Essays on Belief-driven Macroeconomic Volatility and Expectation Formation

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Abstract

This thesis investigates macroeconomic volatility through the lens of expectation formation, emphasising the role of belief-driven mechanisms. In particular, I explore the implications of incorporating Diagnostic Expectations (DE), a recent deviation from the standard rationality assumption, into macroeconomic models.

The first chapter embeds DE into a Small Open Economy framework *à la* Justiniano and Preston (2010), a benchmark model for analysing exchange rate dynamics. Recent studies show that DE generate excess volatility, short-term extrapolative behaviour and predictable shifts in investor sentiment, characteristics that align with puzzles in international macroeconomics, particularly excess exchange rate volatility and exchange rate disconnect (Obstfeld & Rogoff, 2000). For this reason, DE emerge naturally as a possible behavioural explanation for these phenomena. In this chapter, I leverage the international finance nature of the economy to study the interaction between DE and the exchange rate transmission channel, which is otherwise absent in a closed economy. I parameterise the model following the open economy literature and show that when the model is populated with diagnostic agents, the economy exhibits greater volatility *vis à vis* the rational model. Moreover, DE introduce an amplification mechanism through shock extrapolation, which helps to qualitatively account for the excess volatility of the real exchange rate and its disconnection from fundamentals. The degree of departure from Rational Expectations (RE), captured by the diagnostic parameter, plays a central role in this extrapolation mechanism, with larger values amplifying the effect. I also use the model to assess the sensitivity of the results to different parameter values. The main finding highlights that economic openness and DE do not operate in isolation; rather, they amplify each other's effects. In addition, I show that persistence mechanisms, especially interest rate smoothing, are essential for translating and intensifying the amplification effect of DE into short-run macroeconomic dynamics.

The second chapter expands the study of DE within macroeconomic models, now concentrating on the housing sector. Empirical evidence from the U.S. reveals that the housing market exhibits an unusually high degree of volatility, with survey-based expectations displaying biases that challenge the rationality assumption. In addition, traditional

models often depend on volatile housing preference shocks to account for these fluctuations. In this chapter, I argue that the expectations channel plays a key role in driving housing market volatility. I incorporate DE into a Two-Agent New Keynesian (TANK) model featuring a housing and a banking sector to analyse the impact of this departure from rationality. I calibrate some parameters to the U.S. economy for the post-Volker - pre-Covid-19 pandemic period and estimate the remaining parameters using Sequential Monte Carlo methods. I find that DE reduce the volatility of the housing preference shock by more than one-third relative to RE, while still reproducing the observed housing market fluctuations. This result holds regardless of whether agents' imperfect memory is based on recent or on three-year past experiences. When the expectations channel is removed, that is, when agents become rational, the model fails to generate the high volatility in house prices found in the data. These findings emphasise the importance of the expectations formation process for explaining a substantial part of the "unmodeled disturbances that can affect the housing market", which Iacoviello and Neri (2010) attribute to a housing preference shock, and in shaping policy responses.

The third chapter extends the previous analyses by further demonstrating the effects of incorporating DE into macroeconomic models. Survey evidence, first presented by Coibion and Gorodnichenko (2015), sparked a broader discussion on deviations from the Full Information Rational Expectations (FIRE) framework (Fuhrer, 2018; Angeletos, Huo, & Sastry, 2021; Kohlhas & Walther, 2021). Specifically, Coibion and Gorodnichenko (2015) find that Forecast Errors (FE) and Forecast Revisions (FR) are predictable, suggesting that agents do not fully incorporate available information, a challenge to the FIRE hypothesis. In this chapter, I explore the impact of DE on the state-space structure of linear macroeconomic models and the resulting FE and FR across different horizons. In a three-equation specification, I derive testable expressions in terms of the model-parameters and also generalise it to the case of larger models. I find that DE introduce predictability in the form of moving average (MA) processes. To assess whether expectation formation differs across agents, I analyse survey data from the Philadelphia Fed's Survey of Professional Forecasters alongside policymakers' forecasts from the Greenbook/Tealbook. The empirical results indicate that one-period-ahead FE generally follow the MA structures implied by DE, though evidence of overreaction appears only for GDP growth forecasts and primarily when including the post-pandemic period. Mixed results are, however, observed in the case of FR. For longer forecast horizons, FE include autoregressive components deviating from DE, whereas FR align more closely with DE-driven expectations, suggesting stronger revisions in the direction of the shock realisation. While these results provide insights into belief formation, they remain far from conclusive.

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ALTERNATIVE THESIS FORMAT

This is an alternative format Ph.D. Thesis and includes three papers. The papers are presented in the following order:

1. The Effects of Diagnostic Expectations in a Small Open Economy
2. A Diagnostic TANK Model for the Housing Market
3. Forecasting Under Distorted Beliefs: The Impact of Diagnostic Expectations

I declare that this thesis has not been submitted for any other degree at the University of Glasgow or any other institution.

Marcos Gaspar Montenegro Calvimonte

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A mis padres, hermano y abuela.

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Introduction

Motivation

The long-standing assumption that economic agents form expectations rationally has been pivotal in macroeconomic models. However, in recent years, the idea that belief-driven forces shape macroeconomic dynamics has gained increasing attention. An expanding body of empirical evidence and literature challenges the rational expectations (RE) assumption by revealing systematic deviations from Full-Information Rational Expectations (FIRE).[1] Survey-based analyses of forecast errors from consumers, business, and professional forecasters indicate consistent predictable patterns, suggesting that agents do not fully incorporate all available information when forming expectations. This finding contradicts the core premise of FIRE and raises fundamental questions about its validity as a benchmark, yet consensus on a suitable alternative is still lacking (Reis, 2020).

Understanding belief-driven dynamics is essential for improving macroeconomic models as it shifts the focus from exogenous disturbances to the internal mechanisms that shape expectation formation. Traditional models often struggle to replicate observed macroeconomic volatility without relying on large and unrealistic shocks. Chari, Kehoe, and McGrattan (2009) highlight that many of these shocks lack solid microeconomic foundations and exhibit unrealistically high variances. Behavioural deviations offer a potential solution, as they suggest that distortions in expectation formation can amplify macroeconomic dynamics in a way that aligns more closely with observed data.

Many models now deviate from the traditional FIRE framework in macroeconomics. Some examples among these include adaptive learning (Evans & Honkapohja, 2001), sticky information (Mankiw & Reis, 2002), rational inattention (Sims, 2003) and cognitive discounting (Gabaix, 2020). Nevertheless, Diagnostic Expectations (DE) stand out as a significant recent framework in this area. Formalised by Gennaioli and Shleifer

[1] A comprehensive list of studies testing the FIRE assumption would be extensive and it is beyond the scope of this motivation. However, some remarkable examples range from Mishkin (1983), Muth (1961), Fama (1970) to more recent contributions such as Coibion and Gorodnichenko (2015), Bordalo, Gennaioli, Ma, and Shleifer (2020), Born, Enders, and Müller (2023), Farmer, Nakamura, and Steinsson (2024), among others. For a broader literature review, see Coibion, Gorodnichenko, and Kamdar (2018).

(2010), DE are based on the concept of representativeness heuristic from Kahneman and Tversky (1972). This heuristic explains a cognitive bias in the human memory recall mechanism, whereby an individual overestimates the likelihood of certain outcomes based on their perception of current conditions. This distortion introduces exaggerated responses and forward-looking beliefs, which contribute to greater volatility and feedback mechanisms that intensify optimism and pessimism. As a result, DE generate more volatile expectations, amplifying the impact of shocks in macroeconomic models. This approach successfully captures credit cycle characteristics (Bordalo, Gennaioli, & Shleifer, 2018), as well as observed overreactions in forecast data (Bordalo et al., 2020).

This thesis investigates the qualitative and quantitative role of DE within New Keynesian models, moving beyond their initial application in real business cycle frameworks. The main objective is to leverage DE as an amplification mechanism to explain the excessive volatility and deviations from fundamentals observed in macroeconomic data. In particular, this thesis focuses on two core empirical economic patterns: the dynamics of exchange rates in open economies and the high volatility of the U.S. housing market relative to fundamentals. These aspects are worth studying not only because exchange rates shape daily economic interactions and the housing sector reflects broader economic conditions, but also because both are sensitive to monetary policy decisions.

Belief-driven macroeconomic volatility in exchange rates and housing markets can lead to policy misjudgments and inefficiencies if traditional models continue to assume RE. Exchange rate misalignments can distort trade and investment, while housing market volatility amplifies financial instability, highlighting the need to better understand how individuals form expectations. By integrating DE into a New Keynesian framework, this thesis contributes to a growing literature that reassesses economic modelling, policy design, and market dynamics through the lens of expectations-driven distortions, specifically DE (L'Huillier, Singh, & Yoo, 2021; Bounader & Elekdag, 2024; Bianchi, Ilut, & Saijo, 2024). Beyond this, I take a step further by examining how DE affect Forecast Errors (FE) and Forecast Revisions (FR) in a general class of linear models. I derive testable expressions to assess their alignment with survey data and to offer insights into the implications of departures from rationality for macroeconomic analysis and forecasting.

Overall, I find that incorporating DE into a Small Open Economy (SOE) offers a qualitative explanation for the excess volatility of exchange rates and their disconnection from fundamentals. I also find that a DE-augmented New Keynesian model with a housing sector demonstrates a stronger empirical fit than a RE model when estimated on U.S. data. The model successfully captures the housing market fluctuations while relying on a less volatile housing preference shock, normally used by the rational literature to explain these dynamics. Finally, I show that representativeness, as a belief formation bias, introduces predictable components in the FE and FR from the models, however, the empirical results are far from conclusive.

The remainder of this Introduction provides a review of the three chapters and outlines their methodology and results.

Review of Chapters

Chapter 1 explores the role of representativeness, in the form of DE, in shaping macroeconomic dynamics within a Small Open Economy (SOE) framework. Traditional models often struggle to explain why exchange rates exhibit extreme volatility and seem so apparently disconnected from macroeconomic fundamentals, despite their relative stability (Obstfeld & Rogoff, 2000). Moreover, Devereux and Engel (2002) point out that exchange rates may vary dramatically, without being influenced by fluctuations in other macroeconomic variables. This chapter evaluates whether behavioural deviations provide a superior explanation compared to traditional RE models.

I embed DE in a version of the Justiniano and Preston (2010) SOE model, where economic openness activates the exchange rate transmission channel, facilitating an examination of its interaction with DE. Agents within this economy will assign a higher probability to states of the world that are more likely tomorrow in light of what they observe today, relative to what they would have predicted in the previous period. Therefore, to solve this model (and the model in Chapter 2), I integrate insights from Bordalo et al. (2018), assuming that agents misperceive the state of the economy, while also adopting the RE representation of the DE model in terms of its solution structure, as proposed by L'Huillier et al. (2021) and incorporating the naïveté approach from Bianchi et al. (2024).

The results suggest that the presence of diagnostic agents in a SOE could explain why exchange rates are excessively volatile and often appear disconnected from fundamentals. I quantify the impact of DE and its interaction with persistence mechanisms by analysing second-order moments and impulse responses. A comparison between DE and RE reveals that a SOE with diagnostic agents is more volatile than one with rational agents, with the amplification effect varying according to the degree of distortion in agents' beliefs. Overall, the chapter qualitatively demonstrates that including representativeness through DE in a SOE framework provides a robust explanation for the exchange rate puzzle.

Chapter 2 continues the exploration of belief-driven macroeconomic volatility by turning to the housing sector, where expectations appear to influence market behaviour (De Stefani, 2021; Adam, Pfäuti, & Reinelt, 2024; Kuchler, Piazzesi, & Stroebel, 2023).

Similar to exchange rate dynamics, the housing market exhibits characteristics that make it particularly susceptible to shifts in beliefs and sentiment, thus positioning DE as a suitable analytical framework. Building on its effectiveness in explaining credit cycles and recognising the deep interconnection between credit and housing, this chapter moves beyond qualitative discussion by extending and estimating a Two-Agent New Keynesian (TANK) model with diagnostic agents.

The model integrates elements from Iacoviello and Neri (2010) for the housing sector and from Gertler and Karadi (2011) for the banking sector. DE influence beliefs as agents in this model will extrapolate historical patterns resulting in amplified market responses. Unlike standard models that attribute such fluctuations to large housing preference shocks, this approach demonstrates that DE provide a more plausible mechanism. In this chapter, however, I go a step further by considering not only cases where agents rely on the most recent past, but also those that allow for a slow-moving memory of past experiences.

I estimate the housing model using Sequential Monte Carlo methods (see Herbst & Schorfheide, 2014) for the U.S. over the period 1984:Q1 to 2019:Q4. The results confirm that DE reduce the need for large housing preference shocks by over one-third while still replicating observed housing market volatility. Without DE, the model either struggles to generate the pronounced house price fluctuations seen in the data or requires large and unrealistic shifts in housing preferences. The quantitative analysis also highlights the importance of recent experiences in shaping expectations, with evidence favouring the model that includes immediate past events over those that rely on distant memories.

Overall, this chapter reinforces the idea that behavioural distortions, such as DE, are central to housing market cycles. It provides an empirical evaluation of representativeness within a model that includes housing and financial frictions, with an estimated diagnostic parameter value consistent with previous findings. Given the importance of housing in household decision making, failing to account for expectation-driven distortions could undermine the effectiveness of monetary policy, as individuals may not respond to policy changes as intended. Therefore, incorporating DE into policy frameworks would help Central Banks and policymakers better anticipate and respond to market fluctuations.

Chapter 3 builds on the insights of the previous two chapters by studying how incorporating DE affects the state-space representation of a linear macroeconomic model and its implications for Forecast Errors (FE) and Forecast Revisions (FR) across different horizons. The inclusion of DE not only imposes a common structure across model variables but also adds systematic predictability to the FE and FR, which is consistent with survey data that challenge the FIRE assumption (Coibion & Gorodnichenko, 2015;

Kohlhas & Walther, 2021; among others).

The main contribution of this chapter is to develop a general and testable expression for FE and FR in terms of the state-space solution for linear models with DE. First, by means of a three-equation model, I show that DE introduce predictability in FE and FR in the form of a moving average (MA). Second, I extend this result by demonstrating that in larger models FE and FR follow a vector moving average (VMA) process. Using data from the Survey of Professional Forecasters and the Greenbook/Tealbook, I examine two periods: post-Volcker disinflation to pre-pandemic, with an alternative specification that extends into the post-pandemic period. I am interested in studying several questions, for instance, whether expectation formation varies across individuals, variables, and forecast horizons.

Empirical results confirm that FE and FR follow predictable patterns consistent with the structure generated by DE. However, estimates are mixed. In the univariate case, some variables exhibit MA structures, while others do not. In contrast, in the multivariate case, a VMA provides the best fit for both FE and FR, reinforcing the idea that belief distortions play a crucial role in shaping expectations. Nonetheless, capturing extreme volatility during crises, such as the Great Financial Crisis and the COVID-19 pandemic, remains a challenge. In general, this chapter is informative rather than conclusive, opening many avenues for further research. The way agents form expectations appears to vary depending on the individual, the information available, the variables that are forecasted, and the forecast horizon.

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Chapter 1

The Effects of Diagnostic Expectations in a Small Open Economy

1 Introduction

In macro and finance models, deviations from the mainstream hypothesis of rational expectations have blossomed in recent years. Among these alternatives, Diagnostic Expectations (DE) is becoming increasingly popular. Bordalo, Gennaioli, and Shleifer (2018) developed DE by formalising the concept of representativeness that Kahneman and Tversky (1972) introduced in their seminal paper. Representativeness describes a heuristic or shortcut in an individual's judgemental process, which generates a pattern of grouping events by similarity around a reference category. By this process, representativeness produces confusion between representative and likely events, resulting in an overestimation of its likelihood.

DE therefore offer a source of excess volatility, short-term extrapolative behaviour, and predictability of investors' boom-bust sentiment (Bordalo et al., 2018 and Bordalo, Gennaioli, Shleifer, & Terry, 2021). These are emblematic features in the context of exchange rates, which have long been recognised as one of the central puzzles in international macroeconomics. As highlighted by Obstfeld and Rogoff (2000), exchange rates often exhibit excessive volatility and appear disconnected from macroeconomic fundamentals. The ability of DE to amplify short-term fluctuations and generate excess volatility suggests that it may provide a behavioural explanation for these empirical patterns. Furthermore, evidence from forecast errors supports the idea that market participants systematically overreact to exchange rate movements. Following Bordalo et al. (2018), I analyse the predictability of forecast errors using a regression approach in which the independent variable is the logarithm of the nominal exchange rate for the New Zealand dollar against the U.S. dollar at time t, whereas the dependent variable is the quarterly forecast error for the nominal exchange rate, both plotted in Figure 1.1.[1] The estimated coefficient is significant at the 5% level, and its value suggests that a 1% increase in the nominal exchange rate at time t leads to a 0.0949% overestimation of the exchange rate at $t+1$. This finding provides direct evidence against the full-information rational expectations assumption in a developed country, reinforcing the role of DE in shaping exchange rate expectations.[2]

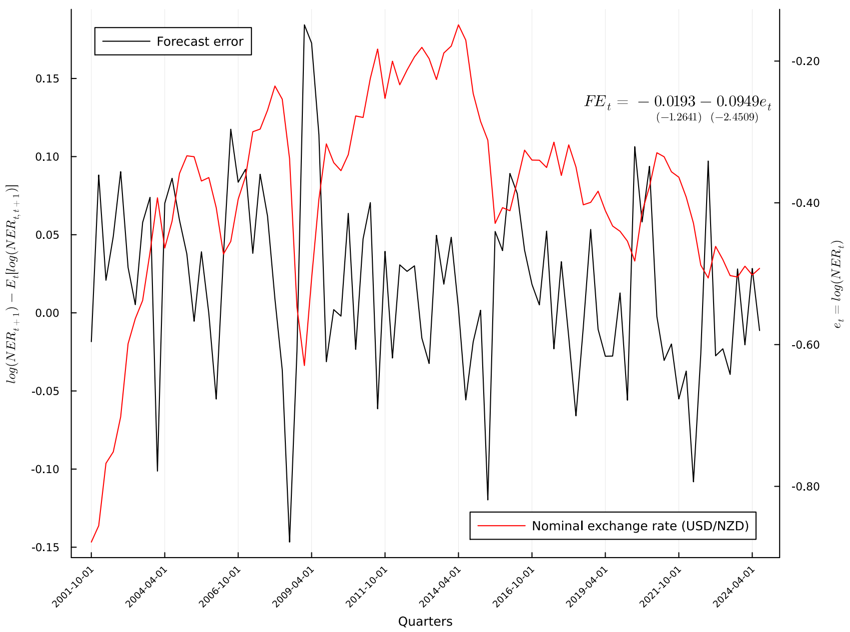

Figure 1.1: Predictable errors in forecasts of nominal exchange rate.
Note: The data series used to construct the variables in this plot were obtained from the Reserve Bank of New Zealand (RBNZ). The nominal exchange rate denotes the rate at which one currency is exchanged for another currency, in this case New Zealand dollar against U.S. dollar. The forecasted exchange rate is obtained from the RBNZ survey of expectations, which asks for the mid-rate at the end of next quarter and publishes the mean of the responses.

In this chapter, I explore the implications of incorporating representativeness in the form of DE into a Small Open Economy (SOE) model *à la* Justiniano and Preston (2010b). The motivation to do so is twofold. First, I conduct my analysis using the

[1]The quarterly forecast error is calculated as the difference between the realised value of the logarithm of the nominal exchange rate one quarter ahead and the logarithm of the one-quarter-ahead mean forecast. The time period covers December 2001 until September 2024.

[2]In developing countries, this also seems to be true, as evidenced in Pozdnyakova (2025).

Justiniano and Preston (2010b) model because this model has become a common benchmark to understand the behaviour of real exchange rates (Dennis, Leitemo, & Söderström, 2006; Alpanda, Kotzé, & Woglom, 2010 and Ca'Zorzi, Kolasa, & Rubaszek, 2017; among others). Second, I incorporate DE since the previous finding and its inherent features suggest that the approach could provide a non-rational explanation for key puzzles in international macroeconomics, particularly those related to exchange rates. Moreover, by opening up the economy, the exchange rate transmission channel becomes relevant, and I can study the interaction between DE and openness, as well as the role of DE in exchange rate volatility and its implications for domestic variables.

This chapter contributes to the literature in three ways. First, in the absence of an application of DE in a SOE, this is the foremost study to embed and analyse the properties of DE in an open economy framework. Prior literature focused on how DE affected closed and medium-scale New Keynesian models (L'Huillier, Singh, & Yoo, 2021; Bianchi, Ilut, & Saijo, 2021). To solve the model, I employ a log-linearisation strategy in line with Bordalo, Gennaioli, Shleifer, and Terry (2021). Moreover, I obtain a representation of the model with diagnostic agents in an alternative way to L'Huillier et al. (2021). Second, this chapter shows that the presence of diagnostic agents in a SOE offers a behavioural explanation for certain international macroeconomic puzzles, specifically the excess exchange rate volatility and the exchange rate disconnect puzzle.[3] Third, this chapter also studies aspects of the interaction between DE and the persistence mechanisms that characterise the model.

Using unconditional variance and impulse responses, I quantify the impact and magnitude of DE in this SOE, as well as their interaction with persistence mechanisms. The results are meaningful. A SOE populated with diagnostic agents is generally more volatile than its counterpart with rational expectations (RE). This amplified response suggests that DE is a helpful mechanism to explain how susceptible an economy is to disturbances. Moreover, the magnitude of this outcome strongly depends on the degree of distortion in the agents' beliefs. As the diagnosticity of agents increases, the economy becomes more sensitive. However, the amplification effect of DE turns out to be neither homogeneous nor linear for the SOE's variables. This emerges as a consequence of persistence mechanisms and the inclusion of endogenous state variables. In addition, I compare the impulse responses under RE and DE. In most cases there is an initial over-reaction in response to the shock and a subsequent reversal towards rationality. Such a reversal in behaviour is not instantaneous due to the presence of persistence mechanisms that propagate DE

[3] Devereux and Engel (2002) explain these puzzles by stating that "while exchange rate volatility is ultimately tied to volatility in fundamental shocks to the economy, the exchange rate can display extremely high volatility without any implications for the volatility of other macroeconomic variables" (p. 4).

effects in the model.

I also study how changes in the model's parameterisation amplify or attenuate the effects of DE. The main result points to the existence of an interaction between the open economy channel and DE. The most influential parameters on DE's amplification mechanism are those related to the openness of the economy, that is, the degree of openness and the elasticity of substitution between domestic and imported goods. Finally, I also show how the persistence mechanisms within the model interact with DE, playing a role in either propagating or muting its effects. Interest rate smoothing interacts with DE and helps to intensify and prolong its effects after a shock hits the economy. Habits, on the other hand, neither intensify nor mute DE's effects in most of the variables. Whereas, when I add both mechanisms to the diagnostic SOE, I find that interest rate smoothing governs almost all variables' reactions, while habits shape the behaviour of consumption, output, and labour.[4]

Related Literature

This chapter is linked to very recent articles including DE in macroeconomic models. The leading works are those of Bordalo et al. (2018), Bordalo, Gennaioli, Shleifer, and Terry (2021), Maxted (2019) and L'Huillier et al. (2021). These authors incorporate DE in macro-finance environments. Bordalo et al. (2018) find that including DE in a macroeconomic model of investment improves its ability to capture and replicate empirical characteristics with regard to credit cycles. Bordalo, Gennaioli, Shleifer, and Terry (2021) and Maxted (2019) combine DE in real business cycle models with financial frictions. Their main results are greater variability in the macroeconomy and the ability to replicate aspects of financial crises, as well as the countercyclicality of credit spreads.

More closely related to my work are the articles of L'Huillier et al. (2021), Bianchi et al. (2021) and Na and Yoo (2025). The first authors derive a general framework to incorporate DE in linear models and demonstrate that DE are a viable behavioural alternative to generate fluctuations in business cycle models with shocks of more realistic size. The second authors analyse the inclusion of DE in linear models, with emphasis on distant memory. In this scenario, they discover that DE create a complex set of dynamics, marked by significant persistence and sudden changes in the way shocks propagate. The last authors build on these efforts and extend an open economy real business cycle with DE. They estimate the model with Argentinean data and find that trend productivity shocks play a less important role as diagnostic agents perceive transitory shocks to be more persistent. This helps the DE model outperform the RE one.

[4] Labour in this model represents hours worked as specified in Justiniano and Preston (2010a). The original Justiniano and Preston (2010b) model also introduces price indexation. In this chapter, I remove this after testing whether its inclusion was meaningful for the results.

This article also relates to an emerging literature that attempts to put together the pieces of international macroeconomic puzzles. Specifically, those trying to explain exchange rate puzzles by introducing behavioural assumptions in macro models. For example, Crucini, Shintani, and Tsuruga (2020) include inattentive firms *à la* Gabaix (2020) in a two-country sticky price model.[5] They show that firm inattentiveness helps account for complementarity between the purchasing power parity and the law of one price, i.e. the former being too persistent and the latter insufficiently persistent. Another example of a SOE model with bounded rationality is the study of Du, Eusepi, and Preston (2021). These authors estimate a SOE as in Justiniano and Preston (2010b) with learning. Their main result is that by including learning, their model is able to generate extrapolation bias. This extrapolation, therefore, helps the model to address the persistence and volatility of the exchange rate, but at the cost of predicting negative international macroeconomic co-movements between domestic and foreign output growth, that contradict empirical data. Another article is Candian and De Leo (2023), where the authors develop and include misperceptions and over-extrapolative beliefs in an open economy model. Their findings suggest that this model reproduces exchange rate dynamics and also accounts for the observed predictability of forecast errors in interest rates.

In addition, another framework that effectively accounts for exchange rate puzzles by inducing meaningful volatility is rare (macroeconomic) disasters as in Barro (2006). Guo (2007) and Farhi and Gabaix (2016) analyse the results of integrating the rare-disasters framework within a standard SOE. They found that such disaster-based SOE introduces a higher volatility in stocks and exchange rates as agents are afraid of the possibility of a disaster happening. The authors conclude that this model is capable of accounting for various macroeconomic puzzles, both quantitatively and qualitatively.

Structure of the chapter

The chapter is organised as follows. Section 2 explains the main idea behind DE. Section 3 outlines a SOE. Section 4 introduces DE into this model and explains how to solve the model. Section 5 discusses the main results. Section 6 presents a sensitivity analysis. Section 7 concludes.

2 Diagnostic expectations

Bordalo et al. (2018) introduce DE, grounded in psychological evidence, as a way to model beliefs. They rely on Gennaioli and Shleifer (2010) formalisation of the Kahneman

[5] In similar lines, Kolasa, Ravgotra, and Zabczyk (2022) also include boundedly rational agents following Gabaix (2020) in an Open Economy New Keynesian model. They use it to analyse monetary policy implications, and they find that their model solves UIP-related puzzles.

and Tversky (1972) representativeness heuristic, which describes a systematic departure on agents' probabilistic judgements from Bayesian updating. Moreover, Kahneman and Tversky (1972) define such heuristic by stating that: "an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in the relevant reference class" (p. 296).

Diagnosticity can be thought of as a distortion in an agent's memory recall mechanism, which provokes a confusion between the concepts of representative and probable. According to this, an agent would make mistakes by misperceiving an uncommon present condition, such as the state of the economy, as typical when compared to a certain reference point. Therefore, she would erroneously assign an inflated probability to the unusual state, inducing exaggerated responses and also forward-looking beliefs. These ultimately yield two key features: the kernel of truth property introduced in the psychology literature and the exemption from the Lucas (1976) critique.

In line with Bordalo et al. (2018), who are the first authors to embed representativeness in a macrodynamic model as a departure from the rationality assumption, I also apply this logic to the formation of agents' beliefs about aggregate economic variables. Likewise, I define the state of the economy, represented by X, to follow an AR(1) process:

$$X_{t+1} = \rho X_t + \epsilon_{t+1}, \tag{1.1}$$

where $\rho \in [0,1)$ is a vector of persistence parameters, and ϵ_{t+1} is normally distributed with mean zero and standard deviation σ_ϵ. There are two reasons not to change the assumption of the state's behaviour as an AR(1) process. First, as Benjamin (2019) suggests, it is very tractable and displays some convenient formal properties. Second, as L'Huillier et al. (2021) state, extending such a process in a Dynamic Stochastic General Equilibrium (DSGE) environment is not crucial given the already complex dynamics introduced by mechanisms such as habits, indexation, and so on.

At time t, the diagnostic agent will forecast the state of the economy at time $t+1$. In doing so, her mind will work searching for and recovering realisations of X_{t+1} given X_t, which are representative relative to some context. This suggests, under the AR(1) process considered, that the context is limited to information held at $t-1$, that is, the agent will overweight the last realisation of X_t.[6] Bordalo et al. (2018) formalise this overweighting "as if"the agent uses a distorted density function such:

[6] Considering past performance of economic variables is important because "it links representativeness to dynamic inference problems that are important in finance and macroeconomics"(Gennaioli & Shleifer, 2020, p. 145).

$$f^{\phi}(X_{t+1}|X_t) = f(X_{t+1}|X_t)\left[\frac{f(X_{t+1}|X_t)}{f(X_{t+1}|\rho X_{t-1})}\right]^{\phi}\frac{1}{Z}, \tag{1.2}$$

where $f(X_{t+1}|.)$ stands for the density of X_{t+1} conditional on two events: first, the current realisation of X_t, and second, the most recent past realisation, ρX_{t-1}, as a reference. Here, it is worth highlighting how representativeness alters the rational agent's beliefs, $f(X_{t+1}|X_t)$, by the likelihood ratio $\frac{f(X_{t+1}|X_t)}{f(X_{t+1}|\rho X_{t-1})}$ to a degree given by the diagnostic parameter ϕ. Moreover, the ratio exhibits that when individuals perceive the state as relatively more frequent, i.e. diagnostic, it will more easily come to their mind. Consequently, they will assign a higher probability to a future occurrence of a state based on the realised X_t in comparison to the past information ρX_{t-1}.[7] Additionally, the constant Z in (1.2) ensures that the probability distribution $f^{\phi}(X_{t+1}|X_t)$ integrates to one, and the diagnostic parameter $\phi \in [0, +\infty)$ measures the extent of agents' belief distortions. The closer ϕ is to zero, the less restricted an agent's memory is, and thus the more proper use of information she makes, forming RE in the limit. On the other hand, the greater the value of ϕ, the more selective the agent's memory becomes, which increases the likelihood of easily retrievable representative states being recalled over non-representative ones.

In line with the above, by assuming that X_{t+1} is log-normally distributed under the true target and the comparison one, i.e. $ln(X_{t+1})|X_t \sim N(\mu_0, \sigma_\epsilon^2)$ and $ln(X_{t+1})|\rho X_{t-1} \sim N(\mu_{-1}, \sigma_\epsilon^2)$, contributes to generate characteristics about the diagnostic distribution in (1.2), summarised by Gennaioli and Shleifer (2010). A key aspect is that the distribution also turns out to be a log-normal distribution, maintaining the same variance, σ_ϵ^2, as the distribution under rationality, but with a distorted mean[8]:

$$\mathbb{E}_t^{\phi}(X_{t+1}) = \mathbb{E}_t(X_{t+1}) + \phi[\mathbb{E}_t(X_{t+1}) - \mathbb{E}_{t-1}(X_{t+1})]. \tag{1.3}$$

This equation embodies the so called "kernel of truth"characteristic, which is represented by the expression $\phi[\mathbb{E}_t(X_{t+1}) - \mathbb{E}_{t-1}(X_{t+1})]$.[9] This summarises how representativeness affects agents' behaviour, who correctly react in the same direction as the new information, but in an excessive manner. The exaggerated response generates a shift in the objective distribution of X_{t+1}. This causes a higher or lower mean compared to the rational case after a positive or negative new information, respectively.

Figure 1.2, taken from Gennaioli and Shleifer (2020), illustrates the previous mecha-

[7] Bordalo, Coffman, Gennaioli, and Shleifer (2019) present an illustrative example applying this framework to stereotypes, more precisely, the assessment of the distribution of hair colour among the Irish.

[8] The formal derivation of this result is presented in Appendix 1.B and follows Bordalo et al. (2018) and L'Huillier et al. (2021).

[9] In Macmillan Dictionary (2020), a kernel of truth is defined as "a very small part of something that is true, wise, etc".

nism. The left-most distribution describes the reference scenario, i.e. agent's beliefs of future values X_{t+1} conditional on current realisations being equal to its expected value. From the assumed AR(1) process, and after applying the expectation operator, this is equal to $X_t = \rho X_{t-1}$, represented by μ_{-1} in the figure.[10] The centre distribution also exhibits agent's beliefs of future values X_{t+1} conditional on the current realisation of X_t. However, this realisation is influenced by good news $\epsilon_t \neq 0$, which explains the shift of the graph to the right. Lastly, the right-most distribution illustrates the diagnostic distribution. This is obtained using the two previous distributions and Equation (1.2). The features introduced by diagnosticity are clearly observed: (i) extrapolation in the direction of the shock, which generates an inflated mean, as described by equation (3); (ii) greater weight on states in the direction of the shock, and (iii) a thinner left tail in line with the base rate neglect.

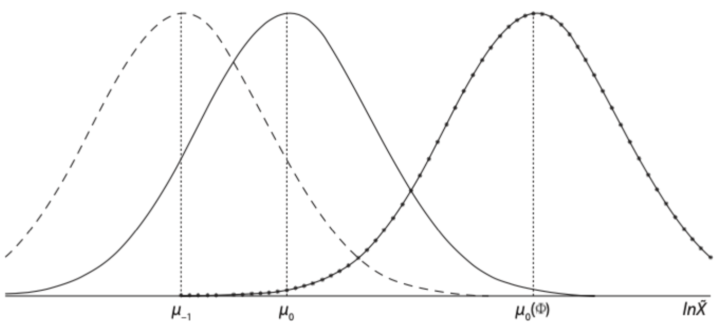

Figure 1.2: Distribution of ln(X) under Rational and Diagnostic Expectations (Gennaioli & Shleifer, 2020)

3 A Small Open Economy Model

In this section, I outline a small open economy framework *á la* Justiniano and Preston (2010b). This economy is populated by a unit mass of identical households with external habits, a continuum of monopolistically competitive domestic and retail firms, a Central Bank, and an exogenous foreign economy. Households supply labour to domestic firms and consume a basket of domestic and foreign goods. Domestic firms hire labour in a perfectly competitive market and their pricing decision presents some degree of inertia through a Calvo-style price rigidity. Retail firms in the SOE are intermediaries that sell foreign goods in the domestic economy and also set prices in a Calvo-style manner. The model is closed with a domestic Central Bank following a Taylor-type rule.

[10]The derivation follows by applying the expectation operator as follows: $\mathbb{E}_t[X_t] = \mathbb{E}_t[\rho X_{t-1}] + \mathbb{E}_t[\epsilon_t]$, and by $\mathbb{E}_t[\epsilon_t] = 0$.

3.1 Households

The economy consists of a unit mass of homogeneous households, as they face the same budget constraint and they have the same preferences. This allows to focus on a representative household who maximises her lifetime utility over consumption and labour, at each point in time:

$$U_0 = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t W(C_t, L_t, \Gamma_t)\right], \tag{1.4}$$

where the momentary utility function is assumed to be strictly concave, twice continuously differentiable, satisfy the Inada conditions, and it takes the following form:

$$W(C_t, L_t, \Gamma_t) = \frac{\Gamma_t(C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi}, \tag{1.5}$$

where $\beta \in (0,1)$ represents the household's rate of time preference; σ, $\varphi > 0$ are the inverse of the inter-temporal elasticity of consumption and labour supply, respectively. L_t denotes hours of labour supplied and Γ_t is a consumption preference shock, which follows $\Gamma_{t+1} = (\Gamma_t)^{\rho_\gamma} e^{(\epsilon^\gamma_{t+1})}$, where $\epsilon^\gamma_{t+1} \sim i.i.d.[0, \sigma^2_{\epsilon_\gamma}]$. C_t denotes agent's consumption in time t and $H_t \equiv hC_{t-1}$ represents the external habit-stock, where $h \in (0,1)$ denotes its degree. Moreover, C_t is a composite index of goods produced domestically and abroad:

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{1.6}$$

where $\alpha \in [0,1]$ is a measure of the economy's degree of openness, also representing the share of foreign goods in domestic consumption. $\eta > 0$ is the elasticity of substitution between domestic and foreign goods. Following Justiniano and Preston (2010b), $C_{H,t}$ and $C_{F,t}$ are Dixit-Stiglitz aggregates of domestic and foreign goods, respectively:

$$C_{H,t} = \left[\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad C_{F,t} = \left[\int_0^1 C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}},$$

here $\epsilon > 1$ measures the substitutability between varieties of domestic or foreign goods.[11] Finally, each household decides how to optimally assign her expenditure between domestic

[11] The demand within each category, optimal for any given expenditure, is obtained by solving a minimisation problem for the good produced domestically, as well as for the good produced overseas, yielding:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}, \qquad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}.$$

and imported goods. In doing so, she optimises and obtains the following standard demand functions:

$$C_{H,t} = (1-\alpha)\Big(\frac{P_{H,t}}{P_t}\Big)^{-\eta} C_t \quad \text{and} \quad C_{F,t} = \alpha\Big(\frac{P_{F,t}}{P_t}\Big)^{-\eta} C_t, \tag{1.7}$$

where $P_t \equiv [(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the domestic consumer price index (CPI).

Households maximise Equation (1.4) while being restricted by an intertemporal budget constraint given by:

$$P_t C_t + B_t + B_t^* s_t = W_t L_t + D_{H,t} + D_{F,t} + T_t + R_{t-1} B_{t-1} + s_t R_{t-1}^* \Phi_t(A_{t-1}, \tilde{\Phi}_t) B_{t-1}^*. \tag{1.8}$$

I assume that asset markets are incomplete. Households have access to one-period domestic bonds (B_t) and one-period foreign bonds (B_t^*), with their risk-free (gross) returns, R_{t-1} and R_{t-1}^*, respectively. Moreover, I follow Schmitt-Grohé and Uribe (2003) and add a debt elastic interest rate premium $\Phi_t(A_{t-1}, \tilde{\Phi}_t)$. This is implemented to induce stationarity in an open economy model. It is assumed to take the functional form:

$$\Phi_t = e^{-[\chi A_{t-1} + \tilde{\Phi}_t]},$$

$A_{t-1} \equiv \frac{s_{t-1} B_{t-1}^*}{\bar{Y} P_{t-1}}$ is the real quantity of outstanding foreign debt expressed in domestic currency as a fraction of steady-state output, as defined in Justiniano and Preston (2010b). $\tilde{\Phi}_t$ is a risk premium shock, which follows $\tilde{\Phi}_{t+1} = (\tilde{\Phi}_t)^{\rho_{\tilde{\Phi}}} e^{(\epsilon_{t+1}^{\tilde{\Phi}})}$. s_t is the nominal exchange rate expressing the price of the foreign currency in terms of the domestic one. Households earn income in the form of nominal wages W_t and dividends from domestic and retail firms, $D_{H,t}$ and $D_{F,t}$, since they are the owners of both firms. Finally, T_t denotes a lump sum government transfer.

The household chooses $\{C_t, L_t, B_t, B_t^*\}_{t=0}^{\infty}$ to solve her optimisation problem, which can be represented using the Lagrangian method as:

$$\begin{aligned} \mathscr{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \Bigg\{ \beta^t \Bigg[\Bigg(\frac{\Gamma_t (C_t - H_t)^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\varphi}}{1+\varphi} \Bigg) + \lambda_t \Big(P_t C_t + B_t + B_t^* s_t - W_t L_t \\ - D_{H,t} - D_{F,t} - T_t - R_{t-1} B_{t-1} - s_t R_{t-1}^* \Phi_t(A_{t-1}, \tilde{\Phi}_t) B_{t-1}^* \Big) \Bigg] \Bigg\}. \end{aligned} \tag{1.9}$$

The optimal conditions of this Lagrangian with respect to C_t, L_t, B_t and B_t^* generate the following intratemporal and intertemporal conditions:

$$\frac{\partial \mathscr{L}}{\partial C_t} : \Gamma_t(C_t - H_t)^{-\sigma} - \lambda_t P_t = 0, \tag{1.10}$$

$$\frac{\partial \mathscr{L}}{\partial L_t} : -L_t^{\varphi} + W_t \lambda_t = 0, \tag{1.11}$$

$$\frac{\partial \mathscr{L}}{\partial B_t} : -\lambda_t + \beta R_t \mathbb{E}_t[\lambda_{t+1}] = 0, \tag{1.12}$$

$$\frac{\partial \mathscr{L}}{\partial B_t^*} : -\lambda_t s_t + \beta R_t^* \mathbb{E}_t[\lambda_{t+1} s_{t+1} \Phi_{t+1}(A_t, \tilde{\Phi}_{t+1})] = 0. \tag{1.13}$$

After some work, these conditions boil down to the typical labour supply equation (1.14) and an Euler equation (1.15):

$$\Gamma_t(C_t - hC_{t-1})^{-\sigma} \frac{W_t}{P_t} = L_t^{\varphi}, \tag{1.14}$$

$$\beta R_t \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \frac{\Gamma_{t+1}}{\Gamma_t} \right] = 1. \tag{1.15}$$

3.2 Firms

3.2.1 Technology

Domestic firms produce domestic goods using the following production technology, where the level of technology Z_t is assumed to be the same among the firms.

$$Y_t(j) = Z_t L_t(j). \tag{1.16}$$

These firms hire labour in a perfectly competitive market, taking wages as given. Z_t follows $Z_{t+1} = (Z_t)^{\rho_\zeta} e^{(\epsilon_{t+1}^{\zeta})}$, and $\epsilon_{t+1}^{\zeta} \sim i.i.d.[0, \sigma_{\epsilon_\zeta}^2]$. A firm's marginal cost of production under this technology is defined as $MC_{H,t} = \frac{W_t}{P_{H,t} Z_t}$. Each firm minimises its cost subject to the good's demand by choosing how much labour to hire.

3.2.2 Price Setting of Domestic firms

Domestic firms set prices in a staggered manner, more specifically, they follow a Calvo-style price setting. They receive a random draw each period from a Bernoulli distribution. This draw indicates whether each firm is able to re-optimise and set a new price with probability $(1 - \theta_H)$, or whether a firm is not able to adjust its price, with probability θ_H. Since all firms able to set their price in period t, after a successful draw, have the same decision problem, they will choose a common price. Hence, the Dixit-Stiglitz aggregate domestic price index evolves as:

$$P_{H,t} = \left[(1-\theta_H) {P'}_{H,t}^{1-\epsilon} + \theta_H P_{H,t-1}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}. \tag{1.17}$$

When optimising a new price $P'_{H,t}$, a firm seeks to maximise the expected present discounted value of profits subject to its demand curve:

$$\max_{P'_{H,t}} V_t^H(i) = \mathbb{E}_t \sum_{T=t}^{\infty} (\theta_H)^{T-t} \Big[\mathcal{Q}_{t,T} Y_{H,T}(i) \Big(P'_{H,t} - P_{H,T} MC_T \Big) \Big]$$
$$\text{subject to: } \mathrm{Y}_{H,t}(i) \leq \left(\frac{P'_{H,t}}{P_{H,T}} \right)^{-\epsilon} (C_{H,T} + C^*_{H,T})$$
$$\mathrm{Q}_{t,T} = \beta^T \left(\tfrac{X_T}{X_t} \right)^{-\sigma} \tfrac{P_t}{P_T},$$

where MC_T is the real marginal cost, as previously specified, and X_T represents the marginal utility at time T. The factor $(\theta_H)^{T-t}$ accounts for a firm's possibility of not being able to reset its price in the next $(T-t)$ periods. The first-order condition of this problem is:

$$\frac{\partial V_t^H(i)}{\partial P'_{H,t}} : \mathbb{E}_t \sum_{T=t}^{\infty} (\theta_H)^{T-t} \left[\mathcal{Q}_{t,T} Y_{H,T}(i) \left(P'_{H,t} - \frac{\epsilon}{\epsilon - 1} P_{H,T} MC_T \right) \right] = 0, \tag{1.18}$$

replacing $\mathcal{Q}_{t,T}$ and rearranging:

$$\frac{\partial V_t^H(i)}{\partial P'_{H,t}} : \mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_H)^{T-t} \left[X_T^{-\sigma} Y_{H,T}(i) \left(P'_{H,t} - \frac{\epsilon}{\epsilon - 1} P_{H,T} MC_T \right) \right] = 0, \tag{1.19}$$

solving the previous expression for $P'_{H,t}$ gives the following condition:

$$P'_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_H)^{T-t} X_T^{-\sigma} Y_{H,T}(i) P_{H,T} MC_T}{\mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_H)^{T-t} X_T^{-\sigma} Y_{H,T}(i)}, \tag{1.20}$$

which I express in terms of two auxiliary variables $K_{1,t}$ and $K_{2,t}$:

$$P'_{H,t} = \frac{\epsilon}{\epsilon - 1} \frac{K_{1,t}}{K_{2,t}}. \tag{1.21}$$

3.2.3 Price setting of Retail firms

In addition to domestic firms, there is a continuum of retail firms importing differentiated goods, the prices of which are assumed to satisfy the law of one price at the docks. These firms have a certain degree of power when setting their prices in the domestic market, given that they are assumed to be monopolistically competitive. This leads to violations of the law of one price in the form of short-run deviations. Moreover, similar to domestic firms, retailers are assumed to set prices *à la* Calvo, i.e. with a probability $(1-\theta_F)$ a firm will optimally set its price, while with a probability θ_F it will not. Analogously to the domestic case, the Dixit-Stiglitz aggregate price index for a retailer's price evolves as follows:

$$P_{F,t} = \left[(1-\theta_F){P'}_{F,t}^{1-\epsilon} + \theta_F P_{F,t-1}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}. \tag{1.22}$$

The optimisation problem of a retailer firm is alike the domestic one. Here, a firm will choose $P'_{F,t}$ to maximise the expected present discounted value of profits, subject to the domestic demand curve of foreign goods:

$$\max_{P'_{F,t}} V_t^F = \mathbb{E}_t \sum_{T=t}^{\infty} (\theta_F)^{T-t} \Big[\mathcal{Q}_{t,T} C_{F,T}(i) \Big(P'_{F,t} - N_T s_T P^*_{F,T}(i)\Big)\Big]$$
$$\text{subject to: } \mathrm{C}_{F,T}(i) \leq \left(\frac{P'_{F,t}}{P_{F,T}}\right)^{-\epsilon} C_{F,T}$$
$$\mathrm{Q}_{t,T} = \beta^T \left(\tfrac{X_T}{X_t}\right)^{-\sigma} \tfrac{P_t}{P_T},$$

where N_t is a shock to the markup of import prices over marginal costs, which follows $N_{t+1} = (N_t)^{\rho_\nu} e^{(\epsilon^\nu_{t+1})}$ and $\epsilon^\nu_{t+1} \sim i.i.d.[0, \sigma^2_{\epsilon_\nu}]$.

The first-order condition of a retailer's problem is:

$$\frac{\partial V_t^F(i)}{\partial P'_{F,t}} : \mathbb{E}_t \sum_{T=t}^{\infty} (\theta_F)^{T-t} \left[\mathcal{Q}_{t,T} C_{F,T} \left(P'_{F,t} - \frac{\epsilon}{\epsilon-1} N_T s_T P^*_{F,T}(i)\right)\right] = 0, \tag{1.23}$$

replacing $\mathcal{Q}_{t,T}$ from the Euler equation and rearranging:

$$\frac{\partial V_t^F}{\partial P'_{F,t}} : \mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_H)^{T-t} \left[X_T^{-\sigma} C_{F,T}(i) \left(P'_{F,t} - \frac{\epsilon}{\epsilon-1} N_T s_T P^*_{F,T}(i)\right)\right] = 0. \tag{1.24}$$

Solving the previous expression for $P'_{F,t}$ gives the following condition:

$$P'_{F,t} = \frac{\epsilon}{\epsilon-1} \frac{\mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_F)^{T-t} X_T^{-\sigma} C_{F,T}(i) N_T s_T P^*_{F,T}}{\mathbb{E}_t \sum_{T=t}^{\infty} (\beta\theta_F)^{T-t} X_T^{-\sigma} C_{F,T}(i)}, \tag{1.25}$$

which I express in terms of two auxiliary variables $J_{1,t}$ and $J_{2,t}$:

$$P'_{F,t} = \frac{\epsilon}{\epsilon-1} \frac{J_{1,t}}{J_{2,t}}. \tag{1.26}$$

3.3 Consumer Price Index, Real Exchange Rate, Terms of Trade, Law of One Price gap

In this section, I introduce key definitions of variables common in open economies, such as Terms of Trade (TOT), CPI, Real Exchange Rate (RER) and Law of One Price gap (LOP).

First, TOT are defined as the ratio of foreign to domestic prices, i.e., how much of foreign goods can be bought with one unit of home good. A greater value for TOT is equivalent to an improvement of the domestic economy's competitiveness, which could be due to an increase in foreign prices and/or to a fall in domestic prices.

$$tot_t = \frac{P_{F,t}}{P_{H,t}}. \tag{1.27}$$

The CPI was introduced previously as part of the household's budget constraint, and it is repeated here:

$$P_t \equiv [(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}}. \tag{1.28}$$

In order to define the RER, I first need to introduce the nominal exchange rate s_t, which expresses the price of a foreign currency in terms of a domestic currency. A depreciation of the domestic currency occurs when s_t increases, which affects the RER in the same direction:

$$Q_t = \frac{s_t P_t^*}{P_t}. \tag{1.29}$$

Next, the LOP gap is defined as a wedge between the world price of foreign goods (numerator) in domestic currency, with respect to the domestic price of these goods (denominator):

$$\Psi_t = \frac{s_t P_t^*}{P_{F,t}}. \tag{1.30}$$

3.4 Uncovered Interest Rate Parity

Under the assumption of incomplete international financial markets, the equilibrium condition among prices in the domestic currency of foreign and domestic bonds yields the Uncovered Interest rate Parity (UIP) condition. To obtain such a condition, divide Equation (1.12) by Equation (1.13):

$$\frac{\lambda_t / R_t}{\lambda_t s_t / R_t^*} = \mathbb{E}_t \left[\frac{\beta \lambda_{t+1}}{\beta \lambda_{t+1} s_{t+1} \Phi_{t+1}(A_t, \tilde{\Phi}_{t+1})} \right], \tag{1.31}$$

which after cancelling and rearranging yields the UIP condition:

$$\mathbb{E}_t\Big[\frac{s_{t+1}}{s_t}R_t^*\Phi_{t+1}(A_t,\tilde{\Phi}_{t+1})\Big] = R_t. \tag{1.32}$$

3.5 Flow Budget Constraint, Trade Balance and Net Capital Account

To obtain the flow budget constraint, first I divide the household's budget constraint (1.8) by P_t and multiply and divide its last right-hand side term by s_{t-1} and P_{t-1}.

$$C_t+\frac{B_t}{P_t}+\frac{B_t^*s_t}{P_t} = \frac{W_tL_t+D_{H,t}+D_{F,t}+T_t}{P_t}+\frac{B_{t-1}R_{t-1}}{P_t}+\frac{B_{t-1}^*s_tR_{t-1}^*\Phi_t(A_{t-1},\tilde{\Phi}_t)}{P_t}\frac{s_{t-1}}{s_{t-1}}\frac{P_{t-1}}{P_{t-1}}. \tag{1.33}$$

Next, I assume that domestic bonds, B_t, are in net zero supply and that lump sum taxes are zero. In addition, using the expressions for profits of the domestic and retailer firms, $\Pi_{H,t} = P_{H,t}(C_{H,t}+C_{H,t}^*) - W_tL_t$ and $\Pi_{F,t} = P_{F,t}C_{F,t} - s_tP_t^*C_{F,t}$, the definition of total domestic output, $Y_t = C_{H,t}+C_{H,t}^*$ and the definition of the real quantity of outstanding foreign debt A_t, I obtain the following:

$$C_t + YA_t = \tilde{P}_{H,t}Y_t + (\tilde{P}_{F,t} - \frac{s_tP_t^*}{P_t})C_{F,t} + \frac{s_t}{s_{t-1}}\frac{R_{t-1}^*}{\pi_t}A_{t-1}\Phi_t(A_{t-1},\tilde{\Phi}_t), \tag{1.34}$$

where I use the following definitions $\tilde{P}_{H,t} \equiv P_{H,t}/P_t$, $\tilde{P}_{F,t} \equiv P_{F,t}/P_t$, as well as the expressions for the RER (1.29) and the demand for imported goods (1.7), which yield:

$$YA_t - \frac{s_t}{s_{t-1}}\frac{R_{t-1}^*}{\pi_t}A_{t-1}\Phi_t(A_{t-1},\tilde{\Phi}_t) = \tilde{P}_{H,t}Y_t - C_t + \alpha(\tilde{P}_{F,t}-Q_t)\tilde{P}_{F,t}^{-\eta}C_t. \tag{1.35}$$

Notice that expression (1.35) is the international balance of payments of the SOE, where the left-hand side represents the net capital account, and the right-hand side the net trade balance.

3.6 Equilibrium-Aggregate demand and output

The equilibrium in the SOE's goods market requires that total domestic production equals the sum of domestic and foreign consumption of home-produced goods:

$$Y_t = C_{H,t} + C_{H,t}^*, \tag{1.36}$$

which after replacing for $C_{H,t}$ and $C_{H,t}^*$, where the latter, the foreign demand for the good produced domestically, is assumed to be $C^*_{H,t} = \left(\frac{s_t P^*_{H,t}}{P^*_t}\right)^{-\eta^*} Y^*_t$[12]:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \left(\frac{s_t P^*_{H,t}}{P^*_t}\right)^{-\eta} Y^*_t. \tag{1.37}$$

3.7 Central Bank interest rate setting

To close this SOE, I specify the behaviour of a domestic Central Bank which follows a Taylor-type rule. This Central Bank aims to stabilise inflation and output growth by reacting to movements in CPI and output growth. Moreover, the interest rate in the current period is also driven by the interest rate in the previous period.

$$R_t = R_{t-1}^{\omega_R}\left(\frac{P_t}{P_{t-1}}\right)^{\omega_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\omega_{\Delta y}} M_t, \tag{1.38}$$

ω_R is the degree of interest rate smoothing, ω_π and ω_{Δ_y} are the feedback coefficients on CPI inflation and real output growth, respectively. Finally, M_t is a monetary policy shock, which follows $M_{t+1} = (M_t)^{\rho_\mu} e^{(\epsilon^\mu_{t+1})}$, and $\epsilon^\mu_{t+1} \sim i.i.d.[0, \sigma^2_{\epsilon_\mu}]$.

3.8 The Foreign Economy

In setting the foreign economy, I follow Gali and Monacelli (2005) assuming that it is exogenous to the SOE. Furthermore, it is characterised by Π^*_t and R^*_t being constant and by world output evolving as:

$$Y^*_{t+1} = (Y^*_t)^{\rho_{y^*}} e^{(\epsilon^{y^*}_{t+1})}, \tag{1.39}$$

where $\epsilon^{y^*}_{t+1} \sim i.i.d.[0, \sigma^2_{\epsilon_{y^*}}]$.

3.9 Log-linear Approximation of the Model

This subsection exhibits the log-linear approximation around a non-stochastic zero-inflation steady state of the optimality conditions previously presented.[13] The variables in this section are defined as log deviations from their respective steady-state values. First, I consider the domestic economy's conditions and then those pertaining to the foreign economy. The result of log-linearly approximating the household's Euler equation (1.15)

[12] For simplicity, I assume that the elasticity of substitution between domestic and foreign produced goods is the same in the SOE and the rest of the world, $\eta = \eta^*$.

[13] Appendix 1.A shows the full derivation of these and complementary conditions.

is:

$$\hat{c}_t - h\hat{c}_{t-1} = \mathbb{E}_t[\hat{c}_{t+1}] - h\hat{c}_t - \frac{1-h}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) + \frac{1-h}{\sigma}(\gamma_t - \mathbb{E}_t[\gamma_{t+1}]), \tag{1.40}$$

where $\rho = \beta^{-1} - 1$ and denotes the rate of time preference.

Next, to obtain an expression of domestic output, I log-linearise the goods market clearing condition equation (1.37) yielding:

$$\hat{y}_t = (1-\alpha)\hat{c}_t + \alpha\eta(2-\alpha)\hat{tot}_t + \alpha\eta\hat{\psi}_t + \alpha\hat{y}_t^*, \tag{1.41}$$

where ψ_t is obtained after log-linearising the LOP gap equation (1.30):

$$\hat{\psi}_t = \hat{q}_t - (1-\alpha)\hat{tot}_t. \tag{1.42}$$

I derive the following expression for TOT after log-linearising Equation (1.27) and taking first differences. This facilitates the interpretation of changes in the TOT as the difference between foreign-goods and domestic-goods inflation.

$$\Delta\hat{tot}_t = \pi_{F,t} - \pi_{H,t}. \tag{1.43}$$

A similar process of log-linearising and taking first differences is applied to the CPI definition in Equation (1.28):

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}, \tag{1.44}$$

which after using the previous expression can be expressed as:

$$\pi_t = \pi_{H,t} + \alpha\Delta\hat{tot}_t. \tag{1.45}$$

Turning to the domestic firms' price-setting optimality condition, after log-linearising Equation (1.21) and some rearrangement, I obtain the following domestic Phillips curve:

$$\pi_{H,t} = \lambda_H\hat{mc}_t + \beta\mathbb{E}_t[\pi_{H,t+1}], \tag{1.46}$$

where $\lambda_H = \frac{(1-\theta)(1-\theta\beta)}{\theta}$ and the log-linear real marginal cost is, after some replacements:

$$\hat{mc}_t = \varphi\hat{y}_t - (1+\varphi)\zeta_t + \alpha\hat{tot}_t + \frac{\sigma}{1-h}(\hat{c}_t - h\hat{c}_{t-1}). \tag{1.47}$$

Retailers' Phillips curve is obtained in a similar way, after log-linearising and rearranging Equation (1.26):

$$\pi_{F,t} = \lambda_F(\hat{\psi}_t + \nu_t) + \beta \mathbb{E}_t[\pi_{F,t+1}], \tag{1.48}$$

where $\lambda_F = \frac{(1-\theta_F)(1-\theta_F \beta)}{\theta_F}$.

Log-linearising the uncovered interest rate parity condition in Equation (1.32) and using the definition of real exchange rate, I obtain:

$$(i_t - \mathbb{E}_t[\pi_{t+1}]) - (i_t^* - \mathbb{E}_t[\pi_{t+1}^*]) = \mathbb{E}_t[\Delta q_{t+1}] - \chi a_t - \mathbb{E}_t[\tilde{\phi}_{t+1}], \tag{1.49}$$

while by log-linearising the expression for the flow budget constraint in Equation (1.35) yields:

$$\hat{a}_t - \beta^{-1}\hat{a}_{t-1} = y_t - \hat{c}_t - \alpha(\hat{tot}_t + \hat{\psi}_t). \tag{1.50}$$

$a_t = log(e_t B_t^* / P_t \bar{Y})$ defines the log real net foreign asset position as a fraction of steady-state output and determines the evolution of debt in the model. The right-hand side is the trade balance:

$$\hat{nx}_t = \hat{y}_t - \hat{c}_t - \alpha(\hat{tot}_t + \hat{\psi}_t). \tag{1.51}$$

Next, I log-linearise the monetary policy rule in Equation (1.38):

$$i_t = \rho + \omega_R i_{t-1} + \omega_\pi \pi_t + \omega_{\Delta y} \Delta \hat{y}_t + \mu_t. \tag{1.52}$$

Finally, I present the log-linear representation of the evolution of foreign output and the shock processes.

The rest of the world's output evolves as:

$$\hat{y}_{t+1}^* = \rho_{y^*} \hat{y}_t^* + \epsilon_{t+1}^{y^*}. \tag{1.53}$$

Productivity shock AR(1) process:

$$\zeta_{t+1} = \rho_\zeta \zeta_t + \epsilon_{t+1}^{\zeta}. \tag{1.54}$$

Preference shock AR(1) process:

$$\gamma_{t+1} = \rho_\gamma \gamma_t + \epsilon_{t+1}^{\gamma}. \tag{1.55}$$

Monetary shock AR(1) process:

$$\mu_{t+1} = \rho_\mu \mu_t + \epsilon^{\mu}_{t+1}. \tag{1.56}$$

Mark-up of import prices over marginal cost shock AR(1) process:

$$\nu_{t+1} = \rho_\nu \nu_t + \epsilon^{\nu}_{t+1}. \tag{1.57}$$

Finally, risk premium shock AR(1) process:

$$\tilde{\phi}_{t+1} = \rho_{\tilde{\phi}} \tilde{\phi}_t + \epsilon^{\tilde{\phi}}_{t+1}. \tag{1.58}$$

4 Diagnostic Small Open Economy Model and Solution Methods

The Diagnostic Small Open Economy (DSOE) is formed by a similar set of agents, households and firms, as well as a Central Bank. Unlike rational agents, these households and firms are assumed to form expectations about future variables diagnostically. In a log-linear context as the one specified in the previous section, in which the true data generating process of the vector of the exogenous state variable follows an AR (1) process such as $X_{t+1} = \rho X_t + \epsilon_{t+1}$, diagnosticity implies that agents perceive this process as an ARMA(1,1), $X_{t+1} = \rho X_t + \rho\phi\epsilon_t + \epsilon_{t+1}$, instead. This result follows the Bordalo, Gennaioli, Shleifer, and Terry (2021) approach when calculating the diagnostic distribution of each state variable applying Equation (1.2), which finally delivers the diagnostic distribution[14]:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(- \frac{1}{2\sigma^2}\Big\{ x^2_{t+1} - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right] + (\rho\bar{x}_t)^2 \\ + \phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right]\Big\}\Bigg) Z, \end{aligned} \tag{1.59}$$

where $\bar{x}_{t-1}$ denotes the realisation of the random variable x in period $t-1$, while $\bar{x}_t$ is the realisation in time t and Z is a normalising constant.

The strategy I follow to solve the log-linear model adopts Bordalo, Gennaioli, Shleifer, and Terry (2021) idea of a diagnostic agent forecasting future state variables "as if" they follow an ARMA(1,1) process, and also the idea of these agents continuing to believe the same in the future. That is, I solve the model as if the true process of the exogenous

[14] The derivation of the expression is presented in Appendix 1.B.

states is an ARMA (1,1) and then eliminate the MA terms. By doing so, the linear solution of such model will not just link the decision variables to the exogenous states and the predetermined variables, it will also link them to the realisations of the shocks. I represent the model by closely following Dennis (2020) as:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} s_{t+1} \\ E_t d_{t+1} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} s_t \\ d_t \end{bmatrix} + \begin{bmatrix} C_1 \\ 0 \end{bmatrix} \begin{bmatrix} \epsilon_{t+1} \end{bmatrix}. \tag{1.60}$$

Here, s_t denotes the vector of state and predetermined variables, while d_t is the vector of decision variables. In this model, the vector s_t will then include exogenous state variables, predetermined variable, as well as realisations of the shocks:

$$\begin{aligned} s_t^{'} = [&\epsilon_{t,\gamma}\ \epsilon_{t,\zeta}\ \epsilon_{t,\mu}\ \epsilon_{t,\nu}\ \epsilon_{t,\phi}\ \epsilon_{t,y^*}\ \gamma_t\ \zeta_t\ \mu_t\ \nu_t\ \phi_t\ y_t^* \\ & y_{t-1}\ c_{t-1}\ \pi_{H,t-1}\ i_{t-1}\ e_{t-1}\ s_{t-1}\ \pi_{F,t-1}\ q_{t-1}\ a_{t-1}], \end{aligned}$$

which can be re-expressed using sub-vectors s_t^{ϵ} for the lagged innovations to the shocks, s_t^{exo} for the exogenous state variables, and s_t^{endo} for the endogenous state variables as:

$$s_t^{'} = [s_t^{\epsilon}\ s_t^{exo}\ s_t^{endo}].$$

The vector of the decision variables, d_t, will include:

$$d_t^{'} = [y_t\ c_t\ \pi_{H,t}\ \pi_t\ i_t\ e_t\ s_t\ q_t\ mc_t\ \psi_t\ \pi_{F,t}\ a_t\ l_t\ w_t\ \Delta e_t].$$

In order to solve the model, first a linear solution of the form $d_t = Fs_t$ is conjectured linking the decision variables to the shocks' realisations, exogenous states, and predetermined variables. Second, after some algebra, an expression for matrix F is obtained and then computed using direct iteration. Third, with this result for matrix F, a solution for matrix M can also be obtained, and thus the law of motion for the vector s_t. At this stage, both the RE and the DE models have been solved. For exposition purposes, I divide matrix M and F into three submatrices, depending on which sub-vector of $s_t{}'$ they multiply. I re-rewrite the solution as:

$$\begin{aligned} s_{t+1}^{'} &= M_{\epsilon} s_t^{\epsilon'} + M_{exo} s_t^{exo'} + M_{endo} s_t^{endo'} + C\epsilon_{t+1}. \\ d_t^{'} &= F_{\epsilon} s_t^{\epsilon'} + F_{exo} s_t^{exo'} + F_{endo} s_t^{endo'}. \end{aligned}$$

The submatrices F_{exo}, F_{endo}, which link the exogenous and the predetermined variables to the decision variables, as well as M_{exo} and M_{endo}, which govern the evolution of exogenous and predetermined variables, remain identical under both RE and DE. The core distinction arises in how agents perceive the shocks. Rational agents correctly interpret the shocks as AR(1), resulting in zero entries in the submatrices F_ϵ and M_ϵ. In contrast, diagnostic agents misperceive the shocks as ARMA(1,1), resulting in non-zero values in M_ϵ that capture the moving average components, and in F_ϵ to reflect the extrapolation of the lagged innovations to the shocks throughout the economy. In the last step, I set the coefficients on the MA terms in M_ϵ to zero, ensuring that any further analysis is performed under the true data-generating process while still assuming that the agents have DE. This dependency on the realisation of the shocks is also obtained by L'Huillier et al. (2021).

5 Results and Numerical Analysis

5.1 Parameterisation

Table 1.1 shows the parameterisation of the model. The discount factor, β, is set to 0.99, a value widely used for quarterly models. With the model then parameterised for a quarterly frequency, this results in an annualised real interest rate of 4%. The value of the inverse of the intertemporal elasticity of consumption, σ, is taken from Hoffmann, Søndergaard, and Westelius (2011) and is equal to 3.5. This value is within the range frequently used in the open economy literature.[15] The inverse elasticity of labour supply, φ, is set to 1/3. The degree of openness, or the share of foreign goods in consumption, α, is set to 0.2, while the elasticity of substitution between domestic and imported goods, η, is equal to 0.75. These values fall within the range of those found using data from small open economies and are further corroborated by the meta-analysis study conducted by Bajzik, Havranek, Irsova, and Schwarz (2019). The elasticity of the risk premium to net foreign assets, χ, is equal to 0.01, as normally found in this literature. The habit parameter is 0.6 following the work of Leith, Moldovan, and Rossi (2012); this value is supported by the meta-analysis of Havranek, Rusnak, and Sokolova (2017).

The probabilities for domestic and retail firms to reset their prices, θ_H and θ_F, respectively, are set in line with the results of Mihailov, Rumler, and Scharler (2011) for European countries and with the values used by Justiniano and Preston (2010b). An unsuccessful re-optimisation probability of 0.75 implies an average duration between price changes of 4 quarters. Turning to the Taylor rule coefficients, ω_i, ω_π and $\omega_{\Delta y}$, their values are chosen to be 0.85, 2.5 and 0.5, as in Hoffmann et al. (2011).

[15] Gali and Monacelli (2005) assign a value of one to this parameter, meaning consumption reacts directly to changes in the real interest rate, while Steinsson (2008) chooses a value of 5, which implies a lower intertemporal elasticity.

Table 1.1: Benchmark calibration

Description	Parameter	Value
Discount factor	β	0.99
Inverse elasticity of inter-temporal substitution	σ	3.5
Inverse elasticity of labour supply	φ	1/3
Elasticity of risk premium to net foreign assets	χ	0.01
Habit parameter	h	0.6
Share of foreign goods in consumption	α	0.2
Elasticity of substitution between domestic and imported goods	η	0.75
Domestic price Calvo parameter	θ_H	0.75
Import price Calvo parameter	θ_F	0.75
Taylor rule smoothing parameter	ω_R	0.85
Taylor rule inflation feedback parameter	ω_π	2.5
Taylor rule output growth feedback parameter	$\omega_{\Delta y}$	0.5
Productivity shock AR(1) parameter	ρ_ζ	0.9
Preference shock AR(1) parameter	ρ_γ	0.9257
Monetary shock AR(1) parameter	ρ_μ	0.5
Cost-push shock AR(1) parameter	ρ_ν	0.9352
Risk premium shock AR(1) parameter	ρ_ϕ	0.94
Foreign output AR(1) parameter	ρ_{y^*}	0.86
Standard deviation of productivity innovation	$100^*\sigma_{\epsilon_\zeta}$	0.37
Standard deviation of preference innovation	$100^*\sigma_{\epsilon_\gamma}$	0.1610
Standard deviation of monetary innovation	$100^*\sigma_{\epsilon_\mu}$	0.25
Standard deviation of cost-push innovation	$100^*\sigma_{\epsilon_\nu}$	1.57
Standard deviation of risk premium shock	$100^*\sigma_{\epsilon_\phi}$	0.25
Standard deviation of foreign output innovation	$100^*\sigma_{\epsilon_{y^*}}$	0.78
Diagnostic parameter	ϕ	0.8

The autoregressive coefficient for the technology shock, ρ_a, is equal to 0.90 and is based on Monacelli (2005). The persistence of the preference and the cost push shock, 0.92 and 0.93 respectively, are taken from Dennis et al. (2006). In addition, the value 0.86 for ρ_y^* is the same as in Gali and Monacelli (2005). The value of the persistence of the risk premium shock is set as in Justiniano and Preston (2010b). Finally, the diagnostic parameter, ϕ, is equal to 0.8, which is taken from Bordalo, Gennaioli, Kwon, and Shleifer (2021), who state that this value is in line with quarterly estimates from macro- and financial data surveys.

5.2 Results

In this section, I present the results from solving the rational and diagnostic models using the above parameterisation. Upon solving the model, I compute unconditional standard deviations. Additionally, I simulate impulse responses to analyse the reactions of variables when subjected to various shocks in the economy under each beliefs formation framework.

5.2.1 Unconditional volatility

Table 1.2 shows unconditional standard deviations for nominal exchange rate growth, annualised imported-goods inflation, annualised domestic-goods inflation, marginal costs, the annualised nominal interest rate, annualised consumer price index inflation, the real wage, the law of one price gap, the real exchange rate, output, consumption, labour, and the terms of trade under rationality and diagnosticity.

Table 1.2: Unconditional standard deviations

Variable	Rational Expectations	Diagnostic Expectations	Percentage increase
Nominal Exchange Rate growth	2.283	4.374	91.6%
Imported-goods Inflation	2.508	3.394	35.3%
Domestic-goods Inflation	0.825	1.099	33.2%
Marginal Cost	0.906	1.158	27.8%
Nominal interest rate	0.944	1.114	18.0%
CPI	0.402	0.465	15.6%
Real wage	1.775	2.026	14.1%
Law of one price	4.637	5.287	14.0%
Real Exchange Rate	7.126	7.838	9.9%
Output	1.422	1.525	7.2%
Consumption	0.572	0.606	6.0%
Labour	1.470	1.554	5.7%
Terms of trade	5.709	5.988	4.8%

By comparing the volatility measures of the SOE under RE and DE, I find that adding DE causes volatility to be generally higher throughout this economy. When ordering the variables in this table by the percentage increase of their unconditional standard deviation, it turns out that the variables most affected are prices, while the least affected are the real variables. Among the former, the nominal exchange rate (NER) growth and imported-good and domestic-good inflation exhibit the greatest effect, with changes of 91.6%, 35.3% and 33.2%, respectively. Among the latter, consumption, labour, and the terms of trade show the smallest impact with changes between 4% and 6%. These increases in volatility are a product of the extrapolation of errors in expectations that diagnosticity introduces in the model. Concretely, the presence of endogenous variables and persistence mechanisms helps to further propagate these errors throughout the economy, generating the final amplification effect, which happens in a heterogeneous manner. These results are qualitatively in line with L'Huillier et al. (2021), but in a different context, as they focus on closed economies, while I analyse an open economy model.

5.2.2 Impulse Response Functions

The SOE is affected, as in Justiniano and Preston (2010b), by six different shocks: (i) a productivity shock, (ii) a monetary policy shock, (iii) an aggregate preference shock,

(iv) a markup of imported prices shock, (v) a shock to foreign output, and (vi) a risk-premium shock. The plots of the impulse responses depict variables' responses in dashed black lines when agents form expectations rationally, while in red solid lines when agents form expectations diagnostically.

5.2.2.1 Technology shock: a positive technology innovation of one standard deviation rises labour productivity, generating an increase in the real wage (panel N), which causes domestic firms to demand less of this input (panel M), thus pushing their marginal costs down (panel O). This drop in marginal costs is reflected as a fall in domestic-goods inflation (panel C). This happens since the proportion of domestic firms able to re-optimise cut prices in response to the marginal costs drop, given their desire to maintain their markup. In consequence, the competitiveness of the SOE is positively affected, i.e. the terms of trade increases (panel I). However, the magnitude of this increase does not outweigh the decrease in domestic-goods inflation and therefore CPI inflation falls (panel E). The Central Bank, which follows a Taylor rule reacting to the last period nominal interest rate, CPI inflation, and output growth, responds by lowering the nominal interest rate (panel F). In response to the shock and the rise in consumption, output (panel A) positively deviates from its steady state. This increase in consumption (panel B) occurs modestly at the beginning given the early positive real interest rate (panel G). As soon as the real interest rate starts falling, households further increase their consumption. This leads, by the UIP condition, to an augmented expected nominal return from the domestic risk-free bonds with respect to its foreign counterpart, causing nominal and real exchange rates to initially depreciate (panel J and panel K). The consequence of this is a positive deviation from the law of one price gap (panel H) and, therefore, an increase in imported-goods inflation (panel D). This reaction contributes to the rise in the TOT and enhances domestic production (substitution effect). By requiring higher production, firms will demand more labour, moving the marginal costs, and the rest of the variables back towards the steady state.

When agents are diagnostic, the impulse responses are mainly characterised by initial overreactions. At the time the productivity shock hits the economy, agents react in the same direction as in the rational case, although sometimes less-so. This reaction is so because diagnostic agents believe that the law of motion of this shock follows an ARMA(1,1) process instead of the true AR(1). Therefore, they assign a higher probability to a scenario in which workers are more productive in the next period after the shock, which would imply a higher wage. Thus, firms will demand less labour tomorrow than today. In addition, as households unrealistically believe that they will be richer, they swap current leisure for future leisure. Altogether, this generates a higher positive response in

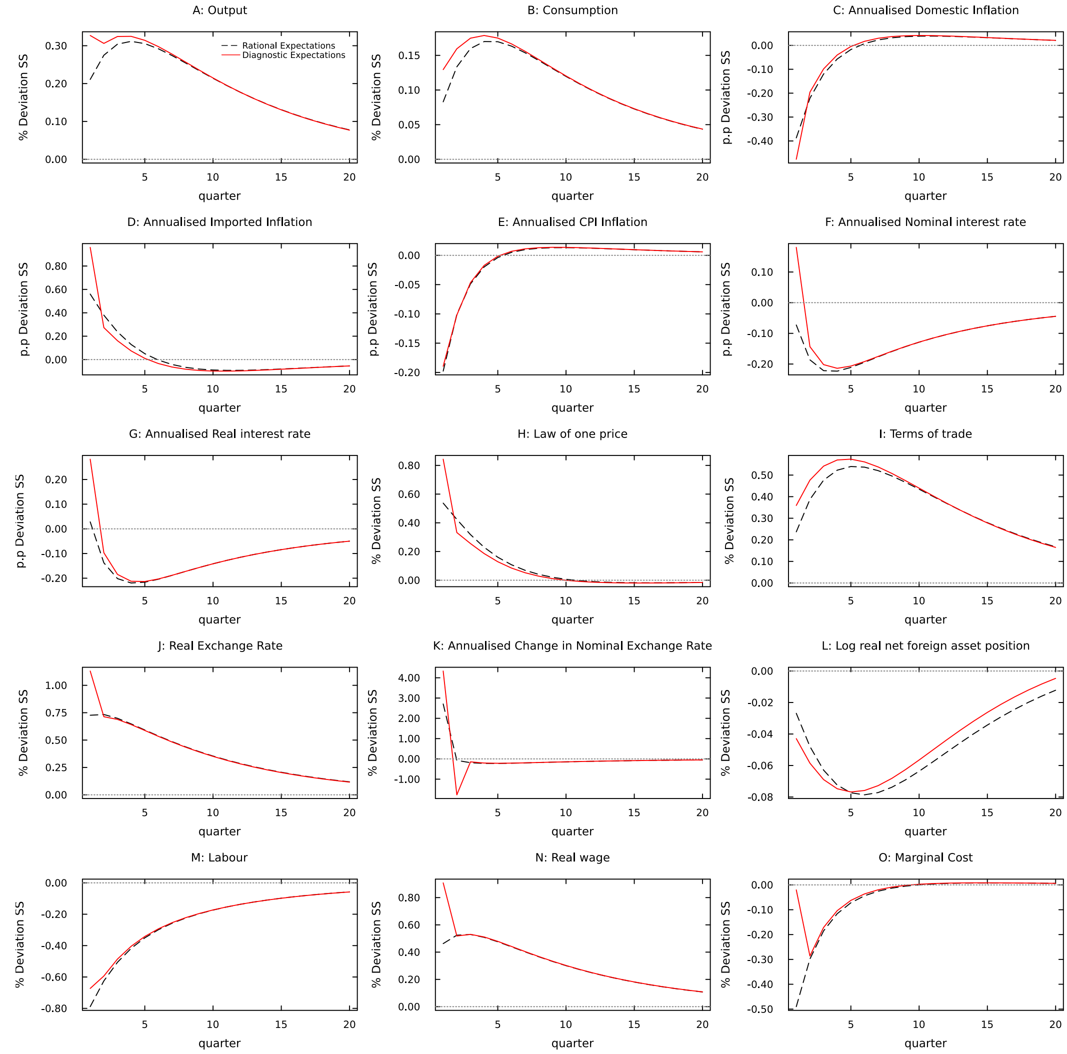

Figure 1.3: Impulse responses to a one standard deviation productivity shock. Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal cost. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

the real wage and a smaller drop in marginal costs, under DE than RE.[16] This is reflected in a higher initial output. Prices fall by more than under rationality, as firms believe that marginal costs will suffer a further decrease next period, and they will probably not be able to re-optimise. This effect further translates into the economy in a similar way to RE. However, as soon as agents realise that their beliefs about the process of the shock are incorrect, they abruptly reverse their behaviour towards rationality. Such a correction is influenced by the presence of more realistic rigidities, such as habits and interest rate smoothing, which propagates DE aspects in the model, mainly in the short term.

[16] L'Huillier et al. (2021) express that general equilibrium mutes some amplified reactions under diagnosticity, in this case the adjustment of wages affects labour and marginal costs reactions.

5.2.2.2 Monetary policy shock: Figure 1.4 exhibits responses to a monetary policy shock under RE and DE. In this case, I consider a shock that tightens monetary policy, i.e. an increase in the nominal interest rate. As a result of this tighter policy, the economy contracts. This follows from an increase in the real interest rate (panel F) as prices are sticky, generating a fall in domestic consumption. Due to habit persistence, the response of consumption behaves in a hump-shaped manner (panel B). Reduced demand for consumption goods generates a fall in output (panel A), which negatively impacts firms' labour demand (panel M) and the real wage (panel N) and thus their marginal costs (panel O). The set of firms able to re-optimise will react by cutting prices, generating a decline in domestic-goods inflation (panel C). The open economy channel works through the exchange rate, both nominal and real. Following the UIP condition, both appreciate in response to a higher real interest rate. The appreciation in the nominal exchange rate (panel K) generates a negative impact on the prices of imported goods (panel D), especially in the case of a high pass-through. The real exchange rate appreciation (panel J) causes a switch in domestic households' expenditure from domestic to foreign goods. The terms of trade decline; therefore, domestic goods become relatively more expensive than foreign goods (panel I), making them less attractive to the rest of the world. This explains the larger drop in output compared to consumption.

Under DE, agents will behave assigning a higher probability, in this case, to a scenario in which monetary policy is further tightened. Households react to this scenario by intertemporally shifting consumption in order to smooth it, as they expect a higher interest rate. The contraction in the economy is also initially stronger. This generates a stronger decrease in the marginal costs of domestic firms, which they transfer to domestic prices. Henceforth, the rest of the variables follow the course of reactions as in RE, but with the characterised overreaction under DE. As soon as agents realise that the scenario to which they have assigned a higher probability has not happened, they strongly adjust towards the rational case. More precisely, their reversal is sharp enough to surpass the rational scenario, thus showing some strong reversal. The full convergence seems to occur around the tenth quarter for most of the variables.

5.2.2.3 Aggregate preference shock: Impulse responses to an aggregate preference shock under a rational and diagnostic framework are exhibited in Figure 1.5. This shock positively affects the marginal utility of present consumption compared to future consumption, as well as the marginal utility of consumption relative to marginal (dis)utility of labour. This stimulates households to substitute consumption intertemporally, developing an increase in current consumption to the detriment of future consumption (panel B). On the other hand, the increase in current marginal utility of consumption generates

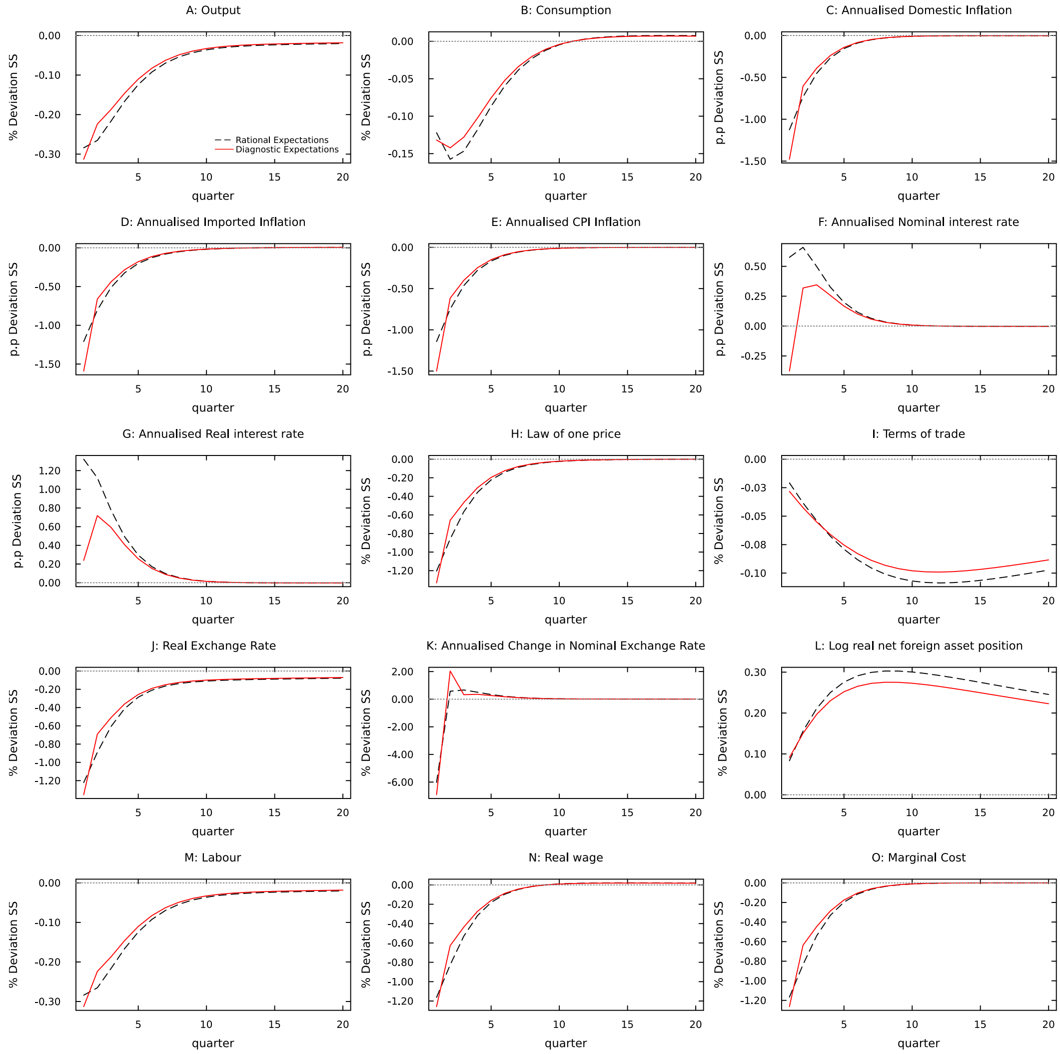

Figure 1.4: Impulse responses to a one standard deviation monetary policy shock. Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal costs. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

a relative decline in current utility of labour, which negatively affects labour and the real wage (panels M and N, respectively). Moreover, as domestic firms are characterised by a linear production function in labour, the fall in labour translates into a fall of the same size in output (panel A). In response to less demand pressures, marginal costs move in the opposite direction, driving up domestic prices (panel C). Together, a contraction in the total production of domestic goods and an increase in its price imply that the boost in total consumption of domestic households is driven by an expansion in their demand for foreign goods. This substitution effect, as well as the drop in foreign demand for domestic goods, is explained by the decline in the terms of trade (panel I). Accordingly, the trade balance deviates negatively from its steady state, leading to a negative response of the asset position (panel L). The Central Bank responds to the higher CPI (panel E) and higher output (panel A) by increasing the nominal interest rate. The real exchange rate, as well as the nominal exchange rate, appreciates in response to this policy tightening. This appreciation affects the domestic prices of foreign goods, generating a deviation in the law of one price gap (panel H). Given a positive level of pass-through, the lower real exchange rate is transmitted to retailers, who will initially decrease their prices, putting a pressure on imported-goods inflation to fall (panel D).

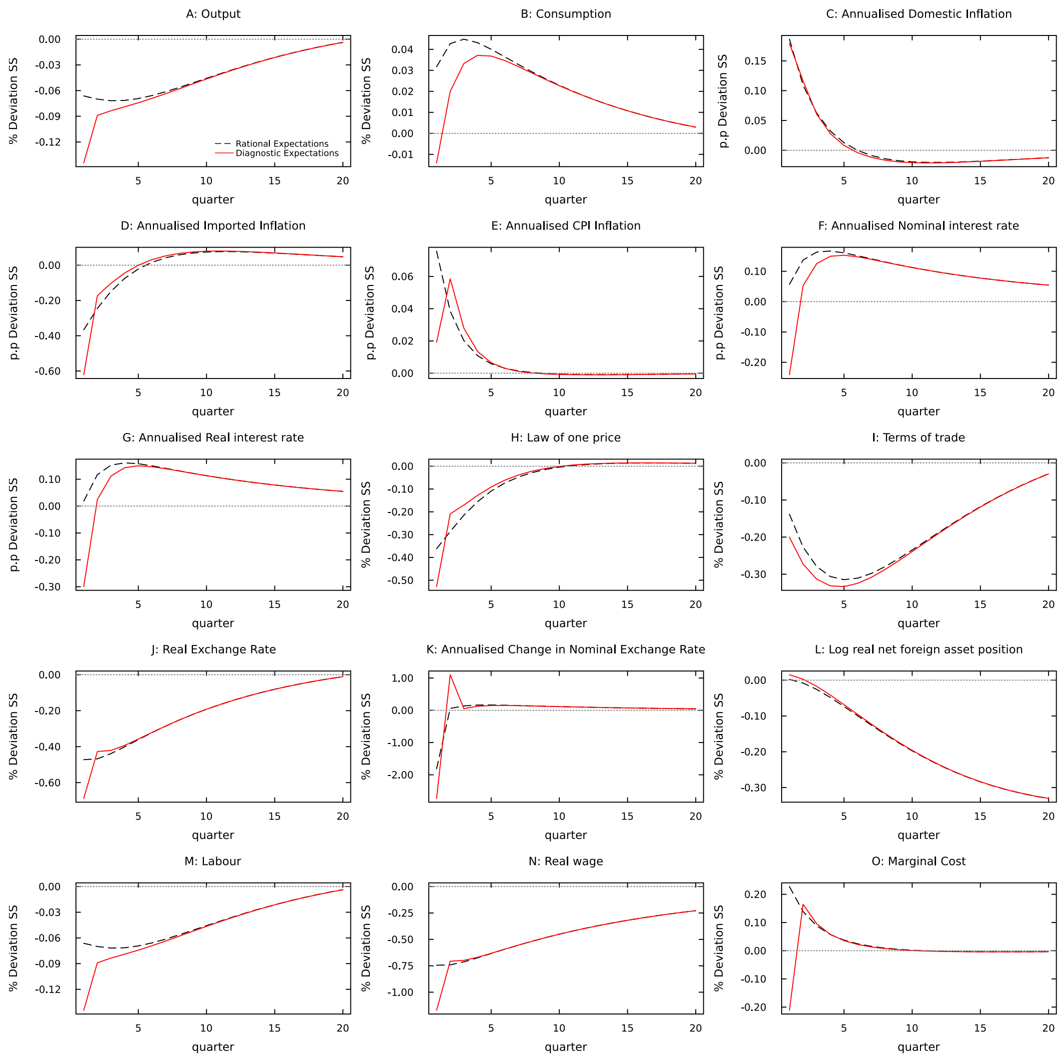

Figure 1.5: Impulse responses to a one standard deviation preference shock. Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal costs. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

In the presence of DE, the response to the aggregate preference shock differs significantly because agents perceive it as an ARMA(1,1) process, believing that it is more persistent than it actually is. This directly alters the dynamics of the Euler equation, as both the current and the expected future value of the aggregate preference shock influence it. Consequently, agents who form beliefs in a diagnostic way will assign a higher probability to a scenario in which the marginal utility of consumption is higher in the period after the shock hits the economy. Therefore, the intertemporal substitution of diagnostic agents is inverse in comparison to the rational case, instead of substituting future consumption for present consumption, households do the opposite. This generates initial responses characterised by its reaction in a direction contrary to the rational case (for example, panel B) or by stronger initial reactions and subsequent sharp convergences to rationality. Total consumption is an exception to the sharp reversal; its behaviour is smooth due to the presence of habits.

5.2.2.4 Cost-push shock or import price mark-up shock: Figure 1.6 shows the reactions of variables to a cost-push shock in the form of increased market power of retail firms. On impact, prices of imported goods increase as retailers have higher desired mark-ups over marginal costs. This generates higher imported-goods inflation (panel D), making domestic goods more competitive (panel I). Households in this economy respond to this increase in prices of foreign goods by importing less, generating a drop in domestic consumption (panel B). As domestic firms produce less output, they will demand less labour (panel M), and thus experience a decrease in their marginal costs (panel O). The reduction in the marginal costs will be translated by re-optimising firms to domestic prices (panel C). Nevertheless, this decline in domestic-goods inflation does not outweigh the effect of accelerated imported-goods inflation. Therefore, the CPI increases (panel E). However, the reaction of the Taylor rule followed by the Central Bank is dominated by the fall in output, causing a decrease in the nominal interest rate (panel F). The nominal exchange rate appreciates (panel K) in response to the increase in the real interest rate (panel G) following the UIP condition. In this economy, the increase in the terms of trade produces an expenditure switching towards domestic goods, which works to expand the home economy and bring it back to steady state.

When agents are diagnostic, more specifically retailers, after the cost-push shock they will believe that a representative state of the economy in the following period will be characterised by their market power expanding more. Consequently, their initial response to the shock is described by an overreactive behaviour. As retailers set prices in a staggered manner, there will be some of them optimising today that will not be able to re-optimise in the second period, thus they charge now higher prices than those under rational expectations. This stronger reaction is further extended to all economic variables in a manner

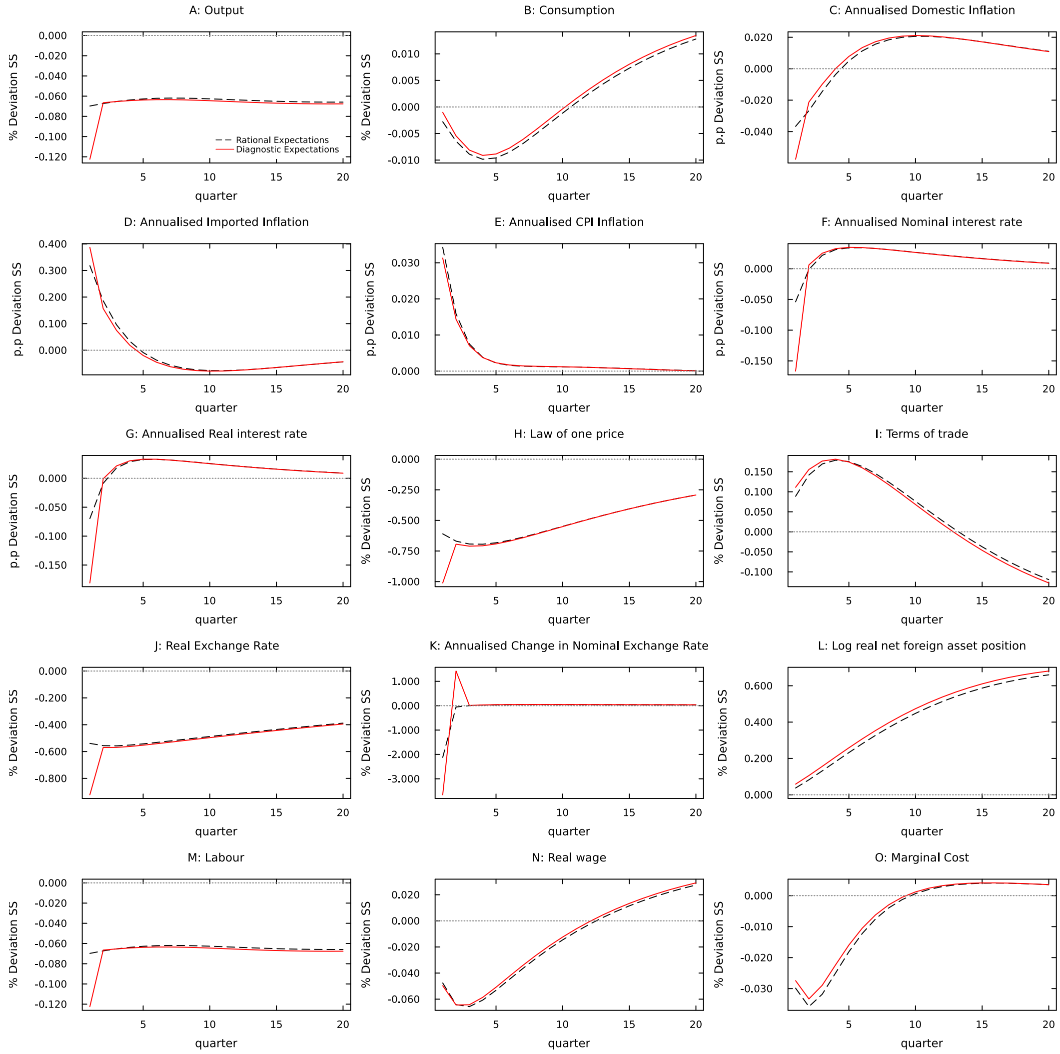

Figure 1.6: Impulse responses to a one standard deviation cost-push shock (markup of import prices).

Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal costs. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

similar to that in RE. Once agents recognise the true law of motion of the shock, they revert to rationality. In some cases, the reversion is sharp enough to surpass the rational response.

5.2.2.5 Foreign output shock: Figure 1.7 exhibits the responses to a foreign output shock. This shock generates a positive impact on both domestic total consumption (panel B), as well as in foreign consumption due to higher incomes. This results in an increase in domestic production (panel A). In order to produce this higher amount of domestic goods, firms will demand more labour (panel M), therefore increasing marginal costs (panel O), which firms translate to prices (panel C). However, the income effect produced by the higher foreign output causes the real and nominal exchange rates to appreciate (panel J and panel K), which creates a negative wedge for the law of one price (panel H), as foreign goods are more expensive in domestic currency than in the rest of the world, pushing import prices downwards (panel D). Such a reaction dominates the increase in domestic-goods inflation, weakening the domestic economy's terms of trade (panel I). All this decreases the CPI (panel E). However, the monetary authority reacts by increasing interest rates as the impact on output outweighs that on CPI inflation in the Taylor rule. Moreover, even though the terms of trade have declined, the stronger increase in output relative to consumption results in a positive deviation from steady state in the trade balance and a larger debt position (panel L). This could reflect the wealthier conditions in the rest of the world in response to the foreign output shock, which positively impacts the demand for the domestic economy's exports.

Quite like previous shocks, the impulse responses of the variables when agents are diagnostic show an initial overreaction and posterior reversal towards rationality. In this particular situation, domestic agents operate diagnostically, leading them to anticipate that international agents will accumulate more wealth. This expectation arises because they attribute a higher likelihood to that outcome. Therefore, domestic producers initially increase their production, requiring more labour, pushing their marginal costs up. This results in a boost in domestic prices. In addition, representativeness exacerbates the income effect of higher foreign output, causing a stronger appreciation of the real and nominal exchange rates. The subsequent reversion to rationality is strong enough for some of the variables to surpass the rational impulse response, further delaying convergence to steady state.

5.2.2.6 Risk Premium shock: Figure 1.8 shows the responses to a negative shock to the risk premium. Looking at the UIP condition, the RER and NER react negatively, i.e. they initially appreciate (panels J and K), which generates a huge drop in the LOP gap (panel H), as well as in the domestic price of foreign goods (panel D). Real debt becomes negative (panel L) as a response to a negative deviation of the trade balance from steady state, which results from a deterioration of the TOT (panel I). A worsening competitiveness generates a shift in domestic households' demand from domestic to foreign goods, seen as a rise in imports, which boosts total consumption (panel B). At the same time, foreign demand for domestic goods also drops, implying a decrease in exports. In addition, domestic firms face higher marginal costs (panel O) as a result of higher real wage (panel N), which is translated into prices (panel C). The rise in the real wage leads to a reduced demand for labour (panel M), subsequently causing a downturn in pro-

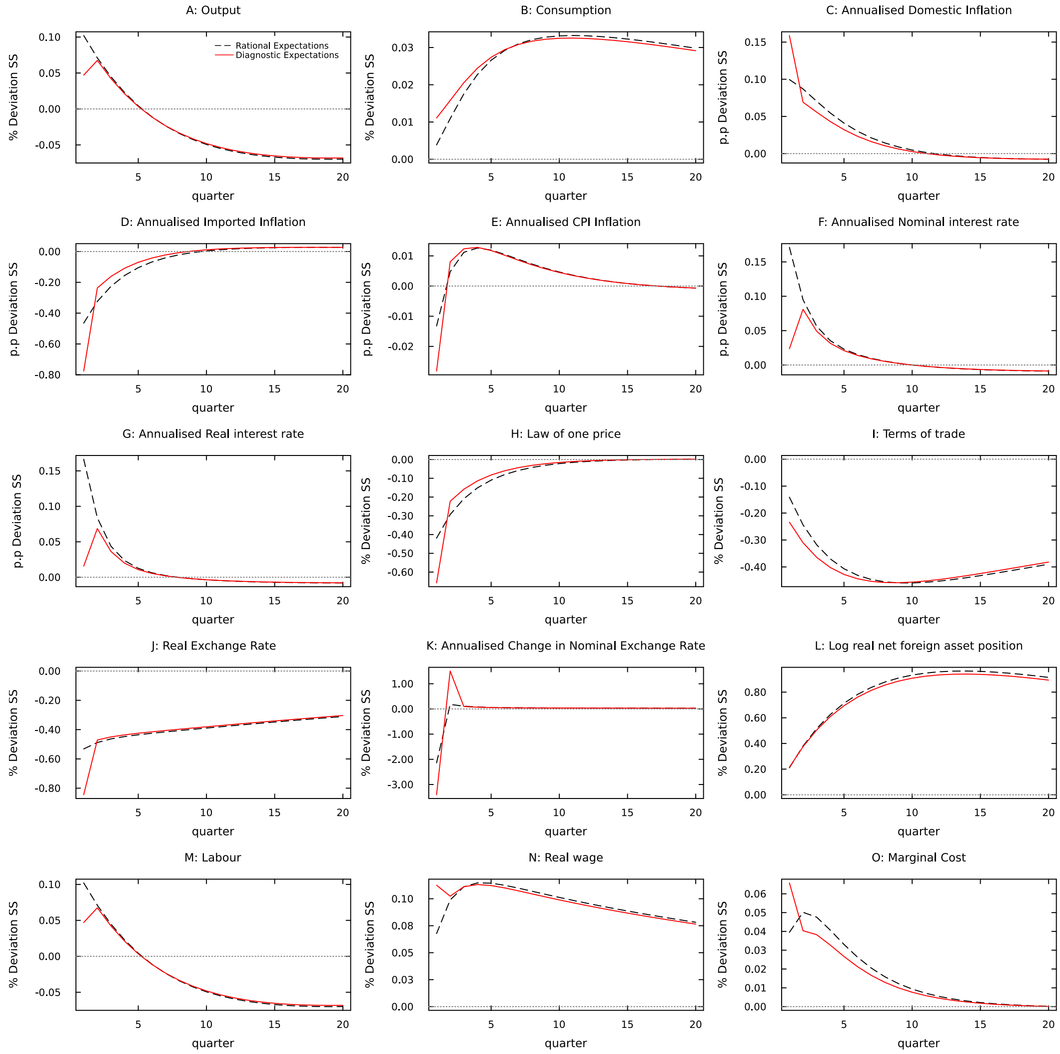

Figure 1.7: Impulse responses to a one standard deviation foreign output shock. Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal costs. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

duction levels (panel A). The sharp decline in output drives the Central Bank to lower the nominal interest rate (panel F), despite the increase in CPI (panel E). The economy adjusts back to steady state as the shock fades, the RER and NER start depreciating, and competitiveness is improved, which drives the recovery of output.

The variables of the DSOE also show exaggerated initial responses to a risk premium shock. In this case, agents believe that the decrease in the risk premium will intensify in the next period; therefore, the RER and NER appreciate more strongly. In response, the nominal interest rate plunges. The bigger drop in competitiveness generates a more pronounced shift in the consumption decisions of diagnostic agents, boosting total con-

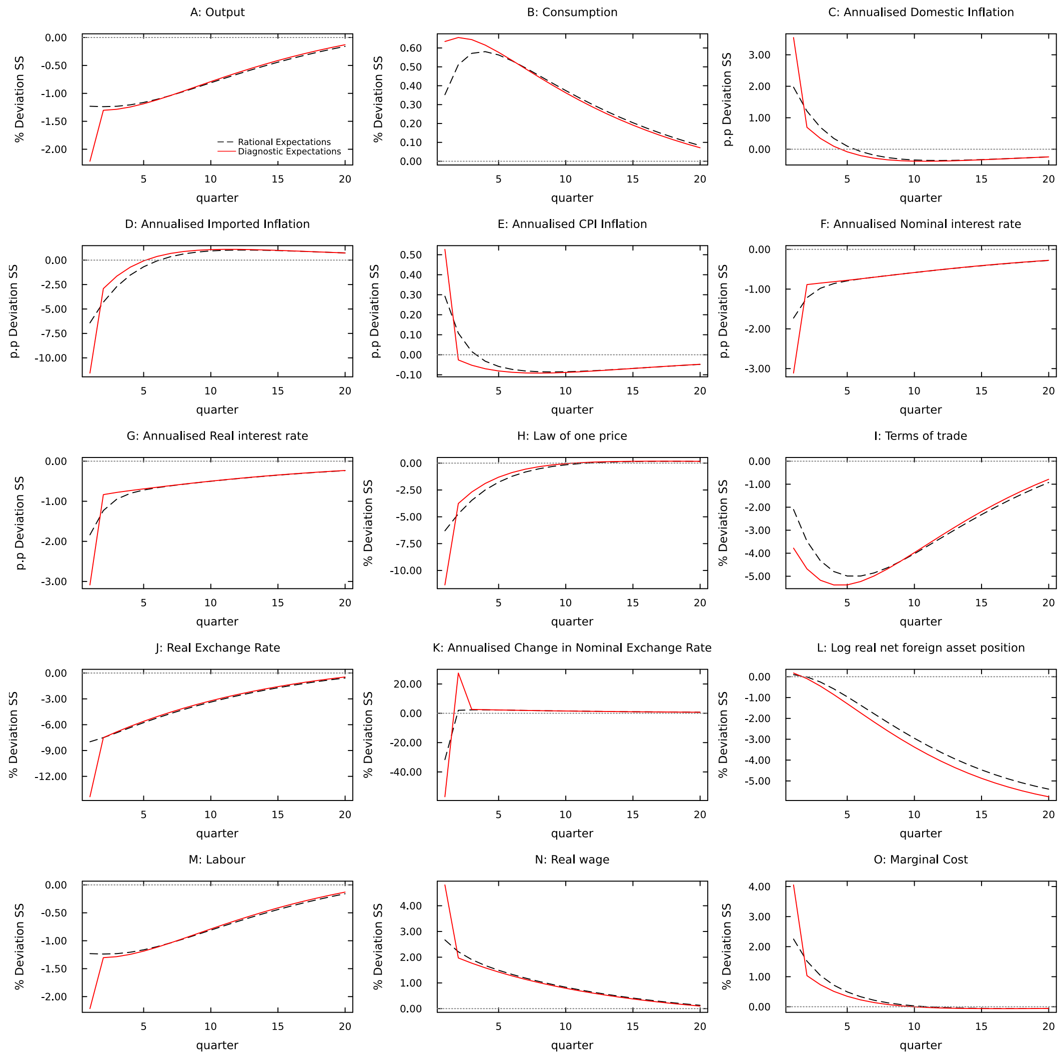

Figure 1.8: Impulse responses to a one standard deviation risk premium shock. Each panel depicts the response of output, consumption, annualised domestic-goods and imported-goods inflation, CPI, annualised nominal and real interest rate, LOP gap, TOT, RER, annualised change in the nominal exchange rate, log real net foreign asset position, labour, real wage and marginal costs. The black dotted lines represent the impulse responses when agents form expectations rationally, whilst the red solid lines exhibit the impulse responses when expectations are formed diagnostically.

sumption. This increase in consumption translates in a larger decline in real debt in response to a worsening trade balance with respect to the steady state. At the same time, rising real wages amplify the contraction in labour demand, generating an almost double fall in output compared to the rational case. However, as soon as the diagnostic agent realises the true process of the shock, these initial overreactions revert to the rational scenario. This occurs more rapidly in most of the variables, with the exception of consumption, due to the presence of habits.

6 Sensitivity analysis

In this section, I run a set of sensitivity analysis of the models' results. First, I focus on the impact of changing the diagnostic parameter ϕ. Second, I study how modifying other parameters affects the impact of DE in a SOE. Third, I analyse how each persistence mechanism added in the model behaves in the diagnostic environment. In order to do so, I isolate their effects by studying one mechanism at a time.

6.1 The role of the diagnostic parameter ϕ

The degree of diagnosticity is the key parameter in the model as it governs the extent to which agents' beliefs are distorted. In this subsection, I study how variation in ϕ affects the volatility of the SOE's variables. Table 1.3 shows the unconditional standard deviations of the main model's variables. The second column of this table exhibits results for the rational model, which can be thought of as a special case when $\phi = 0$. Likewise, columns (3), (4) and (5) show the results for the diagnostic model under different parameterisations for the diagnostic parameter: 0.5, 0.8 and 1.0, respectively.[17]

Table 1.3: Unconditional standard deviations when ϕ varies

Variable	ϕ=0.0	ϕ=0.5	ϕ=0.8	ϕ=1.0
Output	1.422	1.481	1.525	1.557
Consumption	0.572	0.592	0.606	0.617
Domestic-goods Inflation	0.825	0.981	1.099	1.184
CPI Inflation	0.402	0.438	0.465	0.484
Nominal interest rate	0.944	1.026	1.114	1.186
Terms of trade	5.709	5.877	5.988	6.066
Real Exchange Rate	7.126	7.532	7.838	8.065
Marginal Cost	0.906	1.041	1.158	1.247
Law of one price gap	4.637	5.002	5.287	5.500
Imported-goods Inflation	2.508	3.013	3.394	3.671
Labour	1.470	1.518	1.554	1.582
Real wage	1.775	1.916	2.026	2.109
Nominal Exchange Rate growth	2.283	3.539	4.374	4.946

Note: This table illustrates the unconditional standard deviations of the variables within the model as the level of diagnosticity changes.

The main result that emerges from this table is that any degree of diagnosticity produces a higher generalised volatility in the economy. Moreover, it seems that the amplification in the volatility of the variables is not linear with respect to the diagnostic parameter. After examining the change in standard deviations for equal-sized changes in ϕ, it is apparent that the difference is smaller (larger) when transitioning from $\phi = 0$

[17] The case of $\phi = 1.0$ could be considered particular, since such value generates agents' forecast error to be of equal size as the incoming shock.

to $\phi = 0.5$ compared to when ϕ shifts from 0.5 to 1.0. Thus, I can state that the more diagnostic agents become, the more volatile is the economy, ultimately leading to amplified reactions. In addition, another relevant outcome generated by the inclusion of DE in a SOE is the disconnection of the RER's volatility from the volatility of the rest of the variables. This is evident when comparing the outcomes in column (2) with those in column (5). The volatility of all variables increases as ϕ moves from 0 to 1, but the effect is strongest for the RER and the nominal exchange rate, while other variables exhibit a smaller increase. This highlights DE's potential as a behavioural explanation for specific international macroeconomic puzzles, a burgeoning area of research within the modern literature.

6.2 Parameters sensitivity

Table 1.4 presents unconditional standard deviations of the SOE's variables under different parameterisations. This subsection aims to analyse how sensitive the amplification effect of DE is to changes in specific parameters. For comparison purposes, the first line of Table 1.4 repeats the results obtained for the benchmark calibration presented in Table 1.2.

6.2.1 Interest rate smoothing parameter ρ_i

A change in this parameter affects the desire of the Central Bank to smooth the behaviour of its instrument. The most notable result is that the greater the value of ρ_i, i.e. a stronger desire to smooth the nominal interest rate, the weaker the amplification effect of DE. This result is in line with the expected reactions under RE and the Central Bank's willingness to reduce the volatility of its instrument. Additionally, the amplification effect of DE remains roughly the same for the remaining variables.

6.2.2 Habits parameter h

The degree of habits governs the dependence of agents' utility on recent past consumption. When agents are diagnostic, the effects caused by a change in habit persistence are in line with RE. This means that the presence of diagnostic agents that form habits strengthens the exaggeration effect of DE throughout the model in the expected direction, although mildly compared to other mechanisms.

6.2.3 Inverse intertemporal elasticity of substitution parameter σ

This parameter governs the intertemporal sensitivity of household consumption to changes in the real interest rate. It also relates to the degree of risk aversion of a household. When

Table 1.4: Unconditional standard deviation: sensitivity to key parameters

Parameter	Output	Consumption	Domestic Inflation	CPI	Interest rate	RER	Imported inflation	Hours worked
Benchmark	1.525	0.606	1.099	0.465	1.114	7.838	3.394	1.554
Interest rate smoothing								
$\omega_R = 0.75$	1.527	0.606	1.087	0.463	1.175	7.846	3.410	1.556
$\omega_R = 0.95$	1.523	0.607	1.110	0.469	1.059	7.831	3.381	1.553
Habits								
$h = 0.5$	1.516	0.621	1.097	0.464	1.082	7.844	3.400	1.542
$h = 0.7$	1.540	0.587	1.104	0.467	1.149	7.831	3.386	1.574
Inverse IES								
$\sigma = 2.0$	1.284	0.963	1.040	0.450	1.046	7.663	3.387	1.240
$\sigma = 5.0$	1.653	0.443	1.127	0.474	1.145	7.930	3.400	1.708
Degree of openness								
$\alpha = 0.1$	0.904	0.406	0.716	0.446	0.686	9.097	4.268	0.997
$\alpha = 0.3$	2.097	0.812	1.462	0.486	1.485	7.036	2.755	2.101
Elasticity of substitution								
$\eta = 0.65$	1.746	0.746	1.199	0.463	1.099	10.077	3.844	1.763
$\eta = 0.85$	1.494	0.559	1.054	0.469	1.144	6.848	3.142	1.526
Domestic good Calvo								
$\theta_H = 0.65$	1.603	0.566	1.316	0.569	0.911	8.047	3.533	1.619
$\theta_H = 0.85$	1.381	0.718	0.786	0.388	1.454	7.440	3.088	1.445
Imported good Calvo								
$\theta_F = 0.65$	1.505	0.643	1.217	0.457	1.352	7.524	4.405	1.539
$\theta_F = 0.85$	1.577	0.553	0.922	0.486	0.968	8.552	2.234	1.597

Note: This table displays the unconditional standard deviations when the original parameterisation remains unchanged, except for the adjustment of a specific parameter, either increased or decreased.

agents are diagnostic, an increase in this parameter generates a stronger amplification effect of DE. This amplification is appreciated as larger volatilities for almost all variables, specifically domestic variables. An exception is consumption, for which the amplification effect is ameliorated. The lower volatility in consumption happens because the diagnostic agent overreacts by behaving as more risk averse, making their consumption even less responsive to changes in the real interest rate.

6.2.4 Degree of openness α

In this case, I study whether and how a more open economy influences the effects of DE. The results in Table 1.4 show that the more open an economy is, the greater the effects of DE on domestic variables, while its effect on foreign variables decreases, although they remain the most volatile variables. This outcome suggests that the RER has a greater effect on the volatility of domestic variables when α increases. In addition, the amplification power of DE further magnifies the impact of the RER on domestic volatility. Therefore, a SOE populated with diagnostic agents is more sensitive to movements in variables related to the rest of the world.

6.2.5 Elasticity of substitution between domestic and foreign goods η

This parameter governs the degree of substitutability between domestic and foreign produced goods. A higher value for η means that the goods become more substitutable, while a smaller value means that they become more complementary. Therefore, when η increases, the amplification effect of DE is muted by the expenditure switching effect that a higher elasticity generates. This is not the case for the CPI and the nominal interest rate. For the CPI, even though both standard deviations for domestic-goods and imported-goods inflation decrease, the change in their covariance is not big enough to generate the same movement in the CPI. For the nominal interest rate, the volatility is higher as a consequence of the higher standard deviation of the CPI transmitted by the Taylor rule.

6.2.6 Domestic and foreign good Calvo parameters θ_H, θ_F

A change in these parameters affects the probability that a firm will reset its price. Higher values of θ_H or θ_F imply greater price stickiness and thus more price dispersion for domestic or foreign goods, respectively. When agents are diagnostic, increasing or decreasing price sluggishness influences volatility in the same way as under RE. Specifically, the more flexible the prices, the greater the effect of DE on domestic-goods and foreign-goods inflation. This amplified effect is strongly translated into the rest of the economy when θ_H changes than when θ_F changes, as domestic goods make up a larger share of the consumption basket than imported goods. In addition, the opposite effect in the volatility measure of output is the result of open economy channels that work through the RER.

6.3 Persistence mechanism and diagnostic expectations

This section explores whether and how the model's incorporated persistence mechanisms, namely interest rate smoothing and habits, help in transmitting diagnosticity characteristics to the economic variables.

6.3.1 Interest rate smoothing

The model incorporates interest rate smoothing via the Taylor rule followed by the Central Bank in Equation (1.38). It indicates that, besides responding to changes in the CPI and output growth, the Central Bank aims to gradually adjust its instrument over a period of time.

I plot the impulse responses of five different models.[18] When comparing the reactions of the models under RE or DE but without persistence mechanisms, I find evidence that reinforces the central conclusion that DE generates higher volatility, stronger initial reactions, and sharp reversals. However, when interest rate smoothing is included in the model with diagnostic agents, this persistence mechanism generally intensifies the effects of diagnosticity and helps to propagate it throughout the economy. There are, though, two variables for which the effects of DE are rather moderated, consumption and nominal interest rate as a result of the mechanism. Finally, when contrasting the responses of the model including both persistence mechanisms and DE with the model including just interest rate smoothing and DE, I study to what extent the persistence mechanism in the latter model dominates the variables' reaction in the former model. I conclude that interest rate smoothing indeed governs such behaviour for all the variables in the full model, after almost all shocks, with the exception of consumption, output, and labour.

6.3.2 Habits

The introduction of habits in the model is done in the households' utility function by linking current and past consumption, generating persistence in the model. Likewise the previous mechanism, its effects are shown in impulse responses from five different models.[19]

In contrast to the previous case, when the models with diagnostic agents but with and without habits are compared, the inclusion of this persistence mechanism shows that the diagnostic features of the model do not intensify or mute. This can be seen because the responses for almost all variables are very close to each other. Nevertheless, the inclusion of habits influences the behaviour of three variables: output, consumption, and labour. The effect of this persistence mechanism is a smoother reaction, which mutes the effect of a strong initial overreaction. Moreover, it explains the fact that the volatility of consumption, output and labour are among the ones increasing the least when agents are

[18] For exposition purposes, the figures 1.9 to 1.14 that plot the impulse responses of five different models to each of the six shocks are presented in Appendix 1.C. Model 1 (solid green line) represents the full model, which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

[19] Similarly, for exposition purposes, figures 1.15-1.20 are presented in Appendix 1.C. Model 1 (solid green line) represents the full model that includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

diagnostic. Additionally, when contrasting the impulse responses of the model including both persistence mechanisms and DE with the model including just habits and DE, I find evidence that habits mainly interfere in shaping the behaviour of output, consumption, and labour's responses after each shock.

7 Concluding remarks

This chapter studies the effects of DE in a small open economy framework. In particular, it examines the ability of DE to help explain international macroeconomic puzzles, especially those related to exchange rates. These puzzles share similarities with those in finance, which DE have been used to successfully explain. I incorporate DE into a SOE model *à la* Justiniano and Preston (2010b). The features of this model, specifically its openness and persistence mechanisms, enable me to examine the effects of DE within a broader framework that incorporates a wider range of shocks and frictions.

According to the results, the DSOE offers a qualitative explanation for the excess volatility of the real exchange rate and its disconnection from fundamentals. The presence of diagnostic agents generates an amplification mechanism that produces endogenous volatility and amplification of short-term behaviour. Moreover, these effects exhibit some degree of sensitiveness to changes in the parameterisation of the model, especially to the degree of openness and the elasticity of substitution between domestic and imported goods. The results also reveal that persistence mechanisms are important for amplifying and transmitting the effects of DE throughout the model, albeit in a heterogeneous manner.

Future research could progress by empirically testing the validity of DE in exchange rate behaviour, for example, by estimating the model. In addition, another direction could be to study the effects of higher macroeconomic instability generated by DE under different monetary policy rules.

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Appendices

1.A Small Open Economy model log-linearisation

This appendix presents the derivation of the small open economy's log-linear optimality conditions.

1.A.1 Households optimal conditions

I start by log-linearising the intra-temporal condition (1.14). Defining $X_t = C_t - H_t$ and $w_t = W_t/P_t$:

$$L_t^{\varphi}\frac{X_t^{\sigma}}{\Gamma_t} = w_t,$$

$$(e^{ln(L_t)})^{\varphi}(e^{ln(X_t)})^{\sigma}(e^{ln(\Gamma_t)})^{-1} = e^{ln(w_t)}.$$

The first-order Taylor expansion gives:

$$\bar{L}^{\varphi}\bar{X}^{\sigma}\bar{\Gamma}^{-1} + \bar{L}^{\varphi}\bar{X}^{\sigma}\bar{\Gamma}^{-1}\varphi\hat{l}_t + \bar{L}^{\varphi}\bar{X}^{\sigma}\bar{\Gamma}^{-1}\sigma\hat{x}_t - \bar{L}^{\varphi}\bar{X}^{\sigma}\bar{\Gamma}^{-1}\gamma_t = \bar{w} + \bar{w}\hat{w}_t.$$

Simplifying and replacing $\hat{x}_t$:

$$\boxed{\varphi\hat{l}_t + \frac{\sigma}{1-h}(\hat{c}_t - h\hat{c}_{t-1}) - \gamma_t = \hat{w}_t.} \tag{1.61}$$

Similarly for the Euler condition (1.15), I get:

$$X_t^{-\sigma}\Gamma_t = \beta\mathbb{E}_t[X_{t+1}^{-\sigma}(1+\pi_{t+1})^{-1}\Gamma_{t+1}R_t],$$

$$(e^{ln(X_t)})^{-\sigma}e^{ln(\Gamma_t)} = \beta\mathbb{E}_t[(e^{ln(X_{t+1})})^{-\sigma}(e^{ln(1+\pi_{t+1})})^{-1}(e^{ln(R_t)})e^{ln(\Gamma_{t+1})})].$$

The first-order Taylor expansion gives:

$$\bar{X}^{-\sigma}\bar{\Gamma} - \sigma\bar{X}^{-\sigma}\bar{\Gamma}\hat{x}_t + \bar{X}^{-\sigma}\bar{\Gamma}\gamma_t =$$
$$\beta\bar{R}\bar{X}^{-\sigma}\bar{\Gamma} + \mathbb{E}_t[-\beta\bar{R}\sigma\bar{X}^{-\sigma}\bar{\Gamma}\hat{x}_{t+1} - \beta\bar{X}^{-\sigma}\bar{R}\bar{\Gamma}\hat{R}_t - \beta\bar{X}^{-\sigma}\bar{R}\bar{\Gamma}\pi_{t+1} + \beta\bar{X}^{-\sigma}\bar{R}\bar{\Gamma}\gamma_{t+1}],$$

where $1 = \beta\bar{R}$ and $R_t = (1+i_t)$. Substituting and simplifying I obtain:

$$\boxed{\hat{c}_t - h\hat{c}_{t-1} = \mathbb{E}_t[\hat{c}_{t+1} - h\hat{c}_t] - \frac{1-h}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) + \frac{1-h}{\sigma}(\gamma_t - \mathbb{E}_t[\gamma_{t+1}]).} \tag{1.62}$$

1.A.2 Domestic firms' optimisation condition

First, I log-linearise the domestic price index, Equation (1.17) around the zero steady state ($P = P_H = P'_H$), obtaining:

$$p'_{H,t} - p_{H,t-1} = \frac{1}{1-\theta_H}\pi_{H,t}. \tag{1.63}$$

Second, I present the log-linear expression of Equation (1.21):

$$p'_{H,t} = k_{1,t} - k_{2,t}. \tag{1.64}$$

To obtain the right-hand side elements of this expression, I proceed to log-linearise the auxiliary variables $K_{1,t}$ and $K_{2,t}$. The former can be expressed recursively as:

$$K_{1,t} = X_t^{-\sigma} Y_t P_{H,t} MC_t + \theta_H \beta \mathbb{E}_t[K_{1,t+1}],$$

$$e^{ln(K_{1,t})} = (e^{ln(X_t)})^{-\sigma} e^{ln(Y_t)} e^{ln(P_{H,t})} e^{ln(MC_t)} + \theta_H \beta \mathbb{E}_t[e^{ln(K_{1,t+1})}].$$

The first-order Taylor expansion gives:

$$\bar{K}_1 \hat{k}_{1,t} = \bar{X}\bar{Y}\bar{P}_H \bar{MC}(-\sigma\hat{x}_t + \hat{y}_t + \hat{p}_{H,t} + \hat{mc}_t) + \theta_H \beta \bar{K}_1 \mathbb{E}_t[\hat{k}_{1,t+1}].$$

Dividing both sides of the previous equation by $\bar{K}_1$, and from the fact that $\bar{K}_1 = (1-\theta_H\beta)^{-1}\bar{X}^{-\sigma}\bar{Y}\bar{P}_H\bar{MC}$, I obtain the log-linearised expression of this auxiliary variable:

$$\boxed{\hat{k}_{1,t} = (1-\theta_H\beta)(-\sigma\hat{x}_t + \hat{y}_t + \hat{p}_{H,t} + \hat{mc}_t) + \theta_H \beta \mathbb{E}_t[\hat{k}_{1,t+1}].} \tag{1.65}$$

A similar procedure is applied to obtain the log-linearisation of $K_{2,t}$:

$$K_{2,t} = X_t^{-\sigma} Y_t + \theta_H \beta \mathbb{E}_t[K_{2,t+1}],$$

$$e^{ln(K_{2,t})} = (e^{ln(X_t)})^{-\sigma} e^{ln(Y_t)} + \theta_H \beta \mathbb{E}_t[e^{ln(K_{2,t+1})}].$$

The first-order Taylor expansion gives:

$$\bar{K}_2 \hat{k}_{2,t} = \bar{X}\bar{Y}(-\sigma\hat{x}_t + \hat{y}_t) + \theta_H \beta \bar{K}_2 \mathbb{E}_t[\hat{k}_{2,t+1}].$$

Dividing both sides of the previous equation by $\bar{K}_2$ and from the fact that $\bar{K}_2 = (1-\theta_H\beta)^{-1}\bar{X}^{-\sigma}\bar{Y}$, I obtain the log-linearised expression of the denominator in Equation (1.21):

$$\boxed{\hat{k}_{2,t} = (1-\theta_H\beta)(-\sigma\hat{x}_t + \hat{y}_t) + \theta_H \beta \mathbb{E}_t[\hat{k}_{2,t+1}].} \tag{1.66}$$

Substituting the results from Equation (1.65) and Equation (1.66) into Equation (1.64), I obtain:

$$p'_{H,t} = (1-\theta_H\beta)(-\sigma\hat{x}_t + \hat{y}_t + \hat{p}_{H,t} + \hat{mc}_t + \sigma\hat{x}_t - \hat{y}_t) + \theta_H \beta \mathbb{E}_t[\hat{k}_{1,t+1} - \hat{k}_{2,t+1}],$$

which after rearranging and replacing $(\hat{k}_{1,t+1} - \hat{k}_{2,t+1}) = p'_{H,t+1}$:

$$p'_{H,t} = (1-\theta_H\beta)(\hat{p}_{H,t} + \hat{mc}_t) + \theta_H\beta\mathbb{E}_t[p'_{H,t+1}].$$

Subtracting $\hat{p}_{H,t-1}$ from both sides:

$$p'_{H,t} - \hat{p}_{H,t-1} = (1-\theta_H\beta)(\hat{p}_{H,t} + \hat{mc}_t) + \theta_H\beta\mathbb{E}_t[p'_{H,t+1}] - \hat{p}_{H,t-1}.$$

Expanding the first terms on the right-hand side:

$$p'_{H,t} - \hat{p}_{H,t-1} = \hat{p}_{H,t} - \theta_H\beta\hat{p}_{H,t} + (1-\theta_H\beta)\hat{mc}_t + \theta_H\beta\mathbb{E}_t[p'_{H,t+1}] - \hat{p}_{H,t-1}.$$

Re-arranging and cancelling terms out:

$$p'_{H,t} - \hat{p}_{H,t-1} = (1-\theta_H\beta)\hat{mc}_t + \pi_{H,t} + \theta_H\beta\mathbb{E}_t[p'_{H,t+1} - p_{H,t}].$$

Next, using the results of Equation (1.63):

$$\tfrac{1}{1-\theta_H}\pi_{H,t} = (1-\theta_H\beta)\hat{mc}_t + \pi_{H,t} + \theta_H\beta\mathbb{E}_t[\tfrac{1}{1-\theta_H}\pi_{H,t+1}].$$

which after some re-arrangements gives the Phillips curve:

$$\boxed{\pi_{H,t} = \lambda_H\hat{mc}_t + \beta\mathbb{E}_t[\pi_{H,t+1}],} \tag{1.67}$$

where $\lambda_H = \frac{(1-\theta_H\beta)(1-\theta_H)}{\theta_H}$.

1.A.3 Retail firms' optimisation condition

First, I log-linearise the retailer's price index in Equation (1.22) around the zero steady state ($P = P_F = P'_F$), obtaining:

$$p'_{F,t} - \hat{p}_{F,t-1} = \frac{1}{1-\theta_F}\pi_{F,t}. \tag{1.68}$$

Second, I present the log-linear expression of Equation (1.26):

$$p'_{F,t} = \hat{j}_{1,t} - \hat{j}_{2,t}. \tag{1.69}$$

To obtain the right-hand side elements of this expression, I proceed to log-linearise the auxiliary variables $J_{1,t}$ and $J_{2,t}$. The former can be expressed recursively as:

$$J_{1,t} = X_t^{-\sigma}C_{F,t}N_t s_t P^*_{F,t} + \theta_F\beta\mathbb{E}_t[J_{1,t+1}],$$

$$e^{ln(J_{1,t})} = (e^{ln(X_t)})^{-\sigma}e^{ln(C_{F,t})}e^{ln(N_t)}e^{ln(s_t)}e^{ln(P^*_{F,t})} + \theta_F\beta\mathbb{E}_t[e^{ln(J_{1,t+1})}].$$

The first-order Taylor expansion gives:

$$\bar{J}_1\hat{j}_{1,t} = \bar{X}\bar{Y}\bar{N}\bar{s}\bar{P}^*_F(-\sigma\hat{x}_t + \hat{c}_{F,t} + \nu_t + \hat{s}_t + \hat{p}^*_{F,t}) + \theta_F\beta\bar{J}_1\mathbb{E}_t[\hat{j}_{1,t+1}].$$

Dividing both sides of the previous equation by $\bar{J}_1$ and from the fact that $\bar{J}_1 = (1-\theta_F\beta)^{-1}\bar{X}\bar{Y}\bar{N}\bar{s}\bar{P}_F$, I obtain the log-linearised expression of this auxiliary variable:

$$\boxed{\hat{j}_{1,t} = (1-\theta_F\beta)(-\sigma\hat{x}_t + \hat{c}_{F,t} + \nu_t + \hat{s}_t + \hat{p}^*_{F,t}) + \theta_F\beta\mathbb{E}_t[\hat{j}_{1,t+1}].} \tag{1.70}$$

A similar procedure is applied to obtain the log-linearisation of $J_{2,t}$:

$$J_{2,t} = X_t^{-\sigma} C_{F,t} + \theta_F \beta \mathbb{E}_t[J_{2,t+1}],$$

$$e^{ln(J_{2,t})} = (e^{ln(X_t)})^{-\sigma} e^{ln(Y_t)} + \theta_F \beta = \mathbb{E}_t[e^{ln(J_{2,t+1})}].$$

The first-order Taylor expansion gives:

$$\bar{J}_2 \hat{j}_{2,t} = \bar{X}\bar{Y}(-\sigma\hat{x}_t + \hat{c}_{F,t}) + \theta_F \beta \bar{J}_2 \mathbb{E}_t[\hat{j}_{2,t+1}].$$

Dividing both sides of the previous equation by $\bar{J}_2$ and from the fact that $\bar{J}_2 = (1-\theta_F\beta)^{-1}\bar{X}^{-\sigma}\bar{Y}$, I obtain the log-linearised expression of the denominator in Equation (1.26):

$$\boxed{\hat{j}_{2,t} = (1-\theta_F\beta)(-\sigma\hat{x}_t + \hat{c}_{F,t}) + \theta_F\beta\mathbb{E}_t[\hat{j}_{2,t+1}].} \tag{1.71}$$

Substituting the results from Equation (1.70) and Equation (1.71) into Equation (1.69), I obtain:

$$p'_{F,t} = (1-\theta_F\beta)(-\sigma\hat{x}_t + \hat{c}_{F,t} + \nu_t + \hat{s}_t + \hat{p}^*_{F,t} + \sigma\hat{x}_t - \hat{c}_{F,t}) + \theta_F\beta\mathbb{E}_t[\hat{j}_{1,t+1} - \hat{j}_{2,t+1}],$$

which after rearranging and replacing $(\hat{j}_{1,t+1} - \hat{j}_{2,t+1}) = p'_{F,t+1}$:

$$p'_{F,t} = (1-\theta_F\beta)(\nu_t + \hat{s}_t + \hat{p}^*_{F,t}) + \theta_F\beta\mathbb{E}_t[p'_{F,t+1}].$$

Subtracting $\hat{p}_{F,t-1}$ from both sides:

$$p'_{F,t} - \hat{p}_{F,t-1} = (1-\theta_F\beta)(\nu_t + \hat{s}_t + \hat{p}^*_{F,t}) + \theta_F\beta\mathbb{E}_t[p'_{F,t+1} - \hat{p}_{F,t-1}].$$

Expanding the first terms on the right-hand side:

$$p'_{F,t} - \hat{p}_{F,t-1} = \hat{p}_{F,t} - \theta_F\beta\hat{p}_{F,t} + (1-\theta_F\beta)(\hat{s}_t + \nu_t) + \theta_F\beta\mathbb{E}_t[p'_{F,t+1} - \hat{p}_{F,t-1}].$$

Re-arranging:

$$p'_{F,t} - \hat{p}_{F,t-1} = (1-\theta_F\beta)(\hat{s}_t + \nu_t) + \hat{\pi}_{F,t} + \theta_F\beta\mathbb{E}_t[p'_{F,t+1} - \hat{p}_{F,t}].$$

Next, using the results of Equation (1.68):

$$\tfrac{1}{1-\theta_F}\pi_{F,t} = (1-\theta_F\beta)(\hat{s}_t + \nu_t) + \pi_{F,t} + \theta_F\beta\mathbb{E}_t[\tfrac{1}{1-\theta_F}\pi_{F,t+1}],$$

which after some re-arrangements gives the Phillips curve:

$$\boxed{\pi_{F,t} = \lambda_F(\hat{\psi}_t + \nu_t) + \beta\mathbb{E}_t[\pi_{F,t+1}],} \tag{1.72}$$

where $\lambda_F = \frac{(1-\theta_F\beta)(1-\theta_F)}{\theta_F}$, and as in equilibrium $P = P^*_F = P'_F$, the law of one price $\hat{\psi}_t$ is equal to $\hat{s}_t$.

1.A.4 Consumer Price Index, Real Exchange Rate, Terms of Trade and Law of One Price gap

The expression (1.27) shows the terms of trade:

$$tot_t = \tfrac{P_{F,t}}{P_{H,t}},$$

$$e^{ln(tot_t)} = e^{ln(P_{F,t})}e^{ln(P_{H,t})}.$$

The first-order Taylor expansion gives:

$$\overline{tot}\hat{tot}_t = \bar{P_F}\bar{P_H}(\hat{p}_{F,t} - \hat{p}_{H,t}).$$

Resulting, after simplifying and subtracting $\hat{tot}_{t-1}$ from the left-hand side and $(\hat{p}_{F,t-1} - \hat{p}_{H,t-1})$ from the right-hand side:

$$\boxed{\Delta\hat{tot}_t = \pi_{F,t} - \pi_{H,t}\ .} \tag{1.73}$$

Next, I proceed to log-linearise the CPI from Equation (1.28):

$$P_t \equiv [(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}},$$

$$(e^{ln(P_t)})^{1-\eta} = (1-\alpha)(e^{ln(P_{H,t})})^{1-\eta} + \alpha(e^{ln(P_{F,t})})^{1-\eta}.$$

The first-order Taylor expansion gives:

$$\bar{P}\hat{p}_t = (1-\alpha)\bar{P_H}\hat{p}_{H,t} + \alpha\bar{P_F}\hat{p}_{F,t}.$$

As in equilibrium $P = P_H = P_F$, cancelling and subtracting $\hat{p}_{t-1}$ from the left-hand side and $(1-\alpha)\hat{p}_{H,t-1}$ and $\alpha\hat{p}_{F,t-1}$ from the right-hand side, I obtain the following:

$$\boxed{\pi_t = (1-\alpha)\pi_{H,t} - \alpha\pi_{F,t},} \tag{1.74}$$

which can also be expressed, using Equation (1.73) as:

$$\pi_t = \pi_{H,t} + \alpha\Delta\hat{tot}_t.$$

Now, I log-linearise Expression (1.29), the real exchange rate:

$$Q_t = \frac{s_t P_t^*}{P_t},$$

$$e^{ln(Q_t)} = e^{ln(s_t)}e^{ln(P_t^*)}(e^{ln(P_t)})^{-1}.$$

The first-order Taylor expansion gives:

$$\bar{Q}\hat{q}_t = \bar{s}\bar{P}^*\bar{P}(\hat{s}_t + \hat{p}_t^* - \hat{p}_t).$$

After simplifying, this expression becomes:

$$\boxed{\hat{q}_t = \hat{s}_t + \hat{p}_t^* - \hat{p}_t.} \tag{1.75}$$

The log-linear expression for the real exchange rate will be used to solve for the law of one price gap, which I log-linearise from Equation (1.30):

$$\Psi_t = \frac{P_t^* s_t}{P_{F,t}},$$

$$e^{ln(\Psi_t)} = e^{ln(P_t^*)}e^{ln(s_t)}(e^{ln(P_{F,t})})^{-1}.$$

The first-order Taylor expansion gives:

$$\bar{\Psi}\hat{\psi}_t = \bar{P}^* \bar{s} \bar{P}_F(\hat{p}_t^* + \hat{s}_t - \hat{p}_{F,t}).$$

After cancelling terms, given that in steady state $P^* = P_F$:

$$\psi_t = \hat{p}_t^* + \hat{s}_t - \hat{p}_{F,t},$$

where using Equation (1.73), Equation (1.74) and equation (1.75) gives:

$$\boxed{\hat{\psi}_t = \hat{q}_t - (1-\alpha)\hat{tot}_t\ .} \tag{1.76}$$

1.A.5 Uncovered Interest Parity

This condition is a result of the assumption of the incomplete international markets, which log-linearisation follows:

$$\mathbb{E}_t\left[\tfrac{s_{t+1}}{s_t} R_t^* \Phi_{t+1}(A_{t+1})\right] = R_t,$$

$$e^{ln(R_t)} = \mathbb{E}_t[e^{ln(s_{t+1})}(e^{ln(s_t)})^{-1} e^{ln(R_t^*)} e^{ln(\Phi_{t+1}(A_{t+1}))}].$$

The first-order Taylor expansion gives:

$$\bar{R}i_t = \bar{R}^* \bar{s}\bar{s}^{-1}\bar{A}\bar{\phi}\mathbb{E}_t[i_t^* + \hat{s}_{t+1} - \hat{s}_t - \chi a_t - \phi_{t+1}].$$

Which after simplifying:

$$\boxed{i_t - i_t^* = \mathbb{E}_t[\Delta s_{t+1}] - \chi a_t - \mathbb{E}_t[\phi_{t+1}].} \tag{1.77}$$

1.A.6 Flow Budget Constraint

This expression is obtained from the household's budget constraint, assuming that domestic bonds are in net zero supply and using the expressions for profits of the domestic and retailers firms.

$$YA_t - \tfrac{s_t}{s_{t-1}}\tfrac{R_{t-1}^*}{\pi_t} A_{t-1}\Phi_t(A_{t-1}, \tilde{\Phi}_t) = \tilde{P}_{H,t}Y_t - C_t + \alpha(\tilde{P}_{F,t} - Q_t)\tilde{P}_{F,t}^{-\eta}C_t\ ,$$

$$Ye^{ln(A_t)} - e^{ln(s_t)}(e^{ln(s_{t-1})})^{-1}e^{ln(R_{t-1}^*)}(e^{ln(\pi_t)})^{-1}e^{ln(A_{t-1})}e^{ln(\Phi_t(A_{t-1},\tilde{\Phi}_t))} =$$
$$e^{ln(\tilde{P}_{H,t})}e^{ln(Y_t)} - e^{ln(C_t)} + \alpha(e^{ln(\tilde{P}_{F,t})})^{1-\eta}e^{ln(C_t)} - \alpha e^{ln(Q_t)}(e^{ln(\tilde{P}_{F,t})})^{-\eta}e^{ln(C_t)}.$$

The first-order Taylor expansion gives:

$$\bar{Y}\bar{A}_t\hat{a}_t - \bar{s}(\bar{s})^{-1}\bar{R}^*\bar{\pi}\bar{A}\Phi(\bar{A},0)(\hat{s}_t - \hat{s}_{t-1} + \hat{i}_{t-1}^* - \pi_t + \hat{a}_{t-1} + \phi_t(\hat{a}_t, \tilde{\phi}_t)) =$$
$$\bar{P}_H\bar{Y}(\hat{p}_{H,t} + \hat{y}_t) - \bar{C}\hat{c}_t + \alpha\bar{C}(\bar{P}_F)^{1-\eta}((1-\eta)\hat{p}_{F,t} + \hat{c}_t) - \alpha\bar{Q}(\bar{P}_F)^{-\eta}\bar{C}(\hat{q}_t - \eta\hat{p}_{F,t} + \hat{c}_t)$$

Using the results that at the steady state $\bar{Y} = \bar{C}$, $\bar{R} = 1/\beta$ and $\bar{P}_F = \bar{P}_F = \bar{Q} = 1$, as well as the definition of terms of trade previously derived, we obtain, after simplifying and rearranging terms:

$$\boxed{\hat{a}_t - \beta^{-1}\hat{a}_{t-1} = y_t - \hat{c}_t - \alpha(\hat{tot}_t + \hat{\psi}_t)} \tag{1.78}$$

1.A.7 Equilibrium-Aggregate demand and output

The log-linear approximation of Equation (1.37) around the zero inflation steady state is:

$$Y_t = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \left(\frac{s_t P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t,$$

$$e^{ln(Y_t)} = (1-\alpha)(e^{ln(\frac{P_{H,t}}{P_t})})^{-\eta} e^{ln(C_t)} + (e^{ln(\frac{s_t P^*_{H,t}}{P^*_t})})^{-\eta} e^{ln(C^*_t)}.$$

The first-order Taylor expansion gives:

$$\bar{Y}\hat{y}_t = \bar{C}_H(-\eta(\hat{p}_{H,t} - \hat{p}_t) + \hat{c}_t) + \bar{C}^*_H(-\eta(\hat{s}_t + \hat{p}^*_{H,t} - \hat{p}^*_t) + \hat{y}^*_t),$$

which after replacing using the expressions for $\hat{tot}_t$ and for $\hat{\psi}_t$:

$$\bar{Y}\hat{y}_t = \bar{C}_H(-\eta(\hat{p}_{H,t} - \hat{p}_t) + \hat{c}_t) + \bar{C}^*_H(-\eta(\hat{\psi}_t + \hat{tot}_t) + \hat{y}^*_t).$$

As in steady state $C_H = (1-\alpha)C$ and $C_F = \alpha C$, and assuming a balanced trade, i.e. export of domestic economy equal to import ($C^*_H = C_F$), which implies $Y = C$, the log-linearised equilibrium condition yields:

$$\boxed{\hat{y}_t = (1-\alpha)\hat{c}_t + \alpha\eta(2-\alpha)\hat{tot}_t + \alpha\eta\hat{\psi}_t + \alpha\hat{y}^*_t.} \tag{1.79}$$

1.A.8 Central bank interest rate setting

The model exhibits a Central Bank that follows a Taylor rule. In this case, such rule responds to the previous period interest rate, CPI and output growth. To log-linearise it:

$$R_t = R_{t-1}^{\gamma_R}\left(\frac{P_t}{P_{t-1}}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\gamma_{\Delta y}} M_t,$$

$$e^{ln(R_t)} = (e^{ln(R_{t-1})})^{\gamma_R}(e^{ln(\Pi_t)})^{\gamma_\pi}(e^{ln(\frac{Y_t}{Y_{t-1}})})^{\gamma_{\Delta y}} e^{ln(M_t)}.$$

The first-order Taylor expansion gives:

$$\bar{R}i_t = \bar{R}[\gamma_R i_{t-1} + \gamma_\pi \pi_t + \gamma_{\Delta y}\Delta\hat{y}_t + \mu_t],$$

which simplifies to:

$$\boxed{i_t = \rho + \gamma_R i_{t-1} + \gamma_\pi \pi_t + \gamma_{\Delta y}\Delta\hat{y}_t + \mu_t.} \tag{1.80}$$

1.A.9 The Foreign Economy and shocks

The log-linearised versions of the law of motion of world's output and the shocks are straightforward to obtain:

World's output:

$$\hat{y}^*_{t+1} = \rho_{y^*}\hat{y}^*_t + \epsilon^{y^*}_{t+1}. \tag{1.81}$$

Productivity shock AR(1) process:

$$\zeta_{t+1} = \rho_\zeta \zeta_t + \epsilon^\zeta_{t+1}. \tag{1.82}$$

Preference shock AR(1) process:

$$\gamma_{t+1} = \rho_\gamma \gamma_t + \epsilon^\gamma_{t+1}. \tag{1.83}$$

Monetary shock AR(1) process:

$$\mu_{t+1} = \rho_\mu \mu_t + \epsilon^\mu_{t+1}. \tag{1.84}$$

Mark-up of import prices over marginal costs shock AR(1) process:

$$\nu_{t+1} = \rho_\nu \nu_t + \epsilon^\nu_{t+1}. \tag{1.85}$$

Risk-premium shock AR(1) process:

$$\tilde{\phi}_{t+1} = \rho_{\tilde{\phi}} \tilde{\phi}_t + \epsilon^{\tilde{\phi}}_{t+1}. \tag{1.86}$$

1.B Diagnostic probability density function

To obtain the result given by Expression (1.3), I consider the standard probability density function of a normally distributed variable x_{t+1}:

$$f(x_{t+1}|x_t) = \tfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho x_t)^2}{2\sigma^2}}.$$

Recalling the definition of the diagnostic probability density function $f^\phi(x_{t+1}|x_t) = f(x_{t+1}|x_t = \bar{x}_t)\left[\frac{f(x_{t+1}|\bar{x}_t)}{f(x_{t+1}|\rho\bar{x}_{t-1})}\right]^\phi$:

$$f^\phi(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left[\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^2\bar{x}_{t-1})^2}{2\sigma^2}}}\right]^\phi Z, \tag{1.87}$$

Simplifying and rewriting, I obtain:

$$f^\phi(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2} - \frac{1}{2\sigma^2}\phi\left[(x_{t+1}-\rho\bar{x}_t)^2 - (x_{t+1}-\rho^2\bar{x}_{t-1})^2\right]\right\}} Z. \tag{1.88}$$

Expanding the argument of the exponential function, the expression can be rewritten as:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right] + (\rho\bar{x}_t)^2 + \phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right] \Big\}\Bigg) Z. \tag{1.89}$$

The constant Z is given by:

$$Z = exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ -\phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right] + 2\rho\bar{x}_t\phi\left[\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right] + \phi^2\left[\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right]^2 \Big\}\Bigg). \tag{1.90}$$

Therefore, after some algebra, the diagnostic pdf is equal to:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{\left[x_{t+1} - \left(\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right)\right]^2\right\}}. \tag{1.91}$$

This, as Gennaioli and Shleifer (2020) states, contains the kernel of a normal distribution with a distorted mean and the same variance, as specified in the main text of the article.

1.C Extra figures

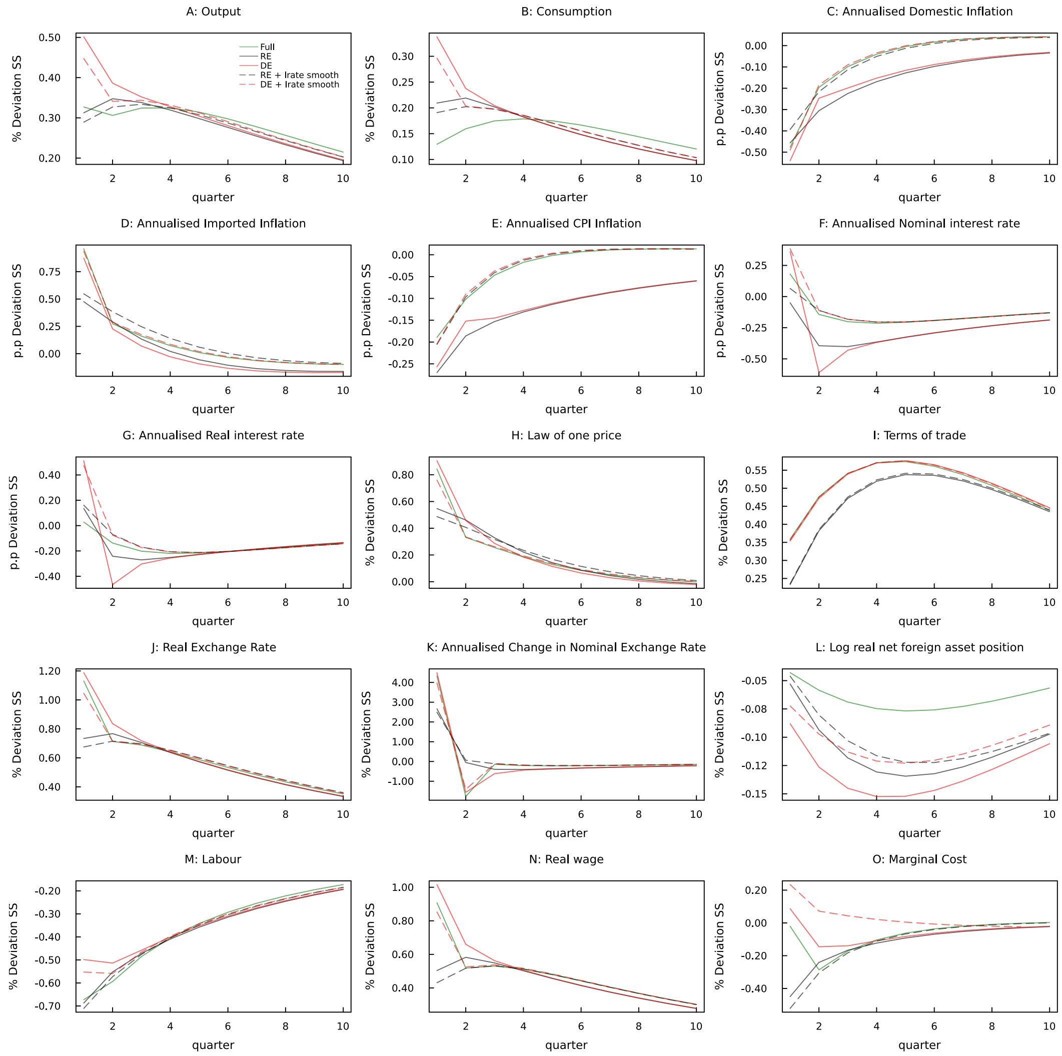


Figure 1.9: Interest rate smoothing and Technology shock.
Each panel depicts the response of the variables to a technology shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

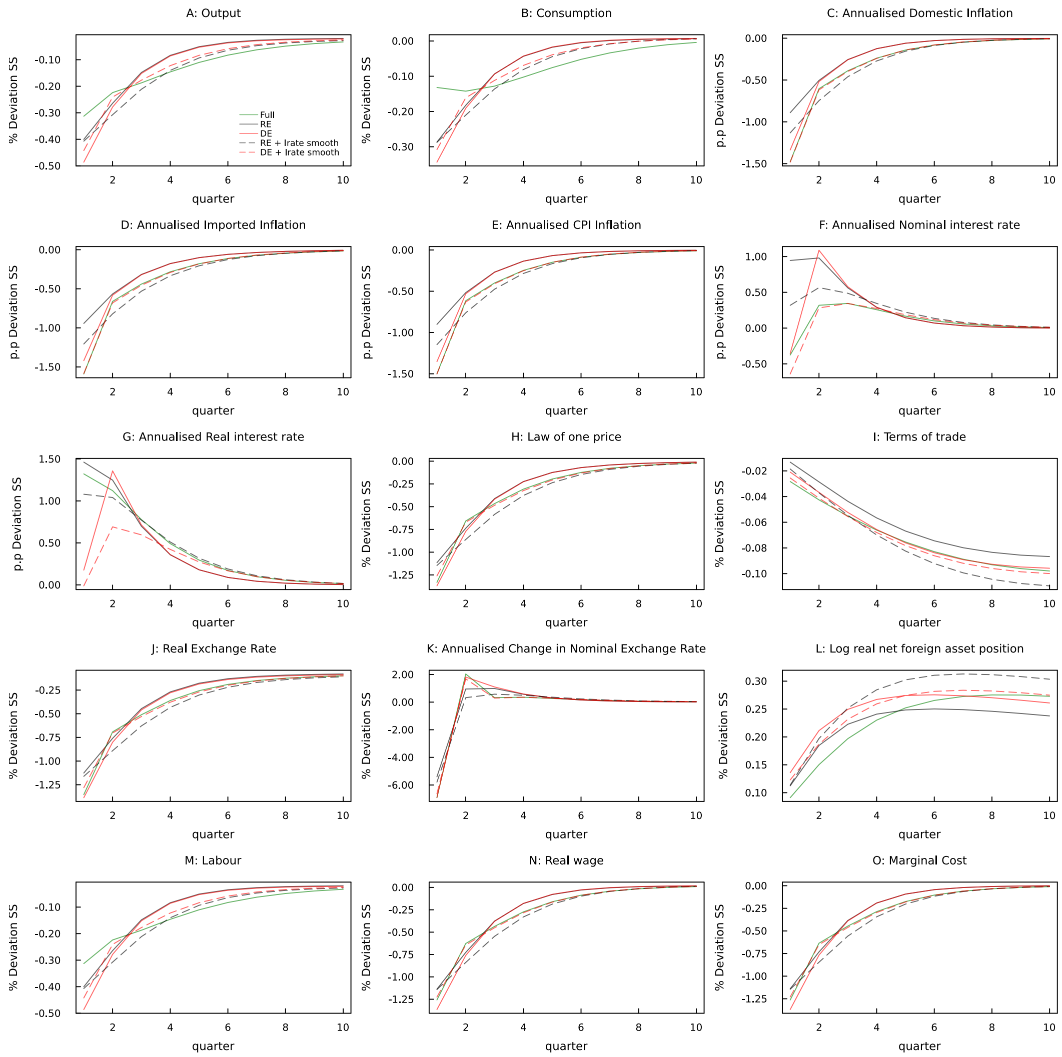


Figure 1.10: Interest rate smoothing and Monetary policy shock.
Each panel depicts the response of the variables to a monetary policy shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

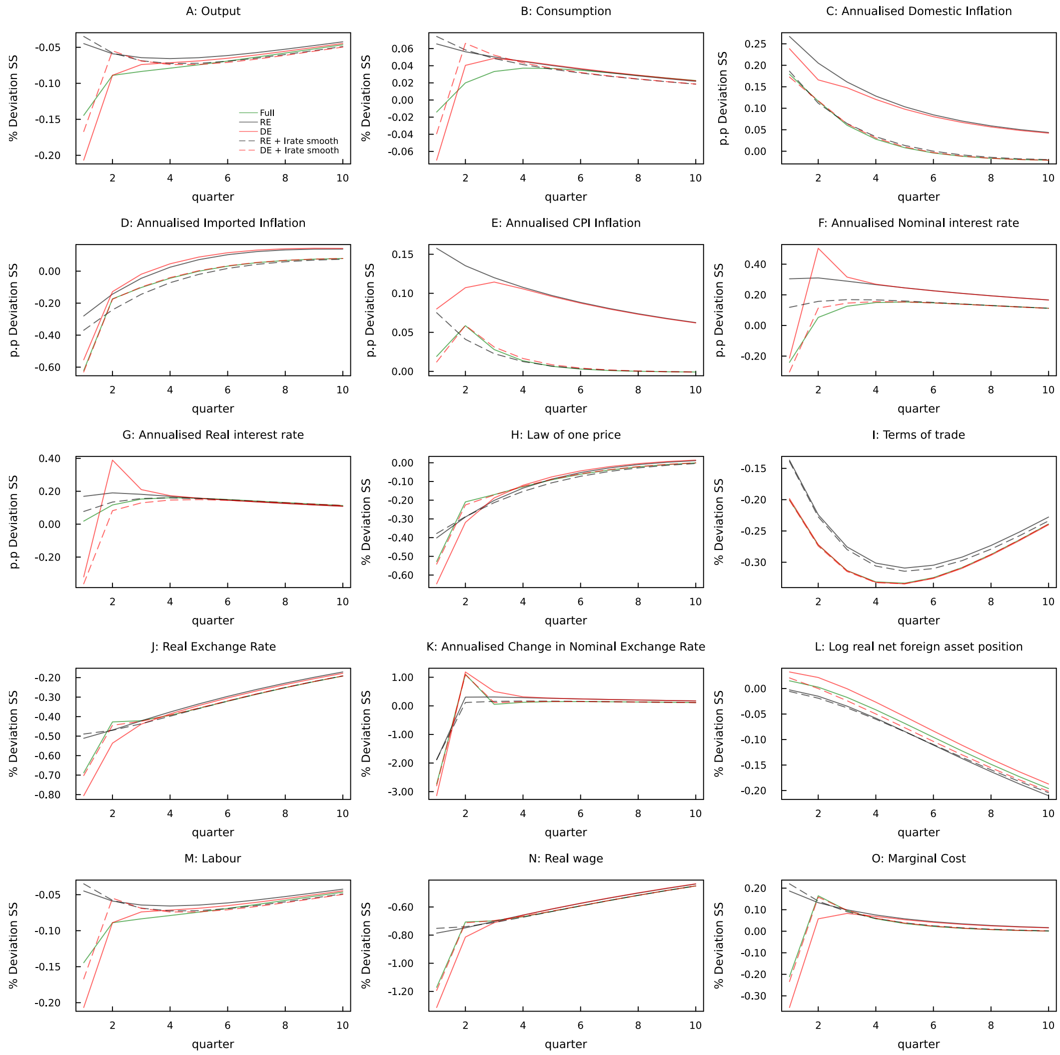


Figure 1.11: Interest rate smoothing and Preference shock.
Each panel depicts the response of the variables to a preference shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

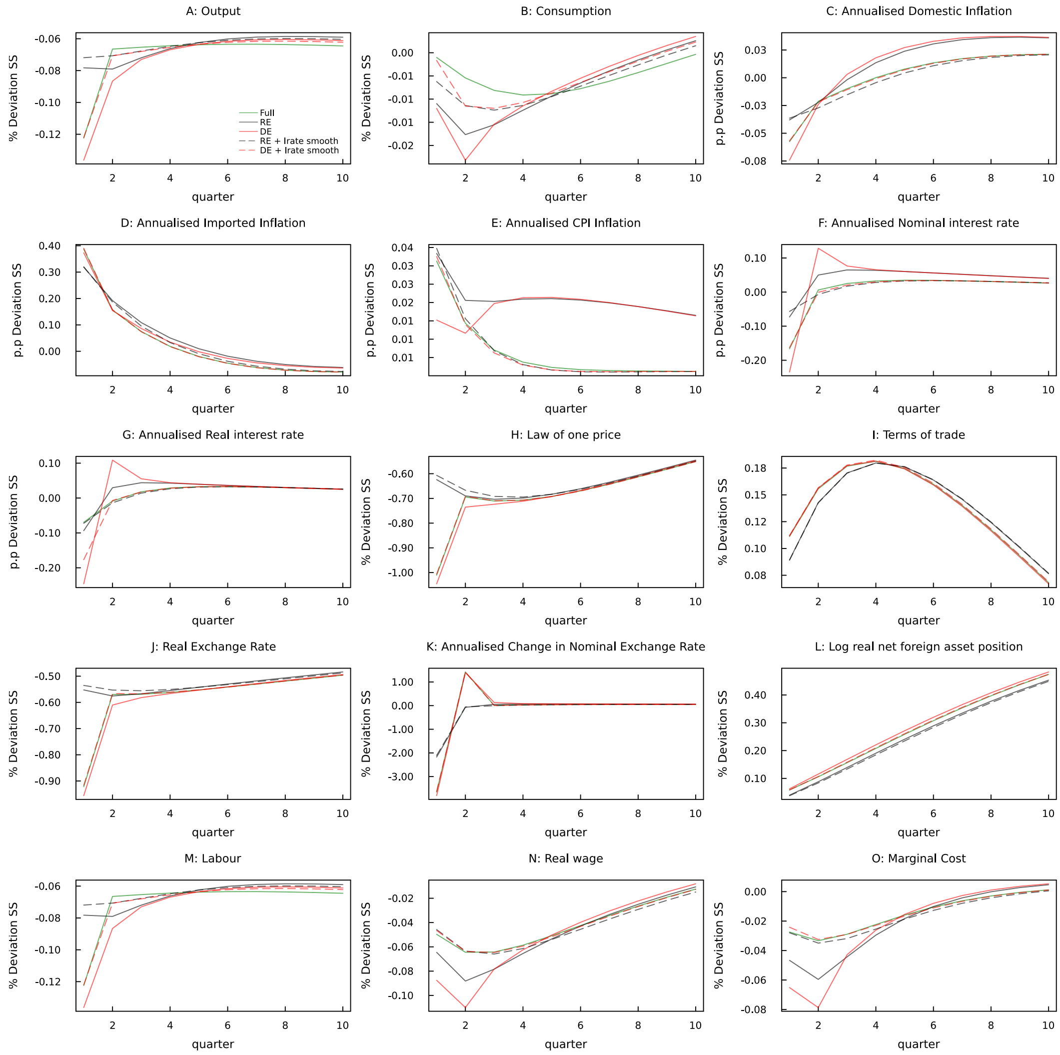


Figure 1.12: Interest rate smoothing and Cost push shock.
Each panel depicts the response of the variables to a cost push shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

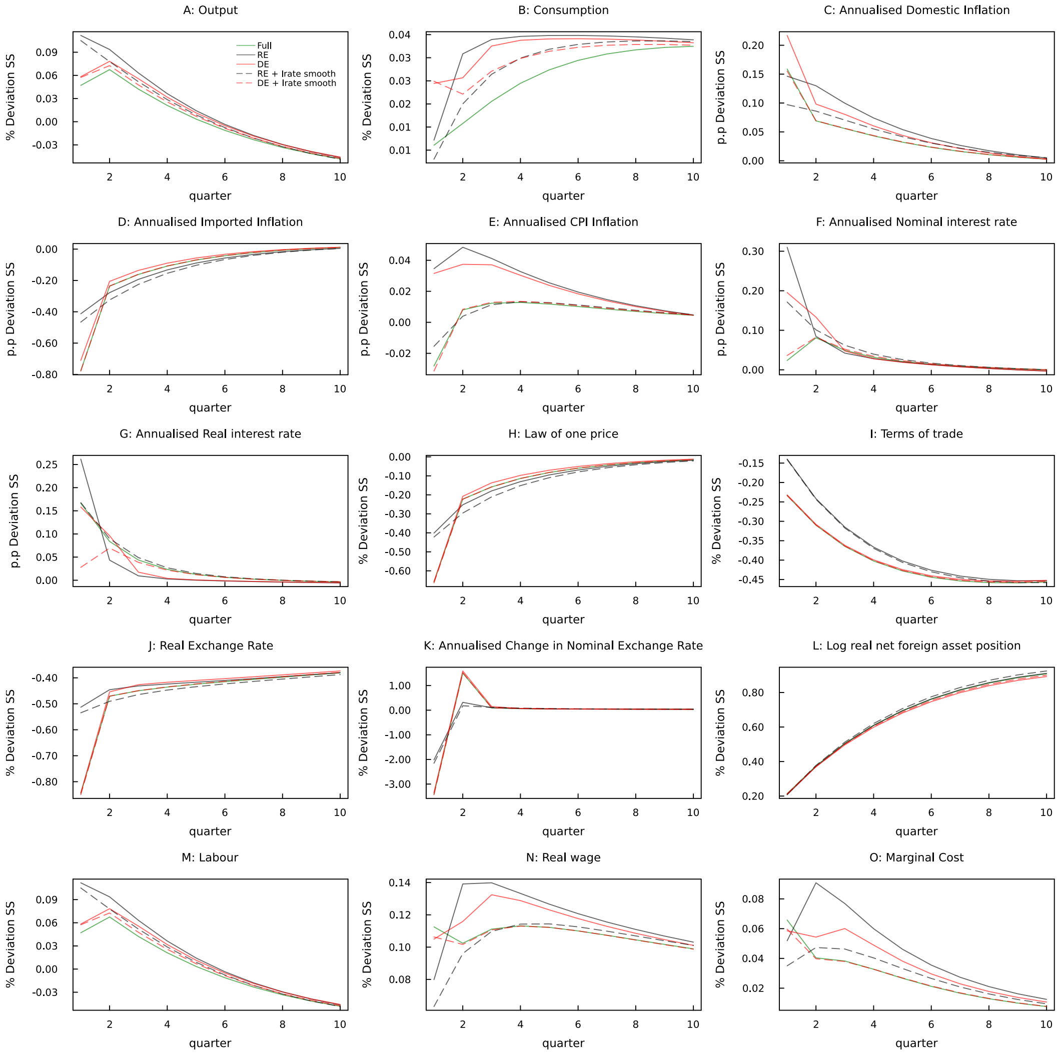


Figure 1.13: Interest rate smoothing and Foreign output shock.
Each panel depicts the response of the variables to a foreign output shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

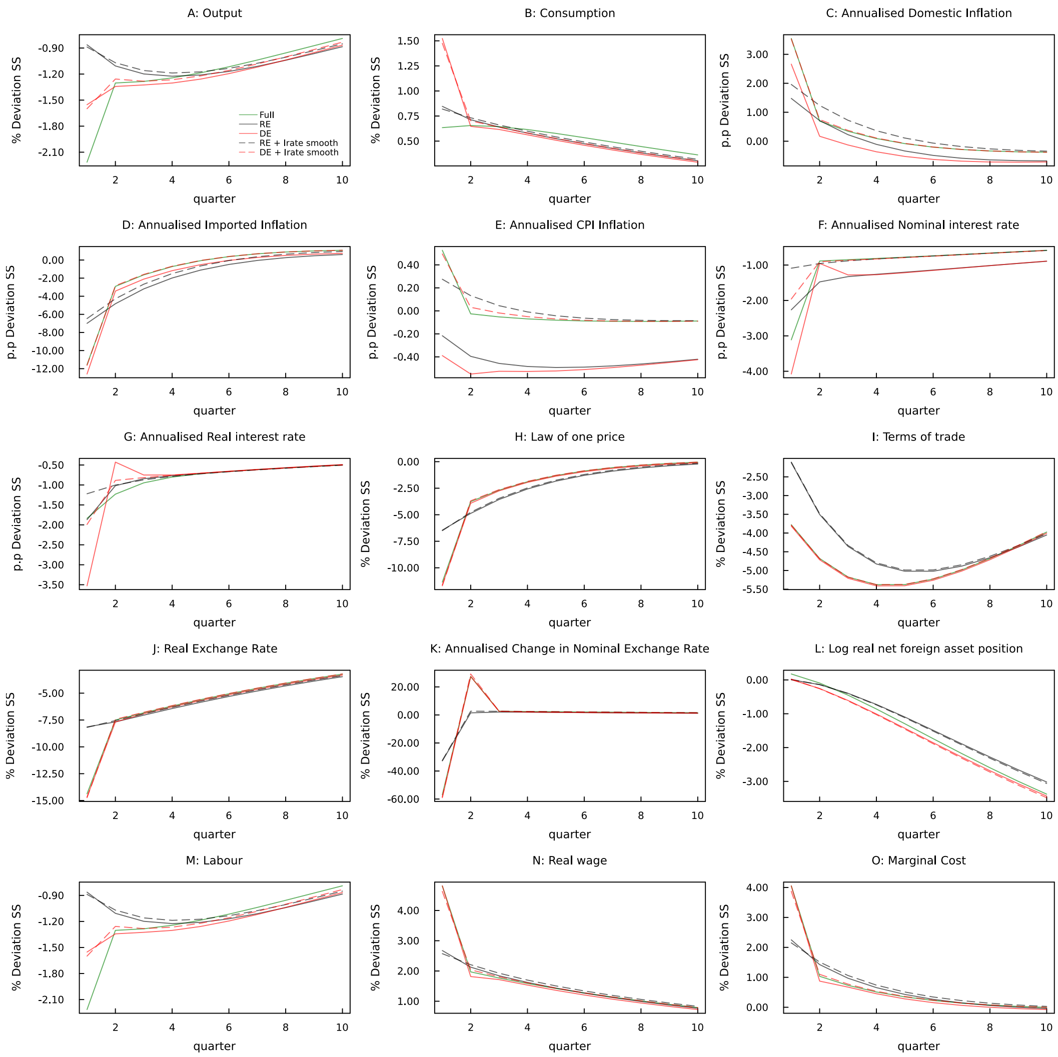


Figure 1.14: Interest rate smoothing and Risk-premium shock.
Each panel depicts the response of the variables to a foreign output shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and the interest rate smoothing mechanism is included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

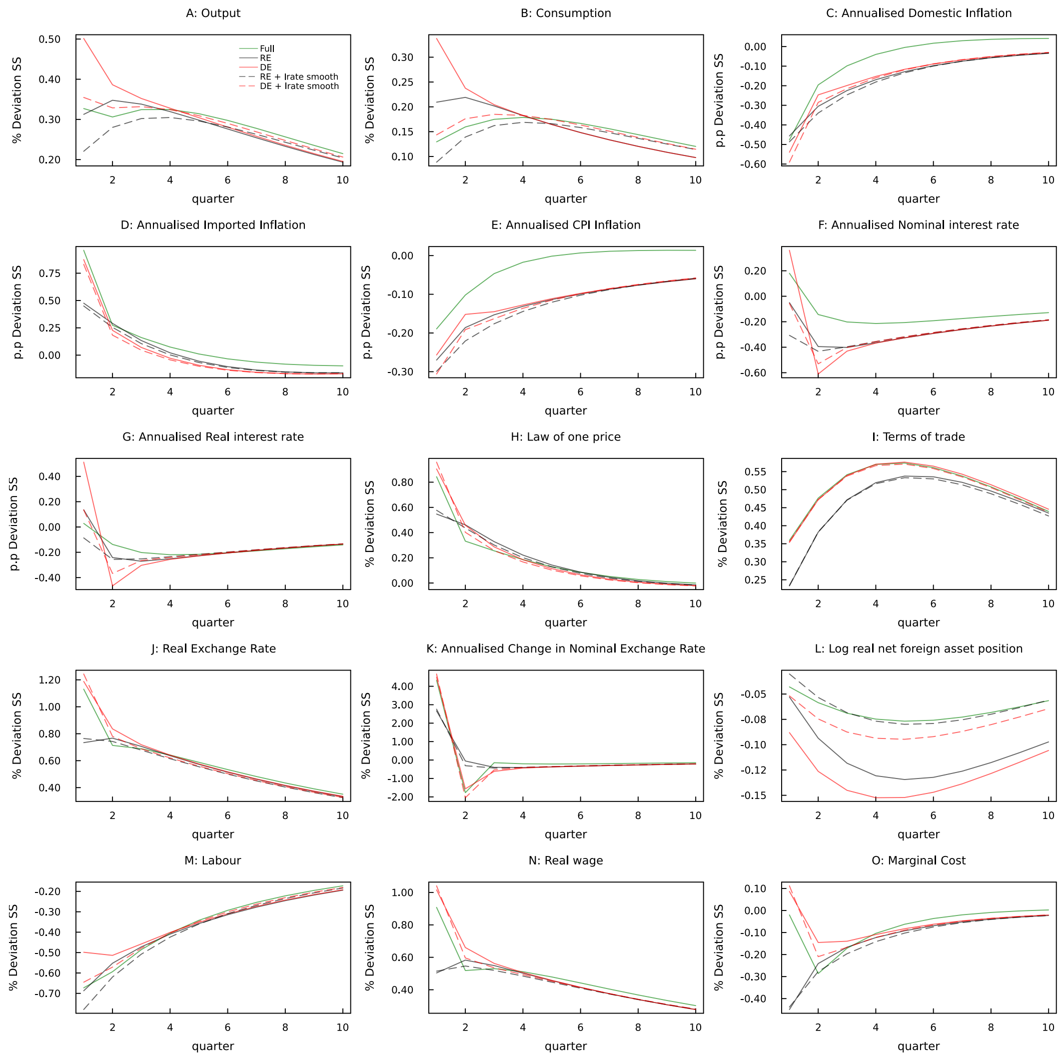


Figure 1.15: Habits and Technology shock.
Each panel depicts the response of the variables to a technology shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

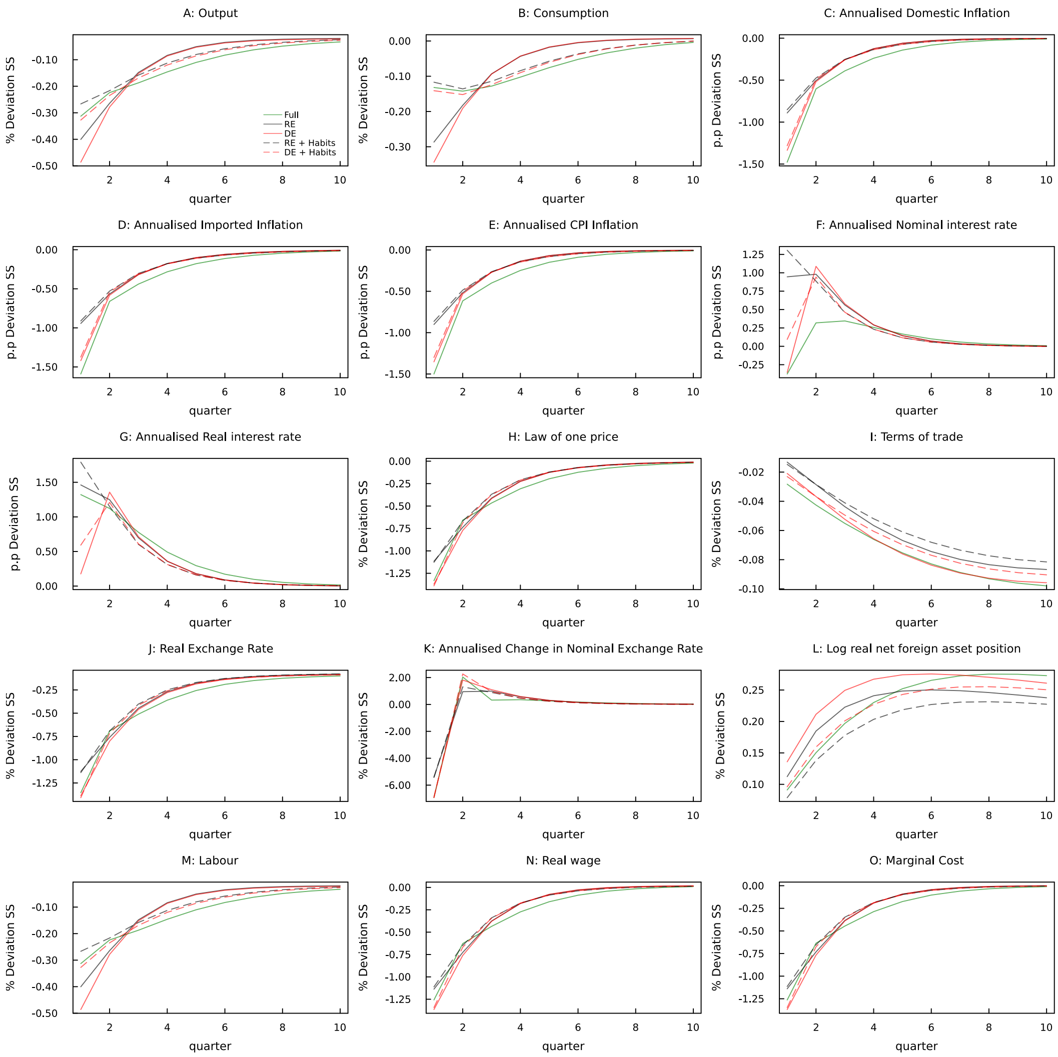


Figure 1.16: Habits and Monetary policy shock.
Each panel depicts the response of the variables to a monetary policy shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

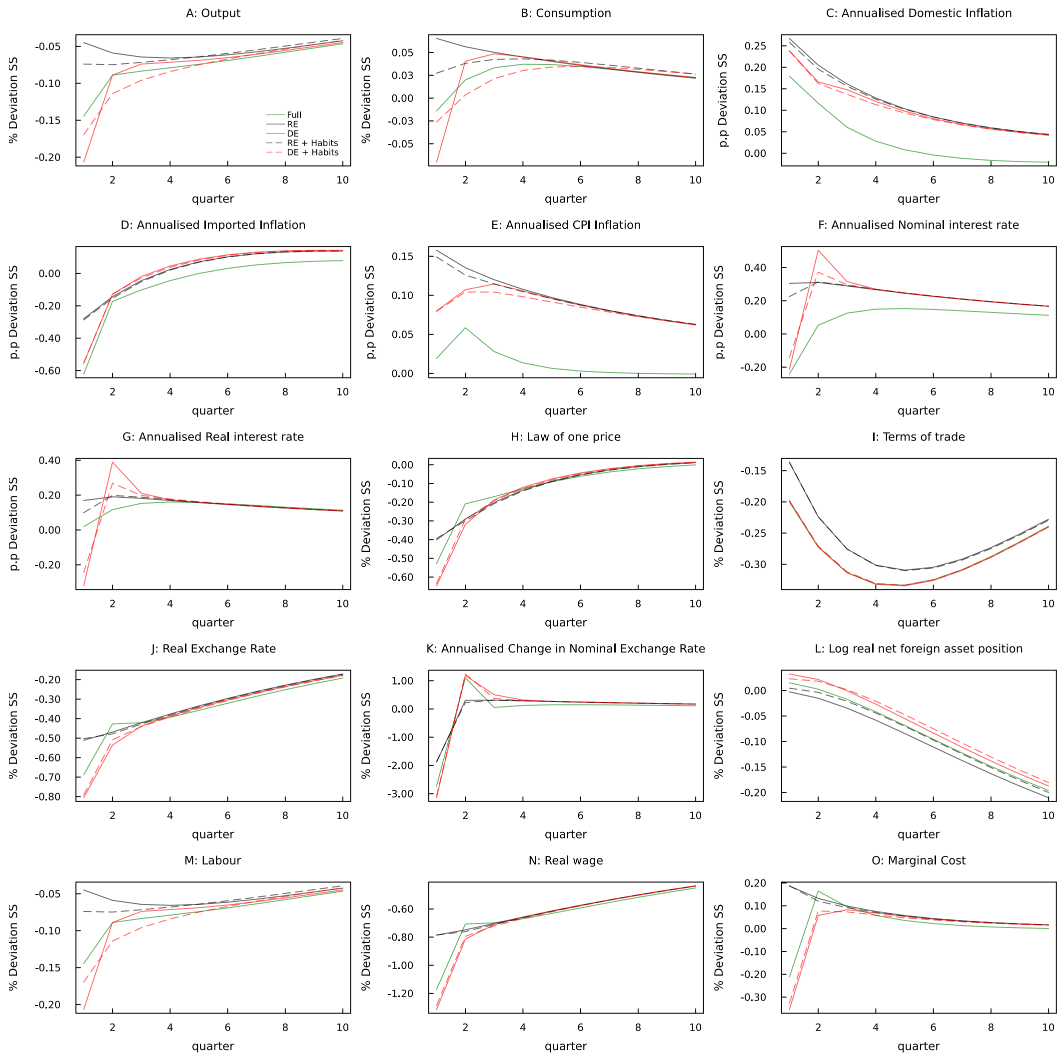


Figure 1.17: Habits and Preference shock.
Each panel depicts the response of the variables to a preference shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

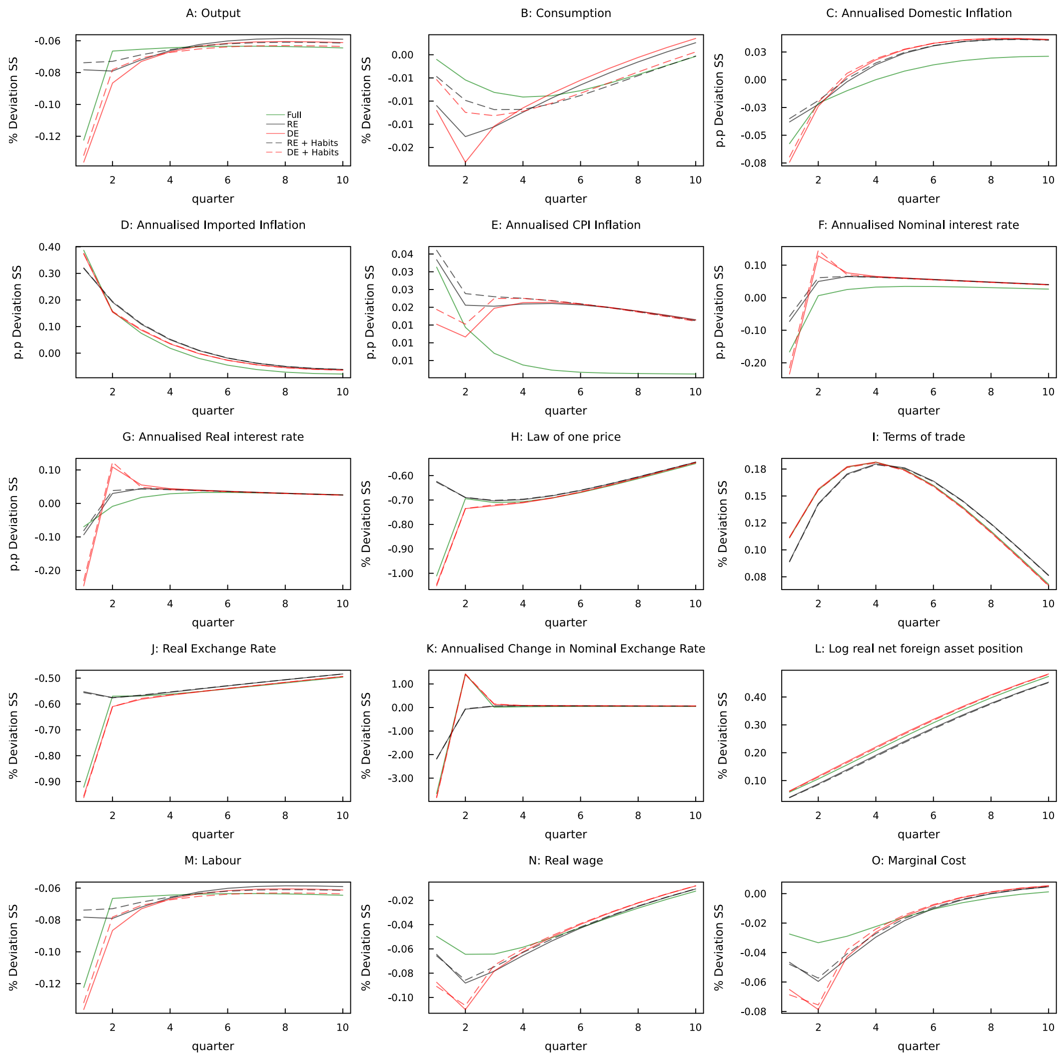


Figure 1.18: Habits and Cost push shock.
Each panel depicts the response of the variables to a cost push shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

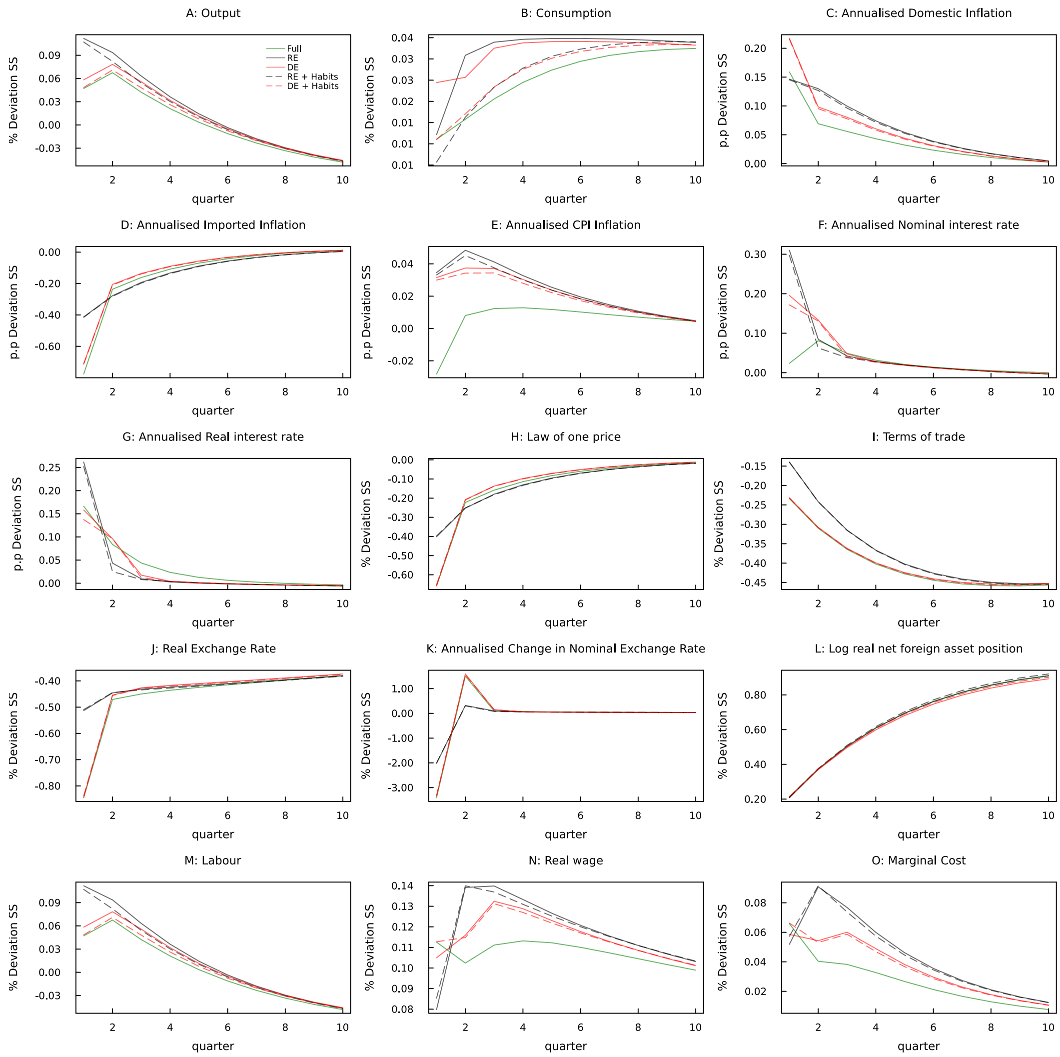


Figure 1.19: Habits and Foreign output shock.
Each panel depicts the response of the variables to a foreign output shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

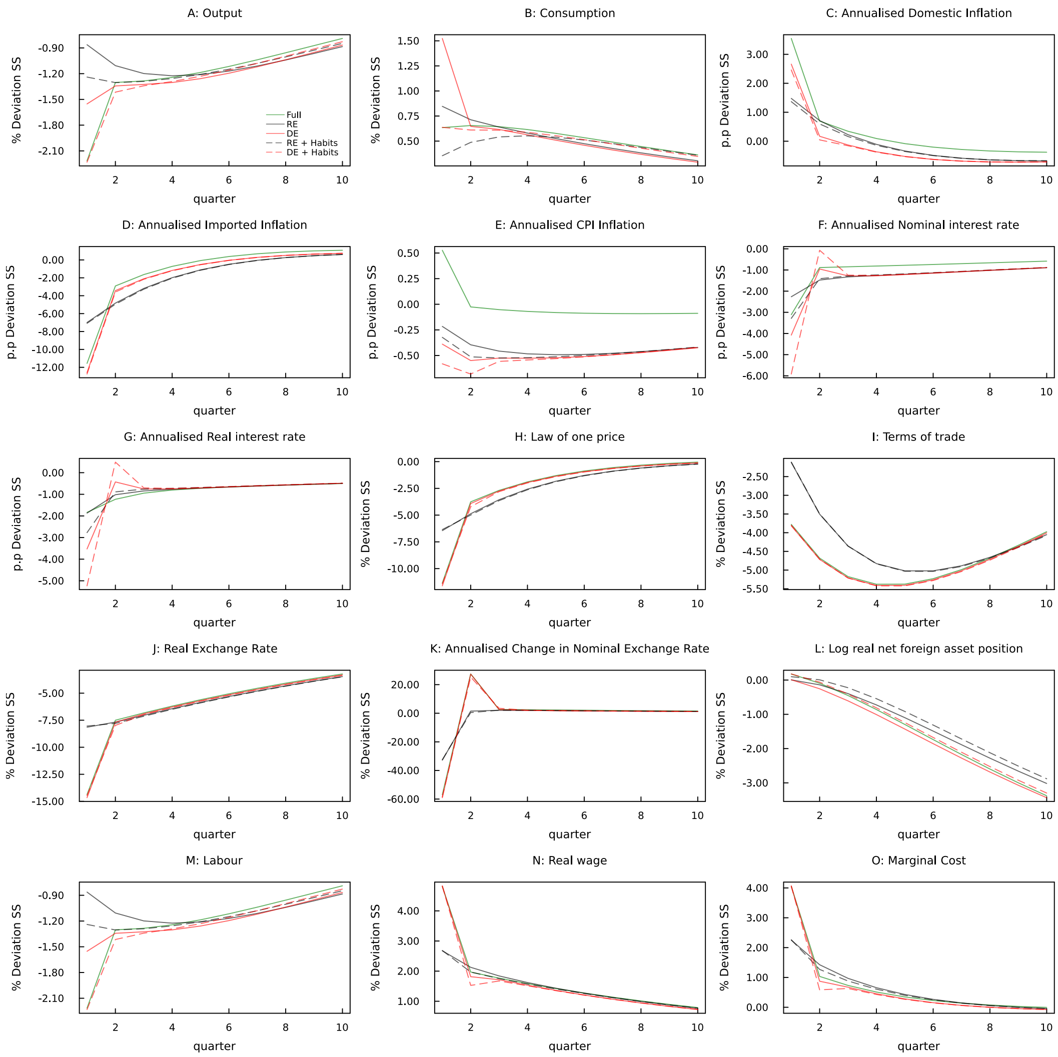


Figure 1.20: Habits and Risk-premium shock.
Each panel depicts the response of the variables to a foreign output shock under different models. Model 1 (solid green line) represent the full model which includes DE and both persistence mechanisms. Model 2 (solid black line) and Model 3 (solid red line) plot the responses of the baseline model where agents are either rational or diagnostic, respectively. Model 4 (dashed black line) exhibits impulse responses when the economy is populated with rational agents and habits are included. The same illustrates Model 5 (dashed red line), though with diagnostic agents.

Chapter 2

A Diagnostic TANK Model for the Housing Market

1 Introduction

During the period spanning from the mid-eighties to the aftermath of the Great Financial Crisis, the U.S. housing market has been defined by its high volatility (Piazzesi & Schneider, 2016). The standard deviations of the real residential investment and the real house price growth rates are 3.452% and 1.723%, respectively, while the real GDP growth rate has a standard deviation of 0.576%. This indicates that residential investment and house price growth rates are six and three times more volatile than GDP, as shown in Figure 2.1. Understanding what drives these volatilities is relevant, given the valuable information that the housing market provides about ongoing changes in economic activity (Chahrour & Gaballo, 2021) and the importance of housing in households' decisions and wealth (Davis & Heathcote, 2005).

Traditional models of the housing sector typically attribute pronounced house price movements to housing preference shocks, but this approach limits the insights offered for policy analysis by overlooking expectation-driven dynamics. For instance, Gelain, Lansing, and Mendicino (2012) suggests that agents' expectations can significantly influence monetary policy responses. Moreover, empirical studies challenge the rationality assumption in the housing market, indicating that expectations are the source of the pronounced fluctuations in the sector. The evidence reveals that housing market expectations strongly track recent observed house price changes (Kuchler, Piazzesi, & Stroebel, 2023; Adam,

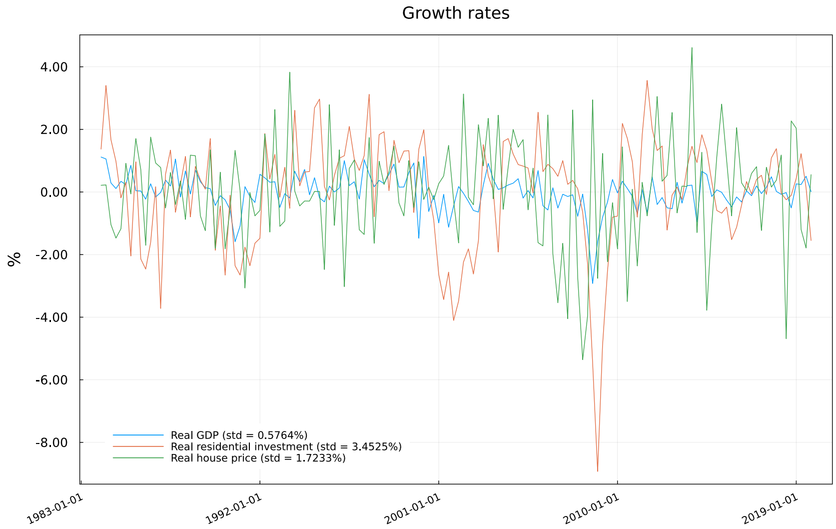


Figure 2.1: Real GDP, real house price and real residential investment in percentage change.

Pfäuti, & Reinelt, 2024), with price expectations showing short-run momentum (Gohl, Haan, Michelsen, & Weinhardt, 2024). Additionally, De Stefani (2021) finds that the risk of a downturn after a long period of growth in house prices is underestimated by consumers, generating predictable errors. Together, these findings position non-rational expectations as a promising explanation for the higher volatility in housing markets, while suggesting potentially different monetary and macro-prudential policy responses.

In this context, I develop a two-agent New Keynesian (TANK) model that incorporates a housing market inspired by Iacoviello and Neri (2010), a banking sector following the framework in Gertler and Karadi (2011), and Diagnostic Expectations (DE), as proposed by Bordalo, Gennaioli, and Shleifer (2018).[1] Diagnostic agents form beliefs influenced by recent (or not so recent) trends. For example, a history of rising (falling) house prices tends to make future prices following the same trend more prominent in diagnostic-agents' minds, but when these projections do not materialise, DE creates feedback loops that amplify optimism or pessimism. By introducing DE into this model, I aim to address the observed volatility in the housing market without relying on large housing preference shocks. The main contribution of this chapter, therefore, is to show that DE can account for approximately thirty to fifty percent of this volatility while achieving a better empirical fit with a smaller housing preference shock. This offers a more compelling alternative to traditional explanations that attribute unexplained demand changes to large shifts in housing preferences. This result has significant implications for policy, as understanding the dynamics of expectations-driven volatility in the housing market enables policymak-

[1]The authors build DE on Kahneman and Tversky (1972) concept of representativeness. This describes a judgemental process where the most distinguished characteristic of an event plays the main role in a human's mind when assigning probabilities.

ers to better anticipate risks of speculative bubbles and refine interventions to address potential financial instability and resource misallocation.

Building on this, I also explore an alternative structure for the memory used by agents forming DE. Instead of relying solely on the immediate past, I extend the framework to examine how more distant memories might affect and shape agents' background context. Following Bordalo et al. (2018), I introduce a slow-moving reference by defining representativeness to be a mixture of current and past likelihood ratios. This approach differs from Bianchi, Ilut, and Saijo (2024), since they consider a weighted average of lagged expectations as a memory-driven benchmark. To the best of my knowledge, this is the first attempt to incorporate and study DE with a slow-moving structure as a reference in a model that features heterogeneous agents, a housing sector and a banking sector. Under this set up, I derive that diagnostic agents using a slow-moving memory misperceive the shock as an ARMA(1,S) process, where S represents the number of periods used to form the memory.

I calibrate and estimate three models for the U.S. economy, using recent advancements in macroeconomic model estimation by Herbst and Schorfheide (2014): one with rational agents, one with diagnostic agents featuring short memory, and one with diagnostic agents and distant memory. The results support the role of DE in driving the U.S. housing market dynamics. Compared to traditional Rational Expectations (RE) models, DE reduce the standard deviation of the housing preference shock by at least one-third, suggesting that DE could be a viable alternative to the "catchall of all the unmodeled disturbances that can affect housing demand" (Iacoviello & Neri, 2010, p. 150). Though the evidence favours the DE model with a one-quarter lag reference, two key takeaways emerge from extending the memory horizon from short to long-term: first, the prominence of recent events in shaping agents' expectations, and second, that most attention beyond this period centres between quarters three and ten.

In addition, a historical shock decomposition analysis indicates that the shock transmission mechanism in the economy remains stable regardless of the agents' expectations formation process. The difference lies in the more volatile expectations intrinsic to DE, which amplify the impact of shocks without altering their transmission through the economy. I also examine the influence of DE on the economy using impulse responses. In general, DE with short and distant memory share similar characteristics: initial overreactions, greater persistence, and pronounced fluctuations. The extrapolation of shocks explains the initial overreactions and subsequent reversals observed in both DE models, but other features are specific to each framework. In the DE model with a one-quarter reference, the economy's rigidities propagate the initial overreaction. However, in the DE model with twelve-quarters slow-moving reference, more pronounced fluctuations emerge as agents may remain overly optimistic (pessimistic) due to the longer span memory in their expectation formation process.

A counterfactual analysis further disentangles the propagation and amplification mechanism of DE. When diagnostic agents, who typically base their beliefs on either recent or distant information, suddenly become rational, the model struggles to replicate house price volatility. This challenge provides further evidence that it is the expectations mechanism, particularly DE, that drives the cycles in the housing market. This chapter thereby contributes to a growing body of research advocating for models that integrate expectation formation more closely aligned with observed economic behaviour, moving beyond preference shocks to examine the dynamics of households and market expectations.

Related literature

This chapter is linked to recent articles that incorporate DE in macroeconomic models. One group of authors incorporates DE in macro-finance environments. Bordalo et al. (2018) find that such an extended macroeconomic model captures the empirical findings regarding credit cycles. Bordalo, Gennaioli, Shleifer, and Terry (2021) and Maxted (2024) combine DE in real business cycle models with financial frictions. Their main results are a greater variability in the macroeconomy and the ability to replicate financial boom-bust credit cycles aspects, as well as the countercyclicality of credit spreads. More recently, L'Huillier, Singh, and Yoo (2024) derive a general framework to incorporate DE in linear models and show that DE are a viable behavioural alternative to generate fluctuations in business cycle models with shocks of more realistic size.

While previous studies explored DE within a one-period reference framework, Bianchi et al. (2024) focus on distant memory and find that DE generate rich dynamics, characterised by significant persistence and sudden changes in the way shocks propagate. Along this line, and closely related to the work here, Qi (2021) and Bounader and Elekdag (2024) introduce DE and distant memory in New Keynesian models with heterogeneity in agents, a housing market and financial frictions. The first author finds higher persistence and significant responses from house prices to a total factor productivity shock in a TANK model. The second authors contribute showing that DE and financial frictions reinforce shock amplification, especially after demand shocks. My work builds on these efforts and contributes to this literature by empirically estimating the diagnostic parameter and the weights assigned to past references. I also incorporate a banking sector, which introduces additional frictions and channels through which expectations can lead to a more volatile economy.

This chapter also contributes to the literature on housing market dynamics in macroeconomic models, particularly focusing on two main approaches: (i) housing preference shocks and (ii) excess volatility driven by expectations arising from rationality departures. The first group of authors concentrates on understanding the nature of shocks and movements in the housing market, as well as the effects that such variations have on the economy. The work of Iacoviello and Neri (2010) represents the cornerstone of this literature. They find that a housing demand shock can explain at least a quarter of housing market fluctuations, estimating the standard deviation of the housing preference shock to be about 4%. In their words, this shock is "spontaneous, primitive and their interpretable characteristics are questionable" (p. 158).

Other authors have estimated similar models for different countries. For example, Gerali, Neri, Sessa, and Signoretti (2010) use European data and estimate a standard deviation of housing preference shock of around 7%, while Funke and Paetz (2013) find that a comparable shock in the Hong Kong housing market has a standard deviation of roughly 10%. Similarly, Mendicino and Punzi (2014) analyse the relative importance of the housing preference shock in a theoretical model, concluding that it accounts for 70% of the volatility in house prices. More recently, Ge, Li, Li, and Liu (2022) examine the Chinese housing market and find that a housing preference shock standard deviation of approximately 7% explains over 80% of the sector's volatility. Lambertini, Mendicino, and Punzi (2017) augment the model of Iacoviello and Neri (2010) with news shocks and shows that this extended model can account for boom-bust patterns in the housing sector. In contrast to these studies, my main contribution here is to provide a more comprehensive alternative to the "catchall of all the unmodeled disturbances that can affect housing demand" (Iacoviello & Neri, 2010, p. 150), captured by the housing preference shock.

In this chapter, I introduce a deviation from rational expectations, aligning with the second group of literature that explores behavioural alternatives such as adaptive expectations and learning. Some researchers, including Gelain et al. (2012), and Granziera and Kozicki (2015), argue that adaptive expectations can increase the volatility of the housing market due to overoptimism and overreaction to fundamentals. Moreover, adaptive expectations have been successful in generating momentum and volatility in models of the stock market, characteristics also present in the housing sector. However, adaptive expectations are an ad hoc, not micro-founded approach, making DE a better choice. In addition, DE has already shown its ability to capture the run-ups and sharp decline behaviour present in the financial markets (Bordalo et al., 2018; Bordalo, Gennaioli, Kwon, & Shleifer, 2021).

Alternatively, including learning suggests that individuals form mechanical backward-looking rules for belief updating. Chahrour and Gaballo (2021), Caines (2020) and Gandré (2022) provide evidence supporting the inclusion of learning about house prices as an amplification and propagation mechanism that helps to account for the dynamics of macro variables, as well as credit and housing. However, this approach assumes that agents do not understand the true data-generating process. In contrast, DE have three advantages over mechanical models of non-rational beliefs: it is forward-looking (immunity to the Lucas (1976) critique), it better accounts for measured expectations of financial analysts and macro forecasters, and its diagnostic parameters have been estimated in some data sets (Bordalo, Gennaioli, Shleifer, & Terry, 2021; L'Huillier et al., 2024; Bianchi et al., 2024). Additionally, DE have been successfully applied not only in macro and finance settings but also in, for example, modelling social stereotypes (Bordalo, Coffman, Gennaioli, & Shleifer, 2016). Therefore, these factors provide a basis for incorporating DE into a macroeconomic model to analyse the housing market behaviour, which represents another contribution of this chapter.

Structure of the chapter

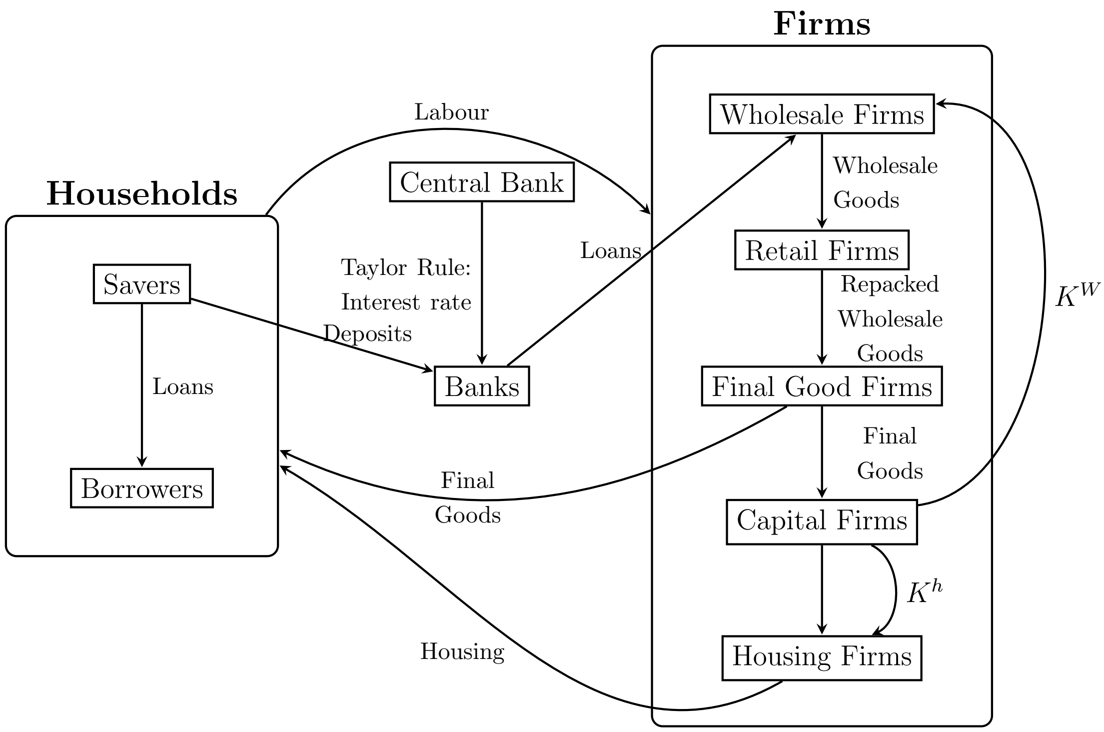
The rest of the chapter is structured as follows. In Section 2, I present the model. Section 3 explains how I include DE and solve the resulting model. The calibration and estimation of the parameters are outlined in Section 4. Section 5 discusses the quantitative results. A counterfactual analysis is done in Section 6, and Section 7 concludes.

2 Model

The basic structure of the model is similar to Iacoviello (2005), Iacoviello and Neri (2010) and Gelain et al. (2012), although I extend it in several ways. First, I include capital producers that sell part of the total capital stock to wholesale firms and rent the rest to housing firms. This allows me to derive an explicit expression for the real price of capital, as well as for the rental rate of capital in the housing sector (Gambacorta & Signoretti, 2014). Second, to model the housing market price and quantity dynamics, I introduce a housing production sector that produces houses using capital and labour services (Iacoviello & Neri, 2010). Finally, I incorporate financial frictions using a banking sector as in Gertler and Karadi (2011). In this section, I present the derivations under RE, whereas in a later section I show how to modify the model to introduce DE.

The model summarised in Figure 2.2 consists of two types of households: patient and impatient, each of mass $1-n$ and n, respectively. The patient households are the savers in the economy. They provide liquidity to impatient households, borrowers, in the form of loans. There are five types of firms: (i) wholesale firms producing wholesale goods, (ii) retailer firms repackaging wholesale goods and introducing a price rigidity *à la* Calvo, (iii) a final good firm producing its output using goods from retailers as inputs, (iv) housing firms producing houses with labour and capital as inputs, and (v) capital good firms

Figure 2.2: Economy model



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combining undepreciated capital and the final good to update and produce new capital. The model also features a banking sector as in Gertler and Karadi (2011). These banks act as financial intermediaries between patient households' deposits and wholesale firms' loans.[2] Finally, there is a Central Bank that sets the nominal interest rate following a simple Taylor-type rule. The model includes habit formation in consumption, investment adjustment costs, and nominal price rigidities. Time is discrete and one period in the model represents one-quarter.

2.1 Households

The economy is populated by two types of households, patient and impatient, denoted with subscripts "p" and "i", respectively. They consume final goods, buy housing and supply labour. The patient households save in the form of deposits in banks and lend money to impatient households, who borrow using their housing as collateral.

[2]This version of the model does not allow for arbitrage between loan and deposit interest rates, this means the banks do not intermediate between households. The main reason behind this choice is to keep the banking problem easy to track. However, in a future version, banks will not only serve as intermediaries between patient households and firms, they will also mediate transactions with impatient households.

2.1.1 Patient households

A representative patient household derives utility from consumption, $c_{p,t}$, and housing, $h_{p,t}$, and disutility from labour $n_{p,t}$. She discounts future utility flows by β_p and her expected discounted lifetime utility is:

$$U_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \left[log(c_{p,t} - \gamma c_{p,t-1}) + \Gamma_t \nu_p^h log(h_{p,t}) - \nu_p^n \frac{n_{p,t}^{1+\varphi}}{1+\varphi} \right], \quad (2.1)$$

where Γ_t is a housing preference shock that follows the AR(1) process $log(\Gamma_{t+1}) = \rho_\Gamma log(\Gamma_t) + \sigma_{\epsilon^\Gamma} \epsilon_{t+1}^\Gamma$, with $\rho_\Gamma \in (0,1)$ and $\epsilon_{t+1}^\Gamma \sim i.i.d.[0, \sigma_{\epsilon^\Gamma}^2]$. The habit formation parameter is $\gamma \in$ (0,1) and ν_p^h and ν_p^n govern the patient household's utility from housing and labour, respectively. The parameter φ is the inverse elasticity of the labour supply.

The patient household maximises her utility subject to the following budget constraint:

$$c_{p,t} + q_t[h_{p,t} - (1-\delta_h)h_{p,t-1}] + d_t^B + d_t^l = \frac{d_{t-1}^B R_{t-1}^d}{\pi_t} + \frac{d_{t-1}^l R_{t-1}^l}{\pi_t} + w_t n_{p,t} + \Pi_{f,t} + \Pi_{B,t}. \quad (2.2)$$

q_t is real house prices, δ_h is the rate at which housing depreciates, and w_t is the real wage from supplying labour. The term d_{t-1}^B represents the deposits held by the patient household in the bank at the end of time $t-1$, which yield a riskless gross nominal return of R_{t-1}^d between periods $t-1$ and t. d_{t-1}^l represents loans that patient households extend to impatient households, yielding a gross nominal return of R_{t-1}^l. π_t is the gross inflation rate and $\Pi_{f,t}$ and $\Pi_{B,t}$ are transfers of profits that households receive from firms and banks.

The resulting first order conditions of the patient household's maximisation problem with respect to $c_{p,t}$, $n_{p,t}$, $h_{p,t}$, d_t^B and d_t^l are:

$$\lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p \gamma}{(c_{p,t+1} - \gamma c_{p,t})}, \quad (2.3)$$

$$\nu_p^n n_{p,t}^{\varphi} = w_t \lambda_{p,t}, \quad (2.4)$$

$$\lambda_{p,t} q_t = \frac{\Gamma_t \nu_p^h}{h_{p,t}} + \beta_p \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{p,t+1} \Big], \quad (2.5)$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \left[\lambda_{p,t+1} \frac{R_t^d}{\pi_{t+1}} \right], \quad (2.6)$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \left[\lambda_{p,t+1} \frac{R_t^l}{\pi_{t+1}} \right], \quad (2.7)$$

where $\lambda_{p,t}$ denotes the marginal utility of consumption.

2.1.2 Impatient households

A representative impatient household also receives utility from consumption, $c_{i,t}$, and housing, $h_{i,t}$, and disutility from labour, $n_{i,t}$. She discounts future utility flows by β_i, which is smaller than the patient household's discount factor, β_p, and her expected discounted lifetime utility is:

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \left[log(c_{i,t} - \gamma c_{i,t-1}) + \Gamma_t \nu_i^h log(h_{i,t}) - \nu_i^n \frac{n_{i,t}^{1+\varphi}}{1+\varphi} \right], \tag{2.8}$$

where $\gamma \in (0,1)$ and φ are the same habit formation and inverse elasticity of labour supply parameters as for the patient household. ν_i^h and ν_i^n govern the utility of housing and labour for the impatient household. She faces the same housing preference shock Γ_t, and maximises her utility subject to the following budget constraint:

$$c_{i,t} + q_t(h_{i,t} - (1-\delta_h)h_{i,t-1}) + \frac{l_{t-1}R_{t-1}^l}{\pi_t} = w_t n_{i,t} + l_t. \tag{2.9}$$

She also faces a limit on her liabilities during period t as a fraction χ of her expected housing value in period $t+1$:

$$l_t \leq \frac{\chi}{R_t^l} \mathbb{E}_t[q_{t+1}\pi_{t+1}h_{i,t}]. \tag{2.10}$$

Loans obtained by the impatient households from the patient households between periods $t-1$ and t are denoted l_{t-1}. The condition $(1-n)d_t^l = nl_t$ must be satisfied for the loan market to clear, as it implies that in aggregate loans extended by patient households correspond to loans obtained by impatient households. The parameter χ denotes the loan-to-value ratio and measures the liquidity degree of housing.

The impatient household's optimisation problem leads to the following first-order conditions with respect to $c_{i,t}$, $n_{i,t}$ $h_{i,t}$ and l_t:

$$\lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_b \gamma}{(c_{i,t+1} - \gamma c_{i,t})}, \tag{2.11}$$

$$\nu_i^n n_{i,t}^{\varphi} = w_t \lambda_{i,t}, \tag{2.12}$$

$$\lambda_{i,t} q_t = \frac{\Gamma_t \nu_i^h}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R_t^l} \mathbb{E}_t [q_{t+1}\pi_{t+1}], \tag{2.13}$$

$$\lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \left[\lambda_{i,t+1} \frac{R_t^l}{\pi_{t+1}} \right], \tag{2.14}$$

where $\lambda_{i,t}$ is the marginal utility of consumption and $\mu_{i,t}$ is the Lagrange multiplier on the collateral constraint (2.10).

2.2 Firms

Firms in this economy are owned by the patient households. There are five types of firms: wholesale firms, retail firms, final good producers, capital producers, and housing producers.

2.2.1 Wholesale firms

Wholesale firms buy capital K^W_{t-1}, at the end of time $t-1$, from capital producers, and hire labour, N^W_t, from patient and impatient households. During period t, they produce wholesale goods, Y^W_t, which they sell to retail firms, using a Cobb-Douglas production function:

$$Y^W_t = A_t {N^W_t}^{1-\alpha} {K^W_{t-1}}^{\alpha}, \tag{2.15}$$

where A_t is total factor productivity shock in the wholesale goods sector. This shock obeys an AR(1) process $log(A_{t+1}) = \rho_A log(A_t) + \sigma_{\epsilon^A}\epsilon^A_{t+1}$, where $\rho_A \in (0,1)$ and $\epsilon^A_{t+1} \sim i.i.d.[0, \sigma^2_{\epsilon^A}]$.

At the end of period t, wholesale firms obtain funds from the banking sector to finance the acquisition of capital K^W_t. In order to do so, they take loans, S_t, equal to the quantity of capital acquired, K^W_t, and price each at the unit price of capital q^K_t as detailed in Gertler and Karadi (2011).

$$q^K_t K^W_t = q^K_t S_t. \tag{2.16}$$

After finishing production in period $t-1$, wholesale firms sell their undepreciated capital in the open market. These firms' profits are the value of their production plus the value of their capital stock left over, net of their total costs which include labour and capital expenses.[3] The profit maximisation problem is:

$$\max_{N^W_t, K^W_{t-1}} \left[P_{m,t} Y^W_t + (1-\delta_k) q^K_t K^W_{t-1} - R^K_t q^K_{t-1} K^W_{t-1} - w_t N^W_t \right],$$

subject to the production function. $P_{m,t}$ is the relative intermediate output price, R^K_t is the state-contingent required gross return on capital during time t. The first-order

[3] Gertler and Karadi (2011) specify that wholesale firms do not face any frictions in the process of obtaining funding from banks. The authors also state that "wholesale firms are able to offer the banks a perfectly state-contingent security, which is best thought of as equity (or perfectly state-contingent debt)"(p. 23).

conditions for this firm, i.e. the demands for labour and capital, are:

$$w_t = P_{m,t}(1-\alpha)A_t\left(\frac{K^W_{t-1}}{N^W_t}\right)^{\alpha}, \tag{2.17}$$

$$q^K_{t-1}R^K_t = r^K_t + (1-\delta_k)q^K_t, \tag{2.18}$$

where $r^K_t = P_{m,t}\alpha A_t\left(\frac{N^W_t}{K^W_{t-1}}\right)^{1-\alpha}$ is the capital rental rate. Solving for the labour-to-capital ratio, replacing it in equation (2.17) and equating the results, I obtain an expression for marginal costs:

$$mc_t = \frac{1}{A_t}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K_t}{\alpha}\right)^{\alpha}. \tag{2.19}$$

2.2.2 Final good firms

Final-good firms aggregate the output of retail firms $y_t(j)$ according to a Dixit-Stiglitz production technology:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}}dj\right]^{\frac{\epsilon}{\epsilon-1}}.$$

These firms sell the final product in a perfectly competitive market. Y_t represents the final good, $y_t(j)$ denotes the $j'th$ retail-firm input used in the production of the final good, and ϵ denotes the elasticity of substitution between any two inputs, assumed to be greater than 1. This firm's profit maximisation is a static problem, and from its first-order condition I obtain the demand equation for each input as:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}Y_t.$$

Since this final good producing firm is competitive, it makes zero profit, and its price is a function of the inputs' prices, i.e. an aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon}dj\right]^{\frac{1}{1-\epsilon}}.$$

2.2.3 Retail firms

Retailers simply repackage intermediate output, that is, wholesale production. It takes one intermediate output unit to make a unit of retail output. The marginal cost is thus the relative wholesale output price $P_{m,t}$. The retailer seeks to maximise its profit solving:

$$\max_{P_t(j)} P_t(j) \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t - mc_t \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t.$$

After optimising with respect to the choice variable $P_t(j)$, I obtain:

$$P_t(j) = \frac{\epsilon}{\epsilon - 1} mc_t.$$

This condition shows the market power that these firms have since they set their price, when there is no price rigidity, as a mark-up of the marginal cost. However, in the presence of some price rigidity, this result changes. Here I assume a price setting style *à la* Calvo. In each period, the firms receive a random draw from a Bernoulli distribution. This indicates that, with a probability $1 - \theta$, $\theta \in [0, 1]$, the firm will be able to change its price. Conversely, with a probability θ, the firm will not be able to set a new price, keeping it unchanged.

$$P_t(j) = P_{t-1}(j), \forall j \in [0, \theta),$$

$$P_t(j) = P_t^*(j), \forall j \in [\theta, 1],$$

where $P_t^*(j)$ is determined by solving the maximisation problem:

$$\max_{P_t^*(j)} V_t(j) = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \left\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{\epsilon} Y_{t+i} \right] \right\}.$$

The result determines that retailers who have obtained a successful draw will set their prices as a constant mark-up on an expression related to their expected discounted nominal total costs, relative to an expression related to their expected discounted real output.

$$P_t^*(j) = \frac{\epsilon}{\epsilon - 1} \left[\frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} mc_{t+i} P_{t+i}^{\epsilon} Y_{t+i}}{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} P_{t+i}^{\epsilon - 1} Y_{t+i}} \right]. \tag{2.20}$$

The above equation does not depend on j, so every retail firm that can set its price in period t will choose the same price. Moreover, in the limiting case of no price rigidity, the familiar expression of a firm's optimal price as a constant mark-up on real marginal costs is obtained. Given the previous result and the price rigidity mechanism, the Dixit-Stiglitz aggregate domestic price index evolves as follows:

$$P_t^{1-\epsilon} = (1 - \theta)(P_t^*(j))^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}.$$

From the last equation and defining the gross inflation as $\left(\frac{P_t}{P_{t-1}}\right) = \pi_t$, I obtain:

$$\pi_t^{1-\epsilon} = (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon} + \theta\left(\frac{P_{t-1}}{P_{t-1}}\right)^{1-\epsilon}.$$

Solving for gross inflation reveals the relationship between inflation and the aggregate price level. Inflation turns out to be a function of the relative price, π_t^*, between the price optimally set by the firms, P_t^*, and the price of the final good.

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\pi_t^*\right)^{1-\epsilon}. \tag{2.21}$$

2.2.4 Capital good firms

Patient households own capital good firms. During period t, they transform the output in the form of investment, I_t, and undepreciated capital, $(1-\delta_k)K_{t-1}$, to produce new capital, K_t. Part of this new capital, K_t^W, is sold to wholesale firms, at the price q_t^K. The rest, K_t^h, is rented to housing firms at the rental rate r_t^h. The undepreciated capital, thus, is equal to the undepreciated capital rented to housing firms and the undepreciated capital bought from wholesale firms.

The representative capital producer maximises its expected discounted profits. At the end of period t, this firm receives income from selling capital to wholesale firms and renting capital to housing firms, while paying the costs of gross investment and undepreciated capital purchases from wholesale firms.

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Big[q_t^K K_t^W - q_t^K(1-\delta_k)K_{t-1}^W + r_t^{K,h} K_t^h - I_t\Big]. \tag{2.22}$$

The maximisation problem is subject to the total capital law of motion and the definition of aggregate capital stock.

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t, \tag{2.23}$$

$$K_t = K_t^W + K_t^h, \tag{2.24}$$

where δ_k is the capital depreciation rate and ψ is a parameter measuring the cost paid for adjusting investment. The law of motion implies that old capital can be converted one-to-one into new capital, while the transformation of general output is subject to a quadratic adjustment cost.

The optimality conditions with respect to K_t^W, K_t^h and I_t are:

$$q_t^K - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)q_{t+1}^K = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}, \tag{2.25}$$

$$r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}, \tag{2.26}$$

$$1 = \lambda_{K,t}\Big[1-\frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}}-1\Big)^2-\psi\Big(\frac{I_t}{I_{t-1}}-1\Big)\Big(\frac{I_t}{I_{t-1}}\Big)\Big]+\beta_p\psi\mathbb{E}_t\Big[\frac{\lambda_{p,t+1}}{\lambda_{p,t}}\lambda_{K,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big(\frac{I_{t+1}}{I_t}-1\Big)\Big], \tag{2.27}$$

where $\lambda_{K,t}$ denotes the Lagrange multiplier on the capital law of motion.

2.2.5 Housing firms

At time t, housing firms produce new houses, I_t^h, using a Cobb-Douglas production technology. This process requires capital, K_{t-1}^h, rented from the capital producer, and labour, N_t^h, hired from patient and impatient households at the real wage w_t.

$$I_t^h = Z_t {N_t^h}^{1-\mu_h} {K_{t-1}^h}^{\mu_h}, \tag{2.28}$$

where μ_h is the income share of the capital used to produce new housing. Z_t is total factor productivity in the housing sector. It obeys an AR(1) process $log(Z_{t+1}) = \rho_Z log(Z_t) + \sigma_{\epsilon^Z}\epsilon_{t+1}^Z$, where $\rho_Z \in (0,1)$ and $\epsilon_{t+1}^Z \sim i.i.d.[0, \sigma_{\epsilon^Z}^2]$.

Housing firms maximise the difference between their earnings from selling new houses and their costs in wages and rent. Denoting the price of new houses by q_t, the representative housing producer maximisation problem is:

$$\max_{N_t^h, K_{t-1}^h} \big[q_t I_t^h - r_t^{K,h} K_{t-1}^h - w_t N_t^h\big],$$

subject to the production technology I_t^h.

The first-order conditions, with respect to N_t^h and K_{t-1}^h, yield the following demands for labour and capital:

$$w_t = (1-\mu_h) q_t \frac{I_t^h}{N_t^h}, \tag{2.29}$$

$$r_t^{K,h} = \mu_h q_t \frac{I_t^h}{K_{t-1}^h}. \tag{2.30}$$

2.3 Banks

This sector closely follows the setting proposed by Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In every period, each bank obtains funds in the form of deposits $D_{\tau,t}$ from patient households, which pay a nominal gross interest rate R_t^d in the next period. The banks transform these funds into loans for wholesale firms. They take the form of equities $S_{\tau,t}$, which yield an ex-post return R^K_{t+1}.

Each bank τ has wealth -or net worth- $NW_{\tau,t}$ at the end of period t, and its balance sheet is given by:

$$q^K_t S_{\tau,t} = NW_{\tau,t} + D_{i,t}. \tag{2.31}$$

Equation 2.31 states that a bank finances loans with newly issued deposits and net worth. Moreover, $D_{\tau,t}$ represents a bank's debt, while $S_{\tau,t}$ a bank's asset. Thus, $NW_{\tau,t}$ will be its equity capital, which evolves over time as the difference between expected earnings on loans to wholesale firms and interest payments on borrowing from patient households[4]:

$$\begin{aligned} NW_{\tau,t+1} &= R^K_{t+1} q^K_t S_{\tau,t} - R^d_t D_{\tau,t}, \\ &= \left(R^K_{t+1} - R^d_t\right) q^K_t S_{\tau,t} + R^d_t NW_{\tau,t}. \end{aligned} \tag{2.32}$$

From expression (2.32), one can appreciate that net worth's growth, above the riskless return R^d_t, depends on the risk premium $(R^K_{t+1} - R^d_t)$ and total loans. Defining $\beta^i_B \frac{\lambda_{p,t+i}}{\lambda_{p,t}}$ as the stochastic discount factor a banker τ at time t applies to earnings at time $t+i$, where $\beta_B = \beta_p \geq \beta_i$ because patient households own the banks, the bank will refuse to fund any loans with a discounted return smaller than the discounted cost of deposits. Therefore, the following inequality must apply for the bank to operate:

$$\beta^i_B \mathbb{E}_t \Big[\frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Big(R^K_{t+1} - R^d_t \Big) \Big] \geq 0, i \geq 0.$$

Gertler and Karadi (2011) summarise this stating: "as long as the bank earns a risk adjusted return greater than or equal to the return the household can earn on its deposits, it pays for the banker to keep building assets until exiting the industry"(p. 20).

Each bank has a probability σ to continue functioning until the next period and a probability to exit $1-\sigma$. This prevents the bank from overcoming its financial constraint by saving indefinitely. In addition, it is assumed that the number of banks entering and exiting the sector is equal, keeping the total constant.

In each period, a banker's objective is to maximise her expected final wealth:

$$V^B_{\tau,t} = \max \mathbb{E}_t \sum_{i=0}^{\infty} (1-\sigma)\sigma^i \beta^{i+1}_B \frac{\lambda_{p,t+i}}{\lambda_{p,t}} NW_{\tau,t+i}, \tag{2.33}$$

subject to its balance sheet (2.31), equity capital law of motion (2.32) and an incentive constraint. This incentive constraint arises from introducing a moral hazard problem to

[4]Gertler and Karadi (2011) assume that banks can only accumulate net worth by retained earnings and do not issue new assets.

limit the bank's ability to issue deposits. Following Gertler and Kiyotaki (2010), at the beginning of a period and after the bank has accepted deposits, it has two options: (i) divert a fraction ζ of its assets to the patient households or (ii) hold its assets until the next period when payoffs are realised, and then pay its deposit obligations.[5] If the bank chooses the first option, it closes, following the default on its debt. The bank will need to afford the costs coming from creditors reclaiming their remaining fraction $(1-\zeta)$ of funds. Therefore, due to the risk that a bank may default on its debts, creditors will be reluctant to lend large amounts to the bank at the beginning of each period. This creates friction and acts as an incentive constraint for the bank when trying to obtain funds.

$$V^B_{\tau,t} \geq \zeta(q^K_t S_{t,\tau}). \tag{2.34}$$

The condition (2.34) suggests that the bank will refrain from diverting funds as long as its franchise value is greater than or equal to the portion it can divert. Thus, I re-write the bank's problem equation (2.33) in a Bellman equation form as:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \{(1-\sigma)NW_{\tau,t} + \sigma \max V^B_{\tau,t+1}(NW_{\tau,t+1})\}, \tag{2.35}$$

which is subject to:

$$q^K_t S_{\tau,t} = NW_{\tau,t} + D_{\tau,t},$$
$$NW_{\tau,t+1} = \left(R^K_{t+1} - R^d_t\right) S_{\tau,t} + R^d_t NW_{\tau,t},$$
$$V^B_{\tau,t} \geq \zeta(q^k_{t,f} S_{\tau,t}).$$

Assuming that the value function $V^B_{\tau,t}$ is linear in $NW_{\tau,t}$, that is, $V^B_{\tau,t} = \nu^B_t NW_{\tau,t}$, where ν^B_t depends only on aggregate quantities; and defining ξ_t as the Lagrange multiplier on the incentive constraint, the first-order conditions for $S_{\tau,t}$ and $NW_{\tau,t}$ are:

$$\frac{\xi_t \zeta}{1+\xi_t} = \mathbb{E}_t \big[(1-\sigma+\sigma\nu^b_{t+1})\left(R^K_{t+1} - R^d_t\right)\big], \tag{2.36}$$

$$\frac{1}{1+\xi_t} = \mathbb{E}_t \big[(1-\sigma+\sigma\nu^b_{t+1}) R^d_t\big], \tag{2.37}$$

where equation (2.36) makes the marginal benefit from increasing assets and the marginal cost of tightening the incentive constraint equal. Defining the bank's net worth adjusted marginal value as $\Omega_{\tau,t+1} = (1-\sigma+\sigma\nu^b_{t+1})$, I re-express the value function:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{ \Omega_{\tau,t+1} \Big[\left(R^K_{t+1} - R^d_t\right) q^K_t S_{\tau,t} + R^d_t NW_{\tau,t} \Big] \Big\}.$$

[5] By assumption, patient households do not deposit funds in the banks they own.

Multiplying and dividing this expression by $NW_{\tau,t}$, I obtain:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Big\{ \Omega_{\tau,t+1} \Big[\Big(R^K_{t+1} - R^d_t \Big) \phi_t + R^d_t \Big] \Big\} NW_{\tau,t}, \tag{2.38}$$

where $\phi_t = \frac{q^K_t S_{\tau,t}}{NW_{\tau,t}}$ and the term between curly brackets is ν^b_t. Therefore, if the incentive constraint is binding, $\nu^b_t = \zeta \phi_t$:

$$q^K_t S_{\tau,t} = \phi_t NW_{\tau,t}. \tag{2.39}$$

Using the result from the previous two equations and after some rearranging, I obtain an expression for the leverage:

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} R^d_t}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} \Big(R^K_{t+1} - R^d_t \Big)}. \tag{2.40}$$

This expression does not depend on any firm-specific factor, making it possible to sum across wholesale firms, obtaining:

$$q^K_t S_t = \phi_t NW_t. \tag{2.41}$$

Finally, I derive a law of motion for NW_t as the sum of the old (existing) and young (new) banks net worth:

$$NW_t = NW_{o,t} + NW_{n,t}. \tag{2.42}$$

Given that a fraction σ of bankers at time $t-1$ survive until time t, $NW_{o,t}$ is:

$$NW_{o,t} = \sigma \Big(R^K_t q^K_{t-1} S_{t-1} - R^d_{t-1} D_{t-1} \Big). \tag{2.43}$$

As I described earlier, new banks receive funds from patient households, following Gertler and Karadi (2011), I assume this transfer equals to a small fraction of the assets intermediated by exiting banks in their final operating period. That is, banks exiting with an i.i.d. probability have assets worth $(1-\sigma)(R^K_t q^K_{t-1} S_{t-1})$, from which a fraction $\omega/(1-\sigma)$ is transferred to the entering banks.

$$NW_{n,t} = \omega (R^K_t q^K_{t-1} S_{t-1}). \tag{2.44}$$

Combining these two conditions, I obtain the following expression for NW_t:

$$NW_t = (\sigma + \omega)(R_t^K q_{t-1}^K S_{t-1}) - \sigma R_{t-1}^d D_{t-1}. \quad (2.45)$$

2.4 Central Bank

To close the model, the Central Bank sets the nominal interest rate, R_t^d, following a Taylor-type rule, which targets inflation and GDP growth stabilisation.

$$\frac{R_t^d}{\bar{R}^d} = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\omega_\pi} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{\omega_y} M_t, \quad (2.46)$$

where the steady state of the policy rate is $\bar{R}^d = (1/\beta_p)$. I follow Iacoviello and Neri (2010) and define GDP as the sum of consumption and investment, both non-residential and residential. That is, $GDP_t = C_t + I_t + \bar{q} I_t^h$, where $\bar{q}$ denotes the steady state value of real housing prices, so that short-run changes in real house prices do not affect GDP growth (Iacoviello & Neri, 2010, p. 132). M_t is a monetary policy shock, which follows an AR(1) process $log(M_{t+1}) = \rho_M log(M_t) + \sigma_{\epsilon^M} \epsilon_{t+1}^M$, where $\rho_M \in (0, 1)$ and $\epsilon_{t+1}^M \sim i.i.d.[0, \sigma_{\epsilon^M}^2]$.

2.5 Market clearing and aggregation

In equilibrium, each household's weighted contribution to consumption, labour and housing will determine the aggregates C_t, N_t and H_t, respectively.

$$C_t = (1 - n)c_{p,t} + (n)c_{i,t}, \quad (2.47)$$

$$N_t = (1 - n)n_{p,t} + (n)n_{i,t}, \quad (2.48)$$

$$H_t = (1 - n)h_{p,t} + (n)h_{i,t}. \quad (2.49)$$

In addition, the amount of total labour demanded by wholesale firms and housing firms should be equal to the total amount of labour supplied by households.

$$N_t = N_t^W + N_t^h. \quad (2.50)$$

Total loans obtained by the impatient households must equal total loans provided by the patient household. Similarly, total deposits in the banking sector needs to equal aggregate deposits from the patient households.

$$(1 - n)d_t^l = nl_t, \quad (2.51)$$

$$D_t = (1-n)d_t^B. \quad (2.52)$$

As specified previously, the total loans issued by the wholesale firms to acquire funding for their capital acquisition must be equal to their demand of capital.

$$S_t = K_t^W. \quad (2.53)$$

From capital producers, the total capital stock should equal the sum of capital supplied to wholesale firms and capital rented to housing firms.

$$K_t = K_t^W + K_t^h. \quad (2.54)$$

And its law of motion is:

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2]I_t. \quad (2.55)$$

New housing or housing investment must also satisfy a law of motion. It establishes that the new housing is equal to the difference between the housing stock at time t net of the undepreciated housing stock from time $t-1$.

$$I_t^h = H_t - (1-\delta_h)H_{t-1}. \quad (2.56)$$

The markets for final goods must clear.

$$Y_t = C_t + I_t. \quad (2.57)$$

In addition, the link between the final good and the wholesale goods is given by[6]:

$$Y_t = \frac{Y_t^W}{\nu_t^j}, \quad (2.58)$$

where ν_t^j is a measure of price dispersion. Finally, I introduce the sum of durable and non-durable goods as GDP:

$$GDP_t = C_t + I_t + \bar{q}I_t^h. \quad (2.59)$$

[6]For the derivation of this condition, see Appendix 2.A.

3 Model solution

In this section, I present the solution method that I use to solve linear models with diagnostic agents. Using this strategy, I aim to obtain a RE representation of the DE model, along similar lines to L'Huillier et al. (2024).

3.1 Including diagnostic expectations

The main difference in this model is that agents are not rational; they are diagnostic. Consequently, when forming expectations, these agents are influenced by a cognitive mechanism that relies on past experiences, either from the near or distant past. This reliance directly impacts the way diagnostic agents assign probabilities to future scenarios, leading to mistakes, corrections, and exaggerated responses. Following Bordalo, Gennaioli, Shleifer, and Terry (2021), I model this departure from RE by assuming that agents misperceive the way the state of the economy evolves over time. Following the shock processes included in the model, I assume that the state of the economy evolves as an AR(1) process, $x_{t+1} = \rho_x x_t + \epsilon_{t+1}$, where $\epsilon_{t+1} \sim \mathcal{N}(\mu, \sigma^2)$ and x_{t+1} has a probability density function (pdf):

$$f(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho x_t)^2}{2\sigma^2}}. \tag{2.60}$$

At time t, diagnostic agents form beliefs about the future state in $t+1$ by recalling past realisations of economic conditions that are at the forefront of their mind. That is, they compare information about the current economic conditions with what they already know or remember about past behaviour. During such a process, they use a distorted density function instead of the rational pdf, as defined by Bordalo et al. (2018):

$$f^{\phi}(x_{t+1}|x_t) = f(x_{t+1}|x_t = \bar{x}_t) \left[\frac{f(x_{t+1}|\bar{x}_t)}{f(x_{t+1}|\rho\bar{x}_{t-1})} \right]^{\phi} Z. \tag{2.61}$$

I denote the realisation of the variable by $\bar{x}_t$, thus the diagnostic distribution depends on realisations of x_t at the current time, $\bar{x}_t$, as well as in the past through the reference event, $\bar{x}_{t-1}$. Here, I assume that the agent only considers the most recent past when forming expectations. Z is a normalising constant and $\phi \geq 0$ is the diagnostic parameter, which embeds the rational case when it is equal to 0. Replacing (2.60) in (2.61), I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left[\frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^2\bar{x}_{t-1})^2}{2\sigma^2}}} \right]^{\phi} Z. \tag{2.62}$$

After simplifying and grouping terms, I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left\{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2} - \frac{1}{2\sigma^2}\phi\left[(x_{t+1}-\rho\bar{x}_t)^2-(x_{t+1}-\rho^2\bar{x}_{t-1})^2\right]\right\}} Z. \tag{2.63}$$

Expanding and re-writing the argument in the exponential:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right] + (\rho\bar{x}_t)^2 \\ + \phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right] \Big\}\Bigg) Z. \end{aligned} \tag{2.64}$$

The constant Z is given by:

$$\begin{aligned} Z = exp\Bigg(-\frac{1}{2\sigma^2}\Big\{ -\phi\left[(\rho\bar{x}_t)^2 - (\rho^2\bar{x}_{t-1})^2\right] + 2\rho\bar{x}_t\phi\left[\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right] \\ + \phi^2\left[\left(\rho\bar{x}_t - \rho^2\bar{x}_{t-1}\right)\right]^2 \Big\}\Bigg). \end{aligned} \tag{2.65}$$

Therefore, after some algebra, the diagnostic pdf when the reference is the recent past is equal to:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{\left[x_{t+1}-\left(\rho\bar{x}_t+\phi\left(\rho\bar{x}_t-\rho^2\bar{x}_{t-1}\right)\right)\right]^2\right\}}. \tag{2.66}$$

Following Gennaioli and Shleifer (2018), Equation (2.66) shows that the diagnostic distribution contains the kernel of a normal distribution with an unchanged variance, σ^2, but a distorted mean. This distortion arises because, instead of relying purely on RE, diagnostic agents overweight recent changes in the economic state, as they rely on the most recent past when forming expectations, leading to a mean shift. Diagnostic expectations can then be expressed as:

$$\mathbb{E}_t^{\theta}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi\left[\mathbb{E}_t(x_{t+1}) - \mathbb{E}_{t-1}(x_{t+1})\right], \tag{2.67}$$

The results in Equation (2.66) can be rewritten in terms of the shock realisation. Using the assumption of x_{t+1} following an AR(1) process, the shock realisation can be obtained as $\epsilon_t = x_t - \rho x_{t-1}$. Substituting this into expression (2.66) yields:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{[x_{t+1}-(\rho\bar{x}_t+\phi\rho\epsilon_t)]^2\right\}}. \tag{2.68}$$

Again, as in Gennaioli and Shleifer (2018) and making use of expression (2.67), this implies the following:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi\rho\epsilon_t. \tag{2.69}$$

This is the key finding. It indicates that when the agents are diagnostic ($\phi > 0$), there is extrapolation in the direction of the shock. This occurs because agents misperceive the shock to exhibit greater persistence than the true data generating process, mistakenly interpreting it as ARMA(1,1) process.

These results can be generalised to the case where remote memories influence the diagnostic agent's reference through a mixture of current and past likelihood ratios.[7] The diagnostic pdf in this case is:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}} \left\{ \left[\prod_{s=1}^{S} \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^s \bar{x}_{t+1-s})^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2}{2\sigma^2}}} \right]^{\alpha_s} \right\}^{\phi} Z, \tag{2.70}$$

where S represents the time span used by the diagnostic agent as memory, while α_s denotes the weights that the agent attaches to present and past representativeness.

Lemma 1: *Using the results for the case in which memory is governed by the most recent past, but now assuming that the agent has a slow-moving reference, the diagnostic pdf in (2.70) is characterised by:*

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi \sum_{s=1}^{S} \alpha_s \left[\mathbb{E}_{t+1-s}(x_{t+1}) - \mathbb{E}_{t-s}(x_{t+1}) \right], \tag{2.71}$$

This can also be rewritten in terms of the realisations of the shock as:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi \sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}. \tag{2.72}$$

Thus, when the agent is diagnostic and relies on a slow-moving reference, they misperceive the shock as an ARMA(1,S) process, where S denotes the memory length.

3.2 Solution procedure

I solve the model to first-order accuracy using Klein (2000). First, I assume that diagnostic agents form beliefs based on a more distant past by using a moving average over the last twelve quarters as memory.[8] Consequently, making use of expression (2.72), agents'

[7] The full derivation for the slow-moving reference is presented in Appendix 2.B.

[8] In the main body of the article, I also present results in which the reference group is the most recent past. In this case all attention is on the previous quarter, that is, α_1 is equal to 1, whereas the remaining

concept of the state of the economy is as if it follows an ARMA(1,12) process instead of the true AR(1):

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi[(\rho\alpha_1\epsilon_t + \rho^2\alpha_2\epsilon_{t-1} + \rho^3\alpha_3\epsilon_{t-2} + \rho^4\alpha_4\epsilon_{t-3} + \rho^5\alpha_5\epsilon_{t-4} + \rho^6\alpha_6\epsilon_{t-5} + \rho^7\alpha_7\epsilon_{t-6} + \rho^8\alpha_8\epsilon_{t-7} + \rho^9\alpha_9\epsilon_{t-8} + \rho^{10}\alpha_{10}\epsilon_{t-9} + \rho^{11}\alpha_{11}\epsilon_{t-10} + \rho^{12}\alpha_{12}\epsilon_{t-11})]. \tag{2.73}$$

Second, I incorporate the MA components into the model as auxiliary variables and rewrite the exogenous shock processes as ARMA(1,12). Third, I compute the non-stochastic steady-state, point at which the model will be perturbed, by finding the fix-point of the system using Newton's method. Forth, I log-linearise the model variables around their steady state and solve the resulting system, which solution takes the following form:

$$\mathbf{x}_{t+1} = \mathbf{h}\mathbf{x_t} + \mathbf{k}\epsilon_{\mathbf{t+1}}$$
$$\mathbf{y_t} = \mathbf{g}\mathbf{x_t},$$

where $\mathbf{y}_t$ denotes an (m x 1) vector of endogenous variables and $\mathbf{x}_t$ denotes an (n x 1) vector of state variables. The latter is comprised of three sub-vectors. The first, of size (n_1 x 1), contains the auxiliary variables for the MA terms in the shock processes. The second, of size (n_2 x 1), includes the exogenous variables; and the third, of size (n_3 x 1), is composed of the predetermined variables. Therefore, the matrix $\mathbf{g}$ linking the decision variables with the states can also be divided into three submatrices:

$$\mathbf{g} = \left[\begin{array}{c|c|c} \mathbf{g_1} & \mathbf{g_2} & \mathbf{g_3} \end{array}\right].$$

The submatrices $\mathbf{g_2}$ and $\mathbf{g_3}$, of size (m x n_2) and (m x n_3), connect the decision variables to the exogenous states and the predetermined variables, respectively. A comparison between the solutions of these submatrices under RE and DE reveals that they remain unchanged since they are independent of the diagnostic parameter. The distinction arises in the submatrix $\mathbf{g_1}$, sized (m x n_1), which links the decision variables to the realised shocks in the MA processes. While the elements of this matrix are zero in the rational solution, in the diagnostic solution, they take on nonzero values. This reflects the source of the additional volatility that DE generate, and it aligns with the findings of L'Huillier

weights are equal to zero. The choice of twelve quarters follows from empirical evidence in the housing market found by Adam et al. (2024) and the estimation results of Bianchi et al. (2024).

et al. (2024).

Since the DE solution is based on agents misperceving the process for the state of the economy as an ARMA(1,12) rather than an AR(1) process, the first-order coefficient matrix in the state-transition equation, $\mathbf{h}$, still includes the parameters associated with the MA terms. The last step, therefore, is to turn off these MA terms, ensuring that any further analysis is performed under the true data-generating process. However, the elements in $\mathbf{g_1}$ that capture the dependence of the decision variables on the MA terms remain operative. These elements reflect the result that diagnostic agents extrapolate past shocks when forming expectations about the state of the economy.

4 Model Estimation

I estimate the rational model, the diagnostic model with short-term memory, as well as the diagnostic model with distant memory using U.S. quarterly data for the period 1984:Q1 to 2019:Q4, which I describe in Subsection 4.1. The estimation approach adopted is Sequential Monte Carlo, as outlined in Subsection 4.2. Subsection 4.3 describes the calibration of the structural parameters, while Subsection 4.4 shows the prior distributions of the parameters that are estimated. Subsection 4.5 exhibits the estimation results.

4.1 Data

I use eight macroeconomic time series to estimate and calibrate the model. All variables are log-transformed using the natural logarithm, detrended using first-differences and demeaned, with the exception of the nominal interest rate which is transformed into a quarterly rate and demeaned. Housing wealth is expressed in real per capita terms as it is adjusted by the population level and the implicit price deflator, while the total amount of loans to households is equal to the sum of residential mortgages and consumer credit of households and non-profit organisations.[9] I obtained the data from the Board of Governors of the Federal Reserve System and the Bureau of Economic Analysis, using the National Accounts and Flow of Funds. I also use the Census Bureau House Price Index. The full set of variables is:

- Real Gross Domestic Product growth: $\Delta GDP_t = ln(GDP_t/GDP_{t-1})$
- GDP implicit price deflator: $\hat{\pi}_t = ln(P_t/P_{t-1})$
- Real Residential Investment growth: $\Delta I^h_t = ln(I^h_t/I^h_{t-1})$
- Real House price growth: $\Delta q_t = ln(q_t/q_{t-1})$

[9] A detailed explanation of the data series can be found in Appendix 2.D.

- Nominal interest rate: $\hat{R_t^d} = ln(R_t/R_{t-1})$
- Real Loans growth: $\Delta l_t = ln(l_t/l_{t-1})$
- Real Non-residential investment growth: $\Delta I_t = ln(I_t/I_{t-1})$
- Real Housing wealth growth: $\Delta(qH_t) = ln(qH_t/qH_{t-1})$

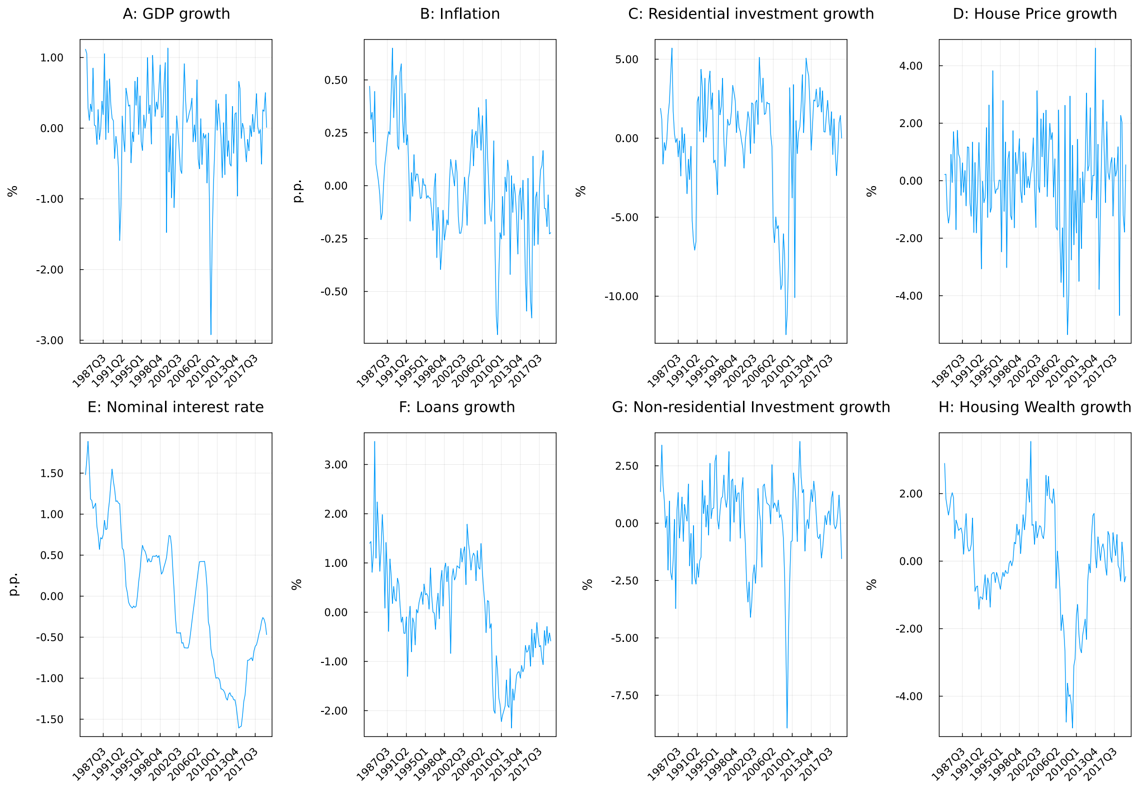

Figure 2.3: U.S. Macroeconomic variables.
Note: Real gross domestic product, real residential investment, real house price, real loans, real non-residential investment and real housing wealth growths are in percentages. Inflation and nominal interest rate are quarterly.

4.2 Methodology

I estimate the log-linearised rational model, diagnostic model with short-term memory and diagnostic model with distant memory using a Bayesian strategy method, drawing on recent advancements in macroeconomic model estimation by Herbst and Schorfheide (2014). Herbst and Schorfheide (2014) introduce an alternative class of algorithms to the traditional Random Walk Metropolis-Hastings (RWMH) method, known as Sequential Monte Carlo (SMC). This estimation method for DSGE models combines features of classic importance sampling and MCMC techniques.

The aim is to infer the parameters' posterior distribution by combining the likelihood function, $p(\theta|Y)$, of a DSGE model with a prior distribution, $p(\theta)$, on its parameters:

$$p(\theta|Y) = \frac{p(Y|\theta)p(\theta)}{p(Y)},$$

where θ indicates a vector of parameters and Y represents the data set. The marginal data density is $p(Y) = \int p(Y|\theta)p(\theta)d\theta$.

In order to do so, the SMC relies on a candidate density, $g(\theta)$, from which it generates sample draws or particles, as it is an importance sampler at its core. Each of these particles has an associated importance weight, $w(\theta)$. This follows from the identity:

$$\mathbb{E}_t[h(\theta)] = \int h(\theta)\frac{p(Y|\theta)p(\theta)}{p(Y)}d\theta = \frac{1}{p(Y)}\int h(\theta)\frac{p(Y|\theta)p(\theta)}{g(\theta)}g(\theta)d\theta,$$

where the weights on the draws are $w(\theta) = \frac{p(Y|\theta)p(\theta)}{g(\theta)}$. This means that at each stage of the algorithm, a set of pairs $\{(\theta^i, w(\theta^i))\}_{i=1}^N$ will be an approximation of $p(\theta|Y)$.

The algorithm initiates by drawing initial particles from the prior distribution, as it represents a distribution that is easy to sample from. The algorithm ends with a sequence of pairs of particles and weights that embody the final importance sample approximation of the posterior. In between, the process recursively generates intermediate or "bridge" distributions. These bridge distributions serve as transitional steps in the iterative process that gradually shifts the initial prior distribution towards the final posterior distribution. Each iteration refines the approximation of the posterior by updating the particle weights and resampling. It can be thought of as an iterative moulding process that refines and transitions the distributions from their initial prior form to their posterior.[10]

I use SMC because it offers advantages over RWMH. For instance, it is suitable for parallel computing during the model's estimation step for many particles, thereby enhancing computational efficiency. Additionally, when new data become available, SMC facilitates the re-estimation of the model by picking up from where it was left off, saving computational resources and time. It also eliminates the need for additional computations since it approximates the marginal likelihood as a by product.

The estimation was performed using Julia 1.7.3 in Atom. I adapted the code for the SMC algorithm from Salazar-Perez and Seoane (2024). The current choice of hyperparameters for the SMC is constrained by available computational resource. The number of particles and stages are set to 500 and 200, respectively. The bending coefficient is within the range used in the literature and it is taken from Salazar-Perez and Seoane (2024).[11]

[10] For a more detailed explanation of the algorithm see Herbst and Schorfheide (2014) and Cai et al. (2021).

[11] The bending coefficient controls the likelihood tempering in the algorithm. In this chapter, I use the

I adjust the scaling factor to target an acceptance rate around 25%, as it is done in Cai et al. (2021).

4.3 Calibration

In this section, I present the calibration of the model. Table 2.1 lists the structural parameters with their respective values and the corresponding source or target. Each period in the model represents one quarter. The discount factor for patient households, β_p, is set to 0.9915. This implies a 3.42% annualised interest rate in steady state, which is close to the average 3.52% over the sample period.

Some parameters are calibrated to match first-order moments in the data. The housing depreciation rate, δ_h, is set to 0.0060 in order to generate an average total housing wealth to GDP ratio of 145.53% as in the period analysed. This parameter value is slightly lower than the one from Iacoviello and Neri (2010) and Mendicino and Punzi (2014), and implies an annual housing depreciation of around 2.5%. The loan-to-value ratio, χ, is calibrated at a value of 0.8016. This choice aims to achieve a household credit to total housing wealth ratio of 35.24%, and it is consistent with the range found in the literature (Iacoviello & Neri, 2010; Gelain et al., 2012; Mendicino & Punzi, 2014, among others). The patient households' housing preference weight is set to 0.2361 so that the share of total housing wealth owned by patient households is 60%.[12] Moreover, the model generates a residential investment to GDP ratio equal to the sample period average of 3.44%, by setting the housing preference weight of the impatient households to 0.0906. Finally, I calibrate the elasticity of wholesale goods with respect to capital, α, to 0.3752, and I set the capital share in housing production, μ_h, at 0.3 to obtain a non-residential investment to GDP ratio of 27%. The elasticity of the wholesale goods with respect to capital has a value that falls within the range commonly used in macroeconomic models, while the capital share in housing production results from the sum of the exponents for all inputs that are not labour in Iacoviello and Neri (2010).

The remaining structural parameters are taken from the literature. For instance, the proportion of impatient households, n, follows Gelain et al. (2012) and targets the top decile of households in the economy. The discount factor for the impatient households, β_i, is 0.9715. This value ensures that while linearising around the steady-state, these households' borrowing constraint is binding. The chosen value for the inverse labour supply elasticity, φ, is 0.1, which, as in Gelain et al. (2012), implies a very flexible labour supply. This article is also the source of the labour disutility parameters for the patient

fixed tempering schedule from Herbst and Schorfheide (2014).

[12] The total housing wealth share hold by the patient household is targeted following Wolff (2016).

Table 2.1: Calibration: Structural parameters

Description	Parameter	Value	Target/Source
Households			
Proportion of impatient households	n	0.9	Gelain et al. (2012)
Inverse elasticity of labour supply	φ	0.1	Gelain et al. (2012)
Patient Households			
Discount factor	β_p	0.9915	Annualised interest rate of 3.52%
Housing preference weight	ν_p^h	0.2361	Patient households share of housing wealth = 60%
Labour disutility	ν_p^n	1.19	Gelain et al. (2012)
Impatient Households			
Discount factor	β_i	0.9715	Borrowing constraint's binding
Housing preference weight	ν_i^h	0.0906	Residential investment/GDP = 3.44%
Labour disutility	ν_i^n	4.54	Gelain et al. (2012)
Loan-to-value ratio	χ	0.8016	Household credit to total housing wealth = 35.24%
Wholesale firms			
Elasticity of final good with respect to capital	α	0.3752	Investment/GDP = 27%
Final firms and Retailers firms			
Elasticity of substitution	ϵ	11	10 % markup
Capital good firms			
Capital depreciation rate	δ_k	0.025	Typical in macroeconomic model literature
Housing firms			
Housing depreciation rate	δ_h	0.0060	Housing wealth/GDP = 143.23%
Elasticity of housing with respect to capital	μ_h	0.3	Iacoviello and Neri (2010)
Banks			
Banks' surviving probability	σ	0.9725	Gertler and Karadi (2011)
Absconding rate of the bankers	ζ	0.383	Gertler and Karadi (2011)
Start up fund for the new bankers	ω	0.003	Gertler and Karadi (2011)

and impatient household, ν_p^n and ν_i^n.

The rate at which capital depreciates, δ_k, equals 0.025. This value is in line with standard values in the literature. Retail firms target a 10% steady state mark-up, thus I set the elasticity of substitution, ϵ, to 11. Finally, the banking sector is characterised exactly as in Gertler and Karadi (2011). The parameters are consistent with Gertler and Karadi (2011)'s goal to achieve an interest rate spread of around one hundred basis points, maintain a steady state leverage ratio at 4, and ensure an average banker lifespan of 10 years. Therefore, the survival probability of the banker, σ, is 0.9725, the fraction of capital that the banker can steal, ζ, is equal to 0.383, and the start-up fund for new bankers is 0.003. This implies a spread close to 1% and a leverage ratio slightly below 4.

4.4 Prior distributions

Table 2.2 summarises the prior distributions for the parameters to estimate. I set the shapes for each prior based on the feasible parameter support and in consistency with previous studies. Accordingly, for the standard errors of the shocks, I use an inverse gamma distribution as in L'Huillier et al. (2024) and Justiniano, Primiceri, and Tambalotti (2010).[13] For the persistence, since these parameters are bounded between 0 and 1, I choose a loose beta prior with mean 0.5 and standard deviation 0.2. The values for the monetary policy rule feedback parameters were set equal to Taylor's original specifications. I choose a normal distribution with prior mean 1.5 for the response to inflation and a 0.125 mean for the response to output growth; their standard deviations are 0.25 and 0.05, respectively. The prior on the investment adjustment costs follows Smets and Wouters (2007), it is normal with mean 4.0 and standard deviation 1.5. Whereas for the habit formation and Calvo parameter, I chose the same beta priors as in Iacoviello and Neri (2010). The priors' means are 0.5 and 0.667, respectively, with standard deviations equal to 0.05 and 0.075.

Table 2.2: Prior distribution of the parameters

Description	Parameter	Distribution	Mean	Std. dev
Structural Parameters				
Inv. adjustment cost	ψ	Normal	4.0	1.5
Habit formation	γ	Beta	0.667	0.05
Calvo parameter	θ	Beta	0.5	0.075
Taylor rule inflation	ω_π	Normal	1.50	0.25
Taylor rule output growth	$\omega_{\Delta y}$	Normal	0.125	0.05
Diagnostic parameters				
Diagnostic parameter	ϕ	Normal	1.0	0.3
1st quarter reference	α_1	Uniform	0.5	0.29
2nd quarter reference	α_2	Uniform	0.5	0.29
3rd quarter reference	α_3	Uniform	0.5	0.29
4th quarter reference	α_4	Uniform	0.5	0.29
5th quarter reference	α_5	Uniform	0.5	0.29
6th quarter reference	α_6	Uniform	0.5	0.29
7th quarter reference	α_7	Uniform	0.5	0.29
8th quarter reference	α_8	Uniform	0.5	0.29
9th quarter reference	α_9	Uniform	0.5	0.29
10th quarter reference	α_{10}	Uniform	0.5	0.29
11th quarter reference	α_{11}	Uniform	0.5	0.29
12th quarter reference	α_{12}	Uniform	0.5	0.29
Autoregressive coefficients				
Goods TFP	ρ_A	Beta	0.5	0.2
Housing TFP	ρ_Z	Beta	0.5	0.2
Monetary policy	ρ_M	Beta	0.5	0.2
Housing demand	ρ_Γ	Beta	0.5	0.2
Standard deviation of shocks				
Good TFP	$100*\sigma_{\epsilon^A}$	Inverse Gamma	0.5	2.0
Housing TFP	$100*\sigma_{\epsilon^Z}$	Inverse Gamma	0.5	2.0
Monetary policy	$100*\sigma_{\epsilon^M}$	Inverse Gamma	0.5	2.0
Housing demand	$100*\sigma_{\epsilon^\Gamma}$	Inverse Gamma	0.5	2.0

Note: The Inverse Gamma priors are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\frac{s}{2\sigma^2}}$. I borrow the function InverseGamma1.jl and *inverse_gamma_1_specification* from the Dynare package for Julia, developed by Adjemian et al. (2024), to obtain the parameters ν and s of and Inverse Gamma distribution characterised as in the table above.

For the diagnostic parameter, I employ a normal distribution with mean 1.0 and standard deviation 0.3, as in L'Huillier et al. (2024). Prior information on this parameter

[13]The Inverse Gamma distribution is typically used as a prior for the variance estimation. However, as noted in Adjemian (2016), priors in practice are often defined over the standard deviation of a structural shock. Following this convention in the literature, I adopt the Type I Inverse Gamma distribution as defined by the Adjemian (2016).

is limited given the scarcity of studies estimating it. A similar situation applies to the weights on the backward references, with Bianchi et al. (2024) providing the only estimation example in the literature.[14] In their study, the approach involves modelling these weights using a beta distribution. They start by estimating the mean and standard deviation of this distribution, then they proceed with rescaling and discretising it. Here, however, I estimate each weight employing a diffuse prior over the range (0,1), allowing the data to inform the analysis. The purpose behind estimating diagnostic and memory parameters is twofold: to support the presence of DE in the housing market, and to demonstrate that DSGE models incorporating DE can fit business cycle data better than those with RE.

4.5 Estimation results

I jointly estimate the remaining parameters to match second-order moments of four U.S. time series: real GDP growth, inflation, real residential investment growth, and real house price growth, for the period 1984-2019. Table 2.3 gathers the results for the three estimated models: the diagnostic model with a twelve-quarter moving reference, the diagnostic model with a one-quarter reference, and the rational expectations model.[15] I report the mean posterior and the 90% high probability density credible interval (HPDI) for each parameter.[16] The last line reports the log of the marginal likelihood for each model.

The parameter values differ among the estimated models. Adjusting investment becomes more costly in the diagnostic model, more than twice as much when the reference period is the immediate past.[17] This tempers the diagnostic firms' overreaction such that the fluctuations in investment are not as pronounced as they could be with a lower adjustment cost. The Calvo parameter estimates are very similar between the models. They indicate that firms' prices are sticky, as they can be reset once every seven quarters. These results are comparable with the values obtained by Iacoviello and Neri (2010). House-

[14] It is worth noting that in this article, I follow Bordalo et al. (2018) when assuming that remote memories affect the way the diagnostic agent forms expectations. Therefore, I define representativeness in terms of current and past likelihood ratios. On the other hand, Bianchi et al. (2024) stipulate that the comparison group the diagnostic agent uses as reference is an average of lagged RE conditional on $t - J$ information, where J is the time span of the lag. In Appendix 2.B I show how these two approaches are related.

[15] Note that the diagnostic framework with a twelve-quarter slow-moving memory encompasses both the diagnostic model using the last quarter as reference and the rational case. In the first, all attention is on the last quarter, meaning that $\alpha_1 = 1$ and the remaining αs are equal to zero. While, in the second, all weights and the diagnostic parameter are equal to zero.

[16] Figures 2.9, 2.10 and 2.11 in Appendix 2.C show the posterior distributions for the variables of each model.

[17] Gabriel and Ghilardi (2012) estimate values within a similar range. They claim that this result arises from an interaction between investment costs and financial frictions.

holds seem to have a somewhat high degree of habit formation with estimated values for this parameter slightly above 0.7. Nevertheless, it is a value close to the 0.6 proposed by Leith, Moldovan, and Rossi (2012) and supported by the meta-analysis in Havranek, Rusnak, and Sokolova (2017). Similarly, there is some variation in the Central Bank's Taylor rule parameters. In comparison to RE, the inflation feedback increases if agents form DE considering the most recent past, whereas it decreases when they use a more distant memory. In contrast, the feedback on output growth shows the opposite behaviour. The Central Bank is less sensitive to output growth volatility when the diagnostic agent memory only includes the immediate past. However, when it includes remoter memories, the Central Bank reacts as strongly as in the rational case. These estimates suggest that agents' behaviour directly influences the Central Bank's trade-off between stabilising inflation and output growth volatility. This is in line with the conclusions of Bounader and Elekdag (2024).

Table 2.3: Estimation

Description	Parameter	DE Ref: Q12		DE Ref: Q1		RE	
		Mean	[0.05, 0.95]	Mean	[0.05, 0.95]	Mean	[0.05, 0.95]
Structural Parameters							
Inv. adjustment cost	ψ	0.8696	[0.5039,1.2422]	2.0600	[1.1548,3.3689]	0.8163	[0.4974,1.2026]
Habit formation	γ	0.7224	[0.6383,0.7896]	0.7415	[0.6558,0.7956]	0.7143	[0.6199,0.7763]
Calvo parameter	ϕ	0.8485	[0.8288,0.8637]	0.8732	[0.8604,0.8827]	0.8593	[0.8424,0.8718]
Taylor rule inflation	ω_π	1.6680	[1.4599,1.8769]	1.7381	[1.4643,2.0024]	1.7183	[1.4661,1.9764]
Taylor rule output growth	$\omega_{\Delta y}$	0.1972	[0.1249,0.2646]	0.1795	[0.0996,0.2455]	0.2029	[0.1102,0.2764]
Diagnostic parameters							
Diagnostic parameter	ϕ	0.1303	[0.0050,0.3265]	0.4555	[0.2819,0.6629]		
1st quarter reference	α_1	0.6714	[0.2689,0.9517]	1.0			
2nd quarter reference	α_2	0.2209	[0.0147,0.5706]				
3rd quarter reference	α_3	0.2054	[0.0055,0.6358]				
4th quarter reference	α_4	0.5226	[0.1065,0.9487]				
5th quarter reference	α_5	0.0990	[0.0057,0.3096]				
6th quarter reference	α_6	0.3797	[0.0380,0.8109]				
7th quarter reference	α_7	0.5930	[0.1513,0.9603]				
8th quarter reference	α_8	0.4963	[0.0910,0.8893]				
9th quarter reference	α_9	0.4775	[0.0829,0.9068]				
10th quarter reference	α_{10}	0.5157	[0.1629,0.8209]				
11th quarter reference	α_{11}	0.5219	[0.1789,0.8178]				
12th quarter reference	α_{12}	0.1340	[0.0087,0.4000]				
Autoregressive coefficients							
Goods TFP	ρ_A	0.8307	[0.7791,0.8906]	0.8691	[0.8153,0.9217]	0.8169	[0.7559,0.8750]
Housing TFP	ρ_Z	0.9413	[0.9212,0.9595]	0.9514	[0.9331,0.9660]	0.9546	[0.9379,0.9679]
Monetary policy	ρ_M	0.6561	[0.5444,0.7373]	0.7625	[0.6807,0.8115]	0.6896	[0.5965,0.7573]
Housing demand	ρ_Γ	0.9614	[0.9400,0.9800]	0.9445	[0.9080,0.9715]	0.9293	[0.8876,0.9633]
Standard deviation of shocks							
Good TFP	100*$\sigma_{\epsilon A}$	1.4435	[1.2829,1.6180]	1.3084	[1.1121,1.4724]	1.6550	[1.4820,1.8551]
Housing TFP	100*$\sigma_{\epsilon Z}$	3.9204	[3.3804,3.9830]	3.7382	[3.3965,4.1451]	3.7089	[3.3996,4.1035]
Monetary policy	100*$\sigma_{\epsilon M}$	0.3107	[0.2416,0.3896]	0.2222	[0.1719,0.2794]	0.2959	[0.2286,0.3771]
Housing demand	100*$\sigma_{\epsilon \Gamma}$	5.3588	[4.0070,6.6326]	7.2345	[4.6284,10.9440]	11.2891	[7.2150,16.4806]
Log marginal likelihood		568.67		598.91		591.63	

Note: The structural parameters include the investment adjustment cost (ψ), the habit formation (γ), the Calvo parameter (θ), the Central Bank Taylor rule inflation feedback (ω_π), and output growth feedback ($\omega_{\Delta y}$). The diagnostic parameters include the diagnosticity (ϕ) and the weights on past quarters as reference ($\alpha_{n=1}^{12}$). The autocorrelation coefficients measure the persistence of the goods TFP shock (ρ_A), housing TFP shock (ρ_Z), monetary shock (ρ_M), and housing demand (preference) shock (ρ_Γ), while $\sigma_{\epsilon_A}, \sigma_{\epsilon_Z}, \sigma_{\epsilon_M}, \sigma_{\epsilon_\Gamma}$ measure the standard deviations.

The key parameter in this analysis is the diagnostic parameter ϕ, which quantifies the size of the departure from rationality. For the DE model with one-quarter lag reference, the estimation places a substantial mass around a value of 0.4555, with a 90% HPDI away from zero, providing strong evidence in favour of DE. This value is consistent with the range found in the literature (L'Huillier et al., 2024; Bordalo, Gennaioli, Shleifer, & Terry, 2021). However, in the case with twelve-quarters lags reference, the posterior mean drops to 0.1303. This finding contrasts with Bianchi et al. (2024), who reported a diagnostic degree magnitude of around 2 when agents rely on distant memories. It is important to note that their estimate is relatively high compared to others in the literature. Bianchi et al. (2024) obtained this under the assumption that the weights assigned to lagged expectations sum up to one, which then requires a higher degree of diagnosticity to match initial overreactions. This is not the case in the current study as I do not impose any constraints on the weights. Instead, I am interested in capturing whether there is a particular specification regarding their rate of decay.

Analysing the weights assigned to lagged representativeness as a reference, the estimates indicate two key findings. First, the reliance of diagnostic agents on past information (as indicated by non-zeros $\alpha's$ values) is inversely related to their degree of diagnosticity. This suggests that the slow-moving memory mechanism plays a crucial role in distributing the DE effects over time. Second, the most immediate quarter has the highest value, emphasising the importance of recent events in shaping agents' expectations. However, quarters three to ten account for approximately 70% of the total weight. This observation is consistent with Bianchi et al. (2024), who found a similar concentration of attention within these quarters in their model.

Turning to the estimates of the shocks, I note that the differences between most of the autoregressive coefficients do not exhibit a clear pattern. ρ_A is shown to be more persistent after the introduction of DE with the immediate past as reference, but when the length of the memory for the diagnostic agent is expanded including distant lags, the value decreases towards the rational benchmark. ρ_M shows a similar outcome. In contrast, the autoregressive coefficients for the housing market behave differently. ρ_Z remains relatively stable, while ρ_Γ turns out higher under both DE models. A different story holds for the standard deviations of the shocks. Overall, the estimated values are smaller in the DE models versus the rational expectations model, except for the housing TFP shock. This is consistent with evidence from previous articles pointing that DE is the channel through which shocks explain fluctuations (L'Huillier et al., 2024).

Here, I focus on the magnitude of the housing preference shock. This shock has been the major driver in rational expectations models that attempt to explain the dynamics of the housing market, with estimates of its standard deviation between 3% and 10%

(Iacoviello & Neri, 2010; Iacoviello, 2015; Ge et al., 2022). Iacoviello and Neri (2010) describe this housing preference shock as either "genuine shifts in tastes for housing, or a catchall for all the unmodeled disturbances that can affect housing demand"(p. 150). The estimated standard deviation of the housing preference shock under RE is 11.2891%. Instead, when agents are diagnostic, the values plummeted to 7.2345% and 5.3588%, contingent on whether their imperfect memory is driven by the immediate past or the last three years. This finding suggests that a significant part of that "catchall" seems to be related to the way agents form their expectations.[18] Specifically, DE help explain housing market volatility while relying on a smaller housing preference shock. Gandré (2022), using the learning framework, reaches a similar conclusion highlighting the necessity for stronger shock variances under rationality compared to a model with behavioural agents.

5 Quantitative Results

This section evaluates the performance of the models in matching second-order moments for selected variables. The bottom line in Table 2.3 summarises these findings, showing a log data density of 598.91 for the diagnostic model with one-quarter lag reference, compared to 591.63 for the rational expectations model. The difference between these measures, called the Bayes factor, is 7.28 in favour of the model with diagnostic agents, implying that its fit is better against the RE model.[19] This section proceeds showing how well the models do in fitting targeted moments of the data series. It also includes an analysis on the drivers of the business cycle.

5.1 Second-order moments

I use the data in section 4.1 to calculate empirical moments. The time series variables are demeaned to make them comparable with their model counterparts, where there is no growth. I simulate series with the same length as the data, i.e. hundred and forty-four observations, ten thousand times.

Table 2.4 compares the standard deviation (in %) of targeted variables in the data with that of the diagnostic models (DE Ref: Q12 and DE Ref: Q1), as well as in the rational expectations model (RE). The three models perform reasonably well. Although real GDP growth appears more volatile in the models than in the data, the DE model

[18] By introducing DE with agents relying on the most recent past, the standard deviation estimate decreases by 35.91%, whereas if they use a longer time-span memory, it drops by 52.53%.

[19] Kass and Raftery (1995) classifies a statistic 2log(Bayes Factor) = 14.56, as in this chapter, to be a very strong evidence towards the diagnostic model over the rational model.

with one-quarter lag memory generates a value closer to the observed target. While the models tend to produce a more stable inflation, the results overall suggest that they successfully capture the excess volatility in the housing market.

Table 2.4: Second-order moments in data and model

	Data	DE Ref:Q12	DE Ref:Q1	RE
Targeted moments				
Standard deviation				
Δ Real GDP	0.5764	0.8625	0.7271	0.8758
Relative standard deviation to GDP growth				
Inflation	0.4262	0.3025	0.3397	0.3057
Δ Real House prices	2.9896	2.4381	3.1882	2.3282
Δ Real Residential Investment	5.9893	5.0424	5.3184	4.7029

Note: Growth rates for real GDP, real house price, real residential investment. Model moments were obtained from averaging over ten thousand simulations of hundred and forty four observations each.

Despite this, the RE model consistently underestimates the magnitude of the relative volatility observed in the housing sector, whereas the evidence from the DE models is more accurate. Specifically, the DE Ref: Q1 model offers the closest fit. It achieves this by relying less on an ad hoc housing preference shock and more on the amplification mechanism inherent to the expectation formation process. This highlights that DE seem to better capture the dynamics of housing market fluctuations.

5.2 Historical shock decomposition

Figure 2.4 displays the historical shock decomposition for the model that better fits the data, the diagnostic model in which agents overweight recent past experiences when forming expectations.[20] This figure illustrates the nearly one-to-one relation between the four variables used in the estimation and the shocks. At each point in time, the bars indicate the proportion of the variable's deviation from its steady state that can be attributed to a particular shock, providing insight into the dynamic effects of these shocks on the variable over time. The orange bar represents the effect from the wholesale TFP shock, the green bar shows the impact of the housing TFP shock, the purple bar reflects the monetary policy shock, and the yellow bar represents the housing preference shock. The initial values, depicted in blue, show the impact of how far the variable is from its steady state at that moment. Since the data series do not begin at this point, the bars start from a value different from zero, but gradually diminish over time.

Real GDP growth is largely explained by the technology shock in the wholesale sector,

[20] Figures 2.12 and 2.13 in Appendix 2.C show the same figure for the other two estimated models.

as well as by monetary policy shocks. Inflation, in contrast, is mainly influenced by monetary policy shocks. The latter is expected since most of the analysis covers the "Great Moderation" period. Housing shocks, both supply and demand, play a relevant part describing the swings in the housing market. Real residential investment growth is mainly driven by the technology shock in the housing sector. Whereas, the housing preference shock explains real house price growth by directly affecting the marginal utility of housing for both agents. Darracq Paries and Notarpietro (2008) report similar results in both the US and the euro area.

The results found here do not show significant differences to the rational expectations case, nor to the diagnostic model with a twelve-quarters reference as shown in the Appendix 2.C. This suggests that shocks impact the economy in similar ways in DE and in RE; however, the amplification of these effects is driven by more volatile expectations inherent to the DE framework.

5.3 Impulse response function analysis

The following sections analyse the impact of the four shocks under the three estimated models using impulse responses. The responses are in log deviations from the steady state. I assume a 1% standard deviation shock in both the housing and wholesale goods sectors, as well as for the housing preference shock. The monetary policy shock, on the other hand, has a size of 25 basis points.

5.3.1 Effects of a wholesale goods productivity shock

Figure 2.5 displays the impulse responses to a positive productivity shock in the wholesale goods sector. The direction is as expected under RE. The shock increases labour and capital productivity, resulting in higher production (panel A) and consumption (panel B). Inflation decreases (panel C) as re-optimising firms adjust their prices in response to the fall in marginal cost. The Central Bank lowers the nominal interest rate (panel D), but the real interest rate increases (panel E). The lower nominal interest rate has a positive effect on house prices (panel F), as they initially jump and gradually converge back to the steady state. Loans (panel H) exhibit a U-shaped response due to the behaviour of the interest rate, which influences two forces: patient households willing to lend and impatient households willing to borrow. Housing investment (panel G) reacts positively, driven by the increase in house prices.

In contrast, the responses under both DE models are characterised by initial overreactions, longer persistence, and more pronounced fluctuations. The initial overreactions are generated through the extrapolation of the shock and are common to both approaches.

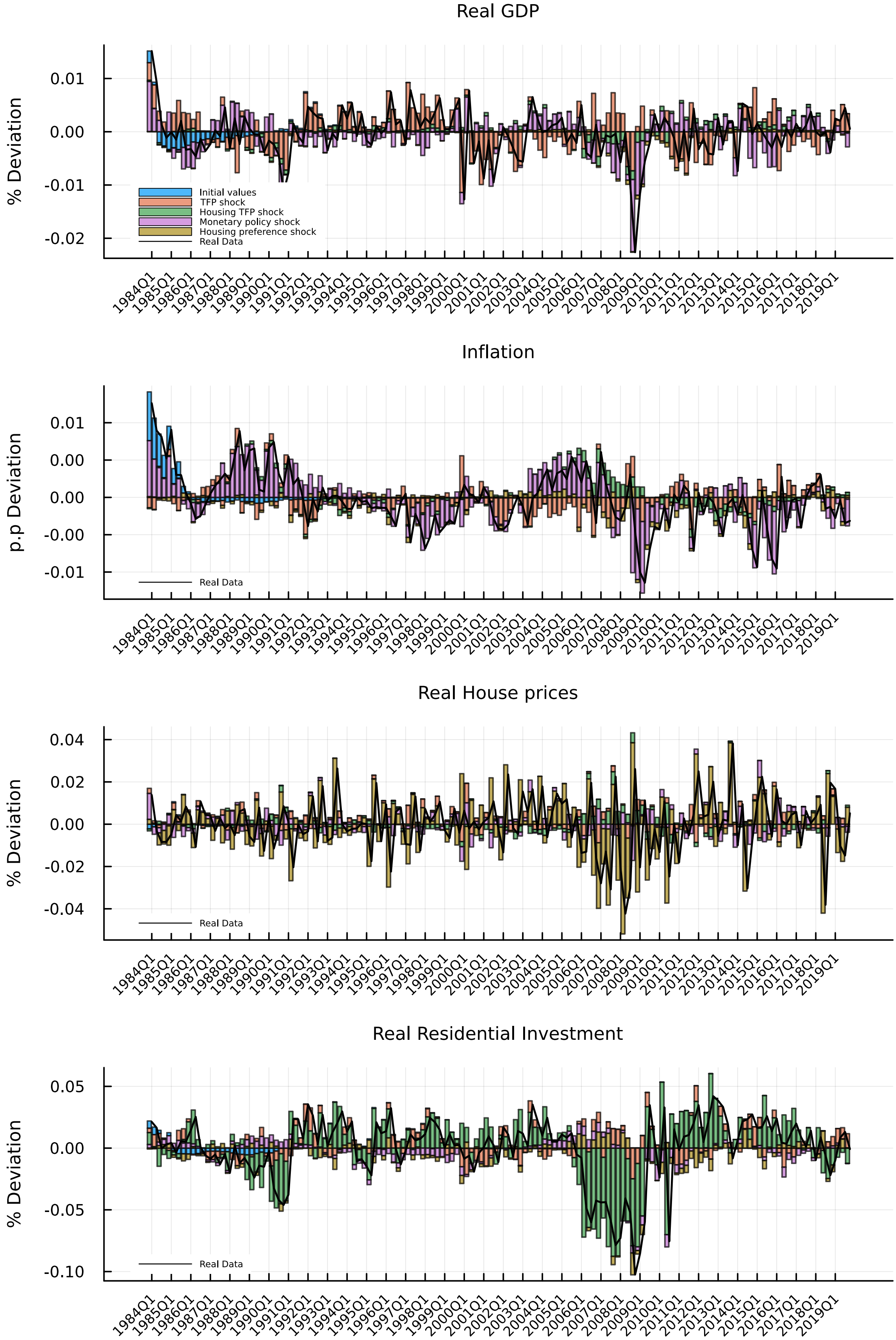

Figure 2.4: Historical shock decomposition under DE model with one-quarter reference.

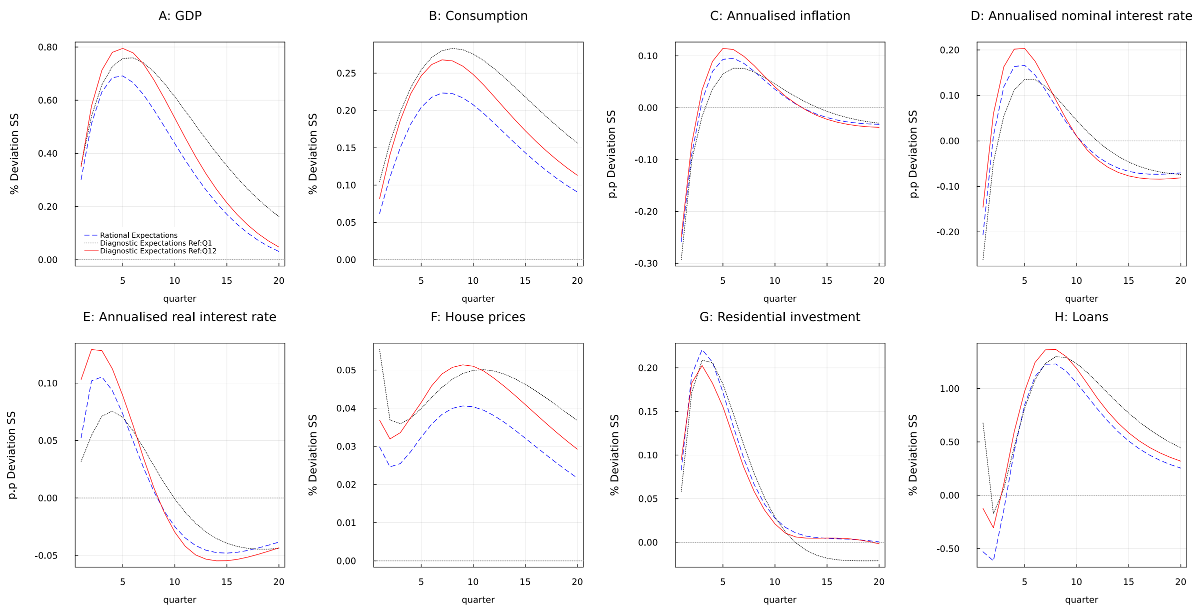

Figure 2.5: Impulse responses to a wholesale goods productivity shock.
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

However, the persistence and pronounced fluctuations are more specific to each framework. In the DE model with a one-quarter reference, the model's rigidities propagate the initial overreaction throughout the economy. Conversely, the DE model, which incorporates a twelve-quarter slow-moving reference, displays more significant fluctuations because of the extended memory involved in their expectation formation process.

Agents who misperceive the shock as an ARMA (1,1) or an ARMA (1,12) generate optimism about their productivity in the future. As a result, households will assign a higher probability to a scenario in which they are richer, leading to demand pressure (panel A and panel B). Firms experiencing higher labour costs will hire less workers and thus decrease their marginal costs. This lower marginal costs are reflected in the drop in domestic prices (panel C); firms cut prices more since they might not be able to re-optimise in the future. The Central Bank lowers the interest rate (panel D) by a larger amount when the diagnostic agent reference is the most recent past as its Taylor rule reacts more strongly to deviations in inflation. These two forces are the core of the noticeable difference in the real interest rate response (panel E). Households extrapolate current surprise disinflation into the future, i.e. they expect inflation to further decrease. However, since the opposite happens, agents realise about their mistake and so does the Central Bank, which hikes nominal interest rates, creating a boom-bust pattern in the real interest rate as L'Huillier et al. (2024). This behaviour of the real interest rate alters the shape of the impulse response of loans (panel H), since now impatient households will be more willing to borrow against their house value. This additional liquidity puts pressure on house prices (panel F). House prices exhibit a boom-bust pattern that aligns with historical interpretations of bubbles, as suggested by Gelain et al. (2012), but this chapter goes one step further by incorporating a micro- and psychologically founded belief formation model.[21] Finally, higher house prices stimulate higher housing investment (panel G).

5.3.2 Effects of a housing sector productivity shock

Impulse responses to a positive productivity shock in the housing sector are shown in Figure 2.6. The impact primarily affects variables related to the housing market. Housing investment exhibits a positive response (panel G) due to the increased productivity of capital and labour in this sector. The increased productivity, in turn, leads to a higher housing supply, resulting in a fairly persistent decline in house prices (panel F). The decline in house prices affects consumption of final goods (panel B) as there is a reallocation of resources, and a fall in annualised inflation (panel C) and nominal interest rate. GDP (panel A), nevertheless, is positively driven by the housing sector.

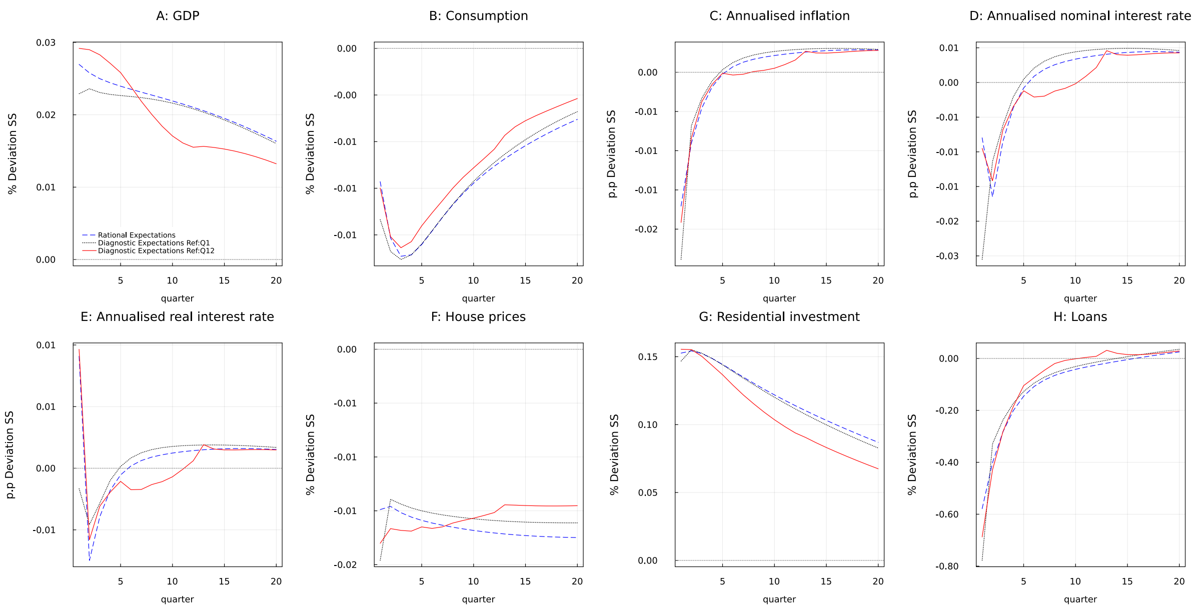

Figure 2.6: Impulse responses to a housing sector productivity shock .
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

[21] Greenspan (2002) defines: “Bubbles are often precipitated by perceptions of real improvements in the productivity and underlying profitability of the corporate economy. But, as history attests, investors then too often exaggerate the extent of the improvement in economic fundamentals. Human psychology being what it is, bubbles tend to feed on themselves, and booms in their later stages are often supported by implausible projections of potential demand.”

In comparison to the RE case, the DE models have a similar response, in magnitude, of housing investment (panel G). Because the diagnostic agent believes that this TFP shock is more persistent than it actually is, they overestimate the future productivity of the housing sector and anticipate a higher housing supply. Consequently, diagnostic-households initially overreact, leading to a more pronounced decline in house prices compared to the rational case (panel F). This overreaction in house prices is stronger for the DE model that has short-term memory agents. However, if agents realise that their beliefs are inconsistent with the shock process, house prices correct and converge faster to steady state. In fact, this faster correction occurs because households experience disappointment when their expectations prove overly optimistic, leading to a lower residential investment under DE compared to RE.

5.3.3 Effects of a tightening monetary policy

Figure 2.7 illustrates the impulse responses of a tightening monetary policy shock. The responses observed under the RE behave as anticipated in standard models. The shock depresses the economy, resulting in a negative deviation of GDP (panel A) and consumption (panel B) from their steady state. Inflation decreases (panel C), and the Central Bank reacts to these movements in output and inflation by lowering the nominal interest rate (panel D). However, the decrease in inflation exceeds the adjustment in the nominal interest rate, resulting in an increased real interest rate (panel E). This higher real interest rate has a negative effect on house prices (panel F), since mortgages for impatient households become more expensive (panel H). Higher cost of borrowing, in turn, depresses housing demand and housing investment (panel G), as the increased cost of capital impacts investment decisions.

When agents are diagnostic, impulse responses exhibit some distinct features. The fall in GDP is relatively larger and more persistent than for RE (panel A), when the reference is the most recent past. This is because agents believe that the Central Bank will further tighten monetary policy in the future. Those beliefs explain the stronger initial fall in prices (panel C), which leads to stronger reactions in the nominal interest rate (panel D) and, therefore, slightly smaller real interest rate (panel E). Agents mistakenly expect the variables to follow this path, but as events unfold and there are no further surprises in monetary policy, they adjust their expectations. This revision in expectations explains the sudden increase in the nominal interest rate and the jump in inflation, which are more pronounced in the diagnostic model where agents rely on short memory. Moreover, the behaviour of consumption (panel B) follows the Euler equation, and the U-shaped reaction is more persistent. A similar story holds for loans. The change in the real interest rate, as well as the decline in house prices (panel F), impacts the borrowing

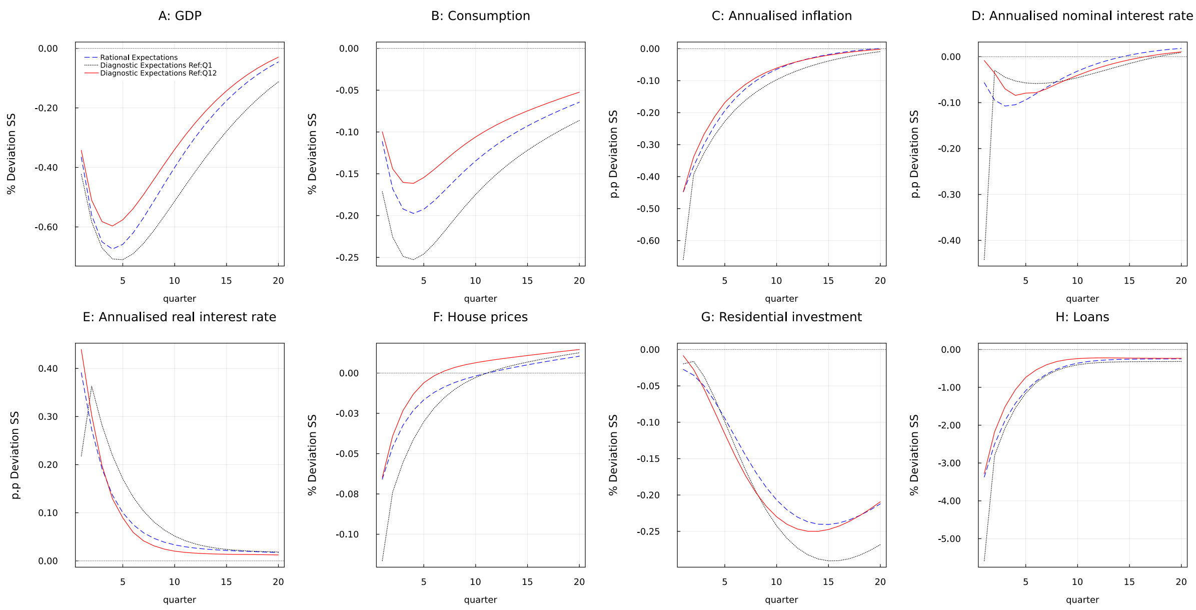

Figure 2.7: Impulse responses to a monetary policy shock.
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

constraint of the impatient household. It decreases her collateral, and so does her ability to obtain funds (panel H). This drags on housing demand and amplifies the fall in house prices. The subsequent recovery follows from the relaxation of the impatient household's collateral constraint. De Stefani (2021) reports empirical results that are consistent with this behaviour. In contrast, the impact of the shock on the economy populated with diagnostic agents with distant memory is ameliorated. This amelioration may stem from the reduced persistence of the shock compared to the other two scenarios, in conjunction with the diagnostic parameter estimate and the agent's attention framework.

5.3.4 Effects of a housing preference shock

Figure 2.8 provides details on the impact of the housing preference shock. Under RE, this shock shifts households' taste towards the housing sector as it directly hits the marginal utility of housing for both agents. As a result, an increase in housing demand places upward pressure on house prices (panel F). Although higher prices would make housing less desirable overall, impatient households experience a loosening in their collateral constraint (panel H), reflecting their willingness to leverage their financial position, as they need to finance higher housing costs. However, this effect is insufficient to offset the decline in the demand for housing from patient agents. Additionally, the rise in interest rates (panel D and panel E) diverts funds away from the housing sector, causing a delayed increase in residential investment (panel G), which, once realised, stimulates GDP (panel

A).

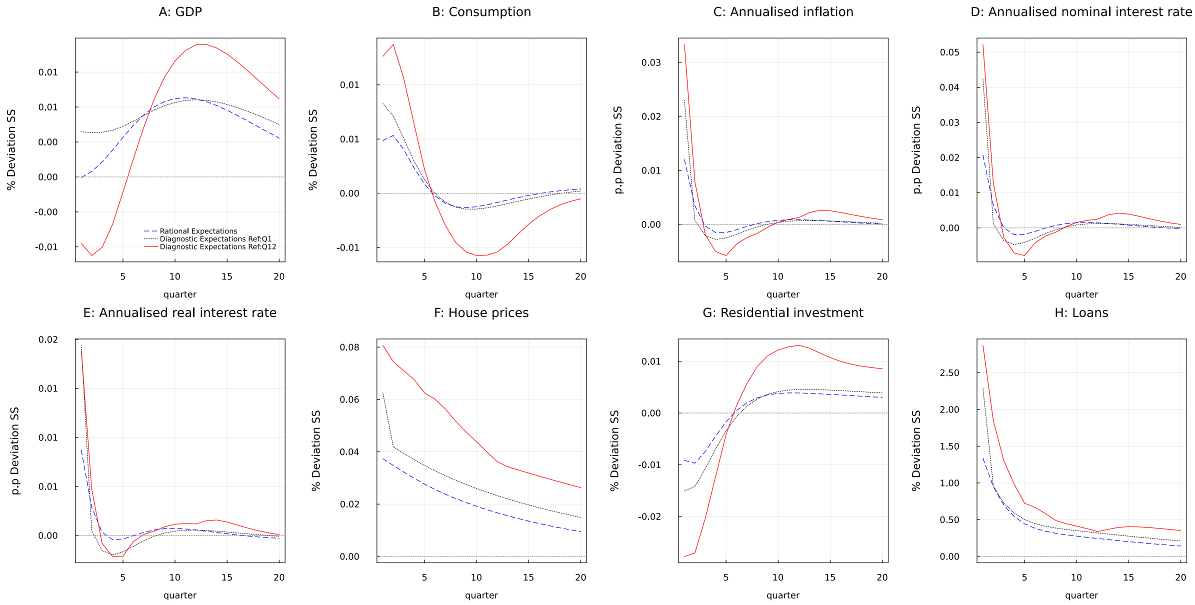

Figure 2.8: Impulse responses to a housing preference shock.
Note: The blue dashed line represents the responses when agents have rational expectations. The solid red line illustrates the impulse responses when agents have DE with a 3-year memory recall, while the black dotted line DE with the last quarter as reference.

The responses under both DE scenarios are in the same direction as in RE, although clearly amplified. When agents are diagnostic, after the shock hits, they expect further pressure in the housing market, anticipating house prices to rise even higher. This pressure leads to an initial overreaction in house prices (panel F), under both short-term and long-term memory. Such significant shift impacts other variables in the economy in the same way as in rational expectations. Impatient households experience a greater loosening of their collateral constraint, but the stronger decline in patient households' housing demand, combined with the hike in interest rates, diverts more funds away from the housing sector (panel G).[22] However, the main difference occurs if the agents realise that the true shock process is AR(1) rather than ARMA(1,1) or ARMA(1,12). In the first case, the rebound occurs faster as agents rapidly revise their expectations, while in the second case, it takes longer. This difference is evident in the impulse responses for house prices, where the drop in period 2 is more pronounced when the reference is the most recent past. This shock also generates stronger cycles in GDP (panel A) and consumption (panel B), with the drop in residential investment driving a recession for the first year and a half. Another key difference lies in the persistence and fluctuations of responses. The

[22] Iacoviello and Neri (2010) argue that, due to the absence of wage rigidities, the housing investment sector could become isolated from monetary and inflation disturbances, therefore leading the model to underestimate the correlation between housing prices and both consumption and housing investment. This may explain the findings observed here.

initial overreaction, particularly when the memory spans twelve-quarters, takes longer to die out. Gandré (2022) suggests that these movements originate in taste-swings in households, directly affecting intra-temporal and intertemporal trade-offs.

6 Counterfactual analysis

In this section, I conclude the analysis by presenting a counterfactual study to explore what happens if agents in the estimated diagnostic models suddenly behave rationally. Specifically, I evaluate a scenario where the expectation channel in the DE models is turned off, and agents adopt rational expectations instead. In this alternative setup, I set the diagnostic parameter and the weights on past quarters to zero, effectively eliminating the diagnostic component of expectation formation, while keeping all other parameters fixed. This analysis allows for a direct comparison between the original DE models and a model where agents rely solely on rational expectations. The comparison helps to assess how the expectation channel in the DE models influences the volatility dynamics in contrast to a purely rational expectations framework.

Table 2.5: Real house price growth second-order moment: data vs counterfactual model

	Data	DE Ref:Q12	DE Ref:Q1	RE Ref:Q12 Counterfactual	RE Ref:Q1 Counterfactual
Volatility relative to GDP					
Real House price growth	2.9896	2.4381	3.1882	1.8877	2.4992

Note: House price growth rate is obtained from averaging over ten thousand simulations of hundred and forty four observations each.

The results in Table 2.5 suggest that rational counterfactual models struggle to amplify house price volatility. Both counterfactual RE models produce a measure that is around 22% lower than their diagnostic counterpart. Notably, the RE Ref: Q1 counterfactual generates a higher measure since the estimated size of the preference shock standard deviation is higher than in the case of the twelve-quarter reference (7.23% vs 5.35%). This finding underscores the significant role that DE play in driving the dynamics of the housing market. In other words, around a third of the volatility in the housing market originates in the expectations channel, through DE.

7 Concluding remarks

This chapter examines expectations as a central driver of housing market volatility by integrating diagnostic expectations with both short-term and long-term memory into a

TANK model featuring housing and banking sectors. The results, based on the diagnostic parameter and reference period weight estimates, empirically support the DE model. Evidence favours the model in which diagnostic agents only consider the immediate past quarter when forming beliefs. This model successfully accounts for economic fluctuations, particularly in the housing market, when conditioned on less volatile shocks. Specifically, the DE model explains housing price and quantity dynamics with a housing preference shock innovation that is two-thirds the size of that under RE, which suggests DE as a more comprehensive alternative to the "catchall of all the unmodeled disturbances that can affect housing demand" (Iacoviello & Neri, 2010, p. 150).

Another noteworthy result is that, when the expectations channel in the DE models is shut down, the models fail to generate the higher volatility in house prices relative to real GDP growth observed in the data. Together with the previous result, this suggests that DE drive cyclical dynamics in the housing market and, given the sector's significance in household decision making, underline the need to consider DE in policy recommendations.

Future work would enhance the analysis. One direction I plan to explore is to allow the banking sector to intermediate between households, which would provide insights about the role of expectations in the housing credit market. Another possible extension would be to allow for heterogeneity in the degree of diagnosticity to capture diverse belief formation across households.

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Appendices

2.A Model Derivations

2.A.1 Households

2.A.1.1 Patient

$$\mathcal{L}_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \Bigg\{ \Big[log(c_{p,t} - \gamma c_{p,t-1}) + \Gamma_t \nu_p^h log(h_{p,t}) - \nu_p^n \frac{n_{p,t}^{1+\varphi}}{1+\varphi} \Big] - \lambda_{p,t} \Big[c_{p,t} + q_t[h_{p,t} - (1-\delta_h)h_{p,t-1}] + d_t^B + d_t^l - \frac{d_{t-1}^B R_{t-1}^d}{\pi_t} - \frac{d_{t-1}^l R_{t-1}^l}{\pi_t} - w_t n_{p,t} - \Pi_{f,t} - \Pi_{B,t} \Big] \Bigg\} \tag{2.74}$$

The optimal conditions of this Lagrangian with respect to $c_{p,t}$, $n_{p,t}$, $h_{p,t}$, d_t^B and d_t^l are:

$$\frac{\partial \mathcal{L}_p}{\partial c_{p,t}} : \lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p \gamma}{(c_{p,t+1} - \gamma c_{p,t})}. \tag{2.75}$$

$$\frac{\partial \mathcal{L}_p}{\partial n_{p,t}} : \nu_p^n n_{p,t}^{\varphi} = w_t \lambda_{p,t}. \tag{2.76}$$

$$\frac{\partial \mathcal{L}_p}{\partial h_{p,t}} : \lambda_{p,t} q_t = \frac{\Gamma_t \nu_p^h}{h_{p,t}} + \beta_p \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{p,t+1} \Big]. \tag{2.77}$$

$$\frac{\partial \mathcal{L}_p}{\partial d_t^B} : \lambda_{p,t} = \beta_p \mathbb{E}_t \left[\lambda_{p,t+1} \frac{R_t^d}{\pi_{t+1}} \right]. \tag{2.78}$$

$$\frac{\partial \mathcal{L}_p}{\partial d_t^l} : \lambda_{p,t} = \beta_p \mathbb{E}_t \left[\lambda_{p,t+1} \frac{R_t^l}{\pi_{t+1}} \right]. \tag{2.79}$$

2.A.1.2 Impatient

$$\mathcal{L}_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_i^t \Bigg\{ \Big[log(c_{i,t} - \gamma c_{i,t-1}) + \Gamma_t \nu_i^h log(h_{i,t}) - \nu_i^n \frac{n_{i,t}^{1+\varphi}}{1+\varphi} \Big] - \\ \lambda_{i,t} \Big[c_{i,t} + q_t(h_{i,t} - (1-\delta_h)h_{i,t-1}) + \frac{l_{t-1}R_{t-1}^l}{\pi_t} - l_t - w_t n_{i,t} \Big] - \\ \mu_{i,t} \Big[l_t - \frac{\chi}{R_t^l}(q_{t+1}\pi_{t+1})h_{i,t} \Big] \Bigg\} \tag{2.80}$$

The optimal conditions of this Lagrangian with respect to $c_{i,t}$, $n_{i,t}$ $h_{i,t}$ and l_t are:

$$\frac{\partial \mathcal{L}_i}{\partial c_{i,t}} : \lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_i \gamma}{(c_{i,t+1} - \gamma c_{i,t})}. \tag{2.81}$$

$$\frac{\partial \mathcal{L}_i}{\partial n_{i,t}} : \nu_i^n n_{i,t}^{\varphi} = w_t \lambda_{i,t}. \tag{2.82}$$

$$\frac{\partial \mathcal{L}_i}{\partial h_{i,t}} : \lambda_{i,t} q_t = \frac{\Gamma_t \nu_i^h}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R_t^l} \mathbb{E}_t [q_{t+1}\pi_{t+1}], \tag{2.83}$$

$$\frac{\partial \mathcal{L}_i}{\partial l_t} : \lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \left[\lambda_{i,t+1} \frac{R_t^l}{\pi_{t+1}} \right] \tag{2.84}$$

2.A.2 Firms

2.A.2.1 Wholesale firms

$$\max_{N_t^W, K_{t-1}^W} \Pi_t^{w,f} = [P_{m,t} Y_t^W + (1-\delta_k) q_t^K K_{t-1}^W - R_t^K q_{t-1}^K K_{t-1}^W - w_t N_t^W \Big] \tag{2.85}$$

subject to:

$$Y_t^W = A_t {N_t^W}^{1-\alpha} {K_{t-1}^W}^{\alpha}, \tag{2.86}$$

The first-order conditions with respect to N_t^W and K_{t-1}^W are:

$$w_t = P_{m,t}(1-\alpha) A_t \left(\frac{K_{t-1}^W}{N_t^W} \right)^{\alpha}, \tag{2.87}$$

$$q_{t-1}^K R_t^K = r_t^K + (1-\delta_k) q_t^K, \tag{2.88}$$

where $r_t^K = P_{m,t} \alpha A_t \left(\frac{N_t^W}{K_{t-1}^W} \right)^{1-\alpha}$ is the rental rate of capital. Obtaining the ratio $\frac{N_t^W}{K_{t-1}^W}$ from the wage and rental rate expressions and equating them, I obtain an equation for the marginal cost:

$$mc_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r_t^K}{\alpha} \right)^{\alpha}. \tag{2.89}$$

2.A.2.2 Final good firm

The final good producer purchases goods repackaged by the retailers and aggregates them according to a Dixit-Stiglitz production technology. After that, they sell the final product in a perfect competitive market at a price P_t.

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{2.90}$$

Y_t represents the final good, $y_t(j)$ denotes the $j'th$ retailer input. This firm's profit maximisation is a static problem and can be stated as:

$$\max_{y_t(j)} P_t \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 p_t(j) y_t(j) dj, \tag{2.91}$$

where Y_t was replaced using its definition. The first-order condition of this decision problem by choosing $\{y_t(j)\}_{j=0}^1$ is given by:

$$P_t \frac{\epsilon}{\epsilon - 1} \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon - 1}{\epsilon} y_t(j)^{\frac{\epsilon-1}{\epsilon}-1} = P_t(j), \forall j$$

$$\Rightarrow P_t \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{1}{\epsilon-1}} y_t(j)^{-\frac{1}{\epsilon}} = P_t(j)$$

$$\Rightarrow y_t(j)^{-\frac{1}{\epsilon}} = \left(\frac{P_t(j)}{P_t} \right) \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{-\frac{1}{\epsilon-1}}$$

$$\Rightarrow y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{-\frac{\epsilon}{\epsilon-1}}. \tag{2.92}$$

Which after using the definition of Y_t, the demand equation for each input turns out to be:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t. \tag{2.93}$$

Since this final good-producing firm acts in a competitive market, it makes zero profit. Replacing the demand equation in the maximisation problem, I obtain:

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

$$\Rightarrow P_t Y_t = \int_0^1 P_t(j) \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t dj$$

$$\Rightarrow P_t Y_t = P_t^{\epsilon} Y_t \int_0^1 P_t(j)^{1-\epsilon} dj$$

$$P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj. \tag{2.94}$$

Rearranging this equation yields an expression of the price of the final good as a function of the intermediate inputs' prices, i.e. an aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}} \tag{2.95}$$

2.A.2.3 Retail firms

In the presence of price rigidity *à la Calvo*, retailers will be able to change their price with a probability $(1-\theta)$, while with a probability θ they will not. To determine the new price $P_t^*(j)$, the retail firms maximise:

$$V_t(j) = \mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \left\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{\epsilon} Y_{t+i} \right] \right\}.$$

The first-order condition of this problem is:

$$\frac{\partial V_t(j)}{\partial P_t^*(j)} : \mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^i \left\{ \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[\left(\frac{P_t^*(j)}{P_{t+i}} \right)^{-\epsilon} - \epsilon \left(\frac{P_t^*(j)}{P_{t+i}} - mc_{t+i} \right) \left(\frac{P_t^*(j)}{P_{t+i}} \right)^{-(\epsilon+1)} \right] \frac{Y_{t+i}}{P_{t+i}} \right\} = 0 \tag{2.96}$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[(1-\epsilon)(P_t^*(j))^{-\epsilon} P_{t+i}^{\epsilon-1} + \epsilon mc_{t+i} (P_t^*(j))^{-(\epsilon-1)} P_{t+i}^{\epsilon} \right] Y_{t+i} = 0$$

$$\mathbb{E}_t \sum_{i=0}^{\infty} (\beta\theta)^t \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \left[(1-\epsilon)(P_t^*(j))^{-\epsilon} P_{t+i}^{\epsilon-1} Y_{t+1} + \epsilon mc_{t+i} P_{t+i}^{\epsilon} Y_{t+i} \right] = 0$$

After rearranging, the result of this maximisation problem determines that retail firms, which have obtained a successful draw, will set their price as a constant mark-up on an expression related to their expected discounted nominal total costs relative to an expression related to their expected discounted real output.

$$P_t^*(j) = \frac{\epsilon}{\epsilon - 1} \left[\frac{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} mc_{t+i} P_{t+i}^{\epsilon} Y_{t+i}}{\mathbb{E}_t \sum_{i=0}^{\infty} (\beta_p \theta)^i \lambda_{p,t+i} P_{t+i}^{\epsilon-1} Y_{t+i}} \right]. \tag{2.97}$$

The above equation does not depend on j, this implies that every retailer firm that is able to set its price in period t will choose the same price. Moreover, in the limiting case, with no price rigidity, the firm's optimal price is a constant markup on real marginal costs. This expression can be written in terms of two auxiliary variables, $x_{1,t}$ and $x_{2,t}$:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1}\frac{x_{1,t}}{x_{2,t}}, \tag{2.98}$$

where the auxiliary variables take the following recursive forms:

$$x_{1,t} = \lambda_{p,t} mc_t Y_t + \theta\beta_p \mathbb{E}_t(\pi_{t+1})^{\epsilon} x_{1,t+1}. \tag{2.99}$$

$$x_{2,t} = \lambda_{p,t} Y_t + \theta\beta_p \mathbb{E}_t(\pi_{t+1})^{\epsilon-1} x_{2,t+1}. \tag{2.100}$$

Next, I define an auxiliary variable ν_t^j for the measure of price dispersion:

$$\nu_t^j = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj. \tag{2.101}$$

Making use of the fact that a proportion of firms are able to reset their price, while others are not, the price dispersion can be re-written as:

$$\nu_t^j = \int_0^{1-\theta} \left(\frac{P_t^*(j)}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_t}\right)^{-\epsilon} dj \tag{2.102}$$

To obtain an expression of the price dispersion in terms of the inflation rate, multiply and divide by powers of P_{t-1} where necessary, given:

$$\nu_t^j = \int_0^{1-\theta} \left(\frac{P_t^*(j)}{P_t}\right)^{-\epsilon} dj + \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} \left(\frac{P_{t-1}}{P_t}\right)^{-\epsilon} dj.$$

Using the definition of π_t^* and of gross inflation π_t, the previous expression becomes:

$$\nu_t^j = (1-\theta)(\pi_t^*)^{-\epsilon} + (\pi_t)^{\epsilon} \int_{1-\theta}^1 \left(\frac{P_{t-1}(j)}{P_{t-1}}\right)^{-\epsilon} dj$$

where the last term, using the definition of the auxiliary variable, is equal to $\theta\nu_{t-1}^j$. Replacement yields the following:

$$\nu_t^j = (1-\theta)(\pi_t^*)^{-\epsilon} + (\pi_t)^{\epsilon}\theta\nu_{t-1}^j. \tag{2.103}$$

Using the definition for the price dispersion, the demand equation for each input, the final good equation, and the fact that it takes one intermediate output unit to make one unit of retail output, I obtain the expression linking aggregate wholesale production with final good production:

$$Y_t = \frac{Y_t^W}{\nu_t^j}$$

2.A.2.4 Capital good firms

The capital good producers maximise:

$$\mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^t \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Big[q_t^K K_t^W - q_t^K (1-\delta_k) K_{t-1}^W + r_t^{K,h} K_t^h - I_t \Big], \tag{2.104}$$

subject to the law of motion of total capital and the definition of aggregate capital.

$$K_t = (1-\delta_k)K_{t-1} + [1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2] I_t, \tag{2.105}$$

$$K_t = K_t^W + K_t^h. \tag{2.106}$$

I write the problem in Lagrangian form as:

$$\begin{aligned} \mathcal{L}_K = \mathbb{E}_0 \sum_{i=0}^{\infty} \beta_p^i \frac{\lambda_{p,t+i}}{\lambda_{p,t}} \Bigg\{ & \Big[q_t^K K_t^W - q_t^K (1-\delta_k) K_{t-1}^W + r_t^{K,h} K_t^h - I_t \Big] - \\ & \lambda_{K,t} \Big[K_t^W + K_t^h - (1-\delta_k)(K_{t-1}^W + K_{t-1}^h) - \Big(1 - \frac{\psi}{2}\big(\frac{I_t}{I_{t-1}} - 1\big)^2\Big) I_t \Big] \Bigg\} \end{aligned} \tag{2.107}$$

The optimality conditions with respect to K_t^W, K_t^h and I_t are:

$$\frac{\partial \mathcal{L}_K}{\partial K_t^W} : q_t^K - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} (1-\delta_k) q_{t+1}^K = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} (1-\delta_k) \lambda_{K,t+1}. \tag{2.108}$$

$$\frac{\partial \mathcal{L}_K}{\partial K_t^h} : r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}} (1-\delta_k) \lambda_{K,t+1}. \tag{2.109}$$

$$\begin{aligned} \frac{\partial \mathcal{L}_K}{\partial I_t} : 1 = \lambda_{K,t} & \Bigg[1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 - \psi\Big(\frac{I_t}{I_{t-1}} - 1\Big)\Big(\frac{I_t}{I_{t-1}}\Big) \Bigg] + \\ & \beta_p \psi \mathbb{E}_t \Bigg[\frac{\lambda_{p,t+1}}{\lambda_{p,t}} \lambda_{K,t+1} \Big(\frac{I_{t+1}}{I_t}\Big)^2 \Big(\frac{I_{t+1}}{I_t} - 1\Big) \Bigg], \end{aligned} \tag{2.110}$$

2.A.2.5 Housing firms

This firm's profit maximisation is a static problem and can be stated as:

$$\max_{N_t^h, K_{t-1}^h} \Pi_t^h = [q_t I_t^h - r_t^{K,h} K_{t-1}^h - w_t N_t^h], \tag{2.111}$$

subject to:

$$I_t^h = Z_t N_t^{h\,1-\mu_h} K_{t-1}^{h\ \ \mu_h}, \tag{2.112}$$

After replacing the production function in the profit expression, I re-write the problem as following:

$$\max_{N_t^h, K_{t-1}^h} \Pi_t^h = [q_t(Z_t N_t^{h\,1-\mu_h} K_{t-1}^{h\ \ \mu_h}) - r_t^{K,h} K_{t-1}^h - w_t N_t^h], \tag{2.113}$$

The first-order conditions of this maximisation problem with respect to N_t^h and K_t^h are:

$$w_t = (1-\mu_h) q_t \frac{I_t^h}{N_t^h}. \tag{2.114}$$

$$r_t^{K,h} = \mu_h q_t \frac{I_t^h}{K_t^h}. \tag{2.115}$$

2.A.3 Banks

To solve the optimisation problem of bank τ, I write it in a Bellman equation form as:

$$V_{\tau,t}^B(NW_{i,t}) = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \{(1-\sigma) NW_{\tau,t} + \sigma \max V_{\tau,t+1}^B(NW_{\tau,t+1})\}, \tag{2.116}$$

is subject to:

$$q_t^K S_{\tau,t} = NW_{\tau,t} + D_{\tau,t},$$
$$NW_{\tau,t+1} = \left(R_{t+1}^K - R_t^d\right) S_{\tau,t} + R_t^d NW_{\tau,t},$$
$$V_{\tau,t}^B \geq \zeta(q_{t,f}^k S_{\tau,t}).$$

I start guessing that the value function $V_{\tau,t}^B$ is linear in $NW_{\tau,t}$, $V_{\tau,t}^B = \nu_t^B NW_{\tau,t}$, where ν_t^B depends only on aggregate quantities. Then, I replace the balance sheet in the evolution of the net worth equation, which then I plug into the Bellman equation. The problem now is to maximise the new Bellman equation subject to the incentive constraint. I re-express the bank's i problem using the Lagrangian as:

$$\mathcal{L}_B = \Big[(1-\sigma+\sigma\nu_{t+1}^B)\Big(\Big(R_{t+1}^K - R_t^d\Big) q_t^K S_{\tau,t} + R_t^d NW_{\tau,t}\Big)\Big](1+\xi_t) - \xi_t(\zeta(q_t^K S_{\tau,t})), \tag{2.117}$$

where ξ_t is the Lagrange multiplier with respect to the incentive constraint, and the first order condition with respect to $S_{\tau,t}$ and $NW_{\tau,t}$ are:

$$\frac{\partial \mathcal{L}_B}{\partial S_{\tau,t}} : \frac{\xi_t \zeta}{(1+\xi_t)} = \mathbb{E}_t\big[(1-\sigma+\sigma\nu_{t+1}^b)\big(R_{t+1}^K - R_t^d\big)\big]. \tag{2.118}$$

$$\frac{\partial \mathcal{L}_B}{\partial NW_{\tau,t}} : \frac{1}{(1+\xi_t)} = \mathbb{E}_t\Big[(1-\sigma+\sigma\nu^b_{t+1})\Big(\frac{R^d_t}{\pi_{t+1}}\Big)\Big]. \tag{2.119}$$

Defining the adjusted marginal value of the net worth as $\Omega_{\tau,t+1} = (1-\sigma+\sigma\nu^b_{t+1})$, the value function can be re-expressed as:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}}\Big\{\Omega_{\tau,t+1}\Big[\Big(R^K_{t+1} - R^d_t\Big)q^K_t S_{\tau,t} + R^d_t NW_{\tau,t}\Big]\Big\}$$

Multiplying and dividing this expression by $NW_{\tau,t}$ I obtain:

$$V^B_{\tau,t} = \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}}\Big\{\Omega_{\tau,t+1}\Big[\Big(R^K_{t+1} - R^d_t\Big)\phi_t + R^d_t\Big]\Big\}NW_{\tau,t}, \tag{2.120}$$

where $\phi_t = \frac{q^K_t S_{\tau,t}}{NW_{\tau,t}}$ and the term between the curly brackets is ν^b_t. Therefore, if the incentive constraint is binding, i.e. $\nu^b_t = \zeta\phi_t$, replacing the previous result:

$$\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}}\Big\{\Omega_{\tau,t+1}\Big[\Big(R^K_{t+1} - R^d_t\Big)\phi_t + R^d_t\Big]\Big\} = \zeta\phi_t.$$

This, after rearranging, implies that the leverage is equal to:

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}}\Omega_{\tau,t+1}R^d_t}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}}\Omega_{\tau,t+1}\Big(R^K_{t+1} - R^d_t\Big)}. \tag{2.121}$$

2.A.4 Equilibrium conditions

The model is characterised by 47 equations, with 43 endogenous variables $\{\lambda_{p,t}, c_{p,t}, n_{p,t}, h_{p,t}, d^B_{p,t}, d^l_{p,t}, \lambda_{i,t}, c_{i,t}, n_{i,t}, h_{i,t}, l_{i,t}, \mu_{i,t}, I_t, K_t, K^W_t, K^h_t, \lambda_{K,t}, q^K_t, I^h_t, H_t, q_t, r^K_t, r^{K,h}_t, w_t, R^d_t, R^l_t, R^K_t, mc_t, N_t, N^W_t, N^h_t, C_t, Y_t, x_{1,t}, x_{2,t}, \pi_t, \pi^*_t, \nu^j_t, \phi_t, \Omega_t, NW_t, D_t, S_t\}$ and 4 exogenous shocks $\{A_t, Z_t, M_t, \Gamma_t\}$.

2.A.4.1 Patient Households

$$\lambda_{p,t} = \frac{1}{(c_{p,t} - \gamma c_{p,t-1})} - \frac{\beta_p\gamma}{(c_{p,t+1} - \gamma c_{p,t})}. \tag{2.122}$$

$$\nu^n_p n^\varphi_{p,t} = w_t\lambda_{p,t}. \tag{2.123}$$

$$\lambda_{p,t}q_t = \frac{\Gamma_t\nu^h_p}{h_{p,t}} + \beta_p\mathbb{E}_t\Big[(1-\delta_h)q_{t+1}\lambda_{p,t+1}\Big]. \tag{2.124}$$

$$\lambda_{p,t} = \beta_p\mathbb{E}_t\left[\lambda_{p,t+1}\frac{R^d_t}{\pi_{t+1}}\right]. \tag{2.125}$$

$$\lambda_{p,t} = \beta_p \mathbb{E}_t \left[\lambda_{p,t+1} \frac{R^l_t}{\pi_{t+1}} \right]. \tag{2.126}$$

$$c_{p,t} + q_t[h_{p,t} - (1-\delta_h)h_{p,t-1}] + d^B_t + d^l_t = \frac{d^B_{t-1}R^d_{t-1}}{\pi_t} + \frac{d^l_{t-1}R^l_{t-1}}{\pi_t} + w_t n_{p,t} + \Pi_{f,t} + \Pi_{B,t}. \tag{2.127}$$

2.A.4.2 Impatient Households

$$\lambda_{i,t} = \frac{1}{(c_{i,t} - \gamma c_{i,t-1})} - \frac{\beta_i \gamma}{(c_{i,t+1} - \gamma c_{i,t})}. \tag{2.128}$$

$$\nu^n_i n^{\varphi}_{i,t} = w_t \lambda_{i,t}. \tag{2.129}$$

$$\lambda_{i,t} q_t = \frac{\Gamma_t \nu^h_i}{h_{i,t}} + \beta_i \mathbb{E}_t \Big[(1-\delta_h) q_{t+1} \lambda_{i,t+1} \Big] + \mu_{i,t} \frac{\chi}{R^l_t} \mathbb{E}_t [q_{t+1} \pi_{t+1}]. \tag{2.130}$$

$$\lambda_{i,t} - \mu_{i,t} = \beta_i \mathbb{E}_t \left[\lambda_{i,t+1} \frac{R^l_t}{\pi_{t+1}} \right], \tag{2.131}$$

$$c_{i,t} + q_t(h_{i,t} - (1-\delta_h)h_{i,t-1}) + \frac{l_{t-1}R^l_{t-1}}{\pi_t} = w_t n_{i,t} + l_t. \tag{2.132}$$

$$l_t \leq \frac{\chi}{R^l_t} \mathbb{E}_t [q_{t+1} \pi_{t+1}] h_{i,t}. \tag{2.133}$$

2.A.4.3 Goods firms

$$Y^W_t = A_t {N^W_t}^{1-\alpha} {K^W_{t-1}}^{\alpha}. \tag{2.134}$$

$$\nu^j_t = (1-\theta)(\pi^*_t)^{-\epsilon} + (\pi_t)^{\epsilon} \theta \nu^j_{t-1}. \tag{2.135}$$

$$mc_t = \frac{1}{A_t} \left(\frac{w_t}{1-\alpha} \right)^{1-\alpha} \left(\frac{r^K_t}{\alpha} \right)^{\alpha}. \tag{2.136}$$

$$\frac{r^K_t}{w_t} = \frac{\alpha N^W_t}{(1-\alpha) K^W_{t-1}} \tag{2.137}$$

$$q^K_{t-1} R^K_t = r^K_t + (1-\delta_k) q^K_t. \tag{2.138}$$

I define two auxiliary variables to reexpress pricing as:

$$\pi^*_t = \frac{\epsilon}{\epsilon - 1} \frac{x_{1,t}}{x_{2,t}}. \tag{2.139}$$

These variables have a recursive representation given by:

$$x_{1,t} = \lambda_{p,t} mc_t Y_t + \theta \beta_p \mathbb{E}_t (\pi_{t+1})^{\epsilon} x_{1,t+1}. \tag{2.140}$$

$$x_{2,t} = \lambda_{p,t} Y_t + \theta \beta_p \mathbb{E}_t (\pi_{t+1})^{\epsilon - 1} x_{2,t+1}. \tag{2.141}$$

$$\pi_t^{1-\epsilon} = \theta + (1-\theta)\left(\pi_t^*\right)^{1-\epsilon}. \tag{2.142}$$

2.A.4.4 Housing firms

$$I_t^h = Z_t {N_t^h}^{1-\mu_h} {K_{t-1}^h}^{\mu_h}, \tag{2.143}$$

$$\frac{r_t^{K,h}}{w_t} = \frac{\mu_h N_t^h}{(1-\mu_h)K_{t-1}^h} \tag{2.144}$$

2.A.4.5 Capital firms

$$q_t^K - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)q_{t+1}^K = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{2.145}$$

$$r_t^{K,h} = \lambda_{K,t} - \beta_p \frac{\lambda_{p,t+1}}{\lambda_{p,t}}(1-\delta_k)\lambda_{K,t+1}. \tag{2.146}$$

$$\frac{1}{\lambda_{K,t}} = 1 - \frac{\psi}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 - \psi\Big(\frac{I_t}{I_{t-1}} - 1\Big)\Big(\frac{I_t}{I_{t-1}}\Big) + \beta_p \psi \mathbb{E}_t\left[\frac{\lambda_{p,t+1}}{\lambda_{p,t}}\lambda_{K,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\Big(\frac{I_{t+1}}{I_t} - 1\Big)\right]. \tag{2.147}$$

2.A.4.6 Banks

$$\Omega_{\tau,t+1} = (1 - \sigma + \sigma\zeta\phi_t). \tag{2.148}$$

$$q_t^K S_t = \phi_t NW_t. \tag{2.149}$$

$$NW_t = (\sigma + \omega)(R_t^K q_{t-1}^K S_{t-1}) - \sigma R_{t-1}^d D_{t-1}. \tag{2.150}$$

$$\phi_t = \frac{\beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1} R_t^d}{\zeta - \beta_B \mathbb{E}_t \frac{\lambda_{p,t+1}}{\lambda_{p,t}} \Omega_{\tau,t+1}\Big(R_{t+1}^K - R_t^d\Big)}. \tag{2.151}$$

$$q_t^K S_t = NW_t + D_t. \tag{2.152}$$

2.A.4.7 Central Bank

$$R_t^d = (1/\beta_p)\left(\frac{\pi_t}{\bar{\pi}}\right)^{\omega_\pi}\left(\frac{GDP_t}{GDP_{t-1}}\right)^{\omega_y} M_t, \tag{2.153}$$

2.A.4.8 Aggregation

$$C_t = (1-n)c_{p,t} + (n)c_{i,t} \tag{2.154}$$

$$N_t = (1-n)n_{p,t} + (n)n_{i,t}. \tag{2.155}$$

$$H_t = (1-n)h_{p,t} + (n)h_{i,t} \tag{2.156}$$

$$N_t = N_t^W + N_t^h. \tag{2.157}$$

$$Y_t = \frac{Y_t^W}{\nu_t^j} \tag{2.158}$$

$$GDP_t = C_t + I_t + \bar{q} I_t^h. \tag{2.159}$$

$$D_t = (1-n) d_t^B \tag{2.160}$$

$$(1-n) d_t^l = n l_t \tag{2.161}$$

$$S_t = K_t^W \tag{2.162}$$

$$K_t = (1-\delta_k) K_{t-1} + \left[1 - \frac{\psi}{2}(I_t/I_{t-1} - 1)^2\right] I_t \tag{2.163}$$

$$K_t = K_t^W + K_t^h \tag{2.164}$$

$$I_t^h = H_t - (1-\delta_h) H_{t-1} \tag{2.165}$$

2.A.4.9 Shocks

$$log(A_{t+1}) = \rho_A log(A_t) + \sigma_{\epsilon^A} \epsilon_{t+1}^A. \tag{2.166}$$

$$log(Z_{t+1}) = \rho_Z log(Z_t) + \sigma_{\epsilon^Z} \epsilon_{t+1}^Z. \tag{2.167}$$

$$log(M_{t+1}) = \rho_M log(M_t) + \sigma_{\epsilon^M} \epsilon_{t+1}^M. \tag{2.168}$$

$$log(\Gamma_{t+1}) = \rho_\Gamma log(\Gamma_t) + \sigma_{\epsilon^\Gamma} \epsilon_{t+1}^\Gamma. \tag{2.169}$$

2.A.5 Steady State

2.A.5.1 Patient

$$R^d = \frac{1}{\beta_p}. \tag{2.170}$$

$$\lambda_p = \frac{(1-\beta_p \gamma)}{(1-\gamma) c_p}. \tag{2.171}$$

$$\nu_p^n n_p^\varphi = w \lambda_p. \tag{2.172}$$

$$\frac{1}{h_p \lambda_p q} = \frac{1 - \beta_p (1-\delta_h)}{\nu_p^h}. \tag{2.173}$$

2.A.5.2 Impatient

$$\lambda_i = \frac{(1-\beta_i \gamma)}{(1-\gamma) c_i}. \tag{2.174}$$

$$\nu_i^n n_i^\varphi = w \lambda_i. \tag{2.175}$$

$$\mu_i = (1 - \beta_i R^l) \lambda_i. \tag{2.176}$$

$$\frac{1}{h_i \lambda_i q} = \frac{1 - \beta_i(1-\delta_h) - (1-\beta_i R^l)\frac{\chi}{R^l}}{\nu_i^h}. \tag{2.177}$$

$$c_i = wn_i + (1 - R^l)l - qh_i\delta_h. \tag{2.178}$$

$$\frac{lR^l}{\chi} = qh_i. \tag{2.179}$$

2.A.5.3 Goods firms

$$\frac{r^K}{w} = \frac{\alpha N^W}{(1-\alpha)K^W}. \tag{2.180}$$

$$R^K = r^K + (1 - \delta_k). \tag{2.181}$$

$$mc = \left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r^K}{\alpha}\right)^{\alpha}. \tag{2.182}$$

$$\frac{Y}{K^W} = \left(\frac{N^W}{K^W}\right)^{1-\alpha}. \tag{2.183}$$

$$x_1 = \frac{\lambda_p mcY}{1-\theta\beta}. \tag{2.184}$$

$$x_2 = \frac{\lambda_p Y}{1-\theta\beta}. \tag{2.185}$$

$$\pi^* = \frac{\epsilon}{\epsilon - 1}\frac{x_1}{x_2}. \tag{2.186}$$

$$\pi^{1-\epsilon} = \theta + (1-\theta)\left(\pi^*\right)^{1-\epsilon}. \tag{2.187}$$

$$\nu^j = \frac{(1-\theta)(\pi^*)^{-\epsilon}}{1-\theta\pi^{\epsilon}}. \tag{2.188}$$

2.A.5.4 Housing firms

$$I^h = N^{h\,1-\mu_h} K^{h\,\mu_h}. \tag{2.189}$$

$$\frac{r^{K,h}}{w} = \frac{\mu_h N^h}{(1-\mu_h)K^h}. \tag{2.190}$$

2.A.5.5 Capital firms

$$q^K = 1. \tag{2.191}$$

$$r^{K,h} = 1 - \beta_p(1 - \delta_k). \tag{2.192}$$

$$\frac{1}{\lambda_K} = 1. \tag{2.193}$$

2.A.5.6 Banks

$$\Omega = (1 - \sigma + \sigma\zeta\phi). \tag{2.194}$$

$$\frac{NW}{K^W} = \frac{1}{\phi}. \tag{2.195}$$

$$\frac{NW}{K^W} = 1 - \frac{D}{K^W}. \tag{2.196}$$

$$\frac{NW}{K^W} = \frac{(\sigma + \omega)R^K - \frac{\sigma}{\beta_p}}{1 - \frac{\sigma}{\beta_p}} = \frac{1}{\phi}. \tag{2.197}$$

$$\phi = \frac{\beta_p \Omega R^d}{\zeta - \beta_p \Omega (R^K - R^d)}. \tag{2.198}$$

2.A.5.7 Central Bank

$$R^d = \frac{1}{\beta_p}. \tag{2.199}$$

2.A.5.8 Aggregation

$$C = (1 - n)c_p + (n)c_i. \tag{2.200}$$

$$N = (1 - n)n_p + (n)n_i. \tag{2.201}$$

$$H = (1 - n)h_p + (n)h_i. \tag{2.202}$$

$$N = N^f + N^h. \tag{2.203}$$

$$Y = Y^W. \tag{2.204}$$

$$GDP = C + I + qI^h. \tag{2.205}$$

$$D = (1 - n)d^B. \tag{2.206}$$

$$(1 - n)d^l = nl. \tag{2.207}$$

$$S = K^W. \tag{2.208}$$

$$\delta_k = \frac{I}{K}. \tag{2.209}$$

$$K = K^W + K^h. \tag{2.210}$$

$$\delta_h = \frac{I^h}{H}. \tag{2.211}$$

2.B Diagnostic probability density function

To obtain the diagnostic probability density function of the economy's state, I use the assumption that it follows an AR(1) process and that a standard probability density function of a normally distributed variable x_{t+1} is:

$$f(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x_{t+1}-\rho x_t)^2}{2\sigma^2}}.$$

Recalling the definition of the diagnostic probability density function under a slow-moving reference group $f^{\phi}(x_{t+1}) =$
$f(x_{t+1}|x_t = \rho\bar{x}_t)\left\{\left[\prod_{s=1}^{S}\frac{f(x_{t+1}|\rho^s\bar{x}_{t+1-s})}{f(x_{t+1}|\rho^{s+1}\bar{x}_{t-s})}\right]^{\alpha_s}\right\}^{\phi} Z$, and using the previous expression:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}}\left\{\left[\prod_{s=1}^{S}\frac{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x_{t+1}-\rho^s\bar{x}_{t+1-s})^2}{2\sigma^2}}}{\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2}{2\sigma^2}}}\right]^{\alpha_s}\right\}^{\phi} Z, \tag{2.212}$$

Simplifying and rewriting, I obtain:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}}e^{\left\{-\frac{(x_{t+1}-\rho\bar{x}_t)^2}{2\sigma^2}-\frac{1}{2\sigma^2}\phi\left[\sum_{s=1}^{S}\alpha_s\left((x_{t+1}-\rho^s\bar{x}_{t+1-s})^2-(x_{t+1}-\rho^{s+1}\bar{x}_{t-s})^2\right)\right]\right\}}Z. \tag{2.213}$$

I expand and re-write the argument of the exponential as:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}}exp\Bigg(-\frac{1}{2\sigma^2}\Big\{&(x_{t+1}^2 - 2x_{t+1}\rho\bar{x}_t + (\rho\bar{x}_t)^2)+ \\ &\phi\Big[\sum_{s=1}^{S}\alpha_s\big((x_{t+1}^2 - 2x_{t+1}\rho^s\bar{x}_{t+1-s} + (\rho^s\bar{x}_{t+1-s})^2)- \\ &(x_{t+1}^2 - 2x_{t+1}\rho^{s+1}\bar{x}_{t-s} + (\rho^{s+1}\bar{x}_{t-s})^2)\big)\Big]\Big\}\Bigg)Z. \end{aligned} \tag{2.214}$$

This can be further expanded and rearranged, after taking $2x$ as common factor:

$$\begin{aligned} f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}}exp\Bigg(-\frac{1}{2\sigma^2}\Bigg\{&x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\left[\sum_{s=1}^{S}\alpha_s\left(\rho^s\bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s}\right)\right]\right]+ \\ &(\rho\bar{x}_t)^2 + \phi\left[\sum_{s=1}^{S}\alpha_s\left((\rho^s\bar{x}_{t+1-s})^2 - (\rho^{s+1}\bar{x}_{t-s})^2\right)\right]\Bigg\}\Bigg)Z, \end{aligned} \tag{2.215}$$

where the constant Z is given by:

$$Z = exp\Bigg(-\frac{1}{2\sigma^2}\Bigg\{ -\phi \left[\sum_{s=1}^{S} \alpha_s \left((\rho^s \bar{x}_{t+1-s})^2 - (\rho^{s+1}\bar{x}_{t-s})^2 \right) \right] + 2\rho\bar{x}_t\phi \left[\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right] + \phi^2 \left[\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right]^2 \Bigg\}\Bigg). \tag{2.216}$$

After some algebra, the diagnostic pdf is equal to:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{ \left[x_{t+1} - \left(\rho\bar{x}_t + \phi\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right) \right]^2 \right\}}. \tag{2.217}$$

This, as Gennaioli and Shleifer (2018) states, contains the kernel of a normal distribution with a distorted mean and the same variance. Therefore:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \mathbb{E}_t(x_{t+1}) + \phi \sum_{s=1}^{S} \alpha_s \left[\mathbb{E}_{t+1-s}(x_{t+1}) - \mathbb{E}_{t-s}(x_{t+1}) \right]. \tag{2.218}$$

Expression (2.217) can be extended in order to re-write it in terms of the realisations of the shocks:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ -\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right] + (\rho\bar{x}_t)^2 + 2\rho\bar{x}_t\phi \left[\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right] + \phi^2 \left[\sum_{s=1}^{S} \alpha_s \left(\rho^s \bar{x}_{t+1-s} - \rho^{s+1}\bar{x}_{t-s} \right) \right]^2 \Big\} \Bigg\},$$

which can be rewritten using the AR(1) process definition as:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ -\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x_{t+1}\left[\rho\bar{x}_t + \phi\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1} \right] + (\rho\bar{x}_t)^2 + 2\rho\bar{x}_t\phi \left[\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1} \right] + \phi^2 \left[\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1} \right]^2 \Bigg\},$$

This can be rearranged as:

$$f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}\left\{ \left[x_{t+1} - \left(\rho\bar{x}_t + \phi\sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1} \right) \right]^2 \right\}}. \tag{2.219}$$

Again, this function contains the kernel of a normal distribution with a distorted mean:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi \sum_{s=1}^{S} \rho^s \alpha_s \epsilon_{t+s-1}. \tag{2.220}$$

This way of modelling DE with slow-moving reference embeds the one from Bianchi et al. (2024) as a special case. This occurs when $\alpha_1 = 1$ and the rest are such that $\sum_{s=1}^{S} \alpha_s' = 1$, where $\alpha_s' = (\alpha_s - \alpha_{s+1})$. In that case, the diagnostic expectation will be defined as:

$$\mathbb{E}_t^{\phi}(X_{t+1}) = \mathbb{E}_t(X_{t+1}) + \phi\left[\mathbb{E}_t(X_{t+1}) - \mathbb{E}_t^{r}(X_{t+1})\right], \tag{2.221}$$

where $\mathbb{E}_t^r(X_{t+1}) = \sum_{s=1}^{S} \alpha_s' \mathbb{E}_{t-s}(X_{t+1})$

2.B.1 Diagnostic distribution using last twelve-quarters as reference

Using the previous result, if the Diagnostic agent form expectations taking into account the last twelve-quarters, I obtain the following probability density function:

$$\begin{gathered}
f^{\phi}(x_{t+1}|x_t) = \frac{1}{\sigma\sqrt{2\pi}} exp\Bigg\{ -\frac{1}{2\sigma^2}\Big\{ x_{t+1}^2 - 2x[\rho\bar{x}_t + \phi[\rho\alpha_1(\bar{x}_t - \rho\bar{x}_{t-1}) + \rho^2\alpha_2(\bar{x}_{t-1} - \rho\bar{x}_{t-2}) + \\
\rho^3\alpha_3(\bar{x}_{t-2} - \rho\bar{x}_{t-3}) + \rho^4\alpha_4(\bar{x}_{t-3} - \rho\bar{x}_{t-4}) + \rho^5\alpha_5(\bar{x}_{t-4} - \rho\bar{x}_{t-5}) + \rho^6\alpha_6(\bar{x}_{t-5} - \rho\bar{x}_{t-6}) + \\
\rho^7\alpha_7(\bar{x}_{t-6} - \rho\bar{x}_{t-7}) + \rho^8\alpha_8(\bar{x}_{t-7} - \rho\bar{x}_{t-8}) + \rho^9\alpha_9(\bar{x}_{t-8} - \rho\bar{x}_{t-9}) + \rho^{10}\alpha_{10}(\bar{x}_{t-9} - \rho\bar{x}_{t-10}) + \\
\rho^{11}\alpha_{11}(\bar{x}_{t-10} - \rho\bar{x}_{t-11}) + \rho^{12}\alpha_{12}(\bar{x}_{t-11} - \rho\bar{x}_{t-12})] + (\rho\bar{x}_t)^2 + 2\rho x_t\phi[\rho\alpha_1(\bar{x}_t - \rho\bar{x}_{t-1}) + \\
\rho^2\alpha_2(\bar{x}_{t-1} - \rho\bar{x}_{t-2}) + \rho^3\alpha_3(\bar{x}_{t-2} - \rho\bar{x}_{t-3}) + \rho^4\alpha_4(\bar{x}_{t-3} - \rho\bar{x}_{t-4}) + \rho^5\alpha_5(\bar{x}_{t-4} - \rho\bar{x}_{t-5}) + \\
\rho^6\alpha_6(\bar{x}_{t-5} - \rho\bar{x}_{t-6}) + \rho^7\alpha_7(\bar{x}_{t-6} - \rho\bar{x}_{t-7}) + \rho^8\alpha_8(\bar{x}_{t-7} - \rho\bar{x}_{t-8}) + \rho^9\alpha_9(\bar{x}_{t-8} - \rho\bar{x}_{t-9}) + \\
\rho^{10}\alpha_{10}(\bar{x}_{t-9} - \rho\bar{x}_{t-10}) + \rho^{11}\alpha_{11}(\bar{x}_{t-10} - \rho\bar{x}_{t-11}) + \rho^{12}\alpha_{12}(\bar{x}_{t-11} - \rho\bar{x}_{t-12})] + \\
\phi^2[\rho\alpha_1(\bar{x}_t - \rho\bar{x}_{t-1}) + \rho^2\alpha_2(\bar{x}_{t-1} - \rho\bar{x}_{t-2}) + \rho^3\alpha_3(\bar{x}_{t-2} - \rho\bar{x}_{t-3}) + \rho^4\alpha_4(\bar{x}_{t-3} - \rho\bar{x}_{t-4}) + \\
\rho^5\alpha_5(\bar{x}_{t-4} - \rho\bar{x}_{t-5}) + \rho^6\alpha_6(\bar{x}_{t-5} - \rho\bar{x}_{t-6}) + \rho^7\alpha_7(\bar{x}_{t-6} - \rho\bar{x}_{t-7}) + \rho^8\alpha_8(\bar{x}_{t-7} - \rho\bar{x}_{t-8}) + \\
\rho^9\alpha_9(\bar{x}_{t-8} - \rho\bar{x}_{t-9}) + \rho^{10}\alpha_{10}(\bar{x}_{t-9} - \rho\bar{x}_{t-10}) + \rho^{11}\alpha_{11}(\bar{x}_{t-10} - \rho\bar{x}_{t-11}) + \\
\rho^{12}\alpha_{12}(\bar{x}_{t-11} - \rho\bar{x}_{t-12})]^2 \Big\} \Bigg\},
\end{gathered}$$

which implies:

$$\begin{aligned}
\mathbb{E}_t^{\phi}(\bar{x}_{t+1}) = \rho\bar{x}_t &+ \phi[\rho\alpha_1(\bar{x}_t - \rho\bar{x}_{t-1}) + \rho^2\alpha_2(\bar{x}_{t-1} - \rho\bar{x}_{t-2}) + \rho^3\alpha_3(\bar{x}_{t-2} - \rho\bar{x}_{t-3}) + \rho^4\alpha_4(\bar{x}_{t-3} - \rho\bar{x}_{t-4}) \\
&+ \rho^5\alpha_5(\bar{x}_{t-4} - \rho\bar{x}_{t-5}) + \rho^6\alpha_6(\bar{x}_{t-5} - \rho\bar{x}_{t-6}) + \rho^7\alpha_7(\bar{x}_{t-6} - \rho\bar{x}_{t-7}) + \rho^8\alpha_8(\bar{x}_{t-7} - \rho\bar{x}_{t-8}) \\
&+ \rho^9\alpha_9(\bar{x}_{t-8} - \rho\bar{x}_{t-9}) + \rho^{10}\alpha_{10}(\bar{x}_{t-9} - \rho\bar{x}_{t-10}) + \rho^{11}\alpha_{11}(\bar{x}_{t-10} - \rho\bar{x}_{t-11}) \\
&+ \rho^{12}\alpha_{12}(\bar{x}_{t-11} - \rho\bar{x}_{t-12})].
\end{aligned} \tag{2.222}$$

After using the definition of the AR(1) process:

$$\mathbb{E}_t^{\phi}(x_{t+1}) = \rho x_t + \phi[(\rho\alpha_1\epsilon_t + \rho^2\alpha_2\epsilon_{t-1} + \rho^3\alpha_3\epsilon_{t-2} + \rho^4\alpha_4\epsilon_{t-3} + \rho^5\alpha_5\epsilon_{t-4} + \rho^6\alpha_6\epsilon_{t-5} + \rho^7\alpha_7\epsilon_{t-6} + \rho^8\alpha_8\epsilon_{t-7} + \rho^9\alpha_9\epsilon_{t-8} + \rho^{10}\alpha_{10}\epsilon_{t-9} + \rho^{11}\alpha_{11}\epsilon_{t-10} + \rho^{12}\alpha_{12}\epsilon_{t-11})] \tag{2.223}$$

Thus, agents mistakenly perceive the AR(1) shock as an ARMA(1,12).

2.C Additional results

2.C.1 Posterior distributions and historical decomposition

This subsection presents figures showing the posterior distributions from the SMC of the DE model with twelve-quarters reference, DE model with one-quarter reference, and the RE model. It also exhibits the historical decomposition for the RE model and the DE model with twelve-quarters reference.

2.D Definition of data variables

I calibrate the model using quarterly data for the United States. I obtained the data from the Board of Governors of the Federal Reserve System and the Bureau of Economic Analysis, using the National Accounts and Flow of Funds. I also use the Census Bureau House Price Index. The sample period begins in 1984:Q1 and ends in 2019:Q4, i.e. pre-pandemic. The variables that I use are:

Output

Data: Real Gross Domestic Product (Billions of chained 2012 Dollars, seasonally adjusted annual rate). The series is adjusted by the civilian non-institutional population. The result is log-transformed, detrended using the first difference, and demeaned. Source: Board of Governors of the Federal Reserve System.

Inflation

Data: Implicit Price Deflator (Index 2012 = 100, seasonally adjusted annual rate). The series is in quarter-on-quarter log differences and is demeaned. Source: Board of Governors of the Federal Reserve System.

Nominal short-term interest rate

Data: Federal funds rate. Quarterly average of the monthly series. During the zero lower bound period, the Wu-Xia shadow federal funds rate is used. Source: Board of Governors of the Federal Reserve System and Wu and Xia (2016).

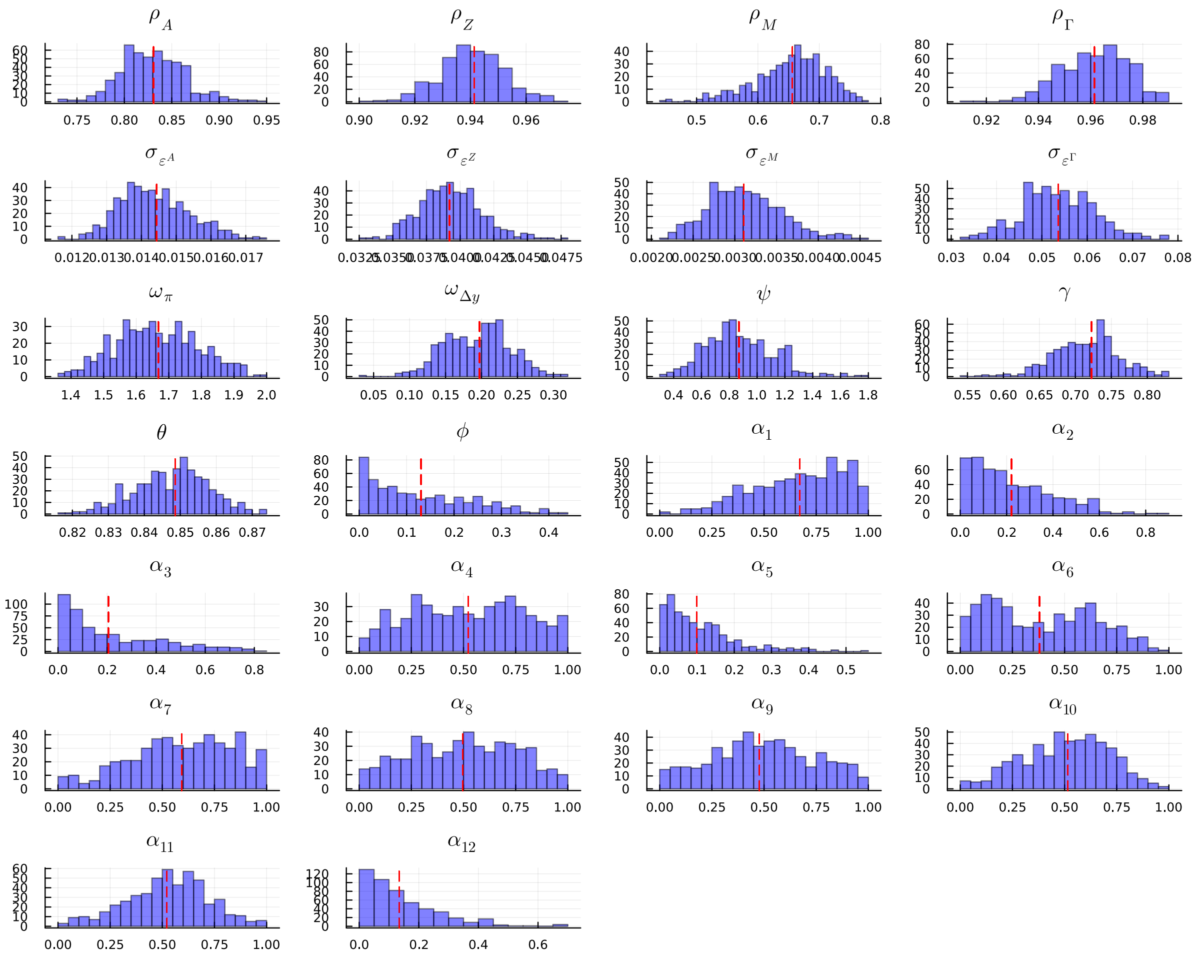

Figure 2.9: Posterior distributions parameters DE model with twelve-quarters reference. Note: The red dashed line represents the mean of each posterior distribution.

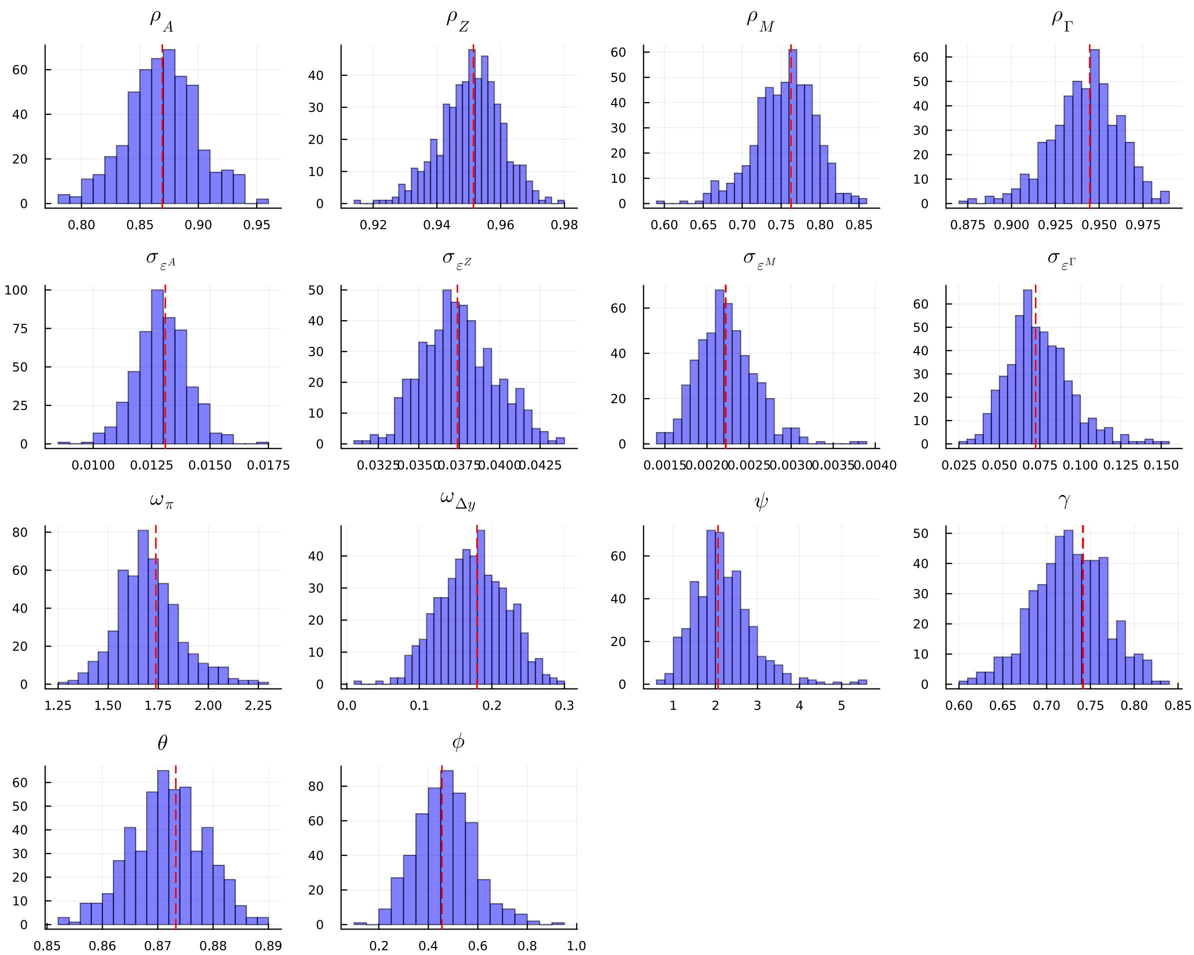

Figure 2.10: Posterior distributions parameters DE model with one-quarter reference.
Note: The red dashed line represents the mean of each posterior distribution.

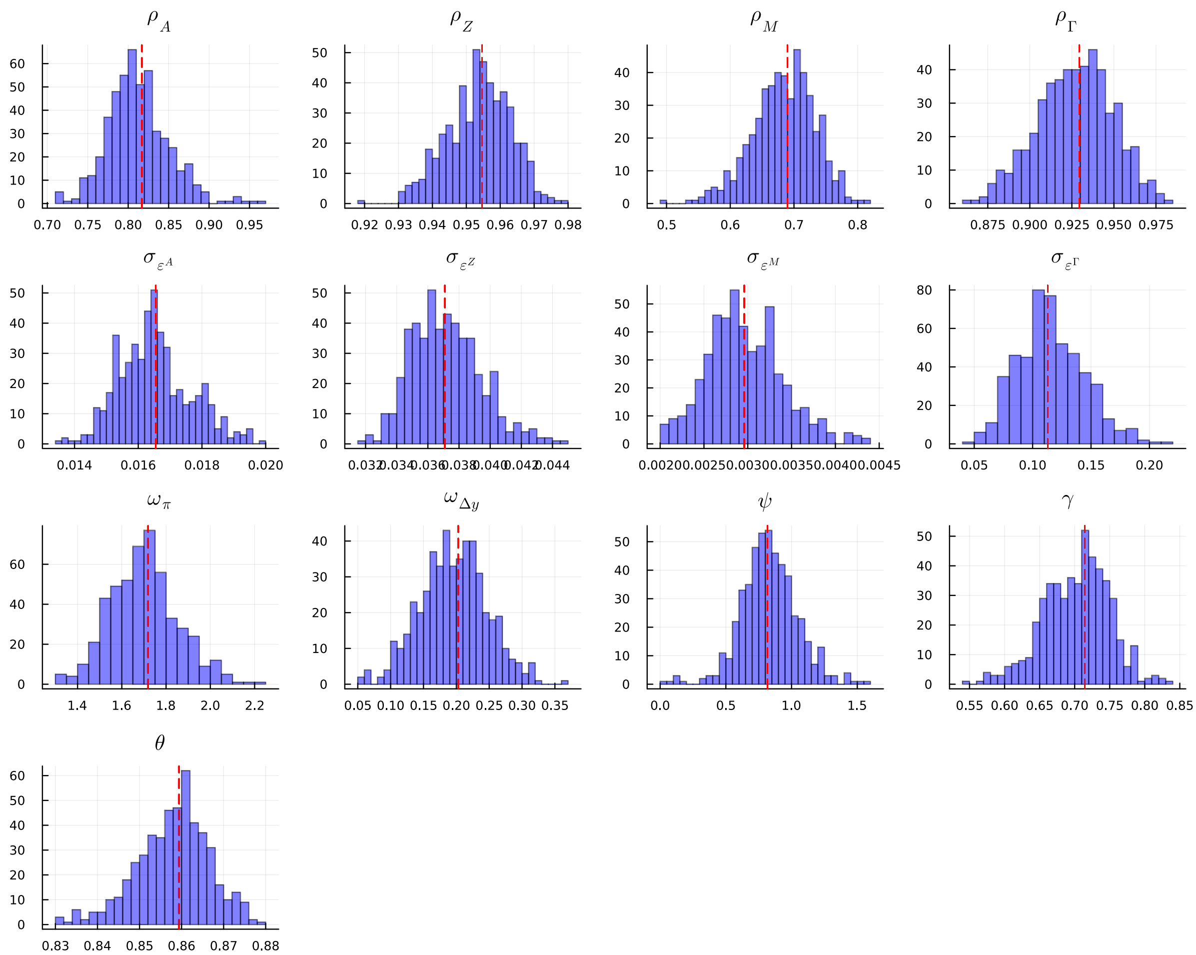

Figure 2.11: Posterior distributions parameters RE model.
Note: The red dashed line represents the mean of each posterior distribution.

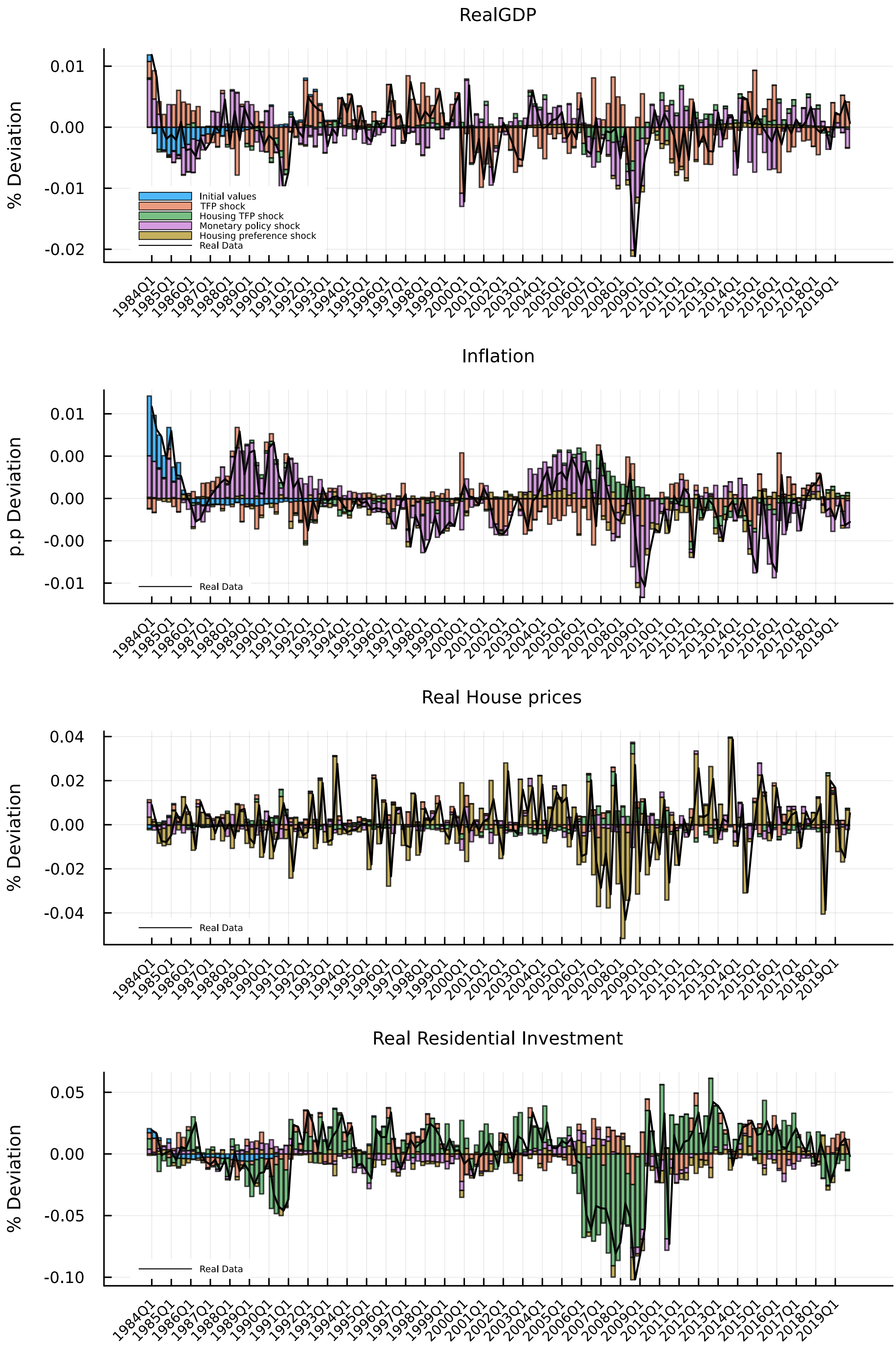

Figure 2.12: Historical shock decomposition under RE model.

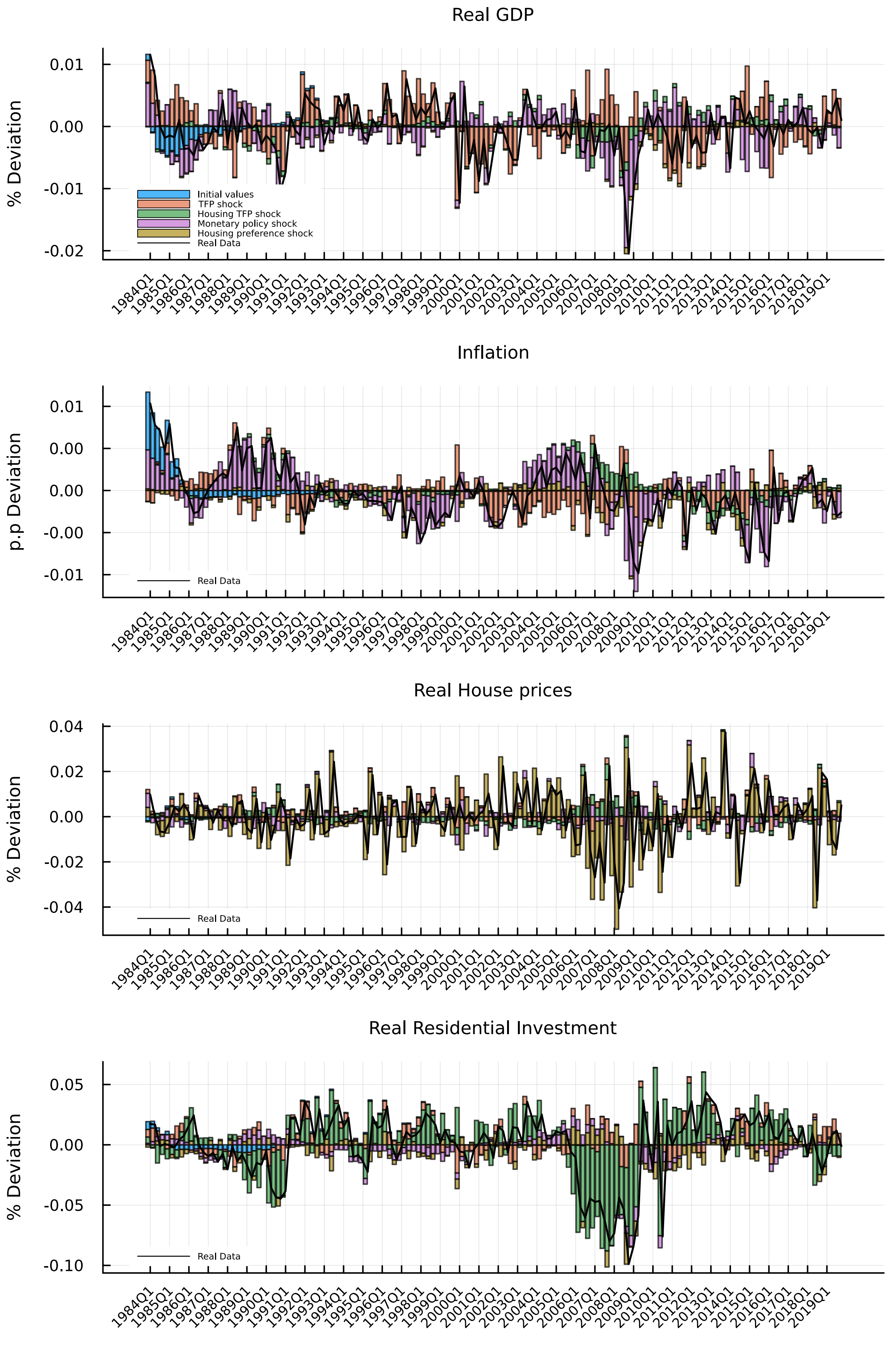

Figure 2.13: Historical shock decomposition under DE model with twelve-quarters reference.

House prices

Data: Census Bureau House Price Index (Index 2012 = 100, quarterly new one-family houses sold including value of lot). This series is deflated using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Census Bureau.

Loans to households

Data: Households and Non-profit Organisations, one-to-four-family residential mortgages (Billions of dollars, seasonally adjusted) and Households and Non-profit Organisations, Consumer credit (Billions of dollars, seasonally adjusted). The total amount of loans to households equals the sum of the two series, which is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Bureau of Economic Analysis.

Nonresidential investment

Data: Private Nonresidential Fixed Investment (Billions of dollars, seasonally adjusted annual rate). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Board of Governors of the Federal Reserve System.

Residential investment

Data: Private Residential Fixed Investment (Billions of dollars, seasonally adjusted annual rate). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Board of Governors of the Federal Reserve System.

Housing wealth

Data: Households and Non-profit Organisations, Real Estate at Market Value (Billions of Dollars, not seasonally adjusted). The series is adjusted by the population level and converted in real terms using the Implicit Price Deflator. The result is log-transformed, detrended using the first difference, and also demeaned. Source: Bureau of Economic Analysis.

Population level

Thousands of Persons. Quarterly average of the monthly series. Not seasonally adjusted. I transformed this series into an index as in Smets and Wouters (2007) but with base year 2012:3.

Chapter 3

Forecasting Under Distorted Beliefs: The Impact of Diagnostic Expectations

1 Introduction

Recent years have witnessed a burgeoning literature questioning the widely used benchmark assumption that agents form expectations rationally. The predictability of survey forecast errors, as noted in data patterns, gives rise to this scepticism.[1] Such predictability suggests that agents do not fully incorporate all available information when forming expectations, contradicting the core premise of the Full Information Rational Expectations (FIRE) theory. Consequently, researchers have developed alternative theories to account for deviations from FIRE. A prominent and recent framework is Diagnostic Expectations (DE).[2] When integrated in macroeconomic models, DE have been shown to generate greater macroeconomic fluctuations, improving the ability to explain certain characteristics associated with credit cycles (Bordalo, Gennaioli, & Shleifer, 2018), as well as to improve understanding of the dynamics of the housing market (Montenegro Calvimonte,

[1] The starting point is the evidence presented in the influential study of Coibion and Gorodnichenko (2015). This work encouraged further analysis. For example: Fuhrer (2018), Angeletos, Huo, and Sastry (2021), Kohlhas and Walther (2021), Eva and Winkler (2023), Born, Enders, and Müller (2023), among others.

[2] Another deviations from the FIRE assumption are adaptive learning (Evans & Honkapohja, 2009, Marcet & Sargent, 1989), inattention (Gabaix, 2020), sticky information (Mankiw & Reis, 2002), heterogeneous expectations (De Grauwe & Ji, 2019), among others.

2024). Additionally, DE are also consistent with overreactions in forecast data (Bordalo, Gennaioli, Ma, & Shleifer, 2020).

In this chapter, I study how the integration of DE affects the spatial and intertemporal dimensions of a macroeconomic model. In particular, I explore the impact of this belief formation process on the model's state-space structure and the resulting Forecast Errors (FE) and Forecast Revisions (FR) across varying horizons. The objective is to determine whether incorporating DE leads to a common structure across model variables, as well as among their FE and FR. I first examine the effects of DE in a simple three-equation macroeconomic model. The derivations reveal that DE introduce a moving average (MA) structure for the equations governing the dynamics of inflation and output gap in this model. Moreover, consistent with Bordalo et al. (2018), FE and FR are predictable in light of the information available in period t, often showing systematic reversals.[3]

Building on this effort, I extend the framework to larger-scale models, deriving general expressions for the forecast errors and forecast revisions that allow for a multivariate analysis of DE as they turn out to follow a vector moving average (VMA) process.[4] This extension demonstrates that the predictability of forecast errors and the overreaction to new information in forecast revisions under DE remain, while offering an empirical tool to also study any cross-correlation effects within FE and FR across variables in the model. To examine how forecast errors and forecast revisions evolve over time and interact across different variables, I assess the presence of DE among professional forecasters and policymakers using data from the Federal Reserve Bank (FED) of Philadelphia Survey of Professional Forecasters (SPF) and Greenbook/Tealbook forecasts. The objective is to determine whether DE influence the forecasts of professional forecasters and policymakers, considering their potential effects on macroeconomic dynamics and the crucial role of the expectation channel in the transmission of monetary policy.[5]

The empirical analysis examines the predictability of forecast errors and forecast revision through the application of MA and VMA models. In the univariate context, the findings suggest that DE introduce predictability by shaping MA structures. However, evidence of overreaction from DE emerges only in the real GDP growth forecast errors of professional forecasters for the period that includes the COVID pandemic.[6] The lag

[3]Analogously, Bordalo, Gennaioli, Shleifer, and Terry (2021) demonstrate that under DE, FE for total factor productivity (TFP) are negatively correlated with the current TFP, suggesting predictability of these errors.

[4]The approach follows Hajdini and Kurmann (2024), in the sense that they show that ex-post FE from any regime-shift FIRE model with a minimum state variable solution are predictable.

[5]Tien, Sinclair, and Gamber (2021) analyse the economic consequences, if any, of mistakes in the FED's forecasts. Their results indicate that, on average, FED's forecast errors have little impact on economic outcomes, whereas during recessions, these errors significantly influence the economy.

[6]Dataset characteristics definitely influence the outcomes since I use average survey responses, which tend to underreact, as shown in Coibion and Gorodnichenko (2015), unlike individual responses that

structure of the MA process is notably sensitive to the particular variables and periods analysed. Different insights emerge for forecast revisions; professional forecasters overcorrect expectations, akin to DE, only for the 3-month Treasury bill. Including the COVID and post-COVID periods significantly affects the results' sensitivity.

In the multivariate context, the findings are consistent with the presence of DE among professional forecasters, since the best-fitting model of both forecast errors and forecast revisions consists of a VMA structure. Forecast errors do not exhibit any cross-variable interactions, as previous shock realisations influence only the specific variable they impact. Real GDP growth forecast errors display overreaction, consistent with DE, while those for inflation and the T-bill rate show underreaction. For forecast revisions, professional forecasters overreact to shocks in the T-bill rate, excessively adjusting their expectations for both interest rates and inflation. A similar, but weaker, response emerges for inflation, whereas GDP growth forecast revisions lack statistical significance. In both forecast errors and forecast revisions, the VMA models struggle to capture extreme volatility during crises, such as the Great Financial Crisis and the COVID-19 pandemic.[7]

Through out-of-sample evaluation, I show that the VMA models generally perform well in predicting one-period-ahead values and align with the predictability structure introduced by DE. While some parameters reflect overreaction in FE and FR, others do not, yet the models still offer valuable insights. In addition, I also explore multiple forecast horizons in both forecast errors and forecast revisions from professional forecasters. The results reveal that forecast errors three periods ahead exhibit greater persistence, potentially due to expectations rigidities or gradual learning dynamics. In contrast, the forecast revisions for the same forecast horizon show a stronger overreaction through larger revisions captured by higher values in the estimates. Although the findings in this chapter are inconclusive; they are informative and offer central banks some guidance on market reactions to signals, leading to possible refinements of monetary policy decisions and communication.

Related literature

This chapter is mainly related to the strand of the literature that studies deviations from the FIRE assumption. Within this broader domain, my research is closely related to articles that integrate DE into macroeconomic models, examining their compatibility with survey data through model forecast errors and forecast revisions, or matching moments from empirical survey evidence.

The first relevant contribution to this field is Bordalo et al. (2018), who identified predictable forecast errors and systematic reversals in credit spread forecasts and showed

often overreact (Bordalo et al., 2020).

[7] These results are also robust to considering the median response from the SPF.

that embedding DE helps reconcile this evidence. In a similar line, Bordalo et al. (2021) investigate a real business cycle model with heterogeneous firms and dispersed information. Their analysis revealed that DE bridge individual-level overreaction with aggregate forecast inertia, mirroring patterns observed in firm-level surveys. Bianchi, Ilut, and Saijo (2024a) expand on this by showing that distant memory within the DE framework effectively replicates untargeted empirical responses from SPF data regarding inflation and GDP growth. In addition, Bianchi, Ilut, and Saijo (2024b) connect DE with uncertainty, illustrating that this framework accounts for the survey pattern in which overreaction increases with extended forecast horizons. This chapter contributes to this literature in a few notable ways. First, it studies the incorporation of DE in macroeconomic models, with a focus on how DE influence spatial and intertemporal dynamics. Second, it explores the impact of DE on the state-space structure of these models. Moreover, my work extends the contribution of Bordalo et al. (2018) by offering a generalised framework that yields expressions for the forecast errors and the forecast revisions across all variables within a model and for different forecast horizons, allowing for a further test of their predictability.

Beyond DE, there is another strand of the literature exploring other deviations from the FIRE assumption in macroeconomic modelling. For example, Rychalovska, Slobodyan, and Wouters (2023) use information from surveys during the estimation of a DSGE model with adaptive learning. Their model-based expectations are consistent with those from the SPF and they achieve superior long-term predictions. Another contribution is Hajdini (2023), who assumes a combination of autoregressive misspecified forecasts and myopia for the expectation formation process. Her findings underscore that the model's inflation predictions align well with three observed empirical patterns in consensus forecasts.[8] In addition, and resonating with my results, she points out that one must rely on both deviations from full information and rational expectations to better fit the expectation formation process of households and firms. Further evidence comes from Hajdini and Kurmann (2024), as they develop a regime-robust test for the FIRE assumption and find that their regime-dependent models' expectations mismatch those from survey forecasts. My contribution to this area is two-fold. First, I study a micro-founded model of expectation formation which is immune to the Lucas (1976) critique and that has shown to be consistent with forecast data (Bordalo et al., 2020). Second, I derive testable general expressions or law of motions for forecast errors and forecast revisions for all variables included in a macroeconomic model populated with diagnostic agents.

Finally, my work relates to the empirical literature that analyses the consistency of

[8] Hajdini (2023) shows that these are (i) delayed overshooting; (ii) under-reaction to ex-ante forecast revisions; and (iii) overreaction to recent events.

different assumptions on the expectation formation process with forecasting data, particularly those considering DE. Ortiz (2020) compares different behavioural deviations that generate overreaction and finds that the SPF data broadly align with DE, although not for all variables.[9] Bürgi and Ortiz (2022) identified offsetting patterns in forecast revisions across horizons, which they reconcile with a model of long-run smoothing relative to DE, although they do not incorporate distant memory in their comparison. Additional evidence supporting the presence of DE comes from the work of Tozzo and Auer (2023). The authors find, using the Bank Lending Survey, that bankers' forecast errors are predictable and aligned with an amplification distortion such as the one generated by DE. Most recently, Wang (2024) uses a forecasting model for inflation and derives the law of motion for forecast errors under DE and DE with noisy information. He finds that, in recent years, households and firms have diverged in terms of under- and overreaction patterns. My contribution to this empirical literature lies in evaluating not only the predictability of forecast errors and the overreaction in forecast revisions across different variables and horizons, but also whether they fully or partially align with DE. Furthermore, I analyse whether the expectation formation process differs between professional forecasters and policymakers, adding an additional layer of depth to understanding heterogeneity in belief formation and its implications for macroeconomic modelling.

Structure of the chapter

The rest of the chapter proceeds as follows. Section 2 briefly describes the concept of DE and, by means of a simple model, analytically illustrates how it affects forecast errors and forecast revisions in that setting. In Section 3, I show the generalisation of the result to larger-scale models in state-space form, where DE similarly generate forecast errors containing predictable components and forecast revisions overly adjusting to new information. The data series used to test the results of the previous two sections are outlined in Section 4. Section 5 presents the findings of the empirical analysis. Section 6 follows with a robustness analysis and Section 7 examines multiple forecast horizons. Finally, Section 8 offers some concluding remarks.

2 Diagnostic expectations, forecast errors and forecasts revisions: A simple example

Diagnostic expectations describe a cognitive bias based on the "representative heuristic"concept introduced by Kahneman and Tversky (1972). It describes a mental shortcut

[9] In another paper, however, Ortiz (2024) provides evidence suggesting that a noisy information rational expectations model, where agents misperceive the true data generating process law of motion, does a better job in matching some of the survey data.

in which the agent overweights future states of the economy that seem more likely in light of current observations relative to what they would have predicted in the previous period. The purpose of this section is to demonstrate the impact of incorporating DE on the spatial and intertemporal dimensions of a macroeconomic model. Specifically, I aim to explore how DE affects the model's state-space structure and the resulting errors and revisions across different forecasting horizons. In addition, I examine whether the inclusion of DE leads to a common state space among the model variables.

I assume that the economy is described by a simple three-equation system:

$$\pi_t = \beta E_t^{\phi} \pi_{t+1} + \kappa \tilde{y}_t + z_t, \tag{3.1}$$

$$\tilde{y}_t = -\delta \pi_t, \tag{3.2}$$

$$z_{t+1} = \rho z_t + \epsilon_{t+1} + \rho\phi\epsilon_t, \tag{3.3}$$

where E_t^{ϕ} is the diagnostic expectations operator, π_t denotes the inflation rate, $\tilde{y}_t$ the output gap, and z_t a cost-push shock. All parameters in the model are positive. Equation (3.3) shows that the true data generating process for the state variable z_t is autoregressive (AR) of order 1. However, under DE the shock process is misperceived as an autoregressive moving average (ARMA) process of order (1,1) by the agents, as they extrapolate in the direction of the shock (blue part).

First, assuming that agents are diagnostic, I solve the model using the method of undetermined coefficients. This solution embeds the rational case when $\phi = 0$. Since under DE the system has two state variables: the cost-push shock z_t and the realisation of the shock ϵ_t, I conjecture a solution taking the following form:

$$\pi_t = A z_t + B\epsilon_t \tag{3.4}$$

$$\tilde{y}_t = C z_t + D\epsilon_t \tag{3.5}$$

I begin by replacing equation (3.2) in (3.1):

$$\pi_t = \beta E_t^{\phi} \pi_{t+1} - \frac{\kappa}{\delta}\pi_t + z_t.$$

I rearrange the terms and obtain:

$$\left(1 + \frac{\kappa}{\delta}\right)\pi_t = \beta E_t^{\phi} \pi_{t+1} + z_t.$$

Using the conjectured solution in (3.4) and replacing it in the previous expression:

$$Az_t + B\epsilon_t = \frac{\beta E_t^{\phi}[Az_{t+1} + B\epsilon_{t+1}] + z_t}{1 + \frac{\kappa}{\delta}}.$$

Applying the DE operator and using expression (3.3):

$$Az_t + B\epsilon_t = \frac{\beta A(\rho z_t + \rho\phi\epsilon_t) + z_t}{1 + \frac{\kappa}{\delta}}.$$

Note that here I use the fact that $E_t^{\phi}[\epsilon_{t+1}] = 0$. Next, I collect the terms common to z_t:

$$A\Big(1 + \frac{\kappa}{\delta}\Big)z_t = \beta A\rho z_t + z_t,$$

which after equating coefficients implies:

$$A\Big(1 + \frac{\kappa}{\delta}\Big) = \beta A\rho + 1.$$

Now, I solve for A and obtain an expression that depends only on the parameters from the model:

$$A = \frac{1}{(1 + \frac{\kappa}{\delta} - \beta\rho)}. \tag{3.6}$$

Using this solution, I obtain B:

$$B = \frac{\beta\rho\phi}{(1 + \frac{\kappa}{\delta})(1 + \frac{\kappa}{\delta} - \beta\rho)}. \tag{3.7}$$

Finally, replacing these two expressions in the conjectured solution and in the relation from equation (3.2), I obtain an expression for inflation and output gap that depends on the two states:

$$\pi_t = \frac{1}{(1 + \frac{\kappa}{\delta} - \beta\rho)}z_t + \frac{\beta\rho\phi}{(1 + \frac{\kappa}{\delta})(1 + \frac{\kappa}{\delta} - \beta\rho)}\epsilon_t. \tag{3.8}$$

$$\tilde{y}_t = -\frac{1}{\delta(1 + \frac{\kappa}{\delta} - \beta\rho)}z_t - \frac{\beta\rho\phi}{\delta(1 + \frac{\kappa}{\delta})(1 + \frac{\kappa}{\delta} - \beta\rho)}\epsilon_t. \tag{3.9}$$

In equations (3.8) and (3.9), it is easy to appreciate that when agents are rational, i.e. $\phi = 0$, the solutions depend only on one state variable. On the other hand, when agents are diagnostic, both variables exhibit dependency on the current shock ϵ_t, captured by the second term in the expressions.

Inflation and output gap equilibrium dynamics can be re-expressed as ARMA(1,1) processes. This occurs because diagnostic agents extrapolate their beliefs about the state of the economy to the variables. The MA(1) term reflects this behaviour, showing a dependency on the realisation of the shock. This leads to overreactions and reversals, as well as higher volatility in the economy. In contrast, if agents are rational, the processes simplify to AR(1). I rewrite expressions (3.8) and (3.9) as:

$$\pi_t = az_t + b\epsilon_t. \tag{3.10}$$

$$\tilde{y}_t = cz_t + d\epsilon_t. \tag{3.11}$$

where $a = \frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}$, $b = \frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}$, $c = -\frac{1}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)}$ and $d = -\frac{\beta\rho\phi}{\delta(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}$

For a clearer presentation, during the remainder of the explanation, I will make use of the lag operator L. Replacing the AR(1) true data generating process for z_t:

$$\pi_t = a(\rho L z_t + \epsilon_t) + b\epsilon_t.$$

$$\tilde{y}_t = c(\rho L z_t + \epsilon_t) + d\epsilon_t.$$

Lagging equations (3.10) and (3.11), and conveniently rearranging:

$$aLz_t = L\pi_t - bL\epsilon_t.$$

$$cLz_t = L\tilde{y}_t - dL\epsilon_t.$$

Replacing in the previous expressions:

$$\pi_t = \rho L\pi_t - \rho bL\epsilon_t + (a+b)\epsilon_t. \tag{3.12}$$

$$\tilde{y}_t = \rho L\tilde{y}_t - \rho dL\epsilon_t + (c+d)\epsilon_t. \tag{3.13}$$

After replacing a, b, c, and d, and reorganising the expressions:

$$(1-\rho L)\pi_t = \left[\left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right) + \left(\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right)(1-\rho L)\right]\epsilon_t. \tag{3.14}$$

$$(1-\rho L)\tilde{y}_t = \left[\left(-\frac{1}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)} - \frac{\beta\rho\phi}{\delta(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right)(1-\rho L)\right]\epsilon_t. \tag{3.15}$$

This result is key. It demonstrates that both variables exhibit an ARMA(1,1) structure, as stated before. More importantly, the variables share the same MA(1) structure, with the only difference being the scaling factor $-1/\delta$ on the output gap compared to inflation. Additionally, the expression also reveals that when $\phi = 0$, the second term in the brackets on the right-hand side of the expressions vanishes, leading to no additional volatility as the solution simplifies to an AR(1) process.

2.1 Forecast Errors

In this subsection, I am interested in obtaining an expression for the one-period-ahead Forecast Error (FE) made by the diagnostic agent in the three-equation model. First, following Coibion and Gorodnichenko (2015) and Bordalo et al. (2020), I define the ex-post one-period-ahead FE as the difference between the $t + 1$ realisation of a variable relative to the forecast at the end of period t and the beginning of period $t + 1$. For clarity, here I only derive the results for the inflation forecast case, noting that the results for the output gap can be obtained in a similar manner.

$$FE_t^{\pi} = \pi_{t+1} - E_t^{\phi}\pi_{t+1}. \tag{3.16}$$

$$FE_t^{\tilde{y}} = \tilde{y}_{t+1} - E_t^{\phi}\tilde{y}_{t+1}. \tag{3.17}$$

Next, I want to derive the expected inflation in period $t + 1$, for that case I recall the solution form of π_t from expression (3.10), I advance it one time and apply the DE operator:

$$E_t^{\phi}\pi_{t+1} = E_t^{\phi}[az_{t+1} + b\epsilon_{t+1}]. \tag{3.18}$$

Here, I use the result that the diagnostic agent misconceives the shock process as an ARMA(1,1) instead of the true data-generating process AR(1), obtaining:

$$E_t^{\phi}\pi_{t+1} = E_t^{\phi}[a(\rho L z_{t+1} + \rho\phi L\epsilon_{t+1} + \epsilon_{t+1})] + bE_t^{\phi}[\epsilon_{t+1}], \tag{3.19}$$

which after using E_t^{ϕ} equals:

$$E_t^{\phi}\pi_{t+1} = a\rho L z_{t+1} + a\rho\phi L\epsilon_{t+1}. \tag{3.20}$$

This equation for inflation expectations can also be written in an AR(1) form following a similar course of action as I did before:

$$E_t^{\phi}\pi_{t+1} = \rho L\pi_{t+1} - \rho b L\epsilon_{t+1} + a\rho\phi L\epsilon_{t+1}. \tag{3.21}$$

After some rearrangement and replacement of parameters, this becomes:

$$E_t^{\phi}\pi_{t+1} - \rho L\pi_{t+1} = \left(-\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)} + \frac{\phi}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho L\epsilon_{t+1}. \tag{3.22}$$

For convenience, I present again expression (3.14), but for period $t+1$:

$$(1-\rho L)\pi_{t+1} = \left[\left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right) + \left(\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right)(1-\rho L)\right]\epsilon_{t+1}. \quad (3.23)$$

Subtracting (3.22) from (3.23):

$$\begin{aligned}(1-\rho L)\pi_{t+1} - (E_t^{\phi}\pi_{t+1} - \rho L\pi_{t+1}) = &\left[\left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\right. \\ &+ \left.\left(\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right)(1-\rho L)\right]\epsilon_{t+1} - \\ &\left(-\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)} + \frac{\phi}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho L\epsilon_{t+1}.\end{aligned} \quad (3.24)$$

The left-hand side of this expression is the forecast error, FE_t^{π}, while the right-hand side simplifies due to the common factor with opposite signs:

$$FE_t^{\pi} = \left[\left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right) + \left(\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\right]\epsilon_{t+1} - \left(\frac{\phi}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\epsilon_t. \quad (3.25)$$

Equation (3.25) shows that the FE_t^{π} is independent and identically distributed (i.i.d.) when the agent is rational. This corresponds to the case where FE_t^{π} depends only on ϵ_{t+1} as $\phi = 0$. However, when $\phi > 0$, households systematically make errors, which means that FE_t^{π} no longer is i.i.d. and now contains a predictable component. The sign of such effect is the opposite to the innovation realisation, since all parameters are positive. For example, a positive innovation would lead the agent to overreact and predict higher inflation in the next period. But, since the true data generating process is AR(1), this higher inflation will not materialise as expected, resulting in an over-prediction which has a negative impact on the FE.

Following the same steps and due to the previous result that the output gap solution structure is a scaled version of the inflation solution, I can write the FE for the output gap as:

$$FE_t^{\tilde{y}} = \left[-\frac{1}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)} - \frac{\beta\rho\phi}{\delta(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)}\right]\epsilon_{t+1} + \left(\frac{\phi}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\epsilon_t. \quad (3.26)$$

2.2 Forecast Revisions

In this subsection, I am interested in obtaining an expression for the one-period-ahead Forecast Revisions (FR) made by the diagnostic agent from the same simple model considered before. Also following Coibion and Gorodnichenko (2015) and Bordalo et al. (2020), I define the ex-ante one-period-ahead FR as reflecting the impact that news known to agents at the time of the forecast have. They are defined as follow:

$$FR_t^{\pi} = E_t^{\phi}\pi_{t+1} - E_{t-1}^{\phi}\pi_{t+1} \tag{3.27}$$

$$FR_t^{\tilde{y}} = E_t^{\phi}\tilde{y}_{t+1} - E_{t-1}^{\phi}\tilde{y}_{t+1} \tag{3.28}$$

The first term on the right-hand side of Equation (3.27) was already derived in the previous subsection, shown in Equation (3.22):

$$E_t^{\phi}\pi_{t+1} - \rho L\pi_{t+1} = \left(-\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)} + \frac{\phi}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho L\epsilon_{t+1}, \tag{3.29}$$

Now using this definition, but under the DE operator with information known until period $t-1$, I obtain:

$$E_{t-1}^{\phi}\pi_{t+1} - \rho^2 L^2\pi_{t+1} = \left(-\frac{\beta\rho\phi}{(1+\frac{\kappa}{\delta})(1+\frac{\kappa}{\delta}-\beta\rho)} + \frac{\phi}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho^2 L^2\epsilon_{t+1}, \tag{3.30}$$

Therefore, implementing the FR definition and making use of Expression (3.14), it follows that:

$$FR_t^{\pi} = \left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\epsilon_t + \left(\frac{1}{(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\phi(\epsilon_t - \rho\epsilon_{t-1}), \tag{3.31}$$

This implies that diagnostic agents incorporate the news they receive between periods $t-1$ and t, as they review their forecasts in the same direction of the shock. For example, if agents face positive news ($\epsilon_t > \epsilon_{t-1}$) between periods $t-1$ and t, they will revise their forecast upwards. The magnitude of this revision is determined by the diagnostic parameter ϕ. If this parameter is equal to zero, the last term of equation 3.31 disappears, indicating that agents follow rational expectations, which means they do not adjust their beliefs based on past information or deviations, unlike diagnostic agents. For the output gap, the expression is:

$$FR_t^{\tilde{y}} = -\left(\frac{1}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\epsilon_t - \left(\frac{1}{\delta(1+\frac{\kappa}{\delta}-\beta\rho)}\right)\rho\phi(\epsilon_t - \rho\epsilon_{t-1}), \tag{3.32}$$

Note that the FR here mirrors the result that scaling the inflation solution by the factor $(-1/\delta)$ yields the solution for the output gap.

3 Generalised framework

In this part, I demonstrate that the previous findings can be extended to larger-scale models structured in state-space form. In these models, DE continue to generate FE containing a predictable component and FR that reflect over adjustments in the direction of new information.

3.1 Environment

I consider the first-order perturbation solution of discrete-time models following the standard form:

$$E_t[\mathbf{f}(\mathbf{z}_t, \mathbf{x}_t, \mathbf{y}_t, \boldsymbol{\epsilon}_t, \mathbf{z}_{t+1}, \mathbf{x}_{t+1}, \mathbf{y}_{t+1}, \boldsymbol{\epsilon}_{t+1})] = \mathbf{0}. \tag{3.33}$$

$\mathbf{z}_t$ denotes a (n_z x 1) vector of exogenous state variables, $\mathbf{x}_t$ denotes a (n_x x 1) vector of endogenous state variables, and $\boldsymbol{\epsilon_t}$ is a (n_ϵ x 1) vector of shock realisations. The total number of state variables is given by n, which is the result of the sum of $n_z+n_x+n_\epsilon$. Furthermore, $\mathbf{y}_t$ denotes an (m x 1) vector of decision variables, where m represents their total number. I include the shocks' realisations as auxiliary variables representing the MA terms from the exogenous state processes. These terms are used when solving the model under DE as agents believe that the exogenous shock processes follow an ARMA(1,S), instead of an AR(1).

The system has a solution form:

$$\underbrace{\begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_{t+1} \\ \boldsymbol{\epsilon}_{t+1} \end{bmatrix}}_{\Gamma_{t+1}} = \underbrace{\begin{bmatrix} \overbrace{\mathbf{h}_{z,z}}^{H_z} & \overbrace{\mathbf{h}_{z,x}}^{H_x} & \overbrace{\mathbf{h}_{z,\epsilon}}^{H_\epsilon} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix}}_{\text{H}} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_{t+1} \end{bmatrix}.$$

$$\mathbf{y}_t = \underbrace{\begin{bmatrix} \overbrace{\mathbf{g}_{1,z}}^{G_z} & \overbrace{\mathbf{g}_{1,x}}^{G_x} & \overbrace{\mathbf{g}_{1,\epsilon}}^{G_\epsilon} \\ \mathbf{g}_{2,z} & \mathbf{g}_{2,x} & \mathbf{g}_{2,\epsilon} \\ . & . & . \\ . & . & . \\ \mathbf{g}_{m,z} & \mathbf{g}_{m,x} & \mathbf{g}_{m,\epsilon} \end{bmatrix}}_{\text{G}} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix}$$

where $\mathbf{\Gamma}_{t+1}$ represents the full vector of state variables which has a size (n x 1). Matrix $\mathbf{H}$ presents the first-order coefficients in the state transition equation and k presents the loading matrix on the shocks. The matrix $\mathbf{G}$ links the decision variables with the vector of states. To better understand the impact of DE, I present both transition matrices divided into submatrices. The submatrix $\mathbf{H}_z$, of size (n x n_z), contains the coefficients governing the dependence of the whole vector of state variables with respect to the exogenous states $\mathbf{z}_t$. Similarly, $\mathbf{H}_x$ represents the transition for the state vector with respect to endogenous state variables $\mathbf{x}_t$, and it has a size (n x n_x). In addition, the (n x n_ϵ) matrix $\mathbf{H}_\epsilon$, shows how the vector of state variables evolves regarding the shocks' realisations $\boldsymbol{\epsilon}_t$. I also slice the matrix $\mathbf{G}$ in three submatrices.[10] $\mathbf{G}_z$ and $\mathbf{G}_x$, with size (m x n_z) and (m x n_x), connect the decision variables to exogenous states and predetermined variables, respectively. Submatrix $\mathbf{G}_\epsilon$, sized (m x n_ϵ), links the decision variables to the realised shocks.

These matrices will differ between the RE and DE solutions. For instance, in matrix $\mathbf{G}$, the difference lies in submatrix $\mathbf{G}_\epsilon$, which will have non-zero values under DE, while it will have all zero values under RE. In the case of matrix $\mathbf{H}$, this difference will be reflected in the elements of $\mathbf{H}_{x,\epsilon}$. Additionally, the entries in $\mathbf{H}_{z,\epsilon}$ that link the vector of exogenous states to the shocks' realisations will be different from zero. These values in matrix $\mathbf{H}$ will be turned off after solving the model under diagnosticity, so future realisations are driven by the true data generating process, but with diagnostic agents.

Using these results and in a similar way as in Subsections 2.1 and 2.2, I can obtain an expression for the FE and FR in a more general setting. On this occasion, when agents form DE, the expression will also exhibit dependence on the realisation of the shocks. This implies, as before, that the forecast errors contain a predictable component and that forecast revisions over-adjust in the direction of the new information.

I start forwarding one period ahead the diagnostic solution for the endogenous variables:

$$\mathbf{y}_{t+1} = \mathbf{G}\mathbf{\Gamma}_{\mathbf{t+1}} \tag{3.34}$$

Next, I replace the law of motion for the vector of state variables including exogenous variables, endogenous variables, and the shocks' realisations. It is important to emphasise that the solution applied here for the exogenous state variables follows RE. Consequently, exogenous shocks will follow the true data-generating process, which in this case is an AR(1). As a result, the entries in the submatrix $\mathbf{h}_{z,\epsilon}$, linking the vector of exogenous states with the shocks' realisations, and $\mathbf{h}_{z,x}$, linking the vector of exogenous states with

[10]Throughout the analysis in this chapter, since I am focused on the behaviour of FE and FR under DE, I will assume that the solution for matrix G is always derived under DE unless explicitly stated otherwise.

the endogenous variables, will be zeros. In addition, since the realisations of the shocks do not depend on either exogenous or endogenous variables and the loading matrix for the shocks is captured by the submatrix $\mathbf{k}_z$, the elements of $\mathbf{h}_{\epsilon,z}$, $\mathbf{h}_{\epsilon,x}$ and $\mathbf{h}_{\epsilon,\epsilon}$ are also zeros.

$$\mathbf{y}_{t+1} = \mathbf{G}\left(\begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{0} & \mathbf{0} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\begin{bmatrix} \boldsymbol{\epsilon}_{t+1} \end{bmatrix}\right). \tag{3.35}$$

Thus, the solution for $\mathbf{y}_{t+1}$ is:

$$\mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{0} \end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x} \\ \mathbf{0} \end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+1} \tag{3.36}$$

Now, I intend to obtain an expression for the diagnostic expected values of the endogenous variables. I apply the diagnostic operator to expression (3.34):

$$E_t^\phi \mathbf{y}_{t+1} = E_t^\phi[\mathbf{G}\boldsymbol{\Gamma}_{t+1}] \tag{3.37}$$

Replacing the **diagnostic** solution for the law of motion of the state vector, in which the only difference with respect to the expression (3.35) is that now $h_{z,\epsilon}$ presents values different from zeros:

$$E_t^\phi \mathbf{y}_{t+1} = \mathbf{G}E_t^\phi\left(\begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{0} & \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\begin{bmatrix} \boldsymbol{\epsilon}_{t+1} \end{bmatrix}\right), \tag{3.38}$$

which after applying the DE operator becomes:

$$E_t^\phi \mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{0} \end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x} \\ \mathbf{0} \end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix}\boldsymbol{\epsilon}_t. \tag{3.39}$$

Finally, subtracting (3.39) from (3.36) as specified by the FE definition in the previous section, and using the partitioned $\mathbf{G}$ matrix, I obtain:

$$\mathbf{y_{t+1}} - E_t^\phi \mathbf{y_{t+1}} = -\mathbf{G}_z\mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+1} \tag{3.40}$$

Expression (3.40) is the general representation of the results in Equations (3.25) and (3.26). The dependency on the realisation of the shocks is captured by the first term, where $\mathbf{G}_z$ has size (m x n_z) and $\mathbf{h}^{DE}_{z,\boldsymbol{\epsilon}}$ (n_z x n_ϵ).

This result can be generalised for the FE h periods ahead as[11]:

$$\mathbf{y_{t+h}} - E^{\phi}_t \mathbf{y_{t+h}} = \left[-\mathbf{G}_z \mathbf{H}^{h-1}_{1,1} \mathbf{h}^{DE}_{z,\epsilon} - \mathbf{G}_x \mathbf{H}^{h-1}_{2,1} \mathbf{h}^{DE}_{z,\epsilon}\right] \boldsymbol{\epsilon}_t + \mathbf{G} \sum_{\tau=1}^{h} \mathbf{H}^{h-\tau} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+\tau}. \quad (3.41)$$

Here, $H^{h-1}_{i,j}$ is used to denote a specific sub-matrix corresponding to each power of matrix H, as longer horizons lead to more complex terms. The dependency of the FE on the realisations of the shocks remains valid for an h-periods ahead horizon; however, the magnitude of the response will differ since the matrix multiplying the vector $\boldsymbol{\epsilon}_t$ is different. Now, this is not only a function of $\mathbf{G}_z$ but also of $\mathbf{G}_x$. As the forecast period extends, the FE must account for the effect that DE have on the endogenous states and how this effect is distributed over time.

Similarly, I can obtain the general expression for the FR for one period and h periods ahead. Recalling expression (3.39) and replacing $\mathbf{z}_t$ using the law of motion for the state variables in the system solution:

$$E^{\phi}_t \mathbf{y}_{t+1} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{0} \end{bmatrix} \left(\mathbf{h}_{z,z}\mathbf{z}_{t-1} + \mathbf{h}_{z,x}\mathbf{x}_{t-1} + \mathbf{h}^{DE}_{z,\epsilon}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t\right) + \mathbf{G} \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x} \\ \mathbf{0} \end{bmatrix} \mathbf{x}_t + \mathbf{G} \begin{bmatrix} \mathbf{h}^{DE}_{z,\epsilon} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\epsilon}_t, \quad (3.42)$$

Using this expression, but applying the DE operator with information until period $t-1$:

$$E^{\phi}_{t-1} \mathbf{y}_{t+1} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{0} \end{bmatrix} \left(\mathbf{h}_{z,z}\mathbf{z}_{t-1} + \mathbf{h}_{z,x}\mathbf{x}_{t-1} + \mathbf{h}^{DE}_{z,\epsilon}\boldsymbol{\epsilon}_{t-1}\right) + \mathbf{G} \begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x} \\ \mathbf{0} \end{bmatrix} \mathbf{x}_t. \quad (3.43)$$

Following the definition of FR, I subtract Equation (3.43) from Equation (3.42) and obtain the following:

$$E^{\phi}_t \mathbf{y}_{t+1} - E^{\phi}_{t-1} \mathbf{y}_{t+1} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{0} \end{bmatrix} \mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G} \begin{bmatrix} \mathbf{h}^{DE}_{z,\epsilon} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} \boldsymbol{\epsilon}_t, \quad (3.44)$$

[11] For a detailed derivation of this result, see Appendix 3.A.

Expression (3.44) shows that the FR of the diagnostic agent will exhibit an overreaction in the direction of the shock, governed by the second term on the right side since the submatrix $\mathbf{h}_{z,\epsilon}^{DE}$ will depend on the diagnostic parameter ϕ. In the case of rational expectations, that is $\phi = 0$, the expression boils down to the first term on the right-hand side of (3.44).

This result can also be generalised for h periods ahead[12]:

$$E_t^{\phi}\mathbf{y_{t+h}} - E_{t-1}^{\phi}\mathbf{y_{t+h}} = \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,1}^h \\ \mathbf{H}_{2,1}^h \\ \mathbf{0}\end{bmatrix}\mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,3}^h \\ \mathbf{H}_{2,3}^h \\ \mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t, \tag{3.45}$$

Analogously to the FE, $H_{i,j}^h$ here refers to a particular submatrix related to each power of the matrix H, as extended horizons introduce more intricate terms. The outcome obtained in the previous section that agents' forecast adjustments will overreact in alignment with the shock continues to hold in expression (3.44).

4 Data

This section describes the data series that I use to test the empirical alignment of Forecast Errors (FE) and Forecast Revisions (FR) with the structures specified by the diagnostic models. I have two main objectives. First, I want to analyse whether these structures are consistent across different variables. Second, I want to assess if this consistency holds across agents. To do this, I calculate the FE and FR time series for real gross domestic product (GDP) growth, headline consumer price index (CPI) inflation, and the 3-month Treasury bill rate (T-bill). I gather expectations data from the Philadelphia FED Survey of Professional Forecasters (SPF) and policymakers from the Greenbook or Tealbook, together with real-time data.[13]

The SPF is a quarterly survey, initially conducted by the American Statistical Association and the National Bureau of Economic Research, and now managed by the Philadelphia Fed. The professional forecasters provide their projections for the next five quarters, as well as for the current and following year. The Greenbook, on the other hand, is produced by the research staff at the Board of Governors before each meeting of the Federal Open Market Committee. It contains projections about future quarters

[12] Appendix 3.B presents a detailed derivation of this result.

[13] The reason I do not include any survey on consumers is because the way the questions elicit expectations, for example for 12-months inflation on a monthly basis, creates an overlapping data problem. This is absent in both the SPF and the Greenbook, given that their forecasts are made quarterly for up to the coming five quarters.

of the U.S. economy. Last, I use the Real-Time Series for Macroeconomists from the Philadelphia FED as real-time observed realisations.[14] The data set includes information as it existed at specific points in time in the past, before any revisions were made. This approach avoids the use of revised data, which may reflect reclassification or additional information that was not available during the time the forecasts were performed (see Croushore, 2010).[15]

4.1 Real GDP Growth

I use the SPF series for the one- and two-quarters ahead growth rate, expressed in annualised percentage points, of the mean forecast for the level of real GDP. The first observation is 1968:Q4, however, due to the unavailability of data for other variables starting that early, I use the series starting in 1981:Q3, which gives the expected annualised growth rate of real GDP from 1981:Q3 to 1981:Q4. The last observation I use is 2023:Q4. From the Greenbook projections, I employ the quarterly growth in real GDP in annualised percentage points. In this case, the series has the same starting point, but it ends in 2018:Q4, as a result of a lag in publishing the books. The real-time data for quarterly growth real GDP are from 1981:Q3 until 2024:Q1. There is a missing value for 1995:Q4, therefore the second available release for that quarter was used instead.

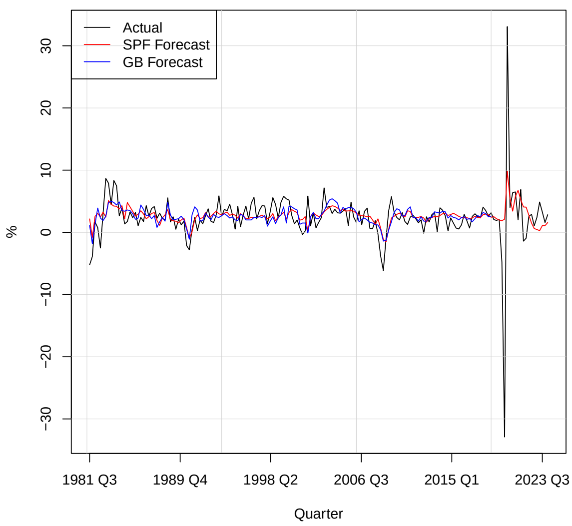

Figure 3.1: Survey expectations and empirical data on annualised real GDP growth rate. Note: The black line illustrates the empirical data on quarter annualised growth in real GDP. The red line represents the mean forecasts for the annualised quarter percent changes of GDP growth rate from SPF. The projections from the Greenbook are shown with a blue line.

[14] Croushore and Stark (2001) provides a detailed explanation about the construction of the data set, as well as the properties of several of the variables in the data set across vintages.

[15] Revisions from initial release to each of the actuals vary substantially, affecting tests of forecasts depending on chosen actuals. (Croushore, 2010).

4.2 Inflation

Regarding the inflation rate, I use the forecast series for the headline CPI inflation rate of the SPF from 1981:Q3 to 2023:Q4. I rely on the annualised quarterly forecast of the percent changes in the consumer price index. When it comes to the Greenbook, I use headline CPI inflation projections made for one quarter into the future. In some quarters, there are multiple observations; this depends on the publication dates of the Greenbook. In these cases, I decided to use the first observation available. The series also spans for a shorter period compared to the SPF due to the lag in publishing the books. I use the available data from 1981:Q3 to 2018:Q4.

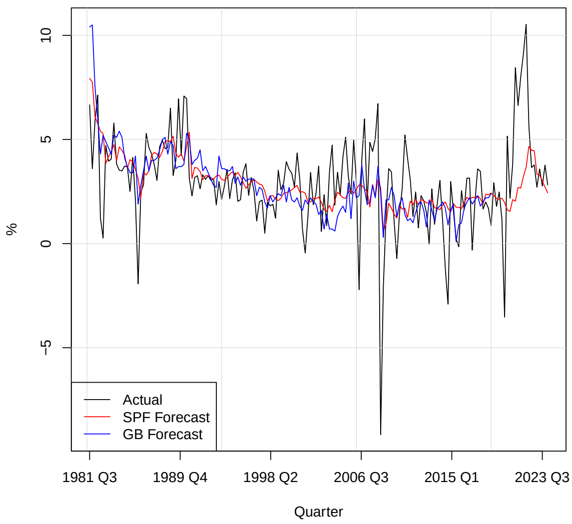

Figure 3.2: Survey expectations and empirical data on annualised quarterly inflation.
Note: The black line denotes the real-time quarterly annualised headline CPI inflation rate. The red line represents the expected change in annualised quarter inflation from the SPF. The projections from the Greenbook are illustrated with a blue line.

The real-time CPI data is provided on a monthly and seasonally adjusted basis. From each vintage quarter, the last entry is used because it represents the publicly available data at that particular time, without any modifications. I calculate the quarterly CPI as the average over a 3-month window. Then, I compute the quarterly annualised inflation rate for the period 1981:Q4 until 2024:Q1.

4.3 3-month Treasury bill rate

The last variable I consider is the 3-month Treasury bill rate. In the SPF, professionals forecast the quarterly average of the underlying daily levels in percentage points. The first observation available is in 1981:Q3, and the last one that I use is 2023:Q4. For real-time data, I rely on the 3-month Treasury bill secondary market rate from the Federal Reserve

Bank of St. Louis, which is at monthly frequency. To obtain the quarterly measure, I calculate the average over a 3-month window. The first available observation is 1934:Q1, however, to match the data coverage from the SPF, I use the period 1981:Q4 to 2024:Q1. This variable is not forecasted in the Greenbook.

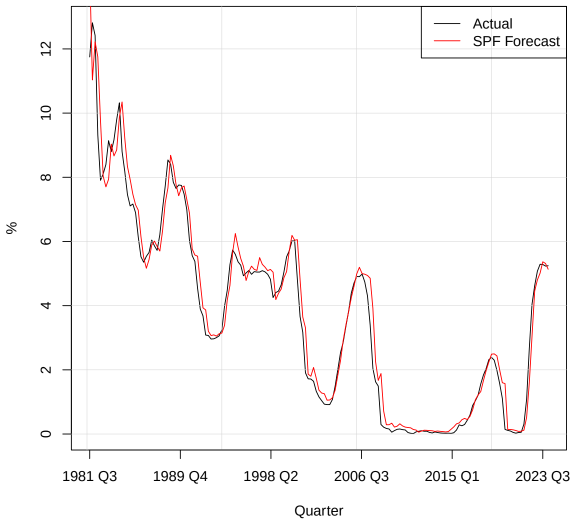

Figure 3.3: Survey expectations and empirical data on 3-month Treasury bill rate.
Note: The black line is the empirical data on quarterly 3-month Treasury bill rate. The red line represents the forecast rate obtained from the SPF.

5 Results

In this section, I present the results of the estimated models to test whether the Forecast Errors (FE) and Forecast Revisions (FR) of real GDP growth, inflation, and the 3-month Treasury bill rate follow the structure predicted by the DE model. First, I calculate the FE for each variable k in each period as:

$$FE^k_{j:Qi} = k_{j:Qi+1} - E_{j:Qi}[k_{j:Qi+1}],$$

and the FR as:

$$FR^k_{j:Qi} = E_{j:Qi}[k_{j:Qi+1}] - E_{j:Qi-1}[k_{j:Qi+1}].$$

where, j represents the year of the surveys or observations and i the quarter. Thus, for example, to determine the professionals' FE in the third quarter of 2020, I subtract the forecast made in 2020:Q3 for next-quarter inflation from the actual inflation rate realised in the fourth quarter of year 2020. In a similar way, to determine the FR for the third quarter of 2020, I subtract the two-quarters-ahead inflation predictions derived from the forecast made in 2020:Q2 from the next-quarter inflation projections from the forecast made in 2020:Q3.

In the next step, I employ the Box-Jenkins approach to identify and estimate the models that best fit these FE and FR for each variable individually. This approach requires estimating multiple ARMA(p,q) models and selecting the model with the lowest value of the information criterion. A similar exercise is performed for the multivariate case. I follow Tsay (2013) in using the extended cross-correlation matrices to specify the order (p, q) of a VARMA model. Thereafter, I estimate the suggested model or models and use an information criterion to choose among them.

5.1 Forecast errors

5.1.1 Univariate

For the univariate analysis, I fit multiple models for the FE of each variable in the data set. Then, I select the model that exhibits the lowest value of the Akaike Information Criterion (AIC).[16] Given that the coverage periods for the SPF and the Greenbook vary, I repeat this process while aligning the time frames of both surveys. This approach serves two purposes: first, to determine if the choice of model structure and estimation method is affected by the period of analysis, and second, to evaluate the robustness of the results in light of post-COVID data included in the SPF forecasts but not in the Greenbook forecasts.

5.1.1.1 Real GDP forecast errors: Figure 3.4 displays the calculated FE for the real GDP growth rate using data from the SPF (panel A) and Greenbook (panel B) projections. The difference in magnitude is driven by the coverage periods. While panel A includes the COVID-19 crisis, panel B does not, as data are not available yet.

The results, shown in Table 3.1, indicate that an MA(1) process is the best-fitting model for both cases. It should be noted that, while the intercepts are negative and not significantly different from zero, the MA(1) coefficients are significant. This result offers evidence on the predictability of forecast errors and quantifies the dominant role that recent shocks play on the current value of FE in both surveys. However, the estimated values differ in sign between the SPF-based analysis in column (1) and the Greenbook-based analysis in column (2). With the entire sample of SPF data (1981:Q3 to 2023:Q4), the coefficient on the MA(1) term is negative, indicating that past shocks affect the FE negatively, implying some degree of overreaction in the expectations of professional

[16] The procedure involves iterating over various ARMA models, evaluating their information criterion values, and ultimately selecting the one with the lowest value.

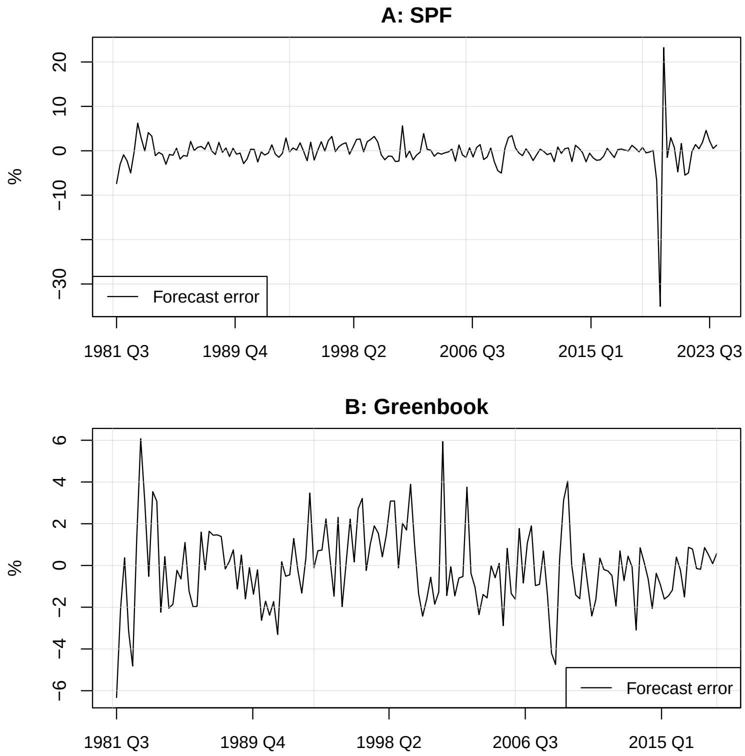

Figure 3.4: Forecast errors - Real GDP growth rate.
Note: The black line is the calculated FE for real GDP growth using the its values from the Real-time Series for Macroeconomists, the SPF (A) and Greenbook (B) forecasts.

forecasters. On the other hand, with the Greenbook data (1981:Q3 to 2018:Q4), the coefficient on the MA(1) term is positive, implying that the previous shock helps predict the FE; however, since the estimate is positive, it suggests that central bank forecasters gradually adjust their expectations, that is, they underreact.

Table 3.1: Best fitting model for real GDP growth forecast errors

	(1) SPF Estimate	(2) Greenbook Estimate	(3) SPF - GB period Estimate
MA(1)	-0.1991 **	0.3047 ***	0.2953 ***
	(0.0780)	(0.0771)	(0.0773)
intercept	-0.2325	-0.1280	-0.1611
	(0.2300)	(0.1954)	(0.1931)
Box-Ljung test	5.350	14.586	16.269
	(0.9995)	(0.7996)	(0.6998)

Note: This table shows the estimates of three MA(1) processes for real GDP growth FE. The results under column (1) are based on the SPF, Philadelphia Fed. The results in column (2), on the other hand, are obtained using the Greenbook, Philadelphia Fed. Finally, column (3) shows the results using SPF for the same period as in the Greenbook. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: “***”0.01, “**”0.05, “*”0.1.

Column (3) examines whether the study period affects the outcomes. Using SPF data for the sub-period 1981:Q3 to 2018:Q4, I fit a MA(1) model and find that the coefficient's estimate not only shares the same sign but is also of similar size to that derived from Greenbook data. This indicates a consistency in the FE for this variable among professional forecasters and policymakers in the US economy, before the COVID pandemic. The last row of the table shows the Ljung-Box test, used to check that there are no significant autocorrelations in the residuals for any of the models. In all cases, the null is not rejected, suggesting that the MA(1) models are well-specified and adequately explain the FE for all three datasets.

The empirical findings for the real GDP growth rate FE generally align with the moving average structure introduced by the DE framework, as specified in previous sections. However, a tendency to overreact emerges solely when considering the forecast period following the COVID pandemic, marked by considerable volatility. A future task would be to assess, once the data become accessible, whether policymakers exhibited similar reactions when this period is considered.

5.1.1.2 Inflation forecast errors: Figure 3.5 shows the calculated Forecast Errors (FE) for the inflation rate of the headline CPI using data from the SPF (panel A) and the Greenbook (panel B) projections. The main difference is again the length of the series. Panel A includes the COVID-19 pandemic and the recent increase in inflation, while panel B does not, as the data are not available yet.

In Table 3.2, I present the estimates of the models with the lowest AIC for the inflation FE. The results in column (1) indicate that a MA(3) structure provides the best fit for the FE calculated using the SPF survey data. In contrast, column (2) reveals that a MA(1) process is more suitable for the FE obtained using Greenbook projections. These results confirm the findings that, in an economy with diagnostic agents, FE will have a predictable component and that this component will be a MA structure. In addition, in both cases, the intercepts are not significantly different from zero, although their signs are the opposite.

The coefficients for the residuals of one and three lags are significant in column (1) (at 1% and 5% , respectively). The larger coefficient for the MA(1) term indicates that the FE is more dependent on the most recent realisation of the residual than on those from a more distant past. The positive values suggest that these shocks lead to positive FE, indicating some degree of under-reaction to past information by professional forecasters. Column (2) shows that the magnitude for the MA(1) term using the Greenbook data is relatively close to that in column (1), thus also capturing some kind of underreaction from policy makers.

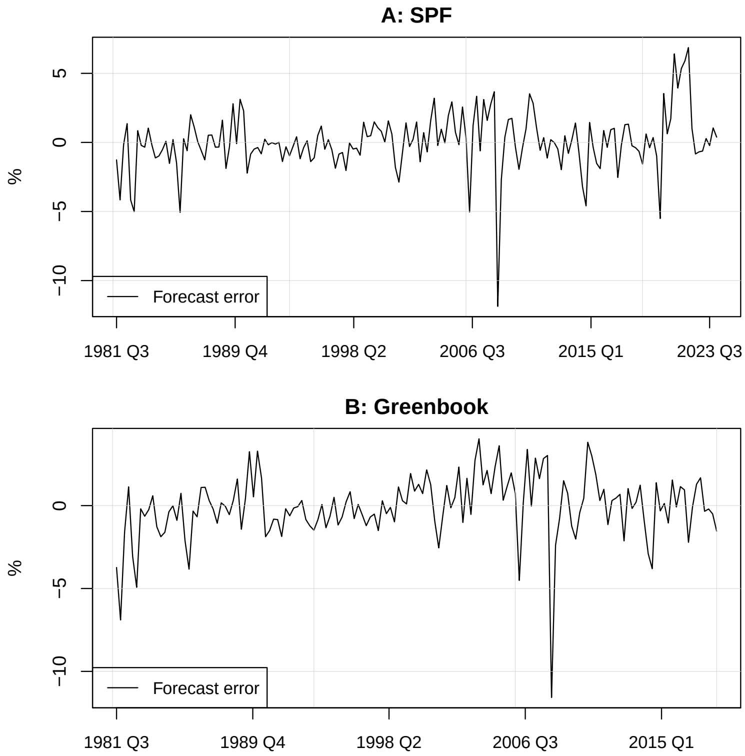

Figure 3.5: Forecast errors - headline CPI inflation.
Note: The black line is the calculated FE for annualised quarter headline CPI inflation using real time CPI from the Real-time Series for Macroeconomists, the SPF (A) and Greebook (B) forecasts.

Table 3.2: Best fitting model for headline consumer price inflation forecast errors

	(1) SPF Estimate	(2) Greenbook Estimate	(3) SPF - GB period Estimate
MA(1)	0.3208 ***	0.2889 ***	0.1958 **
	(0.0762)	(0.0803)	(0.0804)
MA(2)	-0.0262		-0.2112 **
	(0.0848)		(0.0845)
MA(3)	0.1609 **		
	(0.0720)		
intercept	0.0346	-0.0932	-0.1344
	(0.2223)	(0.1963)	(0.1481)
Box-Ljung test	8.6154	11.328	8.7799
	(0.9869)	(0.9373)	(0.9853)

Note: This table shows the estimates of three MA processes for headline CPI inflation FE. The results under column (1) are based on the SPF, Philadelphia Fed. The results in column (2), on the other hand, are obtained using the Greenbook, Philadelphia Fed. Finally, column (3) shows the results using SPF for the same period as in the Greenbook. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: "***"0.01, "**"0.05, "*"0.1.

As for the case of real GDP, another set of models is estimated using the SPF data, but for the same time period as available from the Greenbook (i.e. 1981:Q4 to 2018:Q4). Column (3) displays the results pointing to a MA(2) process for professional forecasters for the period before COVID. Moreover, while the signs are the same as those in column (1), the magnitudes are different. The estimated coefficients suggest a tendency to underreact to the most recent shock, given its positive coefficient, while overreacting to a shock from two periods ago, as denoted by its negative coefficient. This discrepancy may be due to the recent inflation resurgence, as during this period consumer expectations were more accurate than those of professional forecasters. Overall, these findings show that with respect to inflation, the FE of professional forecasters and policy makers in the U.S. exhibit a predictable component; however, the patterns are different between the two groups of forecasters and across sample periods. The results indicate that policy makers overreact and professional forecasters underreact to some extent.[17]

5.1.1.3 3-month Treasury bill rate forecast errors: Figure 3.6 shows the calculated FE for the 3-month Treasury bill rate using only SPF data, as Greenbook projections are not available for this variable.

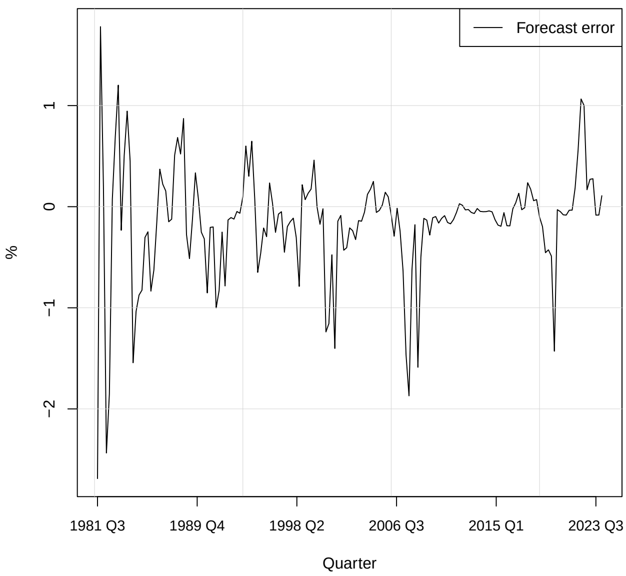

Figure 3.6: Forecast errors - 3-month Treasury bill rate.
Note: The black line is the calculated FE for the 3-month Treasury bill rate using its value from the Federal Reserve Bank of St. Louis and the SPF forecasts.

I followed the same procedure as in the previous two cases and selected the model

[17] Han, Ma, and Mao (2023) analyse AR, MA, and ARMA models for inflation FE using SPF data over a longer period (1969:Q1 - 2019:Q4). They find positive and significant coefficients for the MA(2) model, with magnitudes larger than my estimates, suggesting that the data span influences the final results.

with the lowest AIC. Table 3.3 summarises the results, indicating that a MA(4) is the model that best fits the 3-month Treasury bill FE. In contrast to the models for inflation and real GDP growth, the estimated intercept is negative and significantly different from zero at a 5% level. This implies a systematic overprediction of the interest rate, which can be visually seen in Figure 3.3. The coefficients on the error terms are significant for lags 1, 3 and 4. Aligning with earlier results, the most recent past error exhibits a greater coefficient, indicating that recent experiences significantly influence the FE. With all significant coefficients being positive, this reflects, as in the case of inflation, that professional forecasters exhibit some degree of underreaction to shocks. Nonetheless, when the period studied excludes the post-pandemic, a certain level of overreaction is observed concerning the shocks from two periods before.

Table 3.3: Best fitting model for 3-month Treasury bill rate forecast errors

	(1) SPF Estimate	(2) SPF - GB period Estimate
MA(1)	0.5156 *** (0.0753)	0.5180 *** (0.0792)
MA(2)	-0.1271 (0.0817)	-0.1711 ** (0.0859)
MA(3)	0.1629 * (0.0720)	0.1578 (0.0982)
MA(4)	0.3450 *** (0.0790)	0.3679 *** (0.0846)
intercept	- 0.1679 ** (0.0790)	-0.1894 ** (0.0759)
Box-Ljung test	20.554 (0.4238)	23.83 (0.2499)

Note: This table shows the estimate of an MA(4) process for the 3-month Treasury bill rate FE. The results are based on the SPF, Philadelphia Fed. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: "***"0.01, "**"0.05, "*"0.1.

The empirical findings for the 3-month Treasury bill rate FE also align with the moving average structure introduced by the DE framework. However, there is no evidence of a consistent overreaction from professional forecasters, since the only evidence is when considering the pre-COVID data and solely to the two-lagged shock realisation. The Ljung-Box test here checks that there are no significant autocorrelations in the residuals for any of the models.

In general, the findings presented in this section for the univariate scenarios suggest that DE effectively introduces predictability in FE through the modelling of MA structures. However, it appears that the lags that should be included in the MA processes are sensitive to the specific variables and the periods considered. The dataset characteristics may also influence the results since I use the average of survey responses, which tend to underreact, as shown in Coibion and Gorodnichenko (2015). This underreaction contrasts with each individual forecaster responses, which often overreact (Bordalo et al., 2020).

5.1.2 Multivariate

In this subsection, I use the three variables considered in the univariate case to perform a multivariate analysis.[18] The main objective is to test the result presented in expression (3.41). This specifies that if agents exhibit DE, their FE would contain a predictable component in the form of a moving average process. In a multivariate context, this relationship is represented as a vector moving average (VMA) process:

$$\mathbf{z_t} = \boldsymbol{\mu} + \epsilon_\mathbf{t} - \sum_{\mathbf{i=1}}^{\mathbf{q}} \boldsymbol{\theta}_\mathbf{i} \epsilon_{\mathbf{t-1}},$$

where $\boldsymbol{\mu}$ is a constant vector denoting the mean of $\mathbf{z_t}$, $\boldsymbol{\theta}_i$ are (k x k) matrices, and $\{\epsilon_\mathbf{t}\}$ is a vector of white noise disturbances.

Table 3.4 presents the two-way p-value table of extended cross-correlation matrices for the data set. Following Tsay (2013), the values in Table 3.4 inform statistical significance tests for the cross-correlations between multiple time series at different lags. By comparing the elements of this table with the type I error α, I can identify the appropriate order of the VARMA(p,q) model to estimate. Based on this, the first matrix entry is significant at the 5% level, suggesting that a VAR (1) or a VMA (1) model should be fitted.

Table 3.4: P-values of extended cross-correlation matrices - mean forecast errors

AR/MA	**0**	**1**	**2**	**3**
0	0.0260	0.9202	0.8735	0.9430
1	0.9387	0.9981	0.9890	0.9920
2	0.9911	0.9597	1.0000	0.9995
3	0.9993	0.9965	1.0000	0.9999

A VMA(1) model is preferred over a VAR(1) model based on its lower AIC value, indicating a better fit.[19] The estimation results are presented in Table 3.5. The constant terms for real GDP growth and inflation FE are found to be not significantly different from zero, indicating that there is no systematic over- or under-prediction. In contrast, the constant term for the 3-month T-bill rate is significantly different from zero at the 5%

[18] I only do the multivariate analysis using the SPF data set as policymakers do not forecast future T-bill rates in the Greenbook.

[19] The AIC value for the VMA(1) is 2.723 while for the VAR(1) it is 2.730

level of confidence. This indicates that professional forecasters consistently overpredict the 3-month Treasury bill rate when forming expectations. These observations align with the conclusions drawn from the previous univariate analysis.

Table 3.5: Forecast errors VMA(1) coefficient estimates

Coefficient(s)	**Estimate**	**Std. Error**	**t value**	**Pr**$(>\lvert t\rvert)$
FE_RGDP_constant	-0.2121	0.2267	-0.935	0.3495
FE_π_constant	0.0504	0.2011	0.251	0.8019
FE_TBILL_constant	-0.1431	0.0529	-2.705	0.0068 **
$\epsilon_{t-1}^{RGDP,RGDP}$	-0.1842	0.0871	-2.115	0.0344 *
$\epsilon_{t-1}^{RGDP,\pi}$	-0.0418	0.0428	-0.976	0.3288
$\epsilon_{t-1}^{RGDP,TBill}$	-0.0037	0.0101	-0.365	0.7149
$\epsilon_{t-1}^{\pi,RGDP}$	-0.1644	0.1452	-1.132	0.2574
$\epsilon_{t-1}^{\pi,\pi}$	0.3070	0.0873	3.514	0.0004 ***
$\epsilon_{t-1}^{\pi,TBill}$	-0.0109	0.0181	-0.604	0.5461
$\epsilon_{t-1}^{TBill,RGDP}$	0.0186	0.6012	0.031	0.9752
$\epsilon_{t-1}^{TBill,\pi}$	0.0240	0.3017	0.080	0.9363
$\epsilon_{t-1}^{TBill,TBill}$	0.3992	0.0692	5.765	8.18e-09 ***

Note: This table shows the estimates of a VMA(1) process. The results are based on the mean values from the SPF, Philadelphia Fed. Significance levels: "***"0.01, "**"0.05, "*"0.1.

Examining the matrix that illustrates how each of the three variables is affected by the one-period lagged realisations of the shocks, the significant coefficients are those linking each shock to its respective variable. These coefficients are represented by the values located on the diagonal. This pattern suggests that previous shocks have an impact solely on the forecast error related to the specific variable they affect, rather than influencing others. In other words, there are no cross-variable effects as the other coefficients are not significantly different from zero.

$$\begin{bmatrix} FE_t^{RGDP} \\ FE_t^{\pi} \\ FE_t^{TBill} \end{bmatrix} = \begin{bmatrix} -0.2121 \\ 0.0504 \\ -0.1431^{***} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{RGDP} \\ \epsilon_t^{\pi} \\ \epsilon_t^{TBill} \end{bmatrix} - \begin{bmatrix} 0.1842^{*} & 0.0418 & 0.0037 \\ 0.1645 & -0.3071^{***} & 0.0109 \\ -0.0186 & -0.0241 & -0.3992^{***} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1}^{RGDP} \\ \epsilon_{t-1}^{\pi} \\ \epsilon_{t-1}^{TBill} \end{bmatrix}$$

The signs of the estimates remain consistent with those found in MA(1) terms in the univariate analyses. These show an overreaction for the FE related to real GDP growth rate, whereas both inflation and the T-bill rate show an underreaction. However, the main differences are in the magnitudes of the coefficients. While for the FE in real GDP growth and inflation, the values are relatively close to those found for the most recent shock realisation in the univariate analysis, for the T-bill rate the value is noticeably smaller (0.3992 in the multivariate analysis vs. 0.5180 in the univariate case). This outcome may be influenced by the requirement that VMA lags must be the same for all variables included in the model, which means that each variable is affected by shocks with the same lag structure.

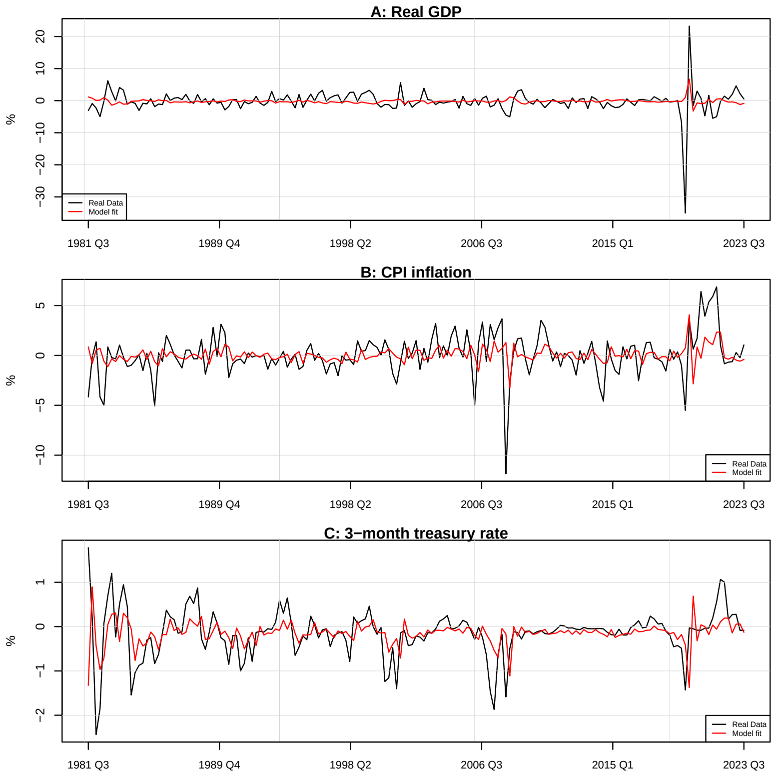

Figure 3.7: Forecast errors - VMA(1) model.
Note: The black line is the real data of the FE. The red line represents the model fit.

Figure 3.7 provides an overview of the performance of the VMA model in fitting the FE to the three variables. It can be observed that the model encounters difficulties in accurately capturing the highly volatile fluctuations that occur during periods of crisis, such as the Great Financial Crisis and the recent COVID-19 pandemic. However, the model appears to perform effectively in aligning the FE associated with the 3-month treasury rate throughout the period. It is important to note that the model does not accurately capture the large variations in real GDP growth and inflation during and post the COVID-19 crisis.

5.2 Forecast revisions

5.2.1 Univariate

In this subsection, I perform the same analysis as in Subsection 5.1 but for the Forecast Revisions (FR). I fit multiple models for each variable in the data set and then select the model exhibiting the lowest AIC value. I also run the analysis for the SPF and Greenbook, as well as for the SPF without including the COVID-pandemic.

5.2.1.1 Real GDP growth forecast revisions: The FR for the real GDP growth rate of professional forecasters (A) and policy makers (B) is shown in Figure 3.8. Here, too, the difference in magnitude is due to the coverage periods, while the SPF include the Covid-19 period, characterised by large revisions in expectations, the Greenbook data only extend until 2018:Q4.

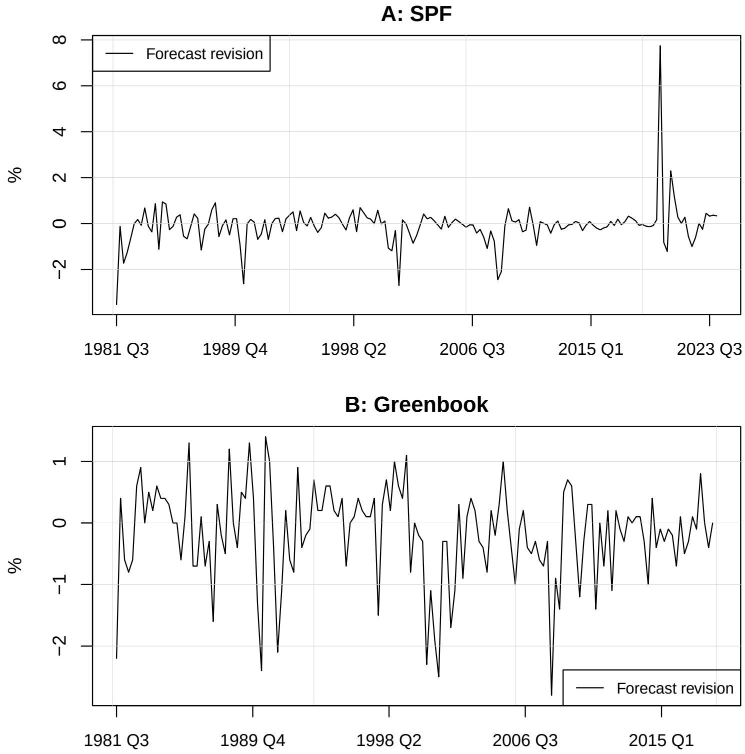

Figure 3.8: Forecast revisions - Real GDP growth rate.
Note: The black line is the calculated FR for real GDP growth using the SPF (A) and Greenbook (B) forecasts.

The findings in this case, as illustrated in Table 3.6, reveal a discrepancy between the predictions of professional forecasters and those of policymakers. These differences are also apparent when I consider different time periods. In the three cases examined, the models present AR components. These do not align with the prediction that, when DE are incorporated into a macroeconomic model, the FR should be predictable based on past shock realisations and, therefore, consist solely of MA terms. The presence of AR terms implies additional sources of predictability beyond what DE suggest.

The best-fitting model for the FR using SPF data for 1981:Q3 to 2023:Q4 is an ARMA(3,3). This infers that the FR at time t can be explained by its three last values and the last three shock realisations, although only the AR and MA terms of order 1 and 3 are statistically significant. Column (2) shows the model that best fits the data from the Greenbook, which is an AR(1), while if I use the SPF data for the same period, column (3), it is and ARMA(1,1). In both cases, the intercept is statistically different from zero

Table 3.6: Best fitting model for real GDP growth forecast revisions

	(1) SPF Estimate	(2) Greenbook Estimate	(3) SPF - GB period Estimate
AR(1)	0.8886 ***	0.2207 ***	0.7327 ***
	(0.1925)	(0.0815)	(0.1556)
AR(2)	0.0485		
	(0.2953)		
AR(3)	-0.4840 ***		
	(0.1775)		
MA(1)	-0.8213 ***		-0.5118 ***
	(0.1612)		(0.1829)
MA(2)	-0.1792		
	(0.2514)		
MA(3)	0.7503 ***		
	(0.1610)		
intercept	-0.0913	-0.1806 **	-0.1667 *
	(0.0876)	(0.0789)	(0.0937)
Box-Ljung test	5.2814	12.703	20.668
	(0.9996)	(0.8897)	(0.4169)

Note: This table shows the estimates of three processes for real GDP growth FR. The results under column (1) are based on the SPF, Philadelphia Fed. The results in column (2), on the other hand, are obtained using the Greenbook, Philadelphia Fed. Finally, column (3) shows the results using SPF for the same period as in the Greenbook. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: "***"0.01, "**"0.05, "*"0.1.

and negative, indicating that on average both agents revise their forecasts downward. In the latter case, the estimate of the autoregressive parameter suggests a high degree of persistence, whereas the coefficient associated with the error term implies that a positive shock leads to a downward adjustment in expectations for the subsequent period.

In general, the forecast revisions for the real GDP growth rate lack the systematic pattern implied by the DE model. In contrast, the results highlight a certain underreaction or slow learning in the expectation formation process of professional forecasters and policy makers.

5.2.1.2 Inflation forecast revisions: The FR data series for inflation are shown in Figure 3.9. In Table 3.7, I present the estimates of the best-fitting model for each scenario. When examining the time series calculated using the SPF between 1981:Q3 and 2023:Q4, an ARMA (1,1) is the best model. This notably contrasts with the DE model's forecast, which points toward merely including MA terms. The coefficients outlined in column (1) reveal a substantial level of persistence, quantified at 0.8077, alongside a negative response to the previous period's shock. The latter implies that a positive shock leads forecasters to revise their expectations downward, while a negative shock results in an upward revision. These results contrast with the DE model, which predicts revisions in the same direction as the shock due to extrapolation.

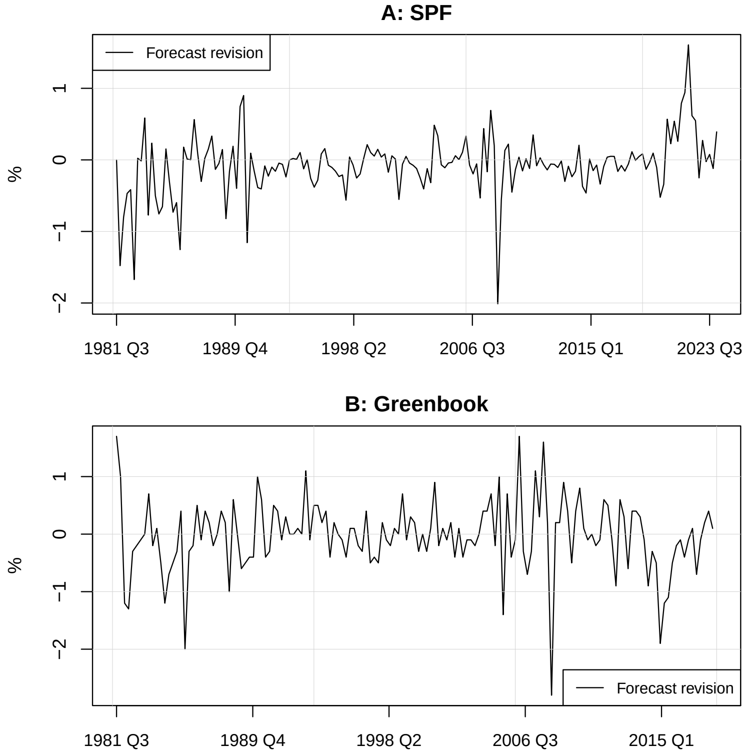

Figure 3.9: Forecast revisions - headline CPI inflation.
Note: The black line is the calculated FR for annualised quarter headline CPI inflation using SPF (A) and Greenbook (B) forecasts.

Concerning the updates in forecasts made by policy makers, a MA(1) process has been selected as the most suitable model. The sign of the estimated coefficient is positive (column (2)), which may indicate an overreaction in the expectations that these agents hold, as predicted by DE. Nonetheless, it is important to note that the estimated coefficient is not statistically significant. When analysing the time series corresponding to the SPF for the identical period covered by the Greenbook, the results show a negative and statistically significant intercept of -0.1247. This finding implies a consistent downward adjustment in inflation expectations as time progresses. Additionally, the estimates for the AR(1) and MA(1) terms both exhibit values close to one, indicating potential concerns related to near-unit root behaviour. In addition, the value estimated for the MA(1) component suggests that, when considering pre-COVID data, there is a tendency for professional forecasters to overreact in the direction of the shock, as predicted by DE.

In general, the evidence on FR for the headline CPI inflation shows variations between different agents and over varying time periods. As DE suggest, the FR has some degree of

Table 3.7: Best fitting model for headline consumer price inflation forecast revisions

	(1) SPF Estimate	(2) Greenbook Estimate	(3) SPF - GB period Estimate
AR(1)	0.8077***		-0.9980 ***
	(0.1704)		(0.0106)
MA(1)	-0.6334 ***	0.1287	0.9932 ***
	(0.2218)	(0.0860)	(0.0208)
intercept	-0.0858	-0.0301	-0.1247 ***
	(0.0579)	(0.0573)	(0.0314)
Box-Ljung test	18.36	15.073	23.728
	(0.5637)	(0.7722)	(0.2545)

Note: This table shows the estimates of three processes for CPI inflation FR. The results under column (1) are based on the Survey of Professional Forecasters, Philadelphia Fed. The results in column (2), on the other hand, are obtained using the Greenbook, Philadelphia Fed. Finally, column (3) shows the results using SPF for the same period as in the Greenbook. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: "***"0.01, "**"0.05, "*"0.1.

predictability through MA terms. Moreover, some evidence points towards autoregressive behaviour.

5.2.1.3 3-month Treasury bill rate forecast revisions: Similarly to the forecast errors, the Forecast Revisions (FR) for the 3-month T-bill rate is only available from the SPF and it is depicted in Figure 3.10.

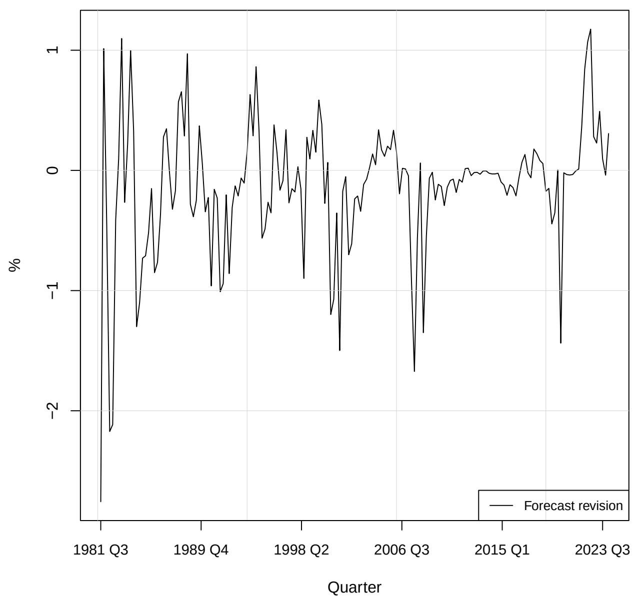

Figure 3.10: Forecast revisions - 3-month Treasury bill rate.
Note: The black line is the calculated FR for the 3-month Treasury bill rate using SPF forecasts.

Table 3.8 details the models that best fit the forecast revisions for the 3-month Treasury bill rate, as derived from the SPF. In the full sample, the process follows a MA(4) structure. The coefficients for the first, third and fourth lags are statistically significant at the 1% level, and their positive signs suggest that the forecasts tend to be revised in the same direction of the shock realisation, as predicted by DE. In contrast, as shown in column (2), during the Great Moderation period, the FR demonstrate marked autoregressive behaviour, with a dominant AR(1) coefficient of 1.3728, followed by significant AR(2) and weaker AR(3) and AR(4) terms. The MA(1) coefficient in this period is very close to -1, indicating a high degree of persistence and potential overcorrection in revisions after a shock. The intercept is negative and statistically significant in both cases, suggesting a systematic downward adjustment in expectations that remained unchanged even after the pandemic. The Box-Ljung test statistics do not indicate strong evidence of serial correlation in the residuals for either model.

Table 3.8: Best fitting model for 3-month Treasury bill forecast revisions

	(1) SPF Estimate	(2) SPF - GB period Estimate
AR(1)		1.3728 ***
		(0.0926)
AR(2)		-0.6322 ***
		(0.1652)
AR(3)		0.4971
		(0.1557)
AR(4)		-0.3258
		(0.0953)
MA(1)	0.3957 ***	-0.9999 ***
	(0.0781)	(0.0212)
MA(2)	-0.0022	
	(0.0819)	
MA(3)	0.2048 **	
	(0.0816)	
MA(4)	0.2666 ***	
	(0.0805)	
intercept	-0.1475 **	-0.1594 ***
	(0.0876)	(0.0937)
Box-Ljung test	9.8365	15.988
	(0.971)	(0.7174)

Note: This table shows the estimates of two processes for the 3-month T-bill rate FR. The results are based on the SPF, Philadelphia Fed. The last row exhibits the Ljung-Box test statistic value, with its p-value in parenthesis. Significance levels: “***”0.01, “**”0.05, “*”0.1.

5.2.2 Multivariate

In this subsection, I use the three variables previously analysed individually to perform a multivariate analysis. In a similar vein to subsection 5.1.2, Table 3.9 exhibits the two-way table of p values of extended cross-correlation matrices. When comparing the elements of this table with the type I error, the significant entries at the 5% level are on row one until column (3), which suggests estimating six different models: VAR(1), VMA(1), VMA(2), VMA(3), VARMA(1,1) and VARMA(1,2). After comparing the AIC values, I chose the VMA(1) model.[20]

Table 3.9: P-values of extended cross-correlation matrices - mean forecast revisions

AR/MA	**0**	**1**	**2**	**3**
0	0.0000	0.0008	0.0028	0.1029
1	0.3597	0.2059	0.7383	0.9904
2	0.6375	0.7784	0.8908	0.9927
3	0.9693	0.9388	0.9211	0.8623

Table 3.10 presents the results. The estimated coefficients for the intercepts are statistically significant at the 5% level for both inflation and T-bill rate, which provides evidence that professional forecasters have a consistent downward bias in adjusting their expectations for these variables. Upon examining the matrix governing how past shocks influence current Forecast Revisions (FR), the lower right corner 2 by 2 submatrix reveals statistically significant values. The impact of previous shocks on the T-bill rate is particularly notable. It appears to incite an excessive adjustment in the forecasts by professionals for inflation and T-bill rate, indicative of overreactive behaviour. Consequently, any perturbation in the interest rate results in excessive revisions by professional forecasters concerning both variables. A parallel scenario is observed for inflation; however, the magnitude of the influence is comparatively reduced. In contrast, estimates for revisions of forecasts related to real GDP growth rate do not exhibit statistically significant values. The direction indicated by the signs suggests a tendency towards overreacting to news, thereby motivating a revision of their current expectations in the same direction of the shocks.

In conclusion, the findings suggest the presence of DE as FR align with the outcome depicted in Equation (3.44). This expression indicates that, accounting for DE, the revisions in forecasts should follow a VMA process with positive coefficients, thus reflecting over-reactive behaviour, manifested as excessive adjustments to forecasts in response to emerging shocks or news.

[20]The AIC values for the models are: $AIC_{VAR(1)}$=-3.530, $AIC_{VMA(1)}$=-3.540, $AIC_{VMA(2)}$=-3.401, $AIC_{VMA(3)}$=-3.462, $AIC_{VARMA(1,1)}$=Inf and $AIC_{VARMA(1,2)}$=NaN.

Table 3.10: Forecast revisions VMA(1) coefficient estimates

Coefficient(s)	**Estimate**	**Std. Error**	**t value**	**Pr**$(>\|t\|)$
FR_RGDP_constant	-0.0575	0.0714	-0.806	0.4202
FR_π_constant	-0.0789	0.0377	-2.091	0.0365 **
FR_TBILL_constant	-0.1215	0.0504	-2.411	0.0158 **
$\epsilon_{t-1}^{RGDP,RGDP}$	0.0813	0.0843	0.964	0.3349
$\epsilon_{t-1}^{RGDP,\pi}$	0.0279	0.0340	0.819	0.4125
$\epsilon_{t-1}^{RGDP,TBill}$	0.0120	0.0402	0.300	0.7640
$\epsilon_{t-1}^{\pi,RGDP}$	0.0847	0.1742	0.487	0.6265
$\epsilon_{t-1}^{\pi,\pi}$	0.1582	0.0766	2.066	0.0388 **
$\epsilon_{t-1}^{\pi,TBill}$	0.2083	0.0925	2.252	0.0243 **
$\epsilon_{t-1}^{TBill,RGDP}$	0.1357	0.1363	0.995	0.3195
$\epsilon_{t-1}^{TBill,\pi}$	0.1862	0.0610	3.051	0.0022 ***
$\epsilon_{t-1}^{TBill,TBill}$	0.3037	0.0704	4.312	1.62e-05 ***

Note: This table shows the estimates of a VMA(1) process. The results are based on the mean values from the SPF, Philadelphia Fed. Significance levels: "***"0.01, "**"0.05, "*"0.1.

$$\begin{bmatrix} FR_t^{RGDP} \\ FR_t^{\pi} \\ FR_t^{TBill} \end{bmatrix} = \begin{bmatrix} -0.0575 \\ -0.0789^{**} \\ -0.1431^{**} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{RGDP} \\ \epsilon_t^{\pi} \\ \epsilon_t^{TBill} \end{bmatrix} - \begin{bmatrix} -0.0813 & -0.0279 & -0.0121 \\ -0.0848 & -0.1583^{**} & -0.2084^{**} \\ -0.1358 & -0.1863^{***} & -0.3037^{***} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1}^{RGDP} \\ \epsilon_{t-1}^{\pi} \\ \epsilon_{t-1}^{TBill} \end{bmatrix}$$

The performance of the VMA (1) model can be visually assessed in Figure 3.11. Similar to the forecast errors multivariate model, the VMA(1) model analysed here encounters difficulties in accurately representing the dynamics of FR during crisis periods. In the context of real GDP growth, the COVID-19 pandemic introduced significant over-revisions. This challenge was further compounded in the post-pandemic period with increased inflation and interest rates, which the model also failed to accurately predict. Despite this, the model supports the VMA structure predicted by DE.

6 Robustness

Within the analytical framework detailed in Section 5, the evaluations were initially conducted using mean values of forecasts derived from the SPF. In this section, I consider median forecasts to perform a robustness analysis and test the reliability of the results. I also evaluate the models' out-of-sample performance for both Forecast Errors (FE) and Forecast Revisions (FR).[21]

[21] The SPF includes forecasts from multiple forecasters, providing both the mean and median of their responses, making it suitable for these evaluations. In contrast, the Greenbook provides a single forecast

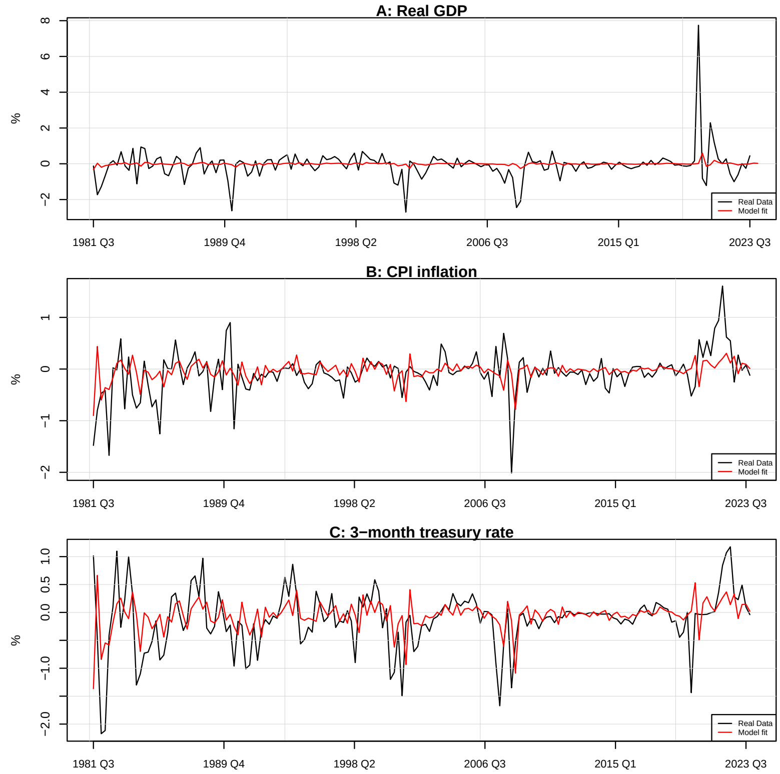

Figure 3.11: Forecast revisions - VMA(1) model.
Note: The black line depicts the real data series of the FR, while the red one is the model fit.

6.1 Use of median responses

6.1.1 Forecast errors

For single-variable analyses, Table 3.11 summarises the results of the univariate models that best fit the FE for real GDP growth, inflation, and the T-bill rate, employing the median forecasts from the SPF survey data from 1981:Q3 to 2023:Q4. Columns (1) and (2) reveal moving average orders, magnitudes, signs, and statistical significance that are very similar to those in column (1) of Tables 3.1 and 3.2. This indicates that the findings related to the FE for the real GDP growth rate and inflation are robust regardless of whether the mean or median forecast is used. When examining the T-bill rate forecasts, the notable difference is that the coefficient for the third lag shock realisation based on the median forecasts becomes significant at the 10% level, a deviation from its insignificance noted in Table 3.3, while the remaining coefficients are in line with those obtained earlier, and so is the MA(4) order. Overall, the results suggest that the findings of the univariate analysis in Section 5 are robust.

that represents the view of the policy makers in the Philadelphia FED. Therefore, the use of the SPF is emphasised here, as it offers a range of forecasts from various professional forecasters.

Table 3.11: Forecast errors robustness analysis using median responses from SPF

	(1) RGDP	(2) Inflation	(3) TBill
MA(1)	-0.1855 **	0.3333 ***	0.4938 ***
	(0.0769)	(0.0763)	(0.0748)
MA(2)		-0.0189	-0.1777 **
		(0.0850)	(0.0806)
MA(3)		0.1529 **	0.1619 *
		(0.0715)	(0.0880)
MA(4)			0.3402 ***
			(0.0785)
intercept	-0.2319	0.0089	-0.1508 **
	(0.2329)	(0.2250)	(0.0695)

Note: This table shows the estimate of an MA(1) process for the real GDP growth rate FE, an MA(3) process for headline CPI inflation FE, and an MA(4) process for the 3-month T-bill rate FE. The results are based on the SPF, Philadelphia Fed. Significance codes: "***"0.01, "**"0.05, "*"0.1.

Using a multivariate analysis, the following expression summarises the results from the VMA(1) process.[22] Here too, the findings of Section 5 are confirmed when using median forecasts, i.e. past shock realisations exclusively impact the FE of the specific variable they affect with the only statistically significant values being those on the diagonal. However, the FE for the real GDP growth rate demonstrate a reduced overreaction, with the parameter decreasing from -0.1842 when using mean forecasts to -0.1690 when using median forecasts. The results for T-bill rate also show a milder underreaction, with its value decreasing to 0.3720 from 0.3992. In contrast, inflation shows a greater underreaction.

$$\begin{bmatrix} FE_t^{RGDP} \\ FE_t^{\pi} \\ FE_t^{TBill} \end{bmatrix} = \begin{bmatrix} -0.2095 \\ 0.0259 \\ -0.1255^{**} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{RGDP} \\ \epsilon_t^{\pi} \\ \epsilon_t^{TBill} \end{bmatrix} - \begin{bmatrix} 0.1690^{**} & 0.0489 & 0.0045 \\ 0.1529 & -0.3297^{***} & 0.0106 \\ -0.0182 & -0.0260 & -0.3720^{***} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1}^{RGDP} \\ \epsilon_{t-1}^{\pi} \\ \epsilon_{t-1}^{TBill} \end{bmatrix}$$

In summary, with median forecasts, FE maintain the predictable MA structure implied by the presence of diagnostic expectations, similar to the case of using the mean SPF. However, in both contexts, an overreaction is found solely in the FE of the real GDP growth rate.

[22] Appendix 3.C.1, Table 3.17 shows the complete results.

6.1.2 Forecast revisions

When considering Forecast Revisions (FR), the results appear to be less robust in some cases. As shown in Table 3.12, the best fit univariate models for FR of real GDP growth and inflation have ARMA structures and different coefficient values when using median forecasts, compared to those based on mean forecasts (in Tables 3.6 and 3.7). In particular, while the optimal models are AR(4) and AR(1) under the SPF median forecasts, when using the SPF mean forecasts, ARMA(3) and MA(1) are identified as the best-fitting models for these two variables. In contrast, the results for the T-bill rate are robust, with the same MA specification as obtained earlier and coefficients of similar magnitudes.

Table 3.12: Forecast revisions robustness analysis using median responses from SPF

	(1) RGDP	(2) Inflation	(3) TBill
AR(1)	0.1895 **	0.3749 ***	
	(0.0803)	(0.0707)	
AR(2)	-0.0904		
	(0.0806)		
AR(3)	0.1782 **		
	(0.0808)		
AR(4)	0.1267		
	(0.1171)		
MA(1)			0.3989 ***
			(0.0774)
MA(2)			-0.0430
			(0.0803)
MA(3)			0.2351 ***
			(0.0829)
MA(4)			0.2659 ***
			(0.0797)
intercept	-0.0890	-0.0645	-0.1455 **
	(0.1241)	(0.0473)	(0.0738)

Note: This table shows the estimate of an AR(4) process for the real GDP growth rate FR, an AR(1) for headline CPI inflation FR, and an MA(4) for the 3-month T-bill rate FR. The results are based on the SPF, Philadelphia Fed. Significance codes: "***"0.01, "**"0.05, "*"0.1.

The robustness analysis continues by estimating a VMA(1) model for the FR data set using the median SPF. The results are expressed in matrix form in the following expression.[23]

$$\begin{bmatrix} FR_t^{RGDP} \\ FR_t^{\pi} \\ FR_t^{TBill} \end{bmatrix} = \begin{bmatrix} -0.0383 \\ 0.0614 \\ -0.1170^{**} \end{bmatrix} + \begin{bmatrix} \epsilon_t^{RGDP} \\ \epsilon_t^{\pi} \\ \epsilon_t^{TBill} \end{bmatrix} - \begin{bmatrix} -0.1586^{*} & -0.0195 & -0.0029 \\ 0.0236 & -0.2557^{***} & -0.1725^{*} \\ -0.1455 & -0.1540^{***} & -0.2973^{***} \end{bmatrix} \begin{bmatrix} \epsilon_{t-1}^{RGDP} \\ \epsilon_{t-1}^{\pi} \\ \epsilon_{t-1}^{TBill} \end{bmatrix}$$

Unlike in the case of forecast errors, the findings here display a certain degree of robustness. The values in the 2 by 2 submatrix located in the lower right corner remain statistically significant. However, the impact of an inflationary shock on the FR in inflation is two thirds greater than when the mean SPF is used (-0.2557 vs -0.1583). Similarly, the effect of a real GDP shock on the FR of real GDP growth has almost doubled in size (increasing from an absolute value of 0.0813 to an absolute value of 0.1586), and has also become significant at 10% confidence level, which was not the case in the analysis of Section 5. In general, the findings suggesting the presence of DE in the FR are robust to the use of the median SPF data.

6.2 Out-of-sample forecast evaluation

One way to evaluate the out-of-sample performance of the VMA models is to study how well they fit future values that are not considered in the estimation process. In this case, since the estimated model is a VMA(1), I study how well the model predicts the value observed in 2024:Q1. I use this method because, after one step, the predictions in a VMA(1) depend solely on the mean vector since future residuals are unknown and assumed to be zero, which means that the model predictions converge to the mean of the process.[24]

In Figure 3.12, I plot the model's one-period-ahead prediction for the three Forecast Errors (FE) with a 95% confidence interval. The new observations, denoted by the black dots in the figure, lie within this interval across all cases. The model effectively captures the next period's inflation FE and the upward trend of the T-bill rate FE. However, for the FE of the real GDP growth, the actual data in 2024:Q1 shows a change of trend relative to the previous quarters, which the model finds hard to predict.

An analogous analysis is conducted for the VMA(1) model with respect to FR, as depicted in Figure 3.13. The confidence interval contains the new data points; however, the model performs less well in point estimation. The prediction for the real GDP growth rate FR is the closest to the actual observation, yet it inaccurately suggests a downward

[23] Appendix 3.C.1, Table 3.19 shows the complete results.

[24] This is a limitation of the VMA model's structure. VMA models are not designed to project beyond the immediate horizon because they lack autoregressive or trend components.

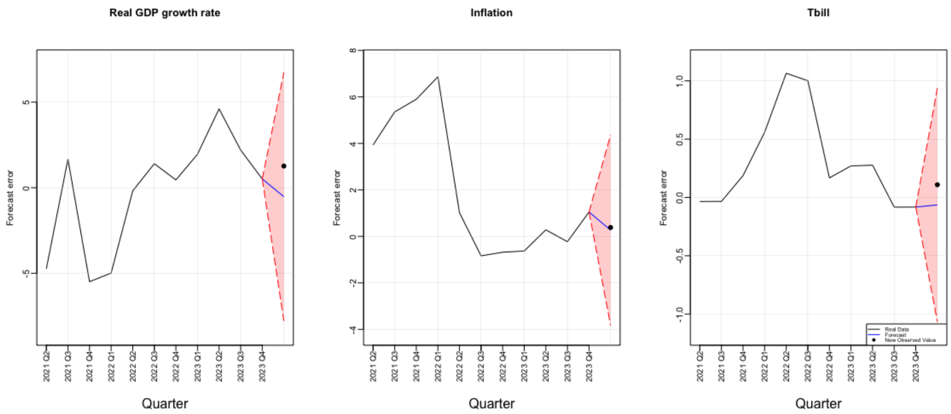

Figure 3.12: Forecast errors prediction - VMA(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FE for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

revision contrary to the actual trend. A similar pattern is observed for the T-bill rate. In contrast, the prediction for the inflation FR accurately identifies the direction but not the extent of the change in FR.[25]

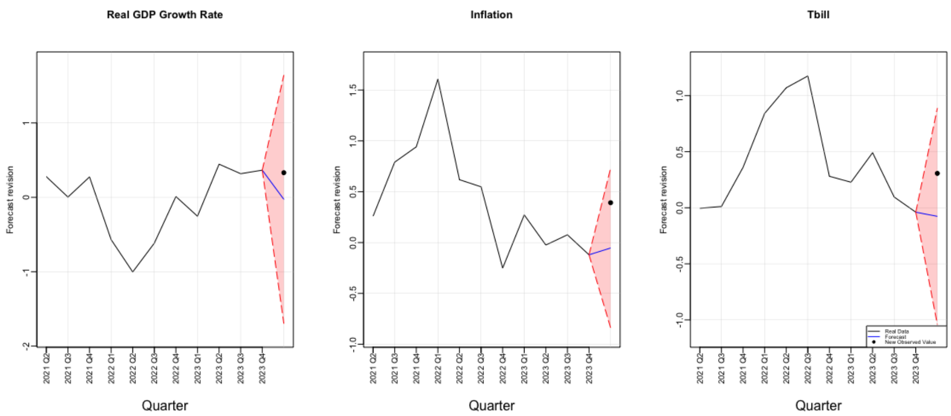

Figure 3.13: Forecast revisions prediction - VMA(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FR for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

To gain a deeper understanding of the VMA models' performance for both FE and FR, I summarise a few accuracy metrics in Table 3.13, namely the Mean Squared Prediction Error (MSPE), the Root Mean Squared Prediction Error (RMSPE) and the Mean Ab-

[25] In Appendix 3.C.2 I present the graphs for the case in which the models are estimated using the median responses of the SPF.

solute Percentage Error (MAPE).[26] In general, the VMA(1) model performs well when forecasting out-of-sample inflation FE one period ahead, as evidenced by its low error measures. Specifically, the model exhibits the smallest prediction errors for inflation FE compared to real GDP growth rate and T-bill rate FE. The VMA(1) model struggles to predict the real GDP growth rate FE, as the metrics for this variable (MSPE = 3.2041, RMSPE = 1.7900) are significantly higher than those for the inflation FE and the T-bill rate FE. For forecast revisions, the VMA(1) model shows relatively consistent performance across the three variables, with similar values for MSPE, RMSPE, and MAPE.[27]

Table 3.13: Forecast errors and revisions metrics

		Real GDP growth rate	**Inflation**	**TBill**
	MSPE	3.2041	0.0128	0.0299
FE	RMSPE	1.7900	0.1132	0.1731
	MAPE	1.7900	0.1132	0.1731
	MSPE	0.1284	0.1985	0.1467
FR	RMSPE	0.3584	0.4455	0.3830
	MAPE	0.3584	0.4455	0.3830

Note: MSPE is Mean Squared Prediction Error, RMSPE is Root Mean Squared Prediction Error, and MAPE is Mean Absolute Percentage Error.

7 Multiple forecast horizons: three-quarters ahead

This section exploits the fact that the SPF contains multiple forecasting horizons to check the general result for the Forecast Errors (FE) and Forecast Revisions (FR) h periods ahead, as obtained in Section 3. In this case, following Coibion and Gorodnichenko (2015), I opt for $h = 3$ since the SPF includes up to four-quarters ahead forecasts and to obtain the FR, the expression calls for an additional forecasting horizon.

7.1 t+3 Forecast errors

Table 3.14 presents the outcome of the multivariate model that most accurately captures professional forecasters' predictions three periods ahead.[28] In this case, the model has

[26]The MSPE gives a sense of the overall error magnitude in the forecast, while the RMSPE is the squared root of the MSPE and provides the error magnitude in the same unit as the data. The MAPE measures the average absolute error as a percentage of the actual values.

[27]In Appendix 3.C.2, Table 3.20 presents these measures for the estimated models using the SPF median. The biggest difference in the results is that, when using the median, the model prediction error of the one-period-ahead real GDP growth rate FE is worst.

[28]Appendix 3.C.2, Table 3.22 shows the p-values of the extended cross-correlation matrix that informs which models might be the best at capturing the behaviour of the variables.

a VAR(1) structure, different from the VMA(1) for the forecasts for one period ahead. This suggests that long-term FE may exhibit greater persistence and serial correlation, potentially due to expectation rigidities or gradual learning dynamics. If forecasters systematically misupdate expectations across multiple periods, errors can become autocorrelated, favouring a VAR representation.

Table 3.14: Forecast errors t+3 VAR(1) coefficient estimates

Coefficient(s)	Estimate	Std. Error	t value	$Pr(>\|t\|)$
FE_RGDP_constant	-0.3332	0.3452	-0.965	0.3344
FE_π_constant	-0.0277	0.1693	-0.164	0.8698
FE_TBILL_constant	-0.0691	0.0498	-1.387	0.1653
$FE\_RGDP_{t-1}^{RGDP,RGDP}$	-0.1467	0.0792	-1.852	0.0641 *
$FE\_RGDP_{t-1}^{RGDP,\pi}$	-0.0441	0.0388	-1.135	0.2566
$FE\_RGDP_{t-1}^{RGDP,TBill}$	0.0120	0.0114	1.049	0.2942
$FE\_\pi_{t-1}^{\pi,RGDP}$	-0.1369	0.1529	-0.895	0.3707
$FE\_\pi_{t-1}^{\pi,\pi}$	0.4141	0.0750	5.521	3.38e-08 ***
$FE\_\pi_{t-1}^{\pi,TBill}$	0.0305	0.0220	1.384	0.1664
$FE\_TBILL_{t-1}^{TBill,RGDP}$	0.0294	0.3003	0.098	0.9218
$FE\_TBILL_{t-1}^{TBill,\pi}$	0.0685	0.1467	0.467	0.6401
$FE\_TBILL_{t-1}^{TBill,TBill}$	0.8056	0.0430	18.731	2e-16 ***

Note: This table shows the estimates of a VAR(1) process. The results are based on the mean values from the SPF, Philadelphia Fed. Significance levels: "***"0.01, "**"0.05, "*"0.1.

Moreover, according to the table, the only statistically significant results are those associating each FE with its own lag, indicating the absence of cross-correlation between the variables' FE. The out-of-sample performance of this VAR(1) is shown in Figure 3.14.[29] The model does a good job overall, considering that the out-of-sample observations lie within the confidence interval and are relatively close to the point estimates. Similarly to the analysis in Subsection 6.2, the models fails to account the trend in the FE of the real GDP growth rate.

7.2 t+3 Forecast revisions

The three-periods ahead FR also presents a VMA(1) structure, similar to the one estimated for the one-period FR in Subsection 5.2. However, the main difference is that FR in multiple period horizons exhibit a more pronounced response to shocks, especially in the real GDP growth rate and the T-bill rate, which is reflected in the higher estimated parameters in Table 3.15.[30] The opposite result holds for inflation. Moreover, recent

[29]The in-sample performance is presented in Appendix 3.C.2, Figure 3.18

[30]This result is in line with Bianchi et al. (2024b) who find higher overreaction for extended forecast horizons.

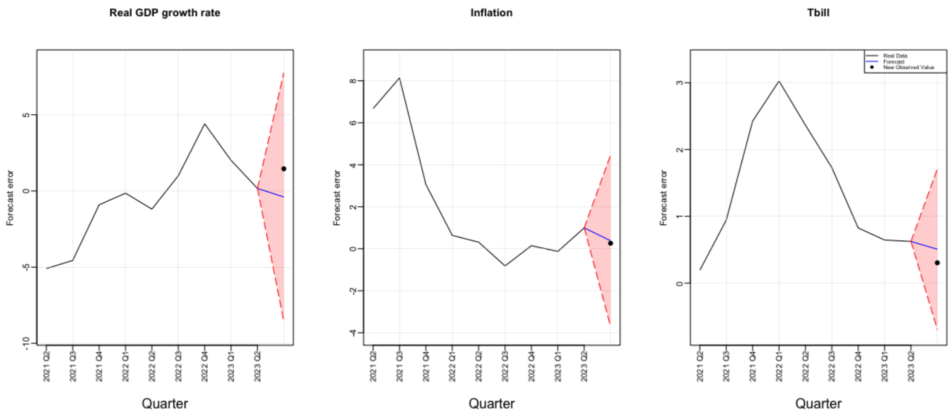

Figure 3.14: t+3 Forecast errors prediction - VAR(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FE for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

shocks seem to significantly influence the FR three periods ahead for real GDP growth, unlike the case of one-period ahead. Professional forecasters, in this context, also have a tendency to overreact to shocks to the T-bill rate, with the impact being about two-thirds greater than when considering one-period ahead.

Table 3.15: Forecast revisions t+3 VMA(1) coefficient estimates

Coefficient(s)	**Estimate**	**Std. Error**	**t value**	**Pr(> $\|t\|$)**
FR_RGDP_constant	-0.0020	0.0440	-0.046	0.9629
FR_π_constant	-0.0771	0.0253	-3.037	0.0023 ***
FR_TBILL_constant	-0.1459	0.0519	-2.811	0.0049 ***
$\epsilon_{t-1}^{RGDP,RGDP}$	0.2095	0.0824	2.540	0.0110 **
$\epsilon_{t-1}^{RGDP,\pi}$	-0.1287	0.0478	-2.693	0.0070 ***
$\epsilon_{t-1}^{RGDP,TBill}$	0.0644	0.0658	0.979	0.3276
$\epsilon_{t-1}^{\pi,RGDP}$	0.0303	0.1403	0.216	0.8288
$\epsilon_{t-1}^{\pi,\pi}$	-0.0287	0.0795	-0.362	0.7176
$\epsilon_{t-1}^{\pi,TBill}$	0.0816	0.1187	0.687	0.4918
$\epsilon_{t-1}^{TBill,RGDP}$	0.0139	0.0851	0.164	0.8696
$\epsilon_{t-1}^{TBill,\pi}$	0.1941	0.0516	3.763	0.0001 ***
$\epsilon_{t-1}^{TBill,TBill}$	0.4839	0.0691	7.003	2.5e-12 ***

Note: This table shows the estimates of a VMA(1) process. The results are based on the mean values from the SPF, Philadelphia Fed. Significance codes: "***"0.01, "**"0.05, "*"0.1.

Overall, the results suggest a stronger revision in the direction of the shock realisation the longer the forecast horizon. Figure 3.15 illustrates the performance of the out-of-sample VMA(1) model. The results are similar to the one-period ahead case, although here the point prediction for the FR of the real GDP growth rate is more accurate in terms of point estimation and direction. The model still struggles to capture the revision for inflation and the T-bill.

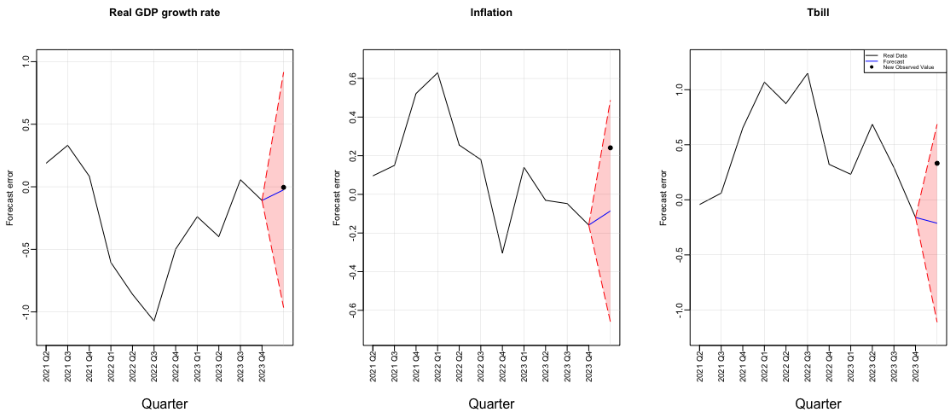

Figure 3.15: t+3 Forecast revisions prediction - VMA(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FE for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

8 Concluding remarks

In this chapter, I explore the impact of DE on the state-space structure of macroeconomic models and the resulting Forecast Errors (FE) and Forecast Revisions (FR). By analytically deriving the expressions for FE and FR in both a simple three-equation model and a generalised framework, I show that representativeness, as a belief formation process, introduces predictability components to the FE and FR, in line with previous empirical findings. Using data from the Philadelphia FED SPF and Greenbook/Tealbook forecasts, I assess whether forecasts by professional forecasters and policy makers are influenced by diagnosticity.

The short-term quantitative results indicate that univariate FE generally follow the expected MA structures as suggested by the analytical results, albeit with different lags across variables. In addition, significant overreaction only appears in the real GDP growth rate FE of professional forecasters when the COVID-19 pandemic is included; otherwise, the evidence largely points to underreactions. In the multivariate case, the MA(1) structure is consistent across all variables, but overreaction remains specific to real GDP growth. On the other hand, the univariate results for FR are inconclusive with respect to DE, whereas the multivariate results provide stronger evidence, particularly showing substantial revisions in inflation and T-bill forecasts.

Considering longer forecast horizons, the estimates suggest that FE three periods ahead may exhibit greater persistence and serial correlation, since the estimated model is a VAR(1). This could potentially be due to rigidities in the expectation formation process or some gradual learning dynamics. The estimates for the FR reveal that professional forecasters, in this context, have a tendency to overreact to shocks to the T-bill rate and real GDP, but present no significant response to shocks to inflation. The results imply that the longer the forecast horizon is, the stronger the revision in the direction of the shock realisation.

In summary, the findings in this chapter are informative but far from conclusive, as the empirical evidence does not seem to suggest a definitive pattern. Therefore, as noted in Reis (2020), there is still little agreement on a suitable non-full information rational expectations benchmark. Further research on expectation formation is needed, as it appears that the process is heterogeneous across households and variables, as well as state-dependent. This is crucial given the importance of the expectations channel for, for example, monetary policy transmission.

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Appendices

3.A Generalised Forecast Errors t+1 and t+h Periods Ahead

The system has a solution form:

$$\underbrace{\begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_{t+1} \\ \boldsymbol{\epsilon}_{t+1} \end{bmatrix}}_{\Gamma_{t+1}} = \underbrace{\begin{bmatrix} \overbrace{\mathbf{h}_{z,z}}^{H_z} & \overbrace{\mathbf{h}_{z,x}}^{H_x} & \overbrace{\mathbf{h}_{z,\epsilon}}^{H_\epsilon} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix}}_{\text{H}} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1}.$$

$$\mathbf{y}_t = \underbrace{\begin{bmatrix} \overbrace{\mathbf{g}_{1,z}}^{G_z} & \overbrace{\mathbf{g}_{1,x}}^{G_x} & \overbrace{\mathbf{g}_{1,\epsilon}}^{G_\epsilon} \\ \mathbf{g}_{2,z} & \mathbf{g}_{2,x} & \mathbf{g}_{2,\epsilon} \\ . & . & . \\ . & . & . \\ \mathbf{g}_{m,z} & \mathbf{g}_{m,x} & \mathbf{g}_{m,\epsilon} \end{bmatrix}}_{\text{G}} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix}$$

In order to obtain the FE, first I forward the diagnostic solution for the endogenous variables one period ahead:

$$\mathbf{y}_{t+1} = \mathbf{G}\boldsymbol{\Gamma}_{\mathbf{t+1}} \tag{3.46}$$

Replacing the law of motion for the vector of the state variables, including exogenous variables, endogenous variables, and the shocks' realisations:

$$\mathbf{y}_{t+1} = \mathbf{G}\left(\begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{h}_{z,x} & \mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1}. \right). \tag{3.47}$$

Thus, the solution for $\mathbf{y}_{t+1}$ is:

$$\mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{h}_{\epsilon,z}\end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,x}\\ \mathbf{h}_{x,x}\\ \mathbf{h}_{\epsilon,x}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{k}_{z}\\ \mathbf{k}_{x}\\ \mathbf{k}_{\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_{t+1}. \tag{3.48}$$

Next, I obtain an expression for the diagnostic expected values of the endogenous variables. I apply the diagnostic operator to the previous expression, obtaining:

$$E_t^{\phi}\mathbf{y}_{t+1} = E_t^{\phi}[\mathbf{G}\boldsymbol{\Gamma}_{t+1}], \tag{3.49}$$

which after replacing the **diagnostic** solution for the law of motion of the state vector:

$$E_t^{\phi}\mathbf{y}_{t+1} = \mathbf{G}E_t^{\phi}\left(\begin{bmatrix}\mathbf{h}_{z,z} & \mathbf{h}_{z,x} & \mathbf{h}_{z,\epsilon}^{DE}\\ \mathbf{h}_{x,z} & \mathbf{h}_{\mathbf{x},\mathbf{x}} & \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\begin{bmatrix}\mathbf{z}_t\\ \mathbf{x}_t\\ \boldsymbol{\epsilon}_t\end{bmatrix} + \begin{bmatrix}\mathbf{k}_{z}\\ \mathbf{k}_{x}\\ \mathbf{k}_{\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_{t+1}\right). \tag{3.50}$$

After applying the diagnostic expectations operator, this becomes:

$$E_t^{\phi}\mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{h}_{\epsilon,z}\end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,x}\\ \mathbf{h}_{x,x}\\ \mathbf{h}_{\epsilon,x}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,\epsilon}^{DE}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_t, \tag{3.51}$$

Using the FE definition:

$$\begin{aligned}\mathbf{y_{t+1}} - E_t^{\phi}\mathbf{y_{t+1}} =& \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{h}_{\epsilon,z}\end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,x}\\ \mathbf{h}_{x,x}\\ \mathbf{h}_{\epsilon,x}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{k}_{z}\\ \mathbf{k}_{x}\\ \mathbf{k}_{\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_{t+1} - \\ &\left(\mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{h}_{\epsilon,z}\end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,x}\\ \mathbf{h}_{x,x}\\ \mathbf{h}_{\epsilon,x}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,\epsilon}^{DE}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_t\right).\end{aligned} \tag{3.52}$$

After cancelling terms, this equals:

$$\mathbf{y_{t+1}} - E_t^{\phi}\mathbf{y_{t+1}} = \mathbf{G}\left(\begin{bmatrix}\mathbf{h}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix} - \begin{bmatrix}\mathbf{h}_{z,\epsilon}^{DE}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\right)\boldsymbol{\epsilon}_{\mathbf{t}} + \mathbf{G}\begin{bmatrix}\mathbf{k}_{z}\\ \mathbf{k}_{x}\\ \mathbf{k}_{\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_{\mathbf{t+1}}. \tag{3.53}$$

Given that, once the model is solved, future realizations are driven by the shocks true data generating process, the entries of $\mathbf{h}_{z,\boldsymbol{\epsilon}}$ are zero. In addition, using the partitioned matrix $\mathbf{G}$ in three-submatrices, and the fact that sub-matrix $\mathbf{h}_{\epsilon,\epsilon}$ has zero elements, the first terms can be written as:

$$\begin{bmatrix} \overbrace{\mathbf{g}_{1,z}}^{G_z} & \overbrace{\mathbf{g}_{1,x}}^{G_x} & \overbrace{\mathbf{g}_{1,\epsilon}}^{G_\epsilon} \\ \mathbf{g}_{2,z} & \mathbf{g}_{2,x} & \mathbf{g}_{2,\epsilon} \\ . & . & . \\ . & . & . \\ \mathbf{g}_{m,z} & \mathbf{g}_{m,x} & \mathbf{g}_{m,\epsilon} \end{bmatrix} \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} \right) \boldsymbol{\epsilon}_t.$$

Here, since the elements in $\mathbf{h}_{x,\epsilon}$ are the same in both brackets, the final expression for the one period ahead FE boils to:

$$\mathbf{y_{t+1}} - E_t^{\phi} \mathbf{y_{t+1}} = -\mathbf{G}_z \mathbf{h}_{z,\epsilon}^{DE} \boldsymbol{\epsilon}_t + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon_{t+1}}. \tag{3.54}$$

In order to obtain the general expression for the FE $t+h$ periods ahead, I calculate the FE for $t+2$ periods ahead:

$$\mathbf{y}_{t+2} = \mathbf{G}\boldsymbol{\Gamma}_{\mathbf{t+2}}. \tag{3.55}$$

Along the sames lines as in the case $t+1$, I replace the law of motion for $\boldsymbol{\Gamma}_{t+2}$[1]:

$$\mathbf{y}_{t+2} = \mathbf{G} \left(\begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{h}_{z,x} & \mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t+1} \\ \mathbf{x}_{t+1} \\ \boldsymbol{\epsilon}_{t+1} \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \right), \tag{3.56}$$

where I further replace the law of motion for $\boldsymbol{\Gamma}_{t+1}$:

$$\mathbf{y}_{t+2} = \mathbf{GH} \left(\begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{h}_{z,x} & \mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{h}_{\epsilon,z} & \mathbf{h}_{\epsilon,x} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{t} \\ \mathbf{x}_{t} \\ \boldsymbol{\epsilon}_{t} \end{bmatrix} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} \right) + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.57}$$

[1]Initially, I present the derivations considering the whole matrices, later on I elaborate on this, and consider their specific elements.

which after opening the parentheses equals:

$$\mathbf{y}_{t+2} = \mathbf{G}\mathbf{H}^2 \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.58}$$

The expected diagnostic values for endogenous variables in period $t+2$ are equal to:

$$E_t^\phi \mathbf{y}_{t+2} = \mathbf{G}\mathbf{H}^{2,DE} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix}. \tag{3.59}$$

Thus, again following its definition, the FE $t+2$ periods ahead is:

$$\mathbf{y}_{t+2} - E_t^\phi \mathbf{y}_{t+2} = \mathbf{G}\left(\mathbf{H}^2 - \mathbf{H}^{2,DE}\right) \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.60}$$

For the case of $t+3$ periods ahead, the FE results:

$$\mathbf{y}_{t+3} - E_t^\phi \mathbf{y}_{t+3} = \mathbf{G}\left(\mathbf{H}^3 - \mathbf{H}^{3,DE}\right) \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G}\mathbf{H}^2 \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+3}. \tag{3.61}$$

Therefore, the generalized expression for the FE $t+h$ periods ahead can be written as:

$$\mathbf{y}_{t+h} - E_t^\phi \mathbf{y}_{t+h} = \mathbf{G}\left(\mathbf{H}^h - \mathbf{H}^{h,DE}\right) \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G} \sum_{\tau=1}^{h} \mathbf{H}^{h-\tau} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+\tau}. \tag{3.62}$$

In what follows, I will show that final expression (3.62) exhibits a predictable component, as it simplifies and depends on the vector of shocks realizations $\boldsymbol{\epsilon}_t$. Nevertheless, the matrix multiplying this vector turns out to be a dense product of sub-matrices as h increases. Here I derive matrix $H^{h,DE}$ since matrix H^h results from setting sub-matrix $\mathbf{h}_{z,\epsilon}^{DE}$ to zero. First, considering $t+2$ periods ahead and the fact that some of these sub-matrices are $\mathbf{0}$ as presented in section 3.1, matrix $H^{2,DE}$ results in:

$$H^2 = \begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{0} & \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix} * \begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{0} & \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$H^2 = \begin{bmatrix} \mathbf{h}_{z,z}^2 & \mathbf{0} & \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z} & \mathbf{h}_{x,x}^2 & \mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon} \\ 0 & 0 & \mathbf{0} \end{bmatrix} \tag{3.63}$$

To obtain matrix $H^{3,DE}$, I multiply (3.63) by H:

$$H^3 = \begin{bmatrix} \mathbf{h}_{z,z}^2 & \mathbf{0} & \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z} & \mathbf{h}_{x,x}^2 & \mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} * \begin{bmatrix} \mathbf{h}_{z,z} & \mathbf{0} & \mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z} & \mathbf{h}_{x,x} & \mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

which results in

$$H^3 = \begin{bmatrix} \mathbf{h}_{z,z}^3 & \mathbf{0} & \mathbf{h}_{z,z}^2\mathbf{h}_{z,\epsilon}^{DE} \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{h}_{z,z} + \mathbf{h}_{x,x}^2\mathbf{h}_{x,z} & \mathbf{h}_{x,x}^3 & (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}^2\mathbf{h}_{x,\epsilon} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{3.64}$$

First, I replace (3.63) in (3.60), cancel terms, and use the fact that for realised values, sub-matrix $\mathbf{h}_{z,\epsilon}^{DE}$ equals $\mathbf{0}$, obtaining:

$$\begin{aligned} \mathbf{y}_{t+2} - E_t^{\phi}\mathbf{y}_{t+2} = \mathbf{G} \left(\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}^{DE} \\ \mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} \right) \boldsymbol{\epsilon}_t \\ + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \end{aligned} \tag{3.65}$$

After solving the parenthesis and using the partitions of matrix $\mathbf{G}$, I obtain that the expression for the FE two periods ahead is:

$$\mathbf{y_{t+2}} - E_t^{\phi}\mathbf{y_{t+2}} = \left(-\mathbf{G}_z \overbrace{\mathbf{h}_{z,z}}^{H_{1,1}^1} \mathbf{h}_{z,\epsilon}^{DE} - \mathbf{G}_x \overbrace{\mathbf{h}_{x,z}}^{H_{2,1}^1} \mathbf{h}_{z,\epsilon}^{DE} \right) \boldsymbol{\epsilon}_t + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.66}$$

Here, I use over-braces above certain elements in the expression, following the notation $H_{1,1}^1$. This highlights the specific sub-matrices from matrix H^1 that are used, with the goal of deriving a generalised expression. Now, substituting (3.64) in (3.61):

$$\mathbf{y}_{t+3} - E_t^{\phi}\mathbf{y}_{t+3} = \mathbf{G}\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{h}_{x,x}^2\mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix} - \begin{bmatrix} \mathbf{h}_{z,z}^2\mathbf{h}_{z,\epsilon}^{DE} \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}^2\mathbf{h}_{x,\epsilon} \\ \mathbf{0} \end{bmatrix}\right)\boldsymbol{\epsilon}_t + \mathbf{G}\left(\mathbf{H}^2\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+1} + \mathbf{H}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+2} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+3}\right). \tag{3.67}$$

After solving the parenthesis and using the partitions of matrix **G**, I obtain that the expression for the FE three periods ahead is:

$$\mathbf{y_{t+3}} - E_t^{\phi}\mathbf{y_{t+3}} = \left(-\mathbf{G}_z\overbrace{\mathbf{h}_{z,z}^2}^{H_{1,1}^2}\mathbf{h}_{z,\epsilon}^{DE} - \mathbf{G}_x\overbrace{(\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z})}^{H_{2,1}^2}\mathbf{h}_{z,\epsilon}^{DE}\right)\boldsymbol{\epsilon}_t + \mathbf{G}\left(\mathbf{H}^2\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+1} + \mathbf{H}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+2} + \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+3}\right). \tag{3.68}$$

Similarly, the notation above the over-braces specifies which element of matrix H^2 is being used in this expression. Using this notation, I can then generalise the result to the case of h periods ahead:

$$\mathbf{y_{t+h}} - E_t^{\phi}\mathbf{y_{t+h}} = \left[-\mathbf{G}_z\mathbf{H}_{1,1}^{h-1}\mathbf{h}_{z,\epsilon}^{DE} - \mathbf{G}_x\mathbf{H}_{2,1}^{h-1}\mathbf{h}_{z,\epsilon}^{DE}\right]\boldsymbol{\epsilon}_t + \mathbf{G}\sum_{\tau=1}^{h}\mathbf{H}^{h-\tau}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+\tau}. \tag{3.69}$$

3.B Generalised Forecast Revisions t+1 and t+h Periods Ahead

I start the derivation for FR 1 and h periods ahead from the solution of y_{t+1}:

$$\mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,z} \\ \mathbf{h}_{x,z} \\ \mathbf{h}_{\epsilon,z} \end{bmatrix}\mathbf{z}_t + \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,x} \\ \mathbf{h}_{x,x} \\ \mathbf{h}_{\epsilon,x} \end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix} \mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{x,\epsilon} \\ \mathbf{h}_{\epsilon,\epsilon} \end{bmatrix}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix}\boldsymbol{\epsilon}_{t+1}. \tag{3.70}$$

I replace z_t using the law of motion for the exogenous state variables in the system solution:

$$\mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{h}_{\epsilon,\mathbf{z}}\end{bmatrix}(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}_{z,\epsilon}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t) + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,x}\\ \mathbf{h}_{x,x}\\ \mathbf{h}_{\epsilon,x}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{h}_{\epsilon,\epsilon}\end{bmatrix}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{k}_z\\ \mathbf{k}_x\\ \mathbf{k}_\epsilon\end{bmatrix}\boldsymbol{\epsilon}_{t+1}, \tag{3.71}$$

where the products are conformable since $\mathbf{G}$ is size (m x n), $\mathbf{H}_z$ has size (n x n_z) and the submatrices $\mathbf{h}_{z,z}$, $\mathbf{h}_{z,x}$ and $\mathbf{h}_{z,\epsilon}$ have sizes (n_z x n_z), (n_z x n_x) and (n_z x n_ϵ), respectively. Finally, the submatrix $\mathbf{k}_z$ has size (n_z x n_ϵ).

Now, using the fact that some elements of this matrix are zeros, and taking the diagnostic expectations operator with information until period t and then until period $t-1$, I obtain:

$$E_t^\phi \mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{0}\end{bmatrix}\left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}^{DE}_{z,\epsilon}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t\right) + \mathbf{G}\begin{bmatrix}\mathbf{0}\\ \mathbf{h}_{x,x}\\ \mathbf{0}\end{bmatrix}\mathbf{x}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}^{DE}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t, \tag{3.72}$$

and

$$E_{t-1}^\phi \mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{0}\end{bmatrix}\left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}^{DE}_{z,\epsilon}\boldsymbol{\epsilon}_{t-1}\right) + \mathbf{G}\begin{bmatrix}\mathbf{0}\\ \mathbf{h}_{x,x}\\ \mathbf{0}\end{bmatrix}\mathbf{x}_t. \tag{3.73}$$

Using the definition of the FR and subtracting equation (3.73) from equation (3.72):

$$E_t^\phi \mathbf{y}_{t+1} - E_{t-1}^\phi \mathbf{y}_{t+1} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}\\ \mathbf{h}_{x,z}\\ \mathbf{0}\end{bmatrix}\mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}^{DE}_{z,\epsilon}\\ \mathbf{h}_{x,\epsilon}\\ \mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t, \tag{3.74}$$

This expression shows that the FR of the diagnostic agent has a predictable component. In addition, the FR will exhibit an overreaction in the direction of the shock, govern by the second term on the right-hand side, since the submatrix $\mathbf{h}^{DE}_{z,\epsilon}$ will depend on the diagnostic parameter ϕ. In the case of rational expectations, that is, $\phi = 0$, the expression boils down to just the first term on the right-hand side.

In order to obtain the general expression for the FR $t+h$ periods ahead, I use Equation (3.58):

$$\mathbf{y}_{t+2} = \mathbf{G}\mathbf{H}^2 \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.75}$$

Replacing H^2 obtained in Equation (3.63):

$$\mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}^2 & \mathbf{0} & \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon} \\ \mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z} & \mathbf{h}_{x,x}^2 & \mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon} \\ 0 & 0 & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{z}_t \\ \mathbf{x}_t \\ \boldsymbol{\epsilon}_t \end{bmatrix} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.76}$$

After multiplying the elements of H^2 by the vector of state variables:

$$\mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}^2\mathbf{z}_t + \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}\boldsymbol{\epsilon}_t \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{z}_t + \mathbf{h}_{x,x}^2\mathbf{x}_t + (\mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon})\boldsymbol{\epsilon}_t \\ \mathbf{0} \end{bmatrix} + \mathbf{G}\mathbf{H} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+1} + \mathbf{G} \begin{bmatrix} \mathbf{k}_z \\ \mathbf{k}_x \\ \mathbf{k}_\epsilon \end{bmatrix} \boldsymbol{\epsilon}_{t+2}. \tag{3.77}$$

Using the law of motion for z_t and applying the diagnostic expectations operator at t:

$$E_t^\phi \mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}^2 \left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t\right) + \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_t \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z}) \left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t\right) + \mathbf{h}_{x,x}^2\mathbf{x}_t + (\mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon})\boldsymbol{\epsilon}_t \\ \mathbf{0} \end{bmatrix}. \tag{3.78}$$

Following the same step, but now taking the diagnostic expectations operator at $t-1$:

$$E_{t-1}^\phi \mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}^2 \left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_{t-1}\right) \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z}) \left(\mathbf{h}_{z,z}z_{t-1} + \mathbf{h}_{z,x}x_{t-1} + \mathbf{h}_{z,\epsilon}^{DE}\boldsymbol{\epsilon}_{t-1} + \mathbf{k_z}\boldsymbol{\epsilon}_t\right) + \mathbf{h}_{x,x}^2\mathbf{x}_t \\ \mathbf{0} \end{bmatrix}. \tag{3.79}$$

Now, using the definition of FR and subtracting (3.79) from (3.78), I obtain the FR for $t+2$ periods ahead:

$$E_t^\phi \mathbf{y}_{t+2} - E_{t-1}^\phi \mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}^2 \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,z} + \mathbf{h}_{x,x}\mathbf{h}_{x,z}) \\ \mathbf{0} \end{bmatrix} \mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G} \begin{bmatrix} \mathbf{h}_{z,z}\mathbf{h}_{z,\epsilon}^{DE} \\ (\mathbf{h}_{x,z}\mathbf{h}_{z,\epsilon}^{DE} + \mathbf{h}_{x,x}\mathbf{h}_{x,\epsilon}) \\ \mathbf{0} \end{bmatrix} \boldsymbol{\epsilon}_t. \tag{3.80}$$

This result can be expressed in terms of the submatrices from (3.63) as:

$$E_t^\phi \mathbf{y}_{t+2} - E_{t-1}^\phi \mathbf{y}_{t+2} = \mathbf{G} \begin{bmatrix} \mathbf{H}_{1,1}^2 \\ \mathbf{H}_{2,1}^2 \\ \mathbf{0} \end{bmatrix} \mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G} \begin{bmatrix} \mathbf{H}_{1,3}^2 \\ \mathbf{H}_{2,3}^2 \\ \mathbf{0} \end{bmatrix} \boldsymbol{\epsilon}_t. \tag{3.81}$$

In the case of the FR $t+3$ periods ahead, the expression is:

$$E_t^{\phi}\mathbf{y}_{t+3} - E_{t-1}^{\phi}\mathbf{y}_{t+3} = \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}^3\\(\mathbf{h}_{x,z}\mathbf{h}_{z,z}+\mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{h}_{z,z}+\mathbf{h}_{x,x}^2\mathbf{h}_{x,z}\\\mathbf{0}\end{bmatrix}\mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{h}_{z,z}^2\mathbf{h}_{z,\epsilon}^{DE}\\(\mathbf{h}_{x,z}\mathbf{h}_{z,z}+\mathbf{h}_{x,x}\mathbf{h}_{x,z})\mathbf{h}_{z,\epsilon}^{DE}+\mathbf{h}_{x,x}^2\mathbf{h}_{x,\epsilon}\\\mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t, \tag{3.82}$$

which generalises to:

$$E_t^{\phi}\mathbf{y}_{t+3} - E_{t-1}^{\phi}\mathbf{y}_{t+3} = \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,1}^3\\\mathbf{H}_{2,1}^3\\\mathbf{0}\end{bmatrix}\mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,3}^3\\\mathbf{H}_{2,3}^3\\\mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t. \tag{3.83}$$

Therefore, the FR for h periods ahead can be written as:

$$E_t^{\phi}\mathbf{y}_{\mathbf{t+h}} - E_{t-1}^{\phi}\mathbf{y}_{\mathbf{t+h}} = \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,1}^h\\\mathbf{H}_{2,1}^h\\\mathbf{0}\end{bmatrix}\mathbf{k_z}\boldsymbol{\epsilon}_t + \mathbf{G}\begin{bmatrix}\mathbf{H}_{1,3}^h\\\mathbf{H}_{2,3}^h\\\mathbf{0}\end{bmatrix}\boldsymbol{\epsilon}_t. \tag{3.84}$$

$H_{i,j}^h$ here refers to a particular submatrix related to each power of the matrix H, as extended horizons introduce more complex terms.

3.C Additional Results

3.C.1 Robustness analysis - Median SPF

Table 3.16: P-values of extended cross-correlation matrices - median forecast errors

AR/MA	0	1	2	3
0	0.0503	0.9325	0.8889	0.9259
1	0.9343	0.9963	0.9765	0.9846
2	0.9902	0.9771	1.0000	0.9999
3	0.9988	0.9934	1.0000	0.9997

3.C.2 Out-of-sample forecast evaluation - Median SPF

Table 3.17: Forecast errors VMA(1) coefficient estimates - median SPF

Coefficient(s)	Estimate	Std. Error	t value	Pr(> |t|)
FE_RGDP_constant	-0.2095	0.2305	-0.909	0.3633
FE_π_constant	0.0259	0.2045	0.127	0.8990
FE_TBILL_constant	-0.1255	0.0525	-2.389	0.0168 **
$\epsilon_{t-1}^{RGDP,RGDP}$	-0.1690	0.0853	-1.981	0.0476 **
$\epsilon_{t-1}^{RGDP,\pi}$	-0.0489	0.0426	-1.147	0.2514
$\epsilon_{t-1}^{RGDP,TBill}$	-0.0045	0.0105	-0.434	0.6642
$\epsilon_{t-1}^{\pi,RGDP}$	-0.1529	0.1449	-1.056	0.2911
$\epsilon_{t-1}^{\pi,\pi}$	0.3297	0.0865	3.811	0.0001 ***
$\epsilon_{t-1}^{\pi,TBill}$	-0.0106	0.0188	-0.567	0.5709
$\epsilon_{t-1}^{TBill,RGDP}$	0.0182	0.5891	0.031	0.9752
$\epsilon_{t-1}^{TBill,\pi}$	0.0260	0.2998	0.087	0.9307
$\epsilon_{t-1}^{TBill,TBill}$	0.3720	0.0765	4.862	1.16e-06 ***

Note: This table shows the estimates of a VMA(1) process. The results are based on the median values from the SPF, Philadelphia Fed. Significance levels: "***"0.01, "**"0.05, "*"0.1.

Table 3.18: P-values of extended cross-correlation matrices - median forecast revisions

AR/MA	0	1	2	3
0	0.0000	0.0004	0.0022	0.0278
1	0.5370	0.1943	0.8686	0.7852
2	0.7805	0.3452	0.9776	0.9804
3	0.9448	0.9906	0.9612	0.9317

Table 3.19: Forecast revisions VMA(1) coefficient estimates - median SPF

Coefficient(s)	Estimate	Std. Error	t value	Pr(> |t|)
FE_RGDP_constant	-0.0383	0.0840	-0.457	0.6479
FE_π_constant	-0.0614	0.0394	-1.557	0.1194
FE_TBILL_constant	-0.1170	0.0518	-2.255	0.0241 **
$\epsilon_{t-1}^{RGDP,RGDP}$	0.1585	0.0835	1.899	0.0576 *
$\epsilon_{t-1}^{RGDP,\pi}$	0.0194	0.0292	0.665	0.5060
$\epsilon_{t-1}^{RGDP,TBill}$	0.0029	0.0376	0.079	0.9368
$\epsilon_{t-1}^{\pi,RGDP}$	-0.0235	0.1974	-0.119	0.9050
$\epsilon_{t-1}^{\pi,\pi}$	0.2556	0.0724	3.530	0.0004 ***
$\epsilon_{t-1}^{\pi,TBill}$	0.1725	0.0999	1.726	0.0843 *
$\epsilon_{t-1}^{TBill,RGDP}$	0.1454	0.1458	0.997	0.3185
$\epsilon_{t-1}^{TBill,\pi}$	0.1539	0.0566	2.720	0.0065 ***
$\epsilon_{t-1}^{TBill,TBill}$	0.2973	0.0744	3.992	6.55e-05 ***

Note: This table shows the estimates of a VMA(1) process. The results are based on the median values from the SPF, Philadelphia Fed. Significance levels: "***"0.01, "**"0.05, "*"0.1.

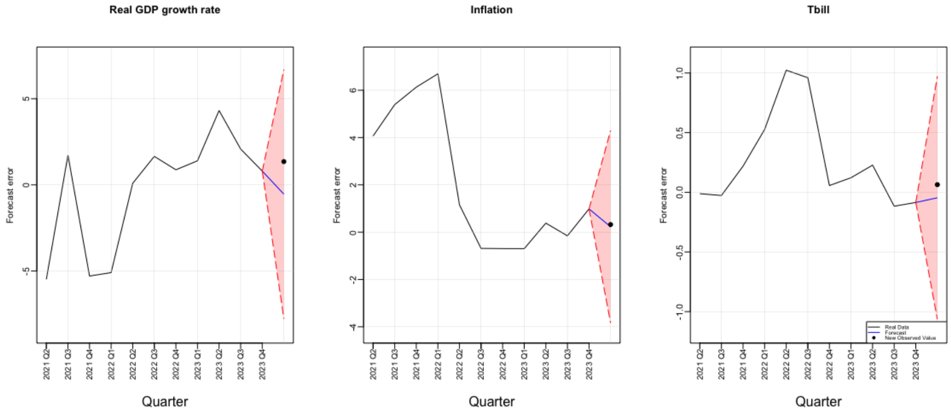

Figure 3.16: Forecast errors prediction - VMA(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FE for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

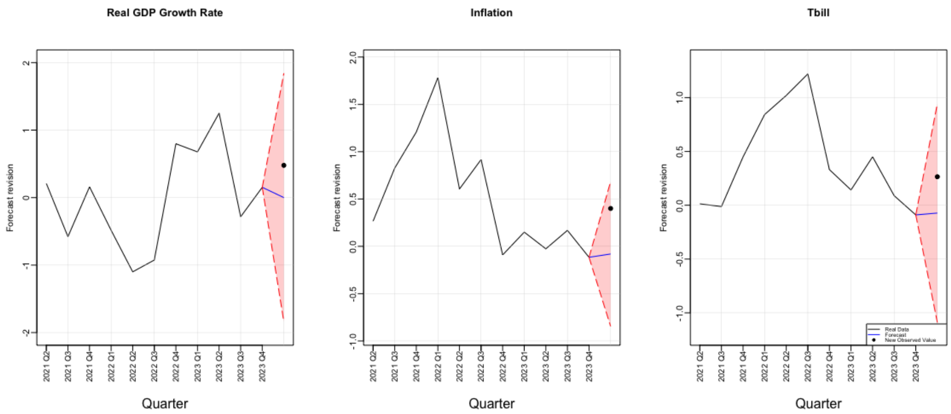

Figure 3.17: Forecast revisions prediction - VMA(1) model.
Note: The black line depicts the real data series from 2021:Q2 to 2023:Q4. The blue line is the prediction of the FE for 2024:Q1. The black dot represents the observation 2024:Q1 used to compare out-of-sample performance of the VMA(1) model.

Table 3.20: Forecast errors metrics - median SPF

		Real GDP growth rate	**Inflation**	**TBill**
FE	MSPE	3.5690	0.0086	0.0121
	RMSPE	1.8891	0.0928	0.1100
	MAPE	1.8891	0.0928	0.1100
FR	MSPE	0.2280	0.2313	0.1140
	RMSPE	0.4775	0.4810	0.3376
	MAPE	0.4775	0.4810	0.3376

Table 3.21: P-values of extended cross-correlation matrices - mean forecast errors t+3 periods ahead

AR/MA	0	1	2	3
0	0.0000	0.0000	0.0631	0.6089
1	0.0077	0.9909	0.9694	0.7310
2	0.6452	0.5861	0.9947	0.9999
3	0.9713	0.8901	0.9993	0.9991

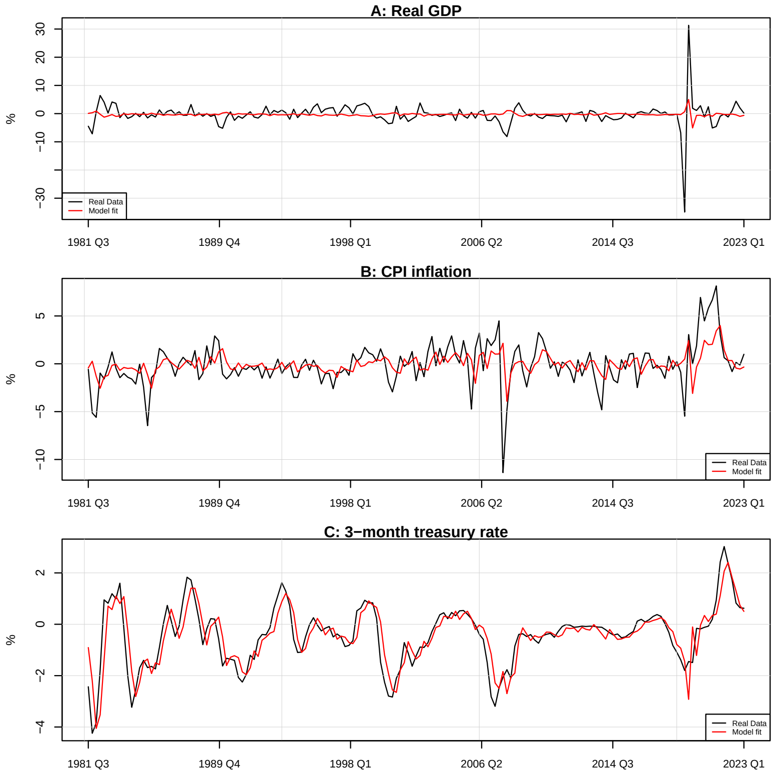

Figure 3.18: Forecast errors t+3 periods ahead - VMA(1) model.
Note: The black line depicts the real data series of the FR, while the red one is the model fit.

Table 3.22: P-values of extended cross-correlation matrices - mean forecast revisions t+3 periods ahead

AR/MA	0	1	2	3
0	0.0000	0.0015	0.0514	0.3335
1	0.1849	0.0853	0.8194	0.3695
2	0.5619	0.8616	0.7068	0.2582
3	0.9417	0.7841	0.9969	0.8648

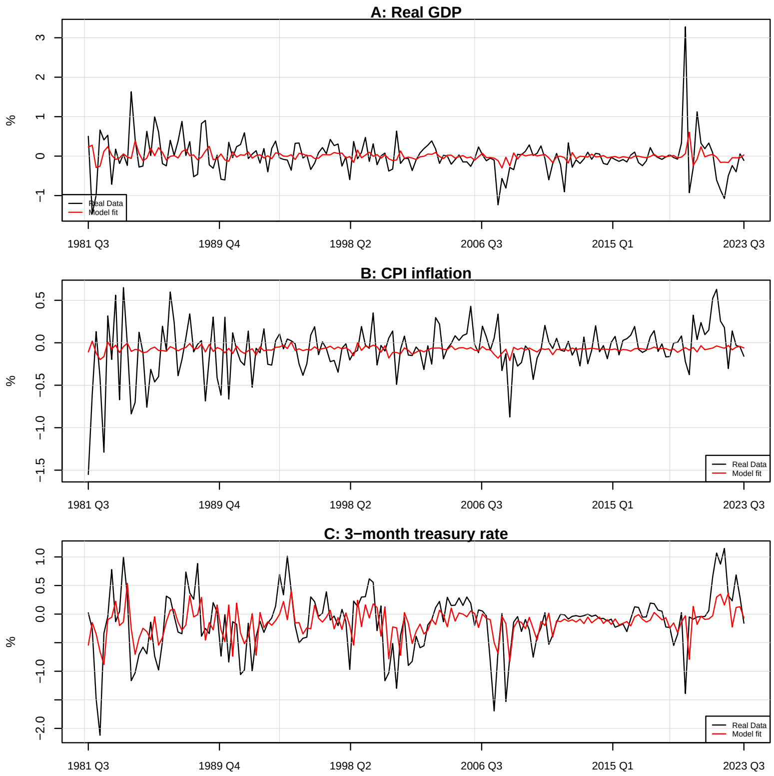

Figure 3.19: Forecast revisions t+3 periods ahead - VMA(1) model.
Note: The black line depicts the real data series of the FR, while the red one is the model fit.

Summary and Future Research

This thesis has studied the dynamics of belief-induced macroeconomic volatility, focusing on the effects of including Diagnostic Expectations (DE) in New Keynesian models. I found that when agents form biased expectations governed by representativeness, exchange rate and housing market fluctuations can be explained with smaller shock innovations, as DE work as an amplification mechanism that generates highly volatile expectations. Moreover, I found that memory plays a crucial role: in the housing market, individuals appear to place greater weight on recent information compared to more distant past events. Finally, I derive general expressions for both Forecast Errors (FE) and Forecast Revisions (FR) in terms of state-space model solutions, making them suitable for empirical testing. In addition, I show that DE introduce predictability in the FE and over-adjustments in the direction of the news in FR, which contradicts the FIRE assumption.

The chapters unfold to build a comprehensive framework for understanding the effects of DE across macroeconomic settings. The first chapter conducts a qualitative analysis about the implications of DE in a Small Open Economy (SOE) model. It highlights how DE interact with persistence mechanisms to create excess volatility, offering a behavioural explanation for exchange rate puzzles. The second chapter takes a quantitative approach, building on the qualitative insights of the first chapter, to show the role of DE in explaining housing market fluctuations. Using a TANK model with housing and banking sectors, it incorporates DE with short- and long-term memory and empirically supports the framework based on estimates of the diagnostic parameters. Finally, the third chapter formalises the predictability introduced by DE, offering a tool to test its presence in real-world forecasting data. Together, these studies provide a structured analysis of how belief distortions shape macroeconomic outcomes, contributing both, theoretically and empirically, to the literature on expectation formation.

Chapter 1

In chapter 1, I employ a DSGE model to examine the impact of DE on international macroeconomic dynamics, particularly in addressing key exchange rate puzzles. Motivated by the parallels between these puzzles and those in finance, where DE has proven effective, I incorporate DE into a SOE framework inspired by Justiniano and Preston (2010). The openness of the model and the built-in persistence mechanisms provide a structured environment to assess how DE influence exchange rate behaviour and contribute to a more realistic representation of belief-driven macroeconomic fluctuations.

The findings suggest that the DE-augmented SOE model (DSOE) provides a qualitative explanation for the excess volatility of exchange rates and their apparent disconnect from fundamentals. The presence of diagnostic agents introduces an amplification mechanism that fuels endogenous volatility and short-term fluctuations. In addition, the results show that the effects of DE depend on key structural parameters, such as openness and the elasticity of substitution, while persistence mechanisms influence how DE propagates and amplifies.

Chapter 2

In chapter 2, I present a TANK model that includes a housing sector and a banking sector, along with diagnostic agents, to examine the role of expectations in driving the volatility of the housing market. In this case, agents rely on both short- and long-term memory when forming expectations, allowing me to study to which information individuals attribute greater significance when forecasting. The solution method follows L'Huillier, Singh, and Yoo (2021) and Bianchi, Ilut, and Saijo (2024). In terms of estimation methods, here I use Sequential Monte Carlo given its advantages with respect to Random Walk Metropolis Hastings and I estimate the model using data from the U.S. for the period 1984:Q1 until 2019:Q4.

The main finding is that the models with DE better capture the high relative volatility in the housing sector, particularly house prices, even when conditioned on less volatile housing preference shocks. As Iacoviello and Neri (2010) note, these shocks serve as a catchall of all unmodeled disturbances that can affect housing demand. In the estimation, I obtain values for the diagnostic parameter and importance weights on past events in support of DE. In particular, the Diagnostic TANK model using a one-quarter reference outperforms both the model including a three-year reference and the rational benchmark. Further analysis of the expectations channel shows that shutting down DE prevents the model from generating the pronounced house price fluctuations observed in the data. Altogether, these findings indicate that expectation formation is a key source of unmodeled disturbances in the housing sector, shaping market cycles, and stress the need to account for it in policy analysis.

Chapter 3

In chapter 3, I extend the analysis of DE by examining their impact on the state-space representation of linear macroeconomic models and the resulting forecast errors and forecast revisions. Building on the previous chapters, I follow Hajdini and Kurmann (2024) and analytically derive expressions for FE and FR in both a three-equation model and a general framework. The purpose is to explore how representativeness, as a cognitive bias, introduces predictability components into FE and over-adjustments in FR, and whether these deviations are observable in survey data. To test this, I examine data from the Philadelphia FED's Survey of Professional Forecasters and the Greenbook/Tealbook forecasts.

The findings show that DE lead to a moving average (MA) structure in the univariate case, and a vector moving average (VMA) structure in the multivariate case. These systematic FE and FR contrast with the predictions of rational expectations. Empirical tests using data from professional forecasters and policymakers confirm that, with some exceptions, a MA or VMA structure best describes FE and FR, particularly for real GDP growth and when the post-Covid-19 period is included. However, estimates do not consistently point in the direction of overreaction. In summary, this chapter offers a framework for assessing the quantitative impact of DE, and, whilst not definitive, the results provide meaningful empirical insight.

Future Research

Here I present some potential extensions based on the chapters of this thesis, as well as new research directions.

One possible extension is to estimate the SOE model from Chapter 1 for Australia, Canada, and New Zealand, following Justiniano and Preston (2010), and compare the results with those in their paper. If the results mirror those in Chapter 2, I would expect a reduction in the size of the shock innovations driving exchange rate fluctuations. Furthermore, estimating this model using data from developing countries could further reveal whether agents with diverse historical economic experiences form expectations that align more or less with rationality.

Another potential extension involves the model in Chapter 2. An important, yet unexplored, channel in that model is financial intermediation between patient and impatient households, particularly in the context of mortgage markets and their role in credit cycles. Since the housing and credit markets are closely intertwined, incorporating this channel could allow DE to explain their co-movement and volatility more effectively, further improving the model's fit. Moreover, since the model already exhibits improved empirical performance over its rational counterpart, it could be a valuable tool for scenario analysis and macroprudential policy evaluation, offering potential benefit for policymakers.

The third chapter opens up numerous avenues for further research, as the expectation formation process appears to vary based on the individual, the forecasted variable, and the time horizon. Incorporating state-dependency or regime-shifts, in line with Hajdini and Kurmann (2024), into a model with diagnostic agents could help reconcile the observed disconnection between individual overreactions and aggregate underreactions, which DE by itself does not fully capture. Once this extension is implemented, it would be interesting to evaluate how adding survey data to the estimation process improves, or not, the ability of the model to reflect observed expectations dynamics.

A promising new research direction is to explore the high volatility in emerging markets, particularly during sudden stops, sovereign debt crises, and default risks, through the lens of DE. This framework could provide new insights into the persistent cycles of volatility and stagnation in emerging markets. DE may explain the "sudden" nature of these stops by capturing how investors' overreactions to short-term negative signals intensify capital outflow and exacerbate crises. Another important research direction is to examine how DE impact the effectiveness of unconventional monetary policy, particularly in explaining the forward guidance exchange rate puzzle identified by Galí (2020). Incorporating DE could help reconcile this discrepancy by generating short-term overreactions and long-term underreactions in response to policy rate announcements in an open economy model.

I leave these and all other possible suggestions for future research.

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