Housing and Labour Market Interactions

Thesis by
Øyvind Masst

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Adam Smith Business School, College of Social Science
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Abstract

As workers switch jobs, they also often choose to move residences to be closer to their new place of work. This thesis investigates the dynamic interactions between housing and labour markets, showing that when housing is scarce, it acts as a barrier to job market transitions and aggregate employment.

While previous research has explored these markets independently, this study contributes to the literature by developing a Dynamic Stochastic General Equilibrium (DSGE) model treating the markets jointly. Chapter 1 introduces the theoretical framework, demonstrating how search frictions and spillover effects between housing and labour markets allow the model to replicate key stylized facts.

The second chapter empirically estimates the model using Bayesian methods and UK time-series data from 1971Q2 to 2020Q1. It quantifies spillover elasticities, monetary policy parameters, and shock decompositions, shedding light on the effects of major housing and labour policy interventions during the Thatcher era. Counterfactual simulations reveal how policy reforms shaped market flexibility and economic resilience. The third chapter extends the analysis to the US economy using data from 1965Q2 to 2020Q1. Comparative analysis highlights structural differences in labour market flexibility between the UK and the US. A counterfactual experiment explores the macroeconomic consequences of a more flexible labour market in the United Kingdom, drawing lessons from the experience of the United States.

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Declaration of the Author

"I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution."

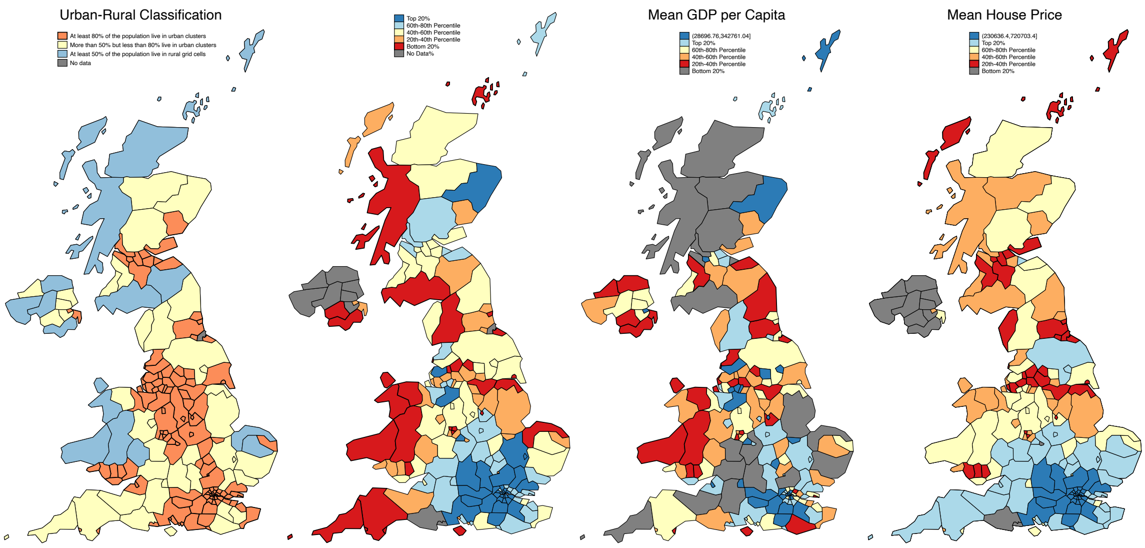
Printed Name: Øyvind Rønning Masst

Signature:

Overview

Based on the observation that the labour market is heterogeneous and spatially concentrated in cities, where the higher wages demanded by the specialised labour force raises the cost of housing, the relationship between the real estate and labour markets have achieved sporadic attention from policymakers and academia over the past 50 years. Within academia, the foundations of the studies into the relationship started from the labour, housing, and urban economics literatures. In labour economics, the key strands of research lie within the geographical local nature of labour markets[1], and insights from the search friction literature[2]. Particularly relevant to this thesis hypothesis that tight housing markets act as a barrier to job market activity is Manning and Petrongolo (2017), who show that costs associated with accessing labour markets increase rapidly with distance, due to limiting the geographical size of job-search areas.

In the housing literature, an early study into the relationship is Kain (1968), who examined how the prevalence of segregation in the housing market led to African Americans being concentrated in specific urban neighborhoods, limiting their ability to relocate closer to employment hubs. Other strands of research, not focusing on explicit discrimination, instead examine how the housing market may be a key input in determining workers' willingness to be geographically mobile. Oswald (1996) explores the link between homeownership and mobility, noting that homeowners display lower levels of mobility than renters. Building on the insights from the labour literature discussed above, more recent papers, such as Head and Lloyd-Ellis (2012), develop a theoretical model where they show that mobility is directly related to the ease with which housing transactions can be undertaken, causing a cyclical dynamic where recessions are associated with low selling probabilities and low levels of labour mobility even when households face extended periods of unemployment.



Urban Map　　Income Map　　Output Map　　House Price Map

Figure 2: From Left to Right: Urban Classification is taken from Eurostat described in its Territorial typologies manual. Average Weekly Earnings computed as the mean earnings per region for the period 2010 to 2020 from the Office National Statistics. Data on output per capita is taken at an annual interval from 2022 to 2023 available from Office National Statistics. House prices are taken from the U.K. Land Registry

[1]See for example Enrico (2011) for a thorough discussion of the topic in the handbook of labour economics.

[2]See for example Rogerson and Shimer (2011) chapter in the handbook of labour economics tracing the key developments in the search literature.

In the urban economics literature, it is well-established that large urban centers give rise to an urban productivity premium arising from agglomeration, creating a concentration of firms and workers who generate positive externalities on one another[3]. This agglomeration combined with localised labour markets results in low unemployment rates and high wages, creating a pull effect on outside labour. However, as demand for relocation increases, it creates upward pressures on real estate prices and housing costs, raising the cost of relocation and commuting, acting as a barrier to job-sector matching. Hinting at such an urban productivity premium translating into higher wages and house prices are illustrated in figure 2, which plots the urban classification and quantile divisions of average weekly income per capita, mean GDP per capita, and average house prices in the UK.

As the ratio of house prices to incomes, plotted in figure 3, reaches increasingly high levels, attention to the negative effects unaffordable housing has on the aggregate economy has increasingly resulted in calls for policy intervention from think tanks, policymakers, and politicians[4], with the newly elected 2024 UK Labour Party government making housing market intervention a major point in their party manifesto to facilitate economic growth[5].

Seeking to contribute to this discussion, this thesis examines the relationship between housing and labour markets through a structural model parameterised for, and applied to the aggregate economies of the United Kingdom and United States.

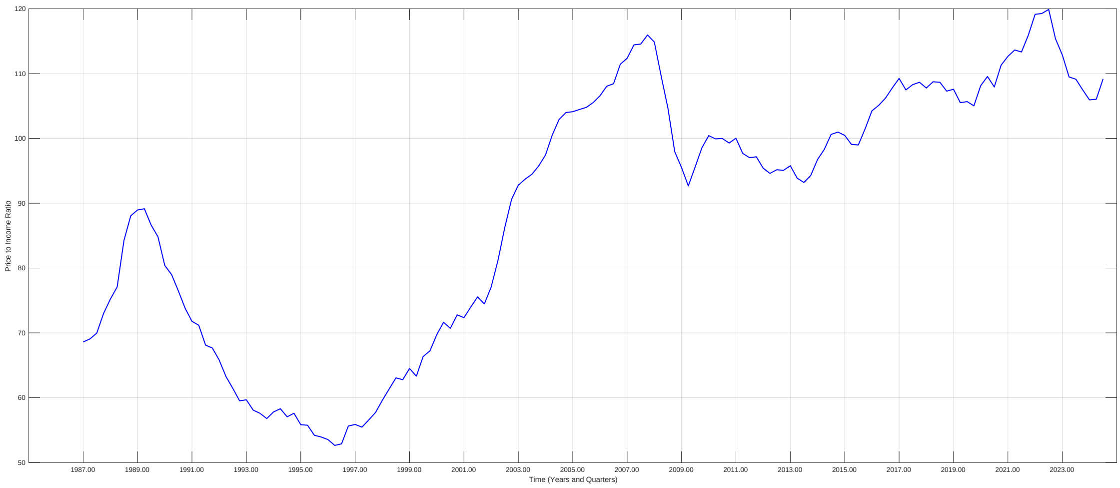


Figure 3: House Price to Income Ratio in the UK, 1987Q1 to 2024Q3, data taken from Organization for Economic Co-operation

The ***first chapter*** presents the model. The model operates with homogeneous households and firms, search and matching frictions in both the housing and labour markets, a monetary policy channel that transmits shocks onto the housing market, and a spillover channel between the two markets. The labour market is modeled in the well-established fashion of Diamond, Mortensen, and Pissarides types of search models[6], with the housing market based on Head, Lloyd-Ellis, and Sun (2014). The spillover channel is guided by the theoretical strands identified above, where tight labour markets, associated with low unemployment and high wages, drive up housing market activity. A tight housing market, on the other hand, is associated with high house prices and difficulties in finding houses, acting as a barrier to labour market activity.

In addition to contributing to the theoretical literature on search and matching in housing and labour markets, ***chapter two*** adds to the empirical literature. This chapter applies

[3] See Behrens and Robert-Nicoud (2015) for a survey of findings in agglomeration economics.

[4] See for example Judge (2019), who provides an overview of the relationship in the UK and the negative effects on the labour market.

[5] See: *Labour Party Manifesto*

[6] See Pissarides (2000)

the structural model to the UK economy using Bayesian techniques and UK time-series data ranging from 1971Q2 to 2020Q1 to quantify the value of the spillover elasticities active in the housing and labour markets, the model's monetary policy parameters, and the exogenous shocks driving the dynamics of the UK economy during the period. This empirical contribution situates the research in the established class of estimated New Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) models over the labour market[7]. It is also broadly related to the existing literature on estimated DSGE models for the housing market, where only a few examples incorporate search frictions[8]. In treating the housing and labour markets jointly, there exists, to my knowledge, only a theoretical literature, highlighting the research's ability to fill a meaningful gap.

To validate the estimation results and inform the ongoing policy debate in the UK, the chapter continues by examining the effect of housing and labour market interventions under Margaret Thatcher's premiership through counterfactual analysis. The chapter also examines the estimated state probabilities and shock decomposition analysis to validate the model's ability to detect periods of high volatility and changes in monetary policy priorities, contextualizing these results within key historical events and policy shifts.

Chapter three studies the implications of labour market mobility on the aggregate economy by comparing the UK labour market to its more flexible US counterpart. The section extends the empirical research by estimating the parameterised economy for the United States using time-series data from 1965Q2 to 2020Q1. These results are then compared to those reported in 2 to highlight structural differences in the housing-labour market relationship in the two countries. Finally, a counterfactual experiment examines the consequences had the UK's housing and labour market interventions of the early 1980s induced greater labour market flexibility and mobility.

[7]See for example Lubik (2009) for the US, or Faccini et al. (2011) for the UK.

[8]The closest is perhaps Carrillo (2012), though the focus of our studies differs significantly.

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Chapter 1

Housing and Labour Market Spillovers

This chapter investigates how the joint dynamics of housing and labour markets influence the aggregate economy. It presents a theoretical framework that integrates search-and-matching frictions in both markets, endogenous housing supply, and a monetary authority responding to inflation and output changes. A key innovation is the interaction between the two search markets, which reproduces the co-movement of vacancies, unemployment, housing vacancies, and homeownership in response to housing and labour market shocks. Through a simulation exercise, the chapter also compares monetary policy outcomes with and without the spillover channel, demonstrating how these interactions amplify and propagate monetary shocks.

1.1 Introduction

Understanding housing and labour market dynamics is key to answering fundamental questions within macroeconomics. However, while these topics have been treated independently, a theoretical framework suited for empirical investigation exploring the joint dynamics of the two markets is lacking in the macroeconomic literature. This chapter addresses this gap by developing a unified framework that captures the joint dynamics of housing and labour markets, showing that the framework is able to replicate the observed co-movement of job vacancies, employment, houses for sale, and homeownership over the business cycle.

The resulting model features search frictions in housing and labour markets, and a spillover channel through which one market affects the other. While the model is aggregate in nature and features no financial frictions or spatial dimensions directly, it still captures key channels of transmission between markets, and does a good job of replicating the aggregate dynamics arising from job changes and residential moves being highly correlated (See for example, Topel and Ward (1992), Kennan and Walker (2003), or Langella and Manning (2022)). I also show that the simulated economy is able to replicate key stylised facts of the business cycle dynamics of the two markets, importantly the joint co-movement of both housing and labour market supply and demand over the business cycle, as plotted on figure 1.1.

This research is related to six strands of previous literature. The first two, studying labour over the business cycle, and studying the relationship between labour markets and monetary policy, are at the very core of macroeconomics, and fits into seminal papers such as Keynes (1937), Kydland and Prescott (1982), Rogerson (1988) on dynamics, and Beveridge (1944), Phillips (1958), and Okun (1963) on policy, see Topel (1999) for a survey on the relationship. In studying housing markets in the macroeconomy, the model is also broadly related to the literature examining the housing markets role in shock propogation and amplification over the business cycle[1].

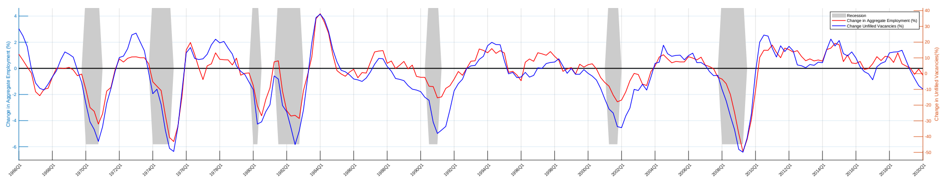
[1]See for example, Monacelli (2009), Iacoviello and Neri (2010), Mian and Sufi (2014), Jordà et al.

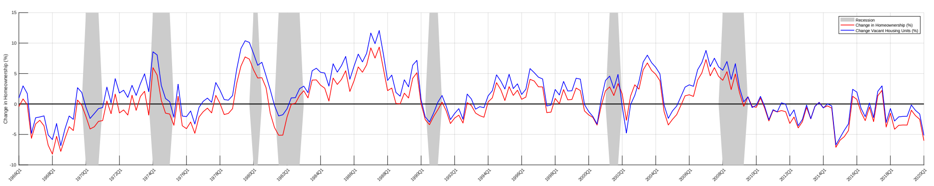
Methodologically more closely related to the proposed model, both housing and labour market research have emphasised the role of search frictions in general equilibrium modelling. This class of models, typically attributed to the joint work of Peter A. Diamond, Dale T. Mortensen, and Christopher Pissarides (DMP for short), was initially applied to labour markets, with Mortensen and Pissarides (1994) showing how search frictions were able to reproduce the key stylised fact of the co-movements in vacancies and unemployment across the business cycle, as illustrated for the United States in Figure 1.1. Further work on the now canonical version of the DMP model summarised in Pissarides (2000) extends the analysis by incorporating Nash Bargaining, with work by Shimer (2005) and Hall (2005) examining the framework's ability to capture the presence of sticky wage contracts and the low volatility of wages relative to employment. In the proposed framework, the labour market is a discrete-time version of Pissarides (2000), suitable for empirical estimation similar to Lubik (2009).

In incorporating search frictions in the housing market, the research is related to Kain (1968) and Krainer (2001). In modeling the housing market with search frictions in a Dynamic Stochstic General Equalibrium (DSGE) setting, the model is also related to Genesove and Han (2012), Head, Lloyd-Ellis, and Sun (2014), and Hedlund (2016) studies into the business cycle dynamics of housing markets[2]. As I will discuss in greater detail in section 1.4, the search frictions are crucial for the housing market to reproduce the co-movement of housing supply and demand over the business cycle analagous to the puzzle answered by the DMP model in the labour market. This business cycle movement is depicted on Figure 1.1[3].

Figure 1.1: The joint movement of: Employment, Vacancies, Homeownership, and Residential Housing for Sale in the United States (1966Q1–2020Q1).

Top: Year-on-Year Growth in Aggregate Employment (Red) and Unfilled Vacancies (Blue) in the United States for the period 1966Q1 to 2020Q1. Aggregate Employment is defined as the product the intensive and extensive margin of labour. Hours worked is taken from U.S. Bureau of Labor Statistics, and employment is taken as a rate and computed as 1 less the unemployment rate. Unemployment and Vacancy data are taken from Brian C Jenkins' extended reproduction files of Shimer (2005) available at: https://github.com/letsgoexploring/economic-data/blob/master/dmp/csv/beveridge_curve_data.csv.





Bottom: Year-on-Year Growth in Homeownership (Red), Vacant Housing Listings (Blue) in the United States for the period 1966Q1 to 2020Q1. Data on housing tenure is taken from U.S. Census Bureau, and normalised by the total housing stock.

(2016), and Garriga, Kydland, et al. (2017)

[2]For a more complete survey of the literature, please see Han and Strange (2015)

[3]Note that there is little reliable data on homelessness, so as a proxy I use homeownership, meaning that there is a positive relationship in the housing market data as opposed to the inverse relationship observed in the labour market data.

Finally, the proposed model is at its core a study into the joint dynamics of housing and labour markets, but the existing literature treating these markets jointly is sparse and not sufficiently developed. Kain (1968) provides an early study describing some theoretical relationships between the two markets but does not present a formal model. Nenov (2015) examines the relationship in a spatial model where workers face moving decisions and make optimal choices based on relocation costs and job market premia. Most similar to my work, Head and Lloyd-Ellis (2012) propose a two-"country" model with search frictions in both the residential and labour markets, and households making moving decisions between the economies based on moving costs, housing market liquidity, and wage premia.

The structure of this chapter is as follows: Section 1.2 and 1.3 describe the model's structural relationships and report parameter calibration. Section 1.4 investigates the model's dynamics in response to exogenous shocks to aggregate supply and demand shocks, and to isolated labour and housing market shocks. Through a comparison of the shocks when the spillover channel is active and inactive, the simulations are used to highlight the importance of the interconnected market in driving cyclical variations in the the housing and labour market jointly. Section 1.5 extends the analysis of the model dynamics by examining the role of the spillover channel for monetary policy transmission. Section 1.6 concludes.

1.2 Model

1.2.1 Setting and Population

Time is discrete and the economy is populated by a measure Q_t individual agents. The aggregate measure of population grows at the the constant and exogenous *net* rate μ:

$$Q_{t+1} = (1 + \mu)Q_t \tag{1.1}$$

All households are infinitely lived, discount time at a rate β_t, own firms, and consume consumables. Households own two types of labour. They supply labour to the the construction sector frictionlessly, and face search frictions when operating in general industry which produces output goods. All agents also require housing which they either rent or own. While the rental market clears in a Walrasian fashion in every period, households face search frictions when buying houses in the housing market.

1.2.2 Households

The population is distributed amongst a constant number of households H. As the aggregate population grows between periods, these μQ_t new entrants are assigned to existing households s.t. each household grows by $\frac{\mu Q_t}{H}$ members per period. There is no mobility between the households once assigned. To avoid creating a high level of heterogeneity and to aid aggregation, we follow Merz (1995) in assuming that each household is made up of an extended family, where some proportion of the household is employed and earn a wage income, and some proportion is unemployed receiving an unemployment benefit. Similarly, some proportion of household members rent and and some own houses. Within the household, there exists perfect consumption risk sharing.

1.2.2.1 The individual Household

The individual household derive utility from consuming the habbit adjusted composite good (X_t), enjoying leisure hours ($1 - h_{c,t}$), and per period consumption of housing services (z_h). They derive disutility from providing labour to general industry ($L_{c,t}$) and to the construction sector ($L_{h,t}$). Their utility function takes the form:

$$U(X_t, \frac{L_{c,t}}{H}h_{c,t}, \frac{L_{h,t}}{H}, \frac{N_t}{H}) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varrho_t \frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H} + \chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h_{i,c,t})^{1-\nu}-1}{1-\nu} - \chi_h L_{i,h,t}\frac{Q_t}{H} + \frac{N_t}{H}z^H \right] \tag{1.2}$$

Where: All variables with subscript "i" represent choice variables belonging to the individual household.

To recreate well-established consumption smoothing behaviour and persistence in consumption, it is assumed that households habit adjusted bundle depend on external habits a la Campbell and Cochrane (1999) and Abel (1990). That is, the consumption bundle is made up of the individual households choice of per period consumption ($c_{i,t}$), and the per-household measure of all households consumption in the previous period (C_{t-1}). That is:

$$X_{i,t} = \frac{c_{i,t}}{A_t} - \theta\frac{c_{t-1}}{A_{t-1}} \tag{1.3}$$

Where: A_t captures the trend level of aggregate productivity, and satisfies:

$$A_{t+1} = (1 + \varnothing_t)A_t \tag{1.4}$$

In (1.2), the first term – $\varrho_t \frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H}$ – represents the utility derived from consumption. Each individual member of the household consumes the composite good $X_{i,t}$ as described by (1.3). Since the consumption adjusted bundle $X_{i,t}$ represents the utility of an individual household member, we multiply the bundle by $\frac{Q_t}{H}$ to develop a representation of the consumption of the household.

The second term – $\chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h^i_{i,c,t})^{1-\nu}-1}{1-\nu}$ – represents the dis-utility of working in the general industry. $\chi_{c,t}$ is a smoothing parameter. $L_{c,t}$ is the aggregate number of people employed in the general industry of the economy, thus $\frac{L_{c,t}}{H}$ represent the proportion of employed members of the household. $h_{i,c,t}$ represent the number of labour hours provided by the individual worker, and ν represent elasticity of substitution with respect to leisure hours.

The third term – $\chi_h L_{h,t}\frac{Q_t}{H}$ – represent the dis-utility associated with providing labour hours to the construction sector. Since construction sector labour only depends on the intensive margin, there is no need to normalise $L_{h,t}$ by the number of households. The final term – $\frac{N_t}{H}z^H$ – is the utility value of home-ownership derived by the $\frac{N_t}{H}$ proportion of the household who are homeowners.

When firms and unemployed workers in the general industry match, they sign contracts where the real wage rate $w_{c,t}$ is obtained through a Nash surplus bargaining process. These employed individuals thus receive a labour income $h_{i,c,t}w_{c,t}$. Unemployed persons receive an unemployment benefit $b_{c,t}$ as a transfer from the government, which is financed through lump-sum taxes[4].

[4]See section: 1.71

Households face a disaggregated budget constraint that states that all household members ($\frac{Q_t}{H}$) face costs associated with their expenditure on per-period consumption spending ($C_{i,t}$), lump sum taxes ($T_{i,t}$), any $\Omega_{i,t}$ housing costs, and, any savings in private bonds which can be brought forwards to the next period ($\mathcal{A}_{i,t+1}$). Household incomes are made up of unemployment benefits ($b_{c,t}$) derived by the $U_{c,t}$ unemployed households, the returns on bonds brought forward from the previous period ($R_t\mathcal{A}_{i,t}$), any profits made from owning firms ($\Phi_{i,t}$), and labour incomes in the two sectors: $w_{c,t}h_{i,c,t}$ derived by the $L_{c,t}$ employed agents in the general industry, and $w_{h,t}$ incomes from the construction sector.

$$C_{i,t}\frac{Q_t}{H} + \frac{Q_t}{H}\mathcal{A}_{i,t+1} + \frac{Q_t}{H}T_{i,t} + \frac{Q_t}{H}\Omega_{i,t} = \frac{Q_t}{H}\Phi_{i,t} + \frac{L_{c,t}}{H}w_{c,t}h_{c,t} + \frac{U_{c,t}}{H}b_{c,t} + \frac{Q_t}{H}w_{h,t}L_{i,h,t} + \frac{Q_t}{H}R_t\mathcal{A}_{i,t} \tag{1.5}$$

Households housing costs are divided into the maintenance/taxation (m_t^h) costs paid by all matched homeowners, rental costs (r_t^h) paid by renters, and transaction costs in the housing market for sales and purchases. These transactions are summarized by the final two terms of expression (1.6) below. Because all agents who are active in the housing market transition between either being a matched homeowner, (N_t) or a searching buyer (B_t), any per-period adjustments in the stock of homeowners must reflect either a successful match, or a separation and sale. I.e: If a member of household "i" has successfully matched with a house previously owned by household "j", there has been a one unit increase in the level of home-ownership in household "i", and a one unit decrease in household "j". The transaction is then carried out at the equilibrium house price P_t^h, which is discussed in further detail in section 1.2.4.6. Due to the presence of perfect risk sharing within the household the, these housing costs can be collected together in the variable ($\Omega_{i,t}$):

$$\frac{Q_t}{H}\Omega_{i,t} = F_{i,t}r_t^h + N_{i,t}m_t^h + P_t^h(N_{i,t} - N_{i,t-1}) - P_t^h(N_{j,t} - N_{j,t-1}) \tag{1.6}$$

Where: Subscript $'i'$ denote variables belonging to household $'i'$, while subscript $'j'$ belongs to "other" households.

1.2.2.2 Individual Households Problem

The households problem is to maximise their utility function (1.2), subject to their habitual consumption bundle (1.3) and their budget constraint (1.5). They have choice variables of consumption, labour supply to the construction sector, and bond savings – $\{C_{i,t}, L_{i,h,t}, \mathcal{A}_{i,t+1}\}$ – and take: $\{\ r_t^h,\ m_t^h,\ w_{c,t},\ h_{c,t},\ b_{c,t},\ w_{h,t},\ Q_t,\ N_t,\ F_t,\ L_{c,t},$ and $H\ \}$ as given. Thus, the households maximisation problem can be described:

$$\max_{\{X_{i,t},C_{i,t},L_{i,h,t},\mathcal{A}_{i,t}\}} U(\cdot)\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\varrho_t\frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H} + \chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h_{i,c,t}^i)^{1-\nu}-1}{1-\nu} - \chi_h L_{i,h,t}\frac{Q_t}{H} + \frac{N_t}{H}z^H\right]$$

$$S.t: X_{i,t} = \frac{C_{i,t}}{A_t} - \theta\frac{C_{t-1}}{A_{t-1}}$$

$$And: \frac{Q_t}{H}C_{i,t} + \frac{Q_t}{H}\mathcal{A}_{i,t+1} + \frac{Q_t}{H}T_{i,t} + \frac{Q_t}{H}\Omega_{i,t} = = \frac{Q_t}{H}\Phi_{i,t} + \frac{L_{c,t}}{H}w_{c,t}h_{c,t} + \frac{U_{c,t}}{H}b_{c,t} + \frac{Q_t}{H}w_{h,t}L_{i,h,t} + \frac{Q_t}{H}R_t\mathcal{A}_{i,t}$$

Yielding the following F.O.C[5]:

[5] See appendix: A:2 for derivations

$$\varrho_t X_{i,t}^{-\sigma} = \lambda_t A_t \tag{1.7}$$

$$\chi_h = \varrho_t X_{i,t}^{-\sigma} \frac{w_{h,t}}{A_t} \tag{1.8}$$

$$\varrho_t X_{i,t}^{-\sigma} \frac{1}{A_t} = \beta \mathbb{E}_t \left[\varrho_{t+1} X_{i,t+1}^{-\sigma} \frac{1}{A_{t+1}} \frac{Q_{t+1}}{Q_t} R_{t+1} \right] \tag{1.9}$$

1.2.2.3 Aggregate Household

Consider the optimal choice of the individual household. Since all households are homogeneous, they all make the same optimal choices $\{X_{i,t}, C_{i,t}, L_{i,h,t}, \mathcal{A}_{i,t+1}\}_{t=0}^{\infty}$, thus we can express the aggregate budget constraint by multiplying by the number of households (H):

$$C_{i,t}Q_t + Q_t\mathcal{A}_{i,t+1} + Q_tT_{i,t} + Q_t\Omega_{i,t} = Q_t\Phi_{i,t} + L_{c,t}w_{c,t}h_{c,t} + U_{c,t}b_{c,t} + Q_tw_{h,t}L_{i,h,t} + Q_tR_t\mathcal{A}_{i,t}$$

And defining the aggregate measure as the realisation of individual variables multiplied by population:

$$C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = \Phi_t + L_{c,t}w_{c,t}h_{c,t} + (Q_t - L_{c,t})b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \tag{1.10}$$

Where: The level of housing investment undertaken by households can be expressed as:

$$\Omega_t = N_t m_t^h + \beta \mathbf{E}_t \left[(H_{t+1} - H_t)(\frac{\lambda_{t+1}}{\lambda_t} \hat{\mathcal{V}}_{t+1}) \right] \tag{1.11}$$

And: $\beta \mathbf{E}_t\{(H_{t+1} - H_t)(\frac{\lambda_{t+1}}{\lambda_t} \tilde{\mathcal{V}}_{t+1})\}$ represent the discounted value of transacting for newly constructed houses[6].

Where: the aggregation of the individual households first order conditions yield[7]:

$$A_t \lambda_t = \varrho_t (\frac{X_t}{Q_t})^{-\sigma} \tag{1.12}$$

$$\chi_h = \varrho_t (\frac{X_t}{Q_t})^{-\sigma} \frac{w_{h,t}}{A_t} \tag{1.13}$$

$$\varrho_t (\frac{X_t}{Q_t})^{-\sigma} \frac{Q_t}{A_t} = \beta \mathbb{E}_t \left[\varrho_{t+1} (\frac{X_{t+1}}{Q_{t+1}})^{-\sigma} \frac{Q_{t+1}}{A_{t+1}} R_{t+1} \right] \tag{1.14}$$

And where: the aggregate consumption bundle is defined as:

$$\frac{X_t}{Q_t} = \frac{C_t}{Q_t A_t} - \theta \frac{C_{t-1}}{A_{t-1} Q_{t-1}} \tag{1.15}$$

1.2.3 Labour Market

1.2.3.1 General Industry

Suppose there exists a continuum of final good producers who operate in perfect competition and frictionlessly package intermediary goods into final goods production – Y_t, which can be converted frictionlessly into consumption goods[8]. There also exists $j \in J$ normalised by measure 1 intermediary firms in the general industry. Such intermediary firms firms produce an intermediary good – $y_t(j)$ – by hiring workers through a costly process of search and matching based on Diamond, Mortensen and Pissarides workhouse model summarised in Pissarides (2000). Once matched, a firm-worker pair negotiate wages and hours through a Nash bargaining process, and a match is destroyed by a stochastic process. Final goods firms sell their output to households, and intermediary producers sell their goods in a monopolistically competitive market.

[6] See section: 1.2.4
[7] See appendix: A:2 for derivations
[8] See section 1.2.7

1.2.3.2 Matching Technology

Suppose that the probability that a worker matches with a vacant job is as typically in the literature dependent on the aggregate number of unemployed workers and vacancies in the economy. Let the total number of matches in the economy be captured by the matching function $M_{c,t}(V_{c,t}, U_{c,t})$, which depend on the level of matching efficiency $(\kappa_{c,t})$, and on $U_{c,t}$ and $V_{c,t}$ – the total number of unemployed workers and vacancies respectively. That is:

$$M_{c,t}(U_{c,t}, V_{c,t}) = \kappa_{c,t} U_{c,t}^{\delta_{c,t}} V_{c,t}^{1-\delta_{c,t}} \tag{1.16}$$

Where: $V_{c,t} = \int_0^1 V_t(j) d\varsigma$ aggregation over all firms, indexed by ς.

Let the labor market tightness be denoted as:

$$\omega_{c,t} \equiv \frac{V_{c,t}}{U_{c,t}} \tag{1.17}$$

We can then express the job filling rate (matching probability for the firm) as $\gamma_{c,t}$[9], ensuring that a tighter labour market reduces the matching probability of firms:

$$\gamma_{c,t} \equiv \frac{M_{c,t}(U_{c,t}, V_{c,t})}{V_{c,t}} = \kappa_{c,t} \omega_{c,t}^{-\delta_{c,t}} \tag{1.18}$$

And the job finding rate of the unemployed (matching probability for unemployed) as $\lambda_{c,t}$[10], ensuring that a tighter labour market raises the matching probability of workers:

$$\lambda_{c,t} \equiv \frac{M_{c,t}(U_{c,t}, V_{c,t})}{U_{c,t}} = \gamma_{c,t}\omega_{c,t} = \kappa_{c,t} \omega_{c,t}^{1-\delta_{c,t}} \tag{1.19}$$

1.2.3.3 Transition Probabilities and Laws of Motion in the Labour Market

At the beginning of period "t", there are $L_{c,t-1}(j)$ employed persons in firm-worker matches carried over from period "$t-1$" within firm "j". During period "t", these existing matches suffers separations by probability: $\vartheta_{c,t} \in (0, 1)$, such that $(1 - \vartheta_{c,t})L_{c,t-1}(j)$ matches survive. The firm also advertise vacancies, of which which $\gamma_{c,t}V_{c,t}(j)$ are filled within period "t". The number of employed persons at the end of period "t" is thus:

$$L_{c,t}(j) = (1 - \vartheta_{c,t})L_{c,t-1}(j) + \gamma_{c,t}V_{c,t}(j) \tag{1.20}$$

It is assumed that the job filling rate is the same for all firms. Aggregating the above equation, we can express the total number of employed households $L_{c,t} = \int_0^1 L_{c,t}(j), dj$, and the total number of vacancies $V_{c,t} = \int_0^1 V_{c,t}(j), dj$ in the economy. That is:

$$L_{c,t} = (1 - \vartheta_{c,t})L_{c,t-1} + \gamma_{c,t}V_{c,t} \tag{1.21}$$

At the beginning of period "t", the total number of unemployed households is defined as being equal to the labour force less those who are not employed at the end of period "$t-1$". During period "t", $\vartheta_t L_{c,t-1}$ persons become unemployed. They are then added to the stock of unemployed, and immediately start searching for a new job. That is: $U_{c,t} = Q_t - L_{c,t-1} + \vartheta_t L_{c,t-1}$. Hence, we can define the aggregate number of unemployed persons in the economy at the end of period "t":

$$U_{c,t} = Q_t - (1 - \vartheta_{c,t})L_{c,t-1} \tag{1.22}$$

[9]See appendix: A:2

[10]See appendix: A:2

The timing described above can be summarised by Figure 3.1, below:

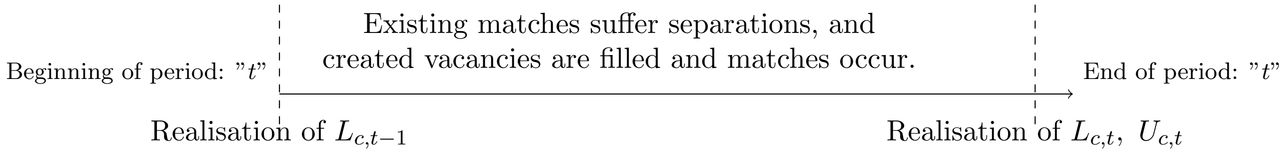


Figure 1.2: Timing of events in the labour market.

1.2.3.4 Final Good Producers

Following Dixit and Stiglitz (1977), there exists a continuum of perfectly competitive final goods producing firms which purchase intermediary inputs and aggregates them according to the production technology described by (1.23), which they then sell to households.

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.23}$$

Where: Y_t and $y_t(j)$ denotes the aggregate and individual output respectively. $\epsilon > 1$ is the stochastic elasticity of substitution between intermediary input goods.

Such final good producing firms face a maximisation problem where they decide how many final goods to produce, and how many intermediary goods to purchase – $\{Y_t, y_t(j)\}$ – taking prices $\{P_t, p_t(j)\}$ as given in order to maximise profits:

$$\Pi_t = P_t Y_t - \int_0^1 p_t(j) y_t(j) \, dj \tag{1.24}$$

Where: P_t and $p_t(j)$ denotes the aggregate and individual price level respectively.

Yielding the aggregate demand curve for intermediary goods[11]:

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \tag{1.25}$$

Where: $\varepsilon > 1$ is the elasticity of substitution between goods.

And path for the aggregate price level[12]:

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}} \tag{1.26}$$

1.2.3.5 Intermediary Good Producers

In the intermediary goods producing industry, firm "j" operates the following technology:

$$y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j) \tag{1.27}$$

Where: $Y_t(j)$ is firm "j" output, z_t is a stationary productivity shock, A_t is trend growth rate of productivity, $L_{c,t}(j)$ is the number of workers hired by firm "j", and $h_{c,t}(j)$ is the number of hours provided by labour to firm "j".

These firms face a cost of ι when posting vacancies, and must compensate labour at a wage rate $w_{c,t}$ determined through Nash surplus bargaining. The individual firms profit function is thus:

$$\Phi_t(j) = \frac{p_t(j)}{P_t} y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \tag{1.28}$$

Intermediary good producers set prices according to Calvo (1983), and keep prices in each period with probability $\varsigma \in [0, 1]$ such that $(1 - \varsigma)$ firms set prices optimally each period.

[11] See appendix: A:2 for derivations

[12] See appendix: A:2 for derivations

Hence, intermediary firms solve a profit maximisation problem described by (1.28) where they must choose the price of their goods, level of labour to hire, the number of vacancies to post – $\{p_t(j), L_{c,t}(j), V_{c,t}(j)\}$ – subject to the law of motion for employment (1.20), their production function (1.27), and their demand curve (1.25). They take $\{P_t,\ w_{c,t},\ Y_t\}_{t=0}^{\infty}$ as given. That is:

$$\max_{\{p_t(j),L_{c,t}(j),V_{c,t}(j)\}} \Phi_t(j) : \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta\varsigma)^t \frac{p_t(j)}{P_t} y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j)\right]$$
$$S.t : L_{c,t}(j) = (1-\vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_{c,t}(j)$$
$$And : Y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j)$$
$$And : Y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$

The first order conditions of the firms optimisation yields[13]:

$$\eta_t = \frac{\iota}{\gamma_{c,t}} \tag{1.29}$$

$$\eta_t = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + \beta\varsigma \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1}\right] (1-\vartheta_{c,t}) \tag{1.30}$$

$$\frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma) \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\vartheta_{c,t}) \frac{\iota}{\gamma_{c,t+1}}\right] \tag{1.31}$$

$$(\pi_t + 1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1 - (1-\varsigma)\left(\frac{\epsilon}{(\epsilon-1)} \frac{K_{1,t}}{K_{2,t}}\right)^{1-\epsilon}\right) \tag{1.32}$$

Where:

$$K_{1,t} = w_{c,t} \frac{Y_t}{z_t A_t} + \varsigma\beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1+\pi_{t+1})^{\epsilon+1} K_{1,t+1}\right] \tag{1.33}$$

$$K_{2,t} = Y_t + \varsigma\beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1+\pi_{t+1})^{\epsilon} K_{2,t+1}\right] \tag{1.34}$$

Where (1.29) is the job-posting condition, and states that firm "j" will post vacancies so long as the value of filling a position is greater or equal to the cost of posting a vacancy: $\eta_t \gamma_{c,t} \geq \iota$. (1.30) is the job-creation condition and states that firm "j" will create jobs so long the value of an unfilled vacancy is equal to the current period profit generated from filling a vacancy, and the discounted expected future value of the vacancy in the next period. (1.31) is a combination of (1.29) and (1.30), and relates the job-posting to job-creation in equilibrium. ξ_t is the Lagrange multiplier associated with the demand curve (1.25), while η_t is the Lagrange multiplier attached to the law of motion for employment (1.20). (1.32) is the Phillips curve and describes the path of path of inflation.

The aggregation of (1.27) and (1.28) for all $j \in J$ yields:

$$Y_t = \frac{z_t A_t}{\Delta_t} L_{c,t} h_{c,t} \tag{1.35}$$

$$\Phi_t = Y_t - w_{c,t} h_{c,t} L_{c,t} - \iota V_{c,t} \tag{1.36}$$

Where, $\Delta_t = (1-\varsigma)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \varsigma \pi_t^{\varepsilon} \cdot \Delta_{t-1}$ is the price dispersion in the economy. Because the system is log-linearised around its deterministic steady state where: $\pi = 1, \Delta_t = 1$, the price dispersion doesn't introduce additional first-order dynamics outside of those described in 1.35.

[13] See appendix: A:2 for derivations

1.2.3.6 Value Functions in the General Industry

1.2.3.6.1 Workers Value Functions Let V_t^E and V_t^U represent the value of employment/unemployment respectively to a worker. If workers are employed in the general industry they receive labour income, and enjoy $(1-h_{c,t})$ leisure hours which provide utility at a rate: $\chi_{c,t}\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}$. We can then express the value to the household of employment as:

$$V_t^E = h_{c,t}w_{c,t} + \frac{\chi_{c,t}}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right) \tag{1.37}$$
$$+\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\vartheta_{c,t+1}(1-\lambda_{c,t+1}))V_{t+1}^E + \vartheta_{c,t+1}(1-\lambda_{c,t+1})V_{t+1}^U\right]\right]$$

Where, the terms inside the expectation is a composite describing the expected pay-off of being either employed or unemployed in the next period. With probability $(1-\vartheta_{c,t+1})$ an employed person does not suffer a separation and remains employed in the next period. Of those $\vartheta_{c,t+1}$ that become unemployed, they find a new job within the same period according to the job finding probability: $\lambda_{c,t+1}$. Alternatively, red$\vartheta_{c,t+1}(1-\lambda_{c,t+1})$ become unemployed and are unable to find a new job in the next period.

Next, consider the value of unemployment. These workers earn no labour income, but receive the unemployment benefit: $b_{c,t}$. These unemployed households remain unemployed in the next period with a probability: $1-\lambda_{c,t+1}$, and become employed by the job finding probability: $\lambda_{c,t+1}$. Thus, the value to the household of unemployment is:

$$V_t^U = b_{c,t} + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\lambda_{c,t+1})V_{t+1}^U + \lambda_{c,t+1}V_{t+1}^E\right]\right] \tag{1.38}$$

We can then express the worker's surplus from becoming employed ($V_t^W = V_t^E - V_t^U$):

$$\Rightarrow V_t^W = h_{c,t}w_{c,t} - b_{c,t} + \frac{\chi_{c,t}}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1}))V_{t+1}^W\right] \tag{1.39}$$

1.2.3.6.2 Firms Value Functions Let V_t^F and V_t^V represent the value of a match/vacancy to the firm. In each period, firms derive revenues from producing output, and has labour costs $h_{c,t}w_{c,t}$ per worker: $V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t})$. In period "$t+1$", the match between the firm and the worker survive to the next period by probability: $(1-\vartheta_{c,t+1})$, and workers and firms are separated from the match by probability: $\vartheta_{c,t+1}$. The present value of a filled job to a firm is thus a combination of this net-revenue and the future expected stream of revenue from the match. That is:

$$V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t}) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}[(1-\vartheta_{c,t+1})V_{t+1}^F + \vartheta_{c,t}V_{t+1}^V]\right]$$

If a separation occurs, the probability of filling the vacancy is $\gamma_{c,t}$, and cause the firm to incur a cost ι of posting the vacancy. By (1.29), firms post vacancies so long as the value of employing a worker to the firm equals the cost of recruitment: $\gamma_{c,t}V_t^F = \iota$. Recognising that the value of a filled vacancy is equal to the R.H.S. of (1.30) with equilibrium described by (1.31), we have: $V_t^F \equiv \eta_t = \frac{\iota}{\gamma_{c,t}}$. Free entry of firms implies that the value of unfilled vacancy will be driven to zero $V_t^V \equiv 0, \forall t$. In other words, the model assumes that the same position cannot be readvertised and refilled, and the match between a particular firm and a particular worker is destroyed with probability $\vartheta_{c,t}$. Thus:

$$V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t}) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right] \tag{1.40}$$

1.2.3.7 Bargaining Problem and Wage Equation

When firms and workers meet to negotiate wages, they divide the total surplus from the match according to a Nash bargaining process where we assume that worker's share of the joint surplus is given by $\epsilon_{c,t}$. Following the efficiency proof established by Hosios (1990), we set the steady state value of $\epsilon_{c,t}$ equal to the matching elasticity in the labour market (δ_c). Total surplus of a match is the sum of the value of the match to the firm and worker, that is: $V^T_{c,t} = V^W_{c,t} + V^F_{c,t}$. Then, maximizing the Nash product of the match requires solving the unconstrained maximisation problem:

$$\max_{V^F_t, V^W_t} : (V^W_t)^{\epsilon_{c,t}}(V^F_t)^{1-\epsilon_{c,t}} - \varphi_t(V^W_t + V^F_t - V^T_{c,t})$$

F.O.C:

$$\frac{\partial L}{\partial V^W_t} \equiv 0 = \epsilon_{c,t}(V^W_t)^{\epsilon_{c,t}-1}(V^F_t)^{1-\epsilon_{c,t}} - \varphi_t$$

$$\frac{\partial L}{\partial V^J_t} \equiv 0 = (1-\epsilon_{c,t})(V^W_t)^{\epsilon_{c,t}}(V^F_t)^{-\epsilon_{c,t}} - \varphi_t$$

Which implies[14]:

$$h_{c,t}w_{c,t} = \epsilon_{c,t}\left[h_{c,t}\xi_t z_t A_t + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\left(1-(1-\lambda_{c,t+1})\frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t+1})\epsilon_{c,t}}\right)\frac{\iota}{\gamma_{c,t+1}}\right]\right] + (1-\epsilon_{c,t})\left[b_{c,t} - \frac{\chi_{c,t}}{\lambda_t}\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}\right] \tag{1.41}$$

Where (1.41) determines the wage as a weighted average between the marginal revenue product of the worker plus the cost of replacing the worker, and the outside option of the worker.

1.2.3.8 Hours Worked

Finally to close the labour market, firms and workers determine how many hours to supply/hire based on an optimisation problem aimed at maximising the joint surplus of the match:

$$\begin{aligned}\max_{\{h_{c,t}\}}(V^T_t) &= \max_{\{h_{c,t}\}}\left(V^W_t + V^{Firm}_t\right) \\ &= \max_{\{h_{c,t}\}}\left[\frac{h_{c,t}w_{c,t} - b_{c,t} + {}_{c,t}}{\lambda_t\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right)}\right. \\ &+\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})V^W_{t+1}\right] \\ &\left.+h_{c,t}(z_t\xi_t A_t - w_{c,t}) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right]\right]\end{aligned}$$

First order conditions:

$$0 = w_{c,t} - \frac{\chi_{c,t}}{\lambda_t}(1-h_{c,t})^{-\nu} + z_t\xi_t A_t - w_{c,t}$$

From where we can express optimal labour hours:

$$z_t\xi_t A_t = \frac{\chi_{c,t}}{\lambda_t}(1-h_{c,t})^{-\nu} \tag{1.42}$$

[14]See appendix: A:2 for derivations

1.2.4 Housing Sector

In the housing sector there is a stock of houses (H_t) which grows endogenously through housing construction described in section: 1.2.4.4. Households require housing in every period, which they either rent or own. To buy a house, searching buyers engage in a costly process of search and matching similar to the friction described in section 1.2.3. Rental contracts are either short- of long-term in nature. Permanent renters have no interest in owning housing and thus opt for long-term contracts, while searching buyers are only renting temporarily and thus opt for short-term contracts.

1.2.4.1 Housing Stock

At time t the city has stock of housing in the economy (H_t) which is either vacant and listed for sale ($V_{h,t}$), or occupied by one of the economies Q_t agents. Thus, total housing stock can be defined:

$$H_t = Q_t + V_{h,t} \tag{1.43}$$

Where: these $V_{h,t}$ vacant houses are made up of a combination of newly constructed houses by property developers, and houses which have been listed for sale by mismatched homeowners.

1.2.4.2 Matching Technology

Matching in the housing market is determined by the matching function $M_{h,t}$, which depends on the matching technology ($\kappa_{h,t}$), and the number of searching buyers (B_t) and houses for sale (S_t).

$$M_{h,t}(B_t, S_t) = \kappa_{h,t} B_t^{\delta_h} S_t^{1-\delta_h} \tag{1.44}$$

Let market tightness, which acts as our proxy for housing market liquidity be defined as the number of searching buyers divided by the number of houses for sale. That is:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} \tag{1.45}$$

Let the house filling rate (matching probability for houses for sale) be denoted by $\gamma_{h,t}$ and defined[15]:

$$\gamma_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{S_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h} \tag{1.46}$$

And, let the house finding rate (matching probability of a searching buyer) be denoted by $\lambda_{h,t}$ and be defined[16]:

$$\lambda_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{B_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h - 1} \tag{1.47}$$

1.2.4.3 Transition Probabilities and Laws of Motion in the Housing Market

1.2.4.3.1 Home Owners At time t, there are N_t home-owning households. These homeowners become mismatched with their house at an exogenous probability $\vartheta_{h,t} \in (0, 1)$. If mismatched, homeowners become unhappy with their home and no longer receive the utility value of home-ownership (z^H). As a consequence, these households seek to sell their house which gets added to the stock of houses for sale – S_t, and become searching buyers – B_t – trying to match with a new house. Their law of motion is thus number of homeowners who did not become mis-matched in the previous period – $(1 - \vartheta_{h,t})N_{t-1}$, and those $\lambda_{h,t} B_t$ searching buyers who successfully matched with a house in the current period:

$$N_t = (1 - \vartheta_{h,t})N_{t-1} + \lambda_{h,t} B_t \tag{1.48}$$

[15] See appendix section: A:2

[16] See appendix section: A:2

1.2.4.3.2 Permanent Renters Permanent renters never change type, and have no interest in owning houses. By (1.1), each period the population grows by a rate μ. Let ψ_t represent the proportion of the μQ_t people who act as searching buyers, while $1-\psi_t$ be the proportion that acts as perpetual renters. The stock evolution of perpetual renters is then given by:

$$F_t = F_{t-1} + (1-\psi_{t-1})\mu Q_{t-1} \tag{1.49}$$

1.2.4.3.3 Searching Buyers Each period, a proportion – $\vartheta_{h,t}N_t$ – homeowners become mismatched and transition into being searching buyers. Next, recall that by (**??**), the population is equal to the stock of renters, buyers and homeowners. The stock of searching buyers is thus given by the difference between per-period population, and per-period renters and homeowners. That is:

$$B_t = Q_t - F_t - (1-\vartheta_{h,t})N_{t-1} \tag{1.50}$$

1.2.4.3.4 Houses for Sale The number of houses for sale are those vacant housing units – $V_{h,t}$ – which are currently unoccupied. There are also "chains", which are those housing units which are listed for sale but still occupied while the seller attempts to find match with a buyer – $C_{h,t}$. This is a feature of multiple housing markets, notably prevalent in the UK[17], where a sequence of houses listed for sale are dependant both upon the buyers receiving the money from selling their houses and on the sellers successfully buying the houses that they intend to move into[18]. The shock thus mimics an implicit form of *on the "job" search* similarly to the process described in Pissarides (2000).

The stock of houses for sale is thus:

$$S_t = V_{h,t} + C_{h,t} \tag{1.51}$$

It is assumed that some proportion – τ – of houses for sale are in chains, that is:

$$C_{h,t} = \tau S_t \tag{1.52}$$

Combining (1.45) with (1.51), and (1.52) the housing market tightness must satisfy[19]:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} = \frac{(1-\tau)B_t}{H_t - Q_t} \tag{1.53}$$

1.2.4.4 Housing Construction

In the construction sector, there are three key stocks: undeveloped land (K_t^L), developed land ($\hat{H}_t$), and constructed housing (H_t). All undeveloped land is owned by the government, which releases it for development to firms in the construction sector. These firms operate under perfect competition and undertake both the development of land and the construction of new housing. To produce housing, firms combine developed land with construction labour ($L_{h,t}$) using a simple technology. They solve a cost minimisation problem dependant these two cost factors:

$$H_{t+1} - H_t = \min\left(\hat{H}_{t+1} - \hat{H}_t,\ \phi_t L_{h,t}\right) \tag{1.54}$$

Where: ϕ_t denotes the productivity of construction labour.

Solving the minimisation implies that land is developed until the cost of development equals the cost of employing labour. That is:

$$\hat{H}_{t+1} - \hat{H}_t = \phi_t L_{h,t} \tag{1.55}$$

[17] See: Office of Fair Trading (2010)

[18] I.e: In a four-household chain, A buys B's house, B uses the money from that sale to buy C's house, and C uses the money from that sale to buy D's house

[19] See appendix section A:2

Unable to store developed land and with free entry into the construction sector, firms construct new houses so long as profitable. They therefore build houses on any developed parcel of land. That is:

$$H_{t+1} - H_t = \phi_t L_{h,t} \tag{1.56}$$

Undeveloped land (K_t^L) is released exogenously at the rate $\varkappa$, intended to capture the rate at which the government releases land for development. Thus, the exogenous law of motion for land satisfies:

$$K_{t+1}^L = (1 + \varkappa) K_t^L \tag{1.57}$$

Such undeveloped land can be sold to construction sector firms at a price $q_{h,t}$. Prior to making the purchase, the firm can costlessly evaluate the development costs associated with the parcel. Reflecting that different parcels of land require different levels of development with different levels of associated costs, these costs are assumed to be heterogenous and draw from the distribution

$$c \sim A(c), \quad c \in [\underline{c}, \overline{c}]$$

With free entry, profits are driven to zero until all units of land with development costs $c \leq q_{h,t}$ are used for construction, ensuring that land development is increasing in $q_{h,t}$. With the level of undeveloped land being the difference between total available land and developed land – $K_t^L - \hat{H}_t$, the quantity of land converted satisfies:

$$\hat{H}_{t+1} - \hat{H}_t = \Lambda \left(\frac{q_{h,t}}{A_t} \right) (K_t^L - \hat{H}_t) \tag{1.58}$$

Where, $\Lambda \left(\frac{q_{h,t}}{A_t} \right)$ is the reduced form land conversion function defined: $\Lambda \left(\frac{q_{h,t}}{A_t} \right)^{\varpi}$, with $0 < \varpi < 1$.

Consider the profit earned by firms operating in the construction sector. To construct a new house, house builders face aggregate labour costs – $L_{h,t} w_{h,t}$ – and buy a unit of developed land for $q_{h,t}$. They earn revenues by selling newly constructed houses. Once a new house is built, it is listed for sale at the option price – $\hat{V}_{t+1}$. Their profit function is thus the difference between the revenues earned from house selling and the cost of land acquisition for the $H_{t+1}^H - H_t^H$ units of houses constructed, and the aggregate costs of hiring labour:

$$\Pi_{const,t} = (H_{t+1} - H_t) \left[\beta \mathbb{E}_t \left[(\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1}) \right] - q_{h,t} \right] - w_{h,t} L_{h,t} \equiv 0 \tag{1.59}$$

Where: $\hat{V}_{t+1}$ is the value function associated with a vacant house not yet listed either for sale or for rent, and can be thought of as the option price of a vacant house. Requiring the profit function to equal zero follows from both developers and house builders opperating in perfect competition.

Per unit of houses constructed, house builders face $\frac{w_{h,t} L_{h,t}}{(H_{t+1} - H_t)}$ labours costs, by (1.56) this implies that developer face per house labour costs: $\frac{w_{h,t} L_{h,t}}{(H_{t+1} - H_t)} \to \frac{w_{h,t} L_{h,t}}{\phi_t L_{h,t}} \to \frac{w_{h,t}}{\phi_t}$. With the zero profit condition described by (1.59), this implies that the wage rate in the housing construction sector is given by:

$$\frac{w_{h,t}}{\phi_t} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right] - q_{h,t} \tag{1.60}$$

1.2.4.5 Value Functions in the Housing Market

1.2.4.5.1 Perpetual renters Renters are never interested in buying a house, and thus never transition. Their value function thus only depends on cost of their long-term rental contracts and their continuation value:

$$V_t^F = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} V_{t+1}^F \right] - r_t^h \tag{1.61}$$

1.2.4.5.2 Homeowners

Each period, homeowners homeowners pay maintenance/housing tax costs – m_t^h, and receive the utility value of homeownership – z^H – normalised by the Lagrange multiplier associated with households maximisation – λ_t. If they suffer a separation shock, they list their house for sale and receive its expected value – $\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}$. If a searching buyer matches with a house for sale, they pay the house price – $\frac{\lambda_{t+1}}{\lambda_t}P_{t+1}^h$.

$$\begin{aligned} V_t^N = & - m_t^h + \frac{z^H}{\lambda_t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \vartheta_{h,t}(1 - \lambda_{h,t+1}))V_{t+1}^N \right.\right. \\ & \left.\left. + \vartheta_{h,t}(\hat{V}_{t+1} - \lambda_{h,t+1}P_{t+1}^h) + \vartheta_{h,t}(1 - \lambda_{h,t+1})V_{t+1}^B \right] \right] \end{aligned} \tag{1.62}$$

Where: The first term captures that each period, homeowners pay maintenance/housing tax costs – m_t^h. They receive the They receive the normalised utility value $\frac{z^H}{\lambda_t}$. The third term is a composite capturing their continuation values. The first part of the composite term captures that with probability $\vartheta_{h,t}$ homeowners suffer a separation, of which $\lambda_{h,t+1}$ match with a new house in the next period. Thus the stock of households who start at homeowners in period "t" and remain as homeowners at the end of period "$t+1$" is: $(1-\vartheta_{h,t}(1-\lambda_{h,t})$, these households then receive the next next period value of homeownership – V_{t+1}^N. The second part captures that of those who suffer the mismatch shock in period "t", all receive the value of a vacant house, and that $\lambda_{h,t+1}$ of them purchase a new house in the period immediately following. The third term captures that with probability: $\vartheta_{h,t}(1-\lambda_{h,t})$, mismatched households do not match with a new house, and thus transition into searching buyers.

1.2.4.5.3 Searching Buyers

In period "t", searching buyers rent through short-term contracts, while searching for a house and pay housing rent – r_t^{h*}. With probability: $\lambda_{h,t+1}$ they match with a house for sale in the next period, and pay the transaction price – $\frac{\lambda_{t+1}}{\lambda_t}P_{t+1}^h$. Their value function is thus:

$$V_t^B = -r_t^{h*} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \lambda_{h,t+1})V_{t+1}^B + \lambda_{h,t+1}(V_{t+1}^N - P_{t+1}^h) \right] \right] \tag{1.63}$$

Where the first term is their rental costs, and the second term is their composite continuation value. The first part of the continuation value captures that with probability: $(1-\lambda_{h,t+1})$ they fail to match with a house for sale and continue to search in future periods. The second part captures that with probability: $\lambda_{h,t+1}$ they successfully match with a house, taking on the value function of a homeowners – V_{t+1}^h and pay the transaction price – P_{t+1}^h.

1.2.4.5.4 Value of a vacant house

At the beginning of each period, an unoccupied house, can either be put on the short-term rental market, or listed for sale. Such vacant houses move costlessly between sale and rental markets. Homeowners wish to maximise profits, so they solve the maximisation problem:

$$\hat{V}_t = \max \left[r_t^{h*} - m_t^h + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right], V_t \right] \tag{1.64}$$

Where: $\hat{V}_t$ is the value of a vacant house. The first argument describes the value of a house listed on the rental market. V_t is the value of house designated for sale. As houses move frictionlessly between the two markets, it follows that:

$$\hat{V}_t = r_t^{h*} - m_t^h + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right] = V_t \tag{1.65}$$

Note that: this is an equilibrium condition - homeowners sell house until they are indifferent between the sale price and rental return, and where rental prices (r_t^{h*}) adjusts to maintain the equality above.

A vacant house on the for sale market matches with a searching buyer in the next period with probability: $\gamma_{h,t+1}$, and transacts at equilibrium housing price – P_{t+1}^h. If no match occurs, the house is valued at the future options value – $\hat{V}_{t+1}$. Thus, the value of a house to a seller, V_t, satisfies:

$$V_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[\gamma_{h,t+1} P^h_{t+1} + (1 - \gamma_{h,t+1}) \hat{V}_{t+1} \right] \right] \tag{1.66}$$

Where: The house transacting price (P^h_t) is determined through Nash bargaining[20].

1.2.4.6 Bargaining Problem, House Price Equation and Rents

1.2.4.6.1 House Prices When a searching buyer meets with a vacant house for sale they determine the transaction price by engaging in Nash bargaining over the total surplus of a match, similarly to how wages are determined in the general industry[21].

Let V^{Buy}_t be the value of successfully buying a house. This value is given by the value of becoming a homeowner – V^N_t – less the cost of purchasing the house – P^h_t – and the value of continuing as a searching buyer – V^B_t.

$$V^{Buy}_t = (V^N_t - P^h_t) - V^B_t \tag{1.67}$$

Let V^{Sell}_t be value of a match to the seller, and be the difference between the value of being a homeowner, and becoming a searching buyer after a sale. That is:

$$V^{Sell}_t = (V^B_t + V_t) - V^N_t \tag{1.68}$$

Let $V^T_{h,t} = V^{Sell}_t + V^{Buy}_t$.

Maximising the Nash product gives rise to the bargaining problem:

$$\begin{aligned} \max_{V^{Sell}_t, V^{Buy}_t} : \quad & (V^{Sell}_t)^{\epsilon'_{h,t}} (V^{Buy}_t)^{1-\epsilon'_{h,t}} \\ S.t. : \quad & V^T_{h,t} = V^{Sell}_t + V^{Buy}_t \end{aligned}$$

Which yields the familiar sharing rule for house prices[22]:

$$P^h_t = (1 - \epsilon_{h,t})(V^N_t - V^B_t) + \epsilon_{h,t} V_t \tag{1.69}$$

Where: $\epsilon_{h,t}$ denote the bargaining power of the buyer.

[20] See section 1.2.4.6
[21] See section: 3.2.3.7
[22] See appendix section: A:2

1.2.4.6.2 Rent Prices Long-term rental contracts are assumed to be related to the short-term rate according to:

$$r_t^h = \upsilon r_{t-1}^h + (1-\upsilon)\, r_t^{h*}. \tag{1.70}$$

When $\upsilon = 0$ both rates are the market-determined optimising flexible rate. When υ is large, while the small proportion of the market is able to adjust the price when the new rental contract is written, the substantial proportion of the market participants face a highly persistent rent that reacts slowly to the market rent hikes and falls. This type of rent dynamics can work as a rough description of a rent control policy where landlords are not able to change the in-contract price, which mostly affects long-term (permanent) renters.

1.2.5 Policy

1.2.5.1 Fiscal Policy

There exists a government whose sole purpose is to finance the unemployment benefit b_t paid to all $U_{c,t}$ unemployed persons. This government raises money through charging lump-sum taxes T_t on all households, levying property taxes/charging maintenance costs m_t^h on all N_t units of owned housing, and from proceeds arising from selling $(K_t^L - \hat{H}_t)$ units of land at the price $(q_{h,t})$. The governments budget constraint is thus:

$$b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(K_t^L - \hat{H}_t)$$

Further, using that the construction sectors zero profit condition (1.59) implies that all developed untis are used for construction, the governments budget constraint becomes:

$$\Rightarrow b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(H_{t+1} - H_t) \tag{1.71}$$

Where: the level of lump sum taxes (T_t) adjust to maintain the equality. The total value of land sales $(q_{h,t}(H_{t+1} - H_t))$ is determined endogenously, while the values of land taxes (m_t^h) and unemployment benefits $(b_{c,t})$ reflect observed data ratios described in section 3.3.

1.2.5.2 Monetary Policy

The monetary authority sets interest rates by considering deviations of interest rates and inflation, and the growth rate of output. That is, they follow the Taylor type rule described below:

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\alpha_i} \left(\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\alpha_\pi} \left(\frac{Y_t}{(1+\mu)(1+\gamma)Y_{t-1}}\right)^{\alpha_y}\right)^{1-\alpha_i} e^{m_t} \tag{1.72}$$

Where: i_{ss} and π_{ss} are steady state values

The monetary authority can follow one of two regimes – Hawkish (H) and dovish (D) – which governs how strongly they respond to inflationary deviations captured through parameter α_π such that $\alpha_\pi^H > \alpha_\pi^D$. The switches of the economy between these two regimes are governed by two-regime Markov chain $\upsilon_M \in \{H, D\}$ with transition matrix

$$T^M = \begin{bmatrix} 1-p_{HD} & p_{HD} \\ p_{DH} & 1-p_{DH} \end{bmatrix}$$

Where: $p_{ij} = P(\upsilon_{M,t+1} = j | \upsilon_{M,t} = i)$.

1.2.6 Spillovers

To capture the spill-overs that occur between housing and labour markets the model allows for for the state of one market to affect the matching and separation probabilities of the other. While the two effects move in opposite directions, the matching effect dominates throughout giving rise to *net* gains or losses to employment/home-ownership in response to adjustments in the other market.

1.2.6.1 Housing Market Spillovers

In the real world, people may separate from their current job due to a number of reasons, both voluntary and involuntary. Following such a separation job seekers may choose to relocate either within a local area or a more broadly to be close available job opportunities. The "looser"[23] the housing market tightness – $\omega_{h,t} \equiv \frac{B_t}{S_t}$ – the easier it is for such unemployed workers to make the relocation both in terms of house finding probabilities and prices. It is therefore assumed that the matching efficiency in the labour market which governs the ability of unemployed workers to match with vacant jobs to be decreasing with housing market tightness:

$$\kappa_{c,t} = \tilde{\kappa}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\kappa} \tag{1.73}$$

Conversely, if the housing market is very loose, currently employed workers may recognise that the barrier of relocation is weak/small, and may respond by quitting/separating more frequently.

$$\vartheta_{c,t} = \tilde{\vartheta}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\vartheta} \tag{1.74}$$

Where: While $\tilde{\kappa}_{c,t}$ and $\tilde{\vartheta}_{c,t}$ may, in principle, capture additional unmodelled variation, their treatment differs across chapters. In Chapters 2 and 3, $\tilde{\kappa}_{c,t}$ is modelled as stochastic and evolves according to an exogenous shock process, allowing labour market matching efficiency to vary in response to structural and policy shocks. In contrast, in Chapter 1, $\tilde{\kappa}_{c,t}$ is treated as a deterministic value computed from the steady state. In all chapters, separation rates—$\tilde{\vartheta}_{c,t}$ and $\tilde{\vartheta}_{h,t}$—are treated as constant and calibrated directly from observed data or steady-state conditions. The elasticities ζ_κ and ζ_ϑ determine how labour market frictions respond to housing market tightness.

1.2.6.2 Labour and Monetary Spillovers

While financial constraints are not explicitly modelled in the economy, financial constraints are a key restriction on households ability to purchase homes. Thus, and noting the pro-cyclicality of both housing and labour markets[24],[25], the decision to move house will be influenced by the ability to find employment in the new area. With greater market tightness in the labour market – $\omega_{c,t} \equiv \frac{V_{c,t}}{U_{c,t}}$ – the easier it is to find a job for an unemployed worker, and by the wage bargaining solution (1.41), the higher the real wage earned by workers. In a similar fashion to in the labour market, the matching efficiency in the housing market is positively related to the the relative market tightness in the labour market, and negatively related to the real interest rate:

$$\kappa_{h,t} = \tilde{\kappa}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\kappa} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{-\theta_\kappa} \tag{1.75}$$

The inclusion of real interest rates are motivated by the fact that while housing wealth represent the majority of households assets, it also represents the largest liability faced by households[26] with the cost of financing directly influenced by the prevailing real interest rate set by the monetary authority. Thus, its inclusion captures the transmission mechanism of monetary policy onto the housing estimated in Iacoviello and Neri (2010) now seminal paper.

As in the labour market, where a tighter housing market acted as a barrier to labour market transitions, affecting both matching and separation rates, housing market separation

[23] That is, the lower the ratio of searching buyers to houses for sale

[24] For housing, see for example Piazzesi and Schneider (2016)

[25] For labour, see for example Ashenfelter and Card (2011)

[26] See for example Causa et al. (2019)

also responds to both interest rates and labour market conditions. A tighter labour market and lower interest rates facilitates moves, speeding up matches as described above, and separations by:

$$\vartheta_{h,t} = \tilde{\vartheta}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\vartheta} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{\theta_\vartheta} \tag{1.76}$$

Where: Analogously, $\tilde{\kappa}_{h,t}$ is treated as stochastic in Chapters 2 and 3, evolving through an exogenous shock process. This allows the housing market's matching efficiency to respond flexibly to real interest rate and labour market developments. In Chapter 1, however, $\tilde{\kappa}_{h,t}$ is calibrated from the steady state and treated as deterministic. Separation in the housing market is modelled deterministically throughout. The elasticities η_κ and η_ϑ govern the responsiveness of housing market frictions to macroeconomic conditions.

1.2.7 Resource Constraint

All consumption undertaken by households are produced endogenously in the economy as described in section 3.2.3.4. Combining the households budget constraint (1.5) with the aggregated profit function (1.36) of intermediary producers, the governments fiscal policy (1.71), the level of housing investment undertaken by households is given by (1.11), the construction sectors labour demand equation (1.60), and the law of motion for housing construction (1.56), we can express the aggregate resource constraint[27]:

$$Y_t = C_t + \iota V_{c,t} \tag{1.77}$$

Where: (1.77) implies that output can be converted costlessly into consumption goods.

[27] See appendix section A:2

1.3 Calibration

Let $\mathcal{P}$ represent the entire set of parameters in the model. This whole set of parameters can be subdivided into three subsets. The first set – $\mathcal{P}_1 = \{\mu, \pi, R, l_c, h_c, \frac{l_h}{l_c}, \frac{w_h}{h_c w_c}, \frac{b_c}{h_c w_c}, \frac{\iota}{h_c w_c}, \frac{1}{\lambda_c^w}, \gamma_c^d, \frac{h}{Q}, \frac{b+f}{Q}, \vartheta_h, \frac{1}{\lambda_h^w}, \frac{r^h}{h_c w_c}, \frac{p^h}{h_c w_c}\}$ – is based on observations in the data and is described in 3.3. The second group described in section 3.3.3 are stored in the subset: $\mathcal{P}_2 = \{\gamma, \sigma, \theta, \nu, \epsilon, \delta_c, \delta_h, \Lambda, s_s^s, \upsilon, \varsigma, \zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta, \theta_\kappa, \theta_\vartheta, \alpha_y, \alpha_i, \alpha_\pi\}$, and is made up calibrated parameters. The parameters contained in $\mathcal{P}_1$ and $\mathcal{P}_2$ allows for the calculation of steady state parameters $\mathcal{P}_3 = \{\beta, \lambda_c, \gamma_c, \xi, u_c, \vartheta_c, \omega_c, w_c, y, c, x, \lambda, \chi_c, \chi_h, \epsilon_c, w_h, z^h, q_h, k, r^h, v_c, s, n, b, f, \omega_h, \lambda_h, \gamma_h, \kappa_h, \phi, \upsilon, m^h, \upsilon^B, \upsilon^N\}$, as discussed in section 3.3.4.

1.3.1 Stationarity and Detrending

The model features two sources of deterministic trend growth: population growth, described by equation (1.1), and productivity growth, described by equation (1.4). To ensure that all variables and their dynamic responses can be interpreted as stationary deviations from a balanced growth path, the model is made stationary by removing these trends from relevant variables. This is achieved by dividing variables by the appropriate trend component to ensure the existence of a stationary representation with constant steady-state values. See the appendix, section A:3.

The model's observation equations, presented in the appendix on data sources and measurement equations, are consistent with this transformation. The same detrending logic underpins the empirical implementation in Chapters 2 and 3.

1.3.2 Empirical Parameters:

Table 1.1 reports the empirical and calibrated parameters imposed on the model described in section: 1.2. The values reported are taken as long run ("steady-state") ratio's observed in data for the period 1971Q2-2020Q1. The full list of sources used in the table is available in the appendix, section A:1.

The growth rate of population (μ), real rate of interest (R), and rate of inflation ($1+\pi$), and the rate of employment (l_c) is set to match the mean of the reported data variable. The intensive margin of labour ($h_{c,t}$) is expressed as a fraction of a 40-hour work week. The ratio of the construction sector to the consumption sector labour is obtained by calculating the ratio of all employees in the construction sector to the total number of employed persons.

The ratio of earnings in the construction to consumption sector is calculated as the aggregate income in the two sectors ($\frac{w_h}{h_c w_c}$), where the notation account for the fact that the consumption sector differentiates between the intensive and extensive margin, while the construction sector only adjusts across the extensive margin.

The ratio of benefits to earnings, ($\frac{b_c}{h_c w_c}$) is taken from the OECD. The provided measure is the long run mean observation from 2001 to 2020, expressed as the proportion of benefits received by a single person without children after five months of unemployment who previously received the average wage. Other benefits such as social payments or housing benefits are excluded. While the aggregate income ($h_c w_c$) can be obtained through the sources discussed in section A:1, there exists no good, published measures of the cost of posting vacancies. This ratio is thus calibrated and uninformed. In parametrising the transition probabilities within the labour market, I follow Pissarides (2000) and define the average unemployment duration as one over the job-finding rate ($\frac{1}{\lambda_c}$). With the data measured in weeks, we use this measure in our parametrisation, and then compute the quarterly job finding rate as: $\lambda_c = 1 - (1 - \lambda_c^w)^{52/4}$.

Parameter	Meaning	Parameter Value
μ	Trend population growth rate	0.0025
$1+\pi$	Steady state level of inflation, quarterly	0.0123
R	Real interest rate, quarterly	0.0045
l_c	Employment rate	0.93
h_c	Hours worked	0.4101
$\frac{l_h}{l_c}$	Ratio of Construction Employment to General Labour Employment	0.036
$\frac{w_h}{h_c w_c}$	Wage in construction to general ratio	1.18
$\frac{b_c}{h_c w_c}$	Unemployment benefit to earnings ratio	0.39
$\frac{\iota}{h_c w_c}$	Vacancy posting to earnings ratio	0.5
$\frac{1}{\lambda_c^w}$	Unemployment duration	40 weeks
γ_c^d	Daily job filling rate	0.05 days
$\frac{h}{Q}$	Housing stock to occupied housing ratio	1.03
$\frac{b+f}{Q}$	Rent to occupied house ratio	0.34
ϑ_h	Average time between house moves, years	13.4 years
$\frac{1}{\lambda_h^w}$	House finding duration	20 weeks
$\frac{r^h}{h_c w_c}$	Rent to earnings ratio	0.356
$\frac{p^h}{h_c w_c}$	House price to earnings ratio	24.03

Table 1.1: Empirical Parameter Values

Following Davis, Faberman, et al. (2013) who shows that treating monthly job openings and hiring flows as outcomes of a daily processes helps address issues relating to time aggregation biases. Specifically, as many vacancies are filled within less than one month, aggregation at a monthly frequency will not account for vacancies that a posted and filled within the reference period will be unrecorded in vacancy stocks, causing an underestimation of vacancy durations. These issues would be even more pronounced in our model, as the data is taken at a quarterly frequency. They report a mean daily job filling rate of $\approx 5.2\%$, and we use this parameter. We then translate the parameter into a quarterly measure through: $\gamma_c = 1-(1-\gamma_c^d)^{365/4}$.

The housing stock ($\frac{h}{Q}$), and the rent to occupied housing ratio ($\frac{b+f}{Q}$) are parametrised relative to the stock of occupied housing given that all agents require housing. For estimates on the average time between house moves and average completion time to complete housing transactions, I rely on industry data due to the lack of published time-series data. We translate the monthly measure into quarterly data through the transformation: $\lambda_h = 1-(1-\lambda_h^w)^{52/4}$. For house prices, I use estimates provided by the Land Registries hedonic model for the UK housing market. For estimates on rental costs, I produce a combined measure of rental costs taken from the OECD which provides a price-to-rent index, and translate the index values to nominal prices through observations on average let agreed prices for a reference period.

1.3.3 Calibrated Parameters:

Table 1.2 reports calibrated parameters for the parametrised economy. The trend growth rate of technology $\varnothing$ is set to zero to allow the discount rate (β) to be established directly from the steady state level of interest. The habit persistence parameter associated with the consumption bundle is set to a reasonable 0.8, the same level reported in Christiano, Eichenbaum, and Evans (2005), and instead use the inter-temporal elasticity of substitution (σ) to alter consumption dynamics.

I set all matching elasticity parameters (δ_c, δ_h) to 0.7. Notice that that the bargaining power for buyers in the housing market (ϵ_h) is set to 0.5, assuming an even split of the surplus between buyers and sellers. With $\delta_h \neq \epsilon_h$, this parameterisation violates Hosios (1990) efficiency condition for the housing market. This choice is motivated by an assumption that the decentralised nature of the the housing market, the heterogeneity of the housing market, and the non-modeled financial constraints on transactions, are all captured by the searching and matching frictions. Because the bargaining power of workers (ϵ_c) is estimated, the labour market also violates the efficiency condition.

Λ and ϖ are parameters associated with the production function of land conversion $\Lambda(\frac{q_{h,t}}{A_t})$, which has the functional form: $\Lambda(\frac{q_{h,t}}{A_t})^{\varpi}$. The scaling parameter, Λ, and the elasticity parameter, ϖ, are set to reflect the assumption that land is scarce, motivated by the observation that the ratio of construction to stock of houses for the reference period is only 0.3 percent[28]. Finally, as the focus of the current study is to understand the spill-overs between the two markets, the parameter governing rental contracts is set to zero, so rental contracts are fully flexible and optimized in each period.

The spill-over elasticities active in the labour and housing market ($\zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta, \theta_\kappa, \theta_\vartheta$), as well as the Taylor rule parameters ($\alpha_y, \alpha_i, \alpha_\pi$) are quantified and estimated in chapters 2 and 3, for the simulations reported in section 1.4 examining the model dynamics, I calibrate these parameters to match the results from the estimation reported in chapter 2 and 3.

Parameter	Meaning	Parameter Value
$\varnothing$	Trend productivity growth rate	0.000
σ	Inter-temporal elasticity of substitution	2.0
θ	Habit persistence parameter	0.8
ν	Elasticity of labour supply	2.0
ε	Elasticity of substitution between intermediary goods	6.0
δ_c	Matching elasticity in labour market	0.7
δ_h	Matching elasticity in housing market	0.7
ϵ_h	Bargaining power of buyers in housing market	0.5
Λ	Production function parameter	0.025
ϖ	Production function parameter	0.5
υ	Rent control parameter	0.0
ς	Calvo Parameter	0.8
ζ_κ	Elasticity of labour matching W.R.T. housing tightness	4.4792
ζ_ϑ	Elasticity of labour separation W.R.T. housing tightness	5.9529
η_κ	Elasticity of housing market matching W.R.T. labor tightness	5.1328
η_ϑ	Elasticity of housing separation W.R.T. labour tightness	0.4069
θ_κ	Elasticity of housing matching W.R.T. interest rates	2.9702
θ_ϑ	Elasticity of housing separation W.R.T. interest rates	1.6315

Table 1.2: Combined Calibrated Parameters

[28] See appendix section A:1

1.3.4 Computed steady-state parameters

Based on the systems steady state relationship[29], the empirical and calibrated parameter values reported in tables 1.1 to 1.2, the models remaining structural parameters: $\{\beta, \lambda_c, \gamma_c, \xi, u_c, \vartheta_c, \omega_c, w_c, y, c, x, \lambda, \chi_c, \chi_h, \epsilon_c, w_h, z^h, q_h, k, r^h, v_c, s, n, b, f, \omega_h, \lambda_h, \gamma_h, \kappa_h, \phi, v, m^h, v^B, v^N\}$, are reported in table 1.3.

SS Variable (Symbol)	Parameter Value	SS Variable (Symbol)	Parameter Value
Discount Rate (β)	0.9955	Number of Homeowners (n)	0.6746
Finding rate (labour) (λ_c)	0.2805	Number of searching buyers (b)	0.0300
Filling rate (labour) (γ_c)	0.9907	Number of permanent renters (f)	0.3100
Mark-up (ξ)	0.8333	Housing market tightness (ω_h)	0.6010
Unemployment rate ($1 - l_c$)	0.07	Housing market finding rate (λ_h)	0.4867
Employment rate (l_c)	0.93	Share of searching workers (u_c)	0.0973
Separation rate (labour) (ϑ_c)	0.0269	Housing market filling rate (γ_h)	0.2925
Labour market tightness (ω_c)	0.2831	Matching efficiency (Housing) (κ_h)	0.4177
Wages (consumption sector) (w_c)	0.8249	Construction sector productivity (ϕ)	0.0769
Output (y)	0.3814	Vacancy Value (Housing) (v)	7.9395
Period Consumption (c)	0.3786	Cost of housing maintenance/tax (m^h)	0.2386
Habitual Consumption (x)	0.0757	Value of being a searching buyer (v^B)	-29.1357
Multiplier on households (λ)	174.4295	Value of homeownership (v^N)	-20.8170
Dis-utility (Consumption sector) (χ_c)	50.5849	Utility Value of Home-ownership (z^h)	17.3952
Dis-utility (Construction sector) (χ_h)	69.6287	Land for construction (q_h)	2.6943
Bargaining power of workers (ϵ_c)	0.1323	Undeveloped land (k)	1.0927
Wages (Construction sector) (w_h)	0.3992	Rent (r^h)	0.2937
Unemployment benefit (b_c)	0.1319	Vacancies (v_c)	0.0198
Cost of posting vacancy (ι)	0.1015	Houses for Sale (s)	0.0300
Matching efficiency (Labour) (κ_c)	0.4095	House Price (p^h)	8.1291
Labour supply (Construction) (l_h)	0.0335		

Table 1.3: Steady-State Variables (Parameter Values)

1.4 Model Dynamics

To better understand the model's dynamics, the system at rest is subject to a number of AR(1) exogenous shocks. While a larger number of shock parameters (ρ_i, σ_i) are examined and estimated in chapters 2 and 3, this section examines the generalized impulse response functions of the system to unit shocks in order to facilitate a comparative analysis of the different shock effects on the system. The process for the shocks examined as part of this chapter is detailed below in table 1.4.

Shock Description	Variable	AR(1) Process	Innovation Term
Labour supply shock	$\chi_{c,t}$	$\chi_{c,t} = \rho_{\chi_c}\chi_{c,t-1} + \epsilon_{\chi_c}$	$\epsilon_{\chi_c} \sim \mathcal{N}(0, \sigma^2_{\chi_c})$
Housing demand shock	ψ_t	$\psi_t = \rho_\psi \psi_{t-1} + \epsilon_\psi$	$\epsilon_\psi \sim \mathcal{N}(0, \sigma^2_\psi)$
Productivity (TFP) shock	z_t	$z_t = \rho_z z_{t-1} + \epsilon_z$	$\epsilon_z \sim \mathcal{N}(0, \sigma^2_z)$
Consumption demand shock	ϱ_t	$\varrho_t = \rho_\varrho \varrho_{t-1} + \epsilon_\varrho$	$\epsilon_\varrho \sim \mathcal{N}(0, \sigma^2_\varrho)$
Cost-push (markup) shock	ϵ_t	$\epsilon_t = \rho_\epsilon \epsilon_{t-1} + \eta_\epsilon$	$\eta_\epsilon \sim \mathcal{N}(0, \sigma^2_\epsilon)$

Table 1.4: Shock processes considered in Chapter 1.

[29] See the appendix, section: C

The structure of this section is as follows, subsection 1.4.1 analyses the model's response to exogenous shocks isolated to labour and housing markets to highlight the propagation mechanism that the spillover mechanism generates between the two markets. This is then followed by an examination of the system's response to shocks aggregate supply and demand shocks in subsection 1.4.2 and 1.4.3 highlighting the role of the labour market in equilibrium adjustments. Subsection 1.5 examines the implications of the spillovers existence for monetary policy modelling.

1.4.1 Labour and Housing Market shocks

To examine the transmission channel between the two markets, I subject the model to two exogenous shocks. The first raises the disutility of being active in the labour market ($\chi_{c,t}$), causing households to supply less labour, cetris paribus. The second shock raises the proportion of new enterants to the economy who wish to buy a house (ψ_t), simulating a housing demand shock in the economy. Figure 1.3 depicts the effect of the supply shock on key variables in the model, the full system response is provided in the appendix, table 7.

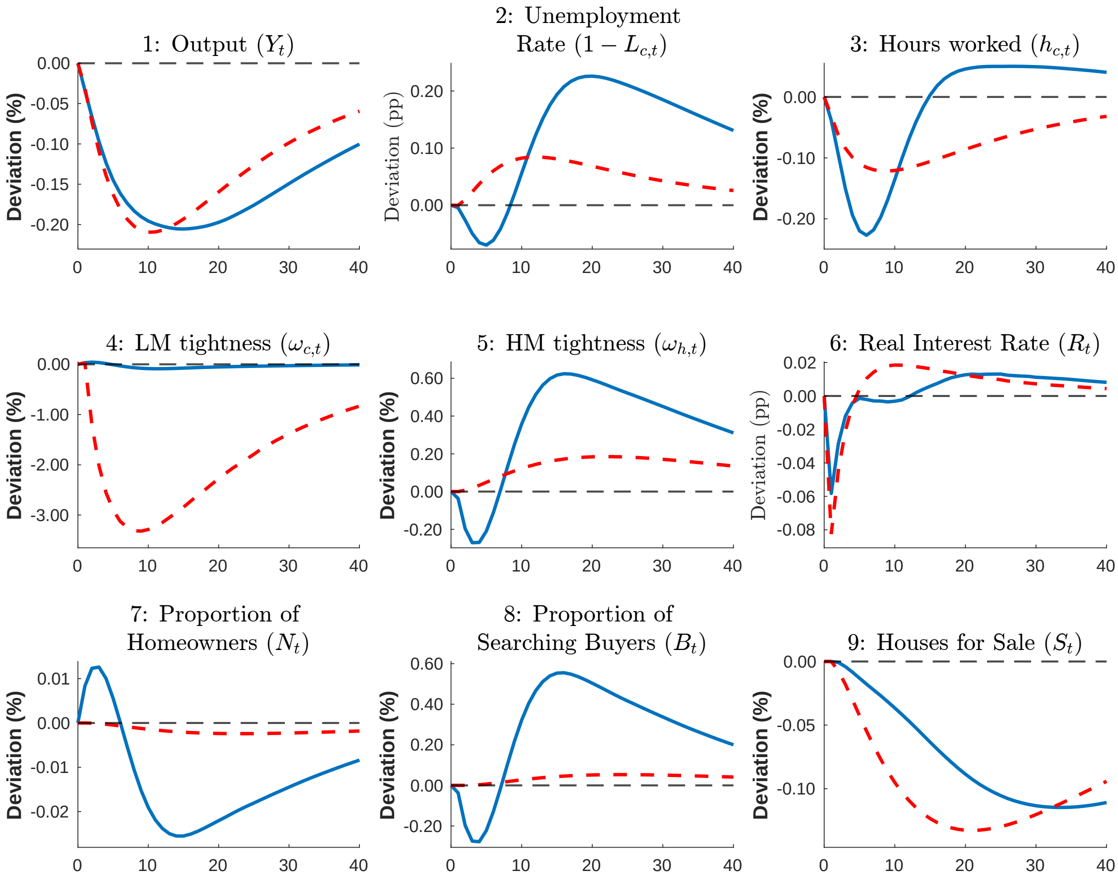


Figure 1.3: Response of the UK parameterised economy to a Labour Supply Shock.

From Left to Right: Panel 1: Output (Y_t), Panel 2: Unemployment Rate ($1 - L_{c,t}$), Panel 3: Hours Worked ($h_{c,t}$), Panel 4: LM tightness ($\omega_{c,t}$), Panel 5: HM tightness ($\omega_{h,t}$), Panel 6: Real Rate of Interest (R_t), Panel 7: Proportion of Homeowners (N_t), Panel 8: Proportion of Searching Buyers (B_t), Panel 9: Houses for Sale (S_t). The system with active spillovers is plotted in **blue**, the isolated system without housing and labour market interactions in dashed **red** lines.

Consider first the response of the system absent the spillover mechanism of the interconnected housing and labour markets. As intended, the shock causes households to reduce their supply of labour in line with their utility maximising behavior in light of the higher disutility of working in the consumption sector. As a consequence, there is a reduction in employment and an increase in unemployment, as shown in panel two of Figure 1.3. Due to the presence of search frictions, the extensive margin of labour can only adjust gradually, and the intensive margin initially contributes to absorbing the shock as shown in panel three. With production in the economy taking only aggregate labour hours worked ($L_{c,t}h_{c,t}$) as an input, the knock on effect of the shock is to reduce output produced in the economy as shown in panel one.

With a higher unemployment rate, the level of labour market tightness falls, as shown in panel four. Notice that absent market spillovers, the labour market effect is isolated and the adjustment in labour market tightness is significantly more prounounced than the effect on unemployment.

Faced with a negative output gap, the monetary authority responds by initially letting real interest rates fall below its steady state level as shown in panel six. Faced with lower real borrowing costs, there is an increase in the number separatations and matches in the housing market, but with the separation effect dominating. As a result, there is an increase in demand as agents transition from being homeowners to searching buyers as shown in panels seven and eight, respectively. With housing market activity falling, the incentive for housing construction falls, resulting a fewer vacant houses being listed for sale, as shown in panel nine. With greater housing demand and lower supply, the housing market tightness increases above its steady-state level, as shown in panel five.

Consider next the system under its baseline specification with spillovers between the two markets, plotted as **blue** lines in Figure 1.3. Upon the realisation of the shock, households utility maximising behaviour still dictates that households provide less aggregate labour ($L_{c,t}h_{c,t}$). As shown in panels two and three, the inclusion of the spillovers between the two markets gives rise to a dynamic where hours worked reacts strongly. Over time, the extensive margin is able to adjusts, and unemployment goes above its steady state level. The aggregate effect on the labour market is to reduce supply, causing a knock on effect on output as shown in panel one. While unemployment reaches its peak at a higher level of deviation from steady state under the baseline specification, we see that the effect on labour market tightness, plotted in panel four, is moderated relative to the baseline specification as consumption sector firms increase vacancy creation, showing how the model reproduces the labour market co-movement of vacancies and unemployment.

Again, we observe in panel six that the monetary authority responds to the shock by allowing real interest rates to become negative. However, while we in the isolated market specification saw that change to borrowing costs drove some minor changes in housing market outcomes, we notice that the baseline specification with interconnected markets gives rise to more dynamic responses as seen in panels seven to nine. Specifically, with a looser labour market reducing, and lower real borrowing increasing housing market activity, the two spillover channels are working in opposite directions. The labour market channel dominates, and housing market activity falls. With lower *net* transition probabilities, housing demand builds up, and the homeownership share falls, and with low transaction proabilities, prices and constructions falls. Ultimately, these spillover effects cause the level of housing market tightness to absorb some of the labour market effect, displaying an amplified change in housing market tightness as shown in panel five.

Next, we will examine the response of the system to the housing demand shock, with auxiliary variables plotted in the appendix, Figure 8. Again, lets first consider the specification of the model where the housing and labour market operate independently plotted as dashed **red** lines of Figure 1.4. As shown in panel *one*, the shock raises the number of searching buyers (my proxy for demand). All other things being equal, higher housing demand translates into a tighter housing market, better selling probabilities and higher house prices. In response to these changes, construction sector firms respond by building more houses, raising the number of houses for sale, as shown in panel *three*. With both demand and supply increasing, there are more matches occurring in the housing market, raising the proportion of homeowners as shown in panel *two*. Absent any spillovers, the labour market operates independently of the housing market, and the variables do not react as shown in panels *five* and *six*.

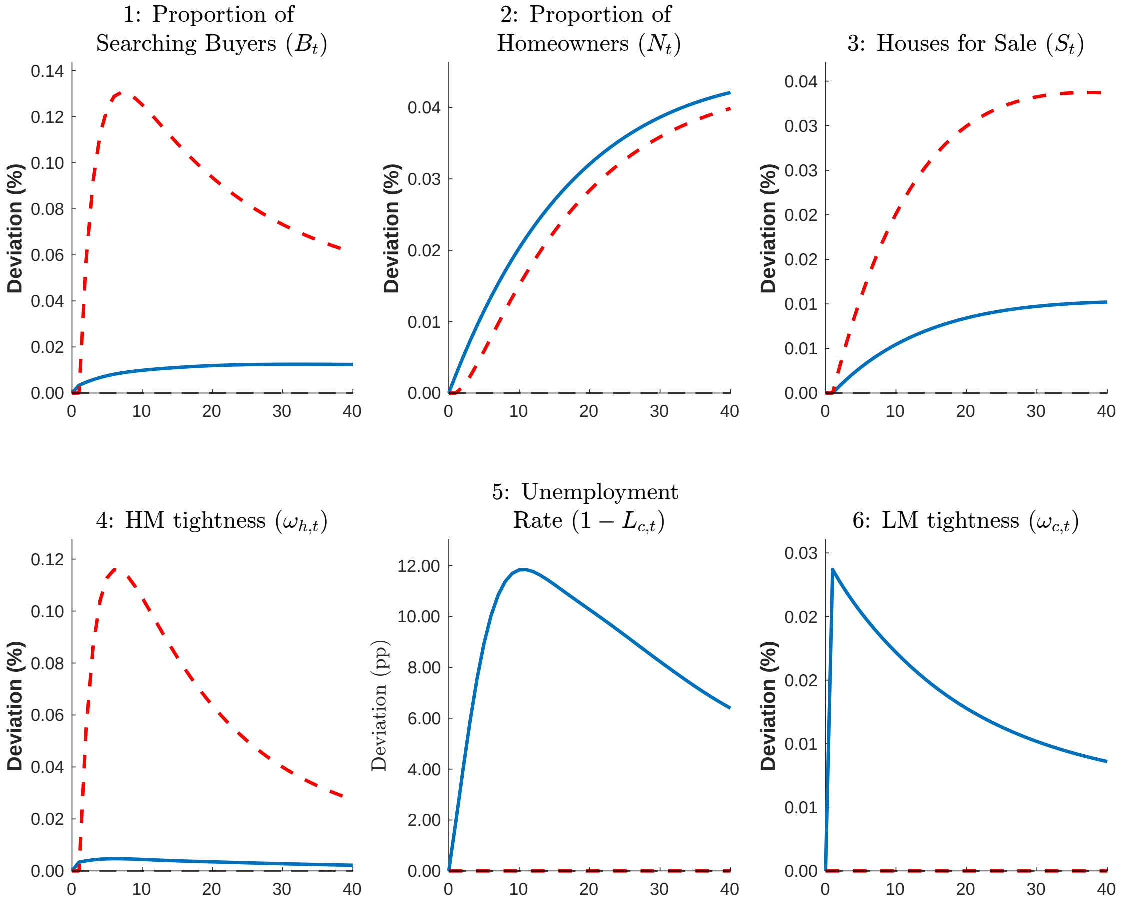


Figure 1.4: Response of the UK parameterised economy to a Housing Demand Shock.

From Left to Right: Panel 1: Proportion of Searching Buyers (B_t), Panel 2: Proportion of Homeowners (N_t), Panel 3: Houses for Sale (S_t), Panel: 4: Output (Y_t), Panel 5: Unemployment ($1 - L_{c,t}$), Panel 6: LM tightness ($\omega_{c,t}$). The system with active spillovers is plotted in **blue**, the isolated system without housing and labour market interactions in dashed **red** lines.

Next, lets examine the systems response to the housing demand shock under the baseline specification with market interactions plotted in solid **blue** lines. The direction of change in the housing market remains unchanged, with the shock raising the number of searching buyers, homeowners, supply, and the housing market tightness. However, as seen in panels *one* to *four*, the magnitude of the effect changes dramatically due to the interaction between the housing and labour market.

With a tighter housing market, the spillovers channel acts to reduce labour market activity. Facing a reduction in both matching and seperation rates, the *net* effect is a small increase in unemployment as shown in panel *five*. Labour market tightness is plotted in panel *six*. We notice that tightness adjusts about 10 times as much as the unemployment rate, suggesting that vacancy creation in the economy must simultaneously be falling.

way nature of the market interaction, the tighter labour market acts to spur housing market activity, driving the alternative response seen in panels *one* to *four*. Specifically, a tighter labour market raises matching and separation probabilities, with the matching probability dominating, resulting in a net increase in matches between searching buyers and vacant houses, explaining the relative lower deviation from steady state observed in supply and demand (panels *one* and *three*), and thus higher proportion of homeowners plotted in panel *two*.

By examining the systems response to isolated shocks under both interconnected and independent markets we see that when the markets we find that when the spillovers are inactive, both shocks have similar effects on their own market, but a negligible effect on the other market. However, once we allow the markets to interact, the spillover mechanism allows the model to recreate the key features of business cycle dynamics, namely the co-movement of vacancies and employment, and houses for sale and home ownership.

1.4.2 Technology

To further highlight the crucial role of the spillover mechanism in propagating shocks and in shaping model dynamics, the model is also subject to two shocks that affect household and firm behavior. The first is a total factor productivity (TFP) shock, which raises the level of output through the production function, $Y_t = z_t A_t L_{c,t} h_{c,t}$, for all levels of aggregate labour ($L_{c,t} h_{c,t}$) employed. Again, lets start by examining the behaviour of the system absent market spillovers. The shock largely behaves as a standard TFP shock in a labour search literature. Able to produce more from each firm-worker pair, output in the economy rises quickly as shown in panel *one* of Figure 1.5[30]. Higher productivity also raises the value of a match between a firm-worker pair, which raises wages negotiated by workers through the Nash bargaining process as shown in panel *two*.

A shortcoming of the model does a poor job of satisfying Okun (1963) law, predicting an increase in unemployment in response to productivity and output increases in contrast to the standard assumptions regarding the pro-cyclicality of the labour market discussed in Rogerson and Shimer (2010). This counterintuitive result arises from households' utility-maximizing behaviour. Higher output raises household consumption, reducing the need to supply labour as leisure becomes more valuable – a standard wealth effect. In response, aggregate labour supply, $h_{c,t} L_{c,t}$ falls below their steady state level. Search frictions ensures that the intensive margin absorbs most of the initial adjustment as shown in panel *three*, with changes in transition probabilities driving adjustments on the extensive margin over time as shown in panel *four*. With unemployment rising, labour market tightness (panel *five*) deviates negatively relative to its steady state, as unemployment increases faster than vacancy creation. Faced with higher than steady state output, the central bank responds by tightening monetary policy as shown in panel *nine*.

As the labour market has no direct effect on the housing market, we observe a similar dynamic to that observed under the labour supply shock discussed in the preceding section. Driven by households utility maximising behaviour, higher productivity and unemployment in the consumption sector causes greater housing construction, raising supply of vacant houses as shown in panel *eight*. As borrowing constraints tighten, both separations and matching probabilities in the housing market suffers, and there is a gradual reduction in demand relative to steady state, cetris paribus. However, as shown in panel *seven*, this is not the case in the dynamic model, since all other things are not equal. As supply growth outpaces demand growth, housing market tightness turns negative (see panel *six*), facilitating matching. The tightness effect, in turn dominates the effect of higher interest rates, ultimately leading to an increase in the number of homeowners as shown in panel *seven*.

Turning now to the the models baseline behavior, the model with full housing-labour market interactions in plotted as solid **blue** lines on Figure 1.5. Output and wages behave similarly to under the independent market specification, increasing as each firm-workerer pair becomes more productive as shown in panels *one* and *two*. Similarly, we also notice that the

[30] Please see auxiliary variables available the appendix, figure 9.

intensive margin of labour absorbs much of the initial periods of the shock effect as plotted in figure *three*. Finally, we also again observe that the positive output gap motivates the monetary authority to set real interest rates higher than their steady state level as shown in panel *nine*.

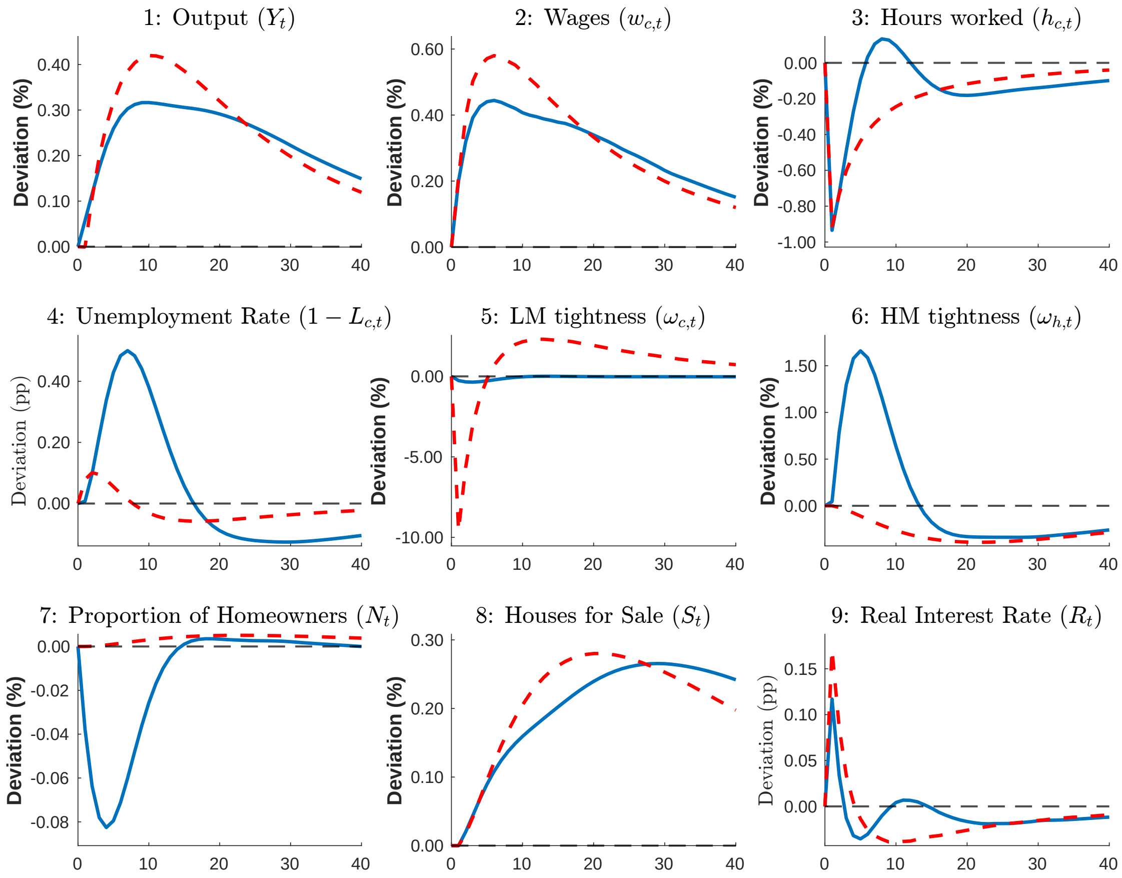


Figure 1.5: Response of the UK parameterised Economy to an Aggregate Productivity Shock.

From Left to Right: Panel 1: Output (Y_t), Panel 2: Wages in Consumption Sector ($W_{c,t}$), Panel 3: Hours Worked ($h_{c,t}$), Panel 4: Unemployment Rate ($1-L_{c,t}$), Panel 5: LM tightness ($\omega_{c,t}$), Panel 6: HM tightness ($\omega_{h,t}$), Panel 7: Proportion of Homeowners (N_t), Panel 8: Houses for Sale (S_t), Panel 9: Real Rate of Interest (R_t). The system with active spillovers is plotted in **blue**, the isolated system without housing and labour market interactions in dashed **red** lines.

Consider the housing market with interactions. In panel *eight*, we again observe that households optimising behaviour causes supply to increase. However, higher interest rates and unemployment now both contribute to reducing housing market activity, and their retarding effect dominates. There is thus a build up of demand unable to match with a vacant house, and the proportion of the economy acting as homeowners fall below its steady state level as shown in panel *seven*. With both demand and supply rising, the two effects work in opposite directions, but the demand effect dominates, and housing market tightness deviates positively relavative to its steady state, as shown in panel *six*.

Finally, consider the behaviour of the extensive margin of labour plotted in panel *four*. With market interactions, the tight housing market has a retarding effect on labour market activity, and there are both less matches and separations than when the market opperated independently. The matching effect dominates, i.e: The reduction in matches is greater than the reduction in separations. As a result, there is a net increase to unemployment relative to only the shock effect as shown in panel *four*, highlighting the amplifying effect of the market interaction.

1.4.3 Consumption Demand Shock

To further investigate the model dynamics and highlight the crucial role of the spillover mechanism in inducing a response in both markets, figure 1.6 reports the model's response to a shock to consumption preferences (ϱ_t)[31]. Irrespective of whether housing and labour markets operate in unison or independently, the shock functions as a standard consumption demand shock. Initially, households, motivated by their utility maximising behavior, respond by raising consumption of the habit adjusted bundle (X_t) as shown in panel *one*. To enable this higher consumption, output expands as shown in panel *two*. In turn, with output produced taking only labour as an input, higher production requires higher levels of aggregate labour. In the presence of search frictions, the intensive margin initially absorbs much of the reaction as shown in panel *three*, before the extensive margin responds after 10-20 periods as shown in panel *four*.

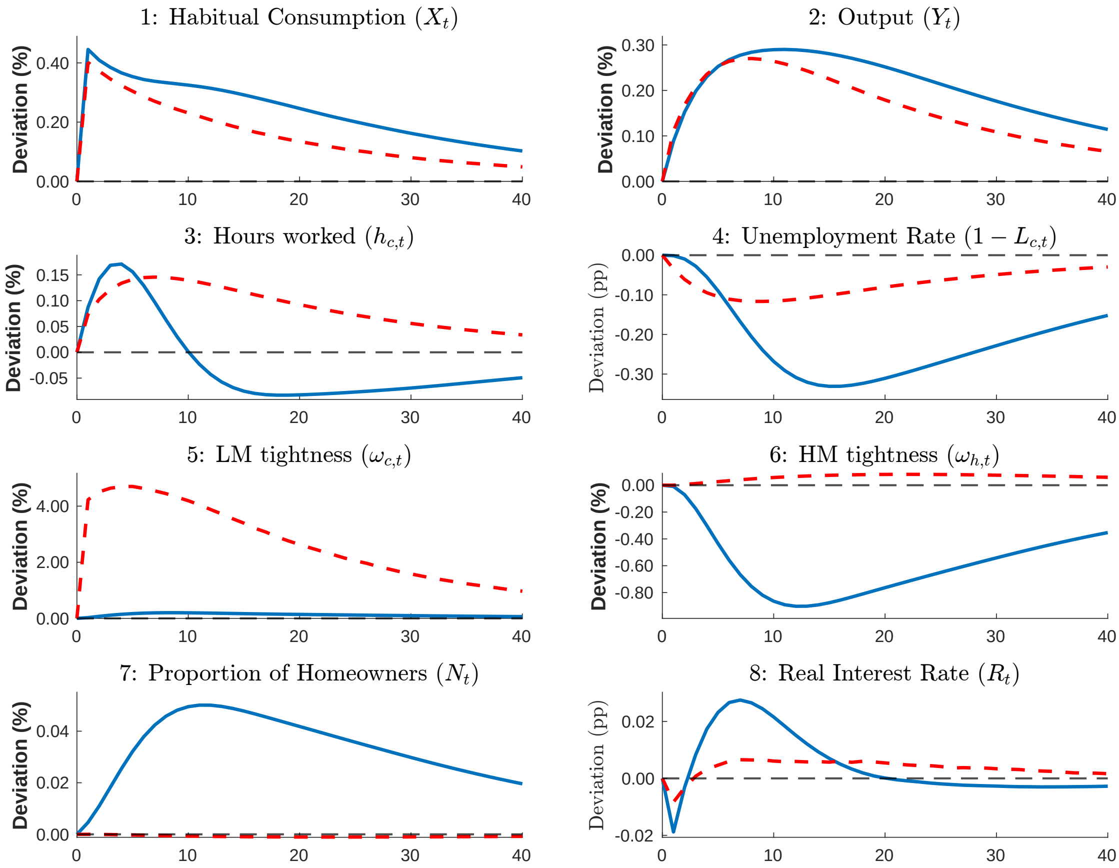


Figure 1.6: Response of the UK parameterised Economy to a Consumption Preference Shock.

From Left to Right: Panel 1: Habitutal Consumption (X_t), Panel 2: Output Y_t, Panel 3: Hours Worked ($h_{c,t}$), Panel 4: Unemployment ($1 - L_{c,t}$), Panel 5: LM tightness ($\omega_{c,t}$), Panel 6: HM tightness ($\omega_{h,t}$). Panel 7: Proportion of Homeowners (N_t), Panel 8: Real Rate of Interest (R_t). The system with active spillovers is plotted in **blue**, the isolated system without housing and labour market interactions in dashed **red** lines.

When the two markets operate independently, we as in the three preceeding cases only observe a small change in the housing market driven by the interest rate channel. As shown in panel *eight*, the monetary authority responds to the output gap by allowing the real rate of interest to turn positive relative to its steady state value. Higher borrowing costs reduce *net* matching probabilities for searching buyers, reducing the number of well-matched homeowners as shown in panel *seven*. With more buyers unable to match, there is a gradual increase in the level of housing market tightness plotted in panel six.

[31]Please find auxiliary variables in figure 10

Turning now to the baseline specification, we again see the role of market interactions in shaping model dynamics. While the initial reactions in consumption (panel *one*), output (panel *two*), and the intensive margin of labour (panel *three*) is unchanged, the knock on effects give rise to different dynamics. As unemployment begins to increase as shown in panel *four* it drives the increase in labour market tightness shown in panel *five*. A tighter labour market encourages housing market activity, and more searching buyers are able to match with a home, raising the level of homeownership shown in panel *seven*. In response, the level of housing market tightness (figure *six*) turns negative, which slows down the labour market, contributing to the stronger response in unemployment shown in panel *four*.

1.5 Monetary Policy Implications

Having established that the spillover gives rise to important business cycle features, this section explores the implications of the spillover for monetary policy formation by examining the system's behaviour with and without the spillover active to shocks to monetary policy variables. Specifically, I show that the inclusion of spillovers amplifies and propagates monetary policy interventions onto the aggregate economy. The main simulation examines the system's response to an unexpected increase in inflation. The response is implemented through a unit shock to the elasticity of substitution between intermediary goods (ϵ_t), which directly enters the economy's Phillips Curve (1.32), resulting in an above steady-state level of inflation, as shown in panel *one* of Figure 1.7.

As before, lets start by considering the response of the system absent the market spillover plotted in dashed **red** lines. As the elasticity of substitution between goods increases, final goods producers reduce prices paid for intermediary goods, lowering the mark-up ($\xi = \frac{\epsilon-1}{\epsilon}$) charged by intermediary producers. As a consequence, the surplus generated by a matched worker-firm pair shrinks, resulting in the job market equilibrium adjusting by reducing vacancy creation. With fewer vacancies created, it becomes harder for unemployed workers to find a job, and there is a reduction in the job-market finding rate shown on panel *three*[32]. As a result of this lower job finding rate, there is a an increase in unemployment, as shown in panel *two*. With higher unemployment and lower vacancy creation, the level of labour market tightness reacts and falls drastically relative to its steady state level as shown in panel *three*. Output is plotted in panel *four*, and is as previously discussed driven by the level of labour being used as an input. We thus observe a reduction in output as unemployment increases.

The monetary authority thus faces both a negative output gap which calls for expansionary monetary policy, while the rate of inflation is lies above its steady state level, calling for nominal rates to adjust upwards. The central bank responds by raising nominal rates as shown in panel *five*, but less so than the rate of inflation. The real rate of interest, plotted in panel *six*, thus falls below its steady state value. As seen in previous simulations, the housing market only reacts marginally when opperating independently of the labour market. While lower interest rates initially helps facilitate housing market matching, the shock causes hosing construction to suffer as shown in panel *eight*. As a consequence, there is a minor reduction in the number of homeowners as shown in panel *seven*, and the market tightens slightly as shown in panel *nine*.

[32]Note that the change in the labour market finding rate under the specification lies between -0.5 and 1, and thus does not show clearly on figure *ten*.

Next, consider the case when the two markets are allowed to interact plotted in **blue**. The unexpected inflation again reduces the value of a firm-worker match, and there is an increase in unemployment and reduction in output as shown in plots *two* and *four*. However, as unemployment rises, there is a reduction in labour market tightness plotted on panel *ten*. This has a cooling effect on the housing market, reducing *net* probabilities for searching buyers to transition into home-ownership, as shown in panel *seven*. As the number of homeowners falls and demand builds up, raising the incentive for housing construction as shown in panel *eight*. However, changes in demand dominates, and the housing market grows increasingly tight for 15 periods as shown in panel *nine*.

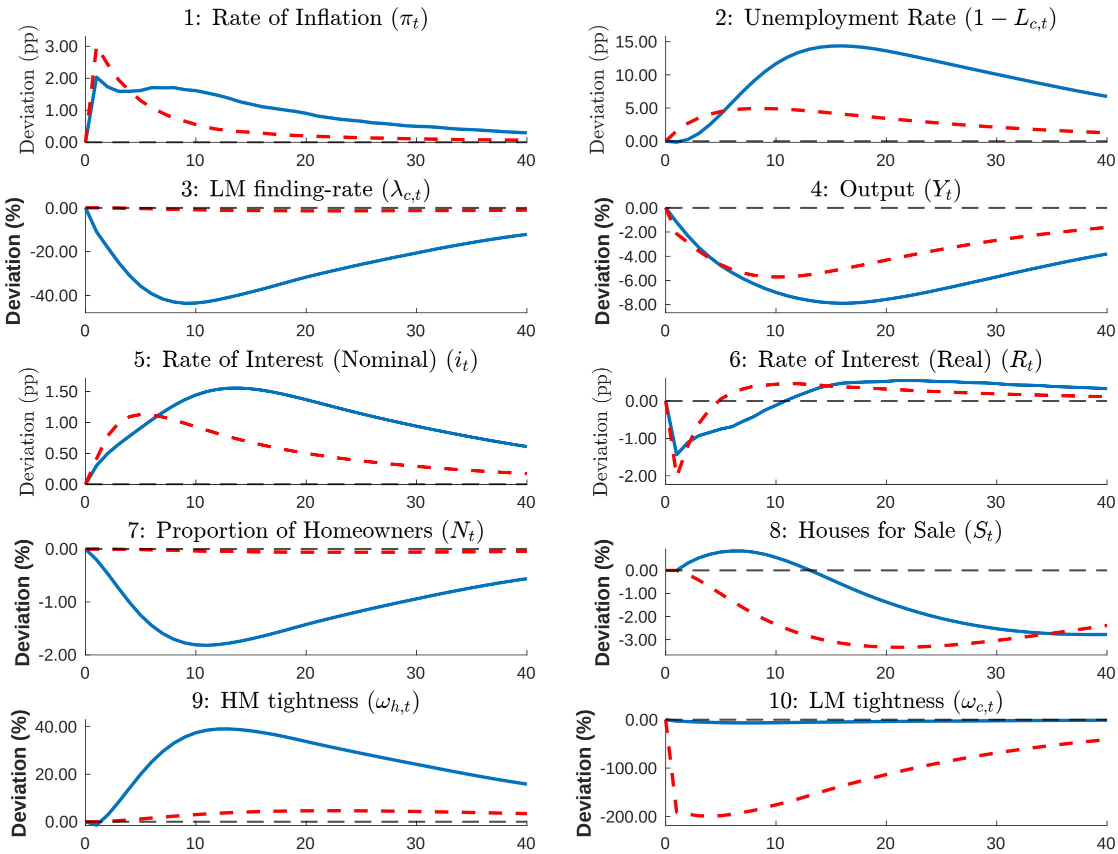


Figure 1.7: Response of the UK parameterised Economy to a Cost Push Shock.

From Left to Right: Panel 1: Rate of inflation (π_t), Panel 2: Unemployment rate ($1-L_{c,t}$), Panel 3: Labour Market Finding Rate ($\lambda_{c,t}$), Panel 4: Output (Y_t), Panel 5: Nominal Rate of Interest (i_t), Panel 6: Real Rate of Interest (R_t), Panel 7: Proportion of Households acting as Homeowners (N_t), Panel 8: Houses for Sale (S_t), Panel 9: Labour Market Tightness ($\omega_{c,t}$), Panel 10: Housing Market Tightness ($\omega_{h,t}$). Baseline plotted in **blue**, and the specification without the spillover channel in **red**.

The tighter housing market, in turn slows down activity in the labour market. The finding rate suffers as shown in panel *three*, and the effect on unemployment plotted in panel *two* displays an amplified effect relative to under the specification absent market interactions. Comparing the response of the model with and without the spillover active, the key takeaway is that the model's output dynamics are strictly determined by the labour market in the absence of spillovers. These model dynamics are further confirmed in an extension of this analysis plotted in Figure 12 in the appendix, which simulates an unexpected implementation of "bad" monetary policy that raises interest rates for one period through a unit AR(1) shock. These two simulations illustrate that even in the absence of financial frictions and collateral constraints, the model identifies a distinct channel through which monetary policy shocks propagate and amplify their effects on output via housing (asset) markets. This dynamic shares similarities with the financial accelerator mechanism of Bernanke et al. (1999), as implemented in Iacoviello and Neri (2010), albeit operating through a different underlying mechanism.

1.6 Concluding Remarks

This study has presented a theoretical framework that integrates the housing and labor markets through search-and-matching frictions and examines their interconnected dynamics under varying economic shocks and monetary policy interventions. A central finding is the critical role of spillover effects in explaining the observed co-movement of key macroeconomic indicators, including employment levels, homeownership rates, labor market vacancies, and housing market tightness across the business cycle.

Without an active spillover mechanism, shocks to either the labor or housing markets remain largely isolated, failing to propagate meaningfully into the other sector. For instance, productivity or consumption preference shocks in the labor market have negligible impacts on housing market outcomes in the absence of spillovers, and vice versa. The inclusion of spillovers, however, enables the model to replicate robust stylized facts, such as the synchronized responses of vacancies, unemployment, and housing market activity to macroeconomic shocks.

Monetary policy responses are also significantly affected by the presence of spillovers. The analysis demonstrates that monetary interventions, have amplified and more persistent effects on the aggregate economy when spillovers are active. Specifically, the interplay between the two markets influences output, employment, and housing activity in ways that simple independent sectoral models fail to capture.

In conclusion, this study underscores the necessity of incorporating spillover effects between housing and labor markets to understand the broader macroeconomic consequences of shocks and policy interventions. Future research should focus on refining the empirical estimation of these spillover channels and exploring their implications under alternative policy frameworks and in the context of different economic structures.

Chapter 2

Searching for flexibility: The Joint Impact of Thatcher's Reforms of UK Labour and Housing Markets*

This chapter applies the theoretical model developed in in Chapter 1 to evaluate the effects of the policy interventions into the housing and labour market implemented in the United Kingdom during the early 1980's. Through eleven time series spanning 1971Q1 to 2020Q1, the model's structural parameters are estimated and analysed in relation to the strength of the spillover parameters, the shift in monetary policy objectives, and to understand the sources of aggregate fluctuations through an examination of the estimated variance decomposition of the models exogenous shocks.

2.1 Introduction

The motivation for our paper lies in far-reaching reforms introduced in the 1980s. Britain, like many other countries, faced serious economic challenges throughout the 1970s. The Thatcher government, which assumed office in 1979, proceeded over three terms to introduce fundamental reforms in many areas of economic and social life. The underlying philosophy was one of freeing up markets and deregulation. A centerpiece of these reforms was a sequence of five laws aimed at shifting the balance of power in the labour market away from trade unions to bolster management's 'right to manage'. This shift in the balance of power proved enduring, undoubtedly assisted by a changing industrial structure moving away from traditionally highly unionized sectors. Notably, the first piece of legislation introduced by the new government, The Employment Act 1980, coincided with the peak of union membership (just over 13 million), which then declined year after year for the next 11 years. By the time Labour returned to government in 1997, membership was under 8 million.

The Thatcher governments also introduced many tax and benefit reforms over the decade of the 1980's[1], aimed at increasing employment. Briefly, one might point to changes to income tax, increases in the relative importance of in-work benefits, and a decrease in out-of-work benefits. For example, in 1982, the Earnings-Related Supplement (ERS) to unemployment benefits was abolished. There were also initiatives to help the unemployed find work by reducing search costs and by initiatives to improve training.[2] In short, trade union legislation, welfare and tax reform and other innovations were aimed at increasing what one would now label search and match efficiency.

*Note that this chapter is based on joint work with Tatiana Kirsanova and Charles Nolan for a paper under the same title.

[1]Some of the more significant changes did not come in until the late 1980's. See MAYHEW (1991).

[2]See Johnson and Stark (1989) for a contemporary assessment of some of those reforms. Muellbauer and Soskice (2022) is a modern and wider perspective on many of those issues. A useful overview is in MAYHEW (1991), while a detailed look at various tax and benefit changes are in Bowen and Mayhew (1990).

Equally striking were the privatization programs, especially the Housing Acts of 1980, which gave millions of council tenants the right to purchase their council houses on very favorable terms (see Jones and Murie (2006)). Interestingly, some economists had argued that the preponderance of council housing before the 1980's made the labour market less flexible as workers might be unwilling to move if that meant joining the end of the housing queue in a new locale. Later assessments on that issue uncovered a complex set of interactions.[3] In any event, housing reform also proved to be an enduring feature of Mrs. Thatcher's legacy. Owner-occupied tenure in 1980 was 55.5%; by 1990, it had risen to 67.1%. Over the same period, the proportion of publicly rented housing fell from 31.1% to 22.1%, and the private rented sector decreased from 12.7% to 7.5%.[4] Moreover, Jones and Murie (2006) note that sales of public sector dwellings under the Right to Buy policy had generated £36.8 billion by the end of the financial year 2002/03 and will amount to about £40 billion over the first 25 years.

Alongside labour and housing reforms were several others that complemented them. Liquidity restrictions on banks were removed and capital controls were lifted enabling banks to lend substantially more than before. Similarly, the building society sector was deregulated. By the late 1970s, building societies had accounted for over 95% of mortgages, protected from bank competition by the liquidity regulations just mentioned, among other tax and regulatory privileges. These mutual institutions, primarily funded by retail savings, were largely prohibited from accessing any form of wholesale funding source. Throughout the 1980s, building societies were permitted to expand their product range, diversify their funding sources, and demutualise. The result of these reforms on banks and building societies was a financial sector that decisively shifted toward mortgage and property finance. Net mortgage lending in 1980 was a little over £7 billion; by 1989, it had risen to over £34 billion (Boleat (1994)).

These structural reforms went ahead simultaneously with a 'paradigm shift' in macroeconomic policy, moving away from the Keynesian objective of full employment towards prioritising price stability in monetary policy, albeit without granting independence to the Bank of England. The Medium Term Financial Strategy, launched in March 1980, aimed for a gradually declining growth rate of the money supply. Over the ensuing four years, the growth rate of money supply (M3) was to be reduced from 7-11 percent in 1980-81 to 4-8 percent in 1983-84. This shift in the monetary policy framework was a crucial backdrop to structural reform since aggregate price stability was seen as central in allowing the price mechanism–the free market–to work efficiently.

The interplay between reforms to monetary policy and labour and housing markets forms the foundation of the complex legacy of Thatcher's economic policies. The central question for us is not merely how each reform performed in isolation but how, collectively, they influenced the dynamics between labour and housing markets and the broader economy. In seeking to contribute to a more robust and adaptable economic environment, did these policies, in fact, operate in isolation? Or did they collectively enhance or mitigate the impact of one another?

To address these questions in a formal way, we build and estimate a small and stylized New Keynesian model with interconnected labour and housing markets. This allows us to study the joint determination of house prices and unemployment, controlling for monetary policy stance. We use a workhorse search and matching model (Mortensen and Pissarides (1994)) in a general equilibrium setting (see, e.g., Christiano, Eichenbaum, and Trabandt (2016), and Lubik (2009), for similar DSGE treatments), combined with a search and matching model of the housing sector, where the modeling approach is most similar to Head, Lloyd-Ellis, and Sun (2014). We introduce a linkage between the two markets, assuming that the matching efficiency and separation rate in one market are affected by the tightness of the

[3]See Muellbauer and Murphy (2008) for a detailed overview of that issue and many others related to housing and the economy.

[4]The data are from the Department of the Environment, quoted in Stephens (1993).

other market. Specifically, we allow for the fact that a tight housing market may inhibit job search, resulting in fewer job quits and fewer job matches. As we mentioned earlier, when council housing was much more significant, some economists worried that the housing market was a significant impediment to labour market flexibility. On the other hand, a tight labour market might incentivise labour to move. In that case, we allow for homeowners receiving a mismatch signal such that they find a housing match more easily elsewhere.

As we discuss later, we also allow the real interest rate to enter directly into the matching functions. This is to indicate if other channels related to issues such as stricter borrowing constraints at times of high interest rates are of significance. Similarly, we allow the separation rate in the labour market to be affected by a higher interest rate (to capture effects such as an increase in the number of bankruptcies which is more likely when interest rates are high). We model monetary policy in terms of simple rules, allowing coefficients to shift to reflect important changes in the monetary policy framework. We then use a Bayesian approach to estimate the model using quarterly UK data from 1971-2020.

The paper is organised as follows. Section 2.2 presents the model, section 2.3 reports the result of Bayesian estimation, section 2.4 discusses implications of empirical findings for three particular reforms. Section 2.5 concludes.

2.2 The Model

In this section, we outline a model of an economy populated by households, firms engaged in production in the 'general industry,' and 'property developers'. Within each household, there are both employed and unemployed individuals, renters and homeowners, with risk sharing occurring at the household level. Credit frictions are not modelled explicitly but their impact will be captured (albeit imperfectly) in the shocks we recover as part of the estimation we undertake later on and via our permitting the real interest rate to impact the matching functions (see later). Individuals participate in labor market activity as described by standard search and matching models à la Mortensen and Pissarides (1994), and their search and matching activity in the housing market is similar to that described by Head and Lloyd-Ellis (2012). We assume, in effect, international risk sharing and employ a closed economy model, based on Gali and Monacelli (2005).

2.2.1 Households

The economy is populated by households, with the population size denoted as Q_t, which grows at the exogenous net rate μ:

$$Q_{t+1} = (1+\mu)Q_t. \tag{1.1}$$

Within this population, which also serves as the labour force, all individuals are infinitely lived and discount time at a rate β. Each individual supplies labour elastically to the construction sector but can be either employed or unemployed in the general industry. Each individual is either a permanent renter or not. People who are not permanent renters are either homeowners who also occupy their own homes or they are 'moving homes', meaning they live in rented accommodations, searching for a home to buy, and may also have a house to sell. In contrast, permanent renters do not want to buy a house and do not have one to sell.

Each individual is a member of one of the households. There is a constant number of households, H, while the size of a household changes over time. When the population rises, people are assigned to existing households, and there is no mobility between households. Each household may have both employed and unemployed individuals in the general and construction industries. It may have homeowners, renters, and buyers. A household is the unit at which consumption risk-sharing occurs, so all members of one household have the same level of consumption.

All individuals derive utility from habit- and productivity-adjusted consumption:

$$X_{i,t} = \frac{C_{i,t}}{A_t} - \theta \frac{C_{t-1}}{A_{t-1}}, \tag{1.3}$$

Where A_t is the trending productivity level of the economy:

$$A_{t+1} = (1 + \varnothing_t) A_t \tag{1.4}$$

With a stationary exogenous process a_t.

In the general industry, households face search frictions and unemployment. If employed, they provide $h_{c,i,t}$ labour hours to the general industry. In the construction industry, individuals derive disutility from supplying elastically $L_{h,i,t}$ units of labour, and this term enters the utility function trivially and additively. Owner-occupiers derive utility z^H from housing services of the owned house.

The total utility of a household can be expressed as:

$$U^H = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varrho_t \frac{(X_{i,t})^{1-\sigma}}{1-\sigma} \frac{Q_t}{H} + \chi_{c,t} \frac{L_{c,t}}{H} \frac{(1-h_{c,i,t})^{1-\nu} - 1}{1-\nu} - \chi_{h,t} L_{h,i,t} \frac{Q_t}{H} + z_t^H \frac{N_t}{H} \right]. \tag{1.2}$$

In this equation, $L_{c,t}$ represents the number of employed individuals, and N_t is the number of owner-occupiers in the economy. The index i is associated with consumption, hours, and labour supply in the construction sector—these are all choice variables. All individuals within one household will make the same decisions.

When firms and households in the general industry match, they enter into contracts where the real wage rate $w_{c,t}$ is determined through a Nash bargaining process. Consequently, an employed individual receives a labour income $h_{c,i,t} w_{c,t}$. An unemployed person receives an unemployment benefit b_t as a government transfer, financed through lump-sum taxes.

The budget constraint for household i states that consumption spending (C_t), lump sum taxes (T_t), investment in private bonds ($\mathcal{A}_{t+1}$), and net investment in housing (Ω_t) must be financed through labour incomes, unemployment benefits, bond returns, and any profits earned by firms (Φ_t):

$$C_{i,t} \frac{Q_t}{H} + \frac{Q_t}{H} \mathcal{A}_{i,t+1} + \frac{Q_t}{H} T_{i,t} + \frac{Q_t}{H} \Omega_{i,t} = \frac{Q_t}{H} \Phi_{i,t} + \frac{L_{c,t}}{H} w_{c,t} h_{c,t} + \frac{U_{c,t}}{H} b_{c,t} + \frac{Q_t}{H} w_{h,t} L_{i,h,t} + \frac{Q_t}{H} R_t \mathcal{A}_{i,t} \tag{1.5}$$

Households housing costs are divided into the maintenance/taxation (m_t^h) costs paid by all matched homeowners, rental costs (r_t^h) paid by renters, and transaction costs in the housing market for sales and purchases. These transactions are summarized by the final two terms of expression (1.6) below. Because all agents who are active in the housing market transition between either being a matched homeowner, (N_t) or a searching buyer (B_t), any per-period adjustments in the stock of homeowners must reflect either a successful match, or a separation and sale. I.e: If a member of household "i" has successfully matched with a house previously owned by household "j", there has been a one unit increase in the level of home-ownership in household "i", and a one unit decrease in household "j". The transaction is then carried out at the equilibrium house price P_t^h, which is discussed in further detail in section 3.2.4.6. Due to the presence of perfect risk sharing within the household the, these housing costs can be collected together in the variable ($\Omega_{i,t}$):

$$\frac{Q_t}{H}\Omega_{i,t} = r_t^h F_{i,t} + N_{i,t}m_t^h + P_t^h\left(N_{i,t} - N_{i,t-1}\right) - P_t^h\left(N_{j,t} - N_{j,t-1}\right),$$

where index j denotes households other than the i-th household.

Aggregation of first-order conditions across all households yields (see Appendix A:2):

$$A_t\lambda_t = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma} \tag{1.12}$$

$$\chi_{h,t} = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma}\frac{w_{h,t}}{A_t} \tag{1.13}$$

$$\left(\frac{X_t}{Q_t}\right)^{-\sigma}\frac{Q_t}{A_t}\varrho_t = \beta\mathbb{E}_t R_{t+1}\left(\frac{X_{t+1}}{Q_{t+1}}\right)^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}\varrho_{t+1} \tag{1.14}$$

$$\frac{X_t}{Q_t} = \frac{C_t}{Q_tA_t} - \theta\frac{C_{t-1}}{Q_{t-1}A_{t-1}} \tag{1.15}$$

(2.1)

and the aggregated budget constraint can be written as:

$$C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = \Phi_t + w_{c,t}h_{c,t}L_{c,t} + b_tU_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t,$$

2.2.2 Production

We use workhorse search and marching model (Mortensen and Pissarides (1994)) in a general equilibrium setting, see e.g. Christiano, Eichenbaum, and Trabandt (2016) and Lubik (2009) for more similar DSGE treatment. There is a unit-continuum of firms that hire new workers at a matching market. Unemployed workers search for a job and fill a vacant position. Firms pay fixed cost of posting a vacancy. Matches are stochastically destroyed with exogenous probability and workers become unemployed. Wages and hours worked are determined through a Nash bargaining process, that takes place between workers and firms. Firms sell goods in a monopolistically competitive market and choose their price subject to their demand curve. Prices are free to adjust.

2.2.2.1 Labour Dynamics

The number of workers employed by the j'th firm, $j \in [0,1]$, at the end of period $t-1$ is denoted $L_{c,t-1}(j)$. New jobs are created at the beginning of period t and some jobs are exogenously dissolved at the end of period t such that by the end of period t the j'th firm's employment level is:

$$L_{c,t}(j) = \left(1 - \vartheta_{c,t}\right)L_{c,t-1}(j) + \gamma_{c,t}V_{c,t}(j), \tag{1.20}$$

where $\vartheta_{c,t} \in (0,1)$ is the separation rate and $\gamma_{c,t}V_{c,t}(j)$ represents the number of new jobs, or matches, formed between the pool of unemployed workers and firm j.

The total number of matches that occur economy-wide is governed by the constant-returns-to-scale matching technology:

$$M_{c,t}(U_{c,t}, V_{c,t}) = \kappa_{c,t}U_{c,t}^{\delta_c}V_{c,t}^{1-\delta_c}, \tag{1.16}$$

where $V_{c,t}$ denotes the the economy-wide number of vacancies, $\delta_c \in (0,1)$ represents the elasticity of matches with respect to the unemployment rate, and $\kappa_{c,t}$ denotes the matching efficiency.

We define the level of labour market tightness, $\omega_{c,t}$, by

$$\omega_{c,t} = \frac{V_{c,t}}{U_{c,t}}, \tag{1.17}$$

so that the labour market is tight ($\omega_{c,t}$ is high) when the size of the unemployment pool is small relative to the number of vacancies. Given the matching technology and the definition of labour market tightness, the economy's job-filling rate is:

$$\gamma_{c,t} = \frac{M_{c,t}(U_{c,t}, V_{c,t})}{V_{c,t}} = \kappa_{c,t}\omega_{c,t}^{-\delta_c} \tag{1.18}$$

and its job-finding rate is:

$$\lambda_{c,t} = \frac{M_{c,t}(U_{c,t}, V_{c,t})}{U_{c,t}} = \kappa_{c,t}\omega_{c,t}^{1-\delta_c} = \omega_{c,t}\gamma_{c,t}. \tag{1.19}$$

We assume that all firms take $\omega_{c,t}$ (and hence $\gamma_{c,t}$ and $\lambda_{c,t}$) as given, and write equation (1.20) as:

$$L_{c,t}(j) = (1 - \vartheta_{c,t}) L_{c,t-1}(j) + V_{c,t}(j)\gamma_{c,t}, \tag{1.20}$$

where $V_{c,t}(j)$ is the number of vacancies posted by the j'th firm.

With economy-wide employment equalling $L_{c,t} = \int_0^1 L_{c,t}(j)\,dj$, the number of people that are unemployed and searching for work at the start of period t is:

$$U_{c,t} = Q_t - (1 - \vartheta_{c,t})L_{c,t-1}. \tag{1.22}$$

2.2.2.2 Firms

Firm j produces according to the production technology

$$Y_t(j) = z_t A_t h_{c,t}(j) L_{c,t}(j), \tag{1.27}$$

where A_t is an aggregate technology shock and z_t stationary productivity shock. To hire more workers, firms must post a vacancy and pay a fixed cost ι. Firms are monopolistically competitive, choosing the price they charge for their good subject to the demand curve:

$$Y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\epsilon} Y_t, \tag{1.25}$$

where $\epsilon > 1$ is the stochastic elasticity of substitution among goods. In equation (1.25), $p_t(j)$ denotes the price of the j'th firm's good, P_t denotes the aggregate price level, and Y_t denotes aggregate output.

Taking $\{P_t, w_t, Y_t, L_{c,t}(j)\}_{t=0}^{\infty}$ as given, the decision problem confronting the j'th firm is:

$$\max_{\{p_t(j), L_{c,t}(j), V_{c,t}(j)\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0}\left(\frac{p_t(j)}{P_t} Y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j)\right)\right], \tag{1.28}$$

subject to the production technology (1.27), the demand curve (1.25), and the law-of-motion for employment (1.21).

Aggregated across firms, first order conditions yield (see Appendix F):

$$\iota = \eta_t \gamma_{c,t}, \tag{1.29}$$

$$\eta_t = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + \beta \mathbb{E}_t \left[(1 - \vartheta_{c,t+1}) \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1}\right] \tag{1.30}$$

$$K_{1t} = w_{c,t} \frac{Y_t}{z_t A_t} + \alpha\beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}(1 + \pi_{t+1})^{\epsilon+1} K_{1t+1}\right] \tag{1.33}$$

$$K_{2t} = Y_t + \alpha\beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t}(1 + \pi_{t+1})^{\epsilon} K_{2t+1}\right] \tag{1.34}$$

$$(\pi_t + 1)^{\epsilon-1} = \frac{1}{\alpha}\left(1 - (1 - \alpha)\left(\frac{\epsilon}{(\epsilon - 1)}\frac{K_{1t}}{K_{2t}}\right)^{1-\epsilon}\right) \tag{1.32}$$

Here ξ_t is the Lagrange multiplier on the demand curve (1.25) which can be interpreted as the real marginal cost of production. Lagrange multiplier η_t is associated with the law of motion for employment and represents the value of filling a vacancy (1.21).

Equation (1.29) is the job-posting condition that says that firms will post vacancies up to the point where the expected payoff from filling a position equals the cost of posting the vacancy. Equation (1.30) is the job creation condition which says that the value of a newly filled position should equal the current-period profit generated from the match plus the expected discounted value of a filled position next period.

The last three equations determine the inflation process under Calvo (1983) price setting.

Finally, the aggregated profit of firms is

$$\Phi_t = Y_t - w_{c,t} h_{c,t} L_{c,t} - \iota V_{c,t}. \tag{1.36}$$

2.2.2.3 Wages and Hours Worked

The real wage and the number of hours worked are determined through Nash bargaining between workers and firms. Expressed in terms of period-t final goods, we denote using $\mathcal{V}_t^E$ and $\mathcal{V}_t^U$ the value to the household of having a member employed and unemployed, respectively.

The value to a household of having a member employed is given by:

$$\begin{aligned} \mathcal{V}_t^E = {} & h_{c,t} w_{c,t} + \chi_{c,t} \frac{(1-h_{c,t})^{1-\nu} - 1}{(1-\nu)\lambda_t} \\ & + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\vartheta_{c,t+1} (1-\lambda_{c,t+1}) \mathcal{V}_{t+1}^U + (1 - \vartheta_{c,t+1}(1-\lambda_{c,t+1})) \mathcal{V}_{t+1}^E \right) \right]. \end{aligned} \tag{1.37}$$

Looking at the terms on the right hand side of equation (1.37), the first term represents the extra goods that the household receives through the worker's labour income. The second term captures the value of the worker's leisure, expressed in terms of period-t final goods. The third term is a composite one that reflects the expected payoffs to being either unemployed or employed next period. For a worker that is employed today, the probability that they are unemployed next period is given by the separation rate, $\vartheta_{c,t+1}$, multiplied by the probability that they are unable to be matched to a new job in period $t+1$, which equals one minus the job-filling rate. The payoff to being unemployed next period in terms of next-period goods is $\mathcal{V}_{t+1}^U$. The next-period payoff to being employed equals $\mathcal{V}_{t+1}^E$, which is multiplied by the probability of being employed. These next-period payoffs are multiplied by the marginal rate of substitution and the discount factor in order to be expressed in terms of period-t final goods.

The value to the household of having a member unemployed is:

$$\mathcal{V}_t^U = b_{c,t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\lambda_{c,t+1}) \mathcal{V}_{t+1}^U + \lambda_{c,t+1} \mathcal{V}_{t+1}^E \right) \right], \tag{1.38}$$

where the first term on the right hand side reflects the real benefits that accrue to being unemployed today and the second term is a composite one reflecting the expected payoffs to being either unemployed or employed next period, which are then expressed in terms of period-t goods by multiplying by the marginal rate of substitution and the discount factor.

Given equations (1.37) and (1.38), the match surplus, $\mathcal{V}_t^S = \mathcal{V}_t^E - \mathcal{V}_t^U$, for the household equals:

$$\mathcal{V}_t^S = h_{c,t} w_{c,t} - b_{c,t} + \chi_{c,t} \frac{(1-h_{c,t})^{1-\nu} - 1}{(1-\nu)\lambda_t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\vartheta_{c,t+1})(1-\lambda_{c,t+1}) \mathcal{V}_{t+1}^S \right]. \tag{1.39}$$

Turning to the representative firm, the value of an unfilled vacancy, $\mathcal{V}_t^V$, equals zero while the value of filling a vacancy, $\mathcal{V}_t^F$, is given by equation (1.30). Recognizing that $\mathcal{V}_t^F = \eta_t$ and making use of equation (1.29), we have:

$$\mathcal{V}_t^F = h_{c,t}\left(\xi_t z_t A_t - w_{c,t}\right) + \beta \mathbb{E}_t\left[\left(1-\vartheta_{c,t+1}\right)\frac{\lambda_{t+1}}{\lambda_t}\frac{\iota}{\gamma_{c,t+1}}\right]. \tag{1.40}$$

The first term on the right hand side captures the real profits obtained from the goods produced from hiring an additional worker (filling a vacancy). The second term reflects the payoff that the firm receives when the match continues next period and the firm does not have to post a vacancy in order to fill a vacant position.

We assume that the real wages are set by Nash bargaining with the worker's share of the joint surplus equal to $\epsilon_{c,t}$, leading to the well-known sharing rule (see Appendix):

$$\mathcal{V}_t^S = \epsilon_{c,t}\left(\mathcal{V}_t^S + \mathcal{V}_t^F\right). \tag{2.3}$$

Substituting equations (1.39) and (1.40) and the one-period lead of equation (1.29) into equation (2.3) yields:

$$h_{c,t}w_{c,t} = \epsilon_{c,t}\left(\xi_t z_t A_t h_{c,t} + \beta\mathbb{E}_t\left[\left(1-\vartheta_{c,t+1}\right)\frac{\lambda_{t+1}}{\lambda_t}\left(1-\left(1-\lambda_{c,t+1}\right)\frac{\left(1-\epsilon_{c,t}\right)\epsilon_{c,t+1}}{\left(1-\epsilon_{c,t+1}\right)\epsilon_{c,t}}\right)\frac{\iota}{\gamma_{c,t+1}}\right]\right)$$
$$+\left(1-\epsilon_{c,t}\right)\left(b_{c,t} - \frac{\chi_{c,t}}{\lambda_t}\frac{\left(1-h_{c,t}\right)^{1-\nu}-1}{(1-\nu)}\right). \tag{1.41}$$

Where (1.41) determines the real wage per worker as a weighted average of a terms equaling the marginal revenue product of the worker plus the value of not having to replace the worker and a term equaling the outside option of the worker.

Finally, hours worked are chosen to maximize the joint surplus of the match, $\mathcal{V}_t^S + \mathcal{V}_t^F$, which gives (see Appendix):

$$\xi_t z_t A_t = \chi_{c,t}\frac{\left(1-h_{c,t}\right)^{-\nu}}{\lambda_t}. \tag{2.4}$$

2.2.3 Housing Sector

2.2.3.1 Housing Stock

At time t the city has stock of housing in the economy (H_t) which is either vacant and listed for sale ($V_{h,t}$), or occupied by one of the economies Q_t agents. Thus, total housing stock can be defined:

$$H_t = Q_t + V_{h,t} \tag{1.43}$$

Where: these $V_{h,t}$ vacant houses are made up of a combination of newly constructed houses by property developers, and houses which have been listed for sale by mismatched homeowners.

2.2.3.2 Matching Technology

Matching in the housing market is determined by the matching function $M_{h,t}$, which depends on the matching technology ($\kappa_{h,t}$), and the number of searching buyers (B_t) and vacant houses for sale (S_t).

$$M_{h,t}(B_t, S_t) = \kappa_{h,t}B_t^{\delta_h}S_t^{1-\delta_h} \tag{1.44}$$

Let market tightness, which acts as our proxy for housing market liquidity be defined as the number of searching buyers divided by the number of vacant houses for sale. That is:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} \tag{1.45}$$

Let the house filling rate (matching probability for vacant houses) be denoted by $\gamma_{h,t}$ and defined[5]:

$$\gamma_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{S_t} = \kappa_{h,t}\omega_{h,t}^{\delta_h} \tag{1.46}$$

And, let the house finding rate (matching probability of a searching buyer) be denoted by $\lambda_{h,t}$ and be defined[6]:

$$\lambda_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{B_t} = \kappa_{h,t}\omega_{h,t}^{\delta_h - 1} \tag{1.47}$$

2.2.3.3 Transition Probabilities and Laws of Motion in the Housing Market

At time t, there are N_t home-owning households. These homeowners become mismatched with their house at an exogenous probability $\vartheta_{h,t} \in (0, 1)$. If mismatched, homeowners become unhappy with their home and no longer receive the utility value of home-ownership (z^H). As a consequence, these households seek to sell their house which gets added to the stock of houses for sale – S_t, and become searching buyers – B_t – trying to match with a new house. Their law of motion is thus number of homeowners who did not become mis-matched in the previous period – $(1 - \vartheta_{h,t})N_{t-1}$, and those $\lambda_{h,t}B_t$ searching buyers who successfully matched with a house in the current period:

$$N_t = (1 - \vartheta_{h,t})N_{t-1} + \lambda_{h,t}B_t \tag{1.48}$$

The $(1 - \psi_t)$ is a proportion of new borns who are perpetual renters. That is:

$$F_t = F_{t-1} + (1 - \psi_{t-1})\mu_{t-1}Q_{t-1}. \tag{1.49}$$

Then, the number of searching buyers at the start of period t is:

$$B_t = Q_t - F_t - (1 - \vartheta_{h,t})\, N_{t-1}. \tag{1.50}$$

The number of houses for sale is the sum of vacant houses and those still occupied as the sellers are in a chain, the latter list their house for sale but remain in their house. In the context of our model they rent from themselves. Therefore,

$$S_t = V_{h,t} + C_{h,t} \tag{1.51}$$

where $C_{h,t}$ is the number of sellers that are in a chain.

We assume that the number of sellers who are in a chain is a proportion of all sellers:

$$C_{h,t} = \alpha_{h,t}S_t \tag{1.52}$$

Then, it follows that

$$\omega_{h,t} = \frac{(1 - \alpha_{h,t})\, B_t}{H_t - Q_t}.$$

2.2.3.4 Housing Construction

In the construction sector, there are three key stocks: undeveloped land (K_t^L), developed land ($\hat{H}_t$), and constructed housing (H_t). All undeveloped land is owned by the government, which releases it for development to firms in the construction sector. These firms operate under perfect competition and undertake both the development of land and the construction of new housing. To produce housing, firms combine developed land with construction labour ($L_{h,t}$) using a simple technology. They solve a cost minimisation problem dependant these two cost factors:

$$H_{t+1} - H_t = \min\left(\hat{H}_{t+1} - \hat{H}_t,\ \phi_t L_{h,t}\right) \tag{1.54}$$

[5]See appendix section: A:2
[6]See appendix section: A:2

Where: ϕ_t denotes the productivity of construction labour.

Solving the minimisation implies that land is developed until the cost of development equals the cost of employing labour. That is:

$$\hat{H}_{t+1} - \hat{H}_t = \phi_t L_{h,t} \tag{1.55}$$

Unable to store developed land and with free entry into the construction sector, firms construct new houses so long as profitable. They therefore build houses on any developed parcel of land. That is:

$$H_{t+1} - H_t = \phi_t L_{h,t} \tag{1.56}$$

Undeveloped land (K_t^L) is released exogenously at the rate $\varkappa$, intended to capture the rate at which the government releases land for development. Thus, the exogenous law of motion for land satisfies:

$$K_{t+1}^L = (1 + \varkappa) K_t^L \tag{1.57}$$

Such undeveloped land can be sold to construction sector firms at a price $q_{h,t}$. Prior to making the purchase, the firm can costlessly evaluate the development costs associated with the parcel. Reflecting that different parcels of land require different levels of development with different levels of associated costs, these costs are assumed to be heterogenous and draw from the distribution

$$c \sim A(c), \quad c \in [\underline{c}, \overline{c}]$$

With free entry, profits are driven to zero until all units of land with development costs $c \leq q_{h,t}$ are used for construction, ensuring that land development is increasing in $q_{h,t}$. With the level of undeveloped land being the difference between total available land and developed land – $K_t^L - \hat{H}_t$, the quantity of land converted satisfies:

$$\hat{H}_{t+1} - \hat{H}_t = \Lambda \left(\frac{q_{h,t}}{A_t} \right) (K_t^L - \hat{H}_t) \tag{1.58}$$

Where, $\Lambda \left(\frac{q_{h,t}}{A_t} \right)$ is the reduced form land conversion function defined: $\Lambda \left(\frac{q_{h,t}}{A_t} \right)^{\varpi}$, with $0 < \varpi < 1$.

Consider the profit earned by firms operating in the construction sector. To construct a new house, house builders face aggregate labour costs – $L_{h,t} w_{h,t}$ – and buy a unit of developed land for $q_{h,t}$. They earn revenues by selling newly constructed houses. Once a new house is built, it is listed for sale at the option price – $\hat{V}_{t+1}$. Their profit function is thus: the difference between the revenues earned from house selling and the cost of land acquisition for the $H_{t+1}^H - H_t^H$ units of houses constructed, and the aggregate costs of hiring labour:

$$\Pi_{const,t} = (H_{t+1} - H_t) \left[\beta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right) \right] - q_{h,t} \right] - w_{h,t} L_{h,t} \equiv 0 \tag{1.59}$$

Where: $\hat{V}_{t+1}$ is the value function associated with a vacant house not yet listed either for sale or for rent, and can be thought of as the option price of a vacant house. Requiring the profit function to equal zero follows from both developers and house builders operating in perfect competition.

With stationary representation:

$$\frac{w_{h,t}}{\phi_t A_t} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}}{A_{t+1}} \right] - \frac{q_{h,t}}{A_t} \tag{2.6}$$

2.2.3.5 Value Functions and the House Price

A permanent renter may move home, but remains a permanent renter. Her value function $\mathcal{V}_t^F$ satisfies the following equation:

$$\mathcal{V}_t^F = -r_t^h + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \mathcal{V}_{t+1}^F \right] \tag{1.61}$$

where r_t^h is a rent rate.

When an owner-occupier receives a mismatch shock with probability $\vartheta_{h,t}$, it causes her vacate the house, move out into a rented accommodation, and either list house for sale or put it on the rental market. The value function of an owner-occupier takes the form:

$$\begin{aligned} \mathcal{V}_t^N &= -m_t^h + \frac{z_t^h}{\lambda_t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} ((1 - \vartheta_{h,t}(1 - \lambda_{h,t+1})) \mathcal{V}_{t+1}^N \right. \\ & \left. + \vartheta_{h,t} \left(\tilde{\mathcal{V}}_{t+1} - \lambda_{h,t+1} P_{t+1}^h \right) + \vartheta_{h,t} \left(1 - \lambda_{h,t+1} \right) \mathcal{V}_{t+1}^B \right) \right]. \end{aligned}$$

In the current period, the homeowner pays maintenance and receives utility from the house. The expected payoff is the following. If there is no mismatch shock then she remains an owner-occupier. When mismatch shock happens, the expected value of the vacant house is $\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \tilde{\mathcal{V}}_{t+1}$as the sale or rent may only happen one period later. The homeowner expects to be the searching buyer with probability $(1 - \lambda_{h,t+1})$ when not matched with a suitable house next period, but she becomes a new owner-occupier if the next-period match realises. In the latter case, the price she pays for new house is P_{t+1}^h. Given the quarterly frequency of the model, it is not realistic to demand that the buyer searches for a house more than one quarter.

A searching buyer pays rent rate r_t^h. Her value function can be described by the following equation:

$$\mathcal{V}_t^B = -r_t^h + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\left(1 - \lambda_{h,t+1} \right) \mathcal{V}_{t+1}^B + \lambda_{h,t+1} \left(\mathcal{V}_{t+1}^N - P_{t+1}^h \right) \right) \right] \tag{1.63}$$

With probability $1 - \lambda_{h,t+1}$ the searching buyer remains a searching buyer, and with probability $\lambda_{h,t+1}$ there is a match so she becomes an owner-occupier, derives the value from homeownership, but price P_{t+1}^h for the house.

At the beginning of each period, an unoccupied house $\tilde{\mathcal{V}}_t$, can either be put on the rental market, or listed for sale. Such vacant houses move frictionlessly between sale and rental markets. Homeowners maximise profits and solve the following maximisation problem:

$$\tilde{\mathcal{V}}_t = \max[r_t^h - m_t^h + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \tilde{\mathcal{V}}_{t+1} \right], \mathcal{V}_t] \tag{1.64}$$

Where the first argument describes the value of a house listed on the rental market and where the house owner is responsible for maintenance payments. $\mathcal{V}_t$ is the value of house designated for sale. As houses move frictionlessly between the two markets, it follows that:

$$\tilde{\mathcal{V}}_t = r_t^h - m_t^h + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \tilde{\mathcal{V}}_{t+1} \right] = \mathcal{V}_t. \tag{1.65}$$

As homeowners will look for a new house, they are likely to sell when conditions seem right, so we assume that they also do not want to wish to be locked to a long-term rental contract.

The value of an unsold house to a seller, $\mathcal{V}_t$, satisfies the following equation:

$$\mathcal{V}_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(\gamma_{h,t+1} P_{t+1}^h + \left(1 - \gamma_{h,t+1} \right) \tilde{\mathcal{V}}_{t+1} \right). \right] \tag{1.66}$$

When a searching buyer meets with a vacant house for sale, the buyer and the seller determine the transaction price by engaging in Nash bargaining over the total surplus of a match in the same way as wages are determined in the general industry.

The match surplus for the searching buyer is:

$$\mathcal{V}_t^S = \mathcal{V}_t^N - \mathcal{V}_t^B - P_t^h, \tag{2.7}$$

while the match surplus for the seller is:

$$\mathcal{V}_t^E = P_t^h - \mathcal{V}_t.$$

Maximisation of the Nash product yields (see Appendix)

$$P_t^h = (1 - \epsilon_{h,t})(\mathcal{V}_t^N - \mathcal{V}_t^B) + \epsilon_{h,t}\mathcal{V}_t \tag{1.69}$$

where $\epsilon_{h,t}$ is the bargaining power of the buyer.

2.2.4 Aggregation and Equilibrium

Aggregation of (1.20) yields

$$L_{c,t} = (1 - \vartheta_c) L_{c,t-1} + V_{c,t}\gamma_{c,t} \tag{1.21}$$

The aggregate production function is

$$Y_t = \frac{z_t A_t}{\Delta_t} L_{c,t} h_{c,t} \tag{1.35}$$

Where, $\Delta_t = (1 - \varsigma)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \varsigma\pi_t^{\varepsilon} \cdot \Delta_{t-1}$ is the price dispersion in the economy. Because the system is log-linearised around its deterministic steady state where: $\pi = 1, \Delta_t = 1$, the price dispersion doesn't introduce additional first-order dynamics outside of those described in 1.35.

There exists a government whose sole purpose is to finance the unemployment benefit b_t paid to all $U_{c,t}$ unemployed persons. This government raises money through charging lump-sum taxes T_t on all households, levying property taxes/charging maintenance costs m_t^h on all N_t units of owned housing, and from proceeds arising from selling $(K_t^L - \hat{H}_t)$ units of land at the price $(q_{h,t})$. The governments budget constraint is thus:

$$b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(K_t^L - \hat{H}_t)$$

Using the fact that all are either employed or unemployed, we can substitute: $U_{c,t} = Q_t - L_{c,t}$ to express the number of unemployed persons. Further, using that the construction sectors zero profit condition (1.59) implies that all developed units are used for construction, the governments budget constraint becomes:

$$\Rightarrow b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(H_{t+1} - H_t) \tag{1.71}$$

Where: the level of lump sum taxes (T_t) adjust to maintain the equality. The total value of land sales $(q_{h,t}(H_{t+1} - H_t))$ is determined endogenously, while the values of land taxes (m_t^h) and unemployment benefits $(b_{c,t})$ reflect observed data ratios described in section 3.3.

And the requirement that net private bonds are in zero net supply $\mathcal{A}_t = 0$ yield the resource constraint:

$$Y_t = C_t + \iota V_{c,t} \tag{1.77}$$

A private sector equilibrium consists of stochastic processes of 26 aggregate endogenous variables $\{R_t, X_t, \lambda_t, w_{h,t}, C_t, \xi_t, L_{c,t}, U_{c,t}, \omega_{c,t}, w_{c,t}, h_{c,t}, Y_t, \omega_{h,t}, \mathcal{V}_t^F, \mathcal{V}_t^N, \mathcal{V}_t^B, \mathcal{V}_t, r_t^h, P_t^h, B_t, S_t, N_t, F_t, L_{h,t}, H_t, q_{h,t}\}$ such that 26 equations ((1.12), (1.13), (1.14), (1.15), (1.21), (1.22), (1.29), (1.30), (1.31), (1.32), (1.35), (1.41), (1.42), (1.43), (1.45), (1.48), (1.50), (1.49), (1.58),(1.56), (1.60), (1.61), (1.62), (1.63), (1.65), (1.66), (1.69), and (1.77).

(1.69) hold, given the system of exogenous processes for population (1.1), land (1.57), trend productivity (1.4) and AR(1) processes for trend to total factor productivity a_t, trend to population growth q_t, stationary productivity shocks z_t in general industry, taste shock ϱ_t, shocks to separation rate in labour $\vartheta_{c,t}$ and housing $\vartheta_{h,t}$ markets, shocks to bargaining power in the labour $\epsilon_{c,t}$ and housing $\epsilon_{h,t}$ markets, shock to land $\varkappa_t$, and shock to the number of permanent renters, ψ_t.

2.2.5 Interconnected Markets

This model does not distinguish between involuntary and voluntary job transitions. Workers seeking to relocate may take housing conditions into account. If the housing market is very tight, meaning there is a high relative number of buyers, this may deter potential job seekers from looking for new jobs, as moving houses might be difficult. This would result in fewer job quits and fewer new job matches:

$$\kappa_{c,t} = \tilde{\kappa}_{c,t} \left(\frac{\omega_{h,t}}{\omega_h} \right)^{-\zeta_\kappa},$$
$$\vartheta_{c,t} = \tilde{\vartheta}_{c,t} \left(\frac{\omega_{h,t}}{\omega_h} \right)^{-\zeta_\vartheta}.$$

Similarly, if the labour market is very tight, meaning there are many vacancies, there might be incentives to move to a better geographical location. Homeowners will receive a mismatch signal more often, and finding a match with a new house may be easier, as many people relocate:

$$\kappa_{h,t} = \tilde{\kappa}_{h,t} \left(\frac{\omega_{c,t}}{\omega_c} \right)^{\eta_\kappa} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{-\theta_\kappa},$$
$$\vartheta_{h,t} = \tilde{\vartheta}_{h,t} \left(\frac{\omega_{c,t}}{\omega_c} \right)^{\eta_\vartheta} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{\theta_\vartheta}.$$

In this chapter, $\tilde{\kappa}_{c,t}$ and $\tilde{\kappa}_{h,t}$ are modelled as stochastic processes, each evolving according to an AR(1) process with exogenous shocks. This allows for time-varying matching efficiency in both labour and housing markets, capturing fluctuations in unobserved market frictions or institutional flexibility. In contrast, separation rates—$\tilde{\vartheta}_{c,t}$ and $\tilde{\vartheta}_{h,t}$—are treated as constant and calibrated from steady-state relationships.

Parameters ζ and η are elasticities of one market matching efficiency and separation rate with respect of the other market tightness. These elasticities affect the speed of the change in either employment or homeownership, i.e. they measure the acceleration of employment or homeownership dynamics over time.

We also assume that activity in the housing market can be directly affected by the households credit market conditions. Cameron et al. (2006) argue that the UK house prices are significantly influenced by a credit conditions index, which is largely determined by the loan to value ratio.[7] However, in our empirical specification we use real interest rate to measure credit conditions. This approach attempts to mimic many transmission mechanisms of monetary policy on the housing market that are missing in this model, and that could help to identify housing matching shocks. Firstly, there are no mortgages. Even if mortgage payments were introduced, they would have a very limited effect on consumption behavior due to risk-sharing at the household level. Consequently, an increase in interest rate payments and lower disposable income of homeowners would have only a limited effect on their consumption. Secondly, the model does not account for first-time buyers who face borrowing constraints and would need to accumulate greater savings to obtain a mortgage with lower interest payments when the interest rate is high. The introduction of the real interest rate into the matching function – with θ_κ and θ_ϑ being relevant elasticities and R being steady state real rate – aims to mimic the effect of stricter borrowing constraints at times of high-interest rates.[8] Similarly, we assume that the separation rate can also be affected by a higher interest rate, likely leading to an increase in the number of bankruptcies when interest rates are high. We investigate the quantitative significance of these linkages.

[7] This index is estimated in Fernandez-Corugedo and Muellbauer (2006).

[8] In our empirical analysis, we were unable to find any strong link between the loan to value ratio and the housing matching channel.

2.2.6 Monetary Policy

To close the model, we assume that monetary policy operates with a simple interest rate rule:

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\alpha_i} \left(\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\alpha_\pi} \left(\frac{Y_t}{(1+\mu)(1+\gamma)Y_{t-1}}\right)^{\alpha_y}\right)^{1-\alpha_r} \exp(m_t)$$

Where: i_{ss} and π_{ss} are steady state values.

To account for changes in the monetary policy framework over time we chose to introduce two distinct monetary policy states: hawkish (H) and dovish (D) state, with different values of parameter α_π such that $\alpha_\pi^H > \alpha_\pi^D$.[9] The switches of the economy between these two regimes are governed by two-regime Markov chain $\upsilon_M \in \{H, D\}$ with transition matrix

$$T^M = \begin{bmatrix} 1-p_{HD} & p_{HD} \\ p_{DH} & 1-p_{DH} \end{bmatrix},$$

Where: $p_{ij} = P(\upsilon_{M,t+1} = j|\upsilon_{M,t} = i)$.

2.2.7 Shock Volatilities

The literature on good luck or good policy demonstrates that shock volatilities change with time and play an important role in explaining inflation dynamics, see e.g. Sims and Zha (2006). We, therefore, assume that shock shifts are governed by two-regime Markov chain $\upsilon_S \in \{T, V\}$, where state T describes relatively low volatility of all shocks ('tranquil' state) and state V describes relatively high volatility of them ('volatile' state). Transition between these state is described by matrix

$$T^S = \begin{bmatrix} 1-q_{TV} & q_{TV} \\ q_{VT} & 1-q_{VT} \end{bmatrix}.$$

where $q_{ij} = P(\upsilon_{S,t+1} = j|\upsilon_{S,t} = i)$.

2.3 Bayesian Estimation

The model has two unit roots: one arising from aggregate productivity growth (1.4) and the other from population growth (1.1). To ensure a well-defined steady state and enable empirical implementation, the model is transformed into a stationary representation by removing these deterministic trends from the equilibrium system, as outlined in Appendix B.

To maintain consistency with this stationary formulation, the macroeconomic time series used in estimation are also transformed into stationary counterparts. For level-based variables, we compute growth rates using log differences and remove sample means to eliminate deterministic drift. For ratio-based variables, we compute changes in the ratio level and similarly demean the series. Hours worked are normalised relative to a 40-hour work week. Inflation and nominal interest rates are left in levels and not demeaned. The data sources and transformation procedures are documented in greater detail in Appendix A:1.

2.3.1 Shocks and Data

We employ Bayesian methods to fit the log-linearized system to the UK data. For the estimation period of 1971Q2-2020Q1, we use the following nine data series: the growth rate of real output, the growth rate of real earnings, the unemployment rate, the growth rate of labour market tightness, the growth rate of the house price-to-rent ratio, the growth rate of the housing stock, house sales and the homeownership rate, see Figure 2.1. Detailed descriptions of the data, data sources and data transformation can be found in Appendix A:1. All computations presented in this paper were implemented using RISE toolbox (Maih

(2015)) for MATLAB, using methods discussed in Hashimzade et al. (2024), see further details in Appendix A:5.

While the model accommodates various shocks, not all of them can be identified with the available data. We have selected the following eleven shocks for estimation: stationary productivity shocks z_t, taste shocks ρ_t, shocks to matching efficiency in the labour market $\kappa_{c,t}$ and in the housing market $\kappa_{h,t}$, shock to bargaining power in the labour market $\epsilon_{c,t}$, shocks to productivity in the construction sector ϕ_t, shock to permanent renters ψ_t, labour supply chock $\chi_{c,t}$, shocks to housing sale chains $\alpha_{h,t}$, shock to elasticity of substitution (cost-push shock) ε_t and monetary policy shock m_t.[10]

2.3.2 Parameter Estimates

Parameters of the model can be divided into three subsets. The first subset includes the structural parameters that can be calibrated using the observed long run (steady-state) ratios. The second subset of parameters is calibrated because the simplicity of the model and the shortage of available data do not allow us to identify them. The third subset includes parameters that are estimated, including the persistence and standard deviations of structural shocks. We will discuss these sets in turn.

2.3.2.1 Observed Ratios and Implied Parameters

This model is relatively tightly parameterised, meaning that a large number of parameters is pinned down by steady state relationships. These parameters are summarised in Table 2.1 and the detailed data sources are given in Appendix A:1.

Table 2.1: Known Steady State Ratios

Data ratio	Notation	Value UK
Trend productivity growth rate, quarterly	$\varnothing$	0.0000
Population growth rate, quarterly	μ	0.0025
Steady state level of inflation, quarterly	$1+\pi_{ss}$	0.0123
Real interest rate, quarterly	$R=\frac{1+\gamma}{\beta}$	1.0045
Employment rate, quarterly	l_c	0.93
Hours of work, normalised	h_c	0.3418
Average duration of unemployment, weeks	$\frac{1}{\lambda_c^w}$	40
Daily filling rate	γ_c^d	0.05
Share of employment in construction sector	$\frac{l_h}{l_c}$	0.036
Earnings ratio	$\frac{w_h}{h_c w_c}$	1.18
Unemployment benefit to earnings ratio	$\frac{b_c}{h_c w_c}$	0.19
Cost of posting vacancy to earnings ratio	$\frac{\iota}{h_c w_c}$	0.5
Houses to occupied houses ratio	h	1.03
Share of rented houses to occupied houses	$b+f$	0.34
Average time to find a house, weeks	$\frac{1}{\lambda_h^w}$	20
Average time between house moves, years	ϑ_h	13
Rent to earnings ratio	$\frac{r^h}{h_c w_c}$	0.356
House price to quarterly earnings	$\frac{p^h}{h_c w_c}$	24.03

[9] Zanetti (2016) chose to use a variable inflation target to account for these changes.

[10] We could have used shocks to the separation rate in the labor market $\vartheta_{c,t}$ and the housing market $\vartheta_{h,t}$ instead of matching efficiency shocks. This would lead to a nearly equivalent model. Similarly, we could have used a shock to the rate of land conversion $\varkappa_t$, instead of the productivity shock in the construction sector. These shocks have nearly identical transmission mechanisms, further results are available upon request.

These ratios come from different sources. Parameters γ, μ, R and l_c are data averages over the sample. We use data for the average weekly hours of work and assumed that the total available time to divide between work and leisure is 16 hours per day for 6 days a week to compute h_c. The average duration of unemployment in UK is about 40 weeks that allows us to calibrate weekly (λ_c^w) and then quarterly job finding rate, $\lambda_c = 1 - (1 - \lambda_c^w)^{\frac{52}{4}} = 0.28$ which is consistent with findings in Hobijn and Şahin (2009).

When working with filling and finding rates one should note that both are probabilities and should be below one. While our model is quarterly, many events in the labour market happen at a higher frequency, resulting in quarterly estimates of these probabilities being close to one. We are unaware of any research done with the UK data to estimate the filling rate, while Davis, Faberman, et al. (2013) argue that one must look at daily filling rates to get an understanding of labour market flows. We chose to calibrate the daily filling rate $\gamma_c^d = 0.05$ following that study, however this does not bear any significant complications, as it implies quarterly filling rate $\gamma_c = 1 - \left(1 - \gamma_c^d\right)^{\frac{365}{4}} = 0.9907$, which is very close to one.

The share of employment in the construction sector is the data average over the sample. The earnings ratio $\frac{w_h}{h_c w_c}$ is computed by dividing weekly earnings in the construction sector (w_h) by weekly earnings in the economy ($h_c w_c$); here we implicitly assume that given that the construction sector constitutes less than 5% by employment, then excluding it from the measure of the 'rest of the economy' would lead to negligible changes.

The replacement ratio is calibrated following the OECD data, to be $\frac{b_c}{h_c w_c} = 0.19$. It is difficult to set the ratio of cost of posting vacancy to earnings. This is hiring cost and its value can be quite large. We set it to 0.5, although values in a large neighborhood of this do not have any material effects on our findings.

The data on vacant, owner-occupied and rented houses come from the Ministry of Housing, Communities & Local Government, we use them to compute the ratio of all housing units to occupied houses ratio, we also assume that the share of rented houses to all occupied houses reflects the share of searching buyers and permanent renters.

The average time between house moves was about 13 years in 2000-2008, as reported by the Savills Estate Agents and Letting Company. It is hard to estimate how long it takes to find a house in the UK. We assume that it takes about 20 weeks between the start of an active search and getting the keys roughly speaking that breaks down as 4.5 month to search 1.5 month to complete.

The steady state ratios presented in Table 2.1 allow us to calibrate structural parameters collected in $\mathcal{P}_3 = \{\beta, \gamma, \mu, \xi, u, \vartheta_c, \epsilon_c, \chi_c, \vartheta_h, \chi_h, \psi, \phi\}$ and some other steady states as discussed in Appendix C. Finally, parameters of utility function σ, θ, ν, elasticity of substitution ϵ, proportion of sellers in chains α_h are calibrated.

2.3.2.2 Calibrated Parameters

We calibrate a number of parameters. They are given in Table 2.2 and the choice is mostly determined by a difficulty in identification. For example, if we estimate matching efficiencies, matching elasticities δ_c and δ_h have to be calibrated. The introduction of taste shock renders θ (and σ) weakly identified. The bargaining power of searching buyers ϵ_h is not identifiable in this model given the availability of data used in estimation and so has to be calibrated. Finally, parameters of the land conversion function, Λ and s determine the curvature of the function, and units of measurement of unobservable land quantity, we cannot estimate them without data on land, so we calibrate them also.

We assume AR(1) stochastic processes for matching efficiency in both markets, and for separation and bargaining power processes in the housing market. The mean values for these

Table 2.2: Calibrated Parameters

Habit persistence	θ	0.8
Elasticity of substitution	ϵ	6
Elasticity of matches, labour	δ_c	0.7
Elasticity of matches, housing	δ_h	0.7
Buyers bargaining power	ϵ_h	0.5
Sellers in chain, HM	ϑ_h	0.5
Production function parameter	Λ	0.025
Production function parameter	s	0.5

processes and structural parameters, collected in $P_2 = \{\delta_c, \delta_h, \vartheta_h, \epsilon_h, \xi, \epsilon, \Lambda, s\}$, are calibrated as reported in Table 2.2. Together with ratios given in Table 2.1 these two sets of parameters determine steady states: $\mathcal{P}_3 = \{u, w_c, y, c, x, \lambda, \lambda_c, \gamma_c, \omega_c, w_h, b_c, \iota, \kappa_c, p^h, l_h, \lambda_h, n, b, f, \omega_h, \gamma_h, \kappa_h, p^h, v, v^B, v^N, v^F, r^h, q_h, z^h\}$. The procedure to obtain these estimates is given in Appendix C.

2.3.2.3 Estimated Parameters

Finally, we estimate $\mathcal{P}_4 = \{\sigma, \nu, \zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta\}$ and shock parameters collected in $\mathcal{P}_4 = \{\rho^j, \sigma_j\}$, where j is the shock index.

2.3.3 Historical Narrative

Figure 2.2 plots the estimated probabilities of being in a particular monetary policy and shock volatility state. Table 2.3 reports the estimated policy coefficients. The estimated monetary policy rule demonstrates a statistically significant difference between the hawkish ($\alpha_\pi^H \simeq 1.83$) and dovish ($\alpha_\pi^D = 1.02$) long-run response to inflation.

Our sample starts in 1971, just as the Bretton Woods system of fixed exchange rates ended. For most of the five-year period following 1971, the UK had a floating exchange rate but lacked a monetary anchor, as detailed in the brief history of UK monetary policy frameworks by HM Treasury (2013) and King (1997).[11] Our model identifies this period as having large shock volatility and a 100% certainty of dovish monetary policy.

While the formal targeting of monetary aggregates began in 1976 with published targets for M3, our model identifies a move towards a hawkish policy only after 1980, when Thatcher's government launched the Medium Term Financial Strategy designed to reduce inflation. The Bank of England faced many difficulties in meeting its M3 target between 1976-79: the targets were frequently overshot, and targets revised or abandoned. The Medium Term Financial Strategy proposed a gradual decline in M3 growth rates. In addition, starting from 1981, the M3 targets were complemented by M1 and later M0 targets, recognizing the destabilizing effect of financial innovations on the relationship between M3 and nominal income, as discussed by e.g. Mishkin (2001). This strategy worked for some time; inflation was steadily falling until 1985 when M3 substantially overshot the target. The M3 target was suspended in 1985 and then dropped altogether in 1987. The steady increase in inflation in the late 1980s occurred during a period when, '...the framework for monetary policy was, at best, opaque', (King (1997)).

Consistent with this narrative, the model indicates a significant increase in dovish monetary policy during the late 1980s. It also highlights a period of high shock volatility in 1988, as shown in Figure 2.2. The Bank of England's base rate rose rapidly from 7.4% at the end of May 1988 to 12.9% in November 1988 and to 14.9% in October 1989, with mortgage rates following the same pattern. This likely contributed to a subsequent decline in real house prices by more than 40% over the following years. Our model — which correctly identifies all recessions — singles out this very distinct market crash as a shock, rather than a change in policy state.

The UK joined the European System of Exchange Rates in 1990 and left in 1992, with a 20% currency devaluation and the associated inflation spike. The first, perhaps implicit, inflation target of 1-4% per year was introduced in 1992, and the Bank of England gained instrument independence in 1997. Consistently, the model shows a quick reduction in the probability of dovish policy from 1992 to 1995. Moreover, the probability of hawkish policy remains close to 100% until the end of the sample: the Great Financial Crisis of 2008 and the associated quantitative easing are not identified as dovish policy.

2.3.4 Interconnected Markets

Table 2.3 reports estimated elasticities. These results suggest that the housing and labour markets in the UK are strongly connected.

First, an increase in labour market activity fuels housing market activity. The estimate of η_κ suggests that if the tightness of the labour market rises by one percent, the housing market will have about 6% more matches. While the separation rate falls, the reduction of half a percent is not substantial, and the matching effect dominates. This is expected, as many separations in the housing market are the first part of a house move and are either not recorded in the data because the corresponding match happens within the same quarter, or

[11] See also Batini and Nelson (2005) for an interesting and wide ranging discussion of British monetary and macroeocnomic policy frameworks.

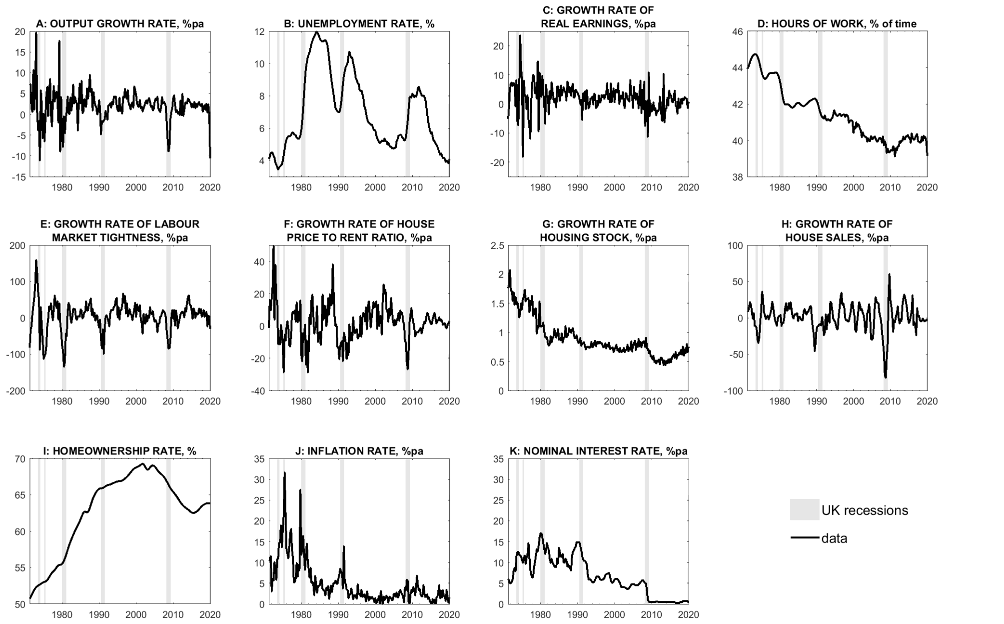


Figure 2.1: Historical Data Used for Estimation. Panels A-K show: The Growth Rate of real GDP, the Unemployment Rate, the Growth Rate of Real Earnings, Hours worked as a proportion of a 40 hours work week, the Growth Rate of Labor Market Tightness, the Growth rate of the House Price to Rent Ratio, the growth rate in the Housing Stock and Sales, the homeownership rate, and the monetary measures of Inflation and Interest Rates.

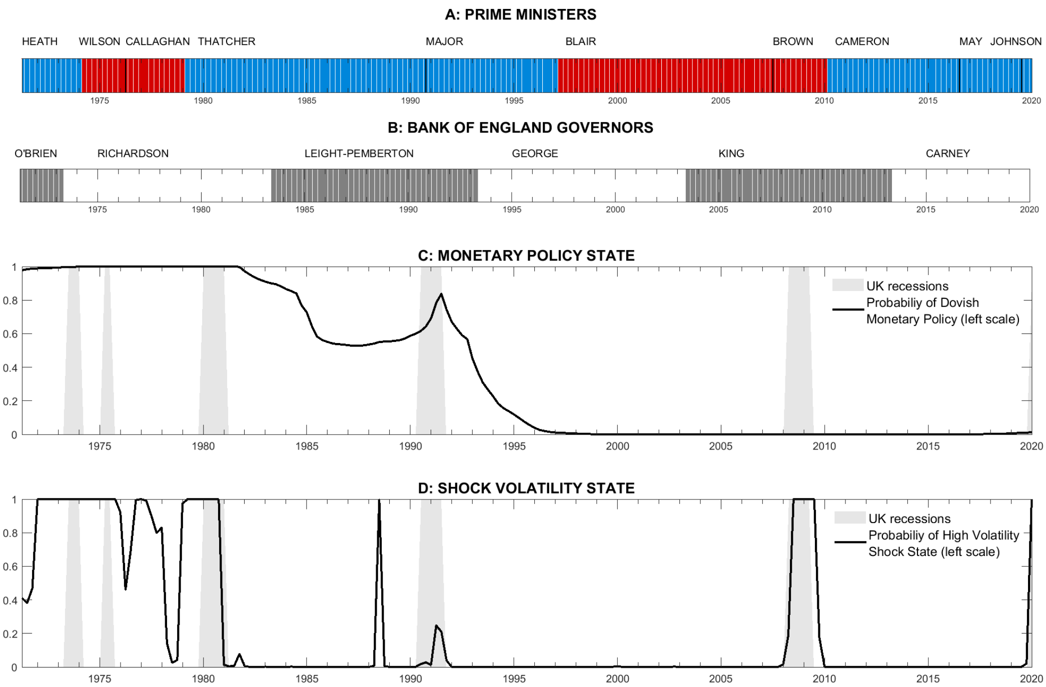


Figure 2.2: State Probabilities

Table 2.3: Estimation Results 1971Q2-2020Q1.

Parameters		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]
		Model Parameters	
Elast. of HM matching eff-cy wrt. LM tightness	η_κ	$N(3.0, 1.0)$	5.1328 [5.0620,5.2238]
Elast. of HM matching eff-cy wrt. Interest rate	θ_κ	$N(2.0, 1.0)$	2.9702 [1.4903,4.2434]
Elast. of LM matching eff-cy wrt. HM tightness	ζ_κ	$N(3.0, 1.0)$	4.4792 [4.4355,4.5159]
Elast. of HM separation rate wrt. LM tightness	η_ϑ	$N(3.0, 1.0)$	0.4069 [0.3815,0.4358]
Elast. of HM separation rate wrt. Interest Rate	θ_ϑ	$N(2.0, 1.0)$	1.6315 [0.6991,2.7304]
Elast. of LM separation rate wrt. HM tightness	ζ_ϑ	$N(3.0, 1.0)$	5.9529 [5.8998,5.9816]
		Shock Processes	
AR(1), technology	ρ^z	$B(0.5, 0.10)$	0.9555 [0.9553,0.9556]
AR(1), taste	ρ^ϱ	$B(0.5, 0.10)$	0.7207 [0.7026,0.7377]
AR(1), housing tech.	ρ^ϕ	$B(0.5, 0.10)$	0.9528 [0.9489,0.9554]
AR(1), labour supply	ρ^{χ_c}	$B(0.5, 0.10)$	0.9448 [0.9369,0.9509]
AR(1), matching LM	ρ^{κ_c}	$B(0.5, 0.10)$	0.8979 [0.8943,0.9029]
AR(1), bargaining LM	ρ^{ϵ_c}	$B(0.5, 0.10)$	0.9527 [0.9465,0.9555]
AR(1), matching HM	ρ^{κ_h}	$B(0.5, 0.10)$	0.9552 [0.9540,0.9556]
AR(1), renters HM	ρ^ψ	$B(0.5, 0.10)$	0.9554 [0.9553,0.9556]
AR(1), sales HM	ρ^{α_h}	$B(0.5, 0.10)$	0.9534 [0.9480,0.9556]
AR(1), elsticity of subst.	ρ^ε	$B(0.5, 0.10)$	0.9415 [0.9311,0.9526]

continued in the next page

there is a corresponding match just one quarter later. Counterfactuals discussed in the next section show that the estimated quantitative values of $\eta-$coefficients capture the imbalance of matching and separation in booms as an important factor to explain the cyclical behavior of house prices.

An active housing market, however, slows down activity in the labour market. A 1% increase in housing market tightness reduces the number of matches in the labour market and the separation rate by 4.5% and 6% respectively. Further counterfactual analysis shows that these coefficients imply that the matching effect dominates: increased housing market tightness results in a higher unemployment rate, though the quantitative effect is not very pronounced.

We find there is some effect of monetary policy—directly via interest rates—on housing market activity. The data indicate relatively high elasticities of housing matching efficiency and the separation rate with respect to real interest rates. An increase in the real interest rate by one percent reduces the matching efficiency by 3% and increases the separation rate by about 1.5%. The combined effect is a reduction in the homeownership rate. As we discuss later, this effect does not quantitatively compensate for the absence of financial frictions and the presence of consumption risk sharing in this model.

Table 2.3: Estimation Results 1971Q2-2020Q1 – continued.

Parameters		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]	
			H	D
Policy	α_π	$N(2, 0.5)$	1.7813 [1.7748,1.7864]	1.0067 [1.0004,1.0132]
Policy	α_y	$N(1, 0.5)$	0.2899 [0.2738,0.3048]	
Policy	α_i	$B(0.5, 0.15)$	0.8398 [0.8337,0.8480]	
			T	V
Std, technology	σ_z	$I(0.01, 0.02)$	0.0061 [0.0060,0.0062]	0.0159 [0.0153,0.0164]
Std, taste	σ_ϱ	$I(0.01, 0.02)$	0.0572 [0.0543,0.0602]	0.1614 [0.1550,0.1660]
Std, housing tech.	σ_ϕ	$I(0.01, 0.02)$	0.0950 [0.0914,0.0976]	0.1947 [0.1903,0.1983]
Std, labour supply	σ_{χ_c}	$I(0.01, 0.02)$	0.0222 [0.0218,0.0227]	0.0352 [0.0328,0.0373]
Std, matching LM	σ_{κ_c}	$I(0.01, 0.02)$	0.0548 [0.0540,0.0554]	0.0693 [0.0584,0.0809]
Std, bargaining LM	σ_{ϵ_c}	$I(0.01, 0.02)$	0.0989 [0.0882,0.1136]	0.2652 [0.2427,0.2914]
Std, matching HM	σ_{κ_h}	$I(0.01, 0.02)$	0.4264 [0.3785,0.4762]	1.3051 [1.2061,1.4067]
Std, renters HM	σ_ψ	$I(0.01, 0.02)$	0.3529 [0.3326,0.3664]	0.7256 [0.7155,0.7851]
Std, sales HM	σ_{α_h}	$I(0.01, 0.02)$	0.0822 [0.0735,0.0920]	0.2097 [0.1658,0.2560]
Std, elsticity	σ_ε		0.0014 [0.0013,0.0015]	0.0031 [0.0026,0.0040]
Std, policy			0.0027 [0.0025,0.0029]	
State Probabilities				
Prob. to move from H to D	p_{HD}	$B(0.05, 0.025)$	0.0022 [0.0011,0.0040]	
Prob. to move from D to H	p_{DH}	$B(0.05, 0.025)$	0.0030 [0.0014,0.0049]	
Prob. to move from T to V	Q_{TV}	$B(0.05, 0.025)$	0.0555 [0.0360,0.0787]	
Prob. to move from V to T	Q_{VT}	$B(0.05, 0.025)$	0.0595 [0.0161,0.1194]	

Note: The posterior distribution is obtained using the Metropolis–Hastings algorithm.

We did not find that credit market conditions, as measured by the households' debt-to-income ratio[12], affected housing market matching efficiency in a quantitatively substantial way (these results are not shown). This is likely because the transmission mechanism of housing matching shocks is different in this model—they not only affect house prices but are also important drivers of other variables that are not directly affected by credit market conditions. We discuss these issues in the next section.

[12] In using these data, we implicitly assume that households typically borrow as much as they can, so these data reflect the loan-to-income ratios that they face. While the data capture the whole mortgage stock, its dynamics—when adjusted for the price-to-income ratio—are similar to those observed in the credit conditions index constructed in Fernandez-Corugedo and Muellbauer (2006) for a comparable time period.

2.3.5 Relative Importance of Shocks

Table 2.3 suggests that all shocks are relatively persistent and there is statistically significant difference between their standard deviations in the high and low volatility states at 95% confidence level. Figure 2.2 shows that the high volatility regime was prevalent in the pre-1980s and then it virtually coincides with the Great Recession and the Covid recession. Finaly, Table 2.4 presents variance decomposition in the ergodic state. This table, together with the estimated elasticities helps to understand the relative importance of shocks in the long run.

The general industry quantity variables are mostly driven by labour market and demand shocks, but there are notable spillovers from the housing market. In particular, shocks to matching efficiency in the housing market explain a substantial part of the volatility in employment and the number of searching workers in the labour market.

Table 2.4: Variance decomposition. Ergodic distribution.

		z_t	ϱ_t	$\chi_{c,t}$	$\kappa_{c,t}$	$\epsilon_{c,t}$	ε_t	$\kappa_{h,t}$	ϕ_t	$\alpha_{h,t}$	ψ_t	m_t
Output	Y	5.7	24.3	8.9	0.3	14.1	24.1	18.2	0.7	2.8	0.4	0.5
Employment	l_c	0.0	1.0	0.5	2.0	33.0	9.2	44.5	1.6	6.7	0.9	0.2
Searching workers	u_c	0.0	0.9	0.4	0.4	26.1	7.5	54.2	2.1	6.8	1.1	0.2
Real wage rate	w_c	4.7	0.4	5.0	0.0	18.7	57.7	10.4	0.4	0.3	0.1	2.1
Work hours	h_c	7.0	28.1	9.5	2.1	14.3	14.4	19.7	0.7	3.0	0.4	0.4
Labour supply	l_h	0.1	0.7	0.1	0.0	15.0	3.0	25.9	51.0	2.8	0.9	0.1
Earnings in construction	w_h	12.5	6.3	17.4	0.8	8.4	38.4	11.5	0.4	1.7	0.2	1.7
Housing stock	H	0.0	0.6	0.3	0.1	27.0	6.4	44.5	10.4	3.9	5.9	0.1
Rent rate	r^h	0.2	1.2	0.4	0.1	29.7	4.4	50.2	3.6	4.3	4.6	0.2
House price	P	0.7	1.6	0.4	0.1	29.7	4.4	50.2	3.6	4.3	4.6	0.2
Homeowners	N	0.0	0.1	0.0	0.0	2.8	0.7	4.4	0.2	0.4	91.2	0.0
Searching buyers	B	0.0	0.8	0.4	0.1	30.4	8.0	50.8	2.0	3.8	3.0	0.2
House sales	$\gamma_c B$	0.0	0.6	0.2	0.1	13.1	3.2	6.3	1.8	1.6	73.1	0.1

Note: The list of shocks: technology z, preference (demand) ϱ, labour supply χ_c, matching efficiency in the labour market κ_c, bargaining power in the labour market ϵ_c, matching efficiency in the housing market κ_h, technology in constriction sector ϕ, house sales α_h, renters ψ, elasticity of substitution (cost-push) ε, monetary policy m.

Housing market variables are driven mostly by housing market shocks, although there are meaningful spillovers from the labour market. Shocks to workers' bargaining power explain a notable part of the long run volatility of house prices, and the number of buyers. Notably, the monetary policy shock explains little in the long run.

To understand the role of shocks in the short run, consider the growth rate of the house price to rent ratio in the top panel of Figure 2.3, which covers the Thatcher period in office. It is apparent that shocks to bargaining power drove the large cyclical reduction in house prices in 1980-81 and explain the peak in 1988, the latter typically associated with relatively low interest rates. In both episodes, bargaining shocks also explain a substantial share of the growth rate of real earnings, consistent with the demand pressure on house prices. Another significant driver of the price to rent ratio—the housing matching efficiency shock—counterweighs the positive bargaining shocks to explain the relatively stable house prices in the second half of 1979 and is nearly completely responsible for the dramatic fall in house prices during 1988-91. This episode is often described in large measure as a consequence of a sharp increase in policy rates[13] and greater financial constraints[14]. As this feature is not in

[13] See, for example, Boleat (1994).

[14] By greater financial constraints we are referring to factors such as those noted by Boleat (1994). He notes: 'Mortgage borrowing has itself become relatively less attractive as a consequence of government actions that have reduced both mortgage tax relief and tax rates and, therefore, the value of tax relief.'. p.261.

our model, stricter financial constraints are captured by a negative housing matching shock.

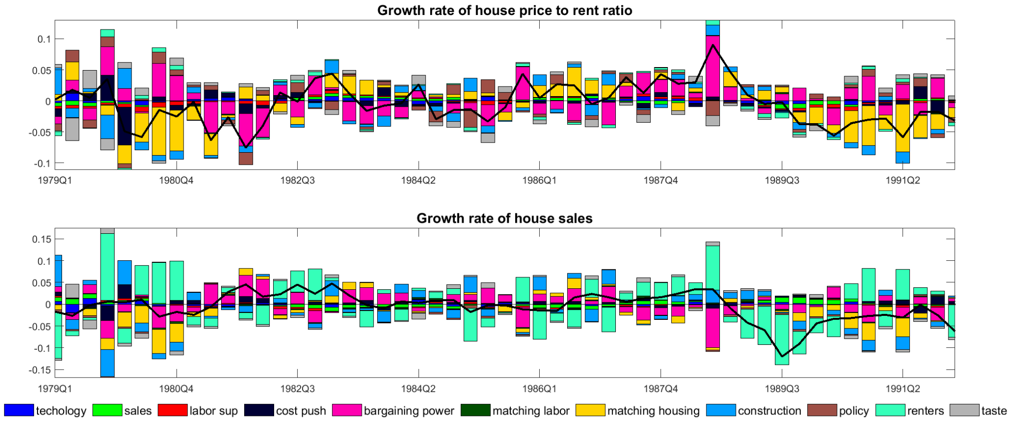


Figure 2.3: Historical decomposition of selected variables.

The bottom panel in Figure 2.3 shows that the main driver of the growth rate of house sales is the shock to renters. In this model, this shock captures the influx of first-time buyers. The figure shows a substantial effect of these shocks in the second half of 1979, which coincides with the start of the Right to Buy privatisation program. This shock also explains the collapse in sales in 1988-1991, consistent with prohibitive borrowing constraints that disproportionately affect first-time buyers.

2.4 Economic Implications

In this section, we consider the economic implications of four substantial changes in economic environment, that were introduced at the start of Mrs Thatcher's term in office. They are: A drastic reduction in housing construction and the large privatisation program; labour market reforms aimed at, amongst other things, reducing the power of trade unions[15]; and a change in core monetary policy objectives.

2.4.1 Fall in Housing Construction

Panel A in Figure 2.4 visualises the dynamics of housing construction in the 1980s. By 1982, three years into the new government, the growth rate of Local Authorities (LA) housing was less than a quarter of a percent, one-tenth the size of its rate in 1972. While there was some increase in private housing construction, the effect was too small to compensate for the loss of LA housing.

To understand the quantitative effects of this loss, we conduct a counterfactual experiment where we apply a positive construction technology shock to increase the level of productivity in the housing construction sector to compensate for the lost LA housing construction. Panel B plots the actual data and the counterfactual line, which maintains the annual growth rate of about 1.5% per annum, consistent with the average construction growth rate over the 1970s.

[15] For a different take on labour market reforms, related to the benefit system, see Zanetti (2016).

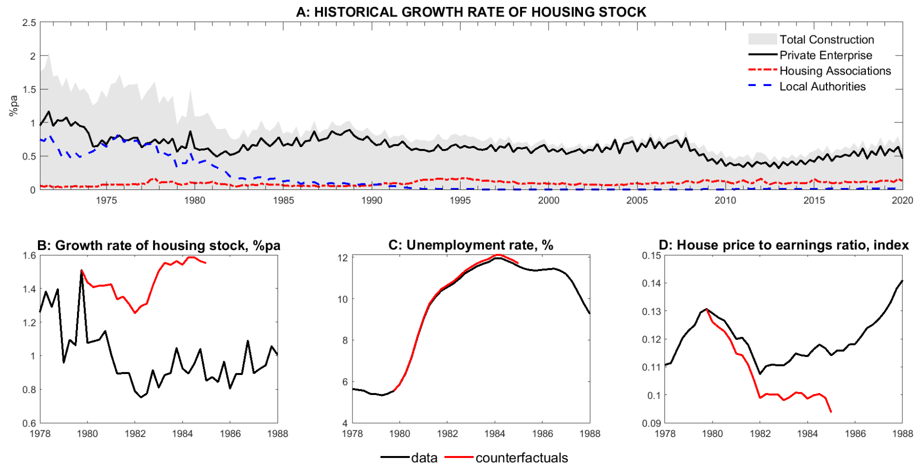


Figure 2.4: Collapse of Local Authorities Housing Construction

A greater supply of empty houses reduces housing market tightness. With spillover to the labour market, the labour market matching efficiency and the separation rate increase. A higher separation rate leads to more searching workers and, generally, a higher unemployment rate, despite an increase in the job finding rate. However, Panel C shows that the quantitative effect on unemployment is very small. A more direct effect on the housing market is, however, substantial: with a greater supply of houses, house prices go down, while real earnings are virtually unchanged. As a result, housing affordability improves (Panel D).

In summary, the counterfactuals suggest that if the construction of houses remained at about 0.4% per quarter for a period of 5 years, the price-to-earnings ratio would decrease by more than 10%, while the effect on unemployment would negligible.

Note that in this experiment, we treat social housing tenants as homeowners rather than permanent renters. This is because permanent renters in this model do not have any frictions in moving a house. Homeowners have frictions, and so do social tenants, perhaps at much greater extent. As we do not distinguish between private and social housing, we are likely to overstate the implied counterfactual effect on the labour market if the attachment to social housing imposes some restrictions on workers' mobility (Hughes and McCormick (1987)). This experiment quantifies the effect of a reduction in housing construction as a whole.

2.4.2 Right to Buy

The Right to Buy programme started in 1980, and the number of sales of Local Authorities houses peaked in 1982, with sales amounting to about 200 thousands dwellings. Panel A in Figure 2.5 shows that from 1979 to 1982, housing construction was shrinking, and the house price-to-earnings ratio was falling.

To understand the quantitative implications of this privatisation programme, we use a positive shock to the number of permanent renters, assuming the proportion of permanent renters in the economy does not fall as sharply, and the number of homeowners rises about 50% slower than what actually occurred (Panel B of Figure 2.5). The counterfactual increase in the homeownership rate is consistent with the increase observed from 1971 to 1979.

With a higher proportion of permanent renters, the number of homeowners would be lower, with lower activity in the housing market, and lower housing market tightness. The direct effect of a lower real house price is an improvement in housing affordability: the house price to real earnings ratio would fall. With a lower house price, housing construction would fall further, reducing the house price. However, the quantitative significance of this shock is not very large: house price falls about 3%, while the change in unemployment is very small.

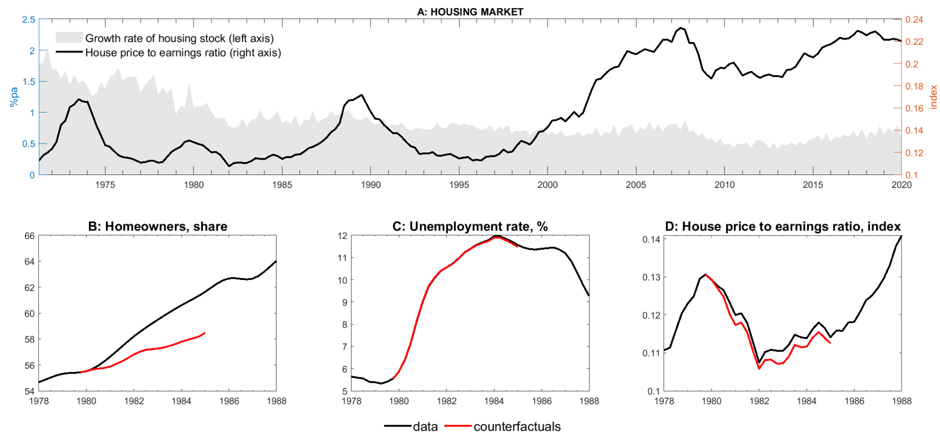


Figure 2.5: Right to Buy

2.4.3 Trade Unions

Panel B in Figure 2.1 shows that at the end of the 1970s, unemployment was relatively low, but went up steeply in the severe recession of 1980-81. By that time, trade union membership was at its peak and the number of labour disputes and consequent days lost was high, notably in 1978-79 during the 'Winter of Discontent', as workers striked to keep wage growth in line with high inflation (see Panel A in Figure 2.6).

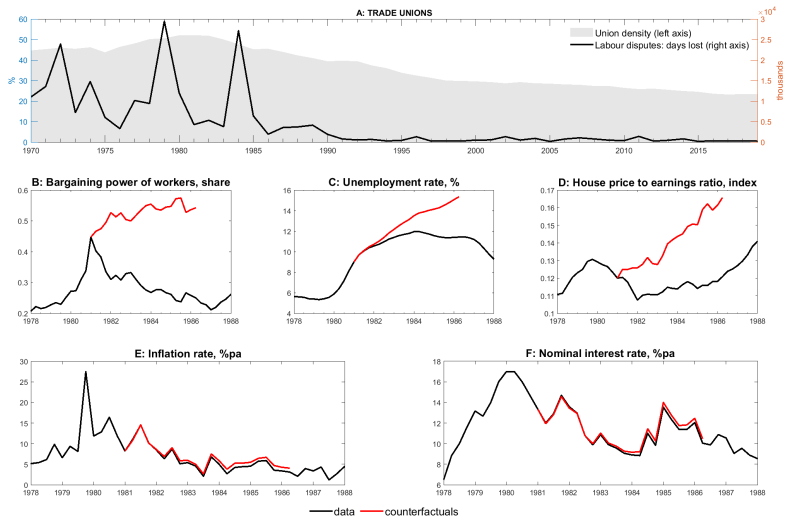


Figure 2.6: Trade Unions

Motivated by a desire to curb what was perceived as excessive union power, which had led to widespread strikes and economic disruptions during the 1970s, the Thatcher administration introduced a series of legislative and policy changes to reduce the power of trade unions. These included the Employment Acts of 1980, 1982, and 1988, the Trade Union Act of 1984, and the Public Order Act of 1986. While confrontations with trade unions were recurrent throughout the 1980s—with the Miners' Strike of 1984-85 a notable example—our estimations show that bargaining power declined post-1980 and remained relatively low for the rest of the administration's period in office. Specifically, Panel B plots the smoothed latent data on bargaining power that we recovered through Bayesian filtering.

To quantify the effect of these reforms, we assume that these policy changes are captured by a shock to workers' bargaining power, $\epsilon_{c,t}$, which is estimated to be negative during the period of interest. Therefore, in the following counterfactual experiment we set this shock to zero. This generates a higher level of bargaining power, consistent with its level at the end of the 1980, and maintains a slight upward trend as if it were following the pre-1980 pattern, see Panel B. A higher bargaining power would lead to higher wages and earnings of those employed in general industry, and to a higher unemployment rate – amounting to about 3 percentage points by the end of the five-year period, as shown in Panel C, and to a further reduction in labour market tightness. A spillover to the housing market would generate lower activity, with lower filling and finding rates, fewer matches and fewer separations. With a substantial reduction in matches and only a small reduction in the separation rate, the number of buyers would be higher, and housing market tightness would not be so subdued. House prices would fall by less, and housing affordability, as measured by the ratio of house price to earnings, would worsen, see Panel D.

2.4.4 Monetary Policy

Monetary policy gradually shifted from dovish in the pre-1980 period to hawkish in the post-ERM period, as illustrated in Figure 2.2. This prompts the question of what the economic impact would have been if monetary policy had adopted a more hawkish stance from the start of the 1980s, similar to the shift observed in the US. Many researchers identify a decisive shift in the US monetary policy framework around 1980 or shortly thereafter.[16]

Figure 2.7 plots the results of a counterfactual experiment in which we assume that monetary policy becomes 100% hawkish starting in 1981Q1 and remains so for the following five years. This figure indicates that inflation would decrease by 2-4 percentage points and, as a consequence, the interest rate would be reduced by about 2 percentage points. A lower interest rate would lead to higher house prices, but the quantitative effect is approximately 2%, which is not very significant.

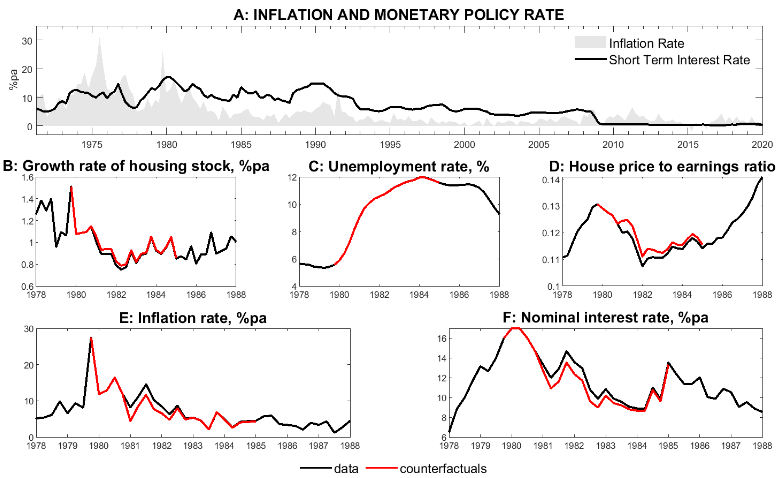


Figure 2.7: Effect of a switch to Hawkish policy

While monetary policy plays an important role in identifying shocks, it has only limited power in explaining the dynamics of house prices in this model. First, the demand effect of lower interest rates is mitigated by the assumption of risk sharing, which is a common feature of representative agent models estimated using time series. Second, the absence of financial frictions and borrowing constraints mitigates the effect of interest rates on first-time buyers. While we compensate for the absence of these features by assuming that housing market

[16]See for example, Goodfriend and King (2005).

activity can be affected by the real interest rate directly, this is not satisfactory. As discussed in Section 2.3.5, borrowing constraints are likely to remain unexplained as part of the housing matching efficiency shock and are not captured by monetary-policy-related factors.

2.5 Concluding Remarks

In a sequence of counterfactual experiments, we study the quantitative implications of three macroeconomic reforms in the 1980s. The model suggests that, during the Thatcher era, the spillovers between the housing and labour markets were significant. Thus, our results indicate that the decision not to continue building council houses impacted negatively on housing affordability but affected conditions in the labour market much less. The Right to Buy privatization programme similarly affected negatively housing affordability, but also resulted in a higher unemployment rate. On the other hand, labour market reforms, that resulted in lower bargaining power of trade unions, generated a significant reduction in the unemployment rate. The overall effect on the economy is plotted in Figure 2.8 and can be summarised as follows: lower unemployment by about one percentage point at the peak of unemployment in 1984, and a sustained reduction in housing affordability – about 10% throughout 1982-85, as measured by house price to earnings ratio.

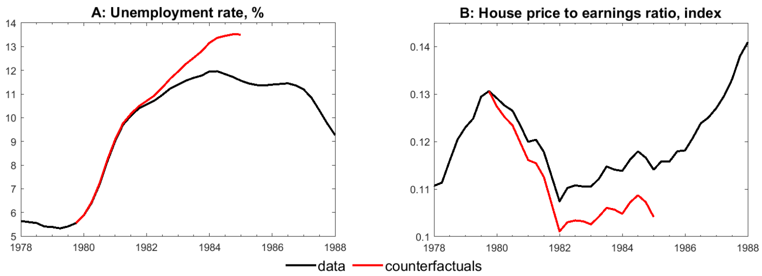


Figure 2.8: Combined effect of three reforms: housing, trade-unions, and privatisation

The model is simple and lacks important features both of the housing and labour markets. It also lacks important financial frictions in mortgage lending. For example, homes provide housing services in the current set-up but they do not serve as collateral for loans. And so, inevitably, this paper is a first pass at the issues it seeks to address. Nevertheless, the interaction of these markets looks to be qualitatively significant and worth pursing in richer environments.

Chapter 3

A More Mobile Labour Market

This chapter studies the role of labour market flexibility in the macroeconomy. Guided by a vast literature establishing that the United States has comparably higher rates of labour mobility than the United Kingdom, I use the US as an alternative against which to evaluate the UK economy. I estimate the model's structural parameters based on eleven timeseries for the two countries, showing that the higher level of labour mobility gives rise to a greater sensitivity in labour markets to changes in housing market dynamics. Based on these findings, I conduct a counterfactual experiment aimed at assessing the implications of a more flexible labour market in the stabilisation of output and employment in the United Kingdom during the early 1980's.

3.1 Introduction:

Tight labour markets pull workers in, loose labour markets push workers away. However, the ability and willingness of workers to move geographically differs significantly between economies. This chapter investigates how labour mobility influences the housing-labour market relationship. To answer this question, I estimate the spillover parameters in two economies shown to have meaningful differences in labour mobility, using their results to conduct a counterfactual simulation, asking the question: *What if Margaret Thatcher's interventions had resulted in greater labour mobility?* The results of this counter-factual is then used to quantify the implications of greater labour market mobility on the UK economy.

The results of the investigation shows that the United States labour market is more sensitive to changes in the housing market than that observed in the United Kingdom. With the spillover elasticity fundamentally capturing how changes to housing supply and demand affect the supply and demand for labour, I argue that this higher estimated sensitivity can be partially attributed to the US greater level of labour mobility. That is, tight housing markets have a stronger retarding effect on labour market relocation, than that observed in the United Kingdom, where workers have been more geographically rigid.

The finding adds to the empirical evidence of labour in the United States displaying comparably high levels of spatial mobility. Born out of Mundell (1961) discussion surrounding labour's role in absorbing region specific shocks, there is a rich literature establishing that labour mobility in the US exceeds that observed in Europe, and the link between mobility, convergence, and the efficient allocation of labour market resources (See for example, Blanchard and Katz (1992), Decressin and Fatas (1995), Beyer and Smets (2015)).

The role of labour mobility in ensuring an efficient allocation of labour has also been emphasized in UK specific studies (See for example, Langella and Manning (2022), or Postel-Vinay and Sepahsalari (2023)), who similarly show that mobility is linked to productivity, output and household income. Judge (2019) and Cominetti et al. (2022) examine the negative consequences arising from UK labour market rigidity, and discuss possible policy interventions. Building on this observation, I contribute to this debate through a counterfactual experiment aimed at understanding the consequences of the UK's higher level of labour market rigidity. Leveraging the insights gained from the parameter estimation, I impose a more flexible labour market on the UK, and examine the effects during the housing and labour market interventions of the early 1980's, as discussed in further detail in Chapter 2.

The structure of this chapter is as follows. Section 3.2 describes the structural model introduced in Chapter 1. Section 3.3 report the empirical and calibrated parameters for the UK and US economies. Based on these empirical parameters, steady state variables are computed and analysed. Section 3.4 describes the investigation strategy, compares the eleven time-series used for the estimation, before reporting, analysing and comparing the parameter estimates. The counter factual expriment is reported in section 3.5, 3.6 concludes.

3.2 Model

3.2.1 Setting and Population

Time is discrete and the economy is populated by a measure Q_t individual agents. The aggregate measure of population grows at the the constant and exogenous *net* rate μ:

$$Q_{t+1} = (1 + \mu)Q_t$$

All households are infinitely lived, discount time at a rate β_t, own firms, and consume consumables. Households own two types of labour. They supply labour to the the construction sector frictionlessly, and face search frictions when operating in general industry which produces output goods. All agents also require housing which they either rent or own. While the rental market clears in a Walrasian fashion in every period, households face search frictions when buying houses in the housing market. F_t agents are permanent renters who do not want to own housing and are inactive in the transaction housing market. B_t individuals are searching buyers who either have never owned a house, or who have already sold their previous house. Such buyers who are active in the search market and attempt to match with a house for sale and who rent in the Walrasian market until they match successfully. Finally, an aggregate measure N_t individuals are homeowners who may suffer a mismatch shock, in which case they list their home for sale and become searching buyers.

3.2.2 Households

The population is distributed amongst a constant number of households H. As the aggregate population grows between periods, these μQ_t new entrants are assigned to existing households s.t. each household grows by $\frac{\mu Q_t}{H}$ members per period. There is no mobility between the households once assigned. To avoid creating a high level of heterogeneity and to aid aggregation, we follow Merz (1995) in assuming that each household is made up of an extended family, where some proportion of the household is employed and earn a wage income, and some proportion is unemployed receiving an unemployment benefit. Similarly, some proportion of household members rent and and some own houses. Within the household, there exists perfect consumption risk sharing.

3.2.2.1 The individual Household

The individual household derive utility from consuming the habbit adjusted composite good (X_t), enjoying leisure hours $(1 - h_{c,t})$, and per period consumption of housing services (z_h). They derive disutility from providing labour to general industry ($L_{c,t}$) and to the construction sector ($L_{h,t}$). Their utility function takes the form:

$$U(X_t, \frac{L_{c,t}}{H}h_{c,t}, \frac{L_{h,t}}{H}, \frac{N_t}{H}) =$$
$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varrho_t \frac{X_{i,t}^{1-\sigma}}{1-\sigma} \frac{Q_t}{H} + \chi_{c,t} \frac{L_{c,t}}{H} \frac{(1-h_{i,c,t})^{1-\nu}-1}{1-\nu} - \chi_h L_{i,h,t} \frac{Q_t}{H} + \frac{N_t}{H} z^H \right] \tag{1.2}$$

Where: All variables with subscript "i" represent choice variables belonging to the individual household.

It is assumed that households habit adjusted bundle depend on external habits. That is, the consumption bundle is made up of the individual households choice of per period consumption ($c_{i,t}$), and the per-household measure of all households consumption in the previous period (C_{t-1}). That is:

$$X_{i,t} = \frac{c_{i,t}}{A_t} - \theta \frac{c_{t-1}}{A_{t-1}} \tag{1.3}$$

Where: A_t captures the trend level of aggregate productivity, and satisfies:

$$A_{t+1} = (1 + \varnothing_t)A_t \tag{1.4}$$

In (1.2), the first term – $\varrho_t \frac{X_{i,t}^{1-\sigma}}{1-\sigma} \frac{Q_t}{H}$ – represents the utility derived from consumption. Each individual member of the household consumes the composite good $X_{i,t}$ as described by (1.3). Since the consumption adjusted bundle $X_{i,t}$ represents the utility of an individual household member, we multiply the bundle by $\frac{Q_t}{H}$ to develop a representation of the consumption of the household.

The second term – $\chi_{c,t} \frac{L_{c,t}}{H} \frac{(1-h^i_{i,c,t})^{1-\nu}-1}{1-\nu}$ – represents the dis-utility of working in the general industry. $\chi_{c,t}$ is a smoothing parameter. $L_{c,t}$ is the aggregate number of people employed in the general industry of the economy, thus $\frac{L_{c,t}}{H}$ represent the proportion of employed members of the household. $h_{i,c,t}$ represent the number of labour hours provided by the individual worker, and ν represent elasticity of substitution with respect to leisure hours.

The third term – $\chi_h L_{h,t} \frac{Q_t}{H}$ – represent the dis-utility associated with providing labour hours to the construction sector. Since construction sector labour only depends on the intensive margin, there is no need to normalise $L_{h,t}$ by the number of households. The final term – $\frac{N_t}{H} z^H$ – is the utility value of home-ownership derived by the $\frac{N_t}{H}$ proportion of the household who are homeowners.

When firms and unemployed workers in the general industry match, they sign contracts where the real wage rate $w_{c,t}$ is obtained through a Nash surplus bargaining process. These employed individuals thus receive a labour income $h_{i,c,t}w_{c,t}$. Unemployed persons receive an unemployment benefit $b_{c,t}$ as a transfer from the government, which is financed through lump-sum taxes[1].

Households face a disaggregated budget constraint that states that all household members ($\frac{Q_t}{H}$) face costs associated with their expenditure on per-period consumption spending ($C_{i,t}$), lump sum taxes ($T_{i,t}$), any $\Omega_{i,t}$ housing costs, and, any savings in private bonds which can be brought forwards to the next period ($\mathcal{A}_{i,t+1}$). Household incomes are made up of unemployment benefits ($b_{c,t}$) derived by the $U_{c,t} = Q_t - L_{c,t}$ unemployed households, the returns on bonds brought forward from the previous period ($R_t\mathcal{A}_{i,t}$), any profits made from owning firms ($\Phi_{i,t}$), and labour incomes in the two sectors: $w_{c,t}h_{i,c,t}$ derived by the $L_{c,t}$ employed agents in the general industry, and $w_{h,t}$ incomes from the construction sector.

[1]See: 1.71

$$C_{i,t}\frac{Q_t}{H} + \frac{Q_t}{H}\mathcal{A}_{i,t+1} + \frac{Q_t}{H}T_{i,t} + \frac{Q_t}{H}\Omega_{i,t} = \frac{Q_t}{H}\Phi_{i,t} + \frac{L_{c,t}}{H}w_{c,t}h_{c,t} + \frac{U_{c,t}}{H}b_{c,t} + \frac{Q_t}{H}w_{h,t}L_{i,h,t} + \frac{Q_t}{H}R_t\mathcal{A}_{i,t} \tag{1.5}$$

Households housing costs are divided into the maintenance/taxation (m_t^h) costs paid by all matched homeowners, rental costs (r_t^h) paid by renters, and transaction costs in the housing market for sales and purchases. These transactions are summarized by the final two terms of expression (1.6) below. Because all agents who are active in the housing market transition between either being a matched homeowner, (N_t) or a searching buyer (B_t), any per-period adjustments in the stock of homeowners must reflect either a successful match, or a separation and sale. I.e: If a member of household "i" has successfully matched with a house previously owned by household "j", there has been a one unit increase in the level of home-ownership in household "i", and a one unit decrease in household "j". The transaction is then carried out at the equilibrium house price P_t^h, which is discussed in further detail in section 3.2.4.6. Due to the presence of perfect risk sharing within the household the, these housing costs can be collected together in the variable ($\Omega_{i,t}$):

$$\frac{Q_t}{H}\Omega_{i,t} = F_{i,t}r_t^h + N_{i,t}m_t^h + P_t^h(N_{i,t} - N_{i,t-1}) - P_t^h(N_{j,t} - N_{j,t-1}) \tag{1.6}$$

Where: Subscript $'i'$ denote variables belonging to household $'i'$, while subscript $'j'$ belongs to "other" households.

3.2.2.2 Individual Households Problem

The households problem is to maximise their utility function (1.2), subject to their habitual consumption bundle (1.3) and their budget constraint (1.5). They have choice variables of consumption, labour supply to the construction sector, and bond savings – $\{C_{i,t}, L_{i,h,t}, \mathcal{A}_{i,t+1}\}$ – and take: { r_t^h, m_t^h, $w_{c,t}$, $h_{c,t}$, $b_{c,t}$, $w_{h,t}$, Q_t, N_t, F_t, $L_{c,t}$, and H } as given. Thus, the households maximisation problem can be described:

$$\max_{\{X_{i,t},C_{i,t},L_{i,h,t},\mathcal{A}_{i,t}\}} U(\cdot) : \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varrho_t \frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H} + \chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h_{i,c,t}^{i})^{1-\nu}-1}{1-\nu} - \chi_h L_{i,h,t}\frac{Q_t}{H} + \frac{N_t}{H}z^H \right]$$

$$S.t : X_{i,t} = \frac{C_{i,t}}{A_t} - \theta\frac{C_{t-1}}{A_{t-1}}$$

$$And : \frac{Q_t}{H}C_{i,t} + \frac{Q_t}{H}\mathcal{A}_{i,t+1} + \frac{Q_t}{H}T_{i,t} + \frac{Q_t}{H}\Omega_{i,t} = = \frac{Q_t}{H}\Phi_{i,t} + \frac{L_{c,t}}{H}w_{c,t}h_{c,t} + \frac{U_{c,t}}{H}b_{c,t} + \frac{Q_t}{H}w_{h,t}L_{i,h,t} + \frac{Q_t}{H}R_t\mathcal{A}_{i,t}$$

Yielding the following F.O.C[2]:

$$\varrho_t X_{i,t}^{-\sigma} = \lambda_t A_t \tag{1.7}$$

$$\chi_h = \varrho_t X_{i,t}^{-\sigma}\frac{w_{h,t}}{A_t} \tag{1.8}$$

$$\varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t} = \beta\mathbb{E}_t\left[\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\right] \tag{1.9}$$

[2]See appendix: A:2 for derivations

3.2.2.3 Aggregate Household

Consider the optimal choice of the individual household. Since all households are homogeneous, they all make the same optimal choices $\{X_{i,t}, C_{i,t}, L_{i,h,t}, \mathcal{A}_{i,t+1}\}_{t=0}^{\infty}$, thus we can express the aggregate budget constraint by multiplying by the number of households (H):

$$C_{i,t}Q_t + Q_t\mathcal{A}_{i,t+1} + Q_tT_{i,t} + Q_t\Omega_{i,t} = Q_t\Phi_{i,t} + L_{c,t}w_{c,t}h_{c,t}U_{c,t}b_{c,t} + Q_tw_{h,t}L_{i,h,t} + Q_tR_t\mathcal{A}_{i,t}$$

And defining the aggregate measure as the realisation of individual variables multiplied by population:

$$C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = \Phi_t + L_{c,t}w_{c,t}h_{c,t}U_{c,t}b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \tag{1.5}$$

Where: The level of housing investment undertaken by households can be expressed as:

$$\Omega_t = N_tm_t^h + \beta\mathbf{E}_t\left[(H_{t+1} - H_t)(\frac{\lambda_{t+1}}{\lambda_t}\hat{\mathcal{V}}_{t+1})\right] \tag{1.11}$$

And: $\beta\mathbf{E}_t\{(H_{t+1} - H_t)(\frac{\lambda_{t+1}}{\lambda_t}\tilde{\mathcal{V}}_{t+1})\}$ represent the discounted value of transacting for newly constructed houses.

Where: the aggregation of the individual households first order conditions yield[3]:

$$A_t\lambda_t = \varrho_t(\frac{X_t}{Q_t})^{-\sigma} \tag{1.12}$$

$$\chi_h = \varrho_t(\frac{X_t}{Q_t})^{-\sigma}\frac{w_{h,t}}{A_t}$$

And where the aggregate consumption bundle is defined as:

$$\frac{X_t}{Q_t} = \frac{C_t}{Q_tA_t} - \theta\frac{C_{t-1}}{A_{t-1}Q_{t-1}} \tag{1.15}$$

3.2.3 Labour Market

3.2.3.1 General Industry

Suppose there exists a continuum of final good producers who operate in perfect competition and frictionlessly package intermediary goods into final goods production – Y_t. There also exists $j \in J$ normalised by measure 1 intermediary firms in the general industry. Such intermediary firms firms produce an intermediary good – $y_t(j)$ – by hiring workers through a costly process of search and matching based on Diamond, Mortensen and Pissarides workhouse model summarised in Pissarides (2000). Once matched, a firm-worker pair negotiate wages and hours through a Nash bargaining process, and a match is destroyed by a stochastic process. Final goods firms sell their output to households, and intermediary producers sell their goods in a monopolistically competitive market.

3.2.3.2 Matching Technology

Suppose that the probability that a worker matches with a vacant job is as typically in the literature dependant on the aggregate number of unemployed workers and vacancies in the economy. Let the total number of matches in the economy be captured by the matching function $M_{c,t}(V_{c,t}, U_{c,t})$, which depend on the level of matching efficiency ($\kappa_{c,t}$), and on $U_{c,t}$ and $V_{c,t}$ – the total number of unemployed workers and vacancies respectively. That is:

$$M_{c,t}(U_{c,t}, V_{c,t}) = \kappa_{c,t}U_{c,t}^{\delta_{c,t}}V_{c,t}^{1-\delta_{c,t}}$$

[3] See appendix: A:2 for derivations

Where: $V_{c,t} = \int_0^1 V_t(j) d\varsigma$ aggregation over all firms, indexed by ς.

Let the labor market tightness be denoted as:

$$\omega_{c,t} \equiv \frac{V_{c,t}}{U_{c,t}} \tag{1.17}$$

We can then express the job filling rate (matching probability for the firm) as $\gamma_{c,t}$[4], ensuring that a tighter labour market reduces the matching probability of firms:

$$\gamma_{c,t} \equiv \frac{M_{c,t}(U_{c,t}, V_{c,t})}{V_{c,t}} = \kappa_{c,t}\omega_{c,t}^{-\delta_{c,t}} \tag{1.18}$$

And the job finding rate of the unemployed (matching probability for unemployed) as $\lambda_{c,t}$[5], ensuring that a tighter labour market raises the matching probability of workers:

$$\lambda_{c,t} \equiv \frac{M_{c,t}(U_{c,t}, V_{c,t})}{U_{c,t}} = \gamma_{c,t}\omega_{c,t} = \kappa_{c,t}\omega_{c,t}^{1-\delta_{c,t}} \tag{1.19}$$

3.2.3.3 Transition Probabilities and Laws of Motion in the Labour Market

At the beginning of period "t", there are $L_{c,t-1}(j)$ employed persons in firm-worker matches carried over from period "$t-1$" within firm "j". During period "t", these existing matches suffers separations by probability: $\vartheta_{c,t} \in (0,1)$, such that $(1-\vartheta_{c,t})L_{c,t-1}(j)$ matches survive. The firm also advertise vacancies, of which which $\gamma_{c,t}V_{c,t}(j)$ are filled within period "t". The number of employed persons at the end of period "t" is thus:

$$L_{c,t}(j) = (1-\vartheta_{c,t})L_{c,t-1}(j) + \gamma_{c,t}V_{c,t}(j) \tag{1.20}$$

It is assumed that the job filling rate is the same for all firms. Aggregating the above equation, we can express the total number of employed households $L_{c,t} = \int_0^1 L_{c,t}(j), dj$, and the total number of vacancies $V_{c,t} = \int_0^1 V_{c,t}(j), dj$ in the economy. That is:

$$L_{c,t} = (1-\vartheta_{c,t})L_{c,t-1} + \gamma_{c,t}V_{c,t} \tag{1.21}$$

Which has stationary representation[6]:

$$\frac{L_{c,t}}{Q_t} = (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \gamma_{c,t}\frac{V_{c,t}}{Q_t} \tag{3.4}$$

At the beginning of period "t", the total number of unemployed households is defined as being equal to the labour force less those who are not employed at the end of period "$t-1$". During period "t", $\vartheta_t L_{c,t-1}$ persons become unemployed. They are then added to the stock of unemployed, and immediately start searching for a new job. That is: $U_{c,t} = Q_t - L_{c,t-1} + \vartheta_t L_{c,t-1}$. Hence, we can define the aggregate number of unemployed persons in the economy at the end of period "t":

$$U_{c,t} = Q_t - (1-\vartheta_{c,t})L_{c,t-1} \tag{1.22}$$

Which has stationary representation[7]:

$$\frac{U_{c,t}}{Q_t} = 1 - (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \tag{3.5}$$

[4] See appendix: A:2
[5] See appendix: A:2
[6] See appendix: A:2
[7] See appendix: A:2

The timing described above can be summarised by Figure 3.1, below:

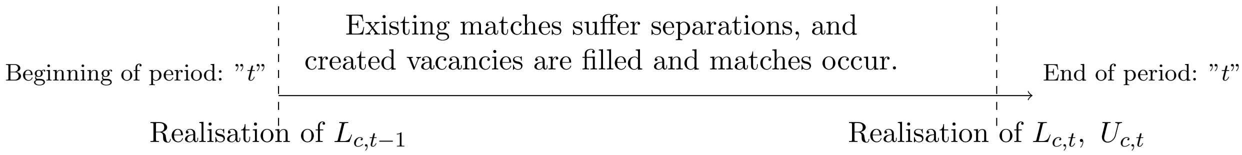


Figure 3.1: Timing of events in the labour market.

3.2.3.4 Final Good Producers

Following Dixit and Stiglitz (1977), there exists a continuum of perfectly competitive final goods producing firms which purchase intermediary inputs and aggregates them according to the production technology described by (1.23), which they then sell to households.

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.23}$$

Where: Y_t and $y_t(j)$ denotes the aggregate and individual output respectively. $\epsilon > 1$ is the stochastic elasticity of substitution between intermediary input goods.

Such final good producing firms face a maximisation problem where they decide how many final goods to produce, and how many intermediary goods to purchase – $\{Y_t, y_t(j)\}$ – taking prices $\{P_t, p_t(j)\}$ as given in order to maximise profits:

$$\Pi_t = P_t Y_t - \int_0^1 p_t(j) y_t(j) \, dj \tag{1.24}$$

Where: P_t and $p_t(j)$ denotes the aggregate and individual price level respectively.

Yielding the aggregate demand curve for intermediary goods[8]:

$$y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \tag{1.25}$$

Where: $\varepsilon > 1$ is the elasticity of substitution between goods.

And path for the aggregate price level[9]:

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} \, dj \right]^{\frac{1}{1-\varepsilon}} \tag{1.26}$$

3.2.3.5 Intermediary Good Producers

In the intermediary goods producing industry, firm "j" operates the following technology:

$$y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j) \tag{1.27}$$

Where: $Y_t(j)$ is firm "j" output, z_t is a stationary productivity shock, A_t is an aggregate productivity shock, $L_{c,t}(j)$ is the number of workers hired by firm "j", and $h_{c,t}(j)$ is the number of hours provided by labour to firm "j".

These firms face a cost of ι when posting vacancies, and must compensate labour at a wage rate $w_{c,t}$ determined through Nash surplus bargaining. The individual firms profit function is thus:

$$\Phi_t(j) = \frac{p_t(j)}{P_t} y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \tag{1.28}$$

[8] See appendix: A:2 for derivations

[9] See appendix: A:2 for derivations

Intermediary good producers set prices according to Calvo (1983), and keep prices in each period with probability $\varsigma \in [0,1]$ such that $(1-\varsigma)$ firms set prices optimally each period.

Hence, intermediary firms solve a profit maximisation problem described by (1.28) where they must choose the price of their goods, level of labour to hire, the number of vacancies to post – $\{p_t(j), L_{c,t}(j), V_{c,t}(j)\}$ – subject to the law of motion for employment (1.20), their production function (1.27), and their demand curve (1.25). They take $\{P_t,\ w_{c,t},\ Y_t\}_{t=0}^{\infty}$ as given. That is:

$$\max_{\{p_t(j),L_{c,t}(j),V_{c,t}(j)\}} \Phi_t(j) : \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta\varsigma)^t \frac{p_t(j)}{P_t} y_t(j) - w_{c,t}h_{c,t}(j)L_{c,t}(j) - \iota V_{c,t}(j)\right]$$
$$S.t : L_{c,t}(j) = (1-\vartheta_{c,t})L_{c,t-1}(j) + \gamma_{c,t}V_{c,t}(j)$$
$$And : Y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j)$$
$$And : Y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$

The first order conditions of the firms optimisation yields[10]:

$$\eta_t = \frac{\iota}{\gamma_{c,t}} \tag{1.29}$$

$$\eta_t = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\eta_{t+1}\right](1-\vartheta_{c,t}) \tag{1.30}$$

$$\frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t})\frac{\iota}{\gamma_{c,t+1}}\right] \tag{1.31}$$

$$(\pi_t+1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1-(1-\varsigma)\left(\frac{\epsilon}{(\epsilon-1)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\epsilon}\right) \tag{1.32}$$

Where:

$$K_{1,t} = w_{c,t}\frac{Y_t}{z_t A_t} + \varsigma\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon+1}K_{1,t+1}\right] \tag{1.33}$$

$$K_{2,t} = Y_t + \varsigma\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon}K_{2,t+1}\right] \tag{1.34}$$

Where (1.29) is the job-posting condition, and states that firm "j" will post vacancies so long as the value of filling a position is greater or equal to the cost of posting a vacancy: $\eta_t\gamma_{c,t} \geq \iota$. (1.30) is the job-creation condition and states that firm "j" will create jobs so long the value of an unfilled vacancy is equal to the current period profit generated from filling a vacancy, and the discounted expected future value of the vacancy in the next period. (1.31) is a combination of (1.29) and (1.30), and relates the job-posting to job-creation in equilibrium. ξ_t is the Lagrange multiplier associated with the demand curve (1.25), while η_t is the Lagrange multiplier attached to the law of motion for employment (1.20). (1.32) is the Phillips curve and describes the path of path of inflation.

The aggregation of (1.27) and (1.28) for all $j \in J$ yields:

$$Y_t = \frac{z_t A_t}{\Delta_t} L_{c,t} h_{c,t} \tag{1.35}$$

$$\Phi_t = Y_t - w_{c,t}h_{c,t}L_{c,t} - \iota V_{c,t} \tag{1.36}$$

Where, $\Delta_t = (1-\varsigma)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \varsigma\pi_t^{\varepsilon}\cdot\Delta_{t-1}$ is the price dispersion in the economy. Because the system is log-linearised around its deterministic steady state where: $\pi = 1, \Delta_t = 1$, the price dispersion doesn't introduce additional first-order dynamics outside of those described in 1.35.

[10] See appendix: A:2 for derivations

3.2.3.6 Value Functions in the General Industry

3.2.3.6.1 Workers Value Functions Let V_t^E and V_t^U represent the value of employment/unemployment respectively to a worker. If workers are employed in the general industry they receive labour income, and enjoy $(1-h_{c,t})$ leisure hours which provide utility at a rate: $\chi_{c,t}\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}$. We can then express the value to the household of employment as:

$$V_t^E = h_{c,t}w_{c,t} + \frac{\chi_{c,t}}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right) \tag{1.37}$$
$$+\beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\vartheta_{c,t+1}(1-\lambda_{c,t+1}))V_{t+1}^E + \vartheta_{c,t+1}(1-\lambda_{c,t+1})V_{t+1}^U\right]\right]$$

Where, the terms inside the expectation is a composite describing the expected pay-off of being either employed or unemployed in the next period. With probability $(1-\vartheta_{c,t+1})$ an employed person does not suffer a separation and remains employed in the next period. Of those $\vartheta_{c,t+1}$ that become unemployed, they find a new job within the same period according to the job finding probability: $\lambda_{c,t+1}$. Alternatively, $\vartheta_{c,t+1}(1-\lambda_{c,t+1})$ become unemployed and are unable to find a new job in the next period.

Next, consider the value of unemployment. These workers earn no labour income, but receive the unemployment benefit: $b_{c,t}$. These unemployed households remain unemployed in the next period with a probability: $1-\lambda_{c,t+1}$, and become employed by the job finding probability: $\lambda_{c,t+1}$. Thus, the value to the household of unemployment is:

$$V_t^U = b_{c,t} + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\lambda_{c,t+1})V_{t+1}^U + \lambda_{c,t+1}V_{t+1}^E\right]\right] \tag{1.38}$$

We can then express the worker's surplus from becoming employed ($V_t^W = V_t^E - V_t^U$):

$$\Rightarrow V_t^W = h_{c,t}w_{c,t} - b_{c,t} + \frac{\chi_{c,t}}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1}))V_{t+1}^W\right] \tag{1.39}$$

3.2.3.6.2 Firms Value Functions Let V_t^F and V_t^V represent the value of a match/vacancy to the firm. In each period, firms derive revenues from producing output, and has labour costs $h_{c,t}w_{c,t}$ per worker: $V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t})$. In period "$t+1$", the match between the firm and the worker survive to the next period by probability: $(1-\vartheta_{c,t+1})$, and workers and firms are separated from the match by probability: $\vartheta_{c,t+1}$. The present value of a filled job to a firm is thus a combination of this net-revenue and the future expected stream of revenue from the match. That is:

$$V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t}) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}[(1-\vartheta_{c,t+1})V_{t+1}^F + \vartheta_{c,t}V_{t+1}^V]\right]$$

If a separation occurs, the probability of filling the vacancy is $\gamma_{c,t}$, and cause the firm to incur a cost ι of posting the vacancy. By (1.29), firms post vacancies so long as the value of employing a worker to the firm equals the cost of recruitment: $\gamma_{c,t}V_t^F = \iota$. Recognising that the value of a filled vacancy is equal to the R.H.S. of (1.30) with equilibrium described by (1.31), we have: $V_t^F \equiv \eta_t = \frac{\iota}{\gamma_{c,t}}$. Free entry of firms implies that the value of unfilled vacancy will be driven to zero $V_t^V \equiv 0, \forall t$. In other words, the model assumes that the same position cannot be readvertised and refilled, and the match between a particular firm and a particular worker is destroyed with probability ϑ. Thus:

$$V_t^F = h_{c,t}(z_t\xi_tA_t - w_{c,t}) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right] \tag{1.40}$$

3.2.3.7 Bargaining Problem and Wage Equation

When firms and workers meet to negotiate wages, they divide the total surplus from the match according to a Nash bargaining process where we assume that worker's share of the joint surplus is given by $\epsilon_{c,t}$. Following the efficiency proof established by Hosios (1990), we set the steady state value of $\epsilon_{c,t}$ equal to the matching elasticity in the labour market (δ_c). Total surplus of a match is the sum of the value of the match to the firm and worker, that is: $V^T_{c,t} = V^W_{c,t} + V^F_{c,t}$. Then, maximizing the Nash product of the match requires solving the unconstrained maximisation problem:

$$\max_{V_t^F, V_t^W} : (V_t^W)^{\epsilon_{c,t}} (V_t^F)^{1-\epsilon_{c,t}} - \varphi_t (V_t^W + V_t^F - V_{c,t}^T)$$

F.O.C:

$$\frac{\partial L}{\partial V_t^W} \equiv 0 = \epsilon_{c,t} (V_t^W)^{\epsilon_{c,t}-1} (V_t^F)^{1-\epsilon_{c,t}} - \varphi_t$$

$$\frac{\partial L}{\partial V_t^J} \equiv 0 = (1-\epsilon_{c,t}) (V_t^W)^{\epsilon_{c,t}} (V_t^F)^{-\epsilon_{c,t}} - \varphi_t$$

Which implies[11]:

$$h_{c,t} w_{c,t} = \epsilon_{c,t} \left[h_{c,t} \xi_t z_t A_t + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1 - \vartheta_{c,t+1}) \left(1 - (1 - \lambda_{c,t+1}) \frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t+1})\epsilon_{c,t}} \right) \frac{\iota}{\gamma_{c,t+1}} \right] \right] + (1-\epsilon_{c,t}) \left[b_{c,t} - \frac{\chi_{c,t}}{\lambda_t} \frac{(1-h_{c,t})^{1-\nu} - 1}{1-\nu} \right] \quad (1.41)$$

Where (1.41) determines the wage as a weighted average between the marginal revenue product of the worker plus the cost of replacing the worker, and the outside option of the worker.

3.2.3.8 Hours Worked

Finally to close the labour market, firms and workers determine how many hours to supply/hire based on an optimisation problem aimed at maximising the joint surplus of the match:

$$\begin{aligned}
\max_{\{h_{c,t}\}} (V_t^T) &= \max_{\{h_{c,t}\}} \left(V_t^W + V_t^{Firm} \right) \\
&= \max_{\{h_{c,t}\}} \Bigg[h_{c,t} w_{c,t} - b_{c,t} + \frac{\chi_{c,t}}{\lambda_t} \left(\frac{(1-h_{c,t})^{1-\nu} - 1}{(1-\nu)} \right) \\
&+ \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\vartheta_{c,t+1})(1-\lambda_{c,t+1}) V_{t+1}^W \right] \\
&+ h_{c,t}(z_t \xi_t A_t - w_{c,t}) + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (1-\vartheta_{c,t+1}) \frac{\iota}{\gamma_{c,t+1}} \right] \Bigg]
\end{aligned}$$

First order conditions:

$$0 = w_{c,t} - \frac{\chi_{c,t}}{\lambda_t} (1-h_{c,t})^{-\nu} + z_t \xi_t A_t - w_{c,t}$$

From where we can express optimal labour hours:

$$z_t \xi_t A_t = \frac{\chi_{c,t}}{\lambda_t} (1-h_{c,t})^{-\nu} \quad (1.42)$$

[11] See appendix: A:2 for derivations

3.2.4 Housing Sector

In the housing sector there is a stock of houses (H_t) which grows endogenously through housing construction described in section: 3.2.4.4. Households require housing in every period, which they either rent or own. To buy a house, searching buyers engage in a costly process of search and matching similar to the friction described in section 3.2.3. Rental contracts are either short- of long-term in nature. Permanent renters have no interest in owning housing and thus opt for long-term contracts, while searching buyers are only renting temporarily and thus opt for short-term contracts.

3.2.4.1 Housing Stock

At time t the city has stock of housing in the economy (H_t) which is either vacant and listed for sale ($V_{h,t}$), or occupied by one of the economies Q_t agents. Thus, total housing stock can be defined:

$$H_t = Q_t + V_{h,t} \tag{1.43}$$

Where: these $V_{h,t}$ vacant houses are made up of a combination of newly constructed houses by property developers, and houses which have been listed for sale by mismatched homeowners.

Where the stationary representation of (1.43) is:

$$\frac{H_t}{Q_t} = 1 + \frac{V_{h,t}}{Q_t} \tag{3.6}$$

3.2.4.2 Matching Technology

Matching in the housing market is determined by the matching function $M_{h,t}$, which depends on the matching technology ($\kappa_{h,t}$), and the number of searching buyers (B_t) and houses for sale (S_t).

$$M_{h,t}(B_t, S_t) = \kappa_{h,t} B_t^{\delta_h} S_t^{1-\delta_h} \tag{1.44}$$

Let market tightness, which acts as our proxy for housing market liquidity be defined as the number of searching buyers divided by the number of houses for sale. That is:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} \tag{1.45}$$

Let the house filling rate (matching probability for houses for sale) be denoted by $\gamma_{h,t}$ and defined[12]:

$$\gamma_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{S_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h} \tag{1.46}$$

And, let the house finding rate (matching probability of a searching buyer) be denoted by $\lambda_{h,t}$ and be defined[13]:

$$\lambda_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{B_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h - 1} \tag{1.47}$$

3.2.4.3 Transition Probabilities and Laws of Motion in the Housing Market

3.2.4.3.1 Home Owners At time t, there are N_t home-owning households. These homeowners become mismatched with their house at an exogenous probability $\vartheta_{h,t} \in (0, 1)$. If mismatched, homeowners become unhappy with their home and no longer receive the utility value of home-ownership (z^H). As a consequence, these households seek to sell their house which gets added to the stock of houses for sale – S_t, and become searching buyers – B_t – trying to match with a new house. Their law of motion is thus number of homeowners

[12] See appendix section: A:2

[13] See appendix section: A:2

who did not become mis-matched in the previous period – $(1-\vartheta_{h,t})N_{t-1}$, and those $\lambda_{h,t}B_t$ searching buyers who successfully matched with a house in the current period:

$$N_t = (1-\vartheta_{h,t})N_{t-1} + \lambda_{h,t}B_t \tag{1.48}$$

With stationary representation:

$$\frac{N_t}{Q_t} = (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \lambda_{h,t}\frac{B_t}{Q_t} \tag{3.7}$$

3.2.4.3.2 Permanent Renters Permanent renters never change type, and have no interest in owning houses. By (1.1), each period the population grows by a rate μ. Let ψ_t represent the proportion of the μQ_t people who act as searching buyers, while $1-\psi_t$ be the proportion that acts as perpetual renters. The stock evolution of perpetual renters is then given by:

$$F_t = F_{t-1} + (1-\psi_{t-1})\mu Q_{t-1} \tag{1.49}$$

With stationary representation:

$$\frac{F_t}{Q_t} = \frac{F_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + (1-\psi_{t-1})\mu\frac{Q_{t-1}}{Q_t} \tag{3.8}$$

3.2.4.3.3 Searching Buyers Each period, a proportion – $\vartheta_{h,t}N_t$ – homeowners become mismatched and transition into being searching buyers. Next, recall that by (**??**), the population is equal to the stock of renters, buyers and homeowners. The stock of searching buyers is thus given by the difference between per-period population, and per-period renters and homeowners. That is:

$$B_t = Q_t - F_t - (1-\vartheta_{h,t})N_{t-1} \tag{1.50}$$

With stationary representation:

$$\frac{B_t}{Q_t} = 1 - \frac{F_t}{Q_t} - (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \tag{3.9}$$

3.2.4.3.4 Houses for Sale The number of houses for sale are those vacant housing units – $V_{h,t}$ – which are currently unoccupied. There are also "chains", which are those housing units which are listed for sale but still occupied while the seller attempts to find match with a buyer – $C_{h,t}$. This is a feature of multiple housing markets, notably prevalent in the UK[14], where a sequence of houses listed for sale are dependant both upon the buyers receiving the money from selling their houses and on the sellers successfully buying the houses that they intend to move into[15]. The shock thus mimics an implicit form of *on the "job" search* similarly to the process described in Pissarides (2000).

The stock of houses for sale is thus:

$$S_t = V_{h,t} + C_{h,t} \tag{1.51}$$

It is assumed that some proportion – τ – of houses for sale are in chains, that is:

$$C_{h,t} = \tau S_t \tag{1.52}$$

Combining (1.45) with (1.51), and (1.52) the housing market tightness must satisfy[16]:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} = \frac{(1-\tau)B_t}{H_t - Q_t} \tag{1.53}$$

[14]See: Office of Fair Trading (2010)

[15]I.e: In a four-household chain, A buys B's house, B uses the money from that sale to buy C's house, and C uses the money from that sale to buy D's house

[16]See appendix section: A:2

3.2.4.4 Housing Construction

In the construction sector, there are three key stocks: undeveloped land (K_t^L), developed land ($\hat{H}_t$), and constructed housing (H_t). All undeveloped land is owned by the government, which releases it for development to firms in the construction sector. These firms operate under perfect competition and undertake both the development of land and the construction of new housing. To produce housing, firms combine developed land with construction labour ($L_{h,t}$) using a simple technology. They solve a cost minimisation problem dependant these two cost factors:

$$H_{t+1} - H_t = \min\left(\hat{H}_{t+1} - \hat{H}_t,\ \phi_t L_{h,t}\right) \tag{1.54}$$

Where: ϕ_t denotes the productivity of construction labour.

Solving the minimisation implies that land is developed until the cost of development equals the cost of employing labour. That is:

$$\hat{H}_{t+1} - \hat{H}_t = \phi_t L_{h,t} \tag{1.55}$$

Unable to store developed land and with free entry into the construction sector, firms construct new houses so long as profitable. They therefore build houses on any developed parcel of land. That is:

$$H_{t+1} - H_t = \phi_t L_{h,t} \tag{1.56}$$

With stationary representation:

$$\frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \phi_t \frac{L_{h,t}}{Q_t} \tag{3.10}$$

Undeveloped land (K_t^L) is released exogenously at the rate $\varkappa$, intended to capture the rate at which the government releases land for development. Thus, the exogenous law of motion for land satisfies:

$$K_{t+1}^L = (1+\varkappa)K_t^L \tag{1.57}$$

Such undeveloped land can be sold to construction sector firms at a price $q_{h,t}$. Prior to making the purchase, the firm can costlessly evaluate the development costs associated with the parcel. Reflecting that different parcels of land require different levels of development with different levels of associated costs, these costs are assumed to be heterogenous and draw from the distribution

$$c \sim A(c), \quad c \in [\underline{c}, \overline{c}]$$

With free entry, profits are driven to zero until all units of land with development costs $c \leq q_{h,t}$ are used for construction, ensuring that land development is increasing in $q_{h,t}$. With the level of undeveloped land being the difference between total available land and developed land – $K_t^L - \hat{H}_t$, the quantity of land converted satisfies:

$$\hat{H}_{t+1} - \hat{H}_t = \Lambda\left(\frac{q_{h,t}}{A_t}\right)(K_t^L - \hat{H}_t) \tag{1.58}$$

With stationary representation:

$$\frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \Lambda\left(\frac{q_{h,t}}{A_t}\right)(\frac{K_t^L}{Q_t} - \frac{H_t}{Q_t}) \tag{3.11}$$

Where, $\Lambda\left(\frac{q_{h,t}}{A_t}\right)$ is the reduced form land conversion function defined: $\Lambda\left(\frac{q_{h,t}}{A_t}\right)^{\varpi}$, with $0 < \varpi < 1$.

Consider the profit earned by firms operating in the construction sector. To construct a new house, house builders face aggregate labour costs – $L_{h,t}w_{h,t}$ – and buy a unit of developed land for $q_{h,t}$. They earn revenues by selling newly constructed houses. Once a new house is built, it is listed for sale at the option price – $\hat{V}_{t+1}$. Their profit function is thus: the difference between the revenues earned from house selling and the cost of land acquisition for the $H^H_{t+1} - H^H_t$ units of houses constructed, and the aggregate costs of hiring labour:

$$\Pi_{const,t} = (H_{t+1} - H_t)\left[\beta\mathbb{E}_t\left[(\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1})\right] - q_{h,t}\right] - w_{h,t}L_{h,t} \equiv 0 \tag{1.59}$$

Where: $\hat{V}_{t+1}$ is the value function associated with a vacant house not yet listed either for sale or for rent, and can be thought of as the option price of a vacant house. Requiring the profit function to equal zero follows from both developers and house builders operating in perfect competition.

With stationary representation:

$$\frac{w_{h,t}}{\phi_t A_t} = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\frac{V_{t+1}}{A_{t+1}}\right] - \frac{q_{h,t}}{A_t} \tag{2.6}$$

3.2.4.5 Value Functions in the Housing Market

3.2.4.5.1 Perpetual renters Renters are never interested in buying a house, and thus never transition. Their value function thus only depends on cost of their long-term rental contracts and their continuation value:

$$V^F_t = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}V^F_{t+1}\right] - r^h_t \tag{1.61}$$

With stationary representation:

$$\frac{V^F_t}{A_t} = \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\frac{V^F_{t+1}}{A_t+1}\right] - \frac{r^h_t}{A_t} \tag{3.12}$$

3.2.4.5.2 Homeowners Each period, homeowners homeowners pay maintenance/housing tax costs – m^h_t, and receive the utility value of homeownership – z^H – normalised by the Lagrange multiplier associated with households maximisation – λ_t. If they suffer a separation shock, they list their house for sale and receive its expected value – $\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}$. If a searching buyer matches with a house for sale, they pay the house price – $\frac{\lambda_{t+1}}{\lambda_t}P^h_{t+1}$.

$$\begin{aligned} V^N_t = &- m^h_t + \frac{z^H}{\lambda_t} + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\vartheta_{h,t}(1-\lambda_{h,t+1}))V^N_{t+1}\right.\right. \\ &\left.\left. + \vartheta_{h,t}(\hat{V}_{t+1} - \lambda_{h,t+1}P^h_{t+1}) + \vartheta_{h,t}(1-\lambda_{h,t+1})V^B_{t+1}\right]\right] \end{aligned} \tag{1.62}$$

Where: The first term captures that each period, homeowners pay maintenance/housing tax costs – m^h_t. They receive the normalised utility value $\frac{z^H}{\lambda_t}$. The third term is a composite capturing their continuation values. The first part of the composite term captures that with probability $\vartheta_{h,t}$ homeowners suffer a separation, of which $\lambda_{h,t+1}$ match with a new house in the next period. Thus the stock of households who start at homeowners in period "t" and remain as homeowners at the end of period "$t+1$" is: $(1-\vartheta_{h,t}(1-\lambda_{h,t})$, these households then receive the next next period value of homeownership – V^N_{t+1}. The second part captures that of those who suffer the mismatch shock in period "t", all receive the value of a vacant house, and that $\lambda_{h,t+1}$ of them purchase a new house in the period immediately following. The third term captures that with probability: $\vartheta_{h,t}(1-\lambda_{h,t})$, mismatched households do not match with a new house, and thus transition into searching buyers.

The stationary representation of (1.62) is:

$$\begin{aligned} \frac{V^N_t}{A_t} = &- \frac{m^h_t}{A_t} + \frac{z^H}{\lambda_t A_t} + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\left[(1-\vartheta_{h,t}(1-\lambda_{h,t+1}))\frac{V^N_{t+1}}{A_{t+1}}\right.\right. \\ &\left.\left. + \vartheta_{h,t}\left(\frac{V_{t+1}}{A_{t+1}} - \lambda_{h,t+1}\frac{P^h_{t+1}}{A_{t+1}}\right) + \vartheta_{h,t}(1-\lambda_{h,t+1})\frac{V^B_{t+1}}{A_{t+1}}\right]\right] \end{aligned} \tag{3.13}$$

3.2.4.5.3 Searching Buyers In period "t", searching buyers rent through short-term contracts, while searching for a house and pay housing rent – r_t^{h*}. With probability: $\lambda_{h,t+1}$ they match with a house for sale in the next period, and pay the transaction price – $\frac{\lambda_{t+1}}{\lambda_t} P_{t+1}^h$. Their value function is thus:

$$V_t^B = -r_t^{h*} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[(1-\lambda_{h,t+1}) V_{t+1}^B + \lambda_{h,t+1} (V_{t+1}^N - P_{t+1}^h) \right] \right] \tag{1.63}$$

Where the first term is their rental costs, and the second term is their composite continuation value. The first part of the continuation value captures that with probability: $(1-\lambda_{h,t+1})$ they fail to match with a house for sale and continue to search in future periods. The second part captures that with probability: $\lambda_{h,t+1}$ they successfully match with a house, taking on the value function of a homeowners – V_{t+1}^h and pay the transaction price – P_{t+1}^h.

The stationary representation of (1.63) is:

$$\frac{V_t^B}{A_t} = -\frac{r_t^{h*}}{A_t} + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[(1-\lambda_{h,t+1}) \frac{V_{t+1}^B}{A_{t+1}} + \lambda_{h,t+1} \left(\frac{V_{t+1}^N}{A_{t+1}} - \frac{P_{t+1}^h}{A_{t+1}} \right) \right] \right] \tag{3.14}$$

3.2.4.5.4 Value of a vacant house At the beginning of each period, an unoccupied house, can either be put on the short-term rental market, or listed for sale. Such vacant houses move costlessly between sale and rental markets. Homeowners wish to maximise profits, so they solve the maximisation problem:

$$\hat{V}_t = \max[r_t^{h*} - m_t^h + \beta E_t \{ \frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \}, V_t] \tag{1.64}$$

Where: $\hat{V}_t$ is the value of a vacant house. The first argument describes the value of a house listed on the rental market. V_t is the value of house designated for sale. As houses move frictionlessly between the two markets, it follows that:

$$\hat{V}_t = r_t^{h*} - m_t^h + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right] = V_t \tag{1.65}$$

Note that: this is an equilibrium condition - homeowners sell house until they are indifferent between the sale price and rental return, and where rental prices (r_t^{h*}) adjusts to maintain the equality above.

Where (1.65) has stationary representation:

$$\frac{V_t}{A_t} = \frac{r_t^{h*}}{A_t} - \frac{m^h}{A_t} + \beta E_t \left[\frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}}{A_{t+1}} \right] \tag{3.15}$$

A vacant house on the for sale market matches with a searching buyer in the next period with probability: $\gamma_{h,t+1}$, and transacts at equilibrium housing price – P^h_{t+1}. If no match occurs, the house is valued at the future options value – $\hat{V}_{t+1}$. Thus, the value of a house to a seller, V_t, satisfies:

$$V_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left[\gamma_{h,t+1} P^h_{t+1} + (1 - \gamma_{h,t+1}) \hat{V}_{t+1} \right] \right] \tag{1.66}$$

Where: The house transacting price (P^h_t) is determined through Nash bargaining[17].

Where the stationary representation of (1.66) is:

$$\frac{V_t}{A_t} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[\gamma_{h,t+1} \frac{P^h_{t+1}}{A_{t+1}} + (1 - \gamma_{h,t+1}) \frac{V_{t+1}}{A_{t+1}} \right] \right] \tag{3.16}$$

3.2.4.6 Bargaining Problem, House Price Equation and Rents

3.2.4.6.1 House Prices When a searching buyer meets with a vacant house for sale they determine the transaction price by engaging in Nash bargaining over the total surplus of a match, similarly to how wages are determined in the general industry[18].

Let V^{Buy}_t be the value of successfully buying a house. This value is given by the value of becoming a homeowner – V^N_t – less the cost of purchasing the house – P^h_t – and the value of continuing as a searching buyer – V^B_t.

$$V^{Buy}_t = (V^N_t - P^h_t) - V^B_t \tag{2.7}$$

Let V^{Sell}_t be value of a match to the seller, and be the difference between the value of being a homeowner, and becoming a searching buyer after a sale. That is:

$$V^{Sell}_t = (V^B_t + V_t) - V^N_t \tag{1.68}$$

Let $V^T_{h,t} = V^{Sell}_t + V^{Buy}_t$.

Maximising the Nash product gives rise to the bargaining problem:

$$\max_{V^{Sell}_t, V^{Buy}_t} : \quad (V^{Sell}_t)^{\epsilon'_{h,t}} (V^{Buy}_t)^{1-\epsilon'_{h,t}}$$
$$S.t. : \quad V^T_{h,t} = V^{Sell}_t + V^{Buy}_t$$

Which yields the familiar sharing rule for house prices[19]:

$$P^h_t = (1 - \epsilon_{h,t})(V^N_t - V^B_t) + \epsilon_{h,t} V_t \tag{1.69}$$

Where: $\epsilon_{h,t}$ denote the bargaining power of the buyer.
And (1.69) has stationary representation:

$$\frac{P^h_t}{A_t} = (1 - \epsilon_{h,t}) \left(\frac{V^N_t}{A_t} - \frac{V^B_t}{A_t} \right) + \epsilon_{h,t} \frac{V_t}{A_t} \tag{3.17}$$

[17]See section 3.2.4.6
[18]See section: 3.2.3.7
[19]See appendix section: A:2

3.2.4.6.2 Rent Prices

Long-term rental contracts are assumed to be related to the short-term rate according to:

$$r_t^h = \upsilon r_{t-1}^h + (1-\upsilon)\, r_t^{h*}. \tag{1.70}$$

When $\upsilon = 0$ both rates are the market-determined optimising flexible rate. When υ is large, while the small proportion of the market is able to adjust the price when the new rental contract is written, the substantial proportion of the market participants face a highly persistent rent that reacts slowly to the market rent hikes and falls. This type of rent dynamics can work as a rough description of a rent control policy where landlords are not able to change the in-contract price, which mostly affects long-term (permanent) renters.

3.2.5 Policy

3.2.5.1 Fiscal Policy

There exists a government whose sole purpose is to finance the unemployment benefit b_t paid to all $U_{c,t}$ unemployed persons. This government raises money through charging lump-sum taxes T_t on all households, levying property taxes/charging maintenance costs m_t^h on all N_t units of owned housing, and from proceeds arising from selling $(K_t^L - \hat{H}_t)$ units of land at the price $(q_{h,t})$. The governments budget constraint is thus:

$$b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(K_t^L - \hat{H}_t)$$

Using the fact that all are either employed or unemployed, we can substitute: $U_{c,t} = Q_t - L_{c,t}$ to express the number of unemployed persons. Further, using that the construction sectors zero profit condition (1.59) implies that all developed units are used for construction, the governments budget constraint becomes:

$$\Rightarrow b_{c,t}U_{c,t} = T_t + N_t m_t^h + q_{h,t}(H_{t+1} - H_t) \tag{1.71}$$

Where: the level of lump sum taxes (T_t) adjust to maintain the equality. The total value of land sales $(q_{h,t}(H_{t+1} - H_t))$ is determined endogenously, while the values of land taxes (m_t^h) and unemployment benefits $(b_{c,t})$ reflect observed data ratios described in section 3.3.

3.2.5.2 Monetary Policy

The monetary authority sets interest rates by considering deviations of interest rates and inflation, and the growth rate of output. That is, they follow the Taylor type rule described below:

$$\frac{1+i_t}{1+i_{ss}} = \left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\alpha_i} \left(\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\alpha_\pi} \left(\frac{Y_t}{(1+\mu)(1+\gamma)Y_{t-1}}\right)^{\alpha_y} \right)^{1-\alpha_i} e^{m_t} \tag{1.72}$$

Where: i_{ss} and π_{ss} are steady state values

The monetary authority can follow one of two regimes – Hawkish (H) and dovish (D) – which governs how strongly they respond to inflationary deviations captured through parameter α_π such that $\alpha_\pi^H > \alpha_\pi^D$. The switches of the economy between these two regimes are governed by two-regime Markov chain $\upsilon_M \in \{H, D\}$ with transition matrix

$$T^M = \begin{bmatrix} 1-p_{HD} & p_{HD} \\ p_{DH} & 1-p_{DH} \end{bmatrix}$$

Where: $p_{ij} = P(\upsilon_{M,t+1} = j | \upsilon_{M,t} = i)$.

The aggregate change in prices in the economy is defined:

3.2.6 Spillovers

To capture the spill-overs that occur between housing and labour markets the model allows for for the state of one market to affect the matching and separation probabilities of the other. While the two effects move in opposite directions, the matching effect dominates throughout giving rise to *net* gains or losses to employment/home-ownership in response to adjustments in the other market.

3.2.6.1 Housing Market Spillovers

In the real world, people may separate from their current job due to a number of reasons, both voluntary and involuntary. Following such a separation job seekers may choose to relocate either within a local area or a more broadly to be close available job opportunities. The "looser"[20] the housing market tightness – $\omega_{h,t} \equiv \frac{B_t}{S_t}$ – the easier it is for such unemployed workers to make the relocation both in terms of house finding probabilities and prices. It is therefore assumed that the matching efficiency in the labour market which governs the ability of unemployed workers to match with vacant jobs to be decreasing with housing market tightness:

$$\kappa_{c,t} = \tilde{\kappa}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\kappa} \tag{1.73}$$

Conversely, if the housing market is very loose, currently employed workers may recognise that the barrier of relocation is weak/small, and may respond by quitting/separating more frequently.

$$\vartheta_{c,t} = \tilde{\vartheta}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\vartheta} \tag{1.74}$$

Where: In this chapter, $\tilde{\kappa}_{c,t}$ is treated as a stochastic process, evolving through an exogenous shock that captures regime-dependent variation in labour market matching efficiency. This allows the model to reflect shifts in institutional flexibility or structural frictions across regimes. By contrast, $\tilde{\vartheta}_{c,t}$ remains deterministic and is calibrated from steady-state separation dynamics. The elasticities ζ_κ and ζ_ϑ determine the sensitivity of labour market frictions to housing market tightness, forming part of the regime-switching spillover mechanisms examined in this chapter.

3.2.6.2 Labour and Monetary Spillovers

While financial constraints are not explicitly modelled in the economy, financial constraints are a key restriction on households ability to purchase homes. Thus, and noting the procyclicality of both housing and labour markets[21],[22], the decision to move house will be influenced by the ability to find employment in the new area. With greater market tightness in the labour market – $\omega_{c,t} \equiv \frac{V_{c,t}}{U_{c,t}}$ – the easier it is to find a job for an unemployed worker, and by the wage bargaining solution (1.41), the higher the real wage earned by workers. In a similar fashion to in the labour market, the matching efficiency in the housing market is positively related to the the relative market tightness in the labour market, and negatively related to the real interest rate:

$$\kappa_{h,t} = \tilde{\kappa}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\kappa} \left(\frac{1 + i_t}{1 + \pi_{t+1}} R \right)^{-\theta_\kappa} \tag{1.75}$$

The inclusion of real interest rates are motivated by the fact that while housing wealth represent the majority of households assets, it also represents the largest liability faced by households[23] with the cost of financing directly influenced by the prevailing real interest rate

[20] That is, the lower the ratio of searching buyers to houses for sale

[21] For housing, see for example Piazzesi and Schneider (2016)

[22] For labour, see for example Ashenfelter and Card (2011)

[23] See for example Causa et al. (2019)

set by the monetary authority. Thus, its inclusion captures the transmission mechanism of monetary policy onto the housing estimated in Iacoviello and Neri (2010) now seminal paper.

As in the labour market, where a tighter housing market acted as a barrier to labour market transitions, affecting both matching and separation rates, housing market separation also responds to both interest rates and labour market conditions. A tighter labour market and lower interest rates facilitates moves, speeding up matches as described above, and separations by:

$$\vartheta_{h,t} = \tilde{\vartheta}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\vartheta} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{\theta_\vartheta} \tag{1.76}$$

Where, R is the steady state value of the real rate of interest. As in Chapter 2, $\tilde{\kappa}_{h,t}$ is treated as a stochastic process, evolving with its own exogenous shock in order to capture time-varying frictions in the housing market. In contrast, $\tilde{\vartheta}_{h,t}$ remains deterministic and calibrated from steady-state separation behaviour. The elasticities η_κ and η_ϑ determine the responsiveness of housing market matching and separation rates to labour market tightness and interest rate fluctuations, shaping the transmission of monetary policy across regimes.

3.2.7 Equilibrium

All consumption undertaken by households are produced endogenously in the economy as described in section 3.2.3.4. Combining the households budget constraint (1.5) with the aggregated profit function (1.36) of intermediary producers, the governments fiscal policy (1.71), the level of housing investment undertaken by households is given by (1.11), the construction sectors labour demand equation (1.60), and the law of motion for housing construction (1.56), we can express the aggregate resource constraint[24]:

$$Y_t = C_t + \iota V_{c,t} \tag{1.77}$$

Where: (1.77) implies that output can be converted costlessly into consumption goods and has stationary representation:

$$\frac{Y_t}{Q_t A_t} = \frac{C_t}{Q_t A_t} + \frac{\iota v_{c,t}}{Q_t A_t} \tag{3.18}$$

3.3 Calibration

3.3.1 Stationarity

The model applied in this chapter retains the same structure and trend properties as described in Chapters 1 and 2, featuring deterministic growth in both productivity and population. To enable a meaningful steady-state representation and ensure compatibility with the estimation framework, the system is transformed into a stationary form by removing these trends. This transformation allows all macroeconomic variables to be interpreted as deviations from a balanced growth path. To maintain consistency with this model specification, the observed data for both the UK and the US are similarly transformed into stationary counterparts. Level variables are converted into growth rates using log differences and demeaned by subtracting their sample mean, while ratio-based variables are differenced and demeaned in the same fashion. Hours worked are normalised relative to a 40-hour work week, and inflation and nominal interest rates are used in levels without demeaning. These transformations are applied consistently across countries to ensure comparability, as detailed in Appendix A:1.

[24] See appendix: A:2

3.3.2 Empirical Parameters:

Figure 3.1 reports the empirical and calibrated parameters imposed on the model described in section 3.2. The empirical values reported relate to the long run ("steady-state") ratio's observed in data for the period of investigation[25]. These ratio's are used to compute steady-state variables and non-calibrated parameters. presented in sections 3.3.3 and 3.3.4. For information about sources and computation, please refer to the appendix section A:1.

Parameter	Meaning	Value (US)	Value (UK)
μ	Trend population growth rate	0.00356	0.0025
$1+\pi$	Steady state level of inflation, quarterly	0.0026	0.0123
R	Real interest rate, quarterly	0.00389	0.0045
l_c	Employment rate	0.94	0.93
h_c	Hours worked	0.4358	0.4101
$\frac{l_h}{l_c}$	Ratio of Construction to General Employment	0.05	0.036
$\frac{w_h}{h_c w_c}$	Wage in construction to general ratio	1.41	1.18
$\frac{b_c}{h_c w_c}$	Unemployment benefit to earnings ratio	0.44	0.39
$\frac{\iota}{h_c w_c}$	Vacancy posting to earnings ratio	0.15	0.5
$\frac{1}{\lambda_c^w}$	Unemployment duration	18 weeks	40 weeks
γ_c^d	Daily job filling rate	0.05 days	0.05 days
$\frac{h}{Q}$	Housing stock to occupied housing ratio	1.13	1.03
$\frac{b+f}{Q}$	Rent to occupied house ratio	0.34	0.34
ϑ_h	Average time between house moves, years	11.9 years	13.4 years
$\frac{1}{\lambda_h^w}$	House finding duration	24 weeks	20 weeks
$\frac{r^h}{h_c w_c}$	Rent to earnings ratio	0.275	0.356
$\frac{p^h}{h_c w_c}$	House price to earnings ratio	16.91	24.03

Table 3.1: Empirical Parameter Values in the US and the UK Economies

The growth rate of population (μ), real rate of interest ($\frac{1+r_t}{\beta}$), rate of inflation ($1+\pi$), and the rate of employment (l_c) is set to match the mean of the reported data variable. The intensive margin of labour ($h_{c,t}$) is expressed as a fraction of a 40 hour work week. The ratio of construction sector to consumption sector labour is obtained by calculating the ratio of all employees in the construction sector to the number of total employed persons in the two economies.

The ratio of earnings in the construction to consumption sector is calculated as the aggregate income in the two sectors ($\frac{w_h}{h_c w_c}$), where the notation account for the fact that the consumption sector differentiates between the intensive and extensive margin, while the construction sector only adjusts across the extensive margin. In the US, we calculate aggregate earnings as the product of average hourly earnings and average weekly hours in the two sectors, while in the UK we use measures from the Office for National Statistics on average weekly earnings.

The ratio of benefits to earnings, ($\frac{b_c}{h_c w_c}$) is taken from the OECD. The provided measure is the long run mean observation from 2001 to 2020, expressed as the proportion of benefits received by a single person without children after five months of unemployment who previously received the average wage. Other benefits such as social payments or housing benefits are excluded. The results indicate that the US net replacement rate is higher than that observed in the UK is however senstive to the choice of including benefits in addition to the direct wage insurance payment, or the period of unemployment[26].

[25] 1965Q2-2020Q1 for the US, and 1971Q2-2020Q1 for the UK

[26] See for example Whiteford (2022) showing how the UK unemployment benefit is is one of the lowest in the OECD if taken by itself after a month period of unemployment, but one of the highest after 5 years of unemployment

While the aggregate income ($h_c w_c$) can be obtained through the sources discussed in A:1, there exists no good, published, quantified measure of the cost of posting vacancies that are comparable in the two countries. This ratio is thus calibrated and uninformed. The motivation to set UK posting costs higher than US posting costs is informed by the fact that the UK labour market suffers from stricter regulation than that in the US, as measured by the OECD's Strictness of employment protection measure[27]. While this measure does not fully isolate the cost of hiring from flow and firing costs, the imperfect proxy still acts to inform the calibration of the variable.

In parameterising the transition probabilities in the two economies, we follow Pissarides (2000) and define the average unemployment duration as one over the job-finding rate ($\frac{1}{\lambda_c}$). With the data measured in weeks, we use this measure in our parameterisation, and then compute the quarterly job finding rate as: $\lambda_c = 1 - (1 - \lambda_c^w)^{52/4}$.

Following Davis, Faberman, et al. (2013), who shows that treating monthly job openings and hiring flows as outcomes of a daily processes helps address issues relating to time aggregation biases. Specifically, as many vacancies are filled within less than one month, aggregation at a monthly frequency will not account for vacancies that a posted and filled within the reference period will be unrecorded in vacancy stocks, causing an underestimation of vacancy durations. These issues would be even more pronounced in our model, as the data is taken at a quarterly frequency. They report a mean daily job filling rate of $\approx 5.2\%$, and we use this parameter for both the US and UK economy. We then translate the parameter into a quarterly measure through: $\gamma_c = 1 - (1 - \gamma_c^d)^{365/4}$.

We parameterise the housing stock relative to the stock of occupied housing ($\frac{h}{Q}$) and rent to occupied housing ration ($\frac{b+f}{Q}$) from the the two economies respective housing surveys. For the UK data, some of the UK national countries (specifically Northern Ireland and Scotland) suffer from discontinuities in their timeseries, as such, the data from the English housing survey is used as a proxy for the United Kingdom as a whole.

For estimates on the average time between house moves, I have to rely on industry data provided, but not published by major real estate firms developers. However, these estimates are sensitive data availability, with alternative estimates estimating average separation rates both higher and lower than those reported in table 3.1. Similarly, we also depend on industry data as there are no academic or governmental sources on the average completion time for housing transactions. Using industry data, we say that in the UK, the average period for a housing transaction is 20 weeks, while for the US the estimate is somewhat higher at 24 weeks. We translate the monthly measure into quarterly data through the transformation: $\lambda_h = 1 - (1 - \lambda_h^w)^{52/4}$.

For house prices, we use the estimate provided by the Land Registries hedonic model for the UK housing market. For estimates on rental costs, we produce a combined measure of rental costs taken from the OECD which provides a price-to-rent index, and translate the index values to nominal prices through observations on average let agreed prices for a reference period. In the US, we use the quarterly estimates for median house and rental prices prices provided by the U.S. Census Bureau, Housing Vacancy and Homeownership, table: 11A/B. To express the ratio of prices/rental costs to incomes, we use the same data on aggregate income discussed above in relation to the labour market.

[27]See: Organization for Economic Co-operation

3.3.3 Calibrated Parameters:

Table 3.2 reports calibrated parameters for the US and UK parameterised economies. The trend growth rate of technology ϕ is set to zero to to allow the discount rate (β) to be established directly from the steady state level of interest. The habit persistence parameter associated with the consumption bundle is set to a reasonable 0.8, the same level reported in Christiano, Eichenbaum, and Evans (2005), and instead use the inter-temporal elasticity of substitution (σ) to alter consumption dynamics between the two countries. We set the elasticity of labour supply is set to capture the greater level of rigidity in the UK labour market, resulting in a smaller variation in the intensive margin or labour ($h_{c,t}$) in response to changes to incomes. The elasticity of substitution between goods (ϵ) which determines the level of mark-ups and price stickiness is set to reflect an assumed greater level of price competition in the US consumption sector than the UK consumption sector.

Parameter	Meaning	Value (US)	Value (UK)
$\varnothing$	Trend productivity growth rate	0.000	0.000
σ	Inter-temporal elasticity of substitution	3.0	2.0
θ	Habit persistence parameter	0.8	0.8
ν	Elasticity of labour supply	3.0	2.0
ε	Elasticity of substitution between intermediary goods	11.0	6.0
δ_c	Matching elasticity in labour market	0.7	0.7
δ_h	Matching elasticity in housing market	0.7	0.7
Λ	Production function parameter	0.025	0.025
s_s^s	Production function parameter	0.5	0.5
υ	Rent control parameter	0.0	0.0
ς	Calvo Parameter	0.8	0.8

Table 3.2: Combined Calibrated Parameters for US and UK

As the level of matching efficiency in the housing (κ_h) and labour (κ_c) market is estimated directly using Bayesian methods, the other parameters associated with the matching functions (1.16) and (1.44) must be calibrated. We thus set all matching elasticity parameters (δ_c, δ_h) to 0.7. Notice that that the bargaining power for buyers in the housing market (ϵ_h) is set to 0.5, assuming an even split of the surplus between buyers and sellers. With $\delta_h \neq \epsilon_h$, this parameterisation violates Hosios (1990) efficiency condition for the housing market. This choice is motivated by an assumption that the decentralised nature of the the housing market, the heterogeneity of the housing market, and the non-modeled financial constraints on transactions, are all captured by the searching and matching frictions. Because the bargaining power of workers (ϵ_c) is estimated, the labour market also violates the efficiency condition.

Λ and s_s^s are parameters associated with the production function of land conversion $\Lambda(q_{h,t})$, which has the functional form: $\Lambda(q_{h,t})^{s_s^s}$. The scaling parameter, Λ, and the elasticity parameter, s_s^s, are set to reflect the assumption that land is scarce, motivated by the observation that the ratio of construction to stock of houses for the reference period is only 0.3 percent[28]. Finally, as the focus of the current study is to understand the spill-overs between the two markets, the parameter governing rental contracts is set to zero, so rental contracts are fully flexible and optimized in each period.

[28]See A:1

3.3.4 Computed steady-state parameters

Based on the systems steady state relationship[29], the empirical and calibrated parameter values reported in tables 3.1 and 3.2, the model's remaining structural parameters and steady state variables: $\{\beta, \lambda_c, \gamma_c, \xi, u_c, \vartheta_c, \omega_c, w_c, y, c, x, \lambda, \chi_c, \chi_h, \epsilon_c, w_h, z^h, q_h, k, r^h, v_c, s, n, b, f, \omega_h, \lambda_h, \gamma_h, \kappa_h, \phi, v, m^h, v^B, v^N\}$, are reported in table: 3.3[30]. While many of the steady-state variables have limited direct economic interpretation, some interesting results emerge when comparing the two countries, hinting at greater labour market flexibility, and a more active housing market in the US than in the UK.

SS Variable (Symbol)	Value (US)	Value (UK)	SS Variable (Symbol)	Value (US)	Value (UK)
Discount Rate (β)	0.9961	0.9955	Number of Homeowners (n)	0.6785	0.6746
Finding rate (labour) (λ_c)	0.5243	0.2805	Number of searching buyers (b)	0.0435	0.0300
Filling rate (labour) (γ_c)	0.9907	0.9907	Number of permanent renters (f)	0.2965	0.3100
Mark-up (ξ)	0.9091	0.8333	Housing market tightness (ω_h)	0.3014	0.6010
Unemployment rate ($1 - l_c$)	0.06	0.07	Housing market finding rate (λ_h)	0.4249	0.4867
Employment rate (l_c)	0.94	0.93	Share of searching workers (u_c)	0.1261	0.0973
Separation rate (labour) (ϑ_c)	0.0671	0.0269	Housing market filling rate (γ_h)	0.1281	0.2925
Labour market tightness (ω_c)	0.5293	0.2831	Matching efficiency (Housing) (κ_h)	0.2965	0.4177
Wages (consumption sector) (w_c)	0.8990	0.8249	Construction sector productivity (ϕ)	0.1189	0.0769
Output (y)	0.4097	0.3814	Vacancy Value (Housing) (v)	6.2607	7.9395
Period Consumption (c)	0.4058	0.3786	Cost of housing maintenance/tax (m^h)	0.2008	0.2386
Habitual Consumption (x)	0.0812	0.0757	Value of being a searching buyer (v^B)	-12.5755	-29.1357
Multiplier on households (λ)	1867.8	174.4295	Value of homeownership (v^N)	-5.5845	-20.8170
Dis-utility (Consumption sector) (χ_c)	305.4169	50.5849	Utility Value of Homeownership (z^h)	323.6985	17.3952
Dis-utility (Construction sector) (χ_h)	1048.4	69.6287	Land for construction (q_h)	1.2613	2.6943
Bargaining power of workers (ϵ_c)	0.5733	0.1323	Undeveloped land (k)	1.5009	1.0927
Wages (Construction sector) (w_h)	0.5603	0.3992	Rent (r^h)	0.2472	0.2937
Unemployment benefit (b_c)	0.1724	0.1319	Vacancies (v_c)	0.0318	0.0198
Cost of posting vacancy (ι)	0.0588	0.1015	Houses for Sale (s)	0.1300	0.0300
Matching efficiency (Labour) (κ_c)	0.6346	0.4095	House Price (p^h)	6.6259	8.1291
Labour supply (Construction) (l_h)	0.0338	0.0335			

Table 3.3: Steady-State Variables

[29]See section: C

[30]Computations are in the appendix, sections B and C

First, the computation of the discount rate (β) reflects the higher long-run observed real interest rate (R) in the UK compared to the US. Similarly, the steady-state finding rate (λ_c) and separation probability (ϑ_c) are estimated to be about 80% and 300% higher in the US than in the UK, reflecting the greater flexibility of the US labour market.

This higher dynamism leads to greater turnover and higher frictional unemployment in the US, consistent with previous studies[31]. Consequently, the steady-state unemployment rate is higher in the US than the UK. This result, however, contrasts with the empirically calibrated long-run employment rates of 94% in the US and 93% in the UK[32]. This discrepancy highlights the influence of observed data in calibrating long-run targets versus the dynamics of the theoretical model. While the steady state units of wages, consumption, output, are higher in the US than the UK, reflecting the higher per-capital earnings, output, and consumption levels observed in the two countries' data.

Comparing the steady state values of the two economies housing markets, many results have little economic meaning, or follows uninterestingly from empirical parametrization – for example the higher observed wage in the construction sector observed in the US. I note that while the steady state estimates for home-ownership (n) is very similar, there level of housing demand (b) is computed to be higher in the US. While the similarity between homeownership is somewhat surprising given the observed behaviour discussed in section 3.3.2, the higher level of housing demand observed in the US – and subsequent lower renting demand – can be seen as reflecting the change observed in UK housing tenure due to the Right to Buy policy discussed in Chapter 2, whereby policy was used to facilitate households transition from subsidized renting to home-ownership.

[31] See for example: Davis and Haltiwanger (1999), who in the handbook for labour economics notes that for the 1980's-90's, the US labour market both has lower unemployment, but greater dynamism and thus had more job destruction and matching than in the UK.

[32] See section: 3.3

3.4 Bayesian Estimation

3.4.1 State Space Representation:

Let $\mathcal{P}$ represent the entire set of parameters. The system described in section 3.2, summarised in the appendix, section A:4, is inherently non-stationary. To ensure tractability and comparability, the system is transformed into a stationary representation by controlling for the growth rates of trend technology (A_t), population (Q_t), and land (K_t^L). The derivations for this transformation are detailed in Appendix B.

The calibration process begins with the empirical parameters described in Section 3.3, which are stored in the set $\mathcal{P}_1 = \{\mu, \pi, R, l_c, h_c, \frac{l_h}{l_c}, \frac{w_h}{h_c w_c}, \frac{b_c}{h_c w_c}, \frac{\iota}{h_c w_c}, \frac{1}{\lambda_c^w}, \gamma_c^d, \frac{h}{Q}, \frac{b+f}{Q}, \vartheta_h, \frac{1}{\lambda_h^w}, \frac{r^h}{h_c w_c}, \frac{p^h}{h_c w_c}\}$.

Next, the calibrated parameters discussed in section: 3.3.3 are stored in the set $\mathcal{P}_2 = \{\gamma, \sigma, \theta, \nu, \epsilon, \delta_c, \delta_h, \Lambda, s_s^s, \upsilon, \varsigma\}$. Using these parameters, the steady-state relationships detailed in Appendix 3.3.4 allow us to compute the remaining steady-state parameters, stored in the set $\mathcal{P}_3 = \{\beta, \lambda_c, \gamma_c, \xi, u_c, \vartheta_c, \omega_c, w_c, y, c, x, \lambda, \chi_c, \chi_h, \epsilon_c, w_h, z^h, q_h, k, r^h, v_c, s, n, b, f, \omega_h, \lambda_h, \gamma_h, \kappa_h, \phi, v, m^h, v^B, v^N\}$.

The final block of parameters is estimated using Bayesian techniques, implemented with the RISE toolbox[33]. These include the structural parameters—spillover elasticities and the monetary policy coefficients associated with the Taylor rule. These estimated parameters are stored in $\mathcal{P}_{4,p} = \{\zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta, \theta_\kappa, \theta_\vartheta, \alpha_y, \alpha_i, \alpha_\pi\}$, and the shock parameters stored in $\mathcal{P}_{4,s} = \{\rho^z, \sigma^z, \rho^\rho, \sigma^\rho, \rho^\phi, \sigma^\phi, \rho^{\chi_c}, \sigma^{\chi_c}, \rho^{\kappa_c}, \sigma^{\kappa_c}, \rho^{\epsilon_c}, \sigma^{\epsilon_c}, \rho^{\kappa_h}, \sigma^{\kappa_h}, \rho^\psi, \sigma^\psi, \rho^{\tau_h}, \sigma^{\tau_h}, \rho^\varepsilon, \sigma^\varepsilon\}$.

The system is then represented in state-space form as follows:

$$\mathbf{X}_t = \mathbf{A}(s_t; \mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_{4,p})\mathbf{X}_{t-1} + \mathbf{B}(s_t; \mathcal{P}_{4,s})\mathbf{u}_t,$$

Where:

- $\mathbf{X}_t$: State vector of latent variables defined: $\mathbf{X}_t = \{R_t, X_t, \lambda_t, w_{h,t}, C_t, \xi_t, L_{c,t}, U_{c,t}, \omega_{c,t}, w_{c,t}, h_{c,t}, Y_t, \omega_{h,t}, V_{F,t}, V_{N,t}, V_{B,t}, V_t, r_{h,t}, P_t^h, B_t, S_t, N_t, F_t, L_{h,t}, H_t, q_{h,t}\}$
- $\mathbf{A}$: Transition matrix, determined by regime index: $s_t \in \{TD, VD, TH, VH\}$, and parameters: $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \mathcal{P}_{4,p}$.
- $\mathbf{B}$: Shock impact matrix, determined by $\mathcal{P}_{4,s}$.
- $\mathbf{u}_t$: Vector of structural shocks: $\mathbf{u}_t = \{z_t, \rho_t, \tilde{\kappa}_{c,t}, \epsilon_{c,t}, \tilde{\kappa}_{h,t}, \phi_t, \psi_t, \chi_{c,t}, \tau_t, \varepsilon_t, m_t\}$

3.4.2 Identificataion Strategy

To estimate the parameters in $\mathcal{P}_4$, we use Bayes' rule:

$$P(\mathcal{P}_4|\mathbf{Y}) \propto P(\mathbf{Y}|\mathcal{P}_4)P(\mathcal{P}_4),$$

Where: $P(\mathbf{Y}|\mathcal{P}_4)$ is the likelihood function, and $P(\mathcal{P}_4)$ represents the prior distributions.

And the identification relies on mapping the log-linearized system to the data of observed variables through the generalized measurement equation:

$$\mathbf{Y}_t = \mathbf{C}\mathbf{X}_t + \mathbf{D}\epsilon_t,$$

Where:

- $\mathbf{Y}_t$: Vector of observed variables, defined as:$\mathbf{Y}_t = \{\text{Output}_t, \text{Earnings}_t, \text{Housing Stock}_t, \text{House Price to Rent Ratio}_t, \text{Labour Market Tightness}_t, \text{Hours Worked}_t, \text{Housing Sales}_t, \text{Homeownership Proportion}_t\}$.

[33] See: Maih (2015)

- **C**: Measurement matrix, mapping states to observables.

- $\mathbf{X}_t$: State vector of latent variables defined: $\mathbf{X}_t = \{R_t, X_t, \lambda_t, w_{h,t}, C_t, \xi_t, L_{c,t}, U_{c,t}, \omega_{c,t}, w_{c,t}, h_{c,t}, Y_t, \omega_{h,t}, V_{F,t}, V_{N,t}, V_{B,t}, V_t, r_{h,t}, P_t^h, B_t, S_t, N_t, F_t, L_{h,t}, H_t, q_{h,t}\}$

- **D**: Measurement error matrix

- ϵ_t: Vector of measurement errors.

The specific mappings between the observed variables $\mathbf{Y}_t$ and the latent states $\mathbf{X}_t$ are defined by the measurement equations summarised in table: 7. For observables on growth rates (Output, Earnings, Housing Stock, House Price to Rent Ratio, Labour Market Tightness, and Housing Sales), growth rates are computed at quarterly rates before the data is demeaned. To recover real earnings from nominal data, the times series are deflated using the quoted measure of inflation. For the the intensive margin of labour, UK measure is computed by dividing the total number of hours worked by the number of persons in employment. A full description of the data is available in the appendix, section: A:1.

The likelihood function $P(\mathbf{Y}|\mathcal{P}_4)$ is computed recursively using RISE's standard Kalman filter[34] to update guesses of latent state variables stored in $\mathbf{X}_t$ using predictions from the state space system and observed realisations. The likelihood optimization has been conducted using the Artificial Bee Colony algorithm of Karaboga and Basturk (2007). Once the likelihood is maximized, the posterior distribution of parameters is explored using the Metropolis-Hastings (MH) algorithm with 100,000 draws are taken from the Markov Chain.

3.4.3 Measurement and Data

The time series of the observables contained in vector $\mathbf{Y}_t$ for the two countries is plotted in figure: 3.2. When estimating parameters in the UK economy, we use time series running from 1971Q2-2020Q1, while for the US the period of study covers the period 1965Q1-2020Q2. The growth rate of output (Panel A) is more volatile in the UK than in the US, both in terms of frequency of fluctuations and magnitude of deviations[35] from the mean growth rate. The UK experiences sharper peaks and troughs in its business cycle, indicating greater variability. However, the US demonstrates stronger reductions during recessions and accelerated recoveries during expansions[36], suggesting a heightened sensitivity to shocks that shape its business cycle dynamics. This difference may stem from structural factors such as greater labor and capital market flexibility in the US, more responsive policy interventions, or differing propagation mechanisms of economic shocks. Thus, while the UK exhibits higher overall variability, the US's sharper short-term responses underscore its greater sensitivity to shocks in driving cyclical adjustments.

Given the well-established stylized fact that labor market variables are typically pro-cyclical in nature[37], and the connection between incomes, productivity, and output, the greater observed fluctuations in UK GDP growth are also reflected in the growth rate of real earnings (Panel B), which exhibits a higher frequency of fluctuations in the UK but stronger business cycle responses in the US.

Turning to the remaining labor market variables, the growth rate of labor market tightness (Panel E) shows high levels of volatility in both countries ($0.0937 \leq \sigma \leq 0.0996$), but the UK displays greater responses in terms of amplitude ($\Delta Y_{UK} = 0.7333 \geq \Delta Y_{US} = 0.5911$). The

[34]See: Maih (2024) for further details about the implementation of the Kalman filter

[35]Determined by comparing standard deviations – $\sigma_{UK} = 0.0093 \geq \sigma_{US} = 0.0055$ – and amplitude ranges – $\Delta Y_{UK} = 0.0769 \geq \Delta Y_{US} = 0.0318$.

[36]For example, see the behavior of the variables during the Great Recession.

[37]See, for example, Rogerson and Shimer (2010), who provide an excellent discussion of these stylized facts in their chapter of the Handbook of Labour Economics, where they that the pro-cyclicality is primarily driven by changes in productivity and the income effect on labor supply.

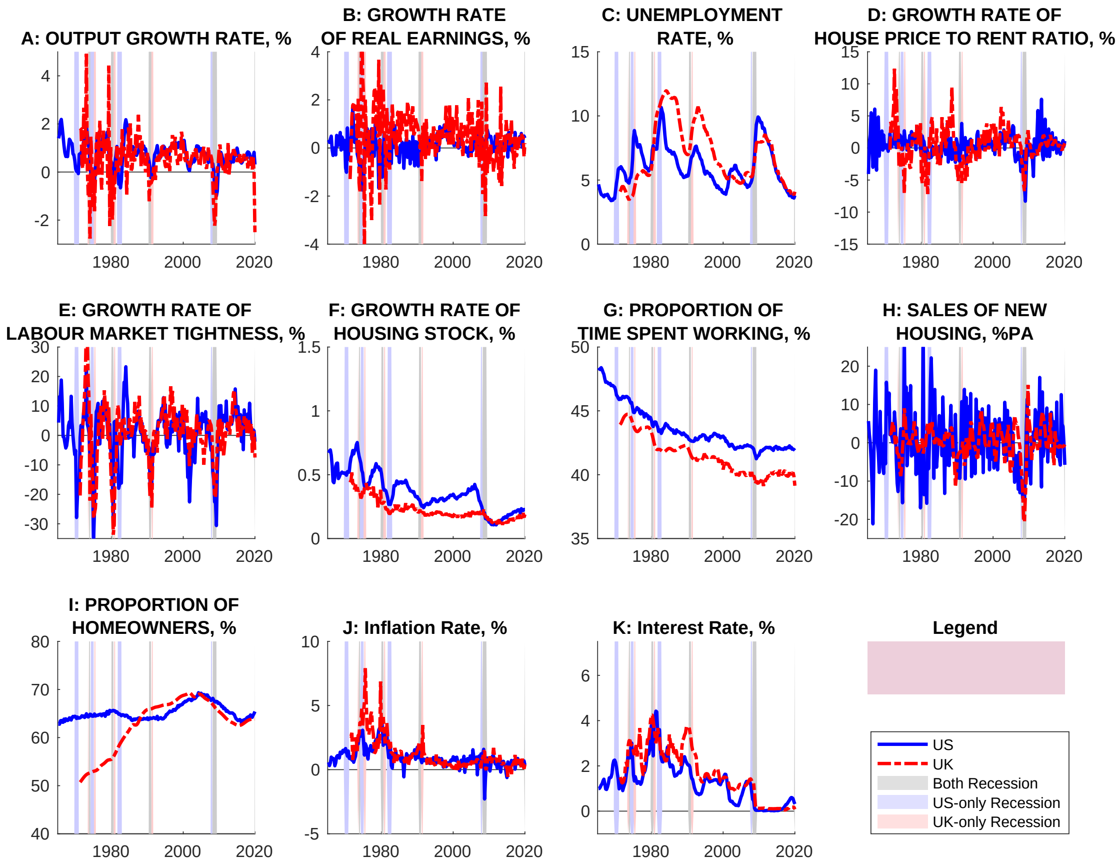


Figure 3.2: Historical Data Used for Estimation. Panels A-K show: The Growth Rate of real GDP, the Growth Rate of Real Earnings, the Unemployment Rate, the Growth rate of the House Price to Rent Ratio, the Growth Rate of Labor Market Tightness, the growth rate in the Housing Stock and Sales, the homeownership rate, and the monetary measures of Inflation and Interest Rates. The United States is plotted on the solid **blue** line, while the UK data is plotted in the dashed **red** line. The US data ranges from 1965Q2-2020Q1, while the UK data is for 1971Q2-2020Q1.

unemployment rate, shown in Panel C, initially lies below the US rate during the 1970-1980 period, while exhibiting similar business cycle dynamics. In the early 1980s, both countries experienced significant increases in unemployment due to contractionary monetary policies aimed at achieving price stability, structural changes linked to de-industrialization, and labor market reforms that reduced workers' bargaining power. Unemployment peaked at 10.8 percent in the US and 11.9 percent in the UK, but the US labor market recovered by 1982Q4, whereas the UK recovery lagged until 1985Q1. From 1980-2000, the UK unemployment rate remained consistently higher than that of the US.

Examining Panel G, the proportion of time spent working (expressed as a percentage of a standard 40-hour workweek), we observe that the mean time spent working has fallen in both countries over the period of study. Notably, beyond this negative trend, the data consistently show that hours worked are lower in the UK than in the US, a well-established stylized fact[38].

Next, consider the housing market. In the growth rate of the price-to-rent ratio (Panel D), we observe a pronounced pro-cyclical behavior with clear perriods of housing housing market booms and busts. Both countries exhibit significant volatility ($\sigma_{UK} = 0.0576$, $\sigma_{US} = 0.0498$), with the UK again showing a greater range of fluctuations ($\Delta Y_{UK} = 0.322 \geq \Delta Y_{US} = 0.284$). The elevated volatility in the UK's price-to-rent ratio is partially explained by the housing market interventions prior to and during Thatchers premiership discussed in Chapter 2 where

[38]See, for example, Alesina et al. (2005), who attribute this difference to institutional factors such as labor laws, union strength, and cultural preferences for leisure in Europe compared to the US.

average house prices were affected by sudden periods of supply and demand increase, as well as reflecting that UK's housing demand is fairly elastic, while supply is very constrained[39].

The growth rate of the housing stock (Panel: F) shows that the trend growth rate is falling in both countries, and that the US consistently has a higher rate of housing construction than the UK. In both countries, there is relatively little volatility, with standard deviations of $\sigma_{UK} = 0.0048$ and $\sigma_{US} = 0.0032$. Finally, the growth rate of housing sales depicted in Panel: H demonstrates substantial volatility in both countries, underscoring the sensitivity of housing transactions to economic and financial conditions. The standard deviations are comparable ($\sigma_{UK} = 0.1234$ and $\sigma_{US} = 0.1125$), with slightly greater amplitude in the UK ($\Delta Y_{UK} = 0.5532 \geq \Delta Y_{US} = 0.4871$), suggests that housing transactions in the UK may be more responsive to changes in (unmodelled) credit conditions and changes to macro variables.

One of the perhaps most interesting time series is the proportion of the economy acting as home-owners (Panel: I). While the US proportion is relatively stable, fluctuating between 62.5 and 69.5 percent and with a mean value of 65.23 percent, the UK starts comparatively low, with a home ownership rate of just over 50 percent. During the period of observation, we notice a significant growth in the measure as Thatchers "Right to Buy" regulation came into effect[40]. Due to this policy intervention, the proportion of home-owners in the UK continues to grow, eventually surpassing the US measure in 1988Q4. After this period, the proportion of homeowners continue to grow in both countries, until they reach a peak before the Great recession, and home ownership again falls to ≈ 65 percent.

Finally, the rate of inflation (Panel: J) is broadly similar in the two countries, apart from the stagflationary period of the 70's which more strongly affected the UK, resulting in higher peak inflationary observations. Similarly, the rate of interest (Panel: K) in the two countries can be split between Three periods. The 70's, where both countries monetary authorities are engaged in monetary targeting, keep interest rates relatively low and stable. The Volcker reforms and shift to inflation targeting resulting in both countries drastically increasing interest rates in the early 80's, before economic conditions allowed rates to come down in both countries. Finally, the post Great Recession period has both countries engaged in very loose monetary policy, with interest rates approaching the Zero Lower Bound.

3.4.4 Estimation Results:

The estimation results for the UK and US economies, covering the periods 1971Q2-2020Q1 and 1965Q1-2020Q2, are presented in Tables 3.4 to 3.6. The tables report prior distributions, posterior means, and 95 percent confidence intervals for the estimated parameters in $\mathcal{P}_4$. For regime-switching parameters, posteriors are separated into (H)awkish and (D)ovish monetary policy regimes, while shock volatilities are categorized by (T)ranquil or (V)olatile states of the world.

The first six rows of Table 3.4 report spill-over elasticities $\{\zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta, \theta_\kappa, \theta_\vartheta\}$, which describe interactions between the housing, labor, and money markets:

$$\kappa_{c,t} = \tilde{\kappa}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\kappa}, \quad \kappa_{h,t} = \tilde{\kappa}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\kappa} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{-\theta_\kappa} \qquad (1.73, 1.75)$$

$$\vartheta_{c,t} = \tilde{\vartheta}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}} \right)^{-\zeta_\vartheta}, \quad \vartheta_{h,t} = \tilde{\vartheta}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}} \right)^{\eta_\vartheta} \left(\frac{1+i_t}{1+\pi_{t+1}} R \right)^{\theta_\vartheta} \qquad (1.74, 1.76)$$

[39] Recall that housing stock to population is significantly higher in the US than in the UK – See: 3.3
[40] See: Chapter 2

Table 3.4: Estimation Results 1971Q2-2020Q1 (UK) and 1965Q1-2020Q1 (US).

Parameters		UK			US		
		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]	
Elast. of LM matching eff-cy wrt. HM tightness	(ζ_κ)	$N(3.0, 1.0)$	4.4792 [4.4355,4.5159]		$N(3.0, 1.0)$	3.1595 [3.1589,3.1600]	
Elast. of LM separation rate wrt. HM tightness	(ζ_ϑ)	$N(3.0, 1.0)$	5.9529 [5.8998,5.9816]		$N(3.0, 1.0)$	5.3634 [5.3615,5.3646]	
Elast. of HM matching eff-cy wrt. LM tightness	(η_κ)	$N(3.0, 1.0)$	5.1328 [5.0620,5.2238]		$N(3.0, 1.0)$	1.6069 [1.5943,1.6180]	
Elast. of HM matching eff-cy wrt. Interest rate	(θ_κ)	$N(2.0, 1.0)$	2.9702 [1.4903,4.2434]		$N(2.0, 1.0)$	0.0646 [-0.0011,0.1173]	
Elast. of HM separation rate wrt. LM tightness	(η_ϑ)	$N(3.0, 1.0)$	0.4069 [0.3815,0.4358]		$N(3.0, 1.0)$	0.6931 [0.6924,0.6938]	
Elast. of HM separation rate wrt. Interest rate	(θ_ϑ)	$N(2.0, 1.0)$	1.6315 [0.6991,2.7304]		$N(2.0, 1.0)$	0.4884 [0.4490,0.5203]	
Policy	(α_y)	$N(1, 0.5)$	0.2899 [0.2738,0.3048]		$N(1, 0.5)$	0.2871 [0.2860,0.2888]	
Policy	(α_i)	$B(0.5, 0.15)$	0.8398 [0.8337,0.8480]		$B(0.5, 0.15)$	0.7049 [0.7002,0.7084]	
Monetary regime			H	D		H	D
Policy	(α_π)	$N(2, 0.5)$	1.7813 [1.7748,1.7864]	1.0067 [1.0004,1.0132]	$N(2, 0.5)$	2.1407 [2.0739,2.2213]	1.1014 [1.0999,1.1035]

Table 3.5: Estimation Results 1971Q2-2020Q1 (UK) and 1965Q1-2020Q1 (US) – continued.

Parameters		UK		US	
		Prior dist.	Posterior dist.	Prior dist.	Posterior dist.
		Mean ∼ Type (mean,std)	Mean [95% conf.int.]	Mean ∼ Type (mean,std)	Mean [95% conf.int.]
AR(1), technology	(ρ^{z})	$B(0.5, 0.10)$	0.9555 [0.9553,0.9556]	$B(0.5, 0.10)$	0.9556 [0.9555,0.9556]
AR(1), taste	(ρ^{ϱ})	$B(0.5, 0.10)$	0.7207 [0.7026,0.7377]	$B(0.5, 0.10)$	0.8275 [0.8131,0.8404]
AR(1), housing tech.	(ρ^{ϕ})	$B(0.5, 0.10)$	0.9528 [0.9489,0.9554]	$B(0.5, 0.10)$	0.9070 [0.9014,0.9091]
AR(1), labour supply	(ρ^{χ_c})	$B(0.5, 0.10)$	0.9448 [0.9369,0.9509]	$B(0.5, 0.10)$	0.9391 [0.9388,0.9393]
AR(1), matching LM	(ρ^{κ_c})	$B(0.5, 0.10)$	0.8979 [0.8943,0.9029]	$B(0.5, 0.10)$	0.9368 [0.9293,0.9454]
AR(1), bargaining LM	(ρ^{ϵ_c})	$B(0.5, 0.10)$	0.9527 [0.9465,0.9555]	$B(0.5, 0.10)$	0.7565 [0.7560,0.7573]
AR(1), matching HM	(ρ^{κ_h})	$B(0.5, 0.10)$	0.9552 [0.9540,0.9556]	$B(0.5, 0.10)$	0.8838 [0.8795,0.8885]
AR(1), renters HM	(ρ^{ψ})	$B(0.5, 0.10)$	0.9554 [0.9553,0.9556]	$B(0.5, 0.10)$	0.9555 [0.9554,0.9556]
AR(1), sales HM	(ρ^{τ_h})	$B(0.5, 0.10)$	0.9534 [0.9480,0.9556]	$B(0.5, 0.10)$	0.9556 [0.9556,0.9556]
AR(1), elsticity of subst.	(ρ^{ε})	$B(0.5, 0.10)$	0.9415 [0.9311,0.9526]	$B(0.5, 0.10)$	0.7346 [0.7343,0.7352]

Table 3.6: Estimation Results 1971Q2-2020Q1 (UK) and 1965Q1-2020Q1 (US) – continued.

Parameters		UK			US		
		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]		Prior dist. Type (mean,std)	Posterior dist. Mean [95% conf.int.]	
			T	V		T	V
Std, technology	(σ_z)	$I(0.01, 0.02)$	0.0061 [0.0060,0.0062]	0.0159 [0.0153,0.0164]	$I(0.01, 0.02)$	0.0064 [0.0059,0.0068]	0.0063 [0.0058,0.0066]
Std, taste	(σ_ϱ)	$I(0.01, 0.02)$	0.0572 [0.0543,0.0602]	0.1614 [0.1550,0.1660]	$I(0.01, 0.02)$	0.0433 [0.0427,0.0438]	0.0519 [0.0511,0.0529]
Std, housing tech.	(σ_ϕ)	$I(0.01, 0.02)$	0.0950 [0.0914,0.0976]	0.1947 [0.1903,0.1983]	$I(0.01, 0.02)$	0.1542 [0.1533,0.1551]	0.3055 [0.2817,0.3281]
Std, labour supply	(σ_{χ_c})	$I(0.01, 0.02)$	0.0222 [0.0218,0.0227]	0.0352 [0.0328,0.0373]	$I(0.01, 0.02)$	0.0143 [0.0141,0.0145]	0.0249 [0.0243,0.0254]
Std, matching LM	(σ_{κ_c})	$I(0.01, 0.02)$	0.0548 [0.0540,0.0554]	0.0693 [0.0584,0.0809]	$I(0.01, 0.02)$	0.0283 [0.0281,0.0286]	0.0337 [0.0316,0.0369]
Std, bargaining LM	(σ_{ϵ_c})	$I(0.01, 0.02)$	0.0989 [0.0882,0.1136]	0.2652 [0.2427,0.2914]	$I(0.01, 0.02)$	0.1824 [0.1776,0.1865]	0.2757 [0.2743,0.2773]
Std, matching HM	(σ_{κ_h})	$I(0.01, 0.02)$	0.4264 [0.3785,0.4762]	1.3051 [1.2061,1.4067]	$I(0.01, 0.02)$	0.1424 [0.1348,0.1489]	0.4051 [0.4035,0.4064]
Std, renters HM	(σ_ψ)	$I(0.01, 0.02)$	0.3529 [0.3326,0.3664]	0.7256 [0.7155,0.7851]	$I(0.01, 0.02)$	0.2606 [0.2594,0.2620]	0.6515 [0.6021,0.6820]
Std, sales HM	(σ_{α_h})	$I(0.01, 0.02)$	0.0822 [0.0735,0.0920]	0.2097 [0.1658,0.2560]	$I(0.01, 0.02)$	0.3089 [0.3072,0.3101]	0.8716 [0.8177,0.9453]]
Std, elasticity	(σ_ε)	$I(0.01, 0.02)$	0.0014 [0.0013,0.0015]	0.0031 [0.0026,0.0040]	$I(0.01, 0.02)$	0.0018 [0.0017,0.0019]	0.0021 [0.0019,0.0023]
State Probabilities							
Prob. to move from H to D	p_{HD}	$B(0.05, 0.025)$	0.0022 [0.0011,0.0040]		$B(0.05, 0.025)$	0.0676 [0.0671,0.0679]	
Prob. to move from D to H	p_{DH}	$B(0.05, 0.025)$	0.0030 [0.0014,0.0049]		$B(0.05, 0.025)$	0.0658 [0.0633,0.0678]	
Prob. to move from T to V	Q_{TV}	$B(0.05, 0.025)$	0.0555 [0.0360,0.0787]		$B(0.05, 0.025)$	0.0987 [0.0951,0.1065]	
Prob. to move from V to T	Q_{VT}	$B(0.05, 0.025)$	0.0595 [0.0161,0.1194]		$B(0.05, 0.025)$	0.0321 [0.0310,0.0338]	

Starting with the spill-over from the housing to the labour market. The estimation results indicate that in both the UK and the US, tight housing markets have an adverse effect on aggregate labour market outcomes. With matching elasticities dominating separation elasticities ($\zeta_\kappa < \zeta_\vartheta$), increases to housing market tightness will result in a net increase in employment, ceteris paribus. That is, during periods when transacting in the housing markets is more difficult, workers face greater barriers to job changes, ultimately resulting in a less dynamic job market and higher unemployment.

One notable finding is that the difference between matching and separation elasticities ($\zeta_\kappa - \zeta_\vartheta$) is larger in the US than in the UK, suggesting that the US labor market is more sensitive to housing market conditions. To show the implications of this on the model, I simulate the behaviour of the estimated US and UK economies to unit shock housing supply and demand shocks similar to the implementation in Chapter 1. As shown in figures 3.3 and 3.4, the higher sensitivity found in the US gives rise to a stronger labour market response.

Given the numerous structural differences between the two economies, the source of this higher sensitivity is difficult to pin down. However, a plausible explanation lies in the greater level of labour mobility in the United States. With workers showing both a higher willingness and ability to relocate during "normal" times, the cooling effect of a tight housing market is more pronounced than that observed in British workers, who are are shown to be more local in the data.

Next, turning to the housing market. Recall that labor market tightness ($\omega_{c,t}$) and interest rates (R_t) jointly influence housing market matching and separation rates. With. higher levels of labor market tightness improving housing market matching efficiency and reducing separation rates, while, higher interest rates negatively impact both metrics, reflecting the dual role of credit conditions in housing dynamics.

In both the UK and US, housing market matching is more sensitive to changes in labor market outcomes than to interest rates ($\eta_\kappa > \theta_\kappa$), indicating that household income constraints are more significant than credit constraints for housing transactions. Elasticities with respect to labor market tightness (η_ϑ) are smaller than one, implying that separation effects in the housing market are dampened during recessions – a finding consistent with the observed decline in housing sales during economic downturns[41], and is consistent with the descriptive statistics in Hedlund (2016) and Garriga and Hedlund (2020).

[41] See Growth Rate of Housing Sales, Section: 3.4.3.

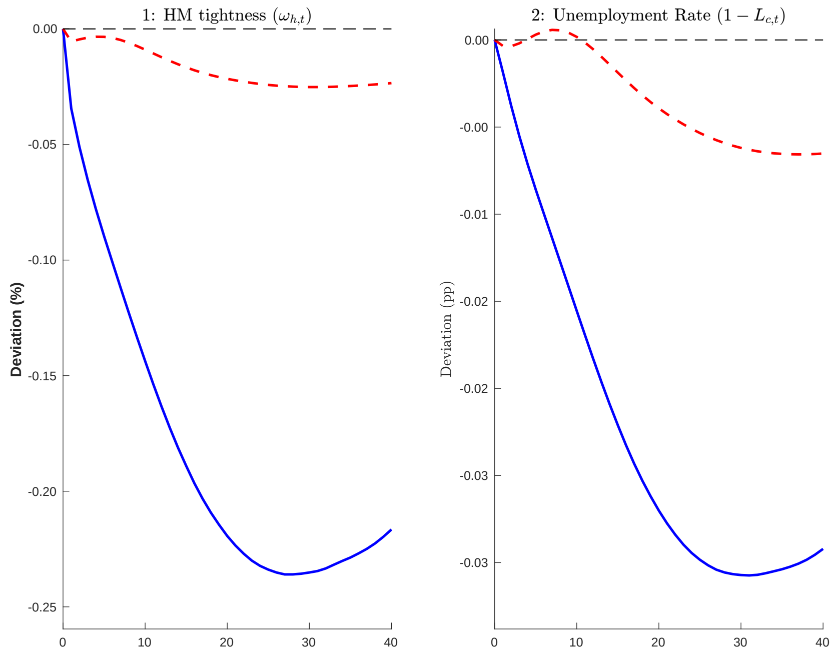


Figure 3.3: Housing Supply Shock

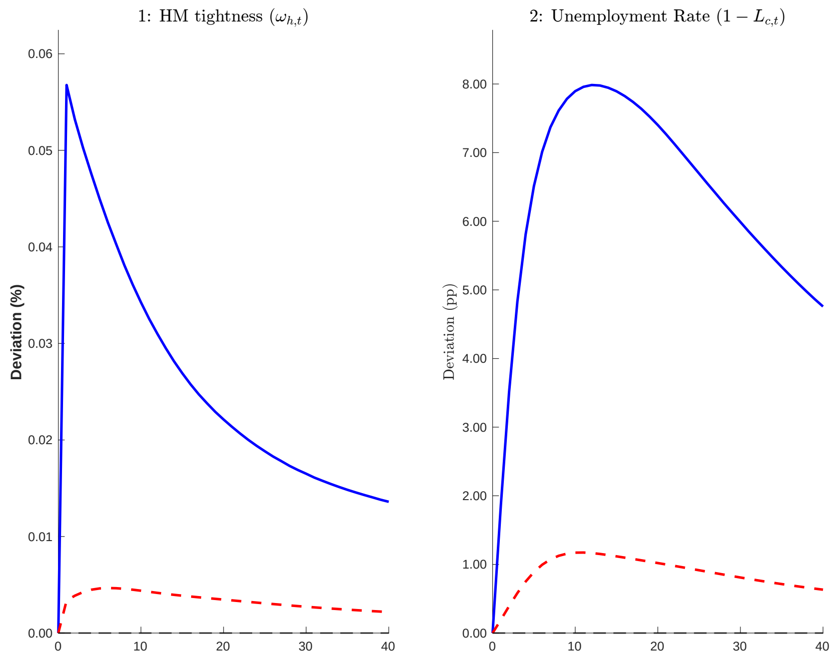


Figure 3.4: Housing Demand Shock

Comparison of Housing Supply and Demand Shocks under Baseline US and UK Parameters Note that in both figures: **LHS:** Housing Market Tightness, **RHS:** Unemployment. The UK is plotted in red and the US in blue. In both simulations shocks are identical unit shocks.

The estimated elasticities picks up on significant differences between the UK and US housing markets. In the UK, elasticities with respect to interest rates ($\theta_{\kappa,UK}$ and $\theta_{\vartheta,UK}$) exceed one, suggesting that credit conditions played a stronger role in housing market dynamics over the period than in the US. With estimates ($\theta_{\kappa,US} < \theta_{\vartheta,US} < 1$), there appears to be only a muted response to interest rate fluctuations, and a weak response to the labour market.

The final three rows of table: 3.4 report the elasticities associated with the Taylor rule. Overall, the results are not significantly different, and were not the primary interredest of this study, so the quantification of the policy rule parameters should not be overstated. For both countries, $\alpha_\pi > 1$. However, the estimated inflation response parameter is consistently higher in the US ($\alpha_{\pi,US} > \alpha_{\pi,UK}$) under both the hawkish and dovish monetary regimes. This aligns with the historical emphasis of the Federal Reserve on combating inflation, particularly following the Volcker reforms of the early 1980s, while the BoE's formal adoption of inflation targeting only began in 1992. Further evidence of the model doing a good job of capturing this historical shift can be seen in figure 3.5, which plots the estimated probability of being in a Dovish Monetary State per period.

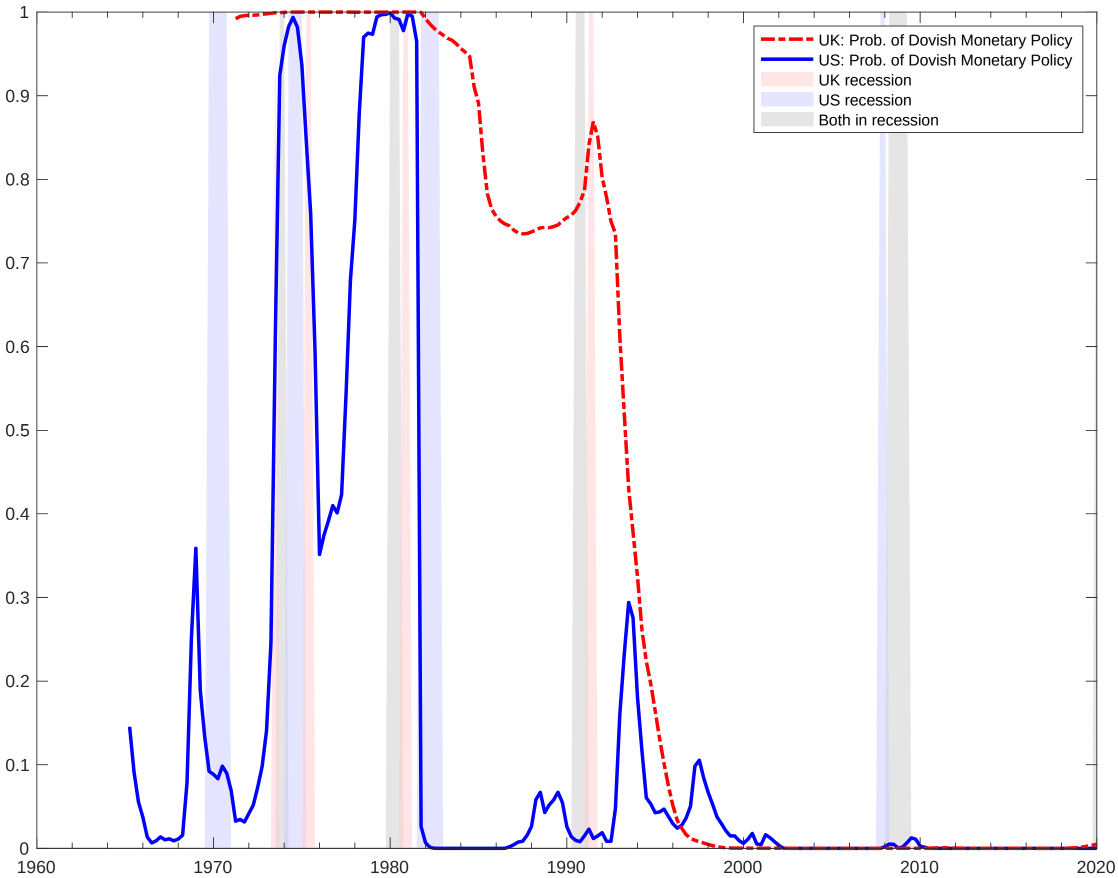


Figure 3.5: Probability of Being in Dovish Monetary State. United States plotted in solid blue, United Kingdom plotted in dashed red.

3.5 Counter-factual

Having established that the stronger sensitivity of labour market activity to housing market changes in the US can, in part, be attributed to a more mobile US labour force[42], the counterfactual explores an alternative scenario: *What if labour mobility in the UK had been encouraged under Thatcher's premiership?* To investigate this, the simulation imposes US parameter estimates for the spill-over elasticities between the housing and labour markets,

[42]See section 3.4.4

specifically $\{\zeta_\kappa, \zeta_\vartheta, \eta_\kappa, \eta_\vartheta\}$. In effect, the state-space representation of the UK economy is recalibrated to reflect this counterfactual flexibility:

$$\mathbf{X}_t = \mathbf{A}(s_t^{UK}; \mathcal{P}_1^{UK}, \mathcal{P}_2^{UK}, \mathcal{P}_3^{UK}, \mathcal{P}_{4,p\{\zeta_\kappa,\zeta_\vartheta,\eta_\kappa,\eta_\vartheta\}^{US}\{\theta_\kappa,\theta_\vartheta,\alpha_y,\alpha_i,\alpha_\pi\}^{UK}})\mathbf{X}_{t-1} + \mathbf{B}(s_t^{UK}; \mathcal{P}_{4,s}^{UK})\mathbf{u}_t$$

This setup isolates the impact of spill-over dynamics in the markets of interest, while holding other structural characteristics constant, including interest rate elasticities. The decision to focus solely on the interaction terms between housing and labour markets is to enable a clearer understanding of the mechanisms driving spill-overs, avoiding the conflation of effects stemming from structural differences in financial and credit markets, which are not the primary focus of this study. Moreover, simulations confirm that including or excluding US interest rate elasticities does not materially affect the qualitative predictions of the counterfactual[43].

As the counter-factual gets implimented in 1980Q1, the UK economy is undergoing various reforms discussed in chapter 2, and the *growth rate* of labour market tightness is positive as discussed shown in panel E of figure 2.1 in section 3.4.3. This positive growth rate translates into the model predicting the latent variable of the *levels* of labour market tightness being above its steady state level[44], encouraging housing market activity. In the housing market, the right to buy programme has resulted in high levels of sales and homeownership, ultimately translating into estimates of above steady state *levels* of housing market tightness, acting as a drag on the labour market.

Recall that the direct effect of altering the spillover parameters on the labour market is a reduction in both the separation and matching elasticities associated with housing market tightness[45]. This ensures that responses in both matching ($\kappa_{c,t}$) and separation ($\vartheta_{c,t}$) are more muted compared to the baseline calibration. However, as discussed in section 3.4.4, the gap between separation and matching elasticities ($\zeta_\vartheta - \zeta_\kappa$) widens. Consequently, tighter housing markets are expected to increase unemployment relative to the baseline, ceteris paribus, as the reduction in matching efficiency outweighs the decline in separation rates. Simulation results validate this assertion, showing that the counterfactual specification has an amplifying effect on the labour market relative to the baseline[46].

In the housing market, the effect of imposing the US estimates for $\{\eta_\kappa, \eta_\vartheta\}$ is to reduce the sensitivity of housing market matching ($\kappa_{h,t}$) to changes in labour market tightness[47]. In terms of separation, sensitivity to labour market tightness increases[48]. However, noting that the change in matching elasticity outweighs the change in separation[49], the net effect, all other things being equal, is to amplify the housing market's response to labour market fluctuations. This is confirmed through simulated shocks to labour supply, demonstrating stronger interactions under the counterfactual specification[50].

As shown in figure 3.6, the counter-factual simulation diverges significantly from the actual data observables upon the imposition of the US spill-over elasticities in 1981Q1[51]. From the backdrop of a tight housing market, and the change in parametrization resulting in

[43]See appendix, figure 15, which shows that the counter-factual simulations, both with and without the US sensitivity to interest rates implemented

[44]See the appendix, figure 15

[45]Specifically, $\zeta_\kappa^{UK} = 4.4792$ exceeds $\zeta_\kappa^{US} = 3.1595$, and $\zeta_\vartheta^{UK} = 5.9529$ exceeds $\zeta_\vartheta^{US} = 5.3634$.

[46]See appendix, figure 13, showing the amplified response of the counter-factual simulation to a housing supply shock

[47]Recall: $\eta_\kappa^{UK} = 5.1328$ while $\eta_\kappa^{US} = 1.6069$.

[48]$\eta_\vartheta^{UK} = 0.4069$, compared to $\eta_\vartheta^{US} = 0.6931$.

[49]$\Delta\eta_\kappa = \eta_\kappa^{UK} - \eta_\kappa^{US} = 3.5259$, while $\Delta\eta_\vartheta = \eta_\vartheta^{UK} - \eta_\vartheta^{US} = -0.2862$

[50]See appendix, figure 14, showing the stronger housing market response to a labour market supply shock

[51]Note that auxiliary variables are printed in the appendix, figure 15. These variables are used to verify certain model dynamics, but are not of primary interest to the counter factual simulation.

the UK labour market showing a greater sensitivity to transaction probabilities in the housing market, we unsurprisingly see a strong response in the labour market to the tight housing market. This is depicted in panel A of figure 3.6, where the weaker drag generated by the housing market translates into improved transition probabilities in the labour market. There is an immediate drop in unemployment from 8.14 to 7.2 percent, before the unemployment rate stabalises at 6.5 to 7 percent over the two year period. This result is likely overstated, since the change in parameterisation is implemented as a shock/exogenous change of the "world", rather than a more likely gradual adjustment.

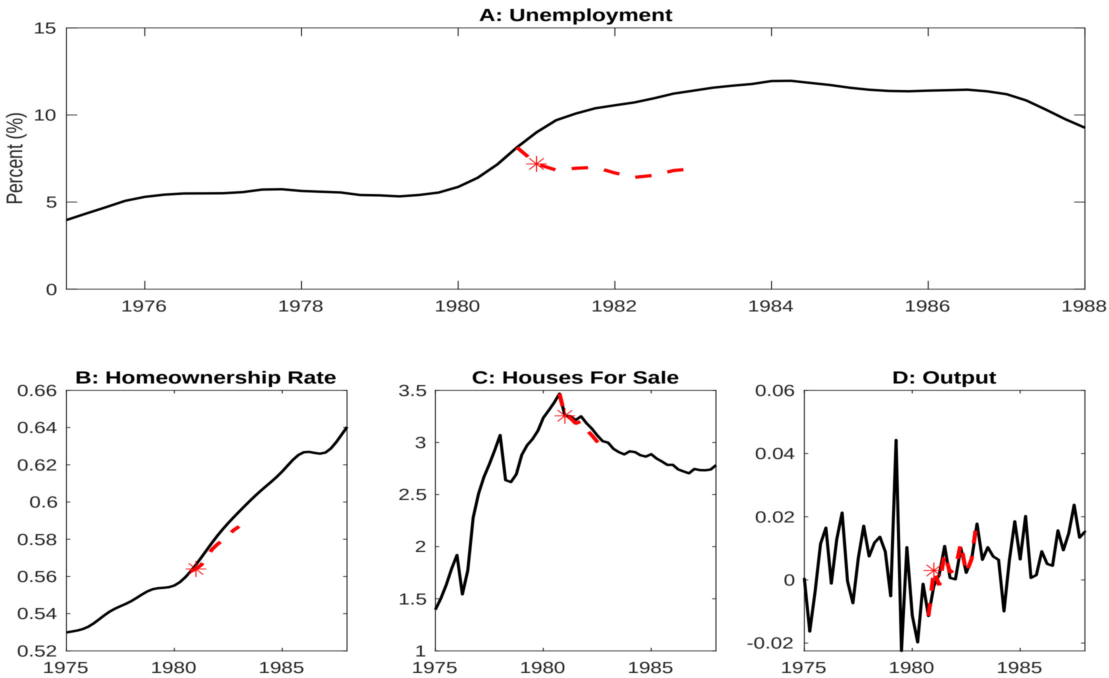


Figure 3.6: Responses of Select Variables to Counter Factual Simulation of the UK economy with US spillovers

In the housing market, the more flexible labour market, together with the starting point of an above-steady-state level of labour market tightness ensures that the amplifying effect on of the labour market on the housing market becomes weaker. Upon the imposition of the counter-factual in 1980Q1, we thus observe that the transition probabilities in the housing market falls, resulting in fewer matches for searching buyers and a lower homeownership rate as shown in panel B.

With lower transaction probabilities, the price of housing falls, reducing the incentive for housing construction and reducing the growth rate of the housing stock. With less new houses added to the economy, and fewer homeowners separating from their matched house, housing supply suffers as shown in panel C. With both increases to demand and decreases to supply, the housing market stabalises through increases to housing market tightness, which further encourages labour market activity and drives down unemployment further.

Output in the economy is dependent on employment. While some gains in employment are offset by the model predicting reductions in the extensive margin of labour, the overall effect on aggregate labor supply ($h_{c,t}L_{c,t}$) is to increase. While the underlying data on output growth is volatile, causing the counter-factual to display similar levels of volatility, the model predicts that the quarterly growth rate of output produced in the economy will increase by a mean of 0.47 *percentage points* over the two year period.

3.6 Concluding Remarks:

In this chapter, I extended my study into the relationship between housing and labour markets by developing and estimating the model described in Chatper 1 for both the United States and United Kingdom. I discuss how the model captures the theorised greater flexibility in the US labour market through a comparison of estimation results, showing that the response in UK labour markets are only about two-thirds as strong as in the United States. I argue that these results can be partially attributed to the Greater level of labour mobility seen in the United US vis a vis the UK.

Finally, an experiment is conducted to examine the counterfactual that, as a consequence of Thatchers interventions into the housing and labour market discussed in Chapter 2, UK workers developed a similar level of housing and labour market mobility to their American counterparts. Based on the counterfactual simulation, I find that a more flexible economy would have had a positive effect on UK employment and output levels, with the unemployment rate dropping approximately 2 percentage points. and a mean improvement in the quarterly growth rate of output of 0.47 percentage points. With more relocations, there is an increase in renting households, and the homoeownership falls as workers undertake more frequent job moves. While these results are in line with the literature on labour flexibility, the counterfactual is implemented as a sudden realized shift, while a more gradual adjustment might allow for the investigation of important transition dynamics.

While this study quantifies the spillover based on the US time series, and thus contributes to the non-existent literature on empircal macro search models over the joint housing-labour market, the US results are primarily used as an alternative against which the UK economy is assessed. In addition to addressing the methodological extensions discussed in the preceding chapters, future research should be applied more directly to evaluate US policy interventions and market dynamics.

APPENDICES

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A:1 Data Sources and Measurement Equations:

A Details on data used in estimation

Measurement Equation	Data Concept
$\text{Output}_t = \log\left((1+\mu)\, q_t\, (1+\gamma)\, a_t \frac{y_t}{y_{t-1}}\right)$	Growth Rate of Real GDP
$\text{Earnings}_t = \log\left((1+\gamma)\, a_t \frac{w_{c,t}h_{c,t}}{w_{c,t-1}h_{c,t-1}}\right)$	Growth Rate of Labour Market Earnings
$\text{Housing Stock}_t = \log\left((1+\mu)\, q_t \frac{H_t}{H_{t-1}}\right)$	Growth Rate of Housing Stock
$\text{Labour Market Tightness}_t = \log\left(\frac{\omega_{c,t}u_{c,t}(1-l_{c,t-1})}{\omega_{c,t-1}u_{c,t-1}(1-l_{c,t})}\right)$	Growth Rate of Labour Market Tightness
$\text{Hours Worked}_t = h_{c,t}$	Growth Rate of Hours Worked
$\text{Housing Sales}_t = \log\left((1+\mu)\, q_t \frac{s_t}{s_{t-1}}\right)$	Growth Rate of Housing Sales
$\text{Homeownership Proportion}_t = n_t + \alpha_{h,t} b_t \omega_{h,t}^{-1}$	Homeownership Proportion
$\text{Unemployment}_t = 1 - l_{c,t}$	Unemployment Rate
$\text{Inflation}_t = \pi_t$	Inflation Rate
$\text{Nominal Interest Rate}_t = (1+i_t)$	Nominal Interest Rate
$\text{House Price to Rent Ratio}_t = \log\left(\frac{P_t r_t^h}{P_{t-1} r_{t-1}^h}\right)$	Growth Rate of House Price to Rent Ratio

Table 7: Measurement Equations and Corresponding Data Concepts Including House Price to Rent Ratio

A.0.1 Output

United Kingdom

As a measure of real Gross Domestic Product (GDP) in the United Kingdom, we use the Office for National Statistics data series: ABNI. The series tracks GDP as a chained volume measure and provides seasonally adjusted data. The quarterly growth rate is computed, before the mean growth rate over the observation is removed from the observations.

United States

We take observations on real GDP in the United States from the U.S. Bureau of Economic Analysis, data series: GDPC1. The time-series is a chain measure of GDP and is seasonaly adjusted. The quarterly growth rate is computed, before the mean growth rate over the observation is removed from the observations.

A.0.2 Labour Market Earnings

United Kingdom

To express the growth rate of earnings in the labour market, we produce an estimate from two time-series. For the period 1971Q1 - 1999Q4, we use ‘Spliced Average Weekly Earnings, 1919-2015’ from the Bank of England publication ‘A millennium of macroeconomic data’. To update this series until 2020Q1, we use the time-series for ’Average Weekly Earnings’ of total pay for the whole economy from the Office for National Statistics, data series: EARN01.

To construct a real measure of earnings, we deflate the earnings data by the Organization for Economic Co-operation and Development dataset for the Consumer Price Index for all items (OECD Descriptor ID: CPALTT01, OECD unit ID: IDX, OECD country ID: GBR). The quarterly growth rate is computed from the real data, before the mean growth rate over the observation is removed from the observations.

United States

To express weekly nominal earnings we compute the measure as the product of average earnings per hour multiplied by the average numbers of hours worked. For data on hourly wages we use the U.S. Bureau of Labor Statistics timeseries over Average Hourly Earnings of Production and Nonsupervisory Employees, Total Private, series id: AHETPI. For observations on hours worked, we use the U.S. Bureau of Labor Statistics estimate of Average Weekly Hours of Production and Nonsupervisory Employees, Total Private, series id: AWHNONAG.

To express the real measure, the computed weekly earnings are deflated by the GDP deflator. For data, we take observations on the deflator from U.S. Bureau of Economic Analysis, series id: GDPDEF. The quarterly growth rate is computed from the deflated data, before the mean growth rate over the observation is removed from the observations.

A.0.3 Housing Stock

United Kingdom

To express the growth rate in the residential housing, we produce a time-series on on the housing stock from two times-series. For the period 1971 - 2014 we take estimates on all dwellings in the United Kingdom from the Ministry of Housing, Communities and Local Government data on housing tenure, table: 101. For the period 2014 - 2020, we reproduce the time-series by adding the data from the three countries of the United Kingdom together. We use data from the **Data˙Houses˙UK˙Eng**, table: 100, for data on English housing tenure. We then add data on housing tenure in Scotland from the Scottish Government, Local Government and Housing Directorate, and for Northern Ireland Department for Communities housing tenure data is available in table: Supply.

We then compute quarterly growth rates from the data. The observations are converted into quarterly frequency by assuming a linear growth rate through the year, and then performing a seasonal adjustment. We then divide the gross increase by the seasonally adjusted total stock of houses. Note that the stock of houses series is heavily rounded, and the number of completed new dwellings is relatively small, so the growth rates are not heavily affected by using the annual data as the base for calculating the growth rate.

United States

For estimates of the growth of the housing stock we use data on New Privately-Owned Housing Units Completed: Total Units, series id: COMPUTSA, provided jointly by the U.S. Census Bureau and U.S. Department of Housing and Urban Development. For estimates on the stock of housing, we use data from the U.S. Census Bureau. The quarterly growth rate is computed, before being demeaned.

A.0.4 House Price to Rent Ratio

United Kingdom

For estimates on house prices, we use U.K. Land Registry data on nominal average transaction prices in the UK, reported in GBP. For estimates on rental cost, the Home Let provide a point estimates in GBP. These point estimates is then combined with time series data from Organization for Economic Co-operation and Development on rental costs as an index value. We then compute the quarterly growth rate and demean the data.

United States

For the United States, two time series are combined before computing the demeaned growth rate. For the period 1964Q1-1969Q4, house price data is taken from industry source "dqydj", available from: https://dqydj.com/historical-home-prices/. For 1970Q1 - 2020Q1, data on house prices are taken from the Bank for International Settlements data on Residential Property Prices for United States, series id: QUSN628BIS. Observations on rental prices are consistant accross the period, and are taken from the U.S. Bureau of Labor Statistics estimate for average rental costs of primary residences in U.S. Cities, series id: CUUR0000SEHA.

A.0.5 Labour Market Tightness

United Kingdom

For the period 1978Q1 - 2001Q1, we use the spliced time-series on unfilled vacancies from the Bank of England publication 'A millennium of macroeconomic data.' To update this series until 2020Q1, we use the Office for National Statistics vacancy data, series id: AP2Y.

For statistics on the number of unemployed persons, we use the Office for National Statistics estimate of the number of unemployed persons aged 16 and over, based on the data from the Labour Force Survey. Series id: MGSC.

Tightness is computed as a ratio, before the growth rate is computed. The data is then demeaned.

United States

For data on labour market tightness we take reproduction files from Brian C Jenkins examination of the Beveridge (1944) curve applied to US data, available from: (https://github.com/letsgoexploring/economicdata/blob/master/dmp/csv/beveridge_curve_data.csv).

Tightness is computed as a ratio, before the growth rate is computed. The data is then demeaned.

A.0.6 Hours Worked

United Kingdom

To obtain the data on average weekly hours worked, we divided 'Total actually hours worked' estimates from Office for National Statistics by 'The number of people in employment' from Office for National Statistics. We then divide the estimate by an assumed 40 hour work week to express the data as a percentage rate.

United States

We take data on average hours worked from the U.S. Bureau of Labor Statistics, who provide an estimate of "Average Weekly Hours of Production and Nonsupervisory Employees, Total Private", series id: AWHNONAG. We again express the measure as a percentage of a 40 hour work week.

A.0.7 Housing Sales

United Kingdom

We take data on the number of residential property transactions from U.K. Land Registry. Prior to 1995 this data is only available as annual measurements. For these periods, quarterly estimated are produced assuming linear yearly growth rates, before seasonal adjustments are applied, and demeaned growth rates are calculated.

United States

To compute demeaned growth rates of housing sales in the United States, we combine two time series.

For 1968Q1 to 2020Q1 we take data from the estimate of quarterly housing sales. This is proprietary data, retrieved through Refinitiv data stream service. Prior to 1968, no estimates exist for sales of existing homes, so we use data on new home sales from the .

A.0.8 Home-ownership

United Kingdom

For estimates on home ownership in the United Kingdom we have to use England as a proxy due to the lack of good data on Scotland and Northern Ireland. We use data from Ministry of Housing, Communities and Local Government, who report the proportion of homeowners in the English Housing Survey in annex table 1.1. Annualized growth rates are computed, before quarterly estimates are produced assuming linear yearly growth rates, before being seasonally adjusted.

United States

For the United States, we use estimates from the U.S. Census Bureau tenure data (Table 8). We then compute quarterly growth rates.

A.0.9 Inflation

United Kingdom

For data on inflation, we use the Office for National Statistics 12 month change in the index value of the Retail Price Index, expressed as a quarterly rate. Series id: CZBH.

United States

We take observations on inflation from the Board of Governors of the Federal Reserve System (US) measure of consumer price inflation, series id: CPIAUCSL.

A.0.10 Nominal Interest Rate

United Kingdom

As a measure of UK interest rates, we use the Bank of England "Bank Rate", available at: https://www.bankofengland.co.uk/-/media/boe/files/monetary-policy/baserate.xls.

United States

We use data on the "Federal Funds Effective Rate" provided by the Board of Governors of the Federal Reserve System (US), series id: DFF.

B Steady State Computation

SS Variable	Computation	Value (US)	Value (UK)
Discount Rate	$\beta = \frac{1}{R}\frac{(1+\gamma)}{(1+\mu)}$	0.9961	0.9955
Finding rate (labour)	$\lambda_c = 1-(1-\lambda_c^w)^{52/4}$	0.5243	0.2805
Filling rate (labour)	$\gamma_c = 1-(1-\gamma_c^d)^{365/4}$	0.06	0.07
Mark-up	$\xi = \frac{\epsilon-1}{\epsilon}$	0.9091	0.8333
Unemployment rate	$u_c = \frac{1-l_c}{1-\lambda_c}$	0.1261	0.0973
Separation rate (labour)	$\vartheta_c = 1-\frac{(1-u)(1+\mu)}{l_c}$	0.0671	0.0269
Labour market tightness	$\omega_c = \frac{\lambda_c}{\gamma_c}$	0.5293	0.2831
Wages (consumption sector)	$w_c = \frac{\xi\gamma_c}{(\frac{\iota}{w_c h_c}(1-\beta(1-\vartheta_c))+\gamma_c)}$	0.8990	0.8249
Output	$y = h_c l_c$	0.4097	0.3814
Period Consumption	$c = y - \frac{\iota}{w_c h_c}\omega_c u w_c h_c$	0.4058	0.3786
Habitual Consumption	$x = (1-\theta)c$	0.0812	0.0757
Multiplier on households	$\lambda = x^{-\sigma}$	1867.8	174.4295
Dis-utility (Consumption sector)	$\chi_c = \xi\lambda(1-h_c)^{\nu}$	305.4169	50.5849
Dis-utility (Construction sector)	$\chi_h = x^{-\sigma}\frac{w_h}{w_c h_c}w_c h_c$	1048.4	69.6287
Bargaining power of workers	$\epsilon_c = \frac{h_c w_c - b_c + \frac{\chi_c}{\lambda}\frac{(1-h_c)^{1-\nu}-1}{1-\nu}}{h_c\xi + \beta(1-\vartheta_c)\lambda_c\frac{\iota}{\gamma_c} - b_c + \frac{\chi_c}{\lambda}\frac{(1-h_c)^{1-\nu}-1}{1-\nu}}$	0.5733	0.1323
Wages (Construction sector)	$w_h = \frac{w_h}{h_c w_c}h_c w_c$	0.5603	0.3992
Unemployment benefit	$b_c = \frac{b_c}{h_c w_c}h_c w_c$	0.1724	0.1319
Cost of posting vacancy	$\iota = \frac{\iota}{h_c w_c}h_c w_c$	0.0588	0.1015
Matching efficiency (Labour)	$\kappa_c = \lambda_c\omega_c^{-1+\delta_c}$	0.6346	0.4095
House Price	$p^h = \frac{p^h}{h_c w_c}h_c w_c$	6.6259	8.1291
Labour supply (Construction)	$l_h = \frac{l_h}{l_c}l_c$	0.0338	0.0335

Table 8: Steady-State Variables (Page 1)

SS Variable	Computation	Value (US)	Value (UK)
Number of Home-owners	$n = \frac{(1-b-f)(1+\mu)}{(1-\vartheta_h)}$	0.6785	0.6746
Number of searching buyers	$b = \frac{(\mu+\vartheta_h)(1-b-f)}{(1-\vartheta_h)\lambda_h}$	0.0435	0.0300
Number of permanent renters	$f = 1 - \frac{\psi}{\frac{(\mu+\vartheta_h)}{(1-\vartheta_h)\lambda_h}+1} - b$	0.2965	0.3100
Housing market tightness	$\omega_h = \frac{(1-\alpha_h)b}{h-1}$	0.3014	0.6010
Housing market finding rate	$\lambda_h = 1 - (1 - \lambda_h^w)^{52/4}$	0.4249	0.4867
Housing market filling rate	$\gamma_h = \omega_h \lambda_h$	0.1281	0.2925
Matching efficiency (Housing)	$\kappa_h = \lambda_h \omega_h^{1-\delta_h}$	0.2965	0.4177
Construction sector productivity	$\phi = \frac{h\mu}{l_h}$	0.1189	0.0769
Vacancy Value (Housing)	$v = \frac{\beta\gamma_h}{(1-\beta(1-\gamma_h))} \frac{p^h}{h_c w_c} h_c w_c$	6.2607	7.9395
Cost of housing maintenance/tax	$m^h = \frac{r^h}{h_c w_c} h_c w_c - (1-\beta)$	0.2008	0.2386
Value of being a searching buyer	$v^B = \frac{-\frac{r^h}{h_c w_c} h_c w_c + \beta\lambda_h v^N - \beta\lambda_h \frac{p^h}{h_c w_c} h_c w_c}{1-\beta+\beta\lambda_h}$	-12.5755	-29.1357
Value of home-ownership	$v^N = v^B + \frac{1}{(1-\epsilon_h)} (1 - \frac{\epsilon_h\beta\gamma_h}{(1-\beta(1-\gamma_h))}) \frac{p^h}{h_c w_c} h_c w_c$	-5.5845	-20.8170
Utility Value of Home-ownership	$z^h = \lambda \left[\frac{(1-\beta)(1-\beta((1-\vartheta_h)-\epsilon_h\lambda_h(1-c\vartheta_h)-(1-\epsilon_h)\gamma_h))}{1-\beta(1-\gamma_h)-\epsilon_h\beta\gamma_h} \left(v^N - v^B\right) - (r^h - m^h) \right]$	323.6985	17.3952
Land for construction	$q_h = \beta v - w_h \frac{1}{\phi}$	1.2613	2.6943
Undeveloped land	$k = \frac{h\mu + \Lambda(q_h)h}{\Lambda(q_h)}$	1.5009	1.0927
Rent	$r^h = w_c \frac{r^h}{w_c h_c}$	0.2472	0.2937
Vacancies	$v_c = \omega_c(1 - l_c)$	0.0318	0.0198
Houses for Sale	$s = \frac{B(1+\tau)}{\omega_h}$	0.1300	0.0300

Table 9: Steady-State Variables (Page 2)

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A:2 Mathematical Appendix

A Households:

A.0.1 Households Problem: The households problem then has choice variables: $\{C_{i,t}, L_{i,h,t}, \mathcal{A}_{i,t+1}\}$, and take: $\{$ r_t^h, m_t^h, $w_{c,t}$, $h_{c,t}$, $b_{c,t}$, $w_{h,t}$, Q_t, N_t, F_t, $L_{c,t}$, and H $\}$ as given. Thus, the maximisation problem:

$$\max_{\{X_{i,t},C_{i,t},L_{i,h,t},\mathcal{A}_{i,t}\}} U(\cdot)\sum_{t=0}^{\infty}\beta^t(\varrho_t\frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H}+\chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h_{i,c,t}^i)^{1-\nu}-1}{1-\nu}-\chi_h L_{i,h,t}\frac{Q_t}{H}+\frac{N_t}{H}z^H)$$

$$S.t : X_{i,t}=\frac{C_{i,t}}{A_t}-\theta\frac{C_{t-1}}{A_{t-1}}$$

$$And : \frac{Q_t}{H}C_{i,t}+\frac{Q_t}{H}\mathcal{A}_{i,t+1}+\frac{Q_t}{H}T_{i,t}+\frac{Q_t}{H}\Omega_{i,t}=$$

$$=\frac{Q_t}{H}\Phi_{i,t}+\frac{L_{c,t}}{H}w_{c,t}h_{c,t}+\frac{U_{c,t}}{H}b_{c,t}+\frac{Q_t}{H}w_{h,t}L_{i,h,t}+\frac{Q_t}{H}R_t\mathcal{A}_{i,t}$$

Solving the optimisation of (1.2) w.r.t. (1.3) and (1.5) gives rise to the following Lagrangian and First Order Conditions:

$$\mathcal{L}=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\varrho_t\frac{X_{i,t}^{1-\sigma}}{1-\sigma}\frac{Q_t}{H}+\chi_{c,t}\frac{L_{c,t}}{H}\frac{(1-h_{i,c,t}^i)^{1-\nu}-1}{1-\nu}-\chi_h L_{h,t}\frac{Q_t}{H}+\frac{N_t}{H}z^H\right.$$

$$-\upsilon_t\left(\frac{C_{i,t}}{A_t}-\theta\frac{C_{t-1}}{A_{t-1}}-X_{i,t}\right)$$

$$-\lambda_t\left(\frac{Q_t}{H}C_{i,t}+\frac{Q_t}{H}\mathcal{A}_{i,t+1}+\frac{Q_t}{H}T_{i,t}+\frac{Q_t}{H}\Omega_{i,t}\right.$$

$$\left.\left.-\frac{Q_t}{H}\Phi_{i,t}-\frac{L_{c,t}}{H}w_{c,t}h_{c,t}-\frac{Q_t-L_{c,t}}{H}b_{c,t}-\frac{Q_t}{H}w_{h,t}L_{i,h,t}-\frac{Q_t}{H}R_t\mathcal{A}_{i,t}\right)\right\}$$

$$\frac{\partial\mathcal{L}}{\partial X_{i,t}}\equiv 0=\beta^t\varrho_t X_{i,t}^{-\sigma}\frac{Q_t}{H}+\beta^t\upsilon_t\rightarrow(-)\upsilon_t=\varrho_t X_{i,t}^{-\sigma}\frac{Q_t}{H} \tag{19}$$

$$\frac{\partial\mathcal{L}}{\partial C_{i,t}}\equiv 0=(-)\beta^t\upsilon_t(\frac{1}{A_t})-\beta^t\lambda_t(\frac{Q_t}{H})\rightarrow(-)\upsilon_t=A_t\lambda_t\frac{Q_t}{H} \tag{20}$$

$$\frac{\partial\mathcal{L}}{\partial L_{i,h,t}}\equiv 0=\beta^t\lambda_t(\frac{Q_t}{H}w_{h,t})-\beta^t(\chi_h\frac{Q_t}{H})\rightarrow\lambda_t=\frac{\chi_h}{w_{h,t}} \tag{21}$$

$$\frac{\partial\mathcal{L}}{\partial\mathcal{A}_{i,t+1}}\equiv 0=\lambda_{t+1}\beta^{t+1}(\frac{Q_{t+1}}{H}R_{t+1})-\lambda_t\beta^t(\frac{Q_t}{H})\rightarrow\lambda_t=\beta\mathbb{E}_t\{\lambda_{t+1}(\frac{Q_{t+1}}{Q_t}R_{t+1})\} \tag{22}$$

Combining (19) and (20):

$$\varrho_t X_{i,t}^{-\sigma}\frac{Q_t}{H}=A_t\lambda_t\frac{Q_t}{H}\rightarrow\varrho_t X_{i,t}^{-\sigma}=A_t\lambda_t\equiv\varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t}=\lambda_t \tag{1.7}$$

Combining (21) and (1.7):

$$\chi_h=\varrho_t X_{i,t}^{-\sigma}\frac{w_{h,t}}{A_t} \tag{1.8}$$

Substituting (22) into (1.7):

$$\varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t}=\beta\mathbb{E}_t\{\lambda_{t+1}(\frac{Q_{t+1}}{Q_t}R_{t+1})\}$$

And the fact that (22) implies that: $\lambda_{t+1}=\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}$:

$$\varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t}=\beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\} \tag{1.9}$$

A.0.2 The Aggregate Household The Aggregate Households Problem:

Consider the aggregation of the individual households optimal choice $\{X_{i,t}, c_{i,t}, l_{i,h,t}, \mathcal{A}_{i,t+1}\}_{t=0}^{\infty}$ as described by: (1.7) - (1.9). Multiplying by population Q_t to express the aggregate choice of the consumption bundle:

$$\varrho_t X_{i,t}^{-\sigma} = \lambda_t A_t \tag{1.7}$$

$$\rightarrow \mathcal{A}_t \lambda_t Q_t = \varrho_t X_{i,t}^{-\sigma} Q_t$$

$$\rightarrow \mathcal{A}_t \lambda_t Q_t^{1+\sigma} Q_t^{-\sigma} = \varrho_t X_{i,t}^{-\sigma} Q_t^{1+\sigma} Q_t^{-\sigma}$$

$$\rightarrow \mathcal{A}_t \lambda_t Q_t^{1+\sigma} Q_t^{-\sigma} = \varrho_t X_t^{-\sigma} Q_t^{1+\sigma}$$

$$\rightarrow \mathcal{A}_t \lambda_t Q_t^{-\sigma} = \varrho_t X_t^{-\sigma}$$

$$\rightarrow \mathcal{A}_t \lambda_t = \varrho_t (\frac{X_t}{Q_t})^{-\sigma} \tag{1.12}$$

$$\chi_h = \varrho_t X_{i,t}^{-\sigma} \frac{w_{h,t}}{A_t} \tag{1.8}$$

$$\rightarrow \chi_h Q_t = \varrho_t X_{i,t}^{-\sigma} \frac{w_{h,t}}{A_t} Q_t$$

$$\rightarrow \chi_h Q_t^{1+\sigma} Q_t^{-\sigma} = \varrho_t X_{i,t}^{-\sigma} \frac{w_{h,t}}{A_t} Q_t^{1+\sigma} Q_t^{-\sigma}$$

$$\rightarrow \chi_h Q_t^{1+\sigma} Q_t^{-\sigma} = \varrho_t X_t^{-\sigma} \frac{w_{h,t}}{A_t} Q_t^{1+\sigma}$$

$$\rightarrow \chi_h Q_t^{-\sigma} = \varrho_t X_t^{-\sigma} \frac{w_{h,t}}{A_t}$$

$$\rightarrow \chi_h = \varrho_t (\frac{X_t}{Q_t})^{-\sigma} \frac{w_{h,t}}{A_t} \tag{1.13}$$

$$\varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\} \quad (1.9)$$

$$\rightarrow \varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t}Q_t = \beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}Q_t$$

$$\rightarrow \varrho_t X_{i,t}^{-\sigma}\frac{1}{A_t}Q_t^{1+\sigma}Q_t^{-\sigma} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}Q_t^{1+\sigma}Q_t^{-\sigma}$$

$$\rightarrow \varrho_t X_t^{-\sigma}\frac{1}{A_t}Q_t^{1+\sigma} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}Q_t^{1+\sigma}Q_t^{-\sigma}$$

$$\rightarrow \varrho_t (\frac{X_t}{Q_t})^{-\sigma}\frac{1}{A_t}\frac{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}}{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{i,t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}\frac{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}}{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}}$$

$$\rightarrow \varrho_t (\frac{X_t}{Q_t})^{-\sigma}\frac{1}{A_t}\frac{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}}{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}\frac{Q_{t+1}^{1+\sigma}}{Q_{t+1}^{1+\sigma}Q_{t+1}^{-\sigma}}$$

$$\rightarrow \varrho_t (\frac{X_t}{Q_t})^{-\sigma}\frac{1}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{Q_t}R_{t+1}\}\frac{Q_{t+1}^{1+\sigma}}{Q_{t+1}}$$

$$\rightarrow \varrho_t (\frac{X_t}{Q_t})^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}X_{t+1}^{-\sigma}\frac{1}{A_{t+1}}\frac{Q_{t+1}}{1}R_{t+1}\}Q_{t+1}^{\sigma}$$

$$\rightarrow \varrho_t (\frac{X_t}{Q_t})^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}(\frac{X_{t+1}}{Q_{t+1}})^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}R_{t+1}\} \quad (1.14)$$

The aggregate consumption bundle:

Consider the households habit adjusted bundle:

$$x_t = \frac{c_{i,t}}{A_t} - \theta\frac{c_{t-1}}{A_{t-1}} \quad (1.3)$$

All households make same choice:

$$x_t = \frac{c_{i,t}}{A_t} - \theta\frac{c_{i,t-1}}{A_{t-1}}$$

The aggregate habit adjusted bundle:

$$\rightarrow X_{i,t}Q_t = \frac{c_{i,t}Q_t}{A_t} - \theta\frac{c_{i,t-1}}{A_{t-1}}Q_t \rightarrow X_t = \frac{C_t}{A_t} - \theta\frac{c_{t-1}}{A_{t-1}}Q_t\frac{Q_{t-1}}{Q_{t-1}}$$

$$\rightarrow \frac{X_t}{Q_t} = \frac{C_t}{Q_tA_t} - \theta\frac{C_{t-1}}{A_{t-1}Q_{t-1}} \quad (1.15)$$

B Matching Technology in the Labour Market:

Recall that labour market tightness is defined:

$$\omega_{c,t} \equiv \frac{V_{c,t}}{U_{c,t}} \tag{1.17}$$

B.0.1 Job Filling Rate in Labour Market:

$$\gamma_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{V_{c,t}} = \frac{\kappa_{c,t} U_{c,t}^{\delta_{c,t}} V_{c,t}^{1-\delta_{c,t}}}{V_{c,t}} = \kappa_{c,t} \left(\frac{U_{c,t}}{V_{c,t}}\right)^{\delta_{c,t}} = \kappa_{c,t} \left(\frac{V_{c,t}}{U_{c,t}}\right)^{-\delta_{c,t}} = \kappa_{c,t} \omega_{c,t}^{-\delta_{c,t}} \tag{1.18}$$

B.0.2 Job Finding Rate in Labour Market:

$$\begin{aligned} \lambda_{c,t} \equiv \frac{M^C(\cdot)}{U_{c,t}} &= \frac{\kappa_{c,t} U_{c,t}^{\delta_{c,t}} V_{c,t}^{(1-\delta_{c,t})}}{U_{c,t}} = \kappa_{c,t} U_{c,t}^{\delta_{c,t}-1} V_{c,t}^{1-\delta_{c,t}} \\ &= \kappa_{c,t} \left(\frac{V_{c,t}}{U_{c,t}}\right)^{1-\delta_{c,t}} = \kappa_{c,t} \omega_{c,t}^{1-\delta_{c,t}} = \gamma_{c,t} \omega_{c,t} \end{aligned} \tag{1.19}$$

C Matching Technology in the Housing Market:

Recall that labour market tightness is defined:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} \tag{1.45}$$

C.0.1 House Filling Rate:

$$\gamma_{h,t} \equiv \frac{M^H(B_t, S_t)}{S_t} = \frac{\kappa_t B_t^{\delta_h} S_t^{1-\delta_h}}{S_t} = \kappa_{h,t} B_t^{\delta_h} S_t^{-\delta_h} = \kappa_{h,t} \left(\frac{B_t}{S_t}\right)^{\delta_h} = \kappa_{h,t} \omega_{h,t}^{\delta_h} \tag{1.46}$$

C.0.2 House Finding Rate:

$$\lambda_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{B_t} = \frac{\kappa_{h,t} B_t^{\delta_h} S_t^{1-\delta_h}}{B_t} = \kappa_{h,t} S_t^{1-\delta_h} B_t^{\delta_h - 1} = \kappa_{h,t} \left(\frac{B_t}{S_t}\right)^{\delta_h - 1} = \kappa_{h,t} \omega_{h,t}^{\delta_h - 1} \tag{1.47}$$

D Housing Market Tightness Equation with Chains:

Recall that labour market tightness is defined:

$$\omega_{h,t} \equiv \frac{B_t}{S_t} \tag{1.45}$$

Re-arrange the definition of stocks of houses for sale (1.51) to isolate vacant houses ($V_{h,t}$):

$$S_t = V_{h,t} + C_{h,t} \Rightarrow V_{h,t} = S_t - C_{h,t} \tag{1.51}$$

Combine the re-arranged version of (1.51) with the definition of sale chains (1.52):

$$C_{h,t} = \tau S_t \tag{1.52}$$

$$\begin{aligned} &\rightarrow V_{h,t} = S_t(1-\tau) \\ &\rightarrow \frac{V_{h,t}}{(1-\tau)} = S_t \end{aligned} \tag{23}$$

Combine (23) with the definition of housing market tightness (1.45):

$$\omega_{h,t} = \frac{B_t}{S_t} \rightarrow \omega_{h,t} = \frac{B_t}{\frac{V_{h,t}}{(1-\tau)}} = \frac{B_t(1-\tau)}{V_{h,t}}$$

Isolate vacant houses ($V_{h,t}$) the housing stock definition (1.43):

$$H_t = Q_t + V_{h,t} \rightarrow V_{h,t} = Q_t - H_t \tag{1.43}$$

Finally, combine (D) and (1.43):

$$\rightarrow \omega_{h,t} = \frac{B_t(1-\tau)}{Q_t - H_t} \tag{1.53}$$

E Final Good Producers

E.0.1 Demand Function:

We have the profit function:

$$\max_{\{Y_t, y_t(j)\}} \Pi_t = P_t Y_t - \int_0^1 p_t(j) y_t(j)\, dj \tag{1.24}$$

Substituting the final good producers production function (1.23) into (1.24) we have:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.23}$$

$$\rightarrow \max_{\{y_t(j)\}} \Pi_t = P_t \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}}\, dj\right]^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 p_t(j) y_t(j)\, dj$$

Solving the unconstrained maximisation:

$$\frac{\delta \Pi_t}{\delta y_t(j)} \equiv 0 = \frac{\epsilon}{\epsilon-1} P_t \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}}\, dj\right]^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon-1}{\epsilon} y_t(j)^{\frac{\epsilon}{\epsilon-1}-1} - p_t(j)$$

$$\rightarrow 0 = P_t \left[\int_0^1 y_t(j)^{\frac{\epsilon-1}{\epsilon}}\, dj\right]^{\frac{1}{\epsilon-1}} y_t(j)^{\frac{-1}{\epsilon}} - p_t(j)$$

$$\rightarrow 0 = P_t Y_t^{\frac{1}{\epsilon}} y_t(j)^{\frac{-1}{\epsilon}} - p_t(j)$$

$$\rightarrow 0 = \frac{y_t(j)}{Y_t}^{\frac{-1}{\epsilon}} - \frac{p_t(j)}{P_t}$$

$$\rightarrow y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t \tag{1.25}$$

F Intermediary Goods producers

F.0.1 Firms Problem:

$$\max_{\{p_t(j),L_{c,t}(j),V_{c,t}(j)\}} \Pi_t \sum_{t=0}^{\infty} \left\{ (\beta\varsigma)^t \left[\frac{p_t(j)}{P_t} y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \right] \right\}$$
$$S.t: L_{c,t}(j) = (1-\vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_t(j)$$
$$And: y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j)$$
$$And: y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$

F.0.2 Job Creation: Consider the optimisation problem faced by firms when making their decisions in the labour market with search frictions. They need to decide on how many workers to hire, and how many vacancies to create to achieve this number of employees – $\{L_{c,t}(j), V_{c,t}(j)\}$. Substituting the demand curve (1.25) into the firms output function (1.27) and profit function (1.28) the optimisation problem becomes:

$$\max_{\{L_{c,t}(j),V_{c,t}(j)\}} \Pi_t \sum_{t=0}^{\infty} \left\{ (\beta\varsigma)^t \left[\left(\frac{p_t(j)}{P_t}\right)^{1-\varepsilon} Y_t - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \right] \right\}$$
$$S.t: L_{c,t}(j) = (1-\vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_t(j)$$
$$And: \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t = z_t A_t L_{c,t}(j) h_{c,t}(j)$$

Yielding the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \Bigg\{ (\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \Bigg[\left(\frac{p_t(j)}{P_t}\right)^{1-\varepsilon} Y_t - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \\ -\eta_t (L_{c,t}(j) - (1-\vartheta_{c,t}) L_{c,t-1}(j) - V_{c,t}(j)\gamma_{c,t}) \\ -\eta_{t+1} (L_{c,t+1}(j) - (1-\vartheta_{c,t}) L_{c,t}(j) - V_{t+1}(j)\gamma_{c,t}) \\ -\xi_t \left(\left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t - z_t A_t L_{c,t}(j) h_{c,t}(j) \right) \Bigg] \Bigg\}\end{aligned}$$

Consider the First Order Condition of the labour market variables:

$$\frac{\partial L}{\partial V_{c,t}(j)} \equiv 0 = -(\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \iota + (\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \eta_t \gamma_{c,t} \Rightarrow \eta_t = \frac{\iota}{\gamma_{c,t}} \tag{1.29}$$

$$\begin{aligned}\frac{\partial L}{\partial L_{c,t}(j)} \equiv 0 = -(\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} w_{c,t} h_{c,t}(j) - (\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \eta_t \\ +\mathbb{E}_t \left\{ (\beta\varsigma)^{t+1} \frac{\lambda_{t+1}}{\lambda_0} \eta_{t+1} \right\} (1-\vartheta_{c,t}) + (\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \xi_t (z_t A_t h_{c,t}(j))\end{aligned}$$

$$\Rightarrow \eta_t = -w_{c,t} h_{c,t}(j) + (\beta\varsigma) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1} \right\} (1-\vartheta_{c,t}) + \xi_t (z_t A_t h_{c,t}(j))$$

$$\Rightarrow \eta_t = h_{c,t}(j)(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1} \right\} (1-\vartheta_{c,t}) \tag{24}$$

Where the aggregation of $j \in J$ in 24 yields:

$$\Rightarrow \eta_t = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \eta_{t+1} \right\} (1-\vartheta_{c,t}) \tag{1.30}$$

Then, using the fact that (1.29) implies that: $\eta_{t+1} = \frac{\iota_{t+1}}{\gamma_{c,t+1}}$, and combining (1.29) and (1.30) we can express the firms job-creation condition:

$$\Rightarrow \frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\iota_{t+1}}{\gamma_{c,t+1}} \right\} (1 - \vartheta_{c,t}) \tag{1.31}$$

F.0.3 Price setting: Consider next the price setting decision faced by the intermediary good producer with maximisation problem:

$$\begin{aligned} \max_{\{p_t(j)\}} \Pi_t \sum_{t=0}^{\infty} &\left\{ (\beta\varsigma)^t \left[\frac{p_t(j)}{P_t} y_t(j) - w_{c,t} h_{c,t}(j) L_{c,t}(j) - \iota V_{c,t}(j) \right] \right\} \\ S.t &: L_{c,t}(j) = (1 - \vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_t(j) \\ And &: y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j) \\ And &: y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \end{aligned}$$

Re-arrange the firms production function (1.27) to isolate $h_{c,t}(j)L_{c,t}(j) = y_t(j)\frac{1}{z_t A_t}$ and introduce into the profit function (1.28):

$$\begin{aligned} \max_{\{p_t(j)\}} \Pi_t \sum_{t=0}^{\infty} &\left\{ (\beta\varsigma)^t \left[\frac{p_t(j)}{P_t} y_t(j) - y_t(j) \frac{w_{c,t}}{z_t A_t} - \iota V_{c,t}(j) \right] \right\} \\ S.t &: L_{c,t}(j) = (1 - \vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_t(j) \\ And &: y_t(j) = \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \end{aligned}$$

Then, substitute in the demand curve (1.25) into the profit function:

$$\begin{aligned} \max_{\{p_t(j)\}} \Pi_t \sum_{t=0}^{\infty} &\left\{ (\beta\varsigma)^t \left[\left(\frac{p_t(j)}{P_t} \right)^{1-\varepsilon} Y_t - \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \frac{w_{c,t}}{z_t A_t} - \iota V_{c,t}(j) \right] \right\} \\ S.t &: L_{c,t}(j) = (1 - \vartheta_{c,t}) L_{c,t-1}(j) + \gamma_{c,t} V_t(j) \end{aligned}$$

Yielding the Lagrangian:

$$\begin{aligned} \mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} &\left\{ (\beta\varsigma)^t \frac{\lambda_t}{\lambda_0} \left[\left(\frac{p_t(j)}{P_t} \right)^{1-\varepsilon} Y_t - \left(\frac{p_t(j)}{P_t} \right)^{-\varepsilon} Y_t \frac{w_{c,t}}{z_t A_t} - \iota V_{c,t}(j) \right. \right. \\ &- \eta_t (L_{c,t}(j) - (1 - \vartheta_{c,t}) L_{c,t-1}(j) - V_{c,t}(j)\gamma_{c,t}) \\ &- \eta_{t+1} (L_{c,t+1}(j) - (1 - \vartheta_{c,t}) L_{c,t}(j) - V_{t+1}(j)\gamma_{c,t})] \} \end{aligned}$$

Maximise with respect to own price:

$$\frac{\partial \mathcal{L}}{p_t(j)} \equiv 0 = \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left[\left((1-\varepsilon) \left(\frac{p_t^*(j)}{P_{t+k}} \right)^{-\varepsilon} + \varepsilon \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) \left(\frac{p_t^*(j)}{P_{t+k}} \right)^{-\varepsilon-1} \right) Y_{t+k} \right] \right\}$$

$$\Rightarrow 0 = \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left[\left((1-\varepsilon) p_t^*(j)^{-\varepsilon} P_{t+k}^{\varepsilon} + \varepsilon \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) p_t^*(j)^{-\varepsilon-1} P_{t+k}^{\varepsilon+1} \right) Y_{t+k} \right] \right\}$$

$$\Rightarrow 0 = \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left[\left((1-\varepsilon) p_t^*(j) P_{t+k}^{\varepsilon} + \varepsilon \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) P_{t+k}^{\varepsilon+1} \right) Y_{t+k} \right] \right\}$$

$$\Rightarrow (\varepsilon-1)\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} p_t^*(j) P_{t+k}^{\varepsilon} Y_{t+k} \right\} = \varepsilon \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) P_{t+k}^{\varepsilon+1} Y_{t+k} \right\}$$

$$\Rightarrow p_t^*(j) = \frac{\varepsilon}{(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) P_{t+k}^{\varepsilon+1} Y_{t+k} \right\}}{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} P_{t+k}^{\varepsilon} Y_{t+k} \right\}}$$

$$\Rightarrow \frac{p_t^*(j)}{P_t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) P_{t+k}^{\varepsilon+1} P_t^{-1} Y_{t+k} \right\}}{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} P_{t+k}^{\varepsilon} P_t^{-1} Y_{t+k} \right\}}$$

$$\Rightarrow \frac{p_t^*(j)}{P_t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) (P_{t+k} P_t^{-1})^{\varepsilon+1} Y_{t+k} \right\}}{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} (P_{t+k} P_t^{-1})^{\varepsilon} Y_{t+k} \right\}}$$

$$\Rightarrow \frac{p_t^*(j)}{P_t} = \frac{\varepsilon}{(\varepsilon-1)} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon+1} Y_{t+k} \right\}}{\mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} Y_{t+k} \right\}}$$

Let $K_{1,t}$ and $K_{2,t}$ be defined:

$$K_{1,t} = \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{w_{c,t+k}}{z_{t+k}A_{t+k}} \right) \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon+1} Y_{t+k} \right\} \tag{25}$$

$$K_{2,t} = \mathbb{E}_t \sum_{k=0}^{\infty} \left\{ (\beta\varsigma)^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon} Y_{t+k} \right\} \tag{26}$$

Where the recursive representation of (25) and (26) is:

$$K_{1,t} = Y_t \left(\frac{w_{c,t}}{z_t A_t} \right) + \beta\varsigma \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1+\pi_{t+1})^{\varepsilon+1} K_{1,t+1} \right\} \tag{1.33}$$

$$K_{2,t} = Y_t + \beta\varsigma \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1+\pi_{t+1})^{\varepsilon} K_{2,t+1} \right\} \tag{1.34}$$

Such that we can express:

$$\frac{p_t^*(j)}{P_t} = \frac{\varepsilon}{\varepsilon-1} \frac{K_{1,t}}{K_{2,t}} \tag{27}$$

F.0.4 Deriving the aggregate price level: Substituting the demand function (1.25) into the final goods producers production function (1.23) we have:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t \tag{1.25}$$

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1.23}$$

$$\rightarrow Y_t = \left[\int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t^{\frac{\epsilon-1}{\epsilon}}\,dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow Y_t = \left[\int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{1-\varepsilon} Y_t^{\frac{\epsilon}{\epsilon-1}}\,dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow Y_t = Y_t\left[\int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{1-\varepsilon} dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow 1 = \left[\int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{1-\varepsilon} dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow 1 = P_t^{\epsilon}\left[\int_0^1 (p_t(j))^{1-\varepsilon}\,dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow P_t^{-\epsilon} = \left[\int_0^1 (p_t(j))^{1-\varepsilon}\,dj\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$\rightarrow P_t = \left[\int_0^1 p_t(j)^{1-\epsilon}\,dj\right]^{\frac{1}{1-\epsilon}} \tag{1.26}$$

Recall that in the presence of Calvo style pricing, $(1-\varsigma)$ firms set prices optimally to $p_t^*(j)$, and ς firms maintain the existing price, thus in each period, the aggregate price level is described:

$$P_t = \left[\int_0^1 p_t(j)^{1-\epsilon}\,dj\right]^{\frac{1}{1-\epsilon}} = \left[\int_0^{1-\varsigma} p_t(j)^{1-\epsilon} + \int_{1-\varsigma}^1 p_t(j)^{1-\epsilon}\,dj\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_t = \left[\int_0^{1-\varsigma} p_t^*(j)^{1-\epsilon} + \int_{1-\varsigma}^1 P_{t-1}^{1-\epsilon}\,dj\right]^{\frac{1}{1-\epsilon}}$$

$$\Rightarrow P_t^{1-\varepsilon} = (1-\varsigma)P_t^{*1-\varepsilon} + \varsigma P_{t-1}^{1-\epsilon} \tag{28}$$

Phillips Curve:
Rewriting (28) we can express:

$$\Rightarrow P_t^{1-\varepsilon} - \varsigma P_{t-1}^{1-\epsilon} = (1-\varsigma) P_t^{*1-\varepsilon}$$

$$\Rightarrow (1-\varsigma)\left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} = 1 - \varsigma\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}$$

$$\Rightarrow \left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} = \frac{1}{(1-\varsigma)}\left(1 - \varsigma\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}\right)$$

$$\Rightarrow \frac{P_t^*}{P_t} = \left[\frac{1}{(1-\varsigma)}\left(1 - \varsigma\left(\frac{P_{t-1}}{P_t}\right)^{1-\varepsilon}\right)\right]^{\frac{1}{1-\varepsilon}}$$

$$\Rightarrow \frac{P_t^*}{P_t} = \left[\frac{1}{(1-\varsigma)}\left(1 - \frac{\varsigma}{(1+\pi_t)^{1-\varepsilon}}\right)\right]^{\frac{1}{1-\varepsilon}} \Leftrightarrow \frac{P_t^*}{P_t} = \left[\frac{1}{(1-\varsigma)}\left(1 - \frac{\varsigma}{(1+\pi_{t+1})^{1-\varepsilon}}\right)\right]^{\frac{1}{1-\varepsilon}}$$

Thus, finally establishing:

$$\frac{P_t^*}{P_t} = \left[\frac{1}{(1-\varsigma)}\left(1 - \frac{\varsigma}{(1+\pi_{t+1})^{1-\varepsilon}}\right)\right]^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1}\frac{K_{1,t}}{K_{2,t}} \tag{29}$$

Yielding the Phillips curve expression:

$$\Rightarrow 1 - \frac{\varsigma}{(1+\pi_t)^{1-\varepsilon}} = \frac{1-\varsigma}{1}\left(\frac{\varepsilon}{(1-\varepsilon)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\varepsilon}$$

$$\Rightarrow \frac{1}{\varsigma} - \frac{1}{(1+\pi_t)^{1-\varepsilon}} = \frac{1-\varsigma}{\varsigma}\left(\frac{\varepsilon}{(1-\varepsilon)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\varepsilon}$$

$$\Rightarrow \frac{1}{(1+\pi_t)^{1-\varepsilon}} = \frac{1}{\varsigma} - \frac{1-\varsigma}{\varsigma}\left(\frac{\varepsilon}{(1-\varepsilon)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\varepsilon}$$

$$\Rightarrow (1+\pi_t)^{\varepsilon-1} = \frac{1}{\varsigma}\left(1 - (1-\varsigma)\left(\frac{\varepsilon}{(1-\varepsilon)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\varepsilon}\right) \tag{1.32}$$

G Output Aggregation

The final goods producer aggregates a continuum of intermediate goods $y_t(j)$ using a CES aggregator:

$$Y_t = \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\, dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.23}$$

Cost minimisation implies demand for each good j is given by:

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} Y_t \tag{1.25}$$

Let the price difference between firm "j's" output and aggregate output be denoted by the price dispertion term: Δ_t:

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} dj$$

Aggregate intermediate output is thus:

$$\Rightarrow \int_0^1 y_t(j), dj = \int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} dj = \Delta_t Y_t$$

Each intermediary firm j uses labour to produce output:

$$y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j) \tag{1.27}$$

With symmetry across firms, it follows that the aggregation of: $L_{c,t}(j) = L_{c,t}$, and $h_{c,t}(j) = h_{c,t}$ for all j. Aggregate aggregate output becomes:

$$y_t(j) = z_t A_t L_{c,t}(j) h_{c,t}(j)$$

$$\Rightarrow \int_0^1 y_t(j), dj = z_t A_t \int_0^1 L_{c,t}(j) h_{c,t}(j), dj$$

$$\Rightarrow Y_t \cdot \Delta_t = z_t A_t L_{c,t} h_{c,t}$$

$$Y_t = \frac{z_t A_t}{\Delta_t} L_{c,t} h_{c,t} \tag{1.35}$$

G.0.1 Price Dispersion

In the presence of Calvo pricing, we know that $1 - \varsigma$ firms set prices optimally, while ς firms unable to do so. Thus, the price dispertion term can be expressed:

$$\Delta_t \equiv \int_0^1 \left(\frac{p_t(j)}{P_t}\right)^{-\varepsilon} dj = \int_0^{1-\varsigma} \left(\frac{p_t^*(j)}{P_t}\right)^{-\varepsilon} dj + \int_{1-\varsigma}^1 \left(\frac{p_{t-1}(j)}{P_t}\right)^{-\varepsilon} dj$$

$$\Rightarrow \Delta_t = (1-\varsigma)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \varsigma \left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} \int_0^1 \left(\frac{p_{t-1}(j)}{P_t}\right)^{-\varepsilon} dj$$

$$\Rightarrow \Delta_t = (1-\varsigma)\left(\frac{P_t^*}{P_t}\right)^{-\varepsilon} + \varsigma \pi_t^{\varepsilon} \cdot \Delta_{t-1} \tag{30}$$

Where 30 is the recursive expression of the price dispertion term quoted in the text.

H Wage Bargaining Problem:

The surplus bargaining problem is:

$$\max_{V_t^F, V_t^W} : \mathcal{L} = (V_t^W)^{\epsilon_{c,t}}(V_t^F)^{1-\epsilon_{c,t}} - \varphi_t(V_t^W + V_t^F - V_{c,t}^T)$$

F.O.C:

$$\frac{\partial \mathcal{L}}{\partial V_t^W} \equiv 0 = \epsilon_{c,t}(V_t^W)^{\epsilon_{c,t}-1}(V_t^F)^{1-\epsilon_{c,t}} - \varphi_t$$

$$\Rightarrow \varphi_t = \epsilon_{c,t}(V_t^W)^{\epsilon_{c,t}-1}(V_t^F)^{1-\epsilon_{c,t}}$$

$$\frac{\partial \mathcal{L}}{\partial V_t^J} \equiv 0 = (1-\epsilon_{c,t})(V_t^W)^{\epsilon_{c,t}}(V_t^F)^{-\epsilon_{c,t}} - \varphi_t$$

$$\Rightarrow \varphi_t = (1-\epsilon_{c,t})(V_t^W)^{\epsilon_{c,t}}(V_t^F)^{-\epsilon_{c,t}}$$

Which implies:

$$\epsilon_{c,t}(V_t^W)^{\epsilon_{c,t}-1}(V_t^F)^{1-\epsilon_{c,t}} = (1-\epsilon_{c,t})(V_t^W)^{\epsilon_{c,t}}(V_t^F)^{-\epsilon_{c,t}} \Rightarrow \epsilon_{c,t}V_t^F = (1-\epsilon_{c,t})V_t^W \tag{31}$$

Substitute into the worker's and firm's surplus expressions: The worker's surplus from a match is given by:

$$V_t^W = h_{c,t}w_{c,t} - b_{c,t} + \frac{\chi_c}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}\right) + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})V_{t+1}^W\right\} \tag{1.39}$$

The firm's surplus from a match is:

$$V_t^F = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right\} \tag{1.40}$$

Substitute the expressions for V_t^F and V_t^W into the surplus-sharing condition (31):

$$\epsilon_{c,t}\lambda_t\left[h_{c,t}(\xi_t z_t A_t - w_{c,t}) + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right\}\right] = (1-\epsilon_{c,t})\lambda_t\left[h_{c,t}w_{c,t} - b_{c,t} + \frac{\chi_c}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}\right)\right.$$
$$\left. + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})V_{t+1}^W\right\}\right]$$

Now cancel the λ_t on both sides and group terms:

$$\epsilon_{c,t}\left[h_{c,t}(\xi_t z_t A_t - w_{c,t}) + \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+1}}\right\}\right] = (1-\epsilon_{c,t})\left[h_{c,t}w_{c,t} - b_{c,t} + \frac{\chi_c}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}\right)\right]$$
$$+ (1-\epsilon_{c,t})\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})V_{t+1}^W\right\}$$

Next, using that (31) implies that $\epsilon_{c,t}V_{t+1}^F = (1-\epsilon_{c,t})V_{t+1}^W$, and substituting in (1.40) with a one period forward lead yields:

$$V_{t+1}^W = \frac{\epsilon_{c,t+1}}{1-\epsilon_{c,t+1}}V_{t+1}^F$$

Substitute the firm's future surplus:

$$V_{t+1}^F = \frac{\iota}{\gamma_{c,t+1}} + \beta\mathbb{E}_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+2}}\right\}$$

Hence:

$$V_{t+1}^W = \frac{\epsilon_{c,t+1}}{1-\epsilon_{c,t+1}}\left[\frac{\iota}{\gamma_{c,t+1}} + \beta\mathbb{E}_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+2}}\right\}\right]$$

Substitute this into the expectation on the right-hand side:

$$\begin{aligned}
&\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})V^W_{t+1}\right\}\\
=&\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})(1-\lambda_{c,t+1})\cdot\frac{\epsilon_{c,t+1}}{1-\epsilon_{c,t+1}}\left[\frac{\iota}{\gamma_{c,t+1}}+\beta\mathbb{E}_{t+1}\left\{\frac{\lambda_{t+2}}{\lambda_{t+1}}(1-\vartheta_{c,t+1})\frac{\iota}{\gamma_{c,t+2}}\right\}\right]\right\}
\end{aligned}$$

Now collect all terms and isolate $h_{c,t}w_{c,t}$:

$$\begin{aligned}
h_{c,t}w_{c,t} = \epsilon_{c,t}&\left[h_{c,t}\xi_t z_t A_t+\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t+1})\left(1-(1-\lambda_{c,t+1})\cdot\frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t+1})\epsilon_{c,t}}\right)\cdot\frac{\iota}{\gamma_{c,t+1}}\right\}\right]\\
&+(1-\epsilon_{c,t})\left[b_{c,t}-\frac{\chi_c}{\lambda_t}\cdot\frac{(1-h_{c,t})^{1-\nu}-1}{1-\nu}\right] \qquad (1.41)
\end{aligned}$$

I House price equation

The maximisation problem:

$$\max_{V_t^{Sell},V_t^{Buy}} : \quad (V_t^{Sell})^{\epsilon'_{h,t}}(V_t^{Buy})^{1-\epsilon'_{h,t}}$$
$$S.t. : \quad V_{h,t}^{T} = V_t^{Sell} + V_t^{Buy}$$

Which gives rise to the Lagrangian:

$$\mathcal{L} = (V_t^S)^{\epsilon'_{h,t}}(V_t^B)^{1-\epsilon'_{h,t}} - \xi_t(V_t^S + V_t^B - V_{h,t}^T)$$

First Order Conditions:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial V_t^{Sell}} \equiv 0 &= \epsilon_{h,t}(V_t^{Sell})^{\epsilon'_{h,t}-1}(V_t^{Buy})^{1-\epsilon'_{h,t}} - \xi_t \\ \Rightarrow \xi_t &= \epsilon_{h,t}(V_t^{Sell})^{\epsilon'_{h,t}-1}(V_t^{Buy})^{1-\epsilon'_{h,t}}\end{aligned} \tag{32}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial V_t^{Buy}} \equiv 0 &= (1-\epsilon'_{h,t})(V_t^S)^{\epsilon'_{h,t}}(V_t^B)^{-\epsilon'_{h,t}} - \xi_t \\ \Rightarrow \xi_t &= (1-\epsilon'_{h,t})(V_t^{Sell})^{\epsilon'_{h,t}}(V_t^{Buy})^{-\epsilon'_{h,t}}\end{aligned} \tag{33}$$

Set $\xi_t = \xi_t$ in (32) and (33):

$$\begin{aligned}\epsilon'_{h,t}(V_t^{Sell})^{\epsilon'_{h,t}-1}(V_t^{Buy})^{1-\epsilon'_{h,t}} &= (1-\epsilon'_{h,t})(V_t^{Sell})^{\epsilon'_{h,t}}(V_t^{Buy})^{-\epsilon'_{h,t}} \\ \Rightarrow (1-\epsilon'_{h,t})V_t^{Sell} &= \epsilon'_{h,t}V_t^{Buy}\end{aligned}$$

Use (2.7) and (1.68) to show:

$$V_t^{Buy} = (V_t^N - P_t^h) - V_t^B \tag{2.7}$$

$$V_t^{Sell} = (V_t^B + V_t) - V_t^N \tag{1.68}$$

$$\Rightarrow (1-\epsilon'_{h,t})[V_t^B + V_t - V_t^N] = \epsilon'_{h,t}[V_t^N - P_t^h - V_t^B]$$

$$\Rightarrow [V_t^B + V_t - V_t^N] - \epsilon'_{h,t}V_t^B - \epsilon'_{h,t}V_t + \epsilon'_{h,t}V_t^N = \epsilon'_{h,t}V_t^N - \epsilon'_{h,t}P_t^h - \epsilon'_{h,t}V_t^B$$

$$\Rightarrow [V_t^B + V_t - V_t^N] - \epsilon'_{h,t}V_t = -\epsilon'_{h,t}P_t^h$$

$$\Rightarrow (\epsilon'_{h,t} + 1)V_t - V_t^B + V_t^N = \epsilon'_{h,t}P_t^h$$

$$\Rightarrow P_t^h = (1 + \frac{1}{\epsilon'_{h,t}})V_t + \frac{1}{\epsilon'_{h,t}}(V_t^N - V_t^B)$$

Let $\epsilon_{h,t} \equiv 1 - \frac{1}{\epsilon'_{h,t}}$, then the house price satisfies:

$$\Rightarrow P_t^h = (1-\epsilon_{h,t})(V_t^N - V_t^B) + \epsilon_{h,t}V_t \tag{1.69}$$

J Resource Constraint

Consider the households budget constraint (1.5):

$$C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = \Phi_t + L_{c,t}w_{c,t}h_{c,t} + (Q_t - L_{c,t})b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \tag{1.5}$$

Introduce the aggregated profit function (1.36):

$$\Phi_t = Y_t - w_{c,t}h_{c,t}L_{c,t} - \iota V_{c,t} \tag{1.36}$$

$$\Rightarrow C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = Y_t - w_{c,t}h_{c,t}L_{c,t} - \iota V_{c,t} + L_{c,t}w_{c,t}h_{c,t} + (Q_t - L_{c,t})b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t$$
$$\Rightarrow C_t + \mathcal{A}_{t+1} + T_t + \Omega_t = Y_t - \iota V_{c,t} + (Q_t - L_{c,t})b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t$$

Introduce the governments fiscal policy (1.71):

$$\begin{aligned} &C_t + \mathcal{A}_{t+1} + T_t + \Omega_t \\ =&Y_t - \iota V_{c,t} + (Q_t - L_{c,t})b_{c,t} + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \end{aligned}$$

$$b_{c,t}(Q_t - L_{c,t}) = T_t + N_t m_t^h + q_{h,t}(H_{t+1} - H_t) \tag{1.71}$$

$$\begin{aligned} \Rightarrow &C_t + \mathcal{A}_{t+1} + \Omega_t \\ =&Y_t - \iota V_{c,t} + N_t m_t^h + q_{h,t}(H_{t+1} - H_t) + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \end{aligned}$$

And the level of housing investment undertaken by households is given by (1.11):

$$\Omega_t = N_t m_t^h + \beta E_t\left\{(H_{t+1} - H_t)\left(\frac{\lambda_{t+1}}{\lambda_t}\hat{\mathcal{V}}_{t+1}\right)\right\} \tag{1.11}$$

$$\begin{aligned} \Rightarrow &C_t + \mathcal{A}_{t+1} + \beta E_t\left\{(H_{t+1} - H_t)\left(\frac{\lambda_{t+1}}{\lambda_t}\hat{\mathcal{V}}_{t+1}\right)\right\} \\ =&Y_t - \iota V_{c,t} + q_{h,t}(H_{t+1} - H_t) + w_{h,t}L_{h,t} + R_t\mathcal{A}_t \end{aligned}$$

Introduce the labour demand equation (1.60):

$$\frac{w_{h,t}}{\phi_t} = \beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}\right\} - q_{h,t} \Rightarrow w_{h,t} = \phi_t\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}\right\} - \phi_t q_{h,t} \tag{1.60}$$

$$\begin{aligned} \Rightarrow &C_t + \mathcal{A}_{t+1} + \beta E_t\left\{(H_{t+1} - H_t)\left(\frac{\lambda_{t+1}}{\lambda_t}\hat{\mathcal{V}}_{t+1}\right)\right\} \\ =&Y_t - \iota V_{c,t} + q_{h,t}(H_{t+1} - H_t) + L_{h,t}\left[\phi_t\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}\right\} - \phi_t q_{h,t}\right] + R_t\mathcal{A}_t \end{aligned}$$

Introduce (1.56):

$$H_{t+1} - H_t = \phi_t L_{h,t} \tag{1.56}$$

$$\begin{aligned} \Rightarrow &C_t + \mathcal{A}_{t+1} + \beta E_t\left\{\phi_t L_{h,t}\left(\frac{\lambda_{t+1}}{\lambda_t}\hat{\mathcal{V}}_{t+1}\right)\right\} \\ =&Y_t - \iota V_{c,t} + q_{h,t}\phi_t L_{h,t} + L_{h,t}\left[\phi_t\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\hat{V}_{t+1}\right\} - \phi_t q_{h,t}\right] + R_t\mathcal{A}_t \end{aligned}$$

$$\Rightarrow C_t + \mathcal{A}_{t+1} = Y_t - \iota V_{c,t} + R_t\mathcal{A}_t$$

Finally, imposing that savings in private bonds ($\mathcal{A}_t$) must be in zero net supply: $A_t = 0, \forall t$, we have:

$$Y_t = C_t + \iota V_{c,t} \tag{3.18}$$

A:3 Stationary representation:

There are two trend growth variables in the model, population growth described by (1.1), and productivity growth (1.4).

$$\frac{Q_{t+1}}{Q_t} = (1+\mu) \tag{1.1}$$

The growth rate of population affects the household behaviour due to aggregation through: (1.12), (1.13), (1.14), (1.15). By the definition of population (**??**), the employment and unemployment $(L_{c,t}, U_{c,t})$ law of motion is affected through: (1.21), (1.22). Similarly, the law of motion for housing market outcomes (F_t, N_t, B_t) is affected through: (1.48), (1.49), (1.50). As production depends on employment through $(z_t A_t L_{c,t} h_{c,t})$, output (1.35) grows with population through $(L_{c,t})$. By extension, the resource constraint grows with population through output in (1.77). Finally, since all agents making up the population require housing, the housing stock is defined as the sum of population and vacant housing units through (1.43), thus, the law of motion for development of land (3.11), and housing (1.56) grows with population.

$$A_{t+1} = (1+\varnothing)A_t \tag{1.4}$$

Meanwhile, the trend growth rate of productivity (A_t), raises output through the production function (1.35). Changes to output affects consumption through the resource constraint (1.77), which in turn affects the habit adjusted consumption bundle (1.15). The change consumption also affects the households optimal choice of (1.12) and the Euler equation (1.14). As the stochastic discount factor (λ_t) grows with productivity through (1.12), we must also control for the trend productivity growth rate in affected equations: the construction sector labour demand equation (1.60), and the value functions associated with the housing market: (1.61), (1.62), (1.63), (3.15), (1.66). And the transaction and house price equation (1.69).

The production function is also part of the intermediary good producing firms maximisation problem, and thus enters into the job-creation equalibrium (1.31). The production function is also part of the Nash bargaining problem faced by workers/firms in the labour market, thus affecting the wage equation (1.41), and by the extension the decision about how many hours to work (1.42).

A Households:

The aggregate representation of the households variables: (1.12), (1.13), (1.14), (1.15) are already stationary through the aggregation described in section: A.0.2.

$$A_t\lambda_t = \varrho_t(\frac{X_t}{Q_t})^{-\sigma} \tag{1.12}$$

$$\chi_h = \varrho_t(\frac{X_t}{Q_t})^{-\sigma}\frac{w_{h,t}}{A_t} \tag{1.13}$$

$$\varrho_t(\frac{X_t}{Q_t})^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}(\frac{X_{t+1}}{Q_{t+1}})^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}R_{t+1}\} \tag{1.14}$$

$$\frac{X_t}{Q_t} = \frac{C_t}{Q_tA_t} - \theta\frac{C_{t-1}}{A_{t-1}Q_{t-1}} \tag{1.15}$$

B Law of Motion for the Labour Market

Recall that by (1.1), population grows each period, to ensure a stationary representation (1.21) and (1.22) is obtained by normalising for population Q_t:

B.0.1 Law of Motion for Employment:

$$L_{c,t} = (1-\vartheta_{c,t})L_{c,t-1} + \gamma_{c,t}V_{c,t} \tag{1.21}$$

$$\rightarrow\frac{L_{c,t}}{Q_t} = (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_t} + \gamma_{c,t}\frac{V_{c,t}}{Q_t}$$

$$\rightarrow\frac{L_{c,t}}{Q_t} = (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \gamma_{c,t}\frac{V_{c,t}}{Q_t} \tag{3.4}$$

B.0.2 Law of Motion for Unemployment:

$$U_{c,t} = Q_t - (1-\vartheta_{c,t})L_{c,t-1} \tag{1.22}$$

$$\rightarrow\frac{U_{c,t}}{Q_t} = \frac{Q_t}{Q_t} - (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_t}$$

$$\rightarrow\frac{U_{c,t}}{Q_t} = 1 - (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \tag{3.5}$$

$$Y_t = z_t A_t L_{c,t} h_{c,t} \tag{1.35}$$

C Law of Motion for the Housing Market:

Recall that by (1.1), population grows each period, while population affects the types in the housing market through (**??**) to ensure a stationary representation of housing types we normalise for population Q_t:

C.0.1 Law of Motion for Homeowners:

$$N_t = (1-\vartheta_{h,t})N_{t-1} + \lambda_{h,t}B_t \tag{1.48}$$

$$\rightarrow \frac{N_t}{Q_t} = (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_t} + \lambda_{h,t}\frac{B_t}{Q_t}$$

$$\rightarrow \frac{N_t}{Q_t} = (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \lambda_{h,t}\frac{B_t}{Q_t} \tag{3.7}$$

C.0.2 Law of Motion for Permanent Renters:

$$F_t = F_{t-1} + (1-\psi_{t-1})\mu Q_{t-1} \tag{1.49}$$

$$\rightarrow \frac{F_t}{Q_t} = \frac{F_{t-1}}{Q_t} + (1-\psi_{t-1})\mu\frac{Q_{t-1}}{Q_t}$$

$$\rightarrow \frac{F_t}{Q_t} = \frac{F_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + (1-\psi_{t-1})\mu\frac{Q_{t-1}}{Q_t} \tag{3.8}$$

C.0.3 Law of Motion for Buyers:

$$B_t = Q_t - F_t - (1-\vartheta_{h,t})N_{t-1} \tag{1.50}$$

$$\rightarrow \frac{B_t}{Q_t} = 1 - \frac{F_t}{Q_t} - (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_t}$$

$$\rightarrow \frac{B_t}{Q_t} = 1 - \frac{F_t}{Q_t} - (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \tag{3.9}$$

D Output:

Since employment ($L_{c,t}$) grows between periods, a stationary representation is obtained by normalising for population Q_t:

$$Y_t = z_t A_t L_{c,t} h_{c,t} \tag{1.35}$$

$$\rightarrow \frac{Y_t}{Q_t} = z_t A_t h_{c,t} \frac{L_{c,t}}{Q_t} \tag{??}$$

E Resource Constraint:

Since output growth with population through ($L_{c,t}$), and directly through trend productivity, a stationary representation of the resource constraint requires normalising for both trend growth variables:

$$Y_t = C_t + \iota V_{c,t} \tag{1.77}$$

$$\rightarrow \frac{Y_t}{Q_t A_t} = \frac{C_t}{Q_t A_t} + \frac{\iota v_{c,t}}{Q_t A_t} \tag{3.18}$$

F Housing Stock:

Housing stock grows with population through the definition of population (**??**). We therefore normalising the expression of the housing stock (1.43) by Q_t to ensure a stationary representation:

$$H_t = Q_t + V_{h,t} \tag{1.43}$$

$$\rightarrow \frac{H_t}{Q_t} = 1 + \frac{V_{h,t}}{Q_t} \tag{3.6}$$

G Production of Developed land:

Due to the free entry into the housing market we know that $\hat{h}_t = h_t$ as discussed in section: **??**. Housing stock grows with population through (**??**). We therefore normalising the expression describing how much land gets developed (1.58) by Q_t to ensure a stationary representation:

$$\hat{H}_{t+1} - \hat{H}_t = \Lambda\left(\frac{q_{h,t}}{A_t}\right)(K_t^L - \hat{H}_t) \tag{1.58}$$

$$\rightarrow \frac{H_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \Lambda\left(\frac{q_{h,t}}{A_t}\right)(\frac{K_t^L}{Q_t} - \frac{H_t}{Q_t})$$

$$\rightarrow \frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \Lambda\left(\frac{q_{h,t}}{A_t}\right)(\frac{K_t^L}{Q_t} - \frac{H_t}{Q_t}) \tag{3.11}$$

H Law of Motion for Housing:

Housing stock grows with population through (**??**). We therefore normalising the expression of the housing law of motion (1.56) by Q_t to ensure a stationary representation:

$$H_{t+1} - H_t = \phi_t L_{h,t} \tag{1.56}$$

$$\rightarrow \frac{H_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \phi_t \frac{L_{h,t}}{Q_t}$$

$$\rightarrow \frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \phi_t \frac{L_{h,t}}{Q_t} \tag{3.10}$$

I Construction sector wage equation:

Since the trend productivity growth rate affects the stochastic discount factor (λ_t), we need to normalise the expression of construction sector wages (1.60) by the trend growth rate (A_t):

$$\frac{w_{h,t}}{\phi_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right\} - q_{h,t} \tag{1.60}$$

$$\rightarrow \frac{w_{h,t}}{\phi_t A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{V_{t+1}}{A_t} \right\} - \frac{q_{h,t}}{A_t}$$

$$\rightarrow \frac{w_{h,t}}{\phi_t A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}}{A_{t+1}} \right\} - \frac{q_{h,t}}{A_t} \tag{2.6}$$

J Value Functions in the Housing Market:

As the trend productivity growth rate affects the stochastic discount factor (λ_t) through (1.12), we need to normalise the expressions of the value functions in the housing market (1.61), (1.62), (1.63), (3.15), and (1.66). Similarly, the transaction and house price equation (1.69) is also made statinoary through normalising the trend ptoductivity growth rate A_t.

J.0.1 Value Function of Permanent Renters:

$$V_t^F = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} V_{t+1}^F \right\} - r_t^h \tag{1.61}$$

$$\rightarrow \frac{V_t^F}{A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{V_{t+1}^F}{A_t} \right\} - \frac{r_t^h}{A_t}$$

$$\rightarrow \frac{V_t^F}{A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}^F}{A_t + 1} \right\} - \frac{r_t^h}{A_t} \tag{3.12}$$

J.0.2 Value Function of Homeowners:

$$\begin{aligned} V_t^N = & -m_t^h + \frac{z_t^h}{\lambda_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \vartheta_{h,t}(1 - \lambda_{h,t+1})) V_{t+1}^N \right. \right. \\ & \left. \left. + \vartheta_{h,t}(\hat{V}_{t+1} - \lambda_{h,t+1} P_{t+1}^h) + \vartheta_{h,t}(1 - \lambda_{h,t+1}) V_{t+1}^B \right] \right\} \end{aligned} \tag{1.62}$$

$$\begin{aligned} \rightarrow \frac{V_t^N}{A_t} = & -\frac{m_t^h}{A_t} + \frac{z_t^h}{\lambda_t A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \vartheta_{h,t}(1 - \lambda_{h,t+1})) \frac{V_{t+1}^N}{A_t} \right. \right. \\ & \left. \left. + \vartheta_{h,t} \left(\frac{V_{t+1}}{A_t} - \lambda_{h,t+1} \frac{P_{t+1}^h}{A_t} \right) + \vartheta_{h,t}(1 - \lambda_{h,t+1}) \frac{V_{t+1}^B}{A_t} \right] \right\} \end{aligned}$$

$$\begin{aligned} \rightarrow \frac{V_t^N}{A_t} = & -\frac{m_t^h}{A_t} + \frac{z_t^h}{\lambda_t A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[(1 - \vartheta_{h,t}(1 - \lambda_{h,t+1})) \frac{V_{t+1}^N}{A_{t+1}} \right. \right. \\ & \left. \left. + \vartheta_{h,t} \left(\frac{V_{t+1}}{A_{t+1}} - \lambda_{h,t+1} \frac{P_{t+1}^h}{A_{t+1}} \right) + \vartheta_{h,t}(1 - \lambda_{h,t+1}) \frac{V_{t+1}^B}{A_{t+1}} \right] \right\} \end{aligned} \tag{3.13}$$

J.0.3 Value Function of Searching Buyers:

$$V_t^B = -r_t^{h*} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\lambda_{h,t+1}) V_{t+1}^B + \lambda_{h,t+1} (V_{t+1}^N - P_{t+1}^h) \right] \right\} \quad (1.63)$$

$$\rightarrow \frac{V_t^B}{A_t} = -\frac{r_t^{h*}}{A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\lambda_{h,t+1}) \frac{V_{t+1}^B}{A_t} + \lambda_{h,t+1} \left(\frac{V_{t+1}^N}{A_t} - \frac{P_{t+1}^h}{A_t} \right) \right] \right\}$$

$$\rightarrow \frac{V_t^B}{A_t} = -\frac{r_t^{h*}}{A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[(1-\lambda_{h,t+1}) \frac{V_{t+1}^B}{A_{t+1}} + \lambda_{h,t+1} \left(\frac{V_{t+1}^N}{A_{t+1}} - \frac{P_{t+1}^h}{A_{t+1}} \right) \right] \right\} \quad (3.14)$$

J.0.4 Value of a Vacant House 1:

$$\hat{V}_t = r_t^{h*} - m_t^h + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right\} = V_t \quad (1.65)$$

$$\rightarrow \frac{V_t}{A_t} = \frac{r_t^{h*}}{A_t} - \frac{m^h}{A_t} + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{V_{t+1}}{A_t} \right\}$$

$$\rightarrow \frac{V_t}{A_t} = \frac{r_t^{h*}}{A_t} - \frac{m^h}{A_t} + \beta E_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}}{A_{t+1}} \right\} \quad (3.15)$$

J.0.5 Value of a Vacant House 2:

$$V_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\gamma_{h,t+1} P_{t+1}^h + (1-\gamma_{h,t+1}) \hat{V}_{t+1} \right] \right\} \quad (1.66)$$

$$\rightarrow \frac{V_t}{A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\gamma_{h,t+1} \frac{P_{t+1}^h}{A_t} + (1-\gamma_{h,t+1}) \frac{V_{t+1}}{A_t} \right] \right\}$$

$$\rightarrow \frac{V_t}{A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[\gamma_{h,t+1} \frac{P_{t+1}^h}{A_{t+1}} + (1-\gamma_{h,t+1}) \frac{V_{t+1}}{A_{t+1}} \right] \right\} \quad (3.16)$$

K House Price Equation:

$$P_t^h = (1-\epsilon_{h,t})(V_t^N - V_t^B) + \epsilon_{h,t} V_t \quad (1.69)$$

$$\rightarrow \frac{P_t^h}{A_t} = (1-\epsilon_{h,t}) \left(\frac{V_t^N}{A_t} - \frac{V_t^B}{A_t} \right) + \epsilon_{h,t} \frac{V_t}{A_t} \quad (3.17)$$

A:4 Full System:

The system has 34 endogenous variables: $\{Y_t, X_t, C_t, \lambda_t, \xi_t, L_{c,t}, U_{c,t}, \omega_{c,t}, w_{c,t}, h_{c,t}, \lambda_{c,t}, \gamma_{c,t}, \vartheta_{c,t}, w_{h,t}, \omega_{h,t}, L_{h,t}, \lambda_{h,t}, \gamma_{h,t}$ $\mathcal{V}_t^N, \mathcal{V}_t^B, r_t^{h*}, r_t^h, P_t^h, \mathcal{V}_t, B_t, N_t, F_t, S_t, H_t, q_{h,t}\ R_t, i_t, \pi_t, \kappa_{c,t}, \vartheta_{c,t}, \kappa_{h,t}, \vartheta_{h,t}\}$. Three exogenous variables: $\{Q_t, A_t, K_t\}$. And, 11 exogenous shock variables: $\{\varrho_t, \chi_{c,t}, \epsilon_t, z_t, \epsilon_{c,t}, \psi_t, \tau_t, \phi_t, \kappa_{\tilde{c},t}, \kappa_{\tilde{h},t}, m_t\}$

A Aggregated Variables

A.1 Endogenous Variables

$$1: A_t\lambda_t = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma} \qquad \rightarrow A_t\lambda_t \tag{1.12}$$

$$2: \chi_h = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma}\frac{w_{h,t}}{A_t} \qquad \rightarrow w_{h,t} \tag{1.13}$$

$$3: \varrho_t(\frac{X_t}{Q_t})^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}(\frac{X_{t+1}}{Q_{t+1}})^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}R_{t+1}\} \qquad \rightarrow R_t \tag{1.14}$$

$$: R_t = \frac{i_t}{\pi_t} \qquad \rightarrow R_t$$

$$4: \frac{X_t}{Q_t} = \frac{C_t}{Q_tA_t} - \theta\frac{C_{t-1}}{A_{t-1}Q_{t-1}} \qquad \rightarrow \frac{X_t}{Q_t} \tag{1.15}$$

$$5: \gamma_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{V_{c,t}} = \kappa_{c,t}\omega_{c,t}^{-\delta_{c,t}} \qquad \rightarrow \gamma_{c,t} \tag{1.18}$$

$$6: \lambda_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{U_{c,t}} = \gamma_{c,t}\omega_{c,t} \qquad \rightarrow \lambda_{c,t} \tag{1.19}$$

$$7: \frac{L_{c,t}}{Q_t} = (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \gamma_{c,t}\frac{V_{c,t}}{Q_t} \qquad \rightarrow \frac{L_{c,t}}{Q_t} \tag{3.4}$$

$$8: \frac{U_{c,t}}{Q_t} = 1 - (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \qquad \rightarrow \frac{U_{c,t}}{Q_t} \tag{1.22}$$

$$9: \frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t})\frac{\iota_{t+1}}{\gamma_{c,t+1}}\right\} \qquad \rightarrow \omega_{c,t} \tag{1.31}$$

$$10: (\pi_t+1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1-(1-\varsigma)\left(\frac{\epsilon}{(\epsilon-1)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\epsilon}\right) \qquad \rightarrow \pi_t \tag{1.32}$$

$$: K_{1,t} = w_{c,t}\frac{Y_t}{z_tA_t} + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon+1}K_{1,t+1}\right\} \tag{1.33}$$

$$: K_{2,t} = Y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon}K_{2,t+1}\right\} \tag{1.34}$$

$$11: \frac{Y_t}{Q_t} = z_t A_t h_{c,t} \frac{L_{c,t}}{Q_t} \qquad \rightarrow \frac{Y_t}{Q_t} \qquad (??)$$

$$\begin{aligned}12: & h_{c,t} w_{c,t} = \epsilon_{c,t} \Big[h_{c,t} z_t \xi_t \\ & + \beta(1-\vartheta_{c,t}) \mathbb{E}_t \left\{ \frac{\lambda_t}{\lambda_{t+1}} \left(1 - (1-\lambda_{c,t+1}) \frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t})\epsilon_{c,t}} \right) \frac{\iota_{t+1}}{\gamma_{c,t+1}} \right\} \Big] \\ & + \beta(1-\epsilon_{c,t}) \left[b_{c,t} + \frac{\chi_c}{\lambda_t} \left(\frac{(1-h_{c,t})^{1-\nu} - 1}{(1-\nu)} \right) \right] \end{aligned} \qquad \rightarrow w_{c,t} \qquad (1.41)$$

$$13: z_t \xi_t A_t = \frac{\chi_c}{\lambda_t} (1-h_{c,t})^{-\nu} \qquad \rightarrow \xi_t \qquad (1.42)$$

$$14: \omega_{h,t} \equiv \frac{B_t}{S_t} \qquad \rightarrow \omega_{h,t} \qquad (1.45)$$

$$15: \gamma_{h,t} \equiv \frac{M^H(B_t, S_t)}{S_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h} \qquad \rightarrow \gamma_{h,t} \qquad (1.46)$$

$$16: \lambda_{h,t} \equiv \frac{M_{h,t}(B_t, S_t)}{B_t} = \kappa_{h,t} \omega_{h,t}^{\delta_h - 1} \qquad \rightarrow \lambda_{h,t} \qquad (1.47)$$

$$17: \frac{N_t}{Q_t} = (1-\vartheta_{h,t}) \frac{N_{t-1}}{Q_{t-1}} \frac{Q_{t-1}}{Q_t} + \lambda_{h,t} \frac{B_t}{Q_t} \qquad \rightarrow \frac{N_t}{Q_t} \qquad (3.7)$$

$$18: \frac{F_t}{Q_t} = \frac{F_{t-1}}{Q_{t-1}} \frac{Q_{t-1}}{Q_t} + (1-\psi_{t-1}) \mu \frac{Q_{t-1}}{Q_t} \qquad \rightarrow \frac{F_t}{Q_t} \qquad (3.8)$$

$$19: \frac{B_t}{Q_t} = 1 - \frac{F_t}{Q_t} - (1-\vartheta_{h,t}) \frac{N_{t-1}}{Q_{t-1}} \frac{Q_{t-1}}{Q_t} \qquad \frac{B_t}{Q_t} \qquad (3.9)$$

$$20: \omega_{h,t} \equiv \frac{B_t}{S_t} = \frac{(1-\tau) B_t}{H_t - Q_t} \qquad \rightarrow S_t \qquad (1.53)$$

$$21: \frac{H_{t+1}}{Q_{t+1}} \frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \phi_t \frac{L_{h,t}}{Q_t} \qquad \rightarrow \frac{L_{h,t}}{Q_t} \qquad (1.56)$$

$$22: \frac{w_{h,t}}{\phi_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \hat{V}_{t+1} \right\} - q_{h,t} \qquad \rightarrow q_{h,t} \qquad (1.60)$$

$$\begin{aligned}23: & \frac{V_t^N}{A_t} = -\frac{m_t^h}{A_t} + \frac{z_t^h}{\lambda_t A_t} + \beta \mathbb{E}_t \Big\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \Big[(1 - \vartheta_{h,t}(1-\lambda_{h,t+1})) \frac{V_{t+1}^N}{A_{t+1}} \\ & + \vartheta_{h,t} \left(\frac{V_{t+1}}{A_{t+1}} - \lambda_{h,t+1} \frac{P_{t+1}^h}{A_{t+1}} \right) + \vartheta_{h,t}(1-\lambda_{h,t+1}) \frac{V_{t+1}^B}{A_{t+1}} \Big] \Big\} \end{aligned} \qquad \rightarrow \frac{V_t^N}{A_t} \qquad (3.13)$$

$$24:\frac{V_t^B}{A_t}=-\frac{r_t^{h*}}{A_t}+\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\left[(1-\lambda_{h,t+1})\frac{V_{t+1}^B}{A_{t+1}}+\lambda_{h,t+1}\left(\frac{V_{t+1}^N}{A_{t+1}}-\frac{P_{t+1}^h}{A_{t+1}}\right)\right]\right\} \quad \rightarrow \frac{V_t^B}{A_t}$$

(3.14)

$$25:\frac{V_t}{A_t}=\frac{r_t^{h*}}{A_t}-\frac{m^h}{A_t}+\beta E_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\frac{V_{t+1}}{A_{t+1}}\right\} \quad \rightarrow \frac{r_t^{h*}}{A_t}$$

(3.15)

$$26:\frac{V_t}{A_t}=\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\left[\gamma_{h,t+1}\frac{P_{t+1}^h}{A_{t+1}}+(1-\gamma_{h,t+1})\frac{V_{t+1}}{A_{t+1}}\right]\right\} \quad \rightarrow \frac{V_t}{A_t}$$

(3.16)

$$27:P_t^h=(1-\epsilon_{h,t})(V_t^N-V_t^B)+\epsilon_{h,t}V_t \quad \rightarrow P_t^h$$

(1.69)

$$28:r_t^h=\upsilon r_{t-1}^h+(1-\upsilon)\,r_t^{h*} \quad \rightarrow r_t^h$$

(1.70)

$$29:\frac{1+i_t}{1+i_{ss}}=\left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\alpha_i}\cdot$$

$$\left(\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\alpha_\pi}\left(\frac{Y_t}{(1+\mu)(1+\gamma)Y_{t-1}}\right)^{\alpha_y}\right)^{1-\alpha_i}\exp(m_t) \quad \rightarrow i_t$$

(1.72)

$$30:\frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t}-\frac{H_t}{Q_t}=\Lambda\left(\frac{q_{h,t}}{A_t}\right)\left(\frac{K_t^L}{Q_t}-\frac{H_t}{Q_t}\right) \quad \rightarrow H_t$$

(3.11)

$$31:\kappa_{c,t}=\tilde{\kappa}_{c,t}\left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\kappa} \quad \rightarrow \kappa_{c,t}$$

(1.73)

$$32:\vartheta_{c,t}=\tilde{\vartheta}_{c,t}\left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\vartheta} \quad \rightarrow \vartheta_{c,t}$$

(1.74)

$$33:\kappa_{h,t}=\tilde{\kappa}_{h,t}\left(\frac{\omega_{c,t}}{\omega_{c,ss}}\right)^{\eta_\kappa}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)^{-\theta_\kappa} \quad \rightarrow \kappa_{h,t}$$

(1.75)

$$34:\vartheta_{h,t}=\tilde{\vartheta}_{h,t}\left(\frac{\omega_{c,t}}{\omega_{c,ss}}\right)^{\eta_\vartheta}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)^{\theta_\vartheta} \quad \rightarrow \vartheta_{h,t}$$

(1.76)

$$35:\frac{P^H}{h_c w_c}=\frac{P_t^H}{h_{c,t}w_{c,t}} \quad \rightarrow h_{c,t}$$

$$36:\frac{Y_t}{Q_t A_t}=\frac{C_t}{Q_t A_t}+\frac{\iota v_{c,t}}{Q_t A_t} \quad \rightarrow \frac{C_t}{Q_t A_t}$$

(3.18)

A.2 Exogenous Variables:

$$35 : \frac{Q_{t+1}}{Q_t} = (1+\mu) \qquad \Rightarrow \frac{Q_{t+1}}{Q_t} \qquad (1.1)$$

$$36 : A_{t+1} = (1+\varnothing)A_t \qquad \Rightarrow A_t \qquad (1.4)$$

$$37 : K^L_{t+1} = (1+\varkappa_t)K^L_t \qquad \Rightarrow K_t \qquad (1.57)$$

A.3 Shocks:

$$38 : (\varrho_t) = \rho_\varrho(\varrho_{t-1}) + \epsilon_\varrho; \epsilon_\varrho \sim (0, \sigma^2_{\varrho,i}); \qquad \rightarrow \varrho_t$$

$$39 : (\chi_{c,t}) = \rho_{\chi_c}(\chi_{c,t-1}) + \epsilon_{\chi_c}; \epsilon_{\chi_c} \sim (0, \sigma^2_{\chi,i}); \qquad \rightarrow \chi_{c,t}$$

$$40 : (\epsilon_t) = \rho_\epsilon(\epsilon_{h,t-1}) + \epsilon_\epsilon; \epsilon_{\epsilon_t} \sim (0, \sigma^2_{\epsilon_i}); \qquad \rightarrow \epsilon_t$$

$$41 : (z_t) = \rho_z(z_{t-1}) + \epsilon_z; \epsilon_z \sim (0, \sigma^2_{z,i}); \qquad \rightarrow z_t$$

$$42 : (\epsilon_{c,t}) = \rho_{\epsilon_{c,t}}(\epsilon_{c,t-1}) + \epsilon_{\epsilon_{c,t}}; \epsilon_{\epsilon_{c,t}} \sim (0, \sigma^2_{\epsilon_{c,i}}); \qquad \rightarrow \epsilon_{c,t}$$

$$43 : (\psi_t) = \rho_\psi(\psi_{t-1}) + \epsilon_\psi; \epsilon_{\psi_{c,t}} \sim (0, \sigma^2_{\epsilon_i}); \qquad \rightarrow \psi_t$$

$$44 : (\tau_t) = \rho_\tau(\tau_{h,t-1}) + \epsilon_\tau; \epsilon_{\tau_t} \sim (0, \sigma^2_{\tau_i}); \qquad \rightarrow \tau_t$$

$$45 : (\phi_t) = \rho_\phi(\phi_{h,t-1}) + \epsilon_{\phi_t}; \epsilon_{\phi_t} \sim (0, \sigma^2_{\phi_t}) \qquad \rightarrow \phi_t$$

$$46 : (\tilde{\kappa}_{c,t}) = \rho_{\tilde{\kappa}_{c,t}}(\tilde{\kappa}_{c,t-1}) + \epsilon_{\tilde{\kappa}_{c,t}}; \epsilon_{\tilde{\kappa}_{c,t}} \sim (0, \sigma^2_{\tilde{\kappa}_{c,i}}); \qquad \rightarrow \tilde{\kappa}_{c,t}$$

$$47 : (\tilde{\kappa}_{h,t}) = \rho_{\tilde{\kappa}_{h,t}}(\tilde{\kappa}_{h,t-1}) + \epsilon_{\tilde{\kappa}_{h,t}}; \epsilon_{\tilde{\kappa}_{h,t}} \sim (0, \sigma^2_{\tilde{\kappa}_{h,t}}); \qquad \rightarrow \tilde{\kappa}_{h,t}$$

$$48 : m_t = \epsilon_{m_t}; \epsilon_{m_t} \sim (0, \sigma^2_{\tilde{\kappa}_{h,t}}); \qquad \rightarrow m_t$$

B Normalised Representation

Let: $x_t = \frac{X_t}{Q_t}, c_t = \frac{C_t}{Q_t A_t}, \lambda_t = \lambda_t A_t, y_t = \frac{Y_t}{Q_t A_t}, \iota = \frac{\iota}{A_t}, l_{c,t} = \frac{L_{c,t}}{Q_t}, v_{c,t} = \frac{V_{c,t}}{Q_t}, u_t = \frac{U_{c,t}}{Q_t}, n_t = \frac{N_t}{Q_t}, f_t = \frac{F_t}{Q_t}, b_t = \frac{B_t}{Q_t}, l_{h,t} = \frac{L_{h,t}}{Q_t}, h_t = \frac{H_t}{Q_t}, k_t^L = \frac{K_t^L}{Q_t}, v_t^N = \frac{\mathcal{V}_t^N}{A_t}, m_t^h = \frac{m_t^h}{A_t}, v_t^B = \frac{\mathcal{V}_t^B}{A_t}, r_t^h = \frac{r_t^h}{A_t}, v_t^F = \frac{\mathcal{V}_t^F}{A_t}, p_t^h = \frac{P_t^h}{A_t}, w_{c,t} = \frac{w_{c,t}}{A_t}, b_{c,t} = \frac{b_{c,t}}{A_t}, w_{h,t} = \frac{w_{h,t}}{A_t}, q_{h,t} = \frac{q_{h,t}}{A_t}$. And, introduce the process for the exogenous variables: $\frac{Q_{t+1}}{Q_t} = (1+\mu)$, $\frac{A_{t+1}}{A_t} = (1+\varnothing)A_t$, $K_{t+1}^L = (1+\varkappa_t)K_t^L$.

B.1 Endogenous Variables

$$1: A_t\lambda_t = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma} \tag{1.12}$$

$$:\lambda_t = \varrho_t x_t^{-\sigma} \qquad \rightarrow \lambda_t \tag{1.12}$$

$$2: \chi_h = \varrho_t\left(\frac{X_t}{Q_t}\right)^{-\sigma}\frac{w_{h,t}}{A_t} \tag{1.13}$$

$$:\chi_h = \varrho_t x_t^{-\sigma} w_{h,t} \qquad \rightarrow w_{h,t} \tag{1.13}$$

$$3: \varrho_t(\frac{X_t}{Q_t})^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}(\frac{X_{t+1}}{Q_{t+1}})^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}R_{t+1}\} \tag{1.14}$$

$$:\varrho_t x_t^{-\sigma}\frac{Q_t}{A_t} = \beta\mathbb{E}_t\{\varrho_{t+1}x_{t+1}^{-\sigma}\frac{Q_{t+1}}{A_{t+1}}R_{t+1}\} \tag{1.14}$$

$$:\varrho_t x_t^{-\sigma} = \beta\mathbb{E}_t\{\varrho_{t+1}x_{t+1}^{-\sigma}\frac{(1+\mu)}{(1+\varnothing)}R_{t+1}\} \qquad \rightarrow R_t \tag{1.14}$$

$$4: \frac{X_t}{Q_t} = \frac{C_t}{Q_t A_t} - \theta\frac{C_{t-1}}{A_{t-1}Q_{t-1}} \tag{1.15}$$

$$:x_t^{-\sigma} = c_t - \theta c_{t-1} \qquad \rightarrow x_t \tag{1.15}$$

$$5: \gamma_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{V_{c,t}} = \kappa_{c,t}\omega_{c,t}^{-\delta_{c,t}} \qquad \rightarrow \gamma_{c,t} \tag{1.18}$$

$$:\kappa_{c,t} = \tilde{\kappa}_{c,t}\left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\kappa} \qquad \rightarrow \kappa_{c,t} \tag{1.73}$$

$$6: \lambda_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{U_{c,t}} = \gamma_{c,t}\omega_{c,t} \qquad \rightarrow \lambda_{c,t} \tag{1.19}$$

$$:\kappa_{c,t} = \tilde{\kappa}_{c,t}\left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\kappa} \qquad \rightarrow \kappa_{c,t} \tag{1.73}$$

$$7: \frac{L_{c,t}}{Q_t} = (1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \gamma_{c,t}\frac{V_{c,t}}{Q_t} \qquad (3.4)$$

$$: l_{c,t} = (1-\vartheta_{c,t})l_{c,t-1}\frac{Q_{t-1}}{Q_t} + \gamma_{c,t}v_{c,t} \qquad (3.4)$$

$$: l_{c,t} = \frac{(1-\vartheta_{c,t})}{(1+\mu)}l_{c,t-1} + \gamma_{c,t}v_{c,t} \qquad \to l_{c,t} \qquad (3.4)$$

$$8: \frac{U_{c,t}}{Q_t} = 1-(1-\vartheta_{c,t})\frac{L_{c,t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \qquad (3.5)$$

$$: u_{c,t} = 1-(1-\vartheta_{c,t})l_{c,t-1}\frac{Q_{t-1}}{Q_t} \qquad (3.5)$$

$$: u_{c,t} = 1-\frac{(1-\vartheta_{c,t})}{(1+\mu)}l_{c,t-1} \qquad \to u_{c,t} \qquad (3.5)$$

$$9: \frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t A_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t})\frac{\iota_{t+1}}{\gamma_{c,t+1}}\right\} \qquad \to \omega_{c,t} \qquad (1.31)$$

$$: \frac{\iota}{\gamma_{c,t}A_t} = h_{c,t}(\xi_t z_t - \frac{w_{c,t}}{A_t}) + (\beta\varsigma)\mathbb{E}_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}(1-\vartheta_{c,t})\frac{\iota_{t+1}}{\gamma_{c,t+1}A_{t+1}}\right\} \qquad \to \omega_{c,t} \qquad (1.31)$$

$$: \frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t - w_{c,t}) + (\beta\varsigma)\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1-\vartheta_{c,t})\frac{\iota_{t+1}}{\gamma_{c,t+1}}\right\} \qquad \to \omega_{c,t} \qquad (1.31)$$

$$10: (\pi_t+1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1-(1-\varsigma)\left(\frac{\epsilon}{(\epsilon-1)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\epsilon}\right) \qquad \to \pi_t \qquad (1.32)$$

$$: K_{1,t} = w_{c,t}\frac{Y_t}{z_t A_t} + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon+1}K_{1,t+1}\right\} \qquad (1.33)$$

$$: K_{1,t} = w_{c,t}y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon+1}K_{1,t+1}\right\} \qquad (1.33)$$

$$: K_{2,t} = Y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon}K_{2,t+1}\right\} \qquad (1.34)$$

$$: K_{2,t} = y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon}K_{2,t+1}\right\} \qquad (1.34)$$

$$11: \frac{Y_t}{Q_t} = z_t A_t h_{c,t}\frac{L_{c,t}}{Q_t} \qquad (??)$$

$$: y_t = z_t h_{c,t} l_{c,t} \qquad \to y_t \qquad (??)$$

$$12: h_{c,t}\frac{w_{c,t}}{A_t} = \epsilon_{c,t}\left[h_{c,t}z_t\xi_t\right.$$
$$+\beta(1-\vartheta_{c,t})\mathbb{E}_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\left(1-(1-\lambda_{c,t+1})\frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t})\epsilon_{c,t}}\right)\frac{\iota_{t+1}}{A_{t+1}\gamma_{c,t+1}}\right\}\Bigg]$$
$$+\beta(1-\epsilon_{c,t})\left[\frac{b_{c,t}}{A_t}+\frac{\chi_c}{A_t\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right)\right] \quad \to w_{c,t} \tag{1.41}$$

$$: h_{c,t}w_{c,t} = \epsilon_{c,t}\left[h_{c,t}z_t\xi_t\right.$$
$$+\beta(1-\vartheta_{c,t})\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(1-(1-\lambda_{c,t+1})\frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t})\epsilon_{c,t}}\right)\frac{\iota_{t+1}}{\gamma_{c,t+1}}\right\}\Bigg]$$
$$+\beta(1-\epsilon_{c,t})\left[b_{c,t}+\frac{\chi_c}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right)\right] \quad \to w_{c,t} \tag{1.41}$$

$$13: z_t\xi_t A_t = \frac{\chi_c}{\lambda_t}(1-h_{c,t})^{-\nu} \quad \to \xi_t \tag{1.42}$$

$$14: \omega_{h,t} \equiv \frac{B_t}{S_t} \quad \to \omega_{h,t} \tag{1.45}$$

$$15: \gamma_{h,t} \equiv \frac{M^H(B_t,S_t)}{S_t} = \kappa_{h,t}\omega_{h,t}^{\delta_h} \quad \to \gamma_{h,t} \tag{1.46}$$

$$: \kappa_{h,t} = \tilde{\kappa}_{h,t}\left(\frac{\omega_{c,t}}{\omega_{c,ss}}\right)^{\eta_\kappa}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)^{-\theta_\kappa} \quad \to \kappa_{h,t} \tag{1.75}$$

$$16: \lambda_{h,t} \equiv \frac{M_{h,t}(B_t,S_t)}{B_t} = \kappa_{h,t}\omega_{h,t}^{\delta_h-1} \quad \to \lambda_{h,t} \tag{1.47}$$

$$17: \frac{N_t}{Q_t} = (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + \lambda_{h,t}\frac{B_t}{Q_t} \quad (1.48)$$

$$: n_t = (1-\vartheta_{h,t})n_{t-1}\frac{Q_{t-1}}{Q_t} + \lambda_{h,t}b_t \quad (1.48)$$

$$: n_t = \frac{(1-\vartheta_{h,t})}{(1+\mu)}n_{t-1} + \lambda_{h,t}b_t \qquad \to n_t \quad (1.48)$$

$$18: \frac{F_t}{Q_t} = \frac{F_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} + (1-\psi_{t-1})\mu\frac{Q_{t-1}}{Q_t} \quad (3.8)$$

$$: f_t = f_{t-1}\frac{Q_{t-1}}{Q_t} + (1-\psi_{t-1})\mu\frac{Q_{t-1}}{Q_t} \quad (3.8)$$

$$: f_t = f_{t-1}\frac{1}{(1+\mu)} + (1-\psi_{t-1})\mu\frac{1}{(1+\mu)} \quad (3.8)$$

$$: f_t(1+\mu) = f_{t-1} + (1-\psi_{t-1})\mu \qquad \to f_t \quad (3.8)$$

$$19: \frac{B_t}{Q_t} = 1 - \frac{F_t}{Q_t} - (1-\vartheta_{h,t})\frac{N_{t-1}}{Q_{t-1}}\frac{Q_{t-1}}{Q_t} \quad (3.9)$$

$$: b_t = 1 - f_t - (1-\vartheta_{h,t})n_t\frac{Q_{t-1}}{Q_t} \quad (3.9)$$

$$: b_t = 1 - f_t - n_t\frac{(1-\vartheta_{h,t})}{(1+\mu)} \qquad \to b_t \quad (3.9)$$

$$20: \omega_{h,t} \equiv \frac{B_t}{S_t} = \frac{(1-\tau)B_t}{H_t - Q_t} \qquad \rightarrow S_t \tag{1.53}$$

$$: \omega_{h,t} \equiv \frac{B_t}{S_t} = \frac{(1-\tau)B_t}{H_t - Q_t} \qquad \rightarrow S_t \tag{1.53}$$

$$21: \frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t} - \frac{H_t}{Q_t} = \phi_t \frac{L_{h,t}}{Q_t} \tag{3.10}$$

$$: h_{t+1}\frac{Q_{t+1}}{Q_t} - h_t = \phi_t l_{h,t} \tag{3.10}$$

$$: h_{t+1}(1+\mu) - h_t = \phi_t l_{h,t} \qquad \rightarrow l_{h,t} \tag{3.10}$$

$$22: \frac{w_{h,t}}{\phi_t A_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \frac{V_{t+1}}{A_{t+1}} \right\} - \frac{q_{h,t}}{A_t} \tag{1.60}$$

$$: \frac{w_{h,t}}{\phi_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} V_{t+1} \right\} - q_{h,t} \qquad \rightarrow q_{h,t} \tag{1.60}$$

$$23: \frac{V_t^N}{A_t} = -\frac{m_t^h}{A_t} + \frac{z_t^h}{\lambda_t A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[(1 - \vartheta_{h,t}(1-\lambda_{h,t+1}))\frac{V_{t+1}^N}{A_{t+1}} \right.\right.$$
$$\left.\left. + \vartheta_{h,t}\left(\frac{V_{t+1}}{A_{t+1}} - \lambda_{h,t+1}\frac{P_{t+1}^h}{A_{t+1}} \right) + \vartheta_{h,t}(1-\lambda_{h,t+1})\frac{V_{t+1}^B}{A_{t+1}} \right] \right\} \tag{3.13}$$

$$: v_t^N = -m_t^h + \frac{z_t^h}{\lambda_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \vartheta_{h,t}(1-\lambda_{h,t+1}))v_{t+1}^N \right.\right.$$
$$\left.\left. + \vartheta_{h,t}(\hat{V}_{t+1} - \lambda_{h,t+1}p_{t+1}^h) + \vartheta_{h,t}(1-\lambda_{h,t+1})v_{t+1}^B \right] \right\} \qquad \rightarrow v_t^N \tag{1.62}$$

$$24: \frac{V_t^B}{A_t} = -\frac{r_t^{h*}}{A_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1} A_{t+1}}{\lambda_t A_t} \left[(1-\lambda_{h,t+1})\frac{V_{t+1}^B}{A_{t+1}} + \lambda_{h,t+1}\left(\frac{V_{t+1}^N}{A_{t+1}} - \frac{P_{t+1}^h}{A_{t+1}} \right) \right] \right\} \tag{3.14}$$

$$: v_t^B = -r_t^{h*} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\lambda_{h,t+1})v_{t+1}^B + \lambda_{h,t+1}\left(v_{t+1}^N - p_{t+1}^h \right) \right] \right\} \qquad \rightarrow v_t^B \tag{3.14}$$

$$25:\frac{V_t}{A_t}=\frac{r_t^{h*}}{A_t}-\frac{m^h}{A_t}+\beta E_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\frac{V_{t+1}}{A_{t+1}}\right\} \quad (3.15)$$

$$:v_t=r_t^{h*}-m^h+\beta E_t\left\{\frac{\lambda_{t+1}}{\lambda_t}v_{t+1}\right\} \quad \to r_t^{h*} \quad (1.65)$$

$$26:\frac{V_t}{A_t}=\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}A_{t+1}}{\lambda_t A_t}\left[\gamma_{h,t+1}\frac{P_{t+1}^h}{A_{t+1}}+(1-\gamma_{h,t+1})\frac{V_{t+1}}{A_{t+1}}\right]\right\} \quad (3.16)$$

$$:v_t=\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[\gamma_{h,t+1}p_{t+1}^h+(1-\gamma_{h,t+1})v_{t+1}\right]\right\} \quad \to v_t \quad (1.66)$$

$$27:\frac{P_t^h}{A_t}=(1-\epsilon_{h,t})\left(\frac{V_t^N}{A_t}-\frac{V_t^B}{A_t}\right)+\epsilon_{h,t}\frac{V_t}{A_t} \quad \to P_t^h \quad (1.69)$$

$$:p_t^h=(1-\epsilon_{h,t})(v_t^N-v_t^B)+\epsilon_{h,t}v_t \quad \to P_t^h \quad (1.69)$$

$$28:r_t^h=\upsilon r_{t-1}^h+(1-\upsilon)\,r_t^{h*} \quad \to r_t^h \quad (1.70)$$

$$29:\frac{1+i_t}{1+i_{ss}}=\left(\frac{1+i_{t-1}}{1+i_{ss}}\right)^{\alpha_i}\cdot$$

$$\left(\left(\frac{1+\pi_t}{1+\pi_{ss}}\right)^{\alpha_\pi}\left(\frac{Y_t}{(1+\mu)(1+\gamma)Y_{t-1}}\right)^{\alpha_y}\right)^{1-\alpha_i}\exp(m_t) \quad \to i_t \quad (1.72)$$

$$30:\frac{H_{t+1}}{Q_{t+1}}\frac{Q_{t+1}}{Q_t}-\frac{H_t}{Q_t}=\Lambda\left(\frac{q_{h,t}}{A_t}\right)\left(\frac{K_t^L}{Q_t}-\frac{H_t}{Q_t}\right)$$

$$:h_{t+1}\frac{Q_{t+1}}{Q_t}-h_t=\Lambda\left(\frac{q_{h,t}}{A_t}\right)(k_t^L-h_t) \quad (3.11)$$

$$:h_{t+1}(1+\mu)-h_t=\Lambda\left(\frac{q_{h,t}}{A_t}\right)(k_t^L-h_t) \quad \to h_t \quad (3.11)$$

$$31:\vartheta_{c,t}=\tilde{\vartheta}_{c,t}\left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\vartheta} \quad \to \vartheta_{c,t} \quad (1.74)$$

$$32:\vartheta_{h,t}=\tilde{\vartheta}_{h,t}\left(\frac{\omega_{c,t}}{\omega_{c,ss}}\right)^{\eta_\vartheta}\left(\frac{1+i_t}{1+\pi_{t+1}}\right)^{\theta_\vartheta} \quad \to \vartheta_{h,t} \quad (1.76)$$

$$33:\frac{P^H}{h_c w_c}=\frac{P_t^H/A_t}{h_{c,t}w_{c,t}/A_t} \quad \to h_{c,t}$$

$$33:\frac{P^H}{h_c w_c}=\frac{p_t^H}{h_{c,t}w_{c,t}} \quad \to h_{c,t}$$

$$34:\frac{Y_t}{Q_tA_t}=\frac{C_t}{Q_tA_t}+\frac{\iota v_{c,t}}{Q_tA_t} \quad \to \frac{C_t}{Q_tA_t} \quad (1.77)$$

$$:y_t=c_t+\iota v_{c,t} \quad \to c_t \quad (1.77)$$

B.2 Shocks:

$$35 : (\varrho_t) = \rho_\varrho(\varrho_{t-1}) + \epsilon_\varrho; \epsilon_\varrho \sim (0, \sigma^2_{\varrho,i}); \qquad \rightarrow \varrho_t$$

$$36 : (\chi_{c,t}) = \rho_{\chi_c}(\chi_{c,t-1}) + \epsilon_{\chi_c}; \epsilon_{\chi_c} \sim (0, \sigma^2_{\chi,i}); \qquad \rightarrow \chi_{c,t}$$

$$37 : (\epsilon_t) = \rho_\epsilon(\epsilon_{h,t-1}) + \epsilon_\epsilon; \epsilon_{\epsilon_t} \sim (0, \sigma^2_{\epsilon_i}); \qquad \rightarrow \epsilon_t$$

$$38 : (z_t) = \rho_z(z_{t-1}) + \epsilon_z; \epsilon_z \sim (0, \sigma^2_{z,i}); \qquad \rightarrow z_t$$

$$39 : (\epsilon_{c,t}) = \rho_{\epsilon_{c,t}}(\epsilon_{c,t-1}) + \epsilon_{\epsilon_{c,t}}; \epsilon_{\epsilon_{c,t}} \sim (0, \sigma^2_{\epsilon_{c,i}}); \qquad \rightarrow \epsilon_{c,t}$$

$$40 : (\psi_t) = \rho_\psi(\psi_{t-1}) + \epsilon_\psi; \epsilon_{\psi_{c,t}} \sim (0, \sigma^2_{\epsilon_i}); \qquad \rightarrow \psi_t$$

$$41 : (\tau_t) = \rho_\tau(\tau_{h,t-1}) + \epsilon_\tau; \epsilon_{\tau_t} \sim (0, \sigma^2_{\tau_i}); \qquad \rightarrow \tau_t$$

$$42 : (\phi_t) = \rho_\phi(\phi_{h,t-1}) + \epsilon_{\phi_t}; \epsilon_{\phi_t} \sim (0, \sigma^2_{\phi_t}) \qquad \rightarrow \phi_t$$

$$43 : (\tilde{\kappa}_{c,t}) = \rho_{\tilde{\kappa}_{c,t}}(\tilde{\kappa}_{c,t-1}) + \epsilon_{\tilde{\kappa}_{c,t}}; \epsilon_{\tilde{\kappa}_{c,t}} \sim (0, \sigma^2_{\tilde{\kappa}_{c,i}}); \qquad \rightarrow \tilde{\kappa}_{c,t}$$

$$44 : (\tilde{\kappa}_{h,t}) = \rho_{\tilde{\kappa}_{h,t}}(\tilde{\kappa}_{h,t-1}) + \epsilon_{\tilde{\kappa}_{h,t}}; \epsilon_{\tilde{\kappa}_{h,t}} \sim (0, \sigma^2_{\tilde{\kappa}_{h,t}}); \qquad \rightarrow \tilde{\kappa}_{h,t}$$

$$45 : m_t = \epsilon_{m_t}; \epsilon_{m_t} \sim (0, \sigma^2_{\tilde{\kappa}_{h,t}}); \qquad \rightarrow m_t$$

C Steady-State

$$1 : \lambda_t = \varrho_t x_t^{-\sigma} \tag{1.12}$$

$$: \lambda = x^{-\sigma} \qquad \to \lambda$$

$$2 : \chi_h = \varrho_t x_t^{-\sigma} w_{h,t} \tag{1.13}$$

$$: \chi_h = x_{-\sigma_{w_h}} \qquad \to w_h$$

$$3 : \varrho_t x_t^{-\sigma} = \beta \mathbb{E}_t \{ \varrho_{t+1} x_{t+1}^{-\sigma} \frac{(1+\mu)}{(1+\varnothing)} R_{t+1} \} \tag{1.14}$$

$$: 0 = \beta \frac{(1+\mu)}{(1+\varnothing)} R$$

$$: R = \frac{(1+\varnothing)}{(1+\mu)\beta} \qquad \to R$$

$$4 : x_t^{-\sigma} = c_t - \theta c_{t-1} \qquad \to x_t \tag{1.15}$$

$$: x^{-\sigma} = (1-\theta)c \qquad \to x$$

$$5 : \gamma_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{V_{c,t}} = \kappa_{c,t} \omega_{c,t}^{-\delta_{c,t}} \qquad \to \gamma_{c,t} \tag{1.18}$$

$$: \gamma_c = \kappa_c \omega_c^{-\delta_c} \qquad \to \gamma_c$$

$$6 : \lambda_{c,t} \equiv \frac{M^C(\cdot,\cdot)}{U_{c,t}} = \gamma_{c,t} \omega_{c,t} \tag{1.19}$$

$$: \lambda_c = \gamma_c \omega_c \qquad \to \lambda_c$$

$$7 : l_{c,t} = \frac{(1-\vartheta_{c,t})}{(1+\mu)} l_{c,t-1} + \gamma_{c,t} v_{c,t} \tag{3.4}$$

$$: l_c = \frac{(1-\vartheta_c)}{(1+\mu)} l_c + \gamma_c v_c \qquad \to l_c$$

$$8 : u_{c,t} = 1 - \frac{(1-\vartheta_{c,t})}{(1+\mu)} l_{c,t-1} \qquad \to u_{c,t} \tag{3.5}$$

$$: u_c = 1 - \frac{(1-\vartheta_c)}{(1+\mu)} l_c \qquad \to u_c$$

$$9 : \frac{\iota}{\gamma_{c,t}} = h_{c,t}(\xi_t z_t - w_{c,t}) + (\beta\varsigma) \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} (1-\vartheta_{c,t}) \frac{\iota_{t+1}}{\gamma_{c,t+1}} \right\} \qquad \to \omega_{c,t} \tag{1.31}$$

$$: \frac{\iota}{\gamma_c} = h_c(\xi - w_c) + (\beta\varsigma)(1-\vartheta_c) \frac{\iota_t}{\gamma_c} \qquad \to \omega_c$$

$$10 :(\pi_t + 1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1-(1-\varsigma)\left(\frac{\epsilon}{(\epsilon-1)}\frac{K_{1,t}}{K_{2,t}}\right)^{1-\epsilon}\right) \quad \to \pi_t \qquad (1.32)$$

$$:K_{1,t} = w_{c,t}y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon+1}K_{1,t+1}\right\} \qquad (1.33)$$

$$:K_{2,t} = y_t + \varsigma\beta\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}(1+\pi_{t+1})^{\epsilon}K_{2,t+1}\right\} \qquad (1.34)$$

:Steady-state simplifications:

$$K_1 = w_c y + \varsigma\beta\frac{1}{1-\varsigma\beta(1+\pi)^{\epsilon+1}}$$

$$K_2 = y + \varsigma\beta\frac{1}{1-\varsigma\beta(1+\pi)^{\epsilon}}$$

$$:(\pi+1)^{\epsilon-1} = \frac{1}{\varsigma}\left(1-(1-\varsigma)\left(\frac{\epsilon}{\epsilon-1}\frac{K_1}{K_2}\right)^{1-\epsilon}\right) \quad \to \pi$$

$$11 :y_t = z_t h_{c,t} l_{c,t} \quad \to y_t \qquad (\textbf{??})$$

$$:y = h_c l_c \quad \to y$$

$$\begin{aligned}12 :h_{c,t}w_{c,t} = \epsilon_{c,t}\Big[&h_{c,t}z_t\xi_t \\ &+\beta(1-\vartheta_{c,t})\mathbb{E}_t\left\{\frac{\lambda_{t+1}}{\lambda_t}\left(1-(1-\lambda_{c,t+1})\frac{(1-\epsilon_{c,t})\epsilon_{c,t+1}}{(1-\epsilon_{c,t})\epsilon_{c,t}}\right)\frac{\iota_{t+1}}{\gamma_{c,t+1}}\right\}\Big] \\ &+\beta(1-\epsilon_{c,t})\left[b_{c,t}+\frac{\chi_c}{\lambda_t}\left(\frac{(1-h_{c,t})^{1-\nu}-1}{(1-\nu)}\right)\right]\end{aligned} \quad \to w_{c,t} \qquad (1.41)$$

$$\begin{aligned}:h_c w_c = \epsilon_c\Big[&h_c\xi \\ &+\beta(1-\vartheta_c)\frac{\lambda}{\lambda}\left(1-(1-\lambda_c)\frac{(1-\epsilon_c)\epsilon_c}{(1-\epsilon_c)\epsilon_c}\right)\frac{\iota}{\gamma_c}\Big] \\ &+\beta(1-\epsilon_c)\left[b_c+\frac{\chi_c}{\lambda}\left(\frac{(1-h_c)^{1-\nu}-1}{(1-\nu)}\right)\right]\end{aligned}$$

$$\begin{aligned}:h_c w_c = \epsilon_c&\left[h_c\xi+\beta(1-\vartheta_c)\left(1-(1-\lambda_c)\right)\frac{\iota}{\gamma_c}\right] \\ &+\beta(1-\epsilon_c)\left[b_c+\frac{\chi_c}{\lambda}\left(\frac{(1-h_c)^{1-\nu}-1}{(1-\nu)}\right)\right]\end{aligned} \quad \to w_c$$

$$13 :z_t\xi_t A_t = \frac{\chi_c}{\lambda_t}(1-h_{c,t})^{-\nu} \qquad (1.42)$$

$$:\xi = \frac{\chi_c}{\lambda}(1-h_c)^{-\nu} \quad \to \xi$$

$$14 :\omega_{h,t} \equiv \frac{B_t}{S_t} \qquad (1.45)$$

$$:\omega_h \equiv \frac{B}{S} \quad \to \omega_h$$

$$15 :\gamma_{h,t} = \kappa_{h,t}\omega_{h,t}^{\delta_h} \qquad (1.46)$$

$$:\gamma_h = \kappa_h\omega_h^{\delta_h} \quad \to \gamma_h$$

$$16 :\lambda_{h,t} = \kappa_{h,t}\omega_{h,t}^{\delta_h-1} \qquad (1.47)$$

$$:\lambda_h = \kappa_h\omega_h^{\delta_h-1} \quad \to \lambda_h$$

$$17: n_t = \frac{(1-\vartheta_{h,t})}{(1+\mu)} n_{t-1} + \lambda_{h,t} b_t \tag{1.48}$$

$$: n(1+\mu) = (1-\vartheta_h)n + \lambda_h b$$

$$: n = \frac{\lambda_h b}{\vartheta_h + \mu} \qquad \to n$$

$$18: f_t(1+\mu) = f_{t-1} + (1-\psi_{t-1})\mu \tag{3.8}$$

$$: f(1+\mu) = f + (1-\psi)\mu$$

$$: f\mu = (1-\psi)\mu$$

$$: f = (1-\psi) \qquad \to f$$

$$19: b_t = 1 - f_t - n_t \frac{(1-\vartheta_{h,t})}{(1+\mu)} \tag{3.9}$$

$$: b = 1 - f - n\frac{(1-\vartheta_h)}{(1+\mu)} \qquad \to b$$

20 :

$$21: h_{t+1}(1+\mu) - h_t = \phi_t l_{h,t} \qquad \to l_{h,t} \tag{3.10}$$

$$: h(1+\mu) - h = \phi l_h$$

$$: h\frac{\mu}{\phi} = l_h \qquad \to l_h$$

$$22: \frac{w_{h,t}}{\phi_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} V_{t+1} \right\} - q_{h,t} \tag{1.60}$$

$$: \frac{w_h}{\phi} = \beta V - q_h$$

$$: q_h = \beta V - \frac{w_h}{\phi} \qquad \to q_h$$

$$23: v_t^N = -m_t^h + \frac{z_t^h}{\lambda_t} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\vartheta_{h,t}(1-\lambda_{h,t+1}))v_{t+1}^N + \vartheta_{h,t}(\hat{V}_{t+1} - \lambda_{h,t+1} p_{t+1}^h) + \vartheta_{h,t}(1-\lambda_{h,t+1}) v_{t+1}^B \right] \right\} \tag{1.62}$$

$$: v^N = -m^h + \frac{z^h}{\lambda} + \beta \left[(1-\vartheta_h(1-\lambda_h))v^N + \vartheta_h(V - \lambda_h p^h) + \vartheta_h(1-\lambda_h)v^B \right] \qquad \to v_t^N$$

$$24: v_t^B = -r_t^{h*} + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[(1-\lambda_{h,t+1}) v_{t+1}^B + \lambda_{h,t+1} \left(v_{t+1}^N - p_{t+1}^h \right) \right] \right\} \quad (3.14)$$

$$: v^B = -r^{h*} + \beta \left[(1-\lambda_h) v^B + \lambda_h \left(v^N - p^h \right) \right] \quad \rightarrow v^B$$

$$25: v_t = r_t^{h*} - m^h + \beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} v_{t+1} \right\} \quad \rightarrow r_t^{h*} \quad (1.65)$$

$$: v = r^{h*} - m^h + \beta v$$

$$: v(1-\beta) = r^{h*} - m^h$$

$$: v = \frac{r^{h*} - m^h}{(1-\beta)} \quad \rightarrow r^{h*}$$

$$26: v_t = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left[\gamma_{h,t+1} p_{t+1}^h + (1-\gamma_{h,t+1}) v_{t+1} \right] \right\} \quad (1.66)$$

$$: v = \beta \gamma_h p^h + \beta(1-\gamma_h) v$$

$$: v = \frac{\beta \gamma_h p^h}{1-\beta(1-\gamma_h)} \quad \rightarrow v$$

$$27: p_t^h = (1-\epsilon_{h,t})(v_t^N - v_t^B) + \epsilon_{h,t} v_t \quad (1.69)$$

$$: p^h = (1-\epsilon_h)(v^N - v^B) + \epsilon_h v \quad \rightarrow p^h$$

$$28: r_t^h = \upsilon r_{t-1}^h + (1-\upsilon)\, r_t^{h*} \quad (1.70)$$

$$: r^h = \upsilon r^h + (1-\upsilon) r^{h*}$$

$$: r^h (1-\upsilon) = (1-\upsilon) r^{h*}$$

$$: r^h = r^{h*} \quad \rightarrow r^h \quad (1.70)$$

29 :

$$30: h_{t+1}(1+\mu) - h_t = \Lambda \left(\frac{q_{h,t}}{A_t} \right) (k_t^L - h_t) \quad (3.11)$$

$$: h(1+\mu) - h = \Lambda \left(\frac{q_h}{A} \right) (k_0^L - h)$$

$$: h\mu = \Lambda \left(\frac{q_h}{A} \right) (k_0^L - h)$$

$$: h(\mu + \Lambda \frac{q_h}{A}) = \Lambda \left(\frac{q_h}{A} \right) k_0^L$$

$$: h = \frac{\Lambda \left(\frac{q_h}{A} \right) k_0^L}{(\mu + \Lambda \frac{q_h}{A})} \quad \rightarrow h$$

$$31 :\vartheta_{c,t} = \tilde{\vartheta}_{c,t} \left(\frac{\omega_{h,t}}{\omega_{h,ss}}\right)^{-\zeta_\vartheta} \tag{1.74}$$

$$:\vartheta_c = \tilde{\vartheta}_c (1)^{-\zeta_\vartheta} \qquad \rightarrow \vartheta_c$$

$$32 :\vartheta_{h,t} = \tilde{\vartheta}_{h,t} \left(\frac{\omega_{c,t}}{\omega_{c,ss}}\right)^{\eta_\vartheta} \left(\frac{1+i_t}{1+\pi_{t+1}}\right)^{\theta_\vartheta} \tag{1.76}$$

$$:\vartheta_h = \tilde{\vartheta}_h (1)^{\eta_\vartheta} \left(\frac{1+i}{1+\pi}\right)^{\theta_\vartheta}$$

$$33 :\xi = \frac{1-\epsilon_t}{\epsilon_t} \qquad \rightarrow \xi_t \tag{27}$$

$$:\xi = \frac{1-\epsilon}{\epsilon} \qquad \rightarrow \xi$$

$$34 :\frac{Y_t}{Q_t A_t} = \frac{C_t}{Q_t A_t} + \frac{\iota v_{c,t}}{Q_t A_t} \qquad \rightarrow \frac{C_t}{Q_t A_t} \tag{1.77}$$

$$:y_t = c_t + \iota v_{c,t} \tag{1.77}$$

$$:\frac{Y_t}{Q_t A_t} = \frac{C_t}{Q_t A_t} + \frac{\iota v_{c,t}}{Q_t A_t} \qquad \rightarrow \frac{C_t}{Q_t A_t} \tag{1.77}$$

$$:y = c + \iota v_c \qquad \rightarrow c$$

A:5 Numerical Implementation

We use RISE toolbox (Maih2015) for MATLAB to perform all parts of the numerical implementation.[52] We code the model in a symbolic form and then solve with perturbation methods. In our estimation we impose relatively wide priors and use the *Artificial Bee Colony* algorithm by **KarabogaBasturk2007** for global optimisation. We use the IMM filter and recover latent variables using the associated regime-switching smoother developed in **HKKM2024**. Variance and historical decompositions are computed using the standard routines in RISE.

[52]RISE stands for 'Rationality in Switching Environments'. The codes and documentation are available at https://github.com/jmaih/RISE_toolbox

A:6 Simulation Appendix for Chapter 1: Housing and Labour Market Spillovers

A Labour Supply Shock

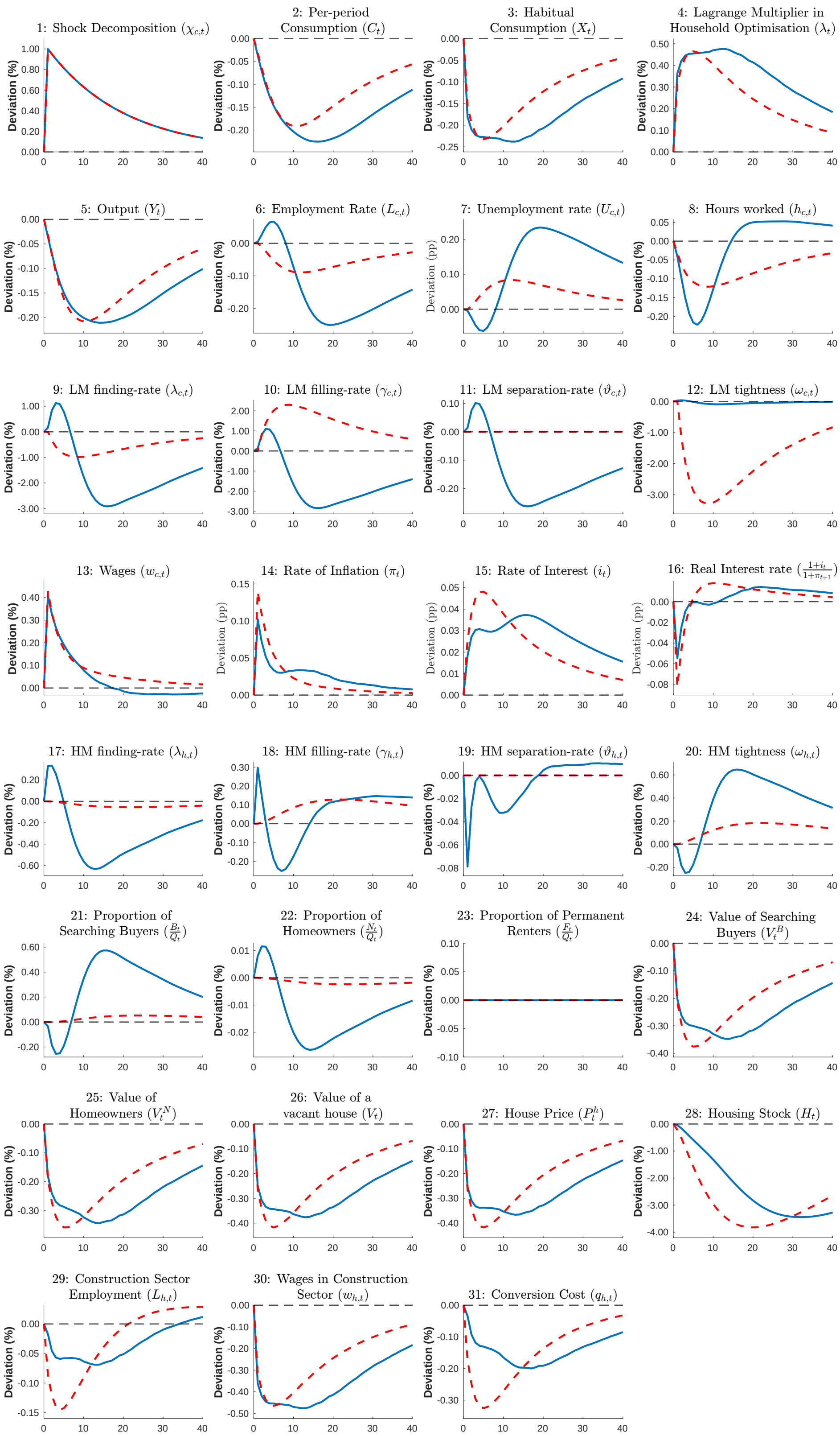


Figure 7: Response of the system to a unit shock to the disutility of working in the consumption sector, with and without active spillovers between the housing and labour market

B Housing Demand Shock

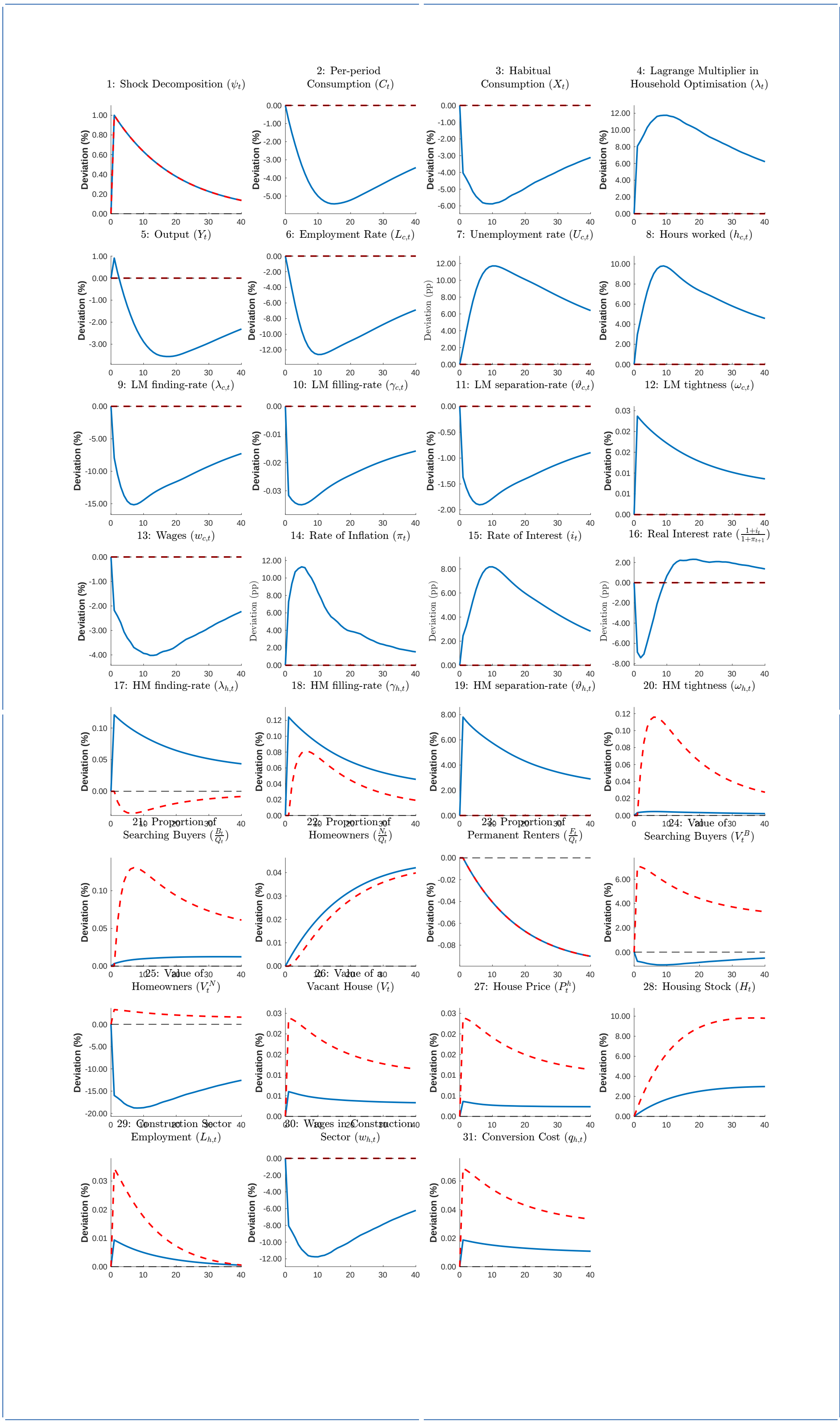


Figure 8: Response of the system to a unit shock to construction sector productivity, with and without active spillovers between the housing and labour market

C Total Factor Productivity Shock

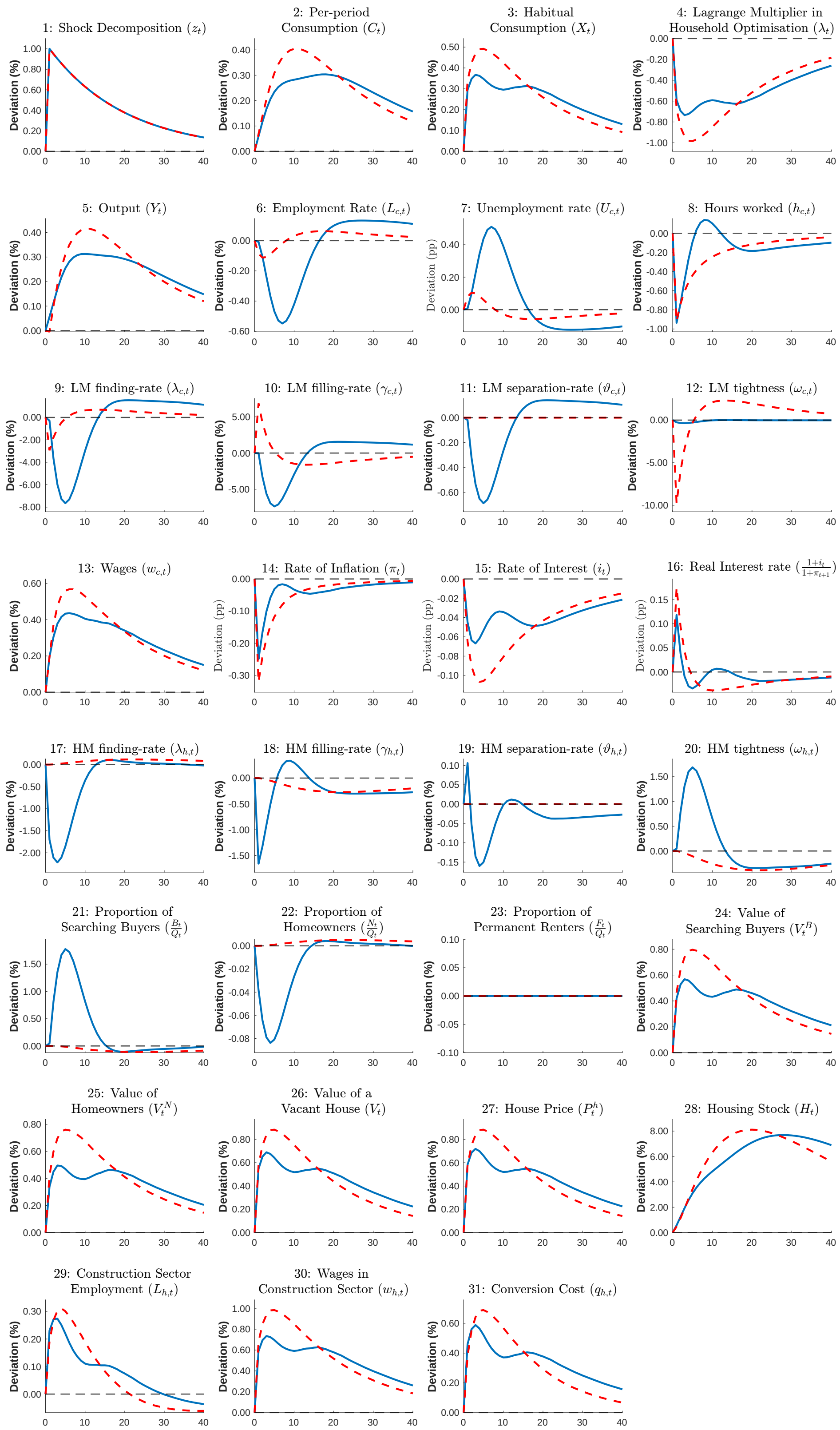


Figure 9: Response of the system to a unit shock to total factor productivity, with and without active spillovers between the housing and labour market

D Consumption Preference Shock

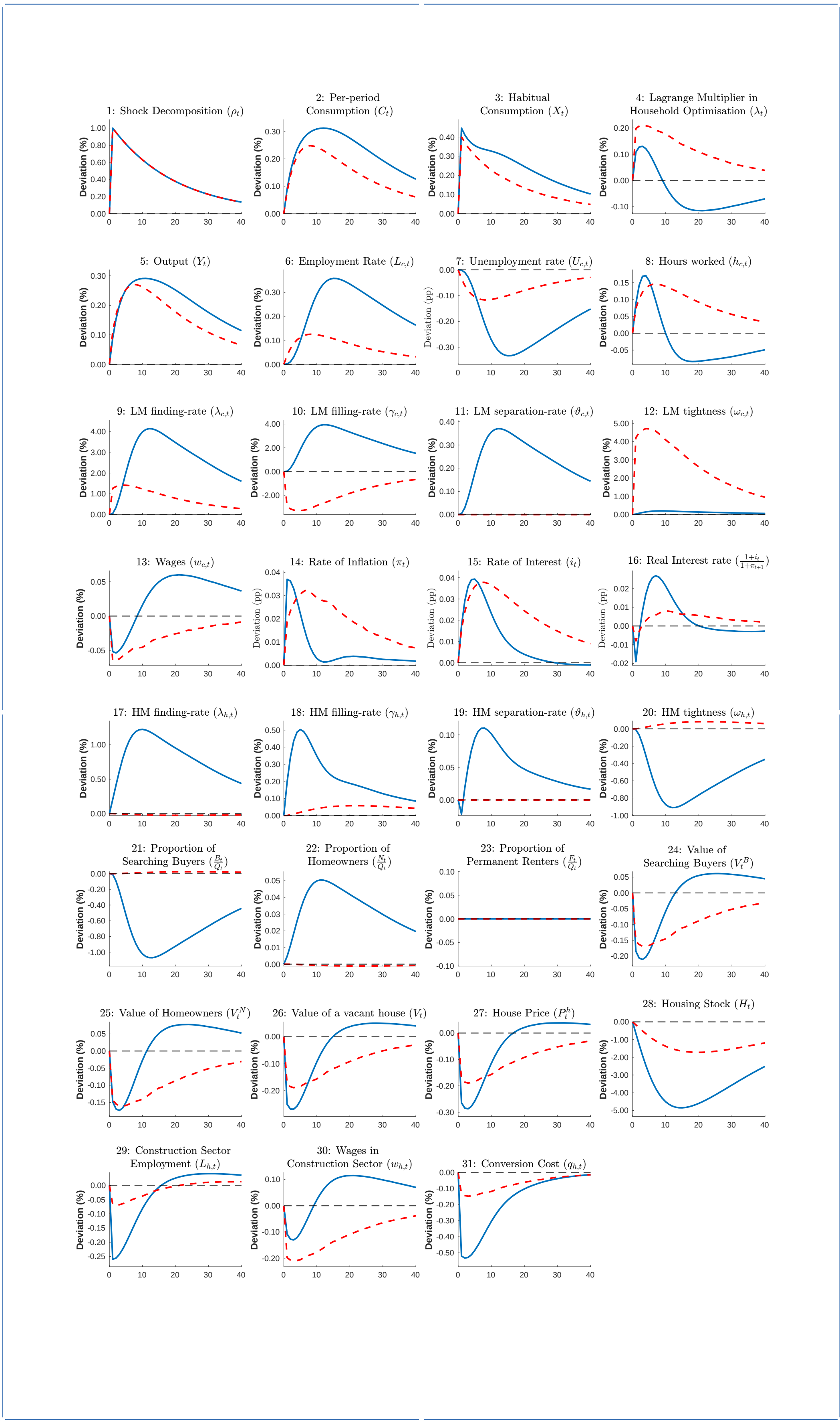


Figure 10: Response of the system to a unit shock to consumption preferences, with and without active spillovers between the housing and labour market

E Cost Push Shock

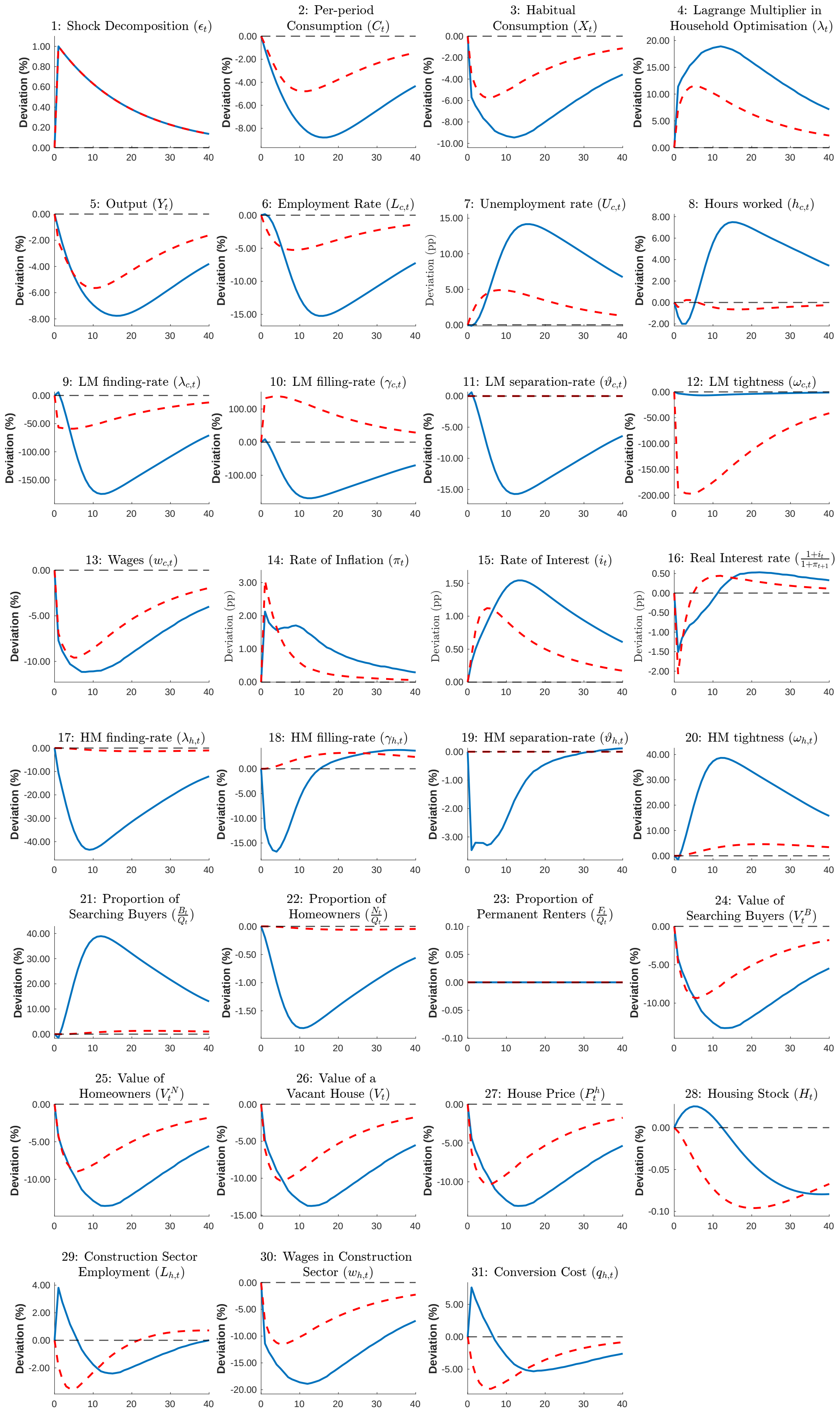


Figure 11: Response of the system to a unit shock to the elasticity of substitution between inputs, with and without active spillovers between the housing and labour market

F Interest Rate Shock

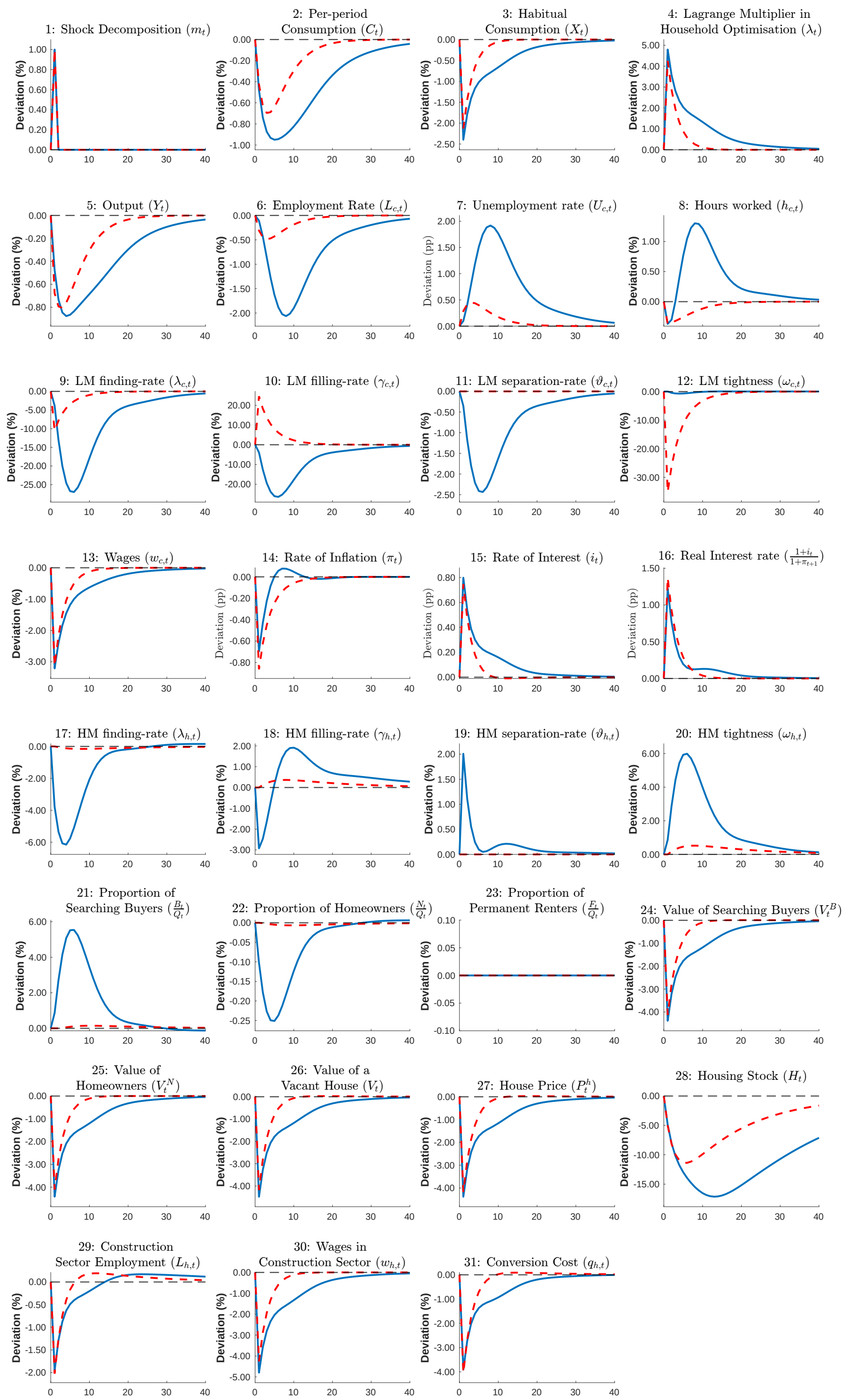


Figure 12: Response of the system to a AR(0) unit shock to monetary policy, with and without active spillovers between the housing and labour market

A:7 Simulation Appendix for Chapter 3: A More Mobile Labour Market

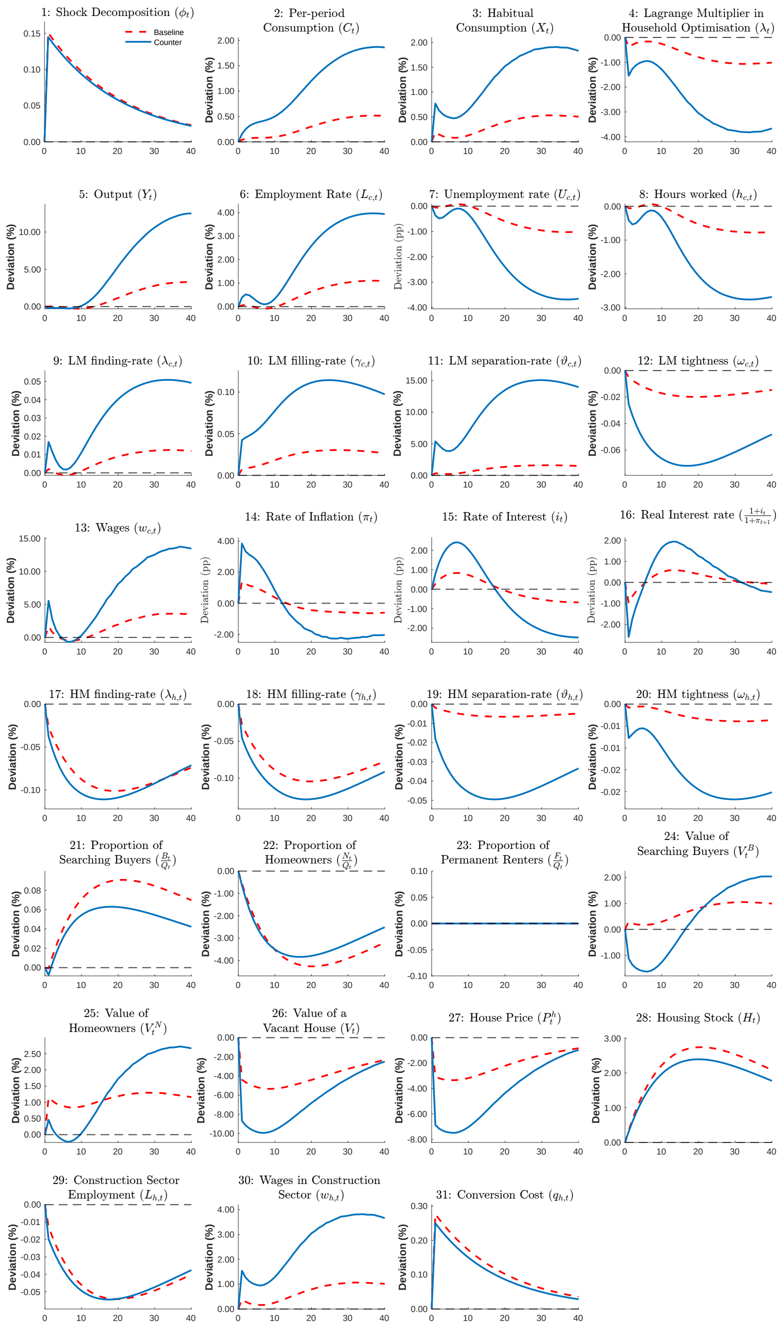


Figure 13: Response of the UK economy under Baseline and counter-factual experiment to a unit shock to housing construction productivity.

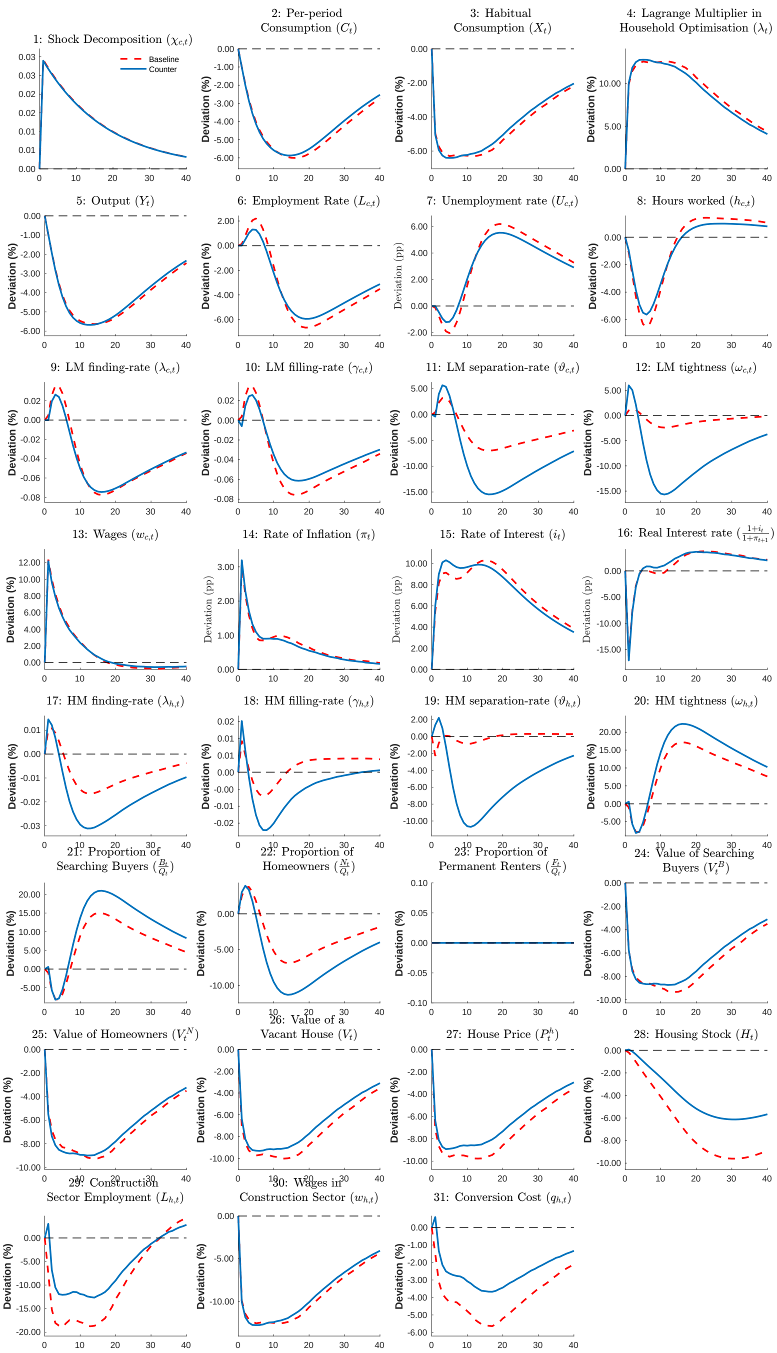
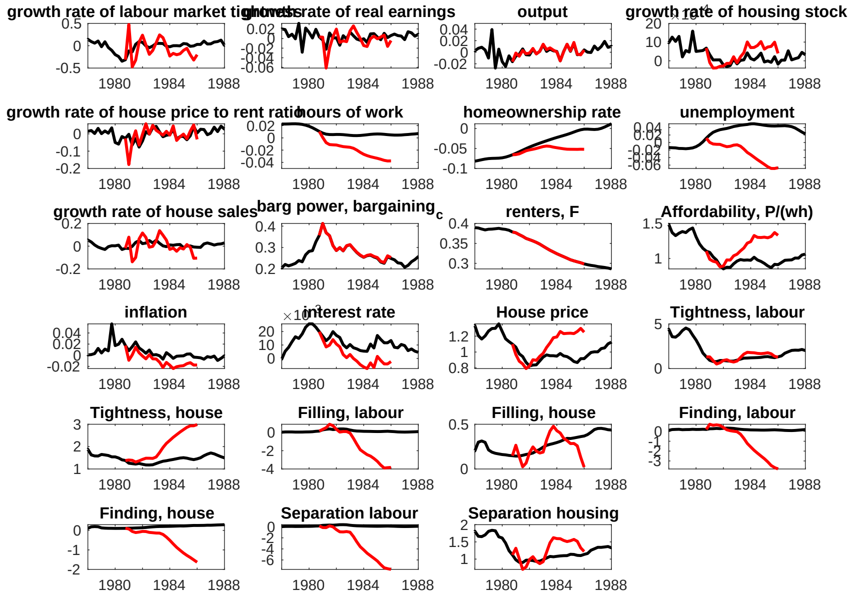
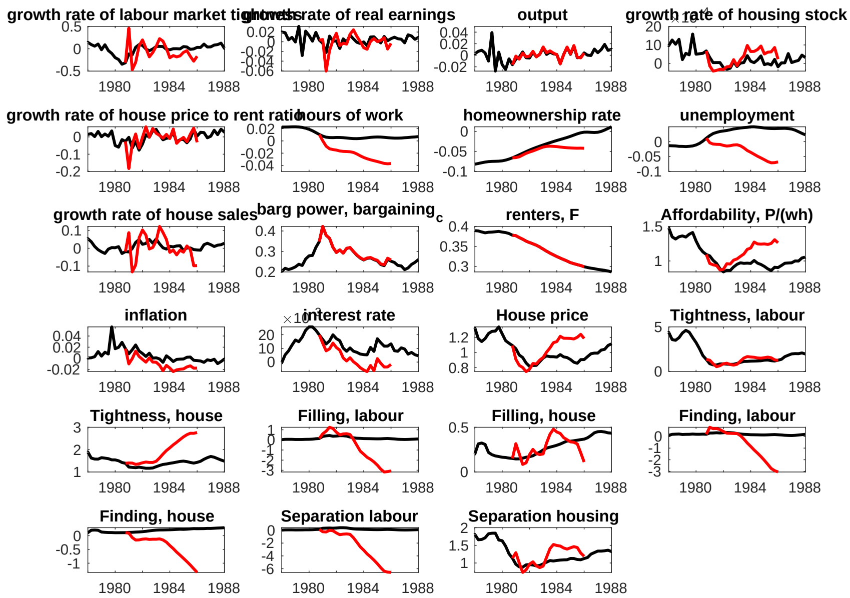


Figure 14: Response of the UK economy under Baseline and counter-factual experiment to a labour supply shock



(a) Simulated UK economy with only US labour market spillovers



(b) Simulated UK economy with full US labour market spillovers

Figure 15: Comparison of simulated UK economies under different US labour market spillover scenarios. Panel (a) shows the effects of only US labour market spillovers, while Panel (b) includes full spillovers.