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Essays on Financial Frictions and Macroeconomic Policy

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A Dissertation

Submitting in fulfilment of the requirements of the Degree of Doctor of Philosophy

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Abstract

This thesis contains of four chapters. The first chapter presents the introduction and all other three chapters look at different aspects of monetary policy in an economy with financial frictions.

The second chapter studies the conditions under which a modest financial shock can trigger a deep recession with a prolonged period of slow recovery. We suggest that two factors can generate such a profile. The first is that the economy has accumulated a moderately high level of private debt by the time the adverse shock occurs. The second factor is when monetary policy is restricted by the zero lower bound. When present, these factors can result in a sharp contraction in output followed by a slow recovery. Perhaps surprisingly, we use a standard DSGE model with financial frictions along the lines of Jermann and Quadrini (2012) to demonstrate this result and so do not need to rely on dysfunctional interbank markets.

The third chapter studies international transmission of financial shocks between two economies under flexible exchange rate regime. We consider different degrees of financial integration and demonstrate that welfare is maximised for an intermediate value of degree of it. Under perfect risk sharing there is large volatility of output during the period of adjustment, while the deleveraging is performed faster. With greater restrictions on international financial flows, the deleveraging is substantially slowed down which leads to longer periods of adjustment and greater costs. We demonstrate that in such world the effect of one country's credit shock has very limited effect on another country. When monetary policymakers cooperate and choose interest rate optimally, the unaffected country can nearly eliminate all aftereffects of the shock to the other country. To some extent, limited financial integration prevents the spread of volatility across the border, however, unconstrained monetary policy is the key to these results.

In the fourth chapter we use two-country model and assume that both countries are locked into a permanently fixed exchange rate regime within a currency union. We demonstrate that the centralised monetary policy alone is unable to stabilise the economy. National fiscal policies must be activated to counteract asymmetric shocks. We demonstrate, however, that the effectiveness of fiscal policy is limited. Even if it is chosen optimally, fiscal policy does not eliminate cyclical patterns in economic adjustment, which is welfare-reducing volatility of economic variables. This model reveals that shocks hitting one economy, result in sharp contraction of consumption in both countries.

Contents

List of Figures

List of Tables

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I cannot end without thanking my family, especially my parents, on whose constant encouragement and love I have relied throughout my time at the Academy. Their unflinching courage will always inspire me in my works.

To my husband Mohammad, all I can say is it would take another thesis to express my deep love for you. Your patience, love and encouragement have upheld me, particularly in those many days in which I spent more time with my computer than with you. I promise! Mohammad, now it's your turn.

Finally, I would like to acknowledge for the scholarship provided by the Adam Smith Business School, and a great thank to all the Business School staff at the University of Glasgow.

It is to my parents and my husband that I dedicate this work.

Maryam Shafiei

September 2017

Author's Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Glasgow or any other institution.

Printed Name: MARYAM SHAFIEI DEH ABAD

Signature:

Chapter 1

Introduction

Nearly a decade has passed since the beginning of the Great Recession in 2008, but for a number of economies recovery is slow and remains weak. In fact, some even worry it has led in a new era of permanently lower trend growth. The Great Recession stands out for at least two reasons. First, in each country the average recession is not as *deep* as the Great Recession. Second, in each country the average recession experiences its lowest output growth rate at the beginning of the recession (period zero in the figure). Thereafter, economies generally return to a more normal growth path within one or two quarters. By way of contrast, following the arrival of the Great Recession growth reached its minimum some two to four quarters later and growth remains at the lower end of the typical post-war experience; recovery has been *slow* by recent historical standards.

The role of 'financial frictions' in explaining these observed patterns has been identified as central by many researchers and policymakers and there is now a large and growing body of research which seeks to provide quantitative macroeconomic models to explain how seemingly well-functioning economies might 'unexpectedly' end up with financial crises. For example, Boissay et al. (2015) demonstrate how an 'innocuous' positive productivity shock can lead to rare but deep financial crises. The role of financial shocks in generating realistically frequent recessions is discussed in Nolan and Thoenissen (2009), Jermann and Quadrini (2012), Christiano et al. (2013) and Mumtaz and Zanetti (2016) to mention only a few. Notably, Jermann and Quadrini (2012) propose a macroeconomic model with financial frictions which is quite closely aligned with the US data since the early 1990s. The authors point out, however, that their model cannot replicate the *large* reduction in hours worked and output observed during the Great Recession. Moreover, so far researchers in this literature have generally focussed less on explaining the second aspect of the Great Recession that we highlighted above: the 'slow recovery'.[1]

[1] This is despite the two-decades of slow growth affecting Japan since the early 1990s. An interesting recent exception is a paper by Benigno and Fornaro (2015) emphasising non-linear features in a growth model to explain 'stagnation traps'.

The Great Recession affected many countries. The transmission of shocks was quick and strong. We need a model to study how this was possible. Recently, Perri and Quadrini (2011) use a two-country model with financial market frictions to demonstrate how financial shocks may transmit from one country to another. It crucially relies on financial integration between countries. Domestic and foreign firms are able to borrow at a global financial market at the same interest rate. They demonstrate how a shock, which originates in one country, creates a shortage of liquidity in both countries and results in an international recession.

In this thesis I investigate how a small financial shock can result in large reduction in output and a prolonged recession. I look at single closed economy setting as well as two-country model.

In the second chapter we demonstrate how a simple log-linear DSGE model with financial frictions a la Jermann and Quadrini (2012) and with nominal rigidities a la Rotemberg (1983) is capable of generating the observed deep recessions, and slow recoveries that we argue are present in the data. In this model, the recession is triggered by a financial shock which reduces the proportion of output which banks will be able to recover when firms default. The implied reduction in bank credit requires substantial deleveraging in the economy and this leads to a recession. This result is interesting as some have argued that the onset of the crisis was essentially an adverse credit event (see e.g., Taylor, 2008), whilst others attribute a large role to adverse selection and moral hazard problems in the interbank/money markets (Boissay et al., 2015). We suspect both explanations likely played a role, but our contribution in this chapter is simply to argue that standard models of financial frictions explain more than perhaps is generally realised. Two assumptions are helpful in generating a deep recession, consistent with that observed during the recent financial crisis. The first is the existence of 'overlending'. An initially high level of debt as indeed was observed prior to the Great Recession in many developed countries, see e.g. Schularick and Taylor (2012). Following the shock the reduction in debt presages a fall in the capital stock. In turn the lower capital stock requires less finance and these two effects reinforce one another and the sluggish adjustment of both stocks results in a much greater reduction of capital, output and hours worked. We show that the higher the initial debt to output ratio, the sharper the subsequent recession. For example, if the stock of lending is 10 percent above its steady state level, a financial shock nearly doubles the consequent reduction in output, compared with the case when debt is initially at steady state. The second assumption is the existence of the zero lower bound (ZLB) on nominal interest rates. We demonstrate that the ZLB, alone, can generate sharp and deep recessions. The inability of the interest rate to help 'spread' the cost of the required deleveraging over many periods, results in an immediate and sharp reduction in the capital stock. Once the capital stock returns to the optimal level, further adjustment is slow. This is why the ZLB scenario facilitates capturing the second stylised fact: Once the initial large reduction in output is corrected, the speed of further recovery is substantially slowed down as compared to the case when monetary policy operates without constraints.

In the third chapter and the fourth chapter, I model interdependent economies to describe channels of transmission of credit shocks between countries.

In the third chapter, we consider different degrees of financial integration between countries and investigate their effect. The model is based on the one presented in the second chapter, but firms can borrow from abroad. When allocating portfolios between home and foreign corporate bonds, households face intermediation costs, which depend on the degree of external indebtedness of the home economy. There is a risk premium if the country is in net borrowing position. We also assume two independent monetary authorities, and therefore, floating exchange rate regime, and the ability of each policymaker to affect its own interest rate. Our main objective is to investigate how the degree of financial integration affects international transmission of credit shocks. We also investigate if the country size matters for the severity of recessions. Our results demonstrate that welfare is maximised for an intermediate value of degree of financial integration. If the intermediation cost is absent, there is large volatility of output during the period of adjustment, while the deleveraging is performed faster. With greater costs on international financial flows, the deleveraging is substantially slowed down which leads to longer periods of adjustment and greater costs. We demonstrate that in two country model under flexible exchange rate and independent monetary authorities, the effect of one country's credit shock has very limited effect on another country. When monetary policymakers cooperate and choose interest rates optimally, the unaffected country can nearly eliminate all aftereffects of the shock to the other country. To some extent, limited financial integration prevents the spread of volatility across the border, however, unconstrained monetary policy is the key to these results.

In the fourth chapter, we investigate the importance of exchange rate regime for international transmission of credit shocks. Specifically, we use the same two-country model as in the previous chapter, but assume that both countries are locked into a permanently fixed exchange rate regime within a currency union. We therefore ignore any issues of imperfect credibility of exchange rate pegs and do not discuss exchange rate crises. We, therefore, also assume that the monetary policymaker has a mandate to stabilise both countries' economies. We demonstrate that, unlike under flexible exchange rate regime studied in the previous chapter, the centralised monetary policy alone is unable to stabilise the economy. National fiscal policies must be activated to counteract asymmetric shocks. We demonstrate, however, that the effectiveness of fiscal policy is limited. Even if it is chosen optimally, fiscal policy does not eliminate cyclical patterns in economic adjustment, which is welfare-reducing volatility of economic variables. This model demonstrates that shocks hitting one economy, result in sharp contraction of consumption in another country. Countercyclical fiscal policy is able to avoid major recession, however. In contrast to results in the previous chapter, the shocks propagation mechanism is much stronger under fixed exchange rate regime. As before, we assume variable degree of financial integration and study its importance for the propagation of credit shocks.

Chapter 2

Deep Recessions

2.1 Introduction

Almost a decade has passed since the onset of the Great Recession in 2008, but for a number of economies recovery is slow and remains fragile. In fact, some even worry it has ushered in a new era of permanently lower trend growth. The origins of some of these concerns are reflected in Figure 2.1, which plots all post-war recessions for the UK, US and Japan.[1] The Great Recession stands out for at least two reasons. First, in each country the average recession is not as *deep* as the Great Recession. Second, in each country the average recession experiences its lowest output growth rate at the onset of the recession (period zero in the figure). Thereafter, economies generally return to a more normal growth path within one or two quarters. By way of contrast, following the onset of the Great Recession growth reached its minimum some two to four quarters later and growth remains at the lower end of the typical post-war experience; recovery has been *slow* by recent historical standards.

The role of 'financial frictions' in explaining these observed patterns has been identified as central by many researchers and policymakers and there is now a large and growing body of research which seeks to provide quantitative macroeconomic models to explain how seemingly well-functioning economies might 'unexpectedly' end up with financial crises. For example, Boissay et al. (2015) demonstrate how an 'innocuous' positive productivity shock can lead to rare but deep financial crises. The role of financial shocks in generating realistically frequent recessions is discussed in Nolan and Thoenissen (2009), Jermann and Quadrini (2012), Christiano et al. (2013) and Mumtaz and Zanetti (2016) to mention only a few. Notably, Jermann and Quadrini (2012) propose a macroeconomic model with financial frictions which is quite closely aligned with the US data since the

[1] Episods in Figure 2.1 can be identified in the available data series in the FRED FRB St. Louis database. The data series are NAEXKP01JPQ657S (Japan 1960Q1-2014Q4), NAEXKP01GBQ652S_PCH (UK 1955Q1-2014Q4) and GDPC1_PCH (US 1947Q1-2015Q1). All series are seasonally adjusted.

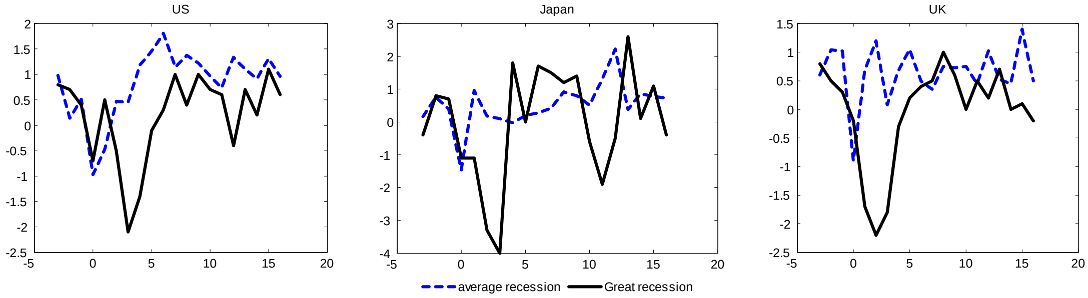
early 1990s. The authors point out, however, that their model cannot replicate the *large* reduction in hours worked and output observed during the Great Recession. Moreover, so far researchers in this literature have generally focussed less on explaining the second aspect of the Great Recession that we highlighted above: the 'slow recovery'.[2]

In this chapter we demonstrate how a simple log-linear DSGE model with financial frictions *a la* Jermann and Quadrini (2012) and with nominal rigidities *a la* Rotemberg (1983) is capable of generating the observed *deep* recessions, and *slow* recoveries that we argue are present in the data. In this model, the recession is triggered by a financial shock which reduces the proportion of output which banks will be able to recover when firms default. The implied reduction in bank credit requires substantial deleveraging in the economy and this leads to a recession. This result is interesting as some have argued that the onset of the crisis was essentially an adverse credit event (see e.g., Taylor, 2008), whilst others attribute a large role to adverse selection and moral hazard problems in the interbank/money markets (Boissay et al., 2015). We suspect both explanations likely played a role, but our contribution in this chapter is simply to argue that standard models of financial frictions explain more than perhaps is generally realised.

Two assumptions are helpful in generating a deep recession, consistent with that observed during the recent financial crisis. The first is the existence of 'overlending'. An initially high level of debt as indeed was observed prior to the Great Recession in many developed countries, see e.g. Schularick and Taylor (2012). Following the shock the reduction in debt presages a fall in the capital stock. In turn the lower capital stock requires less finance and these two effects reinforce one another and the sluggish adjustment of both stocks results in a much greater reduction of capital, output and hours worked. We show that the higher the initial debt to output ratio, the sharper the subsequent recession. For example, if the stock of lending is 10 percent above its steady state level, a financial shock nearly doubles the consequent reduction in output, compared with the case when debt is initially at steady state.

The second assumption is the existence of the zero lower bound (ZLB) on nominal interest rates. We demonstrate that the ZLB, alone, can generate sharp and deep recessions. The inability of the interest rate to help 'spread' the cost of the required deleveraging over many periods, results in an immediate and sharp reduction in the capital stock. Once the capital stock returns to the optimal level, further adjustment is slow. This is why the ZLB scenario facilitates capturing the second stylised fact: Once the initial large reduction in output is corrected, the speed of further recovery is substantially slowed down as compared to the case when monetary policy operates without constraints.

[2] This is despite the two-decades of slow growth affecting Japan since the early 1990s. An interesting recent exception is a paper by Benigno and Fornaro (2015) emphasising non-linear features in a growth model to explain 'stagnation traps'.



The period zero corresponds to the start of each of post-war recessions.[3]

Figure 2.1: Post-war recessions in the US, Japan and UK. The data series are available in the FRED FRB St. Louis database. The data series are NAEXKP01JPQ657S (Japan 1960Q1-2014Q4), NAEXKP01GBQ652S-PCH (UK 1955Q1-2014Q4), GDPC1-PCH (US 1947Q1-2015Q1).

The chapter is organized as follows. In the next section we present the model. We discuss the empirical evidence and the corresponding calibration of the model in section 2.5. In section 2.6 we discuss the sequence of policy experiments, which show how to generate stylized post-crisis dynamics like those with which we motivated this chapter. Section 2.7 concludes.

2.2 The Model

We present a simple model with firms' borrowing constraints *a la* Jermann and Quadrini (2012) and with nominal rigidities *a la* Rotemberg (1983). The economy is populated by households and firms. Firms use labor and capital to produce differentiated goods. Firms issue equity and debt and use intra-period loans to finance working capital. Firms face credit restrictions because financial intermediaries fear they may not repay those loans. The detailed model of the economy is presented in this section.

2.2.1 Households

There is a continuum of homogeneous infinitely-living households of measure one who share identical preferences and technology. Households are indexed by j. The typical household seeks to maximize the following utility function:

$$\max_{C_t^j, n_t, b_{t+1}, s_{t+1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t U\left(C_t^j, n_t^j\right)$$

where $\mathbb{E}_t$ indicates expectations conditional on information available at time t; $0 < \beta < 1$ is the discount factor; C_t and n_t are a consumption aggregate and labor supply in period

t, respectively. The period utility is:

$$U\left(C_t^j, n_t^j\right) = \frac{C_t^{j1-\sigma}}{1-\sigma} - \alpha\frac{n_t^{j1+\psi}}{1+\psi}$$

where σ is elasticity of relative risk aversion, ψ is the elasticity of labour supply and α is a 'preference' parameter. The households are assumed to hold equity, shares and corporate bonds, which are for simplicity assumed to be real bonds. The household's budget constraint in nominal term can be written as:

$$W_t n_t^j + P_t b_t + \Omega_t s_t\left(D_t + \bar{p}_t\right) + P_t\Phi_t = q_t P_t b_{t+1} + s_{t+1}\bar{p}_t + P_t C_t^j + P_t T_t \tag{2.1}$$

where W_t is the nominal wage rate, P_t is the price of goods, $\bar{p}_t$ is the market price of shares, D_t is the dividend, s_t is equity holdings. $P_t b_t$ is the market nominal value of one-period real bonds b_t which mature at period t held by the households, q_t is t-period price of bonds which mature at t+1, $P_t\Phi_t$ is nominal profit from the ownership of final good firms, T_t is a government transfer and the nominal return on bonds is

$$1 + i_t = \frac{\Pi_{t+1}}{q_t}.$$

Finally, Ω_t is a capital quality shock. The capital quality shock will serve as an exogenous trigger of debt dynamics. The random variable Ω_t captures some form of economic obsolescence, as opposed to physical depreciation.[4] We allow for occasional disasters in the form of sharp contractions in quality as we describe later. These disaster shocks serve to initiate financial crises. We assume that the households share the revenues of owning firms in equal proportion. Following Woodford (2003) we consider a cashless economy. Therefore the only explicit role played by money is to serve as a unit of account.

We assume that all households have the same level of initial wealth. As they face the same labour demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use individual index.

The standard optimization of utility with respect to C_t, n_t , b_{t+1} , and s_{t+1}, subject to the budget constraint yields the following system of first order conditions:

$$\alpha n_t^{\psi} = w_t C_t^{-\sigma}; \tag{2.2}$$

$$q_t = \beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}; \tag{2.3}$$

$$\frac{\bar{p}_t}{P_t} = \beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\left(d_{t+1} + \frac{\bar{p}_{t+1}}{P_{t+1}}\right)\Omega_{t+1}; \tag{2.4}$$

$$b_t = q_t b_{t+1} + s_{t+1}\frac{\bar{p}_t}{P_t} + C_t + T_t - \Omega_t s_t\left(d_t + \frac{\bar{p}_t}{P_t}\right) - w_t n_t - \Phi_t. \tag{2.5}$$

[4]Following the finance literature, e.g. Merton (1973), the capital quality shock is a simple way to introduce an exogenous source of variation in the value of capital. Our treatment of this shock follows Gertler et al. (2012).

where we defined All derivations are given in Appendix A.1. These conditions are familiar, reflecting optimal labour supply decisions, bond purchases, equity purchases and intertemporal resource allocation. The budget constraint is written in an aggregated form. Equation (2.3) is the standard Euler equation and determines the consumption smoothing behavior of the households. (2.2) is the standard labour supply condition. It determines the quantity of labor supplied as a function of real wage, given the marginal utility of consumption. Finally equation (2.5) is the aggregate budget constraint.

2.2.2 Firms

There are two types of firms in this economy. There are flexible price intermediate goods producers and monopolistically competitive retailers. We discuss each in turn.

Intermediate goods producers

We assume that there is a continuum of firms of measure one with a standard production technology

$$y_t = F(k_t, n_t) = Ak_t^{\theta} n_t^{1-\theta} \tag{2.6}$$

and all firms are equal. A is a constant productivity shifter, n_t is the labor input which can be flexibly changed at time t, k_t is the capital stock determined at time $t-1$ and predetermined at time t which is consistent with the typical timing convention. θ is the capital share. Capital in process for period $t+1$ is transformed into capital for production after the realisation of a multiplicative shock to capital quality, Ω_{t+1}

$$k_{t+1} = ((1-\delta)\, k_t + I_t)\, \Omega_{t+1} \tag{2.7}$$

where I_t is investment and δ is the depreciation rate. The capital quality shock will serve as an exogenous trigger of debt dynamics. The random variable Ω_{t+1} captures some form of economic obsolescence, as opposed to physical depreciation.[5] We allow for occasional disasters in the form of sharp contractions in quality as we describe later. These disaster shocks serve to initiate financial crises. Firms use equity and debt to finance their operations. They prefer debt, b_t, to equity because of debt's tax advantage, τ_t: see, Jermann and Quadrini (2012). This is also the assumption made in Hennessy and Whited (2005).

The budget constraint can be written as:

$$\frac{P_{mt}}{P_t} F(k_t, n_t) + \frac{b_{t+1}}{R_t} = b_t + \frac{W_t}{P_t} n_t + I_t + \frac{\Psi\left(D_t, D_{t-1}\right)}{P_t} \tag{2.8}$$

[5]Following the finance literature, e.g. Merton (1973), the capital quality shock is a simple way to introduce an exogenous source of variation in the value of capital. Our treatment of this shock follows Gertler et al. (2012).

where P_{mt} is the nominal price of produced intermediate goods, $R_t = 1 + r_t(1 - \tau_t)$ is the after tax return on bonds, and $1 + r_t = \frac{1+i_t}{\Pi_{t+1}}$. $\Psi(D_t, D_{t-1})$ is the nominal payout to shareholders.

We assume that firms raise funds via both intertemporal debt, b_t, and an intra-period loan, L_t , to finance working capital. They pay back the interest-free intra-period loan at the end of the period. Firms start the period with intertemporal debt b_t and they choose labour, investment in capital, the dividend, D_t , and new intertemporal debt, b_{t+1}, *before* producing. Therefore the payments to workers $W_t n_t$, suppliers of investment goods $P_t I_t$, shareholders $\Psi(D_t, D_{t-1})$ and bondholders $P_t b_t$ are made ahead of the realization of revenues. The intra-period loan contracted by the firm will cover these costs as follows:

$$L_t = P_t I_t + W_t n_t + \Psi(D_t, D_{t-1}) + P_t b_t - P_t \frac{b_{t+1}}{R_t}.$$

From here and the budget constraint $L_t = P_{mt}F(k_t, n_t)$ is repaid at the end of the period and is free of interest.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intra-period loan. The total liabilities of the firm at that time are $L_t+ P_t q_t b_{t+1}$, where q_t is t-period price of bonds which mature at t+1, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t = P_{mt}F(k_t, n_t)$. These can be 'diverted' by the firm and so cannot be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_t k_{t+1}$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability Ξe^{ξ_t} the full value $P_t k_{t+1}$ will be recovered, but with probability $1-\Xi e^{\xi_t}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi e^{\xi_t} (P_t k_{t+1} - P_t q_t b_{t+1}) \geq P_{mt}F(k_t, n_t). \tag{2.9}$$

This constraint is derived based on the renegotiation process between the firm and the lender in the case of default. The derivation is given in Appendix A.3. By increasing the level of debt the enforcement constraint becomes tighter. On the other hand, increasing the stock of capital relaxes the enforcement constraint. Most of the enforcement constraint used in the literature shared these properties. The probability Ξe^{ξ_t} is stochastic and depends on uncertain markets conditions.[6] We call this variable as "financial shocks", because it affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. Notice that Ξe^{ξ_t} is the same for all firms. Hence, there are two sources of aggregate uncertainty in our model: productivity e^{z_t} and financial Ξe^{ξ_t} but we do not

[6]The variable Ξe^{ξ_t} could be interpreted as the probability of finding a buyer. Because we assume that the search for a buyer is required for the sale of the firm's capital. The probability increases when the market conditions improve.

include productivity shocks for in our exposition. Since there are no idiosyncratic shocks, we will focus on the symmetric equilibrium where all representative firms are the same.

We can slightly modify (2.9), to see clearly how the shock Ξe^{ξ_t} affects the economy. Suppose the case in which $\tau = 0$ so that $R = 1 + r$. Using the budget constraint (2.8) to substitute for $P_t k_{t+1} - P_t q_t b_{t+1}$ and remembering that the intra-period loan is equal to the revenues, $L_t = P_{mt} F(k_t, n_t)$, the enforcement constraint can be rewritten as

$$\frac{\Xi e^{\xi_t}}{1 - \Xi e^{\xi_t}} \left(P_t \left(1 - \delta\right) k_t - P_t b_t - W_t n_t - \Psi\left(D_t, D_{t-1}\right)\right) \geq P_{mt} F(k_t, n_t)$$

At the beginning of the period k_t and b_t are given. The firm have control only over the input of labor, n_t, and the equity payout, $\Psi\left(D_t, D_{t-1}\right)$. If the firm wishes to keep the production level unchanged, a negative financial shock (lower Ξe^{ξ_t}) requires a reduction in equity payout $\Psi\left(D_t, D_{t-1}\right)$ or employment. In other words, the firm is forced to raise its equity and cut the new intertemporal debt. Thus, the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity will determine if the financial shock affects employment.

The firm's nominal payout to shareholders is assumed to be subject to a quadratic adjustment cost which is a way to formalize the rigidities affecting the substitution between debt and equity:

$$\Psi\left(D_t, D_{t-1}\right) = D_t + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2 D_t$$

where the nominal equity payout D_t is given and $\kappa \geq 0$ is a parameter.[7]

The parameter κ is key for the role of financial shocks. Since when $\kappa = 0$ the economy is almost frictionless, therefore debt adjustments caused by financial shocks can be quickly assisted through changes in firm equity. When $\kappa > 0$, it is costly to substitute debt and equity and firm's readjustment becomes slowly. As a result, financial shocks will have a substantial effect on macroeconomic situation of a country.

Each firm maximizes profit subject to budget constraint (2.8) and enforcement constraint (2.9), so the Lagrangian is:

$$\begin{aligned} L \quad = \quad & \sum_{t=0}^{\infty} m_{0,t} \left(\frac{D_t}{P_t} + \mu_t \left(\Xi e^{\xi_t} \left(k_{t+1} - q_t b_{t+1}\right) - \frac{P_{mt}}{P_t} F(k_t, n_t)\right)\right. \\ & + \lambda_t \left(\left(1 - \delta\right) k_t + \frac{P_{mt}}{P_t} F(k_t, n_t) + \frac{b_{t+1}}{R_t} - b_t - \frac{W_t}{P_t} n_t \right. \\ & \left.\left. - \frac{k_{t+1}}{\Omega_{t+1}} - \left(\frac{D_t}{P_t} + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2 \frac{D_t}{P_t}\right)\right)\right) \end{aligned}$$

[7]One way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact later confirmed by subsequent studies. This could obtain from agency problems.

where $m_{t,t+1}$ is the stochastic discount factor, μ_t and λ_t are Lagrange multipliers.

The first order conditions are (2.8), (2.9) and derivatives with respect to n_t, k_{t+1}, $b_{t+1}, D_t, \mu_t, \lambda_t$:

$$
\begin{aligned}
0 &= (\lambda_t - \mu_t) X_t F_n(k_t, n_t) - \lambda_t w_t && (2.10)\\
0 &= \mu_t \Xi e^{\xi_t} - \frac{\lambda_t}{\Omega_{t+1}} \\
&\quad + \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left((\lambda_{t+1} - \mu_{t+1}) X_{t+1} F_k(k_{t+1}, n_{t+1}) + \lambda_{t+1}(1-\delta) \right) && (2.11)\\
0 &= \frac{\lambda_t}{R_t} - \mu_t \Xi e^{\xi_t} \frac{1}{1+r_t} - \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \lambda_{t+1} && (2.12)\\
0 &= 1 + \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1} C_t^{-\sigma}} \lambda_{t+1} 2\kappa \left(\frac{D_{t+1}}{D_t} - 1 \right) \frac{D_{t+1}^2}{D_t^2} && (2.13)\\
&\quad - \lambda_t \left(1 + 2\kappa \left(\frac{D_t}{D_{t-1}} - 1 \right) \frac{D_t}{D_{t-1}} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \right) \\
0 &= \Xi e^{\xi_t} (k_{t+1} - q_t b_{t+1}) - X_t F(k_t, n_t) && (2.14)\\
0 &= (1-\delta) k_t + X_t F(k_t, n_t) + \frac{b_{t+1}}{R_t} - b_t - w_t n_t \\
&\quad - \frac{k_{t+1}}{\psi_{t+1}} - \left(\frac{D_t}{P_t} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \frac{D_t}{P_t} \right) && (2.15)
\end{aligned}
$$

where $X_t = \dfrac{P_{mt}}{P_t}$, $w_t = \dfrac{W_t}{P_t}$ and $\Pi_t = \dfrac{P_t}{P_{t-1}}$ is gross inflation, F_n and F_k are derivatives of $F(k_t, n_t)$ respect to n and k. All derivations are given in Appendix A.2.

Equation (2.10), the optimal condition for labor indicates that the marginal productivity of labor is equal to the marginal cost ($\frac{\lambda_t w_t}{(\lambda_t - \mu_t) X_t}$). As the enforcement constraint becomes tighter, the effective cost of labor rises and its demand falls. Therefore, financial shocks could transmit to the real sector of the economy through the demand of labor.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa = 0$. In this case $\lambda_t = 1$ (see condition (2.13)) and condition (2.12) becomes $\mu_t \Xi e^{\xi_t} \frac{R_t}{1+r_t} + R_t \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \lambda_{t+1} = 1$. This denotes that there is a negative relation between Ξe^{ξ_t} and the multiplier μ_t taking as given the aggregate prices R_t, r_t, and $m_{t,t+1}$. In other words, lower probability of recovering firm's capital make the enforcement constraint tighter. Then from equation (2.10) we see that a higher μ_t implies a lower demand for labor.

This mechanism is strengthened when $\kappa > 0$. In this case readjusting the financial structure becomes costly, and the change in Ξe^{ξ_t} induces a larger volatility in μ_t. Of course, prices will be affected by the change in the policies of all firms.

Retailers

We introduce nominal rigidities *a la* Rotemberg (1983). Each final good producer i buys goods at price P_{mt}, and repackages them, producing the final good, which may also be costlessly transformed into capital. It sets its optimal price p_t^i and produces quantity $y_t(i)$. The firm faces a familiar demand for its good

$$y_t(i) = \left(\frac{p_t^i}{P_t}\right)^{-\varepsilon} Y_t.$$

Here the elasticity of substitution between any pair of goods is given by $\varepsilon > 1$. Firm chooses price p_t^i which solves the following optimization problem:

$$\begin{aligned} & \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(\frac{p_t^i}{P_t} y_t^i - \frac{P_{mt}}{P_t} y_t^i - \frac{\omega}{2} \left(\frac{p_t^i}{p_{t-1}^i} - 1 \right)^2 Y_t \right) \\ = \; & \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(\left(\frac{p_t^i}{P_t} \right)^{1-\varepsilon} Y_t - \frac{P_{mt}}{P_t} \left(\frac{p_t^i}{P_t} \right)^{-\varepsilon} Y_t - \frac{\omega}{2} \left(\frac{p_t^i}{p_{t-1}^i} - 1 \right)^2 Y_t \right) \end{aligned}$$

where $\frac{\omega}{2}\left(\frac{p_t^i}{p_{t-1}^i} - 1\right)^2 Y_t$ represents the cost of adjusting prices.

The aggregated first order condition is:

$$\omega \left(\Pi_t - 1\right) \Pi_t = (1-\varepsilon) + \varepsilon X_t + \omega \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left(\Pi_{t+1} - 1\right) \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \tag{2.16}$$

The derivation is given in Appendix A.2.2.

Finally, the profit Φ_t in the household budget constraint can be found from the aggregation of firms' budget constraints:

$$P_t \Phi_t = P_t Y_t - P_{mt} Y_t - \frac{\omega}{2} \left(\Pi_t - 1\right)^2 Y_t P_t \tag{2.17}$$

2.2.3 Government

The government collects taxes and pays it as transfers and government spending:

$$T_t = \frac{b_{t+1}}{R_t} - \frac{b_{t+1}}{1+r_t} \tag{2.18}$$

$$G_t = \tau_t^w w_t n_t + \tau_t^d d_t + \tau_t^x Y_t \tag{2.19}$$

2.2.4 Private Sector Equilibrium and Market Clearing

Private Sector Equilibrium is determined by the system (2.2)-(2.5), (2.8), (2.9), (2.10)-(2.13), (2.16). We substitute out equations which determine share prices, and arrive to the following system

$$\begin{aligned}
0 &= (\lambda_t - \mu_t)\, X_t F_n(k_t, n_t) - \lambda_t w_t && (2.20)\\
0 &= -\frac{\lambda_t}{\Omega_{t+1}} + \mu_t \Xi e^{\xi_t} + \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left((\lambda_{t+1} - \mu_{t+1})\, X_{t+1} \theta \frac{Y_{t+1}}{k_{t+1}} + \lambda_{t+1} (1-\delta) \right) && (2.21)\\
0 &= \frac{\lambda_t}{R_t} - \mu_t \Xi e^{\xi_t} \frac{\Pi_{t+1}}{1+i_t} - \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \lambda_{t+1} && (2.22)
\end{aligned}$$

$$\begin{aligned}
0 &= 1 + 2\kappa\beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\lambda_{t+1}}{\Pi_{t+1}} \left(\frac{d_{t+1}}{d_t} \Pi_{t+1} - 1 \right) \left(\frac{d_{t+1}}{d_t} \Pi_{t+1} \right)^2 && (2.23)\\
&\quad - \lambda_t \left(1 + 2\kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right) \frac{d_t}{d_{t-1}} \Pi_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 \right) &&\\
X_t Y_t &= \Xi e^{\xi_t} \left(k_{t+1} - b_{t+1} \frac{\Pi_{t+1}}{1+i_t} \right) && (2.24)\\
\frac{b_{t+1}}{R_t} &= b_t + w_t n_t + \frac{k_{t+1}}{\Omega_{t+1}} - (1-\delta)\, k_t + d_t \left(1 + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 \right) && (2.25)\\
&\quad - X_t Y_t &&\\
\omega\, (\Pi_t - 1)\, \Pi_t &= (1-\varepsilon) + \varepsilon X_t + \omega\beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} (\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \Pi_{t+1} && (2.26)\\
\frac{\Pi_{t+1}}{1+i_t} &= \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} && (2.27)\\
\alpha n_t^{\psi} &= w_t C_t^{-\sigma} && (2.28)
\end{aligned}$$

where $Y_t = A k_t^{\theta} n_t^{1-\theta}$, $R_t = 1 + r_t(1-\tau_t)$.

Finally, the resource constraint yields

$$Y_t = C_t + \frac{k_{t+1}}{\Omega_{t+1}} - (1-\delta)\, k_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 d_t + \frac{\omega}{2} (\Pi_t - 1)^2\, Y_t \qquad (2.29)$$

and system (2.20)-(2.29) is used to determine equilibrium variables for $\lambda_t, \mu_t, X_t, C_t, k_t,$

$\Pi_t, n_t, d_t, w_t, b_t$ given the policy instruments i_t and τ_t. The derivation of resource constraint is given in Appendix A.4.

2.3 Linearisation

For every variable z_t with steady state $z \neq 0$ we denote $\hat{z}_t = \log \frac{z_t}{z}$. We linearise the system around the steady state to yield:

$$
\begin{aligned}
\hat{w}_t &= \frac{\mu}{1-\mu}\left(\hat{\lambda}_t-\hat{\mu}_t\right)+\hat{X}_t+\theta\hat{k}_t-\theta\hat{n}_t && (2.30)\\
\hat{\lambda}_t &= \beta\left(X\theta\frac{Y}{k}\left(\hat{\lambda}_{t+1}-\mu\hat{\mu}_{t+1}\right)+X\theta\frac{Y}{k}(1-\mu)\left(\theta\hat{k}_{t+1}+(1-\theta)\hat{n}_{t+1}-\hat{k}_{t+1}+\hat{X}_{t+1}\right)\right.\\
&\quad\left.-(1-\mu\Xi)\sigma\hat{C}_{t+1}+(1-\delta)\hat{\lambda}_{t+1}\right)+\mu\Xi\left(\hat{\mu}_t+\hat{\xi}_t\right)+\beta(1-\mu\Xi)\sigma\hat{C}_t && (2.31)\\
0 &= \beta\left(\sigma\hat{C}_t-\sigma\hat{C}_{t+1}+\hat{\lambda}_{t+1}\right)-\frac{1}{R}\left(\hat{\lambda}_t-\frac{(1-\tau)}{\beta R}\left(\hat{\imath}_t-\hat{\Pi}_{t+1}\right)+\frac{r\tau}{R}\hat{\tau}_t\right) && (2.32)\\
&\quad+\frac{\Xi\mu}{1+r}\left(\hat{\mu}_t+\hat{\xi}_t-\hat{\imath}_t+\hat{\Pi}_{t+1}\right)\\
\hat{\lambda}_t &= 2\kappa\beta\left(\hat{\Pi}_{t+1}+\hat{d}_{t+1}-\hat{d}_t\right)-2\kappa\left(\hat{\Pi}_t+\hat{d}_t-\hat{d}_{t-1}\right) && (2.33)\\
\theta\hat{k}_t &= \frac{\Xi}{XY}\left(k\left(\hat{k}_{t+1}+\hat{\xi}_t\right)-\frac{b}{1+r}\left(\hat{b}_{t+1}-\hat{\imath}_t+\hat{\Pi}_{t+1}+\hat{\xi}_t\right)\right)-(1-\theta)\hat{n}_t-\hat{X}_t && (2.34)\\
k\hat{k}_{t+1} &= XY\left(\hat{X}_t+\theta\hat{k}_t+(1-\theta)\hat{n}_t\right)+\frac{b}{R}\left(\hat{b}_{t+1}-\frac{(1-\tau)}{\beta R}\left(\hat{\imath}_t-\hat{\Pi}_{t+1}\right)+\frac{r\tau}{R}\hat{\tau}_t\right) && (2.35)\\
&\quad-wn\left(\hat{w}_t+\hat{n}_t\right)-b\hat{b}_t-d\hat{d}_t+(1-\delta)k\hat{k}_t\\
\hat{\Pi}_t &= \frac{\varepsilon X}{\omega}\hat{X}_t+\beta\mathbb{E}_t\hat{\Pi}_{t+1} && (2.36)\\
\hat{C}_t &= \hat{C}_{t+1}-\frac{1}{\sigma}\left(\hat{\imath}_t-\hat{\Pi}_{t+1}\right) && (2.37)\\
\hat{w}_t &= \psi\hat{n}_t+\sigma\hat{C}_t && (2.38)\\
Y\theta\hat{k}_t &= C\hat{C}_t+k\hat{k}_{t+1}-k(1-\delta)\hat{k}_t-Y(1-\theta)\hat{n}_t && (2.39)
\end{aligned}
$$

The linearisation of equations and steady states are given in Appendix A.6 and A.5.

2.4 Policy

Monetary policy is assumed to behave optimally, minimizing an *ad hoc* welfare loss, given by the objective

$$L=\mathbb{E}_0\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\hat{\Pi}_t^2+\varkappa_y\hat{Y}_t^2+\varkappa_\imath\left(\hat{\imath}_t-\hat{\imath}_{t-1}\right)^2\right).$$

Thus we assume that the policymaker has objectives over inflation and output, $\hat{Y}_t=\theta\hat{k}_t+(1-\theta)\hat{n}_t$, as well as over interest rate smoothing. This policy objective has significant empirical support, one recent discussion can be found in Chen et al. (2014).[8][9]

Note that the interest rate may be constrained by the ZLB. We compute numerically the implications of such a restriction using the approach developed in Laséen and Svensson (2011) and extended to the case of discretion in Chen et al. (2014).

[8] See also estimation of policy objectives in e.g. Dennis (2006), Ilbas (2010) and Givens (2012).

[9] We can derive microfounded objectives for some other classes of policies using approach developed by Schmitt-Grohe and Uribe (2004). The comparison of those policies for different specifications of objectives allows demonstrates great robustness of results to specifications of policy objectives.

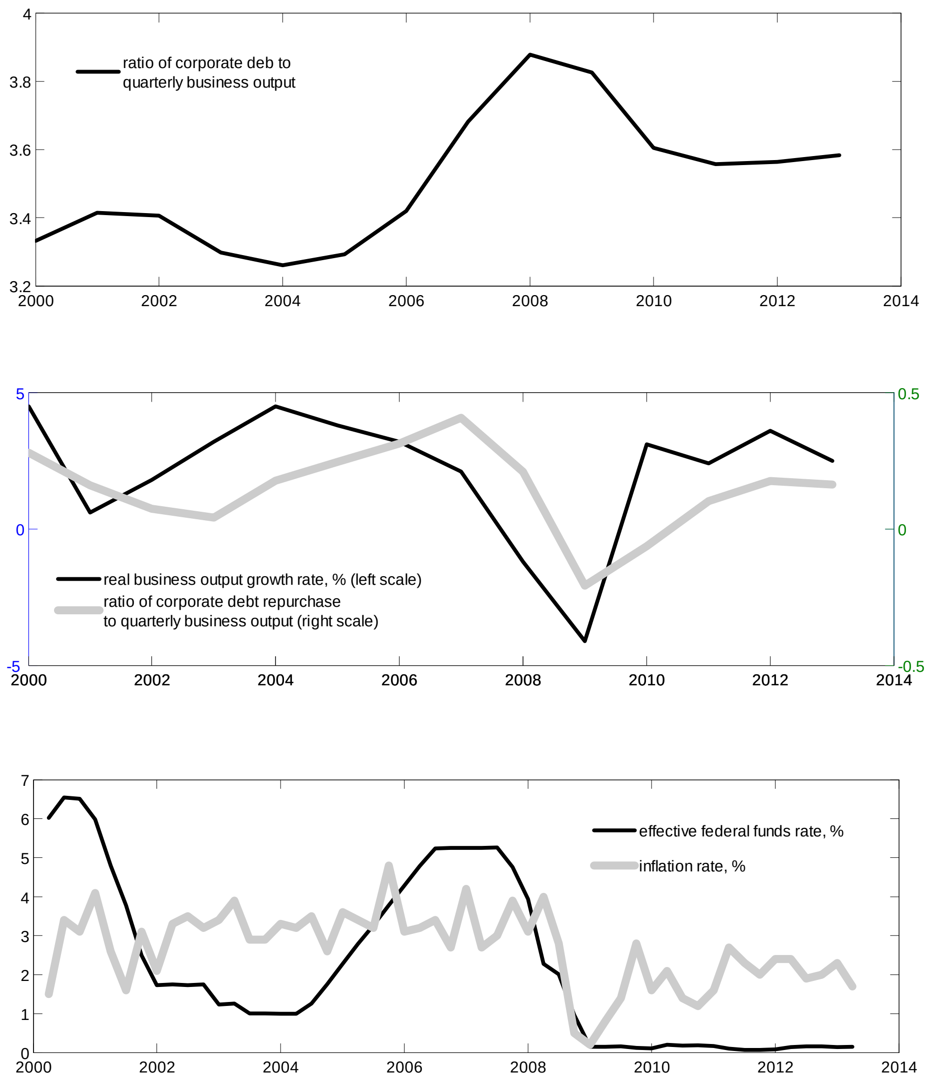


Figure 2.2: Historical data in the US. Data sources: NIPA and FoF tables.

2.5 Calibration

The model is calibrated to a quarterly frequency. We fix $\beta = 0.9825$. The capital depreciation rate is set to $\delta = 0.025$. The capital ratio in production function is set to $\theta = 0.36$, and the mean value of A is normalized to 1. The tax wedge which corresponds to the advantage of debt over equity is determined to be $\tau = 0.35$, and the dividend adjustment cost parameter set to $\kappa = 0.146$ as in Jermann and Quadrini (2012).

We calibrate the steady state debt to output ratio to match the data. The quarterly ratio of debt to output for the non-financial business sector is 3.25 over the sample period

1984:I-2010:II, see the top panel in Figure 2.2. In order to match that, we set the steady state value of the financial variable, Ξ , to 0.1634.[10]

Parameters of the household utility function are determined as follows. The calibration of the Frisch intertemporal elasticity of substitution in labor supply, ψ, is assumed to be equal to 1 and the risk aversion parameter is: $\sigma = 1$. The relative weight on the disutility of labour, $\alpha = 1.8834$, is chosen so as to set steady state hours worked equal to 0.3.

We calibrate the measure of price stickiness, $\omega = 80$, in a way that corresponds to a probability of firms changing prices every 3 quarters (in a corresponding Calvo model). The elasticity of substitution between any pair of goods ε is equal to 11 in steady state which gives a 10% mark up.

Parameters of the policy objective function are chosen to be $\varkappa_y = 0.5$ and $\varkappa_\iota = 0.6$, see Chen et al. (2014).[11]

It remains to calibrate the shock and the initial states to simulate the scenarios of interest. The second panel in Figure 2.2 plots the historical data of corporate debt to output ratio (quarterly). The average value of this ratio during 1984-2009 is 3.25. The peak of 3.87 in 2008 was somewhat above the average value, and the consequent reduction to 3.55 in 2011 constitutes a reduction of about 10% relative to its peak. We use these numbers as a guide to our simulations.

Note that the model suggests the following steady state relationship

$$\frac{b}{Y} = \frac{\theta\left(\varepsilon-1\right)}{\varepsilon\left(2-\frac{1}{\beta R}-\beta\left(1-\delta\right)\right)} - \left(\frac{\theta\left(\varepsilon-1\right)\left(\frac{1}{\beta R}-1\right)}{\varepsilon\left(2-\frac{1}{\beta R}-\beta\left(1-\delta\right)\right)} + \frac{\left(\varepsilon-1\right)}{\beta\varepsilon}\right)\frac{1}{\Xi}$$

so that a reduction in the debt to output ratio can be explained by a reduction in the recovered share of output, Ξ, as all other parameters are structural. Rough calculations indicate that a shock of about 10% is not unreasonable.

Based on this evidence, we consider an AR(1) credit shock $\hat{\xi}_t = \rho\hat{\xi}_{t-1} + \varepsilon_t$ with persistence $\rho = 0.95$, and we examine the dynamic implications of a negative 10% innovation in ε_t.

[10] Data sources: NIPA and FoF tables. The calculations follow Jermann and Quadrini (2012).

[11] The results are very robust to wide range of parameters $\varkappa_y$ and $\varkappa_\iota$ between zero and one.

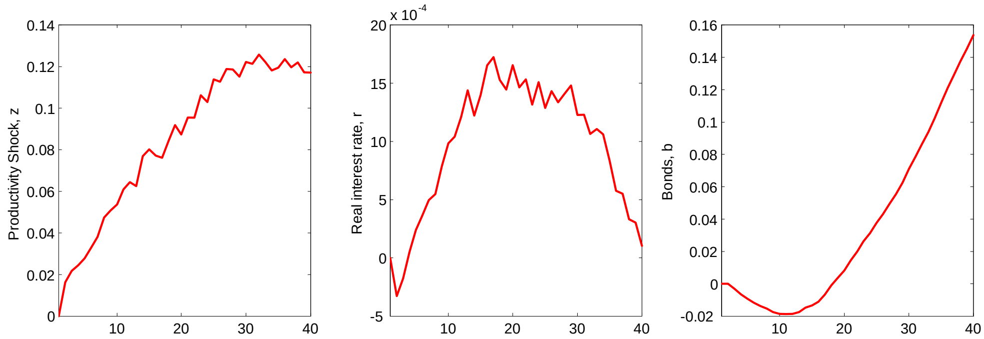


Figure 2.3: Ten years of positive productivity shock

2.6 Discussion

Jermann and Quadrini (2012) demonstrate the ability of an RBC-type model with financial frictions to explain the three episodes of recessions. As the authors note, it is evident that the model fails to account for the depth of the Great Recession.

In this chapter we claim that deep recessions can still be generated by a very similar model. To support this claim we run two numerical experiments. In the first experiment we demonstrate how an initial state with 'excess' lending results in deeper recessions following a financial shock. In the second experiment we demonstrate how ZLB on interest rates may results in a large reduction in output.

2.6.1 The effect of over-lending

We start with the baseline scenario of a negative credit shock, impacting the economy which is initially at steady state. Such a shock reduces the proportion of output which banks will be able to recover in the case of default. Banks lend to firms at the beginning of the period, so that firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the financial shock reduces the probability of recovery, the amount of bank lending falls. Firms which are not able to obtain funds up front have to deleverage or reduce production. Firms reduce their labour demand, produce less output and also pay lower wages, see Figure 2.4. The equilibrium prices of intermediate goods and final goods fall as a result of lower income and lower demand. Firms reduce the amount of borrowing. Both constraints for firms are tightened, as the values of the Lagrange multipliers indicate.

In response to lower inflation and output the central bank reduces the nominal interest rate sufficiently to guarantee a reduction in the real rate. Low real interest rates result in falling consumption profile over time.

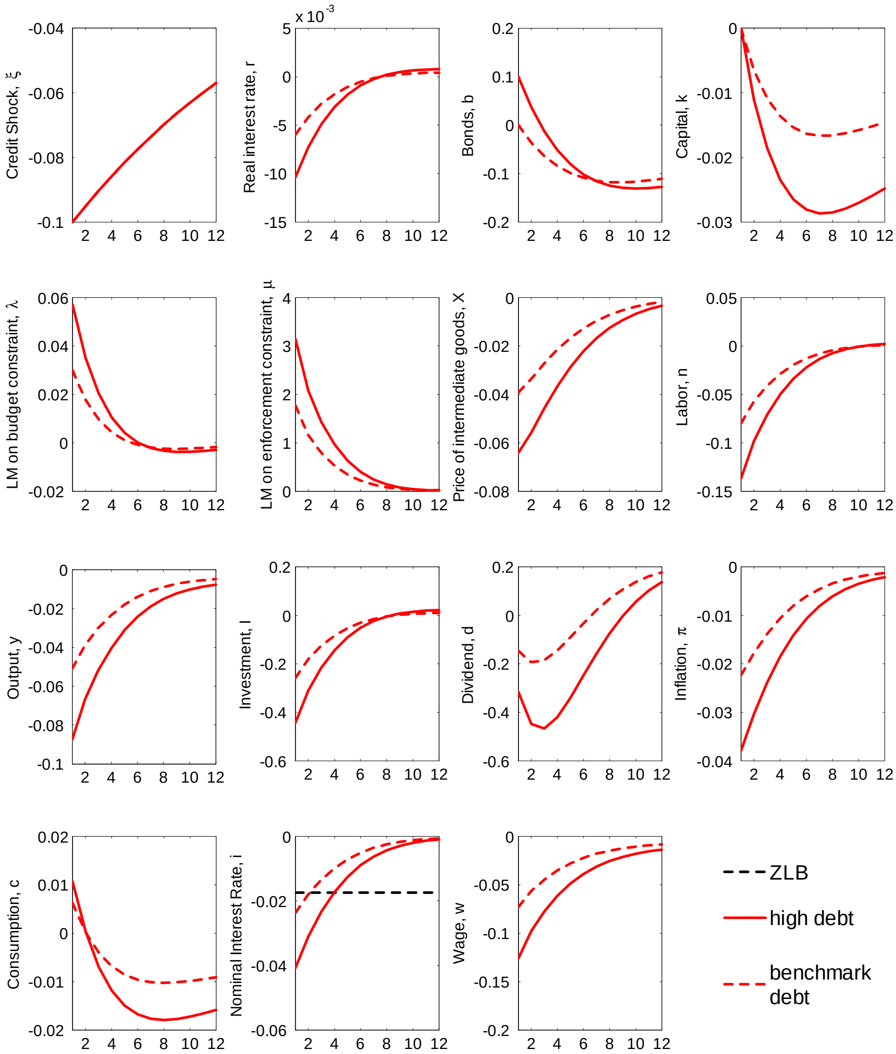


Figure 2.4: The effect of high corporate debt and an AR(1) negative credit shock with persistence $\rho = 0.95$.

Lower interest rates also make it easier for firms to pay out the existing stock of corporate debt, so it helps to reduce the debt stock quickly. In addition, output falls by less than wages, profits of firms fall and so dividends fall.

The reduction of output doubles if the financial shock requires greater deleveraging. We can illustrate this in the following scenario. Suppose the economy suffers from an one-off but permanent 'capital quality' shock where agents suddenly realise that the level of accumulated debt is above the current level of the underlying capital stock. The resulting deleveraging brings the level of debt down towards the steady state level of new, quality-adjusted capital. In figure 2.4 we assume that the over-lending is 10 percent, i.e., as a result of the permanent contraction in capital quality the initial debt to output ratio is exactly 10 percent higher than its steady state level in the economy with quality adjusted capital stock. Such a degree of over-lending may be plausible, see Section 2.5. Moreover, this excess can easily be achieved in this model, see Figure 2.3, which demonstrates that the stock of outstanding corporate debt accumulates more than the required 10 percent above the steady state level in the course of 10 years.[12] This high initial level of debt requires greater deleveraging than in the default scenario. Following the shock the reduction in debt reduces the capital stock and the lower capital stock requires less financing. These two effects reinforce each other and the sluggish adjustment of both stocks results in much overshooting of debt below the steady state in the course of adjustment. As a result, there is greater reduction in labour demand and wages, and much lower inflation. The interest rate falls by more, not only because of inflation, but because it also helps to stabilize debt – and thus all policy-relevant variables – faster. The reduction in output and labour doubles, which is consistent with the evidence presented in Figure 2.4.

However, in the process of this adjustment, the optimal interest rate violates the ZLB. Therefore, next we discuss the implications of the ZLB on the dynamics of the economy.

2.6.2 The effect of ZLB

The effect of ZLB is illustrated in Figure 2.5. We compare two scenarios: the first one is the default case of 'unconstrained discretion', discussed in the section above, where the interest rate can move below the ZLB, and the second scenario, where such movements are prohibited. When the optimal interest rate is constrained by the ZLB, the recession is deeper. Inflation does not fall immediately but adjusts with a delay and converges back to the steady state, and is higher than in the previous scenario without the ZLB. When the financial shock occurs both constraints are tightened and the interest is reduced but

[12] Boissay, Collard, and Smets (2015) discuss that the 'average' development before the deep recession is a period of positive productivity shocks, their model suggest at least 10 years of AR(1) productivity shocks with $\rho_z = 0.9$ and standard error of 0.013. We stick to Jermann and Quadrini (2012) calibration of the model and we hit the economy with positive AR(1) productivity shocks with $\rho_z = 0.95$ and standard error of 0.008.

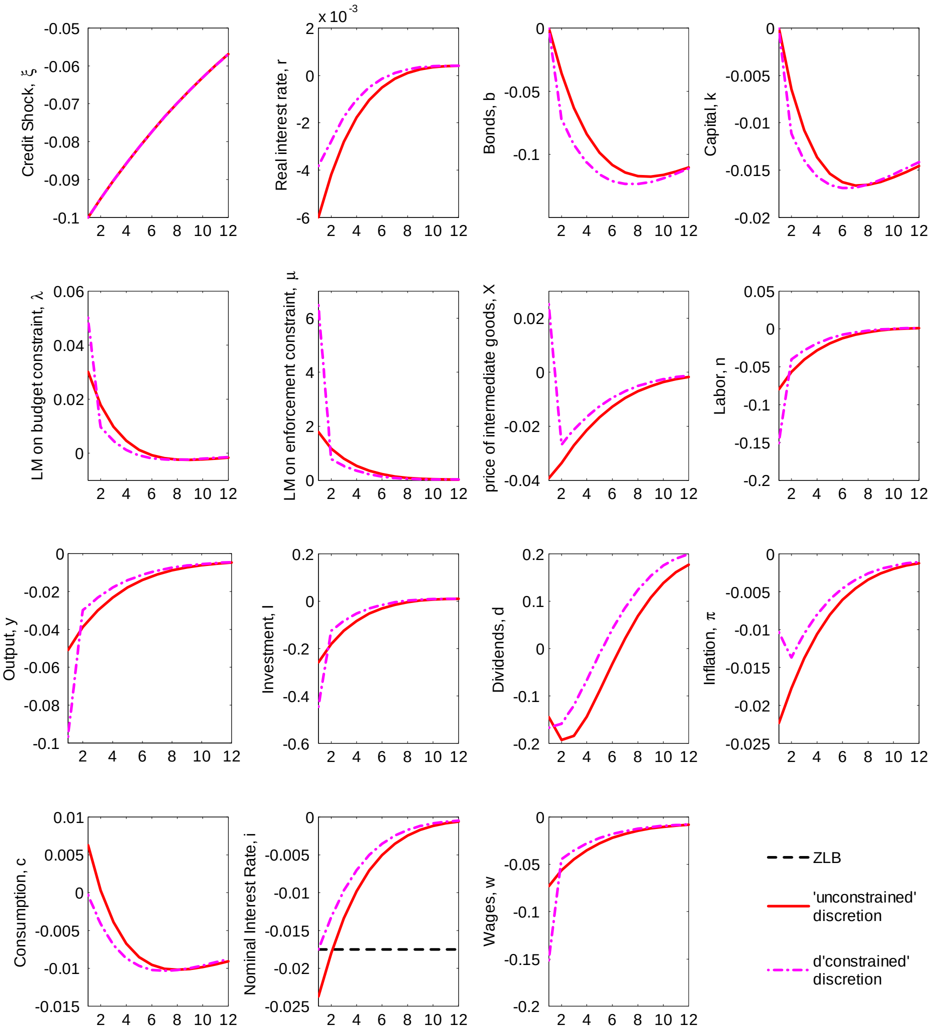


Figure 2.5: The effect of ZLB and an AR(1) negative credit shock with persistence $\rho = 0.95$.

not as much as the policymaker would like. The enforcement constraint is tightened much more than in the 'no ZLB' scenario, see Figure 2.5. As the interest rate on debt remains 'too high', greater deleveraging is required. Consumption drastically falls, and so does output and demand. Both bond and capital stocks fall quickly and by large amounts. Wages and labor fall instantaneously. The absence of monetary intervention results in a deep recession.

As capital and labour fall, the production of intermediate goods fall too. The supply of intermediate goods is greatly reduced, much lower than the demand for final goods. As a result, the initial-periods price of intermediate goods rises, and so does inflation. However, expected inflation remains negative. Together with relatively high interest rate this results in only a small reduction of the real interest rate and so consumption falls only slow over time. As a result, we observe reduction in consumption and investment

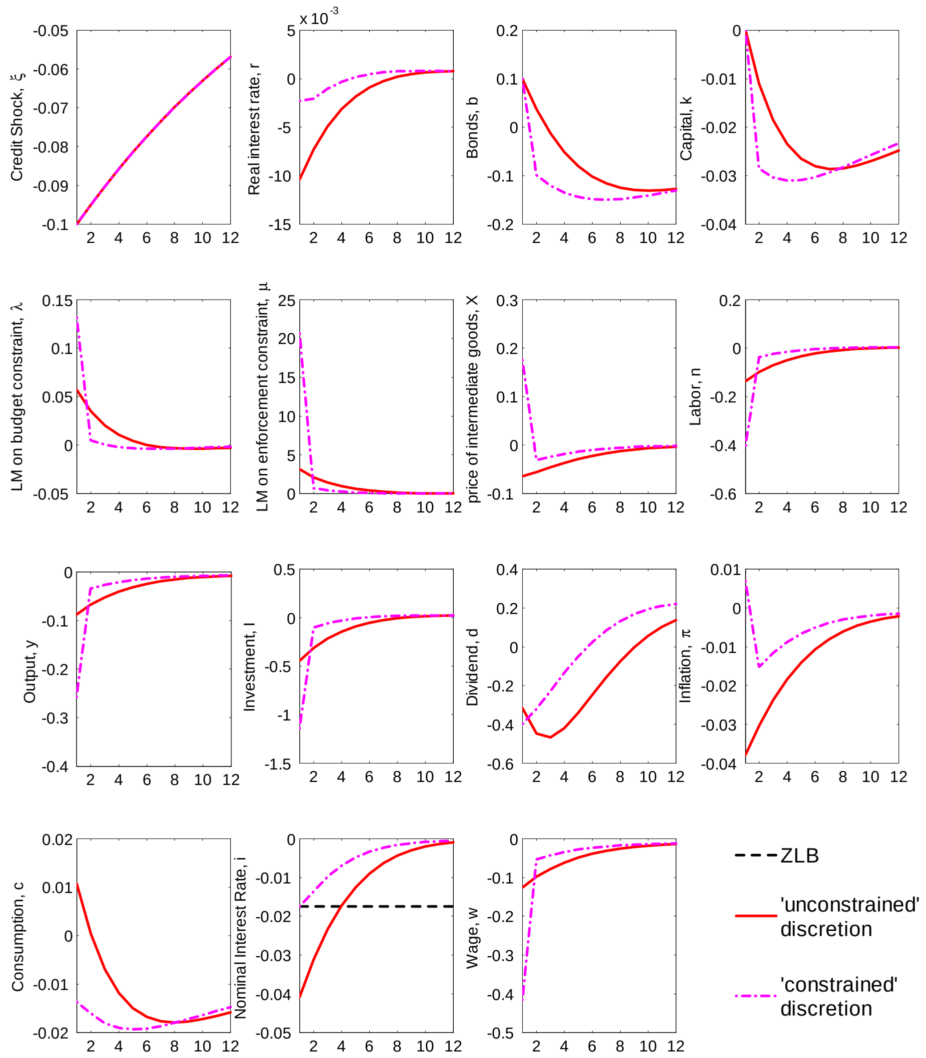


Figure 2.6: The effect of ZLB, over-lending and an AR(1) negative credit shock with persistence $\rho = 0.95$.

reflecting the reduction in output in the first few periods following the shock.

Once the initial-periods capital and debt de-accumulation is done, the constraint is weakened substantially. There is no further need to reduce bonds and capital quickly, and no need to restrain investment as much. Investment remains negative, but somewhat higher than in the first several periods. Intermediate goods firms increase output, and the price of intermediate goods falls to equalize demand and supply. Therefore, inflation falls as costs fall. Inflation is negative and it is optimal to keep the interest rate below the steady state level in order to stabilize the economy, but the interest rate does not need to be below the ZLB. We show that it is optimal to slightly raise the interest rate above the ZLB. The higher interest rate increases the real interest rate, but it still remains below the steady state level. As such, consumption continues falling to match the desired path for capital and supply. At some point the optimal deleveraging is achieved, the real rate rises back to the steady state level and above so that expected future consumption is higher than current consumption. Finally, consumption and demand start rising, prices and inflation rise and the economy converges back to the steady state. The adjustments is however slow and the growth rate of the economy, measured by $\hat{y}_t - \hat{y}_{t-1}$, is noticeably slower than in scenarios with no ZLB.

Note that this model requires convergence to the well-defined steady state. That means that the economy grows faster when recovering from a negative shock than if it were not hit by a shock at all. However, we compare the speed of recovery in different scenarios after a negative financial shock. We have shown that an initial stock of overlending results in higher output growth rate and the ZLB results in lower growth rate along the most of recovery path, excluding several initial periods. The effect of the ZLB dominates, see Figure 2.6. In the Figure the economy is hit by a financial shock when there is 10% overlending, and there is a restriction on interest rate movements below the ZLB. This simple superposition of two scenarios generates a very substantial reduction in output, and slow recovery. The slow recovery is the 'cost' of rapid deleveraging, due to the presence of ZLB.

2.7 Conclusion

In this chapter we demonstrate how a simple model with borrowing constrained firms is able to replicate two empirical facts, observed during the recent financial crisis. We demonstrate that, in response to a moderate financial shock, the economy may generate a very deep recession. Two assumptions are helpful in generating a deep recession, consistent with that observed during the recent financial crisis. The first is the existence of 'overlending'. We show that the higher the initial debt to output ratio, the sharper the subsequent recession. For example, if the stock of lending is 10 percent above its steady state level, a financial shock nearly doubles the consequent reduction in output, compared with the case when debt is initially at steady state. The second assumption is the existence of the zero lower bound (ZLB) on nominal interest rates. We show that if the interest rate is constrained by the zero lower bound the dynamics of the economy involve a 'stagnation' period, when the recovery is very slow.

Chapter 3

Two country model with flexible exchange rate

3.1 Introduction

Financial crisis of 2007 has started as a subprime lending crisis, affecting one sector in one country. It quickly spread to many other countries. Financial interdependence of economies must have played a key role in international transmission of shocks.

In this chapter and the next chapter, I model interdependent economies to describe channels of transmission of credit shocks between countries.

The role of credit shocks for macroeconomic fluctuations has been investigated primarily in closed economy models, see Christiano et al. (2009), Gertler and Karadi (2011), Goldberg (2010), Guerrieri and Lorenzoni (2010), Khan and Thomas (2010), Jermann and Quadrini (2012), and Liu et al. (2013). We have studied closed economy in the second chapter of this thesis.

Recently, Perri and Quadrini (2011) use a two-country model with financial market frictions to demonstrate how financial shocks may transmit from one country to another. It crucially relies on financial integration between countries. Domestic and foreign firms are able to borrow at a global financial market at the same interest rate. They demonstrate how a shock, which originates in one country, creates a shortage of liquidity in both countries and results in an international recession.

In this chapter, we present a different model. We do not assume complete financial integration, but rather consider different degrees of financial integration and investigate their effect. The model is based on the one presented in the second chapter, but firms can borrow from abroad. When allocating portfolios between home and foreign corporate bonds, households face intermediation costs, which depend on the degree of external indebtedness of the home economy. There is a risk premium if the country is in net borrowing position. We also assume two independent monetary authorities, and therefore, floating exchange rate regime, and the ability of each policymaker to affect its own interest rate. Compared to Perri and Quadrini (2011) we have much less of financial integration between countries. Our main objective is to investigate how the degree of financial integration affects international transmission of credit shocks. We also investigate if the country size matters for the severity of recessions.

Our results demonstrate that welfare is maximised for an intermediate value of degree of financial integration. If the intermediation cost is absent, there is large volatility of output during the period of adjustment, while the deleveraging is performed faster. With greater costs on international financial flows, the deleveraging is substantially slowed down which leads to longer periods of adjustment and greater costs. We demonstrate that in two country model under flexible exchange rate and independent monetary authorities, the effect of one country's credit shock has very limited effect on another country. When monetary policymakers cooperate and choose interest rates optimally, the unaffected country can nearly eliminate all aftereffects of the shock to the other country. To some extent, limited financial integration prevents the spread of volatility across the border, however, unconstrained monetary policy is the key to these results.

This chapter is organized as follows. In the next section, we outline the model. Section 3.3 covers the linearised version of the system of equations. Section 3.4 describes the calibration of the model. Section 3.5 describes the social welfare measure. The policy set-up is given in Section 3.6. Section 3.7 discusses results, section 3.8 concludes.

3.2 The Model

We present a simple two-country model with financial frictions and with incorporated nominal rigidities *a la* Rotemberg (1983). The world economy is populated by a continuum of agents on the interval of $[0;1]$. The population on the segment $[0;n)$ belongs to country H (Home), while the segment $[n;1]$ belongs to country F (Foreign). Each economy is populated by households and firms. Firms use labor and capital to produce differentiated goods. Firms issue equity and debt and use intra-period loans to finance working capital. Firms face credit restrictions due to uncertainty of recovering these loans. Preferences reflect home bias in consumption. The detailed model of the economy is presented in this section.

3.2.1 Law of One Price, The Terms of Trade and Relative Prices

We assume that the law of one price holds, implying $p_{Ft}(z) = E_t p^*_{Ft}(z)$, $p_{Ht}(z) = E_t p^*_{Ht}(z)$ for all $z \in [0,1]$ where $E_t = [H]/[F]$ is the nominal exchange rate, that is the price of foreign currency in terms of home currency, and $p^*_{Ft}(z)$ is the price of foreign good z denominated in foreign currency (Of course, the holding of one price does not imply that PPP holds, unless we assume the absence of home bias). We define the terms of trade is the relative price of imported goods:[1]

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

The real exchange rate – the ratio of CPI inflations, expressed in domestic currency – is defined as

$$\mathcal{Q}_t = \frac{E_t P^*_t}{P_t}.$$

3.2.2 Domestic Households

The home economy (H), is populated by a continuum of homogeneous infinitly-living households who share identical preferences and technology and maximise the expected lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, n_t)$, with aggregated period utility

$$U(C_t, n_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha \frac{n_t^{1+\psi}}{1+\psi} \tag{3.1}$$

where C_t is private home consumption, n_t is home labor, β is the discount factor, $\mathbb{E}_0$ is the actuarial expectation at time $t = 0$. Furthermore $\psi \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, α is a preference parameter. We assume home bias in consumption. In more detail, a composite consumption index, C_t, is defined as a Dixit-Stiglitz aggregator of the continuum of goods $i \in [0,1]$ produced in the foreign country and home[2]

$$C_t = \left((1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Parameter $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods. Parameter $\gamma \in [0,1]$ is the weight of imported goods in private home consumption and is inversely related to the degree of home bias in preferences. Another interpretation for γ is as a natural index of openness or the import share. The import share depends on $(1-n)$ which is the relative size of foreign economy, and on ϖ

[1] Let a "*" denote foreign variables.

[2] The same relationship can be written in per capita terms.

which is the degree of trade openness. It yields $\gamma = (1-n)\varpi$. We assume home bias in consumption:

$$1-\gamma = (1-(1-n)\varpi) > \gamma^* = n\varpi^*$$

which implies

$$1-n\varpi^* > (1-n)\varpi$$

Similarly, home bias in foreign preferences requires $1-\gamma^* > \gamma$ which again implies

$$1-n\varpi^* > (1-n)\varpi$$

C_{Ht} and C_{Ft} are domestic consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F given by the CES functions

$$C_{Ht} = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n c_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}, \; C_{Ft} = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 c_{Ft}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}$$

where each consumption bundle C_{Ht} and C_{Ft} is composed of imperfectly substitutable varieties of goods $z \in [0,1]$ produced within a given country with elasticity of substitution $\epsilon > 1$.

The aggregated nominal intertemporal budget constraint at time t for household i belonging to country H is given by

$$\begin{aligned} & \int_0^1 [P_{Ht}(z)C^i_{Ht}(z) + P_{Ft}(z)C^i_{Ft}(z)]dz + \frac{A^i_{t+1}}{1+i_t} + \frac{B^i_{t+1}E_t}{(1+i^*_t)\,\phi\left(\frac{E_tB_{t+1}}{P_t}\right)} \\ \leq \; & A^i_t + B^i_tE_t + (1-\tau^w_t)\,W^i_tn^i_t + \left((1-\tau^d_t)\,D_t + P_{St}\right)s_t - s_{t+1}P_{St} + T^i_t + P_{Ht}\Phi_t \end{aligned}$$

$P_{Ht}(z)$ and $P_{Ft}(z)$ are price indices of domestic and foreign (imported from country F) goods z, where the latter is expressed in domestic currency. A_t is the one-period domestic corporate bond held by domestic households (real bonds, in terms of domestic prices), B_t is the one-period foreign corporate bonds held by domestic households, W^i_t is the nominal wage and T^i_t denotes lump-sum taxes/transfers. τ^w_t denotes a country specific tax on nominal income and E_t is the nominal exchange rate, given as the price of one of unit foreign currency in terms of home currency. $P_{Ht}\Phi_t$ is nominal profit from the ownership of capital-producing firms and retailers (note that $\Phi_t = \Phi^C_t + \Phi^R_t$). Here s_t is the domestic share of equity which is wholly owned by domestic households, D_t denotes the equity payout paid to the shareholders, P_{St} is the market price of domestic shares, τ^d_t denotes the tax on equity payout. We assume that the households share the revenues of owning firms in equal proportion. Following Woodford (2003) we consider a cashless economy. Therefore the only explicit role played by money is to serve as a unit of account.

We introduce incomplete financial markets as in Benigno (2009).[3] Domestic households hold domestic equity shares s_t and noncontingent bonds issued by firms of home and foreign countries. Households of country H can trade in two nominal one-period, risk-free bonds. Bonds A_t are issued by home firms and are denominated in home currency, bonds B_t are issued by foreign firms and are denominated in foreign currency. Households belonging to country H have to pay an intermediation cost, if they want to trade in the foreign bond. These costs are determined by the function $\phi(\cdot)$. Function $\phi(\cdot)$ depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by the domestic households. If a household belongs to a country which is in a 'borrowing position' ($B_{t+1} < 0$), it will be charged with a premium on the foreign interest rate and if the household belongs to a country which is in a 'lending position' ($B_{t+1} > 0$), it receives a rate of return lower than the foreign interest rate. Along with Benigno (2009) we need the following restrictions on $\phi(\cdot)$: $\phi(0) = 1$ and $\phi(\cdot)$ is 1 only if $B_t = 0$. Furthermore $\phi(\cdot)$ has to be a differentiable, decreasing function in the neighborhood of zero. $\phi'(0) = -\chi$.[4]

The intermediation profits $\digamma_t$ are defined analogous to Benigno (2009)

$$\digamma_t = \frac{B^i_{t+1} E_t}{(1 + i^*_t)} \left(\frac{1}{\phi\left(\frac{E_t B_{t+1}}{P_t}\right)} - 1 \right)$$

and shared equally among foreign households. The domestic budget constraint is then given as

$$\begin{aligned} & P_t C_t + s_{t+1} P_{St} + \frac{A^i_{t+1}}{1 + i_t} + \frac{B^i_{t+1} E_t}{(1 + i^*_t)} + \digamma_t \\ \leq \; & A^i_t + B^i_t E_t + (1 - \tau^w_t) W^i_t n^i_t + \left((1 - \tau^d_t) D_t + P_{St}\right) s_t + T^i_t + P_{Ht} \Phi_t \end{aligned}$$

The optimal allocation within each variety of goods z yields per capita relationships

$$c_{Ht}(z) = \frac{1}{n} \left(\frac{p_{Ht}(z)}{P_{Ht}}\right)^{-\epsilon} C_{Ht}, \; c_{Ft}(z) = \frac{1}{1-n} \left(\frac{p_{Ft}(z)}{P_{Ft}}\right)^{-\epsilon} C_{Ft}$$

for all $z \in [0, 1]$, where

$$P_{Ht} = \left(\frac{1}{n} \int_0^n p_{Ht}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}, \; P_{Ft} = \left(\frac{1}{1-n} \int_n^1 p_{Ft}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}$$

are the price indexes for domestic and imported goods, whereby the latter is expressed in domestic currency. Note as ϵ rises, the individual goods become closer substitutes and therefore the individual firms have less market power.

Finally, the optimal condition of expenditures between domestic and imported (foreign) bundles of goods is given by[5]

$$C_{Ht} = (1 - \gamma) \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t \text{ and } C_{Ft} = \gamma \left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t$$

[3] See Benigno (2009) for a generalized asset trading framework, that follows Ghironi et al. (2006).
[4] We assume it is convex and can be approximated by $\phi(x) = 1 - \chi x + \bar{\chi} x^2$.
[5] The same relationship can be written in per capita terms.

where z denotes the good's type or variety and $p_{Ht}(z), p_{Ft}(z)$ are prices of individual home and foreign produced goods.

where $P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, P_t is the consumer price index (CPI) in country H and P_{Ht}, P_{Ft} are domestic and foreign goods price indices. Note that if the economy is closed, $\gamma = 0$, the CPI equals domestic prices. Correspondingly we can write total consumption expenditures by domestic households as $P_t C_t = P_{Ht}C_{Ht} + P_{Ft}C_{Ft}$. The aggregated budget constraint can therefore be rewritten as

$$\begin{aligned} & \left(1-\tau_t^w\right) W_t^i n_t^i + A_t^i + B_t^i E_t + \left(\left(1-\tau_t^d\right) D_t + P_{St}\right) s_t + T_t^i + P_{Ht}\Phi_t \\ \geq \; & \frac{A_{t+1}^i}{1+i_t} + \frac{B_{t+1}^i E_t}{\left(1+i_t^*\right) \phi\left(\frac{E_t B_{t+1}}{P_t}\right)} + s_{t+1}P_{St} + P_t C_t \end{aligned} \tag{3.2}$$

We assume that all households in the same country have the same level of initial wealth. As they face the same labour demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use individual index within each country.

We maximize (3.1) with respect to (3.2) and arrive to the following system

$$\frac{C_t^{-\sigma}}{P_t} = \Delta_t \tag{3.3}$$

$$\frac{W_t}{P_t} = \frac{\alpha n_t^{\psi}}{C_t^{-\sigma}\left(1-\tau_t^w\right)} \tag{3.4}$$

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \tag{3.5}$$

$$\frac{1}{1+i_t^*} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \frac{E_{t+1}}{E_t} \phi\left(\frac{E_t B_{t+1}}{P_t}\right) \tag{3.6}$$

$$P_{St} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \left(\left(1-\tau_{t+1}^d\right) D_{t+1} + P_{St+1}\right) \tag{3.7}$$

$$\begin{aligned} & \left(1-\tau_t^w\right) W_t n_t + A_t + B_t E_t + \left(\left(1-\tau_t^d\right) D_t + P_{St}\right) s_t + T_t \\ = \; & \frac{A_{t+1}}{1+i_t} + \frac{B_{t+1} E_t}{\left(1+i_t^*\right) \phi\left(\frac{E_t B_{t+1}}{P_t}\right)} + s_{t+1}P_{St} + P_t C_t \end{aligned} \tag{3.8}$$

All derivations are given in Appendix B.1. Equation (3.5) is the standard Euler equation and determines the consumption smoothing behavior of the households. Equation (3.6) is the Euler equation derived from the optimal choice of the foreign bond. (3.4) is the standard labour supply condition. It determines the quantity of labor supplied as a function of real wage, given the marginal utility of consumption. Finally equation (3.8) is the aggregate budget constraint.

The incomplete financial market framework generates deviations from the uncovered interest parity (UIP). Combining (3.5) and (3.6) yields the optimal portfolio choice of the households of country H

$$(1+i_t) = \mathbb{E}_t \left(1+i_t^*\right) \frac{E_{t+1}}{E_t} \phi \left(\frac{E_t B_{t+1}}{P_t}\right) \tag{3.9}$$

$\phi\left(\frac{E_t B_{t+1}}{P_t}\right)$ can also be interpreted as a risk premium term on the interest rate. If the economy is a net debtor, the domestic interest rate is above the foreign interest rate and if the economy is a net creditor the domestic interest rate is below the foreign interest rate. Therefore movements in the net foreign asset positions affect the interest differential between the two countries.

3.2.3 Foreign Households

Similarly the foreign economy is populated by a continuum of homogeneous infinitly-living households who share identical preferences and technology and maximise the expected lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^*, n_t^*\right)$, with aggregated period utility

$$U\left(C_t^*, n_t^*\right) = \frac{C_t^{*1-\sigma}}{1-\sigma} - \alpha \frac{n_t^{*1+\psi}}{1+\psi} \tag{3.10}$$

where C_t^* is private foreign consumption, n_t^* is foreign labor, β is the discount factor, $\mathbb{E}_0$ is the actuarial expectation at time $t=0$. Furthermore $\psi \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, α is a preference parameter. We assume home bias in consumption. In more detail, a composite consumption index, C_t^*, is defined as a Dixit-Stiglitz aggregator of the continuum of goods $i \in [0,1]$ produced in the foreign country and home[6]

$$C_t^* = \left(\left(1-\gamma^*\right)^{\frac{1}{\eta}} C_{Ft}^{*\frac{\eta-1}{\eta}} + \gamma^{*\frac{1}{\eta}} C_{Ht}^{*\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

Parameter $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods. Parameter $\gamma^* \in [0,1]$ is the weight of foreign imported goods in private foreign consumption and is inversely related to the degree of foreign bias in preferences. Another interpretation for γ^* is as a natural index of foreign openness or the foreign import share. The foreign import share depends on n which is the relative size of

[6]The same relationship can be written in per capita terms.

home economy, and on ϖ^* which is the degree of trade openness. It yields $\gamma^* = n\varpi^*$. We assume home bias in consumption:

$$1 - \gamma^* > \gamma$$

which implies

$$1 - n\varpi^* > (1-n)\varpi$$

Similarly, home bias in foreign preferences requires $1 - \gamma > \gamma^*$ which again implies

$$\begin{aligned} (1 - (1-n)\varpi) &> n\varpi^* \\ 1 - n\varpi^* &> (1-n)\varpi \end{aligned}$$

C^*_{Ht} and C^*_{Ft} are foreign consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F given by the CES functions

$$C^*_{Ht} = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n c^*_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}, \; C^*_{Ft} = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 c^*_{Ft}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}$$

Each consumption bundle C^*_{Ht} and C^*_{Ft} is composed of imperfectly substitutable varieties of goods $z \in [0,1]$ produced within a given country with elasticity of substitution $\epsilon > 1$.

The aggregated nominal intertemporal budget constraint at time t for household i in foreign currency is given by

$$\begin{aligned} & (1 - \tau^{w*}_t) W^*_t n^*_t + \frac{A^*_t}{E_t} + B^*_t + \left((1 - \tau^{d*}_t) D^*_t + P^*_{St}\right) s^*_t + P^*_{Ft}\Phi^*_t \\ \geq \; & \frac{A^*_{t+1}}{E_t (1+i_t)\, \phi^*\left(\frac{A^*_{t+1}}{E_t P^*_t}\right)} + \frac{B^*_{t+1}}{(1+i^*_t)} + s^*_{t+1}P^*_{St} - T^*_t \\ & + \int_0^1 [P^*_{Ht}(z)C^{i*}_{Ht}(z) + P^*_{Ft}(z)C^{i*}_{Ft}(z)]dz \end{aligned}$$

$P^*_{Ht}(z)$ and $P^*_{Ft}(z)$ are price indices of domestic (exported to country F) and foreign produced goods z, where they are expressed in foreign currency. B^*_t is the one-period foreign corporate bond held by foreign households (real bonds, in terms of domestic prices), A^*_t is the one-period home corporate bonds held by foreign households, W^{*i}_t is the nominal foreign wage and T^{*i}_t denotes lump-sum taxes/transfers. τ^{w*}_t denotes a country specific tax on nominal income and E_t is the nominal exchange rate, given as the price of one of unit foreign currency in terms of home currency. Here $P^*_{Ft}\Phi^*_t$ is nominal profit from the ownership of capital-producing firms and retailers (note that $\Phi^*_t = \Phi^{C*}_t + \Phi^{R*}_t$). Here s^*_t the foreign share of equity which is wholly owned by foreign households, D^*_t the equity payout paid to the shareholders, P^*_{St} is the market price of foreign-owned shares,τ^{d*}_t denotes the tax on equity payout. We assume that the households share the revenues of owning firms in equal proportion. Following Woodford (2003) we consider a cashless economy. Therefore the only explicit role played by money is to serve as a unit of account.

We introduce incomplete financial markets as in Benigno (2009). Foreign households hold foreign equity shares and noncontingent bonds issued by firms of home and foreign countries. Households of country F can trade in two nominal one-period, risk-free bonds. Bonds B_t^* are issued by foreign firms and are denominated in foreign currency, bonds A_t^* are issued by domestic firms and are denominated in foreign currency. Households belonging to country F have to pay an intermediation cost, if they want to trade in the domestic bond. These costs are determined by the function $\phi^*(\cdot)$. Function $\phi^*(\cdot)$ depends on the real holdings of the home assets in the entire economy, and therefore is taken as given by the foreign households. If a household belongs to a country which is in a 'borrowing position' ($A_{t+1}^* < 0$), it will be charged with a premium on the domestic interest rate and if the household belongs to a country which is in a 'lending position' ($A_{t+1}^* > 0$), it receives a rate of return lower than the domestic interest rate. Along with Benigno (2009) we need the following restrictions on $\phi^*(\cdot)$: $\phi^*(0) = 1$ and $\phi^*(\cdot)$ is 1 only if $A_t^* = 0$. Furthermore $\phi^*(\cdot)$ has to be a differentiable, decreasing function in the neighborhood of zero. $\phi^{*\prime}(0) = -\chi^*$.[7]

The intermediation profits $\digamma_t^*$ are defined analogous to Benigno (2009)

$$\digamma_t^* = \frac{A_{t+1}^*}{E_t\left(1+i_t\right)}\left(\frac{1}{\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} - 1\right)$$

and shared equally among foreign households. The domestic budget constraint is then given as

$$\begin{aligned} & \left(1-\tau_t^{w*}\right) W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau_t^{d*}\right) D_t^* + P_{St}^*\right) s_t^* + P_{Ft}^* \Phi_t^* \\ \geq\ & \frac{A_{t+1}^*}{E_t\left(1+i_t\right)} + \frac{B_{t+1}^*}{\left(1+i_t^*\right)} + s_{t+1}^* P_{St}^* - T_t^* + \digamma_t^* + P_t^* C_t^* \end{aligned}$$

The optimal allocation within each variety of goods z yields per capita relationships

$$c_{Ht}^*(z) = \frac{1}{n}\left(\frac{p_{Ht}^*(z)}{P_{Ht}^*}\right)^{-\epsilon} C_{Ht}^* \ ,\ c_{Ft}^*(z) = \frac{1}{1-n}\left(\frac{p_{Ft}^*(z)}{P_{Ft}^*}\right)^{-\epsilon} C_{Ft}^*$$

for all $z \in [0,1]$, where

$$P_{Ht}^* = \left(\frac{1}{n}\int_0^n p_{Ht}^*(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}} \ ,\ P_{Ft}^* = \left(\frac{1}{1-n}\int_n^1 p_{Ft}^*(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}$$

are the price indexes for home and foreign produced goods, where both are expressed in foreign currency. Note as ϵ rises, the individual goods become closer substitutes and therefore the individual firms have less market power.

[7] We assume it is convex and can be approximated by $\phi^*(x) = 1 - \chi^* x + \bar{\chi}^* x^2$.

Finally, the optimal condition of expenditures between home and foreign produced goods is given by[8]

$$C_{Ft}^{*}=(1-\gamma^{*})\left(\frac{P_{Ft}^{*}}{P_{t}^{*}}\right)^{-\eta}C_{t}^{*}\text{ and }C_{Ht}^{*}=\gamma^{*}\left(\frac{P_{Ht}^{*}}{P_{t}^{*}}\right)^{-\eta}C_{t}^{*}$$

where z denotes the good's type or variety and $p_{Ht}^{*}(z), p_{Ft}^{*}(z)$ are prices of individual home and foreign produced goods.

where $P_{t}^{*}=\left((1-\gamma^{*})\,P_{Ft}^{*1-\eta}+\gamma^{*}P_{Ht}^{*1-\eta}\right)^{\frac{1}{1-\eta}}$, P_{t}^{*} is the consumer price index (CPI) in country F and P_{Ht}^{*}, P_{Ft}^{*} are domestic and foreign goods price indices. Note that if the economy is closed, $\gamma^{*}=0$, the CPI equals foreign prices. Correspondingly we can write total consumption expenditures by foreign households as $P_{t}^{*}C_{t}^{*}=P_{Ht}^{*}C_{Ht}^{*}+P_{Ft}^{*}C_{Ft}^{*}$. The aggregated budget constraint can therefore be rewritten as

$$\begin{aligned}&(1-\tau_{t}^{w*})\,W_{t}^{*}n_{t}^{*}+\frac{A_{t}^{*}}{E_{t}}+B_{t}^{*}+\left((1-\tau_{t}^{d*})\,D_{t}^{*}+P_{St}^{*}\right)s_{t}^{*}+P_{Ft}^{*}\Phi_{t}^{*}\\ \geq\;&\frac{A_{t+1}^{*}}{E_{t}\,(1+i_{t})\,\phi^{*}\left(\frac{A_{t+1}^{*}}{E_{t}P_{t}^{*}}\right)}+\frac{B_{t+1}^{*}}{(1+i_{t}^{*})}+s_{t+1}^{*}P_{St}^{*}-T_{t}^{*}+P_{t}^{*}C_{t}^{*}\end{aligned}\tag{3.11}$$

We assume that all households in the same country have the same level of initial wealth. As they face the same labour demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use individual index within each country.

We maximize (3.10) with respect to (3.11) and arrive to the following system

$$\frac{C_{t}^{*-\sigma}}{P_{t}^{*}}=\Delta_{t}^{*}\tag{3.12}$$

$$\frac{W_{t}^{*}}{P_{t}^{*}}=\frac{\alpha n_{t}^{*\psi}}{C_{t}^{*-\sigma}\,(1-\tau_{t}^{*w})}\tag{3.13}$$

$$\frac{1}{(1+i_{t}^{*})}=\beta\mathbb{E}_{t}\frac{C_{t+1}^{*-\sigma}}{C_{t}^{*-\sigma}\Pi_{Ft+1}^{*}}\tag{3.14}$$

$$\frac{1}{(1+i_{t})}=\beta\mathbb{E}_{t}\frac{C_{t+1}^{*-\sigma}}{C_{t}^{*-\sigma}}\frac{E_{t}}{\Pi_{Ft+1}^{*}E_{t+1}}\phi^{*}\left(\frac{A_{t+1}^{*}}{E_{t}P_{t}^{*}}\right)\tag{3.15}$$

$$P_{St}^{*}=\beta\mathbb{E}_{t}\frac{C_{t+1}^{*-\sigma}}{\Pi_{Ft+1}^{*}C_{t}^{*-\sigma}}\left((1-\tau_{t+1}^{*d})\,D_{t+1}^{*}+P_{St+1}^{*}\right)\tag{3.16}$$

$$\begin{aligned}&(1-\tau_{t}^{w*})\,W_{t}^{*}n_{t}^{*}+\frac{A_{t}^{*}}{E_{t}}+B_{t}^{*}+\left((1-\tau_{t}^{d*})\,D_{t}^{*}+P_{St}^{*}\right)s_{t}^{*}+T_{t}^{*}\\ =\;&\frac{A_{t+1}^{*}}{E_{t}\,(1+i_{t})\,\phi^{*}\left(\frac{A_{t+1}^{*}}{E_{t}P_{t}^{*}}\right)}+\frac{B_{t+1}^{*}}{(1+i_{t}^{*})}+s_{t+1}^{*}P_{St}^{*}+P_{t}^{*}C_{t}^{*}\end{aligned}\tag{3.17}$$

[8] The same relationship can be written in per capita terms.

All derivations are given in Appendix B.1. Where the budget constraint is written in an aggregated form. Equation (3.14) is the standard Euler equation and determines the consumption smoothing behavior of the households. Equation (3.15) is the Euler equation derived from the optimal choice of the foreign bond. (3.13) is the standard labour supply condition. It determines the quantity of labor supplied as a function of real wage, given the marginal utility of consumption. Finally equation (3.17) is the aggregate budget constraint.

The incomplete financial market framework generates deviations from the uncovered interest parity (UIP). Combining (3.14) and (3.15) yields the optimal portfolio choice of the households of country F

$$(1 + i_t^*) = \mathbb{E}_t \left(1 + i_t\right) \frac{E_t}{E_{t+1}} \phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right) \tag{3.18}$$

$\phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right)$ can also be interpreted as a risk premium term on the interest rate. If the economy is a net debtor, the foreign interest rate is above the domestic interest rate and if the economy is a net creditor the foreign interest rate is below the domestic interest rate. Therefore movements in the net domestic asset positions affect the interest differential between the two countries.

Combining (3.6, 3.15) with the Euler equation of the foreign country (3.9, 3.18) yields the international risk sharing condition

$$\mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} = \mathbb{E}_t \frac{C_{t+1}^{*-\sigma}}{\Pi_{Ft+1}^* C_t^{*-\sigma}} \frac{E_t}{E_{t+1}} \phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right) \tag{3.19}$$

$$\mathbb{E}_t \frac{C_{t+1}^{*-\sigma}}{\Pi_{Ft+1}^* C_t^{*-\sigma}} = \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \frac{E_{t+1}}{E_t} \phi \left(\frac{E_t B_{t+1}}{P_t} \right) \tag{3.20}$$

Note that if $\frac{A_{t+1}^*}{E_t P_t^*} \equiv \frac{E_t B_{t+1}}{P_t} \equiv 0$ then $\phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right) = \phi \left(\frac{E_t B_{t+1}}{P_t} \right) = 1$ and equations (3.19) and (3.20) simplifies to the standard international risk sharing relationship which is obtained in a complete securities markets setting (see e.g. Galí and Monacelli (2005)).

3.2.4 Domestic Firms

Intermediate goods producers

We assume that there is a continuum of firms $j \in [0, 1]$ in country (H) with a gross production function

$$Y_t = F(e^{z_t}, k_t, n_t) = Z e^{z_t} k_t^{\theta} n_t^{1-\theta}$$

and all firms are equal. Ze^{z_t} is the stochastic productivity, common to all firms, k_t is the capital input and n_t is the labor input. k_t is assumed to be chosen at time $t-1$ and predetermined at time t which is consistent with the typical timing convention. On the contrary, the labor input n_t can be flexibly changed at time t.

Each period firms buy the investment good I_t

$$I_t = k_{t+1} - (1-\delta) k_t \tag{3.21}$$

where δ is the depreciation rate.

Therefore the payments to workers $W_t n_t$, suppliers of investment goods $P_{Ht} Q_t I_t$, shareholders $\Psi(D_t, D_{t-1})$ and bondholders A_t^T are made ahead of the realization of revenues. The intra-period loan contracted by the firm will cover these costs as follows:

$$L_t = P_{Ht} Q_t I_t + W_t n_t + \Psi(D_t, D_{t-1}) + A_t^T - \frac{A_{t+1}^T}{1 + i_t(1-\tau_t)}$$

Firms use equity and debt to finance their operations. They prefer nominal debt, $A_t^T = A_t + A_t^*$, to equity in general because of debt's tax advantage (τ_t). This is also the assumption made in Hennessy and Whited (2005). Given i_t the nominal interest rate, the effective gross interest rate for the firm is $R_t = 1 + i_t(1-\tau_t)$, where τ_t represents the tax benefit.

We assume that firms raise funds by the intertemporal nominal debt A_t^T and the intra-period domestic loan, L_t to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues. They pay back the free-interest intra-period loan at the end of the period.

Firms start the period with intertemporal debt A_t^T and they choose labour n_t, investment in capital I_t, equity payout, D_t , and the new intertemporal debt A_{t+1}^T before producing. Therefore, the aggregated nominal budget constraint of firms can be written as

$$P_{mt} F(e^{z_t}, k_t, n_t) + \frac{A_{t+1}^T}{1 + i_t(1-\tau_t)} \geq A_t^T + W_t n_t + P_{Ht} Q_t I_t + \Psi(D_t, D_{t-1}) \tag{3.22}$$

From the budget constraint $L_t = P_{mt} F(e^{z_t}, k_t, n_t)$ is repaid at the end of the period and is free of interest. Where P_{mt} is the nominal price of produced intermediate goods, $\Psi(D_t, D_{t-1})$ is the nominal payment to shareholders, $P_{Ht} Q_t$ is the price of investment goods, and and W_t is the nominal wage in home country.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intra-period loan. The total liabilities of the firm at that time are $L_t + \frac{A_{t+1}}{1+i_t}$, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t = P_{mt}F(e^{z_t}, k_t, n_t)$. These can be 'diverted' by the firm, and so can not be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_{Ht}Q_t k_{t+1}$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability Ξe^{ξ_t} the full value $P_{Ht}Q_t k_{t+1}$ will be recovered, but with probability $1 - \Xi e^{\xi_t}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi e^{\xi_t}\left(P_{Ht}Q_t k_{t+1} - \frac{A^T_{t+1}}{1+i_t}\right) \geq P_{mt}F(e^{z_t}, k_t, n_t) \tag{3.23}$$

This constraint is derived based on the renegotiation process between the firm and the lender in the case of default. The derivation is given in Appendix A.3.

By increasing the level of debt the enforcement constraint becomes tighter. On the other hand, increasing the stock of capital relaxes the enforcement constraint. Most of the enforcement constraint used in the literature shared these properties. The probability Ξe^{ξ_t} is stochastic and depends on uncertain markets conditions.[9] We call this variable as "financial shocks", because it affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. Notice that Ξe^{ξ_t} is the same for all firms. Hence, there are two sources of aggregate uncertainty in our model: productivity e^{zt} and financial Ξe^{ξ_t}. Since there are no idiosyncratic shocks, we will focus on the symmetric equilibrium where all representative firms are the same.

We can slightly modify (3.23), to see clearly how the shock Ξe^{ξ_t} affects the economy. Suppose the case in which $\tau = 0$ so that $R = 1 + i$. Using the budget constraint (3.22) to substitute for $P_{Ht}Q_t k_{t+1} - \frac{A^T_{t+1}}{1+i_t}$ and remembering that the intra-period loan is equal to the revenues, $L_t = P_{mt}F(e^{z_t}, k_t, n_t)$, the enforcement constraint can be rewritten as

$$\frac{\Xi e^{\xi_t}}{1 - \Xi e^{\xi_t}}\left(P_{Ht}Q_t\left(1-\delta\right)k_t - A^T_t - W_t n_t - \Psi\left(D_t, D_{t-1}\right)\right) \geq P_{mt}F(e^{z_t}, k_t, n_t)$$

At the beginning of the period k_t and A^T_t are given. The firm have control only over the input of labor, n_t, and the equity payout, $\Psi\left(D_t, D_{t-1}\right)$. If the firm wishes to keep the production level unchanged, a negative financial shock (lower Ξe^{ξ_t}) requires a reduction in equity payout $\Psi\left(D_t, D_{t-1}\right)$ or employment. In other words, the firm is forced to raise its equity and cut the new intertemporal debt. Thus, the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity will determine if the financial shock affects employment.

[9] The variable Ξe^{ξ_t} could be interpreted as the probability of finding a buyer. Because we assume that the search for a buyer is required for the sale of the firm's capital. The probability increases when the market conditions improve.

The firm's nominal payout to shareholders assumed to be subject to a quadratic adjustment cost which is a way to formalize the rigidities affecting the substitution between debt and equity:

$$\Psi\left(D_t, D_{t-1}\right) = D_t + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2 D_t$$

where the nominal equity payout D_t is given and $\kappa \geq 0$ is a parameter.[10]

The parameter κ is key for the role of financial shocks. Since when $\kappa = 0$ the economy is almost frictionless, therefore debt adjustments caused by financial shocks can be quickly assisted through changes in firm equity. When $\kappa > 0$, it is costly to substitute debt and equity and firm's readjustment becomes slowly. As a result, financial shocks will have a substantial effect on macroeconomic situation of a country.

The first order conditions with respect to n_t, k_{t+1}, A^T_{t+1}, D_t, μ_t, λ_t can be written as

$$\begin{aligned}
\lambda_t W_t &= F_n(e^{z_t}, k_t, n_t)\left(\lambda_t - \mu_t\right) P_{mt} && (3.24)\\
0 &= \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \left(\left(\lambda_{t+1} - \mu_{t+1}\right) P_{mt+1} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) + \lambda_{t+1} P_{Ht+1} Q_{t+1} \left(1 - \delta\right)\right) \\
&\quad - \left(\lambda_t - \mu_t \Xi e^{\xi_t}\right) P_{Ht} Q_t && (3.25)\\
0 &= \frac{\lambda_t}{1 + i_t(1 - \tau_t)} - \mu_t \Xi e^{\xi_t} \frac{1}{1 + i_t} - \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \lambda_{t+1} && (3.26)\\
1 &= \lambda_t \left(1 + 2\kappa \left(\frac{D_t}{D_{t-1}} - 1\right) \frac{D_t}{D_{t-1}} + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2\right) \\
&\quad - \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \lambda_{t+1} 2\kappa \left(\frac{D_{t+1}}{D_t} - 1\right) \frac{D_{t+1}^2}{D_t^2} && (3.27)\\
0 &= \Xi e^{\xi_t} \left(P_{Ht} Q_t k_{t+1} - \frac{A^T_{t+1}}{1 + i_t}\right) - P_{mt} F(e^{z_t}, k_t, n_t) && (3.28)\\
A^T_t &= P_{mt} F(e^{z_t}, k_t, n_t) + \frac{A^T_{t+1}}{1 + i_t(1 - \tau_t)} - W_t n_t - P_{Ht} Q_t I_t - \Psi\left(D_t, D_{t-1}\right) && (3.29)
\end{aligned}$$

All derivations are given in Appendix B.2. $m_{t,t+1} = \beta \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}}$ is a nominal stochastic discount factor and the budget constraint is written in an aggregated form. The stochastic discount factor $m_{t,t+1}$, the wage W_t and interest rate i_t are determined in the general equilibrium and are taken as given by an individual firm.

Equation (3.24), the optimal condition for labor indicates that the marginal productivity of labor is equal to the marginal cost $\left(\frac{\lambda_t W_t}{(\lambda_t - \mu_t) P_{mt}}\right)$. As the enforcement constraint becomes tighter, the effective cost of labor rises and its demand falls. Therefore, financial shocks could transmit to the real sector of the economy through the demand of labor.

[10] One way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact later confirmed by subsequent studies. This could obtain from agency problems.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa = 0$. In this case $\lambda_t = 1$ (see condition (3.27)) and condition (3.26) becomes $\mu_t \Xi e^{\xi_t} \frac{R_t}{1+i_t} + R_t \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \lambda_{t+1} = 1$. This denotes that there is a negative relation between Ξe^{ξ_t} and the multiplier μ_t taking as given the aggregate prices R_t, i_t, and $\beta \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}}$. In other words, lower probability of recovering firm's capital make the enforcement constraint tighter. Then from equation (3.24) we see that a higher μ_t implies a lower demand for labor.

This mechanism is strengthened when $\kappa > 0$. In this case readjusting the financial structure becomes costly, and the change in Ξe^{ξ_t} induces a larger volatility in μ_t. Of course, prices will be affected by the change in the policies of all firms.

Capital producers

Capital producers belong to households. They make new capital using input of final output and subject to adjustment costs. They sell new capital to firms at price $P_{Ht} Q_t$. Their nominal profit

$$P_{Ht} \Phi_t^C = P_{Ht} Q_t I_t - P_{Ht} I_t \left(1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right)$$

Specifically, they buy I_t of the final good, pay $P_{Ht} I_t \left(1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right)$ as they may need to adjust contracts if the amount of the investment goods changes. They repackage the good into investment good (costlessly) and sell it to firms at price $P_{Ht} Q_t$ and receive $P_{Ht} Q_t I_t$.

The first order condition yields

$$Q_t = 1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 + \frac{I_t}{I_{t-1}} \varrho \left(\frac{I_t}{I_{t-1}} - 1 \right) - \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}} \varrho \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2$$

Retailers

Retailers repackage intermediate output. The marginal cost is P_{mt}. We introduce nominal rigidities *a la* Rotemberg.

Cost minimisation yields

$$y_t^i = \left(\frac{p_{Ht}^i}{P_{Ht}} \right)^{-\varepsilon} Y_t$$

where index i is of retailer i and Y_t is final output. Retailers costlessly brand intermediate output. They have monopolistic power but have adjustment cost.

The firm's profit is

$$p^i_{Ht}\Phi^R_t = p^i_{Ht}y^i_t\left(1-\tau^x_t\right) - P_{mt}y^i_t - \frac{\omega}{2}\left(\frac{p^i_{Ht}}{p^i_{Ht-1}}-1\right)^2 Y_t P_{Ht}$$

we introduce sales tax τ^x_t.

The first order condition yields

$$\omega\left(\Pi_{Ht}-1\right)\Pi_{Ht} = \left(1-\varepsilon\right)\left(1-\tau^x_t\right) + \varepsilon X_t + \omega\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\left(\Pi_{Ht+1}-1\right)\frac{Y_{t+1}}{Y_t}\Pi_{Ht+1} \quad (3.30)$$

where $X_t = \frac{P_{mt}}{P_{Ht}}$. The derivation is given in Appendix B.2.

and the aggregated across firms profit

$$P_{Ht}\Phi^R_t = P_{Ht}Y_t\left(1-\tau^x_t\right) - P_{mt}Y_t - \frac{\omega}{2}\left(\Pi_{Ht}-1\right)^2 Y_t P_{Ht}$$

3.2.5 Foreign Firms

Intermediate Goods Producers

The optimisation problem for foreign firms is symmetric. We assume that there is a continuum of firms $j^* \in [0,1]$ in country (F) with a gross revenue function

$$Y^*_t = F(e^{z^*_t}, k^*_t, n^*_t) = Ze^{z^*_t}k^{*\theta}_t n^{*1-\theta}_t$$

and all firms are equal. $e^{z^*_t}$ is the stochastic productivity, common to all firms, k^*_t is the capital and n^*_t the labor in country F. k^*_t is assumed to be chosen at time $t-1$ and predetermined at time t which is consistent with the typical timing convention. On the contrary, the labor input n^*_t can be flexibly changed at time t.

Each period firms buy the investment good I^*_t

$$I^*_t = k^*_{t+1} - \left(1-\delta\right)k^*_t$$

where I^*_t is investment and δ^* is the depreciation rate in country F.

Therefore the payments to workers $W_t^* n_t^*$, suppliers of investment goods $P_{Ft}^* Q_t^* I_t^*$, shareholders $\Psi^*\left(D_t^*, D_{t-1}^*\right)$ and bondholders B_t^T are made ahead of the realization of revenues. The intra-period loan contracted by the firm will cover these costs as follows:

$$L_t^* = P_{Ft}^* Q_t^* I_t^* + W_t^* n_t^* + \Psi^*\left(D_t^*, D_{t-1}^*\right) + B_t^T - \frac{B_{t+1}^T}{1+i_t^*(1-\tau_t^*)}$$

Firms use equity and debt to finance their operations. They prefer nominal debt, $B_t^T = B_t + B_t^*$, to equity in general because of debt's tax advantage (τ_t^*). This is also the assumption made in Hennessy and Whited (2005). Given i_t^* the nominal interest rate, the effective gross interest rate for the firm is $R_t^* = 1 + i_t^*(1-\tau_t^*)$, where τ_t^* represents the tax benefit.

We assume that firms raise funds by the intertemporal nominal debt B_t^T and the intra-period domestic loan, L_t to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues. They pay back the free-interest intra-period loan at the end of the period.

Firms start the period with intertemporal debt B_t^T and they choose labour n_t^*, investment in capital I_t^*, equity payout, D_t^* , and the new intertemporal debt B_{t+1}^T before producing. Therefore, the firm's aggregated nominal budget constraint can be written as

$$P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{B_{t+1}^T}{1+i_t^*(1-\tau_t^*)} \geq B_t^T + W_t^* n_t^* + P_{Ft}^* Q_t^* I_t^* + \Psi^*\left(D_t^*, D_{t-1}^*\right) \quad (3.31)$$

From the budget constraint $L_t^* = P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*)$ is repaid at the end of the period and is free of interest. Where P_{mt}^* is the nominal price of produced intermediate goods, $\Psi^*\left(D_t^*, D_{t-1}^*\right)$ is the nominal payment to shareholders, $P_{Ft}^* Q_t^*$ is the price of investment goods, and W_t^* is the nominal wage in foreign country.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intra-period loan. The total liabilities of the firm at that time are $L_t^* + \frac{B_{t+1}^T}{1+i_t^*}$, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t^* = P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*)$. These can be 'diverted' by the firm, and so can not be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_{Ft}^* Q_t^* k_{t+1}^*$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability $\Xi^* e^{\xi_t^*}$ the full value $P_{Ft}^* Q_t^* k_{t+1}^*$ will be recovered, but with probability $1 - \Xi^* e^{\xi_t^*}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi^* e^{\xi_t^*} \left(P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*} \right) \geq P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) \quad (3.32)$$

This constraint is derived based on the renegotiation process between the firm and the lender in the case of default. The derivation is given in Appendix A.3.

By increasing the level of debt the enforcement constraint becomes tighter. On the other hand, increasing the stock of capital relaxes the enforcement constraint. Most of the enforcement constraint used in the literature shared these properties. The probability $\Xi^* e^{\xi_t^*}$ is stochastic and depends on uncertain markets conditions.[11] We call this variable as "financial shocks", because it affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. Notice that $\Xi^* e^{\xi_t^*}$ is the same for all firms. Hence, there are two sources of aggregate uncertainty in our model: productivity $e^{z_t^*}$ and financial $\Xi^* e^{\xi_t^*}$. Since there are no idiosyncratic shocks, we will focus on the symmetric equilibrium where all representative firms are the same.

We can slightly modify (3.32), to see clearly how the shock $\Xi^* e^{\xi_t^*}$ affects the economy. Suppose the case in which $\tau^* = 0$ so that $R^* = 1 + i^*$. Using the budget constraint (3.31) to substitute for $P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*}$ and remembering that the intra-period loan is equal to the revenues, $L_t = P_{mt} F(e^{z_t}, k_t, n_t)$, the enforcement constraint can be rewritten as

$$\frac{\Xi^* e^{\xi_t^*}}{1 - \Xi^* e^{\xi_t^*}} \left(P_{Ft}^* Q_t^* (1-\delta) k_t^* - B_t^T - W_t^* n_t^* - \Psi^* \left(D_t^*, D_{t-1}^* \right) \right) \geq P_{mt}^* F^* (e^{z_t *}, k_t^*, n_t^*)$$

At the beginning of the period k_t^* and B_t^T are given. The firm have control only over the input of labor, n_t^*, and the equity payout, $\Psi^* \left(D_t^*, D_{t-1}^* \right)$. If the firm wishes to keep the production level unchanged, a negative financial shock (lower $\Xi^* e^{\xi_t^*}$) requires a reduction in equity payout $\Psi^* \left(D_t^*, D_{t-1}^* \right)$ or employment. In other words, the firm is forced to raise its equity and cut the new intertemporal debt. Thus, the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity will determine if the financial shock affects employment.

The firm's nominal payout to shareholders assumed to be subject to a quadratic adjustment cost which is a way to formalize the rigidities affecting the substitution between debt and equity:

$$\Psi^* \left(D_t^*, D_{t-1}^* \right) = D_t^* + \kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1 \right)^2 D_t^*$$

where the nominal equity payout D_t^* is given and $\kappa^* \geq 0$ is a parameter.[12]

[11] The variable $\Xi^* e^{\xi_t^*}$ could be interpreted as the probability of finding a buyer. Because we assume that the search for a buyer is required for the sale of the firm's capital. The probability increases when the market conditions improve.

[12] One way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact later confirmed by subsequent studies. This could obtain from agency problems.

The parameter κ^* is key for the role of financial shocks. Since when $\kappa^* = 0$ the economy is almost frictionless, therefore debt adjustments caused by financial shocks can be quickly assisted through changes in firm equity. When $\kappa^* > 0$, it is costly to substitute debt and equity and firm's readjustment becomes slowly. As a result, financial shocks will have a substantial effect on macroeconomic situation of a country.

The first order conditions with respect to n_t^*, k_{t+1}^*, B_{t+1}^T, D_t^*, μ_t^*, λ_t^* can be written as

$$\begin{aligned}
\lambda_t^* W_t^* &= \left(\lambda_t^* - \mu_t^*\right) P_{mt}^* F_n(e^{z_t^*}, k_t^*, n_t^*) && (3.33)\\
0 &= \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \left(\left(\lambda_{t+1}^* - \mu_{t+1}^*\right) P_{mt+1}^* F_k(e^{z_{t+1}^*}, k_{t+1}^*, n_{t+1}^*) + \lambda_{t+1}^* P_{Ft+1}^* Q_{t+1}^* (1-\delta)\right) \\
&\quad - \left(\lambda_t^* - \mu_t^* \Xi_t^* e^{\xi_t^*}\right) P_{Ft}^* Q_t^* && (3.34)\\
0 &= \frac{\lambda_t^*}{1+i_t^*(1-\tau_t^*)} - \mu_t^* \Xi^* e^{\xi_t^*} \frac{1}{1+i_t^*} - \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \lambda_{t+1}^* && (3.35)\\
1 &= \lambda_t^* \left(1 + 2\kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1\right) \frac{D_t^*}{D_{t-1}^*} + \kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1\right)^2\right) \\
&\quad - \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \lambda_{t+1}^* 2\kappa^* \left(\frac{D_{t+1}^*}{D_t^*} - 1\right) \frac{D_{t+1}^{*2}}{D_t^{*2}} && (3.36)\\
0 &= \Xi^* e^{\xi_t^*} \left(P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*}\right) - P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) && (3.37)\\
B_t^T &= P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{B_{t+1}^T}{1+i_t^*(1-\tau_t^*)} - W_t^* n_t^* \\
&\quad - P_{Ft}^* Q_t^* \left(k_{t+1}^* - (1-\delta^*) k_t^*\right) - \Psi^* \left(D_t^*, D_{t-1}^*\right) && (3.38)
\end{aligned}$$

All derivations are given in Appendix B.2. $m_{t,t+1} = \beta \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*}$ is an stochastic discount factor and the budget constraint is written in an aggregated form. The stochastic discount factor $m_{t,t+1}$, the wage W_t^* and interest rate i_t^* are determined in the general equilibrium and are taken as given by an individual firm.

Equation (3.33), the optimal condition for labor indicates that the marginal productivity of labor is equal to the marginal cost $\left(\frac{\lambda_t^* W_t^*}{(\lambda_t^* - \mu_t^*) P_{mt}^*}\right)$. As the enforcement constraint becomes tighter, the effective cost of labor rises and its demand falls. Therefore, financial shocks could transmit to the real sector of the economy through the demand of labor.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa^* = 0$. In this case $\lambda_t^* = 1$ (see condition (3.36)) and condition (3.35) becomes $\mu_t^* \Xi^* e^{\xi_t^*} \frac{R_t^*}{1+i_t^*} + R_t^* \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \lambda_{t+1}^* = 1$. This denotes that there is a negative relation between $\Xi^* e^{\xi_t^*}$ and the multiplier μ_t^* taking as given the aggregate prices R_t^*, i_t^*, and $\beta \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*}$. In other words, lower probability of recovering firm's capital make the enforcement constraint tighter. Then from equation (3.33) we see that a higher μ_t^* implies a lower demand for labor.

This mechanism is strengthened when $\kappa^* > 0$. In this case readjusting the financial structure becomes costly, and the change in $\Xi^* e^{\xi_t^*}$ induces a larger volatility in μ_t^*. Of course, prices will be affected by the change in the policies of all firms.

Capital producers

Capital producers belong to households. They make new capital using input of final output and subject to adjustment costs. They sell new capital to firms at price $P_{Ft}^* Q_t^*$. Their nominal profit

$$P_{Ft}^* \Phi_t^{*C} = P_{Ft}^* Q_t^* I_t^* - P_{Ft}^* I_t^* \left(1 + \frac{\varrho}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right)$$

Specifically, they buy I_t^* of the final good, pay $P_{Ft}^* I_t^* \left(1 + \frac{\varrho}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2\right)$ as they may need to adjust contracts if the amount of the investment goods changes. They repackage the good into investment good (costlessly) and sell to firms at price $P_{Ft}^* Q_t^*$ and receive $P_{Ft}^* Q_t^* I_t^*$.

The first order condition yields

$$Q_t^* = 1 + \frac{\varrho}{2}\left(\frac{I_t^*}{I_{t-1}^*} - 1\right)^2 + \frac{I_t^*}{I_{t-1}^*}\varrho\left(\frac{I_t^*}{I_{t-1}^*} - 1\right) - \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^*}\varrho\left(\frac{I_{t+1}^*}{I_t^*} - 1\right)\left(\frac{I_{t+1}^*}{I_t^*}\right)^2$$

Retailers

Retailers repackage intermediate output. The marginal cost is P_{mt}^*. We introduce nominal rigidities *a la* Rotemberg.

Cost minimisation yields

$$y_t^{*i} = \left(\frac{p_{Ft}^{*i}}{P_{Ft}^*}\right)^{-\varepsilon} Y_t^*$$

where index i is of retailer i and Y_t^* is final output. Retailers costlessly brand intermediate output. They have monopolistic power but have adjustment cost.

The firm's profit is

$$p_{Ft}^{*i} \Phi_t^{*R} = p_{Ft}^{*i} y_t^{*i} (1 - \tau_t^{*x}) - P_{mt}^* y_t^{*i} - \frac{\omega}{2}\left(\frac{p_{Ft}^{*i}}{p_{Ft-1}^{*i}} - 1\right)^2 Y_t^* P_{Ft}^*$$

we introduce sales tax τ_t^{*x}.

The first order condition yields

$$\omega\left(\Pi_{Ft}^{*}-1\right)\Pi_{Ft}^{*}=(1-\varepsilon)\left(1-\tau_{t}^{*x}\right)+\varepsilon X_{t}^{*}+\omega\beta\mathbb{E}_{t}\frac{U_{c,t+1}^{*}}{U_{c,t}^{*}}\left(\Pi_{Ft+1}^{*}-1\right)\frac{Y_{t+1}^{*}}{Y_{t}^{*}}\Pi_{Ft+1}^{*} \quad (3.39)$$

where $X_t^* = \frac{P_{mt}^*}{P_{Ft}^*}$. The derivation is given in Appendix B.2.

and the aggregated across firms profit

$$P_{Ft}^{*}\Phi_{t}^{*R}=P_{Ft}^{*}Y_{t}^{*}\left(1-\tau_{t}^{*x}\right)-P_{mt}^{*}Y_{t}^{*}-\frac{\omega}{2}\left(\Pi_{Ft}^{*}-1\right)^{2}Y_{t}^{*}P_{Ft}^{*}$$

3.2.6 Governments

The government in each country collects taxes and pays it as transfers and government spending:

$$\begin{aligned} T_t &= -\frac{A_{t+1}^T}{1+i_t(1-\tau_t)}+\frac{A_{t+1}^T}{1+i_t} && (3.40)\\ P_{Ht}G_t &= \tau_t^w W_t N_t+\tau_t^d D_t+\tau_t^x P_{Ht}Y_t && (3.41)\\ T_t^* &= -\frac{B_{t+1}^T}{1+i_t^*(1-\tau_t^*)}+\frac{B_{t+1}^T}{1+i_t^*} && (3.42)\\ P_{Ft}^*G_t^* &= \tau_t^{*w}W_t^*N_t^*+\tau_t^{*d}D_t^*+\tau_t^{*x}P_{Ft}^*Y_t^* && (3.43) \end{aligned}$$

We assume that the domestic government buys goods (G), taxes sale (with tax rate τ_t^x).

$$\begin{aligned} P_{Ht}G_t &= \tau_t^x P_{Ht}Y_t\\ P_{Ft}^*G_t^* &= \tau_t^{*x}P_{Ft}^*Y_t^* \end{aligned}$$

3.2.7 Market Clearing and Private Sector Equilibrium

In order to close the system, we write down two market clearing constraints:

$$\begin{aligned} Y_t = {} & (1-\gamma)\Upsilon_t^\eta C_t+\gamma^*\Gamma_t^\eta S_t^\eta C_t^*+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_{Ht}-1\right)^2 d_t+G_t+\frac{\omega}{2}\left(\Pi_{Ht}-1\right)^2 Y_t\\ & +I_t\left(1+\frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right)+\left(\frac{1}{\phi\left(\frac{E_tB_{t+1}}{P_t}\right)}-1\right)\frac{B_{t+1}E_t}{\left(1+i_t^*\right)P_{Ht}} \qquad (3.44) \end{aligned}$$

$$Y_t^* = (1-\gamma^*)\,\Gamma_t^\eta C_t^* + \gamma\left(\frac{\Upsilon_t}{S_t}\right)^\eta C_t \quad +\kappa\left(\frac{d_t^*}{d_{t-1}^*}\Pi_{Ft}^* - 1\right)^2 d_t^* + G_t^* + \frac{\omega}{2}\left(\Pi_{Ft}^* - 1\right)^2 Y_t^*$$

$$+I_t^*\left(1+\frac{\varrho}{2}\left(\frac{I_t^*}{I_{t-1}^*}-1\right)^2\right) \quad +\left(\frac{1}{\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)}-1\right)\frac{A_{t+1}^*}{E_t\left(1+i_t\right)P_{Ft}^*}$$

where $Y_t = Ze^{z_t}k_t^\theta N_t^{1-\theta}, Y_t^* = Ze^{z_t^*}k_t^{*\theta}N_t^{*1-\theta}, \Upsilon_t = \left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}$,

$\Gamma_t = \left((1-\gamma^*)+\gamma^* S_t^{\eta-1}\right)^{\frac{1}{1-\eta}}$.

Together with households first order conditions (3.3)-(3.8), (3.12)-(3.17) and firms first order conditions (3.24)-(3.29), (3.33)-(3.38), (3.30), (3.39), government budget constraints (3.40)-(3.43) and one net foreign assets equation

$$0 = \gamma S_t^{1-\eta}\Upsilon_t^\eta C_t - \gamma^*\Gamma_t^\eta S_t^\eta C_t^* - \mathbb{E}_t a_{t+1}^*\frac{\Pi_{Ht+1}}{(1+i_t)} + a_t^* + \mathbb{E}_t b_{t+1}\frac{\Pi_{Ft+1}^*}{(1+i_t^*)}S_t - b_t S_t$$

and the definition of nominal exchange rate

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

They describe the evolution of the economy and determine twenty five variables: C_t, n_t, w_t, λ_t, μ_t, X_t, k_t, Π_{Ht}, d_t, b_t, a_t, Q_t, C_t^*, n_t^*, w_t^*, λ_t^*, μ_t^*, X_t^*, k_t^*, Π_{Ft}^*, d_t^*, b_t^*, a_t^*, Q_t^* and S_t: Appendix B.3 demonstrates that the system is internally consistent. Policy instruments are i_t; i_t^*; τ_t^w; τ_t^{w*}; τ_t^d; τ_t^{d*}; τ_t^x; τ_t^{x*}; τ_t; τ_t^* and it remains to describe policy.

3.3 Linearised system

Let $\hat{X}_t = \log X_t - \log X$ denote the log-deviation of variable X_t from its steady state value X. In line with Benigno (2009) and Paoli (2009) we assume a symmetric steady state, which implies that the net foreign asset position is zero in the steady state.[13] The log-linear approximations for the equilibrium conditions of our model are given as:

$$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \mathbb{E}_t\hat{\Pi}_{Ht+1}\right) + \frac{1}{\sigma}\gamma\left(\mathbb{E}_t\hat{S}_{t+1} - \hat{S}_t\right) \tag{3.45}$$

and

$$\begin{aligned}\hat{\Pi}_{Ht} \;&=\; \beta\mathbb{E}_t\hat{\Pi}_{Ht+1} + \frac{(\varepsilon-1)(1-\tau^x)}{\omega}\left(\frac{\tau^x}{(1-\tau^x)}\hat{\tau}_t^x + \frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w\right.\\ &\quad \left.+\frac{\mu}{(1-\mu)}\left(\hat{\mu}_t - \hat{\lambda}_t\right) + \sigma\hat{C}_t - \hat{z}_t - \theta\hat{k}_t + (\theta+\psi)\,\hat{n}_t + \gamma\hat{S}_t\right)\end{aligned} \tag{3.46}$$

[13] Although non-zero steady state holdings of foreign assets seems to be the empirical case (see eg. Lane and Milesi-Ferretti (2002)) the simplification doesn´t alter our results.

The log-linearized Euler equation and the Phillips curve (PC) of the domestic country (see non-linear (3.30) and (3.5)) state that current consumption is positively related to future consumption, future inflation and future terms of trade and negatively related to current nominal interest rate and current terms of trade. Current inflation is positively related to future inflation, sales tax, income tax, Lagrange multiplier of the borrowing constraint, consumption, labor and terms of trade and negatively related to Lagrange multiplier of the budget constraint, productivity shock and capital.

We linearised the system of FOC's (3.25)-(3.29) and obtain the following equations

$$\begin{aligned}
\hat{\lambda}_t &= \mu\Xi\left(\hat{\mu}_t+\hat{\xi}_t\right)+\sigma\left(1-\mu\Xi\right)\hat{C}_t \\
&+\left(\beta\left(1-\mu\right)X\theta\frac{Y}{k}-\left(1-\mu\Xi\right)\right)\sigma\mathbb{E}_t\hat{C}_{t+1}+\left(1-\mu\Xi\right)\mathbb{E}_t\hat{\Pi}_{Ht+1} \\
&+\beta\left(\left(1-\mu\right)X\theta\frac{Y}{k}+\left(1-\delta\right)\right)\mathbb{E}_t\hat{\lambda}_{t+1}+\beta\left(1-\mu\right)X\theta\frac{Y}{k}\gamma\mathbb{E}_t\hat{S}_{t+1} \\
&+\beta\left(1-\mu\right)X\theta\frac{Y}{k}\left(\psi+1\right)\mathbb{E}_t\hat{n}_{t+1}-\beta\left(1-\mu\right)X\theta\frac{Y}{k}\hat{k}_{t+1}
\end{aligned} \tag{3.47}$$

$$\begin{aligned}
\mathbb{E}_t\hat{\lambda}_{t+1} &= \left(\mu\Xi+1\right)\left(\hat{\lambda}_t+\frac{i\tau}{R}\hat{\tau}_t\right)+\left(\mu\Xi-\frac{\left(\mu\Xi+1\right)\left(1-\tau\right)}{\beta R}\right)\hat{\imath}_t \\
&-\mu\Xi\left(\hat{\mu}_t+\hat{\xi}_t\right)-\sigma\hat{C}_t+\sigma\mathbb{E}_t\hat{C}_{t+1}
\end{aligned} \tag{3.48}$$

$$\hat{\lambda}_t=2\beta\kappa\left(\mathbb{E}_t\hat{d}_{t+1}-\hat{d}_t+\mathbb{E}_t\hat{\Pi}_{Ht+1}\right)-2\kappa\left(\hat{d}_t-\hat{d}_{t-1}+\hat{\Pi}_{Ht}\right) \tag{3.49}$$

$$\begin{aligned}
&\Xi\frac{k}{Y}\hat{k}_{t+1}-\beta\Xi\frac{a}{Y}\hat{a}_{t+1}+\beta\Xi\frac{a}{Y}\frac{1}{a}S^2\frac{\chi}{\chi^*}b_{t+1}-\beta\Xi\frac{a}{Y}\mathbb{E}_t\hat{\Pi}_{Ht+1} \\
=\;&-\beta\Xi\frac{a}{Y}\hat{\imath}_t-\Xi\left(\frac{k}{Y}-\beta\frac{a}{Y}\right)\hat{\xi}_t \\
&+X\left(\frac{\mu}{\left(1-\mu\right)}\left(\hat{\mu}_t-\hat{\lambda}_t\right)+\sigma\hat{C}_t+\frac{\tau^w}{\left(1-\tau^w\right)}\hat{\tau}^w_t+\left(1+\psi\right)\hat{n}_t+\gamma\hat{S}_t\right)
\end{aligned} \tag{3.50}$$

$$\begin{aligned}
-\frac{k}{Y}\hat{k}_{t+1} &= \left(w\frac{n}{Y}-X\right)\left(\sigma\hat{C}_t+\frac{\tau^w}{\left(1-\tau^w\right)}\hat{\tau}^w_t+\left(1+\psi\right)\hat{n}_t+\gamma\hat{S}_t\right) \\
&-X\frac{\mu}{\left(1-\mu\right)}\left(\hat{\mu}_t-\hat{\lambda}_t\right) \\
&+\frac{a}{Y}\hat{a}_t-\frac{\chi}{\chi^*}\frac{b_t}{Y}+\frac{a}{YR}\frac{\left(1+i\right)\left(1-\tau\right)}{R}\hat{\imath}_t-\frac{a}{YR}\frac{i\tau}{R}\hat{\tau}_t \\
&-\left(1-\delta\right)\frac{k}{Y}\hat{k}_t+\frac{d}{Y}\hat{d}_t-\left(\hat{a}_{t+1}-\frac{1}{a}\frac{\chi}{\chi^*}b_{t+1}+\mathbb{E}_t\hat{\Pi}_{Ht+1}\right)\frac{a}{YR}
\end{aligned} \tag{3.51}$$

(3.47) and (3.49) show that Lagrange multiplier λ positively related to credit shock, Lagrange multiplier μ, consumption, future inflation and negatively related to inflation and future consumption. The log-linearized aggregate demand equation is (see (3.44))

$$\hat{Y}_t=\frac{k}{Y}\hat{k}_{t+1}+\left(\frac{\chi}{\chi^*}+1\right)\beta\frac{b_{t+1}}{Y}-\left(\frac{\chi}{\chi^*}+1\right)\frac{b_t}{Y}+\frac{C}{Y}\left(\hat{C}_t+\gamma\hat{S}_t\right)-\left(1-\delta\right)\frac{k}{Y}\hat{k}_t+\frac{G}{Y}\hat{G}_t \tag{3.52}$$

and states that domestic output is positively related to future domestic capital, domestic consumption, terms of trade, domestic government spending and future foreign bonds but negatively to improvements in the foreign bonds and domestic capital.

Due to the cost in trading foreign bonds the uncovered interest rate parity condition is not valid anymore (see (3.9))

$$\hat{\imath}_t = \hat{\imath}_t^* + \mathbb{E}_t\hat{\Pi}_{Ht+1} - \mathbb{E}_t\hat{\Pi}_{Ft+1}^* + \mathbb{E}_t\hat{S}_{t+1} - \hat{S}_t - \chi b_{t+1} \tag{3.53}$$

There is a time varying risk-premium that depends on both the net future foreign asset position of the country b_{t+1} and costs of changing the asset holdings χ. This risk premium could be positive or negative depending on the Home country being a borrower or a lender in the international assets market.

From the resource constraint we get the following extra equation

$$\begin{aligned} 0 \quad = \quad & \gamma\frac{C}{Y}\left(\left((1-\eta)(1-\gamma)+\gamma\right)\hat{S}_t + \hat{C}_t\right) - \gamma^*\frac{C^*}{Y^*}\frac{Y^*}{Y}\left(\eta\left(1-\gamma^*\right)\hat{S}_t + \hat{C}_t^*\right) \\ & + \left(\frac{\chi}{\chi^*}+1\right)\beta\frac{b_{t+1}}{Y} - \left(\frac{\chi}{\chi^*}+1\right)\frac{b_t}{Y} \end{aligned} \tag{3.54}$$

where $R = 1 + i(1-\tau)$.

For the other country the corresponding equations are

$$\hat{C}_t^* = \mathbb{E}_t\hat{C}_{t+1}^* - \frac{1}{\sigma}\left(\hat{\imath}_t^* - \mathbb{E}_t\hat{\Pi}_{Ft+1}^*\right) - \frac{1}{\sigma}\gamma^*\left(\mathbb{E}_t\hat{S}_{t+1} - \hat{S}_t\right) \tag{3.55}$$

$$\begin{aligned} \hat{\Pi}_{Ft}^* \quad = \quad & \beta\mathbb{E}_t\hat{\Pi}_{Ft+1}^* + \frac{(\varepsilon-1)\left(1-\tau^{*x}\right)}{\omega}\left(\frac{\tau^{*x}}{\left(1-\tau^{*x}\right)}\hat{\tau}_t^{*x} + \frac{\mu^*}{\left(1-\mu^*\right)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right)\right. \\ & \left. + (\psi+\theta)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{\left(1-\tau^{*w}\right)}\hat{\tau}_t^{*w} - \hat{z}_t^* - \theta\hat{k}_t^* - \gamma^*\hat{S}_t\right) \end{aligned} \tag{3.56}$$

$$\begin{aligned} \hat{\lambda}_t^* \quad = \quad & \Xi^*\mu^*\left(\hat{\mu}_t^* + \hat{\xi}_t^*\right) + \left(1-\mu^*\Xi^*\right)\sigma\hat{C}_t^* \\ & + \left(\beta\left(1-\mu^*\right)X^*\theta\frac{Y^*}{k^*} - \left(1-\mu^*\Xi^*\right)\right)\sigma\mathbb{E}_t\hat{C}_{t+1}^* + \left(1-\mu^*\Xi^*\right)\mathbb{E}_t\hat{\Pi}_{Ft+1}^* \\ & + \beta\left(\left(1-\mu^*\right)X^*\theta\frac{Y^*}{k^*} + (1-\delta)\right)\mathbb{E}_t\hat{\lambda}_{t+1}^* \\ & + \beta\left(1-\mu^*\right)X^*\theta\frac{Y^*}{k^*}\left((\psi+1)\,\mathbb{E}_t\hat{n}_{t+1}^* - \gamma^*\mathbb{E}_t\hat{S}_{t+1} - \hat{k}_{t+1}^*\right) \end{aligned} \tag{3.57}$$

$$\begin{aligned} \mathbb{E}_t\hat{\lambda}_{t+1}^* \quad = \quad & \left(\mu^*\Xi^*+1\right)\left(\hat{\lambda}_t^* + \frac{i^*\tau^*}{\left(1+\left(1-\tau^*\right)i^*\right)}\hat{\tau}_t^*\right) + \left(\mu^*\Xi^* - \frac{\left(\mu^*\Xi^*+1\right)\left(1-\tau^*\right)}{\beta\left(1+\left(1-\tau^*\right)i^*\right)}\right)\hat{\imath}_t^* \\ & - \mu^*\Xi^*\left(\hat{\mu}_t^* + \hat{\xi}_t^*\right) - \sigma\hat{C}_t^* + \sigma\mathbb{E}_t\hat{C}_{t+1}^* \end{aligned} \tag{3.58}$$

$$\hat{\lambda}_t^* = 2\beta\kappa^*\left(\mathbb{E}_t\hat{d}_{t+1}^* - \hat{d}_t^* + \mathbb{E}_t\hat{\Pi}_{Ft+1}^*\right) - 2\kappa^*\left(\hat{d}_t^* - \hat{d}_{t-1}^* + \hat{\Pi}_{Ft}^*\right) \tag{3.59}$$

$$\begin{aligned}\Xi^*\frac{k^*}{Y^*}\hat{k}_{t+1}^* &= \beta\Xi^*\frac{b^*}{Y^*}\left(\frac{1}{b^*}b_{t+1} + \hat{b}_{t+1}^* + \mathbb{E}_t\hat{\Pi}_{Ft+1}^*\right) - \beta\Xi^*\frac{b^*}{Y^*}\hat{\imath}_t^* - \Xi^*\left(\frac{k^*}{Y^*} - \beta\frac{b^*}{Y^*}\right)\hat{\xi}_t^* \\ &\quad + X^*\left(\frac{\mu^*}{(1-\mu^*)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right) + (\psi+1)\,\hat{n}_t^* + \sigma\hat{C}_t^* - \gamma^*\hat{S}_t\right) \end{aligned} \tag{3.60}$$

$$\begin{aligned}-\frac{k^*}{Y^*}\hat{k}_{t+1}^* &= \left(w^*\frac{N^*}{Y^*} - X^*\right)\left((\psi+1)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w} - \gamma^*\hat{S}_t\right) \\ &\quad - \frac{b^*}{R^*Y^*}\left(\hat{b}_{t+1}^* + \frac{b_{t+1}}{b^*} + \mathbb{E}_t\hat{\Pi}_{Ft+1}^*\right) - (1-\delta)\frac{k^*}{Y^*}\hat{k}_t^* + \frac{d^*}{Y^*}\hat{d}_t^* \\ &\quad + \frac{b^*}{R^*Y^*}\frac{(1+i^*)(1-\tau^*)}{R^*}\hat{\imath}_t^* - \frac{b^*}{R^*Y^*}\frac{i^*\tau^*}{R^*}\hat{\tau}_t^* \\ &\quad - X^*\frac{\mu^*}{(1-\mu^*)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right) + \frac{b^*}{Y^*}\left(\frac{b_t}{b^*} + \hat{b}_t^*\right) \end{aligned} \tag{3.61}$$

$$\begin{aligned}\hat{Y}_t^* &= \frac{k^*}{Y^*}\hat{k}_{t+1}^* - \left(\frac{\chi}{\chi^*} + 1\right)\beta\frac{b_{t+1}}{Y^*} + \left(\frac{\chi}{\chi^*} + 1\right)\frac{b_t}{Y^*} + \frac{C^*}{Y^*}\left(\hat{C}_t^* - \gamma^*\hat{S}_t\right) \\ &\quad - (1-\delta)\frac{k^*}{Y^*}\hat{k}_t^* + \frac{G^*}{Y^*}\hat{G}_t^* \end{aligned} \tag{3.62}$$

$$\hat{\imath}_t^* = \hat{\imath}_t + \mathbb{E}_t\hat{\Pi}_{Ft+1}^* - \mathbb{E}_t\hat{\Pi}_{Ht+1} - \mathbb{E}_t\hat{S}_{t+1} + \hat{S}_t - \chi^* a_{t+1}^* \tag{3.63}$$

$$\begin{aligned}0 &= \gamma\frac{C}{Y}\left(((1-\eta)(1-\gamma)+\gamma)\,\hat{S}_t + \hat{C}_t\right) - \gamma^*\frac{C^*}{Y^*}\frac{Y^*}{Y}\left(\eta(1-\gamma^*)\,\hat{S}_t + \hat{C}_t^*\right) \\ &\quad + \left(\frac{\chi}{\chi^*} + 1\right)\beta\frac{b_{t+1}}{Y} - \left(\frac{\chi}{\chi^*} + 1\right)\frac{b_t}{Y} \end{aligned} \tag{3.64}$$

where $R^* = 1 + i^*(1-\tau^*)$. The linearisation of equations and steady states are given in Appendix B.5 and B.4.

3.4 Calibration

The model is calibrated to a quarterly frequency.[14] We fix $\beta = 0.9825$. The capital depreciation rate is set to $\delta = 0.025$. The capital ratio in production function is set to $\theta = 0.36$, and the mean value of A is normalized to 1. The tax wedge which corresponds to the advantage of debt over equity is determined to be $\tau = 0.35$, and the dividend adjustment cost parameter set to $\kappa = 0.146$ as in Jermann and Quadrini (2012).

[14] Note that both countries are symmetric and we only include calibration of Home country.

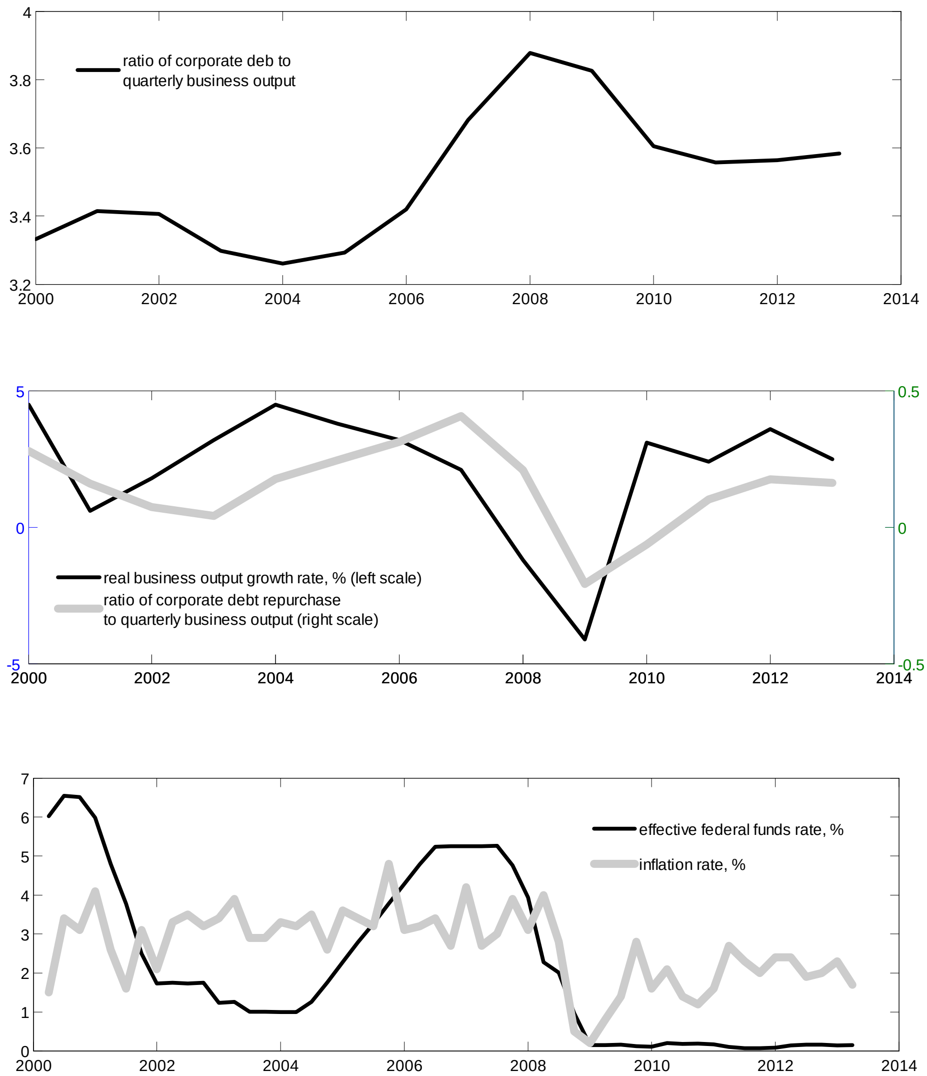


Figure 3.1: Historical data in the US. Data sources: NIPA and FoF tables.

We calibrate the steady state debt to output ratio to match the data. The quarterly ratio of debt to output for the non-financial business sector is 3.25 over the sample period 1984:I-2010:II, see the top panel in Figure 3.1. In order to match that, we set the steady state value of the financial variable, Ξ , to 0.1634.[15]

Parameters of the household utility function are determined as follows. The calibration of the Frisch intertemporal elasticity of substitution in labor supply, ψ, is assumed to be equal to 1 and the risk aversion parameter is: $\sigma = 1$. The relative weight on the disutility of labour, $\alpha = 1.8834$, is chosen so as to set steady state hours worked equal to 0.3.

We calibrate the measure of price stickiness, $\omega = 80$, in a way that corresponds to a probability of firms changing prices every 3 quarters in a corresponding Calvo model. The elasticity of substitution between any pair of goods ε is equal to 11 in steady state which gives a 10% mark up.

Parameters of the policy objective function are chosen to be $\vartheta_y = 0.3$, and $\vartheta_s = 0.1$, see Chen et al. (2014).[16]

It remains to calibrate the shock and the initial states to simulate the scenarios of interest. The second panel in Figure 3.1 plots the historical data of corporate debt to output ratio (quarterly). The average value of this ratio during 1984-2009 is 3.25. The peak of 3.87 in 2008 was somewhat above the average value, and the consequent reduction to 3.55 in 2011 constitutes a reduction of about 10% relative to its peak. We use these numbers as a guide to our simulations.

Based on this evidence, we consider an AR(1) credit shock $\hat{\xi}_t = \rho\hat{\xi}_{t-1} + \varepsilon_t$ with persistence $\rho = 0.95$, and we examine the dynamic implications of a negative 10% innovation in ε_t.

The intertemporal elasticity of substitution between domestic and foreign goods η is set to 2, which lies in the values assumed in the RBC literature $(1 - 2)$ and the degree of trade openness ϖ and ϖ^* are set to 0.3. The relative size of each country is calibrated depending on the nature of the analysis. In the main case of interest, labelled base line case, with 'large foreign' and 'small home' countries the Home country has size $n = 0.3$, as the relative size in terms of population or employment of Greece, Ireland, Italy and Portugal is about one third of the total population of the EMU countries. However, we also consider two identical countries with $n = 0.5$ when we discuss transmission mechanisms. As in Benigno (2009), Ghironi et al. (2006), we assume that the costs of changing the asset holdings with respect to the steady state are such that $\chi = \chi^* = 0.01$.[17]

[15] Data sources: NIPA and FoF tables. The calculations follow Jermann and Quadrini (2012).

[16] The results are very robust to wide range of parameters ϑ_y and ϑ_s between zero and one.

[17] Kollmann (2003) has used Lane and Milesi-Ferretti (2002) estimates on the relationship between real interest rate differentials and net foreign asset position. He assumes a value of 0.0019 in a case in which

3.5 Social Objectives

We assume the following world-wide social welfare function:

$$U = \sum_{t=0}^{\infty} \beta^t V_t \tag{3.65}$$

with the flow objective

$$V_t = n\hat{\Pi}_{Ht}^2 + n\vartheta_y \hat{Y}_t^2 + (1-n)\,\hat{\Pi}_{Ft}^{*2} + (1-n)\,\vartheta_y \hat{Y}_t^{*2} + \vartheta_s \hat{S}_t^2 \tag{3.66}$$

We, therefore, assume that volatilities of inflation, output and the terms of trade are costly for the society. We assume this *ad hoc* objective but we also check robustness of our results to a change of coefficients.

Objective function in this form was used in Benigno and Benigno (2006) and Clarida et al. (2002) and Corsetti et al. (2011).

3.6 Policy set-up

In this section we describe the policy set-up. Particularly, we demonstrate how a financial shock in a country could generate volatility in other country's macroeconomic variables. Given the loss function (equation (3.66)) we solve for the linear–quadratic optimal policy problem. The shock produces a dynamic path of deleveraging which depends endogenously on policy. We study the effects of cooperative monetary policy intervention. We assume that policymakers act under commitment.

The sequence of events and actions within a period is as follows. At the beginning of every period t, the state $\hat{a}_t, \hat{b}_t^*, \hat{b}_t, \hat{k}_t, \hat{k}_t^*, \hat{d}_t$ and $\hat{d}_t^*$ are known and shock $\hat{z}_t, \hat{z}_t^*, \hat{\xi}_t, \hat{\xi}_t^*$ realizes. Then the two policymakers choose the value of $\hat{\imath}_t$ and $\hat{\imath}_t^*$ cooperatively. There is a particular move, when policymakers maximise the world's objective and there are two instruments ($\hat{\imath}_t$, $\hat{\imath}_t^*$). After the policymakers have moved, in the next stage the private sector simultaneously adjusts its choice variables $\hat{\Pi}_{Ht}, \hat{\Pi}_{Ft}^*$ and $\hat{C}_t, \hat{C}_t^*$. The optimal $\hat{\Pi}_{Ht}, \hat{\Pi}_{Ft}^*$,$\hat{C}_t, \hat{C}_t^*$ and policy $\hat{\imath}_t$, $\hat{\imath}_t^*$ result in the new level of $\hat{a}_{t+1}, \hat{b}_{t+1}^*, \hat{b}_{t+1}, \hat{k}_{t+1}, \hat{k}_{t+1}^*, \hat{d}_{t+1}$ and $\hat{d}_{t+1}^*$ by the beginning of the next period $t+1$.

the net foreign asset position is normalized by exports. In our case, since the net foreign asset position is normalized by quarterly GDP, with an export/GDP ratio of 15%, a value of 0.0019 implies a value for χ equal to 0.012, which is consistent the calibration that we use.

3.7 Results

3.7.1 Transmission mechanism

In this section, we examine the transmission mechanism of asymmetric credit shocks. We start with two identical countries ($n = 0.5$), characterized by the same steady state intermediation cost, $\chi = \chi^* = 0.01$. Next we discuss the country-size effects. We perform some robustness analysis along different assumptions on $\chi = \chi^*$. Since our welfare objective function is not micro-founded, we also check the robustness of our results using different values for output and terms of trade coefficients (ϑ_y and ϑ_s). We discuss the welfare consequences of our results as well.

Country size symmetry

In Figure (3.2), we observe the impulse responses of both equal size economies ($n = 0.5$) to a negative 10% credit shock in Home country and a positive 10% credit shock in foreign country. Such negative shock in Home country, reduces the proportion of Home output which banks will be able to recover in case of default. Banks lend to domestic firms at the beginning of the period, so that domestic firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the negative credit shock reduces the probability of recovery in Home country, the amount of bank lending falls. Domestic firms which are not able to obtain funds up front have to deleverage or reduce the value of production. Domestic firms reduce labour, produce less output and also pay lower wages, see Figure (3.2). The equilibrium prices of Home goods fall as a result of lower income and lower demand. Domestic firms reduce the amount of borrowing. In response to lower inflation and output the central bank reduces the nominal interest rate. Initial reduction of consumption is too low to compensate the large initial reduction in output. Domestic households sell assets to finance consumption. Besides, Home goods are now cheaper and foreigners want to buy them. As a result of inflow of funds, Home currency appreciates (see also equation (3.45) and equation (3.53)). Lower interest rate also makes it easier for banks to pay out the existing debt, so it helps to reduce the debt quickly. Output falls by less than wages, profits of firms fall and so dividends fall. Domestic firm's budget constraint becomes tighter but its enforcement constraint is looser at the initial moment then it gets tighter too gradually over time, see equation (3.49) and equation (3.46). The adjustment goes with overshooting as capital changes only slowly and it continues to fall while the effect of persistent shock disappears.

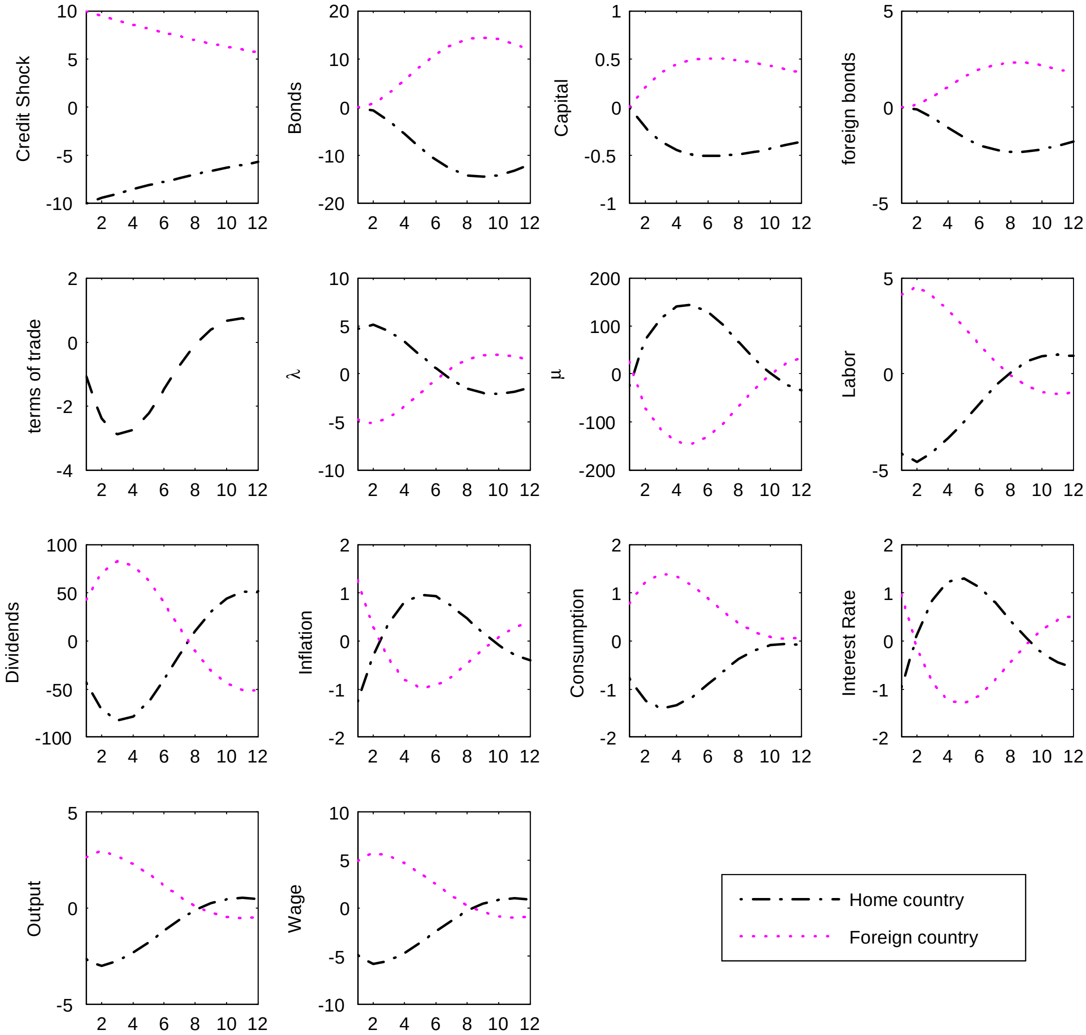


Figure 3.2: Impulse responses of symmetric size countries to an asymmetric credit shock

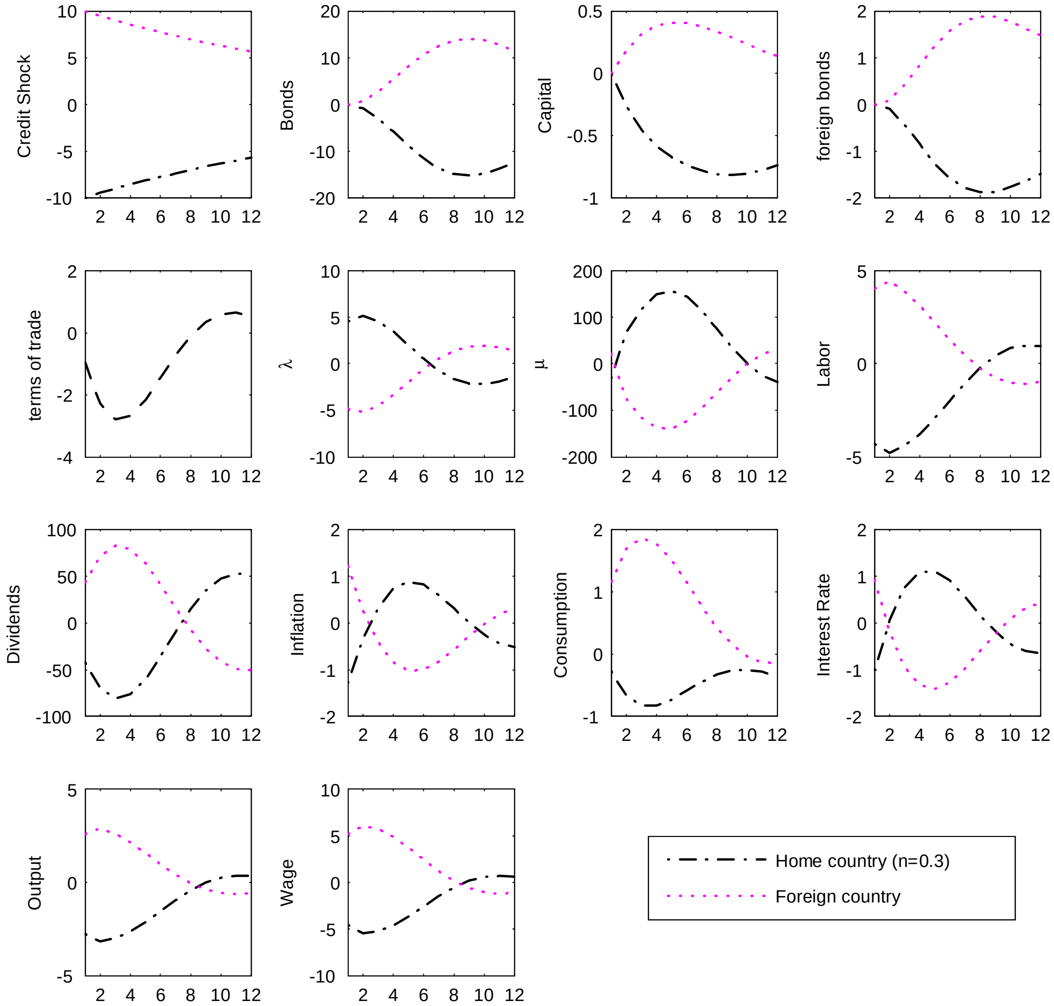


Figure 3.3: Impulse responses of asymmetric size countries to an asymmetric credit shock

Country size asymmetry

In Figure (3.3), we observe the impulse responses of two different size economies ($n = 0.3$) to a negative 10% credit shock in Home country and a positive 10% credit shock in foreign country. In what follows we compare our results with the same scenario assuming two identical countries (see Figure (3.2)). Figure (3.3) shows that country-size do not have any particular effect on any macroeconomic variable except on consumption. Domestic import share ($\gamma = (1-n)\varpi$) is higher for smaller n, and because Home inflation, interest rate and terms of trade are almost unchanged then Home consumption will be higher (see equation (3.45)). And for the same reason, smaller $\gamma^* = n\varpi^*$, foreign consumption is smaller. There are also small changes on foreign assets and terms of trade that we explain them as follows: To finance higher level of consumption, domestic households need to sell assets and foreign assets fall by more. Besides, higher import share, γ, indicates that foreign goods are cheaper (see equation (3.46)). Then as a result of higher import and outflow of funds, Home currency will appreciate by less.

Home welfare loss is slightly greater than the foreign welfare loss (see Table 3.1). The small H country maximises the world objective welfare, but there is a large relative weight to the objectives of the large foreign country. Table 3.1 shows that, welfare relevant variables at Home display higher volatility. Table 3.1 shows that in different size countries, smaller country suffers a bigger loss. In contrast, bigger country and the world benefit a lower level of welfare loss.

Variance	$n = 0.5$	$n = 0.3$
$\hat{Y}_t \times 10^2$	0.62	1.17
$\hat{Y}_t^* \times 10^2$	0.62	0.43
$\hat{\Pi}_{Ht} \times 10^2$	0.13	0.22
$\hat{\Pi}_{Ft}^* \times 10^2$	0.13	0.10
$\hat{S}_t \times 10^2$	0.17	0.16
Loss (W)$\times 10^2$	0.34	0.34
Loss (H)$\times 10^2$	0.34	0.58
Loss (F)$\times 10^2$	0.34	0.24

Table 3.1: Volatilities of output, inflation, terms of trade and unconditional welfare loss due to a credit shock assuming two symmetric (second column) and asymmetric (third column) size countries

3.7.2 Robustness analysis

How do the results change under different intermediation cost, χ? We address this question through different assumptions on χ.

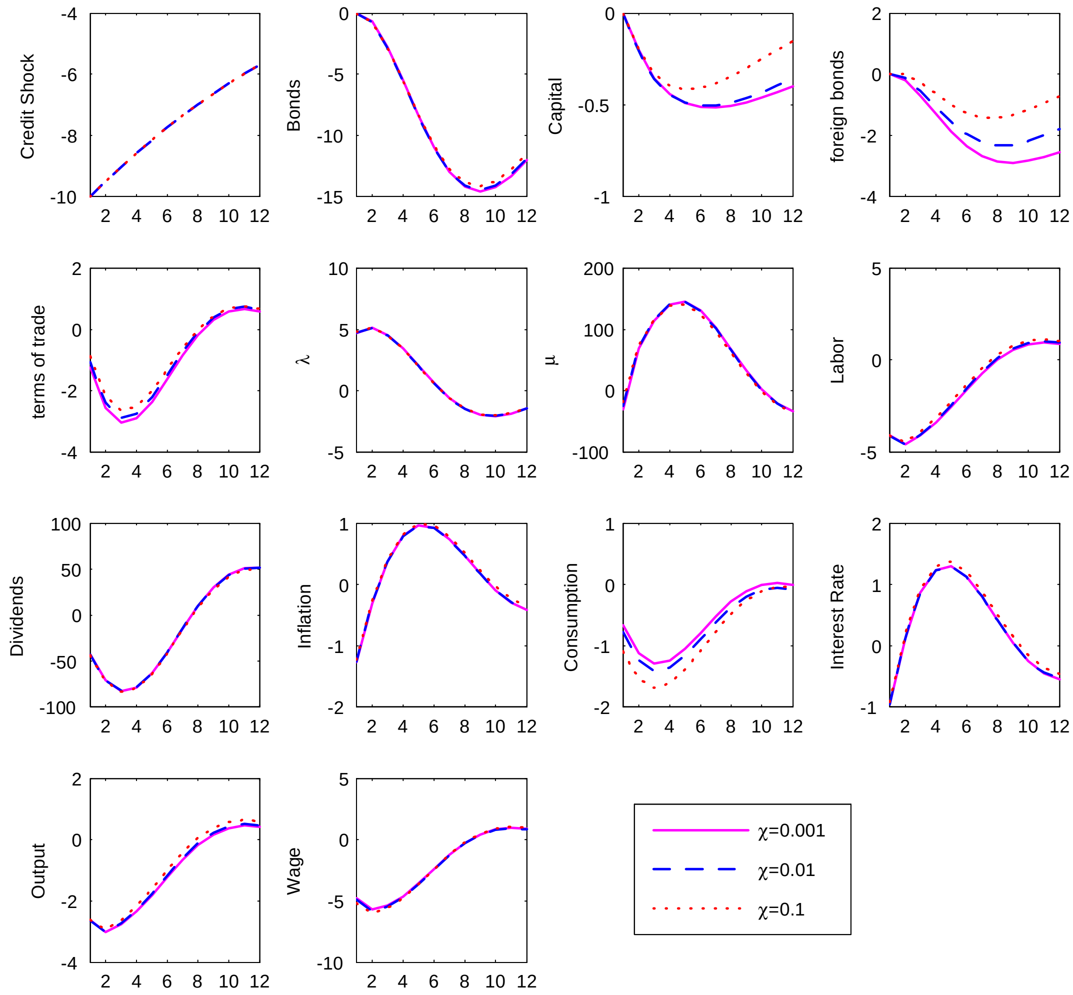


Figure 3.4: Robustness analysis for different value of $\chi = \chi^*$ under asymmetric credit shock.

In Figure (3.4), we observe the impulse responses of both equal size economies ($n = 0.5$) to a negative 10% credit shock in Home country and a positive 10% credit shock in foreign country using different χ's, and how optimal policy changes for alternative values of χ around the benchmark value of 0.01.[18][19] Namely we plot impulse responses under optimal policy for $\chi = \chi^* = 0.001$, 0.01 and 0.1. A value of χ below 0.01 leads to greater volatility of home output which is costly, but also helps to deleverage faster. Output falls by more in the Home country because the terms of trade (P_{Ft}/P_{Ht}) deviates by more. Instead, for a higher $\chi = 0.1$, the real income is less volatile. As we see in Figure (3.4) and Table (3.2), bigger χ corresponds to smaller deviation of terms of trade, and output. Deviation of inflation is almost the same for all three alternative value of χ. Therefore, welfare loss falls as χ rises.

By assuming two different size countries, ($n = 0.3$), welfare loss is the highest when $\chi = 0.001$ and it is the lowest when χ is the benchmark level 0.01.

To summarise, welfare is maximised for an intermediate value of degree of financial integration. If the intermediation cost is absent, there is large volatility of output during the period of adjustment, while the deleveraging is performed faster. With greater costs on international financial flows, the deleveraging is substantially slowed down which leads to longer periods of adjustment and greater costs.

Variance ($n = 0.5$)	$\chi = 0.001$	$\chi = 0.01$	$\chi = 0.1$
$\hat{Y}_t \times 10^2$	0.6200	0.6200	0.6000
$\hat{Y}_t^* \times 10^2$	0.6200	0.6200	0.6000
$\hat{\Pi}_{Ht} \times 10^2$	0.1332	0.1322	0.1334
$\hat{\Pi}_{Ft}^* \times 10^2$	0.1332	0.1322	0.1334
$\hat{S}_t \times 10^2$	0.1900	0.1700	0.1400
Loss (W)$\times 10^2$	0.3400	0.3400	0.3300
Loss (H)$\times 10^2$	0.3400	0.3400	0.3300
Loss (F)$\times 10^2$	0.3400	0.3400	0.3300

Table 3.2: Volatilities of output, inflation, terms of trade and unconditional welfare loss for different value of intermediation cost due to a credit shock assuming two symmetric size countries

Figure (3.5) shows how optimal policy changes for alternative values of ϑ_y and ϑ_s around the benchmark value of $\vartheta_y = 0.3$ and $\vartheta_s = 0.1$. We assumed these coefficients were not microfounded, so it is useful to investigate if they play any important role in our results. When deviation of output penalised by less (smaller ϑ_y, dash line): comparing to the benchmark (dash-dot line), labor, capital and output fall by more because the penalty on output deviation is lower. Consumption is higher and Home firms are more

[18] Note that Foreign responses are completely asymmetric to Domestic responses, so we skip adding Foreign responses into this Figure.

[19] Note that we assume $\chi = \chi^*$.

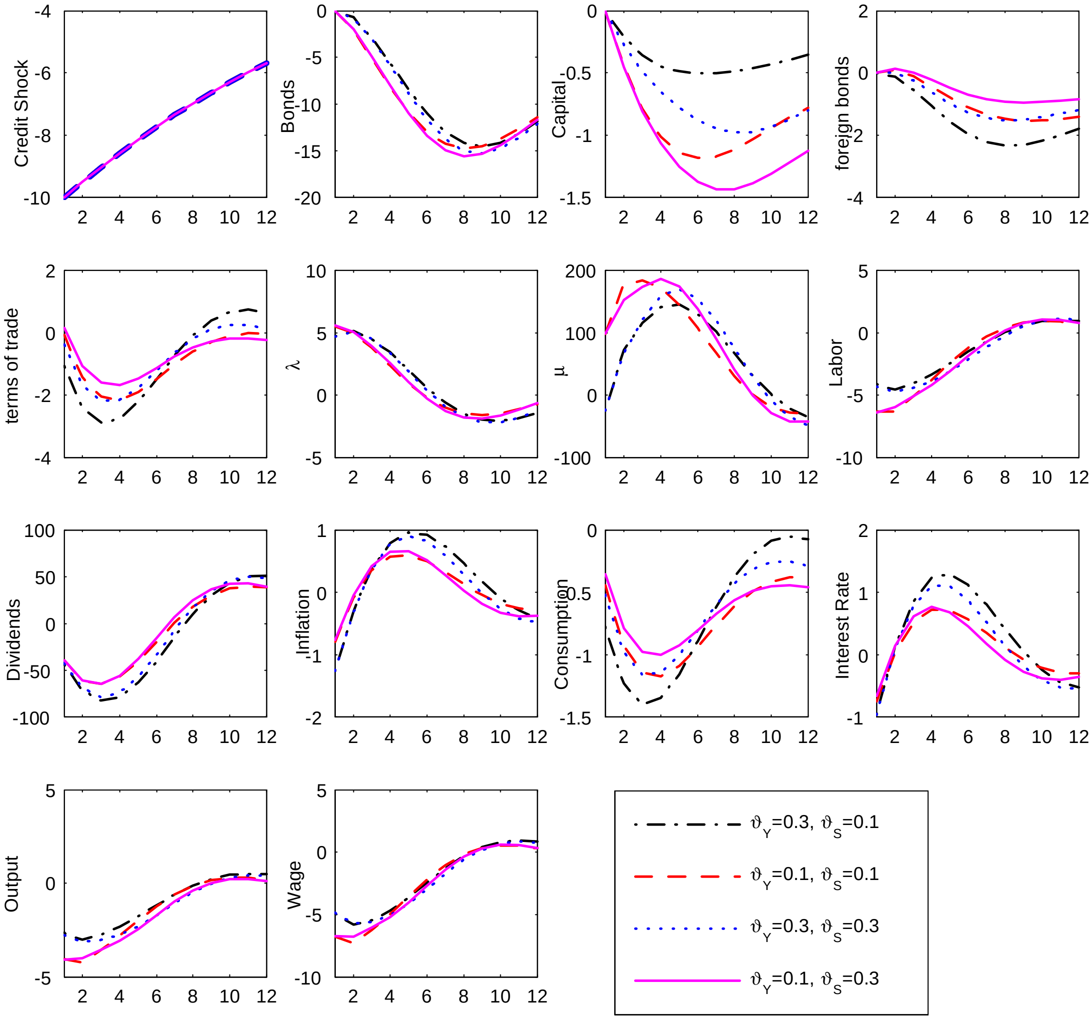


Figure 3.5: Robustness analysis for diffenet values of ϑ_y and ϑ_s under asymmetric credit shock

Variance ($n = 0.3$)	$\chi = 0.001$	$\chi = 0.01$	$\chi = 0.1$
$\hat{Y}_t \times 10^2$	1.1667	1.1667	1.0667
$\hat{Y}^*_t \times 10^2$	0.4286	0.4286	0.4143
$\hat{\Pi}_{Ht} \times 10^2$	0.2177	0.2150	0.2147
$\hat{\Pi}^*_{Ft} \times 10^2$	0.0973	0.0968	0.0974
$\hat{S}_t \times 10^2$	0.1800	0.1600	0.1300
Loss (W)$\times 10^2$	0.3500	0.3400	0.3300
Loss (H)$\times 10^2$	0.5900	0.5800	0.5500
Loss (F)$\times 10^2$	0.2400	0.2400	0.2300

Table 3.3: Volatilities of output, inflation, terms of trade and unconditional welfare loss for different value of intermediation cost due to a credit shock assuming two asymmetric size (n=0.3) countries

interested in foreign bonds because of their higher rate of interest. Then foreign bonds rises and as a result of outflow of funds, Home currency depreciated. Then inflation falls by less (see equation (3.45) and equation (3.53)). When deviation of terms of trade is penalised by more (higher ϑ_s, dot line): Since terms of trade's movement is penalised by more, it falls by less comparing to the benchmark case. It corresponds to the higher level of foreign assets and outflow of funds. Output deviation is the same as the benchmark and consumption is higher. As a result, investment on capital must fall (see equation (3.52)) and welfare loss has its biggest value. Now when we have both effects together (smaller ϑ_y and larger ϑ_s, solid line): Again output falls by more, so does capital and labor. Because of both effects, terms of trade rises to its highest level. Then foreign bonds rises too. Consumption rises and so does the inflation (see equation (3.45) and equation (3.53)). Enforcement constraint is tighter, because of buying higher level of foreign bonds. Budget constraint is tighter because of lower production and employment level.

To summarise, welfare is minimised for the higher volatility of output but lower volatility of inflation and terms of trade. If the penalty on output deviation is absent, there is large volatility of output, while inflation volatility and welfare fall. With greater penalty on terms of trade volatility, its volatility and welfare fall as well.

3.7.3 Contagion

Finally in a two country model under flexible exchange rate and independent monetary authorities, we demonstrate the effect of one country's credit shock on another country.

Figure (3.6) shows that a negative shock in one country regardless of size, has no particular effect on macroeconomic variables of other country except a small change in consumption, and foreign bonds. Foreign bonds in home country falls and foreign bonds

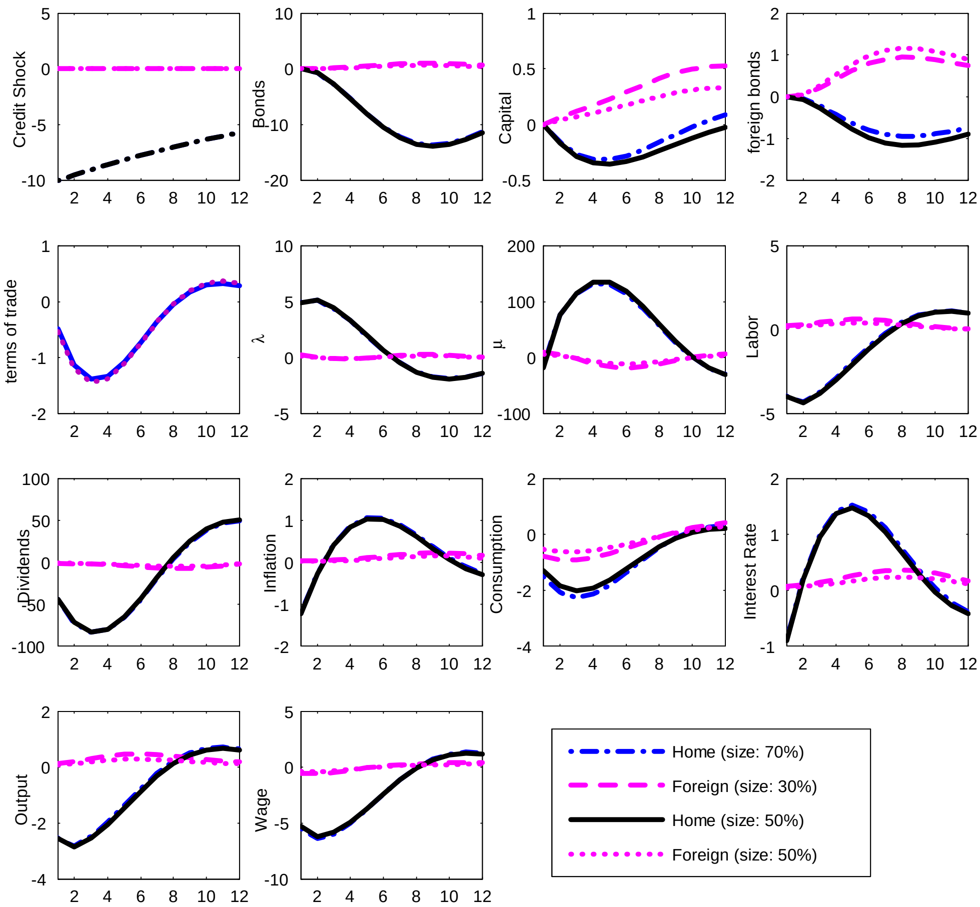


Figure 3.6: Impulse responses of symmetric vs asymmetric size countries to a non-symmetric credit shock

Variance ($\chi = 0.01$)	$\vartheta_y = 0.3$, $\vartheta_s = 0.1$	$\vartheta_y = 0.1$, $\vartheta_s = 0.1$	$\vartheta_y = 0.3$, $\vartheta_s = 0.3$	$\vartheta_y = 0.1$, $\vartheta_s = 0.3$
$\hat{Y}_t \times 10^2$	0.62	1.16	0.72	1.24
$\hat{Y}_t^* \times 10^2$	0.62	1.16	0.72	1.24
$\hat{\Pi}_{Ht} \times 10^2$	0.13	0.04	0.13	0.05
$\hat{\Pi}_{Ft}^* \times 10^2$	0.13	0.04	0.13	0.05
$\hat{S}_t \times 10^2$	0.17	0.098	0.097	0.07
Loss (W)$\times 10^2$	0.34	0.40	0.35	0.42
Loss (H)$\times 10^2$	0.34	0.40	0.35	0.42
Loss (F)$\times 10^2$	0.34	0.40	0.35	0.42

Table 3.4: Volatilities of output, inflation, terms of trade and unconditional welfare loss for different penalties on output and terms of trade deviations due to a credit shock assuming two symmetric size countries

in foreign country rises, then as a result of inflow of funds domestic currency appreciates (see equation (3.53)). Therefore foreign consumption falls (see equation (3.55)).

Foreign welfare loss is slightly greater than the Home welfare loss (see Table (3.1)). The small foreign country maximises the world objective welfare, but there is a large relative weight to the objectives of the large Home country.

To summarise, simulations demonstrate that in two country model under flexible exchange rate and independent monetary authorities, the effect of one country's credit shock has very limited effect on another country. When monetary policymakers cooperate and choose interest rate optimally, the unaffected country can nearly eliminate all aftereffects of the shock to the other country. To some extent, limited financial integration prevents the spread of volatility across the border, however, unconstrained monetary policy is the key to these results, as we shall see in the next chapter of the thesis.

3.8 Conclusion

In this chapter, we try to address several important factors that explain cross-country differences in the effects of the financial crisis. We analyse transmission of shocks from one country to another for two different cases: country size symmetry and asymmetry. Our results demonstrate that welfare is maximised for an intermediate value of degree of financial integration. If the intermediation cost is absent, there is large volatility of output during the period of adjustment, while the deleveraging is performed faster. With greater costs on international financial flows, the deleveraging is substantially slowed down which leads to longer periods of adjustment and greater costs. We demonstrate that in two country model under flexible exchange rate and independent monetary authorities, the effect of one country's credit shock has very limited effect on another country. When monetary policymakers cooperate and choose interest rate optimally, the unaffected country can nearly eliminate all aftereffects of the shock to the other country. To some extent, limited financial integration prevents the spread of volatility across the border, however, unconstrained monetary policy is the key to these results.

We also investigate if the country size matters for the severity of recessions. We show that country-size do not have any particular effect on any macroeconomic variable except on consumption. Smaller country suffers a bigger loss. One reason is the higher rate of import share for smaller country. Since the objective function is not microfounded, we assume variable degree of penalty on output and terms of trade deviations. We find that welfare is minimised for the higher volatility of output but lower volatility of inflation and terms of trade. If the penalty on output deviation is absent, there is large volatility of output, while inflation volatility and welfare fall. With greater penalty on terms of trade volatility, terms of trade volatility and welfare fall as well.

Chapter 4

Monetary Union: Fixed Exchange Rate Regime

4.1 Introduction

Financial crisis of 2007 has started as a subprime lending crisis, affecting one sector in one country. It quickly spread to many other countries. Arguably, the presence of fixed or semi-fixed exchange rate targeters, such as Thailand, Indonesia and South Korea, of countries locked into the European Monetary Union, played an important effect in amplification and spread of financial shocks.

In this chapter, we investigate the importance of exchange rate regime for international transmission of credit shocks. Specifically, we use the same two-country model as in the previous chapter, but assume that both countries are locked into a permanently fixed exchange rate regime within a currency union. We therefore ignore any issues of imperfect credibility of exchange rate pegs and do not discuss exchange rate crises. We, therefore, also assume that the monetary policymaker has a mandate to stabilise both countries' economies.

We demonstrate that, unlike under flexible exchange rate regime studied in the previous chapter, the centralised monetary policy alone is unable to stabilise the economy. National fiscal policies must be activated to counteract asymmetric shocks. We demonstrate, however, that the effectiveness of fiscal policy is nevertheless is limited. Even if it is chosen optimally, fiscal policy does not eliminate cyclical patterns in economic adjustment, which is welfare-reducing volatility of economic variables.

This model demonstrates that shocks hitting one economy, result in sharp contraction of consumption in another country. Countercyclical fiscal policy is able to avoid major recession, however. In contrast to results in the previous chapter, the shocks propagation mechanism is much stronger under fixed exchange rate regime. As before, we assume variable degree of financial integration and study its importance for the propagation of credit shocks.

This chapter is organized as follows. In the next section, we outline the model. Section 4.3 covers the linearised version of the system of equations. Section 4.4 describes calibration of the model. Section 4.5 discusses policy objectives. Section 4.6 discusses the results and section 4.7 concludes.

4.2 The Model

We present a simple two-country model with financial frictions and with incorporated nominal rigidities *a la* Rotemberg (1983). The world economy is populated by a continuum of agents on the interval of $[0;1]$. The population on the segment $[0;n)$ belongs to country H (Home), while the segment $[n;1]$ belongs to country F (Foreign). Each economy is populated by households and firms. Firms use labor and capital to produce differentiated goods. Firms issue equity and debt and use intra-period loans to finance working capital. Firms face credit restrictions due to uncertainty of recovering these loans. Preferences reflect home bias in consumption. The detailed model of the economy is presented in this section.

4.2.1 Law of One Price, The Terms of Trade and Relative Prices

We assume that the law of one price holds, implying $p_{Ft}(z) = E_t p^*_{Ft}(z)$, $p_{Ht}(z) = E_t p^*_{Ht}(z)$ for all $z \in [0,1]$ where $E_t = [H]/[F]$ is the nominal exchange rate, that is the price of foreign currency in terms of home currency, and $p^*_{Ft}(z)$ is the price of foreign good z denominated in foreign currency (Of course, the holding of one price does not imply that PPP holds, unless we assume the absence of home bias).[1] We define the terms of trade is the relative price of imported goods:

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

The real exchange rate – the ratio of CPI inflations, expressed in domestic currency – is defined as

$$\mathcal{Q}_t = \frac{E_t P^*_t}{P_t}.$$

[1] Let a "*" denote foreign variables.

4.2.2 Domestic Households

The home economy (H), is populated by a continuum of homogeneous infinitly-living households who share identical preferences and technology and maximise the expected lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, n_t)$, with aggregated period utility

$$U(C_t, n_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha \frac{n_t^{1+\psi}}{1+\psi} \tag{4.1}$$

where C_t is private home consumption, n_t is home labor, β is the discount factor, $\mathbb{E}_0$ is the actuarial expectation at time $t = 0$. Furthermore $\psi \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, α is a preference parameter. We assume home bias in consumption. In more detail, a composite consumption index, C_t, is defined as a Dixit-Stiglitz aggregator of the continuum of goods $i \in [0,1]$ produced in the foreign country and home[2]

$$C_t = \left((1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

Parameter $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods. Parameter $\gamma \in [0,1]$ is the weight of imported goods in private home consumption and is inversely related to the degree of home bias in preferences. Another interpretation for γ is as a natural index of openness or the import share. The import share depends on $(1-n)$ which is the relative size of foreign economy, and on ϖ which is the degree of trade openness. It yields $\gamma = (1-n)\varpi$. We assume home bias in consumption:

$$1 - \gamma = (1 - (1-n)\varpi) > \gamma^* = n\varpi^*$$

which implies

$$1 - n\varpi^* > (1-n)\varpi$$

Similarly, home bias in foreign preferences requires $1 - \gamma^* > \gamma$ which again implies

$$1 - n\varpi^* > (1-n)\varpi$$

C_{Ht} and C_{Ft} are domestic consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F given by the CES functions

$$C_{Ht} = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n c_{Ht}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}, \quad C_{Ft} = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 c_{Ft}(z)^{\frac{\epsilon-1}{\epsilon}} dz\right)^{\frac{\epsilon}{\epsilon-1}}$$

where each consumption bundle C_{Ht} and C_{Ft} is composed of imperfectly substitutable varieties of goods $z \in [0,1]$ produced within a given country with elasticity of substitution $\epsilon > 1$.

[2]The same relationship can be written in per capita terms.

The aggregated nominal intertemporal budget constraint at time t for household i belonging to country H is given by

$$\int_0^1 [P_{Ht}(z)C^i_{Ht}(z) + P_{Ft}(z)C^i_{Ft}(z)]dz + \frac{A^i_{t+1}}{1+i_t} + \frac{B^i_{t+1}E_t}{(1+i^*_t)\,\phi\left(\frac{E_tB_{t+1}}{P_t}\right)}$$
$$\leq \; A^i_t + B^i_tE_t + (1-\tau^w_t)\,W^i_tn^i_t + \left(\left(1-\tau^d_t\right)D_t + P_{St}\right)s_t - s_{t+1}P_{St} + T^i_t + P_{Ht}\Phi_t$$

$P_{Ht}(z)$ and $P_{Ft}(z)$ are price indices of domestic and foreign (imported from country F) goods z, where the latter is expressed in domestic currency. A_t is the one-period domestic corporate bond held by domestic households (real bonds, in terms of domestic prices), B_t is the one-period foreign corporate bonds held by domestic households, W^i_t is the nominal wage and T^i_t denotes lump-sum taxes/transfers. τ^w_t denotes a country specific tax on nominal income and E_t is the nominal exchange rate, given as the price of one of unit foreign currency in terms of home currency. $P_{Ht}\Phi_t$ is nominal profit from the ownership of capital-producing firms and retailers (note that $\Phi_t = \Phi^C_t + \Phi^R_t$). Here s_t is the domestic share of equity which is wholly owned by domestic households, D_t denotes the equity payout paid to the shareholders, P_{St} is the market price of domestic shares, τ^d_t denotes the tax on equity payout. We assume that the households share the revenues of owning firms in equal proportion. Following Woodford (2003) we consider a cashless economy. Therefore the only explicit role played by money is to serve as a unit of account.

We introduce incomplete financial markets as in Benigno (2009).[3] Domestic households hold domestic equity shares s_t and noncontingent bonds issued by firms of home and foreign countries. Households of country H can trade in two nominal one-period, risk-free bonds. Bonds A_t are issued by home firms and are denominated in home currency, bonds B_t are issued by foreign firms and are denominated in foreign currency. Households belonging to country H have to pay an intermediation cost, if they want to trade in the foreign bond. These costs are determined by the function $\phi(\cdot)$. Function $\phi(\cdot)$ depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by the domestic households. If a household belongs to a country which is in a 'borrowing position' ($B_{t+1} < 0$), it will be charged with a premium on the foreign interest rate and if the household belongs to a country which is in a 'lending position' ($B_{t+1} > 0$), it receives a rate of return lower than the foreign interest rate. Along with Benigno (2009) we need the following restrictions on $\phi(\cdot)$: $\phi(0) = 1$ and $\phi(\cdot)$ is 1 only if $B_t = 0$. Furthermore $\phi(\cdot)$ has to be a differentiable, decreasing function in the neighborhood of zero. $\phi'(0) = -\chi$.[4]

The intermediation profits $\digamma_t$ are defined analogous to Benigno (2009)

$$\digamma_t = \frac{B^i_{t+1}E_t}{(1+i^*_t)}\left(\frac{1}{\phi\left(\frac{E_tB_{t+1}}{P_t}\right)} - 1\right)$$

[3]See Benigno (2009) for a generalized asset trading framework, that follows Ghironi et al. (2006).
[4]We assume it is convex and can be approximated by $\phi(x) = 1 - \chi x + \bar{\chi}x^2$.

and shared equally among foreign households. The domestic budget constraint is then given as

$$\begin{aligned} & P_t C_t + s_{t+1} P_{St} + \frac{A_{t+1}^i}{1+i_t} + \frac{B_{t+1}^i E_t}{(1+i_t^*)} + \digamma_t \\ \leq \; & A_t^i + B_t^i E_t + \left(1-\tau_t^w\right) W_t^i n_t^i + \left(\left(1-\tau_t^d\right) D_t + P_{St}\right) s_t + T_t^i + P_{Ht}\Phi_t \end{aligned}$$

The optimal allocation within each variety of goods z yields per capita relationships

$$c_{Ht}(z) = \frac{1}{n}\left(\frac{p_{Ht}(z)}{P_{Ht}}\right)^{-\epsilon} C_{Ht}, \; c_{Ft}(z) = \frac{1}{1-n}\left(\frac{p_{Ft}(z)}{P_{Ft}}\right)^{-\epsilon} C_{Ft}$$

for all $z \in [0,1]$, where

$$P_{Ht} = \left(\frac{1}{n}\int_0^n p_{Ht}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}, \; P_{Ft} = \left(\frac{1}{1-n}\int_n^1 p_{Ft}(z)^{1-\epsilon} dz\right)^{\frac{1}{1-\epsilon}}$$

are the price indexes for domestic and imported goods, whereby the latter is expressed in domestic currency. Note as ϵ rises, the individual goods become closer substitutes and therefore the individual firms have less market power.

Finally, the optimal condition of expenditures between domestic and imported (foreign) bundles of goods is given by[5]

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t \text{ and } C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t$$

where z denotes the good's type or variety and $p_{Ht}(z), p_{Ft}(z)$ are prices of individual home and foreign produced goods.

where $P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, P_t is the consumer price index (CPI) in country H and P_{Ht}, P_{Ft} are domestic and foreign goods price indices. Note that if the economy is closed, $\gamma = 0$, the CPI equals domestic prices. Correspondingly we can write total consumption expenditures by domestic households as $P_t C_t = P_{Ht}C_{Ht} + P_{Ft}C_{Ft}$. The aggregated budget constraint can therefore be rewritten as

$$\begin{aligned} & \left(1-\tau_t^w\right) W_t^i n_t^i + A_t^i + B_t^i E_t + \left(\left(1-\tau_t^d\right) D_t + P_{St}\right) s_t + T_t^i + P_{Ht}\Phi_t \\ \geq \; & \frac{A_{t+1}^i}{1+i_t} + \frac{B_{t+1}^i E_t}{\left(1+i_t^*\right)\phi\left(\frac{E_t B_{t+1}}{P_t}\right)} + s_{t+1}P_{St} + P_t C_t \end{aligned} \tag{4.2}$$

We assume that all households in the same country have the same level of initial wealth. As they face the same labour demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use individual index within each country.

[5]The same relationship can be written in per capita terms.

We maximize equation (4.1) with respect to equation (4.2) and arrive to the following system

$$\frac{U_{C,t}}{P_t} = \Delta_t \tag{4.3}$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{C,t}\left(1-\tau_t^w\right)} \tag{4.4}$$

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \tag{4.5}$$

$$\frac{1}{1+i_t^*} = \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \frac{E_{t+1}}{E_t} \phi\left(\frac{E_t B_{t+1}}{P_t}\right) \tag{4.6}$$

$$P_{St} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{\Pi_{Ht+1} C_t^{-\sigma}} \left(\left(1-\tau_{t+1}^d\right) D_{t+1} + P_{St+1}\right) \tag{4.7}$$

$$\begin{aligned} & \left(1-\tau_t^w\right) W_t n_t + A_t + B_t E_t + \left(\left(1-\tau_t^d\right) D_t + P_{St}\right) s_t + T_t \\ = \; & \frac{A_{t+1}}{1+i_t} + \frac{B_{t+1} E_t}{\left(1+i_t^*\right) \phi\left(\frac{E_t B_{F,t+1}}{P_t}\right)} + s_{t+1} P_{St} + P_t C_t \end{aligned} \tag{4.8}$$

All derivations are given in Appendix B.1. Equation (4.5) is the standard Euler equation and determines the consumption smoothing behavior of the households. Equation (4.6) is the Euler equation derived from the optimal choice of the foreign bond. equation (4.4) is the standard labour supply condition. It determines the quantity of labor supplied as a function of real wage, given the marginal utility of consumption. Finally equation (4.8) is the aggregate budget constraint.

The incomplete financial market framework generates deviations from the uncovered interest parity (UIP). Combining equation (4.5) and equation (4.6) yields the optimal portfolio choice of the households of country H

$$\left(1+i_t\right) = \mathbb{E}_t \left(1+i_t^*\right) \frac{E_{t+1}}{E_t} \phi\left(\frac{E_t B_{t+1}}{P_t}\right) \tag{4.9}$$

$\phi\left(\frac{E_t B_{t+1}}{P_t}\right)$ can also be interpreted as a risk premium term on the interest rate. If the economy is a net debtor, the domestic interest rate is above the foreign interest rate and if the economy is a net creditor the domestic interest rate is below the foreign interest rate. Therefore movements in the net foreign asset positions affect the interest differential between the two countries.

4.2.3 Foreign Households

Similarly the foreign economy is populated by a continuum of homogeneous infinitly-living households who share identical preferences and technology and maximise the expected lifetime utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^*, n_t^*\right)$, with aggregated period utility

$$U\left(C_t^*, n_t^*\right) = \frac{C_t^{*1-\sigma}}{1-\sigma} - \alpha \frac{n_t^{*1+\psi}}{1+\psi} \tag{4.10}$$

where C_t^* is private foreign consumption, n_t^* is foreign labor, β is the discount factor, $\mathbb{E}_0$ is the actuarial expectation at time $t = 0$. Furthermore $\psi \geqslant 0$ measures the labor supply elasticity, $\sigma \geqslant 0$ measures the elasticity of consumption, α is a preference parameter. We assume home bias in consumption. In more detail, a composite consumption index, C_t^*, is defined as a Dixit-Stiglitz aggregator of the continuum of goods $i \in [0,1]$ produced in the foreign country and home[6]

$$C_t^* = \left((1-\gamma^*)^{\frac{1}{\eta}} C_{Ft}^{*\frac{\eta-1}{\eta}} + \gamma^{*\frac{1}{\eta}} C_{Ht}^{*\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

Parameter $\eta > 0$ is the intratemporal elasticity of substitution between home and foreign consumption goods. Parameter $\gamma^* \in [0,1]$ is the weight of foreign imported goods in private foreign consumption and is inversely related to the degree of foreign bias in preferences. Another interpretation for γ^* is as a natural index of foreign openness or the foreign import share. The foreign import share depends on n which is the relative size of home economy, and on ϖ^* which is the degree of trade openness. It yields $\gamma^* = n\varpi^*$. We assume home bias in consumption:

$$1 - \gamma^* > \gamma$$

which implies

$$1 - n\varpi^* > (1-n)\varpi$$

Similarly, home bias in foreign preferences requires $1 - \gamma > \gamma^*$ which again implies

$$\begin{aligned} (1-(1-n)\varpi) &> n\varpi^* \\ 1 - n\varpi^* &> (1-n)\varpi \end{aligned}$$

C_{Ht}^* and C_{Ft}^* are foreign consumption sub-indexes of the continuum of differentiated goods produced respectively in country H and F given by the CES functions

$$C_{Ht}^* = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n c_{Ht}^*(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}, C_{Ft}^* = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 c_{Ft}^*(z)^{\frac{\epsilon-1}{\epsilon}} dz \right)^{\frac{\epsilon}{\epsilon-1}}$$

[6] The same relationship can be written in per capita terms.

Each consumption bundle C^*_{Ht} and C^*_{Ft} is composed of imperfectly substitutable varieties of goods $z \in [0,1]$ produced within a given country with elasticity of substitution $\epsilon > 1$.

The aggregated nominal intertemporal budget constraint at time t for household i in foreign currency is given by

$$
\begin{aligned}
& \left(1-\tau_t^{w*}\right) W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau_t^{d*}\right) D_t^* + P_{St}^*\right) s_t^* + P_{Ft}^* \Phi_t^* \\
\geq \; & \frac{A_{t+1}^*}{E_t\left(1+i_t\right) \phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} + \frac{B_{t+1}^*}{\left(1+i_t^*\right)} + s_{t+1}^* P_{St}^* - T_t^* \\
& + \int_0^1 [P^*_{Ht}(z) C^{i*}_{Ht}(z) + P^*_{Ft}(z) C^{i*}_{Ft}(z)] dz
\end{aligned}
$$

$P^*_{Ht}(z)$ and $P^*_{Ft}(z)$ are price indices of domestic (exported to country F) and foreign produced goods z, where they are expressed in foreign currency. B^*_t is the one-period foreign corporate bond held by foreign households (real bonds, in terms of domestic prices), A^*_t is the one-period home corporate bonds held by foreign households, W_t^{*i} is the nominal foreign wage and T_t^{*i} denotes lump-sum taxes/transfers. τ_t^{w*} denotes a country specific tax on nominal income and E_t is the nominal exchange rate, given as the price of one of unit foreign currency in terms of home currency. Here $P^*_{Ft}\Phi^*_t$ is nominal profit from the ownership of capital-producing firms and retailers (note that $\Phi^*_t = \Phi_t^{C*} + \Phi_t^{R*}$). Here s^*_t the foreign share of equity which is wholly owned by foreign households, D^*_t the equity payout paid to the shareholders, P^*_{St} is the market price of foreign-owned shares,τ_t^{d*} denotes the tax on equity payout. We assume that the households share the revenues of owning firms in equal proportion. Following Woodford (2003) we consider a cashless economy. Therefore the only explicit role played by money is to serve as a unit of account.

We introduce incomplete financial markets as in Benigno (2009). Foreign households hold foreign equity shares and noncontingent bonds issued by firms of home and foreign countries. Households of country F can trade in two nominal one-period, risk-free bonds. Bonds B^*_t are issued by foreign firms and are denominated in foreign currency, bonds A^*_t are issued by domestic firms and are denominated in foreign currency. Households belonging to country F have to pay an intermediation cost, if they want to trade in the domestic bond. These costs are determined by the function $\phi^*(\cdot)$. Function $\phi^*(\cdot)$ depends on the real holdings of the home assets in the entire economy, and therefore is taken as given by the foreign households. If a household belongs to a country which is in a 'borrowing position' ($A^*_{t+1} < 0$), it will be charged with a premium on the domestic interest rate and if the household belongs to a country which is in a 'lending position' ($A^*_{t+1} > 0$), it receives a rate of return lower than the domestic interest rate. Along with Benigno (2009) we need the following restrictions on $\phi^*(\cdot)$: $\phi^*(0) = 1$ and $\phi^*(\cdot)$ is 1 only if $A^*_t = 0$. Furthermore $\phi^*(\cdot)$ has to be a differentiable, decreasing function in the neighborhood of zero. $\phi^{*\prime}(0) = -\chi^*$.[7]

[7] We assume it is convex and can be approximated by $\phi^*(x) = 1 - \chi^* x + \bar{\chi}^* x^2$.

The intermediation profits $\digamma_t^*$ are defined analogous to Benigno (2009)

$$\digamma_t^* = \frac{A_{t+1}^*}{E_t(1+i_t)}\left(\frac{1}{\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} - 1\right)$$

and shared equally among foreign households. The domestic budget constraint is then given as

$$\begin{aligned}&(1-\tau_t^{w*})W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau_t^{d*}\right)D_t^* + P_{St}^*\right)s_t^* + P_{Ft}^*\Phi_t^* \\ \geq\ & \frac{A_{t+1}^*}{E_t(1+i_t)} + \frac{B_{t+1}^*}{(1+i_t^*)} + s_{t+1}^* P_{St}^* - T_t^* + \digamma_t^* + P_t^* C_t^*\end{aligned}$$

The optimal allocation within each variety of goods z yields per capita relationships

$$c_{Ht}^*(z) = \frac{1}{n}\left(\frac{p_{Ht}^*(z)}{P_{Ht}^*}\right)^{-\epsilon} C_{Ht}^*\ ,\ c_{Ft}^*(z) = \frac{1}{1-n}\left(\frac{p_{Ft}^*(z)}{P_{Ft}^*}\right)^{-\epsilon} C_{Ft}^*$$

for all $z \in [0,1]$, where

$$P_{Ht}^* = \left(\frac{1}{n}\int_0^n p_{Ht}^*(z)^{1-\epsilon}dz\right)^{\frac{1}{1-\epsilon}}\ ,\ P_{Ft}^* = \left(\frac{1}{1-n}\int_n^1 p_{Ft}^*(z)^{1-\epsilon}dz\right)^{\frac{1}{1-\epsilon}}$$

are the price indexes for home and foreign produced goods, where both are expressed in foreign currency. Note as ϵ rises, the individual goods become closer substitutes and therefore the individual firms have less market power.

Finally, the optimal condition of expenditures between home and foreign produced goods is given by[8]

$$C_{Ft}^* = (1-\gamma^*)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^* \text{ and } C_{Ht}^* = \gamma^*\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*$$

where z denotes the good's type or variety and $p_{Ht}^*(z), p_{Ft}^*(z)$ are prices of individual home and foreign produced goods.

where $P_t^* = \left((1-\gamma^*)P_{Ft}^{*1-\eta} + \gamma^* P_{Ht}^{*1-\eta}\right)^{\frac{1}{1-\eta}}$, P_t^* is the consumer price index (CPI) in country F and P_{Ht}^*, P_{Ft}^* are domestic and foreign goods price indices. Note that if the economy is closed, $\gamma^* = 0$, the CPI equals foreign prices. Correspondingly we can write total consumption expenditures by foreign households as $P_t^* C_t^* = P_{Ht}^* C_{Ht}^* + P_{Ft}^* C_{Ft}^*$. The aggregated budget constraint can therefore be rewritten as

$$\begin{aligned}&(1-\tau_t^{w*})W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau_t^{d*}\right)D_t^* + P_{St}^*\right)s_t^* + P_{Ft}^*\Phi_t^* \\ \geq\ & \frac{A_{t+1}^*}{E_t(1+i_t)\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} + \frac{B_{t+1}^*}{(1+i_t^*)} + s_{t+1}^* P_{St}^* - T_t^* + P_t^* C_t^* \end{aligned} \tag{4.11}$$

[8] The same relationship can be written in per capita terms.

We assume that all households in the same country have the same level of initial wealth. As they face the same labour demand and own equal share of all firms, they face identical budget constraints. They all will have identical consumption paths, so we do not use individual index within each country.

We maximize equation (4.10) with respect to equation (4.11) and arrive to the following system

$$\frac{U_C\left(C_t^*, n_t^*\right)}{P_t^*} = \Delta_t^* \tag{4.12}$$

$$\frac{W_t^*}{P_t^*} = -\frac{U_n\left(C_t^*, n_t^*\right)}{U_C\left(C_t^*, n_t^*\right)\left(1-\tau_t^{*w}\right)} \tag{4.13}$$

$$\frac{1}{\left(1+i_t^*\right)} = \beta \mathbb{E}_t \frac{U_C\left(C_{t+1}^*, n_{t+1}^*\right)}{U_C\left(C_t^*, n_t^*\right)\Pi_{Ft+1}^*} \tag{4.14}$$

$$\frac{1}{\left(1+i_t\right)} = \beta \mathbb{E}_t \frac{U_C\left(C_{t+1}^*, n_{t+1}^*\right)}{\Pi_{Ft+1}^* U_C\left(C_t^*, n_t^*\right)} \frac{E_t}{E_{t+1}} \phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right) \tag{4.15}$$

$$P_{St}^* = \beta \mathbb{E}_t \frac{U_C\left(C_{t+1}^*, n_{t+1}^*\right)}{\Pi_{Ft+1}^* U_C\left(C_t^*, n_t^*\right)}\left(\left(1-\tau_{t+1}^{*d}\right) D_{t+1}^* + P_{St+1}^*\right) \tag{4.16}$$

$$\begin{aligned} &\left(1-\tau_t^{w*}\right) W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau_t^{d*}\right) D_t^* + P_{St}^*\right) s_t^* + T_t^* \\ = &\ \frac{A_{t+1}^*}{E_t\left(1+i_t\right)\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} + \frac{B_{t+1}^*}{\left(1+i_t^*\right)} + s_{t+1}^* P_{St}^* + P_t^* C_t^* \end{aligned} \tag{4.17}$$

All derivations are given in Appendix B.1. Equation (4.14) is the standard Euler equation and determines the consumption smoothing behavior of the households. Equation (4.15) is the Euler equation derived from the optimal choice of the foreign bond. equation (4.13) is the standard labour supply condition. It determines the quantity of labor supplied as a function of real wage, given the marginal utility of consumption. Finally equation (4.17) is the aggregate budget constraint.

The incomplete financial market framework generates deviations from the uncovered interest parity (UIP). Combining equation (4.14) and equation (4.15) yields the optimal portfolio choice of the households of country F

$$\left(1+i_t^*\right) = \mathbb{E}_t\left(1+i_t\right)\frac{E_t}{E_{t+1}}\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right) \tag{4.18}$$

$\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)$ can also be interpreted as a risk premium term on the interest rate. If the economy is a net debtor, the foreign interest rate is above the domestic interest rate and if the economy is a net creditor the foreign interest rate is below the domestic interest rate. Therefore movements in the net domestic asset positions affect the interest differential between the two countries.

Combining equations (4.6, 4.15) with the Euler equation of the foreign country (4.9, 4.18) yields the international risk sharing condition

$$\mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} = \mathbb{E}_t \frac{U_C\left(C^*_{t+1}, n^*_{t+1}\right)}{\Pi^*_{Ft+1} U_C\left(C^*_t, n^*_t\right)} \frac{E_t}{E_{t+1}} \phi^* \left(\frac{A^*_{t+1}}{E_t P^*_t}\right) \tag{4.19}$$

$$\mathbb{E}_t \frac{U_C\left(C^*_{t+1}, n^*_{t+1}\right)}{\Pi^*_{Ft+1} U_C\left(C^*_t, n^*_t\right)} = \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \frac{E_{t+1}}{E_t} \phi \left(\frac{E_t B_{t+1}}{P_t}\right) \tag{4.20}$$

Note that if $\frac{A^*_{t+1}}{E_t P^*_t} \equiv \frac{E_t B_{t+1}}{P_t} \equiv 0$ then $\phi^*\left(\frac{A^*_{t+1}}{E_t P^*_t}\right) = \phi\left(\frac{E_t B_{t+1}}{P_t}\right) = 1$ and equations (4.19) and (4.20) simplifies to the standard international risk sharing relationship which is obtained in a complete securities markets setting (see e.g. Galí and Monacelli (2005)).

4.2.4 Domestic Firms

Intermediate goods producers

We assume that there is a continuum of firms $j \in [0,1]$ in country (H) with a gross production function

$$Y_t = F(e^{z_t}, k_t, n_t) = Ze^{z_t} k_t^{\theta} n_t^{1-\theta}$$

and all firms are equal. Ze^{z_t} is the stochastic productivity, common to all firms, k_t is the capital input and n_t is the labor input. k_t is assumed to be chosen at time $t-1$ and predetermined at time t which is consistent with the typical timing convention. On the contrary, the labor input n_t can be flexibly changed at time t.

Each period firms buy the investment good I_t

$$I_t = k_{t+1} - (1-\delta) k_t$$

where δ is the depreciation rate.

Therefore the payments to workers $W_t n_t$, suppliers of investment goods $P_{Ht} Q_t I_t$, shareholders $\Psi\left(D_t, D_{t-1}\right)$ and bondholders A^T_t are made ahead of the realization of revenues. The intra-period loan contracted by the firm will cover these costs as follows:

$$L_t = P_{Ht} Q_t I_t + W_t n_t + \Psi\left(D_t, D_{t-1}\right) + A^T_t - \frac{A^T_{t+1}}{1 + i_t(1-\tau_t)}$$

Firms use equity and debt to finance their operations. They prefer nominal debt, $A^T_t = A_t + A^*_t$, to equity in general because of debt's tax advantage (τ_t). This is also the assumption made in Hennessy and Whited (2005). Given i_t the nominal interest rate, the effective gross interest rate for the firm is $R_t = 1 + i_t(1 - \tau_t)$, where τ_t represents the tax benefit.

We assume that firms raise funds by the intertemporal nominal debt A_t^T and the intra-period domestic loan, L_t to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues. They pay back the free-interest intra-period loan at the end of the period.

Firms start the period with intertemporal debt A_t^T and they choose labour n_t, investment in capital I_t, equity payout, D_t , and the new intertemporal debt A_{t+1}^T before producing. Therefore, the aggregated nominal budget constraint of firms can be written as

$$P_{mt}F(e^{z_t}, k_t, n_t) + \frac{A_{t+1}^T}{1 + i_t(1 - \tau_t)} \geq A_t^T + W_t n_t + P_{Ht}Q_t I_t + \Psi\left(D_t, D_{t-1}\right) \tag{4.21}$$

From the budget constraint $L_t = P_{mt}F(e^{z_t}, k_t, n_t)$ is repaid at the end of the period and is free of interest. Where P_{mt} is the nominal price of produced intermediate goods, $\Psi\left(D_t, D_{t-1}\right)$ is the nominal payment to shareholders, $P_{Ht}Q_t$ is the price of investment goods, and and W_t is the nominal wage in home country.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intra-period loan. The total liabilities of the firm at that time are $L_t + \frac{A_{t+1}}{1+i_t}$, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t = P_{mt}F(e^{z_t}, k_t, n_t)$. These can be 'diverted' by the firm, and so can not be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_{Ht}Q_t k_{t+1}$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability Ξe^{ξ_t} the full value $P_{Ht}Q_t k_{t+1}$ will be recovered, but with probability $1 - \Xi e^{\xi_t}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi e^{\xi_t} \left(P_{Ht}Q_t k_{t+1} - \frac{A_{t+1}^T}{1 + i_t} \right) \geq P_{mt}F(e^{z_t}, k_t, n_t) \tag{4.22}$$

This constraint is derived based on the renegotiation process between the firm and the lender in the case of default. The derivation is given in Appendix A.3.

By increasing the level of debt the enforcement constraint becomes tighter. On the other hand, increasing the stock of capital relaxes the enforcement constraint. Most of the enforcement constraint used in the literature shared these properties. The probability

Ξe^{ξ_t} is stochastic and depends on uncertain markets conditions.[9] We call this variable as "financial shocks", because it affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. Notice that Ξe^{ξ_t} is the same for all firms. Hence, there are two sources of aggregate uncertainty in our model: productivity e^{z_t} and financial Ξe^{ξ_t}. Since there are no idiosyncratic shocks, we will focus on the symmetric equilibrium where all representative firms are the same.

We can slightly modify equation (4.22), to see clearly how the shock Ξe^{ξ_t} affects the economy. Suppose the case in which $\tau = 0$ so that $R = 1 + i$. Using the budget constraint equation (4.21) to substitute for $P_{Ht}Q_t k_{t+1} - \frac{A^T_{t+1}}{1+i_t}$ and remembering that the intra-period loan is equal to the revenues, $L_t = P_{mt}F(e^{z_t}, k_t, n_t)$, the enforcement constraint can be rewritten as

$$\frac{\Xi e^{\xi_t}}{1 - \Xi e^{\xi_t}} \left(P_{Ht}Q_t \left(1-\delta\right) k_t - A^T_t - W_t n_t - \Psi\left(D_t, D_{t-1}\right)\right) \geq P_{mt}F(e^{z_t}, k_t, n_t)$$

At the beginning of the period k_t and A^T_t are given. The firm have control only over the input of labor, n_t, and the equity payout, $\Psi\left(D_t, D_{t-1}\right)$. If the firm wishes to keep the production level unchanged, a negative financial shock (lower Ξe^{ξ_t}) requires a reduction in equity payout $\Psi\left(D_t, D_{t-1}\right)$ or employment. In other words, the firm is forced to raise its equity and cut the new intertemporal debt. Thus, the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity will determine if the financial shock affects employment.

The firm's nominal payout to shareholders assumed to be subject to a quadratic adjustment cost which is a way to formalize the rigidities affecting the substitution between debt and equity:

$$\Psi\left(D_t, D_{t-1}\right) = D_t + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2 D_t$$

where the nominal equity payout D_t is given and $\kappa \geq 0$ is a parameter.[10]

The parameter κ is key for the role of financial shocks. Since when $\kappa = 0$ the economy is almost frictionless, therefore debt adjustments caused by financial shocks can be quickly assisted through changes in firm equity. When $\kappa > 0$, it is costly to substitute debt and equity and firm's readjustment becomes slowly. As a result, financial shocks will have a substantial effect on macroeconomic situation of a country.

The first order conditions with respect to n_t, k_{t+1}, A^T_{t+1}, D_t, μ_t, λ_t can be written as

[9]The variable Ξe^{ξ_t} could be interpreted as the probability of finding a buyer. Because we assume that the search for a buyer is required for the sale of the firm's capital. The probability increases when the market conditions improve.

[10]One way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact later confirmed by subsequent studies. This could obtain from agency problems.

$$
\begin{aligned}
\lambda_t W_t &= F_n(e^{z_t}, k_t, n_t)\left(\lambda_t - \mu_t\right) P_{mt} && (4.23)\\
0 &= \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \left(\left(\lambda_{t+1} - \mu_{t+1}\right) P_{mt+1} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) + \lambda_{t+1} P_{Ht+1} Q_{t+1}\left(1-\delta\right)\right)\\
&\quad - \left(\lambda_t - \mu_t \Xi e^{\xi_t}\right) P_{Ht} Q_t && (4.24)\\
0 &= \frac{\lambda_t}{1 + i_t(1-\tau_t)} - \mu_t \Xi e^{\xi_t} \frac{1}{1+i_t} - \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \lambda_{t+1} && (4.25)\\
1 &= \lambda_t \left(1 + 2\kappa \left(\frac{D_t}{D_{t-1}} - 1\right) \frac{D_t}{D_{t-1}} + \kappa \left(\frac{D_t}{D_{t-1}} - 1\right)^2\right)\\
&\quad - \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \lambda_{t+1} 2\kappa \left(\frac{D_{t+1}}{D_t} - 1\right) \frac{D_{t+1}^2}{D_t^2} && (4.26)\\
0 &= \Xi e^{\xi_t} \left(P_{Ht} Q_t k_{t+1} - \frac{A_{t+1}^T}{1+i_t}\right) - P_{mt} F(e^{z_t}, k_t, n_t) && (4.27)\\
A_t^T &= P_{mt} F(e^{z_t}, k_t, n_t) + \frac{A_{t+1}^T}{1 + i_t(1-\tau_t)} - W_t n_t - P_{Ht} Q_t I_t - \Psi\left(D_t, D_{t-1}\right) && (4.28)
\end{aligned}
$$

All derivations are given in Appendix B.2. $m_{t,t+1} = \beta \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}}$ is an stochastic discount factor and the budget constraint is written in an aggregated form. The stochastic discount factor $m_{t,t+1}$, the wage W_t and interest rate i_t are determined in the general equilibrium and are taken as given by an individual firm.

Equation (4.23), the optimal condition for labor indicates that the marginal productivity of labor is equal to the marginal cost ($\frac{\lambda_t W_t}{(\lambda_t - \mu_t) P_{mt}}$). As the enforcement constraint becomes tighter, the effective cost of labor rises and its demand falls. Therefore, financial shocks could transmit to the real sector of the economy through the demand of labor.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa = 0$. In this case $\lambda_t = 1$ (see condition (4.26)) and condition (4.25) becomes $\mu_t \Xi e^{\xi_t} \frac{R_t}{1+i_t} + R_t \beta \mathbb{E}_t \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}} \lambda_{t+1} = 1$. This denotes that there is a negative relation between Ξe^{ξ_t} and the multiplier μ_t taking as given the aggregate prices R_t, i_t, and $\beta \frac{U_{C,t+1}}{\Pi_{Ht+1} U_{C,t}}$. In other words, lower probability of recovering firm's capital make the enforcement constraint tighter. Then from equation (4.23) we see that a higher μ_t implies a lower demand for labor.

This mechanism is strengthened when $\kappa > 0$. In this case readjusting the financial structure becomes costly, and the change in Ξe^{ξ_t} induces a larger volatility in μ_t. Of course, prices will be affected by the change in the policies of all firms.

Capital producers

Capital producers belong to households. They make new capital using input of final output and subject to adjustment costs. They sell new capital to firms at price $P_{Ht} Q_t$.

Their nominal profit

$$P_{Ht}\Phi_t^C = P_{Ht}Q_tI_t - P_{Ht}I_t\left(1 + \frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$$

Specifically, they buy I_t of the final good, pay $P_{Ht}I_t\left(1 + \frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right)$ as they may need to adjust contracts if the amount of the investment goods changes. They repackage the good into investment good (costlessly) and sell it to firms at price $P_{Ht}Q_t$ and receive $P_{Ht}Q_tI_t$.

The first order condition yields

$$Q_t = 1 + \frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 + \frac{I_t}{I_{t-1}}\varrho\left(\frac{I_t}{I_{t-1}} - 1\right) - \beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\varrho\left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

Retailers

Retailers repackage intermediate output. The marginal cost is P_{mt}. We introduce nominal rigidities *a la* Rotemberg.

Cost minimisation yields

$$y_t^i = \left(\frac{p_{Ht}^i}{P_{Ht}}\right)^{-\varepsilon} Y_t$$

where index i is of retailer i and Y_t is final output. Retailers costlessly brand intermediate output. They have monopolistic power but have adjustment cost.

The firm's profit is

$$p_{Ht}^i\Phi_t^R = p_{Ht}^iy_t^i\left(1 - \tau_t^x\right) - P_{mt}y_t^i - \frac{\omega}{2}\left(\frac{p_{Ht}^i}{p_{Ht-1}^i} - 1\right)^2 Y_tP_{Ht}$$

we introduce sales tax τ_t^x.

The first order condition yields

$$\omega\left(\Pi_{Ht} - 1\right)\Pi_{Ht} = \left(1 - \varepsilon\right)\left(1 - \tau_t^x\right) + \varepsilon X_t + \omega\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\left(\Pi_{Ht+1} - 1\right)\frac{Y_{t+1}}{Y_t}\Pi_{Ht+1} \quad (4.29)$$

where $X_t = \frac{P_{mt}}{P_{Ht}}$. The derivation is given in Appendix B.2.

and the aggregated across firms profit

$$P_{Ht}\Phi_t^R = P_{Ht}Y_t\left(1 - \tau_t^x\right) - P_{mt}Y_t - \frac{\omega}{2}\left(\Pi_{Ht} - 1\right)^2 Y_tP_{Ht}$$

4.2.5 Foreign Firms

Intermediate Goods Producers

The optimisation problem for foreign firms is symmetric. We assume that there is a continuum of firms $j^* \in [0,1]$ in country (F) with a gross revenue function

$$Y_t^* = F(e^{z_t^*}, k_t^*, n_t^*) = Ze^{z_t^*} k_t^{*\theta} n_t^{*1-\theta}$$

and all firms are equal. $e^{z_t^*}$ is the stochastic productivity, common to all firms, k_t^* is the capital and n_t^* the labor in country F. k_t^* is assumed to be chosen at time $t-1$ and predetermined at time t which is consistent with the typical timing convention. On the contrary, the labor input n_t^* can be flexibly changed at time t.

Each period firms buy the investment good I_t^*

$$I_t^* = k_{t+1}^* - (1-\delta)\, k_t^*$$

where I_t^* is investment and δ^* is the depreciation rate in country F.

Therefore the payments to workers $W_t^* n_t^*$, suppliers of investment goods $P_{Ft}^* Q_t^* I_t^*$, shareholders $\Psi^*\left(D_t^*, D_{t-1}^*\right)$ and bondholders B_t^T are made ahead of the realization of revenues. The intra-period loan contracted by the firm will cover these costs as follows:

$$L_t^* = P_{Ft}^* Q_t^* I_t^* + W_t^* n_t^* + \Psi^*\left(D_t^*, D_{t-1}^*\right) + B_t^T - \frac{B_{t+1}^T}{1 + i_t^*(1-\tau_t^*)}$$

Firms use equity and debt to finance their operations. They prefer nominal debt, $B_t^T = B_t + B_t^*$, to equity in general because of debt's tax advantage (τ_t^*). This is also the assumption made in Hennessy and Whited (2005). Given i_t^* the nominal interest rate, the effective gross interest rate for the firm is $R_t^* = 1 + i_t^*(1-\tau_t^*)$, where τ_t^* represents the tax benefit.

We assume that firms raise funds by the intertemporal nominal debt B_t^T and the intra-period domestic loan, L_t to finance working capital. Working capital is required to cover the cash flow mismatch between the payments made at the beginning of the period and the realization of revenues. They pay back the free-interest intra-period loan at the end of the period.

Firms start the period with intertemporal debt B_t^T and they choose labour n_t^*, investment in capital I_t^*, equity payout, D_t^* , and the new intertemporal debt B_{t+1}^T before producing. Therefore, the firm's aggregated nominal budget constraint can be written as

$$P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{B_{t+1}^T}{1 + i_t^*(1-\tau_t^*)} \geq B_t^T + W_t^* n_t^* + P_{Ft}^* Q_t^* I_t^* + \Psi^*\left(D_t^*, D_{t-1}^*\right) \qquad (4.30)$$

From the budget constraint $L_t^* = P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*)$ is repaid at the end of the period and is free of interest. Where P_{mt}^* is the nominal price of produced intermediate goods, $\Psi^*\left(D_t^*, D_{t-1}^*\right)$ is the nominal payment to shareholders, $P_{Ft}^* Q_t^*$ is the price of investment goods, and W_t^* is the nominal wage in foreign country.

The ability of firms to borrow is bounded because they may choose to default on their debt. Default arises after the realization of revenues but before repaying the intra-period loan. The total liabilities of the firm at that time are $L_t^* + \frac{B_{t+1}^T}{1+i_t^*}$, as it will need to pay back the loan and buy back all the bonds. The total liquid resources of the firm are $L_t^* = P_{mt}^{*\$} F(e^{z_t^*}, k_t^*, n_t^*)$. These can be 'diverted' by the firm, and so can not be recovered by the lender after a default. Then, the only asset available to the lender is capital $P_{Ft}^* Q_t^* k_{t+1}^*$. Following Jermann and Quadrini (2012), we assume that the liquidation value of capital is unknown at the moment of contracting the loan. With probability $\Xi^* e^{\xi_t^*}$ the full value $P_{Ft}^* Q_t^* k_{t+1}^*$ will be recovered, but with probability $1 - \Xi^* e^{\xi_t^*}$ the liquidation value is zero. Therefore the enforcement constraint will be as follows:

$$\Xi^* e^{\xi_t^*} \left(P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1 + i_t^*} \right) \geq P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) \tag{4.31}$$

This constraint is derived based on the renegotiation process between the firm and the lender in the case of default. The derivation is given in Appendix A.3.

By increasing the level of debt the enforcement constraint becomes tighter. On the other hand, increasing the stock of capital relaxes the enforcement constraint. Most of the enforcement constraint used in the literature shared these properties. The probability $\Xi^* e^{\xi_t^*}$ is stochastic and depends on uncertain markets conditions.[11] We call this variable as "financial shocks", because it affects the tightness of the enforcement constraint and, therefore, the borrowing capacity of the firm. Notice that $\Xi^* e^{\xi_t^*}$ is the same for all firms. Hence, there are two sources of aggregate uncertainty in our model: productivity $e^{z_t^*}$ and financial $\Xi^* e^{\xi_t^*}$. Since there are no idiosyncratic shocks, we will focus on the symmetric equilibrium where all representative firms are the same.

We can slightly modify equation (4.31), to see clearly how the shock $\Xi^* e^{\xi_t^*}$ affects the economy. Suppose the case in which $\tau^* = 0$ so that $R^* = 1 + i^*$. Using the budget constraint (4.30) to substitute for $P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*}$ and remembering that the intra-period loan is equal to the revenues, $L_t = P_{mt} F(e^{z_t}, k_t, n_t)$, the enforcement constraint can be rewritten as

$$\frac{\Xi^* e^{\xi_t^*}}{1 - \Xi^* e^{\xi_t^*}} \left(P_{Ft}^* Q_t^* \left(1 - \delta\right) k_t^* - B_t^T - W_t^* n_t^* - \Psi^* \left(D_t^*, D_{t-1}^*\right)\right) \geq P_{mt}^* F^*(e^{z_t *}, k_t^*, n_t^*)$$

[11] The variable $\Xi^* e^{\xi_t^*}$ could be interpreted as the probability of finding a buyer. Because we assume that the search for a buyer is required for the sale of the firm's capital. The probability increases when the market conditions improve.

At the beginning of the period k_t^* and B_t^T are given. The firm have control only over the input of labor, n_t^*, and the equity payout, $\Psi^*\left(D_t^*, D_{t-1}^*\right)$. If the firm wishes to keep the production level unchanged, a negative financial shock (lower $\Xi^* e^{\xi_t^*}$) requires a reduction in equity payout $\Psi^*\left(D_t^*, D_{t-1}^*\right)$ or employment. In other words, the firm is forced to raise its equity and cut the new intertemporal debt. Thus, the flexibility with which the firm can change its financial structure, i.e., the composition of debt and equity will determine if the financial shock affects employment.

The firm's nominal payout to shareholders assumed to be subject to a quadratic adjustment cost which is a way to formalize the rigidities affecting the substitution between debt and equity:

$$\Psi^*\left(D_t^*, D_{t-1}^*\right) = D_t^* + \kappa^*\left(\frac{D_t^*}{D_{t-1}^*} - 1\right)^2 D_t^*$$

where the nominal equity payout D_t^* is given and $\kappa^* \geq 0$ is a parameter.[12]

The parameter κ^* is key for the role of financial shocks. Since when $\kappa^* = 0$ the economy is almost frictionless, therefore debt adjustments caused by financial shocks can be quickly assisted through changes in firm equity. When $\kappa^* > 0$, it is costly to substitute debt and equity and firm's readjustment becomes slowly. As a result, financial shocks will have a substantial effect on macroeconomic situation of a country.

The first order conditions with respect to n_t^*, k_{t+1}^*, B_{t+1}^T, D_t^*, μ_t^*, λ_t^* can be written as

$$\begin{aligned}
\lambda_t^* W_t^* &= \left(\lambda_t^* - \mu_t^*\right) P_{mt}^* F_n(e^{z_t^*}, k_t^*, n_t^*) && (4.32)\\
0 &= \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*}\left(\left(\lambda_{t+1}^* - \mu_{t+1}^*\right) P_{mt+1}^* F_k(e^{z_{t+1}^*}, k_{t+1}^*, n_{t+1}^*) + \lambda_{t+1}^* P_{Ft+1}^* Q_{t+1}^* (1-\delta)\right) \\
&\quad - \left(\lambda_t^* - \mu_t^* \Xi_t^* e^{\xi_t^*}\right) P_{Ft}^* Q_t^* && (4.33)\\
0 &= \frac{\lambda_t^*}{1 + i_t^*(1-\tau_t^*)} - \mu_t^* \Xi^* e^{\xi_t^*} \frac{1}{1+i_t^*} - \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \lambda_{t+1}^* && (4.34)\\
1 &= \lambda_t^* \left(1 + 2\kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1\right)\frac{D_t^*}{D_{t-1}^*} + \kappa^*\left(\frac{D_t^*}{D_{t-1}^*} - 1\right)^2\right) \\
&\quad - \beta \mathbb{E}_t \frac{U_{c,t+1}^*}{U_{c,t}^* \Pi_{Ft+1}^*} \lambda_{t+1}^* 2\kappa^* \left(\frac{D_{t+1}^*}{D_t^*} - 1\right) \frac{D_{t+1}^{*2}}{D_t^{*2}} && (4.35)\\
0 &= \Xi^* e^{\xi_t^*}\left(P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*}\right) - P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) && (4.36)\\
B_t^T &= P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{B_{t+1}^T}{1 + i_t^*(1-\tau_t^*)} - W_t^* n_t^* \\
&\quad - P_{Ft}^* Q_t^* \left(k_{t+1}^* - (1-\delta^*) k_t^*\right) - \Psi^*\left(D_t^*, D_{t-1}^*\right) && (4.37)
\end{aligned}$$

[12] One way of thinking about the adjustment cost is that it captures the preferences of managers for dividend smoothing. Lintner (1956) showed that managers are concerned about smoothing dividends over time, a fact later confirmed by subsequent studies. This could obtain from agency problems.

All derivations are given in Appendix B.2. $m^*_{t,t+1} = \beta \frac{U^*_{c,t+1}}{U^*_{c,t}\Pi^*_{Ft+1}}$ is a nominal stochastic discount factor and the budget constraint is written in an aggregated form. The stochastic discount factor $m^*_{t,t+1}$, the wage W^*_t and interest rate i^*_t are determined in the general equilibrium and are taken as given by an individual firm.

Equation (4.32), the optimal condition for labor indicates that the marginal productivity of labor is equal to the marginal cost $(\frac{\lambda^*_t W^*_t}{(\lambda^*_t - \mu^*_t)P^*_{mt}})$. As the enforcement constraint becomes tighter, the effective cost of labor rises and its demand falls. Therefore, financial shocks could transmit to the real sector of the economy through the demand of labor.

To get further insights, it will be convenient to consider the special case in which the cost of equity payout is zero, that is, $\kappa^* = 0$. In this case $\lambda^*_t = 1$ (see condition (4.35)) and condition (4.34) becomes $\mu^*_t \Xi^* e^{\xi^*_t} \frac{R^*_t}{1+i^*_t} + R^*_t \beta \mathbb{E}_t \frac{U^*_{c,t+1}}{U^*_{c,t}\Pi^*_{Ft+1}} \lambda^*_{t+1} = 1$. This denotes that there is a negative relation between $\Xi^* e^{\xi^*_t}$ and the multiplier μ^*_t taking as given the aggregate prices R^*_t, i^*_t, and $\beta \frac{U^*_{c,t+1}}{U^*_{c,t}\Pi^*_{Ft+1}}$. In other words, lower probability of recovering firm's capital make the enforcement constraint tighter. Then from equation (4.32) we see that a higher μ^*_t implies a lower demand for labor.

This mechanism is strengthened when $\kappa^* > 0$. In this case readjusting the financial structure becomes costly, and the change in $\Xi^* e^{\xi^*_t}$ induces a larger volatility in μ^*_t. Of course, prices will be affected by the change in the policies of all firms.

Capital producers

Capital producers belong to households. They make new capital using input of final output and subject to adjustment costs. They sell new capital to firms at price $P^*_{Ft}Q^*_t$. Their nominal profit

$$P^*_{Ft}\Phi^{*C}_t = P^*_{Ft}Q^*_t I^*_t - P^*_{Ft}I^*_t \left(1 + \frac{\varrho}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2\right)$$

Specifically, they buy I^*_t of the final good, pay $P^*_{Ft}I^*_t \left(1 + \frac{\varrho}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2\right)$ as they may need to adjust contracts if the amount of the investment goods changes. They repackage the good into investment good (costlessly) and sell to firms at price $P^*_{Ft}Q^*_t$ and receive $P^*_{Ft}Q^*_t I^*_t$.

The first order condition yields

$$Q^*_t = 1 + \frac{\varrho}{2}\left(\frac{I^*_t}{I^*_{t-1}} - 1\right)^2 + \frac{I^*_t}{I^*_{t-1}}\varrho\left(\frac{I^*_t}{I^*_{t-1}} - 1\right) - \beta \mathbb{E}_t \frac{U^*_{c,t+1}}{U^*_{c,t}}\varrho\left(\frac{I^*_{t+1}}{I^*_t} - 1\right)\left(\frac{I^*_{t+1}}{I^*_t}\right)^2$$

Retailers

Retailers repackage intermediate output. The marginal cost is P^*_{mt}. We introduce nominal rigidities *a la* Rotemberg.

Cost minimisation yields

$$y^{*i}_t = \left(\frac{p^{*i}_{Ft}}{P^*_{Ft}}\right)^{-\varepsilon} Y^*_t$$

where index i is of retailer i and Y^*_t is final output. Retailers costlessly brand intermediate output. They have monopolistic power but have adjustment cost.

The firm's profit is

$$p^{*i}_{Ft}\Phi^{*R}_t = p^{*i}_{Ft} y^{*i}_t \left(1 - \tau^{*x}_t\right) - P^*_{mt} y^{*i}_t - \frac{\omega}{2}\left(\frac{p^{*i}_{Ft}}{p^{*i}_{Ft-1}} - 1\right)^2 Y^*_t P^*_{Ft}$$

we introduce sales tax τ^{*x}_t.

The first order condition yields

$$\omega\left(\Pi^*_{Ft} - 1\right)\Pi^*_{Ft} = (1-\varepsilon)\left(1 - \tau^{*x}_t\right) + \varepsilon X^*_t + \omega\beta\mathbb{E}_t\frac{U^*_{c,t+1}}{U^*_{c,t}}\left(\Pi^*_{Ft+1} - 1\right)\frac{Y^*_{t+1}}{Y^*_t}\Pi^*_{Ft+1} \quad (4.38)$$

where $X^*_t = \frac{P^*_{mt}}{P^*_{Ft}}$. The derivation is given in Appendix B.2.

and the aggregated across firms profit

$$P^*_{Ft}\Phi^{*R}_t = P^*_{Ft} Y^*_t \left(1 - \tau^{*x}_t\right) - P^*_{mt} Y^*_t - \frac{\omega}{2}\left(\Pi^*_{Ft} - 1\right)^2 Y^*_t P^*_{Ft}$$

4.2.6 Governments

The government in each country collects taxes and pays it as transfers and government spending:

$$\begin{aligned}
T_t &= -\frac{A^T_{t+1}}{1+i_t(1-\tau_t)} + \frac{A^T_{t+1}}{1+i_t} && (4.39)\\
P_{Ht}G_t &= \tau^w_t W_t N_t + \tau^d_t D_t + \tau^x_t P_{Ht} Y_t && (4.40)\\
T^*_t &= -\frac{B^T_{t+1}}{1+i^*_t(1-\tau^*_t)} + \frac{B^T_{t+1}}{1+i^*_t} && (4.41)\\
P^*_{Ft}G^*_t &= \tau^{*w}_t W^*_t N^*_t + \tau^{*d}_t D^*_t + \tau^{*x}_t P^*_{Ft} Y^*_t && (4.42)
\end{aligned}$$

We assume that the domestic government buys goods (G), taxes sale (with tax rate τ^x_t).

$$\begin{aligned}
P_{Ht}G_t &= \tau^x_t P_{Ht} Y_t\\
P^*_{Ft}G^*_t &= \tau^{*x}_t P^*_{Ft} Y^*_t
\end{aligned}$$

4.2.7 Market Clearing and Private Sector Equilibrium

In order to close the system, we write down two market clearing constraints:

$$\begin{aligned}
Y_t &= (1-\gamma)\Upsilon^\eta_t C_t + \gamma^*\Gamma^\eta_t S^\eta_t C^*_t + \kappa\left(\frac{d_t}{d_{t-1}}\Pi_{Ht}-1\right)^2 d_t + G_t + \frac{\omega}{2}\left(\Pi_{Ht}-1\right)^2 Y_t\\
&+I_t\left(1+\frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right) + \left(\frac{1}{\phi\left(\frac{E_t B_{t+1}}{P_t}\right)}-1\right)\frac{B_{t+1}E_t}{(1+i^*_t)\,P_{Ht}} && (4.43)
\end{aligned}$$

$$\begin{aligned}
Y^*_t = (1-\gamma^*)\Gamma^\eta_t C^*_t + \gamma\left(\frac{\Upsilon_t}{S_t}\right)^\eta C_t \quad &+\kappa\left(\frac{d^*_t}{d^*_{t-1}}\Pi^*_{Ft}-1\right)^2 d^*_t + G^*_t + \frac{\omega}{2}\left(\Pi^*_{Ft}-1\right)^2 Y^*_t\\
+I^*_t\left(1+\frac{\varrho}{2}\left(\frac{I^*_t}{I^*_{t-1}}-1\right)^2\right) \quad &+\left(\frac{1}{\phi^*\left(\frac{A^*_{t+1}}{E_t P^*_t}\right)}-1\right)\frac{A^*_{t+1}}{E_t\,(1+i_t)\,P^*_{Ft}}
\end{aligned}$$

where $Y_t = Ze^{z_t}k^\theta_t N^{1-\theta}_t, Y^*_t = Ze^{z^*_t}k^{*\theta}_t N^{*1-\theta}_t, \Upsilon_t = \left((1-\gamma)+\gamma S^{1-\eta}_t\right)^{\frac{1}{1-\eta}}$,

$\Gamma_t = \left((1-\gamma^*)+\gamma^* S^{\eta-1}_t\right)^{\frac{1}{1-\eta}}$.

Together with households first order conditions (3.3)-(3.8), (3.12)-(3.17) and firms first order conditions (4.23)-(4.28), (4.32)-(4.37), (4.29), (4.38), government budget constraints (4.39)-(4.42) and one net foreign assets equation

$$0 = \gamma S^{1-\eta}_t \Upsilon^\eta_t C_t - \gamma^*\Gamma^\eta_t S^\eta_t C^*_t - \mathbb{E}_t a^*_{t+1}\frac{\Pi_{Ht+1}}{(1+i_t)} + a^*_t + \mathbb{E}_t b_{t+1}\frac{\Pi^*_{Ft+1}}{(1+i^*_t)}S_t - b_t S_t$$

and the definition of nominal exchange rate

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

They describe the evolution of the economy and determine twenty five variables: C_t, n_t, w_t, λ_t, μ_t, X_t, k_t, Π_{Ht}, d_t, b_t, a_t, Q_t, C_t^*, n_t^*, w_t^*, λ_t^*, μ_t^*, X_t^*, k_t^*, Π_{Ft}^*, d_t^*, b_t^*, a_t^*, Q_t^* and S_t: Appendix B.3 demonstrates that the system is internally consistent. Policy instruments are i_t; i_t^*; τ_t^w; τ_t^{w*}; τ_t^d; τ_t^{d*}; τ_t^x; τ_t^{x*}; τ_t; τ_t^* and it remains to describe policy.

4.2.8 Fixed Exchange Rate Regime and Policy Instruments

We assume that both countries form a currency union, so there is only one central bank and permanently fixed nominal exchange rate. The definition of nominal exchange rate $\hat{E}_t$

$$\hat{S}_t - \hat{S}_{t-1} = \hat{\Pi}_{Ft}^* - \hat{\Pi}_{Ht} + \hat{E}_t - \hat{E}_{t-1}$$

collapses to

$$\hat{S}_t - \hat{S}_{t-1} = \hat{\Pi}_{Ft}^* - \hat{\Pi}_{Ht}$$

while equations (3.53) and (3.63) yield:

$$\hat{\imath}_t^* = \hat{\imath}_t + \chi b_{t+1} \tag{4.44}$$

$$\hat{\imath}_t = \hat{\imath}_t^* + \chi^* a_{t+1}^* \tag{4.45}$$

These equations imply that in a monetary union with incomplete financial markets households face different short-term interest rates. With no loss of generality we assume that the Central Bank controls $\hat{\imath}_t$.

Each of the two independent fiscal authorities in countries H and F controls sale's tax rate and spending, $\{\hat{\tau}_t^x, \hat{G}_t\}$ and $\{\hat{\tau}_t^{x*}, \hat{G}_t^*\}$, respectively.

4.3 Linearised System

Let $\hat{X}_t = \log X_t - \log X$ denote the log-deviation of variable X_t from its steady state value X. In line with Benigno (2009) and Paoli (2009) we assume a symmetric steady state, which implies that the net foreign asset position is zero in the steady state.[13] The log-linear approximations for the equilibrium conditions of our model are given as:

$$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} + \frac{1}{\sigma}\left(\mathbb{E}_t\hat{\Pi}_{Ht+1} - \hat{\imath}_t^a + \frac{\chi}{2}b_{t+1}\right) + \frac{1}{\sigma}\gamma\mathbb{E}_t\left(\hat{\Pi}^*_{Ft+1} - \hat{\Pi}_{Ht+1}\right) \tag{4.46}$$

$$\begin{aligned}\hat{\lambda}_t &= \mu\Xi\left(\hat{\mu}_t + \hat{\xi}_t\right) + (1-\mu\Xi)\,\sigma\hat{C}_t + \mathbb{E}_t\left(\beta(1-\mu)\,X\theta\frac{Y}{k} - (1-\mu\Xi)\right)\sigma\hat{C}_{t+1} \\ &\quad - (1-\mu\Xi)\,\gamma\mathbb{E}_t\left(\hat{\Pi}^*_{Ft+1} - \hat{\Pi}_{Ht+1}\right) + \beta\left(X\theta\frac{Y}{k}(1-\mu) + (1-\delta)\right)\mathbb{E}_t\hat{\lambda}_{t+1} \\ &\quad + \beta(1-\mu)\,X\theta\frac{Y}{k}\left((\psi+1)\,\mathbb{E}_t\hat{n}_{t+1} + \gamma\left(\mathbb{E}_t\hat{\Pi}^*_{Ft+1} - \mathbb{E}_t\hat{\Pi}_{Ht+1} + \hat{S}_t\right) - \hat{k}_{t+1}\right)\end{aligned} \tag{4.47}$$

$$\begin{aligned}&(\mu\Xi+1)\left(\hat{\lambda}_t + \frac{i\tau}{R}\hat{\tau}_t\right) + \left(\mu\Xi - \frac{(\mu\Xi+1)(1-\tau)}{\beta R}\right)\left(\hat{\imath}_t^a - \frac{\chi}{2}b_{t+1}\right) \\ &= \mu\Xi\left(\hat{\mu}_t + \hat{\xi}_t\right) + \sigma\left(\hat{C}_t - \mathbb{E}_t\hat{C}_{t+1}\right) - \gamma\mathbb{E}_t\left(\hat{\Pi}^*_{Ft+1} - \hat{\Pi}_{Ht+1}\right) + \mathbb{E}_t\hat{\lambda}_{t+1} - \mathbb{E}_t\hat{\Pi}_{Ht+1}\end{aligned} \tag{4.48}$$

$$\hat{\lambda}_t = 2\beta\kappa\left(\mathbb{E}_t\hat{d}_{t+1} - \hat{d}_t + \mathbb{E}_t\hat{\Pi}_{Ht+1}\right) - 2\kappa\left(\hat{d}_t - \hat{d}_{t-1} + \hat{\Pi}_{Ht}\right) \tag{4.49}$$

$$\begin{aligned}&\Xi\frac{k}{Y}\left(\hat{\xi}_t + \hat{k}_{t+1}\right) - \beta\Xi\frac{a}{Y}\left(\hat{\xi}_t + \hat{a}_{t+1} - \frac{\chi}{\chi^*}\frac{1}{a}b_{t+1} + \mathbb{E}_t\hat{\Pi}_{Ht+1} - \hat{\imath}_t^a + \frac{\chi}{2}b_{t+1}\right) \\ &= X\left((\psi+1)\,\hat{n}_t + \sigma\hat{C}_t + \frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w + \gamma\hat{S}_t + \frac{\mu}{(1-\mu)}\hat{\mu}_t - \frac{\mu}{(1-\mu)}\hat{\lambda}_t\right)\end{aligned} \tag{4.50}$$

$$\begin{aligned}\hat{\Pi}_{Ht} &= \beta\mathbb{E}_t\hat{\Pi}_{Ht+1} + \frac{(\varepsilon-1)(1-\tau^x)}{\omega}\left(\frac{\tau^x}{(1-\tau^x)}\hat{\tau}_t^x + \frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w\right. \\ &\quad \left. + \frac{\mu}{(1-\mu)}\left(\hat{\mu}_t - \hat{\lambda}_t\right) + \sigma\hat{C}_t - \hat{z}_t - \theta\hat{k}_t + (\theta+\psi)\,\hat{n}_t + \gamma\hat{S}_t\right)\end{aligned} \tag{4.51}$$

$$\hat{Y}_t = \frac{k}{Y}\hat{k}_{t+1} + \beta\left(\frac{\chi}{\chi^*}+1\right)\frac{b_{t+1}}{Y} - \left(\frac{\chi}{\chi^*}+1\right)\frac{b_t}{Y} + \frac{C}{Y}\left(\hat{C}_t + \gamma\hat{S}_t\right) - (1-\delta)\frac{k}{Y}\hat{k}_t + \frac{G}{Y}\hat{G}_t \tag{4.52}$$

$$\begin{aligned}\frac{k}{Y}\hat{k}_{t+1} &= \left(X - w\frac{n}{Y}\right)\left((\psi+1)\,\hat{n}_t + \sigma\hat{C}_t + \frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w + \gamma\hat{S}_t\right) \\ &\quad + \left(\hat{a}_{t+1} - \frac{\chi}{\chi^*}\frac{1}{a}b_{t+1} + \mathbb{E}_t\hat{\Pi}_{Ht+1} - \frac{(1+i)(1-\tau)}{R}\left(\hat{\imath}_t^a - \frac{\chi}{2}b_{t+1}\right) + \frac{i\tau}{R}\hat{\tau}_t\right)\frac{a}{YR} \\ &\quad + X\frac{\mu}{(1-\mu)}\left(\hat{\mu}_t - \hat{\lambda}_t\right) - \frac{a}{Y}\left(\hat{a}_t - \frac{1}{a}\frac{\chi}{\chi^*}b_t\right) + (1-\delta)\frac{k}{Y}\hat{k}_t - \frac{d}{Y}\hat{d}_t\end{aligned} \tag{4.53}$$

Optimal decisions of the household are described by the Euler equation (4.46) and by a standard New Keynesian Phillips curve (4.51).

[13] Although non-zero steady state holdings of foreign assets seems to be the empirical case (see eg. Lane and Milesi-Ferretti (2002)) the simplification doesn´t alter our results.

For the other country the corresponding equations are

$$\hat{C}_t^* = \mathbb{E}_t\hat{C}_{t+1}^* + \frac{1}{\sigma}\left(\mathbb{E}_t\hat{\Pi}_{Ft+1}^* - \hat{\imath}_t^a - \frac{\chi}{2}b_{t+1}\right) - \frac{1}{\sigma}\gamma^*\mathbb{E}_t\left(\hat{\Pi}_{Ft+1}^* - \hat{\Pi}_{Ht+1}\right) \tag{4.54}$$

$$\begin{aligned}
\hat{\lambda}_t^* &= \Xi^*\mu^*\left(\hat{\mu}_t^* + \hat{\xi}_t^*\right) + (1-\mu^*\Xi^*)\,\sigma\hat{C}_t^* \\
&\quad + \mathbb{E}_t\left(\beta(1-\mu^*)X^*\theta\frac{Y^*}{k^*} - (1-\mu^*\Xi^*)\right)\sigma\hat{C}_{t+1}^* \\
&\quad + \gamma^*(1-\mu^*\Xi^*)\,\mathbb{E}_t\left(\hat{\Pi}_{Ft+1}^* - \hat{\Pi}_{Ht+1}\right) + \beta\mathbb{E}_t\left(X^*\theta\frac{Y^*}{k^*}(1-\mu^*) + (1-\delta)\right)\hat{\lambda}_{t+1}^* \\
&\quad + \beta(1-\mu^*)X^*\theta\frac{Y^*}{k^*}\left((\psi+1)\,\mathbb{E}_t\hat{n}_{t+1}^* - \gamma^*\left(\mathbb{E}_t\hat{\Pi}_{Ft+1}^* - \mathbb{E}_t\hat{\Pi}_{Ht+1} + \hat{S}_t\right) - \hat{k}_{t+1}^*\right)
\end{aligned} \tag{4.55}$$

$$\begin{aligned}
\mathbb{E}_t\hat{\lambda}_{t+1}^* &= (\mu^*\Xi^*+1)\left(\hat{\lambda}_t^* + \frac{i^*\tau^*}{R^*}\hat{\tau}_t^*\right) + \left(\mu^*\Xi^* - \frac{(\mu^*\Xi^*+1)(1-\tau^*)}{\beta R^*}\right)\left(\hat{\imath}_t^a + \frac{\chi}{2}b_{t+1}\right) \\
&\quad - \mu^*\Xi^*\left(\hat{\mu}_t^* + \hat{\xi}_t^*\right) + \sigma\left(\mathbb{E}_t\hat{C}_{t+1}^* - \hat{C}_t^*\right) \\
&\quad - \gamma^*\mathbb{E}_t\left(\hat{\Pi}_{Ft+1}^* - \hat{\Pi}_{Ht+1}\right) + \mathbb{E}_t\hat{\Pi}_{Ft+1}^*
\end{aligned} \tag{4.56}$$

$$\hat{\lambda}_t^* = 2\beta\kappa^*\left(\mathbb{E}_t\hat{d}_{t+1}^* - \hat{d}_t^* + \mathbb{E}_t\hat{\Pi}_{Ft+1}^*\right) - 2\kappa^*\left(\hat{d}_t^* - \hat{d}_{t-1}^* + \hat{\Pi}_{Ft}^*\right) \tag{4.57}$$

$$\begin{aligned}
&\Xi^*\frac{k^*}{Y^*}\left(\hat{\xi}_t^* + \hat{k}_{t+1}^*\right) - \beta\Xi^*\frac{b^*}{Y^*}\left(\hat{\xi}_t^* + \hat{b}_{t+1}^* + \left(\frac{1}{b^*} - \frac{\chi}{2}\right)b_{t+1} + \mathbb{E}_t\hat{\Pi}_{Ft+1}^* - \hat{\imath}_t^a\right) \\
&= X^*\left((\psi+1)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w} - \gamma^*\hat{S}_t + \frac{\mu}{(1-\mu)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right)\right)
\end{aligned} \tag{4.58}$$

$$\begin{aligned}
\hat{\Pi}_{Ft}^* &= \beta\mathbb{E}_t\hat{\Pi}_{Ft+1}^* + \frac{(\varepsilon-1)(1-\tau^{*x})}{\omega}\left(\frac{\tau^{*x}}{(1-\tau^{*x})}\hat{\tau}_t^{*x} + \frac{\mu^*}{(1-\mu^*)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right)\right. \\
&\quad \left. + (\psi+\theta)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w} - \hat{z}_t^* - \theta\hat{k}_t^* - \gamma^*\hat{S}_t\right)
\end{aligned} \tag{4.59}$$

$$\hat{Y}_t^* = \frac{k^*}{Y^*}\hat{k}_{t+1}^* - \beta\left(\frac{\chi}{\chi^*}+1\right)\frac{b_{t+1}}{Y^*} + \left(\frac{\chi}{\chi^*}+1\right)\frac{b_t}{Y^*} + \frac{C^*}{Y^*}\left(\hat{C}_t^* - \gamma^*\hat{S}_t\right) - (1-\delta)\frac{k^*}{Y^*}\hat{k}_t^* + \frac{G^*}{Y^*}\hat{G}_t^* \tag{4.60}$$

$$\begin{aligned}
\frac{k^*}{Y^*}\hat{k}_{t+1}^* &= X^*\left((\psi+1)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w} - \gamma^*\hat{S}_t + \frac{\mu}{(1-\mu)}\left(\hat{\mu}_t^* - \hat{\lambda}_t^*\right)\right) \\
&\quad + \frac{b^*}{R^*Y^*}\left(\hat{b}_{t+1}^* + \frac{b_{t+1}}{b^*} + \mathbb{E}_t\hat{\Pi}_{Ft+1}^* - \frac{(1+i^*)(1-\tau^*)}{R^*}\left(\hat{\imath}_t^a + \frac{\chi}{2}b_{t+1}\right) + \frac{i^*\tau^*}{R^*}\hat{\tau}_t^*\right) \\
&\quad - \frac{b^*}{Y^*}\left(\frac{b_t}{b^*} + \hat{b}_t^*\right) + (1-\delta)\frac{k^*}{Y^*}\hat{k}_t^* - \frac{d^*}{Y^*}\hat{d}_t^* \\
&\quad - w^*\frac{n^*}{Y^*}\left((\psi+1)\,\hat{n}_t^* + \sigma\hat{C}_t^* + \frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w} - \gamma^*\hat{S}_t\right)
\end{aligned} \tag{4.61}$$

$$\begin{aligned}
&\left(\eta(1-\gamma^*)\gamma^*\frac{C^*}{Y^*}\frac{Y^*}{Y} - ((1-\eta)(1-\gamma)+\gamma)\gamma\frac{C}{Y}\right)\hat{S}_t \\
&= \gamma\frac{C}{Y}\hat{C}_t - \gamma^*\frac{C^*}{Y^*}\frac{Y^*}{Y}\hat{C}_t^* + \left(\frac{\chi}{\chi^*}+1\right)\beta\frac{b_{t+1}}{Y} - \left(\frac{\chi}{\chi^*}+1\right)\frac{b_t}{Y}
\end{aligned} \tag{4.62}$$

$$\hat{S}_t - \hat{S}_{t-1} = \hat{\Pi}^*_{Ft} - \hat{\Pi}_{Ht} \tag{4.63}$$

$$G\hat{G}_t = \tau^w wN\left(\hat{\tau}^w + \hat{w}_t + \hat{N}_t\right) + \tau^d d\left(\hat{\tau}^d + \hat{d}_t\right) + \tau^x Y\left(\hat{\tau}^x + \hat{Y}_t\right) \tag{4.64}$$

Where $\hat{\imath}^a_t = \dfrac{\hat{\imath}_t + \hat{\imath}^*_t}{2}$. The linearisation of equations and steady states are given in Appendix B.5 and B.4.

4.4 Calibration

The model is calibrated to a quarterly frequency.[14] We fix $\beta = 0.9825$. The capital depreciation rate is set to $\delta = 0.025$. The capital ratio in production function is set to $\theta = 0.36$, and the mean value of A is normalized to 1. The tax wedge which corresponds to the advantage of debt over equity is determined to be $\tau = 0.35$, and the dividend adjustment cost parameter set to $\kappa = 0.146$ as in Jermann and Quadrini (2012).

We calibrate the steady state debt to output ratio to match the data. The quarterly ratio of debt to output for the non-financial business sector is 3.25 over the sample period 1984:I-2010:II, see the top panel in Figure 4.1. In order to match that, we set the steady state value of the financial variable, Ξ , to 0.1634.[15]

Parameters of the household utility function are determined as follows. The calibration of the Frisch intertemporal elasticity of substitution in labor supply, ψ, is assumed to be equal to 1 and the risk aversion parameter is: $\sigma = 1$. The relative weight on the disutility of labour, $\alpha = 1.8834$, is chosen so as to set steady state hours worked equal to 0.3.

We calibrate the measure of price stickiness, $\omega = 80$, in a way that corresponds to a probability of firms changing prices every 3 quarters in a corresponding Calvo model. The elasticity of substitution between any pair of goods ε is equal to 11 in steady state which gives a 10% mark up.

Parameters of the policy objective function are chosen to be $\vartheta_y = 0.3$, $\vartheta_g = 0.01$ and $\vartheta_s = 0.1$, see Chen et al. (2014).[16]

It remains to calibrate the shock and the initial states to simulate the scenarios of interest. The second panel in Figure 4.1 plots the historical data of corporate debt to output ratio (quarterly). The average value of this ratio during 1984-2009 is 3.25. The peak of 3.87 in 2008 was somewhat above the average value, and the consequent reduction

[14]Note that both countries are symmetric and we only include calibration of Home country.
[15]Data sources: NIPA and FoF tables. The calculations follow Jermann and Quadrini (2012).
[16]The results are very robust to wide range of parameters ϑ_y and ϑ_s between zero and one.

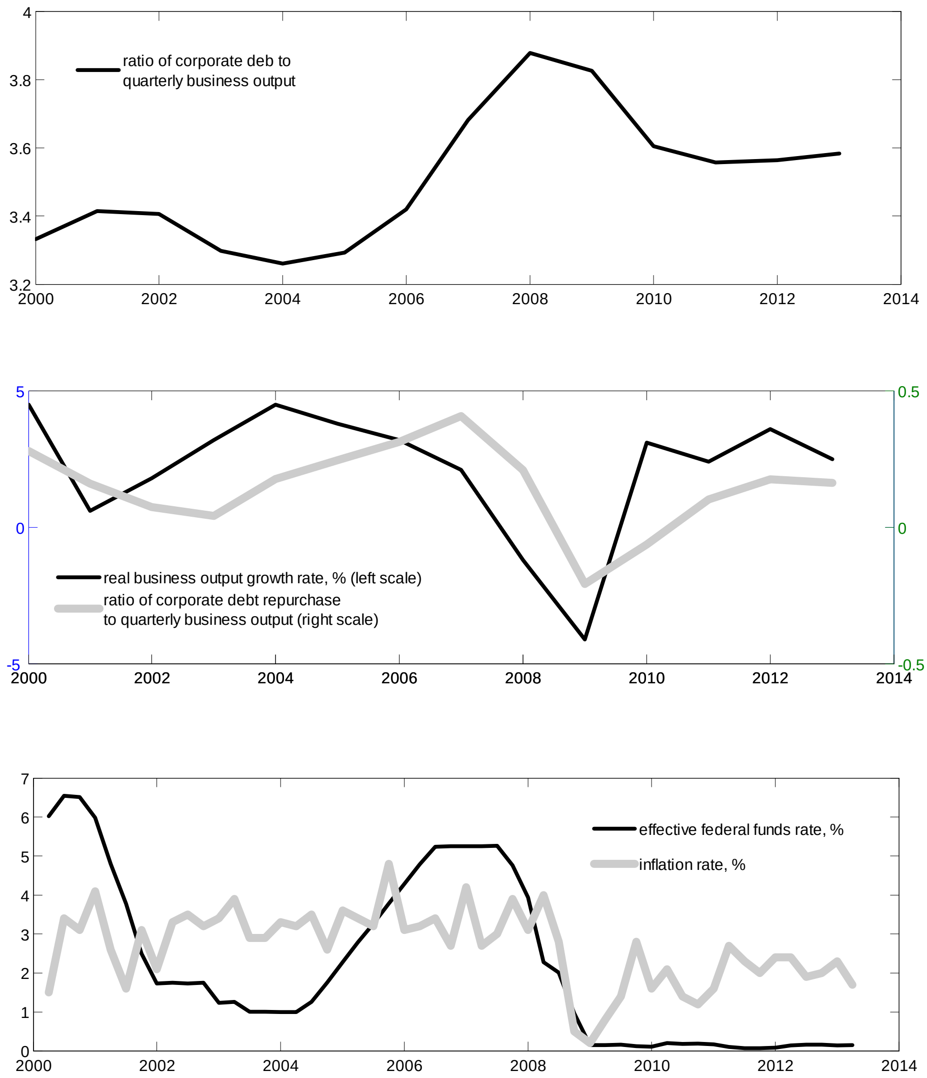


Figure 4.1: Historical data in the US. Data sources: NIPA and FoF tables.

to 3.55 in 2011 constitutes a reduction of about 10% relative to its peak. We use these numbers as a guide to our simulations.

Based on this evidence, we consider an AR(1) credit shock $\hat{\xi}_t = \rho\hat{\xi}_{t-1} + \varepsilon_t$ with persistence $\rho = 0.95$, and we examine the dynamic implications of a negative 10% innovation in ε_t.

The intertemporal elasticity of substitution between domestic and foreign goods η is set to 2, which lies in the values assumed in the RBC literature $(1-2)$ and the degree of trade openness ϖ and ϖ^* are set to 0.3. The share of government spending to GDP, $\frac{G}{Y}$ and $\frac{G^*}{Y^*}$, are set to 0.20 for each country, in line with the EMU data. The relative size of each country is calibrated depending on the nature of the analysis. In the main case of interest, labelled base line case, with 'large foreign' and 'small home' countries the home country has size $n = 0.3$, as the relative size in terms of population or employment of Greece, Ireland, Italy and Portugal is about one third of the total population of the EMU countries. However, we also consider two identical countries with $n = 0.5$ when we discuss transmission mechanisms. As in Benigno (2009), Ghironi et al. (2006), we assume that the costs of changing the asset holdings with respect to the steady state are such that $\chi = \chi^* = 0.01$.[17]

4.5 Policy Objectives

Monetary policy is assumed to behave optimally, minimizing an *ad hoc* welfare loss, given by the objective

$$U = \sum_{t=0}^{\infty} \beta^t V_t \tag{4.65}$$

with the flow objective

$$V_t = n\hat{\Pi}_{Ht}^2 + n\vartheta_g\hat{G}_t^2 + n\vartheta_y\hat{Y}_t^2 + (1-n)\hat{\Pi}_{Ft}^{*2} + (1-n)\vartheta_g\hat{G}_t^{*2} + (1-n)\vartheta_y\hat{Y}_t^{*2} + \vartheta_s\hat{S}_t^2 \tag{4.66}$$

The union-wide monetary policymaker minimises the union-wide inflation, output, terms of trade and there is also volatility of government expenditures. One reason to include terms in G is that we have assumed that the private sector values consumption of public goods, and this is also reflected in policymaker's objectives. Additionally, we will study cooperative monetary and fiscal policy, and restricting volatility of fiscal instrument could be an important stabilising factor. We assume this *ad hoc* objective but we also check

[17] Kollmann (2003) has used Lane and Milesi-Ferretti (2002) estimates on the relationship between real interest rate differentials and net foreign asset position. He assumes a value of 0.0019 in a case in which the net foreign asset position is normalized by exports. In our case, since the net foreign asset position is normalized by quarterly GDP, with an export/GDP ratio of 15%, a value of 0.0019 implies a value for χ equal to 0.012, which is consistent the calibration that we use.

robustness of our results to a change of government spendings coefficients, $\vartheta_g, \vartheta_g^*$ specifically.

Given the loss function (equation (4.66)) we solve for the linear–quadratic optimal policy problem. The shock produces a dynamic path of deleveraging which depends endogenously on policy. Objective function in this form was used in Benigno and Benigno (2006) and Clarida et al. (2002) and Corsetti et al. (2011).

We assume that the monetary authority choose the average interest rates (i_t^a) and fiscal policy makers of both countries choose taxes (τ_t^x , τ_t^{x*}) cooperatively to minimize the welfare loss (4.65) subject to the system (4.46)–(4.62).

4.6 Results

4.6.1 Transmission mechanism

In this section, we examine the transmission mechanism of symmetric and asymmetric credit shocks. We start with two identical countries (e.g. $n = 0.5$). We compare our results with closed economy model. Next we discuss the country-size effects. We also discuss the welfare consequences of our results. Since our welfare objective function is not micro-founded, we check the robustness of our results using different values for government spending coefficient ($\vartheta_g = \vartheta_g^*$).

Country-size symmetry and symmetric credit shock

In Figure (4.2), we observe the impulse responses of both equal size economies ($n = 0.5$) to negative 10% credit shocks in home and foreign countries. Such negative shocks have the same consequences for both home and foreign countries: it reduces the proportion of output which banks will be able to recover in case of default. Banks lend to firms at the beginning of the period, so that firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the negative credit shock reduces the probability of recovery, the amount of bank lending falls. Firms which are not able to obtain funds up front have to deleverage or reduce the value of production. Firms reduce labour, produce less output and also pay lower wages, see Figure (4.2). As a result of a symmetric credit shock, terms of trade and foreign bonds become zero (see equation (4.63) and equation (4.44)). To fight the upcoming inflation sales tax (τ_t^x , τ_t^{x*}) rises and nominal interest rate falls. Consumption falls. Both constraints are tighter as the Figure shows. Inflation falls initially following a shock (see also equation (4.51)). Low interest rates result in falling consumption profile

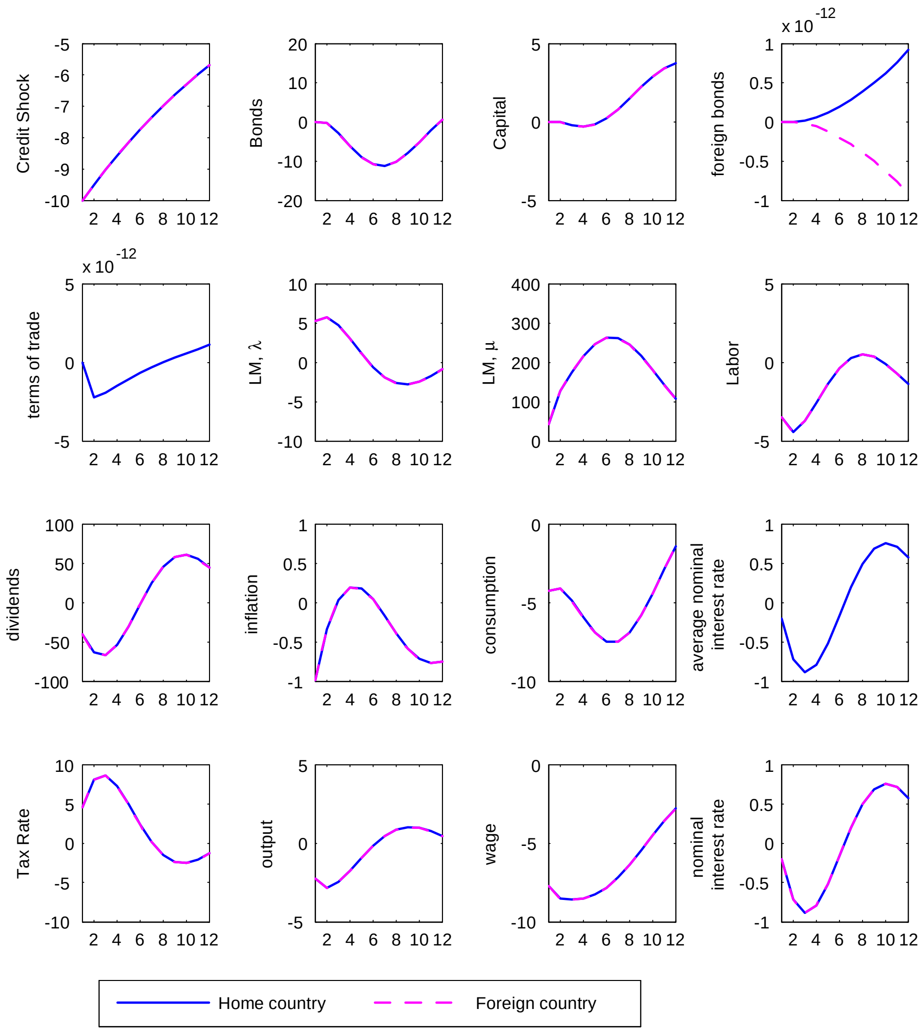


Figure 4.2: Impulse responses of symmetric countries to a symmetric credit shock in fixed exchange rate regime

over time (see also equation (4.52) and equation (4.46)). Firms reduce the amount of borrowing. Lower interest rate also makes it easier for banks to pay out the existing debt, so it helps to reduce the debt quickly. Output falls by less than wages, profits of firms fall and so dividends fall. The adjustment goes with overshooting as capital changes only slowly and it continues to fall while the effect of persistent shock disappears. Results are similar to closed economy model (see the Appendix (B.6)). However, there are several important differences between this model and the one in the second chapter. Here we assume that there is optimising fiscal policy, constrained by balanced budget, while in the second chapter only automatic stabilisers work as spending and taxes are kept at their steady state levels. Second, we study policy under commitment, while the second chapter studies policy under discretion. Both these factors change the timing of adjustment, although the direction remains the same as in closed economy considered in the second chapter.

Country-size symmetry and asymmetric credit shock

In Figure (4.3), we plot the impulse responses of both equal size economies ($n = 0.5$) to negative 10% credit shock in home country and a positive 10% credit shock in foreign country. We explain the impulse responses of home country to a negative shock, since foreign responses is exactly asymmetric to home responses and we skip adding that. Such negative shock reduces the proportion of output which banks will be able to recover in case of default. Banks lend to firms at the beginning of the period, so that firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the negative credit shock reduces the probability of recovery, the amount of bank lending falls. Firms which are not able to obtain funds up front have to deleverage or reduce the value of production. Firms reduce labour, produce less output and also pay lower wages, see Figure (4.3). Because of fixed exchange rate, average interest rate is zero but domestic interest rate falls gradually over time as foreign bonds rises (see equation (4.44)). Domestic consumption falls initially but the reduction is not as low as needed to explain the reduction in output. Domestic inflation rises initially while nominal interest rate is zero (see equation (4.44)). Then terms of trade falls (see equation (4.63)). To fight inflation, domestic sales tax (τ_t^x) rises initially. Both constraints are tighter as the Figure shows. Firms reduce the amount of borrowing. Lower interest rate also makes it easier for banks to pay out the existing debt, so it helps to reduce the debt quickly. Taxes play some of interest rate role. Under floating exchange rate regime interest rate falls to increase inflation and raise consumption, under fixed exchange rate taxes rise. As government budget is balanced, fiscal policy increases spending, increasing domestic output. However, the nominal exchange rate cannot adjust and the terms of trade is only changing because prices change, so the initial appreciation is not as big as under floating exchange rate regime. As a result, there is no reduction in foreign asset holdings. Output falls by less than wages, profits of firms fall and so dividends fall. The

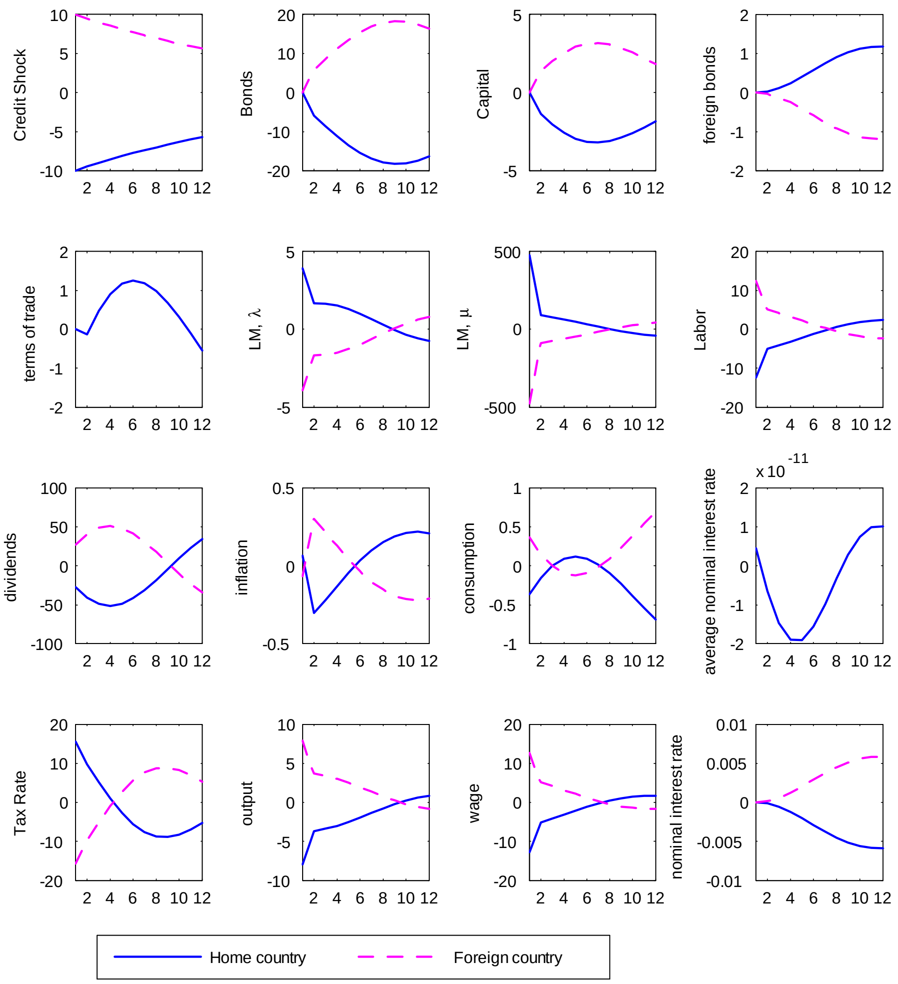


Figure 4.3: Impulse responses of symmetric countries to an asymmetric credit shock in fixed exchange rate regime

adjustment goes with overshooting as capital changes only slowly and it continues to fall while the effect of persistent shock disappears.

Country-size asymmetry and asymmetric credit shock

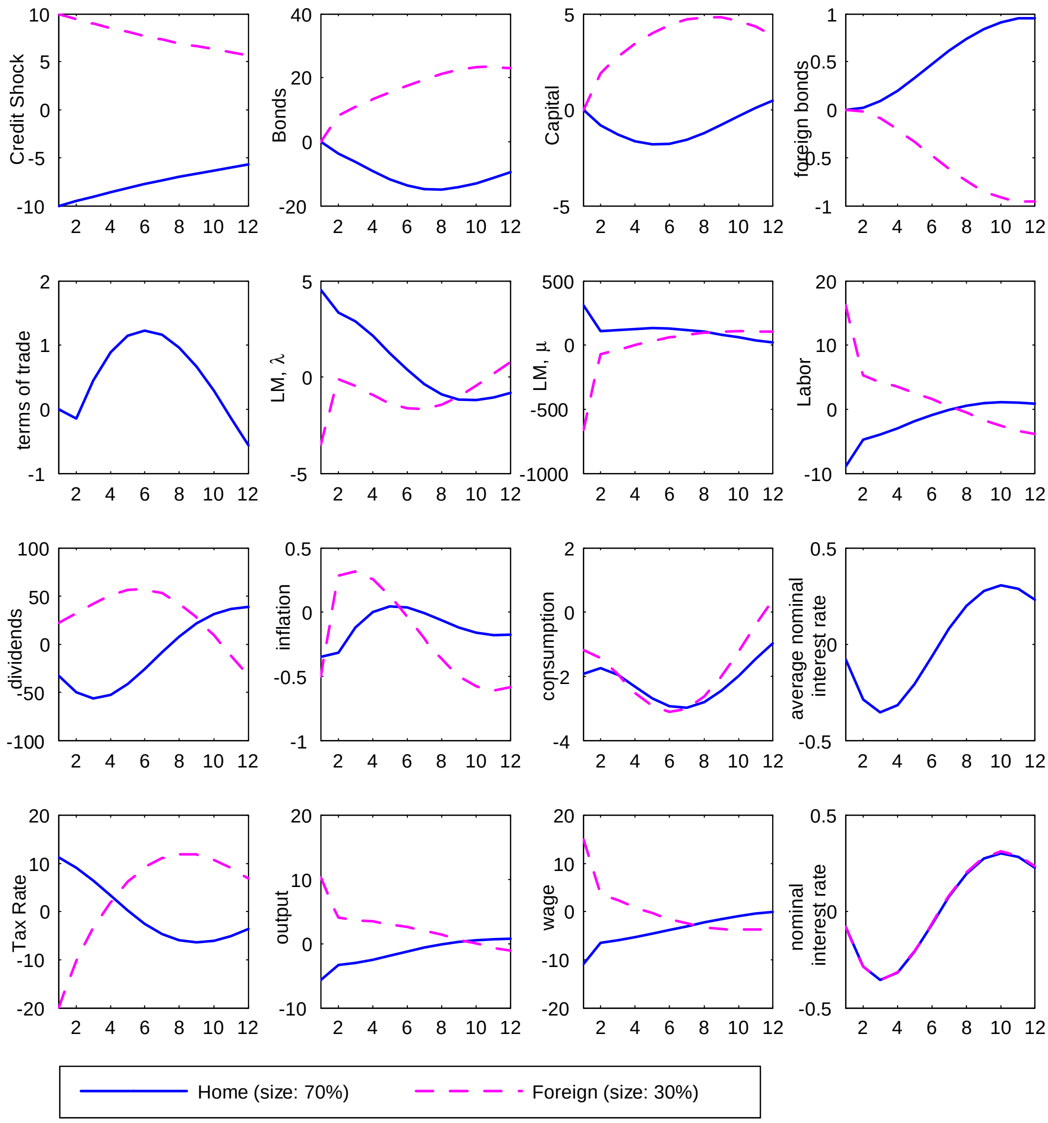


Figure 4.4: Impulse responses of asymmetric-size countries ($n = 0.7$) to asymmetric credit shocks in a fixed exchange rate regime

In Figure (4.4), we observe the impulse responses of two different size economies ($n = 0.7$) to negative 10% credit shock in home country and positive 10% credit shock in foreign country in a fixed exchange rate regime. We compare our results with asymmetric credit shocks in two identical country model (see Figure (4.3)). The smaller country is more affected as a result of these shocks as we see in the Figure (4.4): Domestic import share ($\gamma = (1 - n)\varpi$) is smaller, then a negative credit shock reduces the proportion of

Variances	$n = 0.7$	$n = 0.5$
$\hat{Y}_t \times 10^2$	0.69	1.46
$\hat{Y}_t^* \times 10^2$	3.63	1.46
$\hat{\Pi}_{Ht} \times 10^2$	0.03	0.05
$\hat{\Pi}_{Ft}^* \times 10^2$	0.12	0.05
$\hat{G}_t \times 10^2$	5.94	13.2
$\hat{G}_t^* \times 10^2$	35.0	13.2
$\hat{S}_t \times 10^2$	0.27	0.29
Loss (W)$\times 10^2$	0.70	0.65
Loss (H)$\times 10^2$	0.33	0.65
Loss (F)$\times 10^2$	1.59	0.65

Table 4.1: Volatilities of output, inflation, government spending, terms of trade and unconditional welfare loss for symmetric (third column) and asymmetric size (second column) countries due to a credit shock

domestic output by less.[18] Banks lend to firms at the beginning of the period, so that firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the negative credit shock reduces the probability of recovery, the amount of bank lending also falls by less. Firms which are not able to obtain funds up front have to deleverage or reduce the value of production. Firms reduce labour, produce less output and also pay lower wages, see Figure (4.4). Besides, lower import share, γ, indicates that foreign goods are cheaper (see equation (4.51)). Then as a result of higher import and outflow of funds, terms of trade falls by less. To fight inflation domestic tax rises by less, so domestic inflation and domestic consumption fall by more (see equation (4.51) and equation (4.46) and also Figure (4.3)). Low interest rates result in falling consumption profile over time (see also equation (4.52) and equation (4.46)). Low interest rate also makes it easier for banks to pay out the existing debt, so it helps to reduce the debt quickly. Output falls by less than wages, profits of firms fall and so dividends fall. The adjustment goes with overshooting as capital changes only slowly and it continues to fall while the effect of persistent shock disappears.

And for the same reason, foreign import share $\gamma^* = n\varpi^*$ is higher. Then a positive credit shock increases the proportion of foreign output by more.[19] Banks lend to foreign firms at the beginning of the period, so that foreign firms are able to pay wages. As the enforcement constraint is always binding, the difference between bonds and capital is covered by a loan. As the positive credit shock rises the probability of recovery, the amount of bank lending also rises by more. Foreign firms which have enough funds up front, raise their debt or raise the value of production. Foreign firms raise labour, produce more output and also pay more wages, see Figure (4.4). To fight inflation foreign tax falls by more, so foreign inflation and foreign consumption fall by more (see equation (4.59) and equation (4.54) and also Figure (4.3)). Low interest rates result in falling consumption profile over time (see also equation (4.60) and equation (4.54)). Foreign output rises by

[18] The proportion of domestic output which banks will be able to recover in case of default.
[19] The proportion of foreign output which banks will be able to recover in case of default.

less than wages, profits of firms rise and so dividends rise. The adjustment goes with overshooting as capital changes only slowly and it continues to rise while the effect of persistent shock disappears.

4.6.2 Robustness analysis

Variance	$\vartheta_g = 0$	$\vartheta_g = 0.01$	$\vartheta_g = 1$
Output, $\hat{Y}_t$	0.008	0.015	0.027
Inflation, $\hat{\Pi}_{Ht}$	0.0002	0.0004	0.0006
Terms of trade, $\hat{S}_t$	0.0027	0.0029	0.0071
Consumption, $\hat{C}_t$	0.212	0.041	0.043
Government spending, $\hat{G}$	3.058	0.132	0.008
Domestic assets, $\hat{a}$	0.578	0.43	0.389
Capital, $\hat{k}$	0.019	0.024	0.029
Foreign assets, $\hat{b}$	0.012	0.006	0.018
BC Lagrange multiplier, $\hat{\lambda}$	0.011	0.016	0.023
EC Lagrange multiplier, $\hat{\mu}$	46.00	77.29	186.23
Employment, $\hat{n}$	0.012	0.036	0.069
Dividend payout, $\hat{d}$	13.949	6.532	4.937
Average interest rate, $\hat{\imath}^a \times 10^2$	0.01	0.025	0.029
Loss	0.034	0.007	0.01

Table 4.2: Volatilities of given variables and unconditional welfare loss for different value of penalty on deviation of government spending due to a credit shock assuming two symmetric size countries

Variance	$\chi = 0.001$	$\chi = 0.01$	$\chi = 0.1$
$\hat{Y}_t \times 10^2$	1.48	1.46	1.48
$\hat{Y}_t^* \times 10^2$	1.48	1.46	1.48
$\hat{\Pi}_{Ht} \times 10^2$	0.04862	0.04856	0.04956
$\hat{\Pi}_{Ft}^* \times 10^2$	0.04862	0.04856	0.04956
$\hat{G}_t \times 10^2$	13.94	13.2	13.68
$\hat{G}_t^* \times 10^2$	13.94	13.2	13.68
$\hat{S}_t \times 10^2$	0.2900	0.2900	0.300
Loss (W)$\times 10^2$	0.6600	0.6500	0.6600
Loss (H)$\times 10^2$	0.6600	0.6500	0.6600
Loss (F)$\times 10^2$	0.6600	0.6500	0.6600

Table 4.3: Volatilities of output, inflation, government spending, terms of trade and unconditional welfare loss for different value of intermediation cost due to a credit shock assuming two symmetric size countries

How do the results change under different penalty on government spending deviations, ϑ_g or different intermediation cost, χ? We address this question through different assumptions on the value of penalty on government spending deviations, ϑ_g and on intermediation costs, χ (see Figure (4.5)): The true social welfare loss is with $\vartheta_g = 0.01$. When ϑ_g is 0, then there is very high volatility of G which is penalised by the social objective. When

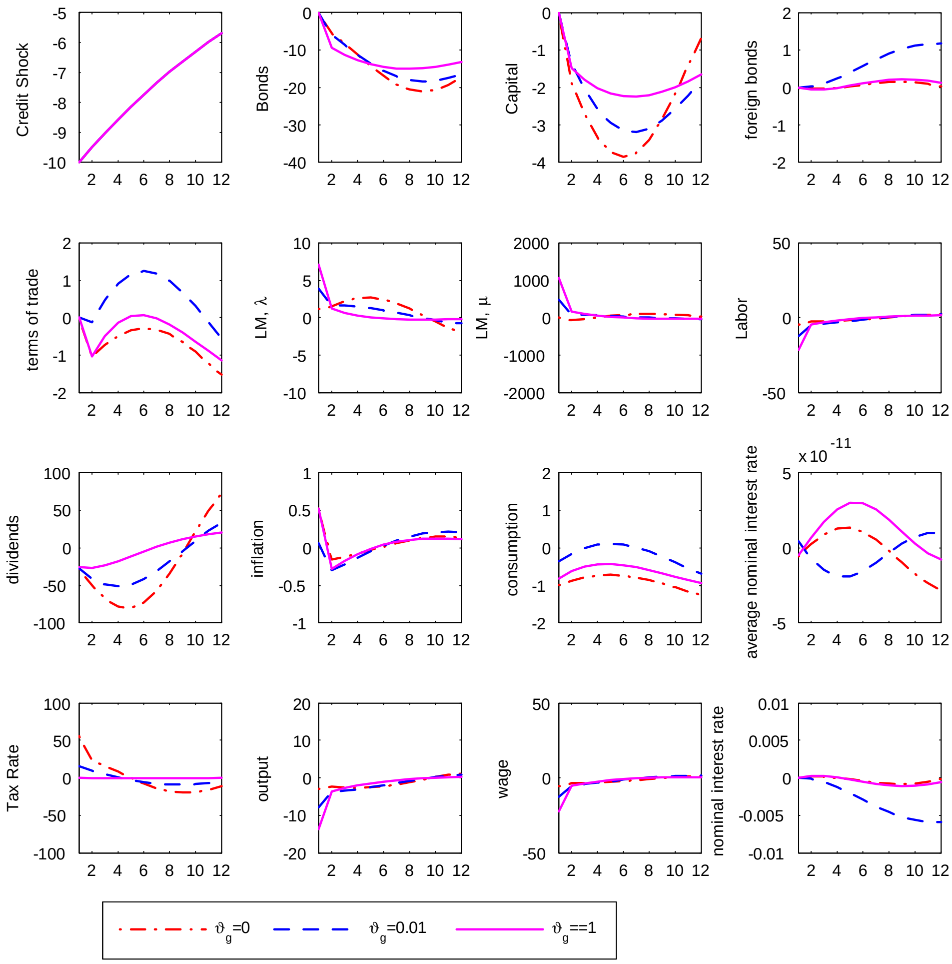


Figure 4.5: Robustness analysis for symmetric-size countries, and asymmetric credit shocks

$\vartheta_g = 1$, then G is not flexible enough to stabilise the economy well. Therefore it is not surprising that $\vartheta_g = 0.01$ delivers the best result as measured by welfare objective. Table (4.2) also demonstrates how volatility of different welfare components change with ϑ_g. With greater penalty fiscal policy is more constrained to stabilise the economy, volatility of most economic variables rises. If the penalty on G is further increased, then at some threshold value the economic behavior does not change. We know that if G is not volatile at all then the economy is unstable, so that a solution which stabilises the economy cannot be found. However, any finite penalty on G allows G to stabilise the economy, although at great costs. Table (4.3) and (4.5) show that the results are similar to the one under the flexible exchange rate. Greater financial integration is more costly under credit shocks.

Variance	$\vartheta_g = 0$	$\vartheta_g = 0.01$	$\vartheta_g = 1$
$\hat{Y}_t \times 10^2$	0.26	0.69	1.17
$\hat{Y}_t^* \times 10^2$	2.73	3.63	6.97
$\hat{\Pi}_{Ht} \times 10^2$	0.01	0.03	0.04
$\hat{\Pi}_{Ft}^* \times 10^2$	0.07	0.1	0.17
$\hat{G}_t \times 10^2$	0.23	0.27	0.69
$\hat{G}_t^* \times 10^2$	213.4	5.94	0.21
$\hat{S}_t \times 10^2$	640.8	34.97	2.7
Loss (W)$\times 10^2$	3.77	0.7	1.03
Loss (H)$\times 10^2$	2.25	0.33	0.47
Loss (F)$\times 10^2$	7.31	1.59	2.34

Table 4.4: Volatilities of output, inflation, government spending, terms of trade and unconditional welfare loss for different value of penalty on deviation of government spending due to a credit shock assuming two asymmetric size (n=0.7) countries

Variance	$\chi = 0.001$	$\chi = 0.01$	$\chi = 0.1$
$\hat{Y}_t \times 10^2$	0.6857	0.6857	0.6857
$\hat{Y}_t^* \times 10^2$	3.67	3.63	3.7
$\hat{\Pi}_{Ht} \times 10^2$	0.0344	0.0343	0.0342
$\hat{\Pi}_{Ft}^* \times 10^2$	0.116	0.117	0.12
$\hat{G}_t \times 10^2$	6.143	5.94	6.13
$\hat{G}_t^* \times 10^2$	38.37	34.97	35.73
$\hat{S}_t \times 10^2$	0.2700	0.2700	0.2800
Loss (W)$\times 10^2$	0.7200	0.7000	0.7200
Loss (H)$\times 10^2$	0.3300	0.3300	0.3300
Loss (F)$\times 10^2$	1.6300	1.5900	1.6200

Table 4.5: Volatilities of output, inflation, government spending, terms of trade and unconditional welfare loss for different value of intermediation cost due to a credit shock assuming two asymmetric size countries, n=0.7

4.6.3 Contagion

In Figure (4.6), we plot the impulse responses of two identical economies ($n = 0.5$) to only a negative 10% credit shock in home country in a fixed exchange rate regime. Compared to

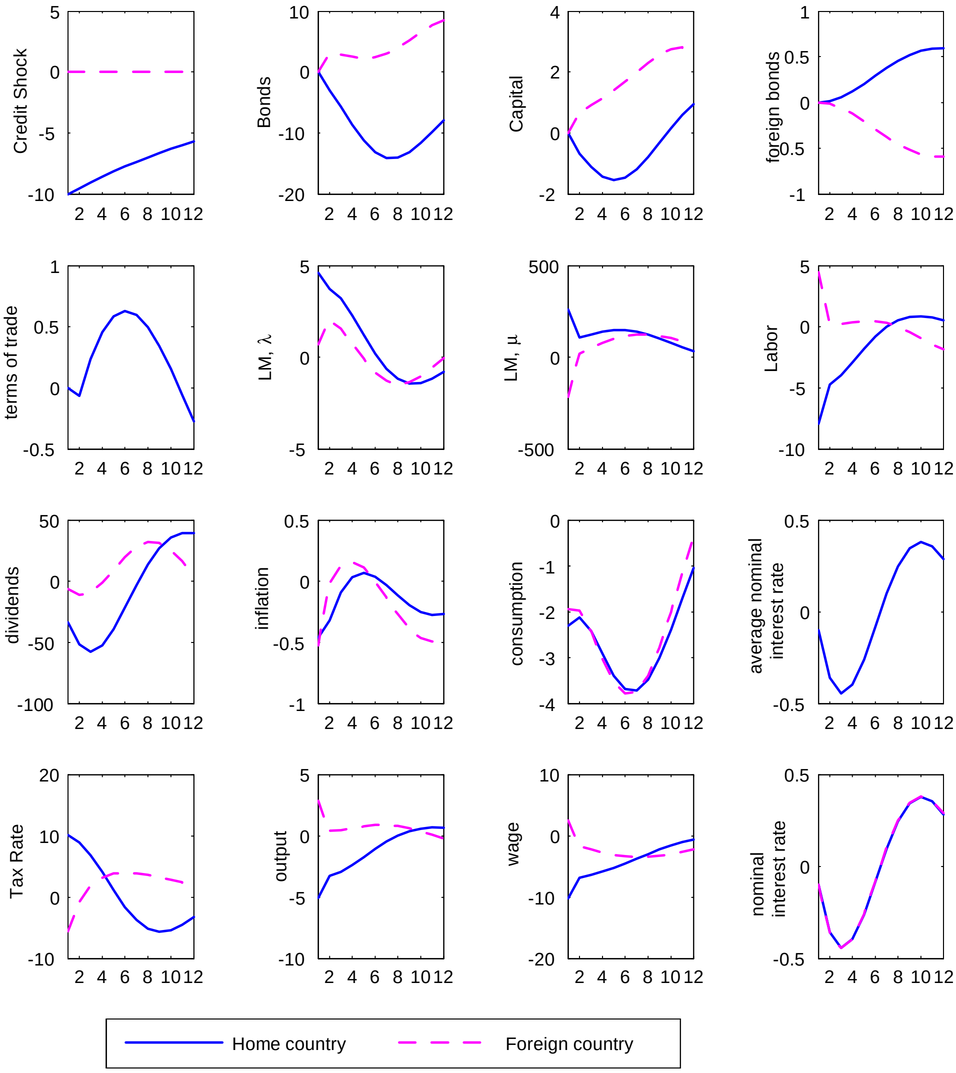


Figure 4.6: Impulse responses of identical countries to a non-symmetric credit shock in fixed exchange rate regime

the floating exchange rate regime there is substantial effect for foreign country. Home and foreign consumption fall at great extent, almost with perfect correlation. Output however is much less correlated, with reduction in foreign output offset by increase in taxes and in government expenditures in the medium-run. The reduction in interest rate helps to stabilise inflation and consumption. Terms of trade appreciates at home, but depreciates in the other country in the first couple of periods after the shock. As a result, consumption of home residents switches to foreign-produced goods, keeping output at foreign country high in first initial periods. Higher output results in higher income abroad.

4.7 Conclusion

We demonstrate that, unlike under flexible exchange rate regime studied in the previous chapter, the centralised monetary policy alone is unable to stabilise the economy. National fiscal policies must be activated to counteract asymmetric shocks. We demonstrate, however, that the effectiveness of fiscal policy is limited. Even if it is chosen optimally, fiscal policy does not eliminate cyclical patterns in economic adjustment, which is welfare-reducing volatility of economic variables.

This model demonstrates that shocks hitting one economy, result in sharp contraction of consumption in another country. Countercyclical fiscal policy is able to avoid major recession, however. In contrast to results in the previous chapter, the shocks propagation mechanism is much stronger under fixed exchange rate regime. As before, we assume variable degree of financial integration and study its importance for the propagation of credit shocks. Since the objective function is not microfounded, we assume variable degree of penalty on government spending deviations. With greater penalty fiscal policy is more constrained to stabilise the economy, volatility of most economic variables rises. If the penalty further increased, then at some threshold value the economic behavior does not change. However, any finite penalty on government spending allows it to stabilise the economy, although at great costs.

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Appendix A

Deep Recession

A.1 Household's Optimisation Problem

The Lagrangian can be written as

$$\begin{aligned}\Gamma_t &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, n_t) + \Lambda_t \left((1-\tau_t^w) \frac{W_t}{P_t} n_t + b_t + s_t \frac{\left((1-\tau_t^d) D_t + \bar{p}_t \right)}{P_t} + \Phi_t \right.\right. \\ &\left.\left. -q_t b_{t+1} - s_{t+1} \frac{\bar{p}_t}{P_t} - C_t - T_t \right) \right\}\end{aligned}$$

And the first-order conditions are taken with respect to C_t, n_t , b_{t+1}, s_{t+1} and Λ_t.

$$U_C(C_t, n_t) = \Lambda_t$$

$$U_n(C_t, n_t) + \Lambda_t (1-\tau_t^w) w_t = 0$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}$$

$$\Lambda_t \frac{\bar{p}_t}{P_t} = \beta \mathbb{E}_t \Lambda_{t+1} \frac{\left((1-\tau_{t+1}^d) D_{t+1} + \bar{p}_{t+1} \right)}{P_{t+1}}$$

$$(1-\tau_t^w) \frac{W_t}{P_t} n_t + b_t + s_t \frac{\left((1-\tau_t^d) D_t + \bar{p}_t \right)}{P_t} + \Phi_t = q_t b_{t+1} + s_{t+1} \frac{\bar{p}_t}{P_t} + C_t + T_t$$

FOCs in real term are

$$U_C(C_t, n_t) = \Lambda_t$$

$$\alpha n_t^{\psi} = (1-\tau_t^w) w_t C_t^{-\sigma}$$

$$\frac{1}{1+r_t} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}$$

$$p_{St} = \frac{\bar{p}_t}{P_t} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \left((1-\tau_{t+1}^d) d_{t+1} + p_{St+1} \right)$$

$$(1-\tau_t^w) w_t n_t + b_t + s_t \left((1-\tau_t^d) d_t + p_{St} \right) + \Phi_t = q_t b_{t+1} + s_{t+1} p_{St} + C_t + T_t$$

where $p_{St} = \frac{\bar{p}_t}{P_t}$ and $d_t = \frac{D_t}{P_t}$ and we normalise the number of shares to be equal to one (See Jermann and Quadrini (2012)).

A.2 Firms'Optimisation Problems

A.2.1 Intermediate Goods Producers

The firm's optimisation problem subject to 2.8 and 2.9 and the Lagrangian can be written as follows:

$$
\begin{aligned}
L \quad = \quad & \sum_{t=0}^{\infty} m_{0,t} \left(\frac{D_t}{P_t} + \mu_t \left(\Xi e^{\xi_t} \left(Q_t k_{t+1} - q_t b_{t+1} \right) - \frac{P_{mt}}{P_t} F(e^{z_t}, k_t, n_t) \right) \right. \\
& + \lambda_t \left(Q_t \left(1 - \delta\right) k_t + \frac{P_{mt}}{P_t} F(e^{z_t}, k_t, n_t) + \frac{b_{t+1}}{R_t} - b_t - \frac{W_t}{P_t} n_t \right. \\
& \left. \left. - Q_t \frac{k_{t+1}}{\Omega_{t+1}} - \left(\frac{D_t}{P_t} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \frac{D_t}{P_t} \right) \right) \right)
\end{aligned}
$$

And the first-order conditions in the text are taken with respect to n_t, k_{t+1}, b_{t+1}, D_t, μ_t, λ_t :

$$
\frac{\partial L}{\partial n_t} = \left(\lambda_t - \mu_t \right) \frac{P_{mt}}{P_t} F_n(e^{z_t}, k_t, n_t) - \lambda_t \frac{W_t}{P_t} = 0
$$

$$
\begin{aligned}
\frac{\partial L}{\partial k_{t+1}} \quad = \quad & \mu_t \Xi e^{\xi_t} Q_t - Q_t \frac{\lambda_t}{\Omega_{t+1}} \\
& + \mathbb{E}_t m_{t+1} \left(\lambda_{t+1} \left(\frac{P_{mt+1}}{P_{t+1}} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) + Q_{t+1} \left(1 - \delta\right) \right) \right. \\
& \left. + \mu_{t+1} \left(- \frac{P_{mt+1}}{P_{t+1}} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) \right) \right)
\end{aligned}
$$

$$
\frac{\partial L}{\partial b_{t+1}} = \frac{\lambda_t}{R_t} - \mu_t \Xi e^{\xi_t} q_t - \mathbb{E}_t m_{t+1} \lambda_{t+1} = 0
$$

$$
\begin{aligned}
\frac{\partial L}{\partial D_t} \quad = \quad & \left(1 - \lambda_t \left(1 + 2\kappa \left(\frac{D_t}{D_{t-1}} - 1 \right) \frac{D_t}{D_{t-1}} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \right) \right. \\
& \left. + \mathbb{E}_t m_{t,t+1} \lambda_{t+1} 2\kappa \left(\frac{D_{t+1}}{D_t} - 1 \right) \frac{D_{t+1}^2}{D_t^2} \frac{1}{\Pi_{t+1}} \right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L}{\partial \mu_t} \quad = \quad & \Xi e^{\xi_t} \left(Q_t k_{t+1} - q_t b_{t+1} \right) - \frac{P_{mt}}{P_t} F(e^{z_t}, k_t, n_t) \\
\frac{\partial L}{\partial \lambda_t} \quad = \quad & \left(1 - \delta\right) k_t + \frac{P_{mt}}{P_t} F(e^{z_t}, k_t, n_t) + \frac{b_{t+1}}{R_t} - b_t - \frac{W_t}{P_t} n_t \\
& - \frac{k_{t+1}}{\Omega_{t+1}} - \left(\frac{D_t}{P_t} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \frac{D_t}{P_t} \right)
\end{aligned}
$$

where $m_{t,s}$ is the stochastic discount factor.

$$\begin{aligned} m_{t,s} &= \beta^{s-t}\frac{U_{C,s}}{U_{C,t}}\frac{P_t}{P_s} \\ m_{t,t+1} &= \beta\frac{U_{C,t+1}}{\Pi_{t+1}U_{C,t}} \end{aligned}$$

We substitute $m_{t,t+1}$ and get the following real system

$$\begin{aligned} 0 &= X_t\left(\lambda_t-\mu_t\right)(1-\theta)Ae^{z_t}k_t^{\theta}n_t^{-\theta}-\lambda_t w_t \\ 0 &= \beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\Pi_{t+1}}\left(\left(\lambda_{t+1}-\mu_{t+1}\right)X_{t+1}\theta\frac{Y_{t+1}}{k_{t+1}}+\lambda_{t+1}Q_{t+1}(1-\delta)\right)-Q_t\lambda_t+\mu_t\Xi e^{\xi_t}Q_t \\ 0 &= -\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\Pi_{t+1}}\lambda_{t+1}+\lambda_t\frac{1}{R_t}-\mu_t\frac{\Xi e^{\xi_t}}{1+r_t} \end{aligned}$$

$$\begin{aligned} 0 &= 1+2\kappa\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{\lambda_{t+1}}{\Pi_{t+1}}\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}-1\right)\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}\right)^2 \\ &\quad -\lambda_t\left(1+2\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)\frac{d_t}{d_{t-1}}\Pi_t+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2\right) \end{aligned}$$

$$\begin{aligned} 0 &= \Xi e^{\xi_t}\left(Q_t k_{t+1}-q_t b_{t+1}\right)-X_t F(e^{z_t},k_t,n_t) \\ 0 &= (1-\delta)k_t+X_t F(e^{z_t},k_t,n_t)+\frac{b_{t+1}}{R_t}-b_t-w_t n_t \\ &\quad -\frac{k_{t+1}}{\Omega_{t+1}}-\left(d_t+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2 d_t\right) \end{aligned}$$

A.2.2 Retailers

The firm's optimisation problem is standard

$$\begin{aligned} &\mathbb{E}_t\sum_{\tau=t}^{\infty}m_{t,\tau}\left(p_\tau^i y_\tau^i\left(1-\tau_\tau^x\right)-P_{m\tau}y_\tau^i-\frac{\omega}{2}\left(\frac{p_\tau^i}{p_{\tau-1}^i}-1\right)^2 Y_\tau P_\tau\right) \\ =\ &\mathbb{E}_t\sum_{\tau=t}^{\infty}m_{t,\tau}\left(\frac{p_\tau^i}{P_\tau}y_\tau^i\left(1-\tau_\tau^x\right)-\frac{P_{m\tau}}{P_\tau}y_\tau^i-\frac{\omega}{2}\left(\frac{p_\tau^i}{p_{\tau-1}^i}-1\right)^2 Y_\tau\right)P_\tau \\ =\ &\mathbb{E}_t\sum_{\tau=t}^{\infty}m_{t,\tau}\left(\left(\frac{p_\tau^i}{P_\tau}\right)^{1-\varepsilon}Y_\tau\left(1-\tau_\tau^x\right)-\frac{P_{m\tau}}{P_\tau}\left(\frac{p_\tau^i}{P_\tau}\right)^{-\varepsilon}Y_\tau-\frac{\omega}{2}\left(\frac{p_\tau^i}{p_{\tau-1}^i}-1\right)^2 Y_\tau\right)P_\tau \end{aligned}$$

where $\frac{\omega}{2}\left(\frac{p_\tau^i}{p_{\tau-1}^i}-1\right)^2 Y_\tau P_\tau$ represents the cost of adjusting prices. The first order condition is:

$$\begin{aligned} \frac{\partial}{\partial p_\tau^i} &: \mathbb{E}_t m_{t,\tau}\left((1-\varepsilon)\left(\frac{p_\tau^i}{P_\tau}\right)^{-\varepsilon}Y_\tau\frac{1}{P_\tau}\left(1-\tau_\tau^x\right)-\omega\left(\frac{p_\tau^i}{p_{\tau-1}^i}-1\right)Y_\tau\frac{1}{p_{\tau-1}^i}\right. \\ &\left.+\varepsilon\frac{P_{m\tau}}{P_\tau}\left(\frac{p_\tau^i}{P_\tau}\right)^{-\varepsilon-1}Y_\tau\frac{1}{P_\tau}\right)+\mathbb{E}_t m_{t,\tau+1}\omega\left(\frac{p_{\tau+1}^i}{p_\tau^i}-1\right)Y_{\tau+1}\frac{p_{\tau+1}^i}{(p_\tau^i)^2} \end{aligned}$$

from where after $p^i_\tau = P_\tau$:

$$\frac{\partial}{\partial p^i_\tau} : \mathbb{E}_t m_{t,\tau} \left((1-\varepsilon) Y_\tau (1-\tau^x_\tau) - \omega (\Pi_\tau - 1) Y_\tau \Pi_\tau \right.$$
$$\left. + \varepsilon \frac{P_{m\tau}}{P_\tau} Y_\tau \right) + \mathbb{E}_t m_{t,\tau+1} \omega (\Pi_{\tau+1} - 1) Y_{\tau+1} \Pi^2_{\tau+1}$$

$$0 = (1-\varepsilon) Y_\tau (1-\tau^x_\tau) - \omega (\Pi_\tau - 1) Y_\tau \Pi_\tau + \varepsilon X_\tau Y_\tau + \omega\beta \mathbb{E}_t \frac{C^{-\sigma}_{\tau+1}}{C^{-\sigma}_\tau} (\Pi_{\tau+1} - 1) Y_{\tau+1} \Pi_{\tau+1}$$

A.3 Derivation of the enforcement constraint

Firms may decide to default after the realization of revenues but before returning the intra-period loan. The total liabilities are $L_t +\ P_t q_t b_{t+1}$, that is, the intra-period loan plus the new intertemporal debt. At this moment the firm also obtains liquidity $L_t = F(e^{z_t}, k_t, n_t)$ from selling its products. In the case of default the lender uses the right to liquidate the firm's capital. We assume that at the moment of contracting the loan the liquidation value of physical capital is unknown. With probability Ξe^{ξ_t} the lender will be able to obtain the whole value $P_t Q_t k_{t+1}$ but with probability $1 - \Xi e^{\xi_t}$ he obtains nothing.[1] Both lender and firm are not able to see the liquidation value before the actual default. Hence, to obtain the result from renegotiation, we have to examine these two cases one at a time. In doing so, suppose that the firm has all the bargaining power and the lender gets only the threat value. We have two possible cases:

First, when the liquidation value is $P_t Q_t k_{t+1}$: Since the lender can claim the whole capital, the firm has to pay the amount that leaves the lender indifferent between liquidation and keeping the firm in operation. This needs the firm to pay $P_t Q_t k_{t+1} - P_t q_t b_{t+1}$ and guarantee to pay $P_t b_{t+1}$ at the beginning of the next period, when the intertemporal debt is due.[2] So, the ex-post value of default is as follows

$$l_t + \mathbb{E}_t m_{t,t+1} \sum_{t=s+1}^{\infty} \beta^{t-s-1} \frac{D_t}{P_t} - P_t Q_t k_{t+1} + P_t q_t b_{t+1}$$

Second, when the liquidation value is zero: In this case, liquidation obviously is not the best choice for the lender. But the best choice is to wait until the next period when $P_t b_{t+1}$ is due. In the current period the lender receives nothing and the firm keeps the liquidity $l_t = F(e^{z_t}, k_t, n_t)$. Therefore, the ex-post value of default is:

$$l_t + \mathbb{E}_t m_{t,t+1} \sum_{t=s+1}^{\infty} \beta^{t-s-1} \frac{D_t}{P_t}$$

[1] In the first chapter we do not include "capital producers", so we can ignore Q_t.

[2] The required value $P_t Q_t k_{t+1} - P_t q_t b_{t+1}$ could be more than the liquidity l_t. In this case we assume that shareholders raise the extra cash without any additional costs.

We expect the following liquidation value when the debt contracted:

$$l_t + \mathbb{E}_t m_{t,t+1} \sum_{t=s+1}^{\infty} \beta^{t-s-1} \frac{D_t}{P_t} - \Xi e^{\xi_t} \left(P_t Q_t k_{t+1} - P_t q_t b_{t+1} \right)$$

So the following constraint enforces that the value of not defaulting must not be smaller than the expected value of defaulting:

$$\mathbb{E}_t m_{t,t+1} \sum_{t=s+1}^{\infty} \beta^{t-s-1} \frac{D_t}{P_t} \geqslant l_t + \mathbb{E}_t m_{t,t+1} \sum_{t=s+1}^{\infty} \beta^{t-s-1} \frac{D_t}{P_t} - \Xi e^{\xi_t} \left(P_t Q_t k_{t+1} - P_t q_t b_{t+1} \right)$$

which can be written as in equation 2.9.

A.4 Market Clearing and Private Sector Equilibrium

We can take the sum of the household's budget constraint (2.1),

$$W_t n_t^j + b_t P_t + s_t \left(D_t + \bar{p}_t \right) + P_t \Phi_t = P_t q_t b_{t+1} + s_{t+1} \bar{p}_t + P_t C_t^j + P_t T_t$$

firm's budget constraint (2.8),

$$P_{mt} F(e^{z_t}, k_t, n_t) + P_t \frac{b_{t+1}}{R_t} = P_t b_t + W_t n_t + P_t I_t + \Psi \left(D_t, D_{t-1} \right)$$

and government's budget constraint (2.18)-(2.19),

$$\begin{aligned} P_t T_t &= P_t \frac{b_{t+1}}{R_t} - P_t \frac{b_{t+1}}{1 + r_t} \\ P_t G_t &= \tau_t^w W_t N_t + \tau_t^d D_t + \tau_t^x P_t Y_t \end{aligned}$$

and obtain the resource constraint

$$Y_t = C_t + k_{t+1} - (1 - \delta) k_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 d_t + \frac{\omega}{2} \left(\Pi_t - 1 \right)^2 Y_t$$

The complete system which determines the private sector equilibrium can be written as:

$$\begin{aligned} 0 &= \left(\lambda_t - \mu_t \right) (1 - \theta) X_t Y_t - \lambda_t w_t n_t \\ 0 &= -\beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma} \Pi_{t+1}} \lambda_{t+1} + \frac{\lambda_t}{R_t} - \mu_t \frac{\Xi e^{\xi_t}}{1 + r_t} \end{aligned}$$

$$\frac{\lambda_t}{\Omega_{t+1}} - \mu_t \Xi e^{\xi_t} = \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma} \Pi_{t+1}} \left(\left(\lambda_{t+1} - \mu_{t+1} \right) X_{t+1} \theta \frac{Y_{t+1}}{k_{t+1}} + \lambda_{t+1} (1 - \delta) \right)$$

$$\begin{aligned} 0 &= 1 + 2\kappa \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\lambda_{t+1}}{\Pi_{t+1}} \left(\frac{d_{t+1}}{d_t} \Pi_{t+1} - 1 \right) \left(\frac{d_{t+1}}{d_t} \Pi_{t+1} \right)^2 \\ &\quad - \lambda_t \left(1 + 2\kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right) \frac{d_t}{d_{t-1}} \Pi_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 \right) \end{aligned}$$

$$
\begin{aligned}
X_t Y_t &= \Xi e^{\xi_t}\left(k_{t+1} - q_t b_{t+1}\right) \\
0 &= (1-\delta) k_t + X_t F(e^{z_t}, k_t, n_t) + \frac{b_{t+1}}{R_t} - b_t - w_t n_t \\
&\quad - \frac{k_{t+1}}{\Omega_{t+1}} - \left(d_t + \kappa \left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2 d_t\right)
\end{aligned}
$$

$$
\begin{aligned}
\omega\left(\Pi_t - 1\right) Y_t \Pi_t &= \left((1-\varepsilon)\left(1-\tau_t^x\right) + \varepsilon X_t\right) Y_t + \omega\beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\left(\Pi_{t+1} - 1\right) Y_{t+1} \\
\frac{1}{1+r_t} &= \beta \mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \\
\alpha n_t^{\psi} &= \left(1-\tau_t^w\right) w_t C_t^{-\sigma}
\end{aligned}
$$

where $Y_t = Z e^{z_t} k_t^{\theta} N_t^{1-\theta}$, $R_t = 1 + i_t(1-\tau_t)$.

There are 10 equations and 10 unknowns: $\lambda_t, \mu_t, X_t, C_t, k_t, d_t, n_t, w_t, \Pi_t$ b_t for given policy instruments i_t; τ_t.

A.5 Steady State

The steady state level of the system is as follows:

$$
\begin{aligned}
r &= \frac{1}{\beta} - 1 \\
\lambda &= 1 \\
\mu &= \left(\frac{1}{\beta R} - 1\right)\frac{1}{\Xi} \\
X &= \frac{(\varepsilon - 1)\left(1 - \tau^x\right)}{\varepsilon} \\
\frac{Y}{k} &= \frac{1 - \mu\Xi - \beta(1-\delta)}{\beta X \theta (1-\mu)} \\
\frac{Y}{n} &= \left(A\left(\frac{k}{Y}\right)^{\theta}\right)^{\frac{1}{1-\theta}}
\end{aligned}
$$

$$
\begin{aligned}
w &= \frac{Y}{n} X (1-\mu)(1-\theta) \\
\frac{b}{Y} &= \left(\frac{k}{Y} - \frac{X}{\Xi}\right)\frac{1}{\beta} \\
\frac{d}{Y} &= X + \left(\frac{1}{R} - 1\right)\frac{b}{Y} - w\frac{n}{Y} - \delta\frac{k}{Y} \\
\frac{C}{Y} &= 1 - \delta\frac{k}{Y} \\
\frac{I}{Y} &= \delta\frac{k}{Y} \\
1 + r &= 1 + i
\end{aligned}
$$

A.6 Reduced form linearised system

For every variable Z_t with steady state $Z \neq 0$ we denote $\hat{Z}_t = \log \frac{Z_t}{Z}$. We linearise the system around the above steady state to yield:

$$
\begin{aligned}
0 &= X_t\left(\lambda_t - \mu_t\right)(1-\theta) A e^{z_t} k_t^{\theta} n_t^{-\theta} - \lambda_t w_t \\
0 &= X\left(1+\hat{X}_t\right)\left(\lambda(1+\hat{\lambda}_t) - \mu(1+\hat{\mu}_t)\right)(1-\theta) A\left(1+\hat{z}_t\right) k^{\theta}\left(1+\hat{k}_t\right)^{\theta} n^{-\theta}(1+\hat{n}_t)^{-\theta} \\
&\quad -\lambda(1+\hat{\lambda}_t) w(1+\hat{w}_t) \\
0 &= (1-\theta)\frac{Y}{n} X\left(\lambda(1+\hat{\lambda}_t+\hat{X}_t+\hat{z}_t+\theta\hat{k}_t-\theta\hat{n}_t) - \mu(1+\hat{\mu}_t+\hat{X}_t+\hat{z}_t+\theta\hat{k}_t-\theta\hat{n}_t)\right) \\
&\quad -\lambda w(1+\hat{\lambda}_t+\hat{w}_t) \\
0 &= X(\lambda-\mu)(1-\theta)\frac{Y}{n} - \lambda w \\
0 &= (1-\theta)\frac{Y}{n} X\left(\hat{\lambda}_t+\hat{X}_t+\hat{z}_t+\theta\hat{k}_t-\theta\hat{n}_t - \mu(\hat{\mu}_t+\hat{X}_t+\hat{z}_t+\theta\hat{k}_t-\theta\hat{n}_t)\right) \\
&\quad -w(\hat{\lambda}_t+\hat{w}_t)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\Pi_{t+1}}\left(\left(\lambda_{t+1}-\mu_{t+1}\right) X_{t+1}\theta\frac{Y_{t+1}}{k_{t+1}} + \lambda_{t+1}(1-\delta)\right) - \lambda_t + \mu_t \Xi e^{\xi_t} \\
0 &= \frac{\beta}{\Pi}\left(1+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}-\hat{\Pi}_{t+1}\right)\begin{pmatrix} \left(\lambda\left(1+\hat{\lambda}_{t+1}\right)-\mu\left(1+\hat{\mu}_{t+1}\right)\right) X\left(1+\hat{X}_{t+1}\right) \\ \theta\frac{Y\left(1+\hat{Y}_{t+1}\right)}{k\left(1+\hat{k}_{t+1}\right)} + \lambda(1-\delta)\left(1+\hat{\lambda}_{t+1}\right)\end{pmatrix} \\
&\quad -\lambda\left(1+\hat{\lambda}_t\right) + \mu\left(1+\hat{\mu}_t\right)\Xi\left(1+\hat{\xi}_t\right) \\
0 &= \beta\left((\lambda-\mu) X\theta\frac{Y}{k} + (1-\delta)\right) - 1 + \mu\Xi \\
0 &= \beta\left(\theta\frac{Y}{k} X\left(\hat{\lambda}_{t+1} - \mu\hat{\mu}_{t+1} + (1-\mu)\left(\hat{X}_{t+1}+\hat{Y}_{t+1}-\hat{k}_{t+1}+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}-\hat{\Pi}_{t+1}\right)\right)\right. \\
&\quad \left. +(1-\delta)\left(\hat{\lambda}_{t+1}+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}-\hat{\Pi}_{t+1}\right)\right) - \hat{\lambda}_t + \Xi\mu\left(\hat{\mu}_t+\hat{\xi}_t\right)
\end{aligned}
$$

We assume that in steady state $\Pi = 1$.

$$
\begin{aligned}
0 &= -\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}\Pi_{t+1}}\lambda_{t+1} + \lambda_t\frac{1}{R_t} - \mu_t\frac{\Xi e^{\xi_t}}{1+r_t} \\
0 &= -\frac{\beta\lambda}{\Pi}\left(1+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}+\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right) + \left(1+\hat{\lambda}_t-\hat{R}_t\right)\frac{\lambda}{R} - \mu\Xi\frac{\left(1+\hat{\mu}_t+\hat{\xi}_t\right)}{1+r\left(1+\hat{r}_t\right)} \\
0 &= -\beta + \frac{1}{R} - \mu\frac{\Xi}{1+r} \\
0 &= -\beta\left(\sigma\hat{C}_t-\sigma\hat{C}_{t+1}+\hat{\lambda}_{t+1}-\hat{\Pi}_{t+1}\right) + \left(\hat{\lambda}_t-\hat{R}_t\right)\frac{1}{R} - \frac{\mu\Xi}{1+r}\left(\hat{\mu}_t+\hat{\xi}_t-\frac{r}{1+r}\hat{r}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= 1 + 2\kappa\beta\mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{\lambda_{t+1}}{\Pi_{t+1}} \left(\frac{d_{t+1}}{d_t}\Pi_{t+1} - 1\right) \left(\frac{d_{t+1}}{d_t}\Pi_{t+1}\right)^2 \\
&\quad -\lambda_t \left(1 + 2\kappa \left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right) \frac{d_t}{d_{t-1}}\Pi_t + \kappa \left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2\right) \\
0 &= 1 + \frac{2\kappa\beta\lambda}{\Pi} \left(1 + \sigma\hat{C}_t - \sigma\hat{C}_{t+1} - \hat{\Pi}_{t+1} + \hat{\lambda}_{t+1}\right) \\
&\quad \left(\Pi\left(1 + \hat{\Pi}_{t+1} + \hat{d}_{t+1} - \hat{d}_t\right) - 1\right) \left(\Pi\left(1 + \hat{\Pi}_{t+1} + \hat{d}_{t+1} - \hat{d}_t\right)\right)^2 \\
&\quad -\lambda\left(1 + \hat{\lambda}_t\right) \left(\begin{array}{c} 1 + 2\kappa\left(\hat{\Pi}_t + \hat{d}_t - \hat{d}_{t-1}\right)\left(1 + \hat{\Pi}_t + \hat{d}_t - \hat{d}_{t-1}\right) \\ +\kappa\left(\hat{\Pi}_t + \hat{d}_t - \hat{d}_{t-1}\right)^2 \end{array}\right) \\
1 &= \lambda \\
0 &= 2\kappa\beta\left(\hat{\Pi}_{t+1} + \hat{d}_{t+1} - \hat{d}_t\right) - \hat{\lambda}_t - 2\kappa\left(\hat{\Pi}_t + \hat{d}_t - \hat{d}_{t-1}\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \Xi e^{\xi_t}\left(k_{t+1} - \frac{b_{t+1}}{1 + r_t}\right) - X_t Y_t \\
0 &= \Xi\left(1 + \hat{\xi}_t\right)\left(k\left(1 + \hat{k}_{t+1}\right) - \frac{b}{1+r}\left(1 + \hat{b}_{t+1} - \frac{r\hat{r}_t}{1+r}\right)\right) \\
&\quad -XY\left(1 + \hat{X}_t + \hat{Y}_t\right) \\
0 &= \Xi\left(k - \frac{b}{1+r}\right) - XY \\
0 &= \Xi\left(k\left(\hat{k}_{t+1} + \hat{\xi}_t\right) - \frac{b}{1+r}\left(\hat{b}_{t+1} - \frac{r\hat{r}_t}{1+r} + \hat{\xi}_t\right)\right) - XY\left(\hat{X}_t + \hat{Y}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= X_t Y_t + \frac{b_{t+1}}{R_t} - w_t n_t - b_t - d_t\left(1 + \kappa\left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2\right) - I_t \\
0 &= XY\left(1 + \hat{X}_t + \hat{Y}_t\right) + \frac{b\left(1 + \hat{b}_{t+1}\right)}{R\left(1 + \hat{R}_t\right)} - wn\left(1 + \hat{w}_t\right)\left(1 + \hat{n}_t\right) - b\left(1 + \hat{b}_t\right) \\
&\quad -d\left(1 + \hat{d}_t\right)\left(1 + \kappa\left(\frac{1 + \hat{d}_t}{1 + \hat{d}_{t-1}}\Pi\left(1 + \hat{\Pi}_t\right) - 1\right)^2\right) \\
&\quad -I\left(1 + \hat{I}_t\right) \\
0 &= XY + \frac{b}{R} - wn - b - d\left(1 + \kappa\left(\Pi - 1\right)^2\right) - I \\
0 &= XY\left(\hat{X}_t + \hat{Y}_t\right) + \frac{b}{R}\left(\hat{b}_{t+1} - \hat{R}_t\right) - wn\left(\hat{w}_t + \hat{n}_t\right) - b\hat{b}_t - d\hat{d}_t - I\hat{I}_t
\end{aligned}
$$

$$
\begin{aligned}
0 &= (1-\varepsilon)\left(1-\tau_t^x\right)+\varepsilon X_t+\omega\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\left(\Pi_{t+1}-1\right)\frac{Y_{t+1}}{Y_t}-\omega\left(\Pi_t-1\right)\Pi_t \\
0 &= (1-\varepsilon)\left(1-\tau^x\left(1+\hat{\tau}_t^x\right)\right)+\varepsilon X\left(1+\hat{X}_t\right)-\omega\left(\Pi\left(1+\hat{\Pi}_t\right)-1\right)\Pi\left(1+\hat{\Pi}_t\right) \\
&\quad +\omega\beta\mathbb{E}_t\left(1+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}\right)\left(\Pi\left(1+\hat{\Pi}_{t+1}\right)-1\right)\frac{Y\left(1+\hat{Y}_{t+1}\right)}{Y\left(1+\hat{Y}_t\right)} \\
0 &= (1-\varepsilon)\left(1-\tau^x\right)+\varepsilon X \\
\omega\hat{\Pi}_t &= (\varepsilon-1)\,\tau^x\hat{\tau}_t^x+\varepsilon X\hat{X}_t+\omega\beta\mathbb{E}_t\hat{\Pi}_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{1+r_t} &= \beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \\
\frac{1}{1+r}\left(1-\frac{r}{1+r}\hat{r}_t\right) &= \beta\mathbb{E}_t\left(1+\sigma\hat{C}_t-\sigma\hat{C}_{t+1}\right) \\
\frac{1}{1+r} &= \beta \\
-r\beta\hat{r}_t &= \sigma\hat{C}_t-\sigma\mathbb{E}_t\hat{C}_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
\alpha n_t^{\psi} &= \left(1-\tau_t^w\right)w_t C_t^{-\sigma} \\
C^{-\sigma}w\left(1-\tau^w\right)\left(1-\frac{\tau^w\hat{\tau}_t^w}{1-\tau^w}\right)\left(1+\hat{w}_t-\sigma\hat{C}_t\right) &= \alpha n^{\psi}\left(1+\psi\hat{n}_t\right) \\
w &= \frac{\alpha n^{\psi}C^{\sigma}}{\left(1-\tau^w\right)} \\
\hat{w}_t-\sigma\hat{C}_t-\frac{\tau^w\hat{\tau}_t^w}{1-\tau^w} &= \psi\hat{n}_t
\end{aligned}
$$

$$
\begin{aligned}
R_t &= 1+r_t\left(1-\tau_t\right) \\
R\left(1+\hat{R}_t\right) &= 1+r\left(1+\hat{r}_t\right)\left(1-\tau\left(1+\hat{\tau}_t\right)\right) \\
R\left(1+\hat{R}_t\right) &= 1+r\left(1-\tau\right)\left(1+\hat{r}_t-\frac{\tau}{1-\tau}\hat{\tau}_t\right) \\
\hat{R}_t &= \frac{-r\left(1-\tau\right)}{R}+\frac{r\left(1-\tau\right)}{R}\left(1+\hat{r}_t-\frac{\tau}{1-\tau}\hat{\tau}_t\right) \\
\hat{R}_t &= \frac{r\left(1-\tau\right)}{R}\hat{r}_t-\frac{r\tau}{R}\hat{\tau}_t
\end{aligned}
$$

$$
\begin{aligned}
1+r_t &= \frac{1+i_t}{\mathbb{E}_t\Pi_{t+1}} \\
1+r+r\hat{r}_t &= \frac{1+i+i\hat{\imath}_t}{\Pi+\Pi\hat{\Pi}_{Ht+1}} \\
1+r &= 1+i \\
\frac{i}{1+i}\hat{r}_t &= \frac{i}{1+i}\hat{\imath}_t-\hat{\Pi}_{t+1} \\
\hat{r}_t &= \hat{\imath}_t-\frac{1}{r\beta}\mathbb{E}_t\hat{\Pi}_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
Y_t &= C_t + I_t \left(1 + \frac{\varrho}{2}\left(\frac{I_\tau}{I_{\tau-1}} - 1\right)^2\right) + \kappa\left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2 d_t + \frac{\omega}{2}\left(\Pi_t - 1\right)^2 Y_t \\
0 &= C\left(1 + \hat{C}_t\right) + I\left(1 + \hat{I}_t\right)\left(1 + \frac{\varrho}{2}\left(\frac{I\left(1 + \hat{I}_t\right)}{I\left(1 + \hat{I}_{t-1}\right)} - 1\right)^2\right) \\
&\quad + \kappa\left(\frac{d\left(1 + \hat{d}_t\right)}{d\left(1 + \hat{d}_{t-1}\right)}\Pi\left(1 + \hat{\Pi}_t\right) - 1\right)^2 d\left(1 + \hat{d}_t\right) - Y\left(1 + \hat{Y}_t\right) \\
&\quad + \frac{\omega}{2}\left(\Pi\left(1 + \hat{\Pi}_t\right) - 1\right)^2 Y\left(1 + \hat{Y}_t\right) \\
Y &= C + I \\
Y\hat{Y}_t &= C\hat{C}_t + I\hat{I}_t
\end{aligned}
$$

Appendix B

Two Open Economies

B.1 Household's Optimisation Problem

B.1.1 Domestic Households

The Lagrangian can be written as

$$\begin{aligned}\Gamma_t &= \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \Big\{ U(C_t, n_t) + \Delta_t \Big((1-\tau^w) W_t n_t + A_t + B_t E_t + \left(\left(1-\tau^d\right) D_t + P_{St} \right) s_t \\ &\quad + T_t - \frac{A_{t+1}}{1+i_t} - \frac{B_{t+1}E_t}{(1+i_t^*)\,\phi\left(\frac{E_t B_{t+1}}{P_t}\right)} - s_{t+1}P_{St} - P_t C_t \Big) \Big\}\end{aligned}$$

And the first-order conditions are taken with respect to C_t, n_t , A_{t+1} , B_{t+1}, s_{t+1} and Δ_t.

$$\frac{U_{C,t}}{P_t} = \Delta_t$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{C,t}(1-\tau^w)}$$

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \frac{P_t U_{C,t+1}}{P_{t+1} U_{C,t}}$$

$$\frac{1}{1+i_t^*} = \beta \mathbb{E}_t \frac{P_t U_{C,t+1}}{P_{t+1} U_{C,t}} \frac{E_{t+1}}{E_t} \phi\left(\frac{E_t B_{t+1}}{P_t}\right)$$

$$P_{St} = \beta \mathbb{E}_t \frac{P_t U_{C,t+1}}{P_{t+1} U_{C,t}} \left(\left(1-\tau^d\right) D_{t+1} + P_{St+1} \right)$$

$$\begin{aligned} & (1-\tau^w) W_t n_t + A_t + B_t E_t + \left(\left(1-\tau^d\right) D_t + P_{St} \right) s_t + T_t \\ = \; & \frac{A_{t+1}}{1+i_t} + \frac{B_{t+1}E_t}{(1+i_t^*)\,\phi\left(\frac{E_t B_{F,t+1}}{P_t}\right)} + s_{t+1}P_{St} + P_t C_t \end{aligned}$$

FOCs in real term are

$$\frac{U_{C,t}}{P_t} = \Delta_t$$

$$\alpha n_t^{\psi}\Upsilon_t = w_t C_t^{-\sigma}\left(1-\tau^{w}\right)$$

$$\frac{1}{(1+i_t)} = \beta\mathbb{E}_t \frac{\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} U_{C,t+1}}{\left((1-\gamma)+\gamma S_{t+1}^{1-\eta}\right)^{\frac{1}{1-\eta}} U_{C,t}\Pi_{Ht+1}}$$

$$\frac{1}{1+i_t^*} = \beta\mathbb{E}_t \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{S_{t+1}}{S_t}\frac{1}{\Pi_{Ft+1}^*}\left(\frac{1-\gamma+\gamma S_t^{1-\eta}}{1-\gamma+\gamma S_{t+1}^{1-\eta}}\right)^{\frac{1}{1-\eta}} \phi\left(\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{-\frac{1}{1-\eta}} S_t b_{t+1}\Pi_{t+1}^*\right)$$

$$p_{St} = \beta\mathbb{E}_t \frac{\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} C_{t+1}^{-\sigma}}{\left((1-\gamma)+\gamma S_{t+1}^{1-\eta}\right)^{\frac{1}{1-\eta}} C_t^{-\sigma}\Pi_{Ht+1}} \left(\left(1-\tau^{d}\right) d_{t+1}\Pi_{Ht+1} + p_{St+1}\Pi_{Ht+1}\right)$$

$$\begin{aligned}
&\left(1-\tau^{w}\right) w_t n_t + a_t + b_t S_t + \left(1-\tau^{d}\right) d_t + \frac{T_t}{P_{Ht}} \\
= \; & \frac{a_t}{(1+i_t)}\Pi_{Ht+1} + \frac{b_{t+1}S_t\Pi_{Ft+1}^*}{\left(1+i_t^*\right)\phi\left(\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{-\frac{1}{1-\eta}} S_t b_{t+1}\Pi_{t+1}^*\right)} \\
&+\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} C_t
\end{aligned}$$

where $p_{St} = \frac{P_{St}}{P_{Ht}}$ and $d_t = \frac{D_t}{P_{Ht}}$. See also Appendix (G) and (F) for more notations. We normalise the number of shares to be equal to one (See Jermann and Quadrini (2012)).

B.1.2 Foreign Households

The Lagrangian can be written as

$$\begin{aligned}
\Gamma_t^* = \; & \mathbb{E}_t\sum_{t=0}^{\infty}\beta^t\Bigg\{U\left(C_t^*, n_t^*\right) + \Delta_t^*\Bigg(\left(1-\tau^{w*}\right)W_t^* n_t^* + \frac{A_t^*}{E_t} + B_t^* + \left(\left(1-\tau^{d*}\right)D_t^* + P_{St}^*\right)s_t^* \\
& -\frac{A_{t+1}^*}{E_t\left(1+i_t\right)\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)} - \frac{B_{t+1}^*}{\left(1+i_t^*\right)} - s_{t+1}^* P_{St}^* - P_t^* C_t^* + T_t^*\Bigg)\Bigg\}
\end{aligned}$$

And the first-order conditions in the text are taken with respect to C_t^*, n_t^* , B_{t+1}^*, A_{t+1}^*, s_{t+1}^* and Δ_t^*.

$$\frac{U_C\left(C_t^*, n_t^*\right)}{P_t^*} = \Delta_t^*$$

$$\frac{W_t^*}{P_t^*} = -\frac{U_n\left(C_t^*, n_t^*\right)}{U_C\left(C_t^*, n_t^*\right)\left(1-\tau^{w*}\right)}$$

$$\frac{1}{\left(1+i_t^*\right)} = \beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^*, n_{t+1}^*\right)P_t^*}{U_C\left(C_t^*, n_t^*\right)P_{t+1}^*}$$

$$\frac{1}{\left(1+i_t\right)} = \beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^*, n_{t+1}^*\right)}{U_C\left(C_t^*, n_t^*\right)}\frac{P_t^* E_t}{P_{t+1}^* E_{t+1}}\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)$$

$$P_{St}^{*}=\beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^{*},n_{t+1}^{*}\right)P_t^{*}}{U_C\left(C_t^{*},n_t^{*}\right)P_{t+1}^{*}}\left(\left(1-\tau^{d*}\right)D_{t+1}^{*}+P_{St+1}^{*}\right)$$

$$\begin{aligned}
&\left(1-\tau^{w*}\right)W_t^{*}n_t^{*}+\frac{A_t^{*}}{E_t}+B_t^{*}+\left(\left(1-\tau^{d*}\right)D_t^{*}+P_{St}^{*}\right)s_t^{*}+T_t^{*}\\
=&\ \frac{A_{t+1}^{*}}{E_t\left(1+i_t\right)\phi^{*}\left(\frac{A_{t+1}^{*}}{E_tP_t^{*}}\right)}+\frac{B_{t+1}^{*}}{\left(1+i_t^{*}\right)}+s_{t+1}^{*}P_{St}^{*}+P_t^{*}C_t^{*}
\end{aligned}$$

and the FOCs in real term are

$$\frac{U_C\left(C_t^{*},n_t^{*}\right)}{P_t^{*}}=\Delta_t^{*}$$

$$w_t^{*}=-\frac{\Gamma_tU_n\left(C_t^{*},n_t^{*}\right)}{U_C\left(C_t^{*},n_t^{*}\right)\left(1-\tau^{w*}\right)}$$

$$\frac{1}{\left(1+i_t^{*}\right)}=\beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^{*},n_{t+1}^{*}\right)\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_t^{\eta-1}\right)^{\frac{1}{1-\eta}}}{\Pi_{Ft+1}^{*}U_C\left(C_t^{*},n_t^{*}\right)\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_{t+1}^{\eta-1}\right)^{\frac{1}{1-\eta}}}$$

$$\begin{aligned}
\frac{1}{\left(1+i_t\right)}\ =&\ \beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^{*},n_{t+1}^{*}\right)}{U_C\left(C_t^{*},n_t^{*}\right)}\frac{\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_t^{\eta-1}\right)^{\frac{1}{1-\eta}}S_t}{\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_{t+1}^{\eta-1}\right)^{\frac{1}{1-\eta}}S_{t+1}}\\
&\frac{1}{\Pi_{Ht+1}}\phi^{*}\left(\left(\left(1-\gamma^{*}\right)S_t^{1-\eta}+\gamma^{*}\right)^{-\frac{1}{1-\eta}}\Pi_{t+1}a_{t+1}^{*}\right)
\end{aligned}$$

$$p_{St}^{*}=\beta\mathbb{E}_t\frac{U_C\left(C_{t+1}^{*},n_{t+1}^{*}\right)\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_t^{\eta-1}\right)^{\frac{1}{1-\eta}}}{U_C\left(C_t^{*},n_t^{*}\right)\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_{t+1}^{\eta-1}\right)^{\frac{1}{1-\eta}}\Pi_{Ft+1}^{*}}\left(\left(1-\tau^{d*}\right)d_{t+1}^{*}\Pi_{Ft+1}^{*}+p_{St+1}^{*}\Pi_{Ft+1}^{*}\right)$$

$$\begin{aligned}
&\left(1-\tau^{w*}\right)w_t^{*}n_t^{*}+\frac{a_t^{*}}{S_t}+b_t^{*}+\left(\left(1-\tau^{d*}\right)d_t^{*}+p_{St}^{*}\right)s_t^{*}\\
&+\frac{T_t^{*}}{P_{Ft}^{*}}-\left(\left(1-\gamma^{*}\right)+\gamma^{*}S_t^{\eta-1}\right)^{\frac{1}{1-\eta}}C_t^{*}\\
=&\ \mathbb{E}_t\frac{a_{t+1}^{*}\Pi_{Ht+1}}{\left(1+i_t\right)\phi^{*}\left(\left(\left(1-\gamma^{*}\right)S_t^{1-\eta}+\gamma^{*}\right)^{-\frac{1}{1-\eta}}\Pi_{t+1}a_{t+1}^{*}\right)S_t}+\mathbb{E}_t\frac{b_{t+1}^{*}\Pi_{Ft+1}^{*}}{\left(1+i_t^{*}\right)}+s_{t+1}^{*}p_{St}^{*}
\end{aligned}$$

where $p_{St}^{*}=\frac{P_{St}^{*}}{P_{Ft}^{*}}$ and $d_t^{*}=\frac{D_t^{*}}{P_{Ft}^{*}}$. See also Appendix (G) and (F) for more notations. We normalise the number of shares to be equal to one (See Jermann and Quadrini (2012)).

B.2 Firms'Optimisation Problems

B.2.1 Domestic Firms

Intermediate Goods Producers

The firm's optimisation problem subject to equations (3.22) and (3.23) and the Lagrangian can be written as follows:

$$
\begin{aligned}
L_0^f \;=\;& \mathbb{E}_0 \sum_{t=0}^{\infty} m_{0,t} \left(D_t + \mu_t \left(\Xi e^{\xi_t} \left(P_{Ht} Q_t k_{t+1} - \frac{A_{t+1}^T}{1+i_t} \right) - P_{mt} F(e^{z_t}, k_t, n_t) \right) \right. \\
& + \lambda_t \left(P_{mt} F(e^{z_t}, k_t, n_t) + \frac{A_{t+1}^T}{1+i_t(1-\tau_t)} - A_t^T - W_t n_t \right. \\
& \left. \left. - P_{Ht} Q_t \left(k_{t+1} - (1-\delta) k_t\right) - \Psi\left(D_t, D_{t-1}\right) \right) \right)
\end{aligned}
$$

And the first-order conditions in the text are taken with respect to n_t, k_{t+1}, A_{t+1}^T, D_t, μ_t, λ_t.

$$
\begin{aligned}
\lambda_t W_t &= \left(\lambda_t - \mu_t\right) P_{mt} F_n(e^{z_t}, k_t, n_t) \\
0 &= \mathbb{E}_t m_{t,t+1} \left(\left(\lambda_{t+1} - \mu_{t+1}\right) P_{mt+1} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) + \lambda_{t+1} P_{Ht+1} Q_{t+1} \left(1-\delta\right) \right) \\
& \quad - \left(\lambda_t - \mu_t \Xi e^{\xi_t}\right) P_{Ht} Q_t \\
0 &= \frac{\lambda_t}{1+i_t(1-\tau_t)} - \mu_t \Xi e^{\xi_t} \frac{1}{1+i_t} - \mathbb{E}_t m_{t,t+1} \lambda_{t+1} \\
1 &= \lambda_t \left(1 + 2\kappa \left(\frac{D_t}{D_{t-1}} - 1 \right) \frac{D_t}{D_{t-1}} + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 \right) \\
& \quad - \mathbb{E}_t m_{t,t+1} \lambda_{t+1} 2\kappa \left(\frac{D_{t+1}}{D_t} - 1 \right) \frac{D_{t+1}^2}{D_t^2} \\
0 &= \Xi e^{\xi_t} \left(P_{Ht} Q_t k_{t+1} - \frac{A_{t+1}^T}{1+i_t} \right) - P_{mt} F(e^{z_t}, k_t, n_t) \\
A_t^T &= P_{mt} F(e^{z_t}, k_t, n_t) + \frac{A_{t+1}^T}{1+i_t(1-\tau_t)} - W_t n_t - P_{Ht} Q_t I_t - \Psi\left(D_t, D_{t-1}\right)
\end{aligned}
$$

where $m_{s,t}$ is the stochastic discount factor.

$$
\begin{aligned}
m_{t,s} &= \beta^{s-t} \frac{U_{C,s}}{U_{C,t}} \frac{P_t}{P_s} \\
m_{t,t+1} &= \beta \frac{U_{C,t+1}}{\Pi_{t+1} U_{C,t}}
\end{aligned}
$$

We substitute $m_{t,t+1}$ and get the following real system

$$\begin{aligned}
\lambda_t w_t &= (\lambda_t - \mu_t) X_t F_n(e^{z_t}, k_t, n_t) \\
0 &= \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}} \frac{\Upsilon_t}{\Upsilon_{t+1}} \left((\lambda_{t+1} - \mu_{t+1}) X_{t+1} F_k(e^{z_{t+1}}, k_{t+1}, n_{t+1}) + \lambda_{t+1} Q_{t+1} (1-\delta) \right) \\
&\quad - \left(\lambda_t - \mu_t \Xi e^{\xi_t} \right) Q_t \\
0 &= \frac{\lambda_t}{1 + i_t(1-\tau_t)} - \mu_t \Xi e^{\xi_t} \frac{1}{1+i_t} - \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}\Pi_{Ht+1}} \frac{\Upsilon_t}{\Upsilon_{t+1}} \lambda_{t+1} \\
1 &= \lambda_t \left(1 + 2\kappa \left(\frac{d_t}{d_{t-1}} \Pi_{Ht} - 1 \right) \frac{d_t}{d_{t-1}} \Pi_{Ht} + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_{Ht} - 1 \right)^2 \right) \\
&\quad - \beta \mathbb{E}_t \frac{U_{C,t+1}}{U_{C,t}\Pi_{Ht+1}} \frac{\Upsilon_t}{\Upsilon_{t+1}} \lambda_{t+1} 2\kappa \left(\frac{d_{t+1}}{d_t} \Pi_{Ht+1} - 1 \right) \frac{d_{t+1}^2}{d_t^2} \Pi_{Ht+1}^2 \\
0 &= \Xi e^{\xi_t} \left(Q_t k_{t+1} - \frac{a_{t+1}^T}{(1+i_t)} \mathbb{E}_t \Pi_{Ht+1} \right) - X_t F(e^{z_t}, k_t, n_t) \\
a_t^T &= X_t F(e^{z_t}, k_t, n_t) + \frac{a_{t+1}^T}{(1 + i_t(1-\tau_t))} \mathbb{E}_t \Pi_{Ht+1} - w_t n_t - Q_t I_t - d_t \\
&\quad - \kappa \left(\frac{d_t}{d_{t-1}} \Pi_{Ht} - 1 \right)^2 d_t
\end{aligned}$$

See also Appendix (G) and (F) for more notations.

Capital producers

The objective of a capital producer is to choose I_t to maximise nominal profit

$$\max_{I_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(P_{m\tau} Q_\tau I_\tau - P_{m\tau} \left(1 + \frac{\varrho}{2} \left(\frac{I_\tau}{I_{\tau-1}} - 1 \right)^2 \right) I_\tau \right)$$

And the first-order condition in the text is taken with respect to I_t.

If no costs $\varrho = 0$ then

$$Q_\tau = 1$$

Retailers

The firm's optimisation problem is standard

$$
\begin{aligned}
& \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(p^i_{H\tau} y^i_\tau \left(1-\tau^x_\tau\right) - P_{m\tau} y^i_\tau - \frac{\omega}{2} \left(\frac{p^i_{H\tau}}{p^i_{H\tau-1}} - 1 \right)^2 Y_\tau P_{H\tau} \right) \\
= \ & \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(\frac{p^i_{H\tau}}{P_{H\tau}} y^i_\tau \left(1-\tau^x_\tau\right) - \frac{P_{m\tau}}{P_{H\tau}} y^i_\tau - \frac{\omega}{2} \left(\frac{p^i_{H\tau}}{p^i_{H\tau-1}} - 1 \right)^2 Y_\tau \right) P_{H\tau} \\
= \ & \mathbb{E}_t \sum_{\tau=t}^{\infty} m_{t,\tau} \left(\begin{array}{c} \left(\frac{p^i_{H\tau}}{P_{H\tau}}\right)^{1-\varepsilon} Y_\tau \left(1-\tau^x_\tau\right) - \frac{P_{m\tau}}{P_{H\tau}} \left(\frac{p^i_{H\tau}}{P_{H\tau}}\right)^{-\varepsilon} Y_\tau \\ -\frac{\omega}{2} \left(\frac{p^i_{H\tau}}{p^i_{H\tau-1}} - 1 \right)^2 Y_\tau \end{array} \right) P_{H\tau}
\end{aligned}
$$

where $\frac{\omega}{2} \left(\frac{p^i_{H\tau}}{p^i_{H\tau-1}} - 1 \right)^2 Y_\tau P_{H\tau}$ represents the cost of adjusting prices. The first order condition is:

$$
\begin{aligned}
\frac{\partial}{\partial p^j_{H\tau}} \ : \ & \mathbb{E}_t m_{t,\tau} \left((1-\varepsilon) \left(\frac{p^i_{H\tau}}{P_{H\tau}} \right)^{-\varepsilon} Y_\tau \left(1-\tau^x_\tau\right) - \omega \left(\frac{p^i_{H\tau}}{p^i_{H\tau-1}} - 1 \right) Y_\tau \frac{P_{H\tau}}{p^i_{H\tau-1}} \right. \\
& \left. + \varepsilon \frac{P_{m\tau}}{P_{H\tau}} \left(\frac{p^i_{H\tau}}{P_{H\tau}} \right)^{-\varepsilon-1} Y_\tau \right) + \mathbb{E}_t m_{t,\tau+1} \omega \left(\frac{p^i_{H\tau+1}}{p^i_{H\tau}} - 1 \right) Y_{\tau+1} \frac{p^i_{H\tau+1}}{\left(p^i_{H\tau}\right)^2} P_{H\tau+1}
\end{aligned}
$$

from where after $p^i_{H\tau} = P_{H\tau}$:

$$
\begin{aligned}
\frac{\partial}{\partial p^j_{H\tau}} \ : \ & \mathbb{E}_t m_{t,\tau} \left((1-\varepsilon) Y_\tau \left(1-\tau^x_\tau\right) - \omega \left(\Pi_\tau - 1\right) Y_\tau \Pi_\tau \right. \\
& \left. + \varepsilon \frac{P_{m\tau}}{P_{H\tau}} Y_\tau \right) + \mathbb{E}_t m_{t,\tau+1} \omega \left(\Pi_{\tau+1} - 1\right) Y_{\tau+1} \Pi^2_{\tau+1}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial p^j_{H\tau}} \ : \ & \mathbb{E}_t m_{t,\tau} \left((1-\varepsilon) Y_\tau \left(1-\tau^x_\tau\right) - \omega \left(\Pi_\tau - 1\right) Y_\tau \Pi_\tau \right. \\
& \left. + \varepsilon \frac{P_{m\tau}}{P_{H\tau}} Y_\tau \right) + \beta \mathbb{E}_t \frac{C^{-\sigma}_{\tau+1}}{C^{-\sigma}_\tau} \frac{\Upsilon_\tau}{\Upsilon_{\tau+1}} \omega \left(\Pi_{\tau+1} - 1\right) Y_{\tau+1} \Pi_{\tau+1}
\end{aligned}
$$

$$
\omega \left(\Pi_\tau - 1\right) Y_\tau \Pi_\tau = (1-\varepsilon) Y_\tau \left(1-\tau^x_\tau\right) + \varepsilon X_\tau Y_\tau + \omega \beta \mathbb{E}_t \frac{C^{-\sigma}_{\tau+1}}{C^{-\sigma}_\tau} \frac{\Upsilon_\tau}{\Upsilon_{\tau+1}} \left(\Pi_{\tau+1} - 1\right) Y_{\tau+1} \Pi_{\tau+1}
$$

B.2.2 Foreign Firms

Intermediate goods producers

The Lagrangian can be written as

$$
\begin{aligned}
L^{f*}_0 \ = \ & \mathbb{E}_0 \sum_{t=0}^{\infty} m^*_{0,t} \left(D^*_t + \mu^*_t \left(\Xi^* e^{\xi^*_t} \left(P^*_{Ft} Q^*_t k^*_{t+1} - \frac{B^T_{t+1}}{1+i^*_t} \right) - P^*_{mt} F(e^{z^*_t}, k^*_t, N^*_t) \right) \right. \\
& + \lambda^*_t \left(P^*_{mt} F(e^{z^*_t}, k^*_t, N^*_t) + \frac{B^T_{t+1}}{1+i^*_t(1-\tau^*_t)} - B^T_t - W^*_t n^*_t \right. \\
& \left. \left. - P^*_{Ft} Q^*_t \left(k^*_{t+1} - (1-\delta) k^*_t\right) - \Psi^* \left(D^*_t, D^*_{t-1}\right) \right) \right)
\end{aligned}
$$

And the first-order conditions in the text are taken with respect to n_t^*, k_{t+1}^*, B_{t+1}^T, D_t^*, μ_t^*, λ_t^*.

$$
\begin{aligned}
\lambda_t^* W_t^* &= \left(\lambda_t^* - \mu_t^*\right) P_{mt}^* F_n(e^{z_t^*}, k_t^*, n_t^*) \\
0 &= \mathbb{E}_t m_{t,t+1}^* \left(\left(\lambda_{t+1}^* - \mu_{t+1}^*\right) P_{mt+1}^* F_k(e^{z_{t+1}^*}, k_{t+1}^*, n_{t+1}^*) + \lambda_{t+1}^* P_{Ft+1}^* Q_{t+1}^* (1-\delta)\right) \\
&\quad - \left(\lambda_t^* - \mu_t^* \Xi_t^* e^{\xi_t^*}\right) P_{Ft}^* Q_t^* \\
0 &= \frac{\lambda_t^*}{1 + i_t^*(1-\tau_t^*)} - \mu_t^* \Xi^* e^{\xi_t^*} \frac{1}{1+i_t^*} - \mathbb{E}_t m_{t,t+1}^* \lambda_{t+1}^* \\
1 &= \lambda_t^* \left(1 + 2\kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1\right) \frac{D_t^*}{D_{t-1}^*} + \kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1\right)^2\right) \\
&\quad - \mathbb{E}_t m_{t,t+1}^* \lambda_{t+1}^* 2\kappa^* \left(\frac{D_{t+1}^*}{D_t^*} - 1\right) \frac{D_{t+1}^{*2}}{D_t^{*2}} \\
0 &= \Xi^* e^{\xi_t^*} \left(P_{Ft}^* Q_t^* k_{t+1}^* - \frac{B_{t+1}^T}{1+i_t^*}\right) - P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) \\
B_t^T &= P_{mt}^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{B_{t+1}^T}{1 + i_t^*(1-\tau_t^*)} - W_t^* n_t^* \\
&\quad - P_{Ft}^* Q_t^* \left(k_{t+1}^* - (1-\delta^*) k_t^*\right) - \Psi^* \left(D_t^*, D_{t-1}^*\right)
\end{aligned}
$$

and the FOCs in real term are

$$
\begin{aligned}
\lambda_t^* w_t^* &= \left(\lambda_t^* - \mu_t^*\right) X_t^* F_n(e^{z_t^*}, k_t^*, n_t^*) \\
0 &= \beta \mathbb{E}_t \frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{\Gamma_t}{\Gamma_{t+1}} \left(\left(\lambda_{t+1}^* - \mu_{t+1}^*\right) X_{t+1}^* F_k(e^{z_{t+1}^*}, k_{t+1}^*, n_{t+1}^*) + \lambda_{t+1}^* Q_{t+1}^* (1-\delta)\right) \\
&\quad - \left(\lambda_t^* - \mu_t^* \Xi_t^* e^{\xi_t^*}\right) Q_t^* \\
0 &= \frac{\lambda_t^*}{1 + i_t^*(1-\tau_t^*)} - \mu_t^* \Xi^* e^{\xi_t^*} \frac{1}{1+i_t^*} - \beta \mathbb{E}_t \frac{C_{t+1}^{*-\sigma}}{\Pi_{Ft+1}^* C_t^{*-\sigma}} \frac{\Gamma_t}{\Gamma_{t+1}} \lambda_{t+1}^* \\
1 &= \lambda_t^* \left(1 + 2\kappa^* \left(\frac{d_t^*}{d_{t-1}^*} \Pi_{Ft}^* - 1\right) \frac{d_t^*}{d_{t-1}^*} \Pi_{Ft}^* + \kappa^* \left(\frac{d_t^*}{d_{t-1}^*} \Pi_{Ft}^* - 1\right)^2\right) \\
&\quad - 2\kappa \beta \mathbb{E}_t \frac{C_{t+1}^{*-\sigma}}{C_t^{*-\sigma}} \frac{\Gamma_t}{\Gamma_{t+1}} \Pi_{Ft+1}^* \lambda_{t+1}^* \left(\frac{d_{t+1}^*}{d_t^*} \Pi_{Ft+1}^* - 1\right) \left(\frac{d_{t+1}^*}{d_t^*}\right)^2 \\
0 &= \Xi^* e^{\xi_t^*} \left(Q_t^* k_{t+1}^* - \frac{b_{t+1}^T}{(1+i_t^*)} \mathbb{E}_t \Pi_{Ft+1}^*\right) - X_t^* F(e^{z_t^*}, k_t^*, n_t^*) \\
b_t^T &= X_t^* F(e^{z_t^*}, k_t^*, n_t^*) + \frac{b_{t+1}^T \mathbb{E}_t \Pi_{Ft+1}^*}{(1 + i_t^*(1-\tau_t^*))} - w_t^* n_t^* \\
&\quad - Q_t^* \left(k_{t+1}^* - (1-\delta^*) k_t^*\right) - d_t^* - \kappa^* \left(\frac{d_t^*}{d_{t-1}^*} \Pi_{Ft}^* - 1\right)^2 d_t^*
\end{aligned}
$$

where $m_{t,t+1}^*$ is the stochastic discount factor.

$$
m_{t,t+1}^* = \beta \frac{C_{t+1}^{*-\sigma}}{\Pi_{t+1}^* C_t^{*-\sigma}}
$$

Capital producers

The objective of a capital producer is to choose I^*_τ to maximise nominal profit

$$\max_{I^*_\tau} \mathbb{E}_t \sum_{\tau=t}^{\infty} m^*_{t,\tau} \left(Q^*_\tau I^*_\tau - \left(1 + \frac{\varrho}{2} \left(\frac{I^*_\tau}{I^*_{\tau-1}} - 1 \right)^2 \right) I^*_\tau \right) P^*_{F,\tau}$$

If no costs $\varrho = 0$ then

$$Q^*_\tau = 1$$

Retailers

The firm's optimisation problem is standard

$$\begin{aligned} & \mathbb{E}_t \sum_{\tau=t}^{\infty} m^*_{t,\tau} \left(\frac{p^{*i}_{F\tau}}{P^*_{F\tau}} y^{*i}_\tau \left(1 - \tau^{*x}_\tau\right) - \frac{P^*_{m\tau}}{P^*_{F\tau}} y^{*i}_\tau - \frac{\omega}{2} \left(\frac{p^{*i}_{F\tau}}{p^{*i}_{F\tau-1}} - 1 \right)^2 Y^*_\tau \right) P^*_{F\tau} \\ = \; & \mathbb{E}_t \sum_{\tau=t}^{\infty} m^*_{t,\tau} \left(\begin{array}{c} \left(\frac{p^{*i}_{F\tau}}{P^*_{F\tau}} \right)^{1-\varepsilon} Y^*_\tau \left(1 - \tau^{*x}_\tau\right) - \frac{P^*_{m\tau}}{P^*_{F\tau}} \left(\frac{p^{*i}_{F\tau}}{P^*_{F\tau}} \right)^{-\varepsilon} Y^*_\tau \\ - \frac{\omega}{2} \left(\frac{p^{*i}_{F\tau}}{p^{*i}_{F\tau-1}} - 1 \right)^2 Y^*_\tau \end{array} \right) P^*_{F\tau} \end{aligned}$$

where $\frac{\omega}{2} \left(\frac{p^{*i}_{F\tau}}{p^{*i}_{F\tau-1}} - 1 \right)^2 Y^*_\tau$ represents the cost of adjusting prices. The first order conditions are:

$$\begin{aligned} \frac{\partial}{\partial p^{*j}_{F\tau}} \quad : \quad & \mathbb{E}_t m^*_{t,\tau} \left((1-\varepsilon) \left(\frac{p^{*i}_{F\tau}}{P^*_{F\tau}} \right)^{-\varepsilon} Y^*_\tau \left(1 - \tau^{*x}_\tau\right) - \omega \left(\frac{p^{*i}_{F\tau}}{p^{*i}_{F\tau-1}} - 1 \right) Y^*_\tau \frac{P^*_{F\tau}}{p^{*i}_{F\tau-1}} \right. \\ & \left. + \varepsilon \frac{P^*_{m\tau}}{P^*_{F\tau}} \left(\frac{p^{*i}_{F\tau}}{P^*_{F\tau}} \right)^{-\varepsilon-1} Y^*_\tau \right) + \mathbb{E}_t m^*_{t,\tau+1} \omega \left(\frac{p^{*i}_{F\tau+1}}{p^{*i}_{F\tau}} - 1 \right) Y^*_{\tau+1} \frac{p^{*i}_{F\tau+1} P^*_{F\tau+1}}{\left(p^{*i}_{F\tau}\right)^2} \end{aligned}$$

from where after $p^{*i}_{F\tau} = P^*_{F\tau}$:

$$\omega \left(\Pi^*_t - 1\right) Y^*_t \Pi^*_t = (1-\varepsilon) Y^*_t \left(1 - \tau^{*x}_t\right) + \varepsilon X^*_t Y^*_t + \beta \mathbb{E}_t \frac{C^{*-\sigma}_{t+1}}{C^{*-\sigma}_t} \frac{\Gamma_t}{\Gamma_{t+1}} \omega \left(\Pi^*_{t+1} - 1\right) Y^*_{t+1} \Pi^*_{t+1}$$

B.3 Market Clearing and Private Sector Equilibrium

For each country we can take the sum of the household budget constraint and the government budget constraint and obtain two financial accounts

$$
\begin{aligned}
FAa \quad : \quad Y_t = {} & \frac{P_t}{P_{Ht}} C_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_{Ht} - 1 \right)^2 d_t + G_t + I_t \left(1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) \\
& + \frac{\omega}{2} \left(\Pi_t - 1 \right)^2 Y_t + \left(\frac{1}{\phi \left(\frac{E_t B_{t+1}}{P_t} \right)} - 1 \right) \frac{B_{t+1} E_t}{\left(1 + i_t^* \right) P_{Ht}} \\
& + \frac{1}{P_{Ht}} \left(A_t^* - B_t E_t + \frac{B_{t+1} E_t}{\left(1 + i_t^* \right)} - \frac{A_{t+1}^*}{\left(1 + i_t \right)} \right)
\end{aligned}
$$

$$
\begin{aligned}
FAb \quad : \quad Y_t^* = {} & \left(\frac{1}{\phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right)} - 1 \right) \frac{A_{t+1}^*}{E_t \left(1 + i_t \right) P_{Ft}^*} \\
& + \frac{1}{P_{Ft}^*} \left(\frac{A_{t+1}^*}{E_t \left(1 + i_t \right)} - \frac{B_{t+1}}{\left(1 + i_t^* \right)} + B_t - \frac{A_t^*}{E_t} \right) + G_t^* + \frac{\omega}{2} \left(\Pi_t^* - 1 \right)^2 Y_t^* \\
& + I_t^* \left(1 + \frac{\varrho}{2} \left(\frac{I_t^*}{I_{t-1}^*} - 1 \right)^2 \right) + \frac{P_t^*}{P_{Ft}^*} C_t^* + \kappa \left(\frac{d_t^*}{d_{t-1}^*} \Pi_{Ft}^* - 1 \right)^2 d_t^*
\end{aligned}
$$

which can be used in the final system instead of household budget constraint.

The two market clearing conditions for countries H and F are

$$
\begin{aligned}
P_{Ht} Y_t = {} & C_{Ht} P_{Ht} + C_{Ht}^* P_{Ht}^* E_t + \kappa \left(\frac{D_t}{D_{t-1}} - 1 \right)^2 D_t + P_{Ht} I_t \left(1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) \\
& + P_{Ht} G_t + P_{Ht} \frac{\omega}{2} \left(\Pi_t - 1 \right)^2 Y_t + \left(\frac{1}{\phi \left(\frac{E_t B_{t+1}}{P_t} \right)} - 1 \right) \frac{B_{t+1} E_t}{\left(1 + i_t^* \right)}
\end{aligned}
$$

$$
\begin{aligned}
P_{Ft}^* Y_t^* = {} & C_{Ft}^* P_{Ft}^* + C_{Ft} \frac{P_{Ft}}{E_t} + \left(\frac{1}{\phi^* \left(\frac{A_{t+1}^*}{E_t P_t^*} \right)} - 1 \right) \frac{A_{t+1}^*}{E_t \left(1 + i_t \right)} + P_{Ft}^* \frac{\omega}{2} \left(\Pi_t^* - 1 \right)^2 Y_t^* \\
& + P_{Ft}^* G_t^* + P_{Ft}^* I_t^* \left(1 + \frac{\varrho}{2} \left(\frac{I_t^*}{I_{t-1}^*} - 1 \right)^2 \right) + P_t^* C_t^* + \kappa^* \left(\frac{D_t^*}{D_{t-1}^*} - 1 \right)^2 D_t^*
\end{aligned}
$$

these are balance of goods. We substitute consumption into the market clearing conditions and obtain

$$
\begin{aligned}
GAa \quad : \quad Y_t = {} & (1 - \gamma) \left((1 - \gamma) + \gamma S_t^{1-\eta} \right)^{\frac{\eta}{1-\eta}} C_t + \gamma^* \left((1 - \gamma^*) S_t^{1-\eta} + \gamma^* \right)^{\frac{\eta}{1-\eta}} C_t^* \\
& + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_{Ht} - 1 \right)^2 d_t + G_t + I_t \left(1 + \frac{\varrho}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) + \frac{\omega}{2} \left(\Pi_t - 1 \right)^2 Y_t \\
& + \left(\frac{1}{\phi \left(\frac{E_t B_{t+1}}{P_t} \right)} - 1 \right) \frac{B_{t+1} E_t}{\left(1 + i_t^* \right) P_{Ht}}
\end{aligned}
$$

$$GAb \ : \ Y_t^* = (1-\gamma^*)\left((1-\gamma^*)+\gamma^* S_t^{\eta-1}\right)^{\frac{\eta}{1-\eta}} C_t^* \ + \gamma\left((1-\gamma)S_t^{\eta-1}+\gamma\right)^{\frac{\eta}{1-\eta}} C_t$$
$$+\left(\frac{1}{\phi^*\left(\frac{A_{t+1}^*}{E_t P_t^*}\right)}-1\right)\frac{A_{t+1}^*}{E_t(1+i_t)P_{Ft}^*}+G_t^*+\frac{\omega}{2}(\Pi_t^*-1)^2 Y_t^*$$
$$+I_t^*\left(1+\frac{\varrho}{2}\left(\frac{I_t^*}{I_{t-1}^*}-1\right)^2\right)+\kappa\left(\frac{d_t^*}{d_{t-1}^*}\Pi_{Ft}^*-1\right)^2 d_t^*$$

Of course, the sum of two financial constraints is equal to the sum of two market clearing conditions, so one equation is redundant. To close the system, we use equations GAa and GAb; and instead of using either FAa or FAb we use FAa-GAa.

The complete system which determines the private sector equilibrium of home country can be written as:

$$0 = (\lambda_t-\mu_t)(1-\theta)X_tY_t-\lambda_t w_t n_t$$
$$0 = \frac{\lambda_t}{R_t}-\mu_t\Xi e^{\xi_t}\frac{1}{1+i_t}-\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\lambda_{t+1}$$

$$\left(\lambda_t-\mu_t\Xi e^{\xi_t}\right)Q_t$$
$$= \beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\Pi_{t+1}\left(\left(\lambda_{t+1}-\mu_{t+1}\right)X_{t+1}F_k(e^{z_{t+1}},k_{t+1},n_{t+1})+\lambda_{t+1}Q_{t+1}(1-\delta)\right)$$

$$1+2\kappa\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\lambda_{t+1}\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}-1\right)\left(\frac{d_{t+1}}{d_t}\Pi_{t+1}\right)^2$$
$$= \lambda_t\left(1+2\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)\frac{d_t}{d_{t-1}}\Pi_t+\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2\right)$$

$$X_tY_t = \Xi e^{\xi_t}\left(Q_tk_{t+1}-\left(a_{t+1}+a_{t+1}^*\right)\frac{\mathbb{E}_t\Pi_{t+1}}{(1+i_t)}\right)$$
$$Q_t = 1+\frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2+\frac{I_t}{I_{t-1}}\varrho\left(\frac{I_t}{I_{t-1}}-1\right)-\beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\varrho\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2$$

$$0 = X_tY_t+\left(a_{t+1}+a_{t+1}^*\right)\frac{\mathbb{E}_t\Pi_{t+1}}{R_t}-(a_t+a_t^*)-w_tn_t\Upsilon_t-Q_tI_t-d_t-\kappa\left(\frac{d_t}{d_{t-1}}\Pi_t-1\right)^2 d_t$$

$$\omega(\Pi_t-1)Y_t\Pi_t = ((1-\varepsilon)(1-\tau_t^x)+\varepsilon X_t)Y_t+\omega\beta\mathbb{E}_t\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}}\frac{\Upsilon_t}{\Upsilon_{t+1}}(\Pi_{t+1}-1)Y_{t+1}\Pi_{t+1}$$
$$\frac{1}{1+i_t} = \beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\frac{\Upsilon_t}{\Upsilon_{t+1}\Pi_{t+1}}$$
$$\frac{1}{1+i_t^*} = \beta\mathbb{E}_t\frac{U_{C,t+1}}{U_{C,t}}\frac{S_{t+1}}{S_t}\frac{\Upsilon_t}{\Upsilon_{t+1}\Pi_{t+1}^*}\phi\left(\frac{S_t}{\Upsilon_t}b_{t+1}\Pi_{t+1}^*\right)$$
$$\alpha n_t^{\psi} = w_t(1-\tau_t^w)\frac{C_t^{-\sigma}}{\Upsilon_t}$$

where $Y_t = Ze^{z_t}k_t^{\theta}N_t^{1-\theta}$, $R_t = 1+i_t(1-\tau_t)$, $\Upsilon_t = \left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}}$.

Finally the resource constraint yields

$$\begin{aligned} Y_t &= C_t \Upsilon_t + I_t + G_t + \kappa \left(\frac{d_t}{d_{t-1}} \Pi_t - 1 \right)^2 d_t + \frac{\omega}{2} \left(\Pi_t - 1 \right)^2 Y_t \\ &+ \mathbb{E}_t \left(\frac{1}{\phi \left(\frac{S_t}{\Upsilon_t} b_{t+1} \Pi^*_{t+1} \right)} - 1 \right) b_{t+1} S_t \frac{\Pi^*_{t+1}}{1 + i^*_t} \\ &+ \left(a^*_t - a^*_{t+1} \frac{\mathbb{E}_t \Pi_{t+1}}{1 + i_t} - S_t b_t + S_t b_{t+1} \frac{\mathbb{E}_t \Pi^*_{t+1}}{1 + i^*_t} \right) \end{aligned}$$

similarly the Private Sector Equilibrium for the foreign country is:

$$\begin{aligned} \lambda^*_t w^*_t &= \left(\lambda^*_t - \mu^*_t \right) X^*_t \frac{S_t}{\Upsilon^*_t} F_n(e^{z^*_t}, k^*_t, n^*_t) \\ \frac{\lambda^*_t}{R^*_t} &= \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \lambda^*_{t+1} + \mu^*_t \Xi^* e^{\xi^*_t} \frac{1}{1 + i^*_t} \end{aligned}$$

$$\begin{aligned} & \left(\lambda^*_t - \mu^*_t \Xi^*_t e^{\xi^*_t} \right) Q^*_t \\ = \; & \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \Pi^*_{t+1} \left(\left(\lambda^*_{t+1} - \mu^*_{t+1} \right) X^*_{t+1} F_k(e^{z^*_{t+1}}, k^*_{t+1}, n^*_{t+1}) + \lambda^*_{t+1} Q^*_{t+1} \left(1 - \delta \right) \right) \end{aligned}$$

$$\begin{aligned} & 1 + 2\kappa^* \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \lambda^*_{t+1} \left(\frac{d^*_{t+1}}{d^*_t} \Pi^*_{t+1} - 1 \right) \left(\frac{d^*_{t+1}}{d^*_t} \Pi^*_{t+1} \right)^2 \\ = \; & \lambda^*_t \left(1 + 2\kappa^* \left(\frac{d^*_t}{d^*_{t-1}} \Pi^*_t - 1 \right) \frac{d^*_t}{d^*_{t-1}} \Pi^*_t + \kappa^* \left(\frac{d^*_t}{d^*_{t-1}} \Pi^*_t - 1 \right)^2 \right) \end{aligned}$$

$$X^*_t Y^*_t = \Xi^* e^{\xi^*_t} \left(Q^*_t k^*_{t+1} - \left(b_{t+1} + b^*_{t+1} \right) \frac{\mathbb{E}_t \Pi^*_{t+1}}{1 + i^*_t} \right)$$

$$0 = X^*_t Y^*_t + \left(b_{t+1} + b^*_{t+1} \right) \frac{\mathbb{E}_t \Pi^*_{t+1}}{R^*_t} - \left(b_t + b^*_t \right) - w^*_t n^*_t \Gamma_t - Q^*_t I^*_t - d^*_t - \kappa^* \left(\frac{d^*_t}{d^*_{t-1}} \Pi^*_t - 1 \right)^2 d^*_t$$

$$Q^*_t = 1 + \frac{\varrho}{2} \left(\frac{I^*_t}{I^*_{t-1}} - 1 \right)^2 + \frac{I^*_t}{I^*_{t-1}} \varrho \left(\frac{I^*_t}{I^*_{t-1}} - 1 \right) - \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \varrho \left(\frac{I^*_{t+1}}{I^*_t} - 1 \right) \left(\frac{I^*_{t+1}}{I^*_t} \right)^2$$

$$\omega \left(\Pi^*_t - 1 \right) Y^*_t \Pi^*_t = \left(1 - \varepsilon \right) Y^*_t \left(1 - \tau^{*x}_t \right) + \varepsilon X^*_t Y^*_t + \beta \mathbb{E}_t \frac{C^{*-\sigma}_{t+1}}{C^{*-\sigma}_t} \frac{\Gamma_t}{\Gamma_{t+1}} \omega \left(\Pi^*_{t+1} - 1 \right) Y^*_{t+1} \Pi^*_{t+1}$$

$$\begin{aligned} \frac{\mathbb{E}_t \Pi^*_{t+1}}{1 + i^*_t} &= \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \frac{\Gamma_t}{\Gamma_{t+1}} \\ \frac{\mathbb{E}_t \Pi_{t+1}}{1 + i_t} &= \beta \mathbb{E}_t \frac{U_{C^*,t+1}}{U_{C^*,t}} \frac{\Gamma_t S_t}{\Gamma_{t+1} S_{t+1}} \phi^* \left(\frac{\Pi_{t+1}}{\Upsilon^*_t} a^*_{t+1} \right) \\ \alpha n^{*\psi}_t &= w^*_t \left(1 - \tau^{*w}_t \right) \frac{C^{*-\sigma}_t}{\Gamma_t} \end{aligned}$$

$$\begin{aligned} Y^*_t &= \Gamma_t C^*_t + I^*_t + G^*_t + \kappa^* \left(\frac{d^*_t}{d^*_{t-1}} \Pi^*_t - 1 \right)^2 d^*_t + \frac{\omega}{2} \left(\Pi^*_t - 1 \right)^2 Y^*_t \\ &+ \mathbb{E}_t \left(\frac{1}{\phi^* \left(\frac{\Pi_{t+1}}{\Upsilon^*_t} a^*_{t+1} \right)} - 1 \right) \frac{a^*_{t+1}}{S_t} \frac{\Pi_{t+1}}{1 + i_t} - \left(a^*_t - a^*_{t+1} \frac{\mathbb{E}_t \Pi_{t+1}}{1 + i_t} \right) \frac{1}{S_t} + b_t - b_{t+1} \frac{\mathbb{E}_t \Pi^*_{t+1}}{1 + i^*_t} \end{aligned}$$

$$
\begin{aligned}
Y_t - Y_t &= \left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} C_t + I_t + G_t + \kappa\left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2 d_t + I_t\frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 \\
&\quad + \frac{\omega}{2}\left(\Pi_t - 1\right)^2 Y_t - \frac{A_{t+1}^*}{P_{Ht}\left(1+i_t\right)} + \frac{A_t^*}{P_{Ht}} - \frac{B_t E_t}{P_{Ht}} + \frac{B_{t+1}E_t}{P_{Ht}\left(1+i_t^*\right)\phi\left(\frac{E_t B_{F,t+1}}{P_t}\right)} \\
&\quad - (1-\gamma)\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{\eta}{1-\eta}} C_t - \gamma^*\left((1-\gamma^*)S_t^{1-\eta}+\gamma^*\right)^{\frac{\eta}{1-\eta}} C_t^* - I_t - G_t \\
&\quad - \frac{E_t B_{t+1}}{P_{Ht}\left(1+i_t^*\right)}\left(\frac{1}{\phi\left(\frac{E_t B_{t+1}}{P_t}\right)} - 1\right) - \kappa\left(\frac{d_t}{d_{t-1}}\Pi_t - 1\right)^2 d_t - I_t\frac{\varrho}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2 \\
&\quad - \frac{\omega}{2}\left(\Pi_t - 1\right)^2 Y_t
\end{aligned}
$$

we get

$$
0 = \gamma S_t^{1-\eta}\Upsilon_t^{\eta} C_t - \gamma^*\Gamma_t^{\eta} S_t^{\eta} C_t^* - a_{t+1}^*\frac{\mathbb{E}_t\Pi_{t+1}}{1+i_t} + a_t^* + b_{t+1}\frac{\mathbb{E}_t\Pi_{t+1}^*}{1+i_t^*}S_t - b_t S_t
$$

where $Y_t^* = Ze^{z_t^*}k_t^{*\theta}N_t^{*1-\theta}$, $R_t^* = 1 + i_t^*(1-\tau_t^*)$, $\Gamma_t S_t = \left((1-\gamma^*)S_t^{1-\eta}+\gamma^*\right)^{\frac{1}{1-\eta}}$ (See also Appendix G for more notations).

There are 25 equations and 25 unknowns: C_t, n_t, w_t, λ_t, μ_t, X_t, k_t, Π_t, d_t, b_t, a_t, Q_t, C_t^*, n_t^*, w_t^*, λ_t^*, μ_t^*, X_t^*, k_t^*, Π_t^*, d_t^*, b_t^*, a_t^*, Q_t^*, and S_t for given policy instruments i_t; i_t^*; τ_t^w; τ_t^{w*}; τ_t^d; τ_t^{d*}; τ_t^x; τ_t^{x*}; τ_t; τ_t^* .

B.4 Steady State

The steady state level of the system is as follows:

$$
\begin{aligned}
S &= 1 \\
\lambda &= 1 \\
\beta &= \frac{1}{1+i} \\
1 &= \phi\left(\left((1-\gamma)+\gamma S^{1-\eta}\right)^{-\frac{1}{1-\eta}} S b_{t+1}\right) \rightarrow b=0 \\
Q &= 1 \\
X &= \frac{(\varepsilon-1)\left(1-\tau^{x}\right)}{\varepsilon} \\
\mu\Xi &= \frac{1}{(1+i(1-\tau))\,\beta}-1 \\
\frac{Y}{k} &= \frac{1-\mu\Xi-\beta(1-\delta)}{\beta(1-\mu)\,X\theta} \\
\frac{Y}{N} &= \left(A\left(\frac{k}{Y}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \\
\beta\frac{a}{Y} &= \left(\frac{k}{Y}-\frac{X}{\Xi}\right) \\
w\Upsilon &= (1-\mu)\,X\,(1-\theta)\,\frac{Y}{N} \\
\frac{d}{Y} &= X+\frac{a}{Y}\left(\frac{1}{(1+i(1-\tau))}-1\right)-\frac{N}{Y}\Upsilon w-\delta\frac{k}{Y} \\
\frac{C}{Y}\Upsilon &= 1-\delta\frac{k}{Y}-\frac{G}{Y} \\
Y &= \left(n^{(\sigma+\psi)}\frac{w\left(1-\tau^{w}\right)}{d\left(\frac{N}{Y}\right)^{\psi}\left(\frac{C}{Y}\right)^{\sigma}}\right)^{\frac{1}{\psi+\sigma}}
\end{aligned}
$$

$$
\begin{aligned}
\lambda^* &= 1 \\
\beta &= \frac{1}{1+i^*} \\
1 &= \phi^*\left(\left((1-\gamma^*)\, S^{1-\eta}+\gamma^*\right)^{-\frac{1}{1-\eta}} a^*\right) \rightarrow a^*=0 \\
Q^* &= 1 \\
w_t^* &= -\frac{U_n\left(C_t^*, n_t^*\right)}{U_c\left(C_t^*, n_t^*\right)\left(1-\tau_t^{*w}\right)} \\
X^* &= \frac{(\varepsilon-1)\left(1-\tau^{*x}\right)}{\varepsilon} \\
\mu^* &= \frac{1}{\left(1+i^*(1-\tau^*)\right)\Xi^*}-\frac{1}{\Xi^*} \\
\frac{Y^*}{k^*} &= \frac{1-\mu^*\Xi^*-\beta\,(1-\delta)}{\beta\,(1-\mu^*)\,\theta X^*} \\
\frac{Y^*}{N^*} &= \left(A\left(\frac{k^*}{Y^*}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \\
\frac{b^*}{Y^*} &= \left(\frac{k^*}{Y^*}-\frac{X^*}{\Xi^*}\right)\frac{1}{\beta} \\
w^*\Gamma &= (1-\mu^*)\, X^*\,(1-\theta)\,\frac{Y^*}{n^*} \\
\frac{d^*}{Y^*} &= X^*+\frac{b^*}{Y^*}\left(\frac{1}{\left(1+i^*(1-\tau^*)\right)}-1\right)-w^*\Gamma\frac{n^*}{Y^*}-\delta\frac{k^*}{Y^*} \\
\Gamma\frac{C^*}{Y^*} &= 1-\delta\frac{k^*}{Y^*}-\frac{G^*}{Y^*} \\
Y^* &= \left((1-n)^{(\sigma+\psi)}\,\frac{w^*\left(1-\tau^{*w}\right)}{d\left(\frac{N^*}{Y^*}\right)^{\psi}\left(\frac{C^*}{Y^*}\right)^{\sigma}}\right)^{\frac{1}{\psi+\sigma}}
\end{aligned}
$$

$$
\begin{aligned}
\left((1-\gamma)+\gamma S^{1-\eta}\right)^{\frac{1}{1-\eta}} &= \Upsilon \\
\left((1-\gamma^*)\, S^{1-\eta}+\gamma^*\right)^{\frac{1}{1-\eta}} &= \Gamma S \\
\left(1-\gamma^*+S^{\eta-1}\gamma^*\right)^{\frac{1}{1-\eta}} &= \Gamma
\end{aligned}
$$

$$
\gamma S^{-\eta}\left((1-\gamma^*)\, S^{1-\eta}+\gamma^*\right)\Gamma^{\frac{1-\sigma}{\psi+\sigma}} = \left[\frac{\left[\frac{C^*}{Y^*}\Gamma\right]}{\left[\frac{C}{Y}\Upsilon\right]}\,\frac{Y^*\Gamma^{\frac{1-\sigma}{\psi+\sigma}}}{Y\Upsilon^{\frac{1-\sigma}{\psi+\sigma}}}\right]\gamma^*\left((1-\gamma)+\gamma S^{1-\eta}\right)\Upsilon^{\frac{1-\sigma}{\psi+\sigma}}
$$

B.5 Reduced form Linearised System

For every variable z_t with steady state $z \neq 0$ we denote $\hat{z}_t = \log\frac{z_t}{z}$. We linearise the system around the steady state to yield:

$$
\begin{aligned}
w\,(1+\hat{w}_t) &= \frac{\Upsilon\alpha n^{\psi}C^{\sigma}\,(1+\psi\hat{n}_t)\left(1+\sigma\hat{C}_t+\hat{\Upsilon}_t\right)}{(1-\tau^w)\left(1-\frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w\right)} \\
\hat{w}_t &= \psi\hat{n}_t+\sigma\hat{C}_t+\hat{\Upsilon}_t+\frac{\tau^w}{(1-\tau^w)}\hat{\tau}_t^w
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{(1+i)(1+\hat{\imath}_t)} &= \beta\mathbb{E}_t\frac{1}{\left(1+\hat{\Pi}_{t+1}\right)}\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}\right)}\\
\hat{C}_t &= \hat{C}_{t+1}+\frac{1}{\sigma}\left(\hat{\Pi}_{t+1}-\hat{\imath}_t\right)+\frac{1}{\sigma}\left(\hat{\Upsilon}_{t+1}-\hat{\Upsilon}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{(1+i^*)(1+\hat{\imath}_t^*)} &= \beta\mathbb{E}_t\frac{\left(1+\hat{S}_{t+1}\right)}{\left(1+\hat{S}_t\right)}\frac{1}{\left(1+\hat{\Pi}^*_{Ft+1}\right)}\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}\right)}\\
&\quad \phi\left(\left(1-\gamma+\gamma S^{1-\eta}\right)^{-\frac{1}{1-\eta}}Sb_{t+1}\right)\\
1 &= \left(1+\hat{S}_{t+1}-\hat{S}_t+\hat{\imath}_t^*-\hat{\Pi}^*_{Ft+1}-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t+\sigma\hat{C}_t-\hat{\Upsilon}_{t+1}\right)\\
&\quad \phi\left(\left(1-\gamma+\gamma S^{1-\eta}\right)^{-\frac{1}{1-\eta}}Sb_{t+1}\right)\\
\hat{C}_t &= \hat{C}_{t+1}-\frac{1}{\sigma}\left(\hat{\imath}_t^*-\hat{\Pi}^*_{Ft+1}\right)-\frac{1}{\sigma}\frac{(1-\gamma)}{(1-\gamma+\gamma S^{1-\eta})}\left(\hat{S}_{t+1}-\hat{S}_t\right)+\frac{\chi}{\sigma}\frac{S}{\Upsilon}b_{t+1}
\end{aligned}
$$

Note that we assume:

$$
\begin{aligned}
\phi(x) &= \phi(0)+\phi'(0)\,x=1-\chi x\\
\varrho &= 0\\
\phi^*(x) &= \phi^*(0)+\phi^{*\prime}(0)\,x=1-\chi^* x\\
\varrho^* &= 0
\end{aligned}
$$

$$
\begin{aligned}
\left(1+\hat{\lambda}_t\right)w(1+\hat{w}_t) &= \left(\left(1+\hat{\lambda}_t\right)-\mu(1+\hat{\mu}_t)\right)X\left(1+\hat{X}_t\right)(1-\theta)\frac{Y\left(1+\hat{Y}_t\right)}{n(1+\hat{n}_t)}\\
\hat{\lambda}_t+\hat{w}_t &= \frac{1}{(1-\mu)}\hat{\lambda}_t-\frac{\mu}{(1-\mu)}\hat{\mu}_t+\hat{X}_t+\hat{Y}_t-\hat{n}_t
\end{aligned}
$$

$$
\begin{aligned}
&\left(1+\hat{\lambda}_t\right)-\mu(1+\hat{\mu}_t)\,\Xi\left(1+\hat{\xi}_t\right)\\
&= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}\right)}\left(\begin{array}{c}\left(\left(1+\hat{\lambda}_{t+1}\right)-\mu\left(1+\hat{\mu}_{t+1}\right)\right)X\left(1+\hat{X}_{t+1}\right)\theta\frac{Y\left(1+\hat{Y}_{t+1}\right)}{k\left(1+\hat{k}_{t+1}\right)}\\ +\left(1+\hat{\lambda}_{t+1}\right)(1-\delta)\end{array}\right)\\
0 &= (1-\mu\Xi)\left(-\sigma\hat{C}_{t+1}+\sigma\hat{C}_t+\hat{\Upsilon}_t-\hat{\Upsilon}_{t+1}\right)-\hat{\lambda}_t+\mu\Xi\left(\hat{\mu}_t+\hat{\xi}_t\right)\\
&\quad +\beta X\theta\frac{Y}{k}\left(\hat{\lambda}_{t+1}-\mu\hat{\mu}_{t+1}\right)+\beta(1-\mu)X\theta\frac{Y}{k}\left(\hat{X}_{t+1}+\hat{Y}_{t+1}-\hat{k}_{t+1}\right)+\beta(1-\delta)\hat{\lambda}_{t+1}
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\left(1+\hat{\lambda}_t\right)}{\tau(1+\hat{\tau}_t)+(1+i)(1+\hat{\imath}_t)(1-\tau(1+\hat{\tau}_t))}-\mu(1+\hat{\mu}_t)\,\Xi\left(1+\hat{\xi}_t\right)\frac{1}{(1+i)(1+\hat{\imath}_t)}\\
&= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\Pi\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}+\hat{\Pi}_{Ht+1}\right)}\left(1+\hat{\lambda}_{t+1}\right)\\
&(\mu\Xi+1)\left(\hat{\lambda}_t+\frac{i\tau}{(1+(1-\tau)i)}\hat{\tau}_t\right)+\left(\mu\Xi-\frac{(\mu\Xi+1)(1-\tau)}{\beta(1+(1-\tau)i)}\right)\hat{\imath}_t-\mu\Xi\left(\hat{\mu}_t+\hat{\xi}_t\right)\\
&= \left(-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t+\hat{\lambda}_{t+1}+\sigma\hat{C}_t-\hat{\Upsilon}_{t+1}-\hat{\Pi}_{Ht+1}\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= 1+2\kappa\beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}\right)}\left(1+\hat{\lambda}_{t+1}\right) \\
&\quad \left(\frac{\left(1+\hat{d}_{t+1}\right)}{\left(1+\hat{d}_t\right)}\left(1+\hat{\Pi}_{Ht+1}\right)-1\right)\frac{\left(1+2\hat{d}_{t+1}\right)}{\left(1+2\hat{d}_t\right)}\left(1+\hat{\Pi}_{Ht+1}\right) \\
&\quad -\left(1+\hat{\lambda}_t\right)\left(\begin{array}{c} 1+2\kappa\left(\frac{\left(1+\hat{d}_t\right)}{\left(1+\hat{d}_{t-1}\right)}\left(1+\hat{\Pi}_{Ht}\right)-1\right)\frac{\left(1+\hat{d}_t\right)}{\left(1+\hat{d}_{t-1}\right)}\left(1+\hat{\Pi}_{Ht}\right) \\ +\kappa\left(\frac{\left(1+\hat{d}_t\right)}{\left(1+\hat{d}_{t-1}\right)}\left(1+\hat{\Pi}_{Ht}\right)-1\right)^2 \end{array}\right) \\
\hat{\lambda}_t &= 2\beta\kappa\left(\hat{d}_{t+1}-\hat{d}_t+\hat{\Pi}_{Ht+1}\right)-2\kappa\left(\hat{d}_t-\hat{d}_{t-1}+\hat{\Pi}_{Ht}\right)
\end{aligned}
$$

$$
\begin{aligned}
&\Xi\left(1+\hat{\xi}_t\right)\left(\left(1+\hat{Q}_t\right)k\left(1+\hat{k}_{t+1}\right)-\left(a\left(1+\hat{a}_{t+1}\right)+a^*_{t+1}\right)\frac{\left(1+\hat{\Pi}_{Ht+1}\right)}{\left(1+i\right)\left(1+\hat{\imath}_t\right)}\right) \\
= \; & X\left(1+\hat{X}_t\right)Y\left(1+\hat{Y}_t\right) \\
&\Xi\frac{k}{Y}\left(\hat{\xi}_t+\hat{k}_{t+1}\right)-\beta\Xi\frac{a}{Y}\left(\hat{\xi}_t+\hat{a}_{t+1}+\frac{1}{a}a^*_{t+1}+\hat{\Pi}_{Ht+1}-\hat{\imath}_t\right) \\
= \; & X\left(\hat{X}_t+\hat{Y}_t\right)
\end{aligned}
$$

$$
\hat{Q}_t=\varrho\left(\hat{I}_t-\hat{I}_{t-1}\right)-\beta\varrho\left(\hat{I}_{t+1}-\hat{I}_t\right)
$$

$$
\begin{aligned}
&\omega\left(\left(1+\hat{\Pi}_{Ht}\right)-1\right)\left(1+\hat{\Pi}_{Ht}\right) \\
&= \left(1-\varepsilon\right)\left(1-\tau^x\left(1+\hat{\tau}^x_t\right)\right)+\varepsilon X\left(1+\hat{X}_t\right) \\
&\quad +\omega\beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}_{t+1}+\hat{\Upsilon}_t\right)}{\left(1-\sigma\hat{C}_t+\hat{\Upsilon}_{t+1}\right)}\left(\left(1+\hat{\Pi}_{Ht+1}\right)-1\right)\frac{\left(1+\hat{Y}_{t+1}\right)}{\left(1+\hat{Y}_t\right)}\left(1+\hat{\Pi}_{Ht+1}\right) \\
\hat{\Pi}_{Ht} &= \frac{\left(\varepsilon-1\right)\left(1-\tau^x\right)}{\omega}\left(\frac{\tau^x}{\left(1-\tau^x\right)}\hat{\tau}^x_t+\hat{X}_t\right)+\beta\hat{\Pi}_{Ht+1}
\end{aligned}
$$

$$
\begin{aligned}
Y\left(1+\hat{Y}_t\right) &= C\left(1+\hat{C}_t\right)\left(1-\gamma+\gamma S^{1-\eta}\right)^{\frac{1}{1-\eta}}\left(1+\frac{\gamma S^{1-\eta}}{\left(1-\gamma+\gamma S^{1-\eta}\right)}\hat{S}_t\right) \\
&+\left(k\left(1+\hat{k}_{t+1}\right)-(1-\delta)\,k\left(1+\hat{k}_t\right)\right)+G\left(1+\hat{G}_t\right) \\
&+\kappa\left(\frac{\left(1+\hat{d}_t\right)}{\left(1+\hat{d}_{t-1}\right)}\left(1+\hat{\Pi}_{Ht}\right)-1\right)^2 d\left(1+\hat{d}_t\right)+\frac{\omega}{2}\left(\Pi_t-1\right)^2 Y\left(1+\hat{Y}_t\right) \\
&+\left(\frac{1}{\phi\left(\left(1-\gamma+\gamma S^{1-\eta}\right)^{-\frac{1}{1-\eta}}Sb_{t+1}\right)}-1\right)b_{t+1}\frac{S}{\left(1+i^*\right)} \\
&+\left(\begin{array}{c} a_t^*-a_{t+1}^*\frac{\left(1+\hat{\Pi}_{Ht+1}\right)}{(1+i)(1+\hat{\imath}_t)}-S\left(1+\hat{S}_t\right)b_t \\ +S\left(1+\hat{S}_t\right)b_{t+1}\frac{\left(1+\hat{\Pi}^*_{Ft+1}\right)}{(1+i^*)(1+\hat{\imath}^*_t)} \end{array}\right) \\
\hat{Y}_t &= \frac{C}{Y}\Upsilon\left(\hat{C}_t+\frac{\gamma S^{1-\eta}}{\left(1-\gamma+\gamma S^{1-\eta}\right)}\hat{S}_t\right)+\frac{k}{Y}\hat{k}_{t+1}-(1-\delta)\,\frac{k}{Y}\hat{k}_t+\frac{G}{Y}\hat{G}_t \\
&+\frac{a_t^*}{Y}-\beta\frac{a_{t+1}^*}{Y}-S\frac{b_t}{Y}+S\beta\frac{b_{t+1}}{Y}
\end{aligned}
$$

$$
\begin{aligned}
0 &= X\left(1+\hat{X}_t\right)Y\left(1+\hat{Y}_t\right) \\
&+\left(a\left(1+\hat{a}_{t+1}\right)+a_{t+1}^*\right)\frac{1+\hat{\Pi}_{Ht+1}}{\tau\left(1+\hat{\tau}_t\right)+(1+i)\left(1+\hat{\imath}_t\right)\left(1-\tau\left(1+\hat{\tau}_t\right)\right)} \\
&-\left(a\left(1+\hat{a}_t\right)+a_t^*\right)-w\left(1+\hat{w}_t\right)n\left(1+\hat{n}_t\right) \\
&-\left(1+\hat{Q}_t\right)\left(k\left(1+\hat{k}_{t+1}\right)-(1-\delta)\,k\left(1+\hat{k}_t\right)\right) \\
&-d\left(1+\hat{d}_t\right)-\kappa\left(\frac{\left(1+\hat{d}_t\right)}{\left(1+\hat{d}_{t-1}\right)}\left(1+\hat{\Pi}_{Ht}\right)-1\right)^2 d\left(1+\hat{d}_t\right) \\
0 &= X\left(\hat{X}_t+\hat{Y}_t\right) \\
&+\left(\hat{a}_{t+1}+\frac{1}{a}a_{t+1}^*+\hat{\Pi}_{Ht+1}-\frac{(1+i)\,(1-\tau)}{R}\hat{\imath}_t+\frac{i\tau}{R}\hat{\tau}_t\right)\frac{a}{YR} \\
&-\frac{a}{Y}\left(\hat{a}_t+\frac{1}{a}a_t^*\right)-\frac{k}{Y}\hat{k}_{t+1}+(1-\delta)\,\frac{k}{Y}\hat{k}_t-\frac{d}{Y}\hat{d}_t-w\frac{n}{Y}\left(\hat{w}_t+\hat{n}_t\right)
\end{aligned}
$$

$$
\begin{aligned}
w^*\left(1+\hat{w}_t^*\right) &= \frac{\Gamma\alpha n^{*\psi}C^{*\sigma}\left(1+\psi\hat{n}_t^*\right)\left(1+\sigma\hat{C}_t^*+\hat{\Gamma}_t\right)}{\left(1-\tau^{*w}\right)\left(1-\frac{\tau^{*w}}{(1-\tau^{*w})}\hat{\tau}_t^{*w}\right)} \\
\hat{w}_t^* &= \psi\hat{n}_t^*+\sigma\hat{C}_t^*+\hat{\Gamma}_t+\frac{\tau^{*w}}{\left(1-\tau^{*w}\right)}\hat{\tau}_t^{*w}
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\left(1+i^*\right)\left(1+\hat{\imath}_t^*\right)} &= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}_{t+1}^*+\hat{\Gamma}_t\right)}{\left(1-\sigma\hat{C}_t^*+\hat{\Gamma}_{t+1}\right)}\frac{1}{\left(1+\hat{\Pi}_{Ft+1}^*\right)} \\
\hat{C}_t^* &= \hat{C}_{t+1}^*+\frac{1}{\sigma}\left(\hat{\Gamma}_{t+1}-\hat{\Gamma}_t+\hat{\Pi}_{Ft+1}^*-\hat{\imath}_t^*\right)
\end{aligned}
$$

$$\begin{aligned}
\frac{1}{(1+i)(1+\hat{\imath}_t)} &= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}^*_{t+1}+\hat{\Gamma}_t+\hat{S}_t\right)}{\left(1-\sigma\hat{C}^*_t+\hat{\Gamma}_{t+1}+\hat{S}_{t+1}\right)}\frac{1}{\left(1+\hat{\Pi}_{Ht+1}\right)} \\
&\quad \phi^*\left(\left((1-\gamma^*)S^{1-\eta}+\gamma^*\right)^{-\frac{1}{1-\eta}}a^*_{t+1}\right) \\
\hat{C}^*_t &= \hat{C}^*_{t+1}-\frac{1}{\sigma}\left(\hat{\imath}_t-\hat{\Pi}_{Ht+1}\right)+\frac{1}{\sigma}(1-\gamma^*)\left(\hat{S}_{t+1}-\hat{S}_t\right)+\frac{\chi^*}{\sigma}a^*_{t+1}
\end{aligned}$$

$$\begin{aligned}
\left(1+\hat{\lambda}^*_t\right)w^*(1+\hat{w}^*_t) &= \left(\left(1+\hat{\lambda}^*_t\right)-\mu^*(1+\hat{\mu}^*_t)\right)X^*\left(1+\hat{X}^*_t\right)(1-\theta)\frac{Y^*\left(1+\hat{Y}^*_t\right)}{n^*(1+\hat{n}^*_t)} \\
\hat{\lambda}^*_t+\hat{w}^*_t &= \frac{1}{(1-\mu^*)}\hat{\lambda}^*_t-\frac{\mu^*}{(1-\mu^*)}\hat{\mu}^*_t+\hat{X}^*_t+\hat{Y}^*_t-\hat{n}^*_t
\end{aligned}$$

$$\begin{aligned}
&\left(\left(1+\hat{\lambda}^*_t\right)-\mu^*(1+\hat{\mu}^*_t)\Xi^*\left(1+\hat{\xi}^*_t\right)\right)\left(1+\hat{Q}^*_t\right) \\
&= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}^*_{t+1}+\hat{\Gamma}_t\right)}{\left(1-\sigma\hat{C}^*_t+\hat{\Gamma}_{t+1}\right)}\left(\begin{array}{c}\left(\left(1+\hat{\lambda}^*_{t+1}\right)-\mu^*\left(1+\hat{\mu}^*_{t+1}\right)\right)X^*\left(1+\hat{X}^*_{t+1}\right) \\ \theta\frac{Y^*\left(1+\hat{Y}^*_{t+1}\right)}{k^*\left(1+\hat{k}^*_{t+1}\right)}+\left(1+\hat{\lambda}^*_{t+1}\right)\left(1+\hat{Q}^*_{t+1}\right)(1-\delta)\end{array}\right) \\
0 &= (1-\mu^*\Xi^*)\left(-\sigma\hat{C}^*_{t+1}+\sigma\hat{C}^*_t+\hat{\Gamma}_t-\hat{\Gamma}_{t+1}\right)-\hat{\lambda}^*_t+\Xi^*\mu^*\left(\hat{\mu}^*_t+\hat{\xi}^*_t\right) \\
&\quad +\beta X^*\theta\frac{Y^*}{k^*}\left(\hat{\lambda}^*_{t+1}-\mu^*\hat{\mu}^*_{t+1}\right)+\beta(1-\mu^*)X^*\theta\frac{Y^*}{k^*}\left(\hat{X}^*_{t+1}+\hat{Y}^*_{t+1}-\hat{k}^*_{t+1}\right) \\
&\quad +\beta(1-\delta)\hat{\lambda}^*_{t+1}
\end{aligned}$$

$$\begin{aligned}
&\frac{\left(1+\hat{\lambda}^*_t\right)}{\tau^*(1+\hat{\tau}^*_t)+(1+i^*)(1+\hat{\imath}^*_t)(1-\tau^*(1+\hat{\tau}^*_t))} \\
&-\mu^*(1+\hat{\mu}^*_t)\Xi^*\left(1+\hat{\xi}^*_t\right)\frac{1}{(1+i^*)(1+\hat{\imath}^*_t)} \\
&= \beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}^*_{t+1}+\hat{\Gamma}_t\right)}{\left(1-\sigma\hat{C}^*_t+\hat{\Gamma}_{t+1}+\hat{\Pi}^*_{Ft+1}\right)}\left(1+\hat{\lambda}^*_{t+1}\right) \\
&(\mu^*\Xi^*+1)\left(\hat{\lambda}^*_t+\frac{i^*\tau^*}{(1+(1-\tau^*)i^*)}\hat{\tau}^*_t\right)+\left(\mu^*\Xi^*-\frac{(\mu^*\Xi^*+1)(1-\tau^*)}{\beta(1+(1-\tau^*)i^*)}\right)\hat{\imath}^*_t \\
&-\mu^*\Xi^*\left(\hat{\mu}^*_t+\hat{\xi}^*_t\right) \\
&= \left(-\sigma\hat{C}^*_{t+1}+\hat{\Gamma}_t-\hat{\Gamma}_{t+1}-\hat{\Pi}^*_{Ft+1}+\sigma\hat{C}^*_t+\hat{\lambda}^*_{t+1}\right)
\end{aligned}$$

$$\begin{aligned}
0 &= 1+\beta\mathbb{E}_t\frac{\left(1-\sigma\hat{C}^*_{t+1}+\hat{\Gamma}_t\right)}{\left(1-\sigma\hat{C}^*_t+\hat{\Gamma}_{t+1}\right)}\left(1+\hat{\lambda}^*_{t+1}\right)2\kappa^*\left(\frac{\left(1+\hat{d}^*_{t+1}\right)}{\left(1+\hat{d}^*_t\right)}\left(1+\hat{\Pi}^*_{Ft+1}\right)-1\right) \\
&\quad \frac{\left(1+2\hat{d}^*_{t+1}\right)}{\left(1+2\hat{d}^*_t\right)}\left(1+\hat{\Pi}^*_{Ft+1}\right) \\
&\quad -\left(1+\hat{\lambda}^*_t\right)\left(\begin{array}{c}1+2\kappa^*\left(\frac{\left(1+\hat{d}^*_t\right)}{\left(1+\hat{d}^*_{t-1}\right)}\left(1+\hat{\Pi}^*_{Ft}\right)-1\right)\frac{\left(1+\hat{d}^*_t\right)}{\left(1+\hat{d}^*_{t-1}\right)}\left(1+\hat{\Pi}^*_{Ft}\right) \\ +\kappa^*\left(\frac{\left(1+\hat{d}^*_t\right)}{\left(1+\hat{d}^*_{t-1}\right)}\left(1+\hat{\Pi}^*_{Ft}\right)-1\right)^2\end{array}\right) \\
\hat{\lambda}^*_t &= 2\beta\kappa^*\left(\hat{d}^*_{t+1}-\hat{d}^*_t+\hat{\Pi}^*_{Ft+1}\right)-2\kappa^*\left(\hat{d}^*_t-\hat{d}^*_{t-1}+\hat{\Pi}^*_{Ft}\right)
\end{aligned}$$

$$
\begin{aligned}
& \Xi^* \left(1+\hat{\xi}_t^*\right)\left(\left(1+\hat{Q}_t^*\right) k^* \left(1+\hat{k}_{t+1}^*\right) - \left(b_{t+1} + b^* \left(1+\hat{b}_{t+1}^*\right)\right) \frac{\left(1+\hat{\Pi}_{Ft+1}^*\right)}{\left(1+i^*\right)\left(1+\hat{\imath}_t^*\right)}\right) \\
= \; & X^* \left(1+\hat{X}_t^*\right) Y^* \left(1+\hat{Y}_t^*\right) \\
& \Xi^* \frac{k^*}{Y^*}\left(\hat{\xi}_t^* + \hat{k}_{t+1}^*\right) - \beta \Xi^* \frac{b^*}{Y^*}\left(\frac{1}{b^*} b_{t+1} + \hat{b}_{t+1}^* + \hat{\Pi}_{Ft+1}^* - \hat{\imath}_t^* + \hat{\xi}_t^*\right) \\
= \; & X^* \left(\hat{X}_t^* + \hat{Y}_t^*\right)
\end{aligned}
$$

$$
\hat{Q}_t^* = \varrho\left(\hat{I}_t^* - \hat{I}_{t-1}^*\right) - \beta \varrho \left(\hat{I}_{t+1}^* - \hat{I}_t^*\right)
$$

$$
\begin{aligned}
& \omega\left(\left(1+\hat{\Pi}_{Ft}^*\right)-1\right)\left(1+\hat{\Pi}_{Ft}^*\right) \\
= \; & (1-\varepsilon)\left(1-\tau^{*x}\left(1+\hat{\tau}_t^{*x}\right)\right) + \varepsilon X^* \left(1+\hat{X}_t^*\right) \\
& +\omega \beta \mathbb{E}_t \frac{\left(1-\sigma \hat{C}_{t+1}^* + \hat{\Gamma}_t\right)}{\left(1-\sigma \hat{C}_t^* + \hat{\Gamma}_{t+1}\right)}\left(\left(1+\hat{\Pi}_{Ft+1}^*\right)-1\right) \frac{\left(1+\hat{Y}_{t+1}^*\right)}{\left(1+\hat{Y}_t^*\right)}\left(1+\hat{\Pi}_{Ft+1}^*\right) \\
\hat{\Pi}_{Ft}^* \; = \; & \frac{(\varepsilon-1)\left(1-\tau^{*x}\right)}{\omega}\left(\frac{\tau^{*x}}{\left(1-\tau^{*x}\right)} \hat{\tau}_t^{*x} + \hat{X}_t^*\right) + \beta \hat{\Pi}_{Ft+1}^*
\end{aligned}
$$

$$
\begin{aligned}
Y^*\left(1+\hat{Y}_t^*\right) \; = \; & \left(\left(1-\gamma^*\right) S^{1-\eta} + \gamma^*\right)^{\frac{1}{1-\eta}} \frac{\left(1+\frac{\left(1-\gamma^*\right) S^{1-\eta}}{\left(\left(1-\gamma^*\right) S^{1-\eta}+\gamma^*\right)} \hat{S}_t\right)}{S\left(1+\hat{S}_t\right)} C^*\left(1+\hat{C}_t^*\right) \\
& +\left(k^*\left(1+\hat{k}_{t+1}^*\right) - (1-\delta) k^*\left(1+\hat{k}_t^*\right)\right) + G^*\left(1+\hat{G}_t^*\right) \\
& +\kappa^*\left(\frac{\left(1+\hat{d}_t^*\right)}{\left(1+\hat{d}_{t-1}^*\right)}\left(1+\hat{\Pi}_{Ft}^*\right)-1\right)^2 d^*\left(1+\hat{d}_t^*\right) \\
& +\frac{\omega}{2}\left(\Pi_t^*-1\right)^2 Y^*\left(1+\hat{Y}_t^*\right) \\
& +\left(\frac{1}{\phi^*\left(\left(\left(1-\gamma^*\right) S^{1-\eta}+\gamma^*\right)^{-\frac{1}{1-\eta}} a_{t+1}^*\right)} - 1\right) \frac{a_{t+1}^*}{S} \frac{1}{(1+i)} \\
& -\frac{1}{S\left(1+\hat{S}_t\right)}\left(a_t^* - a_{t+1}^* \frac{1}{(1+i)} - S b_t + S b_{t+1} \frac{1}{1+i^*}\right) \\
\hat{Y}_t^* \; = \; & \frac{C^*}{Y^*} \Gamma\left(\hat{C}_t^* - \frac{\gamma^*}{\left(\left(1-\gamma^*\right) S^{1-\eta}+\gamma^*\right)} \hat{S}_t\right) + \frac{k^*}{Y^*} \hat{k}_{t+1}^* - (1-\delta) \frac{k^*}{Y^*} \hat{k}_t^* + \frac{G^*}{Y^*} \hat{G}_t^* \\
& -\frac{1}{S} \frac{a_t^*}{Y^*} + \beta \frac{1}{S} \frac{a_{t+1}^*}{Y^*} + \frac{b_t}{Y^*} - \beta \frac{b_{t+1}}{Y^*}
\end{aligned}
$$

$$
\begin{aligned}
0 &= X^* \left(1+\hat{X}_t^*\right) Y^* \left(1+\hat{Y}_t^*\right) + \left(b_{t+1} + b^* \left(1+\hat{b}_{t+1}^*\right)\right) \\
&\quad \frac{1+\hat{\Pi}_{Ft+1}^*}{\tau^* \left(1+\hat{\tau}_t^*\right) + \left(1+i^*\right)\left(1+\hat{\imath}_t^*\right)\left(1-\tau^*\left(1+\hat{\tau}_t^*\right)\right)} \\
&\quad - \left(b_t + b^* \left(1+\hat{b}_t^*\right)\right) - w^* \left(1+\hat{w}_t^*\right) n^* \left(1+\hat{n}_t^*\right) \\
&\quad - \left(1+\hat{Q}_t^*\right) \left(k^* \left(1+\hat{k}_{t+1}^*\right) - (1-\delta) k^* \left(1+\hat{k}_t^*\right)\right) \\
&\quad - d^* \left(1+\hat{d}_t^*\right) - \kappa^* \left(\frac{\left(1+\hat{d}_t^*\right)}{\left(1+\hat{d}_{t-1}^*\right)} \left(1+\hat{\Pi}_{Ft}^*\right) - 1\right)^2 d^* \left(1+\hat{d}_t^*\right) \\
0 &= X^* \left(\hat{X}_t^* + \hat{Y}_t^*\right) + \frac{b^*}{R^* Y^*} \begin{pmatrix} \hat{b}_{t+1}^* + \frac{b_{t+1}}{b^*} + \hat{\Pi}_{Ft+1}^* \\ -\frac{(1+i^*)(1-\tau^*)}{R^*} \hat{\imath}_t^* + \frac{i^* \tau^*}{R^*} \hat{\tau}^*_t \end{pmatrix} \\
&\quad - \frac{b^*}{Y^*} \left(\frac{b_t}{b^*} + \hat{b}_t^*\right) - \frac{k^*}{Y^*} \hat{k}_{t+1}^* + (1-\delta) \frac{k^*}{Y^*} \hat{k}_t^* - \frac{d^*}{Y^*} \hat{d}_t^* - w^* \frac{n^*}{Y^*} \left(\hat{w}_t^* + \hat{n}_t^*\right)
\end{aligned}
$$

$$
\begin{aligned}
0 &= \gamma S^{1-\eta} \left((1-\gamma) + \gamma S^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \left(1 + (1-\eta) \hat{S}_t\right) \\
&\quad \left(1 + \frac{\eta \gamma S^{1-\eta}}{\left((1-\gamma) + \gamma S^{1-\eta}\right)} \hat{S}_t\right) C \left(1+\hat{C}_t\right) \\
&\quad - \gamma^* \left(\left((1-\gamma^*) S^{1-\eta} + \gamma^*\right)^{\frac{\eta}{1-\eta}} \left(1 + \frac{\eta (1-\gamma^*) S^{1-\eta}}{\left((1-\gamma^*) S^{1-\eta} + \gamma^*\right)} \hat{S}_t\right)\right) C^* \left(1+\hat{C}_t^*\right) \\
&\quad - \beta a_{t+1}^* + a_t^* + \beta b_{t+1} S - b_t S \\
0 &= \gamma S^{1-\eta} \left((1-\gamma) + \gamma S^{1-\eta}\right)^{\frac{\eta}{1-\eta}} \frac{C}{Y} \left(\frac{(1-\eta)(1-\gamma) + \gamma S^{1-\eta}}{\left((1-\gamma) + \gamma S^{1-\eta}\right)} \hat{S}_t + \hat{C}_t\right) \\
&\quad - \gamma^* \left((1-\gamma^*) S^{1-\eta} + \gamma^*\right)^{\frac{\eta}{1-\eta}} \frac{C^*}{Y^*} \frac{Y^*}{Y} \left(\frac{\eta (1-\gamma^*) S^{1-\eta}}{\left((1-\gamma^*) S^{1-\eta} + \gamma^*\right)} \hat{S}_t + \hat{C}_t^*\right) \\
&\quad - \beta \frac{a_{t+1}^*}{Y^*} \frac{Y^*}{Y} + \frac{a_t^*}{Y^*} \frac{Y^*}{Y} + \beta \frac{b_{t+1}}{Y} S - \frac{b_t}{Y} S
\end{aligned}
$$

$$
\begin{aligned}
\hat{Y}_t &= \hat{z}_t + \theta \hat{k}_t + (1-\theta) \hat{n}_t \\
\hat{Y}_t^* &= \hat{z}_t^* + \theta \hat{k}_t^* + (1-\theta) \hat{n}_t^*
\end{aligned}
$$

$$
a_{t+1}^* = -\frac{\chi}{\chi^*} b_{t+1}
$$

Proof (for the last equation):

From the following equations

$$
\hat{C}_t = \hat{C}_{t+1} + \frac{1}{\sigma} \left(\hat{\Pi}_{Ht+1} - \hat{\imath}_t\right) + \frac{1}{\sigma} \left(\hat{\Upsilon}_{t+1} - \hat{\Upsilon}_t\right)
$$

$$
\hat{C}_t = \hat{C}_{t+1} - \frac{1}{\sigma} \left(\hat{\imath}_t^* - \hat{\Pi}_{Ft+1}^*\right) - \frac{1}{\sigma} \frac{(1-\gamma)}{\left(1-\gamma+\gamma S^{1-\eta}\right)} \left(\hat{S}_{t+1} - \hat{S}_t\right) + \frac{\chi}{\sigma} \frac{S}{\Upsilon} b_{t+1}
$$

$$
\hat{C}_t^* = \hat{C}_{t+1}^* + \frac{1}{\sigma} \left(\hat{\Gamma}_{t+1} - \hat{\Gamma}_t + \hat{\Pi}_{Ft+1}^* - \hat{\imath}_t^*\right)
$$

$$\hat{C}_t^* = \hat{C}_{t+1}^* - \frac{1}{\sigma}\left(\hat{\imath}_t - \hat{\Pi}_{Ht+1}\right) + \frac{1}{\sigma}\frac{(1-\gamma^*)\,S^{1-\eta}}{((1-\gamma^*)\,S^{1-\eta}+\gamma^*)}\left(\hat{S}_{t+1} - \hat{S}_t\right) + \frac{\chi^*}{\sigma S\Gamma}a_{t+1}^*$$

we get

$$\hat{\imath}_t = \hat{\imath}_t^* - \hat{\Pi}_{Ft+1}^* + \hat{\Pi}_{Ht+1} + \left(\hat{S}_{t+1} - \hat{S}_t\right) - \chi b_{t+1}$$

$$\hat{\imath}_t = \hat{\imath}_t^* - \hat{\Pi}_{Ft+1}^* + \hat{\Pi}_{Ht+1} + \hat{S}_{t+1} - \hat{S}_t + \chi^* a_{t+1}^*$$

so that

$$\chi^* a_{t+1}^* + \chi b_{t+1} = 0$$

B.6 Closed vs open economy model

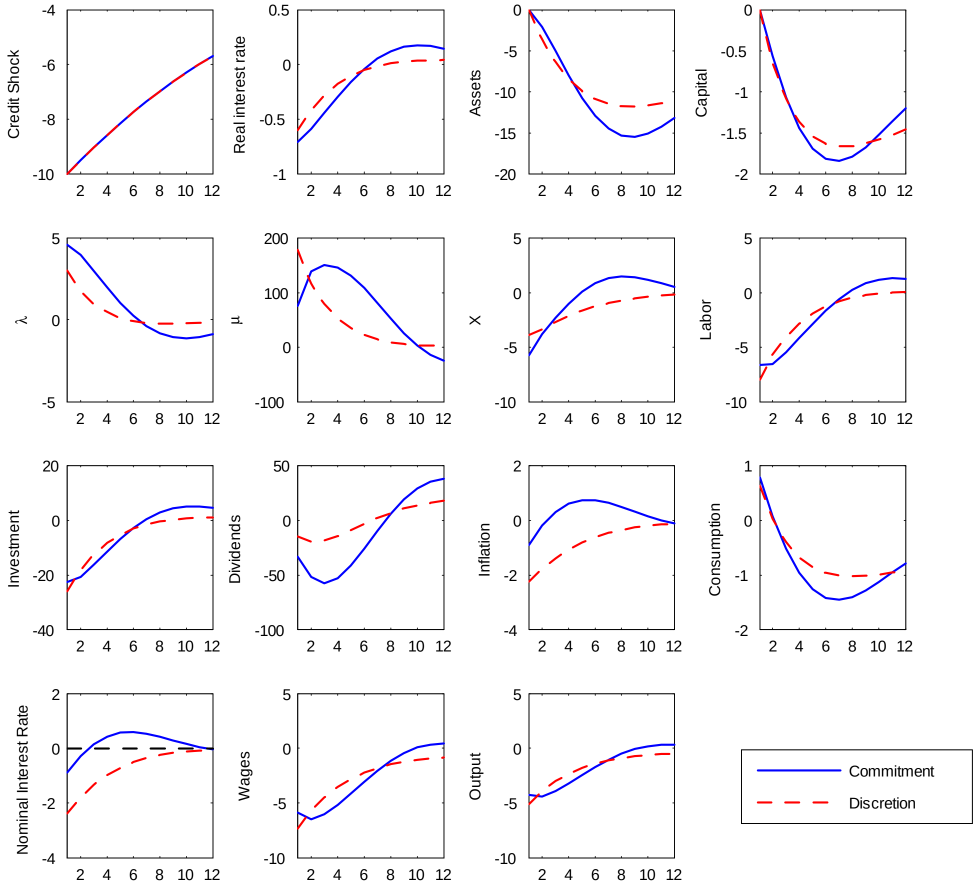

Figure B.1: Impulse responses of a closed economy to a credit shock under commitment and discretion (with no fiscal policy)

Appendix C

Welfare Loss Computation

Consider the following linear constraint

$$Z_{s+1} = MZ_s + B\varepsilon_{s+1}$$

Z_s is predetermined variable

from where, assuming that ε_s are not correlated with Z_s, it follows

$$Z'_{s+1} = Z'_s M' + \varepsilon'_{s+1} B'$$

$$Z_{s+1} Z'_{s+1} = (MZ_s + B\varepsilon_{s+1}) \left(Z'_s M' + \varepsilon'_{s+1} B' \right)$$

$$\sum_{s=0}^{\infty} \omega^s Z_{s+1} Z'_{s+1} = \sum_{s=0}^{\infty} \omega^s (MZ_s + B\varepsilon_{s+1}) \left(Z'_s M' + \varepsilon'_{s+1} B' \right)$$

$$\sum_{s=0}^{\infty} \omega^s Z_{s+1} Z'_{s+1} = \sum_{s=0}^{\infty} \omega^s \left(MZ_s Z'_s M' + B\varepsilon_{s+1} Z'_s M' + MZ_s \varepsilon'_{s+1} B' + B\varepsilon_{s+1} \varepsilon'_{s+1} B' \right)$$

$$\begin{aligned} &\sum_{s=0}^{\infty} \omega^s Z_{s+1} Z'_{s+1} \\ = \;& \sum_{s=0}^{\infty} \omega^s \left(MZ_s Z'_s M' + B \underbrace{\varepsilon_{s+1} Z'_s}_{0,\ \text{uncorrelated}} M' + M \left(Z_s \varepsilon'_{s+1} \right) B' + B\varepsilon_{s+1} \varepsilon'_{s+1} B' \right) \end{aligned}$$

$$\sum_{s=0}^{\infty} \omega^s Z_{s+1} Z'_{s+1} = \sum_{s=0}^{\infty} \omega^s \left(MZ_s Z'_s M' + B\varepsilon_{s+1} \varepsilon'_{s+1} B' \right)$$

$$\sum_{s=0}^{\infty} \omega^s Z_{s+1} Z'_{s+1} = M \left(\sum_{s=0}^{\infty} \omega^s Z_s Z'_s \right) M' + B \left(\sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon'_{s+1} \right) B'$$

$$\frac{1}{\omega} \left(\sum_{s=0}^{\infty} \omega^{s+1} Z_{s+1} Z'_{s+1} + Z'_0 Z_0 \right) = M \left(\sum_{s=0}^{\infty} \omega^s Z_s Z'_s \right) M' + B \left(\sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon'_{s+1} \right) B' + \frac{1}{\omega} Z_0 Z'_0$$

$$\frac{1}{\omega} \left(\sum_{s=0}^{\infty} \omega^s Z_s Z'_s \right) = M \left(\sum_{s=0}^{\infty} \omega^s Z_s Z'_s \right) M' + B \left(\sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon'_{s+1} \right) B' + \frac{1}{\omega} Z_0 Z'_0$$

$$\sum_{s=0}^{\infty} \omega^s Z_s Z_s' = \omega M \left(\sum_{s=0}^{\infty} \omega^s Z_s Z_s' \right) M' + \omega B \left(\sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon_{s+1}' \right) B' + Z_0 Z_0'$$

$$\mathcal{E}_0 \sum_{s=0}^{\infty} \omega^s Z_s Z_s' = \omega M \left(\mathcal{E}_0 \sum_{s=0}^{\infty} \omega^s Z_s Z_s' \right) M' + \omega B \left(\mathcal{E}_0 \sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon_{s+1}' \right) B' + Z_0 Z_0'$$

Denote $V = \sum_{s=0}^{\infty} \omega^s Z_s Z_s'$. For covariance stationary ε_s with covariance matrix Σ

$$\begin{aligned}
\mathcal{E}_0 V &= \omega M \mathcal{E}_0 V M' + \omega B \left(\mathcal{E}_0 \sum_{s=0}^{\infty} \omega^s \varepsilon_{s+1} \varepsilon_{s+1}' \right) B' + Z_0 Z_0' \\
&= \omega M \mathcal{E}_0 V M' + \omega B \left(\sum_{s=0}^{\infty} \omega^s \mathcal{E}_0 \left(\varepsilon_{s+1} \varepsilon_{s+1}' \right) \right) B' + Z_0 Z_0' \\
&= \omega M \mathcal{E}_0 V M' + \frac{\omega}{1-\omega} B \Sigma B' + Z_0 Z_0'
\end{aligned}$$

$$X = \omega M X M' + \frac{\omega}{1-\omega} B \Sigma B' + Z_0 Z_0'$$

$$\begin{aligned}
vec(X) &= \omega vec(M X M') + \frac{\omega}{1-\omega} vec(B \Sigma B') + vec(Z_0 Z_0') \\
vec(X) &= \omega (M \otimes M) vec(X) + \frac{\omega}{1-\omega} (B \otimes B) vec(\Sigma) \\
&\quad + (Z_0 \otimes Z_0) \\
vec(X) - \omega (M \otimes M) vec(X) &= \frac{\omega}{1-\omega} (B \otimes B) vec(\Sigma) + (Z_0 \otimes Z_0) \\
(I - \omega (M \otimes M)) vec(X) &= \frac{\omega}{1-\omega} (B \otimes B) vec(\Sigma) + (Z_0 \otimes Z_0) \\
vec(X) &= (I - \omega (M \otimes M))^{-1} vec\left(\frac{\omega}{1-\omega} B \Sigma B' + Z_0 Z_0' \right)
\end{aligned}$$

Appendix D

ZLB under Discression: application of the Laséen-Svensson (2011) approach

Consider the standard LQ RE model. The policy objective is quadratic

$$L_t = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}g_s'Qg_s = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(y_s'\mathcal{Q}y_s + 2y_s'\mathcal{P}u_s + u_s'\mathcal{R}u_s\right). \tag{D.1}$$

subject to the system of linear constraints

$$\begin{bmatrix} x_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}[u_t] + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}[\epsilon_{t+1}], \tag{D.2}$$

where $y_s = [x_s', X_s']'$. We assume A_{22} is invertible. We consider standard discretionary policy.

D.1 No binding constraints

This section presents the standard discretionary solution.

Suppose that the reaction of the private sector is given by a linear rule

$$X_t = -Nx_t. \tag{D.3}$$

Representation (D.3) can be rewritten in an equivalent form in terms of predetermined variables and controls (as did Oudiz and Sachs, 1985). We one-period lead (D.3) and substitute for x_{t+1} from the first equation (D.2):

$$X_{t+1} = -Nx_{t+1} = -N(A_{11}x_t + A_{12}X_t + B_1u_t).$$

Combining this with the second equation in (D.2) we obtain:

$$\begin{aligned} X_t &= -(A_{22} + NA_{12})^{-1}[(A_{21} + NA_{11})y_t + (B_2 + NB_1)u_t] \\ &= -Jx_t - Ku_t, \end{aligned} \tag{D.4}$$

where

$$J = (A_{22} + NA_{12})^{-1}(A_{21} + NA_{11}), \tag{D.5}$$

$$K = (A_{22} + NA_{12})^{-1}(B_2 + NB_1). \tag{D.6}$$

The policymaker is maximising its objective function with respect to u_t, taking time-consistent reaction X_t as given and recognising dependence of X_t on policy u_s. We define Lagrangian with period term

$$\begin{aligned} H_s &= \frac{1}{2}\beta^{s-t}(y_s'\mathcal{Q}y_s + 2y_s'\mathcal{P}u_s + u_s'\mathcal{R}u_s) + \lambda_{s+1}'(A_{11}x_s + A_{12}X_s + B_1u_s - y_{s+1}) \\ &\quad +\mu_s'\left(X_s + Jx_s + Ku_s\right), \end{aligned}$$

with λ_s and μ_s are Lagrange multipliers.

First order conditions can be written as

$$\begin{aligned} 0 &= \left(\mathcal{P}_1' - K'\mathcal{Q}_{12}'\right)x_s + \left(\mathcal{P}_2' - K'\mathcal{Q}_{22}\right)X_s + \left(\mathcal{R} - K'\mathcal{P}_2\right)u_s + \left(B_1' - K'A_{12}'\right)\beta\xi_{s+1}, \\ 0 &= \left(\mathcal{Q}_{11} - J'\mathcal{Q}_{12}'\right)x_s + \left(\mathcal{Q}_{12} - J'\mathcal{Q}_{22}\right)X_s + \left(\mathcal{P}_1 - J'\mathcal{P}_2\right)u_s - \lambda_s + \left(A_{11}' - J'A_{12}'\right)\beta\xi_{s+1}, \\ \eta_s &= -\mathcal{Q}_{12}'x_s - \mathcal{Q}_{22}X_s - \mathcal{P}_2u_s - A_{12}'\beta\xi_{s+1} \end{aligned}$$

and equations (D.4) and the first equation of (D.2). Here $\xi_s = \beta^{s-t}\lambda_s$, $\eta_s = \beta^{s-t}\mu_s$ and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ are partitioned conformally with $y_s = [x_s', X_s']'$ and u_s.

Substitute out X_s and η_s, and relabelling the matrices we arrive to the following linear system

$$\begin{aligned} 0 &= P^{*\prime}x_s + R^*u_s + B^{*\prime}\beta\xi_{s+1}, \\ 0 &= Q^*x_s + P^*u_s - \lambda_s + A^{*\prime}\beta\xi_{s+1}, \\ 0 &= A^*x_s + B^*u_s - x_{s+1} \end{aligned}$$

which can be written in a matrix form

$$\begin{bmatrix} I & 0 \\ 0 & \Phi_{22} \end{bmatrix}\begin{bmatrix} x_{t+1} \\ \tilde{u}_{t+1} \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix}\begin{bmatrix} x_t \\ \tilde{u}_t \end{bmatrix}, \tag{D.7}$$

where $\tilde{u}_t = [u_t', \xi_t']'$and

$$\begin{aligned} &\Phi_{22} = \begin{bmatrix} 0 & \beta B^{*\prime} \\ 0 & \beta A^{*\prime} \end{bmatrix}, \quad \Psi_{21} = \begin{bmatrix} -P^{*\prime} \\ -Q^* \end{bmatrix}, \quad \Psi_{22} = \begin{bmatrix} -R^* & 0 \\ -P^* & I \end{bmatrix}, \\ &\Psi_{11} = A^*, \quad \Psi_{12} = \begin{bmatrix} B^* & 0 \end{bmatrix}. \end{aligned}$$

A solution to system (D.7) will necessarily have a linear form of

$$\tilde{u}_t = \begin{bmatrix} u_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} -F \\ S \end{bmatrix} y_t \tag{D.8}$$

It is straightforward to show that system matrices in (D.8) satisfy the following well-known Riccati equations describing solution to a discretionary problem

$$S = Q^* + \beta A^{*\prime} S A^* - (P^{*\prime} + \beta B^{*\prime} S A^*) (R^* + \beta B^{*\prime} S B^*)^{-1} (P^{*\prime} + \beta B^{*\prime} S A^*) \tag{D.9}$$

$$F = (R^* + \beta B^{*\prime} S B^*)^{-1} (P^{*\prime} + \beta B^{*\prime} S A^*) \tag{D.10}$$

Practically, the solution can be found with generalised Schur decomposition of (D.7).

D.2 Binding constraint on instrument

Following Laséen and Svensson (2011) we augment the original system by the vector of predetermined state variables $\boldsymbol{z}^t \equiv (z_{t,t}, z_{t+1,t} \dots z_{t+T,t})'$ in order to account for the sequence of anticipated policy shocks. Vector $\boldsymbol{z}^t$ denotes a projection in period t of future realizations of shocks, $z_{t+\tau,t}, \tau = 0, 1, ..., T$. $z_{t,t}$ follows a moving average process

$$z_{t,t} = \eta_{t,t} + \sum_{s=1}^{T} \eta_{t,t-s},$$

where $\eta_{t,t-s}, s = 0, 1, ..T$, are zero-mean $i.i.d.$ shocks. For $T = 0$, $z_{t,t} = \eta_{t,t}$. For $T > 0$, the stochastic shocks following a moving average process:

$$\begin{aligned} z_{t+\tau,t+1} &= z_{t+\tau,t} + \eta_{t+\tau,t+1}, \;\; \tau = 1, ..., T \\ z_{t+T+1,t+1} &= \eta_{t+T+1,t+1}. \end{aligned}$$

The above stochastic shocks process can be rewritten in the following matrix form

$$\boldsymbol{z}^{t+1} = \boldsymbol{A}_z \boldsymbol{z}^t + \boldsymbol{\eta}^{t+1},$$

where $\boldsymbol{\eta}^{t+1} \equiv \left(\eta_{t+1,t+1}, \eta_{t+2,t+1} \dots \eta_{t+T+1,t+1}\right)'$ is a $(T+1)$ vector of $i.i.d.$shocks and A_z is $(n_1+1) \times (n_1+1)$ matrix

$$A_z = \begin{bmatrix} \mathbf{0}_{T\times 1} & \boldsymbol{I}_T \\ 0 & \mathbf{0}_{1\times T} \end{bmatrix}$$

We denote the vector of predetermined state variables $x_t^c = [z_t', x_t']'$, where superscript c stands for ‘constrained’, and vector z_t consists of anticipated shocks.

Matrices in equation (D.2) can be written as

$$A_{11}^c = \begin{bmatrix} A_z & 0 \\ 0 & A_{11} \end{bmatrix}, B_1^c = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}$$

and matrices $\mathcal{Q}$, $\mathcal{P}$ and $\mathcal{R}$ can be redefined to account for additional state variables (we keep the same notation).

Finally, equation for policy instrument has to be augmented to account for reaction to shocks z_t. This is achieved by replacing the top left square submatrix of new Ψ_{21} with R^*. As before, the solution can be found by solving the augmented system (D.7) with Schur decomposition.

Appendix E

Commitment: application of the Laséen-Svensson (2011) approach

Consider the standard LQ RE model. The policy objective is quadratic

$$L_t = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}g_s'Qg_s = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(y_s'\mathcal{Q}y_s + 2y_s'\mathcal{P}u_s + u_s'\mathcal{R}u_s\right). \tag{E.1}$$

subject to the system of linear constraints

$$\begin{bmatrix} x_{t+1} \\ X_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_t \\ X_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}[u_t] + \begin{bmatrix} C \\ 0 \end{bmatrix}[\epsilon_{t+1}], \text{ or} \tag{E.2}$$

$$y_{t+1} = Ay_t + Bu_t + \zeta_{t+1}, \quad \text{and } x_0 \text{ given,} \tag{E.3}$$

where $y_t = [x_t', X_t']'$. We assume A_{22} is invertible. We consider standard discretionary policy.

Set up the Lagrangian,

$$\mathcal{L}_0 = \min_{\{u_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty}\beta^t\left[(y_t'\mathcal{Q}y_t + 2y_t'\mathcal{P}u_t + u_t'\mathcal{R}u_t) + 2\lambda_{t+1}'(Ay_t + Bu_t + \zeta_{t+1} - y_{t+1})\right]$$

The first order conditions for u_t are

$$-B'\mathbb{E}_t\lambda_{t+1} = \mathcal{P}'y_t + \mathcal{R}u_t \tag{E.4}$$

The first order conditions for y_t are

$$\beta A'\mathbb{E}_t\lambda_{t+1} = \lambda_t - \beta\mathcal{Q}y_t - \beta\mathcal{P}u_t \tag{E.5}$$

which can be written in the matrix form

$$\begin{bmatrix} I & 0 & 0 \\ 0 & 0 & \beta A' \\ 0 & 0 & -B' \end{bmatrix} \begin{bmatrix} y_{t+1} \\ u_{t+1} \\ \mathbb{E}_t \lambda_{t+1} \end{bmatrix} = \begin{bmatrix} A & B & 0 \\ -\beta \mathcal{Q} & -\beta \mathcal{P} & I \\ \mathcal{P}' & \mathcal{R} & 0 \end{bmatrix} \begin{bmatrix} y_t \\ u_t \\ \lambda_t \end{bmatrix} + \begin{bmatrix} \zeta_{t+1} \\ 0 \\ 0 \end{bmatrix} \tag{E.6}$$

Partition (E.6) as

$$\begin{aligned} & \begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{12} & \tilde{G}_{13} & \tilde{G}_{14} & \tilde{G}_{15} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ X_{t+1} \\ u_{t+1} \\ \mathbb{E}_t \lambda_{1t+1} \\ \mathbb{E}_t \lambda_{2t+1} \end{bmatrix} \\ = & \begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{12} & \tilde{D}_{13} & \tilde{D}_{14} & \tilde{D}_{15} \end{bmatrix} \begin{bmatrix} x_t \\ X_t \\ u_t \\ \mathbb{E}_t \lambda_{1t} \\ \mathbb{E}_t \lambda_{2t} \end{bmatrix} + \begin{bmatrix} C\epsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

and reshuffle so x_t and λ_{2t} come first (since we have initial values for these)

$$\begin{aligned} & \begin{bmatrix} \tilde{G}_{11} & \tilde{G}_{15} & \tilde{G}_{12} & \tilde{G}_{13} & \tilde{G}_{14} \end{bmatrix} \begin{bmatrix} x_{t+1} \\ \mathbb{E}_t \lambda_{2t+1} \\ X_{t+1} \\ u_{t+1} \\ \mathbb{E}_t \lambda_{1t+1} \end{bmatrix} \\ = & \begin{bmatrix} \tilde{D}_{11} & \tilde{D}_{15} & \tilde{D}_{12} & \tilde{D}_{13} & \tilde{D}_{14} \end{bmatrix} \begin{bmatrix} x_t \\ \mathbb{E}_t \lambda_{2t} \\ X_t \\ u_t \\ \mathbb{E}_t \lambda_{1t} \end{bmatrix} + \begin{bmatrix} C\epsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

let

$$k_t = \begin{bmatrix} x_t \\ \mathbb{E}_t \lambda_{2t} \end{bmatrix} \quad \text{and} \quad \rho_t = \begin{bmatrix} X_t \\ u_t \\ \mathbb{E}_t \lambda_{1t} \end{bmatrix}$$

Then

$$G \mathbb{E}_t \begin{bmatrix} k_{t+1} \\ \rho_{t+1} \end{bmatrix} = D \begin{bmatrix} k_t \\ \rho_t \end{bmatrix}$$

We define the auxiliary variables

$$\begin{bmatrix} \theta_t \\ \delta_t \end{bmatrix} = Z^H \begin{bmatrix} k_t \\ \rho_t \end{bmatrix}$$

where we will associate the stable roots with θ, and the unstable with δ.

Use the generalized Schur decomposition $G = QSZ^H$ and $D = QTZ^H$, premultiply with the non-singular matrix Q^H from the generalized Schur decomposition to get

$$Q^H QSZ^H \mathbb{E}_t \begin{bmatrix} k_{t+1} \\ \rho_{t+1} \end{bmatrix} = Q^H QTZ^H \begin{bmatrix} k_t \\ \rho_t \end{bmatrix}$$

then we get

$$S\mathbb{E}_t\begin{bmatrix}\theta_{t+1}\\ \delta_{t+1}\end{bmatrix}=T\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}$$

where S and T are both upper triangular, then

$$\begin{bmatrix}S_{\theta\theta} & S_{\theta\delta}\\ 0 & S_{\delta\delta}\end{bmatrix}\mathbb{E}_t\begin{bmatrix}\theta_{t+1}\\ \delta_{t+1}\end{bmatrix}=\begin{bmatrix}T_{\theta\theta} & T_{\theta\delta}\\ 0 & T_{\delta\delta}\end{bmatrix}\begin{bmatrix}\theta_t\\ \delta_t\end{bmatrix}$$

The lower right block contains the unstable roots, so a stable solution requires that $\delta_t = 0$ for all t. The remaining equations are then $S_{\theta\theta}\mathbb{E}_t\theta_{t+1} = T_{\theta\theta}\theta_t$, which we solve

$$\mathbb{E}_t\theta_{t+1} = S_{\theta\theta}^{-1}T_{\theta\theta}\theta_t$$

since $S_{\theta\theta}$ is invertible. The reason is that $\det(S_{\theta\theta})$ equals the product of the diagonal elements since $S_{\theta\theta}$ is triangular, and that all diagonal elements are non-zero since $|T_{ii}/S_{ii}| < 1$ are sorted first so there cannot be any zeros in the diagonal of $S_{\theta\theta}$; $\det(S_{\theta\theta})$ is therefore non-zero and $S_{\theta\theta}$ is invertible.

E.1 Matrix Singularity

Instead of (E.2) and (E.3), let the dynamic equations be

$$\begin{aligned}\begin{bmatrix}x_{t+1}\\ H\mathbb{E}_tX_{t+1}\end{bmatrix} &= \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_t\\ X_t\end{bmatrix}+\begin{bmatrix}B_1\\ B_2\end{bmatrix}u_t+\begin{bmatrix}C\\ 0\end{bmatrix}\epsilon_{t+1}, \text{ or} \\ \tilde{H}y_{t+1} &= Ay_t+Bu_t+\zeta_{t+1}, \quad \text{and } x_0 \text{ given, where } \tilde{H}=\begin{bmatrix}I & 0\\ 0 & H\end{bmatrix}\end{aligned} \tag{E.7}$$

where H can be singular (if not, premultiply by H^{-1} to get the system on standard form).

The first order conditions for y_t, corresponding to (E.5), are then

$$\beta A'\mathbb{E}_t\lambda_{t+1} = \tilde{H}'\lambda_t - \beta\mathcal{Q}y_t - \beta\mathcal{P}u_t \tag{E.8}$$

so we can write (E.7), (E.8), and (E.4) as , corresponding to (E.6), as

$$\begin{bmatrix}\tilde{H} & 0 & 0\\ 0 & 0 & \beta A'\\ 0 & 0 & -B'\end{bmatrix}\begin{bmatrix}y_{t+1}\\ u_{t+1}\\ \mathbb{E}_t\lambda_{t+1}\end{bmatrix}=\begin{bmatrix}A & B & 0\\ -\beta\mathcal{Q} & -\beta\mathcal{P} & \tilde{H}'\\ \mathcal{P}' & \mathcal{R} & 0\end{bmatrix}\begin{bmatrix}y_t\\ u_t\\ \lambda_t\end{bmatrix}+\begin{bmatrix}\zeta_{t+1}\\ 0\\ 0\end{bmatrix}$$

We can then apply the same solution algorithm as for to this system.

Appendix F

Parameterisation

$$
\begin{aligned}
U\left(c_t, n_t\right) &= \frac{C_t^{1-\sigma}}{1-\sigma} - \alpha\frac{n_t^{1+\psi}}{1+\psi} \\
U_c\left(c_t, n_t\right) &= C_t^{-\sigma} \\
U_{c^*}\left(c_t^*, n_t^*\right) &= C_t^{*-\sigma} \\
U_n\left(c_t, n_t\right) &= -\alpha n_t^{\psi} \\
U_{n^*}\left(c_t^*, n_t^*\right) &= -\alpha n_t^{*\psi} \\
F(e^{z_t}, k_t, n_t) &= Ae^{z_t}k_t^{\theta}n_t^{1-\theta} = Y_t \\
F_k(e^{z_t}, k_t, n_t) &= \theta Ae^{z_t}k_t^{\theta-1}n_t^{1-\theta} = \theta\frac{Y_t}{k_t} \\
F_n(e^{z_t}, k_t, n_t) &= \left(1-\theta\right)Ae^{z_t}k_t^{\theta}n_t^{-\theta} = \left(1-\theta\right)\frac{Y_t}{n_t}
\end{aligned}
$$

$$
\begin{aligned}
Y_t &= Ze^{z_t}k_t^{\theta}N_t^{1-\theta} \\
R_t &= 1 + i_t(1-\tau_t) \\
Y_t^* &= Ze^{z_t^*}k_t^{*\theta}N_t^{*1-\theta} \\
R_t^* &= 1 + i_t^*(1-\tau_t^*)
\end{aligned}
$$

$$S_t = \frac{P_{Ft}}{P_{Ht}}.$$

$$
\begin{aligned}
\frac{P_t}{P_{Ht}} &= \left((1-\gamma) + \gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
\frac{P_t}{P_{Ft}} &= \left((1-\gamma)\,S_t^{\eta-1} + \gamma\right)^{\frac{1}{1-\eta}} \\
\frac{P_t^*}{P_{Ht}^*} &= \left((1-\gamma^*)\,S_t^{1-\eta} + \gamma^*\right)^{\frac{1}{1-\eta}} \\
\frac{P_t^*}{P_{Ft}^*} &= \left((1-\gamma^*) + \gamma^* S_t^{\eta-1}\right)^{\frac{1}{1-\eta}} \\
p_{Ft}(z) &= E_t p_{Ft}^*(z),\ p_{Ht}(z) = E_t p_{Ht}^*(z)
\end{aligned}
$$

$$
\begin{aligned}
\left((1-\gamma)+\gamma S_t^{1-\eta}\right)^{\frac{1}{1-\eta}} &= \Upsilon_t \\
\left((1-\gamma^*)\, S_t^{1-\eta}+\gamma^*\right)^{\frac{1}{1-\eta}} &= \Upsilon_t^* = \Gamma_t S_t \\
\left(1-\gamma^*+S_t^{\eta-1}\gamma^*\right)^{\frac{1}{1-\eta}} &= \Gamma_t
\end{aligned}
$$

Appendix G

Normalisation

We introduce the following notations

$$
\begin{aligned}
w_t &= \frac{W_t}{P_t} = \frac{W_t}{P_{Ht}} \\
X_t &= \frac{P_{mt}}{P_{Ht}} \\
d_t &= \frac{D_t}{P_{Ht}} \\
a_t &= \frac{A_t}{P_{Ht}} \\
b_t &= \frac{B_t}{P^*_{Ft}} \\
p_{St} &= \frac{P_{St}}{P_{Ht}} \\
s_{t+1} &= s_t = 1
\end{aligned}
$$

$$
\begin{aligned}
w^*_t &= \frac{W^*_t}{P^*_t} = \frac{W^*_t}{P^*_{Ft}} \\
X^*_t &= \frac{P^*_{mt}}{P^*_{Ft}} \\
d^*_t &= \frac{D^*_t}{P^*_{Ft}} \\
b^*_t &= \frac{B^*_t}{P^*_{Ft}} \\
a^*_t &= \frac{A^*_t}{P_{Ht}} \\
p^*_{St} &= \frac{P^*_{St}}{P^*_{Ft}}
\end{aligned}
$$