A statistical and probabilistic approach for improving efficiency in Air Traffic Flow Management

by

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Odi et amo. Quare id faciam, fortasse requiris.
Nescio, sed fieri sentio et excrucior.

Caio Valerio Catullo, Carmen LXXXV

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Abstract

This thesis presents a novel approach based on statistics and probability theory to improve Air Traffic Flow Management (ATFM) efficiency.
This work proposes four main contributions, which are briefly described below.

A procedure for determining the Airport capacity using an estimated probability that a given number of movements can occur is shown. This procedure is a probabilistic approach that employs the statistical information of aircraft arrivals and departures that are collected over a given time interval. The procedure is demonstrated using historical arrival and departure data to estimate the capacity of London Heathrow airport.

A procedure to establish a strategic arrival schedule is developed. An implicit feature of this probabilistic procedure is that it takes into account the uncertainty in the arrival and departure times at a given airport. The methodology has been applied at Glasgow International Airport to design a strategic schedule that reduces either the probability of conflict of the arrivals or the length of the landing slots necessitated by flights. The benefits that are expected through the application of this methodology are a reduction of airborne delays and/or an increase of the airport capacity. Thus a safer and more efficient system is achieved.

A decision support tool for Air Traffic Flow Management (ATFM) is developed. This tool is designed to allow air traffic controllers to organise an air traffic flow pattern using a ground holding strategy. During the daily planning of air traffic flow unpredictable events such as adverse weather conditions and system failures occur, necessitating airborne and ground-hold delays. These delays are used by controllers as a means of avoiding 4D status conflicts. Of the two methods, ground hold delays are preferred because they are safer, less expensive and cause even less pollution. In this thesis a method of estimating the duration of a ground-hold for a given flight is developed. The proposed method is novel in the use of a real time stochastic analysis. The method is demonstrated using Glasgow International Airport. The results presented

show how a ground hold policy at a departure airport can increase capacity and minimise conflicts at a destination airport.

A methodology to regulate the dispatch of aircraft through any congested sector and Terminal Manoeuvring Areas (TMA) in order to reduce conflicts is presented. This methodology, based on statistics and probability theory, presents a new schedule at the strategic planning level that will ensure, with a high probability, that the aircraft will comply with the established separation minima during their route and during their approach holding patterns. Air Traffic Control Centres (ATC) will be less likely to be overloaded and thus a minimisation of the penalty imposed on aircraft by the operators will be achieved.

Contents

List of Figures

List of Tables

List of acronyms

ACAS	Aircraft Collision Avoidance System
ADS	Automatic Dependent Surveillance
ADS-B	ADS-Broadcast
ADS-C	ADS-contract
AOC	Airline Operation Centers
ATA	Air Transport Association
ASAS	Airborne Separation Assurance System
ASM	Airspace Management
ATC	Air Traffic Control
ATCSCC	Air Traffic Control System Centre
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
ATS	Air Traffic Services
CAA	Civil Aviation Authority (U.K. and other countries)
CDF	Cumulative Distribution Function
CFMU	Central Flow Management Unit
CNS/ATM	Communication, Navigation, Surveillance/Air Traffic Management
FAA	Federal Aviation Administration (U.S.)
FANS	Future Air Navigation System
GHP	Ground Holding-Problem
GLA TMA	Glasgow Terminal Manoeuvring Area
GNS	Global Navigation System
GNSS	Global Navigation Satellite System
IATA	International Air Transport Association
ICAO	International Civil Aviation Organization
IFR	Instrument Flight Rules

L1	Distance flown by aircraft 1 until the intersection point
L1	Distance flown by aircraft 1 until the intersection point
LT1	Distance between the departure and the arrival airport of aircraft 1
LT2	Distance between the departure and the arrival airport of aircraft 2
NAS	National Airspace System
NAVAIDS	Navigation Aids
ND	Navigation Display
NMI	Nautical Miles
PDF	Probability Density Function
RA	Resolution Advisories
RADAR	Radio Detection And Ranging
RNAV	Area Navigation
RVSM	Reduced Vertical Separation Minima
SICASP	SSR Improvements and Collision Avoidance Systems Panel
SSR	Secondary Surveillance Radar
SUA	Special Use Airspace
TFM	Traffic Flow Management
TMA	Terminal Manoeuvring Area
UK CAA	United Kingdom Civil Aviation Authority

Nomenclature

Operators and sets

$\triangleq$	is defined to be
$\boldsymbol{N}$	is the set of integer numbers
$\boldsymbol{Z}$	is the set of integer relative numbers
$\boldsymbol{R}$	is the set of real numbers
$\boldsymbol{T}$	is the set of the points belonging to a time period
$\boldsymbol{C}$	is the set of the points belonging either to a volume or to an area
$\boldsymbol{K}$	is the set of airports
$\boldsymbol{F}$	is the set of flights
A	departure airport
B	arrival airport
P	generic point belonging either to a volume or to an area
$d(\cdot,\cdot)$	euclidean distance
Ω	experiment
ζ	outcome of the experiment

Scalars

c^{ai}	airborne delay cost
c^{gh}	ground hold delay cost
c^{sc}	speed control cost
c^{re}	re-routing cost
c^{ea}	earlier arrival cost
t_s^a	scheduled arrival time
t_s^d	scheduled departure time
$PC(\cdot,\cdot)$	probability of conflict
TPC	total probability of conflict
LT1	distance between the departure and the arrival airport of aircraft 1
LT2	distance between the departure and the arrival airport of aircraft 2
φ	relative angle between the routes

L1	distance flown by aircraft 1 until the intersection point
L2	distance flown by aircraft 2 until the intersection point

Random Variables

$\underline{\mathbf{C}}$	Capacity
$\mathbf{D}$	Departure Capacity
$\mathbf{D}^s$	Departure Scheduled Capacity
$\mathbf{A}$	Arrival Capacity
$\mathbf{A}^s$	Arrival scheduled Capacity
$\mathbf{d}^a$	Arrival Delay
$\mathbf{t}^a$	Arrival Time
$\mathbf{d}^d$	Departure Delay
$\mathbf{t}^d$	Departure Time
$\mathbf{t}^t$	Travelling Time
$\underline{\mathbf{v}}$	Velocity
$\underline{\mathbf{s}}$	Position of aircraft
$\Delta\mathbf{T}$	Time of running of the aircraft
Γ	Trajectory of the aircraft

Sets in which the random variables are defined

$\mathcal{K}$	is the set in which the capacity is defined
$\mathcal{A}$	is the set in which the arrival delay is defined
$\mathcal{B}$	is the set in which the arrival time is defined
$\mathcal{C}$	is the set in which the departure delay is defined
$\mathcal{D}$	is the set in which the departure time is defined
$\mathcal{E}$	is the set in which the travelling time is defined
$\mathcal{F}$	is the set in which the velocity is defined
$\mathcal{G}$	is the set in which the position of aircraft is defined
$\mathcal{J}$	is the set in which the time of running of the aircraft is defined
$\mathcal{H}$	is the set in which the trajectory of the aircraft is defined

Chapter 1

Introduction

1.1 Air Traffic Flow Management (ATFM)

This thesis focuses on improving Air Traffic Flow Management (ATFM) efficiency using a statistical and probabilistic approach. ATFM aims to match the demands of aircraft operators to airports and airspace capacity.

Air Traffic Flow Management (ATFM) is [1]:

" (...) the process that allocates traffic flows to scarce capacity resources (e.g. it meters arrival at capacity constrained airports) (...) Traffic flow management services are designed to meter traffic to taxed capacity resources, both to assure that unsafe levels of traffic congestion do not develop and to distribute the associated movement delays equitably among system users."

Three different types of ATFM services aiming at organising the traffic flows through airports can be distinguished:

Strategic Planning: This service commences seven months before the day under consideration and continues until forty-eight hours before the time period being planned. Schedules and flight plans are computed for the flights due to operate in a given network of airports. A solution that aims at minimising airport congestion while maximising safety and reducing costs is sought.

Pre-Tactical Planning: This service is provided during the two days that preceds the day in question. The goal of the pre-tactical planning is to indicate to air traffic controllers where delays can be expected owing to adverse weather conditions, political issues etc. in order to optimise the use of the airport.

Tactical Planning: This service is provided during the day of the operations, and introduces the final modifications to the established schedules and flight plans.

Airport capacity is defined as the number of movements (arrivals and departures) which an airport can manage over a period of time. Currently airport capacity is computed by using a technique based on facilities analysis. It has been empirically observed that this technique gives an airport capacity less than the maximum operational airport capacity. In other words this calculated airport capacity is less than what airport actually achieve in practice. This thesis proposes a new approach to estimate the operational airport capacity based on statistics and probability theory.

Furthermore, this thesis presents a new approach to ATFM at both strategic and tactical planning that aims to:

- reduce the probability of conflict between aircraft arriving at a given airport, which will decrease the likelihood of airborne delay. Thus the benefits expected by the implementation of this approach are a reduction of the costs for the airlines, an improvement of the safety of airports operations and even a reduce pollution.
- or reduce the length of the slots required by flights, which will increase the airport capacity.
- or a combination of these two objectives.

Finally, a strategic schedule that reduces congestion in airspace is proposed. This strategic schedule aims at reducing the probability of aircraft violating the established separation minima either during en route or during approach patterns.

The kernel of this new approach is to model the ATFM parameters as random variables to:

- model airport capacity.
- propose a strategic schedules, which will reduce the probability of conflict between aircraft either arriving at a given airport or crossing a given airspace sector.
- propose a tactical schedule, which will reduce airlines, airports and operators costs.

Air Traffic Flow Management (ATFM) is an activity of the broader issue Air Traffic Management (ATM). In the next section a brief description of ATM will be given.

1.2 Air Traffic Management (ATM)

The International Civil Aviation Organisation (ICAO) Future Air Navigation System (FANS) committee states [2]:

"*The general objective of Air Traffic Management (ATM) is to enable aircraft operators to meet their planned time of departure and arrival and adhere to their preferred flight profiles with minimum constraints without compromising agreed levels of safety*".

The purpose of ATM is the growth of efficiency through the optimisation of the existing resources and facilities. Increasing safety and reducing costs are the main concerns for ATM. Air Traffic Management (ATM) involves two principal functions [1]:

- Air Traffic Flow Management (ATFM). A service whose objective is to enable the throughput of the traffic flow to or through airports and areas while guaranteeing a high level of safety and security.
- Air Traffic Control (ATC). A service which is responsible for sufficient separation between aircraft, and also between aircraft and obstructions on the ground, in order to avoid collisions while assuring a proper flow of traffic.

The purpose of Air Traffic Flow Management (ATFM) is to ensure the optimum traffic flow while guaranteeing a given condition of safety. ATFM has the task to allocate resources and facilities of airports and ground structures. At present [3] [4] [5] [6] [7] [8], the organisations in charge of the air traffic flow management are the Central Flow Management Unit (CFMU) for Europe and the Air Traffic Control System Centre (ATCSCC) for the USA. In the USA the absence of national boundaries means that the administration of the traffic flow is more straightforward than in Europe.

The main goal of Air Traffic Control (ATC) is to guarantee safety and to give aircraft optimal trajectories to fly from one airport to another. There is a simple definition of safety in ATC systems: two aircraft can never be closer than one standard separation. A standard separation is a distance usually given in nautical miles. This distance is usually of 8 or 5 Nautical Miles (NMI) in the horizontal plane while of 1000 or 2000 feet in the vertical plane and depends on the equipment available to control aircraft. Two aircraft are in conflict when both these two standard separations are violated.

1.3 The evolution of the Air Transport

As shown in [9] [10] [6] [11] the global Gross Domestic Product increases at an average of 3.7% during the period 1960-1995. In more detail, during the periods 1960-1970, 1970-1980, 1980-1990, and 1990-1995 the average annual growth rates were respectively 4.8, 3.6, 3.0 and 2.8 per cent. The air transport growth rate increased more than the average of the Gross Domestic Product. The traffic measured in term of passenger-kilometres (PKP), based on internal and international flights, has revealed an average annual increase rate of 8.9 per cent throughout 1960-1995. During the periods 1960-1970, 1970-1980 and 1980-1990, 1990-1995 the traffic grew at an average annual rate of 13.4, 9.0, 5.7 and 3.4 per cent, respectively.

The exponential growth of the demand for air transport is producing congestion. ATM resources and facilities are not growing accordingly. The result is an increase in delays and traffic affecting safety and even comfort for passengers. The main advantage of air travel is the en-route speed. Consequences of the congestion are an increase in terms of the time required to reach the final destination and a reduction in efficiency of the air transport system. As demand for air transport outstrips the supply of air transport infrastructure, passenger frustration has increased as more and more flights are delayed for longer times. Air traffic control, runway space, terminal capacity, and ground access capacity need to be expanded otherwise there will be risk to loose air travel growth in the 21st century.

The University of Glasgow has carried out an analysis of data covering the first week of July 1998 for London Heathrow Airport (see Figure 1.1). This investigation has revealed that 47% of the arrivals and 45% of the departures had a delay greater than 10 minutes. These flights are affected by an average delay of about half an hour and no influence of the day can be pointed out.

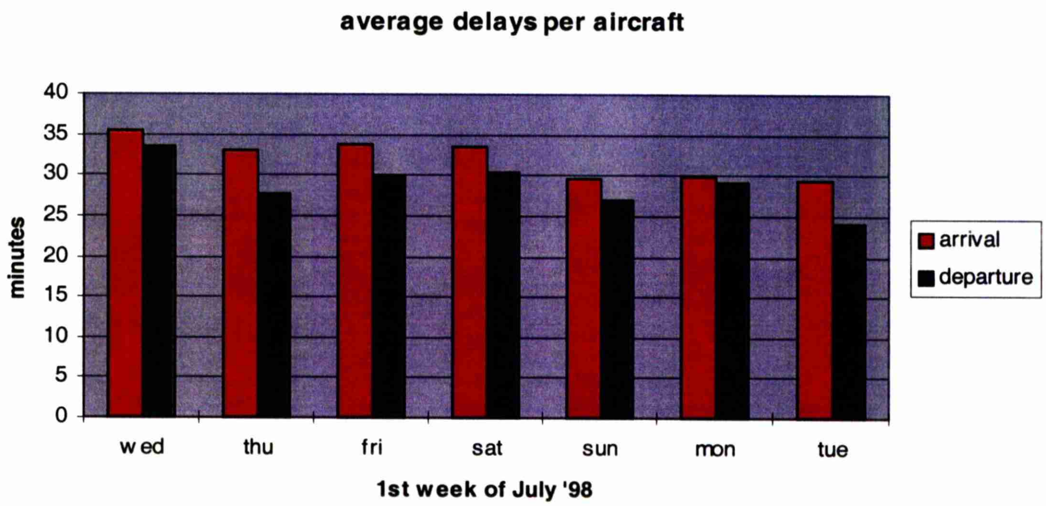


Figure 1.1: Average delay per aircraft during the first week of July 1998 for London Heathrow Airport.

At present, almost 35% of all European flights are delayed by more than 20 minutes, which represents a significant cost for airlines, principally due to fuel consumption. Two minutes delay during the landing approach procedure of typical commercial airline causes the consumption of almost 180 Kg of extra fuel. For a large airline operating 1000 daily flights, the extra fuel cost would amount to 33 million Euro over a year. Furthermore, the environmental cost due to the burning of this extra fuel has to be taken into account.

ICAO, in the document [12] declares:

"Whereas the rapid growth of air traffic places heavy demands on airports and air navigation systems and causes serious congestion problems in some areas of the world (...). Recognising that further measures, including longer term measures, will be required to expand the airport and air navigation system capacity to more efficiently accommodate future air traffic (...)".

And further:

"States are urged to take measures that have positive effects on airport and airspace capacity, in consultation with users and airport operators and without prejudice to safety (...). States are invited to recognise that airports and airspace constitute an integrated system and developments in both areas should be harmonised; (...)".

1.4 Analysis of the Air transport Mode

In this section an analysis of the Air Transport is given. The task is to point out the main bottleneck effecting the system.
Any kind of transport system is divided in three different blocks:

- Vehicle
- Medium
- Infrastructure and management

1.4.1 Vehicle

Two main factors influence the vehicle of the Air Transport system:

- Flight performance, including airframe and engine characteristics.
- Communications, Navigation and Surveillance capabilities. (CNS capabilities)

Since the last decades, aircraft aerodynamics has been considerably improved. Innovative wings shape combined with new aerospace materials have given sensational aerodynamic efficiency to aircraft. Anyway this advantage is totally vain if aircraft will cross fully congested airspace system. A new ATM system and new CNS capabilities are required to cope with the exponential growth of the demand.

1.4.2 Medium

The air is the medium to take into account. Improvements of the air transport system with respect to the air consist mainly to obtain a more accurate mathematical model of this fluid. More accurate aerodynamic computations can be achieved and consequently more efficient aircraft shapes can be designed.

1.4.3 Management and Infrastructure

The current Air Traffic Management (ATM) system already shows its totally inadequacy to fulfil the exponential growth of air travel demand. This increase can be met by building new ground structures resources and facilities. However, this solution is expensive and it needs almost a decade to be implemented. Another option is to improve the efficiency of the current ATM system. To work in safe conditions, airports and ATC centres deal with a number of operations definitely below their real operability. This causes problems of seriously reducing ground structures' efficiency. In conclusion, large economic savings and great environmental benefits can be achieved by enhancing ATM efficiency.

1.5 The future of the CNS/ATM System

In late 1991, the International Civil Aviation Organisation (ICAO) introduced a novel idea: the CNS/ATM. This concept represented a significant departure from the traditional navigation systems, which were based on voice communication and without an effective digital air/ground system to interchange data.

ICAO defines the CNS/ATM systems as [2]: *"Communications, Navigation and Surveillance systems, employing digital technologies, including satellite systems together with various levels of automation, applied in support of a seamless global Air Traffic Management system"*.

This definition embraces different levels of implementation of CNS/ATM. The main concerns, which this future CNS/ATM system is aimed at solving, are listed below:

- Challenge to implement a world wide compatible and consistent protocol, which satisfies and fulfils both ATFM and ATC.
- Limitations in the voice communications.
- Challenge to support aircraft automation on the ground and in the air due to lack of digital air-ground data interchange.

At the same time, this new CNS/ATM system is aimed at fulfilling these Air Traffic Management goals:

- Improve safety for the air transport mainly by reducing congestion in airports and in airspace. This thesis will show a methodology to decrease traffic in airports, airspace and terminal manoeuvring areas.
- Decrease airlines and ground structures expenses by reducing the amount of delays. Chapter 4 and 5 will propose a strategic and tactical schedule, which aims at reducing costs and expenses caused by delays.
- Improve data-link support between aircraft-aircraft and aircraft-ground stations.
- Employ the use of advanced ground-based data processing systems to improve navigation in four dimensions.
- Implement new systems to increase airport and airspace throughput while maintaining the same level of safety. This thesis will present an approach that aims at growing the efficiency of the existing resources.
- Opening new routes.

ATM system will take advantage of the introduction and implementation of this complex and interrelated set of new Communications, Navigation and Surveillance technologies (CNS). This new integrated global system would enable pilots to decide own routes that will be, then, dynamically adjusted in the most cost-efficient manner. Many organisations are working to propose a step-based way to the future system. The Radio Technical Commission for Aeronautics (RTCA) released in 1995 the "Final Report of RTCA Task Force 3; Free Flight Implementation" [13].

Free flight is defined as:

"... a safe and efficient flight operating capability under instrument flight rules (IFR) in which the operators have the freedom to select their path and speed in real time. Air traffic restrictions are only imposed to ensure separation, to preclude exceeding airport capacity, to prevent unauthorised flight through Special Use Airspace (SUA), and to ensure safety of flight..."

The future path of ATM is described in Figure 1.2:

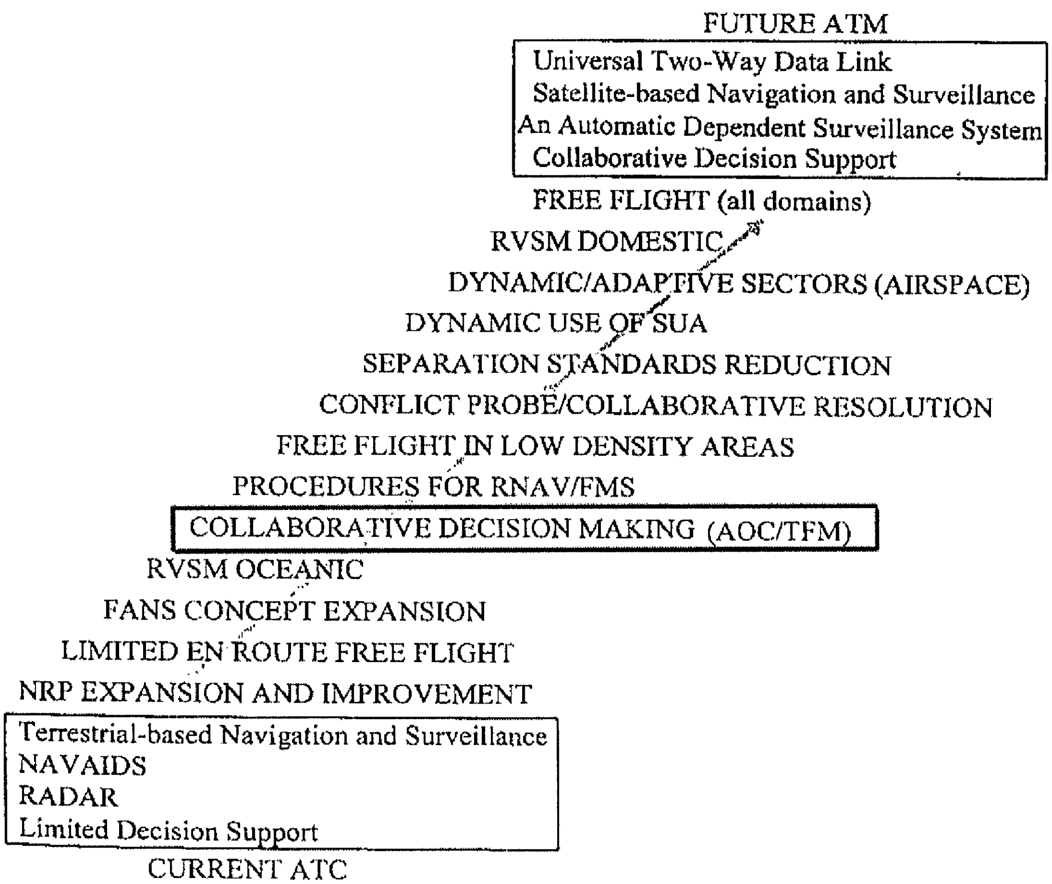


Figure 1.2: The path to ATM: Source [13].

Another strength of the novel approach presented in this thesis is that this proposed model can be implemented both within the current ATC structure and within Free Flight airspace. This methodology represents a flexible and powerful tool to manage and to deal with the current and future ATM path.

1.5.1 Free Flight

Free Flight is an Air Traffic Management operation where airspace users have more freedom to select their own routes and their departure time. Aircraft are no longer restricted to given airways as they are allowed to fly at their optimal altitudes choosing the most suitable routes. As aircraft are no longer required to follow fixed airways, ATFM and ATC systems need deep changes.

- ATFM has to optimise airports and airspace throughput.
- ATC has to monitor and control traffic in a given airspace to ensure minima separation between aircraft.

This thesis establishes a methodology to ensure the efficient and expeditious traffic flow through airports, sectors and terminal manoeuvring areas. This methodology is based on statistics and probability theory and aims to assure that airports and airspace will not be overloaded by a percentage previously fixed by ATFM operators. In this dissertation it will be shown that the proposed methodology provides a statistical reduction of traffic congestion over a fixed period of time. Therefore the methodology does not aim to prevent actual conflicts but to reduce the likelihood of conflicts occurrences. Chapter 6 focuses to reduce the risk of collision between en-route aircraft at strategic planning. To prevent collisions and to provide minima separation between aircraft, airborne separation is proposed as a complementary part of the methodology presented in this thesis.

As reported by [14] "*Currently a pilot establishes a flight plan, or a "contract", with air traffic control. This plan requires the aircraft to fly along a specific route. Any deviation from the designated route must be pre-approved by an air traffic controller. For example, if a thunderstorm is encountered along the flight path, the pilot must notify the air traffic controller of the need to change course, and the controller would designate a course for the plane to avoid the storm. Under the Free Flight concept, the pilot will be able to choose the route, speed and altitude to achieve the desired results, notifying the air traffic controller of the new route.*"

Figures 1.3 and 1.4 (inspired by [15]) show how profoundly Free Flight could change significantly the airspace structure.

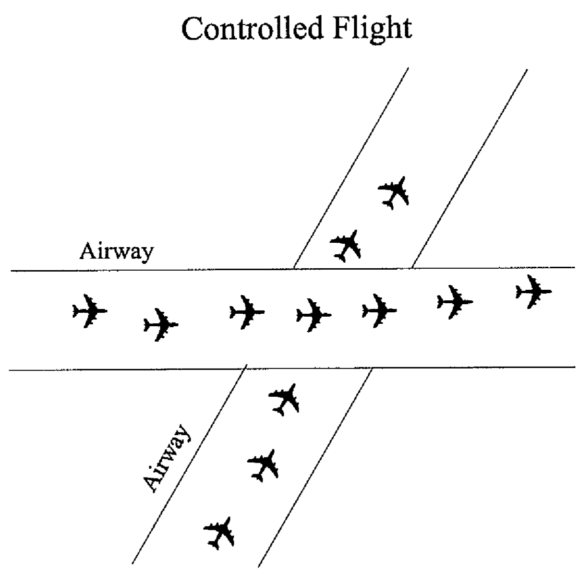


Figure 1.3: Controlled flight in the en-route phase.

Figure 1.3 represents controlled flight airspace in which aircraft are restricted in only some defined areas of the airspace. Aircraft are required to fly along specific and fixed routes called *"airways"*. Figure 1.4 shows the future Free Flight airspace. Aircraft have freedom to fly in every area and in every direction. ATM structure requires to be significantly changed to ensure the minima separation between aircraft.

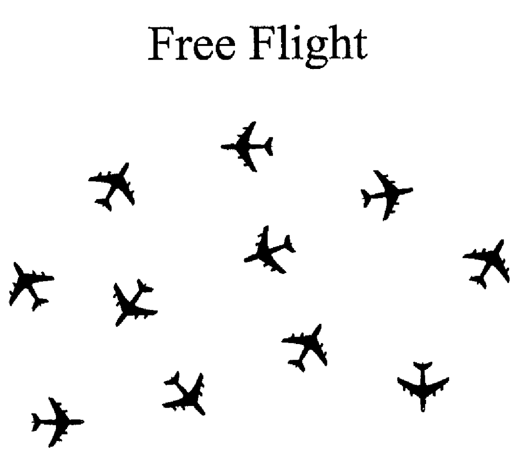


Figure 1.4: Free flight in the en-route phase.

1.5.2 Airborne Separation Assurance System (ASAS)

The ASAS concept is defined by the ICAO Secondary Surveillance Radar Improvements and Collision Avoidance Systems Panel (SICAS Panel) as [16]:

"The equipment, protocols and other aircraft state data, flight crew and ATC procedures which enable the pilot to exercise responsibility, in agreed and appropriate circumstances, for separation of his aircraft from one or more aircraft "

The ASAS concept encompasses two broad categories of proposed applications [17], which are briefly described below:

- *Traffic Situational Awareness Applications*: Provision of information to the flight crew regarding position, identity, flight status and intentions of proximate aircraft.
- *Co-operative Separation Applications*: The pilot uses ASAS equipment to perform an operational procedure that involves complying with defined separation minima with proximate aircraft.

Thus, the ASAS applications range from the mere enhancement of the flight crew's awareness of the surrounding traffic to the transfer of the responsibility for separation assurance from the ground-based control to the cockpit in the appropriate circumstances. Although the ASAS applications are still in the research and development stage, they are foreseen as cornerstones of the future ATM.

The main benefits expected to be delivered by the ASAS applications have been anticipated by the SICASP Panel [18] and are outlined below:

- Improvement of the pilot's situational awareness. Operational safety is expected to improve with the provision of information regarding identity, status, position and intentions of the proximate aircraft.
- Increase in the capacity and improvement of the efficiency of ATC through the active involvement of the aircraft crew in the separation assurance process. The

delegation of the responsibility for separation assurance to the cockpit is expected to reduce the controllers' workload.

- Increase in the airspace capacity by enabling a more accurate compliance to separation minima. Ultimately, ASAS is expected to contribute to the reduction of these separation minima.

On the other hand, the SICAS Panel maintains that ASAS should be kept independent from Airborne Collision Avoidance System (ACAS) ([16], [17], [19]). The SICAS Panel ratifies ACAS as the last recourse safety function that prevents imminent mid-air collisions even if the primary means for separation fails.

Under the assumption that ASAS and ACAS will be available, Chapter 6 proposes a tool to distribute traffic evenly in the airspace to reduce the likelihood that conflict will be detected. The benefit of the implementation of this tool is a decrease of the number of times where the airborne separation assurance systems will be activated.

1.6 Literature review

The following sections present a review of the *state of the art*. All the investigations carried out concerning ATFM are mainly focused on developing deterministic approaches. Many works introduce and develop random models but no author has showed a coherent methodology. The main contribution of this thesis is to propose a systematic procedure for ATFM fully based on statistics and probability theory.

This thesis focuses its studies mainly in three different ATFM issues:

- Estimation of the real airport capacity (Chapter 3)
- Proposing a strategic and tactical scheduling (Chapter 4 and 5)
- Methodology to guarantee separation minima between aircraft at strategic planning (Chapter 6 and 7)

A brief *state of the art* analysis of each of these issues is given in the next sub-sections.

1.6.1 Airport Capacity

As declared in [20]:

"Airport capacity is defined as the maximum number of operations (arrivals and departures) that can be performed during a fixed time interval (e.g., 15 minutes or one hour) at a given airport under given condition."

World-wide airports are more and more congested because of the increased air traffic demand in this last decade. To reduce overloads, long-term approaches include construction of additional airports and additional runways at existing airports. Instead the determination of the real airport capacity is the main goal to achieve in the short-term to improve efficiency. Nowadays airports declare their capacity taking into account infrastructure's configuration such as runways and gates position [7]. Currently the code SIMMOD^{TM} (Airport and Airspace Simulation Model) is available. This is an internationally recognised standard for modelling airport capacities.
As reported in [21]: *"The FAA Runway Capacity Model is an analytic model consisting of a series of equations to compute airport capacity. This model assumes an infinite queue of arrivals and departures, and provides a theoretical maximum throughput for the runways. Capacity is computed by determining the minimum sustainable time between successive arrivals and by inverting this time to find the maximum number of arrivals per hour. The maximum number of departures which can be inserted between the arrivals is then calculated, to give the arrival-priority capacity. If a specific ratio of departures to arrivals is specified, then departure-priority capacity and any intermediate points are calculated. Capacity is calculated by interpolating between points."*

Many times airports succeed in dealing with congested period. To do so, airports show that they can exceed their declared capacity. Thus, the estimation of this real airport capacity is the major requirement for short-term improvements. This real airport capacity is influenced by many different factors such as meteorological conditions, runways and gates configurations, aircraft type, number of landing and taking off, human factor and so on.

[22] describes some methodologies. A queue model is presented to estimate the capacity of a simple runway. The arrival demand is described by a Poisson probability distribution whose parameter is function of the arrival and service rate. This model presents mainly two major limitations as it takes into account only few factors and then it gives a *steady-state* solution even if airports need many hours to reach this condition. Another technique, still described in [22], is based on an analytical approach. The capacity of the runway is estimated taking into account the length of the common approach path, aircraft speeds and minimum aircraft separations as specified by air traffic regulations.

Nowadays the most accurate studies accomplished are described in [20] [23]. The author considers arrivals and departures simultaneously as two processes influencing each other. The number of arrivals is dependent by the number of departures and vice-versa. This relation between these two variables is physically justified by the fact that only a given number of movements (departures and arrivals) can be allowed in a chosen runway in a fixed period of time. Empirically, it is observed the number of departures is slightly higher than the number of arrivals. This can be physically justified by the fact that departures are usually more predictable than arrivals. Control Towers can determine with a great accuracy the departure time of an aircraft while the arrival time is inevitably effected by a higher uncertainty. The function, which links the number of arrivals and departures, is defined to be the airport capacity curve.

To estimate the capacity curve, an empirical method based on real data is developed. For a given period of time, a number of arrivals and departures is observed. This number is assumed to be equal to the real airport capacity during peak hours. A histogram of the relative frequency is plotted. The frequency is determined as the number of occurrences of the same pair of values divided by the total number of observations. The airport capacity is then estimated using some rejection criteria, for instances, including within the curve a fixed percentage of points.

Airport capacity is modelled as Curve 1 of Figure 1.5 if 100% observations are requested to be included. Instead, curve 2 of Figure 1.5 represents a robust estimation that rejects some extreme observations. In [24], the author proposes different possible capacity scenarios. Each of these scenarios has a given probability of occurrences.

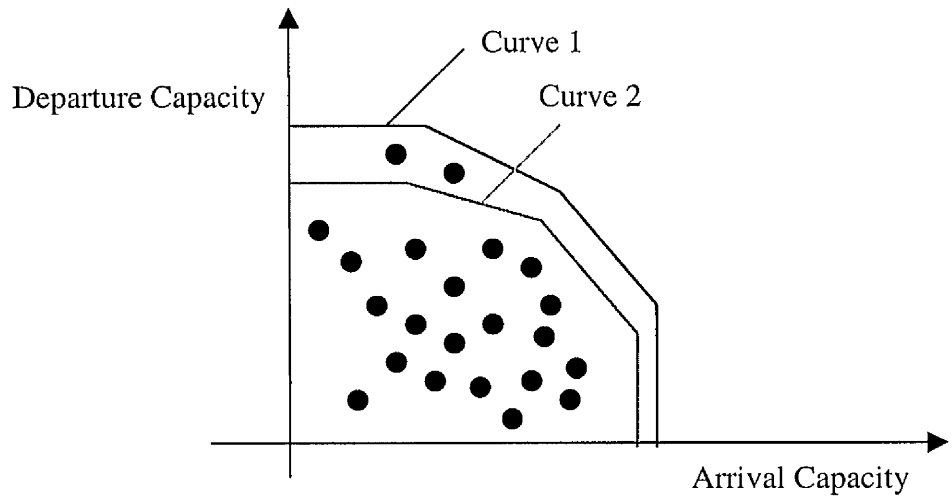


Figure 1.5: Airport arrival/departure capacity curve (general view).

An alternative airport capacity model will be presented in Chapter 3. The approach is fully based on statistics and probability theory. Airport capacity will be defined as a four dimensional discrete random variable. For a given pair of numbers, which expresses the number of arrivals and departures, the probability that the airport is able to deal with such a number of arrivals and departures (movements) will be given.

Once the capacity curve is estimated, airport operators have information pertaining to the maximum number of movements that can be scheduled in a period of time. Then taking into account the airport capacity, air traffic managers allocate the arrival/departure demand in the way to optimise airport's efficiency. To achieve this goal, [25] proposes a collaborative optimisation model based on queue theory and linear programming.

1.6.2 Ground Hold Policy (GHP)

As stated in [26], the problem of assigning to aircraft the necessary ground hold time so that total delay cost is minimised will here after be referred to as the ground holding problem (GHP). Two different versions of GHP can be distinguished according to the moment in which the decisions are taken: *static* and *dynamic*. In the first case, the ground and airborne holds are established once at the beginning of the day. In the second case the amount of ground hold and airborne delay is assigned during the course of the day taking into account new information available real time. The parameters involved in the GHP can be modelled either as *deterministic* or as *random* variables giving respectively a deterministic or a probabilistic version of GHP.

There has been much research concerning GHP, however [27] is the first reference that gives a methodical and a comprehensive description of the GHP problem. [28] proposed a dynamic programming algorithm to solve the single-airport static probabilistic GHP once the probability density function (PDF) of the capacity of the airport involved is known. [29] showed an efficient and powerful algorithm for the single-airport static deterministic GHP. He even proposed several heuristic for the single-airport probabilistic GHP and a closed-network three-airport formulation for the static deterministic GHP. [30] dealt with the single-airport dynamic probabilistic GHP.

The first systematic attempt to examine a network of airports is given by [26], where a static deterministic multi-airport GHP is analysed. A brief description mainly focused in the pure 0-1 integer programming formulation is given.

Let $\boldsymbol{K}$={1,2,...K} and $\boldsymbol{T}$={1,2,...T} be a set of airports and of time periods, respectively. Given a set of flights $\boldsymbol{F}$={1,2,...F}[†], it is assumed to know $\forall f \in \boldsymbol{F}$ the departure airport of the flight $f \in \boldsymbol{F}$ denoted by $k_f^d \in \boldsymbol{K}$, the arrival airport of the flight $f \in \boldsymbol{F}$ denoted by $k_f^a \in \boldsymbol{K}$, the ground delay cost function of the flight $f \in \boldsymbol{F}$ denoted by $c_f^{gh}(t)\boldsymbol{K}\times\boldsymbol{T}\times\boldsymbol{F}$ and the airborne delay cost function of the flight $f \in \boldsymbol{F}$ denoted by $c_f^{ai}(t)\boldsymbol{K}\times\boldsymbol{T}\times\boldsymbol{F}$. Furthermore it is assumed to know $\forall$ (k,t)$\in \boldsymbol{K}\times\boldsymbol{T}$, the departure capacity $D_k(t)$ and the arrival capacity $A_k(t)$ of the departure and arrival airport of the flight $f \in \boldsymbol{F}$. Since this model deals with a deterministic version of GHP, all the parameters involved are fixed numbers rather than random variables. In the next chapters of this thesis all these parameters will be presented as random variables in order to present a coherent stochastic method for GHP. The decision variables are g_f and a_f. These are respectively the amount of ground-hold delay and airborne delay to allocate to the flight $f \in \boldsymbol{F}$. The assignment decision variables are u_{ft} and v_{ft} that are equal to 1 if the flight $f \in \boldsymbol{F}$ is allowed to take-off respectively to land at the period $t \in \boldsymbol{T}$.

[†] The word *flight* means a regular connection between the departure and arrival airport.

The authors propose to minimise the total cost. This quantity can be expressed through the amount of ground and airborne delay as:

$$\min\left(\sum_{f=1}^{F}(c_f^{gh}g_f + c_f^{ai}a_f)\right) \qquad \begin{cases} c_f^{gh} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ c_f^{ai} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \end{cases}$$

Subject to the constraint that the number of flights planned to arrive and to depart does not exceed respectively the arrival and departure capacity:

$$\sum_{f;k_f^d=k} u_{ft} \le D_k(t) \qquad (k,t)\in \boldsymbol{K}\times\boldsymbol{T}$$

$$\sum_{f;k_f^a=k} v_{ft} \le A_k(t) \qquad (k,t)\in \boldsymbol{K}\times\boldsymbol{T}$$

In [31], a multi airport probabilistic formulation for the static and dynamic GHP is given. In this model, different possible scenarios of airport capacities are introduced giving a probabilistic GHP. There are L possible scenarios $\boldsymbol{L}$={1,2,...L} and the scenario l∈$\boldsymbol{L}$ has probability of occurrences equal to $p_l \in [0,1]$. The arrival capacity of the airport k is equal to $A_k(t)$. Assuming infinite departure capacities, the static probabilistic integer programming formulation is given as here it follows:

$$\min\left(\sum_{f=1}^{F}(c_f^{gh}g_f + c_f^{ai}\sum_{l=1}^{L}p_l a_f^l)\right) \qquad \begin{cases} c_f^{gh} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ c_f^{ai} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ p_l \in [0,1]\times\boldsymbol{L} \end{cases}$$

subject to the constraint:

$$\sum_{f;k_f^a=k} v_{ft}^l \le A_k^l(t) \qquad (k,t)\in \boldsymbol{K}\times\boldsymbol{T}, l\in \boldsymbol{L}$$

[31] does not investigate how to compute the probability p_l of each different scenarios. Besides this paper only takes into account a finite number of possible scenarios. Instead, in this thesis airport capacity will be defined as a multi dimensional random variable. Hence, an infinite number of possible scenarios are considered due to the intrinsic

nature of the random variables. Furthermore, the PDF of this multi dimensional random variable is built and, hence, the term p_l is computed. The paper presents even an algorithm for the dynamic GHP policy. This problem is solved by running again the program as better airport capacities estimation become available. In Chapter 3 and in Chapter 4 of this dissertation, the airport capacity and the arrival times will be presented as random variables respectively and a static algorithm will be shown. In Chapter 5, a dynamic probabilistic model for GHP problem will be then discussed.

1.6.3 Separation minima in strategic planning

The airspace environment is divided into controlled zones denoted by sectors. An Air Traffic Centre (ATC) controls each sector.

[32] introduces and defines the en route sector capacity as:

"There is a restriction on the number of airplanes that may fly within a sector at a given time. This number is dependent on the number of aircraft that an air traffic controller can manage at one time, the geographic location, and the weather conditions. We will refer to the restrictions on the number of aircraft in a given sector at a given time as the en route sector capacities".

The capacity of each sector is limited. An ATC centre cannot handle more than a given number of aircraft in the sector while guaranteeing the same level of safety. The maximum number of aircraft that an ATC can deal with is called sector capacity. The goal of ATFM is to avoid overload of these ATC centres and to minimise the penalty imposed by operators to aircraft.

[32] presents a 0-1 integer programming model for the deterministic Air Traffic Flow Management that addresses capacity restrictions on the en route airspace.

Based on the works developed for the Airborne Separation Assurance System (ASAS), this thesis proposes a methodology to solve the en route congestion at the strategic planning using statistics and probability theory. The papers [33] [34] present a methodology how to estimate the probability that a conflict occurs in Free Flight once it is given a pair of predicted trajectory and their level of uncertainty. The trajectory prediction errors are modelled as normally distributed. The work described in [35] presents a conflict resolution algorithm for Autonomous Aircraft operations in the future ATM. The algorithm is based on the calculation of the probability of conflict between aircraft. This calculation relies on an approach to the modelling of the aircraft's projected trajectories that takes into account the uncertainty of the trajectory prediction process. Thus, the aircraft's future route is regarded as a deterministic trajectory perturbed by Brownian motion. The algorithm is designed for two-dimensional encounters involving two or more aircraft flying straight lines at constant speeds.

The aim of Chapter 6 and 7 is to present a methodology for computing a strategic planning schedule that will provide a statistical reduction of air traffic congestion over a fixed period of time. It will be shown that the proposed methodology secures the minima separation between aircraft with a previous given probability. The methodology does not aim to prevent actual conflicts and, hence, ASAS technology will be a necessary complementary tool.

1.7 The proposed statistical and probabilistic approach for improving ATFM efficiency

The contribution of this thesis consists in the fact that it presents a systematic and coherent tool based on statistics and probability theory for Air Traffic Flow Management (ATFM). This tool can be implemented both in controlled airspace and in free flight airspace and thus it seems a novel method even to solve the transition period for free flight's implementation.

Apart from Chapter 2, all the other chapters always start with a section developing the theoretical mathematical background. This section is then followed by a description of the code and finally by showing examples.

Chapter 2 carries out a delay analysis cost in terms of expenses and safety.

Chapter 3 presents airport capacity as a four dimensional discrete random variable. For each pair of number of arrivals and departures, this airport capacity model gives the probability that the airport is able to deal with such a number of movements (arrivals and departures). This new proposed model has been tested with Airport London Heathrow Capacity.

In Chapter 4, a methodology aimed at developing a strategic schedule is showed. A minimum conflict schedule is proposed according to delays estimations achieved by modelling aircraft arrival times as random variables. This schedule has been tested with Glasgow International Airport.

Chapter 5 proposes a real-time tool for tactical planning. A basic example applied to Glasgow International Airport has been developed.

In Chapter 6, an algorithm that ensures separation minima between aircraft at strategic planning is presented. A schedule that reduces the probability of overloading ATC Centres is proposed by modelling aircraft's position as random variables.

Chapter 7 defines a model for air traffic density. This model has been applied for Glasgow International Airport.

Final conclusions are given in Chapter 8.

Appendix A shows an example of the software developed by using Matlab GUI (*Graphic User Interface*).

Appendix B carries out a data correlation analysis to justify the hypothesis assumed in this thesis.

Chapter 2

Delay Cost Analysis

2.1 Introduction

This section presents a delay analysis in terms of cost and safety. Such a study is necessary to give priorities in the decision-making process to determine flights priorities. In [36], [37], [38], [39], [40], [41] and [42], a broad and general delays analysis is carried out. Anyway, a practical and effective cost estimation is impossible to perform because of the difficulty to gain data from airlines. [43] and [44] present a critique and detailed investigation to figure out airlines expenses. However in these works, no concrete data and information are shown because airlines are not willing to publish their personal accounting balance.

Elementary models to estimate the delay cost are presented. The aim is to give an idea of the major costs effecting airlines to determine the most cost-effective scenario. Airport operators can only identify some particular operations to reduce delays cost. First a description of these possible procedures is given, and then an analysis of the cost is presented.

The different procedures are:

- Airborne Delay
- Ground Hold Delay
- Speed Control
- Re-routing

These procedures are detailed described in the following sections.

2.2 Airborne delay

Airborne delay is the procedure that consists in involving delaying aircraft when they are still in the air. The assignment of airborne delay is a policy widely used to avoid that aircraft arrive at the destination airports when these are still busy and overloaded. When this situation occurs, aircraft are obliged to wait in the air until the control tower authorise them to land. The cost of the operation will be, thus, higher than it was expected at the beginning. In terms of safety, airborne delay is the worst circumstance when it is compared with the other procedures. It is always riskier to be in the air than to be in the ground, airborne delay increases the flying time and thus the probability of a failure.

2.2.1 Airborne delay: Flight and cabin crew

Airborne delay implies an increase in flight time and therefore it may result in additional operating costs and relief crews may be necessary to cover airlines operations when the previous crew have exceeded their time on duty as a result of airborne delays.

Flight and cabin crew cost is usually expressed as hourly cost for a given aircraft. Taking into the account that airborne delay increases with flying time, the product between the hourly flight and cabin crew cost and the delay will give the additional expense. Thus, the function describing the airborne delay cost is assumed to be linear on the delay.

The constant of proportionality varies with the aircraft type:

$$C_{fc} = c_{fc} \cdot t$$

where c_{fc} is the average wage of the flight and cabin crew per hour while t is the time always expressed in hour. It is necessary to consider the average as pilots, co-pilots and cabin attendants have different wages. In further work even the significant variations in salary due to the different policy of countries and airlines can be taken into account.

2.2.2 Airborne delay: Fuel consumption

Fuel consumption varies considerably from route to route in relation to sector lengths, aircraft weight, weather conditions and so on. Therefore a hourly fuel cost tends to be only a rough approximation. According to [45], an approximate estimation of the fuel consumption is given as the fuel flow (Kg/min) which is function of the thrust and it depends on the aircraft type and on the flight phase.

Jet and Turboprop:

$$f_{cr} = \eta \cdot T \cdot c_{fcr}$$

$$f_{\min} = c_{f3} \cdot (1 - \frac{h}{c_{f4}})$$

$$f_{nom} = \eta \cdot T$$

where:

f_{cr} [Kg/min]: fuel consumption in cruise conditions;

f_{min} [Kg/min]: fuel consumption for descend, and idle thrust conditions;

f_{nom} [Kg/min]: fuel consumption for all other flight's phases conditions;

T [N]: Thrust;

h [ft]: altitude above sea level.

The thrust specific consumption η [Kg/min/KN], is calculated as:

Jet: $\eta = c_{f1} \cdot (1 + \frac{V_{TAS}}{c_{f2}})$

Turboprop: $\eta = c_{f1} \cdot (1 - \frac{V_{TAS}}{c_{f2}}) \cdot (\frac{V_{TAS}}{1000})$

Piston: $f_{cr} = c_{f1} + c_{fcr}$

$f_{\min} = c_{f3}$

$f_{nom} = c_{f1}$

$c_{f1}, c_{f2}, c_{f3}, c_{f4}, c_{fcr}$ are fuel consumption coefficients calculated from reference conditions.

Therefore, the total fuel consumption can be calculated as follows:

$$F(Kg) = f \cdot t$$

and the cost of fuel consumption is:

$$C_F = c_{fuel} \cdot F$$

where c_{fuel} is the price of the aviation fuel.

2.2.3 Airborne delay: Navigation Fees

The navigation fees consist basically on two factors: airports and en route charges. To use airport facilities, airlines have to pay fees to airport authorities. Runways' utilisation is charged with a landing fee related to aircraft's weight while the use of the terminal facilities is charged with a fee based on the number of passengers boarded at that airport.

Civil aviation authorities impose navigation fees on aircraft flying through their airspace to cover the expenses caused by the air traffic control centres. Air traffic control centres provide a series of disparate services and they enable aircraft to fly safely within the airspace.

In USA air traffic control centres are free public service and charges are not imposed directly to airlines. Costs due to the navigation expenses are recovered by the Federal Aviation Administration with passenger tickets through an adding tax.

In Europe, the Central Route Charges Office (CRCO) is the organisation designed for the navigation fee assignments. A fee, that takes into account the distance flown, is charged for each flight performed under Instrument Flight Rules (IFR) in the Flight Information Regions (FIRs) falling within the competence of the Member States. The total charge per flight R equals the sum of the charges r_i generated in the FIRs of the individual States i concerned:

$$R = \sum_{n} r_i$$

The individual charge r_i is equal to the product of the distance factor d_i, the weight factor p and the unit rate t_i.

$$r_i = d_i \cdot p \cdot t_i$$

the product between d_i and p is defined as the number of service units in State i for this flight. An aircraft delayed in the air will run more distance in the State of the destination airport, according to this taxation airborne delay entails an extra cost in en route charges. The additional expense due to the airborne delay is a linear function of the distance flown in the last State's airspace. As aircraft's speed is supposed to be constant during airborne delay, the distance is proportional to the time through the velocity. With this idea, the cost due to the navigation fees during airborne delay can be considered linear with the time.

The navigation charges can be expressed as follows:

$$C_{tax} = \sum_{n} (\sum_{m} v_j \cdot t_j) \cdot p \cdot t_i$$

where v_j is the constant velocity of the aircraft in the different periods of the flight, t_j is the time the aircraft is flying at that speed and m is the number of periods considered during the flight.

2.3 Ground Hold Delay

Ground-holding policy consists in delaying aircraft's departure even if this airborne is ready to take off. This plane will not be allowed to start the engines and leave the gate or parking area. The objective of Ground-Holding policies is to absorb airborne delay on the ground because ground delays entail crew, maintenance and depreciation costs while airborne delays entails, in addition, fuel and safety costs. In the case the aircraft had already taken off, it is obvious that the Ground-Holding policy cannot be used.

To estimate the cost related to the ground hold delay, the following main points are considered.

2.3.1 Ground hold delay: Flight and cabin crew

Everything shown in section 2.2.1 concerning the flight and cabin crew can be applied in the ground holding issue. Since the flight and cabin crew cost is calculated on a hourly basis, there is no difference if the aircraft is waiting on the air or on the ground.

2.3.2 Ground hold delay: Navigation Fees

The en route charges will not be increased since the aircraft will fly the previous planned distance in each State's airspace. The ground hold delay will cause different airport's charges as the aircraft uses the departure terminal more. In addition to the normal charge, aircraft will pay parking or hangar fees since they stay in the airport beyond the fixed time period. Anyway these adding charges are relatively small when they are compared with the basic landing and passengers fees. As the parking and hangarage fees are built in a hourly basis, the additional cost will be a linear function of time:

$$C_{af} = c_{af} \cdot t$$

where c_{af} is the hourly cost of the additional airport fees. When an aircraft is held on the ground, fuel is not consumed. This is the great advantage of this method in terms of the economical issue. The total cost will be lower than in any other cases as during the ground hold policy there is not any consumption of additional fuel.

2.4 Speed Control

According to this procedure, aircraft modify their velocity while they are maintaining the same route. Safety is prejudiced due to the differing flying times, decreasing speed aircraft will fly longer giving more chances for eventual failures. Increasing speed, aircraft will be in the flying phase for less time decreasing probability of misadventures but components will be more stressed increasing, thus, chances of eventual failures. In conclusion according to each type of aircraft and flight operation, speed control can increase or decrease safety.

2.4.1 Speed Control: Flight and cabin crew

The cost of the flight and cabin crew is computed similarly to the airborne and ground hold delay as shown in the previous sections 2.2.1 and 2.3.1. Thus, the function describing the delay cost due to the speed control is assumed to be linear on the delay. The case when aircraft arrive earlier than scheduled will be investigated in section 2.6.

2.4.2 Speed Control: Fuel cost

Aircraft in normal conditions during their own route fly in cruise with a velocity that is the optimal in term of safety and fuel consumption. Cruise speed differs for each type and kind of aircraft. Increasing the velocity, the airborne will be effected by a major fuel consumption when this is compared with its optimal condition. Decreasing the speed, the fuel burnt can be lower, equal or higher with the amount of fuel consumed when the aircraft fly in its optimal cruise speed. In this thesis the cost caused by the speed control entails in the same methodology previously treated during the airborne delay a shown in section 2.2.2. The velocity of the aircraft is assumed to be constant during the cruise phase, furthermore, the time needed for the speed control action is assumed to be relatively short when it is compared with the cruise period. In this hypothesis, the cost of the speed control method is a linear function of time.

2.4.3 Speed Control: Navigation Fees

As the aircraft changes only its velocity while heading the same route, the total distance flown is identical to the estimated one. The needed time to fly will be shorter or longer depending on the kind of the speed control. The en route and airport charges will be equal as the distance flown and the time of hangaring are the same.

2.4 Re-routing

Re-routing consists on changing the route once the aircraft has left its departure airport. This is an operation implemented to decrease congestion at the arrival airport or in the airspace. In terms of cost it can be compared to the airborne delay.

2.4.1 Re-routing: Flight and cabin crew

The cost of the flight and cabin crew is computed similarly to the airborne and ground hold delay as shown in the previous sections 2.2.1 and 2.3.1. Thus, the function describing the delay cost due to the re-routing control is assumed to be linear on the delay. The case when aircraft arrive earlier than scheduled will be investigated in section 2.6.

2.4.2 Re-routing: Fuel cost

During the cruise phase, the velocity of the aircraft is the optimum in term of fuel consumption. Re-routing causes an inevitable change in the travelling time and, thus, the total fuel burnt during the flight is different when it is compared with the consumption during the standard route.

If the re-routing will oblige the flight to a longer journey, the fuel consumption during the flight will increase. In this circumstance, the fuel cost is supposed to be linear function of the time because it is assumed that the velocity of the aircraft during the re-routing is constant.

2.4.3 Re-routing: Navigation Fees

The airport fees will be basically identical while, as the aircraft will cross different airspace, the en route charges will be different. It is assumed that the velocity is constant during the cruise phase, according to the European route charge method the additional charge can be approximated to a linear function of time.

2.5 Delay cost

In this section, the costs due to the different kind of delays are reassumed:

2.5.1 Airborne delay cost

The factors considered in this case are:

- Flight and cabin crew cost (per aircraft type) $C_{fc} = c_{fc} \cdot t$

- Fuel cost $C_F = c_{fuel} \cdot f \cdot t$:

  Fuel consumption is different for each kind and type of aircraft. The parameter f depends on the phase of the flight, for airborne delay $f = f_{nom}$ as the aircraft change its own speed.

- Navigation Fees $C_{tax} = v_h \cdot t \cdot p \cdot t_i$:

  This is the cost of the airborne delay in the holding area. v_h is the aircraft speed during the airborne delay, t is the time the aircraft is waiting before landing, p is the weight factor (it depends on the aircraft type) and t_i is the unit rate for flights in the FIRs of State (this parameter depends on the State of the destination airport).

The total cost can be expressed as:

$$c^{ai} = (c_{fc} + c_{fuel} \cdot f_{nom} + v_h \cdot p \cdot t_i) \cdot t$$

2.5.2 Ground hold cost

The ground holding cost is due to:

- Flight and cabin crew cost: $C_{fc} = c_{fc} \cdot t$

- Navigation Fees: $C_{af} = c_{af} \cdot t$

  where c_{af} is the cost of the additional airport fees that the airline must pay because of the delay.

The total cost is: $c^{gh} = (c_{fc} + c_{af}) \cdot t$

2.5.3 Speed control delay cost

The factors considered are:

- Flight and cabin crew cost: $C_{fc} = c_{fc} \cdot t$

- Fuel cost: $C_F = c_{fuel} \cdot f \cdot t$

  In this case the parameter $f{=}f_{cr}$ as the aircraft during the re-routing is supposed to keep its own cruise speed.

The total cost in this case is:

$$c^{sc} = (c_{fc} + c_{fuel} \cdot f_{cr}) \cdot t$$

2.5.4 Re-routing delay cost

The total cost is:

- Flight and cabin crew cost: $C_{fc} = c_{fc} \cdot t$

- Fuel cost: $C_F = c_{fuel} \cdot f \cdot t$

  The fuel flow is f_{cr} because the aircraft keep the cruise phase during the re-routing.

- Navigation Fees:

  Referring to the final navigation charges that were estimated in the previous paragraph.

$$C_{tax} = \sum_{n} (\sum_{m} v_j t_j) \cdot p \cdot t_i$$

$$C_{tax}' = \sum_{n} (\sum_{m} v_j t_j') \cdot p \cdot t_i$$

$$\Delta C_{tax} = C_{tax}' - C_{tax} = \sum_{n} p \cdot t_i \cdot (\sum_{m} v_j t_j' - v_j t_j) \approx c_{tax} \cdot t$$

The total cost can be expressed as follows:

$$c^{re} = (c_{fc} + c_{fuel} \cdot f_{cr} + c_{tax}) \cdot t$$

2.5.5 Comparison between delay cost

According to these expressions, all kind of cost are approximated into linear functions of the time. The coefficient of proportionality between delay and cost is different for each case. The most expensive delay is the airborne delay because extra significant fuel consumption is added. This is caused because aircraft wait in the air as at the arrival airport there is too much congestion. Re-routing and speed-control are actions that result in less expenses than the airborne delay if properly and efficiently applied.

The solution, which offers the minimum in term of expenses, is the ground hold delay. Neither extra-fuel is burnt nor navigation fees are added. This method offers even the great advantage to be safer as aircraft held on the ground rather than being in the air. Flying is always a critical phase as eventual failures of the devices can cause terrible disasters. All the aeronautical community considers the ground hold to be the best technique to apply in presence of delays and congestion.

In Figure 2.1, all the cost referred to the different kind of delays are shown.

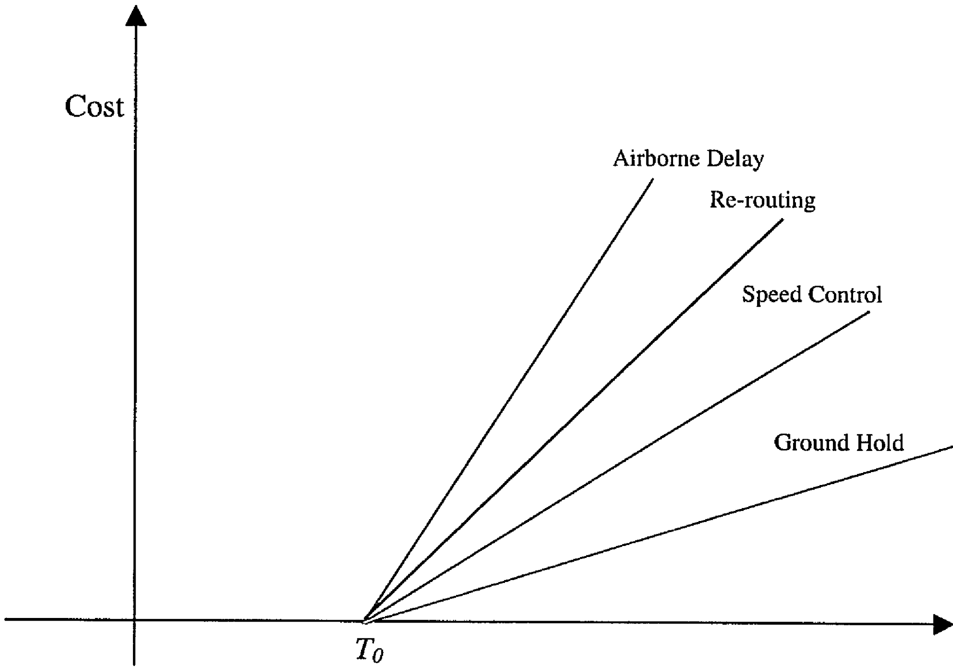


Figure 2.1: Comparison of the costs due to the different kind of delays.

where T_0 is the scheduled arrival time. Figure 2.1 corresponds to a fixed type of aircraft as each aircraft needs a particular flight cabin and has a different rate of fuel consumption.

2.6 Earlier arrival

In the previous sections, only the cost due to delays is analysed. When an aircraft arrives earlier than scheduled, the destination airport could be overloaded. In this situation, the control tower will not allow the new arrival to start its landing procedure and airborne delay will be assigned to it. In this circumstance the additional cost of the flight is mainly due to the fuel consumption. Navigation fees will not change because the route flown in each State's airspace is the same. Only the cost related to the fuel consumption is relevant to estimate the additional expenses for an earlier arrival. The aircraft flies with a higher speed than the cruise velocity (it might be caused by the wind) and, hence, a major quantity of fuel is burnt.

The expense due to the airborne delay is:

$$c^{ea} = c_{fuel} \cdot \left[f_{hs} T_f - f_{cr} T \right] + \left(c_{fc} + c_{fuel} \cdot f_{nom} + v_h \cdot p \cdot t_i \right) \cdot \left(T - T_f \right)$$

where, f_{hs} is the fuel flow when the aircraft is flying with a speed higher than the cruise velocity; T is the duration of the scheduled flight and T_f is the real duration of the flight ($T_f < T$). In a first approximation, it is assumed that the major quantity of fuel is consumed during the cruise phase and thus the cost for an earlier arrival is a linear function of the time. Figure 2.2 shows the cost due to an earlier arrival:

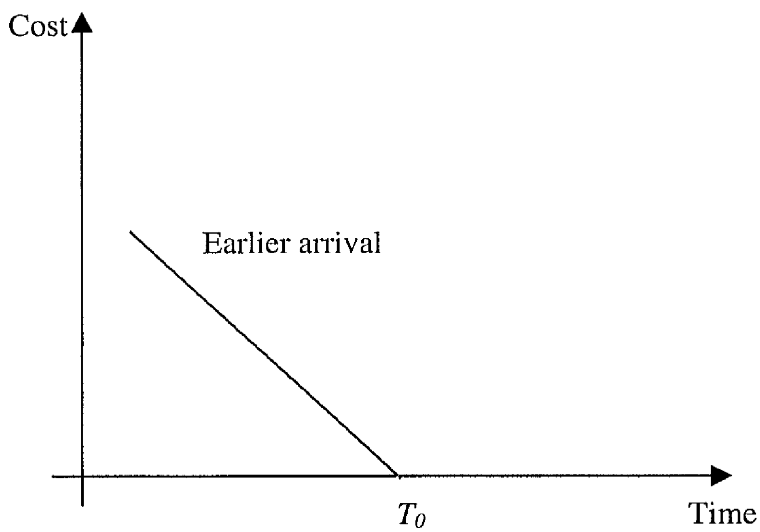


Figure 2.2: Cost for an earlier arrival.

In some circumstances it can be required that an aircraft arrives earlier than it is scheduled. Imagine that it is known that an aircraft arrive late and then this aircraft is planned to depart just soon after for another destination. If an aircraft of the fleet arrives earlier, it can be temporally assigned to this new destination in order to minimise airlines global cost.

Another factor to take into account, when an aircraft arrives earlier, is that maybe the runways are free but the airport facilities are under congestion. For instance the staff, who are in control of the service *"baggage reclaim"*, can be already overloaded for other aircraft previous landed. This last point has an important significance for the gate-

to-gate problem. Although the flight arrived even earlier, passengers might go out from the airport even later than if the flight landed on time.
The model that is proposed in this thesis is only a first approximation to the issue. Many other factors are not included such as the customer's satisfaction. If passengers are not pleased by the offered service than it is likely to have a reduction of the customers for the future.

The cost function for earlier and later arrivals is showed in Figure 2.3.

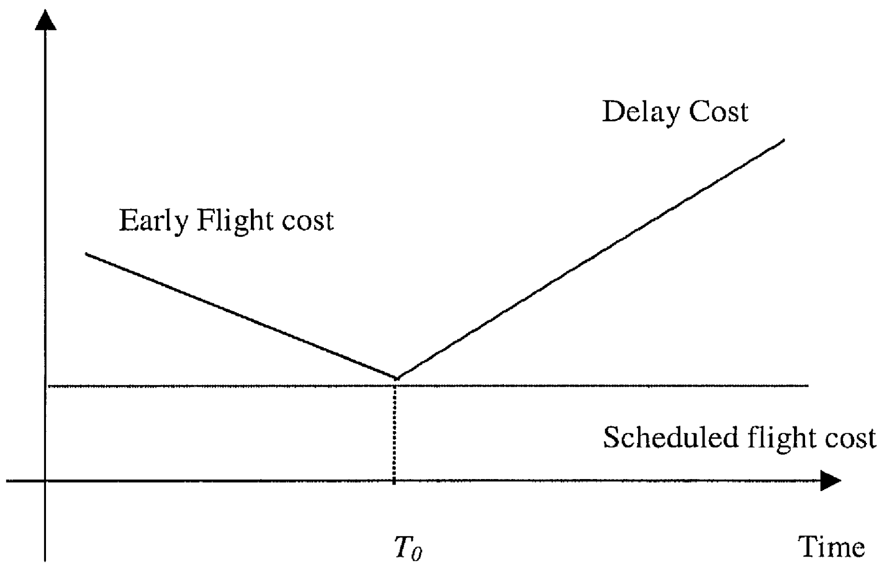


Figure 2.3: Cost for later and earlier arrival.

where T_0 is the scheduled arrival time.

2.7 Missed a connection

When a delay happens, passengers are in danger to miss the successive flight if they have eventually a connection. Hubs are a chosen airport by a given airline where most of its flights arrive to allow connections with other aircraft. With this policy, airlines succeed in increasing the number of destination airports keeping the same number of aircraft for their fleet. One of the major requirements demanded on pilots is to reach hubs on time. Delays increase the risk of missing the connections and these circumstances cause extra expenses for airlines.

When a flight arrives at an airport chosen to be a hub by an airline, some passengers board on another flight to reach their final destination. If the aircraft is on delay, two different options can be presented:

1. The connection flight does not wait for the passengers.
2. The connection flight waits for the passengers.

Airlines can choose between the first and the second option. One of the reasons that influences the choice is the number of passengers travelling in the second flight and even the number of tickets sold in economy and in business class. In the first option, passengers will be accommodated on the next aircraft that is heading to their final destination. This next flight could happen in the same or in another day. Airlines will reimburse the expenses for meals and if it is necessary to host the passengers for one night even in an hotel accommodation. According to each airline policy, even an indemnity can be given to those passengers that have missed the connection. Then even the administrative expenses caused by organising all the circumstances have to be added. The total cost for a missed connection is a function of the number of passengers. It is not directly related to the amount of the delay as if the aircraft arrive so late the connection flight has already departed, the cost will be fixed. The total cost for a missed connection is given adding this fixed cost and the cost due to the amount of delay itself, as shown in Figure 2.4.

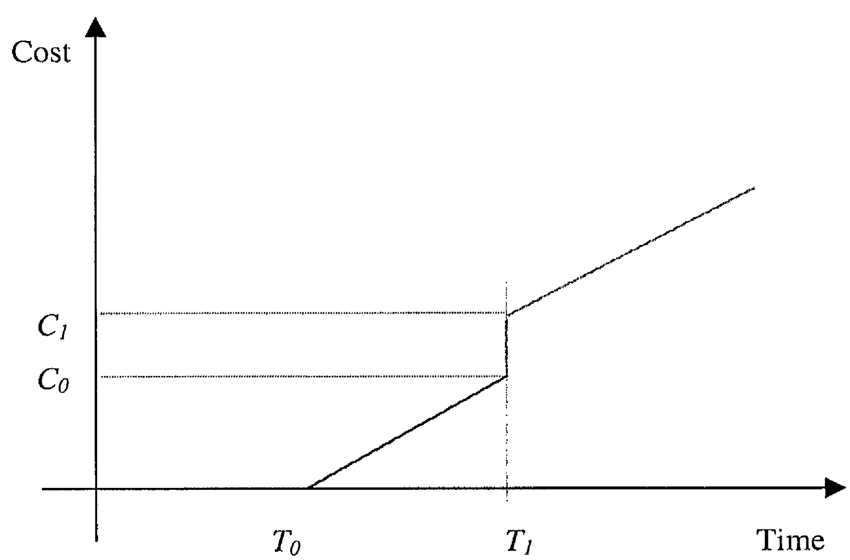


Figure 2.4: Cost function for a missed connection.

where C_1 is the cost of the first missed flight, T_0 is the scheduled arrival time and T_1 is the departure time of the flight planned for the connection. If the delay exceeds the given time T_1, passengers have missed the connection flight. The cost increases of a quantity equals to C_1-C_0 that is directly proportional to the number of passengers that have missed this connection.

In the second option, the connection flight waits until the delayed aircraft arrive at the hub airport and passengers board it. This circumstance is equivalent to a ground hold delay, this cost has been already detailed described in the section 2.3.2.

At the beginning of the day, the number of passengers that will require a connection flight is known. Airlines can estimate the cost of each missed connection. According to the cheapest solution airlines can establish which option applies in case of delays. If the number of the passengers that need the connection is low, the cost for the ground hold delay is higher than the expenses for the missed flight. In this case the departure flight is allowed to take off without any passengers coming from the previous flight.

The optimum criteria to take advantage of the airline fleet consist in using all planes as much as possible. According to this idea the less time aircraft are hold in airports, more profits are gained by airlines. Flights belonging to the same fleet are required to be perfectly co-ordinated and synchronised in order to minimise their parking time.

2.8 Cancellation cost

Airlines try to minimise the parking times of their aircraft in order to optimise the efficiency of the fleet. A plane in the same day is assigned to run different flights, airlines attempt to reduce the time between its arrival and its consecutive departure. Sometimes the delay suffered by an aircraft is so high that the eventual consecutive flight assigned to that plane should be delayed by an amount that it is more economically convenient to cancel it. Both in the departure and in the arrival airport, even if the flight is cancelled a slot has been already assigned. It concerns to airport policy decide whatever or not make the airline pay a fine and the eventual its amount. Airlines have to afford all the expenses for the additional services given to the passengers of the cancelled flight. They have to provide these customer meals and even accommodation if the next flight planned to that destination is planned for the next day. Airlines sometimes give even compensation in money to satisfy these customers.

All these factors are taken into account to estimate the cost due to a cancellation. All these terms are constants, as it is possible to estimate the necessary amount of money for the airport services once the number of passengers is known.

As shown in section 2.3.2, the cost for ground hold delay is linear with the time. The cost for flight cancellation and for ground hold delay are plotted in Figure 2.5. It is possible to observe a point of intersection. At this point the two costs are equal and in economical terms the amount of the expenses are the same in both cases. If T_{canc} is the value of time the cost of the cancellation equals the cost of ground hold delay, then for value of $t<T_{canc}$ it is more cost effective to employ ground hold procedure rather than the cancellation of the flight.

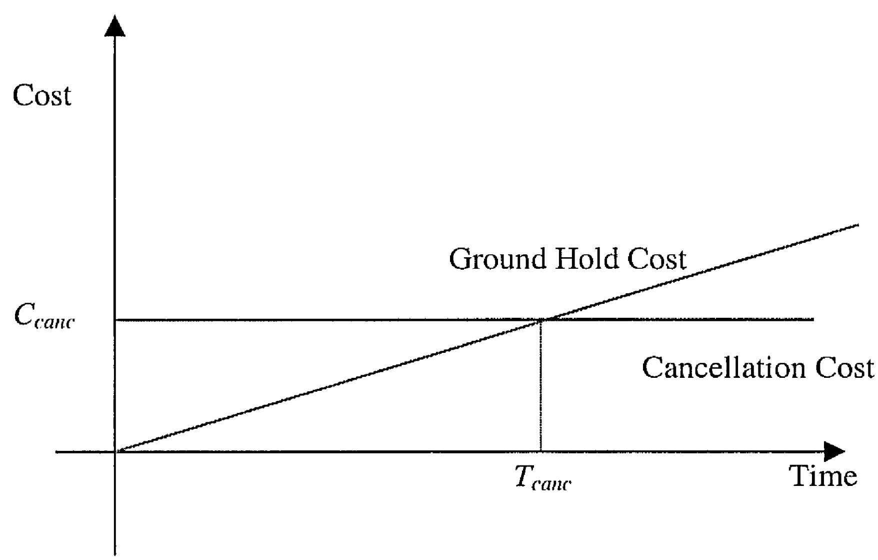


Figure 2.5: Relation between Ground hold Cost and Cancellation Cost.

where C_{canc} is the cancellation cost.

Chapter 3

Airport Capacity Model

3.1 Introduction

Airport capacity is a parameter related to the airport's systems and air traffic control services, which expresses the number of arrivals and departures that the airport can manage successfully ensuring an established level of safety. As already discussed in section 1.6.1, a definition of airport capacity has been given in [20].

Currently each airport declares its own estimated capacity taking into account its own facilities and infrastructures [7]. The number of runways and gates available are examples of parameters effecting the number of aircraft landing and taking off that the airport can manage. In many circumstances it has been empirically observed that the actual operational airport capacity differs from its own declared capacity. Airport capacity is a variable effected by highly unpredictable parameters such as rain, fog, visibility and so on. During adverse weather conditions, the ability of the terminal area operators to efficiently control and maintain safety levels in the airport operations is reduced. During a storm, the air traffic controllers are unable to maintain the normal volume of arrivals and departures causing inevitable delays.

In this chapter a development of a new methodology for estimating the actual operational capacity of an airport is shown. This methodology is based on statistics and probability theory and uses historical arrival and departure data rather than information regarding the airport's infrastructure. The proposed methodology is applied to calculate the capacity of London Heathrow Airport. The following example shows a model of random airport capacity, that is a variable which gives for a fixed number of scheduled arrivals and scheduled departures, the probability that the airport is able to deal with such a number of movements.

3.2 Mathematical model

For clarity, the following editing rules will be followed in all sections of this dissertation concerning the mathematical models:

- scalars will be typed in normal text (e.g. ΔT, n).
- sets, in which scalars are defined, are typefaced in italic boldface letter (e.g. $\boldsymbol{N}$, $\boldsymbol{R}$,$\boldsymbol{T}$).
- all random variables will be written in boldface letters (e.g. $\mathbf{D}$, $\mathbf{D}^s$, $\mathbf{d}^a$).
- all multidimensional random variables in outlined boldface (e.g. $\underline{\mathbf{C}}$, $\underline{\mathbf{v}}$, $\underline{\mathbf{s}}$).
- sets, in which random variables are defined, are typefaced with the following style $\mathcal{K}$, $\mathcal{A}$, $\mathcal{B}$.

3.2.1 Airport Capacity as a Four Dimensional Discrete Random Variable

As defined by [46], a four dimensional discrete random variable consists of an experiment Ω with a probability measure $\Pr\{\cdot\}$ defined on a sample space and a function that assigns a set of four discrete numbers to each outcome of in the sample space of the experiment Ω.

Define:

$$\underline{\mathbf{C}} \stackrel{\Delta}{=} (\mathbf{D}, \mathbf{D}^s, \mathbf{A}, \mathbf{A}^s) \qquad \mathbf{C} \subseteq \mathcal{K}^4$$

where:

$\mathbf{D} \stackrel{\Delta}{=}$ **D**eparture Capacity. This is modelled as a discrete random variable defined in the set $\mathcal{X}$. To every outcome $\zeta \in \mathcal{X}$ of the experiment $\mathbf{\Omega}$, an integer number $\mathbf{D}(\zeta) \in N$ is assigned. The Departure Capacity is the discrete random variable that gives the probability for a fixed number of departures occurring in a fixed interval time $\Delta T \in \boldsymbol{R}$. Given a number of scheduled departures, number of arrivals, number of scheduled arrivals, the number of departures is observed for a fixed interval time $\Delta T \in \boldsymbol{R}$. The ratio between this number of departures and the number of observations gives the relative frequency. According to the law of large number, for an enough large number of observations the relative frequency converges to the probability of the event.

$\mathbf{D}^s \stackrel{\Delta}{=}$ **D**eparture **S**cheduled Capacity. This is modelled as a discrete random variable defined in the set $\mathcal{X}$. To every outcome $\zeta \in \mathcal{X}$ of the experiment $\mathbf{\Omega}$, an integer number $\mathbf{D}^s(\zeta) \in N$ is assigned. The Departure Scheduled Capacity is the discrete random variable that gives the probability for a fixed number of scheduled departures occurring in a fixed interval time $\Delta T \in \boldsymbol{R}$. This variable seems to be a deterministic parameter due to the possibility to count the number of scheduled departures. In section 3.2.3 the nature of this parameter will be extensively clarified. Given a number of departures, number of arrivals, number of scheduled arrivals, the number of scheduled departures is observed for a fixed interval time $\Delta T \in \boldsymbol{R}$. The ratio between this number of scheduled departures and the number of the all observations gives the relative frequency. According to the law of large number, for an enough large number of observations the relative frequency converges to the probability of the event.

$\mathbf{A} \stackrel{\Delta}{=}$ **A**rrival Capacity. This is modelled as a discrete random variable defined in the set $\mathcal{X}$. To every outcome $\zeta \in \mathcal{X}$. of the experiment $\mathbf{\Omega}$, an integer number $\mathbf{A}(\zeta) \in N$ is assigned. Given a number of departures, number of scheduled departure, number of scheduled arrivals, the number of arrivals is observed For a fixed interval time $\Delta T \in \boldsymbol{R}$. The ratio between this number of arrivals and the number of the all observations gives the relative frequency. According to the law of large number, for an enough large number of observations the relative frequency converges to the probability of the event.

$\mathbf{A}^s \stackrel{\Delta}{=}$ Arrival Scheduled Capacity. This is modelled as a discrete random variable defined in the set $\mathcal{X}$. To every outcome $\zeta \in \mathcal{X}$. of the experiment **Ω,** an integer number $\mathbf{A}^s(\zeta) \in N$ is assigned. The Arrival Scheduled Capacity is the discrete random variable that gives the probability for a given number of scheduled arrivals occurring in a fixed interval time $\Delta T \in \boldsymbol{R}$. This variable seems to be a deterministic parameter due to the possibility to count the number of scheduled departures. In section 3.2.3 the nature of this parameter will be extensively clarified. Given a number of departures, number of arrivals, number of scheduled arrivals, the number of scheduled arrivals is observed for a fixed interval time $\Delta T \in \boldsymbol{R}$. The ratio between this number of scheduled arrivals and the number of the all observations gives the relative frequency. According to the law of large number, for an enough large number of observations the relative frequency converges to the probability of the event.

$\underline{\mathbf{C}} \stackrel{\Delta}{=}$ Capacity of the airport. This is a four dimensional discrete random variable defined in the set $\mathcal{X}$. To every outcome $\zeta \in \mathcal{X}$ of the experiment **Ω,** a set of four integer numbers $\mathbf{D}(\zeta) \times \mathbf{D}^s(\zeta) \times \mathbf{A}(\zeta) \times \mathbf{A}^s(\zeta) \in N^4$ is assigned. Capacity of the Airport is a discrete random variable that gives the probability for a given number of departures, scheduled departures, arrivals and scheduled arrivals occurring for a fixed interval time $\Delta T \in \boldsymbol{R}$.

3.2.2 Probability Density Function Building for Airport Capacity

The experiment **Ω** is defined in the following step:

ζ = observation of the number of departures, scheduled departures, arrivals, scheduled arrivals in a time interval ΔT.

To clarify: the outcome ζ of the experiment **Ω** is a set of four integer numbers $\zeta=(n_a, n_{as}$ $n_d, n_{ds})$ where

n_d is the number of departures occurred in the time interval $\Delta T \in \boldsymbol{R}$.

n_{sd} is the number of scheduled departures occurred in the time interval $\Delta T \in \boldsymbol{R}$.

n_a is the number of arrivals occurred in the time interval $\Delta T \in \boldsymbol{R}$.

n_{sa} is the number of scheduled arrivals occurred in the time interval $\Delta T \in \boldsymbol{R}$.

The four dimensional discrete random variable $\underline{C} \subseteq \mathcal{X}$ is defined by:

$$\underline{C}(\zeta) \triangleq \zeta$$

It follows that $\zeta = (n_d, n_{ds}, n_a, n_{sa}) \in \boldsymbol{R}^4$ which indicates the number of arrivals, scheduled arrivals, departures, scheduled departure has a double meaning. $\zeta = (n_d, n_{ds}, n_a, n_{sa}) \in \boldsymbol{R}^4$ is both the outcome of the experiment $\boldsymbol{\Omega}$ and the value of the four dimensional discrete random variable. Thus, the four dimensional discrete random variable $\underline{C} \subseteq \mathcal{X}$ has as domain $\boldsymbol{N}^4$ and range the set N^4. The airport capacity $\underline{C} \subseteq \mathcal{X}$ is a four-dimensional integer random variable.

$$\underline{C}(\zeta) \subseteq \mathcal{X} = N^4$$

Supposing that the experiment $\boldsymbol{\Omega}$ is performed $n \in \boldsymbol{N}$ times, at a given outcome ζ of the experiment, the four dimensional discrete random variable $\underline{C} \subseteq \mathcal{X}$ associates a set of four different values $\underline{C}(\zeta)$. Given a set of four integer numbers $(d, d^s, a, a^s) \in N^4$ and denoting by $n(d, d^s, a, a^s) \in \boldsymbol{N}$ the total number of trials such as

$$\underline{C}\left(\mathbf{D}(\zeta), \mathbf{D}^s(\zeta), \mathbf{A}(\zeta), \mathbf{A}^s(\zeta)\right) = \left(d, d^s, a, a^s\right)$$

the statistics (Probability Density Function PDF and Cumulative Density Function CDF) of the random variable $\underline{C} \subseteq \mathcal{X}$ are possible to be determined by applying the *Law of Large numbers* [47] [48]:

If the probability of an event a in a given experiment Ω equals p and the experiment is repeated n times, then for any $\varepsilon>0$

$$\lim_{n\to\infty} \Pr\left\{\left|\frac{n_a}{n} - p\right| \leq \varepsilon\right\} = 1$$

where n_a equals the number of success of a.

This concept may also be expressed as:

If the experiment Ω is repeated n times and the event a occurs n_a times, then, with a high degree of certainty, the relative frequency n_a/n of the occurrence of a is close to $p=Pr(a)$,

$$\Pr(a) \cong \frac{n_a}{n}$$

In conclusion the PDF of $\underline{\mathbf{C}}$ is computed as ratio between the number of trials such as $\underline{\mathbf{C}}(\mathbf{D}(\zeta), \mathbf{D}^s(\zeta), \mathbf{A}(\zeta), \mathbf{A}^s(\zeta)) = (d, d^s, a, a^s)$ and the total number of trials n.

$$\Pr(\mathbf{D} = d, \mathbf{D}^s = d^s, \mathbf{A} = a, \mathbf{A}^s = a^s) \cong \frac{n(d, d^s, a, a^s)}{n}$$

This is an important result because the PDF of $\underline{\mathbf{C}}$ can be built using real data.

3.2.3 Explanation of Airport Capacity

An airport capacity $\underline{\mathbf{C}} \subseteq \mathcal{X}$, defined as a four dimensional discrete random variable, has been introduced. For a fixed number of arrivals, scheduled arrivals, departures and scheduled departures, the random variable airport capacity $\underline{\mathbf{C}} \subseteq \mathcal{X}$ gives the probability that these fixed numbers of arrivals, scheduled arrivals, departures and scheduled departures occur.

In most of the applications, given a scheduled number of arrivals and a scheduled number of departures, the probability that the airport is able to deal with these given numbers of arrivals and departures is required to compute. The task of airport operators is to schedule *d* (number of departures) and *a* (number of arrivals). These quantities are known because they are fixed previously by operators. The random variable airport capacity will be reduced from a four dimensional random variable to a two dimensional random variable with the use of the conditional probability.

$$\Pr\left\{\begin{array}{l}\text{airport deals with} \\ \quad d \text{ departures} \\ \quad a \text{ arrivals} \\ \quad \text{assumed} \\ \; d \text{ scheduled departures} \\ \; a \text{ scheduled arrivals}\end{array}\right\} = \Pr\{\mathbf{D} = d, \mathbf{A} = a \mid \mathbf{D}^s = d, \mathbf{A}^s = a\} =$$

$$= \frac{\Pr\{\mathbf{D} = d, \mathbf{D}^s = d, \mathbf{A} = a, \mathbf{A}^s = a\}}{\Pr\{\mathbf{D}^s = d, \mathbf{A}^s = a\}} = f_{\underline{\mathrm{C}}(\mathrm{DA})}(d, a \mid \mathbf{D}^s = d, \mathbf{A}^s = a)$$

In terms of frequency interpretations the required probability can be given by considering a sequence of *n* trials and denoting by *n(d,a)* the sub-sequence of this *n* trials such as:

$$\mathbf{D}^s(\zeta) = d, \mathbf{A}^s(\zeta) = a$$

Within this sub-sequence having ignored all others trials, denoting by *n(d,a)* the number of trials such as:

$$\mathbf{D}(\zeta) = d, \mathbf{A}(\zeta) = a$$

the required conditional probability can be calculated as:

$$\frac{\Pr\{\mathbf{D} = d, \mathbf{D}^s = d^{s*}, \mathbf{A} = a, \mathbf{A}^s = a^{s*}\}}{\Pr\{\mathbf{D}^s = d^{s*}, \mathbf{A}^s = a^{s*}\}} \cong \frac{n(d,a)}{n(d^{s*}, a^{s*})}$$

The probability that the airport deal with maximum d departure and a arrivals is given by:

$$\Pr\left\{\begin{array}{l}\text{airport deals with maximum}\\ \quad d \text{ departures}\\ \quad a \text{ arrivals}\\ \quad \text{assumed} \quad \text{exactly}\\ d \text{ scheduled departures}\\ a \text{ scheduled arrivals}\end{array}\right\} = \Pr\{\mathbf{D} \le d, \mathbf{A} \le a \mid \mathbf{D}^s = d, \mathbf{A}^s = a\} =$$

$$= \frac{\Pr\{\mathbf{D} = d, \mathbf{D}^s = d, \mathbf{A} = a, \mathbf{A}^s = a\}}{\Pr\{\mathbf{D}^s = d, \mathbf{A}^s = a\}} = F_{\underline{C}(\mathrm{DA})}(d,a \mid \mathbf{D}^s = d, \mathbf{A}^s = a)$$

3.2.4 Airport Capacity as a set of Bernoulli Random Variables

This presentation of the airport capacity in terms of a four dimensional discrete random variable might seem complex and even tedious. Another way to present intuitively the airport capacity derives by defining for each possible pair of scheduled departures and scheduled arrivals $(d^s,a^s)\in N^2$ a Bernoulli random variable.

$$\textbf{\textit{CBernoulli}} \stackrel{\Delta}{=} \{Bernoulli(p_{d^1a^1}, q_{d^1a^1}), \ldots Bernoulli(p_{d^ia^i}, q_{d^ia^i}) \ldots\} \qquad \{\forall (d^ia^i) \in N^2$$

$\textbf{\textit{CBernoulli}} \stackrel{\Delta}{=}$ This is a set of Bernoulli random variables. A Bernoulli random variable denoted by $Bernoulli(\, p_{d^sa^s}\; q_{d^sa^s})$ is defined for a given pair of scheduled departures and scheduled arrivals $(d^s,a^s)\in N^2$. The event that the airport deals with these fixed numbers of departures and arrivals is labelled *success* while the event that the airport fails to deal with these fixed numbers of departures and arrivals is labelled *failure*.

Denoting by

$p_{d^sa^s}$ = *probability of success*. This is the probability of the event that the airport deals with d^s departures and a^s arrivals.

$q_{d^sa^s}$ = *probability of failure*. This is the probability of the event that the airport fails to deal with d^s departures and a^s arrivals.

the sum of these two terms is equal to one $p_{d^s a^s} + q_{d^s a^s} = 1$ as the certain event is their union event.

To estimate $p_{d^s a^s}$ the ratio between the number of *successes* and the number of trials will be computed. To estimate $q_{d^s a^s}$ the ratio between the number of *failures* and the number of trials has to be computed.

3.2.5 Equivalence between C and C*Bernoulli*

The introductions of airport capacity both as a four dimensional discrete random variable **C** and as a set of Bernoulli random variable C*Bernoulli* seems two different definitions.

Here the proof that they are two equivalent approaches† follows.

Let (d,d^s,a,a^s) be a set a four real number and let **z** be a Bernoulli random variable such as:

$$\mathbf{z}(d,d^s,a,a^s) = \begin{cases} 1 & \text{if} \quad \underline{\mathbf{C}}(\zeta) \le (d,d^s,a,a^s) \\ 0 & \text{if} \quad \underline{\mathbf{C}}(\zeta) > (d,d^s,a,a^s) \end{cases}$$

The expected value of the Beronulli random variable **z** is computed:

$$\begin{aligned} \mathrm{E}\{\mathbf{z}(d,d^s,a,a^s)\} &= 1 \cdot \Pr\{\mathbf{z}(d,d^s,a,a^s) = 1\} = \Pr\{\underline{\mathbf{C}}(\zeta) \le (d,d^s,a,a^s)\} = \\ &= \Pr\{\mathbf{D} \le d, \mathbf{D}^s \le d^s, \mathbf{A} \le a, \mathbf{A}^s \le a^s\} \end{aligned}$$

† In this thesis, all the Probability Density Functions (PDFs) were built through the use of real data. Histogram diagrams of the relative historical occurrences were built. This approach is derived from the direct application of the *Law of large numbers*. This theorem marries Probability and Statistics together as shown extensively in Appendix B.

Applying the law of the large number, for a large sample the expected value of the random variable Bernoulli **z** will be equal to the relative frequency.

$$\mathrm{E}\{\mathbf{z}(d,d^s,a,a^s)\} = \Pr\{\mathbf{D} \le d, \mathbf{D}^s \le d^s, \mathbf{A} \le a, \mathbf{A}^s \le a^s\} \cong \frac{n(d,d^s,a,a^s)}{n}$$

3.3 Study case: London Heathrow Airport Capacity

London Heathrow airport capacity $\underline{\mathbf{C}} \subseteq \mathcal{K}$ were built according to the previous model. Real data has been provided by the United Kingdom Civil Aviation Authority (CAA) to the Air Traffic Management (ATM) Research Group of the University of Glasgow. This information consists of data for London Heathrow Airport. The data set gives the planned arrival and departure times as well as the real arrival and departure times for every flight landing or taking off from London Heathrow airport between 1st of October 1997 and 30th of September 1998. An example of this data is shown in Table 3.1. To build the model for airport capacity, it is necessary to know the number of scheduled/real arrivals $(n_a,n_{as}) \in \boldsymbol{R}^2$ and the number of scheduled/real departures $(n_d,n_{ds}) \in \boldsymbol{R}^2$ that occurs in the airport in a given day and in a fixed interval time $\Delta T \in \boldsymbol{R}$.

The four dimensional discrete random variable airport capacity can be introduced and defined as a stochastic process:

$$\underline{\mathbf{C}}(\zeta;t) = (\mathbf{D}, \mathbf{D}^s, \mathbf{A}, \mathbf{A}^s;t) \qquad \underline{\mathbf{C}} \subseteq \mathcal{K}^4 x\boldsymbol{R}$$

The stochastic process airport capacity gives for a selected month, day and hour, the probability that the airport is able to deal with a given number of movements (arrivals and departures). With this extension for a given instant of time $t \in \boldsymbol{R}$, a new airport capacity, always as a four dimensional random variable, is computed. If for the variable $t \in \boldsymbol{R}$ the month is fixed, then it is possible to say that for each month a new airport capacity is calculated.

Airport	Arr./Dep.	Actual_time	Planned_time	Airline	Flight N.	Other Airports
Heathrow	A	01/01/98 04:17	01/01/98 04:15	Virgin Atlantic Airways	201	Hong Kong (Chep Lap Kok)
Heathrow	A	01/01/98 04:21	01/01/98 04:35	British Airways	32	Manila
Heathrow	A	01/01/98 04:30	01/01/98 04:55	British Airways	134	Riyadh
Heathrow	A	01/01/98 04:30	01/01/98 05:20	British Airways	212	Boston
Heathrow	A	01/01/98 04:33	01/01/98 04:50	British Airways	28	Hong Kong (Chep Lap Kok)
Heathrow	A	01/01/98 04:44	01/01/98 05:15	Cathay Pacific Airways	251	Hong Kong (Chep Lap Kok)
Heathrow	A	01/01/98 04:52	01/01/98 05:25	British Airways	56	Johannesburg
Heathrow	A	01/01/98 05:04	01/01/98 05:35	British Airways	12	Perth
Heathrow	A	01/01/98 05:06	01/01/98 05:10	British Airways	144	Dacca
Heathrow	A	01/01/98 05:10	01/01/98 05:30	Singapore Airlines	322	Singapore
Heathrow	A	01/01/98 05:19	01/01/98 05:55	British Airways	112	New York (Jf Kennedy)
Heathrow	A	01/01/98 05:29	01/01/98 05:40	British Airways	10	Melbourne
Heathrow	A	01/01/98 05:33	01/01/98 05:25	British Airways	34	Jakarta (Soekarno-Hatta Intnl)
Heathrow	A	01/01/98 05:40	01/01/98 05:55	Malaysian Airlines System	2	Kuala Lumpur (Sepang)
Heathrow	A	01/01/98 05:44	01/01/98 05:30	Qantas	9	Sydney
Heathrow	A	01/01/98 06:02	01/01/98 06:30	United Airlines	918	Washington
Heathrow	A	01/01/98 06:04	01/01/98 06:20	American Airlines	86	Chicago
Heathrow	A	01/01/98 06:06	01/01/98 06:45	British Airways	216	Washington
Heathrow	A	01/01/98 06:08	01/01/98 06:30	El Al	317	Tel Aviv
Heathrow	A	01/01/98 06:09	01/01/98 06:40	Air Canada	856	Toronto
Heathrow	A	01/01/98 06:10	01/01/98 06:30	Ghana Airways	730	Accra
Heathrow	A	01/01/98 06:12	01/01/98 06:25	American Airlines	108	Boston
Heathrow	A	01/01/98 06:16	01/01/98 06:20	Gulf Air	7	Abu Dhabi International
Heathrow	A	01/01/98 06:18	01/01/98 07:00	British Airways	174	New York (Jf Kennedy)
Heathrow	A	01/01/98 06:19	01/01/98 06:25	British Airways	128	Dhahran
Heathrow	A	01/01/98 06:21	01/01/98 06:55	Virgin Atlantic Airways	6	Miami International
Heathrow	A	01/01/98 06:22	01/01/98 06:25	British Airways	66	Philadelphia International
Heathrow	A	01/01/98 06:23	01/01/98 06:40	United Airlines	930	San Francisco

Table 3.1: Example of data provided by the UK CAA for London Heathrow Airport .

This is very useful to estimate airport capacity in two difference months, for instance in August and in December because it is very likely that the airport capacity is substantial different due to the different weather conditions. To simplify this application, in this thesis the airport capacity is hypothesised to be a stationary stochastic process. Then it is possible to write:

$$\underline{C}(\zeta;t) = \underline{C}(\zeta) \qquad \forall t \in \boldsymbol{R}$$

This hypothesis assumes that given two any months (e.g. in December and in August) the airport capacity is the same.

To build the airport capacity, Matlab® Program has been used. The code can be divided in these basic steps:

- *Data Loading*

The data are loaded in Matlab® from an Excel Worksheet. The input is the number of scheduled departures $n_{ds} \in \boldsymbol{R}$, number of scheduled arrivals $n_{as} \in \boldsymbol{R}$, number of real arrivals $n_a \in \boldsymbol{R}$, number of real departures $n_d \in \boldsymbol{R}$ for a given day and for a fixed interval time $\Delta T \in \boldsymbol{R}$.

- *PDF construction*

Using the mathematical background explained in section 3.2, the PDF of the airport capacity modelled as a four dimensional random variable can be built. The target is the computation of the conditional distribution with an assumed number of scheduled arrivals and departures. This target is reached passing through the computation of two bi-dimensional PDFs as the run time is definitely shorter. These PDF are the PDF of the scheduled arrivals and departures and the PDF of the real arrivals and departures.
Note that increasing the dimensions of the PDF, increases exponentially the time for its computation. For example, if for a two dimensional PDF the code takes 10 seconds, for a four dimensional it will take 1000 seconds.

- *Computation*

The two bi dimensional PDFs built are the PDF of the scheduled arrivals and departures and the PDF of the real arrivals and departures. Their ratio gives the requested airport's capacity.

In this example, only four months are considered: December 1997, January 1998, August 1998 and September 1998. These months have been selected as they correspond to periods of the year when air traffic demand is high. To build the airport capacity all twenty-four hours of the day should be used, theoretically each of them gives useful information. In practice for those hours having a low number of scheduled arrivals/departures, the probability that the airport succeeds is one. Thus to build airport capacity a busy hour has to be taken into account. In this example the interval time selected is 17:00 until 18:00 o'clock[†] (ΔT= 1 hour) as it is a peak hour during the day.
The data are shown in Table 3.2 and in Table 3.3. From Table 3.2, the maximum number of scheduled operations that is the sum of arrivals and departures is equal to 88, while from Table 3.3, the real maximum number of movements is equal to 85. This difference is due to many causes, such as fog, mist and even human factors. The declared capacity for London Heathrow Airport is estimated by considering the infrastructure, i.e. taking into account the runways and their relative positions. In reality, in some cases the air traffic controllers are able to deal with even more movements than the declared number. In this work a model is proposed giving the probability of success for a given set of two numbers (arrivals and departures).
Using ΔT=1 hour time period, and 4 months, gives about 120 items of data to build the PDF. This number is low to build bi dimensional PDF and the resulting lack of accuracy effects this example. To give a solution to this problem or data coming from more months are added or shorter periods of time can be considered. For example if the length of the interval time is ΔT=15 minutes long, more data are available although always four months are considered. The data will become one hundred and twenty multiplied four.

[†] The Airport Capacity is supposed to be a stationary stochastic process, thus it is not relevant whatever interval time ΔT is chosen.

December 1997				January 1998			
Real		*Scheduled*		*Real*		*Scheduled*	
Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure
37	40	39	39	27	31	35	32
38	38	37	40	39	35	36	36
43	39	37	39	28	24	35	36
37	40	38	39	20	35	33	32
38	39	37	38	41	29	39	34
39	39	**44**	**44**	35	38	37	38
40	40	37	39	38	39	35	38
35	39	40	39	39	41	36	39
33	35	38	39	41	40	36	38
31	34	37	38	34	36	34	34
37	37	38	39	41	39	36	39
28	44	22	37	40	40	39	38
39	37	34	34	37	38	38	39
35	44	38	39	38	37	37	39
36	35	40	39	35	38	37	39
39	38	38	39	40	36	36	38
34	35	35	38	34	33	33	34
35	40	37	39	36	31	38	37
37	36	37	39	41	35	39	39
34	40	34	32	36	41	36	39
28	36	37	36	37	36	36	39
37	37	38	35	32	37	38	39
39	33	37	37	39	33	37	39
33	27	29	24	35	34	34	34
10	11	10	11	41	38	36	38
36	39	31	31	40	32	39	38
38	34	36	34	38	38	38	39
37	40	38	35	41	36	37	38
39	36	38	33	33	32	34	36
36	38	35	32	40	33	36	38

Table 3.2: Number of operations during the months December 1997 and January 1998 in an interval time ΔT=1 hour (from 17:00 until 18:00) for London Heathrow Airport.

August 1998				September 1998			
Real		*Scheduled*		*Real*		*Scheduled*	
Arrival	Departure	Arrival	Departure	Arrival	Departure	Arrival	Departure
31	38	41	37	38	43	39	39
41	39	40	40	39	35	39	40
44	37	41	40	39	42	40	38
42	39	39	40	41	41	40	40
39	37	40	40	43	41	41	36
43	36	40	39	41	39	41	40
39	36	40	40	39	35	41	39
39	37	41	36	42	41	39	40
42	42	42	40	36	38	40	39
39	41	41	40	39	34	40	39
39	41	38	39	40	34	40	40
44	**41**	41	39	44	36	41	37
39	36	40	39	40	38	41	40
40	39	40	40	40	30	41	40
40	39	41	37	40	38	39	40
39	38	42	40	40	39	40	40
42	41	40	40	37	38	39	39
38	42	40	40	36	40	40	40
39	42	40	40	38	35	41	36
40	40	40	39	40	40	40	40
44	39	40	40	43	37	41	40
43	41	41	37	43	42	39	40
35	40	41	40	39	39	40	40
42	39	41	40	44	39	40	39
42	38	39	40	36	41	40	40
38	35	40	40	34	32	41	36
40	41	40	39	36	40	40	40
39	40	40	40	40	41	40	40
39	36	41	37	42	43	39	40
43	36	40	39	40	36	40	40

Table 3.3: Number of operations during the months August 1998 and September 1998 in an interval time $\Delta T=1$ hour (from 17:00 until 18:00) for London Heathrow Airport.

Due to the lack of data some modifications are carried out to the graphs in order to accommodate the information available to the real air traffic behaviour.

In Figure 3.1, the unmodified PDF is plotted. In the x and y axes the number of arrivals and departures are displayed while in the z-axis the correspondence probability is shown. Due to the lack of data, for a big number of arrivals and departures the probability the airport deals with such movements is almost one. Having few samples the estimation of the probability is effected by a big error.

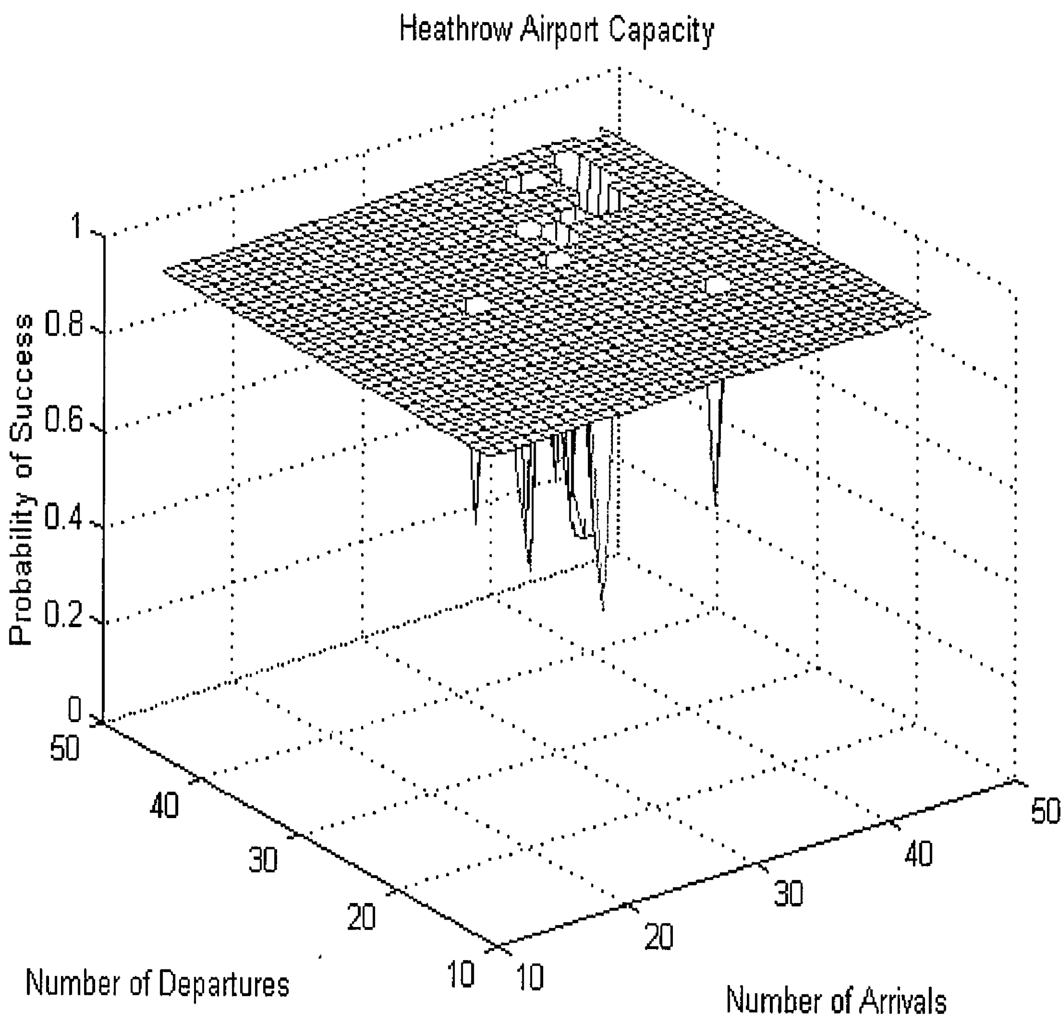


Figure 3.1: London Heathrow Airport Capacity.

In Figure 3.2 the PDF of London Heathrow Airport Capacity is shown. This graph is a modification of the PDF shown in Figure 3.1, computing for each point an average of its surrounding points.

When the number of arrivals and departures is not high, the airport is able to manage all of them and then the associated probability is almost one. This probability decreases when the number of movements increases. Finally, if the number of the operations is too high, the airport has difficulty dealing with them and the probability of success is almost zero. In this situation, some of the flights have to be delayed.

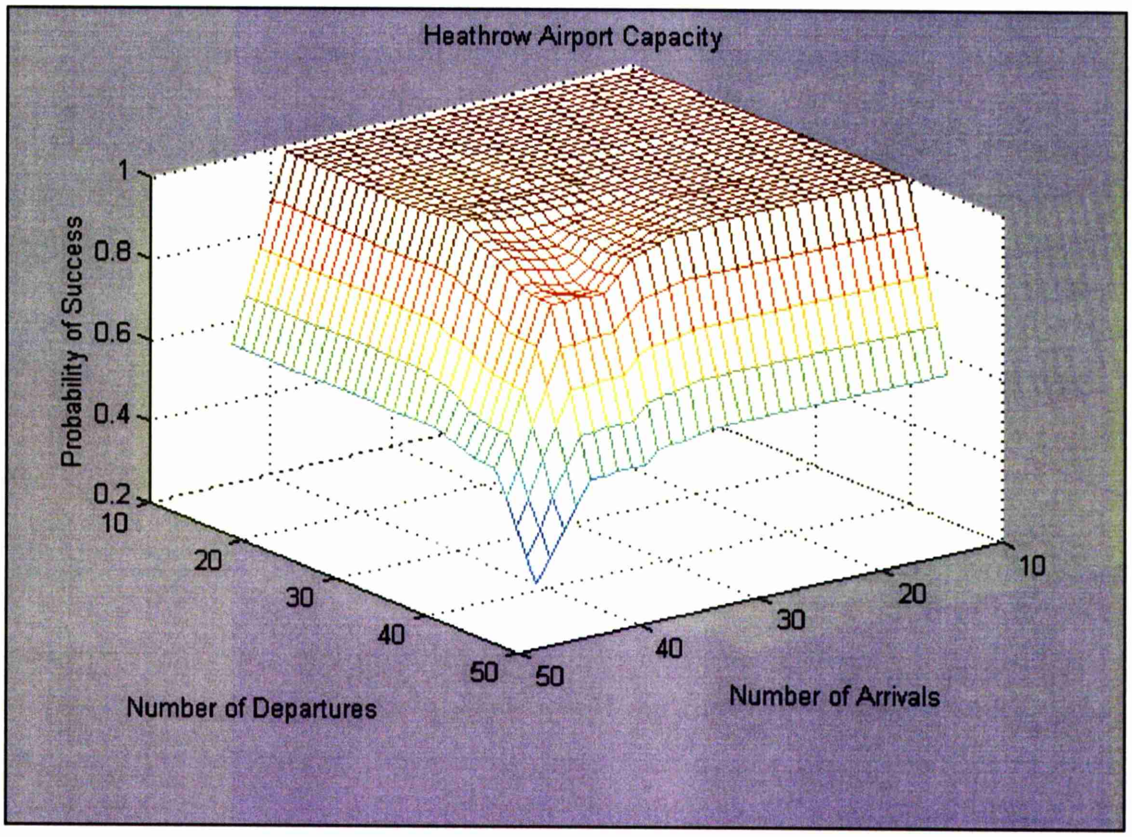


Figure 3.2: London Heathrow Airport Capacity with modified PDF.

Arrivals and departures are linked parameters, increasing the number of arrivals will decrease the number of feasible departures.

In mathematical term $\mathbf{D}\subseteq\mathcal{X}$ and $\mathbf{A}\subseteq\mathcal{X}$ are dependent random variables:

$$\exists(\mathrm{d},\mathrm{a})\in N^2 \quad \text{such as} \quad \Pr\{\mathbf{D}\leq \mathrm{d},\mathbf{A}\leq \mathrm{a}\}\neq \Pr\{\mathbf{D}\leq \mathrm{d}\}\Pr\{\mathbf{A}\leq \mathrm{a}\}$$

Here is an alternate way to see the airport capacity. Considering, for example, if the number of arrivals is always low, the probability the airport could manage all the flights would depend on the number of the departures. Firstly, if this number is also low, the airport will not have any problem and the probability will be almost one. As the number of departures increases, the probability of success decreases. When the number of departures is too high, the airport is not able to deal with that number of arrivals and departure at the same time and, therefore, the probability will be zero. The same reasoning could be followed if a low number of departures is considered.

3.4 Further developments: network of airports with random airport capacities

In [26] a static probabilistic multi-airport GHP is analysed, while in [31] a probabilistic multi-airport GHP is formulated (see section 1.6.2). In this section an extension to the formulations given in those papers is presented. The airport capacity is modelled as a multi dimensional random variable.

A set of airports $\boldsymbol{K}=\{1,2,...K\}$ and an ordered set of time periods $\boldsymbol{T}=\{1,2,...T\}$ are considered. Given a set of flights $\boldsymbol{F}=\{1,2,...F\}$, $\forall f \in \boldsymbol{F}$ the departure airport of the flight $f \in \boldsymbol{F}$ denoted by $k_f^d \in \boldsymbol{K}$, the arrival airport of the flight $f \in \boldsymbol{F}$ denoted by $k_f^a \in \boldsymbol{K}$, the ground delay cost function of the flight $f \in \boldsymbol{F}$ denoted by $c_f^{gh}(t) \in \boldsymbol{R} \times \boldsymbol{T} \times \boldsymbol{F}$ and the airborne delay cost function of the flight $f \in \boldsymbol{F}$ denoted by $c_f^{ai}(t) \in \boldsymbol{R} \times \boldsymbol{T} \times \boldsymbol{F}$ are known.

Furthermore $\forall k \in \boldsymbol{R}$ the capacity $\underline{\mathbf{C}}_k(\mathbf{D},\mathbf{D}^s,\mathbf{A},\mathbf{A}^s) \subseteq \mathcal{K} \times K$ of the departure and arrival airport of the flight $f \in \boldsymbol{F}$ is assumed to be given as a multi dimensional random variable. The decision variables are $g_f \in \boldsymbol{R} \times \boldsymbol{F}$ and $a_f \in \boldsymbol{R} \times \boldsymbol{F}$. These are the amount of ground-hold delay and airborne delay to allocate to the flight $f \in \boldsymbol{F}$. The assignment decision variables are u_{ft} and v_{ft} that are equal to 1 if the flight $f \in \boldsymbol{F}$ is allowed to take-off respectively to land at the period $t \in \boldsymbol{T}$.

The goal is to minimise the total cost expressed as ground and airborne delays and to maximise the probability that the airport deal with the number of flights planned to arrive and to depart:

$$\min\left(\sum_{f=1}^{F}(c_f^{gh}g_f + c_f^{ai}a_f)\right) \qquad \begin{cases} c_f^{gh} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ c_f^{ai} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \end{cases}$$

$$\max\left(\Pr\left\{\underline{\mathbf{C}_k}\left(\mathbf{D}_k \le \sum_{f;k_f^d=k} u_{ft}, \mathbf{A}_k \le \sum_{f;k_f^a=k} v_{ft} \mid \mathbf{D}_k^s = d^s, \mathbf{A}_k^s = a^s\right)\right\}\right) \qquad \begin{cases} \underline{\mathbf{C}_k} \subseteq \mathcal{K}\times K \\ \overline{(d^s, a^s)} \in N^2 \end{cases}$$

subject to the constraints

$$\sum_{t\in T} u_{ft} = 1 \qquad f \in \boldsymbol{F}$$

$$\sum_{t\in T} v_{ft} = 1 \qquad f \in \boldsymbol{F}$$

$$u_{ft}, v_{ft} \in \{0,1\}$$

These constraints mean that for every flight $f\in\boldsymbol{F}$ exactly one of the variables u_{ft} must be equal to 1 and the others must be equal to zero, and similarly for the variables v_{ft}.

3.5 Conclusions

In this chapter a methodology how to estimate the actual operational capacity is presented using historical arrival and departure data rather than information regarding the airport's infrastructure. This methodology is based on statistics and probability theory and it allows air managers to calculate the probability that airports can manage a demanded number of movements.

An example has been shown, London Heathrow Airport has been selected and presented due to its large size. In further work, weather forecast and other parameters can be added to prognosticate congested period.

Chapter 4

Current situation and the Strategic Planning

4.1 Introduction

With the increased growth of aircraft operations forecast for the next decade, the problem of air traffic congestion is a major concern, since the currently available capacity is insufficient in many parts of the world. Air traffic congestion causes delays that undermine operational efficiency and safety.

This chapter considers and introduces a new approach to Air Traffic Flow Management (ATFM) that aims at minimising the probability of conflict between aircraft arriving at a given airport. This minimisation will reduce the likelihood of airborne delay, thereby reducing the costs for the airlines (see section 2.3.5) and improving the safety of airports operations.

The Air Traffic Flow Management function can be considered as consisting of two distinct processes. The first process is concerned with flow management of en route air traffic and the second is concerned with the flow of arriving and departing aircraft at a given airport. This chapter describes the development of a model that addresses the arrival and departure flow management. The model has been implemented for Glasgow International Airport and is shown to reduce the likelihood of two landings being requested within a time interval of 3 minutes. This time interval is regarded as the minimum time spacing between two aircraft at touch-down for safe landing operations.

As far as air traffic flow management is concerned, an airport can be considered to be a system comprising of the following sub-systems:

runways

taxiways

ramps

gates system

Recent investigations such as [49] [50] have shown that runways are the most influential factor in the air traffic flow performance of an airport. The inefficient use of runways constrains the airport throughput and affects negatively the operation of the other sub-systems of the airport.

In this chapter a strategic scheduling (see section 1.1) is proposed which decreases the air traffic demand for airports. Using real data a statistical prediction of the peak hours for Glasgow International Airport is computed. Then through a new schedule an arrival demand evenly distributed in time is achieved.

4.2 Mathematical Model

The core of the methodology is to distribute air traffic demand evenly in time in order to avoid congestion in airports, where system overloads cause inevitable delays and even discomfort for passengers.

Applying statistics and probability theory, firstly for a given airport an estimation of the peak hours is calculated. Then a strategic schedule is achieved distributing evenly in time the arriving aircraft. From real data provided by the *Civil Aviation Authority* CAA, Probability Density Functions (PDFs) are built solving for a selected flight the probability of landing at a given time at the destination airport. Each PDF reports the probability of the delay for a given flight.

Consider an aircraft arriving at the destination airport, because of unpredictable factors such for instance adverse weather conditions and devices failures, its touching-down time is effected by a delay which is the difference between the actual and the planned landing time. Note that this delay, as introduced and defined, can be both positive and negative, positive values of this parameter mean aircraft is arriving later while negative values aircraft is arriving earlier. Observing this delay and performing this operation many times, a function that associates the relative number of occurrences of a given delay is built. Applying the *Law of large number*, increasing the number of samples the frequency of occurrences tends to be equal to the probability of the event. A random variable that solves the flight delay is firstly introduced then it is statistically defined through the computation of its PDF. The task is to decrease airport congestion and high density traffic.

4.2.1 Arrival Time and Departure Time as Real Random Variables

Deterministic approaches to Air Traffic Flow Management (ATFM) require a large and unmanageable set of parameters to model all problems of the complex ATFM issue. In forecasting aircraft movement in terms of arrival and departure time, take-off time and taxiing time, there is always uncertainty due to passengers behaviour, ground system overload, equipment failures, weather factors etc [51].

In this work the arrival times of the aircraft at a given airport are introduced and defined as real random variables. Through this approach, the intrinsic unpredictability that inevitably characterises those parameters is successfully modelled. In this work only a new schedule for the arrivals is presented although a similar procedure can be implemented also for the departures.

As defined by [46], a real random variable consists of an experiment Ω with a probability measure $\Pr\{.\}$ defined on a sample space and a function that assigns a real number to each outcome in the sample space of the experiment Ω.

Scheduled arrival times, arrival delays and arrival times are defined as:

$$\mathbf{t}^a \stackrel{\Delta}{=} t_s^a + \mathbf{d}^a \qquad \begin{cases} \mathbf{t}^a \subseteq \mathcal{B} \\ t_s^a \in \boldsymbol{R} \\ \mathbf{d}^a \subseteq \mathcal{A} \end{cases}$$

where:

$t_s^a \stackrel{\Delta}{=}$ **S**cheduled **A**rrival **T**ime for a selected flight. This is the scheduled arrival time of the flight† in the hypothesis of absence of delay††. This is a deterministic variable having as domain the set of the real numbers $\boldsymbol{R}$.

$\mathbf{d}^a \stackrel{\Delta}{=}$ Arrival **D**elay for a selected flight. This is modelled as a real random variable defined in the set $\mathcal{A}$. This choice has been made due to the intrinsic unpredictability affecting delays. For an aircraft operating a given flight, an arrival delay (positive or negative) can be observed. To every outcome $\zeta \in \mathcal{A}$ of the experiment $\boldsymbol{\Omega}$, a real number $\mathbf{d}^a(\zeta) \in \boldsymbol{R}$ is assigned.

$\mathbf{t}^a \stackrel{\Delta}{=}$ Arrival **T**ime for a selected flight. This variable is the sum of a deterministic and of a real random variable. The result gives a real random variable defined in the set $\mathcal{B}$.

Scheduled departure times, departure delays and departure times are defined as:

$$\mathbf{t}^d \stackrel{\Delta}{=} t_s^d + \mathbf{d}^d \qquad \begin{cases} \mathbf{t}^d \subseteq \mathcal{D} \\ t_s^d \in \boldsymbol{R} \\ \mathbf{d}^d \subseteq \mathcal{C} \end{cases}$$

† Throughout this dissertation the word *aircraft* refers to a single plane that in a given journey goes from the departure to the arrival airport while the word *flight* refers to a regular scheduled service between departure and arrival airport.

†† In this dissertation *delays* can be positive or negative. Delays are negative when the aircraft arrives earlier than scheduled. Delays are positive when the aircraft arrives later than scheduled.

where:

$t_s^d \triangleq$ **S**cheduled **D**eparture **T**ime for a selected flight. This is the scheduled departure time of the flight in the hypothesis of absence of delay. This is a deterministic variable having as domain the set of the real numbers $\boldsymbol{R}$.

$\mathbf{d^d} \triangleq$ **D**eparture **D**elay for a selected flight. This is modelled as a real random variable defined in the set $\mathcal{C}$. This choice has been made due to the intrinsic unpredictability affecting delays. For an aircraft operating a given flight, a departure delay (positive or negative) can be observed. To every outcome $\zeta \in \mathcal{C}$ of the experiment $\mathbf{\Omega}$, a real number $\mathbf{d^d}(\zeta) \in \boldsymbol{R}$ is assigned.

$\mathbf{t^d} \triangleq$ **D**eparture **T**ime for a selected flight. This variable is the sum of a deterministic and of a real random variable. The result gives a real random variable defined in the set $\mathcal{D}$.

Figure 4.1 shows the PDFs of departure $f_{t^d}(t)$ and arrival $f_{t^a}(t)$ times with repsect to the time. PDF formal defnition and construction will be given in the next section.

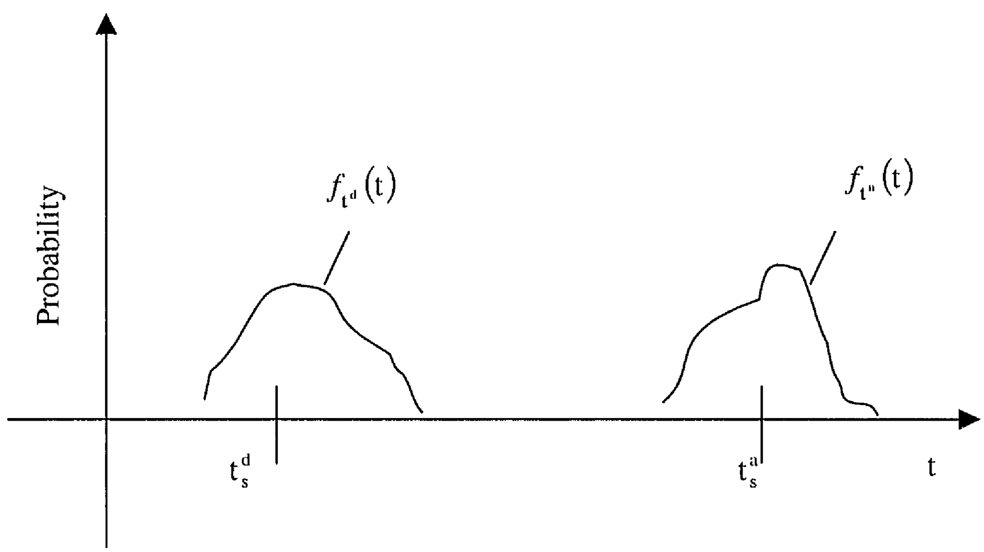


Figure 4.1: Definition of departure and arrival time.

For simplicity, in this thesis only the arrival times will be modelled as real random variables. This methodology could also be applied taking into account both departure and arrival times. In this case, the departure and arrival times are expected to be correlated. Using the correlation between $\mathbf{t}^{d} \subseteq \mathcal{D}$ and $\mathbf{t}^{a} \subseteq \mathcal{B}$, the uncertainty of $\mathbf{t}^{a} \subseteq \mathcal{B}$ could be reduced once $\mathbf{t}^{d} \subseteq \mathcal{B}$ is known.

4.2.2 Building Cumulative and Probability Density Functions (CDF and PDF)

In section 4.2.1, real random variables were introduced to model the Air Traffic Flow Management problem. A methodology for building the statistics (PDF and CDF) of the random variables is presented in this section. A given flight between two airports is considered. This flight is identified by the scheduled arrival time which is the time printed on the travellers ticket. For a fixed aircraft (single journey) during this given flight, a delay both in the departure and in the arrival is observed because of the intrinsic unpredictable aspects of the flight operations. Performing this operation n times and recording in each step the delay observed, a Probability Density Function (PDF) can be built. This PDF represents the frequency of the different time delays for a given flight and include implicitly all the parameters concerned with the punctuality of the flight from departure to arrival.

The experiment $\mathbf{\Omega}$ is defined in the following step:

ζ = difference between actual landing time and scheduled arrival time =
= actual landing time – scheduled arrival time

and the random variable arrival delay is defined $\mathbf{d}^{a}$ by:

$$\mathbf{d}^{a}(\zeta) \stackrel{\Delta}{=} \zeta$$

it follows that ζ, which is the arrival delay of the aircraft, has a double meaning. ζ is both the outcome of the experiment $\mathbf{\Omega}$ and the value of the random variable. Hence the random variable $\mathbf{d}^a \subseteq \mathcal{A}$ has as domain and range the set of real number $\boldsymbol{R}$. The arrival delay $\mathbf{d}^a \subseteq \mathcal{A}$ is a real random variable.

$$\mathbf{d}^a(\zeta) \subseteq \mathcal{A} = \boldsymbol{R} \;\rightarrow\; \boldsymbol{R}$$

Supposing this experiment Ω is performed n times, at a given outcome ζ of the experiment, the real random variable $\mathbf{d}^a \subseteq \mathcal{A}$ associates a value $\mathbf{d}^a(\zeta) \in \boldsymbol{R}$. Given a number x and denoting by $n(x)$ the total number of trials such that $\mathbf{d}^a(\zeta) \leq x$, the statistics (CDF and PDF) of the random variable $\mathbf{d}^a \subseteq \mathcal{A}$ are possible to be determined by applying the *Law of Large numbers.*

Throughout this dissertation, PDFs and CDFs are computed as histograms of the relative frequency of observing a given event. These histograms are considered equivalent to the PDFs and CDFs respectively by applying the *Law of large numbers* [47] [48]:

*If the probability of an event a in a given experiment **Ω** equals p and the experiment is repeated n times, then for any ε>0*

$$\lim_{n \to \infty} \Pr\left\{ \left| \frac{n_a}{n} - p \right| \leq \varepsilon \right\} = 1$$

where n_a equals the number of success of a.

This concept may also be expressed as:

*If the experiment **Ω** is repeated n times and the event a occurs n_a times, then, with a high degree of certainty, the relative frequency n_a/n of the occurrence of a is close to p=Pr(a),*

$$\Pr(a) \cong \frac{n_a}{n}$$

provided n is sufficiently large.

In conclusion, the Cumulative Density Function (CDF) for a given x is computed as ratio between the number of trials such as $\mathbf{d}^a(\zeta) \leq x$ and the total number of trials n.

$$F(x) = \Pr\{\mathbf{d}^a(\zeta) \leq x\} \cong \frac{n(x)}{n}$$

To determine the Probability Density Function (PDF) $f(x)$ for a given x, the experiment Ω is performed n times and the number of trials such as:

$$x \leq \mathbf{d}^a(\zeta) \leq x + \Delta x,$$

is counted.

Denoting by $\Delta n(x)$ the number of these trials, the Probability Density Function can be computed as ratio between the number of trials such as $x \leq \mathbf{d}^a(\zeta) \leq x+\Delta x$ and the total number of trials, provided n sufficiently large and Δx sufficiently small[†].

$$f(x)\Delta x \cong \frac{\Delta n(x)}{n}$$

This is an important result because in this thesis all the PDF of the arrival delays will be built using real data provided by the Civil Aviation Authorithy.

4.2.3 Probability of Conflict and Total Probability of Conflict

4.2.3.1 Probability of Conflict (PC)

In this section the Probability of Conflict is defined. Given two aircraft $i \in N$ and $j \in N$, a conflict occurs if these two aircraft arrive at the destination airport within a temporal difference less than a fixed threshold $\Delta T_{ij} \in \boldsymbol{R}$. The Probability of conflict expresses the likelihood that two aircraft arrive quasi-simultaneously at the destination airport. Quasi-

[†] In Appendix B n sufficiently large and Δx sufficiently small will be properly clarified.

simultaneously means that they arrive within a temporal difference less than a fixed threshold ΔT_{ij}, see Figure 4.2.

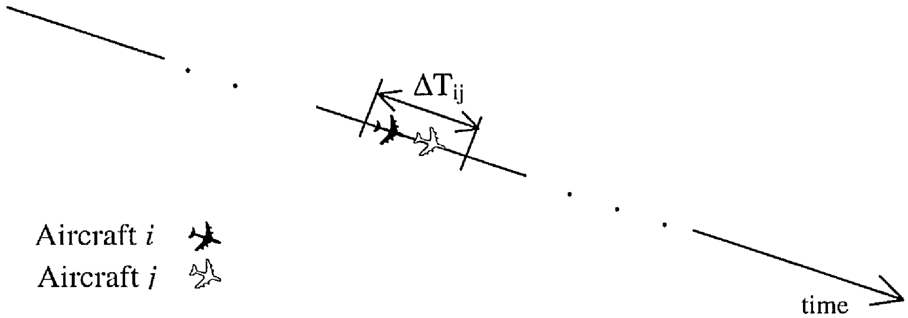


Figure 4.2: A conflict occurs if two aircraft arrive within a temporal distance $\Delta T_{ij} \in \boldsymbol{R}$.

The event "Conflict between the aircraft $i \in N$ and $j \in N$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is defined as:

$A_{ij} \overset{\Delta}{=}$ *"Aircraft $i \in N$ and aircraft $j \in N$ arrive within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".*

The probability of the event "Conflict between the aircraft $i \in N$ and $j \in N$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is defined as:

$PC_{ij}^{\Delta T = \Delta T_{ij}} \overset{\Delta}{=}$ *"The probability that the aircraft $i \in N$ and $j \in N$ arrive within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".*

In the remainder of this dissertation the probability of conflict will be marked briefly as $PC_{ij}^{\Delta T = \Delta T_{ij}}$ or even as PC when there is not any possibilities of misunderstandings between interval times and aircraft involved.

Let $\{t_1^a, t_2^a, \ldots t_n^a\} \subseteq \mathcal{B}^n$ be the arrival times of a set of flights $\{1,2,\ldots n\} \in \boldsymbol{N}^n$. The Probability of Conflict above defined does not give any preference to the arrival order of the aircraft involved. Aircraft $i \in N$ can arrive before or after aircraft $j \in N$. The

Probability of Conflict between aircraft $i \in N$ and $j \in N$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$ can be expressed through the difference between the real random variables $\mathbf{t}_i^a \subseteq \mathcal{B}$ and $\mathbf{t}_j^a \subseteq \mathcal{B}$ as below shown:

$$PC_{ij}^{\Delta T=\Delta T_{ij}} \stackrel{\Delta}{=} \Pr\left\{\begin{array}{l}\text{aircraft } i \text{ and } j \text{ arrive} \\ \text{with a temporal difference} \\ \text{less than } \Delta T_{ij}\end{array}\right\} =$$

$$= \Pr\{\mathbf{t}_i^a - \mathbf{t}_j^a \leq \Delta T_{ij}\} + \Pr\{\mathbf{t}_j^a - \mathbf{t}_i^a \leq \Delta T_{ij}\} =$$

$$= \Pr\{\left|\mathbf{t}_i^a - \mathbf{t}_j^a\right| \leq \Delta T_{ij}\}$$

Denoting by $\mathbf{z}_{ij} = \mathbf{t}_i^a - \mathbf{t}_j^a$, the following equivalence is derived:

$$\Pr\{\left|\mathbf{t}_i^a - \mathbf{t}_j^a\right| \leq \Delta T_{ij}\} \quad \Leftrightarrow \quad \Pr\{\left|\mathbf{z}_{ij}\right| \leq \Delta T_{ij}\}$$

With the previous position the difference between two real random variables $\mathbf{t}_i^a \subseteq \mathcal{B}$, $\mathbf{t}_j^a \subseteq \mathcal{B}$ is leaded in the study of only one real random variable $\mathbf{z}_{ij} \subseteq \mathcal{B}$.

$$f_{z_{ij}}(z_{ij}) = \int_{-\infty}^{+\infty} f_{t_i^a t_j^a}(z_{ij} + t_j)dt_j$$

where $f_{t_i^a t_j^a}\left(t_i^a, t_j^a\right)$ is the joint PDF of the real random variables $\mathbf{t}_i^a \subseteq \mathcal{B}$ and $\mathbf{t}_j^a \subseteq \mathcal{B}$.

The arrival times $\mathbf{t}_i^a \subseteq \mathcal{B}$ and $\mathbf{t}_j^a \subseteq \mathcal{B}$ are supposed to be independent real random variables[†]. Denoting by $f_{t_i^a}(t)$ and $f_{t_j^a}(t)$ the Probability Density Function of the real random variables $\mathbf{t}_i^a \subseteq \mathcal{B}$ and $\mathbf{t}_j^a \subseteq \mathcal{B}$, the independence of these random variables is expressed by:

$$f_{z_{ij}}(z_{ij}) = \int_{-\infty}^{+\infty} f_{t_i^a}(z_{ij} + t_j) f_{t_j^a}(t_j)dt_j = \int_{-\infty}^{+\infty} f_{t_i^a}(t_i) f_{t_j^a}(t_i - z_{ij})\, dt_i$$

† The proof of such a hypothesis is given in section B.3.1.

This is an important result as the integral shown above is very similar to the well-known integral of convolution, see [48] for further details.

In conclusion, under the hypothesis that the arrival times $\{t_1^a, t_2^a, \ldots t_n^a\} \subseteq \mathcal{B}^n$ are independent real random variables, the Probability of Conflict is calculated by this integral:

$$PC_{ij}^{\Delta T=\Delta T_{ij}} \stackrel{\Delta}{=} \Pr\left\{\begin{array}{c}\text{aircraft } i \text{ and } j \text{ arrive} \\ \text{with a temporal difference} \\ \text{less than } \Delta T_{ij}\end{array}\right\} = \int_{-\Delta T_{ij}}^{\Delta T_{ij}} \int_{-\infty}^{+\infty} f_{t_i^a}(z_{ij}+t_j) f_{t_j^a}(t_j) dt_j dz_{ij}$$

4.2.3.2 Total Probability of Conflict (TPC)

Let $\{t_1^a, t_2^a, \ldots t_n^a\} \subseteq \mathcal{B}^n$ be the arrival times of a set of flights $\{1,2,\ldots n\} \in N^n$. The Total Probability of Conflict is the probability of conflict between all the possible pairs of flights of the given set. The Total Probability of Conflict embraces all the possible pairs while the Probability of Conflict defined in the previous section is referred to only one pair of flights.

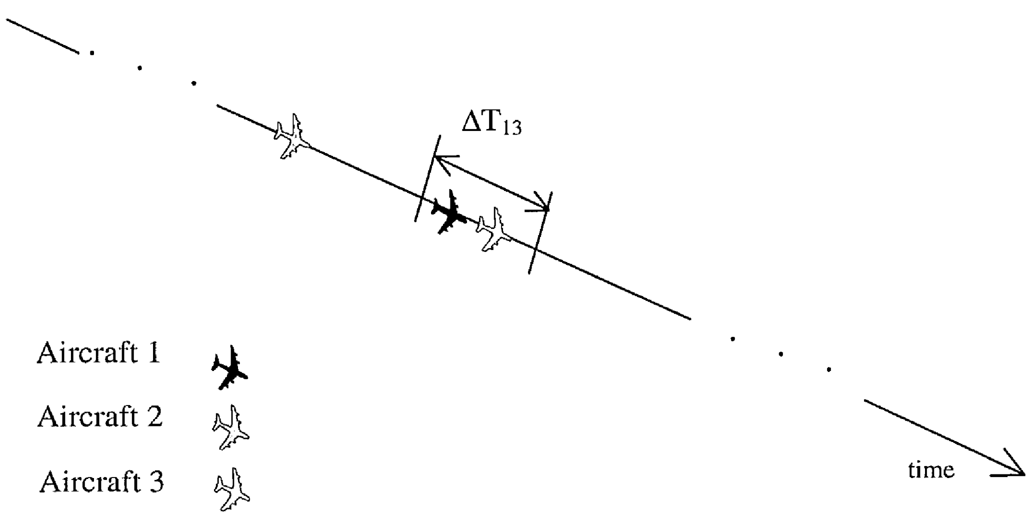


Figure 4.3: TPC is the probability of conflict between all the possible pairs of flights.

The Total Probability of Conflict (TPC) between the aircraft $\{1,2,\ldots n\} \in N^n$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$ is defined as:

$TPC_{1\ldots n}^{\Delta T=\Delta T_{ij}} \stackrel{\Delta}{=}$ *"The probability that at least two aircraft* $(i,j)\in\{1,2,\ldots n\}^2$ *with* $i\neq j$ *arrive within a temporal difference less than* $\Delta T_{ij}\in\boldsymbol{R}$*".*

In the remainder of this work the total probability of conflict will be marked briefly as $TPC_{1\ldots n}^{\Delta T=\Delta T_{ij}}$ or even as TPC when there is not any possibilities of misunderstandings between interval times and aircraft involved.

The Total Probability of Conflict is the probability of the event union of the conflicts between all possible pairs of aircraft:

$$TPC_{1\ldots n}^{\Delta T=\Delta T_{ij}} \stackrel{\Delta}{=} \left\{\begin{array}{l}\text{At least two aircraft } (i,j)\in\{1,2,\ldots n\}\\ \text{with } i\neq j \text{ arrive within a temporal}\\ \text{difference less than } \Delta T_{ij}\in\boldsymbol{R}\end{array}\right\} =$$

$$= \Pr\{A_{12}\cup A_{13}\cup\ldots\cup A_{1n}\cup\ldots\cup A_{(n-1)n}\} = \Pr\left\{\bigcup_{i=1}^{n-1} A_{i(i+1)}\right\}$$

The computation of the probability of the event union requires a large number of terms. Here a basic example is shown, only 3 aircraft are considered. The task is to show how to compute the conditional probabilities involved in the term $\Pr\left\{\bigcup_{i=1}^{n-1} A_{i(i+1)}\right\}$.

Let $\{1,2,3\}$ be a set of flights. Denoting by A_{ij} the event aircraft $i\in\{1,2,3\}$ and $j\in\{1,2,3\}$ (with $i\neq j$) arrive within a temporal difference less than $\Delta T_{ij}=\Delta T^*$, the following equivalence are given:

$$PC_{12}^{\Delta T=\Delta T^*} = \Pr\{A_{12}\} = \alpha_{12}$$

$$PC_{23}^{\Delta T=\Delta T^*} = \Pr\{A_{23}\} = \alpha_{23}$$

$$PC_{13}^{\Delta T=\Delta T^*} = \Pr\{A_{13}\} = \alpha_{13}$$

The probability of the event union or in an equivalent expression the probability that at least two aircraft arrive together is hereby computed:

$$\mathrm{TPC}_{1,2,3}^{\Delta T=\Delta T^*} \stackrel{\Delta}{=} \left\{ \begin{array}{l} \text{At least two aircraft } (i,j) \in \{1,2,3\} \\ \text{with } i \neq j \text{ arrive within a temporal} \\ \text{difference less than } \Delta T^* \in \boldsymbol{R} \end{array} \right\} =$$

$$= \Pr\{A_{12} \cup A_{23} \cup A_{13}\} =$$

$$= \Pr\{A_{12}\} + \Pr\{A_{23}\} + \Pr\{A_{13}\} +$$

$$- \Pr\{A_{12} \mid A_{23}\}\Pr\{A_{23}\} - \Pr\{A_{12} \mid A_{13}\}\Pr\{A_{13}\} - \Pr\{A_{23} \mid A_{13}\}\Pr\{A_{13}\} +$$

$$+ \Pr\{A_{12} \mid A_{23}A_{13}\}\Pr\{A_{23} \mid A_{13}\}\Pr\{A_{13}\}$$

The term $\Pr\{A_{12} \mid A_{23}\}$ expresses the probability of the event *"Aircraft 1, 2 arrive together once aircraft 2 and 3 already arrived"*. This probability is equal to $\Pr\{A_1\}$ where the event[†] *"Aircraft 1 arrives"* is denoted by A_1. If aircraft 2 already arrived, clearly the probability that aircraft 1,2 arrive is due only to aircraft 1.

$$\Pr\{A_{12} \mid A_{23}\} = \Pr\{A_1\} = \alpha_1$$

Similarly the term $\Pr\{A_{12} \mid A_{13}\}$ expresses the probability of the event *"Aircraft 1, 2 arrive together once aircraft 1 and 3 already arrived"*. This probability is equal to $\Pr\{A_2\}$ where the event *"Aircraft 2 arrives"* is denoted by A_2. If aircraft 1 already arrived, clearly the probability that aircraft 1,2 arrive is due only to aircraft 2.

$$\Pr\{A_{12} \mid A_{13}\} = \Pr\{A_2\} = \alpha_2$$

The term $\Pr\{A_{12} \mid A_{23}A_{13}\}$ can be written as $\Pr\{A_{12} \mid A_{123}\}$ where A_{123} is denoted the event *"Aircraft 1, 2 and 3 arrive together"*. The term $\Pr\{A_{12} \mid A_{123}\}$ expresses the probability of the event *"Aircraft 1, 2 arrive together once aircraft 1,2 and 3 already arrived"*. This probability is equal to 1. If aircraft 1,2 and 3 already arrived, aircraft 1 and 2 also arrived certainly together.

$$\Pr\{A_{12} \mid A_{23}A_{13}\} = 1$$

[†] This event do not have any physical meaning as the flight arrive according to its PDF and thus in infinitesimal intervals dt.

Denoting by $\Pr\{A_{123}\} = \alpha_{123}$, the event union can be computed as:

$$\mathrm{TPC}_{1,2,3}^{\Delta T = \Delta T^*} \overset{\Delta}{=} \left\{ \begin{array}{l} \text{At least two aircraft } (i,j) \in \{1,2,3\} \\ \text{with } i \neq j \text{ arrive within a temporal} \\ \text{difference less than } \Delta T^* \in \boldsymbol{R} \end{array} \right\} =$$

$$= \Pr\{A_{12} \cup A_{23} \cup A_{13}\} =$$

$$= \alpha_{12} + \alpha_{23} + \alpha_{13} - \alpha_1 \alpha_{23} - \alpha_2 \alpha_{13} - \alpha_2 \alpha_{13} + 1 \cdot \alpha_2 \alpha_{13} =$$

$$= \alpha_{12} + \alpha_{23} + \alpha_{13} - 2 \cdot \alpha_1 \alpha_{23} =$$

$$= \alpha_{12} + \alpha_{23} + \alpha_{13} - 2 \cdot \alpha_{123}$$

The explanation of this result is straight forward considering Figure 4.4. The Total Probability of Conflict is the probability of the event $\{A_{12} \cup A_{23} \cup A_{13}\}$, which is represented in Figure 4.4 with the red lined area.

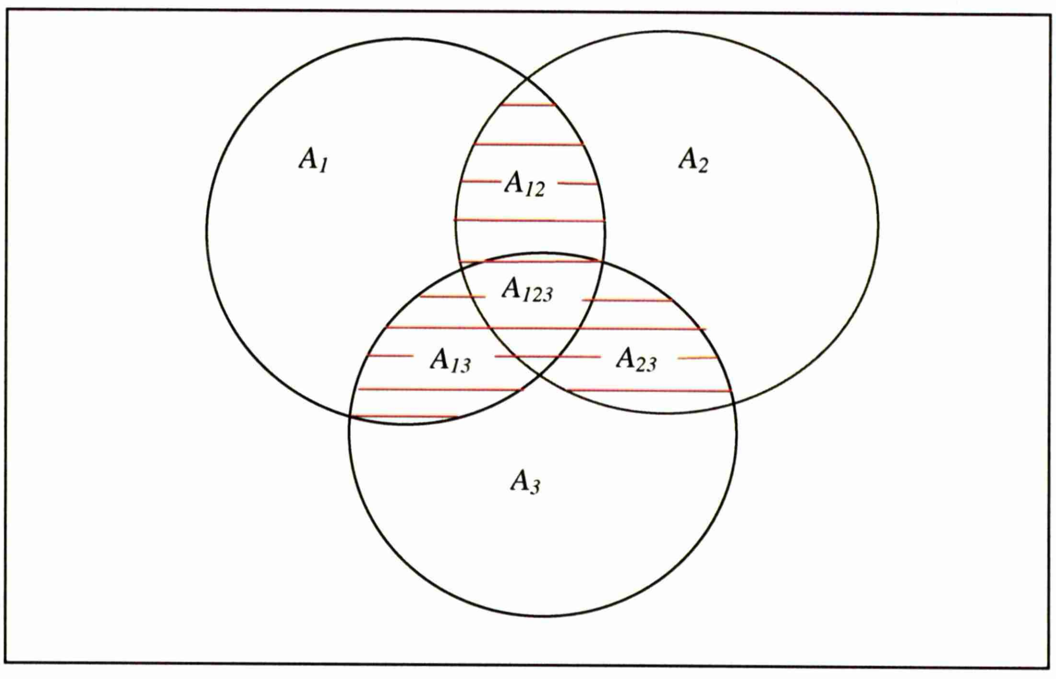


Figure 4.4: Venn diagram showing the Total Probability of Conflict.

Each of the sets A_{12},A_{23} and A_{13} includes the set A_{123} and, hence, each of the terms α_{12}, α_{23} and α_{13} takes into account the probability of the event A_{123} , $\Pr\{A_{123}\} = \alpha_{123}$. To calculate the probability of the event union $\{A_{12} \cup A_{23} \cup A_{13}\}$, the term α_{123} has to be subtracted twice from the sum of the terms α_{12}, α_{23} and α_{13}.

As shown in section 4.2.3.1, if the arrival times $\{t_1^a, t_2^a, t_3^a\} \subseteq \mathfrak{B}^3$ are known then the probability of conflicts $PC_{12}^{\Delta T=\Delta T^*}$, $PC_{23}^{\Delta T=\Delta T^*}$ and $PC_{13}^{\Delta T=\Delta T^*}$ between the pairs of aircraft (1,2) (2,3) (1,3) are given by the following integrals:

$$PC_{12}^{\Delta T=\Delta T^*} = \alpha_{12} = \int_{-\Delta T_{12}}^{\Delta T_{12}} \int_{-\infty}^{+\infty} f_{t_1^a}(z_{12} + t_2) f_{t_2^a}(t_2) dt_2 dz_{12}$$

$$PC_{23}^{\Delta T=\Delta T^*} = \alpha_{23} = \int_{-\Delta T_{23}}^{\Delta T_{23}} \int_{-\infty}^{+\infty} f_{t_2^a}(z_{23} + t_3) f_{t_3^a}(t_3) dt_3 dz_{23}$$

$$PC_{13}^{\Delta T=\Delta T^*} = \alpha_{13} = \int_{-\Delta T_{133}}^{\Delta T_{13}} \int_{-\infty}^{+\infty} f_{t_1^a}(z_{13} + t_3) f_{t_3^a}(t_3) dt_3 dz_{13}$$

The term α_1 is unknown because only the probabilities of conflicts α_{12}, α_{23} are computed using the integrals above described. Assuming that A_1, A_2, A_3 are independent events[†], the following equivalence are given:

$$\begin{cases} \Pr\{A_1 \cap A_2\} = \Pr\{A_1\}\Pr\{A_2\} \\ \Pr\{A_2 \cap A_3\} = \Pr\{A_2\}\Pr\{A_3\} \\ \Pr\{A_1 \cap A_3\} = \Pr\{A_1\}\Pr\{A_3\} \end{cases}$$

Solving the following system, the unknowns $(\alpha_1, \alpha_2, \alpha_3) \in [0,1]^3$ can be computed:

$$\begin{cases} \alpha_{12} = \alpha_1 \alpha_2 \\ \alpha_{23} = \alpha_2 \alpha_3 \\ \alpha_{13} = \alpha_1 \alpha_3 \end{cases}$$

[†] Even if these events do not have any physical meaning, the proof of such a hypothesis is given in Appendix B.3.1. The proof shows that the time-series of the arrival times are made of independent samples.

Finally the term $(\alpha_1, \alpha_2, \alpha_3) \in [0,1]^3$ are expressed through these equivalence:

$$\begin{cases} \alpha_1 = \sqrt{\dfrac{\alpha_{12}\alpha_{13}}{\alpha_{23}}} \\ \alpha_2 = \sqrt{\dfrac{\alpha_{12}\alpha_{23}}{\alpha_{13}}} \\ \alpha_3 = \sqrt{\dfrac{\alpha_{23}\alpha_{13}}{\alpha_{12}}} \end{cases} \Leftrightarrow \begin{cases} \alpha_1 = \sqrt{\dfrac{PC_{12}^{\Delta T=\Delta T^*} \cdot PC_{23}^{\Delta T=\Delta T^*}}{PC_{23}^{\Delta T=\Delta T^*}}} \\ \alpha_2 = \sqrt{\dfrac{PC_{12}^{\Delta T=\Delta T^*} \cdot PC_{23}^{\Delta T=\Delta T^*}}{PC_{13}^{\Delta T=\Delta T^*}}} \\ \alpha_3 = \sqrt{\dfrac{PC_{23}^{\Delta T=\Delta T^*} \cdot PC_{13}^{\Delta T=\Delta T^*}}{PC_{12}^{\Delta T=\Delta T^*}}} \end{cases}$$

These equivalence express the terms $(\alpha_1, \alpha_2, \alpha_3) \in [0,1]^3$ using the probabilities of conflict $PC_{12}^{\Delta T=\Delta T^*}$, $PC_{23}^{\Delta T=\Delta T^*}$ and $PC_{13}^{\Delta T=\Delta T^*}$. These former probabilities are computed using the PDFs of the arrival times which are built using the real data as shown in section 4.2.2.

4.2.4 Interval length (IL)

Let $\{t_1^a, t_2^a, \ldots t_n^a\} \subseteq \mathcal{B}^n$ be the arrival times of a set of flights $\{1,2,\ldots n\} \in N^n$. Interval Length expresses the length of the shortest slot in which the likelihood of arrivals of a given set of flights $\{1,2,\ldots n\} \in N^n$ is high.

Interval Length relative to the set of flight $\{1,2,\ldots n\}$ is defined:

$IL_{1,2\ldots n}^{\alpha=\alpha^*} \overset{\Delta}{=}$ *"The shortest time period in which a fixed percentage* $\alpha^* \in [0,1]$ *of arrivals arrive relative to the flights* $\{1,2,\ldots n\}$."

Interval Length relative to the set of flights $\{1,2,\ldots n\}$ is denoted by $IL_{1,2\ldots n}^{\alpha=\alpha^*}$ or even as IL when there is no misunderstandings between flights and percentage $\alpha^* \in [0,1]$.

Let $\mathbf{t}_1^a \mathbf{t}_2^a \ldots \mathbf{t}_n^a$ be the random variable that represents the arrival times of whatever flight belonged to the set $\{1,2,\ldots n\}$. This is a mono dimensional random variable because the arrival time of any flights belonged to the set {1,2,...n} is considered. All the flights of the set are equally observed because No preference is given to a specific flight.

$\mathbf{t_1^a t_2^a \ldots t_n^a} \stackrel{\Delta}{=}$ This is a real random variable. Consider the set of flights having arrival times $\{\mathbf{t_1^a, t_2^a, \ldots t_n^a}\}$ and observe when at least one of these flights arrives. Performing this operation n times, a real random variable is defined. To every outcome $\zeta \in \mathcal{B}$ of the experiment Ω, a real number $\mathbf{t_1^a t_2^a \ldots t_n^a}(\zeta) \in \boldsymbol{R}$ is assigned.

Denoting by A_n the event "*The flight n arrives in the time period ΔT*", the probability at least one aircraft belonged to the given set arrives is given by:

$$\Pr\{A_1 \cup A_2 \ldots \cup A_n\} = \Pr\left\{\begin{matrix}\text{at least one} \\ \text{aircraft} \\ \text{arrives in } \mathbb{IL}\end{matrix}\right\} = \int_{\Delta T} f_{t_1^a t_2^a \ldots t_n^a}(t)dt$$

The interval length is defined as the shortest time period in which a fixed percentage $\alpha^* \in [0,1]$ of arrivals arrive, thus the minimum ΔT has to be sought. $\mathbb{IL}_{1,2\ldots n}^{\alpha=\alpha^*}$ can be finally computed solving the following equation:

$$\min\left(\int_{\mathbb{IL}_{1,2\ldots n}^{\alpha=\alpha^*}} f_{t_1^a t_2^a \ldots t_n^a}(t)dt = \alpha^*\right)$$

where $\alpha^* \in [0,1]$ is a fixed percentage.

4.2.5 Strategic planning

In section 4.2.1, delays were introduced as random variables. In general, delays can be presented as stochastic processes as they are dependent of the time. The aircraft scheduled to land at $t_s^a \in \boldsymbol{R}$ has a delay $\mathbf{d^a} \subseteq \mathcal{A}$, this scheduled time can be changed into a new scheduled time $t_s^{a'} \in \boldsymbol{R}$. Thus, in general another delay $\mathbf{d^{a'}} \subseteq \mathcal{A}$ is defined.

The task of this chapter is to present a methodology for strategic scheduling. Scheduled arrival times $t_s^a \in \boldsymbol{R}$ will be affected by changes. These adjustments $T \in \boldsymbol{R}$ of the scheduled arrival times will be small when they are compared with the whole interval time required by PDFs. Hence, delays are hypothesised stationary random variables:

$$\mathbf{d}^a\left(t_s^a\right) = \mathbf{d}^a\left(t_s^a + T\right) \qquad \begin{cases} \mathbf{d}^a \subseteq \boldsymbol{\mathcal{A}} \\ t_s^a \in \boldsymbol{R} \\ \forall T \in \boldsymbol{R} \end{cases}$$

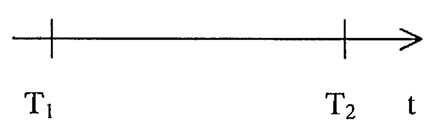
With this hypothesis, the same delays $\mathbf{d}^a \subseteq \boldsymbol{\mathcal{A}}$ are observed even if the scheduled departure times $t_s^d \in \boldsymbol{R}$ are changed.

4.2.5.1 Time Constraint

In some circumstances, airport operators require a schedule where the arrival times of the flights are bounded in a fixed interval time. Such a demand is usually required to permit passenger and luggage connections.

Consider a time interval:

$[T_1,T_2]$ $\qquad$ $(T_1,T_2) \in \boldsymbol{R}^2$



Given a set of n aircraft $\{1,2,\ldots n\} \in \boldsymbol{N}^n$, the goal is to compute $t_{s1}^a, t_{s2}^a \ldots t_{si}^a \ldots t_{sn}^a \in \boldsymbol{R}^n$ such as:

- The maximum probability that aircraft $i \in \boldsymbol{N}$ arrives in the time interval $[T_1,T_2]$. $\forall i \in \{1,2,\ldots n\}$.

- The maximum probability that aircraft $i \in \boldsymbol{N}$ and aircraft $j \in \boldsymbol{N}$ do not arrive within a time difference less than $\Delta T_{ij} \in \boldsymbol{R}$. $\forall (i,j) \in \{1,2,\ldots n\}^2$ with $i \neq j$.

This problem can be solved with the optimisation of a proper built functional. The solution of the defined problem has to be in correspondence of the absolute[†] minimum/maximum of this proper built functional. Hence, the problem is transformed in seeking this minimum/maximum.

Let $\{\mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a\} \subseteq \mathcal{B}^n$ be the arrival times of a set of flights $\{1,2,\ldots n\} \in N^n$. Each aircraft has a scheduled time $t_{sn}^a \in \boldsymbol{R}$ and a delay $\mathbf{d}_n^a \subseteq \mathcal{A}$. The arrival time is a real random variable defined in the set $\mathcal{B}$ because the arrival time $\mathbf{t}_n^a \subseteq \mathcal{B}$ is defined as sum of a mere number and of a random variable (see section 4.2.1).

$$\begin{cases} \mathbf{t}_1^a = t_{s1}^a + \mathbf{d}_1^a \\ \mathbf{t}_2^a = t_{s2}^a + \mathbf{d}_2^a \\ \quad\vdots \\ \mathbf{t}_n^a = t_{sn}^a + \mathbf{d}_n^a \end{cases} \qquad \begin{array}{c} (\mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a) \subseteq \mathcal{B}^n \\ (t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a) \in \boldsymbol{R}^n \\ (\mathbf{d}_1^a, \mathbf{d}_2^a, \ldots \mathbf{d}_n^a) \subseteq \mathcal{A}^n \end{array}$$

A scalar functional in the n-Cartesian product of the sets $\mathcal{B}$ is defined.

$$\Phi(t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a; \mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a) \subseteq \mathcal{B}^n \quad \rightarrow \quad \boldsymbol{R}$$

The goal is to calculate the n-tuple $t_{s1}^a, t_{s2}^a \ldots t_{sn}^a \in \boldsymbol{R}^n$ which gives the optimal solution. This n-tuple can either minimise or maximise this proper built functional. The optimal solution is computed solving the following system of n equations with n unknowns.

$$\begin{cases} \dfrac{\partial \Phi(t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a; \mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a)}{\partial t_{s1}^a} = 0 \\ \dfrac{\partial \Phi(t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a; \mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a)}{\partial t_{s2}^a} = 0 \\ \quad\vdots \\ \quad\vdots \\ \dfrac{\partial \Phi(t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a; \mathbf{t}_1^a, \mathbf{t}_2^a, \ldots \mathbf{t}_n^a)}{\partial t_{sn}^a} = 0 \end{cases}$$

[†] The minimum/maximum of this proper built functional can be also relative. In this case the variables have to be properly constrained.

An example of functional $\Phi\left(t^a_{s1}, t^a_{s2}, \ldots t^a_{sn}; \mathbf{t}^a_1, \mathbf{t}^a_2, \ldots \mathbf{t}^a_n\right)$ whose maximum is in correspondence of the solution of the defined problem is shown below. In this example, the optimal solution is given in correspondence of the maximum of this functional.

$$\Phi\left(t^a_{s1}, t^a_{s2}, \ldots t^a_{sn}; \mathbf{t}^a_1, \mathbf{t}^a_2, \ldots \mathbf{t}^a_n\right) =$$

$$= \sum_{i=1}^{n} \Pr\left\{\begin{matrix}\text{aircraft } i \text{ arrives} \\ \text{in } [T_1, T_2]\end{matrix}\right\} + \Pr\left\{\begin{matrix}\text{No pair of aircraft } (i, j) \in \{1,2,\ldots n\} \\ \text{with } i \neq j \text{ arrive within a temporal} \\ \text{difference less than } \Delta T^* \in \boldsymbol{R}\end{matrix}\right\} =$$

$$= \sum_{i=1}^{n} \Pr\{\mathbf{t}^a_i \in [T_1, T_2]\} + \left(1 - \left\{\begin{matrix}\text{At least two aircraft } (i, j) \in \{1,2,\ldots n\} \\ \text{with } i \neq j \text{ arrive within a temporal} \\ \text{difference less than } \Delta T^* \in \boldsymbol{R}\end{matrix}\right\}\right) =$$

$$= \sum_{i=1}^{n} \int_{T_1}^{T_2} f_{\mathbf{t}^a_i}(t)dt + \left(1 - TPC^{\Delta T = \Delta T^*}_{1\ldots n}\right)$$

The first term guarantees that the *i*-aircraft arrive in the fixed interval time $[T_1,T_2]$. The second term avoids aircraft arriving within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$. Each of these two terms will be analysed in detail.

The first term $\Pr\{\mathbf{t}^a_i \in [T_1, T_2]\}$ gives the *maximum probability* that the *i*-aircraft arrives in the interval $[T_1,T_2]$, see Figure 4.5. *Maximum probability* means that with a high degree of certainty, the *i*-aircraft will land in the time interval previously fixed by operators according to their constraints.

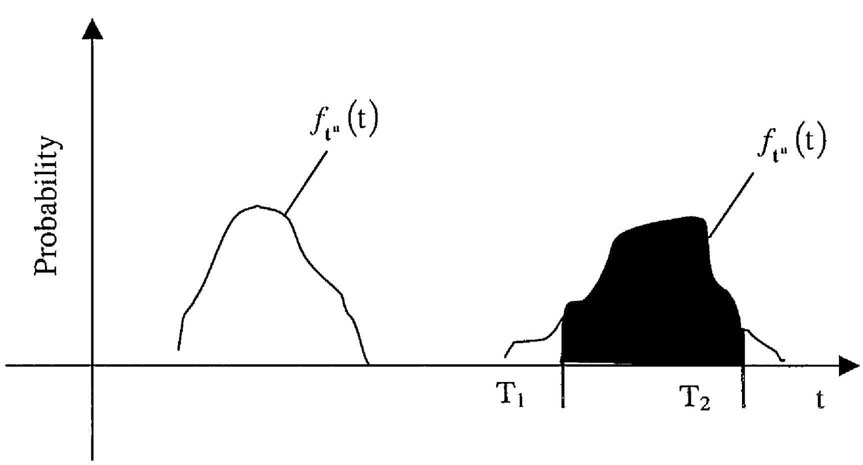


Figure 4.5: Probability that an aircraft arrives in the fixed time interval $[T_1,T_2]$.

The second term $\left(1-\mathrm{TPC}_{1\ldots n}^{\Delta T=\Delta T^*}\right)$ is analysed. $\mathrm{TPC}_{1\ldots n}^{\Delta T=\Delta T^*}$ is the total probability of conflict between the possible pair of aircraft belonging to the set $\{1,2,\ldots n\}$. The term $\left(1-\mathrm{TPC}_{1\ldots n}^{\Delta T=\Delta T^*}\right)$ expresses the opposite of the Total Probability of Conflict, hence, the Total Probability of no conflict. The term $\left(1-\mathrm{TPC}_{1\ldots n}^{\Delta T=\Delta T^*}\right)$ is the probability that aircraft $i\in N$ and aircraft $j\in N$ do not conflict $\forall(i,j)\in\{1,2,\ldots n\}^2$ with $i\neq j$. The second term expresses the constraint that aircraft do not arrive within a temporal difference less than $\Delta T_{ij}\in \boldsymbol{R}$. A Venn diagram is shown in Figure 4.6, where the set no conflict is represented in black lined area.

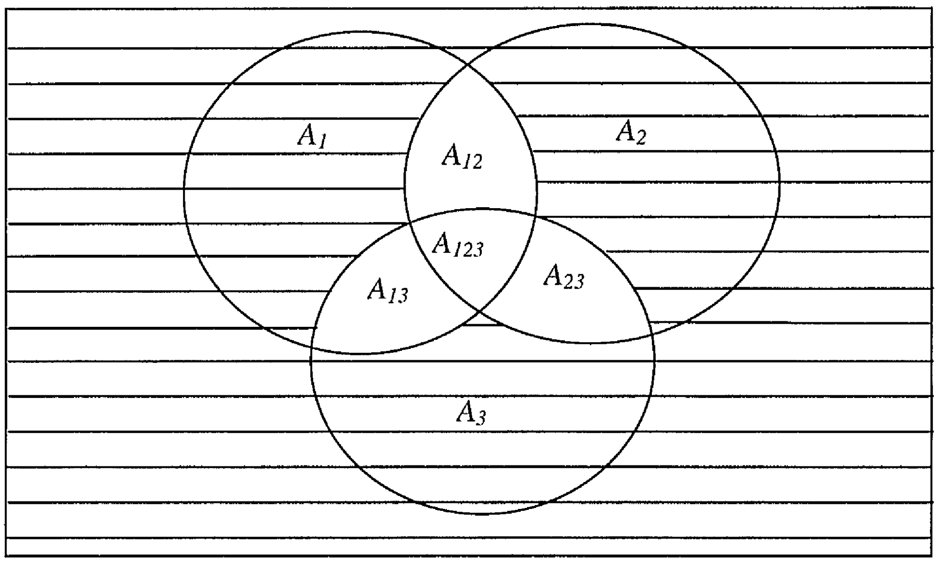


Figure 4.6: Venn diagram showing the Total Probability of No Conflict.

4.2.5.2 Probability Constraint

In some circumstances the main task is to organise a flexible and dynamic schedule imposing only avoidance of simultaneously aircraft arrival. This aim is achieved fixing the probability and computing the shifting times.

Consider two incoming aircraft having arrival time $\mathbf{t}_i^a\subseteq\mathfrak{B}$ and $\mathbf{t}_j^a\subseteq\mathfrak{B}$. The task is to compute the shifting time $T_{ij}\in\boldsymbol{R}$ such as the probability the real random variable $\mathbf{t}_i^a\subseteq\mathfrak{B}$ is greater than the real random variable $\mathbf{t}_j^a\subseteq\mathfrak{B}$ is equal to a fixed percentage $\alpha_{ij}\in[0,1]$.

$$\Pr\{\mathbf{t}_i^a + T_{ij} > \mathbf{t}_j^a\} = \alpha_{ij}$$

As shown in the section 4.2.3.1, the expression above can be simplified in:

$$\Pr\{\mathbf{t}_i^a + \mathrm{T}_{ij} > \mathbf{t}_j^a\} = \int_{-\infty}^{\mathrm{T}_{ij}} \int_{-\infty}^{+\infty} f_{\mathbf{t}_i^a}(\mathrm{z}_{ij} + \mathrm{t}_j) f_{\mathbf{t}_j^a}(\mathrm{t}_j) \mathrm{dt}_j \mathrm{dz}_{ij}$$

The following equation in the only unknown $\mathrm{T}_{ij} \in \boldsymbol{R}$ has to be solved to verify the condition: "*The real random variable* $\mathbf{t}_i^a \subseteq \mathcal{B}$ *is greater than the real random variable* $\mathbf{t}_j^a \subseteq \mathcal{B}$ *of a quantity equals to* $\alpha_{ij} \in [0,1]$. "

$$\int_{-\infty}^{\mathrm{T}_{ij}} \int_{-\infty}^{+\infty} f_{\mathbf{t}_i^a}(\mathrm{z}_{ij} + \mathrm{t}_j) f_{\mathbf{t}_j^a}(\mathrm{t}_j) \mathrm{dt}_j \mathrm{dz}_{ij} = \alpha_{ij}$$

Both the departure and arrival schedule of the aircraft *i* have to be shifted by T_{ij} (see Table 4.1)

	Old departing schedule	**Old arrival schedule**	**New departing schedule**	**New arrival schedule**
Aircraft j	t_{sj}^d	t_{sj}^a	t_{sj}^d	t_{sj}^a
Aircraft i	t_{si}^d	t_{si}^a	$t_{si}^d + T_{ij}$	$t_{si}^a + T_{ij}$

Table 4.1: Departure and arrival schedule for two aircraft.

Let $\left(t^d_{s1}, t^d_{s2,\ldots,} t^d_{sn}\right) \in \boldsymbol{R}^n$ be the departure scheduled time and $\left\{\mathbf{t}^a_1, \mathbf{t}^a_2, \ldots \mathbf{t}^a_n\right\} \subseteq \mathcal{B}^n$ be the arrival times of a set of flights $\{1,2,\ldots n\} \in \boldsymbol{N}^n$. The task is to compute the shifting times $\left(T_{12,} T_{23,} \ldots T_{(n-1)n}\right) \in \boldsymbol{R}^{n-1}$ to satisfy the conditions:

$$\begin{cases} \Pr\left\{\mathbf{t}^a_2 + T_{12} > \mathbf{t}^a_1\right\} = \alpha_{12} \\ \Pr\left\{\mathbf{t}^a_3 + T_{23} > \mathbf{t}^a_2\right\} = \alpha_{23} \\ \quad\vdots \\ \Pr\left\{\mathbf{t}^a_j + T_{ij} > \mathbf{t}^a_i\right\} = \alpha \\ \quad\vdots \\ \Pr\left\{\mathbf{t}^a_n + T_{(n-1)n} > \mathbf{t}^a_{n-1}\right\} = \alpha_{(n-1)} \end{cases}$$

where $\left(T_{12,} T_{23,} \ldots T_{(n-1)n}\right) \in \boldsymbol{R}^{n-1}$ are the required unknowns representing the time to shift the random variable $\left(\mathbf{t}^a_1, \mathbf{t}^a_2, \ldots \mathbf{t}^a_n\right) \subseteq \mathcal{B}^n$.

The term $\Pr\{\mathbf{t}^a_i + T_{ij} > \mathbf{t}^a_j\}$ expresses the probability that one random variable is greater than the other one. To compute the unknowns $\left(T_{12,} T_{23,} \ldots T_{(n-1)n}\right) \in \boldsymbol{R}^{n-1}$, this probability is imposed by fixing the terms $\alpha_{12,}, \alpha_{23,} \ldots \alpha_{(n-1)n} \in [0,1]^{n-1}$.

Solving the following system formed by $n-1$ separated equations with the $n-1$ unknowns $\left(T_{12,} T_{23,} \ldots T_{(n-1)n}\right) \in \boldsymbol{R}^{n-1}$, the required shifting times are calculated.

$$\begin{cases} \displaystyle\int_{-\infty}^{T_{12}} \int_{-\infty}^{+\infty} f_{\mathbf{t}^a_1}(z_{12} + t_2) f_{\mathbf{t}^a_2}(t_2)\, dt_2\, dz_{12} = \alpha_{12} \\ \displaystyle\int_{-\infty}^{T_{23}} \int_{-\infty}^{+\infty} f_{\mathbf{t}^a_2}(z_{23} + t_3) f_{\mathbf{t}^a_3}(t_3)\, dt_3\, dz_{23} = \alpha_{23} \\ \qquad\vdots \\ \displaystyle\int_{-\infty}^{T_{ij}} \int_{-\infty}^{+\infty} f_{\mathbf{t}^a_i}(z_{ij} + t_j) f_{\mathbf{t}^a_j}(t_j)\, dt_j\, dz_{ij} = \alpha_{ij} \\ \qquad\vdots \\ \displaystyle\int_{-\infty}^{T_{(n-1)n}} \int_{-\infty}^{+\infty} f_{\mathbf{t}^a_{n-1}}(z_{(n-1)n} + t_n) f_{\mathbf{t}^a_n}(t_n)\, dt_n\, dz_{(n-1)n} = \alpha_{(n-1)n} \end{cases}$$

Table 4.2 shows the proposed schedule for n aircraft.

This proposed new scheduling keeps the same sequence to justify the hypothesis that the arrival delays are stationary random variables.

$$\mathbf{d}^a\left(t_s^a\right) = \mathbf{d}^a\left(t_s^a + T\right) \qquad \begin{cases} \mathbf{d}^a \subseteq \mathcal{A} \\ t_s^a \in \boldsymbol{R} \\ \forall T \in \boldsymbol{R} \end{cases}$$

This is true for $T \in \boldsymbol{R}$ small when it is compared with the whole interval time required by the PDFs.

	Old departing scheduling	**Old arrival scheduling**	**New departing scheduling**	**New arrival Scheduling**
Aircraft 1	t_{s1}^d	t_{s1}^a	t_{s1}^d	t_{s1}^a
Aircraft 2	t_{s2}^d	t_{s2}^a	$t_{s2}^d + T_{12}$	$t_{s2}^a + T_{12}$
Aircraft 3	t_{s3}^d	t_{s3}^a	$t_{s3}^d + T_{12} + T_{23}$	$t_{s3}^a + T_{12} + T_{23}$
...	...	...	...	...
Aircraft i	t_{si}^d	t_{si}^a	$t_{si}^d + \sum_{k=2}^{i} T_{(k-1)k}$	$t_{si}^a + \sum_{k=2}^{i} T_{(k-1)k}$
...	...	...	...	...
Aircraft n	t_{sn}^d	t_{sn}^a	$t_{sn}^d + \sum_{k=2}^{n} T_{(k-1)k}$	$t_{sn}^a + \sum_{k=2}^{n} T_{(k-1)k}$

Table 4.2: Departure and arrival schedules for n aircraft.

4.3 The Decision Support Tool for the strategic schedule

Using the theoretical model shown in section 4.2, a Decision Support Tool is designed to help airport managers to plan the optimum sequence of arrival flights. A Matlab® programming code was developed to implement this methodology.

Three versions of the program code are designed to simulate each of the cases proposed:

- Current situation
- Strategic Scheduling
- Tactical Scheduling

In this chapter the first two codes are described while the Tactical Scheduling is described in Chapter 5. In all the programs the same layout is followed:

- *Data loading.*

The first part of the code loads the data from an Excel spreadsheet. The input is the difference, measured in minutes, between the observed arrival time and the scheduled arrival time for a given flight[†].

- *PDFs building*

PDFs of the arrival time are built for each flight using the mathematical theory shown in section 4.2. PDFs are computed using real data through building histogram diagrams.

- *Computation*

The minimum aircraft time separation pertaining the landing procedure depends on the aircraft's category. For simplicity, in this dissertation this value is fixed in three minutes

[†] In all this dissertation, the word *flight* means a regular connection between the departure airport and the arrival airport. Do not misunderstand with the single aircraft that in a given journey goes from the origin to the destination.

whatever aircraft are involved. During peak hour, operators allocate airborne and ground hold delays to guarantee such a time distance. Following the definition given in section 4.2.4, define probability of conflict as the probability that aircraft $i \in N$ and aircraft $j \in N$ arrive within a temporal difference less than ΔT_{ij}=3minutes. To increase safety and decrease cost a reduction of such a probability has to be achieved. The probability of conflict is a measure of safety and cost. This part of the program computes the Probability of Conflict between all the possible incoming aircraft that are categorised in pair $PC_{ij}^{\Delta T=3\min} \quad \forall (i,j) \in N^2$ with $i \neq j$. In section 4.2.3, $PC_{ij}^{\Delta T=3\min}$ was calculated through an integral, which was leaded to the well-known integral of convolution.

- *Results: Total Probability of Conflict(TPC) and Interval Length (IL)*

The Total Probability of Conflict (TPC) and the Interval Length (IL) are computed to measure the improvements due to the proposed algorithm, see sections 4.2.4 and 4.2.5 for their mathematical definitions. The first $TPC_{1\ldots n}^{\Delta T=3\min}$ indicates the probability of at least two aircraft arriving within an interval time of three minutes while $IL_{1,2,\ldots n}^{\alpha=\alpha^*}$ indicates the time interval in which a fixed percentage $\alpha^* \in [0,1]$ (e.g. $\alpha^*=0.9$) of arrivals are placed. The Total Probability of Conflict $TPC_{1\ldots n}^{\Delta T=3\min}$ is directly linked to the airborne delay as increasing the probability means runways are free decrease cycling around the airport. The decrease of the Interval Length $IL_{1,2,\ldots n}^{\alpha=\alpha^*}$ permits operators and users to plan more arrivals, consequently increasing efficiency and airport's resources and hence airport capacity.

- *Plot of the results*

The last part of the program plot the results. The first figures are the sample and the PDFs of the arrival time for each flight. The convolutions between consecutive flights are shown, and finally, the relative positions of all the aircraft are displayed.

4.4 Study case: Glasgow International Airport

Using real data provided to the ATM Research Group of Glasgow University by the United Kingdom Civil Aviation Authority (UK CAA), an example pertaining to Glasgow International Airport is solved. The data cover a period commencing the 1st October 1997 through the 30th September 1998. The set of data contains both the planned arrival/departure time and the real arrival/departure time. The airlines involved, the flight numbers, the departure and the destination airports are also listed.

In Table 3.1 an Excel spreadsheet is provided showing data pertaining to London Heathrow Airport as the data given for Glasgow International Airport are equal.

Initially to investigate the validity of the methodology, flight arrivals at Glasgow International Airport during the period 8:00 a.m. and ending at 8:25 a.m. are studied and analysed. The most significant data about the flights considered in this example are presented in Table 4.3. Note that only four flights are planned to land although the model can be applied even for a higher number of aircraft. More flights will increase the computation time without giving any theoretical improvements.

N.	Flight ID	Label	Departure	Scheduled time
1	Aer Lingus 222	AE	Dublin	t^a_{s1} =8:00
2	Air Canada 842	AC	Toronto	t^a_{s2} =8:00
3	British Airways 1902	BA	Birmingham	t^a_{s3} =8:25
4	European Air Charter 121	EAC	Bournemouth	t^a_{s4} =8:25

Table 4.3: Airline, flight number, departure and scheduled time for the four aircraft.

This interval time is chosen for different reasons:

- The interval time is one of the busiest of the day, hence the simulation gives a significant test to analyse the methodology.
- The aircraft are scheduled to arrive in pair at the same time. The scheduled arrival times are $t^a_{s1} = t^a_{s2}$ =8:00 a.m. and $t^a_{s3} = t^a_{s4}$ =8:25 a.m., hence time conflicts are analysed.

- The scheduled arrival times are close enough to allow cross conflicts. If one of the aircraft planned at 8:00 o'clock a.m. arrives late, with high probability[†] it will conflict with one the aircraft scheduled at 8:25 a.m. (or vice-versa).
- Both short and long haul flights are involved in the considered time interval. These two kinds of flights have different statistic properties due to the different length of the journey.

Firstly the Total Probability of Conflict $TPC_{1,2,3,4}^{\Delta T=3\,min}$ and the Interval Length $IL_{1,2,3,4}^{\alpha=.9}$ are analysed and computed in the current situation. Using the historical arrivals times a strategic schedule, based on statistics and probability theory, is developed and proposed. Strategic scheduling is the methodology for long time planning which ranges from two days to six/seven months in advance.

In the proposed algorithm the delay issue is taken into account, further information such as flight priorities, operator's requirements (e.g.) can be easily added. Historical arrival times imply data from the past, given that the knowledge of the behaviour of the aircraft in the past can predict arrival times in the future.

4.4.1 Current situation

From real data Probability Density Functions of the delay are built. In the example at Figure 4.7, British Airways (BA) flight coming from Birmingham (UK) scheduled to arrive at Glasgow International Airport (UK) at 8:25 a.m. is analysed.

The first chart of Figure 4.7 shows the delays $\mathbf{d}_3^a(\zeta_1), \mathbf{d}_3^a(\zeta_2), \ldots \mathbf{d}_3^a(\zeta_n) \in \boldsymbol{R}^n$ that are the difference in minutes between the real landing time $\mathbf{t}_3^a \subseteq \mathfrak{B}$ and the scheduled arrival time $t_{s3}^a \in \boldsymbol{R}$. Note that the delay can be both negative and positive. In the first case the aircraft arrived earlier while in the second case later than the planned time. The sample has almost 130 observations[††] covering a period of four/five months.

[†] In the following sections of this dissertation, the statement *with high probability* will be clarified.

[††] Such a chosen number of observations is statistically justified in Appendix B.

In the second graph of Figure 4.7, the PDF $f_{d_3^a}\left(d_3^a\right)$ is displayed. In the *x*-axis the delay measured in minutes is shown while in the *y*-axis its correspondence probability is plotted. The origin of the PDF is set at the scheduled arrival time, which is t_{s3}^a=8:25 a.m. The points placed before zero refer to aircraft arriving earlier than planned while the observations positioned after the origin are effected by a delay. The usual delay of this flight is shown through the PDF being mostly placed at the right side of the origin.

The vertical axis gives the probability that a fixed delay occurs. Given the fact that this is short haul flight, its arrival times are close to each other and hence the standard deviation of its PDF is relatively small. In the following of this section the standard deviation will be formally defined and calculated for different flights. It will be shown that long distance flight have a greater standard deviation when it is compared with the standard deviation of short distance flight.

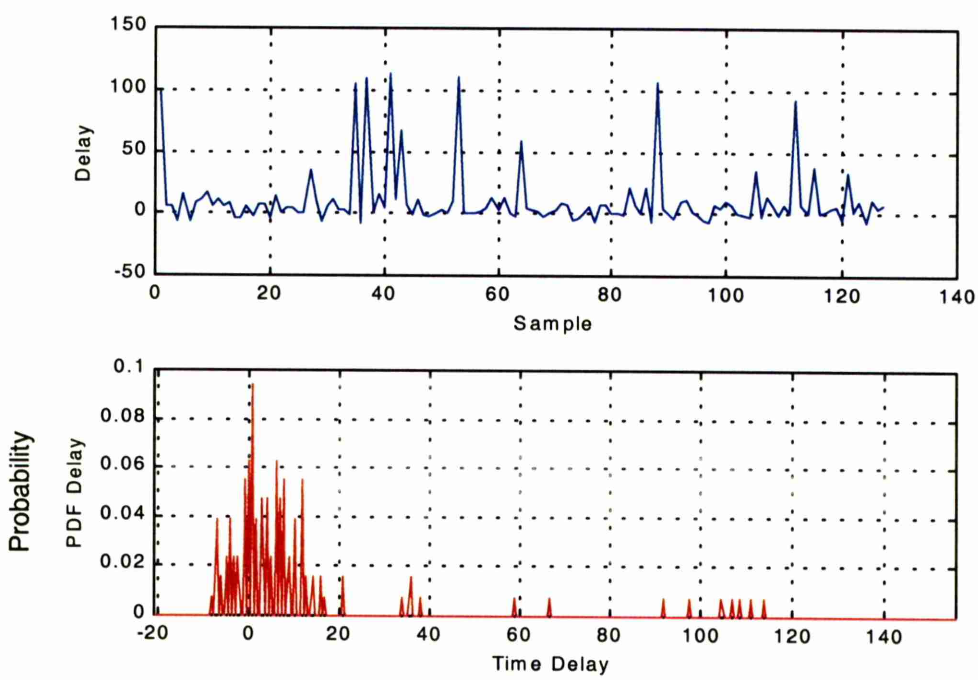


Figure 4.7: Samples and Probability Density Function of the delays for British Airways (BA).

At first sight, the PDF of the delay can be seen as a histogram of the observed arrival times, however, its nature comes directly from Probability's Theory. For a large numbers of samples, the relative frequency of occurring equals the probability of the event. This statement allows the development of the PDFs directly from the historical data marrying Probability's theory and Statistics together as proved in Appendix B.

In Figure 4.8, Air Canada (AC) flight flying from Toronto (CA) to Glasgow (UK) scheduled to arrive at t^a_{s2}=8:00 o'clock a.m. is discussed. Owing to its long trip time, its PDF $f_{\mathbf{d}^a_2}(d^a_2)$ is effected by a greater standard deviation when it is compared to the standard deviation of short distance flight. Its real arrival time is more unpredictable with an uncertainty of between −20 until +50 minutes and thus $IL_2^{\alpha=.9}$=70 minutes.

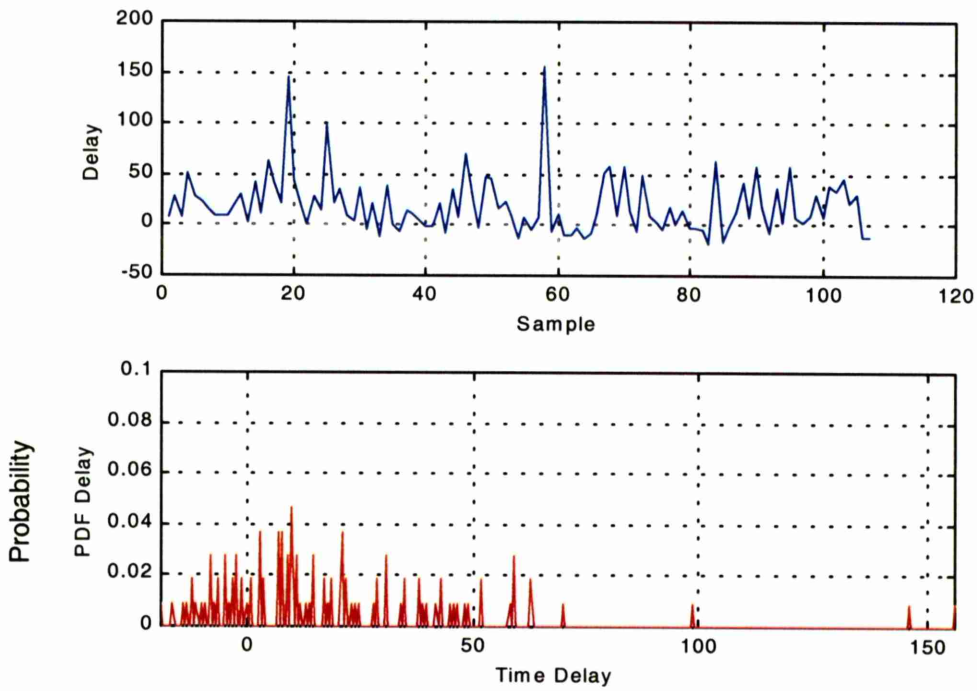


Figure 4.8: Sample delay and Probability Density Function for Air Canada (AC).

Studying PDfs of different flights coming at Glasgow International Airport, it has been revealed that long distant flight are effected by a greater uncertainty with respect to the short distance. This consideration has a physical meaning because there are more factors, which influence and disturb long haul flights.

The expected value η and the variance σ^2 of the random variable arrival delay $\mathbf{d^a} \subseteq \mathcal{A}$ are defined as:

$$\eta = E\{\mathbf{d^a}\} = \int_{-\infty}^{+\infty} d^a f_{\mathbf{d^a}}(d^a) dd^a$$

$$\sigma^2 = E\{(\mathbf{d^a} - \eta)^2\} = \int_{-\infty}^{+\infty} (d^a - \eta)^2 f_{\mathbf{d^a}}(d^a) dd^a$$

The mean η equals the center of gravity of the PDF, while the variance σ^2 equals the moment of inertia of the PDF. The positive square root σ is called standard deviation.

Table 4.5 shows the expected value and the standard deviation of the PDFs referred to the aircraft planned to arrive in the interval 8:00 - 8:25.

Flight ID	**Mean η (min)**	**Standard deviation σ (min)**	**Average arrival time $t_s^a+\eta$**
Aer Lingus 222	-3	11	7:57 a.m.
Air Canada 842	+20	29	8:20 a.m.
British Airways 1902	+11	25	8:36 a.m.
European Air Charter 121	-4	10	8:21 a.m.

Table 4.4: Expected value and variance for the considered flights in the current situation.

The Probability of Conflict $\alpha_{ij} \in [0,1]$ between the aircraft *i* and *j* is computed using the PDFs of the delay. This step is achieved by calculating the difference between the random variables $\mathbf{t}_i^a \subseteq \mathcal{B}$ and $\mathbf{t}_j^a \subseteq \mathcal{B}$. Denoting by:

$$\mathbf{z}_{ij} = \mathbf{t}_i^a - \mathbf{t}_j^a$$

$\mathbf{z}_{ij} \subseteq \mathcal{B}$ is a real random variable. To every outcome $\zeta = (\zeta_i, \zeta_j) \in \boldsymbol{R}^2$ of the experiment $\boldsymbol{\Omega}$, a real number $\mathbf{z}_{ij}\left(\mathbf{t}_i^a(\zeta_i) - \mathbf{t}_j^a(\zeta_j)\right) \in \boldsymbol{R}$ is assigned. As shown in the mathematical section 4.2.3, the PDF of $\mathbf{z}_{ij} \subseteq \mathcal{B}$ is computed using the well-known convolution integral.

In Figure 4.9 the PDF of the random variable difference $\mathbf{z}_{12} \subseteq \mathcal{B}$ between the random variable $\mathbf{t}_1^a \subseteq \mathcal{B}$ of Aer Lingus (AE) and $\mathbf{t}_2^a \subseteq \mathcal{B}$ of Air Canada (AC) is plotted. In the *x*-axis the difference between the arrival times is displayed, while in the *y*-axis the correspondent probability is indicated.

The probability that Aer Lingus arrives one hundred minutes later than Air Canada is equal to the definite integral of $f_{z_{12}}(z_{12})$ between one hundred and positive infinity.

Figure 4.9 shows that this probability is almost zero. Aer Lingus flight will arrive usually earlier than the Air Canada one as the function $f_{z_{12}}(z_{12})$ is mostly displaced at the negative x-axis.

The Probability of Conflict $PC_{12}^{\Delta T=3\,min}$ is defined in section 4.2.3.1 as the probability two aircraft arrive within 3 minutes of difference. As explained before, this probability is calculated by integrating $f_{z_{12}}(z_{12})$ over the closed interval $[-3,+3]$.

$$\Pr\{|\mathbf{t}_1^a > \mathbf{t}_2^a| \leq 3\} = \int_{-3}^{+3} f_{z_{12}}(z_{12})dz_{12}$$

These data are the basis to solve the Total Probability of Conflict $TPC_{1,2,3,4}^{\Delta T=3\,min}$ by combining the $PC^{\Delta T=3\,min}$ of all the flights involved as explained in section 4.2.3.2.

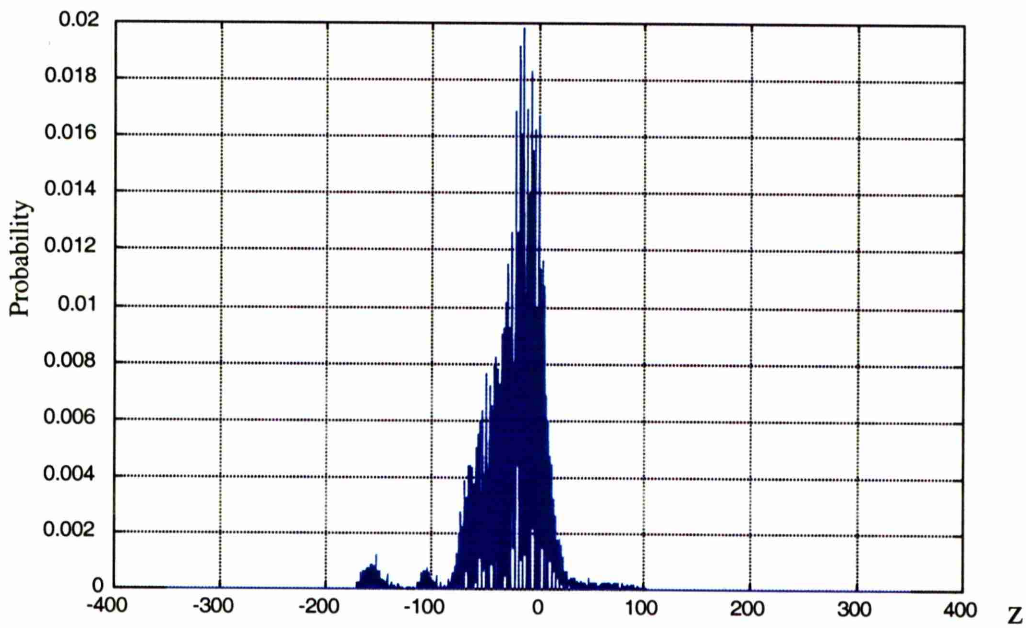


Figure 4.9: Convolution between Aer Lingus and Air Canada.

The relative situation of all the flights pertaining to the selected interval time is shown in Figure 4.10. This graph is obtained by plotting all the PDFs relative to the considered flights.

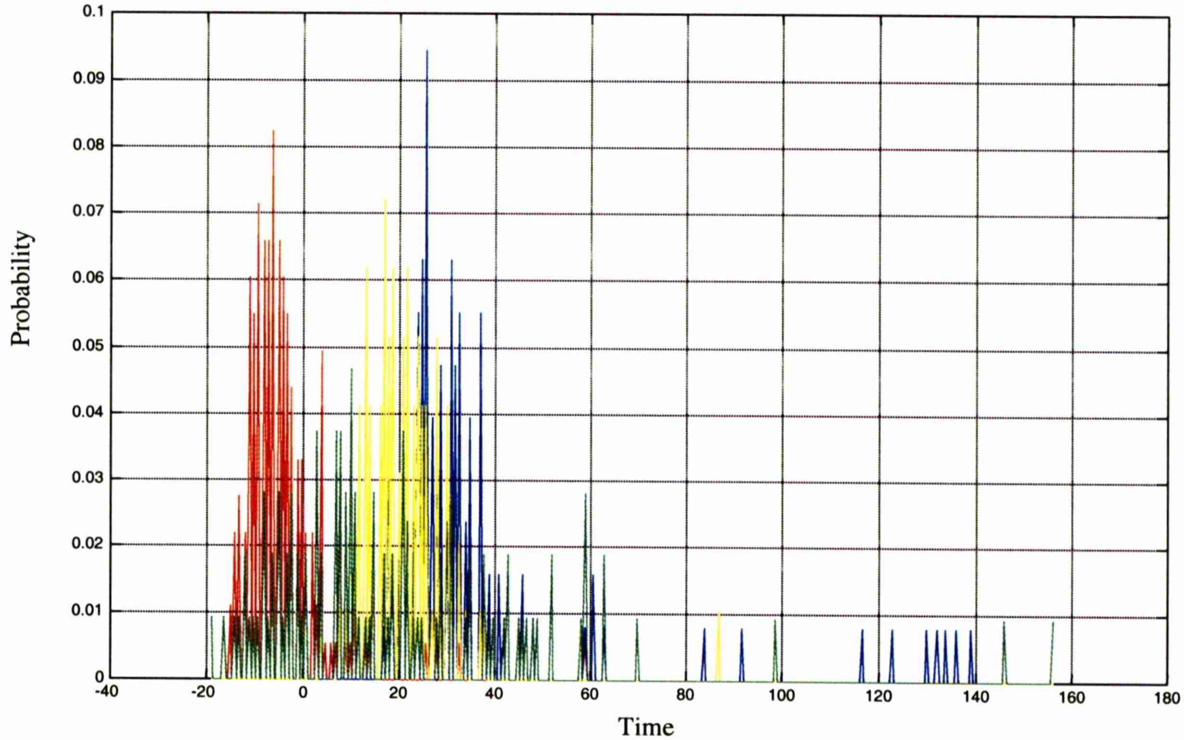


Figure 4.10: PDFs of the arrival times in the current situation.

The origin of the x-axis is set at 8:00 a.m. since it is the starting point of the chosen time interval. The horizontal scale is given in minutes while the vertical axis indicates the probability of the arrival for each flight. This chart is useful in understanding the real landing order and the separation between aircraft.

Owing to the high degree of uncertainty effecting the arrival times, the flights are distributed chaotically giving high traffic density in different moments.

Figure 4.11 shows the total distribution of the arrival time. For a given instant time, this PDF gives the probability of at least one of the four aircraft arrivals and, consequently, this PDF can be interpreted as a measure of the traffic density. Two main peaks are distinguishable, indicating two intervals with high traffic density.

Figure 4.11 shows aircraft are in reality landing even at 8:50 although from scheduling an interval commencing from 8:00 until 8:25 is considered. The interval time to consider is longer than just 25 minutes.

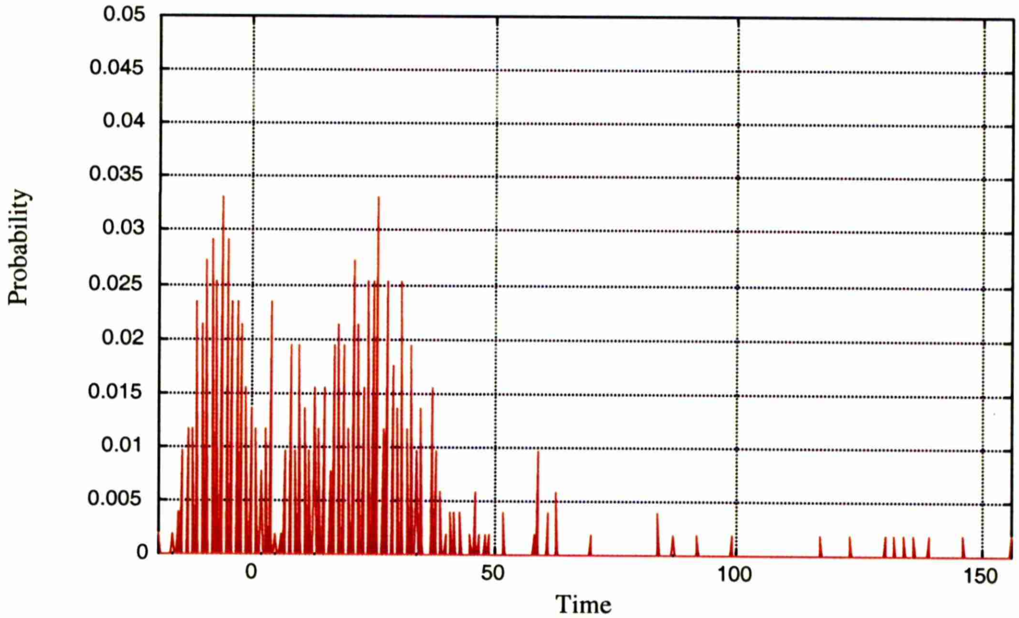


Figure 4.11: PDF of all the data computed together.

The minimum aircraft time separation according to the current aeronautical legislation of landing procedure is equal to 3 minutes. During peak hours operators allocate airborne and ground hold delays to guarantee this temporal distance of 3 minutes. In section 4.2.3.2, the Total Probability of conflict $TPC_{1,2,3,4}^{\Delta T=3\,min}$ is defined as *"The probability that at least two aircraft* $(i,j)\in[1,2,3,4]^2$ *with* $i\neq j$ *arrive within a temporal difference less than* $\Delta T_{ij}=3$ *minutes".* A reduction of $TPC_{1,2,3,4}^{\Delta T=3\,min}$ has to be achieved to increase safety and decrease cost.

The Total Probability of Conflict in the current situation is equal to $TPC_{1,2,3,4}^{\Delta T=3\,min}$ **=35.4%**. This means that in 35 outcomes of 100 observations†, at least two aircraft will request to land in a time interval smaller than 3 minutes. Given the runway is busy, the Probability of Conflict is strongly related to the aircraft having to circling around the airport.

According to the scheduling shown in Table 4.3, the aircraft are planned to land in the interval time, which commences at 8:00 o'clock and stops at 8:25 a.m. In reality, as shown in Figure 4.11, this interval is effected by a greater degree of uncertainty giving a

† A Bernoulli random variable can be defined that it is equals to 1 if at least two aircraft request to land otherwise 0.

starting time at 7:50 a.m. and ending even at 8:50 a.m. Interval Length $IL^{\alpha=.9}_{1,2,3,4}$ is defined in section 4.2.4 as *"The shortest time period in which a fixed percentage $\alpha^* \in [0,1]$ of arrivals arrive relative to the flights* {1,2,...n}*."*.

Airport capacity is strongly linked to this parameter, a decrease of the interval length allows operators to accommodate more flights assuring at the same time the same degree of safety. In the current situation the Interval Length is $IL^{\alpha=.9}_{1,2,3,4}$ **=56** minutes. Table 4.5 lists the most important parameters while Figure 4.12 shows a sketch relative to the current situation.

N.	Flight ID	Label	Scheduled time	TPC (%)	Interval length
1	Aer Lingus 222	AE	t^a_{s1}=8:00	35.4	56 min
2	Air Canada 842	AC	t^a_{s2}=8:00		
3	British Airways 1902	BA	t^a_{s3}=8:25		
4	European Air Charter 121	EAC	t^a_{s4}=8:25		

Table 4.5: Current situation.

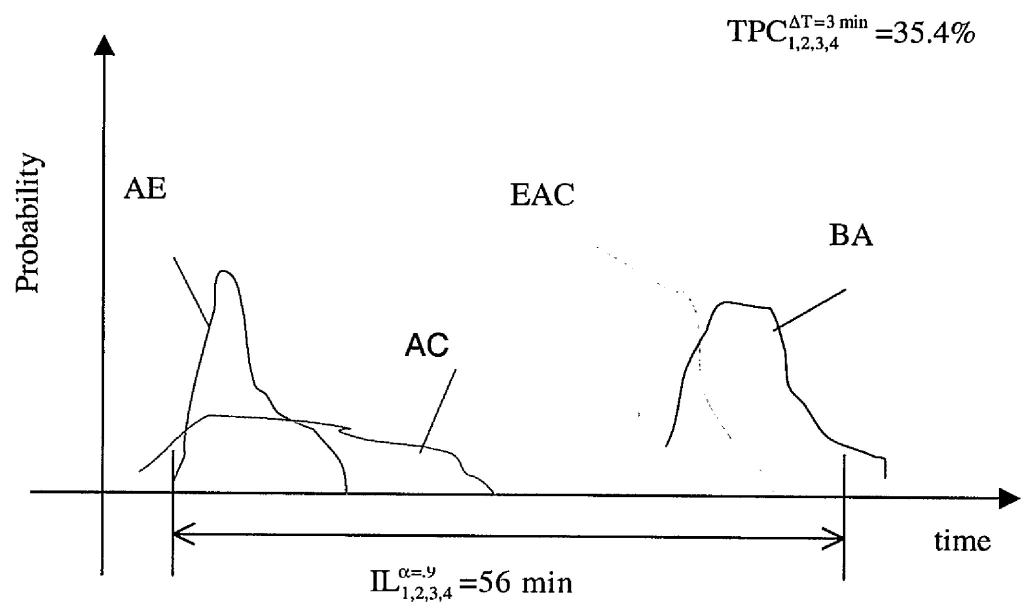


Figure 4.12: Sketch of the PDFs of the arrival times in the current situation.

4.4.2 The Strategic Schedule

The Strategic Schedule is first step towards improving the efficiency of air transport by optimising the airports allocation of the resources. It is called strategic because no real time information is analysed, thus strategic scheduling can be applied for strategic and pre-tactical planning. In Chapter 5, it is proposed that real time information is transmitted to the airports' operators through a data-link message. A dynamic scheduling is developed that can be applied at short notice throughout the day and thus for tactical planning.

The Strategic Schedule optimises the scarce resources of an airport. Through the knowing the behaviour of the flights in the past, the arrival times for the future can be estimated and then a new scheduling is devised which is designed to minimise the conflict at the destination airport is given. This methodology does not investigate the reasons why aircraft are affected by delays, nor does it look at ways in which these could be possible reduced; it simply focuses on optimising the arrivals scheduling given the fact that aircraft are subject to delays. Thus, knowing the expected delay, it will be possible to arrange the arrival times in such a way that traffic and congestion can be minimised.

The objective of these simulations is to achieve better scheduling as compared with the current situation. In this work only those results without a significant change in respect to the current arrival order and scheduled times are considered.

Two different classes of examples are going to be tested:

- The first one sets the Total Probability of Conflict to a value equals to the current status $TPC_{1,2,3,4}^{\Delta T=3\,\min}$ =35.4% and calculates the Interval Length (IL).
- The second one fixes the Interval Length $IL_{1,2,3,4}^{\alpha=.9}$ =56 minutes and computes the TPC.

If priority is given to optimising airport capacity, then TPC is fixed, otherwise if priority is given to reducing the amount of circling around the airport then the Interval Length is fixed. The block diagram shown in Figure 4.13 clarifies the inputs, the outputs and the structure of the simulation.

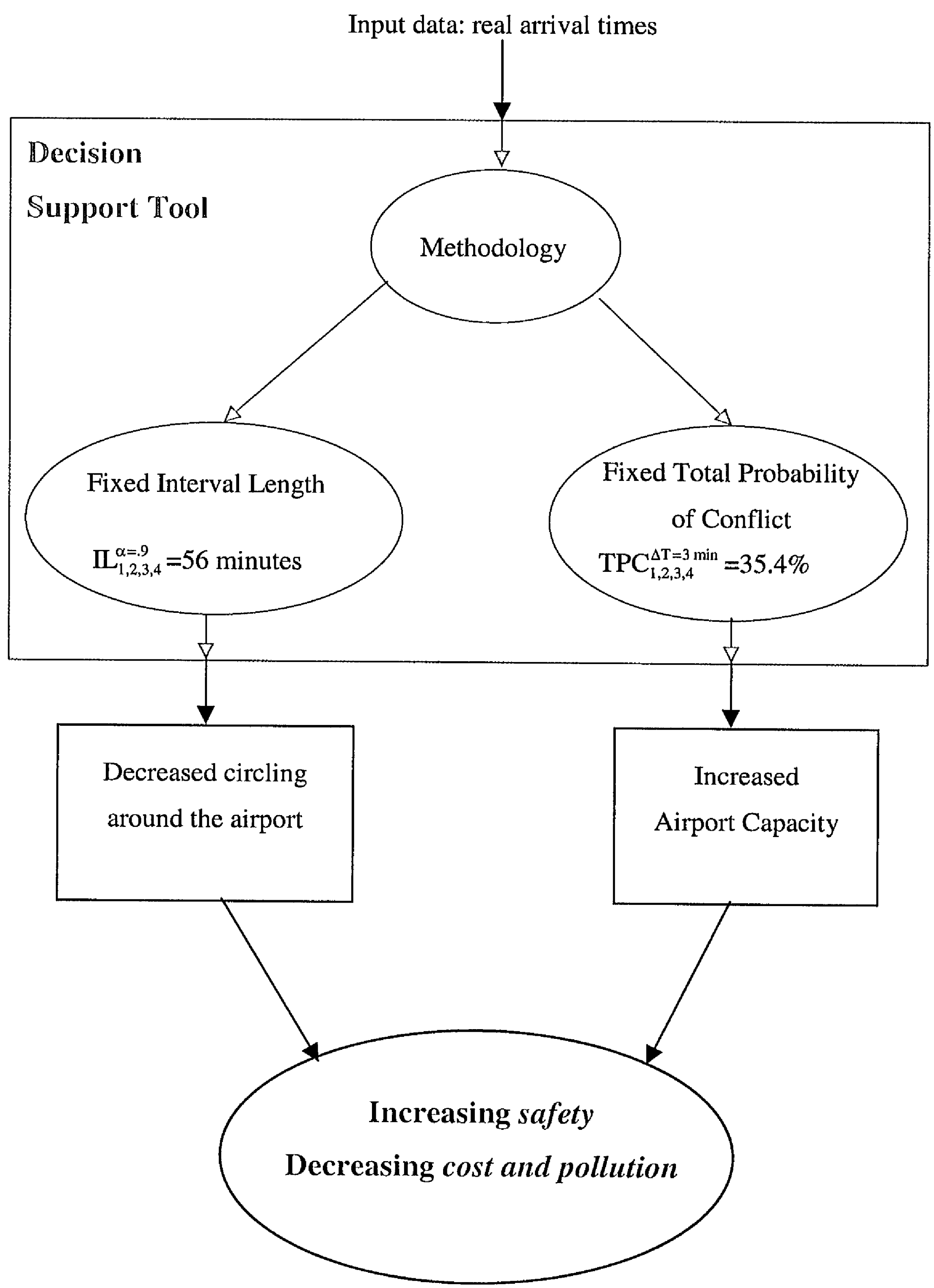


Figure 4.13: Block diagram for the Strategic Arrival Scheduler.

4.4.2.1 Fixed TPC

In these examples, the Total Probability of Conflict (TPC) is set to a value close to the value of the current status 35.4%. This methodology does not investigate the reasons why aircraft are affected by delays, nor does it look at ways in which these could possibly be reduced; it simply focuses on optimising the arrivals scheduling given the fact that aircraft are subject to delays.

Many solutions have been tested; some of them are presented and then explained.

4.4.2.1.1 Solution 1

In this example it is supposed that the airport's operators are mostly willing to respect the arrivals sequence as currently planned. The only change in the scheduling is given to Air Canada flight being located in the last position.
In this instance the output of the program is:

$T_{12} = shifting12 =$	-16.2 min
$T_{23} = shifting23 =$	2.6 min
$T_{34} = shifting34 =$	23.6 min
TPC =	0.34
Interval length =	47.9 min

These data are the necessary information to plan the new arrival strategic schedule. As the Aer Lingus flight is placed in the first position, the time reference is set at 8:00 a.m. The arrival times for the next aircraft are computed as here shown:

European Air Charter: $8{:}00 + shifting12 + 25 = 8{:}09$

British Airways: $8{:}00 + shifting12 + shifting23 + 25 = 8{:}12$

Air Canada: $8{:}00 + shifting12 + shifting23 + shifting34 = 8{:}18$

For clarity, the results are displayed in Table 4.6:

Arrival order	T_{12} (min)	T_{23} (min)	T_{34} (min)	Proposed arrival time AE	EAC	BA	AC
AE, EAC, BA, AC	-16	+3	+24	8:00	8:09	8:12	8:18

Table 4.6: Solution 1: Proposed arrival time.

The Strategic Schedule is the plan for a longer time period, from six months to one day in advance. No real-time data is known. These proposed arrival times are only the schedule for Glasgow International Airport. Corrections of the scheduling in the departure airport have to be taken into account, making whatever alterations are necessary.

The expected arrival times are used to give an approximate idea of the real arrival times. Note the mean is just an indication and it gives a reasonable estimate of the aircraft arrival times. As the arrival time is a random variable, it is impossible to be certain about the actual arrival times.

Flight ID	Scheduled arrival time in the current situation	Expected arrival time in the Strategic Schedule	TPC (%)	Interval length
Aer Lingus 222	t^a_{s1}=8:00	7:57	34.6	48 min
European Air Charter 121	t^a_{s4}=8:25	8:05		
British Airways 1902	t^a_{s3}=8:25	8:33		
Air Canada 842	t^a_{s2}=8:00	8:38		

Table 4.7: Solution 1. Output of the program.

In this example, by maintaining the TPC at its existing value of 34.6%, the interval length is reduced thereby providing the opportunity of accommodating more aircraft arrivals. Only small adjustments to the expected arrival times are required. For instance, the Air Canada flight arrival time appears to have adjusted by 38 minutes but as its mean arrival time is 8:20, the adjustment is, in reality, only 18 minutes.

Figure 4.14 and Figure 4.15 display the arrival demand after the new schedule. The PDFs for each selected flight are shown.

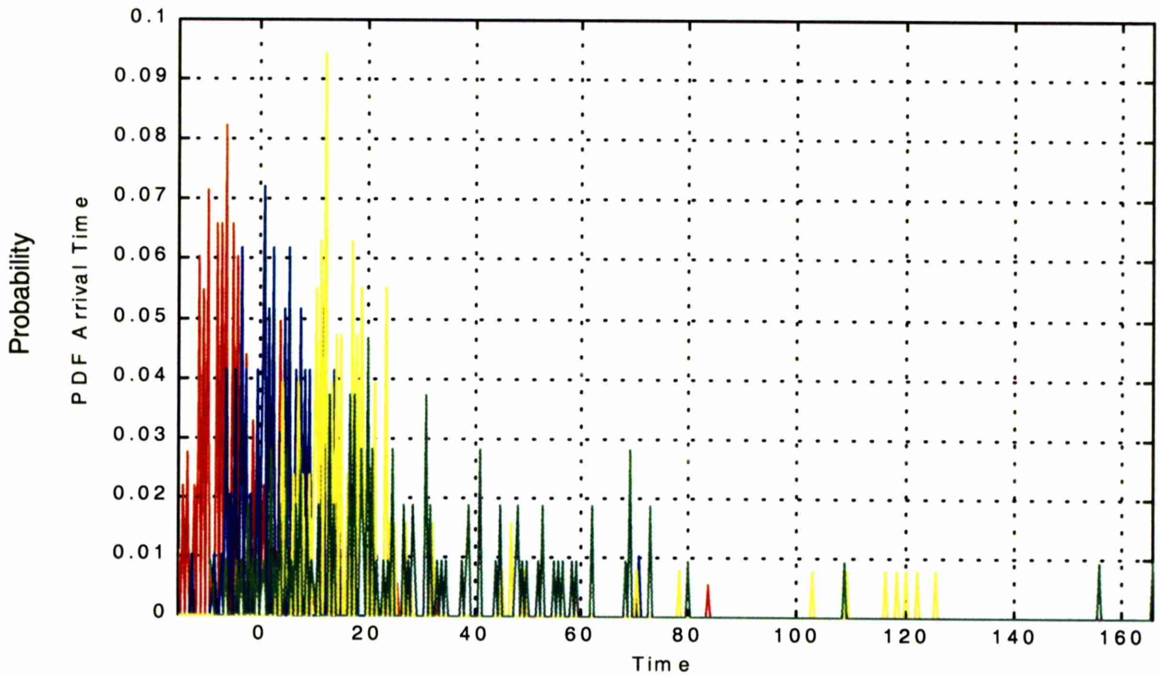


Figure 4.14: Probability Density Functions in the 1st solution.

The first PDF in red belongs to Aer Lingus, the second in blue is the European Air Charter one, the third in yellow corresponds to British Airways while the last one in green is for Air Canada.

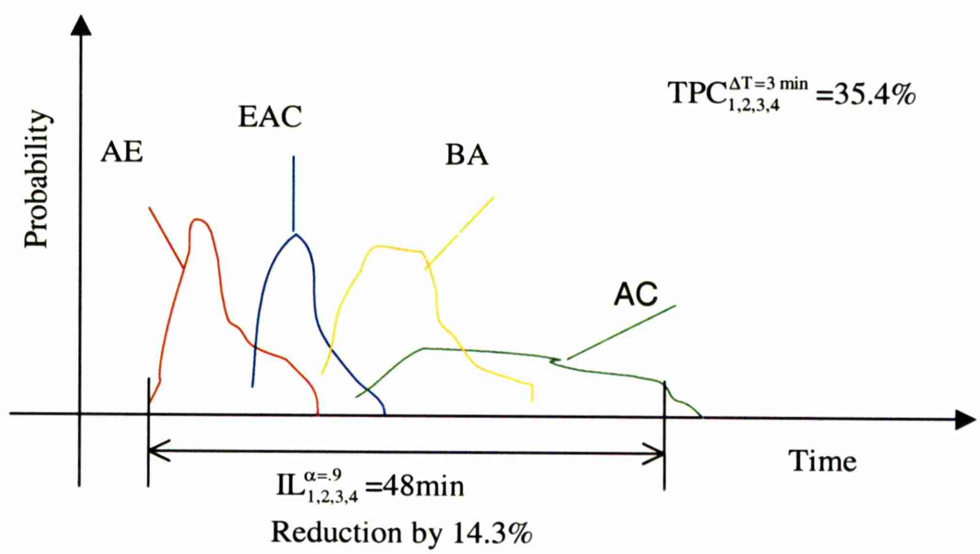


Figure 4.15: Sketch of the PDFs of the arrival times in the in the 1st solution.

4.4.2.1.2 Solution 2

In this simulation it is supposed that the British Airways flight has to arrive on time as (for example) it is a connecting flight. The new order would then be Aer Lingus, British Airways, European Air Charter and Air Canada.
The output of the program is:

T_{12} = *shifting12* =	-25.8 min
T_{23} = *shifting23* =	20.1 min
T_{34} = *shifting34* =	12.2 min
TPC =	0.35
Interval length =	46.39 min

Once again the Aer Lingus flight is placed in the first position, its arrival time is not changed. This is still at 8:00 a.m. The next aircraft is British Airways; the new scheduled times are:

British Airways: $8{:}00 + shifting12 + 25 = 7{:}59$

European Air Charter: $8{:}00 + shifting12 + shifting23 + 25 = 8{:}19$

Air Canada: $8{:}00 + shifting12 + shifting23 + shifting34 = 8{:}06$

Arrival order	T_{12} (min)	T_{23} (min)	T_{34} (min)	Proposed arrival time AE	BA	EAC	AC
AE, BA, EAC, AC	-26	+20	+12	8:00	7:59	8:19	8:06

Table 4.8: Solution 2. Proposed arrival time.

As previously explained, the proposed arrival times may not match the airport manager's expectations. A more reasonable prediction of the real arrival order will be based upon the expected arrival times. Table 4.9 gives the expected arrival times:

Flight ID	Scheduled arrival time in the current situation	Expected arrival time in the Strategic Schedule	TPC (%)	Interval length
Aer Lingus 222	t^a_{s1} =8:00	7:57	35.1	46 min
British Airways 1902	t^a_{s3} =8:25	8:10		
European Air Charter 121	t^a_{s4} =8:25	8:14		
Air Canada 842	t^a_{s2} =8:00	8:26		

Table 4.9: Solution 2. Output of the program.

This second example is slightly better than the first one as, while maintaining the same TPC, the interval length is further reduced by two minutes. Either proposed new schedule could be used depending on the particular requirements of the situation.

Figure 4.16 displays the arrival demand for each selected flight. The first PDF in red is the distribution of the arrival times of Aer Lingus, which is the first flight to land. It is followed then by British Airways in blue, European Air Charter in yellow while Air Canada in green is the last one.

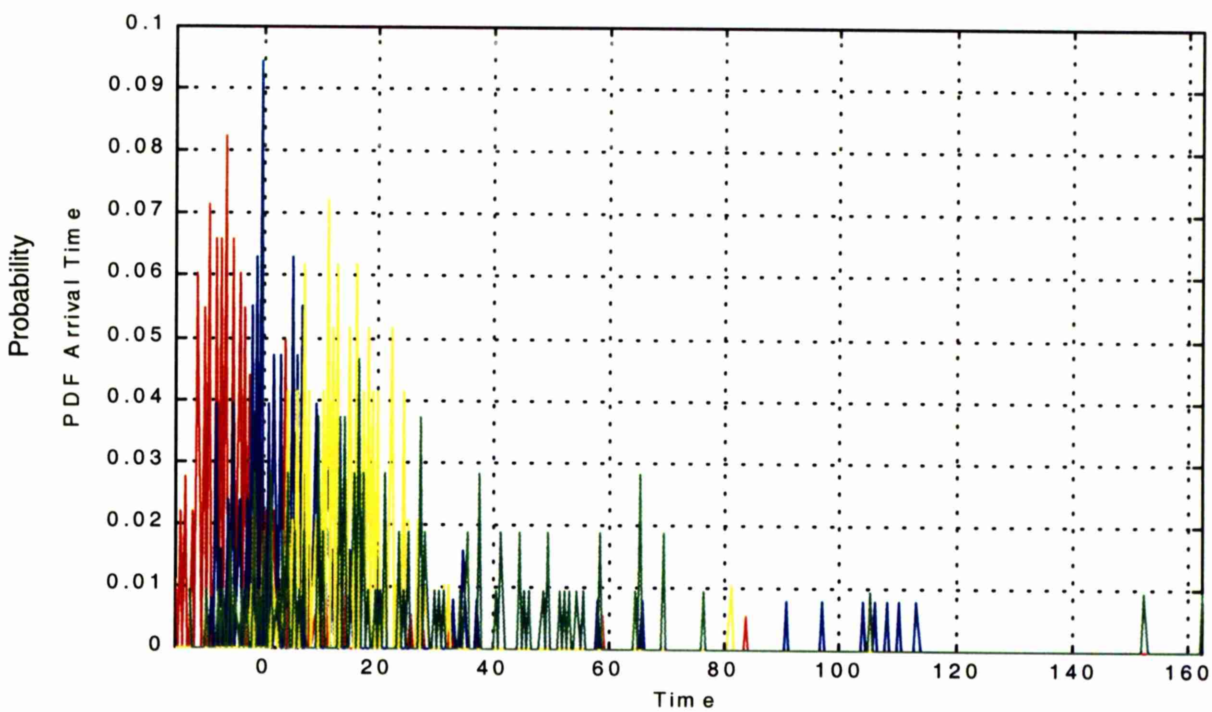


Figure 4.16: PDFs in the 2nd solution.

4.4.2.1.3 Comparison between the 2 solutions

The Strategic Schedule has proved to be a significant advance with respect to the current situation. Any of the solutions reached with the program decreases the expected Interval Length, which means that the capacity of the airport could be raised.
For instance, taking the Solution 1, the improvement achieved with this system is:

$$\text{Relative reduction in the Interval Length} = \frac{56-48}{56}\cdot 100 = \mathbf{14.3\%}$$

4.4.2.2 Fixed Interval length

The aim in this section is to reduce the Probability of Conflict between aircraft while maintaining the same Interval Length as in the current situation. Noting the fact that the Probability of Conflict is an indicator of the time that an aircraft spends circling around the airport while queuing to land, this approach is preferable for aircraft operators, as they can save fuel and time.

The program is organised in the same way as the code developed to fix the TPC, it will no be explained in detail again. The following example illustrates what kind of results can be expected with the strategic schedule.

Working with the same flights as before, the Interval Length will be set to the value corresponding to the current situation: 56 minutes. The airport manager decides in this case to keep the same arrival order as scheduled: Aer Lingus, Air Canada, European Air Charter and British Airways.

The output of the program is:

T_{12} = *shifting12* =	-11.8 min
T_{23} = *shifting23* =	2.2 min
T_{34} = *shifting34* =	-4.4 min
TPC =	0.30
Interval length =	55.78 min

Using this data the Strategic Schedule can be obtained.

Arrival order	**T_{12} (min)**	**T_{23} (min)**	**T_{34} (min)**	**Proposed arrival time** AE	AC	EAC	BA
AE, AC, EAC, BA	-12	+2	-4	8:00	7:48	8:15	8:11

Table 4.10: Proposed arrival time with fixed interval length.

Table 4.11 compares the solution obtained with the Decision Support Tool and the current schedule.

Flight ID	**Scheduled arrival time in the current situation**	**Expected arrival time in the Strategic Schedule**	**TPC (%)**	**Interval length**
Aer Lingus 222	t^a_{s1} =8:00	7:57	30.0	56 min
Air Canada 842	t^a_{s2} =8:00	8:08		
European Air Charter 121	t^a_{s3} =8:25	8:11		
British Airways 1902	t^a_{s4} =8:25	8:22		

Table 4.11: Output of the program with fixed interval length.

The improvement achieved in this case is:

$$\text{Relative reduction in the TPC} = \frac{35.4 - 30.0}{35.4} = \mathbf{15.3\%}$$

Figure 4.10 shows the current situation while Figure 4.17 and Figure 4.18 this strategic schedule. In this strategic schedule smaller gaps exist between the flights and the overlapping is reduced.

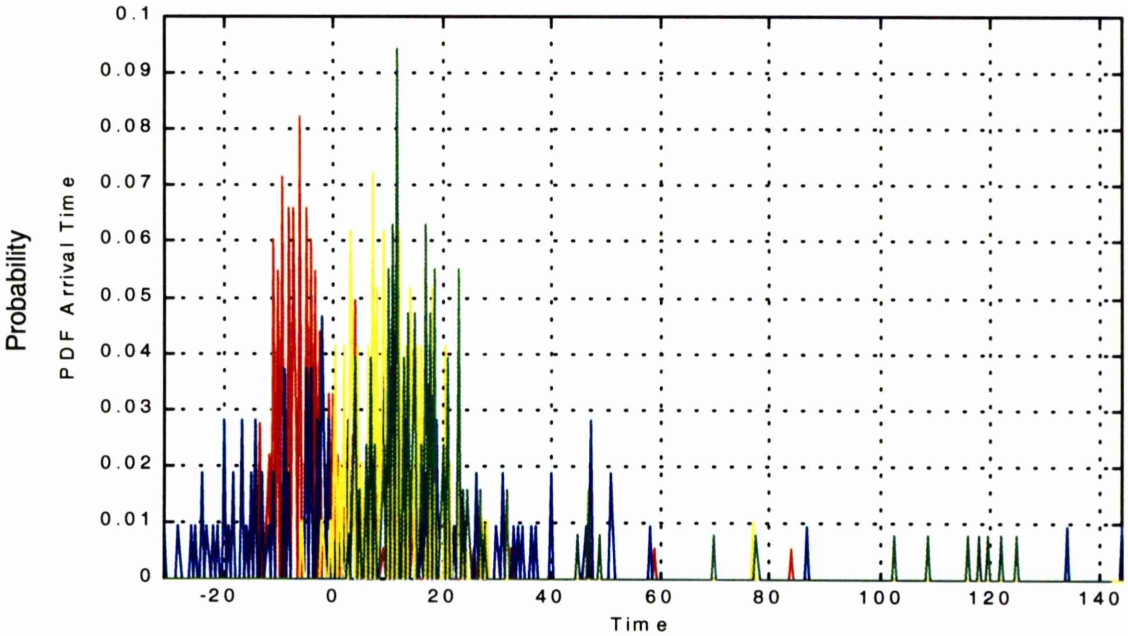


Figure 4.17: PDFs of the arrival times in the Strategic Schedule.

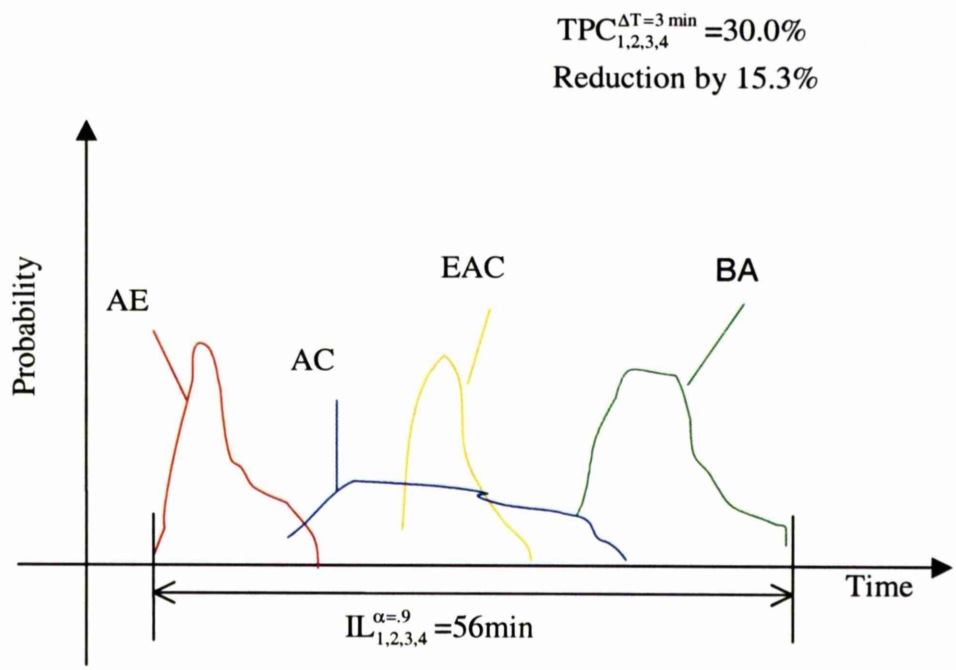


Figure 4.18: Sketch of the PDFs of the arrival times in the Strategic Schedule.

In this Strategic Schedule the flights are better organised, this fact becomes more evident when comparing the overall set of data. The Strategic scheduling, as shown in Figure 4.20, provides a more regularly spaced response to the landing requests than exists in the current situation, see Figure 4.19.

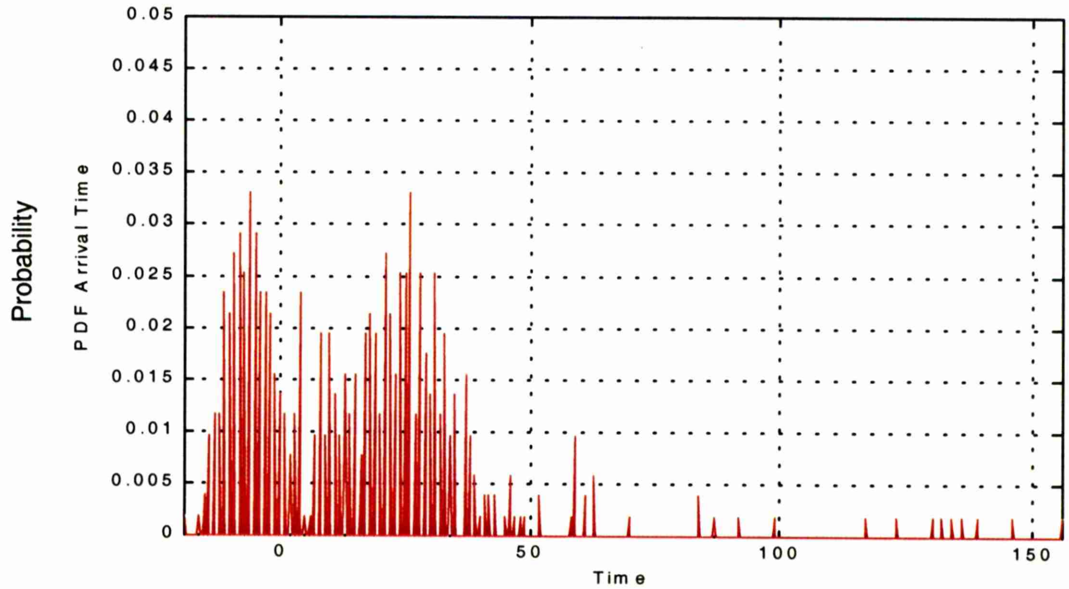


Figure 4.19: PDF of all the data computed together in the current situation.

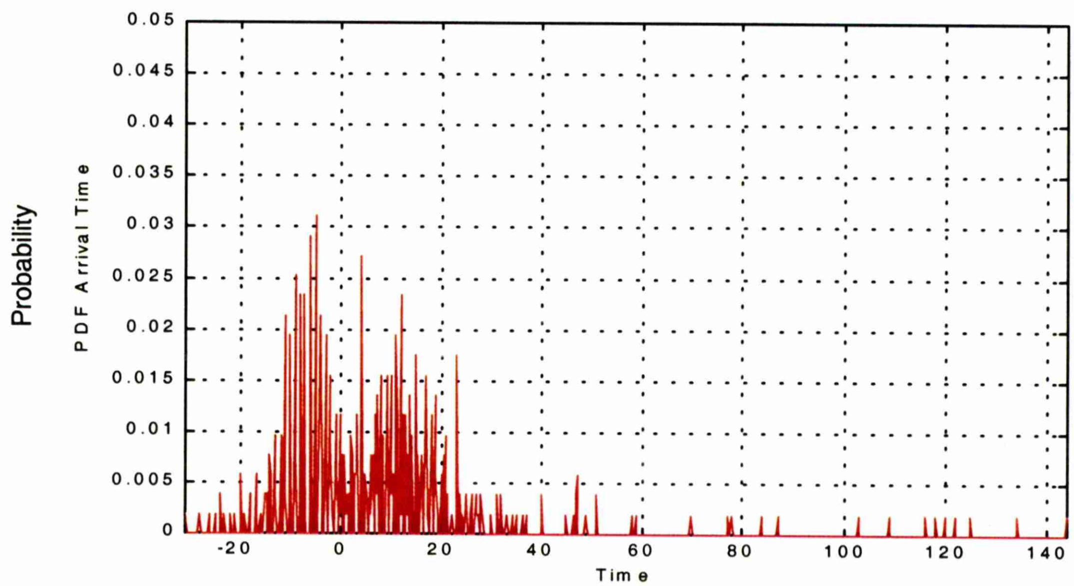


Figure 4.20: PDF of all the data computed together in the Strategic Schedule.

4.4 Conclusions

This chapter has presented a tool for the development of a re-scheduling operational procedure for strategic planning that reduces the probability of landing conflicts. This operational procedure allocates the scheduled arrival times at a given airport in time. It is based on Statistic and Probability Theory and relies on historical data regarding arrival times at the airports.

This new procedure has been applied to improve the schedule of Glasgow International Airport. It has been proved that the consequential benefits include the options of:

- Reducing the interval length (IL) by 14% thereby increasing the airport capacity.
- Reducing the Total Probability of Conflict (TPC) by 15.3% which will decrease airborne delay at the arrival airport, which will reduce fuel consumption both reducing operators costs and environmental pollution.
- A combination of the two options.

Chapter 5

Tactical Planning based on a dynamic probabilistic GHP

5.1 Introduction

Nowadays the international aeronautical community is working to implement new techniques to improve the current Air Traffic Flow Management (ATFM) system. Because of unpredictable events, changes may be required to the planned schedule even at very short notice. As already described in section 1.1, the daily schedule planning is called tactical planning. Based on the availability of new technology such as ADS-B, a tactical planning based on a dynamic probabilistic GHP (see section 2.2.2) is shown in this chapter.

This algorithm uses a data link to transfer information between airports and aircraft. It proposes that a flight during its route will broadcast its current position to the arrival airport. This data will then be computed real-time by the system, which is organising the planning proposing a new scheduling according to users and operators demands. Ground hold and airborne delay will be then allocated and finally communicated to the airports and aircraft.

Currently the Central Flow Management Unit (CFMU) is mainly involved with tactical planning in order to manage and organise the air traffic situation for en-route purposes (ATFM). The CFMU is not directly involved with planning a strategic schedule, as this issue is mainly a concern for airport management. Tactical planning is usually given two hours before departure according to the expected ATM congestion (both en-route and at the arrival airport). It is then revised according to the actual situation (e.g. adverse weather condition, ATC centre overloaded). Flights are only updated in the CFMU systems when they take-off or when they enter a country through an AFTN message issued by ATC.

Currently, further developments are taking place in the collection of actual flight information. The intention is that this will then be distributed to airports and ATC centres in order to optimise ATFM at short notice.

The aim of this chapter is to present a tool to allocate ground hold delay using a data-link support between airports and aircraft using statistics and probability theory.
Imagine a flight that encounters adverse weather condition during its journey. The arrival time will be affected by a delay and inevitable congestion will be observed in the destination airport. Air traffic controllers, to fulfil all landing requests, will assign ground-hold and airborne delay instructions. Ground hold delay is always preferred to airborne delay because it is safer and less expensive. It is better to delay on the ground the other incoming aircraft at their departure airports rather then cause congestion at the arrival airports. Ground hold delay is safer and cheaper due to the fact aircraft are held on the ground rather than kept circling around the destination airport.

5.2 Mathematical model

In this section the mathematical background is developed. Aircraft travelling time will be defined as random variable while velocity and aircraft's position will be introduced as stochastic processes. Using the stochastic process velocity, a real-time estimation of the arrival times will be achieved. The PDF of the stochastic process velocity will be built using a transformation of variable, which involves the travelling time PDF (known by real data). Aircraft's position will be only used to show the general approach for real-time arrival time estimation.

5.2.1 Travelling time as a real random variable. Velocity and position as two dimensional real random variables

The real random variable travelling time is introduced using the same notation of the chapters 3 and 4:

$$\mathbf{t}^{t} \overset{\Delta}{=} \mathbf{t}^{a} - \mathbf{t}^{d} \qquad \begin{cases} \mathbf{t}^{t} \subseteq \mathcal{E} \\ \mathbf{t}^{a} \subseteq \mathcal{B} \\ \mathbf{t}^{d} \subseteq \mathcal{E} \end{cases}$$

$\mathbf{t}^{t} \overset{\Delta}{=}$ Travelling Time of a given flight†. The differences between the arrival and departure times of aircraft operating a given flight are observed. A real random variable is defined assigning a real number to each of these observations. $\mathcal{E}$ is called the set where the travelling time is defined.

During flying phase, cruise velocity is affected by continuous variations because of failures, adverse meteorological conditions and traffic in the airspace environment. Aircraft speed is unpredictable due to the effects of the above factors. Modelling velocity as a two dimensional real random variable is a useful approach getting a powerful tool for aircraft's position estimation, as it will be shown in Chapter 6.

Velocity of the aircraft is defined as:

$$\underline{\mathbf{v}} \overset{\Delta}{=} (\mathbf{v}_x, \mathbf{v}_y) \qquad \underline{\mathbf{v}} \subseteq \mathcal{J}$$

† As in chapter 4, the word *flight* refers to a regular scheduled service between departure and arrival airport. Do not misunderstand with the word *aircraft* that expresses a single plane that in a given journey goes from the departure to the arrival airport.

$\underline{v} \stackrel{\Delta}{=}$ Velocity of the aircraft. This is a two dimensional random variable. The measurements of the velocities in fixed instant of times for a given aircraft operating a selected flight are observed. A two dimensional real random variable is defined assigning a pair of real numbers to each of these observations. $\mathcal{J}$ is called the set where the velocity of aircraft is defined. To every outcome $\zeta \in \mathcal{J}$, a pair of real number are assigned $v_x(\zeta) \in \boldsymbol{R}$, $v_y(\zeta) \in \boldsymbol{R}$.

Position of the aircraft is defined as:

$$\underline{s} \stackrel{\Delta}{=} (s_x, s_y) \qquad \underline{s} \subseteq \mathcal{G}$$

$\underline{s} \stackrel{\Delta}{=}$ Position of the aircraft. This is a bi-dimensional random variable. The positions at fixed times of a given aircraft operating a selected flight are measured. $\mathcal{G}$ is called the set where the position of the aircraft is defined. To every outcome $\zeta \in \mathcal{G}$, a pair of real numbers are assigned $s_x(\zeta) \in \boldsymbol{R}$, $s_y(\zeta) \in \boldsymbol{R}$.

In general, to uniquely determine a *n*-dimensional random variable, it is necessary to determine the joint PDF $f_{\underline{x}}(x_1, x_2, \ldots x_n)$ of the random variables $\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}$ [48]. If a set of axes ξ_1, $\xi_2 \ldots \xi_n$ exists so that the random variables $\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}$ are mutually independent when they are referred to $\xi_1, \xi_2, \ldots \xi_n$, then the joint PDF $f_{\underline{x}}(x_1, x_2, \ldots x_n)$ equals the product of the marginal densities of $\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}$:

$$f_{\xi_1,\xi_2,\ldots\xi_n}(\xi_1,\xi_2,\ldots\xi_n) = f_{\xi_1}(\xi_1) f_{\xi_2}(\xi_2) \ldots f_{\xi_n}(\xi_n)$$

Thus, in this case it is only necessary to determine the marginal PDFs of $\mathbf{x_1}, \mathbf{x_2}, \ldots \mathbf{x_n}$ expressed in the set of axes $\xi_1, \xi_2, \ldots \xi_n$ to determine the joint PDF $f_{\underline{x}}(x_1, x_2, \ldots x_n)$.

Alternatively, if the joint PDF $f_{\underline{x}}(x_1, x_2, \ldots x_n)$ is known, the marginal PDFs $f_{\xi_1}(\xi_1), f_{\xi_2}(\xi_2), \ldots f_{\xi}(\xi_n)$ can be calculated through the computation of the eigenvectors of the covariance matrix of the random variables $\xi_1, \xi_2, \ldots \xi_n$.

5.2.2 Building Cumulative Density Function (CDF) for velocity

A random variable consists of an experiment **Ω** with a probability measure defined on a sample space and a function that assign a number to each outcome in the sample space of the experiment **Ω**, see [46]. In this section the CDF of the two dimensional random variable velocity is built as histogram diagram of the relative frequency of occurrences.

The experiment **Ω** is defined in the following step:

ζ = measurement of aircraft velocity in a given instant of time during a selected flight for a given aircraft. To determine the vector velocity, the measurements of its components are needed. To every outcome ζ of the experiment **Ω**, a pair of real numbers is associated.

To clarify: the outcome ζ of the experiment **Ω** is a pair of real numbers $\zeta=(v_x,v_y)\in \boldsymbol{R}^2$ is defined as:

v_x is the component of the velocity with respect to the x-axis
v_y is the component of the velocity with respect to the y-axis

The two dimensional Random Variable velocity $\underline{\mathbf{v}}$ is defined by:

$$\underline{\mathbf{v}}(\zeta) \stackrel{\Delta}{=} \zeta$$

It follows that $\zeta=(v_x,v_y)$ which indicates the components of the velocity has a double meaning. Hence, $\zeta=(v_x,v_y)$ is both the outcome of the experiment **Ω** and the value of the two dimensional random variable. Therefore the two dimensional random variable $\underline{\mathbf{v}}\subseteq\mathcal{F}$, has as domain $\boldsymbol{R}^2$ and range the set $\boldsymbol{R}^2$. The velocity is a two dimensional real random variable.

$$\underline{\mathbf{v}}(\zeta)\subseteq\mathcal{F} = \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^2$$

The experiment Ω is performed n times. At a given outcome ζ of the experiment, the two dimensional real random variable $\underline{\mathbf{v}} \subseteq \mathcal{F}$ associates a set of two values $\underline{\mathbf{v}}(\zeta)$. Given a set of two real numbers $(v_x, v_y) \in \boldsymbol{R}^2$ and denoting by $n(v_x, v_y)$ the total number of trials such that:

$$\underline{\mathbf{v}}(\mathbf{v}_{\mathbf{x}}(\zeta) \leq v_x, \mathbf{v}_{\mathbf{y}}(\zeta) \leq v_y)$$

the cumulative density function (CDF) of the two dimensional real random variable velocity $\underline{\mathbf{v}} \subseteq \mathcal{F}$ can be determined by applying the *Law of Large numbers.*

Throughout this dissertation, PDFs and CDFs are computed as histograms of the relative frequency of observing a given event. These histograms are considered equivalent to the PDFs and CDFs by applying the *Law of large numbers* [47] [48]:

If the probability of an event a in a given experiment $\boldsymbol{\Omega}$ *equals p and the experiment is repeated n times, then for any* $\varepsilon > 0$

$$\lim_{n \to \infty} \Pr\left\{ \left| \frac{n_a}{n} - p \right| \leq \varepsilon \right\} = 1$$

where n_a *equals the number of success of a.*

This concept may also be expressed as:

If the experiment $\boldsymbol{\Omega}$ *is repeated n times and the event a occurs* n_a *times, then, with a high degree of certainty, the relative frequency* n_a/n *of the occurrence of a is close to* $p = Pr(a)$,

$$\Pr(a) \cong \frac{n_a}{n}$$

provided n is sufficiently large.

The joint cumulative distribution function of the two dimensional real random variable $\underline{\mathbf{v}} \subseteq \mathcal{F}$ is defined by:

$$F_{\underline{\mathrm{v}}}(\mathrm{v}_{\mathrm{x}},\mathrm{v}_{\mathrm{y}}) \stackrel{\Delta}{=} \Pr\{\mathbf{v}_{\mathrm{x}} \leq v_x, \mathbf{v}_{\mathrm{y}} \leq v_y\}$$

In conclusion the joint cumulative distribution function is computed as ratio between the number of trials such as $\underline{\mathbf{v}}(\mathbf{v}_{\mathrm{x}}(\zeta) \leq v_x, \mathbf{v}_{\mathrm{y}}(\zeta) \leq v_y)$ and the total number of trials n.

$$F_{\underline{\mathrm{v}}}(\mathrm{v}_{\mathrm{x}},\mathrm{v}_{\mathrm{y}}) \cong \frac{n(v_x, v_y)}{n}$$

This is an important result because the CDF can be built using real data. However, in this thesis aircraft's velocity CDF will be built using an alternative methodology due to the lack of real data concerning aircraft's velocity. This alternative methodology will be extensively shown in section 5.2.7.

5.2.3 Building Cumulative Density Function (CDF) for aircraft's position

A random variable consists of an experiment **Ω** with a probability measure defined on a sample space and a function that assign a number to each outcome in the sample space of the experiment **Ω**, see [46]. In this section the CDF of the two dimensional random variable aircraft's position is built as histogram diagram of the relative frequency of occurrences.

The experiment **Ω** is defined through the following step:

ζ = measurement of aircraft position in a fixed instant of time during a selected flight for a given aircraft. To determine the vector indicating the aircraft's position, the measurements of its components are needed. To every outcome ζ of the experiment **Ω** a pair of real numbers is associated.

To clarify: the outcome ζ of the experiment Ω is a pair of real numbers $\zeta=(s_x,s_y)\in \boldsymbol{R}^2$ is defined as:

s_x is the component of the vector of the aircraft position with respect to the *x*-axis
s_y is the component of the vector of the aircraft position with respect to the *y*-axis

The two dimensional Random Variable space $\underline{s}$ is defined by:

$$\underline{s}(\zeta) \stackrel{\Delta}{=} \zeta$$

It follows that $\zeta=(s_x,s_y)$ which indicates the components of the aircraft's position has a double meaning, hence, $\zeta=(s_x,s_y)$ is both the outcome of the experiment Ω and the value of the two dimensional random variable. The two dimensional random variable $\underline{s}\subseteq\mathcal{G}$, hence, has as domain $\boldsymbol{R}^2$ and range the set $\boldsymbol{R}^2$. Aircraft's position is a two dimensional real random variable.

$$\underline{s}(\zeta)\subseteq\mathcal{G} = \boldsymbol{R}^2 \rightarrow \boldsymbol{R}^2$$

The experiment Ω is performed n times. At a given outcome ζ of the experiment, the two dimensional real random variable $\underline{s}$ associates a set of two different values $\underline{s}(\zeta)$. Given a set of two real numbers (s_x, s_y) and denoting by $n(s_x, s_y)$ the total number of trials such that:

$$\underline{s}\left(s_x(\zeta)\le s_x, s_y(\zeta)\le s_y\right)$$

the cumulative density function (CDF) of the two dimensional real random variable aircraft's position $\underline{s}\subseteq\mathcal{G}$ is possible to be determined by applying the *Law of Large numbers*.

Throughout this dissertation, PDFs and CDFs are computed as histograms of the relative frequency of observing a given event. These histograms are considered equivalent to PDFs or CDFs by applying the *Law of large numbers* [47] [48]:

If the probability of an event a in a given experiment $\boldsymbol{\Omega}$ *equals p and the experiment is repeated n times, then for any* $\varepsilon>0$

$$\lim_{n\to\infty} \Pr\left\{\left|\frac{n_a}{n} - p\right| \le \varepsilon\right\} = 1$$

where n_a *equals the number of success of a.*

this concept may also be expressed as:

If the experiment $\boldsymbol{\Omega}$ *is repeated n times and the event a occurs* n_a *times, then, with a high degree of certainty, the relative frequency* n_a/n *of the occurrence of a is close to* $p=Pr(a)$,

$$\Pr(a) \cong \frac{n_a}{n}$$

provided n is sufficiently large.

The joint cumulative distribution function of the two dimensional real random variable $\underline{s} \subseteq \mathcal{G}$ is defined by:

$$F_{\underline{s}}(\mathrm{s_x},\mathrm{s_y}) \stackrel{\Delta}{=} \Pr\{\mathbf{s_x} \le s_x, \mathbf{s_y} \le s_y\}$$

The joint cumulative distribution function can be computed as ratio between the number of trials such as $\underline{s}(\mathbf{s_x}(\zeta) \le \mathrm{s_x}, \mathbf{s_y}(\zeta) \le \mathrm{s_y})$ and the total number of trials n.

$$F_{\underline{s}}(\mathrm{s_x},\mathrm{s_y}) \cong \frac{n(s_x, s_y)}{n}$$

This is an important result because the CDF can be built using real data. However, in this thesis aircraft's position CDF will be built using an alternative methodology due to the lack of real data concerning aircraft's position. This alternative methodology will be extensively shown in section 6.2.2.

5.2.4 Velocity and aircraft position as stochastic processes

In sections 5.2.1, 5.2.2 and 5.2.3, velocity and aircraft's position are defined as two dimensional random variables. In this section they will be defined as stochastic processes. The random variable velocity is defined by the experiment **Ω**. The outcome ζ, of the experiment **Ω** is here given:

ζ = measurement of aircraft velocity in a fixed instant of time during a selected flight for a given aircraft.

A two dimensional real random variable $\underline{\mathbf{v}}(\zeta)\subseteq\mathcal{V}$ is defined for a given instant of time. If $t_1,t_2,t_3,\ldots t_n$ $(t_1,t_2,t_3,\ldots t_n)\in \boldsymbol{T}^n$ are instants of time at which the experiment **Ω** is performed, the outcome of **Ω** will be the set of values $\{\underline{\mathbf{v}}(\zeta_1,t_1),\underline{\mathbf{v}}(\zeta_2,t_2), \underline{\mathbf{v}}(\zeta_3,t_3),\ldots\underline{\mathbf{v}}(\zeta_n,t_n)\}$. Therefore, a function of two variables t and ζ is defined. For a fixed t, the two dimensional random variable speed $\underline{\mathbf{v}}(\zeta)\subseteq\mathcal{V}$ is obtained. This indicates the probability of occurrence of a certain speed. For a specific ζ, the time function v(t) indicating the time-record of the speed during the flight is obtained. Hence, the family of random variables $\underline{\mathbf{v}}(\zeta,t)\subseteq\mathcal{V}x\boldsymbol{T}$ is defined to be a stochastic process.

Aircraft's position is defined as stochastic process. The experiment **Ω**, specified by the outcome ζ, is here given:

ζ = measurement of aircraft's position in a fixed instant of time during a selected flight for a given aircraft.

A two dimensional real random variable $\underline{\mathbf{s}}(\zeta)\subseteq\mathcal{G}$ is defined for a given instant of time. If $t_1,t_2,t_3,\ldots t_n$ $(t_1,t_2,t_3,\ldots t_n)\in \boldsymbol{T}^n$ are instants of time at which the experiment **Ω** is performed, the outcome of **Ω** will be the set of values $\{\underline{\mathbf{s}}(\zeta_1,t_1),\underline{\mathbf{s}}(\zeta_2,t_2), \underline{\mathbf{s}}(\zeta_3,t_3),\ldots\underline{\mathbf{s}}(\zeta_n,t_n)\}$. Hence, a function of two variables t and ζ is defined. For a fixed t, the two dimensional random variable position of the aircraft $\underline{\mathbf{s}}(\zeta)\subseteq\mathcal{G}$ is obtained. This indicates the probability of being the aircraft in a given position. For a specific ζ, the time function s(t) indicating the time-record of the position of the aircraft during the flight is obtained. Hence, the family of random variables $\underline{s}(\zeta,t)\subseteq\mathcal{G}x\boldsymbol{T}$ is defined to be a stochastic process.

5.2.5 Velocity as a stationary stochastic process. Aircraft's position as a non-stationary stochastic process

An aircraft which goes from the point $\mathbf{A}(A_x,A_y)\in\boldsymbol{R}^2$ to the point $\mathbf{B}(B_x,B_y)\in\boldsymbol{R}^2$ crossing the point $\mathbf{P}(P_x,P_y)\in\boldsymbol{R}^2$ is given. Let $\underline{v}(\zeta;t)\subseteq\mathcal{L}x\boldsymbol{T}$ be the stochastic process velocity of this aircraft. A set of axes along and cross-track direction of the nominal intended route of the aircraft is considered as shown in Figure 5.1:

- the axis τ, whose unit vector is labelled with $\underline{\tau}$, is along-track direction of the nominal intended route of the aircraft
- the axis n, whose unit vector is labelled with $\underline{n}$, is cross-track direction of the nominal intended route of the aircraft

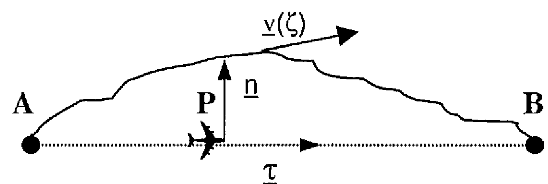


Figure 5.1: Set of axes along and cross-track direction of the nominal intended route of the aircraft.

The marginal PDFs of the two dimensional random variable velocity $\underline{v}(\zeta;t)\subseteq\mathcal{L}x\boldsymbol{T}$ along and across track are defined as:

$$\Pr\left\{\begin{matrix}\text{aircraft}\\ \text{has velocity}\\ v_\tau(t)\end{matrix}\right\}=\Pr\{v_\tau(t)<\underline{v}(t)\cdot\underline{\tau}(t)\le v_\tau(t)+dv_\tau(t)\}=\int_{-\infty}^{+\infty}f_{\underline{v}(v_\tau,v_n)}(v_\tau,v_n;t)dv_n$$

$$\Pr\left\{\begin{matrix}\text{aircraft}\\ \text{has velocity}\\ v_n(t)\end{matrix}\right\}=\Pr\{v_n(t)<\underline{v}(t)\cdot\underline{n}(t)\le v_n(t)+dv_n(t)\}=\int_{-\infty}^{+\infty}f_{\underline{v}(v_\tau,v_n)}(v_\tau,v_n;t)dv_\tau$$

Without considering taking off and landing time (during climbing and descent phase speed is affected by substantial changes), the hypothesis that the speed is time dependent is not necessary. For a given instant of time during aircraft cruise, a velocity distribution indicating the probability of occurrences of a given speed can be computed. Therefore, the stochastic process velocity $\underline{\mathbf{v}}(\zeta,t) \subseteq \mathcal{F}xT$ is assumed to be stationary.

$$\underline{\mathbf{v}}(\zeta;t) = \underline{\mathbf{v}}(\zeta) \quad \forall t \in \boldsymbol{T} \Leftrightarrow \quad \begin{cases} \mathbf{v}_\tau(\zeta;t) = \mathbf{v}_\tau(\zeta) \\ \mathbf{v}_n(\zeta;t) = \mathbf{v}_n(\zeta) \end{cases} \quad \forall t \in \boldsymbol{T}$$

Hypothesising the stochastic process $\underline{\mathbf{v}}(\zeta;t) \subseteq \mathcal{F}xT$ being stationary $\underline{\mathbf{v}}(\zeta) \subseteq \mathcal{F}$, the PDFs, with each of them being calculated for different instant times, are assumed to be identical (see Figure5.2).

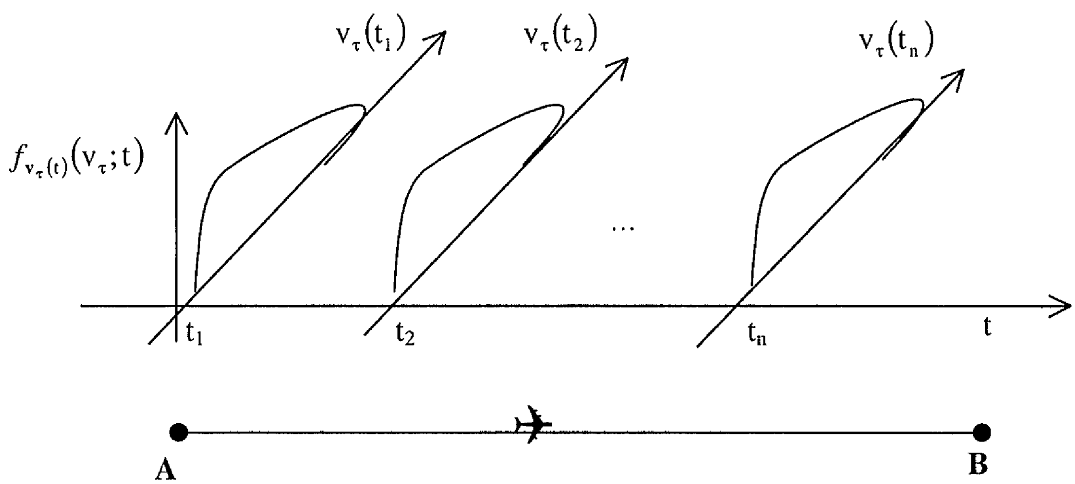


Figure 5.2: The PDF of the velocity along the track $v_\tau \subseteq \mathcal{F}xT$ is a stationary stochastic process.

Similarly, the marginal PDFs along and cross the track for the two dimensional random variable position $\underline{s}(\zeta;t) \subseteq \mathcal{G}xT$ are:

$$\Pr\left\{\begin{matrix}\text{aircraft} \\ \text{is placed in} \\ s_\tau(t)\end{matrix}\right\} = \Pr\{s_\tau(t) < \underline{s}(t)\cdot\underline{\tau}(t) \le s_\tau(t) + ds_\tau(t)\} = \int_{-\infty}^{+\infty} f_{\underline{s}(s_\tau,s_n)}(s_\tau,s_n;t)ds_n$$

$$\Pr\left\{\begin{matrix}\text{aircraft} \\ \text{is placed in} \\ s_n(t)\end{matrix}\right\} = \Pr\{s_n(t) < \underline{s}(t)\cdot\underline{n}(t) \le s_n(t) + ds_n(t)\} = \int_{-\infty}^{+\infty} f_{\underline{s}(s_\tau,s_n)}(s_\tau,s_n;t)ds_\tau$$

For a given instant of time during aircraft cruise, a distribution indicating the probability that the aircraft is placed in a given position can be computed. At the beginning of the journey the aircraft is likely to be close to the departure airport. At the end of the journey the aircraft will be likely to be close to the destination airport. Hence, the space is a no stationary stochastic process. In section 6.2 the nature of this parameter will be extensively clarified.

5.2.6 Theoretical tactical planning

Given a cartesian co-ordinate axes XY, an aircraft going from the point $\mathbf{A}(A_x,A_y)\in \boldsymbol{R}^2$ to the point $\mathbf{B}(B_x,B_y)\in \boldsymbol{R}^2$ crossing the point $\mathbf{P}(P_x,P_y)\in \boldsymbol{R}^2$ is considered.

Let $\underline{s}(\mathbf{P};t)\subseteq \mathcal{G}\text{x}\boldsymbol{T}$ be the stochastic process position of this aircraft, let $\mathbf{P}^*(P_x,P_y)\in \boldsymbol{R}^2$ be a point of the space and let $t^*\in \boldsymbol{T}$ be an instant of time. The probability that the aircraft is placed in $\mathbf{P}^*$ at the time t^* is:

$$\Pr\left\{\begin{array}{c}\text{aircraft}\\ \text{is in } \mathbf{P}^*\\ \text{at the instant}\\ \text{time } t^*\end{array}\right\} = \Pr\left\{\underline{s}(\mathbf{P}^*;t^*) < \underline{s}(\mathbf{P}^*;t^*) \le \underline{s}(\mathbf{P}^*;t^*) + d\underline{s}\right\} =$$

$$= \Pr\left\{\underline{s}(P_x^*;t^*) < \underline{s}(P_x^*;t^*) \le \underline{s}(P_x^*;t^*) + dP_x, \underline{s}(P_y^*;t^*) < \underline{s}(P_y^*;t^*) \le \underline{s}(P_y^*;t^*) + dP_y\right\} =$$

$$= f_{\underline{s}}(\mathbf{P}^*;t^*)d\mathbf{P}$$

Given two instances time $t_1\in \boldsymbol{T}$ and $t_2\in \boldsymbol{T}$, the following two random variables are considered:

$$\underline{s}(\mathbf{P};t_1)\subseteq \mathcal{G}\text{x}\boldsymbol{T} \qquad \text{and} \qquad \underline{s}(\mathbf{P};t_2)\subseteq \mathcal{G}\text{x}\boldsymbol{T}$$

The joint distribution depends on $t_1\in \boldsymbol{T}$ and $t_2\in \boldsymbol{T}$ and is denoted by:

$$F(\underline{s}_1,\underline{s}_2;t_1,t_2) = \Pr\left\{\underline{s}(\mathbf{P};t_1) \le \underline{s}(\mathbf{P};t_1), \underline{s}(\mathbf{P};t_2) \le \underline{s}(\mathbf{P};t_2)\right\}$$

This is called second-order distribution of the process $\underline{s}(\mathbf{P};t)\subseteq\mathcal{G}x\boldsymbol{T}$. The corresponding density is given by:

$$f(\underline{s}_1,\underline{s}_2;t_1,t_2)=\frac{\partial^2 F(\underline{s}_1,\underline{s}_2;t_1,t_2)}{\partial\underline{s}_1\partial\underline{s}_2}$$

the following properties are derived:

$$\lim_{s_2\to\infty} F(\underline{s}_1,\underline{s}_2;t_1,t_2)=F(\underline{s}_1;t_1)$$

$$f(\underline{s}_1;t_1)=\int_{-\infty}^{+\infty} f(\underline{s}_1,\underline{s}_2;t_1,t_2)ds_2$$

The conditional density is given by:

$$f(\underline{s}_1,t_1\mid\underline{s}_2(t_2)=s_2)=\frac{f(\underline{s}_1,\underline{s}_2;t_1,t_2)}{f(\underline{s}_2;t_2)}$$

Given the points $\mathbf{P}_1(P_{x1},P_{y1})\in\boldsymbol{R}^2$, $\mathbf{P}_2(P_{x2},P_{y2})\in\boldsymbol{R}^2$ and two instant of times $t_1\in\boldsymbol{T}$, $t_2\in\boldsymbol{T}$ with $t_1<t_2$; the probability the aircraft is in $\mathbf{P}_1(P_{x1},P_{y1})\in\boldsymbol{R}^2$ at the time $t_1\in\boldsymbol{T}$ and in $\mathbf{P}_2(P_{x2},P_{y2})\in\boldsymbol{R}^2$ at the time $t_2\in\boldsymbol{T}$ is (see Figure 5.3):

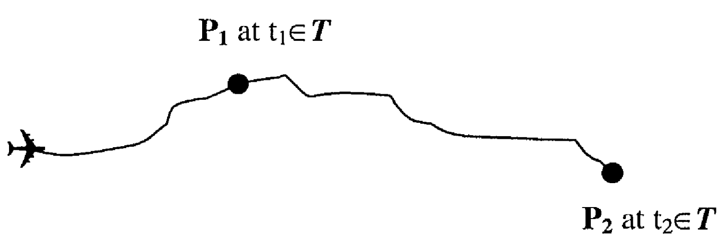


Figure 5.3: aircraft is in $\mathbf{P}_1(P_{x1},P_{y1})\in\boldsymbol{R}^2$ at $t_1\in\boldsymbol{T}$ and in $\mathbf{P}_2(P_{x2},P_{y2})\in\boldsymbol{R}^2$ at $t_2\in\boldsymbol{T}$.

$$\Pr\left\{\begin{array}{c}\text{aircraft is in }\mathbf{P}_1\\ \text{at the instant } t_1\\ \text{and in }\mathbf{P}_2\\ \text{at the instant } t_2\end{array}\right\}=$$

$$=\Pr\{\underline{s}(\mathbf{P}_1;t_1)<\underline{s}(\mathbf{P}_1;t_1)\le\underline{s}(\mathbf{P}_1;t_1)+d\mathbf{P},\underline{s}(\mathbf{P}_2;t_2)<\underline{s}(\mathbf{P}_2;t_2)\le\underline{s}(\mathbf{P}_2;t_2)+d\mathbf{P}\}=$$

$$=f_{\underline{s}}(\mathbf{P}_1,\mathbf{P}_2;t_1,t_2)d\mathbf{P}_1d\mathbf{P}_2$$

The probability the aircraft is in $\mathbf{P}_2(P_{x2},P_{y2})\in \boldsymbol{R}^2$ at the time $t_2\in \boldsymbol{T}$ once it is in $\mathbf{P}_1(P_{x1},P_{y1})\in \boldsymbol{R}^2$ at the time $t_1\in \boldsymbol{T}$ (see Figure 5.4)

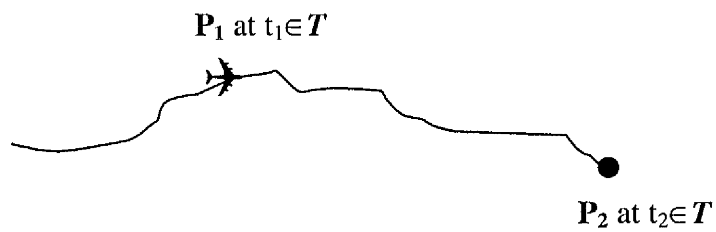


Figure 5.4: Aircraft is in $\mathbf{P}_2(P_{x2},P_{y2})\in \boldsymbol{R}^2$ at $t_2\in \boldsymbol{T}$ once it is in $\mathbf{P}_1(P_{x1},P_{y1})\in \boldsymbol{R}^2$ at $t_1\in \boldsymbol{T}$.

$$\Pr\left\{\begin{array}{c}\text{aircraft is in } \mathbf{P}_2\\ \text{at the instant } t_2\\ \text{once it is in } \mathbf{P}_1\\ \text{at the instant } t_1\end{array}\right\} =$$

$$= \Pr\{\underline{s}(\mathbf{P}_2;t_2) < \underline{s}(\mathbf{P}_2;t_2) \le \underline{s}(\mathbf{P}_2;t_2) + d\mathbf{P} \mid \underline{s}(\mathbf{P}_1;t_1) < \underline{s}(\mathbf{P}_1;t_1) \le \underline{s}(\mathbf{P}_1;t_1) + d\mathbf{P}\} =$$

$$= f(\underline{s}_2, t_2 \mid \underline{s}_1(t_1) = \underline{s}_1) = \frac{f(\underline{s}_1, \underline{s}_2; t_1, t_2)}{f(\underline{s}_1; t_1)}$$

5.2.7 Considerations to build the PDF of the along track velocity

Due to the lack of data regarding the experiments defining the random variable velocity $\underline{v}(\zeta)\subseteq\mathcal{S}$, the PDF of the velocity along the track is obtained through a transformation of variables from related experiments from which data are available.

Considering the work described [52], the PDF of the random variable travelling time will be transformed into the PDF of the velocity along the track assuming certain simplifying hypothesis. The PDF of the travelling time is obtained using the data provided by the Civil Aviation Authority. An example of this data is shown in Table 3.1. The set of data provide only the scheduled and real arrival and departure times for every flight landing and taking off from London Heathrow Airport London and Glasgow International Airport.

The departure and arrival airports are supposed to be placed in **A** and **B**. For a given journey, a travelling time $t^{t}_{AB}(\zeta) \in \boldsymbol{R}$ is observed. The temporal mean of the velocity $\overline{v_\tau}(\zeta)$ is computed through the following equation:

$$\overline{v_\tau}(\zeta) = \frac{d(\mathbf{A},\mathbf{B})}{t^{t}_{AB}(\zeta)} \qquad \begin{cases} d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ t^{t}_{AB}(\zeta) \in \boldsymbol{R} \\ \overline{v_\tau}(\zeta) \in \boldsymbol{R} \end{cases}$$

For a given journey, the temporal mean of the velocity $\overline{v_\tau}(\zeta)$ is observed (see Figure 5.5).

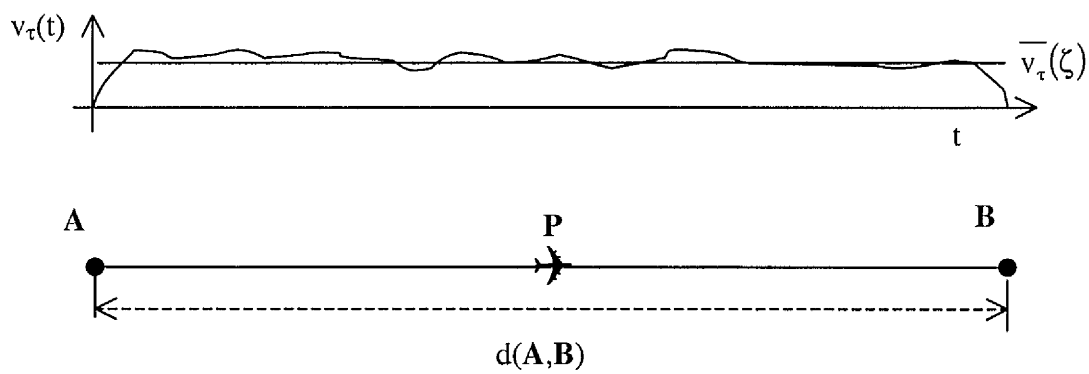


Figure 5.5: For a given journey, the temporal mean of the velocity is observed.

Observing this temporal mean n times a random variable is built that represents the distribution of the mean of the velocity.

$$\overline{\mathbf{v}_\tau} = g\left(\mathbf{t}^{t}_{AB}\right) = \frac{d(\mathbf{A},\mathbf{B})}{\mathbf{t}^{t}_{AB}} \qquad \begin{cases} d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{t}_{AB} \subseteq \mathcal{E} \\ \overline{\mathbf{v}_\tau} \subseteq \mathcal{F} \end{cases}$$

If the stationary stochastic process $\mathbf{v}_\tau \subseteq \mathcal{F}$ is erogodic in the mean, the temporal mean is equal to the expected value of the random variable[†] $\mathbf{v}_\tau \subseteq \mathcal{F}$.

[†] The stationary stochastic process $\mathbf{v}_\tau$ becomes a random variable when an instant of time is fixed. See section 5.2.4.

$$\overline{\mathrm{v}_\tau}(\zeta) = E\{\mathbf{v}_\tau\} \qquad \begin{cases} \overline{\mathrm{v}_\tau}(\zeta) \in \boldsymbol{R} \\ E\{\mathbf{v}_\tau\} \in \boldsymbol{R} \end{cases}$$

The equality written above is between two mere real numbers. In section 5.2.8 this equality will be proven even for the random variables $\overline{\mathbf{v}_\tau}$ and $\mathbf{v}_\tau$. The reader not interested in this proof can skip the next section.

5.2.8 Equivalence between $\overline{\mathrm{v}_\tau}$ and v_τ

Let v_τ be a real number and let **z** be a Bernoulli random variable such as:

$$\mathbf{z}(\mathrm{v}_\tau) = \begin{cases} 1 \quad \text{if} \quad \mathrm{v}_\tau \le \mathbf{v}_\tau < \mathrm{v}_\tau + \mathrm{dv}_\tau \\ \qquad 0 \quad \text{otherwise} \end{cases}$$

The expected value of the Bernoulli random variable **z** is computed:

$$\mathrm{E}\{\mathbf{z}(\mathrm{v}_\tau)\} = 1 \cdot \Pr\{\mathbf{z}(\mathrm{v}_\tau) = 1\} = \Pr\{\mathrm{v}_\tau \le \mathbf{v}_\tau < \mathrm{v}_\tau + \mathrm{dv}_\tau\}$$

Clearly if $\mathbf{v}_\tau$ is ergodic then **z** is also ergodic. The temporal mean of **z** is equal to the expected value of **z**

$$\mathrm{E}\{\mathbf{z}(\mathrm{v}_\tau)\} = \overline{\mathbf{z}(\mathrm{v}_\tau)}$$

as shown before $\mathrm{E}\{\mathbf{z}(\mathrm{v}_\tau)\} = \Pr\{\mathrm{v}_\tau \le \mathbf{v}_\tau < \mathrm{v}_\tau + \mathrm{dv}_\tau\}$

Thus the following equivalence is given

$$\overline{\mathbf{z}(\mathrm{v}_\tau)} = \Pr\{\mathrm{v}_\tau \le \mathbf{v}_\tau < \mathrm{v}_\tau + \mathrm{dv}_\tau\}$$

Clearly if $\overline{\mathbf{z}(\mathrm{v}_\tau)} = \Pr\{\mathrm{v}_\tau \le \mathbf{v}_\tau < \mathrm{v}_\tau + \mathrm{dv}_\tau\}$ is true $\forall \mathrm{v}_\tau \in \boldsymbol{R}$, then $\overline{\mathbf{v}_\tau} = \mathbf{v}_\tau$

5.2.9 Continuation of the considerations to build the PDF of the velocity along the track

In section 5.2.8, the equality $\overline{\mathbf{v}_\tau} = \mathbf{v}_\tau$ was proven under the hypothesis that $\mathbf{v}_\tau$ is a stationary stochastic process ergodic in the mean. The real random variable $\mathbf{v}_\tau \subseteq \mathcal{F}$ is not directly defined for each experimental outcome. Alternatively $\mathbf{v}_\tau$ is defined indirectly via the real random variable $\mathbf{t}^t_{AB} \subseteq \mathcal{E}$ and the function $g(\mathbf{t}^t_{AB})$.

$$\mathbf{v}_\tau = g\left(\mathbf{t}^t_{AB}\right) = \frac{d(\mathbf{A},\mathbf{B})}{\mathbf{t}^t_{AB}} \qquad \begin{cases} d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^t_{AB} \subseteq \mathcal{E} \\ \mathbf{v}_\tau \subseteq \mathcal{F} \end{cases}$$

Similarly for avoiding any preference to the random variable $\mathbf{t}^t_{AB} \subseteq \mathcal{E}$; the following ratio is derived:

$$\mathbf{t}^t_{AB} = g^{-1}(\mathbf{v}_\tau) = \frac{d(\mathbf{A},\mathbf{B})}{\mathbf{v}_\tau} \qquad \begin{cases} d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^t_{AB} \subseteq \mathcal{E} \\ \overline{\mathbf{v}_\tau} \subseteq \mathcal{F} \end{cases}$$

Two different approaches are possible to consider:

1 building $\mathbf{t}^t_{AB} \subseteq \mathcal{E}$ and determine $\mathbf{v}_\tau \subseteq \mathcal{F}$ throughout $g(\mathbf{t}^t_{AB})$

2 building $\mathbf{v}_\tau \subseteq \mathcal{F}$ and determine $\mathbf{t}^t_{AB} \subseteq \mathcal{E}$ throughout $g^{-1}(\mathbf{v}_\tau)$

Since the outcomes $\zeta_1, \zeta_2, \ldots \zeta_n$ of the experiment $\boldsymbol{\Omega}$ for $\mathbf{t}^t_{AB}$ are easier to observe, the first approach will be followed.

The *Fundamental Theorem of transformation of variables* [48] is applied to transform the PDF of the travelling time of the route AB $f_{\mathbf{t}^t_{AB}}\left(t^t_{AB}\right)$ in the PDF of the velocity along the track $f_{\mathbf{v}_\tau}(v_\tau)$

Finally, the PDF of $\mathbf{v}_\tau \subseteq \mathcal{F}$ is given by:

$$f_{v_\tau}(v_\tau) = \frac{|d(\mathbf{A},\mathbf{B})|}{(v_\tau)^2} f_{t^{\iota}_{AB}}\left(\frac{d(\mathbf{A},\mathbf{B})}{v_\tau}\right)$$

The equation $v_\tau = \dfrac{d(\mathbf{A},\mathbf{B})}{t^{\iota}_{AB}}$ has a single solution $t^{\iota}_{AB} = \dfrac{d(\mathbf{A},\mathbf{B})}{v_\tau}$ for every v_τ

Since $\qquad g'(t^{\iota}_{AB}) = -\dfrac{d(\mathbf{A},\mathbf{B})}{(t^{\iota}_{AB})^2} = -\dfrac{(v_\tau)^2}{d(\mathbf{A},\mathbf{B})}$

5.2.10 Applied tactical planning based on a dynamic probabilistic GHP

The velocity along the track $\mathbf{v}_\tau(\zeta) \subseteq \mathcal{F}$ is computed through the ratio between the travelling time $\mathbf{t}^{\iota}_{AB} \subseteq \mathcal{E}$ of the route **AB** and the euclidean† distance $d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R}$ between the point **A** and the point **B**

$$\mathbf{v}_\tau = \frac{d(\mathbf{A},\mathbf{B})}{\mathbf{t}^{\iota}_{AB}} \qquad \begin{cases} \mathbf{v}_\tau \subseteq \mathcal{F} \\ d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{\iota}_{AB} \subseteq \mathcal{E} \end{cases}$$

The travelling time $\mathbf{t}^{\iota}_{PB} \subseteq \mathcal{E}$ of the aircraft for the route **PB** is:

$$\mathbf{t}^{\iota}_{PB} = \frac{d(\mathbf{P},\mathbf{B})}{\mathbf{v}_\tau} \qquad \begin{cases} d(\mathbf{P},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{v}_\tau \subseteq \mathcal{F} \\ \mathbf{t}^{\iota}_{PB} \subseteq \mathcal{E} \end{cases}$$

Substituting the velocity $\mathbf{v}_\tau(\zeta) \subseteq \mathcal{F}$, the travelling time $\mathbf{t}^{\iota}_{PB} \subseteq \mathcal{E}$ of the route **PB** can be expressed as travelling time $\mathbf{t}^{\iota}_{AB} \subseteq \mathcal{E}$ of the route **AB**

$$\mathbf{t}^{\iota}_{PB} = \frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{\iota}_{AB} \qquad \begin{cases} d(\mathbf{P},\mathbf{B}) \in \boldsymbol{R},\ d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{\iota}_{AB} \subseteq \mathcal{E} \\ \mathbf{t}^{\iota}_{PB} \subseteq \mathcal{E} \end{cases}$$

† The euclidean distance can be substituted with the geodetic distance for long haul flights.

The arrival time $\mathbf{t}^{a}_{PB} \subseteq \mathcal{B}$ in **B** once the aircraft has departed from **P** is:

$$\mathbf{t}^{a}_{PB} = \mathbf{t}^{d}_{PB} + \mathbf{t}^{t}_{PB} =$$

$$= \mathbf{t}^{d}_{PB} + \frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{t}_{AB} \qquad \begin{cases} d(\mathbf{P},\mathbf{B}) \in \boldsymbol{R} \\ d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{t}_{AB} \subseteq \mathcal{E} \\ \mathbf{t}^{d}_{PB} \subseteq \mathcal{A} \end{cases}$$

If the departure time from **P** is known, the random variable $\mathbf{t}^{d}_{PB} \subseteq \mathcal{A}$ simplifies to a mere scalar $t^{d}_{PB}(\zeta) \in \boldsymbol{R}$. In a real implementation of this methodology, the departure time from **P** can be known through a data-link message between aircraft and ground.

$$\mathbf{t}^{a}_{PB} = t^{d}_{PB}(\zeta) + \frac{d(\mathbf{P},\mathbf{B})}{d(\mathbf{A},\mathbf{B})}\mathbf{t}^{t}_{AB} \qquad \begin{cases} \mathbf{t}^{a}_{PB} \subseteq \mathcal{B} \\ t^{d}_{PB} \in \boldsymbol{R} \\ d(\mathbf{P},\mathbf{B}) \in \boldsymbol{R} \\ d(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{t}_{AB} \subseteq \mathcal{E} \end{cases}$$

This formula solves the arrival time in the destination airport **B** once the departure time of the aircraft in **P** is known. Clearly, the departure time from **P** is equal to the arrival time at **P** $t^{d}_{PB}(\zeta) = t^{a}_{PB}(\zeta)$ as the aircraft is flying, see Figure 5.6.

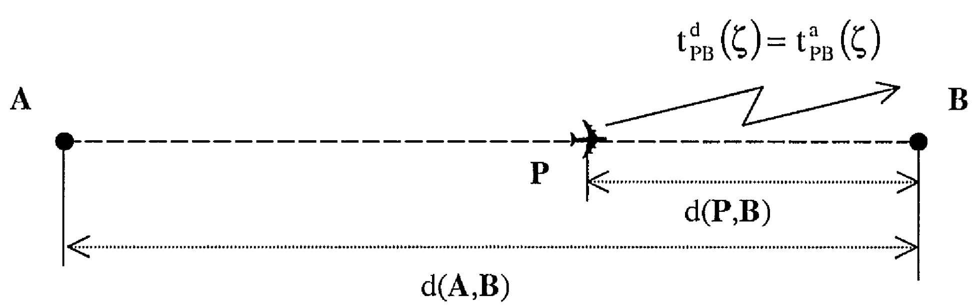


Figure 5.6: The departure time from **P** is supposed to be known with a data-link message.

The parameters involved are the eucledian distances d(**P**,**B**)∈$\boldsymbol{R}$, d(**A**,**B**)∈$\boldsymbol{R}$, the real random variable travelling time $\mathbf{t}^{t}_{AB} \subseteq \mathcal{E}$ and the departure time from **P** $t^{d}_{PB} \in \boldsymbol{R}$.

Applying the *Fundamental Theorem of transformation of variables* [48], the PDF of the arrival time at the destination airport **B** $\mathbf{t}^{a}_{PB} \subseteq \mathcal{B}$ is derived by:

$$f_{t^{a}_{PB}}\left(t^{a}_{PB}\right)=\frac{|d(\mathbf{A},\mathbf{B})|}{|d(\mathbf{P},\mathbf{B})|}f_{t^{t}_{AB}}\left(\left(t^{a}_{PB}-t^{d}_{PB}\right)\frac{d(\mathbf{A},\mathbf{B})}{d(\mathbf{P},\mathbf{B})}\right)$$

This is an important result because the PDF $f_{t^{a}_{PB}}\left(t^{a}_{PB}\right)$ of the arrival time in **B** is given using only:

- the PDF of the travelling time of the route **AB** $\mathbf{t}^{t}_{AB} \subseteq \mathcal{E}$ which is built using real data provided by CAA.
- the departure time from **P**, $t^{d}_{PB} \in \boldsymbol{R}$ which is supposed to be known through a data-link message.

5.3 The Decision Support Tool for the tactical schedule

In this section an application for Glasgow International Airport is shown. Supposing the arrival time at a given point **P** of the route is known, the new Estimated Time of Arrival (ETA)[†] at the destination airport placed in **B** is computed. Then a new schedule is given according to the ground hold strategy.

ADS-C (Contract) technology can be used to communicate the necessary data. An ADS Contract is an ADS reporting plan which establishes the conditions of ADS data reporting (i.e., data required by the ground system and frequency of ADS reports). An ATC Ground System establishes an ADS-Contract with an aircraft to obtain data relating to its position. The aircraft responds automatically and periodically with report contents and rate depending on the request.

There are three possible types of contract possibly established between the ground ATC system and an ADS-C equipped aircraft:

[†] To maintain the current aeronautical notation, ETA is introduced as a fixed time. Because of the probabilistic approach of this work, ETA is the distribution of the arrival time (PDF).

- Periodic: The aircraft transmits its position reports at a regular rate.

- Event: The aircraft transmits its position reports (for example an altitude or speed change) independently of any periodic contract in effect.

- On demand: The aircraft transmits a single position report when requested by the ground ATC system.

According to these definitions, this decision support tool can be seen as an application of ADS-C on demand. The ground ATC requests and receives the aircraft position.

Nevertheless, if the program is run several times, it will work as an ADS-C periodic or ADS-B, updating the position information at a fixed frequency.

The program uses only one reporting point whose spatial co-ordinates are chosen by users. This choice takes into account that it is a compromise between accuracy in ETA and time available to take actions. When the aircraft is close to the destination airport, the arrival time can be predicted with more certainty. However air traffic controllers will have less time and the resulting schedule will be less efficient in terms of safety and expenses. In these circumstances, it would be too late to delay the incoming aircraft on the ground at their departure airports and it would be necessary to impose airborne delays.

On the other hand, if the position is sent too early in the course of the flight, there will be less accuracy in predicting the ETA.

5.4 Study case: Glasgow International Airport

Apart from minor changes, the program layout is very similar to the code used for the Strategic Schedule, so no further explanations will be given. In this first approach, only one flight is assumed to have data link capabilities. It will be the flight with the longest range as its PDF is affected by the greater uncertainty. Furthermore it is likely that this aircraft will be equipped with a more sophisticated avionics system than the others.

To show the new methodology and in order to compare the results, the same example previous selected for the Strategic Scheduling is chosen.

The time interval refers to Glasgow International Airport and it commences at 8:00 a.m. and ends at 8:30 a.m. According to the scheduling, four flights are supposed to land. The flight coming from Toronto is supposed to be equipped with data link capability. It transmits the time when it arrives at a point located 1000 Km away from the airport. This distance is chosen because it gives already satisfying results.

In this example the only information required is the arrival time at a certain point, so a simpler technology could be used instead of ADS. The tool is very flexible and using ADS technology it is possible to modify the program to implement a real time system. Instant by instant the data concerning the position of the aircraft can be broadcast and this data can be simultaneously processed.

5.4.1 Simulation

The program has been developed in Matlab® and it has the same structure as the code designed for the Strategic Scheduling described in Chapter 4. When the Air Canada (AC) flight arrives at a point located **P**=1000 Km away from Glasgow International Airport, the aircraft transmits the time $t^{d}_{PB}(\zeta) \in \boldsymbol{R}$ (reference UTM) to the arrival airport placed in **B**. The traffic manager codes this data into the computer calculating a new ETA.

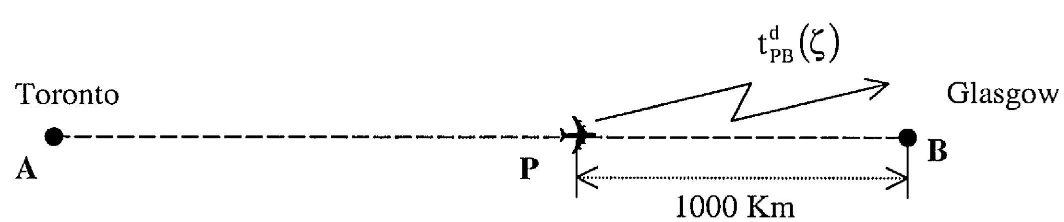


Figure 5.7: Air Canada flight transmits its position information.

If the flight arrives early at the point placed **P**=1000 Km away from the airport, it will be allocated as first in the arrival order while if it is late it will be placed as the last one.

In all the intermediate situations, depending of the amount of the delay, it will be allocated between the flights.

Through the tactical scheduling, the airport managers can have control over flights using data link capabilities. Suggestions to pilots to increase, decrease or keep the same velocity can be given. To reduce complexity, the example to be considered will not use the feed back capability. To study the affect of the data link, it is enough to run the program again simulating the new data.

Updating the information, the PDFs of the arrival time for given flights seem compressed†, as shown comparing Figure 5.8 and Figure 5.9. The uncertainty is clearly reduced so there is more accuracy in predicting the arrival time. The closer the aircraft is to the airport minor uncertainty effects ETA. The following are the PDFs for the Air Canada flight before taking off and when recalculated using the updated information from a point **P**=1000 Km away from Glasgow International Airport.

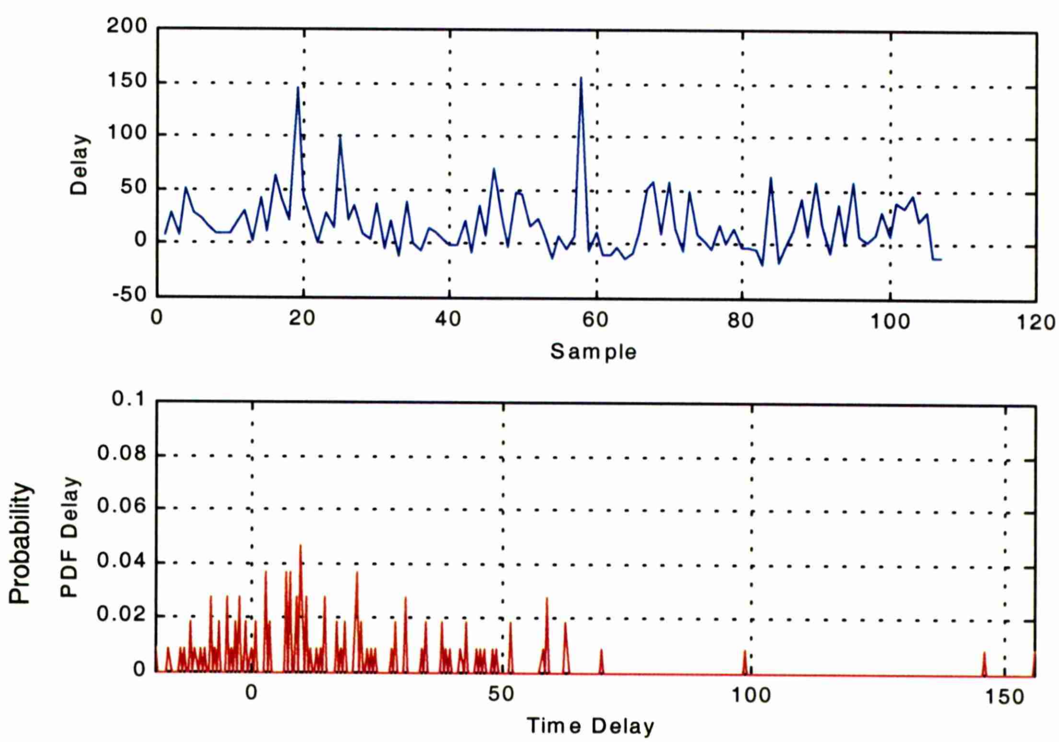


Figure 5.8: PDF of the Air Canada flight without any updated position information.

† The variance of the distribution is decreased.

The PDF shown in Figure 5.9 is referred to the scheduled arrival time of the aircraft hence it is in the form $f_{t^a_{PB}-8:00}\left(t^a_{PB}-8:00\right)$. Comparing this transformed PDF (see Figure 5.9) with the PDF used in the Strategic Scheduling (see Figure 5.8), the difference is clear.

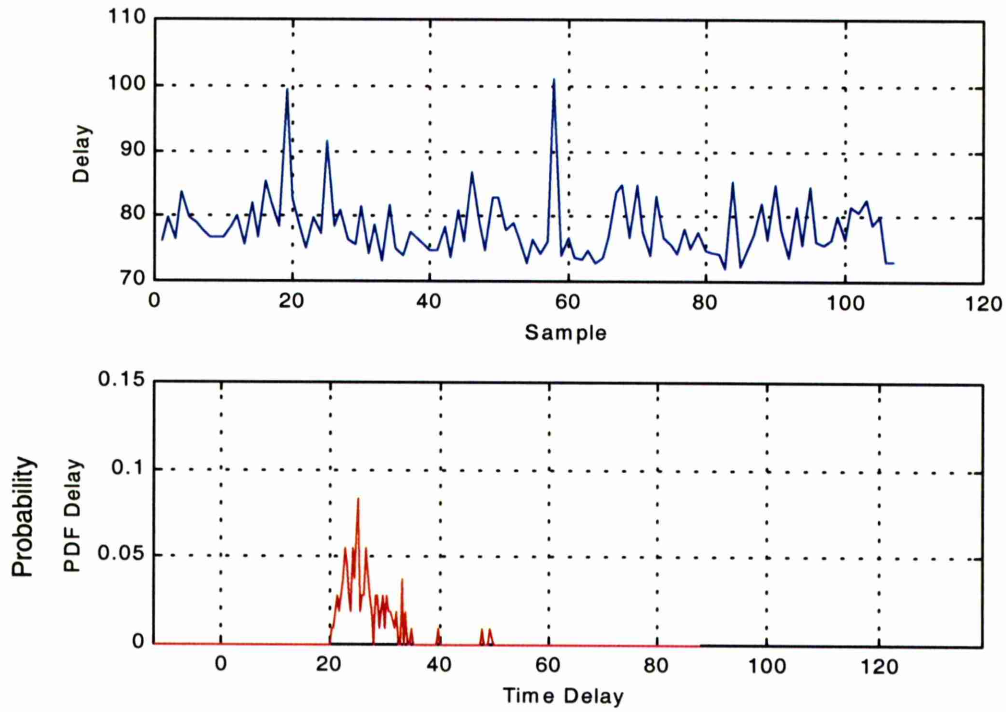


Figure 5.9: PDF of the Air Canada flight with updated information 1000 Km from the airport.

Supposing the updated information is not used then only the departure time is known. In the tactical schedule the significant improvement in determining ETA comes from knowing at what time the aircraft is at the point **P**=1000 Km away from airport. The reduction in the uncertainty of the arrival time allows the airport manager to accommodate the requests of landing more efficiently.

The following example applied to Glasgow International Airport demonstrates the validity of the mathematical theory shown in section 5.2. In this example, a reduced Interval Length (IL) will be considered while maintaining the total Probability of Conflict (TPC) at the value 35.4% (current status).

The arrival time of the aircraft without any data link capabilities will remain unchanged and they are shown in Table 4.3. Since their means are known, the most likely arrival order[†] has been computed, as displayed in Table 5.1:

Label	Flight ID	Scheduled arrival time t_s^a	Mean of the delay η (min)	Average arrival time $t_s^a+\eta$
AE	Aer Lingus 222	8:00 a.m.	-3	7:57 a.m.
EAC	European Air Charter 121	8:25 a.m.	-4	8:21 a.m.
BA	British Airways 1902	8:25 a.m.	+11	8:36 a.m.

Table 5.1: Average order and arrival time of the short haul flights.

The task now is how to allocate the landing of Air Canada flight between the others when the time of reaching the point **P**=1000 Km away from Glasgow International Airport is known. Four cases are analysed as shown in Figure 5.10:

- The aircraft comes very early and lands first, before the Aer Lingus (AE) flight.
- The aircraft is delayed. It lands last after the British Airways (BA) flight.
- The aircraft is nearly on time. To optimise efficiency it is scheduled to land between the others, in the second or in the third position.

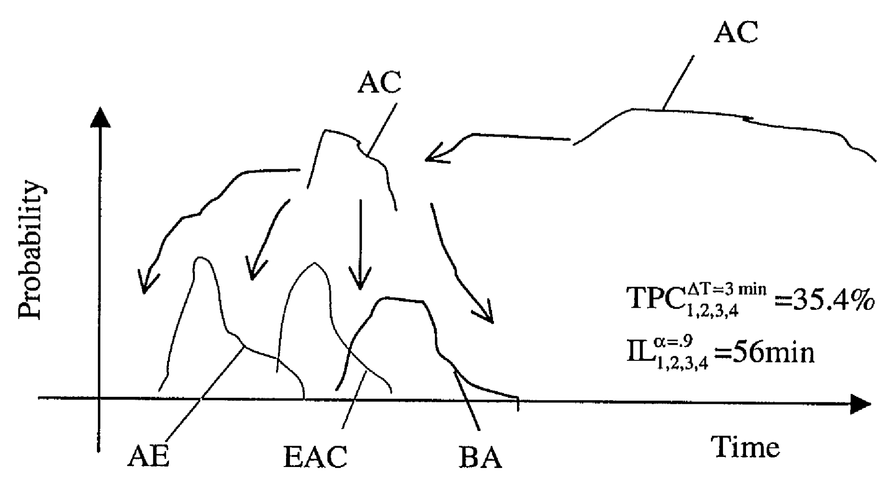


Figure 5.10: Air Canada can be placed in the first, second, third or fourth position.

[†] Note that the mean does not give the most likely arrival order but it does a rank from which is good to start.

The new input required is the time when the aircraft reaches the point located at 1000 Km before Glasgow International Airport. The new estimated time of arrival (ETA) is computed and the new order of the aircraft's arrival is then established.

To estimate the advantages of this real time system, the probability that each of the arrival order occurs is computed. For instance, it is not likely that Air Canada arrives before Aer Lingus. Although both flights are planned to land at 8:00 a.m., the mean delay of Air Canada is 20 minutes while Aer Lingus usually arrives 3 minutes earlier.

To compare the benefits, the program is run keeping the TPC around its value in the current situation. To simulate all the possible arrival orders, the time when the Air Canada flight is 1000 Km away from the airport is modified.

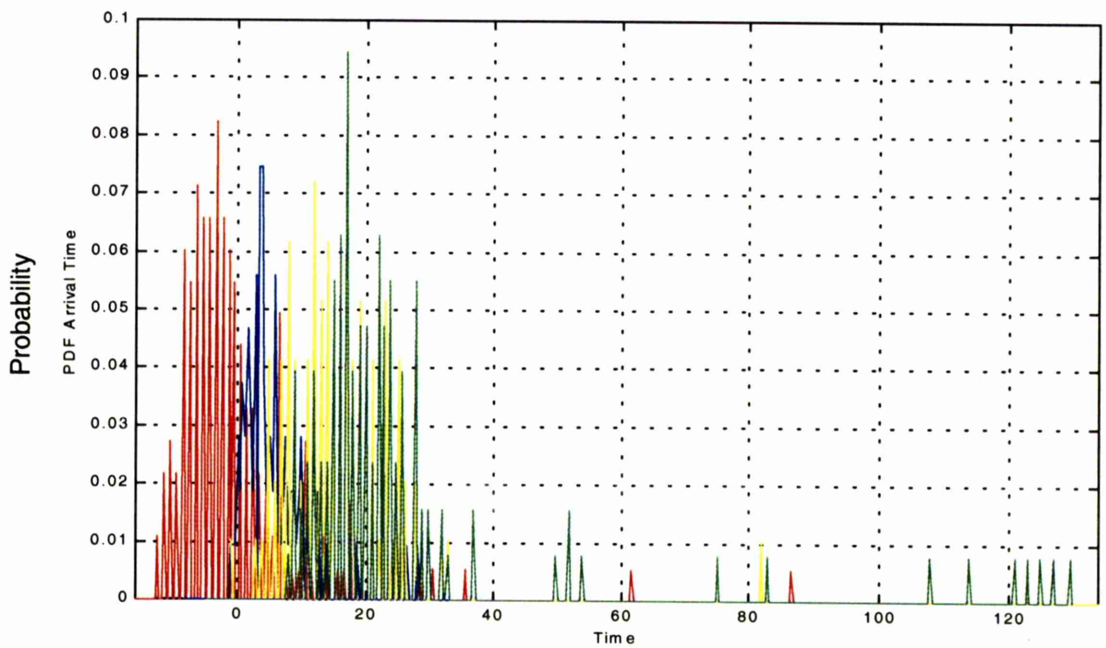


Figure 5.11: PDF of the four flights with Air Canada in the second position.

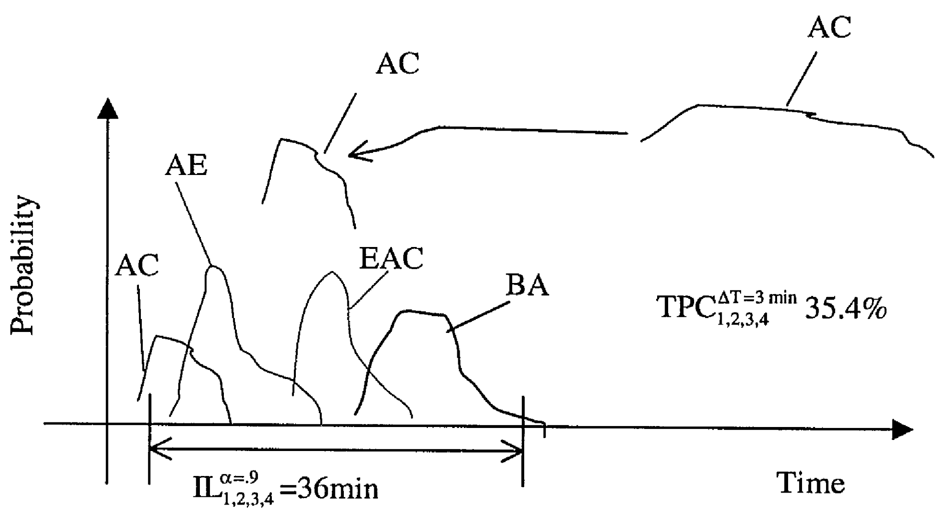
In Figure 5.11 the second PDF in the arrival order corresponds to the Air Canada flight. When the position of the aircraft is known, the reduction of the uncertainty in the arrival time is remarkable, because the Air Canada flight now has a PDF with a lower standard deviation.

Table 5.2 presents the main results obtained from the simulation:

Arrival Order	Time when AC is 1000 Km away from the airport	Outputs of the program TPC	interv. length	AC delay (min)
AC, AE, EAC, BA	$t^{d}_{PB}(\zeta) < 6:34$	35.3%	36 min	-11
AE, AC, EAC, BA	$t^{d}_{PB}(\zeta) < 6:45$	34.5%	38 min	0
AE, EAC, AC, BA	$t^{d}_{PB}(\zeta) < 6:55$	35.3%	39 min	+10
AE, EAC, BA, AC	$t^{d}_{PB}(\zeta) < 7:05$	34.9%	37 min	+20

Table 5.2: Results of the Tactical Schedule.

If the Air Canada flight arrives at the transmitting point (in this case the point **P** placed 1000 Km away from Glasgow International airport) before $t^{d}_{PB}(\zeta) < 6:34$ a.m., there is *high probability*† it will arrive early and it will be accommodate as the first in the landing order (see Figure 5.12).



Reduction by 35.7%.
This event occurs with probability equals to 25.2%

Figure 5.12: Sketch of the PDFs when Air Canada is placed in the first position.

If the Air Canada flight arrives at the transmitting point between 6:34 a.m. and 6:45 a.m., it will be the second in the arrival order having a high probability of arriving after Aer Lingus. European Air Charter will then be shifted into the third position and ground hold delay will be allocated to it.

† *High probability* is meant a given percentage previous fixed by users, in this example 90%.

These times are calculated by imposing the shifting times equal to zero in all cases. If Air Canada is placed in the first position and shifting12 = 0, Aer Lingus flight does not need to be changed. Air Canada has just arrived early enough to avoid conflict with Aer Lingus and no change in the scheduling is required. The same kind of imposition is repeated when Air Canada is accommodated in the second, third and fourth position.

According to the scheduling the Air Canada flight should arrive at Glasgow International Airport at 8:00 a.m. Supposing it keeps a constant cruise speed it should reach the transmitting point at 6:45 a.m. but unpredictable events effect its punctuality. In the last column of Table 5.2 the delay computed respected to this 6:45 a.m. is displayed.

Arrival order	**Accumulative Probability**	**Probability**
AC, AE, EAC, BA	1.9%	1.9%
AE, AC, EAC, BA	27.1%	25.2%
AE, EAC, AC, BA	52.0%	24.9%
AE, EAC, BA, AC	70.1%	18.0%

Table 5.3: Probability of each possible arrival order.

In the last column of Table 5.3 the probabilities corresponding to each arrival order are displayed. The probability, that the arrival order is "AC,AE,EAC,BA", is equal to 1.9 %, while the probability of the event "AE,AC,EAC,BA" is equal to 25.2 %. In the second column are showed the probabilities of the event union.

Hence, the mean of interval length (IL) is:

$$\text{Interval length} = 36 \cdot 0.019 + 38 \cdot 0.252 + 39 \cdot 0.505 + 37 \cdot 0.224 = \textbf{38 minutes}$$

This value has been computed using the interval lengths shown in Table 5.2 with their respective probability of occurrences shown in Table 5.3.

The relative interval length reduction respect to the Strategic schedule shown in section 4.4.2 is:

$$\text{Relative reduction}_{\text{strategic schedule}} = \frac{48-38}{48} = \mathbf{20.8\%}$$

The relative interval length reduction respect to the current situation is:

$$\text{Relative reduction}_{\text{current situation}} = \frac{56-38}{56} = \mathbf{32.1\%}$$

5.4.2 The Tactical Schedule according to the ground hold cost

In this thesis a methodology showing how to allocate ground hold delay is presented. In further works it would be possible to extend this methodology to consider airborne delay and re-routing.

Table 5.4 shows the changing arrival times given by the program in each case and the new schedule proposed according to them.

Arrival order	T_{12} (min)	T_{23} (min)	T_{34} (min)	Proposed arrival time AE	EAC	BA	AC
AC,AE,EAC,BA	0	-11	-1	8:00	8:14	8:13	7:49
AE,**AC**,EAC,BA	0	-9	-1	8:00	8:16	8:15	8:00
AE,EAC,**AC**,BA	-15	0	+7	8:00	8:10	8:17	8:10
AE,EAC,BA,**AC**	-16	+1	0	8:00	8:09	8:10	8:20

Table 5.4: The proposed arrival time in the Tactical Schedule.

In all the cases the Air Canada flight is always scheduled to land at 8:00 a.m. as it has already left its departure airport. To simplify this work only ground hold delays are allocated and no attention is given to airborne delays. As the Air Canada flight is already in the air, no ground hold delay could be imposed upon it. Note that in Table 5.4, there appear some shifting time equal to 0, as it is imposed to compute the determining conditions.

In the first case the arrival order is AC, AE, EAC, BA. The first aircraft is Air Lingus planned to arrive at 8:00 a.m.. The new schedule for BA and EAC is given by the following formulas:

European Air Charter: $8:00 + \mathit{shifting12} + \mathit{shifting23} + 25 = 8:14$

British Airways: $8:00 + \mathit{shifting12} + \mathit{shifting23} + \mathit{shifting34} + 25 = 8:13$

In the second case the arrival order is AE, AC, EAC, BA. The starting aircraft is always Air Lingus planned at 8:00 a.m., then

European Air Charter: $8:00 + \mathit{shifting12} + \mathit{shifting23} + 25 = 8:16$

British Airways: $8:00 + \mathit{shifting12} + \mathit{shifting23} + \mathit{shifting34} + 25 = 8:15$

In the third case the arrival order is AE, EAC, AC, BA. The starting aircraft is always Air Lingus planned at 8:00 a.m., then

European Air Charter: $8:00 + \mathit{shifting12} + 25 = 8:10$

British Airways: $8:00 + \mathit{shifting12} + \mathit{shifting23} + \mathit{shifting34} + 25 = 8:17$

In the next table the mean of the arrival times are shown. For instance, Aer Lingus is scheduled to arrive at 8:00 a.m. but taking into account it expected arrival time it will arrive at 7:57 a.m.

Arrival order	**T_{12} (min)**	**T_{23} (min)**	**T_{34} (min)**	**Expected arrival time**			
				AE	**EAC**	**BA**	**AC**
AC,AE,EAC,BA	0	-11	-1	7:57	8:10	8:24	7:52
AE,**AC**,EAC,BA	0	-9	-1	7:57	8:12	8:26	8:03
AE,EAC,**AC**,BA	-15	0	+7	7:57	8:06	8:28	8:13
AE,EAC,BA,**AC**	-16	+1	0	7:57	8:05	8:21	8:23

Table 5.5: The expected arrival time in the Tactical Schedule.

Through the computation of the mean, the real arrival order chosen by the operators is finally accomplished.

The previous two tables show the schedules in four different situations, depending on the arrival time of the Air Canada flight. Airport operators need to fix only one schedule. If the other aircraft are still on the ground when the position information is received, ground holds can be imposed as this is a safer and cheaper solution than the circling around the airport.

If the Air Canada flight arrives before Aer Lingus, no ground hold has to be imposed upon the other flights as they are already planned to arrive simultaneously. As shown in Table 5.3, the probability of this happening is 1.9%.

If the flight from Toronto arrives at the point **P** located 1000 Km away from the airport between 6:34 a.m. and 6:47 a.m., there is a probability of 25.2% that the Air Canada and Aer Lingus flights will arrive at the same time. To avoid collision, the other last three flights require to be shifted as many minutes as Air Canada arrives at that point after 6:34 a.m. For instance, if it crosses that point at 6:40 a.m., a ground hold delay of 6 minutes has to be imposed to the other flights.

There is a probability of 24.9% corresponding to the event: "Air Canada arrives between Aer Lingus and European Air Charter". In this situation, ground hold delay would be imposed only on the last two flights. The amount of delay is equal to the difference between the arrival time of Air Canada to the reference point and 6:47 a.m.

In 18.0% of cases, Air Canada will be arrive between European Air Charter and British Airways. In this circumstance ground hold delay will be imposed only to the last flight. In the last rows of Table 5.6 no ground hold is required as Air Canada arrives in the last position after British Airways.

Remember only the flights until 8:25 a.m. are analysed excluding all those which are planned to come later. To estimate the improvements achieved with this methodology all the flights with which the Air Canada arrival could conflict have to be included.

Arrival order	Probability of the Event	Time condition	Ground hold for AE	Ground hold for EAC	Ground hold for BA
AC...,AE,EAC,BA	1.9%	t^d_{PB} <6:34	0	0	0
AC,AE,EAC,BA	25.2%	6:34≤ t^d_{PB} <6:47	t^d_{PB} -6:34	t^d_{PB} -6:34	t^d_{PB} -6:34
AE,**AC**,EAC,BA	24.9%	6:47≤ t^d_{PB} <6:55	0	t^d_{PB} -6:47	t^d_{PB} -6:47
AE,EAC,**AC**,BA	18.0%	6:55≤ t^d_{PB} <7:05	0	0	t^d_{PB} -6:55
AE,EAC,BA,...**AC**	30.0%	t^d_{PB} ≥7:05	0	0	0

Table 5.6: Ground delays in each case.

5.5 Further developments: network of airports with random airport capacities and random departure times

In [26] and [31] multi-airport GHP are formulated, see section 1.6.2. In section 3.4, the airport capacity is modelled as a multi dimensional random variable and an extension to those methodologies is given. In this section the departure times are modelled as real random variable proposing a multi airport dynamic probabilistic GHP.

A set of airports $\boldsymbol{K}$={1,2,...K} and an ordered set of time periods $\boldsymbol{T}$={1,2,...T} are considered. Given a set of flights $\boldsymbol{F}$={1,2,...F}, $\forall f \in \boldsymbol{F}$ the departure airport of the flight f∈$\boldsymbol{F}$ (denoted by $k^d_f \in \boldsymbol{K}$), the arrival airport of the flight f∈$\boldsymbol{F}$ (denoted by $k^a_f \in \boldsymbol{K}$) and the ground delay cost function of the flight f∈$\boldsymbol{F}$ (denoted by $c^{gh}_f(t) \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F}$ are assumed to be known. Furthermore $\forall$ k∈$\boldsymbol{K}$, the capacity $\underline{\mathbf{C}}_k(\mathbf{D},\mathbf{D}^s,\mathbf{A},\mathbf{A}^s)\subseteq\mathcal{K}\times K$ of the departure and arrival airport of the flight f∈$\boldsymbol{F}$ is assumed to be known. The decision variable is $t^d_{sf} \in \boldsymbol{R}\times\boldsymbol{F}$, which is the scheduled departure time of the flight f∈$\boldsymbol{F}$. $\mathbf{d}^d_f \subseteq \mathcal{C}\times F$ is the departure delay. This is a real random variable expressing the amount of ground hold delay to assign to the flight f∈$\boldsymbol{F}$. This is a real random variable as the amount of ground hold delay is unknown until the day of the operation. Airborne delay is disappeared because with this tactical planning all the airborne delay is transformed in ground hold delay. As explained in section 4.2.1, departure times $\mathbf{t}_f^d \subseteq \mathcal{D}\times F$ are the sum of departure delays $\mathbf{d}_f^d \subseteq \mathcal{C}\times F$ and of scheduled departure times $t_{sf}^d \in \boldsymbol{R}\times\boldsymbol{F}$. The assignment decision variables are u_{ft} and v_{ft},, which are equal to 1 if the flight f$\in\boldsymbol{F}$ is allowed to take-off or to land at the period t$\in\boldsymbol{T}$. The goal is to minimise the expected value of the total cost expressed as sum of the ground delays allocated to the flights of the set $\boldsymbol{F}$ and to maximise the probability that the airport deal with the number of flights planned to arrive and to depart:

$$\min\left(\sum_{f=1}^{F} E(c_f^{gh}\mathbf{d}_f^d)\right) \qquad \begin{cases} c_f^{gh} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ \mathbf{d}_f^d \subseteq \mathcal{C}\times\boldsymbol{F} \end{cases}$$

$$\max\left(\Pr\left\{\underline{\mathbf{C}_k}\left(\mathbf{D}_k \le \sum_{f;k_f^d=k} u_{ft}, \mathbf{A}_k \le \sum_{f;k_f^a=k} v_{ft} \mid \mathbf{D}_k^s = d^s, \mathbf{A}_k^s = a^s\right)\right\}\right) \qquad \begin{cases} \underline{\mathbf{C}_k} \subseteq \mathcal{K}\times K \\ \overline{(d^s, a^s)} \in N^2 \end{cases}$$

subject to the constraints

$$\mathbf{t}_f^d = t_{sf}^d + \mathbf{d}_f^d \qquad \begin{cases} \mathbf{t}_f^d \subseteq \mathcal{D}\times\boldsymbol{F} \\ t_{sf}^d \in \boldsymbol{R}\times\boldsymbol{F} \\ \mathbf{d}_f^d \subseteq \mathcal{C}\times\boldsymbol{F} \end{cases}$$

$$\sum_{t\in T} u_{ft} = 1 \qquad f \in \boldsymbol{F}$$

$$\sum_{t\in T} v_{ft} = 1 \qquad f \in \boldsymbol{F}$$

$$u_{ft}, v_{ft} \in \{0,1\}$$

These constraints mean that for every flight f$\in\boldsymbol{F}$ exactly one of the variables u_{ft} must be equal to 1 and the others must be equal to zero, and similarly for the variables v_{ft}.

5.6 Conclusions

Along this chapter using statistics and probability theory a Decision Support Tool for Air Traffic Flow Management has been developed. It is a tool designed to help airport operators to allocate ground-hold delays to aircraft. The idea is based on a dynamical statistical algorithm that computes the uncertainty of the arrival time of a given aircraft.

Using real time ATFM information given by an air/ground data link, the prediction of the future arrival time is refined as the event horizon gets closer to the present. Ground hold delay are then allocated to reduce the probability of conflict between the arrival aircraft.

The algorithm has been programmed and tested using the CAA statistics for Glasgow International Airport. The results of the tests have indicated a promising reduction in the interval length (IL) equals to 32.1% and thus a significant improvement in the airport's capacity. Anyway the advantage of such a high percentage improvement in the airport's capacity whereas the disadvantage is an increase in expenses due to the allocation of ground hold delays.

In this example only ground hold delay is assigned while from ATFM centers even other instructions can be imposed as airborne delays, speed controls and re-routing. As further work, if the cost associated to all these operations are known the global cost can be computed and the cheapest solution can be implemented. For instance in some circumstances, it might be better to increase the Air Canada flight's speed rather than hold on the ground all the others.

Chapter 6

Airborne Separation Minima for Strategic Planning

6.1 Introduction

Currently the overall airspace environment is divided into sectors. Air Traffic Controller Centres (ATC) control aircraft flow inside of these sectors [7]. Each sector has a limited capacity. An ATC centre cannot handle more than a given number of aircraft in the sector guaranteeing the same level of safety. The number of aircraft that an ATC can deal with is called sector capacity, see [32] and section 2.2.4. In some circumstances, the aircraft flow crossing a given sector exceeds its capacity and the risk of collision increases. To ensure the same level of safety in such situations, air traffic controllers issue restrictive instructions that may cause losses in term of punctuality and reduce the efficiency of the aircraft operations.

This chapter presents a decision support tool that aims at reducing the probability of the throughput of aircraft crossing a given sector exceeding its capacity. The proposed tool is based on statistics and probability theory. The tool proposes a new schedule at the strategic planning level that will guarantee, with a probability set by the air traffic managers, that the aircraft will not violate the separation minima during their routes. The benefits expected from methodology include the reduction of the probability of the overload of the Air Traffic Control Centres. Thus a safer and more efficient ATM system could be achieved.

In this chapter, aircraft position is described with predicted position plus across-track and along-track uncertainties. These uncertainties are modelled as real random variables whose Probability Density Functions (PDF) are built using real data provided by the Civil Aviation Authority (CAA) under some hypothesis.

6.2 Mathematical Model

6.2.1 Theoretical formulation: from determinism to randomness

In this section a theoretical formulation of the topic is exposed. Firstly a deterministic model is introduced, then a stochastic approach is presented marrying the classical kinematic equation with probability's theory.

6.2.1.1 Deterministic approach

A cartesian co-ordinate axes XY and an aircraft is considered. This aircraft has a time dependant velocity. Here below this time functions is given:

$$\underline{v_1}(t) : t \subseteq \boldsymbol{R} \quad \rightarrow \quad \boldsymbol{R}^2$$

At each instant time $t \in \boldsymbol{R}$, the real vectorial function $\underline{v}(t)$ corresponds a vector velocity $\underline{v}(v_{x,}, v_y) \in \boldsymbol{R}^2$, $v_{x,}, v_y$ are the components of this vector with respect to the cartesian axes XY. The integration of the velocity with respect to the time gives the position of the aircraft. For a fixed instant time $t \in \boldsymbol{R}$, a real vectorial function is computed, which gives the position $\underline{s_1}(s_{x1}, s_{y1}) \in \boldsymbol{R}^2$ of the aircraft.

$$\underline{s_1}(s_{x1}, s_{y1}; t) = \int_{\Delta T} \underline{v_1} dt$$

Denoting by Γ_1 the trajectory runs by the aircraft 1 in the interval time $\Delta T \in \boldsymbol{R}$, the space dependence can be expressed by:

$$\underline{s_1}(\Gamma_1, \Delta T; t) = \int_{\Delta T} \underline{v_1} dt$$

In mathematical topology, the trajectory Γ_1 is the co-domain of the real vectorial function:

$$\underline{s_1}(t) : t \subseteq \Delta T \quad \rightarrow \quad \Gamma_1 \subseteq \boldsymbol{R}^2$$

Aircraft labelled with 1 travels from the point $\mathbf{A}(A_x,A_y) \in \boldsymbol{R}^2$ to the point $\mathbf{B}(B_x,B_y) \in \boldsymbol{R}^2$ crossing the point $\mathbf{P}(P_x,P_y) \in \boldsymbol{R}^2$.

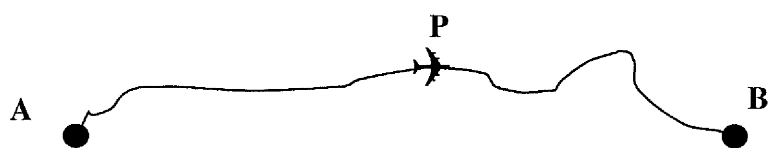


Figure 6.1: Aircraft travels from **A** to **B** crossing **P**.

- For a given trajectory $\Gamma_1^* \subseteq \boldsymbol{R}^2$ and for a fixed time interval $\Delta T^* \in \boldsymbol{R}$; $\underline{s_1}(t)$ solves the time dependence of the co-ordinates to reach the point $\mathbf{B}(B_x,B_y) \in \boldsymbol{R}^2$.
- For a given trajectory $\Gamma_1^* \subseteq \boldsymbol{R}^2$; $\underline{s_1}(\Delta T; t) \subseteq \boldsymbol{R}^2$ is a function of two variables which solves the time dependence of the co-ordinates to reach the point $\mathbf{B}(B_x,B_y) \in \boldsymbol{R}^2$ in the time interval $\Delta T \subseteq \boldsymbol{R}^2$.
- For a given fixed interval time $\Delta T^* \in \boldsymbol{R}$; $\underline{s_1}(\Gamma_1; t)$ is a function of two variables which solves the time dependence of the co-ordinates to reach the point $\mathbf{B}(B_x,B_y) \in \boldsymbol{R}^2$ following the trajectory $\Gamma_1 \subseteq \boldsymbol{R}^2$.

Given two aircraft, the task is to compute the optimal real vectorial functions $\underline{s_1}(t)$, $\underline{s_2}(t)$ in terms of some demands such as:

- maximum mutual distance
- minimum travelling time
- minima lengths of the trajectories run by the aircraft

This problem can be solved with the optimisation of a proper built functional. The solution of the defined problem has to be in correspondence of the absolute† minimum/maximum of this proper built functional. Hence, the problem is recast in seeking this minimum/maximum.

An example of functional is here below shown. The dependencies of the different demands are clearly expressed:

$$\Phi = \Phi\left(\max\left(\left|\underline{s_1} - \underline{s_2}\right|\right), \min(\Delta T), \min\left(l(\Gamma_1), l(\Gamma_2)\right)\right)$$

The terms in the functional are:

- $\max\left|\underline{s_1} - \underline{s_2}\right|$ imposes that the mutual distance between aircraft is maximum
- $\min(\Delta T)$ imposes that the time needed to run the trajectory is minimum
- $\min\left(l(\Gamma_1), l(\Gamma_2)\right)$ imposes that the length of the trajectories is minima

6.2.1.2 Stochastic approach

Time of running of the aircraft and trajectory run by the aircraft are defined as random variables:

$\mathbf{\Delta T} \overset{\Delta}{=}$ Time of running of the aircraft. This is modelled as a random variable defined in the set $\mathcal{J}$. To every outcome $\zeta \in \mathcal{J}$ of the experiment $\mathbf{\Omega}$, the running time of the aircraft is observed. Performing this operation n times and according to the *Law of large numbers*, a real random variable is defined. This random variable gives the probability $\Pr\{\mathbf{\Delta T} = \Delta T\}$ of the interval time $\Delta T \in \boldsymbol{R}$ needed by the aircraft to go from $\mathbf{A}(A_x, A_y) \in \boldsymbol{R}^2$ to $\mathbf{B}(B_x, B_y) \in \boldsymbol{R}^2$.

† The minimum/maximum of this proper built functional can be also relative. In this case the variables have to be properly constrained.

$\Gamma \overset{\Delta}{=}$ Trajectory runs by the aircraft. This is modelled as a random variable defined in the set $\mathcal{S}$. To every outcome $\zeta \in \mathcal{S}$ of the experiment $\mathbf{\Omega}$, the trajectory run by the aircraft $\Gamma(\zeta^*) \subseteq \boldsymbol{R}^2$ is assigned to $\Gamma \subseteq \mathcal{S}$. Performing this operation *n* times and according to the *Law of large numbers*, a real random variable is defined. This random variable gives the probability $\Pr\{\Gamma = \Gamma\}$ of running the trajectory $\Gamma \subseteq \boldsymbol{R}^2$ to go from $\mathbf{A}(A_x, A_y) \in \boldsymbol{R}^2$ to $\mathbf{B}(B_x, B_y) \in \boldsymbol{R}^2$.

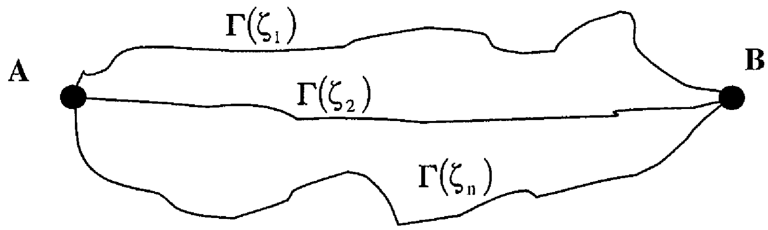


Figure 6.2: To every outcome $\zeta \in \mathcal{S}$, the trajectory run by the aircraft $\Gamma(\zeta^*) \subseteq \boldsymbol{R}^2$ is assigned.

The stochastic equations can be directly derived by the deterministic equations considering velocity and interval time as random variables:

$$\underline{s_1}(s_{x1}, s_{y1}; t) = \int_{\Delta T} \underline{v_1} dt$$

The integral described has random variables as both integrating functions and integrating limits. A stochastic integral is defined. The formal mathematical meaning of the stochastic integral is above the task of this work. The interested reader may find further details in the references [53], [54], [55] and [56].

The random function position $\underline{s}(s_x(\zeta), s_y(\zeta); t) \subseteq \mathcal{G} \times \boldsymbol{T}$ is the random variable that gives the cartesian co-ordinates of the position of the aircraft. The random function position $\underline{s}$ can also be expressed by the trajectory run and the time needed to run this trajectory $\underline{s}(\Gamma(\zeta), \Delta T(\zeta); t) \subseteq \mathcal{J} \times \mathcal{S} \times \boldsymbol{T}$. The random variables $\underline{s}(s_x(\zeta), s_y(\zeta); t) \subseteq \mathcal{G} \times \boldsymbol{T}$ and $\underline{s}(\Gamma(\zeta), \Delta T(\zeta); t) \subseteq \mathcal{J} \times \mathcal{S} \times \boldsymbol{T}$ are expressing the same phenomena, therefore a mathematical dependency† can be sought.

† A function $f \subseteq \mathcal{S} x \mathcal{J} x \boldsymbol{T} \rightarrow \mathcal{G} x \boldsymbol{T}$ such as $\underline{s}(s_x(\zeta), s_y(\zeta); t) = f(\underline{s}(\Gamma(\zeta), \Delta T(\zeta); t))$ exists.

The equation written above can be expressed also as:

$$\underline{\mathbf{s}_1}(\Gamma_1(\zeta),\Delta\mathbf{T}(\zeta);t)=\int_{\Delta T}\underline{\mathbf{v}_1}dt$$

- For a given outcome ζ^* of the experiment $\mathbf{\Omega}$; the function simplifies in the deterministic function treated in section 6.2.1.1

$$\underline{\mathbf{s}_1}(\Gamma_1(\zeta^*),\Delta\mathbf{T}(\zeta^*);t)=\underline{s_1}(\Gamma_1,\Delta T;t)$$

- For a given trajectory $\Gamma_1^*\subseteq\boldsymbol{R}^2$; $\underline{\mathbf{s}_1}(\Delta\mathbf{T}(\zeta);t)\subseteq\mathcal{J}$ is a real random variable that gives the probability of running the given trajectory $\Gamma_1^*\subseteq\boldsymbol{R}^2$ for each interval time $\Delta\mathbf{T}(\zeta)\in\mathfrak{S}$.

$$\Pr\left\{\begin{matrix}\text{aircraft}\\ \text{takes }\Delta T_1\\ \text{assumed}\\ \mathbf{\Gamma}_1=\Gamma_1^*\end{matrix}\right\}=\Pr\{\Delta\mathbf{T}=\Delta T\mid\mathbf{\Gamma}_1=\Gamma_1^*\}=$$

$$=\frac{f_{\underline{s_1}(\Gamma_1,\Delta T;t)}(\Gamma_1^*,\Delta T;t)}{\int_{-\infty}^{+\infty}f_{\underline{s_1}(\Gamma_1,\Delta T;t)}(\Gamma_1,\Delta T;t)d\Delta\Gamma_1}=f_{\underline{s_1}(\Delta T;t)}(\Delta T;t)$$

- For a given time interval $\Delta T^*\in\boldsymbol{R}$; $\underline{\mathbf{s}_1}(\Gamma_1(\zeta);t)\subseteq\mathfrak{S}$ is a real random variable that gives the probability of running the trajectory $\mathbf{\Gamma}_1\subseteq\mathfrak{S}$ in the time interval $\Delta T^*\in\boldsymbol{R}$.

$$\Pr\left\{\begin{matrix}\text{aircraft}\\ \text{follows }\Gamma_1\\ \text{assumed}\\ \Delta\mathbf{T}=\Delta T^*\end{matrix}\right\}=\Pr\{\mathbf{\Gamma}_1=\Gamma_1\mid\Delta\mathbf{T}=\Delta T^*\}=$$

$$=\frac{f_{\underline{s_1}(\Gamma_1,\Delta T;t)}(\Gamma_1,\Delta T^*;t)}{\int_{-\infty}^{+\infty}f_{\underline{s_1}(\Gamma_1,\Delta T;t)}(\Gamma_1,\Delta T;t)d\Delta T}=f_{\underline{s_1}(\Gamma_1;t)}(\Gamma_1;t)$$

The task is to compute the optimal trajectories of two aircraft under the following probabilistic constraints.

- The probability of the mutual distance exceeding a fixed threshold $\Delta D \in \boldsymbol{R}$ is maximum
- The probability of solving the conflict within the fixed interval time $\Delta T \in \boldsymbol{R}$ is maximum
- The probability that the lengths of the trajectories do not exceed $l_{\Gamma_1} \in \boldsymbol{R}$ (for the first aircraft) and $l_{\Gamma_2} \in \boldsymbol{R}$ (for the second aircraft) is maximum.

The maximum of a proper built functional Φ gives the solution to the defined problem.

$$\Phi = \Phi\left(\max \Pr\left\{\left\|\underline{s_1} - \underline{s_2}\right\| > \Delta D\right\} \max \Pr\{\Delta T < \Delta T\}, \max \Pr\left\{l(\Gamma_1) < l_{\Gamma_1}, l(\Gamma_2) < l_{\Gamma_2}\right\}\right)$$

In the next sections, this functional will be solved considering only the mutual distance. The criteria to guarantee a given separation $\Delta D \in \boldsymbol{R}$ between two aircraft during their route will be shown.

6.2.2 Problem approaching for separation minimum in flight planning

If the stochastic processes $\underline{s_1}(\Gamma_1(\zeta), \Delta T(\zeta); t) \subseteq \mathcal{J} \times \mathfrak{F} \times \boldsymbol{T}$ and $\underline{s_2}(\Gamma_2(\zeta), \Delta T(\zeta); t) \subseteq \mathcal{J} \times \mathfrak{F} \times \boldsymbol{T}$ are statistically determined, the problem stated in 6.2.12 can be solved.

To determine the PDF of $\underline{s_1}(\Gamma_1(\zeta), \Delta T(\zeta); t)$ the following experiment Ω has to be performed:

ζ = observation of the trajectory $\Gamma \subseteq \boldsymbol{R}^2$ in the interval time $\Delta T \subseteq \boldsymbol{R}$

This experiment is difficult to perform in real life because the position of the aircraft during its journey has to be observed and recorded.

An alternative methodology to determine the PDF of $\underline{s}_1(\Gamma_1(\zeta),\Delta T(\zeta);t)$ is to compute the space through an integration of the stochastic process velocity with respect to the time. Supposing that the velocity and the space are statistically independent, the space is the limit of the sum of an infinite number of independent random walks. Hence the space converges towards a Gaussian process distribution. Therefore a Brownian Motion can be defined [35] [57]. As a consequence of this definition, the PDF of the arrival time at a given point $\mathbf{P}(P_x,P_y)\in \boldsymbol{R}^2$ can be calculated. However, in this thesis the Brownian motion approach is not adopted since the statistical independence of the velocities as random variables within a stochastic process is not known.

The data of the arrival times at the destination airport are known, see Table 3.1. As explained in section 4.2.2, the PDFs of the arrival times are determined from the experiment $\mathbf{\Omega}$. In section 5.2.7, the PDF of the velocity is calculated from the travelling time.

In this thesis, the space and the velocity are hypothesised to be linked by a linear transformation. This assumption has a physical justification as the PDF of the arrival time at the destination airport computed with this linear transformation corresponds to the PDF of the arrival times computed using the real data. This hypothesis imposes a correlation between velocity and space†. This imposed correlation gives as a result a PDF of the arrival times at the destination airport that is equal to the PDF of the arrival times built using the real data.

$$\underline{s}(\zeta;t) = \underline{v}(\zeta)t \qquad \begin{cases} \underline{s}(\zeta;t) \subseteq \mathcal{G}x\boldsymbol{T} \\ \underline{v}(\zeta) \subseteq \mathcal{J} \end{cases}$$

An instant of time $t\in \boldsymbol{T}$ is considered; the space run by the aircraft is

$$\underline{s}(t,\zeta) = \underline{v}(\zeta)t \qquad \Leftrightarrow \qquad \begin{cases} s_\tau(t,\zeta) = v_\tau(\zeta)t \\ s_n(t,\zeta) = v_n(\zeta)t \end{cases}$$

† In the Brownian motion, the velocities computed in two different instant of times are independent.

The PDF of the space $s_\tau \subseteq \mathcal{G}$ can be computed from the velocity through the *Fundamental Theorem of transformation of variables* [48]:

$$f_{s_\tau}(s_\tau;t) = |1| f_{v_\tau}\left(\frac{s_\tau}{t}\right)$$

Substituting the velocity with the travelling time, the PDF of the space along the track $s_\tau \subseteq \mathcal{G}$ is given by:

$$f_{s_\tau}(s_\tau) = |1| \frac{|d(\mathbf{A},\mathbf{B})|}{\left(\frac{s_\tau}{t}\right)^2} f_{t'_{AB}}\left(\frac{d(\mathbf{A},\mathbf{B})t}{s_\tau}\right)$$

The PDF of the space $s_n \subseteq \mathcal{G}$ is unknown as no data are available concerning the lateral deviation. In the section 6.3.2.2, this PDF will be built under some approximations.

6.2.3 Probability of conflict PC

In this section the probability of a mid-air conflict between two aircraft will be defined. To do so, the definition of Probability of Conflict given in section 4.2.4 will be generalised to account for the aircraft positions.

6.2.3.1 Probability of two aircraft being simultaneously in the same position

Let $\boldsymbol{C}$ be a set of points belonging to a two-dimensional plane and $\boldsymbol{T}$ an interval of time. In a given instant time $t^* \in \boldsymbol{T}$, aircraft 1 has a probability to be in $\mathbf{P}_1 \in \boldsymbol{C}$, $\mathbf{P}_2 \in \boldsymbol{C}$,... $\mathbf{P}^* \in \boldsymbol{C}$. In the other hand, in the same instant time $t^* \in \boldsymbol{T}$, aircraft 2 has a probability to be in $\mathbf{P}_3 \in \boldsymbol{C}$, $\mathbf{P}_4 \in \boldsymbol{C}$,... $\mathbf{P}^* \in \boldsymbol{C}$. In this section the probability that both aircraft 1 and 2 are only at the point $\mathbf{P}^*$ of the region $\boldsymbol{C}$ is computed, see Figure 6.3.

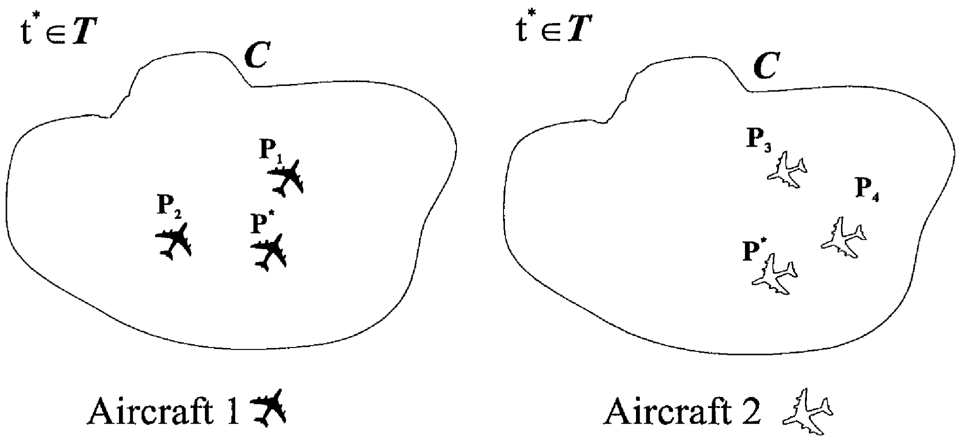


Figure 6.3: In $t^*\in \boldsymbol{T}$, aircraft 1 is in $\mathbf{P}_1\in \boldsymbol{C}$, $\mathbf{P}_2\in \boldsymbol{C}$,… $\mathbf{P}^*\in \boldsymbol{C}$ while aircraft 2 is in $\mathbf{P}_3\in \boldsymbol{C}$, $\mathbf{P}_4\in \boldsymbol{C}$,… $\mathbf{P}^*\in \boldsymbol{C}$.

Given a point $\mathbf{P}^*(P_x,P_y)\in \boldsymbol{C}$ and the instant time of $t^*\in \boldsymbol{T}$, the probability that both aircraft 1 and 2 are in $\mathbf{P}^*\in \boldsymbol{C}$ simultaneously at the time $t^*\in \boldsymbol{T}$ is:

$$\Pr\left\{\begin{array}{c}\text{aircraft 1}\\ \text{is in } \mathbf{P}^*\in \boldsymbol{C}\\ \text{at the instant}\\ \text{time } t^*\in \boldsymbol{T}\end{array}\right\} = \Pr\left\{\underline{s}_1(\mathbf{P}^*;t^*)<\underline{s}_1(\mathbf{P}^*;t^*)\le \underline{s}_1(\mathbf{P}^*;t^*)+d\underline{s}_1\right\}=$$

$$=\Pr\left\{\underline{s}_1(\mathbf{P}^*;t^*)<\underline{s}_1(\mathbf{P}^*;t^*)\le \underline{s}_1(\mathbf{P}^*;t^*)+d\mathbf{P},\underline{s}_1(\mathbf{P}^*;t^*)<\underline{s}_1(\mathbf{P}^*;t^*)\le \underline{s}_1(\mathbf{P}^*;t^*)+dt\right\}$$

$$\Pr\left\{\begin{array}{c}\text{aircraft 2}\\ \text{is in } \mathbf{P}^*\in \boldsymbol{C}\\ \text{at the instant}\\ \text{time } t^*\in \boldsymbol{T}\end{array}\right\} = \Pr\left\{\underline{s}_2(\mathbf{P}^*;t^*)<\underline{s}_1(\mathbf{P}^*;t^*)\le \underline{s}_2(\mathbf{P}^*;t^*)+d\underline{s}_2\right\}$$

The event "Conflict in $\mathbf{P}\in \boldsymbol{C}$ between the aircraft $i\in \boldsymbol{N}$ and $j\in \boldsymbol{N}$ in the instant of time $t^*\in \boldsymbol{T}$" is defined as:

"Aircraft $i\in N$ and aircraft $j\in N$ are at a point $\mathbf{P}\in C$ in the instant of time $t^*\in \boldsymbol{T}$ *".*

The probability of the event "Conflict in $\mathbf{P}\in \boldsymbol{C}$ between the aircraft $i\in \boldsymbol{N}$ and $j\in \boldsymbol{N}$ in the instant of time $t^*\in \boldsymbol{T}$" is marked as $PC_{ij}(\mathbf{P}^*,t^*)$ and is:

"The probability that the aircraft $i \in N$ and $j \in N$ arrive at a point $\mathbf{P} \in C$ in the instant of time $t^* \in T$".

The probability of conflict $PC_{12}(\mathbf{P}^*, t^*)$ in $\mathbf{P}^*(P_x, P_y) \in C$ between aircraft 1 and 2 is given by:

$$PC_{12}\left(\mathbf{P}^*, t^*\right) \stackrel{\Delta}{=} \Pr\left\{\begin{array}{c}\text{aircraft 1}\\ \text{conflicts with}\\ \text{aircraft 2}\\ \text{in } \mathbf{P}^* \in C \text{ at the time } t^* \in T\end{array}\right\} =$$

$$= \Pr\left\{\left(\begin{array}{c}\text{aircraft 1}\\ \text{is in } \mathbf{P}^*\\ \text{at the instant}\\ \text{time } t^*\end{array}\right) \bigcap \left(\begin{array}{c}\text{aircraft 2}\\ \text{is in } \mathbf{P}^*\\ \text{at the instant}\\ \text{time } t^*\end{array}\right)\right\} =$$

$$= \Pr\left\{s_1\left(\mathbf{P}^*; t^*\right) < \underline{s}_1\left(\mathbf{P}^*; t^*\right) \le s_1\left(\mathbf{P}^*; t^*\right) + d\underline{s}_1, s_2\left(\mathbf{P}^*; t^*\right) < \underline{s}_2\left(\mathbf{P}^*; t^*\right) \le \underline{s}_2\left(\mathbf{P}^*; t^*\right) + d\underline{s}_2\right\}$$

If $\underline{s}_1 \subseteq \mathcal{G} x T$ and $\underline{s}_2 \subseteq \mathcal{G} x T$ are supposed to be independent, clearly the following equivalence is derived:

$$PC_{12}\left(\mathbf{P}^*, t^*\right) = \Pr\left\{s_1(\mathbf{P}; t) < \underline{s}_1(\mathbf{P}; t) \le s_1(\mathbf{P}; t) + d\underline{s}_1\right\} \Pr\left\{s_2(\mathbf{P}; t) < \underline{s}_2(\mathbf{P}; t) \le \underline{s}_2(\mathbf{P}; t) + d\underline{s}_2\right\}$$

6.2.3.2 Probability of two aircraft being quasi-simultaneously in the same position within a given interval time

Let C be a set of points belonging to a two-dimensional plane and T an interval of time. Aircraft 1 and aircraft 2 can reach a given point $\mathbf{P}^* \in C$ in two different instant of time $t_1 \in T$ and $t_2 \in T$. Conflict is defined when the difference between $t_1 \in T$ and $t_2 \in T$ is less than a fixed threshold $\Delta T_{12} \in R$, see Figure 6.4.

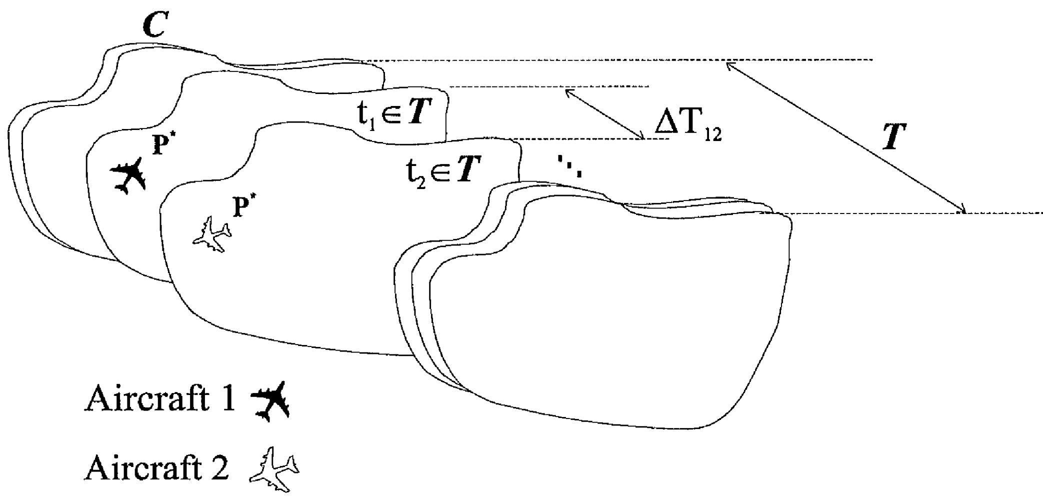


Figure 6.4: Aircraft 1 and 2 reach a point $\mathbf{P}^* \in \boldsymbol{C}$ in two different instant $t_1 \in \boldsymbol{T}$ and $t_2 \in \boldsymbol{T}$.

The event "Conflict in $\mathbf{P}^* \in \boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is define as:

"Aircraft $i \in \boldsymbol{N}$ and aircraft $j \in \boldsymbol{N}$ are at a point $\boldsymbol{P}^* \in \boldsymbol{C}$ in the instant time t^* $\forall\, t^* \in \boldsymbol{T}$ within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".

The probability of the event "Conflict in $\mathbf{P}^* \in \boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is marked as $PC_{ij}^{\Delta T = \Delta T_{ij}}(\mathbf{P}^*, \boldsymbol{T})$ and is:

"The probability that the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ are at a point $\boldsymbol{P}^* \in \boldsymbol{C}$ in the instant time t^* $\forall\, t^* \in \boldsymbol{T}$ within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".

The definition above is equivalent to the definition given in section 4.2.4 if:

- **P** = the arrival airport.
- $\Delta T = \Delta T_{12}$
- $\boldsymbol{T} = \,]-\infty, +\infty[$

A particular case is when the time period is an instant of time $\Delta T = t^* + dt$, the probability of conflict $PC_{12}^{\Delta T = t^* + dt}(\mathbf{P}, \boldsymbol{T})$ at the point $\mathbf{P} \in \boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between aircraft 1 and 2 relative to the time period $\Delta T = t^* + dt$ simplifies in:

$$PC_{12}^{\Delta T = t^* + dt}(\mathbf{P}, \boldsymbol{T}) \stackrel{\Delta}{=} \Pr\left\{ \left(\begin{array}{l} \text{aircraft 1 and} \\ \text{aircraft 2} \\ \text{conflicts in } \mathbf{P} \in \boldsymbol{C} \\ \text{in the instant } t^* \in \boldsymbol{T} \end{array} \right) \cup \ldots \forall t^* \in \boldsymbol{T} \right\}$$

6.2.3.3 Probability of two aircraft being simultaneously in the same position within a given region

Let $\boldsymbol{C}$ be a set of points belonging to a two-dimensional plane and $\boldsymbol{T}$ an interval of time. In a given instant time $t^* \in \boldsymbol{T}$, aircraft 1 has a probability to be in $\mathbf{P_1} \in \boldsymbol{C}$, $\mathbf{P_2} \in \boldsymbol{C}$ … $\mathbf{P}^* \in \boldsymbol{C}$. In the other hand, in the same instant time $t^* \in \boldsymbol{T}$, aircraft 2 has a probability to be in $\mathbf{P_1} \in \boldsymbol{C}$, $\mathbf{P_2} \in \boldsymbol{C}$ … $\mathbf{P}^* \in \boldsymbol{C}$. In this section the probability that both aircraft 1 and 2 are in the points $\mathbf{P_1} \in \boldsymbol{C}$, $\mathbf{P_2} \in \boldsymbol{C}$ … $\mathbf{P}^* \in \boldsymbol{C}$ of the region $\boldsymbol{C}$ is computed, see Figure 6.5.

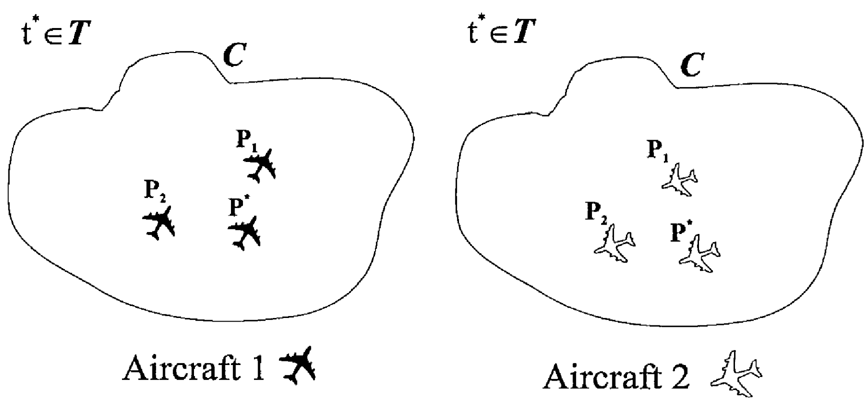


Figure 6.5: In a given instant time $t^* \in \boldsymbol{T}$, aircraft 1 and 2 are in $\mathbf{P_1} \in \boldsymbol{C}$, $\mathbf{P_2} \in \boldsymbol{C}$ … $\mathbf{P}^* \in \boldsymbol{C}$.

Conflict in the region $\boldsymbol{C}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ in the instant of the time $t^* \in \boldsymbol{T}$ is defined as:

"Aircraft $i \in \boldsymbol{N}$ and aircraft $j \in \boldsymbol{N}$ arrive at a point $\boldsymbol{P}$ $\forall \boldsymbol{P} \in \boldsymbol{C}$ in the instant time of time $t^* \in \boldsymbol{T}$".

The probability of the event "Conflict in the region C between the aircraft $i \in N$ and $j \in N$ in the instant of the time $t^* \in T$" is marked as $PC_{ij}(C, t^*)$ and is:

"The probability that the aircraft $i \in N$ and $j \in N$ are in P $\forall P \in C$ in the instant time of time $t^* \in T$".

Before computing the probability of conflict $PC_{12}(C, t^*)$ in the region C in the instant of time $t^* \in T$, the random variable $\underline{s}_1\underline{s}_2$ has to be defined. This random variable expresses simultaneously the position of aircraft 1 and 2†.

$\underline{s}_1\underline{s}_2 \overset{\Delta}{=}$ This is a four dimensional†† random variable. For a given pair of points $\mathbf{P}_1(P_{1x}, P_{2x}) \in C$ and $\mathbf{P}_2(P_{2x}, P_{2y}) \in C$, the PDF $f_{\underline{s}_1\underline{s}_2}(\mathbf{P}_1, \mathbf{P}_2)$ gives the probability aircraft 1 is in $\mathbf{P}_1 \in C$ and aircraft 2 is in $\mathbf{P}_2 \in C$.

If $t_1, t_2, t_3, \ldots t_n$ $(t_1, t_2, t_3, \ldots t_n) \in T^n$ are instants of time, the set of values $\{\underline{s}_1\underline{s}_2(\zeta_1, t_1), \underline{s}_1\underline{s}_2(\zeta_2, t_2), \ldots \underline{s}_1\underline{s}_2(\zeta_n, t_n)\}$ defines a stochastic process.

$$f_{\underline{s}_1\underline{s}_2}(\mathbf{P}_1, \mathbf{P}_2; t)d\mathbf{P}_1 d\mathbf{P}_2 dt = \Pr\left\{\begin{array}{l}\text{aircraft 1 is in } \mathbf{P}_1 \\ \text{aircraft 2 is in } \mathbf{P}_2 \\ \text{at the time t}\end{array}\right\}$$

Finally, the probability of conflict $PC_{12}(C, t^*)$ in the region C in the instant of time $t^* \in T$ is given by the following expression:

$$PC_{12}(C, t^*) \overset{\Delta}{=} \Pr\left\{\begin{array}{c}\text{aircraft 1} \\ \text{conflicts} \\ \text{aircraft 2} \\ \text{in } C \text{ and in } t^* \in T\end{array}\right\} = \iint_C f_{\underline{s}_1\underline{s}_2}(\mathbf{P}, \mathbf{P}; t^*) d\mathbf{P}$$

† Do not confuse with the joint distribution given in section 5.2.6 relative to only one stochastic process

†† $\underline{s}_1\underline{s}_2$ is four dimensional because C is hypothesised to be a subset of a two-dimensional set.

If $\underline{\mathbf{s}_1} \subseteq \mathcal{G}xT$ and $\underline{\mathbf{s}_2} \subseteq \mathcal{G}xT$ are supposed to be independent, the following expression is derived:

$$PC_{12}(C, t^*) = \iint_C f_{\underline{s}_1 \underline{s}_2}(\mathbf{P}, \mathbf{P}; t^*) d\mathbf{P} = \iint_C f_{\underline{s}_1}(\mathbf{P}; t^*) f_{\underline{s}_2}(\mathbf{P}; t^*) d\mathbf{P}$$

6.2.3.4 Probability of two aircraft being quasi-simultaneously in the same position within a given region and a given interval time

Let $\boldsymbol{C}$ be a set of points belonging to a two-dimensional plane and $\boldsymbol{T}$ an interval of time. Aircraft 1 and aircraft 2 can reach given points $\mathbf{P}_1 \in \boldsymbol{C}$, $\mathbf{P}_2 \in \boldsymbol{C}$,... $\mathbf{P}^* \in \boldsymbol{C}$ in different instant of times $t_1 \in \boldsymbol{T}$, $t_2 \in \boldsymbol{T}$,...$t^* \in \boldsymbol{T}$. Conflict is defined when the difference between $t_1 \in \boldsymbol{T}$ and $t_2 \in \boldsymbol{T}$ is less than a fixed threshold $\Delta T_{12} \in \boldsymbol{R}$..

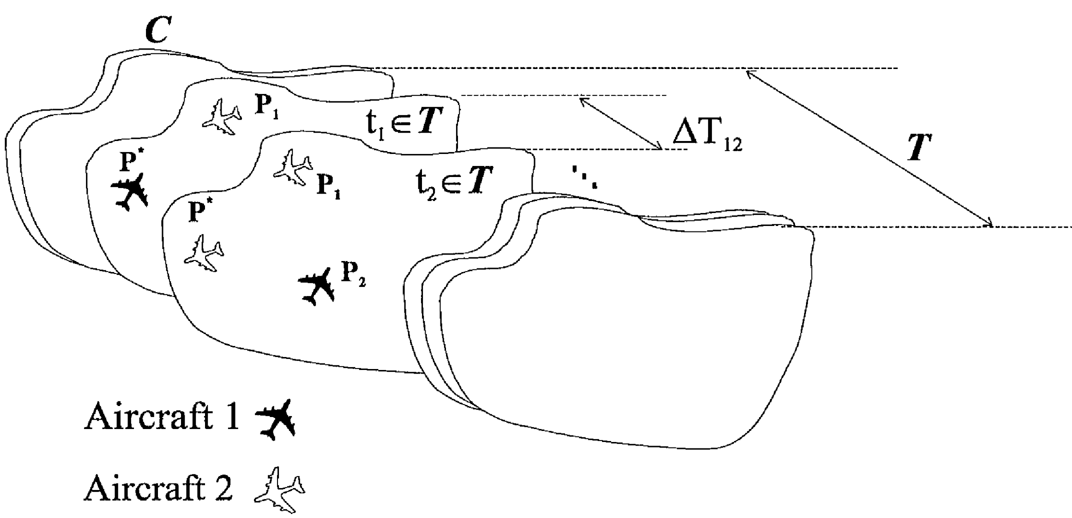


Figure 6.6: Aircraft 1 and aircraft 2 reach the points $\mathbf{P}_1 \in \boldsymbol{C}$, $\mathbf{P}_2 \in \boldsymbol{C}$,... $\mathbf{P}^* \in \boldsymbol{C}$ in $t_1 \in \boldsymbol{T}$, $t_2 \in \boldsymbol{T}$,...$t^* \in \boldsymbol{T}$.

Define "Conflict in the region $\boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" the event:

"Aircraft $i \in \boldsymbol{N}$ and aircraft $j \in \boldsymbol{N}$ are at a point $\boldsymbol{P}$ $\forall \boldsymbol{P} \in \boldsymbol{C}$ in the instant time t^* $\forall t^* \in \boldsymbol{T}$ within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".

The probability of the event "Conflict in the region $\boldsymbol{C}$ and in the interval time T between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is marked as $PC_{ij}^{\Delta T = \Delta T_{ij}}(\boldsymbol{C},\boldsymbol{T})$ and it is:

"The probability that the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ are at a point $\boldsymbol{P}$ $\forall \boldsymbol{P} \in \boldsymbol{C}$ in the instant time t^* $\forall t^* \in \boldsymbol{T}$ within a temporal difference less than $\Delta T_{ij} \in \boldsymbol{R}$".

The definition above is equivalent to the definition given in section 4.2.4 if:

- $\boldsymbol{C} = \mathbf{P}$
- $\mathbf{P}$ = the arrival airport
- $\Delta T = \Delta T_{12}$
- $\boldsymbol{T} = \,]-\infty,+\infty[$

A particular case is when the time period is equals to an instant of time $\Delta T = t^* + dt$, The probability of conflict $PC_{12}^{\Delta T = t^* + dt}(\boldsymbol{C},\boldsymbol{T})$ in the region $\boldsymbol{C}$ and in the interval time T between the aircraft 1 and 2 relative to the time period $\Delta T = t^* + dt$ is:

$$PC_{12}^{\Delta T = t^* + dt}(\boldsymbol{C},\boldsymbol{T}) \stackrel{\Delta}{=} \Pr\left\{ \bigcup \left(\begin{array}{l} \text{aircraft 1 and} \\ \text{aircraft 2} \\ \text{conflicts in } t^* \in \boldsymbol{T} \end{array} \right) \ldots \forall t^* \in \boldsymbol{T} \right\}$$

If the time period is equal to an instant of time than these events are incompatible. If $\underline{s_1} \subseteq \boldsymbol{\mathcal{G}} x\boldsymbol{T}$ and $\underline{s_2} \subseteq \boldsymbol{\mathcal{G}} x\boldsymbol{T}$ are supposed to be independent, the following expression is derived:

$$PC_{12}^{\Delta T = t^* + dt}(\boldsymbol{C},\boldsymbol{T}) = \iiint_{CxT} f_{\underline{s}_1}(\mathbf{P};t) f_{\underline{s}_2}(\mathbf{P};t) d\mathbf{P}dt$$

6.2.3.5 Probability of two aircraft being quasi-simultaneously approximately in the same position within a given region and a given interval time

Let $\boldsymbol{C}$ be a set of points belonging to a two-dimensional plane and $\boldsymbol{T}$ an interval of time. Aircraft 1 reaches a given point $\mathbf{P_1} \in \boldsymbol{C}$ in the instant of times $t_1 \in \boldsymbol{T}$ and aircraft 2 reaches

$\mathbf{P_2} \in \boldsymbol{C}$ in the instant of times $t_2 \in \boldsymbol{T}$. Conflict is defined when the distance between the points $\mathbf{P_1} \in \boldsymbol{C}$ and $\mathbf{P_2} \in \boldsymbol{C}$ is less than a fixed threshold $\Delta D_{12} \in \boldsymbol{R}$ and simultaneously the difference between $t_1 \in \boldsymbol{T}$ and $t_2 \in \boldsymbol{T}$ is less than another fixed threshold $\Delta T_{12} \in \boldsymbol{R}$, see Figure 6.7.

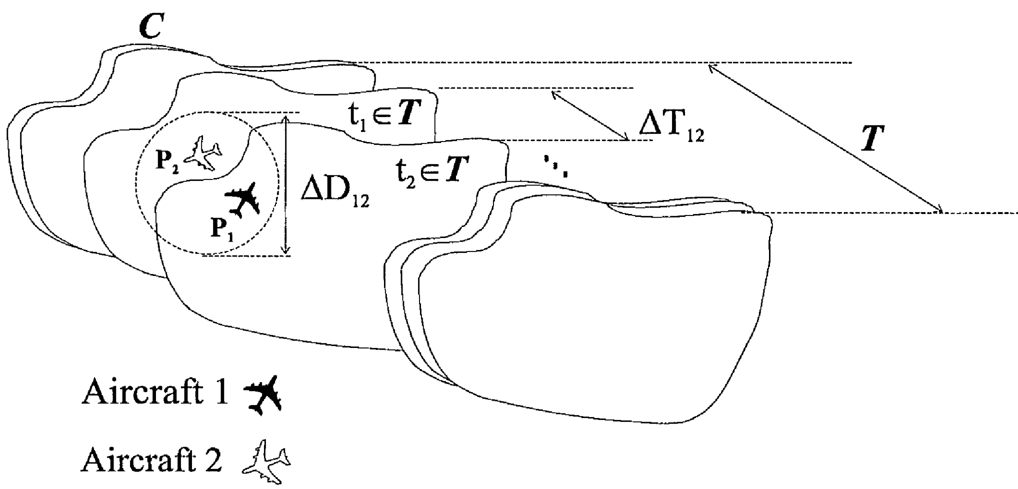


Figure 6.7: Aircraft 1 reaches a point $\mathbf{P_1} \in \boldsymbol{C}$ in $t_1 \in \boldsymbol{T}$ and aircraft 2 reaches $\mathbf{P_2} \in \boldsymbol{C}$ in $t_2 \in \boldsymbol{T}$.

Define "Conflict in the region $\boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the distance $\Delta D_{ij} \in \boldsymbol{R}$ and relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" the event:

"Aircraft $i \in \boldsymbol{N}$ arrive at a point $\boldsymbol{P_i} \in \boldsymbol{C}$ and aircraft $j \in \boldsymbol{N}$ arrive at a point $\boldsymbol{P_j} \in \boldsymbol{C}$ such as $d(\boldsymbol{P_i}, \boldsymbol{P_j}) \leq \Delta D_{ij}$ within a temporal difference less than $t^* \pm \Delta T_{ij}$ with $t^* \in \boldsymbol{T}$ and $\Delta T_{ij} \in \boldsymbol{R}$".

The probability of the event "Conflict in the region $\boldsymbol{C}$ and in the interval time $\boldsymbol{T}$ between the aircraft $i \in \boldsymbol{N}$ and $j \in \boldsymbol{N}$ relative to the distance $\Delta D_{ij} \in \boldsymbol{R}$ and relative to the time period $\Delta T_{ij} \in \boldsymbol{R}$" is:

"The probability that aircraft $i \in \boldsymbol{N}$ arrive at a point $\boldsymbol{P_i} \in \boldsymbol{C}$ and aircraft $j \in \boldsymbol{N}$ arrive at a point $\boldsymbol{P_j} \in \boldsymbol{C}$ such as $d(\boldsymbol{P_i}, \boldsymbol{P_j}) \leq \Delta D_{ij}$ within a temporal difference less than $t^* \pm \Delta T_{ij}$ with $t^* \in \boldsymbol{T}$ and $\Delta T_{ij} \in \boldsymbol{R}$".

The definition above is equivalent to the definition given in section 4.2.4 if:

- $C = \mathbf{P}$
- $\mathbf{P}_1 = \mathbf{P}_2$ = the arrival airport.
- $\Delta T = \Delta T_{12}$
- $\boldsymbol{T} =]-\infty,+\infty[$
- $\Delta D = \Delta D_{12}$

A particular case is when the time period is equals to an instant of time $\Delta T = t^* + dt$. The probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t^*+dt}(C,T)$ in the region C and in the interval time T between the aircraft 1 and 2 relative to the distance $\Delta D = \Delta D_{12}$ and relative to the time period $\Delta T = t^* + dt$ is:

$$PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t^*+dt}(C,T) \stackrel{\Delta}{=} \Pr\left\{ \bigcup_{\substack{\forall \mathbf{P}_1 \in C \\ \forall \mathbf{P}_2 \in C \\ \forall t^* \in T}} \left(\begin{array}{l} \text{aircraft 1 is in } \mathbf{P}_1 \text{ and} \\ \text{aircraft 2 is in } \mathbf{P}_2 \\ \text{at the instant of} \\ \text{time } t^* \in T \text{ with} \\ \|\mathbf{P}_1 - \mathbf{P}_2\| \le \Delta D \end{array} \right) \right\}$$

The expression described above will be used in the following of this thesis. The only requirement imposed to aircraft is that aircraft will maintain a safety distance during their route within a fixed probability.

6.2.4 Criteria for the separation minima

In this section the functional shown in section 6.2.1.2 is expressed. To simplify this functional, the only criteria imposed is:

- the probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(C,T)$ is lower than a fixed value $\alpha \in [0,1]$ with the constraint that $\forall t \in T$ the probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(C,t)$ does not exceed a given probability $\beta \in [0,1]$.

To achieve such a criteria, the shifting time $T_{12} \in \boldsymbol{R}$ is the only control parameter.

As already shown in chapter 4, the departure time of the aircraft 2 will be required to be changed according to the scheduling shown in Table 6.1.

	Old departing schedule	**New departing schedule**
Aircraft 1	t_{s1}^{d}	t_{s1}^{d}
Aircraft 2	t_{s2}^{d}	$t_{s1}^{d}+T_{12}$

Table 6.1: Strategic scheduling for airborne separation minima.

The condition is:

$$PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(C,T)\leq\alpha$$

with the constraint:

$$PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(C,t)\leq\beta \qquad \forall t\in \boldsymbol{T}$$

The condition can be expressed as:

$$\frac{\iiint\limits_{C\times T} f_{\underline{s}_1}(\mathbf{P};t) f_{\underline{s}_2}(\mathbf{P};t+T_{12})d\mathbf{P}dt}{T}\leq\alpha$$

with the constraint:

$$\iint\limits_{C} f_{\underline{s}_1}(\mathbf{P};t) f_{\underline{s}_2}(\mathbf{P};t+T_{12})d\mathbf{P}\leq\beta \qquad \forall(t+T_{12})\in \boldsymbol{T}$$

The unknown is the time $T_{12}\in \boldsymbol{R}$ required to shift the departure time of the aircraft 1.

6.3 Description of the code

The main hypotheses assumed in the Matlab® code developed are:

- aircraft fly at the same altitude level. This thesis is, therefore, based on a two dimensional model.
- aircraft routes are straight lines. This hypothesis is not a real restriction on the use of the code because a more complex trajectory can be always divided in straight segments.

The goals of the programming code developed are:

- to calculate the position of the aircraft $\underline{s}(s_x, s_y; t) \subseteq \mathcal{G} x \boldsymbol{T}$ for every given fixed instant of time $t \in \boldsymbol{T}$.
- to minimise the probability of conflict between two aircraft during their routes.

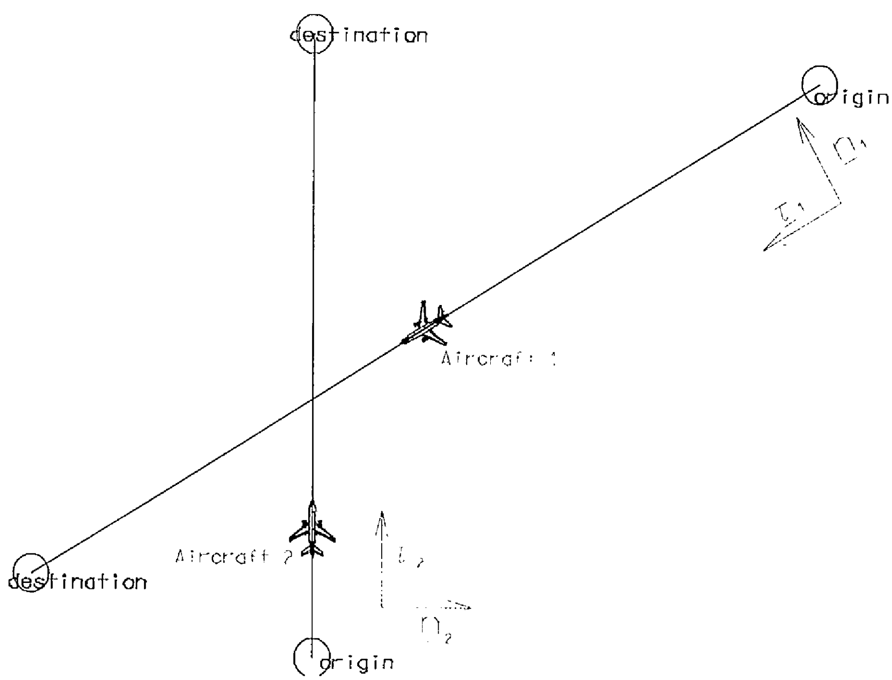


Figure 6.8: Intersection point between two aircraft's routes.

Two aircraft are requested to respect a minimum mutual distance separation while they are flying. Currently, the minimum allowed horizontal separation for en-route airspace is 5 nautical miles (nmi).

The program is developed based on the assumption that no alert is issued between aircraft, which consequently keep maintaining their route and speed. In this code, the starting points of the flights are the departure airports. However these starting points can be set in any other points where the aircraft's positions are perfectly known in an instant of time.

6.3.1 Data

First the code loads the lotus files containing arrival and scheduled arrival times of the involved flights. An example of this information is shown in Table 3.1. Then the code processes the necessary parameters that univocally describe aircraft's routes. The parameters involved are graphically shown in Figure 6.9 and here below listed:

- LT1: distance between departure and arrival airport of aircraft 1.
- LT2: distance between departure and arrival airport of aircraft 2.
- φ: relative angle between the two routes.
- L1: distance flown by aircraft 1 until the intersection point.
- L2: distance flown by aircraft 2 until the intersection point.

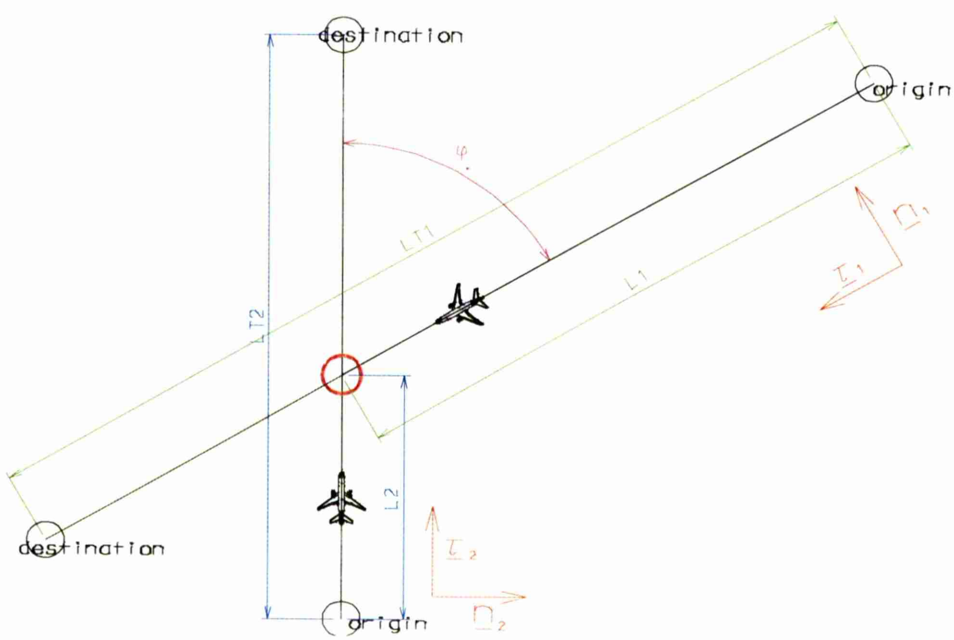


Figure 6.9: Parameters describing the two routes.

6.3.2 PDF construction

In this section, the PDFs of both the along-track and cross-track co-ordinates of the aircraft position will be built for a given flight[†]. Owing to the lack of data concerning aircraft position, worst case approximations are made.

6.3.2.1 PDF of aircraft along track position

To calculate the PDF of the aircraft position along the track $\mathbf{s}_\tau \subseteq \mathcal{G}$, the following approximations are made:

- aircraft's delay[††] is caused exclusively by variations of the velocity along track direction.
- the length of the route is fixed.

Under these two assumptions, the along track velocity $\mathbf{v}_\tau \subseteq \mathcal{F}$ is equal to the ratio between the route length, which is a scalar for hypothesis, and the travelling time which is still considered as a random variable.

$$\mathbf{v}_\tau = \frac{Length(scalar)}{Travelling\ time(random\,variable)} = \frac{\mathrm{d}(\mathbf{A},\mathbf{B})}{\mathbf{t}^{\mathrm{t}}_{\mathrm{AB}}} \qquad \begin{cases} \mathrm{d}(\mathbf{A},\mathbf{B}) \in \boldsymbol{R} \\ \mathbf{t}^{\mathrm{t}}_{\mathrm{AB}} \in \mathcal{E} \end{cases}$$

Once the velocity along the track is known, the space run by the aircraft at the instant time t^* is given by (see section 6.2.2):

$$\mathbf{s}_\tau = \mathbf{v}_\tau \mathrm{t}^* \qquad \begin{cases} \mathbf{s}_\tau \in \mathcal{G} \\ \mathbf{v}_\tau \in \mathcal{F} \\ \mathrm{t}^* \in \boldsymbol{R} \end{cases}$$

An example of PDF of along track space run by aircraft is shown in Figure 6.10. This graph displays the PDF of the position along the track $\mathbf{s}_\tau \subseteq \mathcal{G}$ for the instant time t^*=90 min after aircraft departure.

[†] As in the previous chapters, also here the word *flight* refers to a regular connection between departure and arrival airport. Do not misunderstand with the single aircraft that in a given journey goes from the origin to the destination.

[††] As in the previous chapters, delays can be positive or negative.

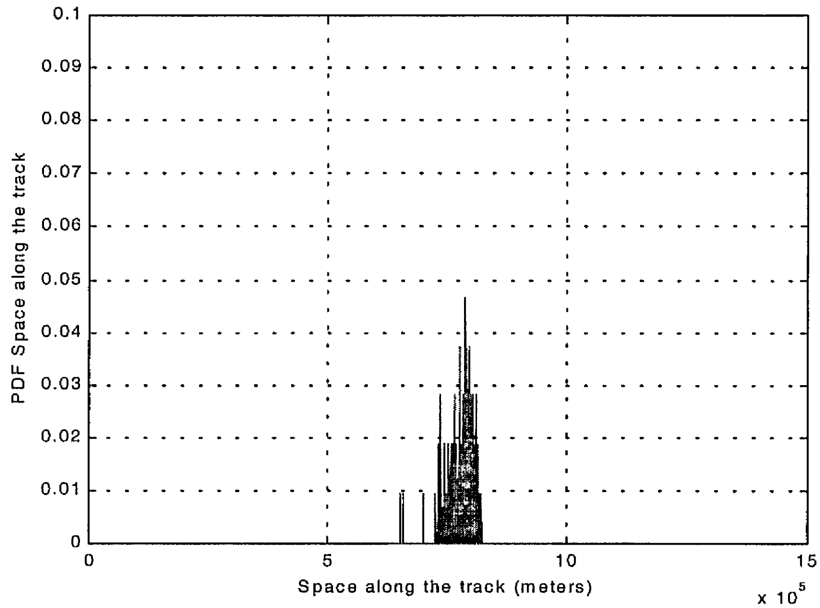


Figure 6.10: PDF of along track position in a fixed instant of time; LT= 800km t*=90min†.

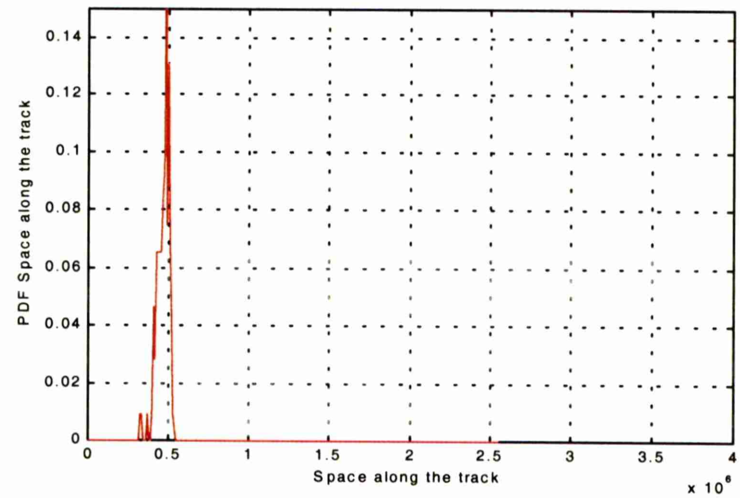
The x-axis represents the run distance†† as the origin of the axis is set at the departure airport. In other words, the x-axis demarcates the along track space run by the aircraft from the departure airport. The y-axis represents the probability that the aircraft is in a given position along the track in the fixed instant of time t^*=90 min.

Observing the time evolving of PDF of aircraft along track position, something intuitively already known is confirmed: the further ahead the prediction is made the more likely the estimation is less certain. Estimation of aircraft position is effected by less uncertainty at the beginning of journey when it is compared with estimation performed at the end of the journey. Figure 6.11 shows the along track PDF for different instant of times and shows that the PDF becomes more and more uncertain in correspondence of times further ahead.

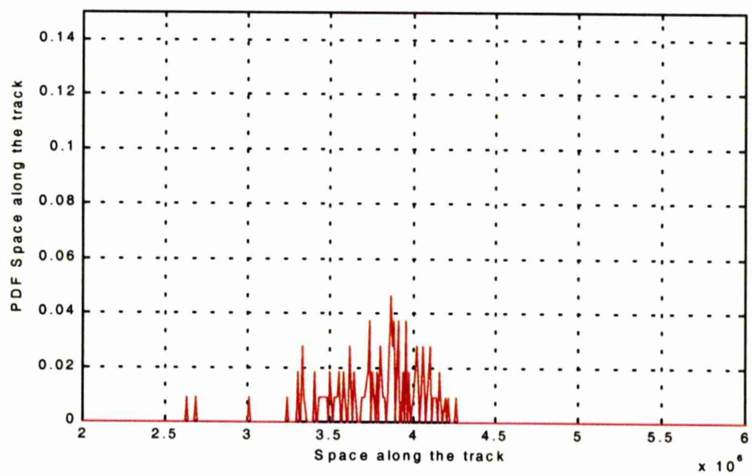
Figure 6.11 shows that in the instant t^* =55min the aircraft is situated within a range of 80km with a probability higher than 90%, while the maximum probability that the aircraft is in a given position is 15%. In the instant of time t^* =9h25min, the uncertainty is higher and the prediction of aircraft position is really difficult. In this case the maximum probability that the aircraft is in a given position is less than 5%.

† Time is referred to the departure time of the aircraft.

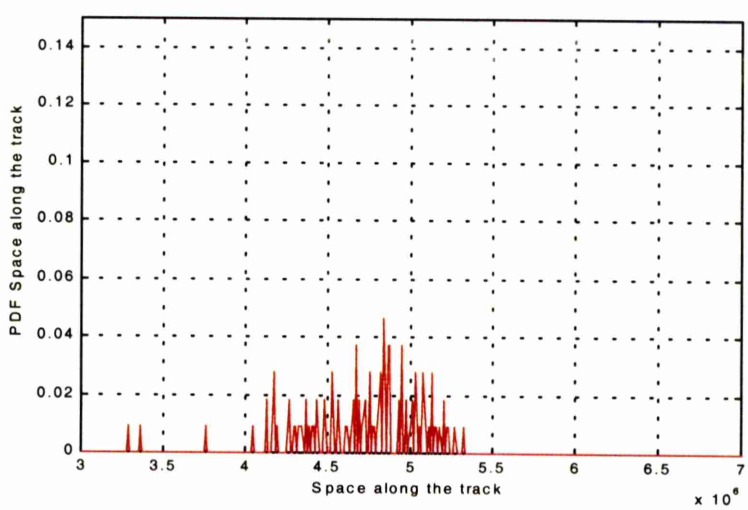
†† Note that the scale of the x-axe is 10^5m=100km.



$t^* = 55$min



$t^* = 5$h40min



$t^* = 9$h25min

Figure 6.11: Time evolving of the PDF of along track position.

6.3.2.2 PDF of aircraft cross track position

Suppose that the speed of the aircraft is fixed. Under this hypothesis, possible variations of the arrival time are only caused by changes of the route length. Arrival delays can be provoked either by aircraft flying in the right or in the left direction of its route, see Figure 6.12.

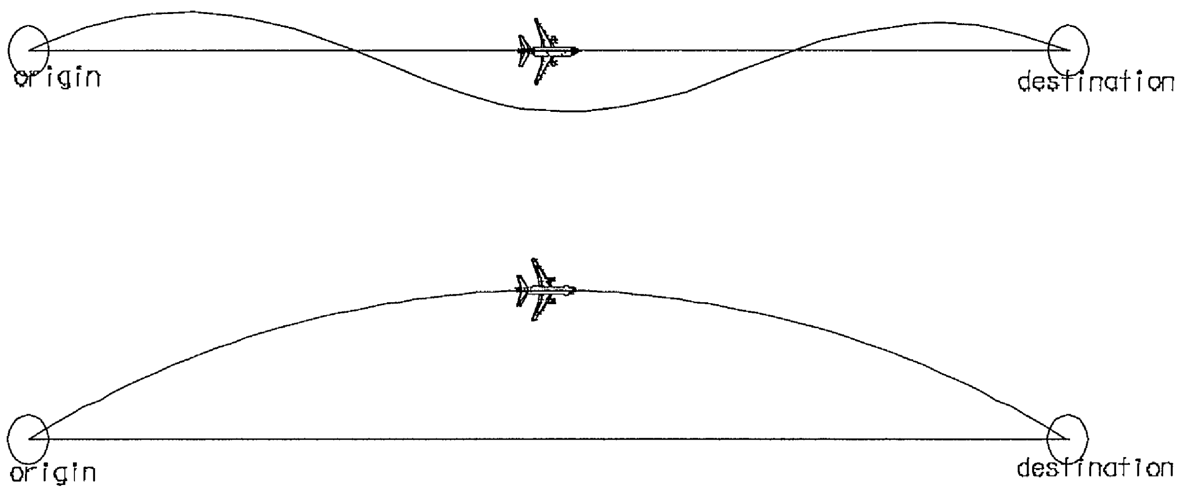


Figure 6.12: Aircraft are effected by lateral deviation during their journeys.

Aircraft during its journey is effected by lateral deviations, which are caused by many unpredictable factors (e.g. wind, adverse weather conditions, tracking and navigation error). This alteration from the planned trajectory causes inevitably an increase in the length route.

Owing to the lack of data describing the lateral deviation, some approximations are assumed:

- The PDF of the cross track position is symmetric with respect to the route and it is modelled as a normal distribution $\mathbf{s}_n = Normal(\mu_{s_n}, \sigma_{s_n})$.
- The mean and the variance of this normal distribution $\mathbf{s}_n = Normal(\mu_{s_n}, \sigma_{s_n})$ are supposed to be equal to the mean and variance of the random variable $\tilde{s}_n \subseteq \mathcal{G}$. Thus, $\mu_{s_n} = \mu_{\tilde{s}_n}$ and $\sigma_{s_n} = \sigma_{\tilde{s}_n}$.

This random variable $\tilde{\mathbf{s}}_{\mathrm{n}}$ is built under the hypothesis that arrival delays are due to extra lateral length run by the aircraft. Aircraft are assumed to run the route **AB** with a speed equals to the expected value of the along track velocity $\mathbf{v}_\tau \subseteq \mathcal{F}$.

$$E\{\mathbf{v}_\tau\} = \frac{\mathrm{d}(\mathbf{A},\mathbf{B})}{E\{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}}\}} \qquad \begin{cases} \mathbf{v}_\tau \subseteq \mathcal{F} \\ \mathrm{d}(\mathbf{A},\mathbf{B}) \in R \\ \mathbf{t}^{\mathbf{t}}_{\mathrm{AB}} \subseteq \mathcal{E} \end{cases}$$

The module† of the vector velocity $\underline{\mathbf{v}} \subseteq \mathcal{F}$ is given by:

$$\|\underline{\mathbf{v}}\| = \frac{\mathit{Length(random\ variable)}}{\mathit{Travelling\ time(scalar)}} = \frac{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}} E\{\mathbf{v}_\tau\}}{\dfrac{\mathrm{d}(\mathbf{A},\mathbf{B})}{E\{\mathbf{v}_\tau\}}} = \frac{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}}\mathrm{d}(\mathbf{A},\mathbf{B})}{E\{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}}\}^2} \qquad \begin{cases} \underline{\mathbf{v}} \subseteq \mathcal{F} \\ \mathrm{d}(\mathbf{A},\mathbf{B}) \in R \\ \mathbf{t}^{\mathbf{t}}_{\mathrm{AB}} \subseteq \mathcal{E} \end{cases}$$

In conservative hypothesis, the module of the lateral velocity can be assumed to be equal to the difference between the module of the velocity and the module of the along track velocity. Considering that the length of each side of a triangle is less than the sum of the lengths of the remaining two sides, for a given outcome ζ of the experiment $\boldsymbol{\Omega}$ the following inequality is derived:

$$\mathbf{v}_{\mathrm{n}}(\zeta) = \underline{\mathbf{v}}(\zeta) - \mathbf{v}_\tau(\zeta) \quad \Rightarrow \quad \|\mathbf{v}_{\mathrm{n}}(\zeta)\| \le \|\underline{\mathbf{v}}(\zeta)\| - \|\mathbf{v}_\tau(\zeta)\|$$

The module of the along track velocity denoted for simplicity by $\mathbf{v}_{\mathbf{n}}$ is equal to:

$$\mathbf{v}_{\mathrm{n}} = \frac{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}} E\{\mathbf{v}_\tau\}}{\dfrac{\mathrm{d}(\mathbf{A},\mathbf{B})}{E\{\mathbf{v}_\tau\}}} - E\{\mathbf{v}_\tau\} = \frac{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}}\mathrm{d}(\mathbf{A},\mathbf{B})}{E\{\mathbf{t}^{\mathbf{t}}_{\mathrm{AB}}\}^2} - E\{\mathbf{v}_\tau\} \qquad \begin{cases} (\mathbf{v}_\tau, \mathbf{v}_{\mathrm{n}}) \subseteq \mathcal{F} \\ \mathrm{d}(\mathbf{A},\mathbf{B}) \in R \\ \mathbf{t}^{\mathbf{t}}_{\mathrm{AB}} \subseteq \mathcal{E} \end{cases}$$

† The module is a random variable defined through the random variable $\underline{\mathbf{v}} \subseteq \mathcal{F}$ and the function $g(\underline{\mathbf{v}}) = \|\underline{\mathbf{v}}\|$.

Taking into account that if the aircraft arrives earlier then zero is assigned to $\tilde{s}_n \subseteq \mathcal{G}$, the cross track position is given by:

$$\tilde{s}_n = \begin{cases} v_n t^* & \text{if} \quad v_n \geq 0 \\ 0 & \text{if} \quad v_n < 0 \end{cases} \qquad \begin{cases} \tilde{s}_n \in \mathcal{G} \\ v_n \in \mathcal{F} \\ t^* \in R \end{cases}$$

$$s_n = Normal\left(\mu_{\tilde{s}_n}, \sigma_{\tilde{s}_n}\right) \quad \text{where} \quad \begin{cases} \mu_{\tilde{s}_n} = 0 \\ \left(\sigma_{\tilde{s}_n}\right)^2 = \mathrm{Var}(\tilde{s}_n) \end{cases}$$

Figure 6.13 shows an example of the PDF of aircraft cross track position $s_n \subseteq \mathcal{G}$ for t^*=2h50min.

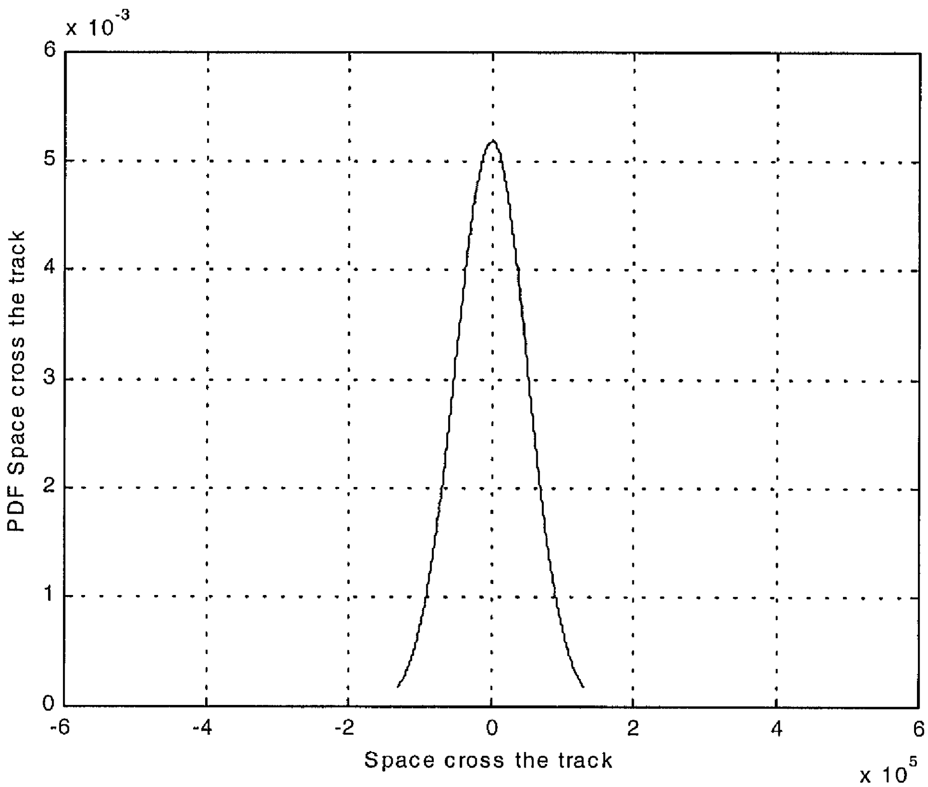


Figure 6.13: PDF of the aircraft cross track position $s_n \subseteq \mathcal{G}$ for t^*=2h50min.

Note that the PDF of the cross track position has less uncertainty when it is compared with the error along the track. In Figure 6.14 for t^*=9h25min the error is $o(10^5)$ while in Figure 6.11 always for t^*=9h25min the error is $o(10^6)$.

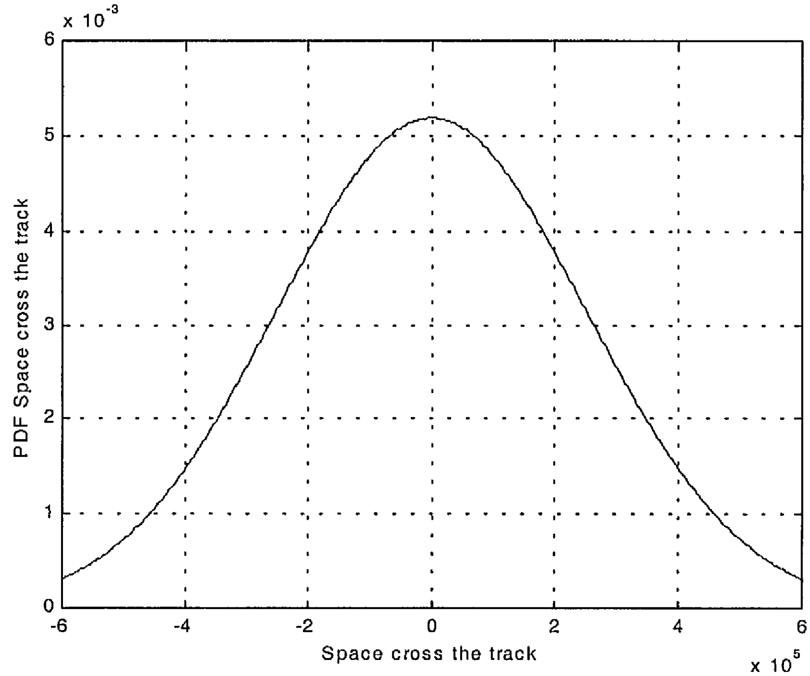


Figure 6.14: PDF of the position cross the track for t^*=9h25min .

6.3.3 PDF of aircraft position as a two dimensional random variable

As shown in section 5.2.3, aircraft position is a two-dimensional random variable $\underline{s}(s_\tau, s_n; t) \subseteq \mathcal{G}xT$, which is characterised by a bi-dimensional PDF. For simplicity to build this bi-dimensional PDF, along and cross track deviations are assumed to be independent events. If the along track position for a fixed $t^* \in \boldsymbol{R}$ is $\mathbf{s}_\tau(\zeta; t^*) = s_\tau^*$, then this value does not influence the cross track position $\mathbf{s}_n(\zeta; t^*) = s_n^*$ and vice-versa.

$$\Pr\{\mathbf{s}_n(\zeta; t^*) = s_n^* \mid \mathbf{s}_\tau(\zeta; t^*) = s_\tau^*\} = \Pr\{\mathbf{s}_n(\zeta; t^*) = s_n^*\} \qquad \forall (s_n^*, s_\tau^*) \in \boldsymbol{R}^2$$

This hypothesis is conservative because the independence of along and cross track positions entails the case where the aircraft follows the longest route en correspondence of the lowest speed. Under this assumption, the bi-dimensional PDF can be computed as product between along and cross track PDFs:

$$f_{\underline{s}(s_\tau, s_n)}(s_\tau, s_n; t) = f_{s_\tau}(s_\tau; t) f_{s_n}(s_n; t) \qquad \begin{cases} \forall (s_\tau, s_n) \in \boldsymbol{R}^2 \\ \forall t \in \boldsymbol{R} \end{cases}$$

Continuous functions are theoretically treated even if for computational application discrete functions are practically built. Given a grid in the airspace, the discrete bi-dimensional PDF solves the probability that the aircraft is in a given cell of the grid.

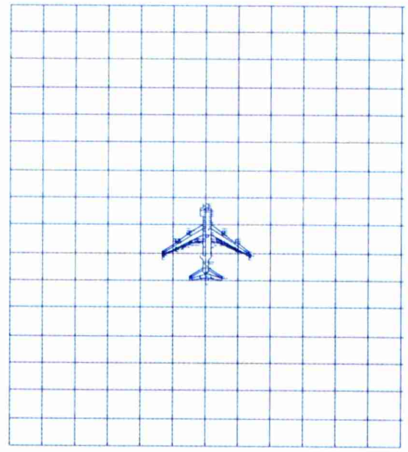


Figure 6.15: Grid in the airspace.

In Figure 6.16 the PDF of aircraft position $\underline{s}(s_\tau, s_n; t) \subseteq \mathcal{G}xT$ is shown.

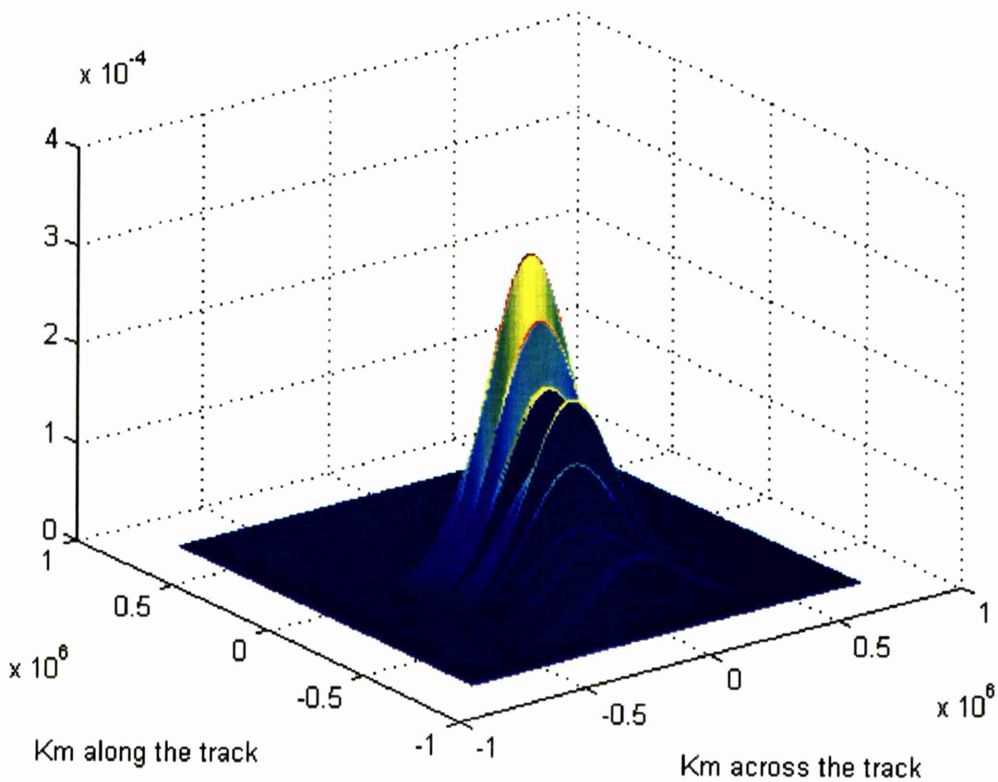


Figure 6.16: PDF of the bi-dimension random variable position of the aircraft.

The axes of this three-dimensional graphic shown in Figure 6.16 represent:

- τ-axis: space along the track run by the aircraft
- n-axis: space cross the track run by the aircraft
- z-axis: probability that in a fixed instant of time the aircraft is in a determinate position of the airspace. In other words, the probability that in a fixed instant of time the aircraft is in a determinate cell of the grid shown in Figure 6.15.

When the given instant time $t=t^*$ changes, the bi-dimensional PDF of aircraft position changes as the further in advance the estimation is made. The time evolving of this bi-dimensional PDF performs as the time evolving of along and cross track PDFs, see Figure 6.17.

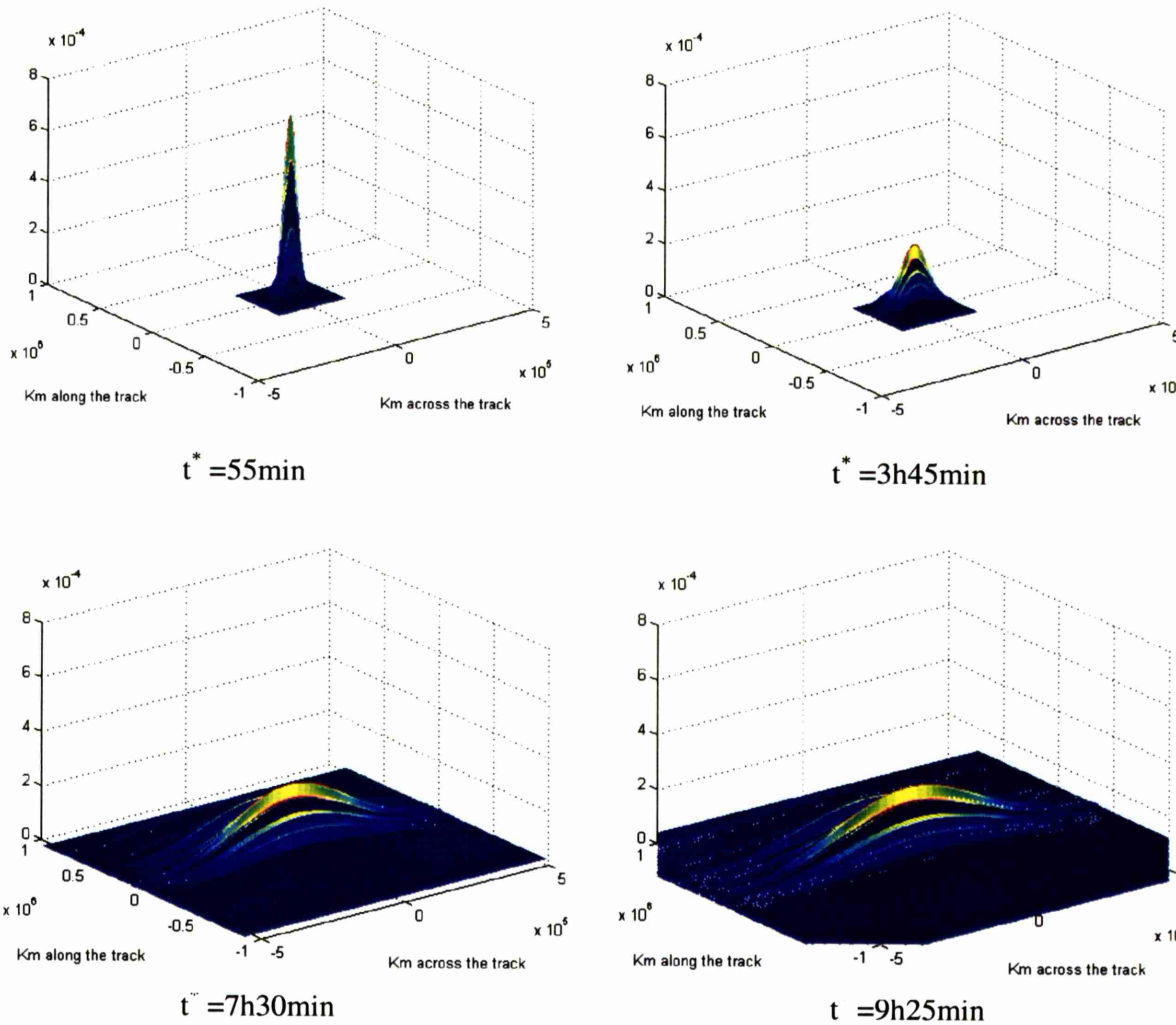


Figure 6.17: PDF of the bi-dimension random variable position of the aircraft for various instants of times during the flight time of the aircraft.

As clearly shown in Figure 6.17, the uncertainty along the track *o(10^6)* is greater than the uncertainty cross the track *o(10^5)*. The shape of the aircraft position PDF is narrower in cross track direction when it is compared with the PDF along track direction.

6.3.4 Probability of Conflict in a fixed instant of time

As shown in section 6.2.3.5, the probability of conflict $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ in a fixed instant of time $\Delta T_{12}=t^*+dt$ is defined as the probability that the mutual distance between two aircraft is less than ΔD=5 nautical miles (9260 meters).

$$\Pr\left\{\left\|\underline{s_1}(s_{1\tau},s_{1n};t^*)-\underline{s_2}(s_{2\tau},s_{2n};t^*)\right\|<\Delta D\right\}\quad\left\{\begin{array}{c}(\underline{s_1},\underline{s_2})\subseteq \mathcal{G}^2 xT^2\\ \Delta D\in \boldsymbol{R}\end{array}\right.$$

To calculate this probability, the developed code makes an approximation in order to simplify this computation. Instead of using the bi-dimensional PDFs of aircraft position for both aircraft, the program considers the bi-dimensional PDF of aircraft 2 and only the PDF along track position for aircraft 1 (see Figure 6.18 and Figure 6.19). In other words, it is assumed that the uncertainty cross the track of aircraft 1 does not exist. As the along track uncertainty is much higher than the cross track uncertainty, this approximation will not imply a significant alteration of results.

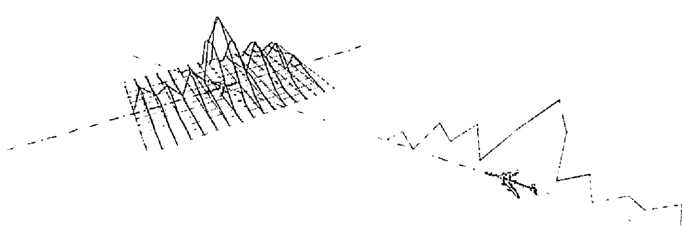


Figure 6.18: PDF used for aircraft 2 and aircraft 1. Prospective view.

The code treats discrete functions rather than continuos functions, hence, the program calculates the probability of conflict $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ using a grid in the airspace.

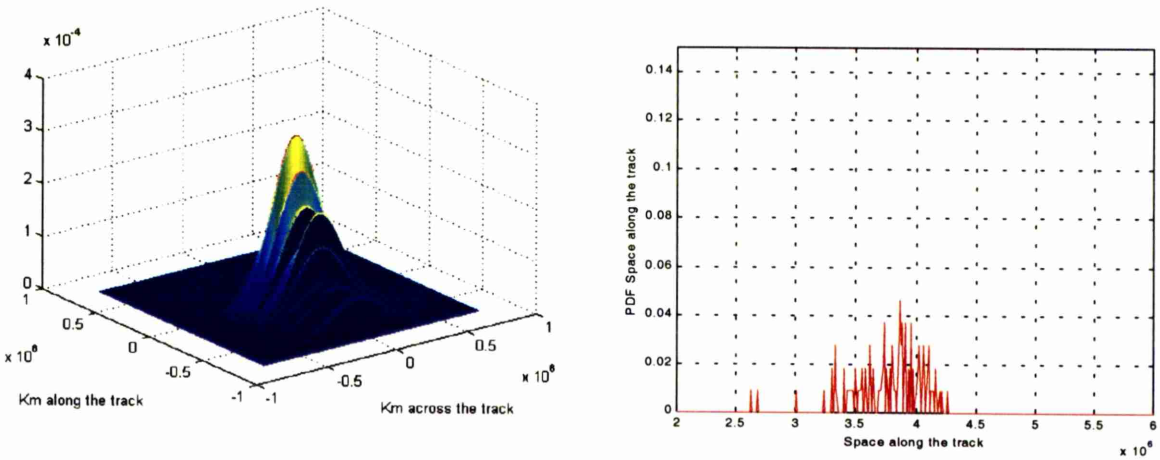


Figure 6.19: PDF used for aircraft 2 and aircraft 1.

The main steps followed by the code are here below listed:

1 First the program calculates the probability that aircraft 1 is in a given cell. The code multiplies this number by the probability that aircraft 2 is in a cell situated within a 5nmi radius circle centred on aircraft 1. As shown in Figure 6.20, a square area approximates the circular area in which the conflict is defined to occur. This additional approximation is made to simplify further the computation.

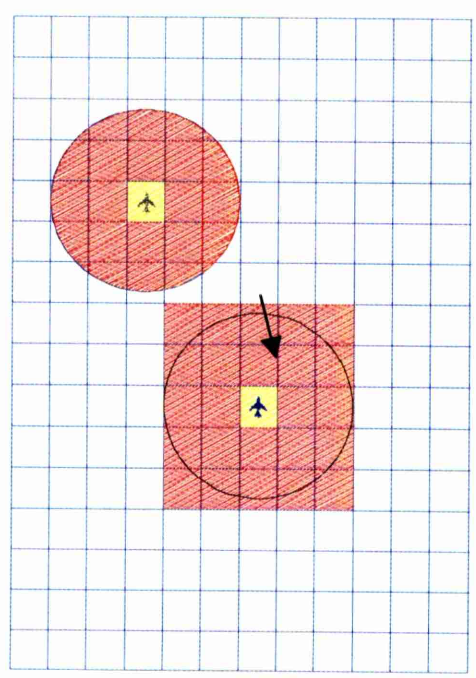


Figure 6.20: Approximation of the conflict area.

2 The operation described in point 1 is repeated for all the cells in which the probability that aircraft 1 is situated in these cells is different than zero.

3 Finally the $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ is obtained as sum of all the terms of the point 2.

The next expression resumes the explanation:

$$PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)=\sum_{i,j\in Z^2}\Pr\left\{\begin{matrix}\textit{aircraft 1 is in cell } (i,j), \textit{aircraft 2}\\ \textit{is in a cell inside a circle}\\ \textit{centred in}(i,j) \textit{ and radius } \Delta D\end{matrix}\right\}=$$

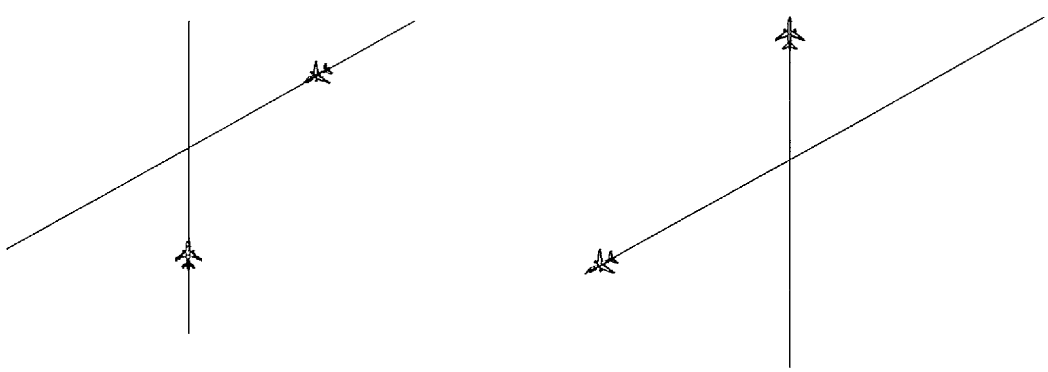
These events are incompatible because if aircraft 1 is in the cell (i,j) at the time t^*, aircraft 1 will not be in another cell in the same instant of time t^*.

$$=\sum_{i,j\in Z^2}\sum_{k,l=-\Delta D}^{\Delta D}\Pr\{\textit{aircraft 1 is in cell } (i,j), \textit{aircraft 2 is in a cell} (i+k,j+l)\}=$$

These events are assumed to be independent, see section B.3.1 for the proof.

$$=\sum_{i,j\in Z^2}\sum_{k,l=-\Delta D}^{\Delta D}\Pr\{\textit{aircraft 1 is in cell } (i,j)\}\Pr\{\textit{aircraft2 is in a cell} (i+k,j+l)\}$$

To shorten the duration of the program, the starting point is set when both aircraft are close to the intersection point and $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ is still zero. Then, the program starts and continues running until $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ becomes again zero. In some aircraft configuration, computer runs take a long time even after both aircraft passed through the intersection point, as shown in Figure 6.21.



a b

Figure 6.21: a) Starting point of the program; b) Final point of the program.

In Figure 6.22, the probability of conflict in a fixed instant of time $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ $\forall t^* \in [0,36$ minutes] is shown. In the *x*-axis the instant of time $t^* \in [0,36$ minutes] is displayed while in the *y*-axes the probability of conflict that occur in that given instant of time is plotted.

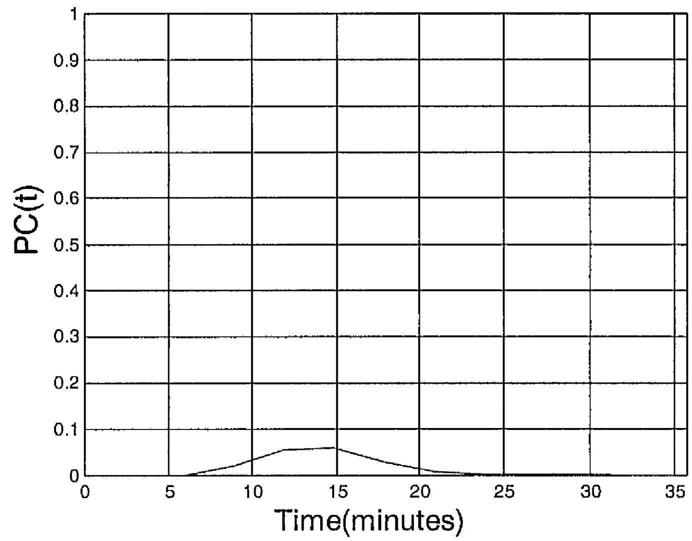


Figure 6.22: PC(t) versus time.

The maximum $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$, which occurs during the encounter, is the most important parameter to take into account. Figure 6.22 shows that the maximum value of { $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ with $t^* \in [0,36$ minutes]} is approximately 6%.

6.3.5 Probability of conflict for the encounter in an interval of time

More important than the probability of conflict in a fixed instant of time $PC_{12}^{\Delta D=5nmi,\Delta T=t^*+dt}(C,t^*)$ $\forall t^* \in \boldsymbol{T}$ is the $PC_{12}^{\Delta D=5nmi,\Delta T=t+dt}(C,\boldsymbol{T})$ for the encounter in the interval of time $\boldsymbol{T}$. The probability of conflict in the interval time $PC(C,\boldsymbol{T})$ is a metric of the risk taken during the total encounter.

The $PC(C,\boldsymbol{T})$ is calculated using the next expression:

$$PC_{12}^{\Delta D=5nmi,\Delta T=t+dt}(C,\boldsymbol{T}) = \Pr\{(conflict\ in\ t_1)\bigcup\ldots\bigcup(conflict\ in\ t_n)\} =$$

These events are incompatible because the conflicts occur in different instant of times. Supposing that ***T***=[0,T], the expression becomes:

$$= \frac{\int_0^T PC_{12}^{\Delta D=5nmi,\Delta T=t+dt}(\boldsymbol{C},t)dt}{T} \approx \frac{\sum_{t_i=0}^{T} PC_{12}^{\Delta D=5nmi,\Delta T=t_i+dt}(\boldsymbol{C},t_i)\Delta t}{T}$$

$$t_i \in (0, \Delta t, 2\Delta t, ..., n\Delta t = T)$$

Δt = is the period of time between the calculations of the $PC_{12}^{\Delta D=5nmi,\Delta T=t_i+dt}(\boldsymbol{C},t_i)$.

T = interval of time that last the encounter.

t_I = instant of time when the $PC_{12}^{\Delta D=5nmi,\Delta T=t_i+dt}(\boldsymbol{C},t_i)$ is calculated.

So the $PC(\boldsymbol{C},\boldsymbol{T})$ can be calculated as the area below the curve $\frac{PC(\boldsymbol{C},t_i)*\Delta t}{\Delta T}$ versus t_i shown in Figure 6.23.

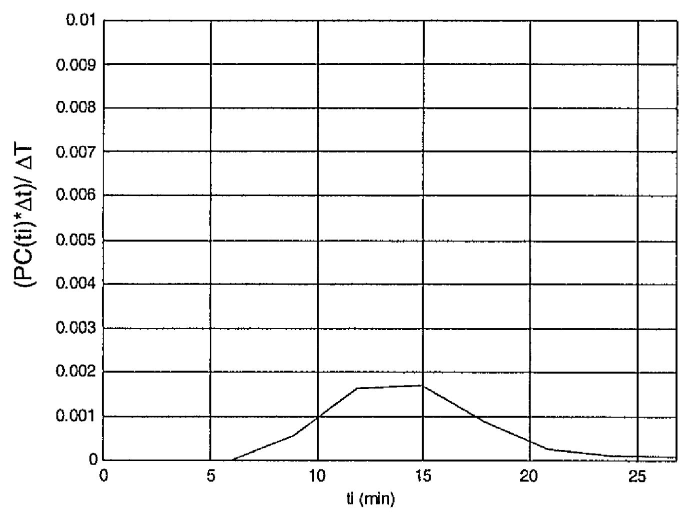


Figure 6.23: $\frac{PC(\boldsymbol{C},t_i)*\Delta t}{\Delta T}$ versus t_i.

6.3.6 Minimising the probability of conflict

As explained in section 6.2.4, the goal of the program is that:

- the probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(\boldsymbol{C},\boldsymbol{T})$ is lower than a fixed value $\alpha \in [0,1]$ with the constraint that $\forall t \in \boldsymbol{T}$ the probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(\boldsymbol{C},t)$ does not exceed a given probability $\beta \in [0,1]$.

The only control parameter used by the program is the mutual distance between aircraft, as clearly shown in Figure 6.24.

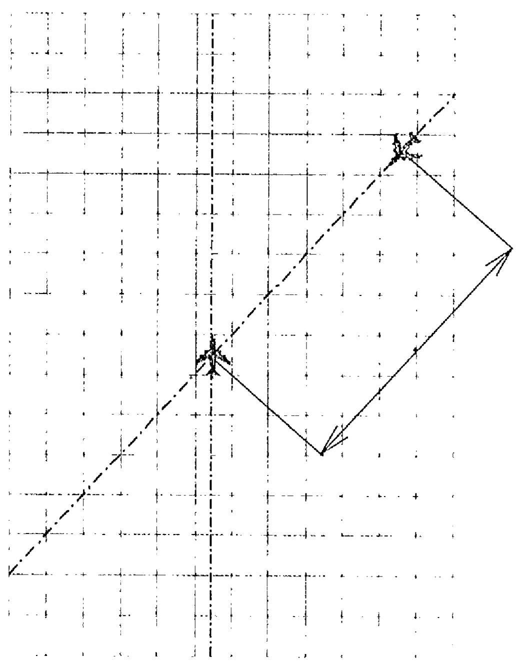


Figure 6.24: Control parameter used by the program.

If this distance increases, aircraft will be further distanced during the whole encounter and consequently $PC(\boldsymbol{C},\boldsymbol{T})$ will decrease. The program calculates the minimum distance to shift aircraft 1 to enable $PC(\boldsymbol{C},\boldsymbol{T})$ be lower than a fixed percentage $\alpha \in [0,1]$. Once this distance is computed, the code calculates $\forall t \in \boldsymbol{T}$ the probability of conflict $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(\boldsymbol{C},t)$. If this probability exceeds the given value $\beta \in [0,1]$, the program will increase the control parameter and then it will compute again $PC(\boldsymbol{C},\boldsymbol{T})$ and $PC_{12}^{\Delta D=\Delta D_{12},\Delta T=t+dt}(\boldsymbol{C},t)$. This procedure will be implemented until the code finds the optimal aircraft distance, which satisfies all defined constraints. Once this optimal distance is found, the new strategic scheduling can be calculated.

6.4 Results

This section presents result achieved analysing an encounter between a long-haul flight and a short-haul flight. These results are theoretical because the data used to build the PDFs come from two flights arriving at Glasgow international airport, which do not have any hazardous of mid-air conflict. However, the task of this thesis is only to propose a methodology for solving airborne separation minima at strategic planning level.

6.4.1 Effect of the distance

The distance between aircraft is the only control parameter used by the code. In this section, distance's effect of changing the probability of conflict will be analysed. As extensively explained in sections 6.3.2 and 6.3.3, aircraft position PDF changes its shape with respect to the time. Consequently if the intersection point is situated near the departure airports (case A of Figure 6.25), the uncertainty would be small and aircraft estimated positions would be accurate. Whereas if the two trajectories intersect at a point close to the arrival airports (case B of Figure 6.25), the uncertainty would be high and consequently would be difficult to predict the occurrence of a conflict.

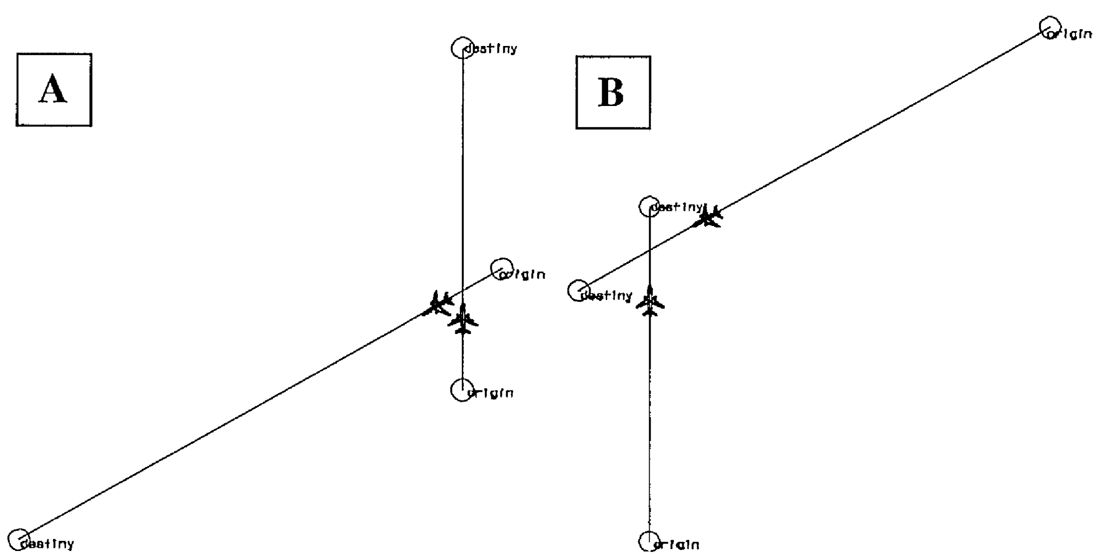


Figure 6.25: Intersection point in two different relative positions.

Figure 6.26 shows an example in which the intersection point is close to both departure airports L1=400Km and L2=400Km (See Figure 6.9 to remind the involved parameters). The graph displays the probability of conflict in the time interval T=[0,50mins] and in the region C PC(C,T) versus the shifted distance.

φ=60°, LT1=6000km, LT2=800km, L1=400km, L2=400km

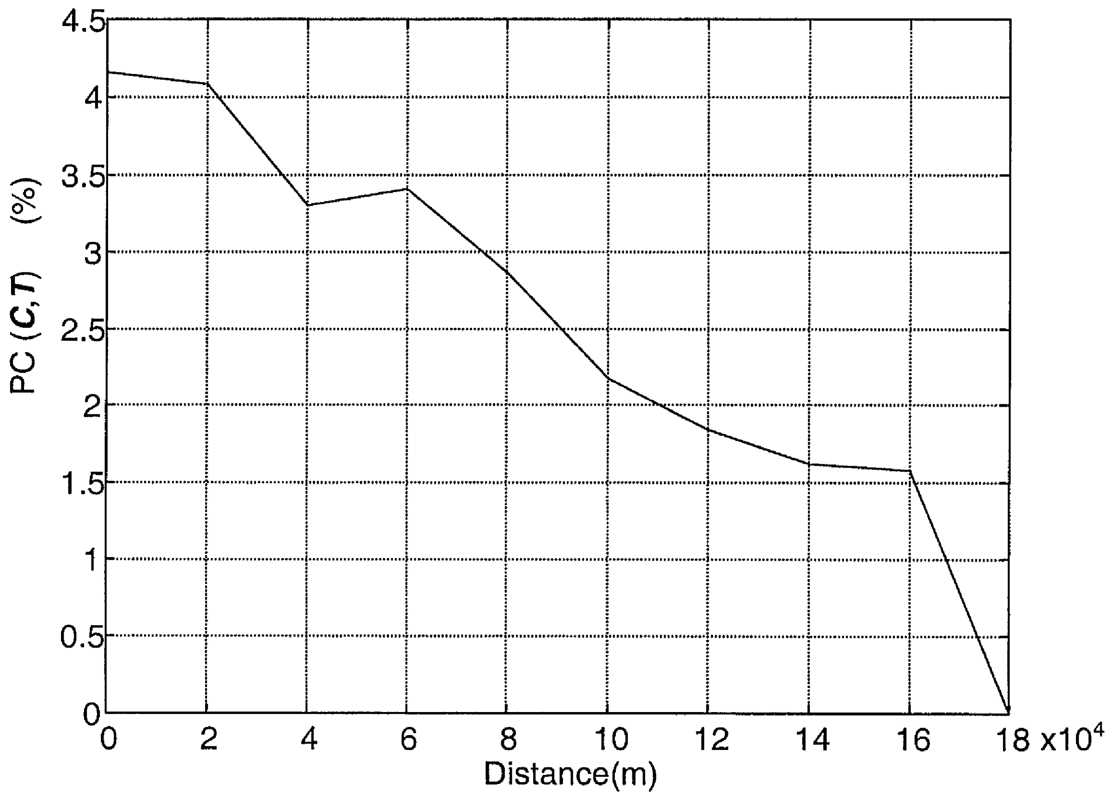


Figure 6.26: PC(C,T) vs. Shifted distance. Case A.

An increase of the shifting distance procures a reduction of PC(C,T), although sometime an opposite tendency is observed because of the irregular PDFs shapes. The unknown shifting distance is computed through Figure 6.26 analysis, for instance the minimum distance is 90 km to achieve PC(C,T) less than 3%. Figure 6.27 shows an example in which the intersection point is far from both departure airports. Some interesting results are achieved and discussed.

φ=60°, LT1=6000km, LT2=800km, L1=5000km, L2=700km

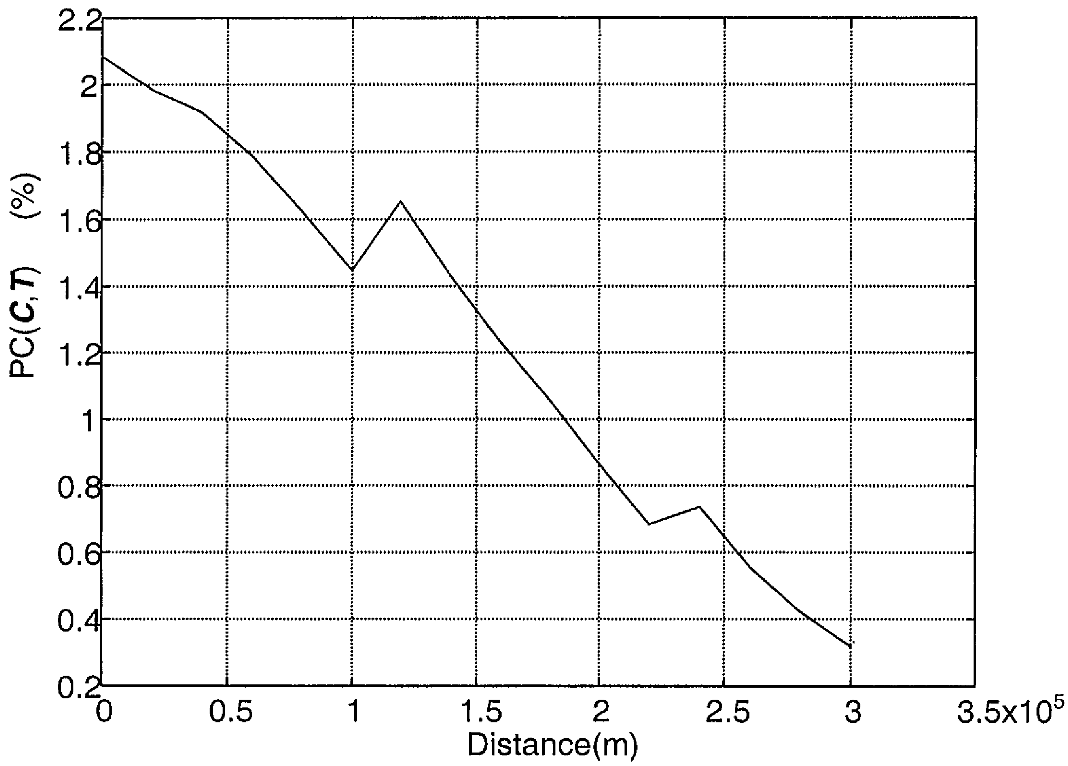


Figure 6.27: PC(C,T) vs. shifted distance. Case B.

In case B of Figure 6.25, PDFs of aircraft position are very broad for both planes. The uncertainties effecting the estimation of aircraft positions are high and, hence, the probability that both aircraft are within a 5 nmi radius circle is low. To achieve PC(C,T) less than 2%, the shifting distance is shorter in case B than in case A. Whereas to reduce PC(C,T) below 1%, the shifting distance is always shorter in case A when it is compared with case B. For instance to achieve PC(C,T) less than 0.2%, the minimum shifted distance is 170 km in case A while above 300 km in case B.

Table 6.2 shows the shifted distance for different L1 while a constant value is assigned to L2. The shifted distance increases as soon as the intersection point gets closer to the arrival airport of aircraft 1.

L1 (km)	L2 (km)	Shifted Distance (km)
3000	400	147
4000	400	163
6000	400	185

φ=60°, LT1=6000km, LT2=800km, $\mathrm{PC}(\boldsymbol{C},\boldsymbol{T})$<0.5%

Table 6.2: L1-L2-D.

In addition to the probability of conflict in a given region and in a given interval time $\mathrm{PC}(\boldsymbol{C},\boldsymbol{T})$, another important parameter to take into account is the probability of conflict in each time instant $\mathrm{PC}(\boldsymbol{C},t^*)$ $\forall$ $t^*\in\boldsymbol{T}$. Figure 6.28 and Figure 6.29 show the maximum value of $\mathrm{PC}(\boldsymbol{C},t^*)$ $\forall$ $t^*\in\boldsymbol{T}$ occurring in case A and case B.

CASE A:

φ=60°, LT1=6000km, LT2=800km, <u>L1=400km, L2=400km</u>

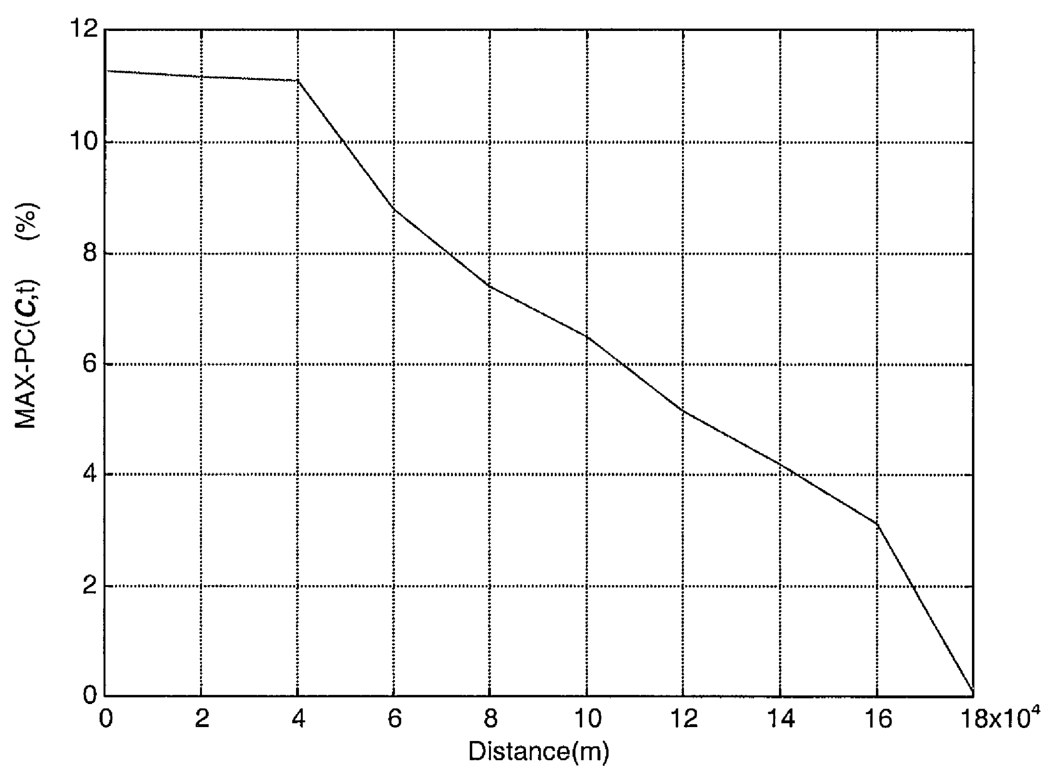


Figure 6.28: MAX-PC($\boldsymbol{C}$,t) vs. shifted distance. Case A.

CASE B:

φ=60°, LT1=6000km, LT2=800km, L1=5000km, L2=700km

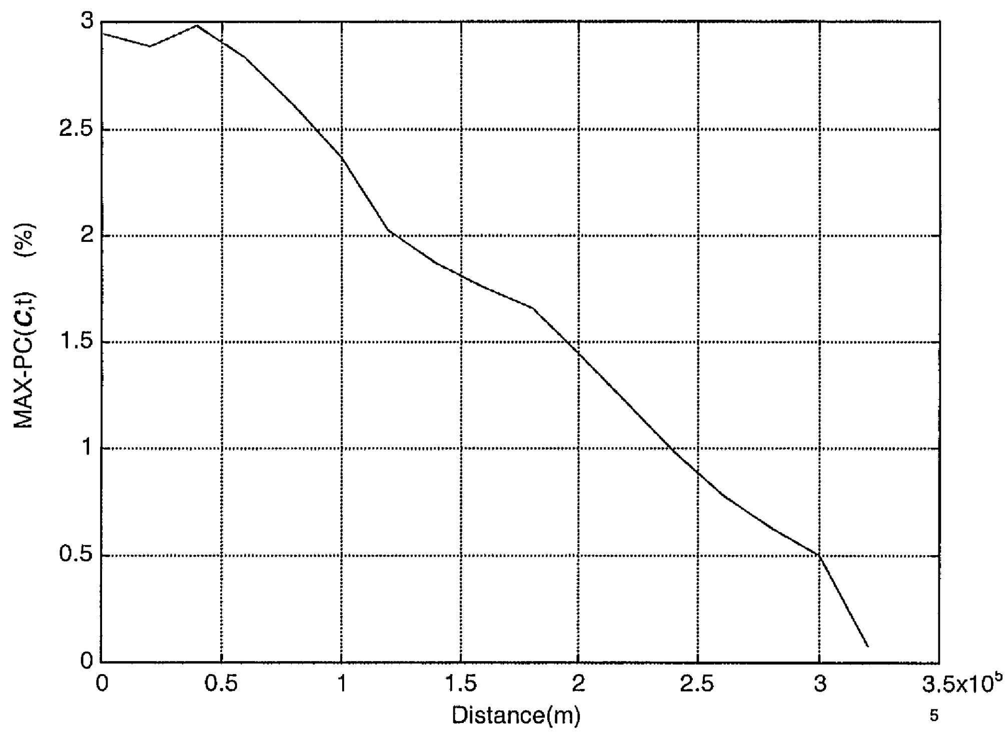


Figure 6.29: MAX-PC(*C*,t) vs. shifted distance. Case B.

Figure 6.28 and Figure 6.29 confirm what explained before. In case B, the prediction of aircraft position is effected by a high uncertainty. In other words, aircraft has a small probability of being in a given position. Therefore even for low value of shifting distance, the maximum probability of conflict in an instant of times is low as shown in Figure 6.29. In case A, the probability of conflict in an instant of times is high for low value of shifting distance in contrast $\mathrm{PC}(C, t^*)$ falls quicker to zero when it is compared with case B as shown in Figure 6.28.

Although $\mathrm{PC}(\boldsymbol{C}, \boldsymbol{T})$ is smaller in case B than in case A, case B is definitely worse for en-route capacity when it is compared with case A. To avoid risk of collision the shifting distance is lower in case A than in case B. These longer shifting distances effecting case B diminish considerably the number of aircraft which can fly in a given airspace region ***C***. In further works, the uncertainty of aircraft position can be reduced through data link messages between aircraft and ground using a similar methodology to the technique shown in Chapter 5.

6.4.2 Effect of the relative angle

The relative angle φ between aircraft trajectories is the second main parameter, which strongly effects $PC(C,T)$. There is symmetry with respect to the along track direction of aircraft 2 because cross track PDF position was modeled as a Gaussian distribution (see section 6.3.2.2). Therefore, to study the effects of the angle only the analysis of φ between 0° and 180° is enough.

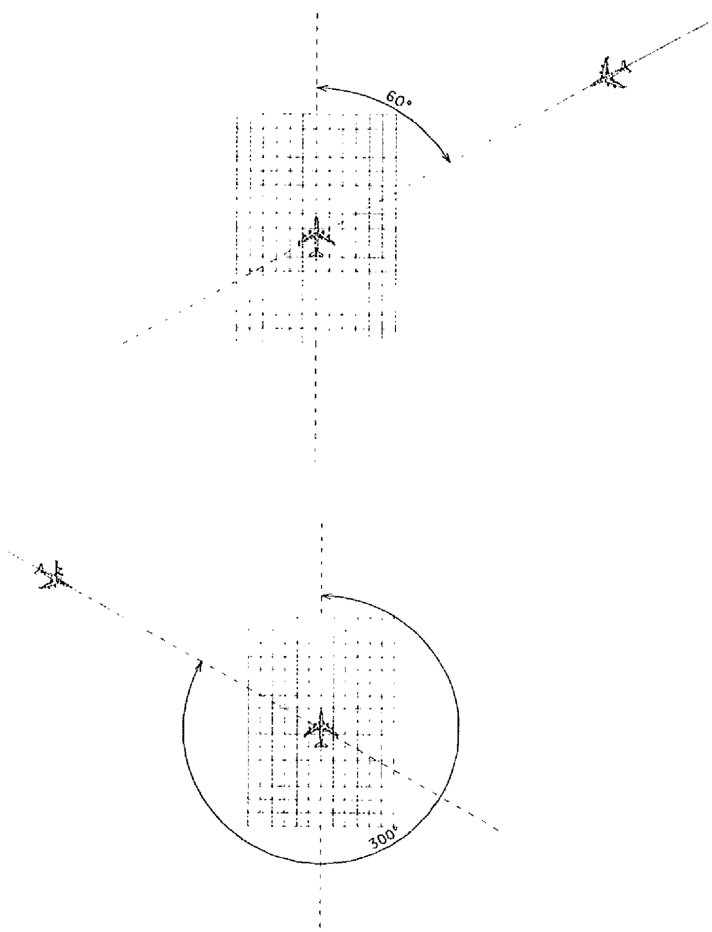


Figure 6.30: There is symmetry with respect to the along track direction of aircraft 2.

Table 6.3 shows the dependence between the shifting distance and the angle between the trajectories.

LT1=6000km, LT2=800km, L1=1000km, L2=400km, $PC(C,T)<0.5$ %.

φ°	**Shifting Distance (km)**
20°	123
40°	150
60°	152
80°	157
100°	132
120°	98
140°	94
160°	101

Table 6.3: φ vs. Shifting distance.

Figures 6.31, Figure 6.32, Figure 6.33 and Figure 6.34 show the dependence between $PC(C,T)$ and the shifting distance for different values of the angle φ.

φ=20°, L1=1000km; L2=400km; LT1=6000km, LT2=800km

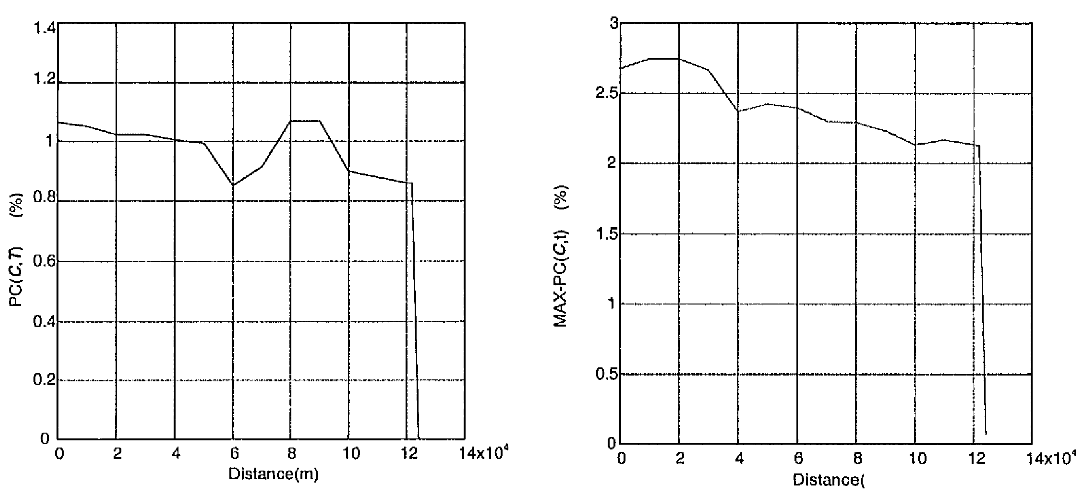


Figure 6.31: $PC(C,T)$ vs. shifted distance and MAX-PC vs. shifted distance for φ=20°.

In Figure 6.31 the relative angle between routes is set to a value equal to 20°. The $PC(C,T)$ and the maximum probability of conflict fall both very quickly to zero. In this case, the minimum distance to shift aircraft 1 is easy to identify. The shapes of the graphs shown in Figure 6.31 have a physically justification. If the angle were 0°, there would always be conflict between aircraft. Little higher value of the angle φ makes this situation be similar apart the fact that now for a given shifted distance there is a possibility to avoid the conflict.

φ=80°, L1=1000km; L2=400km; LT1=6000km, LT2=800km

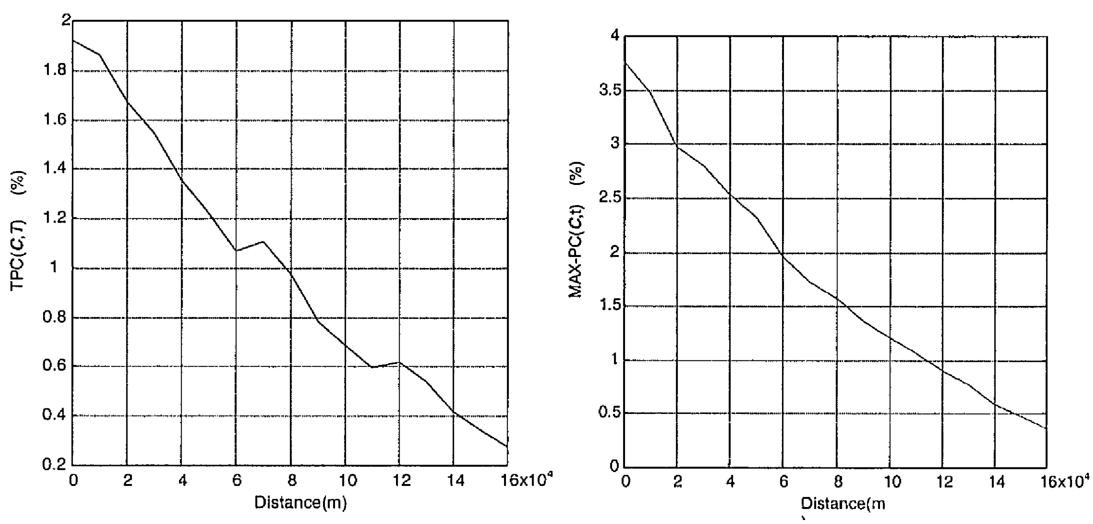


Figure 6.32: $PC(C,T)$ vs. shifted distance and MAX-PC vs. shifted distance φ=80°.

φ=100°, L1=1000km; L2=400km; LT1=6000km, LT2=800km

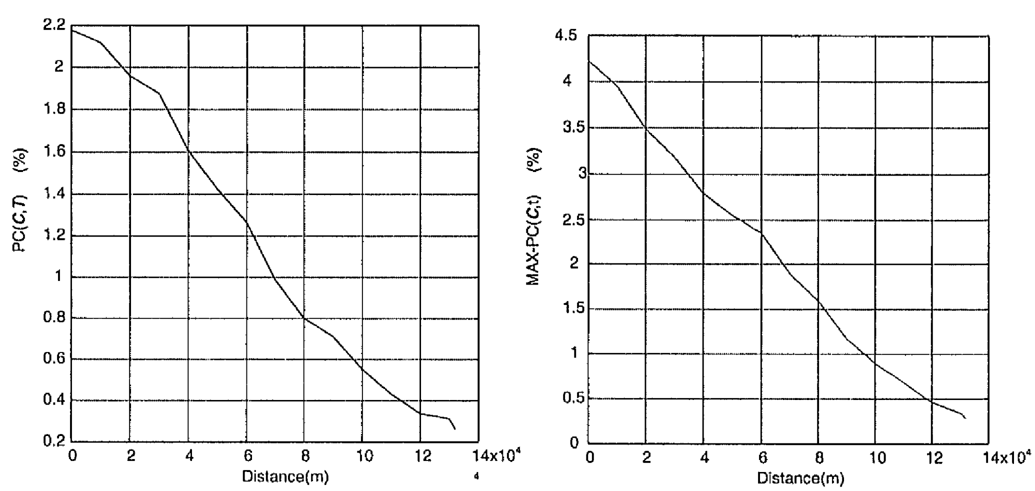


Figure 6.33: $PC(C,T)$ vs. shifted distance and MAX-PC vs. shifted distance φ=100°.

Figure 6.32 and Figure 6.33 show cases in which the angle is 80° and 100°. The graph of $PC(C,T)$ and the graph of the maximum probability of conflict versus shifted distance are different from the case shown in Figure 6.31. The curves decrease smoothly as the shifted distance increases. Depending on the chosen value of $PC(C,T)$, different shifted distances are computed. Therefore, an unequivocal minimum distance does not exist to ensure a safe manoeuvre.

φ=160°, L1=1000km; L2=400km; LT1=6000km, LT2=800km

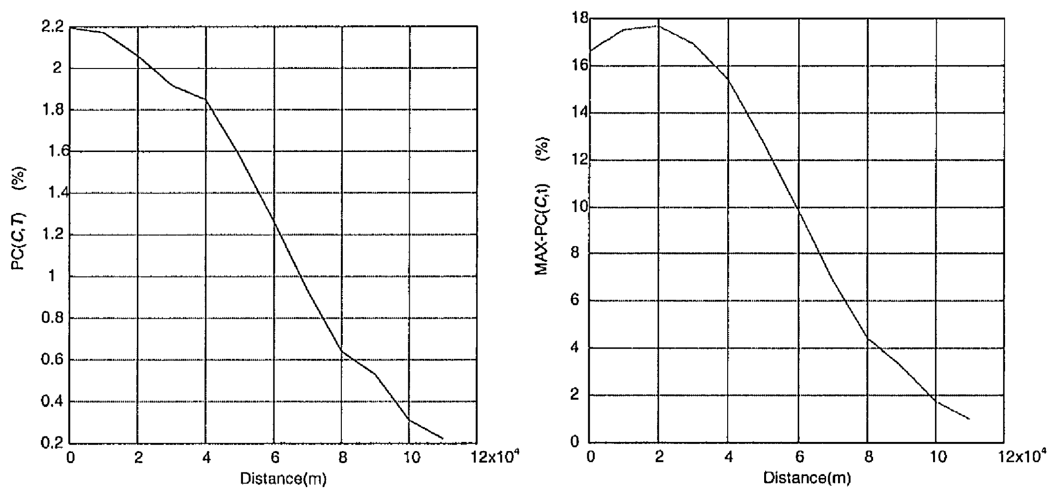


Figure 6.34: $PC(C,T)$ vs. shifted distance and MAX-PC vs. shifted distance φ=160°.

In Figure 6.34 the relative angle between routes is 160°. Consequently the aircraft are almost flying in the same direction. In this example aircraft 2 is faster than aircraft 1, the $PC(C,T)$ decreases very fast as the shifted distance increases.

6.5 Further developments: network of airports and sectors

In the works [26] and [31] multi-airport GHP are formulated while [32] presents a 0-1 integer-programming model to solve en route airspace (see sections 1.6.2 and 1.6.3). In section 3.4, the airport capacity is modelled as a multi dimensional random variable. Section 5.5 presents a methodology in which the departure times are modelled as real random variables proposing a multi airport dynamic probabilistic GHP.

This section will combine

- the model of airport capacity shown in Chapter 3
- the model to solve the strategic and tactical planning presented in Chapter 4 and 5
- the model to reduce congestion in the airspace shown in this Chapter

to statistically increase the throughput of aircraft in a given network of airports and sectors.

Let $\boldsymbol{K}$={1,2,...K} be a set of airports, let $\boldsymbol{T}$={1,2,...T} be an ordered set of time periods and let $\boldsymbol{S}$={1,2,...S} be a set of sectors. Given a set of flights $\boldsymbol{F}$={1,2,...F}, $\forall f \in \boldsymbol{F}$ the departure airport of the flight f$\in\boldsymbol{F}$ (denoted by $k_f^d \in \boldsymbol{K}$), the arrival airport of the flight f$\in\boldsymbol{F}$ (denoted by $k_f^a \in \boldsymbol{K}$), the ground delay cost function of the flight f$\in\boldsymbol{F}$ (denoted by $c_f^{gh}(t) \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F}$ are assumed to be known.

Furthermore $\forall$ s$\in\boldsymbol{S}$, the flights $(i,j) \in \boldsymbol{F}^2$ that will cross a given sector s$\in\boldsymbol{S}$ during the interval time $\boldsymbol{T}$ are known. Furthermore $\forall$ k$\in\boldsymbol{K}$, the capacity $\underline{\mathbf{C}}_k(\mathbf{D},\mathbf{D}^s,\mathbf{A},\mathbf{A}^s) \subseteq \mathcal{K}\times\boldsymbol{K}$ of the departure and arrival airport of the flight f$\in\boldsymbol{F}$ is assumed to be known. The decision variable is $t_{sf}^d \in \boldsymbol{R}\times\boldsymbol{F}$, which is the scheduled departure time of the flight f$\in\boldsymbol{F}$. $\mathbf{d}_f^d \subseteq \mathcal{C}\times\boldsymbol{F}$ is the departure delay. This is a real random variable expressing the amount of ground hold delay to assign to the flight f$\in\boldsymbol{F}$. This is a real random variable as the amount of ground hold delay is unknown until the day of the operation. Airborne delay is disappeared because with this tactical planning all the airborne delay is transformed in ground hold delay. As explained in section 4.2.1, departure times $\mathbf{t}_f^d \subseteq \mathcal{D}\times\boldsymbol{F}$ are the sum of departure delays $\mathbf{d}_f^d \subseteq \mathcal{C}\times\boldsymbol{F}$ and of scheduled departure times $t_{sf}^d \in \boldsymbol{R}\times\boldsymbol{F}$. The assignment decision variables are u_{ft} and v_{ft},, which are equal to 1 if the flight f$\in\boldsymbol{F}$ is allowed to take-off or to land at the period t$\in\boldsymbol{T}$.

The goals are:

- to minimise the expected value of the total cost expressed as sum of the ground delays allocated to the flights of the set $\boldsymbol{F}$.

- to minimise the probability of conflict between aircraft $(i, j) \in \times \boldsymbol{F}^2$ crossing a given sector s∈$\boldsymbol{S}$ in the interval time $\boldsymbol{T}$.
- to maximise the probability that the airport deal with the number of flights planned to arrive and to depart.

$$\min\left(\sum_{\mathrm{f}=1}^{F} E(c_f^{gh}\mathbf{d}_\mathrm{f}^\mathrm{d})\right) \qquad \begin{cases} c_f^{gh} \in \boldsymbol{R}\times\boldsymbol{T}\times\boldsymbol{F} \\ \mathbf{d}_\mathrm{f}^\mathrm{d} \subseteq \boldsymbol{\mathcal{C}}\times\boldsymbol{F} \end{cases}$$

$$\min\left(\Pr\begin{Bmatrix}\text{aircraft } i \text{ and } j \\ \text{conflicts in } \mathrm{s}\in \boldsymbol{S} \\ \text{in } \mathrm{t}\in \boldsymbol{T}\end{Bmatrix}\right) \qquad \begin{cases} (i,j)\in \boldsymbol{F}^2 \\ \forall \mathrm{s}\in \boldsymbol{S} \\ \forall \mathrm{t}\in \boldsymbol{T} \end{cases}$$

$$\max\left(\Pr\left\{\underline{\mathbf{C}_\mathrm{k}}\left(\mathbf{D}_\mathrm{k} \le \sum_{\mathrm{f};\mathrm{k}_\mathrm{f}^\mathrm{d}=\mathrm{k}} u_{\mathrm{ft}}, \mathbf{A}_\mathrm{k} \le \sum_{\mathrm{f};\mathrm{k}_\mathrm{f}^\mathrm{a}=\mathrm{k}} v_{\mathrm{ft}} \mid \mathbf{D}_\mathrm{k}^\mathrm{s} = d^s, \mathbf{A}_\mathrm{k}^\mathrm{s} = a^s\right)\right\}\right) \qquad \begin{cases} \mathbf{C}_\mathrm{k} \subseteq \boldsymbol{\mathcal{K}}\times \boldsymbol{K} \\ \left(d^s, a^s\right)\in \boldsymbol{N}^2 \end{cases}$$

subject to the constraints:

$$\mathbf{t}_\mathrm{f}^\mathrm{d} = \mathrm{t}_\mathrm{sf}^\mathrm{d} + \mathbf{d}_\mathrm{f}^\mathrm{d} \qquad \begin{cases} \mathbf{t}_\mathrm{f}^\mathrm{d} \subseteq \boldsymbol{\mathcal{D}}\times\boldsymbol{F} \\ \mathrm{t}_\mathrm{sf}^\mathrm{d} \in \boldsymbol{R}\times\boldsymbol{F} \\ \mathbf{d}_\mathrm{f}^\mathrm{d} \subseteq \boldsymbol{\mathcal{C}}\times\boldsymbol{F} \end{cases}$$

$$\sum_{\mathrm{t}\in \boldsymbol{T}} u_{\mathrm{ft}} = 1 \qquad \mathrm{f}\in \boldsymbol{F}$$

$$\sum_{\mathrm{t}\in \boldsymbol{T}} v_{\mathrm{ft}} = 1 \qquad \mathrm{f}\in \boldsymbol{F}$$

$$u_{\mathrm{ft}}, v_{\mathrm{ft}} \in \{0,1\}$$

These constraints mean that for every flight f∈$\boldsymbol{F}$ exactly one of the variables u_{ft} must be equal to 1 and the others must be equal to zero, and similarly for the variables v_{ft}.

6.6 Conclusions

In this chapter a tool to ensure separation minimum at strategic planning based on statistics and probability theory has been developed.

It has been demonstrated that this methodology is a powerful tool to reduce congestion in the airspace. The benefits expected by its implementation are a decrease of the workload of air traffic controllers and, hence, an increase of safety and of airspace's capacity.

Chapter 7

Air Traffic Density in a Terminal Manoeuvring Area

7.1 Introduction

Dynamic density is a proposed concept of a metric, which predicts controller workload as a function of air traffic characteristics in a given airspace volume. This concept is essential to develop both air traffic management automation and air traffic procedures. These improvements will inevitably increase ATM flexibility and efficiency.

As reported in [58], the dynamic density metric includes: *"traffic density (a count of aircraft in a volume of airspace) and traffic complexity (a quantitative description of the air traffic complexity in a volume airpsace)."*

This chapter describes a program designed to evaluate the traffic density in terms of number of aircraft in a volume airspace. The approach is novel because of the use of the statistics and probability theory to estimate the number of aircraft. This program has been tested for Glasgow Terminal Manoeuvring Area (GLA TMA) using real data provided by CAA.

7.2 Mathematical model

7.2.1 Air Traffic Density

Given an instant time $t^{*} \in \boldsymbol{T}$, the stochastic process $\underline{\mathrm{s}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*}) \subseteq \mathcal{G} x \boldsymbol{T}$ gives the probability that an aircraft is in a fixed position of an area $\boldsymbol{C}$ (see section 6.2). The integration of this stochastic process $\underline{\mathrm{s}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*}) \subseteq \mathcal{G} x \boldsymbol{T}$ with respect to the area gives the probability of the aircraft to be in $\boldsymbol{C}$.

$$\Pr\left\{\begin{array}{l}\text{aircraft is in } \boldsymbol{C}\\ \text{at the instant}\\ \text{time } \mathrm{t}^{*}\end{array}\right\} = \iint_{C} f_{\underline{\mathrm{s}}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*})\, d\mathrm{s}_x d\mathrm{s}_y = \alpha$$

Let $\{1,2,\ldots \mathrm{n}\} \in \boldsymbol{N}^{n}$ be a set of flights. Let $\underline{\mathrm{s}_{\mathrm{i}}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}) \subseteq \mathcal{G} x \boldsymbol{T}$ with $\mathrm{i} \in [0,1,\ldots \mathrm{n}]$ be the stochastic processes position of the set of flights $\{1,2,\ldots \mathrm{n}\} \in \boldsymbol{N}^{n}$. For a fixed instant of time $\mathrm{t}^{*} \in \boldsymbol{T}$, each aircraft has a probability $\alpha_1, \alpha_2, \ldots \alpha_n$ to be in $\boldsymbol{C}$. Let A_i be the event denoting "*Aircraft i is in* $\boldsymbol{C}$".

$$\Pr\{A_1\} \stackrel{\Delta}{=} \Pr\left\{\begin{array}{l}\text{aircraft 1 is in } \boldsymbol{C}\\ \text{at the instant}\\ \text{time } \mathrm{t}^{*}\end{array}\right\} = \iint_{C} f_{\underline{\mathrm{s}_1}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*})\, d\mathrm{s}_x d\mathrm{s}_y = \alpha_1$$

$$\Pr\{A_2\} \stackrel{\Delta}{=} \Pr\left\{\begin{array}{l}\text{aircraft 2 is in } \boldsymbol{C}\\ \text{at the instant}\\ \text{time } \mathrm{t}^{*}\end{array}\right\} = \iint_{C} f_{\underline{\mathrm{s}_2}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*})\, d\mathrm{s}_x d\mathrm{s}_y = \alpha_2$$

$$\vdots$$

$$\Pr\{A_n\} \stackrel{\Delta}{=} \Pr\left\{\begin{array}{l}\text{aircraft n is in } \boldsymbol{C}\\ \text{at the instant}\\ \text{time } \mathrm{t}^{*}\end{array}\right\} = \iint_{C} f_{\underline{\mathrm{s}_n}}(\mathrm{s}_x, \mathrm{s}_y; \mathrm{t}^{*})\, d\mathrm{s}_x d\mathrm{s}_y = \alpha_n$$

Let $\{1,2\}\in N^2$ be a set of only two flights. Here a basic example involving only two flights is shown. However, this methodology can be applied to whatever number of flights. In section B.3.1 delays will be proven to be independent random variables. As consequence the stochastic processes position will be also independent and, hence, also the events $A_1, A_2, \ldots A_n$. Under this hypothesis, the probability that exactly 0,1 and 2 aircraft are in $\boldsymbol{C}$ in the instant time $t^* \in \boldsymbol{T}$ is given by:

$$\Pr\left\{\begin{matrix}\text{exactly 2} \\ \text{aircraft are } \boldsymbol{C} \\ \text{in the instant} \\ \text{time } t^*\end{matrix}\right\} = \Pr\{A_1 \cap A_2\} = \Pr\{A_1\}\Pr\{A_2\} = \alpha_1\alpha_2$$

The sum of two independent events is equal to the sum of the probability of each event minus the probability of the intersection set. This probability is the probability that at least 1 aircraft is in the area $\boldsymbol{C}$

$$\Pr\left\{\begin{matrix}\text{at least 1} \\ \text{aircraft is in } \boldsymbol{C} \\ \text{in the instant} \\ \text{time } t^*\end{matrix}\right\} = \Pr\{A_1 \cup A_2\} = \Pr\{A_1\} + \Pr\{A_2\} - \Pr\{A_1\}\Pr\{A_2\} =$$
$$= \alpha_1 + \alpha_2 - \alpha_1\alpha_2$$

The probability that exactly 1 aircraft is in $\boldsymbol{C}$ is the subtraction between the probability that at least 1 aircraft is in $\boldsymbol{C}$ and the probability that exactly two aircraft are in $\boldsymbol{C}$.

$$\Pr\left\{\begin{matrix}\text{exactly 1} \\ \text{aircraft is in } \boldsymbol{C} \\ \text{in the instant} \\ \text{time } t^*\end{matrix}\right\} = \Pr\left\{\begin{matrix}\text{at least 1} \\ \text{aircraft is in } \boldsymbol{C} \\ \text{in the instant} \\ \text{time } t^*\end{matrix}\right\} - \Pr\left\{\begin{matrix}\text{exactly 2} \\ \text{aircraft are in } \boldsymbol{C} \\ \text{in the instant} \\ \text{time } t^*\end{matrix}\right\} =$$
$$= \Pr\{(A_1 \cup A_2) - (A_1 \cap A_2)\} = \Pr\{A_1\} + \Pr\{A_2\} - 2\Pr\{A_1\}\Pr\{A_2\} =$$
$$= \alpha_1 + \alpha_2 - 2\alpha_1\alpha_2$$

The probability that no aircraft is in $\boldsymbol{C}$ is equal to the probability of the set complement of the event "*At least one aircraft is in C*"

$$\Pr\left\{\begin{array}{l}\text{exactly } 0\\ \text{aircraft is in } \boldsymbol{C}\\ \text{in the instant}\\ \text{time } t^*\end{array}\right\} = 1 - \Pr\left\{\begin{array}{l}\text{at least } 1\\ \text{aircraft is in } \boldsymbol{C}\\ \text{in the instant}\\ \text{time } t^*\end{array}\right\} =$$

$$= 1 - \alpha_1 - \alpha_2 + \alpha_1\alpha_2$$

This calculation has been performed involving only two aircraft, however this methodology can be extended to n aircraft.

Let $\{1,2,\ldots n\} \in \boldsymbol{N}^n$ be a set of flights and let $\boldsymbol{C}$ be a circle having radius r. Air Traffic Density in the instant time $t^* \in \boldsymbol{T}$ within the circle of radius $r \in \boldsymbol{R}$ is defined as:

"Air Traffic Density is a scalar function, which corresponds the probability that a fixed number $h \leq n$ of aircraft is inside a circle of radius r."

Air Traffic Density is a function of three variables: time $t \in \boldsymbol{T}$, radius of the circle $r \in \boldsymbol{R}$ and number of aircraft inside the circle $h \in \{1,2,\ldots n\}$. The output is the probability that a given number of aircraft is inside the circle of radius r.

$$f(t,r,h): \quad (t,r,h) \subseteq \boldsymbol{T} \times \boldsymbol{R} \times \{0,1,2,\ldots n\} \qquad \rightarrow \qquad [0,1]$$

The instant time $t \in \boldsymbol{T}$ can be extended to an interval time $\Delta t \subseteq \boldsymbol{T}$ through integrating with respect to the time:

$$\Pr\left\{\begin{array}{l}\text{aircraft is in } \boldsymbol{C}\\ \text{in the interval}\\ \text{time } \Delta t\end{array}\right\} = \iiint_{C x \Delta t} f_{\underline{s}}(s_x, s_y; t)\, ds_x\, ds_y\, dt$$

7.3 Description of the code

The code computes the air traffic density of Glasgow TMA for the time interval commencing at 7:00 a.m. and ending at 9:00 a.m. This time interval is chosen because the highest activity in Glasgow International airport is observed from 7:00 a.m. until 9:00 a.m. The GLA TMA is modelled as a circular area around Glasgow airport. The length of the radius of the circle is a parameter of the program which can be changed (for example: radius=60 km).

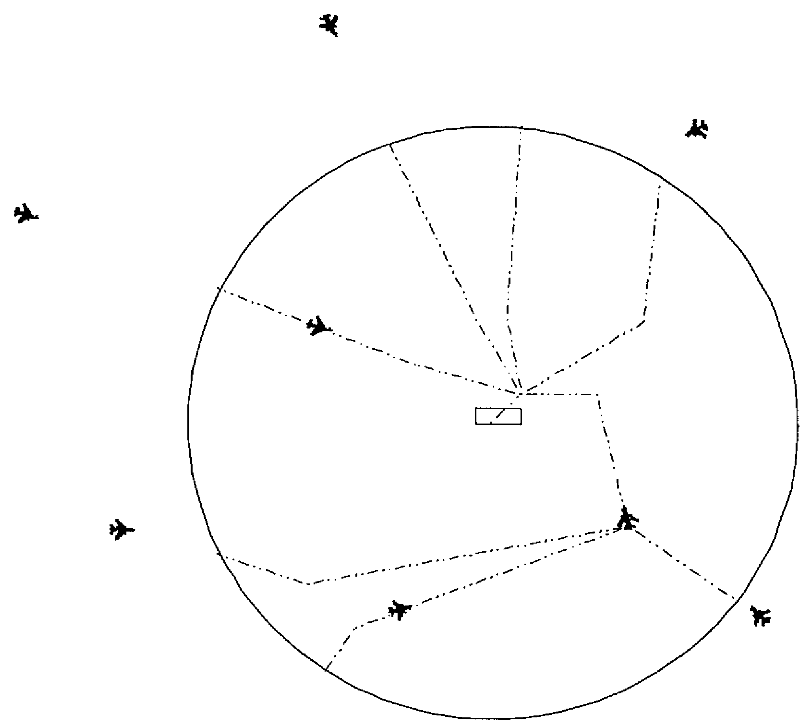


Figure 7.1: GLA TMA.

7.3.1 DATA

The code loads the lotus files containing the real arrival and scheduled arrival times of all aircraft arriving at GLA TMA in the chosen interval time (from 7:00 a.m. until 9:00 a.m.). Anyway, the program can be easily improved by adding the data of all aircraft arriving at Glasgow International airport.

Using these data, the program first calculates the position of the aircraft in each instant of time and then the arrival sequence of the flights.

7.3.2 PDF Construction

The PDFs of aircraft position $\underline{s}(s_\tau, s_n; t) \subseteq \mathcal{G}xT$ were built as extensively explained in chapter 6. In each instant of time, this program only considers simultaneously seven aircraft. This number was checked to be high enough to manage the traffic density in the GLA TMA. In section 7.4, it will be proven that no more than seven aircraft are ever inside the GLA TMA at the same time.

7.3.3 Probability that exactly n aircraft are in GLA TMA in an instant of time

At a fixed instant of time $t^* \in \boldsymbol{T}$, the PDF of the position of the aircraft is calculated $\underline{s}(s_x, s_y; t^*) \subseteq \mathcal{G}x\boldsymbol{T}$. Using this information, the probability that one aircraft is inside the GLA TMA is computed through integrating the stochastic process $\underline{s}(s_x, s_y; t^*) \subseteq \mathcal{G}x\boldsymbol{T}$ with respect to the space, as shown in section 7.2.1.

The program can handle simultaneously only maximum seven aircraft. To do so, the code stops considering a given aircraft when this aircraft has reached a probability of landing higher than 90%. In this circumstance, the program substitutes the landed aircraft with the closest aircraft to GLA TMA.

If the arrival times of the aircraft are independent†, the probability that in an instant of time exactly n=7 aircraft are in GLA TMA is computed as shown in section 7.2.1:

$$\Pr\left\{\begin{array}{l}\text{exactly n} = 7\\ \text{aircraft are}\\ \text{in } \boldsymbol{C} \text{ at } t \in \boldsymbol{T}\end{array}\right\} = \Pr\{n = 7\} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\alpha_7 = \prod_{i=1}^{7} \alpha_i$$

The probability that in an instant of time exactly n=6 aircraft are in GLA TMA is computed as:

† See Appendix B to be proven such an assumption.

$$\Pr\left\{\begin{array}{l}\text{exactly } n=6\\ \text{aircraft are } \boldsymbol{C}\\ \text{in the instant}\\ \text{time } t^{*}\end{array}\right\} = \Pr\{n=6\} = \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_6 + \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5\alpha_7 + \alpha_1\alpha_2\alpha_3\alpha_4\alpha_6\alpha_7 +$$

$$+\alpha_1\alpha_2\alpha_3\alpha_5\alpha_6\alpha_7 + \alpha_1\alpha_2\alpha_4\alpha_5\alpha_6\alpha_7 + \alpha_1\alpha_3\alpha_4\alpha_5\alpha_6\alpha_7 + \alpha_2\alpha_3\alpha_4\alpha_5\alpha_6\alpha_7 - 7\Pr\{n=7\}$$

7.4 Study case: Glasgow Terminal Manoeuvring Area

Figure 7.2 shows the probability that exactly $n \in [1,2\ldots7]$ aircraft are inside the GLA TMA for each instant of time belonging to the interval time commencing at 7:00 a.m. and ending at 9:00 a .m.

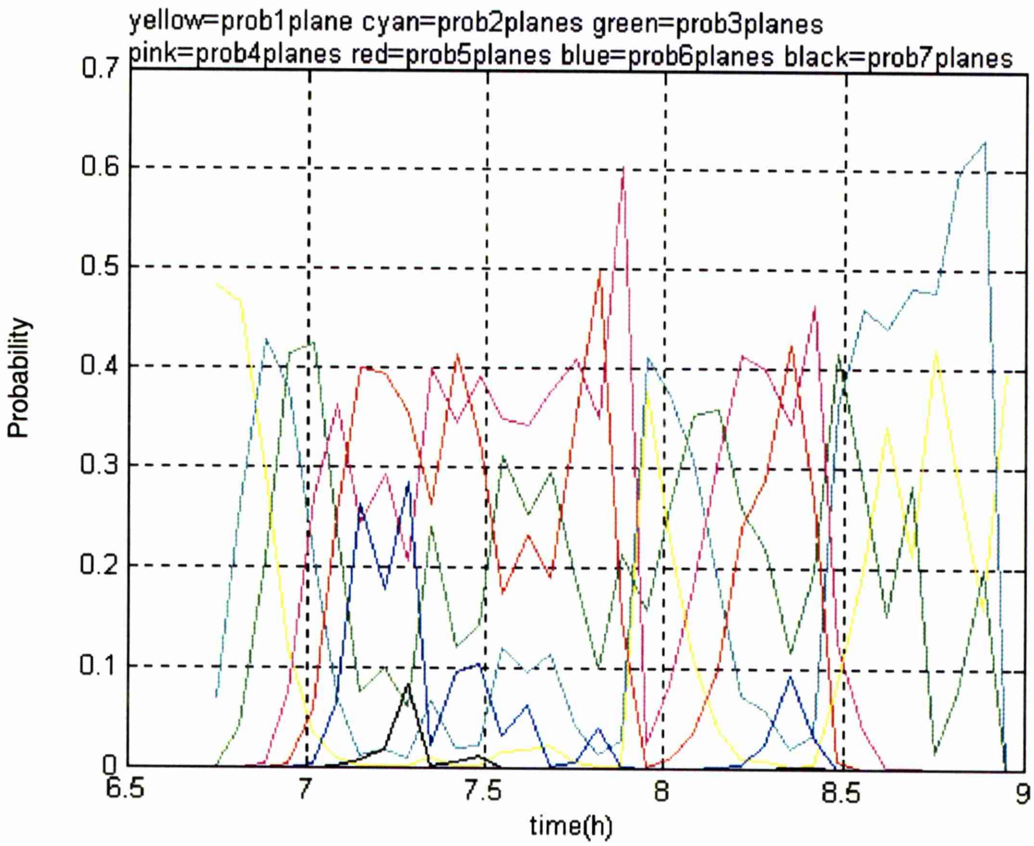


Figure 7.2: Probability that exactly $n \in [1,2\ldots7]$ aircraft are in GLA TMA.

The black and dark blue represent the probability that exactly 7 and 6 planes are inside GLA TMA. Consequently, air traffic density is maximum is at 7:20 a.m. In the other hand, air traffic density has the lower value at eight o'clock. At this time the curves representing the probability that exactly 1(yellow) and 2(cyan) aircraft are inside GLA TMA show their peaks.

Figure 7.3 is equivalent to Figure 7.2 apart the fact that this Figure 7.3 is 3D.

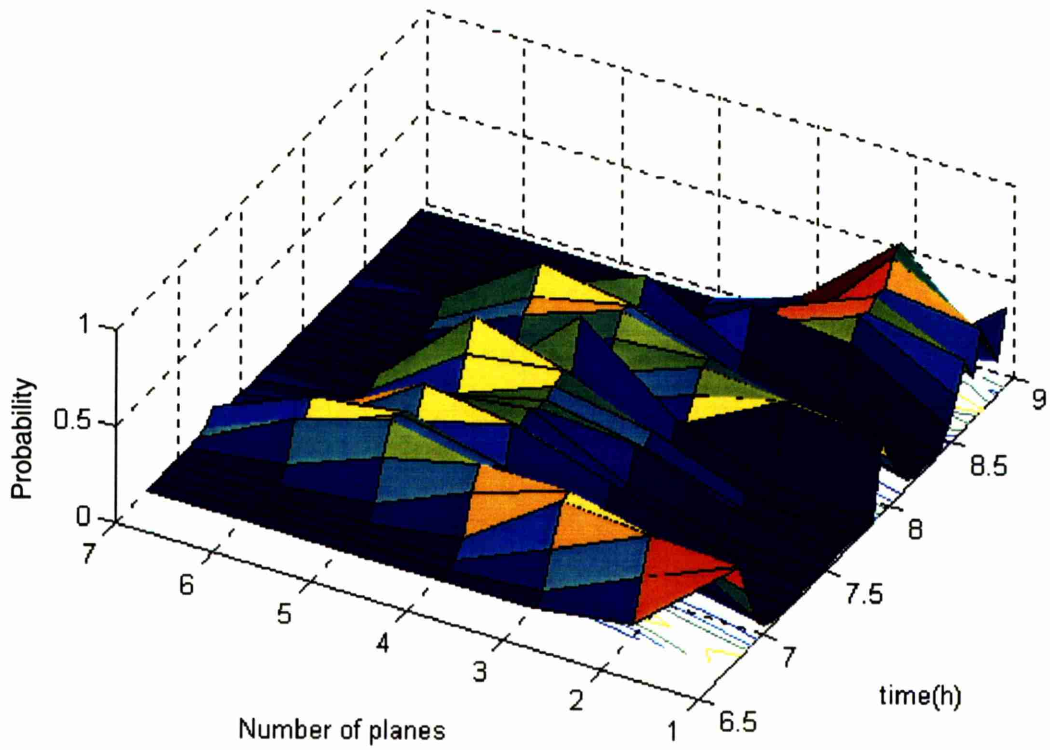


Figure 7.3: Probability that exactly n (n=1,2…7) planes are in GLA TMA-3D.

Figure 7.3 shows clearer how air traffic density evolves along the considered period of time.

7.5 Conclusions

In this chapter a model for air traffic density has been proposed. The goal is to propose a methodology to compute Air Traffic Density as a sort of congestion metric. In future works, this tool can be used to predict and consequently to avoid dispatch of aircraft in Terminal Manoeuvring Area in order to increase safety.

This model has been tested for Glasgow Terminal Manoeuvring Area (GLA TMA) and considers only the number of aircraft presented in a given airspace. In further developments, this tool can be improved adding a quantitative description of the air traffic complexity in a given volume.

Chapter 8

Conclusions & recommendations for further work

8.1 Introduction

This dissertation had presented tools for the development of an operational procedure to improve Air Traffic Flow Management (ATFM) efficiency. The ATFM function can be considered as consisting of two distinct processes. The first process is concerned with the flow of arriving and departing aircraft at a given airport and the second is concerned with the flow management of *en route* airspace. This thesis has developed models that address both the airport and en-route flows management. These tools are based on statistics and probability theory and rely on historical data to model ATFM parameters. This approach implicitly assumes that the probability of the occurrence of an event can be estimated by its frequency of occurrence in the past. This hypothesis has meaning as airport scheduling does not have substantial changes through the year. In this work a theoretical framework was established for the analysis and optimisation of the flight schedule in a network of airports and sectors. The main variables and parameters needed for the analysis of this network (delay cost, airport capacity, etc.) were identified and studied adopting a statistical and probabilistic prospective. Furthermore this theoretical framework was implemented both in controlled and in free flights airspace and, thus, they seemed novel methods to solve even the transition period inevitably required to implement free flight.

8.1.1 Airport Flow Management

A critique of delay analysis in terms of cost and safety was presented in Chapter 2 to give priorities in the decision-making process to identify flights priorities. Elementary models were presented to estimate the delay cost. The aim was present the major expenses effecting airlines to determine the most cost-effective scenario.

Airport capacity is a parameter expressing the maximum number of movements (arrivals and departures) that the airport can manage successfully ensuring an established level of safety. The literature review of Chapter 1 revealed that currently each airport declares its own estimated capacity taking into account its own facilities and infrastructures (e.g. the number of runways, the number of gates and so on). In many circumstances it has been empirically observed that the actual operational airport capacity differs from this declared capacity because of the effects of highly unpredictable parameters (e.g. rain, fog, visibility and so on). Airport capacity was presented in Chapter 3 as a discrete random variable. This model showed to have the advantage of estimating the operational airport capacity using historical arrivals and departures data rather than information concerning airport's infrastructure. This model allows air traffic operators to calculate the probability airports can really deal with a demanded number of movements (arrivals and departures). An example was shown in which the proposed model has been applied to London Heathrow Airport.

A procedure to establish a strategic arrival schedule was developed in Chapter 4. An implicit feature of this probabilistic procedure was that it takes into account the uncertainty in the arrival and departure times at a given airport. The methodology was applied at Glasgow International Airport to design a strategic schedule that reduced either the probability of conflict of the arrivals or the length of the landing slots necessitated by flights. The benefits that were expected through the application of this methodology were a reduction of airborne delays and/or an increase of the airport capacity. Thus a safer and more efficient system was achieved. This new procedure was applied to improve the strategic schedule of Glasgow International Airport using CAA real data. It was proved that the consequential benefits include the options of:

- reducing the Interval Length (IL) by 14.3% thereby increasing the airport capacity.
- reducing the Probability of Conflict (PC) by 15.3% which will decrease airborne delay at the arrival airport, which will reduce fuel consumption thus reducing operators costs and environmental pollution.
- a combination of these two options.

A tactical arrival schedule tool was then presented in Chapter 5. This tool was designed to allow air traffic controllers to organise an air traffic flow pattern using a ground holding strategy. During the daily planning of air traffic flow unpredictable events such as adverse weather conditions and system failures are inevitable and airborne and ground-hold delays are necessitated. These delays are used by controllers as a means of avoiding 4D-status conflicts. Of the two methods, ground hold delays are preferred because they are safer, less expensive and less harmful to the environment. In this dissertation a method of estimating the duration of a ground-hold for a given flight was developed. The proposed method was novel in the use of a real time stochastic analysis. This tactical planning procedure had been tested always for Glasgow International Airport. Using real time information given by an air/ground data link the prediction of the future arrival time was refined as the event horizon got closer to the present. Ground hold delay were then allocated to reduce the probability of conflict between the arrival aircraft The results of the tests had indicated a promising mean reduction of the Interval Length (IL) of 32.1%. The advantage of such a high percentage was a significant improvement in the airport's capacity whereas the disadvantage was an increase in expenses due to the allocation of ground hold delays.

8.1.2 *En route* flow management

Currently the overall airspace environment is divided into sectors. Air Traffic Controller Centres (ATC) control aircraft flow inside of these sectors. Each sector has a limited capacity. An ATC centre cannot handle more than a given number of aircraft in the sector guaranteeing the same level of safety. The number of aircraft that an ATC can deal with is called sector capacity. In some circumstances, the aircraft flow crossing a given sector exceeds its capacity and the risk of collision increases. To ensure the same level of safety in such situations, air traffic controllers issue restrictive instructions that may cause losses in term of punctuality and reduce the efficiency of the aircraft operations. This thesis had presented in Chapter 6 a decision support tool that aims at reducing the probability of the throughput of aircraft crossing a given sector exceeding its capacity. The tool proposed a new schedule at the strategic planning level that guaranteed, with a probability set by the air traffic managers, that the aircraft will not violate the separation minima during their routes. The benefits expected from methodology included the reduction of the probability of the overload of the Air Traffic Control Centres. Furthermore this tool proposed in Chapter 7 a methodology to estimate the air traffic density and, hence, it was used to predict and consequently to avoid dispatch of aircraft in Terminal Manoeuvring Area to increase safety. This model was tested for Glasgow Terminal Manoeuvring Area (GLA TMA).

8.2 Recommendations for further work

The models presented in this thesis focus in improving both safety and economical aspects of aircraft operations. A challenging direction for further research is to adapt these ideas to real air traffic situations. This model can be applied to a real network of airport and sectors using exhaustive data related to airlines' economic policies and operations times in the airports and sectors of the network.

In a free flight scenario the tools presented in this thesis could be combined with an enhanced Airborne Separation Assurance System (ASAS). The expected benefits of the combination of these tools are a probabilistic decrease of the number of times where ASAS will be activated. Furthermore, a deterministic algorithm combined with the statistical tools presented in this work would be necessary to support controllers in dispatching self-separating aircraft into conventionally managed airspace TMA.

Appendix A

Software for strategic planning

A.1 Introduction

In this Appendix an example of the software proposed and designed in Chapter 4 is shown. The program developed is user friendly and its interface allows an easy access to all the potentiality and workability. The system is designed as a helpful tool to reduce congestion in the airspace of the terminal area. Traffic occurs due to landing queue as many planes simultaneously request to land at the arrival airport. In such circumstances, operators allocate airborne delays to aircraft causing an inevitable additional cost and even discomfort for passengers. The aim of this program is to allow the opportunity to reschedule the airport flight arrival times to avoid simultaneous arrivals taking into account statistics and probability theory. The purpose of the program is to enable operators to create a new scheduling for arrivals of aircraft. The software can be easily applied with some variations to schedule departures and/or arrivals for trains, buses etc. As shown and explained in Chapter 4, the algorithm is based on statistics and probability theory. The data used to develop this software are referred to Glasgow International Airport and have been provided by the Civil Aviation Authority UK (CAA). The data covers a period that commences on 1st of October 1997 and ends on

30th of September 1998. An example of the data are shown in Table 3.1, the data provided are referred to the touch down of the aircraft and eventual taxiing time can be added (arrivals 5 minutes; departures 10 minutes according to UK CAA instructions).

A.2 Description of the software

The program has been developed in Matlab® language using Matlab GUI (*Graphic User Interface*). After running Matlab, type "start" in the Command Window. On the screen a panel, as shown in Figure A.1, appears to allow communication between user and program.

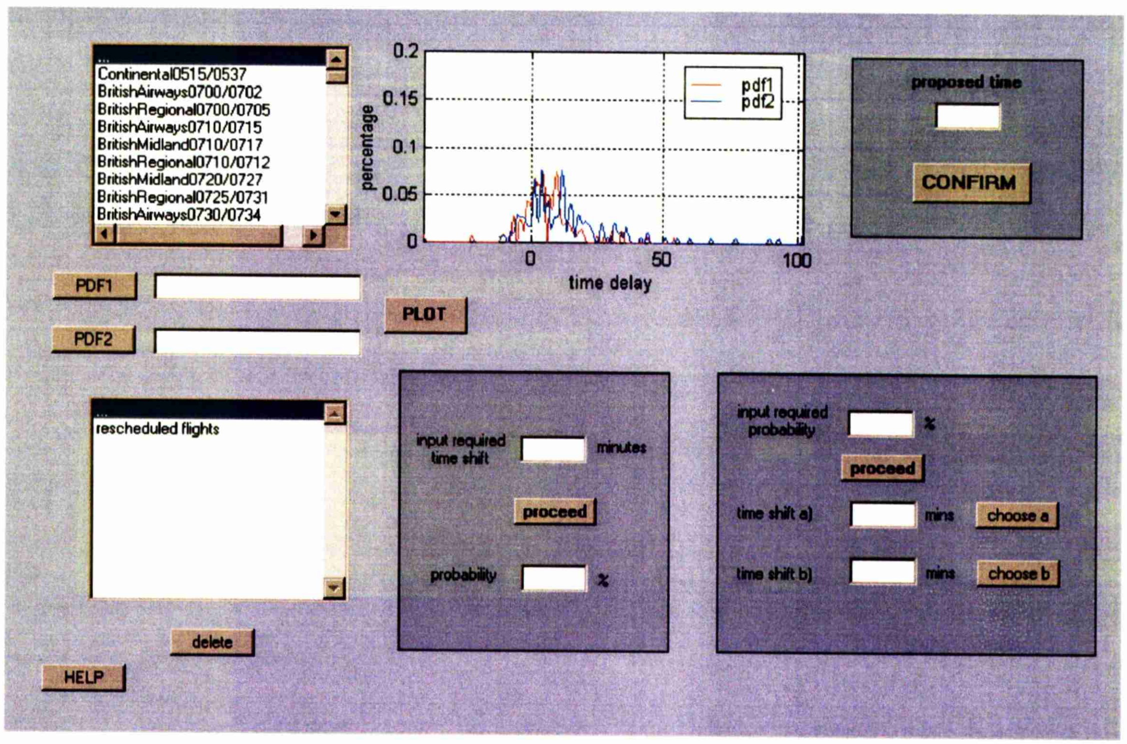


Figure A.1: Starting panel.

Only the list set in the upper left corner contains data in the first run of the program. The notation "BritishAirways0700/0702" denotes the flight of the company "British Airways" that is scheduled to arrive at 7:00 a.m. with an expected arrival time of 7:02 a.m. The average of the arrival times is very useful information to take into account during the planning process because it gives an idea when to expect the flight. This data is helpful in order to plan the new scheduling as many flights are currently scheduled to arrive at the same time. A more real order of arrivals can be sorted out taking into account the difference between their average times. For example, the flights BritishAirways0710/0715, BritishMidland0710/0717 and BritishRegional0710/0712 are scheduled to land at the same time 7:10 a.m. This might appear as a contradiction but the most real arrival order is figured out if the averages of the arrival times are observed.

To select a flight just simply identify and click on it with the mouse, then by pushing the button "PDF1" or "PDF2" it will be inserted in the correspondent list. After a few seconds the selected flight will appear in the box. To decide which flight has to be selected as PDF1 and as PDF2, it is necessary first to establish the flight arrival order. This is an input parameter decided by the user and it gives the operators the freedom to reschedule the airport arrival times according to their preferences. The flight selected as PDF2 will be rescheduled, while PDF1 will keep the current scheduling. The program will generate a new scheduling with the order PDF1, PDF2. After the selection, pushing the button "PLOT" the program will display the corresponding flight PDF in the chart placed in the upper row in the middle of the control panel. This chart is just an aid for operators to ease understanding the delay of the flight.

The software provides the user with the capability that he can imposes two different requirements to achieve the rescheduling:

- impose the time difference between two flights.
- input the probability that the flight labelled with PDF2 arrives after the flight labelled with PDF1 users demand two aircraft's do not conflict.

These two kinds of possibilities make the program more flexible and more suitable for a wider range of users. The first possibility gives as output the probability that the aircraft labelled with PDF2 arrives after the flight labelled with PDF1. The second possibility gives as output the required time difference to guarantee that the flight labelled with PDF2 arrives after than the flight labelled with PDF1 with the probability that has been previously inserted by the user.

When a user wants to reschedule flights imposing only the time difference, the field labelled with "input required time shift" has to be filled with the desired time. Then clicking on the button "proceed" which is situated under this field, the probability that the flight PDF2 arrives after the flight PDF1, expressed in percentage, will appear in the field labelled as "probability". The input time difference has to be expressed in minutes. Positive value means movement of flight selected as PDF2 further in time respect to the flight selected as PDF1, while negative value will move the flight selected as PDF2 closer. New rescheduled time for the flight selected as PDF2 is equal to sum of time for which is scheduled flight selected as PDF1 to arrive plus the shifting time. New rescheduled time for the flight selected as PDF2 will then appear in the field "proposed time" situated in the upper right corner of screen. Clicking the button "CONFIRM", the rescheduled flight will be added to the list located in lower left corner. If required probability does not fulfil user's demand, another value of shifting time can be inserted and then the same procedure described above can be repeated until rescheduling meets requirements.

The second possibility is to input the percentage that the flight labelled with PDF2 arrives after the flight labelled with PDF1. To achieve this demand, the user is first required to insert into the field "input required probability" the desired percentage. Then he is required to click on the button "proceed" that is located under this field. Note that there will be displayed the two possible solutions in the fields labelled with "time shift a)" and "time shift b)". The software computes the shifting time to have the arrival order: flight labelled with PDF1 then flight labelled with PDF2 and vice-versa. The program is designed to choose automatically the solution with the lower absolute value as user usually require to reschedule flights shifting them the minimum in time. This is not always true because it might be more useful to implement a flight combination that causes a greater shifting time in absolute value. The program is designed in the way that the user can choose one of the solutions just clicking the buttons labelled as "choose a" or "choose b". As explained before, positive shifting time means movement in time later and negative shifting time means movement in time earlier. This time shift is related to flight selected as PDF2 and to compute rescheduled time it is necessary to sum the time of flight selected as PDF2 and of the time shift. The rescheduled time will appear in field "proposed time". To confirm this solution, just click on the button "CONFIRM" and rescheduled flight will be added to the list of rescheduled flights.

Note that there is a difference in the notation used to label the original flights and the rescheduled flights. For instance, the original flights are labelled as "BritishMidland0720/0727", this means the flight operated by the company "British Midland" is scheduled to arrive at 7:20 a.m. and is average arrival time is 7:27 a.m. The same rescheduled flight is labelled as "BritishMidland0720/0722". This notation means that the flight operated by the company "British Midland" that was previously scheduled to arrive at 7:20 a.m. is now rescheduled to arrive at 7:22 a.m.

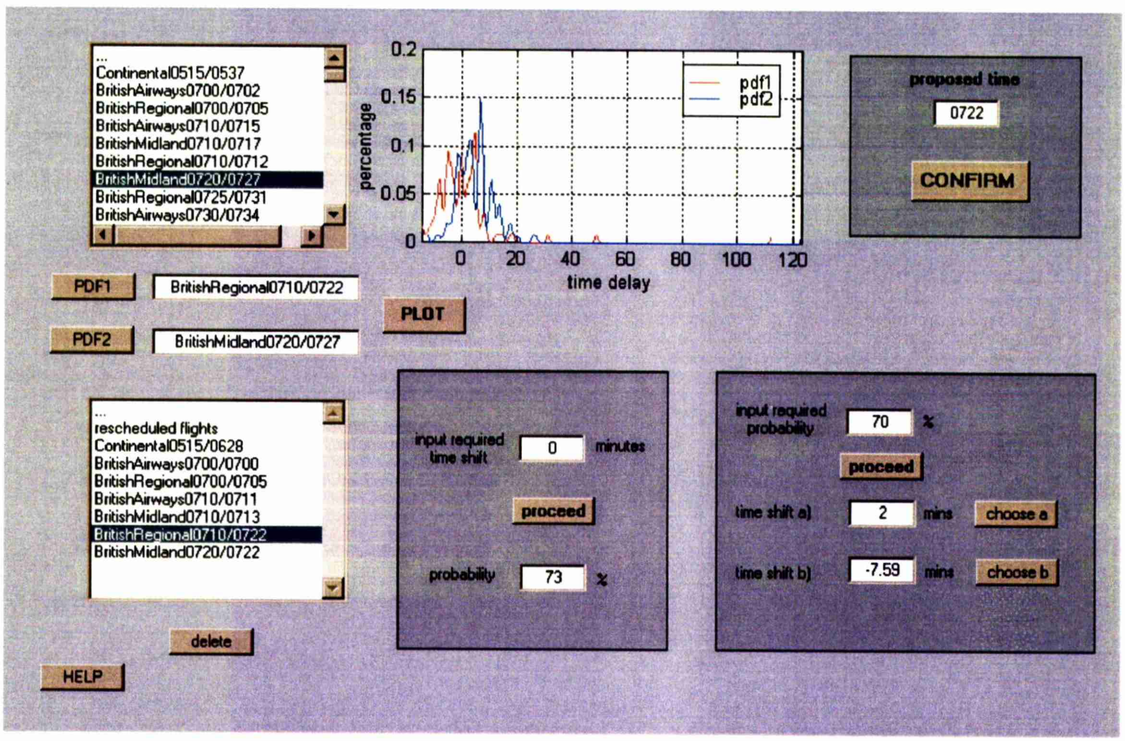


Figure A.2: Example of rescheduling.

As shown in Figure A.2, there are two flights rescheduled to arrive both at 7:22 a.m. "British Regional" flight is scheduled to arrive at 7:10 a.m., trough the examination of its mean it's possible to affirm that usually it is arriving around 7:12 a.m. (+2 minutes difference). The flight operated by the company "British Midland" is scheduled to arrive at 7:20 a.m. and usually it is arriving around 7:27 a.m. (+7 minutes difference). The scheduled time gap between these two flights is 10 minutes, while the interval time through their respective mean is 17 minutes long. As shown in Figure A.2; the necessary time gap to reach probability of 73 % that one aircraft come later than the other one is of 0 minutes. Although the first flight is rescheduled to arrive at 7:22 a.m., according to its PDF it usually comes around 7:24 a.m. The second flight is rescheduled to arrive at 7:22 a.m., according to its PDF it usually comes at 7:29 a.m.

After the first flight is rescheduled, it is possible to proceed through the rescheduling of other flights. Select the rescheduled flight as PDF1 from the new list because a new arrival time is associated to this flight. The following times will be referred to this rescheduled flight and to its new time. From the list of original flights, it is possible to select another flight and then keep proceeding as described above.

If a fixed arrival time is demanded for a given flight, this examined flight has to be selected as PDF2. Write into field "proposed time" the requested scheduled arrival time and then click on button "CONFIRM", this flight will be added to the rescheduled list. Disadvantage of this procedure is that the probability between this flight and its previous flight is not calculated. This probability can be computed later still using the program. This procedure is very useful to allocate the first flight at the starting run in the rescheduled list.

A flight that belongs to the rescheduled list can be deleted just by clicking on the button "delete". Note that if a flight that is not on the bottom of the rescheduling list is deleted, the program automatically will delete even all the flight that are following it.

The program is provided with a help menu, to open it just click on button "HELP" positioned in the lower left corner. The help will appear as shown in Figure A.3. The help is very simple and it is possible to consult it even while the program is running. The help is mostly designed to give to users a brief reminder of the basic instructions concerning the program.

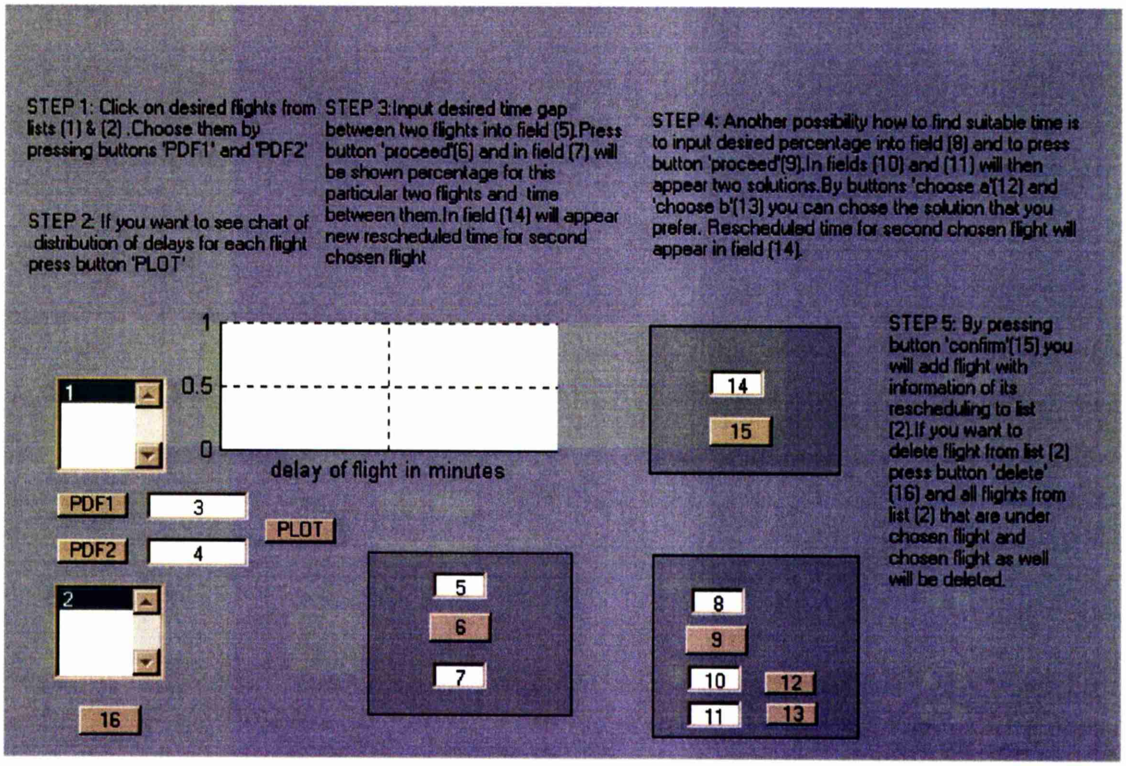


Figure A.3: Help screen.

Appendix B

Justification of the hypotheses

B.1 Introduction

This Appendix provides a mathematical justification for the hypothesis assumed in this thesis.

The work presented in this thesis follows the methodology shown in the block diagram of Figure B.1.

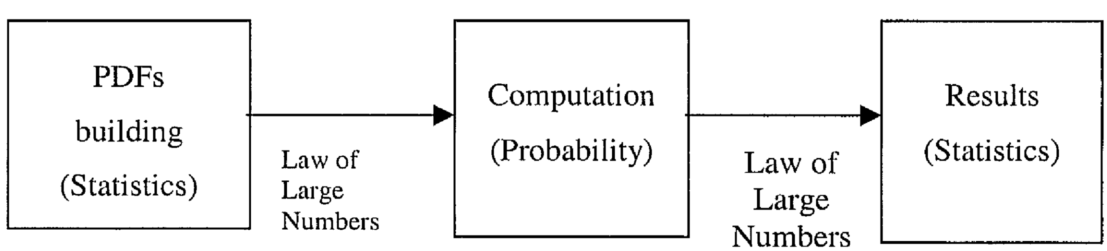


Figure B.1: Block diagram of the methodology.

The first block is based on Statistics and it represents the construction of histogram diagrams of the relative historical occurrences from collected sampled data. Based on the law of large numbers, the obtained histograms are considered to be an approximation of the corresponding PDFs. The approximations are caused by:

- the finite number of samples
- the fact that the collected data are time-dependent

Based on Probability's theory, these built PDFs are manipulated by mathematical operators in the second block. To achieve these manipulations, these PDFs and consequently the collected data are assumed to verify some hypotheses. In conclusion, the second block is affected by approximations due to the fact that the built PDFs verify the assumed hypotheses within a margin of error.

The inputs of the third block are these manipulated PDFs by the second block. Statistics is the process of determining properties of the whole population by collecting and analysing data from a representative segment. Vice-versa given the whole population, statistics determines properties of a sample. The law of large numbers marries Statistics and Probability. These manipulated PDFs give samples applicable to real life after applying the law of large numbers.

B.2 Mathematical background

In this thesis PDFs has been built directly from real data as histogram diagrams of the relative historical occurrences. In this section the mathematical background to justify such a hypothesis is described.

B.2.1 Probability Density Functions (PDFs) as frequency interpretations

The law of large numbers assures that, the expected value of a random variable is equal to the mean of infinite samples. This section proves that $f(x)$ can be found from an infinite sample with similar reasoning, see [48] for further details.

A delay $\mathbf{d}_i$ is considered

$$\mathbf{d}_i(\zeta_1, \zeta_2, \ldots \zeta_i \ldots) = \mathbf{d}_i(\zeta_i) = \mathbf{d}(\zeta_i)$$

where ζ_i is the outcome of the *i*th repetition of the given experiment These random variables are independent and they have the same statistics $f(x)$ and $F(x)$.

Given an $x \in \boldsymbol{R}$, a Bernoulli random variable $\mathbf{z}_i(x)$ is defined:

$$\mathbf{z}_i(x) = \begin{cases} 1 & \text{if } x < \mathbf{d}_i \leq x + dx \\ 0 & \text{if } x < \mathbf{d}_i \leq x + dx \end{cases}$$

The expected value of $\mathbf{z}_i(x)$ is equal to:

$$E\{\mathbf{z}_i(d)\} = 1 \cdot \Pr\{\mathbf{z}_i(d) = 1\} = \Pr\{x < \mathbf{z}_i(d) \leq x + dx\} = f(x)$$

Clearly if the random variables $\mathbf{d}_i$ are independent and $\mathbf{d}_i$ have the same distribution then $\mathbf{z}_i(x)$ are also independent and $\mathbf{z}_i(x)$ have the same distribution. The law of large numbers applied to the random variable $\mathbf{z}_i(x)$ assures that:

$$\lim_{n \to \infty} \frac{\mathbf{z}_1(x) + \ldots + \mathbf{z}_n(x)}{n} = E\{\mathbf{z}_i(x)\}$$

As shown above the expected value of the random variable $\mathbf{z}_i(x)$ is equal to the probability density function of $\mathbf{d}_i$ computed in the point $x \in \boldsymbol{R}$

$$E\{\mathbf{z}_i(x)\} = f_{d_i}(x)$$

This result justifies why the PDFs built in this thesis are computed from histogram diagrams of the relative historical occurrences. Section B.3.2 will show the sample size to consider.

B.2.2 Estimation of p

In the section B.2.1 the PDF $f(x)$ of a real random variable was shown to be a set of Bernoulli random variables $\mathbf{z}_i(x)$.

Let $\mathbf{z}_i(x)$ be a Bernoulli random variable such as:

$$\mathbf{z}_i(x) = \begin{cases} 1 & \text{if } x < \mathbf{d}_i \leq x + dx \\ 0 & \text{if } x < \mathbf{d}_i \leq x + dx \end{cases}$$

The parameters of this Bernoulli random variable $\mathbf{z}_i(x)$ are:

$$p = \Pr\{\mathbf{z}_i(x) = 1\}$$
$$q = \Pr\{\mathbf{z}_i(x) = 0\}$$

The estimator $\hat{p}$ is considered such as:

$$\hat{p} = \frac{\mathbf{z}_1(x) + \ldots + \mathbf{z}_n(x)}{n}$$

This estimator $\hat{p}$ is a Binomial random variable because $\hat{p}$ is the sum of n Bernoulli random variables. For a fixed $x \in \boldsymbol{R}$, a Binomial random variable is built.

B.2.3 Autocorrelation and cross-correlation

Autocorrelation is a mathematical operator to compare time series with itself at successive time positions. The task of this comparison is to detect dependencies through time and, hence, the strength of the self-similarity of the serie.

In chapter 4 arrival delays $\mathbf{d}^a(\zeta)$ are introduced as real random variables. If $\{t_{s1}^a, t_{s2}^a, \ldots t_{sn}^a\} \in \boldsymbol{T}^n$ are different scheduled arrival times, for each arrival scheduled time $t_s^a \in \boldsymbol{T}$ an arrival delay is defined $\mathbf{d}^a(\zeta, t_s^a)$. Thus, the family of random variable $\mathbf{d}^a(\zeta, t_s^a) \subseteq \mathcal{A} \times \boldsymbol{T}$ is defined to be a stochastic process.

The stochastic process $\mathbf{d}^a(\zeta;t)$ is supposed to be stationary

$$\mathbf{d}^a(\zeta;t) = \mathbf{d}^a(\zeta) \qquad \forall t \in \boldsymbol{T}$$

The autocorrelation $R(t_1,t_2)$ of the process $\mathbf{d}^a(\zeta;t)$ is the joint moment of the random variables $\mathbf{d}^a(\zeta;t_1)$ and $\mathbf{d}^a(\zeta;t_2)$

$$R(t_1,t_2) = E\{\mathbf{d}^a(t_1)\mathbf{d}^a(t_2)\} = \int_{-\infty}^{+\infty} d^{a_1} d^{a_2} f(d^{a_1}, d^{a_2}; t_1, t_2) dd^{a_1} dd^{a_2} =$$

The autocovariance $C(t_1,t_2)$ of the process $\mathbf{d}^a(\zeta;t)$ is

$$C(t_1,t_2) = E\{[\mathbf{d}^a(t_1) - \eta(t_1)][\mathbf{d}^a(t_2) - \eta(t_2)]\}$$

Owing to the hypothesis that $\mathbf{d}^a(\zeta;t) \subseteq \mathcal{A} \times \boldsymbol{T}$ is supposed to be stationary, the autocorrelation and the autocovariance of the stochastic process $\mathbf{d}^a(\zeta;t) \subseteq \mathcal{A} \times \boldsymbol{T}$ depend only on t_1-t_2

$$R(t_1,t_2) = E\{\mathbf{d}^a(t+\tau)\mathbf{d}^a(t)\} = R(\tau)$$
$$C(t_1,t_2) = E\{[\mathbf{d}^a(t+\tau) - \eta][\mathbf{d}^a(t) - \eta]\} = C(\tau)$$

For τ=0, the autocovariance simplifies in the variance of the real random variable $\mathbf{d}^a \subseteq \mathcal{A}$.

$$C(0) = E\{[\mathbf{d}^a(t) - \eta][\mathbf{d}^a(t) - \eta]\} = \sigma^2_{d^a}$$

Cross-correlation is a mathematical operator to compare two time series with each other at different time positions to determine eventually pronounced correspondence. The result of this comparison is the strength of the similarity between two series.

Let $\mathbf{d}_1^a(\zeta;t)$ and $\mathbf{d}_2^a(\zeta;t)$ be the arrival delays of two different flights. The cross-correlation $R_{d_1^a d_2^a}(t_1,t_2)$ of the stochastic processes $\mathbf{d}_1^a(\zeta;t)$ and $\mathbf{d}_2^a(\zeta;t)$ is the joint moment of the random variables $\mathbf{d}_1^a(\zeta;t)$ and $\mathbf{d}_2^a(\zeta;t)$.

$$R_{d_1^a d_2^a}(t_1,t_2) = E\{\mathbf{d}_1^a(t_1)\mathbf{d}_2^a(t_2)\} = \int_{-\infty}^{+\infty} d_1^a d_2^a f(d_1^a, d_2^a; t_1, t_2)\, dd_1^a\, dd_2^a =$$

The cross-covariance $C_{d_1^a d_2^a}(t_1,t_2)$ of the processes $\mathbf{d}_1^a(\zeta;t)$ and $\mathbf{d}_2^a(\zeta;t)$ is:

$$C(t_1,t_2) = E\{[\mathbf{d}_1^a(t_1) - \eta(t_1)][\mathbf{d}_2^a(t_2) - \eta(t_2)]\}$$

Owing to the hypothesis that $\mathbf{d}_1^a(\zeta;t)$ and $\mathbf{d}_2^a(\zeta;t)$ are supposed to be stationary, the cross-correlation and the cross-covariance of the stochastic processes $\mathbf{d}_1^a(\zeta;t)$, $\mathbf{d}_2^a(\zeta;t)$ depend only on t_1-t_2

$$R_{d_1^a d_2^a}(t_1,t_2) = E\{\mathbf{d}_1^a(t+\tau)\mathbf{d}_2^a(t)\} = R(\tau)$$

$$C_{d_1^a d_2^a}(t_1,t_2) = E\{[\mathbf{d}_1^a(t+\tau) - \eta(t+\tau)][\mathbf{d}_2^a(t) - \eta(t)]\} = C_{d_1^a d_2^a}(\tau)$$

The autocovariance coefficient is defined:

$$\rho(\tau) = \frac{C(\tau)}{C(0)}$$

B.2.4 Estimation of $C(\tau)$ and $\rho(\tau)$

The sample mean of n times $\mathbf{d}^a$ is defined as:

$$\overline{\mathbf{d}^a}_n = \frac{\sum_1^n \mathbf{d}^a}{n}$$

This is a random variable because $\overline{\mathbf{d}^{a}}_{n}$ is the sum of n random variables. If n experiments are fixed, then the sample mean simplifies in a mere number.

$$\overline{\mathbf{d}^{a}}_{n}(\zeta_1,\zeta_2,\ldots,\zeta_n)=\frac{\mathbf{d}^{a}(\zeta_1)+\mathbf{d}^{a}(\zeta_2)+\cdots+\mathbf{d}^{a}(\zeta_n)}{n}$$

This mere number $\overline{\mathbf{d}^{a}}_{n}(\zeta_1,\zeta_2,\ldots,\zeta_n)$ will be denoted simply by $\overline{d^{a}}_{n}$.

The estimators used for $C(\tau)$ and $\rho(\tau)$ are

$$\hat{C}(\tau)=\frac{1}{n}\sum_{t=1}^{n-h}\left(d_t^{a}-\overline{d^{a}}_{n}\right)\left(d_{t+h}^{a}-\overline{d^{a}}_{n}\right)\qquad 0\le h\le n-1$$

and

$$\hat{\rho}(\tau)=\frac{\hat{C}(\tau)}{\hat{C}(0)}$$

B.3 Delays as time series

A time series is built through the observation of an arrival delay $\mathbf{d}^{a}(\zeta_1;t_1),\mathbf{d}^{a}(\zeta_2;t_2),\ldots\mathbf{d}^{a}(\zeta_n;t_n)$ each day.

The problem is to decide whether the observed time series is from a process of independent random variables. The autocorrelation is a very useful test to proof that a given time series is made by independent samples as shown in [59]. The time series arrival delays are proven to be independent in section B.3.1. This result permits to compute the number of samples required for building PDFs within a fixed interval of confidence, as shown in section B.3.2.

Furthermore, cross-correlation tests are used to detect dependencies between two time series [59]. Cross-correlation test will be used in section B.3.1 to proof that the arrival delays of two different flights are independent. This result is important as in Chapter 4 the arrival times of the aircraft were hypothesis to be independent. If the arrival delays are independent, clearly even the arrival times are independent.

The autocorrelation and cross-correlation tests performed in this appendix use data provided by CAA for Glasgow International Airport and London Heathrow Airport during the years 1997 and 1998. Thus, the results obtained in this appendix are referred only to the described flights and even only to the time period covered by the data. The hypothesis that the time series are made with independent samples or that the time series are independent is not valid anymore if airports or flights or time period are changed.

B.3.1 Independent arrival delays

Figure B.1 shows the autocorrelograms† of the time series of Air Canada Flight n° AC842 whose departure airport is Toronto (CA).

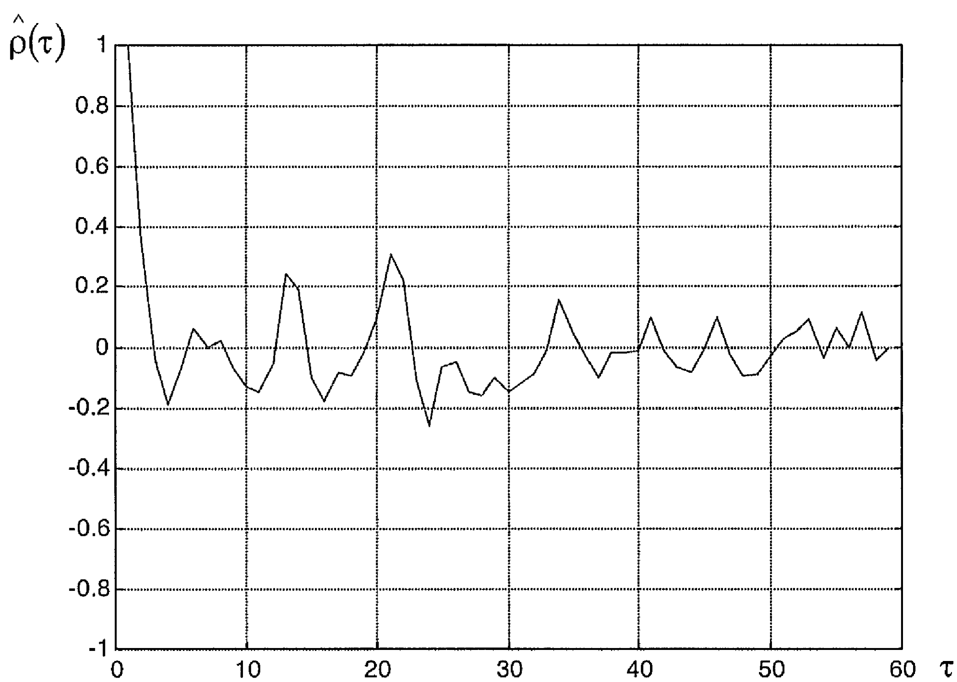


Figure B.2: Autocorrelograms of the time series Air Canada-Flight n°AC842.

† Autocorrelograms is the plot of the autocovariance coefficient for a time series.

Figure B.2 shows the autocorrelograms of the time series of British Flight n° BA84 whose departure airport is London Heathrow Airport.

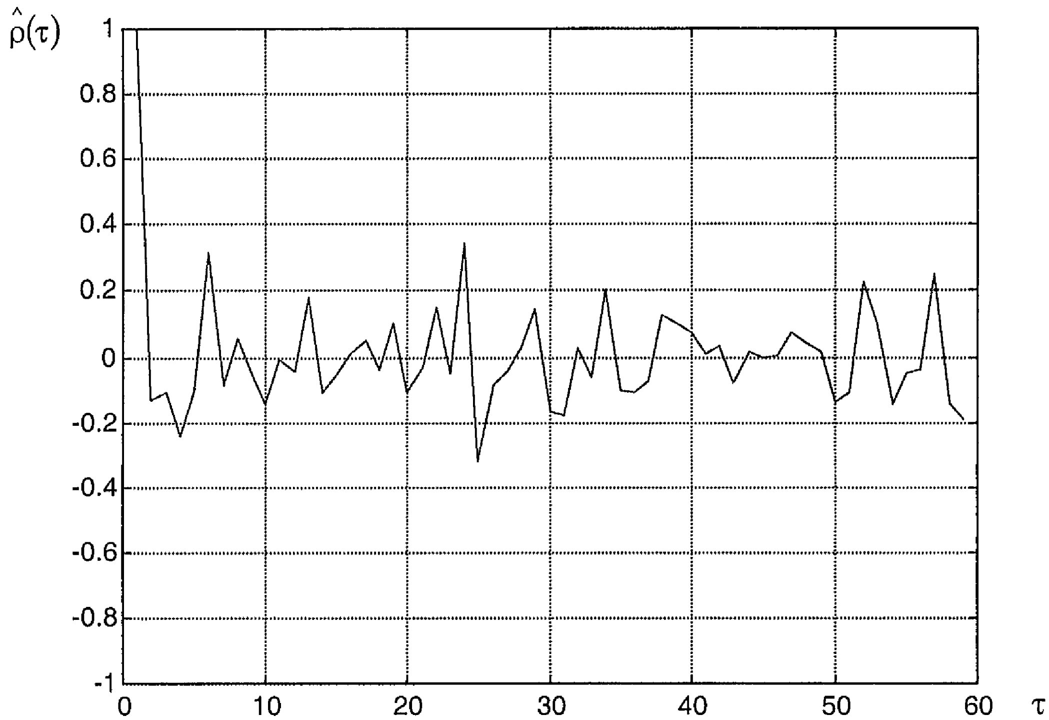


Figure B.3: Autocorrelograms of the time series British Flight-n° BA84. Departure airport: London Heathrow .

Figure B.2 and Figure B.3 shows that the samples of the time series arrival delays can be considered as independent. see [59] for further details.

The physical meaning of this result is that if in a given day an aircraft arrive late, then this fact does no effect the forthcoming days.

The correlogram† shown in Figure B.4 is obtained by calculating the cross-correlation between time series of two different flights. These two flights are chosen according to the fact that they come from different departure airports and they both arrive at Glasgow international airport at different times.

† Correlogram is the plot of the autocovariance coefficient for two time series.

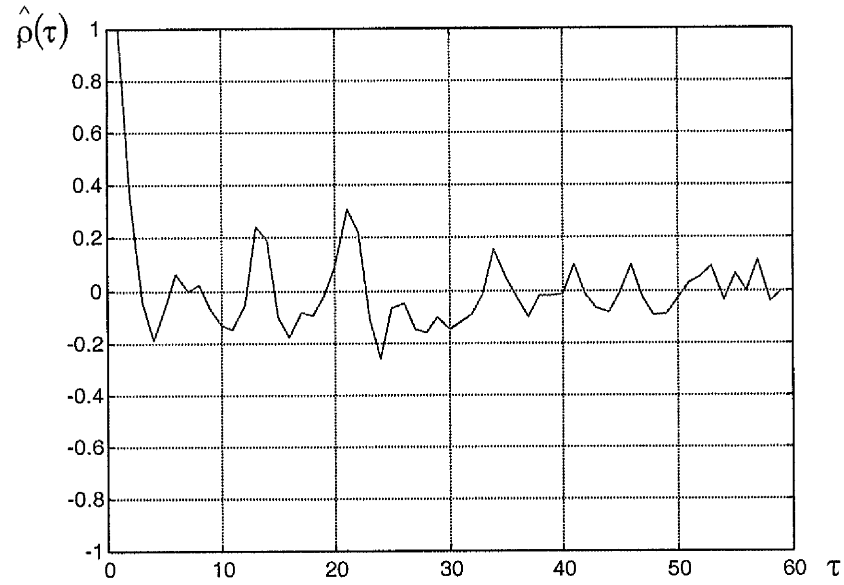


Figure B.4: Cross-correlogram between time series of two flights coming from different airports(AC842-BA84).

The flights considered in Figure B.4 come from different airports, in this case the dependency between the time series is very weak. The physical meaning of this weak dependency is that if an aircraft arrive late at the destination airport then another flight is not likely to arrive late at the same airport. This result is valid even for flights having the same departure and arrival airport.

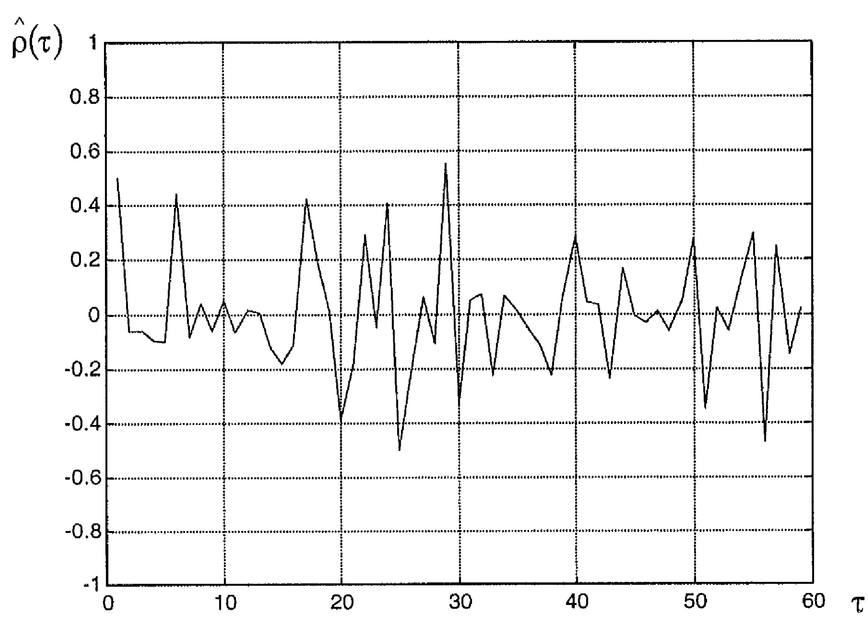


Figure B.5: Cross-correlogram between time series of flights coming from the same airport(BA86-BA84).

Figure B.5 shows the cross-correlogram of two flights scheduled at different times that go from London Heathrow Airport to Glasgow International Airport. In this case the dependence of the time series is also weak even if this dependence is stronger than the case shown in Figure B.4.

B.3.2 Size of the sample

During this thesis, PDFs are built from the histogram diagrams of the relative historical occurrences. In this section how to compute the necessary number of samples required to build these PDFs is shown.

In this thesis all PDFs are built using at least one hundred data $n \geq 100$, this size of the sample is enough for the margin of error required for this work. The goal of this thesis is to individuate the issues where statistics and probability theory can be efficiently applied. In section B.3.1, arrival delays are proven to be independent. The estimator $\hat{p}$ is a Binomial random variable because $\hat{p}$ is the sum of n Bernoulli random variables, see section B.2.2. The PDF of a Binomial random variable of parameters n and p is defined as:

$$f(\mathrm{x}, n, p) = \binom{n}{\mathrm{x}} p^{\mathrm{x}} (1-p)^{n-\mathrm{x}} \qquad \text{with } \mathrm{x} \in \{0,1,2,\ldots n\}$$

This PDF gives the probability of observing exactly x success in n independent trials of where the probability of success in any given trial is p. In this thesis to build the PDFs, at least $n \geq 100$ samples are used. As shown in section B.3.1 these sample and, hence, these trials are independent. If the number of successes in one hundred trials is equal to one, the estimation of $\hat{p}$ is derived by the following expression $\hat{p} = \frac{\mathrm{x}}{n} = \frac{1}{100} = .01$ (see section B.2.2). The PDF in term of p becomes:

$$f(p) = \binom{100}{1} p^{1} (1-p)^{100-1}$$

For a fixed p, this PDF gives the probability to observe $\hat{p}$ =0.01. In Figure B.6 the diagram of the distribution of p is shown for $\hat{p}$ =0.01.

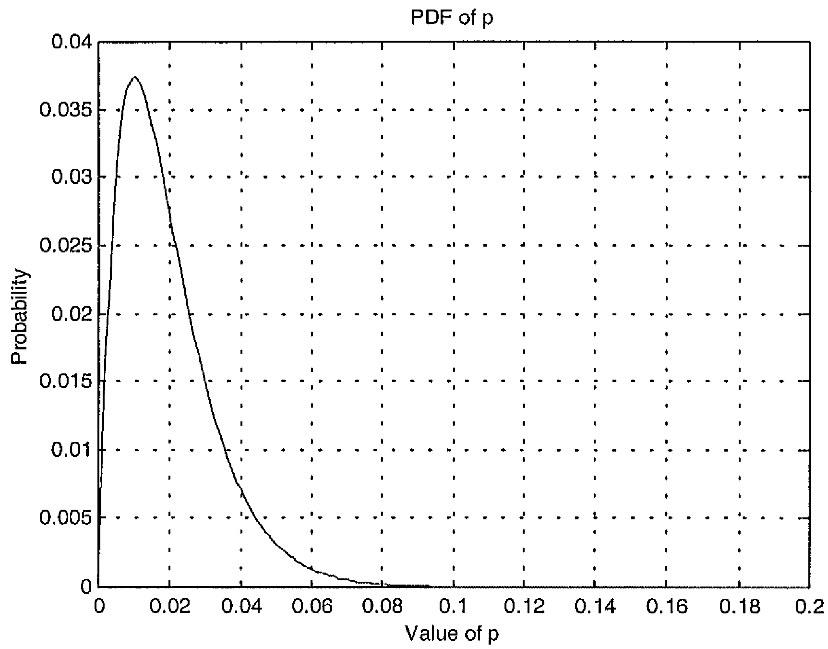


Figure B.6: Binomial test for $\hat{p}$ =0.01.

The 90 % confidence interval for $\hat{p}$ =0.01 is given by:

$$0.003 \leq p \leq 0.040$$

In Figure B.7 the diagram of the distribution of p is showed for $\hat{p}$ =0.03.

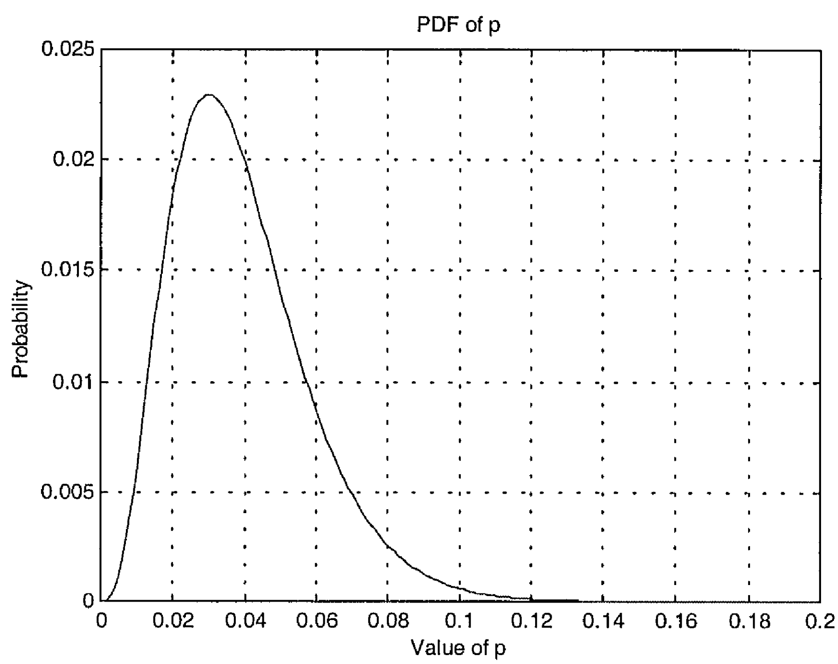


Figure B.7: Binomial test for $\hat{p}$ =0.03.

The 90% confidence interval is given by:

$$0.0120 \leq p \leq 0.070$$

As shown in [60] [61] [62], if the size of the sample is such as

$$\begin{cases} np \geq 5 \\ nq \geq 5 \end{cases}$$

then p is approximately a Normal random variable. The confidence interval is computed through the following expression:

$$\hat{p} - z\sqrt{\frac{\hat{p}\hat{q}}{n}} \leq p \leq \hat{p} + z\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

For an interval of confidence of 90%, the table of the normal standard gives z=1.64.

For an estimation of $\hat{p}$=.05

$$0.0143 \leq p \leq 0.0857$$

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