0) - V^I_{0\sigma\varepsilon\sigma\varepsilon}(x, 0)\right]\sigma_\varepsilon^2 \tag{A.95}$$

which is the welfare measure used in the main text.

A.5 Optimal Policy. Timeless Perspective.

The optimal policy problem is solved from a timeless perspective. The Ramsey planner maximizes the discounted utility function subject to the competitive equilibrium conditions. Following Schmitt-Grohé and Uribe (2011), the portion of the Lagrangian that is relevant for optimal policy from a timeless perspective is

$$\mathcal{L}^R = E_0 \left[\sum_{\tau=0}^{\infty} \beta^{\tau} U_{\tau} \left[C_{\tau}, L_{\tau} \right] + \sum_{\tau=0}^{\infty} \beta^{\tau} Lm'_{\tau} \mathbf{C}_{\tau} \left(\cdot \right) \right] \tag{A.96}$$

where the period t utility function is

$$U_t = \ln \left(C_t - hC_{t-1} \right) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}$$

(β) is the Ramsey planner's discount factor, which I assume to be identical to that of the competitive equilibrium. The vector $\left[Lm_t' \right]$ contains the 29 Lagrange multiplier associated with the 29 equilibrium conditions in period t $\mathbf{C}_t \left(\cdot \right)$. Those equilibrium conditions in period t are:

1. Marginal Utility of Consumption (U_{ct})

$$U_{ct} - E_t \left[\left(C_t - hC_{t-1} \right)^{-1} - \beta h \left(C_{t+1} - hC_t \right)^{-1} \right] = 0.$$

2. Euler Equation. Consumption-Saving (C_t)

$$\beta E_t \Lambda_{t,t+1} R_{t+1} - 1 = 0.$$

3. Stochastic Discount Factor ($\Lambda_{t,t+1}$)

$$E_t \Lambda_{t,t+1} - E_t \frac{U_{ct+1}}{U_{ct}} = 0.$$

4. Labor Market Equilibrium (L_t)

$$\frac{\chi}{(1-\alpha)} \frac{L_t^{\varphi+1}}{U_{ct}} - P_{mt} Y_{mt} = 0.$$

5. Marginal Return on Bank's Assets (v_t)

$$-v_t + E_t \left(1 - \theta_{t+1} \right) \beta \Lambda_{t,t+1} \left(R_{t+1}^k - R_{t+1} \right) + E_t \theta_{t+1} \beta \Lambda_{t,t+1} x_{t,t+1} v_{t+1} = 0.$$

6. Gross Growth of Bank Assets (x_t)

$$x_{t,t+1} - E_t \frac{\phi_{t+1}}{\phi_t} z_{t,t+1} = 0.$$

7. Marginal Return on Bank's Wealth (η_t)

$$-\eta_t + E\left(1-\theta_{t+1}\right)\beta\Lambda_{t,t+1}R_{t+1} + E_t\theta_{t+1}\beta\Lambda_{t,t+1}z_{t,t+1}\eta_{t+1} = 0.$$

8. Gross Growth of Bank Wealth (z_t)

$$z_{t,t+1} - E_t\left[\left(R^k_{t+1} - R_{t+1}\right)\phi_t + R_{t+1}\right] = 0.$$

9. Leverage Ratio (ϕ_t)

$$\phi_t - \frac{\eta_t}{\lambda - \upsilon_t} = 0.$$

10. Aggregate Capital. Loans (K_t)

$$Q_t K_{t+1} - \phi_t N_t = 0.$$

11. Net worth in the banking sector (N_t)

$$-N_t + N_{et} + N_{nt} = 0.$$

12. Existing wealth (N_{et})

$$N_{et} - \theta_{t-1}\left[\left(R^k_t - R_t\right)\phi_{t-1} + R_t\right]N_{t-1} = 0.$$

13. Wealth of new banks (N_{nt})

$$N_{nt} - wQ_t\xi_t K_t = 0$$

14. Return to capital (R_{kt})

$$R^k_t - \frac{\xi_t}{Q_t - 1}\left[\alpha\frac{P_{mt}Y_{mt}}{\xi_t K_t} + Q_t - \delta_t\right] = 0.$$

15. Production of Intermediate goods (Y_{mt})

$$Y_{mt} - A_t\left(U_t\xi_t K_t\right)^{\alpha} L_t^{1-\alpha} = 0.$$

16. Investment (Q_t)

$$\begin{aligned}
&-Q_t + 1 + \frac{\phi_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \phi_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} \\
&-E_t\beta\Lambda_{t,t+1}\phi_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2 = 0.
\end{aligned}$$

17. Depreciation function (U_t)
$$-\delta_t + \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta} = 0.$$
18. Optimal Capacity Utilization (P_{mt})
$$\alpha P_{mt} Y_{mt} - b U_t^{1+\zeta} \xi_t K_t = 0.$$
19. Net Investment (δ_t)
$$-I_{nt} + I_t - \delta_t \xi_t K_t = 0.$$
20. Law of movement of capital (I_{nt})
$$-K_{t+1} + \xi_t K_t + I_{nt} = 0.$$
21. Exogenous government consumption(G_t)
$$-G_t + G g_t = 0.$$
22. Aggregate resources (I_t)
$$-Y_t + C_t + G_t + I_t + \frac{\phi_i}{2} \left(\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}} - 1 \right)^2 (I_{nt} + I_{ss}) = 0.$$
23. Final Production (Y_t)
$$Y_{mt} - Y_t \Delta_t = 0.$$
24. Price Dispersion (Δ_t)
$$-\Delta_t + (1-\gamma) \left[\frac{1 - \gamma \left(\pi_{t-1}^{\gamma^\rho} \pi_t^{-1} \right)^{1-\varepsilon}}{1-\gamma} \right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma \left[\pi_{t-1}^{\gamma^p} \pi_t^{-1} \right]^{-\varepsilon} \Delta_{t-1} = 0.$$
25. Optimal Price Choice (F_t)
$$-F_t + P_{mt} Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{-\gamma^\rho \varepsilon} \pi_{t+1}^{\varepsilon} F_{t+1} = 0.$$
26. (Z_t)
$$-Z_t + (1-\tau_t) Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{\gamma^\rho (1-\varepsilon)} \pi_{t+1}^{-(1-\varepsilon)} Z_{t+1} = 0.$$
27. Optimal choice of price (π_t^*)

$$\pi_t^* - \frac{\varepsilon}{\varepsilon - 1}\frac{F_t}{Z_t}\pi_t = 0.$$

28. Evolution of inflation (π_t)

$$-\pi_t^{1-\varepsilon} + \left[(1-\gamma)\left(\pi_t^*\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\right)^{1-\varepsilon}\right] = 0.$$

29. Fisher Equation (R_t)

$$i_t - E_t R_t \pi_{t+1} = 0.$$

Then the Ramsey planner solves the above problem choosing at period t processes for the 30 endogenous variables U_{ct}, C_t, Λ_t, L_t, v_t, x_t, η_t, z_t, ϕ_t, K_{t+1}, N_t, N_{et}, N_{nt}, R_{kt}, Y_{mt}, Q_t, δ_t, U_t, I_{nt}, P_{mt}, G_t, I_t, Y_t, Δ_t, F_t, Z_t, π_t^*, π_t, R_t, i_t and the 29 Lagrange multipliers associated with the competitive equilibrium relationships. The 5 exogenous processes for the shocks are given by

30. Technology Shock (A_t)

$$\ln A_t = \rho_a \ln A_{t-1} - \varepsilon_{at}.$$

31. Capital Quality Shock (ξ_t)

$$\ln \xi_t = \rho_\varepsilon \ln \xi_{t-1} - \varepsilon_{\xi t}.$$

32. Government Shock (g_t)

$$\ln g_t = \rho_g \ln g_{t-1} - \varepsilon_{gt}.$$

33. Shock to the Probability of dying (θ_t)

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} - \varepsilon_{\theta t}.$$

34. Markup shock (τ_t)

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right)\ln(1-\tau) - \varepsilon_t^\mu.$$

The values for the variables listed above are given dated t<0, and also the values of the Lagrange multipliers associated with the competitive equilibrium constraints are given at t<0. Then, as explained in Schmitt-Grohé and Uribe (2005) the structure of the optimality conditions associated with the Ramsey equilibrium are time invariant.

Appendix B

Appendix Chapter 2

B.1 Set of Equilibrium Conditions. Unconventional Monetary Policy

List of variables (36).

$$U_{ct}, C_t, R_t, \Lambda_{t,t+1}, L_t, P_{mt}, Y_t$$

$$v_t, R^k_{t+1}, x_{t,t+1}, \phi_t, z_t, \eta_t,$$

$$Q_t, K_t, N_t, N_{et}, N_{nt},$$

$$Y_{mt}, U_t,$$

$$I_{nt}, \delta_t, I_t, G_t, \tau_t,$$

$$\xi_t, g_t, A_t, \lambda_t,$$

$$\Delta_t, \pi_t, F_t, Z_t, \pi^*_t,$$

$$i_t, \kappa^{QE}_t$$

Households.

1. Marginal Utility of Consumption (U_{ct})

$$U_{ct} = E_t \left[(C_t - hC_{t-1})^{-1} - \beta h \left(C_{t+1} - hC_t \right)^{-1} \right].$$

2. Euler Equation. Consumption Saving (C_t)

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = 1.$$

3. Stochastic Discount Factor $(\Lambda_{t,t+1})$

$$E_t \Lambda_{t,t+1} = E_t \frac{U_{ct+1}}{U_{ct}}.$$

4. Labor Market Equilibrium (L_t)

$$\frac{\chi}{(1-\alpha)} \frac{L_t^{1+\varphi}}{U_{ct}} = P_{mt} Y_{mt}.$$

Banks.

5. Marginal Return on Bank's Assets (v_t)

$$v_t = E_t (1-\theta) \beta \Lambda_{t,t+1} \left(R_{t+1}^k - R_{t+1} \right) + E_t \theta \beta \Lambda_{t,t+1} x_{t,t+1} v_{t+1}.$$

6. Gross Growth of Bank Assets $(x_{t,t+1})$

$$x_{t,t+1} = E_t \left[\frac{1-\kappa_{t+1}^{QE}}{1-\kappa_t^{QE}} \right] \left[\frac{\phi_{t+1}^T}{\phi_t^T} \right] z_{t,t+1}.$$

7. Marginal Return on Bank's Wealth (η_t)

$$\eta_t = E (1-\theta) \beta \Lambda_{t,t+1} R_{t+1} + E_t \theta \beta \Lambda_{t,t+1} z_{t,t+1} \eta_{t+1}.$$

8. Gross Growth of Bank Wealth $(z_{t,t+1})$

$$z_{t,t+1} = E_t \left[\left(R_{t+1}^k - R_{t+1} \right) \left(1 - \kappa_t^{QE} \right) \phi_t^T + R_{t+1} \right].$$

9. Leverage Ratio (ϕ_t)

$$\phi_t^T = \frac{1}{1-\kappa_t^{QE}} \frac{\eta_t}{\lambda_t - v_t}.$$

10. Aggregate Capital (K_t)

$$Q_t K_{t+1} = \phi_t^T N_t.$$

11. Net worth in the banking sector (N_t)

$$N_t = N_{et} + N_{nt}.$$

12. Existing wealth (N_{et})

$$N_{et} = \theta E_t \left[\left(R_t^k - R_t \right) \left(1 - \kappa_{t-1}^{QE} \right) \phi_{t-1}^T + R_t \right] N_{t-1}.$$

13. Wealth of new banks (N_{nt})

$$N_{nt} = w \left(1 - \kappa_{t-1}^{QE} \right) Q_t \xi_t K_t.$$

Intermediate Producers.

14. Return to capital (R_{kt})

$$E_t R_{t+1}^k = E_t \frac{\xi_{t+1}}{Q_t} \left[\alpha \frac{P_{mt+1} Y_{mt+1}}{\xi_{t+1} K_{t+1}} + Q_{t+1} - \delta_{t+1} \right].$$

15. Production of Intermediate goods (Y_{mt})

$$Y_{mt} = A_t \left(U_t \xi_t K_t \right)^\alpha L_t^{1-\alpha}.$$

Capital Goods Producers.

16. Investment (Q_t)

$$\begin{aligned} Q_t &= 1 + \frac{\phi_i}{2} \left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + \delta K} \right)^2 + \phi_i \left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + \delta K} \right) \frac{I_{nt} + \delta K}{I_{nt-1} + \delta K} \\ &\quad - E_t \beta \Lambda_{t,t+1} \phi_i \left(\frac{I_{nt+1} - I_{nt}}{I_{nt} + \delta K} \right) \left(\frac{I_{nt+1} + \delta K}{I_{nt} + \delta K} \right)^2. \end{aligned}$$

17. Depreciation function (U_t)

$$\delta_t = \delta_c + \frac{b}{1+\zeta} U_t^{1+\zeta}.$$

18. Optimal Capacity Utilization (P_{mt})

$$\alpha P_{mt} Y_{mt} = b U_t^{1+\zeta} \xi_t K_t.$$

19. Net Investment (δ_t)

$$I_{nt} = I_t - \delta_t \xi_t K_t.$$

20. Law of movement of capital (I_{nt})

$$K_{t+1} = \xi_t K_t + I_{nt}.$$

21. Aggregate resources (I_t)

$$Y_t = C_t + G_t + I_t + \frac{\phi_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + \delta K}\right)^2 (I_{nt} + \delta K) + \varsigma_t.$$

Costs of government participation

$$\varsigma_t = \kappa_t^{QE} Q_t K_{t+1} \tau \left[1 + \frac{1}{2}\left[\frac{\kappa_t^{QE} Q_t K_{t+1}}{\kappa_{t-1}^{QE} Q_{t-1} K_t} - 1\right]^2\right]$$

with

$$Q_t S_t^G = \kappa_t^{QE} Q_t K_{t+1}$$

Retailers

22. Final Production (Y_t)

$$Y_{mt} = Y_t \Delta_t.$$

23. Price Dispersion (Δ_t)

$$\Delta_t = (1-\gamma)\left[\frac{1 - \gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma\left[\pi_{t-1}^{\gamma^p}\pi_t^{-1}\right]^{-\varepsilon}\Delta_{t-1}.$$

24. Optimal Price Choice (F_t)

$$F_t = P_{mt} Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{-\gamma^\rho \varepsilon} \pi_{t+1}^{\varepsilon} F_{t+1}.$$

25. (Z_t)

$$Z_t = (1 - \tau_t)\, Y_t + E_t \gamma \beta \Lambda_{t,t+1} \pi_t^{\gamma^\rho (1-\varepsilon)} \pi_{t+1}^{-(1-\varepsilon)} Z_{t+1}.$$

26. Optimal choice of price (π_t^*)

$$\pi_t^* = \frac{\varepsilon}{\varepsilon - 1}\frac{F_t}{Z_t}\pi_t.$$

27. Evolution of inflation (π_t)

$$\pi_t^{1-\varepsilon} = \left[(1-\gamma)\left(\pi_t^*\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\right)^{1-\varepsilon}\right].$$

Policy and Exogenous Variables.

28. Fisher Equation (R_t)

$$i_t = E_t R_t \pi_{t+1}.$$

29. Monetary policy (i_t)

$$\ln\left[\frac{i_t}{i}\right]=\kappa_R\ln\left[\frac{i_{t-1}}{i}\right]+(1-\kappa_R)\left\{\kappa_\pi\left[\ln\frac{\pi_t}{\pi}\right]+\kappa_Y\left[\frac{Y_t}{Y}\right]\right\}+\varepsilon_{it}.$$

30. Credit Policy $\left(\kappa_t^{QE}\right)$

$$\kappa_t^{QE}=\kappa_{QE}E_t\left[\log\left(\frac{R_t^k}{R_t}\right)-\log\left(\frac{R^k}{R}\right)\right].$$

31. Government Spending (G_t)

$$G_t=Gg_t$$

32. Technology Shock (A_t)

$$\ln A_t=\rho_a\ln A_{t-1}-\varepsilon_{at}.$$

33. Capital Quality Shock (ξ_t)

$$\ln\xi_t=\rho_\varepsilon\ln\xi_{t-1}-\varepsilon_{\xi t}.$$

34. Government Shock (g_t)

$$\ln g_t=\rho_g\ln g_{t-1}-\varepsilon_{gt}.$$

35. Shock to the diverting funds (λ_t)

$$\ln\lambda_t=\rho_\lambda\ln\lambda_{t-1}-\varepsilon_{\lambda t}.$$

36. Markup shock (τ_t)

$$\ln(1-\tau_t)=\rho_\mu\ln(1-\tau_{t-1})+\left(1-\rho_\mu\right)\ln(1-\tau)-\varepsilon_t^\mu.$$

Summary of Variables (36) **and Equations** (36).

B.2 Optimal Unconventional Policy. Timeless Perspective.

The optimal policy problem is solved from a timeless perspective. The Ramsey planner maximizes the discounted utility function subject to the competitive equilibrium conditions. Following Schmitt-Grohé and Uribe (2011), the portion of the Lagrangian that is relevant for optimal policy from a timeless perspective is

$$\mathcal{L}^R = E_0 \left[\sum_{\tau=0}^{\infty} \beta^{\tau} U_{\tau} \left[C_{\tau}, L_{\tau} \right] + \sum_{\tau=0}^{\infty} \beta^{\tau} Lm_{\tau}' \mathbf{C}_{\tau} \left(\cdot \right) \right] \tag{B.1}$$

where the period t utility function is

$$U_t = \ln \left(C_t - hC_{t-1} \right) - \frac{\chi}{1+\varphi} L_t^{1+\varphi}$$

(β) is the Ramsey planner's discount factor, which I assume to be identical to that of the competitive equilibrium. The vector $\left[Lm_t^{'} \right]$ contains the 29 Lagrange multiplier associated with the 29 equilibrium conditions in period t $\mathbf{C}_t \left(\cdot \right)$. Those equilibrium conditions in period t are:

1. Marginal Utility of Consumption (U_{ct})

$$U_{ct} - E_t \left[\left(C_t - hC_{t-1} \right)^{-1} - \beta h \left(C_{t+1} - hC_t \right)^{-1} \right] = 0.$$

2. Euler Equation. Consumption-Saving (C_t)

$$\beta E_t \Lambda_{t,t+1} R_{t+1} - 1 = 0.$$

3. Stochastic Discount Factor ($\Lambda_{t,t+1}$)

$$E_t \Lambda_{t,t+1} - E_t \frac{U_{ct+1}}{U_{ct}} = 0.$$

4. Labor Market Equilibrium (L_t)

$$\frac{\chi}{(1-\alpha)} \frac{L_t^{\varphi+1}}{U_{ct}} - P_{mt} Y_{mt} = 0.$$

5. Marginal Return on Bank's Assets (υ_t)

$$-\upsilon_t + E_t \left(1 - \theta_{t+1} \right) \beta \Lambda_{t,t+1} \left(R_{t+1}^k - R_{t+1} \right) + E_t \theta_{t+1} \beta \Lambda_{t,t+1} x_{t,t+1} \upsilon_{t+1} = 0.$$

6. Gross Growth of Bank Assets (x_t)

$$x_{t,t+1} - E_t \frac{\phi_{t+1}}{\phi_t} z_{t,t+1} = 0.$$

7. Marginal Return on Bank's Wealth (η_t)

$$-\eta_t + E\left(1-\theta_{t+1}\right) \beta \Lambda_{t,t+1} R_{t+1} + E_t \theta_{t+1} \beta \Lambda_{t,t+1} z_{t,t+1} \eta_{t+1} = 0.$$

8. Gross Growth of Bank Wealth (z_t)

$$z_{t,t+1} - E_t \left[\left(R^k_{t+1} - R_{t+1} \right) \phi_t + R_{t+1} \right] = 0.$$

9. Leverage Ratio (ϕ_t)

$$\phi_t - \frac{\eta_t}{\lambda - \upsilon_t} = 0.$$

10. Aggregate Capital. Loans (K_t)

$$Q_t K_{t+1} - \phi_t N_t = 0.$$

11. Net worth in the banking sector (N_t)

$$-N_t + N_{et} + N_{nt} = 0.$$

12. Existing wealth (N_{et})

$$N_{et} - \theta_{t-1} \left[\left(R^k_t - R_t \right) \phi_{t-1} + R_t \right] N_{t-1} = 0.$$

13. Wealth of new banks (N_{nt})

$$N_{nt} - w Q_t \xi_t K_t = 0$$

14. Return to capital (R_{kt})

$$R^k_t - \frac{\xi_t}{Q_t - 1} \left[\alpha \frac{P_{mt} Y_{mt}}{\xi_t K_t} + Q_t - \delta_t \right] = 0.$$

15. Production of Intermediate goods (Y_{mt})

$$Y_{mt} - A_t \left(U_t \xi_t K_t \right)^{\alpha} L_t^{1-\alpha} = 0.$$

16. Investment (Q_t)

$$
\begin{aligned}
&-Q_t + 1 + \frac{\phi_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 + \phi_i\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} \\
&-E_t\beta\Lambda_{t,t+1}\phi_i\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}} - 1\right)\left(\frac{I_{nt+1}+I_{ss}}{I_{nt}+I_{ss}}\right)^2 = 0.
\end{aligned}
$$

17. Depreciation function (U_t)

$$
-\delta_t + \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta} = 0.
$$

18. Optimal Capacity Utilization (P_{mt})

$$
\alpha P_{mt} Y_{mt} - bU_t^{1+\zeta}\xi_t K_t = 0.
$$

19. Net Investment (δ_t)

$$
-I_{nt} + I_t - \delta_t \xi_t K_t = 0.
$$

20. Law of movement of capital (I_{nt})

$$
-K_{t+1} + \xi_t K_t + I_{nt} = 0.
$$

21. Exogenous government consumption(G_t)

$$
-G_t + Gg_t = 0.
$$

22. Aggregate resources (I_t)

$$
-Y_t + C_t + G_t + I_t + \frac{\phi_i}{2}\left(\frac{I_{nt}+I_{ss}}{I_{nt-1}+I_{ss}} - 1\right)^2 (I_{nt}+I_{ss}) + \tau\kappa_t^{QE} Q_t K_{t+1}\left[\frac{1}{2}\left[\frac{\kappa_t^{QE} Q_t K_{t+1}}{\kappa_{t-1}^{QE} Q_{t-1} K_t} - 1\right]^2\right] = 0.
$$

23. Final Production (Y_t)

$$
Y_{mt} - Y_t \Delta_t = 0.
$$

24. Price Dispersion (Δ_t)

$$
-\Delta_t + (1-\gamma)\left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma\left[\pi_{t-1}^{\gamma^p}\pi_t^{-1}\right]^{-\varepsilon}\Delta_{t-1} = 0.
$$

25. Optimal Price Choice (F_t)

$$-F_t + P_{mt}Y_t + E_t\gamma\beta\Lambda_{t,t+1}\pi_t^{-\gamma^\rho\varepsilon}\pi_{t+1}^{\varepsilon}F_{t+1} = 0.$$

26. (Z_t)

$$-Z_t + (1-\tau_t)\,Y_t + E_t\gamma\beta\Lambda_{t,t+1}\pi_t^{\gamma^\rho(1-\varepsilon)}\pi_{t+1}^{-(1-\varepsilon)}Z_{t+1} = 0.$$

27. Optimal choice of price (π_t^*)

$$\pi_t^* - \frac{\varepsilon}{\varepsilon-1}\frac{F_t}{Z_t}\pi_t = 0.$$

28. Evolution of inflation (π_t)

$$-\pi_t^{1-\varepsilon} + \left[(1-\gamma)\,(\pi_t^*)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^\rho}\right)^{1-\varepsilon}\right] = 0.$$

29. Fisher Equation (R_t)

$$i_t - E_t R_t \pi_{t+1} = 0.$$

Then the Ramsey planner solves the above problem choosing at period t processes for the 31 endogenous variables U_{ct}, C_t, Λ_t, L_t, v_t, x_t, η_t, z_t, ϕ_t, K_{t+1}, N_t, N_{et}, N_{nt}, R_{kt}, Y_{mt}, Q_t, δ_t, U_t, I_{nt}, P_{mt}, G_t, I_t, Y_t, Δ_t, F_t, Z_t, π_t^*, π_t, R_t, i_t, $\kappa_t^{QE}$and the 29 Lagrange multipliers associated with the competitive equilibrium relationships. The exogenous processes for the shocks are given by

30. Technology Shock (A_t)

$$\ln A_t = \rho_a \ln A_{t-1} - \varepsilon_{at}.$$

31. Capital Quality Shock (ξ_t)

$$\ln \xi_t = \rho_\varepsilon \ln \xi_{t-1} - \varepsilon_{\xi t}.$$

32. Government Shock (g_t)

$$\ln g_t = \rho_g \ln g_{t-1} - \varepsilon_{gt}.$$

33. Shock to the diverting funds (λ_t)

$$\ln \lambda_t = \rho_\lambda \ln \lambda_{t-1} - \varepsilon_{\lambda t}.$$

34. Markup shock (τ_t)

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right)\ln(1-\tau) - \varepsilon_t^\mu.$$

The values for the variables listed above are given dated t<0, and also the values of the Lagrange multipliers associated with the competitive equilibrium constraints are given at t<0. Then, as explained in Schmitt-Grohé and Uribe (2005) the structure of the optimality conditions associated with the Ramsey equilibrium are time invariant.

B.3 Calibration of The Asset Purchases Rule.

The asset purchases rule is:

$$\kappa_t^{QE} = \kappa_0 + \kappa_{QE} E_t \left[\log\left(\frac{R_t^k}{R_t}\right) - \log\left(\frac{R^k}{R}\right) \right], \tag{B.2}$$

given the lack of studies calibrating such a rule, I follow the next strategy in order to calibrate this rule. Once the rule is calibrated, I use it to find the efficiency costs associated with the central bank's intermediation.

The policy rule in (B.2) has two parameters κ_0 and κ_{QE}. I take evidence on the size of the FED's QE1 program and the change in the spread $\left(SP_t = \frac{R_t^k}{R_t} - \frac{R^k}{R}\right)$ associated with it. The evidence is taken for two different observations.

The first point is when the shock hits, and after the FED had implemented QE1 the spread was (SP_t^1). The share of assets held by the FED at that time was κ_t^1. Then the rule (B.2) can be written

$$\kappa_t^1 = \kappa_0 + \kappa_{QE} SP_t^1. \tag{B.3}$$

The empirical evidence (Gagnon et al. (2011)) suggest that the FED increased to 22% its participation in the private assets markets. Hence, κ_t^1 takes that value.

The second points is before the implementation of QE1 but after the shock hit, hence, the policy rule in (B.2) can be written as:

$$\kappa_t^0 = \kappa_0 + \kappa_{QE} SP_t^0, \tag{B.4}$$

at that time the FED held zero percent of private securities. Hence, $\kappa_t^0 = 0$. The system (B.3, B.4) can be solved for the policy coefficients $[\kappa_0,\ \kappa_{QE}]$ using evidence on (SP_t^0 and SP_t^1). I rely on empirical studies that estimated the impact of the QE1 on reducing the spread. In particular, these studies provide information for calculating ($\Delta SP = SP^0 - SP^1$). With that information, I can solve the system (B.3, B.4) and obtain the policy coefficients. Table 2 presents the information

Table B.1. Reduction in the Spread and Calibration of κ_0 and κ_{QE}

		$\Delta^{-}SP$ (*bp.*)	κ_{QE}
1.	BAA-Mortgages (Nov08-March10)	182	12.1
2.	Gagnon et al. (2011)	82	26.8
3.	D'Amico and King (2010)	50	44.0
4.	Hancock and Passmore (2011)	125	17.6
5.	Neely (2010)	94	23.4
6.	Average	107	24.8

The first column in table B.1 shows the reduction in the spread once the FED implemented QE1, $(SP^0 - SP^1)$. The second column shows the feedback coefficient (κ_{QE}) associated with each piece of evidence. For example, take the row 1 in the table, if the FED reduced the spread in 182 basis points after having implemented QE1, the policy coefficients in the rule would be $\kappa_0 = 3.6$ and $\kappa_{QE} = 12.1$. Across the studies presented in table B.1, the average reduction in the spread after implementing QE1 was 107 basis points. After the implementation of QE1, the FED had purchased about 22 percent of long-term, agency debt, MBS and Treasury securities outstanding at the beginning of the asset purchases program (Gagnon et al. (2011)). This implies that average value for the feedback coefficients implied by the calculations in Table B.1, is: κ_{QE} is 24.8 and $\kappa_{QE} = 30.5$.

This value $\kappa_{QE} = 24.8$ means that for each basis point that the spread increased over its long-run level the FED increased its participation in the financial markets on 24.8 basis points. I utilize that value of κ_{QE} to calibrate the rule in the benchmark case.

B.3.1 Efficiency Costs.

In order to calibrate the efficiency costs, I assume that the FED followed a simple rule to implement asset purchases during the past crisis. Additionally, I assume that the FED's reaction was optimal. I utilize the average of the parameters calculated in table 2 ($\kappa_{QE} = 24.8$ and $\kappa_{QE} = 30.5$) in order to calibrate the linear rule:

$$\kappa_t^{QE} = \kappa_0 + \kappa_{QE} E_t \left[\log\left(\frac{R_t^k}{R_t}\right) - \log\left(\frac{R^k}{R}\right) \right]. \tag{B.5}$$

Then, I search for the value of efficiency costs (τ) which make that rule optimal. I assume that if the FED did not increased the QE1 above 22% of assets, was because the efficiency costs impede it.

For example, if the efficiency costs are close to zero $(\tau = 0.0001)$ the feedback coefficient (κ_{QE}) reaches a value of 2500, which is quite far from the evidence suggested by the calibrated rule $(\kappa_{QE} = 24.8)$. However, if implementing this unconventional policy is costlier, then the magnitude of this feedback coefficient decreases.

Table B.2 presents the efficiency costs which make the rule with feedback coefficients $\kappa_{QE} = 24.8$ and $\kappa_{QE} = 30.5$, optimal:

Table B.2. Efficiency Costs and Linear Rule.

	κ_{QE}	τ
1.	24.8	0.0025

this is the efficiency cost used as benchmark.

Appendix C

Appendix Chapter 3

C.1 Summary of Equilibrium Conditions.

Households.

1. Marginal Utility of Consumption. External $\left(U_{ct}^{Ex}\right)$

$$U_{ct}^{Ex} = E_t \left[\frac{1}{\left(C_t^{Ex} - h^{Ext} C_{t-1}^{Ex}\right)^{\sigma}} \right].$$

2. Marginal Utility of Consumption. Internal $\left(U_{ct}^{I}\right)$

$$U_{ct}^{I} = E_t \left[\frac{1}{\left(C_t^{I} - h^{Int} C_{t-1}^{I}\right)^{\sigma}} - \frac{\beta h^{Int}}{\left(C_{t+1}^{I} - h^{Int} C_t^{I}\right)^{\sigma}} \right].$$

3. Euler Equation. Consumption Saving. External $\left(C_t^{Ex}\right)$

$$1 = \beta^{Ext} E_t \frac{U_{ct+1}^{Ex}}{U_{ct}^{Ex}} R_{t+1}^{b}.$$

4. Euler Equation. Consumption Saving. Internal $\left(C_t^{I}\right)$

$$1 = \beta E_t \frac{U_{ct+1}^{I}}{U_{ct}^{I}} R_t.$$

5. Stochastic Discount Factor $(\Lambda_{t,t+1})$

$$\Lambda_{t,t+1} = E_t \left[\Upsilon_{Ext}^{ownership} \beta^{Ext} \frac{U_{ct+1}^{Ex}}{U_{ct}^{Ex}} + \left(1 - \Upsilon_{Ext}^{ownership}\right) \beta \frac{U_{ct+1}^{I}}{U_{ct}^{I}} \right]$$

6. Aggregate Labour (L_t)

$$L_t = \left\{ \left[(\Upsilon) \left(\frac{U_{ct}^{Ex}}{\chi^{Ex}}\right)^{\frac{1}{\varphi}} + (1-\Upsilon) \left(\frac{U_{ct}^{I}}{\chi}\right)^{\frac{1}{\varphi}} \right] \left[(1-\alpha) P_{mt} A_t \left(U_t \xi_t K_t\right)^{\alpha}\right]^{\frac{1}{\varphi}} \right\}^{\frac{\varphi}{\varphi+\alpha}}$$

$$L_t = \Upsilon L_t^{Ex} + (1-\Upsilon) L_t^I$$

7. Labour. External $\left(L_t^{Ex}\right)$

$$L_t^{Ex} = \left[(1-\alpha)\frac{U_{ct}^{Ex}}{\chi^{Ex}}\frac{P_{mt}Y_{mt}}{L_t}\right]^{\frac{1}{\varphi}}.$$

8. Labour. Internal $\left(L_t^I\right)$

$$L_t^I = \left[(1-\alpha)\frac{U_{ct}^I}{\chi^{Int}}\frac{P_{mt}Y_{mt}}{L_t}\right]^{\frac{1}{\varphi}}.$$

Banks.

9. Marginal Return on Bank's Loans to Firms $\left(\upsilon_t^f\right)$

$$\nu_t^f = E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^k - \frac{R_t}{\alpha_t^{res}}\right).$$

10.Arbitrage Between bank's assets. Marginal Return on Bank's Loans to Households $\left(R_t^b\right)$

$$E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^b - \frac{R_t}{\alpha_t^{res}}\right) = \Delta E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^k - \frac{R_t}{\alpha_t^{res}}\right)$$

11. Marginal Return on Bank's Wealth (η_t)

$$\eta_t = E_t\Lambda_{t,t+1}\Omega_{t+1}\frac{R_t}{\alpha_t^{res}}.$$

12. Bank's Discount Factor (Ω_{t+1})

$$\Omega_{t+1} = (1-\theta) + \theta\left(\nu_{t+1}^f\phi_{t+1} + \eta_{t+1}\right).$$

13. Leverage Ratio (ϕ_t)

$$\phi_t = \frac{\eta_t}{\lambda - \upsilon_t^f}.$$

14. Loans to firms $\left(S_t^f\right)$

$$Q_t S_t^f = \phi_t N_t - \Delta q_t S_t^{Ext}.$$

15. Aggregate Capital (K_{t+1})

$$Q_t\xi_t K_{t+1} = Q_t S_t^f$$

16. Loans to Households $\left(S_t^{Ext}\right)$

$$\Delta q_t S_t^{Ext} = \Upsilon B_t^{Ext}$$

17. Households Borrowing $\left(B_t^{Ext}\right)$

$$W_t L_t^{Ex} + \Pi_t^{Ex} - T_t^{Ex} - R_t^b \left[q_{t-1} B_{t-1}^{Ex}\right] + \left[q_t B_t^{Ex}\right] = C_t^{Ex}$$

18. Household Profits $\left(\Pi_t^{Ex}\right)$

$$\begin{aligned}\frac{\Pi_t^{Ex}}{\Upsilon_{Ext}^{Ownership}} &= (1-\theta)\left[\left(R_{t-1}^k - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) Q_{t-1} S_{t-1}^f + \left(R_{t-1}^b - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) q_{t-1} S_{t-1}^{Ext} + \frac{R_{t-1}}{\alpha_{t-1}^{res}} N_{t-1}\right] \\ &- w\left[Q_{t-1} S_{t-1}^f + q_{t-1} \Delta S_{t-1}^{Ext}\right] \\ &+ (Q_t - 1) I_{nt} - \frac{\phi_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 (I_{nt} + I_{ss}) + \\ &[(1-\tau_t)\pi_t^* - P_{mt}] Y_t\end{aligned}$$

19. Transfers $\left(T_t^{Ext}\right)$

$$T_t^{Ext} = \Upsilon_{Ext}^{transfers}\left[G - \tau_t Y_t\right].$$

20. Net worth in the banking sector (N_t)

$$\begin{aligned}N_t = \theta &\left[\left(R_{t-1}^k - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) Q_{t-1} S_{t-1}^f + \left(R_{t-1}^b - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) q_{t-1} S_{t-1}^{Ext} + \frac{R_{t-1}}{\alpha_{t-1}^{res}} N_{t-1}\right] \\ &+ w\left(Q_{t-1} S_{t-1}^f + \Delta q_{t-1} S_{t-1}^{Ext}\right)\end{aligned}$$

21. Aggregate deposits (D_{t-1})

$$D_{t-1} = Q_{t-1} K_t + q_{t-1} \Delta \Upsilon B_{t-1}^{Ext} - N_{t-1}$$

Intermediate Producers.

22. Return to capital (R_{kt})

$$E_t R_{t+1}^k = E_t \frac{\xi_{t+1}}{Q_t}\left[\alpha \frac{P_{mt+1} Y_{mt+1}}{\xi_{t+1} K_{t+1}} + (Q_{t+1} - \delta_{t+1})\right].$$

23. Return to loans to Consumption $\left(q_t^b\right)$

$$E_t R_{t+1}^b = E_t \frac{\xi_{t+1}}{q_t}\left[\alpha \frac{P_{mt+1} Y_{mt+1}}{\xi_{t+1} K_{t+1}} + (q_{t+1} - \delta_{t+1})\right].$$

24. Production of Intermediate goods (Y_{mt})

$$Y_{mt} = A_t \left(U_t \xi_t K_t\right)^{\alpha} L_t^{1-\alpha}.$$

Capital Goods Producers.

25. Investment (Q_t)

$$Q_t = 1 + \frac{\phi_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 + \phi_i\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)\frac{I_{nt} + I_{ss}}{I_{nt-1} + I_{ss}}$$
$$- E_t\beta\Lambda_{t,t+1}\phi_i\left(\frac{I_{nt+1} - I_{nt}}{I_{nt} + I_{ss}}\right)\left(\frac{I_{nt+1} + I_{ss}}{I_{nt} + I_{ss}}\right)^2.$$

26. Depreciation function (δ_t)

$$\delta_t = \delta_c + \frac{b}{1+\zeta}U_t^{1+\zeta}.$$

27. Optimal Capacity Utilization (P_{mt})

$$\alpha P_{mt}Y_{mt} = bU_t^{1+\zeta}\xi_t K_t.$$

28. Net Investment (I_{nt})

$$I_{nt} = I_t - \delta_t\xi_t K_t.$$

29. Law of movement of capital (U_t)

$$K_{t+1} = \xi_t K_t + I_{nt}.$$

30. Aggregate resources (I_t)

$$Y_t = \Upsilon C_t^{Ex} + [1-\Upsilon]\, C_t^I + G + I_t + \frac{\phi_i}{2}\left(\frac{I_{nt} - I_{nt-1}}{I_{nt-1} + I_{ss}}\right)^2 (I_{nt} + I_{ss})\,.$$

Retailers

31. Final Production (Y_t)

$$Y_{mt} = Y_t\Delta_t.$$

32. Price Dispersion (Δ_t)

$$\Delta_t = (1-\gamma)\left[\frac{1-\gamma\left(\pi_{t-1}^{\gamma^\rho}\pi_t^{-1}\right)^{1-\varepsilon}}{1-\gamma}\right]^{\frac{-\varepsilon}{1-\varepsilon}} + \gamma\left[\pi_{t-1}^{\gamma^p}\pi_t^{-1}\right]^{-\varepsilon}\Delta_{t-1}.$$

33. Optimal Price Choice (F_t)

$$F_t = P_{mt}Y_t + E_t\gamma\Lambda_{t,t+1}\pi_t^{-\gamma^\rho\varepsilon}\pi_{t+1}^{\varepsilon}F_{t+1}.$$

34. (Z_t)

$$Z_t = (1-\tau_t)\,Y_t + E_t\gamma\Lambda_{t,t+1}\pi_t^{\gamma^{\rho}(1-\varepsilon)}\pi_{t+1}^{-(1-\varepsilon)}Z_{t+1}.$$

35. Evolution of inflation (π_t)

$$\pi_t^{1-\varepsilon} = \left[(1-\gamma)\left(\frac{\varepsilon}{\varepsilon-1}\frac{F_t}{Z_t}\pi_t\right)^{1-\varepsilon} + \gamma\left(\pi_{t-1}^{\gamma^{\rho}}\right)^{1-\varepsilon}\right].$$

36. Fisher Equation (R_t)

$$i_t = E_t R_t \pi_{t+1}.$$

37. Wage (W_t)

$$W_t = (1-\alpha)\,P_{mt}\frac{Y_{mt}}{L_t}$$

38. Optimal Price (π_t^*)

$$\pi_t^* = \frac{\varepsilon}{\varepsilon-1}\frac{F_t}{Z_t}\pi_t$$

39. Technology Shock (A_t)

$$\ln A_t = \rho_a \ln A_{t-1} - \varepsilon_{at}.$$

40. Capital Quality Shock (ξ_t)

$$\ln \xi_t = \rho_\varepsilon \ln \xi_{t-1} - \varepsilon_{\xi t}.$$

41. Markup shock (τ_t)

$$\ln(1-\tau_t) = \rho_\mu \ln(1-\tau_{t-1}) + \left(1-\rho_\mu\right)\ln(1-\tau) - \varepsilon_t^{\mu}.$$

Summary of Variables (42) **and Equations** (41) .Nominal interest rate and any other policy instrument are determined optimally or via a Taylor Rule.

C.2 Profits and Deposits In Steady State.

C.2.1 Profits of the Firms.

To determine the household deposits in steady state, I need to determine the profits that enter the budget constraint. In the economy three groups of firms produce profits.

Banks. The banks return every period to the households the amount of resources intermediated by the $(1-\theta)$ exiting banks

$$(1-\theta)\left[\left(R_{t-1}^{k}-\frac{R_{t-1}}{\alpha_{t-1}^{res}}\right)Q_{t-1}S_{t-1}^{f}+\left(R_{t-1}^{b}-\frac{R_{t-1}}{\alpha_{t-1}^{res}}\right)q_{t-1}S_{t-1}^{Ext}+\frac{R_{t-1}}{\alpha_{t-1}^{res}}N_{t-1}\right],$$

where $Q_{t-1}S_{t-1}^{f}+q_{t-1}S_{t-1}^{Ext}=Q_{t-1}K_{t}+q_{t-1}\Upsilon B_{t-1}^{Ext}$. At the same time, the household transfer (w) as startup funds. Hence, bank's profits are:

$$\Pi_{t}^{Bank}=(1-\theta)\left[\left(R_{t-1}^{k}-\frac{R_{t-1}}{\alpha_{t-1}^{res}}\right)Q_{t-1}S_{t-1}^{f}+\left(R_{t-1}^{b}-\frac{R_{t-1}}{\alpha_{t-1}^{res}}\right)q_{t-1}S_{t-1}^{Ext}+\frac{R_{t-1}}{\alpha_{t-1}^{res}}N_{t-1}\right] \tag{C.1}$$

$$-w\left[Q_{t-1}K_{t}+q_{t-1}\Delta\Upsilon B_{t-1}^{Ext}\right]. \tag{C.2}$$

Capital Goods Producers. Capital goods producers have profits equal to

$$\Pi_{t}^{Capital}=(Q_{t}-1)\,I_{nt}-\frac{\phi_{i}}{2}\left(\frac{I_{nt}-I_{nt-1}}{I_{nt-1}+I_{ss}}\right)^{2}(I_{nt}+I_{ss}), \tag{C.3}$$

which is the function they maximize every period. In steady state those profits are zero.

Retailers. The retailers maximize every period the function

$$\Pi_{t}^{\mathrm{Ret}}=\left[(1-\tau_{t})\,\frac{P_{t}^{*}}{P_{t}}-P_{mt}\right]Y_{t}^{f}, \tag{C.4}$$

where τ_{t} is assumed to be zero in steady state and it serves only as a cost-push shock. And the aggregate demand is

$$Y_{t}^{f}=\left[\Upsilon C_{t}^{Ex}+[1-\Upsilon]\,C_{t}^{I}+G+I_{t}+\frac{\phi_{i}}{2}\left(\frac{I_{t}}{I_{t-1}}-1\right)^{2}(I_{t})\right] \tag{C.5}$$

C.2.2 Profits in Steady State.

In steady state the profits of the capital produces are zero. Bank's profits are:

$$\Pi^{Bank} = (1-\theta)\left[\left(R^k - \frac{R}{\alpha^{res}}\right)QS^f + \left(R^b - \frac{R}{\alpha^{res}}\right)qS^{Ext} + \frac{R}{\alpha^{res}}N\right] \tag{C.6}$$

$$- w\left[Q_{t-1}K_t + q_{t-1}\Delta\Upsilon B^{Ext}_{t-1}\right]. \tag{C.7}$$

And for the retailers

$$\Pi^{\text{Ret}} = [1-\tau-P_m]\left[\Upsilon C^{Ex} + [1-\Upsilon]\,C^I + G + I\right]. \tag{C.8}$$

C.2.3 Deposits In Steady State.

Total deposits in the economy are

$$D = QK + \Delta\Upsilon B^{Ext} - N. \tag{C.9}$$

The households with external habits have steady-state borrowing of

$$B^{Ex} = \frac{\beta^{Ext}}{\beta^{Ext}-1}\left[C^{Ex} + T^{Ex} - \Pi^{Ex} - WL^{Ex}\right]. \tag{C.10}$$

After substituting the profits in steady state, aggregate borrowing of this group is

$$\begin{aligned}
&\Upsilon B^{Ex}\left[1+\left[\frac{1}{1-R^b}\right]\Upsilon^{Ownership}\left\{q\,(1-\theta)\left(R^b - \frac{R}{\alpha^{res}} + \frac{\Delta R}{\alpha^{res}\phi} - \Delta\frac{w}{1-\theta}\right)\right\}\right] \\
&= \left[\frac{1}{1-R^b}\right]\Upsilon\left[C^{Ex}_t + \frac{T^{Ex}}{\Upsilon} - WL^{Ex}\right] \\
&- \left[\frac{1}{1-R^b}\right]\Upsilon^{Ownership}\left\{(1-\theta)\left(R^k - \frac{R}{\alpha^{res}} + \frac{R}{\alpha^{res}\phi} - \frac{w}{(1-\theta)}\right)K\right\} \\
&- \left[\frac{1}{1-R^b}\right]\Upsilon^{Ownership}\left[(1-\tau-P_m)\,Y\right],
\end{aligned}$$

C.3 Bank's Maximization Problem.

C.3.1 Bank's Maximization.

The banks optimally choose the assets $\left(S_{jt}^{f}\ ,\ S_{jt}^{Ext}\right)$ to maximize

$$N_t = \left(R_{t-1}^{k} - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) Q_{t-1} S_{t-1}^{f} + \left(R_{t-1}^{b} - \frac{R_{t-1}}{\alpha_{t-1}^{res}}\right) q_{t-1} S_{jt-1}^{Ext} + \frac{R_{t-1}}{\alpha_{t-1}^{res}} N_{t-1}$$

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\left[\theta^{i}\right] \Lambda_{t,t+1+i} \left(\begin{array}{c} \left[R_t^{k} - \frac{R_{t-1}}{\alpha_{t-1}^{res}} R_t\right] Q_t S_{jt}^{f} + \left[R_t^{b} - \frac{R_{t-1}}{\alpha_{t-1}^{res}} R_t\right] q_t S_{jt}^{Ext} \\ + \frac{R_{t-1}+\tau_{t-1}^{s}}{\alpha_{t-1}^{res}} N_{jt} \end{array} \right), \quad \text{(C.11)}$$

subject to the incentive constraint (eq. 3.18). The conjectured solution is

$$V_{jt} = \nu_t^{f} Q_t S_{jt}^{f} + \nu_t^{Ext} q_t S_{jt}^{Ext} + \eta_t N_{jt} \quad \text{(C.12)}$$

Using the incentive constraint (eq. 3.18), the Lagrangian for this problem is

$$L = V_{jt} + \mu_t \left[V_{jt} - \lambda\left(Q_t S_{jt}^{f} + \Delta q_t S_{jt}^{Ext}\right)\right],$$

where (μ_t) is the Lagrange multiplier associated with the incentive constraint. Substituting the proposed solution (eq. 3.23)

$$L = [1+\mu_t]\left[\nu_t^{f} Q_t S_{jt}^{f} + \nu_t^{Ext} q_t S_{jt}^{Ext} + \eta_t N_{jt}\right] - \mu_t \lambda \left(Q_t S_{jt}^{f} + \Delta q_t S_{jt}^{Ext}\right),$$

each period the bank chooses $\left(S_{jt}^{f} \text{ and } S_{jt}^{Ext}\right)$. The first order conditions are:

$$S_{jt}^{f} : [1+\mu_t]\, Q_t \nu_t^{f} - \mu_t \lambda Q_t = 0, \quad \text{(C.13)}$$

$$S_{jt}^{Ext} : [1+\mu_t]\, \nu_t^{Ext} q_t - \mu_t \lambda \Delta q_t = 0 \quad \text{(C.14)}$$

and respect to the Lagrange multiplier

$$\nu_t^{f} Q_t S_{jt}^{f} + \nu_t^{Ext} q_t S_{jt}^{Ext} + \eta_t N_{jt} = \lambda\left(Q_t S_{jt}^{f} + \Delta q_t S_{jt}^{Ext}\right) \quad \text{(C.15)}$$

This means that

$$\nu_t^{f} = \left[\frac{\mu_t}{1+\mu_t}\right]\lambda, \quad \text{(C.16)}$$

and that

$$\nu_t^{Ext} = \Delta \left[\frac{\mu_t}{1+\mu_t} \right] \lambda. \tag{C.17}$$

The bank would like to equate the marginal return of its assets, hence from C.16 and C.17

$$\nu_t^{Ext} = \Delta \nu_t^f. \tag{C.18}$$

Substituting the last equation in C.15

$$\nu_t^f \left(Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext} \right) + \eta_t N_{jt} = \lambda \left(Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext} \right). \tag{C.19}$$

Solving C.19 for (η_t)

$$\frac{\eta_t}{\lambda - \nu_t^f} = \frac{Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext}}{N_{jt}}, \tag{C.20}$$

which is the bank's maximum leverage ratio (ϕ_t). And it can be written as

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t^f} \tag{C.21}$$

Now, use the Bellman equation

$$V_{jt} = \max E_t \sum_{i=0}^{\infty} (1-\theta)\, \theta^i \Lambda_{t,t+1+i} \left(N_{jt+1+i} \right),$$

and express it as

$$V_{jt} = E_t \Lambda_{t,t+1} \left[(1-\theta) + \theta \left(\frac{V_{jt+1}}{N_{jt+1}} \right) \right] N_{jt+1}. \tag{C.22}$$

Use the proposed solution (eq. 3.23) in the previous equation

$$\nu_t^f Q_t S_{jt}^f + \nu_t^{Ext} q_t S_{jt}^{Ext} + \eta_t N_{jt} \tag{C.23}$$
$$= E_t \Lambda_{t,t+1} \left[(1-\theta) + \theta \left(\frac{\nu_{t+1}^f Q_{t+1} S_{jt+1}^f + \nu_{t+1}^{Ext} q_{t+1} S_{jt+1}^{Ext} + \eta_{t+1} N_{jt+1}}{N_{jt+1}} \right) \right] N_{jt+1}, \tag{C.24}$$

insert the maximization condition C.18 in C.23 and simplify

$$\nu_t^f Q_t S_{jt}^f + \nu_t^{Ext} q_t S_{jt}^{Ext} + \eta_t N_{jt}$$
$$= E_t \Lambda_{t,t+1} \left[(1-\theta) + \theta \left(\frac{\nu_{t+1}^f Q_{t+1} S_{jt+1}^f + \Delta \nu_t^f q_{t+1} S_{jt+1}^{Ext} + \eta_{t+1} N_{jt+1}}{N_{jt+1}} \right) \right] N_{jt+1}, \tag{C.25}$$

$$\nu_t^f\left(Q_tS_{jt}^f+\Delta q_tS_{jt}^{Ext}\right)+\eta_tN_{jt}$$
$$=E_t\Lambda_{t,t+1}\left[(1-\theta)+\theta\left(\frac{\nu_{t+1}^f\left(Q_{t+1}S_{jt+1}^f+\Delta q_{t+1}S_{jt+1}^{Ext}\right)+\eta_{t+1}N_{jt+1}}{N_{jt+1}}\right)\right]N_{jt+1}, \tag{C.26}$$

$$\nu_t^fQ_tS_{jt}^f+\nu_t^{Ext}q_tS_{jt}^{Ext}+\eta_tN_{jt}$$
$$=E_t\Lambda_{t,t+1}\left[(1-\theta)+\theta\left(\nu_{t+1}^f\frac{Q_{t+1}S_{jt+1}^f+\Delta q_{t+1}S_{jt+1}^{Ext}}{N_{jt+1}}+\eta_{t+1}\right)\right]N_{jt+1}. \tag{C.27}$$

Use the bank's maximum leverage ratio to substitute in the previous equation

$$\begin{aligned}&\nu_t^fQ_tS_{jt}^f+\nu_t^{Ext}q_tS_{jt}^{Ext}+\eta_tN_{jt}\\&=E_t\Lambda_{t,t+1}\left[(1-\theta)+\theta\left(\nu_{t+1}^f\phi_{t+1}+\eta_{t+1}\right)\right]N_{jt+1}.\end{aligned} \tag{C.28}$$

Define the bank's adjusted stochastic discount factor as:

$$\Omega_{t+1}=(1-\theta)+\theta\left(\nu_{t+1}^f\phi_{t+1}+\eta_{t+1}\right). \tag{C.29}$$

Substitute C.29 in C.28

$$\nu_t^fQ_tS_{jt}^f+\nu_t^{Ext}q_tS_{jt}^{Ext}+\eta_tN_{jt}=E_t\Lambda_{t,t+1}\Omega_{t+1}N_{jt+1}. \tag{C.30}$$

Now, insert the evolution of wealth (eq. 3.17) in the previous condition

$$\begin{aligned}&\nu_t^fQ_tS_{jt}^f+\nu_t^{Ext}q_tS_{jt}^{Ext}+\eta_tN_{jt}\\&=E_t\Lambda_{t,t+1}\Omega_{t+1}\left[\left(R_t^k-\frac{R_t}{\alpha_t^{res}}\right)Q_tS_{jt}^f+\left(R_t^b-\frac{R_t}{\alpha_t^{res}}\right)q_tS_{jt}^{Ext}+\frac{R_t}{\alpha_t^{res}}N_{jt}\right],\end{aligned} \tag{C.31}$$

which holds when

$$\nu_t^f=E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^k-\frac{R_t}{\alpha_t^{res}}\right), \tag{C.32}$$

$$\nu_t^{Ext}=E_t\Lambda_{t,t+1}\Omega_{t+1}\left(R_t^b-\frac{R_t}{\alpha_t^{res}}\right), \tag{C.33}$$

and

$$\eta_t = E_t \Lambda_{t,t+1} \Omega_{t+1} \frac{R_t}{\alpha_t^{res}}. \tag{C.34}$$

Use the bank's condition C.18

$$\nu_t^{Ext} = \Delta \nu_t^f. \tag{C.35}$$

$$E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^b - \frac{R_t}{\alpha_t^{res}} - \tau_t^b \right) = \Delta E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^k - \frac{R_t}{\alpha_t^{res}} \right) \tag{C.36}$$

Substitute in C.31

$$\begin{aligned} &\nu_t^f Q_t S_{jt}^f + \Delta \nu_t^f q_t S_{jt}^{Ext} + \eta_t N_{jt} \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} \left[\left(R_t^k - \frac{R_t}{\alpha_t^{res}} \right) Q_t S_{jt}^f + \left(R_t^k - \frac{R_t}{\alpha_t^{res}} \right) \Delta q_t S_{jt}^{Ext} + \frac{R_t}{\alpha_t^{res}} N_{jt} \right], \end{aligned} \tag{C.37}$$

simplifying

$$\begin{aligned} &\nu_t^f \left[Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext} \right] + \eta_t N_{jt} \\ &= E_t \Lambda_{t,t+1} \Omega_{t+1} \left[\left(R_t^k - \frac{R_t}{\alpha_t^{res}} \right) \left(Q_t S_{jt}^f + \Delta q_t S_{jt}^{Ext} \right) + \frac{R_t}{\alpha_t^{res}} N_{jt} \right] \end{aligned} \tag{C.38}$$

which implies

$$\nu_t^f = E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^k - \frac{R_t}{\alpha_t^{res}} \right),$$

$$\eta_t = E_t \Lambda_{t,t+1} \Omega_{t+1} \frac{R_t}{\alpha_t^{res}},$$

$$\phi_t = \frac{\eta_t}{\lambda - \nu_t^f},$$

with

$$\Omega_{t+1} = (1 - \theta) + \theta \left(\nu_{t+1}^f \phi_{t+1} + \eta_{t+1} \right),$$

and

$$\nu_t^{Ext} = \Delta \nu_t^f$$

$$\nu_t^{Ext} = E_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_t^b - \frac{R_t}{\alpha_t^{res}} \right)$$

C.4 Welfare Cost.

Following Schmitt-Grohé and Uribe (2007), I compare the welfare cost of each alternative policy relative to the time invariant equilibrium of the Ramsey policy. The welfare associated with the optimal Ramsey policy conditional on a particular state of the economy in period zero $\left(W_0^R\right)$, is

$$W_0^R = (\Upsilon)\left(1-\beta^{Ext}\right) V_0^{R,B} + (1-\Upsilon)(1-\beta) V_0^{R,S}, \tag{C.39}$$

where (Υ) is the borrower's population. The superscript (R), represents the allocations in the Ramsey regime, while the superscripts B and S, represent the borrowers, and savers allocations, respectively. And,

$$V_0^{R,B} = \left\{ E_0 \sum_{t=0}^{\infty} \left(\beta^{Ext}\right)^t U\left[C_t^{R,B}, L_t^{R,B}\right]\right\}, \tag{C.40}$$

is the welfare function of the borrowers, while

$$V_0^{R,S} = \left\{ E_0 \sum_{t=0}^{\infty} (\beta)^t U\left[C_t^{R,S}, L_t^{R,S}\right]\right\}, \tag{C.41}$$

is the savers' welfare.

If the central bank implements an alternative policy, the social welfare in that regime $\left(W_0^M\right)$, is :

$$W_0^M = (\Upsilon)\left(1-\beta^{Ext}\right) V_0^{M,B} + (1-\Upsilon)(1-\beta) V_0^{M,S} \tag{C.42}$$

where the superscript (M) represents the allocations in the alternative regime. I follow the work of Monacelli (2008), Mendicino and Pescatori (2008), and Rubio and Carrasco (2014) and assume that the Ramsey planner weight the utility of group by their population size and their discount factor, such that, given a constant consumption stream, the two agents achieve the same level of utility.

If the consumption cost, of following the alternative regime instead of the Ramsey policy on a particular state in period zero is represented by the cost of the alternative policy is implicitly defined by

$$\begin{aligned} W_0^M = (\Upsilon)\left(1-\beta^{Ext}\right) &\left\{ E_0 \sum_{t=0}^{\infty} \left(\beta^{Ext}\right)^t U\left[(1-\varkappa) C_t^{R,B}, L_t^{R,B}\right]\right\} \\ &+ (1-\Upsilon)(1-\beta)\left\{ E_0 \sum_{t=0}^{\infty} (\beta)^t U\left[(1-\varkappa) C_t^{R,S}, L_t^{R,S}\right]\right\}, \end{aligned} \tag{C.43}$$

where $[\varkappa]$ is the fraction of consumption of the Ramsey regime that a household is willing to renounce in order to be indifferent between the alternative regime and the Ramsey regime.

The particular utility function of the borrowers is

$$U\left[C_t^{R,B}, L_t^{R,B}\right] = \left[\ln\left(C_t^{R,Ex} - h^{Ex} C_{t-1}^{R,Ex}\right) - \frac{\chi^{Ex}}{1+\varphi}\left(L_t^{R,Ex}\right)^{1+\varphi}\right], \tag{C.44}$$

and that of the savers is:

$$U\left[C_t^{R,S}, L_t^{R,S}\right] = \left[\ln\left(C_t^{R,I} - h^{Int} C_{t-1}^{R,I}\right) - \frac{\chi^{Int}}{1+\varphi}\left(L_t^{R,I}\right)^{1+\varphi}\right]. \tag{C.45}$$

Substituting C.44, and C.45 in C.43

$$\begin{aligned} W_0^M = (\Upsilon)\left\{E_0 \sum_{t=0}^{\infty}\left(\beta^{Ext}\right)^t \left[\begin{array}{c} \ln\left(\begin{array}{c} (1-\varkappa) C_t^{R,Ex} \\ -(1-\varkappa) h^{Ex} C_{t-1}^{R,Ex} \end{array}\right) \\ -\frac{\chi^{Ex}}{1+\varphi}\left(L_t^{R,Ex}\right)^{1+\varphi} \end{array}\right]\right\} \\ +(1-\Upsilon)\left\{E_0 \sum_{t=0}^{\infty}(\beta)^t \left[\begin{array}{c} \ln\left(\begin{array}{c} (1-\varkappa) C_t^{R,I} \\ -(1-\varkappa) h^{Int} C_{t-1}^{R,I} \end{array}\right) \\ -\frac{\chi^{Int}}{1+\varphi}\left(L_t^{R,I}\right)^{1+\varphi} \end{array}\right]\right\}, \end{aligned} \tag{C.46}$$

this can be written as

$$W_0^M = \Upsilon\left[\ln(1-\varkappa)\right] \tag{C.47}$$

$$+(\Upsilon)\left(1-\beta^{Ext}\right)\left\{E_0 \sum_{t=0}^{\infty}\left(\beta^{Ext}\right)^t \left[\begin{array}{c} \ln\left(C_t^{R,Ex} - h^{Ex} C_{t-1}^{R,Ex}\right) \\ -\frac{\chi^{Ex}}{1+\varphi}\left(L_t^{R,Ex}\right)^{1+\varphi} \end{array}\right]\right\} \tag{C.48}$$

$$+(1-\Upsilon)\left[\ln(1-\varkappa)\right]$$

$$+(1-\Upsilon)(1-\beta)\left\{E_0 \sum_{t=0}^{\infty}(\beta)^t \left[\begin{array}{c} \ln\left((1-\varkappa) C_t^{R,I} - (1-\varkappa) h^{Int} C_{t-1}^{R,I}\right) \\ -\frac{\chi^{Int}}{1+\varphi}\left(L_t^{R,I}\right)^{1+\varphi} \end{array}\right]\right\}, \tag{C.49}$$

using C.40, and C.41 in C.47

$$W_0^M = \Upsilon\left[\ln(1-\varkappa)\right] + (\Upsilon)\left(1-\beta^{Ext}\right) V_0^{R,B} \tag{C.50}$$

$$+(1-\Upsilon)\left[\ln(1-\varkappa)\right] + (1-\Upsilon)(1-\beta) V_0^{R,S}, \tag{C.51}$$

which is equal to

$$\Upsilon\left[\ln\left(1-\varkappa\right)\right]+\left(1-\Upsilon\right)\left[\ln\left(1-\varkappa\right)\right] \tag{C.52}$$
$$=\left(\Upsilon\right)\left(1-\beta^{Ext}\right)\left[V_0^{M,B}-V_0^{R,B}\right]+\left(1-\Upsilon\right)\left(1-\beta\right)\left[V_0^{M,S}-V_0^{R,S}\right],$$

Suppose that the economy is populated only by households with external habits ($\Upsilon = 1$). Then, the welfare cost for those households would be

$$\varkappa=\left(1-\beta^{Ext}\right)\left[V_0^{R,B}-V_0^{M,B}\right], \tag{C.53}$$

after noting that:

$$\ln\left(1+\varkappa\right)\approx\varkappa,$$

which is the welfare cost function that is necessary to approximate up to second order in order to conduct accurate welfare comparisons, across regimes, if there are only households with external habits.

C.4.1 Approximating the Welfare Cost up to Second Order.

Equation C.53 can be approximated up to second order around the deterministic Ramsey steady state $(x_0, \sigma_\varepsilon)$ with $[x_0 = x]$ and $\sigma_\varepsilon = 0$. Because in equilibrium $V_0^{R,B}$ and $V_0^{M,B}$ are functions of the initial state vector (x_0) and the parameter scaling the standard deviations of the shocks (σ_ε), the conditional welfare cost can be written as

$$\varkappa\left(x_0,\sigma_\varepsilon\right)=\left(1-\beta^{Ext}\right)\left[V_0^{R,B}\left(x_0,\sigma_\varepsilon\right)-V_0^{M,B}\left(x_0,\sigma_\varepsilon\right)\right]. \tag{C.54}$$

In order to ensure that the different regimes start on the same point, I compare them assuming that their initial point is the same deterministic Ramsey steady state. Hence, only the first and second order derivatives of the cost respect to (σ_ε) have to be considered (see Schmitt-Grohé and Uribe (2007)). Following this, the second order approximation of the previous equation can be written in general terms as

$$\varkappa\approx\varkappa\left(x_0,\sigma_\epsilon\right)+\varkappa_{\sigma_\epsilon}\left(x_0,\sigma_\epsilon\right)\sigma_\epsilon+\frac{1}{2}\varkappa_{\sigma_\epsilon\sigma_\epsilon}\left(x_0,\sigma_\epsilon\right)\sigma_\epsilon^2. \tag{C.55}$$

Now, because all the regimes are approximated across the same deterministic Ramsey steady state, the constant term $[\varkappa\left(x_0,\sigma_\epsilon\right)]$ in eq. C.55 disappears in the comparison. This means that

$$\varkappa\left(x_0,\sigma_\epsilon\right)=0.$$

The terms containing the first order approximation of the policy function $[\varkappa_{\sigma_\epsilon}\left(x_0,\sigma_\epsilon\right)\sigma_\epsilon]$ are zero. This is shown in Schmitt-Grohé and Uribe (2004). Up to a first order of approximation, the derivative of the policy function respect to the parameters scaling the variance of the shocks is zero. For this particular case

$$\varkappa_{\sigma_\epsilon}(x,0)\,\sigma_\epsilon = \left(1-\beta^{Ext}\right)\left[V^{R,B}_{0\sigma\varepsilon}(x,0) - V^{M,B}_{0\sigma\varepsilon}(x,0)\right]\sigma_\epsilon = 0.$$

The term containing the second order approximation is

$$\varkappa_{\sigma\varepsilon\sigma\varepsilon}(x,0) = \frac{1}{2}\left(1-\beta^{Ext}\right)\left[V^{R,B}_{0\sigma\varepsilon\sigma\varepsilon}(x,0) - V^{M,B}_{0\sigma\varepsilon\sigma\varepsilon}(x,0)\right]\sigma^2_\varepsilon. \tag{C.56}$$

Similarly, if there are only households with internal habits, the welfare cost up to second order is

$$\varkappa_{\sigma\varepsilon\sigma\varepsilon}(x,0) = \frac{1}{2}\left(1-\beta\right)\left[V^{R,S}_{0\sigma\varepsilon\sigma\varepsilon}(x,0) - V^{M,S}_{0\sigma\varepsilon\sigma\varepsilon}(x,0)\right]\sigma^2_\varepsilon. \tag{C.57}$$

Given that there are (Υ) borrowers and $(1-\Upsilon)$ savers, the total welfare cost (x) is:

$$\begin{aligned}\varkappa^{Social} &= \Upsilon\left(\frac{1-\beta^{Ext}}{2}\right)\left[V^{R,B}_{0\sigma\varepsilon\sigma\varepsilon}(x,0) - V^{M,B}_{0\sigma\varepsilon\sigma\varepsilon}(x,0)\right]\sigma^2_\varepsilon \\ &+ (1-\Upsilon)\left(\frac{1-\beta}{2}\right)\left[V^{R,S}_{0\sigma\varepsilon\sigma\varepsilon}(x,0) - V^{M,S}_{0\sigma\varepsilon\sigma\varepsilon}(x,0)\right]\sigma^2_\varepsilon,\end{aligned} \tag{C.58}$$

which is the cost function utilized to calculate the convenience of the different policy in terms of the foregone consumption.

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