THREE ESSAYS ON INFORMATION AND BEHAVIORAL ECONOMICS

by

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Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Signed: Tiannan Zhang

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Abstract

This thesis comprises three essays that explore the interplay between strategic behavior, information transmission, and welfare-enhancing policy interventions.

The first chapter develops a multi-receiver incomplete information coordination game with unbiased and biased agents. Unbiased agents aim to align with the underlying state of nature and coordinate with others, while biased agents favor a specific collective outcome. A randomly selected sender observes the state and communicates strategically to the group. I show that truthful communication and full social learning can be sustained in equilibrium provided the degree of conformity among unbiased agents does not exceed one-half and the share of biased agents remains below fifty percent.

The second chapter examines rumor propagation on networks by modifying the communication–coordination game introduced in the previous chapter so that conformity depends on local interactions only. I demonstrate that introducing a small degree of conformity enlarges the parameter space in which truthful communication occurs, thereby relaxing the constraints on biased participation compared to existing models.

The third chapter shifts focus to consumer behavior and welfare by analyzing optimal taxation of sin goods under self-control problems. Using the temptation model of Gul and Pesendorfer (2001) in a monopoly setting, I characterize an endogenous quality–price ceiling and derive welfare-maximizing tax policies. I show that optimal ad valorem taxes decline with market size, potentially turning into subsidies, while specific taxes are not optimal for domestically produced goods. By contrast, for imported goods, both ad valorem and specific taxes improve welfare, with ad valorem taxes

yielding substantially larger gains.

Introduction

Modern economic environments are increasingly shaped by the interplay between information flows, social interactions, and behavioral biases. Individuals do not act in isolation: they exchange information, observe the behavior of peers, and make consumption choices that often involve self-control problems. These dynamics can generate outcomes that deviate from those predicted by models of fully rational, individualized agents. Understanding how conformity, communication, and self-control shape collective behavior is thus essential both for economic theory and for designing policies that improve welfare.

This thesis explores these themes through three essays, each focusing on a distinct but related dimension of individual and collective decision-making. The first essay investigates how conformity affects information transmission in a communication game. The second extends this framework to study rumor propagation in networks, highlighting how conformity amplifies the spread of unverifiable statements. The third essay shifts from information to consumption, analyzing how self-control costs influence the optimal taxation of sin goods under monopoly pricing. Across these contexts, the central concern is how individual behavioral motives interact with structural constraints, such as communication channels, network topologies, or market pricing schemes, to shape welfare outcomes.

The first essay examines the role of conformity in environments where individuals seek both to learn about an underlying state of the world and to coordinate their actions with others. In many real-world contexts, such as political communication, financial markets, or product reviews, agents care not only about the accuracy of information but also about aligning with the majority. To model this, I build on an incomplete information framework in which a sender observes the true state of nature and communicates with multiple receivers. Some receivers may be biased toward a particular outcome, while others are unbiased truth-seekers. The key innovation is to introduce conformity into preferences: receivers value taking actions close to those of their peers.

This modification introduces new strategic tensions. On the one hand, conformity may discourage truthful communication, as individuals prioritize coordination over accuracy. On the other hand, moderate levels of conformity can enhance social learning, as unbiased agents are incentivised to follow when they think others will do so. The analysis shows that the degree of conformity plays a pivotal role in sustaining equilibria with informative communication. This has implications for understanding environments where both truth and alignment matter, such as coordination on policy reforms, adoption of technologies, or information sharing in organizations.

Building on the insights of the first essay, the second turns to the problem of rumor propagation in networks. Rumors are statements whose veracity is uncertain and often unverifiable, yet they spread rapidly within social groups, influencing political, economic, and health-related outcomes. From the perspective of economics, rumors provide a natural laboratory for studying why rational agents might transmit information that is potentially false.

I extend the model of Bloch et al. (2018), who show that rational agents may spread rumors if, on balance, they believe them to be true and stand to benefit if they are. Their model, however, does not account for conformity motives. I introduce a networked setting in the form of an undirected line, in which individuals care about aligning their actions with neighbors. This addition captures the idea that individuals may spread rumors not because they believe them, but because doing so aligns them with their neighbors.

The analysis reveals that conformity expands the set of conditions under which rumors circulate. Even when individuals suspect that a rumor is false, they may transmit it to avoid deviating from peers. This mechanism helps explain why political misinformation can sway elections, why doubts about medical treatments spread despite strong scientific evidence, and why financial rumors propagate through markets. The model highlights conformity as a powerful catalyst for social learning, but also for social mislearning, with significant policy implications for combating misinformation.

The third essay shifts from communication and networks to consumer decision-making under self-control problems. Many goods, such as tobacco, alcohol, and sugary beverages, are associated with temptation and overconsumption. Governments commonly impose sin taxes on such goods to discourage consumption and raise revenue. Yet the optimal design of these taxes is far from straightforward. Sin taxes are often regressive, encourage illicit trade when set too high, and interact in complex ways with consumer behavior.

To analyze these issues, I adopt the temptation model of Gul and Pesendorfer (2001), which formalizes the trade-off between long-term commitment utility and short-term temptation utility. Consumers first choose a menu of options and then select an item within it, with self-control costs arising when temptation conflicts with long-term preferences. I embed this framework in a nonlinear pricing model where a monopolist sells sin goods to heterogeneous consumers. The monopolist cannot observe preferences directly and thus relies on price discrimination through menus.

The analysis distinguishes between consumers facing upward temptation (toward high-quality, high-price goods) and downward temptation (toward low-quality, low-price goods). In such settings, taxation alters not only consumption but also the distribution of self-control costs. I show that specific taxes have no effect on welfare in the case of a domestic monopolist, whereas ad valorem taxes can be welfare-enhancing depending on market size and the distribution of temptation intensities. For imported goods, by contrast, both ad valorem and specific taxes can improve welfare, with optimal ad valorem rates reaching as high as 50 percent.

A central contribution of this essay is to evaluate taxation under three alternative welfare concepts: adjusted-cost welfare (which accounts for self-control costs), normative welfare (which reflects commitment utility as true preferences), and behavioral welfare (which aggregates across selves via ex-post utility). The results highlight how policy prescriptions depend critically on the chosen welfare benchmark. For instance, under normative welfare, higher taxes may be desirable to curb temptation, while under behavioral welfare, lower taxes may be preferred to respect revealed choices. This underscores the theoretical and philosophical challenges of welfare evaluation in behavioral contexts.

Considered jointly, the three essays illuminate how conformity, bias, and self-control interact with institutional and market structures to shape outcomes. The first two chapters demonstrate how social interactions can amplify or suppress the transmission of information, while the third shows how self-control problems alter the design and evaluation of optimal taxation. A unifying theme is that individual motives, ranging from the desire to conform to the pursuit of biased outcomes or the struggle with temptation, exert a profound influence on collective welfare.

The contributions are threefold. First, the thesis extends models of cheap talk and rumor propagation by incorporating conformity, offering new insights into the dynamics of social learning and misinformation. Second, it adapts nonlinear pricing models to account for temptation, yielding novel results on the design of taxation in sin good markets. Third, it brings together these strands to highlight a broader perspective: that behavioral motives must be integrated into economic analysis to understand how communication, networks, and markets function in practice.

By bridging communication games, network economics, and behavioral industrial organization, the thesis contributes to ongoing debates in both theory and policy. It sheds light on how misinformation spreads, how individuals respond to peer pressure, and how governments can design policies in the presence of behavioral biases. Ultimately, the results suggest that effective policy requires not only correcting market failures but also acknowledging the behavioral forces that drive individual and collective decisions.

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Chapter 1

Conformity As A Catalyst For Social Learning

I develop a multi-receiver incomplete information game of coordination with two types of agents: *unbiased agents*, who seek to align their actions with an underlying state of nature and to coordinate with others, and *biased agents*, who favor a specific collective decision. A randomly chosen sender observes the realized state and then engages in strategic communication, after which receivers socially learn through the sender's message. In equilibrium, whether truthful equilibrium and full social learning occur depends jointly on the degree of conformity among unbiased agents and the share of biased agents. In particular, I show that truthful strategic communication to a large group and social learning remain feasible provided the degree of conformity does not exceed one-half and the share of biased agents is below fifty percent.

1.1 Introduction

The rapid development of information technologies has made information more easily accessible and influential in shaping individual decisions. For example, consumers can make independent purchase decisions by watching a review of the product online or through other media. Similarly, voters may decide how to place their ballot in an election based on the opinion of a public figure. In both of these cases, agents may end up choosing the same action because, individually, each agent believes that this action is the best given the information they received. Consider the first of these examples, if consumers are only interested in whether a product is of good quality or not (so that other issues that might affect their decision to purchase, such as budget constraints and other's purchasing decisions are ignored), then purchasing decisions reflect information transfer from the reviewer to the consumers. In this case, consumers have *learned* the quality of the product and purchasing decisions reflect this.

If in the above examples, agents wish to align their actions with other agents then information transfer and learning might not be feasible. This is because agents might hold strong beliefs regarding the actions made by the remaining agents, and thus, even if the reviewer is honest, they coordinate on a given action independently of the review. Alternatively, the reviewer may post a review she considers is most likely to align with the action of some of her viewers. Some agents may then assume that the review is unreliable, which will lead them to decide not to make a purchase.

The willingness of agents to take actions that match those taken by others can be regarded as *conformity*. More specifically, conformity is understood here as willingness to adopt the same action as others regardless of whether, according to the agent's own mind-set, the action is reasonable or correct in some way. This independence on agent's own mind-set can then be interpreted as placing a cost on taking certain actions. This cost may be unconscious (i.e. due to inherent biases) or conscious (the agent may fear the social repercussions of diverging from the norm). Formally, conformity may be represented as a loss function that increases with the extent of divergence from the choices made by others.

The coexistence of conformity in the context of information transmission is main theme of this paper. In particular, this coexistence raises a central question: does conformity allow information to be transferred or does it actually hamper it? This does not have an immediate answer. On the one hand, if agents align with others regardless of the information they receive, then agents may fail to transfer information in favor of conforming to other's. Alternatively, agents may ignore any information they receive. On the other hand, conformity may facilitate information transfer by promoting the dissemination of signals that support preferred actions. Similarly, agents may be more willing to accept unlikely information if doing so results in actions that are consistent with the majority. In the previous example, a consumer may be more likely to be influenced by a positive review if the product is fashionable.

When considering information transfer under conformity, another issue arises: if some agents disregard information and are willing to promote an action independently of the truth, can information still be transferred? If agents seek to conform with others, does the presence of this agent completely erase information transmission? In many applications the existence of such agents is relevant. Alleged health treatments are frequently promoted on social media; similarly, partisan agents may disseminate false information in pursuit of particular political agendas. Interestingly, in some circumstances the existence of these *biased* agents may encourage information transfer if the desire to conform is not too strong. I investigate this possibility in the paper.

To investigate these questions, I model information transfer via a sequential game with imperfect information. The game contains two types of agents *unbiased* agents and *biased* agents. An agent's type is private knowledge to that agent, but the proportion of biased agents is common knowledge. Unbiased agents represent "average" agents who seek the truth but may be influenced by the actions of others. Biased agents, on the other hand, can be thought of as partisan agents who are only interested in a particular outcome. In the reviewer-consumer example, biased agents represent fans of the product who are willing to purchase it and promote independently of its actual quality. Unbiased agents represent standard consumers who are willing to purchase a high-quality product and forgo a low-quality one; however, this decision may be influenced by the behavior of others.

The game proceeds in two phases: first, a randomly chosen *sender* from the population receives a private and perfect signal about the true state of the world. In the reviewer example this might correspond to the reviewer receiving a sample by the manufacturer or making a private purchase. After receiving the signal, the sender takes an action. This action is observed by the remaining agents in the population, referred to as *receivers*, who interpret it as a message about the state of the world (i.e. the reviewer posts a public review of the product on social media). The remaining agents then take their actions independently and privately (in the example this corresponds to making a purchase or not). A *collective outcome* is determined by taking the mean of the receivers' action and the sender's action. Note that communication is costly in my model, the sender commits to his action before the remaining agents take theirs. Moreover, even though the sender has to take an action, she is allowed to lie.

I determine the existence of equilibria where unbiased senders disclose their information truthfully (it is possible, as I show, for them to disclose it by lying, but this is inefficient) and unbiased receivers obey their message. Note that biased senders seek to obfuscate the truth and they always send a message that aligns with their preference in equilibrium. Similarly, they disregard any information and adopt their preferred action. I establish that conformity can in fact encourage social learning even if biased agents are present. This is because from the perspective of unbiased agents, information about an unlike state is very valuable. If biased agents are present and prefer the outcome associated with this state; then receiving a message indicating this is the state becomes less informative since it may come from a biased agent. However, if unbiased agents benefit from conforming and all remaining players choose the action associated to the unlike state when receiving it, then the agent is incentivized to choose this action as well.

More broadly, I show that the possibility of information transfer as follows: if the population contains no biased agents. If the willingness to conform is sufficiently strong, equilibria in which agents coordinate independently of informational content become relevant, even when these equilibria are Pareto inferior. On the other hand, if unbiased agents are unaffected by conformity then social learning is possible if the proportion of biased agents is small. For low prior believes, social learning is thus not very robust to the inclusion of biased agents. As mentioned in the previous paragraph, this is due to some messages not being as informative when biased agents are present. If conformity becomes relevant then information transfer and social learning can survive even for arbitrary proportions of biased agents in small populations. This is because conformity acts as an additional incentive for unbiased agents to take certain actions when receiving the corresponding message since everyone else adopts this action in equilibrium and gains are substantial as result. These gains are however not large enough to discourage deviating from taking actions corresponding to alternative messages. Interestingly, if populations are large then, provided conformity is valued less than truth and biased agents are not a majority, social learning remains feasible; independently of the prior belief.

The above results show that information transfer can be robust when biased agents are part of the population. In the sense that for a fixed prior belief the proportion of biased agents a population can sustain while ensuring that unbiased agents communicate truthfully is larger when biased agents benefit from conforming than when they do not.

The model lends itself to several applications, among which I have already highlighted online reviews and consumer purchasing. In this case my model can help shed light on questions relating to social media motivated consumption trends, for instance the recent trends in matcha tea and Dubai chocolate. In addition, the model might be well suited to study investor herding and political influencing through social media. In these examples, the outcome dependent term can be treated as a formal device to study the interplay of decisions by receiver and sender. On the other hand, this term can have representation in real life; i.e. because there might be externalities in taking a particular action. An example of this is the implementation of prevention policies for infectious diseases. Consider in particular the Covid-19 pandemic when governments needed to make decisions to protect citizens from infection while also considering the burden it places on everyday life activities. An instance of this kind of decision is encouraging the use of masks. The situation of the pandemic at a given time may be viewed as either uncontrolled or controlled by the government by referring to how fast the virus spreads or in some other way. If the virus is uncontrolled encouraging citizens to wear a mask would be the best choice in order to prevent the spread of the virus. On the other hand, if the virus is controlled, mask wearing could be discouraged for citizens to benefit from fuller social interaction.

Assume that the government is informed of the current situation of the pandemic accurately via experts' report or some similar way. The public is unaware as it does not receive this information directly. A government official can then make a policy announcement recommending wearing masks or not. This recommendation need not match the current situation. In the other words, the government may be overcautious and encourage the use of masks when it is in fact not needed; or, on the other hand, it may announce that wearing masks is not needed because it wants to prioritize some social effect or economic factor despite the virus spreading rapidly. After the announcement is released, the public can decide whether to wear a mask or not. Some members of the public may prefer that everyone acts in a way that matches the current pandemic situation (wearing masks when the pandemic is uncontrolled and not wearing them when it is controlled). There may be some others who are very stubborn and only care that society acts in a given way independently of the situation; some may want masks to be worn regardless of the current status of the pandemic to feel safer, while others may not want masks to be worn at all because they see it as too restrictive. Individuals that have such strong preference are unlikely to be affected by the behavior of others, while individuals in the first group (those who do not have a predetermined view) might consider what others are doing to align their own behavior with that of the rest.

1.1.1 *Related Literature*

This paper contributes to the literature on strategic communication, which began with the seminal work of Crawford and Sobel. In Crawford and Sobel (1982), one agent has private information which is allow to communicate to another agent at no cost (*cheap talk*). The last agent then takes an action which determines the payoffs to both. My paper adopts a similar approach by adopting their payoff (although in Crawford and Sobel action spaces are continuous) but I consider multiple receivers instead. Unlike Crawford and

Sobel (1982), senders commit to their message and it affects their payoffs, hence communication is not cheap. Moreover, the environment in my model allows for receivers to not only be concerned with the accuracy of their individual actions but also with coordinating those actions with others.

It is noteworthy that in contrast to standard models of cheap talk with multiple receivers (e.g. Farrell, 1987; and Goltsman and Pavlov, 2011) where the sender has no incentive to misreport to any single receiver in an informative equilibrium, since payoffs depend only on the receiver's action, not on the content of the message itself. Moreover, if the desire to conform is sufficiently large then senders and receivers seek to coordinate naturally, which results in no information being transferred.

A common strand in the literature is to consider different modes of communication, a foundational paper in this regard is Farrell and Gibbons (1989), which discusses the behavior of a sender and two different agents, referred to as audiences. Equilibria in this paper differ depending on whether one or both audiences are present. In my model, communication is public and I therefore do not account for audience-dependent effects, the main focus being the analysis of the willingness to conform to the behavior of others.

Two particularly noteworthy extensions to these models related to this paper are Hagenbach and Koessler (2010), and Galeotti et al. (2013). Both papers reach similar conclusions and allow agents to communicate with multiple other agents. The main focus is to study the network structures involved in honest communication. In contrast, my model allows communication only once by a single agent; nevertheless, I observe similar communication effects. In addition, my model takes inspiration from the payoff function in Hagenbach and Koessler (2010), one of the main differences is that in my model non-coordination payoffs depend on the aggregate actions of all players. In addition, in Hagenbach and Koessler (2010) each agent receives a private signal about the state of nature, with the true state given by the sum of these signals. By contrast, my model features a single fully informed agent, while all other agents are uninformed, rather than each agent possessing partial information.

My paper is closely related to Bloch et al. (2018), who analyze how rumors spread both in public broadcast and through a network. I extend their public broadcast model to account for conformity and assume that the sender commits to his action, whereas in Bloch et al. (2018) agents are allowed to change their actions in the later stages. They focus on the conditions for misinformation transmission in equilibrium. My model implies however that if agents value the truth more than conforming with other, conformity can in fact enhance information transfer and help achieve social learning. While, if the effect is strong enough, it might hamper it in situations where it should have been otherwise possible.

This work is related to Bayesian social learning models, which examine how agents learn from observing others. This literature studies how agents update beliefs and choose actions based on private signals and based on private signals and the observed actions of others (Bikhchandani et al., 1992; Banerjee, 1992). Standard models show that actions may converge over time, but herding or miscoordination can occur, especially when signals are bounded or networks are directed (Smith and Sørensen, 2000; Acemoglu et al., 2011). While this literature emphasizes how information spreads through networks, agents in my model care not only about aligning with the true state, but also about coordinating their actions. This feature, in particular, affects the credibility of communication, as it depends not only on believes about the sender's motives but also on how messages shape collective rather behavior. Unlike models in which repetition is possible, learning in my setting occurs only once.

A large body of political research on collective voting highlights the role of social conformity. Bernheim (1994) shows that individuals may conform to behavioral norms because status depends on how actions signal unobservable predispositions, even when underlying preferences are heterogeneous. Building on this, subsequent studies examine the effect of conformity on collective voting behavior (Coleman, 2004; Moreno et al., 2019). In line with this literature, I incorporate conformity motives into my model.

The paper is organized as follows. The model and equilibrium notions are introduced in detail in section 1.2. Equilibrium characterizations for different parameter values are discussed in section 1.3. Section 1.4 provides a discussion of comparative statistics and robustness of social learning in large populations. Section 1.5 gives a summary and conclusion.

1.2 Model

In this section, I introduce the incomplete information game of coordination mentioned earlier. Broadly speaking, the game is as follows: a given agent (the *sender*) receives information privately, which he can transmit to other agents through his action (*message*). This message is observed by other agents (*receivers*). Immediately after the sender takes his action, the receivers then simultaneously choose their actions based on both the sender's message and what they think the unknown state of the world is.

Information Generation

The receivers' uncertainty, which is central to the game, stems from two private information sources: the sender's type and the state of the world, which determines their preferences. More specifically, consider a population $\mathcal{N}$ of voters $i \in \mathcal{N}$, $|\mathcal{N}| = n \geq 4$, consisting of two types $t \in \mathcal{T} = \{\text{unbiased}, \text{biased}\}$ of agents distinguished by their preferences over messages and outcomes: some do not have predetermined preferences about whether the collective outcome should align with the unknown state of nature and are willing to coordinate with others. Their desire to conform to the behavior of others is measured by a common and publicly known exogenous parameter $\alpha \in [0, 1]$ (0 indicates no desire to conform while 1 indicates a complete desire to conform). There are also agents who have predetermined preferences about what the outcome should be; they are not motivated by the desire to coordinate. The first class of agents are called *unbiased agents*. The set of unbiased agents is $\mathcal{U}$, I use the notation $i^u \in \mathcal{U}$, $|\mathcal{U}| \geq 3$, to indicate that an agent is unbiased. More generally, a superscript "u" refers to unbiased agents. The second class of agents are *biased agents*. The set of biased agents is $\mathcal{B}$, $|\mathcal{B}| \geq 1$, and, just as before, I write $i^b \in \mathcal{B} = \complement_{\mathcal{N}}\mathcal{U}$ to indicate that an agent is unbiased.[1] Similarly, a superscript "b" refers to biased agents.

Each agent knows their type but not that of other agents; however, they know the proportion of types in the population. In other words, individual agent types t are private information, while the proportion of biased agents is common knowledge. For a given unbiased agent $i^u \in \mathcal{U}$ in a population, the *bias ratio* b is the fraction of biased agents among the remaining population, defined by

$$b = \frac{|\mathcal{B}|}{|\mathcal{N}| - 1}.$$

As mentioned at the beginning, the state of nature is also unknown to most agents. It is assumed to be in either one of the two possible states $\Theta = \{0, 1\}$. A particular state is denoted by $\theta \in \Theta$. Each agent's incomplete information is characterized by the pair $(\theta, t) \in \Theta \times \mathcal{T}$ which combines the state of the world and the sender's type.

The assumptions on group size ensure that the population is sufficiently large to study communication effects while preventing biased agents from dominating. For instance, in a group of three agents with one biased agent, an unbiased agent would face either one unbiased and one biased agent, or two biased agents. In such small groups, the bias ratio is at least one-half, meaning that conforming behavior would be disproportionately influenced by the biased agent's presence.

The game proceeds in two distinct phases: (1) message transmission phase: a sender transmits information through a costly action; and (2) collective decision phase: receivers simultaneously choose actions based on the message. I now describe each phase in detail.

Message Transmission

A randomly chosen sender observes the realized state θ, while the remaining agents receive no direct information about the state. Despite θ being unknown to most agents, all unbiased agents are assumed to have a common

[1] $\complement_{\mathcal{N}}$ denotes the complement of the set $\mathcal{U}$ in the set $\mathcal{N}$.

prior belief π about what θ is (the prior belief of biased agents is irrelevant, since their utilities do not depend on the state θ). More precisely, $\pi = \mathbb{P}(\theta = 1) < \frac{1}{2}$, so that π represents the belief that $\theta = 1$ is the true state of nature. The assumption on the belief and its definition are chosen in this way so as to ensure that information transfer plays a meaningful role in the decision process of the unbiased agents.

In the sequel, I analyze the Perfect Bayesian Nash Equilibrium of this game in pure strategies. Each agent chooses one of two actions from the set $A = \{0, 1\}$. Observe that these are labeled in the same way as elements of Θ. As usual, a *strategy* for a given player is a function that specifies an action at each information set of the game.

Once the sender observes θ, he is allowed to decide whether to communicate this signal truthfully or to misreport it by choosing an action in A. Importantly, the sender cannot block or withhold the message. Recall that only he observes θ and takes an action before the receivers (the remaining players). This action is thus the message he sends to the receivers. For a sender of type t and $\theta \in \Theta$ $m^t(\theta)$ is the message he chooses to send. The strategy of the sender i^t can be described by a mapping $m^t(\theta) : \Theta \mapsto A$, $t \in \mathcal{T}$. Specifically, $m^u(\theta)$ denotes the strategy of an unbiased sender while $m^b(\theta)$ denotes the strategy of a biased one. In equilibrium, unbiased senders may truthfully report θ or choose to strategically misreport it depending on their incentives. By contrast, a biased sender strategically sends a fixed message that reflects his predetermined preference, aiming to influence the collective outcome in line with his inflexible objective. This intuitive outcome is analyzed formally later.

Collective Decision

After the sender has taken his action, each unbiased receiver updates his prior belief to a posterior $\rho(m) = \mathbb{P}(\theta = 1|m)$ according to Bayes rule. Note that since unbiased agents initially consider $\theta = 1$ to be less likely, this posterior probability plays a crucial role in determining the actions taken by unbiased receivers. Unbiased agents then take an action simultaneously about what each thinks the value of θ is. A profile of actions for receivers in the population is denoted by $\mathbf{v} = (v_1, ..., v_{n-1})$, with $v_j \in A = \{0, 1\}$, $j = 1, ..., n-1$.

Now, let $x \in [0, 1]$ denote the *collective outcome*. The outcome x is assumed to follow the "rule of the average". In policy terms, the chosen policy is the one proposed by a randomly chosen voter: $x = x(m, \mathbf{v})$: $(m, \mathbf{v}) \mapsto [0, 1]$,

$$x(m, \mathbf{v}) = \frac{1}{n}\left(m + \sum_{j=1}^{n-1} v_j\right).$$

The final outcome x is therefore the average of the actions taken by all agents weighted equally. Note that each agent therefore contributes equally to the final collective outcome, including the sender despite moving first. In addition, the rule of the average is particularly useful from a modeling perspective: when utilities are linear in the collective outcome, taking the simple average allows expectations to be easily computed, preserving tractability while highlighting how communication and coordination shape behavior and avoiding messy nonlinearities while still capturing the idea that more votes causes higher probability. Moreover, the rule makes the marginal contribution of each agent in the outcome transparent: An agent deviating from action 1 changes the collective outcome by exactly $\frac{1}{n}$. This property emphasizes that coordination incentives arise not from asymmetric voting power but from informational and strategic interactions among agents.

A summary of the timeline of the model is provided in figure 1.1 below.

Payoffs

Sender Preferences and Utility

Each sender has *intrinsic* preferences over the set $\Theta = \{0, 1\}$. The utility function of an unbiased sender $i^u \in \mathcal{U}$ choosing strategy m^u is

$$u^S_{i^u}(m^u; \mathbf{v}, \theta, \alpha) = -(1-\alpha)|x(m^u; \mathbf{v}) - \theta| - \frac{\alpha}{n-1}\sum_{j=1}^{n-1} |m^u - v_j(m^u)|. \quad (1.1)$$

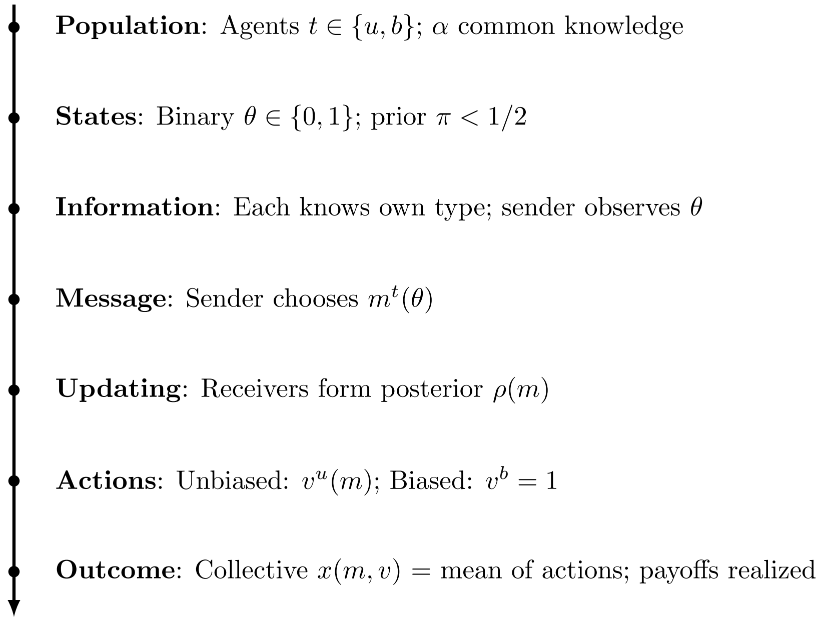

Figure 1.1: Timeline of the model

The first term in this equation corresponds to the loss from the collective outcome x failing to matching the true state θ (scaled by $1 - \alpha$). On the other hand, the second term accounts for losses from failing to coordinate with other agents (scaled by α). The parameter $\alpha \in [0, 1]$ is accordingly the relative weight loss from mis-coordination compared to loss from deviation from the outcome not matching θ. As explained earlier, if $\alpha = 0$ a given unbiased agent does not care about the remaining agents' choices and hence will choose the action that matches what they think the true state is. On the other hand, if $\alpha = 1$, the unbiased agent's utility is completely determined by other agents' actions and therefore they will have a strong incentive to coordinate with the majority.

Note that each unbiased agent shares the same weight α for tractability, even though heterogeneous weights would be more realistic. This simplification should not affect the results significantly.

A biased sender $i^b \in \mathcal{B}$ chooses m^b so as to bring the collective outcome x closer towards his preferred state $\theta = 1$. This is represented by the utility function

$$u^S_{i^b}(m^b; \mathbf{v}) = -|x(m^b; \mathbf{v}) - 1| = \frac{1}{n}\left(m^b + \sum_{j=1}^{n-1} v_j(m^b)\right) - 1. \tag{1.2}$$

Unlike unbiased agents, the biased sender's utility is independent of both the true state of nature θ and the parameter α.

Receiver Preferences and Utility

An unbiased receiver $j^u \in \mathcal{U}$ chooses an action $v_{j^u} \in \{0, 1\}$ to maximize utility by balancing benefits derived from the outcome aligning with the true state and benefits from coordination. The utility function is therefore:

$$u^R_{j^u}(v_{j^u}; \mathbf{v}_{-j^u}, m, \theta, \alpha) = -(1-\alpha)|x(v_{j^u}; \mathbf{v}_{-j^u}, m) - \theta| - \frac{\alpha}{n-1}\left(\sum_{k \notin \{j^u, i\}} |v_{j^u} - v_k| + |v_{j^u} - m|\right), \tag{1.3}$$

where $x(v_{j^u}; \mathbf{v}_{-j^u}, m) = \frac{1}{n}\left(v_{j^u} + m + \sum_{\substack{k=1 \\ k \neq j^u}}^{n-1} v_k\right)$ describes the outcome. Moreover, $\mathbf{v}_{-j^u}$ denotes the action profile of all receivers except j^u; while the sum in this equation is taken over agents other than j^u and the sender i.

Similarly, the utility of a biased receiver j^b from choosing v_{j^b} is

$$u^R_{j^b}(v_{j^b}; \mathbf{v}_{-j^b}, m) = -|x(v_{j^b}; \mathbf{v}_{-j^b}, m) - 1| = \frac{1}{n}\left(v_{j^b} + m + \sum_{\substack{k=1 \\ k \neq j^b}}^{n-1} v_k\right) - 1. \tag{1.4}$$

As stated previously, biased receivers always prefer $x = 1$ regardless of θ. Their utility depends only on the distance between x and 1, and are not affected by coordination. Therefore, $v_{j^b} = 1$ is a dominant strategy for a biased receiver.

1.3 Perfect Bayesian Nash Equilibrium Characterization

I first identify the conditions that determine receivers' optimal actions by requiring that beliefs are consistent. Given a profile of actions $\mathbf{v}$ and a message m, let $x_0^j = x(0; m, \mathbf{v}_{-j})$ denote the expected collective outcome when a given agent j chooses 0; $x_0^j + \frac{1}{n}$ is then the expexted collective outcome if the agent j chooses 1 instead.

For an unbiased receiver j^u, the expected utility from choosing action 0 after receiving message m is then

$$\mathbb{E}[u^R_{j^u}(v_{j^u} = 0; x_0^j)|m] = -(1-\alpha)\left\{\rho(m)(1 - x_0^j) + [1 - \rho(m)]x_0^j\right\} - \alpha\frac{n}{n-1}x_0^j.$$

In this equation, the terms in curly brackets refer to the expected loss from the collective outcome not matching the true state. The second term refers to loss from failing to coordinate with agents who take action 1. These coordination losses do not depend on the posterior beliefs $\rho(m)$ directly, since the actions of other agents are assumed to be known, but they do depend on the (unknown) type of the sender.

The expected utility of $j^u \in \mathcal{U}$ from choosing action 1 is on the other hand,

$$\begin{aligned}\mathbb{E}[u^R_{j^u}(v_{j^u} = 1; x_0^j)|m] = &-(1-\alpha)\left\{\rho(m)\left(1 - x_0^j - \frac{1}{n}\right) + [1-\rho(m)]\left(x_0^j + \frac{1}{n}\right)\right\} \\ &- \alpha\left(1 - \frac{n}{n-1}x_0^j\right).\end{aligned}$$

The previous two equations can be combined to arrive at the conclusion that an unbiased agent j^u chooses action 1 if

$$\rho(m) > \frac{1}{2}\left[1 + \frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m, \mathbf{v}_{-j}))\right], \tag{1.5}$$

while j^u chooses action 0 if

$$\rho(m) < \frac{1}{2}\left[1 + \frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m, \mathbf{v}_{-j}))\right], \tag{1.6}$$

when $\alpha \in [0, 1)$.

If the inequality signs in equations (1.5) and (1.6) are replaced by equal signs; so that

$$\rho(m) = \frac{1}{2}\left[1 + \frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m, \mathbf{v}_{-j}))\right],$$

then the agent is indifferent between both actions. If equality holds, the agent is indifferent between the two actions and may mix. However, since the analysis is restricted to pure-strategy equilibria, I exclude this knife-edge case. Allowing mixed strategies would complicate the analysis by introducing additional coordination considerations, without generating further insight into the mechanisms of interest. Hence, focusing on pure strategies is without loss of generality for the results derived below.

The cutoffs defined by equations (1.5) and (1.6) show that unbiased receivers are more likely to choose action 1 when one (or more) of the following conditions are fulfilled: (i) Their posterior belief $\rho(m)$ is sufficiently high; (ii) the conformity α is stronger (or smaller) when x_0^j is above (or below) the threshold $\frac{n-1}{2n}$; (iii) the expected collective outcome x_0^j is closer to 1. For a given message m and action profile $\mathbf{v}_{-j}$, I denote by $\kappa(m, \mathbf{v}_{-j})$ this cutoff, *the decision threshold*, for agent j. These are discussed in more detail below. Note that this threshold is characterized by two components:

$$\kappa(m, \mathbf{v}_{-j}) = \frac{1}{2}\left[1 + \underbrace{\frac{n\alpha}{1-\alpha}}_{\text{Coordination weight}}\left(1 - \underbrace{\frac{2n}{n-1}x_0^j(m, \mathbf{v}_{-j})}_{\text{Social influence}}\right)\right].$$

Now, consider the expression $\frac{n\alpha}{1-\alpha}$. This ratio appears in the context of determining conditions for truth-telling or equilibrium strategies in such games.

Specifically, it often arises as a threshold in the sender's incentive to tell the truth versus conform to others. The numerator $n\alpha$ scales the conformity weight by the number of players. he denominator $1-\alpha$ is the weight on accuracy. Thus, $\frac{n\alpha}{1-\alpha}$ measures the relative importance of conformity compared to accuracy, adjusted by the group size. I will show that how this ratio affects the threshold of biased ratio telling-truth equilibrium in the next section.

This equation highlights complexity inherent in the bound as it reflects the interaction between coordination and communication. These manifest (as suggested earlier) by the dependence of the bound on three parameters: α, the total population n and the expected collective decision x_0^j. The coordination weight $\frac{n\alpha}{1-\alpha}$ quantifies the value of aligning with others, while the effects of social influence are reflected in the term $\frac{2n}{n-1}x_0^j$, which accounts for the pressure to conform to the expected group behavior. If $\alpha \to 0$ (so that the unbiased agents are interested in the truth only), then the decision threshold satisfies $\kappa(m, \mathbf{v}_{-j}) \to \frac{1}{2}$. In the limiting case where $\alpha = 0$, the decision threshold reduces to $\frac{1}{2}$, meaning that unbiased agents take action 1 only if they think that $\theta = 1$ is likely to be true. On the other hand, if $\alpha \to 1$ (pure coordination), then the threshold $\kappa(m, \mathbf{v}_{-j}) \to +\infty$ when $x_0^j < \frac{n-1}{2n}$. As a result, the action 1 of an unbiased receiver is never chosen in this case. If, by contrast, $x_0^j > \frac{n-1}{2n}$ then $\kappa(m, \mathbf{v}_{-j}) \to -\infty$ and, as a result, an unbiased receiver shuns action 0. More generally, for $\alpha > 0$ the decision bound depends on how many agents are taking action 1, as this number increases the bound becomes less tight and the agent might not need to think that $\theta = 1$ is the most likely state to take action 1. Similarly, if a majority of other agents are taking action 0 the bound may become tighter and the unbiased agent is less incentivised to take action 1.

Given that all biased agents adopt the strategy $v_{j^b} = 1$ in equilibrium, I can restrict the analysis to finding equilibrium strategies for unbiased senders. This allows the following formal simplifications: The receivers' strategy profile $\mathbf{v}$ can be replaced by the vector of unbiased receiver strategies $\{v_{j^u}\}$, $j^u \in \mathcal{U}$. Accordingly, $x_0^j(m, \mathbf{v}_{-j^u})$ replaces $x_0^j(m, \mathbf{v}_{-j})$. Here $\mathbf{v}_{-j^u}$ represents the vector of unbiased receivers except j^u. Furthermore, I will simplify notation in the remainder of the chapter as follows: for a sender i; if he is unbiased, the utility $u^S_{i^u}(m^u; \mathbf{v}, \theta, \alpha)$ is reduced to $u^S_{i^u}(m^u; \mathbf{v}_{j^u}, \theta)$, while if he is biased, the utility $u^S_{i^b}(m^b; \mathbf{v})$ is reduced to $u^S_{i^b}(m^b; \mathbf{v}_{j^u})$. Meanwhile, for a receiver j; if he is unbiased, the utility $u^R_{j^u}(v_{j^u}; \mathbf{v}_{-j^u}, m, \theta, \alpha)$ is reduced to $u^R_{j^u}(x^j_0(m, \mathbf{v}_{-j^u}))$. On the other hand, if he is biased, the utility $u^S_{i^b}(m^b; \mathbf{v})$ is reduced to $u^S_{i^b}(m^b; x^j_0(m, \mathbf{v}_{j^u}))$.

The solution concept in this paper is *Perfect Bayesian Nash Equilibrium* (PBNE, for short). I will consider for simplicity *symmetric* equilibria only. Equilibria is said to be symmetric in the sense that agents of the same type behave identically. In particular, senders of the same type create identical messages, while receivers of the same type follow the same equilibrium strategy. Formally, a profile $(m^u, m^b, \rho, v_{j^u}, v_{j^b})$, with $v_{j^u} : \{0,1\} \mapsto \{0,1\}$; $m^u, m^b : \Theta \mapsto \{0,1\}$ and $\rho \in \Delta(\Theta)$ is a *symmetric PBNE* if it satisfies the next four conditions:

(i) **Senders' strategy** (Optimal Messaging):

For unbiased senders $i^u \in \mathcal{U}$:

$$m^u(\theta) \in \arg\max_{m\in\{0,1\}} \mathbb{E}\left[u^S_{i^u}(m; \mathbf{v}_{-i^u}, \theta, \alpha) \mid \rho(m)\right] \quad \forall \theta \in \Theta,$$

where the expectation takes into accounts receivers' strategies and posterior belief $\rho(m)$. Recall that the sender also knows θ.

For biased senders $i^b \in \mathcal{B}$:

$$m^b(\theta) \in \arg\max_{m\in\{0,1\}} \mathbb{E}\left[u^S_{i^b}(m; \mathbf{v}_{-i^b}, \theta) \mid \rho(m)\right] \quad \forall \theta \in \Theta,$$

where the expectation takes into account receivers' strategies and posterior belief $\rho(m)$.

(ii) **Receivers' strategy** (Optimal Actions):

For an unbiased receiver j^u,

$v_{j^u}(m) = 1$ if $\rho(m) > \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x^j_0(m, v_{-j^u \in \mathcal{U}} = 1)) + 1\right]$;

$v_{j^u}(m) = 0$ if $\rho(m) < \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x^j_0(m, v_{-j^u \in \mathcal{U}} = 0) + 1\right]$;

and for a biased receiver j^b,

$v_{j^b}(m) = 1$.

(iii) **Belief Consistency** (Bayesian Updating):

Whenever possible, the posterior belief $\rho(m)$ is updated via Bayes' rule:

$$\rho(m) = \mathbb{P}(\theta = 1 \mid m) = \frac{b \cdot \mathbb{P}(m^b = m \mid \theta = 1) + (1-b) \cdot \mathbb{P}(m^u = m \mid \theta = 1)}{b \cdot \mathbb{P}(m^b = m) + (1-b) \cdot \mathbb{P}(m^u = m)} \pi,$$

where the prior satisfies $\pi = \mathbb{P}(\theta = 1) < \frac{1}{2}$ and $\mathbb{P}(m^u = m)$ and $\mathbb{P}(m^b = m)$ represent the probabilities of the message received being sent by unbiased sender or biased sender respectively.

Note that it is a dominant strategy for biased receivers to take action 1 so the action of a biased receiver will match that of other biased receivers trivially in equilibrium.

Recall that PBNE can be divided into three categories: i. *separating equilibrium*, where different types of senders $m^t(\theta)$ take different actions and thus all information is transmitted to the receivers; ii. *pooling equilibrium*, where both types of senders take the same action, resulting in no information being transferred to receivers; and, iii. *partially-separating equilibrium*, where one type of agent always takes a certain action while the other separates the actions. A general property of this model is that no *separating equilibria* exists for any value of α however. This is because a biased senders' payoff is irrelevant of the true state of nature. Hence, under most circumstances, they prefer sending a fixed signal 1 rather than sending a message which discloses the true state of nature. Note that the goal of preventing information disclosure does provides an incentive for biased agents send message $m = 0$ in some cases. Partially separating equilibria can be realised in this model however. Naturally, in this case unbiased senders to disclose the state of the world, while biased agents send the same message independently of the signal received. Information transmission by unbiased senders takes one of two forms: an unbiased sender can tell the truth to the remaining agents or lie to them. More formally,

Definition 1.3.1. Truthful equilibrium: *a PBNE in which unbiased senders* *send a message identical to the signal they received, i.e.* $m^u(\theta) = \theta$, $m^b(\theta) = 1$.

Definition 1.3.2. Lying equilibrium: *a PBNE in which unbiased senders send a message opposite to the signal they received, i.e.* $m^u(\theta) = 1 - \theta$, $m^b(\theta) = 0$.

In the following sections, I characterize the equilibria of the model according to different values of the parameters involved beginning with the boundary cases.

Case: Fully Coordinated Unbiased Agents

In this case, $\alpha = 1$ which means the utility of each unbiased agent is independent of the true state, the game reduces to one that resembles a pure coordination one. The utilities include only coordination terms for unbiased agents while they remain the same for biased agents. The utility of a sender $i^u \in \mathcal{U}$ is accordingly:

$$u^S_{i^u}(m^u; \mathbf{v}) = -\frac{1}{n-1}\sum_{j=1}^{n-1} |m^u - v_j|; \tag{1.7}$$

while for an unbiased receiver j^u the utility is

$$u^R_{j^u}(v_{j^u}; \mathbf{v}_{-j^u}) = -\frac{1}{n-1}[\sum_{-j^u \notin \{j^u, i\}} |v_{j^u} - v_{-j^u}| + |v_{j^u} - m|] \tag{1.8}$$

instead. The expected utilities for unbiased agents are easy to compute in this case, since they contain the term coming from coordination only. The utility from choosing action 0 is therefore

$$\mathbb{E}[u^R_{j^u}(v_{j^u} = 0; x^j_0, m)|m] = -\frac{n}{n-1}x^j_0.$$

On the other hand, the utility from choosing action 1 is

$$\mathbb{E}[u^R_{j^u}(v_{j^u} = 1; x^j_0, m)|m] = -(1 - \frac{n}{n-1}x^j_0),$$

regardless of belief $\rho(m)$ since θ no longer affects any type agent's utility. Incomplete information in this case reduces from (θ, t) to simply t.

As a consequence of utility being purely dependent on coordination. Unbiased receivers take their action by following the simple majority rule:

$$v_{j^u} = \begin{cases} 1 & \text{if } x_0^j > \frac{n-1}{2n} \\ 0 & \text{if } x_0^j < \frac{n-1}{2n} \end{cases}$$

In this regime, it is intuitive to conjecture that the rational equilibrium is the one in which all agents take action 1 whenever a single biased agent exists in the population. This is because biased agents create an incentive to take action 1 since they always choose action 1. More formally, when unbiased agents benefit from coordination only then symmetric equilibria can be Pareto ranked. The optimal equilibrium being the one in which all agents take action 1:

Proposition 1.3.1 (Coordination-Dominated Equilibrium)**.** *Given any positive share of biased agents $b > 0$ and state space $\theta \in \{0, 1\}$,*

i. senders' strategies: $m^t(\theta) = 1$ for $t \in \mathcal{T}$; and

ii. receivers' strategies: $v_{j^t}(1) = 1$ for any $t \in \mathcal{T}$ form an equilibrium.

For a proof of this proposition and others, the reader is referred to the appendix at the end of this chapter.

The equilibrium in proposition 1.3.1 constitutes an equilibrium where all agents achieve the maximal possible payoff of 0. Naturally, this outcome is the only stable one whenever $b > 0$. As a result, information transfer is not possible in this regime.

When $b < \frac{1}{2}$, the Pareto efficient outcome is not the only equilibrium. Indeed, if $b < \frac{1}{2}$, unbiased agents can coordinate with each other by taking action 0 while the biased agents persist in sending 1. This leads to an equilibrium in which information about sender type is revealed:

Proposition 1.3.2 (Type-Revealing Equilibrium)**.** *Given $b < \frac{1}{2}$;and $\theta \in \{0, 1\}$,* *i. sender's strategies:* $m^u(\theta) = 0$ *and* $m^b(\theta) = 1$ *for* $i^u \in \mathcal{U}$ *and* $i^b \in \mathcal{B}$*; and*

ii. receivers' strategies: $v_{j^u}(m) = 0$ *and* $v_{j^b}(m) = 1$ *for* $j^u \in \mathcal{U}$ *and* $j^b \in \mathcal{B}$ *on the path.*

The equilibrium in proposition 1.3.2 may arise when unbiased receivers coordinate with each other on voting 0. This forces an unbiased sender to adopt message 0 independently of θ. Receivers coordinating on action 0 may happen when action 1 is seen as risky, for instance. This causes senders' type to be revealed in equilibrium.

Case: Truth-Seeking Unbiased Agents

For $\alpha = 0$, the model coincides with a variant of Bloch et al. (2018), whose work directly inspired the approach taken in this paper. Results in my paper are similar to theirs in this regime. Unlike their paper, I specify that a sender transfers his action as the message. In other words, a sender cannot change his action after sending a message to receivers. Another important finding in my model is the existence, for some particular values of the parameters, of lying equilibria. This is different from Bloch et al. (2018), which focuses exclusively on truthful equilibrium and does not mention lying equilibrium at all.

When $\alpha = 0$, the utilities for biased agents are unchanged while utilities for unbiased agents only contain the term resulting from the outcome being different to the true state θ. Hence, for $i^u \in \mathcal{U}$ if he acts as sender

$$u^S_{i^u}(m^u; \mathbf{v}, \theta) = -|x(m^u; \mathbf{v}) - \theta|; \tag{1.9}$$

while if he acts as receiver

$$u^R_{j^u}(v_{j^u}; \mathbf{v}_{-j^u}, m, \theta) = -|x(v_{j^u}; \mathbf{v}_{-j^u}, m) - \theta|. \tag{1.10}$$

As before, $x^j_0 = x(m, \mathbf{v}_{-j})$ is the collective outcome if agent j chooses 0 and $x^j_0 + \frac{1}{n}$ is the collective outcome if he chooses 1 instead. Observe that for an unbiased receiver $j^u \in \mathcal{U}$ the expected utility of choosing action 0 after receiving the message m is

$$\mathbb{E}[u^R_{j^u}(v_{j^u} = 0; x^j_0, m)|m] = -\rho_j(m)(1 - x^j_0) - [1 - \rho_j(m)]x^j_0;$$

while his expected utility from choosing action 1 is

$$\mathbb{E}[u^R_{j^u}(v_{j^u} = 1; x^j_0, m)|m] = -\rho(m)\left(1 - x^j_0 - \frac{1}{n}\right) - [1 - \rho(m)]\left(x^j_0 + \frac{1}{n}\right)$$

instead. As I discussed earlier, an unbiased agent j^u takes action 1 if $\rho(m) > \frac{1}{2}$, while he takes action 0 if $\rho(m) < \frac{1}{2}$.

There are three PBNE in this case: a truthful equilibrium, a lying equilibrium and a *truthful equilibrium (biased-mimicking)*, where unbiased senders communicate truthfully while all unbiased receivers behave as "biased" agents who prefer a certain choice regardless of the message. The first two types of equilibria occur whenever $b < \frac{\pi}{1-\pi}$; while the truthful equilibrium (biased-mimicking) occurs when $b > \frac{\pi}{1-\pi}$. These are each separately discussed in the next three propositions.

Proposition 1.3.3 (Truthful equilibrium)**.** *Assuming $b < \frac{\pi}{1-\pi}$, for $\theta \in \{0, 1\}$, the message strategies $m^u(\theta) = \theta$ and $m^b(\theta) = 1$ induce the following belief $\rho(m)$ on any unbiased receiver $j^u \in \mathcal{U}$:*

$$\rho(m) = \begin{cases} 0, & \text{if } m = 0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 1. \end{cases} \quad (1.11)$$

Moreover, receivers follow the strategies $v_{j^u}(m) = m$ for any $j^u \in \mathcal{U}$ and $v_{j^b}(m) = 1$ for any $j^b \in \mathcal{B}$.

Proposition 1.3.4 (Lying equilibrium)**.** *Assuming $b < \frac{\pi}{1-\pi}$, for $\theta \in \{0, 1\}$, the message strategies of the sender $m^u(\theta) = 1 - \theta$ and $m^b(\theta) = 0$ induce the following belief $\rho(m)$ on any unbiased receiver $j^u \in \mathcal{U}$:*

$$\rho(m) = \begin{cases} 0, & \text{if } m = 1 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 0. \end{cases} \quad (1.12)$$

Moreover, receivers follow the strategies $v_{j^u}(m) = 1 - m$ *for any* $j^u \in \mathcal{U}$ *and* $v_{j^b}(m) = 1$ *for any* $j^b \in \mathcal{B}$*.*

In a lying equilibrium, unbiased agents reveal their information even though they attempt to hide it. This equilibrium is however not efficient. The fact that this equilibrium is not stable is discussed in section II below. Intuitively, even tough unbiased senders disclose the true state of nature they do it in an inefficient way; by making the mean action be further away from the true state than it should be if they had told the truth instead. In addition, since biased agents wish to conceal the true state to bring the outcome x closer to 1, biased senders are forced to send 0 instead of their preferred choice of 0. This creates further inefficiencies.

The bound of b on these two propositions depends on π. This reflects the fact that even though a large presence of biased agents in the population makes information unreliable, this effect might be mitigated if the prior belief of unbiased agents is high enough.

When b is large enough, the receiving message $m = 1$ does not provide useful information to any unbiased receiver. This is because it is more likely than not that $m = 1$ comes from a biased agent. In these circumstances, unbiased receivers *behave as* if they were another class of "biased" agents in equilibrium. More precisely,

Proposition 1.3.5 (Truthful equilibrium (biased-mimicking))**.** *Assuming* $b > \frac{\pi}{1-\pi}$*, for* $\theta \in \{0, 1\}$*, the message strategies of the sender* $m^u(\theta) = \theta$ *and* $m^b(\theta) = 1$ *induce the following belief* $\rho(m)$ *on any unbiased receiver* $j^u \in \mathcal{U}$*:*

$$\rho(m) = \begin{cases} 0, & \text{if } m = 0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 1. \end{cases} \tag{1.13}$$

Moreover, receivers follow the strategies $v_{j^u}(m) = 0$ *for any* $j^u \in \mathcal{U}$ *and* $v_{j^b}(m) = 1$ *for any* $j^b \in \mathcal{B}$*.*

The nature of equilibria for different choices of the parameters b and π is illustrated in the diagram below.

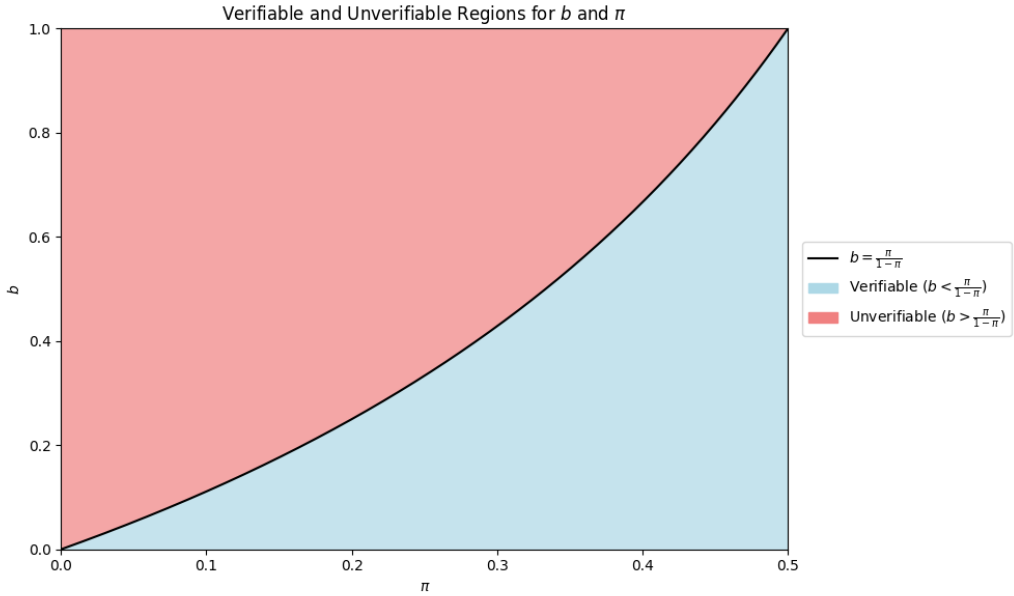

Figure 1.2: The bound b in the equilibrium when $\alpha = 0$

As stated above, as the belief π increases, the threshold between the two equilibrium regimes becomes less tight. Consequently, the population admits a higher proportion of biased agents. In the diagram this can be seen by an increase in the verifiable area (corresponding to both truthful and lying equilibrium, where information transfer takes place) corresponding to the increase of π.

When $\alpha = 0$ and $b < \frac{\pi}{1-\pi}$, truthful equilibrium Pareto dominates the lying one. This is because unbiased senders choose messages that maximize the state-alignment payoff while biased senders follow their preferred action. This motivates an alternative explanation as to why the lying equilibrium appears is that receivers may attempt to punish biased senders by voting 0 when receiving 1. Senders respond to this threat by lying about the true state. However, such a punishment is inefficient as senders have incentives to revert to their original behavior.

Biased-mimicking equilibrium is caused by the proportion of biased agents being too large. Unbiased agents take action 0 when receiving message 1 because this information is unreliable. This in spite of unbiased senders also creating message 1 whenever $\theta = 1$. Note that it is not possible for unbiased agents to punish biased agents since information transfer is not possible; action 0 is, therefore, the only safe option for an unbiased agent.

In summary, the threshold $\frac{\pi}{1-\pi}$ represents a discontinuous transition from a regime where information about θ can be transferred (truthful or lying equilibria) to a regime where information transfer breaks down completely (biased-mimicking equilibrium).

Proposition 1.3.6. *There is no* pooling equilibrium*, i.e. one in which either* $m^t(\theta) = 1$ *for any* $t \in \mathcal{T}$*, or* $m^t(\theta) = 0$ *for any* $t \in \mathcal{T}$*.*

Proposition 1.3.6 is intuitive if $\alpha = 0$. This is because unbiased senders will always attempt to transfer information about the true state to ensure that unbiased agents take actions that match the true state. They are also fully informed about this state. Moreover, biased agents have a strong incentive to create message 1, so a pooling equilibrium where $m^t(\theta) = 0$ for any $t \in \mathcal{T}$ is not possible for this reason.

General Case: Partial Coordinated Unbiased Agents

I consider now the general case when $\alpha \in (0, 1)$ and analyse how the different equilibria depend on both α and b. Recall that since I assumed that there is at least one biased agent and at least 3 unbiased agents in the population the following bounds must hold $\frac{1}{n-1} \leq b \leq \frac{n-3}{n-1} < \frac{n-2}{n-1}$, $|\mathcal{U}| \geq 3$.

Proposition 1.3.7 (Truthful equilibrium)**.** *Let* $\alpha \in (0, 1)$*. For* $\theta \in \{0, 1\}$*, the message strategies of the sender* $m^u(\theta) = \theta$ *and* $m^b(\theta) = 1$ *induce the following belief* $\rho(m)$ *on any unbiased receiver* $j^u \in \mathcal{U}$*:*

$$\rho(m) = \begin{cases} 0, & \text{if } m = 0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 1, \end{cases} \tag{1.14}$$

and receivers follow the strategies $v_{j^u}(m) = m$ *for any* $j^u \in \mathcal{U}$ *and* $v_{j^b}(m) = 1$ *for any* $j^b \in \mathcal{B}$*. This forms an equilibrium if* α *and* b *satisfy one of the following conditions:*

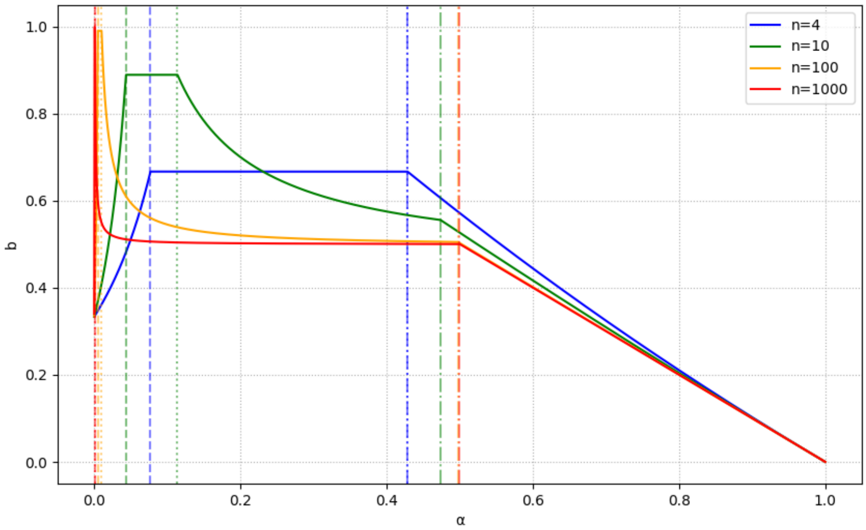

Figure 1.3: The upper bound on b when $\alpha \in (0,1)$ and $\pi = 0.25$ in the truthful equilibrium

(1) $\alpha \in (0, \frac{k-1}{n-1+k(n+1)})$, $b < \frac{\pi}{1-\pi}\left(1 + \frac{2}{R(\alpha,n)-1}\right)$;
(2) $\alpha \in [\frac{k-1}{n-1+k(n+1)}, \frac{n-1}{n^2-2n-1})$, $\forall b \in (0, \frac{n-2}{n-1})$;
(3) $\alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1})$, $b < \frac{1}{2}\left(1 + R(\alpha,n)\right)$;
(4) $\alpha \in [\frac{n-1}{2n-1}, 1)$, $b < \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}$,
where $k = \frac{n-2}{n-1}\frac{1-\pi}{\pi}$ *and* $R(\alpha,n) = \frac{1-\alpha}{n\alpha}$.

When viewed as a function of α the bound on b can be split into four regions (this function is shown in figure 1.3 below for different populations). The first of these regions (i.e $\alpha \in (0, \frac{k-1}{n-1+k(n+1)})$), corresponds to low coordination values. In this case, gains from conformity contribute to information transfer by making the bound on b less tight. Coordination gains are however small and unbiased agents priority is to align with the true state. Note further that when $\alpha = 0$ the threshold $\frac{\pi}{1-\pi}\left(1 + \frac{2}{R(\alpha,n)-1}\right)$ reduces $\frac{\pi}{1-\pi}$, the bound when coordination is absent.

The second region ($\alpha \in [\frac{k-1}{n-1+k(n+1)}, \frac{n-1}{n^2-2n-1})$) corresponds to moderate coordination values. In this regime, gains from conforming to actions taken by others are sufficiently high to ensure unbiased agents are willing to vote 1 when receiving message 1. These gains are not sufficient to provide an incentive for unbiased agents to try to coordinate with biased agents, even if their number is very high. This balance benefits information transfer.

Although conformity can significantly enhance information transfer by allowing for ever increasing numbers of biased agents. The range of α where this takes place has size $\frac{1}{n}$ approximately. This means that for large populations this effect is not robust.

Whenever $\alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1})$ the incentive to coordinate becomes an important factor for unbiased receivers. Naturally, when receiving message $m = 1$ agents conformity ensures biased agents choose action 1. On the other hand, to ensure that unbiased agents vote 0 when receiving message $m = 0$ the number of biased agents cannot be too high so as to ensure belief consistency. This is illustrated in figure 3 by the fast (hyperbolic) reduction in unbiased agents. Since truth seeking is still relevant, the bound on b can exceed $\frac{1}{2}$ slightly.

Finally, if $\alpha \in [\frac{n-1}{2n-1}, 1)$ then coordination incentives are dominant. Since sending message $m = 1$ results in full coordination the sender has a strong incentive to do so. For truthful equilibrium to exist in this range of α, the proportion of biased agents must be very small to ensure that the sender is willing to coordinate with unbiased receivers when $\theta = 0$. Indeed, for large population the bound on b is approximately $1 - \alpha$. Indicating that even a small proportion of biased agents can disrupt truth-telling when agents care only about coordination.

Proposition 1.3.8 (Lying equilibrium)**.** *Let $\alpha \in (0, 1)$. For $\theta \in \{0, 1\}$, the message strategies of the sender $m^u(\theta) = 1 - \theta$ and $m^b(\theta) = 0$ induce the following belief $\rho(m)$ on any unbiased receiver $j^u \in \mathcal{U}$:*

$$\rho(m) = \begin{cases} 0, & \text{if } m = 1 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 0, \end{cases} \tag{1.15}$$

and receivers follow the strategies $v_{j^u}(m) = 1 - m$ for any $j^u \in \mathcal{U}$ and $v_{j^b}(m) = 1$ for any $j^b \in \mathcal{B}$. This forms an equilibrium if α and b satisfy one *of the following conditions:*

(1) $\alpha < (0, \underline{\alpha}(\pi, n))$[2], $b < \frac{\pi}{1-\pi}\left(1 + \frac{2}{\frac{n-3}{n-1}\cdot R(\alpha,n)-1}\right)$;

(2) $\alpha \in [\underline{\alpha}(\pi, n), \frac{n-1}{n^2-2n-1})$, $b < \frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}$;

(3) $\alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1})$, $b < \frac{1}{2}\left(\frac{n-3}{n-1} + R(\alpha, n)\right)$;

(4) $\alpha \in [\frac{n-1}{2n-1}, 1)$, $b < \frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}$,

where $R(\alpha, n) = \frac{1-\alpha}{n\alpha}$.

Similarly to the case $\alpha = 0$ this equilibrium describes a situation where unbiased senders transfer information inefficiently by creating messages opposite to the true state. Biased senders attempt to conceal information by mimicking this behavior (i.e. they follow the strategy $m(\theta) = 0$). Receivers rationally anticipate this deception and decode the messages by choosing the opposite action.

As for truthful equilibrium, the bound on b can be regarded as a function of α four distinct regions appear. A plot of this function for different populations is shown in figure 4 below.

If coordination is small (i.e. if $\alpha < \underline{\alpha}(\pi, n) < \frac{1}{n+1}$) the threshold of b is similar to that in the truthful equilibrium case. The main difference is the presence of a small penalty that arises because the sender does not coordinate with the remaining agents when $\theta = 1$. As in the case of truthful equilibrium, conformity helps relax the bound on b since unbiased receivers benefit slightly from coordinating with biased agents when $m = 0$ is received.

Whenever $\alpha \in [\underline{\alpha}(\pi, n), \frac{n-1}{n^2-2n-1})$ the effect of coordination is strong enough to ensure unbiased receivers take action $v_{j^u} = 1$ when receiving message $m = 0$ but not strong enough for them to deviate from action $v_{j^u} = 0$ when receiving message $m = 1$. Unlike truthful equilibrium, coordination effects are sufficiently high to create an incentive for an unbiased sender to deviate. To prevent this deviation, the number of biased agents must decrease when α increases.

If $\alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1})$ the bound in b resembles again the bound in truthful

[2] where $\frac{\pi}{1-\pi}\left(1 + \frac{2}{\frac{n-3}{n-1}\cdot R(\alpha,n)-1}\right)$ and $\frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}$ intersect at $\underline{\alpha}(\pi, n) \in (0, \frac{1}{n+1})$.

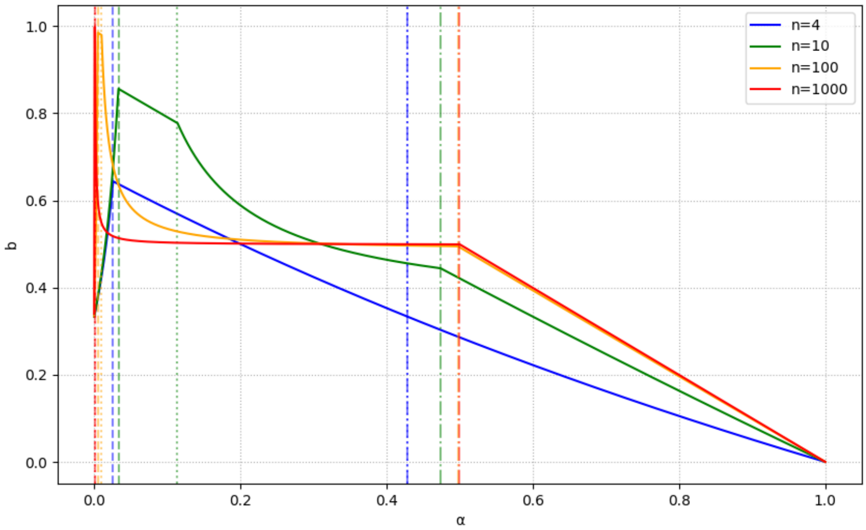

Figure 1.4: The upper bound on b when $\alpha \in (0, 1)$ and $\pi = 0.25$ in the lying equilibrium

equilibrium. The main difference being the factor of $\frac{n-3}{n-1}$. The factor arising because the senders does not coordinate with the remaining agents when $\theta = 1$. As for truthful equilibrium, the effects of conformity are sufficiently strong for unbiased agents to want to deviate from action $v_{j^u} = 0$ when receiving message $m = 1$ (in this case the effect is compounded slightly since the sender also takes action 1). To ensure unbiased receivers do not deviate from their equilibrium strategies the proportion of biased agents must be relatively small. Note however that truth seeking behavior still allows for this proportion to be slightly above $\frac{1}{2}$.

Finally, if $\alpha \in [\frac{n-1}{2n-1}, 1)$ coordination pressures are substantial. To ensure that lying equilibrium remains the proportion of biased agents need to be significantly small. This is because an unbiased sender is now motivated to deviate from sending message $m = 0$ to send message $m = 1$ when $\theta = 1$.

As can be seen in figure 1.4, for large populations the threshold for lying equilibrium to exist reduces to the one for truthful equilibrium.

When $\alpha > 0$ pooling equilibria can exists in the model when coordination becomes sufficiently high.

Proposition 1.3.9 (Pooling equilibrium)**.** *Let $\alpha \in (0,1)$. Given $\alpha > \frac{1}{n+1}$, the following strategies and beliefs form an equilibrium: for $\theta \in \{0,1\}$, the strategies $m^t(\theta) = 1$ for $t \in \mathcal{T}$, induce the following belief $\rho(m)$ on any unbiased receiver $j^u \in \mathcal{U}$:*

$$\rho(m) = \begin{cases} \mu, & \text{if } m = 0 \\ \pi, & \text{if } m = 1, \end{cases} \tag{1.16}$$

i. senders' strategies: $m^t(\theta) = 1$ for $t \in \mathcal{T}$; and

ii. receivers' strategies: $v_{j^t}(1) = 1$ for $t \in \mathcal{T}$ on the path; while

iii. off-the-equilibrium paths:

receivers follow strategies: $v_{j^t}(0) = 1$ for $t \in \mathcal{T}$[3] ; or

$v_{j^u}(0) = 0$ and $v_{j^b}(0) = 1$ for $j^u \in \mathcal{U}$ and $j^b \in \mathcal{B}$ with under the following conditions of α and b: If $b \leq \frac{1}{2}$, then $\alpha > \frac{(1-b)n+b}{n+b}$. While, if $b > \frac{1}{2}$, then $\alpha \in (\frac{(1-b)n+b}{n+b}, \frac{1}{n(2b-1)+1})$. This last condition holds whenever $b \in (\frac{1}{2}, \frac{n}{2(n-1)})$.[4]

The requirement $\alpha > \frac{1}{n+1}$ in proposition 1.3.9 reflects the fact that coordination needs to be strong enough for pooling equilibria to arise. If $b > \frac{1}{2}$ then there exists a unique pooling equilibrium where $v_{j^u}(m) = 1$ whenever $\alpha > \frac{1}{n+1}$. This is because biased agents provide a focal point for agents to coordinate around once conformity is high enough. When $b < \frac{1}{2}$ on the other hand a second inefficient equilibria arises where unbiased agents vote 0 off-the-equilibrium path (i.e. $v_{j^u}(0) = 0$). In this equilibrium stronger coordination is required to ensure an unbiased receiver does not deviate from taking action 1 when $\theta = 0$ where there are additional losses from the final outcome being 1.

If $b < \frac{1}{2}$ pooling equilibria with $v_{j^u}(0) = 0$ should be unstable. This is because either receivers would coordinate with biased agents, resulting in higher pay-offs. Or the receiver would succumb and coordinate with unbiased agents. Interestingly, provided $\alpha > \frac{1}{n+1}$ this results in equilibria where agents type is fully disclosed, but no information about θ is revealed.

[3] The posterior belief is $\rho(0) = \mu > \kappa(m = 0, \mathbf{v}_{-j^u} = 1)$in this case.

[4] In either case, the posterior belief is $\rho(0) = \mu < \kappa(m = 0, \mathbf{v}_{-j^u} = 0)$.

Proposition 1.3.10 (Type-Revealing Equilibrium)**.** *Let* $\alpha > \frac{1}{n+1}$*. Given* $b < \frac{1}{2}(1 - R(\alpha, n))$*, the following strategies form an equilibrium: for* $\theta \in \{0, 1\}$*,*

i. sender's strategies: $m^u(\theta) = 0$ *and* $m^b(\theta) = 1$ *for* $i^u \in \mathcal{U}$ *and* $i^b \in \mathcal{B}$ *respectively;*[5] *and*

ii. receivers strategies: $v_{j^u}(m) = 0$ *and* $v_{j^b}(m) = 1$ *for* $j^u \in \mathcal{U}$ *and* $j^b \in \mathcal{B}$ *respectively.*

Since $\pi < \frac{1}{2}$ unbiased receivers have a preference for action 0 when there is no information. Consequently, when the number of biased agents is small they follow this action, even when coordination is large. To ensure the equilibrium in proposition 1.3.10 exists ensuring the sender does not deviate becomes crucial. This is particularly important when $\theta = 1$ as the sender suffers additional losses form the outcome not matching $\theta = 1$. In this case low number of biased agents ensure the sender does not have an incentive to defect from coordinating with the rest of the unbiased agents.

1.4 Comparative Statics and Equilibrium Robustness

I now discuss how equilibria change as the parameters of the model vary. First, observe that $\alpha = \frac{1}{n+1}$ marks the transition point from a regime where the behavior of unbiased agents is mostly determined by aligning with the true state, to one where coordination dominates. Moreover, as $n \to +\infty$ equilibria where information about θ is transferred becomes less relevant.

Crossing the threshold $\alpha = \frac{1}{n+1}$ gives rise to pooling equilibria, which do not exists for $\alpha < \frac{1}{n+1}$. In the extreme case $\alpha = 1$ where action 1 becomes preferable. Full coordination becomes the optimal outcome. However, for bias thresholds $b < \frac{1}{2}$ less efficient forms of coordination are present: pooling equilibria where unbiased agents take action 0 off-the-equilibrium path, and equilibria where agents coordinate according to their type.

In the general case $\alpha \in (\frac{1}{n+1}, 1)$ if $b \leq \frac{1}{2}$ and $n \to +\infty$ the bound $\alpha > \frac{(1-b)n+b}{n+b}$ can be approximated as $\alpha > 1 - b$. In this regime, a small increase

[5] Note that in this case $\rho(m) = \pi$ for every $m \in \{0, 1\}$.

Δb allows for a reduction in the degree of conformity of roughly the same magnitude to maintain equilibrium (i.e . $\Delta\alpha \approx -\Delta b$). On the other hand, for $b > \frac{1}{2}$ the requirement that $b < \frac{1}{2(1-\frac{1}{n})} \approx \frac{1}{2}(1+\frac{1}{n})$ means that for large populations an equilibrium where unbiased senders take action 0 off-the-equilibrium path requires a population where biased agents exceed unbiased agents by a single agent.

In the other extreme, when $\alpha = 0$ three forms of equilibria exist according to the threshold $b = \frac{\pi}{1-\pi}$. Below this threshold information transferred is possible (either through truthful disclosure by unbiased senders or by indirect inference if senders lie). Above this bound information from senders becomes unreliable and unbiased agents choose action 0 for any message they receive.

The threshold $b = \frac{\pi}{1-\pi}$ becomes less tight as $\pi \uparrow \frac{1}{2}$ where $\frac{\pi}{1-\pi} \to 1$. In fact, for small prior beliefs a small increase in prior belief $\Delta\pi$ results in the population admitting approximately that many more unbiased agents (i.e. $\Delta b \approx \Delta\pi$).

More generally, for $\alpha \in [0,1)$ equilibria where information is transferred exist. For truthful equilibria, for α small ($\alpha < \frac{k-1}{n-1+k(n+1)}$ with $k = \frac{n-2}{n-1}\frac{1-\pi}{\pi}$) the conditions resemble those of pure communication. In fact, the threshold in this region is $\frac{\pi}{1-\pi}\left(1+\frac{2n\alpha}{1-\alpha(1+n)}\right)$. Note that for $\pi \uparrow \frac{1}{2}$ the bound converges to a quantity strictly larger than 1. For large prior beliefs the population admits arbitrary number of agents. In addition, and increase in π leads to a reduction in k which means this region decreases in size. For small values of π and α, a small increase in π, $\Delta\pi$, leads to the population admitting an additional $\Delta b \approx (1+2n\alpha)\cdot\Delta\pi$, the presence of conformity allows an additional $2n^2\pi\alpha$ biased agents compared to when $\alpha = 0$. Similarly, a small increase $\Delta\alpha$ leads to $\Delta b \approx 2n\pi\Delta\alpha$. The intermediate values of α ($\frac{k-1}{n-1+k(n+1)} \leq \alpha < \frac{n-1}{n^2-2n-1}$) do not place any restriction on b.

Once conformity becomes important ($\alpha > \frac{1}{n+1}$) truthful equilibrium bounds depend on α and n only. In particular, for moderately high values ($\frac{n-1}{n^2-2n-1} \leq \alpha < \frac{n-1}{2n-1}$) the b is roughly inversely proportional to the product $n\alpha$, thus if both quantities are multiplied by a common ratio λ, b lowers by approximately $\frac{1}{\lambda^2}$. Observe that the size of this region approaches $\frac{1}{2}$ as $n \to +\infty$. For higher values, ($\frac{n-1}{2n-1} \leq \alpha < 1$) the bound $\frac{n(1-\alpha)}{n-1+\alpha}$ is independent of n for large values of n. In such regime, it is approximately $1-\alpha$. This implies that a small increase $\Delta\alpha$ results in an identical reduction in the bound (i.e. $\Delta b \approx -\Delta\alpha$ for n large).

The behavior of the bound for b lying equilibria is similar to that of truthful equilibria for small values of α and large population sizes. Similar results also hold for moderately high levels of conformity. The main difference is in the intermediate and high regions where the bound is $\frac{(n-2)(1-\alpha)}{n+\alpha-1}$. For large populations, the bound is roughly $1-\alpha$ and independent on n.

Finally, for $\alpha > \frac{1}{n+1}$ and the type-revealing equilibrium has a bound $b < \frac{1}{2}(1-\frac{1-\alpha}{n\alpha}) \approx \frac{1}{2}(1-\frac{1}{n\alpha})$ for $n \to +\infty$. This means that a small change in $\Delta\alpha$ results in the bound becoming tighter by an amount $\Delta b \approx -\frac{\Delta\alpha}{2\alpha^2}$. An analogous reduction takes place with n replacing α.

It follows from this discussion that truthful equilibrium is robust in the sense that for very large populations truthful equilibria can exist, even if biased agents make up to nearly half of the population. This suggests that conformity can help the transfer of information, and thus social learning, in large populations.

Theorem 1.4.1 (Robust Social Learning)**.** *For $n \to \infty$ truthful equilibrium exists whenever $b < \frac{1}{2}$ and $\alpha \in (0, \frac{1}{2})$. If $\alpha > \frac{1}{2}$ then truthful equilibrium exists provided $b < 1-\alpha$.*

1.5 Conclusion

In this paper, I have presented an analysis of a strategic communication game with heterogeneous agents and varying degrees of conformity. I have obtained full equilibrium characterizations across the entire range of conformity levels. If the willingness to conform does not exceed the desire of unbiased agents to take actions that match the true state θ, then conformity can enhance social learning in the sense that for a given prior belief π the amount of biased agents that the population can admit while supporting truthful communication by unbiased agents is larger than when conformity is present. In fact, for small populations for $\alpha < O(\frac{1}{n})$, there exists a regime for which no bound of biased agents exist. More generally, if $n \to \infty$ then in so far as biased agents do not form a majority, truthful equilibrium can survive given any prior belief. On the other hand, if conformity dominates for information transfer to take place the number of biased agents in the population must be substantially lower than it would have been in the absence of conformity.

Despite the relative simplicity of my model, it has several notable applications. For instance, the analysis can be applied to the growing influencer market, Where individuals can post reviews of products or services to other consumers, who in turn make their own purchase decisions. In relation to this, my model could be applied to is the spreading of rumors through the internet. Indeed, followers of a given celebrity can be informed of the person's opinion promptly by looking at posts on social media which may show a given celebrity's support to a certain cause, a promotion of a product or a call to act in a certain way. The follower can also react by expressing support or dislike pretty directly. Attitudes of either celebrity or follower can be "neutral" or "biased", which can affect the direction of the topic being discussed. Additionally, the model may be applied to adoption of technology where agents can observe the deployment of a technology by a socially well-connected individual. The remaining agents can then adopt or not the technology, which might have repercussions for the whole population via network externalities. The model can also be applied to statement on committees. More generally, governmental decision making can also be modeled using the model presented here. This is because often when governments advocate certain policies the final outcome of the policy is mostly determined by the attitudes of the public who might follow it or not depending perhaps on their own biases and preferences. The perceived attitude of public by the government might also have an effect on the policy they wish to propose, preferring a popular one to an unpopular one even if the former might be more beneficial.

As it is a common occurrence with models of strategic communication, multiple equilibria arise for a particular choice of the parameters α and b. Unlike standard models, the presence of conformity effects alters how receivers interpret signals and how the sender evaluates potential deviations. This makes performing equilibrium selection and refinement more challenging formally. Note that in some cases it is intuitively clear that inefficient equilibria such as lying equilibria should not be the outcome of rational interaction, since agents can transfer information truthfully at a lower cost. On the other hand, note that particularly for medium to high values of α, the model contains both separating and pooling equilibria. In the context of signalling games there are two well-known refinements: the Intuitive Criterion introduced in Cho and Kreps (1987) and the Divinity Criterion introduced in Banks and Sobel (1987). Both of these refinements place constraints on the off-path beliefs. It will be interesting to adapt one of these criteria to my model to obtain refinements for some fo the equilibria discussed.

Several extensions could enhance the theoretical realism and empirical applicability of the model. First, the assumption that α is common and known is too restrictive. A more realistic setting would allow each individual to have a privately known value of α, drawn randomly prior distribution. Second, introducing multiple types of biased agents could better capture heterogeneous motivations. Third, in practice, communication spreads though public broadcast amongst a "small group" usually the close relations of different agents. In this regard, I have not considered the social structure underlying information transfer, which is typically represented as a network. In such a setting, the collective outcome may be affected by each agent's friends or close connections, which may have an effect on the agent's belief. In addition, an agent's behavior may depend on the actions of close friends or those in their immediate social or physical proximity.

APPENDIX A

Proof of Proposition 1.3.1. Unbiased receivers: On the equilibrium path: If a message $m = 1$ is released from a sender, which may be either unbiased or biased, then an unbiased receiver votes $v_{j^u}(1) = 1$ (assuming that the remaining unbiased receivers choose 1; i.e. $v_{-j^u}(1) = 1$) since $x_0^j(m = 1, v_{-j^u} = 1)) = \frac{n-1}{n} > \frac{n-1}{2n}$.

Off the equilibrium path: If a message $m = 0$ is released from a sender, which must be unbiased, then :

i. the unbiased receiver votes $v_{j^u}(0) = 1$, assuming the remaining unbiased receivers choose 1 (i.e. $v_{-j^u}(0) = 1$) since $x_0^j(m^u = 0, v_{-j^u} = 1)) = \frac{n-2}{n} > \frac{n-1}{2n}$, when $n \geq 4$.

ii. Whenever $b < \frac{1}{2}$, the unbiased receiver votes $v_{j^u}(0) = 0$, assuming the remaining unbiased receivers choose 0 (i.e.$v_{-j^u}(0) = 0$) since $x_0^j(m^u = 0, v_{-j^u} = 0)) = \frac{n-1}{n}b$.

Biased agents: Both biased receivers and biased senders have no incentive to deviate from their equilibrium actions.

Unbiased sender: An unbiased sender has no incentive to deviate from 1 to 0. Any such deviation only contributes to a loss since the highest pay-off is achieved by following the equilibrium strategy for any value of b.

□

Proof of Proposition 1.3.2. When $b < \frac{1}{2}$:

Unbiased receivers: On the equilibrium path: If a message $m = 1$ is released from a sender, which must be biased, then an unbiased receiver votes $v_{j^u}(1) = 0$ (provided that all other unbiased receivers choose 0, i.e. $v_{-j^u}(1) = 0$) then $x_0^j(m = 1, v_{-j^u} = 0) = \frac{n-1}{n}b < \frac{n-1}{2n}$. If, on the other hand, a message $m = 0$ is released, which must come from an unbiased sender, then an unbiased receiver votes $v_{j^u}(0) = 0$ (all other unbiased receivers choose 0, i.e.$v_{-j^u}(0) = 0$) by the same argument.

Biased agents: Both biased receivers and biased senders have no incentive to deviate from their equilibrium strategies.

Unbiased sender: Any unbiased sender has no incentive to deviate from 0 to 1, since $u^S_{i^u}(m^u = 0; v_{j^u} = 0) = -b > -(1-b) = u^S_{i^u}(m^u = 1; v_{j^u} = 0)$.

□

Proof of Proposition 1.3.3. Let $\alpha = 0$; I check that the truthful equilibrium is a PBNE by showing that neither receivers nor senders deviate from their strategies. The bound on the posterior belief is $\kappa(m, \mathbf{v}_{-j}) = \frac{1}{2}$, for both $m = 0, 1$, and the posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ is

$$\rho(m) = \begin{cases} 0, & \text{if } m = 0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 1. \end{cases} \tag{1.17}$$

Unbiased receivers: I check first the strategy followed by unbiased receivers when the message received is 1. since $\rho(1) = \frac{\pi}{b+(1-b)\pi} > \frac{1}{2}$ due to the condition $b < \frac{\pi}{1-\pi}$, an unbiased receiver chooses 1 when receiving message 1, i.e. $v_{j^u}(1) = 1$. Similarly, the strategy followed by unbiased receivers when the message received is 0 is voting 0 since $\rho(0) = 0 < \frac{1}{2}$, i.e. $v_{j^u}(0) = 0$.

Biased receivers: Any biased receiver strictly benefits from voting 1 as opposed to voting 0, i.e. $v_{j^b}(m) = 1$.

Next, I check that senders have no incentive to deviate from following the strategy $m^u(\theta) = \theta$ (if the sender is unbiased) and $m^b(\theta) = 1$ (if the sender is biased).

Unbiased sender: For unbiased senders, the utility function is

$$\begin{aligned} u^S_{i^u}(m^u; \mathbf{v}, \theta) &= -|x(m^u; \mathbf{v}, \theta) - \theta| = \\ &= -\left| \frac{m^u}{n} + \frac{n-1}{n} b + \frac{n-1}{n}(1-b) v_{j^u}(m^u) - \theta \right| \\ &= \begin{cases} -\frac{m^u}{n} - \frac{n-1}{n} b - \frac{n-1}{n}(1-b) v_{j^u}(m^u), & \text{if } \theta = 0 \\ \frac{m^u}{n} + \frac{n-1}{n} b + \frac{n-1}{n}(1-b) v_{j^u}(m^u) - 1, & \text{if } \theta = 1, \end{cases} \end{aligned}$$

where, as shown earlier, $v_{j^u}(m^u) = m^u$, $v_{j^b}(m^b) = 1$. The utility function is strictly decreasing on the value of message m when $\theta = 0$ while it is strictly increasing on the value of message m when $\theta = 1$. As a result, an unbiased sender sends message 0 when he receives signal 0, while he sends message 1 when he receives signal 1. In short, $m^u(\theta) = \theta$ as required.

Biased sender: For biased senders, the utility function is

$$u^S_{i^b}(m^b; \mathbf{v}, \theta) = -|x(m^b; \mathbf{v}) - 1| = \frac{m^b}{n} + \frac{\sum_{j=1}^{n-1} v_j(m^b)}{n} - 1,$$

where, as before, $v_{j^u}(m^b) = m^b$, $v_{j^b}(m^b) = 1$. $m^b(\theta) = 1$ is the best strategy for a biased sender for any signal he receives.

□

Proof of Proposition 1.3.4. Let $\alpha = 0$; I check that the lying equilibrium is a PBNE by showing that neither receivers nor senders deviate from their strategies. The bound on the posterior belief satisfies $\kappa(m, \mathbf{v}_{-j}) = \frac{1}{2}$ for both $m = 0, 1$ in this case. The posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ is now:

$$\rho(m) = \begin{cases} 0, & \text{if } m = 1 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 0. \end{cases} \tag{1.18}$$

Unbiased receivers: I check the strategy unbiased receivers follow when the message received is 0 first. Since $\rho(0) = \frac{\pi}{b+(1-b)\pi} > \frac{1}{2}$, as per the condition $b < \frac{\pi}{1-\pi}$, an unbiased receiver chooses 1 when receiving message 1, i.e. $v_{j^u}(0) = 1$. Similarly, the strategy followed by unbiased receivers when the message received is 0 is voting 0 since $\rho(1) = 0 < \frac{1}{2}$, i.e. $v_{j^u}(0) = 1$.

Biased receivers: Any biased receiver strictly benefits from voting 1 rather than 0, i.e. $v_{j^b}(m) = 1$.

Next, I check that senders have no incentive to deviate from the equilibrium strategy $m^u(\theta) = 1 - \theta$ (for an unbiased sender) and $m^b(\theta) = 0$ (for a biased sender).

Unbiased sender: For unbiased senders, the utility function is

$$
\begin{aligned}
u^S_{i^u}(m^u;\mathbf{v},\theta) &= -|x(m^u;\mathbf{v},\theta)-\theta| \\
&= -\left|\frac{m^u}{n}+\frac{n-1}{n}b+\frac{n-1}{n}(1-b)v_{j^u}(m^u)-\theta\right| \\
&= \begin{cases} -\frac{m^u}{n}-\frac{n-1}{n}b-\frac{n-1}{n}(1-b)v_{j^u}(m^u), & \text{if } \theta=0 \\ \frac{m^u}{n}+\frac{n-1}{n}b+\frac{n-1}{n}(1-b)v_{j^u}(m^u)-1, & \text{if } \theta=1. \end{cases}
\end{aligned}
$$

Since $v_{j^u}(m^u)=1-m^u$, the utility function of an unbiased sender becomes

$$
u^S_{i^u}(m^u;\mathbf{v},\theta)=\begin{cases} \frac{(n-1)(1-b)-1}{n}m^u-\frac{n-1}{n}, & \text{if } \theta=0 \\ \frac{1-(n-1)(1-b)}{n}m^u+\frac{n-1}{n}-1, & \text{if } \theta=1. \end{cases}
$$

$\frac{(n-1)(1-b)-1}{n}$ is strictly positive since $|\mathcal{U}|\geq 3$. Therefore, the utility function is strictly increasing on the value of message m when $\theta=0$ while it is strictly decreasing on the value of message m when $\theta=1$. As a result, an unbiased sender communicates 1 when he receives signal 0 while communicates 0 when he receives signal 1. In other words, $m^u(\theta)=1-\theta$ as required.

Biased sender: For biased senders, the utility function is

$$
u^S_{i^b}(m^b;\mathbf{v},\theta)=-|x(m^b;\mathbf{v})-1|=\frac{m^b}{n}+\frac{\sum_{j=1}^{n-1}v_j(m^b)}{n}-1,
$$

where $v_{j^u}(m^b)=1-m^b$, $v_{j^b}(m^b)=1$. $m^b(\theta)=0$ is the best strategy for a biased sender for any signal he receives.

□

Proof of Proposition 1.3.5. Let $\alpha=0$; I check that the truthful equilibrium (biased-mimicking) is a PBNE by showing that neither receivers nor senders deviate from their strategies. The bound on the posterior belief $\kappa(m,\mathbf{v}_{-j})=\frac{1}{2}$ for both $m=0,1$. The posterior belief $\rho(m)$ of any unbiased receiver $j^u\in\mathcal{U}$ is:

$$
\rho(m)=\begin{cases} 0, & \text{if } m=0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m=1. \end{cases} \tag{1.19}
$$

Unbiased receivers: I begin by checking the strategy followed by unbiased receivers when the message received is 1. Since $\rho(1) = \frac{\pi}{b+(1-b)\pi} < \frac{1}{2}$, as per the condition $b > \frac{\pi}{1-\pi}$, an unbiased receiver chooses 0 when receiving message 1, i.e. $v_{j^u}(1) = 0$. Similarly, the strategy followed by unbiased receivers when the message received is 0 is voting 0 since $\rho(0) = 0 < \frac{1}{2}$, i.e. $v_{j^u}(0) = 0$.

Biased receivers: Any biased receiver strictly benefits from voting 1 rather than 0, i.e. $v_{j^b}(m) = 1$.

Next, I check that senders have no incentive to deviate from the strategies $m^u(\theta) = \theta$ (if the sender is unbiased) and $m^b(\theta) = 1$ (if the sender is biased instead).

Unbiased sender: For unbiased senders, the utility function is

$$\begin{aligned} u^S_{i^u}(m^u; \mathbf{v}, \theta) &= -|x(m^u; \mathbf{v}, \theta) - \theta| \\ &= -\left| \frac{m^u}{n} + \frac{n-1}{n} b + \frac{n-1}{n}(1-b) v_{j^u}(m^u) - \theta \right| \\ &= \begin{cases} -\frac{m^u}{n} - \frac{n-1}{n} b, & \text{if } \theta = 0 \\ \frac{m^u}{n} + \frac{n-1}{n} b - 1, & \text{if } \theta = 1, \end{cases} \end{aligned}$$

where $v_{j^u}(m^u) = 0$, $v_{j^b}(m^b) = 1$. The utility function is strictly decreasing on the value of message m when $\theta = 0$ while it is strictly increasing on the value of message m when $\theta = 1$. As a result, an unbiased sender delivers message 0 when he receives signal 0, while he delivers message 1 when he receives signal 1. Hence, $m^u(\theta) = \theta$ as required.

Biased sender: For biased senders, the utility function is

$$u^S_{i^b}(m^b; \mathbf{v}, \theta) = -|x(m^b; \mathbf{v}) - 1| = \frac{m^b}{n} + \frac{\sum_{j=1}^{n-1} v_j(m^b)}{n} - 1,$$

where $v_{j^u}(m^b) = 0$ and $v_{j^b}(m^b) = 1$. $m^b(\theta) = 1$ is then the best strategy for a biased sender for any signal he receives.

□

Proof of Proposition 1.3.6. Let $\alpha = 0$; I will prove that there exists no *pooling equilibrium* by considering both scenarios separately. Once again the bound on the posterior belief $\kappa(m, \mathbf{v}_{-j}) = \frac{1}{2}$ independently of the message received.

Claim 1: There is no *pooling equilibrium* where $m^t(\theta) = 1$ for any $t \in \mathcal{T}$.

Proof. By way of contradiction, I assume that $m^t(\theta) = 1$ for any $t \in \mathcal{T}$ is a pooling equilibrium. The posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ becomes:

$$\rho(m) = \begin{cases} \pi, & \text{if } m = 1 \\ \mu, & \text{if } m = 0. \end{cases} \tag{1.20}$$

Note that when message $m = 1$ is transferred unbiased receivers will vote 0 since $\pi < \frac{1}{2}$.

Off-the-equilibrium path: If on the other hand message $m = 0$ reaches the receivers, an unbiased receiver votes 0 if $\mu < \frac{1}{2}$. In that case however an unbiased sender would deviate from sending massage 1 to 0 whenever $\theta = 0$. Meanwhile, an unbiased receiver will vote 1 if $\mu > \frac{1}{2}$. In this case, an unbiased receiver's voting strategy becomes $v_{j^u}(m) = 1 - m$ and an unbiased sender would deviate from sending massage 1 to 0 whenever $\theta = 1$. Additionally, a biased sender can benefit as well by deviating from sending message 1 to sending message 0 instead. This is because the deviation would make the collective outcome closer to 1 since $|\mathcal{U}| \geq 3$. Lastly, if $\mu = \frac{1}{2}$ an unbiased receiver is indifferent between either 0 or 1. He therefore mixes between these two options. Either type of sender has now an incentive to deviate from sending a message 1.

□

Claim 2: There is no *pooling equilibrium* where $m^t(\theta) = 0$ for any $t \in \mathcal{T}$.

Proof. By way of contradiction, I assume that $m^t(\theta) = 0$ for any $t \in \mathcal{T}$ is a pooling equilibrium. The posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ becomes:

$$\rho(m) = \begin{cases} \pi, & \text{if } m = 0 \\ \mu, & \text{if } m = 1. \end{cases} \tag{1.21}$$

Observe that, once again, when message $m = 0$ is transmitted unbiased receivers will vote 0 since $\pi < \frac{1}{2}$.

Off-the-equilibrium path: when message $m = 1$ reaches the public an unbiased receiver will vote 0 if $\mu < \frac{1}{2}$. In this situation a biased receiver profits from sending message $m^b = 1$ since it brings the outcome closer to 1. If instead unbiased receivers vote 1 when receiving 1 (which happens when $\mu > \frac{1}{2}$), biased sender will again deviate to sending message $m^b = 1$. Lastly, if $\mu = \frac{1}{2}$ an unbiased receiver is indifferent between either 0 or 1. He therefore mixes between these two options. Either type of sender has now an incentive to deviate from sending a message 0.

□

□

Proof of Proposition 1.3.7. Let $\alpha \in (0, 1)$; I check that the truthful equilibrium is a PBNE by showing that neither receivers nor senders deviate from their assumed strategies. The bounds on the posterior belief are in this case:

$$\kappa(m = 1, \mathbf{v}_{-j} = 1) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 1, \mathbf{v}_{-j} = 1)) + 1\right],$$
$$\kappa(m = 0, \mathbf{v}_{-j} = 0) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 0, \mathbf{v}_{-j} = 0)) + 1\right].$$

The posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ is:

$$\rho(m) = \begin{cases} 0, & \text{if } m = 0 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 1. \end{cases} \tag{1.22}$$

Unbiased receivers: I begin by confirming the strategy followed by unbiased receivers. An unbiased agent j^u votes 1 when the message received is 1 if $\rho(1) > \frac{1}{2}[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 1, v_{-j^u} = 1)) + 1]$; while he votes 0 when the message received is 0 if $\rho(0) < \frac{1}{2}[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 0, v_{-j^u} = 0)) + 1]$ instead.

Choose an unbiased agent j^u and assume the remaining unbiased agents follow the PBNE symmetric strategies. Then $x_0^j(m = 1, v_{-j^u} = 1) = \frac{n-1}{n}$ and

$x_0^j(m=0, v_{-j^u}=0) = \frac{n-1}{n}b$. The equilibrium conditions become

$$\begin{cases} \frac{\pi}{b+(1-b)\pi} > \frac{1}{2}(1-\frac{n\alpha}{1-\alpha}), \\ 0 < \frac{1}{2}[1+\frac{n\alpha}{1-\alpha}(1-2b)]. \end{cases}$$

The first inequality implies the following bound:

$$b < \frac{\pi}{1-\pi}\left(1+\frac{2}{\frac{1-\alpha}{n\alpha}-1}\right) \quad \text{when } \alpha < \frac{1}{n+1}.$$

Alternatively, when $\alpha \geq \frac{1}{n+1}$, the right hand side of the first inequality is non-positive and the inequality always holds as a result.

The second inequality implies the bound:

$$b < \frac{1}{2}\left(1+\frac{1-\alpha}{n\alpha}\right).$$

Set

$$R(\alpha, n) = \frac{1-\alpha}{n\alpha}.$$

Observe that $R(\alpha, n)$ monotonically decreases as α (resp. n) increases.

Combining both inequalities, b must satisfy

$$b < \min\left\{\frac{\pi}{1-\pi}\left(1+\frac{2}{R(\alpha,n)-1}\right), \frac{1}{2}\left(1+R(\alpha,n)\right)\right\} \quad \text{when } \alpha < \frac{1}{n+1}.$$

Next, I check that senders have no incentive to deviate from the strategies $m^u(\theta) = \theta$ (if the sender is unbiased) and $m^b(\theta) = 1$ (if the sender is biased).

Unbiased sender: For unbiased senders, the utility function is

$$u^S_{i^u}(m^u;\mathbf{v},\theta) = -(1-\alpha)|x(m^u;\mathbf{v},\theta)-\theta| - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|$$

$$= -(1-\alpha)\left|\frac{m^u}{n}+\frac{n-1}{n}b+\frac{n-1}{n}(1-b)v_{j^u}(m^u)-\theta\right| - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|$$

$$= \begin{cases} -(1-\alpha)\left[\frac{m^u}{n}+\frac{n-1}{n}b+\frac{n-1}{n}(1-b)v_{j^u}(m^u)\right] - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|, & \text{if } \theta = 0,\\ (1-\alpha)\left[\frac{m^u}{n}+\frac{n-1}{n}b+\frac{n-1}{n}(1-b)v_{j^u}(m^u)-1\right] - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|, & \text{if } \theta = 1.\end{cases}$$

Recall that $v_{j^u}(m^u) = m^u$. When $\theta = 0$, $u^S_{i^u}(m^u = 0; v_{j^u} = 0, \theta = 0) = -(1-\alpha)\frac{n-1}{n}b - \alpha b > -(1-\alpha)\frac{n-1}{n} - \frac{1-\alpha}{n} = u^S_{i^u}(m^u = 1; v_{j^u} = 1, \theta = 0)$, which requires that the bound

$$b < \frac{n(1-\alpha)}{n+\alpha-1} = \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}$$

must hold. Additionally, the utility function is strictly increasing on the value of message m when $\theta = 1$. As a result, whenever the last bound holds, an unbiased sender will send message 0 when he receives signal 0 while he will send message 1 when he receives signal 1. In short, $m^u(\theta) = \theta$.

Biased sender: For biased senders, the utility function is

$$u^S_{i^b}(m^b;\mathbf{v},\theta) = -|x(m^b;\mathbf{v})-1| = \frac{m^b}{n} + \frac{\sum_{j=1}^{n-1} v_j(m^b)}{n} - 1,$$

where $v_{j^u}(m^b) = m^b$, $v_{j^b}(m^b) = 1$. $m^b(\theta) = 1$ is the best strategy for a biased sender for any signal he receives.

Finding the bounds of b and α: Combining all conditions that unbiased senders and receivers must satisfy as well as the assumption $b < \frac{n-2}{n-1}$; the next bound on b as a function of α follows

$$b < \begin{cases} \min\left\{\frac{\pi}{1-\pi}\left(1+\frac{2}{R(\alpha,n)-1}\right), \frac{1}{2}\left(1+R(\alpha,n)\right), \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}, \frac{n-2}{n-1}\right\}, & \text{when } \alpha \in (0, \frac{1}{n+1}) \\ \min\left\{\frac{1}{2}\left(1+R(\alpha,n)\right), \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}, \frac{n-2}{n-1}\right\}, & \text{when } \alpha \in [\frac{1}{n+1}, 1) \end{cases}$$

Write for each candidate bound b_1, b_2 and b_3 respectively, where

$$b_1 = \frac{\pi}{1-\pi}\left(1+\frac{2}{R(\alpha,n)-1}\right),$$

$$b_2 = \frac{1}{2}\left(1+R(\alpha,n)\right),$$

and

$$b_3 = \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}.$$

i. For $0 < \alpha < \frac{1}{n+1}$, $R(\alpha,n) > 1$. As α strictly increases from 0 to $\frac{1}{n+1}$, $R(\alpha,n)$ strictly decreases from $+\infty$ to 1. Furthermore, b_1 strictly increases from $\frac{\pi}{1-\pi}$ to $+\infty$; b_2 strictly decreases from $+\infty$ to 1, so that $b_2 > 1$; b_3 strictly decreases from $\frac{n}{n-1}$ to 1. The bounds on b are then $b < \frac{\pi}{1-\pi}\left(1+\frac{2}{\frac{1-\alpha}{n\alpha}-1}\right)$ when $\alpha \in (0, \frac{k-1}{n-1+k(n+1)})$. Otherwise, when $\alpha \in (\frac{k-1}{n-1+k(n+1)}, \frac{n-1}{n^2-4n-1})$, $b < \frac{n-2}{n-1}$.

ii. For $\frac{1}{n+1} \leq \alpha < 1$, $0 < R(\alpha,n) < 1$. As α strictly increases from $\frac{1}{n+1}$ to 1, $R(\alpha,n)$ strictly decreases from 1 to 0. Furthermore, b_2 strictly decreases from 1 to $\frac{1}{2}$; b_3 strictly decreases from 1 to 0. Taking the derivatives of b_2 and b_3 separately with respect to α,

$$\frac{d}{d\alpha}\left[\frac{1}{2}\left(1+\frac{1-\alpha}{n\alpha}\right)\right] = -\frac{1}{2n\alpha^2},$$
$$\frac{d}{d\alpha}\left[\frac{n(1-\alpha)}{n+\alpha-1}\right] = -\frac{n^2}{(n+\alpha-1)^2}.$$

Therefore $|-\frac{1}{2n\alpha^2}| > |-\frac{n^2}{(n+\alpha-1)^2}|$ for values of α sufficiently close to $\frac{1}{n+1}$. On the other hand, $|-\frac{1}{2n\alpha^2}| < |-\frac{n^2}{(n+\alpha-1)^2}|$ for α close to 1.

Solving

$$\frac{n(1-\alpha)}{n+\alpha-1} \leq \frac{1}{2}\left(1+\frac{1-\alpha}{n\alpha}\right)$$

I have $\alpha \geq \frac{n-1}{2n-1}$.

Then I check the conditions that ensure each upper bound below $\frac{n-3}{n-1}$ in this region:

$$\frac{1}{2}\left(1+\frac{1-\alpha}{n\alpha}\right) \leq \frac{n-2}{n-1}$$

and

$$\frac{n(1-\alpha)}{n+\alpha-1} \leq \frac{n-2}{n-1}.$$

Solving these two inequalities, I have

$$\alpha \geq \frac{n-1}{n^2-2n-1}$$

and

$$\alpha \geq \frac{2(n-1)}{n^2-2}$$

When $n \geq 4$, $\frac{1}{n+1} < \frac{n-1}{n^2-2n-1} \leq \frac{2(n-1)}{n^2-2} \leq \frac{n-1}{2n-1}$ and the inequalities hold with equality when $n = 4$. As a result, when $\alpha \in (\frac{1}{n+1}, \frac{n-1}{n^2-2n-1})$, $b < \frac{n-2}{n-1}$; when $\alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1})$, $b < \frac{1}{2}\left(1+\frac{1-\alpha}{n\alpha}\right)$; when $\alpha \in [\frac{n-1}{2n-1}, 1)$, $b < \frac{n(1-\alpha)}{n+\alpha-1}$.

In summary, truthful equilibrium exists when b and α satisfy

$$b < \begin{cases} \frac{\pi}{1-\pi}\left(1+\frac{2}{R(\alpha,n)-1}\right), & \text{when } \alpha \in (0, \frac{k-1}{n-1+k(n+1)}) \\ \frac{n-2}{n-1}, & \text{when } \alpha \in [\frac{k-1}{n-1+k(n+1)}, \frac{n-1}{n^2-2n-1}) \\ \frac{1}{2}\left(1+R(\alpha,n)\right), & \text{when } \alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1}) \\ \frac{\frac{1}{\alpha}-1}{\frac{1}{\alpha}-R(\alpha,n)}, & \text{when } \alpha \in [\frac{n-1}{2n-1}, 1) \end{cases}$$

where $k = \frac{n-2}{n-1}\frac{1-\pi}{\pi}$ and $R(\alpha, n) = \frac{1-\alpha}{n\alpha}$.

□

Proof of Proposition 1.3.8. Let $\alpha \in (0,1)$; I check that the lying equilibrium is a PBNE by showing that senders do not have an incentive to deviate from their strategies. The bounds of posterior belief are in this case:

$$\kappa(m = 0, \mathbf{v}_{-j} = 1) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 0, \mathbf{v}_{-j} = 1)) + 1\right],$$
$$\kappa(m = 1, \mathbf{v}_{-j} = 0) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 1, \mathbf{v}_{-j} = 0)) + 1\right].$$

The posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ is:

$$\rho(m) = \begin{cases} 0, & \text{if } m = 1 \\ \frac{\pi}{b+(1-b)\pi}, & \text{if } m = 0. \end{cases} \tag{1.23}$$

Unbiased receivers: Recall that an unbiased agent j^u will vote for 0 when the message received is 1 if $\rho(1) < \frac{1}{2}[\frac{n\alpha}{1-\alpha}(1 - \frac{2n}{n-1}x_0^j(m = 1, v_{-j^u} = 0)) + 1]$ while he votes 1 when the message received is 0 if $\rho(0) > \frac{1}{2}[\frac{n\alpha}{1-\alpha}(1-\frac{2n}{n-1}x_0^j(m = 0, v_{-j^u} = 1))+1]$. As before, assuming that the equilibrium is symmetric, the equilibrium strategies for receivers imply $x_0^j(m = 1, v_{-j^u} = 0) = \frac{(n-1)b+1}{n}$ and $x_0^j(m = 0, v_{-j^u} = 1) = \frac{n-2}{n}$ for any unbiased receiver j^u. The equilibrium conditions are then:

$$\begin{cases} 0 < \frac{1}{2}[1 + \frac{n\alpha}{1-\alpha}(\frac{n-3}{n-1} - 2b)], \\ \frac{\pi}{b+(1-b)\pi} > \frac{1}{2}(1 - \frac{n-3}{n-1}\frac{n\alpha}{1-\alpha}). \end{cases}$$

The first inequality implies the following bound:

$$b < \frac{1}{2}\left(\frac{1-\alpha}{n\alpha} + \frac{n-3}{n-1}\right).$$

The second inequality implies the bound:

$$b < \frac{\pi}{1-\pi}\left(1 + \frac{2}{\frac{1-\alpha}{n\alpha} \cdot \frac{n-3}{n-1} - 1}\right) \quad \text{when } \alpha < \frac{n-1}{n^2-2n-1}.$$

Note that, as in the previous proof, if $\alpha \geq \frac{n-1}{2(n-2)}$ the right hand side first inequality is non-positive and the inequality holds trivially.

Now, combining both inequalities, b must satisfy

$$b < \min\left\{\frac{\pi}{1-\pi}\left(1+\frac{2}{\frac{n-3}{n-1}\cdot R(\alpha,n)-1}\right), \frac{1}{2}\left(\frac{n-3}{n-1}+R(\alpha,n)\right)\right\}, \quad \text{when } \alpha < \frac{n-1}{n^2-2n-1}.$$

As in the previous proof,

$$R(\alpha,n) = \frac{1-\alpha}{n\alpha}.$$

.

Next, I check that senders do not deviate from the strategies $m^u(\theta) = 1-\theta$, whenever $j^u \in \mathcal{U}$, and $m^b(\theta) = 0$, whenever $j^b \in \mathcal{B}$.

Unbiased sender: For unbiased senders, the utility function is

$$\begin{aligned} u^S_{i^u}(m^u; \mathbf{v}, \theta) &= -(1-\alpha)|x(m^u; \mathbf{v}, \theta) - \theta| - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j| \\ &= -(1-\alpha)\left|\frac{m^u}{n} + \frac{n-1}{n}b + \frac{n-1}{n}(1-b)v_{j^u}(m^u) - \theta\right| - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j| \\ &= \begin{cases} -(1-\alpha)[\frac{m^u}{n} + \frac{n-1}{n}b + \frac{n-1}{n}(1-b)v_{j^u}(m^u)] - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|, & \text{if } \theta = 0 \\ (1-\alpha)[\frac{m^u}{n} + \frac{n-1}{n}b + \frac{n-1}{n}(1-b)v_{j^u}(m^u) - 1] - \frac{\alpha}{n-1}\sum_{j=1}^{n-1}|m^u - v_j|, & \text{if } \theta = 1. \end{cases} \end{aligned}$$

When $\theta = 0$, for there to be no profitable deviation it is required that $u^S_{i^u}(m^u = 0; v_{j^u} = 1, \theta = 0) = -(1-\alpha)\frac{n-1}{n} - \alpha < -(1-\alpha)\frac{1+(n-1)b}{n} - \alpha(1-b) = u^S_{i^u}(m^u = 1; v_{j^u} = 0, \theta = 0)$. This condition implies the following bounds

$$\begin{aligned} &\text{If } \alpha < \frac{1}{n+1}, \quad b \in \left(0, \frac{(n-2)(1-\alpha)}{n-\alpha(2n-1)-1}\right) \\ &\text{If } \frac{1}{n+1} \leq \alpha < 1, \quad b \in (0, \frac{n-2}{n-1}) \end{aligned}$$

On the other hand, when $\theta = 1$ for no profitable deviation to be possible $u^S_{i^u}(m^u = 0; v_{j^u} = 1, \theta = 1) = (1-\alpha)(\frac{n-1}{n} - 1) - \alpha > (1-\alpha)(\frac{1+(n-1)b}{n} - 1) - \alpha(1-b) = u^S_{i^u}(m^u = 1; v_{j^u} = 0, \theta = 1)$ must hold. This implies that the requirement

$$b < \frac{(n-2)(1-\alpha)}{n+\alpha-1} = \frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}$$

must hold.

Therefore, an unbiased sender has no incentive to deviate if $b < \frac{(n-2)(1-\alpha)}{n+\alpha-1}$, since $\frac{(n-2)(1-\alpha)}{n+\alpha-1} < \frac{(n-2)(1-\alpha)}{n-\alpha(2n-1)-1}$.

Biased sender: For biased senders, the utility function is

$$u^S_{i^b}(m^b;\mathbf{v},\theta) = -|x(m^b;\mathbf{v})-1| = \frac{m^b}{n} + \frac{\sum_{j=1}^{n-1} v_j(m^b)}{n} - 1,$$

where $v_{j^b}(m^b) = 1-m^b$. $m^b(\theta)=0$ is the best strategy for a biased sender for any signal he receives.

Finding the bounds of b and α: Combining all the conditions derived from requiring no deviations from unbiased senders and receivers, as well as the assumption that $b < \frac{n-2}{n-1}$, the following bound on b as a function α holds:

$$b < \begin{cases} \min\left\{\frac{\pi}{1-\pi}\left(1+\frac{2}{\frac{n-3}{n-1}\cdot R(\alpha,n)-1}\right), \frac{1}{2}\left(\frac{n-3}{n-1}+R(\alpha,n)\right), \\ \frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}, \frac{n-2}{n-1}\right\}, & \text{when } \alpha \in \left(0, \frac{n-1}{n^2-2n-1}\right) \\ \min\left\{\frac{1}{2}\left(\frac{n-3}{n-1}+R(\alpha,n)\right), \frac{\frac{1}{\alpha}-1-2R(\alpha,n)}{\frac{1}{\alpha}-R(\alpha,n)}, \frac{n-2}{n-1}\right\}, & \text{when } \alpha \in \left[\frac{n-1}{n^2-2n-1}, 1\right) \end{cases}$$

Write for each candidate bound b_1, b_2 and b_3 respectively, where

$$b_1 = \frac{\pi}{1-\pi}\left(1+\frac{2}{\frac{n-3}{n-1}\cdot R(\alpha,n)-1}\right),$$

$$b_2 = \frac{1}{2}\left(\frac{n-3}{n-1}+R(\alpha,n)\right),$$

and

$$b_3 = \frac{\frac{1}{\alpha} - 1 - 2R(\alpha, n)}{\frac{1}{\alpha} - R(\alpha, n)}.$$

i. For $0 < \alpha < \frac{1}{n+1}$, $R(\alpha, n) > 1$. As α increases from 0 to $\frac{1}{n+1}$, $R(\alpha, n)$ strictly decreases from $+\infty$ to 1. Furthermore, b_1 strictly increases from $\frac{\pi}{1-\pi}$ to $+\infty$; b_2 strictly decreases from $+\infty$ to $\frac{n-2}{n-1}$; b_3 strictly decreases from $\frac{n-2}{n-1}$ to $\frac{n(n-2)}{n^2-1}$. The values of b are then as follows: when $\alpha \in (0, \underline{\alpha}(\pi, n))$, $b < \frac{\pi}{1-\pi}\left(1 + \frac{2}{\frac{1-\alpha}{n\alpha} - 1}\right)$; while if $\alpha \in (\underline{\alpha}(\pi, n), \frac{n-1}{n^2-4n-1})$, $b < \frac{(n-2)(1-\alpha)}{n+\alpha-1}$.

ii. For $\frac{1}{n+1} \leq \alpha < 1$, $0 < R(\alpha, n) < 1$. As α increases from $\frac{1}{n+1}$ to 1, $R(\alpha, n)$ strictly decreases from 1 to 0. In this range, $b_2 = \frac{1}{2}\left(\frac{n-3}{n-1} + R(\alpha, n)\right) < \frac{n-2}{n-1}$ and $b_3 = \frac{(n-2)(1-\alpha)}{n+\alpha-1} < \frac{n-2}{n-1}$. To find the appropriate bound, I find the lower envelope of b_2 and b_3.

If $b_2 < b_3$, then

$$\frac{1}{2}\left(\frac{n-3}{n-1} + \frac{1-\alpha}{n\alpha}\right) < \frac{(n-2)(1-\alpha)}{n+\alpha-1}.$$

This inequality yields

$$\alpha \in (\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1}).$$

Overall, I have

$$b < \begin{cases} \frac{\pi}{1-\pi}\left(1 + \frac{2}{\frac{n-3}{n-1}R(\alpha,n) - 1}\right), & \text{when } \alpha \in (0, \underline{\alpha}(\pi, n)) \\ \frac{(n-2)(1-\alpha)}{n+\alpha-1}, & \text{when } \alpha \in [\underline{\alpha}(\pi, n), \frac{n-1}{n^2-2n-1}) \\ \frac{1}{2}\left(\frac{n-3}{n-1} + R(\alpha, n)\right), & \text{when } \alpha \in [\frac{n-1}{n^2-2n-1}, \frac{n-1}{2n-1}) \\ \frac{(n-2)(1-\alpha)}{n+\alpha-1}, & \text{when } \alpha \in [\frac{n-1}{2n-1}, 1) \end{cases}$$

where $R(\alpha, n) = \frac{1-\alpha}{n\alpha}$.

□

Proof of Proposition 1.3.9. Let $\alpha \in (0,1)$; I consider the case $m^t(\theta) = 1$ for any $t \in \mathcal{T}$ first. Under these pooling strategies, the posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ becomes:

$$\rho(m) = \begin{cases} \mu, & \text{if } m = 0 \\ \pi, & \text{if } m = 1. \end{cases}$$

As before, choose an unbiased agent j^u and fix the actions of other agents so that $x_0^j(m = 1, \mathbf{v}_{-j^u} = 1) = \frac{n-1}{n}$. Observe that

$$\rho(1) = \pi > \kappa(m = 1, \mathbf{v}_{-j^u} = 1) = \frac{1}{2}\left(1 - \frac{n\alpha}{1-\alpha}\right)$$

always holds if $\alpha > \frac{1-2\pi}{n+1-2\pi}$. Therefore, receivers of both types vote 1 when they receive 1.

Next, I check that senders do not deviate from their strategies either. In this case, $m^u(\theta) = m^b(\theta) = 1$. For a biased sender it is clear that he suffers a loss from a deviating from $m^b = 1$ to 0 regardless of θ since everyone votes 1 on the equilibrium path. When $\theta = 1$, an unbiased sender also has no incentive to deviate from 1 to 0.

Then I just analyse the behavior of unbiased agents when $\theta = 0$. Note that off-the-equilibrium path, $x_0^j(m = 0, \mathbf{v}_{-j^u} = 1) = \frac{n-2}{n}$ or $x_0^j(m = 0, \mathbf{v}_{-j^u} = 0) = \frac{n-1}{n}b$ since any such deviations can only come from an unbiased agent. Biased senders do not deviate from sending message 1, independently of the actions of the receivers. Consequently,

i. If $\rho(0) = \mu > \kappa(m = 0, \mathbf{v}_{-j^u} = 1) = \frac{1}{2}\left(1 - \frac{n-3}{n-1} \cdot \frac{n\alpha}{1-\alpha}\right)$, then receivers also vote 1 off-the-equilibrium path. In this case, if $\theta = 0$, for an unbiased sender to not have any incentive to deviate

$$u_{i^u}^S(m^u = 0; \mathbf{v}_{j^u} = 1, \theta = 0) = -(1-\alpha)\frac{n-1}{n} - \alpha < -(1-\alpha) = u_{i^u}^S(m^u = 1; \mathbf{v}_{j^u} = 1, \theta = 0).$$

This inequality is satisfied provided $\alpha > \frac{1}{n+1}$.

To ensure the off-the-equilibrium belief $\mu \in [0,1]$ exists, I need the condition

$$\frac{1}{2}\left(1-\frac{n-3}{n-1}\cdot\frac{n\alpha}{1-\alpha}\right)<1.$$

Under the requirements $\alpha > \frac{1}{n+1} > \frac{1-2\pi}{n+1-2\pi}$, this condition holds.

ii. If $\rho(0) = \mu < \kappa(m=0, \mathbf{v}_{-j^u}=0) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1-2b)+1\right]$, then unbiased receivers vote 0 off-the-equilibrium path.

$$\frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1-2b)+1\right]>0$$

Solving this inequality, then

$$\begin{cases}\alpha \in (0,1), & \text{if } b \le \frac{1}{2}\\ \alpha < \frac{1}{n(2b-1)+1}, & \text{if } b > \frac{1}{2}\end{cases}$$

Now, if $\theta = 0$ an unbiased sender has no incentive to deviate if

$$u^S_{i^u}(m^u=0;\mathbf{v}_{j^u}=0,\theta=0) = -(1-\alpha)\frac{n-1}{n}b-\alpha b < -(1-\alpha) = u^S_{i^u}(m^u=1;\mathbf{v}_{j^u}=1,\theta=0).$$

This requires

$$\alpha > \frac{(1-b)n+b}{n+b} > \frac{1}{n+1}.$$

When $b \le \frac{1}{2}$, then $\alpha > \frac{(1-b)n+b}{n+b}$; When $b > \frac{1}{2}$, then $\alpha \in (\frac{(1-b)n+b}{n+b}, \frac{1}{n(2b-1)+1})$ holds under the condition that $b \in (\frac{1}{2}, \frac{n}{2(n-1)})$.

□

Proof of Proposition 1.3.10. Let $\alpha \in (0,1)$; I consider the case $m^u(\theta) = 0$ and $m^b(\theta) = 1$ for $i^u \in \mathcal{U}$ and $i^b \in \mathcal{B}$; then the posterior belief $\rho(m)$ of any unbiased receiver $j^u \in \mathcal{U}$ becomes:

$$\rho(m) = \pi, \quad \text{for } m = 0, 1.$$

Receivers follow the strategies $v_{j^u}(m) = 0$ and $v_{j^b}(m) = 1$ for $j^u \in \mathcal{U}$ and $j^b \in \mathcal{B}$ on the path if $\rho(m) = \pi < \kappa(m, \mathbf{v}_{-j^u} = 0)$. As before, choose an unbiased agent j^u and fix the actions of other agents so that $x_0^j(m, \mathbf{v}_{-j^u} = 0) = \frac{n-1}{n}b$. Then the unbiased agent j^u chooses 0 after observing the message assuming that all the unbiased agents choose 0 if

$$\rho(m) = \pi < \kappa(m, \mathbf{v}_{-j^u} = 0) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1-2b) + 1\right].$$

Next, I check that senders do not deviate from their strategies either. In this case, for a biased sender it is clear that he suffers a loss from a deviating from $m^b = 1$ to 0 regardless of θ since receivers take the same action independently of the message.

Next, I consider unbiased senders. For an unbiased sender to not have any incentive to deviate it is required that

$$u_{i^u}^S(m^u = 0; \mathbf{v}_{j^u} = 0, \theta = 0) > u_{i^u}^S(m^u = 1; \mathbf{v}_{j^u} = 0, \theta = 0)$$
$$u_{i^u}^S(m^u = 0; \mathbf{v}_{j^u} = 0, \theta = 1) > u_{i^u}^S(m^u = 1; \mathbf{v}_{j^u} = 0, \theta = 1).$$

Specific,

$$-(1-\alpha)\frac{(n-1)b}{n} - \alpha b > -(1-\alpha)\frac{(n-1)b+1}{n} - \alpha(1-b)$$
$$-(1-\alpha)\frac{(n-1)(1-b)+1}{n} - \alpha b > -(1-\alpha)\frac{(n-1)(1-b)}{n} - \alpha(1-b).$$

By solving these inequalities, I have $b < \frac{1}{2}\left(1 - \frac{1-\alpha}{n\alpha}\right)$. Then $\kappa(m, \mathbf{v}_{-j^u} = 0) = \frac{1}{2}\left[\frac{n\alpha}{1-\alpha}(1-2b) + 1\right] > \frac{1}{2} > \pi$, which ensures that unbiased receivers always choose 0.

□

proof of (no) pooling when $\alpha \in (0,1)$ and $m^t(\theta) = 0$. Let $\alpha \in (0,1)$; By way contradiction suppose that $m^t(\theta) = 0$ for any $t \in \mathcal{T}$ is a pooling equilibrium. In that case, no matter what the actions of the receivers are a biased sender always has a profitable deviation from $m^b = 0$ to $m^b = 1$. □

Chapter 2

Does Conformity Amongst Agents Affect Rumor Propagation In A Network?

This paper studies the effect of conformity on rumor propagation on a simple network. I consider a model that combines a communication and coordination game containing both unbiased and biased agents. Unbiased agents take an action that not only matches the true state of the world but also conforms with the actions of their neighbors, while biased agents take only a specific decision. I show that introducing a small degree of conformity enlarges the parameter region for truthful communication by relaxing the upper bound on the biased share relative to the model in Bloch et al. (2018).

2.1 Introduction

A rumor is a statement whose truth is hard, if not impossible, to verify. In any social group rumors spread across their members when they engage in conversation with one another. Rumors often spread fast through the group, meaning the person who originated the rumor is often not known to others. From the point of economics, rumors are interesting since they provide a good setting to understand what factors make rational agents transfer information that could be false. This can be particularly relevant to policy since rumors can lead to inefficient outcomes. For instance, a rumor regarding the integrity of an otherwise competent politician may tip the results of an election towards a less capable candidate. Another example involves the take-up of medical treatments, some individuals may attempt to discredit a treatment by creating doubt about its safety. I suggest in this paper that rumor propagation is driven by two factors: the likelihood of the rumor being true and the fact that individuals tend to adopt behaviors and accept statements from their family and friends. In this paper I model conforming to the behavior of others by assuming that individuals like to take actions that are close to those of their neighbors when their actions can be observed.

To analyze this problem, I adapt the model in chapter 1 to a simple network, the undirected line. Despite its simplicity, this example can provide substantial insight into more complex cases. The resulting model follows closely the paper of Bloch et al. (2018). The authors introduce a cheap talk model with two classes of individuals, “biased” and “unbiased”. Unbiased agents are interested in the truth, while biased agents are interested in a particular outcome, in the examples above the election of a given candidate independent on his abilities or promoting a particular treatment without regard to its effectiveness. The main innovation I introduce is a term describing conforming with the behavior of neighbors. This is an important feature, as seen in chapter 1 communication can be enhanced or hampered by the willingness of agents to conform to the behavior of the majority.

The main insight in Bloch et al. (2018) is that rumors are propagated because rational agents think on the balance of probabilities they are true and there may be a benefit if they are indeed true. Their model however does not consider that an individual may consider a rumor likely to be false while still be willing to propagate it, since this may misled his neighbors into conforming with the majority. Experimental evidence suggests that individuals may change well-seated opinions such as political ones when they receive information in a social setting where there may be pressure from others to conform with the general rule. Indeed, under certain conditions the desire of individuals to conform to the behavior of others can be so strong that they take decisions based only on the behavior previously observed and not on any private information the individual may have.

My main findings are similar to those in Bloch et al. (2018), communication through a network imposes greater restrictions to the transmission of truthful information.

2.2 Literature Review

This paper relates to a large body of literature on cheap talk. The basic model on which most papers are based was introduced by Crawford and Sobel (1982). A closely related paper that has inspired many subsequent developments is Farrell and Gibbons (1989). The authors introduce a model of cheap talk between a sender and two different agents—referred to as audiences—and discuss equilibria both in private (each audience cannot observe the message sent to the other) and public (both audiences observe the message). These foundational models, however, omit some important features, e.g. in Crawford and Sobel (1982), agents are not allowed to lie.

Rumor propagation influenced by social conformity has also been studied in other fields, including computer science, physics, and epidemiology (Ma et al., 2019; Hung and Plott, 2001; Wan and Wang, 2016; Wang et al., 2017). Common approaches in this literature often draw on epidemiological models in which a rumor is treated as a spreading disease. However, these models differ from those in economics in that they do not account for potential strategic aspects of communication. Moreover, they typically focus on phenomenological features, such as the speed of propagation and the number of individuals exposed to the rumor, rather than on the motivation for its propagation.

Recent years have seen the emergence of new work on strategic communication in networks. Galeotti et al. (2013) introduce a model of multi-player communication in which agents interact in groups, allowing the authors to study the geometry and properties of direct communication networks under decentralized decision-making when agents report truthfully. By contrast, Ambrus et al. (2013) develop a hierarchical cheap-talk model to analyze intermediated communication, where a sender and a receiver interact through a chain of intermediaries. My model differs from these in that it explicitly distinguishes between message creators and message transmitters, with agents potentially serving in both roles. Bravard et al. (2023) similarly extend the framework of Bloch et al. (2018) by assuming that agents do not observe the global network structure but only their local connections, thereby showing how network architecture and limited information shape the diffusion of misinformation. In contrast, I examine how conformity amplifies the spread of inaccurate messages and how conformity rules influence the broader process of information diffusion.

The most significant innovation of my model is the integration of a strategic communication game with a coordination game. The framework combines and adapts two basic models, one due to Bloch et al. (2018) and the other due to Hagenbach and Koessler (2010). The resulting model features two types of agents with conflicting preferences over the truth. As in Bloch et al. (2018), an agent creates a rumor that she shares with her neighbors, after which others decide whether to transmit it. Agents then engage in a coordination game in which preferences again diverge: some individuals want to choose an action close to the true state (Hagenbach and Koessler, 2010) while also coordinating with others.

2.3 Model

2.3.1 *Environment*

There is a finite population $\mathcal{N}$ consisting of n agents, with $n \geq 3$. The state of nature is given by $\theta \in \{0, 1\}$ and is unknown to all but possibly one agent. More precisely, an agent chosen randomly may observe the realization of θ. All agents share the common prior $\pi = \Pr(\theta = 1) < \frac{1}{2}$. Agents form the nodes of a social network $G = (\mathcal{N}, \mathcal{E})$. Agents can communicate information to their neighbors, which in turn can transfer to their neighbors. Once all communication has taken place, each agent submits a vote $v_i \in \{0, 1\}$ simultaneously. A profile of votes for the agents in the population is denoted by $\mathbf{v} = (v_1, ..., v_n)$. An agent's payoff depends both on the realization of the state of nature and the outcome of the collective decision, denoted by $x \in \{0,1\}$.

A pair of agents i and j in the network G share a link, denoted ij ($ij \in \mathcal{E}$), if they have the potential to communicate. In this case, we say that i is a neighbor of j, and vice versa. Although the underlying social network is undirected, communication can occur in either direction, in both directions, or not at all. To represent directional communication, let (i,j) denote the directed link from i to j, and (j,i) the directed link from j to i. The network G and all agents' types are common knowledge. I assume that the network is a *tree*. This implies that it is connected (every agent has at least one neighbor) and that any two agents i and j can be connected by a unique path (a collection of distinct links starting at one agent and ending at the other). This property ensures that there are no cycles which limits possible origins of any message. The structure of the network is independent of the signal $s(\theta)$.

For each agent i, let G_i be the sub-network G that contains all agents other than i with whom agent i seeks to coordinate. Given G_i, $V(G_i)$ denotes its node set. These are the agents i wants to coordinate with. Note that G_i can depend on how information is flowing through the network.

Define the *mean disagreement functional*

$$R(G_i) = \frac{1}{|V(G_i)|} \sum_{j \in V(G_i)} |v_i - v_j|.$$

The model contains two types of agents, some do not have predetermined preferences about the state of nature and are willing to coordinate with others and there are also agents who have predetermined preferences and are unwilling to cooperate. More formally, an agent is *unbiased* if he takes into account the deviation of the outcome of the vote from the true state of nature as well as any loss resulting from not coordinating with others. The payoff for an *unbiased* agent i is then written compactly as

$$u_{i^u}(\mathbf{v}, x; \theta) = -(1-\alpha)\,|x-\theta| - \alpha\, R(G_i),$$

where $\alpha \in (0, 1)$ measures the weight placed on coordination losses. The set of unbiased agents is denoted by $\mathcal{U}$.

On the other hand, an agent is *biased* if she is "stubborn and selfish". She prefers the outcome $x = 1$ independently of the true state and neither gains nor loses from coordinating with others. The payoff for this agent is given by

$$u_{i^b}(x; \theta) = -|x - 1|. \tag{2.1}$$

The set of biased agents is denoted by $\mathcal{B}$. Agent types are common knowledge in the network.

For a set $S \subset \mathcal{N}$ where a message may be created, b_S denotes the fraction of biased agents in S and u_S denotes the fraction of unbiased agents. Clearly, $b_S + u_S = 1$. In particular, for any unbiased agent in S, let

$$b_S \equiv \frac{|\mathcal{B}_S|}{|\mathcal{B}_S| + |\mathcal{U}_S| - 1}$$

denote the fraction of biased agents in the rest of the set S. If $S = \mathcal{N}$, then

$$b \equiv \frac{|\mathcal{B}|}{|N| - 1}$$

is the fraction of biased agents in the remainder of the population.

The game is played in three phases: (1) a message creation phase; (2) a communication phase; and (3) a voting phase.

2.3.2 *Message Creation*

I assume that a perfect signal of the true state, $s(\theta) \in \{0, 1\}$, is generated by nature with probability $p \in (0, 1)$. An agent is then randomly chosen to receive this signal, with all agents being equally likely to be selected. The signal contents is private knowledge of the chosen agent, no-one else observes it. Note that whether a signal has been created or not is also not known by agents who did not receive a signal. In addition, the agent chosen is not known by others, unless he or she has only one neighbour. In that case, the neighbour who receives information from him can infer this.

The chosen agent becomes a *message creator* and she is allowed to decide whether to communicate or remain silent, if she chooses to communicate then this might be done truthfully or not. Formally, the message space is $\{\emptyset, 0, 1\}$ where $\emptyset$ denotes the decision to remain silent. Since I only consider communication strategy profiles and p is not relevant to the posterior belief of other agents, the strategy of the creator can be described by a mapping

$$m_i : \{0, 1\} \mapsto \{\emptyset, 0, 1\},$$

with $m_i(s) = m_i$.

2.3.3 *Communication*

If agent i receives a message m_j from neighbor j, she must decide whether to transmit it to her remaining neighbors or block it. Note that she is not allowed to alter it any way; only to stop it from reaching others. This decision is represented by a mapping

$$t_i : \{m_j\} \mapsto \{m_j, \emptyset\}.$$

If $t_i(m_j) = m_j$, then agent i forwards the message unchanged to all neighbors except j. If $t_i(m_j) = \emptyset$, then agent i *blocks* the message, preventing it from propagating further.

2.3.4 *Collective Vote with Coordination*

After all possible communication has taken place, each unbiased agent i will update his initial belief to a posterior $\rho_i(m) = \Pr(\theta = 1|m)$ (often abbreviated to ρ_i when it is clear from the context) via Bayes rule. Since agents consider 0 more likely, the probability that $\theta = 1$ based on any messages she may have received the from his neighbors is an important quantity. Now, let $x \in [0, 1]$ denote the *collective outcome*. The outcome x is assumed to follow the "rule of the average". Formally, the outcome is given by the average of the actions taken by all agents weighted equally. In other words, $x = x(\mathbf{v})$:

$\mathbf{v} \mapsto [0,1]$,

$$x(\mathbf{v}) = \frac{1}{n}\sum_{j=1}^{n} v_j.$$

2.4 Coordination Networks and Utilities

Recall that for each agent i the set of agents she wishes to coordinate with is G_i. There are several possibilities I consider:

Case 1: Nearest neighbors only For each agent $i \in \mathcal{N}$, let

$$\mathcal{N}_i = \{j \in \mathcal{N} : ij \text{ is a link in the network}\}.$$

The elements of $\mathcal{N}_i$ then correspond to the *neighbours* of agent i. By assumption, the network is connected, so $|\mathcal{N}_i| \geq 1$ for all i.

If agent i coordinates only with her immediate neighbours, let G_i have node set $V(G_i) = \mathcal{N}_i$. The disagreement term is

$$R(G_i) = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} |v_i - v_j|,$$

and the utility is in this case

$$u_{i^u}(\mathbf{v}, x; \theta) = -(1-\alpha)\,|x-\theta| - \frac{\alpha}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i} |v_i - v_j|.$$

Case 2: Entire population

If agent i coordinates with the entire population, let G_i have node set $V(G_i) = \mathcal{N} \setminus \{i\}$. The disagreement measure is

$$R(G_i) = \frac{1}{n-1}\sum_{j\neq i} |v_i - v_j|,$$

so the utility becomes

$$u_{i^u}(\mathbf{v}, x; \theta) = -(1-\alpha)\,|x-\theta| - \frac{\alpha}{n-1}\sum_{j\neq i} |v_i - v_j|.$$

Case 3: Reachability sets $S_i(j)$

In the previous cases, G_i did not depend on message flow. I consider now an example where there is dependence on the way the message travels through the network.

Note that after observing a message from neighbor j, the agent $j \in \mathcal{N}_i$ induces a reachability subgraph $G_i(j)$ with node set

$$V\big(G_i(j)\big) = S_i(j),$$

where $S_i(j)$ denotes the set of agents whose messages could have reached agent i through j. The resulting utility of an unbiased agent i is

$$u_{i^u}(\mathbf{v}, x; \theta) = -(1-\alpha)\,|x-\theta| - \frac{\alpha}{|S_i(j)|} \sum_{k \in S_i(j)} |v_i - v_k|.$$

The three cases differ only in the definition of the coordination group G_i, which determines the normalization factor and the agents contributing to the disagreement term. This functional treats disagreement across agents symmetrically. It is possible to consider a more general disagreement measure by replacing $R(G_i)$ with a weighted average of agents, this reflects that agent i may value agreeing with a given agent more than others,

$$\widetilde{R}(G_i) := \sum_{j \in V(G_i)} w_{ij}\,|v_i - v_j|,$$

where $w_{ij} \geq 0$ and $\sum_{j \in V(G_i)} w_{ij} = 1$.

2.5 Equilibrium

The equilibrium concept of interest is a particular type of Perfect Bayesian Nash Equilibrium (PBNE) in which any unbiased agents behave in a fully informative and truthful manner. Specifically, each unbiased agent transmits any information he receives, creates messages truthfully and votes in accordance with the message. Formally, the messaging and transmission rules for an unbiased agent i^u, are given by $m_{i^u}(s) = s$ and $t_{i^u}(m_j) = m_j$. In the absence of a received message, an unbiased agent votes 0, reflecting the fact that in the absence of information she thinks this is the most likely state. In addition, she may make the inference that a message 0 may have been generated but blocked by a biased agent higher in the network. This assumption is particularly reasonable in large networks. Accordingly, the voting rules in equilibrium are $v_{i^u}(m) = m$ and $v_{i^u}(\emptyset) = 0$, and similarly $v_{i^u}(s) = s$ if the unbiased agent acts as sender.

By contrast, biased agents act strategically to favor their preferred outcome regardless of information: they always create and vote 1, and they block any 0 message encountered. Formally, $m_{i^b}(s) = 1$ and $t_{i^b}(1) = 1$ while $t_{i^b}(0) = \emptyset$. These behaviors correspond to those used in Bloch et al. (2018) for a *Full Communication Equilibrium* (FCE). Moreover, because a biased agent always prefers the collective outcome to move toward 1, voting 1 is a dominant strategy: a biased agent votes 1 regardless of any information or any actions taken by others. Hence $v_{i^b}(m) = 1$ for any $m \in \{0, 1, \emptyset\}$, while $v_{i^b}(s) = 1$ as well.

Given the above strategies, an unbiased agent i updates her belief when receiving a message m from a neighbour j as follows:

1. If she receives message $m = 0$, his posterior belief that the true state is 1 is $\rho_i(m_j = 0) = 0$.

2. If he receives message $m = 1$, he updates her belief using Bayes' rule, taking into account the proportion of biased and unbiased agents in the reachability set. Recall that this is the set of all $k \in \mathcal{N}$ from where the message could have originated and reached i through j. Her posterior in this case reflects the likelihood that message 1 was truthfully generated rather than introduced by a biased agent is

$$\rho_i(m_j = 1) = \frac{\pi}{b_{S_i(j)} + u_{S_i(j)}\pi},$$

Recall that $b_{S_i(j)}$ is the proportion of biased agents in $S_i(j)$, and $u_{S_i(j)}$ is the proportion of unbiased agents in $S_i(j)$.

3. For an agent i who receives no message, his posterior is at most equal to the prior, since silence provides no evidence in favor of the high state. In other words, $\rho_i(\emptyset) \leq \pi$.

Finally, given a profile of actions $\mathbf{v}$ and a message m, let $x_0^i = x(\mathbf{v}_{-i})$ denote the expected collective outcome if agent i chooses 0. If the agent instead chooses 1, the expected collective outcome increases by $\frac{1}{n}$.

Theorem 2.5.1 (Full communication equilibrium)**.** *Consider a network $G = (\mathcal{N}, \mathcal{E})$ and a disagreement functional R, an FCE exists if for every given unbiased agent i and each of her neighbors j:*

$$b_{S_i(j)} \;<\; \frac{\pi}{1-\pi}\left(1 + \frac{2}{\dfrac{1-\alpha}{\Delta_{ij}\alpha} - 1}\right), \qquad \text{whenever } \alpha \in \left(0, \frac{1}{1+\max_{j\in\mathcal{N}_i}\Delta_{ij}}\right).$$

where $\Delta_{ij} = \Delta_{ij}(R\big(G_i(j)\big)) < n$. In addition, the proportion of biased agents in the whole population must satisfy $b < b(\alpha, R)$.

Theorem 2.5.1 establishes sufficient conditions for the existence of a full communication equilibrium (FCE) for a general tree. These conditions depend explicitly on the network architecture, the strength of conformity, and the distribution of biased agents. In particular, for each unbiased agent i and each of her neighbours j, the proportion of biased agents in the relevant reachability set must lie below a threshold that is determined by the prior π, the conformity parameter α, and the strength of coordination within the corresponding coordination group, captured by Δ_{ij}. In addition, the overall share of biased agents in the population must remain below an upper bound $b(\alpha, R)$ implied by the model primitives.

Theorem 2.5.1 and Proposition 1.3.7 both characterise environments in which truthful information transmission can be sustained in equilibrium. Theorem 2.5.1, however, provides more general existence conditions for an FCE that depend explicitly on the *network structure.* These conditions impose upper bounds on both the strength of conformity and the prevalence of biased agents. In particular, the theorem shows that neither conformity nor bias can be too large if relevant information is to be fully disseminated along the network. At the same time, a moderate degree of conformity relaxes the upper bound on how many biased agents a given reachability set may contain while still permitting truthful messages to circulate. For a fixed prior π, the maximum biased share compatible with full communication is therefore (weakly) higher when a non-zero conformity motive is present.

Relative to Proposition 1.3.7, Theorem 2.5.1 is more restrictive in scope: while Proposition 1.3.7 delivers a complete partition into corresponding equilibria of the (α, b) parameter space for the public-broadcast environment, deriving an analogous characterisation for general network structures is considerably more difficult. Nonetheless, the comparison is informative. Proposition 1.3.7 shows that in public broadcast settings, information transmission can be maintained for a wide range of conformity levels and is robust even when biased agents are numerous. By contrast, Theorem 2.5.1 demonstrates that in network settings, sustaining information transmission requires more stringent local conditions that reflect the topology of the underlying network. Although conformity must remain sufficiently weak, its presence still enlarges the set of environments in which truthful communication is feasible by allowing a greater—though still bounded—proportion of biased agents to coexist with information transfer.

2.6 Conclusion

This chapter has examined how conformity motives and the distribution of biased agents interact with network structure to shape the credibility and diffusion of information. Unlike in public-broadcast environments, where messages reach all agents directly, communication in networks proceeds through local interactions. As a result, the sustainability of truthful information transmission is tied not only to global parameters—such as the prior and the overall proportion of biased agents—but also to the topology of the network and coordination motives.

The analysis characterises sufficient conditions under which a full communication equilibrium exists. These conditions require that, for each unbiased agent, the share of biased agents in the relevant reachability sets remains below a threshold that depends on the prior, the conformity parameter, and the underlying coordination architecture. In addition, the overall proportion of biased agents must fall below an endogenous upper bound. The results show that although conformity cannot be too strong, the presence of a small conformity motive can actually enlarge the set of network environments under which truthful communication is feasible. Moderate conformity motivates unbiased agents to align their actions with others which makes them more tolerant to unreliable messages.

Nevertheless, the findings make clear that networks are intrinsically more fragile than the public-broadcast setting. Since information flows through local interactions, distortions introduced by biased agents can be amplified depending on their location in the network. Furthermore, local coordination interests may further disrupt information flow. Consequently, the parameter region supporting full communication is narrower than in broadcast environments, and the feasibility of information transmission is more sensitive to the network structure.

Overall, this chapter highlights the importance of local communication architecture in determining whether truthful information can be sustained in the presence of social conformity and heterogeneous preferences. The results underscore that weak conformity is consistent with information transfer, but effective communication in networks requires careful alignment between incentives, beliefs, and network structure.

In this chapter, I extend the analysis of strategic communication from a public broadcast setting to a networked environment. Specifically, I adapt the model from Chapter 1 to a simple undirected line network, thereby incorporating conformity among neighbors. This framework represents a natural extension of Bloch et al. (2018), allowing for an explicit analysis of how local social interactions influence information transfer.

The results highlight the conditions under which information can propagate across the network. Compared to the public communication setting, the network imposes stricter constraints on the proportion of biased agents that the population can sustain while still supporting truthful communication. This finding aligns with the intuition and results in Bloch et al. (2018), emphasizing that decentralized communication structures generally reduce the robustness of social learning.

This analysis provides insights into how social structures influence the spread of information and rumors, with applications to political communication, online social networks, and public health messaging. By highlighting the interaction between conformity and network topology, the chapter contributes to a deeper understanding of how individual behavior shapes collective learning in realistic social environments.

APPENDIX B

Proof of theorem 2.5.1. **Voting phase:**

The argument mirrors the analysis for the public broadcast environment in Chapter 1. In particular, assuming that a FCE exists, then every agent in the network transfers message 1. This leads to a similar lower bound for $\rho_i(1)$ as in chapter 1. This bound implies that unbiased agents do not have an incentive to deviate from $v_{i^u}(m_j) = 0$ to $v_{i^u}(m_j) = 1$ when $m_j \in \{0, \emptyset\}$.

Transmission phase:

I now provide conditions under which an unbiased agent has no incentive to block a message $m_j = 1$ sent by a biased neighbor. This is the relevant case: if the neighbor were unbiased, the message would be more credible. Blocking a message implies that the agent considers it unreliable and believes that $\theta = 1$ is unlikely; consequently she votes 0 in the next phase. This is the same logic that would generate a deviation in the voting stage.

The interim expected outcome for unbiased agent i^u depends on his own transmission strategy, which will in turn affect his successors' strategies. Therefore, I write $x_0^i(t_i) = x_0^i(|S_i(j)|, |S_j(i)|, t_i)$. Recall that $S_i(j)$ describe the predecessors of i (including j), while $S_j(i)$ describes i's successors (including herself). In equilibrium, the actions of agents in $S_i(j)$ are fixed and determined by message m_j (which is assumed to flow through the set). On the other hand, actions on $S_i(j)$ depend on $t_i \in \{m_j, \emptyset\}$ but are otherwise fixed by equilibrium behavior. The expected utility of $i^u \in \mathcal{U}$ from transmitting message 1 received from her biased neighbor j^b,

$$\mathbb{E}[u_{i^u}(t_{i^u}(m_j^b) = m_j^b; x_0^i(t_{i^u}))|m_j^b = 1] = -(1-\alpha)\left\{\rho_i(m_j^b = 1)\left(1 - x_0^j(t_i = 1) - \frac{1}{n}\right) + [1 - \rho_i(m_j^b = 1)]\left(x_0^i(t_i = 1) + \frac{1}{n}\right)\right\}. \tag{2.2}$$

Case 1: $V(G_i) = \mathcal{N}_i$.

Blocking message 1 now yields

$$\mathbb{E}[u_{i^u}(t_{i^u}(m_j^b) = \emptyset; x_0^i(t_{i^u}))|m_j^b = 1] = -(1-\alpha)\Big\{\rho_i(m_j^b = 1)\left(1 - x_0^j(t_i = \emptyset)\right) + [1 - \rho_i(m_j^b = 1)]x_0^i(t_i = \emptyset)\Big\} - \alpha b_{\mathcal{N}_i}.$$

Combining this with (2.2) implies that an unbiased agent i^u transmits if and only if

$$\rho_i^{Near}(m_j^b = 1) > \frac{1}{2}\left(1 - \frac{b_{\mathcal{N}_i}}{1 - x_0^j(t_i = \emptyset)}\frac{\alpha}{1-\alpha}\right), \tag{2.3}$$

when $\alpha \in [0, 1)$. Note that, once again, $\frac{b_{\mathcal{N}_i}}{1 - x_0^j(t_i = \emptyset)} < n$.

Case 2: $V(G_i) = \mathcal{N} \setminus \{i\}$**.**

Blocking message 1 yields

$$\mathbb{E}[u_{i^u}(t_{i^u}(m_j^b) = \emptyset; x_0^i(t_{i^u}))|m_j^b = 1] = -(1-\alpha)\Big\{\rho_i(m_j^b = 1)\left(1 - x_0^j(t_i = \emptyset)\right) + [1 - \rho_i(m_j^b = 1)]x_0^i(t_i = \emptyset)\Big\} - \alpha\frac{n}{n-1}x_0^j(t_i = \emptyset).$$

Combining this with (2.2) implies that an unbiased agent i^u transmits if and only if

$$\rho_i^{Pop}(m_j^b = 1) > \frac{1}{2}\left(1 - \frac{n}{n-1}\frac{\alpha}{1-\alpha}\frac{x_0^j(t_i = \emptyset)}{1 - x_0^j(t_i = \emptyset)}\right), \tag{2.4}$$

when $\alpha \in [0, 1)$. In addition, $\frac{n}{n-1} < n$.

Case 3: $V(G_i) = S_i(j)$**.**

Blocking the message yields expected payoff

$$\mathbb{E}[u_{i^u}(t_{i^u}(m_j^b) = \emptyset; x_0^i(t_{i^u}))|m_j^b = 1] = -(1-\alpha)\Big\{\rho_i(m_j^b = 1)\left(1 - x_0^j(t_i = \emptyset)\right) + [1 - \rho_i(m_j^b = 1)]x_0^i(t_i = \emptyset)\Big\} - \alpha.$$

Combining this with (2.2) implies that an unbiased agent i^u transmits if and only if

$$\rho_i^{Reach}(m_j^b = 1) > \frac{1}{2}\left(1 - \frac{1}{1 - x_0^j(t_i = \emptyset)}\frac{\alpha}{1-\alpha}\right), \tag{2.5}$$

when $\alpha \in [0, 1)$. Since unbiased successors k^u take action $v_{k^u}(\emptyset) = 0$, $\frac{1}{1-x_0^j(t_i=\emptyset)} < n$.

If instead $m_j = 0$ (so $j \in \mathcal{U}$), then because $v_{k^u}(\emptyset) = v_{k^u}(0) = 0$ for all k^u, blocking yields no benefit for any type of agent.

Message phrase:

Biased agent. A biased agent cannot increase her expected payoff by deviating to either $m(s) = 0$ or $m(s) = \emptyset$.

Unbiased agent. The expected payoff of an unbiased agent who receives a signal $s = 1$ cannot increase by adopting the strategy $m(s) = 0$ or $m(s) = \emptyset$, independently of which coordination rule is applied. This is because the expected payoff from $m_{i^u}(s) = 1$ yields zero loss, whereas any deviation generates a loss.

If instead signal $s = 0$ is received, then s fully reveals the true state is 0.

Case 1: $V(G_i) = \mathcal{N}_i$. For an unbiased agent $i^u \in \mathcal{U}$, the expected utility of $i^u \in \mathcal{U}$ from creating the message 0 from a signal $s = 0$ is

$$\mathbb{E}[u_{i^u}(m_{i^u}(s) = s)|s = 0] = -(1-\alpha)\frac{n-1}{n}b - \alpha b_{\mathcal{N}_i}.$$

Creating the message 1 from a signal $s = 0$ yields

$$\mathbb{E}[u_{i^u}(m_{i^u}(s) = 1)|s = 0] = -(1-\alpha).$$

Truthful messaging is optimal if

$$b_{\mathcal{N}_i} < \frac{1-\alpha}{\alpha}\left(1 - \frac{n-1}{n}b\right). \tag{2.6}$$

Case 2: $V(G_i) = \mathcal{N}$**.** The expected utility of $i^u \in \mathcal{U}$ from creating the message 0 from signal $s = 0$ is

$$\mathbb{E}[u_{i^u}(m_{i^u}(s) = s)|s = 0] = -(1-\alpha)\frac{n-1}{n}b - \alpha b.$$

Creating the message 1 from a signal $s = 0$ yields

$$\mathbb{E}[u_{i^u}(m_{i^u}(s) = 1)|s = 0] = -(1-\alpha).$$

The truthful messaging requires

$$b < \frac{1}{\frac{n-1}{n} + \frac{\alpha}{1-\alpha}}. \tag{2.7}$$

Case 3: $V(G_i) = S_i(j)$**.** $S_i(j) = \emptyset$, identical to the no-coordination case; no deviation is profitable. In each case, the relevant belief thresholds ensure that no profitable deviation exists, either by blocking an incoming message or by misreporting a privately observed signal.

□

Chapter 3

Optimal Sin Taxes When Self-Control Costs Are Present: A Nonlinear Pricing Approach

This paper analyzes optimal taxation of sin goods when consumers exert self-control. This happens, for instance, when consumers struggle balancing current gratification against preserving future health. In the context of a monopoly market, I adopt the temptation model of Gul and Pesendorfer (2001) to characterize the optimal pricing scheme. This scheme contains a quality-price ceiling, determined endogeneously by the market size. Furthermore, I characterize the welfare maximizing tax policy (both for specific and ad valorem taxes) for different behavioral welfare frameworks. In particular, I show that for a domestic monopolist optimal ad valorem tax decreases as the market size grows up, passing from a tax to a subsidy. I show further that imposing a specific tax is not optimal. By contrast, for imported goods both ad valorem and specific tax increases lead to improvements in welfare. Notably, optimal ad valorem tax rates are much higher than optimal specific tax rates.

3.1 Introduction

The UK government currently imposes high "sin taxes" on tobacco, alcohol and sugar-sweetened beverages to curb consumption and raise revenue. However, recently HMRC (His Majesty's Revenue and Customs) receipts from these taxes have been declining as health-conscious trends seem to reduce demand for such products which poses a fiscal challenge. While these taxes are often seen as effective public health interventions and revenue generators they have a complex structure, which combines both specific and ad valorem components, and creates a policy dilemma: should the government maintain high taxes to sustain revenue, or lower them to stimulate consumption and mitigate illicit trade? This tension is further complicated by distributional concerns as sin taxes disproportionately affect low-income households; as well as enforcement challenges due to excessive taxation fueling smuggling and black-market activity. Increasing taxation also creates difficulties beside consumer welfare revenue beyond trade-offs. For instance, as seen in Brazil's 2023 sin tax reform which will phase into a new system by 2032; for imported goods, higher taxes could increase market entry costs, necessitating adjustments in pricing, supplier negotiations, and customs classifications. In light of these issues, age-old questions arise: Should governments tax or subsidize certain goods to maximize social welfare when consumers face a self-control problem? How can policymakers design optimal taxation when demand for harmful products persists? And what are the differential effects on domestic versus imported goods?

To address the above questions with regards to "sin goods", I adopt the approach in Gul and Pesendorfer (2001) (abbreviated to"GP" in the sequel) to incorporate temptation in decision making of consumers. Their model formalizes the ideas of temptation and self-control by characterizing consumer preferences through two utility components: *Commitment utility*, which reflects the consumer's rational, long-term preferences, and *temptation utility* which captures impulsive consumption desires. Moreover, the decision process of the consumer is modeled in two stages. First, the consumer chooses her most preferred menu out of all possible options and subsequently chooses her top ranked item out of the menu. Intuitively, this can be thought as the consumer first choosing which store to buy from and then, once in the store, choosing his highest ranked choice. Unlike standard rational agent models, this framework explicitly accounts for self-control problems (resulting from the consumer consciously restraining herself) while maintaining a single-agent representation. In contrast to time-inconsistent preference models (e.g. Strotz, 1955; Phelps and Pollak, 1968 and Laibson, 1997), it avoids the need to split the consumer into multiple selves and thus make arbitrary assumptions about welfare weights across these selves. Preference reversals in Gul and Pesendorfer are the result of preferences being defined first over consumption sets rather than over consumption sequences. Formally, two sources of utility in the model are present: one denoted by U, the commitment utility, and the other denoted by V; the temptation utility which quantifies how tempting different goods are to the consumer. The consumer's utility in the second stage is given by $U + V$ (his *ex-post utility*); while the utility in the first stage (his *ex-ante utility* W) is given by the difference between the maximum ex-post utility and maximum temptation utility, i.e. for a menu M

$$W(M) = \max_{x \in M}\{U(x) + V(x)\} - \max_{x \in M} V(x). \tag{3.1}$$

If $U + V$ is maximised at $x^\star \in M$ and V is maximised at $y \in M$, then the ex-ante utility is

$$W(M) = U(x^\star) + V(x^\star) - V(y), \tag{3.2}$$

$V(x^\star) - V(y)$ is then the self-control cost.

Sin goods can of course be provided by either a monopoly or a competitive market. I choose to focus on the monopoly case as the welfare implications of taxation are clearer. This is because market competition creates additional interactions between firms and as a result strategic government intervention may create additional equilibrium distortions. My model relies on similar assumptions as the standard non-linear pricing models in Mussa and Rosen

(1978) and Maskin and Riley (1984), where a monopolist sells goods that differ in a single-dimensional quality (or quantity) level. This assumption is well suited to sin goods as there are many instances where firms differentiate their products according to consumer characteristics. For example, luxury spirits (e.g., Johnnie Walker Blue Label, Dom Pérignon champagne) target high-income consumers who are willing to pay for the brand prestige and image. On the other hand, beer firms release low-alcohol products to appeal to a broader health-conscious demographic. As it is standard, the monopolist does not observe consumers' preferences and relies on indirect price discrimination schemes which lead to consumer self-selection. To capture the additional behavioral assumptions, the parameter measuring the intensity of a given consumer preference for quality is replaced by the temptation intensity γ in my model. This temptation intensity only appears in temptation utility.

Following Esteban et al. (2007), I consider two classes of consumers. Consumers facing *upward temptation* and consumers facing *downward temptation*. A consumer facing upward temptation is tempted by high quality high price items. She therefore has a higher willingness to pay when faced with bundles containing such items. By contrast a consumer facing downward temptation is tempted by lower quality lower prices items when faced with bundles having higher price higher quality items. When confronted with consumers of both types the monopolist faces a trade off between offering smaller high quality high price menus that enable him to extract high surpluses from consumers facing upward temptation, but may discourage consumers facing downward temptation, and offering more targeted menus that can ease the self control costs consumers facing downward temptation encounter. The disadvantage of this former approach is that consumers facing upward temptation are disincentivised from purchasing costlier bundles in the menu.

Recall that, I denote by γ a given consumer's temptation intensity. I assume that there is a critical temptation level γ^* where the consumer's marginal value of commitment utility equals that of temptation utility. At this point the consumer behaves as a fully rational agent. The magnitude of the difference between a consumer's γ and γ^* reflects the strength of temptation. When $\gamma > \gamma^*$ (positive difference), the self-control cost arises from higher quality: the larger the difference, the more temptation amplifies the appeal of higher quality. When $\gamma < \gamma^*$ (negative difference), the self-control cost arises from lower price: the larger the difference, the more temptation amplifies the appeal of lower prices—potentially even deterring market entry. My primary interest is the case in which there is population heterogeneity in the degree of temptation γ.

The standard normative welfare analysis of monopoly taxation is presented in Krishna (1984). In particular, Krishna's model analyses the effect of protectionists policies on both a national and foreign monopolist when he faces a population where consumers have different willingness to pay for the goods he provides. Her model builds is based on the model by Mussa and Rosen (1978). My model is closely related to hers, the main differences being the additional behavioral assumptions. Unlike her results, where both specific and ad valorem subsidies are welfare improving for a home monopolist, I show that a specific tax has no effect on welfare in this case. I also characterize the performance of ad valorem taxation according to different population levels. I further show that in the case of a foreign monopolist both ad valorem and specific taxes are welfare improving.

In terms of welfare analysis in my model, I distinguish three types of total welfare: *adjusted-cost welfare*, *normative welfare* and *behavioral welfare*. This is because unlike traditional economics where individuals maximize a well-defined utility function and welfare can be easily defined via this function; behavioral economics incorporates cognitive biases and self-control problems into the utility function. This makes the problem of defining welfare difficult. Indeed, disentangling true utility (a measure of genuine welfare) from revealed preferences (observed choices) is a problem that remains unresolved but seems to be increasingly substantiated by empirical research. There is however much theoretical debate regarding this problem. This debate often intersects deeper philosophical disagreements regarding paternalism versus liberalism. Particularly, when determining optimal policy selection and welfare evaluation.

The three welfare measures are defined as follows: adjusted-cost welfare uses the ex-ante utility of the consumer as a definition of welfare. This is justified with reference to Gul and Pesendorfer (2001) who shows that a dynamically consistent decision-maker benefits when ex-ante undesirable temptations are removed, suggesting welfare should account for both commitment utility and self-control costs in menu-choice contexts. On the other hand, adopting a paternalistic normative perspective demands that welfare should reflect choices made free of temptation (i.e. reflects the "true preferences"). This motivates the choice of commitment utility as welfare measure since, by definition, it is supposed to capture "true" consumer preferences. Finally, a behavioral analysis uses ex-post utility (the sum of commitment and temptation utilities). This is motivated by the interpretation of an agent in behavioral economics as consisting of several selves. The welfare of an agent is thus the sum total of the welfare of each self. In this regard, Strotz (1955) argues welfare must balance conflicting preferences across time (e.g., present vs. future selves). This is further motivated, by noting that behavioral economics challenges the conventional interpretation of reveal preference theory since it is impossible to separate "true" preferences from behavioral distortions when observing agents' choices. In the context of the GP model separating commitment and temptation utility appears unrealistic since only ex-post utility could be inferred by observed choices. Given these theoretical difficulties, I choose to evaluate policy impact across all three welfare frameworks.

My results contribute to the literature in a number of ways. To start with, I characterize an optimal monopoly nonlinear pricing scheme in a behavioral context where agents suffer from temptation. This results in the optimal scheme having price cap at the highest level. Secondly, I characterize the optimal tax design. Third, the most novel contribution results from my analysis of the welfare effects of taxation in this market both domestically and for an importer. In particular, I show that a small tax or a subsidy can improve welfare in the case of a home monopolist. In the case of a foreign monopolist, I show that protectionist policies can result in higher consumer welfare and increase tax revenue.

From the consumer perspective, I show that in equilibrium there are three types of consumers who do not suffer from self-control problem. First, the lowest types always obtains zero utility as they purchase nothing. Second, highest types choose the highest quality highest price product provided by the monopolist. Since these consumers are upward tempted they suffer no self-control costs. Lastly, consumers with temptation intensity at the level of γ^* behave rationally. At this level, the consumer's marginal value of commitment utility and that of temptation utility are equal. Equilibrium structure depends on the upper bound of temptation intensity in two ways: a higher upper bound implies temptation intensity varies over a wider range. Moreover, a higher bound diminishes the fraction of consumers facing self control costs in equilibrium. In geometric terms, the interval of consumers who suffer from self control costs shrinks as the parameter increases.

I show that imposing a tax can mitigate self-control costs by reducing the net utility of temptation-driven choices (e.g., by making high-quality/lower-price temptations less salient). I show that the effectiveness and nature of taxation depends on market size. In particular, I find that larger markets require taxes while smaller market benefit from subsidies.

More broadly, I show that specific tax has no effect on the domestic temptation good while a small ad valorem tax may improve the social welfare when the average level of temptation amongst consumers is not very high. However, when the intensity of temptation varies strongly across consumers a purely ad valorem subsidy policy can improve the welfare. On the other hand for the imported good, both ad valorem and specific tax policy can improve the domestic country's welfare. I find that an optimal ad valorem rate can reach a high level of around 50 per cent. By contrast a small specific tax can be implemented by the government to improve the social welfare.

I also show that the optimal taxation policy depends on the normative and behavioral perspectives. An optimal normative ad valorem tax for domestic goods is a moderate rate between that of the adjusted-cost measure and the behavioral measure, since it ignores the reduction of self-control costs arising from the tax and the lowering of temptation utility. A specific taxation or subsidy policy works the same way for domestic welfare. For the foreign good, the adjusted-cost measure pushes the optimal ad valorem tax above the 50 per cent mark; while the optimal ad valorem tax is lower than 50 per cent in both the normative and behavioral measures. The normative measure shows that a higher tax policy should be implemented by the government as more consumers suffer more pronounced self-control problems when considering a foreign good. The behavioral measure suggests that a lower tax rate is better. It is particularly noteworthy that the optimal normative specific tax rate is much higher than both tax rates as prescribed by the adjusted-cost measure and the behavioral measure. This suggests that self-control costs and temptation utility losses are sensitive to the specific tax rate on imported good.

3.1.1 *Related Literature*

This paper studies optimal taxation in a monopoly market where the monopolist screens agents according to their temptation type. In particular, I focus on how temptation affects consumers' preference over different bundles and how the monopolist accounts for this when providing a menu to the consumers. This differs from the more widely encountered approaches in the literature that rely on dynamics. In particular, present bias and time inconsistency due to hyperbolic discounting. In contrast, consumers in my model are not dynamically inconsistent. This allows for a more straightforward analysis by focusing on how other agents (i.e. the monopolist or, indirectly, the government) account for the consumers behavioral traits in their own decisions. In particular, this permits a more parsimonious comparative statistics analysis in the event a tax is introduced. Similarly, optimal taxation can be easily characterized. Moreover, my model allows for higher heterogeneity in agent types.

Time inconsistency and hyperbolic discounting are well-known in the economic literature. Originally, the seminal work of Strotz (1955) introduced the idea that an agent's future actions may systematically deviate from her initial optimal plan. Such deviation creates a demand for pre-commitment devices, which ensures future decisions are in line with the present optimal path. Building on Strotz's insight, Phelps and Pollak (1968) formalized the notion of "hyperbolic discounting", a time preference structure in which short-run and long-run discount rates differ. However, their application focused on inter generational conflicts; specifically the challenge of second-best national saving when the current generation cannot bind the decisions of its descendants. This idea was then adapted to account for behavioral inconsistencies of a single agent over time in Laibson (1997). The paper also explored how agents behave under imperfect commitment technologies. This was later developed in O'donoghue and Rabin (2001), which provided a rigorous analysis of the welfare and behavioral consequences of present-biased preferences, which provided important insights into phenomena such as procrastination. O'Donoghue and Rabin (2006) incorporates these ideas along with the time-inconsistency assumption into a standard optimal taxation framework. Their conclusions show that imposing a tax on unhealthy items and returning the proceeds to consumers can generally improve total social surplus. In particular, they provide examples showing that taxation can be significantly effective even when agents are afflicted by a relatively small self-control problem. By contrast in my model, if the population are predominantly downwards tempted then introducing a subsidy can improve welfare. On the other hand, if a large proportion of upward tempted consumers are present then taxes are optimal.

A few papers have used the GP model been used to analyze the problem of optimal taxation. For instance, Gul and Pesendorfer (2007) argue that taxing drugs can in fact reduce welfare while prohibitive policies may be an effective way to increase welfare. These results are established by constructing an infinite horizon model of harmful addiction. Similarly, Krusell et al. (2010) were the first to study how linear tax-transfer schemes can be used to improve the welfare in a representative consumer economy where agents are tempted towards current consumption, thereby distorting the incentive to save for future periods. They showed that a savings subsidy improves welfare by making succumbing to temptation less attractive.

Other behavioral models have also been explored in the literature to analyze welfare improvement via taxation. In particular, Haavio and Kotakorpi (2011) discusses how linear sin taxes and transfers can mitigate consumption errors when agents follow a a quasi-hyperbolic discount function (see e.g. Laibson, 1997). This time inconsistency leads to delayed negative effects. More recently, Arvaniti and Sjögren (2023) identify a commitment mechanism that works through endogenous labor choices and affects the design and effectiveness of the optimal tax policy. As can be seen from these examples, existing literature focuses mostly on individual welfare improvement via taxation using general equilibrium frameworks. In this paper, I focus on the normative question of determining whether total welfare can be improved via taxation or subsidy on a monopolistic industry.

On the policy side, research has shown that paternalistic interventions like "sin taxes" can be welfare improving as they can help reduce harms arising from self-control problems. For example, Gruber and Kőszegi (2004) focus on smoking and show that taxation can benefit low income groups. More generally, Gruber and Mullainathan (2005) argue that behavioral effects arising from cigarette taxes are considerably more complex than those predicted by simple rational economic model by referring to behavioral data from both the US and Canada.

This paper also contributes to the normative behavioral welfare analysis. Introducing behavioral effects in economic welfare analysis is a hotly debated topic in the literature. This debate often centers about contesting the foundational assumption of neoclassical economics without questioning the underlying welfare assumptions of policy goals. An often cited example of this approach is Gul and Pesendorfer (2007). On the other hand, a series of recent papers (Bernheim and Rangel, 2007, 2008, 2009) have argued strongly in favor of an alternative normative framework that defines welfare in terms of choice as opposed to well-being or other underlying objectives. Lastly, Chetty (2015) presents a more pragmatic perspective on behavioral economics that emphasizes its role in improving empirical predictions and thus policy decisions. This paper is closer in spirit to this latter perspective, I study and compare how optimal taxation policy is implemented according to different attitudes to welfare.

The welfare effects of taxation under monopoly have been studied extensively. Krishna (1984) show that both specific and ad valorem subsidies can raise welfare in a domestic monopoly market producing a full product line. For a foreign monopoly firm, the outcome of taxation depends on the distribution of consumer types. More recently, McCalman (2010) highlight that optimal trade policy under nonlinear pricing depends not only on terms-of-trade effects but also on consumer heterogeneity and incentive compatibility constraints. In contrast, my model consumer types represent the degree of temptation of each individual consumer. Temptation types are assumed to be uniformly distributed. In this setting, protection policies can improve welfare under both specific and ad valorem taxation. For a domestic monopolist, the welfare impact of ad valorem intervention depends on the upper bound of temptation: taxes enhance welfare when temptation costs are large, while subsidies improve welfare through higher product quality when temptation costs are small. By comparison, specific interventions are welfare-neutral.

The structure of this chapter is as follows: In Section 3.2, the model is described formally in both the simpler case where there are only two consumer types and the more general case where there is a continuum of agents. In section 3.3, I present the optimal taxation and the welfare analysis. The paper is then concluded in Section 3.4 with a discussion of further work and conclusions. Proofs of some of the results in the main body are presented in the Appendix.

3.2 Model

Consumer's type γ is uniformly distributed on $[a, b]$. The cumulative distribution is denoted by $F(\gamma)$ and the density by $f(\gamma)$. Note that γ gives a measure of the degree of temptation the consumer faces. The consumer's commitment utility when offered a bundle of quality q and price p is

$$U(q, p) = q - p, \tag{3.3}$$

and temptation utility is

$$V_\gamma(q, p) = \gamma q - p. \tag{3.4}$$

The ex-ante utility from a menu $M \subset \mathbb{R}^2_+$ is

$$W_\gamma(M) = \max_{(q,p)\in M}\{U(q,p) + V_\gamma(q,p)\} - \max_{(q,p)\in M}\{V_\gamma(q,p)\} \\ = \max_{(q,p)\in M}\{(1+\gamma)q - 2p\} - \max_{(q,p)\in M}\{\gamma q - p\} . \tag{3.5}$$

A menu $M \subset \mathbb{R}^2_+$ is assumed to be a compact set containing the origin $(0,0)$. This reflects the fact that the consumer always has the default choice of not purchasing from the monopolist.

To clarify the properties of commitment utility U and temptation utility V_γ, I define that for any utility function $X_\gamma(q,p)$ and $Y_\gamma(q,p)$, $X_\gamma(q,p) \succsim Y_\gamma(q,p)$ if the marginal value of q is weakly higher for $X_\gamma(q,p)$ than for $Y_\gamma(q,p)$ at any point (q,p).

Naturally, consumers with $\gamma < \gamma^*$ who are tempted downwards to lower quality, cheaper bundles exhibit a lower marginal willingness to pay for additional quality than the committed itself, that is, $V_\gamma \prec U + V_\gamma \prec U$. On the other hand, consumers with $\gamma > \gamma^*$ who are tempted upwards to higher quality, more expensive bundles have a higher marginal willingness to pay for additional quality than the committed itself, that is, $V_\gamma \succ U + V_\gamma \succ U$.

For every menu, it is possible to define an allocation function $x(\gamma) : [a,b] \mapsto M$ that associates with each consumer a bundle $(q(\gamma), p(\gamma))$ according to his type γ that the monopolist hopes the consumer will buy. This bundle might in fact be $(0,0)$. A menu M and allocation x together define a *schedule* (M,x). A schedule is optimal if it maximises profits.

It is possible (see Esteban et al., 2007) to show that an optimal schedule exists under general assumptions, where (i) the monopolist only provides bundles consumers buy, and (ii) all consumers enter the store (I assume that if a consumer is indifferent between entering or not he chooses to enter). Moreover, this schedule does not generate loses to the monopolist at each individual type. I discuss how to characterise this optimal schedule below.

Given an allocation $(q(\gamma), p(\gamma))$, for each consumer's type γ, the firm's profit from the type is

$$\pi(\gamma) = p(\gamma) - C(q(\gamma)),$$

where the cost is

$$C(q(\gamma)) = \frac{1}{2}q(\gamma)^2.$$

The expected profit for the monopoly is $\int_\gamma [p(\gamma) - \frac{1}{2}q(\gamma)^2]dF(\gamma)$. Thus, the firm's problem becomes

$$\max_{q(\gamma),p(\gamma)} \int_\gamma [p(\gamma) - \frac{1}{2}q(\gamma)^2]dF(\gamma)$$

subject to

$$W_\gamma \geq 0, \qquad \text{(ex-ante IR)} \tag{3.6}$$

$$U(q(\gamma), p(\gamma)) + V_\gamma(q(\gamma), p(\gamma)) \geq 0, \qquad \text{(ex-post IR)} \tag{3.7}$$

$$U(q(\gamma), p(\gamma)) + V_\gamma(q(\gamma), p(\gamma)) \geq U(q(\hat{\gamma}), p(\hat{\gamma})) + V_\gamma(q(\hat{\gamma}), p(\hat{\gamma})), \quad \forall\hat{\gamma}. \quad \text{(ex-post IC)} \tag{3.8}$$

The first of these conditions represents ex-ante individual rationality; the consumer cannot be ex-ante worse off by entering the store than she would be if she did not. The second represents ex-post individual rationality; the consumer choice cannot be ex-post worse than he would be if she had not chosen anything. Finally, the third condition represents ex-post incentive compatibility; the consumer choice must be the best given all consumption choices. Here, it is noted that *ex-ante* and *ex-post* utilities refer respectively to the consumer before and after exerting self-control, which differs from their use in standard (non-behavioral) models.

Finally, denote by γ^* the type of consumer who does not have a self-control problem. In this setup, $\gamma^* = 1$. The type γ^* acts as a threshold, consumers with $\gamma < 1$ face downward temptation, while consumers with $\gamma > 1$ face upward temptation.

3.2.1 ***Two-Type Case***

I begin by considering a simpler setting where the consumer population contains only two types; one downward-tempted, labeled as γ_{L}, and another upward tempted, labeled as γ_{H}. More precisely, consumer types satisfy $\gamma_{\text{L}} < 1 < \gamma_{\text{H}}$. By analogy with the continuum case, I assume that the probability that a given consumer type is drawn from the population is $\frac{1}{2}$. This should not significantly alter the results, but makes calculations slightly simpler.

I first consider the complete information case to gain some insight on how the optimal schedule may be characterized under the more realistic assumption of asymmetric information about consumer's types. The monopolist therefore observes the value of each consumer type γ by assumption. This allows the monopolist to offer an individualized menu M to each consumer so as to maximize the monopolist's profit. The consumer then chooses whether to enter the store; if so, she chooses $(q, p) \in M$ to maximize her ex-post utility.

Under complete information, the monopolist maximizes profits under subject only to conditions (3.6) and (3.7), since complete information makes incentive compatibility (3.8) redundant. In this case, the optimal schedule contains a unique non-trivial choice (i.e. the optimal schedule is of the form $(\{(q^*, p^*), (0, 0)\}, (q^*, p^*))$, because offering additional options cannot increase profits and instead risks raising self-control costs for the seller. Therefore, the ex-ante and ex-post IR conditions reduce to

$$\min\{U(q, p), U(q, p) + V_\gamma(q, p)\} \geq 0. \tag{3.9}$$

For the upward-tempted consumers, $U(q, p) \geq 0$ implies $U(q, p) + V_\gamma(q, p) \geq 0$, since $U + V_\gamma \succ U$ when $\gamma > \gamma^*(= 1)$. Thus, condition (3.9) simplifies to $U(q, p) \geq 0$. The profit-maximization problem then reduces to maximizing $\pi(q, p) = p - \frac{1}{2}q^2$ subject to $U(q, p) = 0$. A straightforward calculation then shows that the optimal bundle for the upward tempted consumer is $p^C(\gamma_{\text{H}}) = q^C(\gamma_{\text{H}}) = 1$. The upward-tempted consumer has a stronger willingness to pay for quality because temptation pulls her toward higher bundles. The monopolist can therefore extract the entire surplus by setting $(q, p) = (1, 1)$, leaving the consumer with zero commitment utility but no incentive to opt out.

Conversely, for the downward-tempted consumers, $U(q, p) + V_\gamma(q, p) \geq 0$ implies $U(q, p) \geq 0$, since $U + V_\gamma \prec U$ when $\gamma < \gamma^*$. Then the monopolist now maximizes $\pi(q, p) = p - \frac{1}{2}q^2$ subject to $U(q, p) + V_\gamma(q, p) = 0$. This means that in this case, $q^C(\gamma_\text{L}) = \frac{1+\gamma_\text{L}}{2}$ and $p^C(\gamma_\text{L}) = \frac{(1+\gamma_\text{L})^2}{4}$. For downward-tempted consumers, temptation drags them toward cheaper, lower-quality bundles, effectively lowering their marginal willingness to pay. The monopolist must therefore reduce both quality and price relative to the upward-tempted case in order to satisfy the participation constraint.

Combining the optimal menus for each type of consumer, the monopolist's aggregate profit from both consumers is $\pi^C = \pi^C(\gamma_\text{H}) + \pi^C(\gamma_\text{L}) = \frac{1}{2} + \frac{(1+\gamma_\text{L})^2}{8}$. The first summand comes from the upward-tempted consumer, whilst the second is from the downward-tempted consumer. The monopolist earns a fixed profit of $\frac{1}{2}$ from the high-type consumer, while the profit from the low-type consumer depends on the severity of temptation (through γ_L). When γ_L is small, the consumer is strongly tempted downwards, reducing both the optimal bundle and the monopolist's profit.

Incomplete Information

I now analyze the case of incomplete information where the monopoly does not observe each individual consumer's type. He only knows there is an upward-tempted (also high-temptation type, or "H" type) consumer and a downward-tempted (also low-temptation type, or "L" type) consumer, each occurring with equal probability. The assumption of equal probabilities is without loss of generality, as the distribution does not affect the static analysis of equilibrium outcome. Recall that this means the two consumers satisfy $\gamma_\text{L} < \gamma^* = 1 < \gamma_\text{H}$.

The monopolist will in this case provide a menu

$$\{(q(\gamma_\text{H}), p(\gamma_\text{H})), (q(\gamma_\text{L}), p(\gamma_\text{L})), (0, 0)\}$$

to screen the two consumer types. As in the standard non-linear pricing model, I require that in equilibrium $0 < q(\gamma_{\text{L}}) < q(\gamma_{\text{H}})$.

In this case, the monopolist's profit maximization problem can be rewritten as follows:

$$\max_{\substack{q(\gamma_{\text{H}}),p(\gamma_{\text{H}})\\ q(\gamma_{\text{L}}),p(\gamma_{\text{L}})}} [p(\gamma_{\text{H}}) - \tfrac{1}{2}q(\gamma_{\text{H}})^2] + [p(\gamma_{\text{L}}) - \tfrac{1}{2}q(\gamma_{\text{L}})^2]$$

subject to

$$W_{\gamma_{\text{H}}} \geq 0, \qquad \text{(ex-ante IR for } H) \tag{3.10}$$

$$W_{\gamma_{\text{L}}} \geq 0, \qquad \text{(ex-ante IR for } L) \tag{3.11}$$

$$(1+\gamma_{\text{H}})q(\gamma_{\text{H}}) - 2p(\gamma_{\text{H}}) \geq 0, \qquad \text{(ex-post IR for } H) \tag{3.12}$$

$$(1+\gamma_{\text{L}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}) \geq 0, \qquad \text{(ex-post IR for } L) \tag{3.13}$$

$$(1+\gamma_{\text{H}})q(\gamma_{\text{H}}) - 2p(\gamma_{\text{H}}) \geq (1+\gamma_{\text{H}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}), \quad \text{(ex-post IC for } H) \tag{3.14}$$

$$(1+\gamma_{\text{L}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}) \geq (1+\gamma_{\text{L}})q(\gamma_{\text{H}}) - 2p(\gamma_{\text{H}}). \quad \text{(ex-post IC for } L) \tag{3.15}$$

Before I discuss the solution of this problem, I first show that ex-ante IR constraints are equivalent to non-negative commitment utility, $U \geq 0$.

Proposition 3.2.1. *Taking an arbitrary menu M then ex-ante IR is fulfilled for each $\gamma \in \{\gamma_H, \gamma_L\}$ if and only if, $U(p,q) \geq 0$ for all $(q,p) \in M$.*

Proof. I assume first that ex-ante IR is satisfied for both consumer types then if $(q(\gamma), p(\gamma))$ is a bundle such that maximizes $U + V_\gamma$ for $\gamma \in \{\gamma_{\text{H}}, \gamma_{\text{L}}\}$

$$U(q(\gamma), p(\gamma)) \geq U(q(\gamma), p(\gamma)) + V(q(\gamma), p(\gamma)) - \max_{(q,p)\in M}\{V_\gamma(q,p)\} \geq 0.$$

For the other implication; if $U(q,p) \geq 0$ for every $(q,p) \in M$ and given some type γ the following chain of inequalities holds:

$$
\begin{aligned}
W_\gamma(M) &= \max_{(q,p)\in M}\{U(q,p)+V(q,p)\} - \max_{(q',p')\in M}\{V_\gamma(q,p)\} \\
&\geq U(q',p')+V(q',p')-V(q',p') = U(q',p') \geq 0.
\end{aligned}
$$

In the above inequalities, $(q',p') \in M$ is such that $V(q',p') \geq V(q,p)$ for any $(q,p) \in M$. □

Proposition 3.2.1 implies that the two ex-ante IR constraints for both types (3.10)–(3.11) can be replaced by the simpler conditions $U(q(\gamma_{\text{H}}), p(\gamma_{\text{H}})) \geq 0$ and $U(q(\gamma_{\text{L}}), p(\gamma_{\text{L}})) \geq 0$.

Recall that $\gamma_{\text{L}} < \gamma_{\text{H}}$. Combining ex-post IR (3.13) for the downward-tempted consumer with ex-post IC (3.14) for the upward-tempted consumer yields

$$
(1+\gamma_{\text{H}})q(\gamma_{\text{H}})-2p(\gamma_{\text{H}}) \geq (1+\gamma_{\text{H}})q(\gamma_{\text{L}})-2p(\gamma_{\text{L}}) > (1+\gamma_{\text{L}})q(\gamma_{\text{L}})-2p(\gamma_{\text{L}}) \geq 0.
$$

Hence, ex-post IR (3.12) for γ_{H} never binds and can be ignored. By contrast, ex-post IR (3.13) for γ_{L} must bind, that is $(1+\gamma_{\text{L}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}) = 0$. This is because if this were not so, the monopolist could adjust the menu offered by decreasing the quality, or increasing the price, of a given option. The downward-tempted consumer prefers this option while the upward-tempted one does not, leading to an increase in profit. Similarly, ex-ante IR (3.11) for γ_{L} must be slack. Otherwise this will mean $V_{\gamma_{\text{L}}} < 0$ which would lead to self-control costs for this consumer type. Alternatively,

$$
2q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}) > (1+\gamma_{\text{L}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{L}}) \geq 0
$$

given that $\gamma_{\text{L}} < 1$.

Finally, I substitute the binding ex-post IR (3.13) for γ_{L} into the ex-post IC (3.15) for γ_{L}. This yields $0 \geq (1+\gamma_{\text{L}})q(\gamma_{\text{H}}) - 2p(\gamma_{\text{H}})$, which means that the downward-tempted consumer does not gain from behaving as an upward-tempted consumer. As a consequence, constraint (3.15) must not bind. Either ex-ante IR (3.10) or ex-post IC (3.14) for γ_{H} must bind however. The upwards-tempted consumer cannot obtain simultaneously extra commitment utility or ex-post utility by choosing the lower quality offer if the monopolist is maximizing profits.

From the above arguments it follows that conditions (3.10)-(3.15) can be reduced to the following constraints:

$$q(\gamma_{\rm H}) - p(\gamma_{\rm H}) \geq 0, \tag{3.16}$$
$$(1+\gamma_{\rm L})q(\gamma_{\rm L}) - 2p(\gamma_{\rm L}) = 0, \tag{3.17}$$
$$(1+\gamma_{\rm H})q(\gamma_{\rm H}) - 2p(\gamma_{\rm H}) \geq (1+\gamma_{\rm H})q(\gamma_{\rm L}) - 2p(\gamma_{\rm L}). \tag{3.18}$$

Rearranging the binding constraint (3.17), I have

$$2p(\gamma_{\rm L}) = (1+\gamma_{\rm L})q(\gamma_{\rm L}). \tag{3.19}$$

Substituting into (3.18), the IC constraint (3.18) becomes

$$(1+\gamma_{\rm H})q(\gamma_{\rm H}) - (\gamma_{\rm H} - \gamma_{\rm L})q(\gamma_{\rm L}) - 2p(\gamma_{\rm H}) \geq 0. \tag{3.20}$$

The monopolist's problem is then simplified to choosing a pair of qualities $(q(\gamma_{\rm H}), p(\gamma_{\rm H}), q(\gamma_{\rm L}))$ to maximize the profit function

$$\max_{q(\gamma_{\rm H}),p(\gamma_{\rm H}),q(\gamma_{\rm L})} \big[2p(\gamma_{\rm H}) - q(\gamma_{\rm H})^2\big] + \big[(1+\gamma_{\rm L})q(\gamma_{\rm L}) - q(\gamma_{\rm L})^2\big],$$

subject to (3.16) and (3.18). Denoting $\lambda \geq 0$ and $\mu \geq 0$ as the Kuhn–Tucker multipliers for the inequality conditions, the Lagrangian function is

$$\begin{aligned}\mathcal{L}(q(\gamma_{\rm H}), p(\gamma_{\rm H}), q(\gamma_{\rm L}), \lambda, \mu) = {} & \big[2p(\gamma_{\rm H}) - q(\gamma_{\rm H})^2\big] + \big[(1+\gamma_{\rm L})q(\gamma_{\rm L}) - q(\gamma_{\rm L})^2\big] \\ & + 2\lambda\big[q(\gamma_{\rm H}) - p(\gamma_{\rm H})\big] \\ & + \mu\big[(1+\gamma_{\rm H})q(\gamma_{\rm H}) - (\gamma_{\rm H} - \gamma_{\rm L})q(\gamma_{\rm L}) - 2p(\gamma_{\rm H})\big].\end{aligned}$$

Differentiating with respect to $q(\gamma_{\rm H})$, $q(\gamma_{\rm L})$ and $p(\gamma_{\rm H})$ respectively yields the first-order conditions

$$-2q(\gamma_{\text{H}}) + 2\lambda + \mu(1+\gamma_{\text{H}}) = 0, \tag{3.21}$$
$$(1+\gamma_{\text{L}}) - 2q(\gamma_{\text{L}}) - \mu(\gamma_{\text{H}} - \gamma_{\text{L}}) = 0, \tag{3.22}$$
$$1 - \lambda - \mu = 0. \tag{3.23}$$

These are complemented with the two complementary slackness conditions

$$\lambda \cdot \left[q(\gamma_{\text{H}}) - p(\gamma_{\text{H}})\right] = 0, \tag{3.24}$$
$$\mu \cdot \left[(1+\gamma_{\text{H}})q(\gamma_{\text{H}}) - (\gamma_{\text{H}} - \gamma_{\text{L}})q(\gamma_{\text{L}}) - 2p(\gamma_{\text{H}})\right] = 0. \tag{3.25}$$

From condition (3.23), I have $\lambda = 1 - \mu$. Substituting into conditions (3.21) and (3.22), $q(\gamma_{\text{H}})$ and $q(\gamma_{\text{L}})$ can be expressed as

$$q(\gamma_{\text{H}}) = 1 + \frac{\mu(\gamma_{\text{H}} - 1)}{2},$$

$$q(\gamma_{\text{L}}) = \frac{1 + \gamma_{\text{L}} - \mu(\gamma_{\text{H}} - \gamma_{\text{L}})}{2}.$$

The social welfare W^B is defined as the sum of consumers' ex-ante utilities and the monopolist's profit. Compared to the case where temptation is absent, ex-ante utility is the difference between commitment utility and self-control costs. Aggregate social welfare is therefore the sum of expected value of $U(\gamma) + \pi(\gamma) = p(\gamma) - C(q(\gamma))$ net of any self-control costs. In this two-type case, neither of the consumers suffers from self-control problems. On one hand, the ex-ante utility of downward-tempted consumer is always zero, since the ex-post utility always binds and the maximal temptation is zero. On the other hand, the monopolist ensures that the self-control costs of the upward-tempted consumer are zero, thereby extracting the entirety of her ex-ante surplus. Consequently, aggregate social welfare then reduces to the expected value of $q(\gamma) - C(q(\gamma))$.

Proposition 3.2.2. *Defining the following two bounds* $\gamma_H^- = \frac{1+3\gamma_L+\sqrt{9-2\gamma_L-7\gamma_L^2}}{4}$

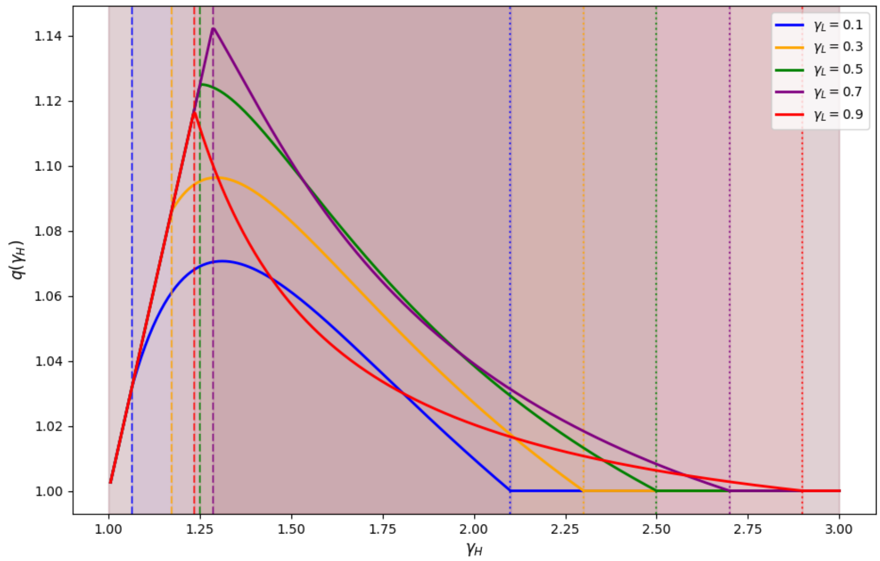

Figure 3.1: Optimal $q(\gamma_\text{H})$ for different γ_L

and $\gamma_H^+ = \gamma_L + 2$, *the optimal bundle for the upwards tempted consumer is* [1]

$$q(\gamma_H) = \begin{cases} \frac{1+\gamma_H}{2}, & \textit{if} \quad 1 < \gamma_H \leq \gamma_H^- \\ 1 + \frac{(\gamma_L-1)(\gamma_H-1)(\gamma_H-\gamma_L-2)}{2(\gamma_H-1)^2+2(\gamma_H-\gamma_L)^2}, & \textit{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ 1, & \textit{if} \quad \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.26}$$

and

$$p(\gamma_H) = \begin{cases} \frac{(1+\gamma_H)^2-(\gamma_H-\gamma_L)(1+2\gamma_L-\gamma_H)}{4}, & \textit{if } 1 < \gamma_H \leq \gamma_H^- \\ 1 + \frac{(\gamma_L-1)(\gamma_H-1)(\gamma_H-\gamma_L-2)}{2(\gamma_H-1)^2+2(\gamma_H-\gamma_L)^2}, & \textit{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ 1. & \textit{if } \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.27}$$

Whereas for the downward-tempted consumer γ_L,

[1] For expositional clarity, the functions are denoted as depending solely on γ_H, for example $q(\gamma_\text{H})$ and $p(\gamma_\text{H})$. Formally, these objects are functions of both γ_H and γ_L. In the analysis that follows, however, γ_L is treated as a fixed parameter, and attention is directed to the comparative statics with respect to γ_H. Similarly, one may treat γ_H as fixed and examine the variation with respect to γ_L.

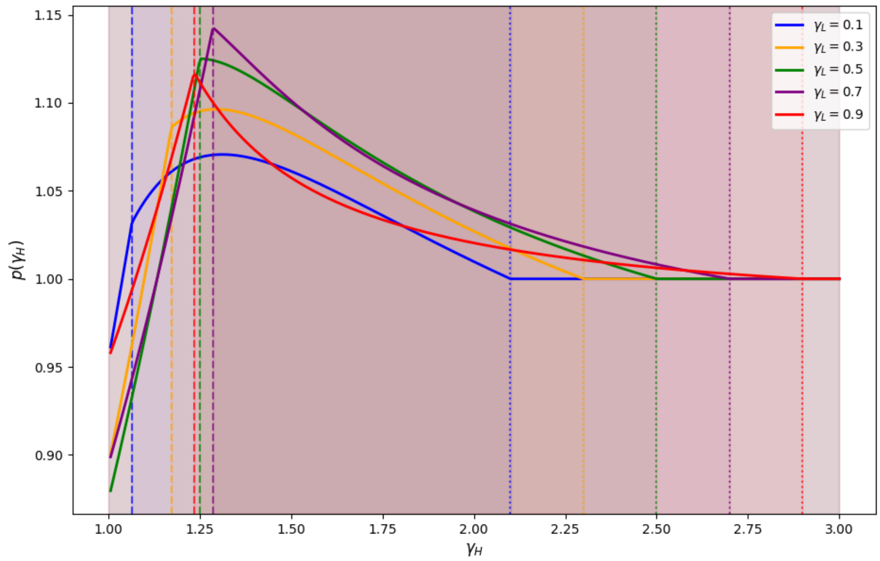

Figure 3.2: Optimal $p(\gamma_{\text{H}})$ for different γ_{L}

$$q(\gamma_L) = \begin{cases} \frac{1+2\gamma_L-\gamma_H}{2}, & \textit{if } 1 < \gamma_H \leq \gamma_H^- \\ \frac{1+\gamma_L}{2} + \frac{(1-\gamma_L)(\gamma_H-\gamma_L)(\gamma_H-\gamma_L-2)}{2(\gamma_H-1)^2+2(\gamma_H-\gamma_L)^2}, & \textit{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ \frac{1+\gamma_L}{2}, & \textit{if } \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.28}$$

and

$$p(\gamma_L) = \begin{cases} \frac{(1+\gamma_L)^2-(1+\gamma_L)(\gamma_H-\gamma_L)}{4}, & \textit{if } 1 < \gamma_H \leq \gamma_H^- \\ \frac{(1+\gamma_L)^2}{4} + \frac{(1-\gamma_L^2)(\gamma_H-\gamma_L)(\gamma_H-\gamma_L-2)}{4(\gamma_H-1)^2+4(\gamma_H-\gamma_L)^2}, & \textit{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ \frac{(1+\gamma_L)^2}{4}, & \textit{if } \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.29}$$

Figures 3.1 and 3.2 illustrate the dependence of $q(\gamma_{\text{H}})$ and $p(\gamma_{\text{H}})$ on γ_{H}, respectively, for a given value of γ_{L}. Figure 3.1 highlights the regime transitions, ranging from a linear quality (resp. pricing) schedule for low values of γ_{H} to a constant schedule. For small values of γ_{H}, the monopolist offers higher qualities and prices than under complete information. This is because IC becomes the only binding constraint and the upward tempted consumer

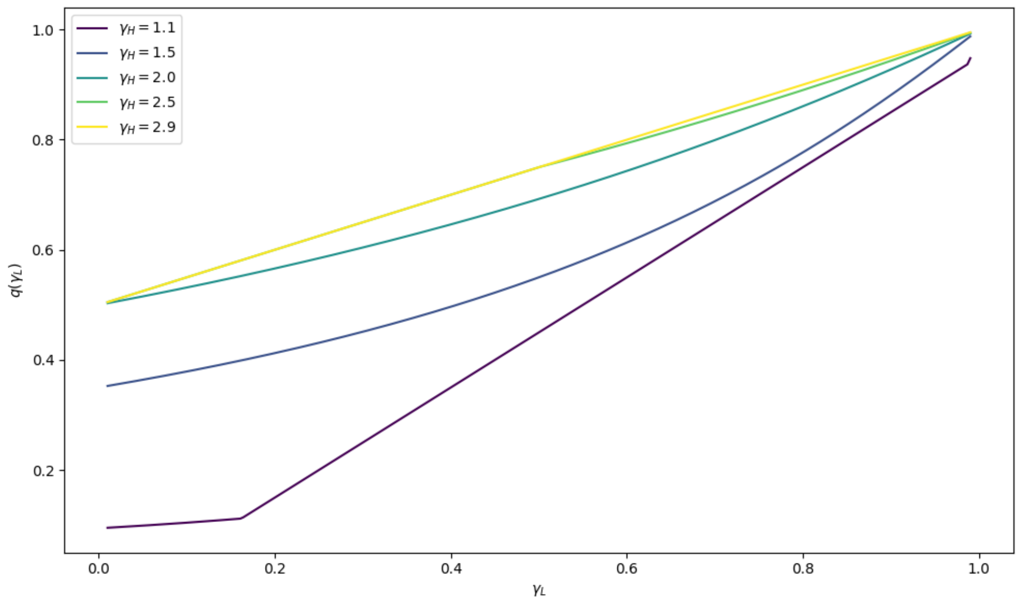

Figure 3.3: Optimal $q(\gamma_L)$ for different γ_H

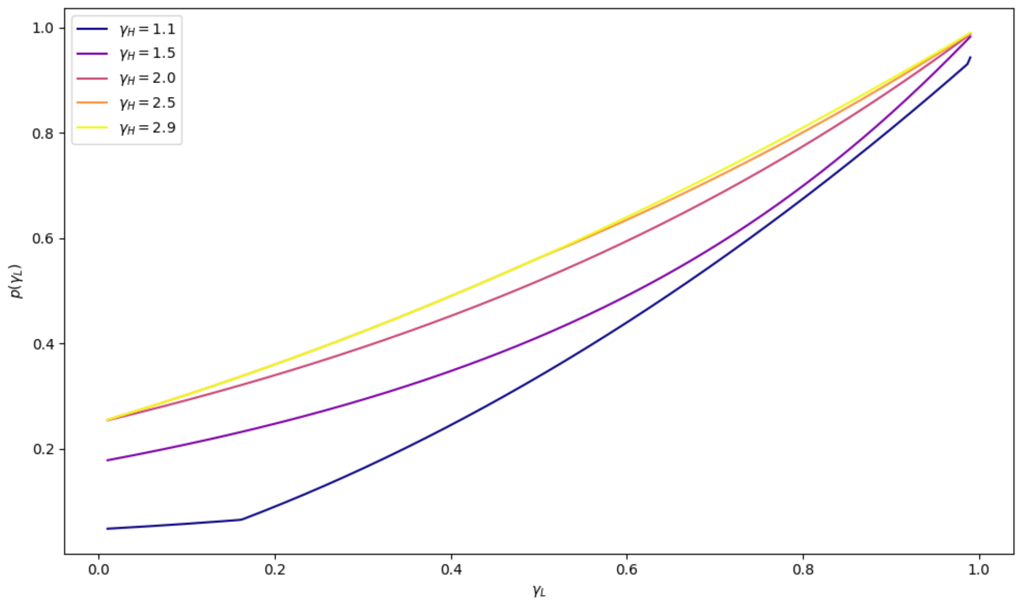

Figure 3.4: Optimal $p(\gamma_L)$ for different γ_H

receives some ex-ante surplus. As γ_H increases ex-ante IR binds as well and the monopolist obtains the highest possible profit in that case. In this regime, the monopolist can continue offering bundles above the perfect information one. By contrast, for sufficiently large values of γ_H, the bundle converges to the one that would be offered under complete information. This is because when the upwards tempted consumer suffers from high temptation ex-ante IR becomes the binding constraint while ex-post IC becomes slack. Due to the possibility of the upward tempted consumer exerting self-control and not entering the store at all, the monopolist cannot attain the same profit he would if the consumer did not have behavioral preferences.

Further to the above details, note that as γ_L raises to 1 the γ_H^+ increases monotonically to 3, its highest value. The behavior of γ_H^- is more complex as it first raises and then decreases. Overall, the region where both constraints bind (i.e. the interval (γ_H^-, γ_H^+)) becomes larger with higher γ_L. Observe that as shown in figures 3.1 and 3.2 the curvature of of both p and q over this interval also changes, from convex to concave as γ_L increases. Moreover for fixed γ_H increases in γ_L result in price reductions when γ_H is low but lead to price increases when γ_H becomes larger, as can be seen from figure 3.2. This is because for low values, increases in γ_L make the consumer types closer, which makes it more difficult for the monopolist to discriminate between consumers.

Figures 3.3 and 3.4 illustrate the dependence of $q(\gamma_L)$ and $p(\gamma_L)$ on γ_L, respectively, for a given value of γ_H. Both figures indicate that this dependence is monotonic: higher values of γ_H induce larger values of quality and price. As γ_L approaches one, the bundle converges to $(1, 1)$, which corresponds to the optimal allocation when the consumer faces no self-control costs. Note further that fixing γ_L and examining how $q(\gamma_L)$ and $p(\gamma_L)$ change with γ_H, figures 3.3 and 3.4 show that increasing γ_H results in higher quality price bundles offered to the downward tempted. This is because in such circumstances the monopolist can offer a much higher quality price bundle to the upward tempted consumer while maintain ex-post IC (and ex-ante IR) and can therefore obtain a higher surplus from the downward tempted consumer.

Proposition 3.2.3. *When there are only two consumer types in the popu-*

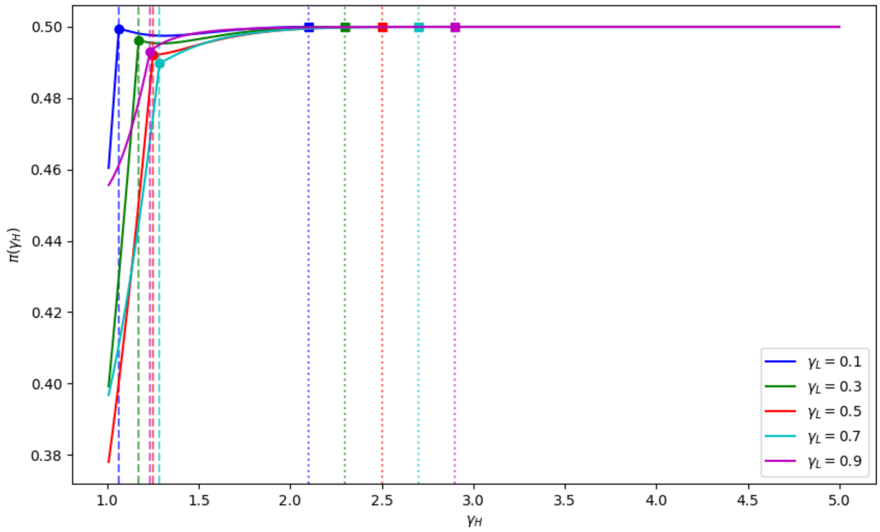

Figure 3.5: Optimal $\pi(\gamma_{\mathrm{H}})$ for different γ_{L}

lation $0 < \gamma_L < 1 < \gamma_H$, with each type being equally like. The monopolist makes profits:

$$\pi(\gamma_H, \gamma_L) = \begin{cases} \frac{1}{2} + \frac{(1+\gamma_L)^2}{8} + \frac{2(\gamma_H-\gamma_L)^2+\gamma_L^2+2\gamma_L-3}{8}, & \text{if } 1 < \gamma_H \leq \gamma_H^- \\ \frac{1}{2} + \frac{(1+\gamma_L)^2}{8} - \frac{(\gamma_L-1)^2(\gamma_H-\gamma_L-2)^2}{8[(\gamma_H-1)^2+(\gamma_H-\gamma_L)^2]}, & \text{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ \frac{1}{2} + \frac{(1+\gamma_L)^2}{8}. & \text{if } \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.30}$$

In particular, the profit from upwards temptation is

$$\pi(\gamma_H) = \begin{cases} \frac{(1+\gamma_H)^2-2(\gamma_H-\gamma_L)(1+2\gamma_L-\gamma_H)}{8}, & \text{if } 1 < \gamma_H \leq \gamma_H^- \\ \frac{1}{2} - \frac{(\gamma_L-1)^2(\gamma_H-\gamma_L-2)^2(\gamma_H-1)^2}{8[(\gamma_H-1)^2+(\gamma_H-\gamma_L)^2]^2}, & \text{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ \frac{1}{2}, & \text{if } \gamma_H \geq \gamma_H^+ \end{cases} \tag{3.31}$$

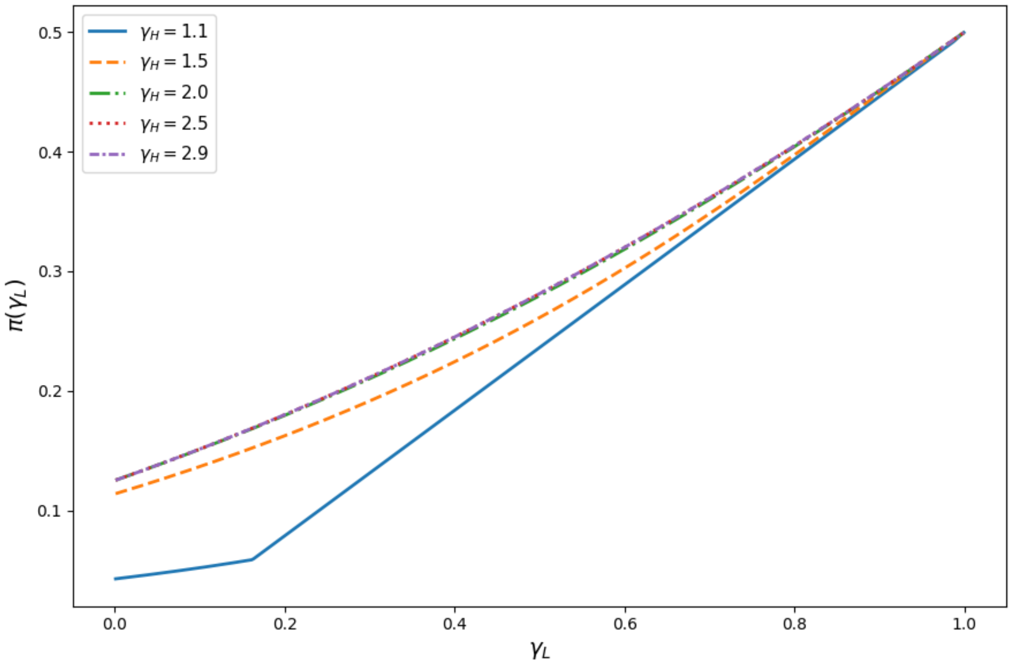

Figure 3.6: Optimal $\pi(\gamma_{\mathrm{L}})$ for different γ_{H}

and the profit from downwards temptation is

$$\pi(\gamma_{\mathrm{L}}) = \begin{cases} \frac{(1+\gamma_{\mathrm{L}})^2-(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})^2}{8}, & \text{if } 1 < \gamma_{\mathrm{H}} \leq \gamma_{\mathrm{H}}^- \\ \frac{(1+\gamma_{\mathrm{L}})^2}{8} - \frac{(\gamma_{\mathrm{L}}-1)^2(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}}-2)^2(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})^2}{8[(\gamma_{\mathrm{H}}-1)^2+(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})^2]^2}, & \text{if } \gamma_{\mathrm{H}} \in (\gamma_{\mathrm{H}}^-, \gamma_{\mathrm{H}}^+) \\ \frac{(1+\gamma_{\mathrm{L}})^2}{8}, & \text{if } \gamma_{\mathrm{H}} \geq \gamma_{\mathrm{H}}^+ \end{cases} \quad (3.32)$$

Figures 3.5 and 3.6 illustrate the profit gains from the upwards and tempted consumer respectively. Note that profits obtained from the upward tempted consumer can raise to at most $\frac{1}{2}$, which corresponds profits under perfect information, where ex-ante IR binds. Note that as for $p(\gamma_{\mathrm{H}})$ and $q(\gamma_{\mathrm{H}})$ increasing γ_{L} leads to changes in curvature in the intermediate region $\gamma_{\mathrm{H}}^- < \gamma_{\mathrm{H}} < \gamma_{\mathrm{H}}^+$. Moreover, for fixed γ_{H} increases in γ_{L} result in lower profits when γ_{L} is low but higher ones when γ_{H} is high, but below γ_{H}^+. On the other hand, profits for γ_{L} are monotonically increasing and the raise to $\frac{1}{2}$, the maximum profit when the consumer faces no self-control costs. As for $p(\gamma_{\mathrm{L}})$ and $q(\gamma_{\mathrm{L}})$ increasing γ_{H}

for fixed γ_L leads to higher monopoly profits, this is because the monopolist can improve the quality and increase the price for the downward tempted consumer without violating ex-post IC.

In addition, the ex-post consumer surplus is

$$w(\gamma_H, \gamma_L) = \begin{cases} \frac{(1+\gamma_H)(2+\gamma_L-\gamma_H)}{4}, & \text{if } 1 < \gamma_H \leq \gamma_H^- \\ \frac{1}{2} + \frac{(1+\gamma_L)^2}{8} - \frac{(\gamma_L-1)^2(\gamma_H-\gamma_L-2)^2}{8[(\gamma_H-1)^2+(\gamma_H-\gamma_L)^2]}, & \text{if } \gamma_H \in (\gamma_H^-, \gamma_H^+) \\ \frac{1}{2} + \frac{(1+\gamma_L)^2}{8}, & \text{if } \gamma_H \geq \gamma_H^+ \end{cases} \quad (3.33)$$

Comparison: With vs. Without Temptation

Consider the standard maximization problem without temptation, ex-ante utility is no different from ex-post utility in this context. I can then think of the temptation type γ as indexing each consumer by her willingness to pay. As in the case with temptation, the seller provides a menu

$$\{(q(\gamma_H), p(\gamma_H)), (q(\gamma_L), p(\gamma_L)), (0, 0)\}$$

to screen each consumer. The monopolist's profit maximization problem is however:

$$\max_{\substack{q(\gamma_H),p(\gamma_H)\\ q(\gamma_L),p(\gamma_L)}} \left[p(\gamma_H) - \tfrac{1}{2}q(\gamma_H)^2\right] + \left[p(\gamma_L) - \tfrac{1}{2}q(\gamma_L)^2\right]$$

subject to

$$(1 + \gamma_H)q(\gamma_H) - 2p(\gamma_H) \geq 0, \quad (3.34)$$

$$(1 + \gamma_L)q(\gamma_L) - 2p(\gamma_L) \geq 0, \quad (3.35)$$

$$(1 + \gamma_H)q(\gamma_H) - 2p(\gamma_H) \geq (1 + \gamma_H)q(\gamma_L) - 2p(\gamma_L), \quad (3.36)$$

$$(1 + \gamma_L)q(\gamma_L) - 2p(\gamma_L) \geq (1 + \gamma_L)q(\gamma_H) - 2p(\gamma_H). \quad (3.37)$$

As in the case where self-control is present, the IR must bind for the downward-tempted consumer while the IC for the upward-tempted consumer implies the IR for this type. Finally, the IC for the downward-tempted consumer always holds.

Note that if the IR bound for the upward-tempted consumer, both upward-tempted and downward-tempted types will behave in the same way. This contradicts the assumption that consumers are separated by their types.

Also note that since the IR binds for the downwards temptation, the IC condition can be written as $(1+\gamma_{\rm H})q(\gamma_{\rm H}) - 2p(\gamma_{\rm H}) \geq (1+\gamma_{\rm H})q(\gamma_{\rm L}) - 2p(\gamma_{\rm L}) = (\gamma_{\rm H} - \gamma_{\rm L})q(\gamma_{\rm L}) \geq 0$, where the last equality holds only when $q(\gamma_{\rm L}) = p(\gamma_{\rm L}) = 0$. If the IC for the upward-tempted consumer binds, so that $(1+\gamma_{\rm H})q(\gamma_{\rm H}) - 2p(\gamma_{\rm H}) = (\gamma_{\rm H} - \gamma_{\rm L})q(\gamma_{\rm L})$, the monopolist's problem is then simplified to choosing a pair of qualities $(q(\gamma_{\rm H}), q(\gamma_{\rm L}))$ to maximize the profit function

$$2\pi = [(1+\gamma_{\rm H})q(\gamma_{\rm H}) + (\gamma_{\rm L} - \gamma_{\rm H})q(\gamma_{\rm L}) - q(\gamma_{\rm H})^2] + [(1+\gamma_{\rm L})q(\gamma_{\rm L}) - q(\gamma_{\rm L})^2].$$

The first-order conditions imply in this case that

$$q(\gamma_{\rm H}) = \frac{1+\gamma_{\rm H}}{2}, \qquad q(\gamma_{\rm L}) = \frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2}.$$

The bounds $1 < \gamma_{\rm H} < 2\gamma_{\rm L} + 1$ and $0 < \gamma_{\rm L} < 1$ guarantee strictly positive quality is provided to both consumers. In this case, the prices offered to each consumer type are

$$p(\gamma_{\rm H}) = \frac{(1+\gamma_{\rm H})^2 + (\gamma_{\rm L}-\gamma_{\rm H})(1+2\gamma_{\rm L}-\gamma_{\rm H})}{4}, \qquad p(\gamma_{\rm L}) = \frac{(1+\gamma_{\rm L})^2 - (1+\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L})}{4}.$$

On the other hand, when $\gamma_{\rm H} \geq 2\gamma_{\rm L} + 1$, the menu always includes $(0, 0)$ for the downwards temptation. In this case,

$$q(\gamma_{\rm L}) = \begin{cases} 0, & \text{if } 0 < \gamma_{\rm L} \leq \frac{\gamma_{\rm H}-1}{2}, \\ \frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2}, & \text{if } \gamma_{\rm L} > \frac{\gamma_{\rm H}-1}{2}, \end{cases} \tag{3.38}$$

and

$$p(\gamma_{\mathrm{L}}) = \begin{cases} 0, & \text{if } 0 < \gamma_{\mathrm{L}} \leq \frac{\gamma_{\mathrm{H}}-1}{2}, \\ \frac{(1+\gamma_{\mathrm{L}})^2-(1+\gamma_{\mathrm{L}})(\gamma_{\mathrm{H}}-\gamma_{\mathrm{L}})}{4}, & \text{if } \gamma_{\mathrm{L}} > \frac{\gamma_{\mathrm{H}}-1}{2}. \end{cases} \tag{3.39}$$

In summary, in the absence of temptation, the monopolist's problem reduces to a standard screening model where the qualities and prices are determined solely by willingness to pay. By contrast, when temptation is present, the allocation differs because the monopolist must additionally account for self-control costs, leading to distortions in both prices and qualities relative to the non-behavioral model.

Taxation Policy

I now extend the model to incorporate taxation. Two common types of taxes are considered: (i) an *ad valorem tax* τ, levied as a proportion of the unit price; and (ii) a *specific tax* s, levied as a fixed amount per unit of output. Both instruments may be negative, in which case they represent subsidies (i.e. $\tau < 0$ is an ad valorem subsidy and $s < 0$ is a per–unit subsidy). For economic relevance and to maintain well-behaved allocations, I assume $\tau \in (-1, 1)$ and $s \in \mathbb{R}$.[2]

When tax policies are embedded with specific tax rate s and ad valorem tax rate τ under incomplete information; the firm problem can be re-written as the following optimization problem:

$$\max_{\substack{q^T(\gamma_{\mathrm{H}}),p^T(\gamma_{\mathrm{H}})\\ q^T(\gamma_{\mathrm{L}}),p^T(\gamma_{\mathrm{L}})}} \frac{1}{2}\cdot\left[p^T(\gamma_{\mathrm{H}})(1-\tau)-\tfrac{1}{2}q^T(\gamma_{\mathrm{H}})^2-s\right]+\frac{1}{2}\cdot\left[p^T(\gamma_{\mathrm{L}})(1-\tau)-\tfrac{1}{2}q^T(\gamma_{\mathrm{L}})^2-s\right],$$

subject to (3.10)–(3.15).

For convenience, I multiply the objective by 4 and rearrange terms, obtaining the following scaled version of the problem:

$$\max_{q^T(\gamma_{\mathrm{H}}),p^T(\gamma_{\mathrm{H}}),q^T(\gamma_{\mathrm{L}})} \left[2(1-\tau)p^T(\gamma_{\mathrm{H}})-q^T(\gamma_{\mathrm{H}})^2\right]+\left[(1-\tau)(1+\gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}})-q^T(\gamma_{\mathrm{L}})^2\right]-4s,$$

[2]The bound $\tau > -1$ ensures $1-\tau > 0$; if $\tau \geq 1$, per-unit revenue becomes nonpositive.

subject to (3.16) and (3.18), which are the rescaled forms of (3.10) and (3.15).

Denoting $\lambda^T \geq 0$ and $\mu^T \geq 0$ by the Kuhn-Tucker multipliers for the inequality constraints, the Lagrangian function is

$$\begin{aligned}\mathcal{L}(q^T(\gamma_{\mathrm{H}}), p^T(\gamma_{\mathrm{H}}), q^T(\gamma_{\mathrm{L}}), \lambda^T, \mu^T) = &\Big[2(1-\tau)p^T(\gamma_{\mathrm{H}}) - q^T(\gamma_{\mathrm{H}})^2\Big] + \Big[(1-\tau)(1+\gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}}) - q^T(\gamma_{\mathrm{L}})^2\Big] \\ &- 4s + 2\lambda^T\Big[q^T(\gamma_{\mathrm{H}}) - p^T(\gamma_{\mathrm{H}})\Big] \\ &+ \mu^T\Big[(1+\gamma_{\mathrm{H}})q^T(\gamma_{\mathrm{H}}) - (\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}}) - 2p^T(\gamma_{\mathrm{H}})\Big].\end{aligned}$$

Following the same method as in the previous section, it can be shown that the ad valorem tax rate τ proportionally reduces equilibrium prices and qualities for each type. Aggregate profit is affected by both the ad valorem tax rate τ and the specific tax rate s.

Proposition 3.2.4. *Under taxation, equilibrium allocations scale as*

$$q^T(\gamma_i) = (1-\tau)q(\gamma_i), \qquad p^T(\gamma_i) = (1-\tau)p(\gamma_i), \qquad i \in \{H, L\}.$$

Aggregate profit satisfies

$$\pi^T = (1-\tau)^2\pi - 2s.$$

The two tax instruments exert distinct effects on monopolist behavior. The ad valorem tax introduces a proportional burden that scales with price and, as a result, quality. Consequently, the effects if ad valorem taxes are more pronounced when high–quality or high–price bundles are considered. In contrast, the specific tax operates as an additive cost shift. When negative, both instruments act as subsidies: an ad valorem subsidy proportionally enhances incentives to increase prices and qualities, while a specific subsidy raises profitability uniformly across all bundles being produced.

Effect of the tax policy on national welfare with self-control cost

This section examines how introducing small ad valorem and specific taxes (or subsidies) affect national welfare, starting from an initial situation of free trade. The benchmark is therefore $\tau = 0$ and $s = 0$. Government revenue is assumed to be rebated to consumers in a lump–sum manner. Quasi-linearity ensures that these lump–sum transfers do not affect demand. National welfare is therefore the sum of consumer commitment utilities $q(\gamma) - p(\gamma)$, government tax revenue $\tau p^T(\gamma) + s$ and monopolist's profits $q^T(\gamma) - \tau p^T(\gamma) - C(q^T(\gamma)) - s$. Formally, national welfare W^B is

$$W^B = q^T(\gamma_{\rm H}) - \frac{1}{2}q^T(\gamma_{\rm H})^2 + q^T(\gamma_{\rm L}) - \frac{1}{2}q^T(\gamma_{\rm L})^2.$$

Here,

$$q^T(\gamma_{\rm H}) = \begin{cases} (1-\tau)\frac{1+\gamma_{\rm H}}{2}, & \text{if } 1 < \gamma_{\rm H} \leq \gamma_{\rm H}^- \\ (1-\tau) + (1-\tau)\frac{(\gamma_{\rm L}-1)(\gamma_{\rm H}-1)(\gamma_{\rm H}-\gamma_{\rm L}-2)}{2(\gamma_{\rm H}-1)^2+2(\gamma_{\rm H}-\gamma_{\rm L})^2}, & \text{if } \gamma_{\rm H} \in (\gamma_{\rm H}^-, \gamma_{\rm H}^+) \\ 1-\tau, & \text{if } \gamma_{\rm H} \geq \gamma_{\rm H}^+ \end{cases} \tag{3.40}$$

$$q^T(\gamma_{\rm L}) = \begin{cases} (1-\tau)\frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2}, & \text{if } 1 < \gamma_{\rm H} \leq \gamma_{\rm H}^- \\ (1-\tau)\frac{1+\gamma_{\rm L}}{2} + (1-\tau)\frac{(1-\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L}-2)}{2(\gamma_{\rm H}-1)^2+2(\gamma_{\rm H}-\gamma_{\rm L})^2}, & \text{if } \gamma_{\rm H} \in (\gamma_{\rm H}^-, \gamma_{\rm H}^+) \\ (1-\tau)\frac{1+\gamma_{\rm L}}{2}, & \text{if } \gamma_{\rm H} \geq \gamma_{\rm H}^+ \end{cases} \tag{3.41}$$

National welfare can be regarded as a piece-wise continuous function of relative magnitudes of $\gamma_{\rm H}$ and $\gamma_{\rm L}$:

$$
W^B(\gamma_{\rm H},\gamma_{\rm L}) = \begin{cases} (1-\tau)(1+\gamma_{\rm L}) \\ \quad -\dfrac{(1-\tau)^2}{4}\big[(\gamma_{\rm H}-\gamma_{\rm L})^2+(1+\gamma_{\rm L})^2\big], & \text{if } 1<\gamma_{\rm H}\le\gamma_{\rm H}^- \\ (1-\tau)\left(1+\dfrac{1+\gamma_{\rm L}}{2}+\Phi+\Psi\right) \\ \quad -\dfrac{(1-\tau)^2}{2}\left[(1+\Phi)^2+\left(\dfrac{1+\gamma_{\rm L}}{2}+\Psi\right)^2\right], & \text{if } \gamma_{\rm H}\in(\gamma_{\rm H}^-,\gamma_{\rm H}^+) \\ (1-\tau)\left(1+\dfrac{1+\gamma_{\rm L}}{2}\right) \\ \quad -\dfrac{(1-\tau)^2}{8}\left[4+(1+\gamma_{\rm L})^2\right], & \text{if } \gamma_{\rm H}\ge\gamma_{\rm H}^+ \end{cases} \tag{3.42}
$$

where

$$
\Phi = \frac{(\gamma_{\rm L}-1)(\gamma_{\rm H}-1)(\gamma_{\rm H}-\gamma_{\rm L}-2)}{2(\gamma_{\rm H}-1)^2+2(\gamma_{\rm H}-\gamma_{\rm L})^2},
$$
$$
\Psi = \frac{(1-\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L}-2)}{2(\gamma_{\rm H}-1)^2+2(\gamma_{\rm H}-\gamma_{\rm L})^2}.
$$

The marginal welfare change due to the imposition of an ad valorem tax can be found by differentiating

$$
q^T(\gamma_{\rm H})-\frac{1}{2}q^T(\gamma_{\rm H})^2+q^T(\gamma_{\rm L})-\frac{1}{2}q^T(\gamma_{\rm L})^2
$$

with respect to τ. Let

$$
S_1 \equiv q(\gamma_{\rm H})+q(\gamma_{\rm L}), \qquad S_2 \equiv q(\gamma_{\rm H})^2+q(\gamma_{\rm L})^2.
$$

Then

$$
\frac{\partial W}{\partial\tau} = -S_1+(1-\tau)\,S_2, \qquad \frac{\partial^2 W}{\partial\tau^2} = -\,S_2\ <\ 0;
$$

where $S_2 < 0$ since at least one consumer type chooses a bundle with a positive quality in equilibrium. $W(\tau)$ is therefore strictly concave in τ for fixed values of γ_{L} and γ_{H}. The first of this derivatives is the *welfare gradient*, the marginal change in welfare due to a change in taxation policy.

Observe that at free trade ($\tau = 0$), the welfare gradient is $S_2 - S_1$. Because both qualities provided by the monopolist never exceed one, it follows that $q(\gamma)^2 \leq q(\gamma)$ for each type, which implies $S_2 \leq S_1$. Hence,

$$\left.\frac{\partial W^B}{\partial \tau}\right|_{\tau=0} = -S_1 + S_2 \leq \ 0,$$

meaning that a small positive tax rate reduces welfare while a small subsidy (a negative tax) increases it.

From the previous relations it follows that the welfare–maximizing ad valorem rate is

$$\tau^\star \ = \ 1 - \frac{S_1}{S_2}.$$

Consider now the optimal ad valorem tax for different ranges of γ_{H} and γ_{L}. When $1 < \gamma_{\text{H}} \leq \gamma_{\text{H}}^-$,

$$\tau^\star = 1 - \frac{2(1+\gamma_{\text{L}})}{(\gamma_{\text{H}} - \gamma_{\text{L}})^2 + (1+\gamma_{\text{L}})^2}.$$

On the other hand, When $\gamma_{\text{H}} \in (\gamma_{\text{H}}^-, \gamma_{\text{H}}^+)$,

$$\tau^\star = 1 - \frac{1 + \frac{1+\gamma_{\text{L}}}{2} + \Phi + \Psi}{(1+\Phi)^2 + \left(\frac{1+\gamma_{\text{L}}}{2} + \Psi\right)^2},$$

where recall that

$$\Phi = \frac{(\gamma_{\text{L}} - 1)(\gamma_{\text{H}} - 1)(\gamma_{\text{H}} - \gamma_{\text{L}} - 2)}{2\big[(\gamma_{\text{H}} - 1)^2 + (\gamma_{\text{H}} - \gamma_{\text{L}})^2\big]},$$

$$\Psi = \frac{(1 - \gamma_{\text{L}})(\gamma_{\text{H}} - \gamma_{\text{L}})(\gamma_{\text{H}} - \gamma_{\text{L}} - 2)}{2\big[(\gamma_{\text{H}} - 1)^2 + (\gamma_{\text{H}} - \gamma_{\text{L}})^2\big]}.$$

Finally, When $\gamma_{\text{H}} \geq \gamma_{\text{H}}^{+}$,

$$\tau^\star = 1 - \frac{\frac{3+\gamma_{\text{L}}}{2}}{1 + \frac{(1+\gamma_{\text{L}})^2}{4}}.$$

The case of specific taxation differs fundamentally. National welfare is independent of the specific tax rate s. This is because a specific tax or subsidy merely transfers resources between firms and the government but does not affect equilibrium allocations. Hence,

$$\frac{\partial W^B}{\partial s} = 0, \qquad \frac{\partial^2 W^B}{\partial s^2} = 0.$$

Meaning specific taxation has no effect on national welfare.

3.2.2 *Continuous-Type Case*

If the population of consumers contains only consumers with upward temptation (so that $a > 1$) or contains only consumers with downward temptation (so that $b < 1$) then it is easy to characterize the optimal menus:

Proposition 3.2.5. *If $a > 1$, let $x^\star$ be a bundle that maximizes profits $\pi(x)$ subject to the constraint $U(x) = 0$ then the menu $M = \{x^\star, (0,0)\}$ and the allocation $x(\gamma) = x^\star$ for every $\gamma \in [a,b]$ form an optimal schedule for the monopolist. This schedule fully extracts each consumer's entire ex-ante surplus.*

Proof. Let $x^\star = (p^\star, q^\star)$ be the bundle that maximises $q - \frac{1}{2}q^2$, and $\gamma \in [a,b]$. Since $\gamma > 1$, the agent is tempted upwards and $V_\gamma(x) \geq U(x) + V_\gamma(x) \geq U(x)$. In particular, $V_\gamma(x^\star) \geq U(x^\star) + V_\gamma(x^\star) \geq U(x^\star)$. This implies that the menu $M = \{x^\star, (0,0)\}$ and allocation satisfy the constraints.

Let M' be a menu and $y : [a,b] \mapsto M'$ be an allocation, both of which satisfy the constraints. By proposition 3.2.1 ex-ante IR is equivalent to the constraint $U(x) \geq 0$ for every bundle $x \in M'$, the constraint $U(y(\gamma)) \geq 0$

must hold for every γ. But for $\gamma \in [a, b]$ and $(p_y(\gamma), q_y(\gamma)) = y(\gamma)$

$$p_y(\gamma) - \frac{1}{2}q_y(\gamma)^2 \leq p^\star - \frac{1}{2}(q^\star)^2.$$

If I integrate over the whole range,

$$\int_a^b p_y(\gamma) - \frac{1}{2}q_y(\gamma)^2 dF(\gamma) \leq p^\star - \frac{1}{2}(q^\star)^2.$$

Hence, the menu $M = \{x^\star, (0, 0)\}$ and the allocation $x(\gamma) = x^\star$ form an optimal schedule.

□

Proposition 3.2.6. *If $b < 1$, (M, x) is an optimal schedule if and only if (M, x) maximizes profits subject to the constraint $U + V_\gamma \geq 0$.*

Proof. For a proof see proposition 3 and its corollary in Esteban et al., 2007.

□

In the more general case where $a < 1 < b$, where both downward and upward temptation are present and tax policies are embedded with specific tax rate s and ad valorem tax rate τ; the monopolist problem can be re-written as the optimal control problem

$$\int_a^b \left\{ \frac{1}{2}(1 - \tau)[(1 + \gamma)q(\gamma) - w(\gamma)] - \frac{1}{2}q(\gamma)^2 - s \right\} dF(\gamma)$$

subject to

$$w'(\gamma) = q(\gamma), \tag{3.43}$$

$$w(\gamma) \geq (\gamma - 1)q(\gamma), \tag{3.44}$$

$$w(a) = 0, \tag{3.45}$$

$$q(\gamma) \geq 0, \tag{3.46}$$

$$q'(\gamma) \geq 0, \tag{3.47}$$

$$\pi(\gamma) \geq 0. \tag{3.48}$$

where $w(\gamma)$ is the ex-post utility of consumer of type γ, defined as $w(\gamma) \equiv (1+\gamma)q(\gamma) - 2p(\gamma)$, and $\pi(\gamma)$ denotes the per-type profit contribution, given by $\pi(\gamma) \equiv \frac{1}{2}(1-\tau)[(1+\gamma)q(\gamma) - w(\gamma)] - \frac{1}{2}q(\gamma)^2 - s$. Here equation (3.43) represents the ex-post incentive compatibility and may be derived using the envelope theorem. Equation (3.44) represents ex-ante individual rationality and might be derived by manipulating the inequality $W_\gamma \geq 0$. Equation (3.45) simply states that it is possible for the monopolist to set the ex-post surplus of the lowest type at zero. Finally, equation (3.46) reflects the fact that quality cannot fall below 0, while equation (3.47) states that the quality function is non-decreasing in type. This last condition ensures that consumers of higher type do not prefer to buy bundles at the lower type.

Proposition 3.2.7. *Equation (3.43) is equivalent to ex-post incentive compatibility while equation (3.44) is equivalent to ex-ante individual rationality.*

The Hamiltonian function for the optimal control problem is

$$H(w,q,\mu,\lambda,\delta,\gamma) = \Big\{\frac{1}{2}(1-\tau)[(1+\gamma)q(\gamma) - w(\gamma)] - \frac{1}{2}q(\gamma)^2 - s\Big\}f(\gamma) + \mu(\gamma)q(\gamma) + \lambda(\gamma)[w(\gamma) - (\gamma-1)q(\gamma)] + \delta(\gamma)q(\gamma),$$

where $\mu(\gamma)$ is the costate variable associated with (3.43), $\lambda(\gamma) \geq 0$ is the multiplier for ex-ante IR (3.44), and $\delta(\gamma) \geq 0$ is the multiplier for the non-negativity constraint (3.46).

The necessary optimality conditions are

$$[\frac{1}{2}(1-\tau)(1+\gamma) - q(\gamma)]\frac{1}{b-a} + \mu(\gamma) - (\gamma-1)\lambda(\gamma) + \delta(\gamma) = 0, \tag{3.49}$$

$$w'(\gamma) = q(\gamma), \tag{3.50}$$

$$\mu'(\gamma) = \frac{1-\tau}{2}\frac{1}{b-a} - \lambda(\gamma), \tag{3.51}$$

$$\lambda(\gamma)[w(\gamma) - (\gamma-1)q(\gamma)] = 0, \qquad \lambda(\gamma) \geq 0, \tag{3.52}$$

$$\delta(\gamma)q(\gamma) = 0, \qquad \delta(\gamma) \geq 0, \tag{3.53}$$

$$\mu(b) = 0, \tag{3.54}$$

$$\pi(\gamma) \geq 0. \tag{3.55}$$

The optimality conditions reveal two distinct channels through which taxation shapes equilibrium allocations. The ad valorem tax rate τ enters multiplicatively in the Hamiltonian and first-order condition (3.49), scaling both the marginal revenue and the costate dynamics. Intuitively, an ad valorem tax reduces the effective marginal revenue from serving each type, thereby proportionally contracting equilibrium quality and transfers across the type distribution. By contrast, the specific tax s enters additively in the Hamiltonian and shifts profits independently of type. This implies that while s reduces overall profitability, it does not directly distort the marginal trade-off between quality provision and information rents. Hence, ad valorem taxation generates allocation distortions across all consumer types, whereas specific taxation acts as a lump-sum extraction from the firm without altering marginal incentives. This distinction mirrors the discrete-type case, but here the distortionary impact of τ applies continuously across the type distribution, while s only shifts the intercept of the profit function.

Theorem 3.2.1. *Suppose that $a < 1$ and $b \in (\frac{5}{3}, 3)$ so that the consumer population contains both upwards and downwards tempted types and assume the quality $q(\gamma)$ is a continuous function of $\gamma \in (a, b)$. Denote by*

$$\underline{\gamma}(\tau, s) = \max\left\{\frac{b-1}{2}, b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right\}$$

and

$$\bar{\gamma} = \frac{5-b}{2}.$$

Impose the following restriction on the specific tax: $s \in (-1, 1)$, and the ad valorem tax: $\tau \in (-1, 1)$. Given such taxes s and τ, the following bound ensures that the solution is well-defined:

$$s < \frac{(1-\tau)^2}{16}\left(-3b^2 + 10b - 3\right) \equiv s_{max}.$$

Moreover, if $|\tau| < 1$ and $-1 < s < s_{\max}$ then $\underline{\gamma}(\tau, s) < \gamma^* < \bar{\gamma}$ (recall that $\gamma^* = 1$). In addition, the quality $q(\gamma)$, ex-post utility $w(\gamma) = (1+\gamma)q(\gamma) - p(\gamma)$

and price $p(\gamma)$ are given as follows:

$$q(\gamma) = \begin{cases} 0, & \textit{if } \gamma < \underline{\gamma}(\tau, s) \\ (1-\tau)(\gamma - \frac{b-1}{2}), & \textit{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ (1-\tau)(3-b), & \textit{if } \gamma > \bar{\gamma} \end{cases} \tag{3.56}$$

$$w(\gamma) = \begin{cases} 0, & \textit{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{1-\tau}{2}(\gamma - \frac{b-1}{2})^2, & \textit{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ (1-\tau)(3-b)(\gamma-1), & \textit{if } \gamma > \bar{\gamma} \end{cases} \tag{3.57}$$

and

$$p(\gamma) = \begin{cases} 0, & \textit{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{1-\tau}{4}[(\gamma+1)^2 - (\frac{b-1}{2}+1)^2], & \textit{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ (1-\tau)(3-b), & \textit{if } \gamma > \bar{\gamma} \end{cases} \tag{3.58}$$

Theorem 3.2.1 establishes that taxation reshapes the optimal schedule determined by the monopolist. On one hand, taxation lowers the participation threshold $\underline{\gamma}(\tau, s)$. Taxation makes serving consumers with temptation parameter γ below $\underline{\gamma}(\tau, s)$ unprofitable. It is therefore optimal for the monopolist to provide zero quality to those consumer types. In addition, The ad valorem tax reduces the slope of the quality schedule, scaling down both $q(\gamma)$ and $w(\gamma)$ across the interior region. Note that specific taxes alone do not affect the bundles of those consumers who are still served.

It should be noted that for types above $\bar{\gamma}$, both quality and price flatten out, implying a bunching at the top. This is due to the additional ex-ante IR constraint due to the presence of self-control. This feature of the optimal quality and price schedules is not present in the problem without self control.

The functions $q(\gamma)$ and $p(\gamma)$ are illustrated in figure 3.7 below when $\tau = 0.1$, $s = 0.1 \cdot s_{\max}$ and $b = 2$ (all the graphs in figs. 3.7-3.10 based on these parameters). Observe that both functions are constant below $\gamma(\tau, s)$ and above $\overline{\gamma}$. The fact that both functions coincide on these intervals implies that a consumer with temptation type in those regions obtains no surplus in commitment utility. In particular, this reflects the fact that ex-ante IR binds for consumers with $\gamma > \overline{\gamma}$. Note that over the interval $[\gamma(\tau, s), \overline{\gamma}]$, $q(\gamma)$ is linear while $p(\gamma)$ is quadratic. Consumers in this interval receive a commitment utility surplus. This is similar to standard non-linear pricing models. Finally, the small jump is due to the fact that $s > 0$. This tax makes serving consumers below $\gamma(\tau, s)$ unprofitable, even if some would have been served if $s = 0$.

Definition 3.2.1 (Size of the market)**.** *The amount of consumers served by the monopolist,* the size of the market*, is given by*

$$|b - \underline{\gamma}| = \min\left\{\frac{b+1}{2}, \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right\}.$$

The first term, $\frac{b+1}{2}$, represents the market when no specific tax is levied. The second term, $\sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}$, captures the reduction in the effective market size caused by the specific tax s and the ad valorem tax τ. Thus, the overall market size is determined by whichever of these two constraints is more restrictive.

Definition 3.2.2 (Size of the temptation region)**.** *Consumers with temptation intensity $\gamma \in (\underline{\gamma}, \bar{\gamma})$, except $\gamma = \gamma^* = 1$, face a self-control problem. The size of this temptation region is defined as*

$$|\bar{\gamma} - \underline{\gamma}| = \min\left\{3, \frac{5-3b}{2} + \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right\}.$$

Here, the bound 3 reflects the maximum possible span of temptation types, given the support of preferences. The second term, $\frac{5-3b}{2} + \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}$, shows how the temptation region expands or contracts depending on market heterogeneity b, the self-control cost s, and the ad valorem tax τ. The actual size of the temptation region is therefore the more restrictive of these two constraints.

The comparative statics are straightforward. As the market parameter

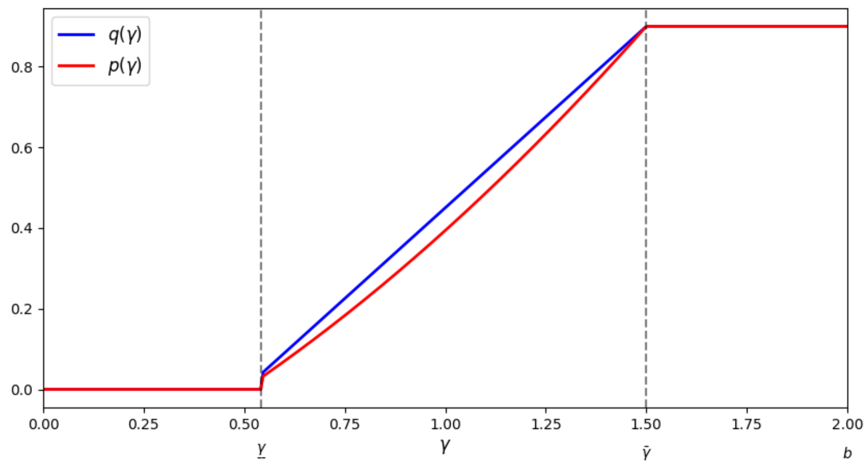

Figure 3.7: The optimal scheme of $q(\gamma)$ and $p(\gamma)$ with $\tau = 0.1$ and $s = 0.1 \cdot s_{\max}$ when $b = 2$

b increases, the size of the market expands while the temptation region contracts. Intuitively, when the consumer base shifts toward higher types, more upwards-tempted consumers can be served without suffering self-control costs. This implies that the monopolist is more likely to implement pooling at higher types in order to maximize expected profit. Increasing b also causes the temptation region to shrink while a larger share of consumers are now located in the temptation-free region. Consequently, the optimal pricing schedule shifts rightward, and the monopolist reduces quality marginally to attract additional consumers by easing the self-control costs of those consumers.

The measure $|\gamma - 1|$ captures the degree of *behavioral divergence*, i.e. the extent of departure from standard utility maximization. A larger value of $|\gamma - 1|$ indicates stronger departure from traditional assumptions. Specifically, when $\gamma - 1 > 0$, the consumer is drawn toward the high-quality option (high-quality temptation); when $\gamma - 1 < 0$, the consumer is attracted by the low-price option (low-price temptation).

As a consequence of proposition 3.2.1 monopoly profits $\pi(\gamma)$ are given by

$$\pi(\gamma) = \begin{cases} 0, & \text{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{(1-\tau)^2}{4}[-(\gamma - b)^2 + \frac{1}{4}(b+1)^2] - s, & \text{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ \frac{(1-\tau)^2}{2}(3-b)(b-1) - s, & \text{if } \gamma > \bar{\gamma} \end{cases} \quad (3.59)$$

Per-type profit $\pi(\gamma)$ is shown in figure 3.8 below. For low types, per-type profit is zero, reflecting the fact that these consumers are excluded from the market. The monopolist finds it unprofitable to serve them since their willingness to pay is too low relative to the effective marginal cost (including the effects of taxation s and τ).

In this intermediate range, profits rise smoothly and concavely with γ. The upward slope reflects that consumers with higher temptation intensity can be charged more aggressively, because the nature of their temptation increases their willingness to accept higher qualities at higher prices. Concavity indicates a diminishing marginal profitability as γ approaches $\bar{\gamma}$. Beyond $\bar{\gamma}$, profits reach a maximum and become flat. Recall that any attempt to further increase profit by distorting the contract would violate the ex-ante individual rationality constraint.

Using the formulas in proposition 3.2.1, the different utilities can be rewritten in terms of γ. Denote by $U(\gamma)$ the commitment utility of type γ; by $V_\gamma(\gamma)$ its temptation utility; by $\max V_\gamma(\gamma)$ the largest temptation faced by type γ; by $\{\max_x V_\gamma(x) - V_\gamma(\gamma)\}$ type γ's temptation cost and by $W(\gamma) = U(\gamma) + V_\gamma(\gamma) - \max V_\gamma(\gamma)$ the ex-ante utility of type γ. Then

$$U(\gamma) = q(\gamma) - p(\gamma) = \begin{cases} 0, & \text{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{(1-\tau)}{4}\left[-(\gamma-1)^2 + \frac{(b-3)^2}{4}\right], & \text{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ 0, & \text{if } \gamma > \bar{\gamma} \end{cases}$$

Note that, as shown in figure 3.9 below, the commitment utility $U(\gamma)$ is increasing in $(\underline{\gamma}(\tau, s), 1)$ and decreasing in $(1, \bar{\gamma})$. This reflects the fact that for consumers facing downwards temptation ex-ante IR is satisfied which means

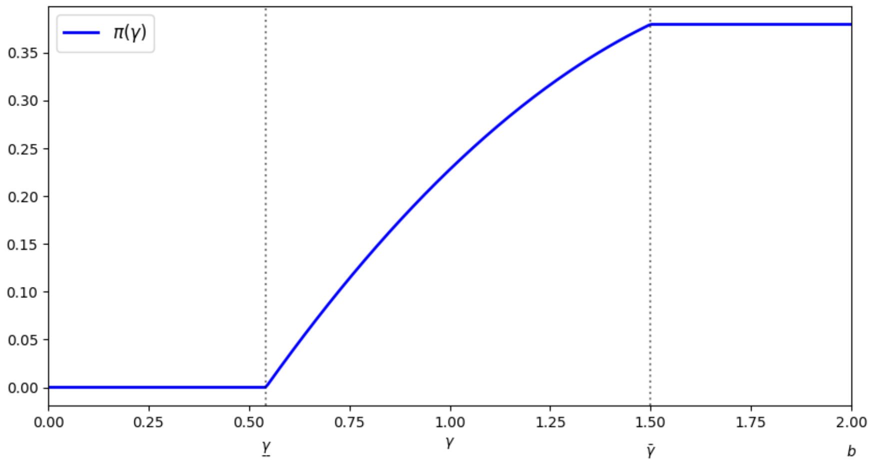

Figure 3.8: The optimal profit $\pi(\gamma)$ with $\tau = 0.1$ and $s = 0.1 \cdot s_{\max}$ when $b = 2$

that since they suffer from self-control costs, they must obtain a commitment utility surplus. This surplus increases as γ approaches 1 (who suffers no self-control problem). On the other hand, for consumers facing upward temptation this surplus is reduced as the monopolist attempts to extract surplus.

$$V_\gamma(\gamma) = \gamma q(\gamma) - p(\gamma) = \begin{cases} 0, & \text{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{(1-\tau)}{4}\left(3\gamma^2 - 2b\gamma + \frac{b^2+2b-3}{4}\right), & \text{if } \gamma \in [\underline{\gamma}(\tau, s), \bar{\gamma}] \\ (1-\tau)(3-b)(\gamma-1), & \text{if } \gamma > \bar{\gamma} \end{cases}$$

As reflected in figure 3.9, the temptation utility V_γ is always increasing for consumers that purchase a bundle. However, it grows linearly above $\overline{\gamma}$ since consumers in this region obtain no commitment utility surplus and price and quantity coincide.

To determine the self-control cost, the maximum temptation must be calculated for each type. For any $\hat{\gamma} \in [\underline{\gamma}, \bar{\gamma}]$ the temptation maximization problem is

$$\max_{q(\gamma),p(\gamma)} \{V_{\hat\gamma}(q(\gamma),p(\gamma))\} = \max_{q(\gamma),p(\gamma)} \{\hat\gamma q(\gamma) - p(\gamma)\}$$
$$= \max_{q(\gamma),p(\gamma)} \left\{\hat\gamma \frac{1-\tau}{2}(2\gamma - b + 1) - \frac{1-\tau}{4}\left[(\gamma+1)^2 - \frac{(b-1)^2}{4} - b\right]\right\}.$$

Taking the first-order derivative of γ, I have

$$\frac{dV}{d\gamma} = (1-\tau)[\hat\gamma - \frac{1}{2}(\gamma+1)] = 0.$$

When $\hat\gamma \in [\frac{\underline{\gamma}+1}{2}, \frac{\bar\gamma+1}{2}]$, $\gamma = 2\hat\gamma - 1 \in [\underline{\gamma}, \bar\gamma]$ the maximum value is $V_{\hat\gamma}(2\hat\gamma-1) = \frac{1-\tau}{4}\left(2\hat\gamma - \frac{b+1}{2}\right)^2$. Meanwhile, when $\hat\gamma < \frac{\underline{\gamma}+1}{2}$, $\frac{dV}{d\gamma} < 0$, $V_{\hat\gamma}$ is maximized at $\underline{\gamma}$ and therefore the maximum temptation is $V_{\hat\gamma}(\underline{\gamma}) = 0$. On the other hand, when $\hat\gamma > \frac{\bar\gamma+1}{2}$, $\frac{dV}{d\gamma} > 0$ and $V_{\hat\gamma}$ is maximized at $\overline{\gamma}$ which means that the maximum temptation satisfies $V_{\hat\gamma}(\bar\gamma) = (1-\tau)(3-b)(\hat\gamma - 1)$. Therefore,

$$\max_x V_\gamma(x) = \max_x \{\gamma q(x) - p(x)\} = \begin{cases} 0, & \text{if } \gamma < \frac{\underline{\gamma}(\tau,s)+1}{2} \\ (1-\tau)\left(\gamma - \frac{b+1}{4}\right)^2, & \text{if } \gamma \in [\frac{\underline{\gamma}(\tau,s)+1}{2}, \frac{\bar\gamma+1}{2}] \\ (1-\tau)(3-b)(\gamma-1), & \text{if } \gamma > \frac{\bar\gamma+1}{2} \end{cases}$$

Using this formula the temptation cost is

$$\max_x V_\gamma(x) - V_\gamma(\gamma) = \begin{cases} 0, & \text{if } \gamma < \underline{\gamma}(\tau,s) \\ -\frac{(1-\tau)}{4}\left(3\gamma^2 - 2b\gamma + \frac{b^2+2b-3}{4}\right), & \text{if } \gamma \in [\underline{\gamma}(\tau,s), \frac{\underline{\gamma}(\tau,s)+1}{2}) \\ \frac{1-\tau}{4}(\gamma-1)^2, & \text{if } \gamma \in [\frac{\underline{\gamma}(\tau,s)+1}{2}, \frac{\bar\gamma+1}{2}) \\ (1-\tau)\left[-\frac{3}{4}\gamma^2 + \left(3 - \frac{b}{2}\right)\gamma + \frac{-b^2+14b-45}{16}\right], & \text{if } \gamma \in [\frac{\bar\gamma+1}{2}, \bar\gamma) \\ 0, & \text{if } \gamma \geq \frac{\bar\gamma+1}{2} \end{cases}$$

As shown in figure 3.9, this function is divided in five regions. The first and last both vanish, since consumers above $\overline{\gamma}$ or below $\gamma(\tau, s)$ face no self-control costs. In the second region, consumers with $\gamma \in [\underline{\gamma}(\tau,s), \frac{\underline{\gamma}(\tau,s)+1}{2})$ are tempted by the bundle $(0,0)$, this cost peaks at $\frac{\underline{\gamma}(\tau,s)+1}{2}$ where consumers begin to be tempted by bundles closer to them. The third region $[\frac{\underline{\gamma}(\tau,s)+1}{2}, \frac{\bar\gamma+1}{2})$ contains

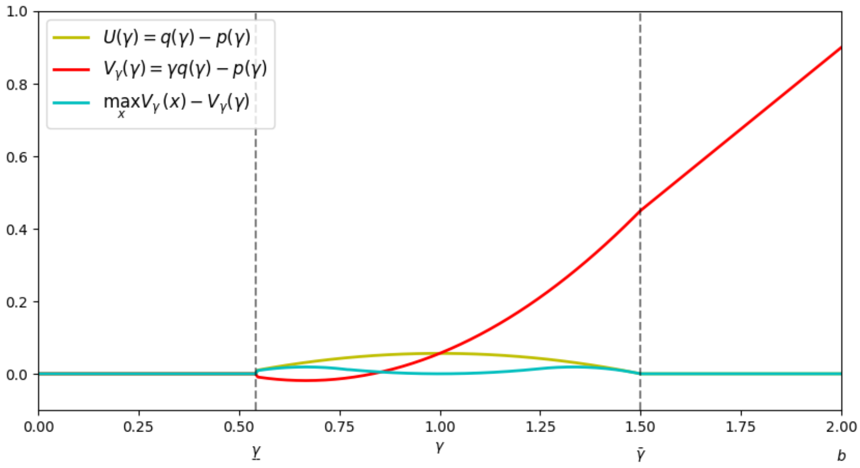

Figure 3.9: Equilibrium $U(\gamma)$, $V_\gamma(\gamma)$ and $\max_x V_\gamma(x) - V_\gamma(\gamma)$ with $\tau = 0.1$ and $s = 0.1 s_{max}$ when $b = 2$

both consumers facing downwards temptation, who pay a low cost as the bundle they are tempted by is close to theirs and the effect of temptation is low in since their type is close to 1. Remember that $\gamma^\star = 1$ faces no self-control by definition. On the other hand, once this transition type is passed self-control costs raise again as consumer are tempted by bundles that are further from theirs, this offsets the fact that the effect of temptation is small for these types. Finally, in the fourth region, types $\gamma \in [\frac{\bar{\gamma}+1}{2}, \bar{\gamma})$ are tempted by the bundle purchased by $\bar{\gamma}$ but since it is relatively close to theirs, the effect is smaller and self-control costs decrease as γ increases.

The ex-ante utility per type γ can be calculated easily from these results

$$W(\gamma) = w(\gamma) - \max_x V_\gamma(x) = \begin{cases} 0, & \text{if } \gamma < \underline{\gamma}(\tau, s) \\ \frac{1-\tau}{2}(\gamma - \frac{b-1}{2})^2, & \text{if } \gamma \in [\underline{\gamma}(\tau, s), \frac{\underline{\gamma}(\tau,s)+1}{2}) \\ (1-\tau)\left(-\frac{\gamma^2}{2} + \gamma + \frac{b^2-6b+1}{16}\right), & \text{if } \gamma \in [\frac{\underline{\gamma}(\tau,s)+1}{2}, \frac{\bar{\gamma}+1}{2}) \\ (1-\tau)\left(\frac{\gamma^2}{2} + \frac{(b-5)\gamma}{2} + \frac{(b-5)^2}{8}\right), & \text{if } \gamma \in [\frac{\bar{\gamma}+1}{2}, \bar{\gamma}) \\ 0, & \text{if } \gamma \geq \bar{\gamma} \end{cases}$$

As illustrated in figure 3.10 below, ex-ante utility is strictly positive only

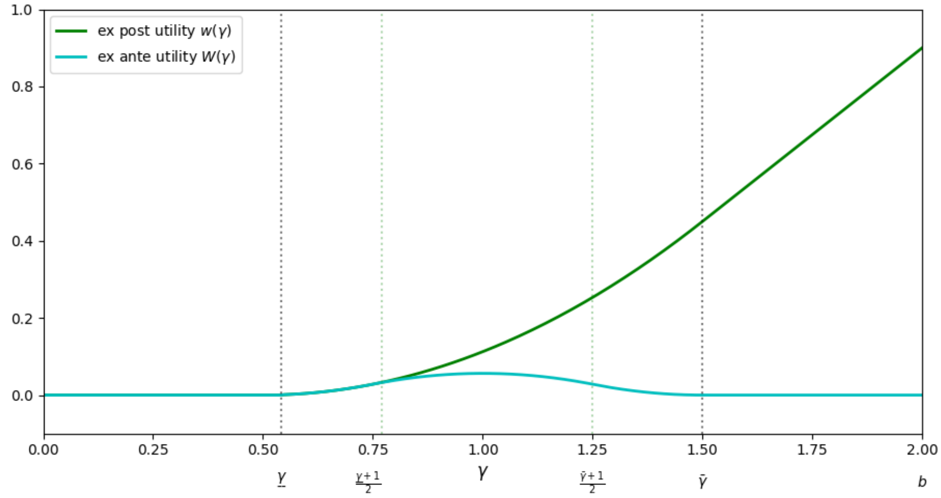

Figure 3.10: Equilibrium $w(\gamma)$ and $W(\gamma)$ with $\tau = 0.1$ and $s = 0.1s_{max}$ when $b = 2$

for types $\gamma \in (\underline{\gamma}(\tau, s), \overline{\gamma})$, which is precisely the region where self-control costs are relevant and ex-ante individual rationality does not bind. For types below $\underline{\gamma}(\tau, s)$ or above $\overline{\gamma}$, $W(\gamma) = 0$, consistent with binding participation constraints. Moreover, $W(\gamma)$ is highest at $\gamma^{\star} = 1$ since this type has standard preferences. Furthermore, for types close to 1 the effects of temptation are small and can therefore benefit from a higher surplus. In contrast, for types farther from 1, self-control costs reduce surplus, explaining the concave shape of $W(\gamma)$ across the temptation region.

Proposition 3.2.8 (Effect of taxation on the bounds of $\underline{\gamma}$ and $\bar{\gamma}$). *The introduction of a small amount of tax, either ad valorem (τ) or specific (s), does not change the upper bound $\bar{\gamma}$. Moreover, the lower bound $\underline{\gamma}(\tau, s)$ never rises to the value of $\bar{\gamma}$. By contrast, the lower bound $\underline{\gamma}(\tau, s)$ is sensitive to taxation, and its behavior depends on the type of tax as follows:*

(1) ***Case of an existing subsidy*** ***(**$s < 0$**):***

Starting from a situation where an specific subsidy has been imposed ($s < 0$) so that that $\underline{\gamma}(\tau, s) = \frac{b-1}{2}$. Neither a reduction of the subsidy s nor the introduction of a tax τ affect this lower bound.

(2) ***Case of a laissez-faire market*** ***(**$s = 0$**):***

i. Imposing a specific tax $s > 0$ increases the lower bound to $\underline{\gamma}(0, s)$ while introducing a specific subsidy $s < 0$ has no effect on the lower bound $\underline{\gamma}(\tau, s)$.

ii. Imposing an ad valorem tax τ has no effect on the lower bound.

(3) ***Case of an existing specific tax ($s > 0$):***

Starting from a situation when a specific tax is being levied ($s > 0$) so that $\underline{\gamma}(\tau, s) = b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}$.

i. Increasing the specific tax s raises the lower bound $\underline{\gamma}(\tau, s)$. This leads to the lowest types leaving the market.

ii. The effect of a change in the ad valorem tax τ depends on whether $\tau < 1$, in which the change in bound is in the same direction as the change in τ; or $\tau > 1$, when the change is in the opposite direction.

In particular, when $\tau = 0$, the lower bound reduces to

$$\underline{\gamma}(s) = \begin{cases} \frac{b-1}{2}, & \text{when } s < 0 \\ b - \frac{\sqrt{(b+1)^2 - 16s}}{2}, & \text{when } s \geq 0 \end{cases}$$

and the derivative of this function is

$$\frac{d\underline{\gamma}(s)}{ds} = \frac{4}{\sqrt{(b+1)^2 - 16s}} > 0 \quad \text{for} \quad s > 0.$$

Finally, recall that the condition $s < s_{\max} = \frac{-3b^2+10b-3}{16}$ ensures that $\underline{\gamma}(s) < 1$ and the optimal control problem is well defined, which leads to a non-empty set of consumers in the market The dependence of $s_{\max}$ on b is shown underneath in figure 3.11. Notice that as b increases $s_{\max}$ approaches 0, meaning that higher-quality goods tolerate only very small specific taxes before the market collapses.

Proposition 3.2.9 (**Effect of specific taxes on market variables**). *An increase in the specific tax s has the following effects:*

(1) It does not affect the quality $q(\gamma)$, the price $p(\gamma)$, ex-ante utility $W(\gamma)$ and ex-post utility $w(\gamma)$ for all of consumers of types $\gamma \in (\underline{\gamma}(\tau, s), b]$;

(2) It reduces the total profit of the monopoly.

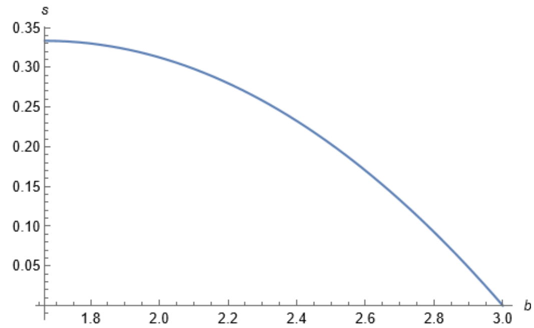

Figure 3.11: The upper bound for the specific policy $s_{\max}$

Proposition 3.2.10 (Effect of ad valorem taxes on market variables). *An increase in the ad valorem tax τ has the following effects:*

(1) It reduces the quality $q(\gamma)$, the price $p(\gamma)$, ex-ante utility $W(\gamma)$ and ex-post utility $w(\gamma)$ to all types of consumers $\gamma \in (\underline{\gamma}(\tau, s), b]$;

(2) It reduces the total profit of the monopoly.

The difference between the two types of taxation lies in how they distort the monopolist's optimal contract design. A specific tax acts as a lump-sum burden on the monopolist, reducing profits without affecting the optimal allocation of quality and prices across types. By contrast, an ad valorem tax directly scales down revenues from each unit sold, making high-quality contracts less profitable. As a result, the monopolist reduces both quality and price, which lowers consumer utilities in addition to profits. This explains why ad valorem taxation has stronger distortive effects on market outcomes compared to specific taxation.

3.3 Welfare Effects

This section examines the impact of introducing a small ad valorem or specific tariff on aggregate welfare when starting from an initial equilibrium where trade is free. The analysis also derives the optimal taxation policy under two alternative market structures, distinguishing between the case of a domestic monopolist and that of a foreign monopolist. As noted in the introduction, multiple theoretical frameworks can be employed to evaluate welfare effects. The analysis proceeds by considering the three primary approaches separately, calculating welfare under each, and subsequently comparing the results across the different measures.

3.3.1 *Welfare Analysis of a Domestic Monopolist with Behavioral Consumers*

Social welfare effect of the ad valorem fiscal policy

For a home monopolist, the total welfare comprises the sum of consumer surplus and expected profits of the monopolist and tariff revenues. However, since tariff revenues represent transfers between consumers and the monopolist, they do not affect aggregate surplus. Accordingly, welfare reduces to the sum of consumer utility (from surplus) and monopolist surplus.

To capture behavioral distortions, I define the self-control cost-adjusted total welfare, denoted by $W_C^H(\tau, s)$. This results from summing (i) ex-ante consumer surplus, (ii) the monopolist's expected profits, (iii) tariff revenues, and (iv) the negative of self-control costs. In other words, the welfare $W_C^H(\tau, s)$ is

$$W_C^H(\tau, s) = \underbrace{\int_a^b \left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma}_{\text{Normative Welfare}} - \underbrace{\int_a^b \Big\{\max\{\gamma q(\gamma) - p(\gamma)\} - [\gamma q(\gamma) - p(\gamma)]\Big\} f(\gamma)d\gamma}_{\text{Self-Control Cost}}.$$

To determine the effect of the introduction of a small intervention from a laissez-faire benchmark where $\tau = s = 0$, I compute the derivative of $W_C^H(\tau, s)$ with respect to τ evaluated at $\tau = s = 0$:

$$\frac{\partial W_C^H(\tau, s)}{\partial \tau}\Big|_{\tau=s=0} = \frac{3-b}{16(b-a)}(b^2 - 30b + 57). \tag{3.60}$$

The sign of this derivative (and thus wether implementing a tax or subsidy) depends on whether the parameter b falls below or exceeds the critical threshold b_C^H :

$$b_C^H = 15 - 2\sqrt{42} \approx 2.04.$$

When b is relatively small, where $b \in \left(\frac{5}{3}, b_C^H\right)$, the welfare gradient is strictly positive:

$$\left.\frac{\partial W_C^H(\tau, s)}{\partial \tau}\right|_{\tau=s=0} > 0.$$

This implies that a marginal tariff raises welfare when there is a large proportion of agents with upwards temptation.

At the boundary $b = b_C^H$, the welfare gradient vanishes:

$$\left.\frac{\partial W_C^H(\tau, s)}{\partial \tau}\right|_{\tau=s=0} = 0.$$

In this regime, abstaining from intervention is optimal.

For b is relatively large, where $b \in (b_C^H, 3)$, the welfare gradient becomes negative:

$$\left.\frac{\partial W_C^H(\tau, s)}{\partial \tau}\right|_{\tau=s=0} < 0.$$

In this case, the introduction of a marginal tax reduces aggregate welfare, indicating that a subsidy would be required to improve efficiency.

Finally, I generalize the analysis by considering the derivative of welfare with respect to τ for arbitrary τ (with $s = 0$). This expression allows me to characterize the optimal ad valorem policy, which balances the trade-off between mitigating temptation-driven inefficiencies and preserving surplus.

To derive the optimal ad valorem taxation policy, I formally evaluate the marginal effect of the ad valorem tax τ on welfare, while setting the specific tax equal to zero. This derivative characterizes the sensitivity of aggregate welfare to incremental changes in τ, thereby allowing the identification of the welfare-maximizing tax (or subsidy) rate.

$$\left.\frac{\partial W_C^H(\tau, s)}{\partial \tau}\right|_{s=0} = \frac{(3-b)}{48(b-a)} \left[3(b^2 - 30b + 57) + 8(b^2 + 6b - 27)\tau\right].$$

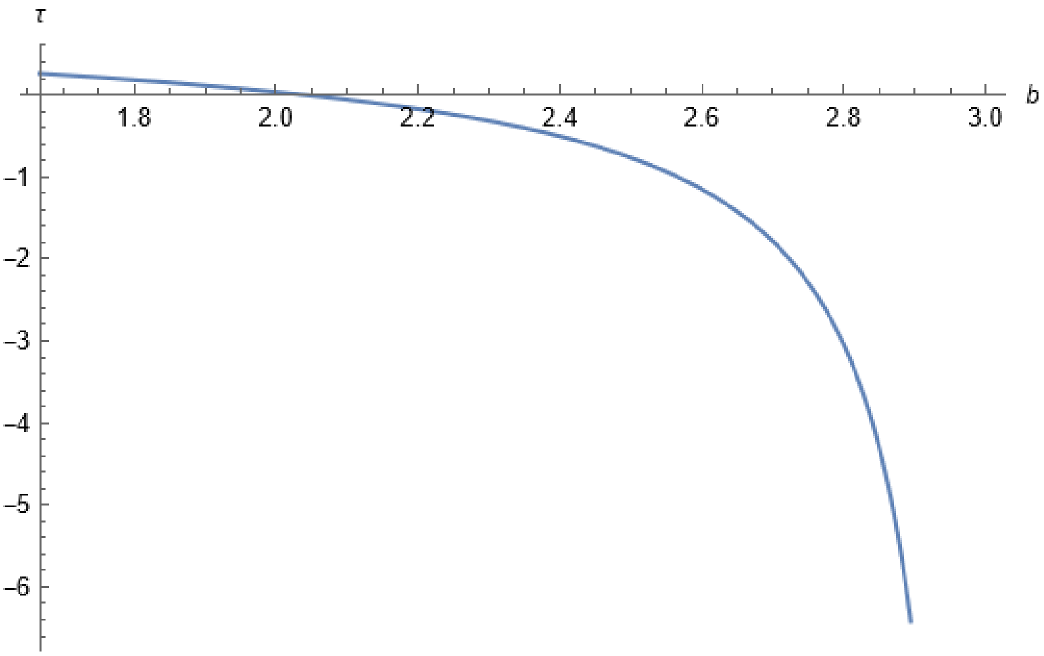

Figure 3.12: The optimal ad valorem policy τ_C^H

From this calculation, the optimal policy is given by the following rule:

$$\tau_C^H = -\frac{3(b^2 - 30b + 57)}{8(b+9)(b-3)}.$$

Proposition 3.3.1 (Optimal Intervention Strategy τ_C^H). *The optimal ad valorem policy is*

$$\tau_C^H = -\frac{3(b^2 - 30b + 57)}{8(b+9)(b-3)}.$$

For populations with

(a) ***Small-market size*** *($b \in (\frac{5}{3}, b_C^H)$): a tax $\tau_C^H \in (0, \frac{33}{128})$ is optimal.*

(b) ***Large-market size*** *($b \in (b_C^H, 3)$): a subsidy $\tau_C^H < 0$ is optimal.*

(c) ***Boundary-market size*** *($b = b_C^H$): no intervention is optimal.*

The intuition behind these results is straightforward. In relatively small markets, where b lies below the critical threshold b_C^H, a tax improves efficiency by counteracting the excessive consumption driven by behavioral biases such as temptation or self-control problems. Conversely, in sufficiently large markets, the distortionary effect of taxation outweighs its corrective benefits, and a subsidy is required to restore efficiency. At the boundary value of $b = b_C^H$, the opposing forces exactly cancel out, so that laissez-faire remains the optimal policy choice.

Next, consider the normative social welfare measure W_U^H, which includes (i) consumer surplus measured in terms of commitment utility, (ii) the monopolist's expected profits, and (iii) tariff revenues, but omits self-control costs:

$$W_U^H(\tau, s) = \underbrace{\int_a^b \left[q(\gamma) - \frac{1}{2} q(\gamma)^2 \right] f(\gamma) d\gamma}_{\text{Normative Welfare}} .$$

As before, the marginal effect of intervention at the laissez-faire benchmark ($\tau = s = 0$) is given by,

$$\left. \frac{\partial W_U^H(\tau, s)}{\partial \tau} \right|_{\tau=s=0} = \frac{3-b}{6(b-a)}(-b^2 - 6b + 15).$$

Define the threshold parameter

$$b_U^H = 2\sqrt{6} - 3 \approx 1.90.$$

For b is relatively small, where $b \in \left(\frac{5}{3}, b_U^H\right)$, the welfare gradient is strictly positive:

$$\left. \frac{\partial W_U^H(\tau, s)}{\partial \tau} \right|_{\tau=s=0} > 0.$$

In this range, a marginal increase in the ad valorem tax rate raises aggregate welfare, indicating that taxation is welfare-improving.

At the boundary $b = b_U^H$, the welfare gradient vanishes:

$$\left.\frac{\partial W_U^H(\tau, s)}{\partial \tau}\right|_{\tau=s=0} = 0.$$

In this transition region, imposing a marginal tax or subsidy has no effect.

For b is relatively large, where $b \in (b_U^H, 3)$, the welfare gradient becomes negative:

$$\left.\frac{\partial W_U^H(\tau, s)}{\partial \tau}\right|_{\tau=s=0} < 0.$$

In this regime, any marginal tax imposition reduces aggregate welfare. In other words, welfare improvement requires a subsidy.

To determine the optimal ad valorem taxation policy, consider derivative of $W_U^H(\tau, s)$ with respect to τ, holding the specific tax equal to zero:

$$\left.\frac{dW_U^H(\tau, s)}{d\tau}\right|_{s=0} = \frac{3-b}{6(b-a)} \left[-(b^2 - 6b + 15) + (b+9)(b-3)\tau\right].$$

Solving the first-order condition:

$$\tau_U^H = \frac{b^2 + 6b - 15}{(b+9)(b-3)},$$

yields the welfare-maximizing ad valorem rate.

Proposition 3.3.2 (Optimal Intervention Strategy τ_U^H). *The optimal ad valorem policy is*

$$\tau_U^H = \frac{b^2 + 6b - 15}{(b+9)(b-3)}.$$

For populations with:

(a) ***Small-market size*** *($b \in (\frac{5}{3}, b_U^H)$): a tax $\tau_U^H \in (0, \frac{5}{32})$ improves welfare.*

(b) ***Large-market size*** *($b \in (b_U^H, 3)$): a subsidy $\tau_U^H < 0$ is welfare-enhancing.*

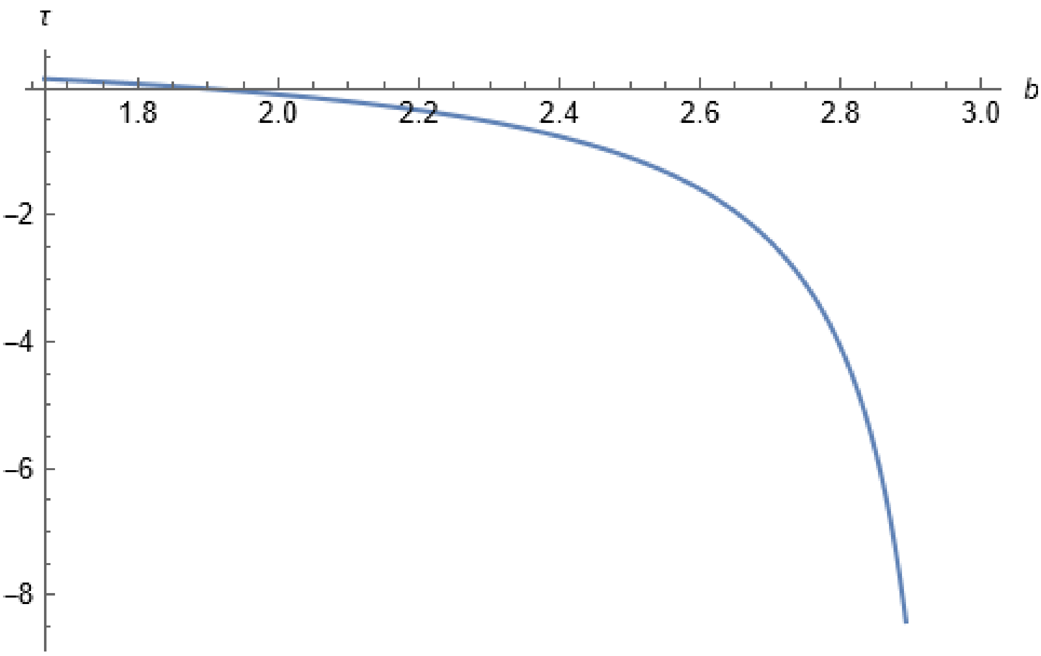

Figure 3.13: The optimal ad valorem policy τ_U^H

(c) ***Boundary-market size*** $(b = b_U^H)$*: (boundary-market size), no intervention is optimal.*

Similarly, in short, the analysis demonstrates that welfare-improving policy is context dependent: small markets call for taxation, large markets require subsidization, and at the critical boundary no intervention is desirable.

The behavioral social welfare W_{U+V}^H, aggregates (i) ex-post consumer surplus, (ii) the firm's expected profits, (iii) tariff revenues. Formally,

$$W_{U+V}^H(\tau, s) = \underbrace{\int_a^b \left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma) d\gamma}_{\text{Normative Welfare}} + \underbrace{\int_a^b \left[\gamma q(\gamma) - p(\gamma)\right] f(\gamma) d\gamma}_{\text{Temptation Surplus}}.$$

The effect of a marginal intervention from the laissez-faire status quo $(\tau = s = 0)$, I compute the welfare gradient $W_C^H(\tau, s)$ with respect to τ:

$$\frac{\partial W_{U+V}^H(\tau, s)}{\partial \tau}\Big|_{\tau=s=0} = \frac{3-b}{96(b-a)}(-43b^2 - 78b + 285).$$

Define the threshold b^H_{U+V} as

$$b^H_{U+V} = \frac{4\sqrt{861} - 39}{43} \approx 1.82.$$

For b is relatively small, where $b \in \left(\frac{5}{3}, b^H_{U+V}\right)$, the welfare gradient is strictly positive:

$$\frac{\partial W^H_{U+V}(\tau, s)}{\partial \tau}\Big|_{\tau=s=0} > 0.$$

The introduction of a marginal tariff results in first-order welfare raises.

At the boundary $b = b^H_{U+V}$, the welfare gradient vanishes:

$$\frac{\partial W^H_{U+V}(\tau, s)}{\partial \tau}\Big|_{\tau=s=0} = 0.$$

This represents a transition point, neither tax nor subsidy yield welfare gains.

For b is relatively large, where $b \in (b^H_{U+V}, 3)$, the welfare gradient becomes negative:

$$\frac{\partial W^H_{U+V}(\tau, s)}{\partial \tau}\Big|_{\tau=s=0} < 0.$$

In this regime, any marginal tax imposition reduces aggregate welfare. In other words, welfare improvement requires a subsidy.

To obtain the optimal ad valorem taxation policy, I differentiate $W^H_U(\tau, s)$ with respect to τ at $s = 0$:

$$\frac{\partial W^H_{U+V}(\tau, s)}{\partial \tau}\Big|_{s=0} = \frac{3-b}{96(b-a)}\left[-(43b^2 + 78b - 285) + 16(b+9)(b-3)\tau\right].$$

From this calculation, the optimal policy is given by the following rule:

$$\tau^H_{U+V} = \frac{43b^2 + 78b - 285}{16(b+9)(b-3)}.$$

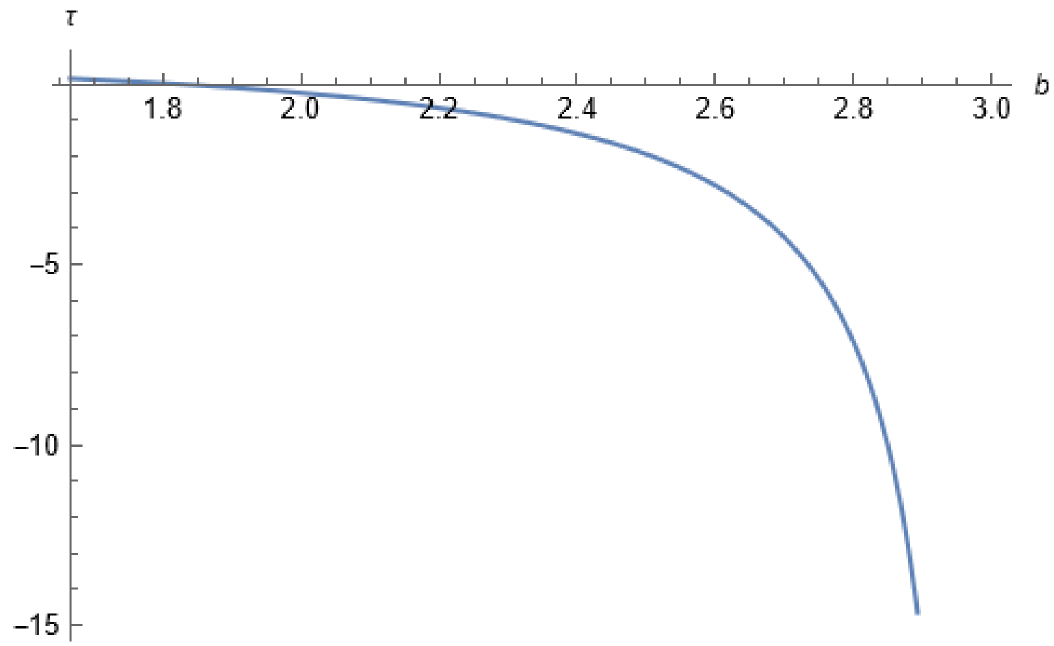

Figure 3.14: The optimal ad valorem policy τ^{H}_{U+V}

Proposition 3.3.3 (Optimal Intervention Strategy τ^{H}_{U+V}). *The optimal ad valorem policy is*

$$\tau^{H}_{U+V} = \frac{43b^2 + 78b - 285}{16(b+9)(b-3)}.$$

For populations with:

(a) ***Small-market size*** $(b \in (\frac{5}{3}, b^{H}_{U+V}))$*: a tax policy* $\tau^{H}_{U+V} \in (0, \frac{5}{32})$ *improves welfare.*

(b) ***Large-market size*** $(b \in (b^{H}_{U+V}, 3))$*: a subsidy policy* $\tau^{H}_{U+V} < 0$ *is welfare-enhancing.*

(c) ***Boundary-market size*** $(b = b^{H}_{U+V})$*: no intervention is optimal.*

Taken together, the analysis across the three welfare criteria establishes a robust pattern for optimal taxation policy. Recall that as the parameter b increases, the market expands while the share of consumers subject to temptation distortions diminishes. In relatively small markets, where the

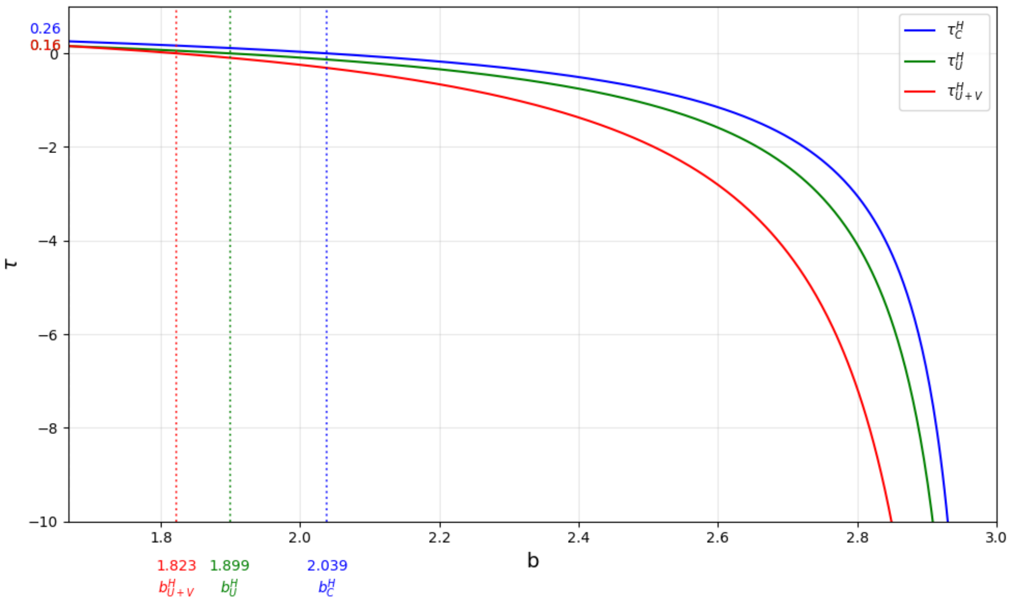

Figure 3.15: The comparison of the optimal ad valorem policy τ^H with a home monopolist

temptation region is sizable, an ad valorem tax is welfare-enhancing because it alleviates self-control problems and improves allocative efficiency. In contrast, in sufficiently large markets, where temptation is less pervasive, a subsidy becomes the welfare-maximizing policy by stimulating additional consumption. Hence, the sign of the optimal intervention is systematically linked to the relative prevalence of temptation in the consumer population.

The welfare effects of an ad valorem tax policy depend critically on the underlying welfare criterion and the share of intermediate consumers in the market. The ordering of optimal policies satisfies

$$\tau^H_C > \tau^H_U > \tau^H_{U+V}$$

as the graph 3.15 shows. When using the adjusted-cost criterion (so that welfare is measured in terms of ex-ante utility), a marginal tax introduction raises welfare whenever the proportion of intermediate consumers is large. Conversely, if this share is small, so that the majority of consumers are upward tempted, a subsidy becomes welfare improving. This result is qualitatively similar to the situation where consumers face no self-control cost. Moreover analogous results hold if the normative (where commitment utility serves a metric of consumer welfare) or behavioral approaches are taken (so ex-post utility is used instead). The key difference lies in the position of the welfare threshold:

$$b^H_C > b^H_U > b^H_{U+V}$$

indicating that the switch between taxation and subsidy occurs at a slightly lower market size when self-control costs are neglected.

The intuition is as follows: Imposing a tax can reduce self-control costs and lower temptation utility. Under the normative perspective, where commitment utility is quadratic in γ, this generates a threshold beyond which the optimal policy switches from taxation to subsidy. Accounting for self-control costs (adjusted-cost welfare) strengthens the case for taxation because higher taxes more effectively reduce temptation-induced distortions. In contrast, under a purely behavioral framework (where only temptation utility matters), higher taxes diminish temptation utility, so a lower tax (or a subsidy in some cases) yields higher welfare.

Social welfare effect of the specific taxation policy

For a home monopolist, the introduction of a specific tax strictly reduces welfare, whereas subsidies are neutral. This result holds under all welfare specifications considered.

Lemma 3.3.1 (Specific Tax Neutrality for W^H_C)**.** *Under the adjusted-cost welfare criterion, any $s > 0$ (specific tax) reduces welfare, while any $s < 0$ (specific subsidy) has no effect.*

Lemma 3.3.2 (Specific Tax Neutrality for W^H_U)**.** *Under the normative welfare criterion, $s > 0$ reduces welfare, and $s < 0$ has no effect.*

Lemma 3.3.3 (Specific Tax Neutrality for W^H_{U+V})**.** *Under the behavioral welfare criterion, $s > 0$ reduces welfare, and $s < 0$ has no effect.*

These lemmas imply that the optimal specific tax for a home monopolist is $s = 0$, independently of the welfare specification;

Proposition 3.3.4 (**Neutrality of Specific Taxes**). *For all welfare specifications (C, U, and U+V), the optimal specific tax rate is zero:*

$$s_C^H = s_U^H = s_{U+V}^H = 0.$$

The intuition for this result is straightforward. If the government provides a specific subsidy, the transfer is fully captured by the monopolist, leaving consumer surplus unaffected because the lowest consumer type served remains unchanged. Conversely, if the government imposes a specific tax, the monopolist raises the cutoff type, so that types in the range $(a, \underline{\gamma}(\tau, s))$ are no longer served in order to maintain non-negative profits at each type. Nevertheless, if self-control costs are considered, then the introduction of a specific tax can help lower this cost.

3.3.2 *Welfare Analysis of a Foreign Monopolist with Behavioral Consumers*

For a foreign monopolist, the total welfare consists of the sum of consumer surplus and tariff revenues.

Social welfare effect of the ad valorem taxation policy

The self-control cost-adjusted welfare is denoted by $W_C^F(\tau, s)$ and consists of the sum of (i) ex-ante consumer surplus and (ii) tariff revenues.

$$W_C^F(\tau, s) = \underbrace{\int_a^b [q(\gamma) - p(\gamma)]\, f(\gamma) d\gamma}_{\text{Commitment Utility Surplus}} - \underbrace{\int_a^b \Big\{ \max\{\gamma q(\gamma) - p(\gamma)\} - [\gamma q(\gamma) - p(\gamma)] \Big\} f(\gamma) d\gamma}_{\text{Self-Control Costs}}$$
$$+ \underbrace{\int_a^b [\tau p(\gamma) + s]\, f(\gamma) d\gamma}_{\text{Tax Revenues}}.$$

Having established the components of self-control cost–adjusted welfare, it is natural to analyze how taxation shapes these terms. In particular, the ad valorem rate τ enters both through its direct contribution to government revenue and indirectly via its influence on consumer surplus and behavioral costs. To isolate the mechanisms at work, it is useful to begin by examining the marginal effect of changes in τ on government revenue. This step provides a benchmark for understanding how fiscal instruments interact with consumer behavior in determining overall welfare.

Proposition 3.3.5 (Marginal effects of ad valorem taxation rates changes)**.** *The marginal effect on government tax revenue effect of an increase in the ad valorem tax rate τ is:*

i. positive when $\tau < \frac{1}{2}$; a small increase in τ yields higher tax revenues.

ii. negative when $\tau > \frac{1}{2}$; a small reduction in τ yields higher tax revenues.

iii. neutral when $\tau = \frac{1}{2}$; a small reduction in τ has no effect on revenue.

Proposition 3.3.5 characterizes the revenue-maximizing properties of the ad valorem tax rate in general terms. To assess the welfare consequences more directly, it is instructive to evaluate the effect of taxation starting from the laissez-faire allocation, where both the ad valorem and specific taxes are set to zero. The following expression computes the derivative of welfare with respect to τ at this point, thereby providing a benchmark for understanding the initial welfare impact of introducing a small ad valorem tax.

At the laissez-faire allocation ($\tau = s = 0$), the derivative of welfare with respect to τ is:

$$\left.\frac{\partial W_C^F(\tau, s)}{\partial \tau}\right|_{\tau=s=0} = -\frac{(b-3)}{48(b-a)}(7b^2 + 30b - 33) > 0.$$

More generally, the derivative of welfare with respect to τ (while keeping $s = 0$) is:

$$\left.\frac{\partial W_C^F}{\partial \tau}\right|_{s=0} = -\frac{(b-3)}{48(b-a)}\left[(7b^2 + 30b - 33) + 4(b^2 - 30b + 33)\tau\right].$$

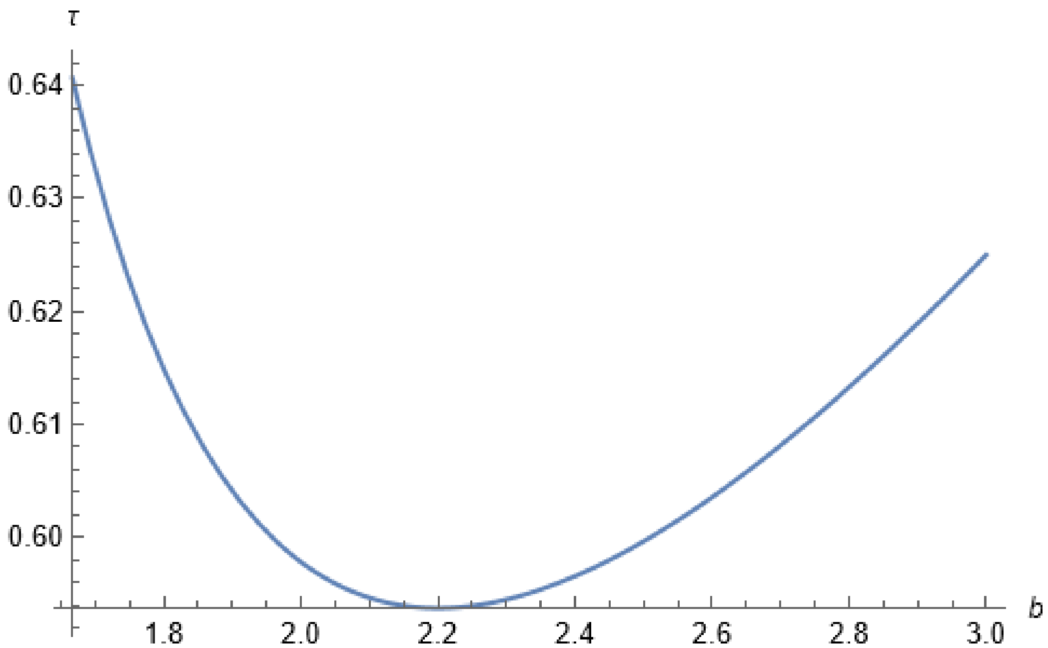

Figure 3.16: The optimal ad valorem policy τ_C^F

Maximizing W_C^F yields the optimal ad valorem tax rate:

$$\tau_C^F = -\frac{7b^2 + 30b - 33}{4(b^2 - 30b + 33)}.$$

Proposition 3.3.6 (U-shaped Optimal Intervention Strategy τ_C^F). *The optimal τ_C^F initially decreases with b, and then increases as b continues to grow, while always remaining above 50per cent.*

From the figure 3.16, the optimal τ_C^F under the adjusted-cost welfare criterion displays a U-shaped with respect to the parameter b. In particular for both relatively small and relatively large values of b a high taxation policy, the optimal policy prescribes a high tax rate, typically exceeding, larger than 50 per cent. In this case, tax increases can offset revenue losses by improving total consumer surplus. When b is close to 2.2, the optimal ad valorem also approaches 0.5. This implies the ad valorem taxation always helps consumers overcome self-control problems when considering a foreign monopolist.

The intuition behind the U-shape is as follows. When the market is relatively small, temptation distortions are severe, and a high tax is required to mitigate excessive consumption induced by the monopolist's pricing strategy. As the market grows, these distortions gradually diminish, lowering the need for heavy taxation; hence the optimal rate falls toward 50 percent around the mid-range of b. However, for very large markets, the monopolist's ability to extract surplus again amplifies the importance of corrective taxation, driving the optimal rate back above 50 percent. Thus, the U-shape captures the trade-off between reducing temptation costs and maintaining sufficient consumer surplus across different market sizes.

The normative social welfare W_U^F, consists of the sum of (i) consumer commitment utility surplus and (ii) tariff revenues, given by

$$W_U^F(\tau, s) = \underbrace{\int_a^b [q(\gamma) - p(\gamma)] f(\gamma) d\gamma}_{\text{Commitment Utility Surplus}} + \underbrace{\int_a^b [\tau p(\gamma) + s] f(\gamma) d\gamma}_{\text{Tax Revenues}}.$$

At the laissez-faire benchmark ($\tau = s = 0$), the derivative of welfare with respect to τ is:

$$\frac{\partial W_U^F(\tau, s)}{\partial \tau}\bigg|_{\tau=s=0} = \frac{(b-3)}{12(b-a)}(b^2 - 18b + 21) > 0.$$

More generally, when holding $s = 0$), the welfare gradient is:

$$\frac{\partial W_U^F(\tau, s)}{\partial \tau}\bigg|_{s=0} = \frac{(b-3)}{12(b-a)} \left[(b^2 - 18b + 21) + (-b^2 + 30b - 33)\tau\right].$$

Accordingly, the optimal ad valorem policy

$$\tau_U^F = \frac{b^2 - 18b + 21}{b^2 - 30b + 33}.$$

Proposition 3.3.7 (**Optimal Intervention Strategy** τ_U^F). *The optimal* τ_C^F *is monotonically increasing in* b*, and remains strictly below* 50 *per cent for all admissible values of* b.

As illustrated in Figure 3.17, the optimal ad valorem policy τ_U^F converges toward 50 per cent as b increases. Note that taxation reduces total standard

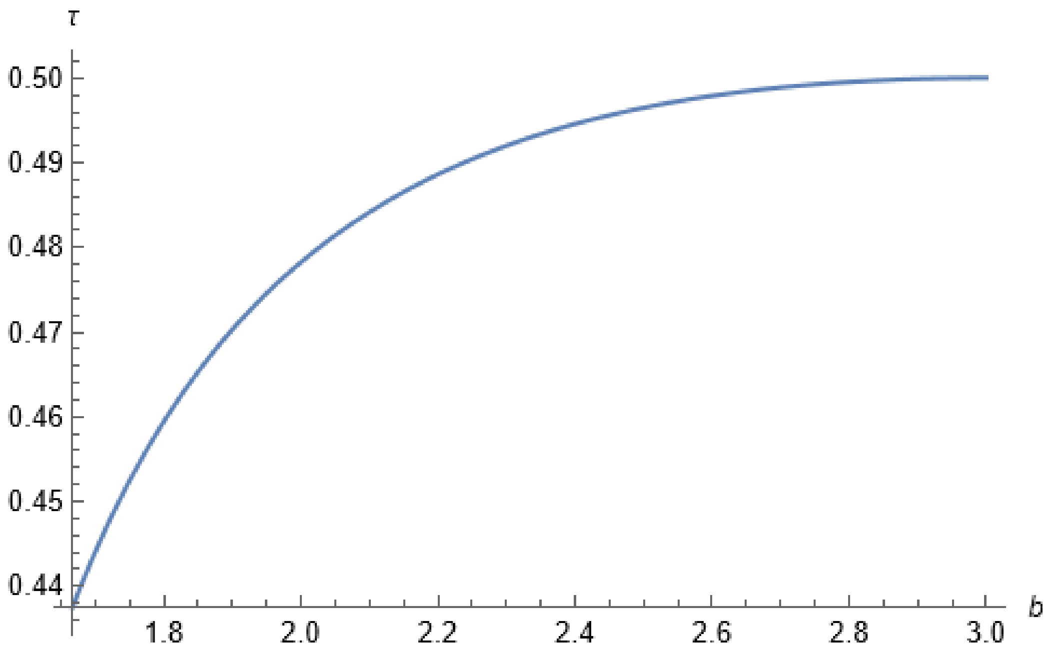

Figure 3.17: The optimal ad valorem tax policy τ_U^F

consumer surplus on one hand while on the other it increases the total tax revenue. In relatively small markets, the reduction in consumer surplus induced by taxation is limited, while the associated increase in tariff revenue is substantial. Consequently, the revenue-raising effect dominates, rendering taxation welfare-enhancing.

The behavioral social welfare W_{U+V}^F, consists of the sum of (i) ex-post consumer surplus and (ii) tariff revenues, given by

$$W_{U+V}^F(\tau, s) = \underbrace{\underbrace{\int_a^b [q(\gamma) - p(\gamma)]\, f(\gamma) d\gamma}_{\text{Commitment Utility Surplus}} + \underbrace{\int_a^b [\gamma q(\gamma) - p(\gamma)]\, f(\gamma) d\gamma}_{\text{Temptation Utility Surplus}}}_{\text{Ex-Post Consumer Surplus}} + \underbrace{\int_a^b [\tau p(\gamma) + s]\, f(\gamma) d\gamma}_{\text{Tax Revenues}}.$$

At the laissez-faire benchmark ($\tau = s = 0$), the derivative of welfare with respect to τ is:

$$\left.\frac{\partial W_{U+V}^F(\tau, s)}{\partial \tau}\right|_{\tau=s=0} = \frac{b-3}{96(b-a)}(35b^2 - 162b + 123) > 0.$$

The derivative of welfare with respect to τ (while keeping $s = 0$) is:

$$\left.\frac{\partial W^F_{U+V}(\tau, s)}{\partial \tau}\right|_{s=0} = \frac{b-3}{96(b-a)} \left[(35b^2 - 162b + 123) + 8(-b^2 + 30b - 33)\tau\right].$$

The optimal ad valorem policy is

$$\tau^F_{U+V} = \frac{35b^2 - 162b + 123}{8(b^2 - 30b + 33)}.$$

Proposition 3.3.8 (**Optimal Intervention Strategy** τ^F_{U+V}). *The optimal* τ^F_C *is monotonically decreasing in* b*, and remains strictly below* 50 *percent for all admissible values of* b.

As shown in Figure 3.18, the optimal ad valorem policy τ^F_U decreases below 50 per cent as b increases. This is because taxation reduces total consumer surplus and the optimal ad valorem policy is below 50 per cent. In smaller markets, by contrast, the reduction in consumer surplus induced by taxation is relatively limited, so that the revenue-enhancing effect dominates. Consequently, the optimal tax remains positive but lies strictly below one-half, highlighting the weaker role of ad valorem taxation in welfare improvement compared to the adjusted-cost or normative frameworks.

The optimal policy satisfies $\tau^F_C > \tau^F_U > \tau^F_{U+V}$ as the graph shows. The results are consistent with the standard model which suggests that market protection policies are always welfare-improving. The main difference is the effect of taxation in reducing consumer's self control costs, which was not taken into account before. The adjusted-cost opinion emphasizes that a heavy tax on a foreign product can improve the country's welfare. For the dependence on market size, paternalism and libertarianism show the opposite results. As the proportion of agents with γ closer to 1 increases, the optimal ad valorem raises when considering normative welfare; while the optimal ad valorem tax reduces when considering behavioral welfare.

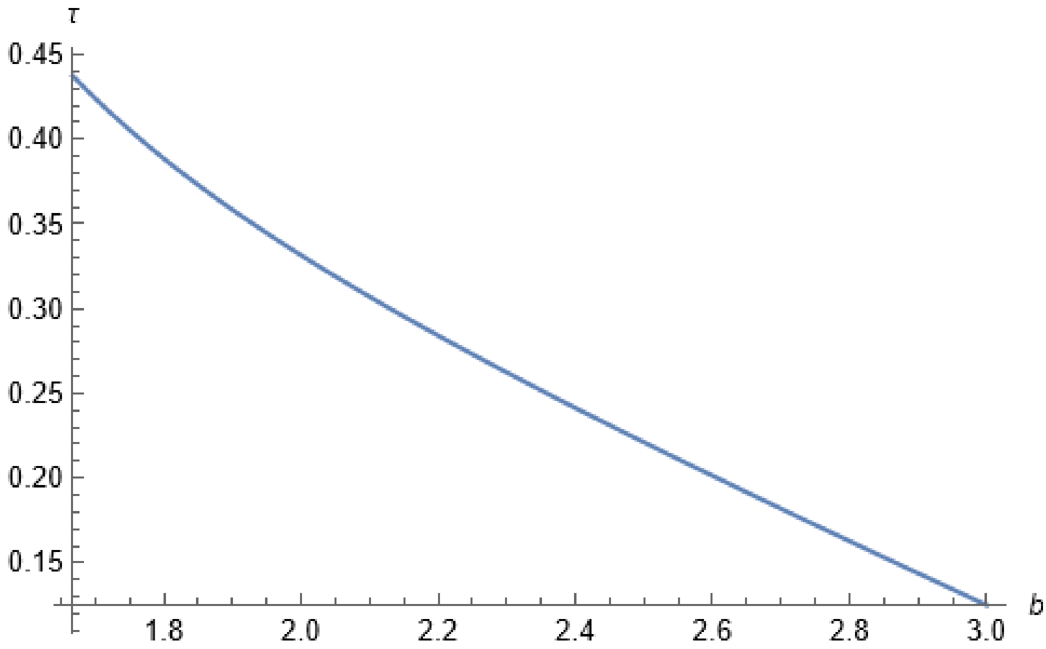

Figure 3.18: The optimal ad valorem tax policy τ^F_{U+V}

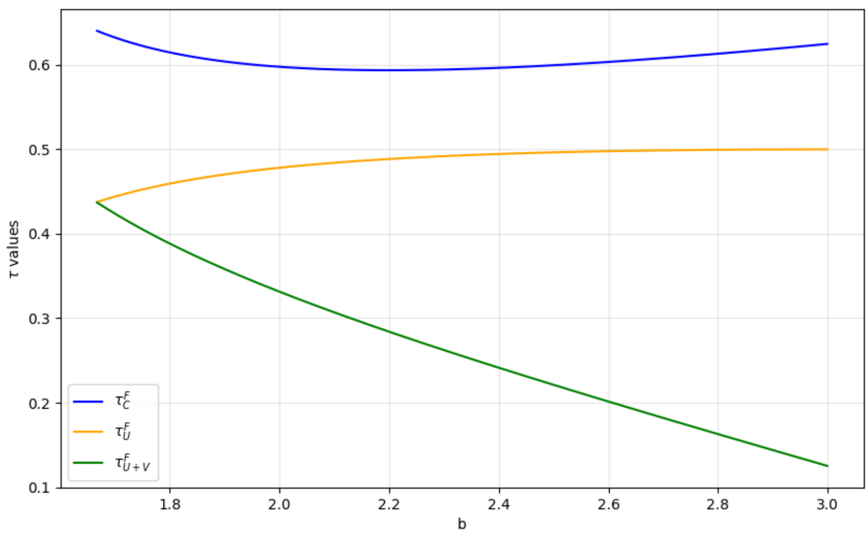

Figure 3.19: The comparison of the optimal ad valorem policy τ^F with a foreign monopolist

Social welfare effect of the specific tax policy

To determine the effect of the introduction of small specific tax ($\tau = s = 0$), I take derivative of $W_C^F(\tau, s)$ with respect to s evaluated at $\tau = s = 0^+$ and $\tau = s = 0^-$:

$$\left.\frac{\partial W_C^F(\tau, s)}{\partial s}\right|_{\tau=0,s=0^+} = \frac{-5b^2 + 2b + 13}{8(b-a)(b+1)},$$
$$\left.\frac{\partial W_C^F(\tau, s)}{\partial s}\right|_{\tau=0,s=0^-} = \frac{b+1}{2(b-a)} > 0.$$

The choice between implementing a tax or subsidy depends on whether the parameter b falls below or exceeds a critical threshold b_C^F :

$$b_C^F = \frac{1+\sqrt{66}}{5} \approx 1.83.$$

For $b \in \left(\frac{5}{3}, b_C^F\right)$, the welfare gradient is strictly positive:

$$\left.\frac{\partial W_C^F(\tau, s)}{\partial s}\right|_{\tau=s=0^+} > 0.$$

This indicates that marginal taxation generates a first-order welfare improvement when initial productivity lies in this interval.

At the boundary $b = b_C^F$, the welfare gradient vanishes:

$$\left.\frac{\partial W_C^F(\tau, s)}{\partial s}\right|_{\tau=s=0^+} = 0.$$

At this point, neither taxation nor subsidy lead to welfare improvements.

For $b \in (b_C^F, 3)$, the welfare gradient becomes negative:

$$\left.\frac{\partial W_C^F(\tau, s)}{\partial s}\right|_{\tau=s=0^+} < 0.$$

In this regime, any small tax imposition reduces aggregate welfare.

To obtain the optimal specific taxation-only policy, I differentiate $W_C^F(\tau, s)$

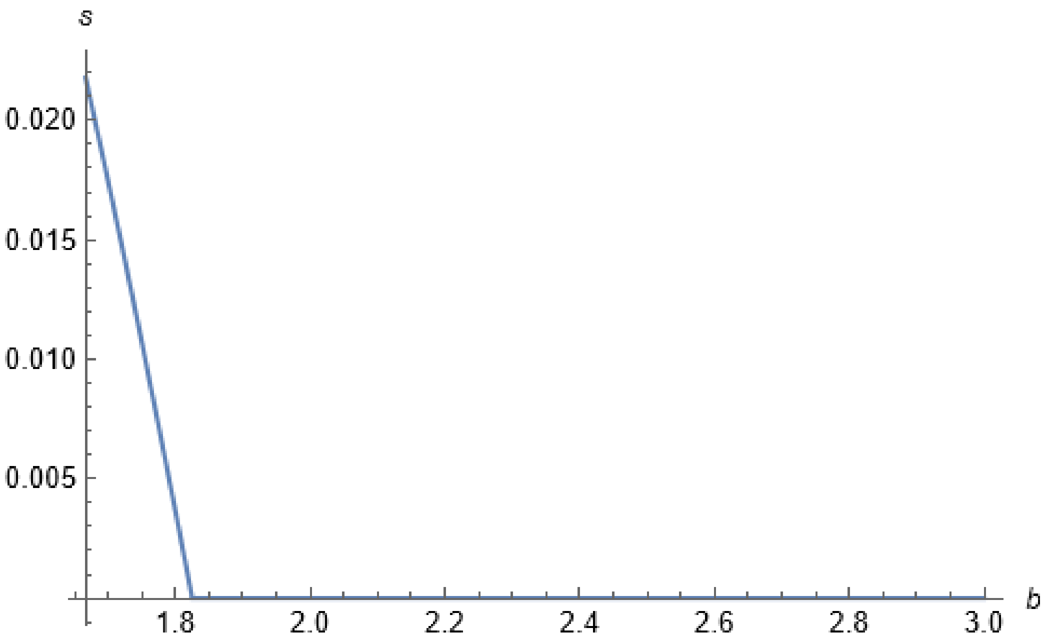

Figure 3.20: The optimal specific tax policy s_C^F

with respect to $s > 0$ and $s < 0$:

$$\frac{\partial W_C^F(\tau, s)}{\partial s}\Big|_{\tau=0,s>0} = \frac{-b^2 - 6b + 1 - 96s + 4(3-b)\sqrt{(b+1)^2 - 16s}}{8(b-a)\sqrt{(b+1)^2 - 16s}},$$
$$\frac{\partial W_C^F(\tau, s)}{\partial s}\Big|_{\tau=0,s<0} = \frac{b+1}{2(b-a)} > 0.$$

Proposition 3.3.9 (**Optimal Intervention Strategy** s_C^F)**.** *The optimal specific policy*

$$s_C^F = \begin{cases} \frac{-7b^2+6b-33}{288} + \frac{(3-b)\sqrt{23b^2+42b+33}}{72\sqrt{2}}, & \text{when } b \in (\frac{5}{3}, b_C^F) \\ 0, & \text{when } b \in [b_C^F, 3) \end{cases}$$

For populations with:

(a) ***Small-market size*** ($b \in (\frac{5}{3}, b_C^F)$)*: a tax policy $s > 0$ is optimal.*

(b) ***Large-market size*** ($b \in [b_C^F, 3)$)*: no intervention is optimal.*

To determine the effect of the introduction of small specific tax ($\tau = s = 0$), I differentiate $W_U^F(\tau, s)$ with respect to s and evaluate the result at

$\tau = s = 0^+$ and $\tau = s = 0^-$:

$$\left.\frac{\partial W^F_U(\tau,s)}{\partial s}\right|_{\tau=0,s=0^+} = \frac{b+1}{2(b-a)} > 0,$$
$$\left.\frac{\partial W^F_U(\tau,s)}{\partial s}\right|_{\tau=0,s=0^-} = \frac{b+1}{2(b-a)} > 0.$$

To obtain the optimal specific taxation-only policy, I take derivative of $W^F_U(\tau, s)$ with respect to $s > 0$ and $s < 0$:

$$\left.\frac{\partial W^F_U(\tau,s)}{\partial s}\right|_{\tau=0,s>0} = \frac{3b^2+2b-1-32s-2(b-1)\sqrt{(b+1)^2-16s}}{2(b-a)\sqrt{(b+1)^2-16s}},$$
$$\left.\frac{\partial W^F_U(\tau,s)}{\partial s}\right|_{\tau=0,s<0} = \frac{b+1}{2(b-a)} > 0.$$

Proposition 3.3.10 (Optimal Intervention Strategy s^F_U). *The optimal specific policy*

$$s^F_U = \min\left\{\frac{2(b^2+2b-1)-(b-1)\sqrt{8-(b-1)^2}}{32}, \frac{-3b^2+10b-3}{16}\right\}$$
$$= \begin{cases} \frac{2(b^2+2b-1)-(b-1)\sqrt{8-(b-1)^2}}{32}, & when\, b \in (\frac{5}{3}, 1+\frac{2}{\sqrt{5}}) \\ \frac{-3b^2+10b-3}{16}, & when\, b \in [1+\frac{2}{\sqrt{5}}, 3) \end{cases}$$

For populations with:

(a) ***Small-market size*** *($b \in (\frac{5}{3}, 1+\frac{2}{\sqrt{5}})$): the optimal tax policy $s > 0$ is increasing on b.*

(b) ***Large-market size*** *($b \in [1+\frac{2}{\sqrt{5}}, 3)$): the optimal tax policy $s > 0$ is decreasing on b.*

To determine the effect of the introduction of small specific tax ($\tau = s = 0$), I take derivative of $W^F_{U+V}(\tau, s)$ with respect to s evaluated at $\tau = s = 0^+$ and $\tau = s = 0^-$:

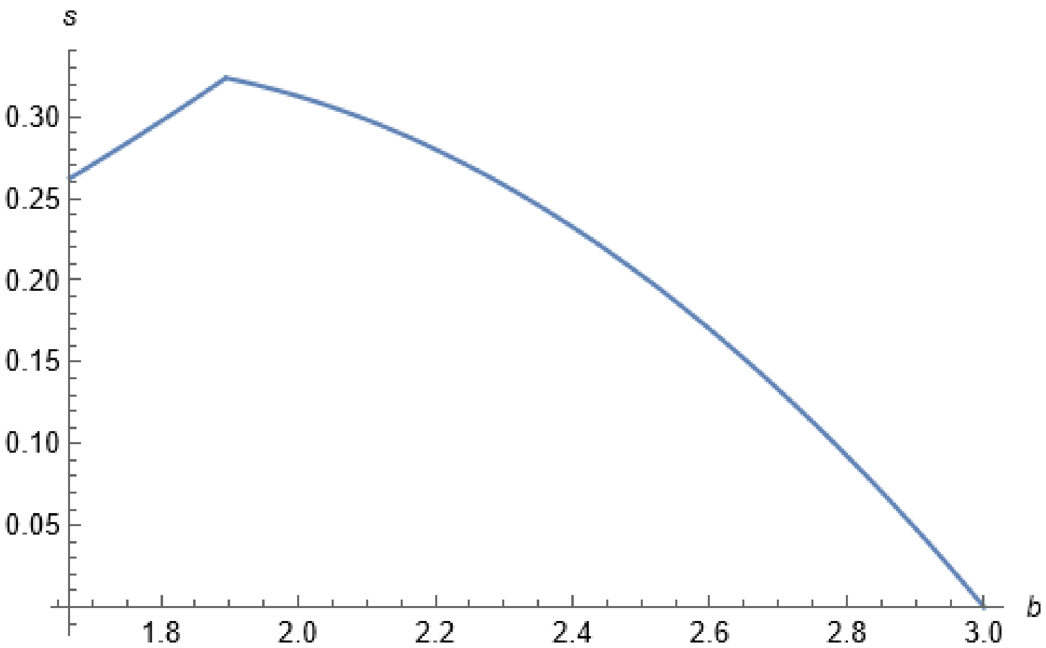

Figure 3.21: The optimal specific policy s^F_U

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} = \frac{-3b^2+4b+2}{4(b-a)(b+1)},$$
$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^-} = \frac{b+1}{2(b-a)} > 0.$$

The choice between implementing a tax or subsidy depends on whether the parameter b falls below or exceeds the critical threshold b^F_{U+V} :

$$b^F_{U+V} = \frac{2+\sqrt{10}}{3} \approx 1.72$$

For $b \in \left(\frac{5}{3}, b^F_{U+V}\right)$, the welfare gradient is strictly positive:

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=s=0^+} > 0.$$

This indicates that marginal taxation generates a first-order welfare improvement when initial productivity lies in this interval.

At the boundary $b = b^F_{U+V}$, the welfare gradient vanishes:

$$\frac{\partial W^F_{U+V}(\tau, s)}{\partial s}\Big|_{\tau=s=0^+} = 0.$$

No intervention yields a welfare gain at this transition point.

For $b \in (b^F_{U+V}, 3)$, the welfare gradient becomes negative:

$$\frac{\partial W^F_{U+V}(\tau, s)}{\partial s}\Big|_{\tau=s=0^+} < 0.$$

In this regime, any marginal tax imposition reduces aggregate welfare.

To obtain the optimal specific taxation-only policy, I take derivative of $W^F_{U+V}(\tau, s)$ with respect to $s > 0$ and $s < 0$:

$$\frac{\partial W^F_{U+V}(\tau, s)}{\partial s}\Big|_{\tau=0,s>0} = \frac{-7b^2 - 4b - 2 - 16s + 4(b+1)\sqrt{(b+1)^2 - 16s}}{4(b-a)\sqrt{(b+1)^2 - 16s}},$$
$$\frac{\partial W^F_{U+V}(\tau, s)}{\partial s}\Big|_{\tau=0,s<0} = \frac{b+1}{2(b-a)} > 0.$$

Proposition 3.3.11 (**Optimal Intervention Strategy** s^F_{U+V})**.** *The optimal specific policy*

$$s^F_{U+V} = \begin{cases} -\frac{5}{16}(3b^2 + 4b + 2) + \frac{b+1}{4}\sqrt{12b^2 + 14b + 7}, & \textit{when } b \in (\frac{5}{3}, b^F_{U+V}) \\ 0, & \textit{when } b \in [b^F_{U+V}, 3) \end{cases}$$

For populations with:

(a) ***Small-market size*** $(b \in (\frac{5}{3}, b^F_{U+V}))$*: a tax policy $s > 0$ is optimal.*

(b) ***Large-market size*** $(b \in [b^F_{U+V}, 3))$*: no intervention is optimal.*

The specific tax policy for a foreign monopolist satisfies $s^F_{U+V} \leq s^F_C < s^F_U$. When $b < b^F_U$, the optimal policy under the utilitarian benchmark s^F_U is above 0.25 and increases in b. This is because a smaller market size causes the total deadweight consumer surplus losses to reduce. Conversely, when $b > b^F_U$, due to the presence of an upper bound on the specific tax s, the optimal policy reaches the maximum value allowed.

Specific taxes exerts a pronounced effect on temptation utility. As a result,

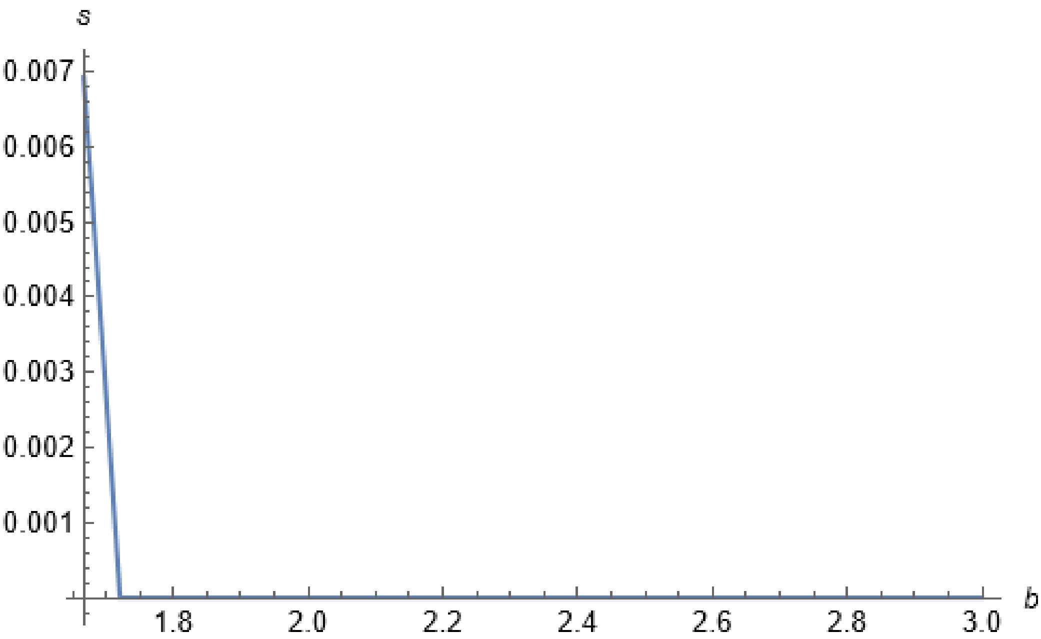

Figure 3.22: The optimal specific tax policy s^F_{U+V}

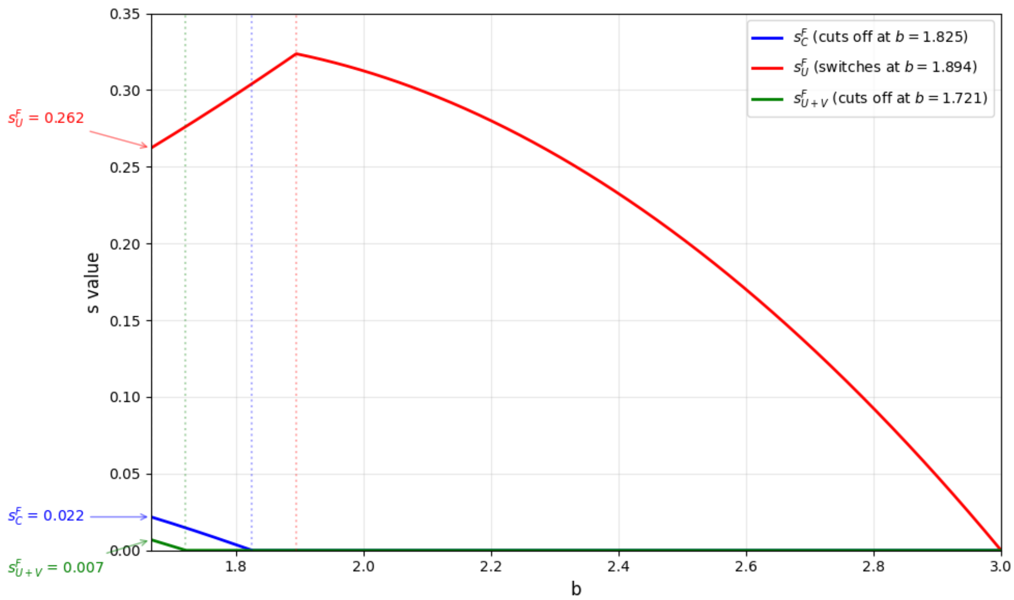

Figure 3.23: The comparison of the optimal specific tax policy s^F with a foreign monopolist

the optimal s^F_{U+V} and s^F_C are very small, as the imposition of a specific tax substantially reduces the maximum level of temptation faced by consumers. The specific tax also reduces the maximum temptation that each consumer faces in the adjusted-cost welfare. The sum of self-control costs are more sensitive to increases when s increases compared to the results of s^F_U.

3.4 Conclusion

In this paper, I have discussed the optimal taxation policy in a monopoly market when consumers face self-control problems over different quality-price bundles. I distinguish consumers by their different degree of temptation and characterize the monopolist profit maximizing menu provided. I conduct behavioral welfare analysis for this model and show that for a domestic firm a small ad valorem tax can increase the social welfare when upward tempted consumers are numerous among the population. On the other hand, an valorem subsidy can increase the welfare when the population consists mostly of downward tempted consumers. Furthermore, in this case no specific policy intervention is optimal. For a foreign firm, both ad valorem and specific taxation policies can increase national welfare. This is a similar conclusion to that of the standard model; optimal policy level may be higher in my model however. The results are robust to the normative framework used.

The findings in my model are in sharp contrast traditional views on taxation policies for "sin goods". It is often argued by policymakers that higher tax rates can lead to welfare improvement by referring to health benefits or lowering any perceived over-consumption of the goods. My findings suggest that the adoption of an ad valorem domestic taxation policy should depend on the temptation range of the population. In populations with mostly downward tempted consumers, there is no justification for imposing a corrective tax; in fact, an ad valorem subsidy could potentially enhance welfare. For instance, a country where alcohol consumption is not deeply embedded in the culture may rationally opt for a lower tax rate on alcohol—or even a subsidy—compared to a nation where some consumers indulge in large alcohol consumption. Even some countries use a complex tax rate sort of mixture of ad valorem tax and specific tax, the sole specific tax will not be effective on the welfare by separating these two taxes.

One advantage of my model is that it clearly identifies the effect on government revenue arising from both ad valorem and specific taxes on a foreign monopolist. This provides clear predictions when behavioral assumptions are included versus when they are not and can consequently help inform policy. In addition, under behavioral assumptions I have shown that higher tax rates are needed to increase the welfare by helping reduce temptation costs.

Although this paper characterizes the optimal taxation policy in the context of a monopoly market where consumers face heterogeneous temptations in a parsimonious way. There are still some important aspects that could be included. First, in line with most work in the literature the model could be extended to a dynamic version, in a similar fashion to Gul and Pesendorfer (2007). Second, the market structure could be replaced by a competitive market. This would allow a comparison between optimal taxation policies and help to identify welfare improvements due to correcting market inefficiencies arising from the monopoly structure.

APPENDIX C

Proof of proposition 3.2.2 - proposition 3.2.3. Now I will confirm the optimal solution in separate situation based on the values of λ and μ. Obviously, $\lambda = \mu = 0$ violates the first-order condition (3.23). I will then discuss the other three possibilities:

i. If $\lambda > 0$ and $\mu = 0$, i.e. $\lambda = 1$, then only ex-ante IR for γ_{H} binds, i.e.

$$q(\gamma_{\text{H}}) - p(\gamma_{\text{H}}) = 0$$

and then

$$(\gamma_{\text{H}} - 1)q(\gamma_{\text{H}}) - (\gamma_{\text{H}} - \gamma_{\text{L}})q(\gamma_{\text{L}}) \geq 0.$$

The first order conditions imply

$$q(\gamma_{\text{H}}) = 1, \qquad q(\gamma_{\text{L}}) = \frac{1 + \gamma_{\text{L}}}{2}, \quad \text{with } \lambda = 1.$$

Therefore,

$$p(\gamma_{\text{H}}) = 1, \qquad p(\gamma_{\text{L}}) = \frac{(1 + \gamma_{\text{L}})^2}{4}.$$

The condition that $\gamma_{\text{H}} \geq \gamma_{\text{L}} + 2$ ensures that incentive compatibility for γ_{H} with the condition $p(\gamma_{\text{H}}) = q(\gamma_{\text{H}})$ holds, i.e. $(\gamma_{\text{H}} - 1)q(\gamma_{\text{H}}) - (\gamma_{\text{H}} - \gamma_{\text{L}})q(\gamma_{\text{L}}) = (\gamma_{\text{H}} - 1) - (\gamma_{\text{H}} - \gamma_{\text{L}})\frac{1+\gamma_{\text{L}}}{2} = \frac{(1-\gamma_{\text{L}})(\gamma_{\text{H}}-\gamma_{\text{L}}-2)}{2} \geq 0$.

In this case, the monopolist's aggregate profit from both consumers is

$$\pi_{\substack{\lambda=1\\ \mu=0}} = \pi_{\substack{\lambda=1\\ \mu=0}}(\gamma_{\text{H}}) + \pi_{\substack{\lambda=1\\ \mu=0}}(\gamma_{\text{L}}) = \frac{1}{2} + \frac{(1 + \gamma_{\text{L}})^2}{8} = \pi^C.$$

Then, the welfare is expressed by

$$W^{B}_{\substack{\lambda=1\\ \mu=0}} = \pi_{\substack{\lambda=1\\ \mu=0}} = \frac{1}{2} + \frac{(1 + \gamma_{\text{L}})^2}{8} = \pi^C.$$

ii. If $\lambda = 0$ and $\mu > 0$, i.e. $\mu = 1$,then then only ex-post IC for γ_{H} binds, i.e.

$$2p(\gamma_{\rm H}) = (1+\gamma_{\rm H})q(\gamma_{\rm H}) - (\gamma_{\rm H}-\gamma_{\rm L})q(\gamma_{\rm L})$$

and then

$$q(\gamma_{\rm H}) - p(\gamma_{\rm H}) \geq 0$$

The first order conditions imply in this case that

$$q(\gamma_{\rm H}) = \frac{1+\gamma_{\rm H}}{2}, \qquad q(\gamma_{\rm L}) = \frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2} \quad \text{with } \mu = 1.$$

The bounds $1 < \gamma_{\rm H} < 2\gamma_{\rm L}+1$ and $0 < \gamma_{\rm L} < 1$ guarantee positive quality is provided to both consumers.

$$p(\gamma_{\rm H}) = \frac{(1+\gamma_{\rm H})^2 - (\gamma_{\rm H}-\gamma_{\rm L})(1+2\gamma_{\rm L}-\gamma_{\rm H})}{4}, \quad p(\gamma_{\rm L}) = \frac{(1+\gamma_{\rm L})^2 - (\gamma_{\rm H}-\gamma_{\rm L})(1+\gamma_{\rm L})}{4}.$$

Then I check if the ex-ante IR for $\gamma_{\rm H}$ is valid: $q(\gamma_{\rm H}) - p(\gamma_{\rm H}) \geq 0$. I have the bound $\gamma_{\rm H} \leq \frac{1+3\gamma_{\rm L}+\sqrt{9-2\gamma_{\rm L}-7\gamma_{\rm L}^2}}{4} < \gamma_{\rm L}+1 < 2\gamma_{\rm L}+1$.

The maximal aggregate profit for the monopolist is then given by

$$\begin{aligned}
\pi_{\substack{\lambda=0\\ \mu=1}} &= \pi_{\substack{\lambda=0\\ \mu=1}}(\gamma_{\rm H}) + \pi_{\substack{\lambda=0\\ \mu=1}}(\gamma_{\rm L}) \\
&= \frac{(1+\gamma_{\rm H})^2 - 2(\gamma_{\rm H}-\gamma_{\rm L})(1+2\gamma_{\rm L}-\gamma_{\rm H})}{8} + \frac{(1+\gamma_{\rm L})^2 - (\gamma_{\rm H}-\gamma_{\rm L})^2}{8} \\
&= \frac{1}{2} + \frac{(1+\gamma_{\rm L})^2}{8} + \frac{2(\gamma_{\rm H}-\gamma_{\rm L})^2 + \gamma_{\rm L}^2 + 2\gamma_{\rm L} - 3}{8} = \pi^C + L_{IR},
\end{aligned}$$

where $L_{IR} = \frac{2(\gamma_{\rm H}-\gamma_{\rm L})^2+\gamma_{\rm L}^2+2\gamma_{\rm L}-3}{8} < 0$ on $0 < \gamma_{\rm L} < 1 < \gamma_{\rm H} \leq \frac{1+3\gamma_{\rm L}+\sqrt{9-2\gamma_{\rm L}-7\gamma_{\rm L}^2}}{4}$, which can be proved by a numerical method. This is because of the profit loss due to providing positive ex-ante utility with $\gamma_{\rm H}$.

$$\begin{aligned} W^{B}_{\substack{\lambda=0\\ \mu=1}} &= q(\gamma_{\rm H}) - \frac{1}{2}q(\gamma_{\rm H})^2 + \pi_{\substack{\lambda=0\\ \mu=1}}(\gamma_{\rm L}) \\ &= \frac{4(1+\gamma_{\rm H}) - (1+\gamma_{\rm H})^2}{8} + \frac{(1+\gamma_{\rm L})^2 - (\gamma_{\rm H}-\gamma_{\rm L})^2}{8} \\ &= \frac{(1+\gamma_{\rm H})(2+\gamma_{\rm L}-\gamma_{\rm H})}{4}. \end{aligned}$$

iii. If both $\lambda, \mu > 0$, then both constraints bind, i.e.

$$q(\gamma_{\rm H}) - p(\gamma_{\rm H}) = 0$$

and

$$(1+\gamma_{\rm H})q(\gamma_{\rm H}) - (\gamma_{\rm H}-\gamma_{\rm L})q(\gamma_{\rm L}) - 2p(\gamma_{\rm H}) = 0.$$

The first order conditions imply

$$q(\gamma_{\rm H}) = p(\gamma_{\rm H}) = 1 + \frac{\mu(\gamma_{\rm H}-1)}{2},$$

$$q(\gamma_{\rm L}) = \frac{1+\gamma_{\rm L} - \mu(\gamma_{\rm H}-\gamma_{\rm L})}{2}, \quad p(\gamma_{\rm L}) = \frac{(1+\gamma_{\rm L})^2 - \mu(1+\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L})}{4}.$$

From the condition $(1+\gamma_{\rm H})q(\gamma_{\rm H}) - (\gamma_{\rm H}-\gamma_{\rm L})q(\gamma_{\rm L}) - 2p(\gamma_{\rm H}) = 0$, I have $\mu = \frac{(\gamma_{\rm L}-1)(\gamma_{\rm H}-\gamma_{\rm L}-2)}{(\gamma_{\rm H}-1)^2+(\gamma_{\rm H}-\gamma_{\rm L})^2}$ where $\gamma_{\rm H}-\gamma_{\rm L} < 2$ to ensure $\mu > 0$.

The maximal aggregate profit of the monopolist in the case $\lambda, \mu > 0$ is thus

$$\begin{aligned} \pi_{\lambda,\mu>0} &= \pi_{\lambda,\mu>0}(\gamma_{\rm H}) + \pi_{\lambda,\mu>0}(\gamma_{\rm L}) \\ &= (\frac{1}{2} - \frac{\mu^2(\gamma_{\rm H}-1)^2}{8}) + \frac{(1+\gamma_{\rm L})^2 - \mu^2(\gamma_{\rm H}-\gamma_{\rm L})^2}{8} \\ &= \frac{1}{2} + \frac{(1+\gamma_{\rm L})^2}{8} - \frac{\mu^2[(\gamma_{\rm H}-1)^2 + (\gamma_{\rm H}-\gamma_{\rm L})^2]}{8} = \pi^C + L_{IC}, \end{aligned}$$

where $L_{IC} = -\frac{\mu^2[(\gamma_{\rm H}-1)^2+(\gamma_{\rm H}-\gamma_{\rm L})^2]}{8} < 0$. This profit loss is caused by the distortion from incentive compatibility where the upwards tempted consumer has no incentive to mimic the downwards temptation.

$$W^B_{\lambda,\mu>0} = \pi_{\lambda,\mu>0} = \frac{1}{2} + \frac{(1+\gamma_{\text{L}})^2}{8} - \frac{\mu^2[(\gamma_{\text{H}}-1)^2 + (\gamma_{\text{H}}-\gamma_{\text{L}})^2]}{8}.$$

Last, I confirm that the solution is global maximum. When $\gamma_{\text{H}} \geq \gamma_{\text{L}} + 2$, the unique solution under the constraints is characterized by the multipliers $\mu = 0$ and $\lambda = 1$. The optimal bundle is therefore $\{(q^*(\gamma_{\text{H}}) = 1, p^*(\gamma_{\text{H}}) = 1), (q^*(\gamma_{\text{L}}) = \frac{1+\gamma_{\text{L}}}{2}, p^*(\gamma_{\text{L}}) = \frac{(1+\gamma_{\text{L}})^2}{4})\}$, and hence the optimal profit is

$$\pi^*_1 = \pi_{\substack{\lambda=1\\ \mu=0}} = \frac{1}{2} + \frac{(1+\gamma_{\text{L}})^2}{8}.$$

When $\frac{1+3\gamma_{\text{L}}+\sqrt{9-2\gamma_{\text{L}}-7\gamma_{\text{L}}^2}}{4} \leq \gamma_{\text{H}} < \gamma_{\text{L}}+2$, the unique solution is characterized by $\lambda, \mu > 0$.

$$q^*(\gamma_{\text{H}}) = p^*(\gamma_{\text{H}}) = 1 + \frac{(\gamma_{\text{L}}-1)(\gamma_{\text{H}}-1)(\gamma_{\text{H}}-\gamma_{\text{L}}-2)}{2(\gamma_{\text{H}}-1)^2 + 2(\gamma_{\text{H}}-\gamma_{\text{L}})^2},$$

$$q^*(\gamma_{\text{L}}) = \frac{1+\gamma_{\text{L}}}{2} + \frac{(1-\gamma_{\text{L}})(\gamma_{\text{H}}-\gamma_{\text{L}})(\gamma_{\text{H}}-\gamma_{\text{L}}-2)}{2(\gamma_{\text{H}}-1)^2 + 2(\gamma_{\text{H}}-\gamma_{\text{L}})^2}$$

$$p^*(\gamma_{\text{L}}) = \frac{(1+\gamma_{\text{L}})^2}{4} + \frac{(1-\gamma_{\text{L}}^2)(\gamma_{\text{H}}-\gamma_{\text{L}})(\gamma_{\text{H}}-\gamma_{\text{L}}-2)}{4(\gamma_{\text{H}}-1)^2 + 4(\gamma_{\text{H}}-\gamma_{\text{L}})^2}.$$

and

$$\pi^*_2 = \pi_{\lambda,\mu>0} = \frac{1}{2} + \frac{(1+\gamma_{\text{L}})^2}{8} - \frac{(\gamma_{\text{L}}-1)^2(\gamma_{\text{H}}-\gamma_{\text{L}}-2)^2}{8[(\gamma_{\text{H}}-1)^2 + (\gamma_{\text{H}}-\gamma_{\text{L}})^2]}$$

.

When $1 < \gamma_{\text{H}} \leq \frac{1+3\gamma_{\text{L}}+\sqrt{9-2\gamma_{\text{L}}-7\gamma_{\text{L}}^2}}{4}$, there are two possible scenarios: the ex-ante IR for γ_{H} may bind or not, while the ex-post IC for γ_{H} binds. For any $\gamma_{\text{L}} \in (0,1)$, $L_{IR} > L_{IC}$. Therefore, the optimal solution is under the slack ex-ante IR and the binding ex-post IC, which is characterized by

$$q^*(\gamma_{\text{H}}) = \frac{1+\gamma_{\text{H}}}{2}, \qquad p^*(\gamma_{\text{H}}) = \frac{(1+\gamma_{\text{H}})^2 + (\gamma_{\text{L}}-\gamma_{\text{H}})(1+2\gamma_{\text{L}}-\gamma_{\text{H}})}{4},$$

$$q^*(\gamma_{\rm L}) = \frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2}, \quad p^*(\gamma_{\rm L}) = \frac{(1+\gamma_{\rm L})^2-(1+\gamma_{\rm L})(\gamma_{\rm H}-\gamma_{\rm L})}{4},$$

and

$$\pi_3^* = \pi_{\substack{\lambda=0\\ \mu=1}} = \frac{1}{2} + \frac{(1+\gamma_{\rm L})^2}{8} + \frac{2(\gamma_{\rm H}-\gamma_{\rm L})^2+\gamma_{\rm L}^2+2\gamma_{\rm L}-3}{8}.$$

□

Proof of proposition 3.2.4. Differentiating with respect to $q^T(\gamma_{\rm H})$, $q^T(\gamma_{\rm L})$ and $p^T(\gamma_{\rm H})$ respectively yields the first order conditions

$$-2q^T(\gamma_{\rm H}) + 2\lambda^T + \mu^T(1+\gamma_{\rm H}) = 0, \tag{3.61}$$

$$(1-\tau)(1+\gamma_{\rm L}) - 2q^T(\gamma_{\rm L}) - \mu^T(\gamma_{\rm H}-\gamma_{\rm L}) = 0, \tag{3.62}$$

$$(1-\tau) - \lambda^T - \mu^T = 0. \tag{3.63}$$

These are complemented with the two complementary slackness conditions

$$\lambda^T[q^T(\gamma_{\rm H}) - p^T(\gamma_{\rm H})] = 0, \tag{3.64}$$

$$\mu^T[(1+\gamma_{\rm H})q^T(\gamma_{\rm H}) - (\gamma_{\rm H}-\gamma_{\rm L})q^T(\gamma_{\rm L}) - 2p^T(\gamma_{\rm H})] = 0. \tag{3.65}$$

From condition (3.63), I have $\lambda^T = (1-\tau) - \mu^T$. Putting is into conditions (3.61) and (3.62), $q(\gamma_{\rm H})$ and $\gamma_{\rm L}$ can be expressed as

$$q^T(\gamma_{\rm H}) = (1-\tau) + \frac{\mu(\gamma_{\rm H}-1)}{2},$$

$$q^T(\gamma_{\rm L}) = \frac{(1-\tau)(1+\gamma_{\rm L}) - \mu(\gamma_{\rm H}-\gamma_{\rm L})}{2}.$$

Now I will confirm the optimal solution in separate situation based on the values of λ^T and μ^T. Obviously, $\lambda^T = \mu^T = 0$ violates the first order condition (3.63). I will then discuss the other three possibilities:

i. If $\lambda^T > 0$ and $\mu^T = 0$, i.e. $\lambda^T = 1 - \tau$, then only ex-ante IR for γ_{H} binds, i.e.

$$q^T(\gamma_{\mathrm{H}}) - p^T(\gamma_{\mathrm{H}}) = 0$$

and then

$$(\gamma_{\mathrm{H}} - 1)q^T(\gamma_{\mathrm{H}}) - (\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}}) \geq 0.$$

The first order conditions imply

$$q^T(\gamma_{\mathrm{H}}) = 1 - \tau, \qquad q^T(\gamma_{\mathrm{L}}) = (1-\tau)\frac{1+\gamma_{\mathrm{L}}}{2}, \quad \text{with } \lambda = 1 - \tau.$$

Therefore,

$$p^T(\gamma_{\mathrm{H}}) = 1 - \tau, \qquad p^T(\gamma_{\mathrm{L}}) = (1-\tau)\frac{(1+\gamma_{\mathrm{L}})^2}{4}.$$

The condition that $\gamma_{\mathrm{H}} \geq \gamma_{\mathrm{L}} + 2$ ensures that incentive compatibility for γ_{H} with the condition $p^T(\gamma_{\mathrm{H}}) = q^T(\gamma_{\mathrm{H}})$ holds, i.e. $(\gamma_{\mathrm{H}} - 1)q^T(\gamma_{\mathrm{H}}) - (\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}}) \geq 0$.

In this case, the monopolist's aggregate profit from both consumers is

$$\Pi_{\substack{\lambda^T=1-\tau \\ \mu^T=0}} = \Pi_{\substack{\lambda^T=1-\tau \\ \mu^T=0}}(\gamma_{\mathrm{H}}) + \Pi_{\substack{\lambda^T=1-\tau \\ \mu^T=0}}(\gamma_{\mathrm{L}}) = (1-\tau)^2\frac{1}{2} + (1-\tau)^2\frac{(1+\gamma_{\mathrm{L}})^2}{8} - 2s = (1-\tau)^2\pi_{\substack{\lambda=1 \\ \mu=0}} - 2s.$$

ii. If $\lambda^T = 0$ and $\mu^T > 0$, i.e. $\mu^T = 1 - \tau$,then then only ex-post IC for γ_{H} binds, i.e.

$$2p^T(\gamma_{\mathrm{H}}) = (1 + \gamma_{\mathrm{H}})q^T(\gamma_{\mathrm{H}}) - (\gamma_{\mathrm{H}} - \gamma_{\mathrm{L}})q^T(\gamma_{\mathrm{L}})$$

and then

$$q^T(\gamma_{\rm H}) - p^T(\gamma_{\rm H}) \geq 0$$

The first order conditions imply in this case that

$$q^T(\gamma_{\rm H}) = (1-\tau)\frac{1+\gamma_{\rm H}}{2}, \qquad q^T(\gamma_{\rm L}) = (1-\tau)\frac{1+2\gamma_{\rm L}-\gamma_{\rm H}}{2} \quad \text{with } \mu = 1-\tau.$$

The bounds $1 < \gamma_{\rm H} < 2\gamma_{\rm L} + 1$ and $0 < \gamma_{\rm L} < 1$ guarantee positive quality is provided to both consumers.

$$p^T(\gamma_{\rm H}) = (1-\tau)\frac{(1+\gamma_{\rm H})^2 - (\gamma_{\rm H}-\gamma_{\rm L})(1+2\gamma_{\rm L}-\gamma_{\rm H})}{4}$$

$$p^T(\gamma_{\rm L}) = (1-\tau)\frac{(1+\gamma_{\rm L})^2 - (\gamma_{\rm H}-\gamma_{\rm L})(1+\gamma_{\rm L})}{4}.$$

Then I check if the ex-ante IR for $\gamma_{\rm H}$ is valid: $q^T(\gamma_{\rm H}) - p^T(\gamma_{\rm H}) \geq 0$. I have the bound

$$\gamma_{\rm H} \leq \frac{1+3\gamma_{\rm L}+\sqrt{9-2\gamma_{\rm L}-7\gamma_{\rm L}^2}}{4}$$

The maximal aggregate profit for the monopolist is then given by

$$\begin{aligned}\Pi_{\substack{\lambda^T=0\\ \mu^T=1-\tau}} &= \Pi_{\substack{\lambda^T=0\\ \mu^T=1-\tau}}(\gamma_{\rm H}) + \Pi_{\substack{\lambda^T=0\\ \mu^T=1-\tau}}(\gamma_{\rm L})\\ &= (1-\tau)^2\frac{(1+\gamma_{\rm H})^2 - 2(\gamma_{\rm H}-\gamma_{\rm L})(1+2\gamma_{\rm L}-\gamma_{\rm H})}{8} + (1-\tau)^2\frac{(1+\gamma_{\rm L})^2-(\gamma_{\rm H}-\gamma_{\rm L})^2}{8} - 2s\\ &= (1-\tau)^2\pi_{\substack{\lambda=0\\ \mu=1}} - 2s.\end{aligned}$$

iii. If both $\lambda^T, \mu^T > 0$, then both constraints bind, i.e.

$$q^T(\gamma_{\rm H}) - p^T(\gamma_{\rm H}) = 0$$

and

$$(1+\gamma_{\rm H})q^T(\gamma_{\rm H}) - (\gamma_{\rm H}-\gamma_{\rm L})q^T(\gamma_{\rm L}) - 2p^T(\gamma_{\rm H}) = 0.$$

The first order conditions imply

$$q^T(\gamma_H) = p^T(\gamma_H) = (1-\tau) + \frac{\mu(\gamma_H - 1)}{2},$$

$$q^T(\gamma_L) = \frac{(1-\tau)(1+\gamma_L) - \mu^T(\gamma_H - \gamma_L)}{2}$$

$$p^T(\gamma_L) = \frac{(1-\tau)(1+\gamma_L)^2 - \mu^T(1+\gamma_L)(\gamma_H - \gamma_L)}{4}.$$

From the condition $(1+\gamma_H)q^T(\gamma_H) - (\gamma_H - \gamma_L)q^T(\gamma_L) - 2p^T(\gamma_H) = 0$, I have

$$\mu^T = \frac{(1-\tau)(\gamma_L - 1)(\gamma_H - \gamma_L - 2)}{(\gamma_H - 1)^2 + (\gamma_H - \gamma_L)^2} = (1-\tau)\mu,$$

where $\gamma_H - \gamma_L < 2$ to ensure $\mu^T > 0$.

The maximal aggregate profit of the monopolist in the case $\lambda, \mu > 0$ is thus

$$\begin{aligned}
\Pi_{\lambda^T,\mu^T>0} &= \Pi_{\lambda^T,\mu^T>0}(\gamma_H) + \Pi_{\lambda^T,\mu^T>0}(\gamma_L) \\
&= (1-\tau)^2\left(\frac{1}{2} - \frac{\mu^2(\gamma_H - 1)^2}{8}\right) + (1-\tau)^2\frac{(1+\gamma_L)^2 - \mu^2(\gamma_H - \gamma_L)^2}{8} - 2s \\
&= (1-\tau)^2\pi_{\lambda,\mu>0} - 2s,
\end{aligned}$$

□

Proof of proposition 3.2.7. Ex-post incentive compatibility means that given an allocation $(q(\gamma), p(\gamma))$ a consumer of type $\hat{\gamma}$ maximises his ex-post utility by selecting bundle $(q(\hat{\gamma}), p(\hat{\gamma}))$. More formally, this bundle must satisfy the first order condition

$$0 = \frac{d}{d\gamma}(U + V_{\hat{\gamma}}) = (1+\hat{\gamma})q'(\hat{\gamma}) - 2p'(\hat{\gamma}). \quad (3.66)$$

The total differential of w at $\hat{\gamma}$ is

$$\frac{dw(\hat{\gamma})}{d\gamma} = (1+\hat{\gamma})q'(\hat{\gamma}) + q(\hat{\gamma}) - 2p'(\hat{\gamma}). \quad (3.67)$$

The previous equation implies $w'(\gamma) = q(\gamma)$. If $w'(\gamma) = q(\gamma)$ the first order condition would follow which means that ex-post incentive compatibility holds.

The other equivalence is easier:

$$w(\gamma) + (1-\gamma)q(\gamma) = 2q(\gamma) - 2p(\gamma) = 2U(q(\gamma), p(\gamma)). \tag{3.68}$$

□

Proof of theorem 3.2.1. Following proposition 7 in Esteban et al. (2007), I can guess that the solution $\underline{\gamma}$ and $\bar{\gamma}$ to the optimal control problem is of the form given in proposition 3.2.1 for some parameters $\underline{\gamma}$ and $\bar{\gamma}$. Moreover, it also follows from that result that $p(\gamma) = 0$ when $\gamma < \underline{\gamma}$, and p is held at a fixed level when $\gamma > \bar{\gamma}$.

I consider first the case $\gamma \in [a, \underline{\gamma})$ so that $q(\gamma) = w(\gamma) = 0$ and find values of $\delta(\gamma)$, $\lambda(\gamma)$ and $\mu(\gamma)$.

Claim 1: $a \leq \gamma < \underline{\gamma}$, then $q(\gamma) = w(\gamma) = 0$ holds.

Proof. There are a number of possible values that $\delta(\gamma)$ and $\lambda(\gamma)$ can take in this situation. It suffices to provide just one set; I provide two possible examples. If $\lambda(\gamma) = 0$ and $\delta(\gamma) > 0$, then condition (3.49) becomes

$$\frac{(1-\tau)(1+\gamma)}{2(b-a)} + \mu(\gamma) + \delta(\gamma) = 0.$$

Integrating condition (3.63) and applying the boundary condition (3.54), I have

$$\mu(\gamma) = \frac{(1-\tau)(\gamma - b)}{2(b-a)}.$$

From the above two equations, we have

$$\delta(\gamma) = -\frac{(1-\tau)(1-b+2\gamma)}{2(b-a)} > 0,$$

which holds implies $\underline{\gamma} \leq \frac{b-1}{2}$. But since $\underline{\gamma}$ is the largest type for which $w(\gamma) = 0$, the equality can hold.

On the other hand, if $\lambda(\gamma) > 0$ and $\delta(\gamma) = 0$ conditions (3.49) and (3.63) become

$$\frac{(1-\tau)(1+\gamma)}{2(b-a)} + \mu(\gamma) - (\gamma - 1)\lambda(\gamma) = 0,$$

$$\mu'(\gamma) = \frac{1-\tau}{2(b-a)} - \lambda(\gamma).$$

From the above two equations, we have

$$\mu(\gamma) = \frac{(1-\tau)(\gamma - b)}{(b-a)(1-\gamma)}$$

$$\lambda(\gamma) = \frac{1-\tau}{2(b-a)} + \frac{(1-\tau)(b-1)}{(b-a)(\gamma-1)^2} > 0.$$

This shows that it is possible to satisfy the necessary conditions in the interval $\gamma \in [a, \underline{\gamma})$ and the guess is verified in this case; $q(\gamma) = w(\gamma) = 0$ holds there.

□

Now, I examine the intermediate case $\underline{\gamma} < \gamma < \bar{\gamma}$,

Claim 2: $\underline{\gamma} < \gamma < \bar{\gamma}$, then $\lambda(\gamma) = \delta(\gamma) = 0$ and $q(\gamma) = (1-\tau)(\gamma - \frac{b-1}{2})$.

Proof. This follows immediately since neither constraint binds in this case and $q(\gamma)$ is continuous so that $q(\frac{b-1}{2}) = 0$.

□

Claim 3: $\underline{\gamma} < \gamma < \bar{\gamma}$ and $w(\gamma)$ is continuous, then $\lambda(\gamma) = \delta(\gamma) = 0$ and $w(\gamma) = \frac{1-\tau}{2}(\gamma - \frac{b-1}{2})^2$.

Proof. This results since $w(\frac{b-1}{2}) = 0$ and $w'(\gamma) = q(\gamma)$.

□

Finally, I consider the last case where $\gamma \in (\bar{\gamma}, b]$. In this situation, the guess requires that $q(\gamma)$ is a constant $C_1 > 0$ and that ex-ante individual rationality (equation 3.44) holds. This condition implies that

$$w(\gamma) = (\gamma - 1)q(\gamma) = C_1(\gamma - 1). \tag{3.69}$$

Claim 4: $\bar{\gamma} = \frac{5-b}{2}$.

Proof. Since $q(\gamma)$ is continuous:

$$(1-\tau)(\bar{\gamma} - \frac{b-1}{2}) = C_1. \tag{3.70}$$

In addition, $w(\gamma)$ is continuous and satisfies individual rationality, which means

$$\frac{(1-\tau)(\bar{\gamma} - \frac{b-1}{2})^2}{2} = C_1(\bar{\gamma} - 1). \tag{3.71}$$

If I combine these two equations, then

$$\bar{\gamma} - \frac{b-1}{2} = 2(\bar{\gamma} - 1). \tag{3.72}$$

Solving this equation $\bar{\gamma} = \frac{5-b}{2}$ and

$$C_1 = (1-\tau)(\bar{\gamma} - \frac{b-1}{2}) = 2(1-\tau)(\bar{\gamma} - 1) = (1-\tau)(3-b). \tag{3.73}$$

□

I just need to check that this gives a consistent solution by finding $\delta(\gamma)$,$\lambda(\gamma)$ and $\mu(\gamma)$:

Claim 5: When $q(\gamma) = 2(1-\tau)(\bar{\gamma}-1)$ and $w(\gamma) = 2(1-\tau)(\bar{\gamma}-1)(\gamma-1)$, $\gamma > \bar{\gamma}$, then $\lambda(\gamma) > 0$ and $\delta(\gamma) = 0$.

Proof. To keep the notation simple, let $q(\gamma) = C_1$ as above. Since this constant is positive $\delta(\gamma) = 0$. In addition, conditions (3.49) and (3.63) together become

$$\frac{(1-\tau)(1+\gamma) - 2C_1}{2(b-a)} + \mu(\gamma) - (\gamma - 1)\lambda(\gamma) = 0.$$

$$\mu'(\gamma) = \frac{1-\tau}{2(b-a)} - \lambda(\gamma).$$

From the two equations above, I have

$$\mu(\gamma) = \frac{[(1-\tau) - C_1](\gamma - b)}{(b-a)(1-\gamma)}$$

$$\lambda(\gamma) = \frac{1-\tau}{2(b-a)} + \frac{(1-\tau)(b-1)(b-2)}{(b-a)(\gamma-1)^2} > 0,$$

since $b > 2$.

□

Claim 6: $\pi(\gamma) \geq 0$, then

$$\gamma \in \left[b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}, b + \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right]$$

Proof. This can be obtained immediately by calculating $\frac{1}{2}(1-\tau)[(1+\gamma)q(\gamma) - w(\gamma)] - \frac{1}{2}q(\gamma)^2 - s \geq 0$.

□

Claim 7: $\underline{\gamma} = \underline{\gamma}(\tau, s)$. When $s < 0$, that is, the monopolist faces a specific subsidy, $\underline{\gamma} = \frac{b-1}{2}$; when $s \geq 0$, that is, the monopolist faces a specific tax, $\underline{\gamma} = b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}$.

Proof. All that remains is to find functions $\delta(\gamma)$, $\lambda(\gamma)$ and $\mu(\gamma)$ that satisfy the necessary conditions and to verify that indeed $\underline{\gamma} = b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}$ and $\overline{\gamma} = \frac{5-b}{2}$.

$$\underline{\gamma} = \max\left\{\frac{b-1}{2}, b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right\} = \begin{cases} \frac{b-1}{2}, & \text{when } s < 0 \\ b - \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}, & \text{when } s \geq 0 \end{cases}$$

and

$$\bar{\gamma} = \min\left\{\frac{5-b}{2}, b + \sqrt{\frac{(b+1)^2}{4} - \frac{4s}{(1-\tau)^2}}\right\} = \frac{5-b}{2},$$

since the minimum of $b+\sqrt{\frac{(b+1)^2}{4}-\frac{4s}{(1-\tau)^2}}$ is b (which is larger than $\frac{5-b}{2}$, when $b>\frac{5}{3}$) at $s=\frac{(b+1)^2}{16}(1-\tau)^2$.

The taxes need to satisfy the following conditions:

$$a < b-\sqrt{\frac{(b+1)^2}{4}-\frac{4s}{(1-\tau)^2}} < 1 < \frac{5-b}{2} < b$$

□

□

Proof of proposition 3.3.1. The social welfare W_C^H is given by

$$\begin{aligned}
W_C^H(\tau,s) &= \int_a^b \left[q(\gamma)-\frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma - \int_a^b \Big\{\max\{\gamma q(\gamma)-p(\gamma)\} - [\gamma q(\gamma)-p(\gamma)]\Big\} f(\gamma)d\gamma \\
&= \underbrace{\int_a^{\underline{\gamma}(\tau,s)} 0f(\gamma)d\gamma + \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \left[\frac{1-\tau}{2}(2\gamma-b+1)-\frac{(1-\tau)^2}{8}(2\gamma-b+1)^2\right] f(\gamma)d\gamma}_{\cdots} \\
&\quad \underbrace{+\int_{\bar{\gamma}}^b \left[(1-\tau)(3-b)-\frac{1}{2}(1-\tau)^2(3-b)^2\right] f(\gamma)d\gamma - \int_a^{\frac{\underline{\gamma}(\tau,s)+1}{2}} 0f(\gamma)d\gamma}_{\text{Total surplus}} \\
&\quad \underbrace{-\int_{\frac{\underline{\gamma}(\tau,s)+1}{2}}^{\frac{\bar{\gamma}+1}{2}} \frac{1-\tau}{4}\left(2\gamma-\frac{b+1}{2}\right)^2 f(\gamma)d\gamma - \int_{\frac{\bar{\gamma}+1}{2}}^b (1-\tau)(3-b)(\gamma-1)f(\gamma)d\gamma}_{\cdots} \\
&\quad \underbrace{+\int_a^{\underline{\gamma}(\tau,s)} 0f(\gamma)d\gamma}_{\cdots} \\
&\quad \underbrace{+\int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \frac{1-\tau}{4}\left(3\gamma^2-2b\gamma+\frac{b^2+2b-3}{4}\right) f(\gamma)d\gamma}_{\text{Self-control costs}} \\
&\quad \underbrace{+\int_{\bar{\gamma}}^b (1-\tau)(3-b)(\gamma-1)f(\gamma)d\gamma}_{\cdots} \\
&\equiv \underbrace{I_1+I_2}_{\text{Normative welfare}} + \underbrace{I_3+I_4}_{\text{(Negative) Maximal temptation}} + \underbrace{I_5+I_6}_{\text{Temptation utility}},
\end{aligned}$$

where the sum of integrals is divided into five parts I_1, I_2, I_3, I_4, I_5 and then I calculate each part separately.

The welfare decomposition comprises three components: (1) normative welfare $(I_1 + I_2)$, representing true preferences; (2) maximal temptation $(I_3 + I_4)$, the highest possible level of temptation in the given menu; and (3) temptation utility $(I_5 + I_6)$, measuring a tempting desire.

$$I_1 = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \left[\frac{1-\tau}{2}(2\gamma - b + 1) - \frac{(1-\tau)^2}{8}(2\gamma - b + 1)^2\right] f(\gamma)d\gamma$$

$$I_2 = \int_{\bar{\gamma}}^{b} \left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma = \int_{\bar{\gamma}}^{b} \left[(1-\tau)(3-b) - \frac{1}{2}(1-\tau)^2(3-b)^2\right] f(\gamma)d\gamma$$

$$I_3 = -\int_{\frac{\underline{\gamma}(\tau,s)+1}{2}}^{\frac{\bar{\gamma}+1}{2}} \max\{\gamma q(\gamma) - p(\gamma)\} f(\gamma)d\gamma = -\int_{\frac{\underline{\gamma}(\tau,s)+1}{2}}^{\frac{\bar{\gamma}+1}{2}} \frac{1-\tau}{4}\left(2\gamma - \frac{b+1}{2}\right)^2 f(\gamma)d\gamma$$

$$I_4 = -\int_{\frac{\bar{\gamma}+1}{2}}^{b} \max\{\gamma q(\gamma) - p(\gamma)\} f(\gamma)d\gamma = -\int_{\frac{\bar{\gamma}+1}{2}}^{b} (1-\tau)(3-b)(\gamma - 1) f(\gamma)d\gamma$$

$$I_5 = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \{\gamma q(\gamma) - p(\gamma)\} f(\gamma)d\gamma = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \frac{1-\tau}{4}\left(3\gamma^2 - 2b\gamma + \frac{b^2 + 2b - 3}{4}\right) f(\gamma)d\gamma$$

$$I_6 = \int_{\bar{\gamma}}^{b} \{\gamma q(\gamma) - p(\gamma)\} f(\gamma)d\gamma = \int_{\bar{\gamma}}^{b} (1-\tau)(3-b)(\gamma - 1) f(\gamma)d\gamma$$

When $s = 0$, the threshold $\underline{\gamma}$ simplifies to:

$$\underline{\gamma} = \frac{b-1}{2}.$$

Consequently, the derivative of $\underline{\gamma}$ with respect to τ is:

$$\frac{\partial \underline{\gamma}}{\partial \tau} = 0,$$

since $\underline{\gamma}$ is independent of τ in this case.

Next, I compute the derivatives of each term I_i $(i = 1, \ldots, 6)$ with respect to τ under the condition $s = 0$. The results are as follows:

$$\frac{\partial I_1}{\partial \tau} = \int_{\frac{b-1}{2}}^{\frac{5-b}{2}} [1 - q(\gamma)] \frac{dq(\gamma)}{d\tau} f(\gamma) d\gamma = \int_{\frac{b-1}{2}}^{\frac{5-b}{2}} \left[1 - (1-\tau)(\gamma - \frac{b-1}{2})\right] \left[-(\gamma - \frac{b-1}{2})\right] f(\gamma) d\gamma$$

$$= \frac{(3-b)^2}{6(b-a)} [2(1-\tau)(3-b) - 3]$$

$$\frac{\partial I_2}{\partial \tau} = \int_{\frac{5-b}{2}}^{b} [1 - q(\gamma)] \frac{dq(\gamma)}{d\tau} f(\gamma) d\gamma = \int_{\frac{b-1}{2}}^{b} [1 - (1-\tau)(3-b)] [-(3-b)] f(\gamma) d\gamma$$

$$= \frac{(3-b)(b+1)}{2(b-a)} [(2-b) - \tau(3-b)]$$

$$\frac{\partial I_3}{\partial \tau} = \int_{\frac{b+1}{4}}^{\frac{7-b}{4}} \frac{1}{4} \left(2\gamma - \frac{b+1}{2}\right)^2 f(\gamma) d\gamma = \frac{(3-b)^3}{24(b-a)} > 0$$

$$\frac{\partial I_4}{\partial \tau} = \int_{\frac{7-b}{4}}^{b} (3-b)(\gamma - 1) f(\gamma) d\gamma = \frac{(3-b)(3b-1)(5b-7)}{32(b-a)} > 0$$

$$\frac{\partial I_5}{\partial \tau} = -\int_{\frac{b-1}{2}}^{\frac{5-b}{2}} \{\gamma q(\gamma) - p(\gamma)\} f(\gamma) d\gamma = -\int_{\frac{b-1}{2}}^{\frac{5-b}{2}} \frac{1}{4} \left(3\gamma^2 - 2b\gamma + \frac{b^2 + 2b - 3}{4}\right) f(\gamma) d\gamma$$

$$= \frac{(3-b)(3b-5)(b+1)}{32(b-a)} > 0$$

$$\frac{\partial I_6}{\partial \tau} = -\int_{\frac{5-b}{2}}^{b} (3-b)(\gamma - 1) f(\gamma) d\gamma = -\frac{(3-b)(3b-5)(b+1)}{8(b-a)} < 0$$

For the commitment utility terms (I_1 and I_2), we derive the following comparative statics with respect to τ:

(a) For I_1:

$$\frac{\partial I_1}{\partial \tau} \begin{cases} > 0 & \text{if } \tau < 1 - \frac{3}{2(3-b)} \\ = 0 & \text{if } \tau = 1 - \frac{3}{2(3-b)} \\ < 0 & \text{if } \tau > 1 - \frac{3}{2(3-b)} \end{cases}$$

(b) For I_2:

$$\frac{\partial I_2}{\partial \tau}\begin{cases} > 0 & \text{if } \tau < 1 - \frac{1}{3-b} \\ = 0 & \text{if } \tau = 1 - \frac{1}{3-b} \\ < 0 & \text{if } \tau > 1 - \frac{1}{3-b} \end{cases}$$

The aggregate effect on commitment utility is:

$$\frac{\partial I_1}{\partial \tau}+\frac{\partial I_2}{\partial \tau} = \frac{(3-b)}{6(b-a)}\left[-(b^2+6b)(1-\tau)-27\tau+15\right] > 0 \quad \text{when } \tau < \frac{b^2+6b-15}{(b+9)(b-3)}$$

For the temptation and self-cost effects:

(a) Negative maximal temptation :

$$\frac{\partial I_3}{\partial \tau} + \frac{\partial I_4}{\partial \tau} = \frac{(3-b)(49b^2-102b+57)}{96(b-a)} > 0.$$

This indicates the resistance cost decreases as τ increases.

(b) Temptation utility:

$$\frac{\partial I_5}{\partial \tau} + \frac{\partial I_6}{\partial \tau} = -\frac{3(3-b)(3b-5)(b+1)}{32(b-a)} < 0$$

showing that temptation utility decreases with higher τ.

The total change in self-control costs (negative maximal temptation minus temptation utility):

$$\sum_{i=3}^{6} \frac{\partial I_i}{\partial \tau} = \frac{(3-b)(37b^2-126b+141)}{192(b-a)} > 0$$

The net effect represents a welfare improvement as τ increases, which suggests τ acts as an effective mechanism for mitigating self-control problems.

The welfare function $W_C^H(\tau, s)$ responds to taxation as:

$$\frac{\partial W_C^H(\tau,s)}{\partial \tau} = \frac{\partial I_1}{\partial \tau} + \frac{\partial I_2}{\partial \tau} + \frac{\partial I_3}{\partial \tau} + \frac{\partial I_4}{\partial \tau} + \frac{\partial I_5}{\partial \tau} + \frac{\partial I_6}{\partial \tau}$$

I will consider the effect on welfare of raising τ or s from zero and look for the optimal tax policies of τ and s.

$$\frac{\partial W_C^H(\tau,s)}{\partial \tau}\Big|_{\tau=s=0} = \frac{3-b}{16(b-a)}(b^2-30b+57)$$

$$\frac{\partial W_C^H}{\partial \tau}\Big|_{s=0} = \frac{(3-b)}{48(b-a)}\left[3(b^2-30b+57)+8(b^2+6b-27)\tau\right].$$

The critical point is $b_C^H = 15-2\sqrt{42} \approx 2.04$.

$$\frac{\partial W_C^H}{\partial \tau}\Big|_{\tau=s=0} \begin{cases} >0 & \text{for } b \in \left(\frac{5}{3}, b_C^H\right), \\ =0 & \text{at } b = b_C^H, \\ <0 & \text{for } b \in (b_C^H, 3). \end{cases}$$

The optimal ad valorem policy $\tau_C^H = -\frac{3(b^2-30b+57)}{8(b+9)(b-3)}$ since $\frac{\partial W_U^H(\tau,s)}{\partial \tau}\Big|_{s=0} > 0$ when $\tau < \tau_C^H$ and $\frac{\partial W_U^H(\tau,s)}{\partial \tau}\Big|_{s=0} < 0$ when $\tau > \tau_C^H$. Specifically, when $b \in (\frac{5}{3}, b_C^H)$, $\tau_C^H \in (0, \frac{33}{128})$; when $b \in (b_C^H, 3)$, $\tau_C^H < 0$.

□

Proof of proposition 3.3.2. The social welfare W_U^H is given by

$$\begin{aligned} W_U^H(\tau,s) &= \int_a^b \left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma \\ &= \int_a^{\underline{\gamma}(\tau,s)} 0 f(\gamma)d\gamma + \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \left[\frac{1-\tau}{2}(2\gamma-b+1) - \frac{(1-\tau)^2}{8}(2\gamma-b+1)^2\right] f(\gamma)d\gamma \\ &+ \int_{\bar{\gamma}}^b \left[(1-\tau)(3-b) - \frac{1}{2}(1-\tau)^2(3-b)^2\right] f(\gamma)d\gamma \\ &= I_1 + I_2. \end{aligned}$$

Therefore,

$$\frac{\partial W_U^H(\tau,s)}{\partial \tau} = \frac{\partial I_1}{\partial \tau} + \frac{\partial I_2}{\partial \tau}.$$

$$\left.\frac{\partial W_{U}^{H}(\tau,s)}{\partial\tau}\right|_{\tau=s=0} = \frac{3-b}{6(b-a)}(-b^2-6b+15)$$
$$\left.\frac{\partial W_{U}^{H}(\tau,s)}{\partial\tau}\right|_{s=0} = \frac{3-b}{6(b-a)}\left[-(b^2-6b+15)+(b+9)(b-3)\tau\right]$$

The critical point is $b_{U}^{H} = 2\sqrt{6}-3 \approx 1.90$.

$$\left.\frac{\partial W_{U}^{H}}{\partial\tau}\right|_{\tau=s=0} \begin{cases} >0 & \text{for } b \in \left(\frac{5}{3}, b_{U}^{H}\right), \\ =0 & \text{at } b = b_{U}^{H}, \\ <0 & \text{for } b \in (b_{U}^{H}, 3). \end{cases}$$

The optimal ad valorem policy $\tau_{U}^{H} = \frac{b^2+6b-15}{(b+9)(b-3)}$ since $\left.\frac{\partial W_{U}^{H}(\tau,s)}{\partial\tau}\right|_{s=0} > 0$ when $\tau < \tau_{U}^{H}$ and $\left.\frac{\partial W_{U}^{H}(\tau,s)}{\partial\tau}\right|_{s=0} < 0$ when $\tau > \tau_{U}^{H}$. Specifically, when $b \in (\frac{5}{3}, b_{U}^{H})$, $\tau_{U}^{H} \in (0, \frac{5}{32})$; when $b \in (b_{U}^{H}, 3)$, $\tau_{U}^{H} < 0$.

□

Proof of proposition 3.3.3. The social welfare W_{U+V}^{H} is given by is

$$\begin{aligned} W_{U+V}^{H}(\tau,s) &= \int_a^b \left[(1+\gamma)q(\gamma) - p(\gamma) - \frac{1}{2}q(\gamma)^2\right] f(\gamma)d\gamma \\ &= \int_a^b \left\{\left[q(\gamma) - \frac{1}{2}q(\gamma)^2\right] + [\gamma q(\gamma) - p(\gamma)]\right\} f(\gamma)d\gamma \\ &= I_1 + I_2 + I_5 + I_6. \end{aligned}$$

Therefore,

$$\frac{\partial W_{U+V}^{H}(\tau,s)}{\partial\tau} = \frac{\partial I_1}{\partial\tau} + \frac{\partial I_2}{\partial\tau} + \frac{\partial I_5}{\partial\tau} + \frac{\partial I_6}{\partial\tau}.$$

The change of $W_{U+V}^{H}(\tau,s)$ on τ from 0 given no specific tax s:

$$\left.\frac{\partial W_{U+V}^{H}(\tau,s)}{\partial\tau}\right|_{\tau=s=0} = \frac{3-b}{96(b-a)}(-43b^2-78b+285).$$

$$\left.\frac{\partial W^H_{U+V}(\tau,s)}{\partial\tau}\right|_{s=0} = \frac{3-b}{96(b-a)}\left[-(43b^2+78b-285)+16(b+9)(b-3)\tau\right].$$

The critical point is $b^H_{U+V} = \frac{4\sqrt{861}-39}{43} \approx 1.82$.

$$\left.\frac{\partial W^H_{U+V}}{\partial\tau}\right|_{\tau=s=0} \begin{cases} >0 & \text{for } b\in\left(\frac{5}{3}, b^H_{U+V}\right), \\ =0 & \text{at } b = b^H_{U+V}, \\ <0 & \text{for } b\in(b^H_{U+V},3). \end{cases}$$

The optimal ad valorem policy $\tau^H_{U+V} = \frac{43b^2+78b-285}{16(b+9)(b-3)}$ since $\left.\frac{\partial W^H_{U+V}(\tau,s)}{\partial\tau}\right|_{s=0} > 0$ when $\tau < \tau^H_{U+V}$ and $\left.\frac{\partial W^H_{U+V}(\tau,s)}{\partial\tau}\right|_{s=0} < 0$ when $\tau > \tau^H_{U+V}$. Specifically, when $b\in(\frac{5}{3}, b^H_{U+V})$, $\tau^H_{U+V}\in(0,\frac{5}{32})$; when $b\in(b^H_{U+V},3)$, $\tau^H_{U+V}<0$.

□

Proof of proposition 3.3.4. When $\tau = 0$,

$$\underline{\gamma}(s) = \begin{cases} \frac{b-1}{2}, & \text{when } s<0 \\ b - \frac{\sqrt{(b+1)^2-16s}}{2}, & \text{when } s\geq 0 \end{cases}$$

Then, $\frac{\partial\underline{\gamma}(s)}{\partial s} = \frac{4}{\sqrt{(b+1)^2-16s}} > 0$ when $s>0$. Additionally, $s < \frac{-3b^2+10b-3}{16}$ ensures that $\underline{\gamma}(s) < 1$.

Taking the derivative of each I_i with respect to s as s approaches from the positive side and as s approaches from the negative side, I have the following:

$$\frac{\partial I_1}{\partial s}\bigg|_{s>0} = -\frac{1}{b-a}\left[q(\underline{\gamma}(s)) - \frac{1}{2}q(\underline{\gamma}(s))^2\right]\frac{\partial\underline{\gamma}(s)}{\partial s}$$
$$= -\frac{1}{b-a}\left[(\underline{\gamma}(s) - \frac{b-1}{2}) - \frac{1}{2}(\underline{\gamma}(s) - \frac{b-1}{2})^2\right]\frac{\partial\underline{\gamma}(s)}{\partial s}$$
$$= -\frac{(b+1-\sqrt{(b+1)^2-16s})(3-b+\sqrt{(b+1)^2-16s})}{2(b-a)\sqrt{(b+1)^2-16s}} < 0$$

$$\frac{\partial I_2}{\partial s}\bigg|_{s>0} = 0$$

$$\frac{\partial I_3}{\partial s}\bigg|_{s>0} = \frac{1}{8(b-a)}\left(2\underline{\gamma}(s) - \frac{b+1}{2}\right)^2\frac{\partial\underline{\gamma}(s)}{\partial s} = \frac{\left(3b-1-2\sqrt{(b+1)^2-16s}\right)^2}{8(b-a)\sqrt{(b+1)^2-16s}} > 0$$

$$\frac{\partial I_4}{\partial s}\bigg|_{s>0} = 0$$

$$\frac{\partial I_5}{\partial s}\bigg|_{s>0} = -\frac{1}{4(b-a)}\left(3\underline{\gamma}(s)^2 - 2b\underline{\gamma}(s) + \frac{b^2+2b-3}{4}\right)\frac{d\underline{\gamma}(s)}{ds}$$
$$= -\frac{1}{4(b-a)}\left(\frac{13b^2+8b-48s-8b\sqrt{(b+1)^2-16s}}{\sqrt{(b+1)^2-16s}}\right)$$

$$\frac{\partial I_6}{\partial s}\bigg|_{s>0} = 0$$

$$\frac{\partial I_i}{\partial s}\bigg|_{s<0} = 0, \quad i = 1,2,3,4,5,6$$

$\frac{\partial I_1}{\partial s}\Big|_{s>0} < 0$, since $0 < \underline{\gamma}(s) - \frac{b-1}{2} < 1$.

The sum change of temptation utility on $s > 0$ is $\frac{\partial I_5}{\partial s}\Big|_{s>0} + \frac{\partial I_6}{\partial s}\Big|_{s>0}$. $\frac{dI_5}{\partial s}\Big|_{s>0} + \frac{\partial I_6}{\partial s}\Big|_{s>0} > 0$ when $\underline{\gamma}(s) \in (\frac{b-1}{2}, \frac{b+3}{6})$ or $s \in (0, \frac{3b-b^2}{9})$; $\frac{\partial I_5}{\partial s}\Big|_{s>0} + \frac{\partial I_6}{\partial s}\Big|_{s>0} = 0$ when $\underline{\gamma}(s) = \frac{b+3}{6}$; $\frac{\partial I_5}{\partial s}\Big|_{s>0} + \frac{dI_6}{ds}\Big|_{s>0} < 0$ when $\underline{\gamma}(s) \in (\frac{b+3}{6}, 1)$ or $s \in$

$\left(\frac{3b-b^2}{9}, \frac{-3b^2+10b-3}{16}\right)$.

$$
\begin{aligned}
&-\frac{\partial \int_a^b \{\max\{\gamma q(\gamma)-p(\gamma)\}-[\gamma q(\gamma)-p(\gamma)]\}f(\gamma)d\gamma}{\partial s}\Big|_{s>0} \\
&=\sum_{i=3}^{6}\frac{\partial I_i}{\partial s}\Big|_{s>0} \\
&=\frac{-13b^2-14b+5+32s+4(b+1)\sqrt{(b+1)^2-16s}}{8(b-a)\sqrt{(b+1)^2-16s}} \\
&=\frac{-9b^2-6b+9-2(\sqrt{(b+1)^2-16s}-b-1)^2}{8(b-a)\sqrt{(b+1)^2-16s}}<0.
\end{aligned}
$$

The sum of self-control cost increases as $s>0$ increases, therefore welfare decreases.

A subsidy cannot increase the welfare through the effect of market size.

Proof of the effect of specific policy s on social welfare W_C^H.

$$
\frac{\partial W_C^H(\tau,s)}{\partial s}=\frac{\partial I_1}{\partial s}+\frac{\partial I_2}{\partial s}+\frac{\partial I_3}{\partial s}+\frac{\partial I_4}{\partial s}+\frac{\partial I_5}{\partial s}+\frac{\partial I_6}{\partial s}.
$$

$$
\begin{aligned}
\frac{\partial W_C^H(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} &= \frac{-9b^2-6b+9}{8(b-a)(b+1)}<0, \\
\frac{\partial W_C^H(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^-} &= 0, \\
\frac{\partial W_C^H(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} &= \sum_{i=1}^{6}\frac{\partial I_i}{\partial s}\Big|_{s>0}<0, \\
\frac{\partial W_C^H(\tau,s)}{\partial s}\Big|_{\tau=0,s<0} &= 0.
\end{aligned}
$$

The differentiates show that $s>0$ is negative while $s<0$ is 0. Therefore, the optimal specific tax is at $s_C^H=0$.

□

Proof of the effect of specific policy s on social welfare W_U^H.

$$\frac{\partial W_U^H(\tau,s)}{ds} = \frac{\partial I_1}{\partial s} + \frac{\partial I_2}{\partial s}.$$
$$\left.\frac{\partial W_U^H(\tau,s)}{\partial s}\right|_{\tau=0,s=0^+} = 0,$$
$$\left.\frac{\partial W_U^H(\tau,s)}{\partial s}\right|_{\tau=0,s=0^-} = 0,$$
$$\left.\frac{\partial W_U^H(\tau,s)}{\partial s}\right|_{\tau=0,s>0} = \left.\frac{\partial I_1}{\partial s}\right|_{s>0} < 0,$$
$$\left.\frac{\partial W_U^H(\tau,s)}{\partial s}\right|_{\tau=0,s<0} = 0.$$

The differentiates show that $s > 0$ is negative while $s < 0$ is 0. Therefore, the optimal specific tax is at $s_U^H = 0$.

□

Proof of the effect of specific policy s on social welfare W_{U+V}^H.

$$\frac{\partial W_{U+V}^H(\tau,s)}{\partial s} = \frac{\partial I_1}{\partial s} + \frac{\partial I_2}{\partial s} + \frac{\partial I_5}{\partial s} + \frac{\partial I_6}{\partial s}.$$
$$\left.\frac{\partial W_{U+V}^H(\tau,s)}{\partial s}\right|_{\tau=0,s=0^+} = \frac{-5b^2}{4(b-a)(b+1)} < 0$$
$$\left.\frac{\partial W_{U+V}^H(\tau,s)}{\partial s}\right|_{\tau=0,s=0^-} = 0$$
$$\left.\frac{\partial W_{U+V}^H(\tau,s)}{\partial s}\right|_{\tau=0,s>0} = \frac{-9b^2-8b-4+16s+4(b+1)\sqrt{(b+1)^2-16s}}{4(b-a)\sqrt{(b+1)^2-16s}} < 0$$
$$\left.\frac{\partial W_{U+V}^H(\tau,s)}{\partial s}\right|_{\tau=0,s<0} = 0$$

$\left.\frac{\partial W_{U+V}^H(\tau,s)}{\partial s}\right|_{\tau=0,s>0} < 0$, since

$$-9b^2-8b-4+16s+4(b+1)\sqrt{(b+1)^2-16s} < -9b^2-8b-4+(b+1)^2 = -8b^2-6b-3 < 0$$

and therefore the numerator is negative.

These value when $s > 0$ is negative while $s < 0$ is 0. Therefore, the optimal specific tax is at $s^H_{U+V} = 0$.

□

□

Proof of proposition 3.3.5. The welfare $W^F_C(\tau, s)$ is given by

$$
\begin{aligned}
W^F_C(\tau,s) &= \int_a^b [q(\gamma)-p(\gamma)]\,f(\gamma)d\gamma - \int_a^b \Big\{\max\{\gamma q(\gamma)-p(\gamma)\} - [\gamma q(\gamma)-p(\gamma)]\Big\} f(\gamma)d\gamma \\
&\quad + \int_a^b [\tau p(\gamma)+s]\,f(\gamma)d\gamma \\
&= \underbrace{\int_a^{\underline{\gamma}(\tau,s)} 0 f(\gamma)d\gamma + \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \frac{(1-\tau)}{4}\left[-(\gamma-1)^2+\frac{(b-3)^2}{4}\right] f(\gamma)d\gamma + \int_{\bar{\gamma}}^{b} 0 f(\gamma)d\gamma}_{\text{Total Commitment Utility Surplus}} \\
&\quad \underbrace{- \int_a^{\frac{\underline{\gamma}(\tau,s)+1}{2}} 0 f(\gamma)d\gamma - \int_{\frac{\underline{\gamma}(\tau,s)+1}{2}}^{\frac{\bar{\gamma}+1}{2}} \frac{1-\tau}{4}\left(2\gamma-\frac{b+1}{2}\right)^2 f(\gamma)d\gamma - \int_{\frac{\bar{\gamma}+1}{2}}^{b} (1-\tau)(3-b)(\gamma-1)f(\gamma)d\gamma + \int_a^{\underline{\gamma}(\tau,s)} 0 f(\gamma)d\gamma}_{\text{Self-Control Cost}} \\
&\quad \underbrace{+ \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \frac{1-\tau}{4}\left(3\gamma^2-2b\gamma+\frac{b^2+2b-3}{4}\right) f(\gamma)d\gamma + \int_{\bar{\gamma}}^{b}(1-\tau)(3-b)(\gamma-1)f(\gamma)d\gamma}_{} \\
&\quad \underbrace{+\frac{(1-\tau)\tau}{4}\int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} [(\gamma+1)^2-(\frac{b-1}{2}+1)^2] f(\gamma)d\gamma + (1-\tau)\tau\int_{\bar{\gamma}}^{b}(3-b)f(\gamma)d\gamma}_{\text{Ad Valorem Tax Revenue}} \\
&\quad \underbrace{+\int_{\underline{\gamma}(\tau,s)}^{b} s f(\gamma)d\gamma}_{\text{Specific Tax Revenue}} \\
&\equiv M + I_3 + I_4 + I_5 + I_6 + T_1 + T_2 + S,
\end{aligned}
$$

where $I_i, i = 3,4,5,6$ are defined as before. I define the other symbols as follows:

$$M = \int_a^b [q(\gamma) - p(\gamma)] f(\gamma) d\gamma = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \frac{(1-\tau)}{4} \left[-(\gamma-1)^2 + \frac{(b-3)^2}{4} \right] f(\gamma) d\gamma$$

$$T_1 = \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} \tau p(\gamma) f(\gamma) d\gamma = \frac{(1-\tau)\tau}{4} \int_{\underline{\gamma}(\tau,s)}^{\bar{\gamma}} [(\gamma+1)^2 - (\frac{b-1}{2}+1)^2] f(\gamma) d\gamma$$

$$T_2 = \int_{\bar{\gamma}}^{b} \tau p(\gamma) f(\gamma) d\gamma = (1-\tau)\tau \int_{\bar{\gamma}}^{b} (3-b) f(\gamma) d\gamma$$

$$S = \int_{\underline{\gamma}(\tau,s)}^{b} s f(\gamma) d\gamma.$$

Taking the derivative of each I_i with respect to τ when $s = 0$, I have the following:

$$\frac{\partial M}{\partial \tau} = \int_{\frac{b-1}{2}}^{\frac{5-b}{2}} \frac{1}{4} \left[(\gamma-1)^2 - \frac{(b-3)^2}{4} \right] f(\gamma) d\gamma = \frac{(b-3)^3}{24(b-a)} < 0$$

$$\frac{\partial T_1}{\partial \tau} = \frac{1-2\tau}{4} \int_{\frac{b-1}{2}}^{\frac{5-b}{2}} [(\gamma+1)^2 - (\frac{b-1}{2}+1)^2] f(\gamma) d\gamma = \frac{(b-3)^2(b+9)}{24(b-a)}(1-2\tau)$$

$$\frac{\partial T_2}{\partial \tau} = (1-2\tau) \int_{\frac{5-b}{2}}^{b} (3-b) f(\gamma) d\gamma = \frac{(3-b)(3b-5)}{2(b-a)}(1-2\tau)$$

$$\frac{\partial S}{\partial \tau} = 0$$

The derivatives of T_1 and T_2 with respect to τ exhibit the following behavior:

For $\tau < \frac{1}{2}$:

$$\frac{\partial T_1}{\partial \tau} > 0 \quad \text{and} \quad \frac{\partial T_2}{\partial \tau} > 0.$$

This means both T_1 and T_2 increase as τ increases when τ is less than $\frac{1}{2}$.

For $\tau > \frac{1}{2}$:

$$\frac{\partial T_1}{\partial \tau} < 0 \quad \text{and} \quad \frac{\partial T_2}{\partial \tau} < 0.$$

This means both T_1 and T_2 decrease as τ increases when τ is greater than $\frac{1}{2}$.

At $\tau = \frac{1}{2}$:

$$\frac{\partial T_1}{\partial \tau} = 0 \quad \text{and} \quad \frac{\partial T_2}{\partial \tau} = 0.$$

This indicates that both T_1 and T_2 have critical points (likely maxima) at $\tau = \frac{1}{2}$.

The total effect is the sum of the individual effects:

$$\frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial \tau} = \frac{(1-2\tau)(b-3)(b^2-30b+33)}{24(b-a)}.$$

The term $(1-2\tau)$ determines the sign of the total effect based on τ: If $\tau < \frac{1}{2}$, $(1-2\tau) > 0$, so the total effect is positive. If $\tau > \frac{1}{2}$, $(1-2\tau) < 0$, so the total effect is negative. If $\tau = \frac{1}{2}$, $(1-2\tau) = 0$, so the total effect is zero. The term $(b-3)(b^2-30b+33)$ is given to be positive for $b \in \left(\frac{5}{3}, 3\right)$. The denominator $24(b-a)$ is obviously positive.

□

Proof of proposition 3.3.6.

$$\frac{\partial W_C^F(\tau,s)}{\partial \tau} = \frac{\partial M}{\partial \tau} + \frac{\partial I_3}{\partial \tau} + \frac{\partial I_4}{\partial \tau} + \frac{\partial I_5}{\partial \tau} + \frac{\partial I_6}{\partial \tau} + \frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{\partial \tau} + \frac{\partial S}{\partial \tau}.$$

I will consider the effect on welfare of raising τ or s from zero and look for the optimal tax policies of τ and s.

$$\left.\frac{\partial W_C^F(\tau,s)}{\partial \tau}\right|_{\tau=s=0} = -\frac{(b-3)}{48(b-a)}(7b^2+30b-33) > 0$$

$$\left.\frac{\partial W_C^F}{\partial \tau}\right|_{s=0} = -\frac{(b-3)}{48(b-a)}\left[(7b^2+30b-33)+4(b^2-30b+33)\tau\right]$$

The optimal ad valorem policy $\tau_C^F = -\frac{7b^2+30b-33}{4(b^2-30b+33)}$ since $\left.\frac{\partial W_C^F(\tau,s)}{\partial \tau}\right|_{s=0} > 0$ when $\tau < \tau_C^F$ and $\left.\frac{\partial W_C^F(\tau,s)}{\partial \tau}\right|_{s=0} < 0$ when $\tau > \tau_C^F$.

□

Proof of proposition 3.3.7. The normative social welfare W_U^F is given by is

$$W_U^F(\tau,s) = \int_a^b [q(\gamma) - p(\gamma)]\, f(\gamma)d\gamma + \int_a^b [\tau p(\gamma) + s]\, f(\gamma)d\gamma$$
$$= M + T_1 + T_2 + S.$$

Therefore,

$$\frac{\partial W_U^F(\tau,s)}{d\tau} = \frac{\partial M}{\partial \tau} + \frac{\partial T_1}{\partial \tau} + \frac{\partial T_2}{d\tau} + \frac{\partial S}{\partial \tau}.$$

$$\left.\frac{\partial W_U^F(\tau,s)}{\partial \tau}\right|_{\tau=s=0} = \frac{(b-3)}{12(b-a)}(b^2 - 18b + 21) > 0$$
$$\left.\frac{\partial W_U^F(\tau,s)}{\partial \tau}\right|_{s=0} = \frac{(b-3)}{12(b-a)}\left[(b^2 - 18b + 21) + (-b^2 + 30b - 33)\tau\right]$$

The optimal ad valorem policy $\tau_U^F = \frac{b^2-18b+21}{b^2-30b+33}$ since $\left.\frac{\partial W_U^F(\tau,s)}{\partial \tau}\right|_{s=0} > 0$ when $\tau < \tau_U^F$ and $\left.\frac{\partial W_U^F(\tau,s)}{\partial \tau}\right|_{s=0} < 0$ when $\tau > \tau_U^F$.

□

Proof of proposition 3.3.8. The behavioral social welfare W_{U+V}^F is given by is

$$W_{U+V}^F(\tau,s) = \int_a^b [q(\gamma) - p(\gamma)]\, f(\gamma)d\gamma + \int_a^b [\gamma q(\gamma) - p(\gamma)]\, f(\gamma)d\gamma$$
$$\equiv M + I_5 + I_6 + T_1 + T_2 + S.$$

Therefore,

$$\frac{\partial W_{U+V}^F(\tau,s)}{\partial \tau} = \frac{\partial M}{\partial \tau} + \frac{\partial I_5}{d\tau} + \frac{\partial I_6}{\partial \tau} + \frac{\partial T_1}{d\tau} + \frac{\partial T_2}{\partial \tau} + \frac{\partial S}{\partial \tau}.$$

The change of $W_{U+V}^F(\tau,s)$ on τ from 0 given no specific tax s:

$$\left.\frac{\partial W_{U+V}^F(\tau,s)}{\partial \tau}\right|_{\tau=s=0} = \frac{b-3}{96(b-a)}(35b^2 - 162b + 123) > 0.$$

$$\left.\frac{\partial W^F_{U+V}(\tau, s)}{\partial \tau}\right|_{s=0} = \frac{b-3}{96(b-a)}\left[(35b^2 - 162b + 123) + 8(-b^2 + 30b - 33)\tau\right].$$

The optimal ad valorem policy $\tau^F_{U+V} = \frac{35b^2-162b+123}{8(b^2-30b+33)}$ since $\left.\frac{\partial W^F_{U+V}(\tau,s)}{\partial \tau}\right|_{s=0} > 0$ when $\tau < \tau^F_{U+V}$ and $\left.\frac{\partial W^F_{U+V}(\tau,s)}{\partial \tau}\right|_{s=0} < 0$ when $\tau > \tau^F_{U+V}$.

□

Proof of proposition 3.3.9. Taking the derivative of each I_i with respect to s as s approaches from the positive side and as s approaches from the negative side, I have the following:

$$\left.\frac{\partial M}{\partial s}\right|_{s>0} = \frac{1}{4(b-a)}\left[(\underline{\gamma}(s)-1)^2 - \frac{(b-3)^2}{4}\right]\frac{d\underline{\gamma}(s)}{ds}$$
$$= \frac{b^2 - 1 - 4s - (b-1)\sqrt{(b+1)^2 - 16s}}{(b-a)\sqrt{(b+1)^2 - 16s}} < 0$$
$$\left.\frac{\partial T_i}{\partial s}\right|_{s>0} = 0, \quad i = 1,2$$
$$\left.\frac{\partial S}{\partial s}\right|_{s>0} = -sf(\underline{\gamma}(s))\frac{\partial \underline{\gamma}(s)}{\partial s} + \int_{\underline{\gamma}(s)}^{b} f(\underline{\gamma}(s))d\gamma = \frac{1}{b-a}\left(-s\frac{d\underline{\gamma}(s)}{ds} + b - \underline{\gamma}(s)\right)$$
$$= \frac{1}{b-a}\left(-\frac{4s}{\sqrt{(b+1)^2-16s}} + \frac{\sqrt{(b+1)^2-16s}}{2}\right) = \frac{(b+1)^2 - 24s}{2(b-a)\sqrt{(b+1)^2-16s}}$$
$$\left.\frac{\partial U}{\partial s}\right|_{s<0} = 0$$
$$\left.\frac{\partial T_i}{\partial s}\right|_{s<0} = 0, \quad i = 1,2$$
$$\left.\frac{\partial S}{\partial s}\right|_{s<0} = \int_{\frac{b-1}{2}}^{b} f(\underline{\gamma}(s))d\gamma = \frac{b+1}{2(b-a)} > 0.$$

$\left.\frac{\partial M}{\partial s}\right|_{s>0} < 0$: increasing specific tax decreases the size of market, and thus the lowest leaving causes the welfare of total commitment to be reduced.

For $\left.\frac{\partial S}{\partial s}\right|_{s>0}$, there two opposite effects of the specific tax policy s. On the one hand, the market size reduces due to the increase of $\underline{\gamma}(s)$ as s increases; on the other hand, the tax revenue increases as s increases. The total effect depends on which effect dominates.

Specifically, when the market size is large enough, $b \in (\frac{5}{3}, \frac{13+4\sqrt{3}}{11})$, the total effect depends on the value of s. A small amount of specific tax causes the increase of tax revenue more than the decrease of market size, then $\frac{\partial S}{\partial s}\Big|_{s>0} > 0$ if $s \in (0, \frac{(b+1)^2}{24})$; While if the amount of tax excesses a threshold, the negative effect of market size dominates the other positive effect, that is, $\frac{\partial S}{\partial s}\Big|_{s>0} < 0$ if $s \in (\frac{(b+1)^2}{24}, \frac{-3b^2+10b-3}{16})$.

When the market size is small, $b \in (\frac{13+4\sqrt{3}}{11}, 3)$, then the effect of tax revenue always dominates among the aggregate effect for any feasible tax policy, $\frac{dS}{ds}\Big|_{s>0} > 0$.

Therefore,

$$\frac{\partial W_C^F(\tau,s)}{\partial s} = \frac{\partial M}{\partial \tau} + \frac{\partial I_3}{\partial s} + \frac{\partial I_4}{\partial s} + \frac{\partial I_5}{\partial s} + \frac{\partial I_6}{\partial s} + \frac{\partial T_1}{\partial s} + \frac{\partial T_2}{\partial s} + \frac{\partial S}{\partial s}.$$

$$\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} = \frac{-5b^2+2b+13}{8(b-a)(b+1)},$$

$$\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^-} = \frac{b+1}{2(b-a)} > 0,$$

$$\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} = \frac{-b^2-6b+1-96s+4(3-b)\sqrt{(b+1)^2-16s}}{8(b-a)\sqrt{(b+1)^2-16s}}$$

$$\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s<0} = \frac{b+1}{2(b-a)} > 0$$

The critical point is $b_C^F = \frac{1+\sqrt{66}}{5} \approx 1.83$.

$$\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} \begin{cases} > 0 & \text{for } b \in \left(\frac{5}{3}, b_C^F\right), \\ = 0 & \text{at } b = b_C^F, \\ < 0 & \text{for } b \in (b_C^F, 3). \end{cases}$$

The optimal specific policy is

$$s_{C}^{F}(\tau, s) = \frac{-7b^2 + 6b - 33}{288} + \frac{(3-b)\sqrt{23b^2 + 42b + 33}}{72\sqrt{2}}$$

$$s_{C}^{F} = \begin{cases} -\frac{5}{16}(3b^2 + 4b + 2) + \frac{b+1}{4}\sqrt{12b^2 + 14b + 7}, & \text{when } b \in (\frac{5}{3}, b_{C}^{F}) \\ 0, & \text{when } b \in [b_{C}^{F}, 3) \end{cases}$$

Since when $b \in (\frac{5}{3}, b_{C}^{F})$, $\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} > 0$ when $s < s_{C}^{F}$ and $\frac{\partial W_C^F(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} < 0$ when $s > s_{C}^{F}$. When $b \in [b_{C}^{F}, 3)$, $\frac{\partial W_C^F(\tau,s)}{\partial s}\Bigg|_{\tau=0,s<0} > 0$ and $\frac{dW_C^F(\tau,s)}{ds}\Bigg|_{\tau=0,s>0} < 0$.

□

Proof of proposition 3.3.10.

$$\begin{aligned}
\frac{\partial W_{U}^{F}(\tau, s)}{\partial s} &= \frac{\partial M}{\partial s} + \frac{vT_1}{\partial s} + \frac{\partial T_2}{\partial s} + \frac{\partial S}{\partial s}. \\
\frac{\partial W_{U}^{F}(\tau, s)}{\partial s}\Big|_{\tau=0,s=0^+} &= \frac{b+1}{2(b-a)} > 0, \\
\frac{\partial W_{U}^{F}(\tau, s)}{\partial s}\Big|_{\tau=0,s=0^-} &= \frac{b+1}{2(b-a)} > 0, \\
\frac{\partial W_{U}^{F}(\tau, s)}{\partial s}\Big|_{\tau=0,s>0} &= \frac{3b^2 + 2b - 1 - 32s - 2(b-1)\sqrt{(b+1)^2 - 16s}}{2(b-a)\sqrt{(b+1)^2 - 16s}}, \\
\frac{\partial W_{U}^{F}(\tau, s)}{\partial s}\Big|_{\tau=0,s<0} &= \frac{b+1}{2(b-a)} > 0.
\end{aligned}$$

$$\begin{aligned}
s_{U}^{F} &= \min\left\{\frac{2(b^2 + 2b - 1) - (b-1)\sqrt{8 - (b-1)^2}}{32}, \frac{-3b^2 + 10b - 3}{16}\right\} \\
&= \begin{cases} \frac{2(b^2+2b-1)-(b-1)\sqrt{8-(b-1)^2}}{32}, & \text{when } b \in (\frac{5}{3}, 1 + \frac{2}{\sqrt{5}}) \\ \frac{-3b^2+10b-3}{16}, & \text{when } b \in [1 + \frac{2}{\sqrt{5}}, 3) \end{cases}
\end{aligned}$$

$\frac{dW^F_U(\tau,s)}{ds}\Big|_{\tau=0,s>0} > 0$, when $s < s^F_U$.
$\frac{dW^F_U(\tau,s)}{ds}\Big|_{\tau=0,s>0} = 0$, when $s = s^F_U$.
$\frac{dW^F_U(\tau,s)}{ds}\Big|_{\tau=0,s>0} < 0$, when $s > s^F_U$.

□

Proof of proposition 3.3.11.

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s} = \frac{\partial M}{\partial s} + \frac{\partial I_5}{\partial s} + \frac{\partial I_6}{\partial s} + \frac{\partial T_1}{\partial s} + \frac{\partial T_2}{\partial s} + \frac{\partial S}{\partial s}.$$

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} = \frac{-3b^2+4b+2}{4(b-a)(b+1)}$$

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^-} = \frac{b+1}{2(b-a)} > 0$$

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} = \frac{-7b^2-4b-2-16s+4(b+1)\sqrt{(b+1)^2-16s}}{4(b-a)\sqrt{(b+1)^2-16s}}$$

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s<0} = \frac{b+1}{2(b-a)} > 0$$

The critical point is $b^F_{U+V} = \frac{2+\sqrt{10}}{3} \approx 1.72$

$$\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s=0^+} \begin{cases} > 0 & \text{for } b \in \left(\frac{5}{3}, b^F_{U+V}\right), \\ = 0 & \text{at } b = b^F_{U+V}, \\ < 0 & \text{for } b \in (b^F_{U+V}, 3). \end{cases}$$

The optimal specific policy

$$s^F_{U+V} = \begin{cases} -\frac{5}{16}(3b^2+4b+2) + \frac{b+1}{4}\sqrt{12b^2+14b+7}, & \text{when } b \in (\frac{5}{3}, b^F_{U+V}) \\ 0, & \text{when } b \in [b^F_{U+V}, 3) \end{cases}$$

Since when $b \in (\frac{5}{3}, b^F_{U+V})$, $\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} > 0$ when $s < s^F_{U+V}$ and $\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s>0} < 0$ when $s > s^F_{U+V}$. When $b \in [b^F_{U+V}, 3)$, $\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\Big|_{\tau=0,s<0} >$

0 and $\left.\frac{\partial W^F_{U+V}(\tau,s)}{\partial s}\right|_{\tau=0,s>0} < 0$.

when $b \in (\frac{5}{3}, b^F_{U+V})$, $s^F_{U+V} \in (0, -\frac{85}{16} + 2\frac{\sqrt{573}}{9})$;

when $b \in (b^F_{U+V}, 3)$, $s^F_{U+V} = 0$.

□

Conclusion

This thesis has examined the interplay between strategic communication, conformity, and self-control in shaping individual behavior and policy-relevant decisions. Across the three essays, I have explored how behavioral motives and structural constraints interact to influence social learning, rumor propagation, and optimal taxation. A unifying theme is that individual preferences in terms of conformity, biased outcomes, or self-control significantly affect information transmission, coordination, and welfare, with implications for both theory and policy.

In Chapter 1, I analyzed a multi-receiver strategic communication game in which agents are heterogeneous, consisting of unbiased truth-seekers and biased agents with partisan preferences. I introduce conformity as a preference for aligning actions with others and examined its impact on social learning. The results show that moderate conformity can enhance information transfer by providing additional incentives for unbiased agents to act on messages they receive, even in the presence of biased agents. Equilibrium outcomes depend on population size, the degree of conformity, and the share of biased agents: in small populations, information can be transferred without upper bounds on biased agents, whereas in large populations, truthful equilibria persist provided biased agents are not a majority. The analysis offers insights into real-world phenomena such as influencer-driven consumption, rumor spread, technology adoption, and government policy communication, where alignment incentives coexist with the desire for accurate information.

Chapter 2 extends this framework to networked environments, adapting the model to a simple undirected line network and incorporating conformity among neighbors. Consistent with prior work (Bloch et al., 2018), the network structure imposes stricter constraints on the proportion of biased agents that allow for truthful communication. While conformity can facilitate coordination locally, the network's decentralized nature reduces the robustness of social learning compared to public broadcast settings. This extension provides a clearer understanding of rumor propagation, political misinformation, and peer-influenced decision-making, demonstrating how local social interactions and network topology affect the balance between truthfulness and conformity.

Chapter 3 shifts the focus from information and networks to consumer behavior and policy design. I analyzed optimal taxation of sin goods in a monopoly market when consumers face heterogeneous self-control problems. By incorporating the temptation framework of Gul and Pesendorfer (2001), I distinguished between upward-tempered and downward-tempered consumers and characterized the monopolist's profit-maximizing menu. Behavioral welfare analysis reveals that domestic ad valorem taxes can improve social welfare when upward-tempered consumers are prevalent, whereas ad valorem subsidies may be preferable for populations dominated by downward-tempered consumers. For a foreign monopolist, both ad valorem and specific taxes enhance national welfare. The results challenge conventional policy heuristics, highlighting that optimal taxation depends on the distribution of consumer temptations and the market context. Furthermore, the analysis demonstrates that welfare evaluations are sensitive to the normative framework, underscoring the importance of distinguishing commitment, temptation, and ex-post utilities in behavioral settings.

Collectively, the three essays contribute to a broader understanding of how behavioral motives influence economic outcomes. Chapters 1 and 2 illuminate the role of conformity in shaping information transfer and social learning, showing that social interactions can either facilitate or hinder the spread of truthful information through public broadcast and network. Chapter 3 demonstrates that behavioral considerations, such as self-control costs, fundamentally alter policy design and its welfare implications. Overall, the findings suggest that economic models and policy interventions must account for both individual biases and the social or institutional context in which decisions occur.

The thesis also identifies several avenues for future research. Incorporating heterogeneity in conformity parameters, multiple types of biased agents, or richer network structures could improve the realism of the communication models. Similarly, extending the taxation framework to dynamic settings or competitive markets would allow for more nuanced comparisons and provide further guidance for policy design. Overall, the results highlight that behavioral motives such as conformity, bias, and self-control are critical determinants of both micro-level decisions and macro-level outcomes, offering valuable insights for theory, empirical research, and public policy.

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